JEE Main 2020 Paper 1 (B.E./B.Tech) Jan 9 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 January 2020, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : [4 marks]

(A) 28

(B) 24

(C) 26

(D) 25

Correct Answer: (D) 25

Solution:

Concept:

- A quadratic with repeated root α has the factorisation $a(x-\alpha)^2$. Comparing coefficients relates the root to its coefficients.
- For a monic quadratic with roots α, β, their product is its constant term. Thus $\alpha\beta = -10$ for the second equation.

Step 1: Use the repeated-root condition Comparing $ax^2 - 2bx + 5 = a(x-\alpha)^2$ gives $a\alpha = b$ and $a\alpha^2 = 5$. In particular, neither a nor α is zero, and multiplication of the first equality by α gives

$$b\alpha = 5.$$

This relation uses both the repeated-root information and the constant term; a root of the first equation alone would not give it.

Step 2: Find the squares of the two roots Substitute the common root into the second equation and then use the relation just obtained:

$$\alpha^2 - 2b\alpha - 10 = 0 \quad \Longrightarrow \quad \alpha^2 = 20.$$

By the product of the roots, $\beta = -10/\alpha$, so

$$\beta^2 = \frac{100}{\alpha^2} = 5, \qquad \alpha^2 + \beta^2 = 20 + 5 = 25.$$

The result does not depend on which sign the real repeated root takes. The other proposed totals, 28, 24 and 26, do not satisfy these two root-square values.

Step 3: Conclusion The required sum is $\boxed{25}$. Hence, the correct option is (D).

Quick Tip to Solve: When a repeated root is shared with another quadratic, compare coefficients of the square first; the product of the roots then gives the other root quickly.

2. Let a function f : [0, 5] $\to$ R be continuous, f(1) = 3 and F be defined as :

$F(x) = \int_1^x t^2 g(t)dt$ **, where** $g(t) = \int_1^t f(u)du$

Then for the function F, the point x = 1 is : [4 marks]

(A) a point of inflection.

(B) a point of local maxima.

(C) a point of local minima.

(D) not a critical point.

Correct Answer: (C) a point of local minima.

Solution:

Concept:

- For a continuous integrand, the fundamental theorem of calculus gives the derivative of an integral with a variable upper limit.
- A stationary point with positive second derivative is a strict local minimum. Continuity of f is enough for the derivatives used here.

Step 1: Differentiate the nested integrals First apply the fundamental theorem to the inner integral. Since f is continuous,

$$g\prime(x) = f(x), \qquad g(1) = \int_1^1 f(u)\,du = 0.$$

The outer integrand $x^2 g(x)$ is continuous, and hence

$$F\prime(x) = x^2 g(x), \qquad F\prime(1) = 0.$$

Thus $x = 1$ is a critical point lying in the interior of the given interval. It cannot be classified merely from the fact that both integrals vanish at their lower limit.

Step 2: Apply the second-derivative test Use the product rule and the given value $f(1) = 3$:

$$F\prime\prime(x) = 2xg(x) + x^2 f(x), \qquad F\prime\prime(1) = 0 + 3 = 3 > 0.$$

The second derivative is continuous near 1, because f and g are continuous. It therefore remains positive in a sufficiently small neighbourhood. The graph is locally concave upward there, so this is not an inflection point or a local maximum. Equivalently, $F\prime$ changes from negative to positive through 1.

Step 3: Conclusion The point $x = 1$ is a strict local minimum of F. Hence, the correct option is (C).

Quick Tip to Solve: For nested integrals, differentiate from the outside inward; an integral whose upper and lower limits coincide immediately becomes zero at that point.

3. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio ℓ_2 : ℓ_1 is equal to : [4 marks]

(A) 8 : 1

(B) 16 : 1

(C) 1 : 8

(D) 1 : 16

Correct Answer: (B) 16 : 1

Solution:

Concept:

- In $(A+B)^{16}$, the general term is $\binom{16}{r}A^{16-r}B^r$. An independent term has zero net exponent of x.
- Use $2\sin\theta\cos\theta=\sin 2\theta$. Minimising a positive reciprocal power requires maximising its positive denominator.

Step 1: Identify the term independent of the variable The general term of the specified expansion is

$$T_{r+1}=\binom{16}{r}\frac{x^{16-2r}}{\cos^{16-r}\theta\,\sin^r\theta}.$$

The exponent vanishes only for $16-2r=0$, namely $r=8$. The independent term is consequently

$$T_9=\binom{16}{8}\frac{1}{(\sin\theta\cos\theta)^8}=\binom{16}{8}\frac{2^8}{\sin^8(2\theta)}.$$

All sines and cosines involved are positive on the stated intervals, so no sign or absolute-value complication arises.

Step 2: Maximise the denominator on each interval For $\pi/8\le\theta\le\pi/4$, the angle 2θ ranges from $\pi/4$ to $\pi/2$, and its sine has maximum 1. Therefore $\ell_1=\binom{16}{8}2^8$. For $\pi/16\le\theta\le\pi/8$, the maximum sine occurs at $2\theta=\pi/4$, where its value is $1/\sqrt{2}$. Thus

$$\frac{\ell_2}{\ell_1}=\frac{1}{(1/\sqrt{2})^8}=2^4=16.$$

The smaller available denominator makes the second minimum larger, ruling out the ratios below one; retaining the eighth power rules out 8.

Step 3: Conclusion The required ratio is $\boxed{16:1}$. Hence, the correct option is (B).

Quick Tip to Solve: Cancel the common binomial coefficient before evaluating the ratio; the ratio of the maximum denominator factors alone determines the answer.

4. Let a – 2b + c = 1.

If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$**, then:** [4 marks]

(A) f(50) = 1

(B) f(–50) = –1

(C) f(50) = –501

(D) f(–50) = 501

Correct Answer: (A) f(50) = 1

Solution:

Concept:

- Adding a multiple of one column to another preserves a determinant. The same is true of the corresponding elementary row operation.
- A column containing one nonzero entry allows immediate cofactor expansion. Keep the cofactor sign $(-1)^{i+j}$ when expanding.

Step 1: Create a constant column The second and third columns differ by 1 in every row. Apply $C_2 \leftarrow C_2 - C_3$ to obtain

$$f(x)=\begin{vmatrix} x+a & 1 & x+1 \\ x+b & 1 & x+2 \\ x+c & 1 & x+3 \end{vmatrix}.$$

This operation preserves the value for every real x; there is no division by an expression that might be zero. Next subtract the original first row from each of the other two rows. The result is

$$f(x)=\begin{vmatrix} x+a & 1 & x+1 \\ b-a & 0 & 1 \\ c-a & 0 & 2 \end{vmatrix}.$$

Step 2: Expand and apply the given coefficient relation Expand along the second column. Its sole nonzero entry is in position $(1,2)$, whose cofactor has a minus sign:

$$f(x)=-\begin{vmatrix} b-a & 1 \\ c-a & 2 \end{vmatrix}=-\{2(b-a)-(c-a)\}=a-2b+c=1.$$

The determinant is therefore independent of x, even though every original entry contains it. In particular, both $f(50)$ and $f(-50)$ equal 1. Thus the proposed value -1 and the values -501 and 501 are all excluded; substituting a large value for x at the start would obscure this cancellation.

Step 3: Conclusion We obtain $f(50)=1$. Hence, the correct option is (A).

Quick Tip to Solve: When two determinant columns differ by a constant vector, subtract them first; row subtraction then often removes the variable entirely.

5. The following system of linear equations

7x + 6y – 2z = 0

3x + 4y + 2z = 0

x – 2y – 6z = 0, has [4 marks]

(A) no solution

(B) infinitely many solutions, (x, y, z) satisfying y = 2z

(C) infinitely many solutions, (x, y, z) satisfying x = 2z

(D) only the trivial solution

Correct Answer: (C) infinitely many solutions, (x, y, z) satisfying x = 2z

Solution:

Concept:

- A homogeneous linear system always has the zero solution. A free parameter in its complete solution produces infinitely many solutions.
- Elimination should be followed by substitution into every equation; a zero determinant by itself does not specify the relation between the variables.

Step 1: Eliminate a variable using the third equation The third equation immediately expresses x in terms of the other two variables:

$$x = 2y + 6z.$$

Substitute this expression into the first equation:

$$7(2y + 6z) + 6y - 2z = 0 \quad \Longrightarrow \quad 20y + 40z = 0.$$

Hence $y = -2z$, and substitution back gives $x = 2(-2z) + 6z = 2z$. These relations are necessary for every solution, including the trivial one.

Step 2: Check the remaining equation and describe all solutions Put $z = t$, where t is any real number. The resulting triple is

$$(x, y, z) = (2t, -2t, t).$$

The second equation becomes $3(2t) + 4(-2t) + 2t = 0$; the first and third are also satisfied by the elimination relations. Therefore every real value of t gives a solution. Choosing $t = 1$ yields the nonzero solution $(2, -2, 1)$, so the solution is not restricted to the zero triple. The relation is $y = -2z$, rather than $y = 2z$, which excludes the competing infinite-solution option. No-solution is impossible for any homogeneous system.

Step 3: Conclusion There are infinitely many solutions satisfying $\boxed{x = 2z}$. Hence, the correct option is (C).

Quick Tip to Solve: For a homogeneous system, display a parameterised family and check it in all equations; this establishes both infinitude and the precise variable relation.

6. Let f and g be differentiable functions on R such that fog is the identity function. If for some a, b $\in$ R, g'(a) = 5 and g(a) = b, then f'(b) is equal to : [4 marks]

(A) 1

(B) 5

(C) $\frac{2}{5}$

(D) $\frac{1}{5}$

Correct Answer: (D) $\frac{1}{5}$

Solution:

Concept:

- The identity function sends x to x and has derivative 1 at every real point.
- The chain rule gives $(f \circ g)\prime(x) = f\prime(g(x))g\prime(x)$. The derivative of the outer function must be evaluated at the inner value.

Step 1: Differentiate the functional identity The stated identity means $f(g(x)) = x$ for every real x. Differentiability of both functions allows us to differentiate both sides:

$$f\prime(g(x))\, g\prime(x) = 1.$$

The product here arises from the chain rule, rather than from ordinary multiplication of the original functions. In particular, the identity does not say that $f(x) = x$ or $g(x) = x$ individually. Their rates of change compensate under composition.

Step 2: Evaluate at the given input At $x = a$, the inner value is $g(a) = b$ and its derivative is $g\prime(a) = 5$. Consequently,

$$f\prime(b) \cdot 5 = 1, \qquad f\prime(b) = \frac{1}{5}.$$

Division is legitimate because 5 is nonzero. The proposed derivative 1 would make the derivative of the composition 5, while the proposed derivative 5 would make it 25. The value $2/5$ would make it 2. None agrees with the required derivative 1, so only the reciprocal value survives this direct check. No information about the numerical values of a or b is needed.

Step 3: Conclusion The derivative is $\boxed{1/5}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the chain-rule equation before substituting: an inverse composition makes the two derivatives reciprocal, but they must be taken at the paired inputs.

7. A random variable X has the following probability distribution :

X:	1	2	3	4	5
P(X):	K^2	2K	K	2K	$5K^2$

Then P(X > 2) is equal to : [4 marks]

(A) $\frac{1}{6}$
(B) $\frac{7}{12}$
(C) $\frac{1}{36}$
(D) $\frac{23}{36}$

Correct Answer: (D) $\frac{23}{36}$

Solution:

Concept:

- The probabilities of all mutually exclusive values of a discrete random variable add to one. Every probability is nonnegative.
- For a requested event, add only the probabilities of the values that satisfy its inequality; here $X > 2$ means $X = 3, 4, 5$.

Step 1: Normalize the distribution The five listed values exhaust the distribution, so their probabilities must satisfy

$$K^2 + 2K + K + 2K + 5K^2 = 1.$$

Collect the quadratic and linear terms before factoring:

$$6K^2 + 5K - 1 = 0, \qquad (6K - 1)(K + 1) = 0.$$

Thus the two algebraic possibilities are $K = 1/6$ and $K = -1$. The second is inadmissible: it would make the probabilities K and $2K$ negative. Therefore the probability parameter is uniquely $K = 1/6$, which also makes every entry between zero and one.

Step 2: Add the probabilities above two The inequality is strict, so omit the entries for both 1 and 2. The required sum is

$$\begin{aligned} P(X > 2) &= P(X = 3) + P(X = 4) + P(X = 5) \\ &= K + 2K + 5K^2 \\ &= \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{23}{36}. \end{aligned}$$

As a separate check, the complementary event has probability

$$P(X \le 2) = K^2 + 2K = \frac{1}{36} + \frac{12}{36} = \frac{13}{36}.$$

Its complement is again $23/36$. Option (A) gives only K, and option (C) gives only K^2; neither adds the required outcomes. Option (B) likewise differs from the event total.

Step 3: Conclusion $P(X > 2) = 23/36$. Hence, the correct option is (D).

Quick Tip to Solve: Normalize first and discard a root that makes any probability negative; then translate the strict inequality into the exact listed outcomes.

8. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ **and** $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$ **for 0** $< \theta < \frac{\pi}{4}$**, then :** [4 marks]

(A) x(1 + y) = 1

(B) y(1 – x) = 1

(C) y(1 + x) = 1

(D) x(1 – y) = 1

Correct Answer: (B) y(1 – x) = 1

Solution:

Concept:

- An infinite geometric series with first term one and common ratio r sums to $1/(1-r)$, provided $|r| < 1$.
- The identities $1 + \tan^2\theta = \sec^2\theta$ and $1 - \cos^2\theta = \sin^2\theta$ simplify the resulting sums.

Step 1: Check convergence and sum both series For $0 < \theta < \pi/4$, we have $0 < \tan^2\theta < 1$. Thus the common ratio of the first series, $-\tan^2\theta$, has modulus less than one. The second ratio is $\cos^2\theta$, which also lies strictly between zero and one. Both sums consequently exist, and their terms begin with one because the index begins at zero:

$$x = \frac{1}{1+\tan^2\theta} = \cos^2\theta, \qquad y = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}.$$

The alternating sign belongs to the common ratio in the first sum; replacing its denominator by $1 - \tan^2\theta$ would be incorrect.

Step 2: Eliminate the angle and test the alternatives Since $1 - x = 1 - \cos^2\theta = \sin^2\theta$, multiplication gives

$$y(1-x) = \frac{1}{\sin^2\theta}\sin^2\theta = 1.$$

This proves the proposed identity for the entire stated interval, rather than at a special angle. The other alternatives can also be excluded directly: $y > 1$ and $x > 0$, so $x(1-y) < 0$ and cannot equal one. Also $y(1+x) > 1$. Finally, writing $c = \cos^2\theta \in (1/2, 1)$,

$$x(1+y) = \frac{c(2-c)}{1-c} > 1,$$

since $3c - c^2 - 1 > 0$ throughout this interval (it is increasing there and equals $1/4$ at $c = 1/2$). Thus only the second relation is valid.

Step 3: Conclusion The required relation is $y(1-x) = 1$. Hence, the correct option is (B).

Quick Tip to Solve: For series indexed from zero, retain the initial term 1. Sum the two geometric series before trying to eliminate the trigonometric parameter.

9. Let [t] denote the greatest integer $\leq$ t and $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$.

Then the function, f(x) = $[x^2]\sin(\pi x)$ is discontinuous, when x is equal to : [4 marks]

(A) $\sqrt{A+1}$

(B) $\sqrt{A+5}$

(C) $\sqrt{A+21}$

(D) $\sqrt{A}$

Correct Answer: (A) $\sqrt{A+1}$

Solution:

Concept:

- For every real t, write $\lfloor t \rfloor = t - \delta$ with $0 \leq \delta < 1$; the bounded error controls limits.
- A jump of $\lfloor x^2 \rfloor$ in its product with a continuous factor is removed if that factor tends to zero at the jump.

Step 1: Evaluate the parameter in the limit Put $\lfloor 4/x \rfloor = 4/x - \delta(x)$, where $0 \leq \delta(x) < 1$, for either sign of nonzero x. Then

$$x\left\lfloor \frac{4}{x} \right\rfloor = 4 - x\delta(x), \qquad |x\delta(x)| \leq |x|.$$

The error tends to zero from both sides, proving $A = 4$. Therefore the four proposed arguments are respectively $\sqrt{5}, 3, 5, 2$. The bounded-error form avoids accidentally reversing a floor inequality when multiplying by a negative x.

Step 2: Check which floor jump survives At $x_0 = \sqrt{5}$, the square increases through 5. For x sufficiently close on the left, $\lfloor x^2 \rfloor = 4$; on the right and at the point it is 5. Consequently

$$\lim_{x\to x_0^-} f(x) = 4\sin(\pi\sqrt{5}), \qquad \lim_{x\to x_0^+} f(x) = 5\sin(\pi\sqrt{5}).$$

Since $\sqrt{5}$ is not an integer, $\sin(\pi\sqrt{5}) \neq 0$, so these limits differ. At each remaining proposed point $m = 2, 3, 5$, the floor term is bounded locally and $\sin(\pi x) \to \sin(\pi m) = 0$. Hence $f(x) \to 0 = f(m)$, even though the floor factor itself jumps there. Thus those three points are continuous points of the product and cannot answer the question.

Step 3: Conclusion The discontinuity occurs at $x = \sqrt{A+1} = \sqrt{5}$. Hence, the correct option is (A).

Quick Tip to Solve: When a floor function is multiplied by a continuous factor, test whether that factor vanishes at the jump before declaring the product discontinuous.

10. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)} = \lambda \tan\theta + 2\log_e |f(\theta)| + C$

where C is a constant of integration, then the ordered pair (λ, f(θ)) is equal to : [4 marks]

(A) (–1, 1 – tanθ)

(B) (1, 1 + tanθ)

(C) (–1, 1 + tanθ)

(D) (1, 1 – tanθ)

Correct Answer: (C) (–1, 1 + tanθ)

Solution:

Concept:

- The substitution $t = \tan\theta$ gives $dt = \sec^2\theta\, d\theta$, matching the factor $d\theta/\cos^2\theta$.
- The double-angle identities give $\tan 2\theta + \sec 2\theta = (1+t)/(1-t)$ wherever the original expression is defined.

Step 1: Convert the integrand into a rational function Work on any interval where the original integrand is defined. Set $t = \tan\theta$, so that

$$\tan 2\theta = \frac{2t}{1-t^2}, \qquad \sec 2\theta = \frac{1+t^2}{1-t^2}.$$

Adding the two expressions and factoring yields

$$\tan 2\theta + \sec 2\theta = \frac{(1+t)^2}{(1-t)(1+t)} = \frac{1+t}{1-t}.$$

The integral therefore becomes

$$I = \int \frac{1-t}{1+t}\, dt.$$

These simplifications are used only on intervals of the original domain; cancellation does not add points at which the original double-angle functions are undefined.

Step 2: Integrate and identify the ordered pair Divide the numerator by the denominator in a way that isolates a logarithmic derivative:

$$\frac{1-t}{1+t} = -1 + \frac{2}{1+t}.$$

Thus

$$I = -t + 2\ln|1+t| + C = -\tan\theta + 2\ln|1+\tan\theta| + C.$$

Comparing coefficients and the logarithm with the required form gives $\lambda = -1$ and $f(\theta) = 1 + \tan\theta$. Differentiating the result gives

$$\sec^2\theta\left(-1 + \frac{2}{1+\tan\theta}\right) = \sec^2\theta\frac{1-\tan\theta}{1+\tan\theta},$$

which is exactly the original integrand after the identity above. Options (B) and (D) have the wrong coefficient of $\tan\theta$; option (A) has the wrong logarithmic factor.

Step 3: Conclusion The ordered pair is $(-1, 1 + \tan\theta)$. Hence, the correct option is (C).

Quick Tip to Solve: When the numerator supplies $\sec^2\theta\, d\theta$, try $t = \tan\theta$; simplify the remaining double-angle terms before integrating.

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11. Let a_n be the n^{th} term of a G.P. of positive terms.

$\sum_{n=1}^{100} a_{2n+1} = 200$ **and** $\sum_{n=1}^{100} a_{2n} = 100$,

then $\sum_{n=1}^{200} a_n$ **is equal to :** [4 marks]

(A) 150

(B) 175

(C) 225

(D) 300

Correct Answer: (A) 150

Solution:

Concept:

- If a geometric progression has first term a and common ratio r, then $a_n = ar^{n-1}$. Positive terms imply $a > 0$ and $r > 0$.
- Consecutive odd and even terms satisfy $a_{2n+1} = ra_{2n}$ and $a_{2n} = ra_{2n-1}$. Summing preserves these relations.

Step 1: Find the common ratio from the given sums Each term in the first given sum is the next term after the corresponding term in the second:

$$a_3 = ra_2, \quad a_5 = ra_4, \quad \ldots, \quad a_{201} = ra_{200}.$$

Add all one hundred identities. The supplied values give

$$\sum_{n=1}^{100} a_{2n+1} = r\sum_{n=1}^{100} a_{2n}, \qquad 200 = 100r.$$

Therefore $r = 2$. No large power or first-term calculation is needed: the two equally sized collections differ only by a shift of one term, which multiplies every summand by the same ratio.

Step 2: Combine the even terms with the correct odd terms The requested sum ends at a_{200}, so its odd terms are $a_1, a_3, \ldots, a_{199}$, not the given collection $a_3, a_5, \ldots, a_{201}$. Compare each desired odd term with the following even term:

$$\sum_{n=1}^{100} a_{2n} = r\sum_{n=1}^{100} a_{2n-1}.$$

Hence the desired odd-term sum is $100/2 = 50$. Splitting the complete sum into odd and even indices

now gives

$$\sum_{n=1}^{200} a_n = \sum_{n=1}^{100} a_{2n-1} + \sum_{n=1}^{100} a_{2n} = 50 + 100 = 150.$$

Adding the two originally supplied sums would give 300, but that total covers terms 2 through 201, not terms 1 through 200. The other numerical options similarly do not respect this index shift.

