JEE Main 2020 Paper 1 (B.E./B.Tech) Jan 8 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 8 January 2020, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then $10A^{-1}$ is equal to : [4 marks]

(A) 6I – A

(B) 4I – A

(C) A – 6I

(D) A – 4I

Correct Answer: (C) A – 6I

Solution:

Concept:

- The inverse of a square matrix exists when its determinant is nonzero. For a two-by-two matrix, the adjugate is formed by interchanging the diagonal entries and changing the signs of the off-diagonal entries.
- The Cayley–Hamilton relation for a two-by-two matrix is $A^2 - (\operatorname{tr} A)A + (\det A)I = 0$. Multiplying this identity by A^{-1} expresses the inverse using A and I.

Step 1: Find the trace and determinant The trace is the sum of the diagonal entries, whereas the determinant subtracts the product of the off-diagonal entries from the product of the diagonal entries. Thus

$$\operatorname{tr} A = 2 + 4 = 6, \qquad \det A = 2(4) - 2(9) = -10.$$

In particular, the determinant is nonzero, so multiplication by the inverse is valid.

Step 2: Use the characteristic identity The characteristic polynomial is $\lambda^2 - 6\lambda - 10$. Substituting the matrix into its characteristic equation gives

$$A^2 - 6A - 10I = 0.$$

Multiply every term by A^{-1}, remembering that $AA^{-1} = I$:

$$A - 6I - 10A^{-1} = 0.$$

Therefore $10A^{-1} = A - 6I$.

Step 3: Check the entries and the other options Direct inversion also gives

$$10A^{-1} = \begin{pmatrix} -4 & 2 \\ 9 & -2 \end{pmatrix} = A - 6I.$$

Option (A) reverses every sign of this matrix. Options (B) and (D) use 4 instead of the trace 6; their diagonal entries therefore disagree. Only option (C) matches both diagonal and off-diagonal entries.

Step 4: Conclusion

$$\boxed{10A^{-1} = A - 6I}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For a two-by-two matrix, compute trace and determinant first; Cayley–Hamilton often gives the requested inverse expression without full inversion.

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2. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is [4 marks]

(A) 3.98

(B) 3.99

(C) 4.01

(D) 4.02

Correct Answer: (B) 3.99

Solution:

Concept:

- For n observations, the population variance is $\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$, where $\bar{x} = \frac{\sum x_i}{n}$. Both sums must be corrected when one observation changes.
- Replacing an incorrect value u by v changes the sum by $v - u$ and the sum of squares by $v^2 - u^2$. The number of observations stays unchanged.

Step 1: Recover the original totals There are 20 observations and their initially calculated mean is 10. Their sum is consequently

$$\sum x_i = 20(10) = 200.$$

Rearranging the variance formula, using the initially calculated variance 4, gives

$$\sum x_i^2 = 20(4 + 10^2) = 2080.$$

These are the totals containing the incorrect observation 9.

Step 2: Correct the sum and the sum of squares Remove 9 and insert 11 in the ordinary sum, and remove 9^2 and insert 11^2 in the squared sum:

$$S = 200 - 9 + 11 = 202,$$

$$Q = 2080 - 81 + 121 = 2120.$$

Hence the correct mean is $202/20 = 10.1$. It would be inconsistent to retain the old mean after correcting the data.

Step 3: Calculate and compare the variance Apply the variance formula to the corrected totals:

$$\sigma^2_{\text{correct}} = \frac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99.$$

The correction increases both the second moment and the square of the mean, by 2 and 2.01 respectively. Their difference therefore decreases by 0.01. Thus 3.98, 4.01 and 4.02 in the other options do not equal the corrected variance.

Step 4: Conclusion The correct variance is $\boxed{3.99}$. Hence, the correct option is (B).

Quick Tip to Solve: When correcting one observation, update both the sum and the sum of squares before recalculating the mean and variance.

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3. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is : [4 marks]

(A) 0.20

(B) 0.02

(C) 0.01

(D) 0.10

Correct Answer: (D) 0.10

Solution:

Concept:

- The event $A \cup B$ means that at least one of the two events occurs. It includes the case in which both events occur.
- The event that exactly one occurs is the disjoint union of $A \setminus B$ and $B \setminus A$. The union separates into these two exclusive cases and $A \cap B$.

Step 1: Partition the event that at least one occurs An outcome in $A \cup B$ belongs to exactly one of three mutually exclusive categories: only A, only B, or both A and B. Adding the probabilities of the first two categories gives the probability of exactly one event. Therefore

$$P(A \cup B) = P(\text{exactly one}) + P(A \cap B).$$

This identity needs no assumption of independence or mutual exclusivity.

Step 2: Substitute the supplied probabilities The question gives $P(A \cup B) = 1/2$ and $P(\text{exactly one}) = 2/5$. Subtract the second probability from the first:

$$P(A \cap B) = \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10} = 0.10.$$

The result is nonnegative and smaller than the union probability, as required for a subset of the union.

Step 3: Check the alternatives Adding 0.10 back to the probability 0.40 of exactly one event recovers the required union probability 0.50. Option (A), 0.20, would instead give 0.60. Option (B), 0.02,

would give 0.42, and option (C), 0.01, would give 0.41. Each contradicts the stated probability of at least one event. No separate values of $P(A)$ or $P(B)$ are needed.

Step 4: Conclusion The probability that both occur together is $\boxed{0.10}$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the probability of exactly one event from the probability of at least one event to isolate the probability of both.

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4. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$.

If $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ and

$b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation : [4 marks]

(A) $x^2 + 101x + 100 = 0$

(B) $x^2 + 102x + 101 = 0$

(C) $x^2 - 102x + 101 = 0$

(D) $x^2 - 101x + 100 = 0$

Correct Answer: (C) $x^2 - 102x + 101 = 0$

Solution:

Concept:

- The number $\alpha = (-1+i\sqrt{3})/2$ is a nonreal cube root of unity. Hence $\alpha^3 = 1$ and $1 + \alpha + \alpha^2 = 0$.
- Powers of α repeat in cycles of three. For roots a, b, the monic quadratic equation is $x^2 - (a+b)x + ab = 0$.

Step 1: Evaluate the sum defining a There are 101 terms, because the summation includes both $k = 0$ and $k = 100$. The terms α^{2k} repeat as $1, \alpha^2, \alpha$, whose sum is zero. The first 99 terms form 33 complete cycles; the last two terms are $\alpha^{198} = 1$ and $\alpha^{200} = \alpha^2$. Thus

$$\sum_{k=0}^{100} \alpha^{2k} = 1 + \alpha^2 = -\alpha.$$

It follows that

$$a = (1+\alpha)(-\alpha) = -(\alpha + \alpha^2) = 1.$$

Step 2: Evaluate b and form the polynomial Every term α^{3k} equals $(\alpha^3)^k = 1$, including the initial term. Therefore

$$b = \sum_{k=0}^{100} 1 = 101.$$

The sum and product of the requested roots are

$$a + b = 102, \qquad ab = 101.$$

Substituting these into the monic quadratic gives

$$x^2 - 102x + 101 = 0.$$

Step 3: Check the signs and remaining choices The polynomial factors as $(x-1)(x-101)$, confirming both roots explicitly. Option (B) has the correct product but its positive middle coefficient gives a negative sum of roots. Options (A) and (D) have product 100 instead of 101, so neither can have the required roots. The inclusive summation endpoint is essential to obtaining 101 rather than 100.

Step 4: Conclusion The required equation is $\boxed{x^2 - 102x + 101 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: For powers of a cube root of unity, group terms into triples and then count the leftover terms, keeping both summation endpoints.

5. Let S be the set of all real roots of the equation,
$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S : [4 marks]

(A) contains exactly two elements.

(B) is an empty set.

(C) is a singleton.

(D) contains at least four elements.

Correct Answer: (C) is a singleton.

Solution:

Concept:

- Set $t = 3^x$. For real x, this substitution gives exactly $t > 0$, and each such t corresponds to one real $x = \log_3 t$.
- The signs inside $|t-1| + |t-2|$ change only at 1 and 2. Solve on the intervals $0 < t < 1$, $1 \le t \le 2$, and $t > 2$, checking that every candidate belongs to its interval.

Step 1: Solve below the first breakpoint The equation becomes $t^2 - t + 2 = |t-1| + |t-2|$. If $0 < t < 1$, the right side is $3 - 2t$, so

$$t^2 + t - 1 = 0, \qquad t = \frac{-1 \pm \sqrt{5}}{2}.$$

The positive root $(\sqrt{5}-1)/2$ lies strictly between 0 and 1. The other root is negative and cannot equal 3^x. Thus this interval contributes one solution.

Step 2: Check the middle interval For $1 \le t \le 2$, the right side is $(t-1) + (2-t) = 1$. Hence

$$t^2 - t + 1 = 0.$$

Its discriminant is $1 - 4 = -3 < 0$, so there are no real solutions in this interval, including its endpoints. Equivalently, the left side of this last equation is $(t - 1/2)^2 + 3/4 > 0$.

Step 3: Check the final interval and count roots For $t > 2$, the absolute values add to $2t - 3$, giving

$$t^2 - 3t + 5 = 0.$$

Its discriminant is $9 - 20 = -11 < 0$, so this interval contributes none. Consequently the original equation has exactly one real root,

$$x = \log_3 \left(\frac{\sqrt{5} - 1}{2} \right).$$

It is negative, consistently with $0 < 3^x < 1$. Thus the set is neither empty nor a two-element set, and it cannot contain four or more elements.

Step 4: Conclusion $S = \{\log_3 ((\sqrt{5} - 1)/2)\}$, so S is a singleton. Hence, the correct option is (C).

Quick Tip to Solve: Replace an exponential by a positive variable, split at every absolute-value breakpoint, and reject roots outside the interval used to derive them.

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6. If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 terms is [4 marks]

(A) 100

(B) $100\frac{1}{2}$

(C) $50\frac{1}{4}$

(D) 50

Correct Answer: (B) $100\frac{1}{2}$

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the nth term is $a_n = a + (n - 1)d$.
- The sum of its first n terms is $S_n = \frac{n}{2}[2a + (n - 1)d]$. The difference of two given terms determines d by dividing by the difference of their indices.

Step 1: Find the common difference The supplied terms give

$$a + 9d = \frac{1}{20}, \qquad a + 19d = \frac{1}{10}.$$

Subtract the first equation from the second. There are ten equal steps from the tenth term to the twentieth term, so

$$10d = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}, \qquad d = \frac{1}{200}.$$

Using only the difference of the terms without dividing by ten would give an incorrect common difference.

Step 2: Recover the first and last terms Insert $d = 1/200$ into the expression for the tenth term:

$$a = \frac{1}{20} - \frac{9}{200} = \frac{1}{200}.$$

Therefore $a_n = n/200$, and the two supplied terms are recovered as $10/200 = 1/20$ and $20/200 = 1/10$. The two-hundredth term is

$$a_{200} = a + 199d = \frac{200}{200} = 1.$$

Step 3: Sum the first two hundred terms The average of the first and last terms equals the average of all the terms in a finite arithmetic progression. Hence

$$S_{200} = \frac{200}{2}\left(\frac{1}{200} + 1\right) = \frac{201}{2} = 100\frac{1}{2}.$$

Option (A) omits the extra half contributed by the nonzero first term. Options (C) and (D) are about half the required total and do not use the correct number of terms. The direct formula $\sum_{n=1}^{200} n/200$ gives the same value.

Step 4: Conclusion The sum is $\boxed{100\frac{1}{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the equations for two known AP terms to find the common difference, then use the first and last terms to calculate the sum.

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7. Let S be the set of all functions f : [0,1] $\to$ R, which are continuous on [0,1] and differentiable on (0,1). Then for every f in S, there exists a c $\in$ (0,1), depending on f, such that [4 marks]

(A) $|f(c) - f(1)| < |f'(c)|$

(B) $|f(c) + f(1)| < (1 + c)\,|f'(c)|$

(C) $|f(c) - f(1)| < (1 - c)\,|f'(c)|$

(D) None

Correct Answer: (D) None

Solution:

Concept:

- A universal assertion about every admissible function is disproved by one function satisfying all its hypotheses. Constant functions are continuous on $[0, 1]$ and differentiable on $(0, 1)$.
- For a constant function $f(x) = k$, the derivative is $f\prime(x) = 0$ everywhere. A strict inequality $0 < 0$ is false; equality cannot be used to establish a strict inequality.

Step 1: Test an admissible constant function Take $f(x) = 1$ for every $x \in [0, 1]$. This is a member of S, so any proposed assertion that holds for every member must hold for this particular function at some interior point. For every $c \in (0, 1)$, we have

$$f(c) = f(1) = 1, \qquad f\prime(c) = 0.$$

There is no choice of the point that changes these values.

Step 2: Check all three proposed inequalities For option (A), the left side is $|1 - 1| = 0$, and the right side is zero, so it requires $0 < 0$, which is false. For option (B), the left side is $|1 + 1| = 2$, while

$(1+c)|f'(c)| = 0$, so it requires $2 < 0$. For option (C), the left side is again zero and the right side is $(1-c)\cdot 0 = 0$, which is also false. Thus each of (A), (B) and (C) fails for this single admissible function. The mean value theorem cannot turn equality into any of these strict inequalities.

Step 3: Conclusion None of the three proposed strict inequalities is guaranteed for every $f \in S$. Hence, the correct option is (D).

Quick Tip to Solve: Before applying a theorem to a universal strict inequality, test a constant function: its zero derivative often settles the question immediately.

8. The system of linear equations

λx + 2y + 2z = 5

2λx + 3y + 5z = 8

4x + λy + 6z = 10 has [4 marks]

(A) a unique solution when λ = –8

(B) no solution when $\lambda = 2$

(C) infinitely many solutions when $\lambda = 2$

(D) no solution when $\lambda = 8$

Correct Answer: (B) no solution when $\lambda = 2$

Solution:

Concept:

- A square linear system has a unique solution when its coefficient determinant is nonzero. A zero determinant alone does not distinguish no solution from infinitely many solutions.
- Elementary row combinations preserve the solution set. At a singular parameter, a contradiction between the transformed equations proves inconsistency.

Step 1: Find the singular parameter values The coefficient matrix and its determinant are

$$A = \begin{pmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{pmatrix}.$$

$$\begin{aligned} \det A &= \lambda(18-5\lambda) - 2(12\lambda - 20) + 2(2\lambda^2 - 12) \\ &= -\lambda^2 - 6\lambda + 16 \\ &= -(\lambda+8)(\lambda-2). \end{aligned}$$

Consequently, both $\lambda = -8$ and $\lambda = 2$ are singular values. In particular, a unique solution at -8, as claimed in (A), is impossible.

Step 2: Check consistency at the parameter in options B and C For $\lambda = 2$, the three equations are

$$2x + 2y + 2z = 5, \quad 4x + 3y + 5z = 8, \quad 4x + 2y + 6z = 10.$$

Subtract twice the first equation from the second and third respectively:

$$-y + z = -2, \qquad -2y + 2z = 0.$$

The second transformed equation requires $-y + z = 0$, contradicting the first. Thus there is no solution, rather than infinitely many solutions. This establishes (B) and excludes (C).

Step 3: Exclude the remaining option At $\lambda = 8$,

$$\det A = -(8 + 8)(8 - 2) = -96 \neq 0.$$

The system therefore has a unique solution at this value, so option (D), which asserts no solution, is false.

Step 4: Conclusion The equations are inconsistent when $\lambda = 2$. Hence, the correct option is (B).

Quick Tip to Solve: After finding a zero determinant, compare the right-hand sides under the same row operations; determinant zero by itself does not imply infinitely many solutions.

9. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$, then [4 marks]

(A) $\alpha + \beta = 60$

(B) $\alpha - \beta = 60$

(C) $\alpha + \beta = -30$

(D) $\alpha - \beta = -132$

Correct Answer: (D) $\alpha - \beta = -132$

Solution:

Concept:

- Adding $(x + y)^6$ and $(x - y)^6$ cancels every term containing an odd power of y. Only the terms with indices $0, 2, 4, 6$ remain.
- Here $y = \sqrt{x^2 - 1}$, so its even powers are polynomials: $y^2 = x^2 - 1$, $y^4 = (x^2 - 1)^2$ and $y^6 = (x^2 - 1)^3$. Coefficients must be collected after these substitutions.

Step 1: Cancel the odd radical terms Write $y = \sqrt{x^2 - 1}$. By the binomial theorem, the given sum is

$$\begin{aligned}(x + y)^6 + (x - y)^6 &= 2\left[x^6 + 15x^4y^2 + 15x^2y^4 + y^6\right] \\ &= 2\left[x^6 + 15x^4(x^2 - 1)\right. \\ &\qquad \left. + 15x^2(x^2 - 1)^2 + (x^2 - 1)^3\right].\end{aligned}$$

The factors 15 are $\binom{6}{2} = \binom{6}{4}$; the entire even-term sum is multiplied by two.

Step 2: Collect the required coefficients Expanding the bracket gives

$$\begin{aligned} & x^6 + 15x^6 - 15x^4 \\ & \quad + 15x^6 - 30x^4 + 15x^2 \\ & \quad + x^6 - 3x^4 + 3x^2 - 1 \\ & = 32x^6 - 48x^4 + 18x^2 - 1. \end{aligned}$$

Thus the complete expression is

$$64x^6 - 96x^4 + 36x^2 - 2.$$

It follows that $\alpha = -96$ and $\beta = 36$. Their sum is -60, which excludes both (A) and (C). Their difference is -132, rather than the positive value 60 stated in (B). Keeping the factor of two is essential when extracting both coefficients.

Step 3: Conclusion $\alpha - \beta = -96 - 36 = -132$. Hence, the correct option is (D).

Quick Tip to Solve: For conjugate binomial sums, cancel odd powers first and substitute the square of the radical before collecting coefficients.

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10. $\lim\limits_{x \to 0} \frac{\int_0^x t \sin(10t) dt}{x}$ **is equal to** [4 marks]

(A) $-\frac{1}{5}$

(B) $-\frac{1}{10}$

(C) 0

(D) $\frac{1}{10}$

Correct Answer: (C) 0

Solution:

Concept:

- If h is continuous near zero, then $F(x) = \int_0^x h(t)\, dt$ is differentiable and $F\prime(x) = h(x)$.
- Since $F(0) = 0$, the quotient $F(x)/x$ tends to $F\prime(0)$. This is exactly the definition of the derivative, so no explicit antiderivative is necessary.

Step 1: Recognise a derivative at zero Define

$$F(x) = \int_0^x t \sin(10t)\, dt.$$

The integrand is continuous on the real line, and the integral with equal limits gives $F(0) = 0$. Hence the requested two-sided limit is

$$\lim_{x \to 0} \frac{F(x) - F(0)}{x - 0} = F\prime(0).$$

This observation applies from both sides of zero, including negative upper limits.

Step 2: Apply the fundamental theorem Differentiating the integral with respect to its upper endpoint

yields

$$F\prime(x) = x\sin(10x).$$

Therefore

$$F\prime(0) = 0\sin 0 = 0.$$

The number 10 is the multiplier inside the sine; there is no extra chain-rule factor in differentiating this integral, because the theorem evaluates the integrand at the upper limit.

Step 3: Check the small-variable behaviour For an independent check, $\sin(10t) = 10t + O(t^3)$ near zero. Thus $t\sin(10t) = 10t^2 + O(t^4)$, and integration gives $F(x) = \frac{10}{3}x^3 + O(x^5)$. Dividing by x leaves a quantity tending to zero. Consequently the nonzero constants $-1/5$, $-1/10$ and $1/10$ in the other options cannot be the limit.

Step 4: Conclusion The required limit is 0. Hence, the correct option is (C).

Quick Tip to Solve: When an integral from zero to x is divided by x, first interpret the quotient as a derivative at zero and evaluate the continuous integrand there.

11. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c}.\vec{a} = 0$, then $\vec{c}.\vec{b}$ is equal to [4 marks]

(A) $-\frac{1}{2}$

(B) $-\frac{3}{2}$

(C) $\frac{1}{2}$

(D) -1

Correct Answer: (A) $-\frac{1}{2}$

Solution:

Concept:

- If $\mathbf{b} \neq \mathbf{0}$ and $\mathbf{b} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{v}$ is parallel to $\mathbf{b}$, so $\mathbf{v} = t\mathbf{b}$ for a scalar t.
- The dot product is bilinear. The perpendicularity condition $\mathbf{c} \cdot \mathbf{a} = 0$ determines the scalar left undetermined by the cross-product condition.

Step 1: Use the equality of cross products Subtract the two cross products to obtain

$$\mathbf{b} \times (\mathbf{c} - \mathbf{a}) = \mathbf{0}.$$

Here $\mathbf{b} = (1, -1, 1) \neq \mathbf{0}$. Therefore there exists a real number t such that

$$\mathbf{c} = \mathbf{a} + t\mathbf{b}.$$

This form automatically satisfies the cross-product condition for every t, because $\mathbf{b} \times \mathbf{b} = \mathbf{0}$.

Step 2: Impose the orthogonality condition The necessary dot products are

$$\mathbf{a} \cdot \mathbf{a} = 1 + 4 + 1 = 6,$$

$$\mathbf{a}\cdot\mathbf{b}=1+2+1=4, \qquad \mathbf{b}\cdot\mathbf{b}=3.$$

Consequently

$$0=(\mathbf{a}+t\mathbf{b})\cdot\mathbf{a}=6+4t, \qquad t=-\frac{3}{2}.$$

This negative scalar is a coefficient of $\mathbf{b}$; it is not itself the dot product asked for.

