JEE Main 2020 Paper 1 (B.E./B.Tech) Jan 8 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 January 2020, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The inverse function of

f(x) = $\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$, x ∈ (-1, 1), is : [4 marks]

(A) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(B) $\frac{1}{4}\left(\log_8 e\right)\log_e\left(\frac{1-x}{1+x}\right)$

(C) $\frac{1}{4}\left(\log_8 e\right)\log_e\left(\frac{1+x}{1-x}\right)$

(D) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

Correct Answer: (C) $\frac{1}{4}\left(\log_8 e\right)\log_e\left(\frac{1+x}{1-x}\right)$

Solution:

Concept:

- To find an inverse, set $y = f(t)$, solve for the original input t, and then replace y by x. The logarithm argument must be positive.
- The change-of-base identity is $\log_8 a = \ln a/\ln 8$, so $\log_8 e = 1/\ln 8$. The original exponential base must remain in the inverse.

Step 1: Rewrite the quotient using one exponential Put $y = f(t)$ and multiply numerator and denominator by 8^{2t}. This does not alter the quotient because the multiplier is strictly positive:

$$y = \frac{8^{4t}-1}{8^{4t}+1}.$$

Cross multiplication gives $y8^{4t} + y = 8^{4t} - 1$. Hence

$$(1-y)8^{4t} = 1+y, \qquad 8^{4t} = \frac{1+y}{1-y}.$$

The exponential is positive, and the output of the original quotient lies between -1 and 1, as required for this logarithm.

Step 2: Take logarithms and compare the options Taking natural logarithms yields

$$4t\ln 8 = \ln\left(\frac{1+y}{1-y}\right), \qquad t = \frac{1}{4\ln 8}\ln\left(\frac{1+y}{1-y}\right).$$

Thus the coefficient is $\frac{1}{4}\log_8 e$. Options (A) and (B) reverse the logarithm argument and give the wrong sign. Option (D) has the right argument but omits $1/\ln 8$. For the stated restriction $-1 < t < 1$, the inverse is restricted to the corresponding image $-4095/4097 < y < 4095/4097$; the same algebraic formula applies there.

Step 3: Conclusion $f^{-1}(x) = \frac{1}{4}(\log_8 e)\ln\left(\frac{1+x}{1-x}\right)$, on the image of the stated domain. Hence, the correct option is (C).

Quick Tip to Solve: Multiply the numerator and denominator by the negative-power exponential first; then solve for the single positive power before taking logarithms.

2. For which of the following ordered pairs (μ, δ), the system of linear equations

x + 2y + 3z = 1

3x + 4y + 5z = μ

4x + 4y + 4z = δ

is inconsistent ? [4 marks]

(A) (1, 0)

(B) (4, 3)

(C) (4, 6)

(D) (3, 4)

Correct Answer: (B) (4, 3)

Solution:

Concept:

- A linear system is inconsistent when elimination produces a row with zero coefficients but a nonzero right-hand side. A zero coefficient determinant alone does not prove inconsistency.
- Any linear dependence between coefficient rows must also hold between their right-hand sides for a solution to exist.

Step 1: Find the dependence between the coefficient rows Let the three equations be E_1, E_2, E_3. Subtracting $3E_1$ from E_2 and $4E_1$ from E_3 gives

$$-2y - 4z = \mu - 3, \qquad -4y - 8z = \delta - 4.$$

The left side of the second reduced equation is twice the left side of the first. Consistency therefore requires

$$\delta - 4 = 2(\mu - 3), \qquad \delta = 2\mu - 2.$$

Equivalently, the coefficient vector of E_3 is twice the difference of the vectors of E_2 and E_1.

Step 2: Test each proposed ordered pair For option (A), $(\mu, \delta) = (1, 0)$, the required value is $2(1) - 2 = 0$, so it is consistent. For option (B), $(4, 3)$, the required value is $2(4) - 2 = 6$, whereas the proposed value is 3. Indeed, elimination produces $0 = 3 - 6 = -3$, which is impossible. For option (C), $(4, 6)$, the relation holds. For option (D), $(3, 4)$, it also holds. The first two coefficient rows are independent, so whenever this relation holds there are two independent equations in three unknowns and at least one solution. Thus those three choices are not inconsistent.

Step 3: Conclusion The only inconsistent ordered pair is $(\mu, \delta) = (4, 3)$. Hence, the correct option is (B).

Quick Tip to Solve: When one coefficient row is a combination of the others, impose exactly the same combination on the constants; a mismatch immediately proves inconsistency.

3. If $\int \frac{\cos x dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = f(x)\left(1+\sin^6 x\right)^{1/\lambda} + c$

where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to [4 marks]

(A) $\frac{9}{8}$

(B) 2

(C) -2

(D) $-\frac{9}{8}$

Correct Answer: (C) -2

Solution:

Concept:

- The factor $\cos x\, dx$ suggests $t = \sin x$. Since the integrand contains $\sin^{-3} x$, the second substitution $u = t^{-2}$ absorbs this negative power.
- After substitution use $\int v^{-2/3}\, dv = 3v^{1/3} + C$. Work on any interval where $\sin x \neq 0$, since the original integrand is undefined at its zeros.

Step 1: Reduce the integral to a single power Set $t = \sin x$, so $dt = \cos x\, dx$, and then put $u = t^{-2} > 0$, giving $du = -2t^{-3}dt$. Because $t^6 = u^{-3}$, we obtain

$$I = \int \frac{dt}{t^3(1+t^6)^{2/3}} = -\frac{1}{2}\int \frac{u^2}{(1+u^3)^{2/3}}\, du.$$

Now set $v = 1 + u^3$, so $dv = 3u^2 du$. Thus

$$I = -\frac{1}{6}\int v^{-2/3}\, dv = -\frac{1}{2}v^{1/3} + C.$$

Step 2: Identify the factor and evaluate the requested value Returning to x, and using $(\sin^6 x)^{1/3} = \sin^2 x$, gives

$$I = -\frac{(1+\sin^6 x)^{1/3}}{2\sin^2 x} + C.$$

Comparison with the stated factorization gives $\lambda = 3$ and $f(x) = -1/(2\sin^2 x)$. At $x = \pi/3$,

$$f(\pi/3) = -\frac{1}{2(3/4)} = -\frac{2}{3}, \qquad \lambda f(\pi/3) = 3\left(-\frac{2}{3}\right) = -2.$$

For a check, differentiating $-\frac{1}{2}(1+t^{-6})^{1/3}$ with respect to t gives $t^{-7}(1+t^{-6})^{-2/3} = t^{-3}(1+t^6)^{-2/3}$. The negative sign rules out (A) and (B), and the exact magnitude rules out (D).

Step 3: Conclusion $\lambda f(\pi/3) = -2$. Hence, the correct option is (C).

Quick Tip to Solve: After replacing sine by a variable, try its reciprocal square: its differential removes the denominator power three and turns the remaining expression into a cube.

4. Let A and B be two independent events such that
P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{6}$.
Then, which of the following is TRUE? [4 marks]

(A) $P\left(\frac{A}{A \cup B}\right) = \frac{1}{4}$
(B) $P\left(\frac{A}{B}\right) = \frac{2}{3}$
(C) $P\left(\frac{A'}{B'}\right) = \frac{1}{3}$
(D) $P\left(\frac{A}{B'}\right) = \frac{1}{3}$

Correct Answer: (D) $P\left(\frac{A}{B'}\right) = \frac{1}{3}$

Solution:

Concept:

- For independent events, $P(A \cap B) = P(A)P(B)$, and conditioning on B does not change $P(A)$ when $P(B) > 0$.
- The conditional probability is $P(E \mid F) = P(E \cap F)/P(F)$. Independence also holds for complementary events, but conditional probabilities involving a union must be calculated separately.

Step 1: Compute intersections, complements and the union The given probabilities yield

$$P(A \cap B) = \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}.$$

By the addition rule,

$$P(A \cup B) = \frac{1}{3} + \frac{1}{6} - \frac{1}{18} = \frac{4}{9}.$$

Also, $P(A') = 2/3$ and $P(B') = 5/6$. Since $A \subseteq A \cup B$, the intersection of A with $A \cup B$ is just A. Hence

$$P(A \mid A \cup B) = \frac{1/3}{4/9} = \frac{3}{4},$$

which is different from the $1/4$ asserted in option (A).

Step 2: Evaluate the remaining conditional probabilities Independence gives

$$P(A \mid B) = \frac{1/18}{1/6} = \frac{1}{3},$$

so option (B), which asserts $2/3$, is false. For the complements,

$$P(A' \cap B') = 1 - P(A \cup B) = \frac{5}{9},$$

therefore $P(A' \mid B') = (5/9)/(5/6) = 2/3$, not the $1/3$ in option (C). Finally,

$$P(A \cap B') = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{18} = \frac{5}{18},$$

so

$$P(A \mid B') = \frac{5/18}{5/6} = \frac{1}{3}.$$

This verifies option (D) directly, with a nonzero conditioning probability. The numerator here is an intersection with B', not with B.

Step 3: Conclusion $P(A \mid B') = 1/3$ is the true statement. Hence, the correct option is (D).

Quick Tip to Solve: Independence survives complementing either event; keep the event before the conditioning bar distinct from its complement to avoid swapping one-third and two-thirds.

5. The locus of a point which divides the line segment joining the point (0, –1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is : [4 marks]

(A) $9x^2 - 3y = 2$

(B) $4x^2 - 3y = 2$

(C) $x^2 - 3y = 2$

(D) $9x^2 - 12y = 8$

Correct Answer: (D) $9x^2 - 12y = 8$

Solution:

Concept:

- If P divides AB internally with $AP : PB = m : n$, then its coordinates are $(nA + mB)/(m + n)$, coordinate by coordinate.
- A locus follows by expressing the coordinates of the moving endpoint in terms of the dividing point and substituting them into the endpoint curve equation.

Step 1: Apply the internal section formula Let the fixed point be $A = (0, -1)$, the moving point on the parabola be $B = (u, v)$, and the dividing point be $P = (x, y)$. The given ratio is $AP : PB = 1 : 2$, so the point is closer to the fixed endpoint. The section formula gives

$$x = \frac{2(0) + u}{3} = \frac{u}{3}, \qquad y = \frac{2(-1) + v}{3} = \frac{v - 2}{3}.$$

Consequently the moving endpoint can be recovered from the dividing point:

$$u = 3x, \qquad v = 3y + 2.$$

The weights are reversed relative to the endpoint order in the ratio; using $(A + 2B)/3$ would describe the other division ratio.

Step 2: Eliminate the moving coordinates and check the choices Since B lies on $u^2 = 4v$, substitution yields

$$(3x)^2 = 4(3y + 2), \qquad 9x^2 = 12y + 8.$$

Dividing by four gives the equivalent equation

$$\frac{9}{4}x^2 - 3y = 2.$$

The displayed choices must now be compared carefully: the equation derived above is exactly

$9x^2 - 12y = 8$, which is option (D), whereas (A), (B), and (C) have different coefficients of x^2 relative to y. For example, taking $B = (2, 1)$ gives $P = (2/3, -1/3)$; substitution gives $9(4/9) - 12(-1/3) = 8$, confirming the ratio and the resulting locus.

Step 3: Conclusion The locus is $9x^2 - 12y = 8$. Hence, the correct option is (D).

Quick Tip to Solve: Write the two endpoint names and the ratio before using the section formula. Test one nonvertex point on the original parabola to catch reversed weights.

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6. Let f(x) = $x\cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true? [4 marks]

(A) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

(B) f '(0) = $-\frac{\pi}{2}$

(C) f is not differentiable at x = 0

(D) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

Correct Answer: (A) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

Solution:

Concept:

- The principal range of $\cos^{-1}$ is $[0, \pi]$. For $0 \le t \le \pi/2$, $\cos(\pi/2 + t) = -\sin t$, so $\cos^{-1}(-\sin t) = \pi/2 + t$.
- Monotonicity of the derivative f' is determined by its slope on each side of zero. Differentiability at the joining point must be checked using the difference quotient.

Step 1: Remove the inverse cosine using its principal range Here $t = |x|$ lies in $[0, \pi/2]$, so $\pi/2 + |x|$ belongs to the principal range of the inverse cosine. Therefore

$$f(x) = x\left(\frac{\pi}{2} + |x|\right) = \begin{cases} \frac{\pi}{2}x - x^2, & -\pi/2 \le x < 0, \\ \frac{\pi}{2}x + x^2, & 0 \le x \le \pi/2. \end{cases}$$

This step is essential: using $\pi/2 - |x|$ would give the wrong sign because its cosine is $+\sin|x|$, while the printed argument is negative.

Step 2: Differentiate each branch and check zero On the open intervals,

$$f'(x) = \begin{cases} \frac{\pi}{2} - 2x, & -\pi/2 < x < 0, \\ \frac{\pi}{2} + 2x, & 0 < x < \pi/2. \end{cases}$$

The slope of f' is -2 on the negative interval and $+2$ on the positive interval. Hence f' decreases on $(-\pi/2, 0)$ and increases on $(0, \pi/2)$, as option (A) states; option (D) reverses these conclusions. At zero, $f(0) = 0$ and

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \left(\frac{\pi}{2} + |h|\right) = \frac{\pi}{2}.$$

Thus the function is differentiable at zero, ruling out (C), and its derivative there is positive, ruling out (B). The cusp in $|x|$ alone does not imply a cusp in its product with x.

Step 3: Conclusion f' decreases on $(-\pi/2, 0)$ and increases on $(0, \pi/2)$. Hence, the correct option is (A).

Quick Tip to Solve: Put the inverse-trigonometric expression into its principal range first. At an absolute-value junction, test the derivative directly rather than assuming nondifferentiability.

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7. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ **is equal to** [4 marks]

(A) e

(B) e^2

(C) $\frac{1}{e^2}$

(D) $\frac{1}{e}$

Correct Answer: (C) $\frac{1}{e^2}$

Solution:

Concept:

- The expression has the indeterminate form 1^∞. For a positive base, take a logarithm so that the exponent becomes a multiplier.
- The standard limit $\lim_{u\to 0}\log(1+u)/u = 1$ converts a small relative change in the base into its exponential limit.

Step 1: Rewrite the base near one Let the required limit be L. Since the numerator and denominator tend to 2, the base is positive near zero. Write

$$\frac{3x^2+2}{7x^2+2} = 1 - \frac{4x^2}{7x^2+2} = 1+u, \qquad u = -\frac{4x^2}{7x^2+2}.$$

Thus $u \to 0$; its leading term is $-2x^2$, not $-4x^2$, because the denominator tends to 2.

Step 2: Evaluate the logarithmic limit Taking logarithms and separating the standard factor gives

$$\log L = \lim_{x\to 0}\frac{\log(1+u)}{u}\frac{u}{x^2} = 1\cdot\lim_{x\to 0}\frac{-4}{7x^2+2} = -2.$$

Exponentiating yields $L = e^{-2}$. The answers e and e^2 have the wrong sign in the exponent: the original base is below one. The answer e^{-1} misses a factor of two in the relative change.

Step 3: Conclusion The limit is $e^{-2} = 1/e^2$. Hence, the correct option is (C).

Quick Tip to Solve: For a ratio of two expressions tending to the same nonzero constant, subtract numerator and denominator before applying the exponential limit.

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8. Let two points be A(1, –1) and B(0, 2). If a point P(x', y') be such that the area of $\triangle$PAB = 5 sq. units and it lies on the line, 3x + y – 4λ = 0, then a value of λ is : [4 marks]

(A) 4

(B) 1

(C) -3

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- The area of a triangle with three known coordinate pairs is half the absolute value of the coordinate determinant.
- The line through $A(1,-1)$ and $B(0,2)$ is $3x+y-2=0$. Lines $3x+y=$ constant are parallel, so the triangle area is constant along each such line.

Step 1: Express the area in terms of the coordinates of P Write $P=(x',y')$. Expanding the determinant using the given order of the vertices gives

$$[\triangle PAB]=\frac{1}{2}\left|x'(-1-2)+1(2-y')+0\right|=\frac{1}{2}|2-3x'-y'|.$$

The absolute value must remain, since P can lie on either side of AB. The stated area of 5 square units therefore requires $|3x'+y'-2|=10$.

Step 2: Use the line containing P and compare the choices Since P lies on the given line, $3x'+y'=4\lambda$. Consequently

$$|4\lambda-2|=10, \qquad 4\lambda-2=10 \text{ or } -10, \qquad \lambda=3 \text{ or } -2.$$

The question asks for a value among the listed choices; only 3 occurs there. For the alternatives 4, 1 and -3, the corresponding triangle areas are respectively 7, 1 and 7 square units, rather than 5.

Step 3: Conclusion A listed possible value is $\lambda=3$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the absolute value in an area equation: it commonly gives two parallel loci, even when only one resulting parameter is listed.

9. Let f : R → R be such that for all x ∈ R
(2^{1+x} + 2^{1-x}), f(x) and (3^x + 3^{-x}) are in A.P.,
then the minimum value of f(x) is [4 marks]

(A) 2
(B) 0
(C) 3
(D) 4

Correct Answer: (C) 3

Solution:

Concept:

- If three expressions are in arithmetic progression, twice the middle expression equals the sum of the outer two.
- For every positive number t, $t + t^{-1} \geq 2$, with equality exactly at $t = 1$. Apply this to exponential terms whose products are one.

Step 1: Obtain the function from the progression The middle term is $f(x)$, so the AP condition gives

$$2f(x) = 2^{1+x} + 2^{1-x} + 3^x + 3^{-x}.$$

Dividing by two and extracting the common factor 2 from the first pair,

$$f(x) = 2^x + 2^{-x} + \frac{1}{2}(3^x + 3^{-x}).$$

This identity holds for every real x; all exponential terms are positive.

Step 2: Find and attain a simultaneous lower bound The two reciprocal pairs satisfy

$$2^x + 2^{-x} \geq 2, \qquad 3^x + 3^{-x} \geq 2.$$

Hence $f(x) \geq 2 + \frac{1}{2} \cdot 2 = 3$. Both inequalities become equalities at $x = 0$, since all four powers then equal one. In particular $f(0) = 3$, so the bound is an attained minimum, not merely an infimum. Options 0 and 2 violate this lower bound; 4 cannot be the minimum because the explicit value at zero is smaller.

Step 3: Conclusion The minimum value of $f(x)$ is 3, attained at $x = 0$. Hence, the correct option is (C).

Quick Tip to Solve: When exponential terms occur in reciprocal pairs, try the inequality t + 1/t at once and check whether their equality conditions coincide.

10. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 resepectively. Each of these 10 observations is multiplied by p and then reduced by q, where p $\neq$ 0 and q $\neq$ 0. If the new mean and new s.d. become half of their original values, then q is equal to [4 marks]

(A) 10

(B) -20

(C) -10

(D) -5

Correct Answer: (B) -20

Solution:

Concept:

- Under $Y = pX - q$, the mean changes to $\overline{Y} = p\overline{X} - q$. Translation affects the mean but does not affect the spread.
- The standard deviation changes to $\sigma_Y = |p|\sigma_X$; the absolute value is essential because a standard deviation is nonnegative.

Step 1: Use the new standard deviation to find p The original standard deviation is 2 and its new value is 1. Therefore

$$1 = |p| \cdot 2, \qquad |p| = \frac{1}{2}, \qquad p = \frac{1}{2} \text{ or } -\frac{1}{2}.$$

A negative multiplier reflects the observations about zero but still halves their distances from the transformed mean, so it cannot be discarded.

Step 2: Apply the mean condition and exclude the prohibited case The new mean is 10. Thus

$$10 = 20p - q, \qquad q = 20p - 10.$$

If $p = 1/2$, this gives $q = 0$, which the question explicitly excludes. The remaining possibility is

$$p = -\frac{1}{2}, \qquad q = -10 - 10 = -20.$$

Checking directly, $Y = -X/2 + 20$ has mean $-20/2 + 20 = 10$ and standard deviation $2/2 = 1$. The other listed values 10, -10, and -5 satisfy neither of the allowed mean equations for $p = \pm 1/2$.

Step 3: Conclusion The required nonzero value is $q = -20$. Hence, the correct option is (B).

Quick Tip to Solve: Always use the absolute value of the multiplier for standard deviation; then let the mean and any nonzero conditions select its sign.

11. For a > 0, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b$ $(0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2, and the area of $\triangle OQR = \frac{1}{2}$, then 'a' satisfies the equation : [4 marks]

(A) $x^6 - 12x^3 + 4 = 0$

(B) $x^6 - 12x^3 - 4 = 0$

(C) $x^6 + 6x^3 - 4 = 0$

(D) $x^6 - 6x^3 + 4 = 0$

Correct Answer: (A) $x^6 - 12x^3 + 4 = 0$

Solution:

Concept:

- The first-quadrant intersections of $y^2 = ax$ and $x^2 = ay$, for $a > 0$, are O(0,0) and P(a,a); hence their common chord is $y = x$.
- Between $x = 0$ and $x = a$, the upper curve is $y = \sqrt{ax}$ and the lower curve is $y = x^2/a$. Integrate their difference to obtain bounded area.

Step 1: Determine the vertical bisector from the triangle The points where $x = b$ meets the chord and the x-axis are $Q = (b, b)$ and $R = (b, 0)$. Thus

$$[\triangle OQR] = \frac{1}{2}b^2 = \frac{1}{2}.$$

Because $b > 0$, we obtain $b = 1$. The given restriction $b < a$ also implies $a > 1$. These facts come from the triangle area, not from an assumption of symmetry of the parabolic region.

Step 2: Equate the area to the left to half the total The total enclosed area is

$$\int_0^a \left(\sqrt{ax} - \frac{x^2}{a}\right) dx = \frac{2}{3}a^2 - \frac{1}{3}a^2 = \frac{a^2}{3}.$$

The area to the left of $x = 1$ must be half of this:

$$\frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6} \quad \Longrightarrow \quad 4a^{3/2} - 2 = a^3.$$

Rearrange to $4a^{3/2} = a^3 + 2$ and square:

$$16a^3 = (a^3 + 2)^2 = a^6 + 4a^3 + 4, \qquad a^6 - 12a^3 + 4 = 0.$$

Both sides before squaring are positive, so the equation is necessary without a sign reversal. The other proposed polynomials have different coefficients or constant signs and do not follow from the bisecting-area condition.

