JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 3 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 3 September 2020, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The function, $f(x) = (3x - 7)x^{2/3}$, $x \in R$, is increasing for all x lying in : [4 marks]

(A) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(B) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(C) $\left(-\infty, \frac{14}{15}\right)$

(D) $\left(-\infty, \frac{14}{15}\right) \cup (0, \infty)$

Correct Answer: (B) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

Solution:

Concept:

- For a real-valued function, a positive derivative on an interval proves that the function is increasing there. Real cube roots are defined for negative arguments as well.
- Use the product rule for $f(x) = (3x-7)x^{2/3}$, and retain the sign of $x^{1/3}$ when testing the derivative.

Step 1: Differentiate away from zero For $x \neq 0$, the product rule gives

$$f'(x) = 3x^{2/3} + \frac{2(3x-7)}{3x^{1/3}} = \frac{9x + 6x - 14}{3x^{1/3}} = \frac{15x - 14}{3x^{1/3}}.$$

The numerator vanishes at $x = 14/15$, whereas the denominator changes sign at zero. The derivative is not defined at zero, so that point must be considered separately rather than cancelled from the sign test.

Step 2: Test the signs on the three intervals For $x < 0$, both numerator and denominator are negative, hence $f'(x) > 0$. For $0 < x < 14/15$, the numerator is negative and the denominator positive, hence $f'(x) < 0$. Finally, for $x > 14/15$, both are positive, so $f'(x) > 0$. Thus the function increases on each of the two intervals separated by the decreasing portion. Option (A) starts its positive interval too early, at $3/7$. Options (C) and (D) include points between zero and $14/15$, where the derivative is negative, and therefore cannot describe the required increasing intervals.

Step 3: Conclusion The required intervals are $(-\infty, 0) \cup (14/15, \infty)$. Hence, the correct option is (B).

Quick Tip to Solve: For fractional powers with odd denominators, keep the real cube root negative when $x < 0$; replacing it by a positive quantity reverses the sign test.

2. For the frequency distribution :

Variate (x) : x_1 x_2 x_3 .... x_{15}

Frequency (f) : f_1 f_2 f_3 ..... f_{15}

where $0 < x_1 < x_2 < x_3 < ... < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be : [4 marks]

(A) 6

(B) 1

(C) 4

(D) 2

Correct Answer: (A) 6

Solution:

Concept:

- For nonnegative frequencies with total $N > 0$, the variance is $\sigma^2 = \sum f_i x_i^2/N - \mu^2$, where $\mu = \sum f_i x_i/N$.
- All observations here lie in $(0, 10]$. A variable confined to an interval of length less than 10 has standard deviation less than 5.

Step 1: Bound the weighted second moment Since $0 < x_i \leq 10$, we have $x_i^2 \leq 10x_i$. Multiplying by the nonnegative frequencies, adding and dividing by $N = \sum f_i$ gives

$$\frac{\sum f_i x_i^2}{N} \leq 10\mu.$$

Consequently,

$$\sigma^2 \leq 10\mu - \mu^2 = 25 - (\mu - 5)^2 \leq 25.$$

In fact equality at 25 would require equal weight at both endpoints 0 and 10, but zero is excluded. Thus $\sigma < 5$; the weaker bound $\sigma \leq 5$ already rules out 6.

Step 2: Check the proposed values The value 6 would require variance 36, which exceeds the upper bound 25, so it is impossible regardless of the frequencies. The other three proposed standard deviations are compatible with observations in this range. For example, equal frequencies at the pairs $(8, 10)$, $(6, 10)$ and $(2, 10)$ give standard deviations 1, 2 and 4 respectively, because two equally weighted values have standard deviation half their separation. Extra distinct listed variates may carry zero frequency; the stated condition only requires a positive total frequency. The largest stated variate is 10, so the bound follows from the given distribution itself.

Step 3: Conclusion The standard deviation cannot be 6. Hence, the correct option is (A).

Quick Tip to Solve: Before computing a weighted variance, compare each proposed standard deviation with half the range; a value exceeding that bound is impossible.

3. The area (in sq. units) of the region

$\{ (x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1,$

$\frac{1}{2} \le x \le 2 \}$ is : [4 marks]

(A) $\frac{79}{16}$

(B) $\frac{79}{24}$

(C) $\frac{23}{6}$

(D) $\frac{23}{16}$

Correct Answer: (B) $\frac{79}{24}$

Solution:

Concept:

- The simultaneous bounds $0 \le y \le x^2 + 1$ and $0 \le y \le x + 1$ mean that the upper boundary is the smaller of the two functions.
- An area with vertical strips is $\int (\text{upper boundary} - \text{lower boundary})\, dx$. Split the integral wherever the upper boundary changes.

Step 1: Locate the switch in the upper boundary Equating the two candidate upper boundaries gives

$$x^2 + 1 = x + 1 \quad \Longrightarrow \quad x(x - 1) = 0.$$

Only the intersection $x = 1$ lies inside the stated range $[1/2, 2]$. For $1/2 \le x \le 1$, $x^2 \le x$, so the parabola $y = x^2 + 1$ lies below the line. For $1 \le x \le 2$, $x \le x^2$, so the line $y = x + 1$ is the lower of the two upper bounds. Both are positive, and the lower boundary remains $y = 0$.

Step 2: Integrate the two positive strip heights Therefore the area is

$$\begin{aligned} A &= \int_{1/2}^{1} (x^2 + 1)\, dx + \int_{1}^{2} (x + 1)\, dx \\ &= \left[\frac{x^3}{3} + x\right]_{1/2}^{1} + \left[\frac{x^2}{2} + x\right]_{1}^{2} \\ &= \left(\frac{4}{3} - \frac{1}{24} - \frac{1}{2}\right) + \left(4 - \frac{3}{2}\right) \\ &= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}. \end{aligned}$$

The first contribution uses the parabola only up to the crossing; extending it to 2 would include points excluded by the line bound. Likewise integrating only the line from $1/2$ would overcount the left-hand portion. The other three fractions do not equal the sum of these two required contributions.

Step 3: Conclusion The area is $79/24$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When two inequalities each give an upper bound for y, integrate their minimum, splitting at intersections within the specified x-interval.

4. Consider the two sets :

A = {m ∈ R : both the roots of

x^2 – (m + 1)x + m + 4 = 0 are real}

and B = [–3, 5).

Which of the following is not true? [4 marks]

(A) A ∩ B = {–3}

(B) B – A = (–3, 5)

(C) A ∪ B = R

(D) $A - B = (-\infty, -3) \cup (5, \infty)$

Correct Answer: (D) $A - B = (-\infty, -3) \cup (5, \infty)$

Solution:

Concept:

- A quadratic with real coefficients has both roots real, including equal roots, exactly when its discriminant is nonnegative.
- For set differences, retain endpoints belonging to the first set but not the second. Square and round brackets must therefore be checked separately.

Step 1: Find the parameter set from the discriminant For the quadratic in x, its discriminant is

$$\begin{aligned} D &= (m+1)^2 - 4(m+4) \\ &= m^2 - 2m - 15 = (m-5)(m+3). \end{aligned}$$

Thus $D \geq 0$ precisely when $m \leq -3$ or $m \geq 5$, and hence

$$A = (-\infty, -3] \cup [5, \infty).$$

The endpoints are included because a zero discriminant gives two equal real roots. In particular, $m = 5$ is not to be discarded merely because the quadratic then has a repeated root.

Step 2: Evaluate each stated set operation With $B = [-3, 5)$, the only common point is -3, so $A \cap B = \{-3\}$; option (A) is true. Removing this point from B gives $B - A = (-3, 5)$, so option (B) is also true. The left ray of A, the middle interval B, and the right ray of A cover every real number, proving $A \cup B = \mathbb{R}$; option (C) is true. For the final difference, -3 is removed because it lies in B, but 5 remains because $5 \notin B$. Therefore

$$A - B = (-\infty, -3) \cup [5, \infty).$$

Option (D) excludes 5 and is the false statement. This endpoint check alone distinguishes it from the correct difference.

Step 3: Conclusion Option (D) is not true: the second interval in $A - B$ must include 5. Hence, the correct option is (D).

Quick Tip to Solve: For a discriminant inequality use closed endpoints at repeated roots, then check each endpoint again when taking a set difference.

5. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to : [4 marks]

(A) $\frac{9}{4}\left(9 - q^2\right)$

(B) $\frac{9}{4}\left(9 + q^2\right)$

(C) $\frac{9}{4}\left(9 - p^2\right)$

(D) $\frac{9}{4}\left(9 + p^2\right)$

Correct Answer: (C) $\frac{9}{4}\left(9 - p^2\right)$

Solution:

Concept:

- Vieta's relations for $x^2 + px + 2 = 0$ give $\alpha + \beta = -p$ and $\alpha\beta = 2$, so both roots are nonzero.
- A symmetric expression in two roots can be simplified using their sum and product, without solving for either root explicitly.

Step 1: Relate the two coefficients For the second quadratic $2x^2 + 2qx + 1 = 0$, the sum of its roots is $-q$. Its roots are the reciprocals, so

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -\frac{p}{2} = -q.$$

Thus $q = p/2$. Also,

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 4.$$

These identities hold for complex as well as real roots, so no extra assumption about the discriminant is required.

Step 2: Simplify the two pairs of factors Pair the two minus factors first:

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha^2 + \beta^2}{\alpha\beta}$$
$$= 2 + \frac{1}{2} - \frac{p^2 - 4}{2} = \frac{9 - p^2}{2}.$$

For the remaining pair,

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = \frac{9}{2}.$$

Multiplying gives $9(9 - p^2)/4$. Options (B) and (D) have the wrong sign of the squared coefficient. Option (A) uses q^2 without the necessary conversion $p^2 = 4q^2$, so it is not the general expression.

Step 3: Conclusion The expression equals $\frac{9}{4}(9 - p^2)$. Hence, the correct option is (C).

Quick Tip to Solve: Group symmetric factors before expanding all four brackets; the identities for the root sum and product usually reduce the algebra to two lines.

6. $\int_{-\pi}^{\pi} |\pi - |x||\, dx$ **is equal to :** [4 marks]

(A) π^2

(B) $2\pi^2$

(C) $\sqrt{2}\pi^2$

(D) $\frac{\pi^2}{2}$

Correct Answer: (A) π^2

Solution:

Concept:

- An even integrand satisfies $\int_{-a}^{a} g(x)\, dx = 2\int_{0}^{a} g(x)\, dx$. Expressions depending only on $|x|$ are even.
- Before removing a modulus, check the sign of its contents over the entire integration interval; here $|x| \le \pi$.

Step 1: Remove the outer modulus using its sign Throughout $[-\pi, \pi]$, the inequality $|x| \le \pi$ implies $\pi - |x| \ge 0$. Therefore

$$|\pi - |x|| = \pi - |x|.$$

The integrand is even, since changing x to $-x$ leaves $|x|$ unchanged. On the positive half-interval, $|x| = x$, and hence the required integral reduces to

$$I = 2\int_{0}^{\pi} (\pi - x)\, dx.$$

There is no cancellation between the negative and positive halves: the integrand is nonnegative on both, rather than odd.

Step 2: Integrate and compare the alternatives Since π is constant with respect to x, direct integration gives

$$I = 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$$
$$= 2\left(\pi^2 - \frac{\pi^2}{2}\right) = \pi^2.$$

As a separate check, on each half the function decreases linearly from height π to zero, producing a triangular area $\pi^2/2$. The two equal contributions add to π^2. Option (D) counts only one half of the interval; option (B) doubles the already complete area. Option (C) would introduce a factor $\sqrt{2}$ that does not arise from either the limits or the linear height. Thus only the first option matches the calculation and the nonnegative area check.

Step 3: Conclusion The integral is π^2. Hence, the correct option is (A).

Quick Tip to Solve: For nested absolute values, determine the sign of the outer argument first, then use evenness to reduce the integration to the positive half-interval.

7. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then : [4 marks]

(A) MQ = $\frac{1}{3}$

(B) PN = 4

(C) PN = 3

(D) MQ = $\frac{1}{4}$

Correct Answer: (D) MQ = $\frac{1}{4}$

Solution:

Concept:

- The axis of $y^2 = 12x$ is the x-axis. A perpendicular to the axis is vertical, while a parallel to the axis is horizontal.
- For two points (x_1, y_1), (x_2, y_2), use slope $m = (y_2 - y_1)/(x_2 - x_1)$ and intercept $c = y_1 - mx_1$. Distances along horizontal or vertical lines are absolute coordinate differences.

Step 1: Assign coordinates to the construction Write $P = (t^2/12, t)$, where $t \neq 0$, since the given nonzero intercept excludes the vertex. Its perpendicular foot is $N = (t^2/12, 0)$. The midpoint is therefore $M = (t^2/12, t/2)$. The horizontal line through M meets the parabola at a point whose ordinate is $t/2$. Substituting into the parabola gives

$$x_Q = \frac{(t/2)^2}{12} = \frac{t^2}{48}, \qquad Q = \left(\frac{t^2}{48}, \frac{t}{2}\right).$$

Step 2: Use the intercept and compare the lengths The slope of NQ and its intercept are

$$m = \frac{t/2}{t^2/48 - t^2/12} = -\frac{8}{t}, \qquad c = 0 - \left(-\frac{8}{t}\right)\frac{t^2}{12} = \frac{2t}{3}.$$

Hence $2t/3 = 4/3$, so $t = 2$. Now

$$PN = |t| = 2, \qquad MQ = \left|\frac{t^2}{12} - \frac{t^2}{48}\right| = \frac{t^2}{16} = \frac{1}{4}.$$

The two proposed values of PN, namely 4 and 3, are both excluded. The horizontal distance is $1/4$, rather than $1/3$; thus only the fourth statement agrees with the construction.

Step 3: Conclusion $MQ = \frac{1}{4}$ and $PN = 2$. Hence, the correct option is (D).

Quick Tip to Solve: Parameterise the point using its ordinate: halving the ordinate divides the abscissa on $y^2 = 12x$ by four, making the midpoint construction immediate.

8. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$ $Ax^3 + Bx^2 + Cx + D$, then B + C is equal to : [4 marks]

(A) -1

(B) -3

(C) 9

(D) 1

Correct Answer: (B) -3

Solution:

Concept:

- Adding a multiple of another column to a column leaves the determinant unchanged. Expand along a column with two zeros to reduce a third-order determinant to a second-order one.
- After factorisation, expand in descending powers of x and compare coefficients with $Ax^3 + Bx^2 + Cx + D$. A negative sign in the minor must be retained.

Step 1: Create a column with two zero entries Use the single replacement $C_3 \leftarrow C_3 - 2C_2 + C_1$, with the original first and second columns. Its entries become

$$(3x-4) - 2(2x-3) + (x-2) = 0,$$
$$(4x-5) - 2(3x-4) + (2x-3) = 0,$$
$$(10x-17) - 2(5x-8) + (3x-5) = 3x-6.$$

Thus expansion along the new third column uses only the bottom entry. The cofactor sign there is positive because $(-1)^{3+3} = 1$.

Step 2: Factor the minor and read the coefficients The unchanged first two rows and first two columns give

$$\begin{aligned} \Delta &= (3x-6)\{(x-2)(3x-4) - (2x-3)^2\} \\ &= 3(x-2)\{3x^2 - 10x + 8 - (4x^2 - 12x + 9)\} \\ &= -3(x-2)(x-1)^2 \\ &= -3x^3 + 12x^2 - 15x + 6. \end{aligned}$$

Therefore $B = 12$ and $C = -15$. Their sum is negative, which already excludes the positive choices 9 and 1. Direct addition gives -3, excluding -1 as well. As a check on the expansion, at $x = 1$ the first and second rows become equal and the determinant vanishes, as the factor $(x-1)^2$ predicts.

Step 3: Conclusion $B + C = 12 - 15 = -3$. Hence, the correct option is (B).

Quick Tip to Solve: Look for constant first differences or zero second differences across columns before expanding a determinant with entries linear in x.

9. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is : [4 marks]

(A) 264

(B) 256

(C) 128

(D) 248

Correct Answer: (B) 256

Solution:

Concept:

- The general term in $(a+b)^n$ is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$, where $r = 0, 1, \ldots, n$. The binomial coefficient is an integer.
- Powers of distinct primes produce a rational integer only when their exponents are nonnegative integers. Thus the fractional powers of 3 and 5 must each disappear; one prime cannot cancel a fractional power of the other.

Step 1: Find the permitted term indices For the given binomial the general term is

$$T_{r+1} = \binom{n}{r} 3^{(n-r)/2} 5^{r/8}.$$

The exponent of 5 is integral precisely when r is a multiple of 8. Such an r is even, so the exponent of 3 is integral precisely when n is also even. Multiplication by the integer coefficient does not remove a nonintegral prime exponent: its prime factorisation adds only integers to these exponents. Therefore odd n yields no integral terms, and for even n the allowed indices are exactly $0, 8, 16, \ldots, 8\lfloor n/8 \rfloor$.

Step 2: Count the terms and choose the least value The initial index $r = 0$ is included, so the number of terms is

$$\left\lfloor \frac{n}{8} \right\rfloor + 1 = 33.$$

Consequently $\lfloor n/8 \rfloor = 32$, which means $256 \le n < 264$. In conjunction with the evenness condition, the possible values are $256, 258, 260, 262$. The smallest is 256. Among the alternatives, 264 gives 34 integral terms, 128 gives 17, and 248 gives 32. These checks also show why the extra endpoint term must not be omitted when counting multiples of eight.

Step 3: Conclusion The least value is $n = 256$. Hence, the correct option is (B).

Quick Tip to Solve: Count admissible indices starting at zero: the number of multiples of 8 in $[0, n]$ is $\lfloor n/8 \rfloor + 1$, not merely $n/8$.

10. The value of ($2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2$ .... up to 51^{th} term) + (1! – 2! + 3! – ..... up to 51^{th} term) is equal to : [4 marks]

(A) 1

(B) 1 + (51)!

(C) 1 – 51(51)!

(D) 1 + (52)!

Correct Answer: (D) 1 + (52)!

Solution:

Concept:

- For positive integer k, ${}^kP_{k-1} = k!/(k-(k-1))! = k!$. Both printed sums have 51 terms with alternating signs starting positive.
- Telescoping occurs when the positive factorial from one index cancels the negative factorial at the next index. Combine the two sums at equal indices before evaluating any large factorial.

Step 1: Rewrite and combine the matching terms Let k denote the term number, from 1 to 51. The first sum contributes $(-1)^{k-1}(k+1)^kP_{k-1}$, while the second contributes $(-1)^{k-1}k!$. Since ${}^kP_{k-1} = k!$, their combined contribution is

$$(-1)^{k-1}\{(k+1)k! + k!\} = (-1)^{k-1}\{(k+1)! + k!\}.$$

For instance, the first paired term is $2 \cdot 1 + 1 = 3 = 2! + 1!$, and the next paired term is negative. This confirms both the coefficient pattern and the starting sign.

Step 2: Cancel the interior factorials Writing the combined sum explicitly gives

$$S = (2! + 1!) - (3! + 2!) + (4! + 3!) - \cdots + (52! + 51!).$$

Every factorial from $2!$ through $51!$ appears once with each sign and cancels. The first surviving term is $1!$; the last surviving term is $52!$, positive because the last term number 51 is odd. Hence $S = 1 + 52!$. This is greater than $1 + 51!$, so the second alternative cannot agree. It is also greater than 1 and positive, ruling out the first alternative and the negative expression $1 - 51(51)!$. No numerical approximation of a large factorial is needed.

Step 3: Conclusion The value is $1 + 52!$. Hence, the correct option is (D).

Quick Tip to Solve: Pair corresponding terms before summing; $(k+2)k! = (k+1)! + k!$ exposes the cancellation of adjacent factorials.

11. The solution curve of the differential equation,
$(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$,
which passes through the point (0, 1), is : [4 marks]

(A) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right) + 2\right)$

(B) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{x}}{2}\right) + 2\right)$

(C) $y^2 = 1 + y\log_e\left(\frac{1+e^{-x}}{2}\right)$

(D) $y^2 = 1 + y\log_e\left(\frac{1+e^{x}}{2}\right)$

Correct Answer: (D) $y^2 = 1 + y\log_e\left(\frac{1+e^{x}}{2}\right)$

Solution:

Concept:

- A separable differential equation can be integrated after putting all functions of y with dy and all functions of x with dx. Use the initial condition to determine the integration constant.
- Near the initial point $y = 1$, division by y^2 is valid. Also $1/(1 + e^{-x}) = e^x/(1 + e^x)$, whose integral is $\ln(1 + e^x)$.

