

JEE Main 2020 Paper 1 (B.E./B.Tech) Jan 7 Shift 1

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 7 January 2020, Shift 1. The total duration is 3 hours.
- (ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, no negative marking and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is : [4 marks]

- (A) $\sqrt{3}$
- (B) $3\sqrt{2}$
- (C) $\frac{3}{\sqrt{2}}$
- (D) $2\sqrt{3}$

Correct Answer: (B) $3\sqrt{2}$

Solution:

Concept:

- For an ellipse with semi-major axis a , semi-minor axis b and eccentricity e , the focal distance from the centre is $c = ae$. Thus the distance between the two foci is $2ae$.
- The directrices lie at distances a/e on opposite sides of the centre. Their separation is $2a/e$, and the full latus rectum has length $2b^2/a$, where $b^2 = a^2(1 - e^2)$.

Step 1: Translate the two distances The stated distances refer to separations of pairs, so each must first be halved. The given focal separation gives $ae = 3$, while the directrix separation gives $a/e = 6$. Dividing these equations eliminates the semi-major axis and determines the eccentricity:

$$e^2 = \frac{ae}{a/e} = \frac{3}{6} = \frac{1}{2}.$$

Since eccentricity is positive, $e = 1/\sqrt{2}$. This is less than one, as an ellipse requires.

Step 2: Calculate the latus rectum Use $a/e = 6$ to obtain $a = 6e = 3\sqrt{2}$. Next, compute the square of the minor semi-axis:

$$b^2 = a^2(1 - e^2) = 18 \left(1 - \frac{1}{2}\right) = 9.$$

The full latus rectum is therefore

$$L = \frac{2b^2}{a} = \frac{18}{3\sqrt{2}} = 3\sqrt{2}.$$

The value in (C) is half this full chord and would result from omitting the factor two. Neither (A) nor (D) satisfies the latus-rectum formula with the given two separations.

Step 3: Conclusion The length of the latus rectum is $3\sqrt{2}$ length units. Hence, the correct option is (B).

Quick Tip to Solve: Use the ratio of the focal separation to the directrix separation to get e^2 immediately; remember that the latus rectum is the full chord $2b^2/a$.

2. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is : [4 marks]

(A) $\frac{5}{2} (6!)$

(B) $6!$

(C) 5^6

(D) $\frac{1}{2} (6!)$

Correct Answer: (A) $\frac{5}{2} (6!)$

Solution:

Concept:

- Every one of the five specified digits must occur, and no other digit is allowed. With six positions, exactly one of these digits occurs twice and the other four occur once.
- The number of distinct permutations of six objects with one identical pair is $6!/2!$. Choices of the repeated digit create disjoint cases, so their counts are added.

Step 1: Choose the repeated digit The digits are 1, 3, 5, 7, 9. Because all five must appear, placing one occurrence of each uses five of the six available places. The remaining place repeats precisely one digit; a triple occurrence would leave insufficient places for the four other required digits. There are therefore five possible choices for the repeated digit, and each admissible number belongs to exactly one such choice. All available digits are nonzero, so there is no leading-zero restriction to subtract.

Step 2: Arrange each multiset and compare the options For a fixed repeated digit, arrange the six entries in the six positions. Interchanging the two identical copies does not change the number, so the ordinary factorial count must be divided by $2!$:

$$N_{\text{fixed}} = \frac{6!}{2!} = 360.$$

Multiplying by the five choices gives

$$N = 5 \frac{6!}{2!} = \frac{5}{2} (6!) = 1800.$$

Option (C), 5^6 , counts strings that may omit some required digits. Option (D) fixes the repeated digit instead of choosing it. Option (B) is also incorrect because it does not implement both the five choices and the identical-pair correction.

Step 3: Conclusion There are $1800 = \frac{5}{2} (6!)$ such six-digit numbers. Hence, the correct option is (A).

Quick Tip to Solve: When a length is one more than the number of required distinct symbols, choose the unique repeated symbol first, then divide the permutation count by $2!$.

3. If $y(\alpha) = \sqrt{2 \left(\frac{\tan \alpha + \cot \alpha}{1 + \tan^2 \alpha} \right) + \frac{1}{\sin^2 \alpha}}$, $\alpha \in \left(\frac{3\pi}{4}, \pi \right)$

$\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is :

[4 marks]

- (A) 4
(B) -4
(C) $\frac{4}{3}$
(D) $-\frac{1}{4}$

Correct Answer: (A) 4

Solution:

Concept:

- The identity $1 + \tan^2 \alpha = \sec^2 \alpha$ simplifies the quotient under the radical, while $\csc^2 \alpha = 1 + \cot^2 \alpha$.
- The principal square root obeys $\sqrt{u^2} = |u|$. The sign of u must be determined on the stated interval before differentiating.

Step 1: Simplify the expression under the square root Throughout the open interval, neither sine nor cosine vanishes, so the following divisions are valid:

$$\frac{\tan \alpha + \cot \alpha}{1 + \tan^2 \alpha} = \frac{1/(\sin \alpha \cos \alpha)}{1/\cos^2 \alpha} = \cot \alpha.$$

Consequently the complete radicand becomes

$$2 \cot \alpha + \csc^2 \alpha = \cot^2 \alpha + 2 \cot \alpha + 1 = (\cot \alpha + 1)^2.$$

Thus $y = |\cot \alpha + 1|$; dropping the absolute value without checking its sign can reverse the derivative.

Step 2: Resolve the sign and differentiate At $3\pi/4$, cotangent equals -1 , and it decreases towards negative infinity as the angle approaches π . On the given open interval, therefore, $\cot \alpha + 1 < 0$. It follows that

$$y = -\cot \alpha - 1, \quad \frac{dy}{d\alpha} = \csc^2 \alpha.$$

At $\alpha = 5\pi/6$, the sine is $1/2$, giving

$$\left. \frac{dy}{d\alpha} \right|_{5\pi/6} = \frac{1}{(1/2)^2} = 4.$$

The negative options have the wrong sign; $4/3$ would correspond to a different trigonometric value and does not follow from the stated angle.

Step 3: Conclusion The required derivative equals 4. Hence, the correct option is (A).

Quick Tip to Solve: Whenever a trigonometric radical becomes a perfect square, use the supplied angle interval to resolve the absolute value before taking its derivative.

4. Let α be a root of the equation $x^2 + x + 1 = 0$ and the

$$\text{matrix } A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$$

then the matrix A^{31} is equal to

[4 marks]

(A) A^2

(B) A

(C) I_3

(D) A^3

Correct Answer: (D) A^3

Solution:

Concept:

- The equation $\alpha^2 + \alpha + 1 = 0$ implies $\alpha^3 = 1$ and $\alpha \neq 1$. Hence powers of α reduce modulo three, and $1 + \alpha + \alpha^2 = 0$.
- To simplify a large matrix power, first seek a small positive power equal to the identity. Matrix multiplication uses row-column scalar products, without complex conjugation.

Step 1: Compute the square using the root identity Write $A = M/\sqrt{3}$ and replace α^4 by α in M . Its rows are then $(1, 1, 1)$, $(1, \alpha, \alpha^2)$, and $(1, \alpha^2, \alpha)$. The first row-column product is 3. Products involving the first row and either other column vanish by $1 + \alpha + \alpha^2 = 0$. The second diagonal product is also zero, since $1 + \alpha^2 + \alpha^4 = 0$, whereas the product of row two and column three is $1 + \alpha^3 + \alpha^3 = 3$. Similarly, the remaining entries give

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = P.$$

Step 2: Reduce the exponent modulo four The matrix P interchanges the second and third coordinates. Applying it twice restores every coordinate, so $P^2 = I_3$. Thus

$$A^4 = (A^2)^2 = P^2 = I_3.$$

Since $31 = 4 \cdot 7 + 3$, associative matrix multiplication gives

$$A^{31} = (A^4)^7 A^3 = A^3.$$

The residues corresponding to A , A^2 , and I_3 are 1, 2, and 0, respectively, whereas 31 has residue 3. In particular, $A^2 \neq I_3$, so reducing the exponent modulo two would be invalid.

Step 3: Conclusion The required power is $A^{31} = A^3$. Hence, the correct option is (D).

Quick Tip to Solve: For a normalized matrix built from cube roots of unity, compute its square first: a coordinate-permutation square often makes its fourth power the identity.

5. If $y = y(x)$ is the solution of the differential equation, $e^y \left(\frac{dy}{dx} - 1 \right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to: [4 marks]

- (A) $2 + \log_e 2$
- (B) $\log_e 2$
- (C) $1 + \log_e 2$
- (D) $2e$

Correct Answer: (C) $1 + \log_e 2$

Solution:

Concept:

- The substitution $u = e^y$ gives $u' = e^y y'$, converting the given equation into a first-order linear equation in u .
- For $u' - u = e^x$, the integrating factor is e^{-x} . Multiplying by it makes the left side the derivative of ue^{-x} .

Step 1: Transform and integrate the differential equation Expand the given equation as $e^y y' - e^y = e^x$. Set $u = e^y$, so that

$$\frac{du}{dx} - u = e^x.$$

Multiplication by e^{-x} yields

$$e^{-x} \frac{du}{dx} - e^{-x} u = 1, \quad \frac{d}{dx}(ue^{-x}) = 1.$$

Integrating with respect to x gives $ue^{-x} = x + C$, where C is a real integration constant. Therefore $u = e^x(x + C)$. This substitution keeps the original exponential relation intact instead of treating the nonlinear equation in y as separable.

Step 2: Apply the initial condition and evaluate The condition $y(0) = 0$ implies $u(0) = e^0 = 1$, and hence $C = 1$. Thus

$$e^y = e^x(x + 1), \quad y = x + \ln(x + 1).$$

This real solution is defined for $x > -1$, which includes both the initial point and the requested point. Substituting $x = 1$ gives $y(1) = 1 + \ln 2$. As a check, $y' - 1 = 1/(x + 1)$, so multiplying by $e^y = e^x(x + 1)$ recovers e^x . Option (B) omits the x term, (A) adds an extra unit, and (D) equals $e^{y(1)}$ rather than $y(1)$ itself.

Step 3: Conclusion The required value is $y(1) = 1 + \log_e 2$. Hence, the correct option is (C).

Quick Tip to Solve: When $e^y y'$ and e^y appear together, substitute $u = e^y$ before choosing an integration method; take the logarithm only at the end.

6. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value of k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1 . Then the expected value of X , is :

[4 marks]

(A) $-\frac{3}{16}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{8}$

(D) $\frac{3}{16}$

Correct Answer: (C) $\frac{1}{8}$

Solution:

Concept:

- The five fair, independent tosses give $2^5 = 32$ equally likely sequences. The expectation is $E[X] = \sum_x xP(X = x)$.
- Classify a sequence by its longest consecutive head run: a run of four or five must not also be counted in the class whose assigned value is three. All other sequences receive -1 .

Step 1: Count the sequences with long head runs Exactly five heads gives only HHHHH, so one sequence has $X = 5$. A longest run of four gives HHHHT or THHHH, so two sequences have $X = 4$. For a longest run of exactly three, place its block HHH at the beginning, middle, or end. At the beginning the fourth toss must be T and the fifth can be H or T: HHHTH, HHHTT. In the middle both adjoining tosses must be T: THHHT. At the end the second toss must be T and the first can be H or T: HTHHH, TTHHH. These are five distinct sequences, none containing four consecutive heads.

Step 2: Form the weighted average and check its sign The long-run classes contain $1 + 2 + 5 = 8$ sequences. The other $32 - 8 = 24$ sequences contain no three consecutive heads and hence have $X = -1$. Their probabilities sum to one:

$$\frac{1}{32} + \frac{2}{32} + \frac{5}{32} + \frac{24}{32} = 1.$$

The expectation is

$$E[X] = \frac{5(1) + 4(2) + 3(5) - 1(24)}{32} = \frac{28 - 24}{32} = \frac{1}{8}.$$

The positive contribution exceeds the negative contribution by four, so the negative options are excluded. The remaining positive alternative $3/16$ would require a weighted numerator of six rather than the actual four.

Step 3: Conclusion The expected value is $E[X] = 1/8$. Hence, the correct option is (C).

Quick Tip to Solve: Count the longest runs first, from five down to three, so sequences containing four heads are never counted again as sequences with exactly three consecutive heads.

7. Five numbers are in A.P. whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is: [4 marks]

- (A) $\frac{21}{2}$
- (B) 27
- (C) 7
- (D) 16

Correct Answer: (D) 16

Solution:

Concept:

- For five terms in an arithmetic progression, the middle term equals their arithmetic mean. Reversing their order changes the sign of the common difference but leaves the numbers unchanged.
- Write the terms symmetrically as $m - 2d, m - d, m, m + d, m + 2d$, with $d \geq 0$. Their product is $m(m^2 - d^2)(m^2 - 4d^2)$.

Step 1: Find the middle term The sum is $5m = 25$, so $m = 5$. Thus the five numbers, arranged in increasing order, are $5 - 2d, 5 - d, 5, 5 + d, 5 + 2d$. Since $-1/2 < 5$, it must be one of the first two terms; the middle and final two terms cannot equal the given negative number.

Step 2: Test both possible positions If $5 - 2d = -1/2$, then $d = 11/4$. Only the first term is negative, and all the other terms are positive, giving a negative product, contrary to $2520 > 0$. Hence $5 - d = -1/2$, which gives $d = 11/2$. The terms are then $-6, -1/2, 5, 21/2, 16$.

Step 3: Check the product and compare the options The proposed terms satisfy both original conditions: their sum is 25, and their product is

$$(-6)(-1/2)(5)(21/2)(16) = 3 \cdot 5 \cdot 168 = 2520.$$

Their greatest value is 16. The number $21/2$ in (A) is a term but is not the greatest; 27 and 7 in (B) and (C) do not belong to this progression.

Step 4: Conclusion The greatest of the five numbers is 16. Hence, the correct option is (D).

Quick Tip to Solve: For an odd number of equally spaced terms, start at the mean and place terms symmetrically on either side; the sign of the product can reject a candidate immediately.

8. The greatest positive integer k , for which $49^k + 1$ is a factor of the sum

$49^{125} + 49^{124} + \dots + 49^2 + 49 + 1$, is:

[4 marks]

(A) 32

(B) 60

(C) 63

(D) 65

Correct Answer: (C) 63

Solution:

Concept:

- The sum of powers $1 + r + \dots + r^{n-1}$ is $(r^n - 1)/(r - 1)$. Factorisation at an even exponent gives $r^{2m} - 1 = (r^m - 1)(r^m + 1)$.
- To prove that a candidate is the greatest, first establish divisibility and then exclude every larger integer, not merely the larger options.

Step 1: Exhibit the factor for sixty-three Put $r = 49$ and let the given sum be S . It contains 126 terms, so

$$S = \frac{r^{126} - 1}{r - 1} = (r^{63} + 1) \frac{r^{63} - 1}{r - 1}.$$

The second factor is the integer $1 + r + \dots + r^{62}$. Thus $49^{63} + 1$ is indeed a factor of S .

Step 2: Rule out larger exponents For $64 \leq k \leq 125$, let $t = 126 - k$, so $1 \leq t < k$. If $r^k + 1$ divided S , it would divide $(r - 1)S = r^{126} - 1$. But modulo $r^k + 1$, $r^k \equiv -1$, giving remainder $-r^t - 1$. This is nonzero, with absolute value strictly less than $r^k + 1$; divisibility is impossible. For $k \geq 126$, the proposed divisor exceeds $S = (r^{126} - 1)/48$, so these values are impossible too.

Step 3: Select the greatest admissible value The factorisation proves that $k = 63$ works, and the remainder argument excludes every larger integer. Consequently the smaller listed values 32 and 60 cannot be the greatest, while the larger listed value 65 is excluded explicitly.

Step 4: Conclusion The greatest positive integer is $k = 63$. Hence, the correct option is (C).

Quick Tip to Solve: Count the terms first: an even number of powers lets you split the geometric sum into two equal blocks and immediately reveals a factor of the form $r^{n/2} + 1$.

9. If $\operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a :

[4 marks]

- (A) straight line whose slope is $\frac{3}{2}$
(B) straight line whose slope is $-\frac{2}{3}$
(C) circle whose diameter is $\frac{\sqrt{5}}{2}$
(D) circle whose centre is at $(-\frac{1}{2}, -\frac{3}{2})$

Correct Answer: (C) circle whose diameter is $\frac{\sqrt{5}}{2}$

Solution:

Concept:

- For a quotient of complex numbers, multiply numerator and denominator by the conjugate of the denominator to extract its real part.
- The equation $(x-h)^2 + (y-k)^2 = R^2$ represents a circle with centre (h, k) , radius R and diameter $2R$. Points where the original quotient is undefined must be excluded.

Step 1: Express the real part in Cartesian coordinates With $z = x + iy$, the numerator is $(x-1) + iy$ and the denominator is $2x + i(2y+1)$. Therefore, provided the denominator is nonzero,

$$\operatorname{Re} \frac{z-1}{2z+i} = \frac{2x(x-1) + y(2y+1)}{4x^2 + (2y+1)^2}.$$

The condition excludes $(x, y) = (0, -1/2)$.

Step 2: Complete the squares Setting the real part equal to one gives

$$2x^2 - 2x + 2y^2 + y = 4x^2 + 4y^2 + 4y + 1.$$

Collect terms and divide by two:

$$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0.$$

Completing squares yields

$$\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{3}{4}\right)^2 = \frac{5}{16}.$$

Step 3: Identify the diameter and reject the alternatives The centre is $(-1/2, -3/4)$, and the radius is $\sqrt{5}/4$, so the diameter is $\sqrt{5}/2$. The excluded point lies on the circle and must be omitted from the exact locus, but all allowed points still lie on this circle. The quadratic equation rejects both straight-line descriptions (A) and (B); the centre in (D) has the incorrect second coordinate.

Step 4: Conclusion The point lies on the circle of diameter $\sqrt{5}/2$, excluding the undefined point $(0, -1/2)$. Hence, the correct option is (C).

Quick Tip to Solve: After extracting the real part, record the denominator restriction before multiplying through; completing squares then gives the centre and radius without geometric guessing.

10. Let $x^k + y^k = a^k$, ($a, k > 0$) and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is:

[4 marks]

- (A) $\frac{1}{3}$
- (B) $\frac{2}{3}$
- (C) $\frac{4}{3}$
- (D) $\frac{3}{2}$

Correct Answer: (B) $\frac{2}{3}$

Solution:

Concept:

- Implicit differentiation of $x^k + y^k = a^k$, with constants a, k , gives $kx^{k-1} + ky^{k-1}y' = 0$.
- When powers of a variable ratio agree along a nonconstant curve, compare their exponents. It is sufficient to compare on the positive branch, where all fractional powers are unambiguous.

Step 1: Differentiate the given curve Treat a and k as constants and apply the chain rule to y^k . Since $k > 0$, cancel k to obtain

$$x^{k-1} + y^{k-1} \frac{dy}{dx} = 0, \quad \frac{dy}{dx} = -\frac{x^{k-1}}{y^{k-1}} = -\left(\frac{y}{x}\right)^{1-k}.$$

This comparison is made where $x, y > 0$; it does not require division at an intercept.

Step 2: Match the required slope The second condition states

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}.$$

As the ratio y/x varies along the curve, the powers must coincide. Thus $1 - k = 1/3$, giving $k = 2/3$. Equivalently, the differentiated curve requires exponent $k - 1 = -1/3$ when written in terms of x/y .

Step 3: Verify and eliminate the other powers For $k = 2/3$, direct differentiation produces $y' = -x^{-1/3}/y^{-1/3} = -(y/x)^{1/3}$, exactly as requested. For the listed values $1/3, 4/3, 3/2$, the exponent $1 - k$ would be $2/3, -1/3, -1/2$, respectively, and none equals $1/3$. Equality at one special point such as $x = y$ is insufficient for a differential relation along the curve.

Step 4: Conclusion The required value is $k = 2/3$. Hence, the correct option is (B).

Quick Tip to Solve: Write the derivative using the same ratio as the prescribed slope; converting $(x/y)^{k-1}$ to $(y/x)^{1-k}$ prevents a sign error in the exponent.

11. If $f(a + b + 1 - x) = f(x)$, for all x , where a and b are fixed positive real numbers, then

$\frac{1}{a+b} \int_a^b x (f(x) + f(x+1)) dx$ is equal to:

[4 marks]

- (A) $\int_{a-1}^{b-1} f(x+1) dx$
- (B) $\int_{a+1}^{b+1} f(x+1) dx$
- (C) $\int_{a-1}^{b-1} f(x) dx$
- (D) $\int_{a+1}^{b+1} f(x) dx$

Correct Answer: Both (A) and (D) are equal to the required integral.

Solution:

Concept:

- For an integrable function symmetric on an interval, $g(a + b - x) = g(x)$, reflection gives $\int_a^b x g(x) dx = (a + b) \int_a^b g(x) dx / 2$.
- The supplied symmetry is about $(a + b + 1)/2$. In particular, it relates $f(x)$ to $f(a + b + 1 - x)$, and $f(x + 1)$ to $f(a + b - x)$.