Step 3: Conclusion The parameter satisfies $a^6 - 12a^3 + 4 = 0$, with the original restriction $a > 1$. Hence, the correct option is (A).

Quick Tip to Solve: Use the chord triangle first: it fixes b immediately and reduces the area condition to a single equation in a.

12. Let f(x) = (sin(tan^{-1}x) + sin(cot^{-1}x))^2 – 1, |x| > 1.

If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}\left(f\left(x\right)\right)\right)$ and $y\left(\sqrt{3}\right) = \frac{\pi}{6}$, then y($-\sqrt{3}$) is equal to : [4 marks]

(A) $\frac{5\pi}{6}$

(B) $-\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Correct Answer: The initial value is on x > 1 and does not fix the independent integration constant on x < -1; every listed value is possible.

Solution:

Concept:

- Use principal branches: $\sin(\tan^{-1} x) = x/\sqrt{1+x^2}$, while $\cot^{-1} x \in (0, \pi)$ gives $\sin(\cot^{-1} x) = 1/\sqrt{1+x^2}$.
- If two differentiable functions have equal derivatives, their difference is constant on each connected interval of their domain. Here $|x| > 1$ gives two separate intervals.

Step 1: Simplify the inverse sine expression Substituting the two sine expressions gives

$$f(x) = \frac{(x+1)^2}{1+x^2} - 1 = \frac{2x}{1+x^2}.$$

At $x = \sqrt{3}$ and $x = -\sqrt{3}$, its values are respectively $\sqrt{3}/2$ and $-\sqrt{3}/2$. Their principal inverse sines are therefore $\pi/3$ and $-\pi/3$. On the stated domain these arguments are strictly between -1 and 1, so the differentiation used in the question is legitimate.

Step 2: Integrate separately on the two intervals The differential equation implies

$$y(x) = \tfrac{1}{2}\sin^{-1}(f(x)) + C_+ \quad (x > 1),$$

$$y(x) = \tfrac{1}{2}\sin^{-1}(f(x)) + C_- \quad (x < -1).$$

The initial value $y(\sqrt{3}) = \pi/6$ fixes $C_+ = 0$, but imposes no restriction on C_-. Consequently $y(-\sqrt{3}) = -\pi/6 + C_-$ is undetermined. For any one of the four listed values, choosing the corresponding C_- gives a differentiable function on the stated domain satisfying every printed condition. If one additionally takes the same zero constant on both components, the conventional intended result is $-\pi/6$; that extra condition is not supplied.

Step 3: Conclusion As printed, the value is not uniquely determined because the domain is disconnected. The conventional answer (B), $-\pi/6$, follows only with an additional common-constant condition; the initial value alone does not prove it.

Quick Tip to Solve: An initial condition determines an integration constant only on the connected interval containing its initial point; inspect the domain before transferring it elsewhere.

13. The shortest distance between the lines

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ **and**

$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ **is :** [4 marks]

(A) 3

(B) $\frac{7}{2}\sqrt{30}$

(C) $3\sqrt{30}$

(D) $2\sqrt{30}$

Correct Answer: (C) $3\sqrt{30}$

Solution:

Concept:

- The direction ratios of a line in symmetric form are its three denominators. A point on the line is obtained by setting the common ratio equal to zero.
- For nonparallel lines through position vectors $\mathbf{a}_1, \mathbf{a}_2$, with direction vectors $\mathbf{d}_1, \mathbf{d}_2$, their shortest distance is $D = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)| / |\mathbf{d}_1 \times \mathbf{d}_2|$. The cross product provides a common perpendicular.

Step 1: Identify points and directions The first line passes through $A = (3, 8, 3)$ and has direction $\mathbf{d}_1 = (3, -1, 1)$. The second passes through $B = (-3, -7, 6)$ and has direction $\mathbf{d}_2 = (-3, 2, 4)$. Therefore the connecting vector is

$$\overrightarrow{AB} = (-6, -15, 3).$$

The negative denominators must retain their signs; they describe directions rather than distances.

Step 2: Calculate the common perpendicular Expand the cross product component by component:

$$\mathbf{d}_1 \times \mathbf{d}_2 = ((-1)4 - 1(2),\ 1(-3) - 3(4),\ 3(2) - (-1)(-3)) = (-6, -15, 3).$$

Its magnitude is

$$\sqrt{36 + 225 + 9} = \sqrt{270} = 3\sqrt{30}.$$

It is nonzero, so the lines are not parallel. Here the connecting vector itself equals this cross product and is perpendicular to both lines.

Step 3: Evaluate the distance and compare the options The scalar triple product is $36 + 225 + 9 = 270$. Hence

$$D = \frac{270}{3\sqrt{30}} = 3\sqrt{30}.$$

The positive value rules out an intersection. Option (A) omits the factor $\sqrt{30}$; options (B) and (D) have coefficients $7/2$ and 2 instead of the calculated coefficient 3.

Step 4: Conclusion The shortest distance is $\boxed{3\sqrt{30}}$. Hence, the correct option is (C).

Quick Tip to Solve: Check whether the vector joining the selected points is perpendicular to both line directions. If it is, its length already gives the shortest distance.

14. If a, b and c are the greatest value of $^{19}C_p$, $^{20}C_q$ and $^{21}C_r$ respectively, then : [4 marks]

(A) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$
(B) $\frac{a}{10} = \frac{b}{22} = \frac{c}{21}$
(C) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$
(D) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

Correct Answer: (D) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

Solution:

Concept:

- In row n of the binomial coefficients, adjacent terms satisfy $\binom{n}{k+1}/\binom{n}{k} = (n-k)/(k+1)$. The terms increase towards the middle and decrease afterwards.
- For even n, the largest coefficient is the unique central coefficient $\binom{n}{n/2}$. For odd n, the two equal central coefficients are the largest. Factorial ratios compare consecutive rows without evaluating large integers.

Step 1: Locate the largest coefficients For $n = 19$, the central indices are 9 and 10; for $n = 20$, the central index is 10; for $n = 21$, the central indices are 10 and 11. Thus we may write

$$a = \binom{19}{9}, \qquad b = \binom{20}{10}, \qquad c = \binom{21}{10}.$$

The choices of one of the equal middle indices for odd rows do not change the maximum values.

Step 2: Compare consecutive maximum values Using factorial definitions, the common factorials cancel:

$$\frac{b}{a} = \frac{20!}{10!10!}\frac{9!10!}{19!} = \frac{20}{10} = 2.$$

Similarly,

$$\frac{c}{b} = \frac{21!}{10!11!}\frac{10!10!}{20!} = \frac{21}{11}.$$

Consequently $b = 2a$ and $c = 42a/11$. These relations compare the coefficients exactly and avoid rounding errors.

Step 3: Convert the ratios into the proposed equalities From the first ratio,

$$\frac{a}{11} = \frac{b}{22}.$$

From the second relation,

$$\frac{c}{42} = \frac{a}{11}.$$

Together they give $a/11 = b/22 = c/42$. Option (A) would require $c/b = 21/22$, contradicting $21/11$. Option (B) would require $b/a = 22/10$, while option (C) requires $b/a = 11/10$; neither equals 2.

Step 4: Conclusion $\boxed{\frac{a}{11} = \frac{b}{22} = \frac{c}{42}}$. Hence, the correct option is (D).

Quick Tip to Solve: For maxima of consecutive binomial rows, identify the middle indices first and cancel factorials in ratios; computing the actual coefficients is unnecessary.

15. Let y = y(x) be a solution of the differential equation,
$\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$, **|x| < 1.**
If $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, **then** $y\left(-\frac{1}{\sqrt{2}}\right)$ **is equal to :** [4 marks]

(A) $-\frac{\sqrt{3}}{2}$
(B) None of those
(C) $\frac{1}{\sqrt{2}}$
(D) $-\frac{1}{\sqrt{2}}$

Correct Answer: (B) None of those

Solution:

Concept:

- For a real solution, both square roots are nonnegative and $|y| \le 1$. Since $\sqrt{1-x^2} > 0$ for $|x| < 1$, the equation implies $y' \le 0$: every real solution is nonincreasing.
- On an interval where $|y| < 1$, separation gives $\arcsin y + \arcsin x = C$. Division by $\sqrt{1-y^2}$ excludes the boundary solutions $y = \pm 1$, so these must be checked separately when extending a solution.

Step 1: Use the initial value in the separated solution Near $x = 1/2$, the initial value is strictly between -1 and 1, so separation is valid:

$$\frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}, \qquad \arcsin y + \arcsin x = C.$$

The condition gives

$$C = \arcsin\frac{\sqrt{3}}{2} + \arcsin\frac{1}{2} = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}.$$

For $0 < x < 1$, this yields $y = \cos(\arcsin x) = \sqrt{1-x^2}$, and its limit as $x \to 0^+$ is 1.

Step 2: Extend through the boundary without losing the sign A solution defined at the requested negative argument must pass continuously through $y(0) = 1$. Because it is nonincreasing as x increases, for $x < 0$ it must satisfy $y(x) \ge y(0) = 1$. The real-domain restriction $y(x) \le 1$ therefore forces $y(x) = 1$ throughout $-1 < x \le 0$. This constant branch satisfies the original equation, and its derivative zero matches the right derivative at zero.

Step 3: Evaluate the requested value and exclude the distractors Thus

$$y(x) = \begin{cases} 1, & -1 < x \le 0, \\ \sqrt{1-x^2}, & 0 < x < 1, \end{cases} \qquad y\left(-\frac{1}{\sqrt{2}}\right) = 1.$$

Blindly substituting a negative x into $\sqrt{1-x^2}$ would produce option (C), but that branch has positive derivative there and fails the original equation. The negative numbers in (A) and (D) also contradict monotonicity and $y(0) = 1$.

Step 4: Conclusion The required value is $\boxed{1}$, which is not any of the three listed numerical values. This differs from the supplied key: its positive reciprocal-root value would come from a branch having

positive derivative at negative arguments, contrary to the original differential equation. Hence, the correct option is (B).

Quick Tip to Solve: After separating an equation containing square roots, recheck signs and restore boundary solutions lost by division. A trigonometric rearrangement can otherwise introduce an invalid branch.

16. If the equation, $x^2 + bx + 45 = 0$ ($b \in R$) has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then : [4 marks]

(A) $b^2 - b = 42$

(B) $b^2 + b = 12$

(C) $b^2 + b = 72$

(D) $b^2 - b = 30$

Correct Answer: (D) $b^2 - b = 30$

Solution:

Concept:

- For a quadratic with real coefficients, nonreal roots occur as a conjugate pair $z, \bar{z}$. Vieta's relations give $z + \bar{z} = -b$ and $z\bar{z} = 45$.
- The modulus is calculated by multiplying a complex number by its conjugate: $|z+1|^2 = (z+1)(\bar{z}+1)$. This expresses the condition directly in terms of the sum and product of the roots.

Step 1: Translate the modulus condition into root relations Expand the product before substituting the given value:

$$|z+1|^2 = z\bar{z} + z + \bar{z} + 1 = 45 - b + 1 = 46 - b.$$

Both conjugate roots have the same modulus after adding the real number 1, so the relation applies to either root. Squaring the specified positive modulus $2\sqrt{10}$ gives 40, without a choice of sign.

Step 2: Determine and check the coefficient Equating the two expressions gives

$$46 - b = 40 \quad \Longrightarrow \quad b = 6.$$

This value must also meet the requirement of nonreal conjugate roots. Its discriminant is

$$b^2 - 4(45) = 36 - 180 = -144 < 0.$$

Indeed the roots are $-3 \pm 6i$, and adding 1 gives $-2 \pm 6i$, each of modulus $\sqrt{4+36} = 2\sqrt{10}$. Thus the derived value satisfies every condition in the question.

Step 3: Test the numerical statements With $b = 6$, the two relevant combinations are

$$b^2 - b = 36 - 6 = 30, \qquad b^2 + b = 36 + 6 = 42.$$

Option (A) incorrectly assigns 42 to the difference. Option (B) assigns 12 to the sum, and option (C) assigns 72 to the sum, whereas its value is 42. Only option (D) states the correct difference.

Step 4: Conclusion $b = 6$, so $\boxed{b^2 - b = 30}$. Hence, the correct option is (D).

Quick Tip to Solve: When a modulus condition involves a conjugate pair, expand the squared modulus and substitute the root sum and product instead of solving for the roots first.

17. If c is a point at which Rolle's theorem holds for the function, f(x) = $\log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval [3, 4], where a $\in$ R, then f"(c) is equal to [4 marks]

(A) $\frac{1}{12}$
(B) $\frac{\sqrt{3}}{7}$
(C) $-\frac{1}{12}$
(D) $-\frac{1}{24}$

Correct Answer: (A) $\frac{1}{12}$

Solution:

Concept:

- Rolle's theorem requires continuity on the closed interval, differentiability inside it, and equality of the endpoint values. Its interior point c satisfies $f\prime(c) = 0$.
- For a positive logarithm argument, write $f(x) = \log(x^2 + \alpha) - \log x - \log 7$. This separates the derivative into elementary terms and makes the stationary-point equation transparent.

Step 1: Use the endpoint equality to find the parameter Since the logarithm is one-to-one for positive arguments, $f(3) = f(4)$ implies

$$\frac{9+\alpha}{21} = \frac{16+\alpha}{28}.$$

Cross-multiplication gives $28(9 + \alpha) = 21(16 + \alpha)$, hence $7\alpha = 84$ and $\alpha = 12$. On $[3, 4]$, both x and $x^2 + 12$ are positive, so the logarithm is defined, continuous, and differentiable as required.

Step 2: Find the Rolle point Differentiate the separated logarithms:

$$f\prime(x) = \frac{2x}{x^2+12} - \frac{1}{x} = \frac{x^2-12}{x(x^2+12)}.$$

The denominator is positive throughout the interval, so the stationary point has $c^2 = 12$. The interior root is $c = \sqrt{12} = 2\sqrt{3}$, which lies strictly between 3 and 4; the negative root is outside the interval.

Step 3: Evaluate the second derivative Differentiating once more gives

$$f\prime\prime(x) = \frac{2(12-x^2)}{(x^2+12)^2} + \frac{1}{x^2}.$$

At $c^2 = 12$, the numerator of the first term vanishes, leaving

$$f\prime\prime(c) = 0 + \frac{1}{12} = \frac{1}{12}.$$

The sign is positive, excluding the two negative choices (C) and (D). The value $\sqrt{3}/7$ in (B) is

different from the exact rational result; the constant 7 inside the original logarithm disappears on differentiation.

Step 4: Conclusion $\boxed{f''(2\sqrt{3}) = \frac{1}{12}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the endpoint equality before differentiating. At the stationary point, substitute its squared value immediately to cancel terms in the second derivative.

18. Let the line y = mx and the ellipse $2x^2 + y^2 = 1$ intersect at a ponit P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and (0, β), then β is equal to : [4 marks]

(A) $\frac{\sqrt{2}}{3}$

(B) $\frac{2}{3}$

(C) $\frac{2\sqrt{2}}{3}$

(D) $\frac{2}{\sqrt{3}}$

Correct Answer: (A) $\frac{\sqrt{2}}{3}$

Solution:

Concept:

- Implicit differentiation of $2x^2 + y^2 = 1$ gives the tangent slope $dy/dx = -2x/y$. At a first-quadrant point (h, k), the normal therefore has slope $k/(2h)$.
- The normal through the point is $y - k = (k/(2h))(x - h)$. Axis intercepts follow by setting $y = 0$ or $x = 0$; the condition $2h^2 + k^2 = 1$ then determines the coordinates.

Step 1: Write the normal in intercept-friendly form Let $P = (h, k)$, where $h > 0$ and $k > 0$. Rearranging the normal equation gives

$$y = \frac{k}{2h}x + \frac{k}{2}.$$

The first-quadrant hypothesis ensures that neither h nor k is zero, so divisions by them are legitimate. The normal has positive slope; using the negative tangent slope instead would yield incorrect intercept signs.

Step 2: Use the given horizontal intercept Put $y = 0$ in the normal:

$$0 = \frac{k}{2h}x + \frac{k}{2} \quad \Longrightarrow \quad x = -h.$$

Comparing with the stated intercept $-1/(3\sqrt{2})$, we obtain

$$h = \frac{1}{3\sqrt{2}}, \qquad h^2 = \frac{1}{18}.$$

Now use the equation of the ellipse:

$$k^2 = 1 - 2h^2 = 1 - \frac{1}{9} = \frac{8}{9}.$$

Since P is in the first quadrant, the positive square root must be taken, giving $k = 2\sqrt{2}/3$.

Step 3: Read the vertical intercept and compare the options At $x = 0$, the normal equation gives $\beta = k/2$. Thus

$$\beta = \frac{1}{2}\frac{2\sqrt{2}}{3} = \frac{\sqrt{2}}{3}.$$

Option (C) is the point's ordinate k, not the intercept. Options (B) and (D) do not satisfy the calculated intercept relation. The line $y = mx$ only confirms the radial line through this first-quadrant point; its slope is not the normal slope.

Step 4: Conclusion $\boxed{\beta = \frac{\sqrt{2}}{3}}$. Hence, the correct option is (A).

Quick Tip to Solve: For the ellipse $2x^2 + y^2 = 1$, its normal at (h, k) has intercepts $(-h, 0)$ and $(0, k/2)$. Derive these once to avoid unnecessary slope algebra.

19. Which one of the following is a tautology? [4 marks]

(A) P $\wedge$ (P $\vee$ Q)

(B) P $\vee$ (P $\wedge$ Q)

(C) Q $\rightarrow$ (P $\wedge$ (P $\rightarrow$ Q))

(D) (P $\wedge$ (P $\rightarrow$ Q)) $\rightarrow$ Q

Correct Answer: (D) (P $\wedge$ (P $\rightarrow$ Q)) $\rightarrow$ Q

Solution:

Concept:

- A tautology is a compound proposition that is true for every possible assignment of truth values to its component propositions.
- The implication $P \rightarrow Q$ is equivalent to $\neg P \vee Q$; an implication is false only when its antecedent is true and its consequent is false.

Step 1: Simplify the antecedent in option D First combine the statement P with the implication $P \rightarrow Q$. By the distributive law,

$$P \wedge (P \rightarrow Q) = P \wedge (\neg P \vee Q) = (P \wedge \neg P) \vee (P \wedge Q) = P \wedge Q.$$

Consequently option D is $(P \wedge Q) \rightarrow Q$. If the antecedent is true, Q is necessarily true; if the antecedent is false, the implication is true automatically. This already establishes truth in all cases.

Step 2: Verify the result algebraically and reject the other options The same implication can be

reduced to the constant true proposition:

$$\neg(P \wedge Q) \vee Q = \neg P \vee \neg Q \vee Q = \text{True}.$$

By absorption, option A reduces to P, as does option B, so both fail whenever P is false. For option C choose $P = \text{False}$ and $Q = \text{True}$. Its antecedent Q is true, but $P \wedge (P \to Q)$ is false because P is false. Thus C is not always true either.

Step 3: Conclusion Only $(P \wedge (P \to Q)) \to Q$ is a tautology. Hence, the correct option is (D).

Quick Tip to Solve: Look for modus ponens: P together with $P \to Q$ forces Q, so the implication from those two premises to Q is always true.

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20. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then cosθ can be : [4 marks]

(A) $\frac{7}{6\sqrt{3}}$

(B) $\frac{7}{6\sqrt{6}}$

(C) $\frac{5}{7}$

(D) $\frac{5}{3\sqrt{3}}$

Correct Answer: (A) $\frac{7}{6\sqrt{3}}$

Solution:

Concept:

- The volume of the parallelepiped with coterminous edge vectors is the absolute scalar triple product: $V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$. The absolute value allows both orientations.
- The angle between two nonzero vectors satisfies $\cos\theta = (\vec{u} \cdot \vec{w})/(|\vec{u}||\vec{w}|)$.

Step 1: Determine the permitted values of lambda With $\vec{v} = (1, 1, 3)$ and $\vec{w} = (2, 1, 1)$, the cross product is

$$\vec{v} \times \vec{w} = (-2, 5, -1).$$

Taking its dot product with $\vec{u} = (1, 1, \lambda)$ gives

$$V = |-2 + 5 - \lambda| = |3 - \lambda|.$$

The given volume is 1 cubic unit, so $|3 - \lambda| = 1$, which gives $\lambda = 2$ or $\lambda = 4$. Both must be checked because the question asks which cosine can occur.

Step 2: Calculate the two possible cosines For general λ, the dot product is $3 + \lambda$, whereas the lengths are $\sqrt{2 + \lambda^2}$ and $\sqrt{6}$. Therefore

$$\cos\theta = \frac{3 + \lambda}{\sqrt{2 + \lambda^2}\sqrt{6}}.$$

For $\lambda = 2$ this is $5/6$. For $\lambda = 4$, it is

$$\frac{7}{\sqrt{18}\sqrt{6}} = \frac{7}{\sqrt{108}} = \frac{7}{6\sqrt{3}}.$$

Only option A equals either permitted value. In particular B has an extra factor $\sqrt{2}$ in its denominator relative to A, C is $5/7$, and D is $5/(3\sqrt{3})$; none equals either $5/6$ or $7/(6\sqrt{3})$.

Step 3: Conclusion The possible values are $5/6$ and $7/(6\sqrt{3})$; the listed attainable value is $7/(6\sqrt{3})$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the absolute value in the volume equation: the two values of the parameter can lead to two different angles, and only one need be listed.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The least positive value of 'a' for which the equation
$2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is [4 marks]

Solution:

Concept:

- A quadratic $Ax^2 + Bx + C = 0$, with real coefficients and $A \neq 0$, has real roots exactly when its discriminant $B^2 - 4AC$ is nonnegative.
- The equality case gives a repeated real root and must be included when the problem asks for the least allowed parameter.

Step 1: Put the equation in standard quadratic form Transfer the right-hand side to the left without changing the coefficient of x:

$$2x^2 + (a - 10)x + \left(\frac{33}{2} - 2a\right) = 0.$$

Thus the quadratic coefficients are $A = 2$, $B = a - 10$, and $C = 33/2 - 2a$. The leading coefficient is always nonzero, so there is no exceptional linear case to examine.

