JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 3 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 3 September 2020, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If
$\sum_{i=1}^{10}(x_i - p) = 3$ **and** $\sum_{i=1}^{10}(x_i - p)^2 = 9$
where $0 \ne p \in R$, then the standard deviation of these observations is : [4 marks]

(A) $\frac{7}{10}$
(B) $\frac{9}{10}$
(C) $\frac{4}{5}$
(D) $\sqrt{\frac{3}{5}}$

Correct Answer: (B) $\frac{9}{10}$

Solution:

Concept:

- For ten observations, the population variance is $\sigma^2 = \frac{1}{10}\sum_{i=1}^{10}(x_i - \bar{x})^2$, where $\bar{x}$ is their arithmetic mean.
- Subtracting the same constant from every observation changes the mean but leaves all deviations from the mean, and hence the standard deviation, unchanged.

Step 1: Introduce the shifted observations Put $y_i = x_i - p$. The supplied sums are $\sum y_i = 3$ and $\sum y_i^2 = 9$. Therefore $\bar{y} = 3/10$, while the original mean is $\bar{x} = p + 3/10$. In particular, $y_i - \bar{y} = x_i - \bar{x}$; the arbitrary nonzero value of p does not have to be determined.

Step 2: Calculate the variance and take its square root Use the mean of the squares minus the square of the mean, with divisor ten because these are the observations whose standard deviation is required:

$$\sigma^2 = \frac{9}{10} - \left(\frac{3}{10}\right)^2 = \frac{90-9}{100} = \frac{81}{100}.$$

Hence $\sigma = \sqrt{81/100} = 9/10$; the nonnegative square root is required. The values $7/10$, $4/5$, and $\sqrt{3/5}$ have squares $49/100$, $64/100$, and $60/100$, respectively, so none reproduces the calculated variance. Using $9/10$ directly as the variance would also omit the mean correction.

Step 3: Conclusion The standard deviation is $\boxed{9/10}$. Hence, the correct option is (B).

Quick Tip to Solve: When sums are given about an arbitrary origin, use the shifted data and subtract the square of their mean before taking the square root.

2. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is : [4 marks]

(A) 9

(B) 10

(C) 18

(D) 20

Correct Answer: (A) 9

Solution:

Concept:

- A cube of side s has total surface area $A = 6s^2$ and volume $V = s^3$; retaining its shape means that all edges change together.
- The chain rule connects each time derivative to the same side-length rate ds/dt. Area rate and volume rate have different dimensions and cannot be equated.

Step 1: Find the instantaneous rate of the side Differentiate the area with respect to time, keeping the numerical factor for all six faces:

$$\frac{dA}{dt} = 12s\frac{ds}{dt}.$$

At the stated instant $s = 10\,\text{cm}$ and $dA/dt = 3.6\,\text{cm}^2\,\text{s}^{-1}$. Thus

$$\frac{ds}{dt} = \frac{3.6}{12 \times 10} = 0.03\,\text{cm}\,\text{s}^{-1}.$$

The edge is increasing, in agreement with the positive area rate.

Step 2: Differentiate the volume and compare the options Now differentiate $V = s^3$ and insert the instantaneous values:

$$\frac{dV}{dt} = 3s^2\frac{ds}{dt} = 3(10)^2(0.03) = 9\,\text{cm}^3\,\text{s}^{-1}.$$

As a direct check, eliminating ds/dt gives $dV/dt = (s/4)(dA/dt) = (10/4)(3.6) = 9$. The values 10, 18 and 20 do not satisfy this relation: at side 10 they would require area rates 4, 7.2 and 8 in the same area-rate units. The second value, 18, can arise from dropping a factor of two when differentiating the area.

Step 3: Conclusion The volume increases at $\boxed{9\,\text{cm}^3\,\text{s}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a cube retaining its shape, eliminate the edge rate first: the volume rate equals one quarter of the side times the total surface-area rate.

3. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is : [4 marks]

(A) 8

(B) 12

(C) $8\sqrt{5}$

(D) $4\sqrt{5}$

Correct Answer: (A) 8

Solution:

Concept:

- For $y^2 = 4ax$, the latus rectum is the chord through the focus $(a, 0)$ perpendicular to the axis; its endpoints are $(a, \pm 2a)$.
- The perpendicular from a circle centre bisects a chord. For radius r, half-chord h, and distance d from the centre to its midpoint, $d^2 + h^2 = r^2$.

Step 1: Locate the common chord and its midpoint Comparing $y^2 = 4x$ with $y^2 = 4ax$ gives $a = 1$. The latus rectum lies on $x = 1$. Substitution gives $y^2 = 4$, so its endpoints are $P = (1, 2)$ and $Q = (1, -2)$. Their midpoint is $M = (1, 0)$, and the half-chord length is $MP = 2$. Both circle centres must lie on the perpendicular bisector $y = 0$, since each is equally distant from P and Q.

Step 2: Find both possible centres Each radius is $2\sqrt{5}$, so its square is 20. Applying the right-triangle relation gives

$$CM = \sqrt{20 - 2^2} = 4.$$

Consequently the only centres of circles of this radius through the two endpoints are $(1-4, 0) = (-3, 0)$ and $(1 + 4, 0) = (5, 0)$. As the two distinct circles have the given common chord, their centres occupy these opposite positions. Their separation is therefore $4 + 4 = 8$. The alternatives 12, $8\sqrt{5}$, and $4\sqrt{5}$ confuse other lengths with the centre separation; in particular the separation is less than twice the radius, as intersecting circles require.

Step 3: Conclusion The distance between the centres is $\boxed{8}$ units. Hence, the correct option is (A).

Quick Tip to Solve: For equal-radius circles sharing a chord, find the half-chord first; the centre separation is twice the perpendicular distance from either centre to the chord.

4. Let R_1 and R_2 be two relation defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers. Then : [4 marks]

(A) Neither R_1 nor R_2 is transitive.

(B) R_2 is transitive but R_1 is not transitive.

(C) R_1 and R_2 are both transitive.

(D) R_1 is transitive but R_2 is not transitive.

Correct Answer: (A) Neither R_1 nor R_2 is transitive.

Solution:

Concept:

- A relation R is transitive if $(a, b) \in R$ and $(b, c) \in R$ imply $(a, c) \in R$ for every allowed triple of real numbers.
- To disprove transitivity, one explicit triple suffices. Rationality of two sums does not imply rationality of a third sum when individual squared terms may be irrational.

Step 1: Disprove transitivity of the first relation Choose real numbers $a = c = \sqrt{\sqrt{2}}$ and $b = \sqrt{2 - \sqrt{2}}$; these radicals are real because $0 < \sqrt{2} < 2$. Then

$$a^2 + b^2 = b^2 + c^2 = \sqrt{2} + (2 - \sqrt{2}) = 2 \in \mathbb{Q}.$$

Thus both (a, b) and (b, c) belong to R_1. But $a^2 + c^2 = 2\sqrt{2} \notin \mathbb{Q}$, so $(a, c) \notin R_1$. This is a counterexample to transitivity of R_1.