Step 3: Conclusion The required sum of the first 200 terms is 150. Hence, the correct option is (A).

Quick Tip to Solve: Write the first and last indices in each sum. Equal-length shifted sums of a G.P. differ by a power of the common ratio.

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12. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is : [4 marks]

(A) $\frac{3}{8}$

(B) $\frac{3}{2}$

(C) $\frac{3}{4}$

(D) $-\frac{3}{4}$

Correct Answer: (A) $\frac{3}{8}$

Solution:

Concept:

- For a parametric curve with $x'(\theta) \neq 0$, the derivative is $dy/dx = y'/x'$.
- The second derivative is $d^2y/dx^2 = (y''x' - y'x'')/(x')^3$; differentiation with respect to x requires division by x'.

Step 1: Compute the parameter derivatives Differentiate both coordinates with respect to θ, applying the chain rule to the double-angle terms:

$$\begin{aligned} x' &= 2\cos\theta - 2\cos 2\theta, \\ x'' &= -2\sin\theta + 4\sin 2\theta, \\ y' &= -2\sin\theta + 2\sin 2\theta, \\ y'' &= -2\cos\theta + 4\cos 2\theta. \end{aligned}$$

At $\theta = \pi$, use $\cos\pi = -1$, $\cos 2\pi = 1$, and both sine values zero. This gives

$$x' = -4, \quad x'' = 0, \quad y' = 0, \quad y'' = 6.$$

Since $x' \neq 0$, the curve can be treated locally as a function of x, so the ordinary parametric derivative formula applies.

Step 2: Apply the second-derivative formula First differentiate the quotient y'/x' with respect to the

parameter and then convert to differentiation with respect to x:

$$\frac{d^2y}{dx^2} = \frac{1}{x'}\frac{d}{d\theta}\left(\frac{y'}{x'}\right) = \frac{y''x' - y'x''}{(x')^3}.$$

Substitution gives

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi} = \frac{6(-4) - 0}{(-4)^3} = \frac{-24}{-64} = \frac{3}{8}.$$

Alternatively, because $y' = 0$ at this point, the formula reduces there to $y''/(x')^2 = 6/16$, confirming the same positive value. Taking just y''/x' would miss the second chain-rule factor; none of the other listed values equals the correctly computed curvature derivative.

Step 3: Conclusion At $\theta = \pi$, $d^2y/dx^2 = 3/8$. Hence, the correct option is (A).

Quick Tip to Solve: For a second parametric derivative, divide by $dx/d\theta$ twice overall: once in dy/dx, then again after differentiating that quotient.

13. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is : [4 marks]

(A) $\frac{965}{2^{11}}$

(B) $\frac{965}{2^{10}}$

(C) $\frac{945}{2^{11}}$

(D) $\frac{945}{2^{10}}$

Correct Answer: For the literal event stated, the probability is $\boxed{16065/32768}$. None of options (A), (B), (C), or (D) is correct; the printed question has no defensible listed answer.

Solution:

Concept:

- Each of the ten distinct balls independently enters one of four distinct boxes, so there are 4^{10} equally likely assignments.
- The event that two boxes contain exactly two and three balls means that the occupancy vector contains both 2 and 3; repeated occupancies must be counted only once.

Step 1: List disjoint occupancy patterns After allocating two and three balls, five balls remain for the other boxes. Ignoring the order of boxes temporarily gives exactly three possibilities: $(0, 2, 3, 5)$, $(1, 2, 3, 4)$, and $(2, 2, 3, 3)$. These patterns are disjoint and exhaust all assignments having a box of size two and a different box of size three. The last pattern is especially important because it has two boxes of each required size.

Step 2: Count assignments for each pattern For a fixed occupancy vector (n_1, n_2, n_3, n_4), the number of assignments is $10!/(n_1!n_2!n_3!n_4!)$. The first two patterns have $4! = 24$ orders, while the last has $4!/(2!2!) = 6$. Thus the three contributions are respectively

$$24\frac{10!}{0!2!3!5!} = 60480, \qquad 24\frac{10!}{1!2!3!4!} = 302400,$$

$$6\frac{10!}{2!2!3!3!} = 151200.$$

Step 3: Divide by the sample space and check the options The favourable count is $60480 + 302400 + 151200 = 514080$, giving

$$P = \frac{514080}{4^{10}} = \frac{16065}{32768}.$$

None of the four printed fractions equals this value. A shortcut that chooses boxes and then distributes the remaining balls unrestricted counts some assignments repeatedly, since the remaining boxes can themselves contain two or three balls. Such a count cannot establish the probability of the stated event.

Step 4: Conclusion For the literal event stated, the probability is $\boxed{16065/32768}$. None of options (A), (B), (C), or (D) is correct; the printed question has no defensible listed answer. The supplied answer key selects (D), $945/1024$, but that count treats each assignment of occupancy $(2, 2, 3, 3)$ four times. Subtracting the three excess counts gives $(967680 - 3 \cdot 151200)/4^{10} = 16065/32768$; therefore the literal printed event proves the different value.

Quick Tip to Solve: In occupancy questions, separate disjoint occupancy patterns first; repeated box sizes require division by the factorials of their multiplicities.

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14. Given : $f(x) = \begin{cases} x \ , & 0 \le x < \frac{1}{2} \\ \frac{1}{2} \ , & x = \frac{1}{2} \\ 1 - x \ , & \frac{1}{2} < x \le 1 \end{cases}$

and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in R$

Then the area (in sq. units) of the region bounded by the curves, y = f(x) and y = g(x) between the lines, 2x = 1 and 2x = $\sqrt{3}$, is : [4 marks]

(A) $\frac{1}{2} + \frac{\sqrt{3}}{4}$

(B) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

(C) $\frac{1}{3} + \frac{\sqrt{3}}{4}$

(D) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

Correct Answer: (D) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

Solution:

Concept:

- The area between two curves is the integral of upper ordinate minus lower ordinate over the specified interval.
- On $[1/2, \sqrt{3}/2]$, the applicable branch of the given function is $f(x) = 1 - x$, including its value at the left endpoint.

Step 1: Select the correct branch and compare ordinates The two vertical boundaries are $x = 1/2$ and $x = \sqrt{3}/2$. This entire interval lies between $1/2$ and 1, so the branch $f(x) = x$ does not contribute.

Subtract the parabola from the straight line:

$$f(x) - g(x) = 1 - x - \left(x - \frac{1}{2}\right)^2 = \frac{3}{4} - x^2.$$

This quantity is nonnegative throughout the interval and vanishes at $x = \sqrt{3}/2$. Therefore the line is the upper curve, and no subdivision or reversal of sign is needed.

Step 2: Integrate the positive difference The required area is

$$A = \int_{1/2}^{\sqrt{3}/2} \left(\frac{3}{4} - x^2\right) dx = \left[\frac{3x}{4} - \frac{x^3}{3}\right]_{1/2}^{\sqrt{3}/2}.$$

At the upper endpoint, the antiderivative equals $3\sqrt{3}/8 - \sqrt{3}/8 = \sqrt{3}/4$. At the lower endpoint, it equals $3/8 - 1/24 = 1/3$. Thus

$$A = \frac{\sqrt{3}}{4} - \frac{1}{3}.$$

It is positive and approximately 0.0997 square units, consistent with a narrow region whose height decreases from one-half to zero. The other three expressions do not equal this integral.

Step 3: Conclusion The area is $\boxed{\sqrt{3}/4 - 1/3}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Choose the piecewise branch from the integration interval before subtracting the curves; then test the sign of the difference.

15. If z be a complex number satisfying |Re(z)| + |Im(z)| = 4, then |z| cannot be : [4 marks]

(A) $\sqrt{10}$
(B) $\sqrt{7}$
(C) $\sqrt{\frac{17}{2}}$
(D) $\sqrt{8}$

Correct Answer: (B) $\sqrt{7}$

Solution:

Concept:

- For $z = x + iy$, the modulus satisfies $|z|^2 = x^2 + y^2$. Put $u = |x|$, $v = |y|$, so $u, v \geq 0$ and $u + v = 4$.
- With a fixed sum, the sum of two squares is smallest when the two nonnegative numbers are equal.

Step 1: Find the possible range of the squared modulus Using $v = 4 - u$, where $0 \leq u \leq 4$, gives

$$|z|^2 = u^2 + (4 - u)^2 = 2(u - 2)^2 + 8.$$

Its minimum is 8 at $u = v = 2$. Its maximum on this closed interval is 16, attained at $(u, v) = (4, 0)$ or $(0, 4)$. Since the expression varies continuously, every squared modulus between 8 and 16 occurs. Equivalently, $2\sqrt{2} \leq |z| \leq 4$.

Step 2: Test the four proposed moduli The squared values corresponding to (A), (B), (C), and (D) are respectively $10, 7, 17/2, 8$. Only 7 lies outside $[8, 16]$, so $\sqrt{7}$ is impossible. To verify that the other choices really occur, take $(u, v) = (3, 1)$ for $\sqrt{10}$, $(5/2, 3/2)$ for $\sqrt{17/2}$, and $(2, 2)$ for $\sqrt{8}$. Each pair sums to four and gives the required sum of squares. The condition concerns absolute real and imaginary components, so using nonnegative examples loses no generality.

Step 3: Conclusion The value $\boxed{\sqrt{7}}$ cannot be the modulus. Hence, the correct option is (B).

Quick Tip to Solve: Square the candidate moduli and compare with the exact interval for the sum of squares; this avoids unnecessary square-root estimates.

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16. If $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$; y(1) = 1; then a value of x satisfying y(x) = e is : [4 marks]

(A) $\sqrt{2}e$

(B) $\frac{1}{2}\sqrt{3}e$

(C) $\frac{e}{\sqrt{2}}$

(D) $\sqrt{3}e$

Correct Answer: (D) $\sqrt{3}e$

Solution:

Concept:

- It is convenient to regard x as a function of y, because multiplying dx/dy by $2x$ produces the derivative of x^2.
- A linear equation $du/dy + P(y)u = Q(y)$ is solved by the integrating factor $\exp(\int P(y)dy)$.

Step 1: Invert the equation and make it linear Near $(1, 1)$, both variables and the given derivative are nonzero, so inversion is valid. From the differential equation,

$$\frac{dx}{dy} = \frac{x^2 + y^2}{xy}.$$

Set $u = x^2$; then

$$\frac{du}{dy} = 2x\frac{dx}{dy} = \frac{2u}{y} + 2y, \qquad \frac{du}{dy} - \frac{2}{y}u = 2y.$$

The integrating factor for positive y is y^{-2}.

Step 2: Integrate and apply the initial condition Multiplying through by y^{-2} gives

$$\frac{d}{dy}\left(\frac{u}{y^2}\right) = \frac{2}{y}.$$

Hence

$$\frac{x^2}{y^2} = 2\ln y + C.$$

At $x = 1, y = 1$, the left side is one and $\ln 1 = 0$, so $C = 1$. The solution relation on the positive branch

is therefore

$$x^2 = y^2(1 + 2\ln y).$$

Step 3: Set the required value of y For $y = e$, $\ln y = 1$, so $x^2 = 3e^2$. The branch through $(1, 1)$ has positive x, giving $x = \sqrt{3}\,e$. This branch reaches $y = e$ continuously: differentiating $x = y\sqrt{1 + 2\ln y}$ gives a positive derivative for $y \geq 1$. The other listed positive multiples of e have squares different from $3e^2$, so they do not satisfy the initial-condition relation.

Step 4: Conclusion A value satisfying $y(x) = e$ is $\boxed{x = \sqrt{3}\,e}$. Hence, the correct option is (D).

Quick Tip to Solve: When the inverse derivative contains x squared divided by xy, try u=x squared after treating x as a function of y.

17. If A = {x ∈ R : |x| < 2} and B = {x ∈ R : |x – 2| ≥ 3}; then : [4 marks]

(A) A – B = [–1, 2)

(B) A ∪ B = R – (2, 5)

(C) A ∩ B = (–2, –1)

(D) B – A = R – (–2, 5)

Correct Answer: (D) B – A = R – (–2, 5)

Solution:

Concept:

- The condition $|x| < 2$ gives the open interval $A = (-2, 2)$.
- The inequality $|x-2| \geq 3$ splits into $x \leq -1$ or $x \geq 5$; endpoints are included because the inequality is non-strict.

Step 1: Write both sets and their union and intersection In interval notation,

$$A = (-2, 2), \qquad B = (-\infty, -1] \cup [5, \infty).$$

Their common elements lie in $(-2, -1]$, including -1 but excluding -2. Their union is

$$A \cup B = (-\infty, 2) \cup [5, \infty) = \mathbb{R} \setminus [2, 5).$$

Thus option (B) incorrectly includes 2, and option (C) incorrectly excludes -1. Checking these boundary points is necessary; the interiors alone cannot distinguish the printed choices.

Step 2: Compute the two set differences Removing B from A leaves

$$A \setminus B = (-1, 2).$$

The point -1 belongs to B, so option (A), which includes it, is false. Conversely, removing A from B gives

$$B \setminus A = (-\infty, -2] \cup [5, \infty) = \mathbb{R} \setminus (-2, 5).$$

The endpoint -2 remains because it is in B but not in A, while 5 remains because it is in B and lies

outside A. This is exactly the expression in option (D).

Step 3: Conclusion $\boxed{B \setminus A = \mathbb{R} \setminus (-2, 5)}$. Hence, the correct option is (D).

Quick Tip to Solve: Test the boundary points -2, -1, 2, and 5 explicitly; open and closed endpoints decide all four choices here.

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18. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, x + 6y = 8; then its eccentricity is : [4 marks]

(A) $\frac{1}{3}\sqrt{\frac{11}{3}}$

(B) $\frac{1}{2}\sqrt{\frac{5}{3}}$

(C) $\sqrt{\frac{5}{6}}$

(D) $\frac{1}{2}\sqrt{\frac{11}{3}}$

Correct Answer: (D) $\frac{1}{2}\sqrt{\frac{11}{3}}$

Solution:

Concept:

- The standard ellipse with major axis along x is $x^2/a^2 + y^2/b^2 = 1$, where $a > b > 0$; its minor-axis length is $2b$.
- A line $lx + my = n$ is tangent to this ellipse exactly when $n^2 = a^2l^2 + b^2m^2$. Its eccentricity is $e = \sqrt{1 - b^2/a^2}$.

Step 1: Determine the semi-minor axis The stated minor-axis length is the full length, so

$$2b = \frac{4}{\sqrt{3}}, \qquad b = \frac{2}{\sqrt{3}}, \qquad b^2 = \frac{4}{3}.$$

Using the full length as b would square an extra factor of two and give the wrong ellipse. The minor axis is explicitly along the y-axis, fixing the placement of a^2 and b^2 in the standard equation.

Step 2: Apply the tangent condition For $x + 6y = 8$, take $l = 1, m = 6, n = 8$. The tangency condition gives

$$64 = a^2 + 36b^2 = a^2 + 36\left(\frac{4}{3}\right) = a^2 + 48.$$

Hence $a^2 = 16$, which satisfies $a^2 > b^2$ as required. One can also verify the condition by substituting $x = 8 - 6y$ in the ellipse and requiring the resulting quadratic in y to have zero discriminant.

Step 3: Calculate the eccentricity and compare options Now

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{4/3}{16} = 1 - \frac{1}{12} = \frac{11}{12}.$$

Consequently

$$e = \sqrt{\frac{11}{12}} = \frac{1}{2}\sqrt{\frac{11}{3}}.$$

Eccentricity is nonnegative and less than one for an ellipse. Squaring the four printed choices gives $11/27, 5/12, 5/6, 11/12$, respectively, so only the last choice has the value obtained from the tangent condition.

Step 4: Conclusion The eccentricity is $\boxed{\frac{1}{2}\sqrt{\frac{11}{3}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Halve the minor-axis length first, then use n squared equals a squared l squared plus b squared m squared for a tangent line.

19. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of the tangent to it at B is : [4 marks]

(A) 2x – y – 24 = 0

(B) x – 2y + 8 = 0

(C) x + 2y + 8 = 0

(D) 2x + y – 24 = 0

Correct Answer: (B) x – 2y + 8 = 0

Solution:

Concept:

- For the parabola $y^2 = 4ax$, a point with parameter t is $(at^2, 2at)$, and its tangent is $ty = x + at^2$.
- The parameters of the two ends of a focal chord satisfy $t_1 t_2 = -1$, because their joining line passes through the focus $(a, 0)$.

Step 1: Identify the parameter at A Comparing $y^2 = 8x$ with $y^2 = 4ax$ gives $a = 2$. At the given point $A(1/2, -2)$, the ordinate equation gives

$$2at_1 = -2 \quad \Rightarrow \quad 4t_1 = -2 \quad \Rightarrow \quad t_1 = -\frac{1}{2}.$$

Its abscissa is also consistent: $at_1^2 = 2(1/4) = 1/2$. Thus the sign of the parameter is fixed by the negative ordinate, rather than by taking a positive square root of the abscissa.

Step 2: Find the other end of the focal chord The focus is $(2, 0)$. For two parametric points the chord equation is $(t_1 + t_2)y = 2x + 2at_1t_2$. Putting the focus into it yields $0 = 2a + 2at_1t_2$, hence $t_1t_2 = -1$. Therefore

$$t_2 = -\frac{1}{t_1} = 2, \qquad B = (2 \cdot 2^2, 4 \cdot 2) = (8, 8).$$

Step 3: Write and check the tangent at B Using $ty = x + at^2$ with $t = 2$ gives

$$2y = x + 8 \quad \Rightarrow \quad x - 2y + 8 = 0.$$

This line passes through $(8, 8)$, as a tangent at B must. Its slope is $1/2$, also obtained by differentiating $y^2 = 8x$: $dy/dx = 4/y = 1/2$ at B. The other listed lines either do not pass through B or have the wrong tangent slope.

Step 4: Conclusion The tangent at the other end is $\boxed{x - 2y + 8 = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: For a focal chord of $y^2 = 4ax$, use $t_2 = -1/t_1$ before writing the tangent; the ordinate determines the parameter sign.

20. If p $\rightarrow$ (p $\wedge$ ~q) is false, then the truth values of p and q are respectively : [4 marks]

(A) T, T

(B) T, F

(C) F, T

(D) F, F

Correct Answer: (A) T, T

Solution:

Concept:

- An implication $p \rightarrow r$ is false only when its antecedent p is true and its consequent r is false.
- A conjunction is true only when both components are true; negation reverses a truth value. Here the consequent is $p \wedge \neg q$.

Step 1: Apply the condition for a false implication Treat the complete bracket $p \wedge \neg q$ as the consequent. Since the given implication is false, its antecedent cannot be false: an implication with a false antecedent is true regardless of its consequent. We therefore require

$$p = T, \qquad p \wedge \neg q = F.$$

This immediately excludes both choices in which p is F, namely (C) and (D).

Step 2: Determine q using the conjunction With $p = T$, the expression $p \wedge \neg q$ has precisely the same truth value as $\neg q$. For this conjunction to be false, $\neg q$ must be false. Thus $q = T$. Notice that setting $q = F$ instead would make $\neg q = T$, hence would make the consequent true and the whole implication true; this rules out option (B).

Step 3: Verify all possible pairs The short truth table checks the logical grouping directly:

p	q	$\neg q$	$p \wedge \neg q$	$p \rightarrow (p \wedge \neg q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

Exactly one row makes the stated implication false. In particular, the negation belongs only to q, while the outer implication has the complete conjunction as its consequent.

Step 4: Conclusion The truth values of p and q are respectively $\boxed{T, T}$. Hence, the correct option is (A).

Quick Tip to Solve: For a false implication, first write antecedent T and consequent F; only then unpack any conjunction or negation inside the consequent.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is _____. [4 marks]

Solution:

Concept:

- A term common to two arithmetic progressions must satisfy both of their congruence conditions and must lie within both finite ranges.
- When the common differences are coprime, consecutive common terms differ by their least common multiple. Here $\text{lcm}(4, 7) = 28$.

Step 1: Express the common-term condition Write a term of the first progression as $3 + 4r$, with integer $r \geq 0$, and a term of the second as $2 + 7s$, with integer $s \geq 0$. Equating them gives

$$3 + 4r = 2 + 7s \quad \Rightarrow \quad 4r - 7s = -1.$$

Working modulo 7, this says $4r \equiv 6 \pmod 7$. Since the inverse of 4 modulo 7 is 2, $r \equiv 12 \equiv 5 \pmod 7$.

Step 2: Generate the common progression The smallest nonnegative parameter is $r = 5$, giving the first common value $3+4(5) = 23$. It also equals $2+7(3)$, so it really belongs to the second progression. Put $r = 5 + 7j$; the common values are then

$$3 + 4(5 + 7j) = 23 + 28j, \qquad j = 0, 1, 2, \ldots.$$

The gap 28 is consistent with being a multiple of both original common differences.

Step 3: Impose the finite upper bounds A common term cannot exceed the smaller last term, namely 407, even though the second progression continues to 709. Therefore

$$23 + 28j \leq 407 \quad \Rightarrow \quad j \leq \frac{384}{28} = 13 + \frac{5}{7}.$$

The allowed integers are $j = 0, 1, \ldots, 13$, giving 14 values. The last is $23 + 28(13) = 387$; the next, 415, is already beyond the first progression. The lower bound is also satisfied for every listed term.

Step 4: Conclusion The number of common terms is $\boxed{14}$.

Quick Tip to Solve: Find the first common term by congruences, then count with the LCM of the common differences, stopping at the smaller upper endpoint.

22. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $\left|\vec{b}\right| = 5$, $\vec{b}.\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $\left|\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right|$ is equal to _____.

[4 marks]

Solution:

Concept:

- For vectors with included angle θ, $\vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}| \cos\theta$, while $|\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}| \sin\theta$.
- The magnitude of $\vec{a} \times \vec{d}$ is $|\vec{a}||\vec{d}| \sin\phi$, where ϕ is their angle. Perpendicular vectors have $\sin\phi = 1$.

Step 1: Determine the magnitude of c The supplied dot product and angle determine the unknown magnitude without needing coordinates. Substitute $|\vec{b}| = 5$, $\theta = \pi/3$, and $\vec{b} \cdot \vec{c} = 10$:

$$10 = 5|\vec{c}| \cos\frac{\pi}{3} = \frac{5}{2}|\vec{c}|.$$

Hence $|\vec{c}| = 4$. The positive value is appropriate because it is a magnitude, and the positive dot product agrees with the acute included angle.

Step 2: Find the inner cross-product magnitude For the same two vectors use the sine of their included angle:

$$|\vec{b} \times \vec{c}| = 5 \cdot 4 \sin\frac{\pi}{3} = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3}.$$

The dot product uses the cosine and the cross product uses the sine; interchanging them would miss the required factor of $\sqrt{3}$. Since the angle is not zero, this cross-product vector is nonzero.

Step 3: Use the stated perpendicularity Let $\vec{d} = \vec{b} \times \vec{c}$. The question states that $\vec{a}$ is perpendicular to this vector, so their included angle is $\pi/2$, regardless of the angle between $\vec{a}$ and either of the original vectors. Thus

$$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{d}| \sin\frac{\pi}{2} = \sqrt{3}(10\sqrt{3}) = 30.$$

No vector direction or component construction is necessary because only the magnitude is requested.

Step 4: Conclusion The required magnitude is $\boxed{30}$.

Quick Tip to Solve: Use the given dot product to find $|\vec{c}|$, then use the sine formula; perpendicularity to the inner cross product makes the last sine factor one.

23. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is _____.

[4 marks]

Solution:

Concept:

- Completing squares identifies a circle's centre and radius. Two circles touch when the centre distance equals the sum of the radii (external contact) or their absolute difference (internal contact).

- Both contact conditions must be examined when the problem asks for the largest possible parameter and does not specify the type of tangency.

Step 1: Identify the two centres and radii Complete the square in the first equation:

$$x^2 - 6x + y^2 + 8 = 0 \quad \Rightarrow \quad (x-3)^2 + y^2 = 1.$$

Its centre is $C_1 = (3, 0)$ and radius is $r_1 = 1$. Similarly, the second equation becomes

$$x^2 + y^2 - 8y + 16 - k = 0 \quad \Rightarrow \quad x^2 + (y-4)^2 = k.$$

Its centre is $C_2 = (0, 4)$ and radius is $r_2 = \sqrt{k} > 0$.

Step 2: Compute the fixed centre distance The distance between the centres is

$$d = \sqrt{(3-0)^2 + (0-4)^2} = \sqrt{9+16} = 5.$$

Therefore the circles have distinct centres. A single common point can arise from either external tangency or internal tangency, and the positive-radius condition alone does not select between them.

Step 3: Solve both tangency conditions For external contact, $d = r_1 + r_2$, so $5 = 1 + \sqrt{k}$, yielding $\sqrt{k} = 4$ and $k = 16$. For internal contact, $d = |r_2 - r_1|$, so $|\sqrt{k} - 1| = 5$. The equation $\sqrt{k} - 1 = 5$ gives $\sqrt{k} = 6$, hence $k = 36$; the other sign gives the impossible $\sqrt{k} = -4$. At $k = 36$, the larger circle contains the smaller and touches it because $6 - 1 = 5$. Comparing the two admissible values, 36 is larger.

Step 4: Conclusion The largest possible value is $\boxed{k = 36}$.

Quick Tip to Solve: For unspecified circle tangency, check both radius sum and radius difference; using only external tangency can miss the larger parameter.

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24. If the distance between the plane, 23x – 10y – 2z + 48 = 0 and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ $(\lambda \in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to _____. [4 marks]

Solution:

Concept:

- The normal of a plane containing a line must be perpendicular to that line's direction and to any vector joining two points in the plane.
- Parallel planes $ax+by+cz+d_1 = 0$ and $ax+by+cz+d_2 = 0$ have separation $|d_1 - d_2|/\sqrt{a^2+b^2+c^2}$.

Step 1: Construct a normal from the given lines The first line contains $P = (-1, 3, -1)$ and has direction $\vec{u} = (2, 4, 3)$. The second contains $Q = (-3, -2, 1)$. Hence $\overrightarrow{PQ} = (-2, -5, 2)$. Any plane

containing both lines must contain these two points and the first direction. A normal is therefore

$$\vec{n} = \vec{u} \times \overrightarrow{PQ} = (23, -10, -2).$$

These vectors are not parallel, so they determine a unique candidate plane.

Step 2: Write the containing plane and check the second direction Through P, this plane is

$$23(x+1) - 10(y-3) - 2(z+1) = 0,$$

or $23x - 10y - 2z + 51 = 0$. For the second line to lie wholly in it, its direction $(2, 6, \lambda)$ must be perpendicular to the normal:

$$23(2) - 10(6) - 2\lambda = 0 \quad \Rightarrow \quad \lambda = -7.$$

Its point Q also satisfies the plane: $-69 + 20 - 2 + 51 = 0$. Thus the plane indeed contains both lines for the required real value of the parameter.

Step 3: Calculate the distance and identify k The given plane has the same normal coefficients and constant 48. Thus the planes are parallel, and the required distance is

$$d = \frac{|51 - 48|}{\sqrt{23^2 + (-10)^2 + (-2)^2}} = \frac{3}{\sqrt{529 + 100 + 4}} = \frac{3}{\sqrt{633}}.$$

Comparing with $k/\sqrt{633}$ gives k directly. The absolute value is required since a geometric distance cannot be negative.

Step 4: Conclusion The required value is $\boxed{k = 3}$.

Quick Tip to Solve: Take the cross product of one line direction with a vector between the two lines; it often reveals the common plane normal before solving the parameter.

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25. If $C_r \equiv {}^{25}C_r$ and

$C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to _____. [4 marks]

Solution:

Concept:

- The coefficient is $C_r = \binom{25}{r}$, and its multiplier follows the arithmetic sequence $1, 5, 9, \ldots, 101$, whose general term is $4r + 1$.
- The binomial identities $\sum_{r=0}^{n} \binom{n}{r} = 2^n$ and $\sum_{r=0}^{n} r\binom{n}{r} = n2^{n-1}$ separate the constant and linear weights.

Step 1: Express the given sum with its general term Let the entire left-hand side be S. At $r = 0$ the multiplier is 1, and at $r = 25$ it is $4(25) + 1 = 101$, so both endpoints agree with the question.

Therefore

$$S = \sum_{r=0}^{25}(4r+1)\binom{25}{r} = 4\sum_{r=0}^{25} r\binom{25}{r} + \sum_{r=0}^{25}\binom{25}{r}.$$

The sum includes all coefficients from the zeroth to the twenty-fifth term; neither endpoint is omitted.

Step 2: Evaluate the two binomial sums Putting $x = 1$ in $(1+x)^{25}$ gives the unweighted sum 2^{25}. Differentiating that expansion and then putting $x = 1$ gives

$$\sum_{r=0}^{25} r\binom{25}{r} = 25(1+1)^{24} = 25 \cdot 2^{24}.$$

Hence

$$S = 4 \cdot 25 \cdot 2^{24} + 2^{25} = 50 \cdot 2^{25} + 2^{25} = 51 \cdot 2^{25}.$$

Comparison with the given expression $2^{25}k$ determines the required dimensionless constant. The factor of two in converting 2^{24} to 2^{25} is essential.

Step 3: Conclusion The value is $\boxed{k = 51}$.

Quick Tip to Solve: For a coefficient multiplied by a linear expression in its index, split the sum into the standard sums of $\binom{n}{r}$ and $r\binom{n}{r}$.

Physics – Section A: Multiple Choice Questions (80 marks)

26. An electron of mass m and magnitude of charge |e| initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t ignoring relativistic effects is : [4 marks]

(A) $\frac{-h}{|e|Et}$

(B) $\frac{-h}{|e|E\sqrt{t}}$

(C) $\frac{-h}{|e|Et^2}$

(D) $\frac{|e|Et}{h}$

Correct Answer: (C) $\frac{-h}{|e|Et^2}$

Solution:

Concept:

- An electric field gives an electron a constant force of magnitude $|e|E$. Starting from rest, its momentum magnitude is $p = |e|Et$.
- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant. The requested rate is a time derivative, not the wavelength itself.

Step 1: Obtain momentum as a function of time The electron accelerates opposite to the electric field because its charge is negative. Its speed and momentum magnitudes nevertheless increase with time.

Ignoring relativistic effects, Newton's law gives

$$ma = |e|E, \qquad v = at = \frac{|e|Et}{m}.$$

Multiplying the speed by the mass gives $p = mv = |e|Et$. Equivalently, constant force multiplied by elapsed time gives the same impulse. The cancellation of mass means that the wavelength at a given elapsed time does not depend explicitly on the electron mass.

Step 2: Differentiate the wavelength For $t > 0$, substitute the momentum into the wavelength relation:

$$\lambda(t) = \frac{h}{|e|Et}.$$

The quantities h, $|e|$ and E are constant, so

$$\frac{d\lambda}{dt} = \frac{h}{|e|E}\frac{d}{dt}(t^{-1}) = -\frac{h}{|e|Et^2}.$$

The negative sign expresses a decreasing wavelength as the momentum increases. Option (A) is the negative wavelength rather than its derivative; (B) has the wrong time dependence. Option (D) is the reciprocal wavelength and is positive, so it cannot describe this rate. At precisely zero time the ideal de Broglie expression diverges, and the displayed result applies after acceleration has begun.

Step 3: Conclusion $\boxed{d\lambda/dt = -h/(|e|Et^2)}$. Hence, the correct option is (C).

Quick Tip to Solve: First find momentum from impulse $p = Ft$; then differentiate h/p. This avoids carrying an unnecessary mass through the calculation.

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27. There is a small source of light at some depth below the surface of water (refractive index = $\frac{4}{3}$) in a tank of large cross sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly) :

[Use the fact that surface area of a spherical cap of height h and radius of curvature r is 2πrh]:

[4 marks]

(A) 17%

(B) 34%

(C) 50%

(D) 21%

Correct Answer: (A) 17%

Solution:

Concept:

- For water to air, the critical angle satisfies $\sin\theta_c = 1/\mu$. Only rays in the upward cone with incidence angle at most θ_c emerge.
- An isotropic point source distributes its light over a full sphere of area $4\pi r^2$. The escaping cone cuts out a spherical cap of area $2\pi r^2(1 - \cos\theta_c)$.

Step 1: Calculate the critical angle Taking the refractive index of air as 1 and that of water as $4/3$, Snell's law at grazing emergence gives

$$\sin\theta_c = \frac{1}{4/3} = \frac{3}{4}, \qquad \cos\theta_c = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}.$$

Angles are measured from the upward normal to the water surface. Rays outside this upward cone undergo total internal reflection instead of passing directly into air. The source depth changes the radius of the illuminated patch on the surface but does not change the cone angle or fraction of total emission.

Step 2: Compare the cap with the entire sphere Choose a sphere of arbitrary radius r centred at the source. The cone intersects it in a cap of height $h = r - r\cos\theta_c$. Using the supplied cap-area formula, the fraction of the total light in this cone is

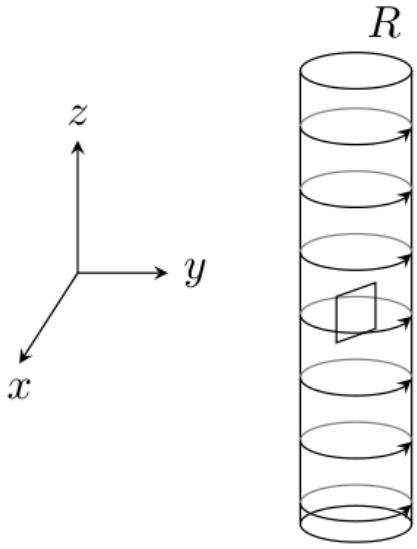
$$f = \frac{2\pi r h}{4\pi r^2} = \frac{1 - \cos\theta_c}{2} = \frac{1}{2}\left(1 - \frac{\sqrt{7}}{4}\right) \approx 0.1693.$$

Thus the emerging percentage is about 16.93%. Dividing by an upward hemisphere alone would give about 34%, but the question asks for the fraction of all light emitted by the source. The 50% choice ignores total internal reflection, while 21% does not follow from the critical-angle solid angle.

Step 3: Conclusion The emerging light is nearly $\boxed{17\%}$ of the total emission. Hence, the correct option is (A).

Quick Tip to Solve: For an isotropic submerged point source, use the solid-angle fraction $(1 - \cos\theta_c)/2$; the factor 2 accounts for emission into the full sphere.

28. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning) : [4 marks]



(A) $\frac{e\mu_0 nIR}{4m}$
(B) $\frac{e\mu_0 nIR}{m}$
(C) $\frac{e\mu_0 nIR}{2m}$
(D) $\frac{2e\mu_0 nIR}{m}$

Correct Answer: (C) $\frac{e\mu_0 nIR}{2m}$

Solution:

Concept:

- The magnetic field inside a long solenoid is $B = \mu_0 nI$, directed along its axis. The radial launch velocity is perpendicular to this field.
- The electron follows a circular orbit of radius $\rho = mv/(eB)$, where e denotes the positive magnitude of its charge. Magnetic force changes direction without changing speed.

Step 1: Determine the orbit radius Inside the solenoid the uniform magnetic force supplies the centripetal force:

$$evB = \frac{mv^2}{\rho}, \qquad \rho = \frac{mv}{e\mu_0 nI}.$$

The electron starts at the axis of the solenoid, not at the centre of its own circular orbit. The centre of its orbit is displaced a distance ρ from the solenoid axis in the plane perpendicular to the field. Reversing the electron charge reverses the sense of rotation, but it does not change the radius or maximum distance from the axis.

Step 2: Apply the geometric clearance condition The trajectory is a circle passing through the launch point on the solenoid axis. Its farthest point from that axis lies one diameter away, so the maximum

radial excursion is 2ρ. To remain inside a solenoid of radius R, the limiting condition is

$$2\rho = R.$$

Consequently the limiting speed is

$$v_{\text{lim}} = \frac{eBR}{2m} = \frac{e\mu_0 nIR}{2m}.$$

Strict clearance corresponds to speeds below this limiting value; the equality is the usual maximum-speed boundary intended in the options. Setting $\rho = R$ instead would incorrectly treat the launch point as the orbit centre and yield (B). Choice (A) is unnecessarily restrictive, while (D) produces a radial excursion much larger than the solenoid radius.

Step 3: Conclusion The maximum allowed limiting speed is $\boxed{e\mu_0 nIR/(2m)}$. Hence, the correct option is (C).

Quick Tip to Solve: When a particle starts on the solenoid axis, compare the orbit diameter, not its radius, with the available solenoid radius.

29. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is: [4 marks]

(A) 1 : 2

(B) 2 : 1

(C) $1 : \sqrt{2}$

(D) $\sqrt{2} : 1$

Correct Answer: (D) $\sqrt{2} : 1$

Solution:

Concept:

- For a wire obeying Hooke's law, elastic energy per unit volume is $u = \frac{1}{2}\sigma\epsilon = \sigma^2/(2Y)$, with stress σ, strain ϵ, and Young's modulus Y.
- For the same tensile load F, stress is F/A. A circular wire has cross-sectional area $A = \pi d^2/4$, so energy density varies as d^{-4}.

Step 1: Write the energy density in terms of diameter Both wires are steel, so they have the same Young's modulus. They also carry the same load, which means equal force rather than equal stress. Substitution gives

$$u = \frac{F^2}{2YA^2} = \frac{F^2}{2Y}\left(\frac{4}{\pi d^2}\right)^2 = \frac{8F^2}{Y\pi^2 d^4}.$$

Although the wires have equal lengths, length cancels from energy per unit volume. The thinner wire therefore stores more energy per unit volume because its stress and strain are both larger under the same force.

Step 2: Use the given density ratio Let the diameters corresponding to the given density order be d_1

and d_2. Then

$$\frac{u_1}{u_2} = \left(\frac{d_2}{d_1}\right)^4 = \frac{1}{4}.$$

Taking fourth roots, or first finding the area ratio, gives

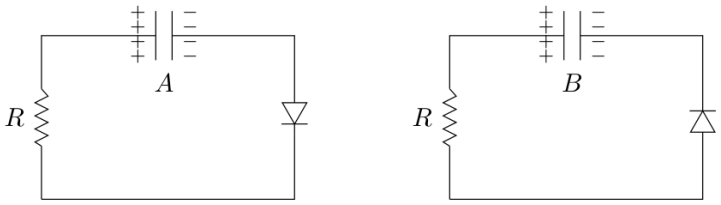
$$\frac{d_1}{d_2} = 4^{1/4} = \sqrt{2}.$$

The first wire has lower energy density, so it must have the larger diameter, which also checks the order of the result. The ratios $1:2$ and $1:\sqrt{2}$ put the thinner wire first and give the opposite density trend. The ratio $2:1$ would yield an energy-density ratio $1:16$, not $1:4$. Confusing total energy with energy density would miss one power of area.

Step 3: Conclusion The ratio of diameters is $\boxed{\sqrt{2}:1}$. Hence, the correct option is (D).

Quick Tip to Solve: Under equal loads, strain-energy density is proportional to inverse diameter to the fourth power; take a fourth root when converting its ratio to a diameter ratio.

30. Two identical capacitors A and B, charged to the same potential 5V are connected in two different circuits as shown below at time t = 0. If the charge on capacitors A and B at time t = CR is Q_A and Q_B respectively, then (Here e is the base of natural logarithm) [4 marks]



(A) $Q_A = \frac{CV}{e}$, $Q_B = \frac{VC}{2}$
(B) $Q_A = \frac{CV}{2}$, $Q_B = \frac{VC}{e}$
(C) $Q_A = VC$, $Q_B = \frac{VC}{e}$
(D) $Q_A = VC$, $Q_B = CV$

Correct Answer: (C) $Q_A = VC$, $Q_B = \frac{VC}{e}$

Solution:

Concept:

- An ideal forward-biased diode permits discharge, whereas a reverse-biased diode blocks current. The initial capacitor charge magnitude is $Q_0 = CV$, with $V = 5\,\text{V}$.
- A capacitor discharging through resistance R has $Q(t) = Q_0 e^{-t/(RC)}$. If the diode prevents a conducting path, the ideal capacitor retains its initial charge.

Step 1: Identify which circuit can discharge In both figures the left capacitor plate is initially positive and the right plate negative. Conventional discharge current would leave the left plate, travel down through the resistor, and return upward through the diode to the negative plate. In circuit A the

diode permits the opposite direction along its vertical branch, so it is reverse biased for this discharge current. No closed conducting path exists, and

$$Q_A(t) = Q_0 = CV.$$

In circuit B the diode is reversed relative to A and permits the required upward current. It is forward biased, so the capacitor B discharges through the resistor. This uses the ideal-diode behaviour implicit in the options; no finite diode threshold is specified.

Step 2: Evaluate the conducting circuit at one time constant For circuit B, the resistor voltage equals the instantaneous capacitor voltage while the diode conducts. Combining $I = Q/(RC)$ with $I = -dQ/dt$ yields

$$Q_B(t) = CV \exp\left(-\frac{t}{RC}\right).$$

At $t = CR = RC$, exactly one time constant has elapsed, and hence

$$Q_B = \frac{CV}{e}, \qquad Q_A = CV.$$

The decaying charge after one time constant is about $0.368CV$, rather than $CV/2$. Thus (A) and (B) both assign an unjustified half-charge and fail to respect diode orientation. Choice (D) would require both diodes to block discharge, whereas B has a forward conducting path.

Step 3: Conclusion $\boxed{Q_A = CV, \quad Q_B = CV/e}$, where $V = 5\,\text{V}$. Hence, the correct option is (C).

Quick Tip to Solve: Trace conventional current from the initially positive plate before applying the exponential law; only the forward-biased diode circuit has an RC discharge.

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31. A plane electromagnetic wave is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$, with its polarization along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant) : [4 marks]

(A) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(B) $B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(C) $B_0\frac{\hat{j}-\hat{i}}{\sqrt{2}}\cos\left(\omega t + k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(D) $B_0\hat{k}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

Correct Answer: (A) $B_0\frac{\hat{i}-\hat{j}}{\sqrt{2}}\cos\left(\omega t - k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

Solution:

Concept:

- In a plane electromagnetic wave, the electric field, magnetic field and propagation direction are mutually perpendicular.
- The field directions obey $\mathbf{B} = c^{-1}\hat{\mathbf{n}} \times \mathbf{E}$, so $\mathbf{E} \times \mathbf{B}$ points along the propagation direction.

Step 1: Identify the electric-field direction Polarization specifies the direction of the electric field. Here $\mathbf{E}$ is along $\hat{k}$, while the unit propagation vector is $\hat{\mathbf{n}} = (\hat{i}+\hat{j})/\sqrt{2}$. Thus the magnetic field lies in the xy-plane and must be perpendicular to both of these directions.