Step 1: Separate variables and integrate The equation gives

$$\frac{1 + y^2}{y^2}\,dy = \frac{dx}{1 + e^{-x}}.$$

Therefore

$$\int (1 + y^{-2})\,dy = \int \frac{e^x}{1 + e^x}\,dx, \qquad y - \frac{1}{y} = \ln(1 + e^x) + C.$$

On the right, the substitution $u = 1 + e^x$ gives $du = e^x dx$. On the left, the integral of y^{-2} is $-y^{-1}$, so the minus sign is essential.

Step 2: Apply the point and match the implicit curve At $(x, y) = (0, 1)$, the left side is zero and the logarithm is $\ln 2$. Hence $C = -\ln 2$, and

$$y - \frac{1}{y} = \ln\left(\frac{1 + e^x}{2}\right).$$

Multiplying by y yields

$$y^2 = 1 + y\ln\left(\frac{1 + e^x}{2}\right).$$

This is the fourth alternative. The first two alternatives have $y^2 + 1$ on the left and an added constant 2 inside the factor; they correspond to $y + 1/y$, whose derivative has the wrong sign for the y^{-2} term. The third uses e^{-x}, whose logarithmic derivative is negative and has the wrong denominator. Differentiating our relation instead gives $(1 + y^{-2})y' = e^x/(1 + e^x)$, recovering the given equation exactly.

Step 3: Conclusion The curve through $(0, 1)$ satisfies $y^2 = 1 + y\ln((1 + e^x)/2)$. Hence, the correct option is (D).

Quick Tip to Solve: Multiply numerator and denominator by e^x before integrating $1/(1 + e^{-x})$; the numerator then becomes the derivative of the denominator.

12. If $y^2 + \log_e(\cos^2 x) = y$,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, **then :** [4 marks]

(A) $|y''(0)| = 2$

(B) $|y'(0)| + |y''(0)| = 3$

(C) $y''(0) = 0$

(D) $|y'(0)| + |y''(0)| = 1$

Correct Answer: (A) $|y''(0)| = 2$

Solution:

Concept:

- Differentiate an implicit relation using the chain rule. Here $d[\ln(\cos^2 x)]/dx = -2\tan x$, valid throughout the stated interval where $\cos x > 0$.
- At $x = 0$, first determine every possible value of y. The denominator $2y - 1$ is nonzero at both possible initial values, so implicit differentiation describes smooth local branches.

Step 1: Determine the first derivative at zero The relation at zero is $y^2 = y$, since $\ln(\cos^2 0) = 0$. Hence $y(0) = 0$ or $y(0) = 1$. Differentiating the full relation gives

$$2yy' - 2\tan x = y', \qquad (2y - 1)y' = 2\tan x.$$

At zero the factor $2y(0) - 1$ is either -1 or $+1$. The right side is zero, so $y'(0) = 0$ for either branch. It would be incorrect to choose just one root of the original quadratic without checking the other.

Step 2: Differentiate again and compare absolute values Applying the product rule to the first-derivative identity yields

$$2(y')^2 + (2y - 1)y'' = 2\sec^2 x.$$

Setting $x = 0$ and using $y'(0) = 0$ gives

$$(2y(0) - 1)y''(0) = 2.$$

Thus $y''(0) = -2$ on the branch through $y = 0$, and $y''(0) = 2$ on the branch through $y = 1$. In either case $|y''(0)| = 2$, while $|y'(0)| + |y''(0)| = 2$. Therefore the claimed sums 3 and 1 are false, and the second derivative is never zero at this point. The absolute value is what makes the first statement valid for both branches.

Step 3: Conclusion For either branch, $|y''(0)| = 2$. Hence, the correct option is (A).

Quick Tip to Solve: When an implicit quadratic permits two values at the evaluation point, check both branches; an absolute-value statement may hold for both even when the signed derivatives differ.

13. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points? [4 marks]

(A) $\left(1, -\frac{1}{\sqrt{2}}\right)$

(B) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

(D) $\left(\frac{1}{\sqrt{2}}, 0\right)$

Correct Answer: (B) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

Solution:

Concept:

- For an ellipse $x^2/a_e^2 + y^2/b_e^2 = 1$, with $a_e > b_e$, the foci lie on the x-axis and satisfy $c^2 = a_e^2 - b_e^2$. Confocal conics have the same centre and the same focal distance c.
- For a hyperbola with transverse axis on the x-axis, $x^2/a_h^2 - y^2/b_h^2 = 1$, the transverse axis has length $2a_h$, and $c^2 = a_h^2 + b_h^2$. A point lies on it only if it satisfies this equation.

Step 1: Find the common focal distance Divide the ellipse equation by 12 to obtain its standard form. Its major axis is horizontal because the larger squared semiaxis is under x^2:

$$\frac{x^2}{4} + \frac{y^2}{3} = 1, \qquad c^2 = 4 - 3 = 1.$$

Thus its foci are $(\pm 1, 0)$, which must also be the foci of the required hyperbola.

Step 2: Construct the hyperbola The given transverse-axis length is $\sqrt{2}$, so $a_h = \sqrt{2}/2$ and $a_h^2 = 1/2$. Therefore

$$b_h^2 = c^2 - a_h^2 = 1 - \frac{1}{2} = \frac{1}{2}.$$

Its equation is consequently $2x^2 - 2y^2 = 1$, or $x^2 - y^2 = 1/2$.

Step 3: Test every listed point For (A), $x^2 - y^2 = 1 - 1/2 = 1/2$. For (B), it is $3/2 - 1/2 = 1$, which fails the equation. For (C), it is $3/2 - 1 = 1/2$; the negative x-coordinate does not affect the square. For (D), it is $1/2 - 0 = 1/2$. Thus (A), (C) and (D) all lie on the hyperbola, whereas (B) does not.

Step 4: Conclusion The hyperbola does not pass through $\left(\sqrt{3/2}, 1/\sqrt{2}\right)$. Hence, the correct option is (B).

Quick Tip to Solve: For confocal ellipse and hyperbola problems, compute the focal distance first; then use the difference of squares for the ellipse and the sum for the hyperbola.

14. The lines
$\vec{r} = \left(\hat{i} - \hat{j}\right) + l\left(2\hat{i} + \hat{k}\right)$ **and**

$$\vec{r} = \left(2\hat{i} - \hat{j}\right) + m\left(\hat{i} + \hat{j} + \hat{k}\right)$$

[4 marks]

(A) do not intersect for any values of l and m

(B) intersect for all values of l and m

(C) intersect when $l = 2$ and m $= \frac{1}{2}$

(D) intersect when $l = 1$ and m $= 2$

Correct Answer: (A) do not intersect for any values of l and m

Solution:

Concept:

- Two parametrised lines intersect only when one value of each parameter makes all three coordinates equal simultaneously. The parameters on the two lines are independent; equal parameters cannot be assumed.
- For a vector equation $\vec{r} = \vec{a} + t\vec{b}$, the components of $\vec{a}$ give a fixed point and those of $\vec{b}$ give the direction. A contradiction between coordinate equations proves that no common point exists.

Step 1: Write the component equations On the first line the coordinates are $(1 + 2l, -1, l)$. On the second they are $(2 + m, -1 + m, m)$. If the lines intersect, their common point must therefore satisfy all of

$$1 + 2l = 2 + m, \qquad -1 = -1 + m, \qquad l = m.$$

We must test these equations together, since agreement of only one or two coordinates does not establish intersection.

Step 2: Use the y- and z-coordinates The y-coordinate equation forces $m = 0$, since the first line always has y-coordinate -1. Substitution into the z-coordinate equation then forces $l = 0$. Hence the only possible parameter pair compatible with both these coordinates is $(l, m) = (0, 0)$. There is no freedom left to choose another pair.

Step 3: Check the remaining coordinate and the options At $l = m = 0$, the x-coordinate equation would require $1 = 2$, which is false. In fact the respective points are $(1, -1, 0)$ and $(2, -1, 0)$. Thus option (B) cannot hold. The pairs in (C) and (D) also fail the y-coordinate requirement $m = 0$, so neither gives an intersection. The direction vectors $(2, 0, 1)$ and $(1, 1, 1)$ are not proportional, so these nonintersecting lines are skew rather than parallel.

Step 4: Conclusion There are no values of l and m for which the two lines intersect. Hence, the correct option is (A).

Quick Tip to Solve: When one line has a constant coordinate, equate that coordinate first. It often fixes a parameter immediately and makes a three-dimensional intersection test very short.

15. 2π - $\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right)$ **is equal to :** [4 marks]

(A) $\frac{7\pi}{4}$

(B) $\frac{5\pi}{4}$

(C) $\frac{3\pi}{2}$

(D) $\frac{\pi}{2}$

Correct Answer: (C) $\frac{3\pi}{2}$

Solution:

Concept:

- For $0<u<1$, the principal value $\sin^{-1}u$ is an acute angle. Its cosine is the positive root $\sqrt{1-u^2}$; this sign is essential when using angle-addition identities.
- The identities $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ and $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ determine both the value and quadrant of a sum.

Step 1: Introduce the two initial angles Put $\alpha=\sin^{-1}(4/5)$ and $\beta=\sin^{-1}(5/13)$. Both are acute, giving $\cos\alpha=3/5$ and $\cos\beta=12/13$. Apply the addition formulas:

$$\sin(\alpha+\beta)=\frac{4\cdot12+3\cdot5}{65}=\frac{63}{65},$$

$$\cos(\alpha+\beta)=\frac{3\cdot12-4\cdot5}{65}=\frac{16}{65}.$$

Both sine and cosine are positive. Since $0<\alpha+\beta<\pi$, this places the sum in the first quadrant.

Step 2: Identify the complementary angle Let $\gamma=\sin^{-1}(16/65)$, also an acute angle. The acute angle $\pi/2-(\alpha+\beta)$ has sine $\cos(\alpha+\beta)=16/65$. Uniqueness of the principal inverse sine on its interval gives

$$\gamma=\frac{\pi}{2}-(\alpha+\beta),\qquad \alpha+\beta+\gamma=\frac{\pi}{2}.$$

This branch check prevents replacing the sum by an angle with the same sine in a different quadrant.

Step 3: Evaluate the required expression Subtract the complete inverse-sine sum from 2π:

$$2\pi-(\alpha+\beta+\gamma)=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}.$$

The values $7\pi/4$ and $5\pi/4$ would require inverse-sine sums of $\pi/4$ and $3\pi/4$, respectively, which contradict the established complementary relation. Option (D) gives the sum itself rather than its subtraction from 2π.

Step 4: Conclusion The given expression equals $3\pi/2$. Hence, the correct option is (C).

Quick Tip to Solve: With inverse sines of Pythagorean ratios, calculate both sine and cosine of the first two angles. A simple cosine often reveals the complementary third angle.

16. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is : [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{1}{5}$
(C) $\frac{1}{7}$
(D) $\frac{1}{6}$

Correct Answer: (D) $\frac{1}{6}$

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the nth term is $a+(n-1)d$, and $S_n = \frac{n}{2}[2a + (n-1)d]$ is the sum of the first n terms.
- The next 15 terms after the first 25 are terms 26 through 40. Their sum is $S_{40} - S_{25}$, not S_{15}. Thus the given equality is equivalently $2S_{25} = S_{40}$.

Step 1: Evaluate the first block sum Here $a = 3$. The first 25 terms have sum

$$S_{25} = \frac{25}{2}[6 + 24d] = 75 + 300d.$$

The first and last terms of this block are 3 and $3 + 24d$, respectively, which also verify the same sum by the average-of-endpoints formula.

Step 2: Evaluate the following block sum The next block starts at term 26, whose value is $3 + 25d$, and ends at term 40, whose value is $3 + 39d$. It has exactly 15 terms. Its sum is

$$\frac{15}{2}[(3 + 25d) + (3 + 39d)] = 45 + 480d.$$

The larger coefficients of d reflect that these are later terms of the progression, even though the block is shorter.

Step 3: Equate and verify the sums Use the stated equality to obtain

$$75 + 300d = 45 + 480d, \qquad 30 = 180d, \qquad d = \frac{1}{6}.$$

For this value, the first sum is $75 + 50 = 125$, and the next sum is $45 + 80 = 125$, confirming the result directly. The other options $1/4$, $1/5$ and $1/7$ do not satisfy the necessary linear equation $180d = 30$, so they cannot give equal block sums.

Step 4: Conclusion The common difference is $d = 1/6$. Hence, the correct option is (D).

Quick Tip to Solve: For equal sums of consecutive blocks, identify the first and last term of each block before applying the sum formula; this avoids treating a later block as a fresh progression starting at the original first term.

17. A dice is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is : [4 marks]

(A) $\frac{1}{8}$

(B) $\frac{1}{9}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{9}$

Solution:

Concept:

- For events A and B with $P(B) > 0$, conditional probability is $P(A \mid B) = P(A \cap B)/P(B)$. The observation changes the relevant sample space to the outcomes in B.
- Two throws of a fair die give 36 equally likely ordered pairs (i, j). Counts of ordered pairs may be used in the conditional ratio because the common factor $1/36$ cancels.

Step 1: List the outcomes allowed by the observation Let B be the event that the sum is a multiple of 4. Within the possible sum range 2 to 12, these sums are 4, 8 and 12. For sum 4 the pairs are $(1, 3), (2, 2), (3, 1)$. For sum 8 they are $(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)$. Sum 12 has only $(6, 6)$. Thus B contains $3 + 5 + 1 = 9$ equally likely ordered outcomes.

Step 2: Impose the condition that a 4 appeared Let A mean that at least one throw has score 4. If one score is 4, a sum divisible by 4 requires the other score to be divisible by 4 too. Among the die faces 1 through 6, only 4 has this property. Consequently $A \cap B$ consists solely of $(4, 4)$. This outcome is counted once, not twice: the phrase at least once includes two occurrences but does not duplicate a single ordered outcome.

Step 3: Form the conditional ratio Therefore

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{9/36} = \frac{1}{9}.$$

Option (A) would incorrectly use eight allowed outcomes; there are nine. Options (C) and (D) would require a different favourable count or a different denominator. The multiples 4, 8 and 12 are not equally likely sums, so assigning each sum probability $1/3$ is also invalid.

Step 4: Conclusion The required conditional probability is $1/9$. Hence, the correct option is (B).

Quick Tip to Solve: For conditional dice problems, retain ordered pairs that satisfy the observation first, and then count the desired pairs inside that reduced set. Never treat distinct sum values as equally likely.

18. Let [t] denote the greatest integer $\leq$ t. If for some $\lambda \in$ R - {1, 0}, $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right|$ = L, then L is equal to : [4 marks]

(A) 1

(B) 2

(C) 0

(D) $\frac{1}{2}$

Correct Answer: (B) 2

Solution:

Concept:

- For $0 < x < 1$, the greatest integer function is $[x] = 0$; for $-1 < x < 0$, it is $[x] = -1$. Also $|x| = x$ on the right of zero and $|x| = -x$ on the left.
- A finite two-sided limit exists only when the left-hand and right-hand limits agree. The excluded values $\lambda = 0, 1$ ensure that the limiting denominators on their respective sides are nonzero.

Step 1: Evaluate the right-hand limit For $x \to 0^+$, use $|x| = x$ and $[x] = 0$. The numerator simplifies to $1 - x + x = 1$, so

$$\lim_{x\to 0^+}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = \lim_{x\to 0^+}\left|\frac{1}{\lambda-x}\right| = \frac{1}{|\lambda|}.$$

The absolute value belongs to the full quotient and must be retained when taking this limit.

Step 2: Evaluate the left-hand limit For $x \to 0^-$, use $|x| = -x$ and $[x] = -1$. The numerator is now $1 - 2x$, while the denominator is $\lambda - 1 - x$. Hence

$$\lim_{x\to 0^-}\left|\frac{1-2x}{\lambda-1-x}\right| = \frac{1}{|\lambda-1|}.$$

Although the numerator has different formulas on the two sides, it tends to 1 on each side. The discontinuity in the floor function is what constrains the parameter.

Step 3: Determine the parameter and common limit The stated existence of L requires $|\lambda| = |\lambda - 1|$. Squaring this equality introduces no sign problem since both sides are nonnegative:

$$\lambda^2 = (\lambda-1)^2, \qquad 2\lambda = 1, \qquad \lambda = \frac{1}{2}.$$

This parameter is permitted by the exclusions. Substitution gives $L = 1/|1/2| = 2$. Thus 1, 0 and $1/2$ are not the common limiting value; in particular $1/2$ is the parameter rather than the limit.

Step 4: Conclusion The common two-sided limit is $L = 2$. Hence, the correct option is (B).

Quick Tip to Solve: Near zero, write the floor function separately on each side before simplifying. When reciprocals of absolute distances agree, solve the equality of those distances to find the parameter.

19. The proposition p $\rightarrow \sim$ (p $\wedge \sim$q) is equivalent to : [4 marks]

(A) ($\sim$p) $\vee$ q

(B) q

(C) ($\sim$p) $\wedge$ q

(D) ($\sim$p) $\vee$ ($\sim$q)

Correct Answer: (A) ($\sim$p) $\vee$ q

Solution:

Concept:

- An implication $u \rightarrow v$ is equivalent to $(\sim u) \vee v$: it is false only when its antecedent is true and its consequent false.
- De Morgan's law gives $\sim (u \wedge v) = (\sim u) \vee (\sim v)$. Double negation removes two successive negations; disjunction is associative and idempotent.

Step 1: Remove the implication For the given statement, the antecedent is p and the entire expression $\sim (p \wedge \sim q)$ is its consequent. Therefore

$$p \rightarrow \sim (p \wedge \sim q) \equiv (\sim p) \vee \sim (p \wedge \sim q).$$

The outer negation applies to the complete conjunction, not just to its first member.

Step 2: Apply De Morgan's law Negating the conjunction changes it to a disjunction:

$$\sim (p \wedge \sim q) = (\sim p) \vee q.$$

Consequently,

$$(\sim p) \vee ((\sim p) \vee q) = (\sim p) \vee q.$$

The repeated term $\sim p$ does not change the truth value.

Step 3: Eliminate the other options When p and q are both false, the original implication is true because its antecedent is false. Options (B) and (C) are both false in this case. When both are true, the original implication is true but option (D) is false. These counterexamples exclude all three alternatives; the simplification above holds for every assignment.

Step 4: Conclusion The proposition is equivalent to $(\sim p) \vee q$. Hence, the correct option is (A).

Quick Tip to Solve: First remove the implication, then apply De Morgan's law to the whole bracket; never distribute a negation without changing the connective.

20. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane : [4 marks]

(A) x – 2y + z = 1

(B) x + 2y – z = 1

(C) x – y – 2y = 1

(D) 2x + y – z = 1

Correct Answer: (D) 2x + y – z = 1

Solution:

Concept:

- A general point on the line through A with direction $\mathbf{d}$ is $A+t\mathbf{d}$. The perpendicular foot H satisfies $(P-H)\cdot\mathbf{d}=0$.
- A point lies on a plane exactly when substitution of its coordinates satisfies the plane equation. Every printed option must therefore be tested separately.

Step 1: Parameterise the line Put $A=(1,-2,3)$, $B=(1,1,0)$ and $P=(4,2,3)$. The direction vector is $B-A=(0,3,-3)$, so

$$H=(1,-2+3t,3-3t).$$

The fixed first coordinate is consistent with the fact that the entire line lies in $x=1$.

Step 2: Impose perpendicularity The displacement from the foot to the external point is $P-H=(3,4-3t,3t)$. Thus

$$0=(P-H)\cdot(0,3,-3)=12-18t.$$

It follows that $t=2/3$ and $H=(1,0,1)$. Also $P-H=(3,2,2)$, whose dot product with the direction vector is zero, confirming the projection.

Step 3: Check the printed planes At $H=(1,0,1)$, the left sides of (A) and (B) are respectively 2 and 0, so neither equals 1. The left side of (D) is $2(1)+0-1=1$. However, option (C) as printed is $x-y-2y=1$; its left side is also $1-0-0=1$. Hence both (C) and (D) contain the foot. The repeated y in (C) must be read literally rather than silently replaced by another variable.

Step 4: Conclusion The perpendicular foot is $\boxed{(1,0,1)}$. Option (D) is the keyed answer, but the printed option (C), $x-y-2y=1$, is also satisfied. Thus the literal options admit both (C) and (D), rather than a unique correct choice.

Quick Tip to Solve: Find the foot once with a dot product, then substitute it into every plane. A printed repeated variable can make more than one option valid.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x = 3 and y = 2, is _____ . [4 marks]

Solution:

Concept:

- The distance of a circle's centre from a tangent line equals its radius. For the vertical line $x = 3$ and horizontal line $y = 2$, these distances are $|h - 3|$ and $|k - 2|$.
- The centre (h, k) lies in the first quadrant on $h + k = 2$, so both coordinates are positive and less than 2. The diameter is twice the radius.

Step 1: Use the centre constraint Write the centre as (h, k), with $h + k = 2$, $h > 0$, $k > 0$. It follows that $0 < h < 2$ and $0 < k < 2$. In particular, the centre is to the left of $x = 3$ and below $y = 2$, so the absolute values have fixed signs.

