JEE Main 2019 Paper 1 (B.E./B.Tech) Apr 8 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 360**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2019, Session 2 (April), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 April 2019, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 360.

(iii) Each multiple choice question (MCQ) has four options, of which only one is correct.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics (120 marks)

1. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? [4 marks]

(A) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(B) d, e, f are in A.P

(C) d, e, f are in G.P

(D) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Correct Answer: (D) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Solution:

Concept:

- Three nonzero numbers in geometric progression satisfy $b^2 = ac$. Thus the discriminant of $ax^2 + 2bx + c$ is zero.
- Three numbers u, v, w form an arithmetic progression precisely when $u + w = 2v$. A common root must satisfy both equations.

Step 1: Find the repeated root Since $b^2 = ac$, the first equation has discriminant $4b^2 - 4ac = 0$. Its only root is therefore $r = -b/a$. The distinctness in the given geometric progression rules out a zero common ratio or zero first term; hence divisions by a, b, c below are valid. The common root of the two quadratics must be this repeated root.

Step 2: Substitute into the second equation Using $r = -b/a$ in $dr^2 + 2er + f = 0$ gives

$$\frac{db^2}{a^2} - \frac{2eb}{a} + f = 0.$$

Replace b^2 by ac, and divide the resulting equation by c:

$$\frac{d}{a} - \frac{2eb}{ac} + \frac{f}{c} = 0.$$

Since $ac = b^2$, this becomes

$$\frac{d}{a} + \frac{f}{c} = 2\frac{e}{b}.$$

Step 3: Check the progression and exclude the distractors The last identity is exactly the arithmetic-progression criterion for the three ratios in their stated order. It does not require the coefficients themselves to be in either progression. For example, take $(a, b, c) = (1, 2, 4)$ and $(d, e, f) = (1, 3, 8)$. Both quadratics vanish at -2. The ratios are $1, 3/2, 2$, which are in A.P. but not G.P.; also $1, 3, 8$ is neither an A.P. nor a G.P. Thus the other statements are not necessary.

Step 4: Conclusion The ratios $d/a, e/b, f/c$ are in A.P. Hence, the correct option is (D).

Quick Tip to Solve: When quadratic coefficients are in G.P., first use the zero discriminant: the repeated root makes a common-root problem a single substitution.

2. If the system of linear equations

x – 2y + kz = 1

2x + y + z = 2

3x – y – kz = 3

has a solution (x,y,z), z $\neq$ 0, then (x,y) lies on the straight line whose equation is : [4 marks]

(A) 4x – 3y – 4 = 0

(B) 3x – 4y – 1 = 0

(C) 4x – 3y – 1 = 0

(D) 3x – 4y – 4 = 0

Correct Answer: (A) 4x – 3y – 4 = 0

Solution:

Concept:

- Adding two linear equations can eliminate a variable whose coefficients are opposites. Here the first and third equations contain $+kz$ and $-kz$.
- The required line is a necessary relation between x and y for every admissible solution; solving separately for all three variables is unnecessary.

Step 1: Eliminate the parameter term Add the first equation $x - 2y + kz = 1$ to the third equation $3x - y - kz = 3$. Their z-terms cancel without dividing by k or by z. Thus

$$(x - 2y + kz) + (3x - y - kz) = 1 + 3,$$

so every solution satisfies

$$4x - 3y = 4.$$

This already gives the required straight-line equation.

Step 2: Check the condition on the third coordinate The sum of the first two equations, minus the third, gives

$$(2k + 1)z = 0.$$

Since the question specifies $z \neq 0$, consistency requires $k = -1/2$. This condition does not change the line obtained above. For example, choosing $z = 10$ and using the second equation together with $4x - 3y = 4$ gives $x = -2, y = -4$. Substitution with $k = -1/2$ verifies all three original equations.

Step 3: Compare the four line equations Rearranging the necessary relation gives $4x - 3y - 4 = 0$, exactly option (A). Option (C) has the same slope but a different constant, so it is a distinct parallel line. Options (B) and (D) interchange the coefficients of x and y and have different slopes. The checked admissible point $(-2, -4)$ lies on (A) and on none of those three distractor lines.

Step 4: Conclusion The point (x, y) lies on $4x - 3y - 4 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Before using determinants on a parameter-dependent system, look for two equations with opposite parameter terms; their sum may give the requested locus immediately.

3. Let $f(x) = \int_0^x g(t)dt$ where g is a non-zero even function. If f(x + 5) = g(x), then $\int_0^x f(t)dt$ equals-

[4 marks]

(A) $5\int_{x+5}^{5} g(t)dt$

(B) $\int_{x+5}^{5} g(t)dt$

(C) $\int_{5}^{x+5} g(t)dt$

(D) $2\int_{5}^{x+5} g(t)dt$

Correct Answer: (B) $\int_{x+5}^{5} g(t)dt$

Solution:

Concept:

- If g is even, then its integral from zero, $f(x) = \int_0^x g(t)\,dt$, is odd: $f(-x) = -f(x)$.
- Reversing integration limits changes the sign. A translation of the integration variable must shift both limits by the same amount.

Step 1: Use the parity of the given functions Changing variables $t = -u$ in the definition gives

$$f(-x) = -\int_0^x g(-u)\,du = -f(x),$$

since $g(-u) = g(u)$. The identity $f(x + 5) = g(x)$, together with evenness of g, also gives

$$f(5 + x) = g(x) = g(-x) = f(5 - x).$$

Thus f is symmetric about the argument 5, even though it is odd about zero.

Step 2: Find the sign after a shift of ten Replace x in the last symmetry identity by $t + 5$. Then

$$f(t + 10) = f(-t) = -f(t).$$

Now apply the given shifted relation at the argument $t + 5$:

$$g(t + 5) = f(t + 10) = -f(t).$$

This sign is essential; simply shifting the limits while leaving the integrand unchanged would miss it.

Step 3: Translate the integral and compare options Use $u = t + 5$, so $du = dt$, and the endpoints $0, x$ become $5, x + 5$. We obtain

$$\int_0^x f(t)\,dt = -\int_0^x g(t + 5)\,dt = -\int_5^{x+5} g(u)\,du = \int_{x+5}^{5} g(u)\,du.$$

Option (B) has these limits and coefficient. Option (C) reverses the sign; (A) adds an unsupported

factor of five, and (D) both reverses the limits and adds a factor of two. No scale factor arises from a translation, because its derivative is one.

Step 4: Conclusion The required integral is $\int_{x+5}^{5} g(t)\,dt$. Hence, the correct option is (B).

Quick Tip to Solve: First establish the parity of an integral from zero. Combining oddness with a shifted even-function identity often produces an antiperiod and fixes the sign.

4. Let S(α) = {(x, y) : $y^2 \le x$, $0 \le x \le \alpha$} and A(α) is area of the region S(α). If for a λ, $0 < \lambda < 4$, A(λ) : A(4) = 2 : 5, then λ equals [4 marks]

(A) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(B) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$

(C) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(D) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

Correct Answer: (C) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

Solution:

Concept:

- The inequality $y^2 \le x$, for fixed $x \ge 0$, describes the vertical interval $-\sqrt{x} \le y \le \sqrt{x}$.
- Area is the integral of upper boundary minus lower boundary. Here its dependence on the endpoint is proportional to the endpoint raised to $3/2$.

Step 1: Set up the full vertical width The region includes points both above and below the x-axis. At a given x, its upper ordinate is $\sqrt{x}$ and its lower ordinate is $-\sqrt{x}$. Therefore the width of a vertical strip is $2\sqrt{x}$, not just $\sqrt{x}$. For any positive endpoint α,

$$A(\alpha) = \int_0^{\alpha} 2\sqrt{x}\,dx.$$

Step 2: Evaluate the area and form the ratio Integrating the power gives

$$A(\alpha) = 2\left[\frac{2}{3}x^{3/2}\right]_0^{\alpha} = \frac{4}{3}\alpha^{3/2}.$$

In particular, $A(4) = \frac{4}{3}\cdot 8 = 32/3$. The stated ratio therefore means

$$\frac{A(\lambda)}{A(4)} = \frac{\lambda^{3/2}}{8} = \frac{2}{5},$$

so $\lambda^{3/2} = 16/5$. The common area coefficient cancels in the ratio.

Step 3: Solve for the positive endpoint Because $\lambda > 0$, raising both sides to the power $2/3$ is

unambiguous:

$$\lambda = \left(\frac{16}{5}\right)^{2/3} = 4\left(\frac{2}{5}\right)^{2/3} = 4\left(\frac{4}{25}\right)^{1/3}.$$

This lies between zero and four, as required. Option (C) matches it. The two options beginning with two have the wrong scale, while (D) uses a cube root of $2/5$ instead of its two-thirds power. Substituting the derived value into $(\lambda/4)^{3/2}$ returns $2/5$.

Step 4: Conclusion The endpoint is $\lambda = 4(4/25)^{1/3}$. Hence, the correct option is (C).

Quick Tip to Solve: For a parabolic region cut off at $x = \alpha$, area scales as $\alpha^{3/2}$; use that power directly when comparing two such areas.

5. Suppose that the points (h,k), (1,2) and (–3,4) lie on the line L_1 . If a line L_2 passing through the points (h,k) and (4,3) is perpendicular to L_1 , then $\frac{k}{h}$ equals : [4 marks]

(A) $\frac{1}{3}$

(B) 3

(C) 0

(D) -$\frac{1}{7}$

Correct Answer: (A) $\frac{1}{3}$

Solution:

Concept:

- The slope through (x_1, y_1) and (x_2, y_2) is $(y_2 - y_1)/(x_2 - x_1)$. Perpendicular nonvertical lines have slopes whose product is -1.
- The unknown point (h, k) belongs to both lines, so it is obtained by solving their two equations simultaneously.

Step 1: Determine the first line Through $(1, 2)$ and $(-3, 4)$, the slope is

$$m_1 = \frac{4-2}{-3-1} = -\frac{1}{2}.$$

Using point-slope form through $(1, 2)$,

$$y - 2 = -\frac{1}{2}(x - 1),$$

which simplifies to $x + 2y = 5$. Consequently the unknown point satisfies $h + 2k = 5$. Both given points satisfy this equation, checking the slope and constant.

Step 2: Determine the perpendicular line The slope of L_2 is the negative reciprocal of $-1/2$, namely $m_2 = 2$. Since L_2 passes through $(4, 3)$, its equation is

$$y - 3 = 2(x - 4), \qquad y = 2x - 5.$$

It follows that $k = 2h - 5$. The point (h, k) is the intersection; it is not an arbitrary point on either line.

Step 3: Solve and evaluate the requested quotient Substitute the second relation into the first:

$$h + 2(2h - 5) = 5, \qquad 5h = 15.$$

Hence $h = 3$ and $k = 1$, giving $k/h = 1/3$. As a check, the slope from $(3, 1)$ to $(4, 3)$ is 2, whose product with $-1/2$ is -1. Option (B) gives the reciprocal h/k, while (C) would require $k = 0$, and (D) has an incompatible negative sign. Only (A) agrees with the intersection.

Step 4: Conclusion The intersection is $(h, k) = (3, 1)$, so $k/h = 1/3$. Hence, the correct option is (A).

Quick Tip to Solve: Write each line in point-slope form, then solve their intersection before taking the requested ratio; this avoids confusing the ratio with a slope.

---

6. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is : [4 marks]

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Concept:

- The complement of observing at least one head is observing no heads, which means every toss is a tail.
- Independent fair tosses have probability $(1/2)^n$ of producing n tails. The number of tosses n must be a positive integer.

Step 1: Express the desired probability Let H be the event of at least one head in n tosses. There is exactly one all-tail outcome among the 2^n equally likely ordered sequences. Using the complement therefore counts all outcomes with one or more heads at once:

$$P(H) = 1 - P(\text{all tails}) = 1 - 2^{-n}.$$

Adding probabilities for exactly one head alone would omit outcomes with two or more heads.

Step 2: Translate the percentage into an inequality The target 90% is $9/10$, so

$$1 - 2^{-n} \geq \frac{9}{10} \quad \Longleftrightarrow \quad 2^{-n} \leq \frac{1}{10}.$$

Equivalently, $2^n \geq 10$. The neighbouring integer powers are $2^3 = 8 < 10$ and $2^4 = 16 \geq 10$. Therefore the first possible integer is $n = 4$. The inequality is inclusive, but neither neighbouring probability is exactly the target.

Step 3: Check minimality and all options For two tosses the probability is $1 - 1/4 = 3/4 = 75\%$,

and for three it is $1 - 1/8 = 7/8 = 87.5\%$; hence options (A) and (B) fail the threshold. Four tosses give $1 - 1/16 = 15/16 = 93.75\%$, satisfying it. Five tosses give $31/32 = 96.875\%$, but that is not the minimum because four already suffice. The success probability increases with n, so checking the boundary integers establishes minimality.

Step 4: Conclusion The minimum required number of tosses is 4. Hence, the correct option is (C).

Quick Tip to Solve: For at least one success in independent trials, use one minus the probability of zero successes, then round the trial count upward to an integer.

---

7. Let the number 2,b,c be in an A.P. and

A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$**. If det(A) ∈ [2, 16], then c lies in the interval :** [4 marks]

(A) [2, 3)

(B) [4, 6]

(C) $(2 + 2^{3/4}, 4)$

(D) $[3, 2 + 2^{3/4}]$

Correct Answer: (B) [4, 6]

Solution:

Concept:

- The Vandermonde determinant with columns corresponding to u, v, w is $(v - u)(w - u)(w - v)$. The order of the differences fixes its sign.
- Write three terms of an arithmetic progression as $2, 2+d, 2+2d$, where d is their common difference; do not assume d is positive before using the determinant condition.

Step 1: Express the determinant using column differences Since $2, b, c$ are in A.P., put $b = 2 + d$ and $c = 2 + 2d$. The given matrix is a Vandermonde matrix with successive values $2, b, c$. Subtracting its first column from the second and third, or applying the determinant formula, gives

$$\det(A) = (b - 2)(c - 2)(c - b).$$

In terms of the common difference, the three factors are $d, 2d, d$, respectively.

Step 2: Apply the determinant bounds Thus

$$\det(A) = 2d^3.$$

The stated inclusive interval yields

$$2 \le 2d^3 \le 16, \qquad 1 \le d^3 \le 8.$$

The real cube function is strictly increasing, so taking real cube roots gives $1 \le d \le 2$. In particular, positivity of d is a consequence of the positive determinant; negative common differences cannot

satisfy the lower bound.

Step 3: Translate the bounds back to the last term Use $c = 2 + 2d$ to obtain

$$4 \le c \le 6.$$

Both endpoints are included: $d = 1$ gives $(2, b, c) = (2, 3, 4)$ and determinant 2, whereas $d = 2$ gives $(2, 4, 6)$ and determinant 16. Option (B) is exactly this interval. The other three intervals have upper endpoints at most four and contain values below four, where $d < 1$ and the determinant falls below two. Thus none represents the complete permissible range.

Step 4: Conclusion The last term satisfies $c \in [4, 6]$. Hence, the correct option is (B).

Quick Tip to Solve: When a Vandermonde matrix uses terms in A.P., replace them by a first term and a common difference; its determinant then becomes a simple cube.

8. If the fourth term in the binomial expansion of $\left(\sqrt{x^{\left(\frac{1}{1+\log_{10} x}\right)}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is : [4 marks]

(A) 100

(B) 10^3

(C) 10

(D) 10^4

Correct Answer: (C) 10

Solution:

Concept:

- The fourth term of $(a + b)^6$ is $\binom{6}{3}a^3b^3$. The term number is one more than the exponent of the second summand.
- For $x > 1$, write $t = \log_{10} x > 0$. Then powers of x can be compared by taking logarithms to base ten.

Step 1: Identify the fourth term Here $a = x^{1/[2(1+\log_{10} x)]}$ and $b = x^{1/12}$. Therefore the given condition is

$$20x^{\frac{3}{2(1+\log_{10} x)}+\frac{1}{4}} = 200.$$

The square root halves the exponent of the first summand. Both summands occur cubed in the fourth term; using an exponent of four for either would select the wrong term.

Step 2: Solve the logarithmic equation Divide by 20 and take logarithms. With $t = \log_{10} x$, this gives

$$t\left(\frac{3}{2(1+t)} + \frac{1}{4}\right) = 1.$$

Since $t > 0$, the denominator $1 + t$ is nonzero. Multiplication by $4(1 + t)$ yields

$$6t + t(1 + t) = 4(1 + t), \qquad t^2 + 3t - 4 = 0.$$

Thus $(t - 1)(t + 4) = 0$. The root $t = -4$ would give $x < 1$, so only $t = 1$ is admissible.

Step 3: Check the listed values For $x = 10$, the exponent in the fourth term is $3/4 + 1/4 = 1$, so the term is indeed $20 \times 10 = 200$. The other options have $t = 2, 3, 4$, none of which satisfies $(t - 1)(t + 4) = 0$. They therefore cannot produce the required fourth term.

Step 4: Conclusion The admissible value is $x = 10$. Hence, the correct option is (C).

Quick Tip to Solve: Introduce $t = \log_{10} x$ before clearing exponents, and retain the restriction $t > 0$ when selecting the quadratic root.

9. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is [4 marks]

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) $\sqrt{6}$

(D) $\frac{2}{3}\sqrt{3}$

Correct Answer: (B) $2\sqrt{3}$

Solution:

Concept:

- For a cylinder of radius r and full height h inscribed centrally in a sphere of radius 3, the axial cross-section gives $r^2 + (h/2)^2 = 9$.
- The volume is $V = \pi r^2 h$. A maximum inside the feasible interval must satisfy $V\prime(h) = 0$, and it must also be compared with the endpoints.

Step 1: Express volume using the full height The sphere passes through the rim of each circular end. The radius of the cylinder and half its height are the perpendicular legs of a right triangle whose hypotenuse is the sphere radius. Hence

$$r^2 = 9 - \frac{h^2}{4}, \qquad V(h) = \pi\left(9h - \frac{h^3}{4}\right), \quad 0 \le h \le 6.$$

It is essential to use $h/2$ in this relation because the centre of the sphere is midway between the two end faces.

Step 2: Differentiate and establish the maximum Differentiation gives

$$V\prime(h) = \pi\left(9 - \frac{3h^2}{4}\right) = 0 \quad \Longrightarrow \quad h^2 = 12.$$

The positive critical height is $h = 2\sqrt{3}$. Also $V\prime\prime(h) = -3\pi h/2 < 0$ at this point. Volume vanishes at

$h = 0$ and $h = 6$, whereas the critical point has positive volume. It is consequently the maximum over the whole feasible interval.

Step 3: Distinguish the other geometric quantities At the optimum, $r^2 = 9 - 12/4 = 6$, so $\sqrt{6}$ is the cylinder radius, not its height. The value $\sqrt{3}$ is half the optimal height. The remaining value $2\sqrt{3}/3$ does not satisfy the stationary equation $h^2 = 12$. This separates all four listed choices.

Step 4: Conclusion The cylinder of maximum volume has full height $2\sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: For an inscribed cylinder, write the sphere relation with half the full height; $\sqrt{6}$ here is the optimal radius, a tempting distractor.

10. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then the distance of R from the origin is : [4 marks]

(A) $2\sqrt{14}$

(B) $\sqrt{53}$

(C) $2\sqrt{21}$

(D) 6

Correct Answer: (A) $2\sqrt{14}$

Solution:

Concept:

- A point on the segment from P to Q can be written as $R = P + t(Q - P)$, where $0 \le t \le 1$. The same parameter must be used in all three coordinates.
- The distance from (x, y, z) to the origin is $\sqrt{x^2 + y^2 + z^2}$, not the sum of its coordinates or the distance from an endpoint.

Step 1: Find the segment parameter The displacement between the endpoints is

$$Q - P = (8 - 2,\ 0 - (-3),\ 10 - 4) = (6, 3, 6).$$

Thus a general point of the segment is $(2 + 6t, -3 + 3t, 4 + 6t)$. The specified first coordinate of R is 4, so

$$2 + 6t = 4 \quad \Longrightarrow \quad t = \frac{1}{3}.$$

This value lies between zero and one, confirming that the point is on the segment itself, rather than only on its extended line.

Step 2: Determine the remaining coordinates Use this same value of t in the other coordinates:

$$y = -3 + 3\left(\frac{1}{3}\right) = -2, \qquad z = 4 + 6\left(\frac{1}{3}\right) = 6.$$

Therefore $R = (4, -2, 6)$. As a check, $R - P = (2, 1, 2)$, exactly one third of $Q - P = (6, 3, 6)$. The coordinate changes therefore have the required common ratio.

Step 3: Calculate the requested distance Apply the three-dimensional distance formula to the origin:

$$OR = \sqrt{4^2 + (-2)^2 + 6^2} = \sqrt{16 + 4 + 36} = \sqrt{56} = 2\sqrt{14}.$$

The other listed distances square to 53, 84 and 36, respectively, whereas the coordinates of R require squared distance 56. In particular, the coordinate $z = 6$ alone is not the distance because both other coordinates are nonzero.

Step 4: Conclusion The distance of R from the origin is $2\sqrt{14}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the known coordinate to find one segment parameter, then reuse it in every coordinate before applying the distance formula.

---

11. A student scores the following marks in five tests :

45, 54, 41, 57, 43.

His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is [4 marks]

(A) $\frac{100}{\sqrt{3}}$

(B) $\frac{10}{\sqrt{3}}$

(C) $\frac{10}{3}$

(D) $\frac{100}{3}$

Correct Answer: (B) $\frac{10}{\sqrt{3}}$

Solution:

Concept:

- For six scores with mean $\bar{x}$, the sum is $6\bar{x}$; this determines the missing score before dispersion is computed.
- The standard deviation of this complete set of six scores is $\sigma = \sqrt{\frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2}$. Divide by six, not by five: the question asks for the dispersion of the listed data.

Step 1: Recover the sixth score The total required by the given mean is $6 \times 48 = 288$. The five known scores add to

$$45 + 54 + 41 + 57 + 43 = 240.$$

Consequently the sixth score is $288 - 240 = 48$. It is exactly the mean and therefore contributes zero to the sum of squared deviations, although it still contributes to the number of observations in the denominator.

Step 2: Compute the variance about the given mean The six deviations from 48 are $-3, 6, -7, 9, -5, 0$. Squaring eliminates the signs, so

$$\sum (x_i - 48)^2 = 9 + 36 + 49 + 81 + 25 + 0 = 200.$$

The variance is therefore

$$\sigma^2 = \frac{200}{6} = \frac{100}{3}.$$

A zero deviation of one observation does not justify dropping that observation; the data still contain six tests.

Step 3: Take the square root and compare options The requested quantity is standard deviation rather than variance. Thus

$$\sigma = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}}.$$

The option $100/3$ is the variance and omits the final square root. The value $100/\sqrt{3}$ incorrectly retains 100 after taking the square root, while $10/3$ fails to take the square root of the denominator. Only one choice matches the computed standard deviation.

Step 4: Conclusion The standard deviation is $10/\sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: First recover the missing score using the mean; then distinguish variance $100/3$ from its square root, the requested standard deviation.

12. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is : [4 marks]

(A) 306

(B) 288

(C) 310

(D) 360

Correct Answer: (C) 310

Solution:

Concept:

- Compare digits from left to right: a number exceeds 4321 at its first position that differs from 4321, provided that differing digit is larger.
- Once a larger digit has been chosen, every remaining place has six independent choices because repetition is allowed. Separate cases by the first differing position to avoid overlap.

Step 1: Count numbers with a larger first or second digit A larger thousands digit must be 5. The other three positions may each contain any of the six allowed digits, including zero, giving $6^3 = 216$ numbers. If the thousands digit is 4, a larger hundreds digit than 3 is either 4 or 5. The last two positions are free, giving $2 \cdot 6^2 = 72$ more numbers. These cases are disjoint because their thousands digits differ.

Step 2: Continue along the common prefix For the prefix 43, the tens digit must exceed 2 to make the first difference there. There are three choices, namely 3, 4 or 5, followed by six choices for the units digit. This contributes $3 \cdot 6 = 18$. Finally, for the prefix 432, only the units digit can decide the inequality. It must be one of 2, 3, 4 or 5, giving four numbers. The value 4321 itself is excluded because the inequality is strict.

Step 3: Add and check the cases Every admissible number has exactly one first differing position, so no number is counted twice and no additional case is needed. The total is

$$216 + 72 + 18 + 4 = 310.$$

Numbers beginning with 0, 1, 2 or 3 cannot exceed 4321, regardless of their later digits. The counts 306, 288 and 360 do not include exactly these four disjoint contributions; in particular, 306 omits the final four numbers with prefix 432.

Step 4: Conclusion There are 310 such four-digit numbers. Hence, the correct option is (C).

Quick Tip to Solve: Count by the first digit that is larger than the corresponding digit of 4321; after that position, allow all six digits independently.

---

13. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$,
then $(1 + iz + z^5 + iz^8)^9$ is equal to : [4 marks]

(A) 1

(B) –1

(C) 0

(D) $(-1 + 2i)^9$

Correct Answer: (B) –1

Solution:

Concept:

- The number $z = \sqrt{3}/2 + i/2$ has modulus one and argument $\pi/6$; hence $z^n = \cos(n\pi/6) + i\sin(n\pi/6)$.
- Multiplying a complex number by i rotates it by $\pi/2$. Simplify the entire expression inside the parentheses before raising it to the ninth power.