Step 1: Build a function symmetric on the integration interval Set $g(x) = f(x) + f(x + 1)$. The given identity yields $f(a + b - x) = f(x + 1)$ and $f(a + b + 1 - x) = f(x)$. Hence $g(a + b - x) = g(x)$. If $J = \int_a^b x g(x) dx$, reflection gives

$$J = \int_a^b (a + b - x) g(x) dx.$$

Adding the two expressions for J shows

$$2J = (a + b) \int_a^b g(x) dx.$$

Step 2: Simplify the unweighted integral By the substitution $u = a + b - x$,

$$\int_a^b f(x + 1) dx = \int_a^b f(a + b - x) dx = \int_a^b f(u) du.$$

Therefore the expression asked for is

$$\frac{J}{a + b} = \frac{1}{2} \int_a^b [f(x) + f(x + 1)] dx = \int_a^b f(x) dx.$$

Step 3: Check every printed option In (A), setting $u = x + 1$ gives $\int_{a-1}^{b-1} f(x + 1) dx = \int_a^b f(u) du$. In (D), setting $u = a + b + 1 - x$ and using the supplied symmetry also gives $\int_{a+1}^{b+1} f(x) dx = \int_a^b f(u) du$. Thus both (A) and (D) are equal to the required value. Options (B) and (C) are not generally equal: with $a = 1, b = 2$ and $f(x) = (x - 2)^2$, the required value is $1/3$, whereas both those options give $7/3$.

Step 4: Conclusion The required integral is $\int_a^b f(x) dx$. The supplied answer key selects (A), which is correct, but the printed symmetry proves that (D) is also correct. Thus the printed choices do not have a unique correct option: both (A) and (D) are valid.

Quick Tip to Solve: When a reflected function appears with a one-unit shift, test each option by substitution: two differently written integration intervals can represent exactly the same integral.

12. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is: [4 marks]

(A) $\frac{1}{6}(24\pi - 1)$

(B) $\frac{1}{3}(12\pi - 1)$

(C) $\frac{1}{3}(6\pi - 1)$

(D) $\frac{1}{6}(12\pi - 1)$

Correct Answer: (D) $\frac{1}{6}(12\pi - 1)$

Solution:

Concept:

- The area between curves written as $x = x_{\text{right}}(y)$ and $x = x_{\text{left}}(y)$ is $\int [x_{\text{right}}(y) - x_{\text{left}}(y)] dy$.
- The circle $x^2 + y^2 = 2$ has radius $\sqrt{2}$, so its area is 2π . To subtract the whole parabolic segment, first confirm it lies inside the circle.

Step 1: Locate the bounded segment The line $y = x$ meets $y^2 = x$ where $y^2 = y$, namely at $(0, 0)$ and $(1, 1)$. For $0 \leq y \leq 1$, the parabola is the left boundary $x = y^2$ and the line is the right boundary $x = y$, because $y^2 \leq y$. These two arcs enclose a single finite region in the first quadrant.

Step 2: Verify that the segment is entirely inside the circle Every point of the segment has $0 \leq y \leq 1$ and $y^2 \leq x \leq y$. Consequently

$$x^2 + y^2 \leq y^2 + y^2 = 2y^2 \leq 2.$$

Thus every point in the region bounded by the parabola and line belongs to the circular disk, and the common area is exactly that whole region. There is no extra outside portion to account for.

Step 3: Subtract the common area from the circular area The common area is

$$A_{\text{common}} = \int_0^1 (y - y^2) dy = \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1 = \frac{1}{6}.$$

The area of the circle not common to this segment is therefore

$$A = 2\pi - \frac{1}{6} = \frac{12\pi - 1}{6}.$$

Options (A) and (B) start with circular area 4π , which is too large. Option (C) subtracts $1/3$ rather than the actual common area $1/6$.

Step 4: Conclusion The required area is $\boxed{(12\pi - 1)/6}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Integrate horizontally here: the boundaries are simply $x = y$ and $x = y^2$. Prove containment in the disk before subtracting the full segment area.

13. If the system of linear equations

$$2x + 2ay + az = 0$$

$$2x + 3by + bz = 0$$

$$2x + 4cy + cz = 0,$$

where $a, b, c \in \mathbb{R}$ are non-zero distinct; has a non-zero solution, then:

[4 marks]

(A) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(B) $a + b + c = 0$

(C) a, b, c are in G.P.

(D) a, b, c are in A.P.

Correct Answer: (A) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Solution:

Concept:

- A homogeneous system always has the zero solution. A non-zero solution exists exactly when its coefficient matrix is singular, so its determinant vanishes.
- For three quantities to be in arithmetic progression, twice the middle quantity must equal the sum of the other two. Since a, b, c are non-zero, division by abc is valid.

Step 1: Apply the condition for a non-zero solution The coefficients of x, y, z , in that order, give the matrix below. Setting its determinant to zero is necessary and sufficient; merely observing that all right-hand sides vanish would establish only the trivial solution.

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0.$$

Expanding along the first row gives

$$2(3bc - 4bc) - 2a(2c - 2b) + a(8c - 6b) = 0.$$

Step 2: Simplify and identify the progression Collect the products carefully:

$$-2bc - 4ac + 4ab + 8ac - 6ab = 0,$$

$$-2bc + 4ac - 2ab = 0.$$

Divide by $-2abc$, which is non-zero by hypothesis:

$$\frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0 \implies \frac{1}{a} + \frac{1}{c} = \frac{2}{b}.$$

This is precisely the arithmetic-progression condition for the reciprocals. The relation is not a sum-zero, geometric-progression or arithmetic-progression condition on the original numbers. For instance, $(a, b, c) = (3, 4, 6)$ satisfies the derived relation, but has non-zero sum, $b^2 \neq ac$, and $a + c \neq 2b$; thus none of the other proposed relations follows.

Step 3: Conclusion $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. Hence, the correct option is (A).

Quick Tip to Solve: For a homogeneous system with a non-zero solution, set the coefficient determinant to zero before trying to solve for the variables.

14. Let α and β be two real roots of the equation

$(k + 1)\tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. if $\tan^2(\alpha + \beta) = 50$, then a value of λ is:

[4 marks]

- (A) $5\sqrt{2}$
- (B) 10
- (C) 5
- (D) $10\sqrt{2}$

Correct Answer: (B) 10

Solution:

Concept:

- Use $t = \tan x$ to turn the equation into a quadratic. Its roots are $\tan \alpha$ and $\tan \beta$, because α, β satisfy the original trigonometric equation.
- For $At^2 + Bt + C = 0$, the sum and product of roots are $-B/A$ and C/A . The angle addition identity is $\tan(\alpha + \beta) = (\tan \alpha + \tan \beta)/(1 - \tan \alpha \tan \beta)$.

Step 1: Find the sum and product of the tangents Bring the right side to the left:

$$(k + 1)t^2 - \sqrt{2}\lambda t + (k - 1) = 0.$$

Since $k \neq -1$, the leading coefficient is non-zero and the quadratic formulae apply. Therefore

$$\tan \alpha + \tan \beta = \frac{\sqrt{2}\lambda}{k + 1}, \quad \tan \alpha \tan \beta = \frac{k - 1}{k + 1}.$$

The roots used here are tangents of the angles, rather than the angles themselves.

Step 2: Use the given square and compare the options The denominator in the addition formula simplifies to

$$1 - \frac{k - 1}{k + 1} = \frac{2}{k + 1} \neq 0.$$

Hence

$$\tan(\alpha + \beta) = \frac{\sqrt{2}\lambda/(k + 1)}{2/(k + 1)} = \frac{\lambda}{\sqrt{2}}.$$

Squaring and imposing the stated value gives

$$\frac{\lambda^2}{2} = 50 \implies \lambda^2 = 100 \implies \lambda = \pm 10.$$

The question asks for a value, and only 10 occurs among the options. The other positive candidates

$5\sqrt{2}, 5, 10\sqrt{2}$ would give squared tangents 25, 25/2, 100, respectively, rather than 50. The positive value is attainable, for example at $k = 1$, when the quadratic has real roots 0 and $5\sqrt{2}$.

Step 3: Conclusion A permitted value is $\lambda = 10$. Hence, the correct option is (B).

Quick Tip to Solve: Apply root formulae to the tangents, then form the tangent of the angle sum; the parameter k cancels before squaring.

15. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ ($\alpha, \beta \in R$) **lies in the plane of the vectors,** $\vec{b} = \hat{i} + \hat{j}$ **and** $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. **If** $\vec{a}$ **bisects the angle between** $\vec{b}$ **and** $\vec{c}$, **then:** [4 marks]

- (A) $\vec{a} \cdot \hat{i} + 3 = 0$
 (B) $\vec{a} \cdot \hat{k} - 4 = 0$
 (C) $\vec{a} \cdot \hat{i} + 1 = 0$
 (D) $\vec{a} \cdot \hat{k} + 2 = 0$

Correct Answer: (B) $\vec{a} \cdot \hat{k} - 4 = 0$

Solution:

Concept:

- The internal bisector of the angle between two non-zero vectors has the direction of the sum of their unit vectors. Adding the original vectors without normalising generally gives a wrong direction.
- The coefficients of $\hat{i}, \hat{j}, \hat{k}$ are the corresponding scalar components. Dotting with a coordinate unit vector selects that component.

Step 1: Normalise the two given vectors The magnitudes are

$$|\vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}, \quad |\vec{c}| = \sqrt{1^2 + (-1)^2 + 4^2} = 3\sqrt{2}.$$

Thus the internal bisector direction is

$$\frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|} = \frac{3(\hat{i} + \hat{j}) + (\hat{i} - \hat{j} + 4\hat{k})}{3\sqrt{2}} = \frac{4\hat{i} + 2\hat{j} + 4\hat{k}}{3\sqrt{2}}.$$

This direction lies in the plane of the given vectors, as required, since it is their linear combination.

Step 2: Use the fixed j component and test the relations Write $\vec{a} = s(4\hat{i} + 2\hat{j} + 4\hat{k})$. Comparing its j component with the given value 2 gives $2s = 2$, so $s = 1$. Consequently

$$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}, \quad \alpha = 4, \quad \beta = 4.$$

It follows that $\vec{a} \cdot \hat{k} - 4 = 4 - 4 = 0$. The left sides in the other three alternatives evaluate to $4 + 3 = 7$, $4 + 1 = 5$, and $4 + 2 = 6$, so those equations are false. The difference of the unit vectors would produce an external bisector; it does not bisect the angle between the two given directed vectors internally. As a direct check, the cosines of the angles made by $(4, 2, 4)$ with $\vec{b}$ and $\vec{c}$ are both $1/\sqrt{2}$.

Step 3: Conclusion $\vec{a} \cdot \hat{k} = 4$, so $\vec{a} \cdot \hat{k} - 4 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: Normalise both vectors before adding them for an internal angle bisector, then use the known component to fix its scale.

16. If $g(x) = x^2 + x - 1$ and

$(g \circ f)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to:

[4 marks]

- (A) $\frac{1}{2}$
- (B) $\frac{3}{2}$
- (C) $-\frac{1}{2}$
- (D) $-\frac{3}{2}$

Correct Answer: (C) $-\frac{1}{2}$

Solution:

Concept:

- The composition $(g \circ f)(x)$ means $g(f(x))$. Thus the formula for g is evaluated at the unknown value $f(x)$, rather than multiplying the functions.
- To obtain one requested function value, substitute its input directly into the composition. Completing the square can show that the resulting quadratic has a unique root.

Step 1: Evaluate the composition at the requested input Let $u = f(5/4)$. Using the given composition first gives

$$g(u) = 4\left(\frac{5}{4}\right)^2 - 10\left(\frac{5}{4}\right) + 5.$$

Simplify using a common denominator:

$$g(u) = \frac{25}{4} - \frac{50}{4} + \frac{20}{4} = -\frac{5}{4}.$$

It is unnecessary to choose a formula for the entire function f ; the requested input alone gives a complete equation for its value.

Step 2: Solve for the function value and reject the other choices Now apply $g(u) = u^2 + u - 1$:

$$u^2 + u - 1 = -\frac{5}{4} \implies u^2 + u + \frac{1}{4} = 0.$$

Completing the square yields

$$\left(u + \frac{1}{2}\right)^2 = 0, \quad u = -\frac{1}{2}.$$

The repeated root makes the answer unique even though solving the composition for general x could involve a choice between two branches. Substitution checks it directly:

$$g\left(-\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} - 1 = -\frac{5}{4}.$$

In contrast, the other proposed inputs $1/2, 3/2, -3/2$ give g -values $-1/4, 11/4, -1/4$, respectively, so none agrees with the composition at $5/4$. One can also see uniqueness from $g(u) = (u + 1/2)^2 - 5/4$: the required output is its minimum value, attained only at $-1/2$.

Step 3: Conclusion $f(5/4) = -1/2$. Hence, the correct option is (C).

Quick Tip to Solve: Substitute the requested input into the composition first; at the minimum of the outer quadratic, its inverse value is unique.

17. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

[4 marks]

- (A) q
- (B) $\sim p$
- (C) p
- (D) $\sim q$

Correct Answer: (B) $\sim p$

Solution:

Concept:

- The implication $p \Rightarrow q$ is equivalent to $\neg p \vee q$. It is false only when its antecedent is true and its consequent is false.
- The distributive identity $(A \vee B) \wedge (A \vee C) = A \vee (B \wedge C)$, together with $q \wedge \neg q = \text{False}$, simplifies the conjunction of implications.

Step 1: Replace both implications by disjunctions Write the statement as $S = (p \Rightarrow q) \wedge (q \Rightarrow \neg p)$. Using the implication identity separately on each part gives

$$S = (\neg p \vee q) \wedge (\neg q \vee \neg p).$$

Reorder the second disjunction, which does not affect its truth value:

$$S = (\neg p \vee q) \wedge (\neg p \vee \neg q).$$

Both brackets now have the same first term, so the distributive identity applies directly.

Step 2: Simplify and verify all truth assignments The expression reduces to

$$S = \neg p \vee (q \wedge \neg q) = \neg p \vee \text{False} = \neg p.$$

For an independent check, when p is false, the first implication is automatically true and the second has the true consequent $\neg p$; hence the conjunction is true for either q . When p is true, the first implication requires q to be true, whereas the second requires q to be false. They cannot both hold, so the conjunction is false for either q . Thus its truth value depends only on the negation of p . It cannot equal q : setting p false and q false gives S true but q false. It cannot equal p : their truth values are opposite for both choices of p . It cannot equal $\neg q$: setting p false and q true gives S true but $\neg q$ false. These checks exclude all three competing alternatives.

Step 3: Conclusion The statement is equivalent to $\neg p$. Hence, the correct option is (B).

Quick Tip to Solve: Convert implications to disjunctions and look for a common term; a remaining conjunction of a statement with its negation is false.

18. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is : [4 marks]

- (A) (4, 3, 2)
- (B) (6, 5, -2)
- (C) (3, 4, -2)
- (D) (6, 5, 2)

Correct Answer: (B) (6, 5, -2)

Solution:

Concept:

- A normal to a plane through three points is obtained by crossing two independent displacement vectors within the plane. The plane equation uses that normal and any one of the given points.
- For a plane $\vec{n} \cdot \vec{r} = d$, reflection of $\vec{r}_0$ is $\vec{r}_0 - 2(\vec{n} \cdot \vec{r}_0 - d)\vec{n}/|\vec{n}|^2$. The midpoint of a point and its image lies on the plane.

Step 1: Find the equation of the plane Use the point $A = (2, 1, 0)$, with $B = (4, 1, 1)$ and $C = (5, 0, 1)$. The two in-plane displacement vectors are

$$\overrightarrow{AB} = (2, 0, 1), \quad \overrightarrow{AC} = (3, -1, 1).$$

Their cross product is

$$\overrightarrow{AB} \times \overrightarrow{AC} = (1, 1, -2).$$

Thus a normal is $\vec{n} = (1, 1, -2)$, and using A gives

$$(x - 2) + (y - 1) - 2z = 0 \implies x + y - 2z - 3 = 0.$$

Each of the other given points satisfies this equation, checking its constant term.

Step 2: Reflect R and check the midpoint For $R = (2, 1, 6)$, the signed plane expression is

$$2 + 1 - 2(6) - 3 = -12, \quad |\vec{n}|^2 = 1 + 1 + 4 = 6.$$

Therefore its image is

$$R' = (2, 1, 6) - \frac{2(-12)}{6}(1, 1, -2) = (6, 5, -2).$$

The midpoint is (4, 3, 2), which satisfies $4 + 3 - 2(2) - 3 = 0$, and the joining vector is parallel to the normal. These are the geometric reflection checks. In particular, option (A) is the perpendicular foot, only halfway to the image. Option (C) has a joining vector not parallel to the normal, and option (D) does not put the midpoint on the plane. Neither can be the reflection.

Step 3: Conclusion The image of R is (6, 5, -2). Hence, the correct option is (B).

Quick Tip to Solve: The perpendicular foot is the midpoint of the point and its reflection; remember the factor of two in the reflection formula.

19. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to : [4 marks]

- (A) -128
- (B) 128
- (C) -64
- (D) -32

Correct Answer: (A) -128

Solution:

Concept:

- A line is tangent to a parabola when substitution of its equation gives a quadratic with a repeated root. Thus its discriminant must be zero.
- For a quadratic $Au^2 + Bu + C = 0$, the repeated-root condition is $B^2 - 4AC = 0$. Apply this separately to the two parabolas, using the same slope m .

Step 1: Use tangency to the first parabola Write the line as $x = (y - 4)/m$. Its slope cannot vanish, since $y = 4$ intersects $y^2 = 4x$ transversely. Substitution gives

$$my^2 - 4y + 16 = 0.$$

A repeated intersection requires

$$(-4)^2 - 4(m)(16) = 0, \quad 16 - 64m = 0.$$

Therefore $m = 1/4$. This also agrees with the standard tangent form $y = mx + 1/m$ for $y^2 = 4x$.

Step 2: Apply the same condition to the second parabola Substitute $y = mx + 4$ into $x^2 = 2by$:

$$x^2 - 2bmx - 8b = 0.$$

Its discriminant is

$$(-2bm)^2 - 4(1)(-8b) = 4b^2m^2 + 32b = 0.$$

Since a parabola requires $b \neq 0$, divide by $4b$ to obtain $bm^2 + 8 = 0$. Hence

$$b = -\frac{8}{m^2} = -\frac{8}{1/16} = -128.$$

The positive option cannot satisfy the tangency equation, while -64 and -32 give the wrong magnitude for the already determined slope.

Step 3: Conclusion $b = -128$. Hence, the correct option is (A).

Quick Tip to Solve: Find the slope using the simpler parabola first, then impose a zero discriminant on the other one.

20. Let the function, $f: [-7, 0] \rightarrow \mathbb{R}$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f , $f(-1) + f(0)$ lies in the interval: [4 marks]

- (A) $[-6, 20]$
- (B) $(-\infty, 20]$
- (C) $[-3, 11]$
- (D) $(-\infty, 11]$

Correct Answer: (B) $(-\infty, 20]$

Solution:

Concept:

- The mean value theorem gives $f(v) - f(u) = f'(c)(v - u)$ for some $c \in (u, v)$, provided continuity and differentiability hold.
- An upper bound on the derivative gives an upper bound on increases of the function. It does not impose a lower bound on the derivative or on function values.

Step 1: Bound the two required values Apply the mean value theorem on $[-7, -1]$. Its hypotheses are satisfied by the stated assumptions. For some $c_1 \in (-7, -1)$,

$$f(-1) - f(-7) = 6f'(c_1) \leq 12.$$

Since $f(-7) = -3$, this gives $f(-1) \leq 9$. Similarly, on $[-7, 0]$,

$$f(0) - f(-7) = 7f'(c_2) \leq 14,$$

so $f(0) \leq 11$. Adding these two inequalities gives

$$f(-1) + f(0) \leq 20.$$

Step 2: Check attainability and the lower end To determine the full interval, consider the family $f_k(x) = -3 + k(x + 7)$, with any real $k \leq 2$. Each member is continuous and differentiable as required, satisfies $f_k(-7) = -3$, and has derivative $k \leq 2$. For this family,

$$f_k(-1) + f_k(0) = -6 + 13k.$$

At $k = 2$, the sum is exactly 20; as k decreases without bound, the sum does likewise. Moreover every real sum at most 20 is obtained by choosing $k = (s + 6)/13$. Thus there is no finite lower endpoint. Options (A) and (C) impose unjustified lower bounds, and (D) excludes the attainable value 20.

Step 3: Conclusion The required interval is $(-\infty, 20]$. Hence, the correct option is (B).