Step 2: Equate the tangent distances Tangency to both given lines requires a single common radius:

$$r = 3 - h = 2 - k.$$

Therefore $h - k = 1$. Solving this together with $h + k = 2$ gives

$$2h = 3, \qquad h = \frac{3}{2}, \qquad k = \frac{1}{2}.$$

Both coordinates are positive, as required. The radius is $r = 3 - 3/2 = 3/2$.

Step 3: Verify the circle and diameter The resulting circle can be written as

$$\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}.$$

Its distances to $x = 3$ and $y = 2$ are each $3/2$. It touches these lines at $(3, 1/2)$ and $(3/2, 2)$, respectively. Thus the same circle satisfies both tangencies and the centre condition. Finally, the requested quantity is the diameter, not the radius, so multiply by two.

Step 4: Conclusion The diameter is $2r = 2 \times \frac{3}{2} = \boxed{3}$ coordinate units.

Quick Tip to Solve: Use the first-quadrant condition before removing absolute values; after finding a tangent distance, double it when the question asks for the diameter.

22. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$.

If $a_{11} = 109$, then a_{22} is equal to _____ . [4 marks]

Solution:

Concept:

- To obtain A^4, first calculate A^2 and then square that result. Matrix multiplication uses row-by-column products rather than entrywise powers.
- Because x is real, $u = x^2 \geq 0$. This nonnegative constraint selects the admissible root after the known matrix entry is substituted.

Step 1: Compute the square of the matrix For $A = \begin{pmatrix} x & 1 \\ 1 & 0 \end{pmatrix}$, direct multiplication gives

$$A^2 = \begin{pmatrix} x^2+1 & x \\ x & 1 \end{pmatrix}.$$

For example, its upper-left entry is $x \cdot x + 1 \cdot 1 = x^2 + 1$, while its lower-right entry is $1 \cdot 1 + 0 \cdot 0 = 1$.

Step 2: Calculate the required fourth-power entries Squaring A^2, the diagonal entries are

$$a_{11} = (x^2+1)^2 + x^2 = x^4 + 3x^2 + 1,$$

$$a_{22} = x^2 + 1.$$

For completeness, both off-diagonal entries are $x(x^2+2)$. We only need the two diagonal formulas, so solving for the sign of x is unnecessary.

Step 3: Use the given entry and reject the inadmissible root The condition $a_{11} = 109$ gives

$$x^4 + 3x^2 - 108 = 0.$$

Set $u = x^2$. Then

$$u^2 + 3u - 108 = (u+12)(u-9) = 0.$$

Since $u \geq 0$, the root -12 is impossible and $u = 9$. Either real value $x = 3$ or $x = -3$ gives the same diagonal entries. Checking, $81 + 27 + 1 = 109$, so the given condition is recovered exactly.

Step 4: Conclusion Therefore $a_{22} = x^2 + 1 = 9 + 1 = \boxed{10}$. This is a dimensionless matrix entry.

Quick Tip to Solve: Square the matrix twice and solve for x^2, not for x; diagonal entries here depend only on even powers.

23. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+....to\,\infty\right)}$ is equal to _____. [4 marks]

Solution:

Concept:

- An infinite geometric progression with first term a and common ratio r, where $|r| < 1$, has sum $a/(1-r)$.
- For a positive logarithm base $b \neq 1$, the identity $(b^m)^{\log_b u} = u^m$ holds for $u > 0$. Expressing the outer base as a power of the logarithm base simplifies the expression.

Step 1: Sum the infinite progression The series inside the logarithm has first term $1/3$ and common ratio $1/3$. Since its ratio has magnitude less than one, it converges and

$$S = \frac{1/3}{1-1/3} = \frac{1}{2}.$$

The argument of the logarithm is therefore positive, so the real logarithm is defined.

Step 2: Rewrite the outer base exactly Use fractions rather than a decimal approximation:

$$0.16 = \frac{16}{100} = \frac{4}{25} = \left(\frac{5}{2}\right)^{-2} = (2.5)^{-2}.$$

Thus the expression becomes

$$\left((2.5)^{-2}\right)^{\log_{2.5}(1/2)}.$$

Both the outer base and the logarithm base are positive; the usual real exponent laws apply.

Step 3: Apply the inverse logarithm identity Rearranging the powers now gives

$$\left((2.5)^{\log_{2.5}(1/2)}\right)^{-2} = \left(\frac{1}{2}\right)^{-2} = 4.$$

As a sign check, $\log_{2.5}(1/2) < 0$, because the base is greater than one and its argument is below one. Raising 0.16, itself below one, to this negative power must produce a number greater than one, in agreement with the result.

Step 4: Conclusion The value of the given expression is $\boxed{4}$, a dimensionless number.

Quick Tip to Solve: When a decimal base appears outside a logarithmic exponent, convert it to a fraction and look for a power of the logarithm base.

24. **If** $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}$ **= 2^{-k} then the value of k is _____ .** [4 marks]

Solution:

Concept:

- The algebraic identity $1-a-b+ab=(1-a)(1-b)$ exposes a product of two small factors. Each cosine here approaches one as $x\to 0$.
- The standard limit $\lim_{t\to 0}(1-\cos t)/t^2=1/2$ follows from $1-\cos t=2\sin^2(t/2)$. Trigonometric limit arguments are in radians.

Step 1: Factor the numerator Let $u=x^2/2$ and $v=x^2/4$. The numerator is exactly

$$1-\cos u-\cos v+\cos u\cos v=(1-\cos u)(1-\cos v).$$

This factorisation avoids subtracting several terms with the same constant value; it also makes the overall order x^8 explicit.

Step 2: Separate the standard limit factors For $x\neq 0$, write

$$\frac{(1-\cos u)(1-\cos v)}{x^8}=\frac{1-\cos u}{u^2}\,\frac{1-\cos v}{v^2}\,\frac{u^2v^2}{x^8}.$$

Here $u^2=x^4/4$ and $v^2=x^4/16$, so $u^2v^2/x^8=1/64$ exactly. Both standard-limit factors approach $1/2$.

Step 3: Evaluate and compare powers of two The desired limit is therefore

$$L=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{64}=\frac{1}{256}=2^{-8}.$$

Equivalently, the leading terms are $1-\cos(x^2/2)\sim x^4/8$ and $1-\cos(x^2/4)\sim x^4/32$; their product has coefficient $1/256$. This independently checks both powers of x and the numerical coefficient. Comparing 2^{-k} with 2^{-8}, injectivity of the exponential fixes k uniquely.

Step 4: Conclusion The limit equals 2^{-8}, hence $\boxed{k=8}$. The requested exponent is dimensionless.

Quick Tip to Solve: Factor the cosine expression before taking limits. Each factor contributes order x^4, so their product naturally matches the denominator x^8.

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25. **If** $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}}=\left(\frac{1+i}{1-i}\right)^{\frac{n}{3}}=1$, **(m, n $\in$ N) then the greatest common divisor of the least values of m and n is _____ .** [4 marks]

Solution:

Concept:

- The quotient can be simplified by multiplying numerator and denominator by the conjugate of the denominator. Here the resulting complex number lies on the unit circle.
- For the principal argument, $i=e^{i\pi/2}$, so $i^a=e^{ia\pi/2}$. This is equal to 1 when its argument is an integer multiple of 2π. Natural-number exponents parameters are positive.

Step 1: Simplify the common base Multiply by $1+i$ above and below:

$$\frac{1+i}{1-i}=\frac{(1+i)^2}{(1-i)(1+i)}=\frac{1+2i+i^2}{2}=i.$$

Thus the two requirements become $i^{m/2}=1$ and $i^{n/3}=1$. The conditions must be imposed separately; equality of the two powers alone would not ensure that either equals 1.

Step 2: Find the least admissible parameters Using the argument of i, the first condition gives

$$e^{im\pi/4}=1 \quad\Longrightarrow\quad \frac{m\pi}{4}=2r\pi \quad\Longrightarrow\quad m=8r.$$

For positive m, the least choice is $r=1$, hence $m_{\min}=8$. Similarly,

$$e^{in\pi/6}=1 \quad\Longrightarrow\quad \frac{n\pi}{6}=2s\pi \quad\Longrightarrow\quad n=12s,$$

so $n_{\min}=12$. Substituting these values gives $i^4=1$ in both original expressions, confirming both requirements.

Step 3: Take the greatest common divisor The positive divisors common to 8 and 12 are 1, 2 and 4. Equivalently, $8=2^3$ and $12=2^2\times 3$, so the highest shared power of 2 is 2^2. Therefore,

$$\gcd(m_{\min},n_{\min})=\gcd(8,12)=4.$$

Step 4: Conclusion The least values are $m=8$ and $n=12$, and their greatest common divisor is $\boxed{4}$. This is a dimensionless integer.

Quick Tip to Solve: Reduce the complex base first, then require each power to complete a whole number of revolutions before finding the gcd.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively : [4 marks]

(A) 4.5 μC on both

(B) +4.5 μC and –4.5 μC

(C) 6 μC and 3 μC

(D) 3 μC and 6 μC

Correct Answer: (C) 6 μC and 3 μC

Solution:

Concept:

- Connected conductors reach electrostatic equilibrium at equal electric potential, because any potential difference would continue to drive charge through the wire.

- For an isolated sphere of radius r, its surface potential is $V = kQ/r$. Since the spheres are far apart, mutual contributions can be neglected. The total charge of the isolated pair is conserved.

Step 1: Conserve the total charge Let the final charges on the larger and smaller spheres be Q_1 and Q_2. The connecting wire permits redistribution but supplies no charge to the isolated system. Therefore,

$$Q_1 + Q_2 = 12\,\mu\text{C} - 3\,\mu\text{C} = 9\,\mu\text{C}.$$

The initially negative charge on the smaller sphere need not remain negative: electrons can move until equilibrium is reached.

Step 2: Apply equality of potentials The radii are $r_1 = 2R/3$ and $r_2 = R/3$. Equal final potentials imply

$$\frac{kQ_1}{2R/3} = \frac{kQ_2}{R/3} \quad \Longrightarrow \quad \frac{Q_1}{Q_2} = 2.$$

Thus the larger sphere must hold twice the final charge of the smaller sphere. Using $Q_1 = 2Q_2$ in charge conservation gives

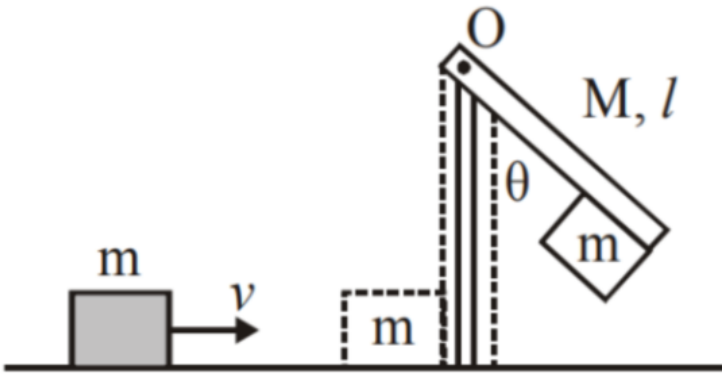
$$3Q_2 = 9\,\mu\text{C}, \qquad Q_2 = 3\,\mu\text{C}, \qquad Q_1 = 6\,\mu\text{C}.$$

Step 3: Check the alternatives Equal charges of $4.5\,\mu\text{C}$ conserve charge but give unequal potentials because the radii differ. The pair $+4.5, -4.5\,\mu\text{C}$ has zero net charge and is impossible here. Reversing the correct charges to $3, 6\,\mu\text{C}$ also violates equal potential. Only the pair $6, 3\,\mu\text{C}$ satisfies both physical conditions.

Step 4: Conclusion The final charges are $\boxed{Q_1 = 6\,\mu\text{C},\ Q_2 = 3\,\mu\text{C}}$. Hence, the correct option is (C).

Quick Tip to Solve: For well-separated spheres connected by a wire, divide the conserved total charge in the ratio of their radii, not equally.

27. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg, and length l = 1 m, the value of θ is approximately :
(take g = 10 m/s^2) [4 marks]



(A) 63°
(B) 69°
(C) 55°
(D) 49

Correct Answer: (A) 63°

Solution:

Concept:

- During the brief sticking collision, angular momentum about the pivot is conserved: the pivot impulse has zero moment about the pivot. Kinetic energy is not conserved in this inelastic collision.
- After impact, the combined rod and block swing without friction, so their rotational kinetic energy becomes gravitational potential energy. For a uniform rod about one end, $I_{\text{rod}} = Ml^2/3$.

Step 1: Find the angular speed just after impact The figure shows the block striking and sticking to the lower end of the rod, at distance l from O. Its incident angular momentum about O is mvl. The combined moment of inertia is

$$I = \frac{Ml^2}{3} + ml^2 = \frac{2}{3} + 1 = \frac{5}{3}\,\text{kg m}^2.$$

Conservation of angular momentum gives

$$mvl = I\omega \quad \Longrightarrow \quad \omega = \frac{1 \times 6 \times 1}{5/3} = \frac{18}{5}\,\text{rad s}^{-1}.$$

The kinetic energy available for the subsequent swing is consequently

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{5}{3}\left(\frac{18}{5}\right)^2 = 10.8\,\text{J}.$$

Step 2: Relate the swing to the potential-energy rise When the system has rotated through θ from the

downward vertical, the rod's centre of mass rises by $(l/2)(1-\cos\theta)$, and the block rises by $l(1-\cos\theta)$. At the turning point the angular speed is zero. Hence,

$$10.8 = \left(\frac{Mgl}{2} + mgl\right)(1-\cos\theta) = 20(1-\cos\theta).$$

It follows that $\cos\theta = 1 - 10.8/20 = 0.46$, giving $\theta \simeq 62.6^\circ$.

Step 3: Choose the closest angle Rounding the calculated turning angle gives approximately 63°. The alternatives 69°, 55° and 49° have cosines inconsistent with 0.46. Using the initial block kinetic energy directly in the swing would be incorrect because energy is lost while it sticks to the rod.

Step 4: Conclusion The rod turns through $\boxed{\theta \approx 63^\circ}$ before momentarily stopping. Hence, the correct option is (A).

Quick Tip to Solve: Use angular momentum only during impact, then conserve mechanical energy for the swing; keeping these stages separate avoids an inelastic-collision error.

28. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is [4 marks]

(A) 50 Ω

(B) 200 Ω

(C) 300 Ω

(D) 100 Ω

Correct Answer: (D) 100 Ω

Solution:

Concept:

- Kirchhoff's voltage law gives the resistor voltage as the battery emf minus the forward voltage across the diode. The diode and resistor carry the same current because they are in series.
- Ohm's law gives $I = V_R/R$. For a fixed positive resistor voltage, current decreases as resistance increases, so the smallest safe resistance occurs at the maximum permitted current.

Step 1: Find the voltage across the resistor The source supplies $1.5\,\text{V}$, while the forward-biased diode consumes a voltage drop of $0.5\,\text{V}$. The remaining voltage must appear across the series resistor:

$$V_R = 1.5 - 0.5 = 1.0\,\text{V}.$$

Using the full battery voltage in Ohm's law would incorrectly count the voltage that already appears across the diode.

Step 2: Impose the safe-current condition Convert the specified limit before substitution:

$$I_{\max} = 10\,\text{mA} = 10 \times 10^{-3}\,\text{A} = 0.010\,\text{A}.$$

Since the resistor is in series with the diode,

$$I = \frac{1.0}{R} \leq 0.010 \quad \Longrightarrow \quad R \geq \frac{1.0}{0.010} = 100\,\Omega.$$

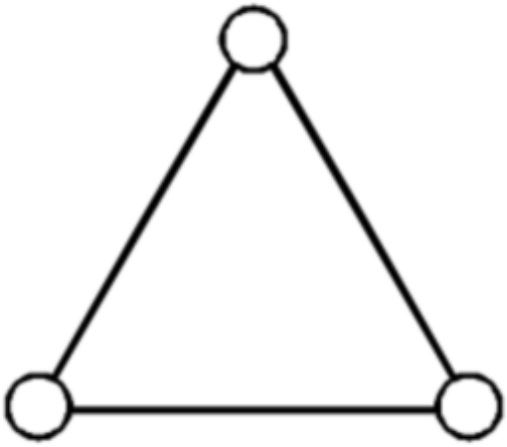
The minimum allowed value is therefore $100\,\Omega$; any larger resistor also protects the diode under the stated constant-drop model.

Step 3: Check which listed value is the minimum For $50\,\Omega$, the current would be $1/50 = 0.020\,\text{A} = 20\,\text{mA}$, exceeding the safe limit. For $200\,\Omega$ and $300\,\Omega$, the currents are $5\,\text{mA}$ and about $3.33\,\text{mA}$. Those values are safe but larger than necessary. The $100\,\Omega$ resistor gives exactly $10\,\text{mA}$.

Step 4: Conclusion The minimum series resistance is $\boxed{100\,\Omega}$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the diode forward drop before calculating the limiting resistor, and convert milliamperes to amperes first.

29. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is : [4 marks]



(A) $\frac{3}{2}RT$

(B) $\frac{9}{2}RT$

(C) $\frac{5}{2}RT$

(D) 3RT

Correct Answer: (D) 3RT

Solution:

Concept:

- Equipartition assigns mean energy $\frac{1}{2}k_BT$ per molecule to each independent quadratic degree of freedom. For one mole, replace k_B by the gas constant R.
- A rigid nonlinear molecule has three translational and three rotational degrees of freedom. The rigid rods fix all interatomic distances, excluding vibrational modes from this model.

Step 1: Count the translational modes The centre of mass of the triangular molecule can move independently along three perpendicular directions. Its translational kinetic energy is the sum of three squared velocity components, so the translational contribution per mole is

$$U_{\text{trans}} = 3\left(\frac{1}{2}RT\right) = \frac{3}{2}RT.$$

The masses being concentrated at the vertices does not remove any direction of translation; the rods being massless only means that the rods add no separate mass.

Step 2: Count the rotational modes The three atoms occupy the corners of a triangle, so they are not collinear. The molecule can rotate about three independent axes through its centre of mass, with a nonzero moment of inertia about each. This differs from the ideal linear-molecule model, which has only two active rotational modes. Thus,

$$U_{\text{rot}} = 3\left(\frac{1}{2}RT\right) = \frac{3}{2}RT.$$

There is no stretching or bending energy in the assumed perfectly rigid framework.

Step 3: Add the contributions and test the options The total number of active degrees of freedom is $f = 3 + 3 = 6$. Therefore,

$$U = \frac{f}{2}RT = \frac{6}{2}RT = 3RT.$$

The value $3RT/2$ counts translation only. The value $5RT/2$ describes a rigid linear molecule rather than a triangle. The value $9RT/2$ would require additional active modes that are excluded by the rigidity assumption. This leaves only $3RT$.

Step 4: Conclusion The internal energy of one mole of the rigid triangular gas is $\boxed{U = 3RT}$. Hence, the correct option is (D).

Quick Tip to Solve: Check molecular shape before counting rotations: a rigid triangular molecule is nonlinear and has three rotational degrees of freedom.

30. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to [4 marks]

(A) 0.17°

(B) 1.7°

(C) 0.57°

(D) 0.07°

Correct Answer: (C) 0.57°

Solution:

Concept:

- In Young's double-slit experiment, bright fringes satisfy $d\sin\theta = n\lambda$. For small angular separations, the angular fringe width is $\Delta\theta \simeq \lambda/d$, measured in radians.
- The linear fringe width is $\beta = \lambda D/d$, so dividing by the screen distance D gives the same angular width. Conversion to degrees requires multiplication by $180/\pi$.

Step 1: Express wavelength and slit separation in common units The specified wavelength and slit separation are

$$\lambda = 500\,\text{nm} = 5.00 \times 10^{-7}\,\text{m},$$

$$d = 0.05\,\text{mm} = 5.00 \times 10^{-5}\,\text{m}.$$

Their ratio is dimensionless. Converting both quantities to metres avoids mixing nanometres with millimetres and introducing an erroneous power of ten.

Step 2: Compute the angular width in radians For adjacent interference fringes near the centre, the path difference changes by one wavelength. With the distant-screen approximation, $\sin\theta \simeq \theta$, so

$$\Delta\theta = \frac{\lambda}{d} = \frac{5.00 \times 10^{-7}}{5.00 \times 10^{-5}} = 1.00 \times 10^{-2}\,\text{rad}.$$

The small value 0.010 radians justifies using the small-angle relation. The distance of the screen does not enter the final angular width, even though it affects the linear width.