Step 1: Evaluate the required powers By the angle representation,

$$z^5 = -\frac{\sqrt{3}}{2} + \frac{i}{2}, \qquad z^8 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i.$$

Multiplication by i, with $i^2 = -1$, also gives

$$iz = -\frac{1}{2} + \frac{\sqrt{3}}{2}i, \qquad iz^8 = \frac{\sqrt{3}}{2} - \frac{i}{2}.$$

These signs matter: in particular, the real part of iz^8 is positive because it comes from multiplying the negative imaginary part of z^8 by i.

Step 2: Combine real and imaginary parts Substitute the values and group corresponding components:

$$1 + iz + z^5 + iz^8 = 1 - \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + i\left(\frac{\sqrt{3}}{2} + \frac{1}{2} - \frac{1}{2}\right)$$
$$= \frac{1}{2} + \frac{\sqrt{3}}{2}i.$$

The result is a unit complex number with argument $\pi/3$; it is neither zero nor $-1 + 2i$.

Step 3: Raise the simplified value to the ninth power Apply De Moivre's theorem once more:

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^9 = \cos(3\pi) + i\sin(3\pi) = -1.$$

The result cannot be 0 because the base has modulus one. It is not 1 because its final argument is an odd multiple of π. The last option has modulus $(\sqrt{5})^9$, rather than one, and is also excluded.

Step 4: Conclusion The expression equals -1. Hence, the correct option is (B).

Quick Tip to Solve: For a unit complex number, compute powers through their angles, then add real and imaginary parts before applying the final exponent.

14. Let f(x) = a^x (a > 0) be written as
f(x) = f_1 (x) + f_2 (x), where f_1 (x) is an even function of f_2 (x) is an odd function.
Then f_1 (x + y) + f_1 (x – y) equals [4 marks]

(A) 2f_1 (x)f_1 (y)
(B) 2f_1 (x + y)f_1 (x – y)
(C) 2f_1 (x)f_2 (y)
(D) 2f_1 (x + y)f_2 (x – y)
Correct Answer: (A) 2f_1 (x)f_1 (y)

Solution:

Concept:

- Every real-domain function has unique even and odd parts: $f_1(x) = [f(x) + f(-x)]/2$ and $f_2(x) = [f(x) - f(-x)]/2$.
- For $f(x) = a^x$, where $a > 0$, exponent laws give $a^{x+y} = a^x a^y$ and $a^{-x} = 1/a^x$. Both identities hold for real arguments.

Step 1: Obtain the even part explicitly From $f(x) = f_1(x) + f_2(x)$, replace x by $-x$. Evenness and oddness give

$$a^{-x} = f_1(x) - f_2(x).$$

Adding this to $a^x = f_1(x) + f_2(x)$ yields

$$f_1(x) = \frac{a^x + a^{-x}}{2}, \qquad f_2(x) = \frac{a^x - a^{-x}}{2}.$$

This determines the functions from the stated decomposition, with no need to guess either part.

Step 2: Add the two shifted even parts Use the expression for f_1 at the two arguments:

$$\begin{aligned} f_1(x+y) + f_1(x-y) &= \frac{a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y}}{2} \\ &= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2} \\ &= 2f_1(x)f_1(y). \end{aligned}$$

The middle factorisation contains all four terms, so there are no omitted cross terms and the factor of two is retained correctly.

Step 3: Check that the other expressions are not identities Take $a = 2$, $x = 0$ and $y = 1$. Then $f_1(1) = 5/4$, $f_2(1) = 3/4$, and the required sum is $5/2$. The other three choices respectively give $25/8$, $3/2$ and $-15/8$, so none is a general identity. Special coincidences for a particular value, such as $a = 1$, do not establish an identity for all allowed values.

Step 4: Conclusion The sum is $2f_1(x)f_1(y)$. Hence, the correct option is (A).

Quick Tip to Solve: Find the even part by averaging $f(x)$ and $f(-x)$; the shifted sum then factors into one expression in x and one in y.

15. If f(1) = 1, f'(1) = 3, then the derivative of f(f(f(x))) + (f(x))^2 at x = 1 is : [4 marks]

(A) 33

(B) 12

(C) 9

(D) 15

Correct Answer: (A) 33

Solution:

Concept:

- The chain rule differentiates a composition by multiplying the derivatives of its successive layers: $(f \circ g)\prime(x) = f\prime(g(x))g\prime(x)$.
- The power rule combined with the chain rule gives $\frac{d}{dx}[f(x)]^2 = 2f(x)f\prime(x)$. The two terms in the question must be differentiated separately.

Step 1: Differentiate all three layers Put $G(x) = f(f(f(x))) + [f(x)]^2$. Starting with the outermost function and then moving inward gives

$$G\prime(x) = f\prime(f(f(x)))f\prime(f(x))f\prime(x) + 2f(x)f\prime(x).$$

There are three derivative factors in the first term because the composition contains three occurrences of the function. Omitting the innermost factor would give the derivative of a different expression.

Step 2: Evaluate the function arguments at one Since $f(1) = 1$, we also have $f(f(1)) = f(1) = 1$. Thus every derivative factor in the first term is evaluated at 1, where its value is 3. The second term uses both the given value of the function and its derivative:

$$G\prime(1) = 3 \cdot 3 \cdot 3 + 2 \cdot 1 \cdot 3 = 27 + 6 = 33.$$

The choices 12, 9 and 15 do not contain the required contribution 27 from the triple composition; in particular, 9 would come from only two derivative factors and also omit the square term.

Step 3: Conclusion The required derivative at $x = 1$ is 33. Hence, the correct option is (A).

Quick Tip to Solve: When a point is fixed by the function, evaluate every nested argument there first; the chain rule then becomes a product of identical derivative values.

16. Let f : R → R be a differentiable function satisfying f'(3) + f'(2) = 0.

Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to [4 marks]

(A) e

(B) e^2

(C) e^{-1}

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- Differentiability at a gives $f(a+h) - f(a) = hf\prime(a) + o(h)$, where $o(h)/h \to 0$.
- For a positive base tending to 1, take logarithms to evaluate a power with exponent tending to infinity. The basic limit is $\log(1+u)/u \to 1$ as $u \to 0$.

Step 1: Take the logarithm of the expression For sufficiently small x, both numerator and denominator approach 1 and are positive. If the displayed expression is $Y(x)$, then

$$\log Y(x) = \frac{\log(1 + f(3+x) - f(3)) - \log(1 + f(2-x) - f(2))}{x}.$$

This difference keeps the sign from the argument $2 - x$, which is essential; replacing that argument with $2 + x$ would change the limit.

Step 2: Use the two derivatives and the given relation Differentiability gives $f(3+x) - f(3) = xf\prime(3) + o(x)$ and $f(2-x) - f(2) = -xf\prime(2) + o(x)$. Applying the logarithm limit to each term therefore yields

$$\lim_{x\to 0} \log Y(x) = f\prime(3) + f\prime(2) = 0.$$

Exponentiation is continuous, so $Y(x) \to e^0 = 1$. The other choices e, e^2, e^{-1} would require logarith-

mic limits 1, 2 and -1 respectively, whereas the hypothesis forces that limit to be zero.

Step 3: Conclusion The value of the given limit is 1. Hence, the correct option is (D).

Quick Tip to Solve: In a quotient raised to $1/x$, subtract the logarithmic first-order changes; an argument $a - x$ introduces an additional minus sign.

17. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$

where C is a constant of integration, then the function f(x) is equal to [4 marks]

(A) $\frac{3}{x^2}$

(B) $-\frac{1}{6x^3}$

(C) $-\frac{1}{2x^3}$

(D) $-\frac{1}{2x^2}$

Correct Answer: (C) $-\frac{1}{2x^3}$

Solution:

Concept:

- The power rule for integration is $\int u^n\,du = u^{n+1}/(n+1) + C$, provided $n \neq -1$.
- Here the substitution $u = x^{-6} + 1$ absorbs the factor x^{-7}. Work on an interval excluding $x = 0$, where the printed integrand is undefined.

Step 1: Rewrite the integrand to expose a substitution Because $x^6 > 0$ for nonzero real x, we have $(1+x^6)^{2/3} = x^4(1+x^{-6})^{2/3}$. Thus

$$I = \int x^{-7}(1+x^{-6})^{-2/3}\,dx.$$

Set $u = 1 + x^{-6}$, so $du = -6x^{-7}\,dx$. Substitution now gives an ordinary power integral:

$$I = -\frac{1}{6}\int u^{-2/3}\,du = -\frac{1}{2}u^{1/3} + C.$$

Step 2: Compare with the required product Returning to x, note that $(x^6)^{1/3} = x^2$ for real nonzero x. Hence

$$I = -\frac{(1+x^6)^{1/3}}{2x^2} + C.$$

Comparison with $xf(x)(1+x^6)^{1/3} + C$ gives $xf(x) = -1/(2x^2)$, and therefore $f(x) = -1/(2x^3)$. Option B has the correct power but a coefficient smaller by a factor of three. Options A and D have an incorrect power of x, and A also has the wrong sign. Differentiating the antiderivative gives $x^{-3}(1+x^6)^{-2/3}$, confirming the result.

Step 3: Conclusion The listed function is $f(x) = -\frac{1}{2x^3}$, for $x \neq 0$. Hence, the correct option is (C).

Quick Tip to Solve: For a binomial involving x^6, factor the sixth power before substituting; its cube root is x^2, even when x is negative.

18. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0,5\sqrt{3})$, then the length of its latus rectum is : [4 marks]

(A) 5

(B) 8

(C) 10

(D) 6

Correct Answer: (A) 5

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b and focal distance c, $a^2 - b^2 = c^2$, with $a > b > 0$.
- The full major and minor axes have lengths $2a$ and $2b$. The full length of a latus rectum is $2b^2/a$, regardless of whether the major axis is horizontal or vertical.

Step 1: Find the two semi-axis lengths The given difference concerns full axis lengths, so

$$2a - 2b = 10 \quad \Longrightarrow \quad a - b = 5.$$

The focus $(0, 5\sqrt{3})$ lies on the vertical major axis and gives $c = 5\sqrt{3}$, hence $c^2 = 75$. Factoring the ellipse identity,

$$(a - b)(a + b) = 75 \quad \Longrightarrow \quad a + b = 15.$$

Solving the sum and difference yields $a = 10$ and $b = 5$. These are semi-axis lengths, not the full axes 20 and 10.

Step 2: Calculate the full focal chord length At the focus of the vertical ellipse $x^2/b^2 + y^2/a^2 = 1$, set $y = c$. Then $x^2 = b^2(1 - c^2/a^2) = b^4/a^2$. The two endpoints have $x = \pm b^2/a$, so their separation is

$$\ell = \frac{2b^2}{a} = \frac{2 \cdot 25}{10} = 5.$$

Thus 8 and 6 disagree with the focal-chord formula. The choice 10 is the full minor-axis length, not the latus rectum. Using a semi-latus rectum would instead give 2.5 and would miss the factor of two.

Step 3: Conclusion The length of the latus rectum is 5 units. Hence, the correct option is (A).

Quick Tip to Solve: Translate a difference of full axes into $a - b$ by dividing by two, then combine it with $(a - b)(a + b) = c^2$.

19. The number of integral values of m for which the equation

$(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is : [4 marks]

(A) 2

(B) infinitely many

(C) 1

(D) 3

Correct Answer: (B) infinitely many

Solution:

Concept:

- A real quadratic $Ax^2 + Bx + C = 0$, with $A \neq 0$, has no real roots precisely when its discriminant $D = B^2 - 4AC$ is negative.
- Here $A = 1 + m^2$ is strictly positive for every real m, so the equation remains quadratic for all integral parameters. A perfect-square factor simplifies the sign test.

Step 1: Expand and factor the discriminant From the printed equation, $B = -2(1 + 3m)$ and $C = 1 + 8m$. Dividing the discriminant by the positive number 4 does not change its sign:

$$\begin{aligned}\frac{D}{4} &= (1 + 3m)^2 - (1 + m^2)(1 + 8m) \\ &= 1 + 6m + 9m^2 - (1 + 8m + m^2 + 8m^3) \\ &= -2m + 8m^2 - 8m^3 = -2m(2m - 1)^2.\end{aligned}$$

Keeping the cubic term is necessary; dropping it would turn the parameter condition into an incorrect bounded interval.

Step 2: Apply the strict sign condition to integers For an integer m, the factor $(2m - 1)^2$ is always strictly positive because its only real zero is $m = 1/2$. Consequently,

$$D < 0 \quad \Longleftrightarrow \quad m > 0.$$

Every positive integer $1, 2, 3, \ldots$ gives no real root. At $m = 0$, the discriminant is zero, so that case has a repeated real root. At every negative integer it is positive and there are two real roots. The admissible integers have no upper bound, ruling out all three finite counts 1, 2 and 3.

Step 3: Conclusion There are infinitely many integral values of m, namely all positive integers. Hence, the correct option is (B).

Quick Tip to Solve: Factor the discriminant before solving its inequality; for an integer parameter, a square with a nonintegral zero has a fixed positive sign.

20. Let f : [–1,3] → R be defined as

$$f(x) = \begin{cases} |x| + [x] & , \quad -1 \le x < 1 \\ x + |x| & , \quad 1 \le x < 2 \\ x + [x] & , \quad 2 \le x \le 3 \end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at: [4 marks]

(A) only three points

(B) four or more points

(C) only two points

(D) only one point

Correct Answer: (A) only three points

Solution:

Concept:

- A function is continuous at an interior point when both one-sided limits equal its value. At a domain endpoint, continuity uses only the limit from inside the domain.
- The floor function $\lfloor x \rfloor$ is constant between successive integers and jumps at an integer. Also $|x| = -x$ for negative x, and $|x| = x$ for nonnegative x.

Step 1: Simplify the formula between the possible jump points On the domain, the function becomes

$$f(x) = \begin{cases} -x - 1, & -1 \le x < 0, \\ x, & 0 \le x < 1, \\ 2x, & 1 \le x < 2, \\ x + 2, & 2 \le x < 3, \\ 6, & x = 3. \end{cases}$$

Each open interval in this list has a linear formula and is continuous. Thus only $-1, 0, 1, 2, 3$ require separate checks; no noninteger point can introduce another discontinuity.

Step 2: Compare values with the one-sided limits At -1, the right limit and value are both 0, so the left endpoint is continuous. At 0, the left limit is -1 but the right limit and value are 0, giving a jump. At 1, the left limit is 1 while the right limit and value are 2, giving another jump. At 2, both one-sided limits and the value are 4, so the change of formula is continuous. Finally, at the right endpoint 3,

$$\lim_{x \to 3^-} f(x) = 5 \neq f(3) = 6.$$

Therefore 3 is discontinuous relative to the given closed domain. The complete set is $\{0, 1, 3\}$: two points would omit the endpoint, one would miss an interior jump, and four or more would count a continuous junction.

Step 3: Conclusion The function is discontinuous at exactly three points, $0, 1, 3$. Hence, the correct option is (A).

Quick Tip to Solve: On a closed domain, check the endpoint value against its inward limit; a floor-function jump at the last endpoint still creates a discontinuity.

21. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $\left|\vec{a} \times \vec{b}\right|$ = r is possible if : [4 marks]

(A) $0 < r < \sqrt{\frac{3}{2}}$

(B) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$

(C) $r \geq 5\sqrt{\frac{3}{2}}$

(D) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$

Correct Answer: (C) $r \geq 5\sqrt{\frac{3}{2}}$

Solution:

Concept:

- The cross product of vectors (a_1, a_2, a_3) and (b_1, b_2, b_3) has components $(a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1)$.
- The square of a vector magnitude is the sum of the squares of its components. Completing the square in a real parameter gives its exact minimum and hence the attainable range.

Step 1: Calculate the cross product and its squared magnitude Using $\vec{a} = (3, 2, x)$ and $\vec{b} = (1, -1, 1)$, we get

$$\vec{a} \times \vec{b} = (x + 2)\hat{i} + (x - 3)\hat{j} - 5\hat{k}.$$

Therefore

$$\begin{aligned} r^2 &= (x + 2)^2 + (x - 3)^2 + 25 \\ &= 2x^2 - 2x + 38 \\ &= 2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}. \end{aligned}$$

All three components contribute; the constant -5 component prevents the cross product from ever vanishing.

Step 2: Find and verify the attainable range The square term is nonnegative, with equality when $x = 1/2$. Hence $r^2 \geq 75/2$, and because a magnitude is nonnegative,

$$r \geq \sqrt{\frac{75}{2}} = 5\sqrt{\frac{3}{2}}.$$

This is also sufficient: for any r satisfying this inequality, the real values $x = 1/2 \pm \sqrt{(r^2 - 75/2)/2}$ produce that magnitude. Thus the lower endpoint is included, and there is no upper bound. Every value in the intervals in A, B and D lies below the required minimum, so none of those alternatives is possible.

Step 3: Conclusion The exact condition is $r \geq 5\sqrt{\frac{3}{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Minimise the square of a cross-product magnitude instead of its square root; completing the square also shows whether the endpoint is attained.

22. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to [4 marks]

(A) $2-\frac{11}{2^{19}}$

(B) $2-\frac{3}{2^{17}}$

(C) $1-\frac{11}{2^{20}}$

(D) $2-\frac{21}{2^{20}}$

Correct Answer: (A) $2-\frac{11}{2^{19}}$

Solution:

Concept:

- The summand is a product of the arithmetic factor k and the geometric factor 2^{-k}. Subtracting a shifted half of the finite sum cancels these arithmetic coefficients.
- For a finite geometric progression, $\sum_{k=1}^{n} r^k = r(1-r^n)/(1-r)$, for $r \neq 1$. The final uncancelled term must be retained.

Step 1: Shift the finite sum Write $S = \frac{1}{2}+\frac{2}{2^2}+\cdots+\frac{20}{2^{20}}$. Dividing by two gives $S/2 = \frac{1}{2^2}+\frac{2}{2^3}+\cdots+\frac{20}{2^{21}}$. Subtract the second expression from the first, aligning terms with the same denominator. The coefficient of each power from 2^{-1} through 2^{-20} becomes one, while the last shifted term remains negative. Thus

$$\frac{S}{2} = \sum_{k=1}^{20} \frac{1}{2^k} - \frac{20}{2^{21}}.$$

Step 2: Evaluate the remaining geometric progression The geometric sum is $1-2^{-20}$. Consequently,

$$\begin{aligned}\frac{S}{2} &= 1-\frac{1}{2^{20}}-\frac{20}{2^{21}}\\ &= 1-\frac{22}{2^{21}},\\ S &= 2-\frac{22}{2^{20}} = 2-\frac{11}{2^{19}}.\end{aligned}$$

Option (B) subtracts $24/2^{20}$, and option (D) subtracts $21/2^{20}$, whereas the exact correction is $22/2^{20}$. Option (C) is below one, although the first two terms already sum to one and all remaining terms are positive.

Step 3: Conclusion The required sum is $\boxed{2-\frac{11}{2^{19}}}$. Hence, the correct option is (A).

Quick Tip to Solve: For $\sum kr^k$, subtract r times the sum and check the final boundary term before simplifying.

23. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point : [4 marks]

(A) $\left(-\frac{1}{4}, \frac{1}{2}\right)$
(B) $\left(-\frac{1}{3}, \frac{4}{3}\right)$
(C) $\left(\frac{3}{4}, \frac{7}{4}\right)$
(D) $\left(\frac{1}{4}, \frac{3}{4}\right)$

Correct Answer: (C) $\left(\frac{3}{4}, \frac{7}{4}\right)$

Solution:

Concept:

- A common point of two curves satisfies both equations. The first-quadrant condition means that both its coordinates are positive.
- For $y^2 = 4ax$, the tangent at (x_1, y_1) is $yy_1 = 2a(x + x_1)$. Here $a = 1$.

Step 1: Find the first-quadrant intersection Substitute the parabola equation $y^2 = 4x$ into the circle equation. This gives

$$x^2 + 4x = 5, \qquad (x - 1)(x + 5) = 0.$$

The root $x = -5$ cannot belong to the first quadrant, and it would also give $y^2 = -20$. Therefore $x = 1$, and then $y^2 = 4$. The positive coordinate is $y = 2$, so the required point of contact is $(1, 2)$. The negative value -2 corresponds to the fourth-quadrant point and must be excluded.

Step 2: Form the tangent and test the choices Using $(x_1, y_1) = (1, 2)$, the tangent is $2y = 2(x + 1)$, or

$$y = x + 1.$$

For option (C), $x + 1 = 3/4 + 1 = 7/4 = y$, so that point lies on the tangent. For (A), $x + 1 = 3/4$, not $1/2$; for (B), $x + 1 = 2/3$, not $4/3$; for (D), $x + 1 = 5/4$, not $3/4$. Thus only (C) satisfies the equation. As a separate slope check, implicit differentiation gives $2y\, dy/dx = 4$, so the tangent slope at $(1, 2)$ is indeed one.

Step 3: Conclusion The tangent passes through $\boxed{(3/4, 7/4)}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the point of contact first; then substitute each proposed point into the tangent equation rather than into the parabola.

24. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is : [4 marks]

(A) 5 : 9 : 13

(B) 5 : 6 : 7

(C) 4 : 5 : 6

(D) 3 : 4 : 5

Correct Answer: (C) 4 : 5 : 6

Solution:

Concept:

- Let $a < b < c$ be opposite $A < B < C$. By the sine rule and $C = 2A$, $c/a = \sin(2A)/\sin A = 2\cos A$.
- The cosine rule gives $\cos A = (b^2 + c^2 - a^2)/(2bc)$. An arithmetic progression also gives $c = 2b - a$.

Step 1: Convert the angle condition into an algebraic equation Combining the sine-rule and cosine-rule expressions yields

$$\frac{c}{a} = \frac{b^2 + c^2 - a^2}{bc}, \qquad bc^2 = a(b^2 + c^2 - a^2).$$

Put $r = b/a$, so $c/a = 2r - 1$. Since the triangle has unequal smallest and greatest angles under $C = 2A$, we require $r > 1$. Dividing the equation by a^3 now gives

$$r(2r - 1)^2 = r^2 + (2r - 1)^2 - 1.$$

Step 2: Solve the ratio and verify the angles Expanding and factoring gives

$$4r^3 - 9r^2 + 5r = r(r - 1)(4r - 5) = 0.$$

Positivity excludes $r = 0$; $r = 1$ would be equilateral and does not obey $C = 2A$. Hence $r = 5/4$, and $a : b : c = 4 : 5 : 6$. This is a genuine triangle since $4 + 5 > 6$. For these sides, $\cos A = (25 + 36 - 16)/60 = 3/4$ and $\cos C = (16 + 25 - 36)/40 = 1/8$. Also $\cos 2A = 2(3/4)^2 - 1 = 1/8$, proving the required double-angle condition. The other listed side ratios are different from the unique admissible ratio just obtained.

Step 3: Conclusion The required side ratio is $\boxed{4 : 5 : 6}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the sine rule to turn a double-angle condition into $c = 2a\cos A$, then apply the cosine rule and the A.P. relation.

25. The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y+ 4z = 5 which is perpendicular to the plane x – y + z = 0 is : [4 marks]

(A) $\vec{r} \times \left(\hat{i} - \hat{k}\right) - 2 = 0$

(B) $\vec{r} \cdot \left(\hat{i} + \hat{k}\right) + 2 = 0$

(C) $\vec{r} \cdot \left(\hat{i} - \hat{k}\right) + 2 = 0$

(D) $\vec{r} \times \left(\hat{i} - \hat{k}\right) + 2 = 0$

Correct Answer: (C) $\vec{r} \cdot \left(\hat{i} - \hat{k}\right) + 2 = 0$

Solution:

Concept:

- Every plane through the intersection of $P_1 = 0$ and $P_2 = 0$ can be written $P_1 + \lambda P_2 = 0$, with the omitted limiting member checked if needed.
- Two planes are perpendicular precisely when their normal vectors have zero dot product. A vector equation of a plane uses a scalar dot product with its normal.

Step 1: Write the family and impose perpendicularity Take $P_1 = x+y+z-1$ and $P_2 = 2x+3y+4z-5$. The family is

$$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - (1 + 5\lambda) = 0.$$

Its normal is $(1 + 2\lambda, 1 + 3\lambda, 1 + 4\lambda)$. The given plane $x - y + z = 0$ has normal $(1, -1, 1)$. Their dot product must vanish, giving

$$(1 + 2\lambda) - (1 + 3\lambda) + (1 + 4\lambda) = 1 + 3\lambda = 0.$$

Thus $\lambda = -1/3$.

Step 2: Simplify and identify the vector equation Substitute $\lambda = -1/3$ into the family. The result is $x/3 - z/3 + 2/3 = 0$, or $x - z + 2 = 0$. With position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, this becomes

$$\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0.$$

Its normal $(1, 0, -1)$ has dot product zero with $(1, -1, 1)$, as required. Option (B) has normal $(1, 0, 1)$, whose dot product is two and therefore fails perpendicularity. Options (A) and (D) use a cross product and do not represent this scalar plane equation. The family construction guarantees that the common line lies in the resulting plane.

Step 3: Conclusion The plane is $\boxed{\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: For a plane through the intersection of two given planes, use their linear combination; impose perpendicularity on the normals, not on arbitrary points.