Step 2: Use the right-handed cross product Calculate the required direction without interchanging the order of the vectors:

$$\hat{\mathbf{n}} \times \hat{k} = \frac{\hat{i} \times \hat{k} + \hat{j} \times \hat{k}}{\sqrt{2}} = \frac{-\hat{j} + \hat{i}}{\sqrt{2}}.$$

As a check, $\hat{k} \times (\hat{i} - \hat{j}) = \hat{i} + \hat{j}$, giving the stipulated energy-flow direction.

Step 3: Compare the transverse directions and phase signs Option (A) has this magnetic-field direction and the phase sign corresponding to forward propagation. In standard scalar phase notation the argument is $\omega t - k\hat{\mathbf{n}} \cdot \mathbf{r}$. The directional expression in the choices represents this propagation dependence. Option (B) makes the magnetic field longitudinal; (D) makes it parallel to the electric field. Option (C) reverses both the displayed transverse direction and the sign of the spatial phase, so it describes the opposite propagation sense.

Step 4: Conclusion The magnetic field is along $(\hat{i} - \hat{j})/\sqrt{2}$, with the forward-travelling phase shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: Find the magnetic-field direction using propagation direction crossed with electric-field direction, then check it with the Poynting vector.

32. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The raito of their respective mean free times is closest to : [4 marks]

(A) 2.3

(B) 1.83

(C) 4.67

(D) 3.67

Correct Answer: (B) 1.83

Solution:

Concept:

- The mean free path of hard-sphere molecules is $\lambda = 1/(\sqrt{2}\, n\pi D^2)$, where n is number density and D is collision diameter.
- The mean free time is $\tau = \lambda/\overline{v}$. At fixed temperature, $\overline{v} = \sqrt{8k_BT/(\pi m)}$, so $\tau \propto \sqrt{m}/r^2$.

Step 1: Form the mean-free-path ratio The collision diameter is twice the atomic radius, but this common factor cancels in the ratio. Since the gases have equal number density,

$$\frac{\lambda_{\mathrm{Ar}}}{\lambda_{\mathrm{Xe}}} = \left(\frac{0.10}{0.07}\right)^2 = \frac{100}{49}.$$

The larger xenon atom has a larger collision cross-section and therefore a smaller mean free path.

Step 2: Include the different thermal speeds At the same temperature the lighter argon atom moves faster. Mean free time is a path divided by a speed, not merely a path. Therefore

$$\frac{\tau_{\mathrm{Ar}}}{\tau_{\mathrm{Xe}}} = \frac{100}{49}\sqrt{\frac{40}{140}} = \frac{100}{49}\sqrt{\frac{2}{7}} \simeq 1.091.$$

The mass factor acts opposite to the mean-free-path factor, which is why neither the squared-radius ratio alone nor the speed ratio alone is the required answer.

Step 3: Select the closest listed value The question asks which value is closest. The absolute differences from the calculated ratio are approximately 1.209, 0.739, 3.579 and 2.579 for (A), (B), (C) and (D), respectively. Thus (B) is the nearest choice, although it is not a close numerical approximation to the ratio obtained from the stated data. In particular, do not reverse the mass ratio just to obtain a more familiar option.

Step 4: Conclusion The stated data give $\tau_{\mathrm{Ar}}/\tau_{\mathrm{Xe}} \simeq 1.09$; among the listed values, 1.83 is closest. Hence, the correct option is (B).

Quick Tip to Solve: For mean free time at fixed number density and temperature, combine both effects: $\tau \propto \sqrt{m}/r^2$.

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33. A wire of length L and mass per unit length 6.0 $\times$ 10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are : 420 Hz and 490 Hz. Then L in meters is : [4 marks]

(A) 5.1 m

(B) 2.1 m

(C) 1.1 m

(D) 8.1 m

Correct Answer: (B) 2.1 m

Solution:

Concept:

- The speed of transverse waves on a stretched wire is $v = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length.
- For a wire fixed at both ends, $f_n = nv/(2L)$; consecutive resonance frequencies differ by the fundamental frequency $v/(2L)$.

Step 1: Calculate the wave speed Use the given linear mass density, rather than the total mass of the wire:

$$v = \sqrt{\frac{540}{6.0 \times 10^{-3}}} = \sqrt{9.0 \times 10^4} = 300\,\mathrm{m\,s^{-1}}.$$

The ratio of tension to mass per unit length has units of speed squared. This speed is a property of the tension and wire density; it does not depend on which harmonic is excited.

Step 2: Find the fundamental from the spacing The two observed resonances are consecutive. Their

difference is therefore one fundamental frequency:

$$f_1 = 490 - 420 = 70\,\text{Hz}.$$

The lower observed frequency is not the fundamental. Indeed, $420/70 = 6$ and $490/70 = 7$, so the observations are consistent with the sixth and seventh harmonics.

Step 3: Solve for the length and check the choices Now use the fundamental-frequency relation:

$$L = \frac{v}{2f_1} = \frac{300}{2 \times 70} = \frac{15}{7}\,\text{m} \simeq 2.14\,\text{m}.$$

To the precision used in the choices this is $2.1\,\text{m}$. The other lengths would give fundamental spacings of about 29.4, 136.4 and $18.5\,\text{Hz}$, respectively, rather than the observed spacing of $70\,\text{Hz}$.

Step 4: Conclusion The wire length is approximately $2.1\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: For consecutive string resonances, subtract the frequencies first; the difference immediately gives the fundamental.

34. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is : [4 marks]

(A) 1

(B) 2

(C) 4

(D) 3

Correct Answer: (C) 4

Solution:

Concept:

- The escape speed from a spherical planet of mass M and radius R is $v_e = \sqrt{2GM/R}$, ignoring atmospheric resistance and rotation.
- The escape-speed ratio depends on the ratio of M/R, not on mass or surface gravity alone. The escaping body's mass cancels from energy conservation.

Step 1: Write the energy condition for escape The minimum launch kinetic energy must equal the magnitude of the initial gravitational potential energy, with zero final kinetic energy at infinity:

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}.$$

Cancelling the test mass gives the escape-speed formula. Both the planet mass and its launch radius must be included when comparing the planets.

Step 2: Compare the two mass-to-radius ratios For planet A, the ratio is M/R. For planet B, the

specified mass and radius are $M/2$ and $R/2$. Thus

$$\frac{M_B}{R_B} = \frac{M/2}{R/2} = \frac{M}{R}, \qquad v_B = \sqrt{\frac{2G(M/2)}{R/2}} = \sqrt{\frac{2GM}{R}} = v_A.$$

Although B has a different surface gravity, its gravitational potential per unit mass at the surface has the same magnitude as A's.

Step 3: Use the definition of n The speed ratio is unity, so the relation printed in the question gives

$$\frac{v_A}{v_B} = 1 = \frac{n}{4} \quad \Longrightarrow \quad n = 4.$$

Values 1, 2 and 3 would imply unequal escape speeds and therefore fail the direct substitution into the mass-to-radius expression. A common incorrect approach is to halve the escape speed when the mass is halved while overlooking that the radius is also halved.

Step 4: Conclusion The two escape speeds are equal, giving $n = 4$. Hence, the correct option is (C).

Quick Tip to Solve: Whenever mass and radius change by the same factor, M/R stays fixed and the escape speed stays unchanged.

35. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet) : [4 marks]

(A) $r = \sqrt{\frac{T}{(d-\rho)g}}$

(B) $r = \sqrt{\frac{2T}{3(d+\rho)g}}$

(C) $r = \sqrt{\frac{T}{(d+\rho)g}}$

(D) $r = \sqrt{\frac{3T}{(2d-\rho)g}}$

Correct Answer: (D) $r = \sqrt{\frac{3T}{(2d-\rho)g}}$

Solution:

Concept:

- A floating droplet is in vertical force balance: its weight equals the sum of buoyancy and the upward surface-tension force.
- For a sphere half immersed, the displaced volume is $2\pi r^3/3$, and its circular contact line has length $2\pi r$. The given upward capillary force is $2\pi rT$.

Step 1: Calculate the weight and buoyancy The complete spherical droplet has volume $4\pi r^3/3$, so its downward weight is

$$W = \frac{4}{3}\pi r^3 dg.$$

Only the immersed hemisphere displaces liquid. Archimedes' principle therefore gives the upward

buoyant force

$$F_b = \frac{2}{3}\pi r^3 \rho g.$$

Using the whole sphere for displaced volume would incorrectly replace the density combination in the final answer by $d - \rho$.

Step 2: Include the upward surface force The liquid surface meets the droplet around the equator, whose circumference is $2\pi r$. With the upward surface-tension action specified in the question, the capillary contribution is $F_T = 2\pi r T$. Vertical equilibrium then requires

$$\frac{4}{3}\pi r^3 dg = \frac{2}{3}\pi r^3 \rho g + 2\pi r T.$$

The two upward forces add; surface tension must not be assigned a downward sign here.

Step 3: Isolate the positive radius Cancel $2\pi r$, rearrange and take the positive square root:

$$\frac{1}{3}r^2(2d - \rho)g = T, \qquad r = \sqrt{\frac{3T}{(2d - \rho)g}}.$$

A finite positive radius in this force model requires $2d > \rho$. The units are correct because $T/(dg)$ has dimensions of area. Options with $d + \rho$ use the wrong buoyancy sign, and (A) also uses the wrong displaced volume.

Step 4: Conclusion The required radius is $r = \sqrt{3T/[(2d - \rho)g]}$. Hence, the correct option is (D).

Quick Tip to Solve: Write weight = buoyancy + capillary force before cancelling factors; half immersion means half the sphere displaces liquid.

36. For the four sets of three measured physical quantities as given below. Which of the following options is correct ?

(i) A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2

(ii) A_2 = 24.44, B_2 = 16.082, C_2 = 240.2

(iii) A_3 = 25.2, B_3 = 19.2812, C_3 = 236.183

(iv) A_4 = 25, B_4 = 236.191, C_4 = 19.5 [4 marks]

(A) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

(B) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2$

(C) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$

(D) $A_4 + B_4 + C_4 > A_3 + B_3 + C_3 = A_2 + B_2 + C_2 > A_1 + B_1 + C_1$

Correct Answer: (D) $A_4 + B_4 + C_4 > A_3 + B_3 + C_3 = A_2 + B_2 + C_2 > A_1 + B_1 + C_1$

Solution:

Concept:

- For addition of measured quantities, round the final sum to the least number of decimal places present among the terms.

- Decimal-place precision, rather than the smallest number of significant figures, controls addition. Carry extra digits during calculation and round only once at the end.

Step 1: Add the first three sets without early rounding Denote the four reported sums by S_1, S_2, S_3, S_4. Their unrounded values are

$$S_1 = 24.36 + 0.0724 + 256.2 = 280.6324,$$
$$S_2 = 24.44 + 16.082 + 240.2 = 280.722,$$
$$S_3 = 25.2 + 19.2812 + 236.183 = 280.6642.$$

In each set, at least one measurement is recorded only to one decimal place. Therefore each of these sums must be reported to one decimal place.

Step 2: Round each sum at its own limiting precision The properly reported first three results are

$$S_1 = 280.6, \qquad S_2 = 280.7, \qquad S_3 = 280.7.$$

For the fourth set,

$$S_4 = 25 + 236.191 + 19.5 = 280.691.$$

Here the measurement 25 has no decimal places. Consequently the fourth sum is rounded to the nearest integer, giving $S_4 = 281$, rather than 280.7.

Step 3: Compare the reported measured results The consistent comparison is therefore

$$S_4 > S_3 = S_2 > S_1.$$

Option (D) states this relation. Option (A) ignores the different final rounding; (B) compares the unrounded arithmetic sums as though every digit were significant; (C) incorrectly treats the first three reported sums as equal. The equality between the second and third results follows from their reporting precision, even though their unrounded arithmetic values differ.

Step 4: Conclusion After applying the addition rule for measured quantities, $S_4 > S_3 = S_2 > S_1$. Hence, the correct option is (D).

Quick Tip to Solve: Underline the least precise decimal place in each set before adding; the term 25 forces the fourth sum to an integer.

37. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then : [4 marks]

(A) $T = \sqrt{\frac{2m}{IB}}$

(B) $T = \sqrt{\frac{\pi m}{IB}}$

(C) $T = \sqrt{\frac{\pi m}{2IB}}$

(D) $T = \sqrt{\frac{2\pi m}{IB}}$

Correct Answer: (D) $T = \sqrt{\frac{2\pi m}{IB}}$

Solution:

Concept:

- A current loop has magnetic dipole moment $\mu = IA = I\pi a^2$, directed along the normal to its plane.
- Near stable alignment with a uniform field, magnetic torque is restoring: $\tau = -\mu B \sin\theta \simeq -\mu B\theta$.
- For angular SHM, $T = 2\pi\sqrt{J/(\mu B)}$, where J is the moment of inertia about the actual rotation axis.

Step 1: Identify the stable equilibrium The field is perpendicular to the plane, so the magnetic moment is initially parallel to the field in the stable orientation. A small rotation through θ about a diameter tilts the normal by the same angle. The field then supplies a torque opposing that displacement. The stipulated oscillation selects the stable orientation; an antiparallel magnetic moment would instead be unstable.

Step 2: Use the diameter moment of inertia The conductor is a thin circular loop, rather than a solid disc. Its moment of inertia about the perpendicular central axis is ma^2. Symmetry and the perpendicular-axis theorem therefore give

$$J = \frac{ma^2}{2}.$$

The rotational equation is

$$\frac{ma^2}{2}\ddot{\theta} = -I\pi a^2 B\theta.$$

Thus the radius cancels: a larger loop has both a greater restoring torque and a greater rotational inertia in the same proportion.

Step 3: Calculate the period and compare Comparing with $\ddot{\theta} + \omega_0^2\theta = 0$,

$$\omega_0^2 = \frac{2\pi IB}{m}.$$

Hence

$$T = 2\pi\sqrt{\frac{m}{2\pi IB}} = \sqrt{\frac{2\pi m}{IB}}.$$

Options (A), (B) and (C) have incorrect numerical factors. In particular, the factor 2π linking angular frequency and period must not be dropped.

Step 4: Conclusion The small-oscillation period is $\boxed{T = \sqrt{2\pi m/(IB)}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the moment of inertia about the diameter, not about the normal; only then substitute the loop magnetic moment.

38. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is: [4 marks]

(A) $2\sqrt{2}\frac{u^2}{g}$

(B) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$

(C) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

(D) $\frac{5}{8}\frac{u^2}{g}$

Correct Answer: (B) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$

Solution:

Concept:

- At the highest point of a projectile, its vertical velocity is zero while its horizontal velocity remains $u\cos\theta$.
- Momentum is conserved during the brief collision, but kinetic energy is not conserved in a completely inelastic collision.
- After sticking, the combined mass falls from height H with zero initial vertical velocity, so $t = \sqrt{2H/g}$.

Step 1: Find the state immediately before impact For $\theta = \pi/3$, the first particle has horizontal velocity $u/2$. Its initial vertical component is $\sqrt{3}u/2$, so the maximum height is

$$H = \frac{(u\sin\theta)^2}{2g} = \frac{3u^2}{8g}.$$

The second particle moves horizontally with speed u. Both vertical velocities are zero at the instant of impact. The height, rather than the earlier flight time, determines the subsequent fall.

Step 2: Conserve horizontal momentum The two equal masses stick together. If their common horizontal speed is V, conservation of momentum during impact gives

$$m\frac{u}{2} + mu = (2m)V, \qquad V = \frac{3u}{4}.$$

Their common vertical velocity remains zero. Using energy conservation across this impact would be incorrect because deformation and other internal processes dissipate kinetic energy.

Step 3: Calculate the distance after the collision The time taken by the combined mass to reach the

ground is

$$t = \sqrt{\frac{2H}{g}} = \frac{\sqrt{3}u}{2g}.$$

Its horizontal displacement during this interval is

$$x = Vt = \frac{3u}{4}\frac{\sqrt{3}u}{2g} = \frac{3\sqrt{3}u^2}{8g}.$$

The question asks for the distance covered by the combined mass, so the distance travelled before impact is excluded. Options (A), (C) and (D) do not equal this product of the common speed and fall time.

Step 4: Conclusion The combined mass covers $\boxed{3\sqrt{3}u^2/(8g)}$ horizontally before reaching the ground. Hence, the correct option is (B).

Quick Tip to Solve: Split collision-projectile problems at the impact: use momentum at impact and vertical kinematics for the fall afterwards.

39. In LC circuit the inductance L = 40 mH and capacitance C = 100 μF. If a voltage V(t) = 10sin(314t) is applied to the circuit, the current in the circuit is given as : [4 marks]

(A) 0.52 cos 314 t

(B) 5.2 cos 314 t

(C) 0.52 sin 314 t

(D) 10 cos 314 t

Correct Answer: (A) 0.52 cos 314 t

Solution:

Concept:

- The inductive and capacitive reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, respectively.
- In a series LC circuit the net reactance is $X_L - X_C$. A negative value means capacitive behaviour, with current leading voltage by $\pi/2$.
- The sinusoidal steady-state current amplitude is $I_0 = V_0/|X_L - X_C|$.

Step 1: Convert the data and calculate reactances From $V(t) = 10\sin(314t)$, the voltage amplitude is $10\,\text{V}$ and angular frequency is $314\,\text{rad}\,\text{s}^{-1}$. Convert the component values before substituting:

$$L = 0.040\,\text{H}, \qquad C = 100 \times 10^{-6}\,\text{F}.$$

Therefore

$$X_L = 314(0.040) = 12.56\,\Omega,$$

$$X_C = \frac{1}{314(100 \times 10^{-6})} \simeq 31.85\,\Omega.$$

The capacitor reactance exceeds the inductor reactance, so this is not a resonance condition.

Step 2: Find the current amplitude The magnitude of the net series reactance is

$$|X_L - X_C| = 31.85 - 12.56 = 19.29\,\Omega.$$

Thus

$$I_0 = \frac{10}{19.29} \simeq 0.518\,\text{A} \simeq 0.52\,\text{A}.$$

Adding the reactance magnitudes would be wrong because the inductor and capacitor voltages are opposite in phase.

Step 3: Include the phase of the current As the circuit is net capacitive, current leads the applied sine voltage by a quarter cycle:

$$i(t) = I_0 \sin(314t + \pi/2) = 0.52\cos(314t)\,\text{A}.$$

Option (C) has the right amplitude but the wrong phase. Options (B) and (D) have incorrect amplitudes. The expression represents the AC steady-state response specified by the alternatives, rather than an arbitrary free transient of an ideal LC circuit.

Step 4: Conclusion The current is $\boxed{i(t) = 0.52\cos(314t)\,\text{A}}$. Hence, the correct option is (A).

Quick Tip to Solve: Compare reactances before choosing sine or cosine: a net capacitive circuit makes current lead the applied voltage.

40. A rod of length L has non-uniform linear mass density given by ρ(x) = $a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and 0 $\leq$ x $\leq$ L. The value of x for the centre of mass of the rod is at : [4 marks]

(A) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

(B) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

(C) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

(D) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

Correct Answer: (C) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

Solution:

Concept:

- For a rod on the x-axis, $x_{\text{CM}} = \int x\,dm / \int dm$; unequal linear density requires a mass-weighted average.

- A small length dx at coordinate x has mass $dm = \rho(x)\,dx$, with $\rho(x) = a + bx^2/L^2$.

Step 1: Integrate the total mass Take the origin at the stated end of the rod and integrate from 0 to L. The total mass is

$$M = \int_0^L \left(a + \frac{bx^2}{L^2}\right) dx = aL + \frac{bL}{3} = \frac{L(3a+b)}{3}.$$

Both density contributions must be included. Averaging the endpoint densities and multiplying by the length would not be exact for a quadratic density distribution.

Step 2: Calculate the first moment of mass Weight each mass element by its distance from the origin:

$$\begin{aligned}\int_0^L x\,dm &= \int_0^L \left(ax + \frac{bx^3}{L^2}\right) dx \\ &= \frac{aL^2}{2} + \frac{bL^2}{4} \\ &= \frac{L^2(2a+b)}{4}.\end{aligned}$$

The additional factor x changes the integrals from those used for the total mass. This is why the two density coefficients receive different relative weights.

Step 3: Divide and check the result Dividing the first moment by total mass yields

$$x_{\text{CM}} = \frac{L^2(2a+b)/4}{L(3a+b)/3} = \frac{3}{4}\left(\frac{2a+b}{3a+b}\right) L.$$

For $b = 0$, this reduces to $L/2$, as a uniform rod must. If $a = 0$, it gives $3L/4$, toward the denser end. Option (A) gives $3L/4$ even for a uniform rod; option (B) gives $2L/3$, and option (D) gives L. These fail the uniform-density check.

Step 4: Conclusion The centre of mass is at $\boxed{x_{\text{CM}} = \frac{3}{4}\frac{2a+b}{3a+b}L}$. Hence, the correct option is (C).

Quick Tip to Solve: Always calculate total mass and first moment separately; then test your formula by setting the nonuniform coefficient to zero.

41. A particle starts from the origin at t = 0 with an initial velocity of 3.0 $\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $\left(6\hat{i}+4\hat{j}\right)$ m/s^2 . The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is :- [4 marks]

(A) 40

(B) 32

(C) 50

(D) 60

Correct Answer: (D) 60

Solution:

Concept:

- With constant acceleration, each Cartesian coordinate obeys $s = ut + \frac{1}{2}at^2$ independently.
- The initial velocity $3\hat{i}$ has components $u_x = 3\,\mathrm{m\,s^{-1}}$ and $u_y = 0$; acceleration components are $a_x = 6$ and $a_y = 4\,\mathrm{m\,s^{-2}}$.