Step 3: Evaluate the requested dot product Using the same representation,

$$\mathbf{c}\cdot\mathbf{b}=\mathbf{a}\cdot\mathbf{b}+t(\mathbf{b}\cdot\mathbf{b})=4-\frac{3}{2}(3)=-\frac{1}{2}.$$

Explicitly, $\mathbf{c}=(-1/2,-1/2,-1/2)$, whose dot product with $\mathbf{a}$ is zero and with $\mathbf{b}$ is $-1/2$. Thus $-3/2$ in option (B) confuses the scalar parameter with the required product, and the values in (C) and (D) fail the direct dot-product check.

Step 4: Conclusion $\mathbf{c}\cdot\mathbf{b}=-\frac{1}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Equal cross products with the same nonzero vector mean the two vectors differ by a multiple of that vector; use the dot-product constraint to find the multiple.

12. Let f : (1, 3) → R be a function defined by

$f(x)=\frac{x[x]}{1+x^2}$ **, where [x] denotes the greatest integer ≤ x. Then the range of f is** [4 marks]

(A) $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{4},\frac{4}{5}\right]$

(B) $\left(\frac{3}{5},\frac{4}{5}\right)$

(C) $\left(\frac{2}{5},\frac{4}{5}\right]$

(D) $\left(\frac{2}{5},\frac{3}{5}\right]\cup\left(\frac{3}{4},\frac{4}{5}\right)$

Correct Answer: No listed option equals (2/5,1/2) union (3/5,4/5].

Solution:

Concept:

- The greatest-integer function is constant on each interval between consecutive integers. On $(1,2)$, $\lfloor x\rfloor=1$, whereas on $[2,3)$, $\lfloor x\rfloor=2$.
- For a positive constant k, the derivative of $kx/(1+x^2)$ is $k(1-x^2)/(1+x^2)^2$, which is negative for $x>1$. Each branch is therefore strictly decreasing.

Step 1: Find the range below the jump For $1<x<2$,

$$f(x)=\frac{x}{1+x^2}, \qquad f\prime(x)=\frac{1-x^2}{(1+x^2)^2}<0.$$

The endpoint limits are $1/2$ as $x\to1^+$ and $2/5$ as $x\to2^-$. Neither endpoint belongs to this branch, so its range is

$$\left(\frac{2}{5},\frac{1}{2}\right).$$

At $x=2$, the greatest integer changes; the preceding formula must not be extended to that point.

Step 2: Find the range on the second branch For $2 \le x < 3$,

$$f(x) = \frac{2x}{1+x^2}, \qquad f\prime(x) = \frac{2(1-x^2)}{(1+x^2)^2} < 0.$$

The included left endpoint gives $f(2) = 4/5$, and the excluded right endpoint has limit $\lim_{x\to 3^-} f(x) = 6/10 = 3/5$. Therefore this branch has range $(3/5, 4/5]$. Combining the two branches gives

$$\text{Range}(f) = \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right].$$

Step 3: Compare every printed option Option (A) instead begins its second interval at $3/4$, so it omits attainable values such as $f(5/2) = 20/29$. Option (B) omits the entire first branch and excludes the attained value $4/5$. Option (C) includes the unattainable gap between $1/2$ and $3/5$. Option (D) also includes that gap and incorrectly excludes $4/5$. Thus none equals the range proved directly from the given function.

Step 4: Conclusion The range is $(2/5, 1/2) \cup (3/5, 4/5]$. No printed option is correct: in particular, option (A) has $3/4$ where the branch calculation gives $3/5$. A different keyed option cannot override these endpoint values.

Quick Tip to Solve: Split the domain at every integer first. Keep the integer endpoint with the branch using its new floor value, then track open and closed endpoints separately.

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13. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then : [4 marks]

(A) $\frac{1}{16} < I^2 < \frac{1}{9}$
(B) $\frac{1}{8} < I^2 < \frac{1}{4}$
(C) $\frac{1}{9} < I^2 < \frac{1}{8}$
(D) $\frac{1}{6} < I^2 < \frac{1}{2}$

Correct Answer: (C) $\frac{1}{9} < I^2 < \frac{1}{8}$

Solution:

Concept:

- For a positive integrand, pointwise bounds give integral bounds: if $m \le g(x) \le M$ on $[a, b]$, then $m(b-a) \le \int_a^b g(x)\,dx \le M(b-a)$.
- The sign of the derivative locates the extreme values of the cubic inside the square root. Since the square root is positive, reciprocation reverses its inequalities.

Step 1: Find the range of the cubic Put $f(x) = 2x^3 - 9x^2 + 12x + 4$. Differentiate and factor:

$$f'(x) = 6x^2 - 18x + 12 = 6(x-1)(x-2).$$

For $1 < x < 2$, the first factor is positive and the second is negative, so the cubic is strictly decreasing.

Its endpoint values are

$$f(1) = 2 - 9 + 12 + 4 = 9, \qquad f(2) = 16 - 36 + 24 + 4 = 8.$$

Thus $8 < f(x) < 9$ throughout the interior of the interval. In particular the radicand is positive everywhere needed.

Step 2: Integrate the reciprocal bounds Taking square roots and reciprocals gives

$$\frac{1}{3} < \frac{1}{\sqrt{f(x)}} < \frac{1}{\sqrt{8}} \qquad (1 < x < 2).$$

Integrate from 1 to 2. Although equality holds at individual endpoints, each strict inequality holds throughout the interior, so the integral inequalities are strict:

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}}.$$

Both sides and I are positive, hence squaring preserves the order:

$$\frac{1}{9} < I^2 < \frac{1}{8}.$$

Option A places the value below $1/9$, while B places it above $1/8$. Option D also fails, since $1/6 > 1/8$.

Step 3: Conclusion Therefore $\frac{1}{9} < I^2 < \frac{1}{8}$. Hence, the correct option is (C).

Quick Tip to Solve: For an integral asking only for a range, bound the integrand using the derivative of its radicand; an explicit antiderivative is unnecessary.

14. The area (in sq. units) of the region
$\{(x,y) \in R^2 : x^2 \le y \le 3 - 2x\}$, is : [4 marks]

(A) $\frac{34}{3}$
(B) $\frac{29}{3}$
(C) $\frac{31}{3}$
(D) $\frac{32}{3}$

Correct Answer: (D) $\frac{32}{3}$

Solution:

Concept:

- For vertical strips bounded below by $y = f(x)$ and above by $y = g(x)$, the area is $\int_a^b [g(x) - f(x)]\,dx$, where a, b are intersection abscissae.
- The inequality $x^2 \le y \le 3 - 2x$ directly identifies the parabola as the lower boundary and the line as the upper boundary wherever the region exists.

Step 1: Locate the horizontal extent of the region A vertical strip exists precisely when its proposed

upper ordinate is at least its lower ordinate. Solve the boundary equation:

$$x^2 = 3 - 2x \quad \Longleftrightarrow \quad x^2 + 2x - 3 = 0.$$

Factoring gives

$$(x+3)(x-1) = 0, \qquad x = -3,\ 1.$$

The quadratic is nonpositive between these roots, so the region extends from $x = -3$ to $x = 1$. For instance, at $x = 0$ the line has height 3 and the parabola height 0, confirming the order of the boundaries. Outside this interval there are no points satisfying both inequalities.

Step 2: Integrate the strip height The required area is

$$A = \int_{-3}^{1} (3 - 2x - x^2)\, dx.$$

An antiderivative is $3x - x^2 - x^3/3$. Evaluate both limits, retaining the signs of the negative lower limit:

$$A = \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1}.$$

$$A = \left(3 - 1 - \frac{1}{3}\right) - (-9 - 9 + 9) = \frac{5}{3} + 9 = \frac{32}{3}.$$

The integrand is nonnegative over the entire integration interval, so this integral is the area without any further sign changes. The other offered values $34/3, 29/3, 31/3$ do not equal the evaluated integral.

Step 3: Conclusion The area is $\frac{32}{3}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Find the intersections first and integrate upper curve minus lower curve. At a negative lower limit, substitute every power before subtracting the bracket.

15. The mirror image of the point (1, 2, 3) in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane ? [4 marks]

(A) (1, –1, 1)

(B) (–1, –1, –1)

(C) (–1, –1, 1)

(D) (1, 1, 1)

Correct Answer: (A) (1, –1, 1)

Solution:

Concept:

- If P and P' are mirror images across a plane, the plane passes through their midpoint and its normal is parallel to $\overrightarrow{PP'}$.
- A plane with normal (a, b, c) through (x_0, y_0, z_0) has equation $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

A point lies on it exactly when it satisfies this equation.

Step 1: Calculate a normal and a point on the plane Write the given point and its image as

$$P = (1, 2, 3), \qquad P' = \left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right).$$

Their joining vector is

$$\overrightarrow{PP'} = \left(-\frac{10}{3}, -\frac{10}{3}, -\frac{10}{3}\right).$$

All three components are equal, so we may use the simpler parallel normal $(1, 1, 1)$. The midpoint is

$$M = \left(\frac{1 - 7/3}{2}, \frac{2 - 4/3}{2}, \frac{3 - 1/3}{2}\right) = \left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right).$$

Reflection makes this midpoint the perpendicular foot from either point to the reflecting plane.

Step 2: Form the plane and test the choices The point-normal equation through M is

$$\left(x + \frac{2}{3}\right) + \left(y - \frac{1}{3}\right) + \left(z - \frac{4}{3}\right) = 0, \qquad x + y + z = 1.$$

For the choices, compute the sum of coordinates. Option A gives $1 - 1 + 1 = 1$, so it lies on the plane. Option B gives -3, option C gives -1, and option D gives 3; none satisfies the required sum. As a consistency check, the coordinate sums of P and P' are 6 and -4, respectively, lying equally far on opposite sides of the value 1.

Step 3: Conclusion The plane is $x + y + z = 1$, and the point $(1, -1, 1)$ lies on it. Hence, the correct option is (A).

Quick Tip to Solve: For a reflection plane, use the midpoint for its position and the difference of the two point vectors for its normal; then substitute each candidate point.

16. The differential equation of the family of curves, $x^2 = 4b(y + b)$, $b \in R$, is : [4 marks]

(A) $x(y')^2 = x - 2yy'$

(B) $x(y')^2 = 2yy' - x$

(C) $xy'' = y'$

(D) $x(y')^2 = x + 2yy'$

Correct Answer: (D) $x(y')^2 = x + 2yy'$

Solution:

Concept:

- To form the differential equation of a one-parameter family, differentiate once with the parameter held constant and eliminate that parameter using the original equation.
- Write $p = y' = dy/dx$. The resulting first-order relation must retain both the original family constraint and the differentiated relation.

Step 1: Differentiate with the parameter fixed The family is

$$x^2 = 4b(y + b),$$

where b is constant along any one curve. Differentiating with respect to x gives

$$2x = 4bp, \qquad x = 2bp.$$

For nonzero x on a nondegenerate graph of the family this gives $b = x/(2p)$. Instead of dividing immediately, we can eliminate b by multiplication, which makes the algebra transparent and avoids an unnecessary division by the slope.

Step 2: Eliminate the parameter Multiply the original equation by p^2 and substitute $2bp = x$:

$$x^2p^2 = 4byp^2 + 4b^2p^2 = 2xyp + x^2.$$

On the ordinary graph portions, cancel x to obtain

$$xp^2 = x + 2yp.$$

The resulting identity also holds at any regular point with $x = 0$ by continuity. Restoring the derivative notation yields

$$x(y')^2 = x + 2yy'.$$

The signs in A and B disagree with this elimination. Option C is only a second-order consequence and admits an extra independent constant: integrating $xy'' = y'$ gives $y = Cx^2 + D$, with no required relation between C and D. The original family has linked coefficients $y = x^2/(4b) - b$, so that broader second-order family is insufficient.

Step 3: Conclusion The differential equation is $x(y')^2 = x + 2yy'$. Hence, the correct option is (D).

Quick Tip to Solve: Keep a family parameter constant while differentiating. With one arbitrary parameter, eliminate it after one differentiation and check signs against the original equation.

17. Which of the following statements is a tautology? [4 marks]

(A) $\sim(p \wedge \sim q) \rightarrow p \vee q$

(B) $\sim(p \vee \sim q) \rightarrow p \vee q$

(C) $\sim(p \vee \sim q) \rightarrow p \wedge q$

(D) $p \vee (\sim q) \rightarrow p \wedge q$

Correct Answer: (B) $\sim(p \vee \sim q) \rightarrow p \vee q$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its component propositions. The implication $A \rightarrow B$ is equivalent to $\neg A \vee B$, and is false only when A is true and B is false.

- De Morgan's laws give $\neg(p \vee \neg q) = \neg p \wedge q$. Also $q \vee \neg q$ is always true.

Step 1: Simplify and prove the second statement In option B take $A = \neg(p \vee \neg q)$ and $B = p \vee q$. Replacing the implication by its equivalent disjunction gives

$$\neg[\neg(p \vee \neg q)] \vee (p \vee q) = (p \vee \neg q) \vee (p \vee q).$$

Associativity and idempotence reduce this to

$$p \vee (\neg q \vee q) = p \vee \text{True} = \text{True}.$$

Thus every assignment of p and q makes option B true. Equivalently, whenever its antecedent $\neg p \wedge q$ is true, q is true and so its consequent $p \vee q$ must be true.

Step 2: Reject the remaining statements with counterexamples A single false assignment suffices to show that a statement is not a tautology. For option A, let p and q both be false. Then $p \wedge \neg q$ is false, its negation is true, and $p \vee q$ is false. The resulting implication is false. For option C, let p be false and q true. Its antecedent $\neg(p \vee \neg q)$ is true but its consequent $p \wedge q$ is false. For option D, let p be true and q false. Then $p \vee \neg q$ is true while $p \wedge q$ is false. These explicit failures reject A, C and D without needing a complete truth table.

Step 3: Conclusion Only $\neg(p \vee \neg q) \rightarrow (p \vee q)$ is true for all truth assignments. Hence, the correct option is (B).

Quick Tip to Solve: Replace an implication by negated antecedent OR consequent. For rejecting a candidate, search for an assignment making its antecedent true and its consequent false.

18. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2,2) is [4 marks]

(A) $\sqrt{2}$

(B) $4\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

Correct Answer: (D) $2\sqrt{2}$

Solution:

Concept:

- Implicit differentiation treats y as a function of x; the product rule gives $d(xy)/dx = y + xy'$. The normal is perpendicular to the tangent, so its slope is $-1/y'$ when the tangent slope is nonzero and finite.
- The perpendicular distance from (x_0, y_0) to $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$.

Step 1: Find the tangent and normal slopes Differentiate the equation

$$x^2 + 2xy - 3y^2 = 0$$

to obtain

$$2x + 2y + 2xy' - 6yy' = 0.$$

Collect the derivative terms and solve:

$$(2x - 6y)y' = -2x - 2y, \qquad y' = \frac{x + y}{3y - x}.$$

At $(2, 2)$, the denominator is $6 - 2 = 4$, so the tangent slope is $4/4 = 1$. Hence the normal slope is -1. The point does lie on the original curve, since $4 + 8 - 12 = 0$, so the requested normal is well defined at this point.

Step 2: Form the normal and calculate the distance The normal passes through $(2, 2)$ with slope -1, giving

$$y - 2 = -(x - 2), \qquad x + y - 4 = 0.$$

Its distance from the origin is therefore

$$d = \frac{|1(0) + 1(0) - 4|}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

The perpendicular foot is $(2, 2)$: it lies on the normal and the segment from the origin has slope 1, perpendicular to that normal. Its length $\sqrt{2^2 + 2^2}$ independently confirms the result. Thus $\sqrt{2}$, $4\sqrt{2}$, and 2 in the other options are not the required distance.

Step 3: Conclusion The perpendicular distance is $2\sqrt{2}$ units. Hence, the correct option is (D).

Quick Tip to Solve: Find the normal equation before using the point-to-line distance formula. Using the tangent equation instead would give the distance to the wrong line.

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19. If a line, y = mx + c is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then : [4 marks]

(A) $c^2 + 6c + 7 = 0$

(B) $c^2 - 7c + 6 = 0$

(C) $c^2 - 6c + 7 = 0$

(D) $c^2 + 7c + 6 = 0$

Correct Answer: (A) $c^2 + 6c + 7 = 0$

Solution:

Concept:

- The tangent to the unit circle at (x_1, y_1) has equation $xx_1 + yy_1 = 1$; its radius is perpendicular to the tangent.
- A line $Ax + By + C = 0$ touches a circle of centre (h, k) and radius r exactly when $|Ah + Bk + C|/\sqrt{A^2 + B^2} = r$.

Step 1: Find the slope of the required tangent At the given point, the tangent to the unit circle is

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 1, \qquad x + y = \sqrt{2}.$$

Its slope is -1. The required line is perpendicular to it, so its slope satisfies $m(-1) = -1$, giving $m = 1$. Thus its equation is $y = x + c$, or $x - y + c = 0$.

Step 2: Apply the centre-to-line distance condition The second circle has centre $(3, 0)$ and radius 1. Therefore the perpendicular distance from its centre to the line must equal one:

$$\frac{|3 + c|}{\sqrt{1^2 + (-1)^2}} = 1.$$

Squaring gives

$$(c + 3)^2 = 2 \quad \Longrightarrow \quad c^2 + 6c + 7 = 0.$$

The two tangents have intercepts $-3 + \sqrt{2}$ and $-3 - \sqrt{2}$, both of which obey this quadratic.

Step 3: Check the alternatives Option (A) is exactly the quadratic obtained from the tangency condition. Option (C) reverses the sign of the linear coefficient and would correspond to a centre on the opposite side. Options (B) and (D) have constant term 6, whereas subtracting the squared distance term gives $9 - 2 = 7$; hence neither can describe these intercepts.

Step 4: Conclusion The required condition is $\boxed{c^2 + 6c + 7 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the slope from the first circle before using the distance formula on the second; retain the absolute value until squaring.

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20. If a hyperbola passes through the point P(10, 16) and it has vertices at ($\pm$ 6, 0), then the equation of the normal to it at P is : [4 marks]

(A) 2x + 5y = 100

(B) x + 3y = 58

(C) x + 2y = 42

(D) 3x + 4y = 94

Correct Answer: (A) 2x + 5y = 100

Solution:

Concept:

- A hyperbola with vertices $(\pm a, 0)$, centred at the origin with coordinate axes as its axes, has standard form $x^2/a^2 - y^2/b^2 = 1$.
- Implicit differentiation gives tangent slope $dy/dx = b^2x/(a^2y)$. The normal slope is its negative reciprocal at the specified point.

Step 1: Determine the hyperbola parameter The vertices give $a = 6$, so $a^2 = 36$. Since $P(10, 16)$ lies

on the hyperbola,

$$\frac{100}{36} - \frac{256}{b^2} = 1.$$

Consequently

$$\frac{256}{b^2} = \frac{64}{36} = \frac{16}{9}, \qquad b^2 = 144.$$

Hence the required standard hyperbola is $x^2/36 - y^2/144 = 1$.

Step 2: Find the slope and write the normal Differentiate the equation with respect to x:

$$\frac{2x}{36} - \frac{2y}{144}\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = \frac{4x}{y}.$$

At $(10, 16)$, the tangent slope is $40/16 = 5/2$, so the normal slope is $-2/5$. The point-slope equation becomes

$$y - 16 = -\frac{2}{5}(x - 10), \qquad 2x + 5y = 100.$$

Step 3: Compare all listed lines All four alternatives pass through P, so substitution of the point alone does not identify the normal. Their respective slopes are $-2/5, -1/3, -1/2, -3/4$. Only the first is the negative reciprocal of $5/2$. The other three therefore fail the perpendicularity condition.

Step 4: Conclusion The normal is $\boxed{2x + 5y = 100}$. Hence, the correct option is (A).

Quick Tip to Solve: When every offered line passes through the given point, compare slopes: the normal must have slope equal to the negative reciprocal of the derivative.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let f(x) be a polynomial of degree 3 such that f(–1) = 10, f(1) = –6, f(x) has a critical point at x = –1 and f'(x) has a critical point at x = 1. Then f(x) has a local minima at x = _____. [4 marks]

Solution:

Concept:

- For a differentiable polynomial, a critical point of f satisfies $f'(x) = 0$. A critical point of f' satisfies $f''(x) = 0$.
- A stationary point with positive second derivative is a strict local minimum. The prescribed function values determine the sign and scale of the cubic.

Step 1: Translate both critical-point conditions Write $f(x) = ax^3 + bx^2 + cx + d$, with $a \neq 0$. Then

$$f'(x) = 3ax^2 + 2bx + c, \qquad f''(x) = 6ax + 2b.$$

Since f' has a critical point at 1, $6a + 2b = 0$, giving $b = -3a$. Also $f'(-1) = 0$, so $3a - 2b + c = 0$, giving $c = -9a$. Thus $f'(x) = 3a(x + 1)(x - 3)$.

Step 2: Use the given values to determine the sign The value conditions now read

$$f(-1) = 5a + d = 10, \qquad f(1) = -11a + d = -6.$$

Subtracting the second from the first gives $16a = 16$, hence $a = 1$ and $d = 5$. In particular the leading coefficient is positive; the classification cannot be decided from the two derivative roots without this sign information.

Step 3: Classify both stationary points We have $f''(x) = 6x - 6$. At $x = -1$, $f''(-1) = -12 < 0$, so this point is a local maximum. At $x = 3$, $f''(3) = 12 > 0$, so this point is the local minimum. Equivalently, $3(x+1)(x-3)$ changes from negative to positive as x crosses 3.