26. Two vertical poles of heights, 20 m and 80 m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is : [4 marks]

(A) 12

(B) 16

(C) 15

(D) 18

Correct Answer: (B) 16

Solution:

Concept:

- A line joining two points can be written using its intercepts. With pole separation $d > 0$, the crossing height can be found without knowing d.
- For crossing segments between tops and opposite feet of poles of heights h_1, h_2, equating the two line heights gives $h = h_1h_2/(h_1 + h_2)$.

Step 1: Assign coordinates and write both lines Choose the foot of the shorter pole as $(0, 0)$, its top as $(0, 20)$, the other foot as $(d, 0)$, and its top as $(d, 80)$. Here d is the unknown positive horizontal separation in metres. The line from $(0, 20)$ to $(d, 0)$ is

$$y = 20\left(1 - \frac{x}{d}\right).$$

The line from $(0, 0)$ to $(d, 80)$ is $y = 80x/d$. Both expressions give height above the same horizontal ground plane.

Step 2: Equate the heights at the intersection At their common point,

$$20 - \frac{20x}{d} = \frac{80x}{d}, \qquad \frac{x}{d} = \frac{1}{5}.$$

Substituting into the rising line yields

$$y = 80\left(\frac{1}{5}\right) = 16\,\text{m}.$$

The separation cancels, so no missing distance is needed. Equivalently, $y = 20 \times 80/(20 + 80) = 16$. The point is inside the interval between the poles since $0 < x/d < 1$, and its height is below the shorter pole, as expected. Options 12, 15 and 18 do not satisfy both line equations at a common value of x/d; only 16 does.

Step 3: Conclusion The height above the horizontal plane is $\boxed{16\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: For crossed lines joining opposite pole tops and feet, use $1/h = 1/h_1 + 1/h_2$; the horizontal pole separation cancels.

27. Which one of the following statements is not a tautology? [4 marks]

(A) $(p \wedge q) \rightarrow (\sim p) \vee q$

(B) $(p \wedge q) \rightarrow p$

(C) $(p \vee q) \rightarrow (p \vee (\sim q))$

(D) $p \rightarrow (p \vee q)$

Correct Answer: (C) $(p \vee q) \rightarrow (p \vee (\sim q))$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its component propositions. One false row is enough to disprove tautologicity.
- An implication $u \rightarrow v$ is false only when u is true and v is false; equivalently it is $\neg u \vee v$.

Step 1: Test the suspect statement with a counterexample Consider option (C), $(p \vee q) \rightarrow (p \vee \neg q)$. Choose $p = \mathrm{F}$ and $q = \mathrm{T}$. Its antecedent becomes $\mathrm{F} \vee \mathrm{T} = \mathrm{T}$, while its consequent is $\mathrm{F} \vee \mathrm{F} = \mathrm{F}$. Thus the implication is false on this assignment. This single counterexample proves that (C) is not a tautology; it is unnecessary to show that it is always false.

Step 2: Confirm that the other three choices are tautologies For (A), if $p \wedge q$ is false then the implication is true automatically. If $p \wedge q$ is true, both p and q are true, so its consequent $\neg p \vee q$ is true because q is true. Thus (A) is true in every case. For (B), whenever $p \wedge q$ is true, p is true, so $(p \wedge q) \rightarrow p$ is always true. For (D), a true p ensures that $p \vee q$ is true, and a false p makes the implication true automatically. Thus (B) and (D) are also tautologies. These arguments cover all possible truth assignments without relying on an incomplete truth table.

Step 3: Conclusion $\boxed{(p \vee q) \rightarrow (p \vee \neg q)}$ is not a tautology. Hence, the correct option is (C).

Quick Tip to Solve: To disprove a tautology involving implication, force its antecedent true and its consequent false; a single valid truth assignment settles it.

---

28. Given that the slope of the tangent to a curve y = y(x) at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is : [4 marks]

(A) $x \log_e |y| = 2(x - 1)$

(B) $x^2 \log_e |y| = -2(x - 1)$

(C) $x \log_e |y| = x - 1$

(D) $x \log_e |y| = -2(x - 1)$

Correct Answer: (A) $x \log_e |y| = 2(x - 1)$

Solution:

Concept:

- The slope of the tangent is dy/dx. A differential equation $dy/dx = 2y/x^2$ is separable wherever $x \neq 0$ and $y \neq 0$.
- Complete the squares to locate a circle's centre. Then use the point on the required curve to

determine the integration constant.

Step 1: Find the given point and integrate the slope equation The circle equation becomes

$$(x-1)^2+(y-1)^2=2,$$

so its centre is $(1,1)$. The required curve passes through this point. Separate the slope equation as

$$\frac{dy}{y}=\frac{2}{x^2}\,dx.$$

Integration gives $\ln|y|=-2/x+C$. The logarithm is valid for the solution through $(1,1)$, since y is positive locally; the zero solution of the differential equation cannot pass through that point.

Step 2: Use the initial point and compare the choices At $(1,1)$, $\ln 1=0=-2+C$, hence $C=2$. Therefore

$$\ln|y|=2-\frac{2}{x},\qquad x\ln|y|=2(x-1).$$

On the interval containing $x=1$, the explicit solution is $y=e^{2-2/x}>0$. Differentiating gives $y\cdot 2/x^2$, which is the stipulated slope. Option (C) would give $dy/dx=y/x^2$, and option (D) would give the negative of the required slope. Option (B) has an extra factor of x, producing a different derivative. Thus these three alternatives fail the differential equation even though the common point alone cannot distinguish them.

Step 3: Conclusion The required equation is $\boxed{x\log_e|y|=2(x-1)}$, with $x\neq 0$. Hence, the correct option is (A).

Quick Tip to Solve: After integrating a slope equation, substitute the specified point before comparing options, then differentiate the result to check its sign and powers.

---

29. If the eccentricity of the standard hyperbola passing through the point (4,6) is 2, then the equation of the tangent to the hyperbola at (4,6) is : [4 marks]

(A) 2x – y – 2 = 0

(B) 3x – 2y = 0

(C) 2x – 3y + 10 = 0

(D) x – 2y + 8 = 0

Correct Answer: (A) 2x – y – 2 = 0

Solution:

Concept:

- For the standard hyperbola $x^2/a^2-y^2/b^2=1$, eccentricity satisfies $e^2=1+b^2/a^2$. Here $a,b>0$ are the semi-axis parameters.
- The tangent at a point (x_1,y_1) on this hyperbola is $xx_1/a^2-yy_1/b^2=1$. A candidate tangent must both pass through that point and have the derivative slope there.

Step 1: Use the eccentricity to relate the axes The given eccentricity is $e=2$, so $4=1+b^2/a^2$, giving

$b^2 = 3a^2$. Substituting the given point $(4, 6)$ into the hyperbola determines the scale, rather than just its shape:

$$\frac{16}{a^2} - \frac{36}{3a^2} = 1 \quad \Rightarrow \quad \frac{4}{a^2} = 1.$$

Thus $a^2 = 4$ and $b^2 = 12$, and the required standard hyperbola is $x^2/4 - y^2/12 = 1$.

Step 2: Form and check the tangent Insert $x_1 = 4$, $y_1 = 6$ and the calculated squared semi-axes into the tangent formula:

$$\frac{4x}{4} - \frac{6y}{12} = 1 \quad \Rightarrow \quad x - \frac{y}{2} = 1.$$

Multiplying by 2 gives $2x - y - 2 = 0$. As an independent check, implicit differentiation gives $y\prime = 3x/y$, whose value at $(4, 6)$ is 2. Our line has slope 2 and passes through the point. All four listed lines pass through the point, so incidence alone cannot select the answer: the other slopes are $3/2$, $2/3$ and $1/2$, none of which equals the tangent slope.

Step 3: Conclusion The tangent is $\boxed{2x - y - 2 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: When all candidate lines pass through the specified point, use the derivative slope to distinguish the tangent; merely substituting the point is insufficient.

30. The tangent and the normal lines at the point ($\sqrt{3}$, 1) to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is : [4 marks]

(A) $\frac{4}{\sqrt{3}}$
(B) $\frac{1}{\sqrt{3}}$
(C) $\frac{2}{\sqrt{3}}$
(D) $\frac{1}{3}$

Correct Answer: (C) $\frac{2}{\sqrt{3}}$

Solution:

Concept:

- The tangent to $x^2 + y^2 = r^2$ at (x_1, y_1) is $xx_1 + yy_1 = r^2$. The normal is the radius through the centre and the point of contact.
- A triangle with its base on the x-axis has area $\frac{1}{2} \times$ base length $\times |y_P|$, where P is its third vertex. The perpendicular height is a vertical distance, not the length of either sloping side.

Step 1: Find the tangent and normal intercepts The point $P = (\sqrt{3}, 1)$ lies on the circle because $3 + 1 = 4$. Its tangent is therefore

$$\sqrt{3}x + y = 4.$$

At $y = 0$, this meets the x-axis at $T = (4/\sqrt{3}, 0)$. The circle is centred at $O = (0, 0)$; hence the normal is $y = x/\sqrt{3}$ and meets the x-axis at O. The tangent and normal intersect at P, so the three requested lines enclose triangle OTP.

Step 2: Calculate the base and perpendicular height The base along the x-axis has length $OT = 4/\sqrt{3}$,

and the perpendicular distance of P from that axis is its ordinate, 1. Consequently,

$$\text{Area}(OTP) = \frac{1}{2}\left(\frac{4}{\sqrt{3}}\right)(1) = \frac{2}{\sqrt{3}}.$$

The option $4/\sqrt{3}$ omits the factor $1/2$ from triangle area. The value $1/\sqrt{3}$ halves the result once too often, while $1/3$ is unrelated to the correct base-height product. The units are square units because both the base and height are coordinate lengths.

Step 3: Conclusion The triangle has area $\boxed{2/\sqrt{3}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a circle centred at the origin, the normal passes through the origin. With the axis as base, use the contact point's ordinate directly as the triangle height.

Physics (120 marks)

31. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :- [4 marks]

(A) 1.7 mm

(B) 1.9 mm

(C) 1.3 mm

(D) 1.5 mm

Correct Answer: (A) 1.7 mm

Solution:

Concept:

- Young's modulus is $Y = (F/A)/(\Delta L/L)$, so a uniform wire stretches by $\Delta L = FL/(AY)$ under an axial force F.
- A circular wire of radius r has area $A = \pi r^2$. For the same load and the same extension, $L/(r^2Y)$ must be equal for both wires; the radii therefore enter quadratically.

Step 1: Equate the two extensions Use $L_A = 2\,\text{m}$, $L_B = 1.5\,\text{m}$, $r_A = R$ and $r_B = 2\,\text{mm}$. Since the load and extension are the same,

$$\frac{FL_A}{\pi R^2 Y_A} = \frac{FL_B}{\pi r_B^2 Y_B}.$$

Cancel the common force and π, then solve for the radius ratio:

$$\frac{R^2}{r_B^2} = \frac{L_A}{L_B}\frac{Y_B}{Y_A}.$$

The modulus ratio must be inverted here: the stiffer wire requires less cross-sectional area for a fixed length and extension.

Step 2: Substitute the ratios and select the nearest value The stated ratio is $Y_A/Y_B = 7/4$. Therefore,

$$\frac{R^2}{(2\,\text{mm})^2} = \frac{2}{1.5}\frac{4}{7} = \frac{16}{21},$$

$$R = 2\sqrt{\frac{16}{21}}\,\text{mm} = \frac{8}{\sqrt{21}}\,\text{mm} \simeq 1.746\,\text{mm}.$$

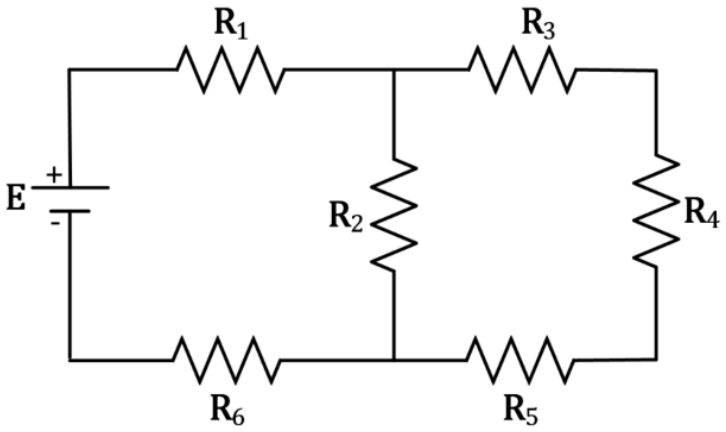
The closest offered radius is $1.7\,\text{mm}$. Its difference is approximately $0.046\,\text{mm}$, compared with 0.154, 0.446, and $0.246\,\text{mm}$ for 1.9, 1.3, and 1.5 respectively. Thus rounding cannot favour any of the other listed choices. Lengths cancel as a ratio, so retaining millimetres for both radii is consistent.

Step 3: Conclusion $R \simeq \boxed{1.7\,\text{mm}}$ among the given choices. Hence, the correct option is (A).

Quick Tip to Solve: Write the extension formula before taking ratios: for equal extensions, r^2 is proportional to L/Y, and take the square root only at the end.

32. In the figure shown, what is the current (in Ampere) drawn from the battery ?

You are given: R_1 = 15Ω, R_2 = 10 Ω, R_3 = 20 Ω, R_4 = 5Ω, R_5 = 25Ω, R_6 = 30 Ω, E = 15 V [4 marks]



(A) 13/24

(B) 7/18

(C) 9/32

(D) 20/3

Correct Answer: (C) 9/32

Solution:

Concept:

- Resistors are in series when the same current must pass through each with no junction between them. Their resistances add. Parallel branches share the same two end nodes and the same potential difference.
- For two parallel resistances R_a, R_b, $R_p = R_aR_b/(R_a + R_b)$. Once the network is reduced, the battery current is $I = E/R_{\text{eq}}$, using the full equivalent resistance.

Step 1: Reduce the right-hand path between the central nodes Between the top and bottom central junctions, the right-hand path contains R_3, R_4, R_5 consecutively, with no other branch at either right-hand corner. Its resistance is

$$R_{345} = R_3 + R_4 + R_5 = 20 + 5 + 25 = 50\,\Omega.$$

That entire path is in parallel with $R_2 = 10\,\Omega$; the individual right-hand resistors are not separately parallel to R_2. Thus

$$R_p = \frac{10 \times 50}{10 + 50} = \frac{25}{3}\,\Omega.$$

This value is below $10\,\Omega$, as any parallel combination must be below its smaller branch resistance.

Step 2: Include the outer series resistors and use Ohm's law Every current drawn from the battery passes through R_1 before splitting and through R_6 after recombination. Hence

$$R_{\text{eq}} = 15 + \frac{25}{3} + 30 = \frac{160}{3}\,\Omega,$$

$$I = \frac{15}{160/3} = \frac{45}{160} = \frac{9}{32}\,\text{A}.$$

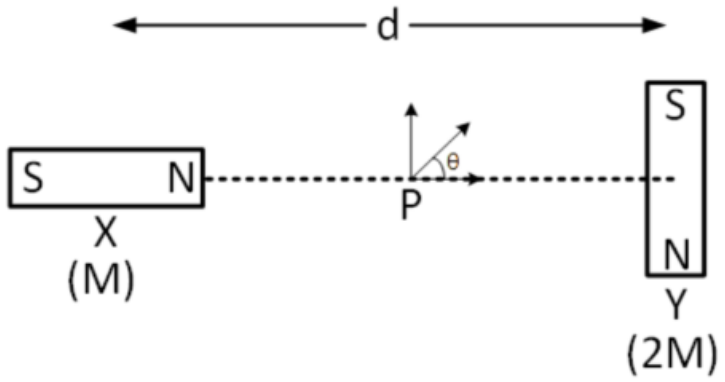
The other current choices do not satisfy the total resistance: $13/24$ and $7/18\,\text{A}$ would each give a resistance below $R_1 + R_6 = 45\,\Omega$, and $20/3\,\text{A}$ is still larger. All are incompatible with the two compulsory outer series resistors alone.

Step 3: Conclusion The battery supplies $\boxed{9/32\,\text{A}}$. Hence, the correct option is (C).

Quick Tip to Solve: Mark the two central junctions first: everything on the right is one series path in parallel with R_2; only then add R_1 and R_6.

33. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing, through their midpoint P, at angle q = 45° with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant ?

(d is much larger than the dimensions of the dipole) [4 marks]



(A) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}\times qv$

(B) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(C) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- Far from a magnetic dipole, its axial field has magnitude $2(\mu_0/4\pi)m/r^3$ and points along its magnetic moment. Its equatorial field has magnitude $(\mu_0/4\pi)m/r^3$ and points opposite the moment.
- Magnetic force is $\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$, with magnitude $|q|vB\sin\phi$. A nonzero field can exert zero force when the velocity is parallel or antiparallel to the resultant field.

Step 1: Resolve the two dipole fields at the midpoint Let $r = d/2$ and $k = \mu_0/(4\pi)$. In the figure, X has moment M directed horizontally right, from S to N. P is on its axial line to the right, so

$$\mathbf{B}_X = \frac{2kM}{r^3}\,\hat{\mathbf{i}}.$$

Y has moment $2M$ vertically downward, from its upper S pole to its lower N pole. P is on Y's equatorial line, so the field there is upward, opposite that moment:

$$\mathbf{B}_Y = \frac{2kM}{r^3}\,\hat{\mathbf{j}}.$$

The axial factor of 2 for X compensates exactly for Y's double dipole moment.

Step 2: Compare the resultant field with the velocity The two perpendicular components are equal;

their sum is

$$\mathbf{B} = \frac{2kM}{r^3}(\hat{\mathbf{i}} + \hat{\mathbf{j}}).$$

It makes 45° above the rightward horizontal. The particle's velocity shown in the figure has precisely that direction. The angle in the force formula is therefore the angle between velocity and the resultant field, namely zero, not the angle to the horizontal. Thus

$$F = |q|vB \sin 0 = 0.$$

The three nonzero choices result from using one component or a resultant magnitude without the required cross-product angle. Neither dipole field vanishes; it is their resultant's alignment with the motion that makes the magnetic force vanish.

Step 3: Conclusion The instantaneous magnetic force has magnitude $\boxed{0}$. Hence, the correct option is (D).

Quick Tip to Solve: Find field directions before magnitudes. Here the axial factor for X and doubled moment of Y give equal perpendicular fields, placing the resultant along the particle's path.

34. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is:- [4 marks]

(A) 1V

(B) 5V

(C) 2V

(D) 3V

Correct Answer: (A) 1V

Solution:

Concept:

- For an ideal parallel-plate capacitor, the potential difference depends on the opposite charges on the facing surfaces: $V = Q_{\text{cap}}/C$. A common charge added to both plates does not create an additional interplate field.
- If the total charges of the two identical plates are Q_1 and Q_2, the differential charge magnitude is $Q_{\text{cap}} = |Q_2 - Q_1|/2$. The remaining equal charge component resides on the outer surfaces in the ideal large-plate approximation.

Step 1: Separate common and differential charges The plate charges are $Q_1 = +2\,\mu\text{C}$ and $Q_2 = +4\,\mu\text{C}$. Their mean is

$$Q_{\text{common}} = \frac{Q_1 + Q_2}{2} = 3\,\mu\text{C}.$$

Write the actual charges as $Q_1 = 3 - 1$ and $Q_2 = 3 + 1$, in microcoulombs. This separates an equal common component of $+3\,\mu\text{C}$ on each plate from the capacitor component $-1\,\mu\text{C}$ and $+1\,\mu\text{C}$. It is the latter pair that produces the field between the plates. The same result follows by subtracting the fields of the two charged sheets in the gap.

Step 2: Convert the differential charge into a voltage The effective magnitude is

$$Q_{\text{cap}} = \frac{|4-2|}{2} = 1\,\mu\text{C}.$$

With $C = 1\,\mu\text{F}$, the voltage magnitude is

$$|V_2 - V_1| = \frac{1\,\mu\text{C}}{1\,\mu\text{F}} = 1\,\text{V}.$$

The plate carrying $+4\,\mu\text{C}$ is at the higher potential. Using $2\,\mu\text{C}$ directly would incorrectly give 2 V, and using the common charge would incorrectly give 3 V. The 5 V option also fails the charge-difference relation. In particular the sum of the two positive charges does not play the role of the opposite plate charge in $Q = CV$.

Step 3: Conclusion The potential difference has magnitude $\boxed{1\,\text{V}}$. Hence, the correct option is (A).

Quick Tip to Solve: For unequal charges on the two capacitor plates, use half their difference in $V = Q/C$, rather than either total plate charge or their sum.

35. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio h_{sph}/h_{cyl} is given by :- [4 marks]



(A) 1

(B) 14/15

(C) 4/5

(D) $2/\sqrt{5}$

Correct Answer: (B) 14/15

Solution:

Concept:

- For rolling without slipping, $\omega = v/R$. The total kinetic energy is translational plus rotational: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.
- A solid sphere has $I = \frac{2}{5}mR^2$, while a solid cylinder has $I = \frac{1}{2}mR^2$ about its symmetry axis. Ideal static friction does no work at the stationary contact point, so mechanical energy is conserved during the climb.

Step 1: Express the maximum height for a rolling body Write $I = kmR^2$, where k depends on the mass distribution. At the greatest height the linear velocity becomes zero. Because rolling without slipping holds throughout, the angular velocity also becomes zero at that instant. Thus all the initial kinetic energy becomes gravitational potential energy:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(kmR^2)\frac{v^2}{R^2} = \frac{1}{2}mv^2(1+k).$$

Cancelling mass gives $h = (1+k)v^2/(2g)$. The identical radii cancel from the rotational energy, and equality of masses is not needed.

Step 2: Insert the sphere and cylinder inertia factors For the solid sphere, $k = 2/5$, giving

$$h_{\text{sph}} = \frac{7v^2}{10g}.$$

For the solid cylinder, $k = 1/2$, giving

$$h_{\text{cyl}} = \frac{3v^2}{4g}.$$

Both start with the same linear speed, so

$$\frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{7/10}{3/4} = \frac{14}{15}.$$

The ratio is less than 1 because the cylinder has more rotational energy per unit mass at a given speed. A ratio of 1 ignores this rotational contribution. The alternatives $4/5$ and $2/\sqrt{5}$ do not equal the ratio of the full energy factors $(1+2/5)/(1+1/2)$; taking ratios of moments alone or square roots of energy factors is inappropriate for heights.

Step 3: Conclusion $\boxed{h_{\text{sph}}/h_{\text{cyl}} = 14/15}$. Hence, the correct option is (B).

Quick Tip to Solve: For a rolling body climbing to rest, compare $1 + I/(mR^2)$, not just the moments of inertia; the translational energy contributes the common 1.

36. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :- [4 marks]

(A) 25 cm

(B) 10 cm

(C) 20 cm

(D) 30 cm

Correct Answer: (B) 10 cm

Solution:

Concept:

- The thin-lens formula is $1/f = 1/v - 1/u$, with distances measured from the optical centre using the Cartesian sign convention.
- A concave mirror returns rays along their incident paths when the object for the mirror is at its centre of curvature. A real object gives a virtual mirror image only when it lies between the pole and focus.

Step 1: Locate the image formed by the lens alone Take the lens at the origin and positive distances to the right. The object has $u = -30\,\text{cm}$, while the convex lens has $f = 20\,\text{cm}$. Thus

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}, \qquad v = 60\,\text{cm}.$$

This real image is to the left of the mirror, whose pole is at $x = 80\,\text{cm}$. The separation between that image and the mirror is therefore $80 - 60 = 20\,\text{cm}$.

Step 2: Apply the unchanged-image condition For the mirror to leave this image position unchanged, the image point must be the centre of curvature of the mirror. The rays pass through this point, reach the mirror normally, and are reflected back through the same point. Hence its radius of curvature is $R = 20\,\text{cm}$, giving

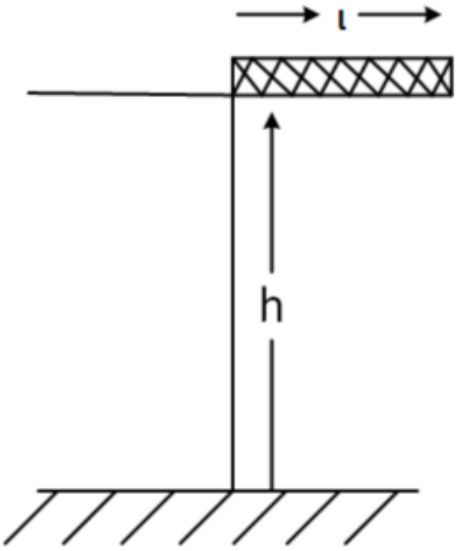
$$|f_m| = R/2 = 10\,\text{cm}.$$

Used by itself, this concave mirror forms a virtual image for real objects at distances less than $10\,\text{cm}$. The requested maximum is this limiting distance; at the focus the image tends to infinity. The alternatives 20, 25 and 30 cm lie outside the virtual-image range.

Step 3: Conclusion The limiting object distance is $10\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: First find the lens image, then measure its distance from the mirror; an unchanged mirror image identifies the centre of curvature.

37. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time t = 0.01s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :- [4 marks]



(A) 0.28

(B) 0.02

(C) 0.3

(D) 0.5

Correct Answer: (D) 0.5

Solution:

Concept:

- While the box is briefly supported at the platform edge, gravity produces a torque $mgl/2$ about the edge.
- For a uniform slender box, the moment of inertia about the supporting edge is $I = ml^2/3$. Once detached, gravity acts through the centre of mass and produces no torque about it.

