JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 2 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 2 September 2020, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

[4 marks]

(A) [-3, ∞)

(B) (-∞, 9]

(C) (-∞, -9] $\cup$ [-3, ∞)

(D) (-∞, -3] $\cup$ [9, ∞)

Correct Answer: (D) (-∞, -3] $\cup$ [9, ∞)

Solution:

Concept:

- For three real consecutive terms of a geometric progression, use $u/r, u, ur$, where $r \neq 0$. Their product is u^3, independent of the common ratio.
- For a nonzero real number r, $r + r^{-1} \geq 2$ when $r > 0$, whereas $r + r^{-1} \leq -2$ when $r < 0$. Equality occurs at $r = 1$ or $r = -1$, respectively.

Step 1: Find the middle term The product of the three terms is positive and equals 27. Therefore their middle term satisfies

$$u^3 = 27 \quad \Rightarrow \quad u = 3.$$

Thus the three terms can be written as $3/r, 3, 3r$, and their sum is

$$S = 3\left(1 + r + \frac{1}{r}\right).$$

Negative common ratios are allowed: a positive product does not mean that all three terms are positive.

Step 2: Find both branches of the range If $r > 0$, the inequality $r + r^{-1} \geq 2$ gives $S \geq 9$. If $r < 0$, write $r = -t$, where $t > 0$. Then

$$S = 3\left(1 - t - \frac{1}{t}\right) \leq -3.$$

As $t + t^{-1}$ takes every value in $[2, \infty)$, both intervals are fully attained. The endpoints occur for the progressions $(3, 3, 3)$ and $(-3, 3, -3)$. The other options either admit unattainable sums between these intervals or omit an attainable branch.

Step 3: Conclusion All possible sums lie in $\boxed{(-\infty, -3] \cup [9, \infty)}$. Hence, the correct option is (D).

Quick Tip to Solve: Fix the middle term from the product first; then split the common ratio into positive and negative cases before using the reciprocal inequality.

2. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ **is :** [4 marks]

(A) $\frac{1}{2}\left(\sqrt{3}-i\right)$

(B) -$\frac{1}{2}\left(\sqrt{3}-i\right)$

(C) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$

(D) $\frac{1}{2}\left(1-i\sqrt{3}\right)$

Correct Answer: (B) -$\frac{1}{2}\left(\sqrt{3}-i\right)$

Solution:

Concept:

- For real A, B, conjugate complex numbers $A+iB$ and $A-iB$ have equal modulus and opposite arguments. Their quotient has modulus one and twice the numerator argument.
- The identity $\cos\theta/(1+\sin\theta)=\tan(\pi/4-\theta/2)$ converts the quotient into polar form; De Moivre's theorem multiplies its argument by the power.

Step 1: Identify the argument of the quotient Put $\theta=2\pi/9$. Since $\theta=40^\circ$, both $1+\sin\theta$ and $\cos\theta$ are positive, so the numerator is in the first quadrant. If its argument is ϕ, then

$$\tan\phi=\frac{\cos\theta}{1+\sin\theta}=\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right).$$

Consequently $\phi=5\pi/36$, or 25°, without a quadrant ambiguity. Division by the conjugate doubles this argument and cancels the moduli.

Step 2: Cube and compare the options The quotient is $\cos(5\pi/18)+i\sin(5\pi/18)$. Its cube is therefore

$$\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}+\frac{i}{2}.$$

This number lies in the second quadrant. Option A has the opposite signs, and options C and D interchange the magnitudes of the real and imaginary parts. Only the negative of $(\sqrt{3}-i)/2$ has the required components. This also checks the unit modulus inherited from the original quotient.

Step 3: Conclusion The required value is $\boxed{-\frac{1}{2}(\sqrt{3}-i)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a complex number divided by its conjugate, double the argument first; apply the outer power only after that doubling.

3. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is : [4 marks]

(A) $\frac{8}{17}$
(B) $\frac{2}{3}$
(C) $\frac{2}{5}$
(D) $\frac{4}{17}$

Correct Answer: (A) $\frac{8}{17}$

Solution:

Concept:

- Bayes's theorem updates the probability of a selected box using the likelihood of the observed event. The two boxes are equally likely, but individual cards across the boxes are not.
- A non-prime positive integer is any positive integer that is not prime. In particular, 1 is non-prime even though it is not composite.

Step 1: Count the non-prime cards in each box Let B_1, B_2 denote the box choices and N the event of a non-prime number. The primes from 1 to 30 are

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29,$$

so Box I contains $30 - 10 = 20$ non-prime cards. Between 31 and 50 the primes are $31, 37, 41, 43, 47$; hence Box II contains $20 - 5 = 15$ non-prime cards. Thus $P(N \mid B_1) = 20/30 = 2/3$ and $P(N \mid B_2) = 15/20 = 3/4$.

Step 2: Apply the equal box-selection probabilities Both box probabilities equal $1/2$. The total probability of observing a non-prime is

$$P(N) = \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{3}{4} = \frac{17}{24}.$$

Bayes's theorem now gives

$$P(B_1 \mid N) = \frac{(1/2)(2/3)}{17/24} = \frac{8}{17}.$$

The value $2/3$ is only the forward likelihood within Box I, not the requested posterior. Counting all non-prime cards with equal weights would also be incorrect, since a card in the smaller box has a larger initial chance of selection.

Step 3: Conclusion The probability that the selected card came from Box I is $\boxed{8/17}$. Hence, the correct option is (A).

Quick Tip to Solve: When a box is chosen first, weight its fraction of favourable cards by its box probability; do not pool the cards as equally likely outcomes.

4. Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is : [4 marks]

(A) $6\,(4-\pi)$

(B) $3\,(4-\pi)$

(C) $6\,(\pi-2)$

(D) $3\,(\pi-2)$

Correct Answer: (C) $6\,(\pi-2)$

Solution:

Concept:

- The ellipse $x^2/a^2+y^2/b^2=1$ encloses area πab. Here its semi-axes are $a=2$ and $b=3$.
- The curve $|x|/a+|y|/b=1$ is a rhombus with vertices $(\pm a,0)$ and $(0,\pm b)$. Its area is half the product of its diagonals.

Step 1: Identify the inner region and check containment The absolute-value equation describes four straight segments connecting $(2,0),(0,3),(-2,0),(0,-3)$. Its diagonals have lengths 4 and 6. In the first quadrant set $u=x/2$, $v=y/3$. On the segment, $u+v=1$ with $u,v\geq 0$, so

$$u^2+v^2=(u+v)^2-2uv\leq 1.$$

Thus the segment lies inside the ellipse, and symmetry gives the same result in every quadrant. The whole rhombus is contained in the ellipse.

Step 2: Subtract the rhombus area from the ellipse area The ellipse area is

$$A_E=\pi(2)(3)=6\pi.$$

The rhombus area is

$$A_R=\frac{1}{2}(4)(6)=12.$$

Therefore the region inside the ellipse but outside the rhombus has area

$$A=A_E-A_R=6\pi-12=6(\pi-2).$$

There are four congruent curved portions, already included in this subtraction. Option D is half the full area, while the expressions involving $4-\pi$ correspond to a different subtraction and do not match these two enclosing figures.

Step 3: Conclusion The requested area is $\boxed{6(\pi-2)}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Read the intercepts from both equations: matching intercepts identify an inscribed rhombus, so subtract its diagonal-product area from the ellipse area.

5. Let α and β be the roots of the equation

$5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3...., then : [4 marks]

(A) $5S_6 + 6S_5 = 2S_4$

(B) $5S_6 + 6S_5 + 2S_4 = 0$

(C) $6S_6 + 5S_5 + 2S_4 = 0$

(D) $6S_6 + 5S_5 = 2S_4$

Correct Answer: (A) $5S_6 + 6S_5 = 2S_4$

Solution:

Concept:

- Every root of $5x^2 + 6x - 2 = 0$ satisfies $5x^2 + 6x = 2$. Multiplying this identity by a power of that same root gives a recurrence.
- For $S_n = \alpha^n + \beta^n$, add the identities for α and β. This avoids calculating irrational roots or their high powers explicitly.

Step 1: Raise the root identity to the required indices Substitute $x = \alpha$ into the quadratic and multiply by α^4:

$$5\alpha^6 + 6\alpha^5 - 2\alpha^4 = 0.$$

The identical argument for the other root gives

$$5\beta^6 + 6\beta^5 - 2\beta^4 = 0.$$

Multiplication by the fourth power is chosen because the proposed answers involve the sixth, fifth and fourth powers. No division by a possibly zero expression is used.

Step 2: Add and express the result using the power sums Adding the two equations and grouping like powers gives

$$5(\alpha^6 + \beta^6) + 6(\alpha^5 + \beta^5) - 2(\alpha^4 + \beta^4) = 0.$$

Therefore

$$5S_6 + 6S_5 = 2S_4.$$

More generally, the same method yields $5S_n + 6S_{n-1} = 2S_{n-2}$ for $n \geq 2$, with $S_0 = 2$. Option B changes the sign of the last term. Options C and D swap the leading coefficients, even though multiplication of the root equation preserves their order. These alterations do not follow from the given quadratic.

Step 3: Conclusion The correct relation is $\boxed{5S_6 + 6S_5 = 2S_4}$. Hence, the correct option is (A).

Quick Tip to Solve: To connect consecutive power sums, multiply the original polynomial identity by the lowest power needed, then add over all its roots.

6. If a function f(x) defined by

$$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$$

be continuous for some a, b, c $\in$ R and f'(0) + f'(2) = e, then the value of of a is : [4 marks]

(A) $\frac{e}{e^2-3e-13}$
(B) $\frac{1}{e^2-3e+13}$
(C) $\frac{e}{e^2-3e+13}$
(D) $\frac{e}{e^2+3e+13}$

Correct Answer: (C) $\frac{e}{e^2-3e+13}$

Solution:

Concept:

- A piecewise function is continuous at a junction when its left limit, its assigned value and its right limit agree. Apply this at both $x = 1$ and $x = 3$.
- The points 0 and 2 lie strictly inside their respective branches, so calculate the requested derivatives from those branches directly: $f\prime(0) = a - b$ and $f\prime(2) = 4c$.

Step 1: Use continuity to express the constants in terms of a At $x = 3$, continuity between the polynomial branches gives

$$9c = 9a + 6c \quad \Rightarrow \quad c = 3a.$$

At $x = 1$, the exponential branch approaches $ae + b/e$, while the middle branch has value c. Hence

$$ae + \frac{b}{e} = c = 3a \quad \Rightarrow \quad b = 3ae - ae^2.$$

Continuity alone supplies these two equations; differentiability at the junctions is neither given nor needed.

Step 2: Impose the derivative condition Within the first branch $f\prime(x) = ae^x - be^{-x}$, and within the middle branch $f\prime(x) = 2cx$. The given derivative sum becomes

$$a - b + 4c = e.$$

Substituting the continuity relations yields

$$a - (3ae - ae^2) + 12a = e,$$

and thus

$$a(e^2 - 3e + 13) = e.$$

The denominator is nonzero because $e^2 - 3e + 13 = (e - 3/2)^2 + 43/4 > 0$. Option B loses the numerator e; options A and D have incorrect constant or linear signs in the denominator.

Step 3: Conclusion The required constant is $\boxed{a = \dfrac{e}{e^2 - 3e + 13}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the simpler polynomial junction first to eliminate one constant; evaluate derivatives only in the branches containing the specified interior points.

7. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to : [4 marks]

(A) 176

(B) 336

(C) 352

(D) 84

Correct Answer: (B) 336

Solution:

Concept:

- The general term in $(u+v)^{10}$ is $T_{r+1} = \binom{10}{r} u^{10-r} v^r$. A term is independent of x exactly when its total exponent is zero.
- For positive numbers with a fixed sum, their product is largest when they are equal: $UV \le ((U+V)/2)^2$. Equality must be checked against the stated positivity condition.

Step 1: Locate the constant term Select r factors of the second term and $10 - r$ factors of the first. Then

$$T_{r+1} = \binom{10}{r} \alpha^{10-r} \beta^r x^{(10-r)/9 - r/6}.$$

The exponent vanishes when $2(10 - r) - 3r = 0$, so $r = 4$. The required term is therefore

$$T_5 = \binom{10}{4} \alpha^6 \beta^4 = 210(\alpha^3 \beta^2)^2.$$

There is just one constant term, since the exponent is a strictly decreasing linear function of r.

Step 2: Maximise using the constraint Put $U = \alpha^3$ and $V = \beta^2$. Both are positive and $U + V = 4$. Thus

$$UV \le \left(\frac{4}{2}\right)^2 = 4.$$

Equality holds at $U = V = 2$, which gives allowed positive values $\alpha = \sqrt[3]{2}$, $\beta = \sqrt{2}$. Consequently, the maximum constant term is

$$210(4)^2 = 3360.$$

Since the question calls this maximum $10k$, divide by ten rather than identifying k with the constant term itself. The other listed values do not satisfy $10k = 3360$.

Step 3: Conclusion The maximum constant term is $3360 = 10k$, so $\boxed{k = 336}$. Hence, the correct option is (B).

Quick Tip to Solve: First set the power of x equal to zero; only then use the constraint to maximise the coefficient.

8. If R = {(x, y) : x, y ∈ Z, $x^2 + 3y^2 \leq 8$} is a relation on the set of integers Z, then the domain of R^{-1} is : [4 marks]

(A) {0, 1}

(B) {–2, –1, 1, 2}

(C) {–1, 0, 1}

(D) {–2, –1, 0, 1, 2}

Correct Answer: (C) {–1, 0, 1}

Solution:

Concept:

- The inverse relation reverses ordered pairs: $(y, x) \in R^{-1}$ exactly when $(x, y) \in R$. Thus the domain of R^{-1} is the range of R.
- An integer belongs to the range precisely when at least one integer first coordinate makes the defining inequality true. Nonnegative squares give a useful necessary bound.

Step 1: Bound the possible second coordinates We need all integers y for which there exists an integer x satisfying

$$x^2 + 3y^2 \leq 8.$$

Since $x^2 \geq 0$, it is necessary that $3y^2 \leq 8$. Hence $|y| \leq \sqrt{8/3} < 2$. The only possible integer values are therefore $-1, 0, 1$. In particular, the values 2 and -2 are impossible even if $x = 0$, because their contribution $3y^2 = 12$ already exceeds eight.

Step 2: Check existence for every candidate The bound alone must be supplemented by an actual first coordinate. Choose $x = 0$, an allowed integer. For $y = 0$, the left side is zero; for $y = 1$ or $y = -1$, it equals three. All satisfy the inequality. Thus $(0, -1), (0, 0), (0, 1) \in R$, and reversing these pairs proves that all three candidates are in the desired domain. Option (A) misses the negative value -1. Option (B) both omits zero and includes impossible values, while option (D) also includes the impossible values ± 2.

Step 3: Conclusion Therefore $\mathrm{Dom}(R^{-1}) = \{-1, 0, 1\}$. Hence, the correct option is (C).

Quick Tip to Solve: To find the domain of an inverse relation, bound the original second coordinate and then exhibit a valid first coordinate.

9. Let y = y(x) be the solution of the differential equation,

$\frac{2+\sin x}{y+1}\cdot\frac{dy}{dx} = -\cos x$, **y > 0,y(0) = 1.**

If y(π) = a and $\frac{dy}{dx}$ at x = π is b, then the ordered pair (a, b) is equal to : [4 marks]

(A) (2, 1)

(B) $\left(2, \frac{3}{2}\right)$

(C) (1, -1)

(D) (1, 1)

Correct Answer: (D) (1, 1)

Solution:

Concept:

- A separable equation can be integrated after collecting the dependent-variable factor with dy and the independent-variable factor with dx.
- The derivative of $\log(2+\sin x)$ is $\cos x/(2+\sin x)$. Here $2+\sin x > 0$, and $y > 0$ also ensures $y+1 > 0$.

Step 1: Integrate and impose the initial value Rearrange the given equation as

$$\frac{1}{y+1}\frac{dy}{dx} = -\frac{\cos x}{2+\sin x}.$$

Integrating gives $\log(y+1) = -\log(2+\sin x) + C$. The positive factors allow us to combine the logarithms and exponentiate:

$$(y+1)(2+\sin x) = K.$$

At $x = 0$, the condition $y(0) = 1$ yields $K = 2\cdot 2 = 4$. Hence the solution on the stated positive branch is

$$y(x) = \frac{4}{2+\sin x} - 1.$$

This also shows directly that the initial condition fixes the integration constant uniquely.

Step 2: Evaluate the value and slope at pi Since $\sin\pi = 0$,

$$a = y(\pi) = \frac{4}{2} - 1 = 1.$$

Differentiate the explicit solution:

$$y\prime(x) = -\frac{4\cos x}{(2+\sin x)^2}.$$

At $x = \pi$, use $\cos\pi = -1$, giving

$$b = -\frac{4(-1)}{2^2} = 1.$$

As a check, substituting $y = 1$, $y\prime = 1$ into the original equation at pi gives $(2/2)\cdot 1 = -\cos\pi = 1$. Options (A) and (B) have the wrong solution value, while option (C) has the slope sign reversed.

Step 3: Conclusion The ordered pair is $\boxed{(a,b) = (1,1)}$. Hence, the correct option is (D).

Quick Tip to Solve: After integration, retain the product (y+1)(2+sin x); use the initial value before differentiating for the required slope.

10. Let X = {x ∈ N : 1 ≤ x ≤ 17} and Y = {ax + b: x ∈ X and a, b ∈ R, a > 0}. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to : [4 marks]

(A) 7

(B) 9

(C) -7

(D) -27

Correct Answer: (C) -7

Solution:

Concept:

- For observations transformed by $Y = aX + b$, the mean becomes $a\overline{X} + b$, while the variance becomes $a^2 \operatorname{Var}(X)$. Translation changes the mean but not the spread.
- For the equally weighted integers $1, 2, \ldots, n$, the mean is $(n+1)/2$ and the population variance is $(n^2 - 1)/12$. The condition $a > 0$ fixes the sign of the scale factor.

Step 1: Find the statistics of the original set The set contains all seventeen consecutive integers from one through seventeen, with each used once. Their mean is

$$\overline{X} = \frac{1 + 17}{2} = 9.$$

Their variance, with division by the number of observations, is

$$\operatorname{Var}(X) = \frac{17^2 - 1}{12} = 24.$$

For example, the sum-of-squares formula gives $\overline{X^2} = 105$, and $105 - 9^2 = 24$, confirming the same value. The sample-variance denominator is not appropriate here: the question asks for the variance of all elements of the set.

Step 2: Use variance first, then the mean The stated variance of the transformed elements gives

$$216 = a^2(24), \qquad a^2 = 9.$$

Because $a > 0$, select $a = 3$, not -3. Now impose the stated mean:

$$17 = 9a + b = 27 + b, \qquad b = -10.$$

Consequently, $a + b = 3 - 10 = -7$. Checking the mean returns $27 - 10 = 17$, and checking the variance returns $9 \cdot 24 = 216$. The choices $7, 9, -27$ do not equal the required sum; the number nine in particular is a^2, an intermediate value rather than the answer.

Step 3: Conclusion We obtain $a = 3$, $b = -10$, and $\boxed{a + b = -7}$. Hence, the correct option is (C).

Quick Tip to Solve: Use variance to determine the positive scale factor, then use the mean to determine the translation.

11. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S : [4 marks]

(A) contains more than two elements.

(B) contains exactly two elements.

(C) is a singleton.

(D) is an empty set.

Correct Answer: (B) contains exactly two elements.

Solution:

Concept:

- A linear system is inconsistent when row reduction produces an equation with zero left side and a nonzero right side. A zero determinant alone does not distinguish inconsistency from infinitely many solutions.
- Eliminating a variable with a coefficient independent of the parameter reduces the problem to two equations. Special parameter values must be tested in those equations before division.

Step 1: Eliminate x without dividing by the parameter The first equation gives $x = 1 + y/2 - z$. Substitute this expression into the third and second equations, respectively:

$$(2\lambda + 1)y = 6, \qquad -3y + 2(\lambda - 1)z = -10.$$

These transformations are reversible, since we only used the nonzero coefficient of x in the first equation. If $\lambda \neq -1/2$, the first reduced equation fixes y. If also $\lambda \neq 1$, the second fixes z, and the expression above then fixes x. Thus every parameter outside these two exceptional values gives a unique solution.

Step 2: Test both exceptional values for contradiction At $\lambda = -1/2$, the first reduced equation becomes $0 = 6$; hence there is no solution. At $\lambda = 1$, it gives $3y = 6$, so $y = 2$, whereas the second reduced equation becomes $-3y = -10$, requiring $y = 10/3$. These demands contradict one another, so this value also gives no solution. Therefore

$$S = \left\{-\frac{1}{2}, 1\right\}.$$

Both exceptional values have been checked rather than automatically declared inconsistent from a vanishing coefficient. There are no other candidates because the preceding elimination gives a solution whenever both reduced coefficients are nonzero. Thus S is neither empty nor a singleton, and it cannot contain more than two elements.