Quick Tip to Solve: Test a linear family with the largest allowed slope to check the upper endpoint and with negative slopes to test any claimed lower bound.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let $A(1, 0)$, $B(6, 2)$ and $C(\frac{3}{2}, 6)$ be the vertices of a triangle ABC . If P is a Point inside the triangle ABC such that the triangles APC , APB and BPC have equal areas, then the length of the line segment PQ , where Q is the point $(-\frac{7}{6}, -\frac{1}{3})$, is _____. [4 marks]

Solution:

Concept:

- An interior point divides a triangle into three triangles of equal area precisely when its barycentric coordinates are equal; it is the centroid.
- The centroid is the average of the three vertex coordinates. Use the distance formula to find its separation from the given point.

Step 1: Identify the equal-area point The three smaller triangles exhaust ABC because P is interior. Their equal areas are therefore each one third of the total. In barycentric coordinates, the weights of A, B, C are the area ratios of the opposite triangles BPC, CPA, APB . All three weights are $1/3$, so P is their equally weighted average. Equivalently, the medians meet at this point and divide the whole triangle into three equal-area triangles when joined to the vertices.

$$P = \frac{A + B + C}{3}.$$

Step 2: Calculate the coordinates and separation Substitute the coordinates carefully, retaining the fractional coordinate of C :

$$x_P = \frac{1 + 6 + 3/2}{3} = \frac{17}{6}, \quad y_P = \frac{0 + 2 + 6}{3} = \frac{8}{3}.$$

For $Q = (-7/6, -1/3)$, the coordinate differences are

$$x_P - x_Q = \frac{17}{6} + \frac{7}{6} = 4, \quad y_P - y_Q = \frac{8}{3} + \frac{1}{3} = 3.$$

The distance formula is

$$PQ = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}.$$

Consequently,

$$PQ = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

The negative coordinates of Q must be subtracted with their signs; subtracting their magnitudes instead would give an incorrect distance. The result is nonnegative as a length must be.

Step 3: Conclusion The length of PQ is 5 coordinate units. The numerical answer is 5.

Quick Tip to Solve: Equal areas around an interior point identify the centroid immediately; average the vertices before using the distance formula.

22. $\lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to _____.

[4 marks]

Solution:

Concept:

- When numerator and denominator both tend to zero, factor them after a suitable substitution rather than substituting zero directly.
- The substitution $t = 3^{x/2}$ gives $3^x = t^2$, $3^{3-x} = 27/t^2$, $3^{-x/2} = 1/t$, and $3^{1-x} = 3/t^2$.

Step 1: Convert the limit to a rational expression As $x \rightarrow 2$, the positive variable $t = 3^{x/2}$ tends to 3. Direct substitution in the original expression gives $9 + 3 - 12 = 0$ in the numerator and $1/3 - 1/3 = 0$ in the denominator, so cancellation is necessary. In the new variable the expression becomes

$$\frac{t^2 + 27/t^2 - 12}{1/t - 3/t^2} = \frac{t^4 - 12t^2 + 27}{t - 3}.$$

Multiplication by t^2 is legitimate because t is positive, hence never zero.

Step 2: Factor and evaluate the continuous remainder Factor the numerator as a quadratic in t^2 :

$$t^4 - 12t^2 + 27 = (t^2 - 9)(t^2 - 3) = (t - 3)(t + 3)(t^2 - 3).$$

For $t \neq 3$, cancel the common factor to get $(t + 3)(t^2 - 3)$. The limit depends on values arbitrarily near the point, so this cancellation is valid even though the original quotient is undefined at the point itself. The remaining polynomial is continuous there, giving

$$\lim_{t \rightarrow 3} (t + 3)(t^2 - 3) = (3 + 3)(9 - 3) = 6 \cdot 6 = 36.$$

This algebraic method preserves the negative exponents and avoids confusing 3^{3-x} with $3^3 - 3^x$.

Step 3: Conclusion The required limit is 36, a dimensionless number.

Quick Tip to Solve: Choose a substitution matching the smallest exponent step; here $3^{x/2}$ turns every exponential term into a rational power expression.

23. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to _____.

[4 marks]

Solution:

Concept:

- For equally weighted observations, variance is $\sigma^2 = \frac{1}{N} \sum x_i^2 - \left(\frac{1}{N} \sum x_i\right)^2$.
- The variance of $1, 2, \dots, N$ is $(N^2 - 1)/12$. Multiplying every observation by a multiplies the variance by a^2 .

Step 1: Find the number of natural-number observations For the first n natural numbers, the sum is

$n(n+1)/2$ and the sum of squares is $n(n+1)(2n+1)/6$. Thus

$$\sigma_n^2 = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}.$$

Set this equal to the supplied variance:

$$\frac{n^2-1}{12} = 10 \implies n^2 = 121.$$

Since n counts observations, take the positive integer root $n = 11$, rather than the negative algebraic root.

Step 2: Account for the scaling of even numbers The first m even natural numbers are $2, 4, \dots, 2m$, obtained by doubling $1, 2, \dots, m$. Their deviations from their mean are also doubled, so their squared deviations and variance are multiplied by four. Therefore

$$\sigma_{\text{even}}^2 = 4 \frac{m^2-1}{12} = \frac{m^2-1}{3}.$$

Using the given value yields

$$\frac{m^2-1}{3} = 16 \implies m^2 = 49 \implies m = 7.$$

Finally, $m+n = 7+11 = 18$. These are population variances of the listed finite sets, so no sample-variance denominator $N-1$ is appropriate.

Step 3: Conclusion The required numerical value is $m+n = 18$, a dimensionless count.

Quick Tip to Solve: When every data value is multiplied by two, multiply its variance by four, not by two.

24. Let S be the set of points where the function, $f(x) = |2-|x-3||$, $x \in \mathbb{R}$ is not differentiable. Then $\sum_{x \in S} f(f(x))$ is equal to ____. [4 marks]

Solution:

Concept:

- An absolute value can create a corner where its argument vanishes. For nested absolute values, check both the inner corner and the zeros of the outer argument.
- A piecewise linear formula makes differentiability explicit: unequal slopes on the two sides of a junction imply failure of differentiability there.

Step 1: Locate and verify every corner The inner expression $|x-3|$ has its junction at $x = 3$. The

outer argument vanishes when $|x - 3| = 2$, giving $x = 1, 5$. Resolving both absolute values gives

$$f(x) = \begin{cases} 1 - x, & x < 1, \\ x - 1, & 1 \leq x < 3, \\ 5 - x, & 3 \leq x < 5, \\ x - 5, & x \geq 5. \end{cases}$$

The consecutive slopes are $-1, +1, -1, +1$. They differ across each of $1, 3, 5$, so all three are genuine nondifferentiable points. On every other point the function agrees locally with a linear function and is differentiable. Hence $S = \{1, 3, 5\}$.

Step 2: Evaluate the composition at those points First evaluate the function once at each member of S :

$$f(1) = 0, \quad f(3) = 2, \quad f(5) = 0.$$

The question asks for $f(f(x))$, so these outputs must be inserted back into the same function. In particular,

$$f(0) = |2 - |-3|| = |2 - 3| = 1, \quad f(2) = |2 - |-1|| = |2 - 1| = 1.$$

Therefore

$$\sum_{x \in S} f(f(x)) = f(0) + f(2) + f(0) = 1 + 1 + 1 = 3.$$

The repeated output zero still contributes twice, because the summation is over the three distinct input points in S , rather than over distinct intermediate outputs.

Step 3: Conclusion The required sum is 3, a dimensionless number.

Quick Tip to Solve: List corners from both absolute values, verify their slopes, then keep repeated intermediate outputs when summing a composition.

25. If the sum of the coefficients of all even powers of x in the product

$(1 + x + x^2 + \dots + x^{2n})(1 - x + x^2 - x^3 + \dots + x^{2n})$ is 61, then n is equal to _____.

[4 marks]

Solution:

Concept:

- For a polynomial $P(x) = \sum a_r x^r$, the sum of its even-power coefficients is $[P(1) + P(-1)]/2$. Adding these values cancels every odd-power term.
- The two factors here have $2n + 1$ terms. The all-positive factor has value $2n + 1$ at 1 and value 1 at -1 ; the alternating factor interchanges these two values.

Step 1: Define the product polynomial Write $A(x) = 1 + x + x^2 + \dots + x^{2n}$ and $P(x) = A(x)A(-x)$. Replacing x by $-x$ only exchanges the factors, so $P(-x) = P(x)$. Thus the product is even: its odd-power coefficients vanish. This symmetry also explains why the sum of even-power coefficients can be obtained directly by evaluation at 1, without expanding the product.

Step 2: Evaluate at the two useful points At $x = 1$, the positive sum counts $2n + 1$ ones. The

alternating sum starts and ends with +1, and its other terms cancel in pairs. Hence

$$A(1) = 2n + 1, \quad A(-1) = 1.$$

Consequently,

$$P(1) = (2n + 1) \cdot 1 = 2n + 1, \quad P(-1) = 1 \cdot (2n + 1) = 2n + 1.$$

The parity-extraction formula gives $S_{\text{even}} = (P(1) + P(-1))/2 = 2n + 1$.

Step 3: Use the specified coefficient sum The stated sum is 61, so

$$2n + 1 = 61 \implies 2n = 60 \implies n = 30.$$

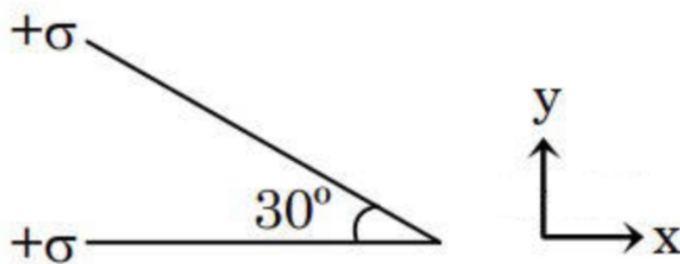
As a check, the two factors then end at x^{60} , each contains 61 terms, and the alternating evaluation is still 1. The result concerns a sum of coefficients, not the number of even powers in the degree-120 product.

Step 4: Conclusion The required dimensionless integer is $n = 30$.

Quick Tip to Solve: Use values at 1 and -1 to separate even and odd coefficients; never expand a long product merely to add coefficients.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Two infinite planes each with uniform surface charge density σ are kept in such a way that the angle between them is 30° . The electric field in the region shown between them is given by : [4 marks]



(A) $\frac{\sigma}{\epsilon_0} \left[\left(1 + \frac{\sqrt{3}}{2}\right) \hat{y} + \frac{\hat{x}}{2} \right]$

(B) $\frac{\sigma}{2\epsilon_0} \left[(1 + \sqrt{3}) \hat{y} + \frac{\hat{x}}{2} \right]$

(C) $\frac{\sigma}{2\epsilon_0} \left[(1 + \sqrt{3}) \hat{y} - \frac{\hat{x}}{2} \right]$

(D) $\frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2}\right) \hat{y} - \frac{\hat{x}}{2} \right]$

Correct Answer: (D) $\frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2}\right) \hat{y} - \frac{\hat{x}}{2} \right]$

Solution:

Concept:

- An infinite uniformly charged nonconducting plane produces field $E_0 = \sigma/(2\epsilon_0)$ on either side, normal to the plane. For positive charge the field points away from the plane.
- The resultant electric field is the vector sum of the two fields. Resolve both normals using the positive x and y axes in the figure; the location inside the wedge determines each sign.

Step 1: Determine each outward normal The observation region lies above the lower horizontal positively charged sheet. Its contribution is therefore $\vec{E}_1 = E_0 \hat{y}$. The other sheet slopes down towards the vertex at an angle of 30° below its leftward direction. The region between the sheets lies on its lower-left side, so its outward normal points down and left. It is not directed up and right: that normal applies to the opposite side of that sheet.

Step 2: Resolve and add the components The upper-sheet normal makes 30° with the downward vertical. Its contribution is

$$\vec{E}_2 = -E_0 \sin 30^\circ \hat{x} - E_0 \cos 30^\circ \hat{y}.$$

Adding $\vec{E}_1$ and using $\sin 30^\circ = 1/2$, $\cos 30^\circ = \sqrt{3}/2$, we obtain

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \left[-\frac{\hat{x}}{2} + \left(1 - \frac{\sqrt{3}}{2}\right) \hat{y} \right].$$

The small positive y component is the difference of two upward/downward contributions, whereas the x component is negative.

Step 3: Check the listed expressions Option (D) has exactly these components and the correct single-sheet factor. Options (A) and (B) incorrectly give positive x components. Option (C) has the correct negative x sign but adds the magnitudes of the vertical fields instead of subtracting them; its coefficient $1 + \sqrt{3}$ also fails the required projection. These sign checks are essential because the fields are vectors, even though the two surface charge densities are equal.

Step 4: Conclusion

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2} \right) \hat{y} - \frac{\hat{x}}{2} \right]$$

The field is measured in N C^{-1} . Hence, the correct option is (D).

Quick Tip to Solve: Draw the outward normal from each positive sheet towards the selected region before resolving its components.

27. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.40$, the work done by air is : ($3^{1.4} = 4.6555$) [Take air to be an ideal gas] [4 marks]

- (A) 60.7 J
- (B) 100.8 J
- (C) 90.5 J
- (D) 48 J

Correct Answer: (C) 90.5 J

Solution:

Concept:

- For a reversible adiabatic change of an ideal gas, PV^γ remains constant. The work done by the gas is $W = (P_1V_1 - P_2V_2)/(\gamma - 1)$.
- At STP use $P_1 = 1.013 \times 10^5 \text{ Pa}$. Convert litres into cubic metres so that pressure times volume gives joules; an expansion must give positive work.

Step 1: Relate the final pressure-volume product The initial and final volumes are $V_1 = 10^{-3} \text{ m}^3$ and $V_2 = 3 \times 10^{-3} \text{ m}^3$. From the adiabatic law,

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{1.4} = \frac{P_1}{3^{1.4}}.$$

Multiplying by $V_2 = 3V_1$ gives

$$P_2V_2 = P_1V_1 \frac{3}{3^{1.4}} = P_1V_1 \frac{3}{4.6555}.$$

This is smaller than P_1V_1 , as the gas cools during an adiabatic expansion.

Step 2: Calculate the work in joules The initial product is $P_1V_1 = (1.013 \times 10^5)(10^{-3}) = 101.3 \text{ J}$.

Therefore

$$W = \frac{101.3}{0.40} \left(1 - \frac{3}{4.6555} \right).$$

Since $3/4.6555 \simeq 0.6444$, this becomes

$$W \simeq 253.25(0.3556) = 90.1 \text{ J}.$$

The closest listed value is 90.5 J. The small difference from the calculation using standard atmospheric pressure is much smaller than the separation between the options, so it does not create a competing choice.

Step 3: Distinguish the available choices The work is close to 90 J, so option (C) is the only consistent choice. The values 60.7 J and 48 J are too small and 100.8 J is too large. In particular, neither the initial PV product itself nor the constant-pressure expression $P_1(V_2 - V_1)$ is the adiabatic work, because the pressure decreases continuously during the expansion. The small rounding difference between the computed value and 90.5 J does not alter the selected option.

Step 4: Conclusion The work done by air is approximately 90.5 J among the given choices. Hence, the correct option is (C).

Quick Tip to Solve: Compute the final PV product as the initial PV product times the volume ratio raised to 1 minus gamma.

28. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16} s. The frequency of revolution of the electron in its first excited state (in s^{-1}) is : [4 marks]

- (A) 5.6×10^{12}
- (B) 1.6×10^{14}
- (C) 7.8×10^{14}
- (D) 6.2×10^{15}

Correct Answer: (C) 7.8×10^{14}

Solution:

Concept:

- In the Bohr model for a fixed hydrogen atom, orbital radius varies as $r_n \propto n^2$ and speed as $v_n \propto 1/n$.
- The orbital period is $T_n = 2\pi r_n / v_n$, hence $T_n \propto n^3$; the revolution frequency is its reciprocal and varies as n^{-3} .

Step 1: Identify the correct excited orbit The ground state has principal quantum number $n = 1$. The first excited state has $n = 2$, rather than $n = 1$. Combining the radius and velocity dependences gives

$$\frac{T_2}{T_1} = \frac{r_2}{r_1} \frac{v_1}{v_2} = 2^2 \times 2 = 8.$$

The electron travels a circumference four times larger at half the speed; therefore its period increases

eightfold. This concerns revolution of the electron, not the frequency of a photon emitted during a transition.

Step 2: Find the new period and its reciprocal Using the given ground-state period,

$$T_2 = 8(1.6 \times 10^{-16}) = 1.28 \times 10^{-15} \text{ s.}$$

The frequency is therefore

$$f_2 = \frac{1}{T_2} = \frac{1}{1.28 \times 10^{-15}} = 7.8125 \times 10^{14} \text{ s}^{-1}.$$

Rounding to the two significant figures used in the data gives $7.8 \times 10^{14} \text{ s}^{-1}$. As a separate check, the ground-state frequency is $6.25 \times 10^{15} \text{ s}^{-1}$, and dividing it by eight gives the same result.

Step 3: Compare all alternatives Option (C) agrees with the reciprocal-period calculation. Option (D), about $6.2 \times 10^{15} \text{ s}^{-1}$, is essentially the ground-state frequency and neglects the change of orbit. Option (A) is two orders of magnitude too small, and option (B) does not follow the required factor of eight. A scaling based only on the radius would also be incorrect because the electron speed changes between Bohr orbits.

Step 4: Conclusion The frequency in the first excited state is $7.8 \times 10^{14} \text{ s}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: For Bohr orbits remember radius, speed and period scale as n^2 , $1/n$ and n^3 respectively.

29. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be: [4 marks]

- (A) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$
- (B) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$
- (C) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$
- (D) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

Correct Answer: (C) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

Solution:

Concept:

- For a freely oscillating series LCR circuit, Kirchhoff's voltage law gives $L\ddot{q} + R\dot{q} + q/C = 0$, where charge q is the electrical displacement variable and $\dot{q}$ is current.
- A damped spring-mass oscillator satisfies $m\ddot{x} + b\dot{x} + kx = 0$. Match coefficients of the second derivative, first derivative and displacement in these equations.

Step 1: Write the electrical equation Let $q(t)$ be the instantaneous capacitor charge. The inductor voltage is $L dI/dt = L\dot{q}$, the resistor voltage is $RI = R\dot{q}$, and the capacitor voltage is q/C . With no

external driving source the algebraic voltage sum vanishes:

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = 0.$$

Inductance resists a change of current, resistance dissipates electrical energy, and the charged capacitor supplies the restoring effect. These are the roles to compare with inertia, damping and spring force.

Step 2: Match each term with the mechanical equation For displacement x of the mass, the inertial, damping and restoring terms give

$$m\ddot{x} + b\dot{x} + kx = 0.$$

Under the correspondence $q \leftrightarrow x$, matching derivative order yields

$$L \leftrightarrow m, \quad R \leftrightarrow b, \quad \frac{1}{C} \leftrightarrow k.$$

The last relation is equivalently $C \leftrightarrow 1/k$, which is the compliance of the spring. The reciprocal matters: capacitance itself does not correspond to stiffness.

Step 3: Check the alternatives using frequency and damping The natural frequencies satisfy $\omega_{0,\text{elec}}^2 = 1/(LC)$ and $\omega_{0,\text{mech}}^2 = k/m$, agreeing with the derived mapping. Option (C) preserves both the restoring and damping terms. Option (B) gets inertia and damping right but incorrectly matches C directly to k. Option (A) exchanges the physical roles of inertia, stiffness and damping. Option (D) uses reciprocal mass, damping and stiffness in the wrong positions and cannot reproduce either differential equation.

Step 4: Conclusion The correspondence is $L \leftrightarrow m, \quad C \leftrightarrow 1/k, \quad R \leftrightarrow b$. Hence, the correct option is (C).

Quick Tip to Solve: Match coefficients in the differential equations; capacitance corresponds to spring compliance, the reciprocal of stiffness.

30. If the magnetic field in a plane electromagnetic wave is given by

$\vec{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{j} \text{ T}$, then what will be expression for electric field ?

[4 marks]

- (A) $\vec{E} = (9\sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} \text{ V/m})$
 (B) $\vec{E} = (60\sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} \text{ V/m})$
 (C) $\vec{E} = (3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{i} \text{ V/m})$
 (D) $\vec{E} = (3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{j} \text{ V/m})$

Correct Answer: (A) $\vec{E} = (9\sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} \text{ V/m})$

Solution:

Concept:

- For a plane wave, electric and magnetic fields are perpendicular, in phase, and satisfy $E_0 = vB_0$, where v is the wave speed.
- The direction of energy propagation is along $\vec{E} \times \vec{B}$. A phase $kx + \omega t$ travels in the negative x

direction, with speed $v = \omega/k$.