Step 4: Conclusion The local minimum occurs at $\boxed{x = 3}$. The required numerical entry is 3 (dimensionless).

Quick Tip to Solve: Distinguish carefully between a critical point of the function and one of its derivative: the latter fixes where the second derivative vanishes.

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22. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$
$\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$ then tan(α + 2β) is equal to _____. [4 marks]

Solution:

Concept:

- The double-angle identities are $1 + \cos 2t = 2\cos^2 t$ and $1 - \cos 2t = 2\sin^2 t$. Square roots initially give absolute values.
- For acute angles, sine and cosine are positive. Use $\tan 2t = 2\tan t/(1 - \tan^2 t)$ and the tangent addition formula after finding the individual ratios.

Step 1: Simplify the condition on alpha Because $0 < \alpha < \pi/2$, its cosine is positive and

$$\sqrt{1 + \cos 2\alpha} = \sqrt{2\cos^2\alpha} = \sqrt{2}\cos\alpha.$$

The first equation therefore becomes

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \tan\alpha = \frac{1}{7}.$$

The interval condition justifies removing the absolute value; it is essential to this simplification.

Step 2: Find the double-angle tangent for beta The other square root is $\sqrt{\sin^2\beta} = \sin\beta$, since β is acute. Hence

$$\sin\beta = \frac{1}{\sqrt{10}}, \qquad \cos\beta = \sqrt{1 - \frac{1}{10}} = \frac{3}{\sqrt{10}}, \qquad \tan\beta = \frac{1}{3}.$$

It follows that

$$\tan 2\beta = \frac{2/3}{1 - 1/9} = \frac{3}{4}.$$

Step 3: Apply the addition formula Now combine the two ratios directly:

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7}\frac{3}{4}} = \frac{25/28}{25/28} = 1.$$

The denominator is $25/28$, which is nonzero, so the tangent is well-defined. No angle evaluation or decimal approximation is necessary.

Step 4: Conclusion $\boxed{\tan(\alpha + 2\beta) = 1}$. The required numerical entry is 1 (dimensionless).

Quick Tip to Solve: Use the acute-angle information before simplifying square roots of squares; then work with rational tangent values instead of calculating angles.

23. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _____. [4 marks]

Solution:

Concept:

- EXAMINATION has eight distinct letter types: E, X, A, M, I, N, T and O. The letters A, I and N occur twice each; the other five occur once.
- Count distinct four-letter arrangements by repetition pattern. Equal copies of a letter are indistinguishable, so a selected repeated pair contributes a divisor of $2!$.

Step 1: Count words with four different letters Choose four different types from the eight available and arrange them in all possible orders. This gives

$$N_{1111} = \binom{8}{4} 4! = 70 \times 24 = 1680.$$

Selecting the letter types first avoids treating the two physical copies of A, I or N as different choices when only one copy is used.

Step 2: Count words containing exactly one pair The paired type must be A, I or N, giving three choices. Choose two different additional types from the remaining seven; each is used only once. The arrangement count for a chosen set is $4!/2! = 12$. Therefore

$$N_{211} = 3\binom{7}{2}\frac{4!}{2!} = 3 \times 21 \times 12 = 756.$$

This category excludes a second pair by explicitly choosing two distinct single types.

Step 3: Count two pairs and combine the cases Choose two of the three repeated types and arrange their two copies each:

$$N_{22} = \binom{3}{2}\frac{4!}{2!2!} = 3 \times 6 = 18.$$

A triple or a quadruple of one letter is impossible because no type occurs more than twice. Thus these three repetition patterns are disjoint and exhaustive. Adding them yields

$$N = 1680 + 756 + 18 = 2454.$$

This counts words whether meaningful or not, as required.

Step 4: Conclusion The number of distinct four-letter words is $\boxed{2454}$. This is a count of words, with

no physical unit.

Quick Tip to Solve: For short words drawn from a repeated-letter pool, classify by multiplicities first: here only four singles, one pair plus two singles, and two pairs are possible.

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24. The sum, $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to _____. [4 marks]

Solution:

Concept:

- Expand the summand into powers of the index: $n(n+1)(2n+1) = 2n^3 + 3n^2 + n$. Linearity lets us sum these three terms separately.
- The standard identities are $\sum_{n=1}^{N} n = N(N+1)/2$, $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$, and $\sum_{n=1}^{N} n^3 = [N(N+1)/2]^2$.

Step 1: Rewrite the sum in standard form Let the required sum be S. Keeping the denominator 4 outside the summation and expanding the numerator gives

$$S = \frac{1}{4}\sum_{n=1}^{7}(2n^3 + 3n^2 + n).$$

By linearity, this is

$$S = \frac{1}{4}\left(2\sum_{n=1}^{7} n^3 + 3\sum_{n=1}^{7} n^2 + \sum_{n=1}^{7} n\right).$$

The upper limit is seven in every sum, and there is no index shift.

Step 2: Evaluate each power sum For $N = 7$, the first-power sum is

$$\sum_{n=1}^{7} n = \frac{7 \cdot 8}{2} = 28.$$

The second-power sum is

$$\sum_{n=1}^{7} n^2 = \frac{7 \cdot 8 \cdot 15}{6} = 140,$$

and the third-power sum is

$$\sum_{n=1}^{7} n^3 = \left(\frac{7 \cdot 8}{2}\right)^2 = 28^2 = 784.$$

Step 3: Substitute and check the arithmetic Substitution yields

$$S = \frac{1}{4}(2 \cdot 784 + 3 \cdot 140 + 28) = \frac{1568 + 420 + 28}{4} = \frac{2016}{4} = 504.$$

As a direct check, the seven summands are $3/2, 15/2, 21, 45, 165/2, 273/2, 210$. Grouping the half-integer terms gives $9 + 219$, while the remaining terms total 276, again producing 504.

Step 4: Conclusion The required sum equals $\boxed{504}$, a dimensionless numerical value.

Quick Tip to Solve: Expand a product of consecutive linear factors into powers of the index, then apply the standard sums; preserve the outside denominator throughout.

25. Let a line y = mx (m > 0) intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area (△OPQ) = 4 sq. units, then m is equal to _____. [4 marks]

Solution:

Concept:

- The nonzero intersection of a line and a parabola is obtained by substituting the line equation into the parabola equation. The condition $m > 0$ fixes the signs of the coordinates.
- For $y^2 = x$, implicit differentiation gives tangent slope $dy/dx = 1/(2y)$. With a horizontal base, a triangle has area one half the base times the perpendicular height.

Step 1: Find the point of intersection Substitute $y = mx$ into $y^2 = x$:

$$m^2x^2 = x \quad \Rightarrow \quad x(m^2x - 1) = 0.$$

The origin is excluded, so the other intersection is

$$P = \left(\frac{1}{m^2}, \frac{1}{m}\right).$$

Both coordinates are positive because the specified slope is positive.

Step 2: Locate the tangent intercept At P, the derivative is $m/2$. The point-slope equation of the tangent is therefore

$$y - \frac{1}{m} = \frac{m}{2}\left(x - \frac{1}{m^2}\right).$$

Put $y = 0$ to locate its meeting point with the horizontal axis:

$$-\frac{1}{m} = \frac{m}{2}x - \frac{1}{2m} \quad \Rightarrow \quad x = -\frac{1}{m^2}.$$

Thus $Q = (-1/m^2, 0)$, lying to the left of the origin.

Step 3: Apply the specified area Take OQ as the base. Its length is positive, $1/m^2$, even though the coordinate of Q is negative. The perpendicular height of P above the base is $1/m$. Hence

$$4 = \frac{1}{2}\frac{1}{m^2}\frac{1}{m} = \frac{1}{2m^3}.$$

This gives $m^3 = 1/8$; the required positive real cube root is $m = 1/2$.

Step 4: Conclusion The required slope is $\boxed{m = 0.5}$, a dimensionless number.

Quick Tip to Solve: For a triangle whose two vertices lie on the horizontal axis, use the length of that axis segment as the base; a negative intercept still gives a positive base length.

26. Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P/C_V value will be : [4 marks]

(A) 23/15

(B) 67/45

(C) 40/27

(D) 19/13

Correct Answer: (D) 19/13

Solution:

Concept:

- Heat capacities of nonreacting ideal-gas components add when the components undergo the same temperature change. The mole fractions must weight the heat capacities, rather than their ratios.
- Equipartition gives $C_{V,m} = fR/2$ and $C_{P,m} = C_{V,m} + R$, where f is the number of active molecular degrees of freedom and R is the gas constant.

Step 1: Assign the molecular degrees of freedom Helium is monatomic, with three translational degrees of freedom. Thus its molar heat capacities are $3R/2$ and $5R/2$. Rigid oxygen is diatomic: three translational and two rotational degrees of freedom contribute, while vibrations are excluded by the rigid-molecule condition. Its molar heat capacities are $5R/2$ and $7R/2$.

Step 2: Add the heat capacities of the samples There are n moles of helium but $2n$ moles of oxygen. The extensive heat capacities of the mixture are

$$C_V = n\frac{3R}{2} + 2n\frac{5R}{2} = \frac{13nR}{2},$$

$$C_P = n\frac{5R}{2} + 2n\frac{7R}{2} = \frac{19nR}{2}.$$

Their difference is $3nR$, as required by Mayer's relation for the total of $3n$ moles.

Step 3: Form the ratio and check the options The common factor $nR/2$ cancels, giving

$$\gamma = \frac{C_P}{C_V} = \frac{19}{13} \simeq 1.462.$$

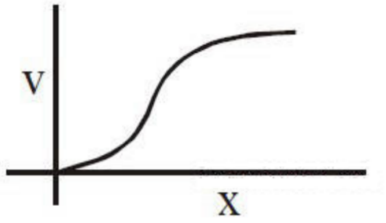
This lies between the oxygen value $7/5$ and the helium value $5/3$, closer to oxygen because oxygen contributes more heat capacity. The other three fractions do not equal the ratio of the correctly weighted sums; directly averaging the individual gamma values would not be valid.

Step 4: Conclusion The mixture has $\boxed{C_P/C_V = 19/13}$. Hence, the correct option is (D).

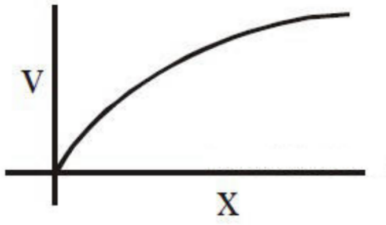
Quick Tip to Solve: For a gas mixture, first add each component's heat capacity using its number of moles, then divide; do not average the individual values of gamma.

27. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale) [4 marks]

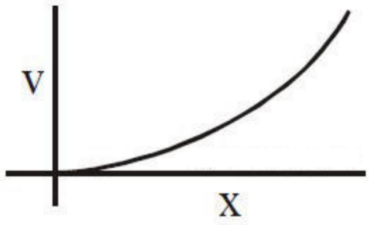
(A)



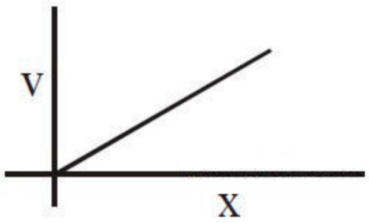
(B)



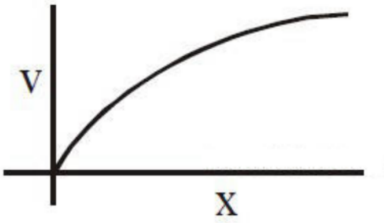
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- A uniform electric field exerts a constant force of magnitude $|q|E$ on a particle of charge q. With no other force, its acceleration has constant magnitude $a = |q|E/m$.
- For rectilinear motion from rest with constant acceleration, $v^2 = 2ax$, where x is the distance travelled. Equivalently, the work done by the electric force becomes kinetic energy.

Step 1: Relate the electric work to kinetic energy The particle starts at rest, so its initial kinetic energy is zero. Its subsequent motion is along the electric force; if the charge is negative, both the acceleration and displacement point opposite to the electric field. In either case the work over the distance x is positive:

$$|q|Ex = \frac{1}{2}mv^2.$$

Therefore

$$v = \sqrt{\frac{2|q|E}{m}}\sqrt{x}.$$

The coefficient is constant for this particle and field.

Step 2: Identify the shape of the curve The square-root law means that the graph passes through the origin and increases for positive x, but becomes progressively less steep. For $x > 0$,

$$\frac{dv}{dx} = \frac{1}{2}\sqrt{\frac{2|q|E}{m}}x^{-1/2} > 0,$$

while its second derivative is negative. Thus the curve is concave downward throughout, with a very steep initial tangent rather than a horizontal one.

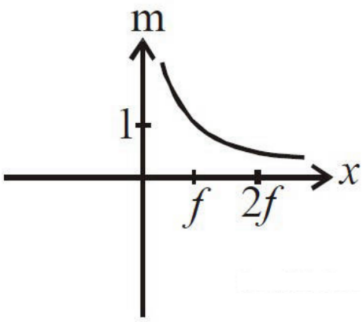
Step 3: Compare all four schematic graphs Graph (B) rises from the origin with continuously decreasing slope, as the square-root dependence requires. Graph (A) changes curvature and initially bends upward, which is incompatible with constant acceleration from rest. Graph (C) bends upward throughout and would require the slope to increase with distance. Graph (D) makes speed proportional to distance rather than its square root. Only (B) has the required origin and curvature.

Step 4: Conclusion The dependence is $\boxed{v = \sqrt{2|q|Ex/m}}$, represented by graph (B). Hence, the correct option is (B).

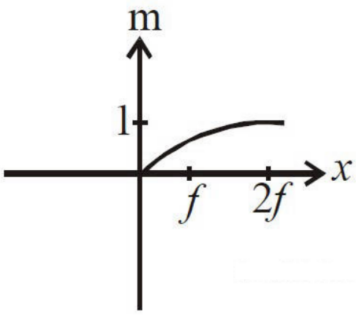
Quick Tip to Solve: A constant force makes velocity linear in time, but speed proportional to the square root of distance when the particle starts from rest.

28. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by :
(Graphs are drawn schematically and are not to scale) [4 marks]

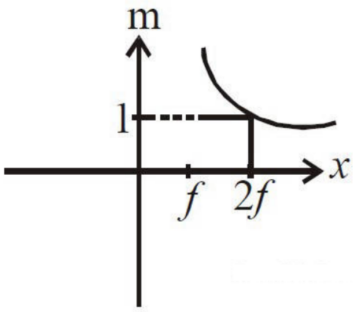
(A)



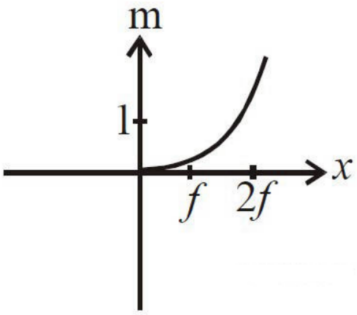
(B)



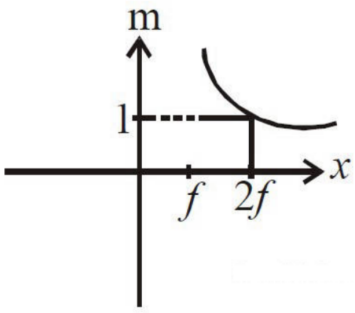
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- For a concave mirror, the mirror formula is $1/f_s = 1/u + 1/v$, using signed Cartesian distances, and transverse magnification is $-v/u$.
- Writing $F = |f_s|$ for the positive focal-length magnitude and $x > F$ for the object distance beyond the focus, the magnitude of magnification is $M = F/(x - F)$.

Step 1: Derive the magnification dependence The object moves outward from the focus, so its distance increases from just above F. With $u = -x$ and $f_s = -F$, the mirror formula gives

$$-\frac{1}{F} = -\frac{1}{x} + \frac{1}{v} \quad \Rightarrow \quad v = -\frac{Fx}{x - F}.$$

The image is real and inverted. Taking the magnitude of $-v/u$ removes its negative sign:

$$M = \frac{|v|}{x} = \frac{F}{x - F}.$$

The graph requested concerns this positive magnitude, not the signed magnification.

Step 2: Check the focus and centre of curvature As the object approaches the focus from beyond it, $x \to F^+$, the denominator tends to zero and $M \to \infty$. At the centre of curvature, $x = 2F$,

$$M = \frac{F}{2F - F} = 1.$$

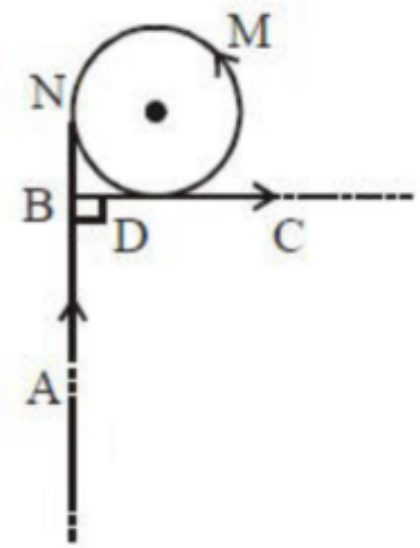
For still larger distances, $M < 1$ and it tends to zero as $x \to \infty$. Throughout this domain $dM/dx = -F/(x - F)^2 < 0$, so the curve decreases.

Step 3: Select the graph with the correct landmarks Graph (C) shows a decreasing curve on the side beyond the focus and explicitly passes through the point $(2f, 1)$, matching the unit magnification at the centre of curvature. Graph (A) does not place that unit-magnification point at $2f$. Graphs (B) and (D) increase with object distance and therefore contradict the negative derivative. These landmark checks decide the answer even though the graphs are schematic.

Step 4: Conclusion The magnitude follows $\boxed{M = F/(x - F)}$ for $x > F$. Hence, the correct option is (C).

Quick Tip to Solve: For concave-mirror magnification graphs, test two landmarks first: infinite magnification at the focus and unit magnification at twice the focal length.

29. A very long wire ABDMNDC is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is : [4 marks]



(A) $\frac{\mu_0 I}{2\pi R}(\pi + 1)$

(B) $\frac{\mu_0 I}{2\pi R}\left(\pi - \frac{1}{\sqrt{2}}\right)$

(C) $\frac{\mu_0 I}{2R}$

(D) $\frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$

Correct Answer: (D) $\frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$

Solution:

Concept:

- The magnetic field of a complete circular loop at its centre has magnitude $B_{\text{loop}} = \mu_0 I/(2R)$, with direction fixed by the right-hand rule.
- Fields from different portions of a wire add vectorially. Straight portions at equal perpendicular distances and with equal endpoint geometry produce equal field magnitudes, even when their directions differ.

Step 1: Separate the loop and straight portions Treat the wire as the full circular turn $DMND$, the vertical straight portion AB, and the horizontal straight portion BC. The centre lies a perpendicular distance R from each tangent. The perpendicular feet are N and D. Both BN and BD equal R, so the line joining the corner to the centre makes an angle of $45°$ with either perpendicular. The vertical segment ends at B, below N; the horizontal segment passes through its perpendicular foot D.

Step 2: Combine the fields of the straight portions A finite straight wire gives $\mu_0 I/(4\pi R)$ times the appropriate difference of endpoint direction cosines. For the vertical portion, the perpendicular foot lies beyond its endpoint, giving

$$B_{AB} = \frac{\mu_0 I}{4\pi R}\left(1 - \frac{1}{\sqrt{2}}\right).$$

Its field is into the page, since current flows upward and the centre is to its right. The horizontal portion extends from the corner past its perpendicular foot to infinity, so

$$B_{BC} = \frac{\mu_0 I}{4\pi R}\left(1 + \frac{1}{\sqrt{2}}\right),$$

directed out of the page. Consequently their net field is out of the page and has magnitude

$$B_{BC} - B_{AB} = \frac{\mu_0 I}{2\pi R\sqrt{2}}.$$

Step 3: Add the complete circular-turn field The circular turn carries counterclockwise current, so its field also points out of the page. Add this field to the net straight-wire field:

$$B_{\text{net}} = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R\sqrt{2}} = \frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right).$$

Option (A) uses an incorrect coefficient for the straight portions. Option (B) subtracts their resultant instead of adding it to the loop field. Option (C) omits the nonzero resultant of the straight portions. Only option (D) includes the correct magnitude and relative direction.

Step 4: Conclusion The field is $\boxed{\frac{\mu_0 I}{2\pi R}(\pi + 1/\sqrt{2})}$, out of the plane of the figure. Hence, the correct option is (D).

Quick Tip to Solve: For each straight segment, first locate the perpendicular foot: endpoint angles subtract if the foot lies outside that segment and add if it lies inside.

30. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06 $\times$ 10^4 N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to : [4 marks]

(A) 30.5 $\times$ 10^4 N

(B) 2.50 $\times$ 10^4 N

(C) 10.2 $\times$ 10^2 N

(D) 5.15 $\times$ 10^3 N

Correct Answer: (D) 5.15 $\times$ 10^3 N

Solution:

Concept:

- The speed of a small transverse wave on a taut wire is $v = \sqrt{T/\mu}$, where T is the tension and μ is the mass per unit length.
- For the same wire, with its linear mass density treated as constant, the square of the wave speed is proportional to tension: $v^2 \propto T$.

