JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 2 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 2 September 2020, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval : [4 marks]

(A) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$
(B) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$
(C) $\left(-\frac{5}{4}, -1\right]$
(D) $\left[-1, -\frac{1}{2}\right]$

Correct Answer: (D) $\left[-1, -\frac{1}{2}\right]$

Solution:

Concept:

- The identity $\sin^2\theta + \cos^2\theta = 1$ converts the fourth powers into a square of a double-angle function.
- For real θ, $0 \le \sin^2(2\theta) \le 1$. Both endpoints occur, so the resulting interval is closed.

Step 1: Rewrite the expression using an identity Square the sum of the two squared trigonometric functions and subtract the product term:

$$\cos^4\theta + \sin^4\theta = (\cos^2\theta + \sin^2\theta)^2 - 2\sin^2\theta\cos^2\theta$$
$$= 1 - \frac{1}{2}\sin^2(2\theta).$$

The given equation is therefore equivalent to

$$\lambda = -1 + \frac{1}{2}\sin^2(2\theta).$$

This form exposes the complete dependence on the real angle.

Step 2: Find the attainable values and check the options Using the bounds of the squared sine gives

$$-1 \le \lambda \le -\frac{1}{2}.$$

The left endpoint occurs at $\theta = 0$, while the right endpoint occurs at $\theta = \pi/4$. Every intermediate value is attained because $\sin^2(2\theta)$ takes every value between zero and one. Thus this condition is both necessary and sufficient for a real solution. Options (A) and (C) include values below -1, which would require a negative squared sine. Option (B) lies above $-1/2$ and would require a squared sine larger than one.

Step 3: Conclusion The required interval is $\boxed{[-1, -1/2]}$. Hence, the correct option is (D).

Quick Tip to Solve: Express a sum of fourth powers through the squared sum first; then check that the endpoint angles exist before choosing open or closed brackets.

2. Let A = {X = (x, y, z)^T: PX = 0 and $x^2 + y^2 + z^2 = 1$} where

$$P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix},$$

then the set A : [4 marks]

(A) is an empty set.

(B) contains more than two elements.

(C) contains exactly two elements.

(D) is a singleton.

Correct Answer: (C) contains exactly two elements.

Solution:

Concept:

- The equation $PX = 0$ describes the null space of P, where $X = (x, y, z)^{\mathrm{T}}$. Solve the linear equations before imposing the unit-length condition.
- A one-dimensional null space is a line through the origin. Its intersection with $x^2 + y^2 + z^2 = 1$ consists of the two opposite unit vectors on that line.

Step 1: Find all solutions of the homogeneous equations The first two rows give

$$x + 2y + z = 0, \qquad -2x + 3y - 4z = 0.$$

Substituting $x = -2y - z$ into the second equation yields

$$7y - 2z = 0.$$

Writing $z = 7t$, we obtain $y = 2t$ and $x = -11t$. The third row adds no restriction, since

$$x + 9y - z = (-11 + 18 - 7)t = 0.$$

Consequently every solution is $X = t(-11, 2, 7)^{\mathrm{T}}$, with real t. The first two equations are independent, so there is no further free direction.

Step 2: Impose the unit-length condition The squared norm of this vector is

$$x^2 + y^2 + z^2 = (121 + 4 + 49)t^2 = 174t^2.$$

The additional constraint therefore requires

$$t = \frac{1}{\sqrt{174}} \quad \text{or} \quad t = -\frac{1}{\sqrt{174}}.$$

Both parameters give valid, distinct vectors. The zero vector does not satisfy the unit-sphere equation. Hence the set is not empty or a singleton, ruling out (A) and (D). Although the homogeneous system alone has infinitely many solutions, the norm constraint retains only two, ruling out (B).

Step 3: Conclusion The set is $A = \{\pm(-11, 2, 7)^{\mathrm{T}}/\sqrt{174}\}$, containing exactly two elements. Hence, the correct option is (C).

Quick Tip to Solve: For a homogeneous system with a unit-norm constraint, determine the dimension of the null space first; a single free direction gives two opposite unit vectors.

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3. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is : [4 marks]

(A) $\sqrt{30}$

(B) $2\sqrt{6}$

(C) $\frac{4\sqrt{5}}{3}$

(D) $\frac{2\sqrt{5}}{3}$

Correct Answer: (C) $\frac{4\sqrt{5}}{3}$

Solution:

Concept:

- For $x^2/a_h^2 - y^2/b_h^2 = 1$, the hyperbola eccentricity satisfies $e_h^2 = 1 + b_h^2/a_h^2$.
- For an ellipse with major semiaxis a and minor semiaxis b, $e_e^2 = 1 - b^2/a^2$ and the full latus rectum has length $2b^2/a$.

Step 1: Put both conics in standard form Dividing the hyperbola equation by 10 gives

$$\frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1, \qquad e_h^2 = 1 + \cos^2\theta.$$

The ellipse becomes

$$\frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1.$$

Because $0 < \theta < \pi/2$, we have $0 < \cos^2\theta < 1$. Thus the ellipse has its major axis along the y-axis: $a^2 = 5$, $b^2 = 5\cos^2\theta$, and $e_e^2 = 1 - \cos^2\theta$. Choosing the larger denominator as a^2 is essential here.

Step 2: Use the eccentricity condition and calculate the length The given relation is $e_h = \sqrt{5}\,e_e$. Squaring it gives

$$1 + \cos^2\theta = 5(1 - \cos^2\theta), \qquad \cos^2\theta = \frac{2}{3}.$$

This lies within the permitted interval. Therefore $b^2 = 10/3$, while $a = \sqrt{5}$, and

$$L = \frac{2b^2}{a} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}.$$

Option (D) is only half this length, the semilatus rectum. The values in (A) and (B) do not equal $2b^2/a$ for these semiaxes and are ruled out by direct substitution.

Step 3: Conclusion The full latus rectum of the ellipse has length $\boxed{4\sqrt{5}/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Identify the major axis from the larger denominator before using eccentricity or latus-rectum formulae; the major axis need not be the x-axis.

4. If the sum of first 11 terms of an A.P.,
a_1, a_2, a_3, ... is 0 ($a \neq 0$), then the sum of the A.P.,
a_1 , a_3 , a_5 ,....., a_{23} is ka_1 , where k is equal to : [4 marks]

(A) $\frac{121}{10}$
(B) $-\frac{121}{10}$
(C) $\frac{72}{5}$
(D) $-\frac{72}{5}$

Correct Answer: (D) $-\frac{72}{5}$

Solution:

Concept:

- For an A.P. with first term a_1 and common difference d, $a_n = a_1 + (n-1)d$ and $S_n = n[2a_1 + (n-1)d]/2$.
- The odd-indexed terms $a_1, a_3, \ldots, a_{23}$ themselves form an A.P. with common difference $2d$. Count their number from their indices.

Step 1: Determine the common difference from the first sum The given zero sum of the first eleven terms implies

$$0 = S_{11} = \frac{11}{2}(2a_1 + 10d).$$

It follows that

$$2a_1 + 10d = 0, \qquad d = -\frac{a_1}{5}.$$

Equivalently, the middle term $a_6 = a_1 + 5d$ is zero. The nonzero first term ensures that the requested coefficient multiplying a_1 is uniquely determined.

Step 2: Count and sum the odd-indexed terms The indices are $1, 3, 5, \ldots, 23$, giving

$$n = \frac{23-1}{2} + 1 = 12$$

terms. Their last term is $a_{23} = a_1 + 22d$. Hence their sum is

$$\begin{aligned} T &= \frac{12}{2}(a_1 + a_{23}) \\ &= 6(2a_1 + 22d) = 12(a_1 + 11d) \\ &= 12a_1\left(1 - \frac{11}{5}\right) = -\frac{72}{5}a_1. \end{aligned}$$

Thus the sum is opposite in sign to the first term. This excludes the positive coefficients (A) and (C). Option (B) has the wrong magnitude; it does not use the twelve odd-indexed terms ending at index 23. The explicit endpoint calculation fixes both the count and the coefficient.

Step 3: Conclusion Comparing $T = ka_1$ with the calculated sum gives $\boxed{k = -72/5}$. Hence, the correct option is (D).

Quick Tip to Solve: An A.P. made from every second term has twice the original common difference. Count selected indices explicitly; the final index is not the number of terms.

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5. Let f : (–1, ∞) $\to$ R be defined by f(0) = 1 and f(x) = $\frac{1}{x}\log_e(1+x)$, x $\neq$ 0. Then the function f : [4 marks]

(A) decreases in (–1, ∞)

(B) decreases in (–1, 0) and increases in (0, ∞)

(C) increases in (–1, ∞)

(D) increases in (–1, 0) and decreases in (0, ∞)

Correct Answer: (A) decreases in (–1, ∞)

Solution:

Concept:

- A function is strictly decreasing on an interval if its derivative is negative there. A separately defined value at a removable singularity must also be checked.
- The inequality $\ln u < u - 1$ holds for $u > 0$, $u \neq 1$; it follows by studying $u - 1 - \ln u$.

Step 1: Determine the derivative sign on both sides of zero For $x \neq 0$, differentiate the quotient:

$$f'(x) = \frac{x/(1+x) - \ln(1+x)}{x^2}.$$

To examine the numerator, put $u = 1/(1+x) > 0$. Since $x/(1+x) = 1 - u$ and $\ln(1+x) = -\ln u$, it becomes

$$1 - u + \ln u < 0 \qquad (u \neq 1).$$

The strict inequality follows from $\ln u < u - 1$. The denominator x^2 is positive, so $f'(x) < 0$ throughout both $(-1, 0)$ and $(0, \infty)$. Thus there can be no increase on either of these intervals.

Step 2: Check the joining value and the global behavior The standard logarithmic limit gives

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1 = f(0).$$

The two decreasing branches therefore join continuously. More explicitly, $\ln(1+x) < x$ when $x \neq 0$ and $x > -1$. For negative x, division reverses the inequality, giving $f(x) > 1$; for positive x, it gives $f(x) < 1$. Hence any point to the left of zero has a larger value than zero or any point to its right. Together with the derivative signs, this proves strict decrease on the entire domain. Option (B) incorrectly claims an increase to the right; (C) claims increase everywhere; (D) incorrectly claims

increase to the left.

Step 3: Conclusion The function decreases throughout $\boxed{(-1, \infty)}$, including across $x = 0$. Hence, the correct option is (A).

Quick Tip to Solve: For a logarithmic quotient defined separately at zero, check the derivative on each side and then compare both branches with the limiting value at zero.

6. If a curve y = f(x), passing through the point (1, 2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to : [4 marks]

(A) $\frac{1}{1-\log_e 2}$

(B) $\frac{1}{1+\log_e 2}$

(C) $\frac{-1}{1+\log_e 2}$

(D) $1 + \log_e 2$

Correct Answer: (B) $\frac{1}{1+\log_e 2}$

Solution:

Concept:

- A differential equation whose right-hand side depends only on y/x is homogeneous. Substitute $y = vx$, so that $dy/dx = v + x\,dv/dx$.
- The initial point fixes the integration constant. Work on the connected positive-x interval through $x = 1$ when using $\ln x$.

Step 1: Reduce the equation to separable form Dividing the equation by $2x^2\,dx$ gives

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2}.$$

Set $v = y/x$. Substitution cancels the linear term:

$$v + x\frac{dv}{dx} = v + \frac{v^2}{2}, \qquad \frac{dv}{v^2} = \frac{dx}{2x}.$$

The initial point has $v = 2$, so this solution is not the separate zero solution excluded by division by v^2. Integrating gives

$$-\frac{1}{v} = \frac{1}{2}\ln x + C.$$

Step 2: Apply the initial condition and evaluate the function At $x = 1, y = 2$, we have $v = 2$, hence $C = -1/2$. Consequently

$$-\frac{2}{v} = \ln x - 1, \qquad v = \frac{2}{1 - \ln x}, \qquad y = \frac{2x}{1 - \ln x}.$$

The interval $0 < x < e$ contains both the initial point and the point to be evaluated, with no zero

denominator. Substituting $x = 1/2$ yields

$$f(1/2) = \frac{1}{1 - \ln(1/2)} = \frac{1}{1 + \ln 2}.$$

Option (A) uses the wrong sign for $\ln(1/2)$. Option (C) has an incorrect overall negative sign; the obtained solution is positive at $1/2$. Option (D) replaces the required reciprocal by the denominator itself.

Step 3: Conclusion The requested value is $\boxed{f(1/2) = 1/(1 + \ln 2)}$. Hence, the correct option is (B).

Quick Tip to Solve: After the substitution y=vx, cancel the common v term before separating variables. At x=1/2, remember that the natural logarithm is negative.

7. Consider a region R = {(x, y) ∈ R : $x^2 \leq y \leq 2x$}. if a line y = α divides the area of region R into two equal parts, then which of the following is true? [4 marks]

(A) $3\alpha^2$ - 8α + 8 = 0

(B) α^3 - $6\alpha^{3/2}$ - 16 = 0

(C) $3\alpha^2$ - $8\alpha^{3/2}$ + 8 = 0

(D) α^3 - $6\alpha^2$ + 16 = 0

Correct Answer: (C) $3\alpha^2$ - $8\alpha^{3/2}$ + 8 = 0

Solution:

Concept:

- The area between two curves can be computed with vertical or horizontal strips. For a horizontal strip, the width is the right-hand boundary minus the left-hand boundary.
- A horizontal line bisects a region when the area below it equals half of the total area. Here the curves meet at $x = 0$ and $x = 2$, so $0 \leq y \leq 4$.

Step 1: Find the total bounded area The intersections satisfy $x^2 = 2x$, giving $x = 0, 2$. The line lies above the parabola between these values, and hence the total area is

$$A = \int_0^2 (2x - x^2)\, dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = \frac{4}{3}.$$

Thus each part must have area $2/3$. The cutting height must lie strictly between 0 and 4.

Step 2: Integrate the horizontal width At height y, the inequalities $x^2 \leq y \leq 2x$ imply $y/2 \leq x \leq \sqrt{y}$. Therefore the area below $y = \alpha$ is

$$A_{\text{below}} = \int_0^{\alpha} \left(\sqrt{y} - \frac{y}{2}\right) dy = \frac{2}{3}\alpha^{3/2} - \frac{1}{4}\alpha^2.$$

The square root boundary is crucial: replacing it by a linear expression would give the wrong power of the cutting height.

Step 3: Impose equality of the two parts Set this area equal to $2/3$ and multiply by 12:

$$8\alpha^{3/2} - 3\alpha^2 = 8.$$

Moving all terms to one side gives $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$. This is option (C). Options (A), (B), and (D) have powers or coefficients inconsistent with the horizontal-strip integral. In particular, neither boundary integration produces an α^3 term.

Step 4: Conclusion The bisecting height satisfies $\boxed{3\alpha^2 - 8\alpha^{3/2} + 8 = 0}$, with $0 < \alpha < 4$. Hence, the correct option is (C).

Quick Tip to Solve: For a horizontal area bisector, express both boundaries as x-functions of y before integrating; here the strip width is $\sqrt{y} - y/2$.

8. The imaginary part of

$\left(3 + 2\sqrt{-54}\right)^{\frac{1}{2}} - \left(3 - 2\sqrt{-54}\right)^{\frac{1}{2}}$ **can be :** [4 marks]

(A) -$2\sqrt{6}$

(B) 6

(C) $\sqrt{6}$

(D) -$\sqrt{6}$

Correct Answer: (A) -$2\sqrt{6}$

Solution:

Concept:

- If $(a + ib)^2 = u + iv$, comparison gives $a^2 - b^2 = u$ and $2ab = v$. Each nonzero complex number has two square roots, which differ by a minus sign.
- The phrase "can be" asks for a possible imaginary part among the choices. Both signs of the square roots must therefore be considered, rather than silently restricting to principal values.

Step 1: Find the roots of the conjugate numbers Use $\sqrt{-54} = 3i\sqrt{6}$, so the two radicands are $3+6i\sqrt{6}$ and $3 - 6i\sqrt{6}$. Notice that

$$(3 + i\sqrt{6})^2 = 9 - 6 + 6i\sqrt{6} = 3 + 6i\sqrt{6}.$$

Taking conjugates gives $(3 - i\sqrt{6})^2 = 3 - 6i\sqrt{6}$. Hence the respective square roots are $\pm(3 + i\sqrt{6})$ and $\pm(3 - i\sqrt{6})$.

Step 2: Compare the possible imaginary parts With the positive signs for both roots, their difference is $2i\sqrt{6}$. With the negative signs for both roots, the difference becomes

$$(-3 - i\sqrt{6}) - (-3 + i\sqrt{6}) = -2i\sqrt{6}.$$

If opposite signs are chosen, the difference is real, equal to 6 or -6, so its imaginary part is zero. Thus the possible imaginary parts are $2\sqrt{6}, -2\sqrt{6}, 0$.

Step 3: Select the listed possibility Among the displayed choices, only $-2\sqrt{6}$ occurs in this set. The number 6 is a possible real difference for one sign choice, but it is not an imaginary part. Neither $\sqrt{6}$ nor $-\sqrt{6}$ can occur, since the imaginary components either add in magnitude or cancel. If principal square roots alone were stipulated, the imaginary part would instead be $2\sqrt{6}$; the wording "can be" permits the listed negative value.

Step 4: Conclusion A possible imaginary part is $\boxed{-2\sqrt{6}}$, obtained by choosing the negative root for each radicand. Hence, the correct option is (A).

Quick Tip to Solve: After finding one complex square root, retain its negative as well. Distinguish the imaginary part of a difference from its real part.

9. Let f : R $\to$ R be a function which satisfies
f(x + y) = f(x) + f(y) $\forall$ x, y $\in$ R. If f(1) = 2 and
g(n) = $\sum_{k=1}^{(n-1)} f(k)$, n $\in$ N then the value of n, for which g(n) = 20, is : [4 marks]

(A) 20

(B) 9

(C) 5

(D) 4

Correct Answer: (C) 5

Solution:

Concept:

- An additive function satisfies $f(x+y) = f(x) + f(y)$. Repeated addition determines its values at positive integers without requiring continuity or any further assumption.
- The first m positive integers sum to $m(m+1)/2$. The upper limit in this problem is $n-1$, not n.

Step 1: Determine the required integer values of the function Since $f(1) = 2$, additivity gives $f(2) = f(1+1) = 4$, $f(3) = f(2+1) = 6$, and so on. More formally, induction yields

$$f(k) = kf(1) = 2k \qquad (k \in \mathbb{N}).$$

Only these integer arguments occur in the sum, so there is no need to infer a formula for the function at every real input. This avoids assuming regularity that the question never states.

Step 2: Evaluate the finite sum Substitute the integer values into the definition of g:

$$g(n) = \sum_{k=1}^{n-1} 2k = 2\frac{(n-1)n}{2} = n(n-1).$$

The factor of 2 in $f(k)$ cancels the denominator of the arithmetic-series formula. At $n = 1$ the sum is empty and equals zero, also agreeing with this expression.

Step 3: Solve the equation and test the choices The condition $g(n) = 20$ becomes

$$n^2 - n - 20 = 0, \qquad (n-5)(n+4) = 0.$$

The roots are 5 and -4, but only 5 is a natural number. A direct check gives $f(1)+f(2)+f(3)+f(4) = 2+4+6+8 = 20$. The other listed values fail: $g(20) = 380$, $g(9) = 72$, and $g(4) = 12$. Thus the upper limit has been handled consistently.

Step 4: Conclusion The admissible value is $\boxed{n = 5}$. Hence, the correct option is (C).

Quick Tip to Solve: For sums of an additive function at integers, first replace $f(k)$ by $kf(1)$, then check whether the last index is n or n minus one.

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10. Let E^C denote the complement of an event E. Let E_1 , E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$
and $P(E_1 \cap E_2 \cap E_3) = 0$.
Then $P(E_2^C \cap E_3^C / E_1)$ is equal to : [4 marks]

(A) $P\left(E_3^C\right)$ - $P(E_2)$
(B) $P\left(E_2^C\right)$ + $P(E_3)$
(C) $P\left(E_3^C\right)$ - $P\left(E_2^C\right)$
(D) $P(E_3)$ - $P\left(E_2^C\right)$
Correct Answer: (A) $P\left(E_3^C\right)$ - $P(E_2)$

Solution:

Concept:

- For $P(E_1) > 0$, conditional probability is $P(F \mid E_1) = P(F \cap E_1)/P(E_1)$. Complementation converts the event into the complement of a union inside E_1.
- Pairwise independence gives $P(E_1 \cap E_2) = P(E_1)P(E_2)$ and $P(E_1 \cap E_3) = P(E_1)P(E_3)$. It does not imply mutual independence of all three events.

Step 1: Write the numerator by inclusion and exclusion Inside E_1, remove the outcomes in E_2 or E_3. Inclusion and exclusion gives

$$P(E_1 \cap E_2^C \cap E_3^C) = P(E_1) - P(E_1 \cap E_2) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3).$$

The triple intersection has probability zero by the explicit hypothesis. It must not be replaced by a product of three probabilities.

Step 2: Use pairwise independence and condition Writing $p_j = P(E_j)$, the numerator is $p_1 - p_1p_2 - p_1p_3$. Since $p_1 > 0$, division is valid:

$$P(E_2^C \cap E_3^C \mid E_1) = \frac{p_1(1-p_2-p_3)}{p_1} = 1 - p_2 - p_3.$$

The hypotheses ensure this expression is nonnegative; it is the probability of the remaining part of E_1.

Step 3: Match the expression to an option Using $P(E_3^C) = 1 - P(E_3)$, the result is $P(E_3^C) - P(E_2)$, exactly option (A). Option (B) instead equals $1 - p_2 + p_3$, option (C) equals $p_2 - p_3$, and option (D) equals $p_2 + p_3 - 1$. These are not the required general expression. Treating conditional independence as automatic would incorrectly introduce an extra p_2p_3 term.

Step 4: Conclusion $\boxed{P(E_2^C \cap E_3^C \mid E_1) = P(E_3^C) - P(E_2)}$. Hence, the correct option is (A).

