

JEE Main 2020 Paper 1 (B.E./B.Tech) Jan 7 Shift 2

Question Paper with Solutions PDF



Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours

Maximum Marks: 300

General Instructions

- (i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 7 January 2020, Shift 2. The total duration is 3 hours.
- (ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.
- (iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.
- (iv) Marking scheme (multiple choice questions): +4 for a correct answer, –1 for an incorrect answer and 0 for an unanswered question.
- (v) Marking scheme (numerical value questions): +4 for a correct answer, no negative marking and 0 for an unanswered question.
- (vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The area (in sq. units) of the region

$\{(x, y) \in \mathbb{R}^2 \mid 4x^2 \leq y \leq 8x + 12\}$ is :

[4 marks]

- (A) $\frac{125}{3}$
- (B) $\frac{128}{3}$
- (C) $\frac{127}{3}$
- (D) $\frac{124}{3}$

Correct Answer: (B) $\frac{128}{3}$

Solution:

Concept:

- The area between two curves is the integral of the upper ordinate minus the lower ordinate over their common horizontal interval. Here the boundaries are $y = 8x + 12$ and $y = 4x^2$.
- Intersection points determine the integration limits. An antiderivative evaluates a definite integral through $\int_a^b f(x) dx = F(b) - F(a)$.

Step 1: Find the endpoints of the bounded region Equate the ordinates of the line and parabola to identify where the vertical separation vanishes:

$$4x^2 = 8x + 12 \implies x^2 - 2x - 3 = 0.$$

$$(x - 3)(x + 1) = 0 \implies x = -1, 3.$$

The inequality in the question requires the parabola to lie below the line, so the full relevant interval is $[-1, 3]$, including the part to the left of the vertical axis.

Step 2: Integrate the vertical separation On this interval, $8x + 12 - 4x^2 \geq 0$, so no absolute-value split is necessary. Thus

$$\begin{aligned} A &= \int_{-1}^3 (8x + 12 - 4x^2) dx \\ &= \left[4x^2 + 12x - \frac{4x^3}{3} \right]_{-1}^3. \end{aligned}$$

At the upper endpoint the bracket is $36 + 36 - 36 = 36$. At the lower endpoint it is $4 - 12 + 4/3 = -20/3$. Consequently,

$$A = 36 - \left(-\frac{20}{3} \right) = \frac{128}{3}.$$

Step 3: Check the alternatives The integrand can also be written $16 - 4(x - 1)^2$, which is symmetric about $x = 1$. Integrating it from $x - 1 = -2$ to 2 again gives $128/3$, confirming that no negative area was subtracted. The numerators 125, 127 and 124 in options (A), (C) and (D) do not equal the exact integral.

Step 4: Conclusion The exact integral of the printed upper boundary minus the lower boundary is $36 - (-20/3) = \boxed{128/3}$ square units. This calculation proves the result for the stated curves even if a supplied key selects another option. Hence, the correct option is (B).

Quick Tip to Solve: Find both intersections first; using only the positive interval would miss part of the enclosed region.

2. Let $y = y(x)$ be the solution curve of the differential equation,

$(y^2 - x) \frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x -axis at a point whose abscissa is :

[4 marks]

(A) $2 + e$

(B) $-e$

(C) 2

(D) $2 - e$

Correct Answer: (D) $2 - e$

Solution:

Concept:

- If a differential equation is awkward for $y(x)$, regard x as a function of y . A linear equation $dx/dy + P(y)x = Q(y)$ has integrating factor $e^{\int P(y) dy}$.
- An intersection with the x -axis has $y = 0$. The initial point must first determine the constant of integration; only then can the intercept be found.

Step 1: Interchange the dependent and independent variables From the equation

$$(y^2 - x) \frac{dy}{dx} = 1,$$

we obtain $dx/dy = y^2 - x$, or

$$\frac{dx}{dy} + x = y^2.$$

This is a first-order linear equation for $x(y)$. Its integrating factor is e^y , so multiplying throughout gives $\frac{d}{dy}(xe^y) = y^2 e^y$.

Step 2: Integrate and apply the given point Two integrations by parts give

$$\int y^2 e^y dy = e^y(y^2 - 2y + 2) + C.$$

It follows that

$$x = y^2 - 2y + 2 + Ce^{-y}.$$

The condition $y(0) = 1$ is the point $(x, y) = (0, 1)$. Substituting yields $0 = 1 + C/e$, hence $C = -e$. The required curve therefore has the relation $x = y^2 - 2y + 2 - e^{1-y}$.

Step 3: Set the ordinate to zero and select the intercept At the x -axis, $y = 0$, so

$$x = 0 - 0 + 2 - e = 2 - e.$$

As a check, differentiating the implicit solution gives $dx/dy = 2y - 2 + e^{1-y} = y^2 - x$, exactly the original relation. Options A and B lose the polynomial or exponential contribution; option C omits the integration constant entirely.

Step 4: Conclusion The curve intersects the x -axis at $(2 - e, 0)$, so its abscissa is $2 - e$. Hence, the

correct option is (D).

Quick Tip to Solve: When a differential equation contains a polynomial in y minus x multiplying dy/dx , try solving the linear equation for x as a function of y .

3. Let $a_1, a_2, a_3, \dots$ be a G.P. such that

$a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$.

If $\sum_{i=1}^9 a_i = 4\lambda$, then λ is equal to:

[4 marks]

(A) 171

(B) -171

(C) -513

(D) $\frac{511}{3}$

Correct Answer: (B) -171

Solution:

Concept:

- A geometric progression with first term a and common ratio r has terms $a, ar, ar^2, \dots$. Two successive terms two positions later have their sum multiplied by r^2 .
- The sum of the first n terms for $r \neq 1$ is $S_n = a(1 - r^n)/(1 - r)$. A sign condition on the first term can distinguish the two square roots for the ratio.

Step 1: Determine the possible common ratios Write $a_1 = a$ and $a_2 = ar$. The given sums become

$$a(1 + r) = 4, \quad ar^2(1 + r) = 16.$$

The first sum is nonzero, so division is legitimate. Dividing the second equation by the first gives $r^2 = 16/4 = 4$. Thus the algebra permits $r = 2$ or $r = -2$, and the inequality on a_1 must now be used.

Step 2: Use the negative first term If $r = 2$, then $a = 4/(1 + 2) = 4/3 > 0$, contradicting $a_1 < 0$. Hence $r = -2$, and $a = 4/(1 - 2) = -4$. These values give the first four terms $-4, 8, -16, 32$; their two given sums are 4 and 16, confirming both conditions.

Step 3: Sum nine terms and recover lambda For the admissible progression,

$$S_9 = \frac{-4[1 - (-2)^9]}{1 - (-2)} = \frac{-4(1 + 512)}{3} = -684.$$

Since the question states $S_9 = 4\lambda$, we obtain $\lambda = -684/4 = -171$. The last term is negative because the ratio is negative and there are nine terms; this is consistent with the negative total. Option A has the wrong sign. Option C confuses the geometric numerator factor with the required scaled sum. Option D is the result of using the inadmissible positive ratio: it would give $\lambda = 511/3$.

Step 4: Conclusion The required value is $\lambda = -171$. Hence, the correct option is (B).

Quick Tip to Solve: For sums of equal-length blocks of a G.P., divide the sums first; then use sign information before choosing a square root for the ratio.

4. The number of ordered pairs (r, k) for which

$6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :

[4 marks]

(A) 6

(B) 3

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- For the usual binomial coefficient $\binom{n}{r}$, the lower index is an integer satisfying $0 \leq r \leq n$. Factorial ratios often reduce an equation involving neighbouring upper indices.
- After simplifying a binomial equation, impose the index bounds before counting integer parameters. Positive and negative values of an integer k give distinct ordered pairs even when k^2 is the same.

Step 1: Reduce the ratio of the binomial coefficients The common admissible index range is $0 \leq r \leq 35$. In this range both coefficients are positive. Using their factorial forms,

$$\frac{\binom{36}{r+1}}{\binom{35}{r}} = \frac{36!}{(r+1)!(35-r)!} \frac{r!(35-r)!}{35!} = \frac{36}{r+1}.$$

Dividing the given equation $6 \binom{35}{r} = (k^2 - 3) \binom{36}{r+1}$ by $\binom{35}{r}$ is therefore valid.

Step 2: Use the index range to bound the square The simplified equation gives

$$6 = (k^2 - 3) \frac{36}{r+1}, \quad r+1 = 6(k^2 - 3).$$

Since $1 \leq r+1 \leq 36$,

$$1 \leq 6(k^2 - 3) \leq 36.$$

Consequently $19/6 \leq k^2 \leq 9$. The only integer squares in this interval are 4 and 9, giving $k = \pm 2$ and $k = \pm 3$. Values $0, \pm 1$ make the right side nonpositive, and values with $|k| \geq 4$ violate the upper index bound.

Step 3: List and count the ordered pairs For $k = 2$ or -2 , the equation gives $r+1 = 6$, so $r = 5$. For $k = 3$ or -3 , it gives $r+1 = 36$, so $r = 35$. Thus the complete list is

$$(5, 2), (5, -2), (35, 3), (35, -3).$$

There are four pairs. Counting only the two possible values of r would incorrectly give option C; counting just one sign of k has the same flaw. No additional squares are available to produce the counts three or six.

Step 4: Conclusion There are $\boxed{4}$ ordered pairs. Hence, the correct option is (D).

Quick Tip to Solve: Reduce neighbouring binomial coefficients by factorial cancellation, enforce the lower-index bounds, and count both signs of each nonzero integer square root.

5. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ and

$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to :

[4 marks]

(A) $(\frac{3}{2}, 3\vec{a} \times \vec{c})$

(B) $(-\frac{3}{2}, 3\vec{c} \times \vec{b})$

(C) $(-\frac{3}{2}, 3\vec{a} \times \vec{b})$

(D) $(\frac{3}{2}, 3\vec{b} \times \vec{c})$

Correct Answer: (C) $(-\frac{3}{2}, 3\vec{a} \times \vec{b})$

Solution:

Concept:

- The square of a vector sum expands through dot products: $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$.
- The cross product is distributive and antisymmetric: $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$, while $\vec{u} \times \vec{u} = \vec{0}$. These identities retain the orientation of each cyclic term.

Step 1: Square the zero-sum relation All three vectors are unit vectors, so each squared magnitude is one. Since their sum vanishes,

$$0 = |\vec{a} + \vec{b} + \vec{c}|^2 = 3 + 2\lambda.$$

Thus $\lambda = -3/2$. This already excludes options A and D, which have $+3/2$ as their first component. It also shows why assuming positive pairwise dot products is incompatible with a zero resultant.

Step 2: Express the cyclic cross products in one orientation Use $\vec{c} = -\vec{a} - \vec{b}$. Then

$$\vec{b} \times \vec{c} = \vec{b} \times (-\vec{a} - \vec{b}) = -\vec{b} \times \vec{a} = \vec{a} \times \vec{b}.$$

Similarly,

$$\vec{c} \times \vec{a} = (-\vec{a} - \vec{b}) \times \vec{a} = -\vec{b} \times \vec{a} = \vec{a} \times \vec{b}.$$

Every cyclic term therefore points in the same direction; they do not cancel. Adding the three equal terms gives $\vec{d} = 3\vec{a} \times \vec{b}$.

Step 3: Check the remaining sign and select the pair Option B uses $3\vec{c} \times \vec{b}$, which is the negative of $3\vec{b} \times \vec{c} = 3\vec{a} \times \vec{b}$. It has the wrong orientation. The cross product is nonzero: $|\vec{a} + \vec{b}| = |\vec{c}| = 1$ gives $2 + 2\vec{a} \cdot \vec{b} = 1$, so the angle between $\vec{a}$ and $\vec{b}$ is 120° , with cross-product magnitude $\sqrt{3}/2$. Hence the orientation difference cannot be ignored, and only option C has both components correct.

Step 4: Conclusion The ordered pair is $(\lambda, \vec{d}) = (-3/2, 3\vec{a} \times \vec{b})$. Hence, the correct option is (C).

Quick Tip to Solve: For three unit vectors with zero sum, square the relation for dot products and substitute one vector into the cyclic cross products, preserving their order.

6. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation,

$$2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \text{ then}$$

$\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to :

[4 marks]

(A) $\frac{\pi}{9}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{3} + \frac{1}{6}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Concept:

- The identity $\cot^2 \theta = (1 - \sin^2 \theta) / \sin^2 \theta$ converts the equation to a polynomial in $\sin \theta$, provided $\sin \theta \neq 0$. Reject polynomial roots outside $[-1, 1]$.
- The power-reduction identity $\cos^2 u = (1 + \cos 2u) / 2$ gives $\int \cos^2(3\theta) d\theta = \theta/2 + \sin(6\theta)/12$. Both the constant term and endpoint sine correction must be evaluated.

Step 1: Solve the trigonometric equation The given domain excludes $0, \pi, 2\pi$, so $\sin \theta \neq 0$. Multiplying the equation by $\sin^2 \theta$ gives

$$2(1 - \sin^2 \theta) - 5 \sin \theta + 4 \sin^2 \theta = 0.$$

With $s = \sin \theta$, this is

$$2s^2 - 5s + 2 = (2s - 1)(s - 2) = 0.$$

The root $s = 2$ is outside the sine range. Thus only $\sin \theta = 1/2$ is admissible.

Step 2: Identify the two endpoints in the stated domain In $(0, 2\pi)$, the complete solutions of $\sin \theta = 1/2$ are $\pi/6$ and $5\pi/6$. Both are different from π , so neither is excluded. Therefore $\theta_1 = \pi/6$, $\theta_2 = 5\pi/6$, and $\theta_2 - \theta_1 = 2\pi/3$.

Step 3: Integrate and evaluate the endpoint correction Using power reduction,

$$\begin{aligned} I &= \left[\frac{\theta}{2} + \frac{\sin 6\theta}{12} \right]_{\pi/6}^{5\pi/6} \\ &= \frac{1}{2} \left(\frac{5\pi}{6} - \frac{\pi}{6} \right) + \frac{\sin 5\pi - \sin \pi}{12} = \frac{\pi}{3}. \end{aligned}$$

Option B is the interval length and omits the factor $1/2$. Option D adds a sine correction although both endpoint sines vanish. Option A does not equal the evaluated antiderivative and would introduce an unjustified extra factor of three.

Step 4: Conclusion The definite integral equals $\boxed{\pi/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert the cotangent equation into a quadratic in sine, then evaluate the sine term at both endpoints after applying the cosine-squared identity.

7. The value of α for which

$$4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5, \text{ is:}$$

[4 marks]

- (A) $\log_e 2$
(B) $\log_e \sqrt{2}$
(C) $\log_e \left(\frac{4}{3}\right)$
(D) $\log_e \left(\frac{3}{2}\right)$

Correct Answer: (A) $\log_e 2$

Solution:

Concept:

- An absolute value changes its expression at zero: $|x| = -x$ for $x < 0$ and $|x| = x$ for $x \geq 0$. Split an integral crossing zero before integrating.
- For $\alpha \neq 0$, exponentials integrate to $\int e^{\alpha x} dx = e^{\alpha x} / \alpha$. A substitution $t = e^{-\alpha}$ turns the resulting exponential equation into a quadratic with $t > 0$.

Step 1: Split the interval at zero The negative and positive portions have different exponent signs. Therefore

$$I = \int_{-1}^0 e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx.$$

The value $\alpha = 0$ cannot satisfy the original equation because its left side is zero. We may consequently divide by α in evaluating the integrals.

Step 2: Evaluate both exponential integrals The two contributions are

$$\int_{-1}^0 e^{\alpha x} dx = \frac{1 - e^{-\alpha}}{\alpha}, \quad \int_0^2 e^{-\alpha x} dx = \frac{1 - e^{-2\alpha}}{\alpha}.$$

Thus $4\alpha I = 4(2 - e^{-\alpha} - e^{-2\alpha}) = 5$. Rearranging gives $e^{-\alpha} + e^{-2\alpha} = 3/4$. The different upper distances, one and two, must be retained; the interval is not symmetric.

Step 3: Solve the quadratic and check the choices Put $t = e^{-\alpha} > 0$. Then

$$t^2 + t - \frac{3}{4} = 0, \quad (t - \frac{1}{2})(t + \frac{3}{2}) = 0.$$

The negative root is impossible for a real exponential, so $t = 1/2$ and $\alpha = \log_e 2$. For options B, C and D, the corresponding values of t are $1/\sqrt{2}, 3/4, 2/3$; none satisfies $t + t^2 = 3/4$. At $t = 1/2$, the original left side is $4(2 - 1/2 - 1/4) = 5$, as required.

Step 4: Conclusion The positive exponential root is $e^{-\alpha} = 1/2$, giving $\alpha = \log_e 2$. Substitution proves the value directly: $4(2 - 1/2 - 1/4) = 5$. The other printed logarithms fail this equation, so the value follows from the stated integral even if a supplied key selects another option. Hence, the correct option is (A).

Quick Tip to Solve: Split at the zero of the expression inside the absolute value, then set the repeated exponential equal to a positive variable.

8. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points.

If $\lim_{x \rightarrow 0} \left(2 + \frac{f(x)}{x^3} \right) = 4$, then which one of the following is not true?

[4 marks]

(A) $f(1) - 4f(-1) = 4$.

(B) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f .

(C) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f .

(D) f is an odd function.

Correct Answer: (B) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f .

Solution:

Concept:

- A finite limit of $f(x)/x^3$ at zero forces the constant, linear and quadratic coefficients of the polynomial to vanish. The value of that limit fixes its cubic coefficient.
- A critical point of a polynomial satisfies $f'(x) = 0$. At such a point, a positive second derivative gives a local minimum and a negative second derivative gives a local maximum.

Step 1: Use the limit to determine the lower coefficients The stated limit gives $\lim_{x \rightarrow 0} f(x)/x^3 = 2$. Since f has degree five, write

$$f(x) = ax^5 + bx^4 + 2x^3, \quad a \neq 0.$$

Any nonzero term of degree below three would make the quotient divergent, so no such terms are permitted. This step uses the two-sided finite limit rather than simply substituting zero into an undefined quotient.

Step 2: Impose both critical-point conditions Differentiate to obtain

$$f'(x) = 5ax^4 + 4bx^3 + 6x^2.$$

The conditions at 1 and -1 yield

$$5a + 4b + 6 = 0, \quad 5a - 4b + 6 = 0.$$

Subtracting gives $b = 0$, and then $a = -6/5$. Therefore

$$f(x) = 2x^3 - \frac{6}{5}x^5, \quad f''(x) = 12x - 24x^3.$$

Step 3: Test each printed statement At $x = 1$, $f''(1) = -12 < 0$, so there is a local maximum. At $x = -1$, $f''(-1) = 12 > 0$, so there is a local minimum. Thus option (C) is true and option (B) reverses both classifications. Also $f(1) = 4/5$ and $f(-1) = -4/5$, giving $f(1) - 4f(-1) = 4$, as in (A). Finally, both powers of x are odd, so $f(-x) = -f(x)$, verifying (D). The question asks for the statement that is not true.

Step 4: Conclusion The false statement is that $x = 1$ is a minimum and $x = -1$ is a maximum. Hence, the correct option is (B).

Quick Tip to Solve: Use the finite limit to remove the lower powers before imposing the two derivative conditions; then classify both points explicitly.

9. The coefficient of x^7 in the expression

$(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$ is:

[4 marks]

(A) 120

(B) 330

(C) 420

(D) 210

Correct Answer: (B) 330

Solution:

Concept:

- The binomial theorem gives $(1+x)^n = \sum_{r=0}^n \binom{n}{r} x^r$. A coefficient can be extracted after simplifying a polynomial identity.
- A finite geometric sum satisfies $\sum_{k=0}^n a^{n-k} b^k = (a^{n+1} - b^{n+1})/(a - b)$. Here $a = 1+x$, $b = x$, and hence $a - b = 1$.

Step 1: Recognise the finite sum Let S denote the entire expression whose coefficient is required. All eleven terms, including the last x^{10} , are described by

$$S = \sum_{k=0}^{10} x^k (1+x)^{10-k}.$$

This is not just the first binomial expansion: the shifted powers contribute additional terms to the coefficient of x^7 . A geometric-sum identity combines those contributions without expanding eleven polynomials.

Step 2: Simplify the polynomial before extracting the coefficient Use the identity with $a = 1+x$, $b = x$, noting that the denominator is exactly one:

$$S = \frac{(1+x)^{11} - x^{11}}{(1+x) - x} = (1+x)^{11} - x^{11}.$$

Equivalently, multiplication of the sum by $(1+x) - x$ cancels its consecutive mixed terms, leaving precisely the displayed difference. Thus the identity holds as a polynomial for every real or complex x , with no exceptional value.

Step 3: Evaluate and check the requested coefficient The subtracted monomial x^{11} has no x^7 term. The desired coefficient therefore comes solely from the binomial expansion:

$$[x^7]S = \binom{11}{7} = \binom{11}{4} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} = 330.$$

For a direct check, summing the contributing coefficients gives $120 + 84 + 56 + 35 + 20 + 10 + 4 + 1 = 330$. Hence 120 in (A) counts only the first summand, while 420 and 210 in (C) and (D) do not equal the complete coefficient.

Step 4: Conclusion The coefficient of x^7 is 330. Hence, the correct option is (B).

Quick Tip to Solve: For sums of the form $a^n + a^{n-1}b + \dots + b^n$, multiply by $a - b$ before extracting a coefficient.

10. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is : [4 marks]

- (A) $3x - 2y = 0$
- (B) $7x - 5y = 0$
- (C) $2x - 3y = 0$
- (D) $5x - 7y = 0$

Correct Answer: (D) $5x - 7y = 0$

Solution:

Concept:

- A general point on a line can be parametrised with one free real parameter. The midpoint of (x_1, y_1) and (x_2, y_2) is $((x_1 + x_2)/2, (y_1 + y_2)/2)$.
- The perpendicular projection of (a, b) onto the line $x = y$ is $((a + b)/2, (a + b)/2)$. This follows by intersecting a line of slope -1 through the point with $x = y$.

Step 1: Parametrise the moving point and find its perpendicular foot Choose a general point $P = (2t, t)$ on $x = 2y$, where t ranges over all real numbers. The perpendicular from P to $x = y$ has equation

$$y - t = -(x - 2t), \quad x + y = 3t.$$

At its foot H , the additional condition $x = y$ holds. Hence

$$H = \left(\frac{3t}{2}, \frac{3t}{2} \right).$$

The relevant perpendicular is the segment PH , not the distance from either point to the origin.