Step 3: Conclusion The set $S = \{-1/2, 1\}$ contains exactly two elements. Hence, the correct option is (B).

Quick Tip to Solve: After locating singular parameter values, substitute each into the reduced equations to distinguish a contradiction from a free variable.

12. Let A be a 2 × 2 real matrix with entries from {0, 1} and |A| ≠ 0. Consider the following two statements :

(P) If A ≠ I_2 , then |A| = –1

(Q) If |A| = 1, then tr(A) = 2,

where I_2 denotes 2 × 2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then : [4 marks]

(A) (P) is true and (Q) is false

(B) Both (P) and (Q) are false

(C) Both (P) and (Q) are true

(D) (P) is false and (Q) is true

Correct Answer: (D) (P) is false and (Q) is true

Solution:

Concept:

- Write $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $a, b, c, d \in \{0, 1\}$. Then $|A| = ad - bc$, and the trace is $a + d$.
- A universal implication is disproved by one admissible counterexample. To prove the other implication, exploit that the products ad and bc can only be zero or one.

Step 1: Test statement P with a nonidentity matrix Consider the admissible matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Its entries are zero or one, it is not I_2, and its determinant is $1 \cdot 1 - 1 \cdot 0 = 1 \neq 0$. Thus it meets every premise of statement (P), but its determinant is positive one rather than negative one. This single counterexample proves that (P) is false. The condition of being different from the identity does not force a binary matrix to have determinant minus one; adding an off-diagonal entry can preserve determinant one.

Step 2: Prove statement Q from the possible products Suppose $|A| = 1$. The determinant equation reads

$$ad - bc = 1.$$

Because each product is either zero or one, this equation is possible only if $ad = 1$ and $bc = 0$. The first equality forces both diagonal entries to equal one: $a = d = 1$. Hence

$$\mathrm{tr}(A) = a + d = 2.$$

The off-diagonal entries need not both vanish, but their product must vanish, which does not affect the trace. Thus statement (Q) is true for every allowed matrix. Options (A) and (C) incorrectly assert (P), and option (B) incorrectly denies (Q).

Step 3: Conclusion Statement (P) is false and statement (Q) is true. Hence, the correct option is (D).

Quick Tip to Solve: For zero-one matrices, reduce determinant conditions to the two binary products ad and bc; use a triangular counterexample for a universal claim.

13. If p(x) be a polynomial of degree three that has a local maximum value 8 at x = 1 and a local minimum value 4 at x = 2; then p(0) is equal to : [4 marks]

(A) 6

(B) 12

(C) -12

(D) -24

Correct Answer: (C) -12

Solution:

Concept:

- At a local maximum or minimum of a differentiable function, its first derivative vanishes. Thus the two stationary abscissae determine the roots of the quadratic derivative.
- If p is cubic and its stationary points occur at 1 and 2, write $p\prime(x) = k(x-1)(x-2)$. The two function values then determine the scale and integration constant.

Step 1: Construct and integrate the derivative The stationary conditions give $p\prime(1) = p\prime(2) = 0$. Since the polynomial has degree three, its derivative has degree two and a nonzero leading coefficient. Therefore

$$p\prime(x) = k(x^2 - 3x + 2), \qquad p(x) = k\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + C.$$

This form incorporates both stationary points without introducing three unrelated coefficients.

Step 2: Use the two values to determine the constants Substituting the local maximum and minimum values gives

$$p(1) = \frac{5k}{6} + C = 8, \qquad p(2) = \frac{2k}{3} + C = 4.$$

Subtract the second equation from the first:

$$\frac{k}{6} = 4 \quad \Longrightarrow \quad k = 24, \qquad C = 8 - 20 = -12.$$

Consequently $p(x) = 8x^3 - 36x^2 + 48x - 12$. As a check, $p\prime\prime(x) = 48x - 72$, which is negative at 1 and positive at 2, so the extrema have the required types. The other proposed constants would not satisfy both specified values simultaneously.

Step 3: Conclusion At zero all the positive powers vanish, so $p(0) = C = -12$. Hence, the correct option is (C).

Quick Tip to Solve: For a cubic with two known stationary points, factor its derivative first; the difference of the two function values eliminates the integration constant.

14. The domain of the function

f(x) = $\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a]\cup[a, \infty)$. Then a is equal to : [4 marks]

(A) $\frac{\sqrt{17}-1}{2}$

(B) $\frac{1+\sqrt{17}}{2}$

(C) $\frac{\sqrt{17}}{2}+1$

(D) $\frac{\sqrt{17}}{2}$

Correct Answer: (B) $\frac{1+\sqrt{17}}{2}$

Solution:

Concept:

- The real function $\sin^{-1} u$ is defined exactly when $-1 \le u \le 1$. The denominator x^2+1 is strictly positive for every real x.
- An expression involving only $|x|$ and x^2 is symmetric about zero. Put $t = |x| \ge 0$ to reduce the domain calculation to one quadratic inequality.

Step 1: Apply the inverse-sine restriction Here the numerator $|x|+5$ is positive and so is the denominator. Therefore the lower restriction on the argument is automatic, and the only remaining requirement is

$$\frac{|x|+5}{x^2+1} \le 1.$$

Multiplication by the positive denominator does not reverse the inequality. Hence

$$x^2 - |x| - 4 \ge 0.$$

There are no excluded denominator zeros to consider.

Step 2: Solve the quadratic in the absolute value Let $t = |x|$. The boundary values are obtained from

$$t^2 - t - 4 = 0, \qquad t = \frac{1 \pm \sqrt{17}}{2}.$$

The quadratic opens upwards, so it is nonnegative outside its two roots. Its negative root cannot be a value of t, and consequently

$$t \ge \frac{1+\sqrt{17}}{2}.$$

This yields two closed rays in x. The endpoints are included because the inverse-sine argument equals 1 there. Values between the rays give an argument greater than 1. The smaller positive numbers in (A) and (D) fail the required inequality, while (C) is larger than the actual boundary and would exclude valid inputs.

Step 3: Conclusion The domain is $\left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$. Thus $a = \frac{1+\sqrt{17}}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Check the sign of an inverse-trigonometric argument before solving two inequalities: positivity makes the lower inverse-sine bound automatic here.

15. The contrapositive of the statement

"If I reach the station in time, then I will catch the train" is : [4 marks]

(A) If I will catch the train, then I reach the station in time.

(B) If I do not reach the station in time, then I will not catch the train.

(C) If I will not catch the train, then I do not reach the station in time.

(D) If I do not reach the station in time, then I will catch the train.

Correct Answer: (C) If I will not catch the train, then I do not reach the station in time.

Solution:

Concept:

- For propositions P and Q, the contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$: reverse the order and negate both statements.
- The contrapositive is logically equivalent to the original implication. The converse $Q \Rightarrow P$ and inverse $\neg P \Rightarrow \neg Q$ are different constructions.

Step 1: Identify the hypothesis and conclusion Set P equal to "I reach the station in time" and Q equal to "I will catch the train". The given sentence has the form $P \Rightarrow Q$. Its hypothesis is arrival in time, whereas catching the train is its conclusion. We must negate the conclusion first, rather than simply negating the words in their existing order. Thus $\neg Q$ says "I will not catch the train", and $\neg P$ says "I do not reach the station in time". No extra condition about the journey is introduced.

Step 2: Reverse and negate the implication Using the rule gives $\neg Q \Rightarrow \neg P$: "If I will not catch the train, then I do not reach the station in time." This matches (C) exactly. Option (A) reverses the original without negating it, so it is the converse. Option (B) negates both parts without reversing them, so it is the inverse. Option (D) negates only the hypothesis and retains the original conclusion, and is also different. The logical equivalence can be checked from the only false case of an implication: the original fails when P is true and Q false, and the contrapositive fails under precisely the same assignment.

Step 3: Conclusion The required contrapositive is: "If I will not catch the train, then I do not reach the station in time. Hence, the correct option is (C).

Quick Tip to Solve: Write an implication as P implies Q before working with sentences; the contrapositive always begins with the negation of Q.

16. A line parallel to the straight line 2x – y = 0 is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to : [4 marks]

(A) 5

(B) 6

(C) 10

(D) 8

Correct Answer: (B) 6

Solution:

Concept:

- The slope of a tangent to a differentiable implicit curve is found by differentiating its equation with respect to x, treating y as a function of x.
- Parallel lines have the same slope. The contact point must satisfy both the slope condition and the original hyperbola equation; squares can avoid an unnecessary choice of sign.

Step 1: Find the tangent-slope condition The line $2x - y = 0$ can be written $y = 2x$, so its slope is 2. Differentiating the hyperbola gives

$$\frac{x}{2} - y\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = \frac{x}{2y}.$$

At a contact point with the specified finite slope,

$$\frac{x_1}{2y_1} = 2 \quad \Longrightarrow \quad x_1 = 4y_1.$$

Here $y_1 \neq 0$; a point with $y_1 = 0$ would give a vertical tangent, which is not parallel to the given line.

Step 2: Substitute the contact relation into the curve Because the contact point lies on the hyperbola,

$$\frac{(4y_1)^2}{4} - \frac{y_1^2}{2} = 1.$$

It follows that

$$\frac{7}{2}y_1^2 = 1, \qquad y_1^2 = \frac{2}{7}, \qquad x_1^2 = \frac{32}{7}.$$

The required combination is therefore

$$x_1^2 + 5y_1^2 = \frac{32}{7} + \frac{10}{7} = 6.$$

There are two possible contact points with opposite coordinate signs, but this expression has the same value for both. Thus choosing either branch does not change the result. Values 5, 10 and 8 do not satisfy the combined curve and slope constraints.

Step 3: Conclusion The value is $x_1^2 + 5y_1^2 = 6$. Hence, the correct option is (B).

Quick Tip to Solve: When the requested answer contains coordinate squares, combine the slope relation with the conic equation directly instead of solving separately for signed coordinates.

17. If the tangent to the curve y = x + sin y at a point (a, b) is parallel to the line joining $(0, \frac{3}{2})$ and $(\frac{1}{2}, 2)$, then : [4 marks]

(A) b = a

(B) |b - a| = 1

(C) $b = \frac{\pi}{2}$ + a

(D) |a + b| = 1

Correct Answer: (B) |b - a| = 1

Solution:

Concept:

- The slope of a line joining two points is the change in its vertical coordinate divided by the change in its horizontal coordinate.
- Implicit differentiation of $\sin y$ with respect to x gives $\cos y\, dy/dx$. A point of tangency must also satisfy the original curve equation.

Step 1: Match the tangent slope with the given line The slope of the line through the two specified points is

$$m = \frac{2 - 3/2}{1/2 - 0} = 1.$$

Differentiate $y = x + \sin y$ implicitly:

$$\frac{dy}{dx} = 1 + \cos y \frac{dy}{dx}, \qquad (1 - \cos y)\frac{dy}{dx} = 1.$$

At (a, b), the tangent is parallel to that line, so $dy/dx = 1$. Substitution gives $1 - \cos b = 1$, or $\cos b = 0$. The slope condition therefore leads to a trigonometric constraint on the ordinate, not directly to an equality between the coordinates.

Step 2: Use the curve equation at the point Since $\cos b = 0$, the identity $\sin^2 b + \cos^2 b = 1$ gives $\sin b = \pm 1$. Also (a, b) lies on the curve, hence

$$b = a + \sin b, \qquad b - a = \sin b = \pm 1.$$

Thus $|b - a| = 1$. Option (A) would require $\sin b = 0$, which contradicts the slope condition. Option (C) would require $b - a = \pi/2$, exceeding the possible magnitude of a sine. The slope condition places no general restriction $|a + b| = 1$, so (D) does not follow. For example, $b = \pi/2$ and $a = \pi/2 - 1$ satisfy the curve and the slope, but their sum is $\pi - 1$.

Step 3: Conclusion The coordinates satisfy $|b - a| = 1$. Hence, the correct option is (B).

Quick Tip to Solve: After implicit differentiation gives a condition on the ordinate, return to the original curve to obtain the required relation between the two coordinates.

18. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also passes through the point : [4 marks]

(A) (0, 6, –2)

(B) (–2, 0, 1)

(C) (0, –6, 2)

(D) (2, 0 –1)

Correct Answer: (B) (–2, 0, 1)

Solution:

Concept:

- A plane contains the direction joining any two of its points. It also contains every direction parallel to it, so two nonparallel such vectors determine its normal by a cross product.
- The line $2x = 3y,\ z = 1$ has constant z; writing $x = 3t, y = 2t$ shows that its direction vector is $(3, 2, 0)$.

Step 1: Construct two directions in the plane Let $A = (1, 2, 1)$ and $B = (2, 1, 2)$. Their displacement vector is

$$\overrightarrow{AB} = (1, -1, 1).$$

A direction parallel to the specified line is $\boldsymbol{d} = (3, 2, 0)$. These vectors are not parallel, so a normal to the plane is

$$\boldsymbol{n} = (1, -1, 1) \times (3, 2, 0) = (-2, 3, 5).$$

Indeed its dot product with each direction vanishes: $-2-3+5 = 0$ and $-6+6 = 0$. This independently checks both signs in the cross product.

Step 2: Form the equation and test the candidate points Using A, the plane equation is

$$-2(x - 1) + 3(y - 2) + 5(z - 1) = 0, \qquad -2x + 3y + 5z = 9.$$

At $(-2, 0, 1)$, its left side is $4 + 0 + 5 = 9$, so that point belongs to the plane. The other candidates give different values: at $(0, 6, -2)$ it is $18 - 10 = 8$; at $(0, -6, 2)$ it is $-18 + 10 = -8$; and at $(2, 0, -1)$ it is $-4-5 = -9$. None equals 9. As a final consistency check, the second given point gives $-4+3+10 = 9$, as required. The fixed coordinate $z = 1$ describes the given line, not every point of the plane.

Step 3: Conclusion The plane also passes through $(-2, 0, 1)$. Hence, the correct option is (B).

Quick Tip to Solve: Treat a line given by two equations parametrically first; a constant coordinate contributes zero to its direction vector, not its constant value.

19. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x + y) + (x^2+xy+y^2) + (x^3+x^2y + xy^2+y^3) + ...$ [4 marks]

(A) $\frac{x+y-xy}{(1+x)(1+y)}$

(B) $\frac{x+y-xy}{(1-x)(1-y)}$

(C) $\frac{x+y+xy}{(1+x)(1+y)}$

(D) $\frac{x+y+xy}{(1-x)(1-y)}$

Correct Answer: (B) $\frac{x+y-xy}{(1-x)(1-y)}$

Solution:

Concept:

- For $|r| < 1$, a geometric series satisfies $\sum_{n=1}^{\infty} r^n = r/(1-r)$. The bounds on both variables ensure convergence.
- The homogeneous sum of degree n is $x^n + x^{n-1}y + \cdots + y^n = (x^{n+1} - y^{n+1})/(x-y)$, valid because $x \neq y$.

Step 1: Express a general bracket Let S denote the required sum. The first bracket has degree one, the next degree two, and so on. Multiplying a degree-n bracket by $x - y$ cancels all its intermediate terms, leaving $x^{n+1} - y^{n+1}$. Thus the index starts at $n = 1$, not zero; a zero-degree bracket would introduce an unwanted constant term.

Step 2: Sum the two geometric series Apply the identity separately to the powers of x and y:

$$S = \frac{1}{x-y}\left(\frac{x^2}{1-x} - \frac{y^2}{1-y}\right).$$

Both series converge absolutely. Indeed, the sum of the absolute values of all monomials is bounded by the product of the geometric series in $|x|$ and $|y|$, so this subtraction and summation are legitimate.

Step 3: Factor and compare the options Put the fractions over a common denominator:

$$S = \frac{x^2 - y^2 - x^2y + xy^2}{(x-y)(1-x)(1-y)} = \frac{(x-y)(x+y-xy)}{(x-y)(1-x)(1-y)}.$$

Cancel $x - y$. The denominator contains minus signs because the geometric ratios are x, y, rather than $-x, -y$; this rules out (A) and (C). Option (D) has the wrong sign of the product term in the numerator.

Step 4: Conclusion The sum is $\frac{x+y-xy}{(1-x)(1-y)}$. Hence, the correct option is (B).

Quick Tip to Solve: Write each homogeneous bracket as a difference of powers; check the starting index before summing the geometric series.

20. Let P(h, k) be a point on the curve
$y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$.
Then the equation of the normal to the curve at P is : [4 marks]

(A) $x - 3y - 11 = 0$

(B) $x - 3y + 22 = 0$

(C) $x + 3y - 62 = 0$

(D) $x + 3y + 26 = 0$

Correct Answer: (D) $x + 3y + 26 = 0$

Solution:

Concept:

- The distance of (x, y) from $3x - y - 3 = 0$ is $|3x - y - 3|/\sqrt{10}$. Minimise this expression after substituting the curve.
- The tangent slope is dy/dx; when it is nonzero, the normal slope is its negative reciprocal.

Step 1: Minimise the distance to the line On the parabola $y = x^2 + 7x + 2$, the distance becomes

$$d(x) = \frac{|3x - (x^2 + 7x + 2) - 3|}{\sqrt{10}} = \frac{|x^2 + 4x + 5|}{\sqrt{10}}.$$

Complete the square: $x^2 + 4x + 5 = (x + 2)^2 + 1$. This is strictly positive for every real x, so the absolute value can be removed. It also shows that the line never meets the parabola. The smallest possible numerator is therefore 1, attained uniquely at $x = -2$. This proves a global minimum, rather than merely finding a stationary distance.

Step 2: Locate the point and find the normal slope Here $h = -2$, and substitution into the original curve gives

$$k = (-2)^2 + 7(-2) + 2 = -8.$$

The derivative is $y\prime = 2x + 7$, so at $P = (-2, -8)$ the tangent slope is 3. Accordingly the normal slope is $-1/3$. The tangent is parallel to the given line at the nearest point, consistent with the perpendicular segment that measures the distance.

Step 3: Write the line through the nearest point Use the point-slope form for the normal:

$$y + 8 = -\frac{1}{3}(x + 2) \quad \Longrightarrow \quad x + 3y + 26 = 0.$$

Options (A) and (B) have slope $1/3$, which is not perpendicular to the tangent. Option (C) has the correct slope but fails the point test: at $(-2, -8)$, its left side is -88, not zero. Only (D) has both the required slope and the required point.

Step 4: Conclusion The required normal is $\boxed{x + 3y + 26 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: Complete the square in the point-to-line distance to prove the nearest point is a global minimum, then take the negative reciprocal tangent slope.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The integral $\int_0^2 ||x-1|-x|\,dx$

is equal to_____. [4 marks]

Solution:

Concept:

- An absolute value must be resolved according to the sign of its argument on each integration interval.
- For a nested modulus, first resolve the inner expression and then find the zeros of the resulting outer argument.

Step 1: Find every change of formula Write $I = \int_0^2 \big||x-1|-x\big|\,dx$. The inner modulus changes form at $x = 1$. For $0 \le x \le 1$, it equals $1 - x$, so the outer expression is $|1 - 2x|$. This introduces a second breakpoint at $x = 1/2$. For $1 \le x \le 2$, the inner modulus equals $x - 1$, hence the argument of the outer modulus is -1, whose modulus is 1. No further zero occurs on this second interval.

Step 2: Resolve the integrand on three intervals The complete piecewise expression is

$$\big||x-1|-x\big| = \begin{cases} 1-2x, & 0 \le x \le \frac{1}{2}, \\ 2x-1, & \frac{1}{2} \le x \le 1, \\ 1, & 1 \le x \le 2. \end{cases}$$

The formulas agree at the two joining points. In particular, resolving just the inner modulus is insufficient: on the first half of the unit interval the outer argument is positive, whereas on its second half it is negative.

Step 3: Integrate each nonnegative piece Add the three integrals with their respective limits:

$$\begin{aligned} I &= [x - x^2]_0^{1/2} + [x^2 - x]_{1/2}^1 + [x]_1^2 \\ &= \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}. \end{aligned}$$

As a geometric check, the first two contributions are triangular areas of base $1/2$ and height 1; the last is the area of a unit rectangle. The integrand never exceeds 1, so the result is also consistent with the interval length 2.

Step 4: Conclusion The integral equals $\boxed{\frac{3}{2} = 1.50}$.

Quick Tip to Solve: For nested moduli, add breakpoints created by the outer expression as well as the zero of the inner expression.

22. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\left|\vec{a}-\vec{b}\right|^2 + |\vec{a}-\vec{c}|^2 = 8$.

Then $\left|\vec{a}+2\vec{b}\right|^2 + |\vec{a}+2\vec{c}|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The squared norm satisfies $|\mathbf{u}-\mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2\mathbf{u}\cdot\mathbf{v}$.
- Every given vector is a unit vector, so each squared norm is 1. Expand the requested squared sums with the same scalar products.