Step 2: Solve the discriminant inequality The discriminant is

$$\begin{aligned}\Delta &= (a - 10)^2 - 4(2)\left(\frac{33}{2} - 2a\right)\\ &= a^2 - 20a + 100 - 132 + 16a\\ &= a^2 - 4a - 32 = (a - 8)(a + 4).\end{aligned}$$

The upward-opening quadratic in a is nonnegative outside its two zeros. Therefore real roots require $a \leq -4$ or $a \geq 8$. Intersecting this set with $a > 0$ leaves $a \geq 8$, whose least member is 8.

Step 3: Check the endpoint directly At $a = 8$, the original equation becomes $2x^2 - 2x + 33/2 = 16$, or

$$2x^2 - 2x + \frac{1}{2} = 2\left(x - \frac{1}{2}\right)^2 = 0.$$

It has the repeated real root $x = 1/2$. Thus the endpoint is attained rather than merely approached, confirming that the least positive value exists.

Step 4: Conclusion The least positive value of the parameter is $\boxed{a = 8}$.

Quick Tip to Solve: For a parameter-dependent quadratic, factor the discriminant first and only then impose positivity of the parameter; do not discard the zero-discriminant endpoint.

22. The number of all 3 × 3 matrices A, with enteries from the set {–1, 0, 1} such that the sum of the diagonal elements of AA^T is 3, is [4 marks]

Solution:

Concept:

- For any real matrix A, the trace $\mathrm{tr}(AA^T)$ equals the sum of the squares of all entries of A, not just its diagonal entries.
- Each permitted entry is -1, 0 or 1, so its square is respectively 1, 0 or 1. The trace condition therefore counts the nonzero positions.

Step 1: Translate the diagonal sum into an entry count Write $A = (a_{ij})$, where $1 \le i, j \le 3$. The i-th diagonal entry of AA^T is the dot product of row i with itself:

$$(AA^T)_{ii} = a_{i1}^2 + a_{i2}^2 + a_{i3}^2.$$

Summing these three diagonal entries gives

$$\mathrm{tr}(AA^T) = \sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij}^2 = 3.$$

Since every squared entry is 0 or 1, exactly three of the nine entries of A must be nonzero. The other six must be zero.

Step 2: Choose positions and assign signs The three nonzero positions can be chosen from the nine distinct matrix positions in $\binom{9}{3}$ ways. Once the positions are fixed, each entry independently has two choices, +1 or -1. Hence each selected set of positions produces 2^3 matrices. There is no symmetry requirement on A and no restriction that the nonzero entries lie on its diagonal. Therefore

$$N = \binom{9}{3} 2^3 = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} \cdot 8 = 84 \cdot 8 = 672.$$

Each resulting matrix determines a unique set of positions and signs, so the count has no duplication.

Step 3: Conclusion The required number of matrices is $\boxed{672}$.

Quick Tip to Solve: When a condition involves $\mathrm{tr}(AA^T)$, replace it with the sum of all squared entries before doing any matrix multiplication.

23. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is _____. [4 marks]

Solution:

Concept:

- Choosing an unordered collection of r distinct objects from n available objects gives $\binom{n}{r}$ possibilities. Drawing marbles here does not record their order.
- The complement of "at most three red marbles" in a four-marble selection is "all four marbles are red". Counting the complement avoids splitting into several colour cases.

Step 1: Count all selections of four marbles There are $5+4+3=12$ marbles in the urn. Treating the individual marbles as the objects being selected, the number of four-marble subsets is

$$\binom{12}{4} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 495.$$

The black and white marbles need not be separated for this total. No factorial for the drawing order is included because a selection is unchanged when the same four marbles are taken in another order.

Step 2: Subtract exactly the forbidden selections A selection of four marbles can contain 0, 1, 2, 3 or 4 red marbles. Only the last possibility violates the requirement. All-red selections use four of the five red marbles, giving

$$\binom{5}{4} = 5.$$

Every other selection has at least one non-red marble and hence has at most three reds. Thus the requested count is

$$\binom{12}{4} - \binom{5}{4} = 495 - 5 = 490.$$

Equivalently, the allowed cases are $\sum_{r=0}^{3} \binom{5}{r}\binom{7}{4-r}$, where the seven non-red marbles comprise the four black and three white marbles. This sum confirms that every allowed colour mixture is included exactly once.

Step 3: Conclusion The number of permitted four-marble selections is $\boxed{490}$.

Quick Tip to Solve: In a draw of four objects, "at most three of one colour" excludes only the all-four case; subtract it from the unrestricted combinations.

24. Let the normal at a point P on the curve
$y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$.
If m is the slope of the tangent at P to the curve, then $|m|$ is equal to [4 marks]

Solution:

Concept:

- Implicit differentiation of a curve equation gives the tangent slope $m = dy/dx$ at a regular point. For a finite nonzero tangent slope, the normal slope is $-1/m$.

- A normal through (x, y) with slope n meets the y-axis at $(0, y - nx)$. The specified intercept provides a second equation for the point.

Step 1: Differentiate the curve and write the normal Differentiating $y^2 - 3x^2 + y + 10 = 0$ gives

$$(2y + 1)\frac{dy}{dx} = 6x, \qquad m = \frac{6x}{2y + 1}.$$

At P, the normal slope is $-(2y + 1)/(6x)$. The case $x = 0$ gives $y^2 + y + 10 = 0$, which has no real solution, so x is nonzero at any real point on this curve. A vertical tangent would have a horizontal normal at $y = -1/2$, which cannot pass through the given intercept $y = 3/2$.

Step 2: Use the given y-intercept to find P The equation of the normal through P and $(0, 3/2)$ yields

$$\frac{3}{2} - y = -\frac{2y + 1}{6x}(0 - x) = \frac{2y + 1}{6}.$$

Multiplying by 6 gives $9 - 6y = 2y + 1$, hence $y = 1$. Substitute this into the original curve:

$$1 - 3x^2 + 1 + 10 = 0 \quad \Rightarrow \quad x^2 = 4.$$

Thus the two possible points are $(2, 1)$ and $(-2, 1)$.

Step 3: Evaluate the tangent slope at both points At either point, $2y + 1 = 3$, so

$$m = \frac{6x}{3} = 2x = 4 \text{ or } -4.$$

Both points have a normal with the required intercept: its intercept is $1 + (2y + 1)/6 = 3/2$. The tangent directions differ in sign, but the absolute slopes coincide, as required by the question.

Step 4: Conclusion The magnitude of the tangent slope is $\boxed{|m| = 4}$.

Quick Tip to Solve: Use the normal intercept before substituting into the curve: here the x factor cancels immediately and determines y without solving a quadratic.

25. The sum $\sum_{k=1}^{20} (1 + 2 + 3 + ... + k)$ is : [4 marks]

Solution:

Concept:

- The inner sum is an arithmetic progression: $1 + 2 + \cdots + k = k(k + 1)/2$. Each value of k gives a different triangular number.
- For the first n positive integers, $\sum k = n(n+1)/2$ and $\sum k^2 = n(n+1)(2n+1)/6$. Linearity allows a sum of terms to be separated into these standard sums.

Step 1: Replace the inner arithmetic sum Let the required sum be S. For a fixed k, there are k terms

with first term 1 and last term k. Hence

$$S = \sum_{k=1}^{20} \frac{k(k+1)}{2} = \frac{1}{2}\left(\sum_{k=1}^{20} k^2 + \sum_{k=1}^{20} k\right).$$

The outer summation must still be performed: the triangular number for $k = 20$, namely 210, is only the last summand, not the requested total.

Step 2: Evaluate both standard sums Substitute $n = 20$ in the two formulae:

$$\sum_{k=1}^{20} k = \frac{20 \times 21}{2} = 210, \qquad \sum_{k=1}^{20} k^2 = \frac{20 \times 21 \times 41}{6} = 2870.$$

Therefore

$$S = \frac{2870 + 210}{2} = \frac{3080}{2} = 1540.$$

As a check, reversing the order of counting gives $20 \times 1 + 19 \times 2 + \cdots + 1 \times 20$, since the integer j appears in all inner sums from $k = j$ to 20. This counts the same terms and also yields 1540.

Step 3: Conclusion The required sum is $\boxed{1540}$. This is a dimensionless numerical value.

Quick Tip to Solve: The sum of the first n triangular numbers is $n(n+1)(n+2)/6$; here use $20 \times 21 \times 22/6$.

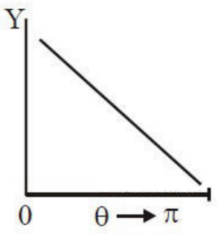
Physics – Section A: Multiple Choice Questions (80 marks)

26. The graph which depicts the results of Rutherford gold foil experiment with α-particales is :

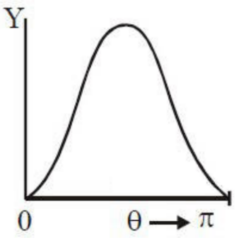
θ : Scattering angle

Y : Number of scattered α-particles detected (Plots are schematic and not to scale) [4 marks]

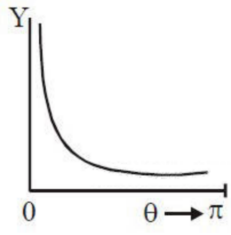
(A)



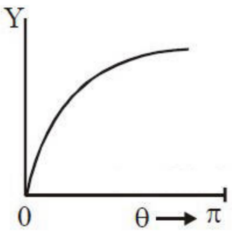
(B)



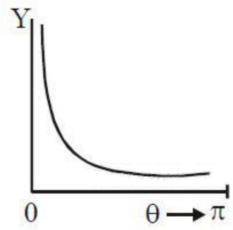
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- In Rutherford scattering, positively charged alpha particles are repelled by the small positively charged nucleus. Most incident particles pass with little deviation because most of the atom is empty space.
- For equal small detector solid angles and fixed incident energy, the Rutherford angular distribution is $Y \propto \csc^4(\theta/2)$. Thus the count is strongly concentrated in the forward direction and becomes small at large angles.

Step 1: Determine the angular trend For $0 < \theta \leq \pi$, $\sin(\theta/2)$ increases with θ, so its inverse fourth power decreases. For example, comparing 30° and 90°,

$$\frac{Y(30^\circ)}{Y(90^\circ)} = \left(\frac{\sin 45^\circ}{\sin 15^\circ}\right)^4 \gg 1.$$

The graph should therefore start very high near zero angle, fall rapidly, and approach a small count at large angles. The ideal formula is singular exactly at zero; the schematic graph only represents the sharply peaked forward distribution.

Step 2: Match the schematic curves Option (C) shows a steep initial decrease followed by a low, nearly flat tail, matching Rutherford scattering. Option (A) assumes a linear fall all the way to zero, which is not the inverse fourth-power dependence. Option (B) peaks at an intermediate angle, incorrectly predicting fewer forward-scattered particles. Option (D) increases with angle, contradicting the observation that large deflections are rare. The tiny rounded feature near the end of the schematic (C) does not change its identifying steep-fall shape.

Step 3: Conclusion The forward-peaked, rapidly decreasing distribution is represented by graph (C). Hence, the correct option is (C).

Quick Tip to Solve: For Rutherford scattering, remember $Y \propto 1/\sin^4(\theta/2)$: small scattering angles give the largest counts.

27. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be : [4 marks]

(A) 45°

(B) 15°

(C) 30°

(D) 60°

Correct Answer: (C) 30°

Solution:

Concept:

- For a transparent medium, the refractive index relative to vacuum is $n = \sqrt{\varepsilon_r \mu_r}$, where ε_r and μ_r are its relative permittivity and relative permeability.
- The critical angle for light travelling from this medium into air satisfies $n \sin\theta_c = 1$, because the refracted ray is at $90°$. Both material parameters must be evaluated at the specified wavelength.

Step 1: Calculate the refractive index The question gives $\varepsilon_r = 3$ and $\mu_r = 4/3$. Their product is

$$\varepsilon_r \mu_r = 3 \times \frac{4}{3} = 4.$$

Hence

$$n = \sqrt{4} = 2.$$

Using only $\sqrt{\varepsilon_r}$ would be inappropriate here: the common optical approximation $\mu_r = 1$ is not available because a different value is explicitly given. Equivalently, the wave speed in the medium is $v = c/2$.

Step 2: Apply Snell's law at the critical angle At the boundary with air, the critical condition gives

$$2\sin\theta_c = \sin 90° = 1, \qquad \sin\theta_c = \frac{1}{2}.$$

The physical critical angle lies between zero and $90°$, so $\theta_c = 30°$. Checking the distractors, $2\sin 45° = \sqrt{2}$, $2\sin 15° \approx 0.518$, and $2\sin 60° = \sqrt{3}$; none equals 1. Only $30°$ satisfies the critical-refraction condition.

Step 3: Conclusion The critical angle is $\boxed{30°}$. Hence, the correct option is (C).

Quick Tip to Solve: First find n using both ε_r and μ_r, then use $\theta_c = \sin^{-1}(1/n)$ for emergence into air.

28. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be : [4 marks]

(A) 4.2 μF

(B) 8.4 μF

(C) 1.6 μF

(D) 3.2 μF

Correct Answer: (C) 1.6 μF

Solution:

Concept:

- For capacitors in parallel, capacitances add: $C_p = C_1 + C_2$. For two capacitors in series, $C_s = C_1C_2/(C_1 + C_2)$.
- The energy of a capacitor maintained at potential difference V is $U = \frac{1}{2}CV^2$. At the same voltage, the ratio of energies equals the ratio of capacitances, not the inverse ratio.

Step 1: Use the energy ratio to find each capacitance Each capacitor is separately connected to the same $1\,\text{V}$ source. Thus the common factors in the energy ratio cancel:

$$\frac{U_2}{U_1} = \frac{\frac{1}{2}C_2(1)^2}{\frac{1}{2}C_1(1)^2} = \frac{C_2}{C_1} = 4.$$

Therefore $C_2 = 4C_1$. The given parallel combination then gives

$$C_1 + 4C_1 = 10\,\mu\text{F}, \qquad C_1 = 2\,\mu\text{F}, \quad C_2 = 8\,\mu\text{F}.$$

The capacitors need not have equal charges in this individual fixed-voltage test; their charges are proportional to their capacitances.

Step 2: Calculate the equivalent series capacitance When reconnected in series, use the reciprocal-sum relation:

$$\frac{1}{C_s} = \frac{1}{2\,\mu\text{F}} + \frac{1}{8\,\mu\text{F}}, \qquad C_s = \frac{2 \times 8}{2 + 8}\,\mu\text{F} = 1.6\,\mu\text{F}.$$

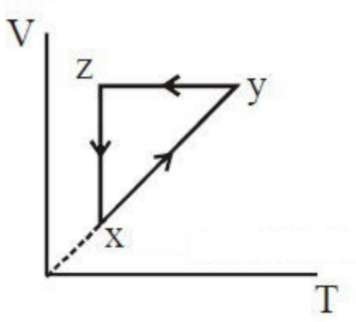
A series equivalent must be smaller than either individual capacitance. Since the smaller capacitor is $2\,\mu\text{F}$, the listed values 4.2, 8.4, and $3.2\,\mu\text{F}$ are all impossible. The calculated $1.6\,\mu\text{F}$ passes this independent check.

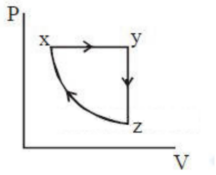
Step 3: Conclusion The effective series capacitance is $\boxed{1.6\,\mu\text{F}}$. Hence, the correct option is (C).

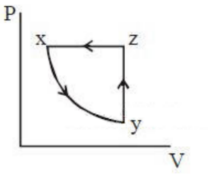
Quick Tip to Solve: At equal voltage, energy ratios directly give capacitance ratios. After calculating a series equivalent, check it is below the smaller capacitor.

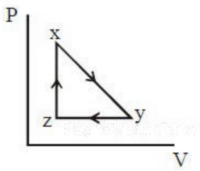
29. A thermodynamic cycle xyzx is shown on a V-T diagram. The P-V diagram that best describes this cycle is :

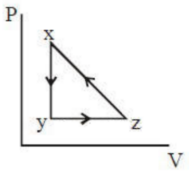
(Diagrams are schematic and not to scale) [4 marks]



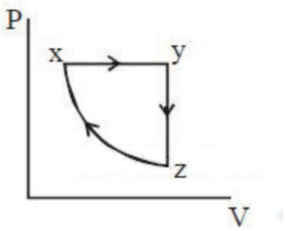
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$. A straight line through the origin on a volume-temperature plot has constant V/T, and therefore constant pressure.
- On a V-T plot, horizontal segments have constant volume and vertical segments have constant temperature. On a P-V plot, these correspond respectively to a vertical line and a rectangular hyperbola.

Step 1: Identify the three processes on the given plot The path $x \to y$ lies along a line whose extension passes through the origin. Both V and T increase in the same ratio, so

$$P = \frac{nRT}{V} = \text{constant}.$$

It is an isobaric expansion. The path $y \to z$ is horizontal in the given plot: volume stays fixed while temperature decreases, so pressure decreases. Finally $z \to x$ is vertical: temperature stays fixed while volume decreases, so pressure increases during isothermal compression.

Step 2: Locate the corresponding states on the pressure-volume plot Since $V_y = V_z > V_x$, states y, z share the right-hand vertical line on a P-V plot. Also $P_x = P_y > P_z$, so x, y share the upper horizontal line. The directions are therefore rightward from x to y, downward from y to z, and back along the isotherm $PV = nRT_x$ from z to x. Option (A) has all these features. Option (B) makes $x \to y$ isothermal instead of isobaric. Options (C) and (D) contain straight sloping paths rather than the required isothermal hyperbola and do not preserve the required equal-volume states.

Step 3: Conclusion The correct pressure-volume cycle is graph (A). Hence, the correct option is (A).

Quick Tip to Solve: Mark constant-temperature and constant-volume legs first; use a line through the origin in a V-T plot to identify the constant-pressure leg.

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30. In finding the electric field using Gauss Law the formula $\left|\vec{E}\right| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula ε_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. The equation can be used in which of the following situation? [4 marks]

(A) Only when $\left|\vec{E}\right|$ = constant on the surface.

(B) For any choice of Gaussian surface.

(C) Only when the Gaussian surface is an equipotential surface.

(D) Only when the Gaussian surface is an equipotential surface and $\left|\vec{E}\right|$ is constant on the surface.

Correct Answer: (D) Only when the Gaussian surface is an equipotential surface and $\left|\vec{E}\right|$ is constant on the surface.

Solution:

Concept:

- Gauss's law always holds in its integral form: $\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$. The flux integrand contains both the field magnitude and its angle to the outward normal.

- To reduce this integral to EA, the field must be normal to the surface with uniform magnitude. An equipotential surface has no tangential electric field, providing the normal-direction condition.

Step 1: Explain the two simplifications required Write the electric flux as

$$\Phi_E = \oint E \cos\alpha \, dA,$$

where α is the angle between the electric field and the local outward area vector. Constancy of E allows its magnitude to be taken outside the integral, but does not by itself remove the factor $\cos\alpha$. For an equipotential surface, $\vec{E}$ is perpendicular to the surface, so the normal-direction condition is satisfied. For the usual outward-field case this gives $\cos\alpha = 1$; an inward field gives the corresponding signed flux.

Step 2: Require both conditions and compare the options When the Gaussian surface is equipotential and the field magnitude is constant, the flux magnitude is

$$|\Phi_E| = EA, \qquad E = \frac{|q_{\text{enc}}|}{\varepsilon_0 A}.$$

For the positive enclosed charge convention of the displayed expression, this is the stated formula. Option (A) supplies only uniform magnitude; it does not ensure a normal field. Option (C) supplies normality but permits the magnitude to vary from point to point. Option (B) confuses the universally valid integral law with this restricted algebraic simplification. Option (D) combines the two necessary simplifying conditions, as for a sphere centered on an isolated point charge.

Step 3: Conclusion Use the simplified expression when the Gaussian surface is equipotential and the field magnitude is constant over it. Hence, the correct option is (D).

Quick Tip to Solve: Before replacing the flux integral by EA, check both uniform field magnitude and a field normal to the Gaussian surface.

31. Consider a solid sphere of radius R and mass density

$\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$, $0 < r \leq R$

The minimum density of a liquid in which it will float is : [4 marks]

(A) $\frac{2\rho_0}{3}$

(B) $\frac{2\rho_0}{5}$

(C) $\frac{\rho_0}{5}$

(D) $\frac{\rho_0}{3}$

Correct Answer: (B) $\frac{2\rho_0}{5}$

Solution:

Concept:

- For a spherically symmetric density distribution, a thin shell of radius r and thickness dr has volume $dV = 4\pi r^2 dr$, so its mass is $dm = \rho(r)\, dV$.

- Archimedes' principle gives maximum buoyancy $\rho_L V g$ for a fully submerged body of volume V. The threshold liquid density is the body's average density M/V, rather than its central density.

Step 1: Integrate the mass of the spherical shells The density decreases from its central value to zero at the surface, so equal radial intervals do not contribute equal mass. Weight each shell by its volume before integrating over the radius:

$$M = 4\pi\rho_0 \int_0^R \left(r^2 - \frac{r^4}{R^2}\right) dr = 4\pi\rho_0 \left(\frac{R^3}{3} - \frac{R^3}{5}\right) = \frac{8\pi\rho_0 R^3}{15}.$$

Step 2: Compare weight with the greatest possible buoyancy The external volume is $V = 4\pi R^3/3$. At the limiting density the whole sphere displaces liquid, so $\rho_L V g = Mg$:

$$\rho_{L,\min} = \frac{M}{V} = \frac{8\pi\rho_0 R^3/15}{4\pi R^3/3} = \frac{2\rho_0}{5}.$$

For any lower density, even complete immersion cannot balance the weight and the sphere sinks. At equality it is neutrally buoyant; for a larger liquid density it can float with only part submerged. Options (C) and (D) are below the required average density. Option (A) permits floating but is not the minimum, so it does not answer the question.

Step 3: Conclusion The limiting liquid density is $\boxed{2\rho_0/5}$. Hence, the correct option is (B).