Quick Tip to Solve: With pairwise independence, expand an intersection of complements first. Keep the given triple-intersection probability explicit until the last substitution.

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11. Let f(x) be a quadratic polynomial such that f(–1) + f(2) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in : [4 marks]

(A) (–3, –1)

(B) (1, 3)

(C) (–1, 0)

(D) (0, 1)

Correct Answer: (C) (–1, 0)

Solution:

Concept:

- A quadratic with roots 3 and r can be written $f(x) = a(x - 3)(x - r)$, where $a \neq 0$. This factor form uses the known root immediately.
- A relation between polynomial values can determine the unknown root without finding the leading coefficient, because the common nonzero factor a cancels.

Step 1: Express both polynomial values in terms of the other root Let the other root be r and write $f(x) = a(x - 3)(x - r)$. Substituting the two arguments gives

$$f(-1) = a(-4)(-1 - r) = 4a(1 + r),$$

$$f(2) = a(-1)(2 - r) = a(r - 2).$$

The negative factors at $x = -1$ multiply to a positive expression. Keeping this sign is essential to locating the root in the correct interval.

Step 2: Apply the given relation The condition $f(-1) + f(2) = 0$ now reads

$$4a(1 + r) + a(r - 2) = 0.$$

Combine the constant and r terms to obtain $a(2 + 5r) = 0$. Because the polynomial is quadratic, its

leading coefficient cannot vanish, and hence

$$2 + 5r = 0, \qquad r = -\frac{2}{5}.$$

This determines the root itself, rather than merely giving a bound for it.

Step 3: Locate and verify the root Since $-1 < -2/5 < 0$, the root belongs to $(-1, 0)$. It lies in neither $(-3, -1)$, $(1, 3)$, nor $(0, 1)$, so the other options are excluded. For a substitution check, choose $a = 1$: then $f(x) = (x - 3)(x + 2/5)$, and $f(-1) = 12/5$, whereas $f(2) = -12/5$. Their sum is zero, as required. The arbitrary scale of the polynomial therefore does not affect the conclusion.

Step 4: Conclusion The other root is $\boxed{-2/5}$, which lies in $\boxed{(-1, 0)}$. Hence, the correct option is (C).

Quick Tip to Solve: When one root is given and polynomial values are related, use factor form. It usually removes the leading coefficient and leaves a linear equation for the other root.

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12. $\lim_{x \to 0} \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{\frac{1}{x}}$ **is equal to :** [4 marks]

(A) 2

(B) 1

(C) e

(D) e^2

Correct Answer: (D) e^2

Solution:

Concept:

- For a positive base approaching 1, take logarithms of an expression with a variable exponent. If the logarithm tends to a finite number L, the original expression tends to e^L.
- The derivative of $\log(\tan u)$ is $\sec^2 u / \tan u$ wherever the tangent is positive. A difference quotient can evaluate the resulting logarithmic limit.

Step 1: Check the base and take logarithms Near $x = 0$, the angle $\pi/4 + x$ lies between 0 and $\pi/2$, so its tangent is positive and the real logarithm is defined. Set

$$Y(x) = \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x}.$$

The base tends to 1 and the exponent is unbounded, so direct substitution is indeterminate. Instead,

$$\log Y(x) = \frac{\log \tan(\pi/4 + x)}{x}.$$

Step 2: Evaluate the logarithmic difference quotient Let $h(x) = \log \tan(\pi/4 + x)$. Since $h(0) =$

$\log 1 = 0$, the last ratio is $(h(x) - h(0))/x$, whose limit is $h\prime(0)$. Differentiate:

$$h\prime(x) = \frac{\sec^2(\pi/4 + x)}{\tan(\pi/4 + x)}.$$

At zero, $\sec^2(\pi/4) = 2$ and $\tan(\pi/4) = 1$, yielding $h\prime(0) = 2$. The derivative is well defined from both sides in the stated neighbourhood.

Step 3: Return from logarithms and exclude the alternatives We have $\log Y(x) \to 2$. Continuity of the exponential therefore gives $Y(x) \to e^2$. The value 2 is the limit of the logarithm, not the requested limit. The value 1 would require a logarithmic limit of zero, while e would require a logarithmic limit of 1; neither matches the derivative just obtained. Thus the logarithmic transformation must be reversed before choosing an answer.

Step 4: Conclusion The required limit equals $\boxed{e^2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a positive expression of the form $b(x)^{1/x}$ with $b(0) = 1$, compute the derivative of $\log b(x)$ at zero and exponentiate the result.

13. Let n > 2 be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is : [4 marks]

(A) 201

(B) 199

(C) 101

(D) 200

Correct Answer: (A) 201

Solution:

Concept:

- A track is determined by an unordered pair of distinct stations, so the total number is $\binom{n}{2} = n(n-1)/2$. Counting ordered pairs would count every track twice.
- The blue tracks join consecutive stations on a closed circular route. There are exactly n such tracks, including the track joining the last station to the first. All other tracks are red.

Step 1: Count the two colours Let B and R denote the blue and red track counts. Each station has two neighbours, but counting these at every station counts each blue track twice. Consequently,

$$B = \frac{2n}{2} = n, \qquad R = \binom{n}{2} - n = \frac{n(n-3)}{2}.$$

The subtraction removes precisely the sides of the circular polygon; its remaining chords are the red tracks.

Step 2: Apply the stated ratio The condition is $R = 99B$, rather than a condition on the total number

of tracks. Therefore,

$$\frac{n(n-3)}{2} = 99n.$$

Since $n > 2$, division by n is valid. It gives $n - 3 = 198$, and hence $n = 201$.

Step 3: Check the offered values For $n = 201$, the blue count is 201 and the red count is $201 \cdot 198/2 = 19899 = 99 \cdot 201$. More generally the ratio is $(n-3)/2$. The choices $199, 101, 200$ instead give $98, 49, 98.5$, respectively, so none has the required ratio. The count also includes the closing blue track, which is essential for this circular route.

Step 4: Conclusion The number of stations is $\boxed{201}$. Hence, the correct option is (A).

Quick Tip to Solve: For a complete track network on a circular arrangement, red-to-blue ratio is $(n-3)/2$; remember the closing side when counting the blue tracks.

14. Let a, b, c $\in$ R be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

A = $\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$

satisfies $A^TA = I$, then a value of abc can be : [4 marks]

(A) 3

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) $\frac{2}{3}$

Correct Answer: No correct option: orthogonality implies the cubic sum is at most 1, whereas the question requires 2.

Solution:

Concept:

- The diagonal entries of $A^TA = I$ are squared column lengths. Hence the first column gives $a^2 + b^2 + c^2 = 1$. Its off-diagonal entries also give $ab + bc + ca = 0$.
- For real numbers whose squares sum to one, each absolute value is at most one. Thus $|a|^3 \le a^2$, and the analogous inequalities hold for b, c. These constraints must hold before any candidate product is accepted.

Step 1: Test the cubic sum against the column length Orthogonality requires

$$a^2 + b^2 + c^2 = 1.$$

It follows that $|a|, |b|, |c| \le 1$. The triangle inequality and the preceding bounds therefore yield

$$a^3 + b^3 + c^3 \le |a|^3 + |b|^3 + |c|^3 \le a^2 + b^2 + c^2 = 1.$$

The stipulated value 2 contradicts this necessary inequality. The extra requirement that all three entries are non-zero does not remove that contradiction.

Step 2: Explain why the apparent algebraic choice is invalid Writing $s = a + b + c$, the off-diagonal condition gives $s^2 = 1$. The identity

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

would formally give $2 - 3abc = s$. Using $s = 1$ produces $abc = 1/3$, the listed option (B), but this calculation has used incompatible hypotheses and does not establish existence. Indeed AM-GM applied to a^2, b^2, c^2 gives $|abc| \leq 1/(3\sqrt{3}) < 1/3$. All four listed products have absolute value at least $1/3$, so each is impossible for a real orthogonal matrix of the stated form.

Step 3: Conclusion As printed, no real triple (a, b, c) satisfies all the conditions, so none of (A)–(D) is defensible. The formal value $1/3$ in (B) does not satisfy the necessary real-variable bound; a unique correct option cannot be selected.

Quick Tip to Solve: Before using a symmetric identity, test orthogonality through column norms. A norm bound can expose inconsistent real hypotheses that formal algebra conceals.

15. The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1, 2) and (sin θ, cos θ) lie on the same side of the line x + y = 1 is : [4 marks]

(A) $\left(0, \frac{\pi}{4}\right)$
(B) $\left(0, \frac{3\pi}{4}\right)$
(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
(D) $\left(0, \frac{\pi}{2}\right)$

Correct Answer: (D) $\left(0, \frac{\pi}{2}\right)$

Solution:

Concept:

- For the line $L(x, y) = x + y - 1 = 0$, two points lie strictly on the same side exactly when the two values of L have the same non-zero sign.
- The identity $\sin\theta + \cos\theta = \sqrt{2}\sin(\theta + \pi/4)$ converts the condition to a standard sine inequality. The original interval $0 < \theta < \pi$ must be retained.

Step 1: Determine the required side At the fixed point $(1, 2)$,

$$L(1, 2) = 1 + 2 - 1 = 2 > 0.$$

Thus the moving point must satisfy $L(\sin\theta, \cos\theta) > 0$. Equality is excluded because it would put the moving point on the line, rather than on the same side as the fixed point. The required inequality is consequently

$$\sin\theta + \cos\theta > 1.$$

Step 2: Solve within the prescribed interval Set $u = \theta + \pi/4$. Then $u \in (\pi/4, 5\pi/4)$, and the inequality becomes $\sin u > 1/\sqrt{2}$. In this range it holds precisely when

$$\frac{\pi}{4} < u < \frac{3\pi}{4}.$$

Subtracting $\pi/4$ gives $0 < \theta < \pi/2$. At $\theta = \pi/2$, the moving point is $(1, 0)$, lying on the line, so that endpoint cannot be included.

Step 3: Compare the alternatives Option (A) discards valid angles such as $\theta = \pi/3$, where the sum is $(\sqrt{3}+1)/2 > 1$. Option (B) admits angles greater than $\pi/2$, where the inequality fails. Option (C) both omits some valid small angles and includes invalid angles beyond $\pi/2$. Only (D) gives the entire required open interval.

Step 4: Conclusion The required set is $\boxed{(0, \pi/2)}$. Hence, the correct option is (D).

Quick Tip to Solve: Evaluate the line expression at the fixed point first; its sign tells you which strict trigonometric inequality to solve.

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16. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is : [4 marks]

(A) $256\sqrt{3}$

(B) $64\sqrt{3}$

(C) $128\sqrt{3}$

(D) $192\sqrt{3}$

Correct Answer: (D) $192\sqrt{3}$

Solution:

Concept:

- If an equilateral triangle has one vertex at the origin, its other two position vectors have equal length and enclose an angle of $60°$. On $y^2 = 8x$, all non-vertex points have $x > 0$.
- For a point (x, y) on this parabola, $x^2 + y^2 = x^2 + 8x$, which is strictly increasing for $x \geq 0$. Equal distances from the origin therefore force equal abscissae of the other vertices.

Step 1: Locate the other two vertices Let the vertex at the origin be O, and write the other vertices as P and Q. Since $OP = OQ$, the monotonicity of $x^2 + 8x$ makes their abscissae equal. Distinct points on the parabola with the same abscissa must have opposite ordinates. Thus put

$$P = (h, k), \qquad Q = (h, -k), \qquad h > 0, \quad k > 0,$$

where $k^2 = 8h$. This establishes the symmetry from the equilateral condition rather than assuming an arbitrary inscribed triangle is symmetric.

Step 2: Impose equality of the sides The base has length $PQ = 2k$, and $OP = \sqrt{h^2 + k^2}$. Hence

$$h^2 + k^2 = 4k^2, \qquad h^2 = 3k^2 = 24h.$$

The non-degenerate triangle has $h \neq 0$, so $h = 24$. It follows that $k^2 = 192$, or $k = 8\sqrt{3}$. The side length is therefore $16\sqrt{3}$.

Step 3: Calculate and compare the areas The perpendicular distance from O to the vertical base $x = 24$ is 24. Consequently,

$$\mathcal{A} = \frac{1}{2}(16\sqrt{3})(24) = 192\sqrt{3}.$$

As a check, the equilateral formula gives $\sqrt{3}(16\sqrt{3})^2/4 = 192\sqrt{3}$ as well. The coefficients $256, 64, 128$ in the other options do not equal the computed coefficient 192.

Step 4: Conclusion The area is $\boxed{192\sqrt{3}}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Equal distances from the vertex force equal x-coordinates on a right-opening parabola; then use side equality before calculating the area.

17. A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to: [4 marks]

(A) -10

(B) 10

(C) 5

(D) -5

Correct Answer: (C) 5

Solution:

Concept:

- A normal vector to a plane is perpendicular to every direction vector lying in the plane. The cross product of two non-parallel in-plane direction vectors supplies such a normal.
- If a plane has normal (l, m, n) and passes through (x_0, y_0, z_0), its equation is $l(x - x_0) + m(y - y_0) + n(z - z_0) = 0$. Every specified point must satisfy that equation.

Step 1: Find a normal vector Take $\mathbf{u} = (1, -2, 2)$ and $\mathbf{v} = (2, 3, -1)$. Their cross product is

$$\mathbf{u} \times \mathbf{v} = \big(2 - 6,\ 4 + 1,\ 3 + 4\big) = (-4, 5, 7).$$

It is non-zero, so the two directions really do determine the plane orientation. Checking the dot products gives $-4 - 10 + 14 = 0$ and $-8 + 15 - 7 = 0$; thus this normal is perpendicular to both directions, as required.

Step 2: Use the point to write the plane Substitute the point $(3, 1, 1)$ into the point-normal form:

$$-4(x - 3) + 5(y - 1) + 7(z - 1) = 0.$$

The constant terms sum to $12 - 5 - 7 = 0$. The plane therefore has the particularly simple equation

$$-4x + 5y + 7z = 0.$$

The given point satisfies it because $-12 + 5 + 7 = 0$.

Step 3: Substitute the second point and test the options For $(\alpha, -3, 5)$, the equation yields

$$-4\alpha - 15 + 35 = 0, \qquad -4\alpha + 20 = 0,$$

so $\alpha = 5$. Direct substitution gives zero for this choice. The other proposed values $-10, 10, -5$ give residuals $60, -20, 40$, respectively, and therefore put the point outside the plane.

Step 4: Conclusion The required value is $\boxed{\alpha = 5}$. Hence, the correct option is (C).

Quick Tip to Solve: After a cross product, verify the normal by two quick dot products; this catches the common sign error in its middle component.

18. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is : [4 marks]

(A) $y = 4x + 2$

(B) $x + 4y = 8$

(C) $y + 4x = 2$

(D) $2y + x = 4$

Correct Answer: (B) $x + 4y = 8$

Solution:

Concept:

- The identity $\cos^2(\sin^{-1} x) = 1 - x^2$ holds for $-1 \le x \le 1$. It simplifies the curve near the required point $x = 0$.
- For $(1 + x)^{2y}$, both the base and exponent vary. Write it as $\exp(2y \ln(1 + x))$ and differentiate with the chain rule. A normal has slope $-1/m$ when the tangent slope $m \neq 0$.

Step 1: Find the point on the curve At $x = 0$, the power term is $1^{2y} = 1$, and $\cos^2(\sin^{-1} 0) = 1$. Therefore the point is $(0, 2)$. In a neighbourhood of this point the curve can be written as

$$y = (1 + x)^{2y} + 1 - x^2.$$

The base $1 + x$ is positive there, so the logarithmic differentiation used next is valid.

Step 2: Differentiate implicitly and evaluate The derivative of the power term is

$$\frac{d}{dx}(1 + x)^{2y} = (1 + x)^{2y} \left(2y' \ln(1 + x) + \frac{2y}{1 + x} \right).$$

Thus

$$y' = (1+x)^{2y}\left(2y'\ln(1+x) + \frac{2y}{1+x}\right) - 2x.$$

At $(0,2)$, the term containing $y'\ln(1+x)$ vanishes, giving $y' = 4$. Hence the tangent slope is positive and equal to four.

Step 3: Write the normal and compare the options The normal slope is $-1/4$, so its equation through $(0,2)$ is

$$y - 2 = -\frac{1}{4}x, \qquad x + 4y = 8.$$

All offered lines pass through $(0,2)$, so checking only the point is insufficient. Option (A) has the tangent slope 4; (C) has slope -4, and (D) has slope $-1/2$. Only (B) has the required negative reciprocal slope.

Step 4: Conclusion The normal is $\boxed{x + 4y = 8}$. Hence, the correct option is (B).

Quick Tip to Solve: When both base and exponent vary, differentiate via an exponential of a logarithm; at x=0 the logarithm term often vanishes.

19. Let S be the sum of the first 9 terms of the series :

$\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\}$

$+ \{x^4 + (k+6)a\} +$ .... where $a \neq 0$ and $x \neq 1$.

If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to : [4 marks]

(A) -3

(B) 1

(C) -5

(D) 3

Correct Answer: (A) -3

Solution:

Concept:

- The sum can be separated into a geometric progression in x and an arithmetic progression multiplied by a. For $x \neq 1$, $\sum_{r=1}^{n} x^r = x(x^n - 1)/(x-1)$.
- The coefficients of a are $k, k+2, k+4, \ldots$. Their sum is $n[2k + (n-1)2]/2$. Since $a \neq 0$, equality of these contributions determines k.

Step 1: Separate the two contributions The first nine powers run from x through x^9, whereas the ninth coefficient of a is $k + 16$. Thus

$$S = \sum_{r=1}^{9} x^r + a\sum_{r=0}^{8}(k+2r).$$

The common ratio in the first sum is x, and the coefficient progression has common difference 2. These are distinct progressions; the whole displayed series need not itself be geometric or arithmetic.

Step 2: Evaluate both sums The geometric part and arithmetic part respectively give

$$\sum_{r=1}^{9} x^r = \frac{x^{10} - x}{x - 1}, \qquad \sum_{r=0}^{8} (k + 2r) = 9k + 2\frac{8 \cdot 9}{2} = 9k + 72.$$

Consequently, $S = (x^{10} - x)/(x - 1) + (9k + 72)a$. On separating the fraction printed in the question, its second contribution is $45a$, because the factor $x - 1$ cancels.

Step 3: Compare the remaining coefficients Subtract the common geometric sum from both expressions and divide by the nonzero a:

$$9k + 72 = 45 \quad \Longrightarrow \quad 9k = -27 \quad \Longrightarrow \quad k = -3.$$

As a direct check, the nine coefficients then run from -3 to 13 and sum to $9(-3 + 13)/2 = 45$. The other proposed values $1, -5, 3$ give coefficient sums $81, 27, 99$, respectively, so none gives the required sum.

Step 4: Conclusion The required value is $\boxed{k = -3}$. Hence, the correct option is (A).

Quick Tip to Solve: Split a mixed series into separate sums before applying a formula; here only the coefficients of the nonzero parameter need to be compared.

20. Which of the following is a tautology ? [4 marks]

(A) $(\sim p) \wedge (p \vee q) \to q$

(B) $(q \to p) \vee \sim (p \to q)$

(C) $(p \to q) \wedge (q \to p)$

(D) $(\sim q) \vee (p \wedge q) \to q$

Correct Answer: (A) $(\sim p) \wedge (p \vee q) \to q$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its component statements. An implication $r \to s$ is false only when r is true and s is false.
- Use distributivity and $p \wedge \neg p \equiv \mathrm{F}$ to simplify the antecedent in option (A). A single false assignment is enough to reject any other proposed tautology.

Step 1: Simplify the antecedent of option (A) Interpret the displayed implication as having antecedent $(\neg p) \wedge (p \vee q)$. Distribution yields

$$(\neg p) \wedge (p \vee q) \equiv [(\neg p) \wedge p] \vee [(\neg p) \wedge q] \equiv (\neg p) \wedge q.$$

Whenever this antecedent is true, q is already true. If the antecedent is false, the implication is true by definition. Therefore the implication is true in both possible cases, regardless of the value of p.

Step 2: Reject options (B) and (C) For option (B), choose $p = \mathrm{F}$ and $q = \mathrm{T}$. Then $q \to p$ is false,

while $p \to q$ is true and its negation is false. Their disjunction is therefore false. For option (C), choose $p = \mathrm{T}$, $q = \mathrm{F}$. The implication $p \to q$ is false, so the conjunction of the two implications is false. In particular, a biconditional is not automatically a tautology.

Step 3: Reject option (D) Take $p = \mathrm{F}$ and $q = \mathrm{F}$. The antecedent $(\neg q) \vee (p \wedge q)$ is true because $\neg q$ is true, but the consequent q is false. Hence this implication is false. This counterexample also shows why merely seeing q in part of an antecedent does not guarantee a tautology: the other part of the disjunction can make the antecedent true when q is false.

Step 4: Conclusion Only $\boxed{[(\neg p) \wedge (p \vee q)] \to q}$ is true for every truth assignment. Hence, the correct option is (A).

Quick Tip to Solve: To test an implication for tautology, try setting its consequent false; then check whether its antecedent can still be true.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If $\mathbf{y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}}$,

then $\frac{dy}{dx}$ **at x = 0 is _____.** [4 marks]

Solution:

Concept:

- For a differentiable function $u(x)$ with $-1 < u(x) < 1$, $\frac{d}{dx}\cos^{-1}u(x) = -u'(x)/\sqrt{1-u(x)^2}$. The square root is the positive one.
- A finite sum can be differentiated term by term. The coefficient k outside each inverse cosine must be retained as well as the factor k produced by differentiating $\sin kx$ or $\cos kx$.

Step 1: Differentiate the inner expression Set $u_k(x) = \frac{3}{5}\cos kx - \frac{4}{5}\sin kx$. Then

$$u_k(0) = \frac{3}{5}, \qquad u_k'(0) = -\frac{4k}{5},$$
$$u_k'(x) = -\frac{3k}{5}\sin kx - \frac{4k}{5}\cos kx.$$

Since $3/5$ lies strictly inside $(-1, 1)$, the inverse cosine is differentiable in a neighbourhood of zero for each of the six terms. No endpoint or branch singularity is encountered at the point requested.