Step 1: Find the angular speed at detachment During the short interval $t = 0.01\,\text{s}$, the box remains essentially horizontal, so its gravitational torque is approximately constant. With $l = 0.3\,\text{m}$,

$$\alpha = \frac{mgl/2}{ml^2/3} = \frac{3g}{2l} \simeq \frac{3(10)}{2(0.3)} = 50\,\text{rad}\,\text{s}^{-2}.$$

Starting from rest, its angular speed when it leaves the edge is

$$\omega = \alpha t = 50(0.01) = 0.5\,\text{rad}\,\text{s}^{-1}.$$

Its rotation in this initial interval is only $\alpha t^2/2 = 0.0025\,\text{rad}$, consistent with the stated nearly horizontal position.

Step 2: Calculate the subsequent angle of rotation The centre of mass falls through approximately $h = 5\,\mathrm{m}$. The small vertical speed acquired during detachment and the finite thickness of the box can be neglected at the requested accuracy. Its flight time is

$$T \simeq \sqrt{\frac{2h}{g}} = \sqrt{\frac{10}{10}} = 1\,\mathrm{s}.$$

With no torque about the freely falling centre of mass, the angular speed stays constant. Thus

$$\theta \simeq \omega T = 0.5(1) = 0.5\,\mathrm{rad}.$$

Including the tiny initial angle does not change the nearest option. The values 0.02, 0.28 and 0.3 rad are inconsistent with the angular impulse and the approximately one-second fall.

Step 3: Conclusion The box rotates through approximately $0.5\,\mathrm{rad}$. Hence, the correct option is (D).

Quick Tip to Solve: Use torque only during contact with the edge; after detachment the box keeps its angular speed while its centre of mass falls.

38. The magnetic field of an electromagnetic wave is given by :-

$$\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right) \left(2\hat{i} + \hat{j}\right) \frac{Wb}{m^2}$$

The associated electric field will be :- [4 marks]

(A) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z - 6 \times 10^{15} t\right) \left(-2\hat{i} + \hat{j}\right) \frac{V}{m}$

(B) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z - 6 \times 10^{15} t\right) \left(2\hat{i} + \hat{j}\right) \frac{V}{m}$

(C) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right) \left(\hat{i} - 2\hat{j}\right) \frac{V}{m}$

(D) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right) \left(-\hat{i} + 2\hat{j}\right) \frac{V}{m}$

Correct Answer: (D) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right) \left(-\hat{i} + 2\hat{j}\right) \frac{V}{m}$

Solution:

Concept:

- The phase $kz + \omega t$ describes propagation in the negative z-direction. The electric and magnetic fields of a plane electromagnetic wave have the same phase.
- The fields satisfy $E_0 = cB_0$, are mutually perpendicular, and $\mathbf{E} \times \mathbf{B}$ points along the direction of propagation.

Step 1: Find the propagation direction and electric amplitude Keeping the phase fixed gives $z =$

$-(\omega/k)t +$ constant, so the wave travels along $-\hat{\mathbf{k}}$. Its speed is

$$c = \frac{\omega}{k} = \frac{6 \times 10^{15}}{2 \times 10^{7}} = 3 \times 10^{8}\,\mathrm{m\,s^{-1}}.$$

Multiplying the magnetic-field coefficient by this speed gives

$$(3 \times 10^{8})(1.6 \times 10^{-6}) = 4.8 \times 10^{2}\,\mathrm{V\,m^{-1}}.$$

The vector multiplying the coefficient must have the same magnitude $\sqrt{5}$ as $2\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

Step 2: Choose the perpendicular vector with the correct sign The vector $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ is perpendicular to the magnetic-field direction because $(-1)(2) + (2)(1) = 0$. Its cross product with that direction is

$$(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) = -5\hat{\mathbf{k}},$$

as required. Therefore

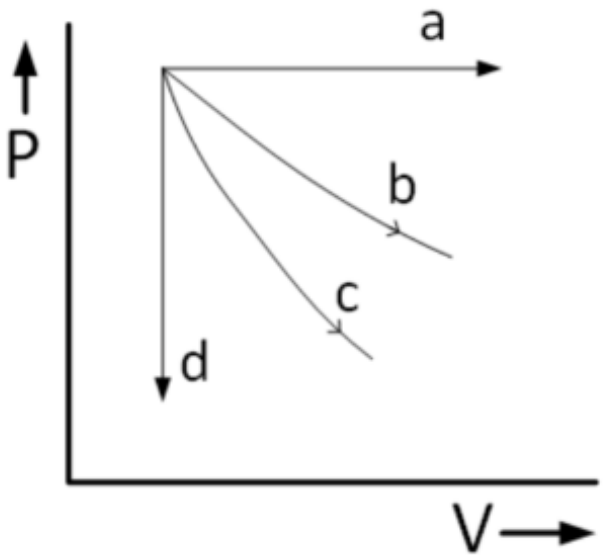
$$\mathbf{E} = 4.8 \times 10^{2} \cos(2 \times 10^{7} z + 6 \times 10^{15} t)(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\,\mathrm{V\,m^{-1}}.$$

Options A and B use the wrong phase; B is also parallel to the magnetic field. Option C is perpendicular but gives energy flow along positive z, opposite to the given wave.

Step 3: Conclusion The electric field has direction $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and the same phase as the given magnetic field. The question-sheet key selects the opposite electric-field vector, but that vector gives energy flow along positive z, whereas the printed plus sign in the phase requires negative z. Hence, the correct option is (D).

Quick Tip to Solve: Check the phase first, then use the cross product to choose between the two possible perpendicular electric-field directions.

39. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :- [4 marks]



(A) d a b c

(B) a d b c

(C) d a c b

(D) a d c b

Correct Answer: (A) d a b c

Solution:

Concept:

- An isochoric process has constant volume, whereas an isobaric process has constant pressure. They appear respectively as vertical and horizontal lines on a P-V plot.
- For an ideal gas, an isotherm obeys $PV =$ constant, whereas a reversible adiabat obeys $PV^{\gamma} =$ constant, where $\gamma = C_P/C_V > 1$.

Step 1: Identify the constant-volume and constant-pressure paths The horizontal coordinate of the diagram is volume V, and its vertical coordinate is pressure P. Along path d the line goes vertically down, so volume stays fixed while pressure decreases. Path d is therefore isochoric. Along path a the line goes horizontally to the right, so pressure stays fixed while volume increases. Path a is therefore isobaric. Consequently, the first two letters in the requested order must be d, a. Options B and D interchange these two basic processes and are excluded.

Step 2: Compare the slopes of the curved paths Both curved paths begin at the same pressure and volume and represent expansion. Differentiating their equations gives

$$\left(\frac{dP}{dV}\right)_T = -\frac{P}{V}, \qquad \left(\frac{dP}{dV}\right)_{\text{ad}} = -\gamma\frac{P}{V}.$$

Since $\gamma > 1$, the adiabat initially falls more steeply and lies below the isotherm on expansion. In the figure c is the steeper, lower curve, while b is the gentler, upper curve. Thus b is isothermal and c is

adiabatic. Option C reverses these two curves and is excluded. This comparison uses the common initial state; it is not based merely on whether a curve bends downward.

Step 3: Conclusion The order is isochoric d, isobaric a, isothermal b, adiabatic c: $d\ a\ b\ c$. Hence, the correct option is (A).

Quick Tip to Solve: On a pressure-volume diagram, identify vertical and horizontal lines first; the steeper curve through a common state is the adiabat.

---

40. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is :- [4 marks]

(A) $v_4 - \frac{v_2}{2}$

(B) $v_4 - \frac{v_2}{4}$

(C) $v_4 - v_2$

(D) $v_4 + v_2$

Correct Answer: (C) $v_4 - v_2$

Solution:

Concept:

- In a short collision with negligible external impulse, the total linear momentum before and after collision is the same.
- All velocities are along $\hat{\mathbf{i}}$, so their signed scalar components can be used directly. Conservation of kinetic energy is not needed unless the collision is specified to be elastic.

Step 1: Write momentum conservation along the collision axis The initial momentum is $m_1v_1 + m_2v_2$, and the final momentum is $m_1v_3 + m_2v_4$. Thus

$$m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4.$$

Substitute the two relations supplied in the question, $m_2 = 0.5m_1$ and $v_3 = 0.5v_1$. Dividing the resulting equation by the nonzero mass m_1 gives

$$v_1 + \frac{v_2}{2} = \frac{v_1}{2} + \frac{v_4}{2}.$$

The mass cancels, so no numerical value of either mass is necessary.

Step 2: Isolate and check the unknown velocity Multiply the reduced equation by 2 and collect terms:

$$2v_1 + v_2 = v_1 + v_4, \qquad v_1 = v_4 - v_2.$$

A direct check is obtained by substituting $v_4 = v_1 + v_2$ into the final momentum: it becomes $m_1v_1/2 + m_1(v_1 + v_2)/2 = m_1v_1 + m_1v_2/2$, exactly the initial value. Options A and B leave an incorrect coefficient of v_2, while option D has the wrong sign. The result uses signed components, so

it also applies when one component is negative. No assumption about a coefficient of restitution has been introduced.

Step 3: Conclusion The unknown velocity is $v_1 = v_4 - v_2$. Hence, the correct option is (C).

Quick Tip to Solve: Cancel the common mass before rearranging momentum conservation; do not add an elastic-collision equation when elasticity is not given.

41. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to :- [4 marks]

(A) 0.2%

(B) 3.5%

(C) 0.7%

(D) 6.8%

Correct Answer: (D) 6.8%

Solution:

Concept:

- For a simple pendulum at small amplitude, $g = 4\pi^2 L/T^2$, where L is the pendulum length and T its period.
- The maximum fractional error for a product of powers is the sum of the absolute fractional errors multiplied by their powers: $\Delta g/g = \Delta L/L + 2\Delta T/T$.

Step 1: Find the length and period uncertainties The measured length is $L = 55.0\,\text{cm}$, and the meter-scale least count is $1\,\text{mm} = 0.1\,\text{cm}$. Therefore

$$\frac{\Delta L}{L} = \frac{0.1}{55.0} = 0.001818.$$

The watch measures the total time $t = 30\,\text{s}$ for 20 oscillations with uncertainty $\Delta t = 1\,\text{s}$. As $T = t/20$ and the oscillation count is exact,

$$T = 1.5\,\text{s}, \qquad \Delta T = 0.05\,\text{s}, \qquad \frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{30}.$$

Dividing the total time by 20 does not reduce its fractional uncertainty.

Step 2: Propagate the uncertainties into gravity The factor $4\pi^2$ has no measurement uncertainty here. Since the period is squared in the denominator, its relative uncertainty contributes twice:

$$\frac{\Delta g}{g} = \frac{0.1}{55.0} + 2\frac{1}{30} = 0.068485.$$

Hence

$$\frac{\Delta g}{g} \times 100\% = 6.8485\% \simeq 6.8\%.$$

The 0.2 percent option reflects approximately the length contribution alone. The 3.5 percent option approximately omits the factor of two for timing, and 0.7 percent is too small for the given one-second timing uncertainty. We add positive uncertainty contributions, rather than subtracting them because the period occurs in the denominator.

Step 3: Conclusion The percentage error in g is approximately 6.8%. Hence, the correct option is (D).

Quick Tip to Solve: For timing many oscillations, use the fractional error of the total recorded time; multiply it by two when finding the error in gravity.

42. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon ? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon :- [4 marks]

(A) E/32

(B) E/16

(C) E/4

(D) E/64

Correct Answer: (B) E/16

Solution:

Concept:

- The minimum mechanical energy supplied to a rocket initially at rest on a spherical body of mass M and radius R is GMm/R, taking gravitational potential energy as zero at infinity.
- For bodies of equal density ρ, mass is proportional to volume, and $M = (4\pi/3)\rho R^3$. For the same rocket, escape energy therefore scales as R^2.

Step 1: Find the mass and radius ratios Let Earth and Moon quantities carry subscripts e and m respectively. Equal density makes the mass ratio equal the given volume ratio:

$$\frac{M_e}{M_m} = \frac{V_e}{V_m} = 64.$$

Volume is proportional to the cube of radius, so

$$\frac{R_e}{R_m} = 64^{1/3} = 4.$$

The rocket mass is the same in both launches. The calculation uses the equal-density condition stated in this problem, rather than the actual astronomical density ratio.

Step 2: Compare the escape energies For minimum escape, the rocket reaches infinity with vanishing

limiting speed. Its required launch energy compensates the negative gravitational potential energy at the surface:

$$E_e = \frac{GM_e m}{R_e} = E, \qquad E_m = \frac{GM_m m}{R_m}.$$

On dividing, the gravitational constant and rocket mass cancel:

$$\frac{E_m}{E_e} = \frac{M_m}{M_e}\frac{R_e}{R_m} = \frac{1}{64} \times 4 = \frac{1}{16}.$$

Thus the smaller mass reduces the energy, but the smaller radius partly offsets that reduction. Option D uses only the mass ratio and misses the radius factor. Option C uses only the radius ratio; option A does not follow from either the mass-radius scaling or the energy expression.

Step 3: Conclusion The minimum launcher energy on the Moon is $E/16$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-density spherical bodies, escape energy for a fixed rocket scales as radius squared, or as volume to the power two-thirds.

43. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is :- [4 marks]

(A) $\sqrt{\frac{qE}{md}}$

(B) $\sqrt{\frac{qE}{2md}}$

(C) $\sqrt{\frac{qE}{-2md}}$

(D) $\sqrt{\frac{2qE}{md}}$

Correct Answer: (D) $\sqrt{\frac{2qE}{md}}$

Solution:

Concept:

- A dipole of moment $p = qd$ experiences torque $\tau = -pE\sin\theta$, where θ is its displacement from stable alignment with the field.
- For small angular displacements, $\sin\theta \simeq \theta$, so $I\ddot{\theta} + pE\theta = 0$ and $\omega^2 = pE/I$.

Step 1: Locate the rotation axis The uniform field exerts equal and opposite forces on the two charges, giving zero resultant force. Their centre of mass remains at rest while the dipole rotates about an axis through its midpoint perpendicular to its plane of rotation. Each charge has mass m, and each is at distance $d/2$ from this axis. Therefore the moment of inertia is

$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}.$$

Using just one mass, or taking the distance of each charge from the midpoint as d, would give the

wrong inertia.

Step 2: Apply the restoring-torque equation Stable equilibrium has the dipole parallel to the field. A small rotation produces a torque opposite to the displacement:

$$\frac{md^2}{2}\ddot{\theta} = -qdE\theta.$$

Comparing this with $\ddot{\theta} + \omega^2\theta = 0$ gives

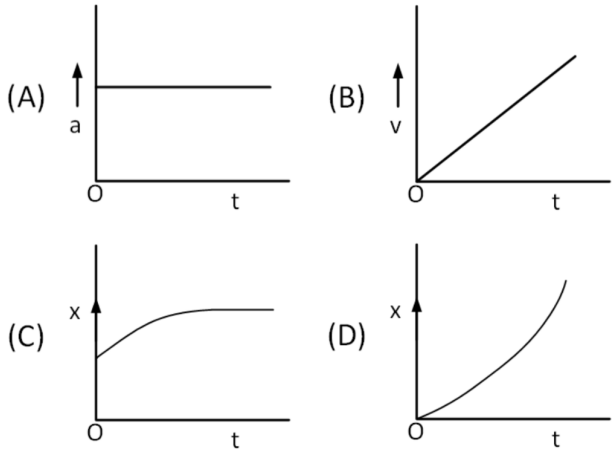
$$\omega^2 = \frac{qdE}{md^2/2} = \frac{2qE}{md}.$$

The positive coefficient confirms stable oscillations. Option (C) would give a negative squared frequency, while (A) and (B) have incorrect factors from the rotational inertia.

Step 3: Conclusion The angular frequency is $\boxed{\omega = \sqrt{2qE/(md)}}$. Hence, the correct option is (D).

Quick Tip to Solve: For two equal endpoint masses, calculate $I = 2m(d/2)^2$ before using $\omega^2 = pE/I$; the separation is not the rotation radius.

44. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time) [4 marks]



(A) (B), (C)

(B) (A)

(C) (A), (B), (C)

(D) (A), (B), (D)

Correct Answer: (D) (A), (B), (D)

Solution:

Concept:

- With initial position and velocity both zero and constant positive acceleration a, the equations are $v = at$ and $x = \frac{1}{2}at^2$.
- The slope of a displacement-time graph equals velocity; its curvature shows how that velocity changes.

Step 1: Check acceleration and velocity Uniform acceleration means that a is independent of time. Since the motion is along the positive axis from rest, this constant is positive, so the acceleration-time graph is horizontal above the time axis. Figure (A) has exactly this form. Integrating the acceleration with the initial condition $v(0) = 0$ gives

$$v(t) = \int_0^t a\, dt = at.$$

Thus the velocity-time graph must be a straight line through the origin with positive slope. Figure (B) satisfies both the initial condition and the constant-slope condition.

Step 2: Check the two displacement graphs Integrating velocity with $x(0) = 0$ gives

$$x(t) = \int_0^t at\, dt = \frac{1}{2}at^2.$$

Consequently the displacement starts at zero, has zero initial slope and then has an increasing positive slope. Figure (D) is the upward-curving graph required. Figure (C) starts above zero and its slope decreases towards zero; it fails the stated starting position and acceleration. Hence every answer combination containing (C) is excluded, and a combination containing only (A) omits the valid velocity and displacement graphs.

Step 3: Conclusion The correct figures are the constant-acceleration, linear-velocity and upward-curving displacement graphs. Hence, the correct option is (D).

Quick Tip to Solve: Check the starting point and slope as well as the curve shape: from rest at the origin, the displacement-time parabola starts with a horizontal tangent.

45. If surface tension (S), Moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be :- [4 marks]

(A) $S^{1/2}I^{1/2}h^{0}$

(B) $S^{3/2}I^{1/2}h^{0}$

(C) $S^{1/2}I^{1/2}h^{-1}$

(D) $S^{1/2}I^{3/2}h^{-1}$

Correct Answer: (A) $S^{1/2}I^{1/2}h^{0}$

Solution:

Concept:

- Linear momentum has dimensions $[p] = MLT^{-1}$. Surface tension is force per length, so $[S] = MT^{-2}$.
- Moment of inertia has dimensions $[I] = ML^2$, and Planck's constant has dimensions $[h] = ML^2T^{-1}$.

Step 1: Set up the exponent equations Let the dimensional formula of momentum in the proposed base quantities be $[p] = [S]^{\alpha}[I]^{\beta}[h]^{\gamma}$. Substituting their ordinary mass, length and time dimensions gives

$$MLT^{-1} = M^{\alpha+\beta+\gamma}L^{2\beta+2\gamma}T^{-2\alpha-\gamma}.$$

Dimensions must match separately, so

$$\alpha + \beta + \gamma = 1, \qquad \beta + \gamma = \frac{1}{2}, \qquad 2\alpha + \gamma = 1.$$

The second equation comes from length and the third from time; it is essential to retain the sign of the time exponent.

Step 2: Solve and verify the exponents Subtracting the length equation from the mass equation gives $\alpha = 1/2$. The time equation then gives $\gamma = 0$, and the length equation gives $\beta = 1/2$. Thus

$$[p] = [S^{1/2}I^{1/2}h^{0}].$$

As a direct check, $\sqrt{SI}$ has dimensions $\sqrt{(MT^{-2})(ML^2)} = MLT^{-1}$. Option (B) has excess mass and time powers because it uses $S^{3/2}$. Options (C) and (D) include h^{-1}, which changes the dimensions and does not yield momentum.

Step 3: Conclusion The dimensional formula is $\boxed{S^{1/2}I^{1/2}h^{0}}$. Hence, the correct option is (A).

Quick Tip to Solve: Surface tension is force divided by length, not force divided by area; writing its dimensions correctly makes the exponent equations straightforward.

46. Let $\left|\vec{A_1}\right| = 3$, $\left|\vec{A_2}\right| = 5$ and $\left|\vec{A_1} + \vec{A_2}\right| = 5$. The value of $\left(2\vec{A_1} + 3\vec{A_2}\right)\left(3\vec{A_1} - 2\vec{A_2}\right)$ is :- [4 marks]

(A) –118.5

(B) –112.5

(C) –99.5

(D) –106.5

Correct Answer: (A) –118.5

Solution:

Concept:

- For any two vectors, $|\vec{A}_1 + \vec{A}_2|^2 = |\vec{A}_1|^2 + |\vec{A}_2|^2 + 2\vec{A}_1 \cdot \vec{A}_2$.
- The scalar product distributes over sums, with $\vec{A} \cdot \vec{A} = |\vec{A}|^2$ and $\vec{A}_1 \cdot \vec{A}_2 = \vec{A}_2 \cdot \vec{A}_1$.

Step 1: Determine the mutual scalar product The given magnitudes are $|\vec{A}_1| = 3$, $|\vec{A}_2| = 5$, and $|\vec{A}_1 + \vec{A}_2| = 5$. Squaring the last relation gives

$$25 = 9 + 25 + 2\vec{A}_1 \cdot \vec{A}_2.$$

Therefore

$$\vec{A}_1 \cdot \vec{A}_2 = -\frac{9}{2} = -4.5.$$

The negative result means that the angle between the two vectors is obtuse. We need no components or explicit angle, because all terms in the requested scalar expression can be obtained directly from this value and the magnitudes.

Step 2: Expand the requested expression Distribute the scalar product carefully, retaining the signs of both cross terms:

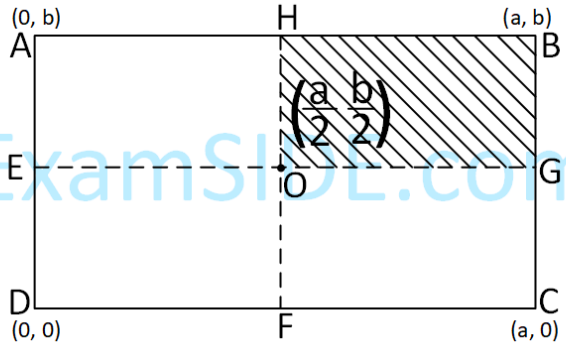
$$\begin{aligned}(2\vec{A}_1 + 3\vec{A}_2) \cdot (3\vec{A}_1 - 2\vec{A}_2) &= 6|\vec{A}_1|^2 - 4\vec{A}_1 \cdot \vec{A}_2 \\ &\quad + 9\vec{A}_2 \cdot \vec{A}_1 - 6|\vec{A}_2|^2 \\ &= 6(9) + 5(-4.5) - 6(25) \\ &= 54 - 22.5 - 150 = -118.5.\end{aligned}$$

The cross-term coefficient is $-4 + 9 = 5$, not $-4 - 9$. The other three numerical options do not equal this expansion using the mutual product fixed by the data.

Step 3: Conclusion The required scalar product is $\boxed{-118.5}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the mutual dot product from the squared resultant first; then collect the cross terms before substituting numerical values.

47. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be :- [4 marks]



ExamSIDE.com

(A) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$
(B) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$
(C) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$
(D) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

Correct Answer: (C) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

Solution:

Concept:

- A uniform lamina has mass proportional to area, and the centre of mass of a rectangle is its geometric centre.
- A removed part can be treated as negative mass: $\vec{r}_{\text{rem}} = (M\vec{r}_{\text{full}} - m\vec{r}_{\text{cut}})/(M-m)$.

Step 1: Find the mass and centre of the removed part The original rectangle extends from $(0,0)$ to (a,b), so its centre is $(a/2, b/2)$. The shaded rectangle HBGO occupies the upper-right quarter: its horizontal limits are $a/2$ and a, and its vertical limits are $b/2$ and b. Its area is $ab/4$, giving removed mass $M/4$, and its centre is

$$\left(\frac{a/2+a}{2}, \frac{b/2+b}{2}\right) = \left(\frac{3a}{4}, \frac{3b}{4}\right).$$

The remaining mass is therefore $3M/4$, not M.

Step 2: Subtract the removed first moments For the horizontal coordinate, subtract the first moment of the removed material from that of the full sheet:

$$x_{\text{rem}} = \frac{M(a/2) - (M/4)(3a/4)}{3M/4} = \frac{Ma(1/2 - 3/16)}{3M/4} = \frac{5a}{12}.$$

The same calculation in the vertical direction gives

$$y_{\text{rem}} = \frac{M(b/2) - (M/4)(3b/4)}{3M/4} = \frac{5b}{12}.$$

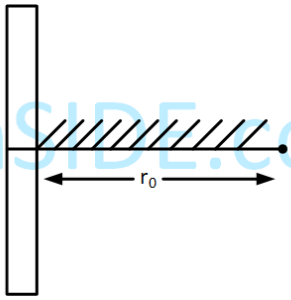
Removing the upper-right material must shift the centre left and downward from $(a/2, b/2)$. This rules out options (B) and (D), both above the midpoint in each coordinate; (A) even lies outside the

original rectangle.

Step 3: Conclusion The remaining centre of mass is $\boxed{(5a/12,\ 5b/12)}$. Hence, the correct option is (C).

Quick Tip to Solve: After subtracting the removed first moment, divide by the remaining mass. Check that the centre shifts away from the removed material.

48. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :- [4 marks]



ExamSIDE.com

(A) $v \propto \left(\frac{r}{r_0}\right)$

(B) $v \propto \ln\left(\frac{r}{r_0}\right)$

(C) $v \propto e^{+r/r_0}$

(D) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

Correct Answer: (D) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

Solution:

Concept:

- The electric field at distance r from a long uniform line charge of density λ is $E(r) = \lambda/(2\pi\varepsilon_0 r)$, directed radially outward for positive λ.
- The work done by the electric field equals the increase in kinetic energy: $\Delta K = \int qE\,dr$.

Step 1: Integrate the electric force Let the positive point charge have charge q and mass m. It is repelled by the positive line charge and moves away from it, so $r \geq r_0$. Since the electric field decreases as the inverse distance, the force is not constant. The work from the release position to distance r is

$$W = \int_{r_0}^{r} \frac{q\lambda}{2\pi\varepsilon_0 r'}\,dr' = \frac{q\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r}{r_0}\right).$$

The logarithm contains a dimensionless ratio and is positive as the charge moves outward.

Step 2: Convert the work into speed The charge starts from rest, so its initial kinetic energy is zero.

Conservation of energy therefore gives

$$\frac{1}{2}mv^2 = \frac{q\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r}{r_0}\right),$$

$$v = \sqrt{\frac{q\lambda}{\pi\varepsilon_0 m}}\sqrt{\ln\left(\frac{r}{r_0}\right)}.$$

The constants under the first square root do not depend on r. Option (B) describes the distance dependence of v^2, not v. Options (A) and (C) also fail the initial condition: they are nonzero at $r = r_0$, whereas the charge is released from rest.

Step 3: Conclusion The speed varies as $\boxed{v \propto \sqrt{\ln(r/r_0)}}$. Hence, the correct option is (D).

Quick Tip to Solve: For an inverse-distance electric force, work is logarithmic. Take the square root only after equating this work to kinetic energy.

49. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star :- [4 marks]

(A) 610 $\times$ 10^{-9} radian

(B) 457.5 $\times$ 10^{-9} radian

(C) 305 $\times$ 10^{-9} radian

(D) 152.5 $\times$ 10^{-9} radian

Correct Answer: (C) 305 $\times$ 10^{-9} radian

Solution:

Concept:

- The Rayleigh limit for a circular objective is the minimum angular separation $\theta_{\min} = 1.22\lambda/D$, in radians, where D is its diameter.
- Wavelength and aperture diameter must be expressed in the same length unit before taking their ratio.

Step 1: Identify the aperture and convert units The question gives the diameter of the objective, not its radius. Thus

$$D = 200\,\text{cm} = 2.00\,\text{m}, \qquad \lambda = 500\,\text{nm} = 500 \times 10^{-9}\,\text{m}.$$

The circular aperture produces an Airy diffraction pattern. According to the Rayleigh criterion, two equally bright point sources are just resolved when the central maximum of one coincides with the first minimum of the other. This introduces the factor 1.22; the single-slit expression λ/D is not the correct numerical criterion here.

Step 2: Evaluate the minimum angle Substitute the stated diameter and wavelength:

$$\theta_{\min} = \frac{1.22(500 \times 10^{-9})}{2.00} = 305 \times 10^{-9}\,\text{rad} = 3.05 \times 10^{-7}\,\text{rad}.$$

The ratio of the two lengths is dimensionless and is interpreted as an angle in radians in the small-angle approximation. Option (A) is twice the required angle and would result from using the radius $1\,\mathrm{m}$ in place of the diameter. Option (D) is half the correct angle, while option (B) also does not follow from the specified aperture and Rayleigh factor.

Step 3: Conclusion The angular limit of resolution is $\boxed{305 \times 10^{-9}\,\mathrm{rad}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the diameter in $1.22\lambda/D$, and remember that angular resolution improves as this minimum resolvable angle becomes smaller.

50. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when :- [4 marks]

(A) R = 1000 r

(B) R = r

(C) R = 2r

(D) R = 0.001 r

Correct Answer: (B) R = r

Solution:

Concept:

- The current supplied by a cell of emf $\mathcal{E}$, internal resistance r, and load R is $I = \mathcal{E}/(R+r)$. Internal and external resistances are in series.
- The useful power delivered to the external resistance alone is $P = I^2R$. The emf and internal resistance are fixed while the external resistance varies.

Step 1: Express the load power in terms of resistance Substitute the circuit current into the load-power formula:

$$P(R) = \frac{\mathcal{E}^2 R}{(R+r)^2}.$$

Increasing the load resistance increases the factor R but also reduces the current. Therefore maximizing current alone does not maximize the useful load power. In particular, a short circuit delivers no power to an external resistance despite its large current.

Step 2: Locate and check the maximum Differentiate with respect to the variable load resistance:

$$\frac{dP}{dR} = \mathcal{E}^2 \frac{r-R}{(R+r)^3}.$$

For positive resistances the denominator is positive. Thus the power increases for R less than r, has zero slope at R=r, and decreases for R greater than r. This confirms a maximum, rather than a minimum. The maximum value is

$$P_{\max} = \frac{\mathcal{E}^2}{4r}.$$

Both 1000r and 0.001r are far from matching the internal resistance; 2r is also on the decreasing side

of the maximum. None gives the maximum load power.

Step 3: Conclusion The external resistance must equal the internal resistance: $R = r$. Hence, the correct option is (B).

Quick Tip to Solve: For maximum power in a resistive load, match the load resistance to the source resistance; do not confuse this condition with maximum current or maximum efficiency.

51. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to :- [4 marks]

(A) 1

(B) 5

(C) 0.1

(D) 2

Correct Answer: (A) 1

Solution:

Concept:

- The empirical nuclear-radius relation is $R = R_0 A^{1/3}$, where A is the mass number and R_0 is approximately the same for different nuclei.
- A nucleus contains A nucleons, so its mass is approximately Am_N. Nuclear density is nuclear mass divided by nuclear volume, not the density of the neutral atom.

Step 1: Find the dependence of nuclear volume on mass number Treat a nucleus as approximately spherical. Its volume is

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} R_0^3 A.$$

Thus a nucleus with a larger mass number has a proportionately larger volume. Calcium-40 has more nucleons than oxygen-16, but it is not correspondingly more dense: the additional nucleons also increase its nuclear volume. Using a radius proportional to A instead of its cube root would give an incorrect dependence.

Step 2: Cancel the common mass-number factor The density for either nucleus is

$$\rho_A = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The mass number cancels, so the approximate ratio is

$$\frac{\rho(^{40}\mathrm{Ca})}{\rho(^{16}\mathrm{O})} = 1.$$

Equivalently, their masses are in the ratio 40:16 and their volumes have the same ratio. Small binding-energy and nuclear-structure corrections do not change the answer closest to unity. The values 5, 0.1 and 2 would all require a substantial dependence of density on A, contrary to the radius law.

Step 3: Conclusion The two nuclear mass densities are approximately equal, giving a ratio of 1. Hence, the correct option is (A).

Quick Tip to Solve: Cube the nuclear-radius law first: nuclear volume and nuclear mass are both proportional to A, so their ratio is nearly constant.

52. A circuit connected to an ac source of emf e = $e_0$sin(100t) with t in seconds, gives a phase difference of $\pi/4$ between the emf e and current i. Which of the following circuits will exhibit this ? [4 marks]

(A) RC circuit with R = 1 kΩ and C = 1μF

(B) RL circuit with R = 1kΩ and L = 1mH

(C) RC circuit with R = 1kΩ and C = 10 μF

(D) RL circuit with R = 1 kΩ and L = 10 mH

Correct Answer: (C) RC circuit with R = 1kΩ and C = 10 μF

Solution:

Concept:

- For a series RC circuit, the phase-angle magnitude obeys $\tan|\phi| = X_C/R$, where $X_C = 1/(\omega C)$. For a series RL circuit it obeys $\tan|\phi| = X_L/R$, where $X_L = \omega L$.
- The coefficient of t in the sinusoidal emf is the angular frequency: $\omega = 100\,\mathrm{rad\,s^{-1}}$. A phase-angle magnitude $\pi/4$ requires reactance equal to resistance.

Step 1: Convert the given component values to SI units All four circuits have

$$R = 1\,\mathrm{k\Omega} = 1000\,\Omega.$$

Since $\tan(\pi/4) = 1$, the required reactance is also 1000 ohms. Use the angular frequency directly; inserting an extra factor of 2π would be incorrect because 100 is already the coefficient of time in the sine function.

Step 2: Check every proposed circuit For the two RC choices,

$$X_C(1\,\mu\mathrm{F}) = \frac{1}{100(10^{-6})} = 10^4\,\Omega,$$

$$X_C(10\,\mu\mathrm{F}) = \frac{1}{100(10^{-5})} = 10^3\,\Omega.$$

Their reactance-to-resistance ratios are respectively 10 and 1. For the two RL choices,

$$X_L(1\,\mathrm{mH}) = 100(10^{-3}) = 0.1\,\Omega,$$

$$X_L(10\,\mathrm{mH}) = 100(10^{-2}) = 1\,\Omega.$$

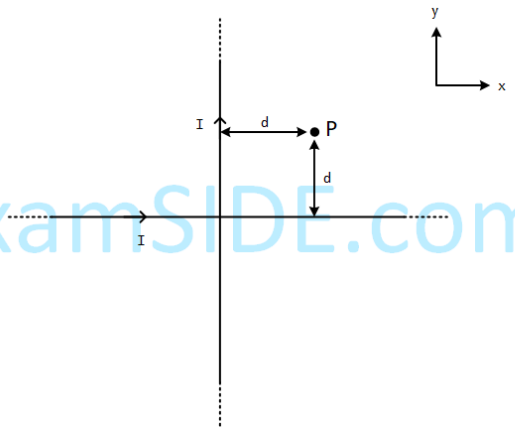
Their ratios are 0.0001 and 0.001, both far below unity. Thus only the RC circuit with the larger capacitance has the required phase magnitude. Its current leads the emf by $\pi/4$; the question specifies

the difference, not a lagging direction.

Step 3: Conclusion The required circuit is RC with $R = 1\,\text{k}\Omega$ and $C = 10\,\mu\text{F}$. Hence, the correct option is (C).

Quick Tip to Solve: At a 45-degree phase difference in a two-component series circuit, set the magnitude of the reactance equal to R, then check the SI prefixes carefully.

53. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure. These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be : [4 marks]



ExamSIDE.com

(A) $\frac{\pm\mu_0 I}{\pi d}\left(\hat{z}\right)$

(B) $-\frac{\mu_0 I}{2\pi d}\left(\hat{x}+\hat{y}\right)$

(C) Zero

(D) $\frac{\mu_0 I}{2\pi d}\left(\hat{x}+\hat{y}\right)$

Correct Answer: (C) Zero

Solution:

Concept:

- The magnetic field magnitude at perpendicular distance d from a very long straight wire carrying current I is $B = \mu_0 I/(2\pi d)$. Its direction follows the right-hand rule.
- Magnetic fields add as vectors. For currents along the coordinate axes in the xy-plane, the field at a point of that plane is perpendicular to the plane.

Step 1: Read the current directions and distances The horizontal wire carries current along the positive x-direction and the vertical wire carries current along the positive y-direction. Point P is in the first quadrant, at perpendicular distance d from each wire. Consequently both fields have the same magnitude,

$$B_x = B_y = \frac{\mu_0 I}{2\pi d},$$

where the subscripts identify the source wire, rather than the direction of the field. Equal magnitude

by itself is insufficient: the right-hand rule decides whether these fields reinforce or cancel.

Step 2: Apply the right-hand rule and superpose the fields For the horizontal wire, P lies above it. With the thumb along positive x, the field is along positive z, out of the xy-plane:

$$\mathbf{B}_{\text{horizontal}} = \frac{\mu_0 I}{2\pi d}\,\hat{\mathbf{z}}.$$

For the vertical wire, P lies to its right. With the thumb along positive y, the field is along negative z:

$$\mathbf{B}_{\text{vertical}} = -\frac{\mu_0 I}{2\pi d}\,\hat{\mathbf{z}}.$$

Adding gives zero. Option (A) would require both contributions to point along positive z, which they do not. Options (B) and (D) put the resultant in the xy-plane, whereas each individual field here is normal to that plane.

Step 3: Conclusion The equal magnetic fields are opposite and cancel: $\mathbf{B}_{\text{net}} = \mathbf{0}$. Hence, the correct option is (C).

Quick Tip to Solve: Determine each wire field direction separately before adding magnitudes; perpendicular wires do not imply perpendicular magnetic fields at a point in their common plane.

---

54. A nucleus A, with a finite de-broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C are respectively : [4 marks]

(A) λ_A, $2\lambda_A$

(B) $2\lambda_A$, λ_A

(C) λ_A, $\lambda_A/2$

(D) $\lambda_A/2$, λ_A

Correct Answer: (D) $\lambda_A/2$, λ_A

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/|p|$: it depends on momentum magnitude, although momentum conservation must use signed vectors.
- In spontaneous fission without an external impulse, total momentum is conserved. The two daughter nuclei have equal mass, so their momentum magnitudes have the same ratio as their speeds.

Step 1: Write the signed daughter momenta Choose the original motion of A as positive. Let each daughter have mass m, and let B have speed v. Then C moves with signed velocity $-v/2$, so

$$p_B = mv, \qquad p_C = -\frac{mv}{2}.$$

Conservation of momentum gives

$$p_A = p_B + p_C = \frac{mv}{2}.$$

Thus B has momentum magnitude $2p_A$, while C has momentum magnitude p_A. The nonzero initial momentum is essential; treating the original nucleus as stationary would contradict its finite de Broglie wavelength and the given unequal daughter speeds.

Step 2: Convert momentum ratios into wavelength ratios Apply the reciprocal relation between wavelength and momentum magnitude:

$$\lambda_A = \frac{h}{mv/2} = \frac{2h}{mv},$$

$$\lambda_B = \frac{h}{mv} = \frac{\lambda_A}{2}, \qquad \lambda_C = \frac{h}{mv/2} = \lambda_A.$$

C has the smaller speed, so it must have the longer wavelength of the two equal-mass daughters. The sign of its momentum does not make its wavelength negative. Options (B) and (C) reverse the required ordering, and option (A) has the right relative ordering but scales both wavelengths incorrectly relative to the moving parent.

Step 3: Conclusion The wavelengths are $\lambda_B = \lambda_A/2$ and $\lambda_C = \lambda_A$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Use signed momenta to conserve momentum, then take their magnitudes when applying the de Broglie formula; do not add daughter momentum magnitudes.

55. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :

[Boltzmann Constant k_B = 1.38 $\times$ 10^{-23} J/K Avogadro Number N_A = 6.02 $\times$ 10^{26} /kg Radius of Earth : 6.4 $\times$ 10^6 m Gravitational acceleration on Earth = 10ms^{-2}] [4 marks]

(A) 3×10^5 K

(B) 10^4 K

(C) 650 K

(D) 800 K

Correct Answer: (B) 10^4 K

Solution:

Concept:

- The root mean square speed of molecules of mass m is $v_{\text{rms}} = \sqrt{3k_BT/m}$. Hydrogen molecules are diatomic, so their molecular mass is approximately twice the atomic mass unit.
- The escape speed from the surface of Earth is $v_e = \sqrt{2gR_E}$. Equating the squares of the two speeds avoids unnecessary square-root calculations.

Step 1: Find the molecular mass and required squared speed The supplied Avogadro-number form

$6.02 \times 10^{26}\,\mathrm{kg}^{-1}$ gives an atomic mass unit as its reciprocal. A hydrogen molecule therefore has mass

$$m = \frac{2}{6.02 \times 10^{26}}\,\mathrm{kg} \approx 3.32 \times 10^{-27}\,\mathrm{kg}.$$

For Earth, the squared escape speed is

$$v_e^2 = 2gR_E = 2(10)(6.4 \times 10^6) = 1.28 \times 10^8\,\mathrm{m^2\,s^{-2}}.$$

Use the molecular mass of hydrogen, not the mass of a single hydrogen atom, because the question concerns hydrogen molecules.

Step 2: Equate the speeds and calculate the temperature Setting $3k_BT/m = 2gR_E$ gives

$$T = \frac{2mgR_E}{3k_B} = \frac{(3.32 \times 10^{-27})(1.28 \times 10^8)}{3(1.38 \times 10^{-23})}\,\mathrm{K}.$$

Simplifying the powers and numerical factors,

$$T \approx \frac{4.25 \times 10^{-19}}{4.14 \times 10^{-23}}\,\mathrm{K} \approx 1.03 \times 10^4\,\mathrm{K}.$$

This is close to ten thousand kelvin. The temperatures 650 K and 800 K are much too small to give the required RMS speed, while 3×10^5 K is about thirty times too large. The equality here is a comparison of characteristic speeds, not a statement that every molecule has the RMS speed.

Step 3: Conclusion The required temperature is approximately 1.0×10^4 K. Hence, the correct option is (B).

Quick Tip to Solve: Equate squared speeds directly and use the mass of a hydrogen molecule; forgetting the factor of two in its mass halves the calculated temperature.

56. The electric field in a region is given by $\vec{E} = (Ax + B)\,\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = –5 is V_2, then $V_1 - V_2$ is :- [4 marks]

(A) –520 V

(B) 180 V

(C) –48 V

(D) 320 V

Correct Answer: (B) 180 V

Solution:

Concept:

- For a one-dimensional electrostatic field, $E_x = -dV/dx$. Hence the potential difference from an initial point to a final point is minus the line integral of the electric field.
- The given field is $E_x = Ax + B = 20x + 10$ in SI units. Because the field changes with position, its

value at just one endpoint cannot be multiplied by the whole distance.

Step 1: Set the limits in the order of the requested potential difference The required result is $V_1 - V_2 = V(1) - V(-5)$. Therefore the integral starts at x=-5 and ends at x=1:

$$V_1 - V_2 = -\int_{-5}^{1} (20x + 10)\, dx.$$

This includes the minus sign in the field-potential relation. Reversing the limits without changing that sign would reverse the physical answer. The field itself changes sign at x=-0.5, so signed integration is necessary rather than adding the magnitudes of contributions.

Step 2: Evaluate the antiderivative at both endpoints Integrating gives

$$V_1 - V_2 = -\left[10x^2 + 10x\right]_{-5}^{1}.$$

At the upper limit the bracket is $10 + 10 = 20$; at the lower limit it is $10(25) + 10(-5) = 200$. Thus

$$V_1 - V_2 = -[20 - 200] = +180\,\text{V}.$$

As a check, the field is linear, so its average between the endpoints is $[E(-5) + E(1)]/2 = (-90 + 30)/2 = -30\,\text{N C}^{-1}$. Multiplying by the displacement 6 m gives a field integral of -180 V, whose negative is the required potential difference. This rules out the two negative choices and also 320 V.

Step 3: Conclusion The potential difference is $V_1 - V_2 = 180\,\text{V}$. Hence, the correct option is (B).

Quick Tip to Solve: Write V(final) minus V(initial) before choosing integration limits, and keep the minus sign in the electric-field relation until the final calculation.

---

57. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70m, then the minimum height of the transmitting antenna should be :

(Radius of the Earth = 6.4 × 10^6 m). [4 marks]

(A) 40 m

(B) 20 m

(C) 51 m

(D) 32 m

Correct Answer: (D) 32 m

Solution:

Concept:

- For an antenna of height $h \ll R$, the distance to the horizon is $d = \sqrt{2Rh}$, where R is the radius of the Earth.
- The maximum separation for direct communication is the sum of the two horizon distances: $D = \sqrt{2Rh_T} + \sqrt{2Rh_R}$. At the minimum transmitting height, this range just equals the required

separation.

Step 1: Find the receiving antenna's horizon distance The receiving antenna is already $70\,\mathrm{m}$ above ground. Its contribution to the line-of-sight range must be subtracted before finding the other height. Use metres throughout:

$$d_R = \sqrt{2(6.4 \times 10^6)(70)} = \sqrt{8.96 \times 10^8} \simeq 2.9933 \times 10^4\,\mathrm{m}.$$

Thus the receiver covers approximately $29.93\,\mathrm{km}$ of the total separation.

Step 2: Calculate the remaining horizon distance and height The transmitter must cover the remaining distance, not the full $50\,\mathrm{km}$ by itself:

$$d_T = 50000 - 29933 \simeq 20067\,\mathrm{m}.$$

Squaring only after this subtraction gives

$$h_T = \frac{d_T^2}{2R} = \frac{(20067)^2}{1.28 \times 10^7} \simeq 31.46\,\mathrm{m}.$$

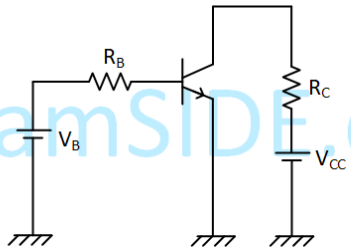
The approximations are justified because both antenna heights are extremely small compared with the Earth's radius.

Step 3: Choose the smallest listed height that meets the range A height of $32\,\mathrm{m}$ gives a total range slightly above $50\,\mathrm{km}$, whereas $20\,\mathrm{m}$ gives only about $45.93\,\mathrm{km}$. The heights $40\,\mathrm{m}$ and $51\,\mathrm{m}$ also provide communication but exceed the minimum required height. Thus they cannot be the requested minimum among the choices.

Step 4: Conclusion The required height is approximately $31.5\,\mathrm{m}$, so the appropriate listed minimum is $32\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the receiver's horizon distance from the required separation before squaring; subtracting antenna heights does not give the range.

58. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, R_C = 1kΩ and V_{CC} = 10 V. What is the minimum base current for V_{CE} to reach saturation ? [4 marks]



ExamSIDE.com

(A) 100 μA

(B) 7 μA

(C) 10 μA

(D) 40 μA

Correct Answer: (D) 40 μA

Solution:

Concept:

- In the active region of a common-emitter transistor, $I_C = \beta I_B$, where β is the dc current gain.
- The collector load limits the current to approximately $I_{C,\text{sat}} = V_{CC}/R_C$ when $V_{CE} \simeq 0$. The minimum base current is the current at the boundary between active operation and saturation.

Step 1: Find the load-limited collector current The emitter is grounded and the collector is connected to the supply through R_C. Apply Kirchhoff's voltage law to that loop:

$$V_{CC} = I_C R_C + V_{CE}.$$

At saturation, take the ideal limiting value $V_{CE} \simeq 0$, so the load receives essentially the full supply voltage:

$$I_{C,\text{sat}} = \frac{10}{1.0 \times 10^3} = 1.0 \times 10^{-2}\,\text{A} = 10\,\text{mA}.$$

Increasing base current further cannot make the collector current exceed this load limit.

Step 2: Calculate the threshold base current Just before saturation, the active-region gain relation still determines the required drive. With $\beta = 250$,

$$I_{B,\text{min}} = \frac{I_{C,\text{sat}}}{\beta} = \frac{10 \times 10^{-3}}{250} = 4.0 \times 10^{-5}\,\text{A} = 40\,\mu\text{A}.$$

For a slightly nonzero practical saturation voltage the estimate changes only slightly; the listed ideal value remains $40\,\mu$A.

Step 3: Check the other base-current choices At 7 μA and 10 μA, the gain relation predicts collector currents of only 1.75 mA and 2.5 mA, leaving substantial collector-emitter voltage. They cannot saturate the transistor. A drive of 100 μA is more than enough but is not the minimum; in saturation the actual current gain is no longer fixed at 250.

Step 4: Conclusion The minimum base drive for saturation is approximately $40\,\mu$A. Hence, the

correct option is (D).

Quick Tip to Solve: Find the collector current set by the supply and load first, then divide by the active-region gain to find the onset of saturation.

59. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to 1/1000 of the original amplitude is close to :- [4 marks]

(A) 100 s

(B) 10 s

(C) 20 s

(D) 50 s

Correct Answer: (C) 20 s

Solution:

Concept:

- The amplitude of a damped harmonic oscillator decays exponentially: $A(t) = A_0 e^{-\lambda t}$, with positive damping constant λ.
- Equal numbers of oscillations take equal times at the stated damped frequency. If the amplitude halves in time $t_{1/2}$, then $A/A_0 = 2^{-t/t_{1/2}}$. This is amplitude decay, rather than energy decay.

Step 1: Convert ten oscillations into the amplitude half-life The frequency is $f = 5\,\mathrm{s}^{-1}$, so the period is $T = 1/f = 0.2\,\mathrm{s}$. The amplitude drops by one factor of two after ten complete periods:

$$t_{1/2} = 10T = \frac{10}{5} = 2\,\mathrm{s}.$$

The information concerns the amplitude directly; using an energy half-life would introduce an incorrect factor of two.

Step 2: Find how many halvings produce the required ratio After n such intervals, the amplitude is $A_0(1/2)^n$. Set this equal to the requested amplitude:

$$2^{-n} = \frac{1}{1000}, \qquad n = \frac{\ln 1000}{\ln 2} \simeq 9.966.$$

Therefore

$$t = nt_{1/2} = 2\frac{\ln 1000}{\ln 2} \simeq 19.93\,\mathrm{s}.$$

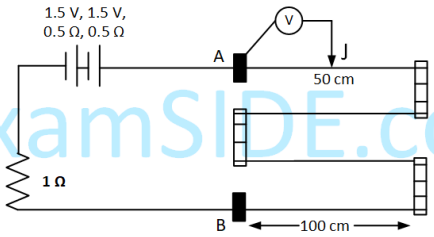
As a quick numerical check, ten halvings give $1/1024$, already extremely close to $1/1000$.