Quick Tip to Solve: For a nonuniform floating body, integrate its mass first and divide by its full external volume; do not average the central and surface densities.

---

32. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is : [4 marks]

(A) $h^{1/3}\ G^{2/3}\ c^{1/3}\ A^{-1}$

(B) $h^{0}\ c^{5}\ G^{-1}\ A^{-1}$

(C) $h^{2/3}\ c^{5/3}\ G^{1/3}\ A^{-1}$

(D) $h^{2}\ G^{3/2}\ c^{1/3}\ A^{-1}$

Correct Answer: (B) $h^{0}\ c^{5}\ G^{-1}\ A^{-1}$

Solution:

Concept:

- Potential difference is energy per unit charge: $[V_0] = ML^2T^{-3}I^{-1}$, where I denotes the dimension of electric current. One ampere A has dimension I.
- The dimensions of the three constants are $[h] = ML^2T^{-1}$, $[c] = LT^{-1}$ and $[G] = M^{-1}L^3T^{-2}$. Dimensional equality fixes the powers independently of the photoelectric material.

Step 1: Write equations for the unknown exponents Let $[V_0] = [h^a c^b G^d A^{-1}]$. The inverse power of ampere already supplies the required current dimension; the remaining constants must supply mass,

length and time. Multiplication of their dimensions gives

$$[h^a c^b G^d A^{-1}] = M^{a-d} L^{2a+b+3d} T^{-a-b-2d} I^{-1}.$$

Matching the exponents with voltage yields $a - d = 1$, $2a + b + 3d = 2$ and $-a - b - 2d = -3$.

Step 2: Solve and check the proposed combinations From the mass equation $d = a - 1$. The length equation then gives $b = 5 - 5a$, while the time equation gives $b = 5 - 3a$. Thus $a = 0$, $d = -1$ and $b = 5$. Directly,

$$[c^5 G^{-1} A^{-1}] = (L^5 T^{-5})(M L^{-3} T^2) I^{-1} = M L^2 T^{-3} I^{-1}.$$

The other choices already fail the mass exponent test: (A) has $a - d = -1/3$, (C) has $a - d = 1/3$, and (D) has $a - d = 1/2$, whereas voltage requires mass exponent one. A zero power of Planck's constant is entirely permissible in a dimensional representation.

Step 3: Conclusion $\boxed{[V_0] = [h^0 c^5 G^{-1} A^{-1}]}$. Hence, the correct option is (B).

Quick Tip to Solve: Check mass exponents first: the power of mass in $h^a G^d$ is $a - d$, which must equal one for a voltage.

---

33. Consider a uniform rod of mass M = 4m and length ℓ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is : [4 marks]

(A) $\frac{3\sqrt{2}}{7}\frac{v}{\ell}$

(B) $\frac{3}{7}\frac{v}{\ell}$

(C) $\frac{3}{7\sqrt{2}}\frac{v}{\ell}$

(D) $\frac{4}{7}\frac{v}{\ell}$

Correct Answer: (A) $\frac{3\sqrt{2}}{7}\frac{v}{\ell}$

Solution:

Concept:

- During the brief collision, angular momentum about the fixed central pivot is conserved because the pivot impulse has zero moment about that point. Linear momentum need not be conserved.
- A uniform rod has central moment of inertia $I_{\text{rod}} = M\ell^2/12$. A particle stuck at distance $\ell/2$ adds $m\ell^2/4$. Its initial angular momentum is $mvr\sin\theta$.

Step 1: Calculate the incoming angular momentum The position vector from the pivot to the struck end lies along the rod and has magnitude $r = \ell/2$. Only the velocity component perpendicular to this vector contributes to angular momentum. Therefore

$$L_i = mv\frac{\ell}{2}\sin\frac{\pi}{4} = \frac{mv\ell}{2\sqrt{2}}.$$

The parallel component of the incident velocity cannot rotate the rod about the pivot, so using the entire speed without the sine factor would overestimate the initial angular momentum.

Step 2: Use the combined moment of inertia After impact the particle and rod rotate together, since the particle sticks. With $M = 4m$, their common moment of inertia is

$$I_f = \frac{4m\ell^2}{12} + \frac{m\ell^2}{4} = \frac{7m\ell^2}{12}.$$

Thus $I_f\omega = L_i$ gives

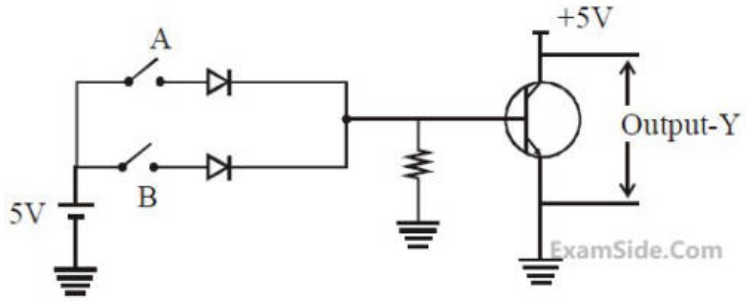
$$\omega = \frac{mv\ell}{2\sqrt{2}}\frac{12}{7m\ell^2} = \frac{6}{7\sqrt{2}}\frac{v}{\ell} = \frac{3\sqrt{2}}{7}\frac{v}{\ell}.$$

The factor v/ℓ has units of inverse time as an angular speed should. Option (B) omits the correct angular factor, (C) is smaller by a factor two, and (D) has an incompatible coefficient. Kinetic energy is not conserved because the sticking collision is inelastic.

Step 3: Conclusion The angular speed just after collision is $\boxed{\omega = \frac{3\sqrt{2}}{7}\frac{v}{\ell}}$. Hence, the correct option is (A).

Quick Tip to Solve: In a pinned collision, conserve angular momentum about the pin and include the stuck particle in the final moment of inertia.

34. Boolean relation at the output stage-Y for the following circuit is : [4 marks]



(A) A.B

(B) $\overline{A} + \overline{B}$

(C) $\overline{A}.\overline{B}$

(D) A + B

Correct Answer: (C) $\overline{A}.\overline{B}$

Solution:

Concept:

- A closed input switch represents logic 1 and an open switch represents logic 0. The two forward-directed diode paths feed the transistor base, so either high input can supply base current.
- The grounded-emitter transistor acts as an inverter: a high base drive turns it on and lowers the collector-emitter output voltage; without base drive it is off and the output is high.

Step 1: Determine the base-drive logic When both switches are open, neither diode supplies current

from the 5 V input source. The base resistor holds the base near ground and the transistor remains off. Closing switch A or switch B supplies positive base current through the corresponding diode; the diode network therefore implements the inclusive OR operation at the base. Both switches closed also drive the base high, rather than producing a different logic state.

Step 2: Invert the base signal and test the choices For $(A, B) = (0, 0)$, the transistor is off and the output is high. For $(0, 1)$, $(1, 0)$ or $(1, 1)$, it conducts and the collector-emitter output is low. The truth table is consequently the NOR truth table:

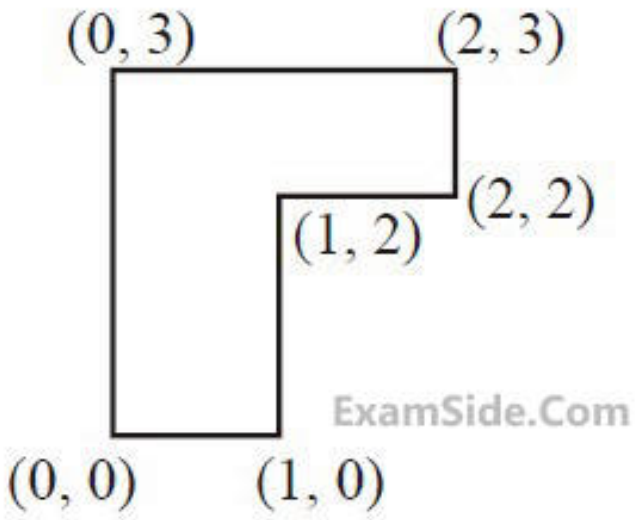
$$Y = \overline{A + B} = \overline{A}\,\overline{B}.$$

Here the last equality is De Morgan's law. Option (B), $\overline{A} + \overline{B}$, is a NAND expression and is high for either one-high input, contrary to this circuit. Option (A), AB, is high for two high inputs, also contrary to the transistor inversion. Option (D), $A + B$, describes the base-drive logic before inversion. Only option (C) gives the required high output exclusively when both inputs are low.

Step 3: Conclusion The output is $\boxed{Y = \overline{A}\,\overline{B}}$, the NOR function. Hence, the correct option is (C).

Quick Tip to Solve: Separate the diode input operation from the transistor inversion: an OR network followed by an inverting transistor produces NOR.

35. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are : [4 marks]



(A) (0.75m, 1.75m)

(B) (1m, 1.75m)

(C) (0.75m, 0.75m)

(D) (1.25m, 1.50m)

Correct Answer: (A) (0.75m, 1.75m)

Solution:

Concept:

- For a uniform lamina, mass is proportional to area. Its centre of mass is the area-weighted average of the centroids of nonoverlapping simple shapes.
- A rectangle's centroid lies at the midpoint of each coordinate interval. For pieces of masses m_i, $x_{\text{cm}} = \sum m_i x_i / \sum m_i$ and $y_{\text{cm}} = \sum m_i y_i / \sum m_i$.

Step 1: Split the flag into two rectangles Use the lower rectangle $0 \le x \le 1$, $0 \le y \le 2$, whose area is $2\,\text{m}^2$ and centroid is $(0.5, 1)\,\text{m}$. The upper rectangle occupies $0 \le x \le 2$, $2 \le y \le 3$; it also has area $2\,\text{m}^2$, with centroid $(1, 2.5)\,\text{m}$. The rectangles share only an edge, so their areas do not overlap. Their total area is $4\,\text{m}^2$.

Step 2: Take the mass-weighted coordinate averages The total mass is $4\,\text{kg}$, and the two equal-area rectangles have the same uniform areal density. Each therefore has mass $2\,\text{kg}$. Hence

$$x_{\text{cm}} = \frac{2(0.5) + 2(1)}{4} = 0.75\,\text{m}, \qquad y_{\text{cm}} = \frac{2(1) + 2(2.5)}{4} = 1.75\,\text{m}.$$

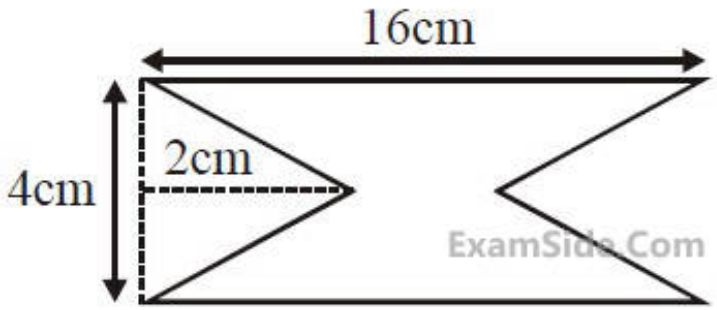
The high horizontal part shifts the centroid upward and to the right of the lower rectangle's centre, as these values show. Option (B) places it at the upper rectangle's horizontal centroid despite the lower piece pulling it left; option (C) is too low, and option (D) is too far right. Treating the outline as a wire would require length weights, but the question specifies a lamina and therefore requires area weights.

Step 3: Conclusion The centre of mass is $\boxed{(0.75\,\text{m},\ 1.75\,\text{m})}$. Hence, the correct option is (A).

Quick Tip to Solve: Choose a nonoverlapping rectangle decomposition with equal areas when possible; equal-area pieces of a uniform lamina have equal centroid weights.

---

36. At time t = 0 magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is : [4 marks]



(A) 48 μV

(B) 28 μV

(C) 56 μV

(D) 36 μV

Correct Answer: (C) 56 μV

Solution:

Concept:

- For a uniform magnetic field perpendicular to a single closed planar loop, the magnetic flux is $\Phi = BA$, where A is its enclosed area. The induced emf magnitude is $|\mathcal{E}| = A|dB/dt|$.
- The hourglass-like outline encloses the outer rectangle minus the two triangular indentations. Use $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$ and $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$ when converting to SI units.

Step 1: Find the actual enclosed area The outer rectangle measures $16\,\mathrm{cm}$ by $4\,\mathrm{cm}$, giving $64\,\mathrm{cm}^2$. Each triangular indentation has vertical base $4\,\mathrm{cm}$ and horizontal depth $2\,\mathrm{cm}$, so its area is $\frac{1}{2}(4)(2) = 4\,\mathrm{cm}^2$. There are two such indentations. Therefore

$$A = 64 - 2(4) = 56\,\mathrm{cm}^2 = 56 \times 10^{-4}\,\mathrm{m}^2.$$

Subtract the cutouts, rather than adding their areas: they lie outside the region bounded by the wire.

Step 2: Apply the constant rate of change of field The field falls linearly, so its magnitude changes at a constant rate throughout the five-second interval:

$$\left|\frac{dB}{dt}\right| = \frac{(1000 - 500)10^{-4}}{5} = 10^{-2}\,\mathrm{T\,s}^{-1}.$$

Consequently

$$|\mathcal{E}| = (56 \times 10^{-4})(10^{-2}) = 56 \times 10^{-6}\,\mathrm{V} = 56\,\mu\mathrm{V}.$$

Only the change of field matters, not its initial value alone. The loop has one turn, so no extra turn factor occurs. The other choices, 48, 28 and 36 microvolts, would require different enclosed areas or field-change rates and do not follow from the indicated dimensions. Lenz's law determines the sense opposing the flux decrease; the question asks only for the emf magnitude.

Step 3: Conclusion The induced emf magnitude is $\boxed{56\,\mu\mathrm{V}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a concave loop, subtract the notches from a surrounding rectangle before using Faraday's law; convert both gauss and square centimetres to SI units.

37. The magnifying power of a telescope with tube 60 cm is 5. What is the focal length of its eye piece ? [4 marks]

(A) 40 cm

(B) 10 cm

(C) 30 cm

(D) 20 cm

Correct Answer: (B) 10 cm

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the final image lies at infinity and the objective and eyepiece focal planes coincide. Its tube length is $L = f_o + f_e$, where f_o and f_e are the positive focal lengths of the objective and eyepiece.
- The magnitude of the angular magnifying power in this adjustment is $M = f_o/f_e$. Image inversion changes the sign of angular magnification, but the stated magnifying power 5 is its magnitude.

Step 1: Express both focal lengths using the magnification The objective forms a real intermediate image, which the eyepiece views as a magnifier. Using the given magnifying power gives

$$\frac{f_o}{f_e} = 5 \quad \Rightarrow \quad f_o = 5f_e.$$

The two focal lengths must add to the given tube length; the tube length itself is not the focal length of the objective.

Step 2: Use the tube length and compare the choices Substitute the focal-length ratio into the normal-adjustment length:

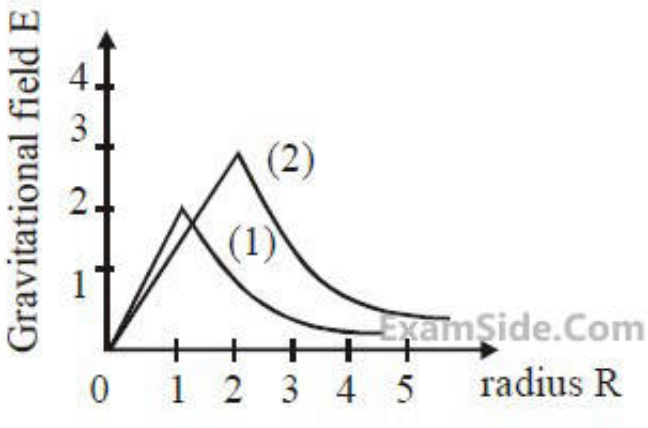
$$60 = 5f_e + f_e = 6f_e, \qquad f_e = 10\,\text{cm}.$$

Then $f_o = 50\,\text{cm}$, so both checks work: $50 + 10 = 60$ and $50/10 = 5$. If the eyepiece had focal length 40, 30 or 20 cm, the same length constraint would give magnification magnitudes 0.5, 1 or 2, respectively. Thus none of the other choices satisfies the stated magnifying power.

Step 3: Conclusion The focal length of the eyepiece is $\boxed{10\,\text{cm}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a telescope in normal adjustment, combine the length and power directly as $f_e = L/(M+1)$; then check that the objective has the longer focal length.

38. Consider two solid spheres of radii R_1 = 1m, R_2 = 2m and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is : [4 marks]



(A) $\frac{2}{3}$
(B) $\frac{1}{6}$
(C) $\frac{1}{2}$
(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{6}$

Solution:

Concept:

- At the surface of a spherically symmetric sphere, its gravitational field has magnitude $g_s = GM/R^2$, with G the gravitational constant. A field ratio alone is therefore insufficient to determine a mass ratio unless the radii are also included.
- For a uniform solid sphere the field grows linearly with distance from the centre inside it and decreases as $1/r^2$ outside it. The peak in each plotted curve is at the surface, so its ordinate supplies the relevant surface field.

Step 1: Read the two surface fields from the graph Sphere (1) has its peak at $R_1 = 1\,\text{m}$, at field ordinate 2. Sphere (2) has its peak at $R_2 = 2\,\text{m}$, at field ordinate 3. The same field scale is used for both curves, hence

$$\frac{g_1}{g_2} = \frac{2}{3}.$$

Compare these two peaks, rather than field values at an arbitrary common horizontal coordinate; the peaks correspond to the specified radii.

Step 2: Convert the field ratio into the mass ratio Rearrange the surface-field law for each sphere:

$$M_1 = \frac{g_1 R_1^2}{G}, \qquad M_2 = \frac{g_2 R_2^2}{G}.$$

Taking the ratio cancels G:

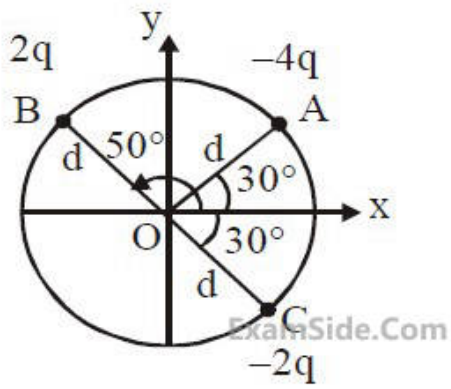
$$\frac{M_1}{M_2} = \frac{g_1}{g_2}\left(\frac{R_1}{R_2}\right)^2 = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}.$$

Option (A) omits the squared-radius correction. Options (C) and (D) also fail the surface-field check: multiplying their mass ratios by $R_2^2/R_1^2 = 4$ would not give the observed field ratio $2/3$.

Step 3: Conclusion The mass ratio is $\boxed{M_1/M_2 = 1/6}$. Hence, the correct option is (B).

Quick Tip to Solve: At each surface peak read g and R together: the mass is proportional to gR^2, not simply to the height of the curve.

39. Three charged particle A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. the charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is : [4 marks]



(A) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

(B) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

(C) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

(D) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$

Correct Answer: With the printed $50°$ angle, $\boxed{E_x = (3\sqrt{3} + 2\sin 50°)q/(4\pi\varepsilon_0 d^2)}$ along +x. No listed option is exactly correct. The answer key selects (C), but this corresponds to a $60°$ angular label, rather than the printed $50°$.

Solution:

Concept:

- The field at the centre due to a charge Q a distance d away has magnitude $k|Q|/d^2$, where $k = 1/(4\pi\varepsilon_0)$. It points towards a negative charge and away from a positive charge.
- Add the signed horizontal components. The two negative charges are at angles $+30°$ and $-30°$ to the positive x-axis. The printed angle between OB and the positive y-axis is $50°$, so OB is at $140°$ to the positive x-axis.

Step 1: Find the contributions of the two negative charges For A, charge $-4q$ attracts a positive test

charge towards A. For C, charge $-2q$ attracts it towards C. Both horizontal components are positive:

$$E_{Ax} = \frac{4kq}{d^2}\cos 30^\circ, \qquad E_{Cx} = \frac{2kq}{d^2}\cos 30^\circ.$$

Their sum is $3\sqrt{3}\,kq/d^2$. This uses the explicitly printed equilateral geometry of AOC, whose central angle is 60°.

Step 2: Include B using its printed angular label The positive charge at B repels the test charge towards the lower right, opposite OB. Its horizontal component is

$$E_{Bx} = -\frac{2kq}{d^2}\cos 140^\circ = \frac{2kq}{d^2}\sin 50^\circ.$$

Consequently the literal printed diagram gives

$$E_x = \frac{kq}{d^2}\left(3\sqrt{3} + 2\sin 50^\circ\right) \approx \frac{6.72824\,kq}{d^2}.$$

In units of kq/d^2, options (A), (B), (C) and (D) have coefficients $3\sqrt{3}$, $\sqrt{3}$, $4\sqrt{3}$ and $8\sqrt{3}$, respectively. None equals the coefficient above. If the angular label were 60°, B would instead contribute $\sqrt{3}kq/d^2$ and option (C) would result; that is a different angle from the one printed.

Step 3: Conclusion With the printed 50° angle, $\boxed{E_x = (3\sqrt{3} + 2\sin 50^\circ)q/(4\pi\varepsilon_0 d^2)}$ along +x. No listed option is exactly correct. The answer key selects (C), but this corresponds to a 60° angular label, rather than the printed 50°.

Quick Tip to Solve: For each charge, choose the field direction from its sign before projecting it. A small change in a labelled angle can change the sum even when the drawing looks symmetric.

---

40. A leak proof cylinder of length 1m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to : [4 marks]

(A) 1.04

(B) 1.26

(C) 1.01

(D) 1.03

Correct Answer: (C) 1.01

Solution:

Concept:

- For a floating body, Archimedes' principle gives $mg = \rho V_{\text{immersed}} g$, where m is the unchanged cylinder mass and ρ is the water density. The displaced water has the same weight as the floating cylinder.
- The cylinder has negligible thermal expansion, so its total length and cross-sectional area A remain

effectively constant. For vertical floating, $V_{\text{immersed}} = Ah$, where h is its submerged length. Thus the product ρh is constant.