Step 2: Apply the chain rule to one summand For the summand $k\cos^{-1}(u_k(x))$, the derivative at zero is

$$-k\frac{u_k'(0)}{\sqrt{1-u_k(0)^2}} = -k\frac{-4k/5}{\sqrt{1-9/25}} = -k\frac{-4k/5}{4/5} = k^2.$$

The two negative signs cancel. In particular, the multiplier outside the inverse cosine is not differentiated away: it is constant with respect to x, and multiplies the entire chain-rule derivative.

Step 3: Sum the six contributions Termwise differentiation now gives

$$\left.\frac{dy}{dx}\right|_{x=0} = \sum_{k=1}^{6} k^2 = \frac{6 \cdot 7 \cdot 13}{6} = 91.$$

Direct addition provides an arithmetic check: $1 + 4 + 9 + 16 + 25 + 36 = 91$. Summing k instead would miss one factor from the chain rule. The positive result is also consistent with the fact that the inner expressions decrease at zero while inverse cosine is decreasing on its domain.

Step 4: Conclusion The required dimensionless derivative is $\boxed{91}$.

Quick Tip to Solve: When an indexed coefficient appears both outside a function and inside its argument, mark both factors before differentiating; here they produce a sum of squares.

22. If the variance of the terms in an increasing A.P.,
b_1 , b_2 , b_3 ,....,b_{11} is 90, then the common difference of this A.P. is_____. [4 marks]

Solution:

Concept:

- For n equally weighted observations, the population variance is $\sigma^2 = \frac{1}{n}\sum_{j=1}^{n}(b_j - \bar{b})^2$. The divisor here is n, not $n - 1$.
- In an arithmetic progression of odd length, the middle term is its arithmetic mean. Writing every deviation as an integer multiple of the common difference removes the unknown first term.

Step 1: Centre the eleven terms at their mean Let $b_j = b_1 + (j - 1)d$, where the common difference d is positive because the progression is increasing. The mean of the eleven terms is

$$\bar{b} = \frac{b_1 + b_{11}}{2} = b_1 + 5d = b_6.$$

Their deviations from the mean are therefore $-5d, -4d, -3d, -2d, -d, 0, d, 2d, 3d, 4d, 5d$. This list contains eleven values, arranged in opposite pairs about zero, as expected for equally spaced observations.

Step 2: Calculate the variance Substitute the deviations into the variance formula:

$$\begin{aligned}\sigma^2 &= \frac{d^2}{11}\left[2(1^2 + 2^2 + 3^2 + 4^2 + 5^2)\right] \\ &= \frac{2d^2}{11}(1 + 4 + 9 + 16 + 25) = \frac{110d^2}{11} = 10d^2.\end{aligned}$$

Shifting the entire progression changes its mean but leaves these deviations unchanged. Thus no information about the first term is needed to determine the common difference.

Step 3: Use the stated variance and the increasing condition The given variance 90 yields

$$10d^2 = 90 \quad \Longrightarrow \quad d^2 = 9 \quad \Longrightarrow \quad d = \pm 3.$$

The negative root would describe a decreasing progression and must be discarded. The positive root

gives a variance $10(3)^2 = 90$, verifying the result. The variance has the square of the units of the terms; its positive square-root scale therefore correctly determines the units of the common difference.

Step 4: Conclusion The common difference of the increasing progression is $\boxed{3}$, in the same units as its terms.

Quick Tip to Solve: For an odd number of A.P. terms, subtract the middle term first; the symmetric deviations make the variance sum short and independent of the first term.

23. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio λ : 1 ($\lambda > 0$). If O is the origin and $\overrightarrow{OB}.\overrightarrow{OP} - 3\left|\overrightarrow{OA} \times \overrightarrow{OP}\right|^2 = 6$, then λ is equal to_____. [4 marks]

Solution:

Concept:

- If P divides AB internally in the ratio $\lambda : 1$, then $\mathbf{p} = (\mathbf{a} + \lambda\mathbf{b})/(1 + \lambda)$, where $\mathbf{a}, \mathbf{b}, \mathbf{p}$ are the respective position vectors.
- Equivalently write $\mathbf{p} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$ with $t = \lambda/(1 + \lambda)$. Internal division with $\lambda > 0$ implies $0 < t < 1$. Evaluate the dot product and squared cross-product magnitude using Cartesian components.

Step 1: Parameterise the point on the segment The vectors are $\mathbf{a} = (1, 1, 1)$ and $\mathbf{b} = (2, 1, 3)$, so $\mathbf{b} - \mathbf{a} = (1, 0, 2)$. Introduce $t = \lambda/(1 + \lambda)$. Then

$$\mathbf{p} = (1 + t, 1, 1 + 2t), \qquad 0 < t < 1.$$

This parameterisation gives $P = A$ at $t = 0$ and $P = B$ at $t = 1$, confirming the orientation of the ratio before using the equation in the question.

Step 2: Evaluate the two vector expressions The dot product is

$$\mathbf{b} \cdot \mathbf{p} = 2(1 + t) + 1 + 3(1 + 2t) = 6 + 8t.$$

For the cross product, direct component evaluation gives

$$\mathbf{a} \times \mathbf{p} = (2t, -t, -t), \qquad |\mathbf{a} \times \mathbf{p}|^2 = 4t^2 + t^2 + t^2 = 6t^2.$$

The magnitude must be squared before applying the coefficient 3 in the given condition; replacing it with the unsquared magnitude would produce a different equation.

Step 3: Solve the condition and recover the ratio Substitution into the printed condition yields

$$6 + 8t - 18t^2 = 6 \quad \Longrightarrow \quad 2t(4 - 9t) = 0.$$

The root $t = 0$ would give $\lambda = 0$, contrary to the condition $\lambda > 0$. Thus $t = 4/9$, and

$$\lambda = \frac{t}{1-t} = \frac{4/9}{5/9} = \frac{4}{5} = 0.8.$$

This lies in the permitted range for an internal division. Checking the original equation gives $6 + 32/9 - 18(16/81) = 6$, as required.

Step 4: Conclusion The required ratio parameter is $\boxed{\lambda = 0.8}$. It is dimensionless.

Quick Tip to Solve: Use a segment parameter between 0 and 1 to simplify vector products, then convert back to the stated division ratio at the end.

24. Let [t] denote the greatest integer less than or equal to t.

Then the value of $\int_1^2 |2x - [3x]|\, dx$ is _____. [4 marks]

Solution:

Concept:

- The greatest-integer function $[3x]$ is constant between consecutive points where $3x$ is an integer. Split the integration interval at $x = 4/3$ and $x = 5/3$.
- For each piece, evaluate the sign of $2x - [3x]$ before removing the absolute value. Values at isolated endpoints do not affect a definite integral.

Step 1: Locate all the integer-function intervals As x increases from 1 to 2, the quantity $3x$ runs from 3 to 6. Except for right endpoints, its greatest integer is 3 on $[1, 4/3)$, 4 on $[4/3, 5/3)$, and 5 on $[5/3, 2)$. At $x = 2$, the value becomes 6, but a change at this single point contributes no area. These are all the necessary breakpoints in the interval.

Step 2: Remove the modulus on each interval On the first interval, $2x - 3 < 0$, since $x < 4/3 < 3/2$. On the second, $2x - 4 < 0$, since $x < 5/3 < 2$. On the third, $2x - 5 < 0$, since $x < 2 < 5/2$. Hence the integrand is respectively $3 - 2x$, $4 - 2x$, and $5 - 2x$. None of these three linear pieces has an additional zero inside its own interval, so no further splitting is needed.

Step 3: Integrate the three linear pieces Denote the required integral by I. The interval decomposition gives

$$\begin{aligned} I &= \int_1^{4/3} (3 - 2x)\, dx + \int_{4/3}^{5/3} (4 - 2x)\, dx + \int_{5/3}^{2} (5 - 2x)\, dx \\ &= [3x - x^2]_1^{4/3} + [4x - x^2]_{4/3}^{5/3} + [5x - x^2]_{5/3}^{2} \\ &= \frac{2}{9} + \frac{1}{3} + \frac{4}{9} = 1. \end{aligned}$$

Every contribution is nonnegative, as required by the original absolute value. The exact fractions also avoid numerical errors near the discontinuities of the greatest-integer function.

Step 4: Conclusion The dimensionless value of the integral is $\boxed{1}$.

Quick Tip to Solve: For a floor-function integral, first split where its argument is an integer, then check the modulus separately on each resulting interval.

25. For a positive integer n,
$\left(1+\frac{1}{x}\right)^n$ is expanded
in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to_____. [4 marks]

Solution:

Concept:

- The binomial expansion gives coefficients $\binom{n}{r}$. Writing the expression in increasing powers of x, starting at x^{-n}, produces the same coefficient sequence because $\binom{n}{r} = \binom{n}{n-r}$.
- For consecutive coefficients, $\binom{n}{r+1}/\binom{n}{r} = (n-r)/(r+1)$. Applying the ratio twice determines the coefficient position and the positive integer n.

Step 1: Represent the three coefficients Let the three consecutive coefficients in increasing-power order be $\binom{n}{r}, \binom{n}{r+1}, \binom{n}{r+2}$, where $0 \le r \le n-2$. Their given ratio is $2:5:12$. Dividing the second by the first cancels the common proportionality factor and gives

$$\frac{n-r}{r+1} = \frac{5}{2} \quad \Longrightarrow \quad 2n - 7r = 5.$$

The next consecutive ratio must be $12/5$, not $12/2$, since its denominator is the middle coefficient.

Step 2: Solve the two ratio equations The third coefficient divided by the second gives

$$\frac{n-r-1}{r+2} = \frac{12}{5} \quad \Longrightarrow \quad 5n - 17r = 29.$$

Multiply the first linear equation by 5 and the second by 2. Subtracting yields

$$\begin{aligned} 10n - 35r &= 25, \\ 10n - 34r &= 58, \\ r &= 33, \qquad n = \frac{7(33)+5}{2} = 118. \end{aligned}$$

The resulting positions lie inside the permitted range. As a direct check, the two successive ratios are $85/34 = 5/2$ and $84/35 = 12/5$, so all three coefficients have exactly the stated ratio.

Step 3: Conclusion The required positive integer is $\boxed{n = 118}$. This is a dimensionless integer, as expected for an exponent in a binomial expansion.

Quick Tip to Solve: Use ratios of adjacent binomial coefficients: the factorials cancel immediately, leaving two linear equations for the exponent and the coefficient position.

26. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of a wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be [4 marks]

(A) 28

(B) 24

(C) 30

(D) 18

Correct Answer: (A) 28

Solution:

Concept:

- In Young's experiment, the fringe width is $\beta = \lambda D/d$, where D is the screen distance and d is the slit separation. These geometrical quantities remain fixed when only the wavelength changes.
- The number of fringes in a fixed screen length L is $N = L/\beta$. Hence $N\lambda$ is constant for that segment: shorter wavelengths produce narrower fringes and a larger fringe count.

Step 1: Relate the two fringe counts For the original and changed wavelengths, the same segment has length

$$L = N_1\frac{\lambda_1 D}{d} = N_2\frac{\lambda_2 D}{d}.$$

Cancelling the common geometry gives $N_2 = N_1\lambda_1/\lambda_2$. The count must increase when the wavelength drops from 700 nm to 400 nm, because more fringe spacings fit into the same length.

Step 2: Substitute and compare the options Using the given original count of 16,

$$N_2 = 16 \times \frac{700}{400} = 16 \times \frac{7}{4} = 28.$$

Both wavelengths are in nanometres, so their units cancel in the ratio. Check the result by comparing the products: $16 \times 700 = 11200$ and $28 \times 400 = 11200$. The alternative counts 24, 30 and 18 would give products 9600, 12000 and 7200 respectively, so none represents the same fixed segment with unchanged slit geometry.

Step 3: Conclusion The same segment contains $\boxed{28}$ fringes after the wavelength change. Hence, the correct option is (A).

Quick Tip to Solve: For the same screen segment in an unchanged double-slit arrangement, multiply fringe count by wavelength; that product stays constant.

27. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is : [4 marks]

(A) 20 μF

(B) 15 μF

(C) 10 μF

(D) 30 μF

Correct Answer: (B) 15 μF

Solution:

Concept:

- The charge on a capacitor is $Q = CV$. Once the voltage source is removed, the two-capacitor system cannot gain net charge from a battery.
- Capacitors connected in parallel have a common final voltage and equivalent capacitance $C_1 + C_2$. Charge is conserved during redistribution, even though some stored electrostatic energy is dissipated.

Step 1: Calculate the charge initially stored The first capacitor has $C_1 = 10\,\mu\text{F}$ and initial voltage $V_i = 50\,\text{V}$. The second capacitor starts uncharged. Therefore the total initial charge available for redistribution is

$$Q_i = C_1 V_i = 10 \times 50 = 500\,\mu\text{C}.$$

Removing the source before making the parallel connection is essential: a connected ideal battery would hold the voltage at its original value.

Step 2: Apply charge conservation at the final voltage At equilibrium both capacitors have the stated voltage of $20\,\text{V}$. Thus

$$(10 + C_2) \times 20 = 500,$$
$$10 + C_2 = 25,$$
$$C_2 = 15\,\mu\text{F}.$$

Here capacitances in the algebra are measured in microfarads. The first capacitor then holds $200\,\mu\text{C}$ and the second $300\,\mu\text{C}$, adding to the original $500\,\mu\text{C}$. For the other offered capacitances 20, 10 and 30 microfarads, charge conservation would predict final voltages $50/3$, 25 and 12.5 volts respectively, rather than 20 volts.

Step 3: Conclusion The capacitance of the initially uncharged capacitor is $\boxed{15\,\mu\text{F}}$. Hence, the correct option is (B).

Quick Tip to Solve: For isolated capacitor charge sharing, conserve charge rather than electrostatic energy; first find the combined capacitance from initial charge divided by final voltage.

28. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is [4 marks]

(A) $[P^2AT^{-2}]$

(B) $\left[P^{\frac{1}{2}}AT^{-1}\right]$

(C) $\left[PA^{\frac{1}{2}}T^{-1}\right]$

(D) $[PA^{-1}T^{-2}]$

Correct Answer: (C) $\left[PA^{\frac{1}{2}}T^{-1}\right]$

Solution:

Concept:

- In the usual mass-length-time basis, momentum has dimension MLT^{-1}, area has dimension L^2, and energy has dimension ML^2T^{-2}.
- Express energy as $P^aA^bT^c$, then equate the powers of mass, length and time. A dimensional change of basis does not change the physical dimension of the quantity.

Step 1: Set up the dimensional identity Let the required formula be $[E] = [P^aA^bT^c]$, with unknown exponents a, b, c. Substitute the familiar dimensions:

$$ML^2T^{-2} = (MLT^{-1})^a(L^2)^bT^c = M^aL^{a+2b}T^{-a+c}.$$

The capital T denotes the chosen time quantity, while the exponent equations simply compare the independent time dimensions on both sides.

Step 2: Match the exponents and test the options Equating each independent dimension gives

$$a = 1, \qquad a + 2b = 2, \qquad -a + c = -2.$$

Consequently $b = 1/2$ and $c = -1$. Therefore

$$[E] = [PA^{1/2}T^{-1}].$$

This also follows physically from energy having the dimensions of momentum times velocity, with velocity represented by area to the power one-half divided by time. Option A contains P^2 and therefore has mass exponent 2; option B contains $P^{1/2}$ and has mass exponent one-half. Both fail immediately. Option D gives length exponent $1 - 2 = -1$ and time exponent $-1 - 2 = -3$, so it fails as well.

Step 3: Conclusion The dimensional formula for energy in the stated basis is $\boxed{[PA^{1/2}T^{-1}]}$. Hence, the correct option is (C).

Quick Tip to Solve: Area supplies length squared: replace a required single length by the square root of area, then match the momentum and time factors.

29. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) [4 marks]

(A) $\frac{R}{2}$

(B) $\frac{\sqrt{5}R-R}{2}$

(C) $\frac{\sqrt{3}R-R}{2}$

(D) $\frac{\sqrt{5}}{2}R - R$

Correct Answer: (B) $\frac{\sqrt{5}R-R}{2}$

Solution:

Concept:

- At altitude h above a spherical Earth of radius R, the acceleration is $g_h = g_0R^2/(R+h)^2$, where g_0 is the surface value.
- In the standard uniform-density Earth model, at depth h the acceleration is $g_d = g_0(1 - h/R)$. Weight equals mass times gravitational acceleration, so equality of weights requires equality of these accelerations.

Step 1: Equate the altitude and depth expressions The same body is used at the two locations, so its mass cancels. Set $x = h/R$, where a positive physical depth requires $0 < x \leq 1$. Equality of the two weights becomes

$$\frac{1}{(1+x)^2} = 1 - x.$$

Use the exact inverse-square altitude formula here. The first-order approximation $g_h \simeq g_0(1 - 2h/R)$ is insufficient, since the nonzero solution need not have $h \ll R$.

Step 2: Factor and choose the physical root Multiplying by $(1+x)^2$ and expanding gives

$$1 = (1-x)(1+2x+x^2) = 1 + x - x^2 - x^3.$$

Therefore

$$x(1 - x - x^2) = 0.$$

The root $x = 0$ only describes the surface. For a positive height, solve $x^2 + x - 1 = 0$ to obtain $x = (-1+\sqrt{5})/2$. The other quadratic root is negative and cannot represent a positive height or depth. The accepted root is about 0.618, within the permitted depth range. Option A gives $x = 0.5$, for which the two sides are $4/9$ and $1/2$; option C also fails the quadratic, and option D gives a different positive value, about 0.118, which fails it too.

Step 3: Conclusion The nonzero height is $\boxed{h = \frac{(\sqrt{5}-1)R}{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: When equal altitude and depth are compared, keep the exact altitude formula, introduce the ratio h/R, and discard the trivial surface solution only after factoring.

30. When the temperature of a metal wire is increased from 0°C to 10°C, its length increases by 0.02%. The percentage change in its mass density will be closest to : [4 marks]

(A) 0.008

(B) 0.06

(C) 0.8

(D) 2.3

Correct Answer: (B) 0.06

Solution:

Concept:

- For a freely expanding isotropic metal, the fractional volume expansion is approximately three times the fractional linear expansion when the change is small.
- Density is mass divided by volume. The mass of the wire remains fixed during heating, so an increase in its volume causes a decrease in density.

Step 1: Convert the linear expansion into volume expansion The stated length increase is 0.02 percent. Its fractional value is

$$\epsilon = \frac{0.02}{100} = 2 \times 10^{-4}.$$

All three linear dimensions increase by the factor $1 + \epsilon$, so

$$\frac{V_f}{V_i} = (1+\epsilon)^3 \simeq 1 + 3\epsilon = 1.0006.$$

The isotropic expansion applies to transverse dimensions as well as the length, even though the sample is a thin wire. The neglected quadratic and cubic contributions are very small compared with the linear term.

Step 2: Calculate the fractional density change Because mass is unchanged,

$$\frac{\rho_f}{\rho_i} = \frac{V_i}{V_f} = (1.0002)^{-3}.$$

Thus the signed percentage change is

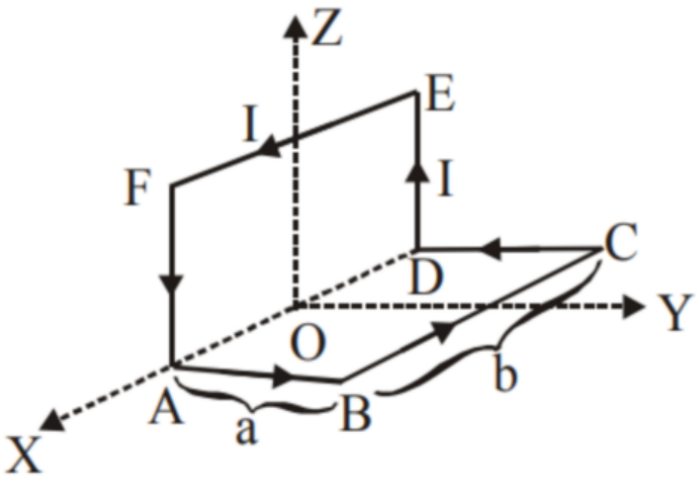
$$100\left[(1.0002)^{-3} - 1\right] \simeq -0.05998\%.$$

Equivalently, the first-order relation gives $\Delta\rho/\rho \simeq -3\epsilon$, or a decrease of 0.06 percent. The options list positive magnitudes, so compare the magnitude of the change. Values 0.008, 0.8 and 2.3 percent are inconsistent with three times the small fractional linear change; 0.06 percent is closest.

Step 3: Conclusion The density decreases by approximately $\boxed{0.06\%}$; its signed percentage change is approximately -0.06%. Hence, the correct option is (B).

Quick Tip to Solve: For a small isotropic expansion at fixed mass, density changes by approximately minus three times the fractional length change. Keep percentage and fractional values distinct.

31. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is [4 marks]



(A) $\sqrt{2}$abI, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

(B) abI, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$

(C) $\sqrt{2}$abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

(D) abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

Correct Answer: (C) $\sqrt{2}$abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$

Solution:

Concept:

- The magnetic moment of a plane current loop is $\boldsymbol{\mu} = I\mathbf{A}$, where the area vector is normal to its plane and its direction follows the right-hand rule for the current.
- For a nonplanar wire loop, add the vector areas of plane loops whose shared edges carry opposite currents. This leaves exactly the current in the original boundary.

Step 1: Resolve the boundary into two rectangles Introduce the common edge AD in the two opposite directions. The loop ABCDA and the loop ADEFA then add to the path ABCDEFA: their currents along DA and AD cancel. Each rectangle has side lengths a and b, so each has area ab. The two magnetic moments must therefore have equal magnitude Iab; adding their magnitudes directly would ignore that their planes are perpendicular.