Step 1: Extract the scalar-product sum Set $s = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$. Expanding both given terms and using the unit lengths gives

$$|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = (2-2\vec{a}\cdot\vec{b}) + (2-2\vec{a}\cdot\vec{c}) = 4-2s.$$

Since this expression equals 8, we obtain $4-2s=8$, hence $s=-2$. The negative sign reflects that the other two vectors point against $\vec{a}$, not along it.

Step 2: Expand the required quantity For the coefficient 2, remember to square the coefficient in the norm term and double it in the cross term:

$$|\vec{a}+2\vec{b}|^2 = 1+4+4\vec{a}\cdot\vec{b}, \qquad |\vec{a}+2\vec{c}|^2 = 1+4+4\vec{a}\cdot\vec{c}.$$

Adding these identities eliminates the need to determine the two scalar products separately. The required expression is $10+4s$, which becomes $10-8=2$.

Step 3: Check the extremal geometry A scalar product of two unit vectors cannot be less than -1. Their sum is -2, so both scalar products must equal -1. Consequently $\vec{b}=\vec{c}=-\vec{a}$. Each vector in the required expression then equals $-\vec{a}$, with squared length 1, giving $1+1=2$ again. Equivalently, the original two squared differences each attain their maximum value 4. This independently verifies the expansion and the sign of the cross terms.

Step 4: Conclusion The required numerical value is $\boxed{2}$.

Quick Tip to Solve: Expand both the given and required norms using the same dot-product sum; a maximum distance between unit vectors signals opposite directions.

23. If $\lim\limits_{x\to1}\frac{x+x^2+x^3+\ldots+x^n-n}{x-1}$ = 820, ($n \in N$) then the value of n is equal to _____. [4 marks]

Solution:

Concept:

- For a positive integer k, $(x^k-1)/(x-1) = 1+x+\cdots+x^{k-1}$ whenever $x\neq1$. Its limit at 1 is k.
- A finite sum may be evaluated term by term in a limit. The first n positive integers sum to $n(n+1)/2$.

Step 1: Separate the numerator into vanishing terms There are exactly n powers in the numerator. Therefore subtracting n is the same as subtracting 1 from each power:

$$x + x^2 + \cdots + x^n - n = \sum_{k=1}^{n} (x^k - 1).$$

This is a finite sum because n is fixed when x approaches 1. Direct substitution would produce $0/0$, so we first cancel the common factor $x - 1$ in each term for $x \neq 1$.

Step 2: Evaluate the limit algebraically The factorisation of a difference of powers gives

$$\lim_{x \to 1} \frac{x^k - 1}{x - 1} = \lim_{x \to 1} (1 + x + \cdots + x^{k-1}) = k.$$

All k terms tend to 1; hence the required limit is

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = 820.$$

This argument needs no infinite-series exchange and avoids differentiating a quotient at its undefined point.

Step 3: Solve for the natural number Multiply by 2 and rearrange to obtain

$$n^2 + n - 1640 = 0 = (n - 40)(n + 41).$$

The two algebraic candidates are 40 and -41. The latter is excluded because $n \in \mathbb{N}$. Substitution verifies $40 \cdot 41/2 = 820$. As a further check, if $f(x) = x + x^2 + \cdots + x^n$, the original quotient is $(f(x) - f(1))/(x - 1)$, so its limit is $f\prime(1) = 1 + 2 + \cdots + n$, agreeing with the factorisation method.

Step 4: Conclusion The required natural number is $\boxed{n = 40}$.

Quick Tip to Solve: Recognise a power sum minus its number of terms as a sum of differences $x^k - 1$; each difference quotient tends to its exponent.

24. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is _____. [4 marks]

Solution:

Concept:

- Dictionary order compares the first unequal letter. Count words with an earlier letter at each position while keeping the earlier prefix fixed.
- All six letters of MOTHER are distinct. After fixing a prefix, the remaining r distinct letters can be arranged in $r!$ ways.

Step 1: Count words before the initial M The alphabetical list of letters is E, H, M, O, R, T. The first letter of a word before MOTHER can be E or H. For each of these two choices the remaining five

letters can occur in any order, giving $2 \cdot 5! = 240$ words. These blocks are disjoint because their first letters differ, and every word in either block precedes every word starting with M.

Step 2: Count earlier choices along the prefix Keep M fixed. The remaining letters are E, H, O, R, T; before the second letter O there are two possibilities E and H, giving $2 \cdot 4! = 48$. Keep MO fixed: among E, H, R, T, the three letters E, H, R precede T, contributing $3 \cdot 3! = 18$. Keep MOT fixed: among E, H, R, only E precedes H, giving $1 \cdot 2! = 2$. These counts refer to the first position at which a word differs from MOTHER, so none is counted twice.

Step 3: Finish the rank calculation After MOTH, the unused letters are E and R. The next required letter E is already the smaller one, so this position contributes zero. Once MOTHE is fixed, R is forced and also contributes zero. The total number of words strictly before MOTHER is thus

$$2(5!) + 2(4!) + 3(3!) + 1(2!) + 0(1!) + 0(0!) = 240 + 48 + 18 + 2 = 308.$$

Dictionary positions start at 1, whereas this calculation counted only earlier words. Add the word MOTHER itself once: its position is $308 + 1 = 309$. No factorial correction for repeated letters is required, since none repeats.

Step 4: Conclusion The position of MOTHER in the dictionary list is $\boxed{309}$.

Quick Tip to Solve: At each letter count unused smaller letters, multiply by the factorial of the remaining positions, then add 1 to convert an earlier-word count to rank.

25. The number of integral values of k for which the line, 3x + 4y = k intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is _____. [4 marks]

Solution:

Concept:

- A line cuts a circle at two distinct points exactly when its perpendicular distance from the centre is strictly less than the radius.
- Completing squares gives the standard circle form $(x-a)^2 + (y-b)^2 = R^2$. The distance from (a, b) to $Ax + By + C = 0$ is $|Aa + Bb + C|/\sqrt{A^2 + B^2}$.

Step 1: Identify the centre and radius Complete both squares in the given equation:

$$x^2 - 2x + y^2 - 4y + 4 = 0 \quad \Longrightarrow \quad (x-1)^2 + (y-2)^2 = 1.$$

Thus the centre is $C = (1, 2)$ and the radius is $R = 1$. The constants added during completing the squares must be balanced; the radius is not 2.

Step 2: Apply the strict secant condition Write the line as $3x + 4y - k = 0$. Its perpendicular distance from the centre is

$$d = \frac{|3(1) + 4(2) - k|}{\sqrt{3^2 + 4^2}} = \frac{|11 - k|}{5}.$$

Two distinct intersections require $d < 1$, so

$$|11 - k| < 5 \quad \Longleftrightarrow \quad 6 < k < 16.$$

Equality would produce a tangent and only one intersection, so neither boundary is included.

Step 3: Count the admissible integers The allowed integral values are $7, 8, 9, 10, 11, 12, 13, 14, 15$. Their number is

$$15 - 7 + 1 = 9.$$

Values outside this interval make the perpendicular distance exceed the radius, while $k = 6$ and $k = 16$ give tangents.

Step 4: Conclusion The required number of integral values of k is $\boxed{9}$.

Quick Tip to Solve: Use a strict distance inequality for two distinct circle intersections; including the tangent endpoints overcounts by two.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Two identical strings X and Z made of same material have tension T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_X/T_Z is [4 marks]

(A) 2.25

(B) 0.44

(C) 1.25

(D) 1.5

Correct Answer: (A) 2.25

Solution:

Concept:

- The fundamental frequency of a string fixed at both ends is $f = (1/2L)\sqrt{T/\mu}$, where L is its vibrating length and μ its mass per unit length.
- Identical strings have the same L and μ. Hence their fundamental-frequency ratio is the square root of their tension ratio.

Step 1: Cancel the identical string parameters For strings X and Z, write

$$f_X = \frac{1}{2L}\sqrt{\frac{T_X}{\mu}}, \qquad f_Z = \frac{1}{2L}\sqrt{\frac{T_Z}{\mu}}.$$

Taking the ratio eliminates both the common length and the common linear density:

$$\frac{f_X}{f_Z} = \sqrt{\frac{T_X}{T_Z}}.$$

The same material alone would not fix linear density, but the statement that the strings are identical

supplies the equal geometry as well.

Step 2: Square the measured frequency ratio Substitute the given frequencies and square:

$$\frac{T_X}{T_Z} = \left(\frac{450}{300}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4} = 2.25.$$

The tension ratio is dimensionless, as a ratio of like quantities must be. Since X has the higher frequency, its tension must be the greater one.

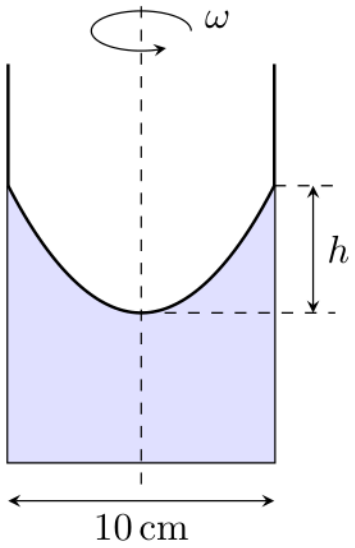
Step 3: Check the alternative ratios The value 1.5 is the frequency ratio, not the tension ratio; the square is essential. The value 0.44 is approximately the inverse tension ratio and would incorrectly make the higher-frequency string less taut. The value 1.25 does not satisfy the frequency law: its square root is not $450/300$.

Step 4: Conclusion $\boxed{T_X/T_Z = 2.25}$. Hence, the correct option is (A).

Quick Tip to Solve: For identical strings, first form the frequency ratio and then square it; frequency varies as the square root of tension.

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27. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h (in cm) of liquid at the centre of vessel and at the side will be : [4 marks]



(A) $\frac{2\omega^2}{25g}$

(B) $\frac{5\omega^2}{2g}$

(C) $\frac{25\omega^2}{2g}$

(D) $\frac{2\omega^2}{5g}$

Correct Answer: (C) $\frac{25\omega^2}{2g}$

Solution:

Concept:

- In steady rigid-body rotation, liquid at radius r moves with speed ωr; its pressure gradients satisfy $\partial p/\partial r = \rho\omega^2 r$ and $\partial p/\partial z = -\rho g$.
- The free surface has constant atmospheric pressure, so its slope is $dz/dr = \omega^2 r/g$. The density cancels from this relation.

Step 1: Relate surface slope to rotation Let z measure height upward and r distance from the vertical axis. Along the free surface $dp = 0$, giving

$$0 = \rho\omega^2 r\, dr - \rho g\, dz \quad \Longrightarrow \quad dz = \frac{\omega^2 r}{g}\, dr.$$

The positive slope means the liquid surface rises toward the cylindrical wall, agreeing with the figure. It is the radial acceleration, not a constant acceleration everywhere, that determines the curved surface.

Step 2: Integrate from the centre to the wall At the centre $r = 0$; at the wall $r = R$. Therefore the height difference is

$$h = z(R) - z(0) = \int_0^R \frac{\omega^2 r}{g}\, dr = \frac{\omega^2 R^2}{2g}.$$

The factor $1/2$ arises from integration. The absolute starting level and total liquid volume do not enter the difference between these two heights.

Step 3: Use consistent centimetre units With $R = 5\,\text{cm}$, use g in $\text{cm}\,\text{s}^{-2}$ when expressing h in centimetres:

$$h = \frac{25\omega^2}{2g}\ \text{cm}.$$

Equivalently, using SI g gives $h = 0.00125\omega^2/g$ metres. The listed expression in (C) is the centimetre-unit form. The other options have the wrong dependence on the radius: the numerator must contain $R^2 = 25$, not its reciprocal or only one power of R.

Step 4: Conclusion $\boxed{h = 25\omega^2/(2g)}$ in consistent centimetre units. Hence, the correct option is (C).

Quick Tip to Solve: For a rotating liquid use the difference $\omega^2 R^2/(2g)$, and keep the units of radius, height and gravitational acceleration consistent.

28. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be [4 marks]

(A) $FA^{-1}V^0$

(B) FA^2V^{-1}

(C) FA^2V^{-2}

(D) FA^2V^{-3}

Correct Answer: (A) $FA^{-1}V^0$

Solution:

Concept:

- Young's modulus is longitudinal stress divided by longitudinal strain. Stress is force per cross-sectional area; strain is extension divided by original length.
- Strain is dimensionless, so the dimensions of Young's modulus are precisely those of force divided by area, irrespective of the chosen base quantities.

Step 1: Express the defining relation For a wire under a longitudinal force F_ℓ, with area A_c, original length L, and extension ΔL,

$$Y = \frac{F_\ell/A_c}{\Delta L/L} = \frac{F_\ell L}{A_c \Delta L}.$$

The symbols for the applied force and cross-section here denote physical quantities, while the question uses F, A, V as the selected fundamental dimensions. Since L and ΔL have equal dimensions, their ratio contributes no dimensional factor.

Step 2: Rewrite in the chosen fundamental units The defining relation immediately gives

$$[Y] = [F][A]^{-1} = F^1A^{-1}V^0.$$

No speed factor is needed. This can also be checked with ordinary dimensions: force per area is $(MLT^{-2})/L^2 = ML^{-1}T^{-2}$, which is the familiar pressure dimension. Changing the basis of fundamental units changes the notation, not the physical dimensions of modulus.

Step 3: Eliminate the other choices Options (B), (C), and (D) all contain A^2 instead of A^{-1}, and negative powers of speed cannot turn any of them into force per unit area. Explicitly, FA^2V^{-n} has ordinary dimensions $ML^{5-n}T^{n-2}$, for $n = 1, 2, 3$; none gives $ML^{-1}T^{-2}$. Only (A) matches both length and time powers.

Step 4: Conclusion $\boxed{[Y] = FA^{-1}V^0}$. Hence, the correct option is (A).

Quick Tip to Solve: Start with the definition stress divided by strain: a modulus always has pressure dimensions because strain carries no units.

29. A plane electromagnetic wave, has frequency of 2.0 $\times$ 10^{10} Hz and its energy density is 1.02 $\times$ 10^{-8} J/m^3 in vacuum. The amplitude of the magnetic field of the wave is close to
($\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$ and speed of light = 3 $\times$ 10^8 ms^{-1}) [4 marks]

(A) 190 nT

(B) 150 nT

(C) 160 nT

(D) 180 nT

Correct Answer: (C) 160 nT

Solution:

Concept:

- For a sinusoidal electromagnetic wave, the time-averaged total energy density is $\overline{u} = B_0^2/(2\mu_0) = \varepsilon_0 E_0^2/2$, where the subscript 0 denotes field amplitude.
- The electric and magnetic contributions to the energy density are equal. The vacuum constants obey $\mu_0\varepsilon_0 c^2 = 1$.

Step 1: Determine the magnetic permeability From the supplied Coulomb constant $k_e = 1/(4\pi\varepsilon_0) = 9 \times 10^9$ and $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$\mu_0 = \frac{1}{\varepsilon_0 c^2} = \frac{4\pi k_e}{c^2} = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}.$$

The wave frequency does not enter the energy-density-to-amplitude relation in vacuum. It would be useful for wavelength, but is unnecessary for this calculation.

Step 2: Obtain the amplitude from the energy density The stated wave energy density is used as its time average. Because $\overline{B^2} = B_0^2/2$, solve

$$B_0 = \sqrt{2\mu_0\overline{u}} = \sqrt{2(4\pi \times 10^{-7})(1.02 \times 10^{-8})}.$$

Thus

$$B_0^2 \simeq 2.5635 \times 10^{-14}\,\mathrm{T}^2, \qquad B_0 \simeq 1.601 \times 10^{-7}\,\mathrm{T} = 160.1\,\mathrm{nT}.$$

The amplitude is larger than the root-mean-square magnetic field by $\sqrt{2}$.

Step 3: Compare the listed values The calculated value is closest to $160\,\mathrm{nT}$. A value of $150\,\mathrm{nT}$ gives average density about $8.95 \times 10^{-9}\,\mathrm{J\,m^{-3}}$, too small. Values 180 and $190\,\mathrm{nT}$ give about 1.29×10^{-8} and $1.44 \times 10^{-8}\,\mathrm{J\,m^{-3}}$, respectively, too large. Keeping both field contributions and the averaging factor gives the listed match.

Step 4: Conclusion The magnetic-field amplitude is approximately $\boxed{160\,\mathrm{nT}}$. Hence, the correct option is (C).

Quick Tip to Solve: For average wave energy density use $B_0 = \sqrt{2\mu_0\overline{u}}$; omitting the 2 confuses amplitude with the rms field.

30. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close is [4 marks]

(A) 1.27 μm

(B) 2.05 μm

(C) 2.87 nm

(D) 2 nm

Correct Answer: (A) 1.27 μm

Solution:

Concept:

- For slit separation d, screen distance D, and a small transverse displacement y, the path difference is $\delta = d\sin\theta \simeq dy/D$.
- Bright interference occurs when $\delta = m\lambda$, with integer order m. The fringe width is $\beta = \lambda D/d$.

Step 1: Convert the geometrical data to SI units The separation is $d = 1\,\text{mm} = 10^{-3}\,\text{m}$, the screen distance is $D = 100\,\text{cm} = 1\,\text{m}$, and the displacement is $y = 1.27\,\text{mm} = 1.27 \times 10^{-3}\,\text{m}$. Since $y/D = 0.00127 \ll 1$, the usual small-angle expression for the path difference is accurate. The screen displacement itself is not the path difference between the two rays.

Step 2: Calculate the path difference Apply the geometry:

$$\delta \simeq \frac{dy}{D} = \frac{(10^{-3})(1.27 \times 10^{-3})}{1} = 1.27 \times 10^{-6}\,\text{m}.$$

Since $1\,\mu\text{m} = 10^{-6}\,\text{m}$, this is $1.27\,\mu\text{m}$. The factors of 10^{-3} from the two millimetre quantities must both be retained.

Step 3: Check the bright-fringe order and alternatives The wavelength is $632.8\,\text{nm} = 0.6328\,\mu\text{m}$. The inferred order is

$$m \simeq \frac{1.27}{0.6328} \simeq 2.007.$$

This identifies the second bright fringe, whose exact displacement would be $1.2656\,\text{mm}$, consistent with the rounded $1.27\,\text{mm}$. Its path difference $2\lambda = 1.2656\,\mu\text{m}$ also rounds near the result. The alternative $2.05\,\mu\text{m}$ disagrees with the geometry, while the nanometre options are hundreds of times too small.

Step 4: Conclusion The path difference is close to $\boxed{1.27\,\mu\text{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: Calculate dy/D in one consistent unit system, then use δ/λ to verify the integer bright-fringe order.

31. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by : [4 marks]

(A) $v = \sqrt{\frac{2}{3}}u$

(B) $v = \frac{u}{\sqrt{3}}$

(C) $v = \frac{u}{\sqrt{2}}$

(D) $v = \frac{1}{\sqrt{6}}u$

Correct Answer: (C) $v = \frac{u}{\sqrt{2}}$

Solution:

Concept:

- Linear momentum is conserved separately along perpendicular axes because the two-particle system has no external impulse.
- In a perfectly elastic collision, total kinetic energy is also conserved. The initially stationary particle can acquire both horizontal and vertical velocity components.

Step 1: Resolve the final momentum Let the velocity of the particle of mass $3m$ after collision be $V_x\hat{i}+V_y\hat{j}$. The smaller particle finally has velocity $v\hat{j}$, so its horizontal momentum is zero. Conservation in the two directions gives

$$mu = 3mV_x, \qquad 0 = mv + 3mV_y.$$

Thus $V_x = u/3$ and $V_y = -v/3$. Its speed squared is therefore

$$V^2 = V_x^2 + V_y^2 = \frac{u^2+v^2}{9}.$$

The minus sign in the vertical component is necessary to keep the total vertical momentum zero.

Step 2: Apply kinetic-energy conservation Use the initial energy of the moving particle and the final energies of both particles:

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)\frac{u^2+v^2}{9}.$$

Cancelling $m/2$ and multiplying by three gives

$$3u^2 = 3v^2 + u^2 + v^2, \qquad 2u^2 = 4v^2.$$

Therefore $v^2 = u^2/2$. This is a two-dimensional collision; applying the familiar one-dimensional final-velocity formula would impose an incorrect direction constraint. Substituting any of the other three proposed speeds violates this simultaneous momentum-and-energy equation.

Step 3: Conclusion The speed is $v = u/\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: For an elastic collision with a perpendicular final velocity, resolve momentum first and substitute the other particle's speed squared into energy conservation.

32. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is : [4 marks]

(A) 11

(B) 20

(C) 15

(D) 13

Correct Answer: (C) 15

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, $U = (f/2)nRT$, where n is its amount in moles.
- A rigid diatomic molecule has three translational and two rotational degrees of freedom. A monatomic gas has only three translational degrees of freedom.