Step 1: Find the submerged lengths at both temperatures The total length is $1\,\text{m} = 100\,\text{cm}$. Subtract the height above the water in each case:

$$h_0 = 100 - 20 = 80\,\text{cm}, \qquad h_4 = 100 - 21 = 79\,\text{cm}.$$

Use the underwater lengths, since only that volume displaces water. The 1 cm increase above the surface is the corresponding 1 cm decrease in immersion.

Step 2: Equate the buoyant forces and estimate the ratio Writing equilibrium separately at the two temperatures and cancelling the common area and gravity gives

$$\rho_0 A h_0 g = mg = \rho_4 A h_4 g,$$

$$\frac{\rho_4}{\rho_0} = \frac{h_0}{h_4} = \frac{80}{79} = 1.01266\ldots.$$

The relative density is dimensionless and is closest to 1.01, option (C). The alternatives 1.04, 1.26 and 1.03 all exceed the calculated value by more. Its being slightly greater than unity also agrees with the increased density of water on heating from zero to four degrees Celsius; the body must displace less volume to support the same weight.

Step 3: Conclusion The required relative density is $\boxed{\rho_4/\rho_0 \approx 1.01}$. Hence, the correct option is (C).

Quick Tip to Solve: For an unchanged floating cylinder, use $\rho_2/\rho_1 = h_1/h_2$ with submerged heights; above-water heights must first be subtracted from the total length.

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41. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length ℓ. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is : [4 marks]

(A) $\frac{m\ell\omega^2}{k-m\omega^2}$

(B) $\frac{m\ell\omega^2}{k-m\omega}$

(C) $\frac{m\ell\omega^2}{k+m\omega^2}$

(D) $\frac{m\ell\omega^2}{k+m\omega}$

Correct Answer: (A) $\frac{m\ell\omega^2}{k-m\omega^2}$

Solution:

Concept:

- Hooke's law gives the inward tension $T = kx$, where x is the extension beyond the natural spring length ℓ. The spring is light, so its own mass does not contribute to the rotating mass.
- Uniform circular motion with angular speed ω requires inward force $m\omega^2 r$. The radius here is the stretched length $r = \ell + x$, because the particle rotates about the fixed end of the spring.

Step 1: Write Newton's law in the radial direction In gravity-free space there is no weight to balance or resolve. The spring tension is the sole radial force and supplies all the required centripetal acceleration:

$$kx = m\omega^2(\ell + x).$$

The centripetal force is the name of this resultant radial force, not an additional outward or inward force to be added separately. In particular, replacing the radius by ℓ would ignore the extension being determined.

Step 2: Solve for the extension and check the options Collect the terms proportional to x on the left:

$$kx - m\omega^2 x = m\ell\omega^2,$$

$$x = \frac{m\ell\omega^2}{k - m\omega^2}.$$

A finite positive extension requires $k > m\omega^2$; otherwise this assumed steady stretched circular configuration cannot exist. Both denominator terms have units of force per length. Options (B) and (D), containing $m\omega$, fail this dimensional test. Option (C) has a plus sign and would contradict the radial equation because an increased radius increases the necessary tension. At small ω, the answer reduces to $x \approx m\ell\omega^2/k$, as expected.

Step 3: Conclusion The spring stretches by $\boxed{x = m\ell\omega^2/(k - m\omega^2)}$, for $k > m\omega^2$. Hence, the correct option is (A).

Quick Tip to Solve: Use the final radius $\ell + x$ in the centripetal acceleration; then bring the term $m\omega^2 x$ to the same side as kx.

42. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV end de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is : [4 marks]

(A) 1.5eV

(B) 4eV

(C) 2eV

(D) 3eV

Correct Answer: (B) 4eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_\gamma - \phi$, where E_γ is the incident photon energy and ϕ is the metal's work function. Energies may all be kept in electronvolts.
- For nonrelativistic electrons, $\lambda = h/p$ and $K = p^2/(2m_e) = h^2/(2m_e\lambda^2)$. The same electron mass and Planck constant occur for both metals, so the kinetic-energy ratio is the inverse square of the

wavelength ratio.

Step 1: Convert the wavelength relation into an energy relation The given wavelength $\lambda_B = 2\lambda_A$ means that the fastest electrons from B have half the momentum and one quarter of the kinetic energy of those from A:

$$\frac{T_B}{T_A} = \left(\frac{\lambda_A}{\lambda_B}\right)^2 = \frac{1}{4}, \qquad T_B = \frac{T_A}{4}.$$

The factor of four comes from the square in kinetic energy; it must not be replaced by the wavelength ratio itself.

Step 2: Find the kinetic energies and work function of B Combine the ratio with the stated energy difference, using numerical values in eV:

$$\frac{T_A}{4} = T_A - 1.5 \quad \Rightarrow \quad \frac{3T_A}{4} = 1.5.$$

Thus $T_A = 2.0\,\text{eV}$ and $T_B = 0.50\,\text{eV}$. Applying the photoelectric equation to metal B gives

$$\phi_B = 4.50 - 0.50 = 4.0\,\text{eV}.$$

For comparison, $\phi_A = 4.0 - 2.0 = 2.0\,\text{eV}$; option (C) is the other metal's work function. Options (A) and (D) would give B kinetic energies 3.0 and 1.5 eV, respectively, neither satisfying the derived wavelength and energy-difference constraints.

Step 3: Conclusion The work function of metal B is $\boxed{4.0\,\text{eV}}$. Hence, the correct option is (B).

Quick Tip to Solve: For electrons from different metals, first use $K\lambda^2 =$ constant, then apply each metal's photon energy separately in Einstein's equation.

---

43. Photon with kinetic energy of 1MeV moves from south to north. It gets an acceleration of 10^{12} m/s^2 by an applied magnetic field (west to east). The value of magnetic field : (Rest mass of proton is 1.6×10^{-27} kg) : [4 marks]

(A) 0.71mT

(B) 7.1mT

(C) 0.071mT

(D) 71mT

Correct Answer: No option is valid for a photon; the proton interpretation gives 0.71 mT, option (A).

Solution:

Concept:

- A uniform magnetic field exerts the Lorentz force $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$ on a charged particle. A photon has zero electric charge and zero rest mass.
- For a nonrelativistic proton moving perpendicular to the field, $ma = qvB$ and $K = mv^2/2$. The proton data supplied allow a conditional calculation, but do not make a photon a proton.

Step 1: Check the particle named in the question The question explicitly says photon. A photon

cannot possess a proton rest mass, and an applied uniform magnetic field does not give a photon the stated Lorentz acceleration. Thus, taken literally, the stated motion is incompatible with every nonzero field option. No value among (A)–(D) solves the photon problem.

Step 2: Evaluate the charged-particle interpretation separately If the particle were a proton, the supplied mass and energy would give, using $e = 1.6 \times 10^{-19}\,\mathrm{C}$,

$$K = 10^6(1.6 \times 10^{-19}) = 1.6 \times 10^{-13}\,\mathrm{J}, \qquad v = \sqrt{\frac{2K}{m}} = \sqrt{2 \times 10^{14}} = 1.414 \times 10^7\,\mathrm{m\,s^{-1}}.$$

South-to-north motion is perpendicular to a west-to-east field, so $\sin\theta = 1$. Consequently,

$$B = \frac{ma}{ev} = \frac{(1.6 \times 10^{-27})(10^{12})}{(1.6 \times 10^{-19})(1.414 \times 10^7)} = 7.07 \times 10^{-4}\,\mathrm{T} = 0.707\,\mathrm{mT}.$$

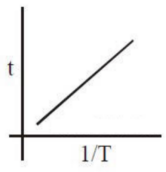
This conditional value matches (A); (B), (C) and (D) differ by factors of ten or one hundred. It is not a solution for an electrically neutral photon.

Step 3: Conclusion As worded with a photon, no listed option is physically valid. If proton was intended, the result would be $B \simeq 0.71\,\mathrm{mT}$, corresponding to option (A).

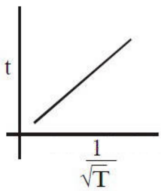
Quick Tip to Solve: Before using $qvB = ma$, check that the named particle has electric charge and that its rest mass is consistent with the supplied data.

44. The plot that depicts the behavior of the mean free time t (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is:
(Graphs are schematic and not drawn to scale) [4 marks]

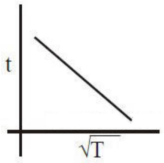
(A)



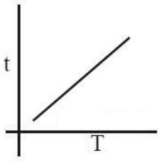
(B)



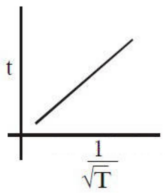
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- The mean free time is $\tau = \lambda/\bar{v}$, where λ is the mean free path and $\bar{v}$ is the mean molecular speed.
- For a fixed amount of ideal gas in a fixed volume, the number density n is constant. With molecular collision diameter d, $\lambda = 1/(\sqrt{2}\pi d^2 n)$, while $\bar{v} = \sqrt{8k_BT/(\pi m)}$.

Step 1: Find the temperature dependence At fixed number density and with the hard-sphere molecular diameter taken as constant, the mean free path does not change with temperature. Raising temperature increases molecular speed as $\sqrt{T}$, so the same mean distance between collisions is travelled in less time. Substitution gives

$$\tau = \frac{1}{\sqrt{2}\pi d^2 n}\sqrt{\frac{\pi m}{8k_BT}} = C\,T^{-1/2},$$

where C is positive and independent of T for these conditions. The symbol t on the plotted vertical axes represents this collision time, rather than elapsed experimental time.

Step 2: Read the axes of each graph Write $X = 1/\sqrt{T}$. Then $\tau = CX$, a straight line with positive slope, which is exactly the qualitative behavior shown in (B). Option (A) plots a straight line against $1/T$, implying $\tau \propto T^{-1}$, which is the wrong power. Option (C) makes τ linear with a negative slope against $\sqrt{T}$, whereas the actual relation is an inverse curve in that variable. Option (D) increases linearly with T, contrary to the decreasing collision time. The schematic axes, not just the apparent direction of a line, decide the choice.

Step 3: Conclusion For constant number density, $t \propto 1/\sqrt{T}$; the straight-line graph against $1/\sqrt{T}$ is (B). Hence, the correct option is (B).

Quick Tip to Solve: Use $\tau = \lambda/\bar{v}$ and inspect the horizontal-axis variable carefully: a straight line against $1/\sqrt{T}$ is not a straight line against T.

45. The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5V and internal resistance of 20Ω, the null point on it is found to be a 1000 cm. The resistance of whole wire is : [4 marks]

(A) 80Ω

(B) 60Ω

(C) 120Ω

(D) 100Ω

Correct Answer: (D) 100Ω

Solution:

Concept:

- At the null point of a potentiometer, no current flows through the test cell. Its emf E equals the potential drop along the balancing length l.
- For a uniform wire of total length L, resistance R and steady current I, the potential gradient is $k = IR/L$. Therefore $E = IRl/L$.

Step 1: Use the balance to find the potential gradient The balancing length is $1000\,\text{cm}$, whereas the whole wire is $1200\,\text{cm}$ long. The emf is balanced by only the first of these lengths:

$$k = \frac{E}{l} = \frac{5}{1000} = 0.005\,\text{V cm}^{-1}.$$

The test cell has internal resistance $20\,\Omega$, but it supplies no current through the galvanometer branch at balance. Its internal voltage drop is therefore zero; using a loaded terminal-voltage formula here would introduce a spurious correction.

Step 2: Find the total drop and resistance The potential gradient is uniform along the entire potentiometer wire, so

$$V_{\text{wire}} = kL = 0.005(1200) = 6\,\text{V}.$$

Convert the given current before applying Ohm's law:

$$I = 60\,\text{mA} = 0.060\,\text{A}, \qquad R = \frac{V_{\text{wire}}}{I} = \frac{6}{0.060} = 100\,\Omega.$$

As a check, the balancing portion has resistance $(1000/1200)100 = 83.33\,\Omega$; its drop is $0.060(83.33) = 5\,\text{V}$, as required. The full-wire resistances 80, 60 and $120\,\Omega$ would give balancing drops of 4, 3 and $6\,\text{V}$, respectively, and thus fail the stated balance.

Step 3: Conclusion The resistance of the complete wire is $\boxed{100\,\Omega}$. Hence, the correct option is (D).

Quick Tip to Solve: At potentiometer balance the test cell carries no current: first find E/l, then multiply by the full wire length before dividing by the wire current.

Physics – Section B: Numerical Value Questions (20 marks)

46. A particle is moving along the x-axis with its coordinate with the time 't' given be $x(t) = 10 + 8t - 3t^2$. Another particle is moving the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$.

At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is _____. [4 marks]

Solution:

Concept:

- Instantaneous velocity is the time derivative of position, evaluated at the specified instant. Constant offsets in position do not contribute to velocity.
- The velocity of particle 2 in the translating frame of particle 1 is $\mathbf{v}_{21} = \mathbf{v}_2 - \mathbf{v}_1$. For perpendicular components, the relative speed is their Euclidean magnitude.

Step 1: Differentiate the two position functions Particle 1 moves on the x-axis, with

$$x(t) = 10 + 8t - 3t^2, \qquad \mathbf{v}_1 = (8 - 6t)\hat{\mathbf{i}}.$$

Particle 2 moves on the y-axis, with

$$y(t) = 5 - 8t^3, \qquad \mathbf{v}_2 = -24t^2\hat{\mathbf{j}}.$$

At $t = 1\,\mathrm{s}$, these become $\mathbf{v}_1 = 2\hat{\mathbf{i}}\,\mathrm{m\,s^{-1}}$ and $\mathbf{v}_2 = -24\hat{\mathbf{j}}\,\mathrm{m\,s^{-1}}$. The negative sign means particle 2 is moving toward decreasing y.

Step 2: Subtract vectors and identify the requested parameter The observer attached to particle 1 has its own instantaneous velocity, which must be subtracted vectorially:

$$\mathbf{v}_{21} = (-2\hat{\mathbf{i}} - 24\hat{\mathbf{j}})\,\mathrm{m\,s^{-1}}.$$

Therefore

$$|\mathbf{v}_{21}| = \sqrt{(-2)^2 + (-24)^2}\,\mathrm{m\,s^{-1}} = \sqrt{580}\,\mathrm{m\,s^{-1}}.$$

The two components are perpendicular, so subtracting their scalar speeds as $24 - 2$ would be incorrect. The question calls the relative speed $\sqrt{v}$; it requests the radicand, rather than $\sqrt{580} \simeq 24.08$. Dimensionally the radicand is a squared speed if units are attached to it; the numerical entry is obtained with speed measured in $\mathrm{m\,s^{-1}}$.

Step 3: Conclusion The relative speed is $\sqrt{580}\,\mathrm{m\,s^{-1}}$, so the requested numerical value is $\boxed{v = 580}$ (radicand in $\mathrm{m^2\,s^{-2}}$).

Quick Tip to Solve: Differentiate positions first, subtract velocity vectors second, and square their perpendicular components only after the vector subtraction.

47. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is _____. [4 marks]

Solution:

Concept:

- For a thin lens in air, the lens maker relation is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the material refractive index and the radii carry Cartesian signs.
- A plane surface has infinite radius of curvature and hence zero reciprocal curvature. A plano-convex lens in air has positive power and positive focal length.

Step 1: Assign the curvature signs Take light to travel from left to right, entering the curved convex face first as stated. Its centre of curvature lies to the right of the front surface, giving $R_1 = +30\,\text{cm}$. The second face is plane, so $R_2 = \infty$ and $1/R_2 = 0$. A plane face still refracts rays arriving from the glass, but its contribution to the thin-lens curvature term is zero; this does not mean the material refractive index is to be ignored.

Step 2: Substitute in the lens maker relation Using $\mu = 1.5$,

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{30} - 0\right) = \frac{0.5}{30} = \frac{1}{60}\,\text{cm}^{-1}.$$

Thus the lens power is positive and

$$f = +60\,\text{cm} = 0.60\,\text{m}.$$

Equivalently, in SI units its power is $1/0.60 = 5/3\,\text{D}$, which confirms the same focal length. One must not use both radii as $30\,\text{cm}$: there is only one curved surface. The location of the particular object would be needed to find its image distance, but it is irrelevant to this lens property. Reversing a thin plano-convex lens in air would also preserve its focal length.

Step 3: Conclusion The focal length is $\boxed{60\,\text{cm}}$, positive because the plano-convex lens is converging in air.

Quick Tip to Solve: For a thin plano-convex lens in air, immediately reduce lens maker's equation to $f = R/(\mu - 1)$; the plane face contributes zero reciprocal radius.

48. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is _____ Hz. [4 marks]

Solution:

Concept:

- For a pipe open at both ends and neglecting end correction, the allowed frequencies are $f_n = nc/(2L)$, with $n = 1, 2, \ldots$, sound speed c and length L.
- The sound speed is $c = \sqrt{B/\rho}$, where B is the adiabatic bulk modulus. With the same elastic

modulus in the comparison, doubling density reduces speed by $\sqrt{2}$.

Step 1: Find the sound speed in the gas The standard density comparison uses unchanged elastic modulus, so

$$\frac{c_{\text{gas}}}{c_{\text{air}}} = \sqrt{\frac{\rho_{\text{air}}}{\rho_{\text{gas}}}} = \frac{1}{\sqrt{2}}.$$

With $c_{\text{air}} = 300\,\text{m}\,\text{s}^{-1}$,

$$c_{\text{gas}} = \frac{300}{\sqrt{2}} = 150\sqrt{2}\,\text{m}\,\text{s}^{-1}.$$

For ideal gases at the same pressure this comparison corresponds to equal ratios of specific heats, since $B = \gamma P$. Density alone would not determine the speed if those elastic properties were allowed to differ; the stated density-based comparison uses this usual approximation.

Step 2: Subtract the two open-pipe harmonic frequencies Here $L = 1\,\text{m}$. Both ends are open, so the fundamental has half a wavelength inside the pipe, and the second harmonic has a full wavelength:

$$f_1 = \frac{c_{\text{gas}}}{2L} = 75\sqrt{2}\,\text{Hz}, \qquad f_2 = \frac{2c_{\text{gas}}}{2L} = 150\sqrt{2}\,\text{Hz}.$$

Subtracting them gives

$$\Delta f = f_2 - f_1 = \frac{c_{\text{gas}}}{2L} = 75\sqrt{2}\,\text{Hz} \simeq 106.066\,\text{Hz}.$$

Do not use the odd-harmonic series of a pipe closed at one end: an open pipe contains the second harmonic, and its adjacent harmonic spacing equals its fundamental frequency.

Step 3: Conclusion The required frequency difference is $\boxed{75\sqrt{2}\,\text{Hz} \simeq 106.07\,\text{Hz}}$.

Quick Tip to Solve: For a pipe open at both ends, adjacent harmonics are separated by the fundamental frequency; first adjust the sound speed for the gas, then use $c/(2L)$.

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49. A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}$ ms^{-1} . It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}$ ms^{-1}. After collision, A moves with a velocity $\vec{v} = 4\left(\hat{i}+\hat{j}\right)$. The energy of B after collision is written as $\frac{x}{10}$. The value of x is _____. [4 marks]

Solution:

Concept:

- In an elastic collision, the total kinetic energy of the two colliding bodies is conserved. Each kinetic energy is $K = \frac{1}{2}m|\vec{v}|^2$, where the square of a vector speed is the sum of the squared Cartesian components.
- Total linear momentum is conserved as a vector. Since the masses are equal, $\vec{u}_A + \vec{u}_B = \vec{v}_A + \vec{v}_B$; this gives an independent check of the energy calculation.

Step 1: Calculate the initial kinetic energy The initial velocities are perpendicular, but kinetic energies

are scalars and must be added. The initial speeds are $3\,\mathrm{m\,s^{-1}}$ and $5\,\mathrm{m\,s^{-1}}$, so

$$K_i = \frac{1}{2}(0.1)(3^2 + 5^2) = 0.05(34) = 1.7\,\mathrm{J}.$$

Do not add the two speeds first: the kinetic energy of each body must be calculated separately.

Step 2: Find the final energy of A and subtract For A, the final velocity has two components, each equal to $4\,\mathrm{m\,s^{-1}}$. Thus $|\vec{v}_A|^2 = 4^2 + 4^2 = 32\,\mathrm{m^2 s^{-2}}$. Its final kinetic energy is

$$K_{Af} = \frac{1}{2}(0.1)(32) = 1.6\,\mathrm{J}.$$

Conservation of energy now gives

$$K_{Bf} = K_i - K_{Af} = 1.7 - 1.6 = 0.1\,\mathrm{J}.$$

Step 3: Check momentum and identify x Conservation of momentum yields $\vec{v}_B = 3\hat{i} + 5\hat{j} - 4\hat{i} - 4\hat{j} = -\hat{i} + \hat{j}$. Therefore $K_{Bf} = \frac{1}{2}(0.1)(1+1) = 0.1\,\mathrm{J}$, agreeing with the energy result. Comparing $K_{Bf} = x/10$ with $0.1 = 1/10$ gives $x = 1$.

Step 4: Conclusion The energy of B after collision is $0.1\,\mathrm{J}$; hence the requested numerical coefficient is $\boxed{x = 1}$.

Quick Tip to Solve: For a velocity given in unit vectors, square and add its components before using kinetic energy; momentum provides a useful independent check.

50. Four resistances of 15Ω, 12Ω, 4Ω and 10Ω respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10Ω to balance the network is _____Ω. [4 marks]

Solution:

Concept:

- A Wheatstone bridge is balanced when the products of the resistances in opposite arms are equal. If the four cyclic arms are P, Q, R, S, the balance condition is $PR = QS$.
- For two positive resistances R_1, R_2 in parallel, $R_{\text{eq}} = R_1 R_2/(R_1 + R_2)$. The equivalent resistance is smaller than either constituent resistance.

Step 1: Apply the cyclic-arm balance condition Let the resistance added in parallel with the $10\,\Omega$ arm be r, and denote that arm's new effective resistance by R_e. The cyclic sequence is $15, 12, 4, R_e$. The opposite-arm products therefore give

$$15 \times 4 = 12 R_e.$$

Consequently,

$$R_e = \frac{60}{12} = 5\,\Omega.$$

Using a ratio without first respecting the cyclic positions could incorrectly pair adjacent rather than

opposite arms.