Step 2: Determine the two normal directions For ABCDA the arrows run from A to B to C to D. Using the axes shown, the right-hand rule gives the area vector along $+\hat{k}$. For ADEFA, the boundary runs from A to D to E to F and the corresponding normal points along $+\hat{j}$. Hence

$$\boldsymbol{\mu}_1 = Iab\hat{k}, \qquad \boldsymbol{\mu}_2 = Iab\hat{j}.$$

The common-edge construction preserves both the size and the sense of the original current.

Step 3: Add the perpendicular magnetic moments The resultant is

$$\boldsymbol{\mu} = Iab(\hat{j} + \hat{k}), \qquad |\boldsymbol{\mu}| = Iab\sqrt{1^2 + 1^2} = \sqrt{2}Iab.$$

Dividing by its magnitude gives the unit direction $(\hat{j} + \hat{k})/\sqrt{2}$. Options A and B give unequal components along two equal-area normals; D has the correct direction but omits the factor $\sqrt{2}$ in the magnitude.

Step 4: Conclusion The magnetic moment has magnitude $\sqrt{2}\,abI$ and direction $(\hat{j} + \hat{k})/\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Close each rectangular face with opposite currents along its shared edge, then add the two area vectors using the right-hand rule.

32. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is : [4 marks]

(A) $\frac{20}{3}J$

(B) $\frac{5}{3}J$

(C) $\frac{10}{3}J$

(D) $\frac{2}{3}J$

Correct Answer: (A) $\frac{20}{3}J$

Solution:

Concept:

- With no external torque about the common axis, angular momentum is conserved: $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$. The same sense of rotation gives both initial terms positive signs.
- The rotational kinetic energy of a rigid system is $K = \frac{1}{2}I\omega^2$. During sticking, internal friction can dissipate energy even though total angular momentum remains unchanged.

Step 1: Calculate the initial angular momentum The discs rotate about the same fixed axis, so their angular momenta add algebraically. Substituting the given moments of inertia and angular speeds,

$$L = (0.1)(10) + (0.2)(5) = 1 + 1 = 2\,\text{kg m}^2\,\text{s}^{-1}.$$

The initial kinetic energy is $\frac{1}{2}(0.1)(10)^2 + \frac{1}{2}(0.2)(5)^2 = 7.5\,\text{J}$, but it is not the quantity conserved when the discs stick.

Step 2: Find the common final angular speed After sticking, both discs have the same angular velocity and their moments of inertia add:

$$I = 0.1 + 0.2 = 0.3\,\text{kg m}^2.$$

Conservation of angular momentum therefore gives

$$\omega = \frac{L}{I} = \frac{2}{0.3} = \frac{20}{3}\,\mathrm{rad\,s^{-1}}.$$

This lies between the original speeds of 5 and 10, as required when friction brings co-rotating discs to a common speed.

Step 3: Evaluate the final energy and compare options Use the combined inertia, not either disc alone:

$$K_f = \frac{1}{2}(0.3)\left(\frac{20}{3}\right)^2 = \frac{20}{3}\,\mathrm{J}.$$

The lost energy is $7.5 - 20/3 = 5/6\,\mathrm{J} > 0$, consistent with friction. The other proposed energies do not satisfy $K_f = L^2/[2(I_1 + I_2)]$ for the conserved value $L = 2$.

Step 4: Conclusion The kinetic energy of the combined system is $\boxed{20/3\,\mathrm{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: For discs that stick together, first conserve angular momentum; calculate the energy only after finding the common angular speed.

33. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?

(A) the mean free path of the molecules decreases.

(B) the mean collision time between the molecules decreases.

(C) the mean free path remains unchanged.

(D) the mean collision time remains unchanged. [4 marks]

(A) (C) and (D)

(B) (A) and (D)

(C) (B) and (C)

(D) (A) and (B)

Correct Answer: (C) (B) and (C)

Solution:

Concept:

- For gas molecules of collision diameter d, the mean free path is $\lambda = 1/(\sqrt{2}\pi d^2 n)$, where $n = N/V$ is the number density.
- The mean molecular speed varies as $\bar{v} = \sqrt{8k_BT/(\pi m)}$. The mean collision time is $\tau = \lambda/\bar{v}$, so both distance and speed must be considered.

Step 1: Identify what the closed container keeps fixed The gas is heated in its container, whose volume is fixed in the usual rigid-container interpretation. Because the container is closed, no molecules escape; thus both N and V remain constant. The number density $n = N/V$ consequently does not change during heating. The molecular size d is also treated as constant in this kinetic-theory model.

Step 2: Examine the mean free path Insert the constant number density and molecular diameter into

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}.$$

There is no independent temperature dependence when n is fixed. Although the pressure rises with temperature, that does not compress the gas in a rigid container. Therefore the mean free path remains unchanged: statement C is true and statement A is false.

Step 3: Examine the time between collisions Heating increases the average molecular speed, since $\bar{v} \propto \sqrt{T}$. The same average free-path distance is covered more quickly, giving

$$\tau = \frac{\lambda}{\bar{v}} \propto T^{-1/2}.$$

The collision time decreases, so statement B is true and statement D is false. Thus option C alone contains both true statements. A includes the false statement D, B includes two false statements, and D includes the false statement A.

Step 4: Conclusion Collision time decreases while mean free path stays unchanged; the true statements are the second and third. Hence, the correct option is (C).

Quick Tip to Solve: In a fixed-volume container, mean free path depends on number density; heating changes molecular speed and therefore the time between collisions.

34. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to (g = 10 ms^{-2}) [4 marks]

(A) 0.049 m

(B) 0.087 m

(C) 0.137 m

(D) 0.172 m

Correct Answer: (B) 0.087 m

Solution:

Concept:

- For a liquid rising in a capillary of radius r, vertical force balance gives $2\pi r S\cos\theta = \pi r^2 h\rho g$, where S is surface tension and θ is the contact angle measured through the liquid.
- The rise is $h = 2S\cos\theta/(\rho g r)$. The two symmetric tangent directions at the opposite walls determine the contact angle; their included angle is not itself θ.

Step 1: Infer the contact angle from the tangents At the two opposite walls the meniscus is symmetric. The tangent lines meet at an acute angle of $60°$; each tangent therefore makes $30°$ with the vertical symmetry line. Since the glass walls are vertical, the contact angle for this rising meniscus is $\theta = 30°$.

Hence

$$\cos\theta = \frac{\sqrt{3}}{2}.$$

Using 60° directly would mistake the angle between two tangents for the angle between one tangent and a wall.

Step 2: Convert the radius and balance forces The radius must be expressed in metres:

$$r = 0.15\,\text{mm} = 1.5 \times 10^{-4}\,\text{m}.$$

The upward surface-tension force acts around the circular contact perimeter. Equating its vertical component to the weight of the raised liquid gives

$$h = \frac{2S\cos\theta}{\rho g r}.$$

The given density is used exactly as stated, $\rho = 667\,\text{kg}\,\text{m}^{-3}$.

Step 3: Substitute and select the nearest height With $S = 0.05\,\text{N}\,\text{m}^{-1}$ and $g = 10\,\text{m}\,\text{s}^{-2}$,

$$h = \frac{2(0.05)(\sqrt{3}/2)}{667(10)(1.5 \times 10^{-4})} = \frac{0.08660}{1.0005}\,\text{m} \approx 0.08656\,\text{m}.$$

This rounds to $0.087\,\text{m}$. The values 0.049, 0.137 and 0.172 m do not agree with the vertical-force balance for the specified radius and contact angle.

Step 4: Conclusion The rise is approximately $h = 0.087\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the angle with the vertical wall before using the capillary-rise formula, and convert the tube radius from millimetres to metres.

35. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is : [4 marks]



(A) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}$Q

(B) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

Correct Answer: (A) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}$Q

Solution:

Concept:

- The potential everywhere inside a uniformly charged thin spherical shell of radius s and total charge q is $V = q/(4\pi\varepsilon_0 s)$, taking zero potential at infinity.
- Equal surface charge densities imply charges proportional to surface areas, so $q_r : q_R = r^2 : R^2$. Potentials are scalars and add directly even where the net electric field vanishes.

Step 1: Express the charge on each shell Let the common surface charge density be σ. For the inner and outer shells respectively,

$$q_r = 4\pi r^2\sigma, \qquad q_R = 4\pi R^2\sigma.$$

The total charge is distributed between the two shells, so

$$Q = q_r + q_R = 4\pi\sigma(r^2 + R^2).$$

Consequently $\sigma = Q/[4\pi(r^2 + R^2)]$. Equal density must not be confused with equal charge: the larger shell carries more charge.

Step 2: Add the centre potentials of the shells The common centre lies inside both spherical surfaces.

Each shell gives the same potential as on its own surface, hence

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_r}{r} + \frac{q_R}{R}\right).$$

Substituting the charges in terms of σ,

$$V = \frac{1}{4\pi\varepsilon_0}\, 4\pi\sigma(r + R).$$

A zero field at the centre does not imply zero potential; potential is measured relative to infinity.

Step 3: Eliminate the density and compare formulas Using the expression obtained from the fixed total charge,

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{R + r}{R^2 + r^2}.$$

This expression treats the radii symmetrically, as equal densities require. Option B introduces an unjustified factor of two. Options C and D give unequal coefficients to the radii despite identical density treatment. Only A reproduces the direct sum of the shell potentials.

Step 4: Conclusion The potential is $\boxed{V = \frac{Q(R + r)}{4\pi\varepsilon_0(R^2 + r^2)}}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal surface densities, first divide the total charge in the ratio of squared radii; then add each shell contribution as charge divided by radius.

36. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is [4 marks]

(A) 6.7×10^{-7} H

(B) 1.1×10^{-1} H

(C) 5.5×10^{-5} H

(D) 1.1×10^{-2} H

Correct Answer: The stated reactance gives 1.59×10^{-2} H, absent from the options; keyed (D) follows only if 100 ohms means impedance.

Solution:

Concept:

- For a coil of inductance L, the inductive reactance is $X_L = \omega L = 2\pi f L$. Reactance is the imaginary component of impedance, not its magnitude.
- For a coil with resistance R, $|Z| = \sqrt{R^2 + X_L^2}$ and $\tan\phi = X_L/R$. The phase 45° therefore gives $R = X_L$.

Step 1: Apply the reactance stated in the question The printed quantity is explicitly called reactance

and has value $X_L = 100\,\Omega$. With $f = 1000\,\text{Hz}$, the inductance follows directly:

$$L = \frac{X_L}{2\pi f} = \frac{100}{2\pi(1000)} = \frac{1}{20\pi}\,\text{H} \approx 1.59 \times 10^{-2}\,\text{H}.$$

The phase angle does not introduce an extra factor into this relation; it supplies information about the resistance.

Step 2: Check consistency with the phase angle The applied voltage leads current by 45°, so

$$\tan 45^\circ = \frac{X_L}{R} = 1, \qquad R = 100\,\Omega.$$

The coil would then have impedance magnitude $|Z| = 100\sqrt{2}\,\Omega$. This is fully consistent with the stated reactance and phase. None of the four listed inductances equals $1.59 \times 10^{-2}\,\text{H}$, so the wording does not support an exact listed answer.

Step 3: Explain the value associated with option D If the stated $100\,\Omega$ were instead the impedance magnitude, the phase relation would give

$$X_L = |Z| \sin 45^\circ = \frac{100}{\sqrt{2}}\,\Omega,$$

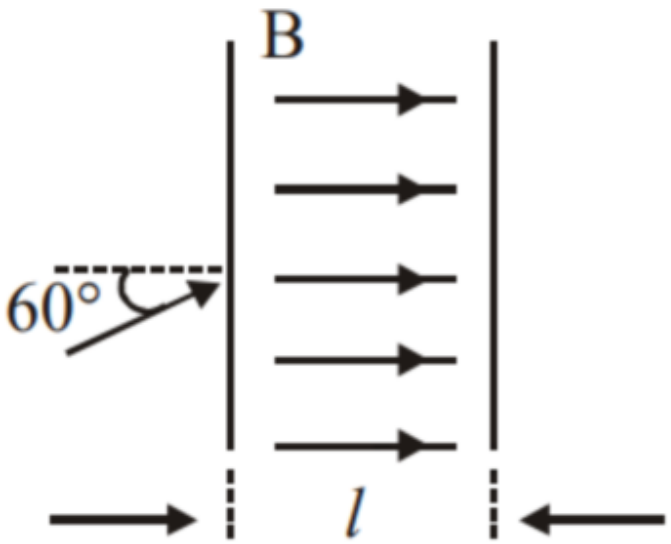
$$L = \frac{100/\sqrt{2}}{2\pi(1000)} \approx 1.13 \times 10^{-2}\,\text{H}.$$

That rounds to option D, which is the keyed choice. It requires interpreting the stated reactance as impedance. Options A, B and C agree with neither calculation; D matches only this alternate interpretation.

Step 4: Conclusion As printed, $L = 1.59 \times 10^{-2}\,\text{H}$, so none of the options is exact. The key gives (D), $1.1 \times 10^{-2}\,\text{H}$, which follows if $100\,\Omega$ denotes impedance rather than the stated reactance.

Quick Tip to Solve: Distinguish reactance from impedance: use $X_L = 2\pi f L$ directly for reactance, and use $X_L = |Z| \sin\phi$ only when the given value is impedance.

37. The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to
(mass of proton = 1.67×10^{-27} kg, charge of the proton = 1.6×10^{-19} C) [4 marks]



(A) 0.22 m

(B) 0.11 m

(C) 0.88 m

(D) 0.44 m

Correct Answer: (D) 0.44 m

Solution:

Concept:

- A magnetic force acts perpendicular to velocity and does no work. Resolve the speed into $v_{\parallel} = v\cos\theta$ along the field and $v_{\perp} = v\sin\theta$ perpendicular to it.
- The perpendicular component produces circular motion with period $T = 2\pi m/(qB)$. The parallel component stays constant, so the pitch of the helix is $v_{\parallel}T$.

Step 1: Identify the speed across the region The field and the length of the region are horizontal in the figure. Therefore the transit distance is determined by the component parallel to the field, not by the speed around the circular path. With the stated angle,

$$v_{\parallel} = 4 \times 10^5 \cos 60^\circ = 2 \times 10^5 \,\mathrm{m\,s^{-1}}.$$

The perpendicular component changes direction continuously but does not contribute to the net distance along the field.

Step 2: Calculate the time for ten revolutions For a proton the magnetic force supplies the centripetal

force: $qv_\perp B = mv_\perp^2/r$. Consequently

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi(1.67\times10^{-27})}{(1.6\times10^{-19})(0.3)} \simeq 2.19\times10^{-7}\,\text{s}.$$

Ten complete revolutions take $10T = 2.19\times10^{-6}\,\text{s}$. The period is independent of the pitch angle.

Step 3: Find the required length and compare The parallel motion is uniform, hence

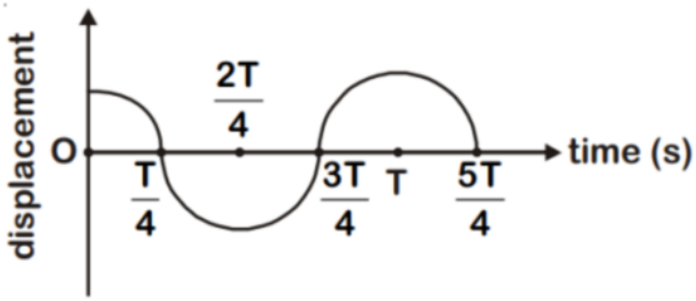
$$l = v_\parallel(10T) = (2\times10^5)(2.19\times10^{-6}) \simeq 0.437\,\text{m}.$$

This is close to 0.44 m. The smaller lengths 0.11 m and 0.22 m would correspond to too few turns at this parallel speed; 0.88 m is about twice the required distance.

Step 4: Conclusion The region has length $l \simeq 0.44\,\text{m}$. Hence, the correct option is (D).

Quick Tip to Solve: For a helix crossing a field region, multiply the number of turns by $v\cos\theta\, 2\pi m/(qB)$; the circular speed controls the radius, not the transit length.

38. The displacement time graph of a particle executing S.H.M is given in figure :
(sketch is schematic and not to scale)
Which of the following statements is/are true for this motion?
(A) The force is zero at t = $\frac{3T}{4}$
(B) The acceleration is maximum at t = T
(C) The speed is maximum at t = $\frac{T}{4}$
(D) The P.E. is equal to K.E. of the oscillation at t = $\frac{T}{2}$ [4 marks]



(A) (B), (C) and (D)
(B) (A), (B) and (C)
(C) (A) and (D)
(D) (A), (B) and (D)
Correct Answer: (B) (A), (B) and (C)

Solution:

Concept:

- For simple harmonic motion, restoring force and acceleration obey $F = -kx$ and $a = -\omega^2 x$, where $\omega = 2\pi/T$. Their magnitudes are largest at an extreme displacement.
- The speed satisfies $v^2 = \omega^2(A^2 - x^2)$, while $U = \frac{1}{2}kx^2$ and $K = \frac{1}{2}k(A^2 - x^2)$. Equal energies require $|x| = A/\sqrt{2}$.

Step 1: Read the equilibrium crossings The schematic graph crosses the time axis at $T/4$ and $3T/4$. Thus $x = 0$ at both instants. At $t = 3T/4$, $F = -kx = 0$, so statement (A) is true. At $t = T/4$, all of the mechanical energy is kinetic and

$$|v| = \omega\sqrt{A^2 - 0} = \omega A.$$

This is the maximum speed, so statement (C) is true. The graph need not be drawn to scale to identify these crossings.

Step 2: Check acceleration at the extreme At $t = T$, the particle is at its positive extreme displacement, $x = A$. Its acceleration is directed toward equilibrium:

$$a = -\omega^2 A, \qquad |a| = \omega^2 A.$$

Thus the acceleration has its maximum magnitude, which is the meaning of maximum acceleration in statement (B). Statement (B) is true; its direction is opposite to the displacement.

Step 3: Compare the energies at half a period At $t = T/2$, the graph reaches the negative extreme, $x = -A$, so the particle is instantaneously at rest. Therefore

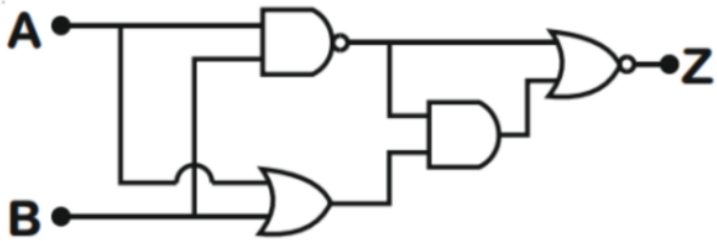
$$U = \tfrac{1}{2}kA^2, \qquad K = 0.$$

They are not equal, making statement (D) false. Every other listed combination includes (D), so all three of those options are eliminated.

Step 4: Conclusion The true statements are those concerning zero force, maximum acceleration magnitude and maximum speed. The equal-energy statement is false. Hence, the correct option is (B).

Quick Tip to Solve: At a displacement-axis crossing, force and potential energy vanish and speed is greatest; at an extreme, speed vanishes and acceleration magnitude is greatest.

39. In the following digital circuit, what will be the output at 'Z', when the input (A, B) are (1, 0), (0, 0), (1, 1,), (0, 1) [4 marks]



(A) 1, 1, 0, 1

(B) 0, 1, 0, 0

(C) 1, 0, 1, 1

(D) 0, 0, 1, 0

Correct Answer: (D) 0, 0, 1, 0

Solution:

Concept:

- An AND gate gives 1 only when both inputs are 1; an OR gate gives 1 when either input is 1. A small circle on an output represents logical inversion.
- The Boolean absorption law is $X + XY = X$. Apply it to the signals entering the final NOR gate before evaluating the input sequence.

Step 1: Name the intermediate signals The upper left gate has the AND outline and an inverted output, so it is a NAND gate. Set

$$X = \overline{AB}.$$

The lower left gate is an OR gate with output

$$Y = A + B.$$

The curved bridge at the wire crossing means that the crossing wires are not connected there; each gate still receives the inputs shown in the circuit. The middle gate is AND, so its output is XY.

Step 2: Simplify the final output The final gate is an OR gate with a small output circle, hence a NOR gate. Its inputs are X directly and XY from the middle gate. Thus

$$Z = \overline{X + XY} = \overline{X} = \overline{\overline{AB}} = AB.$$

The equality $X + XY = X$ follows because if $X = 1$, the OR output is already 1, while if $X = 0$, both terms are zero. The whole network therefore behaves as one AND gate.

Step 3: Evaluate the inputs in their stated order For $(A, B) = (1, 0), (0, 0), (1, 1), (0, 1)$, the products are

$$1 \cdot 0 = 0, \quad 0 \cdot 0 = 0, \quad 1 \cdot 1 = 1, \quad 0 \cdot 1 = 0.$$

Thus only the third input pair produces a high output. Option (A) is high for the wrong pairs, option (B) is high only for the second pair, and option (C) is high for three pairs; none represents the derived

AND response.

Step 4: Conclusion The output sequence is $0, 0, 1, 0$. Hence, the correct option is (D).

Quick Tip to Solve: When one input of an OR gate is X and the other contains X as an AND factor, use absorption before making a truth table.

40. In a hydrogen atom the electron makes a transition from (n + 1)th level to the nth level. If n >> 1, the frequency of radiation emitted is proportional to : [4 marks]

(A) $\frac{1}{n}$
(B) $\frac{1}{n^2}$
(C) $\frac{1}{n^3}$
(D) $\frac{1}{n^4}$

Correct Answer: (C) $\frac{1}{n^3}$

Solution:

Concept:

- The bound-state energy of hydrogen is $E_n = -E_H/n^2$, where $E_H = 13.6\,\text{eV}$. The emitted photon carries the positive energy difference $h\nu = E_{n+1} - E_n$.
- For $n \gg 1$, retain the leading power of n in an exact expression. The frequency of a transition between adjacent levels depends on their spacing, not on the energy of one level alone.