Step 1: Use the vertical coordinate to find time The particle begins at the origin, so its vertical coordinate is

$$y(t) = u_y t + \frac{1}{2}a_y t^2 = 2t^2.$$

Setting the specified coordinate equal to $32\,\mathrm{m}$ gives

$$32 = 2t^2, \qquad t^2 = 16\,\mathrm{s}^2.$$

Since the motion starts at $t = 0$, the physical time is $t = 4\,\mathrm{s}$. The negative algebraic root would describe a time before the specified initial state and must be discarded.

Step 2: Calculate the horizontal coordinate at that instant The same elapsed time must be used in the horizontal equation. Both the initial horizontal velocity and horizontal acceleration contribute:

$$x(t) = u_x t + \frac{1}{2}a_x t^2 = 3t + 3t^2.$$

Therefore

$$x(4) = 3(4) + 3(4)^2 = 12 + 48 = 60\,\mathrm{m}.$$

The value $32\,\mathrm{m}$ belongs to the vertical coordinate only; it is not the displacement in every direction.

Step 3: Check the option against both components The position vector can be written

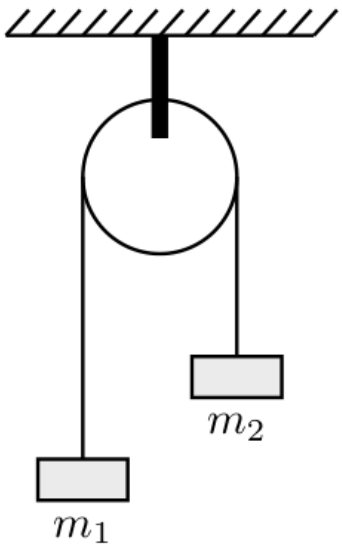
$$\boldsymbol{r}(t) = (3t + 3t^2)\hat{i} + 2t^2\hat{j}.$$

At $t = 4\,\mathrm{s}$ it is $60\hat{i} + 32\hat{j}$ metres, which satisfies the stated vertical condition exactly. Options (A), (B) and (C) give horizontal coordinates that do not follow from the common time and the given horizontal acceleration. The acceleration is constant, so no approximation or trajectory fitting is required.

Step 4: Conclusion The required value is $\boxed{D = 60}$, corresponding to $x = 60\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: When one coordinate is specified, find the time from that component first, then substitute it into the other component equation.

42. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is : [4 marks]



(A) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

(B) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$

(C) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}gh$

(D) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

Correct Answer: (D) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

Solution:

Concept:

- An inextensible string makes both blocks move equal distances with equal speed v. With no slipping at the rim, $v = R\omega$.
- The net loss of gravitational potential energy equals the translational kinetic energy of both blocks plus the rotational kinetic energy of the wheel.
- The wheel rotates about a fixed centre, so its kinetic energy is $\frac{1}{2}I\omega^2$, without any centre-of-mass translation.

Step 1: Find the net potential-energy decrease The heavier block m_1 moves down by h, while m_2 moves up by the same distance. Thus their individual potential-energy changes have opposite signs. The energy released is

$$\Delta U_{\text{released}} = m_1gh - m_2gh = (m_1 - m_2)gh.$$

The wheel centre remains fixed, so it has no gravitational potential-energy change. Using the sum of the masses here would count the rising block incorrectly.

Step 2: Include every kinetic-energy term The system starts from rest. At the specified displacement, the kinetic energy is

$$K = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2.$$

Substitute the no-slip relation $v = R\omega$:

$$K = \frac{1}{2}\left[(m_1 + m_2)R^2 + I\right]\omega^2.$$

Both masses contribute positively to inertia even though their potential-energy changes oppose each other. No energy is lost in the ideal string-wheel model.

Step 3: Solve for the angular speed Conservation of mechanical energy gives

$$(m_1 - m_2)gh = \frac{1}{2}\left[(m_1 + m_2)R^2 + I\right]\omega^2.$$

Taking the positive square root for speed,

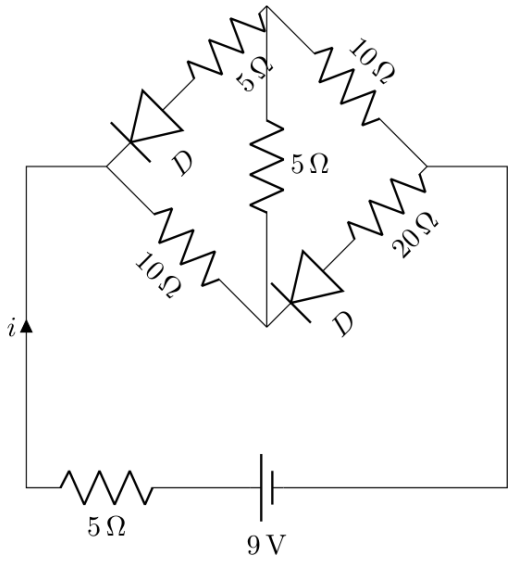
$$\omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}.$$

Option (A) has the wrong driving mass combination and would predict motion for equal masses. Options (B) and (C) put gh outside the square root and are dimensionally inconsistent with angular speed.

Step 4: Conclusion The wheel has angular speed $\boxed{\omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use a difference of masses for the gravitational energy released, but a sum of masses for the translational kinetic energy.

43. The current i in the network is : [4 marks]



(A) 0.6 A

(B) 0.3 A

(C) 0 A

(D) 0.2 A

Correct Answer: (B) 0.3 A

Solution:

Concept:

- An ideal diode conducts only from its anode towards its cathode when forward biased; a reverse-biased diode is an open circuit.
- For resistors carrying the same current in series, the equivalent resistance is their sum. The source current follows Ohm's law, $i = V/R_{\text{eq}}$.

Step 1: Determine the diode states Let the left and right junctions of the diamond be L and R, and its upper and lower junctions be T and B. The long plate of the 9 V cell is on the left, so the external current reaches L from the series resistor. Both diode cathode bars face towards the left or lower side: the upper-left diode would conduct from T towards L, and the lower-right diode would conduct from R towards B. For the applied polarity these two branches are reverse biased, so neither branch carries current.

Step 2: Identify the surviving conducting path Current can still pass from L through the lower-left $10\,\Omega$ resistor to B, then through the central $5\,\Omega$ resistor to T, and finally through the upper-right $10\,\Omega$ resistor to R. Thus the diamond is not an open circuit. Its resistance is

$$R_{\text{diamond}} = 10 + 5 + 10 = 25\,\Omega.$$

This also verifies the assumed diode states: the successive voltage drops make L higher in potential than T, and B higher than R.

Step 3: Include the external series resistor The $5\,\Omega$ resistor outside the diamond also carries the source current. Therefore

$$R_{\text{eq}} = 25 + 5 = 30\,\Omega, \qquad i = \frac{9}{30} = 0.3\,\text{A}.$$

The value 0 A incorrectly removes the remaining resistive path. The other nonzero choices do not include the correct series path and its total resistance.

Step 4: Conclusion The current in the indicated direction is $\boxed{0.3\,\text{A}}$. Hence, the correct option is (B).

Quick Tip to Solve: Mark the cell polarity and each diode cathode first; remove reverse-biased branches before combining any resistors.

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44. A spring mass system (mass m, spring constant k and natural length l) rest in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, (k $\gg m\omega^2$) the relative change in the length of the spring is best given by the option : [4 marks]

(A) $\frac{m\omega^2}{3k}$

(B) $\frac{m\omega^2}{k}$

(C) $\frac{2m\omega^2}{k}$

(D) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$

Correct Answer: (B) $\frac{m\omega^2}{k}$

Solution:

Concept:

- For a spring of constant k, an extension x produces a restoring force kx, directed towards its fixed end.
- A mass rotating at angular speed ω and radius r requires inward force $m\omega^2 r$. Here the actual radius is the stretched length $r = l + x$.

Step 1: Write the radial force balance Initially the spring has its natural length l. When the mass rotates steadily with the disc, the spring stretches by x, so its tension supplies the required centripetal force. Consequently

$$kx = m\omega^2(l + x).$$

Weight and normal reaction balance vertically.

Step 2: Solve for the relative extension Collect the extension terms:

$$(k - m\omega^2)x = m\omega^2 l.$$

Hence the exact relative change is

$$\frac{x}{l} = \frac{m\omega^2}{k - m\omega^2} = \frac{m\omega^2/k}{1 - m\omega^2/k}.$$

This requires $k > m\omega^2$, as given.

Step 3: Use the stated small-parameter limit Let $\epsilon = m\omega^2/k$. Since $\epsilon \ll 1$, the denominator $1 - \epsilon$ differs from unity only by a small correction. Thus

$$\frac{x}{l} = \epsilon + O(\epsilon^2) \simeq \frac{m\omega^2}{k}.$$

No factor of 2, 1/3 or $\sqrt{2/3}$ occurs in the leading term; those alternatives fail the force-balance equation.

Step 4: Conclusion The relative change in spring length is $\boxed{m\omega^2/k}$ to the stated approximation. Hence, the correct option is (B).

Quick Tip to Solve: Use the stretched spring length as the orbit radius first, and apply the small-extension approximation only after solving the force equation.

45. The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state ? [4 marks]

(A) 35.8 nm

(B) 11.4 nm

(C) 8.6 nm

(D) 24.2 nm

Correct Answer: (B) 11.4 nm

Solution:

Concept:

- The Bohr energy levels of a hydrogen-like ion are $E_n = -Z^2(1\,\mathrm{Ry})/n^2$, where one Rydberg of energy is approximately $13.6\,\mathrm{eV}$.
- A photon emitted in a downward transition has energy $\Delta E = E_{\text{initial}} - E_{\text{final}} = hc/\lambda$. The second excited state corresponds to $n = 3$.

Step 1: Find the energy scale of the ion The ground-state ionisation energy is the energy needed to move the electron from $n = 1$ to zero energy at infinity. It is therefore $Z^2\,\mathrm{Ry}$. The given value is 9 Rydbergs, so

$$Z^2 = 9.$$

We need only this energy scale; identifying the ion is unnecessary for calculating the wavelength. The stated ionisation energy is an energy, not the wavenumber Rydberg constant.

Step 2: Calculate the transition energy The ground state is $n = 1$, the first excited state is $n = 2$, and the second excited state is $n = 3$. Therefore the emitted photon comes from $3 \to 1$. Its energy is

$$\Delta E = 9\left(1 - \frac{1}{3^2}\right)\mathrm{Ry} = 8\,\mathrm{Ry} = 108.8\,\mathrm{eV}.$$

The sign is positive because the electron loses energy to the emitted radiation.

Step 3: Convert energy into wavelength Using $hc \simeq 1240\,\text{eV nm}$,

$$\lambda = \frac{1240}{108.8}\,\text{nm} \simeq 11.4\,\text{nm}.$$

As a check, this transition emits less than the full 9-Rydberg ionisation energy. The wavelengths 35.8 nm, 8.6 nm and 24.2 nm correspond to different photon energies and do not satisfy the computed 8-Rydberg difference.

Step 4: Conclusion The emitted wavelength is $\boxed{11.4\,\text{nm}}$. Hence, the correct option is (B).

Quick Tip to Solve: Translate excited-state language into the principal quantum number before substituting in the spectral formula: second excited means n = 3.

Physics – Section B: Numerical Value Questions (20 marks)

46. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm) _____. [4 marks]

Solution:

Concept:

- In Young's double-slit experiment the fringe width is $\beta = \lambda D/d$, where D is the screen distance and d is the slit separation.
- For a fixed small screen interval of length L, the number of fringes is proportional to L/β. With unchanged geometry, this makes the fringe count inversely proportional to the wavelength.

Step 1: Relate each fringe count to the same screen interval Let the first and second fringe widths be β_1 and β_2. The observed portion of the screen is unchanged, so its length can be written in terms of either fringe pattern:

$$L = 15\beta_1 = 10\beta_2.$$

Thus

$$\frac{\beta_2}{\beta_1} = \frac{15}{10} = \frac{3}{2}.$$

Fewer fringes in the same portion mean wider fringes, rather than a smaller fringe width.

Step 2: Use the unchanged experimental geometry Only the light source changes; both D and d remain the same. Consequently the constant geometric factor cancels in the ratio:

$$\frac{\beta_2}{\beta_1} = \frac{\lambda_2 D/d}{\lambda_1 D/d} = \frac{\lambda_2}{\lambda_1}.$$

Taking $\lambda_1 = 500\,\text{nm}$ gives

$$\lambda_2 = \frac{3}{2}(500\,\text{nm}) = 750\,\text{nm}.$$

No conversion to metres is necessary because this ratio is dimensionless.

Step 3: Check the direction and the count The wavelength increases from 500 nm to 750 nm, so the fringe spacing increases by a factor of 1.5. In a fixed interval, the count must therefore decrease by the reciprocal factor:

$$15 \times \frac{500}{750} = 10.$$

This reproduces the second observation and rules out multiplying 500 nm by the inverse count ratio. The question asks for the numerical wavelength in nm.

Step 4: Conclusion The required wavelength is $\boxed{750\,\text{nm}}$, so the numerical entry is $\boxed{750}$.

Quick Tip to Solve: For the same screen portion and slit geometry, keep the product of fringe count and wavelength constant.

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47. Starting at temperature 300 K, one mole of an ideal diatomic gas (γ = 1.4) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) _____. [4 marks]

Solution:

Concept:

- For a quasi-static adiabatic process of an ideal gas with constant heat-capacity ratio γ, the temperature-volume relation is $TV^{\gamma-1} = \text{constant}$.
- For a fixed amount of ideal gas at constant pressure, $PV = nRT$ gives $T/V = \text{constant}$. Apply each relation only to its own stage.

Step 1: Find the temperature after adiabatic compression Denote the initial temperature by $T_1 = 300\,\text{K}$ and the temperature after compression by T_2. The volume decreases from V_1 to $V_2 = V_1/16$, while $\gamma - 1 = 0.4$. Hence

$$T_1 V_1^{0.4} = T_2 V_2^{0.4}, \qquad T_2 = 300 \left(\frac{V_1}{V_2}\right)^{0.4} = 300\,16^{0.4}\,\text{K}.$$

Compression raises the temperature, as expected from work done on a thermally isolated gas.

Step 2: Apply the isobaric expansion relation In the second stage the pressure is held fixed and the gas expands from V_2 to $V_3 = 2V_2$. The number of moles is unchanged. Therefore

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = 2, \qquad T_3 = 600\,16^{0.4}\,\text{K}.$$

The adiabatic relation must not be applied to this stage: heat is supplied during the isobaric expansion, so its temperature rises in direct proportion to volume.

Step 3: Evaluate and round at the end Use $16^{0.4} = 2^{1.6} \simeq 3.031433$. This gives

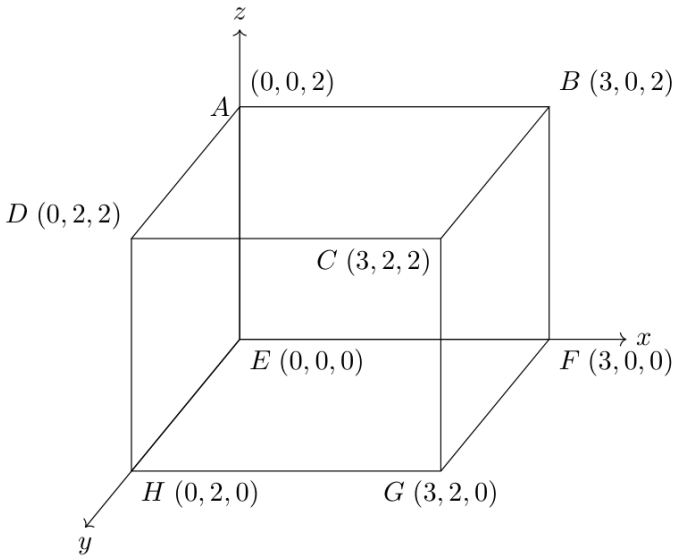
$$T_2 \simeq 909.430\,\text{K}, \qquad T_3 \simeq 1818.860\,\text{K}.$$

Rounding the final value to the nearest integer yields 1819. Keeping the intermediate temperature unrounded avoids an avoidable one-unit error. Kelvin is the absolute temperature scale, and the result is consistent with both stages increasing the temperature.

Step 4: Conclusion The final temperature to the nearest integer is $\boxed{1819\,\mathrm{K}}$; the required numerical entry is $\boxed{1819}$.

Quick Tip to Solve: Label the intermediate state, use the adiabatic temperature-volume law first, and then multiply by the isobaric volume ratio.

48. An electric field $\vec{E} = 4x\hat{i} - (y^2+1)\,\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in Nm^2/C) _____. [4 marks]



Solution:

Concept:

- Electric flux through a face is $\phi = \int \mathbf{E}\cdot\hat{\mathbf{n}}\,dA$, with the outward unit normal of the closed box.
- Only the electric-field component perpendicular to a face contributes. A varying component tangential to that face contributes zero to its flux.

Step 1: Read the face geometry and normal directions The coordinates show a box extending from $x = 0$ to 3, $y = 0$ to 2 and $z = 0$ to 2, in metres. Face ABCD is the top face $z = 2$, with outward normal $+\hat{\mathbf{k}}$. Face BCGF is the right face $x = 3$, with outward normal $+\hat{\mathbf{i}}$. Its dimensions are 2 along y and 2 along z, so

$$A_{\mathrm{BCGF}} = (2)(2) = 4\,\mathrm{m}^2.$$

Step 2: Calculate the flux through ABCD The field is

$$\mathbf{E} = 4x\hat{\mathbf{i}} - (y^2 + 1)\hat{\mathbf{j}} \quad \mathrm{N/C}.$$

It has no z component anywhere. Both displayed components lie parallel to the top face, so

$$\phi_I = \int_{\mathrm{ABCD}} \mathbf{E} \cdot \hat{\mathbf{k}}\, dA = 0.$$

The nonzero magnitude of the field does not imply nonzero flux through every face.

Step 3: Calculate the flux through BCGF and subtract On the face $x = 3$, the outward normal component is constant:

$$\mathbf{E} \cdot \hat{\mathbf{i}} = 4x = 12\,\mathrm{N/C}.$$

The y component is tangential and drops out even though it varies over the face. Therefore

$$\phi_{II} = 12 \times 4 = 48\,\mathrm{N\,m^2/C}.$$

Taking the difference in the order requested gives

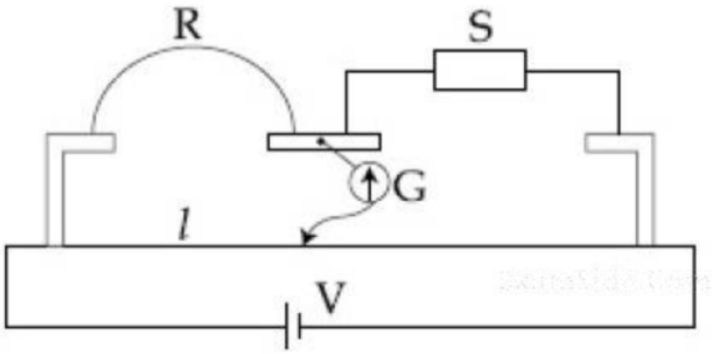
$$\phi_I - \phi_{II} = 0 - 48 = -48\,\mathrm{N\,m^2/C}.$$

The negative sign belongs to the difference, whereas the outward flux through the right face itself is positive.

Step 4: Conclusion The requested difference is $\boxed{-48\,\mathrm{N\,m^2/C}}$, so the numerical entry is $\boxed{-48}$.

Quick Tip to Solve: Write the outward normal of each named face before substituting coordinates; this immediately identifies the contributing field component.

49. In a meter bridge experiment S is a standard resistance. R is a resistance wire. It is found that balancing length is l = 25 cm. If R is replaced by a wire of half length and half diameter that of R of same material, then the balancing distance l' (in cm) will now be_____. [4 marks]



Solution:

Concept:

- At the balance point of a metre bridge, no current passes through the galvanometer and the two resistance ratios are equal. With resistance R in the left gap and S in the right gap, $R/S = l/(100 - l)$, where the lengths are in centimetres.
- The resistance of a uniform wire is $R = \rho L/A = 4\rho L/(\pi d^2)$. The resistivity ρ remains unchanged when the material and temperature remain the same; both the length and the cross-sectional area must be considered.

Step 1: Find the initial resistance ratio The diagram places the wire resistance in the left gap. Hence the balancing length measured from the left end corresponds to that resistance. Initially,

$$\frac{R}{S} = \frac{25}{100 - 25} = \frac{25}{75} = \frac{1}{3}.$$

The remaining length is 75 cm; using 25/100 would incorrectly compare a bridge arm with the entire bridge wire.

Step 2: Calculate the resistance of the replacement wire The new wire has length $L/2$ and diameter $d/2$. Its area is therefore $A/4$, since area varies as the square of diameter. Thus

$$R' = \frac{\rho(L/2)}{A/4} = 2\frac{\rho L}{A} = 2R.$$

Although the wire is shorter, the stronger reduction in its area makes its resistance twice as large.

Step 3: Apply the new balance condition The standard resistance is unchanged, so $R'/S = 2/3$. Therefore

$$\frac{l'}{100 - l'} = \frac{2}{3}.$$

$$3l' = 200 - 2l', \qquad 5l' = 200.$$

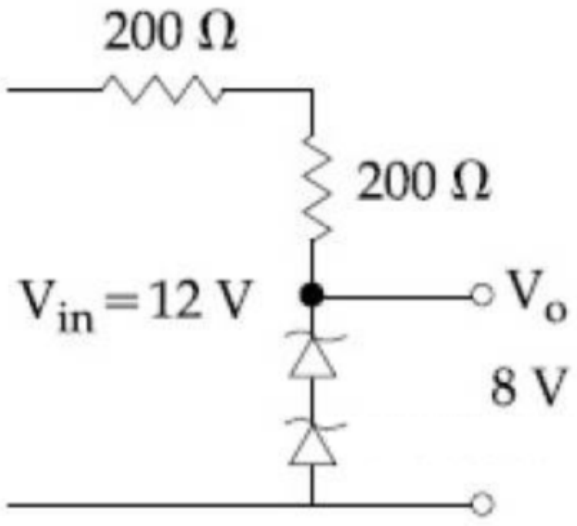
The increase in the left-gap resistance must increase the left balancing length, consistent with the result below.