Step 1: Find the propagation speed and electric amplitude The angular frequency and wave number are $\omega = 48 \times 10^{10} \text{ s}^{-1}$ and $k = 1.6 \times 10^3 \text{ m}^{-1}$. Therefore

$$v = \frac{\omega}{k} = \frac{48 \times 10^{10}}{1.6 \times 10^3} = 3 \times 10^8 \text{ m s}^{-1}.$$

Using the given magnetic amplitude $B_0 = 3 \times 10^{-8} \text{ T}$,

$$E_0 = vB_0 = (3 \times 10^8)(3 \times 10^{-8}) = 9 \text{ V m}^{-1}.$$

This is the usual electromagnetic wave speed, and the electric amplitude is not numerically equal to the magnetic amplitude.

Step 2: Determine the electric-field direction Keeping the phase constant gives $kx + \omega t = \text{constant}$, or $dx/dt = -\omega/k$. Thus the wave propagates along $-\hat{i}$. The magnetic field points along $\hat{j}$ for a positive sine factor. We require $\vec{E} \times \vec{B}$ along $-\hat{i}$. Since $\hat{k} \times \hat{j} = -\hat{i}$, the electric field has the positive $\hat{k}$ direction with the same sine phase as B.

Step 3: Construct the wave and exclude other options Combining the amplitude, direction and phase gives

$$\vec{E} = 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} \text{ V m}^{-1}.$$

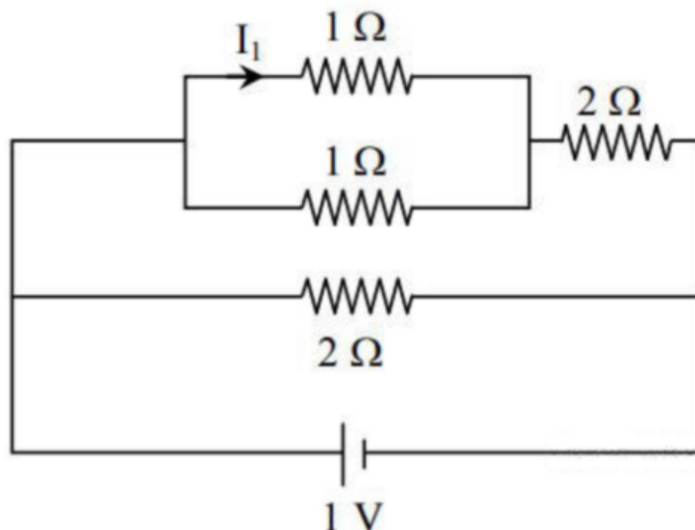
Option (A) has all three correct. Option (B) has the correct direction but its amplitude 60 is inconsistent with $E_0 = vB_0$. Option (C) points along the propagation direction instead of perpendicular to it and also has the wrong amplitude. Option (D) is parallel to the magnetic field, violating transversality and giving zero cross product.

Step 4: Conclusion The electric field is $\boxed{\vec{E} = 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} \text{ V m}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the phase sign to find propagation direction, then choose E so that E cross B points along propagation.

31. The current I_1 (in A) flowing through $1\ \Omega$ resistor in the following circuit is :

[4 marks]



- (A) 0.4
- (B) 0.25
- (C) 0.2
- (D) 0.5

Correct Answer: (C) 0.2

Solution:

Concept:

- Resistors connected across the same two nodes have the same potential difference. The ideal battery fixes the left-to-right node voltage at 1 V.
- The two $1\ \Omega$ resistors form a parallel combination, and this combination is in series with the upper $2\ \Omega$ resistor. Ohm's law is $I = V/R$.

Step 1: Reduce the upper branch The two equal resistors connect between the left node and the intermediate upper node, so their equivalent resistance is

$$R_p = \frac{1 \times 1}{1 + 1} = \frac{1}{2}\ \Omega.$$

The upper branch therefore has resistance

$$R_u = R_p + 2 = \frac{5}{2}\ \Omega.$$

The separate middle $2\ \Omega$ resistor is across the battery, not in series with this branch. Consequently it must not be added to R_u .

Step 2: Divide the upper-branch current The battery applies its full voltage to the upper branch. Its current is

$$I_u = \frac{1}{5/2} = 0.4\ \text{A}.$$

This current reaches the parallel pair and divides equally because its resistances are identical. Thus

$$I_1 = \frac{I_u}{2} = 0.2 \text{ A.}$$

As a voltage check, the parallel pair has drop $0.4 \times 0.5 = 0.2 \text{ V}$, giving $0.2/1 = 0.2 \text{ A}$ through each resistor. Option (A) is the entire upper-branch current; option (D) is the middle-branch current $1/2$. Option (B) follows neither branch resistance and does not satisfy the common voltage condition.

Step 3: Conclusion The current in the indicated direction is $I_1 = 0.2 \text{ A}$. Hence, the correct option is (C).

Quick Tip to Solve: Identify the battery terminals as two fixed-voltage nodes, reduce only the upper branch, and halve its current at the equal parallel pair.

32. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) \ll 1$, the total capacitance of the system is best given by the expression : [4 marks]



(A) $\frac{A\epsilon_0 K}{d} \left(1 + \left(\frac{\alpha d}{2}\right)^2\right)$

(B) $\frac{A\epsilon_0 K}{d} \left(1 + \frac{\alpha d}{2}\right)$

(C) $\frac{A\epsilon_0 K}{d} \left(1 + \frac{\alpha^2 d^2}{2}\right)$

(D) $\frac{A\epsilon_0 K}{d} (1 + \alpha d)$

Correct Answer: (B) $\frac{A\epsilon_0 K}{d} \left(1 + \frac{\alpha d}{2}\right)$

Solution:**Concept:**

- A dielectric varying along the plate separation behaves as infinitesimal capacitors in series. Each layer has capacitance $dC = \epsilon_0 k(x) A / dx$.
- For series layers, reciprocal capacitances add: $1/C = \int_0^d dx / [\epsilon_0 A k(x)]$. Expand consistently to first order in the small dimensionless parameter αd .

Step 1: Integrate the reciprocal capacitance Choose a layer of thickness dx , parallel to the plates. Every layer carries the same free charge, while their potential drops add. With $k(x) = K(1 + \alpha x)$,

$$\frac{1}{C} = \frac{1}{\epsilon_0 AK} \int_0^d \frac{dx}{1 + \alpha x} = \frac{\ln(1 + \alpha d)}{\epsilon_0 AK \alpha}.$$

Therefore the exact expression, with its continuous uniform-dielectric limit, is

$$C = \frac{\epsilon_0 AK \alpha}{\ln(1 + \alpha d)}.$$

Step 2: Expand for the small variation Writing $z = \alpha d$, use $\ln(1 + z) = z - z^2/2 + O(z^3)$. The factor z cancels before the reciprocal is expanded:

$$C = \frac{\epsilon_0 AK}{d} \frac{1}{1 - z/2 + O(z^2)} = \frac{\epsilon_0 AK}{d} \left[1 + \frac{z}{2} + O(z^2) \right].$$

Options (A) and (C) omit the nonzero linear correction. Option (D) has twice the required linear coefficient: the dielectric varies over the separation, so its end value cannot be used throughout. The result reduces to $\epsilon_0 AK/d$ as α tends to zero.

Step 3: Conclusion To first order, $C \simeq \frac{A\epsilon_0 K}{d} (1 + \alpha d/2)$. Hence, the correct option is (B).

Quick Tip to Solve: When the dielectric varies normal to the plates, integrate reciprocal capacitance first; cancel the small parameter before expanding.

33. Which of the following gives a reversible operation?

[4 marks]

(A)



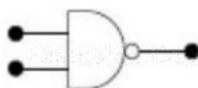
(B)



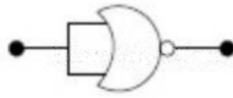
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- A logic operation is reversible when its output uniquely determines its input. For a single binary input, either the identity or the NOT operation is one-to-one.
- A two-input gate with one output generally loses information: four input pairs are compressed into just two possible output states.

Step 1: Read the gate symbols Option (A) is a two-input NOR gate: the OR-shaped body has a small inversion circle at the output. Option (C) is an ordinary two-input OR gate. Option (D) is a two-input NAND gate, recognised by the AND-shaped body and the output inversion circle. In option (B), a single input is split and joined to both inputs of the NOR gate. These are not two independent input bits; both carry the same bit X . The inversion circle changes the common-input OR result into its complement.

Step 2: Test whether the input can be recovered For option (B), Boolean idempotence gives

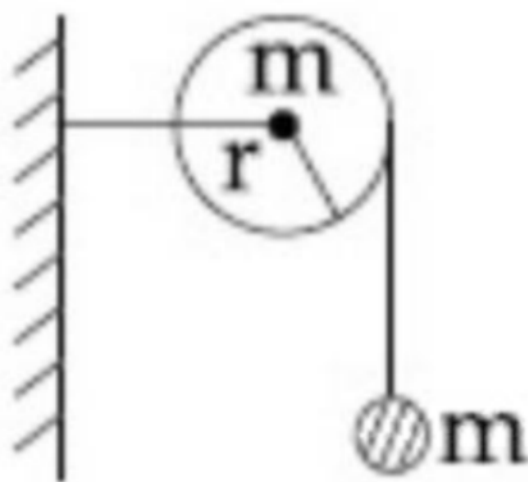
$$Y = \overline{X + X} = \overline{X}.$$

Its complete truth table is $X = 0 \Rightarrow Y = 1$ and $X = 1 \Rightarrow Y = 0$. Thus knowing Y uniquely gives $X = \overline{Y}$, and applying the operation twice restores the original bit. The other gates fail this test: NOR gives output zero for inputs $(0, 1)$, $(1, 0)$, $(1, 1)$; OR gives output one for those same three pairs; NAND gives output one for $(0, 0)$, $(0, 1)$, $(1, 0)$. Their outputs cannot distinguish the original input pairs. Logical reversibility here concerns recovery of the bit, rather than a claim about the energy consumption of a physical gate.

Step 3: Conclusion The tied-input NOR gate acts as a NOT gate and is reversible. Hence, the correct option is (B).

Quick Tip to Solve: Count independent inputs, not input terminals: tying both NOR inputs together converts the gate into a reversible one-bit inverter.

34. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m . When released from rest the bob starts falling vertically. When it has covered a distance of h , the angular speed of the wheel will be: [4 marks]



(A) $r\sqrt{\frac{3}{2gh}}$

(B) $r\sqrt{\frac{3}{4gh}}$

(C) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

(D) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

Correct Answer: (C) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

Solution:

Concept:

- The gravitational potential energy lost by the bob becomes its translational kinetic energy plus the rotational kinetic energy of the disc.
- For a uniform disc about its fixed central axis, $I = mr^2/2$. A taut string unwinding without slipping satisfies $v = r\omega$.

Step 1: Write the mechanical-energy balance The bob and the wheel each have the mass m given in the question. The wheel centre is fixed, so the wheel has no translational kinetic energy. Starting from rest and neglecting axle friction, the loss of gravitational potential energy after a fall h is mgh . Tension transfers energy from the bob to the wheel and is internal to the chosen system. Thus

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

The fixed support does no work because its point of application does not move.

Step 2: Apply the unwinding constraint Substitute the disc inertia and the no-slip relation:

$$mgh = \frac{1}{2}m(r\omega)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = \frac{3}{4}mr^2\omega^2.$$

Cancelling the common mass and rearranging gives

$$\omega^2 = \frac{4gh}{3r^2}, \quad \omega = \frac{1}{r}\sqrt{\frac{4gh}{3}}.$$

The positive root gives angular speed. Options (A) and (B) have dimensions of time rather than inverse time, so cannot be angular speeds. Option (D) underestimates the available kinetic energy by a factor of two in ω^2 . As another check, the bob has $v^2 = 4gh/3 < 2gh$, as expected because some energy rotates the wheel.

Step 3: Conclusion The wheel acquires angular speed $\omega = \frac{1}{r}\sqrt{4gh/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Include both kinetic energies and substitute $v = r\omega$; the fixed wheel has rotational energy but no translational energy.

35. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth) :

[4 marks]

- (A) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$
 (B) $\frac{m}{20}\left(u^2 + \frac{113}{100}\frac{GM}{R}\right)$
 (C) $5m\left(u^2 - \frac{119}{100}\frac{GM}{R}\right)$
 (D) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

Correct Answer: For the literal printed question, the rocket kinetic energy is $5m(u^2 - 119GM/(200R))$, in joules for SI data. None of the printed options equals this result. The supplied key selects (C), but its printed denominator 100 contradicts conservation of energy and vector momentum; a denominator 200 would give the derived result.

Solution:

Concept:

- Outside a spherical Earth, the gravitational potential is $-GM/r$ per unit mass. At height R above the surface the distance from the centre is $2R$.
- During a sudden ejection, vector momentum is conserved. A circular orbit at radius $2R$ requires a tangential velocity of magnitude $v_c = \sqrt{GM/(2R)}$, perpendicular to the initial radial velocity.

Step 1: Find the speed before separation Let the upward radial speed immediately before ejection be

v. Conservation of mechanical energy during the rise gives

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{2R}, \quad v^2 = u^2 - \frac{GM}{R}.$$

This uses height R , not a geocentric radius R . The remaining satellite mass is $9m/10$, while the ejected rocket mass is $m/10$. The satellite must have zero radial velocity after ejection to follow a circular orbit.

Step 2: Conserve both momentum components Take the radial direction and the final satellite tangent as perpendicular coordinate axes. If the rocket velocity components are V_r, V_t , momentum conservation yields

$$mv = \frac{m}{10}V_r, \quad 0 = \frac{9m}{10}v_c + \frac{m}{10}V_t.$$

Hence $V_r = 10v$ and $V_t = -9v_c$, so

$$\begin{aligned} K_r &= \frac{m}{20}(100v^2 + 81v_c^2) \\ &= \frac{m}{20} \left[100u^2 - \frac{100GM}{R} + \frac{81GM}{2R} \right] \\ &= 5m \left(u^2 - \frac{119GM}{200R} \right). \end{aligned}$$

The velocities must be added as perpendicular components, not as scalar speeds. Options (A) and (D) introduce inappropriate cross terms in u ; (B) has the wrong coefficient of u^2 . The listed option (C) has the right coefficient of u^2 , but prints 119/100 rather than the required 119/200.

Step 3: Conclusion For the literal printed question, the rocket kinetic energy is $5m(u^2 - 119GM/(200R))$, in joules for SI data. None of the printed options equals this result. The supplied key selects (C), but its printed denominator 100 contradicts conservation of energy and vector momentum; a denominator 200 would give the derived result.

Quick Tip to Solve: At height R , use orbital radius $2R$, then conserve radial and tangential momentum separately during ejection.

36. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to : [4 marks]

- (A) 22 mm
- (B) 12 mm
- (C) 33 mm
- (D) 2 mm

Correct Answer: (A) 22 mm

Solution:

Concept:

- The magnification magnitude of a compound microscope is the product of objective magnification and eyepiece angular magnification.
- With tube length L , objective focal length f_o , and least distance of distinct vision $D = 250$ mm, use $M \simeq (L/f_o)(1 + D/f_e)$ for a final image at the near point.

Step 1: Determine the objective contribution The supplied tube length and objective focal length are $L = 150$ mm and $f_o = 5$ mm. In the usual long-tube approximation, the objective gives magnification magnitude

$$M_o \simeq \frac{L}{f_o} = \frac{150}{5} = 30.$$

The objective image is inverted, but the question asks for the magnitude of magnification, so the negative image-orientation sign is not needed. The required eyepiece magnification is therefore

$$M_e = \frac{M}{M_o} = \frac{375}{30} = 12.5.$$

Step 2: Find and compare the eyepiece focal length For maximum angular magnification with the final image at the near point,

$$12.5 = 1 + \frac{250}{f_e}, \quad f_e = \frac{250}{11.5} \text{ mm} \simeq 21.74 \text{ mm}.$$

This is closest to 22 mm. Even if normal adjustment is used, $M_e = D/f_e$ gives $f_e = 20$ mm, whose nearest listed value is again 22 mm. Thus the choice is robust to the customary viewing convention implicit in the word "close". Values 12 mm and 2 mm give much larger eyepiece powers, whereas 33 mm gives too little magnification. Keep all lengths in the same unit before taking these ratios.

Step 3: Conclusion The required eyepiece focal length is close to 22 mm. Hence, the correct option is (A).

Quick Tip to Solve: Divide total magnification by L/f_o first; use the resulting eyepiece power to obtain f_e with consistent length units.

37. Two moles of an ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of $\frac{C_P}{C_V}$ for the mixture is : [4 marks]

(A) 1.50

(B) 1.45

(C) 1.47

(D) 1.42

Correct Answer: (D) 1.42

Solution:

Concept:

- For an ideal gas, the molar heat capacities obey Mayer's relation $C_P - C_V = R$. Therefore, if $\gamma = C_P/C_V$, then $C_V = R/(\gamma - 1)$ and $C_P = \gamma R/(\gamma - 1)$.
- Heat capacities are additive for a non-reacting ideal-gas mixture. Its ratio is $\gamma_m = (n_1 C_{P1} + n_2 C_{P2})/(n_1 C_{V1} + n_2 C_{V2})$; one must add the capacities before taking their ratio.

Step 1: Find the capacities of each component For the first gas, $\gamma_1 = 5/3$, so $C_{V1} = 3R/2$ and $C_{P1} = 5R/2$. For the second gas, $\gamma_2 = 4/3$, so $C_{V2} = 3R$ and $C_{P2} = 4R$. These are molar capacities, so the two mole numbers have not yet been included. The second gas has a larger constant-volume capacity per mole and hence contributes more strongly to the denominator.

Step 2: Add the capacities and calculate the ratio The total constant-volume heat capacity of the sample is

$$C_{V,\text{tot}} = 2 \left(\frac{3R}{2} \right) + 3(3R) = 12R.$$

Its total constant-pressure heat capacity is

$$C_{P,\text{tot}} = 2 \left(\frac{5R}{2} \right) + 3(4R) = 17R.$$

Thus $\gamma_m = 17/12 = 1.4167$, which rounds to 1.42. The difference $17R - 12R = 5R$ also correctly equals the total number of moles times R . The choices 1.50, 1.45 and 1.47 do not equal the ratio of these additive capacities; directly averaging the two gamma values is not the mixture rule.

Step 3: Conclusion The ratio for the mixture is $C_P/C_V \simeq 1.42$, a dimensionless number. Hence, the correct option is (D).

Quick Tip to Solve: Convert each gamma into $C_V/R = 1/(\gamma - 1)$, multiply by its mole number, and use $\gamma_m = 1 + n_{\text{tot}}R/C_{V,\text{tot}}$.

38. Consider a circular coil of wire carrying constant current I , forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i . The magnetic flux through the area of the circular coil is given by ϕ_0 . Which of the following option is correct? [4 marks]

- (A) $\phi_i = \phi_0$
- (B) $\phi_i < \phi_0$
- (C) $\phi_i > \phi_0$
- (D) $\phi_i = -\phi_0$

Correct Answer: (D) $\phi_i = -\phi_0$

Solution:

Concept:

- Gauss's law for magnetism states $\oint \mathbf{B} \cdot d\mathbf{A} = 0$: magnetic field lines have no isolated starting or ending points. The fluxes must be treated as signed quantities relative to one chosen normal.
- The field of a localized current loop falls as the inverse cube of distance far away. Thus the magnetic flux through a hemisphere at infinite radius vanishes, allowing the infinite plane to be used as the remaining boundary.

Step 1: Close the plane with a distant hemisphere Choose one normal for the entire plane containing the coil. Temporarily cap a large disk in that plane with a hemisphere to form a closed surface enclosing one half-space. Gauss's law gives the plane flux plus the curved-surface flux as zero. At large radius r , the loop field is of order r^{-3} , whereas the hemispherical area is of order r^2 . Its flux consequently tends to zero as r^{-1} . The net flux through the infinite plane is therefore zero.

Step 2: Separate the inside and outside regions The infinite plane is the union of the circular area bounded by the wire and the region outside it. With the same normal in both regions, their fluxes add:

$$\phi_{\text{plane}} = \phi_0 + \phi_i = 0.$$

Hence $\phi_i = -\phi_0$. Physically, field passing through the loop returns through the exterior region with the opposite normal component; the return flux has equal magnitude. Option (A) misses this sign. Options (B) and (C) propose inequalities instead of the exact signed equality required by the zero net flux. Comparing only magnitudes would conceal the direction information tested here.

Step 3: Conclusion The exterior flux is equal in magnitude and opposite in sign to the interior flux: $\phi_i = -\phi_0$. Hence, the correct option is (D).

Quick Tip to Solve: Use one common normal for both parts of the plane. Magnetic flux leaving through the loop must return through its exterior, so the signed sum is zero.

39. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is :

[4 marks]

- (A) 71.6°
- (B) 90°
- (C) 18.4°
- (D) 45°

Correct Answer: (C) 18.4°

Solution:

Concept:

- Malus's law for the analyser is $I = I_{\max} \cos^2 \theta$, where θ is the angle between the transmission axes and $I_{\max}$ is the aligned-axis intensity.
- Under the stated no-absorption convention, the original reference intensity is $I_{\max}$. Extinction occurs when the transmission axes are perpendicular, at $\theta = 90^\circ$. The question asks for an additional rotation, not the present separation.