Step 1: Write the photon energy with the correct sign The initial level is $n+1$, which lies above level n. Subtracting the final energy from the initial energy gives

$$h\nu = E_H \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right).$$

The bracket is positive, as expected for emission. The constants E_H and h do not depend on n, so they cannot affect the required proportionality.

Step 2: Combine the fractions before approximating The difference simplifies exactly to

$$\nu = \frac{E_H}{h} \frac{(n+1)^2 - n^2}{n^2(n+1)^2} = \frac{E_H}{h} \frac{2n+1}{n^2(n+1)^2}.$$

For a large quantum number, $2n + 1 \simeq 2n$ and $(n+1)^2 \simeq n^2$. Hence

$$\nu \simeq \frac{2E_H}{h} \frac{n}{n^4} = \frac{2E_H}{hn^3}.$$

The two energies are each of order n^{-2}, but their small difference is of order n^{-3}.

Step 3: Check the exponent against the options The numerator contributes one power of n, whereas the denominator contributes four, leaving the inverse third power. An inverse-square answer would describe the binding-energy scale rather than the adjacent-level spacing. The inverse first and fourth powers also disagree with the exact fraction. Equivalently, differentiating $-E_H/n^2$ gives an

adjacent-level spacing approaching $2E_H/n^3$.

Step 4: Conclusion For $n \gg 1$, the emitted frequency satisfies $\nu \propto n^{-3}$. Hence, the correct option is (C).

Quick Tip to Solve: For transitions between neighboring high Bohr levels, take the difference first; differentiating the energy dependence gives the same leading inverse-cube spacing.

41. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i} - 2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave?

[4 marks]

(A) $\frac{1}{\sqrt{5}}\left(\hat{i} + 2\hat{j}\right)$

(B) $\frac{1}{\sqrt{5}}\left(2\hat{i} + \hat{j}\right)$

(C) $\frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

(D) $\frac{1}{\sqrt{2}}\left(\hat{j} + \hat{k}\right)$

Correct Answer: (C) $\frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

Solution:

Concept:

- The energy flux of an electromagnetic wave points along the Poynting vector $\mathbf{S} = (\mathbf{E} \times \mathbf{B})/\mu_0$. The propagation direction therefore uses the ordered cross product $\mathbf{E} \times \mathbf{B}$.
- A unit direction vector is obtained by dividing a nonzero direction vector by its magnitude. In a transverse wave it is perpendicular to both electric and magnetic fields.

Step 1: Compute the ordered cross product The electric field points along $\hat{k}$, while the magnetic field points along $2\hat{i} - 2\hat{j}$. Their positive magnitudes do not change the direction. Using the right-handed Cartesian identities $\hat{k} \times \hat{i} = \hat{j}$ and $\hat{k} \times \hat{j} = -\hat{i}$,

$$\hat{k} \times (2\hat{i} - 2\hat{j}) = 2\hat{j} + 2\hat{i}.$$

Reversing the order would give the opposite direction and would describe energy traveling backward.

Step 2: Normalize and verify perpendicularity The magnitude of $2\hat{i} + 2\hat{j}$ is $\sqrt{2^2 + 2^2} = 2\sqrt{2}$. Thus

$$\hat{n} = \frac{2\hat{i} + 2\hat{j}}{2\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}.$$

Its squared magnitude is $1/2 + 1/2 = 1$. Its dot product with $\hat{k}$ vanishes, and its dot product with $2\hat{i} - 2\hat{j}$ is $(2 - 2)/\sqrt{2} = 0$, so both transverse conditions hold.

Step 3: Eliminate the other direction vectors Options (A) and (B) have unequal in-plane components, so their dot products with $2\hat{i} - 2\hat{j}$ are nonzero. Option (D) has a nonzero $\hat{k}$ component and therefore is not perpendicular to the electric field. Only the equal positive $\hat{i}$ and $\hat{j}$ components agree with the right-hand rule and normalization.

Step 4: Conclusion The unit propagation vector is $\hat{n} = (\hat{i} + \hat{j})/\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use electric field cross magnetic field, then normalize. A quick dot-product check against both fields detects a wrong direction immediately.

42. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to [4 marks]

(A) 1.2×10^{-28} kg

(B) 9.1×10^{-31} kg

(C) 4.8×10^{-27} kg

(D) 9.7×10^{-28} kg

Correct Answer: (D) 9.7×10^{-28} kg

Solution:

Concept:

- For nonrelativistic matter waves, $\lambda = h/p = h/(mv)$. A shorter wavelength corresponds to a larger momentum, rather than necessarily a larger speed alone.
- When comparing two particles, Planck's constant cancels: $\lambda_p/\lambda_e = m_e v_e/(m_p v_p)$. Use the speed ratio and the wavelength ratio together to determine the mass.

Step 1: Translate the speed information into the wavelength ratio Let m and v_p denote the mass and speed of the particle. The electron has mass m_e and speed v_e, with $v_p = 5v_e$. Therefore

$$\frac{\lambda_p}{\lambda_e} = \frac{h/(mv_p)}{h/(m_e v_e)} = \frac{m_e v_e}{m(5v_e)} = \frac{m_e}{5m}.$$

The electron speed cancels, so its numerical value is not needed. The inverse dependence on momentum must be retained when rearranging.

Step 2: Calculate the particle mass Insert the given ratio 1.878×10^{-4} and the electron mass 9.1×10^{-31} kg:

$$m = \frac{9.1 \times 10^{-31}}{5(1.878 \times 10^{-4})}\,\text{kg} = \frac{9.1 \times 10^{-31}}{9.39 \times 10^{-4}}\,\text{kg} \simeq 9.69 \times 10^{-28}\,\text{kg}.$$

Thus the mass is approximately 1.065×10^{3} electron masses. Multiplication by the speed ratio then gives the required momentum ratio.

Step 3: Check the alternatives by substitution For $m = 9.7 \times 10^{-28}$ kg, the predicted ratio is

$$\frac{9.1 \times 10^{-31}}{5(9.7 \times 10^{-28})} \simeq 1.876 \times 10^{-4},$$

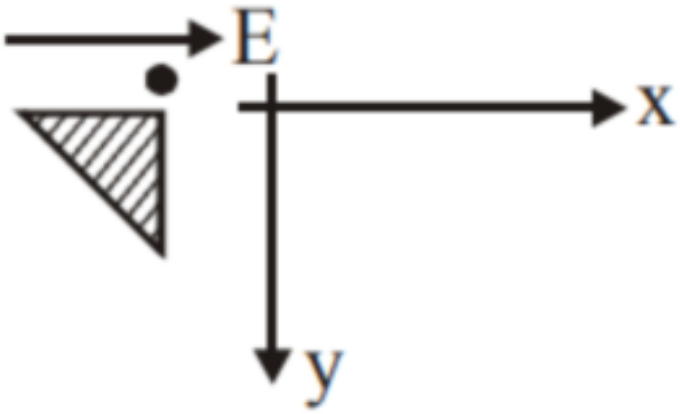
agreeing to rounding. Options (A), (B) and (C) instead give about 1.52×10^{-3}, 0.2, and 3.79×10^{-5}, respectively. None matches the stated wavelength ratio.

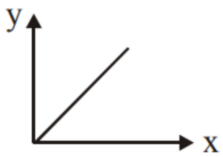
Step 4: Conclusion The particle mass is close to 9.7×10^{-28} kg. Hence, the correct option is (D).

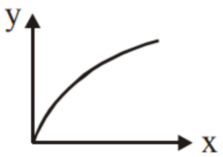
Quick Tip to Solve: Write the wavelength ratio as the inverse momentum ratio first. The factor of five belongs with the unknown mass in the denominator, preventing a factor-of-25 error.

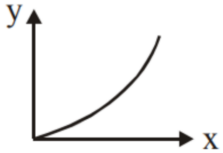
43. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass?

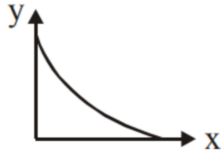
(Curves are drawn schematically and are not to scale). [4 marks]

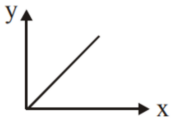


(A) 

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- A charge $q > 0$ in a uniform electric field E experiences a constant force qE along the field. Gravity supplies the constant downward force mg.
- The particle is released from rest, so its initial velocity is zero in both directions. For constant acceleration, displacement from release is $\frac{1}{2}at^2$.

Step 1: Resolve the constant acceleration Take the positive horizontal direction along the electric field and the positive vertical direction downward, as in the figure. The independent acceleration components are

$$a_x = \frac{qE}{m}, \qquad a_y = g.$$

Both components remain constant throughout the motion. There is no initial horizontal velocity: the word released means that the mass is allowed to start from rest rather than projected from the table.

Step 2: Eliminate time from the two displacements Measure x and y from the release point. At time t,

$$x = \frac{qE}{2m}t^2, \qquad y = \frac{g}{2}t^2.$$

Dividing these equations eliminates the common t^2 factor and gives

$$y = \frac{mg}{qE}x.$$

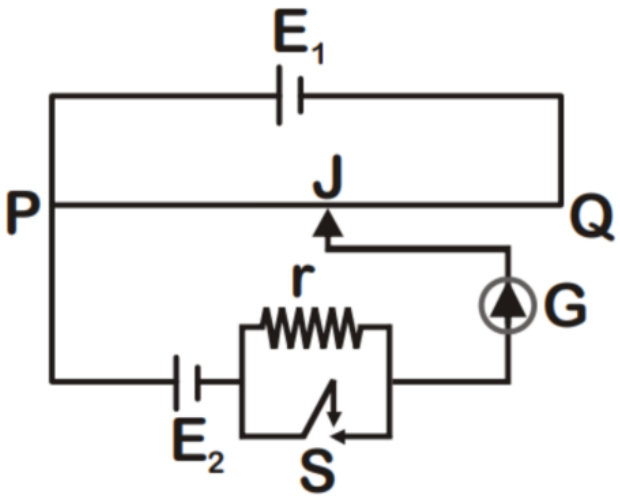
The ratio y/x is positive and constant, so the trajectory is a straight line through the origin. Its speed increases along that line; increasing speed does not by itself make a path curved.

Step 3: Compare the schematic alternatives Option (A) shows a straight line through the release point. Options (B) and (C) show a changing slope, which would require a varying ratio of vertical to horizontal displacement. Option (D) has decreasing y as x increases, inconsistent with the downward acceleration and zero initial velocity. The familiar parabolic projectile trajectory applies when an independent initial velocity is present, not to release from rest under one constant resultant acceleration.

Step 4: Conclusion The mass follows a straight line in the direction of the constant resultant acceleration, with $y = (mg/qE)x$. Hence, the correct option is (A).

Quick Tip to Solve: For release from rest under several constant forces, first combine their accelerations: the displacement vector always remains parallel to that resultant.

44. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is : [4 marks]



(A) 0.04 V/cm

(B) 0.01 V/cm

(C) 0.02 V/cm

(D) 0.03 V/cm

Correct Answer: (C) 0.02 V/cm

Solution:

Concept:

- At a potentiometer balance point no current flows through the galvanometer branch. The balancing potential drop equals the emf $\mathcal{E}_2$ of the test cell.
- For a uniform wire, potential gradient k is potential drop per unit length, so $\mathcal{E}_2 = k\ell$, where ℓ is measured from the connected high-potential end.

Step 1: Identify the balancing length from the diagram The positive terminal of the test cell is connected to end P of the potentiometer wire, while the other terminal reaches the jockey J through the galvanometer. Thus the cell emf is balanced by the drop along PJ. The supplied 49 cm is measured from Q, which is the opposite end. Since the whole wire is 1 m long,

$$PQ = 100\,\text{cm}, \qquad PJ = 100 - 49 = 51\,\text{cm}.$$

The Q label marks the far end: using QJ = 49 cm directly would use the wrong segment.

Step 2: Apply the null-point condition With switch S open, the resistance r lies in the test branch. However, the galvanometer current is zero at balance, so the potential drop across that resistance is zero. It therefore does not modify the balancing emf. We obtain

$$k = \frac{\mathcal{E}_2}{PJ} = \frac{1.02\,\text{V}}{51\,\text{cm}} = 0.020\,\text{V cm}^{-1}.$$

In SI units this is $2.0\,\text{V m}^{-1}$; the numerical value changes because a metre contains 100 centimetres.

Step 3: Check the listed values The balance segment is 51 cm. A gradient of 0.02 V/cm gives precisely $0.02 \times 51 = 1.02$ V. The other offered gradients give 2.04 V, 0.51 V and 1.53 V over this same segment, so none equals the given test-cell emf. The standard cell establishes the gradient, but its emf need not be separately known when a balance length and test-cell emf are provided.

Step 4: Conclusion The potential gradient is $0.02\,\text{V cm}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Locate the terminal to which the test cell is connected before choosing a balance length; a distance quoted from the other end must be subtracted from the full wire length.

45. A heat engine is involved with exchange of heat of 1915 J, – 40J, + 125 J and –Q J, during one cycle achieving an efficiency of 50.0%. The value of Q is [4 marks]

(A) 980 J

(B) 40 J

(C) 400 J

(D) 640 J

Correct Answer: (A) 980 J

Solution:

Concept:

- For a complete cycle the working substance returns to its initial state, so $\Delta U = 0$. The net work output equals the algebraic sum of the heat exchanges.
- Engine efficiency is $\eta = W/Q_{\text{in}} = 1 - Q_{\text{out}}/Q_{\text{in}}$. Here Q_{in} includes every positive heat input; Q_{out} is the magnitude of all rejected heat.

Step 1: Group absorbed and rejected heat separately The positive heat exchanges are 1915 J and 125 J. Both represent energy entering the engine, so

$$Q_{\text{in}} = 1915 + 125 = 2040\,\text{J}.$$

The negative exchanges are -40 J and $-Q$ J, representing heat leaving the engine. Their total positive magnitude is

$$Q_{\text{out}} = 40 + Q.$$

The 125 J must not be subtracted from the input, and the 40 J must not be counted as an absorbed quantity.

Step 2: Use the stated efficiency The efficiency is $50.0\% = 0.500$. Thus the work delivered in one cycle is

$$W = 0.500(2040) = 1020\,\text{J}.$$

From the first law for the cycle,

$$W = 1915 - 40 + 125 - Q = 2000 - Q.$$

Equating the two expressions yields

$$2000 - Q = 1020, \qquad Q = 980\,\text{J}.$$

Equivalently, the rejected heat must be half of 2040 J, giving $40 + Q = 1020$ J.

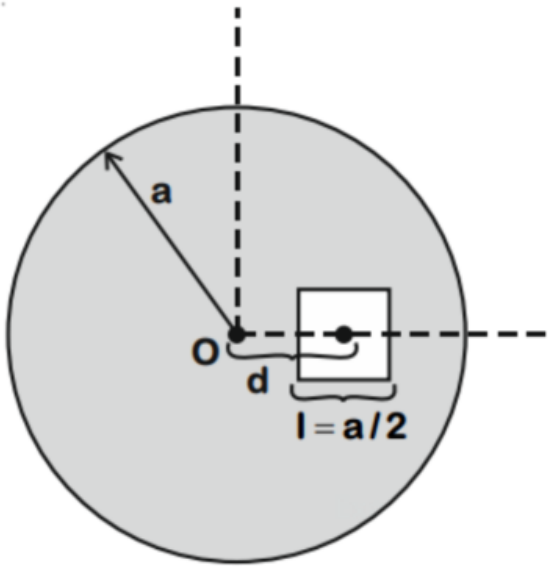
Step 3: Check the alternatives against the energy balance With $Q = 980$ J, the net heat is 1020 J and the efficiency is exactly $1020/2040 = 0.5$. If Q were 40 J, 400 J or 640 J, the corresponding work outputs would be 1960 J, 1600 J or 1360 J, respectively. None is half the absorbed heat. The desired Q is a positive magnitude; the actual associated signed heat exchange is -980 J.

Step 4: Conclusion The magnitude of the final rejected heat exchange is $Q = 980\,\text{J}$. Hence, the correct option is (A).

Quick Tip to Solve: When an engine has several heat exchanges, add all positive inputs first; efficiency uses their total, not just the largest heat input.

Physics – Section B: Numerical Value Questions (20 marks)

46. A square shaped hole of side l = $\frac{a}{2}$ is carved out at a distance d = $\frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion form O is $-\frac{a}{X}$, value of X (to the nearest integer) is : [4 marks]



Solution:

Concept:

- A uniform lamina has mass proportional to area. A removed piece can be treated as a negative mass at the centre of that piece when finding the remaining centre of mass.
- For collinear centres, $x_{\text{cm}} = (Mx_0 - mx_h)/(M - m)$, where M is the mass of the full disc and m is the removed mass.

Step 1: Express both masses using a common areal density Let the constant mass per unit area be σ. The full disc has radius a and mass

$$M = \sigma\pi a^2.$$

The square hole has side $a/2$, so the removed mass is

$$m = \sigma\left(\frac{a}{2}\right)^2 = \frac{\sigma a^2}{4}.$$

The diagram places the square centre at $x_h = d = a/2$ to the right of O. This distance is to its centre, not to its nearer edge.

Step 2: Subtract the removed first moment The full disc is symmetric about O, so its first moment about O is zero. Removing the square subtracts its positive first moment. The remaining centre lies on the horizontal symmetry axis and satisfies

$$x_{\text{cm}} = \frac{0 - m(a/2)}{M - m}.$$

Substituting the areas gives

$$x_{\text{cm}} = -\frac{(\sigma a^2/4)(a/2)}{\sigma\pi a^2 - \sigma a^2/4} = -\frac{a}{8\pi - 2}.$$

The density and the common area factor cancel, as expected for a purely geometric result.

Step 3: Match the required form and round only at the end The stated signed displacement is $-a/X$. Comparing denominators gives

$$X = 8\pi - 2 = 23.1327\ldots.$$

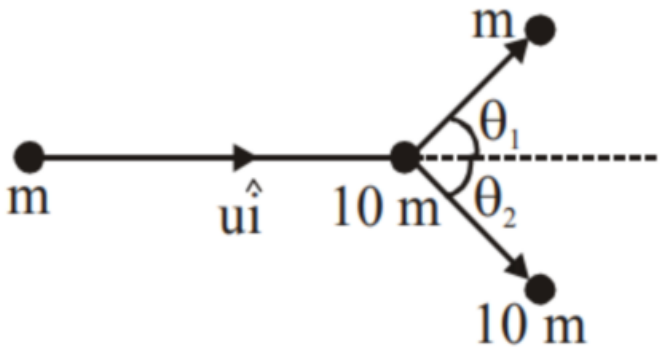
Therefore the nearest integer is 23. The negative displacement is physically consistent: removing material on the right shifts the centre of mass to the left. Its magnitude is about $0.0432a$, much smaller than the hole-centre distance because the removed square has only a small fraction of the full disc mass.

Step 4: Conclusion $X = 8\pi - 2$, so the requested nearest-integer value is $\boxed{23}$. The parameter X is dimensionless.

Quick Tip to Solve: For a hole in a uniform lamina, subtract its area times its centre coordinate and divide by the remaining area; keep the sign to identify the direction of the shift.

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47. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is _____. [4 marks]



Solution:

Concept:

- In an elastic collision, total kinetic energy and vector momentum are conserved. The initially stationary particle contributes no initial momentum or kinetic energy.
- The incident direction is the x-axis, so initial y-momentum is zero. The outgoing y-components in the figure have opposite signs and equal magnitudes.

Step 1: Find the outgoing speed of the lighter particle Write v_1 and v_2 for the final speeds of the

masses m and $10m$, respectively. The lighter particle retains half its initial kinetic energy, so

$$\frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right), \qquad v_1^2 = \frac{u^2}{2}.$$

Hence $v_1 = u/\sqrt{2}$. The half refers to kinetic energy, so it does not mean that the final speed is $u/2$.

Step 2: Use elastic energy conservation for the heavier particle The initial total kinetic energy is $\frac{1}{2}mu^2$. The lighter particle retains $\frac{1}{4}mu^2$, leaving the other half for the heavier particle:

$$\frac{1}{2}(10m)v_2^2 = \frac{1}{4}mu^2.$$

Cancelling m gives

$$v_2^2 = \frac{u^2}{20}, \qquad v_2 = \frac{u}{\sqrt{20}}.$$

Thus the two final kinetic energies are equal, although their speeds and momenta are different because their masses differ.

Step 3: Conserve the transverse momentum and identify n The light particle moves above the incident axis and the heavy particle below it. Conservation of y-momentum gives

$$mv_1 \sin\theta_1 - 10mv_2 \sin\theta_2 = 0.$$

Therefore

$$\sin\theta_1 = \frac{10v_2}{v_1}\sin\theta_2 = \frac{10(u/\sqrt{20})}{u/\sqrt{2}}\sin\theta_2 = \sqrt{10}\sin\theta_2.$$

Comparison with $\sin\theta_1 = \sqrt{n}\sin\theta_2$ yields $n = 10$. Solving either scattering angle individually is unnecessary for this ratio.

Step 4: Conclusion The relation is $\sin\theta_1 = \sqrt{10}\sin\theta_2$, so $\boxed{n = 10}$. The parameter n is dimensionless.

Quick Tip to Solve: When a two-body collision splits kinetic energy equally, use energy to find the speed ratio and transverse momentum to connect the scattering angles.

---

48. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is : (Young's modulus of wire Y = 9×10^{10} Nm^{-2}), (to the nearest integer), _____ [4 marks]

Solution:

Concept:

- For a wire fixed at both ends, the fundamental mode has wavelength $2L$, so $f_1 = v/(2L)$, with wave speed $v = \sqrt{T/\mu}$.
- Young's modulus relates stress and strain by $Y = (T/A)/\epsilon$. For cross-sectional area A and volume density ρ, the linear density is $\mu = \rho A$.

Step 1: Convert the density into SI units The quoted density is per cubic centimetre. Since $1\,\text{cm}^3 =$

$10^{-6}\,\mathrm{m}^3$,

$$\rho = 9 \times 10^{-3}\,\mathrm{kg\,cm^{-3}} = 9 \times 10^{3}\,\mathrm{kg\,m^{-3}}.$$

Use $L = 1\,\mathrm{m}$, $Y = 9 \times 10^{10}\,\mathrm{N\,m^{-2}}$, and the dimensionless strain $\epsilon = 4.9 \times 10^{-4}$. Keeping density in kg/cm cubed with the other SI data would give an incorrect frequency by a large factor.