Step 2: Disprove transitivity of the second relation For R_2, use a different triple: $a = c = 0$ and $b = \sqrt{\sqrt{2}}$. Now

$$a^2 + b^2 = b^2 + c^2 = \sqrt{2} \notin \mathbb{Q}, \qquad a^2 + c^2 = 0 \in \mathbb{Q}.$$

The first two ordered pairs are in R_2, whereas the third is not. Hence R_2 also fails transitivity. The examples must be checked in this order: two hypotheses in the relation followed by a conclusion outside it. Options (B) and (D) each assert transitivity of one relation contradicted by an explicit triple, while (C) asserts both and is contradicted twice.

Step 3: Conclusion Neither R_1 nor R_2 is transitive. Hence, the correct option is (A).

Quick Tip to Solve: For rational-versus-irrational conditions, construct counterexamples using squared terms that cancel an irrational contribution in the two hypothesis sums.

5. Let a, b c $\in$ R be such that $a^2 + b^2 + c^2 = 1$. If

$a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$,

where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is : [4 marks]

(A) 0

(B) $\frac{\pi}{9}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Concept:

- For $\mathbf{u} = (a, b, c)$ and $\mathbf{v} = (b, c, a)$, their scalar product is $ab + bc + ca$, and both squared magnitudes equal $a^2 + b^2 + c^2 = 1$.
- The cosine values at three angles spaced by $2\pi/3$ sum to zero: $\cos\theta + \cos(\theta + 2\pi/3) + \cos(\theta + 4\pi/3) = 0$.

Step 1: Use the common value to obtain a relation among coefficients Call the common value in the question k. At $\theta = \pi/9$, the angles are $\pi/9$, $7\pi/9$, and $13\pi/9$; none has zero cosine. If $k = 0$, the three stated equalities would force $a = b = c = 0$, contradicting their squared sum of one. Thus $k \neq 0$, and $a, b, c \neq 0$. Write $\cos\theta = k/a$, $\cos(\theta + 2\pi/3) = k/b$, and $\cos(\theta + 4\pi/3) = k/c$.

Step 2: Add the cosine identities and evaluate the angle The three cosines sum to zero because expanding the two shifted cosines cancels the sine terms and gives a combined $-\cos\theta$. Therefore

$$k\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 0 \quad \Longrightarrow \quad ab + bc + ca = 0.$$

For the required angle $\phi \in [0, \pi]$,

$$\cos\phi = \frac{ab + bc + ca}{\sqrt{a^2 + b^2 + c^2}\sqrt{b^2 + c^2 + a^2}} = 0.$$

Thus the vectors are perpendicular. The other proposed angles have cosines 1, $\cos(\pi/9) > 0$, and $-1/2$, respectively, and cannot give the zero scalar product just established.

Step 3: Conclusion The angle between the vectors is $\boxed{\pi/2}$ radians. Hence, the correct option is (D).

Quick Tip to Solve: When cyclic vector components multiply three equally spaced cosine values to the same constant, add the cosines to obtain the needed pairwise-product sum.

6. $\lim_{x\to a} \frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}$ **($a \neq 0$) is equal to :** [4 marks]

(A) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

(B) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

(C) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(D) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$

Correct Answer: (B) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

Solution:

Concept:

- For a differentiable real cube-root expression at a nonzero argument, $\frac{d}{dx}\sqrt[3]{u(x)} = \frac{u\prime(x)}{3(\sqrt[3]{u(x)})^2}$. This formula is valid for positive and negative arguments.
- If $f(a) = g(a) = 0$, both functions are differentiable at a, and $g\prime(a) \neq 0$, then their quotient tends to $f\prime(a)/g\prime(a)$, by dividing each first-order difference by $x - a$.

Step 1: Identify the vanishing numerator and denominator Let $f(x) = \sqrt[3]{a+2x} - \sqrt[3]{3x}$ and $g(x) = \sqrt[3]{3a+x} - \sqrt[3]{4x}$. At $x = a$, both are zero. Since $a \neq 0$, the arguments $3a$ and $4a$ are nonzero, so all terms are differentiable in a neighbourhood of the limit point. We may therefore compare their first-order changes rather than substitute directly into the indeterminate quotient.

Step 2: Calculate the derivatives and simplify to an option The numerator derivative at the limit point is

$$f\prime(a) = \frac{2-3}{3(\sqrt[3]{3a})^2} = -\frac{1}{3(\sqrt[3]{3a})^2}.$$

Similarly, $g\prime(a) = (1-4)/(3(\sqrt[3]{4a})^2) = -1/(\sqrt[3]{4a})^2 \neq 0$. Thus

$$L = \frac{f\prime(a)}{g\prime(a)} = \frac{1}{3}\left(\frac{4}{3}\right)^{2/3} = \frac{2^{4/3}}{3^{5/3}} = \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}.$$

The nonzero parameter cancels, including when it is negative, since the cube roots are squared. To distinguish the alternatives exactly, cube the positive result: $L^3 = 16/243$. Options (A), (C), and (D) cube to $16/2187$, $16/81$, and $16/6561$, so none agrees.

Step 3: Conclusion The limit is $\boxed{\frac{2}{3}\left(\frac{2}{9}\right)^{1/3}}$. Hence, the correct option is (B).

Quick Tip to Solve: For two cube-root differences that vanish at the same point, compare their first derivatives and keep the inner-function derivative factors before simplifying powers.

7. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is : [4 marks]

(A) (–3, –1)

(B) (2, 4]

(C) (0, 2)

(D) (1, 3]

Correct Answer: (D) (1, 3]

Solution:

Concept:

- For an upward-opening quadratic $F(x) = ax^2 + bx + c$, opposite signs at two endpoints guarantee exactly one root between them. Here $a = \lambda^2 + 1 > 0$, so the parabola always opens upwards.
- The interval $(0, 1)$ is open: a root equal to 0 or 1 must not be counted. Endpoint cases must therefore be checked separately from the strict sign inequality.

Step 1: Apply the endpoint sign test Write the polynomial and evaluate its endpoints:

$$F(x) = (\lambda^2 + 1)x^2 - 4\lambda x + 2, \qquad F(0) = 2.$$

$$F(1) = \lambda^2 - 4\lambda + 3 = (\lambda - 1)(\lambda - 3).$$

Since $F(0) > 0$, the strict condition $F(1) < 0$ gives one root in $(0, 1)$, while the other lies beyond 1. Solving the parameter inequality gives $1 < \lambda < 3$. Two roots cannot lie inside when the endpoint signs are opposite.

Step 2: Check the boundary and exclude other parameters At $\lambda = 1$, the equation is $2(x - 1)^2 = 0$; its only root is excluded. At $\lambda = 3$, it factors as

$$10x^2 - 12x + 2 = 2(5x - 1)(x - 1) = 0.$$

Thus $x = 1/5$ is inside, while $x = 1$ is excluded, so 3 is included. For $\lambda \leq 0$, every term of $F(x)$ is positive for $x > 0$. For $0 < \lambda < 1$, the discriminant $8(\lambda^2 - 1)$ is negative. For $\lambda > 3$, two positive roots exist, their vertex is before 1, and $F(1) > 0$, so both roots lie before 1. These parameters give two roots, not one. Consequently none of the intervals in (A), (B), or (C) describes the required set.