Step 1: Assign degrees of freedom to each gas Oxygen is diatomic. Since its bond is specified to be rigid, vibration of the bond is excluded, while rotation about the two axes perpendicular to the bond remains possible. Consequently $f_{O_2} = 5$. Argon is monatomic, so it has no molecular rotational or vibrational contribution in this ideal-gas model and $f_{Ar} = 3$. Do not use the same molar internal energy for both components merely because their temperatures are equal.

Step 2: Add the internal energies Both components are at the common temperature T, and the internal energy of an ideal mixture is the sum of their separate internal energies. Thus

$$U_{O_2} = \frac{5}{2}(3)RT = \frac{15}{2}RT,$$

$$U_{Ar} = \frac{3}{2}(5)RT = \frac{15}{2}RT.$$

Adding gives

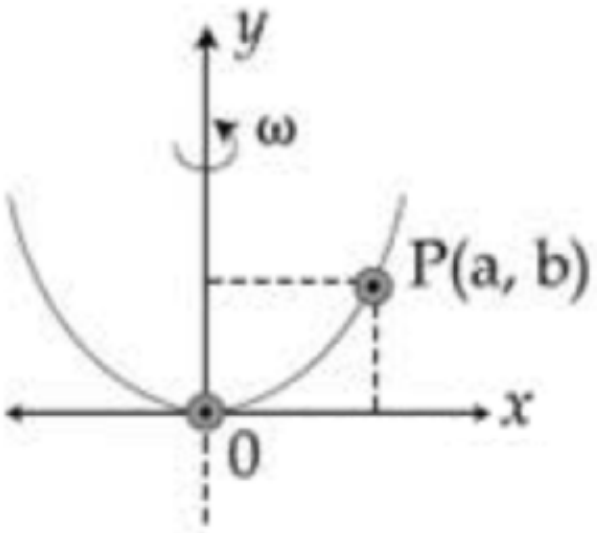
$$U = \left(\frac{15}{2} + \frac{15}{2}\right)RT = 15RT.$$

The numerical coefficient is therefore 15. Counting all eight moles as monatomic would give 12 rather than the required result; counting them all as rigid diatomic would give 20, which is the tempting option (B). The values 11 and 13 do not follow from the stated amounts and active degrees of freedom.

Step 3: Conclusion The total internal energy in units of RT is 15. Hence, the correct option is (C).

Quick Tip to Solve: Compute $nf/2$ separately for each component, then add; a common temperature does not mean a common number of degrees of freedom.

33. A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) : [4 marks]



(A) $2\sqrt{2gC}$

(B) $2\sqrt{gC}$

(C) $\sqrt{\frac{2gC}{ab}}$

(D) $\sqrt{\frac{2g}{C}}$

Correct Answer: (A) $2\sqrt{2gC}$

Solution:

Concept:

- For a bead stationary relative to a smooth rotating wire, the sum of gravity and centrifugal force has zero component along the wire.
- The tangent slope of $y = 4Cx^2$ is $dy/dx = 8Cx$. The wire rotates about the vertical axis, so the distance from that axis at P is a.

Step 1: Write tangential equilibrium Work in the frame rotating with the wire. At $P(a, b)$, centrifugal force is horizontal and outward with magnitude $m\omega^2 a$, while gravity is vertically downward with magnitude mg. The normal reaction is perpendicular to the wire and therefore has no tangential component. For a small tangential displacement (dx, dy), equilibrium requires zero work by these two effective forces:

$$m\omega^2 a\, dx - mg\, dy = 0.$$

Hence

$$\omega^2 a = g\frac{dy}{dx}\bigg|_{x=a}.$$

No Coriolis force acts because the bead is at rest relative to the rotating wire.

Step 2: Use the parabola slope and compare the options Differentiate the given equation rather than

using the secant slope b/a:

$$\frac{dy}{dx} = 8Cx, \qquad \left.\frac{dy}{dx}\right|_P = 8Ca.$$

For the illustrated nonzero value of a, substitution gives

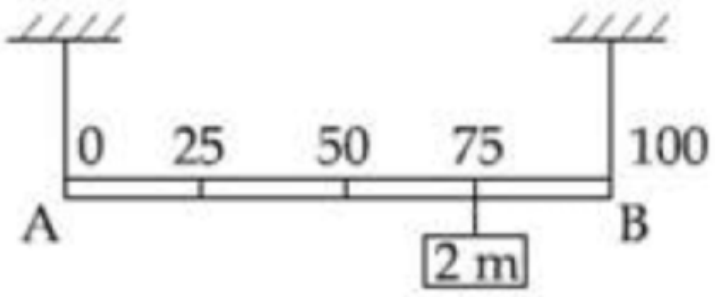
$$\omega^2 a = 8gCa, \qquad \omega^2 = 8gC.$$

The positive angular speed is $\sqrt{8gC} = 2\sqrt{2gC}$. Option (B) would give only half the required centrifugal acceleration. The coordinates cancel, excluding option (C); option (D) also has the wrong dimensions since C has dimensions of inverse length.

Step 3: Conclusion The required angular speed is $\omega = 2\sqrt{2gC}$. Hence, the correct option is (A).

Quick Tip to Solve: For a smooth rotating curve, use $\omega^2 x = g\,dy/dx$; the tangent slope, not y/x, determines equilibrium.

34. Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is : [4 marks]



(A) 0.5 mg

(B) 2 mg

(C) 0.75 mg

(D) 1 mg

Correct Answer: (D) 1 mg

Solution:

Concept:

- A rod in static equilibrium has zero net vertical force and zero net torque about any chosen point.
- The weight of a uniform rod acts through its centre of mass, at the midpoint. A suspended point load acts at its stated attachment point.

Step 1: Balance torques about the right end Let the upward tensions at the ends be T_A and T_B. Choose B as the pivot so that T_B contributes no torque. The rod weight mg acts 0.50 m to the left of B. The suspended mass has weight $2mg$ and is 0.25 m to the left of B, because it is 0.75 m from A.

All forces are vertical and the rod is horizontal, so these distances are their perpendicular lever arms. Torque balance gives

$$T_A(1.00) = mg(0.50) + 2mg(0.25).$$

Both weights turn the rod in the same sense about B; their torques must be added.

Step 2: Calculate the tension and verify force balance Evaluate the right-hand side:

$$T_A = 0.50mg + 0.50mg = mg.$$

The total downward force is $mg + 2mg = 3mg$, so vertical equilibrium also gives

$$T_A + T_B = 3mg, \qquad T_B = 2mg.$$

This is consistent with the heavier load being nearer B. Option (B), $2mg$, is the tension at the opposite end, not at A. Option (A) omits the suspended load's torque, while option (C) fails the torque balance. Both tension forces remain positive, as required for supporting strings.

Step 3: Conclusion The tension in the string at A is $T_A = mg$. Hence, the correct option is (D).

Quick Tip to Solve: Take moments about the other support to isolate the requested tension, and locate the uniform rod's own weight at its midpoint.

35. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/ hour. Speed (in ms^{-1}) of this person as observed from train B will be close to :

(take the distance between the tracks as negligible) [4 marks]

(A) 30.5 ms^{-1}

(B) 29.5 ms^{-1}

(C) 31.5 ms^{-1}

(D) 28.5 ms^{-1}

Correct Answer: (B) 29.5 ms^{-1}

Solution:

Concept:

- Velocities add algebraically along a common straight line: person relative to ground equals person relative to train A plus train A relative to ground.
- The velocity observed from train B is $v_{P/B} = v_{P/G} - v_{B/G}$. Use signed velocities before taking the magnitude for speed.

Step 1: Find the person's ground velocity Choose the direction of train A as positive. Then

$$v_{A/G} = +36 \text{ km h}^{-1}, \qquad v_{B/G} = -72 \text{ km h}^{-1}.$$

Because the person walks opposite to A's motion, the velocity relative to A is $v_{P/A} = -1.8\ \mathrm{km\,h^{-1}}$, not positive. Therefore

$$v_{P/G} = v_{P/A} + v_{A/G} = -1.8 + 36 = 34.2\ \mathrm{km\,h^{-1}}.$$

The person still moves in A's direction relative to the ground because the walking speed is much smaller than the train speed.

Step 2: Transform to train B and convert units Subtract the signed velocity of the observing train:

$$v_{P/B} = 34.2 - (-72) = 106.2\ \mathrm{km\,h^{-1}}.$$

Convert using $1\ \mathrm{km\,h^{-1}} = 5/18\ \mathrm{m\,s^{-1}}$:

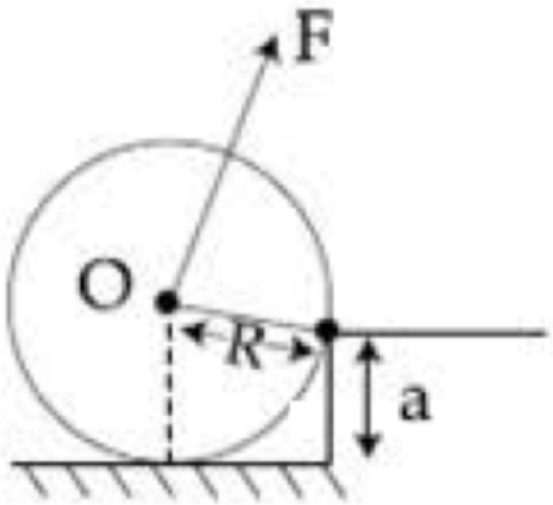
$$|v_{P/B}| = 106.2\frac{5}{18} = 29.5\ \mathrm{m\,s^{-1}}.$$

Equivalently, the trains approach at $10 + 20 = 30\ \mathrm{m\,s^{-1}}$, and the person's backward walk reduces this by $0.5\ \mathrm{m\,s^{-1}}$. Adding that walking speed instead would produce option (A), 30.5, with the wrong sign. The other listed values do not match the stated velocities.

Step 3: Conclusion The observed speed is $29.5\ \mathrm{m\,s^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Assign a positive direction first: subtract the observer's signed velocity, and convert to speed only at the end.

36. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is : [4 marks]



(A) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

(B) $Mg\sqrt{1-\frac{a^2}{R^2}}$

(C) $Mg\frac{a}{R}$

(D) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

Correct Answer: (A) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

Solution:

Concept:

- At the threshold of climbing, the cylinder pivots about the step edge and the ground reaction falls to zero.
- Torque about the edge eliminates the contact reaction. The applied force is perpendicular to the radius joining the edge to the centre, so its moment arm is R.

Step 1: Determine the weight's moment arm Call the step edge P. The distance OP equals the cylinder radius R. Initially the centre is height R above the lower ground, whereas the edge is at height a. Their vertical separation is therefore $R - a$. Let d be the horizontal separation between the centre and the edge. The right triangle gives

$$d^2 + (R - a)^2 = R^2, \qquad d = \sqrt{R^2 - (R - a)^2}.$$

Since gravity acts vertically through O, its perpendicular distance from P is precisely this horizontal distance d. It is not the step height a.

Step 2: Balance the limiting torques The shown force is perpendicular to OP, which maximises its turning effect for a given magnitude at the centre. At impending rotation, its torque just balances the opposing torque of gravity:

$$FR = Mg\,d.$$

Substituting the geometric result gives

$$F_{\min} = \frac{Mg}{R}\sqrt{R^2 - (R - a)^2} = Mg\sqrt{1 - \left(\frac{R - a}{R}\right)^2}.$$

The force tends to zero for a vanishing step, as expected. Option (B) instead tends to Mg. Option (C) incorrectly uses a as the weight lever arm. Option (D) uses $R - a$ as the force arm, appropriate to a horizontal pull rather than the perpendicular pull specified here.

Step 3: Conclusion The minimum force is $Mg\sqrt{1 - ((R - a)/R)^2}$. Hence, the correct option is (A).

Quick Tip to Solve: At a step, use the edge as pivot: find the horizontal gravity arm by Pythagoras, then use the stated direction to find the force arm.

37. A beam of protons with speed 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to :
(Mass of the proton = 1.67×10^{-27} kg, charge of the proton = 1.69×10^{-19} C) [4 marks]

(A) 2 cm

(B) 12 cm

(C) 5 cm

(D) 4 cm

Correct Answer: (D) 4 cm

Solution:

Concept:

- The velocity component parallel to a uniform magnetic field remains unchanged because the magnetic force is perpendicular to the field: $v_{\parallel} = v\cos\theta$.
- The perpendicular component produces circular motion with period $T = 2\pi m/(qB)$. The pitch is the distance travelled along the field in one complete revolution, $p = v_{\parallel}T$.

Step 1: Resolve the initial velocity The angle is measured from the magnetic field, so the axial component uses the cosine, not the sine. Thus

$$v_{\parallel} = (4 \times 10^5)\cos 60^\circ = 2 \times 10^5\,\mathrm{m\,s^{-1}}.$$

The other component controls the radius of the helix, but is not the velocity needed to calculate its pitch.

Step 2: Calculate the revolution time The magnetic force supplies the transverse centripetal force. Consequently the revolution time is independent of the speed and of the helix radius. Using the stated charge rather than substituting a different rounded constant gives

$$T = \frac{2\pi(1.67 \times 10^{-27})}{(1.69 \times 10^{-19})(0.3)} \simeq 2.07 \times 10^{-7}\,\mathrm{s}.$$

Step 3: Find the axial distance in one turn Multiply the constant axial speed by this time:

$$p = (2 \times 10^5)(2.07 \times 10^{-7}) = 4.14 \times 10^{-2}\,\mathrm{m} = 4.14\,\mathrm{cm}.$$

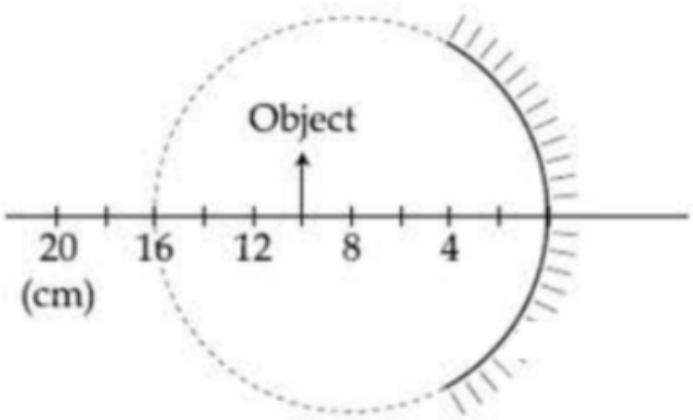
This is closest to 4 cm. The values 2 cm, 12 cm and 5 cm do not match the axial distance computed from the revolution period and the given field.

Step 4: Conclusion The pitch is approximately $4.14\,\mathrm{cm}$, close to $4\,\mathrm{cm}$. Hence, the correct option is (D).

Quick Tip to Solve: For a helix, use cosine for pitch and sine for radius when the angle is given relative to the magnetic field.

38. A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object?

(Figure drawn as schematic and not to scale) [4 marks]



(A) Inverted, real and unmagnified

(B) Inverted, real and magnified

(C) Erect, virtual and magnified

(D) Erect, virtual and unmagnified

Correct Answer: (A) Inverted, real and unmagnified

Solution:

Concept:

- The reflecting face in the figure is the inner face of the spherical surface, so the mirror is concave. Its focal length has magnitude half the radius of curvature.
- For a concave mirror, an object beyond the centre of curvature gives an image between the focus and centre: real, inverted and diminished. The mirror formula is $1/f = 1/v + 1/u$, with magnification $m = -v/u$.

Step 1: Read the geometry from the marked axis The pole is at the right-hand end of the axis. The opposite end of the complete sphere is marked 16 cm away, so the diameter is 16 cm and the radius of curvature is 8 cm. The focus is therefore 4 cm from the pole. The object is at the 10 cm position, as shown midway between the 8 cm and 12 cm labels. Thus it is beyond the centre of curvature.

Step 2: Apply the mirror formula with signs Take the incident direction to be positive. The object and the centre are on the incident side, so $u = -10\,\text{cm}$ and $f = -4\,\text{cm}$. Therefore

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{4} + \frac{1}{10} = -\frac{3}{20}, \qquad v = -\frac{20}{3}\,\text{cm}.$$

The negative image distance places the image in front of the mirror, confirming a real image.

Step 3: Determine orientation and relative size The transverse magnification is

$$m = -\frac{v}{u} = -\frac{-20/3}{-10} = -\frac{2}{3}.$$

Its negative sign means inversion, while its magnitude less than unity means the image is smaller than the object. Thus the stated option uses unmagnified in the sense of not enlarged. A magnified real image would require the object between the focus and centre, and erect virtual images require the object inside the focus.

Step 4: Conclusion The image is inverted, real and unmagnified (diminished), with $m = -2/3$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish the sphere diameter from the mirror radius before locating the focus; then compare the object position with the centre of curvature.

39. Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity ρ_C, ρ_T, ρ_M and ρ_A respectively. Then : [4 marks]

(A) $\rho_C > \rho_A > \rho_T$

(B) $\rho_M > \rho_A > \rho_C$

(C) $\rho_A > \rho_T > \rho_C$

(D) $\rho_A > \rho_M > \rho_C$

Correct Answer: (B) $\rho_M > \rho_A > \rho_C$

Solution:

Concept:

- Resistivity ρ is a material property at a specified temperature. Resistance also depends on geometry through $R = \rho L/A$, so comparisons of material resistivities do not require equal wire dimensions.
- At ordinary room temperature, the approximate resistivities in $\Omega\,\mathrm{m}$ are copper 1.7×10^{-8}, aluminium 2.7×10^{-8}, tungsten 5.6×10^{-8}, and mercury 9.6×10^{-7}.

Step 1: Order the given materials Copper is the best conductor among these four, so it has the lowest resistivity. Aluminium has a somewhat larger resistivity than copper, while tungsten is larger still. Mercury, although a conducting liquid, has a much larger resistivity than these common solid conductors. Thus the complete increasing order is

$$\rho_C < \rho_A < \rho_T < \rho_M.$$

Step 2: Compare the proposed inequalities Option (A) puts copper above aluminium, reversing their actual order. Option (C) puts aluminium above tungsten, also reversing the actual order. Option (D) puts aluminium above mercury, even though mercury has a resistivity more than an order of magnitude greater. Option (B) states only the valid three-material comparison

$$\rho_M > \rho_A > \rho_C.$$

The omission of tungsten from this particular inequality does not make it incorrect.

Step 3: Check the physical interpretation A larger resistivity means a smaller conductivity, since $\sigma = 1/\rho$. This ordering is consistent with the common use of copper and aluminium for low-resistance

conductors and tungsten for high-temperature filaments. These applications are a useful consistency check, though the numerical material-property ordering is what directly decides the answer.

Step 4: Conclusion The correct relationship is $\rho_M > \rho_A > \rho_C$. Hence, the correct option is (B).

Quick Tip to Solve: Remember the room-temperature order copper, aluminium, tungsten, mercury in increasing resistivity, and check each inequality one adjacent pair at a time.

40. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as : [4 marks]

(A) $T^2 \propto R$

(B) $T^2 \propto R^3$

(C) $T \propto R$

(D) $T^2 \propto \frac{1}{R^3}$

Correct Answer: (A) $T^2 \propto R$

Solution:

Concept:

- For a spherically symmetric distribution, only the enclosed mass contributes to the gravitational field at radius R: $g(R) = GM(R)/R^2$. Outer spherical shells exert no net force on the star.
- A shell of radius r and thickness dr has mass $dM = 4\pi r^2 \rho(r) dr$. Circular motion requires $4\pi^2 R/T^2 = g(R)$.

Step 1: Find how the enclosed mass varies The density is not constant, so the enclosed mass must be integrated rather than written as a constant density times the volume. Using $\rho(r) = K/r$,

$$dM = 4\pi K r\, dr, \qquad M(R) = \int_0^R 4\pi K r\, dr = 2\pi K R^2.$$

The density singularity at the centre has a finite integrated mass, since the shell mass tends to zero linearly with radius.

Step 2: Calculate the field in the specified region Insert the enclosed mass into the spherical gravitational-field expression:

$$g(R) = \frac{G(2\pi K R^2)}{R^2} = 2\pi G K.$$

The radial dependence cancels. This is the key difference from motion about a fixed central point mass, for which the field decreases as the inverse square of radius.

Step 3: Relate the field to the orbital period The gravitational acceleration supplies the centripetal acceleration. Hence

$$\frac{4\pi^2 R}{T^2} = 2\pi G K, \qquad T^2 = \frac{2\pi}{GK} R.$$

Since G and K are fixed, T^2 is proportional to R. The cubic dependence would apply to a fixed central mass, and neither a linear dependence of T itself nor an inverse cubic dependence follows here.