Step 3: Compare with the listed times The closest time is $20\,\mathrm{s}$. At $10\,\mathrm{s}$, only five halvings have occurred and the amplitude is still $A_0/32$, much too large. At $50\,\mathrm{s}$ and $100\,\mathrm{s}$, there are 25 and 50 halvings respectively, so the amplitude is far smaller than the target. A constant linear decrease cannot be used because the same fraction, rather than the same absolute amplitude, is lost each interval.

Step 4: Conclusion The time is approximately $19.9\,\mathrm{s}$, closest to $20\,\mathrm{s}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember that $2^{10} = 1024$: a thousandfold amplitude reduction takes almost ten amplitude half-lives.

60. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be :- [4 marks]



ExamSIDE.com

(A) 0.25 V

(B) 0.20 V

(C) 0.50V

(D) 0.75 V

Correct Answer: (A) 0.25 V

Solution:

Concept:

- A uniform potentiometer wire has resistance $R_w = rL$. The potential drop over a length l carrying steady current I is $V = Irl$.
- Cells connected in series add their emfs and internal resistances. An ideal voltmeter draws no current, so its connection does not change the potentiometer current or the potential gradient.

Step 1: Calculate the resistance and total driving emf The four successive wire lengths form one continuous $400\,\text{cm}$ wire between A and B; folding it does not put the four lengths in parallel. Thus

$$R_w = (0.01\,\Omega/\text{cm})(400\,\text{cm}) = 4\,\Omega.$$

The figure shows two aiding cells, each of emf $1.5\,\text{V}$ and internal resistance $0.5\,\Omega$, together with a $1\,\Omega$ external resistor. Therefore

$$E = 3\,\text{V}, \qquad R_{\text{total}} = 4 + 0.5 + 0.5 + 1 = 6\,\Omega.$$

Step 2: Find the current and the drop over AJ The same current flows through every part of the potentiometer wire:

$$I = \frac{E}{R_{\text{total}}} = \frac{3}{6} = 0.5\,\text{A}.$$

The jockey is $50\,\text{cm}$ from A along the first wire segment, whose resistance is

$$R_{AJ} = 0.01 \times 50 = 0.5\,\Omega.$$

The ideal voltmeter measures the potential difference between A and J:

$$V_{AJ} = IR_{AJ} = 0.5 \times 0.5 = 0.25\,\text{V}.$$

Step 3: Check the result against the full wire voltage The full potentiometer wire has a drop $IR_w = 2\,\text{V}$. Since AJ is $50/400 = 1/8$ of its length, its drop must be $2/8 = 0.25\,\text{V}$, consistent with the direct calculation. The choices $0.20\,\text{V}$, $0.50\,\text{V}$ and $0.75\,\text{V}$ do not follow from this series resistance and the stated tapped length; ignoring internal resistance or treating a fold as a parallel branch changes the current incorrectly.

Step 4: Conclusion The expected voltmeter reading is $0.25\,\text{V}$. Hence, the correct option is (A).

Quick Tip to Solve: Unfold the potentiometer mentally: all four lengths carry the same series current, and an ideal voltmeter does not load the tapped segment.

Chemistry (120 marks)

61. Among the following molecules / ions, $C_2^{2-}, N_2^{2-}, O_2^{2-}, O_2$ which one is diamagnetic and has the shortest bond length? [4 marks]

(A) O_2
(B) C_2^{2-}
(C) N_2^{2-}
(D) O_2^{2-}

Correct Answer: (B) C_2^{2-}

Solution:

Concept:

- A species is diamagnetic when all occupied molecular orbitals contain paired electrons. The bond order is $(N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons.
- For these second-period diatomic species, higher bond order indicates the shorter, stronger bond. Both the electron pairing condition and the bond order comparison must be checked.

Step 1: Examine the carbide ion The ion $\text{C}_2{}^{2-}$ has ten valence electrons, the same number as N_2. Its valence molecular-orbital occupation is

$$(\sigma_{2s})^2\,(\sigma^*_{2s})^2\,(\pi_{2p_x})^2\,(\pi_{2p_y})^2\,(\sigma_{2p_z})^2\,.$$

Every electron is paired. There are eight bonding and two antibonding valence electrons, giving bond order $(8-2)/2 = 3$. The filled inner shells cancel out of the bond-order calculation.

Step 2: Compare the other three species Neutral O_2 has twelve valence electrons and two singly occupied degenerate π^* orbitals. It is paramagnetic with bond order 2. The ion $\text{N}_2{}^{2-}$ also has twelve valence electrons: its added electrons occupy the two π^* orbitals separately by Hund's rule. It is therefore paramagnetic with bond order 2. The peroxide ion $\text{O}_2{}^{2-}$ has fourteen valence electrons; both π^* orbitals are fully paired, so it is diamagnetic, but its bond order is only 1.

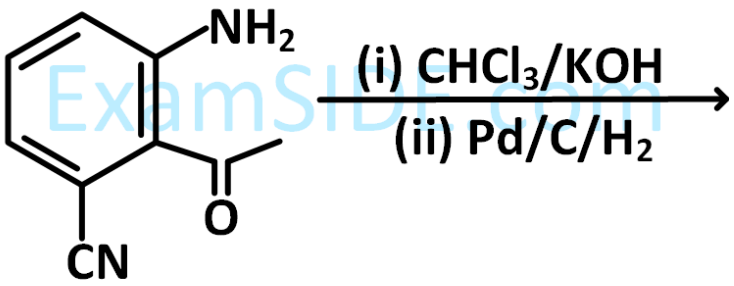
Step 3: Apply both requirements together The oxygen molecule and the nitrogen dianion fail the diamagnetism requirement. Peroxide meets that requirement but has only a single-bond bond order. The carbide ion combines complete electron pairing with the largest bond order, 3, and hence is the

species with the shortest bond among the given choices. Counting only unpaired electrons would not distinguish carbide from peroxide; the second bond-length condition is essential.

Step 4: Conclusion C_2^{2-} is diamagnetic and has bond order 3, the largest here and therefore the shortest bond. Hence, the correct option is (B).

Quick Tip to Solve: First eliminate species with unpaired π^* electrons; then compare the bond orders of the remaining diamagnetic species.

62. The major product of the following reaction is: [4 marks]



(A) Benzene bearing $NHCH_3$, $COCH_3$, and CN at positions 1, 2, and 3, respectively.

(B) Benzene bearing $NHCHCl_2$, $CH(OH)CH_3$, and CN at positions 1, 2, and 3, respectively.

(C) Benzene bearing $NHCH_3$, $CH(OH)CH_3$, and CN at positions 1, 2, and 3, respectively.

(D) Benzene bearing $NHCH_3$, $CH(OH)CH_3$, and CH_2NH_2 at positions 1, 2, and 3, respectively.

Correct Answer: (D) Benzene bearing $NHCH_3$, $CH(OH)CH_3$, and CH_2NH_2 at positions 1, 2, and 3, respectively.

Solution:

Concept:

- A primary amine reacts with chloroform and alcoholic potassium hydroxide in the carbylamine reaction: $RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O$. The product is an isocyanide, with nitrogen still bonded to the original group.
- Catalytic hydrogenation reduces an isocyanide RNC to $RNHCH_3$. Under the stated hydrogenation sequence the ketone becomes a secondary alcohol and the nitrile substituent becomes an aminomethyl group.

Step 1: Follow the original amino group through the first reaction Label the aromatic carbon bearing the original NH_2 as position 1. The acetyl group $COCH_3$ is at position 2 and the nitrile CN at position 3. The primary aromatic amine first forms the corresponding aryl isocyanide:

$$ArNH_2 \xrightarrow{CHCl_3/KOH} ArNC.$$

The carbon introduced by chloroform is attached beyond nitrogen. This connectivity matters: the reaction does not convert the aryl amino group into an aryl nitrile.

Step 2: Reduce each reducible substituent With H_2 and Pd/C, the isocyanide group is reduced to an N-methylamino group, $ArNHCH_3$. The existing side-chain groups also undergo reduction:

$$ArCOCH_3 + H_2 \longrightarrow ArCH(OH)CH_3,$$

$$ArCN + 2\,H_2 \longrightarrow ArCH_2NH_2.$$

The aromatic ring and the relative positions of its three substituents remain the same in the product shown in the choices.

Step 3: Match all three substituents to an option The final ring must therefore carry $NHCH_3$, $CH(OH)CH_3$ and CH_2NH_2 at the original positions 1, 2 and 3. Option (D) alone shows all three changes. Option (A) leaves both the carbonyl and nitrile unchanged. Option (B) retains chlorine on the nitrogen-bound carbon and also leaves the nitrile unchanged. Option (C) reduces the carbonyl and forms the N-methyl group, but incorrectly retains CN instead of CH_2NH_2.

Step 4: Conclusion The major product contains an N-methylamino group, a 1-hydroxyethyl group and an aminomethyl group at the corresponding ring positions. Hence, the correct option is (D).

Quick Tip to Solve: Track the newly introduced isocyanide carbon separately from the pre-existing nitrile carbon: reduction gives $NHCH_3$ from the former and CH_2NH_2 from the latter.

63. For the following reactions, equilibrium constants are given :

$S(s) + O_2(g) \rightleftharpoons SO_2(g)$; $K_1 = 10^{52}$

$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g)$; $K_2 = 10^{129}$

The equilibrium constant for the reaction,

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ is : [4 marks]

(A) 10^{181}

(B) 10^{25}

(C) 10^{77}

(D) 10^{154}

Correct Answer: (B) 10^{25}

Solution:

Concept:

- When a reaction is multiplied by a factor n, its equilibrium constant becomes K^n. Reversing a reaction replaces its constant by $1/K$.
- Adding chemical equations multiplies their equilibrium constants. Pure solid sulphur has unit activity and therefore does not appear in the equilibrium-constant expression.

Step 1: Generate the equation needed to cancel sulphur The desired reaction consumes two moles of SO_2, whereas the first given reaction produces only one. Double the first reaction and then reverse it:

$$2\,SO_2(g) \rightleftharpoons 2\,S\,(s) + 2\,O_2(g).$$

The corresponding constant is

$$K_{\text{rev}} = \frac{1}{K_1^2} = \frac{1}{(10^{52})^2} = 10^{-104}.$$

Doubling the equation squares the equilibrium constant; it does not multiply the constant by two.

Step 2: Add to the sulphur trioxide formation equation Now add the reversed doubled equation to the second given equation,

$$2\,\text{S}\,(\text{s}) + 3\,\text{O}_2(\text{g}) \rightleftharpoons 2\,\text{SO}_3(\text{g}).$$

The two sulphur atoms cancel, and two of the three oxygen molecules cancel. The net equation is exactly

$$2\,\text{SO}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2\,\text{SO}_3(\text{g}).$$

Since the reactions have been added, multiply their constants:

$$K = K_2 K_{\text{rev}} = 10^{129} \times 10^{-104} = 10^{25}.$$

Step 3: Verify the exponent and exclude the other choices The exponent is $129 - 2(52) = 25$. The value 10^{77} would use $129 - 52$ and forget that the first equation must be doubled. The value 10^{181} adds 52 rather than reversing and doubling the first reaction. The exponent 154 is not obtained by the required stoichiometric combination. As an algebraic check, the two SO_3 factors in K_2 are retained while dividing by K_1^2 introduces the required two SO_2 factors in the denominator.

Step 4: Conclusion The equilibrium constant for the requested oxidation reaction is $K = 10^{25}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the target equation as reaction 2 minus twice reaction 1 before operating on the constants: $K = K_2/K_1^2$.

---

64. The correct statement about ICl_5 and ICl_4^- is [4 marks]

(A) ICl_5 is square pyramidal and ICl_4^- is square planar.

(B) ICl_5 is trigonal bipyramidal and is tetrahedral.

(C) ICl_5 is square pyramidal and ICl_4^- is tetrahedral

(D) Both are isostructural.

Correct Answer: (A) ICl_5 is square pyramidal and ICl_4^- is square planar.

Solution:

Concept:

- VSEPR theory counts electron domains around the central atom. Each bond and each lone pair occupies one domain; a multiple bond would still count as one domain.
- The molecular shape describes the positions of bonded atoms, whereas the electron-pair geometry also includes lone pairs. Six domains give an octahedral electron-pair arrangement.

Step 1: Count the domains in iodine pentachloride Iodine has seven valence electrons. In ICl_5, it

forms five single bonds to chlorine, leaving two nonbonding electrons, or one lone pair, on iodine. Thus the central atom has five bond pairs and one lone pair: AX_5E. Its steric number is six, not five. The five bonds therefore cannot form a trigonal bipyramid, which requires only five electron domains and no lone pair.

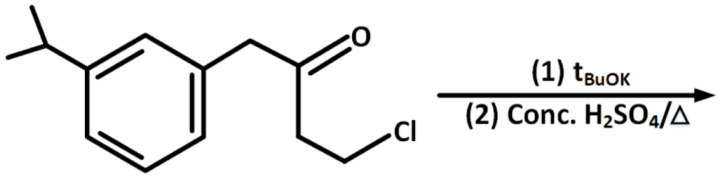
Step 2: Find the shapes of both species In the octahedral arrangement for ICl_5, one position is occupied by a lone pair. The other five positions form a square pyramid: four chlorine atoms in a square plane and one above it. In ICl_4^-, the extra negative charge adds one electron to iodine. After four bonds are formed, four electrons remain as two lone pairs. This gives AX_4E_2, again with six domains. The two lone pairs occupy opposite positions to minimize repulsions; the four chlorine atoms occupy the remaining square plane.

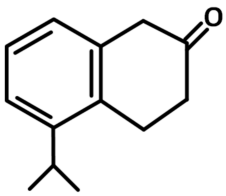
Step 3: Compare the alternatives The molecular shapes are therefore square pyramidal and square planar, respectively. Option B assigns the wrong shape to both species, and C incorrectly makes the four-coordinate anion tetrahedral by ignoring its two lone pairs. Option D confuses their common octahedral electron-pair geometry with their different molecular shapes; they are not isostructural.

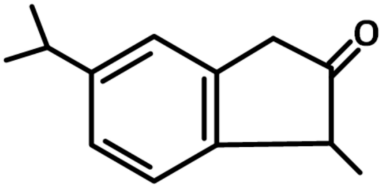
Step 4: Conclusion ICl_5 is square pyramidal and ICl_4^- is square planar. Hence, the correct option is (A).

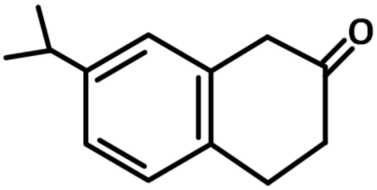
Quick Tip to Solve: For iodine halides, count the negative charge before assigning lone pairs: five bonds leave one lone pair, while four bonds plus one extra electron leave two.

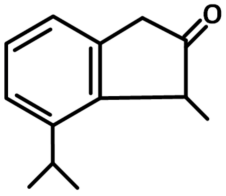
65. The major product obtained in the following reaction is : [4 marks]



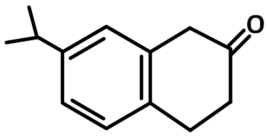
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Potassium tert-butoxide is a strong, bulky base that promotes dehydrohalogenation. Removing hydrogen and chlorine from adjacent carbons can generate an alpha,beta-unsaturated ketone.
- An activated aromatic ring can attack an acid-activated enone intramolecularly. Ring size and the ortho/para directing effect of the isopropyl group determine the major cyclic product.

Step 1: Eliminate hydrogen chloride Write the substrate side chain as $ArCH_2COCH_2CH_2Cl$, with the isopropyl-substituted phenyl group denoted Ar. Potassium tert-butoxide removes a hydrogen from the carbon next to the carbonyl while chloride leaves from the terminal carbon. This gives the conjugated enone $ArCH_2COCH{=}CH_2$. Formation of a conjugated double bond supports this elimination. No carbon atom is lost, and the carbonyl stays in the original side-chain position.

Step 2: Close a six-membered ring Concentrated sulfuric acid activates the enone for intramolecular electrophilic aromatic substitution: its terminal beta carbon is an electrophilic site. An aromatic carbon ortho to the side chain attacks this terminal carbon. Count the new ring from that aromatic carbon through the side-chain-bearing aromatic carbon, benzylic carbon, carbonyl carbon, alpha carbon and terminal beta carbon: it contains six atoms. Rearomatization and proton transfers give the fused six-membered cyclic ketone. B and D show five-membered methyl-substituted rings and therefore do not match this tether closure.

Step 3: Select the less hindered aromatic site The two positions ortho to the side chain are not equivalent because of isopropyl. One closure site is ortho to isopropyl and lies between the two substituents; the other is para to isopropyl and is less crowded. The electron-releasing isopropyl group activates both ortho and para positions, so closure at the less hindered para site is favoured. This gives the substituent arrangement drawn in C. A is the six-membered product of closure at the more crowded site. Thus the first reagent makes the enone, and the second reagent selects the favourable intramolecular aromatic ring closure.

Step 4: Conclusion The major product is the six-membered fused cyclic ketone with the substituent arrangement shown in option C. Hence, the correct option is (C).

Quick Tip to Solve: First turn the beta-chloro ketone into an enone, count six atoms in the closure, then prefer the aromatic site para to isopropyl.

66. The statement that is INCORRECT about the interstitial compounds is : [4 marks]

(A) They are chemically reactive

(B) They have metallic conductivity

(C) They have high melting points

(D) They are very hard

Correct Answer: (A) They are chemically reactive

Solution:

Concept:

- Interstitial compounds form when small atoms such as hydrogen, carbon or nitrogen occupy spaces in a transition-metal lattice. They commonly retain important properties of the parent metallic framework.
- These compounds are characteristically hard, high-melting and chemically inert; their metallic conductivity is retained because the metal framework still supports mobile electrons.

Step 1: Identify the physical properties Small atoms fit into the interstitial holes between the larger metal atoms rather than producing a simple separate molecular lattice. Their presence makes deformation of the metal framework more difficult. Consequently these materials are very hard, as stated in D. Strong interactions involving the inserted atoms and the metal framework also make their melting points high, consistent with C. These are characteristic properties of interstitial compounds, not reasons for marking either statement incorrect.

Step 2: Check electrical and chemical behaviour separately The metallic framework is not replaced by a collection of discrete covalent molecules. Delocalized electrons remain available, so the material can retain metallic conductivity. Thus B is also a correct statement. Electrical conduction does not by itself imply that a solid is chemically reactive: the ability of electrons to move through a lattice and the ease of chemical attack are different properties. Interstitial compounds are generally chemically inert under ordinary conditions, rather than characteristically reactive.

Step 3: Apply the negative wording The question asks for the statement that is INCORRECT. Statements B, C and D all reproduce recognized properties of this class: conductivity, high melting point and hardness. Statement A reverses its usual chemical behaviour by calling it chemically reactive. Selecting a true physical property would answer the opposite of the printed question. The classification is a general characteristic of these materials; it does not assert that no member can react under any extreme condition.

Step 4: Conclusion The incorrect statement is that interstitial compounds are chemically reactive; they are characteristically chemically inert. Hence, the correct option is (A).

Quick Tip to Solve: For interstitial compounds remember the combination: hard, high-melting, conducting and relatively inert. Then check whether the question asks for the false statement.

67. The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are : [4 marks]

(A) 2.84 and 5.92

(B) 4.9 and 0

(C) 0 and 4.9

(D) 0 and 5.92

Correct Answer: (C) 0 and 4.9

Solution:

Concept:

- The spin-only moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons. Determine oxidation state and ligand-field pairing before using the formula.
- Water is a weak-field ligand and cyanide is a strong-field ligand. For an octahedral d^6 ion, these give high-spin and low-spin configurations, respectively.

Step 1: Identify the two iron complex ions The wording prints the two bracketed units with "and", without ionic charges. Consequently their oxidation states are not unambiguously specified literally. The chemically consistent intended neutral salt is $[Fe(H_2O)_6]_2[Fe(CN)_6]$. Under this interpretation the cation is $[Fe(H_2O)_6]^{2+}$ and the anion is $[Fe(CN)_6]^{4-}$. Water is neutral and each cyanide ligand carries charge -1, so iron is $+2$ in both complexes. Iron(II) is $3d^6$. The following calculation explicitly uses that intended salt interpretation rather than treating missing charges as information supplied by the question.

Step 2: Count the unpaired electrons For the cyanide complex, the strong octahedral field favours pairing in the lower-energy t_{2g} orbitals. Its configuration is $t_{2g}^6 e_g^0$, with all electrons paired, so $n = 0$. For the water complex, the ligand field is weaker and the high-spin arrangement is $t_{2g}^4 e_g^2$. The three lower orbitals contain one pair and two unpaired electrons, while the two upper orbitals each contain one electron. Hence the cation has four unpaired electrons.

Step 3: Calculate in the requested order For the anion, $\mu = \sqrt{0(0+2)} = 0\,\text{BM}$. For the cation,

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.90\,\text{BM}.$$

Thus the ordered pair is $(0, 4.9)$. Option B reverses the two species. The value $5.92\,\text{BM}$ belongs to five unpaired electrons, not this high-spin d^6 cation, eliminating D; A has neither the correct pairing pattern nor the requested moments.

Step 4: Conclusion With the intended iron(II) salt interpretation, the anion and cation have moments 0 and $4.9\,\text{BM}$, respectively, corresponding to option C. The question-sheet key gives B, but that reverses the explicitly requested anion-first order. The literal printed wording omits charges, so this answer is conditional on the stated interpretation.

Quick Tip to Solve: Write anion and cation as separate headings before counting spins; cyanide pairs the six d electrons, whereas water leaves four unpaired.

68. The compound that inhibits the growth of tumors is : [4 marks]

(A) cis-$[Pd(Cl)_2(NH_3)_2]$

(B) trans-$[Pd(Cl)_2(NH_3)_2]$

(C) cis-$[Pt(Cl)_2(NH_3)_2]$

(D) trans-$[Pt(Cl)_2(NH_3)_2]$

Correct Answer: (C) cis-$[Pt(Cl)_2(NH_3)_2]$

Solution:

Concept:

- Cisplatin is the antitumour coordination compound cis-$[PtCl_2(NH_3)_2]$. It is a square-planar platinum(II) complex with two ammine and two chlorido ligands.
- Geometrical isomerism changes the relative positions of ligands. In the cis isomer the identical ligands are adjacent; in the trans isomer they are opposite, producing different biological behaviour.

Step 1: Identify the known antitumour compound The coordination compound used to inhibit tumour growth is cisplatin, whose metal centre is platinum rather than palladium. The two chlorido ligands each have charge -1, while each ammine ligand is neutral. The neutral complex consequently contains Pt^{2+}, which commonly forms a square-planar d^8 complex. The formula and geometry must both be specified: simply choosing any complex with two ammonia and two chloride ligands is insufficient.

Step 2: Use the cis arrangement In cisplatin, the two chlorido ligands occupy adjacent sites in the square plane. Following ligand substitution in the cellular environment, platinum can coordinate to DNA bases, producing cross-links that disrupt replication and cell division. This is why the relative positions of the reactive coordination sites matter. The trans isomer has opposite chlorido ligands and is not the standard antitumour compound named in this question. Similar gross formulas therefore do not imply interchangeable medicinal action.

Step 3: Eliminate the other three choices Option C has both required features: platinum as the central metal and the cis arrangement. Option D contains platinum but has the trans geometry, so it is transplatin rather than cisplatin. Options A and B contain palladium and thus do not represent the platinum drug, irrespective of their cis or trans arrangement. The distinction is a specific identification of the recognized compound; it does not require assuming that every compound of another metal has identical chemical or biological properties.

Step 4: Conclusion The tumour-growth inhibitor is cis-$[PtCl_2(NH_3)_2]$, commonly called cisplatin. Hence, the correct option is (C).

Quick Tip to Solve: Check both parts of the name cisplatin: cis fixes the ligand geometry and platin identifies platinum, eliminating the palladium analogues.

69. 5 moles of an ideal gas at 100 K are allowed to undergo reversible compression till its temperature becomes 200 K. If C_V = 28 $JK^{-1}mol^{-1}$, calculate ΔU and ΔpV for this process. (R = 8.0 JK^{-1} mol^{-1}]

[4 marks]

(A) ΔU = 14 kJ; Δ(pV) = 4 kJ

(B) ΔU = 2.8 kJ; Δ(pV) = 0.8 kJ

(C) ΔU = 14 kJ; Δ(pV) = 18 kJ

(D) ΔU = 14 kJ; Δ(pV) = 0.8 kJ

Correct Answer: (A) ΔU = 14 kJ; Δ(pV) = 4 kJ

Solution:

Concept:

- For a fixed amount of ideal gas, internal energy depends only on temperature: $\Delta U = nC_V\Delta T$ when the molar heat capacity C_V is constant.
- The equation of state is $pV = nRT$, so $\Delta(pV) = nR\Delta T$. Both quantities are state-function changes; reversible compression does not make the temperature constant.

Step 1: Calculate the temperature rise The amount is $n = 5\,\text{mol}$, and the temperature increases from $100\,\text{K}$ to $200\,\text{K}$. Therefore $\Delta T = 200 - 100 = 100\,\text{K}$. The quoted $C_V = 28\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$ is a molar heat capacity, so it must be multiplied by all five moles. Reversibility describes the path; it does not permit replacing this stated temperature rise by zero or treating the compression as isothermal.