Step 3: Convert to degrees and compare Using $\pi\,\text{rad} = 180°$,

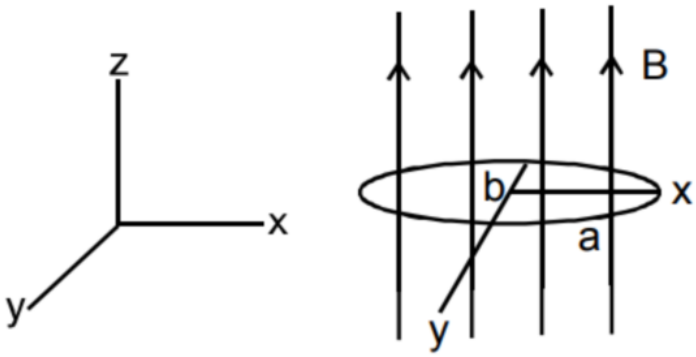
$$\Delta\theta = 0.010 \times \frac{180}{\pi} \simeq 0.573°.$$

The closest listed value is $0.57°$. The values $0.17°$, $1.7°$ and $0.07°$ do not agree with this ratio and conversion. In particular, the intermediate number 0.010 is an angle in radians, so it cannot be compared directly with the options in degrees.

Step 4: Conclusion The angular width is approximately $\boxed{0.57°}$. Hence, the correct option is (C).

Quick Tip to Solve: For angular fringe width, use wavelength divided by slit separation; convert the resulting radians to degrees only at the end.

31. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is : [4 marks]



(A) $\frac{\pi abB\omega}{R}$

(B) $\frac{\pi^2a^2b^2B^2\omega^2}{R}$

(C) $\frac{\pi^2a^2b^2B^2\omega^2}{2R}$

(D) Zero

Correct Answer: (C) $\frac{\pi^2a^2b^2B^2\omega^2}{2R}$

Solution:

Concept:

- The magnetic flux through a plane loop is $\Phi = BA\cos\theta$, where A is its area and θ is the angle between its normal and the magnetic field.
- Faraday's law gives $\mathcal{E} = -d\Phi/dt$. For a resistive loop, the instantaneous heating power is $p = \mathcal{E}^2/R$, and the cycle average of $\sin^2\omega t$ is $1/2$.

Step 1: Find the flux and induced emf The elliptical loop has area $A = \pi ab$. Initially its plane is perpendicular to the vertical magnetic field, as shown. Rotation about the in-plane x-axis changes the angle of its normal relative to the field at the rate ω. Taking the initial angle as zero,

$$\Phi(t) = B\pi ab\cos\omega t.$$

Differentiating with respect to time gives

$$\mathcal{E}(t) = B\pi ab\omega\sin\omega t.$$

The sign fixes the direction of induced current; heating depends on its square, so the sign does not affect the required average.

Step 2: Average the resistive power The peak emf is $\mathcal{E}_0 = \pi abB\omega$, and the induced current is $i(t) = \mathcal{E}(t)/R$. Therefore

$$p(t) = \frac{\pi^2a^2b^2B^2\omega^2}{R}\sin^2\omega t.$$

Averaging over one complete rotation gives

$$\langle p \rangle = \frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}.$$

Option (B) is the peak power and misses the averaging factor. Option (A) has the form of a current rather than power. Option (D) is incorrect because the flux changes during rotation even though its average over a cycle is zero.

Step 3: Conclusion The average power loss is $\boxed{\pi^2 a^2 b^2 B^2 \omega^2/(2R)}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the peak induced emf first, then use $P_{\text{av}} = \mathcal{E}_0^2/(2R)$; averaging the emf itself would incorrectly give zero.

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32. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ? [4 marks]

(A) 12

(B) 3

(C) 9

(D) 6

Correct Answer: (A) 12

Solution:

Concept:

- The transverse wave speed on a stretched rope is $v = \sqrt{T/\mu}$, where T is the local tension and μ is mass per unit length.
- In a stationary rope, the wave frequency remains fixed as the wave moves through regions of different tension. Consequently $\lambda = v/f$, so wavelength is proportional to wave speed.

Step 1: Calculate the tensions at the two ends The rope is uniform, so its linear mass density is

$$\mu = \frac{6}{12} = 0.5\,\text{kg m}^{-1}.$$

Just above the lower end, the rope supports only the attached block, giving $T_{\text{lower}} = 2g$. At the top, it supports both the entire rope and the block, giving $T_{\text{top}} = (6+2)g = 8g$. The rope's own weight therefore makes the tension increase upwards; treating it as massless would miss the change in wavelength.

Step 2: Use the speed ratio at constant frequency The same value of μ applies at both ends. Thus

$$\frac{v_{\text{top}}}{v_{\text{lower}}} = \sqrt{\frac{8g/\mu}{2g/\mu}} = \sqrt{4} = 2.$$

Since the source frequency of the wavetrain does not change as it propagates,

$$\frac{\lambda_{\text{top}}}{\lambda_{\text{lower}}} = 2, \qquad \lambda_{\text{top}} = 2 \times 6 = 12\,\text{cm}.$$

The wavelength increases rather than decreases, so 3 cm is excluded. The unchanged value 6 cm would require equal tensions. The value 9 cm corresponds to a speed ratio 1.5, whereas the actual ratio is 2. The length of the rope is needed to specify its density but cancels from this ratio.

Step 3: Conclusion The wavelength at the top of the rope is $\boxed{12\,\text{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a hanging rope, count all the mass below each point to find tension; then take the square root of the tension ratio, not the ratio itself.

33. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is : [4 marks]

(A) $250\sqrt{3}$

(B) $5\sqrt{3}$

(C) $500\sqrt{3}$

(D) $50\sqrt{3}$

Correct Answer: (C) $500\sqrt{3}$

Solution:

Concept:

- A straight wire segment produces $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi d)$ at a point a perpendicular distance d away; the angles are measured from the perpendicular.
- At the centre of a regular polygon, the magnetic fields of all its sides point along the same normal. Multiply the field of one side by the number of sides and then by the number of turns.

Step 1: Find the geometry for one side Let the side of the regular hexagon be $s = 10\,\text{cm} = 0.10\,\text{m}$. Joining the centre to the endpoints and midpoint of a side gives a right triangle with angle 30° at the centre. The perpendicular distance from the centre to that side is the apothem,

$$d = \frac{s}{2\tan 30^\circ} = \frac{\sqrt{3}\,s}{2}.$$

Both endpoint angles in the finite-wire field formula are 30°, so their sine sum is $2\sin 30^\circ = 1$. It is the apothem, rather than the circumradius s, that belongs in the denominator.

Step 2: Add the fields of six sides and fifty turns For one side of one turn, $B_1 = \mu_0 I/(4\pi d)$. Applying the right-hand rule shows that every side contributes in the same axial direction, hence

$$B = 50 \times 6\frac{\mu_0 I}{4\pi d} = \frac{\mu_0 I}{\pi}\frac{300}{4(\sqrt{3} \times 0.10/2)}.$$

The numerical coefficient when SI lengths are inserted is

$$\frac{300}{0.20\sqrt{3}} = \frac{1500}{\sqrt{3}} = 500\sqrt{3}.$$

Thus the requested field, in units of $\mu_0 I/\pi$, is $500\sqrt{3}$. The coefficient $5\sqrt{3}$ misses the centimetre-to-metre conversion; $50\sqrt{3}$ is ten times too small, while $250\sqrt{3}$ loses a factor of two in the wire or apothem calculation.

Step 3: Conclusion The magnetic field is $B = (500\sqrt{3})\mu_0 I/\pi$, so the required coefficient is $\boxed{500\sqrt{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: For polygon coils, compute the perpendicular distance to a side before using the finite-wire formula, and convert the side length to metres before multiplying by turns.

34. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C. (assume no loss of heat to the surroudnings) is close to : [4 marks]

(A) 348 s

(B) 418 s

(C) 245 s

(D) 365 s

Correct Answer: (A) 348 s

Solution:

Concept:

- For a series RLC circuit, $Z^2 = R^2 + (\omega L - 1/(\omega C))^2$, with $\omega = 2\pi f$. The rms current is $I_{\text{rms}} = V_{\text{rms}}/Z$.
- Only the resistor dissipates average power: $P = I_{\text{rms}}^2 R$. A body of heat capacity H needs heat $Q = H\Delta T$, and with no heat loss $t = Q/P$.

Step 1: Find the reactances and rms current Use $f = 750\,\text{Hz}$, $L = 0.1803\,\text{H}$, and $C = 10 \times 10^{-6}\,\text{F}$. The angular frequency and reactances are

$$\omega = 1500\pi \simeq 4712.39\,\text{s}^{-1},$$

$$X_L = \omega L \simeq 849.64\,\Omega, \qquad X_C = \frac{1}{\omega C} \simeq 21.22\,\Omega.$$

The net reactance is therefore about $828.42\,\Omega$, and

$$Z^2 = 100^2 + (828.42)^2 \simeq 6.963 \times 10^5\,\Omega^2.$$

With a 20 V rms source, $I_{\text{rms}}^2 = 400/Z^2$. No extra factor of two is required because the specified voltage is already an rms value.

Step 2: Equate electrical heat to the required thermal energy The average resistive power is

$$P = \frac{20^2 \times 100}{6.963 \times 10^5} \simeq 0.05745\,\text{W}.$$

The resistor's given heat capacity is for the entire resistor, so no additional mass factor is needed:

$$Q = (2\,\text{J}/^\circ\text{C})(10\,^\circ\text{C}) = 20\,\text{J}.$$

Since no heat escapes to the surroundings,

$$t = \frac{20}{0.05745} \simeq 348\,\text{s}.$$

The other proposed times, 418 s, 245 s and 365 s, are farther from the computed value. In particular, adding reactances instead of subtracting them or treating 20 V as a peak value changes the heating rate incorrectly.

Step 3: Conclusion The required heating time is approximately $\boxed{348\,\text{s}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the rms voltage directly in $P = V_{\text{rms}}^2 R/Z^2$, and remember that inductive and capacitive reactances subtract in a series circuit.

35. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e . By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is : [4 marks]

(A) $2R_e$

(B) $3R_e$

(C) $4R_e$

(D) $2.5R_e$

Correct Answer: (B) $3R_e$

Solution:

Concept:

- For a circular orbit of radius r around the Earth, $v_c^2 = GM/r$. After an instantaneous burn, the new specific energy is $\varepsilon = v^2/2 - GM/r$.
- A bound elliptic orbit has $\varepsilon = -GM/(2a)$, where a is the semi-major axis. Its closest and farthest radii satisfy $r_p + r_a = 2a$.

Step 1: Find the orbital energy immediately after the burn The original orbit is low, so take its radius as R_e, as stipulated. Its circular speed obeys $v_c^2 = GM/R_e$. The rocket increases the tangential speed to $v = \sqrt{3/2}\,v_c$, giving

$$v^2 = \frac{3GM}{2R_e}.$$

The burn changes the speed instantly but not the satellite's position. Thus its new energy per unit mass is

$$\varepsilon = \frac{1}{2}\frac{3GM}{2R_e} - \frac{GM}{R_e} = -\frac{GM}{4R_e}.$$

This is negative, so the orbit remains bound. The new speed is below escape speed because $3GM/(2R_e) < 2GM/R_e$.

Step 2: Locate the farthest point of the new ellipse Equate the computed energy to the elliptic-orbit expression:

$$-\frac{GM}{2a} = -\frac{GM}{4R_e} \quad \Rightarrow \quad a = 2R_e.$$

At the burn the velocity is entirely tangential, so the radial velocity is zero. Since its speed exceeds the local circular speed, this turning point is the perigee, not the apogee: $r_p = R_e$. Therefore

$$R = r_a = 2a - r_p = 4R_e - R_e = 3R_e.$$

Option (A) confuses the semi-major axis with the farthest radius; option (C) gives the full major-axis length $2a$, whereas the question asks for distance from the Earth's centre. The value $2.5R_e$ also fails $r_p + r_a = 4R_e$.

Step 3: Conclusion The farthest distance from the centre of the Earth is $\boxed{R = 3R_e}$. Hence, the correct option is (B).

Quick Tip to Solve: After a tangential speed increase above circular speed, the firing point becomes perigee; use energy for the semi-major axis and then subtract perigee from $2a$.

36. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as [4 marks]

(A) 2.123 cm

(B) 2.124 cm

(C) 2.125 cm

(D) 2.121 cm

Correct Answer: (B) 2.124 cm

Solution:

Concept:

- The least count of a screw gauge is the pitch divided by the number of circular-scale divisions: $\mathrm{LC} = p/N$. It is the displacement for one circular-scale division.
- A directly recorded reading must be compatible with whole multiples of the least count. Decimal places alone are not enough: the last digits must correspond to a possible scale increment.

Step 1: Calculate the instrument resolution The pitch is $0.1\,\mathrm{cm}$ per complete revolution and the

circular scale has 50 equal divisions. Consequently,

$$\text{LC} = \frac{0.1\,\text{cm}}{50} = 0.002\,\text{cm}.$$

Each step of the circular scale therefore changes the recorded thickness by $0.002\,\text{cm}$, which is $0.02\,\text{mm}$. The instrument can display thousandths of a centimetre, but only in steps of two thousandths. Treating the least count as $0.001\,\text{cm}$ merely because three decimal places are shown would overstate the resolution.

Step 2: Check which proposed reading lies on the scale grid Express each value as a number of least-count intervals:

$$\frac{2.123}{0.002} = 1061.5, \qquad \frac{2.124}{0.002} = 1062,$$
$$\frac{2.125}{0.002} = 1062.5, \qquad \frac{2.121}{0.002} = 1060.5.$$

Only 2.124 cm corresponds to an integer number of scale increments. It can, for example, be formed from an axial reading of 2.1 cm and 12 circular-scale divisions:

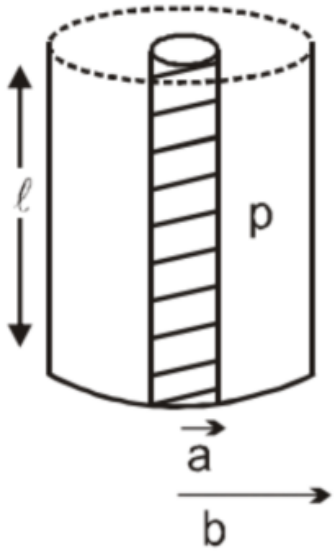
$$2.1 + 12(0.002) = 2.124\,\text{cm}.$$

The other three choices require half a circular-scale division and so do not represent the specified instrument's directly resolved reading. No zero correction or half-division interpolation is supplied in the question, so it should not be introduced.

Step 3: Conclusion The thickness should be recorded as $\boxed{2.124\,\text{cm}}$, consistent with a least count of $0.002\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: After calculating least count, divide each candidate by it: a directly readable value must give an integer, not just have an appropriate number of decimal places.

37. Model a torch battery of length l to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum Joule heating in R will take place for : [4 marks]



(A) $R = \frac{2\rho}{\pi l}\ln\left(\frac{b}{a}\right)$
(B) $R = \frac{\rho}{\pi l}\ln\left(\frac{b}{a}\right)$
(C) $R = \frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right)$
(D) $R = \frac{\rho}{2\pi l}\left(\frac{b}{a}\right)$

Correct Answer: (C) $R = \frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right)$

Solution:

Concept:

- For a conducting layer, resistance is $dr = \rho\, dx/A$, where ρ is resistivity and A is the area normal to current.
- A battery of emf $\mathcal{E}$ and internal resistance r supplies maximum power to a variable load when $R = r$.

Step 1: Identify the direction of current The inner cylindrical electrode and outer shell are concentric. Current crosses the electrolyte radially from radius a to radius b, rather than flowing along the battery length. At radius x, the area perpendicular to radial current is the curved area $A = 2\pi x l$. A thin layer of radial thickness dx therefore has resistance

$$dr = \frac{\rho\, dx}{2\pi x l}.$$

Successive radial layers are crossed one after another, so their resistances add in series.

Step 2: Integrate the internal resistance Integrating over the electrolyte gives

$$r = \int_a^b \frac{\rho\, dx}{2\pi l x} = \frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right).$$

The logarithm is dimensionless, and ρ/l has units of resistance. The factor 2π follows from the cylindrical current area; using a constant cross-section would miss the logarithm.

Step 3: Maximise the load heating The current through the load is $I = \mathcal{E}/(R+r)$, hence its heating power is

$$P_R = I^2 R = \frac{\mathcal{E}^2 R}{(R+r)^2}.$$

Differentiation gives

$$\frac{dP_R}{dR} = \frac{\mathcal{E}^2(r-R)}{(R+r)^3}.$$

It changes from positive to negative at $R = r$. Options (A) and (B) have incorrect numerical factors, while (D) replaces the required logarithm by the radius ratio.

Step 4: Conclusion The maximum heating condition is

$$R = \frac{\rho}{2\pi l} \ln\left(\frac{b}{a}\right).$$

Hence, the correct option is (C).

Quick Tip to Solve: For radial conduction between coaxial cylinders, use the curved area $2\pi x l$ inside the resistance integral before applying load matching.

38. A charged particle carrying charge 1 μC is moving with velocity $\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ ms^{-1}. If an external magnetic field of $\left(5\hat{i}+3\hat{j}-6\hat{k}\right)\times$ 10^{-3} T exists in the region where the particle is moving then the force on the particle is $\vec{F}\times$ 10^{-9} N. The vector $\vec{F}$ is : [4 marks]

(A) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$

(B) $-300\hat{i}+320\hat{j}-90\hat{k}$

(C) $-30\hat{i}+32\hat{j}-9\hat{k}$

(D) $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$

Correct Answer: (C) $-30\hat{i}+32\hat{j}-9\hat{k}$

Solution:

Concept:

- The magnetic Lorentz force is $\mathbf{F}_{\text{mag}} = q(\mathbf{v}\times\mathbf{B})$; its direction depends on the order of the cross product.
- Here $q = 10^{-6}$ C and the field has a common factor 10^{-3} T, giving a force factor 10^{-9} N.

Step 1: Separate the powers of ten The velocity coefficients are $(2,3,4)$ in $\mathrm{m\,s^{-1}}$, and the magnetic-field coefficients are $(5,3,-6)$ times 10^{-3} T. The particle carries positive charge $1\,\mu$C, so the force follows $\mathbf{v}\times\mathbf{B}$, with no reversal for negative charge. Thus

$$\mathbf{F}_{\text{mag}} = 10^{-9}\left[(2\hat{i}+3\hat{j}+4\hat{k})\times(5\hat{i}+3\hat{j}-6\hat{k})\right]\text{N}.$$

Step 2: Evaluate the three components The component formula is $(v_yB_z - v_zB_y,\ v_zB_x - v_xB_z,\ v_xB_y - v_yB_x)$. Therefore

$$F_x = 3(-6) - 4(3) = -30,$$
$$F_y = 4(5) - 2(-6) = 32,$$
$$F_z = 2(3) - 3(5) = -9.$$

These are the components of the coefficient vector requested in the question, since its $10^{-9}\,\mathrm{N}$ factor has already been taken outside.

Step 3: Check direction and scale The coefficient vector is perpendicular to velocity:

$$2(-30) + 3(32) + 4(-9) = 0.$$

It is also perpendicular to the field coefficients since $5(-30) + 3(32) + (-6)(-9) = 0$. These provide checks on the signs in the cross product. Options (A), (B), and (D) have the same component ratios but incorrect overall scales; the charge and field factors together are exactly 10^{-9}, not an additional decimal factor.

Step 4: Conclusion The vector is $\overrightarrow{F} = -30\hat{i} + 32\hat{j} - 9\hat{k}$, corresponding to a physical force $(-30\hat{i} + 32\hat{j} - 9\hat{k}) \times 10^{-9}\,\mathrm{N}$. Hence, the correct option is (C).

Quick Tip to Solve: Extract the charge and field powers of ten first, then compute $\mathbf{v} \times \mathbf{B}$; a zero dot product with velocity checks the result.

39. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as [4 marks]

(A) Irreversible adiabatic

(B) Reversible adiabatic

(C) Irreversible isothermal

(D) Reversible isothermal

Correct Answer: (A) Irreversible adiabatic

Solution:

Concept:

- An adiabatic process has negligible heat transfer, $Q \simeq 0$, during the time interval being considered; insulation is not its only possible cause.
- A reversible process proceeds through equilibrium states under infinitesimal driving differences. A sudden finite pressure imbalance produces an irreversible expansion.

Step 1: Use the short time scale The question concerns the expansion immediately after the balloon bursts. The helium escapes and expands rapidly. Heat transfer through the surrounding air requires time, so negligible heat can be exchanged during this initial rapid event. Consequently the immediate expansion is treated as adiabatic, $Q \simeq 0$. The initial temperature $32^{\circ}\mathrm{C}$ does not impose a constant gas temperature during expansion; it merely specifies the starting state.