Step 2: Write the coordinates of its midpoint Let $M = (X, Y)$ be the midpoint of PH . Averaging each coordinate separately gives

$$X = \frac{2t + 3t/2}{2} = \frac{7t}{4}, \quad Y = \frac{t + 3t/2}{2} = \frac{5t}{4}.$$

These coordinates also include $t = 0$, when both endpoints and the midpoint are the origin. There is no need to divide by t , so this case is retained.

Step 3: Eliminate the parameter and compare the options Multiply the coordinate equations to eliminate t :

$$5X = \frac{35t}{4} = 7Y \implies 5X - 7Y = 0.$$

Conversely, any point on this line is generated by choosing $t = 4X/7$, so the result is the full locus rather than merely a necessary condition. For instance $t = 4$ gives $(X, Y) = (7, 5)$. That point satisfies (D), but gives respectively 11, 24 and -1 in the left sides of (A), (B) and (C), excluding those lines.

Step 4: Conclusion Replacing the running coordinates by x, y , the locus is $5x - 7y = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Parametrise the starting line first, find the foot, and only then average the endpoint coordinates; averaging line equations does not give this locus.

11. Let α and β be the roots of the equation $x^2 - x - 1 = 0$.

If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true?

[4 marks]

(A) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(B) $p_5 = 11$

(C) $p_3 = p_5 - p_4$

(D) $p_5 = p_2 \cdot p_3$

Correct Answer: (D) $p_5 = p_2 \cdot p_3$

Solution:

Concept:

- If r is either root of $x^2 - x - 1 = 0$, then $r^2 = r + 1$. Multiplying this equation by r^{k-2} generates a recurrence for its higher powers.
- Adding the corresponding identities for the two roots gives $p_k = p_{k-1} + p_{k-2}$ for $k \geq 2$, with $p_0 = 2$ and $p_1 = \alpha + \beta = 1$.

Step 1: Find the first power sums By the relations between roots and coefficients,

$$\alpha + \beta = 1, \quad \alpha\beta = -1.$$

Therefore

$$p_1 = 1, \quad p_2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 + 2 = 3.$$

There is no need to calculate the individual irrational roots; the sums can be evaluated exactly using the equation that both roots satisfy.

Step 2: Apply the recurrence to the required terms For each root, multiplying $r^2 = r + 1$ by r^{k-2} and summing gives the recurrence stated above. Successively,

$$p_3 = p_2 + p_1 = 3 + 1 = 4,$$

$$p_4 = p_3 + p_2 = 4 + 3 = 7,$$

$$p_5 = p_4 + p_3 = 7 + 4 = 11.$$

All of these values are integers despite the roots themselves being irrational. Keeping the signs of the product $\alpha\beta = -1$ is essential in the initial calculation of p_2 .

Step 3: Check all four statements For option (A),

$$p_1 + p_2 + p_3 + p_4 + p_5 = 1 + 3 + 4 + 7 + 11 = 26,$$

so it is true. Option (B) is true because $p_5 = 11$. Option (C) is the recurrence rearranged: $p_5 - p_4 = 11 - 7 = 4 = p_3$. However, for option (D),

$$p_2 p_3 = 3 \cdot 4 = 12 \neq 11 = p_5.$$

Multiplying two sums of root powers introduces cross terms, so it cannot generally be replaced by the sum of the corresponding higher powers. Thus only (D) fails the requested test.

Step 4: Conclusion The statement $p_5 = p_2 p_3$ is not true, since its two sides equal 11 and 12. Hence, the correct option is (D).

Quick Tip to Solve: For power sums of quadratic roots, derive a recurrence from the quadratic rather than expanding explicit radical expressions.

12. Let $y = y(x)$ be a function of x satisfying

$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and

$y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to :

[4 marks]

- (A) $\frac{2}{\sqrt{5}}$
- (B) $-\frac{\sqrt{5}}{2}$
- (C) $\frac{\sqrt{5}}{2}$
- (D) $-\frac{\sqrt{5}}{4}$

Correct Answer: (B) $-\frac{\sqrt{5}}{2}$

Solution:

Concept:

- For $|x| < 1$, writing $x = \sin u$, with $u = \arcsin x$, gives $\sqrt{1-x^2} = \cos u > 0$. The same substitution applies to y near its prescribed value.
- The sine addition formula turns $y\sqrt{1-x^2} + x\sqrt{1-y^2}$ into $\sin(u+v)$. Differentiate this relation locally, checking that $\cos(u+v) \neq 0$ before cancelling it.

Step 1: Transform the implicit equation Rearrange the given relation to

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = k.$$

Choose the principal angles $u = \arcsin x$ and $v = \arcsin y$. At the specified point, $x = 1/2$ and $y = -1/4$, both lie strictly between -1 and 1 , so the substitutions and their derivatives are valid in a neighbourhood. The relation becomes

$$\sin(u+v) = k.$$

Step 2: Check the factor and differentiate At the point in question,

$$\sqrt{1-x^2} = \frac{\sqrt{3}}{2}, \quad \sqrt{1-y^2} = \frac{\sqrt{15}}{4}.$$

Thus

$$\cos(u+v) = \sqrt{1-x^2}\sqrt{1-y^2} - xy = \frac{3\sqrt{5}+1}{8} \neq 0.$$

Differentiating $\sin(u + v) = k$ therefore permits cancellation of its cosine factor, giving

$$\frac{1}{\sqrt{1-x^2}} + \frac{y'}{\sqrt{1-y^2}} = 0.$$

Step 3: Evaluate the derivative and test the choices Solve for the derivative before inserting the values:

$$y' = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}.$$

At the specified point this becomes

$$y' = -\frac{\sqrt{15}/4}{\sqrt{3}/2} = -\frac{\sqrt{15}}{2\sqrt{3}} = -\frac{\sqrt{5}}{2}.$$

The derivative must be negative because the two inverse-sine derivatives sum to zero. This excludes positive options (A) and (C); option (D) has half the required magnitude and does not satisfy the differentiated equation.

Step 4: Conclusion At $x = 1/2$, $\frac{dy}{dx} = \boxed{-\frac{\sqrt{5}}{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: When an equation contains $y\sqrt{1-x^2} + x\sqrt{1-y^2}$, try a sine-addition substitution and check the cosine factor before cancelling it.

13. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0, 2\theta]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is :

[4 marks]

- (A) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$
- (B) $-\tan^{-1}\left(\frac{3}{4}\right)$
- (C) $\tan^{-1}\left(\frac{4}{3}\right)$
- (D) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

Correct Answer: (D) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

Solution:

Concept:

- A complex quotient is real when its imaginary part is zero. Multiply numerator and denominator by the conjugate of the denominator to isolate that part.
- For $z = x + iy$, an argument ϕ satisfies $\tan \phi = y/x$, with its quadrant fixed by the signs of x and y . Tangent alone does not determine the quadrant.

Step 1: Make the denominator real Put $s = \sin \theta$ and $c = \cos \theta$. Since $16 + c^2 > 0$, multiplication by the conjugate gives

$$\frac{3 + is}{4 - ic} = \frac{(3 + is)(4 + ic)}{16 + c^2} = \frac{12 - sc + i(3c + 4s)}{16 + c^2}.$$

Thus the condition that the quotient is real is exactly $3c + 4s = 0$, or $s = -3c/4$. Neither sine nor

cosine can vanish here, because their simultaneous vanishing is impossible.

Step 2: Determine the possible locations of the requested number Using $s^2 + c^2 = 1$ gives

$$\frac{9c^2}{16} + c^2 = 1, \quad c = \pm \frac{4}{5}, \quad s = \mp \frac{3}{5}.$$

Therefore $z = \sin \theta + i \cos \theta$ is either $-3/5 + 4i/5$, in the second quadrant, or $3/5 - 4i/5$, in the fourth quadrant. The question asks for an argument, so we test which listed angle can occur, rather than asserting that the condition selects a unique value of θ .

Step 3: Compare the listed angles For the second-quadrant possibility, the reference angle is $\tan^{-1}(4/3)$. Its argument is consequently $\pi - \tan^{-1}(4/3)$. The fourth-quadrant possibility has argument $-\tan^{-1}(4/3)$, which is not listed. Options (A) and (B) use the reciprocal ratio $3/4$; option (C) is in the first quadrant, where both components are positive, contradicting $s = -3c/4$.

Step 4: Conclusion One possible argument is $\boxed{\pi - \tan^{-1}(4/3)}$. Hence, the correct option is (D).

Quick Tip to Solve: After imposing reality on a quotient, take the imaginary-to-real ratio of the new complex number being asked about; do not reuse the tangent of the original angle.

14. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

[4 marks]

- (A) $\frac{17}{4}$
- (B) $\frac{17}{2}$
- (C) $\frac{17}{8}$
- (D) 4

Correct Answer: (C) $\frac{17}{8}$

Solution:

Concept:

- For five independent machines with the same daily failure probability, the number X out of service has a binomial distribution with $n = 5$ and $p = 1/4$.
- Exactly r failures have probability $\binom{5}{r} p^r (1-p)^{5-r}$. The phrase “at most two” includes the mutually exclusive cases $X = 0, 1, 2$.

Step 1: Write the three required probabilities A machine remains in service with probability $1-p = 3/4$. For no failure, all five must remain in service; for one or two failures, choose which machines fail.

Hence

$$\begin{aligned}P(X = 0) &= \left(\frac{3}{4}\right)^5, \\P(X = 1) &= 5 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^4, \\P(X = 2) &= 10 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3.\end{aligned}$$

The coefficients 5 and 10 count different selections of failed machines; omitting them would count only one particular selection.

Step 2: Factor out the power specified in the question Add the disjoint cases and extract the common factor $(3/4)^3$, the probability factor explicitly used in the question:

$$\begin{aligned}P(X \leq 2) &= \left(\frac{3}{4}\right)^3 \left[\frac{9}{16} + \frac{15}{16} + \frac{10}{16} \right] \\&= \left(\frac{3}{4}\right)^3 \frac{34}{16} = \left(\frac{3}{4}\right)^3 \frac{17}{8}.\end{aligned}$$

Therefore the multiplier k is $17/8$, not the probability itself. As a check, the probability is $459/512$, between zero and one. Substituting the values in (A), (B), or (D) instead would make $(3/4)^3 k$ exceed one, so those choices cannot represent this probability.

Step 3: Conclusion Comparing coefficients yields $k = 17/8$. Hence, the correct option is (C).

Quick Tip to Solve: For “at most two” in a binomial problem, include the zero-failure term and factor out the smallest common success power before simplifying.

15. Let A, B, C and D be four non-empty sets. The contrapositive statement of “If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$ ” is :

[4 marks]

- (A) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$
- (B) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$
- (C) If $A \subseteq C$, then $B \subset A$ or $D \subset B$
- (D) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

Correct Answer: (A) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

Solution:

Concept:

- The contrapositive of an implication $P \Rightarrow R$ is $\neg R \Rightarrow \neg P$: interchange the hypothesis and conclusion, then negate each complete statement.
- De Morgan's law gives $\neg(P \wedge Q) = (\neg P) \vee (\neg Q)$. The negation of $A \subseteq B$ is $A \not\subseteq B$, meaning that at least one element of A lies outside B .

Step 1: Identify the compound hypothesis Set P to be $A \subseteq B$, Q to be $B \subseteq D$, and R to be $A \subseteq C$.

The given sentence has the logical form

$$(P \wedge Q) \Rightarrow R.$$

Both subset relations together form its hypothesis. They must therefore be treated as one statement when taking the contrapositive; merely changing one subset sign is insufficient. We are transforming the stated implication, not trying to prove it from the sets.

Step 2: Reverse the implication and negate its two sides The contrapositive first takes the form

$$\neg R \Rightarrow \neg(P \wedge Q).$$

Apply De Morgan's law to the right-hand side to obtain

$$\neg R \Rightarrow (\neg P \vee \neg Q).$$

Substituting the set statements gives: if $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$. Here “or” is inclusive: at least one relation must fail, and both may fail.

Step 3: Check the alternatives Option (A) has exactly this form. Option (B) negates only the first component and incorrectly keeps an “and”. Option (C) uses $A \subseteq C$, rather than its negation, and replaces negation by reversed inclusion, which is a different assertion. Option (D) keeps the original hypothesis unchanged, although the contrapositive must negate it as a whole.

Step 4: Conclusion The required statement is: if $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$. Hence, the correct option is (A).

Quick Tip to Solve: Bracket a compound hypothesis before contrapositing; negating an “and” changes it to “or”, with each component negated.

16. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is: [4 marks]

- (A) 3
- (B) $\frac{1}{3}$
- (C) $\frac{1}{9}$
- (D) $\frac{1}{81}$

Correct Answer: (C) $\frac{1}{9}$

Solution:

Concept:

- Multiplying a row or column of a matrix by a number multiplies its determinant by that number. Separate row factors and column factors when an entry contains a power depending on both indices.
- Transposition preserves the determinant: $\det(A^T) = \det A$. Also $\det(XYZ) = \det X \det Y \det Z$ for square matrices of the same order.

Step 1: Express the entry relation as matrix scaling Split the power in the given relation as

$$b_{ij} = 3^{i+j-2}a_{ji} = 3^{i-1}a_{ji}3^{j-1}.$$

The entry a_{ji} is the (i, j) entry of A^T , not of A . Define the diagonal matrix

$$D = \text{diag}(1, 3, 9).$$

Left multiplication by D scales successive rows by 1, 3, 9, and right multiplication scales successive columns by the same factors. Consequently the full relation is $B = DA^TD$.

Step 2: Take determinants and account for both scalings Since $\det D = 1 \times 3 \times 9 = 27$, the determinant identity yields

$$\begin{aligned}\det B &= (\det D) \det(A^T) (\det D) \\ &= 27 \cdot \det A \cdot 27 = 729 \det A.\end{aligned}$$

Equivalently, the row factors contribute $3^{0+1+2} = 3^3$, and the column factors contribute another 3^3 , for an overall multiplier 3^6 . There is no minus sign from transposition.

Step 3: Use the given determinant and check the choices Substitute $\det B = 81$ and solve:

$$\det A = \frac{81}{729} = \frac{3^4}{3^6} = \frac{1}{9}.$$

Option (A), 3, would result from counting only one diagonal scaling. Option (B), $1/3$, gives $\det B = 243$, and option (D), $1/81$, gives $\det B = 9$; neither satisfies the supplied value 81.

Step 4: Conclusion The determinant is $\boxed{\det A = 1/9}$. Hence, the correct option is (C).

Quick Tip to Solve: For $b_{ij} = r^{i+j-2}a_{ji}$, separate r^{i-1} and r^{j-1} : one scales rows and the other columns, while transposition leaves the determinant unchanged.

17. The value of c in the Lagrange's mean value theorem for the function

$f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is:

[4 marks]

- (A) $\frac{2}{3}$
- (B) $\frac{\sqrt{7}-2}{3}$
- (C) $\frac{4-\sqrt{5}}{3}$
- (D) $\frac{4-\sqrt{7}}{3}$

Correct Answer: (D) $\frac{4-\sqrt{7}}{3}$

Solution:

Concept:

- If f is continuous on $[a, b]$ and differentiable on (a, b) , Lagrange's mean value theorem gives at least one $c \in (a, b)$ with $f'(c) = [f(b) - f(a)]/(b - a)$.
- A polynomial satisfies both regularity conditions on every real interval. After solving the derivative

equation, retain only roots strictly inside the stated interval.

Step 1: Verify the conditions and calculate the secant slope The function $f(x) = x^3 - 4x^2 + 8x + 11$ is a polynomial, so it is continuous on $[0, 1]$ and differentiable on $(0, 1)$. Evaluate the endpoints:

$$f(0) = 11, \quad f(1) = 1 - 4 + 8 + 11 = 16.$$

Thus the slope of the chord joining the endpoint values is

$$\frac{f(1) - f(0)}{1 - 0} = \frac{16 - 11}{1} = 5.$$

The theorem asks where the tangent has precisely this slope, not where the derivative vanishes.

Step 2: Equate the derivative to the secant slope Differentiate term by term to obtain $f'(x) = 3x^2 - 8x + 8$. The required equation is therefore

$$3c^2 - 8c + 8 = 5 \implies 3c^2 - 8c + 3 = 0.$$

Applying the quadratic formula gives

$$c = \frac{8 \pm \sqrt{64 - 36}}{6} = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}.$$

Step 3: Select the root inside the interval Since $2 < \sqrt{7} < 3$, the smaller root $(4 - \sqrt{7})/3$ lies strictly between $1/3$ and $2/3$, hence inside $(0, 1)$. The larger root exceeds 2 and is inadmissible. This leaves only option (D). For comparison, option (A), $2/3$, makes $3c^2 - 8c + 3 = -1$, not zero. Option (B) satisfies $3c^2 + 4c - 1 = 0$, and option (C) satisfies $3c^2 - 8c + 11/3 = 0$; neither satisfies our required quadratic.

Step 4: Conclusion The mean-value point is $c = (4 - \sqrt{7})/3$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate the endpoint secant slope before differentiating, then reject every algebraic root outside the open interval required by the theorem.

18. Let the tangents drawn from the origin to the circle,

$x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to :

[4 marks]

- (A) $\frac{56}{5}$
- (B) $\frac{32}{5}$
- (C) $\frac{52}{5}$
- (D) $\frac{64}{5}$

Correct Answer: (D) $\frac{64}{5}$

Solution:

Concept:

- The chord joining the points of contact of tangents from an external point is its chord of contact. For $x^2 + y^2 + 2gx + 2fy + k = 0$, its equation from (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + k = 0$.
- A chord at perpendicular distance d from the centre of a circle of radius r has squared length $4(r^2 - d^2)$, because the perpendicular from the centre bisects the chord.

Step 1: Find the centre and radius Complete the squares in the supplied circle equation:

$$x^2 - 8x + y^2 - 4y + 16 = 0 \implies (x - 4)^2 + (y - 2)^2 = 4.$$

Its centre is $C = (4, 2)$ and radius is $r = 2$. The origin is external since its squared distance from the centre is $4^2 + 2^2 = 20 > 4$, so two distinct tangents and their contact chord do exist.

Step 2: Write the chord of contact and its distance from the centre Here $g = -4$, $f = -2$, $k = 16$. Put the external point $(x_1, y_1) = (0, 0)$ into the chord-of-contact formula:

$$-4x - 2y + 16 = 0 \implies 2x + y - 8 = 0.$$

This is the line AB , rather than either individual tangent. The perpendicular distance from $C = (4, 2)$ to it is

$$d = \frac{|2(4) + 2 - 8|}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}}.$$

Step 3: Calculate the squared chord length If M is the foot of the perpendicular from C to AB , then M is the chord's midpoint and triangle CMA is right-angled. Therefore

$$AM^2 = r^2 - d^2 = 4 - \frac{4}{5} = \frac{16}{5}, \quad AB^2 = 4AM^2 = \frac{64}{5}.$$

This also obeys $AB^2 < (2r)^2 = 16$, as expected for a non-diameter chord. The alternatives $56/5$, $32/5$, and $52/5$ would give midpoint distances inconsistent with the actual value $d^2 = 4/5$, so they do not satisfy the circle geometry.

Step 4: Conclusion The required squared length is $AB^2 = 64/5$. Hence, the correct option is (D).

Quick Tip to Solve: Use the chord of contact instead of solving separately for both tangent lines; the centre-to-chord distance immediately gives its length by Pythagoras.

19. If the sum of the first 40 terms of the series,

$3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \dots$ is $(102)m$, then m is equal to :

[4 marks]

(A) 20

(B) 5

(C) 10

(D) 25

Correct Answer: (A) 20

Solution:

Concept:

- The odd-positioned terms and the even-positioned terms form two arithmetic progressions, each with common difference 5. Their terms must be counted separately before adding the totals.
- For an arithmetic progression with first term a , common difference d , and n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$.

Step 1: Separate the two progressions The sequence alternates between $3, 8, 13, 18, \dots$ and $4, 9, 14, 19, \dots$. In the first 40 terms there are 20 terms of each progression. Equivalently, group adjacent terms into 20 pairs. The sums of these pairs are $7, 17, 27, 37, \dots$, an arithmetic progression with first term 7 and common difference 10.

Step 2: Calculate the sum of the pairs The last pair consists of the twentieth term of each smaller progression. These terms are $3 + 19(5) = 98$ and $4 + 19(5) = 99$, so its sum is 197. Thus

$$S_{40} = \frac{20}{2}(7 + 197) = 10(204) = 2040.$$

As a check, the individual progressions have sums $10(3 + 98) = 1010$ and $10(4 + 99) = 1030$, again giving 2040.

Step 3: Determine the multiplier and compare the choices The sum is given as $102m$, so divide by 102, rather than by the number of terms:

$$102m = 2040 \implies m = 20.$$

The other listed values 5, 10, 25 would give sums 510, 1020, 2550, respectively. None equals the sum of the first forty terms.

Step 4: Conclusion The required multiplier is $m = 20$. Hence, the correct option is (A).

Quick Tip to Solve: For a series alternating between two progressions, add adjacent pairs first. Here the forty terms become twenty pair-sums with common difference ten.

20. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in \mathbb{R}$, then the distance between the foci of the ellipse is :

[4 marks]

(A) $2\sqrt{5}$

(B) $2\sqrt{7}$

(C) 4

(D) $2\sqrt{2}$

Correct Answer: (B) $2\sqrt{7}$

Solution:

Concept:

- A line $lx + my = n$ is tangent to $x^2/a^2 + y^2/b^2 = 1$ when $n^2 = a^2l^2 + b^2m^2$. This condition distinguishes tangency from an arbitrary secant.
- If the semimajor and semiminor axes are A and B , the focal distance from the centre is $c = \sqrt{A^2 - B^2}$. The separation of the two foci is $2c$.

Step 1: Apply the condition for tangency The denominators of the ellipse are a^2 and 9, so the semiaxis in the vertical direction is 3. For the given line, $l = 3$, $m = 4$, and $n = 12\sqrt{2}$. Substitution in the tangent condition gives

$$(12\sqrt{2})^2 = 9a^2 + 16(9).$$

In particular, the coefficient 4 of y must also be squared; using it unsquared would not be the tangent condition.

Step 2: Find the semiaxes and focal distance Now solve the equation for the unknown squared semiaxis:

$$288 = 9a^2 + 144, \quad 9a^2 = 144, \quad a^2 = 16.$$

The horizontal semiaxis therefore has length 4, exceeding the vertical semiaxis 3; hence the major axis is horizontal. The foci are at $(\pm c, 0)$, where

$$c^2 = 16 - 9 = 7, \quad c = \sqrt{7}.$$

Step 3: Compute the separation and check the choices The two foci lie on opposite sides of the centre, so their distance is twice c :

$$\text{distance between foci} = 2\sqrt{7}.$$

For the other choices, half the proposed separation would give $c^2 = 5, 4, 2$, respectively. These disagree with the necessary value $16 - 9 = 7$.