Step 1: Determine the present axis separation Write the given output as $I = 0.10I_{\max}$. Then

$$\cos^2 \theta = 0.10, \quad \cos \theta = \sqrt{0.10} = 0.31623.$$

On the branch approaching the first crossed position, $\theta = \cos^{-1}(0.31623) \simeq 71.6^\circ$. The axes are already nearly perpendicular because only a small fraction of the reference intensity is transmitted. There is no extra absorption factor to introduce under the convention specified in the question.

Step 2: Find the remaining rotation to extinction Zero output requires $\cos^2 \theta_{\text{final}} = 0$, so the first extinction position is 90° . Rotating farther toward it gives

$$\Delta\theta = 90^\circ - 71.6^\circ = 18.4^\circ.$$

Equivalently, $\sin \Delta\theta = \sqrt{0.10}$. The value 71.6 degrees in option (A) is the existing angle, not the increment. A further 90 degrees would rotate past the nearby crossed position; 45 degrees also fails to make the final axes perpendicular. These checks distinguish the requested rotation from the angle calculated directly using Malus's law.

Step 3: Conclusion The analyser must be rotated a further 18.4° toward the crossed position. Hence, the correct option is (C).

Quick Tip to Solve: For a transmitted fraction f relative to aligned axes, the remaining rotation to extinction is $\sin^{-1} \sqrt{f}$, not $\cos^{-1} \sqrt{f}$.

40. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to :

(1 HP = 746 W, $g = 10 \text{ ms}^{-2}$)

[4 marks]

(A) 1.5 ms^{-1}

(B) 1.7 ms^{-1}

(C) 2.0 ms^{-1}

(D) 1.9 ms^{-1}

Correct Answer: (D) 1.9 ms^{-1}

Solution:

Concept:

- At steady lifting speed, the motor must balance both the load's weight and the opposing friction. The upward driving force is $F = mg + f$, where m is the total lifted mass.
- Mechanical power for force parallel to velocity is $P = Fv$. Convert the motor rating to watts before dividing by force so the resulting speed is in ms^{-1} .

Step 1: Calculate the power and resisting force The given conversion yields

$$P = 60 \times 746 = 44760 \text{ W}.$$

The weight of the elevator at its maximum total load is

$$mg = 2000 \times 10 = 20000 \text{ N}.$$

Since friction opposes upward motion, it adds to the weight rather than subtracting from it. Therefore,

$$F = 20000 + 4000 = 24000 \text{ N}.$$

The stated capacity is already the total mass, so no extra elevator mass is added.

Step 2: Use the steady power balance At the sustainable full-load speed there is no change in kinetic energy, and the motor power supplies gravitational potential energy and work against friction:

$$P = mgv + fv = (mg + f)v.$$

Thus

$$v = \frac{44760}{24000} = 1.865 \text{ ms}^{-1} \simeq 1.9 \text{ ms}^{-1}.$$

Among the listed values, 1.9 is closest. The values 1.5 and 1.7 are too small for the stated full motor power, while 2.0 would require $24000 \times 2.0 = 48000 \text{ W}$, exceeding the available power. Neglecting friction would incorrectly give a speed larger than the correct value.

Step 3: Conclusion The elevator speed is approximately 1.9 ms^{-1} . Hence, the correct option is (D).

Quick Tip to Solve: For steady vertical lifting with friction, write $P = (mg + f)v$. Convert horsepower to watts first, and remember that friction increases the required force.

41. A long solenoid of radius R carries a time (t) - dependent current

$I(t) = I_0 t(1 - t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time interval $0 \leq t \leq 1$, the induced current (I_R) and the induced EMF (V_R) in the ring change as : [4 marks]

- (A) Direction of I_R remains unchanged and V_R is zero at $t = 0.25$
(B) Direction of I_R remains unchanged and V_R is maximum at $t = 0.5$
(C) At $t = 0.25$ direction of I_R reverses and V_R is maximum
(D) At $t = 0.5$ direction of I_R reverses and V_R is zero

Correct Answer: (D) At $t = 0.5$ direction of I_R reverses and V_R is zero

Solution:

Concept:

- Near the middle of a long solenoid, $B = \mu_0 n I$ inside its radius R , while the external magnetic field is negligible. A larger coaxial ring links flux through area πR^2 , not its full area $4\pi R^2$.
- Faraday's law gives the induced emf $V_R = -d\Phi/dt$. For the usual resistive ring, the induced current has the sign of this emf. Lenz's law makes it oppose the change of linked flux.

Step 1: Differentiate the linked flux The solenoid current is $I(t) = I_0 t(1 - t)$. Hence the flux through the enclosing ring is

$$\Phi(t) = \mu_0 n \pi R^2 I_0 t(1 - t).$$

Defining the positive constant $K = \mu_0 n \pi R^2 I_0$, differentiation gives

$$V_R = -K(1 - 2t).$$

Thus the emf depends on the slope of the solenoid current, not on the instantaneous value of the current itself. The fact that the ring is twice as large does not alter the zero of this derivative.

Step 2: Locate the zero and establish reversal For $0 < t < 0.5$, the solenoid flux increases and $1 - 2t > 0$. For $0.5 < t < 1$, it decreases and $1 - 2t < 0$. The emf therefore changes sign at $t = 0.5$, where it vanishes. The induced current reverses across this instant, opposing the increase before it and the decrease after it. At $t = 0.25$, $V_R = -K/2$, so it is nonzero and there is no reversal. Options (A) and (B) incorrectly keep the direction unchanged; (B) also confuses maximum flux with maximum emf. Option (C) places the reversal at the wrong time.

Step 3: Conclusion At $t = 0.5$, the induced emf is zero and the induced current changes direction across that instant. Hence, the correct option is (D).

Quick Tip to Solve: Find the turning point of the source current. Maximum flux corresponds to zero induced emf, and a change from increasing to decreasing flux reverses its sign.

42. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1 , then θ_1 , is close to :

[4 marks]

- (A) 45°
- (B) 30°
- (C) 25°
- (D) 20°

Correct Answer: (C) 25°

Solution:

Concept:

- For a slit of width a at normal incidence, the dark fringes satisfy $a \sin \theta_m = m\lambda$, with $m = 1, 2, \dots$. The central maximum lies between the first minima on either side.
- Use the exact sine relation when diffraction angles are large. Taking the ratio for two minima eliminates both the slit width and the wavelength; the angles themselves are not proportional to order.

Step 1: Relate the first and second minima The second minimum occurs at $\theta_2 = 60^\circ$, so

$$a \sin 60^\circ = 2\lambda.$$

The first minimum satisfies

$$a \sin \theta_1 = \lambda.$$

Divide the second equation by the first to obtain

$$\frac{\sin \theta_1}{\sin 60^\circ} = \frac{1}{2}.$$

The stated wavelength is 6000×10^{-8} cm = 6.0×10^{-7} m, but it cancels in the ratio and need not be substituted.

Step 2: Evaluate the angle and compare choices Using $\sin 60^\circ = \sqrt{3}/2$,

$$\sin \theta_1 = \frac{\sqrt{3}}{4} \simeq 0.4330, \quad \theta_1 \simeq 25.66^\circ.$$

This is closest to 25 degrees. Option (B), 30 degrees, would follow from halving 60 degrees directly, which is not allowed at such large angles. Option (A) has sine about 0.707 and option (D) has sine about 0.342; neither satisfies the first-minimum equation. For a check, the slit width inferred from the second minimum is $a = 2\lambda / \sin 60^\circ$, and inserting the computed θ_1 gives exactly one wavelength of path difference across the slit.

Step 3: Conclusion The first minimum is at approximately 25.66° , closest to 25° . Hence, the correct option is (C).

Quick Tip to Solve: Orders scale with $\sin \theta$, not with θ . For the first and second single-slit minima, use $\sin \theta_1 = \frac{1}{2} \sin \theta_2$.

43. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1} . If the Young's modulus of wire is $16 \times 10^{11} \text{ Nm}^{-2}$, the extension of wire over its natural length is : [4 marks]

- (A) 0.03 mm
- (B) 0.04 mm
- (C) 0.02 mm
- (D) 0.01 mm

Correct Answer: (A) 0.03 mm

Solution:

Concept:

- A transverse wave on a stretched wire travels with speed $v = \sqrt{T/\mu}$, where T is the tension and $\mu = m/L$ is mass per unit length.
- Young's modulus relates axial stress to longitudinal strain: $Y = (T/A)/(\Delta L/L)$. Thus $\Delta L = TL/(AY)$.

Step 1: Convert the wire data to SI units The mass is $m = 6.0 \times 10^{-3} \text{ kg}$, length $L = 0.60 \text{ m}$, and cross-sectional area $A = 1.0 \times 10^{-6} \text{ m}^2$. The area conversion is squared, so a square millimetre is a millionth of a square metre. Hence the mass per unit length is

$$\mu = \frac{6.0 \times 10^{-3}}{0.60} = 1.0 \times 10^{-2} \text{ kg m}^{-1}.$$

Step 2: Find the tension from the wave speed Squaring the wave-speed relation gives $T = \mu v^2$. With $v = 90 \text{ m s}^{-1}$,

$$T = (1.0 \times 10^{-2})(90)^2 = 81 \text{ N}.$$

This is the axial stretching force. It is not the stress, which still requires division by the cross-sectional area.

Step 3: Use the elastic strain to obtain the extension The modulus is $Y = 16 \times 10^{11} \text{ N m}^{-2}$. Substituting the force and dimensions gives

$$\Delta L = \frac{81(0.60)}{(1.0 \times 10^{-6})(16 \times 10^{11})} = 3.0375 \times 10^{-5} \text{ m} = 0.030375 \text{ mm}.$$

To the precision of the choices this is 0.03 mm. The other extensions, 0.04 mm, 0.02 mm and 0.01 mm, would require different tensions and therefore different wave speeds; they do not satisfy both given relations.

Step 4: Conclusion The extension is approximately 0.03 mm. Hence, the correct option is (A).

Quick Tip to Solve: Combine the two relations first: $\Delta L = mv^2/(AY)$. The length cancels, providing a useful check on the substitution.

44. The radius of gyration of a uniform rod of length l , about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is : [4 marks]

- (A) $\frac{1}{8}l$
(B) $\frac{1}{4}l$
(C) $\sqrt{\frac{7}{48}}l$
(D) $\sqrt{\frac{3}{8}}l$

Correct Answer: (C) $\sqrt{\frac{7}{48}}l$

Solution:

Concept:

- The radius of gyration k is defined by $I = Mk^2$; it represents the distance at which the whole mass would give the same moment of inertia.
- For a uniform thin rod, the moment of inertia about an axis through its centre and perpendicular to the rod is $I_{\text{cm}} = Ml^2/12$. A parallel axis at distance d has $I = I_{\text{cm}} + Md^2$.

Step 1: Identify the perpendicular parallel axes The required axis is perpendicular to the rod, just like the central transverse axis. Their separation is explicitly measured from the centre, so $d = l/4$. It is not the distance from an end. The parallel-axis theorem therefore applies directly, without using an end-axis formula or changing the rod length.

Step 2: Add the translation contribution For a rod of total mass M ,

$$I = \frac{Ml^2}{12} + M \left(\frac{l}{4} \right)^2.$$

Use denominator 48 to add the two positive contributions:

$$I = Ml^2 \left(\frac{4}{48} + \frac{3}{48} \right) = \frac{7Ml^2}{48}.$$

Moving away from the centre increases the inertia, so the result must exceed $Ml^2/12$. The expression obtained satisfies that physical check.

Step 3: Convert the inertia into a gyration radius Divide by M and then take the positive square root because a radius is nonnegative:

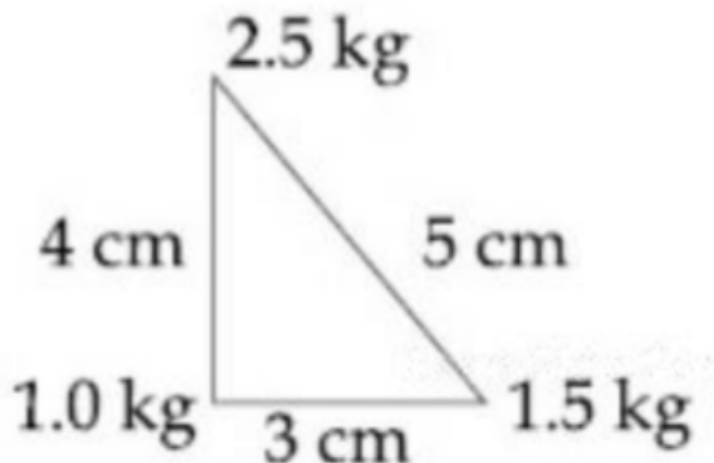
$$k^2 = \frac{7l^2}{48}, \quad k = l\sqrt{\frac{7}{48}}.$$

Options (A) and (B) have radii smaller than the central radius $l/\sqrt{12}$, which is impossible for a displaced parallel axis. Option (D) would give $I = 3Ml^2/8$, a larger translation contribution than the specified $M(l/4)^2$. Only option (C) matches the calculated inertia.

Step 4: Conclusion The radius of gyration is $k = l\sqrt{7/48}$. Hence, the correct option is (C).

Quick Tip to Solve: Apply the parallel-axis theorem to k^2 : $k^2 = k_{\text{cm}}^2 + d^2$. Add squared distances before taking a square root.

45. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point: [4 marks]



- (A) 2.0 cm right and 0.9 cm above 1 kg mass
- (B) 0.9 cm right and 2.0 cm above 1 kg mass
- (C) 0.6 cm right and 2.0 cm above 1 kg mass
- (D) 1.5 cm right and 1.2 cm above 1 kg mass

Correct Answer: (B) 0.9 cm right and 2.0 cm above 1 kg mass

Solution:

Concept:

- For discrete masses, the centre-of-mass coordinates are mass-weighted averages: $x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i}$ and $y_{\text{cm}} = \frac{\sum m_i y_i}{\sum m_i}$.
- Choose the origin at the 1.0 kg mass and use the two perpendicular sides as coordinate axes. Distances may remain in centimetres because all coordinates use the same unit.

Step 1: Assign coordinates from the figure Take positive x horizontally to the right and positive y vertically upward. The 1.0 kg mass is at (0, 0), the 1.5 kg mass at (3.0, 0) cm, and the 2.5 kg mass at (0, 4.0) cm. The 5.0 cm side is the hypotenuse; it is not an additional horizontal or vertical coordinate. The total mass is $M = 1.0 + 1.5 + 2.5 = 5.0$ kg.

Step 2: Calculate the horizontal weighted average Only the 1.5 kg mass has a nonzero horizontal coordinate, so

$$x_{\text{cm}} = \frac{(1.0)(0) + (1.5)(3.0) + (2.5)(0)}{5.0} = 0.9 \text{ cm.}$$

The coordinate is positive, meaning the centre of mass lies to the right of the chosen origin. An unweighted midpoint would ignore the unequal masses and is not applicable.

Step 3: Calculate the height and compare both coordinates Only the 2.5 kg mass contributes to the vertical first moment:

$$y_{\text{cm}} = \frac{(1.0)(0) + (1.5)(0) + (2.5)(4.0)}{5.0} = 2.0 \text{ cm.}$$

The pair (0.9, 2.0) cm lies inside the triangle, as expected for positive masses at its vertices. Option (A) interchanges the two coordinates; option (C) gives the wrong horizontal first moment; option (D) gives neither of the calculated weighted averages.

Step 4: Conclusion The centre of mass is 0.9 cm right and 2.0 cm above the 1 kg mass. Hence, the correct option is (B).

Quick Tip to Solve: Place the origin at the right-angle vertex: each of the other two masses then contributes to only one coordinate, simplifying both weighted sums.

Physics – Section B: Numerical Value Questions (20 marks)

46. A loop ABCDEFA of straight edges has six corner points A(0, 0, 0), B(5, 0, 0), C(5, 5, 0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})$ T. The quantity of flux through the loop ABCDEFA (in Wb) is _____. [4 marks]

Solution:

Concept:

- For a uniform magnetic field, flux through an oriented spanning surface is $\Phi = \vec{B} \cdot \vec{A}$, with vector area $\vec{A} = \int \hat{n} dA$.
- A nonplanar boundary may be spanned by several flat faces. Their oriented vector areas add; flux is independent of the chosen spanning surface because magnetic field has zero divergence.

Step 1: Split the boundary into two square faces Insert the common edge from D to A. One face has boundary A-B-C-D-A in the xy -plane, and the other has boundary A-D-E-F-A in the yz -plane. The artificial edge is traversed in opposite directions on the two faces and cancels, leaving exactly A-B-C-D-E-F-A. Both squares have side 5 m and area 25 m^2 .

Step 2: Determine the oriented area vectors On A-B-C-D-A, the initial edges point along $+\hat{i}$ and $+\hat{j}$, giving normal $+\hat{k}$. On A-D-E-F-A they point along $+\hat{j}$ and $+\hat{k}$, giving normal $+\hat{i}$. Thus

$$\vec{A}_{xy} = 25\hat{k} \text{ m}^2, \quad \vec{A}_{yz} = 25\hat{i} \text{ m}^2.$$

The normals are fixed by the same directed boundary, so their contributions add rather than subtract.

Step 3: Sum the magnetic fluxes The magnetic field is $\vec{B} = (3\hat{i} + 4\hat{k})$ T. Its perpendicular component on the horizontal square is 4 T, while that on the vertical square is 3 T. Therefore

$$\Phi = \vec{B} \cdot (\vec{A}_{xy} + \vec{A}_{yz}) = 4(25) + 3(25) = 100 + 75 = 175 \text{ Wb}.$$

Using the field magnitude with the sum of scalar areas would be incorrect because each face has its own normal. Reversing the entire boundary would reverse the signed flux; its requested quantity remains 175 Wb.

Step 4: Conclusion The flux through the loop is 175 Wb; the numerical entry is 175.

Quick Tip to Solve: For a bent polygonal loop in a uniform field, split it into planar faces and add $\vec{B} \cdot \vec{A}$ for each face with consistent boundary orientation.

47. A beam of electromagnetic radiation of intensity $6.4 \times 10^{-5} \text{ W/cm}^2$ is comprised of wavelength, $\lambda = 310 \text{ nm}$. It falls normally on a metal (work function $\phi = 2\text{eV}$) of surface area of 1 cm^2 . If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x . ($hc = 1240 \text{ eVnm}$, $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$), then x is _____. [4 marks]

Solution:

Concept:

- Photon energy is $E_\gamma = hc/\lambda$. Radiation of intensity I incident normally on area A for time t delivers energy IAt .
- The photon count is $N_\gamma = IAt/E_\gamma$. If only one photon in 10^3 ejects an electron, the electron count is $N_e = N_\gamma/10^3$, provided the photon energy exceeds the work function.

Step 1: Check that photoemission is energetically allowed Using the supplied product $hc = 1240 \text{ eV nm}$ with $\lambda = 310 \text{ nm}$,

$$E_\gamma = \frac{1240}{310} = 4.0 \text{ eV} = 6.4 \times 10^{-19} \text{ J}.$$

This exceeds the 2 eV work function, so emission is possible. The maximum electron kinetic energy would be 2 eV , but the photon count must use the full incident photon energy, not that kinetic energy.

Step 2: Calculate the incident photon count in one second The intensity and area are both given using square centimetres, so those area units cancel directly. There is no need to introduce a separate square-metre conversion:

$$E_{\text{incident}} = (6.4 \times 10^{-5})(1)(1) = 6.4 \times 10^{-5} \text{ J}.$$

Dividing this energy by the energy of one photon gives

$$N_\gamma = \frac{6.4 \times 10^{-5}}{6.4 \times 10^{-19}} = 10^{14}.$$

Step 3: Apply the stated emission efficiency One out of every thousand incident photons ejects an electron. Hence the number emitted in the specified one-second interval is

$$N_e = \frac{10^{14}}{10^3} = 10^{11}.$$

Comparing with the form 10^x yields $x = 11$. The efficiency is a count fraction, not an energy loss to be subtracted from the work function. Both the count and its exponent are dimensionless; the time interval has already been included.

Step 4: Conclusion The number emitted in 1 s is 10^{11} electrons, so $x = 11$ (dimensionless).

Quick Tip to Solve: Check the threshold first, then divide total incident energy by full photon energy and multiply by the quantum efficiency; do not use the photoelectron kinetic energy for photon counting.

48. A non-isotropic solid metal cube has coefficients of linear expansion as :

$5 \times 10^{-5}/^{\circ}\text{C}$ along the x-axis and $5 \times 10^{-6}/^{\circ}\text{C}$ along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}/^{\circ}\text{C}$ then the value of C is

[4 marks]

Solution:

Concept:

- Linear expansion along each principal axis obeys $L_i = L_{i0}(1 + \alpha_i \Delta T)$, where α_i may differ with direction in an anisotropic material.
- The volume is the product of the three side lengths. To first order in temperature change, its expansion coefficient is $\beta = \alpha_x + \alpha_y + \alpha_z$, rather than three times any arbitrarily chosen coefficient.