Step 3: Conclusion The required set is $\boxed{(1, 3]}$. Hence, the correct option is (D).

Quick Tip to Solve: Use endpoint signs first, then test each zero of the endpoint expression separately because the root interval is open.

8. Suppose f(x) is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is : [4 marks]

(A) 6

(B) 2

(C) 8

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- A critical point of a polynomial is a zero of its derivative. A quartic has a cubic derivative, so three distinct critical points determine that derivative up to a nonzero constant factor.
- The condition $f(x) = f(0)$ removes the integration constant. The requested set contains distinct real solutions; repeated roots are not counted repeatedly.

Step 1: Construct the derivative and integrate Because the three critical points are $-1, 0, 1$, the derivative must be

$$f\prime(x) = a(x+1)x(x-1) = a(x^3 - x), \qquad a \neq 0.$$

There is no additional factor: the derivative has degree exactly three and already has three distinct roots. Integrating term by term gives

$$f(x) = a\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C.$$

In particular, $f(0) = C$. The sign or magnitude of a can change the shape and vertical scale of the graph, but cannot change the solutions of the required equality.

Step 2: Find the set and add its squared elements Substitute the integrated expression in the given equality and cancel C. Since $a \neq 0$, divide by it:

$$\frac{x^4}{4} - \frac{x^2}{2} = 0 \quad \Longleftrightarrow \quad x^2(x^2 - 2) = 0.$$

Hence

$$T = \{-\sqrt{2},\ 0,\ \sqrt{2}\}.$$

Although zero is a double root of the equation, it is just one member of the set. Therefore the desired sum is

$$(-\sqrt{2})^2 + 0^2 + (\sqrt{2})^2 = 2 + 0 + 2 = 4.$$

Option (B) counts only one nonzero member, while (A) and (C) are not the sum over this set. The original critical points themselves are not the elements being summed; they only determine the derivative.

Step 3: Conclusion The sum of squares of all elements of T is $\boxed{4}$. Hence, the correct option is (D).

Quick Tip to Solve: When all critical points of a polynomial are given, factor the derivative first; subtracting f(0) then eliminates the integration constant.

9. If a $\triangle ABC$ has vertices A(–1, 7), B(–7, 1) and C(5, –5), then its orthocentre has coordinates :

[4 marks]

(A) (–3, 3)

(B) (3, –3)

(C) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

(D) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

Correct Answer: (A) (–3, 3)

Solution:

Concept:

- The orthocentre is the common intersection of the three altitudes of a triangle. Each altitude passes through a vertex and is perpendicular to the opposite side.
- For two nonvertical perpendicular lines with slopes m_1, m_2, the relation is $m_1 m_2 = -1$. Two independent altitude equations are sufficient to find the orthocentre.

Step 1: Write the altitude through C Use the coordinates $A(-1, 7)$, $B(-7, 1)$, and $C(5, -5)$. The slope of side AB is

$$m_{AB} = \frac{1-7}{-7-(-1)} = \frac{-6}{-6} = 1.$$

The altitude from C therefore has slope -1. Its point-slope equation is

$$y-(-5) = -(x-5), \qquad \text{so} \qquad y = -x.$$

This equation also immediately shows that any candidate orthocentre must have coordinates whose sum is zero. All listed points satisfy this first check, so a second altitude is essential.

Step 2: Intersect with the altitude through A and verify The slope of BC is

$$m_{BC} = \frac{-5-1}{5-(-7)} = \frac{-6}{12} = -\frac{1}{2}.$$

Thus the altitude through A has slope 2:

$$y-7 = 2(x+1), \qquad y = 2x+9.$$

Intersecting with $y = -x$ gives $-x = 2x + 9$, hence $x = -3$ and $y = 3$. As a check, side AC has slope -2, and the line from $B(-7, 1)$ to $(-3, 3)$ has slope $2/4 = 1/2$, perpendicular to AC. Options (B), (C), and (D) lie on the first altitude but do not satisfy $y = 2x + 9$; they cannot be the intersection of the altitudes.

Step 3: Conclusion The orthocentre is $\boxed{(-3, 3)}$. Hence, the correct option is (A).

Quick Tip to Solve: Find two altitude equations and use the third altitude only as a quick check against a sign or coordinate error.

10. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18 k is equal to : [4 marks]

(A) 5

(B) 9

(C) 7

(D) 11

Correct Answer: (C) 7

Solution:

Concept:

- The general term of $(u+v)^n$, with r factors of v, is $T_{r+1} = \binom{n}{r}u^{n-r}v^r$, where $r = 0, 1, \ldots, n$.
- A term independent of x has total exponent zero. Both the power of x^2 and the negative power from $1/x$ must be included when forming that exponent.

Step 1: Determine which term is constant Take $u = \frac{3}{2}x^2$, $v = -\frac{1}{3x}$, and $n = 9$. The general term is

$$T_{r+1} = \binom{9}{r}\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r.$$

Its exponent of x is $2(9-r) - r = 18 - 3r$. Therefore the constant term occurs when

$$18 - 3r = 0, \qquad r = 6.$$

This admissible integer lies between 0 and 9, so there is exactly one constant term. It is the seventh term, since the term index is $r+1$, not r.

Step 2: Evaluate its coefficient and the requested multiple Substitute $r = 6$ in the general term. The negative sign has an even exponent, so its contribution is positive:

$$k = \binom{9}{6}\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6.$$

Now $\binom{9}{6} = 84$, so

$$k = \frac{84 \cdot 27}{8 \cdot 729} = \frac{84}{216} = \frac{7}{18}.$$

Multiplying by the factor explicitly requested in the question gives $18k = 7$. This is a useful check that the answer requested is not the constant coefficient itself. Options 5, 9, and 11 do not equal the coefficient multiplied by 18; choosing them would require an incorrect binomial coefficient or exponent balance.

Step 3: Conclusion The constant coefficient is $k = 7/18$, and $\boxed{18k = 7}$. Hence, the correct option is (C).

Quick Tip to Solve: First set the net exponent of x equal to zero; only then substitute its integer index into the binomial coefficient and numerical factors.

11. If the value of the integral

$$\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{\frac{3}{2}}}\,dx$$

is $\frac{k}{6}$, then k is equal to : [4 marks]

(A) $2\sqrt{3}+\pi$

(B) $3\sqrt{2}-\pi$

(C) $3\sqrt{2}+\pi$

(D) $2\sqrt{3}-\pi$

Correct Answer: (D) $2\sqrt{3}-\pi$

Solution:

Concept:

- For a denominator involving $(1-x^2)^{3/2}$, substitute $x=\sin\theta$. On $0 \le x \le 1/2$, the angle lies in $[0, \pi/6]$, where $\cos\theta > 0$.
- The identities $1-\sin^2\theta = \cos^2\theta$ and $\tan^2\theta = \sec^2\theta - 1$ reduce the integrand to elementary trigonometric terms.