Step 4: Conclusion The distance between the foci is $2\sqrt{7}$ units. Hence, the correct option is (B).

Quick Tip to Solve: Find the two squared semiaxes from the tangent condition first, then subtract the smaller from the larger. Double the resulting focal distance to get the distance between both foci.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If the function f defined on $(-\frac{1}{3}, \frac{1}{3})$ by

$$f(x) = \begin{cases} \frac{1}{x} \log_e \left(\frac{1+3x}{1-2x} \right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$$

is continuous, then k is equal to _____.

[4 marks]

Solution:

Concept:

- Continuity at $x = 0$ requires $f(0) = \lim_{x \rightarrow 0} f(x)$. Thus the assigned value k must equal the two-sided limit of the nonzero branch.
- The standard logarithmic limit is $\lim_{u \rightarrow 0} \frac{\ln(1+u)}{u} = 1$. Also, the logarithm of a positive quotient is the difference of its logarithms.

Step 1: Check the logarithms and split the expression For $-\frac{1}{3} < x < \frac{1}{3}$, both $1 + 3x$ and $1 - 2x$ are positive. The given real logarithm is therefore defined throughout this interval. For $x \neq 0$, use the quotient identity to write

$$f(x) = \frac{\ln(1+3x)}{x} - \frac{\ln(1-2x)}{x}.$$

Both terms have standard logarithmic limits at zero; splitting the logarithm keeps track of the minus sign in its denominator.

Step 2: Evaluate each standard limit In the first term, introduce $3x$ in the denominator; in the second introduce $-2x$. Then

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= 3 \lim_{x \rightarrow 0} \frac{\ln(1+3x)}{3x} + 2 \lim_{x \rightarrow 0} \frac{\ln(1-2x)}{-2x} \\ &= 3(1) + 2(1) = 5. \end{aligned}$$

The contribution from the denominator logarithm is positive: its own limit after division by x is -2 , and this is subtracted.

Step 3: Enforce continuity at the origin The limit exists from both sides, since the standard logarithmic limit applies for either sign of the small arguments. The nonzero branch is already continuous away from zero, being a quotient and composition of continuous functions with nonzero denominators. Consequently, the only remaining requirement for continuity on the stated interval is

$$k = f(0) = \lim_{x \rightarrow 0} f(x) = 5.$$

Step 4: Conclusion The continuous extension requires $k = 5$. This is a dimensionless numerical value.

Quick Tip to Solve: For $\ln[(1+ax)/(1+bx)]/x$ near zero, split the logarithm: the limit is $a - b$. Here $a = 3$ and $b = -2$, so the result is five.

22. If the system of linear equations,

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$3x + 2y + \lambda z = \mu$$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- The solution set of a consistent linear system is either a single point or an affine set with infinitely many points. More than two solutions therefore requires at least one free parameter.
- When two independent equations express the unknowns in terms of a free parameter, the remaining equation must become an identity for infinitely many solutions to remain.

Step 1: Solve the first two equations with a free parameter Subtract the first equation from the second to eliminate x :

$$y + 2z = 4, \quad y = 4 - 2z.$$

Now substitute in $x + y + z = 6$:

$$x = 6 - (4 - 2z) - z = 2 + z.$$

The first two coefficient rows are not multiples, so they are independent and leave exactly one free parameter. Put $z = t$; every common solution of the first two equations is then $(x, y, z) = (2 + t, 4 - 2t, t)$.

Step 2: Impose the third equation for more than two solutions Substitute these expressions into $3x + 2y + \lambda z = \mu$:

$$\begin{aligned} 3(2 + t) + 2(4 - 2t) + \lambda t &= \mu, \\ 14 + (\lambda - 1)t &= \mu. \end{aligned}$$

If $\lambda - 1 \neq 0$, this fixes exactly one t , giving one solution. If $\lambda = 1$ but $\mu \neq 14$, it gives no solution. Only $\lambda = 1$ and $\mu = 14$ make this an identity and allow every real t .

Step 3: Evaluate the requested combination Under these values there are infinitely many solutions, and hence certainly more than two. As a consistency check, four times the first equation minus the second has left side $3x + 2y + z$ and right side $24 - 10 = 14$, exactly the third equation. Therefore

$$\mu - \lambda^2 = 14 - 1^2 = 13.$$

Step 4: Conclusion The required value is 13, with $\lambda = 1$ and $\mu = 14$.

Quick Tip to Solve: Eliminate using the two fixed equations first. The remaining equation must lose its free-parameter coefficient and have the matching constant for infinitely many solutions.

23. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x.y is equal to _____. [4 marks]

Solution:

Concept:

- For n observations, the arithmetic mean is $\bar{x} = \frac{1}{n} \sum x_i$. A specified mean fixes the sum of all observations.
- The variance of the given observations is $\sigma^2 = \frac{1}{n} \sum x_i^2 - \bar{x}^2$. This is the population variance, with divisor n , not the sample-variance divisor $n - 1$.

Step 1: Use the mean to obtain the unknown sum There are eight observations and their mean is 10. Thus their total is $8(10) = 80$. The six known observations have sum

$$3 + 7 + 9 + 12 + 13 + 20 = 64.$$

It follows that

$$x + y = 80 - 64 = 16.$$

This relation alone does not determine the two unknowns, so the variance provides the required second relation.

Step 2: Use the variance to obtain the sum of squares The stated variance is 25. Apply the variance identity with the already known mean:

$$25 = \frac{\sum x_i^2}{8} - 100 \implies \sum x_i^2 = 1000.$$

The known squared observations contribute

$$3^2 + 7^2 + 9^2 + 12^2 + 13^2 + 20^2 = 852.$$

Consequently, the two unknown squares satisfy $x^2 + y^2 = 1000 - 852 = 148$.

Step 3: Extract the product without solving separately Square the relation $x + y = 16$ and use $(x + y)^2 = x^2 + y^2 + 2xy$:

$$256 = 148 + 2xy, \quad 2xy = 108, \quad xy = 54.$$

The values would be $8 \pm \sqrt{10}$, in either order, since their sum is 16 and their product is 54. They are real and reproduce the required squared sum, so the derived constraints are consistent.

Step 4: Conclusion The product of the two unknown observations is $xy = 54$.

Quick Tip to Solve: When a mean and variance are given, find the unknown sum and squared sum first. The identity $2xy = (x + y)^2 - (x^2 + y^2)$ then gives the product directly.

24. If the foot of the perpendicular drawn from the point $(1, 0, 3)$ on a line passing through $(\alpha, 7, 1)$ is $(\frac{5}{3}, \frac{7}{3}, \frac{17}{3})$, then α is equal to _____. [4 marks]

Solution:

Concept:

- The segment joining a point to its perpendicular foot on a line is orthogonal to every direction vector of that line. Orthogonality is expressed by a zero dot product.
- If H is the foot and A is any other point on the line, $\overrightarrow{AH}$ is a direction vector. Therefore $\overrightarrow{PH} \cdot \overrightarrow{AH} = 0$ for the given point P .

Step 1: Form the perpendicular displacement Write the given points as

$$P = (1, 0, 3), \quad H = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right), \quad A = (\alpha, 7, 1).$$

The displacement from the external point to its foot is

$$\overrightarrow{PH} = H - P = \left(\frac{2}{3}, \frac{7}{3}, \frac{8}{3}\right).$$

In particular, the final component is $17/3 - 3 = 8/3$, so the same common denominator can be used in the dot product.

Step 2: Form the line direction and impose orthogonality Because A and H lie on the required line, a direction vector is

$$\overrightarrow{AH} = H - A = \left(\frac{5}{3} - \alpha, -\frac{14}{3}, \frac{14}{3}\right).$$

Its dot product with $\overrightarrow{PH}$ must vanish. Multiplying by 9 to remove denominators gives

$$2(5 - 3\alpha) + 7(-14) + 8(14) = 0.$$

Thus $10 - 6\alpha - 98 + 112 = 0$, or $24 - 6\alpha = 0$.

Step 3: Solve and check the geometry The equation yields $\alpha = 4$. At this value, a scaled direction vector $3\overrightarrow{AH}$ is $(-7, -14, 14)$, while $3\overrightarrow{PH} = (2, 7, 8)$. Their dot product is

$$2(-7) + 7(-14) + 8(14) = -14 - 98 + 112 = 0.$$

The line direction is nonzero, so this is a valid line and the zero dot product verifies the stated perpendicular foot.

Step 4: Conclusion The required coordinate is $\boxed{\alpha = 4}$.

Quick Tip to Solve: For a perpendicular-foot problem, subtract coordinates to get two vectors: point-to-foot and line-point-to-foot. Their dot product gives the unknown coordinate immediately.

25. Let $X = \{n \in \mathbb{N} : 1 \leq n \leq 50\}$. If

$A = \{n \in X : n \text{ is a multiple of } 2\}$ and

$B = \{n \in X : n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is _____.

[4 marks]

Solution:

Concept:

- The smallest set containing both sets A and B is their union $A \cup B$. An element belonging to both sets is included only once.
- For finite sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Multiples common to 2 and 7 are multiples of their least common multiple, 14.

Step 1: Identify the required subset Every element of A and every element of B must belong to the required subset of X . Thus $A \cup B$ is contained in any eligible subset. Conversely, $A \cup B$ itself is a subset of X containing both sets, so it is the smallest such subset. We need its cardinality, rather than the intersection or the sum with repetitions.

Step 2: Count the multiples in each set The even integers run from 2 through 50, while the multiples of 7 run from 7 through 49. Hence

$$n(A) = \left\lfloor \frac{50}{2} \right\rfloor = 25, \quad n(B) = \left\lfloor \frac{50}{7} \right\rfloor = 7.$$

The upper bound 50 is included, so the last even integer must be counted.

Step 3: Remove the double counting An integer in both sets is divisible by 14. Within X , these integers are 14, 28 and 42; the next multiple, 56, exceeds 50. Therefore

$$n(A \cap B) = \left\lfloor \frac{50}{14} \right\rfloor = 3.$$

Adding the two separate counts counts each of these three integers twice. Subtracting their count once gives

$$n(A \cup B) = 25 + 7 - 3 = 29.$$

No other integer needs to be added, because membership of the union requires divisibility by at least one of 2 and 7.

Step 4: Conclusion The smallest subset is $A \cup B$, and it contains 29 elements. The numerical entry is 29, a dimensionless count.

Quick Tip to Solve: For multiples of two integers in a bounded set, count each list and subtract the multiples of their least common multiple once.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is: [4 marks]

- (A) $2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right)$
 (B) $\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right)$
 (C) $2\pi a^4 \left(\frac{aA}{4} + \frac{B}{5} \right)$
 (D) $2\pi a^4 \left(\frac{A}{4} + \frac{B}{5} \right)$

Correct Answer: (A) $2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right)$

Solution:

Concept:

- About the central perpendicular axis, a mass element at distance r contributes $dI = r^2 dm$. Surface density must be included inside the integral when it varies with radius.
- A thin circular ring of radius r and width dr has area $dS = 2\pi r dr$, so $dm = \sigma(r)dS = (A + Br)2\pi r dr$.

Step 1: Choose concentric ring elements All points on a thin concentric ring are the same distance from the stated axis. This lets us use its entire mass at radius r . The disc is made up of such rings between $r = 0$ and $r = a$. The local density is $A + Br$, rather than a constant average density; using the uniform-disc formula without accounting for this dependence would lose the radial weighting.

Step 2: Integrate the moment of inertia Multiply the ring mass by the square of its radius:

$$dI = r^2(A + Br)2\pi r dr = 2\pi(Ar^3 + Br^4) dr.$$

Adding all rings gives

$$I = 2\pi \int_0^a (Ar^3 + Br^4) dr = 2\pi \left(\frac{Aa^4}{4} + \frac{Ba^5}{5} \right).$$

Factoring out a^4 produces

$$I = 2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right).$$

The two terms represent the constant-density and linearly varying contributions, respectively.

Step 3: Check the alternatives If $B = 0$, the mass is $M = \pi Aa^2$, and the expression becomes $\pi Aa^4/2 = Ma^2/2$, as required for a uniform disc. Option (B) gives half this value. In options (C) and (D), the terms inside the brackets do not even have consistent dimensions: A has units of mass per area and B has units of mass per volume, so aB , rather than B , must accompany A .

Step 4: Conclusion

$$I = 2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right).$$

Its dimensions are mass times length squared. Hence, the correct option is (A).

Quick Tip to Solve: For a disc with radial surface density, start with $dm = 2\pi r\sigma(r)dr$; the extra r^2 in inertia makes the integrand proportional to $r^3\sigma(r)$.

27. An emf of 20 V is applied at time $t = 0$ to a circuit containing in series 10 mH inductor and $5\ \Omega$ resistor. The ratio of the currents at time $t = \infty$ and at $t = 40$ s is close to : (Take $e^2 = 7.389$) [4 marks]

- (A) 1.06
- (B) 0.84
- (C) 1.15
- (D) 1.46

Correct Answer: (A) 1.06

Solution:

Concept:

- For an initially unenergised series RL circuit connected to a constant voltage V , the current grows as $i(t) = \frac{V}{R}(1 - e^{-t/\tau})$, where the time constant is $\tau = L/R$.
- The steady current is $i(\infty) = V/R$. Thus the requested steady-to-transient current ratio is $1/(1 - e^{-Rt/L})$; the applied voltage cancels.

Step 1: Convert the inductance and obtain the time constant The stated inductance is 10 mH = 0.010 H, and the resistance is $5\ \Omega$. Therefore

$$\tau = \frac{0.010}{5} = 0.002\text{ s.}$$

The final current is

$$i(\infty) = \frac{20}{5} = 4\text{ A.}$$

The inductor delays the initial current but acts as a zero-voltage element after the current becomes steady.

Step 2: Evaluate the ratio at the stated time Using the printed time $t = 40$ s, rather than changing its unit, gives

$$\frac{t}{\tau} = \frac{40}{0.002} = 20000.$$

Consequently

$$i(40\text{ s}) = 4(1 - e^{-20000})\text{ A}, \quad \frac{i(\infty)}{i(40\text{ s})} = \frac{1}{1 - e^{-20000}} \approx 1.$$

The exponential is negligible, so 40 seconds is effectively the steady state for this circuit. The supplied value of e^2 is unnecessary for these stated parameters.

Step 3: Compare the listed approximations The ratio must be at least one because the growing current approaches its limiting value from below; this immediately excludes 0.84. Of the remaining values, the distances from one are 0.06, 0.15 and 0.46. Thus 1.06 is the closest listed number, although the calculated ratio itself is practically unity. Neither 1.15 nor 1.46 is as close. This comparison follows the wording "close to" without substituting an unstated inductance or time.

Step 4: Conclusion For the printed values, the ratio is $1/(1 - e^{-20000}) \approx 1$, whose closest listed approximation is 1.06. Hence, the correct option is (A).

Quick Tip to Solve: Calculate $t/\tau = Rt/L$ before evaluating an RL exponential. When this number is very large, the transient current is essentially its steady value.

28. The electric field of a plane electromagnetic wave is given by

$$\vec{E} = E_0 \frac{\hat{i} + \hat{j}}{\sqrt{2}} \cos(kz + \omega t)$$

At $t = 0$, a positively charged particle is at the point $(x, y, z) = (0, 0, \frac{\pi}{k})$.

If its instantaneous velocity at $(t = 0)$ is $v_0 \hat{k}$, the force acting on it due to the wave is : [4 marks]

- (A) parallel to $\hat{k}$
- (B) parallel to $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$
- (C) antiparallel to $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$
- (D) zero

Correct Answer: (C) antiparallel to $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Solution:

Concept:

- For a plane wave, $\mathbf{B} = (1/c)\hat{n} \times \mathbf{E}$, where $\hat{n}$ is the propagation direction. A phase $kz + \omega t$ propagates towards decreasing z .
- The force on a charge q is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Both electric and magnetic parts must be evaluated at the specified position and time.

Step 1: Evaluate the phase and electric field Write $\mathbf{u} = (\hat{i} + \hat{j})/\sqrt{2}$. At $t = 0$ and $z = \pi/k$, the phase is π , so

$$\mathbf{E} = -E_0 \mathbf{u}.$$

The electric force on a positive charge therefore points opposite to $\mathbf{u}$. The plus sign between kz and ωt means the wave travels along $-\hat{k}$, since a surface of constant phase satisfies $dz/dt = -\omega/k$.

Step 2: Include the magnetic part of the force Using $\hat{n} = -\hat{k}$, the magnetic field at the event is

$$\mathbf{B} = \frac{1}{c}(-\hat{k}) \times (-E_0 \mathbf{u}) = \frac{E_0}{c} \hat{k} \times \mathbf{u}.$$

For the given velocity,

$$\mathbf{v} \times \mathbf{B} = \frac{v_0 E_0}{c} \hat{k} \times (\hat{k} \times \mathbf{u}) = -\frac{v_0 E_0}{c} \mathbf{u}.$$

Here $\hat{k} \cdot \mathbf{u} = 0$; the vector triple-product identity supplies the negative sign.

Step 3: Determine the net direction The two contributions reinforce:

$$\mathbf{F} = -qE_0 \left(1 + \frac{v_0}{c}\right) \mathbf{u}.$$

For a positive charge and the stated velocity along positive z , the scalar multiplying $-\mathbf{u}$ is positive. The force is transverse, so it cannot point along $\hat{k}$, excluding (A). It is opposite, rather than parallel, to $\mathbf{u}$, excluding (B). The electric and magnetic forces add rather than cancel, so (D) is also excluded.

Step 4: Conclusion The force is antiparallel to $(\hat{i} + \hat{j})/\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Read the wave direction from its phase first: $kz + \omega t$ travels along negative z . Then use the vector triple product to determine the magnetic-force sign.

29. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equal : (Take $g = 10 \text{ ms}^{-2}$ and the rope to be massless) [4 marks]

- (A) 100 N
- (B) 75 N
- (C) 90 N
- (D) 70 N

Correct Answer: (A) 100 N

Solution:

Concept:

- A suspended mass in static equilibrium has zero resultant force. The lower rope segment transmits a tension equal to the weight mg .
- At a massless junction, resolve the upper tension into vertical and horizontal components. A horizontal applied force balances the horizontal component, while the vertical component balances the lower tension.

Step 1: Find the tension below the midpoint The mass is acted upon by its weight downwards and the tension of the bottom half of the rope upwards. With no horizontal force applied directly to the mass, this lower rope segment is vertical. Equilibrium gives

$$T_{\text{lower}} = mg = 10 \times 10 = 100 \text{ N}.$$

The midpoint pulls upwards on the lower segment, so the lower segment exerts a downward force of 100 N on the midpoint.

Step 2: Resolve the upper tension at the junction Let T be the tension in the top half, which is inclined at 45° to the vertical. Balancing vertical forces at the midpoint gives

$$T \cos 45^\circ = 100 \text{ N}.$$

Its horizontal component is balanced by the applied force:

$$F = T \sin 45^\circ.$$

Dividing these two equations avoids calculating T separately:

$$\frac{F}{100} = \tan 45^\circ = 1.$$

Thus $F = 100 \text{ N}$. The upper tension itself is $100\sqrt{2} \text{ N}$, so it should not be mistaken for the horizontal force.

Step 3: Check which data affect the balance The rope length fixes the geometry of the deflected point but does not enter the force balance once the angle is specified. Because the rope is massless, no extra rope weight is added. Options 75 N, 90 N and 70 N would make the ratio of horizontal force to the downward force less than one, corresponding to an angle smaller than 45° . They cannot maintain the

given inclination.

Step 4: Conclusion The required horizontal force is 100 N . Hence, the correct option is (A).

Quick Tip to Solve: At a massless pulled junction, divide horizontal balance by vertical balance to obtain $F = mg \tan \theta$, with θ measured from the vertical.

30. In a Young's double slit experiment, the separation between the slits is 0.15 mm . In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is : [4 marks]

- (A) 4.9 mm
- (B) 5.9 mm
- (C) 6.9 mm
- (D) 3.9 mm

Correct Answer: (B) 5.9 mm

Solution:

Concept:

- In Young's double-slit experiment, the separation of successive bright fringes is the fringe width $\beta = \lambda D/d$, where D is the screen distance and d is the slit separation.
- The formula uses a common system of units. Nanometres and millimetres must be converted before substituting along with a screen distance expressed in metres.

Step 1: Convert all lengths to metres The given quantities are

$$\lambda = 589 \times 10^{-9}\text{ m}, \quad D = 1.5\text{ m}, \quad d = 0.15 \times 10^{-3}\text{ m}.$$

The screen distance is much larger than the slit separation, so the small-angle approximation underlying the usual fringe-width formula is appropriate. Successive bright fringes differ by one unit of interference order, not by half an order.

Step 2: Calculate the spacing of adjacent bright fringes The bright-fringe positions are $y_n = n\lambda D/d$, measured relative to the central bright fringe. Thus

$$y_{n+1} - y_n = \frac{\lambda D}{d}.$$

Substitution gives

$$\beta = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}}\text{ m} = 5890 \times 10^{-6}\text{ m}.$$

Therefore

$$\beta = 5.89 \times 10^{-3}\text{ m} = 5.89\text{ mm}.$$

This spacing is independent of the fringe order within the small-angle approximation.

Step 3: Round and compare the choices Rounding 5.89 mm to one decimal place gives 5.9 mm . The values 4.9 mm , 6.9 mm and 3.9 mm do not follow from the stated wavelength, screen distance and

slit separation. A bright-to-adjacent-dark separation would be half the fringe width, but the question explicitly asks for successive bright fringes, so no factor of one-half is required. The dimensions (m)(m)/(m) correctly reduce to a length.

Step 4: Conclusion The separation is $5.89 \text{ mm} \approx 5.9 \text{ mm}$. Hence, the correct option is (B).

Quick Tip to Solve: Adjacent bright fringes are separated by $\lambda D/d$; only a bright-to-next-dark spacing is half this value. Convert slit separation to metres before calculating.