Step 2: Compute the internal-energy increase Using the ideal-gas temperature dependence of internal energy,

$$\Delta U = nC_V\Delta T = (5)(28)(100) = 14000\,\text{J} = 14\,\text{kJ}.$$

The sign is positive because the final temperature is higher. The value $2.8\,\text{kJ}$ in B uses the heat capacity for only one mole and misses the factor of five. The compression work or exchanged heat is not needed separately to calculate this state-function change.

Step 3: Compute the change in the pressure-volume product Apply the ideal-gas equation at the initial and final states and subtract:

$$\Delta(pV) = nR(T_2 - T_1) = (5)(8.0)(100) = 4000\,\text{J} = 4\,\text{kJ}.$$

Pressure times volume has units of energy. The requested quantity is the change of the product, not $p\Delta V$ at a fixed pressure and not the work integral. The $0.8\,\text{kJ}$ values in B and D again omit the amount of gas. The $18\,\text{kJ}$ in C instead equals $\Delta H = \Delta U + \Delta(pV)$, which was not requested.

Step 4: Conclusion $\Delta U = +14\,\text{kJ}$ and $\Delta(pV) = +4\,\text{kJ}$. Hence, the correct option is (A).

Quick Tip to Solve: For an ideal gas, calculate both changes directly from the same temperature rise: use nC_V for internal energy and nR for the product pV.

70. The covalent alkaline earth metal halide (X = Cl, Br, I) is : [4 marks]

(A) CaX_2

(B) SrX_2

(C) MgX_2

(D) BeX_2

Correct Answer: (D) BeX_2

Solution:

Concept:

- Fajans' rule relates covalent character to polarization: a small, highly charged cation distorts the electron cloud of an anion strongly, increasing electron sharing.
- All the alkaline earth metal cations here have charge $+2$. For the same halide ion, their different sizes therefore control the comparison of polarizing power.

Step 1: Hold the anion and charge fixed The question compares metal halides of the form MX_2, with X a chloride, bromide or iodide. In comparing the metals for a chosen X, both the anion and the $+2$ cation charge are fixed. A smaller metal ion concentrates the same positive charge into a smaller volume and has a stronger electric field at the neighbouring anion. It can distort the negative electron cloud more strongly than a large ion can.

Step 2: Compare the cation sizes The ionic size increases down group 2:

$$Be^{2+} < Mg^{2+} < Ca^{2+} < Sr^{2+}.$$

Accordingly, beryllium has the greatest polarizing power among the choices. Its small ion strongly polarizes the larger halide anion and gives the bond appreciable covalent character. Beryllium halides are consequently the covalent alkaline earth halides singled out by this question. Moving to the larger cations reduces polarization and makes the bonding more predominantly ionic.

Step 3: Check all four formulas Option D, BeX_2, uses the smallest cation and is the required choice. Magnesium halides in C can have some covalent character, especially with a polarizable anion, but magnesium has less polarizing power than beryllium. Calcium and strontium in A and B are still larger and have more ionic halides. The rule compares a continuum of bond character; it does not require pretending that the other bonds have exactly zero covalent contribution. The same ordering of cation polarizing power applies for each of the listed halides.

Step 4: Conclusion The covalent alkaline earth metal halide is BeX_2, where $X = Cl, Br, I$. Hence, the correct option is (D).

Quick Tip to Solve: When cation charges are equal, choose the smallest cation for greatest covalent character; here that is beryllium rather than magnesium, calcium or strontium.

71. The IUPAC symbol for the element with atomic number 119 would be : [4 marks]

(A) uun

(B) une

(C) uue

(D) unh

Correct Answer: (C) uue

Solution:

Concept:

- The temporary IUPAC name for an element is obtained by assigning a numerical root to each digit of its atomic number, in order. For the digits needed here, 1 is un and 9 is enn.
- The systematic name ends in ium. Its three-letter symbol consists of the initial letters of the three numerical roots: only the first letter of an element symbol is capitalised.

Step 1: Split the atomic number into digits The atomic number is 119, whose digits are $1, 1, 9$. Accordingly the sequence of roots is un, un, enn. Their order matters: the symbol is a code for the digits themselves, rather than an abbreviation chosen from a chemical property or from the position in a group.

Step 2: Construct the name and symbol Joining the roots and the ending gives un-un-enn-ium, or ununennium. Taking the initials gives Uue. The options use lower-case typography, so uue is the matching printed entry. The ending ium is not an additional digit-root and does not add another symbol letter.

Step 3: Reject the other digit codes The last letter n in uun denotes nil, the root for zero, so that code corresponds to 110 rather than 119. The code une contains the root nil in the second position and does not represent the repeated initial 1. The last letter h in unh refers to hex, the root for six, and also fails to represent the final digit nine. Thus only the sequence u-u-e preserves all three given digits.

Step 4: Conclusion The systematic name is ununennium and its symbol is Uue, printed as uue in the options. Hence, the correct option is (C).

Quick Tip to Solve: Convert each atomic-number digit into its IUPAC root before taking initials; for 119 the repeated 1 must produce two consecutive u letters.

72. If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength λ, then for 1.5 p momentum of the photoelectron, the wavelength of the light should be: (Assume kinetic energy of ejected photoelectron to be very high in comparison to work function) [4 marks]

(A) 1/2 λ

(B) 3/4 λ

(C) 4/9 λ

(D) 2/3 λ

Correct Answer: (C) 4/9 λ

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function and h, c have their usual meanings.
- For a nonrelativistic electron of mass m_e, $K = p^2/(2m_e)$. Under the stated condition $K \gg \phi$, the photon energy is approximately the electron kinetic energy, so $p^2 \propto 1/\lambda$.

Step 1: Relate the two kinetic energies Let the new wavelength and momentum be λ' and $p' = 1.5p$. The mass is unchanged, so the squared momentum ratio determines the kinetic-energy ratio:

$$\frac{K'}{K} = \frac{(p')^2}{p^2} = (1.5)^2 = \frac{9}{4}.$$

Increasing momentum by fifty percent therefore requires more than a fifty-percent increase in energy.

Step 2: Use the negligible work-function approximation The question explicitly permits neglecting the work function compared with the kinetic energy. Thus $K \simeq hc/\lambda$ and $K' \simeq hc/\lambda'$. Taking the ratio and cancelling the common constants gives

$$\frac{K'}{K} \simeq \frac{\lambda}{\lambda'} = \frac{9}{4}, \qquad \lambda' = \frac{4}{9}\lambda.$$

Step 3: Check the proposed wavelengths A wavelength $\lambda/2$ would increase momentum by $\sqrt{2}$, not $3/2$. A wavelength $3\lambda/4$ gives a factor $\sqrt{4/3}$, while $2\lambda/3$ gives $\sqrt{3/2}$. Only $4\lambda/9$ gives $\sqrt{9/4} = 3/2$. The shorter wavelength also agrees with the need for a larger photon energy.

Step 4: Conclusion The required light has wavelength $\lambda' = 4\lambda/9$ in the stated approximation. Hence, the correct option is (C).

Quick Tip to Solve: When the work function is negligible, square the momentum factor and invert it to obtain the wavelength factor.

73. The percentage composition of carbon by mole in methane is : [4 marks]

(A) 80%

(B) 20%

(C) 75%

(D) 25%

Correct Answer: (B) 20%

Solution:

Concept:

- Mole percentage of an element in a compound counts the moles of its atoms relative to the total moles of constituent atoms. It is different from mass percentage.
- Methane has formula CH_4. Each molecule contains one carbon atom and four hydrogen atoms; equivalently one mole of methane contains one mole of carbon atoms and four moles of hydrogen atoms.

Step 1: Count the constituent atoms on a mole basis Take one mole of methane as a convenient basis. Its carbon contribution is $1\,\text{mol}$ of atoms and its hydrogen contribution is $4\,\text{mol}$ of atoms. Therefore the total is $1 + 4 = 5\,\text{mol}$ of constituent atoms. This denominator counts atoms, not the one mole of methane molecules.

Step 2: Calculate the carbon fraction The mole fraction of carbon among all atoms is the carbon-atom amount divided by the total atomic amount:

$$x_{\text{C}} = \frac{1}{1+4} = \frac{1}{5}.$$

Converting a fraction into a percentage gives

$$\text{Carbon mole percentage} = \frac{1}{5} \times 100\% = 20\%.$$

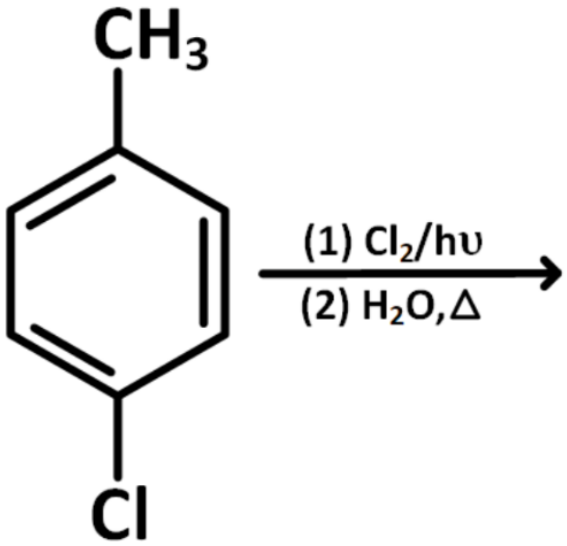
The same result follows directly by counting one carbon in every five atoms of a methane molecule.

Step 3: Distinguish the distractors The complementary 80% is the mole percentage of hydrogen atoms, because hydrogen contributes four of the five atoms. The 75% choice instead comes from the carbon mass contribution, approximately $12/(12+4)$, and answers a different question. The 25% choice incorrectly compares carbon only with hydrogen, $1/4$, instead of with the total number of atoms.

Step 4: Conclusion The percentage composition of carbon by mole in methane is 20%. Hence, the correct option is (B).

Quick Tip to Solve: For mole percentage in a molecular formula, use subscripts as atom counts; introduce atomic masses only when mass percentage is requested.

74. The major product of the following reaction is: [4 marks]



(A) p-ClC_6H_4CHO

(B) p-$ClC_6H_4CO_2H$

(C) p-$ClC_6H_4CH_2OH$

(D) p-$ClC_6H_4CHCl_2$

Correct Answer: (C) p-$ClC_6H_4CH_2OH$

Solution:

Concept:

- Chlorine in ultraviolet light reacts with an alkylbenzene by free-radical substitution at the benzylic side chain. The aryl carbon–chlorine bond does not undergo ordinary hydrolysis with hot water.
- A benzyl chloride undergoes nucleophilic substitution on heating with water to give the corresponding benzyl alcohol. The aromatic ring and its para chlorine substituent remain intact.

Step 1: Identify the reactive side chain The starting compound is para-chlorotoluene: the ring bears CH_3 and Cl in para positions. With Cl_2 under light, a benzylic hydrogen is replaced by chlorine through a radical chain reaction. The benzyl radical intermediate is resonance stabilised, which directs the substitution to the side chain rather than to the aromatic ring.

Step 2: Write the chlorination and hydrolysis sequence The side-chain sequence for the reagents shown is

$$\text{p-}ClC_6H_4CH_3 + Cl_2 \xrightarrow{h\nu} \text{p-}ClC_6H_4CH_2Cl + HCl.$$

Heating the benzyl chloride with water replaces the benzylic chlorine by hydroxyl:

$$\text{p-}ClC_6H_4CH_2Cl + H_2O \xrightarrow{\Delta} \text{p-}ClC_6H_4CH_2OH + HCl.$$

Thus the side chain changes from methyl to hydroxymethyl while the chlorine directly attached to the ring is retained.

Step 3: Compare the four products Option C is para-chlorobenzyl alcohol, matching CH_2OH at

the original methyl position. Option A, the aldehyde, would correspond to hydrolysis of a benzal chloride bearing $CHCl_2$, requiring further side-chain chlorination. Option B, the carboxylic acid, similarly requires a more extensively chlorinated precursor or oxidation. Option D is a dichloromethyl derivative and cannot be the hydrolysed alcohol product of the sequence shown. No excess chlorine or exhaustive chlorination condition is specified.

Step 4: Conclusion The major product is para-chlorobenzyl alcohol, p-$ClC_6H_4CH_2OH$. Hence, the correct option is (C).

Quick Tip to Solve: For an alkylbenzene, distinguish light-induced side-chain chlorination from Lewis-acid-catalysed ring chlorination, then apply the second reagent to that side-chain halide.

75. Fructose and glucose can be distinguished by : [4 marks]

(A) Barfoed's test

(B) Fehling's test

(C) Seliwanoff's test

(D) Benedict's test

Correct Answer: (C) Seliwanoff's test

Solution:

Concept:

- Glucose is an aldohexose and fructose is a ketohexose. A distinguishing test must respond differently to their carbonyl type, rather than merely detect a reducing monosaccharide.
- Seliwanoff's reagent contains resorcinol in concentrated hydrochloric acid. Ketohexoses dehydrate rapidly to a hydroxymethylfurfural derivative that condenses with resorcinol to give a cherry-red colour.

Step 1: Apply Seliwanoff's test On warming with the reagent, fructose, a ketose, is dehydrated more rapidly than glucose. The dehydration product reacts with resorcinol and develops the characteristic deep cherry-red colour quickly. Glucose, an aldose, reacts much more slowly; the speed of colour development under the standard test conditions provides the distinction.

Step 2: Check the reducing-sugar tests Fehling's and Benedict's reagents detect reducing sugars through reduction of copper(II) to copper(I) oxide. Glucose gives these tests, but fructose also gives them: in alkaline solution it can rearrange through an enediol intermediate to aldoses. Therefore a positive copper reduction is not evidence that the original sugar was specifically glucose, and options B and D cannot distinguish this pair.

Step 3: Check Barfoed's test Barfoed's test uses copper(II) in an acidic medium and chiefly distinguishes rapidly reacting monosaccharides from more slowly reacting disaccharides. Both glucose and fructose are monosaccharides, so both respond under the appropriate conditions. This rules out option A. In contrast, the rapid ketose dehydration on which Seliwanoff's test depends directly probes the structural difference between the two sugars.

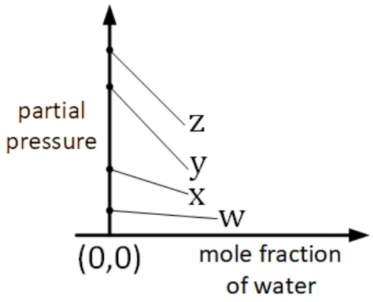
Step 4: Conclusion Glucose and fructose are distinguished by Seliwanoff's test: fructose gives the

rapid cherry-red response. Hence, the correct option is (C).

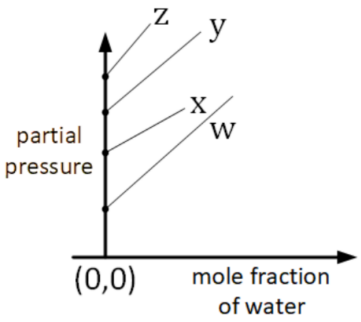
Quick Tip to Solve: Remember the target distinction: Barfoed tests mono- versus disaccharides, while Seliwanoff tests rapid ketose versus slower aldose dehydration.

76. For the solution of the gases w, x, y and z in water at 298K, the Henrys law constants (K_H) are 0.5, 2, 35 and 40 kbar, respectively. The correct plot for the given data is :- [4 marks]

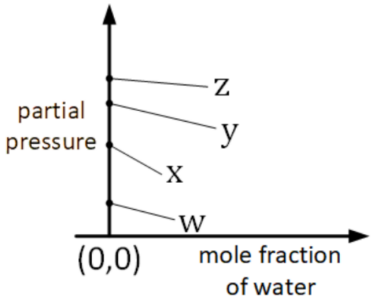
(A)



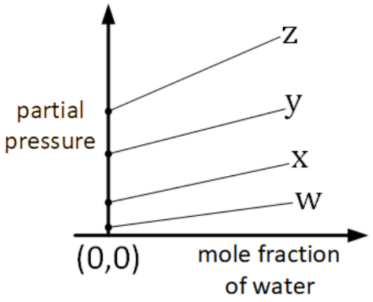
(B)



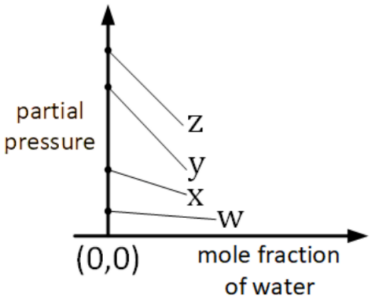
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Henry's law is $p_i = K_{H,i}x_i$, where p_i is gas partial pressure and x_i is the dissolved gas mole fraction. The constant here is expressed in kbar.
- The horizontal axis in the options is the mole fraction of water, not of the gas. For a binary gas–water solution, $x_i = 1 - x_{\text{water}}$, so $p_i = K_{H,i}(1 - x_{\text{water}})$.

Step 1: Determine the line direction Writing $X = x_{\text{water}}$, each plot must obey

$$p_i = K_{H,i} - K_{H,i}X.$$

Its slope is $-K_{H,i}$, which is negative for every gas. Thus pressure decreases as the water mole fraction increases. The positive-slope plots in options B and D use the wrong direction for this horizontal-axis variable and are excluded.

Step 2: Order the intercepts and slope magnitudes The constants are $K_{H,w} = 0.5$, $K_{H,x} = 2$, $K_{H,y} = 35$, and $K_{H,z} = 40$, all in kbar. Therefore the pressure ordering at a common water mole fraction is

$$p_z > p_y > p_x > p_w.$$

The magnitudes of the negative slopes have exactly the same ordering: $40 > 35 > 2 > 0.5$. The z line must decrease most steeply and the w line least steeply.

Step 3: Match both features to a single option Option A has negative slopes, with z and y descending more steeply than x and w, and the intercept ordering z, y, x, w. Option C has the high-intercept z line almost flat and a lower-intercept line steeper, inconsistent with the requirement that each slope magnitude equal its own Henry constant. The lines are schematic portions of the dilute-solution plots; their ordering and relative slopes are the deciding information.

Step 4: Conclusion The correct schematic pressure plot against the mole fraction of water is option A. Hence, the correct option is (A).

Quick Tip to Solve: Always read the mole-fraction axis: using solvent mole fraction changes Henry's law from a positive slope to a negative slope.

77. For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$,
if the rate of formation of B is set to be zero then the concentration of B is given by : [4 marks]

(A) $k_1 k_2 [A]$
(B) $\left(\frac{k_1}{k_2}\right)[A]$
(C) $(k_1 + k_2)[A]$
(D) $(k_1 - k_2)[A]$

Correct Answer: (B) $\left(\frac{k_1}{k_2}\right)[A]$

Solution:

Concept:

- For consecutive first-order steps A $\longrightarrow$ B $\longrightarrow$ C, B is formed from A and simultaneously consumed to form C. Its net rate is the formation rate minus the consumption rate.
- The steady-state condition for the intermediate is $d[B]/dt = 0$. It means the two opposing rates balance, not that both reactions have stopped or that the concentration of B is zero.

Step 1: Write the two contributions to the rate of B The first elementary first-order step supplies B at rate $k_1[A]$, while the second step removes B at rate $k_2[B]$. Since one B is formed or consumed in each event, no additional stoichiometric multiplier is needed. The net rate of change of its concentration is therefore

$$\frac{d[B]}{dt} = k_1[A] - k_2[B].$$

Step 2: Apply the specified zero net rate Setting the net rate to zero gives

$$0 = k_1[A] - k_2[B], \qquad k_2[B] = k_1[A].$$

For a nonzero second-step rate constant, divide by k_2:

$$[B] = \frac{k_1}{k_2}[A].$$

This relates the intermediate concentration to the instantaneous concentration of A under the stated balance condition.

Step 3: Check the dimensions and the alternatives Both first-order rate constants have units of inverse time, so k_1/k_2 is dimensionless and multiplying it by $[A]$ correctly gives a concentration. The product $k_1 k_2[A]$ in option A has concentration per time squared. The sum and difference in options C and D give concentration per time, which is a rate rather than a concentration. They also fail the direct formation–consumption balance.

Step 4: Conclusion The concentration is $[B] = (k_1/k_2)[A]$. Hence, the correct option is (B).

Quick Tip to Solve: For an intermediate, write every route that forms it and every route that consumes it before setting its net rate to zero.

78. Calculate the standard cell potential in (V) of the cell in which following reaction takes place :
$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$
Given that

$$E^{o}_{Ag^{+}/Ag} = xV$$

$$E^{o}_{Fe^{2+}/Fe} = yV$$

$$E^{o}_{Fe^{3+}/Fe} = zV$$

[4 marks]

(A) x + 2y - 3z

(B) x - z

(C) x - y

(D) x + y - z

Correct Answer: (A) x + 2y - 3z

Solution:

Concept:

- Standard Gibbs energies are additive, whereas electrode potentials are not. For a reduction involving n electrons, $\Delta G^\circ = -nFE^\circ$, where F is the Faraday constant.
- The cell reaction reduces Ag^+ at the cathode and oxidises Fe^{2+} at the anode. Thus $E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$, using reduction potentials for both electrodes.

Step 1: Find the missing iron reduction potential Let $u = E^\circ_{Fe^{3+}/Fe^{2+}}$. The reduction of Fe^{3+} to iron can be divided into two reductions:

$$Fe^{3+} + e^- \longrightarrow Fe^{2+}, \qquad Fe^{2+} + 2\,e^- \longrightarrow Fe.$$

The first step transfers one electron and the second transfers two. Their standard Gibbs energies therefore add to the three-electron reduction energy:

$$-3Fz = -Fu - 2Fy.$$

Dividing by $-F$ gives $u = 3z - 2y$. Multiplying a half-reaction does not multiply its electrode potential; the electron factors enter through Gibbs energy.

Step 2: Combine the cathode and anode reactions Silver gains the electron released by iron(II), so the silver reduction potential is x, while the iron reduction potential to be subtracted is u. Therefore

$$E^\circ_{\text{cell}} = x - u = x - (3z - 2y) = x + 2y - 3z.$$

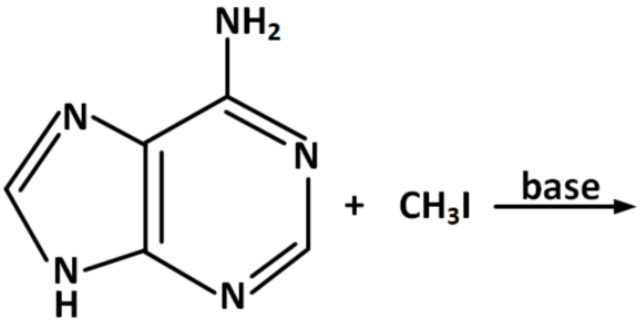
Option (B) incorrectly uses the three-electron iron-to-metal potential directly. Option (C) uses the iron(II)-to-metal couple, which is also absent from the net reaction. Option (D) omits the required

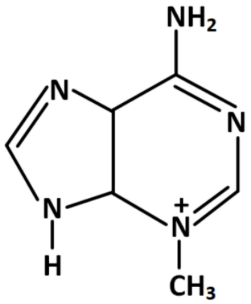
electron-number weighting.

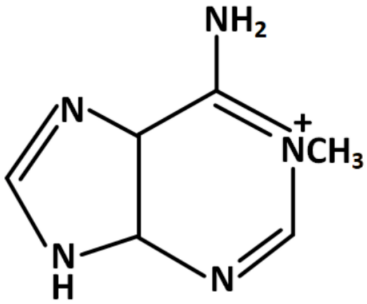
Step 3: Conclusion The standard cell potential is $\boxed{(x+2y-3z)\,\text{V}}$. Hence, the correct option is (A).

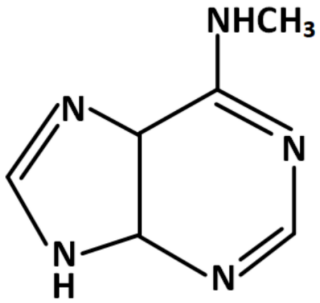
Quick Tip to Solve: Convert each given potential to $-nFE^\circ$ before combining iron couples with different electron transfers.

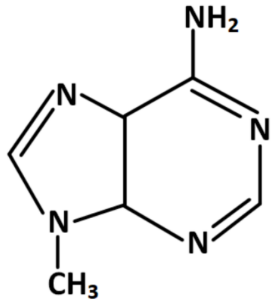
79. The major product in the following reaction is : [4 marks]



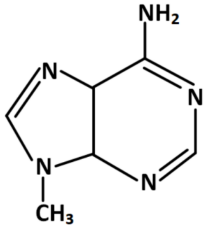
(A)


(B)


(C)


(D)


Correct Answer: (D)



Solution:

Concept:

- Methyl iodide, CH_3I, is an alkylating agent: a nucleophilic nitrogen attacks its methyl carbon by an S_N2 process, expelling I^-.
- In the presence of base, the ring $N-H$ group can lose its proton. Alkylation of the resulting anionic nitrogen replaces that hydrogen by methyl and gives a neutral N-alkylated product.