Step 1: Write the speed equation in both states Let the initial tension be $T_1 = 2.06 \times 10^4\,\text{N}$ and the

final tension be $T_2 = T$. The initial and final wave speeds satisfy

$$v^2 = \frac{T_1}{\mu}, \qquad \left(\frac{v}{2}\right)^2 = \frac{T_2}{\mu}.$$

Both equations use the same linear density because it is the same taut wire. The question asks for a change in the propagation speed, rather than a change in the particle velocity of the vibrating material.

Step 2: Eliminate the wire density Divide the final equation by the initial one. This removes the unknown μ and the actual value of v:

$$\frac{T_2}{T_1} = \frac{(v/2)^2}{v^2} = \frac{1}{4}.$$

Consequently a factor of two reduction in wave speed requires a factor of four reduction in tension; it does not require simply halving the tension.

Step 3: Calculate and compare the numerical values Substitute the given initial tension:

$$T_2 = \frac{2.06 \times 10^4}{4} = 0.515 \times 10^4\,\text{N} = 5.15 \times 10^3\,\text{N}.$$

The result has units of force, as expected for tension. Options (A) and (B) exceed the initial tension and would increase the speed. Option (C) is much smaller than a quarter of the initial tension and would reduce the speed by more than the specified factor. Only (D) gives the required quarter-tension value.

Step 4: Conclusion The required tension is $\boxed{T = 5.15 \times 10^3\,\text{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: When comparing transverse-wave speeds on the same wire, square the speed ratio to obtain the tension ratio; halving speed means quartering tension.

31. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\vec{B} = 5 \times 10^{-8}\hat{j}T$. The corresponding electric field $\vec{E}$ is (speed of light c = 3 × 10^8 ms^{-1}) [4 marks]

(A) 15 $\hat{i}$V / m

(B) -15 $\hat{i}$V / m

(C) 1.66 × 10^{-16} $\hat{i}$V / m

(D) -1.66 × 10^{-16} $\hat{i}$V / m

Correct Answer: (A) 15 $\hat{i}$V / m

Solution:

Concept:

- For an electromagnetic wave in vacuum, the instantaneous field magnitudes satisfy $E = cB$. The electric field, magnetic field and propagation direction are mutually perpendicular.

- The direction of energy transport is the direction of the Poynting vector $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$. Thus both the magnitude and the sign of the electric field must be checked.

Step 1: Calculate the electric-field magnitude The magnetic field at the specified event has magnitude $B = 5 \times 10^{-8}\,\mathrm{T}$. Applying the vacuum field relation gives

$$E = cB = (3 \times 10^{8})(5 \times 10^{-8}) = 15\,\mathrm{V\,m^{-1}}.$$

The powers of ten cancel. This relation is between instantaneous magnitudes at the same place and time, so no phase averaging or root-mean-square conversion is required. The stated frequency does not enter this magnitude relation.

Step 2: Choose the direction from the cross product The wave travels along $+\hat{k}$, while the magnetic field points along $+\hat{j}$. Since $\hat{i} \times \hat{j} = \hat{k}$, the electric field must point along $+\hat{i}$. A field along $-\hat{i}$ would instead give energy transport along $-\hat{k}$, contradicting the stated propagation direction. Hence option (B) has the wrong sign. Options (C) and (D) have the wrong magnitude: they use a value proportional to B/c, whereas a vacuum wave requires $E = cB$.

Step 3: Conclusion

$$\boxed{\vec{E} = 15\hat{i}\,\mathrm{V\,m^{-1}}}$$

Hence, the correct option is (A).

Quick Tip to Solve: First use $E = cB$ for the size, then apply $\vec{E} \times \vec{B}$ to check the sign; the frequency is unnecessary here.

32. A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V? [4 marks]

(A) 8.9 kΩ

(B) 10 kΩ

(C) 9.9 kΩ

(D) 7.9 kΩ

Correct Answer: (C) 9.9 kΩ

Solution:

Concept:

- A galvanometer becomes a voltmeter when a high resistance is connected in series with its coil. The same current then flows through the added resistance and the galvanometer.
- At the full-scale voltage V, the current must equal the galvanometer full-scale current I_g. Therefore $V = I_g(R_s + G)$, where G is the coil resistance and R_s is the added series resistance.

Step 1: Find the required total resistance Convert the current to amperes before substituting:

$I_g = 1\,\text{mA} = 10^{-3}\,\text{A}$. For a full-scale range of $10\,\text{V}$, the combined resistance must be

$$R_{\text{total}} = \frac{V}{I_g} = \frac{10}{10^{-3}} = 10^4\,\Omega = 10\,\text{k}\Omega.$$

This total includes the existing coil resistance; it is not the resistance that must be added externally.

Step 2: Subtract the coil resistance and verify the range The coil already contributes $G = 100\,\Omega$. Thus the required multiplier resistance is

$$R_s = 10^4 - 100 = 9900\,\Omega = 9.9\,\text{k}\Omega.$$

As a check, the coil voltage at full scale is $I_g G = 0.1\,\text{V}$, and the added resistor drops $9.9\,\text{V}$; together these give $10\,\text{V}$. Option (B) confuses total resistance with added resistance and would give a range of $10.1\,\text{V}$. Options (A) and (D) would give full-scale ranges of $9.0\,\text{V}$ and $8.0\,\text{V}$, respectively, so neither matches the required range.

Step 3: Conclusion The galvanometer requires a series resistance of $\boxed{9.9\,\text{k}\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: For a voltmeter multiplier, calculate V/I_g first and subtract the galvanometer resistance; V/I_g alone is the total resistance.

33. Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio V_1 (on S_1) / V_2 (on S_2) of the electrostatic potentials on each sphere is : [4 marks]

(A) $(R_1/R_2)^2$

(B) (R_2/R_1)

(C) $(R_1/R_2)^3$

(D) R_1/R_2

Correct Answer: (A) $(R_1/R_2)^2$

Solution:

Concept:

- For a charged conducting sphere of radius R, the surface potential relative to infinity is $V = kQ/R$, and the field immediately outside the surface is $E = kQ/R^2$.
- Eliminating the charge between these expressions gives $V = ER$. Apply this relation separately to each sphere before using the specified field ratio.

Step 1: Relate each potential to its surface field Write the standard surface expressions for the first sphere:

$$E_1 = \frac{kQ_1}{R_1^2}, \qquad V_1 = \frac{kQ_1}{R_1} = E_1 R_1.$$

Similarly, for the second sphere, $V_2 = E_2 R_2$. This step avoids any unjustified assumption that the two spheres have equal charges. Equal charge is not stated; the information supplied is a relation between

their surface fields.

Step 2: Use the given field ratio Take the ratio and substitute $E_1/E_2 = R_1/R_2$:

$$\frac{V_1}{V_2} = \frac{E_1 R_1}{E_2 R_2} = \left(\frac{R_1}{R_2}\right)\left(\frac{R_1}{R_2}\right) = \left(\frac{R_1}{R_2}\right)^2.$$

The extra radius factor arises because potential decreases as inverse radius whereas field decreases as inverse radius squared for a fixed charge. Option (D) retains only the supplied field ratio and omits this extra factor. Option (B) reverses the radius dependence. Option (C) gives a cubic ratio, which is actually the corresponding charge ratio: $Q_1/Q_2 = (E_1/E_2)(R_1/R_2)^2 = (R_1/R_2)^3$, rather than the potential ratio.

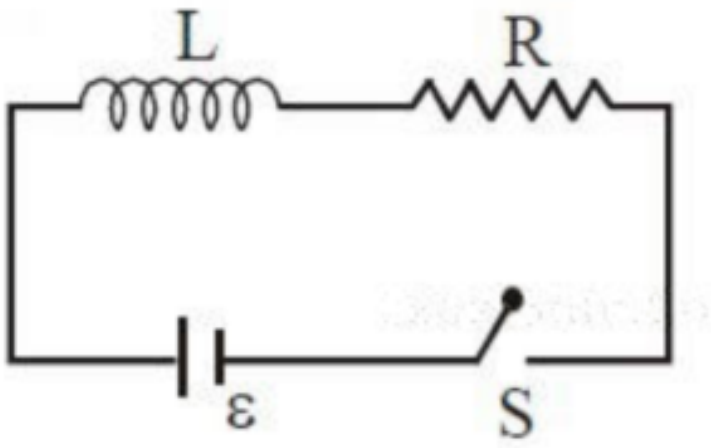
Step 3: Conclusion

$$\boxed{\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^2}$$

Hence, the correct option is (A).

Quick Tip to Solve: At a conducting sphere surface, use $V = ER$; do not assume equal charges when the problem supplies a field ratio instead.

34. A shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and t = t_c (t_c is the time constant of the circuit) is : [4 marks]



(A) $\frac{\varepsilon L}{eR^2}$
(B) $\frac{\varepsilon L}{R^2}$
(C) $\frac{\varepsilon L}{R^2}\left(1 - \frac{1}{e}\right)$
(D) $\frac{\varepsilon R}{eL^2}$

Correct Answer: (A) $\frac{\varepsilon L}{eR^2}$

Solution:

Concept:

- After a constant emf ε is applied to a series LR circuit initially carrying no current, $i(t) = (\varepsilon/R)(1 - e^{-t/\tau})$, where $\tau = L/R$.
- The charge passing through the battery during a time interval is the time integral of the current: $Q = \int i(t)\,dt$. Current at one instant multiplied by the interval is not valid for a changing current.

Step 1: Set up the charge integral The requested upper limit is one time constant, $t_c = \tau = L/R$. The current starts at zero because an ideal inductor prevents an instantaneous change in current. Therefore

$$Q = \int_0^{\tau} \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)\,dt.$$

Here the same current passes through the battery, inductor and resistor, as they are connected in series. The charge delivered by the battery is consequently the same charge crossing any section of this circuit.

Step 2: Evaluate the integral at one time constant Integrate with the exponential sign carefully retained:

$$\begin{aligned} Q &= \frac{\varepsilon}{R}\left[t + \tau e^{-t/\tau}\right]_0^{\tau} \\ &= \frac{\varepsilon}{R}\left(\tau + \frac{\tau}{e} - \tau\right) \\ &= \frac{\varepsilon\tau}{eR} = \frac{\varepsilon L}{eR^2}. \end{aligned}$$

Option (B) would be obtained by treating the current as its final steady value throughout the interval. Option (C) equals $i(\tau)\tau$, which incorrectly treats the final current as constant. Option (D) also fails dimensional analysis: interchanging the powers of resistance and inductance does not give units of charge.

Step 3: Conclusion The charge supplied in the first time constant is $\boxed{Q = \varepsilon L/(eR^2)}$. Hence, the correct option is (A).

Quick Tip to Solve: In an LR charging problem, integrate the growing current. At one time constant, the charge is $1/e$ of the steady-current rectangle $(\varepsilon/R)\tau$.

35. A particle moves such that its position vector $\overrightarrow{r}(t) = \cos\omega t\hat{i} + \sin\omega t\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\overrightarrow{v}(t)$ and acceleration $\overrightarrow{a}(t)$ of the particle : [4 marks]

(A) $\overrightarrow{v}$ and $\overrightarrow{a}$ both are perpendicular to $\overrightarrow{r}$

(B) $\overrightarrow{v}$ and $\overrightarrow{a}$ both are parallel to $\overrightarrow{r}$

(C) $\overrightarrow{v}$ is perpendicular to $\overrightarrow{r}$ and $\overrightarrow{a}$ is directed towards the origin

(D) $\overrightarrow{v}$ is perpendicular to $\overrightarrow{r}$ and $\overrightarrow{a}$ is directed away from the origin

Correct Answer: (C) $\overrightarrow{v}$ is perpendicular to $\overrightarrow{r}$ and $\overrightarrow{a}$ is directed towards the origin

Solution:

Concept:

- Velocity is the time derivative of position, $\vec{v} = d\vec{r}/dt$, and acceleration is $\vec{a} = d\vec{v}/dt$. Differentiate both components rather than inferring directions from their signs individually.
- Two nonzero vectors are perpendicular when their scalar product vanishes. An acceleration proportional to $-\vec{r}$ points towards the origin and is centripetal.

Step 1: Differentiate the position vector The components satisfy $x^2 + y^2 = \cos^2\omega t + \sin^2\omega t = 1$, so the path is a circle of unit radius centred at the origin. Differentiation gives

$$\vec{v}(t) = -\omega\sin\omega t\,\hat{i} + \omega\cos\omega t\,\hat{j}.$$

Its magnitude is the constant $|\omega|$, which is consistent with uniform circular motion. The dot product with position is

$$\vec{r}\cdot\vec{v} = -\omega\cos\omega t\sin\omega t + \omega\sin\omega t\cos\omega t = 0.$$

Thus velocity is tangential and perpendicular to the radius vector.

Step 2: Find the acceleration direction and test the choices Differentiate velocity once again:

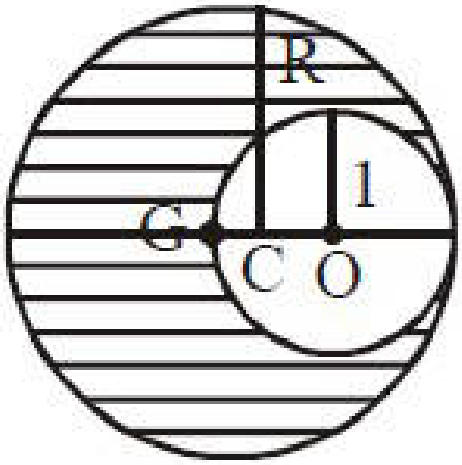
$$\vec{a}(t) = -\omega^2\cos\omega t\,\hat{i} - \omega^2\sin\omega t\,\hat{j} = -\omega^2\vec{r}(t).$$

For circular motion with nonzero ω, the negative scalar coefficient means acceleration points opposite to the position vector, towards the origin. Option (A) is false because acceleration is radial rather than perpendicular to the radius. Option (B) is false because velocity is tangential rather than parallel to the radius. Option (D) has the correct velocity direction but incorrectly makes acceleration outward. Only option (C) combines the two required directions.

Step 3: Conclusion The velocity is perpendicular to $\vec{r}$, and the acceleration is directed towards the origin. Hence, the correct option is (C).

Quick Tip to Solve: For $\cos\omega t$ and $\sin\omega t$ position components, two derivatives give $-\omega^2\vec{r}$, immediately identifying inward acceleration.

36. As shown in figure, when a spherical cavity (centered at O) of radius 1 is cut out of a uniform sphere of radius R (centered at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e, on the surface of the cavity. R can be detemined by the equation : [4 marks]



(A) $(R^2 + R - 1)(2 - R) = 1$

(B) $(R^2 - R - 1)(2 - R) = 1$

(C) $(R^2 - R + 1)(2 - R) = 1$

(D) $(R^2 + R + 1)(2 - R) = 1$

Correct Answer: (D) $(R^2 + R + 1)(2 - R) = 1$

Solution:

Concept:

- A removed portion may be treated as a negative mass when calculating the centre of mass of the remaining body. Uniform density makes the mass of each sphere proportional to the cube of its radius.
- The figure shows a cavity of radius 1 internally tangent to the outer sphere on the right. With the outer centre C as origin and rightwards positive, the cavity centre is at $x_O = R - 1$.

Step 1: Express the remaining centre of mass Let the mass of a sphere of unit radius at this density be m. The full sphere has mass mR^3, located at $x_C = 0$; the removed cavity has mass m, located at $x_O = R - 1$. Subtraction of its moment gives

$$x_G = \frac{mR^3(0) - m(R-1)}{mR^3 - m} = -\frac{R-1}{R^3-1} = -\frac{1}{R^2+R+1}.$$

The cancellation uses $R^3 - 1 = (R-1)(R^2+R+1)$, with $R > 1$ for a nonempty remaining sphere. The negative sign correctly places the remaining centre of mass to the left of C, away from the removed material.

Step 2: Impose the cavity-surface condition According to the figure, G lies on the left surface of the cavity along the line of centres. Its coordinate is therefore

$$x_G = x_O - 1 = (R-1) - 1 = R - 2.$$

Equate this geometric coordinate to the mass calculation:

$$R - 2 = -\frac{1}{R^2 + R + 1} \quad \Longrightarrow \quad (R^2 + R + 1)(2 - R) = 1.$$

This also implies $R < 2$, consistent with G lying left of C. Options (A), (B) and (C) have the wrong quadratic factor: subtracting a unit-radius spherical mass produces the difference of cubes $R^3 - 1$, whose quotient by $R - 1$ is specifically $R^2 + R + 1$.

Step 3: Conclusion The required equation is $\boxed{(R^2 + R + 1)(2 - R) = 1}$. Hence, the correct option is (D).

Quick Tip to Solve: Treat the cavity as negative mass, then use internal tangency to write $CO = R - 1$ and the left cavity surface coordinate as $R - 2$.

37. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is : [4 marks]

(A) 8.75×10^{-3} J

(B) 1.13×10^{-3} J

(C) 8.75×10^{-4} J

(D) 6.25×10^{-4} J

Correct Answer: (C) 8.75×10^{-4} J

Solution:

Concept:

- For pure rolling, the kinetic energy is the sum of translation of the centre of mass and rotation about it: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.
- A uniform solid sphere has $I = \frac{2}{5}mR^2$, and rolling without slipping gives $\omega = v/R$. The radius therefore cancels from the answer.

Step 1: Convert the data to SI units The speed is in centimetres per second, so it must be divided by 100 before squaring. The mass must also be expressed in kilograms. Thus

$$m = 500\,\mathrm{g} = 0.500\,\mathrm{kg}, \qquad v = 5.00\,\mathrm{cm\,s^{-1}} = 0.0500\,\mathrm{m\,s^{-1}}.$$

The translational contribution alone is

$$K_{\mathrm{tr}} = \frac{1}{2}(0.500)(0.0500)^2 = 6.25 \times 10^{-4}\,\mathrm{J}.$$

Step 2: Include rotation and compare the options Substituting the solid-sphere moment of inertia and the rolling condition gives

$$K = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2.$$

Consequently

$$K = \frac{7}{10}(0.500)(0.0500)^2 = 8.75 \times 10^{-4}\,\text{J}.$$

The rotational part is 2.50×10^{-4} J, so it must increase the translational result. Option (D) omits rotation, while option (A) is ten times the required value. Option (B) also differs from the solid-sphere rolling result; it does not follow from the specified moment of inertia.

Step 3: Conclusion The total kinetic energy is 8.75×10^{-4} J. Hence, the correct option is (C).

Quick Tip to Solve: For a rolling solid sphere, use $K = (7/10)mv^2$, and convert centimetres per second before squaring the speed.

38. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is : [4 marks]

(A) $\sqrt{\frac{1}{2}}$

(B) $\frac{1}{2}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\sqrt{\frac{3}{4}}$

Correct Answer: (C) $\sqrt{\frac{3}{2}}$

Solution:

Concept:

- Before impact both particles accelerate downwards at g, so their relative acceleration is zero. Their separation closes at their initial relative speed.
- During the short collision conserve linear momentum. Afterwards the particles move together as one body under gravity; kinetic energy need not be conserved in the collision.

Step 1: Find the collision position and velocities Take upward as positive and write $u = \sqrt{2gh}$ for the launched particle. At time t_c, the heights are $y_1 = h - gt_c^2/2$ and $y_2 = ut_c - gt_c^2/2$. Equating them gives

$$t_c = \frac{h}{u} = \sqrt{\frac{h}{2g}}.$$

Hence the collision height is

$$y_c = h - \frac{1}{2}gt_c^2 = \frac{3h}{4}.$$

Just before collision, $v_1 = -gt_c = -u/2$ and $v_2 = u - gt_c = u/2$. Their equal masses therefore have equal and opposite momenta.

Step 2: Calculate the time after the particles stick together Momentum conservation gives

$$2mV = m(-u/2) + m(u/2) = 0, \qquad V = 0.$$

The combined mass starts its downward journey from rest at height $3h/4$. If τ is its time to reach the ground, then

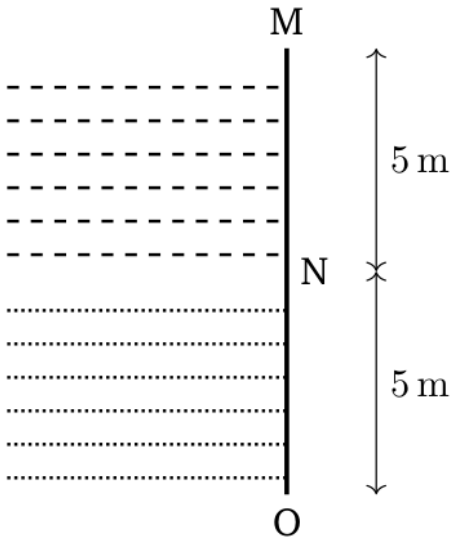
$$\frac{3h}{4} = \frac{1}{2}g\tau^2, \qquad \tau = \sqrt{\frac{3h}{2g}}.$$

The requested time for the combined mass is measured after collision, since that combined body exists only then. Option (A) instead equals the time to collision. Options (B) and (D) do not satisfy the fall-distance equation for the collision height.

Step 3: Conclusion After collision the combined mass takes $\sqrt{3/2}$ units of $\sqrt{h/g}$ to reach the ground. Hence, the correct option is (C).

Quick Tip to Solve: Equal masses meeting with equal and opposite velocities stick at rest; calculate their remaining fall from the collision height.

39. Two liquids of densities ρ_1 an ρ_2 (ρ_2 = 2ρ_1) are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing) [4 marks]



(A) 1/3

(B) 1/2

(C) 1/4

(D) 2/3

Correct Answer: (C) 1/4

Solution:

Concept:

- Gauge pressure in a liquid is the weight of liquid above unit area. In a layered system, the pressure due to every overlying layer must be included.
- The force on a vertical strip is $dF = p(y)b\,dy$, where $b = 10\,\mathrm{m}$ is the wall width. With pressure varying linearly, use mean pressure times area.