31. A particle of mass m and charge q has an initial velocity $\vec{v} = v_0 \hat{j}$. If an electric field $\vec{E} = E_0 \hat{i}$ and magnetic field $\vec{B} = B_0 \hat{i}$ act on the particle, its speed will double after a time: [4 marks]

- (A) $\frac{3mv_0}{qE_0}$
 (B) $\frac{\sqrt{2}mv_0}{qE_0}$
 (C) $\frac{\sqrt{3}mv_0}{qE_0}$
 (D) $\frac{2mv_0}{qE_0}$

Correct Answer: (C) $\frac{\sqrt{3}mv_0}{qE_0}$

Solution:

Concept:

- The Lorentz force is $m d\vec{v}/dt = q(\vec{E} + \vec{v} \times \vec{B})$. A magnetic force is perpendicular to velocity and does no work.
- Here both fields are along the x -axis. The electric field changes v_x , while the magnetic field rotates the velocity in the perpendicular yz -plane without changing its magnitude.

Step 1: Separate the parallel and perpendicular velocities Initially $v_x = 0$ and the speed perpendicular to the field is v_0 . The magnetic force has no x -component, so the parallel equation of motion is $m dv_x/dt = qE_0$. Thus

$$v_x = \frac{qE_0 t}{m}.$$

In the perpendicular plane, the two magnetic-force equations give

$$\frac{d}{dt}(v_y^2 + v_z^2) = 2v_y \frac{qB_0 v_z}{m} + 2v_z \frac{-qB_0 v_y}{m} = 0.$$

Consequently $v_y^2 + v_z^2 = v_0^2$ throughout the motion.

Step 2: Impose the doubled-speed condition The parallel velocity is perpendicular to the rotating transverse velocity. Their magnitudes must therefore be combined using squares, not added directly:

$$v^2 = v_0^2 + \left(\frac{qE_0 t}{m}\right)^2.$$

Setting $v = 2v_0$ gives

$$4v_0^2 - v_0^2 = \left(\frac{qE_0 t}{m}\right)^2, \quad t = \frac{\sqrt{3}mv_0}{qE_0}.$$

The printed options use q as the charge magnitude; for a signed charge the positive time contains $|q|$. Option A would give $\sqrt{10}v_0$, option B gives $\sqrt{3}v_0$, and option D gives $\sqrt{5}v_0$, rather than $2v_0$.

Step 3: Conclusion The required time is $t = \sqrt{3}mv_0/(qE_0)$, with q understood as its magnitude. Hence, the correct option is (C).

Quick Tip to Solve: For parallel electric and magnetic fields, preserve the initial transverse speed and add the electrically acquired parallel velocity in quadrature.

32. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum) [4 marks]

- (A) $\frac{1}{c} \left(\frac{2E}{m} \right)^{\frac{1}{2}}$
 (B) $\frac{1}{c} \left(\frac{E}{2m} \right)^{\frac{1}{2}}$
 (C) $\left(\frac{E}{2m} \right)^{\frac{1}{2}}$
 (D) $c(2mE)^{\frac{1}{2}}$

Correct Answer: (B) $\frac{1}{c} \left(\frac{E}{2m} \right)^{\frac{1}{2}}$

Solution:

Concept:

- The de Broglie relation for a particle is $\lambda = h/p$, where h is Planck's constant and p is momentum.
- For an electron with kinetic energy of a few electronvolts, $E \ll mc^2$, so $E = p_e^2/(2m)$. A photon instead obeys $E = p_\gamma c = hc/\lambda_\gamma$.

Step 1: Express both wavelengths using their common energy The electron energy is its kinetic energy. The nonrelativistic approximation is justified because the electron rest energy is about 511 keV, much larger than a few eV. Solving its energy-momentum relation gives

$$p_e = \sqrt{2mE}, \quad \lambda_e = \frac{h}{\sqrt{2mE}}.$$

A photon has zero rest mass and cannot be treated with $p^2/(2m)$. For the same energy its wavelength is

$$\lambda_\gamma = \frac{hc}{E}.$$

Both expressions use the same energy units, so no electronvolt-to-joule factor is needed when simplifying the symbolic ratio.

Step 2: Divide and check the alternatives The requested order is electron wavelength divided by photon wavelength. Substituting and cancelling h ,

$$\frac{\lambda_e}{\lambda_\gamma} = \frac{h}{\sqrt{2mE}} \frac{E}{hc} = \frac{1}{c} \sqrt{\frac{E}{2m}}.$$

The square root has the units of speed, so division by c makes the ratio dimensionless. Option A is

twice this value, because its factor inside the root is four times too large. Option C lacks the required division by c and has dimensions of speed. Option D has dimensions of energy, so neither C nor D can be a wavelength ratio. Also the ratio is much less than one at a few electronvolts, as expected.

Step 3: Conclusion The wavelength ratio is $\lambda_e/\lambda_\gamma = (1/c)\sqrt{E/(2m)}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal energies, write the two momenta first: $p_e = \sqrt{2mE}$ and $p_\gamma = E/c$. The wavelength ratio is the inverse momentum ratio.

33. A box weight 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10 \text{ ms}^{-2}$ at the north pole and the radius of the earth = 6400 km) : [4 marks]

- (A) 194.32 N
- (B) 195.66 N
- (C) 195.32 N
- (D) 194.66 N

Correct Answer: (C) 195.32 N

Solution:

Concept:

- A spring balance measures the normal supporting force, which is the apparent weight. At the pole there is no centripetal acceleration due to Earth's daily rotation.
- In the spherical-Earth approximation, the equatorial reading is $N = m(g - \omega^2 R)$, where $\omega = 2\pi/T$, R is Earth's radius, and T is one day.

Step 1: Find the mass and equatorial centripetal acceleration The polar reading and the given polar gravitational acceleration determine the mass:

$$m = \frac{196}{10} = 19.6 \text{ kg.}$$

Use the radius in SI units, $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$, and $T = 24 \times 60 \times 60 = 86400 \text{ s}$. Then

$$\omega = \frac{2\pi}{86400} \simeq 7.27 \times 10^{-5} \text{ s}^{-1},$$

$$\omega^2 R \simeq (7.27 \times 10^{-5})^2 (6.4 \times 10^6) \simeq 0.0338 \text{ m s}^{-2}.$$

At the equator the box moves in a circle about Earth's rotation axis, so part of gravity supplies this inward centripetal acceleration.

Step 2: Subtract the rotational reduction from the polar weight Taking inward towards Earth's centre as positive gives $mg - N = m\omega^2 R$, not $N - mg = m\omega^2 R$. Therefore the reading is smaller than the polar value:

$$N = 196 - (19.6)(0.0338) \simeq 195.34 \text{ N.}$$

The question asks for the closest listed value. The difference from 195.32 N is only about 0.02 N, whereas the differences from 194.32, 195.66, and 194.66 N are approximately 1.02, 0.32, and 0.68 N, respectively. Hence the third value is the best match. The small difference between 195.34 and the listed value is within the intended approximate calculation.

Step 3: Conclusion The equatorial spring-balance reading is approximately 195.34 N, closest to 195.32 N. Hence, the correct option is (C).

Quick Tip to Solve: At the pole first use the stated g to find the mass. At the equator reduce the polar reading by $m\omega^2 R$, with the radius expressed in metres.

34. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is : [4 marks]

- (A) ML^2T^{-2}
- (B) MLT^{-2}
- (C) $ML^{-1}T^{-2}$
- (D) ML^2T^{-1}

Correct Answer: (C) $ML^{-1}T^{-2}$

Solution:

Concept:

- The magnetic field stores energy per unit volume $u_B = B^2/(2\mu_0)$, where μ_0 is the permeability of vacuum. Energy density has the same dimensions as pressure.
- Dimensions can also be obtained directly from magnetic force $F = qvB$ and the force per unit length between parallel current-carrying wires.

Step 1: Identify the energy-density dimensions The numerator and permeability in the given expression combine into magnetic energy density; the numerical factor 2 has no dimensions. Mechanical energy is force multiplied by distance, so

$$[\text{energy}] = [MLT^{-2}][L] = ML^2T^{-2}.$$

Dividing by volume gives

$$\left[\frac{B^2}{2\mu_0}\right] = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}.$$

This route also explains why the same expression can represent magnetic pressure: pressure is force divided by area and thus has dimensions $ML^{-1}T^{-2}$.

Step 2: Verify by the dimensions of the electromagnetic quantities Let I denote the dimension of electric current. From $F = qvB$, with $[q] = IT$,

$$[B] = \frac{MLT^{-2}}{(IT)(LT^{-1})} = MT^{-2}I^{-1}.$$

The parallel-wire relation $F/\ell = \mu_0 I_1 I_2 / (2\pi r)$ gives

$$[\mu_0] = MLT^{-2}I^{-2}.$$

Hence

$$\frac{[B]^2}{[\mu_0]} = \frac{M^2 T^{-4} I^{-2}}{MLT^{-2}I^{-2}} = ML^{-1}T^{-2}.$$

The current dimension cancels, agreeing with the mechanical energy-density result. Option A is the dimension of energy rather than energy per volume. Option B is force, and option D is angular momentum. None has the required inverse-length factor of a pressure.

Step 3: Conclusion The required dimension is $ML^{-1}T^{-2}$. Hence, the correct option is (C).

Quick Tip to Solve: Recognise $B^2/(2\mu_0)$ as energy density. Divide the dimensions of energy by volume, or use its equality in dimensions to pressure.

35. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is : [4 marks]

- (A) $\frac{3}{4}$
- (B) $\frac{9}{16}$
- (C) $\frac{\sqrt{3}}{2}$
- (D) $\frac{81}{256}$

Correct Answer: (B) $\frac{9}{16}$

Solution:

Concept:

- For steady flow of an incompressible ideal fluid, conservation of mass gives $\rho Av = \text{constant}$. Since density ρ is unchanged, the volume flow rate Av is constant.
- The cross-sectional area of a circular pipe is $A = \pi d^2/4$, so the flow speed is inversely proportional to the square of the pipe diameter, not to the diameter itself.

Step 1: Associate each limiting speed with its diameter In steady flow the same fluid volume crosses every cross-section per second. A larger cross-section therefore needs a smaller speed to carry that volume. Thus $v_{\min}$ occurs at $d_{\max} = 6.4$ cm, while $v_{\max}$ occurs at $d_{\min} = 4.8$ cm. Apply continuity between these two locations:

$$A_{\max} v_{\min} = A_{\min} v_{\max}.$$

It is essential to pair the maximum area with the minimum speed. Reversing that pairing would produce a ratio larger than one, inconsistent with the requested minimum-to-maximum order.

Step 2: Calculate the area ratio and compare the options Rearranging continuity and inserting the

circular areas gives

$$\frac{v_{\min}}{v_{\max}} = \frac{A_{\min}}{A_{\max}} = \left(\frac{d_{\min}}{d_{\max}}\right)^2 = \left(\frac{4.8}{6.4}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}.$$

Both diameters are in centimetres, so their units cancel without a conversion. Option A, $3/4$, is the diameter ratio and misses the square. Option C, $\sqrt{3}/2$, is the square root of that diameter ratio and has no continuity-law basis. Option D, $81/256$, squares the correct area ratio once more and would correspond to a fourth power of the diameter ratio. Bernoulli's equation is unnecessary because no pressure difference or height is required.

Step 3: Conclusion The minimum-to-maximum speed ratio is $v_{\min}/v_{\max} = 9/16$. Hence, the correct option is (B).

Quick Tip to Solve: Write the wide-pipe and narrow-pipe continuity equation before substituting numbers. For circular pipes the speed ratio uses squared diameters.

36. A planar loop of wire rotates in a uniform magnetic field. Initially at $t = 0$, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at : [4 marks]

- (A) 2.5 s and 7.5 s
- (B) 5.0 s and 10.0 s
- (C) 5.0 s and 7.5 s
- (D) 2.5 s and 5.0 s

Correct Answer: (D) 2.5 s and 5.0 s

Solution:

Concept:

- The magnetic flux through a loop is $\Phi = BA \cos \theta$, where θ is the angle between the field and the normal to the loop plane.
- Faraday's law gives the induced emf $\mathcal{E} = -d\Phi/dt$. For uniform rotation with angular speed $\omega = 2\pi/T$, the magnitude varies as $BA\omega|\sin \omega t|$.

Step 1: Use the initial orientation to write the flux At $t = 0$, the plane of the loop is perpendicular to the magnetic field. Its normal is consequently parallel to the field and the initial flux has maximum magnitude, not zero. Taking that normal as positive,

$$\Phi(t) = BA \cos(\omega t), \quad \omega = \frac{2\pi}{10} = \frac{\pi}{5} \text{ rad s}^{-1}.$$

The rotation axis lies in the loop plane, so the normal rotates relative to the field. Differentiating the flux gives

$$\mathcal{E}(t) = BA\omega \sin(\omega t).$$

A maximum flux at the initial instant therefore corresponds to zero emf, since the instantaneous rate

of change of flux is then zero.

Step 2: Locate the maximum and minimum magnitudes The magnitude is greatest when $|\sin \omega t| = 1$, first at a quarter-period:

$$t_{\max} = \frac{T}{4} = 2.5 \text{ s.}$$

Its magnitude is least, namely zero, when $\sin \omega t = 0$. After the initial instant, the first such minimum occurs at a half-period:

$$t_{\min} = \frac{T}{2} = 5.0 \text{ s.}$$

More generally maxima occur at 2.5, 7.5, 12.5, ... s, and minima at 0, 5, 10, ... s. Option A names two maxima, so its second time is incorrect. Option B names two minima, so its first time is incorrect. Option C places a minimum first and a maximum second, reversing the requested order. Only option D gives a maximum followed by a minimum at the indicated times.

Step 3: Conclusion The induced-emf magnitude is maximum at $\boxed{2.5 \text{ s}}$ and minimum at $\boxed{5.0 \text{ s}}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish the loop plane from its normal. Initial maximum flux means zero emf; the first maximum emf follows after one quarter of a rotation.

37. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg, The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10 \text{ m/s}^2$) must be at least : [4 marks]

- (A) 48000 W
- (B) 62360 W
- (C) 56300 W
- (D) 66000 W

Correct Answer: (D) 66000 W

Solution:

Concept:

- For upward motion at constant speed, the acceleration is zero. The upward lifting force must therefore balance both the weight of the fully loaded elevator and the downward frictional force.
- The mechanical power delivered by a force parallel to velocity is $P = Fv$. Here F is the total motor force and v is the steady upward speed, not an acceleration.

Step 1: Find the fully loaded mass The passengers contribute $10 \times 68 = 680 \text{ kg}$. Adding the elevator itself gives the total mass

$$M = 920 + 680 = 1600 \text{ kg.}$$

The motor must lift this entire mass; using only the passengers or only the elevator would underestimate the required force.

Step 2: Balance all downward forces With $g = 10 \text{ m s}^{-2}$, the weight is $Mg = 16000 \text{ N}$. Friction opposes the upward motion, so it acts downward as well. The steady-motion force balance is

$$F - Mg - 6000 = 0, \quad F = 22000 \text{ N}.$$

Constant speed means zero net force, but it does not mean that the lifting force or the power is zero.

Step 3: Calculate the power and check the choices The minimum mechanical power at the stated full-load operating speed is

$$P = Fv = 22000 \times 3 = 66000 \text{ W}.$$

Weight alone would require 48000 W , the first option, but this omits the 18000 W dissipated against friction. Both 62360 W and 56300 W are smaller than the required total and cannot maintain the specified steady motion.

Step 4: Conclusion The motor must deliver at least $6.6 \times 10^4 \text{ W} = 66 \text{ kW}$ to the elevator. Hence, the correct option is (D).

Quick Tip to Solve: For a lift travelling upward steadily, first add the load to the cage mass, then use $P = (Mg + F_{\text{friction}})v$.

38. A thin lens made of glass (refractive index = 1.5) of focal length $f = 16 \text{ cm}$ is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1 , then the ratio $\frac{f_1}{f}$ is closest to the integer : [4 marks]

- (A) 17
- (B) 1
- (C) 9
- (D) 5

Correct Answer: (C) 9

Solution:

Concept:

- The thin-lens maker formula in a surrounding medium is $1/f_m = (n_g/n_m - 1)(1/R_1 - 1/R_2)$, where n_g and n_m are the glass and medium refractive indices.
- Immersion changes the relative refractive index, while the lens shape and its radii remain fixed. Therefore the curvature factor cancels when the two focal lengths are compared.

Step 1: Write the power in air Let $K = 1/R_1 - 1/R_2$, with the usual signed radii. Taking the refractive index of air as unity,

$$\frac{1}{f} = (1.5 - 1)K = 0.5K.$$

This relates the given focal length $f = 16 \text{ cm}$ to the unchanged lens geometry.

Step 2: Use the refractive index relative to the liquid In the liquid, the relevant index is $1.5/1.42$,

rather than the absolute index 1.5. Thus

$$\frac{1}{f_1} = \left(\frac{1.5}{1.42} - 1 \right) K = \frac{0.08}{1.42} K.$$

The glass still has a larger refractive index than the liquid, so the sign of the lens power is unchanged, but its magnitude is reduced substantially.

Step 3: Divide and select the nearest integer Dividing the air power by the immersed power gives the required focal-length ratio:

$$\frac{f_1}{f} = \frac{0.5}{0.08/1.42} = \frac{0.71}{0.08} = 8.875.$$

Equivalently, $f_1 = 142$ cm. The nearest integer is 9. The value 1 would imply no immersion effect, 5 is too small, and 17 is too large; none matches the numerical ratio obtained from the relative index.

Step 4: Conclusion The ratio is $f_1/f = 8.875$, closest to the integer 9. Hence, the correct option is (C).

Quick Tip to Solve: When a lens is immersed, replace its refractive index by glass index divided by medium index; the ratio of powers immediately gives the inverse ratio of focal lengths.

39. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be :

[4 marks]

- (A) 15 A
- (B) 20 A
- (C) 25 A
- (D) 10 A

Correct Answer: (B) 20 A

Solution:

Concept:

- Appliances connected to the mains operate in parallel, so their rated powers add. The total normal current drawn from a supply voltage V is $I = P_{\text{total}}/V$.
- A fuse must carry the full normal operating current without melting. From the offered ratings, choose the smallest one that is at least as large as the calculated current.

Step 1: Add the power of each group The first group of bulbs consumes $15 \times 45 = 675$ W. The second consumes $15 \times 100 = 1500$ W. The small fans consume $15 \times 10 = 150$ W, and the two heaters consume $2 \times 1000 = 2000$ W. Converting kilowatts to watts before addition avoids a unit error.

Step 2: Determine the total mains current With every listed appliance operating, the load power is

$$P_{\text{total}} = 675 + 1500 + 150 + 2000 = 4325 \text{ W}.$$

At the stated mains voltage,

$$I = \frac{4325}{220} \text{ A} = 19.659 \dots \text{ A}.$$

This is the current through the common supply fuse, not the current through any one appliance branch.

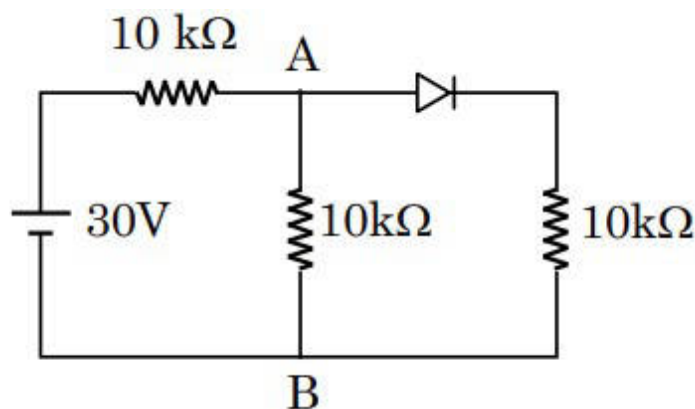
Step 3: Choose the minimum adequate rating The 10 A and 15 A fuses are below the normal demand and would not support the full load. A 20 A fuse is just above 19.66 A and is therefore adequate in this rated-load calculation. A 25 A fuse can carry the current too, but the question asks for the minimum available rating, so it is not selected.

Step 4: Conclusion The load draws approximately 19.66 A, so the minimum offered fuse rating is 20 A. Hence, the correct option is (B).

Quick Tip to Solve: Add appliance powers before dividing by the mains voltage, and round the fuse choice upward to an available rating, never downward.

40. In the figure, potential difference between A and B is :

[4 marks]



- (A) 10 V
- (B) 5 V
- (C) zero
- (D) 15 V

Correct Answer: (A) 10 V

Solution:

Concept:

- An ideal diode conducts as a short circuit when its anode is at higher potential than its cathode, and acts as an open circuit under reverse bias.
- For a conducting diode, the two 10 kΩ vertical resistors are parallel between A and B. Their equivalent resistance forms a voltage divider with the left 10 kΩ resistor.

Step 1: Determine the bias of the diode The upper terminal of the 30 V battery is positive. The diode has its anode at A and its cathode toward the right resistor. To test an open diode, the left and middle

resistors would divide the supply equally, giving $V_A - V_B = 15 \text{ V}$, while the cathode is connected to B through an otherwise current-free resistor. The anode is therefore positive relative to the cathode, so the open-state assumption is inconsistent: the ideal diode conducts.

Step 2: Reduce the conducting network Replace the forward-biased ideal diode by a wire. The middle and right resistors then have the same upper and lower nodes:

$$R_{AB} = \frac{10 \times 10}{10 + 10} \text{ k}\Omega = 5 \text{ k}\Omega.$$

This parallel combination is in series with the left resistor, giving $R_{\text{total}} = 15 \text{ k}\Omega$.

Step 3: Calculate the voltage across A and B The battery current is

$$I = \frac{30}{15000} \text{ A} = 2 \text{ mA}.$$

The potential difference across the parallel portion is

$$V_A - V_B = IR_{AB} = 0.002 \times 5000 = 10 \text{ V}.$$

Each vertical resistor carries 1 mA, consistent with the diode current flowing from A to the right. The 15 V choice incorrectly treats the diode as open; zero and 5 V do not satisfy the divider relation.

Step 4: Conclusion A is 10 V above B, so the potential difference is 10 V. Hence, the correct option is (A).

Quick Tip to Solve: Test diode bias before combining resistors. A forward-biased ideal diode makes the two vertical resistors parallel, so use a 10 k Ω -5 k Ω divider.

41. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2 . If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by : [4 marks]

- (A) $(2)^{\frac{1+\gamma}{2}}$
- (B) 2
- (C) $(\frac{1}{2})^{\frac{1+\gamma}{2}}$
- (D) $(\frac{1}{2})^\gamma$

Correct Answer: (A) $(2)^{\frac{1+\gamma}{2}}$

Solution:

Concept:

- For a fixed quantity of dilute gas with fixed molecular collision cross-section, the mean free path is $\ell = 1/(\sqrt{2}n\sigma)$, so $\ell \propto V$, where $n = N/V$.
- The mean molecular speed varies as $\bar{v} \propto \sqrt{T}$, and the mean collision time is $\tau \sim \ell/\bar{v}$. For the stated adiabatic change, $TV^{\gamma-1}$ is constant.