Step 1: Transform the integrand and both limits Let the integral be I. Put $x=\sin\theta$, so $dx=\cos\theta\,d\theta$. Because the cosine is positive throughout the interval, $(1-x^2)^{3/2}=\cos^3\theta$, without an absolute-value ambiguity. The lower limit becomes 0 and the upper limit becomes $\pi/6$. Hence

$$I=\int_0^{\pi/6}\frac{\sin^2\theta\cos\theta}{\cos^3\theta}\,d\theta=\int_0^{\pi/6}\tan^2\theta\,d\theta.$$

The transformed integral has the same positive value as the original one, since both integrands are nonnegative on their respective intervals.

Step 2: Integrate and compare with k over six Using $\tan^2\theta=\sec^2\theta-1$, integrate directly:

$$I=[\tan\theta-\theta]_0^{\pi/6}=\frac{1}{\sqrt{3}}-\frac{\pi}{6}=\frac{2\sqrt{3}-\pi}{6}.$$

Comparing this exact expression with $I=k/6$ gives $k=2\sqrt{3}-\pi$. The minus sign follows from integrating the negative constant in $\sec^2\theta-1$; thus options (A) and (C), which add π, have the wrong sign. Option (B) has $3\sqrt{2}$, inconsistent with $\tan(\pi/6)=1/\sqrt{3}$. Numerically k is positive and small, in agreement with integrating a nonnegative function over a short interval.

Step 3: Conclusion Thus $\boxed{k=2\sqrt{3}-\pi}$. Hence, the correct option is (D).

Quick Tip to Solve: After a trigonometric substitution in a definite integral, change the limits immediately and use the positive cosine on this interval.

12. The probability that a randomly chosen 5-digit number is made from exactly two digits is :

[4 marks]

(A) $\frac{150}{10^4}$
(B) $\frac{134}{10^4}$
(C) $\frac{121}{10^4}$
(D) $\frac{135}{10^4}$

Correct Answer: (D) $\frac{135}{10^4}$

Solution:

Concept:

- There are $9 \times 10^4 = 90000$ five-digit positive integers, all equally likely. Their first digit cannot be zero.
- Exactly two distinct digits means that both selected digit values occur at least once. Separate pairs containing zero from pairs containing only nonzero digits, because their leading-digit restrictions differ.

Step 1: Count numbers using two nonzero digits Choose an unordered pair of distinct digits from $1, 2, \ldots, 9$. There are $\binom{9}{2} = 36$ choices. For a fixed pair, each of five positions has two choices, giving $2^5 = 32$ strings. Remove the two constant strings, since they use just one digit. Every remaining string is a valid five-digit number, because neither selected digit is zero. Therefore this case contributes

$$N_1 = \binom{9}{2}(2^5 - 2) = 36 \cdot 30 = 1080.$$

Using an unordered pair avoids counting the same number again with its two digit names exchanged.

Step 2: Count pairs containing zero and form the probability Choose the other digit in 9 ways. The leading digit must be this nonzero digit; each of the remaining four places can be zero or that digit. Remove the single all-nonzero string so that zero actually occurs. Hence

$$N_2 = 9(2^4 - 1) = 9 \cdot 15 = 135.$$

The cases are disjoint, giving $N = N_1 + N_2 = 1215$. Thus

$$P = \frac{1215}{90000} = \frac{135}{10000}.$$

Option (A) gives 1350 favourable numbers, (B) gives 1206, and (C) gives 1089, none equal to the count 1215. The leading-zero restriction is the reason one cannot simply use $\binom{10}{2}(2^5 - 2)$ for all pairs.

Step 3: Conclusion The probability is $\boxed{135/10^4}$. Hence, the correct option is (D).

Quick Tip to Solve: For digit-counting problems, split off the cases involving zero before counting strings; then enforce that every chosen digit occurs.

13. If $x^3dy + xy\,dx = x^2dy + 2y\,dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to : [4 marks]

(A) $\frac{\sqrt{e}}{2}$

(B) $\frac{1}{2} + \sqrt{e}$

(C) $\frac{3}{2} + \sqrt{e}$

(D) $\frac{3}{2}\sqrt{e}$

Correct Answer: (D) $\frac{3}{2}\sqrt{e}$

Solution:

Concept:

- A first-order differential equation is separable when all terms involving y can be put beside dy, leaving a function of x beside dx.
- Partial fractions reduce the rational function to terms with elementary logarithmic and power antiderivatives. The condition $x > 1$ makes x and $x - 1$ positive.

Step 1: Separate the variables Collect the differential terms on opposite sides:

$$x^2(x-1)\,dy = y(2-x)\,dx.$$

Since $y(2) = e > 0$, the nonzero solution through this initial point can be obtained by dividing by y:

$$\frac{dy}{y} = \frac{2-x}{x^2(x-1)}\,dx.$$

The excluded values $x = 0, 1$ do not belong to the given interval.

Step 2: Integrate using partial fractions The required decomposition is

$$\frac{2-x}{x^2(x-1)} = -\frac{1}{x} - \frac{2}{x^2} + \frac{1}{x-1}.$$

Multiplication by $x^2(x-1)$ recovers $2 - x$, so the signs are fixed. Integration gives

$$\ln y = -\ln x + \frac{2}{x} + \ln(x-1) + C.$$

Thus the positive solution has the form

$$y = K\frac{x-1}{x}e^{2/x}.$$

Step 3: Apply the initial condition and evaluate Substituting $x = 2$ gives $e = Ke/2$, hence $K = 2$. Therefore

$$y(4) = 2\frac{3}{4}e^{1/2} = \frac{3}{2}\sqrt{e}.$$

Option A has the wrong multiplicative factor, while B and C add a constant to $\sqrt{e}$; neither has the value obtained from the initial condition.

Step 4: Conclusion $y(4) = \frac{3}{2}\sqrt{e}$. Hence, the correct option is (D).

Quick Tip to Solve: Before integrating, factor the coefficient of dy as $x^2(x-1)$; then use the initial condition to determine the multiplicative constant.

14. If z_1 , z_2 are complex numbers such that $\mathrm{Re}(z_1) = |z_1 - 1|$, $\mathrm{Re}(z_2) = |z_2 - 1|$, and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\mathrm{Im}(z_1 + z_2)$ is equal to : [4 marks]

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{2}{\sqrt{3}}$

(D) $2\sqrt{3}$

Correct Answer: (D) $2\sqrt{3}$

Solution:

Concept:

- Write $z_j = x_j + iy_j$, so $\mathrm{Re}\, z_j = x_j$, $\mathrm{Im}\, z_j = y_j$, and $|z_j - 1| = \sqrt{(x_j - 1)^2 + y_j^2}$.
- The argument of a nonzero complex number gives the slope of its direction: $\tan\arg(z_1 - z_2) = (y_1 - y_2)/(x_1 - x_2)$ when the real part is nonzero.