Step 1: Write the volume as the product of expanded lengths Let the initial side lengths be L_{x0}, L_{y0}, L_{z0} , with initial volume $V_0 = L_{x0}L_{y0}L_{z0}$. After a small temperature rise,

$$V = V_0(1 + \alpha_x \Delta T)(1 + \alpha_y \Delta T)(1 + \alpha_z \Delta T).$$

Although the solid starts as a cube, unequal directional coefficients mean its three lengths need not remain equal on heating.

Step 2: Identify the coefficient of the first-order volume change Expanding the product and retaining the terms linear in ΔT gives

$$\frac{\Delta V}{V_0} = (\alpha_x + \alpha_y + \alpha_z) \Delta T.$$

Thus $\beta = \alpha_x + \alpha_y + \alpha_z$. Products such as $\alpha_x \alpha_y (\Delta T)^2$ are higher-order corrections and do not enter the coefficient defined at the initial temperature. The familiar expression 3α follows only when all three linear coefficients are equal.

Step 3: Substitute the three directional coefficients The given values are $\alpha_x = 5 \times 10^{-5} ^{\circ}\text{C}^{-1}$ and $\alpha_y = \alpha_z = 5 \times 10^{-6} ^{\circ}\text{C}^{-1}$. Expressing all terms with the same power of ten,

$$\beta = (50 + 5 + 5) \times 10^{-6} = 60 \times 10^{-6} ^{\circ}\text{C}^{-1}.$$

Comparing with $\beta = C \times 10^{-6} ^{\circ}\text{C}^{-1}$ gives $C = 60$. The coefficient β has inverse-temperature units, while the numerical multiplier C has no unit.

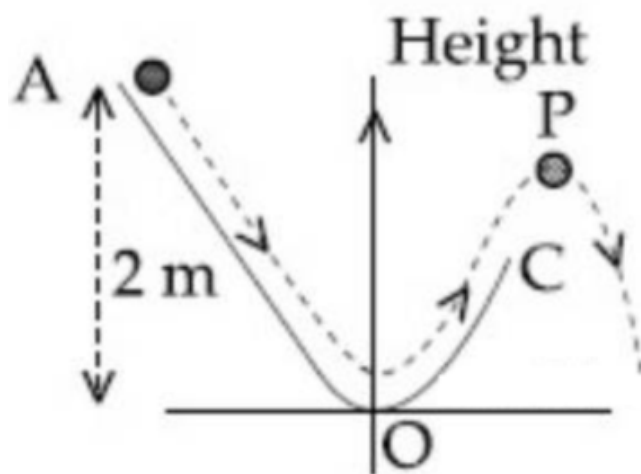
Step 4: Conclusion The volume-expansion coefficient is $60 \times 10^{-6} ^{\circ}\text{C}^{-1}$, so $C = 60$ (dimensionless).

Quick Tip to Solve: For anisotropic expansion, add the three directional linear coefficients after expressing them with a common power of ten. Use 3α only for an isotropic solid.

49. A particle ($m = 1 \text{ kg}$) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P (height 1 m), the kinetic energy of the particle (in J) is :

(Figure drawn is schematic and not to scale; take $g = 10 \text{ ms}^{-2}$)

[4 marks]



Solution:

Concept:

- The mechanical energy is the sum of kinetic energy and gravitational potential energy: $E = K + mgh$, where the reference height is the same at every point.
- On a frictionless track the normal reaction does no work, and in projectile motion gravity is conservative. Thus mechanical energy remains constant from A to P.

Step 1: Choose the energy reference Take the level through O as zero gravitational potential energy. The particle starts from rest at A, so its initial kinetic energy is zero. With the stated mass and initial height, its total energy is

$$E_A = 0 + (1)(10)(2) = 20 \text{ J.}$$

The track can change the direction of velocity, but its frictionless constraint cannot remove mechanical energy.

Step 2: Evaluate the energy at the highest point At P the height is 1 m , so the potential energy there is

$$U_P = mgh_P = (1)(10)(1) = 10 \text{ J.}$$

Apply conservation between A and P, without needing the launch angle or the height of C:

$$K_P + 10 = 20 \Rightarrow K_P = 10 \text{ J.}$$

At the highest point only the vertical velocity vanishes. The horizontal velocity remains, so the kinetic energy is not zero. The loss of gravitational potential energy from A to P supplies exactly this remaining kinetic energy.

Step 3: Conclusion The kinetic energy at P is $\boxed{10 \text{ J}}$. The numerical entry is 10.

Quick Tip to Solve: Use the height difference directly: $K_P = mg(h_A - h_P)$. A projectile at its highest point still has horizontal kinetic energy.

50. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is _____. [4 marks]

Solution:

Concept:

- For a reversible Carnot engine, the efficiency is $\eta = 1 - T_C/T_H$, with both reservoir temperatures in kelvin.
- Over one complete cycle the working substance returns to its initial state. Therefore $W = Q_H - Q_C$, where Q_H is absorbed and Q_C is rejected as a positive heat magnitude.

Step 1: Find the efficiency and heat input The hot and cold reservoir temperatures are 900 K and 300 K, respectively. Their ratio is $T_C/T_H = 1/3$. Hence

$$\eta = 1 - \frac{300}{900} = \frac{2}{3}.$$

The engine converts this fraction of the absorbed heat into useful work. Since the work output per cycle is 1200 J,

$$Q_H = \frac{W}{\eta} = \frac{1200}{2/3} = 1800 \text{ J}.$$

Step 2: Calculate the heat rejected Apply the first law to the complete cycle. There is no net increase in internal energy, so the difference between heat absorbed and heat rejected equals the work delivered:

$$Q_C = Q_H - W = 1800 - 1200 = 600 \text{ J}.$$

The same result follows from the reversible heat ratio:

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} = \frac{1}{3}.$$

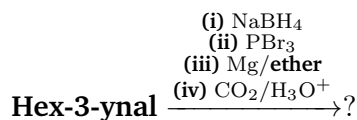
Thus one third of the input heat reaches the cold reservoir and two thirds becomes work. The requested quantity is the heat delivered to the low-temperature reservoir, rather than the total heat absorbed from the hot reservoir.

Step 3: Conclusion The engine delivers $\boxed{600 \text{ J}}$ to the low-temperature reservoir per cycle.

Quick Tip to Solve: For a Carnot cycle, compute the rejected heat directly as $Q_C = WT_C/(T_H - T_C)$; keep heat magnitudes positive.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. What is the product of following reaction?



[4 marks]

- (A) $\text{CH}_3\text{CH}_2\text{CH}=\text{CHCH}_2\text{COOH}$
 (B) $\text{CH}_3\text{CH}_2\text{C}\equiv\text{CCH}_2\text{CH}_2\text{COOH}$
 (C) $\text{CH}_3(\text{CH}_2)_4\text{COOH}$
 (D) $\text{CH}_3\text{CH}_2\text{CH}=\text{CHCH}_2\text{CH}_2\text{COOH}$

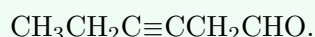
Correct Answer: (B) $\text{CH}_3\text{CH}_2\text{C}\equiv\text{CCH}_2\text{CH}_2\text{COOH}$

Solution:

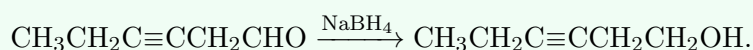
Concept:

- NaBH_4 reduces an aldehyde group to a primary alcohol under the stated conditions. It does not hydrogenate an isolated carbon-carbon triple bond.
- PBr_3 replaces the alcohol's hydroxyl group by bromine. Magnesium in dry ether then converts the alkyl bromide into a Grignard reagent.
- A Grignard reagent reacts with CO_2 , and acidic work-up gives a carboxylic acid with one additional carbon atom. Track the original aldehyde carbon through each step.

Step 1: Reduce only the aldehyde group Number the starting chain from its aldehyde carbon. Hex-3-ynal has six carbons and a triple bond between carbons 3 and 4:

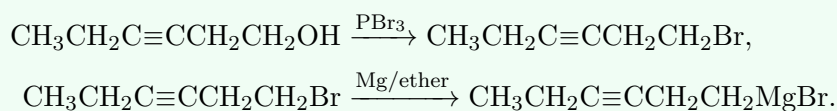


Sodium borohydride converts the terminal aldehyde into an alcohol while leaving the triple bond intact:



Thus the product of the first step is hex-3-yn-1-ol, not an alkene or a saturated alcohol.

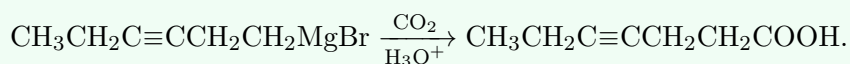
Step 2: Form the organomagnesium intermediate Phosphorus tribromide replaces the hydroxyl group with bromine. Magnesium then inserts into the carbon-bromine bond in dry ether:



The triple bond is internal, so there is no terminal acetylenic hydrogen to consume the Grignard reagent by an acid-base reaction.

Step 3: Add the extra carbon through carbon dioxide The carbon bonded to magnesium attacks the

electrophilic carbon of carbon dioxide. Acidic work-up protonates the resulting carboxylate:



The final chain has seven carbon atoms. Counting from the new carboxyl carbon puts the unchanged triple bond between carbons 4 and 5, giving hept-4-ynoic acid.

Step 4: Compare all four products Option (B) has both required features: one extra carbon and the preserved triple bond. Options (A) and (C) retain only six carbons, inconsistent with carboxylation. Options (A) and (D) also show a double bond, while (C) shows a saturated chain; none of the listed reagents converts the original triple bond into either of these. In particular, sodium borohydride must not be mistaken for a reagent for partial hydrogenation of the alkyne.

Step 5: Conclusion The product is $\boxed{\text{CH}_3\text{CH}_2\text{C}\equiv\text{CCH}_2\text{CH}_2\text{COOH}}$, hept-4-ynoic acid. Hence, the correct option is (B). Although this differs from the supplied answer key, the printed reagents preserve the triple bond and add one carbon, proving (B) for this reaction.

Quick Tip to Solve: Track the original aldehyde carbon as $\text{CHO} \longrightarrow \text{CH}_2\text{OH} \longrightarrow \text{CH}_2\text{Br} \longrightarrow \text{CH}_2\text{MgBr} \longrightarrow \text{CH}_2\text{COOH}$; only the final carboxylation adds a carbon, and the internal triple bond survives.

52. At 35°C, the vapour pressure of CS₂ is 512 mm. Hg and that of acetone is 344 mm Hg. A solution of CS₂ in acetone has a total vapour pressure of 600 mm Hg. The false statement amongst the following is :

[4 marks]

- (A) CS₂ and acetone are less attracted to each other than to themselves.
- (B) a mixture of 100 ml. CS₂ and 100 ml. acetone has a volume < 200 ml.
- (C) Heat must be absorbed in order to produce the solution 35°C
- (D) Raoult's law is not obeyed by this system.

Correct Answer: (B) a mixture of 100 ml. CS₂ and 100 ml. acetone has a volume < 200 ml.

Solution:

Concept:

- For an ideal binary liquid mixture, Raoult's law gives $P = x_A P_A^0 + x_B P_B^0$, a mole-fraction-weighted average of the pure vapour pressures.
- Positive deviation occurs when unlike intermolecular attractions are weaker than like attractions. Mixing then tends to be endothermic and accompanied by expansion.

Step 1: Test whether the pressure can be ideal At the given temperature the pure-component vapour pressures are 512 mm Hg and 344 mm Hg. Since the mole fractions are nonnegative and add to one, any ideal-mixture pressure must lie between those two values. However, the measured pressure is 600 mm Hg, greater even than 512 mm Hg. The mixture therefore displays positive deviation from Raoult's law: molecules escape into the vapour more readily than an ideal-mixture model predicts.

Step 2: Identify the false volume statement The positive deviation is explained by weaker attraction between CS₂ and acetone molecules than within the pure liquids. Thus (A) correctly describes the

intermolecular interactions. Energy is required to disrupt the stronger like interactions, so mixing absorbs heat and (C) is true. The same weakening produces expansion, $\Delta V_{\text{mix}} > 0$, rather than contraction. Accordingly, mixing 100 mL of each liquid gives a volume greater than 200 mL, contrary to (B). Finally, (D) is true because the observed pressure already proves that Raoult's law is not obeyed.

Step 3: Conclusion The false statement is that the final volume is less than 200 mL. Hence, the correct option is (B).

Quick Tip to Solve: If the mixture pressure exceeds both pure-liquid pressures, immediately infer positive deviation: weaker unlike attractions, heat absorption and expansion.

53. The number of orbitals associated with quantum number $n = 5$, $m_s = +\frac{1}{2}$ is :

[4 marks]

- (A) 11
- (B) 25
- (C) 15
- (D) 50

Correct Answer: (B) 25

Solution:

Concept:

- For a shell of principal quantum number n , the allowed azimuthal quantum numbers are $l = 0, 1, \dots, n - 1$. Each subshell contains $2l + 1$ spatial orbitals.
- The spin quantum number m_s specifies an electron spin state, not a new spatial orbital. Each spatial orbital permits one electron with $m_s = +1/2$.

Step 1: List the subshells of the fifth shell For $n = 5$, the allowed values of l are 0, 1, 2, 3, 4, corresponding to the $5s, 5p, 5d, 5f, 5g$ subshells. For each l , the magnetic quantum number ranges from $-l$ through zero to $+l$. Consequently these subshells contain 1, 3, 5, 7, 9 orbitals respectively. Counting them gives

$$N = 1 + 3 + 5 + 7 + 9 = 25.$$

This is also the familiar shell result $N = n^2 = 5^2$.

Step 2: Apply the specified spin without doubling The condition $m_s = +1/2$ selects one allowed spin orientation in each of the 25 orbitals. It does not eliminate any value of l or m_l , and it does not double the spatial-orbital count. The unrestricted electron capacity of the shell would be $2n^2 = 50$, which explains the tempting option (D), but that counts both spins and is not the requested number of orbitals. The counts 11 and 15 in (A) and (C) do not equal the sum of all allowed $2l + 1$ values for the fifth shell. Thus neither is compatible with the complete set of permitted orbital quantum numbers.

Step 3: Conclusion The number of orbitals is 25. Hence, the correct option is (B).

Quick Tip to Solve: Use n^2 for orbitals and $2n^2$ for total electron capacity. Fixing one spin orientation leaves one electron state per orbital.

54. Given that the standard potentials (E°) of Cu^{2+}/Cu and Cu^+/Cu are 0.34 V and 0.522 V respectively, the E° of $\text{Cu}^{2+}/\text{Cu}^+$:

[4 marks]

- (A) - 0.182 V
- (B) - 0.158 V
- (C) 0.182 V
- (D) +0.158 V

Correct Answer: (D) +0.158 V

Solution:

Concept:

- Standard electrode potentials are intensive quantities and cannot simply be added or subtracted when electron counts differ. Use the additive standard Gibbs energy, $\Delta G^\circ = -nFE^\circ$.
- The reduction $\text{Cu}^{2+} + 2e^- \longrightarrow \text{Cu}$ is the sum of the two successive one-electron reductions through Cu^+ .

Step 1: Write the successive reduction steps Let E_1° be the desired potential for $\text{Cu}^{2+} + e^- \longrightarrow \text{Cu}^+$. The second step is $\text{Cu}^+ + e^- \longrightarrow \text{Cu}$, with potential 0.522 V. Adding these reactions cancels the intermediate Cu^+ and gives the overall two-electron reduction $\text{Cu}^{2+} + 2e^- \longrightarrow \text{Cu}$, whose potential is 0.34 V. Because the overall reaction transfers two electrons, its Gibbs energy is $-2F(0.34)$, not $-F(0.34)$.

Step 2: Add Gibbs energies and solve The Gibbs-energy changes of the consecutive steps add:

$$-2F(0.34) = -FE_1^\circ - F(0.522).$$

Cancel the common factor $-F$ to obtain

$$0.680 = E_1^\circ + 0.522, \quad E_1^\circ = 0.158 \text{ V}.$$

The positive sign follows from the subtraction $0.680 - 0.522$. Directly subtracting the two supplied potentials gives -0.182 V , option (A), but wrongly ignores the two-electron factor. Reversing that subtraction gives (C), while (B) has the correct magnitude with the wrong sign. Only (D) respects both the electron count and the reduction directions.

Step 3: Conclusion The standard potential for $\text{Cu}^{2+}/\text{Cu}^+$ is $\boxed{+0.158 \text{ V}}$. Hence, the correct option is (D).

Quick Tip to Solve: Combine nE° , not E° , when adding redox reactions: here $2E_{2+/0}^\circ = E_{2+/+}^\circ + E_{+/0}^\circ$.

55. Match the following :

(i) Riboflavin (a) Beriberi

(ii) Thiamine (b) Scurvy

(iii) Pyridoxine (c) Cheilosis

(iv) Ascorbic acid (d) Convulsions

[4 marks]

(A) (i)-(c), (ii)-(d), (iii)-(a), (iv)-(b)

(B) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(c)

(C) (i)-c, (ii)-(a), (iii)-(d), (iv)-(b)

(D) (i)-(a), (ii)-(d), (iii)-(c), (iv)-(b)

Correct Answer: (C) (i)-c, (ii)-(a), (iii)-(d), (iv)-(b)

Solution:

Concept:

- Vitamins are required in small quantities for normal metabolism; characteristic deficiency symptoms identify the vitamin concerned.
- Riboflavin, thiamine and pyridoxine are vitamins B_2 , B_1 and B_6 , respectively, while ascorbic acid is vitamin C. Match the chemical name before matching the deficiency.

Step 1: Match the B vitamins Riboflavin deficiency causes cheilosis, the cracking or inflammation at the corners of the mouth. Thus (i) matches (c). Thiamine deficiency causes beriberi, involving the nervous and cardiovascular systems, so (ii) matches (a). Pyridoxine participates in amino-acid metabolism and neurotransmitter synthesis; its deficiency can produce convulsions. Therefore (iii) matches (d).

Step 2: Match vitamin C and check the complete code Ascorbic acid is required for normal collagen formation. Its deficiency produces scurvy, with symptoms such as bleeding gums and poor wound healing, so (iv) matches (b). Combining these independent matches gives (i) – (c), (ii) – (a), (iii) – (d), (iv) – (b). Option (A) incorrectly assigns convulsions to thiamine and beriberi to pyridoxine. Option (B) interchanges several deficiencies, including scurvy and cheilosis. Option (D) incorrectly assigns beriberi to riboflavin. Only (C) retains all four correct associations.

Step 3: Conclusion The required code is the one matching riboflavin with cheilosis, thiamine with beriberi, pyridoxine with convulsions and ascorbic acid with scurvy. Hence, the correct option is (C).

Quick Tip to Solve: First lock in thiamine–beriberi and ascorbic acid–scurvy; then use riboflavin–cheilosis to distinguish the remaining codes.

56. The relative strength of interionic/intermolecular forces in decreasing order is :

[4 marks]

- (A) dipole-dipole > ion-dipole > ion-ion
- (B) ion-dipole > dipole-dipole > ion-ion
- (C) ion-dipole > ion-ion > dipole-dipole
- (D) ion-ion > ion-dipole > dipole-dipole

Correct Answer: (D) ion-ion > ion-dipole > dipole-dipole

Solution:

Concept:

- Electrostatic attraction becomes stronger when the interacting charge magnitudes are larger, other conditions such as separation being comparable.
- Ions carry full charges; permanent molecular dipoles contain separated partial charges. Ion-ion, ion-dipole and dipole-dipole interactions therefore have different characteristic strengths.

Step 1: Compare interactions involving ions An ion-ion attraction acts between two full opposite charges, as in an ionic lattice. Its electrostatic potential energy is proportional to $-|q_1 q_2|/r$, so it is generally the strongest of the three interactions listed. An ion-dipole attraction involves one full ionic charge and the oppositely charged end of a polar molecule. It is weaker in the usual comparison because that molecular end carries a partial charge rather than another full ionic charge.

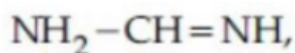
Step 2: Place the dipole interaction and eliminate the other orders Dipole-dipole attraction involves the partial positive end of one polar molecule and the partial negative end of another. With neither participant carrying a full ionic charge, it is generally weaker than ion-dipole attraction. Thus the decreasing order is ion-ion, ion-dipole, dipole-dipole. Option (A) reverses the trend completely. Option (B) places ion-ion last, and option (C) puts ion-dipole above ion-ion. Each contradicts the distinction between full and partial charges; option (D) gives the appropriate hierarchy.

Step 3: Conclusion The decreasing order is ion-ion > ion-dipole > dipole-dipole. Hence, the correct option is (D).

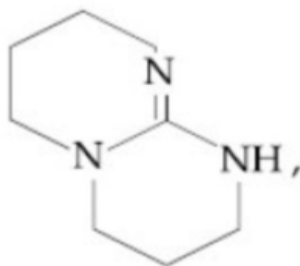
Quick Tip to Solve: Count full ionic charges in the interacting pair: two, one, then none gives the usual decreasing strength in this comparison.