Step 1: Identify the site activated by the base The fused heterocycle shown is adenine, a 6-aminopurine. In its five-membered ring the lower nitrogen bears the explicit hydrogen; this is the N9 position. The exocyclic NH_2 group and the other ring nitrogens are different possible sites, but the base creates the strongest nucleophile by removing the ring $N-H$ proton. The resulting conjugate base is resonance-stabilised over the heterocyclic system. This activation makes substitution at N9 the appropriate major reaction under the stated basic alkylation conditions.

Step 2: Attach methyl and compare the structures The N9 anion attacks the methyl carbon of CH_3I, with simultaneous loss of iodide. Its new bond is N_9-CH_3; the amino substituent remains NH_2. Option (D) shows precisely this replacement of the five-membered-ring N-H by N-methyl. Options (A) and (B) instead place methyl on nitrogens in the six-membered ring and show positively charged products while retaining the original N-H. Option (C) methylates the exocyclic amino group, which is not the site selected by deprotonation of the ring N-H.

Step 3: Conclusion The major product is neutral N9-methyladenine, with CH_3 on the nitrogen originally bearing H in the five-membered ring. Hence, the correct option is (D).

Quick Tip to Solve: With base plus methyl iodide, first locate an explicit heterocyclic N-H; replace that H by methyl before considering quaternisation of other nitrogens.

80. The ion that has sp^3d^2 hybridization for the central atom, is : [4 marks]

(A) $[ICl_2]^-$

(B) $[ICl_4]^-$

(C) $[IF_6]^-$

(D) $[BrF_2]^-$

Correct Answer: (B) $[ICl_4]^-$

Solution:

Concept:

- In the conventional VSEPR treatment, sp^3d^2 hybridisation corresponds to six electron domains around the central atom: bonding pairs plus lone pairs.
- For these singly charged interhalogen anions, count seven central-halogen valence electrons, one electron from each monovalent ligand and one extra electron for the negative charge. Half this total gives the steric number.

Step 1: Calculate the steric number of each ion For ICl_2^-, the count is $(7+2+1)/2 = 5$: two bonding

pairs and three lone pairs, corresponding to sp^3d. For ICl_4^-, the count is

$$\frac{7+4+1}{2} = 6.$$

There are four I-Cl bonding domains and two lone pairs on iodine. For IF_6^-, the count is $(7+6+1)/2 = 7$, with six bonding domains and one lone pair. Finally, BrF_2^- again gives $(7+2+1)/2 = 5$, with two bonds and three lone pairs.

Step 2: Distinguish electron geometry from molecular shape Only ICl_4^- has six electron domains and therefore the required conventional sp^3d^2 assignment. Its electron-domain arrangement is octahedral; the two lone pairs occupy opposite positions and the four chlorine atoms form a square plane. Thus its square-planar molecular shape does not mean that it has only four hybrid orbitals. Options (A) and (D) each have steric number five, while option (C) has steric number seven, so none matches the six-domain requirement.

Step 3: Conclusion $\boxed{ICl_4^-}$ has four bond pairs and two lone pairs on iodine and is assigned sp^3d^2 hybridisation. Hence, the correct option is (B).

Quick Tip to Solve: For interhalogen anions, include the negative-charge electron and count lone pairs; molecular shape alone does not give the steric number.

81. Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product? [4 marks]

(A) Cl – CH = CH_2

(B) F_3C – CH = CH_2

(C) CH_3O – CH = CH_2

(D) H_2N – CH = CH_2

Correct Answer: (B) F_3C – CH = CH_2

Solution:

Concept:

- Addition of HCl to an unsymmetrical alkene is normally governed by the relative stability of the carbocationic intermediates formed on protonation.
- A CF_3 group has a strong electron-withdrawing inductive effect and destabilises an adjacent positive charge. A directly attached heteroatom with a lone pair can instead stabilise a neighbouring carbocation by resonance donation.

Step 1: Compare the two protonation orientations For $CF_3{-}CH{=}CH_2$, ordinary Markovnikov protonation would add H to the terminal carbon, producing $CF_3{-}CH^+{-}CH_3$. The positive charge would then be directly adjacent to the strongly electron-withdrawing CF_3 group, an unfavourable arrangement. Protonation at the internal alkene carbon instead places the cationic centre on the terminal carbon, farther from CF_3. This avoids the especially strong destabilisation of the adjacent carbocation and reverses the usual regioselectivity.

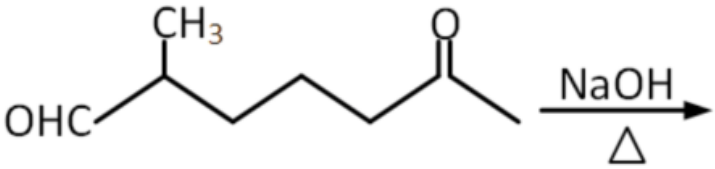
Step 2: Locate chloride and rule out the other substituents Chloride traps the terminal cationic centre,

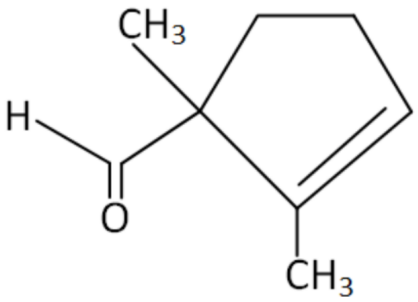
giving the major anti-Markovnikov orientation $CF_3-CH_2-CH_2Cl$. In options (C) and (D), directly attached OCH_3 and NH_2 have lone pairs that stabilise the carbocation formed at the carbon bonded to the heteroatom. Chlorine in option (A) can also donate a lone pair into an adjacent empty p orbital, so it does not act like the purely strongly withdrawing CF_3 substituent. The reversal here is an electronic substituent effect, not the peroxide effect; HCl does not exhibit the usual peroxide-induced anti-Markovnikov addition of HBr.

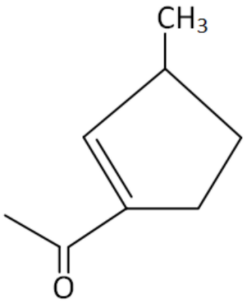
Step 3: Conclusion The alkene is $\boxed{CF_3-CH=CH_2}$, option (B). Its major product places Cl on the terminal carbon. Hence, the correct option is (B).

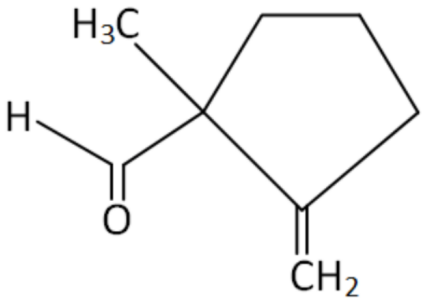
Quick Tip to Solve: When a strong $-I$ group such as CF_3 is present, compare the distance of each possible carbocation from it before applying Markovnikov orientation.

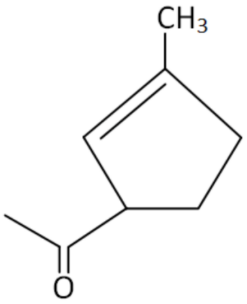
82. The major product obtained in the following reaction is [4 marks]



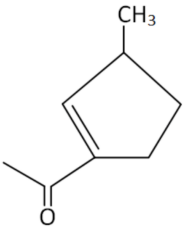
(A)


(B)


(C)


(D)


Correct Answer: (B)



Solution:

Concept:

- Base forms an enolate from an alpha carbon of a carbonyl compound. In an intramolecular aldol reaction this nucleophile attacks another carbonyl group within the same molecule.
- Five- and six-membered ring closures are favoured. Heating the beta-hydroxy carbonyl product promotes dehydration to an alpha,beta-unsaturated carbonyl compound, stabilised by conjugation.

Step 1: Select the favourable ring closure Write the substrate connectivity as follows:

$$OHC-CH(CH_3)-CH_2-CH_2-CH_2-CO-CH_3.$$

Number the aldehyde carbon C1 and the successive chain atoms C2 through C7; the ketone carbonyl is C6. Deprotonation at C5, alpha to the ketone, gives an enolate that can attack the more electrophilic aldehyde carbon C1. The new C5-C1 bond creates the five-membered ring C1-C2-C3-C4-C5. The original ketone remains outside the ring as an acetyl substituent on C5, while the original methyl on C2 remains attached to that ring carbon.

Step 2: Dehydrate and identify the conjugated product The aldol addition gives OH at C1 and a remaining alpha hydrogen at C5. Heating eliminates water across C1-C5, producing the double bond C1=C5 conjugated with the retained acetyl carbonyl. In the options, (B) has this five-membered ring, the acetyl group on a double-bond carbon, and the original methyl at the correct position. Option (D) has a nonconjugated double bond in the wrong position. Options (A) and (C) retain the aldehyde substituent and consume the ketone carbonyl; they do not represent the favoured ketone-enolate attack on the aldehyde. Option (C) also shows an exocyclic double bond rather than the conjugated enone obtained here.

Step 3: Conclusion The major product is the conjugated cyclopentene bearing an acetyl group at one alkene carbon and the original methyl substituent, as drawn in option (B). Hence, the correct option is (B).

Quick Tip to Solve: For an intramolecular aldol, count the ring atoms first, then require the dehydration double bond to lie next to the carbonyl that remains.

83. Which of the following compounds will show the maximum 'enol' content? [4 marks]

(A) CH_3COCH_3

(B) $CH_3COCH_2COCH_3$

(C) $CH_3COCH_2COOC_2H_5$

(D) $CH_3COCH_2CONH_2$

Correct Answer: (B) $CH_3COCH_2COCH_3$

Solution:

Concept:

- The equilibrium enol content depends on the relative stability of the keto and enol tautomers, not simply on the number of alpha hydrogens present.

- An enol is especially stabilised when its double bond is conjugated with another carbonyl group and its OH forms a six-membered intramolecular hydrogen-bonded arrangement with that carbonyl oxygen.

Step 1: Identify the strongest stabilisation of the enol Option (B), $CH_3COCH_2COCH_3$, is a beta-diketone. Enolisation of its central methylene gives $CH_3C(OH)=CHCOCH_3$, or the equivalent tautomer formed from the other end. The resulting enol has conjugation across the $C=C-C=O$ system. Its OH can also hydrogen-bond to the other carbonyl oxygen through a favourable six-membered chelate arrangement. These two effects stabilise the enol substantially relative to the diketo form.

Step 2: Compare the ketone, ester and amide alternatives Acetone in (A) has only one carbonyl, so its enol does not obtain the extra carbonyl conjugation and intramolecular chelation available to a beta-dicarbonyl compound. Options (C) and (D) contain a beta-keto ester and a beta-keto amide, respectively. They can also enolise, but resonance donation from OR or NH_2 strongly stabilises the ester or amide carbonyl in the keto form and reduces the advantage of the corresponding chelated enol compared with a beta-diketone. Thus these do not exceed acetylacetone in enol content. The comparison is based on tautomer stability rather than a claim that the other compounds cannot form enols.

Step 3: Conclusion $\boxed{CH_3COCH_2COCH_3}$, acetylacetone, has the maximum enol content among the listed compounds. Hence, the correct option is (B).

Quick Tip to Solve: Look for a beta-diketone: conjugation plus a six-membered intramolecular hydrogen bond makes its enol unusually stable.

84. Polysubstitution is a major drawback in: [4 marks]

(A) Reimer Tiemann reaction

(B) Friedel Craft's alkylation

(C) Acetylation of aniline

(D) Friedel Craft's acylation

Correct Answer: (B) Friedel Craft's alkylation

Solution:

Concept:

- In Friedel-Crafts alkylation, an alkyl electrophile replaces a hydrogen of an aromatic ring by an alkyl group, generally using a Lewis-acid catalyst.
- An alkyl group activates an aromatic ring toward further electrophilic substitution through electron donation. An acyl group has the opposite effect because its carbonyl withdraws electron density.

Step 1: Explain why alkylation continues After the first Friedel-Crafts alkylation, benzene becomes an alkylbenzene. The introduced alkyl group increases ring electron density through its positive inductive effect and hyperconjugation. Consequently, the monoalkylated product reacts with further alkyl electrophile at least as readily as, and usually more readily than, the starting benzene. Continued

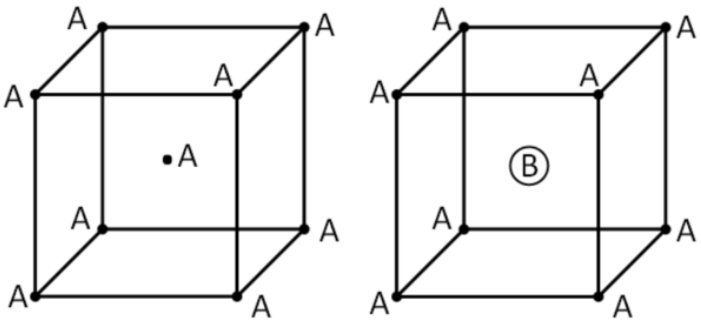
reaction therefore gives di- and higher alkylated products. This makes selective preparation of only the monoalkylated product difficult unless suitable measures, such as using excess aromatic substrate, suppress subsequent substitution.

Step 2: Distinguish the remaining reactions Friedel-Crafts acylation in (D) installs $-COR$, which withdraws electron density by resonance and induction. The acylated aromatic product is deactivated and further acylation is strongly discouraged. Acetylation of aniline in (C) is acylation at the amino nitrogen to form an amide; it is not the activating ring-alkylation sequence responsible for this drawback. The Reimer-Tiemann reaction in (A) introduces a formyl group into phenol and is not the standard reaction limited by progressive alkyl-group activation. Thus the defining explanation, a first alkyl group accelerating further substitution, applies to (B).

Step 3: Conclusion Polysubstitution is a major drawback of Friedel-Crafts alkylation. Hence, the correct option is (B).

Quick Tip to Solve: Alkyl groups activate the ring, so alkylation can repeat; acyl groups deactivate it, so Friedel-Crafts acylation usually stops after one substitution.

85. Consider the bcc unit cells of the solids 1 and 2 with the position of atoms as shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency in solid 2? [4 marks]



(A) 45%

(B) 90%

(C) 75%

(D) 65%

Correct Answer: (B) 90%

Solution:

Concept:

- Packing efficiency is the volume occupied by atoms in one unit cell divided by the volume of the unit cell. Corner atoms contribute one eighth each, whereas a body-centred atom contributes one whole atom.

- For a bcc solid containing identical touching atoms of radius r, the body diagonal gives $\sqrt{3}a_1 = 4r$. Thus its packing fraction is $\pi\sqrt{3}/8$, approximately 0.68.

Step 1: Count the atoms in the two cells In solid 1, the eight corners contribute one A atom and the centre contributes another A atom. Its occupied volume is therefore $2(4\pi r^3/3)$, where r is the radius of A. In solid 2, the corner contribution remains one A atom but the central B atom has radius $2r$. Its volume is eight times the volume of one A atom. Consequently, solid 2 contains an occupied volume equivalent to nine A atoms, rather than two.

Step 2: Compare occupied volume and cell volume The occupied-volume ratio is $V_{\text{atoms},2}/V_{\text{atoms},1} = 9/2$. The edge increases by 50 percent, so $a_2 = 1.5a_1$ and the cell-volume ratio is $(1.5)^3 = 27/8$. Hence

$$\eta_2 = \frac{\pi\sqrt{3}}{8}\frac{9/2}{27/8} = \frac{\pi\sqrt{3}}{6} \approx 0.907.$$

The approximate percentage is 90 percent. Values 45, 75 and 65 percent do not include both the cubic increase of atomic volume with radius and the cubic increase of the cell volume.

Step 3: Conclusion The approximate packing efficiency in solid 2 is 90%. Hence, the correct option is (B).

Quick Tip to Solve: Compare volume ratios before substituting numbers: doubling a radius multiplies that atom's volume by eight, while increasing an edge by 50 percent multiplies the cell volume by 3.375.

86. The Mond process is used for the [4 marks]

(A) Extraction of Zn

(B) extraction of Mo

(C) Purification of Zr and Ti

(D) Purification of Ni

Correct Answer: (D) Purification of Ni

Solution:

Concept:

- The Mond process is a refining process for nickel. It uses the ability of nickel to form a volatile carbonyl with carbon monoxide at a relatively low temperature.
- Purification requires two reversible operations: selectively form a volatile compound from the impure metal, then decompose that compound to deposit pure metal.

Step 1: Form volatile nickel tetracarbonyl Impure nickel is treated with carbon monoxide at about 330–350 K. Nickel forms the volatile compound nickel tetracarbonyl:

$$\mathrm{Ni} + 4\,\mathrm{CO} \longrightarrow \mathrm{Ni(CO)_4}.$$

The carbonyl vapour separates from nonvolatile impurities. Formation of this molecular compound allows the nickel to be transferred away from the original impure metal, so this is purification rather than extraction of an unrelated element.

Step 2: Decompose the carbonyl and identify the metal On heating the separated carbonyl to about 450–470 K, it decomposes:

$$\mathrm{Ni(CO)_4} \longrightarrow \mathrm{Ni} + 4\,\mathrm{CO}.$$

Pure nickel is deposited and carbon monoxide is released. This distinctive carbonyl route identifies the Mond process. Zinc extraction does not use this nickel-carbonyl cycle, and molybdenum extraction is also not the Mond process. Zirconium and titanium are purified by the van Arkel process, in which volatile iodides are formed and then decomposed on a hot filament; that route must not be confused with carbonyl refining.

Step 3: Conclusion The Mond process is used for the purification of Ni. Hence, the correct option is (D).

Quick Tip to Solve: Associate Mond with nickel carbonyl; associate van Arkel with volatile iodides of titanium and zirconium.

87. The maximum prescribed concentration of copper in drinking water is : [4 marks]

(A) 3 ppm

(B) 5 ppm

(C) 0.5 ppm

(D) 0.05 ppm

Correct Answer: (A) 3 ppm

Solution:

Concept:

- The prescribed concentration of a contaminant is an empirical water-quality value. It must be matched to the particular ion or metal named, rather than inferred from atomic mass or periodic trends.
- For the environmental-chemistry values used in this question, the maximum prescribed concentration of copper in drinking water is $3\,\mathrm{ppm}$. In dilute water, ppm corresponds approximately to milligrams per litre.

Step 1: Identify the substance and the relevant prescribed value The question asks specifically about copper in drinking water. The relevant prescribed concentration in the environmental-chemistry table is $3\,\mathrm{ppm}$. Since water has a density close to $1\,\mathrm{g\,mL^{-1}}$, this is approximately $3\,\mathrm{mg}$ of copper per litre of water. This unit interpretation helps distinguish a concentration limit from an amount of metal or a percentage concentration.

Step 2: Match the tabulated concentration with the options Option (A) gives exactly $3\,\mathrm{ppm}$, so it matches the copper entry. The value $5\,\mathrm{ppm}$ in option (B) belongs to a different contaminant entry, commonly zinc, and is not the copper value. Neither $0.5\,\mathrm{ppm}$ nor $0.05\,\mathrm{ppm}$ is the copper concentration requested here; selecting a small value merely because a metal can be toxic would not identify the prescribed entry. These numerical limits are substance-specific, so the table value rather than an assumed relative toxicity decides the answer.

Step 3: Conclusion The maximum prescribed copper concentration asked for is $3\,\text{ppm}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the contaminant paired with its tabulated value: copper corresponds to 3 ppm in this environmental-chemistry question.

88. 0.27 g of a long chain fatty acid was dissolved in 100 cm^3 of hexane. 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaporates and a monolayer is formed. The distance from edge to centre of the watch glass is 10 cm. What is the height of the monolayer?

[Density of fatty acid = 0.9 g cm^{-3}, π = 3] [4 marks]

(A) 10^{-8} m

(B) 10^{-2} m

(C) 10^{-4} m

(D) 10^{-6} m

Correct Answer: (D) 10^{-6} m

Solution:

Concept:

- A uniform monolayer has volume $V = Ah$, where A is its surface area and h is its height. The fatty acid remains after the volatile hexane evaporates.
- The mass delivered by an aliquot is the total dissolved mass multiplied by the aliquot-volume fraction. Convert that mass to volume with $V = m/\rho$, using consistent centimetre units.

Step 1: Find the volume of fatty acid deposited The entire 100 cubic centimetres of solution contains 0.27 g of fatty acid. Since $10\,\text{mL} = 10\,\text{cm}^3$, the deposited aliquot contains

$$m = 0.27\frac{10}{100} = 0.027\,\text{g}.$$

Hexane evaporates and is not part of the final film volume. Using the given fatty-acid density,

$$V = \frac{0.027}{0.9} = 0.030\,\text{cm}^3.$$

Using the full 0.27 g would incorrectly count ten times the material actually delivered.

Step 2: Divide the film volume by the circular area The distance from the centre to the edge is the radius, so $r = 10\,\text{cm}$. With the specified $\pi = 3$,

$$A = \pi r^2 = 3(10)^2 = 300\,\text{cm}^2.$$

The monolayer height is therefore

$$h = \frac{0.030}{300} = 10^{-4}\,\text{cm} = 10^{-6}\,\text{m}.$$

Option (C) retains the numerical value in centimetres but incorrectly labels it metres. Options (A) and (B) also fail the volume-over-area calculation with the given aliquot and radius.

Step 3: Conclusion The monolayer height is $10^{-6}\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: First scale the dissolved mass to the aliquot, then calculate thickness in centimetres and multiply by 10^{-2} to obtain metres.

89. The strength of 11.2 volume solution of H_2O_2 is : [Given that molar mass of H = $1\,\mathrm{g\,mol^{-1}}$ and O = $16\,\mathrm{g\,mol^{-1}}$] [4 marks]

(A) 3.4%

(B) 34%

(C) 13.6%

(D) 1.7%

Correct Answer: (A) 3.4%

Solution:

Concept:

- The volume strength of hydrogen peroxide states the volume of oxygen at STP liberated by complete decomposition of one unit volume of the solution.
- The decomposition is $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. At STP one mole of gas occupies $22.4\,\mathrm{L}$, so one mole of peroxide produces $11.2\,\mathrm{L}$ of oxygen.

Step 1: Convert volume strength into molarity Take one litre of the 11.2-volume solution. It liberates $11.2\,\mathrm{L}$ of oxygen at STP, corresponding to

$$n(O_2) = \frac{11.2}{22.4} = 0.50\,\mathrm{mol}.$$

The balanced reaction requires two moles of peroxide for every mole of oxygen. Thus the litre of solution contains $2(0.50) = 1.00\,\mathrm{mol}$ of H_2O_2, and its molarity is $1.00\,\mathrm{mol\,L^{-1}}$. The factor of two is essential: ignoring it would give half the correct concentration.

Step 2: Express the peroxide mass per 100 millilitres From the supplied atomic masses,

$$M(H_2O_2) = 2(1) + 2(16) = 34\,\mathrm{g\,mol^{-1}}.$$

Therefore one litre of solution contains 34 g of peroxide. In 100 mL the mass is

$$34\frac{100}{1000} = 3.4\,\mathrm{g}.$$

The strength is consequently 3.4% in the usual mass-per-volume expression. The 34 percent choice omits the conversion from a litre to 100 mL; 1.7 percent misses the reaction factor of two, while 13.6 percent is inconsistent with the oxygen yield.

Step 3: Conclusion The strength is 3.4% (mass per volume). Hence, the correct option is (A).

Quick Tip to Solve: For peroxide, volume strength equals 11.2 times molarity; then multiply molarity by 3.4 to obtain its mass-per-volume percentage.

90. The structure of Nylon-6 is : [4 marks]

(A) $[-(CH_2)_4-CO-NH-]_n$

(B) $[-CO-(CH_2)_6-NH-]_n$

(C) $[-CO-(CH_2)_5-NH-]_n$

(D) $[-(CH_2)_6-CO-NH-]_n$

Correct Answer: (C) $[-CO-(CH_2)_5-NH-]_n$

Solution:

Concept:

- Nylon-6 is a polyamide produced from caprolactam, or equivalently from the six-carbon amino acid $H_2N(CH_2)_5COOH$. Its repeating chain contains an amide linkage.
- The number 6 counts all six carbon atoms in the monomer-derived repeating unit, including the carbonyl carbon; only five of those carbons occur in methylene groups.

Step 1: Construct the six-carbon polyamide repeat Opening the caprolactam ring gives a chain segment with five methylene groups between the nitrogen and the carbonyl carbon. The repeat is

$$[-NH-(CH_2)_5-CO-]_n .$$

Reading this same chain in the reverse direction gives $[-CO-(CH_2)_5-NH-]_n$, which is identical connectivity. The terminal bonds shown at the brackets join neighbouring repeats, and the carbonyl carbon of one repeat bonds to the nitrogen of the next through an amide bond.

Step 2: Count methylene groups in each printed structure Option (C) shows one carbonyl group, five methylene groups and one NH group in each repeat. Its carbon count is therefore $1 + 5 = 6$, as required. Option (A) has only four methylene groups, giving five carbons in its repeat. Options (B) and (D) each have six methylene groups in addition to the carbonyl carbon, giving seven carbons rather than six. Changing the direction in which the repeat is written does not change the polymer, but changing the number of methylene groups does. Thus the carbon count, together with the amide linkage, uniquely selects the correct structure.

Step 3: Conclusion Nylon-6 has the repeating unit $[-CO-(CH_2)_5-NH-]_n$. Hence, the correct option is (C).

Quick Tip to Solve: Count the carbonyl carbon in the nylon number: Nylon-6 needs five methylene carbons plus one carbonyl carbon, not six methylene groups.