Step 1: Find the force on the upper half MN The lighter liquid of density ρ_1 lies above the denser liquid, with each layer of height $a = 5\,\mathrm{m}$. On MN the gauge pressure rises from zero at M to $\rho_1 ga$ at N. The mean pressure on this rectangular half is therefore $\rho_1 ga/2$. Its force is

$$F_{MN} = \frac{\rho_1 ga}{2}(ba) = \frac{1}{2}\rho_1 gba^2.$$

Atmospheric pressure acts on the exposed side too and cancels; the force due to the liquids is found with gauge pressure.

Step 2: Include the upper layer pressure on NO At depth z below N, the pressure is

$$p(z) = \rho_1 ga + \rho_2 gz.$$

Thus the lower-half force has a rectangular contribution from the entire upper layer and a triangular contribution from the lower layer:

$$F_{NO} = \rho_1 gba^2 + \frac{1}{2}\rho_2 gba^2.$$

Using $\rho_2 = 2\rho_1$, this becomes $F_{NO} = 2\rho_1 gba^2$. Hence

$$\frac{F_{MN}}{F_{NO}} = \frac{(1/2)\rho_1 gba^2}{2\rho_1 gba^2} = \frac{1}{4}.$$

Option (B) would ignore the upper layer pressure on NO. Options (A) and (D) likewise do not include the correct pressure distribution over the two halves.

Step 3: Conclusion The ratio of liquid forces on MN and NO is $1/4$. Hence, the correct option is (C).

Quick Tip to Solve: For the lower liquid, add the constant pressure of the full upper layer before averaging the pressure across the lower wall.

40. In a double slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is : [4 marks]

(A) 0.853

(B) 0.568

(C) 0.672

(D) 0.760

Correct Answer: (A) 0.853

Solution:

Concept:

- The phase difference produced by a path difference Δ is $\phi = 2\pi\Delta/\lambda$, where λ is the wavelength.
- For the equal-intensity slits of the standard double-slit experiment, $I = 4I_s\cos^2(\phi/2)$. The intensity at a bright-fringe centre is $I_{\max} = 4I_s$.

Step 1: Convert the given path difference to phase At the specified point the path difference is $\Delta = \lambda/8$, so

$$\phi = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}.$$

This is a phase difference of $45°$. The amplitude factor uses half this phase, so its argument is $\pi/8$, not $\pi/4$. At a bright-fringe centre the waves arrive in phase and the amplitude is the sum of the two individual amplitudes.

Step 2: Calculate and check the intensity ratio Dividing by the maximum intensity eliminates the unknown intensity of either slit:

$$\frac{I}{I_{\max}} = \cos^2\left(\frac{\pi}{8}\right) = \frac{1+\cos(\pi/4)}{2}.$$

Therefore

$$\frac{I}{I_{\max}} = \frac{1+1/\sqrt{2}}{2} = \frac{2+\sqrt{2}}{4} \simeq 0.85355.$$

The listed value 0.853 is the corresponding option. The ratio is less than unity but still close to the maximum, as expected for a small path difference. The values 0.568, 0.672 and 0.760 are all inconsistent with the given phase difference and the interference formula; neither the cosine alone nor the cosine of the full phase is the required intensity ratio.

Step 3: Conclusion The intensity ratio is approximately 0.8536, corresponding to 0.853 among the choices. Hence, the correct option is (A).

Quick Tip to Solve: Normalize by the bright-fringe intensity first: $I/I_{\max} = \cos^2(\pi\Delta/\lambda)$. This avoids unknown slit intensities.

41. An electron (mass m) with initial velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$ is in an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, its de-Broglie wave length at time t is given by : [4 marks]

(A) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{m^2v_0^2}}}$

(B) $\frac{\lambda_0\sqrt{2}}{\sqrt{1+\frac{e^2E^2t^2}{m^2v_0^2}}}$

(C) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{2m^2v_0^2}}}$

(D) $\frac{\lambda_0}{\sqrt{2+\frac{e^2E^2t^2}{m^2v_0^2}}}$

Correct Answer: (C) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{2m^2v_0^2}}}$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h_P/p = h_P/(mv)$ for the nonrelativistic electron, where h_P denotes Planck's constant.
- The force is $\boldsymbol{F} = q\boldsymbol{E}$. An electron has charge $-e$, so the given negative z-directed field produces a positive z-directed acceleration.

Step 1: Obtain all velocity components The initial components are $v_x = v_0$, $v_y = v_0$, and $v_z = 0$, giving initial speed $v_i = \sqrt{2}v_0$. The force has no x or y component, so these two velocities remain unchanged. Since $\boldsymbol{E} = -E_0\hat{k}$,

$$\boldsymbol{a} = \frac{(-e)(-E_0\hat{k})}{m} = \frac{eE_0}{m}\hat{k}.$$

At time t,

$$v_z = \frac{eE_0t}{m}, \qquad v(t)^2 = 2v_0^2 + \frac{e^2E_0^2t^2}{m^2}.$$

The velocities in perpendicular directions must be combined by adding their squares.

Step 2: Express the wavelength relative to its initial value Using $\lambda_0 = h_P/(m\sqrt{2}v_0)$, the ratio is

$$\frac{\lambda(t)}{\lambda_0} = \frac{\sqrt{2}v_0}{\sqrt{2v_0^2 + e^2E_0^2t^2/m^2}}.$$

Thus

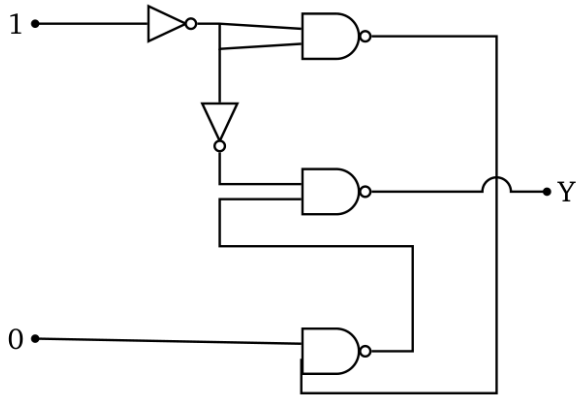
$$\lambda(t) = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}}.$$

Here the E in the options denotes the field magnitude E_0. Option (A) misses the initial two-component speed. Options (B) and (D) even fail the initial condition $\lambda(0) = \lambda_0$. The wavelength decreases as the field increases the electron's momentum.

Step 3: Conclusion The wavelength is $\lambda_0/\sqrt{1 + e^2E_0^2t^2/(2m^2v_0^2)}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the magnitude of the initial velocity vector first; two equal perpendicular components give $\sqrt{2}v_0$, not v_0.

42. In the given circuit, value of Y is : [4 marks]



(A) toggles between 0 and 1

(B) 1

(C) will not execute

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- A NOT gate complements its input: $\overline{1} = 0$, $\overline{0} = 1$. A NAND gate outputs $\overline{AB}$, and hence has output 1 whenever either input is 0.
- Trace the diagram wire by wire. A wire crossing with a bridge is not a connection, so the upper NAND output and the middle NAND output remain distinct signals.

Step 1: Evaluate the upper NOT and NAND gates The external upper input is 1. After the first NOT gate it becomes $a = \overline{1} = 0$. This signal feeds both inputs of the upper NAND, whose output is therefore

$$b = \overline{a \cdot a} = \overline{0 \cdot 0} = 1.$$

The downward branch from a passes through another NOT gate, giving $c = \overline{a} = 1$ at the upper input of the middle NAND. Follow the outer wire from the upper NAND: it crosses the middle output with a bridge and reaches the lower NAND input without joining Y.

Step 2: Evaluate the lower NAND and then Y The lower NAND receives the external input 0 and the upper NAND output $b = 1$, so its output is

$$d = \overline{0 \cdot b} = \overline{0 \cdot 1} = 1.$$

This goes to the lower input of the middle NAND. Its upper input was $c = 1$, giving

$$Y = \overline{c \cdot d} = \overline{1 \cdot 1} = 0.$$

All intermediate inputs are fixed; there is no feedback loop that could cause toggling. Thus option (A) is excluded. Option (B) misses the final NAND inversion, while option (C) is unnecessary because the gates have a consistent, fully determined output.

Step 3: Conclusion The circuit gives $Y = 0$. Hence, the correct option is (D).

Quick Tip to Solve: A zero input forces a NAND output to 1. Evaluate that gate immediately, then trace the remaining fixed signals to the final NAND.

43. A simple pendulum is being used to determine th value of gravitational acceleration g at a certain place. Th length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is : [4 marks]

(A) 4.40%

(B) 3.40%

(C) 2.40%

(D) 5.40%

Correct Answer: (A) 4.40%

Solution:

Concept:

- For a simple pendulum, $T = 2\pi\sqrt{l/g}$, so $g = 4\pi^2 l/T^2$. Here l is its length and T its period.
- For a product or quotient, maximum fractional errors add with the absolute powers: $\Delta g/g = \Delta l/l + 2\Delta T/T$. Counting a fixed number of oscillations introduces no additional timing uncertainty.

Step 1: Find the fractional timing uncertainty The measured interval is the time for $N = 40$ oscillations. With the stopwatch resolution taken as $\Delta t = 1\,\text{s}$,

$$T = \frac{50}{40} = 1.25\,\text{s}, \qquad \Delta T = \frac{1}{40}\,\text{s}.$$

Therefore $\Delta T/T = \Delta t/t = 1/50 = 0.02$. Dividing by 40 changes both the measured time and its absolute uncertainty by the same factor.

Step 2: Include the length error and combine the contributions The length is stated as $25.0\,\text{cm}$, so its last recorded decimal place corresponds to $\Delta l = 0.1\,\text{cm}$. Thus

$$\frac{\Delta l}{l} = \frac{0.1}{25.0} = 0.004.$$

Using the squared dependence on period,

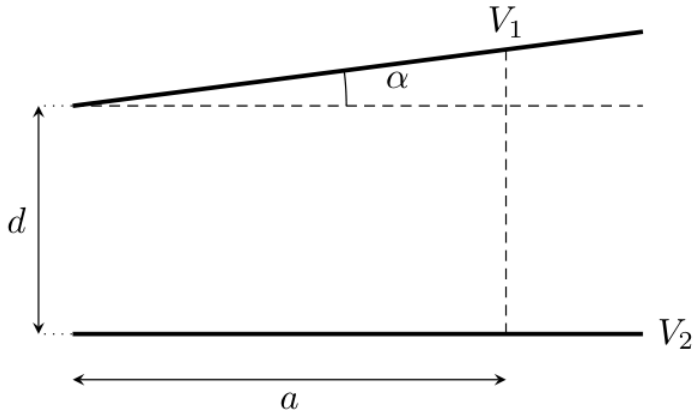
$$\frac{\Delta g}{g} \times 100 = (0.004 + 2 \times 0.02)\,100 = 4.40\%.$$

Options (B) and (C) underestimate the combined error, while (D) overestimates it; none equals the sum of the length contribution, 0.40%, and timing contribution, 4.00%.

Step 3: Conclusion The accuracy requested, expressed as the maximum percentage uncertainty in g, is 4.40%. Hence, the correct option is (A).

Quick Tip to Solve: When timing many oscillations together, use the uncertainty of the whole measured interval; its fractional uncertainty is also that of one period. Double it when calculating the error in g.

44. A capacitor is made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to : [4 marks]



(A) $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$
(B) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$
(C) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$
(D) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$

Correct Answer: (C) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$

Solution:

Concept:

- For a narrow plate strip of area dA and local separation s, $dC = \varepsilon_0 dA/s$, neglecting edge effects. The strips share the same two conducting plates and are therefore in parallel.
- The diagram gives the smallest gap as d. At distance x along the plate, $s(x) = d + x\tan\alpha \simeq d + \alpha x$. For the first-order approximation, $\alpha a/d$ is small.

Step 1: Add the strip capacitances in parallel Take strips of width dx running across the full side length a, so $dA = a\,dx$. Each strip has the same potential difference $V_1 - V_2$, hence

$$C \simeq \varepsilon_0 a \int_0^a \frac{dx}{d+\alpha x} = \frac{\varepsilon_0 a}{\alpha}\ln\left(1+\frac{\alpha a}{d}\right).$$

This integral accounts for the continuously increasing separation. Treating all strips as a series combination would incorrectly assign different voltage drops to portions of the same plates.

Step 2: Expand to first order and compare the options Put $u = \alpha a/d$ and use $\ln(1+u) = u - u^2/2 + \cdots$. The leading correction is

$$C \simeq \frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right).$$

The negative correction is physically necessary because most of the area has separation greater than d; option (A) wrongly increases the capacitance. The average first-order increase in gap is $\alpha a/2$, which fixes the coefficient to $1/2$, excluding the coefficients $1/4$ in (B) and $3/2$ in (D). As $\alpha \to 0$, the ordinary parallel-plate result is recovered.

Step 3: Conclusion The capacitance is close to $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: For a small linear variation of plate spacing, the first-order correction can be checked using the mean extra gap, $\alpha a/2$; the capacitance must decrease when the plates tilt apart.

45. A carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is : [4 marks]

(A) 90 J

(B) 99 J

(C) 1 J

(D) 100 J

Correct Answer: (A) 90 J

Solution:

Concept:

- A reversible Carnot engine has efficiency $\eta = W/Q_H = 1 - T_C/T_H$, where Q_H is heat received from the hot reservoir and Q_C heat rejected to the cold reservoir.
- Reversing the cycle preserves the heat and work magnitudes but reverses their directions. The refrigerator absorbs Q_C, receives work W, and rejects $Q_H = Q_C + W$.

Step 1: Find the heat associated with the hot reservoir The stated engine efficiency is $\eta = 1/10$. For the same reversible cycle operated as a refrigerator, the work input is $W = 10\,\text{J}$, so

$$Q_H = \frac{W}{\eta} = \frac{10}{1/10} = 100\,\text{J}.$$

This is the heat delivered to the hot reservoir in refrigerator operation, rather than the heat removed from the cold reservoir. The reversal changes the directions of energy transfer, not the ratio between their magnitudes.

Step 2: Apply energy conservation to find the absorbed heat Over a complete cycle the working substance returns to its initial state, so its net internal-energy change is zero. Therefore

$$Q_C = Q_H - W = 100 - 10 = 90\,\text{J}.$$

Equivalently, the refrigerator coefficient of performance is $\beta = Q_C/W = (1-\eta)/\eta = 9$, giving $Q_C = 9 \times 10\,\text{J}$. Option (D), $100\,\text{J}$, is the rejected heat, not the absorbed heat; (B) and (C) fail the energy balance with the stated efficiency and work.

Step 3: Conclusion The refrigerator absorbs $90\,\text{J}$ from the lower-temperature reservoir. Hence, the correct option is (A).

Quick Tip to Solve: For a Carnot engine reversed into a refrigerator, use $\beta = 1/\eta - 1$. Multiplying this by the work gives the heat removed from the cold reservoir, not the heat delivered to the hot one.

Physics – Section B: Numerical Value Questions (20 marks)

46. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is: [4 marks]

Solution:

Concept:

- For hydrogen, the Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$. The Balmer series contains transitions ending at $n_f = 2$.
- The first and second members correspond to $n_i = 3$ and $n_i = 4$, respectively. Taking the ratio of their wavenumbers eliminates the Rydberg constant and avoids needing an additional physical constant.

Step 1: Write the two Balmer wavenumbers The first member is the $3 \to 2$ transition, whereas the second member is $4 \to 2$. Thus

$$\frac{1}{\lambda_1} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36},$$

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R_H}{16}.$$

The second transition has a greater photon energy, and hence should have a shorter wavelength than the first; this provides a useful check on the ratio.

Step 2: Take the ratio and convert the wavelength to nanometres Because wavelength is the reciprocal of wavenumber,

$$\frac{\lambda_2}{\lambda_1} = \frac{5/36}{3/16} = \frac{20}{27}.$$

Using the given $\lambda_1 = 6561\,\text{Å}$,

$$\lambda_2 = 6561 \times \frac{20}{27}\,\text{Å} = 4860\,\text{Å}.$$

Since $1\,\text{nm} = 10\,\text{Å}$, this gives $\lambda_2 = 486\,\text{nm}$. Both conversion and the expected decrease in wavelength are satisfied. The answer is a wavelength, so it must retain its unit rather than be reported as an energy or wavenumber.

Step 3: Conclusion The second member of the Balmer series has wavelength $\boxed{486\,\text{nm}}$. The required numerical value is 486.

Quick Tip to Solve: In the Balmer series the lower level stays at 2. Increase the upper level from 3 to 4 for the second member, then take a ratio to cancel the Rydberg constant.

47. The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20Ω and 5Ω, is connected to the parallel combination of two resistors 30Ω and R Ω. The voltage difference across the battery of internal resistance 20Ω is zero, the value of R (in Ω) is : _____

[4 marks]

Solution:

Concept:

- For a cell delivering current I, the terminal voltage is $V = E - Ir$, where E is emf and r its internal resistance. A zero terminal voltage can occur even though the emf is nonzero.
- Series-aiding cells have emfs and internal resistances that add. For external resistors in parallel, $R_L = 30R/(30 + R)$, with every resistance expressed in ohms.

Step 1: Use the zero terminal voltage to determine the current For the battery with $E = 10\,\text{V}$ and $r = 20\,\Omega$, the terminal-voltage condition gives

$$0 = 10 - 20I, \qquad I = 0.50\,\text{A}.$$

Both cells carry this same current because they are in series. Their total emf is $20\,\text{V}$ and total internal resistance is $25\,\Omega$. The terminal voltage of the other cell is $10 - (0.50)(5) = 7.5\,\text{V}$, so the external parallel network must also have a voltage of $7.5\,\text{V}$.

Step 2: Find the load resistance and solve for the unknown resistor Applying the complete circuit relation yields

$$I = \frac{20}{25 + R_L}, \qquad 25 + R_L = \frac{20}{0.50} = 40,$$

so $R_L = 15\,\Omega$. Therefore

$$\frac{30R}{30 + R} = 15, \qquad 30R = 450 + 15R, \qquad R = 30\,\Omega.$$

As a check, two $30\,\Omega$ resistors in parallel give $15\,\Omega$, and their individual currents are $7.5/30 = 0.25\,\text{A}$. These add to the required $0.50\,\text{A}$, which makes the $20\,\Omega$ cell's terminal voltage exactly zero.

Step 3: Conclusion The unknown resistance is $\boxed{R = 30\,\Omega}$. The numerical value to enter is 30.

Quick Tip to Solve: Use the special terminal-voltage condition before solving the full circuit: zero terminal voltage immediately gives $I = E/r$, after which the parallel load is easy to determine.

48. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}s$ before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _____.

[4 marks]

Solution:

Concept:

- A body dropped from rest through a height H under constant gravitational acceleration obeys $H = \frac{1}{2}gT^2$, where T is the total fall time.

- The distance covered in the last interval τ is the difference between positions at T and $T-\tau$: $\Delta s = gT\tau - \frac{1}{2}g\tau^2$. The acceleration is taken constant over the tower height.

Step 1: Express the total fall and the final half-second Let the total fall time be T seconds. Dropped from rest through $100\,\mathrm{m}$, the ball satisfies

$$100 = \frac{1}{2}gT^2, \qquad gT^2 = 200.$$

In the final $\tau = 0.5\,\mathrm{s}$, it travels $19\,\mathrm{m}$, so

$$19 = \frac{1}{2}g\left[T^2 - (T-0.5)^2\right] = \frac{gT}{2} - \frac{g}{8}.$$

Thus $152 = g(4T-1)$. Subtracting the earlier displacement is essential: $19\,\mathrm{m}$ is not the displacement during the first half-second.

Step 2: Solve and select the physically allowed fall time Substitute $g = 200/T^2$ into the interval equation:

$$152T^2 = 800T - 200, \qquad 19T^2 - 100T + 25 = 0.$$

The roots are $T = 5\,\mathrm{s}$ and $T = 5/19\,\mathrm{s}$. The latter is less than $0.5\,\mathrm{s}$ and cannot contain a final half-second of falling, so reject it. Consequently

$$g = \frac{200}{5^2} = 8\,\mathrm{m\,s^{-2}}.$$

For verification, the ball has fallen $\frac{1}{2}(8)(4.5)^2 = 81\,\mathrm{m}$ after $4.5\,\mathrm{s}$, leaving exactly $100 - 81 = 19\,\mathrm{m}$ for the last half-second.

Step 3: Conclusion The acceleration due to gravity on the planet is $\boxed{8\,\mathrm{m\,s^{-2}}}$. The required numerical value is 8.

Quick Tip to Solve: For a last-interval free-fall question, subtract two displacement formulas and check that the total time exceeds the specified interval; an algebraic root can otherwise be physically impossible.

49. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in litres) are taken from each containers and mixed (assume no loss of heat during the process) The value of θ (in °C to the nearest integer) is .....

[4 marks]

C_1	C_2	C_3	T
1*l*	2*l*	–	60°C
–	1*l*	2*l*	30°C
2*l*	–	1*l*	60°C
1*l*	1*l*	1*l*	θ

Solution:

Concept:

- When water samples are mixed without heat loss, the heat lost by the warmer samples equals the heat gained by the cooler samples. Their final temperature is a mass-weighted mean of the initial temperatures.
- For water of the same density and specific heat, these common factors cancel. Thus, with volumes V_i and temperatures t_i, $T = \dfrac{\sum V_i t_i}{\sum V_i}$.