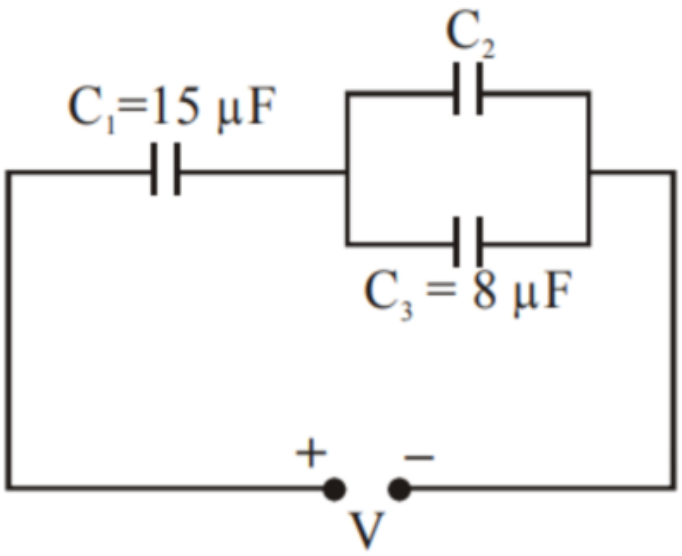
Step 2: Test reversibility The gas begins at a pressure of $1.7\,\mathrm{atm}$ inside the balloon. When the wall ruptures, the pressure confinement is abruptly removed and gas motion follows a finite pressure difference. The expansion is not a sequence of mechanically balanced, quasi-static states. Gas jets, turbulence and mixing make it impossible to reverse the event by an infinitesimal change in external pressure. Therefore the expansion is irreversible.

Step 3: Rule out the alternatives Option (B) has the right heat-transfer approximation but wrongly calls a sudden bursting event reversible. Options (C) and (D) both assume an isothermal process, which would require the temperature to be maintained by suitable thermal exchange rather than inferred from the given initial temperature. Option (D) additionally has the same reversibility error. It is unnecessary to apply the reversible-adiabatic formula $PV^{\gamma} = \text{constant}$: that relation does not describe this abrupt non-equilibrium expansion.

Step 4: Conclusion Immediately after bursting, the expansion is rapid enough to be adiabatic and has a finite pressure imbalance that makes it irreversible. Hence, the correct option is (A).

Quick Tip to Solve: In a sudden bursting or escape process, check heat-transfer time and pressure balance separately: rapid suggests adiabatic, while finite imbalance implies irreversible.

40. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is : [4 marks]



(A) 160 μC

(B) 450 μC

(C) 590 μC

(D) 650 μC

Correct Answer: (C) 590 μC

Solution:

Concept:

- Capacitors in parallel have the same potential difference, and the magnitude of charge on each is $Q = CV$.
- A capacitor in series with a parallel group carries the same charge magnitude as the total charge of that group: $Q_1 = Q_2 + Q_3$.

Step 1: Read the circuit connections The diagram shows $C_1 = 15\,\mu\text{F}$ in series with the parallel combination of C_2 and $C_3 = 8\,\mu\text{F}$. The stated total charge $750\,\mu\text{C}$ is the charge supplied to this series network. Hence C_1 carries $750\,\mu\text{C}$, and the combined charge on the parallel capacitors is also $750\,\mu\text{C}$. It is not necessary to know C_2 beforehand.

Step 2: Find the charge on the known parallel capacitor Because C_2 and C_3 share their two nodes, the given voltage $V_2 = 20\,\text{V}$ is also V_3. Therefore

$$Q_3 = C_3 V_3 = (8\,\mu\text{F})(20\,\text{V}) = 160\,\mu\text{C}.$$

This is the charge on C_3, so option (A) confuses the known capacitor with the one requested.

Step 3: Apply charge conservation at the junction The charge divides between the two parallel branches:

$$Q_2 + Q_3 = 750\,\mu\text{C}.$$

Consequently

$$Q_2 = 750 - 160 = 590\,\mu\text{C}.$$

As a check, $C_2 = Q_2/V_2 = 29.5\,\mu\text{F}$, giving a parallel capacitance of $37.5\,\mu\text{F}$ and charge $37.5 \times 20 = 750\,\mu\text{C}$. The voltage on C_1 would be $750/15 = 50\,\text{V}$, consistent with a total applied voltage of $70\,\text{V}$. Options (B) and (D) do not satisfy the branch-charge sum.

Step 4: Conclusion The charge on capacitor C_2 is $590\,\mu\text{C}$. Hence, the correct option is (C).

Quick Tip to Solve: First identify the parallel pair: equal voltage gives the known branch charge, then subtract it from the series charge to find the other branch.

41. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is
I = $M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio $\frac{L}{R}$ for it to have minimum possible I is [4 marks]

(A) $\frac{3}{2}$

(B) $\sqrt{\frac{3}{2}}$

(C) $\sqrt{\frac{2}{3}}$

(D) $\frac{2}{3}$

Correct Answer: (B) $\sqrt{\frac{3}{2}}$

Solution:

Concept:

- The mass and material density are fixed, so the cylinder volume is fixed: $\pi R^2 L = V$. Its radius and length cannot vary independently.
- For the stated transverse central axis, $I = M(R^2/4 + L^2/12)$. Minimise this expression subject to the fixed-volume constraint.

Step 1: Eliminate one dimension using fixed volume Define the positive constant $K = V/\pi$, so $R^2 L = K$ and $L = K/R^2$. Substitution in the given expression gives

$$\frac{I}{M} = \frac{R^2}{4} + \frac{K^2}{12R^4}.$$

Increasing the radius raises the first term but reduces the second because the length must shrink. This competition is why there is a finite optimal shape; keeping length fixed would ignore the specified fixed mass of material.

Step 2: Locate and verify the minimum Differentiate with respect to the positive radius:

$$\frac{d(I/M)}{dR} = \frac{R}{2} - \frac{K^2}{3R^5}.$$

At the stationary point,

$$\frac{R}{2} = \frac{K^2}{3R^5} \quad \Rightarrow \quad K^2 = \frac{3}{2}R^6.$$

Also

$$\frac{d^2(I/M)}{dR^2} = \frac{1}{2} + \frac{5K^2}{3R^6} > 0,$$

so the stationary point is a minimum. The inertia tends to infinity as R approaches zero or infinity under the constraint, confirming that this is the global minimum.

Step 3: Recover the requested dimensionless ratio Since $K = R^2 L$, the stationary condition becomes

$$R^4 L^2 = \frac{3}{2}R^6 \quad \Rightarrow \quad \left(\frac{L}{R}\right)^2 = \frac{3}{2}.$$

Both lengths are positive, so $L/R = \sqrt{3/2}$. Option (A) gives the squared ratio rather than its positive square root. Options (C) and (D) use the reciprocal ratio or its square and therefore do not obey the derived condition.

Step 4: Conclusion The minimum possible moment of inertia occurs for $L/R = \sqrt{3/2}$. Hence, the correct option is (B).

Quick Tip to Solve: When mass and material are fixed, replace one cylinder dimension using $R^2L =$ constant before differentiating the inertia.

42. The magnetic field of a plane electromagnetic wave is

$\vec{B} = 3 \times 10^{-8} \sin[200\pi\,(y + ct)]\,\hat{i}$ **T**

where c = 3 × 10^8 ms^{-1} is the speed of light. The corresponding electric field is : [4 marks]

(A) $\vec{E} = -10^{-6} \sin[200\pi\,(y + ct)]\,\hat{k}$ V/m

(B) $\vec{E} = -9 \sin[200\pi\,(y + ct)]\,\hat{k}$ V/m

(C) $\vec{E} = 9 \sin[200\pi\,(y + ct)]\,\hat{k}$ V/m

(D) $\vec{E} = 3 \times 10^{-8} \sin[200\pi\,(y + ct)]\,\hat{k}$

Correct Answer: (B) $\vec{E} = -9 \sin[200\pi\,(y + ct)]\,\hat{k}$ V/m

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the field amplitudes satisfy $E_0 = cB_0$, and both fields have the same travelling phase.
- The direction of propagation is along $\mathbf{E} \times \mathbf{B}$. A phase $ky + \omega t$ moves in the negative y-direction.

Step 1: Determine the propagation direction The phase is $\phi = 200\pi(y + ct)$. Following a point of constant phase requires $y + ct =$ constant, which gives $dy/dt = -c$. Thus the wave travels along $-\hat{j}$. The magnetic field is along the x-axis, so the electric field must lie along the z-axis to be perpendicular to both the propagation direction and magnetic field.

Step 2: Calculate the electric amplitude The magnetic amplitude is $B_0 = 3 \times 10^{-8}$ T. Therefore

$$E_0 = cB_0 = (3 \times 10^8)(3 \times 10^{-8}) = 9\ \text{V m}^{-1}.$$

The factors of ten cancel, so options (A) and (D) have the wrong amplitude. Electric and magnetic amplitudes cannot be assigned the same numerical value in SI units; their ratio is the speed of light.

Step 3: Fix the sign of the electric field At a phase where the sine is positive, $\mathbf{B}$ points along $+\hat{i}$. Since $\hat{k} \times \hat{i} = \hat{j}$, a field along $+\hat{k}$ would send energy in the positive y-direction, contradicting the phase. The electric field must instead point along $-\hat{k}$:

$$(-\hat{k}) \times \hat{i} = -\hat{j}.$$

It therefore carries a minus sign with the same sine phase. Option (C) has the correct magnitude but the opposite energy-flow direction. The sign remains consistent when the sine changes sign because

both fields reverse together.

Step 4: Conclusion The electric field is

$$\mathbf{E} = -9\sin[200\pi(y+ct)]\hat{k}\,\mathrm{V\,m^{-1}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Use $E_0 = cB_0$ for magnitude, then check $\mathbf{E}\times\mathbf{B}$ against the travelling-phase direction to choose the sign.

43. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to : [4 marks]

(A) 1.02 eV

(B) 0.81 eV

(C) 0.61 eV

(D) 0.52 eV

Correct Answer: (C) 0.61 eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the metal's work function. The incident photon energy must exceed ϕ for emission.
- For photon energy in electronvolts and wavelength in nanometres, use $hc \simeq 1240\,\mathrm{eV\,nm}$. Changing wavelength changes photon energy, while the work function remains the same for the same metal.

Step 1: Write the two energy balances Let the original maximum kinetic energy be K. The shorter wavelength has greater photon energy and gives the stated kinetic energy $3K$. Thus

$$K = \frac{1240}{500} - \phi = 2.48 - \phi, \qquad 3K = \frac{1240}{200} - \phi = 6.20 - \phi.$$

All energies in these equations are in electronvolts. The threefold factor applies to the kinetic energy, not to the incident photon energy.

Step 2: Eliminate the kinetic energy Multiply the first equation by three and compare it with the second:

$$3(2.48 - \phi) = 6.20 - \phi.$$

Hence

$$7.44 - 3\phi = 6.20 - \phi, \qquad 2\phi = 1.24, \qquad \phi = 0.62\,\mathrm{eV}.$$

This gives $K = 1.86\,\mathrm{eV}$ initially and $5.58\,\mathrm{eV}$ finally, confirming the required factor of three.

Step 3: Compare the listed values The nearest option to $0.62\,\mathrm{eV}$ is $0.61\,\mathrm{eV}$. The other values, 1.02, 0.81 and $0.52\,\mathrm{eV}$, do not reproduce the stated kinetic-energy ratio with these wavelengths. The small

difference between 0.62 and 0.61 is consistent with the question asking for a close value.

Step 4: Conclusion The work function is approximately $0.62\,\text{eV}$, closest to $0.61\,\text{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: When a kinetic-energy ratio is given, write Einstein's equation at both wavelengths and eliminate the unknown kinetic energy before substituting the options.

44. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is : [4 marks]

(A) 4 : 1

(B) 8 : 1

(C) 2 : 1

(D) 0.8 : 1

Correct Answer: (B) 8 : 1

Solution:

Concept:

- A soap bubble has two liquid surfaces, so its excess pressure is $\Delta P = P_{\text{inside}} - P_{\text{outside}} = 4T/r$, where T is surface tension and r is its radius.
- For bubbles of the same soap solution, T is common and radius is inversely proportional to excess pressure. A spherical bubble has volume $V = 4\pi r^3/3$, so the radius ratio must be cubed to obtain the volume ratio.

Step 1: Calculate the excess pressures Take the atmospheric pressure surrounding the bubbles as $1.00\,\text{atm}$. The given pressures are the total internal pressures, so the pressure differences supported by surface tension are

$$\Delta P_1 = 1.01 - 1.00 = 0.01\,\text{atm},$$

$$\Delta P_2 = 1.02 - 1.00 = 0.02\,\text{atm}.$$

Converting atmospheres into pascals is unnecessary because only their ratio is required.

Step 2: Find the radii and volume ratios Using the same surface tension for both bubbles,

$$\frac{r_1}{r_2} = \frac{4T/\Delta P_1}{4T/\Delta P_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{0.02}{0.01} = 2.$$

Therefore

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = 2^3 = 8.$$

The lower internal excess pressure belongs to the larger bubble, as the inverse-radius law predicts.

Step 3: Distinguish the alternatives The ratio $2 : 1$ is the radius ratio, not the volume ratio. The ratio $4 : 1$ would correspond to an area ratio. A ratio smaller than one, such as $0.8 : 1$, reverses the physical order: the first bubble has the lower excess pressure and therefore the larger volume. Only cubing the radius ratio gives the required volume comparison.

Step 4: Conclusion The volumes of the first and second soap bubbles are in the ratio $8:1$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the surrounding atmospheric pressure first; then use inverse excess pressure for radius and cube the radius ratio for volume.

45. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is [4 marks]

(A) $\frac{3}{4}$
(B) $\frac{4}{5}$
(C) $\frac{3}{5}$
(D) $\frac{7}{8}$

Correct Answer: (A) $\frac{3}{4}$

Solution:

Concept:

- Radioactive decay obeys $N(t) = N_0 e^{-\lambda t}$, where N_0 is the initial number of undecayed nuclei and $\lambda > 0$ is the decay constant.
- The surviving fraction is $f(t) = N(t)/N_0 = e^{-\lambda t}$. Equal intervals retain equal fractions of the material present at the start of each interval; they do not remove equal absolute amounts.

Step 1: Relate the two time intervals Let x be the fraction remaining after $t/2$, measured relative to the original sample. The same decay factor applies during the next interval of length $t/2$. Hence the fraction at time t is $x \times x = x^2$. In exponential form,

$$x = e^{-\lambda t/2}, \qquad x^2 = e^{-\lambda t} = \frac{9}{16}.$$

This avoids needing either the actual time or the half-life.

Step 2: Take the physically valid square root Since a surviving fraction cannot be negative,

$$x = \sqrt{\frac{9}{16}} = \frac{3}{4}.$$

The negative mathematical root is excluded because N/N_0 is non-negative. As a check, multiplying the surviving fractions for the two equal intervals gives

$$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16},$$

exactly the fraction supplied in the question.

Step 3: Check the other proposed fractions The other options would give full-time surviving fractions $(4/5)^2 = 16/25$, $(3/5)^2 = 9/25$ and $(7/8)^2 = 49/64$. None equals $9/16$. Simply halving the amount decayed would be incorrect because radioactive decay is exponential rather than linear in elapsed time.

Step 4: Conclusion The fraction of the original active material remaining at time $t/2$ is $\boxed{3/4}$. Hence, the correct option is (A).

Quick Tip to Solve: For exponential decay, the surviving fraction at half the elapsed time is the positive square root of the surviving fraction at the full time.

Physics – Section B: Numerical Value Questions (20 marks)

46. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _____. [4 marks]

Solution:

Concept:

- When the net external torque about the vertical rotation axis is zero, the total angular momentum of the person and platform is conserved: $I_i\omega_i = I_f\omega_f$. Internal forces can change the individual angular momenta but not their sum.
- A uniform circular platform of mass M and radius R has moment of inertia $I_p = MR^2/2$ about its central axis. Treat the person as a mass m located at distance r, contributing mr^2.

Step 1: Write the initial and final moments of inertia Initially the person stands on the rim, so the distance from the axis is R. The total initial moment of inertia is

$$I_i = \frac{1}{2}(200)R^2 + 80R^2 = 180R^2.$$

At the centre, the person has zero radial distance in the point-mass model, giving

$$I_f = \frac{1}{2}(200)R^2 = 100R^2.$$

The radius itself need not be known because both moments contain the same factor R^2.

Step 2: Apply angular-momentum conservation Let n_f be the final rotation rate in revolutions per minute. Since $\omega = 2\pi n/60$, the conversion factor cancels between the initial and final rates:

$$180R^2(5) = 100R^2 n_f.$$

Therefore

$$n_f = \frac{180}{100} \times 5 = 9\,\text{rpm}.$$

The person and platform continue turning in the same sense; the rate increases as mass moves closer to the axis.

Step 3: Check the physical interpretation The initial inertia is 1.8 times the final inertia, so conservation requires the final speed to be 1.8 times the initial speed. Rotational kinetic energy need not remain constant: the person does work while walking inward. Equating rotational kinetic energies

instead of angular momenta would therefore use the wrong conservation law for this process.

Step 4: Conclusion The final rotational speed of the platform is $\boxed{9\,\text{rpm}}$.

Quick Tip to Solve: For a person moving radially on a freely rotating disc, conserve total angular momentum and include both the disc and the person in the initial inertia.

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47. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is (g = 10 ms^{-2}) ____. [4 marks]

Solution:

Concept:

- After the ball leaves the machine, its launch kinetic energy becomes gravitational potential energy at the highest point: $K_{\text{launch}} = mgh$, neglecting air resistance.
- The work of a constant driving force along its displacement is $W = Fs$. The work-energy theorem equates the net work done during launch to the increase in the ball's kinetic energy.

Step 1: Determine the launch energy from the rise The ball rises through $h = 20\,\text{m}$ after release and comes momentarily to rest at the top. Thus

$$\frac{1}{2}mv^2 = mgh$$

and the kinetic energy supplied at launch is

$$K_{\text{launch}} = (0.15)(10)(20) = 30\,\text{J}.$$

Equivalently, $v^2 = 2gh = 400\,\text{m}^2\text{s}^{-2}$, giving $v = 20\,\text{m}\,\text{s}^{-1}$. The energy method is sufficient without explicitly using this speed.

Step 2: Relate the machine work to the supplied energy The pushing part travels $s = 0.20\,\text{m}$ while applying the constant driving force. The work supplied by the machine is

$$W = Fs = F(0.20).$$

During the stated horizontal driving displacement there is no change of gravitational potential energy, so the machine work supplies the launch kinetic energy. Hence

$$0.20F = 30, \qquad F = \frac{30}{0.20} = 150\,\text{N}.$$

Step 3: Check the dimensions and the force scale The energy divided by distance has units $\text{J}/\text{m} = \text{N}$, as required for a force. The weight $mg = 1.5\,\text{N}$ is much smaller than the pushing force because the machine gives the ball substantial kinetic energy over a short stroke. Adding mg to the result would correspond to an additional vertical rise during the stroke, whereas the given driving displacement is

horizontal.

Step 4: Conclusion The constant force applied by the pushing part is $\boxed{150\,\mathrm{N}}$.

Quick Tip to Solve: Use the free-flight height to find launch energy first, then divide by the driving stroke length; this avoids an unnecessary acceleration calculation.

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48. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with Vm volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{\text{(beaker)}} = 6 \times 10^{-6}\ ^{\circ}\text{C}^{-1}$ and $\gamma_{\text{(mercury)}} = 1.5 \times 10^{-4}\ ^{\circ}\text{C}^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to ____. [4 marks]

Solution:

Concept:

- For a small temperature change ΔT, a volume expands as $V(T) = V_0(1 + \gamma\Delta T)$, where γ is the coefficient of volume expansion and V_0 is measured at the reference temperature.
- The empty volume equals the beaker capacity minus the mercury volume. A constant empty volume requires the absolute increase of beaker capacity to equal the absolute increase of mercury volume.

Step 1: Express the unfilled volume at a new temperature At $30^{\circ}\mathrm{C}$, let the mercury volume be V_m, in cubic centimetres. For a temperature change ΔT from this reference, the unfilled volume is

$$V_{\text{empty}} = 500(1 + \gamma_b\Delta T) - V_m(1 + \gamma_m\Delta T).$$

Separating the constant and temperature-dependent terms gives

$$V_{\text{empty}} = (500 - V_m) + (500\gamma_b - V_m\gamma_m)\Delta T.$$

Step 2: Set the temperature coefficient to zero The empty volume remains unchanged as temperature varies, so its coefficient of ΔT must vanish:

$$500\gamma_b - V_m\gamma_m = 0.$$

Therefore

$$V_m = \frac{500\gamma_b}{\gamma_m} = \frac{500(6 \times 10^{-6})}{1.5 \times 10^{-4}} = 20\,\mathrm{cm}^3.$$

The coefficients have the same inverse-temperature unit, which cancels in the ratio, leaving the required volume unit.