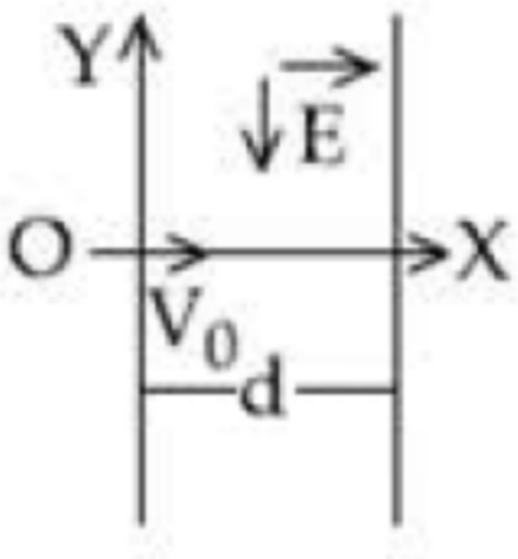
Step 4: Conclusion The orbital period obeys $T^2 \propto R$, or equivalently $T \propto \sqrt{R}$. Hence, the correct option is (A).

Quick Tip to Solve: For a spherical galaxy, integrate the density to obtain enclosed mass before applying the orbital-period formula; the central mass need not be constant.

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41. A charged particle (mass m and charge q) moves along X-axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d.

Equation of path of electron in the region x > d is [4 marks]



(A) y = $\frac{qEd}{mV_0^2}(x-d)$

(B) y = $\frac{qEd}{mV_0^2}\left(\frac{d}{2}-x\right)$

(C) y = $\frac{qEd}{mV_0^2}x$

(D) y = $\frac{qEd^2}{mV_0^2}x$

Correct Answer: (B) y = $\frac{qEd}{mV_0^2}\left(\frac{d}{2}-x\right)$

Solution:

Concept:

- The electric force is $\mathbf{F} = q\mathbf{E}$. With $\mathbf{E} = -E\hat{\mathbf{j}}$, the vertical acceleration is $a_y = -qE/m$, while the horizontal speed remains V_0.
- After leaving the field at $x = d$, there is no acceleration. The trajectory is the straight line tangent to the parabolic path at the exit point, with both exit position and exit velocity retained.

Step 1: Find the transit time and exit displacement The particle enters at the origin with no vertical

velocity. As no horizontal force acts, $x = V_0 t$, and the transit time to the boundary is $t_d = d/V_0$. Constant vertical acceleration then gives

$$y_d = \frac{1}{2}\left(-\frac{qE}{m}\right) t_d^2 = -\frac{qEd^2}{2mV_0^2}.$$

The sign of q determines which side of the axis contains the path.

Step 2: Find the direction of the emerging particle At the exit, the vertical velocity is

$$v_{yd} = -\frac{qE}{m}\frac{d}{V_0}.$$

Thus the slope of the subsequent straight path is

$$\frac{v_{yd}}{V_0} = -\frac{qEd}{mV_0^2}.$$

Using only this slope without the displacement would incorrectly force the final straight line to pass through the origin or the boundary on the axis.

Step 3: Write the line through the actual exit point For $x > d$, the time since exit is $(x - d)/V_0$. Therefore

$$\begin{aligned} y &= y_d + \frac{v_{yd}}{V_0}(x - d) \\ &= -\frac{qEd^2}{2mV_0^2} - \frac{qEd}{mV_0^2}(x - d) \\ &= \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right). \end{aligned}$$

This expression is continuous at $x = d$ and has the required exit slope. The other expressions fail one of these two boundary checks.

Step 4: Conclusion For $x > d$, $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$, with q treated algebraically. Hence, the correct option is (B).

Quick Tip to Solve: Beyond a field boundary use the exit point and exit velocity together; the outgoing straight line is tangent to the path inside the field.

42. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is : (VSD is vernier scale division) [4 marks]

(A) 3.21 cm

(B) 2.99 cm

(C) 3.07 cm

(D) 3.2 cm

Correct Answer: (C) 3.07 cm

Solution:

Concept:

- For a direct vernier, the least count is the difference between one main-scale division and one vernier-scale division: $\mathrm{LC} = 1\,\mathrm{MSD} - 1\,\mathrm{VSD}$.
- The observed reading is main-scale reading plus coinciding vernier division times least count. A positive zero error must be subtracted from the observed reading to obtain the true length.

Step 1: Calculate the least count Ten vernier divisions coincide with nine main-scale divisions. Since one main-scale division is 1 mm,

$$1\,\mathrm{VSD} = \frac{9}{10}\,\mathrm{mm} = 0.9\,\mathrm{mm}.$$

Therefore

$$\mathrm{LC} = 1 - 0.9 = 0.1\,\mathrm{mm} = 0.01\,\mathrm{cm}.$$

This conversion is needed because the measured main-scale reading is given in centimetres.

Step 2: Determine the sign and amount of zero error With the jaws closed, the vernier zero lies to the right of the main-scale zero. The instrument therefore indicates a positive length when the true length is zero. The seventh coinciding vernier division gives

$$e = +7(0.01) = +0.07\,\mathrm{cm}.$$

The zero correction is the negative of this error, namely $-0.07\,\mathrm{cm}$. The rule involving ten minus the coinciding division would apply to a negative zero error, not this case.

Step 3: Correct the measured reading The main-scale mark immediately before the vernier zero is 3.1 cm. With the fourth vernier division coinciding,

$$L_{\mathrm{observed}} = 3.1 + 4(0.01) = 3.14\,\mathrm{cm}.$$

Subtract the positive zero error:

$$L_{\mathrm{true}} = 3.14 - 0.07 = 3.07\,\mathrm{cm}.$$

Adding the error would produce 3.21 cm, while 2.99 cm and 3.2 cm are inconsistent with the stated readings and correction.

Step 4: Conclusion The corrected length of the cylinder is $3.07\,\mathrm{cm}$. Hence, the correct option is (C).

Quick Tip to Solve: Write true reading = observed reading minus zero error, keeping the error sign explicit before doing the arithmetic.

43. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, N = 6.023 $\times$ 10^{26} per kilo mole and 1 eV = 1.6 $\times$ 10^{-19} J. The power output of the reactor is close to [4 marks]

(A) 125 MW

(B) 60 MW

(C) 54 MW

(D) 35 MW

Correct Answer: (B) 60 MW

Solution:

Concept:

- Each uranium nucleus undergoing fission releases the stated energy. The number of nuclei is the fuel mass divided by the mass of one kilomole, multiplied by Avogadro's number.
- Average power is total energy divided by elapsed time: $P = N_f E_f / t$. Convert MeV to joules and days to seconds before substituting.

Step 1: Count the uranium nuclei The molar mass of uranium-235 is $235\,\mathrm{kg\,kmol^{-1}}$. Thus the 2 kg sample contains

$$n = \frac{2}{235}\,\mathrm{kmol}, \qquad N_f = \frac{2}{235}(6.023 \times 10^{26}) = 5.126 \times 10^{24}.$$

Complete use of the fuel means that this many fissions contribute to the specified energy output; the isotope number 235 is the relevant molar mass, whereas 92 is the atomic number.

Step 2: Convert the energy and operating time The energy per fission is

$$E_f = 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11}\,\mathrm{J}.$$

The operating interval is

$$t = 30 \times 24 \times 3600 = 2.592 \times 10^6\,\mathrm{s}.$$

Therefore the total released energy is $N_f E_f = 1.640 \times 10^{14}\,\mathrm{J}$. The statement supplies no efficiency factor, so the requested output is the mean released energy per second.

Step 3: Evaluate and compare the powers

$$P = \frac{1.640 \times 10^{14}}{2.592 \times 10^6} = 6.33 \times 10^7\,\mathrm{W} = 63.3\,\mathrm{MW}.$$

Among the offered values, 60 MW is closest: it differs by about 3.3 MW, compared with about 9.3 MW for 54 MW. The 125 MW and 35 MW values are substantially further away. Keeping the given Avogadro constant per kilomole avoids a spurious factor of one thousand.

Step 4: Conclusion The mean reactor power is approximately $63\,\mathrm{MW}$, closest to $60\,\mathrm{MW}$. Hence, the correct option is (B).

Quick Tip to Solve: Use 235 kg per kilomole with the given kilomolar Avogadro constant; then divide the total fission energy by the full 30-day interval.

44. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required? [4 marks]

(A) T : Large retentivity, small coercivity

(B) P : Large retentivity, large coercivity

(C) P : Small retentivity, large coercivity

(D) T : Large retentivity, large coercivity

Correct Answer: (B) P : Large retentivity, large coercivity

Solution:

Concept:

- Retentivity measures the magnetisation retained after the magnetising field is removed; large retentivity allows a permanent magnet to remain strongly magnetised.
- Coercivity measures the reverse field needed to remove magnetisation. A permanent magnet needs large coercivity; a transformer core needs an easily reversible magnetisation and a narrow hysteresis loop.

Step 1: Identify the permanent-magnet requirements A permanent magnet must continue providing a strong magnetic field after the external magnetising field is switched off. Hence its material must retain a large residual magnetisation: its retentivity must be large. During use it may experience stray fields, mechanical disturbances and opposing magnetic fields. A large coercive field helps it resist demagnetisation, so large coercivity is also desirable. These two properties play different roles: retaining magnetisation alone does not ensure resistance to reversal.

Step 2: Contrast the transformer-core requirements In a transformer the alternating current reverses the magnetising field every half-cycle. The core must therefore magnetise and demagnetise readily instead of preserving its previous magnetic state against the reversal. A soft magnetic material with high permeability and small coercivity is used. Its narrow hysteresis loop reduces the energy dissipated as heat in each cycle. Large coercivity, useful in a permanent magnet, would make reversal difficult and increase hysteresis loss here.

Step 3: Test the stated combinations Option (B) assigns both large retentivity and large coercivity to P, matching the two independent requirements. Option (C) gives P small retentivity, so its residual field would be weak. Option (D) gives T large coercivity, opposing easy cyclic reversal. Option (A) includes small coercivity for T, but the asserted large retentivity is not the best distinguishing requirement for a transformer core; the unambiguously suitable permanent-magnet combination is (B).

Step 4: Conclusion A permanent magnet requires large retentivity and large coercivity. Hence, the correct option is (B).

Quick Tip to Solve: Remember the jobs separately: a permanent magnet must retain and resist reversal; a transformer core must reverse its magnetisation easily.

45. An amplitude modulated wave is represented by the expression

$v_m = 5(1 + 0.6 \cos 6280t) \sin(211 \times 10^4\, t)$ volts

The minimum and maximum amplitudes of the amplitude modulated wave are, respectively [4 marks]

(A) $\frac{3}{2}$ V, 5 V

(B) $\frac{5}{2}$ V, 8 V

(C) 5 V, 8 V

(D) 3 V, 5 V

Correct Answer: The waveform gives 2 V and 8 V; none of the listed pairs matches.

Solution:

Concept:

- For an AM signal $v = A_c[1 + m\cos(\omega_m t)]\sin(\omega_c t)$, the slowly varying envelope is $A(t) = A_c[1 + m\cos(\omega_m t)]$.
- When $0 \le m < 1$, the envelope remains positive and has extrema $A_{\min} = A_c(1 - m)$ and $A_{\max} = A_c(1 + m)$. These are different from the instantaneous voltage extrema over carrier oscillations.

Step 1: Identify the carrier amplitude and modulation index Comparing the given expression with the standard amplitude-modulated wave gives $A_c = 5\,\text{V}$ and $m = 0.6$. The factor $\cos(6280t)$ is the message variation, and the high-frequency sine is the carrier. The angular frequencies determine the rates of these oscillations, but do not alter the extremal values of the envelope. Since $m < 1$, the modulation is not overmodulated and no absolute-value reversal of the envelope is needed.

Step 2: Calculate both envelope amplitudes The cosine lies between -1 and +1. At its minimum, the prefactor of the carrier sine is

$$A_{\min} = 5(1 - 0.6) = 2\,\text{V}.$$

At its maximum, it is

$$A_{\max} = 5(1 + 0.6) = 8\,\text{V}.$$

As a check, the mean of these extrema is $(8 + 2)/2 = 5\,\text{V}$, reproducing the carrier amplitude, and their difference divided by their sum is $(8 - 2)/(8 + 2) = 0.6$, reproducing the modulation index.

Step 3: Check the printed choices The pair demanded in minimum-then-maximum order is therefore $(2\,\text{V}, 8\,\text{V})$. Option (A) has neither extremum correct. Option (B) has the correct maximum but its minimum is 2.5 V rather than 2 V. Option (C) mistakes the carrier amplitude for the minimum. Option (D) gives the wrong maximum and minimum. Consequently none of the four printed pairs satisfies the given waveform; choosing the nearest pair would not solve the expression.

Step 4: Conclusion The minimum and maximum envelope amplitudes are $2\,\text{V}$ and $8\,\text{V}$, respectively. None of the printed options is correct for the stated expression.

Quick Tip to Solve: Compute both envelope extrema and verify that their mean is the carrier amplitude before comparing the offered pairs.

Physics – Section B: Numerical Value Questions (20 marks)

46. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}J$ then value of X to the nearest integer is _____. [4 marks]

Solution:

Concept:

- The charge and stored energy of a capacitor are $Q = CV$ and $U = \frac{1}{2}CV^2$. During isolated charge sharing, the total charge on the connected like-polarity nodes is conserved.
- For two capacitors connected across the same two nodes, the final potential is $V_f = Q/(C_1 + C_2)$ and the energy lost is $\Delta U = \frac{1}{2}[C_1C_2/(C_1+C_2)]V_0^2$. A mere series junction is not this charge-sharing circuit.

Step 1: Calculate the initial stored energy The initially charged capacitor has $C_1 = 5 \times 10^{-6}\,\mathrm{F}$ and $V_0 = 220\,\mathrm{V}$. Thus

$$Q_0 = C_1V_0 = 1.10 \times 10^{-3}\,\mathrm{C}, \qquad U_i = \frac{1}{2}C_1V_0^2 = 0.121\,\mathrm{J}.$$

The second capacitor, $C_2 = 2.5 \times 10^{-6}\,\mathrm{F}$, initially has zero voltage and zero energy. Disconnecting the battery means that no subsequent work is supplied by it.

Step 2: Evaluate the charge-sharing energy change For the charge redistribution implied by the numerical key, the capacitors share a common final voltage. Conservation of charge then gives

$$V_f = \frac{C_1V_0}{C_1 + C_2} = \frac{5(220)}{7.5} = 146.67\,\mathrm{V}.$$

The final field energy is

$$U_f = \frac{1}{2}(C_1 + C_2)V_f^2 = 0.080667\,\mathrm{J}.$$

The energy decrease is therefore

$$U_i - U_f = 0.040333\,\mathrm{J} = \frac{4.0333}{100}\,\mathrm{J}.$$

This energy is dissipated during the transient, rather than being lost through a failure of charge conservation.

Step 3: Distinguish the connection stated in the question The word series in the question does not describe the common-voltage connection used above. Connecting only one plate of the charged capacitor to one plate of the uncharged capacitor, with the other plates isolated, does not provide the closed charge-sharing path assumed in that calculation. A literal isolated series arrangement therefore does not justify the stated redistribution result. The accepted numerical value 4 corresponds to the standard common-voltage charge-sharing interpretation; an actual circuit diagram or a parallel connection specification would remove this inconsistency.

Step 4: Conclusion For common-voltage charge redistribution, $X = 4.0333$, giving the accepted nearest-integer answer $\boxed{4}$. The wording in series is inconsistent with that interpretation; a literal isolated series connection does not produce this charge-sharing energy loss.

Quick Tip to Solve: Before applying the charge-sharing formula, check that both capacitors end across the same two nodes; a single series junction is insufficient.

47. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabatically to 1/10th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is_____. [4 marks]

Solution:

Concept:

- A rigid diatomic ideal gas has five active degrees of freedom, so $C_V = 5R/2$, $C_P = 7R/2$ and $\gamma = C_P/C_V = 7/5$.
- For a reversible adiabatic ideal-gas process, $TV^{\gamma-1}$ is constant. The internal energy change depends only on temperature: $\Delta U = nC_V(T_f - T_i)$.

Step 1: Find the initial absolute temperature and heat capacity The initial temperature must be converted from Celsius to kelvin before using a gas relation:

$$T_i = 20 + 273.15 = 293.15\,\text{K}.$$

With vibration excluded by the rigid-molecule model, air has three translational and two rotational degrees of freedom. Consequently

$$C_V = \frac{5}{2}R, \qquad \gamma = \frac{7}{5} = 1.4.$$

The initial pressure is not needed once the volume ratio and initial temperature are specified.

Step 2: Apply the adiabatic temperature-volume relation The final volume is one tenth of the initial volume, so $V_i/V_f = 10$. Thus

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 293.15 \times 10^{0.4} \approx 736.36\,\text{K}.$$

Compression raises the temperature because work is done on the gas while no heat escapes. Using the isothermal relation here would incorrectly force the temperature and internal energy to remain unchanged.

Step 3: Calculate the rise in internal energy For $n = 5$ moles and $R = 8.314\,\text{J mol}^{-1}\,\text{K}^{-1}$,

$$\Delta U = 5\left(\frac{5}{2}\right)(8.314)(736.36 - 293.15) \approx 4.61 \times 10^4\,\text{J} = 46.1\,\text{kJ}.$$

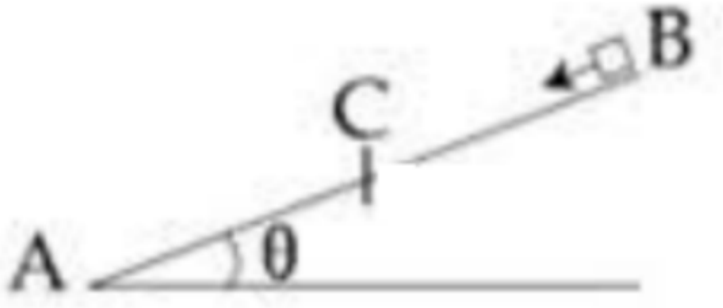
The sign is positive: with $Q = 0$, the first law equates the increase in internal energy to the work done on the gas. Retaining kelvin throughout and converting joules to kilojoules only at the end gives the requested numerical scale.

Step 4: Conclusion The internal energy increases by approximately $46.1\,\text{kJ}$. Therefore X, to the nearest integer, is $\boxed{46}$.

Quick Tip to Solve: For rigid diatomic air use gamma = 7/5, then obtain the temperature rise from the volume ratio before calculating the energy.

48. A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by μ = ktan θ . The value of k is _____.

[4 marks]



Solution:

Concept:

- Gravity does work $mg\sin\theta$ per unit distance down the incline. Its total work equals the loss of gravitational potential energy over the full path.
- Kinetic friction acts only on the rough portion CA and has magnitude $\mu mg\cos\theta$. Since the block starts and ends at rest, total gravitational work equals the energy dissipated by friction.

Step 1: Express the total descent in terms of the rough length Let the rough segment $AC = \ell$. The given relation $BC = 2AC$ makes the smooth segment $BC = 2\ell$, so the full distance down the incline is $BA = 3\ell$. The corresponding vertical drop from B to A is $3\ell\sin\theta$. Therefore gravity releases

$$W_g = 3mg\ell\sin\theta.$$

The normal force is perpendicular to the displacement and does no work.

Step 2: Write the frictional energy loss The block encounters no friction on BC, so it gains kinetic energy there. On CA, the normal reaction remains $N = mg\cos\theta$, giving a frictional force $f = \mu mg\cos\theta$ opposite the downward motion. It acts over only ℓ, hence its work is

$$W_f = -\mu mg\ell\cos\theta.$$

Applying friction over all of BA would count resistance on the smooth part and give an incorrect coefficient.

Step 3: Use the equal initial and final kinetic energies The block starts slipping from rest at B and comes to rest at A. The work-energy theorem therefore gives

$$0 = \Delta K = W_g + W_f = 3mg\ell\sin\theta - \mu mg\ell\cos\theta.$$

Cancelling the positive factors $mg\ell$ yields

$$\mu = 3\tan\theta.$$

This is dynamically consistent: on CA, friction exceeds the downhill component of gravity by $2mg\sin\theta$, removing exactly the kinetic energy accumulated on the twice-as-long smooth segment.

Step 4: Conclusion Comparing $\mu = 3\tan\theta$ with $\mu = k\tan\theta$ gives $\boxed{k = 3}$.

Quick Tip to Solve: Use gravity over the entire path but friction only over AC; the required factor is total incline length divided by rough length.

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49. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $\frac{V}{4}$. If the threshold wavelength for the metallic surface is nλ then value of n will be _____. [4 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where V_s is the stopping potential and ϕ is the metal's work function.
- The threshold wavelength λ_0 satisfies $\phi = hc/\lambda_0$. The work function stays unchanged when the incident wavelength changes.

Step 1: Write the two energy balances Let $X = hc/\lambda$. For the first illumination, the maximum photoelectron kinetic energy is eV. For the second, the photon energy is $X/3$, while the maximum kinetic energy is $eV/4$. Thus the equations for the same surface are

$$eV = X - \phi, \qquad \frac{eV}{4} = \frac{X}{3} - \phi.$$

The factor of three divides photon energy because energy is inversely proportional to wavelength.

Step 2: Eliminate the stopping potential Substitute the first equation into the second, multiply by twelve, and collect terms:

$$\frac{X-\phi}{4} = \frac{X}{3} - \phi, \qquad 3X - 3\phi = 4X - 12\phi.$$

Consequently $9\phi = X$, so $\phi = hc/(9\lambda)$. This is positive and smaller than both incident photon energies, consistent with photoemission in both experiments.