Step 2: Calculate the required parallel resistance The $10\,\Omega$ resistance and the added resistance r must together give $5\,\Omega$. Hence

$$\frac{10r}{10+r} = 5.$$

Cross-multiplying and collecting the terms containing r,

$$10r = 50 + 5r, \qquad 5r = 50, \qquad r = 10\,\Omega.$$

The value is positive and physically possible because a parallel connection reduces the effective resistance.

Step 3: Verify the network is balanced Two equal $10\,\Omega$ resistances in parallel give $5\,\Omega$. Substituting this value, the two opposite products are $15 \times 4 = 60\,\Omega^2$ and $12 \times 5 = 60\,\Omega^2$. Their equality confirms zero potential difference across the bridge detector, as required for balance.

Step 4: Conclusion The resistance to be connected in parallel is $\boxed{10\,\Omega}$.

Quick Tip to Solve: For four arms specified in cyclic order, equate opposite products first; only then replace the altered arm using the parallel-resistance formula.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Arrange the following compounds in increasing order of C–OH bond length :
methanol, phenol, p-ethoxyphenol [4 marks]

(A) phenol $<$ methanol $<$ p-ethoxyphenol

(B) methanol $<$ p-ethoxyphenol $<$ phenol

(C) methanol $<$ phenol $<$ p-ethoxyphenol

(D) phenol $<$ p-ethoxyphenol $<$ methanol

Correct Answer: (D) phenol $<$ p-ethoxyphenol $<$ methanol

Solution:

Concept:

- Bond length decreases as bond order increases. Conjugation of an oxygen lone pair with an aromatic ring gives the phenolic C–O bond partial double-bond character.
- The carbon bonded to oxygen is sp^2-hybridised in phenol but sp^3-hybridised in methanol. Greater s-character also favours a shorter phenolic C–O bond.

Step 1: Compare phenol with methanol In methanol, CH_3OH, the C–OH bond is an ordinary single bond to a saturated carbon. There is no neighbouring aromatic π system into which the hydroxyl oxygen can donate a lone pair. In phenol, oxygen donation into the benzene ring produces resonance contributors with increased C–O bond order. Thus phenol has the shorter C–OH bond and methanol the longer one.

Step 2: Assess the para ethoxy substituent The ethoxy group in p-ethoxyphenol is electron donating

by resonance. Its donation increases electron density in the ring and competes with lone-pair donation from the hydroxyl oxygen. The hydroxyl C–O bond consequently has less partial double-bond character than in unsubstituted phenol and is slightly longer. It still belongs to an aromatic, conjugated system and remains shorter than the C–OH bond in methanol.

Step 3: Assemble the increasing order and test the options The increasing bond-length order is therefore phenol, followed by p-ethoxyphenol, followed by methanol. Options (B) and (C) incorrectly make methanol the shortest. Option (A) puts p-ethoxyphenol beyond methanol, overlooking the shortening associated with its aromatic C–O bond. Only option (D) retains both comparisons consistently.

Step 4: Conclusion The increasing order of C–OH bond length is phenol $<$ p-ethoxyphenol $<$ methanol. Hence, the correct option is (D).

Quick Tip to Solve: First separate aromatic phenols from saturated alcohols; then compare how a ring substituent changes the hydroxyl oxygen's resonance donation.

52. The complex that can show fac- and mer-isomers is : [4 marks]

(A) $[CoCl_2(en)_2]$

(B) $[Pt(NH_3)_2Cl_2]$

(C) $[Co(NH_3)_3(NO_2)_3]$

(D) $[Co(NH_3)_4Cl_2]^+$

Correct Answer: (C) $[Co(NH_3)_3(NO_2)_3]$

Solution:

Concept:

- Facial and meridional geometrical isomerism occurs in an octahedral complex of the form $[MA_3B_3]$, containing three monodentate ligands of each of two types.
- In the fac form, three identical ligands occupy the corners of one triangular face and are mutually cis. In the mer form, they lie on a plane through the metal, with one identical-ligand pair trans.

Step 1: Identify the matching octahedral composition The complex $[Co(NH_3)_3(NO_2)_3]$ has three ammine and three nitro ligands. All six donor positions around cobalt are occupied by monodentate ligands, giving the required octahedral MA_3B_3 arrangement. The three ammine ligands can occupy a single triangular face or positions along a meridian, so two distinct fac and mer arrangements are possible.

Step 2: Exclude the other coordination patterns In option (A), ethylenediamine, en, is bidentate, so $[CoCl_2(en)_2]$ has two chelate rings and two chlorido ligands. Its geometrical alternatives are cis and trans, rather than the facial/meridional classification for three identical monodentate ligands. Option (B), $[Pt(NH_3)_2Cl_2]$, is a four-coordinate square-planar complex; it can be cis or trans but has no octahedral triangular faces.

Step 3: Distinguish the four-plus-two octahedral case Option (D), $[Co(NH_3)_4Cl_2]^+$, contains four ligands of one kind and two of the other. Its two chlorido ligands are either adjacent (cis) or opposite (trans). Since it lacks the three-plus-three ligand grouping, its isomers are not called fac and mer.

These exclusions leave the cobalt complex in option (C).

Step 4: Conclusion $\boxed{[Co(NH_3)_3(NO_2)_3]}$ can show facial and meridional isomerism. Hence, the correct option is (C).

Quick Tip to Solve: Look for octahedral MA_3B_3: three identical ligands on one face give fac, while a trans pair among them identifies mer.

53. The rate of a certain biochemical reaction at physiological temperature (T) occurs 10^6 times faster with enzyme than without. The change in the activation energy upon adding enzyme is : [4 marks]

(A) + 6RT

(B) – 6 (2.303)RT

(C) – 6RT

(D) + 6(2.303)RT

Correct Answer: (B) – 6 (2.303)RT

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where E_a is molar activation energy, R is the gas constant and T is absolute temperature.
- At the same temperature, using the usual comparison with unchanged pre-exponential factor, the ratio of catalysed to uncatalysed rate constants measures the reduction in activation energy. Enzymes increase rate by providing a lower-barrier pathway.

Step 1: Express the rate enhancement as a ratio Let $E_{a,e}$ denote the activation energy with the enzyme and $E_{a,0}$ that without it. At the same reactant conditions, the given rate enhancement means $k_e/k_0 = 10^6$. Dividing the two Arrhenius expressions cancels the common pre-exponential factor:

$$\frac{k_e}{k_0} = \exp\left(\frac{E_{a,0} - E_{a,e}}{RT}\right).$$

The exponent is positive because the catalysed reaction is faster.

Step 2: Take logarithms and calculate the change Define the requested change as final minus initial: $\Delta E_a = E_{a,e} - E_{a,0}$. Taking natural logarithms gives

$$\ln(10^6) = -\frac{\Delta E_a}{RT}.$$

Since $\ln(10^6) = 6\ln 10 = 6(2.303)$, we obtain

$$\Delta E_a = -6(2.303)RT.$$

The dimensions are those of molar energy, as RT has units $\mathrm{J\,mol^{-1}}$.

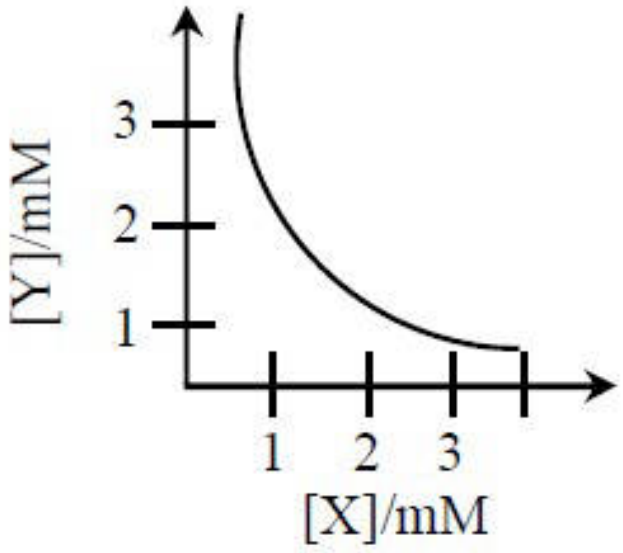
Step 3: Check the sign and the logarithmic factor Positive changes, as in (A) and (D), would raise the activation barrier and slow the reaction, contrary to the observation. Option (C) has the right sign

but would give a factor e^6, not 10^6. The conversion from base-ten powers to the natural exponential requires the extra factor $\ln 10 = 2.303$, selecting (B).

Step 4: Conclusion The activation energy changes by $\boxed{-6(2.303)RT}$; it decreases upon adding the enzyme. Hence, the correct option is (B).

Quick Tip to Solve: Write the energy change as catalysed minus uncatalysed before taking logarithms; a tenfold rate factor contributes $2.303RT$ to the barrier reduction.

54. The stoichiometry and solubility product of a salt with the solubility curve given below is, respectively : [4 marks]



(A) XY, 2×10^{-6} M^3

(B) XY_2, 1×10^{-9} M^3

(C) XY_2, 4×10^{-9} M^3

(D) X_2Y, 2×10^{-9} M^3

Correct Answer: (C) XY_2, 4×10^{-9} M^3

Solution:

Concept:

- For a sparingly soluble salt $XY_2(s) \rightleftharpoons X^{2+}(aq) + 2\,Y^{-}(aq)$, the solubility product is $K_{sp} = [X][Y]^2$. Concentrations enter with powers equal to their stoichiometric coefficients.
- The horizontal and vertical coordinates represent equilibrium ion concentrations, expressed in millimolar units. Before evaluating K_{sp}, convert each plotted concentration using $1\,\text{mM} = 10^{-3}\,\text{M}$.

Step 1: Read the concentration dependence The curve shows that the concentration of Y decreases as that of X increases. Its form corresponds to $[Y]$ varying as $[X]^{-1/2}$: increasing X requires a decrease in the square of the Y concentration so that $[X][Y]^2$ stays constant. This is the equilibrium relation for a salt with one X ion and two Y ions per formula unit, namely XY_2, rather than the relation

$[X]^2[Y]$ = constant for X_2Y.

Step 2: Evaluate the solubility product from the curve Using the plotted concentration pair near $[X] = 1\,\text{mM}$ and $[Y] = 2\,\text{mM}$, convert to molarity and substitute:

$$K_{sp} = (1 \times 10^{-3})(2 \times 10^{-3})^2.$$

Squaring the Y concentration first gives $4 \times 10^{-6}\,\text{M}^2$, so

$$K_{sp} = 4 \times 10^{-9}\,\text{M}^3.$$

The powers of ten must be included for all three concentration factors, not just once.

Step 3: Match both stoichiometry and value Option (C) contains both XY_2 and $4 \times 10^{-9}\,\text{M}^3$. Option (B) has the right stoichiometry but misses the factor $2^2 = 4$ from Y. Option (D) reverses the stoichiometric powers, while (A) gives a one-to-one salt and a value incompatible with the cubic concentration product.

Step 4: Conclusion The salt is $\boxed{XY_2}$ and its solubility product is $\boxed{4 \times 10^{-9}\,\text{M}^3}$. Hence, the correct option is (C).

Quick Tip to Solve: Read the axes as ion concentrations, write the product with stoichiometric powers, and convert mM to M before raising any concentration to a power.

55. The third ionization enthalpy is minimum for : [4 marks]

(A) Ni

(B) Co

(C) Mn

(D) Fe

Correct Answer: (D) Fe

Solution:

Concept:

- The third ionization enthalpy is the energy for the gas-phase process $M^{2+}(g) \longrightarrow M^{3+}(g) + e^-$. Thus compare the configurations of the doubly and triply charged ions, rather than the neutral atoms.
- A half-filled $3d^5$ subshell is particularly stable. Forming this arrangement favours ionization, whereas removing an electron from it requires extra energy.

Step 1: Write the relevant ionic configurations The two outer $4s$ electrons are removed before the $3d$ electrons in these transition metals. The doubly charged ions are therefore $Mn^{2+} : 3d^5$, $Fe^{2+} : 3d^6$, $Co^{2+} : 3d^7$, and $Ni^{2+} : 3d^8$. The third ionization removes a $3d$ electron in every case, so the key distinction is the stability of the resulting ion.

Step 2: Compare the products of third ionization For iron the change is $3d^6 \rightarrow 3d^5$, producing the especially stable half-filled configuration of Fe^{3+}. For manganese it is $3d^5 \rightarrow 3d^4$, destroying a

half-filled arrangement; hence manganese is not the minimum. Cobalt gives $3d^6$ and nickel gives $3d^7$, neither of which gains the half-filled stabilization that iron gains. Their greater nuclear charges also oppose electron removal. Thus Ni and Co do not provide the favourable third-electron removal found for Fe.

Step 3: Conclusion The third ionization enthalpy is minimum for Fe, because Fe^{3+} has the stable $3d^5$ configuration. Hence, the correct option is (D).

Quick Tip to Solve: For a third-ionization comparison, write the M^{2+} configuration first: forming d^5 favours removal, while breaking d^5 resists it.

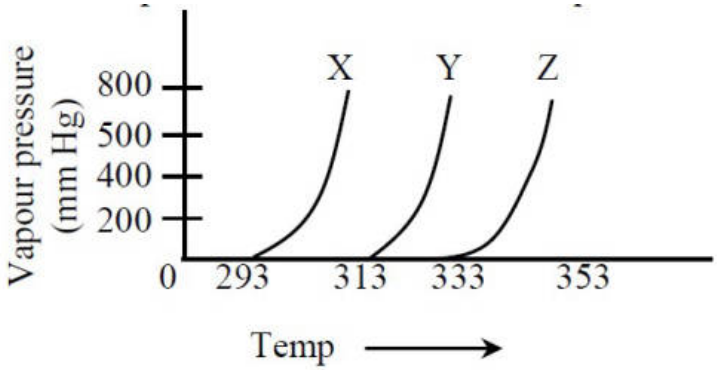
56. A graph of vapour pressure and temperature for three different liquids X, Y, and Z is shown below : The following inferences are made :

(A) X has higher intermolecular interactions compared to Y.

(B) X has lower intermolecular interactions compared to Y.

(C) Z has lower intermolecular interactions compared to Y.

The correct inference (s) is / are : [4 marks]



(A) (B)

(B) (A)

(C) (C)

(D) (A) and (C)

Correct Answer: (A) (B)

Solution:

Concept:

- At a fixed temperature, weaker attractive forces allow molecules to escape more readily from the liquid and generally give a higher equilibrium vapour pressure.
- At a fixed external pressure, the liquid that reaches that pressure at a lower temperature has the lower boiling point. The horizontal positions of the vapour-pressure curves therefore compare the strength of intermolecular attractions.

Step 1: Read the order from the three curves The curve for X lies to the left of the curve for Y, while that for Z lies to its right. A horizontal line at a common vapour pressure therefore meets the curves

in the temperature order

$$T_X < T_Y < T_Z.$$

In particular, X reaches a given pressure with less thermal energy than Y, and Z needs more thermal energy than Y. Equivalently, at an appropriate common temperature, X has the largest vapour pressure and Z has the smallest.

Step 2: Test each stated inference The strength of intermolecular interactions consequently increases in the order

$$X < Y < Z.$$

Statement (A), which assigns stronger interactions to X than to Y, reverses this order and is false. Statement (B), which assigns weaker interactions to X than to Y, is true. Statement (C) is false because Z has stronger, rather than weaker, attractions than Y. Thus only inference (B) survives. The choices containing only (A), only (C), or both (A) and (C) each include a false inference.

Step 3: Conclusion Only the second inference is correct: X has lower intermolecular interactions than Y. This is the first listed choice. Hence, the correct option is (A).

Quick Tip to Solve: Compare the temperatures at the same vapour pressure: the leftmost curve reaches that pressure first and corresponds to weaker attractions.

57. Which of the following statement is not true for glucose? [4 marks]

(A) Glucose gives Schiff's test for aldehyde

(B) Glucosse reacts with hydroxylamine to form oxime

(C) The pentaacetate of glucose does not react with hydroxylamine to give oxime

(D) Glucose exists in two crystalline forms α and β

Correct Answer: (A) Glucose gives Schiff's test for aldehyde

Solution:

Concept:

- Glucose is an aldohexose, but in solution it exists predominantly as cyclic hemiacetal forms in equilibrium with a small proportion of the open-chain aldehyde.
- The α and β forms are anomers differing at the anomeric carbon. Reactions of glucose must be considered together with this cyclic/open-chain equilibrium, rather than treating every molecule as a permanently free aldehyde.

Step 1: Identify the exceptional aldehyde test Although its open-chain structure contains an aldehyde group, glucose does not give the usual Schiff's test for aldehydes. Its predominant cyclic hemiacetal structure accounts for this exception in the standard properties of glucose. Thus the assertion that glucose gives Schiff's test is the false statement asked for. It is not enough to reason that every substance described as an aldose must give every conventional aldehyde test.

Step 2: Check the other three statements Glucose reacts with hydroxylamine, NH_2OH, to form an oxime: reaction with the small open-chain fraction is accompanied by replenishment through the ring-chain equilibrium. Hence (B) is true. In glucose pentaacetate, the anomeric hydroxyl group is

acetylated and a free aldehyde group is not available by ordinary ring opening; it therefore does not form an oxime with hydroxylamine, making (C) true. Finally, glucose does exist as two crystalline anomers, α-glucose and β-glucose, so (D) is also true. These observations are consistent with the cyclic structure rather than contradictions of it.

Step 3: Conclusion The statement "Glucose gives Schiff's test for aldehyde" is not true. The other three listed statements are true. Hence, the correct option is (A).

Quick Tip to Solve: Keep the glucose exceptions together: no Schiff's test, but oxime formation by glucose; the pentaacetate does not form that oxime.

58. The predominant intermolecular forces present in ethyl acetate, a liquid, are: [4 marks]

(A) Dipole-dipole and hydrogen bonding

(B) hydrogen bonding and London dispersion

(C) London dispersion, dipole-dipole and hydrogen bonding

(D) London dispersion and dipole-dipole

Correct Answer: (D) London dispersion and dipole-dipole

Solution:

Concept:

- London dispersion forces arise from instantaneous and induced dipoles and occur between all molecules, including molecules that also have permanent dipoles.
- Conventional intermolecular hydrogen bonding in a pure liquid requires a suitable hydrogen-bond donor, typically an $O-H$, $N-H$, or $F-H$ group. An electronegative atom with lone pairs alone is not sufficient.

Step 1: Inspect the ester molecule Ethyl acetate has the structure $CH_3COOCH_2CH_3$. Its polar carbonyl bond and ester group give it a permanent molecular dipole, so neighbouring molecules exhibit dipole-dipole attractions. Its electron cloud is also polarizable, producing London dispersion attractions. These two mechanisms can act at the same time; calling a molecule polar does not eliminate dispersion forces.

Step 2: Check whether the pure liquid can hydrogen-bond to itself The ester contains oxygen atoms that can accept hydrogen bonds from water or another donor, but none of its hydrogen atoms is bonded directly to oxygen, nitrogen, or fluorine. All its hydrogens are attached to carbon. Hence pure ethyl acetate lacks the donor required for the usual strong intermolecular hydrogen bonding between its own molecules. Option (A) wrongly includes hydrogen bonding and omits dispersion. Options (B) and (C) also wrongly include hydrogen bonding. Option (D) includes exactly the dispersion and permanent dipole interactions present in the liquid.

Step 3: Conclusion The predominant intermolecular forces are London dispersion and dipole-dipole interactions. Hence, the correct option is (D).

Quick Tip to Solve: For a pure ester, look for an O-H or N-H donor before adding hydrogen bonding; ester oxygen can accept a hydrogen bond without supplying a donor.

59. A flask contains a mixture of isohexane and 3-methylpentane. One of the liquids boils at 63°C while the other boils at 60°C What is the best way to separate the two liquids and which one will be distilled out first ? [4 marks]

(A) fractional distillation, 3-methylpentane

(B) simple distillation, 3-methylpentane

(C) fractional distillation, isohexane

(D) simple distillation, isohexane

Correct Answer: (C) fractional distillation, isohexane

Solution:

Concept:

- Fractional distillation separates miscible liquids with close boiling points by repeated vaporization and condensation in a fractionating column.
- Among isomeric alkanes, greater branching generally reduces effective surface contact and London attractions, lowering the boiling point. The more volatile, lower-boiling liquid is collected first.

Step 1: Choose the separation method The stated boiling points differ by only

$$63 - 60 = 3\,^{\circ}\mathrm{C}.$$

Such a small difference makes ordinary simple distillation inefficient: the vapour contains appreciable quantities of both components. A fractionating column supplies repeated vapour-liquid equilibration stages, enriching the rising vapour in the lower-boiling component. Therefore fractional distillation, rather than simple distillation, is the best of the listed methods.

Step 2: Identify the component collected first Isohexane is 2-methylpentane, while the other component is 3-methylpentane. In these two isomers the former has the lower boiling point, approximately $60\,^{\circ}\mathrm{C}$, while 3-methylpentane boils near $63\,^{\circ}\mathrm{C}$. The difference reflects the more compact molecular shape and weaker effective dispersion attractions of isohexane. Its higher volatility makes the early distillate richer in isohexane. Thus the fractional-distillation choice naming 3-methylpentane first is wrong, and both simple-distillation choices use an unsuitable method for this close-boiling pair.

Step 3: Conclusion Use fractional distillation; isohexane is distilled out first because it is the lower-boiling component. Hence, the correct option is (C).

Quick Tip to Solve: Answer close-boiling separation questions in two stages: choose fractional distillation from the small boiling-point gap, then identify the lower-boiling first fraction.

60. For the Balmer series in the spectrum of H atom,
$\overline{\nu} = R_H \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\}$**, the correct statements among (I) to (IV) are :**
(I) As wavelength decreases, the lines in the series converge
(II) The integer n_1 is equal to 2
(III) The lines of longest wavelength corresponds to $n_2 = 3$
(IV) The ionization energy of hydrogen can be calculated from wave number of these lines [4 marks]

(A) (II), (III), (IV)

(B) (I), (II), (III)

(C) (I), (III), (IV)

(D) (I), (II), (IV)

Correct Answer: All four statements are literally true; no printed option lists all four. The expected key is (B), using a direct-transition interpretation of (IV).