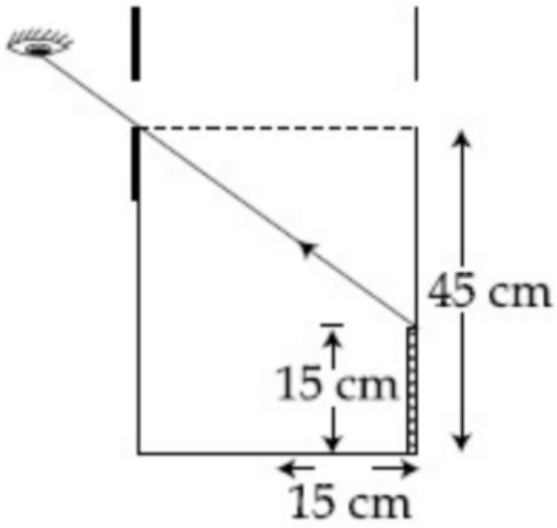
Step 3: Verify the matching volume changes Per degree of temperature increase, the capacity rises by $500(6 \times 10^{-6}) = 0.003\,\mathrm{cm}^3$, and $20\,\mathrm{cm}^3$ of mercury expands by $20(1.5 \times 10^{-4}) = 0.003\,\mathrm{cm}^3$. Their difference is unchanged. Mercury has a much larger expansion coefficient, so only a small fraction of the beaker must be filled to match its absolute increase in capacity.

Step 4: Conclusion The mercury volume at $30^{\circ}\mathrm{C}$ is $\boxed{20\,\mathrm{cc}}$.

Quick Tip to Solve: For constant empty space, equate absolute volume expansions, $V_b\gamma_b = V_m\gamma_m$; equating fractional expansions would be incorrect.

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49. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is N/100, where N is an integer, the value of N is _____. [4 marks]



Solution:

Concept:

- The ray emerging from the liquid into air obeys Snell's law: $\mu \sin i = \sin r$, with both angles measured from the normal to the horizontal liquid surface.
- The observer and the hole fix the emergent direction. The jar radius is $15\,\text{cm}$, so its horizontal width in the cross-section is $30\,\text{cm}$.

Step 1: Locate the surface intersection In the empty jar the straight line from the hole at height $45\,\text{cm}$ reaches the opposite wall at height $15\,\text{cm}$. Its vertical drop and horizontal travel are both $30\,\text{cm}$. Thus its angle with the vertical is $45°$. When the liquid surface is at height $30\,\text{cm}$, that same line meets the surface halfway across the jar, $15\,\text{cm}$ from either wall. The air segment still has $r = 45°$.

Step 2: Find the angle inside the liquid The newly visible bottom edge is at the opposite wall, height zero. From that edge to the surface intersection the ray rises $30\,\text{cm}$ and travels horizontally $15\,\text{cm}$. Therefore

$$\tan i = \frac{15}{30} = \frac{1}{2}, \qquad \sin i = \frac{1}{\sqrt{5}}.$$

The incidence angle is smaller than the emergence angle, as expected for light going from liquid to air.

Step 3: Apply refraction and round the requested integer Using air refractive index unity gives

$$\mu = \frac{\sin 45°}{1/\sqrt{5}} = \sqrt{\frac{5}{2}} = 1.5811\ldots.$$

Since $\mu = N/100$, multiply by 100: $N = 158.11\ldots$. The requested integer is obtained by rounding this result, rather than by rounding the intermediate angles.

Step 4: Conclusion The refractive index is approximately 1.58, and the required dimensionless integer is $\boxed{N = 158}$.

Quick Tip to Solve: Keep the air ray fixed by the observer. Locate its surface intersection first, and measure each angle from the vertical normal, not from the surface.

50. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m–1, is [$\rho_{(\text{liquid})}$ = 900 kgm^{-3}, g = 10 ms^{-2}]
(Give answer in closest integer) _____. [4 marks]

Solution:

Concept:

- For a wetting liquid in a cylindrical tube, the upward surface-tension force is $2\pi r T\cos\theta$, where T is surface tension and r is tube radius.
- The liquid column has weight $\rho\pi r^2 hg$. Balancing the forces gives $h = 2T\cos\theta/(\rho g r)$; the contact angle is essentially zero here.

Step 1: Convert the radius and rise to SI units The radius is given in centimetres, not millimetres. Convert both lengths before substitution:

$$r = 0.015\times10^{-2} = 1.5\times10^{-4}\,\text{m}, \qquad h = 15\times10^{-2} = 0.15\,\text{m}.$$

Use $\rho = 900\,\text{kg}\,\text{m}^{-3}$, $g = 10\,\text{m}\,\text{s}^{-2}$ and $\cos\theta \simeq 1$. The quoted tube dimension is already a radius, so no further division by two is needed.

Step 2: Balance the vertical forces The surface-tension force acts around the complete circular contact line. At equilibrium,

$$2\pi r T\cos\theta = \rho\pi r^2 hg.$$

Cancel πr and solve for the unknown:

$$T = \frac{\rho g r h}{2\cos\theta} = \frac{900(10)(1.5\times10^{-4})(0.15)}{2} = 0.10125\,\text{N}\,\text{m}^{-1}.$$

The positive value is consistent with a wetting liquid that rises above the external surface.

Step 3: Express the result in the requested unit One newton is 10^3 millinewtons. Therefore

$$T = 101.25\,\text{mN}\,\text{m}^{-1}.$$

Rounding to the closest integer gives 101. Rounding only at the final stage avoids losing the quarter millinewton in the intermediate calculation. The capillary-rise relation uses the static column height, so no flow-speed or viscous correction enters this force balance.

Step 4: Conclusion The surface tension is $\boxed{101\,\text{mN}\,\text{m}^{-1}}$ to the closest integer; enter $\boxed{101}$.

Quick Tip to Solve: Use the capillary radius in metres and retain the factor 2 in the circumference force. Convert N/m to mN/m only after calculating the SI result.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The atomic number of the element unnilennium is : [4 marks]

(A) 109

(B) 102

(C) 119

(D) 108

Correct Answer: (A) 109

Solution:

Concept:

- The temporary systematic name encodes the digits of the atomic number, followed by the ending "ium". Read the numerical roots in their original order.
- The roots relevant here are un for 1, nil for 0 and enn for 9. The ending identifies the name as an element name and contributes no extra numerical digit.

Step 1: Separate the numerical roots Break "unnilennium" into its standard naming pieces: un–nil–enn–ium. The first root un represents the hundreds digit 1; nil gives the tens digit 0; enn gives the units digit 9. Combining the roots in sequence gives

$$\text{un} \to 1, \qquad \text{nil} \to 0, \qquad \text{enn} \to 9.$$

The spelling with adjacent n letters follows from joining the roots and the ending; those letters must not be counted as separate digits.

Step 2: Construct the atomic number and check each option The numerical sequence is 109, so

$$Z = 1 \times 100 + 0 \times 10 + 9 = 109.$$

This matches option (A). Option (B), 102, would end in the root bi for the digit 2, not enn. Option (C), 119, would require two successive un roots and would not contain nil. Option (D), 108, requires oct for its last digit. Thus none of these alternatives reproduces all three roots of the printed name.

Step 3: Interpret the result chemically The atomic number counts the protons in the nucleus. Consequently the named element has 109 protons; a neutral atom would also have 109 electrons. It is the encoded atomic number that the question asks for, not an isotope mass number. The systematic roots do not specify a neutron count or an atomic mass, so those quantities cannot change the decoded value.

Step 4: Conclusion The atomic number of unnilennium is $\boxed{109}$. Hence, the correct option is (A).

Quick Tip to Solve: Decode the roots from left to right: un = 1, nil = 0, enn = 9. Ignore the grammatical ending "ium" when reconstructing the number.

52. The mechanism of S_N1 reaction is given as :

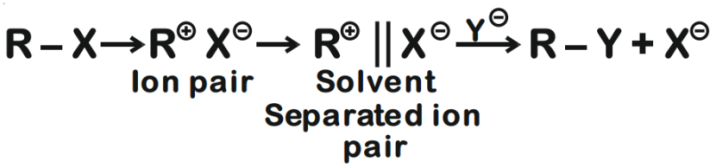
A student writes general characteristics based on the given mechanism as :

(a) The reaction is favoured by weak nucleophiles.

(b) R^+ would be easily formed if the substituents are bulky.

(c) The reaction is accompanied by racemization.

(d) The reaction is favoured by non-polar solvents. [4 marks]



(A) (a) and (b)

(B) (a), (b) and (c)

(C) (a) and (c)

(D) (b) and (d)

Correct Answer: (B) (a), (b) and (c)

Solution:

Concept:

- An S_N1 reaction begins with slow ionization of the carbon–halogen bond. The rate depends on the substrate concentration because carbocation formation precedes nucleophile attack.
- A carbocation is planar at its charged carbon. Polar ionizing solvents stabilize the developing ions, while alkyl substitution can stabilize the carbocation through hyperconjugation and the inductive effect.

Step 1: Assess statements (a) and (b) Statement (a) is accepted: a strong nucleophile is not essential for the slow step, and weak nucleophiles such as water can capture the carbocation. This distinguishes the pathway from a concerted substitution requiring nucleophilic participation in bond breaking. Statement (b) refers to the usual bulky, more highly alkyl-substituted substrates. Their carbocations are stabilized by alkyl groups, and their steric crowding also suppresses backside S_N2 attack. Thus (b) describes the usual favourable substrate trend.

Step 2: Assess stereochemistry and solvent polarity Statement (c) is correct. Once a planar carbocation is formed, the nucleophile can approach either face, giving racemization when the reacting centre is initially chiral. Ion-pair effects can make the two products unequal, so complete 50:50 racemization is not required by this statement. Statement (d) is false: non-polar solvents do not stabilize the separated ionic species effectively. Polar protic or strongly ionizing media favour ionization instead.

Step 3: Compare the offered combinations The accepted statements are (a), (b) and (c). Option (A) omits the stereochemical consequence (c). Option (C) omits the favourable alkyl-substitution trend (b). Option (D) includes the incorrect non-polar-solvent claim (d) and omits other correct features. Only option (B) includes all three accepted characteristics without adding the false solvent statement.

Step 4: Conclusion The weak-nucleophile, substituted-carbocation and racemization statements are correct; the non-polar-solvent statement is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For an SN1 question, check ionization, carbocation stability, planar attack and polar solvent stabilization separately before comparing statement combinations.

53. Henry's constant (in kbar) for four gases α, β, γ and δ in water at 298 K is given below :

	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

(density of water = 10^3 kg m^{-3} at 298 K)

This table implies that : [4 marks]

(A) solubility of γ at 308 K is lower than at 298 K

(B) The pressure of a 55.5 molal solution of δ is 250 bar

(C) α has the highest solubility in water at a given pressure

(D) The pressure of a 55.5 molal solution of γ is 1 bar

Correct Answer: (B) The pressure of a 55.5 molal solution of δ is 250 bar

Solution:

Concept:

- Henry's law in the convention used here is $p = K_H x$, where x is the mole fraction of dissolved gas. At fixed pressure, a smaller K_H means a larger dissolved mole fraction.
- Molality is moles of solute per kilogram of solvent, whereas Henry's law uses mole fraction. Gas dissolution in water generally becomes less favourable as temperature increases.

Step 1: Convert the quoted molality to mole fraction Take $1\,\text{kg}$ water as the solvent basis. It contains approximately $1000/18 = 55.5$ mol water; a 55.5 molal gas solution therefore contains about 55.5 mol gas as well. Hence

$$x = \frac{55.5}{55.5 + 55.5} = \frac{1}{2}.$$

The gas mole fraction is not 55.5, and it is not obtained by dividing the gas moles only by the water moles.

Step 2: Test the pressure and solubility statements For δ, $K_H = 0.5\,\text{kbar} = 500\,\text{bar}$, giving

$$p_\delta = 500 \times \frac{1}{2} = 250\,\text{bar}.$$

Thus (B) is correct. For γ, $K_H = 2 \times 10^{-5}\,\text{kbar} = 0.02\,\text{bar}$, so

$$p_\gamma = 0.02 \times \frac{1}{2} = 0.01\,\text{bar},$$

not 1 bar; (D) is false. The largest constant is that of α, so it is the least soluble at a fixed pressure; (C) is false.

Step 3: Resolve the temperature statement and accepted choices The usual decrease in gas solubility with increasing temperature makes statement (A) acceptable: the solubility of γ at 308 K is lower than at 298 K. The numerical table alone supplies values at 298 K, so this temperature comparison also invokes the standard physical trend, whereas (B) follows directly by calculation. The accepted key includes both statements (A) and (B), and the pressure calculation establishes (B) without needing a temperature extrapolation.

Step 4: Conclusion For δ, the required pressure is $250\,\text{bar}$. Hence, the correct option is (B). The answer key accepts both options (A) and (B).

Quick Tip to Solve: Convert molality to mole fraction before applying p = KH x. For 55.5 mol gas in 1 kg water, gas and solvent have equal mole counts, so x is one half.

54. An organic compound [A], molecular formula $C_{10}H_{20}O_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and an alcohol [C]. Oxidation of [C] with CrO_3 - H_2SO_4 produced [B]. Which of the following strucutres are not possible for [A]? [4 marks]

(A) $(CH_3)_3C-COOCH_2C(CH_3)_3$

(B) $CH_3CH_2CH_2COOCH_2CH_2CH_2CH_3$

(C) $CH_3CH_2CH(CH_3)COOCH_2CH(CH_3)CH_2CH_3$

(D) $CH_3CH_2CH(CH_3)OCOCH_2CH(CH_3)CH_2CH_3$

Correct Answer: (B) $CH_3CH_2CH_2COOCH_2CH_2CH_2CH_3$

Solution:

Concept:

- Acid hydrolysis of an ester $RCOOR'$ gives a carboxylic acid and an alcohol. Oxidation of a primary alcohol RCH_2OH gives $RCOOH$, retaining its carbon skeleton.
- If the alcohol produces the same acid that hydrolysis gives, the ester must have matching acid and primary-alcohol carbon skeletons. A secondary alcohol instead produces a ketone under the stated oxidation.

Step 1: Use the formula and the identity requirement The ten carbons in $C_{10}H_{20}O_2$ must be shared between the two hydrolysis products. Since oxidation of the alcohol must give the same acid, those products have equal carbon counts: each must contain five carbons. For a suitable ester, the alcohol-bearing fragment must also end in $-CH_2O-$, so hydrolysis gives a primary alcohol rather than a secondary one.

Step 2: Check the two possible structures Option (A) hydrolyses to pivalic acid, $(CH_3)_3CCOOH$, and neopentyl alcohol, $(CH_3)_3CCH_2OH$. Oxidation of the latter gives that same acid, and the ester has ten carbons. Option (C) gives 2-methylbutanoic acid and 2-methylbutan-1-ol; oxidation of that primary alcohol also gives 2-methylbutanoic acid. Hence (A) and (C) satisfy both the formula and product-identity conditions.

Step 3: Identify the impossible structures Option (B) is butyl butanoate. Its acid and alcohol each have four carbons, so its ester formula is $C_8H_{16}O_2$, contradicting the specified ten-carbon formula even though oxidation gives the matching acid. In option (D), hydrolysis releases butan-2-ol, a secondary alcohol, together with 3-methylpentanoic acid. Oxidation of butan-2-ol produces butan-2-one, not the acid released by hydrolysis. Thus (D) fails the required oxidation relationship.

Step 4: Conclusion The structures that are not possible are (B) and (D). Hence, the correct option is (B). The answer key accepts both options (B) and (D).

Quick Tip to Solve: For an ester whose alcohol oxidizes to its own acid fragment, compare carbon skeletons across the ester oxygen and verify that the alcohol carbon is primary.

55. The complex that can show optical activity is : [4 marks]

(A) cis-$[Fe(NH_3)_2(CN)_4]^-$

(B) trans-$[Cr(Cl_2)(ox)_2]^{3-}$

(C) trans-$[Fe(NH_3)_2(CN)_4]^-$

(D) cis-$[CrCl_2(ox)_2]^{3-}$ (ox = oxalate)

Correct Answer: (D) cis-$[CrCl_2(ox)_2]^{3-}$ (ox = oxalate)

Solution:

Concept:

- Optical activity requires chirality: a coordination entity and its mirror image must be non-superimposable. A plane or centre of symmetry excludes chirality.
- Oxalate, ox^{2-}, is a bidentate ligand; its two donor atoms occupy adjacent sites of an octahedron. The connectivity of each chelate ring must be retained when comparing mirror images.

Step 1: Examine the complexes containing monodentate ligands Both cis and trans forms of $[Fe(NH_3)_2(CN)_4]^-$ have the octahedral MA_2B_4 arrangement. In the cis form a reflection plane contains the two ammine positions and the metal; in the trans form symmetry is even more apparent. Their mirror images can be superimposed. Thus options (A) and (C) cannot show optical activity.

Step 2: Compare the two bis-oxalato arrangements For $[CrCl_2(ox)_2]^{3-}$, the two oxalate chelate rings impose a definite three-dimensional arrangement. The trans form retains a symmetry element relating its opposite chloride sites and is achiral, eliminating (B). The cis form has the two chloride ligands adjacent and permits two opposite helical arrangements of the chelate rings, conventionally called Δ and Λ. Reflection interchanges these arrangements, and rotation cannot superimpose them while preserving the chelate connections. Thus the cis bis-chelate complex is chiral.

Step 3: Conclusion The optically active complex is cis-$[CrCl_2(ox)_2]^{3-}$, which occurs as a pair of non-superimposable mirror images. Hence, the correct option is (D).

Quick Tip to Solve: For an octahedral $[M(AA)_2B_2]$ complex, test the cis form for optical isomers; do not apply the achirality of MA_2B_4 to chelating ligands.

56. It is true that : [4 marks]

(A) A first order reaction is always a single step reaction

(B) A zero order reaction is a multistep reaction

(C) A zero order reaction is a single step reaction

(D) A second order reaction is always a multistep reaction

Correct Answer: (B) A zero order reaction is a multistep reaction

Solution:

Concept:

- The order of a reaction is obtained from its experimental rate law, whereas molecularity counts the reacting particles in one elementary step.

- For an elementary reaction the rate law follows the participating reactants and its order equals its positive integral molecularity. An overall rate law need not reveal the number of elementary steps.

Step 1: Apply the elementary-step criterion A zero-order law is $r = k[A]^0 = k$: changing the bulk reactant concentration does not change the observed rate in the specified regime. A single elementary event consuming reactant cannot have zero reacting particles. Consequently a zero-order overall process is not an elementary single-step reaction. It commonly arises through a mechanism with adsorption or saturation followed by a slower conversion; the surface or intermediate concentration remains effectively constant over that regime.

Step 2: Test all four statements Option (B) is consistent with zero order being an overall kinetic behaviour produced by a mechanism rather than a zero-molecularity elementary event. Option (C) contradicts this distinction. Option (A) is too strong: first-order kinetics can arise from a unimolecular elementary step, but can also arise from a sequence with a first-order rate-determining step. Option (D) is also too strong: a bimolecular elementary step can directly give second-order kinetics, so a second-order reaction need not always be multistep. Neither a first-order nor a second-order experimental law uniquely establishes a mechanism.

Step 3: Conclusion A zero-order reaction is a multistep overall process in the elementary-reaction classification used here; zero order does not mean zero reacting molecules in an elementary collision. Hence, the correct option is (B).

Quick Tip to Solve: When a statement says "always", distinguish experimental order from elementary-step molecularity before inferring a reaction mechanism.

57. Let C_{NaCl} and C_{BaSO_4} be the conductances (in S) measured for saturated aqueous solutions of NaCl and $BaSO_4$, respectively, at a temperature T. Which of the following is false? [4 marks]

(A) Ionic mobilities of ions from both salts increase with T.

(B) $C_{NaCl}(T_2) > C_{NaCl}(T_1)$ for $T_2 > T_1$

(C) $C_{BaSO_4}(T_2) > C_{BaSO_4}(T_1)$ for $T_2 > T_1$

(D) $C_{NaCl} >> C_{BaSO_4}$ at a given T

Correct Answer: (D) $C_{NaCl} >> C_{BaSO_4}$ at a given T

Solution:

Concept:

- Conductance C, measured in siemens, depends on both the electrolyte and the cell geometry: $C = \kappa A/l$, where κ is conductivity, A is electrode area and l is electrode separation.
- For an aqueous electrolyte, increased temperature generally increases ionic mobility by decreasing solvent viscosity. A temperature comparison for the same cell keeps its geometrical factor unchanged.

Step 1: Assess the temperature statements Ions of both salts move more readily in warmer water, so statement (A) is true. In a fixed cell the saturated sodium chloride solution has a greater conductance at the higher temperature: ionic mobility increases, and its saturation concentration does not decrease

in this range. Thus (B) is true. Saturated barium sulfate remains sparingly soluble, but its ionic mobility and its solubility increase with temperature in the regime considered. Its measured conductance in its fixed cell therefore also rises, making (C) true.