Step 1: Translate the three measured mixtures into equations Let the initial temperatures in containers C_1, C_2, C_3 be t_1, t_2, t_3, respectively, in degrees Celsius. The first row mixes one litre from the first container with two litres from the second, giving three litres at $60°\text{C}$. Applying the same heat balance to the next two rows gives

$$t_1 + 2t_2 = 3(60) = 180,$$
$$t_2 + 2t_3 = 3(30) = 90,$$
$$2t_1 + t_3 = 3(60) = 180.$$

The dash denotes that no water is taken from that container; it does not mean that its temperature is zero.

Step 2: Add the equations to find the equal-volume mean Adding the three equations gives each initial temperature a coefficient of three:

$$3t_1 + 3t_2 + 3t_3 = 180 + 90 + 180 = 450.$$

Therefore, $t_1 + t_2 + t_3 = 150$. The final row uses one litre from each container, so all three masses have equal weights:

$$\theta = \frac{t_1 + t_2 + t_3}{3} = \frac{150}{3} = 50°\text{C}.$$

As a check, solving individually gives $t_1 = 80^\circ\text{C}$, $t_2 = 50^\circ\text{C}$, and $t_3 = 20^\circ\text{C}$; these reproduce all three measured mixture temperatures.

Step 3: Conclusion The required temperature, to the nearest integer, is $\boxed{50^\circ\text{C}}$.

Quick Tip to Solve: When several mixture equations are given, first add them: equal coefficients of the unknown temperatures can reveal the desired equal-volume mean directly.

50. An asteroid is moving directly towards the centre of the earth. When at a distance of 10R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s) ? Give your answer to the nearest integer in kilometer/s _____.

[4 marks]

Solution:

Concept:

- Outside a spherical Earth, gravitational potential energy is $U = -GMm/r$, taking zero potential at infinity. With atmospheric resistance neglected, the sum of kinetic and potential energies stays constant.
- The escape speed at the surface obeys $v_e^2 = 2GM/R$. This relation replaces the unknown combination of Earth mass and radius with the given escape speed.

Step 1: Write energy conservation between the two radii Let $u = 12\,\text{km}\,\text{s}^{-1}$ be the initial speed at distance $10R$ from Earth's centre, and let v be the impact speed at R. The asteroid loses gravitational potential energy as it approaches Earth; hence its speed must increase. The conservation equation is

$$\frac{1}{2}mu^2 - \frac{GMm}{10R} = \frac{1}{2}mv^2 - \frac{GMm}{R}.$$

Cancel the asteroid's mass and rearrange to obtain

$$v^2 = u^2 + 2GM\left(\frac{1}{R} - \frac{1}{10R}\right).$$

The starting distance is measured from the centre, so its potential is evaluated at $10R$, rather than $11R$.

Step 2: Use the escape speed and round the result Using $2GM/R = v_e^2$, the increase in squared speed is $(9/10)v_e^2$. Keep both given speeds in the same units:

$$\begin{aligned} v^2 &= 12^2 + \frac{9}{10}(11.2)^2 \\ &= 144 + 112.896 \\ &= 256.896\ \text{km}^2\,\text{s}^{-2}. \end{aligned}$$

Taking the positive square root gives $v \approx 16.028\,\text{km}\,\text{s}^{-1}$. This exceeds the initial speed, as the gravitational energy balance requires. Round only the final speed, because rounding the intermediate

escape-speed contribution would reduce accuracy unnecessarily.

Step 3: Conclusion The speed when the asteroid strikes Earth is $\boxed{16\,\mathrm{km\,s^{-1}}}$, to the nearest integer.

Quick Tip to Solve: Use the potential-energy difference between the actual initial and final radii; the escape speed enters through its square, so speeds cannot be added directly.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Arrange the following bonds according to their average bond energies in descending order :
C–Cl, C–Br, C–F, C–I [4 marks]

(A) C–Br > C–I > C–Cl > C–F

(B) C–Cl > C–Br > C–I > C–F

(C) C–I > C–Br > C–Cl > C–F

(D) C–F > C–Cl > C–Br > C–I

Correct Answer: (D) C–F > C–Cl > C–Br > C–I

Solution:

Concept:

- The average bond energy measures the energy needed to break one mole of a specified type of covalent bond in the gaseous state. A stronger bond has a larger positive bond energy.
- For comparable carbon-halogen single bonds, increasing the halogen size increases bond length and reduces effective overlap with the carbon bonding orbital. The usual bond-strength trend is therefore downward from fluorine to iodine.

Step 1: Compare the halogen sizes and carbon-halogen overlap The halogen atoms occur down Group 17 in the order $\mathrm{F, Cl, Br, I}$. Their valence shells become progressively larger and more diffuse, so the carbon-halogen distance increases in the same sequence. The compact fluorine valence orbital gives a short and strong carbon-fluorine bond. Chlorine, bromine and iodine form progressively longer bonds with carbon, with less effective overlap. Thus the energy required to separate the bonded atoms decreases down this series. This comparison concerns carbon-halogen bonds; it should not be confused with the separate trend for halogen-halogen bonds.

Step 2: Arrange the bond energies and check every proposed order The descending bond-energy order is

$$\mathrm{C{-}F > C{-}Cl > C{-}Br > C{-}I.}$$

Option (D) reproduces all four positions in this sequence. Options (A) and (B) put the carbon-fluorine bond last, even though it is the strongest in the given family. Option (C) reverses the physical trend and puts the longest carbon-iodine bond first. Each of these alternatives also fails at least one comparison among chlorine, bromine and iodine. The requested order is descending energy, so the strongest bond must be written at the left.

Step 3: Conclusion The correct descending order is $\mathrm{C{-}F > C{-}Cl > C{-}Br > C{-}I}$. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed carbon atom bonded to different halogens, compare halogen size: the shorter bond is generally stronger, so write fluorine first and iodine last.

52. The radius of the second Bohr orbit, in terms of the Bohr radius, a_0, in Li^{2+} is : [4 marks]

(A) $\frac{2a_0}{9}$

(B) $\frac{2a_0}{3}$

(C) $\frac{4a_0}{9}$

(D) $\frac{4a_0}{3}$

Correct Answer: (D) $\frac{4a_0}{3}$

Solution:

Concept:

- A hydrogen-like ion contains only one electron and is described by the Bohr orbit formula $r_n = a_0 n^2/Z$, where n is the principal quantum number, Z is nuclear charge number, and a_0 is the first-orbit radius of hydrogen.
- The radius grows as n^2 but decreases as Z. Increasing nuclear charge attracts the same single electron more strongly; ionic charge is not the nuclear charge number.

Step 1: Identify the appropriate nuclear charge and orbit Lithium has atomic number $Z = 3$. Neutral lithium has three electrons; removing two leaves the one-electron ion Li^{2+}, so the hydrogen-like formula applies. The second Bohr orbit means $n = 2$, not $n = 1$. The nucleus still contains three protons after the electrons have been removed. Therefore, the denominator in the radius formula is three. Using the ionic charge two in that denominator would describe neither the Coulomb force nor the Bohr orbit of this ion.

Step 2: Substitute into the radius formula and compare the options The Bohr model combines Coulomb attraction with centripetal force and the quantization condition $mvr = n\hbar$; eliminating v gives the stated n^2/Z dependence. Thus,

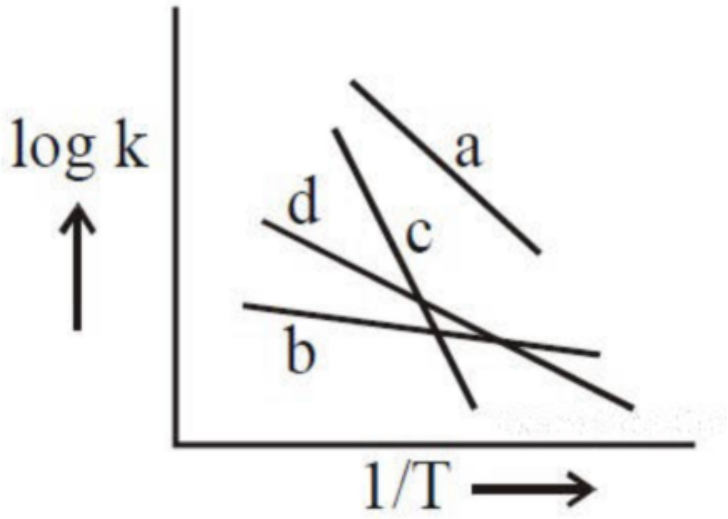
$$r_2 = a_0 \frac{2^2}{3} = \frac{4a_0}{3}.$$

This also equals four times the first-orbit radius of the same ion, since $r_1 = a_0/3$. Options (A) and (C) incorrectly use a denominator of nine, corresponding to a spurious Z^2 dependence of radius. Option (B) has the right denominator but uses n in place of n^2. Only option (D) has both dependences correct. The result is a length expressed as a multiple of a_0, as requested.

Step 3: Conclusion The second-orbit radius of Li^{2+} is $\boxed{4a_0/3}$. Hence, the correct option is (D).

Quick Tip to Solve: For a one-electron ion, write $r_n/a_0 = n^2/Z$ first; use the atomic number for Z, and square only the orbit number.

53. Consider the following plots of rate constant versus $\frac{1}{T}$ for four different reactions. Which of the following orders is correct for the activation energies of these reactions? [4 marks]



(A) $E_c > E_a > E_d > E_b$

(B) $E_a > E_c > E_d > E_b$

(C) $E_b > E_a > E_d > E_c$

(D) $E_b > E_d > E_c > E_a$

Correct Answer: (A) $E_c > E_a > E_d > E_b$

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E/(RT)}$, where E is activation energy, A the pre-exponential factor, and R the gas constant. For common logarithms it becomes $\log k = \log A - E/(2.303RT)$.
- A plot of $\log k$ against $1/T$ has slope $m = -E/(2.303R)$. Consequently the magnitude of its negative slope, rather than the vertical position of the line, determines the activation energy.

Step 1: Read the axes and rank the slope magnitudes The vertical axis in the displayed graph is $\log k$, and the horizontal axis is $1/T$. All four lines slope downwards, consistent with positive activation energies. Line c is the steepest descending line, followed by a, then d; line b is nearly horizontal and has the smallest downward inclination. Therefore, the magnitudes satisfy

$$|m_c| > |m_a| > |m_d| > |m_b|.$$

Line crossings do not change this ranking: they compare rate constants at particular temperatures, whereas activation energy controls the change in logarithmic rate constant as reciprocal temperature changes.

Step 2: Convert slope magnitudes into the energy order Because the same positive factor $2.303R$ multiplies each slope magnitude, $E_i = 2.303R|m_i|$ preserves that order. It follows that

$$E_c > E_a > E_d > E_b.$$

This is exactly option (A). Option (B) interchanges a and c, despite c having the greater inclination magnitude. Options (C) and (D) place b first, although its almost horizontal line shows it has the smallest activation energy. Comparing signed slopes directly would reverse the interpretation, since the most negative slope corresponds to the largest positive energy.

Step 3: Conclusion The activation energies obey $\boxed{E_c > E_a > E_d > E_b}$. Hence, the correct option is (A).

Quick Tip to Solve: On a $\log k$ versus $1/T$ graph, rank downward steepness, ignoring intercepts: the steepest line has the greatest activation energy.

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54. Hydrogen has three isotopes (A), (B) and (C). If the number of neutron(s) in (A), (B) and (C) respectively, are (x), (y) and (z), the sum of (x), (y) an (z) is : [4 marks]

(A) 3

(B) 1

(C) 4

(D) 2

Correct Answer: (A) 3

Solution:

Concept:

- Isotopes are atoms of the same element with equal atomic number but different mass numbers. The number of neutrons is $N = A - Z$, where A is mass number and Z is proton number.
- The three isotopes of hydrogen are protium ${}^{1}_{1}\mathrm{H}$, deuterium ${}^{2}_{1}\mathrm{H}$, and tritium ${}^{3}_{1}\mathrm{H}$. Each has one proton, because its atomic number is one.

Step 1: Find the neutron count for each hydrogen isotope Apply the neutron-count relation separately to the three nuclei. Protium has mass number one, so its single nucleon is the proton and it has no neutron. Deuterium has two nucleons, one proton and one neutron. Tritium has three nucleons, one proton and two neutrons. In equations,

$$\begin{aligned} N(\text{protium}) &= 1 - 1 = 0, \\ N(\text{deuterium}) &= 2 - 1 = 1, \\ N(\text{tritium}) &= 3 - 1 = 2. \end{aligned}$$

The labels (A), (B) and (C) may assign these isotopes in any order, but their sum is independent of that assignment.

Step 2: Add the neutron counts and test the alternatives The required total is therefore

$$x + y + z = 0 + 1 + 2 = 3.$$

The calculation counts only neutrons, so neither the three electrons of the neutral atoms nor their three protons enter the sum. As an independent check, the total mass number is $1 + 2 + 3 = 6$;

subtracting the total proton count $1 + 1 + 1 = 3$ again gives three neutrons. Option (A) gives this value. Option (B) counts only the neutron in deuterium, option (D) gives only the tritium neutron count, and option (C) is inconsistent with both the individual isotope counts and the total-nucleon check.

Step 3: Conclusion The sum of the neutron counts is $\boxed{3}$. Hence, the correct option is (A).

Quick Tip to Solve: For isotopes, keep the proton number fixed and subtract it from each mass number before adding the neutron counts; hydrogen-1 contributes zero.

55. The increasing order of the atomic radii of the following elements is :-

(a) C (b) O

(c) F (d) Cl

(e) Br [4 marks]

(A) (a) < (b) < (c) < (d) < (e)

(B) (c) < (b) < (a) < (d) < (e)

(C) (b) < (c) < (d) < (a) < (e)

(D) (d) < (c) < (b) < (a) < (e)

Correct Answer: (B) (c) < (b) < (a) < (d) < (e)

Solution:

Concept:

- Atomic radius generally decreases from left to right across a period because effective nuclear charge increases while the principal shell remains the same.
- Atomic radius generally increases down a group because a new occupied electron shell is added. Compare elements within one period first and then place the heavier halogens.

Step 1: Order the second-period atoms Carbon, oxygen and fluorine all have their outer electrons in the second shell. From carbon to fluorine, nuclear charge rises, while shielding by inner electrons changes little. The outer electron cloud contracts. Thus the second-period order is

$$r(\mathrm{F}) < r(\mathrm{O}) < r(\mathrm{C}).$$

In the question's labels this gives (c) < (b) < (a). The reverse ordering in option (A) therefore contradicts the across-period trend.

Step 2: Place chlorine and bromine Chlorine has three occupied shells and bromine has four. Their radii exceed those of the listed second-period atoms, and bromine is larger than chlorine. In particular, the usual atomic/covalent radius comparison places carbon below chlorine. Combining the comparisons gives

$$r(\mathrm{F}) < r(\mathrm{O}) < r(\mathrm{C}) < r(\mathrm{Cl}) < r(\mathrm{Br}).$$

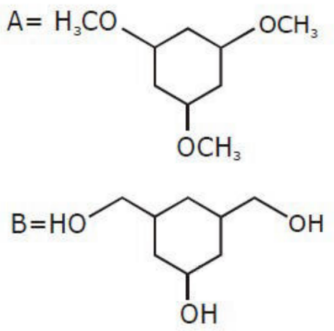
Option (C) incorrectly puts oxygen below fluorine and carbon above chlorine; option (D) incorrectly puts chlorine below the second-period atoms. Only option (B) respects both comparisons.

Step 3: Conclusion The increasing atomic-radius order is $F < O < C < Cl < Br$. Hence, the correct option is (B).

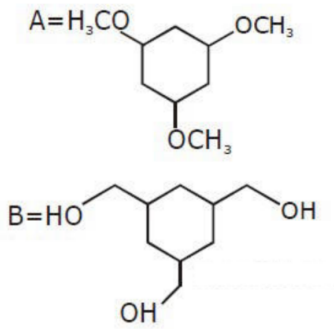
Quick Tip to Solve: First order C, O and F within period 2; then extend the halogen sequence downward from F to Cl to Br.

56. Among the compounds A and B with molecular formula $C_9H_{18}O_3$, A is having higher boiling point the B. The possible structures of A and B are : [4 marks]

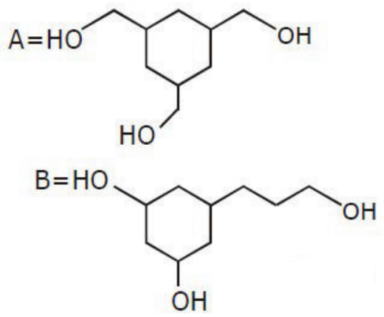
(A) A: 1,3,5-trimethoxycyclohexane; B: 3,5-bis(hydroxymethyl)cyclohexan-1-ol.



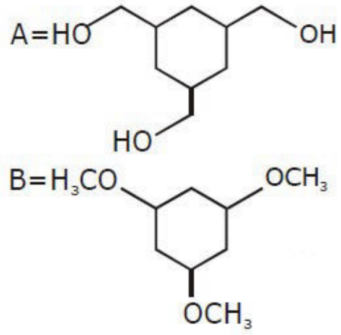
(B) A: 1,3,5-trimethoxycyclohexane; B: cyclohexane-1,3,5-triyltrimethanol.



(C) A: cyclohexane-1,3,5-triyltrimethanol; B: 5-(3-hydroxypropyl)cyclohexane-1,3-diol.

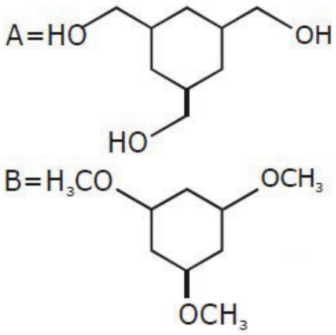


(D) A: cyclohexane-1,3,5-triyltrimethanol; B: 1,3,5-trimethoxycyclohexane.



Correct Answer: (D)

A: cyclohexane-1,3,5-triyltrimethanol; B: 1,3,5-trimethoxycyclohexane.



Solution:

Concept:

- For isomers of the same molecular formula, boiling point depends strongly on intermolecular attractions. Alcohol groups can both donate and accept hydrogen bonds, whereas ether groups cannot donate them.
- A proposed pair must satisfy both the molecular formula $C_9H_{18}O_3$ and the direction of the boiling-point comparison. Stronger intermolecular association generally requires more energy for vaporization.

Step 1: Check the two structures in option D Compound A is a cyclohexane ring carrying three hydroxymethyl groups, $-CH_2OH$. Counting six ring carbons and three substituent carbons gives nine carbons. The substituted ring contributes C_6H_9, and the three hydroxymethyl groups together contribute $C_3H_9O_3$, giving $C_9H_{18}O_3$. Compound B is a cyclohexane ring with three methoxy groups, $-OCH_3$; the corresponding count also gives $C_9H_{18}O_3$. Thus both structures meet the formula.

Step 2: Compare the intermolecular attractions A has three hydroxyl groups and can form an extended intermolecular hydrogen-bond network with other A molecules. B has three ether oxygens but no O-H bond, so molecules of pure B cannot donate hydrogen bonds to one another. Dispersion and dipolar attractions remain, but the strong alcohol hydrogen-bond association makes A harder to vaporize. Therefore option (D) has the required higher boiling point for A.

Step 3: Reject the remaining pairs In option (A), the proposed B has two hydroxymethyl groups and one ring hydroxyl group, giving only eight carbons; the pair fails the stated common formula. Option

(B) reverses the needed comparison by making A the triether and B the hydrogen-bonding triol. In option (C), B has two hydroxyl groups directly on the ring and a flexible terminal hydroxypropyl chain; it is not the non-hydrogen-bonding triether comparison that establishes the stated order. Relative to A, its less compact structure and possible intermolecular association do not support assigning A the higher boiling point. The clear matching alcohol-versus-ether pair is (D).

Step 4: Conclusion The pair with the hydroxymethyl triol as A and the trimethoxy compound as B satisfies the formula and the higher boiling point of A through intermolecular hydrogen bonding. Hence, the correct option is (D).

Quick Tip to Solve: Count carbon atoms before comparing boiling points, then look for O-H donors: ether oxygens alone cannot form a pure-liquid hydrogen-bond network.

57. Two monomers in maltose are : [4 marks]

(A) α-D-glucose and α-D-galactose

(B) α-D-glucose and α-D-Fructose

(C) α-D-glucose and α-D-glucose

(D) α-D-glucose and β-D-glucose

Correct Answer: (C) α-D-glucose and α-D-glucose

Solution:

Concept:

- Maltose is a disaccharide built from two D-glucose residues. Hydrolysis breaks its glycosidic bond and releases the constituent monosaccharides.
- The glycosidic linkage in maltose is $\alpha(1 \rightarrow 4)$: the anomeric carbon of the first glucose is linked to the hydroxyl group at C-4 of the second glucose.

Step 1: Identify the sugar residues Maltose is formed by condensation of two glucose units with loss of one molecule of water. Its hydrolysis can be represented as

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow 2\,C_6H_{12}O_6.$$

Both products are D-glucose, rather than galactose or fructose. Therefore option (A), which contains galactose, and option (B), which contains fructose, fail on the identity of one of the component sugars. The fructose-containing familiar disaccharide is sucrose, not maltose.