57. The increasing order to P_{kb} for the following compounds will be :

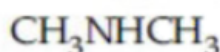
[4 marks]



(A)



(B)



(C)

(A) $(A) < (B) < (C)$

(B) $(C) < (A) < (B)$

(C) $(B) < (A) < (C)$

(D) $(B) < (C) < (A)$

Correct Answer: (C) $(B) < (A) < (C)$

Solution:

Concept:

- The base dissociation constant K_b measures basic strength, and $pK_b = -\log_{10} K_b$. A stronger base consequently has a smaller pK_b .
- Protonation is favoured when the conjugate acid is strongly resonance-stabilised. Guanidine and amidine systems are especially basic because their positive charge can be delocalised over nitrogen atoms.

Step 1: Compare the basic strengths of the three compounds Compound (B) contains a guanidine-type unit: its central carbon is bonded to three nitrogen atoms. Protonation at the imine nitrogen gives a conjugate acid whose positive charge is delocalised over the three nitrogens. This extensive stabilisation makes (B) the strongest base. Compound (A), $\text{NH}_2-\text{CH}=\text{NH}$, is an amidine; its conjugate acid has charge delocalised over two nitrogens and is also strongly stabilised. Compound (C), $(\text{CH}_3)_2\text{NH}$, is dimethylamine. Although the methyl groups donate electron density and the amine is basic, its conjugate acid lacks the corresponding amidinium resonance delocalisation. Hence the basic strength decreases as $(B) > (A) > (C)$.

Step 2: Reverse the order when writing pK_b Because pK_b is the negative logarithm of K_b , increasing pK_b runs from the strongest base to the weakest. The required order is therefore $(B) < (A) < (C)$. Option (A) incorrectly puts the guanidine system after the amidine. Option (B) starts with dimethylamine, which is the weakest base of this set. Option (D) places dimethylamine below the amidine in pK_b , reversing their relative basic strengths. Only option (C) is consistent with both

conjugate-acid stabilisation and the logarithmic definition.

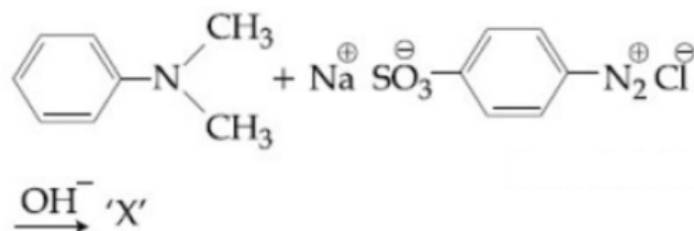
Step 3: Conclusion The guanidine-type compound has the lowest pK_b , followed by the amidine, then dimethylamine. Hence, the correct option is (C).

Quick Tip to Solve: Rank the bases by conjugate-acid resonance first, then reverse the inequality when converting basic strength into increasing pK_b .

58. Consider the following reaction :

The product 'X' is used :

[4 marks]



- (A) as food grade colourant
- (B) in acid base titration as an indicator.
- (C) in laboratory test for phenols
- (D) in protein estimation as an alternative to ninhydrin.

Correct Answer: (B) in acid base titration as an indicator.

Solution:

Concept:

- Aromatic diazonium ions undergo electrophilic azo coupling with strongly activated aromatic rings, forming a coloured $-N=N-$ linkage.
- The dimethylamino group activates its benzene ring and favours coupling at the para position. A conjugated azo compound can change colour when protonation changes its electronic structure.

Step 1: Identify the coupling product The reactants shown are N,N -dimethylaniline and the diazonium derivative of sulfanilic acid, carrying a sulfonate group. The dimethylamino substituent activates the aromatic ring for attack by the diazonium electrophile. Coupling occurs predominantly para to the dimethylamino group, joining the two aromatic rings through an azo linkage, $-N=N-$. The resulting sodium salt of the sulfonated dimethylaminoazobenzene dye is methyl orange. Recognising both the dimethylamino group and the sulfonate-substituted diazonium component distinguishes this product from unrelated coloured compounds.

Step 2: Connect its acid-base response to the stated use Methyl orange has differently coloured acid and base forms because protonation changes conjugation in its chromophore. It is red in acidic solution and yellow in the less acidic or alkaline form, with its transition occurring in the acidic pH range. This makes it useful as an indicator in suitable acid-base titrations. A dye being coloured alone does not establish that it is a food-grade colourant, so (A) does not describe the intended use. The

ferric-chloride test, rather than methyl orange, is the usual laboratory test for phenols, ruling out (C). Methyl orange is not the reagent that substitutes for ninhydrin in protein estimation, so (D) is also incorrect.

Step 3: Conclusion Product X is methyl orange, used as an acid–base titration indicator. Hence, the correct option is (B).

Quick Tip to Solve: The pair dimethylaniline plus diazotised sulfanilic acid identifies methyl orange; then recall its red-to-yellow acid-base colour change.

59. The dipole of CCl_4 , CHCl_3 and CH_4 are in the order :

[4 marks]

- (A) $\text{CCl}_4 < \text{CH}_4 < \text{CHCl}_3$
- (B) $\text{CH}_4 = \text{CCl}_4 < \text{CHCl}_3$
- (C) $\text{CH}_4 < \text{CCl}_4 < \text{CHCl}_3$
- (D) $\text{CHCl}_3 < \text{CH}_4 = \text{CCl}_4$

Correct Answer: (B) $\text{CH}_4 = \text{CCl}_4 < \text{CHCl}_3$

Solution:

Concept:

- The molecular dipole moment is the vector sum of the individual bond dipole moments, not their scalar sum.
- In a regular tetrahedron, four equal bond-dipole vectors cancel by symmetry. Replacing only one substituent breaks that cancellation.

Step 1: Use tetrahedral symmetry for the two symmetric molecules Both CH_4 and CCl_4 have tetrahedral geometries with four identical surrounding atoms. In methane the four equivalent C–H bond dipoles cancel, giving zero molecular dipole moment. In carbon tetrachloride the C–Cl bonds are polar, but their four equal dipole vectors also cancel because of the tetrahedral arrangement. Thus polar bonds do not by themselves imply a polar molecule:

$$\mu(\text{CH}_4) = \mu(\text{CCl}_4) = 0.$$

Step 2: Identify the nonzero resultant in chloroform In CHCl_3 , three bonds are C–Cl bonds and one is a C–H bond. The fourth dipole is no longer identical to the other three, so tetrahedral cancellation is incomplete and the molecule has a nonzero resultant dipole. Consequently both symmetric molecules have a smaller dipole moment than chloroform. Options (A) and (C) incorrectly assign different nonzero rankings to methane and carbon tetrachloride, even though both are nonpolar. Option (D) incorrectly makes chloroform the least polar. Option (B) alone correctly groups the two zero-dipole molecules together and places chloroform above them.

Step 3: Conclusion

$$\mu(\text{CH}_4) = \mu(\text{CCl}_4) < \mu(\text{CHCl}_3).$$

Hence, the correct option is (B).

Quick Tip to Solve: Check symmetry before bond polarity: four identical tetrahedral bonds cancel, whereas three identical bonds and one different bond generally do not.

60. The theory that can completely/properly explain the nature of bonding in $[\text{Ni}(\text{CO})_4]$ is : [4 marks]

- (A) Werner's theory
- (B) Valence bond theory
- (C) Crystal field theory
- (D) Molecular orbital theory

Correct Answer: (D) Molecular orbital theory

Solution:

Concept:

- Bonding in metal carbonyls is synergic: the carbonyl ligand donates an electron pair to the metal through a sigma bond, while the metal returns electron density through pi back-bonding.
- Molecular orbital theory describes overlap between metal orbitals and ligand donor or acceptor orbitals, including the empty π^* orbitals of carbon monoxide.

Step 1: Identify both components of the metal-carbon bond In nickel tetracarbonyl, $\text{Ni}(\text{CO})_4$, the carbon end of each carbon monoxide ligand donates a lone pair into a suitable metal acceptor orbital. This is ligand-to-metal sigma donation. Filled nickel orbitals can simultaneously donate electron density into the empty antibonding π^* orbitals of carbon monoxide, producing metal-to-ligand pi back-bonding. These two interactions reinforce each other: sigma donation increases the availability of metal electron density for back-donation, while back-donation stabilises metal-carbon bonding and weakens the internal carbon-oxygen bond.

Step 2: Select the theory that accommodates the full interaction Molecular orbital theory explicitly includes metal and ligand orbitals of appropriate symmetry, so it can describe both sigma donation and pi back-bonding. Werner's theory explains primary and secondary valencies and coordination arrangements but does not supply this electronic bonding description. Valence bond theory can give a hybridisation and geometry description, yet its simple treatment does not properly account for the complete synergic interaction. Crystal field theory treats ligands primarily as an electrostatic field and does not fully describe the covalent orbital mixing and back-bonding. Therefore only molecular orbital theory among the listed choices properly explains the bonding in this carbonyl.

Step 3: Conclusion Molecular orbital theory describes both sigma donation and pi back-bonding in $\text{Ni}(\text{CO})_4$. Hence, the correct option is (D).

Quick Tip to Solve: Whenever a metal-carbonyl question asks for a complete bonding theory, look for a description that includes CO pi-star acceptor orbitals as well as sigma donation.

61. 1-methylethylene oxide when treated with an excess of HBr produces :

[4 marks]

- (A) $\text{BrCH}_2\text{CHBrCH}_3$
- (B) $\text{BrCH}=\text{CHCH}_3$
- (C) $\text{CH}_3\text{CBr}_2\text{CH}_3$
- (D) $\text{CH}_2=\text{C}(\text{Br})\text{CH}_3$

Correct Answer: (A) $\text{BrCH}_2\text{CHBrCH}_3$

Solution:

Concept:

- An epoxide is a three-membered cyclic ether. Protonating its oxygen activates the strained ring towards nucleophilic opening by bromide.
- With excess HBr, the initially formed bromohydrin reacts further: its hydroxyl group is replaced by bromine. The carbon skeleton remains unchanged.

Step 1: Open the protonated epoxide 1-Methylethylene oxide is propylene oxide, whose ring carbons are a terminal CH_2 carbon and a $\text{CH}(\text{CH}_3)$ carbon. First the oxygen accepts a proton from HBr. Bromide attacks a ring carbon and breaks a carbon–oxygen bond, relieving the ring strain. This produces a bromohydrin. Irrespective of which carbon initially retains oxygen, the two neighbouring ring carbons now carry one bromine and one hydroxyl group.

Step 2: Use the excess hydrogen bromide The word excess means that the reaction does not stop at the bromohydrin. Protonation converts its hydroxyl group into a water leaving group, and bromide replaces it. Thus both former ring carbons acquire bromine. The overall transformation can be represented by the carbon-chain product:



The oxygen leaves as water; no carbon atom is added or lost.

Step 3: Eliminate the other structures Option (B), $\text{BrCH}=\text{CHCH}_3$, and option (D), $\text{CH}_2=\text{C}(\text{Br})\text{CH}_3$, contain a double bond and only one bromine. They do not represent complete cleavage with excess acid. Option (C), $\text{CH}_3\text{CBr}_2\text{CH}_3$, places both bromines on one carbon; epoxide cleavage instead gives a vicinal dibromide with bromines on adjacent carbons.

Step 4: Conclusion The product is 1,2-dibromopropane, $\text{CH}_3\text{CHBrCH}_2\text{Br}$. Hence, the correct option is (A).

Quick Tip to Solve: For an epoxide with excess hydrogen halide, replace the ring oxygen by a halogen on each of the two ring carbons; first check that the carbon skeleton is conserved.

62. The electron gain enthalpy (in KJ/mol) of fluorine, chlorine, bromine and iodine, respectively are: [4 marks]

(A) -296, -325, -333 and -349

(B) 349, -333, -325 and -296

(C) -333, -349, -325 and -296

(D) -333, -325, -349 and -296

Correct Answer: (C) -333, -349, -325 and -296

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for $X(g) + e^- \longrightarrow X^-(g)$. A negative value means that electron attachment releases energy.
- Chlorine has a more negative electron gain enthalpy than fluorine because the compact fluorine $2p$ shell has stronger electron–electron repulsion.

Step 1: Identify the exceptional fluorine–chlorine order Fluorine attracts electrons strongly, but an incoming electron must enter its very small $2p$ shell. Repulsion from electrons already occupying this compact region reduces the energy released. In chlorine the electron enters a more spacious $3p$ orbital, so electron attachment is more exothermic. Therefore chlorine, not fluorine, must have the most negative value among the listed halogens.

Step 2: Assign the numerical values The standard approximate electron gain enthalpies in the order requested are

$$\begin{aligned}\Delta_{\text{eg}}H(\text{F}) &= -333 \text{ kJ mol}^{-1}, \\ \Delta_{\text{eg}}H(\text{Cl}) &= -349 \text{ kJ mol}^{-1}, \\ \Delta_{\text{eg}}H(\text{Br}) &= -325 \text{ kJ mol}^{-1}, \\ \Delta_{\text{eg}}H(\text{I}) &= -296 \text{ kJ mol}^{-1}.\end{aligned}$$

After chlorine the increasing atomic size reduces the attraction for the incoming electron, so bromine and then iodine have progressively less negative values.

Step 3: Compare the four sequences Option (A) incorrectly makes iodine the most exothermic member and assigns the least negative value to fluorine. Option (B) has a positive first value as printed and assigns the wrong magnitudes to fluorine and chlorine. Option (D) interchanges the chlorine and bromine values, incorrectly making bromine the strongest electron acceptor by this enthalpy criterion. Only option (C) preserves all four assignments.

Step 4: Conclusion The sequence is $-333, -349, -325, -296 \text{ kJ mol}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the chlorine exception first: chlorine has the most negative electron gain enthalpy; then the values become less negative through bromine and iodine.

63. Amongst the following statements, that which was not proposed by Dalton was :

[4 marks]

- (A) Matter consists of indivisible atoms all the atoms of a given element have.
- (B) Chemical reactions involve reorganization of atoms. These are neither created nor destroyed in a chemical reaction.
- (C) When gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are the same T & P.
- (D) Identical properties including identical mass. Atoms of different element differ in mass.

Correct Answer: (C) When gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are the same T & P.

Solution:

Concept:

- Dalton described matter as atoms and chemical reactions as rearrangements of atoms, which are neither created nor destroyed in ordinary chemical changes.
- Gay-Lussac's law of combining volumes states that reacting gases and gaseous products have simple whole-number volume ratios at the same temperature and pressure.

Step 1: Recognise the atomic postulates Dalton proposed that matter consists of indivisible atoms, that atoms of one element have identical masses and properties, and that atoms of different elements differ. These are historical postulates of his model. Although modern discoveries of subatomic particles and isotopes qualify them, the question asks what Dalton proposed, rather than which statements are universally valid today. Options (A) and (D) contain these familiar postulates, with their sentence awkwardly divided in the printed options.

Step 2: Check the reaction statement Option (B) says that a chemical reaction reorganises atoms and that atoms are not created or destroyed during the reaction. This is another essential part of Dalton's theory. It explains conservation of mass: the atoms present before a chemical change are still present afterward, though combined or arranged differently. Therefore option (B) cannot be the required statement that Dalton did not propose.

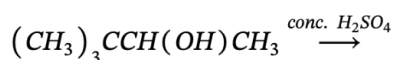
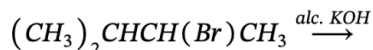
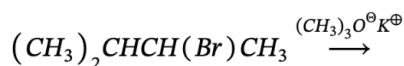
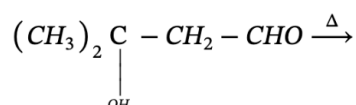
Step 3: Identify the law about gas volumes Option (C) concerns volumes of gaseous reactants and products measured under the same temperature and pressure. This is the law of combining volumes associated with Gay-Lussac. It is a macroscopic relation between measured gas volumes, not a postulate of Dalton's atomic theory. Simple volume ratios must not be confused with Dalton's statement that atoms combine in small whole-number ratios; the quantities involved are different.

Step 4: Conclusion The statement about simple ratios of gaseous volumes at the same temperature and pressure was not proposed by Dalton. Hence, the correct option is (C).

Quick Tip to Solve: Look at the quantity in the statement: atomic numbers and masses suggest Dalton, while combining gas volumes at equal temperature and pressure identify Gay-Lussac.

64. Consider the following reaction: Which of these reaction(s) will not produce Saytzeff product?

[4 marks]



- (A) (d) only
 (B) (b) and (d)
 (C) (a), (c) and (d)
 (D) (c) only

Correct Answer: (D) (c) only

Solution:

Concept:

- Saytzeff elimination favours the more substituted alkene. Small bases generally favour this product, whereas a bulky base can remove a more accessible hydrogen to give the less substituted Hofmann alkene.
- Acid-catalysed alcohol dehydration may involve carbocation rearrangement. Dehydration of a beta-hydroxy carbonyl compound favours the alkene conjugated with the carbonyl group.

Step 1: Examine reactions (a) and (b) In (a), concentrated H_2SO_4 protonates the alcohol and permits loss of water. The initially formed secondary carbocation lies beside a quaternary carbon. A 1,2-methyl shift gives a more stable tertiary carbocation, and subsequent elimination gives the highly substituted alkene $(CH_3)_2C=C(CH_3)_2$. Thus (a) does not constitute a Hofmann exception; rearrangement allows the Saytzeff product to form. In (b), alcoholic KOH acts as a relatively unhindered base. Removing the beta hydrogen from the methine side gives $(CH_3)_2C=CHCH_3$, the more substituted alkene, rather than the terminal alkene. Hence (b) also produces the Saytzeff major product.

Step 2: Distinguish the bulky-base and carbonyl eliminations Reaction (c) uses potassium tert-butoxide, $(CH_3)_3CO^- K^+$. Its bulky alkoxide ion preferentially abstracts a more accessible beta hydrogen from the terminal methyl group. The major product is therefore the less substituted alkene $(CH_3)_2CHCH=CH_2$, instead of the more substituted internal alkene. This is Hofmann orientation

and is the required exception. In (d), heating the beta-hydroxy aldehyde removes water to give $(\text{CH}_3)_2\text{C}=\text{CHCHO}$. This product has both the more substituted double bond and conjugation with the aldehyde carbonyl; elimination towards a methyl group would lose that conjugation and give a less substituted alkene. Thus (d) is not an exception. Options (A), (B) and (C) incorrectly include reaction (d); (B) also includes the ordinary small-base elimination (b), and (C) incorrectly includes the rearranging dehydration (a).

Step 3: Conclusion Only the third reaction gives the Hofmann major product because tert-butoxide preferentially removes the less hindered beta hydrogen. Hence, the correct option is (D).

Quick Tip to Solve: Check the base before applying Saytzeff's rule: tert-butoxide is bulky and favours terminal elimination. For acid dehydration also check rearrangement, and for beta-hydroxy carbonyl compounds check conjugation.

65. Oxidation number of potassium in K_2O , K_2O_2 and KO_2 respectively is :

[4 marks]

- (A) +1, +2 and + 4
- (B) +1, +1 and + 1
- (C) +2, +1 and $+\frac{1}{2}$
- (D) +1, +4, and +2

Correct Answer: (B) +1, +1 and + 1

Solution:

Concept:

- Potassium is a Group 1 metal and has oxidation number +1 in its ordinary ionic compounds, including oxides, peroxides and superoxides.
- The sum of oxidation numbers in a neutral compound is zero. Oxygen has different oxidation numbers in oxide, peroxide and superoxide ions.

Step 1: Evaluate potassium oxide In K_2O , oxygen is an oxide ion with oxidation number -2 . Let each potassium have oxidation number x . Electrical neutrality gives

$$2x - 2 = 0 \implies x = +1.$$

The subscript 2 counts potassium atoms; it does not mean that an individual potassium atom has oxidation number +2.

Step 2: Evaluate the peroxide and superoxide In K_2O_2 , the anion is peroxide, O_2^{2-} , with oxidation number -1 for each oxygen. Therefore

$$2x + 2(-1) = 0 \implies x = +1.$$

In KO_2 , the anion is superoxide, O_2^- , with total charge -1 and average oxygen oxidation number $-\frac{1}{2}$. Hence

$$x + 2\left(-\frac{1}{2}\right) = 0 \implies x = +1.$$

It is oxygen that changes oxidation state across the three compounds.

Step 3: Reject the misleading alternatives Option (A) incorrectly increases potassium's oxidation number as more oxygen is present. Options (C) and (D) likewise assign potassium values such as +2, +4, or $+\frac{1}{2}$, which are inconsistent with both charge balance and its alkali-metal chemistry. Fractional average oxidation numbers belong to oxygen in the superoxide, not to potassium. Option (B) alone gives one positive unit for each potassium throughout.