Step 3: Identify the threshold wavelength Using the definition of threshold gives

$$\frac{hc}{\lambda_0} = \frac{hc}{9\lambda} \quad \Rightarrow \quad \lambda_0 = 9\lambda.$$

As a check, the first maximum kinetic energy is $8X/9$, and the second is $2X/9$; their ratio is exactly four as required.

Step 4: Conclusion The threshold wavelength is 9λ. Therefore the required dimensionless value is $\boxed{n = 9}$.

Quick Tip to Solve: For two photoelectric measurements on one metal, eliminate the unchanged work function or stopping potential before substituting constants.

50. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2 s. The maximum value of EMF induced (in μV) in the coil will be close to the integer _____. [4 marks]

Solution:

Concept:

- Magnetic flux through a single circular coil is $\Phi = BA\cos\theta$, where $A = \pi r^2$ and θ is the angle between the field and area normal.
- For uniform rotation, Faraday's law gives $\varepsilon = BA\omega\sin\omega t$, so its maximum magnitude is $BA\omega$.

Step 1: Convert the radius and find the area The given radius is $10\,\text{cm} = 0.10\,\text{m}$. The area enclosed by the circular coil is therefore

$$A = \pi(0.10)^2 = 0.01\pi\,\text{m}^2.$$

Initially the plane is perpendicular to the field, so the area normal is parallel to it and the initial flux has its maximum magnitude. The rotation about the stated diameter changes this flux sinusoidally.

Step 2: Calculate the angular speed A half rotation corresponds to an angular displacement of π radians, not 2π radians. Because this takes $0.2\,\text{s}$ at constant speed,

$$\omega = \frac{\pi}{0.2} = 5\pi\,\text{rad}\,\text{s}^{-1}.$$

The maximum emf occurs when the coil plane is parallel to the magnetic field; this is where the flux itself is zero but its rate of change is greatest.

Step 3: Evaluate the peak induced emf Using the specified magnetic field and the single coil,

$$\varepsilon_{\max} = (3.0 \times 10^{-5})(0.01\pi)(5\pi) = 1.5\pi^2 \times 10^{-6}\,\text{V}.$$

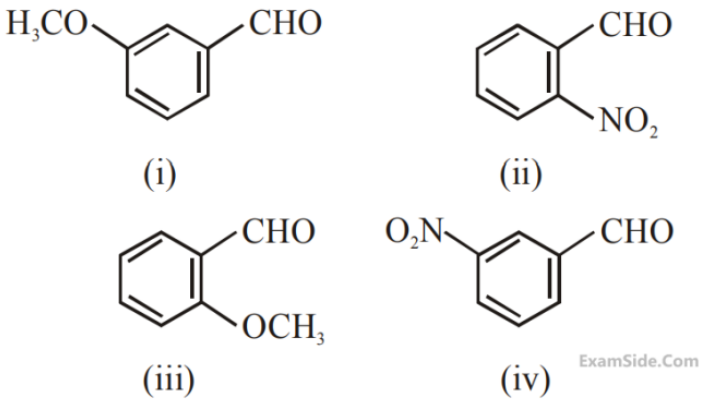
Since $1\,\mu\text{V} = 10^{-6}\,\text{V}$, this is approximately $14.80\,\mu\text{V}$. The question asks for the closest integer, so round only after evaluating the full expression. The average emf over the half turn is different and must not replace this peak value.

Step 4: Conclusion The maximum induced emf is approximately $14.80\,\mu\text{V}$, giving the required integer $\boxed{15}$.

Quick Tip to Solve: For a uniformly rotating coil use the peak value BA times angular speed; flux change divided by time instead gives an average emf.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The increasing order of the following compounds towards HCN addition is : [4 marks]



(A) (iii) < (iv) < (ii) < (i)

(B) (i) < (iii) < (iv) < (ii)

(C) (iii) < (i) < (iv) < (ii)

(D) (iii) < (iv) < (i) < (ii)

Correct Answer: (C) (iii) < (i) < (iv) < (ii)

Solution:

Concept:

- HCN adds to a carbonyl compound by nucleophilic attack of CN^- on the electrophilic carbonyl carbon, followed by protonation.
- Electron-withdrawing substituents increase carbonyl electrophilicity and favour addition; electron donation and steric crowding oppose the attack.

Step 1: Compare the methoxy-substituted aldehydes Compound (i) is meta-methoxybenzaldehyde, whereas compound (iii) is ortho-methoxybenzaldehyde. At the ortho position the methoxy group can donate electron density by resonance into the aromatic system conjugated with the aldehyde, reducing the positive character of the carbonyl carbon. It also crowds the approaching nucleophile. At the meta position its resonance donation does not directly operate at the aldehyde-bearing carbon; its inductive withdrawal remains. Hence (iii) is less reactive than (i).

Step 2: Compare the nitro-substituted aldehydes Compound (iv) has a meta nitro group, which withdraws by the inductive effect and makes the aldehyde carbon more electrophilic than in (i). Compound (ii) has an ortho nitro group: its strong electron-withdrawing influence includes resonance withdrawal through the ring as well as the inductive effect. For the stated comparison this activation gives (ii) the greatest tendency to undergo cyanide addition. Thus the nitro derivatives lie above the methoxy derivatives.

Step 3: Assemble and check the order The increasing order is

$$(\text{iii}) < (\text{i}) < (\text{iv}) < (\text{ii}).$$

Option (A) incorrectly places (i) above the nitro derivatives. Option (B) puts the ortho methoxy compound above the meta methoxy compound. Option (D) incorrectly places the meta methoxy compound above the meta nitro compound. Only option (C) retains both correct pairwise comparisons.

Step 4: Conclusion The increasing order towards HCN addition is $\boxed{(iii) < (i) < (iv) < (ii)}$. Hence, the correct option is (C).

Quick Tip to Solve: First compare ortho and meta methoxy donation, then place nitro-substituted aldehydes higher because withdrawal activates the carbonyl.

52. Consider that a d^6 metal ion (M^{2+}) forms a complex with aqua ligands, and the spin only magnetic moment of the complex is 4.90 BM. The geometry and the crystal field stabilization energy of the complex is [4 marks]

(A) tetrahedral and – 1.6 Δ_t + 1P

(B) octahedral and –2.4 Δ_0 + 2P

(C) tetrahedral and –0.6 Δ_t

(D) octahedral and –1.6 Δ_0

Correct Answer: (C) tetrahedral and –0.6 Δ_t

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{u(u+2)}\,\text{BM}$, where u is the number of unpaired electrons.
- In a tetrahedral field the lower e orbitals contribute $-0.6\Delta_t$ per electron and the upper t_2 orbitals contribute $+0.4\Delta_t$ per electron.

Step 1: Determine the unpaired-electron count The measured moment is $4.90\,\text{BM}$. For four unpaired electrons,

$$\mu = \sqrt{4(4+2)} = \sqrt{24} \simeq 4.90\,\text{BM}.$$

Thus the ion is high spin with four unpaired electrons. An octahedral low-spin d^6 configuration would be t_{2g}^6, with no unpaired electrons, so the low-spin stabilization shown in option (B) cannot apply.

Step 2: Calculate the tetrahedral stabilization For the tetrahedral high-spin d^6 arrangement, the occupancy is $e^3t_2^3$. There is one paired orbital and four singly occupied orbitals. Its crystal-field contribution is

$$E_{\text{CF}} = 3(-0.6\Delta_t) + 3(0.4\Delta_t) = -0.6\Delta_t.$$

The single electron pair is also present in the free high-spin d^6 ion, so no additional pairing-energy cost is needed in this stabilization comparison.

Step 3: Exclude the competing entries A high-spin octahedral d^6 complex would have $t_{2g}^4e_g^2$, giving $4(-0.4\Delta_o) + 2(0.6\Delta_o) = -0.4\Delta_o$, not the $-1.6\Delta_o$ listed in (D). Option (A) gives an incorrect tetrahedral splitting contribution. Although the moment alone does not distinguish both high-spin geometries, only (C) supplies a geometry and energy that are mutually consistent with the given moment.

Step 4: Conclusion The consistent pair is tetrahedral geometry with crystal-field stabilization energy $\boxed{-0.6\Delta_t}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the magnetic moment to count unpaired electrons, then calculate the orbital energy sum independently rather than guessing the geometry from weak ligands.

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53. For the following Assertion and Reason, the correct option is

Assertion (A): When Cu (II) and sulphide ions are mixed, they react together extremely quickly to give a solid.

Reason (R): The equilibrium constant of

$Cu^{2+}(aq) + S^{2-}(aq) \rightleftharpoons CuS(s)$ is high because the solubility product is low. [4 marks]

(A) is false and (R) is true.

(B) Both (A) and (R) are true but (R) is not the explanation for (A).

(C) Both (A) and (R) are true and (R) is the explanation for (A).

(D) Both (A) and (R) are false.

Correct Answer: (B) Both (A) and (R) are true but (R) is not the explanation for (A).

Solution:

Concept:

- For $\mathrm{CuS(s)} \rightleftharpoons \mathrm{Cu^{2+}(aq) + S^{2-}(aq)}$, the solubility product is $K_{sp} = [\mathrm{Cu^{2+}}][\mathrm{S^{2-}}]$.
- Reversing an equilibrium replaces its equilibrium constant by the reciprocal. Equilibrium constants describe the equilibrium position, whereas reaction speeds depend on kinetics.

Step 1: Evaluate the assertion When aqueous copper(II) ions and sulphide ions are mixed under ordinary precipitation conditions, a solid copper(II) sulphide precipitate forms rapidly:

$$\mathrm{Cu^{2+}(aq) + S^{2-}(aq) \longrightarrow CuS(s).}$$

This is the familiar rapid precipitation of an extremely sparingly soluble ionic compound. The assertion describes that observed behaviour and is true.

Step 2: Evaluate the reason The reason refers to the reverse of the dissolution equilibrium. Therefore the precipitation equilibrium constant is

$$K_{\text{precip}} = \frac{1}{K_{sp}}.$$

Copper(II) sulphide has a very small solubility product, so its reciprocal is very large. The reason correctly states that the equilibrium strongly favours solid formation. It is therefore also true.

Step 3: Test whether the reason explains the speed A large equilibrium constant establishes that products are strongly favoured at equilibrium; it does not specify how quickly equilibrium is reached. The activation barrier and reaction mechanism determine the rate. Hence the reason explains the strong tendency to precipitate but does not, by itself, explain the words extremely quickly in the assertion. Option (A) rejects a true assertion, option (C) confuses thermodynamic favourability with

kinetic speed, and option (D) incorrectly rejects both true statements.

Step 4: Conclusion Both the assertion and the reason are true, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B). The supplied final key selects the explanation as correct, but a large equilibrium constant alone cannot establish the speed explicitly stated in the assertion; this kinetic distinction supports (B) for the wording printed here.

Quick Tip to Solve: When an assertion mentions how quickly a reaction occurs, ask whether the reason supplies kinetic information rather than only an equilibrium constant.

54. Which of the following compunds will show retention in configuration on nucleophilic substitution by OH^- ion? [4 marks]

(A) $CH_3-CH(Br)-C_6H_5$

(B) $CH_3-CH(Br)-CH_3$

(C) $CH_3-CH(C_2H_5)-CH_2Br$

(D) $CH_3-CH(Br)-C_6H_{13}$

Correct Answer: (C) $CH_3-CH(C_2H_5)-CH_2Br$

Solution:

Concept:

- An S_N2 substitution inverts the geometry at the carbon directly attacked by the nucleophile; it does not invert a separate neighbouring stereocentre.
- A carbon is stereogenic when its four substituents differ. Substitution at an adjacent primary carbon can leave the arrangement around an existing stereocentre unchanged.

Step 1: Locate the two centres Option (C) has the following structure:

$$CH_3-CH(C_2H_5)-CH_2Br.$$

The central CH carbon has four different substituents: hydrogen, methyl, ethyl, and bromomethyl. It is therefore stereogenic. The bromine, however, is bonded to the adjacent terminal CH_2 carbon, not to this stereocentre.

Step 2: Follow the hydroxide substitution Hydroxide replaces bromide at the primary bromomethyl carbon:

$$R-CH_2Br + OH^- \longrightarrow R-CH_2OH + Br^-.$$

Here $R = CH(CH_3)(C_2H_5)$. The carbon undergoing backside attack bears two identical hydrogen atoms, so inversion at that carbon is not an observable stereochemical change. None of the four bonds to the neighbouring stereogenic carbon is broken. Its spatial arrangement is retained as the bromomethyl group becomes hydroxymethyl.

Step 3: Check the other compounds In (A), bromine is directly attached to the stereogenic benzylic carbon; substitution there can give inversion or racemization rather than the required retention. Compound (B), $CH_3-CHBr-CH_3$, is achiral because the reactive carbon has two methyl groups, so it

does not illustrate retention at a stereocentre. In (D), bromine is attached directly to a stereogenic carbon bearing hydrogen, methyl and hexyl groups; direct substitution again changes or scrambles the arrangement at that centre. Thus only (C) places the leaving group away from the preserved stereocentre.

Step 4: Conclusion Option (C), $CH_3-CH(C_2H_5)-CH_2Br$, retains the arrangement at its existing stereocentre during substitution. Hence, the correct option is (C).

Quick Tip to Solve: Locate the carbon bonded to the leaving group before applying inversion: a neighbouring stereocentre is preserved when none of its bonds is broken.

55. For octahedral Mn(II) and tetrahedral Ni(II) complexes, consider the following statements:

(I) both the complexes can be high spin.

(II) Ni(II) complex can very rarely be low spin.

(III) with strong field ligands, Mn(II) complexes can be low spin.

(IV)aqueous solution of Mn(II) ions is yellow in colour.

The correct statements are : [4 marks]

(A) (I), (III) and (IV) only

(B) (I) and (II) only

(C) (II), (III) and (IV) only

(D) (I), (II) and (III) only

Correct Answer: (D) (I), (II) and (III) only

Solution:

Concept:

- Manganese has atomic number 25, so Mn^{2+} is $3d^5$; nickel has atomic number 28, so Ni^{2+} is $3d^8$. Spin behaviour depends on the ligand-field splitting relative to the pairing energy.
- For an octahedral field the lower and upper sets are t_{2g} and e_g. A weak field favours unpaired electrons, while a sufficiently strong field can favour pairing. Tetrahedral splitting is ordinarily small, so tetrahedral complexes are usually high spin.

Step 1: Check the high-spin statement For weak-field octahedral Mn^{2+}, the five electrons occupy $t_{2g}^3e_g^2$, giving five unpaired electrons. Tetrahedral Ni^{2+} ordinarily has configuration $e^4t_2^4$, with two unpaired electrons. Thus both types can be high spin and statement (I) is correct.

Step 2: Examine the low-spin qualifications An ideal tetrahedral d^8 ion has $e^4t_2^4$, with two unpaired electrons; merely increasing the tetrahedral splitting does not make this configuration diamagnetic. Statement (II) must therefore be read as referring to nickel(II) complexes generally: low-spin nickel(II) can occur with a change to square-planar geometry, as in $[Ni(CN)_4]^{2-}$. It is not a low-spin tetrahedral configuration. On this reading, (II) is accepted in the keyed combination. For octahedral manganese(II), strong-field ligands can produce $t_{2g}^5e_g^0$, with one unpaired electron, so (III) is correct.

Step 3: Reject the colour statement and compare combinations Aqueous manganese(II) is very pale pink, not yellow. Its high-spin half-filled configuration makes its visible transitions weak, so the colour

is faint. Statement (IV) is false. Options (A) and (C) include this false statement; option (B) omits the true strong-field statement (III). Only option (D) gives the required combination.

Step 4: Conclusion Reading (II) as a statement about nickel(II) complexes generally, statements (I), (II) and (III) are accepted, whereas (IV) is false. Hence, the correct option is (D). A strictly tetrahedral interpretation of (II) would be incorrect: tetrahedral d^8 retains two unpaired electrons.

Quick Tip to Solve: Write the metal-ion d count first; then test each statement separately, especially qualifiers such as "can" and "very rarely".

56. In general the property (magnitudes only) that show an opposite trend in comparison to other properties across a period is [4 marks]

(A) Electron gain enthalpy

(B) Electronegativity

(C) Ionization enthalpy

(D) Atomic radius

Correct Answer: (D) Atomic radius

Solution:

Concept:

- Across a period, the nuclear charge increases while the differentiating electrons enter the same principal shell. The shielding does not fully offset the increased nuclear charge.
- The resulting increase in effective nuclear attraction generally reduces atomic radius and strengthens attraction for electrons. The question asks for general trends in magnitudes, not every individual exception.

Step 1: Compare the three increasing properties Ionization enthalpy generally increases from left to right because a more strongly bound electron is harder to remove. Electronegativity generally increases because the atom attracts shared bonding electrons more strongly. Electron gain enthalpy generally becomes more negative across a representative period; its magnitude therefore generally increases. The sign of electron gain enthalpy must not be confused with its magnitude.

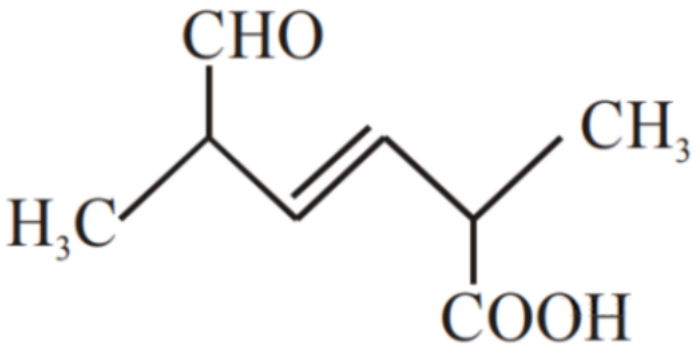
Step 2: Identify the decreasing property As effective nuclear attraction increases, the electron cloud is pulled closer to the nucleus. Because there is no additional principal shell on moving across a period, atomic radius generally decreases. This is the opposite direction to the general increase in the other three listed magnitudes. Local irregularities in ionization and electron gain enthalpies do not reverse the broad comparison requested here.

Step 3: Eliminate the other choices Options (A), (B) and (C) all describe properties associated with the attraction or binding of electrons, and their magnitudes generally rise across a period. Option (D) describes the size of the electron cloud, which contracts as that attraction rises. Noble-gas size conventions and subshell exceptions are not the general trend being asked.

Step 4: Conclusion Atomic radius generally decreases across a period, opposite to the other listed magnitudes. Hence, the correct option is (D).

Quick Tip to Solve: Across a period, remember stronger nuclear attraction: smaller size but generally larger electron-attracting and electron-removal properties.

57. The IUPAC name for the following compound is [4 marks]



(A) 2, 5-dimethyl-6-carboxy-hex-3-enal

(B) 6-formyl-2-methyl-hex-3-enoic acid

(C) 2, 5-dimethyl-5-carboxy-hex-3-enal

(D) 2, 5-dimethyl-6-oxo-hex-3-enoic acid

Correct Answer: (D) 2, 5-dimethyl-6-oxo-hex-3-enoic acid

Solution:

Concept:

- When a molecule contains both carboxylic acid and aldehyde groups, the carboxylic acid has higher priority and supplies the principal suffix, $-$oic acid.
- Choose the longest suitable parent chain containing the carboxyl carbon and the double bond, then number from the carboxyl carbon. An aldehyde carbon included in this parent chain is represented by the prefix oxo.

Step 1: Select and number the parent chain Start with the carbon of the COOH group as carbon 1. Follow the chain through the adjacent substituted carbon, the two alkene carbons, the next substituted carbon and finally the aldehyde carbon. This gives a six-carbon parent. The double bond lies between carbons 3 and 4, so the parent unsaturated acid is hex-3-enoic acid.

Step 2: Locate the substituents and carbonyl group The methyl branch on the carbon next to the acid group is at position 2. The methyl branch on the carbon next to the aldehyde terminus is at position 5. The aldehyde carbon is carbon 6 of the selected parent chain. Its carbonyl oxygen is indicated by 6-oxo because the acid, rather than the aldehyde, supplies the suffix. Thus the prefixes are 2,5-dimethyl and 6-oxo.

Step 3: Assemble the name and eliminate alternatives Combining the prefixes and parent gives 2,5-dimethyl-6-oxohex-3-enoic acid, written in option (D) with separating hyphens. Options (A) and (C) incorrectly make aldehyde the principal group despite the presence of a higher-priority acid. Option (B) uses a formyl branch and fails to select the appropriate six-carbon chain through the aldehyde carbon; it also fails to account for the two methyl substituents in the selected parent.

Step 4: Conclusion The compound is 2,5-dimethyl-6-oxohex-3-enoic acid. Hence, the correct option is (D).

Quick Tip to Solve: For an acid plus aldehyde, number from COOH; if the CHO carbon joins the parent chain, describe its oxygen with oxo.