Step 2: Eliminate the unknown cross-sectional area From the definition of Young's modulus, the stretching tension is $T = YA\epsilon$. Also $\mu = \rho A$. Therefore

$$v = \sqrt{\frac{YA\epsilon}{\rho A}} = \sqrt{\frac{Y\epsilon}{\rho}}.$$

The area cancels, so no diameter is needed. Substitution gives

$$v = \sqrt{\frac{(9 \times 10^{10})(4.9 \times 10^{-4})}{9 \times 10^{3}}} = \sqrt{4900} = 70\,\mathrm{m\,s^{-1}}.$$

The quantity under the square root has units of speed squared.

Step 3: Apply the fixed-end fundamental-mode condition The two clamps are displacement nodes. The lowest nonzero mode has one antinode between them, corresponding to half a wavelength across the wire:

$$\lambda_1 = 2L = 2\,\mathrm{m}.$$

Thus

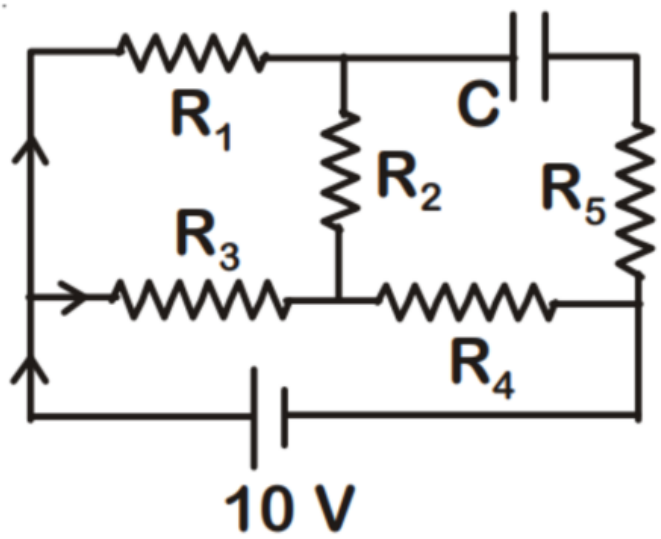
$$f_1 = \frac{v}{\lambda_1} = \frac{70}{2} = 35\,\mathrm{Hz}.$$

The value is already an integer. Higher harmonics are integer multiples of this frequency, so 70 Hz would be the second harmonic rather than the lowest frequency.

Step 4: Conclusion The lowest transverse vibration frequency, to the nearest integer, is $\boxed{35\,\mathrm{Hz}}$.

Quick Tip to Solve: If strain and Young's modulus are supplied instead of tension, combine $T = YA\epsilon$ and $\mu = \rho A$ before substitution; the unknown area cancels.

49. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2 Ω. The potential difference (in V) across the capacitor when it is fully charged is _____. [4 marks]



Solution:

Concept:

- A fully charged ideal capacitor carries no steady direct current: $I_C = C\,dV_C/dt = 0$. Its two plates may nevertheless remain at different potentials.
- Ohm's law is $V = IR$. Resistors in series add, while two parallel resistors obey $R_p = R_a R_b/(R_a + R_b)$.

Step 1: Identify the open branch and choose node potentials Let the right-hand lower junction be at $0\,\text{V}$ and the left battery terminal at $10\,\text{V}$, as indicated by the cell polarity. Call the junction of R_1, R_2 and the left capacitor plate A, and the junction of R_2, R_3, R_4 B. At full charge no current flows through the capacitor branch. Consequently R_5 also carries zero current, its voltage drop is zero, and the right capacitor plate is at $0\,\text{V}$.

Step 2: Reduce the conducting resistor network Between the left node and B, R_1 and R_2 form a series path of $4\,\Omega$, in parallel with $R_3 = 2\,\Omega$. This combination is in series with R_4:

$$R_{LB} = \frac{4 \times 2}{4+2} = \frac{4}{3}\,\Omega, \qquad R_{\text{total}} = \frac{4}{3} + 2 = \frac{10}{3}\,\Omega.$$

The cell current is therefore $I = 10/(10/3) = 3\,\text{A}$. The drop across R_4 is $3 \times 2 = 6\,\text{V}$, so B is at $6\,\text{V}$.

Step 3: Calculate the potential of the left plate The potential drop from the left battery terminal to B is $10 - 6 = 4\,\text{V}$. The current in the R_1, R_2 series path is $4/4 = 1\,\text{A}$. Thus R_1 drops $2\,\text{V}$, leaving A at $10 - 2 = 8\,\text{V}$. The capacitor voltage is the difference between this node and its zero-potential right plate: $V_C = 8 - 0 = 8\,\text{V}$.

Step 4: Conclusion The potential difference across the fully charged capacitor is $\boxed{8\,\text{V}}$. The numerical entry is 8.

Quick Tip to Solve: Open the capacitor branch first; a resistor in that open branch has zero voltage drop, so its outer node fixes the plate potential.

50. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is _____. [4 marks]

Solution:

Concept:

- Snell's law at an air–glass boundary is $\sin i = \mu \sin r$, where the angles are measured from the local normal.
- The normal to a spherical surface is its radius. The two radii drawn to the entry and exit points form an isosceles triangle with the internal ray chord.

Step 1: Find the refracted angle at entry Use the given incidence angle 60° and refractive index $\sqrt{3}$. The refraction angle inside the sphere satisfies

$$\sin r = \frac{\sin 60^\circ}{\sqrt{3}} = \frac{\sqrt{3}/2}{\sqrt{3}} = \frac{1}{2}.$$

The physical refraction angle lies between 0° and 90°, hence $r = 30^\circ$. The ray therefore travels at 30° to the radius through its entry point.

Step 2: Use the spherical geometry at the farther surface Join the sphere's centre to the entry and exit points. Both segments are radii and have equal lengths, so the two base angles of the triangle are equal. The chord makes equal angles with these radii. Thus the angle of incidence at the farther surface is also 30°, rather than the original external angle 60°. The reflected ray makes 30° with the normal on the glass side.

Step 3: Find the emerging angle and the separation For refraction from glass into air,

$$\sin t = \mu \sin 30^\circ = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}, \qquad t = 60^\circ.$$

Reflection and refraction occur on opposite sides of the boundary. The separation of the outgoing reflected and transmitted rays is therefore the supplement of their two angles with the normal:

$$\theta = 180^\circ - (30^\circ + 60^\circ) = 90^\circ.$$

The refraction angle exists, so this is not a total-internal-reflection situation.

Step 4: Conclusion The angle between the reflected and refracted rays at the farther surface is $\boxed{90^\circ}$. The numerical entry in degrees is 90.

Quick Tip to Solve: For a sphere, the straight internal chord meets the two radii at equal angles; apply Snell's law separately at both surfaces.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Two compounds A and B with same molecular formula (C_3H_6O) undergo Grignard's reaction with methylmagnesium bromide to give products C and D. Products C and D show following chemical tests.

C and D respectively are : [4 marks]

Test	C	D
Ceric ammonium nitrate Test	Positive	Positive
Lucas Test	Turbidity obtained after five minutes	Turbidity obtained immediately
Iodoform Test	Positive	Negative

(A) $C = (CH_3)_3COH$; $D = CH_3CH_2CH(OH)CH_3$

(B) $C = CH_3CH_2CH_2CH_2OH$; $D = (CH_3)_3COH$

(C) $C = CH_3CH_2CH_2CH_2OH$; $D = CH_3CH_2CH(OH)CH_3$

(D) $C = CH_3CH_2CH(OH)CH_3$; $D = (CH_3)_3COH$

Correct Answer: (D) $C = CH_3CH_2CH(OH)CH_3$; $D = (CH_3)_3COH$

Solution:

Concept:

- A Grignard reagent adds its alkyl group to a carbonyl carbon. Acidic work-up converts the alkoxide into an alcohol; an aldehyde other than methanal gives a secondary alcohol, while a ketone gives a tertiary alcohol.
- Lucas reagent distinguishes alcohol classes by the rate of turbidity: tertiary alcohols react immediately and secondary alcohols react after a few minutes. A positive iodoform test identifies the $CH_3CH(OH)^-$ unit in these alcohols.

Step 1: Identify the carbonyl isomers The two carbonyl compounds with formula C_3H_6O are propanal, CH_3CH_2CHO, and propanone, CH_3COCH_3. Addition of CH_3MgBr introduces one methyl group, so the alcohol products each contain four carbon atoms. The ceric ammonium nitrate results confirm that both products are alcohols, but do not by themselves distinguish their structures.

Step 2: Use the Lucas and iodoform tests for C C gives turbidity after five minutes, identifying a secondary alcohol. Propanal gives this product on methyl addition and hydrolysis:

$$CH_3CH_2CHO \xrightarrow[H_3O^+]{CH_3MgBr} CH_3CH_2CH(OH)CH_3.$$

This is butan-2-ol. Its $CH_3CH(OH)^-$ group is oxidised to a methyl ketone in the iodoform test,

accounting for the positive result.

Step 3: Identify D and compare all options D produces immediate turbidity and is therefore tertiary. Propanone gives 2-methylpropan-2-ol:

$$CH_3COCH_3 \xrightarrow[H_3O^+]{CH_3MgBr} (CH_3)_3COH.$$

It lacks a hydrogen on the carbon bearing OH, and gives the stated negative iodoform test. Option (A) reverses the required alcohol classes. Options (B) and (C) give primary butan-1-ol as C, contradicting its delayed Lucas turbidity and positive iodoform test. Only (D) assigns both structures correctly.

Step 4: Conclusion C is butan-2-ol, $CH_3CH_2CH(OH)CH_3$, and D is 2-methylpropan-2-ol, $(CH_3)_3COH$. Hence, the correct option is (D).

Quick Tip to Solve: Use Lucas timing to classify each alcohol, then use the iodoform result to check for the methyl secondary-alcohol group.

52. The shape / structure of $[XeF_5]^-$ and XeO_3F_2, respectively, are [4 marks]

(A) Pentagonal planar and trigonal bipyramidal

(B) Trigonal bipyramidal and pentagonal planar

(C) Octahedral and square pyramidal

(D) Trigonal bipyramidal and trigonal bipyramidal

Correct Answer: (A) Pentagonal planar and trigonal bipyramidal

Solution:

Concept:

- VSEPR theory counts electron domains around the central atom. Each bond, whether single or multiple, occupies one domain; each lone pair occupies another.
- Molecular shape describes the positions of atoms only. Electron-domain geometry includes lone pairs, so the two descriptions need not be the same.

Step 1: Count the domains in the pentafluoroxenate ion Xenon supplies eight valence electrons and the negative charge supplies one more. Five single $Xe-F$ bonds use five of the central atom's electrons in bonding. The remaining four electrons make two lone pairs. Thus $[XeF_5]^-$ has five bonding domains and two lone-pair domains:

$$AX_5E_2: \qquad 5+2=7 \text{ electron domains.}$$

The electron-domain arrangement is pentagonal bipyramidal, with two axial positions and five positions in an equatorial pentagon.

Step 2: Place the lone pairs and read the molecular shape The two lone pairs occupy the two opposite axial sites of this seven-domain arrangement. This placement separates them by 180° and leaves the five fluorine atoms in the equatorial plane. The observed shape, ignoring lone pairs, is consequently pentagonal planar. It must not be called trigonal bipyramidal merely because there are five fluorine

atoms: that would neglect the two lone pairs.

Step 3: Count the domains in xenon trioxydifluoride and select For XeO_3F_2, the three $Xe{=}O$ bonds and two $Xe{-}F$ bonds form five bonding domains with no lone pair on xenon. Therefore it is an AX_5 species and has trigonal-bipyramidal geometry. The three multiple-bonded oxygen atoms occupy equatorial sites and the fluorine atoms axial sites. Option (B) reverses the two shapes. Option (C) assigns six-domain shapes incorrectly. Option (D) neglects the lone pairs in the first ion.

Step 4: Conclusion The shapes of $[XeF_5]^-$ and XeO_3F_2 are pentagonal planar and trigonal bipyramidal, respectively. Hence, the correct option is (A).

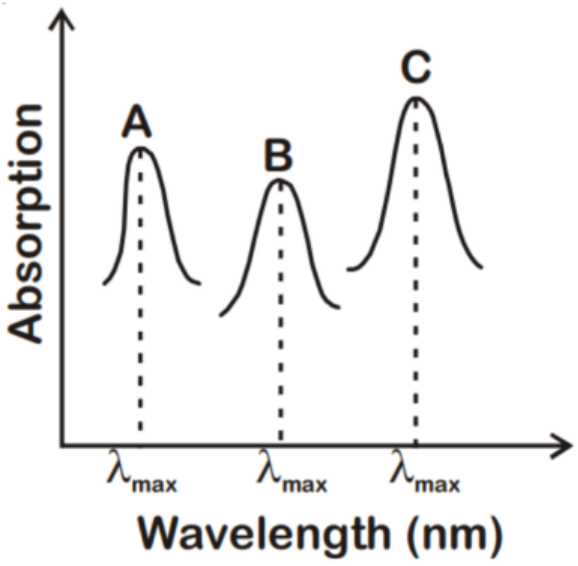
Quick Tip to Solve: Count lone pairs before naming a xenon compound's shape, and count a double bond as one VSEPR domain.

53. Simplified absorption spectra of three complexes ((i), (ii) and (iii)) of Mn^+ ion are provided below; their λ_{max} values are marked as A, B and C respectively. The correct match between the complexes and their λ_{max} values is

(i) $[M(NCS)_6]^{(-6+n)}$

(ii) $[MF_6]^{(-6+n)}$

(iii) $[M(NH_3)_6]^{n+}$ [4 marks]



(A) A-(i), B-(ii), C-(iii)

(B) A-(ii), B-(iii), C-(i)

(C) A-(ii), B-(i), C-(iii)

(D) A-(iii), B-(i), C-(ii)

Correct Answer: (D) A-(iii), B-(i), C-(ii)

Solution:

Concept:

- An octahedral complex absorbs a photon whose energy matches the crystal-field splitting: $\Delta_o = h\nu = hc/\lambda_{\max}$. A larger splitting therefore corresponds to a shorter absorption wavelength.
- For the same metal ion and oxidation state, ligand strength follows the relevant spectrochemical-series order $F^- < NCS^- < NH_3$, with N-bound thiocyanate between fluoride and ammonia.

Step 1: Read the wavelength order from the graph The wavelength axis increases towards the right. The absorption peak marked A is leftmost, B is in the middle and C is rightmost, so

$$\lambda_A < \lambda_B < \lambda_C.$$

Peak height does not decide the splitting; it is the horizontal position of the absorption maximum that matters. Since wavelength and transition energy are inversely related, the corresponding splitting order is $\Delta_A > \Delta_B > \Delta_C$.

Step 2: Rank the three ligand fields All three species have six ligands and the same central metal ion, so the comparison is a ligand-field comparison. Fluoride is the weakest ligand among the three and gives the smallest Δ_o. N-bound thiocyanate gives an intermediate splitting, while ammonia gives the largest:

$$\Delta_o\big((\text{ii})\big) < \Delta_o\big((\text{i})\big) < \Delta_o\big((\text{iii})\big).$$

The wavelength order is the reverse of this sequence. The complex charges shown do not change this conclusion because the metal oxidation state is held fixed.

Step 3: Match the peaks and eliminate alternatives Complex (iii), containing ammonia, must absorb at the shortest wavelength, so it belongs to A. Complex (i), containing thiocyanate, belongs to the intermediate peak B. Complex (ii), containing fluoride, belongs to the longest-wavelength peak C. Option (A) puts ammonia at the longest wavelength, reversing its field-strength consequence. Options (B) and (C) put fluoride at the shortest wavelength even though it gives the smallest splitting. These fail the inverse energy–wavelength relation.

Step 4: Conclusion The correct assignment is A–(iii), B–(i), C–(ii). Hence, the correct option is (D).

Quick Tip to Solve: First rank ligand strengths, then reverse that order for absorption wavelengths; do not rank complexes by peak heights.

54. The number of subshells associated with n = 4 and m = –2 quantum numbers is [4 marks]

(A) 8

(B) 2

(C) 16

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- For principal quantum number n, the azimuthal quantum number takes the integer values $\ell = 0, 1, \ldots, n-1$. These label subshells $s, p, d, f, \ldots$.
- For a given ℓ, the magnetic quantum number satisfies $-\ell \leq m_\ell \leq \ell$ in integer steps. Therefore a specified m_ℓ requires $\ell \geq |m_\ell|$.

Step 1: List the subshells permitted by the principal number The principal quantum number is $n = 4$, which allows exactly four azimuthal values before the magnetic-number restriction is applied:

$$\ell = 0, 1, 2, 3.$$

These represent the $4s$, $4p$, $4d$ and $4f$ subshells. The problem asks how many of these subshells can contain a state with the specified magnetic quantum number; it does not ask for the total number of orbitals in the fourth shell.

Step 2: Apply the magnetic-number restriction to each subshell The specified value is $m_\ell = -2$, so its magnitude is two. In an s subshell $\ell = 0$, only $m_\ell = 0$ is possible. In a p subshell $\ell = 1$, only $-1, 0, 1$ are possible. Both are excluded. For $\ell = 2$, the allowed values $-2, -1, 0, 1, 2$ include the required value. For $\ell = 3$, the allowed values from -3 to 3 also include it.

Step 3: Count qualifying subshells and check the alternatives The qualifying subshells are therefore $4d$ and $4f$, giving a count of two. The two spin orientations of an electron do not create additional subshells. Thus 4 would incorrectly count all four subshells without applying the magnetic restriction, or would confuse spin states with subshells. The number 16 is the total number of orbitals in the $n = 4$ shell, equal to n^2, and 8 likewise does not count the allowed azimuthal values here.

Step 4: Conclusion There are $\boxed{2}$ subshells, namely $4d$ and $4f$. Hence, the correct option is (B).

Quick Tip to Solve: For fixed n and m_ℓ, count integers ℓ from $|m_\ell|$ to $n-1$; the count is $n - |m_\ell|$.

55. The size of a raw mango shrinks to a much smaller size when kept in a concentrated salt solution. Which one of the following processes can explain this? [4 marks]

(A) Osmosis

(B) Reverse osmosis

(C) Diffusion

(D) Dialysis

Correct Answer: (A) Osmosis

Solution:

Concept:

- Osmosis is the movement of solvent through a semipermeable membrane from higher solvent chemical potential to lower solvent chemical potential. In a dilute-solution description, water moves from the less concentrated solution to the more concentrated one.
- The cell membrane is selectively permeable. Loss of water from a cell to a hypertonic external solution is called exosmosis and reduces cell volume.

Step 1: Compare the two solutions The concentrated salt solution outside the mango cells has a lower water chemical potential than the cell sap. The cell membranes separate these two solutions and allow water to move much more readily than the dissolved solutes. Thus the concentration difference drives water out of the cells; it does not drive water into them.

Step 2: Relate water loss to shrinkage As water leaves the cells, their vacuoles lose water and the cells lose turgor. The mango tissue consequently contracts and the fruit becomes smaller. This is ordinary osmosis in the outward direction, or exosmosis. The movement requires no externally imposed pressure.

Step 3: Exclude the other mechanisms Reverse osmosis requires applied pressure opposing the natural osmotic flow, which is absent here. Diffusion alone describes general particle spreading but does not specify the selective membrane and solvent transfer responsible for shrinking living tissue. Dialysis separates small dissolved species from larger ones across a membrane; it is not the process identified by this water-loss observation.

Step 4: Conclusion The mango shrinks because water leaves its cells by exosmosis, a form of osmosis. Hence, the correct option is (A).

Quick Tip to Solve: For a cell in concentrated salt solution, first decide the direction of water flow: water leaving the cell means exosmosis and shrinkage.

56. The one that is not expected to show isomerism is : [4 marks]

(A) $[Pt(NH_3)_2Cl_2]$

(B) $[Ni(NH_3)_4(H_2O)_2]^{2+}$

(C) $[Ni(en)_3]^{2+}$

(D) $[Ni(NH_3)_2Cl_2]$

Correct Answer: (D) $[Ni(NH_3)_2Cl_2]$

Solution:

Concept:

- Geometrical isomerism depends on coordination geometry. Square-planar complexes of the type MA_2B_2 can have cis and trans forms, whereas tetrahedral complexes of this type have only one arrangement.
- An octahedral complex with three identical bidentate ligands can show optical isomerism as a pair of non-superimposable mirror images. Here en denotes ethane-1,2-diamine.

Step 1: Check the platinum and six-coordinate nickel complexes In $[Pt(NH_3)_2Cl_2]$, platinum(II) has square-planar coordination. The two chloride ligands can be adjacent (cis) or opposite (trans), so option (A) shows geometrical isomerism. The six-coordinate complex $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is octahedral. Its two water ligands can likewise occupy adjacent or opposite sites, so option (B) also shows cis-trans isomerism.

Step 2: Check the chelate and tetrahedral complexes In $[Ni(en)_3]^{2+}$, the three bidentate en ligands give octahedral coordination and the two optical configurations conventionally called Δ and Λ. Thus option (C) shows optical isomerism. By contrast, the four-coordinate nickel(II) complex $[Ni(NH_3)_2Cl_2]$ is expected to be tetrahedral. Every pair of vertices of a tetrahedron is equivalent, so placing the two chloride ligands on any pair gives the same arrangement after rotation. There is no distinct cis-trans pair and no optical pair for this ligand pattern.

Step 3: Conclusion The tetrahedral complex $[Ni(NH_3)_2Cl_2]$ is not expected to show isomerism. Hence, the correct option is (D).

Quick Tip to Solve: Count coordination sites and establish the geometry before applying a cis-trans rule: tetrahedral and square-planar complexes behave differently.