Solution:

Concept:

- Balmer emission lines terminate at $n_1 = 2$, with $n_2 = 3, 4, 5, \ldots$. Their wave numbers obey $\overline{\nu} = R_H(1/4 - 1/n_2^2)$, and wavelength is $\lambda = 1/\overline{\nu}$.
- The smallest energy difference gives the longest wavelength. Successive levels become closer in energy as the upper principal quantum number increases, producing a series limit.

Step 1: Check the convergence and the longest wavelength As n_2 increases, $1/n_2^2$ decreases, so the wave number increases towards $R_H/4$ and the wavelength decreases towards $4/R_H$. The difference between successive wave numbers becomes smaller, which means the spectral lines converge at the short-wavelength end. Statement (I) is therefore true. The defining lower level of the Balmer series is $n_1 = 2$, establishing (II). The smallest allowed upper level is $n_2 = 3$; its transition to 2 has the smallest photon energy and the longest wavelength, establishing (III).

Step 2: Distinguish the series limit from ground-state ionization The Balmer limit corresponds to removal from $n = 2$, with $hc\overline{\nu}_{\text{limit}} = hcR_H/4$. This is not itself the ground-state ionization energy hcR_H. That distinction explains why the expected key selects (I), (II), and (III), excluding (IV) in a direct-photon-energy interpretation. However, the actual wording says the ionization energy "can be calculated" from the wave numbers. Literally this is true: a known Balmer transition yields

$$R_H = \frac{\overline{\nu}}{1/4 - 1/n_2^2}, \qquad E_{\text{ion}} = hcR_H.$$

Equivalently, the series limit gives $E_{\text{ion}} = 4hc\overline{\nu}_{\text{limit}}$. Thus (IV) is also defensible as written. All four statements are then true, and no listed option supplies their complete set. Options (A), (C), and (D) each additionally omit a true statement among (I)-(III).

Step 3: Conclusion The expected key is option (B), meaning (I), (II), and (III), when (IV) is interpreted as directly identifying a Balmer photon energy with ground-state ionization. Literally, (IV) is also true because the measured wave numbers determine R_H and hence hcR_H; all four statements are true and no printed combination is fully correct. The question therefore has a wording ambiguity.

Quick Tip to Solve: For Balmer lines fix the lower level at 2. The first line is $3 \to 2$; the series limit is $\infty \to 2$, whose energy is one quarter of ground-state ionization.

61. The first ionization energy (in kJ/mol) of Na, Mg, Al and Si respectively, are : [4 marks]

(A) 496, 577, 786, 737

(B) 496, 737, 577, 786

(C) 786, 737, 577, 496

(D) 496, 577, 737, 786

Correct Answer: (B) 496, 737, 577, 786

Solution:

Concept:

- The first ionization enthalpy is the energy required to remove one electron from each gaseous atom: $\mathrm{M(g)} \longrightarrow \mathrm{M^+(g) + e^-}$. It is quoted in $\mathrm{kJ\,mol^{-1}}$.
- Across the third period, effective nuclear charge generally increases and atomic size decreases. However, removing a $3p$ electron from aluminium is easier than removing a $3s$ electron from magnesium.

Step 1: Write the relevant outer configurations The outer configurations are $\mathrm{Na} : 3s^1$, $\mathrm{Mg} : 3s^2$, $\mathrm{Al} : 3s^2 3p^1$ and $\mathrm{Si} : 3s^2 3p^2$. Sodium loses its single, relatively weakly held outer electron most easily. Magnesium has a filled $3s$ subshell, so its first ionization energy rises appreciably.

Step 2: Apply the subshell exception and assign the values The aluminium electron removed is in the higher-energy, less penetrating $3p$ subshell, whereas magnesium loses a $3s$ electron. Hence aluminium has the lower value despite coming later in the period. Silicon has a larger effective nuclear charge than aluminium, giving the order

$$\mathrm{Na} < \mathrm{Al} < \mathrm{Mg} < \mathrm{Si}.$$

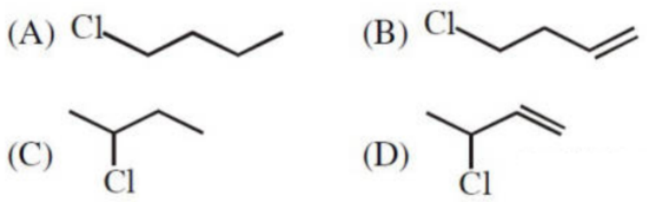
Thus the four increasing numerical values must be assigned as Na = 496, Al = 577, Mg = 737 and Si = 786.

Step 3: Check the sequences offered In the requested order Na, Mg, Al, Si, only (B) keeps Mg above Al while placing Na lowest and Si highest. Options (A) and (D) incorrectly put magnesium below aluminium; option (C) puts sodium highest, opposite to its large size and single $3s$ electron.

Step 4: Conclusion The first ionization energies of Na, Mg, Al and Si are respectively $496, 737, 577, 786\ \mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a third-period ionization-energy list, check the Mg–Al dip before applying the general left-to-right increase.

62. The decreasing order of reactivity towards dehydrohalogenation (E_1) reaction of the following compounds is : [4 marks]



(A) B > D > A > C

(B) D > B > C > A

(C) B > A > D > C

(D) B > D > C > A

Correct Answer: (B) D > B > C > A

Solution:

Concept:

- An E1 elimination begins with slow heterolytic cleavage of the carbon–halogen bond. Reactivity is therefore governed mainly by the stability of the developing carbocation.
- Resonance stabilization of an allylic carbocation is especially effective. A nearby double bond can also assist ionization of a homoallylic halide, while an ordinary secondary carbocation is more stable than an ordinary primary one.

Step 1: Identify the four carbon skeletons Compound A is a primary saturated chloride, $ClCH_2CH_2CH_2CH_3$; C is the secondary chloride $CH_3CH(Cl)CH_2CH_3$. B is the homoallylic chloride $ClCH_2CH_2CH{=}CH_2$, while D is the secondary allylic chloride $CH_3CH(Cl)CH{=}CH_2$. The position of the double bond relative to the chlorine-bearing carbon is crucial; B and D are not equivalent unsaturated halides.

Step 2: Compare the ease of ionization D loses chloride to give an allylic cation whose charge is delocalized:

$$CH_3CH^+CH{=}CH_2 \longleftrightarrow CH_3CH{=}CHCH_2{}^+.$$

It therefore reacts fastest. In B, the nearby double bond assists departure through neighbouring π-bond participation and a delocalized, bridged cationic intermediate. This assistance makes B more reactive than the unassisted saturated secondary chloride C. A would require an unstabilized primary cation and is slowest.

Step 3: Match the complete ordering The comparison gives $D > B > C > A$. Option (A) wrongly puts B above the directly allylic D and A above C. Option (C) also places B above D and raises A too far. Option (D) correctly keeps C above A but still incorrectly makes B faster than D. Only (B) preserves all three comparisons.

Step 4: Conclusion The decreasing order of E1 reactivity is $\boxed{D > B > C > A}$. Hence, the correct option is (B).

Quick Tip to Solve: Locate the carbon bearing the leaving group first: allylic resonance and neighbouring double-bond assistance can outweigh a simple primary/secondary classification.

63. The strength of an aqueous NaOH solution is most accurately determined by titrating :
(Note : consider that an appropriate indicator is used) [4 marks]

(A) Aq. NaOH in a pipette and aqueous oxalic acid in a burette

(B) Aq. NaOH in a burette and aqueous oxalic acid in a conical flask

(C) Aq. NaOH in a volumetric flask and concentrated H_2SO_4 in a conical flask

(D) Aq. NaOH in a burette and concentrated H_2SO_4 in a conical flask

Correct Answer: (B) Aq. NaOH in a burette and aqueous oxalic acid in a conical flask

Solution:

Concept:

- A sodium hydroxide solution is standardized against a known quantity of a suitable primary standard such as oxalic acid. A pipette delivers a fixed aliquot, while a burette measures the variable volume needed to reach the endpoint.
- Oxalic acid is diprotic. Complete neutralization obeys $\mathrm{H_2C_2O_4 + 2\,NaOH \longrightarrow Na_2C_2O_4 + 2\,H_2O}$, so one mole of acid reacts with two moles of base.

Step 1: Choose the standard and the correct vessels Take a measured aliquot of standard aqueous oxalic acid in the conical flask, add an appropriate indicator, and deliver the unknown aqueous NaOH from the burette while swirling. The conical flask permits safe mixing without loss, and the burette gives the volume of the unknown base consumed. Oxalic acid is suitable for preparing an accurately known standard; sodium hydroxide itself is hygroscopic and absorbs atmospheric carbon dioxide.

Step 2: Relate the endpoint volume to concentration If the acid concentration is C_a, its aliquot volume is V_a, and the sodium hydroxide titre is V_b, then

$$C_bV_b = 2C_aV_a, \qquad C_b = \frac{2C_aV_a}{V_b}.$$

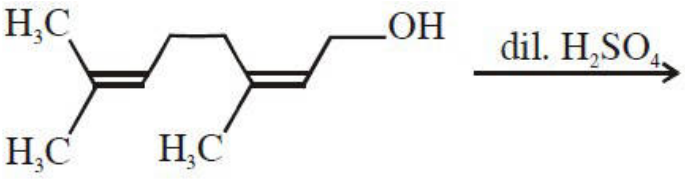
Use the same volume unit throughout. Multiplying C_b by the molar mass $40\ \mathrm{g\,mol^{-1}}$ gives strength in $\mathrm{g\,L^{-1}}$. With phenolphthalein, adding alkali to acid gives a clear first persistent pale-pink endpoint.

Step 3: Reject the less suitable arrangements Option (A) reverses the preferred standardization arrangement and measures variable acid against a fixed aliquot of the unknown. Options (C) and (D) propose concentrated sulfuric acid rather than an accurately prepared aqueous primary-standard solution; (C) also uses a volumetric flask, which is for preparing a fixed volume, as if it were a titration delivery vessel. The standard laboratory arrangement described in (B) is the most appropriate.

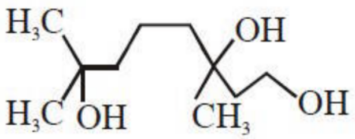
Step 4: Conclusion Place aqueous NaOH in the burette and a measured quantity of standard aqueous oxalic acid in the conical flask. Hence, the correct option is (B).

Quick Tip to Solve: For standardizing NaOH, remember: standard oxalic-acid aliquot in the flask, unknown alkali in the burette, and a 1:2 acid-to-base mole ratio.

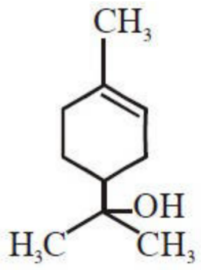
64. The major product of the following reaction is : [4 marks]



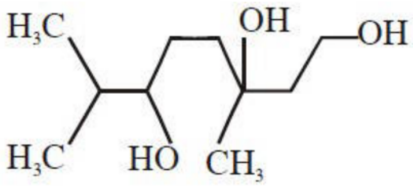
(A) Acyclic 3,7-dimethyloctane-1,3,7-triol (structure shown).



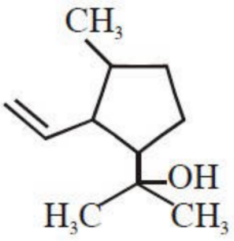
(B) Alpha-terpineol: a cyclohexene bearing methyl and $C(OH)(CH_3)_2$ substitutents (structure shown).



(C) Acyclic 3,7-dimethyloctane-1,3,6-triol (structure shown).

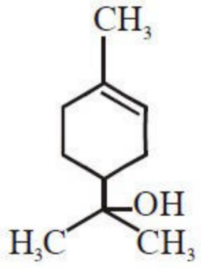


(D) Five-membered cyclic alcohol bearing methyl and vinyl substituents (structure shown).



Correct Answer: (B)

Alpha-terpineol: a cyclohexene bearing methyl and $C(OH)(CH_3)_2$ substitutents (structure shown).



Solution:

Concept:

- In dilute acid, an allylic alcohol can generate a resonance-stabilized allylic carbocation after protonation of its hydroxyl group and loss of water.
- Intramolecular attack by the second double bond can close a favourable six-membered ring. Water then traps the resulting tertiary carbocation and deprotonation produces an alcohol.

Step 1: Generate the allylic electrophile The substrate is the acyclic terpene alcohol

$$\mathrm{HOCH_2CH{=}C(CH_3)CH_2CH_2CH{=}C(CH_3)_2}.$$

Protonation changes the hydroxyl group into a water leaving group. Its departure gives an allylic cation, and resonance allows the positive charge to appear at the substituted allylic position. This permits the two separated alkene regions to interact within the same molecule rather than requiring two independent hydration events.

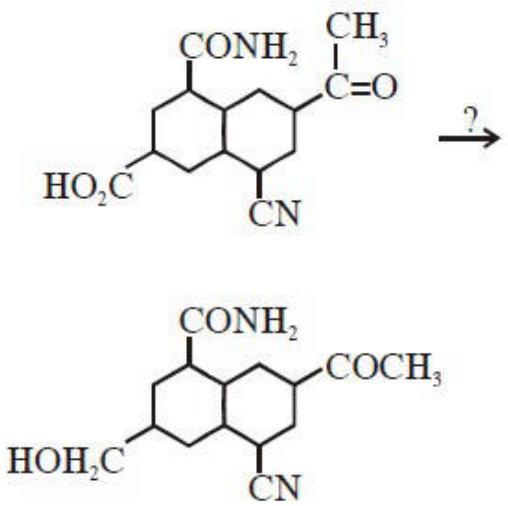
Step 2: Close the six-membered carbon ring The remote alkene attacks the allylic electrophilic end intramolecularly. A new carbon–carbon bond forms a six-membered ring, while the positive charge is transferred to the carbon bearing the two methyl groups outside the ring. That tertiary carbocation is strongly stabilized by its alkyl groups. The ring retains one carbon–carbon double bond; cyclization has consumed the other alkene in making the new sigma bond.

Step 3: Trap the cation and select the product Water attacks this tertiary carbocation, and loss of a proton regenerates the acid catalyst. The product has a cyclohexene ring bearing a methyl group and a $\mathrm{C(OH)(CH_3)_2}$ substituent: the structure in (B), alpha-terpineol. Structures (A) and (C) are acyclic polyols and do not represent the favourable intramolecular ring closure. Structure (D) has a five-membered ring and a vinyl substituent, inconsistent with the six-membered cyclization pathway available here.

Step 4: Conclusion The major product is alpha-terpineol, the six-membered cyclic alcohol shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: When a terpene alcohol contains a suitably spaced second double bond, look for acid-induced six-membered ring closure before treating each double bond as an isolated hydration site.

65. The most suitable reagent for the given conversion is : [4 marks]



(A) $LiAlH_4$

(B) B_2H_6

(C) $NaBH_4$

(D) H_2/Pd

Correct Answer: (B) B_2H_6

Solution:

Concept:

- Chemoselectivity means reducing the required functional group while retaining other reducible groups. The product drawing shows which bonds must change and which must remain intact.
- Diborane, B_2H_6, provides borane, which reacts preferentially with carboxylic acids and reduces $-COOH$ to $-CH_2OH$. Under suitable controlled conditions it leaves the other displayed groups unchanged.

Step 1: Identify the exact change in the structure The starting compound contains a carboxylic acid, a ketone substituent $-COCH_3$, an amide $-CONH_2$, and a nitrile $-CN$. In the product, only the acid group has become $-CH_2OH$. The ketone, amide, nitrile and fused-ring carbon skeleton are all retained. Thus the task is selective acid reduction, not a general reduction of every carbonyl-containing group.

Step 2: Choose the reagent with the required selectivity Borane first interacts strongly with the acidic carboxyl group and effects hydride reduction. After hydrolytic work-up, the original carboxyl carbon becomes the carbon of the primary alcohol:

$$RCOOH \xrightarrow[work-up]{B_2H_6} RCH_2OH.$$

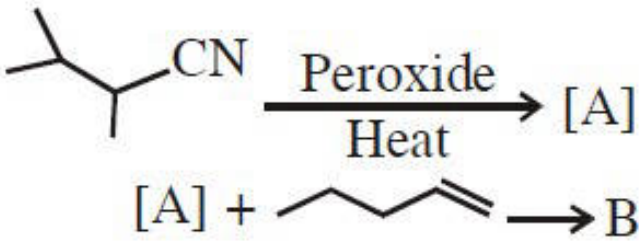
No carbon is removed, and no rearrangement of the bicyclic skeleton is required. Diborane is therefore the reagent that matches the before-and-after drawings.

Step 3: Eliminate the other reducing agents $LiAlH_4$ is too powerful and insufficiently selective: it can reduce the ketone and also the amide and nitrile groups. $NaBH_4$ usually reduces aldehydes and ketones but does not perform the required reduction of an ordinary carboxylic acid, so it targets the wrong group. Catalytic H_2/Pd is not the selective reagent for this acid-to-primary-alcohol conversion in the presence of the other functional groups. These considerations exclude (A), (C) and (D).

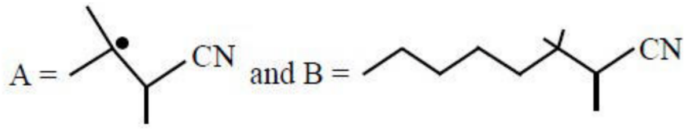
Step 4: Conclusion The most suitable reagent is diborane, $\boxed{B_2H_6}$. Hence, the correct option is (B).

Quick Tip to Solve: Compare the functional groups in reactant and product before choosing a hydride: selective reduction of COOH beside a ketone is a classic borane application.

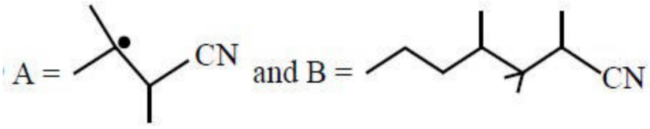
66. The major products A and B in the following reactions are : [4 marks]



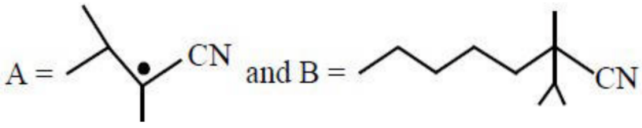
(A) A: radical on the isopropyl methine carbon; B: terminal addition product (structures shown).



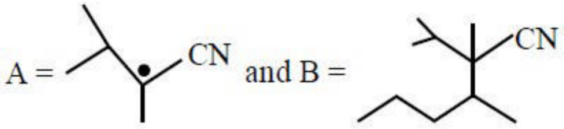
(B) A: radical on the isopropyl methine carbon; B: internal addition product (structures shown).



(C) A: radical on the carbon alpha to CN; B: terminal addition product (structures shown).

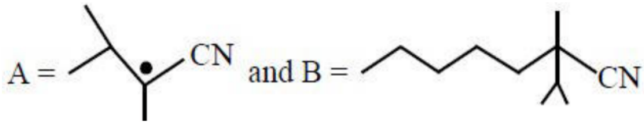


(D) A: radical on the carbon alpha to CN; B: internal addition product (structures shown).



Correct Answer: (C)

A: radical on the carbon alpha to CN; B: terminal addition product (structures shown).



Solution:

Concept:

- Heating a peroxide generates radicals that abstract hydrogen. A hydrogen on the carbon alpha to a nitrile is favoured because the resulting carbon radical is stabilized by the adjacent $-\mathrm{CN}$ group.
- In radical addition to a terminal alkene, bond formation at its terminal carbon leaves the more stable secondary radical on the internal carbon. Subsequent hydrogen abstraction gives the saturated addition product.

Step 1: Locate the radical in intermediate A The starting nitrile is $(\mathrm{CH_3})_2\mathrm{CHCH}(\mathrm{CH_3})\mathrm{CN}$. Abstraction of the hydrogen from the carbon directly attached to $-\mathrm{CN}$ gives

$$\mathrm{A} = (\mathrm{CH_3})_2\mathrm{CH} - \mathrm{C}^{\bullet}(\mathrm{CH_3}) - \mathrm{CN}.$$

This alpha-cyano radical is resonance stabilized. Abstraction at the neighbouring isopropyl methine would instead give a radical not directly conjugated with the nitrile, as drawn in options (A) and (B), so those intermediates are not preferred.

Step 2: Add A to the terminal double bond The alkene is pent-1-ene, $\mathrm{CH_2{=}CHCH_2CH_2CH_3}$. The alpha-cyano radical bonds to its terminal $\mathrm{CH_2}$ carbon. The unpaired electron is thereby placed on the second carbon, giving a secondary radical rather than a primary radical. Hydrogen abstraction then changes this radical centre into $\mathrm{CH_2}$; the five-carbon alkene chain becomes a straight pentyl substituent on the original alpha-cyano carbon.

Step 3: Identify the connectivity of product B The saturated product is

$$\mathrm{B} = (\mathrm{CH_3})_2\mathrm{CH} - \mathrm{C}(\mathrm{CH_3})(\mathrm{CN}) - \mathrm{CH_2CH_2CH_2CH_2CH_3}.$$

It retains the original isopropyl and methyl substituents at the alpha-cyano carbon and adds the pentyl chain there. This matches option (C). Option (D) uses the correct initial radical but bonds it to the internal alkene carbon, leaving the less stable primary radical in the addition step and giving the wrong branched connectivity.

Step 4: Conclusion Intermediate A is the alpha-cyano carbon radical, and product B has the straight pentyl substituent shown in (C). Hence, the correct option is (C).

Quick Tip to Solve: Check two separate preferences: radical formation next to CN, then terminal-alkene attack that leaves a secondary radical. Both must match the chosen option.

67. When gypsum is heated to 393 K, it forms : [4 marks]

(A) $CaSO_4$. $0.5H_2O$

(B) $CaSO_4$. $5H_2O$

(C) Dead burnt plaster

(D) Anhydrous $CaSO_4$

Correct Answer: (A) $CaSO_4$. $0.5H_2O$

Solution:

Concept:

- Gypsum is calcium sulphate dihydrate, $\mathrm{CaSO_4 \cdot 2\,H_2O}$. Heating removes water of crystallisation without decomposing the sulphate ion.
- Controlled heating to $393\,\mathrm{K}$ gives calcium sulphate hemihydrate, called plaster of Paris. A hemihydrate has one water molecule per two formula units of the salt.