Step 1: Find the change in mean free path The number of molecules stays fixed while the volume doubles: $V_2/V_1 = 2$. Hence the number density halves. Molecules have twice the mean distance to travel between successive collisions, giving

$$\frac{\ell_2}{\ell_1} = \frac{V_2}{V_1} = 2.$$

This density effect alone would double the collision time if the temperature were unchanged.

Step 2: Include the adiabatic cooling The adiabatic relation gives

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 2^{1-\gamma}.$$

Therefore the speed ratio is

$$\frac{\bar{v}_2}{\bar{v}_1} = \sqrt{\frac{T_2}{T_1}} = 2^{(1-\gamma)/2}.$$

Since $\gamma > 1$ for an ideal gas, the temperature and molecular speeds decrease during this expansion.

Step 3: Combine distance and speed ratios Using $\tau \sim \ell/\bar{v}$,

$$\frac{\tau_2}{\tau_1} = \frac{\ell_2}{\ell_1} \frac{\bar{v}_1}{\bar{v}_2} = 2 \cdot 2^{(\gamma-1)/2} = 2^{(1+\gamma)/2}.$$

The answer is greater than 2 because paths become longer and molecules become slower. Option (B) omits the cooling, while options (C) and (D) predict a decrease and contradict both physical effects.

Step 4: Conclusion The estimated mean collision-time ratio is $\tau_2/\tau_1 = 2^{(1+\gamma)/2}$. Hence, the correct option is (A).

Quick Tip to Solve: Write $\tau \propto V/\sqrt{T}$ first, then insert the adiabatic temperature ratio; this prevents reversing the speed factor.

42. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half life is close to:

[4 marks]

- (A) 62 min
- (B) 66 min
- (C) 72 min
- (D) 52 min

Correct Answer: (A) 62 min

Solution:

Concept:

- For a single radioactive sample with decay constant λ , the activity follows $A(t) = A_0 e^{-\lambda t}$, because activity is proportional to the number of undecayed nuclei.
- The half-life is $T_{1/2} = \ln 2/\lambda$. The ratio of activities determines λt , so the absolute initial number

of nuclei is not needed.

Step 1: Relate the activity ratio to elapsed time The initial and final activities are 700 s^{-1} and 500 s^{-1} . Their ratio is dimensionless:

$$\frac{A_0}{A} = \frac{700}{500} = 1.4 = e^{\lambda t}.$$

Take natural logarithms to obtain $\lambda t = \ln 1.4$. The fall in activity is exponential rather than a constant decrease per minute.

Step 2: Calculate the decay constant consistently Using the elapsed time in minutes keeps the resulting half-life in minutes:

$$\lambda = \frac{\ln 1.4}{30\text{ min}} = \frac{0.336472\dots}{30}\text{ min}^{-1} \approx 0.0112157\text{ min}^{-1}.$$

Although activity itself is measured in inverse seconds, those units cancel in the ratio, so there is no need to convert the time to seconds unless one also converts the final result back.

Step 3: Find the half-life and compare the choices Substitute into the half-life relation:

$$T_{1/2} = \frac{\ln 2}{\lambda} = 30 \frac{0.693147\dots}{0.336472\dots}\text{ min} \approx 61.8\text{ min}.$$

Thus 62 min is closest. The alternatives 66, 72, and 52 min are farther away and do not reproduce the measured activity ratio over 30 minutes.

Step 4: Conclusion The radioactive sample has half-life approximately 61.8 min, closest to 62 min. Hence, the correct option is (A).

Quick Tip to Solve: Use $T_{1/2} = t \ln 2 / \ln(A_0/A)$ directly when two activities of the same sample and their time separation are given.

43. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperature, T_1 and T_2 . The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2 . T is temperature of the sink of first engine which is also the source for the second which is also the source for the second engine. How is T related to T_1 and T_2 . If both engines perform equal amount of work? [4 marks]

- (A) $T = \frac{2T_1T_2}{T_1+T_2}$
- (B) $T = \sqrt{T_1T_2}$
- (C) $T = \frac{T_1+T_2}{2}$
- (D) $T = 0$

Correct Answer: (C) $T = \frac{T_1+T_2}{2}$

Solution:

Concept:

- A reversible Carnot engine operating between absolute temperatures T_h and T_c has efficiency $\eta = 1 - T_c/T_h$. Its rejected heat and absorbed heat obey $Q_c/Q_h = T_c/T_h$.
- In a cascade, the heat rejected by the first engine is precisely the heat absorbed by the second. Work is the difference between absorbed and rejected heat; equal work is not the same as equal efficiency.

Step 1: Express the heat transfers through the cascade Let the first engine absorb heat Q_1 from the reservoir at T_1 , and let it reject Q at the intermediate temperature T . Carnot reversibility gives

$$Q = Q_1 \frac{T}{T_1}.$$

The second engine absorbs this same Q , not Q_1 . If it rejects Q_2 to the reservoir at T_2 , then

$$Q_2 = Q \frac{T_2}{T} = Q_1 \frac{T_2}{T_1}.$$

Step 2: Equate the two work outputs The work outputs are

$$W_1 = Q_1 - Q = \frac{Q_1}{T_1}(T_1 - T), \quad W_2 = Q - Q_2 = \frac{Q_1}{T_1}(T - T_2).$$

Since both engines perform equal work, cancel the common positive factor Q_1/T_1 :

$$T_1 - T = T - T_2 \implies 2T = T_1 + T_2.$$

The geometric mean in option (B) instead makes the two efficiencies equal, a different condition. The harmonic mean in (A) does not equate the temperature drops and therefore does not give equal work. Option (D) cannot be an intermediate temperature between the two positive-temperature reservoirs.

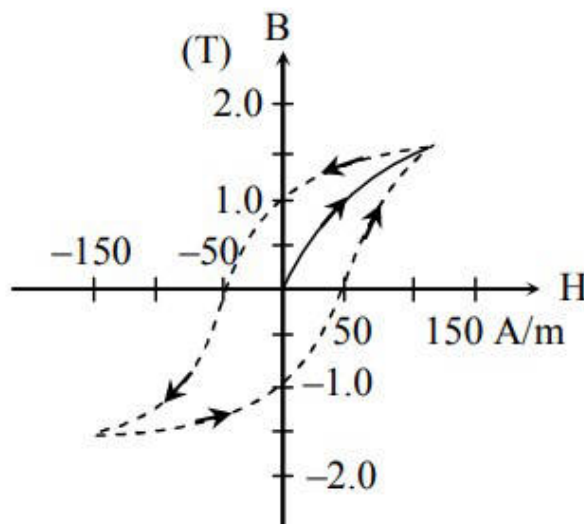
Step 3: Conclusion The intermediate absolute temperature is

$$T = \frac{T_1 + T_2}{2}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For reversible engines in cascade, write every heat transfer in terms of the first heat input. Equal works then mean equal absolute-temperature drops, not equal efficiencies.

44. The figure gives experimentally measured B vs H variation in a ferromagnetic material. The retentivity, coercivity and saturation, respectively, of the material are : [4 marks]



- (A) 1.0 T, 50 A/m and 1.5 T
- (B) 1.5 T, 50 A/m and 1.0 T
- (C) 1.5 T, 50 A/m and 2.0 T
- (D) 150 A/m, 1.0 T and 1.5 T

Correct Answer: (A) 1.0 T, 50 A/m and 1.5 T

Solution:

Concept:

- A ferromagnetic material follows a hysteresis loop: its magnetic flux density B depends on both the applied magnetising field H and its previous magnetisation.
- Retentivity is the magnitude of B remaining when $H = 0$. Coercivity is the reverse-field magnitude needed to reduce B to zero. Saturation is the limiting B reached at large applied field.

Step 1: Read the two intercepts of the hysteresis loop Follow the upper branch after the specimen has been magnetised strongly. At the vertical axis, the applied field is zero but the flux density remains 1.0 T. Therefore the retentivity is

$$B_r = 1.0 \text{ T.}$$

Continue along this branch towards negative H . The curve crosses $B = 0$ at $H = -50 \text{ A m}^{-1}$. Coercivity is quoted as a positive magnitude, so

$$H_c = 50 \text{ A m}^{-1}.$$

The symmetric intercept on the opposite branch gives the same magnitude.

Step 2: Read saturation and compare the ordered triples At sufficiently large positive field the loop approaches approximately $B = 1.5 \text{ T}$. Its negative counterpart is approximately -1.5 T , so the saturation magnitude is 1.5 T. The axis marking 2.0 T is only a scale label; the curve does not reach it.

Option (B) interchanges retentivity and saturation. Option (C) mistakes the 2.0 T scale marking for saturation and also overstates retentivity. Option (D) gives field units for retentivity and flux-density units for coercivity, which is dimensionally incorrect.

Step 3: Conclusion The retentivity, coercivity and saturation, respectively, are

$$1.0 \text{ T}, \quad 50 \text{ A m}^{-1}, \quad 1.5 \text{ T}.$$

Hence, the correct option is (A).

Quick Tip to Solve: Read hysteresis quantities in this order: vertical intercept for retentivity, horizontal intercept magnitude for coercivity, and the plateau height for saturation. Check their units before choosing.

45. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is $\nu_0 = 1400 \text{ Hz}$ and the velocity of sound in air is 350 m/s . The speed of each tuning fork is close to : [4 marks]

- (A) 1 m/s
- (B) $\frac{1}{8} \text{ m/s}$
- (C) $\frac{1}{4} \text{ m/s}$
- (D) $\frac{1}{2} \text{ m/s}$

Correct Answer: (C) $\frac{1}{4} \text{ m/s}$

Solution:

Concept:

- For a stationary observer and a source moving through still air at speed u , the received frequencies are $f_+ = f_0 v / (v - u)$ for approach and $f_- = f_0 v / (v + u)$ for recession, where v is sound speed.
- The beat frequency is the positive difference between the two received frequencies. When $u \ll v$, each Doppler shift is approximately $f_0 u / v$, so the difference is approximately twice one shift.

Step 1: Find the difference of the received frequencies Both tuning forks emit $f_0 = 1400 \text{ Hz}$, but one frequency is shifted up and the other down. With $v = 350 \text{ m s}^{-1}$, their beat frequency is

$$f_b = f_+ - f_- = f_0 v \left(\frac{1}{v - u} - \frac{1}{v + u} \right) = \frac{2 f_0 v u}{v^2 - u^2}.$$

There is no motion of the observer, so the numerator is the same v in both Doppler formulae. Using only one of the shifts would underestimate the frequency difference by a factor of two.

Step 2: Use the small-speed condition and test the options Since the problem states that u is much smaller than v , neglect u^2 compared with v^2 . Thus

$$f_b \simeq \frac{2 f_0 u}{v}, \quad u \simeq \frac{f_b v}{2 f_0} = \frac{2 \times 350}{2 \times 1400} = 0.25 \text{ m s}^{-1}.$$

Here $u/v = 1/1400$, so the omitted relative correction is only about 5.1×10^{-7} . As a direct option

check, speeds 1, 1/8, 1/4 and 1/2 m s⁻¹ give approximately 8, 1, 2 and 4 beats per second, respectively. Only the third agrees with the observation.

Step 3: Conclusion The speed of each tuning fork is close to

$$u = \frac{1}{4} \text{ m s}^{-1}.$$

Hence, the correct option is (C).

Quick Tip to Solve: When identical sources move oppositely at equal small speeds, the beat frequency is twice the individual Doppler shift: use $f_b \simeq 2f_0 u/v$.

Physics – Section B: Numerical Value Questions (20 marks)

46. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is , [4 marks]

Solution:

Concept:

- The magnitude of a vector sum is found by taking its scalar product with itself: $|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2\vec{A} \cdot \vec{B}$.
- Two nonzero vectors are perpendicular when their scalar product is zero. This test finds the angle without requiring the separate magnitudes or components of the given forces.

Step 1: Convert the equal-magnitude condition into a dot product The given resultant is $\vec{R} = \vec{P} + \vec{Q}$, with $|\vec{R}| = |\vec{P}|$. Square this equality; both sides are nonnegative, so this preserves the condition:

$$|\vec{P} + \vec{Q}|^2 = P^2.$$

Expanding the left-hand side gives

$$P^2 + Q^2 + 2\vec{P} \cdot \vec{Q} = P^2 \implies 2\vec{P} \cdot \vec{Q} = -Q^2.$$

Thus the two original forces have a negative scalar product. They are not generally perpendicular; it is the new resultant whose angle must be determined.

Step 2: Test the new resultant against the given force Let $\vec{S} = 2\vec{P} + \vec{Q}$ be the resultant requested in the question. Its scalar product with $\vec{Q}$ is

$$\vec{S} \cdot \vec{Q} = (2\vec{P} + \vec{Q}) \cdot \vec{Q} = 2\vec{P} \cdot \vec{Q} + Q^2 = -Q^2 + Q^2 = 0.$$

Using $\vec{S} \cdot \vec{Q} = |\vec{S}| |\vec{Q}| \cos \theta$, the zero dot product implies $\cos \theta = 0$ for the nonzero force and resultant for which the angle is defined. No numerical choice of force magnitudes is required: the cancellation follows directly from the stated equality of magnitudes.

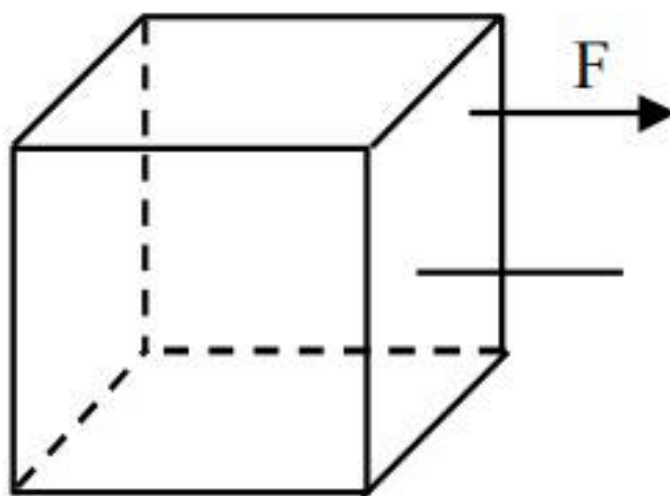
Step 3: Conclusion The resultant $2\vec{P} + \vec{Q}$ is perpendicular to $\vec{Q}$. Therefore

$$\theta = 90^\circ.$$

The numerical entry, with the angle measured in degrees as requested, is 90.

Quick Tip to Solve: When a vector-sum magnitude equals one original magnitude, square it first. The resulting scalar-product identity often makes the angle of another linear combination immediate.

47. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is [4 marks]



Solution:

Concept:

- For the horizontal force shown, vertical equilibrium gives $N = mg$; impending sliding therefore requires $F_s = \mu mg$.
- Toppling begins when the normal reaction reaches the downstream bottom edge. Taking moments about that edge, the applied force at height h must satisfy $F_t h = mg(a/2)$.
- The application point is on the cubical box. Since its centre of mass lies at height $a/2$, a point b above it has height $h = a/2 + b$, with $0 \leq b \leq a/2$.

Step 1: Compare the sliding and toppling thresholds The friction coefficient is $\mu = 0.4$, hence the force required to initiate sliding is

$$F_s = 0.4mg.$$

The weight acts through the centre, a horizontal distance $a/2$ from the possible pivot edge. At impending toppling,

$$F_t \left(\frac{a}{2} + b \right) = mg \frac{a}{2}, \quad F_t = \frac{mga}{a + 2b}.$$

For sliding to occur before toppling, require $F_s \leq F_t$. This gives

$$0.4 \leq \frac{a}{a+2b} \implies \frac{b}{a} \leq \frac{3}{4}.$$

Step 2: Apply the geometric restriction of the box The stability inequality alone would permit b/a up to $3/4$, but that would place the force above the top surface. A point on this cube cannot lie more than $a/2$ above its centre of mass. Therefore the physical restriction $b/a \leq 1/2$ is stronger. At the highest available point, $b = a/2$ and $h = a$, the toppling threshold is $F_t = mg/2$, whereas the sliding threshold is only $0.4mg$. Thus the cube does indeed begin to slide first even at its maximum possible application height; no lower height is needed for stability.

Step 3: Conclusion The greatest allowed value is $b/a = 1/2$, so

$$100 \frac{b}{a} = 50.$$

This requested ratio is dimensionless.

Quick Tip to Solve: After comparing sliding and toppling thresholds, also enforce where the force can actually be applied. A point on a cube lies at most half a side above its centre of mass.

48. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produced liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g] the value of M is ____ [4 marks]

Solution:

Concept:

- In a thermally insulated container, heat released by the hotter substance equals heat absorbed by the colder substance; no heat is transferred to the surroundings.
- At a phase change, heat is mL without temperature change. For liquid water, use $Q = mc\Delta T$, with $c = 1 \text{ cal g}^{-1} ^\circ\text{C}^{-1}$ in the calorie system.

Step 1: Calculate heat required by the initial ice The 200 g of ice starts at its melting point, 0°C . It first melts, absorbing latent heat $L_f = 80 \text{ cal g}^{-1}$, and then the resulting water warms from 0 to 40°C :

$$Q_{\text{gained}} = 200(80) + 200(1)(40 - 0) = 16000 + 8000 = 24000 \text{ cal}.$$

No term for warming ice below zero is required, because the ice is already at its melting point. Since the final state is liquid water at 40°C , all the initial ice has melted.

Step 2: Calculate heat released by steam and balance Each gram of steam first condenses at 100°C , releasing 540 cal, and the condensed water then cools to 40°C , releasing another $1(100 - 40) = 60 \text{ cal}$. Thus each gram releases 600 cal, giving

$$Q_{\text{released}} = M[540 + 1(100 - 40)] = 600M \text{ cal}.$$

Equating the heat transfers yields

$$600M = 24000 \implies M = 40 \text{ g}.$$

Both latent and sensible heat are needed on each side of the balance. The final temperature lies below the steam condensation point and above the ice melting point, consistent with the stated final liquid phase.

Step 3: Conclusion The mass of steam needed is

$$M = 40 \text{ g}.$$

Since M is specified in grams, the numerical entry is 40.

Quick Tip to Solve: Write a separate heat term for every stage: ice melting plus water warming equals steam condensation plus condensate cooling. The final 40°C determines both temperature changes.

49. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) _____ [4 marks]

Solution:

Concept:

- The charge on an isolated capacitor system is conserved. For a capacitor of capacitance C , charge and voltage are related by $Q = CV$.
- The stored electrostatic energy is $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$. Capacitors connected in parallel have the same final potential difference.

Step 1: Find the initial charge and energy Initially only the first capacitor is charged. Convert picofarads to farads before substituting:

$$C = 60 \times 10^{-12} \text{ F}, \quad V = 20 \text{ V}.$$

The isolated system therefore contains charge

$$Q = CV = 1.2 \times 10^{-9} \text{ C}.$$

Its initial energy is

$$U_i = \frac{1}{2}(60 \times 10^{-12})(20)^2 = 12 \times 10^{-9} \text{ J} = 12 \text{ nJ}.$$

Step 2: Use charge conservation after connection The second capacitor is initially uncharged and has the same capacitance. The combined capacitance is $2C = 120 \text{ pF}$, while the total charge remains Q , since the supply has been disconnected. Thus

$$V_f = \frac{Q}{2C} = 10 \text{ V}.$$

Both capacitors finally have this voltage. Their combined energy, rather than the energy of only one of them, is

$$U_f = \frac{1}{2}(2C)V_f^2 = \frac{1}{2}(120 \times 10^{-12})(10)^2 = 6 \text{ nJ}.$$

Step 3: Calculate the dissipated energy The decrease of stored electric-field energy is

$$U_{\text{lost}} = U_i - U_f = 12 - 6 = 6 \text{ nJ}.$$

Charge conservation does not imply conservation of the electrostatic energy alone. During redistribution, the difference is dissipated in the connecting circuit, including resistance and electromagnetic radiation.

Step 4: Conclusion The electrostatic energy lost is $\boxed{6 \text{ nJ}}$. The required numerical entry in nJ is $\boxed{6}$.

Quick Tip to Solve: For a charged capacitor connected to an identical uncharged one, the common voltage becomes half and the total stored energy becomes half of its initial value.

50. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10} \Omega$, where N is an integer then value of N is [4 marks]

Solution:

Concept:

- With an unchanged potentiometer potential gradient k , a balancing length l measures a potential difference kl . The open-circuit length measures emf E , whereas the loaded length measures terminal voltage V .
- If an external resistance R draws current from a cell of internal resistance r , then $E/V = (R+r)/R$, giving $r = R(E/V - 1)$.

Step 1: Determine the loaded balancing length The open-circuit balancing length is $l_1 = 560 \text{ cm}$. Connecting the external resistance draws a current from the cell, producing an internal voltage drop. Its terminal voltage is therefore smaller than its emf, so the balancing length decreases, not increases:

$$l_2 = 560 - 60 = 500 \text{ cm}.$$

Since the potentiometer gradient is unchanged, centimetres may be used directly in the ratio:

$$\frac{E}{V} = \frac{kl_1}{kl_2} = \frac{560}{500} = 1.12.$$

Step 2: Calculate the internal resistance The current through the external 10Ω resistance is $I = V/R$. Using the cell equation $E = V + Ir$ gives

$$E = V \left(1 + \frac{r}{R} \right).$$

Therefore

$$r = R \left(\frac{l_1}{l_2} - 1 \right) = 10 \left(\frac{560}{500} - 1 \right) = 10 \left(\frac{60}{500} \right) = 1.2 \Omega.$$

The denominator is the loaded length, 500 cm, not the original open-circuit length.

Step 3: Match the requested parameter The question expresses the internal resistance as $N/10 \Omega$. Equating this with the calculated physical resistance gives

$$\frac{N}{10} = 1.2, \quad N = 12.$$

The positive resistance also confirms that choosing a longer loaded balancing length would have been physically inconsistent.

Step 4: Conclusion The internal resistance is 1.2Ω , so the required dimensionless integer is $N = 12$.