Step 4: Conclusion The new balancing distance is $\boxed{l' = 40\,\text{cm}}$. The numerical entry is $\boxed{40}$.

Quick Tip to Solve: For a wire changed in both length and diameter, calculate $R'/R = (L'/L)(d/d')^2$ before using the metre-bridge ratio.

50. The circuit shown below is working as a 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is; (considering both zener diodes are identical) _____.

[4 marks]



Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains its Zener voltage across itself. The total regulated output across series Zener diodes is the sum of their breakdown voltages.
- For a resistor the current is $I = V/R$, and a diode dissipates power $P = V_Z I_Z$. Identical series diodes carry the same current and divide the total regulated voltage equally.

Step 1: Identify the voltage across each diode The output is taken across the two identical Zener diodes connected in series, and is regulated at 8 V. Their identical breakdown voltages therefore satisfy

$$2V_Z = 8\,\text{V}, \qquad V_Z = 4\,\text{V}.$$

The 8 V label refers to the pair, so assigning 8 V separately to each diode would overestimate their individual power.

Step 2: Find the current through the series circuit The diagram has two 200-ohm resistors in series between the 12 V input and the regulated output node. Their equivalent resistance is

$$R_s = 200 + 200 = 400\,\Omega.$$

The voltage dropped across this resistance is the input voltage less the regulated output:

$$V_s = 12 - 8 = 4\,\text{V}.$$

$$I = \frac{V_s}{R_s} = \frac{4}{400} = 0.010\,\text{A}.$$

The output terminal is unloaded in the shown circuit, so this whole current flows through the Zener pair.

Step 3: Calculate the power of either diode Each diode carries 10 mA at a voltage of 4 V. Hence

$$P_Z = V_Z I_Z = 4(0.010) = 0.040\,\text{W}.$$

$$P_Z = 0.040 \times 1000 = 40\,\text{mW}.$$

As a check, both diodes together dissipate 80 mW, equal to the output voltage times the circuit current.

Step 4: Conclusion The power dissipated in each Zener diode is $\boxed{40\,\text{mW}}$. The numerical entry is $\boxed{40}$.

Quick Tip to Solve: First add the series resistors, then divide the regulated voltage equally between identical series Zener diodes; use each diode's own voltage in its power.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The first and second ionisation enthalpies of a metal are 496 and 4560 kJ mol^{-1}, respectively. How many moles of HCl and H_2SO_4, respectively, will be needed to react completely with 1 mole of the metal hydroxide ? [4 marks]

(A) 1 and 2

(B) 1 and 0.5

(C) 1 and 1

(D) 2 and 0.5

Correct Answer: (B) 1 and 0.5

Solution:

Concept:

- Ionisation enthalpy is the energy required to remove an electron from a gaseous atom or ion. A very large rise after the first ionisation indicates that the atom has only one readily removable valence electron.
- A monovalent metal forms MOH. Complete neutralisation depends on the number of hydroxide groups and the number of replaceable acidic hydrogen ions, rather than on the numerical magnitude of an ionisation enthalpy.

Step 1: Infer the usual oxidation state of the metal The first ionisation enthalpy is only $496\,\text{kJ mol}^{-1}$, whereas the second is $4560\,\text{kJ mol}^{-1}$. The second value is more than nine times the first. Removing one electron therefore produces a particularly stable closed-shell ion; removing another would involve

an inner shell. The metal is consequently an alkali-type, monovalent metal, and its hydroxide has formula MOH, not $M(OH)_2$.

Step 2: Balance neutralisation with hydrochloric acid Hydrochloric acid provides one acidic hydrogen per molecule. The balanced reaction is

$$MOH + HCl \longrightarrow MCl + H_2O.$$

One mole of hydroxide thus needs one mole of HCl. The option requiring two moles of hydrochloric acid incorrectly treats the metal as divalent.

Step 3: Balance neutralisation with sulfuric acid Sulfuric acid supplies two acidic hydrogens per molecule in complete neutralisation:

$$2\,MOH + H_2SO_4 \longrightarrow M_2SO_4 + 2\,H_2O.$$

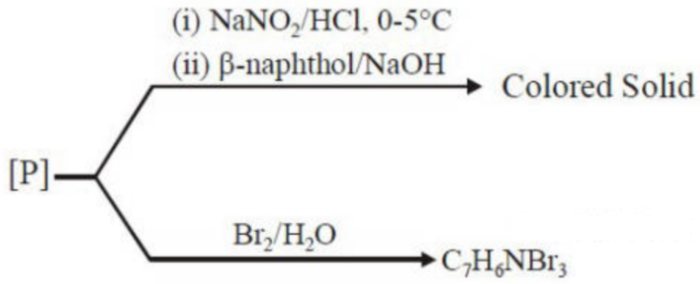
Two moles of hydroxide consume one mole of acid, so one mole consumes $1/2 = 0.5$ mole. The alternatives giving one or two moles of sulfuric acid supply more acid equivalents than the hydroxide needs.

Step 4: Conclusion The required amounts of HCl and H_2SO_4 are respectively $\boxed{1 \text{ mol and } 0.5 \text{ mol}}$. Hence, the correct option is (B).

Quick Tip to Solve: A sudden jump from the first to second ionisation enthalpy signals a monovalent metal; then balance acid and hydroxide equivalents, not acid molecules alone.

52. Consider the following reactions,

The compound [P] is : [4 marks]



(A) 2-Methylaniline (o-toluidine)

(B) 3-Methylaniline (m-toluidine)

(C) N,3-Dimethylaniline (N-methyl-m-toluidine)

(D) 4-Methylaniline (p-toluidine)

Correct Answer: (B) 3-Methylaniline (m-toluidine)

Solution:

Concept:

- A primary aromatic amine reacts with $NaNO_2/HCl$ at 0–$5\,^\circ C$ to form an arenediazonium salt. Coupling with alkaline beta-naphthol gives a coloured azo compound.
- The amino group strongly activates a benzene ring and directs electrophilic substitution to its two ortho positions and its para position. With bromine water, an aromatic primary amine having all three positions free can undergo tribromination.

Step 1: Use the azo-dye reaction to identify the amino group The upper reaction requires a primary aromatic amino group, $ArNH_2$, which forms a diazonium salt before coupling with beta-naphthol. The secondary aromatic amine in option (C) instead undergoes nitrosation and cannot give the stated diazotisation-coupling sequence. Options (A), (B) and (D) are the three toluidine isomers and must be distinguished by the second reaction.

Step 2: Count the available bromination positions For 2-methylaniline, one ortho position relative to NH_2 is occupied by methyl. For 4-methylaniline, the para position is occupied. Each therefore lacks one of the three strongly activated sites. In 3-methylaniline the methyl group occupies a meta position, leaving positions 2, 4 and 6 relative to the amino group all vacant. Bromine water can replace a hydrogen at every one of these sites.

Step 3: Check the product formula The starting formula of 3-methylaniline is C_7H_9N. Substituting three ring hydrogens by bromine gives

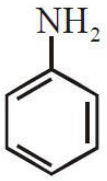
$$C_7H_9N + 3\,Br_2 \longrightarrow C_7H_6NBr_3 + 3\,HBr.$$

The product formula exactly matches the stated lower reaction. It is the 2,4,6-tribromo derivative of 3-methylaniline. Both the primary-amino-group test and the number of available ring positions are satisfied together only by the meta isomer.

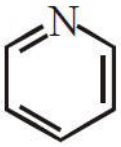
Step 4: Conclusion Compound [P] is $\boxed{\text{3-methylaniline (m-toluidine)}}$. Hence, the correct option is (B).

Quick Tip to Solve: For bromine-water tests on anilines, mark both ortho sites and the para site relative to NH_2; a substituent already occupying one blocks that usual substitution.

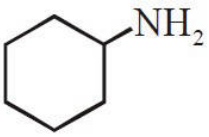
53. **The decreasing order of basicity of the following amines is :** [4 marks]



(I)



(II)



(III)



(IV)

(A) (III) > (II) > (I) > (IV)

(B) (I) > (III) > (IV) > (II)

(C) (III) > (I) > (II) > (IV)

(D) (II) > (III) > (IV) > (I)

Correct Answer: (A) (III) > (II) > (I) > (IV)

Solution:

Concept:

- Basicity depends on how readily the nitrogen lone pair accepts a proton. Electron donation by an alkyl group increases its availability, while resonance delocalisation into an aromatic ring decreases it.
- A lone pair that is part of an aromatic sextet is especially reluctant to bind a proton because protonation at that nitrogen removes the aromatic stabilisation. A pyridine lone pair lies outside the aromatic pi system.

Step 1: Identify the four nitrogen environments Structure (I) is aniline, with an amino group directly bonded to benzene. Structure (II) is pyridine, whose nitrogen is part of a six-membered aromatic ring. Structure (III) is cyclohexylamine, an aliphatic primary amine bonded to a saturated ring. Structure (IV) is pyrrole, with an N-H nitrogen in a five-membered aromatic ring. The word cyclic alone does not determine basicity; lone-pair participation does.

Step 2: Place the strongest and weakest bases In cyclohexylamine the nitrogen lone pair is localised and the cyclohexyl group exerts a positive inductive effect. It is therefore the most readily protonated of these four. In pyrrole the lone pair supplies two electrons to the aromatic sextet; protonating that lone pair disrupts aromaticity. Pyrrole is consequently the weakest base. This already rules out the alternatives that place pyridine above cyclohexylamine or aniline at the top.

Step 3: Compare pyridine with aniline The pyridine lone pair occupies an orbital outside its aromatic pi system, so accepting a proton does not use the ring's aromatic electrons. In aniline the amino lone pair is delocalised into the benzene ring, lowering its availability for protonation. Accordingly pyridine is more basic than aniline. Combining this comparison with the two extremes gives

$$\boxed{(\text{III}) > (\text{II}) > (\text{I}) > (\text{IV})}.$$

The alternative with aniline above pyridine reverses this crucial resonance comparison.

Step 4: Conclusion The decreasing basicity order is $(\text{III}) > (\text{II}) > (\text{I}) > (\text{IV})$. Hence, the correct option is (A).

Quick Tip to Solve: Ask whether the nitrogen lone pair is localised, conjugated with benzene, or essential for aromaticity; pyridine and pyrrole differ precisely in this last point.

54. The reaction of $H_3N_3B_3Cl_3$ (A) with $LiBH_4$ in tetrahydrofuran gives inorganic benzene (B). Further, the reaction of (A) with (C) leads to $H_3N_3B_3(Me)_3$. Compounds (B) and (C) respectively, are :

[4 marks]

(A) Borazine and MeBr

(B) Diborane and MeMgBr

(C) Boron nitride and MeBr

(D) Borazine and MeMgBr

Correct Answer: (D) Borazine and MeMgBr

Solution:

Concept:

- Borazine, $B_3N_3H_6$, is a six-membered ring with alternating boron and nitrogen atoms. It is called inorganic benzene because its ring framework resembles that of benzene.
- Boron-bound chlorine in chloroborazine can be replaced by hydrogen using a hydride reagent, or by an alkyl group using an organomagnesium reagent. A Grignard reagent acts as a source of the carbon group in the latter substitution.

Step 1: Identify the inorganic benzene product The given compound $H_3N_3B_3Cl_3$ has the borazine

ring with chlorine attached to the boron sites and hydrogen attached to the nitrogen sites. Treatment with $LiBH_4$ replaces the boron-bound chlorine atoms by hydrogen without changing the alternating B-N ring. Product (B) is therefore $B_3N_3H_6$, borazine. Diborane has formula B_2H_6 and does not have a B-N ring; boron nitride is likewise not the specified molecular inorganic benzene.

Step 2: Choose the reagent that introduces methyl groups The second product retains the B-N ring but has three methyl groups in place of the three boron-bound chlorine atoms. The required reagent must therefore transfer methyl to boron. Methylmagnesium bromide, CH_3MgBr, is the appropriate Grignard reagent. Schematically the substitution is

$$H_3N_3B_3Cl_3 + 3\,CH_3MgBr \longrightarrow H_3N_3B_3(CH_3)_3 + 3\,MgBrCl.$$

Each methyl group replaces one chlorine atom.

Step 3: Check the paired alternatives Methyl bromide, CH_3Br, is an electrophilic organic halide, rather than the methyl-donating organometallic reagent needed here. Thus the pair containing borazine and methyl bromide fails the second reaction. The pairs containing diborane or boron nitride fail the first reaction. Only the pair borazine and methylmagnesium bromide satisfies both transformations while preserving the original ring.

Step 4: Conclusion The two compounds are respectively $\boxed{\text{borazine and } MeMgBr}$. Hence, the correct option is (D).

Quick Tip to Solve: In chloroborazine reactions, distinguish a hydride donor from an alkyl donor: hydride gives B-H bonds, while a Grignard reagent gives B-C bonds.

55. A, B and C are three biomolecules. The results of the tests performed on them are given below:

	Molisch's Test	Barfoed Test	Biuret Test
A	Positive	Negative	Negative
B	Positive	Positive	Negative
C	Negative	Negative	Positive

A, B and C are respectively : [4 marks]

(A) A = Lactose, B = Glucose, C = Albumin

(B) A = Lactose, B = Fructose, C = Alanine

(C) A = Lactose, B = Glucose, C = Alanine

(D) A = Glucose, B = Fructose, C = Albumin

Correct Answer: (A) A = Lactose, B = Glucose, C = Albumin

Solution:

Concept:

- Molisch's test detects carbohydrates; a positive result identifies a carbohydrate but does not distinguish a monosaccharide from a disaccharide.

- Barfoed's test distinguishes rapidly reacting monosaccharides from disaccharides under the standard test conditions. Biuret's test detects multiple peptide bonds in proteins.

Step 1: Identify A from the carbohydrate tests A gives a positive Molisch test and therefore belongs to carbohydrates. Its negative Barfoed result excludes the monosaccharides glucose and fructose under the prescribed test conditions. Lactose is a disaccharide, so it satisfies both observations. Although lactose is a reducing sugar, that fact does not make its Barfoed test positive in the time used to distinguish monosaccharides.

Step 2: Identify B and C using the remaining results B gives positive Molisch and Barfoed tests, consistent with a monosaccharide such as glucose. Fructose can also respond to Barfoed's test, so identify the complete combination rather than selecting on B alone. C is negative in the carbohydrate tests but positive in the Biuret test. Albumin is a protein with many peptide linkages and meets these conditions. Alanine is a single amino acid: it lacks the required peptide bonds and cannot give the protein-type Biuret response.

Step 3: Check all the proposed combinations Option (A) assigns lactose, glucose and albumin and matches every row. Options (B) and (C) assign alanine to C, contradicting its positive Biuret test. Option (D) assigns glucose to A, contradicting A's negative Barfoed result. Thus the entire pattern, rather than a single positive test, fixes the answer uniquely.

Step 4: Conclusion A is lactose, B is glucose and C is albumin. Hence, the correct option is (A).

Quick Tip to Solve: Use the positive Biuret result to identify the protein first; then use Barfoed's test to separate the two carbohydrates.

56. 5 g of zinc is treated separately with an excess of
(a) dilute hydrochloric acid and
(b) aqueous sodium hydroxide.
The ratio of the volumes of H_2 evolved in these two reactions is : [4 marks]

(A) 1 : 2
(B) 1 : 1
(C) 1 : 4
(D) 2 : 1

Correct Answer: (B) 1 : 1

Solution:

Concept:

- Zinc is amphoteric and reacts with both dilute acid and aqueous alkali to release H_2. In each reaction zinc is oxidised from oxidation state zero to +2.
- At the same temperature and pressure, gas volume is proportional to the amount in moles: $V = nRT/P$. Compare stoichiometric moles of hydrogen, rather than the reagent formula alone.

Step 1: Balance the reaction with hydrochloric acid With excess dilute acid, all the zinc reacts

according to

$$\mathrm{Zn} + 2\,\mathrm{HCl} \longrightarrow \mathrm{ZnCl_2} + \mathrm{H_2}.$$

One mole of zinc therefore produces one mole of hydrogen. If the molar mass of zinc is denoted by M_{Zn}, the hydrogen amount obtained from the given sample is $5/M_{\mathrm{Zn}}$ moles. The coefficient 2 of the acid does not mean two moles of hydrogen gas.

Step 2: Balance the reaction with aqueous alkali In aqueous sodium hydroxide the soluble zincate complex can be written as

$$\mathrm{Zn} + 2\,\mathrm{NaOH} + 2\,\mathrm{H_2O} \longrightarrow \mathrm{Na_2[Zn(OH)_4]} + \mathrm{H_2}.$$

Again, one mole of zinc yields one mole of hydrogen. The alternative school-level sodium zincate representation gives the same gas stoichiometry. Because alkali is in excess, it is zinc, not hydroxide, that limits hydrogen production. The zinc mass is the same in the two separate experiments.

Step 3: Compare the gas amounts and volumes For the two equal zinc samples,

$$n_a(\mathrm{H_2}) : n_b(\mathrm{H_2}) = \frac{5}{M_{\mathrm{Zn}}} : \frac{5}{M_{\mathrm{Zn}}} = 1 : 1.$$

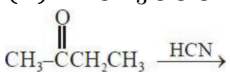
The common gas-volume conversion cancels, giving the same volume ratio. Ratios 1:2, 1:4 and 2:1 would require different hydrogen yields per zinc atom, whereas both balanced equations give exactly one hydrogen molecule per zinc atom.

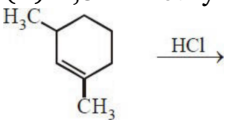
Step 4: Conclusion The ratio of hydrogen volumes is $V_a : V_b = 1 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: Balance both reactions before comparing gas volumes: zinc's +2 oxidation in either medium produces the same amount of hydrogen.

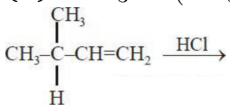
57. Which of the following reactions will not produce a racemic product? [4 marks]

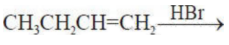
(A) $CH_3COCH_2CH_3$ treated with HCN. (B) 1,3-Dimethylcyclohex-1-ene treated with HCl.

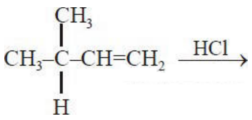




(C) $CH_3CH(CH_3)CH{=}CH_2$ treated with HCl. (D) $CH_3CH_2CH{=}CH_2$ treated with HBr.





Correct Answer: (C) $CH_3CH(CH_3)CH{=}CH_2$ treated with HCl. 

Solution:

Concept:

- A racemic product contains equal amounts of a pair of enantiomers. A product with no stereogenic centre cannot be racemic merely because it forms through a planar carbocation.

- Electrophilic addition to an alkene can involve carbocation rearrangement. A 1,2-hydride shift is favoured when it converts a secondary carbocation into a tertiary carbocation.

Step 1: Follow protonation in option C The alkene in (C) is $CH_3CH(CH_3)CH{=}CH_2$, or 3-methylbut-1-ene. Markovnikov protonation initially places the positive charge on the internal alkene carbon, giving the secondary cation $CH_3CH(CH_3)CH^+CH_3$. The neighbouring carbon bears a hydrogen and can transfer it with its bonding electron pair to this electron-deficient carbon.

Step 2: Rearrange and examine the product A 1,2-hydride shift produces the more stable tertiary cation $(CH_3)_2C^+CH_2CH_3$. Attack by chloride then gives

$$(CH_3)_2C(Cl)CH_2CH_3.$$

This is 2-chloro-2-methylbutane. The carbon bonded to chlorine has two identical methyl groups; the remaining carbons also do not provide a stereogenic centre. Consequently, attack from the two faces cannot generate an enantiomeric pair: both approaches give the same achiral product.

Step 3: Distinguish the other reactions In (A), cyanide addition to planar butan-2-one makes a cyanohydrin carbon bonded to hydroxyl, cyano, methyl and ethyl groups. Its two faces give enantiomers. Addition to the substituted cyclic alkene in (B) allows stereoisomeric chiral products rather than the achiral rearranged product above. In (D), addition of HBr to but-1-ene gives 2-bromobutane with four different groups on its substituted carbon, allowing the two enantiomers. Thus the carbocation's planarity alone is not sufficient: product symmetry must also be tested.

Step 4: Conclusion Reaction (C) gives the achiral rearranged product 2-chloro-2-methylbutane and does not produce a racemic product. Hence, the correct option is (C).

Quick Tip to Solve: Before declaring an alkene addition racemic, check for a hydride shift and then count the different groups on the final tetrahedral carbon.

58. The true statement amongst the following is : [4 marks]

(A) S is a function of temperature but ΔS is not a function of temperature.

(B) Both S and ΔS are not functions of temperature.

(C) Both ΔS and S are functions of temperature.

(D) S is not a function of temperature but ΔS is a function of temperature.

Correct Answer: (C) Both ΔS and S are functions of temperature.

Solution:

Concept:

- Entropy S is a state function. Calling it a state function does not imply that it is independent of temperature; temperature is one of the variables describing the state.
- For reversible heating at constant pressure without a phase change, $dS = C_p\,dT/T$. The entropy change is the difference between the final and initial entropies.