58. Consider the following rections:

'x', 'y' and 'z' in these reactions are respectively. [4 marks]

(i) Glucose + ROH $\xrightarrow{\text{dry HCl}}$ Acetal $\xrightarrow[(CH_3CO)_2O]{\text{x eq. of}}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2}$ A $\xrightarrow[(CH_3CO)_2O]{\text{y eq. of}}$ acetyl derivative

(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{\text{z eq. of}}$ acetyl derivative

(A) 5, 6, & 5

(B) 4, 5 & 5

(C) 4, 6 & 5

(D) 5, 4 & 5

Correct Answer: (C) 4, 6 & 5

Solution:

Concept:

- Acetic anhydride acetylates free hydroxyl groups: one equivalent is consumed for each hydroxyl group converted into an acetate ester.
- Glucose contains five hydroxyl groups. Formation of an alkyl glucoside converts its anomeric hydroxyl into an acetal linkage, while reduction of its aldehyde group creates an additional hydroxyl group.

Step 1: Count hydroxyl groups after acetal formation In reaction (i), glucose reacts with an alcohol in dry hydrogen chloride to form an alkyl glucoside. The anomeric hydroxyl is replaced by an alkoxy group, giving an acetal at that carbon. The remaining hydroxyl groups at the other positions are still available for acetylation. There are four such groups, so the subsequent acetylation requires $x = 4$ equivalents of acetic anhydride.

Step 2: Count hydroxyl groups after hydrogenation In reaction (ii), hydrogen in the presence of nickel reduces the aldehyde functionality of glucose to a primary alcohol. The product A is sorbitol, a six-carbon polyhydric alcohol. The original five hydroxyl groups remain, and reduction introduces one more. Hence A has six hydroxyl groups and consumes $y = 6$ equivalents in complete acetylation.

Step 3: Count the groups in unchanged glucose In reaction (iii), glucose is acetylated directly and forms its pentaacetate because it has five hydroxyl groups. Thus $z = 5$. Option (A) incorrectly retains five acetylatable hydroxyl groups after glucoside formation; option (B) misses the additional hydroxyl created by reduction; option (D) gives both of those counts incorrectly. The carbonyl oxygen itself is not counted as a free hydroxyl in this calculation.

Step 4: Conclusion The required values are $x = 4$, $y = 6$ and $z = 5$. Hence, the correct option is (C).

Quick Tip to Solve: Count free OH groups after each reaction: glucoside formation removes one, hydrogenation adds one, and direct acetylation counts five.

59. An open beaker of water in equilibrium with water vapour is in a sealed container. When a few grams of glucose are added to the beaker of water, the rate at which water molecules : [4 marks]

(A) leaves the solution increases

(B) leaves the vapour increases

(C) leaves the vapour decreases

(D) leaves the solution decreases

Correct Answer: (D) leaves the solution decreases

Solution:

Concept:

- At liquid-vapour equilibrium, evaporation and condensation occur at equal rates. The equilibrium vapour pressure of water depends on temperature and its activity in the liquid.
- Glucose is a nonvolatile solute. For an ideal dilute solution, Raoult's law gives $p_{\text{water}} = x_{\text{water}} p^{\circ}_{\text{water}}$, where the water mole fraction becomes less than one on adding glucose.

Step 1: Determine what glucose changes at the surface Before adding glucose, pure water is in dynamic equilibrium with its vapour. Adding dissolved glucose lowers the mole fraction and escaping tendency of water in the liquid. At a fixed temperature, fewer water molecules leave the liquid surface per unit time than from pure water. Thus the evaporation rate, described as the rate at which water molecules leave the solution, decreases.

Step 2: Separate the immediate change from the new equilibrium Immediately after addition, the vapour population has not yet changed, so the condensation rate is initially approximately its previous value. Because evaporation has decreased, there is net condensation until a new equilibrium with lower water vapour pressure is reached. At that equilibrium the two rates are again equal. The primary effect of adding glucose is the reduction in escape from the solution; it is not an increased escape of water from either phase.

Step 3: Compare the options Option (A) gives the reverse of the nonvolatile-solute effect. Option (B) would describe an increased condensation rate and does not express the initial lowering of water's escaping tendency. Option (C) concerns departure from the vapour rather than the direct effect at the solution surface. The standard explanation of vapour-pressure lowering is therefore exactly the decrease in the rate named in option (D).

Step 4: Conclusion Adding glucose decreases the rate at which water molecules leave the solution. Hence, the correct option is (D).

Quick Tip to Solve: For a nonvolatile solute, first focus on evaporation from the liquid surface; its decrease explains the lower equilibrium vapour pressure.

60. While titrating dilute HCl solution with aqueous NaOH, which of the following will not be required? [4 marks]

(A) Pipette and distilled water

(B) Clamp and phenolphthalein

(C) Burette and porcelain tile

(D) Bunsen burner and measuring cylinder

Correct Answer: (D) Bunsen burner and measuring cylinder

Solution:

Concept:

- A strong acid and a strong base react by neutralization: $HCl + NaOH \longrightarrow NaCl + H_2O$. Their dilute aqueous titration is performed at ordinary laboratory temperature.
- Titration requires accurately measured volumes and a visible endpoint. A pipette measures a fixed aliquot and a burette delivers a variable volume accurately; an indicator identifies completion.

Step 1: Identify the volume-measuring equipment A pipette transfers a known volume of the dilute acid into the conical flask. A burette contains the sodium hydroxide solution and measures the volume delivered during the titration. A measuring cylinder has much coarser graduations and is not the apparatus used for accurately measuring the titre or the fixed acid aliquot. Distilled water is useful for washing the flask walls and preparing or rinsing equipment without introducing interfering dissolved substances.

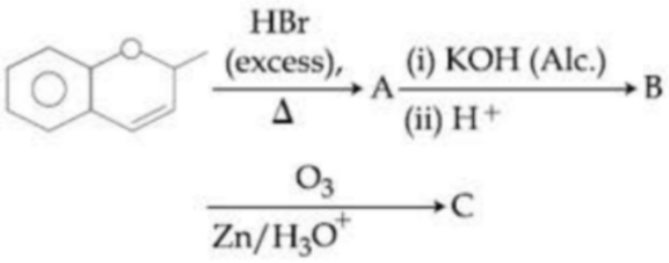
Step 2: Identify the endpoint and supporting equipment A clamp holds the burette vertically and securely. Phenolphthalein is suitable for this strong acid-strong base titration: with acid initially in the flask, the endpoint is the first persistent pale pink colour after adding the base. A white porcelain tile under the flask makes this faint colour easier to distinguish. Thus the pairs in options (A), (B) and (C) consist of equipment or materials with direct practical roles.

Step 3: Reject unnecessary heating and rough measurement No heating is needed to bring about the rapid neutralization of dilute hydrochloric acid by aqueous sodium hydroxide. A Bunsen burner therefore has no role, and heating could introduce avoidable experimental errors. A measuring cylinder also does not replace the accurately calibrated pipette and burette. The pair in option (D), rather than any of the other listed pairs, is not required for carrying out this titration.

Step 4: Conclusion A Bunsen burner and measuring cylinder are not required for the titration. Hence, the correct option is (D).

Quick Tip to Solve: Match each apparatus to its role: pipette for aliquot, burette for titre, indicator for endpoint; a dilute neutralization needs no burner.

61. The major aromatic product C in the following reaction sequence will be : [4 marks]



(A) o-$HOC_6H_4CH_2CO_2H$

(B) o-HOC_6H_4CHO

(C) o-BrC_6H_4CHO

(D) o-$BrC_6H_4CH_2CO_2H$

Correct Answer: (B) o-HOC_6H_4CHO

Solution:

Concept:

- Hot concentrated HBr cleaves the alkyl–oxygen bond of an aryl alkyl ether; the aryl–oxygen bond is retained as a phenolic group.
- Alcoholic KOH removes hydrogen bromide from an alkyl bromide. Reductive ozonolysis cleaves a carbon–carbon double bond to aldehydes or ketones.

Step 1: Open the oxygen-containing ring The starting compound is a methyl-substituted chromene. Protonation of its ether oxygen makes cleavage of the bond to the saturated carbon possible. Bromide attacks on the alkyl side, while the oxygen attached directly to the benzene ring becomes a phenolic hydroxyl group. The aromatic carbon–oxygen bond is not replaced by an aromatic carbon–bromine bond. Excess hydrogen bromide can also add across the side-chain double bond, but the next elimination step removes the added hydrogen bromide.

Step 2: Identify the alkene before ozonolysis Treatment with alcoholic KOH, followed by acidification, gives an ortho-hydroxy aromatic compound with an unsaturated side chain. In the conjugated side-chain alkene, the carbon bonded directly to the aromatic ring bears a hydrogen. This is the structural feature that decides the aromatic ozonolysis product: cleavage at that carbon creates an aldehyde group directly attached to the ring.

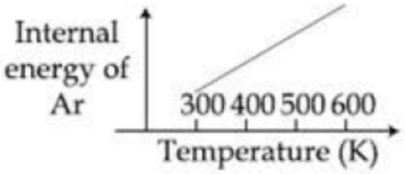
Step 3: Apply reductive cleavage and compare the options With O_3 followed by Zn/H_3O^+, each alkene carbon becomes a carbonyl carbon. The aromatic fragment is therefore o-HOC_6H_4CHO, salicylaldehyde. The zinc work-up is reductive, so an aldehyde is retained rather than oxidised to a carboxylic acid. Options with aromatic bromine incorrectly assume cleavage of the aryl–oxygen bond; the phenylacetic acid option also has the wrong side-chain length and oxidation state.

Step 4: Conclusion The major aromatic product is 2-hydroxybenzaldehyde (salicylaldehyde), o-HOC_6H_4CHO. Hence, the correct option is (B).

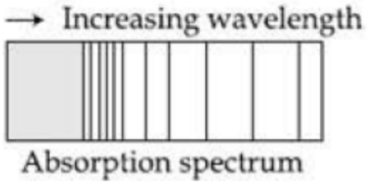
Quick Tip to Solve: For an aryl alkyl ether, retain the oxygen on the aromatic ring; then track which alkene carbon becomes the aromatic carbonyl.

62. The figure that is not a direct manifestation of the quantum nature of atoms is : [4 marks]

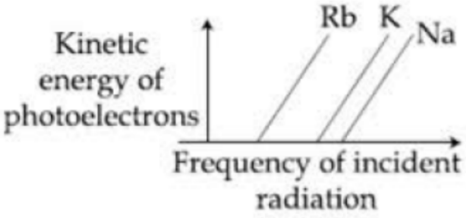
(A)



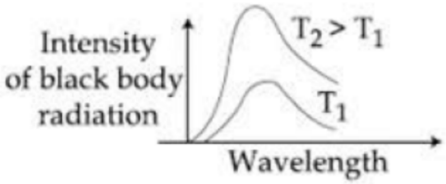
(B)



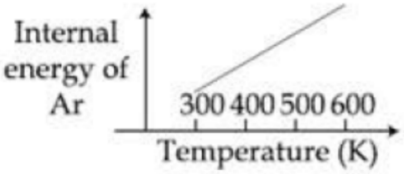
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- Classical equipartition predicts the translational internal energy of an ideal monatomic gas: $U = \frac{3}{2}nRT$.
- Atomic spectral lines, the photoelectric effect and the black-body spectrum require quantisation to explain their characteristic observations.

Step 1: Test the argon internal-energy graph Argon is monatomic. At the temperatures shown, its internal energy is dominated by translational kinetic energy. Classical kinetic theory assigns an

average energy of $\frac{1}{2}k_{\mathrm{B}}T$ to each of the three translational degrees of freedom. Hence

$$U = 3\left(\frac{1}{2}nRT\right) = \frac{3}{2}nRT.$$

A straight-line increase of internal energy with temperature therefore follows without using discrete electronic levels or photon energy packets.

Step 2: Compare the explicitly quantum observations The absorption spectrum contains separated dark lines because atoms absorb only energies equal to differences between their allowed electronic energy levels: $h\nu = E_2 - E_1$. The photoelectron graph has a threshold frequency and obeys $K_{\max} = h\nu - \phi$; a classical wave description cannot explain the threshold and the frequency dependence. These observations directly expose quantised energy transfer.

Step 3: Check the black-body alternative The finite black-body intensity distribution and its wavelength-dependent maximum are explained by Planck's quantisation of oscillator energies. Classical equipartition instead produces the ultraviolet catastrophe. Thus the spectrum, photoelectric plot and black-body curves all represent the failure of a wholly classical account, whereas the argon energy–temperature line is already a classical result. The word "not" selects this latter graph.

Step 4: Conclusion The linear internal-energy versus temperature graph of argon is not a direct manifestation of quantum behaviour. Hence, the correct option is (A).

Quick Tip to Solve: A linear thermal-energy graph can follow from classical equipartition; discrete lines and threshold effects are stronger quantum signatures.

63. If AB_4 molecule is a polar molecule, a possible geometry of AB_4 is [4 marks]

(A) Tetrahedral

(B) see-saw

(C) Square pyramidal

(D) Square planar

Correct Answer: (B) see-saw

Solution:

Concept:

- A molecular dipole is the vector sum of the individual bond dipoles. Identical bonds can cancel only when the molecular geometry has the required symmetry.
- An AB_4 species with one central lone pair has the AX_4E arrangement: trigonal-bipyramidal electron geometry and see-saw molecular geometry.

Step 1: Examine the symmetric four-coordinate shapes In a regular tetrahedral AB_4 molecule, four equal bond-dipole vectors are symmetrically directed towards the vertices of a tetrahedron. Their resultant is zero. In a square-planar molecule, each bond dipole is cancelled by the identical bond directly opposite it. Thus these two ideal geometries do not account for a polar molecule with four identical surrounding atoms.

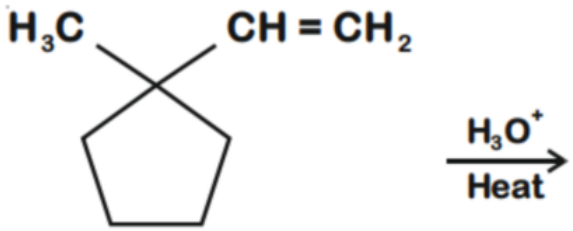
Step 2: Place the lone pair in the five-domain arrangement For five electron domains, the parent electron geometry is trigonal bipyramidal. A lone pair prefers an equatorial site because it then experiences fewer right-angle interactions than at an axial site. The four bonds occupy the two axial positions and the two remaining equatorial positions, giving a see-saw shape. The axial bond dipoles largely cancel, but the equatorial pair does not have an opposite partner to cancel its resultant.

Step 3: Identify the possible polar geometry The asymmetric see-saw arrangement can consequently have a nonzero molecular dipole, as illustrated by SF_4. Square pyramidal is a five-bond molecular geometry, conventionally AX_5E, so it is not a geometry for a central atom bonded to only four B atoms. Among the listed possibilities, see-saw is therefore the one compatible with both the formula and the stated polarity.

Step 4: Conclusion A possible geometry is see-saw, for which the bond dipoles need not cancel. Hence, the correct option is (B).

Quick Tip to Solve: Count bonded atoms separately from lone pairs before using polarity: see-saw has four bonds, whereas square pyramidal has five.

64. The major product in the following reaction is : [4 marks]



(A) 1-(Propan-2-yl)cyclopent-1-ene

(B) 1-(1-Methylcyclopentyl)ethan-1-ol

(C) 1,2-Dimethylcyclohex-1-ene

(D) 3,3-Dimethylcyclohex-1-ene

Correct Answer: (C) 1,2-Dimethylcyclohex-1-ene

Solution:

Concept:

- Acid protonates an alkene to produce the more stable accessible carbocation; a neighbouring carbon–carbon bond can migrate by a 1,2-shift.
- Ring expansion from a five-membered ring to a six-membered ring can relieve strain. Heating favours elimination and formation of the more substituted alkene.

Step 1: Form the initial carbocation The substrate is 1-methyl-1-vinylcyclopentane. Proton addition to the terminal CH_2 end of its vinyl group places the positive charge on the adjacent carbon: the side chain becomes $-CH^+-CH_3$. This secondary carbocation lies immediately beside the quaternary carbon of the cyclopentane ring. That neighbouring ring carbon cannot stabilise it by hydride

migration because it has no hydrogen.

Step 2: Expand the ring by carbon migration One of the adjacent cyclopentane carbon–carbon bonds migrates to the cationic side-chain carbon. The migrating bond supplies its electron pair to form a new bond, incorporating that side-chain carbon into the ring. The ring therefore expands to six members, and the positive charge moves to the original substituted ring carbon. The new carbocation is tertiary; the rearrangement also reduces the strain associated with the smaller ring.

Step 3: Eliminate to obtain the stable alkene Under the heated acidic conditions, loss of a neighbouring proton creates the double bond between the two adjacent methyl-bearing ring carbons. The product is 1,2-dimethylcyclohex-1-ene, a tetrasubstituted alkene. The unrearranged alcohol ignores rearrangement and subsequent elimination. The cyclopentene retains the smaller ring, and 3,3-dimethylcyclohex-1-ene has the wrong methyl positions and a less substituted double bond.

Step 4: Conclusion The major product is 1,2-dimethylcyclohex-1-ene. Hence, the correct option is (C).

Quick Tip to Solve: When a vinyl group adjoins a substituted small ring, check for ring expansion before selecting a simple hydration product.

65. In Carius method of estimation of halogen, 0.172 g of an organic compound showed presence of 0.08 g of bromine. Which of these is the correct structure of the compound? [4 marks]

(A) CH_3CH_2Br

(B) CH_3Br

(C) 4-Bromoaniline

(D) 2,4-Dibromoaniline

Correct Answer: (C) 4-Bromoaniline

Solution:

Concept:

- The bromine mass fraction is the mass of bromine divided by the mass of the organic sample.
- For a compound containing r bromine atoms per molecule, $w_{Br} = rM_{Br}/M$, where M is its molar mass.

Step 1: Calculate the experimental bromine percentage The sample has mass $0.172\,g$, and the bromine present has mass $0.080\,g$. Therefore

$$\%Br = \frac{0.080}{0.172} \times 100 = 46.51\%.$$

The question already gives the bromine mass, not the mass of silver bromide, so no $AgBr$-to-bromine conversion is required.

Step 2: Check the monobromo compounds Taking the atomic masses $C = 12$, $H = 1$, $N = 14$ and $Br = 80$, bromoethane has molar mass $109\,g\,mol^{-1}$, giving $80/109 \times 100 = 73.39\%$ bromine. Bromomethane has molar mass $95\,g\,mol^{-1}$, giving 84.21%. Both percentages greatly exceed the

measured value. For 4-bromoaniline, C_6H_6BrN, the molar mass is

$$M = 6(12) + 6(1) + 80 + 14 = 172\,\text{g mol}^{-1}.$$

Its bromine percentage is exactly $80/172 \times 100 = 46.51\%$.

Step 3: Reject the dibromo alternative 2,4-Dibromoaniline has formula $C_6H_5Br_2N$ and molar mass $251\,\text{g mol}^{-1}$. It contains $160\,\text{g}$ of bromine per mole, so its bromine percentage is $160/251 \times 100 = 63.75\%$, not the observed value. Equivalently, $0.172\,\text{g}$ of 4-bromoaniline is $0.001\,\text{mol}$, containing $0.001 \times 80 = 0.080\,\text{g}$ bromine.

Step 4: Conclusion The compound is 4-bromoaniline, C_6H_6BrN. Hence, the correct option is (C).

Quick Tip to Solve: First decide whether the given analytical mass is elemental halogen or silver halide; only the latter needs a conversion factor.

66. The statement that is not true about ozone is : [4 marks]

(A) In the stratosphere, CFCs release chlorine free radicals (Cl) which reacts with O_3 to give chlorine dioxide radicals

(B) It is a toxic gas and its reaction with NO gives NO_2.

(C) In the atmosphere, it is depleted by CFCs

(D) In the stratosphere, it forms a protective shield against UV radiation

Correct Answer: (A) In the stratosphere, CFCs release chlorine free radicals (Cl) which reacts with O_3 to give chlorine dioxide radicals

Solution:

Concept:

- Ultraviolet radiation decomposes chlorofluorocarbons in the stratosphere, releasing chlorine radicals that catalyse ozone destruction.
- The first chlorine-radical reaction produces chlorine monoxide radical, $ClO^{\bullet}$, not chlorine dioxide radical, $ClO_2^{\bullet}$.

Step 1: Write the chlorine-catalysed reaction correctly The relevant propagation step is

$$Cl^{\bullet} + O_3 \longrightarrow ClO^{\bullet} + O_2.$$

A chlorine atom removes one oxygen atom from ozone and forms chlorine monoxide radical. The statement in option (A) instead names chlorine dioxide radicals. That changes the chemical species and makes the statement incorrect, although its preceding claim that CFCs release chlorine radicals is correct.