Step 1: Convert the modulus conditions to coordinates For each $j = 1, 2$, the given equality becomes

$$x_j = \sqrt{(x_j - 1)^2 + y_j^2}.$$

In particular $x_j \geq 0$. Squaring and cancelling x_j^2 gives

$$2x_j = 1 + y_j^2, \qquad x_j = \frac{1 + y_j^2}{2}.$$

This expression is positive and therefore is consistent with the unsquared equation; no negative-real-part branch has been introduced.

Step 2: Use the direction of the difference Subtract the two coordinate identities and factor:

$$x_1 - x_2 = \frac{(y_1 - y_2)(y_1 + y_2)}{2}.$$

The argument $\pi/6$ implies a nonzero difference with positive real and imaginary parts. In particular, the differences used below are nonzero. Hence

$$\frac{1}{\sqrt{3}} = \tan\frac{\pi}{6} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2}{y_1 + y_2}.$$

Step 3: Extract the requested imaginary part It follows that $y_1 + y_2 = 2\sqrt{3}$. Addition of complex numbers adds their imaginary components, so this sum is exactly $\mathrm{Im}(z_1 + z_2)$. The alternatives $\sqrt{3}/2$, $1/\sqrt{3}$, and $2/\sqrt{3}$ do not satisfy $2/(y_1 + y_2) = 1/\sqrt{3}$; they confuse the slope with the coordinate sum.

Step 4: Conclusion $\mathrm{Im}(z_1 + z_2) = 2\sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: After converting the modulus equation to $2x = 1 + y^2$, subtract the two instances. Factoring the difference of squares exposes the required sum immediately.

15. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ ($b<5$) and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2=1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to : [4 marks]

(A) (8, 10)

(B) (8, 12)

(C) $\left(\frac{24}{5}, 10\right)$

(D) $\left(\frac{20}{3}, 12\right)$

Correct Answer: (A) (8, 10)

Solution:

Concept:

- For an ellipse with major semiaxis a along the x-axis, $c^2=a^2-b^2$, eccentricity $e=c/a$, and the distance between its foci is $2c$.
- For a hyperbola $x^2/a^2-y^2/b^2=1$, $c^2=a^2+b^2$ and $e=c/a$. Both semiaxes are nonzero for the conics in the question.

Step 1: Express both eccentricities The ellipse has major semiaxis 5, since $b<5$, whereas the hyperbola has transverse semiaxis 4. Consequently

$$e_1^2=1-\frac{b^2}{25}, \qquad e_2^2=1+\frac{b^2}{16}.$$

Their eccentricities are positive. Therefore squaring the given product $e_1e_2=1$ preserves its meaning.

Step 2: Determine the common parameter Let $t=b^2$. Substitution gives

$$\left(1-\frac{t}{25}\right)\left(1+\frac{t}{16}\right)=1.$$

Expanding and collecting the terms yields

$$1+\frac{9t-t^2}{400}=1, \qquad t(9-t)=0.$$

The value $t=0$ would make the denominators involving b^2 vanish and does not define the stated conics. Thus $b^2=9$, which also satisfies $b^2<25$.

Step 3: Calculate the two focal separations For the ellipse,

$$c_1=\sqrt{25-9}=4, \qquad \alpha=2c_1=8.$$

For the hyperbola,

$$c_2=\sqrt{16+9}=5, \qquad \beta=2c_2=10.$$

As a check, $e_1=4/5$ and $e_2=5/4$, whose product is one. Options B and D give an incorrect hyperbola

separation of 12, and option C gives an incorrect ellipse separation despite its correct second entry.

Step 4: Conclusion The ordered pair is $(\alpha, \beta) = (8, 10)$. Hence, the correct option is (A).

Quick Tip to Solve: Keep focal distance c distinct from the distance $2c$ between the two foci. Use a minus sign for the ellipse and a plus sign for the hyperbola.

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16. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx$ = A(x)$\tan^{-1}(\sqrt{x})$ + B(x) + C,
where C is a constant of integration, then the ordered pair (A(x), B(x)) can be : [4 marks]

(A) (x + 1, -$\sqrt{x}$)
(B) (x + 1, $\sqrt{x}$)
(C) (x - 1, -$\sqrt{x}$)
(D) (x - 1, $\sqrt{x}$)

Correct Answer: (A) (x + 1, -$\sqrt{x}$)

Solution:

Concept:

- For $x \geq 0$, if $\theta = \tan^{-1}\sqrt{x}$, then $0 \leq \theta < \pi/2$ and $\sin\theta = \sqrt{x/(1+x)}$. Principal inverse trigonometric values therefore agree.
- Integration by parts uses $\int u\,dv = uv - \int v\,du$. Here differentiating the inverse tangent produces a rational expression after the substitution $t = \sqrt{x}$.

Step 1: Simplify the inverse sine The square root in the integrand is real and lies in the domain of principal inverse sine for $x \geq 0$. On this domain,

$$\sin^{-1}\sqrt{\frac{x}{1+x}} = \tan^{-1}\sqrt{x}.$$

Thus it is enough to integrate $\tan^{-1}\sqrt{x}$. This replacement uses the first-quadrant angle and introduces no extra multiple of π.

Step 2: Integrate by parts and substitute For $x > 0$, choose $u = \tan^{-1}\sqrt{x}$, $dv = dx$. Since

$$du = \frac{dx}{2\sqrt{x}(1+x)},$$

we obtain

$$I = x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx.$$

Set $t = \sqrt{x}$, so $dx = 2t\,dt$. Then

$$\frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx = \int \frac{t^2}{1+t^2}\,dt = t - \tan^{-1}t.$$

Step 3: Identify and check the coefficient functions Combining the terms gives

$$I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C.$$

Indeed, differentiation gives

$$I' = \tan^{-1}\sqrt{x} + \frac{x+1}{2\sqrt{x}(1+x)} - \frac{1}{2\sqrt{x}} = \tan^{-1}\sqrt{x}.$$

The positive square-root choice in B has the wrong sign; C and D also have the wrong coefficient of the inverse tangent. The formula extends continuously to $x = 0$.

Step 4: Conclusion One possible pair is $(A(x), B(x)) = (x+1, -\sqrt{x})$. Hence, the correct option is (A).

Quick Tip to Solve: Replace the inverse sine by $\tan^{-1}\sqrt{x}$ first. After integration by parts, $t = \sqrt{x}$ turns the remaining integral into $\int (1 - 1/(1+t^2))\,dt$.

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17. Let A be a 3 × 3 matrix such that

adj A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ **and B = adj(adj A).**

If |A| = λ **and** $|(B^{-1})^T| = \mu$ **, then the ordered pair,**

($|\lambda|$**,** μ**) is equal to :** [4 marks]

(A) (3, 81)

(B) $\left(9, \frac{1}{9}\right)$

(C) $\left(3, \frac{1}{81}\right)$

(D) $\left(9, \frac{1}{81}\right)$

Correct Answer: (C) $\left(3, \frac{1}{81}\right)$

Solution:

Concept:

- For a 3×3 matrix M, $\det(\text{adj}\, M) = (\det M)^2$. This determinant identity avoids finding the original matrix explicitly.
- For an invertible matrix B, $\det(B^{-1}) = 1/\det B$, while transposition leaves a determinant unchanged. The absolute value of $\det A$ is enough here.