Step 1: Establish the temperature dependence of S For a substance with positive heat capacity,

heating changes its entropy. At fixed pressure,

$$S(T_2) - S(T_1) = \int_{T_1}^{T_2} \frac{C_p(T)}{T}\, dT.$$

If the heat capacity is approximately constant, this becomes $C_p \ln(T_2/T_1)$. Thus even in the simplest example, entropy explicitly depends on temperature. Statements (B) and (D), which deny this dependence of S, cannot be generally correct.

Step 2: Test the temperature dependence of the entropy change For a process or reaction compared at different temperatures, both the product and reactant entropies vary with temperature. Their difference varies according to

$$\left(\frac{d\Delta S}{dT}\right)_p = \frac{\Delta C_p}{T}.$$

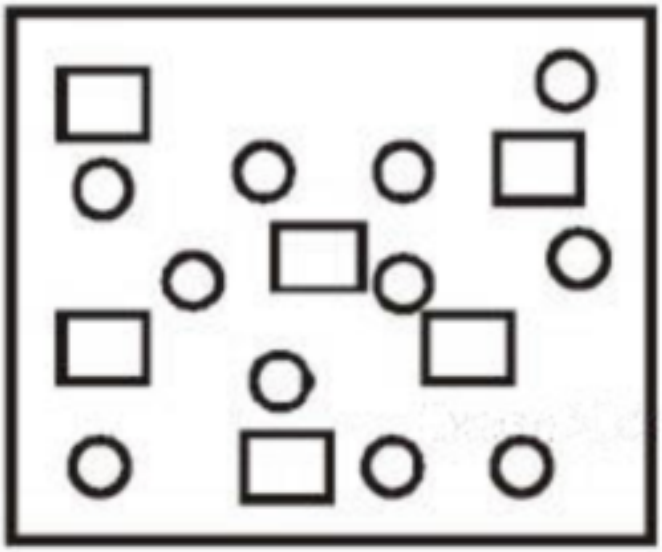
Equivalently, $\Delta S(T_2) = \Delta S(T_1) + \int_{T_1}^{T_2} \Delta C_p(T)\, dT/T$. A special case with zero heat-capacity difference may give an approximately constant value over an interval, but it does not establish a general independence from temperature.

Step 3: Select the general thermodynamic statement Both the entropy of a specified substance and the entropy change of a specified transformation can depend on temperature. Option (A) wrongly treats taking a difference as removing all temperature dependence. Option (B) denies both dependences and option (D) denies that of entropy itself. Option (C) correctly states the general behaviour; no numerical data are required for this conceptual choice.

Step 4: Conclusion Both S and ΔS are functions of temperature. Hence, the correct option is (C).

Quick Tip to Solve: Use the heat-capacity relations: S changes through an integral of C/T, while a reaction's entropy change involves the corresponding heat-capacity difference.

59. In the figure shown below reactant A (represented by square) is in equilibrium with product B (represented by circle). The equilibrium constant is : [4 marks]



(A) 1

(B) 2

(C) 8

(D) 4

Correct Answer: The diagram contains eleven circles and six squares, giving 11/6, which is not among the options.

Solution:

Concept:

- For the interconversion A $\rightleftharpoons$ B, the equilibrium constant is $K_c = [B]_{\text{eq}}/[A]_{\text{eq}}$. The product concentration belongs in the numerator.
- In a particle diagram of one container, concentration is proportional to the number of particles of that species because both counts refer to the same volume.

Step 1: Count the reactant particles The squares represent A. Count them by position to avoid counting any twice: upper-left, upper-right, central, left-lower, right-lower and bottom-central. This gives $N_A = 6$. All six lie within the same boundary and therefore refer to the same equilibrium sample. The squares are reactant particles, so their count must enter the denominator when writing the equilibrium expression for A changing into B. No particle outside the box contributes to the count.

Step 2: Count products and form the concentration ratio Counting the circles gives eleven: one at the top right, three on the next horizontal band, three in the middle band, one below the centre and three along the bottom. Thus $N_B = 11$. Let a single drawn particle correspond to an amount n_0, with container volume V. Then $[A] = 6n_0/V$ and $[B] = 11n_0/V$. The particle scale and volume cancel in the ratio, since both counts belong to the same sample.

Step 3: Evaluate the equilibrium constant For the one-to-one interconversion represented by the

labels,

$$K_c = \frac{[B]}{[A]} = \frac{11n_0/V}{6n_0/V} = \frac{11}{6} \approx 1.83.$$

None of the four printed options equals this value: 1 would require equal populations, 2 would require twelve circles for six squares, and 4 or 8 would require still larger product populations. The supplied key selects 2, but that appears to be the intended ratio rather than the exact ratio in the displayed particle diagram. There is no stated instruction to round the equilibrium constant to an integer, so choosing the nearest option would conceal the discrepancy.

Step 4: Conclusion The displayed counts give $K_c = 11/6 \approx 1.83$, with no exact correct printed option. The supplied key accepts (B), 2, but the literal diagram does not give that exact value.

Quick Tip to Solve: Count each shape once, then divide product particles by reactant particles; the common container volume cancels.

60. The isomer(s) of $[Co(NH_3)_4Cl_2]$ that has/have a Cl–Co–Cl angle of 90°, is/are : [4 marks]

(A) cis only

(B) cis and trans

(C) meridional and trans

(D) trans only

Correct Answer: (A) cis only

Solution:

Concept:

- The coordination number of $[Co(NH_3)_4Cl_2]$ is six, giving the octahedral arrangement considered here. The four ammonia ligands and two chlorido ligands have the pattern MA_4B_2.
- For an octahedral MA_4B_2 complex, the two identical B ligands are either adjacent (cis) or opposite (trans). The ligand-metal-ligand angle distinguishes these arrangements.

Step 1: Place the two chlorido ligands in the cis isomer An octahedron has six ligand positions along three mutually perpendicular axes through the metal. For the cis arrangement, choose two adjacent positions, for example the positive x and positive y directions, for the chlorido ligands. These two cobalt-chlorine bonds are perpendicular, so

$$\angle \mathrm{Cl{-}Co{-}Cl} = 90^\circ.$$

The other four positions are occupied by ammonia ligands. This arrangement satisfies precisely the angle asked for in the question.

Step 2: Compare the trans arrangement For the trans isomer, the chlorido ligands occupy opposite ends of one axis through cobalt, such as the positive and negative x positions. Their bonds are collinear with cobalt between the chlorine atoms, giving

$$\angle \mathrm{Cl{-}Co{-}Cl} = 180^\circ.$$

Although an octahedral complex also has many right angles between other pairs of ligands, those are irrelevant: the question specifies the angle between its two chlorido ligands.

Step 3: Exclude the remaining isomer descriptions Only cis has the requested chlorine-cobalt-chlorine angle. Hence trans alone is excluded, and the combination cis and trans is also excluded. Facial and meridional descriptions apply to the relative positions of a set of three identical ligands, as in an octahedral MA_3B_3 complex; meridional is not the alternative geometrical label for the two chlorido ligands in this MA_4B_2 formula. The requested pair of ligands therefore singles out the cis isomer.

Step 4: Conclusion Only the cis isomer has $\angle \mathrm{Cl}-\mathrm{Co}-\mathrm{Cl} = 90^\circ$. Hence, the correct option is (A).

Quick Tip to Solve: For octahedral complexes with two identical ligands, remember adjacent = cis = 90 degrees and opposite = trans = 180 degrees.

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61. The solubility product of $Cr(OH)_3$ at 298 K is 6.0×10^{-31}. The concentration of hydroxide ions in a saturated solution of $Cr(OH)_3$ will be : [4 marks]

(A) $(2.22 \times 10^{-31})^{1/4}$

(B) $(4.86 \times 10^{-29})^{1/4}$

(C) $(18 \times 10^{-31})^{1/4}$

(D) $(18 \times 10^{-31})^{1/2}$

Correct Answer: (C) $(18 \times 10^{-31})^{1/4}$

Solution:

Concept:

- For a sparingly soluble hydroxide, write the ionic dissolution equation before forming the solubility product. The powers in the expression are its stoichiometric coefficients.
- If the molar solubility of $\mathrm{Cr(OH)_3}$ is s, the dissolution calculation gives $[\mathrm{Cr^{3+}}] = s$ and $[\mathrm{OH^-}] = 3s$.

Step 1: Write the dissolution equilibrium Chromium(III) hydroxide supplies one chromium ion and three hydroxide ions per formula unit:

$$\mathrm{Cr(OH)_3(s)} \rightleftharpoons \mathrm{Cr^{3+}(aq)} + 3\,\mathrm{OH^-(aq)}.$$

The pure solid is omitted from the equilibrium expression. Thus

$$K_{sp} = [\mathrm{Cr^{3+}}][\mathrm{OH^-}]^3.$$

The hydroxide concentration is three times the molar solubility, so using the solubility directly as the hydroxide concentration would lose a factor of three.

Step 2: Express the product in terms of hydroxide concentration Put $x = [\mathrm{OH^-}]$. Using the dissolution stoichiometry, $[\mathrm{Cr^{3+}}] = x/3$, so

$$6.0 \times 10^{-31} = \frac{x}{3}x^3 = \frac{x^4}{3}.$$

Hence

$$x^4 = 18 \times 10^{-31}, \qquad x = (18 \times 10^{-31})^{1/4}.$$

This is the conventional solubility-product calculation intended by the options. Option A instead gives the molar solubility $s = (K_{sp}/27)^{1/4}$; option B has an incorrect numerical factor. Option D uses a square root even though the product contains four powers of concentration.

Step 3: Conclusion The hydroxide-ion concentration from the stated dissolution calculation is $(18 \times 10^{-31})^{1/4}\,\mathrm{mol\,L^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For $M(OH)_3$, eliminate the metal concentration using $[M^{3+}] = [OH^-]/3$; then take the fourth root of $3K_{sp}$.

62. Which of the following has the shortest C-Cl bond? [4 marks]

(A) Cl–CH=CH–CH_3

(B) Cl–CH=CH–NO_2

(C) Cl–CH=CH–OCH_3

(D) Cl–CH=CH_2

Correct Answer: (B) Cl–CH=CH–NO_2

Solution:

Concept:

- The carbon bonded to chlorine is sp^2-hybridised in all four compounds. Differences in C–Cl length therefore depend mainly on the extent of conjugative lone-pair donation from chlorine.
- Greater contribution of a resonance form with a C=Cl bond gives the C–Cl linkage more partial double-bond character and makes it shorter. An electron-withdrawing conjugated group assists this donation.

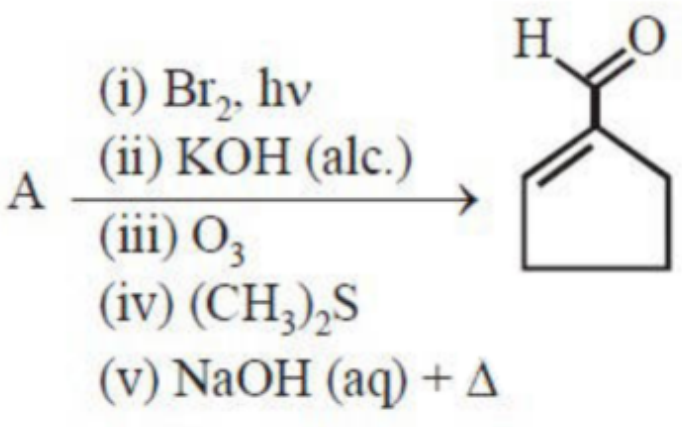
Step 1: Compare the common vinyl chloride framework Each option contains the unit $Cl-CH=CH-R$. A lone pair on chlorine can overlap with the adjacent carbon p orbital. In the resulting charge-separated resonance contribution, chlorine bears positive charge while electron density is displaced through the double bond towards the other carbon. Consequently the C–Cl bond has some double-bond character rather than being an ordinary alkyl C–Cl single bond. Since the immediate hybridisation is the same in every option, it does not by itself select the answer.

Step 2: Identify the group that strengthens chlorine donation In $Cl-CH=CH-NO_2$, the nitro group withdraws electron density by resonance as well as induction. It stabilises electron density displaced towards its end of the conjugated system, enhancing the contribution in which chlorine donates its lone pair. The resulting larger partial C=Cl character shortens the C–Cl bond most. The methyl group in option A is electron releasing, and methoxy in option C donates strongly by resonance; neither provides the nitro group's electron-accepting effect. Unsubstituted vinyl chloride in option D lacks that additional acceptor. Thus option B has the strongest bond-shortening conjugative interaction among the choices.

Step 3: Conclusion $Cl-CH=CH-NO_2$ has the shortest C–Cl bond. Hence, the correct option is (B).

Quick Tip to Solve: When the bonded carbon has the same hybridisation in every choice, compare resonance: an acceptor at the far end of a conjugated system can increase halogen donation and shorten C–halogen.

63. In the following reaction A is : [4 marks]



(A) Methylcyclopentane

(B) Cyclohexane

(C) 1,2-Dimethylcyclobutane

(D) Ethylcyclopentane

Correct Answer: (B) Cyclohexane

Solution:

Concept:

- Free-radical bromination followed by alcoholic KOH converts a cycloalkane to a cycloalkene by substitution and elimination.
- Reductive ozonolysis cleaves the alkene to carbonyl compounds. A dialdehyde can then undergo intramolecular aldol addition and dehydration in hot aqueous base.

Step 1: Work backwards from the five-membered enal The product shown is cyclopent-1-ene-1-carbaldehyde: a five-membered ring containing a double bond, with a formyl group on a double-bond carbon. Such an enal is produced by intramolecular aldol condensation of hexane-1,6-dial, $OHC-(CH_2)_4-CHO$. An enolate at a carbon adjacent to one aldehyde attacks the other terminal aldehyde. The newly formed carbon–carbon bond closes a five-membered ring; dehydration leaves the conjugated carbon–carbon double bond.

Step 2: Identify the alkene that supplies the dialdehyde Reductive ozonolysis of cyclohexene breaks its double bond and opens the six-membered ring. Both double-bond carbons carry hydrogen, so both become aldehyde groups, giving hexane-1,6-dial. Cyclohexane, option B, supplies cyclohexene:

$$\text{cyclohexane} \xrightarrow{Br_2, h\nu} \text{bromocyclohexane} \xrightarrow{\text{alc. KOH}} \text{cyclohexene.}$$

The subsequent O_3, $(CH_3)_2S$ sequence supplies the dialdehyde just identified. Hot aqueous NaOH then produces the shown enal. The methylcyclopentane, dimethylcyclobutane and ethylcyclopentane skeletons in the other options do not give this unsubstituted six-carbon dialdehyde through the specified sequence.

Step 3: Conclusion The starting compound A is cyclohexane. Hence, the correct option is (B).

Quick Tip to Solve: For an intramolecular aldol product, disconnect the newly formed bond next to the carbonyl group; the resulting open-chain dicarbonyl often reveals the alkene before ozonolysis.

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64. The correct order of the spin-only magnetic moments of the following complexes is :

(I) $[Cr(H_2O)_6]Br_2$

(II) $Na_4[Fe(CN)_6]$

(III) $Na_3[Fe(C_2O_4)_3]$ ($\Delta_0 > P$)

(IV) $(Et_4N)_2[CoCl_4]$ [4 marks]

(A) (III) > (I) > (II) > (IV)

(B) (II) ≈ (I) > (IV) > (III)

(C) (III) > (I) > (IV) > (II)

(D) (I) > (IV) > (III) > (II)

Correct Answer: (D) (I) > (IV) > (III) > (II)

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired d electrons. It increases with n.
- Find the oxidation state first, then the d count. Octahedral strong-field complexes can be low spin; tetrahedral chloride complexes are high spin.

Step 1: Determine the d configurations and spin states In I, neutral water and two bromide counterions make chromium $+2$, hence d^4. Water is a weak-field ligand, giving high-spin octahedral $t_{2g}^3 e_g^1$ with four unpaired electrons. In II, six cyanide ions and an overall -4 complex charge make iron $+2$, hence d^6. Strong-field cyanide gives low-spin t_{2g}^6, with no unpaired electrons. In III, three oxalate ions contribute -6; the complex has charge -3, so iron is $+3$, or d^5. The question explicitly states $\Delta_o > P$; use low-spin t_{2g}^5, with one unpaired electron.

Step 2: Count the cobalt electrons and compare moments In IV, two tetraethylammonium cations require $[CoCl_4]^{2-}$. Cobalt is therefore $+2$, or d^7. The tetrahedral chloride complex is high spin, with three unpaired electrons. The counts for I, II, III and IV are respectively $4, 0, 1, 3$, yielding

$$\mu_I = \sqrt{24}\mu_B, \quad \mu_{II} = 0, \quad \mu_{III} = \sqrt{3}\mu_B, \quad \mu_{IV} = \sqrt{15}\mu_B.$$

Thus I exceeds IV, then III, then II. Options A and C incorrectly place the low-spin ferric complex first, while B incorrectly makes low-spin ferrocyanide strongly paramagnetic.

Step 3: Conclusion The order is (I) > (IV) > (III) > (II). Hence, the correct option is (D).

Quick Tip to Solve: Obey an explicitly given $\Delta_o > P$ before using a ligand's usual field-strength reputation; it directly requires electron pairing in the lower octahedral level.

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65. The number of sp^2 hybrid orbitals in a molecule of benzene is : [4 marks]

(A) 24

(B) 12

(C) 6

(D) 18

Correct Answer: (D) 18

Solution:

Concept:

- Each carbon atom in benzene is sp^2-hybridised: one s orbital and two p orbitals mix to give three equivalent hybrid orbitals in a plane.
- The remaining unhybridised p orbital on each carbon participates in the delocalised pi system. It is not an sp^2 hybrid orbital.

Step 1: Count the hybrid orbitals on one carbon A carbon in benzene forms three sigma bonds, two to neighbouring carbon atoms and one to hydrogen. It supplies one sp^2 orbital to each of those sigma bonds. Its three hybrid orbitals point approximately 120° apart, explaining the planar hexagonal sigma framework. The fourth valence orbital is an unhybridised p orbital perpendicular to the ring plane, reserved for pi overlap. Thus the number requested is three hybrid orbitals per carbon, rather than one hybridisation label per carbon.

Step 2: Multiply by the number of carbon atoms Benzene has molecular formula C_6H_6 and contains six equivalent sp^2-hybridised carbon atoms. Consequently

$$N(sp^2 \text{ hybrid orbitals}) = 6 \times 3 = 18.$$

The six hydrogen atoms contribute their 1s orbitals; these are not carbon hybrid orbitals and are not counted. The value 6 counts the carbons or their unhybridised p orbitals, not their hybrid orbitals. The value 12 can arise from counting carbon contributions only to ring sigma bonds while overlooking the C–H bonds. The value 24 counts all four carbon valence orbitals, incorrectly including the six unhybridised p orbitals. Therefore only 18 answers the stated orbital count.

Step 3: Conclusion There are 18 sp^2 hybrid orbitals in one molecule of benzene. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish hybridised atoms from hybrid orbitals: multiply the number of sp^2 atoms by three, and keep the unhybridised p orbitals separate.

66. Which polymer has 'chiral' monomer(s) ? [4 marks]

(A) Nylon 6,6

(B) Buna-N

(C) PHBV

(D) Neoprene

Correct Answer: (C) PHBV

Solution:

Concept:

- A tetrahedral carbon is a stereogenic centre when its four attached groups are different. A monomer containing such a centre can be chiral.
- PHBV is the copolyester of 3-hydroxybutanoic acid and 3-hydroxypentanoic acid. Their hydroxy-bearing carbon atoms provide the required stereogenic centres.

Step 1: Inspect the monomers of PHBV Write the two hydroxy acids as $HOOC-CH_2-CH(OH)-CH_3$ and $HOOC-CH_2-CH(OH)-CH_2-CH_3$. In 3-hydroxybutanoic acid, the $CH(OH)$ carbon is attached to H, OH, CH_3, and CH_2COOH. These four groups are different. In 3-hydroxypentanoic acid, that carbon is attached to H, OH, C_2H_5, and CH_2COOH, again four different groups. Both monomers therefore possess a stereogenic carbon. Ester formation between their alcohol and acid groups gives PHBV.

Step 2: Exclude the other polymer choices Nylon 6,6 is made from hexane-1,6-diamine and adipic acid, whose chains contain methylene groups without a tetrahedral carbon bonded to four different substituents. Buna-N is formed from buta-1,3-diene and acrylonitrile; these starting monomers contain unsaturated carbons and have no stereogenic centre. Neoprene is formed from chloroprene, $CH_2=C(Cl)-CH=CH_2$, which is also achiral. The possibility of stereochemical arrangements within a polymer chain is a different question from chirality of the starting monomer. Among the listed polymers, PHBV is therefore the one whose monomers satisfy the stated requirement.

Step 3: Conclusion PHBV has chiral monomers, 3-hydroxybutanoic acid and 3-hydroxypentanoic acid. Hence, the correct option is (C).

Quick Tip to Solve: Write each monomer before judging chirality; for hydroxy acids, check the carbon bearing OH for four different attached groups.

67. Amongst the following, the form of water with the lowest ionic conductance at 298 K is : [4 marks]

(A) saline water used for intravenous injection

(B) sea water

(C) distilled water

(D) water from a well

Correct Answer: (C) distilled water

Solution:

Concept:

- Ionic conductance requires mobile dissolved ions; molecular water itself is only very weakly ionised.
- At 298 K, pure water has $K_w = [\mathrm{H^+}][\mathrm{OH^-}] = 10^{-14}$, so each ion concentration is only $10^{-7}\,\mathrm{mol\,L^{-1}}$.

Step 1: Identify the charge carriers An electric current in water is carried mainly by dissolved cations and anions. Neutral water molecules do not transport net charge through the liquid. The comparison concerns ionic conductance, so the important distinction is the concentration of dissolved electrolytes rather than the source or medical use of the sample.