Step 2: Check ozone toxicity and its reaction with nitric oxide Ozone is a strong oxidant and is harmful to living tissue when present near ground level. It oxidises nitric oxide to nitrogen dioxide according to

$$NO + O_3 \longrightarrow NO_2 + O_2.$$

Thus the two claims made in option (B) are both true. The toxicity of ozone in the lower atmosphere is compatible with its beneficial role higher in the atmosphere; the location matters.

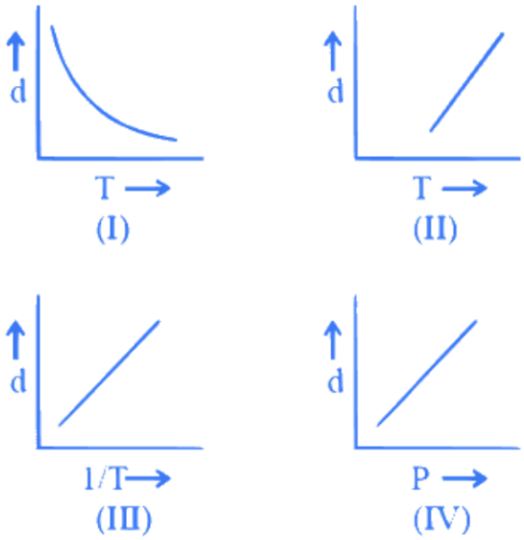
Step 3: Verify the atmospheric statements Chlorine monoxide radicals can react with atomic oxygen to regenerate chlorine radicals: $ClO^{\bullet} + O \longrightarrow Cl^{\bullet} + O_2$. The regenerated radical can destroy further ozone molecules, explaining why CFCs deplete ozone catalytically and why option (C) is true. Stratospheric ozone absorbs harmful ultraviolet radiation and acts as a protective shield, so option (D) is also true. Only the incorrect radical name in option (A) fails the test.

Step 4: Conclusion The false statement is the claim that chlorine radicals react with ozone to give chlorine dioxide radicals; the product is chlorine monoxide radical. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish chlorine monoxide, ClO, from chlorine dioxide, ClO2: the ozone-depletion propagation step makes ClO.

67. Which one of the following graphs is not correct for ideal gas?

d = Density, P = Pressure, T = Temperature [4 marks]



(A) II

(B) III

(C) I

(D) IV

Correct Answer: (A) II

Solution:

Concept:

- For an ideal gas, $PV = nRT$. With molar mass M, mass m, and density $d = m/V$, this becomes $d = PM/(RT)$.
- Temperature in gas-law graphs is absolute temperature in kelvin. Compare two variables while

holding the other independent variable fixed.

Step 1: Derive the density relations Substitute $n = m/M$ into the ideal-gas equation and divide by volume.

$$P = \frac{m}{V}\frac{RT}{M} = \frac{dRT}{M}, \qquad d = \frac{PM}{RT}.$$

At fixed pressure and fixed gas composition, the density is inversely proportional to absolute temperature. Heating expands a fixed mass of gas at constant pressure, so its mass per unit volume decreases. A positive linear increase of density with temperature would contradict this relation.

Step 2: Test each graph Graph I shows density falling hyperbolically as T increases; this agrees with $dT = PM/R$ at constant pressure. Graph II instead shows density increasing linearly with T, so it is incorrect. Graph III plots d against $1/T$: the proportionality $d = (PM/R)(1/T)$ gives a straight line with positive slope. Graph IV plots density against pressure at fixed temperature: $d = (M/RT)P$ again gives a straight line with positive slope. Thus I, III and IV all represent the appropriate ideal-gas dependence, whereas II does not.

Step 3: Conclusion Graph II is not correct for an ideal gas. Hence, the correct option is (A).

Quick Tip to Solve: First write $d = PM/(RT)$; then identify which quantity is constant before judging the shape or slope of a graph.

68. Which of the following is used for the preparation of colloids? [4 marks]

(A) Van Arkel Method

(B) Bredig's Arc Method

(C) Mond Process

(D) Ostwald Process

Correct Answer: (B) Bredig's Arc Method

Solution:

Concept:

- Colloidal particles have sizes intermediate between molecules and coarse suspended particles. A dispersion method breaks bulk material into particles of colloidal dimensions.
- Bredig's arc method prepares metal sols by striking an electric arc between metal electrodes immersed in a suitable cold dispersion medium.

Step 1: Identify the colloid preparation method In Bredig's arc method the metal whose sol is required forms the electrodes. An electric arc produces intense local heating and vaporises some metal. The vapour condenses in the surrounding cold dispersion medium to form fine particles. The process therefore involves both dispersion and condensation and is especially useful for sols of metals such as gold, silver and platinum. Cooling limits excessive aggregation, while a suitable stabilising agent can help preserve the dispersed state.

Step 2: Distinguish the other named processes The van Arkel method is a purification method for metals such as titanium and zirconium. The impure metal is converted into a volatile iodide, which decomposes on a hot filament to deposit pure metal. The Mond process purifies nickel through volatile

nickel carbonyl, followed by its decomposition. The Ostwald process manufactures nitric acid by catalytic oxidation of ammonia, followed by further oxidation and absorption. None of these three is the standard method asked for preparing a colloidal sol. The electric-arc dispersion of metal in Bredig's method directly produces the required colloidal particles.

Step 3: Conclusion Bredig's arc method is used for preparing colloids. Hence, the correct option is (B).

Quick Tip to Solve: Associate Bredig with metal sols, Mond with nickel purification, van Arkel with iodides, and Ostwald with nitric acid.

69. On heating compound (A) gives a gas (B) which is a constituent of air. This gas when treated with H_2 in the presence of a catalyst gives another gas (C) which is basic in nature. (A) should not be : [4 marks]

(A) $Pb(NO_3)_2$

(B) $(NH_4)_2Cr_2O_7$

(C) NH_4NO_2

(D) NaN_3

Correct Answer: (A) $Pb(NO_3)_2$

Solution:

Concept:

- Heating several ammonium salts and metal azides liberates nitrogen, N_2, a major constituent of air.
- In the Haber reaction, nitrogen reacts with hydrogen over an iron catalyst to form ammonia: $N_2 + 3\,H_2 \rightleftharpoons 2\,NH_3$. Ammonia is basic because it accepts a proton.

Step 1: Identify gases B and C The clue that the gas from air combines with hydrogen in the presence of a catalyst to produce a basic gas identifies B as nitrogen and C as ammonia. In water, $NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$, explaining the basic nature. Thus the permitted compound must produce molecular nitrogen on heating; merely producing any gas that occurs in air is not enough to satisfy the whole sequence.

Step 2: Check all four decompositions The relevant balanced thermal decompositions are

$$(NH_4)_2Cr_2O_7 \longrightarrow Cr_2O_3 + N_2 + 4\,H_2O,$$
$$NH_4NO_2 \longrightarrow N_2 + 2\,H_2O,$$
$$2\,NaN_3 \longrightarrow 2\,Na + 3\,N_2.$$

All three release nitrogen and therefore fit the route to ammonia. Lead nitrate behaves differently:

$$2\,Pb(NO_3)_2 \longrightarrow 2\,PbO + 4\,NO_2 + O_2.$$

It liberates nitrogen dioxide and oxygen, rather than nitrogen gas. Neither product is the nitrogen required in the stated Haber synthesis. Consequently lead nitrate is the compound that should not be selected as A.

Step 3: Conclusion Compound A should not be $Pb(NO_3)_2$. Hence, the correct option is (A).

Quick Tip to Solve: Use the final product to work backwards: a basic gas made from a constituent of air and hydrogen points to ammonia and nitrogen.

70. The metal mainly used in devising photoelectric cells is : [4 marks]

(A) Na

(B) Rb

(C) Li

(D) Cs

Correct Answer: (D) Cs

Solution:

Concept:

- The photoelectric effect ejects electrons from a metal surface when incident photons have sufficient energy to overcome the work function ϕ. The threshold condition is $h\nu \geq \phi$.
- Alkali metals lose their outermost electron readily. Caesium has a very low ionisation energy and a low work function, making it useful as a photosensitive material.

Step 1: Relate electron emission to the metal property A photoelectric cell requires a photosensitive surface that emits electrons efficiently under illumination. The energy balance for an emitted electron is $K_{max} = h\nu - \phi$. A smaller work function permits emission for lower photon energies, corresponding to a lower threshold frequency and a longer threshold wavelength. Therefore a metal that holds its valence electron weakly is favoured when choosing among the listed alkali metals. The selection concerns ease of photoemission, rather than mechanical strength or melting point.

Step 2: Compare the listed alkali metals In the alkali-metal group, increasing atomic size and shielding down the group generally make the outermost electron easier to remove. Lithium has a small atom and holds its valence electron relatively tightly. Sodium has a lower ionisation energy than lithium, but is not the preferred answer among these choices. Rubidium releases electrons readily, yet caesium lies below it and has an especially low electron-removal energy. Caesium is consequently widely associated with photoelectric cells and photosensitive cathodes. This comparison eliminates Li, Na and Rb in favour of Cs; it does not require ionisation energy and solid-state work function to be numerically identical.

Step 3: Conclusion The metal mainly used in devising photoelectric cells is caesium, Cs. Hence, the correct option is (D).

Quick Tip to Solve: For this standard application, connect caesium with easy electron emission and a low photoelectric threshold frequency.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The Gibbs change (in J) for the given reaction at
$[Cu^{2+}] = [Sn^{2+}] = 1$ M and 298K is :
$Cu(s) + Sn^{2+}(aq.) \rightarrow Cu^{2+}(aq.) + Sn(s)$;
($E^0_{Sn^{2+}|Sn} = -0.16\,V$,
$E^0_{Cu^{2+}|Cu} = 0.34\,V$)
Take F = 96500 C mol^{-1}) [4 marks]

Solution:

Concept:

- For a reaction transferring n moles of electrons, $\Delta G = -nFE$, where F is the Faraday constant and E is the cell potential for the reaction as written.
- Using reduction potentials, $E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$. Unit activities make the reaction quotient unity, so $E = E^\circ$.

Step 1: Find the potential in the stated direction Copper is oxidised in the given reaction, whereas tin(II) is reduced:

$$\mathrm{Cu\,(s)} \longrightarrow \mathrm{Cu^{2+}(aq)} + 2\,\mathrm{e^-},$$
$$\mathrm{Sn^{2+}(aq)} + 2\,\mathrm{e^-} \longrightarrow \mathrm{Sn\,(s)}.$$

The electron-transfer number is therefore $n = 2$, not four: the same two electrons cancel when the half-reactions are added. With the given reduction potentials,

$$E^\circ_{\text{cell}} = (-0.16) - (+0.34) = -0.50\,\mathrm{V}.$$

The negative sign is appropriate because the reaction is written opposite to the spontaneous copper-deposition direction.

Step 2: Calculate the Gibbs energy change The reaction quotient is $Q = [\mathrm{Cu^{2+}}]/[\mathrm{Sn^{2+}}] = 1$; the pure solids have unit activity. Hence the Nernst logarithmic term vanishes at the stated concentrations and $E = -0.50\,\mathrm{V}$. Substitute into the Gibbs-energy relation:

$$\begin{aligned}\Delta G &= -nFE \\ &= -(2)(96500)(-0.50) \\ &= +96500\,\mathrm{J}.\end{aligned}$$

The positive result signifies that the reaction as written is non-spontaneous under these conditions. Reversing the reaction would reverse the sign, but the question requires the stated direction. The numerical value is per reaction amount represented by the balanced equation.

Step 3: Conclusion The Gibbs energy change for the reaction as written is $\boxed{+96500\,\mathrm{J}}$. The required numerical entry is 96500.

Quick Tip to Solve: Write the oxidation and reduction half-reactions first; then use their direction to fix the sign of E before applying $\Delta G = -nFE$.

72. The internal energy change (in J) When 90 g of water undergoes complete evaporation at 100°C is _____.

(Given : ΔH_{vap} for water at 373 K = 41 kJ/mol,

R = 8.314 JK^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- Enthalpy is defined by $H = U + PV$. For vaporisation to an ideal gas with negligible liquid volume, $\Delta H = \Delta U + \Delta n_g RT$.
- The quoted enthalpy of vaporisation is per mole. Convert the water mass to moles and use consistent joule units before subtracting the expansion term.

Step 1: Calculate the amount and enthalpy change The molar mass of water is $18\,\mathrm{g\,mol^{-1}}$. Therefore

$$n = \frac{90}{18} = 5\,\mathrm{mol}.$$

Each mole of liquid becomes one mole of water vapour, so the increase in gaseous amount is $\Delta n_g = 5\,\mathrm{mol}$. The enthalpy absorbed in complete evaporation is

$$\Delta H = 5(41 \times 10^3) = 205000\,\mathrm{J}.$$

The temperature remains at the boiling temperature during evaporation; the supplied energy changes the phase rather than producing a temperature rise.

Step 2: Subtract the pressure-volume contribution The final water vapour contributes approximately nRT to PV, while the initial liquid volume is negligible in the standard vaporisation relation. Thus

$$\begin{aligned}\Delta U &= \Delta H - \Delta n_g RT \\ &= 205000 - 5(8.314)(373) \\ &= 205000 - 15505.61 \\ &= 189494.39\,\mathrm{J}.\end{aligned}$$

Internal energy rises, but by less than the enthalpy because part of the supplied heat is associated with expansion against the surroundings. The calculation uses $373\,\mathrm{K}$, as specified with the enthalpy data, rather than substituting the Celsius value 100 in the ideal-gas term. Rounding the numerical answer to the nearest joule gives 189494.

Step 3: Conclusion The internal energy change is $\boxed{\Delta U = +189494.39\,\mathrm{J} \approx +189494\,\mathrm{J}}$.

Quick Tip to Solve: For vaporisation, remember $\Delta U = \Delta H - nRT$; multiply both the molar enthalpy and the gas term by the number of moles.

73. The oxidation states of iron atoms in compounds (A), (B) and (C), respectively, are x, y and z. The sum of x, y and z is _____.

$Na_4[Fe(CN)_5(NOS)]$

(A)

$Na_4[FeO_4]$

(B)

$[Fe_2(CO)_9]$

(C)

[4 marks]

Solution:

Concept:

- The sum of oxidation numbers in an ion equals its charge. Sodium is present as Na^{+}, cyanide as CN^{-}, and oxide as O^{2-}.
- In compound (A), the combined NOS ligand has charge -1. Carbon monoxide is a neutral ligand, and an iron–iron bond does not change either iron atom's oxidation number.

Step 1: Balance the charge in compound (A) The four sodium ions require the complex ion to have charge -4. The five cyanide ligands contribute -5, and the NOS^{-} ligand contributes -1. Thus

$$x + 5(-1) + (-1) = -4, \qquad x = +2.$$

Here NOS must be treated as a combined ligand, rather than assigning an independent oxide charge to its oxygen. The charge of a ligand as a whole is what enters the coordination-ion balance.

Step 2: Calculate the oxidation number in compound (B) The four sodium ions again contribute $+4$, so the $\mathrm{FeO_4}$ ion has charge -4. Each of its four oxide atoms has oxidation number -2:

$$y + 4(-2) = -4, \qquad y = +4.$$

The sodium subscript is essential: the given formula contains four sodium atoms, so this charge balance must not be replaced by that of a different ferrate salt.

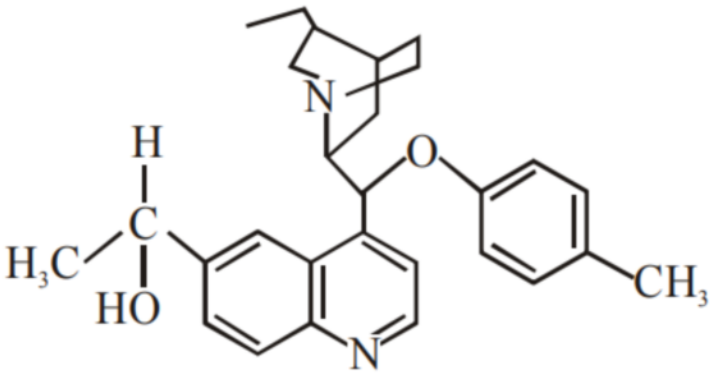
Step 3: Evaluate the neutral carbonyl and add In $\mathrm{Fe_2(CO)_9}$, all nine carbonyl ligands are neutral. The molecule is also neutral, so the two iron atoms have oxidation number zero. The metal–metal bond is between identical atoms and contributes nothing to an oxidation-number assignment. Hence $z = 0$, and

$$x + y + z = 2 + 4 + 0 = 6.$$

Step 4: Conclusion The oxidation states are $x = +2$, $y = +4$ and $z = 0$. Therefore the required numerical answer is $\boxed{6}$.

Quick Tip to Solve: First find the complex ion's charge from its counterions; count neutral CO as zero and use the charge of each combined ligand as a whole.

74. The number of chiral carbons present in the molecule given below is _____ . [4 marks]



Solution:

Concept:

- A chiral carbon is a tetrahedral sp^3 carbon attached to four distinguishable substituents. In a ring, the two directions around the ring must be compared as complete paths.
- Aromatic sp^2 carbons and carbons bearing two identical hydrogen atoms cannot be chiral centres. A nitrogen atom is not included when the question asks specifically for chiral carbons.

Step 1: Identify the two centres outside the bicyclic ring On the left, the carbon bearing OH is attached to H, OH, CH_3, and the substituted aromatic ring. These are four different groups, so it is one chiral carbon. The central carbon joining the aromatic system to the bicyclic amine and to the oxygen-linked aryl group is also tetrahedral. Its fourth substituent is an implicit hydrogen. Its three non-hydrogen groups are different, so it supplies a second chiral carbon.

Step 2: Trace the distinguishable paths in the bicyclic amine The saturated bicyclic part contains three more qualifying carbons: the upper carbon bearing the ethyl side chain, the carbon bridgehead at the right-hand junction, and the lower carbon adjacent to nitrogen that carries the external substituted methine group. At each of these positions, the attached ring paths are inequivalent. One path reaches nitrogen or the substituted carbon in a different sequence from the other paths; therefore the ring bonds do not represent identical substituents. Each of these three carbons has four distinct attachments.

Step 3: Exclude the remaining atoms and total the count The aromatic carbons are planar sp^2 centres, while the unsubstituted methylene and methyl carbons have repeated hydrogens. Neither category contributes. The amine nitrogen is also excluded from a count of carbon stereocentres. Thus the total is

$$N_{\text{chiral carbons}} = 2 + 3 = 5.$$

Step 4: Conclusion There are two chiral carbons outside the bicyclic amine and three within it. The required answer is $\boxed{5}$.

Quick Tip to Solve: At a bicyclic carbon, trace each ring path until its first difference; different routes through the same ring can be different substituents.

75. The mass of gas adsorbed, x, per unit mass of adsorbate, m, was measured at various pressures, p. A graph between $\log \frac{x}{m}$ and log p gives a straight line with slope equal to 2 and the intercept equal to 0.4771. The value of $\frac{x}{m}$ at a pressure of 4 atm is :

(Given log 3 = 0.4771) [4 marks]

Solution:

Concept:

- A straight-line plot of $Y = \log(x/m)$ against $X = \log p$ has the equation $Y = aX + b$, where a is the slope and b is the vertical intercept.
- For a power law $x/m = kp^a$, taking common logarithms gives $\log(x/m) = \log k + a \log p$. Thus the intercept determines k, while the slope itself is the exponent of pressure.

Step 1: Write the equation of the measured straight line Use the horizontal variable $\log p$ and the vertical variable $\log(x/m)$ exactly as specified. With slope 2 and intercept 0.4771, the line is

$$\log\left(\frac{x}{m}\right) = 2\log p + 0.4771.$$

The intercept is the logarithm of the coefficient, not the coefficient itself. Also, no reciprocal is needed for the given slope: it multiplies $\log p$ directly.

Step 2: Recover the power law The supplied value $\log 3 = 0.4771$ identifies the antilogarithm of the intercept. Applying the laws of logarithms,

$$\begin{aligned}\log\left(\frac{x}{m}\right) &= \log(p^2) + \log 3 \\ &= \log(3p^2).\end{aligned}$$

Therefore $x/m = 3p^2$, with pressure expressed in the stated atmosphere convention. This relation follows directly from the measured graph, irrespective of the names sometimes used for the power-law parameters.

Step 3: Substitute the required pressure At $p = 4\,\text{atm}$, substitute the numerical pressure in the fitted relation:

$$\frac{x}{m} = 3(4)^2 = 3 \times 16 = 48.$$

Squaring the pressure before multiplying by 3 is essential. Using 2 as a multiplier instead of the exponent, or using 0.4771 as the coefficient, would not reproduce the given slope and intercept.

Step 4: Conclusion At a pressure of $4\,\text{atm}$, the required numerical value is $\boxed{x/m = 48}$, in the mass-ratio convention used for the graph.

Quick Tip to Solve: For a log–log plot, take the antilogarithm of the intercept to obtain the coefficient, and use the slope as the exponent.