Step 1: Evaluate the determinant of the given adjugate Write $D = \text{adj}\, A$. Expanding along its first row, with alternating cofactor signs, gives

$$\begin{aligned} \det D &= 2\{0(-1) - 2(-2)\} \\ &\quad - (-1)\{(-1)(-1) - 2(1)\} \\ &\quad + 1\{(-1)(-2) - 0(1)\} \\ &= 8 - 1 + 2 = 9. \end{aligned}$$

The nonzero value also establishes that the matrices whose inverses are used below are invertible.

Step 2: Obtain the magnitude of the original determinant The matrix A has order three, so

$$\det(\text{adj } A) = (\det A)^2 = \lambda^2.$$

Hence $\lambda^2 = 9$ and $|\lambda| = 3$. The data need not distinguish $\lambda = 3$ from $\lambda = -3$, because the question explicitly asks for its magnitude. In particular, using 9 as $|\lambda|$ would omit the required square root.

Step 3: Apply the adjugate identity once more Since $B = \text{adj } D$, another application of the order-three identity gives

$$\det B = (\det D)^2 = 9^2 = 81.$$

Therefore

$$\mu = \det\left((B^{-1})^T\right) = \det(B^{-1}) = \frac{1}{81}.$$

Option A misses the reciprocal caused by inversion. Options B and D use an incorrect first coordinate of 9, and B also uses an incorrect second coordinate.

Step 4: Conclusion $(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: For a matrix of order three, taking an adjugate squares the determinant. Apply that twice, then take the reciprocal for the inverse; transposition changes nothing.

18. The plane which bisects the line joining, the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point : [4 marks]

(A) (4, 0, 1)

(B) (0, –1, 1)

(C) (0, 1, –1)

(D) (4, 0, –1)

Correct Answer: (D) (4, 0, –1)

Solution:

Concept:

- The plane bisecting a segment at right angles passes through its midpoint, and the segment direction is a normal vector to that plane.
- A plane through (x_0, y_0, z_0) with normal (a, b, c) has equation $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$. A point lies on it precisely when it satisfies this equation.

Step 1: Find the midpoint and a normal vector Let the endpoints be $P = (4, -2, 3)$ and $Q = (2, 4, -1)$. Their midpoint is

$$M = \left(\frac{4+2}{2}, \frac{-2+4}{2}, \frac{3-1}{2}\right) = (3, 1, 1).$$

A direction vector of the line is

$$\overrightarrow{PQ} = (-2, 6, -4) = -2(1, -3, 2).$$

The required plane is perpendicular to this line, so $(1, -3, 2)$ is a valid normal. Multiplying a normal vector by a nonzero scalar gives the same plane.

Step 2: Write the perpendicular-bisector plane Using M and the normal vector, the point-normal equation becomes

$$(x - 3) - 3(y - 1) + 2(z - 1) = 0.$$

Collecting its constant terms gives

$$x - 3y + 2z - 2 = 0.$$

The midpoint satisfies it, as required. Also the normal is parallel to PQ, confirming the right-angle condition in addition to the midpoint condition.

Step 3: Test every listed point Evaluate the left side $L = x - 3y + 2z - 2$ at each choice:

$$\begin{aligned} L(4, 0, 1) &= 4 + 2 - 2 = 4, \\ L(0, -1, 1) &= 3 + 2 - 2 = 3, \\ L(0, 1, -1) &= -3 - 2 - 2 = -7, \\ L(4, 0, -1) &= 4 - 2 - 2 = 0. \end{aligned}$$

Only the final point produces zero. The first three choices therefore fail the plane equation, rather than merely looking different from the midpoint.

Step 4: Conclusion The plane passes through $(4, 0, -1)$. Hence, the correct option is (D).

Quick Tip to Solve: For a perpendicular-bisector plane, use the segment vector directly as the normal and its midpoint as the point. Test the options only after simplifying the constant term.

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19. If the sum of the series

20 + $19\frac{3}{5}$ + $19\frac{1}{5}$ + $18\frac{4}{5}$ + ...

upto nth term is 488 and the n^{th} term is negative, then : [4 marks]

(A) n = 41

(B) n = 60

(C) n^{th} term is –4

(D) n^{th} term is $-4\frac{2}{5}$

Correct Answer: (C) n^{th} term is –4

Solution:

Concept:

- An arithmetic progression has constant common difference d. Its term and sum formulae are $a_n = a + (n - 1)d$ and $S_n = \frac{n}{2}[2a + (n - 1)d]$, where a is the first term.

- The sum can correspond to two positive lengths when later terms are negative. The additional condition on the last term selects the required length; a root of the sum equation alone is not sufficient.

Step 1: Identify the common difference The first three terms are $20,\ 19\frac{3}{5},\ 19\frac{1}{5}$. Each successive term decreases by $\frac{2}{5}$, so $a = 20$ and $d = -\frac{2}{5}$. Therefore the final term is

$$a_n = 20 - \frac{2}{5}(n-1) = \frac{102 - 2n}{5}.$$

Step 2: Solve the sum equation Substitute the given sum into the arithmetic-series formula:

$$488 = \frac{n}{2}\left[40 - \frac{2}{5}(n-1)\right] = \frac{n(101-n)}{5}.$$

Thus

$$n^2 - 101n + 2440 = 0, \qquad (n-40)(n-61) = 0.$$

The candidate lengths are 40 and 61, both positive integers.

Step 3: Apply the sign condition and check the options For $n = 40$, the last term is $\frac{102-80}{5} = \frac{22}{5} > 0$, so this root violates the stated condition. For $n = 61$,

$$a_{61} = \frac{102 - 122}{5} = -4 < 0.$$

As a direct check, $S_{61} = \frac{61}{2}(20 - 4) = 488$. Options (A) and (B) give neither valid length: the admissible length is 61, not 41 or 60. Option (D) would require a last term of $-\frac{22}{5}$, which is not the computed value.

Step 4: Conclusion The negative final term occurs for $n = 61$, and its value is $\boxed{-4}$. Hence, the correct option is (C).

Quick Tip to Solve: When a decreasing arithmetic series gives two lengths with the same sum, evaluate the last term for both roots before selecting the answer.

20. Let p, q, r be three statements such that the truth value of
$(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively : [4 marks]

(A) T, F, T

(B) F, T, F

(C) T, T, T

(D) T, T, F

Correct Answer: (D) T, T, F

Solution:

Concept:

- An implication $A \rightarrow B$ is false exactly when A is true and B is false. It is true for every other pair of truth values.
- A conjunction is true only if both statements are true. A disjunction is false only if both statements are false. Negation reverses the truth value of its statement.