Step 1: Identify the starting hydrate The two water molecules in gypsum belong to its crystal structure. They are not the oxygen atoms of the sulphate group. Thus a dehydration product retains the $\mathrm{CaSO_4}$ unit, while its water content decreases. The stated temperature is about $120\,^{\circ}\mathrm{C}$, the range used for controlled manufacture of plaster of Paris.

Step 2: Balance the dehydration The change can be written without fractional coefficients as

$$2\,\mathrm{CaSO_4 \cdot 2\,H_2O} \xrightarrow{393\ \mathrm{K}} 2\,\mathrm{CaSO_4 \cdot \tfrac{1}{2}H_2O} + 3\,\mathrm{H_2O}.$$

Equivalently, each mole of gypsum loses one and a half moles of water and retains half a mole. This explains the formula $\mathrm{CaSO_4 \cdot 0.5\,H_2O}$, rather than a pentahydrate.

Step 3: Distinguish the other products Option (B) has more water than gypsum, so it cannot represent this dehydration. More vigorous heating removes the remaining water and produces anhydrous calcium sulphate; strongly heated material is commonly called dead burnt plaster. Therefore neither (C) nor (D) is the product selected by the stated controlled heating condition.

Step 4: Conclusion The product at $393\,\mathrm{K}$ is plaster of Paris, $\mathrm{CaSO_4 \cdot 0.5\,H_2O}$. Hence, the correct option is (A).

Quick Tip to Solve: Remember the hydration sequence: gypsum has 2 waters, plaster of Paris has half a water, and further heating gives anhydrous calcium sulphate.

68. The number of bonds between sulphur and oxygen atoms in $S_2O_8^{2-}$ and the number of bonds between sulphur and sulphur atoms in rhombic sulphur, respectively, are : [4 marks]

(A) 4 and 8

(B) 8 and 6

(C) 4 and 6

(D) 8 and 8

Correct Answer: (D) 8 and 8

Solution:

Concept:

- Peroxodisulphate, $S_2O_8^{2-}$, contains two sulphur centres joined through a peroxide $O-O$ link. Each sulphur is bonded to four oxygen atoms.
- Rhombic sulphur consists of puckered S_8 rings. Each sulphur atom has two sulphur neighbours, and each connection must be counted once.

Step 1: Count the sulphur-oxygen connections The skeleton of the peroxodisulphate ion is $O_3S-O-O-SO_3^{2-}$. On either side, the sulphur centre is attached to three terminal oxygen atoms and to one peroxide oxygen. Hence each sulphur contributes four sulphur-oxygen atom connections:

$$N_{S-O} = 2 \times 4 = 8.$$

The middle peroxide connection is oxygen-oxygen, so it is not an additional sulphur-oxygen bond.

Step 2: Count the bonds around the sulphur ring In an eight-membered closed ring, the sequence of sulphur atoms has eight links, including the link that closes the ring. Equivalently, counting two neighbours for each of eight sulphur atoms counts every link twice. Therefore

$$N_{S-S} = \frac{8 \times 2}{2} = 8.$$

The ring is puckered rather than planar, but puckering does not change the number of bonds.

Step 3: Check the alternatives The first count is eight atom-to-atom sulphur-oxygen connections (the sigma framework), not just the four terminal double bonds in a conventional Lewis drawing. Counting pi components separately is not the bond-connectivity count used by the choices. Options (A) and (C) incorrectly give four for the first count, and options (B) and (C) incorrectly give six links for an eight-membered sulphur ring.

Step 4: Conclusion The required numbers are 8 and 8, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Count each atom-to-atom connection once. A closed ring containing eight atoms has eight edges, irrespective of its puckered shape.

69. As per Hardy-Schulze formulation, the flocculation values of the following for ferric hydroxide sol are in the order : [4 marks]

(A) $AlCl_3 > K_3[Fe(CN)_6] > K_2CrO_4 > KBr = KNO_3$

(B) $K_3[Fe(CN)_6] < K_2CrO_4 < KBr = KNO_3 = AlCl_3$

(C) $K_3[Fe(CN)_6] < K_2CrO_4 < AlCl_3 < KBr < KNO_3$

(D) $K_3[Fe(CN)_6] > AlCl_3 > K_2CrO_4 > KBr > KNO_3$

Correct Answer: (B) $K_3[Fe(CN)_6] < K_2CrO_4 < KBr = KNO_3 = AlCl_3$

Solution:

Concept:

- Ferric hydroxide sol is positively charged. Its coagulation is controlled by oppositely charged ions, which are the anions of the added electrolytes.
- According to the Hardy-Schulze rule, higher counter-ion charge gives greater coagulating power. The flocculation value is the minimum concentration required for coagulation and varies in the opposite order.

Step 1: Identify the effective ions For the five electrolytes the effective counter-ions are $[Fe(CN)_6]^{3-}$ from potassium ferricyanide, $CrO_4{}^{2-}$ from potassium chromate, Br^- from potassium bromide, $NO_3{}^-$ from potassium nitrate, and Cl^- from aluminium chloride. In particular, Al^{3+} is a co-ion of the positive sol; its high positive charge does not make it the active coagulating ion.

Step 2: Order the flocculation values The counter-ion valencies are three, two, one, one and one. Thus the ferricyanide ion has the greatest coagulating power and needs the smallest flocculation concentration; chromate comes next. In the valency-based Hardy-Schulze formulation the three monovalent anions form one class. Consequently

$$K_3[Fe(CN)_6] < K_2CrO_4 < KBr = KNO_3 = AlCl_3.$$

This is the order of flocculation values, not of coagulating powers.

Step 3: Reject the misleading orders Options (A) and (D) fail the inverse relation by placing highly effective ferricyanide at a larger flocculation value. Option (C) distinguishes the three monovalent counter-ions, whereas the stated valency formulation groups them together. The comparison concerns counter-ion valency and does not rank salts by the charge on their cations.

Step 4: Conclusion The required flocculation-value order is ferricyanide $<$ chromate $<$ bromide $=$ nitrate $=$ aluminium chloride. Hence, the correct option is (B).

Quick Tip to Solve: First identify the sol charge, then choose the opposite ion. Reverse the order of coagulating power when the question asks for flocculation value.

70. Among the gases (a) – (e), the gases that cause greenhouse effect are :

(a) CO_2

(b) H_2O

(c) CFCs

(d) O_2

(e) O_3 [4 marks]

(A) (a), (b), (c) and (d)

(B) (a) and (d)

(C) (a), (b), (c) and (e)

(D) (a), (c), (d) and (e)

Correct Answer: (C) (a), (b), (c) and (e)

Solution:

Concept:

- The greenhouse effect results when atmospheric gases absorb outgoing terrestrial infrared radiation and re-emit part of that energy back towards the surface.
- Carbon dioxide, water vapour, chlorofluorocarbons and ozone have infrared-active absorptions. Ordinary molecular oxygen is not counted among greenhouse gases in this classification.

Step 1: Classify the first three gases Carbon dioxide has vibrations that absorb infrared radiation, so (a) is included. Water vapour is also a strong infrared absorber and makes an important natural contribution to the greenhouse effect, so (b) is included. Chlorofluorocarbons have infrared-active molecular vibrations and can absorb in atmospheric spectral windows, so (c) is included as well. A greenhouse gas need not be present in a high concentration to have this effect.

Step 2: Distinguish oxygen from ozone Molecular oxygen, O_2, is a homonuclear diatomic molecule. Its ordinary vibration does not produce the changing electric dipole needed for strong infrared absorption; hence (d) is excluded in the standard classification. Ozone, O_3, is a bent molecule with infrared-active vibrations, so (e) is included. Ozone can absorb ultraviolet light as well, but that additional role does not remove its greenhouse action.

Step 3: Match the complete set The included labels are therefore (a), (b), (c) and (e). Option (A) substitutes oxygen for ozone. Option (B) both includes oxygen and leaves out several greenhouse gases. Option (D) also includes oxygen and omits water vapour. Only the third combination contains all four required species while excluding O_2.

Step 4: Conclusion The greenhouse gases listed are CO_2, H_2O, CFCs and O_3. Hence, the correct option is (C).

Quick Tip to Solve: Do not confuse O_2 with O_3: ozone is infrared active and is a greenhouse gas, whereas ordinary oxygen is excluded from this list.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. What would be the electrode potential for the given half cell reaction at pH = 5? _____.

$2H_2O \rightarrow O_2 + 4H^{\oplus} + 4e^-$; E^0_{red} = 1.23 V

(R = 8.314 J mol^{-1} K^{-1} ; Temp = 298 k;

oxygen under std. atm. pressure of 1 bar) [4 marks]

Solution:

Concept:

- The Nernst equation for a reduction is $E_{\text{red}} = E^\circ_{\text{red}} - (RT/nF)\ln Q$, where Q is written for the reduction reaction and n is its electron count.
- Reversing a half-reaction reverses the sign of its electrode potential. A supplied standard reduction potential must be paired with the reduction direction before evaluating its reaction quotient.

Step 1: Write a consistent reduction reaction The displayed half-reaction evolves oxygen and electrons, so it is an oxidation. The stated $E^\circ_{\text{red}} = 1.23\,\text{V}$ refers to its reverse:

$$\text{O}_2 + 4\,\text{H}^+ + 4\,\text{e}^- \longrightarrow 2\,\text{H}_2\text{O}.$$

For this reduction, with pure liquid water and oxygen at unit standard pressure,

$$Q = \frac{1}{a^4_{\text{H}^+}}.$$

At pH 5 the hydrogen-ion activity is 10^{-5}, giving $Q = 10^{20}$.

Step 2: Apply the Nernst equation Using $n = 4$, $T = 298\,\text{K}$, $R = 8.314\,\text{J mol}^{-1}\text{K}^{-1}$ and $F \simeq 96485\,\text{C mol}^{-1}$,

$$\begin{aligned} E_{\text{red}} &= 1.23 - \frac{8.314 \times 298}{4 \times 96485}\ln(10^{20}) \\ &= 1.23 - 0.2956 \\ &= 0.9344\,\text{V}. \end{aligned}$$

The corresponding potential for the oxidation as displayed is the negative of this, $-0.9344\,\text{V}$.

Step 3: Explain the sign conflict with the supplied key The value near $1.525\,\text{V}$ results from adding the pH correction to $+1.23\,\text{V}$. That mixes the oxidation reaction quotient with a reduction standard potential. For the displayed oxidation one must instead use $E^\circ_{\text{ox}} = -1.23\,\text{V}$; its quotient is 10^{-20}, which gives $-1.23 + 0.2956 = -0.9344\,\text{V}$. Both consistent conventions thus agree on the physical electrode.

Step 4: Conclusion The reduction electrode potential at pH 5 is $\boxed{0.934\,\text{V}}$; for the oxidation exactly as written it is $\boxed{-0.934\,\text{V}}$. The supplied key range 1.52–1.53 V is incompatible with the printed $E^\circ_{\text{red}} = 1.23\,\text{V}$: it uses the oxidation quotient without reversing the standard-potential sign.

Quick Tip to Solve: For an oxygen electrode at unit oxygen pressure, remember $E_{\text{red}} = 1.23 - 0.0591\,\text{pH}$ at 298 K. Reverse the final sign only when reporting oxidation potential.

72. Ferrous sulphate heptahydrate is used to fortify foods with iron. The amount (in grams) of the salt required to achieve 10 ppm of iron in 100 kg of wheat is _____.

Atomic weight : Fe = 55.85; S = 32.00; O = 16.00 [4 marks]

Solution:

Concept:

- For a solid food, ppm by mass means milligrams of the specified substance per kilogram of food. Thus 10 ppm iron means $10\,\mathrm{mg}$ of elemental iron per kilogram of wheat.
- One mole of ferrous sulphate heptahydrate, $\mathrm{FeSO_4 \cdot 7\,H_2O}$, contains one mole of iron. Convert the required iron mass to salt mass using the ratio of their molar masses.

Step 1: Find the iron requirement The concentration refers to elemental iron, not to the whole hydrated salt. For $100\,\mathrm{kg}$ of wheat,

$$m_{\mathrm{Fe}} = 10\,\mathrm{mg\,kg^{-1}} \times 100\,\mathrm{kg} = 1000\,\mathrm{mg} = 1.000\,\mathrm{g}.$$

The same result follows from multiplying $100000\,\mathrm{g}$ by the mass fraction $10/10^6$. This unit conversion is essential because the desired answer is in grams of salt.

Step 2: Calculate the molar mass of the hydrate Include all seven waters of crystallisation. With the given atomic weights and hydrogen of atomic weight 1,

$$\begin{aligned} M(\mathrm{FeSO_4 \cdot 7\,H_2O}) &= 55.85 + 32.00 + 4(16.00) \\ &\quad + 7[2(1.00) + 16.00] \\ &= 277.85\,\mathrm{g\,mol^{-1}}. \end{aligned}$$

Of this mass, $55.85\,\mathrm{g}$ is iron. Water increases the amount of salt needed without increasing its iron content.

Step 3: Convert iron mass into salt mass The moles of salt required equal the moles of iron required, because every formula unit contains one iron atom. Hence

$$m_{\mathrm{salt}} = \frac{1.000}{55.85} \times 277.85 = 4.9749\,\mathrm{g}.$$

The iron mass fraction of the salt is $55.85/277.85$, about 0.201. Thus nearly five grams of hydrate are needed for one gram of iron, which also checks the scale of the answer. Rounding the calculated mass to two decimal places gives $4.97\,\mathrm{g}$, within the supplied accepted range. Using the molar mass of anhydrous ferrous sulphate instead would incorrectly omit the water mass and underestimate the required amount.

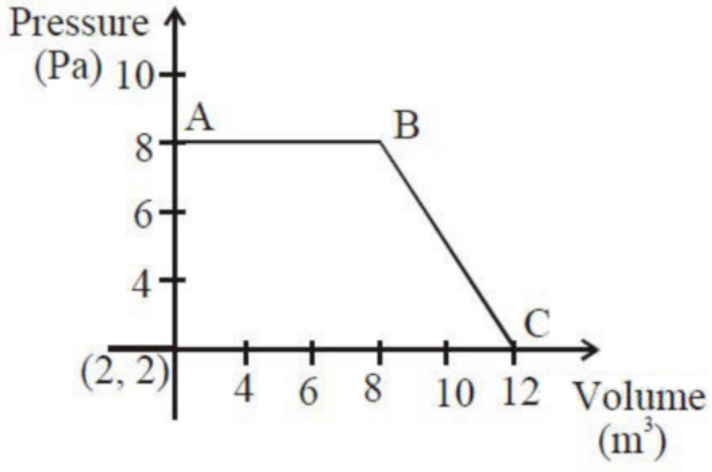
Step 4: Conclusion The required mass is

$$\boxed{m(\mathrm{FeSO_4 \cdot 7\,H_2O}) = 4.97\,\mathrm{g}}.$$

This is $4.9749\,\mathrm{g}$ rounded to two decimal places using the supplied atomic weights, and supplies $1.000\,\mathrm{g}$ of iron in $100\,\mathrm{kg}$ of wheat.

Quick Tip to Solve: Convert ppm to elemental iron mass first, then multiply by $M(\text{hydrated salt})/M(\mathrm{Fe})$. Do not forget the seven waters in the salt molar mass.

73. The magnitude of work done by a gas that undergoes a reversible expansion along the path ABC shown in the figure is _____. [4 marks]



Solution:

Concept:

- For reversible expansion, the work magnitude is $W = \int P\,dV$. Pressure is measured from zero, even when a graph uses a shifted horizontal baseline.
- A constant-pressure segment gives a rectangular area; a straight sloping segment gives a trapezium. In SI units, $1\,\text{Pa}\,\text{m}^3 = 1\,\text{J}$.

Step 1: Read the coordinates and baseline The displayed axes intersect at $(2, 2)$, so their horizontal baseline is at $P = 2\,\text{Pa}$. The states are $A = (2\,\text{m}^3, 8\,\text{Pa})$, $B = (8\,\text{m}^3, 8\,\text{Pa})$, and $C = (12\,\text{m}^3, 2\,\text{Pa})$. Volume increases on both segments, so both contributions to work done by the gas are positive. The axis intersection must not silently be treated as zero pressure.

Step 2: Integrate over the two segments AB is isobaric, whereas pressure decreases linearly on BC. Thus

$$W_{AB} = 8(8-2) = 48\,\text{J}, \qquad W_{BC} = \frac{8+2}{2}(12-8) = 20\,\text{J}.$$

Hence the total is $48 + 20 = 68\,\text{J}$. In the chemical sign convention work on the gas is negative, $w = -68\,\text{J}$, but its magnitude remains positive.

Step 3: Check the shifted-area interpretation The area between ABC and the displayed baseline is

$$(8-2)(8-2) + \tfrac{1}{2}(12-8)(8-2) = 36 + 12 = 48\,\text{J}.$$

This computes $\int (P-2)\,dV$, rather than the required $\int P\,dV$. The missing strip contributes $2(12-2) = 20\,\text{J}$. Restoring it gives $68\,\text{J}$. Therefore the visible enclosed area alone omits a nonzero pressure contribution.

Step 4: Conclusion The printed coordinates give $\boxed{68\,\text{J}}$. The answer key gives 48, the area above the shifted $P = 2\,\text{Pa}$ baseline; the literal work integral requires the additional $20\,\text{J}$ strip.

Quick Tip to Solve: Before using geometric area on a pressure-volume graph, check the baseline pressure. A nonzero baseline adds $P_0\Delta V$ to the visible area.

74. The volume (in mL) of 0.125 M $AgNO_3$ required to quantitatively precipitate chloride ions in 0.3 g of $[Co(NH_3)_6]Cl_3$ is _____.

$M_{[Co(NH_3)_6Cl_3]}$ = 267.46 g/mol

M_{AgNO_3} = 169.87 g/mol [4 marks]

Solution:

Concept:

- In $[Co(NH_3)_6]Cl_3$, all three chloride ions are outside the coordination sphere. Each formula unit releases three free Cl^- ions; the ammonia ligands remain coordinated.
- Precipitation follows $Ag^+ + Cl^- \longrightarrow AgCl(s)$, a 1:1 mole ratio. Molarity is $M = n/V$, with V in litres.

Step 1: Calculate moles of the coordination salt Use the supplied molar mass of the entire cobalt salt, rather than that of silver nitrate:

$$n_{\text{salt}} = \frac{0.300\,\text{g}}{267.46\,\text{g mol}^{-1}} = 1.12166 \times 10^{-3}\,\text{mol}.$$

The square brackets enclose cobalt and all six ammonia ligands. None of the three chloride ions is bonded inside the coordination sphere, so all three are available for precipitation.

Step 2: Find the silver nitrate requirement The dissociation and precipitation equations are

$$[Co(NH_3)_6]Cl_3 \longrightarrow [Co(NH_3)_6]^{3+} + 3\,Cl^-, \qquad Ag^+ + Cl^- \longrightarrow AgCl(s).$$

Consequently,

$$n_{AgNO_3} = n_{Cl^-} = 3n_{\text{salt}} = 3.36499 \times 10^{-3}\,\text{mol}.$$

The factor of three is essential: each mole of the cobalt salt requires three moles of silver nitrate for complete precipitation.

Step 3: Convert the amount into solution volume At $0.125\,\text{mol L}^{-1}$, the volume needed is

$$V = \frac{3.36499 \times 10^{-3}}{0.125}\,\text{L} = 0.0269199\,\text{L} = 26.9199\,\text{mL}.$$

The given molar mass of $AgNO_3$ would be useful for calculating its mass. It is unnecessary here because the solution concentration is already given as molarity, which directly relates moles and volume.

Step 4: Conclusion The required silver nitrate solution volume is $\boxed{26.92\,\text{mL}}$, approximately $26.9\,\text{mL}$.

Quick Tip to Solve: Count the chloride ions outside the square brackets, multiply the salt moles by that count, and divide by silver nitrate molarity before converting litres to millilitres.

75. The number of chiral centres in penicillin is _____. [4 marks]

Solution:

Concept:

- A tetrahedral carbon is a chiral centre if its four attached groups differ. For a ring carbon, compare the two ring paths atom by atom; two carbon neighbours need not be identical groups.
- The penicillin nucleus has fused beta-lactam and sulfur-containing rings. Count tetrahedral stereogenic carbons, excluding planar carbonyl atoms and carbons bearing identical substituents.

Step 1: Identify the two stereogenic beta-lactam carbons The two tetrahedral carbons of the beta-lactam framework, one at the fused-ring junction and the other carrying the acylamino substituent, each have four distinct attachments. Each has hydrogen and three bonds leading into chemically different environments. The ring-junction carbon connects to nitrogen, sulfur and the neighbouring carbon; the acylamino-bearing carbon connects to the side-chain nitrogen, the carbonyl carbon and the ring-junction carbon. They therefore give two chiral centres.

Step 2: Examine the carboxyl-bearing ring carbon The five-membered ring has another tetrahedral carbon carrying hydrogen and a carboxyl group. Its two remaining bonds enter unequal ring environments: one goes toward the ring nitrogen and beta-lactam portion, while the other goes toward the carbon bearing two methyl groups and then sulfur. These paths differ in connectivity and attached atoms. Together with hydrogen and carboxyl, they give four different groups, so this is the third centre.

Step 3: Exclude the other atoms The carbon carrying two methyl groups is not chiral because its methyl groups are identical. The carbonyl carbons in the beta-lactam, carboxyl and side-chain amide groups are trigonal planar, not tetrahedral. Methyl carbons carry repeated hydrogen substituents. The amide nitrogen is not an additional stable tetrahedral carbon centre. Thus exactly the three identified ring carbons contribute; the usual penicillin side chain adds no further centre.

Step 4: Conclusion The number of chiral centres in penicillin is $\boxed{3}$.

Quick Tip to Solve: Trace both ring paths before deciding a ring carbon is symmetric. First exclude carbonyl carbons and the carbon bearing two identical methyl groups.