Step 2: Distinguish a comparison of conductances from conductivities For the two measurements the ratio is

$$\frac{C_{\mathrm{NaCl}}}{C_{\mathrm{BaSO_4}}} = \frac{\kappa_{\mathrm{NaCl}}}{\kappa_{\mathrm{BaSO_4}}} \frac{(A/l)_{\mathrm{NaCl}}}{(A/l)_{\mathrm{BaSO_4}}}.$$

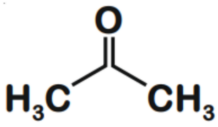
Saturated sodium chloride has a much larger ionic concentration and conductivity than saturated barium sulfate. However, the question specifies conductances in siemens and does not state that the two solutions were measured with the same cell constant. Electrode dimensions can change their measured conductances, so the unconditional inequality in (D) does not follow. It would be justified as the usual conductivity comparison, or for matched cell geometries, but not from the given conductance data alone.

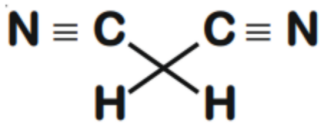
Step 3: Conclusion The false unconditional statement is $C_{\mathrm{NaCl}} \gg C_{\mathrm{BaSO_4}}$ at a given temperature, because conductance also contains the unspecified cell geometry. Hence, the correct option is (D).

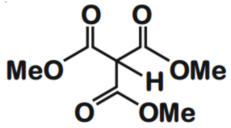
Quick Tip to Solve: Check the unit: siemens denotes conductance, which depends on the cell; conductivity additionally removes the geometrical factor.

58. Which one of the following compounds possesses the most acidic hydrogen? [4 marks]

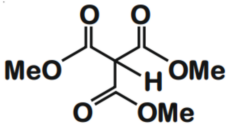
(A) $CH_3-C\equiv C-H$

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- The more stable the conjugate base formed on loss of H^+, the more acidic the original hydrogen. Electron withdrawal and resonance delocalisation stabilise a carbanion.
- An acidic hydrogen between electron-withdrawing groups is activated because the negative charge

can be distributed into neighbouring groups rather than remaining localised on carbon.

Step 1: Compare the first three conjugate bases Option (A), $CH_3C{\equiv}CH$, loses its terminal hydrogen to give an acetylide ion. Its charge is stabilised by the large s character of the carbon orbital, but there is no oxygen-containing resonance system. Option (B), acetone, forms an enolate whose charge is delocalised over one carbonyl group. Option (C), $NC{-}CH_2{-}CN$, forms a conjugate base stabilised by two cyano groups, through their electron-withdrawing effect and delocalisation. It is substantially more acidic than an ordinary terminal alkyne or the alpha hydrogen of acetone.

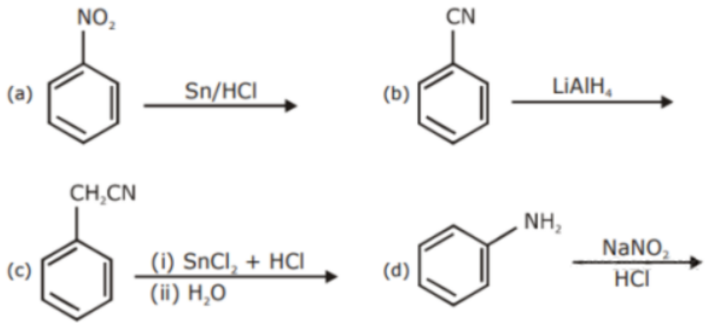
Step 2: Identify the strongest activation in the fourth compound Option (D) is $HC(CO_2CH_3)_3$: the central carbon bears one hydrogen and three methoxycarbonyl groups. Removing that hydrogen gives $[C(CO_2CH_3)_3]^-$. The negative charge can delocalise into each of the three adjacent carbonyl systems, giving contributors with negative charge on oxygen. The combined resonance and inductive stabilisation makes this central hydrogen the most acidic among the choices. Although an ester group is moderated by donation from its alkoxy oxygen, three adjacent ester carbonyls provide stronger overall stabilisation here than the two cyano groups in (C).

Step 3: Conclusion The central hydrogen of $HC(CO_2CH_3)_3$, the compound drawn in (D), is the most acidic because its conjugate base is stabilised by three adjacent carbonyl groups. Hence, the correct option is (D).

Quick Tip to Solve: Draw the conjugate base mentally and count the adjacent electron-withdrawing groups that can actually share its negative charge; do not judge acidity from the original carbon hybridisation alone.

59. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products?

[4 marks]



(A) (a), (c) and (d)

(B) (b) and (c)

(C) (c) and (d)

(D) (a) and (d)

Correct Answer: (C) (c) and (d)

Solution:

Concept:

- Kjeldahl estimation requires the nitrogen in the analysed organic compound to be converted quantitatively to ammonium sulfate during digestion with concentrated sulfuric acid.
- The method is unsuitable for nitrogen in nitro, azo and diazo groups or in certain heterocyclic rings. Determine the reaction products before applying this limitation.

Step 1: Find the products of reductions (a) and (b) In (a), tin and hydrochloric acid reduce nitrobenzene to aniline: $C_6H_5NO_2 \longrightarrow C_6H_5NH_2$. Although the starting nitro compound is unsuitable for ordinary Kjeldahl estimation, its amine product is suitable. In (b), lithium aluminium hydride reduces benzonitrile to benzylamine, $C_6H_5CN \longrightarrow C_6H_5CH_2NH_2$. Its amino nitrogen can also be estimated. Thus neither (a) nor (b) belongs to the requested failing pair.

Step 2: Find the products of (c) and (d) Reaction (c) is Stephen reduction followed by hydrolysis. The nitrile group of $C_6H_5CH_2CN$ becomes an aldehyde group, giving $C_6H_5CH_2CHO$. This organic product contains no nitrogen, so it supplies no organic nitrogen for Kjeldahl estimation; the original nitrile nitrogen has been removed on hydrolysis. In (d), aniline undergoes diazotisation with nitrous acid generated from $NaNO_2/HCl$, forming benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$. Diazonium nitrogen is not converted quantitatively to ammonium sulfate in the Kjeldahl procedure. Therefore the products that do not permit the requested nitrogen estimation are (c) and (d).

Step 3: Eliminate the other combinations Option (A) incorrectly includes the estimable aniline product (a); option (B) incorrectly includes benzylamine (b); and option (D) again includes aniline while omitting the nitrogen-free aldehyde (c). Only the pair in (C) matches both product analyses.

Step 4: Conclusion The products of reactions c and d are respectively a nitrogen-free aldehyde and a diazonium salt, so Kjeldahl nitrogen estimation fails for the stated pair. Hence, the correct option is (C).

Quick Tip to Solve: Apply the Kjeldahl restriction to the reaction product, not to the functional group present in the starting material.

60. Of the species, NO, NO^+, NO^{2+} and NO^- , the one with minimum bond strength is : [4 marks]

(A) NO^-

(B) NO

(C) NO^+

(D) NO^{2+}

Correct Answer: (A) NO^-

Solution:

Concept:

- Molecular orbital bond order is $BO = (N_b - N_a)/2$, where N_b and N_a count electrons in bonding and antibonding molecular orbitals.
- For this closely related series, the smaller bond order corresponds to the weaker bond. Adding an

antibonding electron lowers bond order by one-half; removing one raises it by one-half.

Step 1: Start from neutral nitrogen monoxide Neutral NO has eleven valence electrons. Its occupied bonding orbitals contain eight electrons, while its occupied antibonding orbitals contain three. Thus

$$\mathrm{BO(NO)} = \frac{8-3}{2} = 2.5.$$

Its highest occupied electron is in an antibonding π^* orbital. Removing this electron gives NO^+, with ten valence electrons and bond order 3, analogous in electron count to N_2.

Step 2: Compare electron addition and further removal Adding one electron to NO puts it into the antibonding π^* set. Therefore

$$\mathrm{BO}(NO^-) = \frac{8-4}{2} = 2.$$

Removing a second electron from NO^+ removes a bonding electron, giving

$$\mathrm{BO}(NO^{2+}) = \frac{7-2}{2} = 2.5.$$

Consequently the bond-order sequence is NO^- : 2, NO : 2.5, NO^+ : 3, and NO^{2+} : 2.5. Option (A) alone has the smallest bond order. The two species in (B) and (D) have larger values, and (C) has the largest value, so none of those gives the minimum bond strength.

Step 3: Conclusion NO^- has bond order 2, the minimum in the given series, and hence the weakest bond. Hence, the correct option is (A).

Quick Tip to Solve: On ionising NO, identify whether the removed electron is antibonding or bonding; bond order does not continue increasing after all its highest antibonding electrons have been removed.

61. An acidic buffer is obtained on mixing : [4 marks]

(A) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa

(B) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl

(C) 100 mL of 0.1 M CH_3COOH and 100 mL of 0.1 M NaOH

(D) 100 mL of 0.1 M CH_3COOH and 200 mL of 0.1 M NaOH

Correct Answer: (A) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa

Solution:

Concept:

- An acidic buffer contains a weak acid and an appreciable amount of its conjugate base, usually supplied by its salt. The pair here is CH_3COOH/CH_3COO^-.
- First complete any reaction involving a strong acid or strong base. Use $n = CV$, with volume V in litres; then identify the species left after neutralisation.

Step 1: Calculate the mixture in option A The amounts mixed are $0.100(0.1) = 0.010\,\mathrm{mol}$ of HCl and

$0.200(0.1) = 0.020\,\text{mol}$ of CH_3COONa. Strong acid protonates acetate completely:

$$CH_3COO^- + H^+ \longrightarrow CH_3COOH.$$

Thus $0.010\,\text{mol}$ of acetic acid forms and $0.010\,\text{mol}$ of acetate remains. Both members of the conjugate pair are present, with no excess strong acid. Their equal concentrations give $\text{pH} = pK_a$, an acidic value for acetic acid.

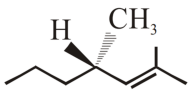
Step 2: Check why the other mixtures do not buffer In B, HCl remains a strong acid, while $NaCl$ supplies no weak-acid conjugate pair. In C, the acid and base amounts are each $0.010\,\text{mol}$, so complete neutralisation leaves sodium acetate without an appreciable amount of acetic acid. In D, $0.020\,\text{mol}$ of hydroxide reacts with only $0.010\,\text{mol}$ of acetic acid, leaving $0.010\,\text{mol}$ of excess strong base. Neither C nor D contains the required acid–salt mixture.

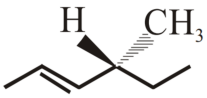
Step 3: Conclusion Option A leaves equal amounts of CH_3COOH and CH_3COO^-, so it produces an acidic buffer. Hence, the correct option is (A).

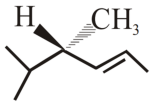
Quick Tip to Solve: Do a mole balance before applying a buffer formula: partial neutralisation gives a buffer, whereas exact neutralisation or excess strong reagent does not.

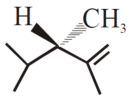
62. Which of the following compounds produces an optically inactive compound on hydrogenation?

[4 marks]

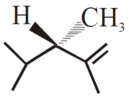
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- Hydrogenation replaces an alkene $C{=}C$ bond by a single bond through addition of one hydrogen atom to each alkene carbon. Compare the complete carbon groups after this change.
- A tetrahedral carbon is stereogenic only when its four substituents are different. If hydrogenation makes two substituents identical and creates no other stereogenic centre, the product is achiral and optically inactive.

Step 1: Follow the two carbon groups in option D The wedge and dashed bonds identify the carbon bearing H and CH_3. On one side it already bears an isopropyl group, $-CH(CH_3)_2$. On the other side the unsaturated substituent is $-C(CH_3){=}CH_2$. Adding hydrogen converts that substituent into $-CH(CH_3)-CH_3$, another isopropyl group. The original centre therefore becomes attached to H, methyl and two identical isopropyl groups. It is no longer stereogenic; each neighbouring methine also bears two identical methyl groups, so there is no new stereogenic centre.

Step 2: Compare the remaining products In A, hydrogenation leaves an n-propyl group on one side of the original centre and an isobutyl group on the other; these remain different. In B, the unsaturated side becomes n-propyl, while the other side is ethyl. In C, one side is isopropyl while the hydrogenated side becomes n-propyl. In each of A, B and C the carbon bearing H and methyl retains four different substituents. Thus those choices retain the stereogenic centre indicated in the question and do not give the achiral product sought.

Step 3: Conclusion Hydrogenation of D makes the two carbon substituents at the indicated centre identical and gives an achiral, optically inactive product. Hence, the correct option is (D).

Quick Tip to Solve: Redraw only the two groups attached to the marked centre after hydrogenation. Loss of the double bond alone does not imply loss of chirality; identical substituents do.

63. The electronic spectrum of $[Ti(H_2O)_6]^{3+}$ shows a single broad peak with a maximum at 20,300 cm^{-1} . The crystal field stabilization energy (CFSE) of the complex ion, in kJ mol^{-1}, is : [4 marks]

(A) 83.7

(B) 242.5

(C) 145.5

(D) 97

Correct Answer: (D) 97

Solution:

Concept:

- In $[Ti(H_2O)_6]^{3+}$, water is neutral, so titanium is $+3$. Ti^{3+} has a $3d^1$ configuration, with its electron in the lower t_{2g} set of an octahedral field.
- The absorption maximum gives the octahedral splitting $\Delta_o = hc\tilde{\nu}$ per ion. For $t_{2g}^1 e_g^0$, the CFSE is $-0.4\Delta_o$; the listed positive answer is its stabilisation magnitude.

Step 1: Convert the spectral splitting to molar energy The wavenumber is $20{,}300\,cm^{-1} = 2.0300 \times$

$10^6\,\mathrm{m}^{-1}$. With $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$, $c = 2.998 \times 10^8\,\mathrm{m\,s}^{-1}$ and $N_A = 6.022 \times 10^{23}\,\mathrm{mol}^{-1}$,

$$\begin{aligned}\Delta_o N_A &= hc\tilde{\nu}N_A \\ &\simeq 2.429 \times 10^5\,\mathrm{J\,mol}^{-1} \\ &\simeq 242.9\,\mathrm{kJ\,mol}^{-1}.\end{aligned}$$

The photon promotes the single electron from the lower to the upper set, so this transition energy is the full splitting, not the CFSE.

Step 2: Apply the stabilisation factor One electron occupies t_{2g}, and none occupies e_g. Hence

$$\mathrm{CFSE} = (1)(-0.4\Delta_o) + (0)(0.6\Delta_o) = -0.4\Delta_o.$$

Its magnitude is $0.4(242.9) \simeq 97.2\,\mathrm{kJ\,mol}^{-1}$, rounded to 97 in the options. B is approximately the entire splitting rather than the stabilisation; C corresponds approximately to $0.6\Delta_o$, the upper-orbital energy. A does not follow the d^1 occupancy.

Step 3: Conclusion The stabilisation energy is about $-97\,\mathrm{kJ\,mol}^{-1}$, with magnitude $97\,\mathrm{kJ\,mol}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For an octahedral d^1 ion, first convert the absorption wavenumber to Δ_o, then multiply by 0.4; do not identify the absorption energy itself as the CFSE.

64. Glycerol is separated in soap industries by : [4 marks]

(A) Fractional distillation

(B) Distillation under reduced pressure

(C) Differential extraction

(D) Steam distillation

Correct Answer: (B) Distillation under reduced pressure

Solution:

Concept:

- A liquid boils when its vapour pressure equals the external pressure. Lowering the external pressure therefore lowers its boiling temperature.
- Vacuum distillation is used for high-boiling liquids that may decompose when strongly heated at ordinary pressure. Glycerol is a standard example of this separation method.

Step 1: Identify the difficulty with ordinary distillation Glycerol is formed during the alkaline hydrolysis of fats in soap manufacture. The soap is separated and glycerol is recovered from the remaining liquid mixture. Glycerol has a very high normal boiling point, approximately $563\,\mathrm{K}$. Heating a glycerol-containing liquid to such a high temperature at atmospheric pressure risks decomposition and makes ordinary high-temperature distillation unsuitable for obtaining it cleanly.

Step 2: Choose the method that lowers the temperature Distillation under reduced pressure allows glycerol to vaporise at a lower temperature. Its vapour is then condensed, separating it from

nonvolatile residues while reducing thermal damage. Fractional distillation primarily separates volatile liquids through repeated vaporisation and condensation; at normal pressure it does not remove the high-temperature problem here. Differential extraction relies on distribution between immiscible solvents rather than the required vacuum recovery. Steam distillation is appropriate for suitably steam-volatile, water-immiscible substances; glycerol is highly water-miscible and is not recovered by that method.

Step 3: Conclusion Glycerol is separated in soap industries by distillation under reduced pressure. Hence, the correct option is (B).

Quick Tip to Solve: When a high-boiling liquid decomposes near its normal boiling temperature, choose reduced pressure: lower external pressure means a lower boiling temperature.

65. Aqua regia is used for dissolving noble metals (Au, Pt, etc.). The gas evolved in this process is : [4 marks]

(A) N_2O_3

(B) N_2

(C) N_2O_5

(D) NO

Correct Answer: (D) NO

Solution:

Concept:

- Aqua regia is a freshly prepared mixture of concentrated HCl and concentrated HNO_3 in a $3:1$ volume ratio. Nitric acid supplies oxidising action, while chloride stabilises dissolved metal complexes.
- During the oxidation of a noble metal, nitrate can be reduced to nitric oxide, NO. Track the oxidation-state change of nitrogen from $+5$ in nitrate to $+2$ in nitric oxide.

Step 1: Write a representative dissolution reaction For gold, the metal is oxidised from zero to $+3$, and chloride binds the resulting gold(III) ion as tetrachloroaurate. A balanced overall equation is

$$Au + HNO_3 + 4\,HCl \longrightarrow HAuCl_4 + NO + 2\,H_2O.$$

Gold loses three electrons, while the nitrogen atom in nitrate gains three electrons on becoming NO. Complex formation helps maintain the dissolved metal in solution and accounts for the effectiveness of the acid mixture.

Step 2: Identify the evolved gas and reject the other choices The gaseous nitrogen-containing product in this dissolution equation is nitric oxide, NO. N_2O_3 and N_2O_5 are different nitrogen oxides and are not the reduction product in the reaction above; N_2 would require a different, much deeper reduction. Fresh nitric oxide can subsequently react with oxygen in air to give brown NO_2, but that secondary oxidation does not change the gas identified by the dissolution reaction or make any of the other listed choices correct.

Step 3: Conclusion The gas evolved in noble-metal dissolution by aqua regia is nitric oxide, NO. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish metal dissolution from subsequent exposure of the gas to air: nitrate reduction gives NO, which can later oxidise to NO_2.

66. If the boiling point of H_2O is 373 K, the boiling point of H_2S will be : [4 marks]

(A) greater than 300 K but less than 373 K

(B) equal to 373 K

(C) more than 373 K

(D) less than 300 K

Correct Answer: (D) less than 300 K

Solution:

Concept:

- Boiling separates molecules by overcoming intermolecular attractions; it does not break the covalent O–H or S–H bonds inside each molecule.
- Water molecules form an extensive hydrogen-bonded network because oxygen is small and strongly electronegative. H_2S lacks comparable intermolecular hydrogen bonding and is held together mainly by much weaker dipolar and dispersion forces.

Step 1: Explain the anomalously high boiling point of water The boiling point $373\,K$ reflects the energy needed to disrupt the strong hydrogen-bond network of liquid water. The large difference from the heavier group 16 hydrides is therefore an intermolecular effect. It is not valid to predict the boiling point simply from molecular mass or from the strength of the covalent bonds in an isolated molecule. The hydrogen-bond contribution makes water exceptional within this series.

Step 2: Place hydrogen sulphide in the correct temperature range Although H_2S is heavier than H_2O, its weaker intermolecular attractions give a much lower boiling temperature, approximately $213\,K$. It is therefore gaseous at ordinary room temperature and its boiling point is less than $300\,K$. Option A still places its boiling point above room temperature and is inconsistent with this value. B would require boiling at the same temperature as the strongly hydrogen-bonded water, while C incorrectly makes it higher. These choices overlook the absence of water-like hydrogen bonding in hydrogen sulphide.

Step 3: Conclusion The boiling point of H_2S is below $300\,K$. Hence, the correct option is (D).

Quick Tip to Solve: For group 16 hydrides, treat water separately because of hydrogen bonding; the usual increase of dispersion forces with mass does not place H_2S above water.