Step 1: Force the antecedent to be true Let $A = p \wedge q$ and $B = \sim q \vee r$. The question specifies that $A \rightarrow B$ is false, so necessarily $A = \mathrm{T}$ and $B = \mathrm{F}$. From the first condition,

$$p \wedge q = \mathrm{T} \quad \Longrightarrow \quad p = \mathrm{T}, \quad q = \mathrm{T}.$$

A false antecedent could never make the implication false; therefore both p and q are fixed, rather than merely one of them.

Step 2: Force the consequent to be false The remaining condition is

$$\sim q \vee r = \mathrm{F}.$$

Each member of this disjunction must be false. Hence $\sim q = \mathrm{F}$, which agrees with $q = \mathrm{T}$, and $r = \mathrm{F}$. We obtain the ordered triple

$$(p, q, r) = (\mathrm{T}, \mathrm{T}, \mathrm{F}).$$

Substitution gives $(\mathrm{T} \wedge \mathrm{T}) \rightarrow (\mathrm{F} \vee \mathrm{F}) = \mathrm{T} \rightarrow \mathrm{F} = \mathrm{F}$, as required.

Step 3: Reject the other listed triples In option (A), $q = \mathrm{F}$, so the antecedent is false and the implication is true. In option (B), $p = \mathrm{F}$, again making the antecedent false and the implication true. In option (C), the antecedent is true but $r = \mathrm{T}$ makes the consequent true, so the implication is also true. Only option (D) has a true antecedent and a false consequent.

Step 4: Conclusion The truth values of p, q, r, respectively, are $\boxed{\mathrm{T}, \mathrm{T}, \mathrm{F}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a false implication, immediately write antecedent true and consequent false; then unpack conjunctions and disjunctions separately.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then, the number of elements in the set S is equal to _____ .

[4 marks]

Solution:

Concept:

- Elimination reduces a homogeneous linear system to its independent constraints. Every solution must satisfy all original equations, including any dependent equation.
- After parameterising integer solutions, the bound on $x^2 + y^2 + z^2$ becomes an inequality for the integer parameter. Positive and negative parameter values give distinct ordered triples.

Step 1: Eliminate the first two equations Write the first two equations as $x - 2y + 5z = 0$ and $-2x + 4y + z = 0$. Add twice the first equation to the second:

$$2(x - 2y + 5z) + (-2x + 4y + z) = 0,$$

so $11z = 0$, hence $z = 0$. Substitution into the first equation then gives $x = 2y$. These deductions are necessary for every solution.

Step 2: Parameterise and verify the full system Because y is an integer, set $y = t$, where $t \in \mathbb{Z}$. Every possible triple has the form

$$(x, y, z) = (2t, t, 0).$$

The third equation is satisfied identically:

$$-7(2t) + 14t + 9(0) = 0.$$

The first two equations also hold for each such triple. Thus this parameterisation is both necessary and sufficient, and it does not omit or introduce any integer solutions.

Step 3: Count the admissible parameter values The squared-length condition becomes

$$15 \leq (2t)^2 + t^2 + 0^2 \leq 150 \quad \Longleftrightarrow \quad 3 \leq t^2 \leq 30.$$

Since t is integral, its absolute value can be $2, 3, 4, 5$. The values $0, \pm 1$ fail the lower bound, while $|t| \geq 6$ fails the upper bound because $36 > 30$. There are two signs for each of four allowed absolute values. Also, different values of t produce different second coordinates, so there is no double counting.

Step 4: Conclusion The parameter values are $t = \pm 2, \pm 3, \pm 4, \pm 5$. Therefore the number of elements of S is $\boxed{8}$.

Quick Tip to Solve: For homogeneous equations with integer solutions, parameterise first and count parameter values only after applying both bounds on the squared length.

22. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to _____ . [4 marks]

Solution:

Concept:

- Inserting m arithmetic means between two endpoints creates $m+2$ terms and $m+1$ equal gaps. If the endpoints are a, b, the common difference is $(b-a)/(m+1)$.
- Inserting three geometric means creates a five-term geometric progression. The second inserted mean is the middle term, whose square is the product of the two endpoints.

Step 1: Determine the second geometric mean The five geometric terms can be written as

$$3,\ 3r,\ 3r^2,\ 3r^3,\ 3r^4 = 243.$$

Therefore $r^4 = 81$. For a real geometric progression, $r = 3$ or $r = -3$; either value gives $r^2 = 9$. Consequently the second inserted geometric mean is

$$G_2 = 3r^2 = 27.$$

Equivalently, the central term satisfies $G_2^2 = 3 \cdot 243 = 729$, and the representation $3r^2$ fixes its positive sign.

Step 2: Express the fourth arithmetic mean The arithmetic progression begins with 3, followed by the inserted means. Its common difference is

$$d = \frac{243-3}{m+1} = \frac{240}{m+1}.$$

The first inserted mean is $3+d$, so the fourth inserted mean is $3+4d$, not $3+3d$. Using the given equality to the second geometric mean gives

$$3 + 4\left(\frac{240}{m+1}\right) = 27.$$

Step 3: Solve and verify the number of means Rearranging,

$$\frac{960}{m+1} = 24, \qquad m+1 = 40, \qquad m = 39.$$

For verification, 39 inserted means give 40 gaps, each of size $240/40 = 6$. The fourth mean is then $3 + 4(6) = 27$, exactly the required geometric mean. The final endpoint is also recovered: $3 + 40(6) = 243$. Thus the count is consistent with both endpoint and matching-mean conditions.

Step 4: Conclusion The required number of inserted arithmetic means is $\boxed{39}$.

Quick Tip to Solve: Count gaps rather than terms: inserting m means makes m+1 gaps, and the kth inserted arithmetic mean lies k gaps beyond the first endpoint.

23. The total number of 3-digit numbers, whose sum of digits is 10, is _____. [4 marks]

Solution:

Concept:

- A three-digit number has hundreds digit $a \in \{1, \ldots, 9\}$ and tens and units digits $b, c \in \{0, \ldots, 9\}$. The leading digit cannot be zero.
- The number of nonnegative integer solutions of $u + v + w = N$ is $\binom{N+2}{2}$, by the stars-and-bars method. Digit upper bounds must still be checked after counting unrestricted solutions.

Step 1: Remove the lower bound on the hundreds digit The condition on the digits is

$$a + b + c = 10, \qquad 1 \le a \le 9, \quad 0 \le b, c \le 9.$$

Set $u = a - 1$. Then $u \ge 0$, and the equation becomes

$$u + b + c = 9.$$

The upper bound for u is 8, because the hundreds digit must not exceed 9. The bounds for b and c remain 9.

Step 2: Count unrestricted triples and remove the excess Initially ignore the upper bound on u. The number of nonnegative solutions is

$$\binom{9+3-1}{3-1} = \binom{11}{2} = 55.$$

Since the total is only 9, neither b nor c can exceed 9. The only possible violation is $u = 9$, which forces $b = c = 0$. This would mean $a = 10$, not a digit. There is exactly one such invalid triple, so the required number is $55 - 1 = 54$.