Quick Tip to Solve: Loading a cell reduces its terminal voltage: subtract the length change first, then use $r = R(l_1 - l_2)/l_2$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Among the statements(a)-(d) the incorrect ones are :

- (a) Octahedral CO(III) complexes with strong fields ligands have very high magnetic moments.
(b) When $\Delta_o < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{eg}^4 e_g^2$
(c) Wavelength of light absorbed by $[\text{Co(en)}_3]^{3+}$ is lower than that of $[\text{CoF}_6]^{3-}$
(d) If the Δ_o for an octahedral complex of CO(III) is $18,000 \text{ cm}^{-1}$, the Δ_t for its tetrahedral complex with the same ligand be $16,000 \text{ cm}^{-1}$

[4 marks]

(A) (a) and (b) only

(B) (b) and (c) only

(C) (c) and (d) only

(D) (a) and (d) only

Correct Answer: (D) (a) and (d) only

Solution:

Concept:

- Cobalt(III) has a d^6 configuration. In an octahedral field, the lower set is t_{2g} and the upper set is e_g . Whether electrons pair depends on the splitting Δ_o compared with the pairing energy P .
- The absorbed photon satisfies $\Delta E = hc/\lambda$. For the same metal and ligands, the approximate tetrahedral splitting is $\Delta_t = \frac{4}{9}\Delta_o$.

Step 1: Test statements (a) and (b) A strong-field ligand gives $\Delta_o > P$. For octahedral Co(III), the six d electrons occupy $t_{2g}^6 e_g^0$, with no unpaired electron. Its spin-only magnetic moment is zero, rather than very high. Statement (a) is therefore incorrect.

When $\Delta_o < P$, promoting electrons to e_g costs less than additional pairing. The high-spin d^6 occupancy

is $t_{2g}^4 e_g^2$, with four unpaired electrons. Thus the high-spin occupancy intended in statement (b) is correct; its lower-level label denotes the octahedral t_{2g} set.

Step 2: Test statements (c) and (d) Ethylenediamine, en, is stronger-field than fluoride. Consequently $[\text{Co(en)}_3]^{3+}$ has a larger splitting than $[\text{CoF}_6]^{3-}$. Since wavelength varies inversely with the photon energy, the en complex absorbs shorter-wavelength light. Statement (c) is correct.

For the stated octahedral splitting,

$$\Delta_t = \frac{4}{9}(18000) = 8000 \text{ cm}^{-1},$$

not 16000 cm^{-1} . Hence statement (d) is incorrect.

Step 3: Select the pair of incorrect statements The incorrect statements are exactly (a) and (d). Option (A) wrongly includes the correct high-spin configuration in (b), option (B) contains the two correct statements, and option (C) wrongly includes the correct wavelength comparison in (c). Only option (D) contains both incorrect statements without a correct one.

Step 4: Conclusion The first and fourth statements are incorrect. Hence, the correct option is (D).

Quick Tip to Solve: For octahedral Co(III), remember low-spin $d^6 = t_{2g}^6$; also use $\Delta_t/\Delta_o = 4/9$, not its reciprocal.

52. Within each pair of elements F & Cl, S & Se, and Li & Na, respectively, the elements that release more energy upon and electron gain are :

[4 marks]

- (A) F, S and Li
- (B) Cl, Se and Na
- (C) Cl, S and Li
- (D) F, Se and Na

Correct Answer: (C) Cl, S and Li

Solution:

Concept:

- Energy released on adding an electron to an isolated gaseous atom is the magnitude of its negative electron gain enthalpy. More energy released means a more negative value, not a numerically larger signed enthalpy.
- Atomic size, nuclear attraction and repulsion within the accepting orbital all matter. Compact second-period p orbitals can repel an incoming electron strongly, so a simple size trend has exceptions.

Step 1: Compare fluorine with chlorine Fluorine accepts an electron into a very compact $2p$ shell, where electron-electron repulsion is relatively large. Chlorine accepts it into a more spacious $3p$ shell, with a smaller repulsion penalty. Thus chlorine has the more negative electron gain enthalpy even though fluorine is more electronegative.

Representative magnitudes are about 328 kJ mol^{-1} for F and 349 kJ mol^{-1} for Cl. The element

releasing more energy in this pair is therefore Cl.

Step 2: Compare the other two pairs Sulphur has a more negative electron gain enthalpy than selenium: the released energies are approximately 200 and 195 kJ mol⁻¹, respectively. The added electron is attracted more strongly in the smaller sulphur atom, so choose S.

For lithium and sodium, adding an electron completes the outer ns^2 arrangement. The smaller lithium atom binds the added electron more strongly: the energy-release magnitudes are approximately 60 and 53 kJ mol⁻¹, respectively. Thus choose Li.

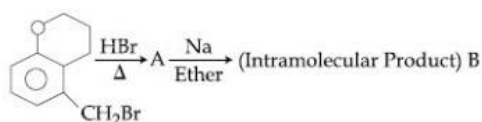
Step 3: Match the three selections to an option In the order requested, the elements are Cl, S and Li. Option (A) uses F instead of Cl, option (B) incorrectly chooses Se and Na, and option (D) makes all three of these selections incorrectly. The combination Cl, S and Li occurs only in option (C).

Step 4: Conclusion The elements that release more energy are Cl, S, Li, respectively. Hence, the correct option is (C).

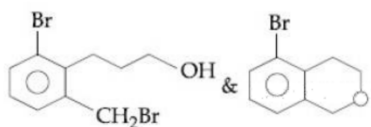
Quick Tip to Solve: Distinguish electronegativity from electron gain enthalpy: chlorine releases more energy than fluorine because its larger accepting orbital reduces electron repulsion.

53. In the following reactions sequence, structure of A and B respectively will be :

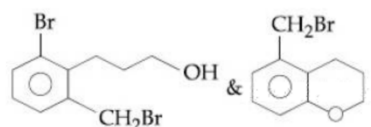
[4 marks]



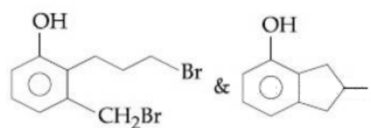
(A)



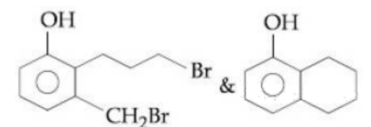
(B)



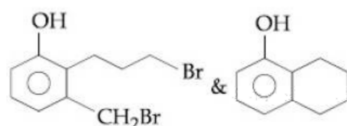
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- On heating an aryl-alkyl ether with HBr, protonation of oxygen is followed by cleavage of the alkyl C–O bond. The aryl C–O bond remains, producing a phenolic group and an alkyl bromide.
- Sodium in dry ether couples carbon atoms bearing halogens in a Wurtz reaction. An intramolecular coupling connects the two halogen-bearing side-chain ends within the same molecule.

Step 1: Open the oxygen-containing ring The starting compound contains an aryl-alkyl ether in its fused six-membered ring. HBr protonates the ether oxygen, after which bromide attacks the primary alkyl carbon bonded to oxygen. The aromatic carbon–oxygen bond cannot undergo the corresponding ordinary substitution at its sp^2 carbon.

Thus oxygen remains on the aromatic ring as –OH. The three-carbon chain that was part of the heterocycle now ends in –CH₂Br, so it is an aryl –CH₂CH₂CH₂Br substituent. The adjacent original –CH₂Br substituent remains. This is structure A shown in options (C) and (D).

Step 2: Couple the two brominated chain ends Sodium in ether induces the specified intramolecular Wurtz coupling between the terminal carbon of –CH₂CH₂CH₂Br and the carbon of the neighbouring –CH₂Br group. The two carbon–bromine bonds are replaced by one carbon–carbon bond, with removal of bromide as sodium bromide.

Count the resulting ring carefully: it contains the two adjacent aromatic attachment carbons, the three carbons of the propyl chain, and the one benzylic carbon of the bromomethyl group. There are $2 + 3 + 1 = 6$ ring atoms. The phenolic OH is retained.

Step 3: Eliminate the other products Options (A) and (B) have an aromatic bromine and an alcoholic chain in A; they require cleavage of the aryl–oxygen bond and therefore give the wrong ether-cleavage product. Option (C) has the right A but a methyl-substituted five-membered ring in B, which requires a different carbon connection. The six-membered carbocycle fused to benzene, with OH retained, is B in option (D).

Step 4: Conclusion A is the phenol bearing adjacent 3-bromopropyl and bromomethyl substituents, and B is the corresponding hydroxy-substituted tetralin shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: In an aryl-alkyl ether, leave oxygen on the aromatic carbon during HBr cleavage; then count both aromatic attachment carbons when finding the Wurtz ring size.

54. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non volatile solute are together sealed in a container. Over time : [4 marks]

- (A) The volume of the solution increases and the volume of the solvent decreases.
(B) The volume of solution does not change and the volume of the solvent decreases.
(C) The volume of the solution and the solvent does not change.
(D) The volume of the solution decreases and the volume of the solvent increases.

Correct Answer: (A) The volume of the solution increases and the volume of the solvent decreases.

Solution:

Concept:

- A non-volatile solute lowers the equilibrium vapour pressure of its solvent. For an ideal solution, Raoult's law gives $p_{\text{solution}} = x_{\text{solvent}}p^0$, where $x_{\text{solvent}} < 1$ and p^0 is the pure solvent vapour pressure.
- Both beakers share the vapour space of the sealed container. Solvent molecules can evaporate from one liquid and condense into the other without the beakers being connected by a liquid pathway.

Step 1: Compare the vapour pressures At the same temperature the pure solvent has equilibrium vapour pressure p^0 . The solution contains a non-volatile solute, so its solvent mole fraction is smaller than unity and

$$p_{\text{solution}} = x_{\text{solvent}}p^0 < p^0.$$

The non-volatile solute does not produce a compensating vapour pressure of its own. Consequently the two exposed liquids do not initially have the same solvent escaping tendency.

Step 2: Determine the direction of net solvent transfer The pure solvent evaporates more readily. Its vapour enters the shared container atmosphere and some condenses into the solution, where the equilibrium solvent vapour pressure is lower. Thus the net transfer is from the pure-solvent beaker to the solution beaker.

The solution is diluted as it receives solvent: its solute amount remains fixed while its solvent amount increases. The pure-solvent beaker loses liquid. In the continuing process the solution volume increases and the volume of pure solvent decreases.

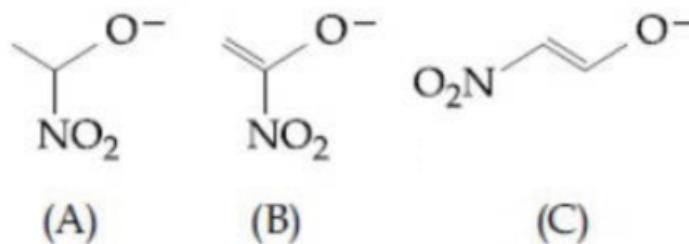
Step 3: Check all four proposed changes Option (A) states exactly this opposing volume change. Option (B) would require the received solvent not to increase the solution volume, contrary to the net transfer. Option (C) would require no net transfer despite the initial vapour-pressure difference. Option (D) reverses the thermodynamically favoured direction. Therefore the physically consistent change is the one in option (A).

Step 4: Conclusion The volume of the solution increases and the volume of the pure solvent decreases. Hence, the correct option is (A).

Quick Tip to Solve: In a shared sealed vapour space, solvent moves from the liquid with higher vapour pressure to the liquid with lower vapour pressure; the non-volatile solute stays in its beaker.

55. The correct order of stability for the following alkoxides is :

[4 marks]



(A) $(C) > (B) > (A)$

(B) $(C) > (A) > (B)$

(C) $(B) > (C) > (A)$

(D) $(B) > (A) > (C)$

Correct Answer: (A) $(C) > (B) > (A)$

Solution:

Concept:

- A negative ion becomes more stable when its negative charge can spread over a conjugated system rather than remaining localised on one atom. Resonance stabilisation is generally more decisive here than an inductive effect alone.
- A nitro group withdraws electron density by its inductive effect and, when a continuous conjugated path is available, by its resonance effect. Compare the actual connectivity before assigning these effects.

Step 1: Identify the stabilisation available in A and B In structure (A), the negatively charged oxygen is bonded to a saturated carbon bearing the nitro group. That saturated carbon interrupts a conjugated path, so the nitro group stabilises the oxygen anion mainly through its electron-withdrawing inductive effect. The charge cannot be delocalised through an uninterrupted chain of overlapping p orbitals.

In (B), the oxygen is attached to a carbon of a carbon-carbon double bond. Its lone pair can participate in the enolate-type resonance system: electron donation from oxygen forms a carbonyl bond while the double-bond electrons shift to the terminal carbon. Thus (B) gains resonance stabilisation unavailable to (A), giving $(B) > (A)$.

Step 2: Compare the extended conjugation in C In (C), oxygen is at one end of the conjugated carbon-carbon double bond and the nitro group is at the other end. The oxygen lone pair can donate along this path, shifting the carbon-carbon double bond and then spreading electron density into the nitro group. The terminal nitro group therefore acts as a resonance acceptor for the conjugated anion. This extended delocalisation is more effective than the arrangement in (B), where the nitro group is attached to the same carbon as oxygen and does not provide the same end-to-end conjugated charge-transfer path. Consequently, (C) is the most stable. Options (B), (C) and (D) each contradict at least one of these comparisons: they either put (A) above (B) or put (B) above (C).

Step 3: Conclusion The order of stability is $C > B > A$. Hence, the correct option is (A).

Quick Tip to Solve: Trace a continuous p-orbital path from the negatively charged oxygen to the nitro group. Extended resonance stabilisation outranks an inductive effect acting through a saturated carbon.

56. The bond order and the magnetic characteristics of CN^- are :

[4 marks]

- (A) 3, paramagnetic
- (B) $2\frac{1}{2}$, paramagnetic
- (C) 3, diamagnetic
- (D) $2\frac{1}{2}$, diamagnetic

Correct Answer: (C) 3, diamagnetic

Solution:

Concept:

- The bond order is one half of the difference between bonding and antibonding electron populations: $\text{BO} = (N_b - N_a)/2$. An electron in an antibonding orbital offsets the contribution of one bonding electron.
- A species is paramagnetic if it contains an unpaired electron and diamagnetic if all occupied molecular orbitals have paired electrons. Count the extra electron associated with a negative ionic charge.

Step 1: Count the electrons of the cyanide ion Carbon contributes six electrons, nitrogen seven, and the negative charge contributes one more. Thus CN^- has $6 + 7 + 1 = 14$ electrons in all, or ten valence electrons. It is isoelectronic with N_2 and CO . The two filled core orbitals have equal bonding and antibonding populations, so their effects on bond order cancel.

Step 2: Fill the valence orbitals and calculate the bond order The ten valence electrons occupy the bonding and antibonding orbitals as

$$(\sigma_{2s})^2(\sigma_{2s}^*)^2(\pi_{2p_x})^2(\pi_{2p_y})^2(\sigma_{2p_z})^2.$$

There are eight electrons in bonding valence orbitals and two in antibonding valence orbitals. Therefore

$$\text{BO} = \frac{8 - 2}{2} = 3.$$

The filled π orbitals each contain an electron pair, rather than a single electron.

Step 3: Determine the magnetic behaviour and compare the choices Every occupied orbital is doubly occupied, so the number of unpaired electrons is zero. The ion is consequently diamagnetic. Option (A) gives the correct bond order but the wrong magnetic behaviour. Options (B) and (D) incorrectly use a half-integral bond order, as if one electron had been omitted or removed from the relevant molecular-orbital occupancy.

Step 4: Conclusion CN^- has bond order 3 and is diamagnetic. Hence, the correct option is (C).

Quick Tip to Solve: Count the ionic charge before filling orbitals: CN^- has 14 electrons, so its closed-shell pattern is the same as that of N_2 .

57. Which of the following statements is correct ?

[4 marks]

- (A) Gluconic acid can form cyclic (acetal/hemiacetal) structure
- (B) Gluconic acid is a partial oxidation product of glucose
- (C) Gluconic acid is obtained by oxidation of glucose with (HNO₃)
- (D) Gluconic acid is a dicarboxylic acid.

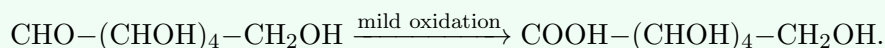
Correct Answer: (B) Gluconic acid is a partial oxidation product of glucose

Solution:

Concept:

- Glucose contains an aldehyde group at one end and a primary alcohol group at the other in its open-chain form. A mild oxidant can oxidise the aldehyde selectively without oxidising the terminal primary alcohol.
- Gluconic acid is the monocarboxylic acid obtained when only the aldehyde end of glucose is oxidised. Strong nitric acid oxidises both terminal groups and produces the dicarboxylic acid, saccharic or glucaric acid.

Step 1: Identify the group converted in forming gluconic acid The change from glucose to gluconic acid is the conversion of $-\text{CHO}$ into $-\text{COOH}$, while the $-\text{CH}_2\text{OH}$ end remains unchanged. In condensed form,



Bromine water is a suitable mild oxidant. Since just one of the two oxidisable terminal groups changes, gluconic acid is a partial oxidation product of glucose.

Step 2: Distinguish mild oxidation from nitric-acid oxidation Concentrated HNO₃ oxidises the aldehyde and the primary alcohol ends. The resulting product is HOOC-(CHOH)₄-COOH, a dicarboxylic acid called saccharic acid. Thus option (C) confuses the mild-oxidation product with the strong-oxidation product, and option (D) incorrectly assigns two carboxyl groups to gluconic acid.

Step 3: Check the statement about a cyclic acetal or hemiacetal A cyclic hemiacetal of glucose forms by addition of an internal hydroxyl group to its aldehyde carbonyl. Gluconic acid no longer has that aldehyde group. It can form a cyclic ester, called a lactone, through its carboxyl and hydroxyl groups, but a lactone is not an acetal or hemiacetal. Hence the cyclic-structure claim in option (A), with the specified acetal/hemiacetal description, is incorrect.

Step 4: Conclusion Gluconic acid is a partial oxidation product of glucose, with the aldehyde end oxidised and the primary alcohol end retained. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the endings: gluconic acid has one carboxyl group; saccharic acid has two. A gluconic-acid ring is a lactone, not a hemiacetal.

58. The number of possible optical isomers for the complexes MA_2B_2 with sp^3 and dsp^2 hybridized metal atom, respectively, is :

Note : A and B are unidentate neutral and unidentate monoanionic ligands, respectively. [4 marks]

(A) 0 and 2

(B) 0 and 0

(C) 0 and 1

(D) 2 and 2

Correct Answer: (B) 0 and 0

Solution:

Concept:

- Optical isomerism requires a chiral arrangement: the complex and its mirror image must be non-superimposable. A plane of symmetry makes an arrangement achiral.
- Four-coordinate sp^3 and dsp^2 descriptions correspond to tetrahedral and square-planar geometries, respectively. The question concerns optical isomers, rather than the total number of geometrical arrangements.

Step 1: Examine the tetrahedral complex For tetrahedral MA_2B_2 , two vertices carry identical A ligands and the other two carry identical B ligands. The geometry has a mirror plane through M and the two A positions, which interchanges the two B positions. Another such plane can be chosen through the B ligands. Its mirror image is therefore superimposable on the original arrangement. A tetrahedral complex with two pairs of identical unidentate ligands does not form an enantiomeric pair.

Step 2: Examine the square-planar complex A square-planar MA_2B_2 complex may have the A ligands adjacent, giving the cis form, or opposite, giving the trans form. However, in each case all four donor atoms and the metal lie in one plane. That coordination plane itself is a mirror plane. Both arrangements are consequently achiral, and neither gives optically active isomers in the stated model with simple unidentate ligands.

Step 3: Separate optical isomerism from geometrical isomerism The two square-planar arrangements must not be counted as two optical isomers: cis and trans are geometrical isomers, not mirror-image partners. Likewise, assigning one ordinary tetrahedral arrangement does not mean it is one optical isomer. Here the requested count is zero for each geometry. Options (A), (C) and (D) assign a nonzero optical count to at least one of these achiral cases, and are rejected.

Step 4: Conclusion The numbers of possible optical isomers for the tetrahedral and square-planar MA_2B_2 complexes are 0 and 0, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Check for a symmetry plane before counting optical isomers. Square-planar cis/trans isomerism is geometrical; it does not by itself create chirality.

59. The redox reaction among the following is :

[4 marks]

- (A) reaction of H_2SO_4 with NaOH .
- (B) formation of ozone from atmospheric oxygen in the presence of sunlight.
- (C) combination of dinitrogen with dioxygen at 2000 K
- (D) reaction of $[\text{Co}(\text{H}_2\text{O})_6]\text{Cl}_3$ with AgNO_3

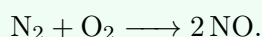
Correct Answer: (C) combination of dinitrogen with dioxygen at 2000 K

Solution:

Concept:

- A redox reaction has simultaneous oxidation and reduction: the oxidation number of at least one element increases and that of another decreases. A change of composition alone does not establish electron transfer.
- An element in its uncombined elemental form has oxidation number zero, including different allotropes of that element. Acid-base neutralisation and precipitation need not change any oxidation number.

Step 1: Analyse the combination of nitrogen and oxygen At high temperature, dinitrogen and dioxygen combine to form nitric oxide:



Both reactants contain elements with oxidation number zero. In NO , oxygen has oxidation number -2 , so nitrogen must have oxidation number $+2$. Nitrogen therefore changes from 0 to $+2$, which is oxidation; oxygen changes from 0 to -2 , which is reduction. The electron loss and gain occur together, confirming a redox reaction.

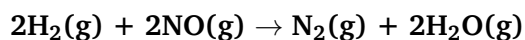
Step 2: Exclude neutralisation and allotrope conversion The reaction $\text{H}_2\text{SO}_4 + 2\text{NaOH} \longrightarrow \text{Na}_2\text{SO}_4 + 2\text{H}_2\text{O}$ is an acid-base neutralisation. Sodium remains $+1$, sulfur remains $+6$, hydrogen remains $+1$, and oxygen remains -2 . In the conversion $3\text{O}_2 \longrightarrow 2\text{O}_3$, oxygen remains in elemental form with oxidation number zero in both dioxygen and ozone. Thus neither option (A) nor option (B) describes a net redox change.

Step 3: Exclude precipitation of the chloride ions Adding AgNO_3 to $[\text{Co}(\text{H}_2\text{O})_6]\text{Cl}_3$ precipitates its outside-sphere chloride ions as AgCl . The net ionic change is $\text{Ag}^+ + \text{Cl}^- \longrightarrow \text{AgCl}(\text{s})$. Silver remains $+1$, chloride remains -1 , and cobalt remains $+3$ in the aqua complex. This is a precipitation process, so option (D) is not a redox reaction.

Step 4: Conclusion Combination of dinitrogen with dioxygen at 2000 K is the redox reaction, because nitrogen is oxidised and oxygen is reduced. Hence, the correct option is (C).