Step 2: Apply the linkage description In the standard description used for maltose formation, the two monomers are two α-D-glucose units, joined through an $\alpha(1 \rightarrow 4)$ bond. Thus option (C) is the stated pair. The second glucose retains a free anomeric carbon, so dissolved maltose can mutarotate at its reducing end; this does not change the identity of its two glucose residues or make its glycosidic linkage a beta linkage. Option (D) is not the standard pair requested here. The alpha descriptor of the glycosidic bond is fixed by the first, bonded anomeric carbon.

Step 3: Conclusion Maltose is described as being formed from α-D-glucose and α-D-glucose with an $\alpha(1 \rightarrow 4)$ glycosidic linkage. Hence, the correct option is (C).

Quick Tip to Solve: Remember maltose as glucose + glucose with an alpha-1,4 link; use the identity of both sugars before checking the linkage.

58. For the following Assertion and Reason, the correct option is :

Assertion : The pH of water increases with increase in temperature.

Reason : The dissociation of water into H^+ and OH^- is an exothermic reaction. [4 marks]

(A) Both assertion and reason are false.

(B) Both assertion and reason are true, but the reason is not the correct explanation for the assertion.

(C) Both assertion and reason are true, and the reason is the correct explanation for the assertion.

(D) Assertion is not true, but reason is true.

Correct Answer: (A) Both assertion and reason are false.

Solution:

Concept:

- For pure water, $K_w = [H^+][OH^-]$, and neutrality means $[H^+] = [OH^-]$. Therefore $pH = \frac{1}{2}pK_w$.
- Autoionization of water is endothermic. Raising temperature increases K_w, so the neutral pH decreases rather than remaining fixed at 7.

Step 1: Test the assertion In pure water write $[H^+] = [OH^-] = x$. The ionic product gives $x^2 = K_w$, hence

$$pH = -\log\sqrt{K_w} = \frac{1}{2}pK_w.$$

When temperature increases, water ionizes more and K_w increases. Consequently $[H^+]$ increases and pH decreases. Thus the assertion that the pH of water increases with increasing temperature is false. Water still remains neutral because the hydrogen and hydroxide ion concentrations increase equally.

Step 2: Test the reason The equilibrium

$$H_2O\,(l) \rightleftharpoons H^+(aq) + OH^-(aq)$$

absorbs heat in the forward direction. Its ionization enthalpy is positive, so autoionization is endothermic, not exothermic. By Le Chatelier's principle, heating favours this forward process. The reason is therefore also false.

Step 3: Compare the truth combinations Since both statements are false, the combinations in (B) and (C), which require both to be true, are excluded. Option (D) is excluded because it calls the reason true. The false-false combination appears only in (A); no claim about a correct explanation can rescue a false physical statement.

Step 4: Conclusion Both the assertion and the reason are false: heating lowers the neutral pH, and water ionization is endothermic. Hence, the correct option is (A).

Quick Tip to Solve: Neutrality means equal ion concentrations, not an invariant pH of 7. Use neutral pH = half of pKw at the stated temperature.

59. The correct order of the calculated spin-only magnetic moments of complexs (A) to (D) is:

(A) $Ni(CO)_4$

(B) $[Ni(H_2O)_6]Cl_2$

(C) $Na_2[Ni(CN)_4]$

(D) $PdCl_2(PPh_3)_2$ [4 marks]

(A) $(C) < (D) < (B) < (A)$

(B) $(C) \approx (D) < (B) < (A)$

(C) $(A) \approx (C) \approx (D) < (B)$

(D) $(A) \approx (C) < (B) \approx (D)$

Correct Answer: (C) $(A) \approx (C) \approx (D) < (B)$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton.
- Count d electrons from the metal oxidation state, then use the ligand field and geometry to decide whether those electrons are paired. A d-electron count alone does not determine magnetism.

Step 1: Count unpaired electrons in the nickel complexes In (A), $Ni(CO)_4$, neutral CO gives nickel oxidation state zero, hence a $3d^{10}$ configuration. All electrons are paired and $n = 0$. In (B), $[Ni(H_2O)_6]Cl_2$, nickel is $+2$, so it is $3d^8$. Its octahedral aqua complex has $t_{2g}^6 e_g^2$ with two unpaired electrons:

$$\mu_B^{(\text{complex B})} = \sqrt{2(2+2)}\,\mu_B = \sqrt{8}\,\mu_B.$$

Here the μ_B on the right is the unit, not the label of the complex.

Step 2: Check cyanide and palladium complexes In (C), $Na_2[Ni(CN)_4]$, nickel is also $+2$, but strong-field cyanide produces a square-planar d^8 complex with paired electrons. Thus its spin-only moment is zero. In (D), neutral $PdCl_2(PPh_3)_2$, the two chloride ligands make palladium $+2$. This $4d^8$ ion forms a square-planar complex, again with all electrons paired. Therefore (D) also has zero spin-only moment.

Step 3: Assemble the order The unpaired-electron counts for (A), (B), (C) and (D) are respectively 0, 2, 0 and 0. Thus

$$\mu(A) = \mu(C) = \mu(D) = 0 < \mu(B) = \sqrt{8}\,\mu_B.$$

Options (A) and (B) wrongly make the carbonyl complex the most magnetic, and option (D) wrongly groups the palladium complex with the paramagnetic aqua complex.

Step 4: Conclusion The nickel carbonyl, nickel cyanide and palladium complexes are diamagnetic; only the octahedral nickel aqua complex has a nonzero spin-only moment, $\sqrt{8}\,\mu_B$. Hence, the correct option is (C).

Quick Tip to Solve: For d8 complexes check geometry: octahedral nickel(II) has two unpaired electrons, while square-planar nickel(II) and palladium(II) are paired.

60. Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?

[4 marks]

(A) $C_6H_5NH_2$

(B) $NH_2 - \overset{O}{\overset{\|}{C}} - NH_2$

(C) $C_6H_5NO_2$

(D) CH_3CH_2-C $\equiv$ N

Correct Answer: (C) $C_6H_5NO_2$

Solution:

Concept:

- Kjeldahl analysis converts suitable organic nitrogen into ammonium sulfate by digestion with concentrated sulfuric acid. Alkali then liberates ammonia for quantitative estimation.
- The method is unsuitable for nitro and azo nitrogen and for nitrogen present in a heterocyclic ring, because ordinary digestion does not quantitatively convert those forms into ammonium sulfate.

Step 1: Identify the nitrogen environments Option (A), $C_6H_5NH_2$, is aniline: its nitrogen is in an amino group attached to a benzene carbon, not a member of the ring. Option (B), NH_2CONH_2, is urea, with amide-type nitrogen. Option (C), $C_6H_5NO_2$, is nitrobenzene, containing a nitro group. Option (D), CH_3CH_2CN, is a nitrile. Distinguishing the functional group is essential; merely containing an aromatic ring does not make aniline nitrogen heterocyclic.

Step 2: Apply the digestion requirement Aniline and urea are standard substrates for Kjeldahl nitrogen estimation, as their nitrogen can be recovered as ammonium ions on digestion. Nitrile nitrogen can also be converted through hydrolytic digestion into ammonium ions and is not the excluded class here. Nitro nitrogen is the decisive exception: in the usual Kjeldahl procedure it is not quantitatively changed into ammonium sulfate. Therefore measuring the ammonia subsequently released would not reliably measure all the nitrogen of nitrobenzene.

Step 3: Eliminate the alternatives Options (A), (B) and (D) do not contain the nitro, azo or ring-nitrogen arrangements responsible for the stated limitation. Option (C) explicitly contains $-NO_2$, so it is the compound to reject for the ordinary method. Special modified reduction procedures are outside the usual Kjeldahl method asked in this question.

Step 4: Conclusion Kjeldahl's method cannot be used in its ordinary form to estimate nitrogen in nitrobenzene, $C_6H_5NO_2$. Hence, the correct option is (C).

Quick Tip to Solve: Check the nitrogen functional group first: nitro, azo and heterocyclic ring nitrogen are the familiar exclusions from ordinary Kjeldahl analysis.

61. The major product [B] in the following sequence of reactions is :- [4 marks]

$$CH_3-C(=CH-CH_2CH_3)-CH(CH_3)_2 \xrightarrow[\text{(ii) } H_2O_2, OH^-]{\text{(i) } B_2H_6} [A] \xrightarrow[\Delta]{\text{dil. } H_2SO_4} [B]$$

(A) $CH_2=C(CH(CH_3)_2)CH_2CH_2CH_3$

(B) $CH_3C(CH(CH_3)_2)=CHCH_2CH_3$

(C) $(CH_3)_2C=C(CH_3)CH_2CH_2CH_3$

(D) $CH_3CH(CH(CH_3)_2)CH=CHCH_3$

Correct Answer: (C) $(CH_3)_2C=C(CH_3)CH_2CH_2CH_3$

Solution:

Concept:

- Hydroboration followed by alkaline peroxide gives anti-Markovnikov hydration: the OH group goes to the less substituted carbon of the alkene without carbocation rearrangement.
- Acid-catalysed dehydration can proceed through a carbocation. A hydride shift that makes a more stable carbocation is allowed; the major alkene generally has the more substituted double bond.

Step 1: Locate the alcohol group In $CH_3C(CH(CH_3)_2)=CHCH_2CH_3$, the right-hand alkene carbon has one hydrogen and is less substituted. Hydroboration–oxidation therefore gives $CH_3CH(CH(CH_3)_2)CH(OH)CH_2CH_3$. The carbon skeleton is preserved in this addition, so the alcohol is secondary.

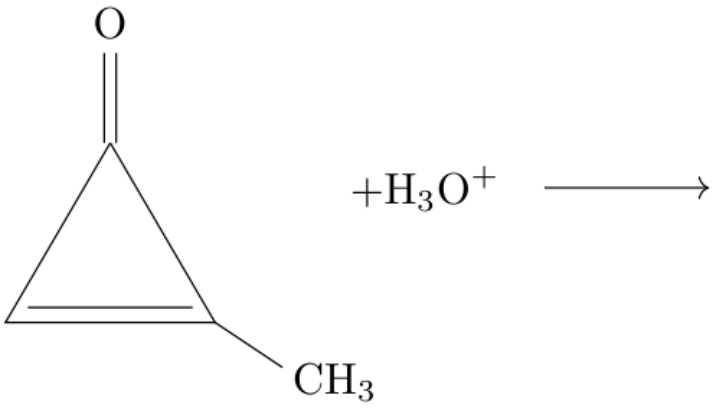
Step 2: Form and rearrange the carbocation On heating with acid, protonation turns OH into a good leaving group. Water departs from the OH-bearing carbon to give a secondary carbocation. A 1,2-hydride shift from the adjacent branched carbon moves the charge onto that carbon and gives a tertiary carbocation. The shift also changes the former OH-bearing unit into a CH_2 unit of an n-propyl side chain.

Step 3: Choose the most substituted elimination product Loss of the beta hydrogen from the isopropyl methine carbon then forms $(CH_3)_2C=C(CH_3)CH_2CH_2CH_3$, a tetrasubstituted alkene. Option A is terminal and less substituted; B is the trisubstituted starting alkene, while D has a disubstituted double bond. These alternatives lack the maximum alkyl substitution available after rearrangement.

Step 4: Conclusion The major product [B] is $(CH_3)_2C=C(CH_3)CH_2CH_2CH_3$. Hence, the correct option is (C).

Quick Tip to Solve: In hydration followed by acidic dehydration, locate OH first, then check hydride shifts before selecting the most substituted alkene.

62. The major product in the following reaction is: [4 marks]



(A) 2-Methylcycloprop-2-ene-1,1-diol (the geminal diol at the original carbonyl carbon).

(B) 3-Hydroxy-2-methylcyclopropan-1-one (OH on the alkene carbon without methyl).

(C) 2-Hydroxy-2-methylcyclopropan-1-one (OH and methyl on the same ring carbon).

(D) The oxygen-protonated cyclopropenone: OH at the original carbonyl carbon, with a positive charge on that ring carbon and the original ring C=C bond retained.

Correct Answer: (D) The oxygen-protonated cyclopropenone: OH at the original carbonyl carbon, with a positive charge on that ring carbon and the original ring C=C bond retained.

Solution:

Concept:

- An oxygen lone pair of a carbonyl group accepts a proton from H_3O^+. The resulting conjugate acid has resonance contributors with positive charge on the carbonyl carbon.
- A planar cyclic system with $4n + 2$ pi electrons is aromatic. The three-membered cyclopropenyl cation framework has two pi electrons and an empty p orbital, permitting cyclic delocalisation.

Step 1: Protonate the carbonyl oxygen The reactant is a methyl-substituted cyclopropenone. Its carbonyl oxygen is the basic site. Proton transfer from H_3O^+ gives an oxygen-protonated carbonyl, with water as the other product. Resonance can move the carbonyl pi-electron pair onto oxygen, leaving an OH group and a positive charge on the ring carbon that originally bore oxygen.

Step 2: Recognise the stable ring cation This contributor retains the C=C bond between the other two ring carbons. Its two pi electrons can delocalise around the three p orbitals of the ring, giving an aromatic cyclopropenyl cation framework. Option D depicts precisely this OH-substituted cation. The methyl substituent does not interrupt the cyclic conjugation.

Step 3: Reject the addition products Option A is the geminal diol formed by adding water across C=O, rather than the aromatic protonated species. B and C add water across the ring C=C bond, destroying the favourable conjugated cation framework; their different OH positions do not provide aromatic stabilisation. The protonated ring species is therefore selected over these neutral hydration structures.

Step 4: Conclusion The major species is the resonance-stabilised oxygen-protonated cyclopropenone represented by the OH-bearing cyclopropenyl cation. Hence, the correct option is (D).

Quick Tip to Solve: For cyclopropenones in acid, examine the aromatic two-pi-electron resonance form before applying ordinary alkene hydration rules.

63. A metal (A) on heating in nitrogen gas gives compound B. B on treatment with H_2O gives a colourless gas which when passed through $CuSO_4$ solution gives a dark blue-violet coloured solution. A and B respectively, are : [4 marks]

(A) Na and $NaNO_3$

(B) Na and Na_3N

(C) Mg and Mg_3N_2

(D) Mg and $Mg(NO_3)_2$

Correct Answer: (C) Mg and Mg_3N_2

Solution:

Concept:

- A reactive metal can combine with nitrogen on heating to give its nitride. Hydrolysis of a metal nitride releases ammonia.
- Excess ammonia coordinates to aqueous copper(II), producing the intensely blue tetraamminecopper(II) complex. The colour therefore identifies ammonia, not a nitrogen oxide.

Step 1: Identify the colourless gas The dark blue-violet colour obtained with copper(II) sulfate and excess gas is characteristic of ammonia. Although a small amount of ammonia first produces copper(II) hydroxide through its basicity, excess ammonia dissolves that precipitate by complex formation. The final coloured solution contains $[Cu(NH_3)_4(H_2O)_2]^{2+}$.

Step 2: Find the metal and the hydrolysable compound Magnesium burns in nitrogen to form magnesium nitride. The balanced reaction and its hydrolysis are

$$3\,Mg + N_2 \longrightarrow Mg_3N_2$$

$$Mg_3N_2 + 6\,H_2O \longrightarrow 3\,Mg(OH)_2 + 2\,NH_3$$

Thus A is magnesium and B is its nitride, not its nitrate.

Step 3: Check each offered pair Option A proposes sodium nitrate, which is not formed by direct heating of sodium in nitrogen and does not hydrolyse to ammonia. Option B incorrectly proposes the direct formation of sodium nitride; lithium is the alkali metal that readily combines directly with nitrogen. Option D also proposes a nitrate, requiring oxygen and failing the hydrolysis test. Only the magnesium–magnesium nitride pair meets every observation.

Step 4: Conclusion $A = Mg$ and $B = Mg_3N_2$. Hydrolysis releases NH_3, which gives the blue copper(II) ammine complex. Hence, the correct option is (C).

Quick Tip to Solve: Work backwards from the deep blue copper–ammonia complex: the gas is ammonia and its water-reactive precursor is a nitride.

64. An unsaturated hydrocarbon X absorbs two hydrogen molecules on catalytic hydrogenation and also gives following reaction : B(3 - oxo - hexanedicarboxylic acid) X will be :- [4 marks]

$$X \xrightarrow[Zn/H_2O]{O_3} A \xrightarrow{[Ag(NH_3)_2]^+}$$

(A) 1,3-Dimethylidenecyclohexane (two exocyclic double bonds).

(B) 3-Methylidenecyclohex-1-ene (the exocyclic carbon is adjacent to an end of the ring double bond).

(C) 4-Methylidenecyclohex-1-ene (the exocyclic carbon is separated by one saturated ring carbon from an end of the ring double bond).

(D) Cyclohex-2-en-1-one.

Correct Answer: (C) 4-Methylidenecyclohex-1-ene (the exocyclic carbon is separated by one saturated ring carbon from an end of the ring double bond).

Solution:

Concept:

- Catalytic hydrogenation consumes one H_2 per C=C bond. Reductive ozonolysis cleaves each C=C bond to carbonyl groups, with alkene hydrogens determining aldehyde formation.
- Tollens reagent oxidises aldehydes to carboxylate groups (giving carboxylic acids after work-up), while an ordinary ketone group remains unchanged.

Step 1: Use the hydrogen uptake Two hydrogen molecules indicate two reducible C=C bonds in the hydrocarbon. Options A, B and C have two double bonds. D is already an oxygen-containing ketone, so it fails the stated hydrocarbon requirement before any detailed cleavage analysis.

Step 2: Open the ring and locate the ketone Cleavage of a ring double bond bearing a hydrogen on each carbon produces a chain with an aldehyde at each end. Cleavage of an exocyclic $C{=}CH_2$ unit leaves a ketone at that ring carbon and gives a separate formaldehyde molecule. In option C, the exocyclic carbon lies two bonds along the ring from the nearer end of the internal double bond. The main-chain dialdehyde is therefore

$$OHC{-}CH_2{-}CO{-}CH_2{-}CH_2{-}CHO.$$

Step 3: Oxidise the two aldehyde ends Tollens reagent converts the terminal aldehyde groups to acid groups, retaining the central ketone:

$$HOOC{-}CH_2{-}CO{-}CH_2{-}CH_2{-}COOH.$$

Numbering from the nearer acid end places the oxo group at carbon 3, as specified. Option B instead gives the ketone adjacent to one acid end, at carbon 2. Option A has no internal ring double bond to open the ring and yields a cyclic diketone after exocyclic cleavage, so it cannot give this open-chain diacid.

Step 4: Conclusion The hydrocarbon is 4-methylidenecyclohex-1-ene, whose ozonolysis and subsequent aldehyde oxidation give the required 3-oxo diacid skeleton. Hence, the correct option is (C).

Quick Tip to Solve: In ozonolysis reconstructions, keep track of the distance from each cleaved ring bond to the carbon that becomes the ketone.

65. Among (a) – (d) the complexes that can display geometrical isomerism are :

(a) $[Pt(NH_3)_3Cl]^+$

(b) $[Pt(NH_3)Cl_5]^-$

(c) $[Pt(NH_3)_2Cl(NO_2)]$

(d) $[Pt(NH_3)_4ClBr]^{2+}$ [4 marks]

(A) (a) and (b)

(B) (c) and (d)

(C) (d) and (a)

(D) (b) and (c)

Correct Answer: (B) (c) and (d)

Solution:

Concept:

- Geometrical isomers have identical connectivity but different relative positions of ligands. Square-planar complexes can have adjacent (cis) or opposite (trans) ligand arrangements.
- Octahedral complexes of the form MA_4BC can have B and C cis or trans. By contrast, MA_5B has only one geometrical arrangement.

Step 1: Classify the four complexes In (a), platinum is +2 and has four ligands, giving the usual square-planar geometry. Its MA_3B pattern has only one arrangement because every position of the single chloride is equivalent. In (b), platinum is +4 and coordination number six gives an octahedral MA_5B pattern; the single ammonia ligand again has no distinct relative arrangement.

Step 2: Test the two remaining ligand patterns Complex (c) is square-planar platinum(II), with two ammonia ligands and two different anions. The ammonia ligands can lie next to each other or opposite each other, giving geometrical isomers. Complex (d) is octahedral platinum(IV), of the form MA_4BC. Chloride and bromide can be adjacent or opposite, so this complex also has cis and trans forms.

Step 3: Compare the option pairs The positive cases are exactly (c) and (d), which is option B. Option A contains only the two negative cases. Option C includes (a), which cannot show geometrical isomerism, and option D includes (b), which also cannot. Linkage possibilities of nitrite in (c) are a separate kind of isomerism and are not needed to establish its geometrical isomers.

Step 4: Conclusion The third and fourth listed complexes display geometrical isomerism: square-planar cis/trans arrangements for the third and octahedral cis/trans arrangements for the fourth. Hence, the correct option is (B).

Quick Tip to Solve: Identify geometry and ligand pattern first: square-planar MA3B and octahedral MA5B each give only one geometrical arrangement.

66. For the following Assertion and Reason, the correct option is

Assertion : For hydrogenation reactions, the catalytic activity increases from Group 5 to Group 11 metals with maximum activity shown by Group 7-9 elements.

Reason : The reactants are most strongly adsorbed on group 7-9 elements. [4 marks]

(A) Both assertion and reason are true and the reason is the correct explanation for the assertion.

(B) The assertion is true, but the reason is false.

(C) Both assertion and reason are true but the reason is not the correct explanation for the assertion.

(D) Both assertion and reason are false.

Correct Answer: (B) The assertion is true, but the reason is false.