67. Thermal power plants can lead to : [4 marks]

(A) Ozone layer depletion

(B) Blue baby syndrome

(C) Eutrophication

(D) Acid rain

Correct Answer: (D) Acid rain

Solution:

Concept:

- Thermal power stations commonly burn coal or other fossil fuels. Sulfur impurities in the fuel and high-temperature combustion produce acidic oxides, particularly SO_2 and NO_x.
- Acid rain is precipitation made unusually acidic by atmospheric sulfuric and nitric acids. The pollutant must be linked to its source and chemical transformation rather than merely to an environmental effect.

Step 1: Identify the emissions Coal contains sulfur, which burns to sulfur dioxide:

$$S + O_2 \longrightarrow SO_2.$$

Some sulfur dioxide is oxidised further to SO_3. Nitrogen and oxygen can also combine at the high combustion temperature to form nitrogen oxides. These gases leave the station in flue gases and enter the atmosphere.

Step 2: Connect the gases to rainwater Atmospheric oxidation and hydration convert the oxides into acids. Representative reactions are

$$2\,SO_2 + O_2 \longrightarrow 2\,SO_3, \qquad SO_3 + H_2O \longrightarrow H_2SO_4.$$

Nitrogen oxides similarly contribute nitric acid. Dissolution of these acids in cloud droplets lowers the pH of rain, producing acid rain, which damages plants, aquatic habitats and buildings.

Step 3: Eliminate the other effects Ozone depletion is primarily associated with catalytic destruction of stratospheric ozone by radicals from substances such as chlorofluorocarbons. Blue baby syndrome results from nitrate contamination of drinking water. Eutrophication is nutrient enrichment of water, mainly by nitrates and phosphates. These do not describe the direct atmospheric consequence of sulfur and nitrogen oxide emissions from thermal plants.

Step 4: Conclusion Thermal power plants can lead to acid rain through emissions of acidic oxides. Hence, the correct option is (D).

Quick Tip to Solve: For fossil-fuel combustion questions, connect sulfur and nitrogen oxides with acid rain; connect dissolved nitrates and phosphates with water-pollution effects.

68. Tyndall effect is observed when: [4 marks]

(A) The diameter of dispersed particles is similar to the wavelenght of light used.

(B) The diameter of dispersed particles is much smaller than the wavelength of light used.

(C) The diameter of dispersed particles is much larger than the wavelength of light used.

(D) The refractive index of dispersed phase is greater than that of the dispersion medium.

Correct Answer: (A) The diameter of dispersed particles is similar to the wavelenght of light used.

Solution:

Concept:

- The Tyndall effect is scattering of light by colloidal particles, making the path of a beam visible when viewed from the side.
- For an appreciable Tyndall effect, the dispersed particles must have sizes comparable with the wavelength of the light, and there must be sufficient optical contrast between the dispersed phase and the dispersion medium.

Step 1: Apply the particle-size condition Visible light has wavelengths of a few hundred nanometres. Colloidal particles can scatter such light appreciably because their dimensions are of a comparable order. The scattered light enters the observer's eye from the beam path and makes the illuminated cone visible. Thus the size condition stated in option (A) is the appropriate one.

Step 2: Distinguish the other size regimes When particles are much smaller than the wavelength, as molecular or ionic species in a true solution, their scattering is too weak to produce the characteristic visible Tyndall cone under ordinary conditions. This rules out (B). Much larger particles describe a coarse dispersion or suspension and are not the standard colloidal size criterion asked here, so (C) does not give the required condition.

Step 3: Check the refractive-index statement The refractive indices of the two phases should differ sufficiently, but the dispersed phase need not always have the greater refractive index. A difference in either direction can produce optical contrast. Therefore the one-sided inequality in option (D) is not a universal criterion. Particle-size comparability gives the correct statement among the alternatives.

Step 4: Conclusion The dispersed-particle diameter should be comparable with the wavelength of the light used. Hence, the correct option is (A).

Quick Tip to Solve: Remember two requirements for the Tyndall cone: comparable particle size and a refractive-index contrast. Do not replace contrast by a compulsory greater-than relation.

69. In a molecule of pyrophosphoric acid, the number of P - OH, P = O and P - O - P bonds/moiety(ies) respectively are : [4 marks]

(A) 4, 2 and 0

(B) 4, 2 and 1

(C) 3, 3 and 3

(D) 2, 4 and 1

Correct Answer: (B) 4, 2 and 1

Solution:

Concept:

- Pyrophosphoric acid has molecular formula $H_4P_2O_7$. It is formed by condensation of two orthophosphoric acid molecules with loss of one water molecule.
- In its usual oxoacid structure, each phosphorus atom has one terminal $P{=}O$ group, two $P{-}OH$ groups and one bond to the oxygen bridge joining the phosphorus atoms.

Step 1: Construct the condensed connectivity The condensation is

$$2\,H_3PO_4 \longrightarrow H_4P_2O_7 + H_2O.$$

One hydroxyl group from each original acid participates in making the common oxygen bridge. The connectivity can be written in condensed form as $(HO)_2P(=O){-}O{-}P(=O)(OH)_2$. This expression contains two phosphorus centres and retains four acidic hydroxyl hydrogens.

Step 2: Count the three requested features Each phosphorus centre has two hydroxyl groups, so the total number of $P{-}OH$ bonds is $2 \times 2 = 4$. Each centre has one terminal oxo group, giving 2 $P{=}O$ bonds. The single oxygen atom between the two phosphorus atoms constitutes one $P{-}O{-}P$ moiety. Its two individual P–O bonds must not be mistaken for two separate bridging moieties.

Step 3: Compare the alternatives The ordered triple is therefore $(4, 2, 1)$. Option (A) incorrectly removes the oxygen bridge, while (C) assigns too many oxo and bridging groups and too few hydroxyl groups. Option (D) interchanges the hydroxyl and terminal-oxo counts. As a formula check, four hydroxyl oxygens, two oxo oxygens and one bridge give all seven oxygen atoms.

Step 4: Conclusion Pyrophosphoric acid contains four $P{-}OH$ bonds, two $P{=}O$ bonds and one $P{-}O{-}P$ moiety. Hence, the correct option is (B).

Quick Tip to Solve: For condensed phosphorus oxoacids, count bridging oxygen atoms separately from the two P–O bonds that meet at each bridge.

70. The antifertility drug "Novestrol" can react with : [4 marks]

(A) Br_2 /water; $ZnCl_2$ /HCl; $FeCl_3$

(B) Br_2 /water; $ZnCl_2$ /HCl; NaOCl

(C) Alcoholic HCN; NaOCl; $ZnCl_2$ /HCl

(D) $ZnCl_2$ /HCl; $FeCl_3$; Alcoholic HCN

Correct Answer: (A) Br_2 /water; $ZnCl_2$ /HCl; $FeCl_3$

Solution:

Concept:

- Novestrol is an estrogenic antifertility drug with a phenolic hydroxyl group, an alcoholic hydroxyl group at a tertiary carbon and an ethynyl substituent.
- Reagent selection follows functional groups: bromine water reacts readily with an activated phenolic ring; neutral ferric chloride tests phenols; Lucas reagent, $ZnCl_2/HCl$, reacts with suitable alcohols.

Step 1: Test bromine water and ferric chloride The phenolic hydroxyl group activates the aromatic ring towards electrophilic bromination, so Novestrol reacts with Br_2 in water. Phenols also give a characteristic coloured complex with $FeCl_3$. Thus both bromine water and ferric chloride are appropriate reagents for the phenolic part of the molecule. The unsaturated substituent supplies an additional site at which halogen addition is possible.

Step 2: Test Lucas reagent The nonphenolic hydroxyl group is attached to a tertiary carbon in the steroid framework. Lucas reagent supplies concentrated hydrochloric acid and zinc chloride, which facilitates replacement of the alcoholic hydroxyl group by chlorine. Tertiary alcohols respond readily through a stabilised carbocation. This response must be assigned to the alcoholic hydroxyl group, not to displacement of the phenolic hydroxyl group.

Step 3: Reject the incompatible reagent sets Alcoholic hydrogen cyanide is used for cyanohydrin formation at an aldehyde or ketone carbonyl group; Novestrol does not possess such a carbonyl. The standard hypochlorite haloform test requires a methyl ketone or an alcohol oxidisable to one, and the tertiary alcohol here does not provide that group. Consequently the sets containing alcoholic HCN or the haloform reagent do not fit its functional groups. Only set (A) contains all three suitable reagents.

Step 4: Conclusion Novestrol reacts with bromine water, $ZnCl_2/HCl$ and $FeCl_3$. Hence, the correct option is (A).

Quick Tip to Solve: Identify phenolic OH and alcoholic OH separately in steroid drugs: they give different qualitative tests, even though both are written as hydroxyl groups.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The photoelectric current from Na (Work function, w_0 = 2.3 eV) is stopped by the output voltage of the cell

Pt(s) | H_2 (g, 1 Bar) | HCl (aq., pH =1) | AgCl(s) | Ag(s).

The pH of aq. HCl required to stop the photoelectric current form K(w_0 = 2.25 eV), all other conditions remaining the same, is _____ $\times 10^{-2}$ (to the nearest integer).

Given, $2.303\frac{RT}{F}$ = 0.06 V;

$E^0_{AgCl|Ag|Cl^-}$ = 0.22 V [4 marks]

Solution:

Concept:

- The stopping potential satisfies $eV_s = h\nu - w_0$. At fixed incident frequency, lowering the work function by $0.05\,\text{eV}$ increases the required stopping potential by $0.05\,\text{V}$.
- For the cell reaction $H_2 + 2\,AgCl \longrightarrow 2\,H^+ + 2\,Cl^- + 2\,Ag$, the Nernst quotient involves both hydrogen-ion and chloride-ion concentrations. For aqueous HCl, both concentrations are $10^{-\text{pH}}$.

Step 1: Find the dependence of cell voltage on pH At unit hydrogen pressure, the hydrogen electrode reduction potential is $E_{\text{H}} = -0.06\,\text{pH}$. For $AgCl + e^- \longrightarrow Ag + Cl^-$,

$$E_{\text{AgCl}} = 0.22 - 0.06\log[\text{Cl}^-] = 0.22 + 0.06\,\text{pH}.$$

Therefore the right-minus-left cell voltage is

$$E_{\text{cell}} = 0.22 + 0.12\,\text{pH}.$$

The factor 0.12 arises because both electrode potentials vary with the HCl concentration.

Step 2: Determine the common photon energy Initially $\text{pH} = 1$, giving $E_{\text{cell}} = 0.22 + 0.12 = 0.34\,\text{V}$. This stops the electrons from sodium, so their maximum kinetic energy is $0.34\,\text{eV}$. The common photon energy is consequently

$$h\nu = 2.30 + 0.34 = 2.64\,\text{eV}.$$

For potassium, the maximum kinetic energy becomes $2.64 - 2.25 = 0.39\,\text{eV}$, requiring a stopping voltage of $0.39\,\text{V}$.

Step 3: Calculate and scale the new pH Equate the new cell voltage to the required stopping potential:

$$0.22 + 0.12\,\text{pH} = 0.39.$$

Hence

$$\text{pH} = \frac{0.17}{0.12} = 1.41666\ldots = 141.666\ldots \times 10^{-2}.$$

Rounding the coefficient, rather than the pH itself, to the nearest integer gives 142. The higher pH correctly increases the cell voltage to compensate for potassium's smaller work function.

Step 4: Conclusion The required dimensionless pH is $1.41666\ldots$, or approximately 142×10^{-2}. The

numerical entry is $\boxed{142}$.

Quick Tip to Solve: In this cell, changing HCl changes both $[H^+]$ and $[Cl^-]$; include both Nernst contributions before calculating the new pH.

72. The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is _____. (R = 0.0821 L atm K^{-1} mol^{-1}) (rounded off ot the nearest integer) [4 marks]

Solution:

Concept:

- Volume strength is the volume of oxygen, measured at the specified temperature and pressure, liberated by complete decomposition of one unit volume of hydrogen peroxide solution.
- The decomposition is $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. Thus two moles of peroxide liberate one mole of oxygen. Use $V = nRT/P$ with the given gas constant to convert that oxygen amount to its gas volume.

Step 1: Choose one litre of solution A molarity of $8.9\,\text{mol}\,\text{L}^{-1}$ means that one litre of solution contains $8.9\,\text{mol}$ of H_2O_2. The balanced decomposition gives

$$n(O_2) = \frac{8.9}{2} = 4.45\,\text{mol}.$$

The denominator two is essential: only half as many moles of oxygen are obtained as the moles of peroxide consumed. The water produced is not included in the volume-strength gas measurement.

Step 2: Calculate oxygen volume under the stated conditions At $273\,\text{K}$ and $1\,\text{atm}$, substitute the supplied constant directly:

$$V(O_2) = \frac{nRT}{P} = \frac{4.45 \times 0.0821 \times 273}{1}\,\text{L}.$$

Since $0.0821 \times 273 = 22.4133\,\text{L}\,\text{mol}^{-1}$, the oxygen volume is

$$V(O_2) = 99.739185\,\text{L}.$$

The gas volume refers to the specified external conditions, rather than to the original liquid volume.

Step 3: Form and round the volume ratio The chosen one litre of solution liberates 99.739185 litres of oxygen. Therefore its volume strength is

$$\frac{99.739185\,\text{L of oxygen}}{1\,\text{L of solution}} = 99.739185.$$

This is a volume ratio and is conventionally stated as a number of volumes. Rounding to the nearest integer gives 100. No extra multiplication by the concentration is needed, since the concentration was already used to find the peroxide moles.

Step 4: Conclusion The solution is approximately 100-volume hydrogen peroxide. The requested numerical entry is $\boxed{100}$; volume strength is a dimensionless volume ratio.

Quick Tip to Solve: At 273 K and 1 atm, volume strength is approximately $11.2M$, because each mole of peroxide generates half a mole of oxygen.

73. The mole fraction of glucose ($C_6H_{12}O_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is _____. [4 marks]

Solution:

Concept:

- Mole fraction is the ratio of the moles of one component to the total moles: $x_i = n_i/(n_{\text{glucose}} + n_{\text{water}})$. The mole fractions of the two components of a binary solution add to one.
- Mass percentage uses masses, rather than mole numbers: $\%$ water $= 100m_{\text{water}}/(m_{\text{water}} + m_{\text{glucose}})$. Convert each amount to mass with its own molar mass.

Step 1: Choose a convenient mole basis Take exactly $1\,\text{mol}$ of the binary solution. Since the glucose mole fraction is 0.1, this basis contains $0.1\,\text{mol}$ of glucose and $0.9\,\text{mol}$ of water. The absolute amount chosen does not affect a percentage, because scaling both components multiplies both the numerator and denominator by the same factor.

Step 2: Convert the component amounts to masses Using the atomic masses C = 12, H = 1 and O = 16, the molar mass of $C_6H_{12}O_6$ is $6(12) + 12(1) + 6(16) = 180\,\text{g mol}^{-1}$. For H_2O, it is $18\,\text{g mol}^{-1}$. Therefore,

$$m_{\text{glucose}} = 0.1(180) = 18.0\,\text{g}, \qquad m_{\text{water}} = 0.9(18) = 16.2\,\text{g}.$$

The total solution mass is $18.0 + 16.2 = 34.2\,\text{g}$.

Step 3: Calculate and round the water percentage Use water mass in the numerator and the entire solution mass in the denominator:

$$\%\ \text{water} = \frac{16.2}{34.2} \times 100 = 47.368\ldots\%.$$

Although water supplies 90 percent of the molecules on this mole basis, it supplies less than half the mass because each glucose molecule is ten times as heavy as a water molecule.

Step 4: Conclusion The mass percentage of water is approximately 47.37%, so the requested nearest-integer answer is $\boxed{47}$.

Quick Tip to Solve: For a binary mixture, write $100(1-x)M_{\text{water}}/[xM_{\text{glucose}} + (1-x)M_{\text{water}}]$; never read mole percent directly as mass percent.

74. The total number of monohalogenated organic products in the following (including stereoisomers) reaction is _____. [4 marks]

$$\underset{\substack{\text{(simplest optically}\\ \text{active alkene)}}}{\text{A}} \xrightarrow[\text{(ii) } X_2/\Delta]{\text{(i) } H_2/Ni/\Delta}$$

Solution:

Concept:

- The simplest optically active alkene must contain a tetrahedral carbon bonded to four different groups. An alkene carbon itself is trigonal planar and cannot be this asymmetric carbon.
- Catalytic hydrogenation saturates the double bond without changing the carbon skeleton. Subsequent reaction with X_2 under heat substitutes one hydrogen by a halogen; count distinct substitution sites and then the stereoisomers of each product.

Step 1: Identify and hydrogenate the alkene The simplest suitable alkene is 3-methylpent-1-ene, $CH_2{=}CH{-}CH(CH_3){-}CH_2{-}CH_3$. Its third carbon has H, methyl, ethyl and vinyl groups, all different. Hydrogenation gives $CH_3CH_2CH(CH_3)CH_2CH_3$, or 3-methylpentane. This alkane is achiral because its middle carbon now bears two identical ethyl groups; the original configuration therefore does not restrict the product count.

Step 2: Separate the four types of hydrogen sites In 3-methylpentane, the two ethyl arms are equivalent. Thus there are four constitutional substitution types: a terminal methyl of an ethyl arm, a methylene of an ethyl arm, the central methine, and the methyl branch attached directly to the central carbon. Count each pair of equivalent arm positions once, rather than treating the left and right arms as different constitutional products.

Step 3: Count configurations for each substitution type Terminal-arm substitution gives $XCH_2CH_2CH(CH_3)CH_2CH_3$: its middle carbon has four different groups, producing 2 enantiomers. Arm-methylene substitution gives $CH_3CH(X)CH(CH_3)CH_2CH_3$: both the substituted carbon and the middle carbon are stereogenic. There is no internal symmetry to create a meso form, so this type gives $2^2 = 4$ stereoisomers. Substitution of the central methine gives $(CH_3CH_2)_2C(X)CH_3$, which is achiral because its ethyl groups are identical, giving 1 product. Substitution on the methyl branch gives $(CH_3CH_2)_2CHCH_2X$, also achiral and giving 1 product.

Step 4: Conclusion Including stereoisomers, the total number of monohalogenated organic products is $2 + 4 + 1 + 1 = \boxed{8}$. This is a count of distinct products, not their relative yields.

Quick Tip to Solve: First use the symmetry of the hydrogenated alkane to count substitution sites. Then check whether replacing H by X makes an originally non-stereogenic neighbouring carbon stereogenic.

75. An element with molar mass 2.7×10^{-2} kg mol^{-1} forms a cubic unit cell with edge length 405 pm. If its density is 2.7×10^{3} kg m^{-3}, the radius of the element is approximately _____ $\times 10^{-12}$ m (to the nearest integer). [4 marks]

Solution:

Concept:

- For a cubic unit cell, $\rho = ZM/(N_A a^3)$, where Z is the number of atoms per cell, M is molar mass, N_A is the Avogadro constant and a is the cell edge.
- The cubic packing type determines the contact relation between edge length and atomic radius. For a face-centred cubic cell, atoms touch along a face diagonal, so $4r = \sqrt{2}a$.

Step 1: Convert the edge length and compute the cell volume The density and molar mass are already in SI units. Convert picometres to metres before cubing:

$$a = 405 \times 10^{-12}\,\text{m} = 4.05 \times 10^{-10}\,\text{m}.$$

$$a^3 = (4.05 \times 10^{-10})^3 = 6.6430125 \times 10^{-29}\,\text{m}^3.$$

This is the volume of one unit cell, not a molar volume.

Step 2: Find the number of atoms in the cell Rearrange the density formula and use $N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$:

$$Z = \frac{\rho a^3 N_A}{M} = \frac{(2.7 \times 10^3)(6.6430125 \times 10^{-29})(6.022 \times 10^{23})}{2.7 \times 10^{-2}}.$$

$$Z = 4.000\ldots \approx 4.$$

The near-integer value identifies a face-centred cubic cell. A simple cubic cell has $Z = 1$ and a body-centred cubic cell has $Z = 2$, so their edge-to-radius relations would be inappropriate here.

Step 3: Use face-diagonal contact to obtain the radius In an fcc cell the face diagonal runs through a corner atom, a face-centred atom and the next corner atom. Its total contact length is four atomic radii. Hence

$$r = \frac{\sqrt{2}\,a}{4} = \frac{405}{2\sqrt{2}}\,\text{pm} = 143.189\ldots\ \text{pm}.$$

Since $1\,\text{pm} = 10^{-12}\,\text{m}$, the coefficient requested in the question is 143.189 before rounding.

Step 4: Conclusion The atomic radius is approximately $\boxed{143 \times 10^{-12}\,\text{m}}$. The nearest-integer entry for the blank is $\boxed{143}$.

Quick Tip to Solve: Identify Z from density before choosing a radius relation: simple cubic, bcc and fcc cells use different contact diagonals. Keep SI units together when evaluating Z.