Quick Tip to Solve: Assign oxidation numbers before classifying a reaction. Oxygen stays at zero during $\text{O}_2 \longrightarrow \text{O}_3$, despite its change of molecular form.

60. For the reaction



the observed rate expression is, $\text{rate} = K_f[\text{NO}]^2[\text{H}_2]$. The rate expression for the reverse reaction is :

[4 marks]

- (A) $K_b[\text{N}_2][\text{H}_2\text{O}]$
- (B) $K_b[\text{N}_2][\text{H}_2\text{O}]^2/[\text{H}_2]$
- (C) $K_b[\text{N}_2][\text{H}_2\text{O}]^2/[\text{NO}]$
- (D) $K_b[\text{N}_2][\text{H}_2\text{O}]^2$

Correct Answer: (B) $K_b[\text{N}_2][\text{H}_2\text{O}]^2/[\text{H}_2]$

Solution:

Concept:

- For a reversible reaction at equilibrium, forward and reverse rates are equal. The equilibrium concentration quotient for the written reaction uses its stoichiometric powers, even when the observed rate law uses different powers.
- For the forward and reverse kinetic laws consistent with the equilibrium quotient, their ratio is the equilibrium constant: $K_c = k_f/k_b$. Do not assume that the observed overall reaction is an elementary step.

Step 1: Write the equilibrium concentration quotient For $2\text{H}_2(\text{g}) + 2\text{NO}(\text{g}) \rightleftharpoons \text{N}_2(\text{g}) + 2\text{H}_2\text{O}(\text{g})$, the concentration quotient at equilibrium is

$$K_c = \frac{[\text{N}_2][\text{H}_2\text{O}]^2}{[\text{H}_2]^2[\text{NO}]^2}.$$

The square on hydrogen in this expression comes from the balanced equation. It must be retained even though the experimentally observed forward rate is only first order in hydrogen.

Step 2: Combine the quotient with the observed forward rate The given law is $r_f = k_f[\text{NO}]^2[\text{H}_2]$. The forward and reverse laws must give equal rates when the concentrations satisfy the equilibrium relation. With $k_b = k_f/K_c$, the compatible reverse expression is

$$r_b = k_b \frac{[\text{N}_2][\text{H}_2\text{O}]^2}{[\text{H}_2]}.$$

Indeed, dividing the given forward law by this expression gives

$$\frac{r_f}{r_b} = \frac{k_f [\text{NO}]^2 [\text{H}_2]^2}{k_b [\text{N}_2] [\text{H}_2\text{O}]^2},$$

which is unity at equilibrium.

Step 3: Compare the concentration powers in the options Option (A) omits the required square on water. Option (C) places nitric oxide in the denominator, so its rate ratio cannot reproduce the equilibrium quotient. Option (D) omits the hydrogen denominator and would leave only the first power of hydrogen in the ratio. The denominator in option (B) supplies the additional hydrogen factor needed to match the stoichiometric equilibrium relation. Merely reversing the balanced equation and

using product coefficients as orders would miss this feature.

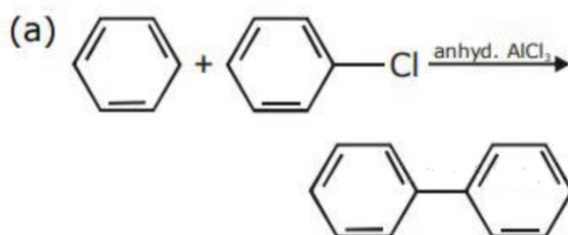
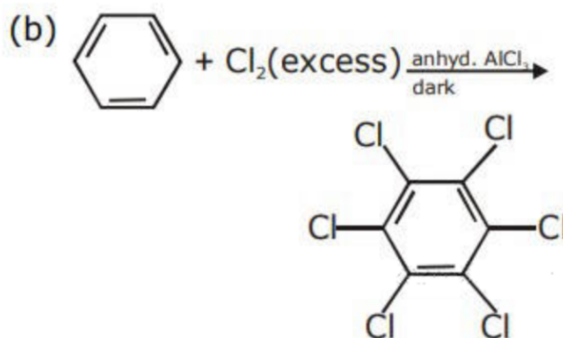
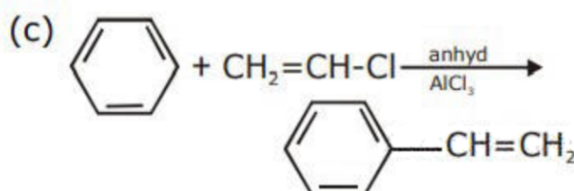
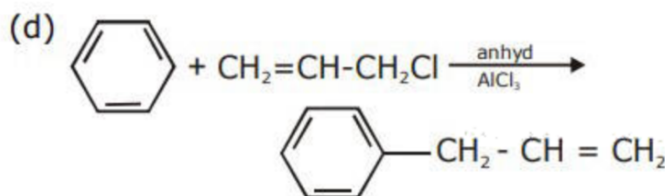
Step 4: Conclusion The reverse rate expression is $r_b = K_b[N_2][H_2O]^2/[H_2]$. Hence, the correct option is (B).

Quick Tip to Solve: For a non-elementary reaction, compare the forward/reverse rate ratio with the equilibrium quotient; do not equate all kinetic orders directly to coefficients.

61. Consider the following reactions :

Which of these reactions are possible ?

[4 marks]



(A) (b), (c) and (d)

(B) (a) and (b)

(C) (a) and (d)

(D) (b) and (d)

Correct Answer: (D) (b) and (d)

Solution:

Concept:

- Friedel–Crafts alkylation needs an electrophile that can be generated from the halide by a Lewis acid such as anhydrous AlCl_3 . Aryl and vinyl halides do not provide the required aryl or vinyl carbocations under these conditions.
- Halogenation of benzene in the dark with a Lewis acid proceeds by electrophilic aromatic substitution. Allylic halides can give resonance-stabilised allylic electrophiles and participate in Friedel–Crafts alkylation.

Step 1: Test reactions (a) and (c) In (a), chlorobenzene would have to supply a phenyl electrophile to benzene. Its aryl carbon–chlorine bond has partial double-bond character, and the proposed phenyl carbocation is highly unstable. Thus benzene and chlorobenzene do not form biphenyl by the indicated Friedel–Crafts route. In (c), chlorine is attached directly to a double-bonded carbon in vinyl chloride, $\text{CH}_2=\text{CHCl}$. This substrate similarly cannot generate the necessary vinyl electrophile with AlCl_3 , so the proposed styrene formation is not possible by that reaction.

Step 2: Test reactions (b) and (d) In (b), excess chlorine and anhydrous AlCl_3 , in the dark, allow successive substitution of ring hydrogen atoms. The depicted product is hexachlorobenzene, C_6Cl_6 , which retains the aromatic ring. It is not the saturated addition product benzene hexachloride. In (d), allyl chloride, $\text{CH}_2=\text{CHCH}_2\text{Cl}$, supplies a resonance-stabilised allylic electrophile. Benzene attacks it and loses a proton to give allylbenzene, $\text{C}_6\text{H}_5\text{CH}_2\text{CH}=\text{CH}_2$. Therefore (b) and (d) are possible. Options containing (a) or (c) include an invalid Friedel–Crafts conversion.

Step 3: Conclusion Only the reactions labelled b and d are possible under the indicated conditions. Hence, the correct option is (D).

Quick Tip to Solve: Locate the carbon bonded to chlorine: aryl and vinyl chlorides fail in ordinary Friedel–Crafts alkylation, whereas allyl chloride can react.

62. The equation that is incorrect is :

[4 marks]

(A) $(\Lambda_m^0)_{\text{KCl}} - (\Lambda_m^0)_{\text{NaCl}} = (\Lambda_m^0)_{\text{KBr}} - (\Lambda_m^0)_{\text{NaBr}}$

(B) $(\Lambda_m^0)_{\text{NaBr}} - (\Lambda_m^0)_{\text{NaI}} = (\Lambda_m^0)_{\text{KBr}} - (\Lambda_m^0)_{\text{NaBr}}$

(C) $(\Lambda_m^0)_{\text{NaBr}} - (\Lambda_m^0)_{\text{NaCl}} = (\Lambda_m^0)_{\text{KBr}} - (\Lambda_m^0)_{\text{KCl}}$

(D) $(\Lambda_m^0)_{\text{H}_2\text{O}} = (\Lambda_m^0)_{\text{HCl}} + (\Lambda_m^0)_{\text{NaOH}} - (\Lambda_m^0)_{\text{NaCl}}$

Correct Answer: (B) $(\Lambda_m^0)_{\text{NaBr}} - (\Lambda_m^0)_{\text{NaI}} = (\Lambda_m^0)_{\text{KBr}} - (\Lambda_m^0)_{\text{NaBr}}$

Solution:

Concept:

- At infinite dilution, the limiting molar conductivity of an electrolyte is the sum of the independent contributions of its ions, each multiplied by its stoichiometric coefficient.
- For a 1:1 electrolyte MX , write $\Lambda_m^0(\text{MX}) = \lambda_{\text{M}^+}^0 + \lambda_{\text{X}^-}^0$. Subtracting salts with a common ion cancels that ion's contribution exactly.

Step 1: Check the two simple difference identities Let k, n, c, b, i denote the limiting ionic conductivity

ties of K^+ , Na^+ , Cl^- , Br^- and I^- , respectively. For option A the two sides reduce to

$$(k + c) - (n + c) = k - n = (k + b) - (n + b).$$

Thus A is correct. For option C the two sides reduce to

$$(n + b) - (n + c) = b - c = (k + b) - (k + c).$$

Thus C is also correct. These are direct consequences of independent ionic migration and do not need numerical conductivity data.

Step 2: Examine the remaining equations In option B, the left-hand side cancels sodium ion, whereas the right-hand side cancels bromide ion:

$$\begin{aligned}\Lambda_m^0(\text{NaBr}) - \Lambda_m^0(\text{NaI}) &= b - i, \\ \Lambda_m^0(\text{KBr}) - \Lambda_m^0(\text{NaBr}) &= k - n.\end{aligned}$$

The anion difference $b - i$ is not the cation difference $k - n$; this is not a valid Kohlrausch identity. In D, adding the conductivities of hydrochloric acid and sodium hydroxide and subtracting sodium chloride cancels both spectator ions:

$$(\lambda_{H^+}^0 + c) + (n + \lambda_{OH^-}^0) - (n + c) = \lambda_{H^+}^0 + \lambda_{OH^-}^0.$$

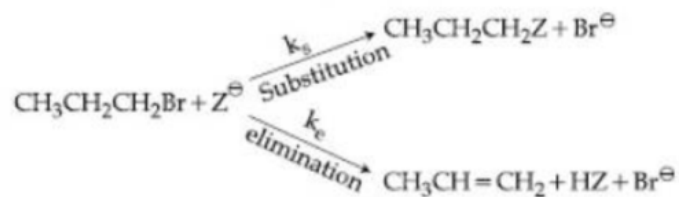
This is the limiting molar conductivity associated with water's constituent ions, so D is valid in the stated limiting-conductivity sense.

Step 3: Conclusion The second equation equates unrelated anion and cation differences and is incorrect. Hence, the correct option is (B).

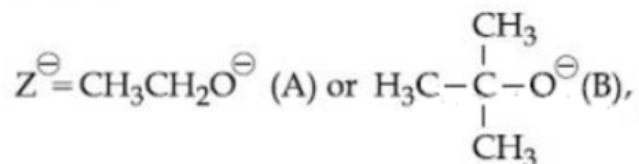
Quick Tip to Solve: Replace each salt by its two ionic conductivity terms, then cancel common ions before judging a conductivity identity.

63. For the following reactions

K_s and K_e , are respectively, the rate constants for substitution and elimination and $\mu = \frac{k_s}{k_e}$ the correct options is [4 marks]



where,



(A) $\mu_B > \mu_A$ and $K_e(\text{A}) > K_e(\text{B})$

(B) $\mu_A > \mu_B$ and $K_e(\text{B}) > K_e(\text{A})$

(C) $\mu_B > \mu_A$ and $K_e(\text{B}) > K_e(\text{A})$

(D) $\mu_A > \mu_B$ and $K_e(\text{A}) > K_e(\text{B})$

Correct Answer: (B) $\mu_A > \mu_B$ and $K_e(\text{B}) > K_e(\text{A})$

Solution:

Concept:

- A primary alkyl bromide can undergo bimolecular nucleophilic substitution, S_N2 , or base-induced bimolecular elimination, $E2$. Substitution requires a relatively unhindered approach to the carbon carrying bromine.
- A bulky strong base is sterically hindered as a nucleophile but can abstract a peripheral beta hydrogen. The ratio $\mu = k_s/k_e$ measures the tendency toward substitution relative to elimination for the same substrate.

Step 1: Compare the substitution tendencies of the two reagents Reagent A is ethoxide, $\text{CH}_3\text{CH}_2\text{O}^\ominus$, whereas B is tert-butoxide, $(\text{CH}_3)_3\text{CO}^\ominus$. Ethoxide has less crowding near its negatively charged oxygen and can approach the primary carbon of 1-bromopropane from the back side. It therefore promotes substitution efficiently, producing the corresponding ether. Tert-butoxide carries three methyl groups around the carbon attached to oxygen. This bulk obstructs approach to the electrophilic carbon, reducing substitution relative to elimination.

Step 2: Compare elimination and form the ratio Tert-butoxide is a stronger, bulky base and favours abstraction of a beta hydrogen instead of attack at the bromine-bearing carbon. The resulting concerted elimination produces propene. Consequently the elimination rate constant is larger for B than for A:

$$k_e(\text{B}) > k_e(\text{A}).$$

Since A favours substitution more strongly and B favours elimination, their selectivity ratios satisfy

$$\mu_A = \frac{k_s(A)}{k_e(A)} > \frac{k_s(B)}{k_e(B)} = \mu_B.$$

The option with both these inequalities is the second one. The first and third options incorrectly give the larger substitution-to-elimination ratio to the bulky base; the fourth incorrectly reverses the elimination tendency.

Step 3: Conclusion The substitution-to-elimination ratio is higher for ethoxide, whereas the elimination rate constant is higher for tert-butoxide. Hence, the correct option is (B).

Quick Tip to Solve: For an alkoxide acting on a primary halide, bulky tert-butoxide shifts the competition toward elimination and lowers the substitution-to-elimination ratio.

64. A chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (A) benzanilide (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvents, hexane : ethylacetate (20:80), the sequence of obtained compounds is :

[4 marks]

- (A) (A), (B) and (C)
- (B) (B), (A) and (C)
- (C) (C), (A) and (B)
- (D) (B), (C) and (A)

Correct Answer: (C) (C), (A) and (B)

Solution:

Concept:

- Silica gel is a polar stationary phase with surface silanol groups. In normal-phase column chromatography, a solute that interacts less strongly with silica moves faster with the mobile phase and is collected earlier.
- Adsorption depends on functional groups and their interactions with the surface, including hydrogen bonding and acid–base interactions. The elution order must therefore compare the ketone, amide and amine groups rather than merely count aromatic rings.

Step 1: Compare acetophenone and benzanilide Compound C, acetophenone, contains a ketone carbonyl. It can accept hydrogen bonds from the silica surface, but it has no nitrogen–hydrogen group to donate a hydrogen bond. Its adsorption is weaker than that of the other two compounds in this mixture, so it travels through the column and is obtained first.

Compound A, benzanilide, contains an amide group. Its carbonyl oxygen can accept a hydrogen bond, while its nitrogen–hydrogen group can donate one. These interactions retain benzanilide more strongly than acetophenone. The amide nitrogen lone pair is delocalised towards the carbonyl, so it is much less available for basic interaction than the lone pair of aniline.

Step 2: Place aniline and read the elution sequence Compound B, aniline, has a basic amino group. It interacts particularly strongly with the acidic silanol sites of silica gel, in addition to its hydrogen-

bonding interactions. It is consequently retained most strongly and comes out last under the stated conditions.

The adsorption-strength order is acetophenone less than benzanilide less than aniline. Since collection order is the reverse of retention strength, the elution sequence is C, then A, then B. The stated hexane–ethyl acetate mixture is the mobile phase used to carry the compounds; it does not make the strongly retained amino compound the first fraction. Options A and B incorrectly collect acetophenone last, and option D incorrectly collects aniline first.

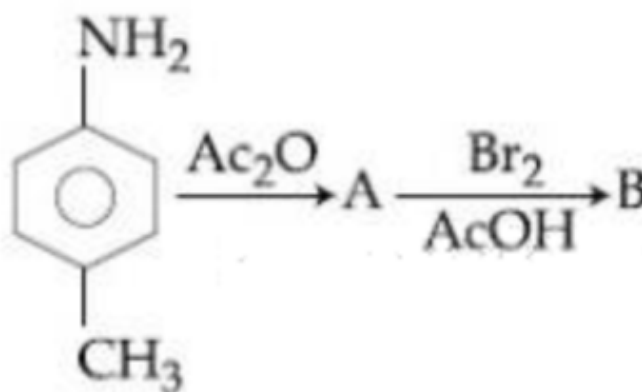
Step 3: Conclusion The compounds are obtained in the sequence acetophenone, benzanilide, aniline, corresponding to C, A, B. Hence, the correct option is (C).

Quick Tip to Solve: On polar silica, collect the least strongly adsorbed compound first. Watch for an amine: its strong interaction with acidic silanol groups can make it elute after a neutral amide.

65. In the following reaction sequence,

The major product B is :

[4 marks]



- (A) Benzene bearing NHCOCH_3 at position 1, Br at position 3 and CH_3 at position 4.
(B) Benzene bearing NHCOCH_3 at position 1, Br at position 2 and CH_3 at position 4.
(C) Benzene bearing NHCOCH_3 at position 1, COCH_3 at position 2 and CH_3 at position 4.
(D) Benzene bearing NHCOCH_3 at position 1, Br at position 2 and CH_2Br at position 4.

Correct Answer: (B) Benzene bearing NHCOCH_3 at position 1, Br at position 2 and CH_3 at position 4.

Solution:

Concept:

- Acetic anhydride acetylates a primary aromatic amino group to give an amide. This protects the amino group and reduces its very strong activating effect without removing its ortho/para-directing character.
- In electrophilic aromatic bromination, the acetamido group, $-\text{NHCOCH}_3$, directs the entering bromine mainly to an ortho or para ring position. A substituent already occupying the para position leaves the equivalent ortho sites available.

Step 1: Identify the product of acetylation The starting compound is para-toluidine: the amino and methyl groups are opposite one another on the benzene ring. Reaction with acetic anhydride converts $-\text{NH}_2$ into $-\text{NHCOCH}_3$, producing para-methylacetanilide A. The ring methyl group remains $-\text{CH}_3$, and no carbonyl group is introduced directly onto a ring carbon by this protection step. The two original substituents therefore remain at positions 1 and 4.

Step 2: Place bromine at the preferred ring position Bromine in acetic acid gives electrophilic aromatic substitution. The acetamido group is the stronger activating director compared with methyl. Its para position is already occupied by methyl, so bromination occurs at either of its two equivalent ortho positions, numbered 2 and 6. Choosing position 2 gives N-(2-bromo-4-methylphenyl)acetamide:

ring substituents: 1: NHCOCH_3 , 2: Br, 4: CH_3 .

The first structure has bromine meta to the acetamido group and is not the major product. The third introduces an acetyl group rather than bromine. The fourth additionally changes the methyl group into $-\text{CH}_2\text{Br}$, which would require side-chain radical bromination conditions rather than the given bromine/acetic-acid treatment.

Step 3: Conclusion Product B is N-(2-bromo-4-methylphenyl)acetamide, with ring bromine ortho to the acetamido group and the para methyl group unchanged. Hence, the correct option is (B).

Quick Tip to Solve: Acetylate the amino group first, then use the acetamido group's ortho/para direction; an occupied para site makes ortho bromination the preferred route.

66. The ammonia (NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide (NaOH) can be neutralized by : [4 marks]

- (A) 200 ml of 0.02 N HCl
- (B) 100 ml of 0.2 N HCl
- (C) 100 ml of 0.1 HCl
- (D) 200 ml of 0.4 N HCl

Correct Answer: (B) 100 ml of 0.2 N HCl

Solution:

Concept:

- Balanced chemical equations give the mole ratio between reactants and products. One mole of urea contains two nitrogen atoms and releases two moles of ammonia on complete reaction with sodium hydroxide.
- Ammonia accepts one proton per molecule: $\text{NH}_3 + \text{HCl} \longrightarrow \text{NH}_4\text{Cl}$. The number of acid equivalents supplied by hydrochloric acid is normality multiplied by volume in litres.

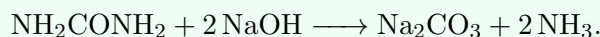
Step 1: Calculate the ammonia liberated The molar mass of urea, NH_2CONH_2 , is

$$M = 12 + 16 + 2(14) + 4(1) = 60 \text{ g mol}^{-1}.$$

Therefore the given amount of urea is

$$n(\text{urea}) = \frac{0.6}{60} = 0.010 \text{ mol.}$$

Its quantitative alkaline reaction is



Thus the ammonia released is $2 \times 0.010 = 0.020 \text{ mol}$. All the given urea is used because the reaction is stated to be quantitative.

Step 2: Compare the acid equivalents in the options Each mole of ammonia requires one mole of hydrochloric acid. Hence 0.020 equivalent of acid is needed. For the second option,

$$\text{acid equivalents} = NV = 0.2 \times 0.100 = 0.020.$$

It therefore neutralises exactly the ammonia liberated. The first option supplies $0.02 \times 0.200 = 0.004$ equivalent, which is insufficient. If the concentration 0.1 in the third option is read in the usual normal or molar sense for monoprotic hydrochloric acid, it supplies only $0.1 \times 0.100 = 0.010$ equivalent and is insufficient. The fourth option supplies $0.4 \times 0.200 = 0.080$ equivalent, leaving acid in excess rather than matching the quantitative neutralisation requirement.

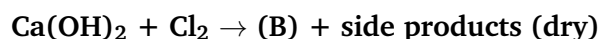
Step 3: Conclusion The ammonia released requires 0.020 mol of hydrochloric acid, supplied by 100 mL of 0.2 N hydrochloric acid. Hence, the correct option is (B).

Quick Tip to Solve: Count the two nitrogen atoms in urea before neutralising: one mole of urea releases two moles of ammonia, so it consumes two acid equivalents.

67. In the following reaction, products (A) and (B) respectively, are :



(hot and conc.)