Solution:

Concept:

- Heterogeneous catalysis involves adsorption of reactants, surface reaction and desorption of products. High activity requires adsorption strong enough to activate molecules but weak enough to release products.
- Transition-metal hydrogenation activity follows an optimum near Groups 7–9. Maximum catalytic activity is associated with intermediate, favourable adsorption strength rather than the strongest possible adsorption.

Step 1: Evaluate the assertion The stated transition-metal trend identifies the middle transition-metal groups, particularly Groups 7–9, as highly active hydrogenation catalysts. In the intended activity trend across Groups 5–11, activity rises towards this central maximum and then falls beyond it. This is the familiar optimum in the balance between activation and product release, so the assertion is accepted as true.

Step 2: Evaluate the reason The claim that the reactants are most strongly adsorbed on Groups 7–9 is false. Excessively strong adsorption occupies active sites and can prevent products from leaving the surface. A catalyst must instead bind reactants with a suitable intermediate strength. Strongest binding and highest catalytic activity are therefore different criteria.

Step 3: Select the logical combination The reason does not describe the adsorption condition required for the asserted maximum: it substitutes maximum strength for optimum strength. Thus A and C fail because both call the reason true. D fails because it declares the assertion false as well. The available combination that preserves the correct trend while rejecting the faulty explanation is B.

Step 4: Conclusion The assertion is true, but the reason is false: Groups 7–9 provide suitable adsorption, not the strongest adsorption. Hence, the correct option is (B).

Quick Tip to Solve: For adsorption-based catalysis, use an optimum-strength test: binding too weakly prevents activation, while binding too strongly prevents desorption.

67. White Phosphorus on reaction with concentrated NaOH solution in an inert atmosphere of CO_2 gives phosphine and compound (X). (X) on acidification with HCl gives compound (Y). The basicity of compound (Y) is : [4 marks]

(A) 2

(B) 4

(C) 1

(D) 3

Correct Answer: (C) 1

Solution:

Concept:

- White phosphorus disproportionates in hot concentrated alkali: some phosphorus is reduced to phosphine and the remainder is oxidised to hypophosphite.
- The basicity of an oxoacid is the number of ionisable hydrogen atoms bonded to oxygen. Hydrogen directly bonded to phosphorus is not counted as an acidic proton.

Step 1: Identify the salt formed in alkali The balanced reaction with sodium hydroxide and water is

$$P_4 + 3\,NaOH + 3\,H_2O \longrightarrow PH_3 + 3\,NaH_2PO_2.$$

Thus compound X is sodium hypophosphite, NaH_2PO_2. Phosphorus changes from oxidation state zero to -3 in phosphine and $+1$ in hypophosphite; one reduced atom and three oxidised atoms balance the electron transfer. The inert atmosphere prevents competing oxidation and does not change the acid obtained from this salt.

Step 2: Acidify the salt and count acidic hydrogens Acidification replaces the sodium ion with hydrogen:

$$NaH_2PO_2 + HCl \longrightarrow H_3PO_2 + NaCl.$$

Compound Y is hypophosphorous acid. Its structure contains one $P{=}O$ bond, one $P{-}OH$ bond and two $P{-}H$ bonds. Only the hydrogen of the single hydroxyl group can ionise in water. Although the molecular formula has three hydrogen atoms, the two P–H hydrogens do not contribute to basicity. Therefore its basicity is one, rather than two, three or four. Counting all hydrogen atoms would incorrectly select option D.

Step 3: Conclusion Y is hypophosphorous acid, H_3PO_2, and has basicity $\boxed{1}$. Hence, the correct option is (C).

Quick Tip to Solve: For phosphorus oxoacids, count P–OH groups, not the total hydrogens in the molecular formula.

68. Preparation of Bakelite proceeds via reactions. [4 marks]

(A) Electrophilic substitution and dehydration

(B) Condensation and elimination

(C) Nucleophilic addition and dehydration

(D) Electrophilic addition and dehydration

Correct Answer: (A) Electrophilic substitution and dehydration

Solution:

Concept:

- Bakelite is a cross-linked phenol-formaldehyde polymer. The phenolic hydroxyl group activates the aromatic ring and directs substitution mainly to the ortho and para positions.
- Formation of hydroxymethyl phenols is followed by condensation with loss of water, joining aromatic units through methylene bridges.

Step 1: Identify the first reaction on the phenol ring Phenol reacts with formaldehyde at the activated ortho and para ring positions, producing hydroxymethyl-substituted phenols. In the acid-catalysed description, protonation makes formaldehyde an electrophile; attack by the aromatic ring replaces a ring hydrogen while aromaticity is restored. The ring reaction is therefore electrophilic substitution. It is not electrophilic addition, because the final aromatic nucleus retains its aromatic character rather than becoming a simple addition product.

Step 2: Follow formation of the polymer bridges The hydroxymethyl groups react further with phenolic ring sites. A hydroxyl group and hydrogen are removed as water while a methylene linkage, $-CH_2-$, connects two phenolic units. Continued linking at multiple ortho and para sites ultimately gives the three-dimensional network called Bakelite. This water-removing step is dehydration. The first option therefore gives the appropriate sequence of reaction types. Option C wrongly calls the ring reaction nucleophilic addition. Option D wrongly calls it electrophilic addition. Option B gives broad condensation terminology but does not identify the initial aromatic electrophilic substitution; the specific mechanistic pair asked for is substitution followed by dehydration.

Step 3: Conclusion Bakelite preparation proceeds by electrophilic substitution followed by dehydration during bridge formation. Hence, the correct option is (A).

Quick Tip to Solve: In phenol-formaldehyde chemistry, first identify substitution on the activated aromatic ring, then look for water loss as methylene bridges form.

69. Which of the following compounds is likely to show both Frenkel and Schottky defects in its crystalline form? [4 marks]

(A) CsCl

(B) AgBr

(C) ZnS

(D) KBr

Correct Answer: (B) AgBr

Solution:

Concept:

- A Frenkel defect is a vacancy-interstitial pair: an ion leaves its regular lattice site and occupies an interstitial site without leaving the crystal.
- A Schottky defect consists of a charge-balanced set of cation and anion vacancies. It lowers the density because ions are absent from the crystal.

Step 1: Distinguish the conditions for the two defects Frenkel disorder is favoured when a relatively small cation can occupy an interstitial position within the anion framework. The number of ions in the solid remains unchanged, so the ideal density is essentially unchanged. Schottky disorder involves removal of cations and anions in the stoichiometric ratio required for electrical neutrality. This introduces paired vacancies and reduces mass for a given crystal volume. A crystal can show both kinds of disorder; the definitions are not mutually exclusive.

Step 2: Compare the four compounds Silver bromide, AgBr, is the standard example that exhibits both Frenkel and Schottky defects. Silver ions can move into interstitial positions, giving Frenkel pairs, while charge-balanced silver and bromide vacancies can also occur. CsCl and KBr are typical examples of Schottky disorder rather than the usual both-defects example. ZnS, with a substantially smaller cation in the anion framework, is a familiar example of Frenkel disorder. Therefore A and D identify salts associated principally with Schottky defects, while C identifies a Frenkel example; none is the required standard compound showing both. The distinction concerns the defect type and does not mean that any ionic solid must be perfectly defect-free in all other respects.

Step 3: Conclusion $\boxed{\mathrm{AgBr}}$ can exhibit both Frenkel and Schottky defects. Hence, the correct option is (B).

Quick Tip to Solve: Remember AgBr as the shared example: a silver-ion interstitial gives Frenkel disorder, while balanced ion vacancies give Schottky disorder.

70. Among the reactions (a) - (d), the reaction(s) that does/do not occur in the blast furnace during the extraction of iron is/are :

(a) $CaO + SiO_2 \rightarrow CaSiO_3$

(b) $3Fe_2O_3 + CO \rightarrow 2Fe_3O_4 + CO_2$

(c) $FeO + SiO_2 \rightarrow FeSiO_3$

(d) $FeO \rightarrow Fe + \frac{1}{2}O_2$ [4 marks]

(A) (a) and (d)

(B) (c) and (d)

(C) (a)

(D) (d)

Correct Answer: (B) (c) and (d)

Solution:

Concept:

- The blast furnace uses carbon monoxide and coke as reducing agents. Iron oxides are reduced stepwise, rather than by simple thermal release of oxygen.
- In iron extraction, basic lime removes silica as calcium silicate slag. Ferrous silicate slag formation is associated with copper extraction, not the stated iron-smelting sequence.

Step 1: Check slag formation and the first oxide reduction Reaction (a), $CaO + SiO_2 \longrightarrow CaSiO_3$, is the normal removal of silica by basic flux. Lime is obtained by decomposition of limestone, and calcium silicate forms slag. Reaction (b), $3\,Fe_2O_3 + CO \longrightarrow 2\,Fe_3O_4 + CO_2$, is a valid initial reduction of haematite by carbon monoxide. Both reactions therefore occur in the furnace, so choices A and C cannot represent the reactions that do not occur.

Step 2: Check ferrous silicate formation and metallic iron production Reaction (c), $FeO + SiO_2 \longrightarrow FeSiO_3$, is the ferrous-silicate slag reaction familiar from copper metallurgy. It is not part of the normal blast-furnace sequence for iron: the limestone flux supplies lime, and silica is removed as calcium silicate, while ferrous oxide is reduced to recover iron. Thus (c) is excluded in the extraction process specified here. Reaction (d), $FeO \longrightarrow Fe + \frac{1}{2}\,O_2$, is not the furnace reduction route: ferrous oxide does not simply decompose to liberate oxygen under these conditions. Its reduction uses reactions such as $FeO + CO \longrightarrow Fe + CO_2$, or reduction by carbon at sufficiently high temperature. Thus (c) and (d) are the excluded reactions. Options A and C wrongly exclude the genuine calcium-silicate slag reaction; option D omits the exclusion of (c).

Step 3: Conclusion The excluded processes are ferrous-silicate formation and direct thermal decomposition of ferrous oxide. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the calcium-silicate slag of iron extraction from the ferrous-silicate slag of copper extraction; FeO is reduced by CO or carbon.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. For an electrochemical cell

Sn(s) | Sn^{2+} (aq,1M)||Pb^{2+} (aq,1M)|Pb(s)

the ratio $\frac{[Sn^{2+}]}{[Pb^{2+}]}$ when this cell attains equilibrium is _____.

(Given $E^0_{Sn^{2+}|Sn} = -0.14V$,

$E^0_{Pb^{2+}|Pb} = -0.13V$, $\frac{2.303RT}{F} = 0.06$) [4 marks]

Solution:

Concept:

- For the cell reaction, $E_{\text{cell}} = E^{\circ}_{\text{cell}} - (0.06/n)\log_{10} Q$, using the numerical value of $2.303RT/F$ supplied in the question.
- At chemical equilibrium the cell emf is zero. Pure solid metals have unit activity, so only the aqueous ionic concentrations enter the reaction quotient.

Step 1: Write the reaction and find its standard emf The left electrode is the oxidation half-cell and the right electrode is the reduction half-cell. Tin loses two electrons and lead ions gain them:

$$\mathrm{Sn(s)} + \mathrm{Pb^{2+}(aq)} \longrightarrow \mathrm{Sn^{2+}(aq)} + \mathrm{Pb(s)}.$$

The number of electrons transferred is $n = 2$. Both given electrode potentials are reduction potentials, hence

$$E^{\circ}_{\text{cell}} = E^{\circ}_{\mathrm{Pb^{2+}/Pb}} - E^{\circ}_{\mathrm{Sn^{2+}/Sn}} = (-0.13) - (-0.14) = 0.01\,\mathrm{V}.$$

Taking their sum instead of their difference would give the wrong driving force.

Step 2: Apply the zero-emf condition at equilibrium For this reaction the quotient is $Q = [\mathrm{Sn^{2+}}]/[\mathrm{Pb^{2+}}]$, since the activities of solid tin and lead are unity. Set the emf to zero and use the supplied constant:

$$0 = 0.01 - \frac{0.06}{2}\log_{10} Q.$$

$$\log_{10} Q = \frac{2(0.01)}{0.06} = \frac{1}{3}, \qquad Q = 10^{1/3} = 2.1544\ldots.$$

The concentrations stated as 1 M specify the standard starting condition; they need not remain equal as the cell reaction proceeds towards equilibrium. The positive standard emf correctly predicts a forward reaction and an equilibrium quotient greater than one.

Step 3: Conclusion At equilibrium, $\boxed{[\mathrm{Sn^{2+}}]/[\mathrm{Pb^{2+}}] \approx 2.15}$. This is a dimensionless concentration ratio.

Quick Tip to Solve: Write the balanced cell reaction before forming Q; at equilibrium use E = 0, not the initial standard emf.

72. Complexes (ML_5) of metals Ni and Fe have ideal square pyramidal and trigonal bipyramidal grometries, respectively. The sum of the 90°, 120° and 180° L-M-L angles in the two complexes is _____. [4 marks]

Solution:

Concept:

- Five ligand directions form $\binom{5}{2} = 10$ distinct ligand–metal–ligand angles. Each unordered pair of ligands is counted once.
- An ideal square pyramid has four basal ligands and one axial ligand; an ideal trigonal bipyramid has three equatorial ligands and two opposite axial ligands.

Step 1: Count all angles in the square pyramid The four basal ligands lie at the corners of a square with the metal at its centre. Adjacent basal directions give four angles of $90°$, and opposite basal directions give two angles of $180°$. The one axial direction is perpendicular to every basal direction, adding four more $90°$ angles. Thus the square-pyramidal complex contributes eight $90°$ angles, zero $120°$ angles and two $180°$ angles. The total is $8 + 0 + 2 = 10$, as required by the ten distinct ligand pairs.

Step 2: Count all angles in the trigonal bipyramid The three equatorial ligand directions form three pairs, each making $120°$. Each of the two axial ligands makes $90°$ with each of the three equatorial ligands, giving $2 \times 3 = 6$ right angles. The two axial directions are opposite, giving one $180°$ angle. This complex contributes $6+3+1 = 10$ angles. Combining the two complexes, the counts are fourteen right angles, three $120°$ angles and three straight angles, so

$$N = 14 + 3 + 3 = 20.$$

Here the question asks for the combined number of angles of the listed types, rather than adding their numerical degree measures. Every ligand pair in these ideal geometries belongs to one of those three angle types.

Step 3: Conclusion The combined number of $90°$, $120°$ and $180°$ L–M–L angles is $\boxed{20}$, a dimensionless count.

Quick Tip to Solve: Use five choose two as a check for each five-coordinate complex, then classify the ten pairs by their axial or equatorial positions.

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73. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500K changes its internal energy by 5000 J. The molar heat capacity at constant volume is _____. [4 marks]

Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature. Its change is $\Delta U = nC_{V,m}\Delta T$, where n is the amount of gas and $C_{V,m}$ is the molar heat capacity at constant volume.
- Molar heat capacity is the energy needed per mole per kelvin. It must be distinguished from the heat capacity of the whole sample, $C_V = nC_{V,m}$, whose unit is $\mathrm{J\,K^{-1}}$.

Step 1: Identify the temperature change and energy supplied The final and initial temperatures are given on the kelvin scale, so their difference needs no unit conversion:

$$\Delta T = T_f - T_i = 500 - 300 = 200 \text{ K}.$$

The sample contains $n = 4$ mol, and its internal energy increases by $\Delta U = 5000$ J. At constant volume the expansion work is zero. Thus, by the first law, the heat absorbed equals this increase in internal energy; no pressure-volume work term has to be subtracted.

Step 2: Calculate the heat capacity per mole Rearrange the internal-energy relation, retaining both the amount of gas and the temperature interval in the denominator:

$$C_{V,m} = \frac{\Delta U}{n\Delta T} = \frac{5000}{4 \times 200} = 6.25 \text{ J mol}^{-1} \text{ K}^{-1}.$$

As a consistency check, the entire sample has heat capacity $5000/200 = 25$ J K^{-1}; dividing this by four moles gives the same molar result. The positive value is consistent with the energy increase on heating. The calculation uses the measured energy change, rather than presuming a particular monatomic or diatomic gas model.

Step 3: Conclusion The molar heat capacity at constant volume is $\boxed{6.25 \text{ J mol}^{-1} \text{ K}^{-1}}$. The numerical entry is 6.25.

Quick Tip to Solve: Divide the energy change by both the number of moles and the temperature rise. Dividing only by the rise gives the sample's heat capacity, not the molar heat capacity.

74. In the following sequence of reactions the maximum number of atoms present in molecule 'C' in one plane is _____. (A is a lowest molecular weight alkyne) [4 marks]

$$\text{A} \xrightarrow[\text{Cu tube}]{\text{Red hot}} \text{B} \xrightarrow[\text{Anhydrous } AlCl_3]{CH_3Cl(1.eq.)} \text{C}$$

Solution:

Concept:

- The lowest molecular weight alkyne is ethyne, HC≡CH. Passing ethyne through a red-hot copper tube causes cyclic trimerisation to benzene.
- Benzene undergoes Friedel–Crafts alkylation with CH_3Cl and anhydrous $AlCl_3$. The ring carbons are trigonal planar, whereas the carbon of an attached methyl group is tetrahedral.

Step 1: Identify the products of the reaction sequence Since an alkyne needs at least two carbon atoms, A is ethyne. Three ethyne molecules form one benzene molecule in the heated copper tube:

$$3\,\text{HC}{\equiv}\text{CH} \xrightarrow{\text{red-hot Cu}} C_6H_6.$$

Hence B is benzene. Methyl chloride, with the Lewis-acid catalyst, replaces a ring hydrogen by a

methyl group:

$$C_6H_6 + CH_3Cl \xrightarrow{AlCl_3} C_6H_5CH_3 + HCl.$$

Thus the intended monosubstituted product C is toluene. The one equivalent of methyl chloride identifies the single methyl substitution in the stated sequence.

Step 2: Count the atoms in the aromatic ring plane All six ring carbon atoms and the five hydrogens directly attached to them lie in the benzene-ring plane. The bond from the ring to the methyl carbon also lies in this plane, because it is one of the three coplanar bonds at the substituted sp^2 carbon. These give $6 + 5 + 1 = 12$ coplanar atoms. Rotation about the ring–methyl bond can place one methyl hydrogen in the same plane. The methyl carbon is sp^3-hybridised: its three C–H bonds have tetrahedral geometry, so the other two methyl hydrogens then lie on opposite sides of that plane. They cannot both be included by treating the entire methyl group as planar. Therefore the maximum count is

$$N_{\text{coplanar}} = 6 + 5 + 1 + 1 = 13.$$

Step 3: Conclusion C is toluene, and the maximum number of its atoms that can lie in one plane is $\boxed{13}$. The numerical entry is 13 atoms.

Quick Tip to Solve: Count the planar aromatic framework first. For an attached methyl group, include its carbon and at most one hydrogen in the ring plane; tetrahedral geometry excludes its other two hydrogens.

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75. $NaClO_3$ is used, even in spacecrafts, to produce O_2. The daily consumption of pure O_2 by a person is 492L at 1 atm, 300K. How much amount of $NaClO_3$, in grams, is required to produce O_2 for the daily consumption of a person at 1 atm, 300 K?

$NaClO_3(s)$ + Fe(s) $\rightarrow$ $O_2(g)$ + NaCl(s) + FeO(s)

R = 0.082 L atm mol^{-1} K^{-1} [4 marks]

Solution:

Concept:

- The ideal gas equation $PV = nRT$ converts a measured gas volume into an amount in moles. Pressure, volume and the gas constant must use compatible units.
- Stoichiometric ratios come from the balanced reaction actually given. Here iron consumes part of the oxygen from sodium chlorate to form FeO, so the oxygen yield differs from simple thermal decomposition of the chlorate alone.

Step 1: Balance the oxygen-generating reaction With sodium and chlorine balanced by one NaCl, one iron atom gives one FeO. Of the three oxygen atoms in one chlorate formula unit, one enters iron(II) oxide and the other two form one oxygen molecule:

$$NaClO_3(s) + Fe(s) \longrightarrow O_2(g) + NaCl(s) + FeO(s).$$

The equation is therefore already balanced with unit coefficients. Consequently, one mole of $NaClO_3$ supplies one mole of O_2 in this stated reaction. Using the ratio for chlorate decomposition without iron would give an incorrect mass for this problem.

Step 2: Convert the daily oxygen requirement into moles Substitute the given conditions directly; the litre-atmosphere units match the specified value of R:

$$n(\mathrm{O_2}) = \frac{PV}{RT} = \frac{1 \times 492}{0.082 \times 300} = \frac{492}{24.6} = 20 \text{ mol}.$$

Thus the required amount of sodium chlorate is also 20 mol. The 300 K temperature must be retained instead of using the standard-temperature molar volume.

Step 3: Calculate the required mass of sodium chlorate Using the usual atomic masses $\mathrm{Na} = 23$, $\mathrm{Cl} = 35.5$ and $\mathrm{O} = 16$, its molar mass is

$$M(\mathrm{NaClO_3}) = 23 + 35.5 + 3(16) = 106.5 \text{ g mol}^{-1}.$$

Multiply this by the stoichiometrically required amount:

$$m = 20 \times 106.5 = 2130 \text{ g}.$$

Step 4: Conclusion The sodium chlorate required for one person's daily oxygen consumption is $\boxed{2130 \text{ g}}$. The numerical entry is 2130.

Quick Tip to Solve: Balance the complete reaction before using a gas-volume calculation: formation of FeO leaves only one mole of oxygen gas per mole of sodium chlorate in this equation.