Step 2: Compare the four samples Saline water for intravenous injection contains dissolved sodium chloride, which supplies $\mathrm{Na^+}$ and $\mathrm{Cl^-}$. Sea water contains substantial dissolved salts, and well water ordinarily contains mineral ions such as calcium, magnesium and bicarbonate. Distillation removes these nonvolatile dissolved salts. Distilled water therefore has far fewer ionic charge carriers than the other three samples, with only the very small concentrations arising from water ionisation in an ideal pure sample.

Step 3: Select the least conducting sample At the same temperature, the very low concentration of ions in distilled water makes its conductance the smallest among the listed waters. Saline, sea and well water are eliminated because each provides extra dissolved electrolytes; low salt concentration is more relevant here than biological suitability or visual clarity.

Step 4: Conclusion Distilled water has the lowest ionic conductance at 298 K. Hence, the correct option is (C).

Quick Tip to Solve: For water-conductance comparisons, rank the dissolved-ion content first: distillation removes salts, whereas clear-looking mineral or saline water still conducts.

68. Biochemical Oxygen Demand (BOD) is the amount of oxygen required (in ppm) : [4 marks]

(A) for sustaining life in a water body.

(B) by anaerobic bacteria to breakdown inorganic waste present in a water body.

(C) for the photochemical breakdown of waste present in 1 m^3 volume of a water body.

(D) by bacteria to break-down organic waste in a certain volume of a water sample.

Correct Answer: (D) by bacteria to break-down organic waste in a certain volume of a water sample.

Solution:

Concept:

- Biochemical oxygen demand (BOD) measures the dissolved oxygen consumed by aerobic microorganisms while decomposing biodegradable organic matter.
- For dilute aqueous samples, oxygen concentration expressed in ppm is approximately the same numerical concentration as $\mathrm{mg\,L^{-1}}$.

Step 1: Identify the process measured Organic waste serves as food for microorganisms. During aerobic decomposition, microorganisms use dissolved oxygen to oxidise biodegradable organic material. BOD describes the oxygen required for that biochemical breakdown in a specified water sample under specified test conditions; it is an indicator of biodegradable organic pollution.

Step 2: Check the alternatives Option (D) identifies both the biological agents, bacteria, and their substrate, organic waste, so it matches the definition. Option (A) is too broad: the oxygen needed to sustain all aquatic life is not the definition of BOD. Option (B) incorrectly specifies anaerobic bacteria and inorganic waste; anaerobic metabolism does not consume dissolved molecular oxygen in the manner measured by BOD. Option (C) refers to a photochemical process, whereas BOD concerns microbial decomposition, not light-induced degradation.

Step 3: Interpret the significance of the demand A larger BOD means that more dissolved oxygen can be removed as biodegradable waste decomposes. This can leave less oxygen for fish and other aerobic aquatic organisms. The demand is a concentration-based property of the sample, not merely the amount of oxygen already dissolved at the start. It therefore distinguishes oxygen consumption by decomposition from the initial oxygen supply.

Step 4: Conclusion BOD is the oxygen required by bacteria to break down organic waste in a certain volume of a water sample. Hence, the correct option is (D).

Quick Tip to Solve: Associate BOD with three words: biological, oxygen-consuming and organic. Anaerobic, inorganic and photochemical descriptions fail this test.

69. Among the statements (a)-(d) the correct ones are :

(a) Lithium has the highest hydration enthalpy among the alkali metals.

(b) Lithium chloride is insoluble in pyridine.

(c) Lithium cannot form ethynide upon its reaction with ethyne.

(d) Both lithium and magnesium react slowly with H_2O. [4 marks]

(A) (b) and (c) only

(B) (a) and (d) only

(C) (a), (b) and (d) only

(D) (a), (c) and (d) only

Correct Answer: (D) (a), (c) and (d) only

Solution:

Concept:

- The small size and high charge density of Li^+ give it the greatest magnitude of hydration enthalpy among alkali-metal ions.
- Lithium shows a diagonal relationship with magnesium, including comparatively slow reaction with water; it does not form an ethynide by direct reaction with ethyne under the stated comparison.

Step 1: Evaluate statement (a) The lithium ion is the smallest alkali-metal cation. Its concentrated positive charge strongly attracts polar water molecules, releasing more energy on hydration than the larger alkali-metal ions. Thus the usual statement that lithium has the highest hydration enthalpy means the largest magnitude of the negative hydration enthalpy. Statement (a) is correct.

Step 2: Evaluate statements (b) and (c) Lithium chloride has appreciable covalent character because the small lithium ion strongly polarises the chloride ion. It is soluble in organic solvents including pyridine, so the assertion that it is insoluble in pyridine is false. In the standard anomalous-behaviour comparison, lithium does not form an ethynide by direct reaction with ethyne, unlike the heavier alkali metals. Statement (c) therefore correctly describes this reaction behaviour. This statement concerns the specified direct reaction and should not be confused with the existence of lithium acetylide reagents obtained by other preparative routes.

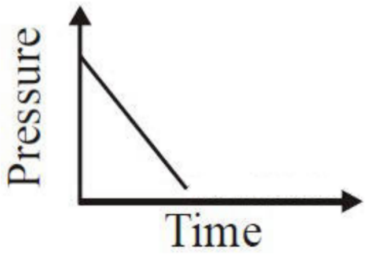
Step 3: Evaluate statement (d) and combine Lithium reacts with water less vigorously than the heavier alkali metals. Magnesium also reacts slowly with cold water because its surface layer inhibits reaction; reaction becomes easier on heating. Hence statement (d), in the ordinary comparison of these metals with water, is correct. We retain statements a, c and d, while rejecting statement b. The alternatives that contain b are excluded because lithium chloride is soluble in pyridine; the alternative listing only a and d incorrectly omits the true statement c.

Step 4: Conclusion The correct statements are a, c and d only. Hence, the correct option is (D).

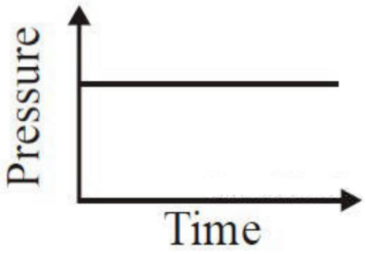
Quick Tip to Solve: For lithium questions, use its small ion size and diagonal relationship with magnesium; do not assign it every reaction trend of the heavier alkali metals.

70. **A mixture of gases O_2, H_2 and CO are taken in a closed vessel containing charcoal. The graph that represents the correct behaviour of pressure with time is :** [4 marks]

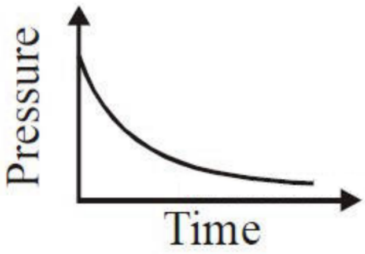
(A)



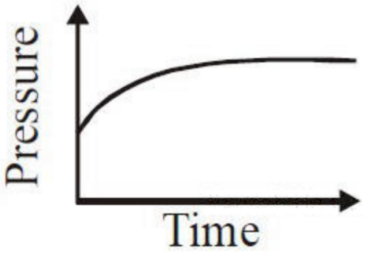
(B)



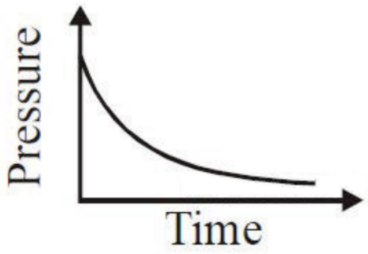
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- Charcoal adsorbs gas molecules on its surface, reducing the number of molecules remaining in the gas phase.
- At fixed temperature and vessel volume, $P = n_{\text{gas}}RT/V$, so removal of gas-phase molecules lowers the total pressure.

Step 1: Determine the initial change Fresh charcoal exposes adsorption sites to the mixture of oxygen, hydrogen and carbon monoxide. Molecules transfer from the free gas to the charcoal surface. Since the vessel is closed, the total number of molecules in the complete vessel is conserved, but the number in the gas phase decreases. It is this gas-phase amount that determines the measured gas pressure, so the pressure initially falls.

Step 2: Determine the long-time behaviour As adsorption proceeds, fewer vacant sites remain and the gas pressure decreases. Adsorption progressively slows, while desorption opposes it. Eventually adsorption and desorption reach dynamic equilibrium. The pressure then approaches a steady residual value rather than continuing to fall at a fixed rate. The different gases may be adsorbed to different extents, but their combined pressure follows the same qualitative decreasing-to-equilibrium behaviour.

Step 3: Compare the plotted curves Graph (C) shows a rapid initial decrease followed by a progressively flatter curve, matching adsorption toward equilibrium. Graph (A) shows a constant-rate linear fall rather than the slowing approach. Graph (B) shows no change despite adsorption, and graph (D) shows increasing pressure, which is opposite to the removal of molecules from the gas phase. Therefore only graph (C) gives the expected qualitative trend.

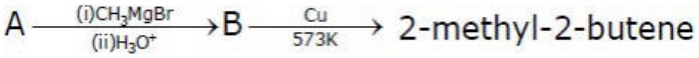
Step 4: Conclusion The pressure decreases and approaches an equilibrium value with time. Hence, the correct option is (C).

Quick Tip to Solve: In a closed constant-volume gas vessel, adsorption lowers gas-phase mole number: choose a falling curve that levels off, rather than a straight fall to zero.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. Consider the following reactions

The mass percentage of carbon in A is _____. [4 marks]



Solution:

Concept:

- Methylmagnesium bromide adds a methyl group to a carbonyl carbon; acidic work-up converts the intermediate alkoxide into an alcohol.
- Passing a tertiary alcohol over heated copper favours dehydration to an alkene. Carbon mass percentage equals the carbon mass per mole divided by the molar mass, multiplied by 100.

Step 1: Identify alcohol B from the alkene The final alkene is 2-methyl-2-butene, $(CH_3)_2C{=}CHCH_3$. Its formation by dehydration corresponds to the tertiary alcohol 2-methylbutan-2-ol. Its structure is

$$(CH_3)_2C(OH)CH_2CH_3.$$

Removal of water from this alcohol gives the more substituted alkene shown. The hydroxyl-bearing carbon in B is attached to two methyl groups and one ethyl group.

Step 2: Reverse the Grignard addition The reagent CH_3MgBr supplies one of those methyl groups. Removing the added methyl group and restoring the carbonyl bond leaves methyl ethyl ketone, butan-2-one, as A. The reaction sequence is therefore

$$CH_3COCH_2CH_3 \xrightarrow[H_3O^+]{CH_3MgBr} (CH_3)_2C(OH)CH_2CH_3.$$

A ketone is required because Grignard addition gives a tertiary alcohol; an aldehyde would instead give a secondary alcohol in this methyl addition.

Step 3: Calculate the carbon percentage Butan-2-one has molecular formula C_4H_8O. Using atomic masses C = 12, H = 1 and O = 16, its molar mass is

$$M = 4(12) + 8(1) + 16 = 72\,\text{g mol}^{-1}.$$

The carbon contribution is $4(12) = 48\,\text{g mol}^{-1}$, hence

$$\%C = \frac{48}{72} \times 100 = 66.666\ldots\%.$$

The methyl group introduced by the reagent belongs to B, so it must not be included when calculating the composition of A.

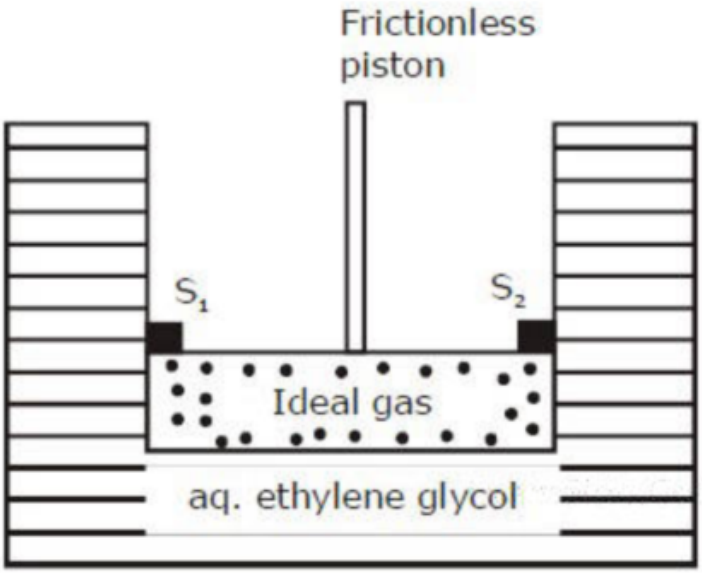
Step 4: Conclusion A is butan-2-one, C_4H_8O, and the mass percentage of carbon is $\boxed{66.67\%}$.

Quick Tip to Solve: Work backward from the alcohol: remove the carbon group introduced by the Grignard reagent before calculating the formula and composition of the original carbonyl compound.

72. A cylinder containing an ideal gas (0.1 mol of 1.0 dm^3) is in thermal equilibrium with a large volume of 0.5 molal aqueous solution of ethylene glycol at its freezing point. If the stoppers S_1 and S_2 (as shown in the figure) are suddenly withdrawn, the volume of the gas in litres after equilibrium is achieved will be____.

(Given, K_f (water) = 2.0 K kg mol^{-1},

R = 0.08 dm^3 atm K^{-1} mol^{-1}) [4 marks]



Solution:

Concept:

- For a non-electrolyte, freezing-point depression is $\Delta T_f = K_f m$; ethylene glycol has van't Hoff factor $i = 1$.
- The frictionless piston reaches mechanical equilibrium with atmospheric pressure, and the large surrounding solution fixes the final gas temperature.

Step 1: Find the temperature of the surroundings The solution is 0.5 molal ethylene glycol in water. Since ethylene glycol does not dissociate into ions,

$$\Delta T_f = K_f m = (2.0)(0.5) = 1.0\,\mathrm{K}.$$

Taking the freezing point of pure water as 273 K, the solution freezes at

$$T_f = 273 - 1 = 272\,\mathrm{K}.$$

The gas is initially in thermal equilibrium with this large bath, and after its expansion the same bath restores its temperature to 272 K.

Step 2: Apply the final mechanical condition The stoppers initially restrain the frictionless piston. Once both stoppers are removed, it can move until the gas pressure balances the external atmospheric pressure. Thus the final equilibrium pressure is $1\,\mathrm{atm}$. The initial volume of $1.0\,\mathrm{dm}^3$ is not the final volume: it is the constrained volume before release. Because the gas is in contact with a large thermal reservoir, the final calculation is isothermal after equilibration, rather than an isolated adiabatic

end-state calculation.

Step 3: Use the ideal-gas equation Use the given gas constant in litre-atmosphere units and $n = 0.1\,\text{mol}$:

$$V = \frac{nRT}{P} = \frac{(0.1)(0.08)(272)}{1} = 2.176\,\text{dm}^3.$$

Since $1\,\text{dm}^3 = 1\,\text{L}$, this is approximately $2.18\,\text{L}$, or $2.2\,\text{L}$ to two significant figures. The final volume exceeds the initial litre, consistent with release of a gas whose initial pressure exceeds atmospheric pressure.

Step 4: Conclusion The equilibrium volume of the gas is $\boxed{2.176\,\text{L} \approx 2.18\,\text{L}}$ (about $2.2\,\text{L}$).

Quick Tip to Solve: Find the bath temperature from freezing-point depression first, then use the final piston pressure in $PV = nRT$; the initial restrained volume does not set the final volume.

73. A sample of milk splits after 60 min. at 300 K and after 40 min. at 400 K when the population of lactobacillus acidophilus in it doubles. The activa tion energy (in kJ/ mol) for this process is closest to_____.

(Given, R = 8.3 J mol^{-1} K^{-1}, $\ln\left(\frac{3}{2}\right) = 0.4$, e^{-3} = 4.0) [4 marks]

Solution:

Concept:

- The bacterial population doubles at each temperature, so the same extent of change is reached. The rate constant is inversely proportional to the time required for that fixed change: $k \propto 1/t$.
- The two-temperature Arrhenius equation is $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$, where E_a is activation energy and R is the gas constant. Temperatures must be in kelvin.

Step 1: Form the ratio of rate constants Label the observations at 300 K and 400 K by subscripts 1 and 2. Because both observations correspond to the population doubling, the kinetic factor associated with the extent of change cancels. Thus

$$\frac{k_2}{k_1} = \frac{t_1}{t_2} = \frac{60}{40} = \frac{3}{2}.$$

The higher temperature gives a shorter doubling time and hence the larger rate constant. The times can remain in minutes because their ratio is dimensionless; converting both to seconds gives the same ratio.

Step 2: Substitute into the Arrhenius equation Use the supplied logarithm, $\ln(3/2) = 0.4$, and $R = 8.3\,\text{J mol}^{-1}\text{K}^{-1}$:

$$0.4 = \frac{E_a}{8.3}\left(\frac{1}{300} - \frac{1}{400}\right) = \frac{E_a}{8.3}\frac{1}{1200}.$$

Therefore,

$$E_a = 0.4(8.3)(1200) = 3984\,\text{J mol}^{-1} = 3.984\,\text{kJ mol}^{-1}.$$

The positive result is consistent with the observed increase in rate on heating. No additional exponential datum is needed because the logarithm of the required rate ratio is already given.

Step 3: Conclusion The activation energy is $\boxed{3.984\,\text{kJ mol}^{-1}}$, approximately $3.98\,\text{kJ mol}^{-1}$.

Quick Tip to Solve: For the same fixed extent of reaction, invert the time ratio before using Arrhenius: the shorter time corresponds to the larger rate constant.

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74. 10.30 mg of O_2 is dissolved into a liter of sea water of density 1.03 g/mL. The concentration of O_2 in ppm is_____. [4 marks]

Solution:

Concept:

- For a mass-based concentration, parts per million is the mass fraction multiplied by 10^6: $\text{ppm} = (m_{\text{solute}}/m_{\text{solution}})10^6$. Both masses must use the same units.
- Density converts the stated solution volume to mass: $m = \rho V$. For aqueous samples, $1\,\text{mg kg}^{-1} = 1\,\text{ppm}$; the density must be included when it differs from $1\,\text{g mL}^{-1}$.

Step 1: Calculate the mass of the sea-water sample The given volume is one litre, or $1000\,\text{mL}$. Use the stated density rather than replacing it by that of pure water:

$$m_{\text{solution}} = 1.03\,\text{g mL}^{-1} \times 1000\,\text{mL} = 1030\,\text{g} = 1.030\,\text{kg}.$$

Thus the dissolved oxygen is distributed through a sample whose mass is 1.030 kg. The density relates to the sea-water sample, so this is the mass entering the concentration calculation. The small oxygen mass does not require a separate density calculation.

Step 2: Convert the oxygen concentration to ppm The mass of dissolved oxygen is $10.30\,\text{mg}$. Dividing by the mass of the solution in kilograms gives the numerical value in ppm:

$$\text{ppm} = \frac{10.30\,\text{mg}}{1.030\,\text{kg}} = 10.00\,\text{mg kg}^{-1} = 10.00.$$

Equivalently, in grams the mass fraction is $0.01030/1030 = 1.00 \times 10^{-5}$; multiplication by 10^6 again gives 10. Using 10.30 directly would give the concentration in mg per litre, not the requested mass-based ppm. Oxygen's molar mass is unnecessary because the requested quantity is a mass ratio, not a molar concentration.

Step 3: Conclusion The concentration of dissolved O_2 is $\boxed{10\,\text{ppm}}$.

Quick Tip to Solve: Convert litres to kilograms with the stated density first; mg per litre and ppm coincide only when the solution density is approximately one kilogram per litre.

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75. The sum of the total number of bonds between chromium and oxygen atoms in chromate and dichromate ions is _____. [4 marks]

Solution:

Concept:

- Chromate is CrO_4^{2-}, containing one tetrahedral chromium centre. In a conventional Lewis representation, two chromium–oxygen links are double bonds and two are single bonds.
- Dichromate is $Cr_2O_7^{2-}$, made from two tetrahedral units sharing a bridging oxygen. Count a single bond once and a double bond twice when determining the total number of bonds, including sigma and pi bonds.

Step 1: Count bonds in chromate In one conventional resonance contributor of the chromate ion, chromium is attached to four oxygen atoms: two are shown by $Cr{=}O$ bonds and two by single $Cr{-}O$ bonds. Each double bond consists of one sigma and one pi bond. Therefore,

$$N_{\text{chromate}} = 2(2) + 2(1) = 6.$$

Equivalently, this contributor contains four sigma bonds and two pi bonds. Resonance distributes the bonding among the oxygen atoms, but does not change this total bond count. Counting only four chromium–oxygen connections would omit the pi bonds asked for in the total.

Step 2: Count bonds in dichromate and add The dichromate ion has a $Cr{-}O{-}Cr$ bridge, contributing two single bonds. Its six terminal oxygen atoms contribute four double bonds and two single bonds in a conventional Lewis contributor. Thus

$$N_{\text{dichromate}} = 4(2) + 2(1) + 2(1) = 12.$$

There are eight sigma bonds in all, since each chromium has four oxygen neighbours, together with four pi bonds. The shared oxygen is a single atom but has two separate chromium–oxygen bonds; it must not be counted as just one bond. The required sum for the two different ions is consequently

$$N_{\text{total}} = 6 + 12 = 18.$$

This count includes bond multiplicity; it is not simply the number of oxygen atoms in the formulae.

Step 3: Conclusion The sum of the total numbers of chromium–oxygen bonds in chromate and dichromate ions is $\boxed{18}$.

Quick Tip to Solve: Count sigma and pi bonds separately, and remember that the bridging oxygen in dichromate makes two single chromium–oxygen bonds.