[4 marks]

(A) NaOCl and Ca(OCl)_2

(B) NaClO_3 and $\text{Ca(ClO}_3)_2$

(C) NaOCl and $\text{Ca(ClO}_3)_2$

(D) NaClO_3 and Ca(OCl)_2

Correct Answer: (D) NaClO_3 and Ca(OCl)_2

Solution:

Concept:

- Chlorine undergoes disproportionation in alkali: the same element is reduced to chloride and oxidised to an oxygen-containing chlorine anion. The oxidation product depends strongly on the reaction conditions.
- Hot, concentrated sodium hydroxide gives chlorate, ClO_3^- , whereas dry slaked lime gives bleaching

material containing hypochlorite, ClO^- . Chlorine has oxidation numbers +5 in chlorate, +1 in hypochlorite and -1 in chloride.

Step 1: Apply the hot, concentrated alkali condition For the first reaction, balance chlorine disproportionation as



Thus the oxygen-containing sodium salt denoted by (A) is sodium chlorate, NaClO_3 . Sodium hypochlorite, NaOCl , instead characterises cold, dilute alkali. Ignoring the words hot and concentrated would therefore select the wrong first product.

Step 2: Identify the hypochlorite formed with lime Chlorine reacts with dry slaked lime to give bleaching powder, whose bleaching component is hypochlorite. In the calcium-hypochlorite representation used by these options, the balanced overall equation is



Consequently (B) corresponds to Ca(OCl)_2 , not calcium chlorate. This representation identifies the hypochlorite component rather than claiming that commercial bleaching powder is a single pure salt.

Step 3: Eliminate the remaining combinations Options (A) and (C) contain NaOCl , so they do not obey the stated hot, concentrated condition. Option (B) has the correct sodium chlorate but incorrectly pairs it with calcium chlorate. Only option (D) combines the chlorate formed in the first reaction with the hypochlorite component in the second.

Step 4: Conclusion The two products in the given order are NaClO_3 and Ca(OCl)_2 , respectively. Hence, the correct option is (D).

Quick Tip to Solve: Underline hot and concentrated before identifying a chlorine-alkali product: it changes hypochlorite into chlorate.

68. Among the statements (a) - (d), the correct ones are :

(a) Decomposition of hydrogen peroxide gives dioxygen.

(b) Like hydrogen peroxide, compounds, such as KClO_3 , $\text{Pb}(\text{NO}_3)_2$ and NaNO_3 when heated liberate dioxygen.

(c) 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide.

(d) Hydrogen peroxide is used for the manufacture of sodium perborate.

[4 marks]

(A) (a), (b), (c) and (d)

(B) (a) and (c) only

(C) (a), (b) and (c) only

(D) (a), (c) and (d) only

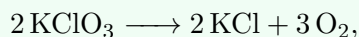
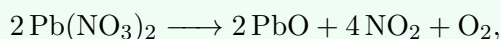
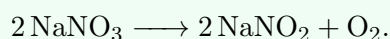
Correct Answer: (A) (a), (b), (c) and (d)

Solution:

Concept:

- Hydrogen peroxide is an unstable peroxide that decomposes into water and dioxygen: $2\text{H}_2\text{O}_2 \longrightarrow 2\text{H}_2\text{O} + \text{O}_2$. Its oxygen atoms have oxidation number -1 .
- Oxygen can also be prepared by thermal decomposition of certain chlorates and nitrates. Industrial peroxide manufacture uses the anthraquinone cycle, and peroxide is a reagent in the preparation of perborate bleaching agents.

Step 1: Verify the oxygen-producing decompositions Statement (a) is correct because the decomposition equation above explicitly liberates dioxygen. Statement (b) is also correct. The three named compounds release oxygen on heating, as shown by the balanced equations



The other products differ, but the common feature asked about is liberation of O_2 , not formation of identical residues.

Step 2: Check the preparation and use of peroxide Statement (c) is correct. In the industrial anthraquinone process, 2-ethylanthraquinone is hydrogenated to its reduced anthrahydroquinone form. Oxidation of that reduced compound by air regenerates the anthraquinone and produces hydrogen peroxide, which is then extracted. Thus the organic compound serves in a repeating preparation cycle rather than being the final peroxide product.

Statement (d) is correct as well: hydrogen peroxide is used to prepare sodium perborate, an oxygen-releasing bleaching material. This is a recognised application of peroxide alongside its own bleaching and disinfecting uses.

Step 3: Compare the sets of statements Every one of the four statements has been verified. Option (B) wrongly omits (b) and (d); option (C) wrongly omits (d); and option (D) wrongly omits (b). There is therefore no justification for excluding either the nitrate/chlorate decompositions or the sodium perborate application.

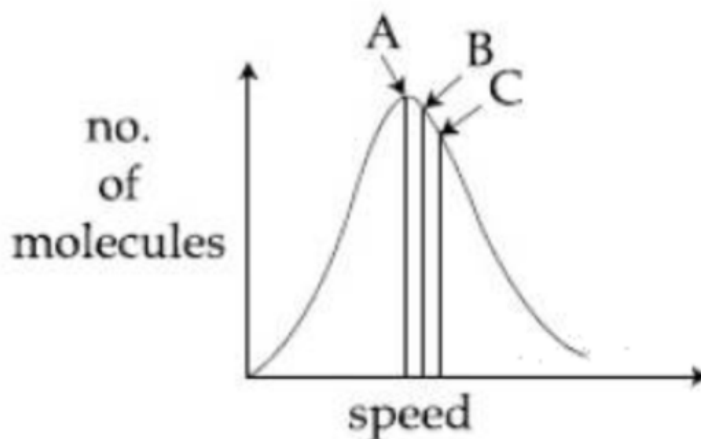
Step 4: Conclusion All four listed statements are correct. Hence, the correct option is (A).

Quick Tip to Solve: For a list of oxygen sources, write each decomposition equation: sodium nitrate gives nitrite, but it still releases oxygen.

69. Identify the correct labels of A,B and C in the following graph from the options given below :

Root mean square speed (V_{rms}); most probable speed (V_{mp}); Average speed (V_{av})

[4 marks]



(A) A - V_{av} ; B - V_{rms} ; C - V_{mp}

(B) A - V_{rms} ; B - V_{mp} ; C - V_{av}

(C) A - V_{mp} ; B - V_{av} ; C - V_{rms}

(D) A - V_{mp} ; B - V_{rms} ; C - V_{av}

Correct Answer: (C) A - V_{mp} ; B - V_{av} ; C - V_{rms}

Solution:

Concept:

- For gas molecules of molar mass M at absolute temperature T , the most probable speed, mean speed and root mean square speed have different definitions. The most probable speed is the position of the peak of a Maxwell speed-distribution curve.
- The formulae are $v_{mp} = \sqrt{2RT/M}$, $v_{av} = \sqrt{8RT/(\pi M)}$ and $v_{rms} = \sqrt{3RT/M}$, with R the gas constant. All use the same positive factor $\sqrt{RT/M}$.

Step 1: Order the three speeds using their formulae Compare their squared numerical coefficients rather than trying to infer the ordering from the heights of the three marked points. Since

$$2 < \frac{8}{\pi} < 3,$$

taking positive square roots gives

$$v_{mp} < v_{av} < v_{rms}.$$

Numerically the coefficients are approximately 1.414, 1.596 and 1.732, respectively. The ordering holds for the same gas and temperature irrespective of the particular values of M and T .

Step 2: Read the horizontal positions of the marked lines The graph has speed on its horizontal axis and number of molecules on its vertical axis. The marked vertical lines occur from left to right as A, B and C. A lies at the maximum of the distribution and so represents the most probable speed. The middle line B represents the average speed, while the rightmost line C represents the root mean square speed. It is the horizontal position, not the curve height at a marker, that identifies the value of a speed.

Step 3: Reject the incorrect assignments Option (A) places the most probable speed at the rightmost line, contradicting both the peak and the formulae. Option (B) makes the root mean square speed the smallest, which is impossible. Option (D) correctly places the most probable speed first but reverses the mean and root mean square speeds. Only option (C) preserves the required increasing order.

Step 4: Conclusion The labels are A: v_{mp} , B: v_{av} , C: v_{rms} . Hence, the correct option is (C).

Quick Tip to Solve: Locate the peak first for the most probable speed; then remember mean comes next and root mean square lies farthest right.

70. The refining method used when the metal and the impurities have low and high melting temperatures, respectively, is :

[4 marks]

- (A) liquation
- (B) vapour phase refining
- (C) distillation
- (D) zone refining

Correct Answer: (A) liquation

Solution:

Concept:

- Liquation separates a readily fusible metal from less fusible impurities by selective melting. The useful temperature lies above the melting point of the metal but below that of the solid impurities.
- The molten metal can flow away from the solid residue on a sloping hearth. This method uses a difference in melting temperatures, whereas distillation primarily uses a difference in volatility or boiling behaviour.

Step 1: Match the stated physical property to the process Let the melting temperatures of the metal and impurities be T_m and T_i , respectively, with $T_m < T_i$. Choose a working temperature T satisfying

$$T_m < T < T_i.$$

At this temperature the metal becomes liquid while the higher-melting impurities remain solid. The metal is allowed to drain or flow away from the solid impurities, so it is physically separated without converting it into a gaseous compound. This selective melting and drainage is the defining operation of liquation.

Step 2: Distinguish liquation from the other methods Vapour phase refining requires formation of

a volatile compound of the desired metal followed by decomposition of that compound to recover the pure metal; the prompt supplies no such chemical selectivity. Distillation requires the metal to vaporise and then condense away from less volatile impurities; low melting temperature alone does not guarantee that behaviour. Zone refining moves a narrow molten zone along a solid ingot and relies on the preferential solubility of impurities in the melt. Its basis is distribution between solid and liquid phases, rather than the simple selective melting described here.

Step 3: Choose the direct separation method The question explicitly contrasts a low-melting metal with high-melting impurities. That information directly supplies the temperature interval needed for liquation. Options (B), (C) and (D) introduce different separation principles that are not established by the given melting-temperature difference.

Step 4: Conclusion The appropriate refining method is liquation. Hence, the correct option is (A).

Quick Tip to Solve: Use melting-point contrast to recognise liquation; use boiling-point or volatility contrast to recognise distillation.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The flocculation value of HCl for arsenic sulphide sol.

is 30 mmol L^{-1} . If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount in grams, of H_2SO_4 in 250 ml required for the above purpose is _____.

(molecular mass of $\text{H}_2\text{SO}_4 = 98 \text{ g/mol}$)

[4 marks]

Solution:

Concept:

- Arsenic sulphide sol is negatively charged, so its effective coagulating ion is the positive counter-ion H^+ . Both hydrochloric acid and sulphuric acid therefore supply counter-ions of the same valency.
- The flocculation value is the minimum electrolyte concentration that coagulates the sol. For this comparison, equate the concentration of hydrogen ions supplied: one mole of HCl supplies one mole of H^+ , whereas one mole of H_2SO_4 supplies two.

Step 1: Convert the given flocculation value The concentration of hydrochloric acid required for coagulation is

$$c(\text{HCl}) = 30 \text{ mmol L}^{-1} = 0.030 \text{ mol L}^{-1}.$$

Since hydrochloric acid is monoprotic, this corresponds to 0.030 mol L^{-1} of the coagulating hydrogen ions. The chloride ion is a co-ion for this negative sol and is not the ion whose valency determines the coagulation comparison.

Step 2: Find the required sulphuric acid concentration Count both acidic hydrogen ions per formula unit of sulphuric acid for the stoichiometric electrolyte comparison:

$$2c(\text{H}_2\text{SO}_4) = 0.030.$$

Therefore

$$c(\text{H}_2\text{SO}_4) = 0.015 \text{ mol L}^{-1}.$$

The sulphate ion is divalent but negative, so applying a divalent-counter-ion coagulation rule to sulphate here would be incorrect. The relevant counter-ion remains monovalent hydrogen ion in both acids.

Step 3: Calculate the amount in the stated volume Convert 250 mL into 0.250 L, then use $n = cV$:

$$n = 0.015 \times 0.250 = 0.00375 \text{ mol}.$$

With the supplied molar mass 98 g mol^{-1} ,

$$m = nM = 0.00375 \times 98 = 0.3675 \text{ g}.$$

This is half the mass that would be obtained by incorrectly using the same molar concentration for both acids, because each sulphuric acid formula unit supplies twice as many hydrogen ions.

Step 4: Conclusion The required mass of H_2SO_4 in 250 mL is 0.3675 g, approximately 0.37 g.

Quick Tip to Solve: Identify the sol charge first: for negative arsenic sulphide, compare hydrogen-ion equivalents rather than the sulphate charge.

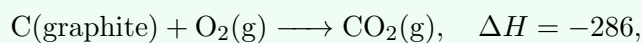
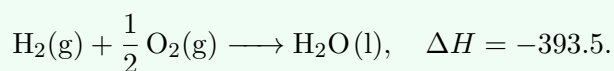
72. The standard heat of formation ($\Delta_f H_{298}^0$) of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are - 1560, -393.5 and -286 KJ/mol, respectively is : [4 marks]

Solution:

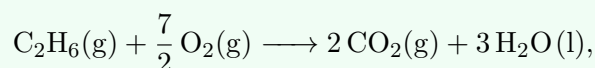
Concept:

- Hess's law states that the enthalpy change depends only on the initial and final states. Subtracting the combustion of ethane from the combustion of its constituent elements gives its formation reaction.
- The standard formation reaction is $2 \text{C}(\text{graphite}) + 3 \text{H}_2(\text{g}) \longrightarrow \text{C}_2\text{H}_6(\text{g})$. Use the combustion enthalpies attached to each substance in the question, including their signs and the stoichiometric factors two and three.

Step 1: Write the combustion equations with the supplied values The data associate $-393.5 \text{ kJ mol}^{-1}$ with hydrogen and -286 kJ mol^{-1} with graphite, respectively. Retain those assignments when applying Hess's law:



For one mole of ethane, the supplied combustion reaction is



with $\Delta H = -1560 \text{ kJ mol}^{-1}$.

Step 2: Form the Hess-law cycle Multiply the graphite combustion by two and the hydrogen combustion by three. Their sum burns the elemental ingredients to the same products as ethane combustion, giving

$$2(-286) + 3(-393.5) = -1752.5 \text{ kJ mol}^{-1}.$$

The alternative path first forms one mole of ethane, with enthalpy change $\Delta_f H_{298}^\circ$, and then burns it. Equating these two paths gives

$$\Delta_f H_{298}^\circ + (-1560) = -1752.5.$$

Step 3: Isolate the formation enthalpy and check the cycle Rearrange without losing the negative combustion sign:

$$\Delta_f H_{298}^\circ = -1752.5 + 1560 = -192.5 \text{ kJ mol}^{-1}.$$

As a check, adding this formation step to the ethane combustion gives $-192.5 - 1560 = -1752.5$, exactly the elemental-combustion sum. The negative value denotes exothermic formation for the numerical data supplied. Replacing the stated hydrogen and graphite values by remembered values would answer a different numerical problem.

Step 4: Conclusion Using the given assignments of the combustion enthalpies, $\Delta_f H_{298}^\circ(\text{C}_2\text{H}_6) = -192.5 \text{ kJ mol}^{-1}$.

Quick Tip to Solve: For Hess-law data given respectively, attach each value to its named reactant before multiplying by stoichiometric coefficients.

73. 3 g of acetic acid is added to 250 mL of 0.1 M HCL and the solution made up to 500 mL. To 20 mL of this solutions $\frac{1}{2}$ mL of 5 M NaOH is added. The pH of the solution is _____.

[Given : pKa of acetic acid = 4.75, molar mass of acetic of acid = 60 g/mol, $\log 3 = 0.4771$] Neglect any changes in volume. [4 marks]

Solution:

Concept:

- A strong base first neutralises the strong acid, HCl, and then converts part of the weak acid, CH_3COOH , into its conjugate base, CH_3COO^- . Calculate these stoichiometric amounts before applying an equilibrium formula.
- For a buffer containing a weak acid and its conjugate base, the Henderson–Hasselbalch relation is $\text{pH} = \text{p}K_a + \log(n_{\text{salt}}/n_{\text{acid}})$. The common solution volume cancels in this mole ratio.

Step 1: Find the acid amounts in the aliquot The initial acetic acid amount is $3/60 = 0.050 \text{ mol}$, while the hydrochloric acid amount is $0.250 \times 0.1 = 0.025 \text{ mol}$. Both are present in the final 500 mL solution. Therefore the 20 mL aliquot contains

$$n(\text{CH}_3\text{COOH}) = 0.050 \frac{20}{500} = 0.0020 \text{ mol},$$

$$n(\text{HCl}) = 0.025 \frac{20}{500} = 0.0010 \text{ mol.}$$

Step 2: Neutralise the strong acid and then the weak acid The added sodium hydroxide supplies

$$n(\text{NaOH}) = 5 \times 0.0005 = 0.0025 \text{ mol.}$$

After 0.0010 mol neutralises all the hydrochloric acid, 0.0015 mol remains to react by



Thus the final mixture contains 0.0015 mol acetate and $0.0020 - 0.0015 = 0.0005$ mol unreacted acetic acid. No excess strong acid or strong base remains; this is an acidic buffer.

Step 3: Calculate the buffer pH Use the remaining acid and the acetate formed, rather than the original acetic acid amount. Their ratio is $0.0015/0.0005 = 3$, so

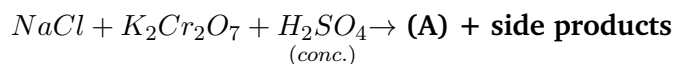
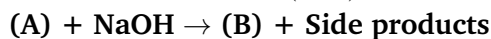
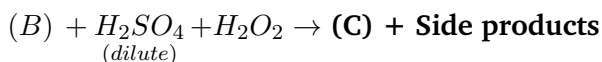
$$\text{pH} = 4.75 + \log 3 = 4.75 + 0.4771 = 5.2271.$$

The ratio greater than one also checks the sign: the pH must be above the given $\text{p}K_a$.

Step 4: Conclusion The solution has $\boxed{\text{pH} \approx 5.23}$. The pH is dimensionless.

Quick Tip to Solve: For a mixture of strong and weak acids, subtract the strong acid from the added hydroxide first; only the remaining hydroxide determines the salt-to-weak-acid ratio.

74. Consider the following reactions :



The sum of the total number of atoms in one molecule each of (A) and (B) and (C) is

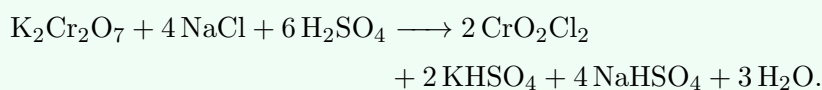
[4 marks]

Solution:

Concept:

- Chloride salts heated with dichromate and concentrated sulphuric acid form chromyl chloride, CrO_2Cl_2 . This is the characteristic volatile compound involved in the chromyl chloride test.
- Alkali converts chromyl chloride into a chromate. Acidified chromate reacts with hydrogen peroxide to give the blue chromium peroxide species CrO_5 . Count atoms from the complete formula of each named product, including sodium in the salt.

Step 1: Identify product A The first reaction is the chromyl chloride test. Consequently $\text{A} = \text{CrO}_2\text{Cl}_2$. A balanced representation with excess concentrated acid is



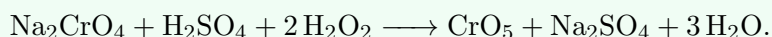
One molecule of A contains one chromium, two oxygen and two chlorine atoms, for $1 + 2 + 2 = 5$ atoms.

Step 2: Identify product B Chromyl chloride reacts with sodium hydroxide to form sodium chromate:



Therefore $B = \text{Na}_2\text{CrO}_4$. Its formula unit has two sodium, one chromium and four oxygen atoms, giving $2 + 1 + 4 = 7$ atoms. The sodium atoms must be included; counting only the chromate ion would omit part of B.

Step 3: Identify product C and add the atom counts On acidification in the presence of hydrogen peroxide, chromate gives the blue peroxide $C = \text{CrO}_5$. A balanced overall equation is



The peroxide contains one terminal oxo oxygen and two peroxide groups, so its formula indeed contains five oxygen atoms. The atom count of C is $1 + 5 = 6$. The requested sum for one unit of each product is

$$5 + 7 + 6 = 18.$$

Step 4: Conclusion $A = \text{CrO}_2\text{Cl}_2$, $B = \text{Na}_2\text{CrO}_4$ and $C = \text{CrO}_5$. The required total number of atoms is 18, a dimensionless count.

Quick Tip to Solve: Recognise the sequence chromyl chloride to yellow chromate to blue chromium peroxide; then count every subscript, including the sodium atoms in the middle product.

75. The number of sp^2 hybridised carbons present in "Aspartame" is

[4 marks]

Solution:

Concept:

- Aspartame is the methyl ester of the dipeptide L-aspartyl-L-phenylalanine. Its structural units include a phenyl ring, one free carboxylic acid, one peptide linkage and one methyl ester.
- A carbon in a benzene ring is sp^2 -hybridised, as is a carbonyl carbon. A saturated carbon with four single bonds is sp^3 -hybridised and must not be included in the requested count.

Step 1: Locate the carbon skeleton of aspartame Write the condensed connectivity so that all relevant carbon environments are visible:



The left fragment is the aspartyl residue, whose side chain retains a free carboxylic acid group. The central $\text{CO}-\text{NH}$ is the peptide linkage, and the right-hand COOCH_3 is the methyl ester of the phenylalanine residue. The phenyl ring is attached through a saturated CH_2 group.

Step 2: Count the aromatic and carbonyl carbons All six carbon atoms of the phenyl ring are trigonal-

planar sp^2 centres; aromatic delocalisation does not change this count to only three. Outside that ring, there are three carbonyl carbons: one in the free --COOH group, one in the peptide --CO--NH-- group, and one in the ester --COOCH_3 group. Each has three sigma-bond directions and a p orbital participating in its carbonyl pi bond. Hence

$$N(sp^2 \text{ carbon}) = 6 + 1 + 1 + 1 = 9.$$

Step 3: Exclude the saturated carbons and check the total The two amino-acid alpha carbons, the two methylene carbons and the ester methyl carbon have only single bonds and are sp^3 -hybridised. These five carbons are excluded. Together with the nine sp^2 carbons they account for all fourteen carbons of aspartame, providing a useful check that neither a carbonyl group nor a saturated side-chain carbon was missed.

Step 4: Conclusion Aspartame contains 9 sp^2 -hybridised carbon atoms. This is a dimensionless atom count.

Quick Tip to Solve: Count six carbons for the entire phenyl ring and then add every carbonyl carbon; exclude the ester methyl carbon and the saturated carbons between the functional groups.