57. The correct observation in the following reactions is : [4 marks]

$$\text{Sucrose} \xrightarrow[\text{Cleavage}]{\text{Glycosidic bond}} \underset{\text{(Hydrolysis)}}{\text{A + B}} \xrightarrow{\text{Seliwanoff's reagent}} ?$$

(A) Formation of blue colour

(B) Formation of violet colour

(C) Formation of red colour

(D) Gives no colour

Correct Answer: (C) Formation of red colour

Solution:

Concept:

- Sucrose is a disaccharide whose glycosidic bond joins glucose and fructose units. Hydrolysis cleaves that bond and produces one molecule of each monosaccharide.
- Seliwanoff reagent contains resorcinol in concentrated hydrochloric acid. Ketohexoses undergo acid dehydration rapidly and give a characteristic red condensation product with resorcinol.

Step 1: Identify the hydrolysis products The first transformation in the diagram is cleavage of the glycosidic bond of sucrose. The products A and B are glucose and fructose, in either naming order. The balanced molecular equation is

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow C_6H_{12}O_6 + C_6H_{12}O_6.$$

Glucose is an aldohexose, while fructose is a ketohexose. Therefore the mixture contains a sugar that responds rapidly to Seliwanoff reagent.

Step 2: Apply the specific colour test Under the strongly acidic conditions, fructose is dehydrated to 5-hydroxymethylfurfural. That product condenses with resorcinol, producing the characteristic cherry-red colour. The presence of glucose does not remove this positive response from fructose; the question concerns the observation for the hydrolysis mixture.

Step 3: Compare the offered observations A blue colour is not the diagnostic positive result of Seliwanoff test; it should not be confused with colours associated with copper-containing carbohydrate reagents. Violet is also not the characteristic result for this test. No colour would ignore the fructose released by sucrose hydrolysis, so that option is excluded as well.

Step 4: Conclusion The hydrolysate contains fructose and therefore gives a red colour with Seliwanoff reagent. Hence, the correct option is (C).

Quick Tip to Solve: When sucrose is hydrolysed before a carbohydrate test, test glucose and fructose separately in your reasoning; fructose controls the Seliwanoff result.

58. Match the type of interaction in column A with the distance dependence of their interaction energy in column B

A	B
(i) ion-ion	(a) $\frac{1}{r}$
(ii) dipole-dipole	(b) $\frac{1}{r^2}$
(iii) London dispersion	(c) $\frac{1}{r^3}$
	(d) $\frac{1}{r^6}$

[4 marks]

(A) (I)-(a), (II)-(b), (III)-(d)

(B) (I)-(b), (II)-(d), (III)-(c)

(C) (I)-(a), (II)-(b), (III)-(c)

(D) (I)-(a), (II)-(c), (III)-(d)

Correct Answer: (D) (I)-(a), (II)-(c), (III)-(d)

Solution:

Concept:

- Interaction energy and force have different powers of separation. For two ions, Coulomb potential energy varies as r^{-1}, while the corresponding force varies as r^{-2}.
- For permanent dipoles in a fixed relative orientation, electrostatic interaction energy varies as r^{-3}. London dispersion is an induced-dipole attraction with energy proportional to $-r^{-6}$.

Step 1: Match the ion-ion and dipole-dipole energies For charges q_1 and q_2, the potential energy is

$$U_{\text{ion-ion}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r}.$$

Hence (i) matches (a), not (b). For two fixed dipoles, the orientation-dependent factor multiplies $p_1 p_2/(4\pi\varepsilon_0 r^3)$. Consequently the dipole-dipole entry (ii) matches (c). The r^{-2} entry would describe, for example, an ion interacting with a suitably oriented permanent dipole, rather than two permanent dipoles.

Step 2: Match dispersion and select the full combination London attraction arises from correlated instantaneous and induced dipoles. Its long-range interaction energy is

$$U_{\text{London}} = -\frac{C_6}{r^6}, \qquad C_6 > 0.$$

Thus (iii) matches (d). Option (A) incorrectly assigns r^{-2} to dipole-dipole energy; option (B) also gives the wrong ion-ion power and dispersion power; option (C) incorrectly assigns both the dipole and dispersion powers. Only (D) gives all three matches. The dipole entry here uses the fixed-orientation dependence; thermal orientational averaging is a distinct situation.

Step 3: Conclusion The energy dependences are respectively r^{-1}, r^{-3}, and r^{-6}. Hence, the correct option is (D).

Quick Tip to Solve: Check whether the question asks for energy or force before matching powers of distance; differentiating an energy introduces one extra inverse power.

59. The results given in the below table were obtained during kinetic studies of the following reaction

2A + B $\rightarrow$ C + D

X and Y in the given table are respectively : [4 marks]

Experiment	[A]/molL^{-1}	[B]/molL^{-1}	Initial rate/molL^{-1} min^{-1}
I	0.1	0.1	6.00×10^{-3}
II	0.1	0.2	2.40×10^{-2}
III	0.2	0.1	1.20×10^{-2}
IV	X	0.2	7.20×10^{-2}
V	0.3	Y	2.88×10^{-1}

(A) 0.3, 0.4

(B) 0.4, 0.3

(C) 0.4, 0.4

(D) 0.3, 0.3

Correct Answer: (A) 0.3, 0.4

Solution:

Concept:

- For a rate law $v = k[A]^m[B]^n$, compare experiments in which only one reactant concentration changes. Reaction orders must be measured; they are not inferred from stoichiometric coefficients.
- At fixed temperature the rate constant is the same for every experiment, so ratios of rates eliminate k. Concentrations in the table are in mol L^{-1}.

Step 1: Find the orders in A and B Experiments I and II keep $[A] = 0.1$ and double $[B]$ from 0.1 to 0.2. The rate rises from 6.00×10^{-3} to 2.40×10^{-2}, a factor of four:

$$2^n = 4 \quad \Rightarrow \quad n = 2.$$

Experiments I and III keep $[B] = 0.1$ and double $[A]$; the rate doubles. Thus

$$2^m = 2 \quad \Rightarrow \quad m = 1, \qquad v = k[A][B]^2.$$

Step 2: Calculate X and Y Compare IV with II: the B concentration is unchanged, whereas the rate triples. Since the rate is first order in A,

$$\frac{X}{0.1} = \frac{7.20 \times 10^{-2}}{2.40 \times 10^{-2}} = 3, \qquad X = 0.3.$$

Compare V with IV: both now have $[A] = 0.3$. The rate increases fourfold, so

$$\left(\frac{Y}{0.2}\right)^2 = \frac{2.88 \times 10^{-1}}{7.20 \times 10^{-2}} = 4, \qquad Y = 0.4.$$

The positive root is required because concentration cannot be negative. As a check, I gives $k = 6\,\mathrm{L^2\,mol^{-2}\,min^{-1}}$, which reproduces IV and V with these concentrations. Options (B) and (C) have the wrong X, and (D) has the wrong Y.

Step 3: Conclusion $X = 0.3\,\mathrm{mol\,L^{-1}}$ and $Y = 0.4\,\mathrm{mol\,L^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Find both orders from the controlled pairs first, then use IV against II for X and V against IV for Y; this avoids repeatedly calculating k.

60. If you spill a chemical toilet cleaning liquid on your hand, your first aid would be [4 marks]

(A) aqueous NaOH

(B) aqueous $NaHCO_3$

(C) aqueous NH_3

(D) vinegar

Correct Answer: (B) aqueous $NaHCO_3$

Solution:

Concept:

- Common acidic toilet-cleaning liquids contain an acid such as hydrochloric acid. The chemical principle tested in this multiple-choice question is neutralisation of an acid by a mild base.
- Sodium hydrogencarbonate, $NaHCO_3$, is a weakly alkaline salt. A strong alkali such as sodium hydroxide is itself corrosive, so strength of basicity alone does not determine the appropriate answer.

Step 1: Identify the acid-base character being tested The toilet-cleaning liquid in the question is treated as acidic. Among the given choices, aqueous sodium hydrogencarbonate is the mild base that reacts with acid. For hydrochloric acid, the chemical reaction is

$$HCl + NaHCO_3 \longrightarrow NaCl + CO_2 + H_2O.$$

This reaction consumes acid; carbon dioxide formation is consistent with the carbonate-family neutralisation reaction. The question asks which listed substance fits this chemical role.

Step 2: Eliminate the other listed substances Aqueous $NaOH$ is a strongly caustic alkali and is not the mild neutralising agent sought. Aqueous NH_3, although basic, is an irritant and is not the preferred reagent in this textbook comparison. Vinegar contains acetic acid, so it adds another acid rather than supplying a base to neutralise the acidic cleaner. Thus the bicarbonate choice alone combines the required alkaline character with the mildness distinguished by the options. This identifies the examination answer through the stated acid-base comparison, rather than treating all alkaline substances as interchangeable.

Step 3: Conclusion Among the listed choices, aqueous $NaHCO_3$ is the intended mild alkaline neutralising agent. Hence, the correct option is (B).

Quick Tip to Solve: In this acid-base MCQ, distinguish a mild bicarbonate solution from a caustic strong alkali; an acidic option cannot neutralise the acidic cleaner.

61. Three elements X, Y and Z are in the 3rd period of the periodic table. The oxides of X, Y and Z, respectively, are basic, amphoteric and acidic. The correct order of the atomic numbers of X, Y and Z is : [4 marks]

(A) X < Z < Y

(B) Y < X < Z

(C) Z < Y < X

(D) X < Y < Z

Correct Answer: (D) X < Y < Z

Solution:

Concept:

- Across a period, increasing nuclear charge generally decreases metallic character and increases non-metallic character. Oxides change correspondingly from basic through amphoteric to acidic.
- A basic oxide reacts with acids, an acidic oxide reacts with bases, and an amphoteric oxide reacts with both. The classification refers to oxide chemistry rather than simply the valency of its element.

Step 1: Locate the amphoteric oxide in period 3 In the third period, aluminium forms amphoteric Al_2O_3. It reacts with acid to form an aluminium salt and with strong base to give aluminate. Thus Y lies at aluminium, of atomic number 13. The oxides of the metals to its left, notably sodium and magnesium, are basic.

Step 2: Place the basic and acidic oxide formers X is to the left of aluminium: Na_2O and MgO are representative basic oxides. Z is to its right: SiO_2, phosphorus oxides, sulphur oxides and chlorine oxides are acidic. Consequently increasing atomic number gives X, then Y, then Z. This progression also agrees with the decrease of electropositive character from left to right.

Step 3: Check the proposed orders Option (A) incorrectly puts the acidic oxide former before the amphoteric one; (B) incorrectly puts the basic oxide former after aluminium; and (C) reverses the entire trend. Option (D) alone keeps the basic, amphoteric and acidic oxide formers in their proper left-to-right sequence.

Step 4: Conclusion The required atomic-number order is $\boxed{X < Y < Z}$. Hence, the correct option is (D).

Quick Tip to Solve: In period 3, anchor the oxide trend at amphoteric aluminium oxide: basic oxides lie to its left and acidic oxides to its right.

62. The molecular geometry of SF_6 is octahedral. What is the geometry of SF_4 (including lone pair(s) of electrons, if any)? [4 marks]

(A) Tetrahedral

(B) Trigonal bipyramidal

(C) Square planar

(D) Pyramidal

Correct Answer: (B) Trigonal bipyramidal

Solution:

Concept:

- VSEPR theory assigns the arrangement of all electron domains around the central atom. Each bond counts as one domain and each lone pair counts as another.
- Five electron domains adopt a trigonal-bipyramidal arrangement. Molecular shape describes atom positions alone, whereas electron-pair geometry includes the lone pairs explicitly requested here.

Step 1: Count bonding pairs and lone pairs Sulphur has six valence electrons. In SF_4, four sulphur electrons participate in four S–F single bonds, leaving two electrons as one lone pair on sulphur. Therefore the central atom has four bonding domains and one lone-pair domain:

$$\text{Steric number} = 4 + 1 = 5.$$

Lone pairs on the fluorine atoms do not contribute to the steric number at sulphur.

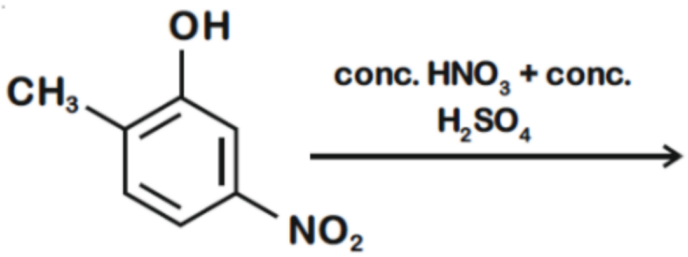
Step 2: Arrange the five electron domains The five domains occupy a trigonal bipyramid. The lone pair favours an equatorial position because this gives only two 90-degree interactions with axial bonding pairs, instead of the three such interactions it would have axially. Omitting the lone pair from the picture leaves a seesaw arrangement of the four fluorine atoms.

Step 3: Use the wording to choose the geometry The question explicitly includes lone pairs, so the required geometry is trigonal bipyramidal, not the seesaw molecular shape. Tetrahedral would require four domains, square planar is associated with six-domain arrangements having two opposite lone pairs, and pyramidal does not describe this five-domain arrangement. The octahedral geometry of SF_6 follows its separate count of six bonding domains.

Step 4: Conclusion Including the lone pair, SF_4 has $\boxed{\text{trigonal-bipyramidal geometry}}$. Hence, the correct option is (B).

Quick Tip to Solve: Always distinguish electron-pair geometry from molecular shape: SF_4 is trigonal bipyramidal when its lone pair is counted, but seesaw when only atoms are counted.

63. The major product of the following reaction is : [4 marks]



(A) 6-Methyl-2, 3, 4-trinitrophenol

(B) 6-Methyl-2, 3-dinitrophenol

(C) 6-Methyl-3, 4-dinitrophenol

(D) 6-Methyl-3, 5-dinitrophenol

Correct Answer: (A) 6-Methyl-2, 3, 4-trinitrophenol

Solution:

Concept:

- The phenolic $-\mathrm{OH}$ group activates its aromatic ring by resonance donation and directs electrophilic substitution to its ortho and para positions.
- Concentrated nitric acid with concentrated sulphuric acid provides strong nitrating conditions through the nitronium electrophile, $\mathrm{NO_2}^+$. Phenols can undergo multiple nitration under these conditions.

Step 1: Identify the available activated positions Number the carbon bearing OH as position 1 and take the methyl-bearing ortho carbon as position 6. The original nitro group then occupies position 3. The OH-directed sites are positions 2 and 6 (ortho) and position 4 (para). Position 6 is already occupied by methyl, leaving positions 2 and 4 available for nitration.

Step 2: Apply the concentrated nitrating mixture The reagents are concentrated $\mathrm{HNO_3}$ and concentrated $\mathrm{H_2SO_4}$, rather than the mild dilute nitric acid used for limited mononitration. The strongly activated phenolic ring undergoes further nitration at both remaining OH-directed positions. The existing methyl and nitro substituents are retained. Thus new nitro groups enter at positions 2 and 4, in addition to the original nitro group at position 3.

Step 3: Compare each product structure Option (A) shows nitro groups at positions 2, 3 and 4 with the original OH and methyl groups intact. Option (B) adds only the ortho nitro group, and option (C) adds only the para nitro group; these stop short of the exhaustive nitration favoured by the concentrated mixture. Option (D) instead introduces a group at position 5, meta to OH, and leaves both strongly OH-directed free positions unfilled.

Step 4: Conclusion The major product is the structure with three nitro groups shown in option (A), equivalently 6-methyl-2, 3, 4-trinitrophenol in the numbering used above. Hence, the correct option is (A).

Quick Tip to Solve: For nitration of a phenol, first mark its free ortho and para positions, then use the reagent strength to distinguish limited nitration from multiple substitution.

64. An organic compound 'A' ($C_9H_{10}O$) when treated with conc. HI undergoes cleavage to yield compounds 'B' and 'C'. 'B' gives yellow precipitate with $AgNO_3$ where as 'C' tautomerizes to 'D'. 'D' gives positive iodoform test. 'A' could be [4 marks]

(A) p-$CH_3C_6H_4-O-CH=CH_2$

(B) $C_6H_5-O-CH_2-CH=CH_2$

(C) $C_6H_5-CH_2-O-CH=CH_2$

(D) $C_6H_5-O-CH=CH-CH_3$

Correct Answer: (C) $C_6H_5-CH_2-O-CH=CH_2$

Solution:

Concept:

- Concentrated HI protonates ether oxygen; iodide can then cleave an accessible alkyl–oxygen bond. Cleavage at a benzyl carbon is especially favourable, whereas an aryl or vinylic carbon resists ordinary nucleophilic substitution.
- A vinyl alcohol fragment, $CH_2=CHOH$, tautomerizes to ethanal, CH_3CHO. Ethanal gives the iodoform test, producing yellow CHI_3.

Step 1: Use the tautomerization clue C must be an enol because it changes to D by tautomerization. A vinyl ether containing $-O-CH=CH_2$ can furnish $HO-CH=CH_2$, whose carbonyl tautomer is ethanal. This explains both the change to D and the positive iodoform test without changing the carbon skeleton.

Step 2: Test cleavage of the benzyl vinyl ether Option (C) is $C_6H_5CH_2-O-CH=CH_2$. Protonation followed by iodide attack at its benzyl carbon gives

$$C_6H_5CH_2-O-CH=CH_2 + HI \longrightarrow C_6H_5CH_2I + HO-CH=CH_2.$$

Thus B is benzyl iodide, which supplies iodide to form yellow AgI with silver nitrate. The other product undergoes

$$CH_2=CHOH \rightleftharpoons CH_3CHO.$$

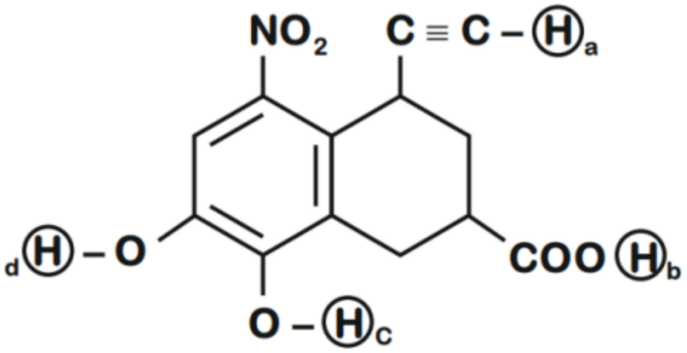
The formula also checks: six ring carbons plus one benzyl carbon plus two vinyl carbons give $C_9H_{10}O$.

Step 3: Exclude the remaining ethers Option (B) cleaves to allyl iodide and phenol; phenol does not tautomerize to an iodoform-positive carbonyl compound in this reaction. In (A) and (D), oxygen is directly attached to an aryl carbon on one side and a vinylic carbon on the other, so there is no benzyl alkyl–oxygen bond to provide the required alkyl iodide and vinyl alcohol pair. They therefore do not satisfy the complete sequence of observations.

Step 4: Conclusion A is benzyl vinyl ether, $\boxed{C_6H_5CH_2-O-CH=CH_2}$, yielding benzyl iodide and an enol that becomes ethanal. Hence, the correct option is (C).

Quick Tip to Solve: Work backwards from tautomerization and the iodoform test: vinyl alcohol suggests ethanal, while easy HI cleavage identifies an alkyl or benzyl group on the other side of oxygen.

65. Arrange the following labelled hydrogens in decreasing order of acidity : [4 marks]



(A) b > c > d > a

(B) b > a > c > d

(C) c > b > d > a

(D) c > b > a > d

Correct Answer: (A) b > c > d > a

Solution:

Concept:

- Acidity increases as the conjugate base becomes more stable. Resonance delocalization and electron-withdrawing groups stabilize negative charge and favour loss of a proton.
- Carboxylic acids are generally more acidic than phenols, while a terminal alkyne is substantially less acidic than either. Nitro substitution further increases phenolic acidity most effectively at ortho or para positions.

Step 1: Identify the four acidic sites The labelled proton b belongs to a carboxyl group, c and d belong to phenolic OH groups, and a is attached to a terminal alkyne carbon. Removing b produces a carboxylate ion whose charge is delocalized over two equivalent oxygen atoms. Removing a instead produces an acetylide ion; although the sp carbon helps stabilize it, this is much less favourable than forming an oxygen-based conjugate base.

Step 2: Compare the two phenolic protons The OH bearing c is para to the nitro group on the aromatic ring. Its phenoxide ion can delocalize negative charge through the ring towards the para nitro substituent, gaining additional resonance stabilization. The OH bearing d is meta to the nitro group, so it benefits from the inductive electron withdrawal but not the same direct resonance withdrawal. Consequently c is more acidic than d.

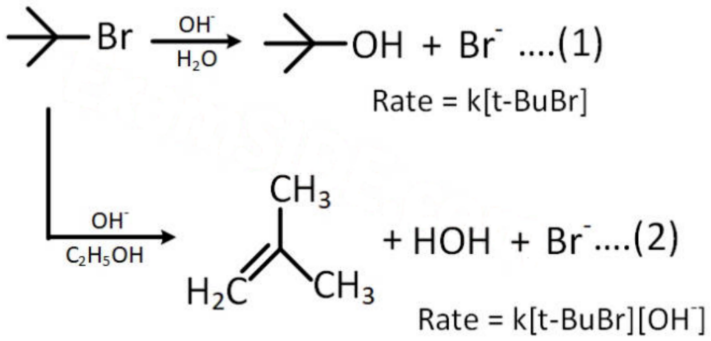
Step 3: Combine the comparisons and inspect the options The carboxyl proton b is strongest; the para-nitro-activated phenolic proton c follows, then the meta-related phenolic proton d, and finally terminal alkyne proton a. Option (B) incorrectly makes the alkyne more acidic than phenols. Options (C) and (D) incorrectly place phenolic c above the carboxylic proton; (D) also reverses the phenol-versus-alkyne ordering for d and a. Only option (A) preserves every comparison.

Step 4: Conclusion The decreasing acidity order is $\boxed{b > c > d > a}$. Hence, the correct option is (A).