Step 4: Conclusion The oxidation numbers of potassium in K_2O , K_2O_2 , and KO_2 are +1, +1, +1, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Keep the alkali metal fixed at +1; use charge balance to determine whether oxygen is present as oxide, peroxide or superoxide.

66. The atomic radius of Ag is closed to :

[4 marks]

- (A) Au
- (B) Cu
- (C) Hg
- (D) Ni

Correct Answer: (A) Au

Solution:

Concept:

- Silver and gold are successive Group 11 transition metals with outer configurations of the general form $d^{10}s^1$. Their metallic atomic radii are very similar.
- Lanthanoid contraction offsets much of the expected size increase on going from a $4d$ element to the corresponding $5d$ element.

Step 1: Compare silver with the next heavier group member Silver belongs to the $4d$ series and gold to the $5d$ series. A simple shell-counting argument would predict that gold is substantially larger because it has an additional principal shell. However, the lanthanoid elements intervene before the $5d$ series. Their $4f$ electrons shield the rising nuclear charge poorly, causing an increased attraction of the outer electrons towards the nucleus.

Step 2: Apply the lanthanoid contraction This contraction largely compensates for the extra shell in gold. Consequently the metallic atomic radii of silver and gold are approximately

$$r(\text{Ag}) \simeq 144 \text{ pm}, \quad r(\text{Au}) \simeq 144 \text{ pm}.$$

The precise tabulated numbers can vary slightly with the radius convention, but the near equality of the two corresponding $4d$ and $5d$ metals is the decisive periodic trend being tested.

Step 3: Compare the alternatives Copper, above silver in Group 11, is a $3d$ metal and is appreciably smaller, with a metallic radius around 128 pm. Nickel is also a smaller $3d$ metal, around 125 pm. Mercury is a neighbouring $5d$ element in Group 12, rather than the corresponding Group 11 element, and its commonly quoted metallic radius is around 151 pm. Thus none gives as close a match to silver

as gold does.

Step 4: Conclusion The atomic radius of silver is closest to that of gold, Au , owing to lanthanoid contraction. Hence, the correct option is (A).

Quick Tip to Solve: For corresponding 4d and 5d transition metals, test lanthanoid contraction before assuming that the heavier element must have a much larger radius.

67. A solution of m-chloronailimne, m-chlorophenol and m-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of NaHCO_3 to give fraction A. The left over organic phase was extracted with dilute NaOH solution to give fraction B. The final organic layer was labelled as fraction C. Fractions A,B and C, contain respectively : [4 marks]

- (A) m-chlorobenzoic acid, m-chloroaniline and m-chlorophenol.
- (B) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline.
- (C) m-chlorophenol, m-chlorobenzoic acid and m-chloronailine.
- (D) m-chloroaniline m-chlorobenzoic acid and m-chlorophenol.

Correct Answer: (B) m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline.

Solution:

Concept:

- Acid-base extraction separates compounds by converting an acidic component into a water-soluble ionic salt while neutral compounds remain mainly in the organic phase.
- Sodium hydrogen carbonate deprotonates carboxylic acids, but is not sufficiently basic to deprotonate ordinary phenols appreciably. Sodium hydroxide can deprotonate phenols.

Step 1: Identify the component extracted first The strongest acid present is m-chlorobenzoic acid. It reacts with saturated sodium hydrogen carbonate to form its water-soluble sodium carboxylate, with evolution of carbon dioxide. Thus fraction A contains the carboxylic-acid component in its salt form. The phenol is too weak an acid for effective extraction by bicarbonate, and the aniline derivative is a base rather than an acid.

Step 2: Identify the second extract and the residue On treating the remaining organic phase with dilute sodium hydroxide, m-chlorophenol forms water-soluble sodium m-chlorophenoxide and enters fraction B. The neutral m-chloroaniline remains in ethyl acetate and therefore constitutes fraction C. The names in the options identify the parent compounds; the alkaline aqueous extracts actually contain their sodium salts.

Step 3: Check the proposed sequences Option A incorrectly leaves phenol until the final organic layer and places the amine in the hydroxide extract. Option C incorrectly extracts the weaker phenol before the carboxylic acid. Option D incorrectly puts the basic amine in the bicarbonate extract. Only option B follows the order dictated by acidity and salt formation.

Step 4: Conclusion Fractions A, B and C correspond respectively to m-chlorobenzoic acid, m-chlorophenol and m-chloroaniline. Hence, the correct option is (B).

Quick Tip to Solve: For successive bicarbonate and hydroxide extractions, remove the carboxylic acid first, then the phenol; a neutral amine remains organic until an acid wash.

68. The IUPAC name of complex $[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NH}_2\text{CH}_3)]\text{Cl}$ is:

[4 marks]

- (A) Diamminechlorido (methanamine) platinum(II)chloride.
- (B) Biasmmine (methanamine)chlorido platinum(II)chloride.
- (C) Diamminechlorido (amminomethane) platinum(II) chloride.
- (D) Diammine (methanamine) chlorido platinum(II)Chloride.

Correct Answer: (A) Diamminechlorido (methanamine) platinum(II)chloride.

Solution:

Concept:

- Name ligands alphabetically, ignoring multiplicative prefixes such as di. Neutral ammonia is named ammine, coordinated chloride is chlorido, and the neutral ligand methanamine is named methanamine.
- The metal oxidation state is obtained from the total charge on the coordination entity; an outside counter-ion is named after the coordination entity.

Step 1: Determine the oxidation state of platinum The chloride outside the square brackets is a singly charged anion, so the complex inside the brackets has charge +1. Both ammonia ligands and methanamine are neutral, while the coordinated chlorido ligand has charge -1. If the oxidation number of platinum is x , charge balance gives

$$x + 2(0) - 1 + 0 = +1, \quad x = +2.$$

The metal is therefore named platinum(II), rather than using a different oxidation number or an anionic-metal ending.

Step 2: Arrange and name the ligands There are two ammonia ligands, giving diammine, one chlorido ligand and one methanamine ligand. Ignoring the prefix di for alphabetisation, ammine precedes chlorido, which precedes methanamine. Follow this ligand sequence by platinum(II), then chloride for the outside counter-ion.

Step 3: Eliminate the other names Option B uses an incorrect multiplicative form for the simple ammine ligand and also misorders chlorido. Option C uses amminomethane instead of the ligand name methanamine. Option D puts methanamine before chlorido, violating alphabetical ligand ordering. These distinctions concern ligand naming and order, even though all the displayed choices indicate platinum(II).

Step 4: Conclusion The name is diamminechlorido(methanamine)platinum(II) chloride. Hence, the correct option is (A).

Quick Tip to Solve: First find the complex charge from the outside ion; then alphabetise ligand names without counting di, tri or tetra.

69. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is : [4 marks]

- (A) less efficient as the resins cannot be regenerated.
- (B) more efficient as it can exchange only cations.
- (C) more efficient as it can exchange both cations as well as anions.
- (D) less efficient as it exchanges only anions.

Correct Answer: (C) more efficient as it can exchange both cations as well as anions.

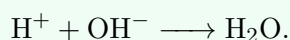
Solution:

Concept:

- Hardness is mainly due to dissolved calcium and magnesium ions. A sodium zeolite softener exchanges these cations for sodium ions, but leaves dissolved anions in the water.
- A complete synthetic-resin deionisation system uses both a hydrogen-form cation exchanger and a hydroxide-form anion exchanger. The released hydrogen and hydroxide ions form water.

Step 1: Compare what zeolite removes In the zeolite process, a calcium or magnesium ion replaces sodium held by the zeolite. This removes the ions responsible for hardness but does not remove the corresponding chloride, sulfate or other anions. The treated water consequently still contains dissolved sodium salts; softening is not the same as complete demineralisation.

Step 2: Explain the two resin stages The cation-exchange resin replaces dissolved positive ions with hydrogen ions, while the anion-exchange resin replaces dissolved negative ions with hydroxide ions. These released ions combine according to



The combined treatment can therefore remove both classes of dissolved ions and gives water of much lower ionic content than sodium-zeolite treatment.

Step 3: Test every alternative Option A is incorrect because exhausted synthetic resins can be regenerated: the cation resin with acid and the anion resin with alkali. Option B incorrectly limits the method to cation exchange, overlooking its anion-exchange stage. Option D incorrectly claims that it exchanges only anions and is less efficient. Option C correctly describes the wider ion-removal capability of the resin method.

Step 4: Conclusion The synthetic-resin method is more efficient because it can exchange both cations and anions. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish softening from deionisation: sodium zeolite removes hardness cations, whereas paired synthetic resins remove cations and anions.

70. The purest form of commercial iron is :

[4 marks]

- (A) pig iron
- (B) wrought iron
- (C) cast iron
- (D) scrap iron and pig iron

Correct Answer: (B) wrought iron

Solution:

Concept:

- The purity of commercial iron depends on its carbon content and other impurities introduced or retained during extraction and refining.
- Wrought iron is the nearly pure commercial form of iron. Pig iron and cast iron retain appreciably more carbon, so they are not the purest forms.

Step 1: Identify the refined form Wrought iron is obtained by refining iron so that most carbon and oxidisable impurities are removed. It is commonly called the purest commercial iron and generally contains less than about 0.5 percent total impurities. Its low carbon content makes it malleable and suitable for shaping, in contrast to the relatively brittle high-carbon forms.

Step 2: Compare pig iron and cast iron Pig iron is the initial high-carbon product obtained from the blast furnace. Besides several percent carbon, it contains impurities such as silicon, phosphorus, sulfur and manganese. Cast iron also contains a few percent carbon and is suitable for casting because it melts readily and flows into moulds. Neither description corresponds to the nearly pure, low-carbon commercial form requested.

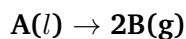
Step 3: Reject the remaining choices Option A is therefore ruled out by the substantial carbon and impurity content of pig iron. Option C is ruled out by the high carbon content of cast iron. Option D merely names scrap iron together with pig iron: scrap is recovered material of variable composition, and combining it with pig iron does not define the purest commercial grade. The standard classification singles out wrought iron.

Step 4: Conclusion Wrought iron is the purest form of commercial iron. Hence, the correct option is (B).

Quick Tip to Solve: When comparing commercial irons, remember the purity order through carbon removal: wrought iron is the nearly pure low-carbon form.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. For the reaction :



$$\Delta U = 2.1 \text{ kcal}, \Delta S = 20 \text{ cal K}^{-1} \text{ at } 300 \text{ K}$$

Hence ΔG in kcal is :

[4 marks]

Solution:

Concept:

- Gibbs energy is calculated from $\Delta G = \Delta H - T\Delta S$, so internal energy must first be converted into enthalpy.
- For ideal gaseous components, $\Delta H = \Delta U + \Delta n_g RT$. Only gaseous stoichiometric coefficients enter Δn_g ; the liquid reactant is excluded.

Step 1: Count the change in gaseous moles For the reaction as written, one liquid reactant produces two gaseous product molecules in stoichiometric terms. There are no gaseous reactants, hence

$$\Delta n_g = 2 - 0 = 2.$$

Taking the gas constant in calories, $R \simeq 2 \text{ cal mol}^{-1} \text{K}^{-1}$, keeps all energy calculations consistent with the supplied caloric units.

Step 2: Convert internal energy to enthalpy At 300 K, the gas-expansion contribution for the reaction is

$$\Delta n_g RT = 2 \times 2 \times 300 = 1200 \text{ cal} = 1.2 \text{ kcal}.$$

Consequently,

$$\Delta H = 2.1 + 1.2 = 3.3 \text{ kcal}.$$

Using a more precise gas constant changes only the last small decimal; the expected precision here uses the conventional rounded value.

Step 3: Calculate the Gibbs energy The entropy contribution is

$$T\Delta S = 300 \times 20 = 6000 \text{ cal} = 6.0 \text{ kcal}.$$

Therefore

$$\Delta G = 3.3 - 6.0 = -2.7 \text{ kcal}.$$

The negative sign means the forward reaction is thermodynamically favourable at the stated temperature. Substituting internal energy directly into the Gibbs-energy expression would incorrectly omit the gas-expansion term.

Step 4: Conclusion The Gibbs energy change for the reaction as written is $\Delta G = -2.7 \text{ kcal}$.

Quick Tip to Solve: Before applying Gibbs energy, check whether the supplied energy is enthalpy or internal energy; a change in gaseous moles requires the correction $\Delta n_g RT$.

72. During the nuclear explosion, one of the products is ^{90}Sr with half life of 6.93 years. If $1\ \mu\text{g}$ of ^{90}Sr was absorbed in the bones of newly born baby in place of Ca, how much time, in years, is required to reduce it by 90% if it is not lost metabolically. [4 marks]

Solution:

Concept:

- Radioactive decay is a first-order process: $N = N_0 e^{-kt}$, where k is the decay constant and N/N_0 is the surviving fraction.
- The half-life is related to the decay constant by $k = \ln 2/t_{1/2}$. If there is no metabolic loss, radioactivity alone determines the remaining amount.

Step 1: Translate the percentage reduction A reduction by 90 percent leaves 10 percent of the original amount, not 90 percent. Starting with $1\ \mu\text{g}$, the required final amount is therefore $0.1\ \mu\text{g}$, so

$$\frac{N}{N_0} = \frac{0.1}{1} = 0.10.$$

The mass ratio equals the number-of-nuclei ratio because the same isotope is being compared before and after decay.

Step 2: Find the decay constant Use the stated half-life of 6.93 years in the first-order half-life relation:

$$k = \frac{0.693}{6.93} = 0.1000\ \text{year}^{-1}.$$

The unit of the decay constant fixes the time unit: using years for the half-life yields years for the required time.

Step 3: Solve for the required time Taking logarithms of the decay law gives

$$t = \frac{1}{k} \ln \frac{N_0}{N} = \frac{1}{0.1000} \ln 10.$$

Hence

$$t = 10(2.3026) = 23.026\ \text{years} \simeq 23.03\ \text{years}.$$

This corresponds to about 3.32 half-lives, consistent with the fact that three half-lives leave 12.5 percent and a little more time is needed to reach 10 percent. The initial microgram quantity cancels from the fractional-decay equation.

Step 4: Conclusion The required time is 23.03 years, approximately 23 years.

Quick Tip to Solve: For a stated percentage loss, calculate the fraction left first. A 90 percent loss means use $N_0/N = 10$, not $1/0.9$.

73. Chlorine reacts with hot and concentrated NaOH and produces compounds (X) and (Y). Compound (X) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (Y) is. [4 marks]

Solution:

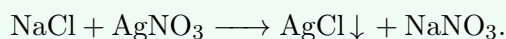
Concept:

- Hot, concentrated alkali disproportionates chlorine into chloride and chlorate. Chlorine is reduced from oxidation state zero to minus one in chloride and oxidised to plus five in chlorate.
- Equivalent resonance contributors distribute the bonding over the three chlorine–oxygen connections in ClO_3^- . Average bond order is the sum of the formal bond orders in one contributor divided by the number of equivalent bonds.

Step 1: Identify both reaction products Write the balanced disproportionation reaction for the specified hot, concentrated conditions:



Sodium chloride gives the stated white precipitate of silver chloride:



Therefore X is NaCl , while Y is sodium chlorate, NaClO_3 . The temperature qualification matters: cold, dilute alkali would give hypochlorite instead of chlorate, which would be a different oxygen-containing ion.

Step 2: Average the chlorine–oxygen bond orders In the conventional resonance representation of the chlorate ion, chlorine has two double bonds to oxygen and one single bond to an oxygen bearing the negative charge. There are three equivalent contributors, because each oxygen can occupy the singly bonded position. Thus no particular oxygen retains the single bond in the resonance hybrid.

$$\text{Total formal bond order} = 2 + 2 + 1 = 5.$$

Dividing by the three equivalent chlorine–oxygen bonds gives

$$\overline{\text{BO}} = \frac{5}{3} = 1.666 \dots$$

This is an average bond order; it must not be confused with chlorine's oxidation number, which is plus five in chlorate.

Step 3: Conclusion The average chlorine–oxygen bond order in $Y = \text{NaClO}_3$ is $\boxed{5/3 \approx 1.67}$, a dimensionless quantity.

Quick Tip to Solve: First use the reaction conditions to distinguish chlorate from hypochlorite; then average the three resonance bond orders rather than using chlorine's oxidation number.

74. Two solutions, A and B, each of 100L was made by dissolving 4g of NaOH and 9.8 g of H₂SO₄ in water, respectively. The pH of the resultant solution obtained from mixing 40L of solution A and 10L of solution B is : [4 marks]

Solution:

Concept:

- Neutralisation consumes equal amounts of hydrogen and hydroxide ions: $\text{H}^+ + \text{OH}^- \longrightarrow \text{H}_2\text{O}$. Count acid equivalents, since each mole of H₂SO₄ supplies two acidic protons in this neutralisation.
- After finding the excess hydroxide, divide its moles by the total mixed volume. At 25°C, $\text{pOH} = -\log_{10}[\text{OH}^-]$ and $\text{pH} + \text{pOH} = 14$.

Step 1: Calculate the concentrations of the stock solutions The molar masses of NaOH and H₂SO₄ are respectively 40 and 98 g mol⁻¹. Thus the moles initially dissolved in each separate 100-litre solution are

$$n_A = \frac{4}{40} = 0.100 \text{ mol}, \quad n_B = \frac{9.8}{98} = 0.100 \text{ mol}.$$

Both stock molarities are $0.100/100 = 0.00100 \text{ mol L}^{-1}$. Equal molarities do not mean equal neutralising capacities: the acid is diprotic and the base contributes one hydroxide per formula unit.

Step 2: Subtract the reacting acid equivalents The chosen aliquots contain

$$n(\text{OH}^-) = 40(0.00100) = 0.0400 \text{ mol},$$

$$n(\text{H}_2\text{SO}_4) = 10(0.00100) = 0.0100 \text{ mol}.$$

From $\text{H}_2\text{SO}_4 + 2 \text{NaOH} \longrightarrow \text{Na}_2\text{SO}_4 + 2 \text{H}_2\text{O}$, the acid consumes $2(0.0100) = 0.0200 \text{ mol}$ hydroxide. Consequently the excess is $0.0400 - 0.0200 = 0.0200 \text{ mol}$, so the mixture is basic.

Step 3: Find the hydroxide concentration and pH Taking the aqueous volumes as additive, the final volume is $40 + 10 = 50 \text{ L}$. Therefore

$$[\text{OH}^-] = \frac{0.0200}{50} = 4.00 \times 10^{-4} \text{ mol L}^{-1}.$$

$$\text{pOH} = 4 - \log_{10} 4 = 3.39794, \quad \text{pH} = 14 - 3.39794 = 10.60206.$$

The remaining strong base controls the pH; averaging the original pH values would ignore both neutralisation and logarithmic concentration dependence.

Step 4: Conclusion The resultant solution has $\boxed{\text{pH} \approx 10.6}$. pH is dimensionless, and its value above seven agrees with the calculated excess hydroxide.

Quick Tip to Solve: Convert the acid aliquot to proton equivalents before subtracting, and use the combined 50-litre volume only after neutralisation.

Solution:**Concept:**

- A chiral carbon is a tetrahedral carbon attached to four different substituents. Compare entire substituent paths, rather than merely the identities of the atoms directly bonded to the carbon.
- An aromatic or carbonyl carbon is trigonal planar and is not a chiral carbon. A tetrahedral carbon with two identical substituents, such as two hydrogens or two chlorines, is also not chiral.

Step 1: Write the connectivity of chloramphenicol Let Ar represent the para-nitrophenyl group. The side-chain connectivity of chloramphenicol can be written compactly as



This identifies the two neighbouring methine carbons, the terminal hydroxymethyl carbon and the dichloromethyl carbon of the amide side chain. The para-nitro substituent belongs to the aromatic ring, so it does not add a tetrahedral carbon centre.

Step 2: Test the two methine carbons The benzylic $\text{CH}(\text{OH})$ carbon is bonded to hydrogen, hydroxyl, the para-nitrophenyl group and the remaining substituted carbon chain. These are four different groups, so this carbon is chiral. The neighbouring $\text{CH}(\text{NHCOCHCl}_2)$ carbon is bonded to hydrogen, the nitrogen-containing amide group, CH_2OH , and $\text{CH}(\text{OH})\text{Ar}$. The two carbon-linked groups differ in their further attachments, so they are distinct substituents. This carbon is chiral as well.

Step 3: Exclude the remaining carbon atoms The terminal CH_2OH carbon has two identical hydrogen atoms and therefore fails the four-different-groups test. The CHCl_2 carbon has two identical chlorine atoms and also fails it. All six aromatic-ring carbons and the amide carbonyl carbon are trigonal planar. None of these can be counted as an additional chiral carbon. Thus exactly the two adjacent side-chain methine carbons satisfy the criterion.

Step 4: Conclusion Chloramphenicol contains 2 chiral carbon atoms. This is a dimensionless count of stereogenic carbon centres, not the number of possible stereoisomers.

Quick Tip to Solve: Inspect the two side-chain methine carbons first; immediately exclude every CH_2 , CHCl_2 , aromatic and carbonyl carbon.