Quick Tip to Solve: Rank the functional groups first, then compare substituent positions: carboxyl OH comes before phenolic OH, and para nitro stabilizes phenoxide more than meta nitro.

66. Consider the reaction sequence given below:

Which of the following statements is true? [4 marks]



(A) Changing the concentration of base will have no effect on reaction (1)

(B) Changing the concentration of base will have no effect on reaction (2)

(C) Changing the base from OH^- to ^-OR will have no effect on reaction (2)

(D) Doubling the concentration of base will double the rate of both the reactions

Correct Answer: (A) Changing the concentration of base will have no effect on reaction (1)

Solution:

Concept:

- In an S_N1 substitution, slow ionization of the substrate determines the rate. The rate is first order in the alkyl halide and independent of the nucleophile concentration.
- An E2 elimination requires substrate and base together in its rate-determining event. Its rate law contains both concentrations; base identity can also affect the rate constant.

Step 1: Interpret reaction (1) The first reaction converts tert-butyl bromide to tert-butyl alcohol in aqueous medium. The rate law printed with it is

$$r_1 = k_1[\text{t-BuBr}].$$

This is the unimolecular substitution route: carbon–bromine bond ionization forms a tertiary carbocation before rapid reaction with the nucleophile. Hydroxide concentration is absent from the given rate law. At fixed substrate concentration and other conditions, changing base concentration therefore does not change this rate.

Step 2: Interpret reaction (2) The second reaction produces 2-methylpropene in alcoholic base. Its printed rate law is

$$r_2 = k_2[\text{t-BuBr}][OH^-].$$

Base removes a beta hydrogen as the leaving group departs, so doubling hydroxide concentration doubles this elimination rate when substrate concentration is fixed. Altering the base from hydroxide to an alkoxide can alter its basicity, steric effects and the rate constant, so unchanged behaviour cannot be asserted.

Step 3: Check the four claims Claim (A) follows directly from the first rate law. Claim (B) contradicts the explicit base term in the second rate law. Claim (C) wrongly assumes that changing the chemical

identity of a reagent cannot affect elimination. Claim (D) is false because doubling base doubles reaction (2) only, while reaction (1) retains its original rate under the conditions of the stated comparison.

Step 4: Conclusion Changing the concentration of base has no effect on the rate of reaction (1), as described by its given rate law. Hence, the correct option is (A).

Quick Tip to Solve: For concentration changes, read the powers in the rate law first: a missing base term means zero order in base, while a single base term means first order in base.

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67. The major product obtained from E_2 - elimination of 3-bromo-2-fluoropentane is [4 marks]

(A) $CH_3CH_2CH(Br)CH{=}CH_2$

(B) $CH_3CH_2C(Br){=}CHCH_3$

(C) $CH_3CH_2CH{=}C(F)CH_3$

(D) $CH_3CH{=}CHCH(F)CH_3$

Correct Answer: (C) $CH_3CH_2CH{=}C(F)CH_3$

Solution:

Concept:

- An E2 reaction removes a beta hydrogen and a leaving group from adjacent carbon atoms in one concerted step; the new pi bond joins those two carbons.
- Bromide is a much better leaving group than fluoride because the carbon–fluorine bond is unusually strong. The inductive effect of fluorine also favours removal of a hydrogen from its carbon.

Step 1: Identify the carbon bearing the leaving group Write the starting chain as

$$CH_3CH(F)CH(Br)CH_2CH_3.$$

Numbering from the nearer end puts fluorine on C2 and bromine on C3. Loss of bromide therefore makes C3 the alpha carbon. Its beta carbons are C2 and C4; both have a hydrogen that can be removed after rotation into the required anti-periplanar arrangement. An alkene that retains bromine would instead require loss of fluoride, the less favourable leaving group.

Step 2: Compare the two eliminations of hydrogen bromide Removing the hydrogen from C2 gives a double bond between C2 and C3:

$$CH_3CH(F)CH(Br)CH_2CH_3 \longrightarrow CH_3C(F){=}CHCH_2CH_3 + HBr.$$

The fluorine substituent withdraws electron density and makes this beta hydrogen easier to remove. Both possible alkenes have two alkyl substituents, so ordinary alkyl-substitution counting alone does not distinguish them. Fluorine must not be counted as an extra alkyl group; its inductive activation of the C2 hydrogen distinguishes the preferred pathway. Removing a hydrogen from C4 instead gives $CH_3CH(F)CH{=}CHCH_3$, the product in (D), which is not the major elimination product. Options (A) and (B) retain bromine and require the less favourable loss of fluoride. Reversing the writing direction of the preferred product gives $CH_3CH_2CH{=}C(F)CH_3$, exactly option (C).

Step 3: Conclusion The major product is 2-fluoropent-2-ene, formed by eliminating hydrogen bromide across C2–C3. Hence, the correct option is (C).

Quick Tip to Solve: First decide which halogen leaves: bromide leaves far more readily than fluoride. Then compare the beta hydrogens adjacent to the brominated carbon.

68. Cast iron is used for the manufacture of : [4 marks]

(A) wrought iron, pig iron and steel

(B) wrought iron and pig iron

(C) wrought iron and steel

(D) pig iron, scrap iron and steel

Correct Answer: (C) wrought iron and steel

Solution:

Concept:

- Cast iron is an iron alloy with appreciable carbon and impurities. Refining it by oxidation lowers the carbon content and removes impurities.
- Wrought iron is nearly pure commercial iron; steel contains a controlled intermediate proportion of carbon. Pig iron is an earlier blast-furnace product.

Step 1: Place the iron products in their manufacturing sequence Iron ore is reduced in the blast furnace to give molten pig iron, which contains a high proportion of carbon together with silicon, phosphorus, sulphur and manganese. Cast iron is obtained by melting pig iron with scrap iron and coke and casting it. Thus pig iron is a starting material in this sequence; it is not normally described as a product manufactured from cast iron. Scrap iron is discarded or recovered iron material, rather than a distinct product of refining cast iron.

Step 2: Identify what controlled oxidation produces When cast iron is heated in a reverberatory furnace lined with iron oxide, its carbon and oxidisable impurities are removed. The resulting low-carbon iron is wrought iron. A representative carbon-removal reaction is

$$\mathrm{Fe_2O_3 + 3\,C \longrightarrow 2\,Fe + 3\,CO.}$$

In steelmaking the carbon content is lowered and then adjusted to a desired value, and other alloying elements may be added for suitable properties. Cast iron can therefore supply both wrought iron and steel. Options (A) and (B) incorrectly include pig iron among the products; (D) includes pig iron and scrap iron as well. Only option (C) lists the two appropriate manufactured materials.

Step 3: Conclusion Cast iron is used for the manufacture of wrought iron and steel. Hence, the correct option is (C).

Quick Tip to Solve: Remember the direction of refining: high-carbon blast-furnace iron comes first; controlled removal of carbon leads towards steel and wrought iron.

69. Amongst the following statements regarding adsorption, those that are valid are

(a) ΔH becomes less negative as adsorption proceeds

(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas

(c) On adsorption, the residual force acting along the surface of the adsorbent increases

(d) With increase in temperature, the equilibrium concentration of adsorbate increases [4 marks]

(A) (c) and (d)

(B) (a) and (b)

(C) (d) and (a)

(D) (b) and (c)

Correct Answer: (B) (a) and (b)

Solution:

Concept:

- Adsorption binds particles to a surface and releases energy, so the enthalpy change is negative. High-energy surface sites are occupied before weaker sites.
- Physical adsorption is favoured by stronger intermolecular attractions and lower temperature. Adsorption reduces the unsatisfied residual forces of the surface.

Step 1: Test the enthalpy and gas-comparison statements Statement (a) is valid. Initially the incoming particles occupy the most energetic adsorption sites and release more heat. As coverage increases, the remaining sites bind less strongly, so the incremental heat evolved decreases: ΔH becomes less negative as adsorption proceeds. Statement (b) is also valid. Ammonia is polar and much more readily liquefied than nitrogen, indicating stronger attractive interactions. Under comparable conditions on the same suitable adsorbent, NH_3 is therefore adsorbed more strongly than N_2.

Step 2: Test the surface-force and temperature statements Statement (c) reverses the effect of adsorption. Surface atoms initially have unbalanced attractions; binding adsorbate molecules satisfies part of these residual forces and lowers the surface energy. The residual force decreases, rather than increases. Statement (d), referring to the equilibrium amount of adsorbate held on the surface, is false for the usual exothermic adsorption process. Heating favours desorption, so at a given gas pressure less material remains adsorbed. This is distinct from saying that the concentration of unadsorbed gas can rise when gas is released. Since (c) and (d) are invalid, options (A), (C) and (D) each contain an invalid statement.

Step 3: Conclusion Only the first two statements are valid. Hence, the correct option is (B).

Quick Tip to Solve: Use three checks for adsorption: heat is released, residual surface forces are satisfied, and heating usually decreases surface coverage.

70. Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are : [4 marks]

(A) Li and Mg

(B) Cs and Ba

(C) Na and Rb

(D) Na and Ca

Correct Answer: (A) Li and Mg

Solution:

Concept:

- Lithium and magnesium show a diagonal relationship: similar ionic size and charge density produce several similar chemical properties.
- Both lithium and magnesium form nitrides directly with nitrogen, and their hydrogencarbonates are stable only in solution rather than as isolable solids.

Step 1: Apply the nitride-formation condition Among the alkali metals, lithium is exceptional in combining directly with nitrogen to form a stable nitride. Magnesium, like the other commonly studied alkaline-earth nitride-forming metals, also combines with nitrogen. Their balanced reactions are

$$6\,\mathrm{Li} + \mathrm{N_2} \longrightarrow 2\,\mathrm{Li_3N},$$

$$3\,\mathrm{Mg} + \mathrm{N_2} \longrightarrow \mathrm{Mg_3N_2}.$$

Sodium, rubidium and caesium do not show this characteristic direct nitride formation under the usual conditions. This already makes the pair lithium and magnesium the appropriate choice.

Step 2: Check the hydrogencarbonate clue and reject the remaining pairs The hydrogencarbonates of lithium and magnesium are known in aqueous solution, but they cannot be isolated as stable solid hydrogencarbonates. On heating their solutions, carbonates are obtained with release of carbon dioxide and water. This is a familiar similarity associated with the diagonal relationship between the first member of Group 1 and the second member of Group 2. The pair Cs and Ba in (B) does not have this combination of properties: caesium has a stable solid hydrogencarbonate and lacks lithium-like direct nitride formation. The sodium-containing pairs (C) and (D) fail because sodium forms solid hydrogencarbonate and does not directly form the corresponding stable nitride in this way.

Step 3: Conclusion The elements A and B are lithium and magnesium, respectively. Hence, the correct option is (A).

Quick Tip to Solve: The combination of direct nitride formation and absence of solid hydrogencarbonates is a strong clue to the diagonal pair Li–Mg.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The oxidation states of transition metal atoms in $K_2Cr_2O_7$, $KMnO_4$ and K_2FeO_4, respectively, are x, y and z. The sum of x, y and z is _____. [4 marks]

Solution:

Concept:

- The sum of the oxidation numbers of all atoms in an electrically neutral compound is zero. Multiplicity must be included for every element.
- Potassium has oxidation number $+1$, while oxygen has oxidation number -2 in these three oxo salts; none contains peroxide oxygen.

Step 1: Calculate the chromium and manganese oxidation numbers For $K_2Cr_2O_7$, let the oxidation number of each chromium atom be x. Charge balance gives

$$2(+1) + 2x + 7(-2) = 0.$$

$$2x = 12 \quad \Rightarrow \quad x = +6.$$

The factor of two is essential because the formula contains two chromium atoms. For $KMnO_4$, let the manganese oxidation number be y. There is one potassium atom and four oxygen atoms, giving

$$(+1) + y + 4(-2) = 0.$$

$$y = +7.$$

These are formal oxidation numbers found by electron-counting rules, not claims that isolated metal ions with these charges occur in the salts.

Step 2: Calculate the iron oxidation number and add the three values For K_2FeO_4, let the iron oxidation number be z. The two potassium atoms contribute $+2$, and four oxygen atoms contribute -8. Therefore

$$2(+1) + z + 4(-2) = 0,$$

$$z = +6.$$

Now add the oxidation number per transition-metal atom from each compound, as requested:

$$x + y + z = 6 + 7 + 6 = 19.$$

Do not add twice the chromium value: the unknown x is the oxidation state of a chromium atom, rather than the combined contribution of both chromium atoms in the formula. Oxidation numbers and their sum are dimensionless.

Step 3: Conclusion The three oxidation states are $+6, +7, +6$, respectively. Their required sum is $\boxed{19}$ (dimensionless).

Quick Tip to Solve: Write one neutrality equation per formula and include each subscript. For dichromate divide the total chromium contribution by two before taking the sum.

72. The heat of combustion of ethanol into carbon dioxide and water is – 327 kcal at constant pressure. The heat evolved (in cal) at constant volume and 27°C (if all gases behave ideally) is (R = 2 cal mol^{-1} K^{-1}) _____. [4 marks]

Solution:

Concept:

- At constant pressure the reaction heat equals ΔH; at constant volume it equals ΔU. For ideal gases, $\Delta H = \Delta U + \Delta n_g RT$.
- In the gas correction, Δn_g counts only gaseous products minus gaseous reactants. At $27^\circ\mathrm{C}$, ethanol and the water formed are liquids.

Step 1: Balance combustion and count gaseous moles Write the reaction for one mole of liquid ethanol at the stated temperature:

$$\mathrm{C_2H_5OH\,(l) + 3\,O_2(g) \longrightarrow 2\,CO_2(g) + 3\,H_2O\,(l).}$$

Only oxygen and carbon dioxide enter the gas-mole count. Liquid water must not be counted as a gas merely because it is a combustion product. Thus

$$\Delta n_g = 2 - 3 = -1.$$

The constant-pressure heat is an enthalpy change of $-327\,\mathrm{kcal\,mol^{-1}}$, or $-327000\,\mathrm{cal\,mol^{-1}}$. The negative sign denotes heat released by the reacting system.

Step 2: Convert the enthalpy change to the constant-volume heat Use $T = 27 + 273 = 300\,\mathrm{K}$ and the given $R = 2\,\mathrm{cal\,mol^{-1}\,K^{-1}}$. Rearranging the ideal-gas relation gives

$$\Delta U = \Delta H - \Delta n_g RT.$$

$$\Delta U = -327000 - (-1)(2)(300).$$

$$\Delta U = -326400\,\mathrm{cal\,mol^{-1}}.$$

Since the gaseous mole number decreases, the enthalpy change is more negative than the internal-energy change by $600\,\mathrm{cal\,mol^{-1}}$. The question asks for heat evolved, so report the positive magnitude rather than the negative thermodynamic change. For the combustion amount corresponding to the stated heat, the released energy is therefore $326400\,\mathrm{cal}$.

Step 3: Conclusion The constant-volume heat change is $-326400\,\mathrm{cal}$; hence the heat evolved is $\boxed{326400\,\mathrm{cal}}$.

Quick Tip to Solve: Balance the phases before counting gas moles. Use calories throughout, and distinguish a negative reaction heat from the positive amount of heat evolved.

73. For the disproportionation reaction
$2Cu^{+}(aq) \rightleftharpoons Cu(s) + Cu^{2+}(aq)$ at 298 K. ln K
(where K is the equilibrium constant) is
_____ $\times 10^{-1}$.
Given :
($E^{0}_{Cu^{2+}/Cu^{+}} = 0.16V$

$$E^{0}_{Cu^{+}/Cu} = 0.52V$$

$\frac{RT}{F} = 0.025$) [4 marks]

Solution:

Concept:

- Disproportionation oxidises and reduces the same species. Here one Cu^{+} ion loses an electron while another gains it, so the balanced reaction transfers $n = 1$ electron.
- For standard reduction potentials, $E^{\circ}_{\text{cell}} = E^{\circ}_{\text{cathode}} - E^{\circ}_{\text{anode}}$. Combining $\Delta G^{\circ} = -nFE^{\circ}_{\text{cell}}$ with $\Delta G^{\circ} = -RT\ln K$ gives $\ln K = nE^{\circ}_{\text{cell}}/(RT/F)$.

Step 1: Identify the two half-reactions The reduction at the cathode is $Cu^{+}(aq) + e^{-} \longrightarrow Cu(s)$, with reduction potential $0.52\,V$. The oxidation is $Cu^{+}(aq) \longrightarrow Cu^{2+}(aq) + e^{-}$; its reverse has the supplied reduction potential $0.16\,V$. Adding the half-reactions gives exactly the stated disproportionation reaction, with one electron cancelled. Thus $n = 1$, even though two copper ions appear on the reactant side.

Step 2: Calculate the standard cell potential Subtract the anode's reduction potential from the cathode's reduction potential:

$$E^{\circ}_{\text{cell}} = 0.52 - 0.16 = 0.36\,V.$$

The positive potential corresponds to a negative standard Gibbs energy change and an equilibrium constant greater than unity.

Step 3: Evaluate the natural logarithm and the requested coefficient Use the supplied value of RT/F directly, since the question asks for a natural logarithm:

$$\ln K = \frac{1 \times 0.36}{0.025} = 14.4 = 144 \times 10^{-1}.$$

No factor of 2.303 is needed; that conversion applies only when using $\log_{10} K$.

Step 4: Conclusion $\ln K = 14.4 = 144 \times 10^{-1}$. Therefore, the dimensionless number to enter in the blank is $\boxed{144}$.

Quick Tip to Solve: Count electrons from the balanced half-reactions, not from the number of copper ions. For a requested $\ln K$, use nFE°/RT without a base-ten conversion factor.

74. The work function of sodium metal is 4.41 $\times$ 10^{-19} J. If photons of wavelength 300 nm are incident on the metal, the kinetic energy of the ejected electrons will be (h = 6.63 $\times$ 10^{-34} J s; c = 3 $\times$ 10^{8} m/s) _____ $\times$ 10^{-21} J. [4 marks]

Solution:

Concept:

- A photon of wavelength λ has energy $E = hc/\lambda$, where h is Planck's constant and c is the speed of light. The wavelength must be expressed in metres with the supplied SI constants.
- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the metal's work function. Emission is possible only when the photon energy is at least ϕ; the excess appears as the maximum electron kinetic energy.

Step 1: Convert the wavelength and calculate the photon energy The incident wavelength is

$$\lambda = 300\,\text{nm} = 300 \times 10^{-9}\,\text{m} = 3.00 \times 10^{-7}\,\text{m}.$$

Therefore, using precisely the values given in the question,

$$E = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{3.00 \times 10^{-7}} = 6.63 \times 10^{-19}\,\text{J}.$$

This is the energy of one photon, so it can be compared directly with the work function in joules per emitted electron.

Step 2: Subtract the energy needed to release the electron The work function is $\phi = 4.41 \times 10^{-19}$ J. Since $E > \phi$, photoelectric emission occurs. Conservation of energy gives

$$K_{\max} = (6.63 - 4.41) \times 10^{-19} = 2.22 \times 10^{-19}\,\text{J}.$$

The work function is subtracted, rather than added, because that part of the photon energy is used to liberate the electron from the surface.

Step 3: Express the answer in the requested power of ten The blank multiplies 10^{-21} J, so rewrite the same kinetic energy as

$$2.22 \times 10^{-19}\,\text{J} = 222 \times 10^{-21}\,\text{J}.$$

Only the numerical coefficient changes when the power of ten is changed; the physical energy remains the same.

Step 4: Conclusion The emitted electrons have maximum kinetic energy $\boxed{222 \times 10^{-21}\,\text{J}}$, so the number to enter is $\boxed{222}$.

Quick Tip to Solve: First evaluate $hc/\lambda - \phi$ in a single consistent energy unit. Convert to the power of ten printed beside the blank only at the end.

75. The ratio of the mass percentages of 'C & H' and 'C & O' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is _____. [4 marks]

Solution:

Concept:

- Ratios of elemental mass percentages are also ratios of the elemental masses in one mole. For a compound $C_xH_yO_z$, these masses are proportional to $12x$, y and $16z$, respectively.
- A saturated acyclic compound containing carbon, hydrogen and divalent oxygen has $y = 2x + 2$. Oxygen atoms do not change this hydrogen relation because they introduce neither a ring nor a multiple bond.
- Complete combustion converts each carbon atom to CO_2 and each pair of hydrogen atoms to H_2O. The external oxygen requirement per mole is $x + y/4 - z/2$ moles of O_2.

Step 1: Translate the two mass ratios into atom ratios Write the formula as $C_xH_yO_z$. The carbon-to-hydrogen mass ratio gives

$$\frac{12x}{y} = 4 \quad \Rightarrow \quad y = 3x.$$

Independently, the carbon-to-oxygen mass ratio gives

$$\frac{12x}{16z} = \frac{3}{4} \quad \Rightarrow \quad z = x.$$

Thus the empirical atomic ratio is CH_3O; it is not yet the molecular formula.

Step 2: Use saturation and the absence of rings to find the formula Combine $y = 3x$ with the saturated acyclic relation $y = 2x + 2$:

$$3x = 2x + 2 \quad \Rightarrow \quad x = 2, \qquad y = 6, \qquad z = 2.$$

Hence the molecular formula is $C_2H_6O_2$. Its carbon, hydrogen and oxygen masses are $24 : 6 : 32$, confirming both given mass ratios $4 : 1$ and $3 : 4$.

Step 3: Balance combustion for two moles of the compound Two moles contain four moles of carbon atoms and twelve moles of hydrogen atoms, giving four moles of carbon dioxide and six moles of water:

$$2\,C_2H_6O_2 + 5\,O_2 \longrightarrow 4\,CO_2 + 6\,H_2O.$$

The products contain fourteen moles of oxygen atoms. Four are already present in the compound, leaving ten to come from O_2, which requires five moles of the gas.

Step 4: Conclusion Complete combustion of two moles of X requires $\boxed{5\,\text{mol}}$ of oxygen gas. The number to enter is $\boxed{5}$.

Quick Tip to Solve: Use mass ratios to obtain an empirical formula, then apply the saturated acyclic hydrogen relation to find the molecular formula before balancing combustion.