Step 3: Verify the count by fixing the hundreds digit For $a = 1$, the remaining sum is $b + c = 9$, giving ten ordered pairs from $(0, 9)$ through $(9, 0)$. For $a = 2$, there are nine pairs; continuing to $a = 9$, there are two pairs with $b + c = 1$. Thus an independent count gives

$$10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 = 54.$$

Each ordered digit triple determines one three-digit number, so no division for permutations or repeated digits is needed.

Step 4: Conclusion The total number of three-digit numbers with digit sum 10 is $\boxed{54}$.

Quick Tip to Solve: Shift the leading digit down by one before using stars and bars, then exclude solutions that would make that shifted digit exceed eight.

24. Let a plane P contain two lines

$\vec{r} = \hat{i} + \lambda\left(\hat{i} + \hat{j}\right)$, $\lambda \in R$ **and**

$\vec{r} = -\hat{j} + \mu\left(\hat{j} - \hat{k}\right)$, $\mu \in R$

If Q(α, β, γ) is the foot of the perpendicular drawn from the point M(1, 0, 1) to P, then 3(α + β + γ) equals _____. [4 marks]

Solution:

Concept:

- A normal vector to a plane containing two nonparallel line directions is their cross product. A point $\mathbf{r}$ lies in the plane through $\mathbf{a}$ with normal $\mathbf{n}$ when $\mathbf{n} \cdot (\mathbf{r} - \mathbf{a}) = 0$.
- The perpendicular from a point to a plane follows the normal direction. Its intersection with the plane is the foot of the perpendicular; substituting a normal-line parameter gives its coordinates.

Step 1: Find a normal and the equation of the plane The line directions are $\mathbf{u} = (1, 1, 0)$ and $\mathbf{v} = (0, 1, -1)$. Their cross product is $(-1, 1, 1)$, so an equivalent normal is $\mathbf{n} = (1, -1, -1)$. The first line contains $A = (1, 0, 0)$. Therefore the plane has equation

$$(x - 1) - y - z = 0, \qquad x - y - z = 1.$$

To check the second line, substitute $(x, y, z) = (0, -1 + \mu, -\mu)$; the left side is 1 for every μ. Thus the equation contains both given lines.

Step 2: Intersect the normal through M with the plane The perpendicular line through $M = (1, 0, 1)$ is

$$(x, y, z) = (1, 0, 1) + t(1, -1, -1) = (1 + t, -t, 1 - t).$$

Insert this point into the plane equation:

$$(1 + t) - (-t) - (1 - t) = 1 \quad \Longrightarrow \quad 3t = 1.$$

Hence $t = \frac{1}{3}$, and the foot is

$$Q = (\alpha, \beta, \gamma) = \left(\frac{4}{3}, -\frac{1}{3}, \frac{2}{3}\right).$$

Step 3: Evaluate the requested combination and verify The coordinate sum is

$$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}.$$

It follows that $3(\alpha + \beta + \gamma) = 5$. As checks, the coordinates satisfy $\frac{4}{3} - (-\frac{1}{3}) - \frac{2}{3} = 1$, and the displacement $Q - M = (\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$ is parallel to the normal. Both the plane-membership and perpendicularity requirements therefore hold.

Step 4: Conclusion The required value is $3(\alpha + \beta + \gamma) = \boxed{5}$.

Quick Tip to Solve: Find the plane normal from the two line directions, then move from the given point along that normal until the plane equation is satisfied.

25. If the tangent to the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x-axis, then the value of c is _____ . [4 marks]

Solution:

Concept:

- The tangent slope to a differentiable curve is dy/dx. The slope of its normal is the negative reciprocal of the tangent slope when the latter is nonzero.
- The point where a line meets the x-axis has ordinate zero. Find this intercept for the normal and the tangent separately, then equate their abscissae.

Step 1: Find the normal to the parabola Differentiate the parabola implicitly, retaining the factor arising from y^2:

$$2y\frac{dy}{dx} = 4, \qquad \frac{dy}{dx} = \frac{2}{y}.$$

At $(1, 2)$, its tangent has slope 1, so its normal has slope -1. The point-slope equation of this normal is

$$y - 2 = -(x - 1), \qquad y = 3 - x.$$

Putting $y = 0$ shows that the required common intercept is $(3, 0)$. The normal does not pass through $(1, 0)$; its given point is above the axis.

Step 2: Find the exponential tangent intercept For $y = e^x$, differentiation gives $dy/dx = e^x$. At (c, e^c), both the ordinate and tangent slope equal e^c, so

$$y - e^c = e^c(x - c).$$

At the intercept on the horizontal axis, set $y = 0$. Since e^c is strictly positive for every real c, division by it is valid:

$$-1 = x - c, \qquad x = c - 1.$$

The common intercept condition therefore becomes

$$c - 1 = 3, \qquad c = 4.$$

As a check, the tangent for this value is $y = e^4(x - 3)$, which indeed vanishes at $x = 3$.

Step 3: Conclusion The required dimensionless value is $\boxed{c = 4}$. Both lines pass through $(3, 0)$, as required.

Quick Tip to Solve: For the exponential curve $y = e^x$, the tangent at abscissa c always cuts the horizontal axis at $c - 1$.

26. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to : [4 marks]

(A) 2 : 1

(B) 1 : 2

(C) 5 : 7

(D) 10 : 7

Correct Answer: (A) 2 : 1

Solution:

Concept:

- An ion accelerated from rest through potential difference V gains kinetic energy qV, so $\frac{1}{2}mv^2 = qV$. Here q denotes the magnitude of its positive charge.
- A hydrogen ion and a singly ionized helium atom each carry charge $+e$. Their masses are approximately m_H and $4m_H$, because removing an electron scarcely changes a nuclear mass.

Step 1: Express each speed using energy conservation The electric field performs work on the positive ions. Starting from rest, their final kinetic energies are equal because both their charges and accelerating potential differences are equal:

$$\frac{1}{2}m_H v_H^2 = eV, \qquad \frac{1}{2}m_{He} v_{He}^2 = eV.$$

Thus

$$v_H = \sqrt{\frac{2eV}{m_H}}, \qquad v_{He} = \sqrt{\frac{2eV}{m_{He}}}.$$

Equal kinetic energies do not imply equal speeds when masses differ. The lighter hydrogen ion must move faster.

Step 2: Calculate and compare the speed ratio Divide the speed expressions so that the unknown accelerating voltage and elementary charge cancel:

$$\frac{v_H}{v_{He}} = \sqrt{\frac{m_{He}}{m_H}} \simeq \sqrt{4} = 2.$$

Therefore the ordered ratio of hydrogen speed to helium speed is $2 : 1$. Option (B) reverses the ratio and would make the heavier ion faster. Options (C) and (D) do not satisfy the equal-energy condition with the approximately fourfold mass ratio. In particular, helium is singly ionized, so its ionic charge is $+e$, not the $+2e$ charge of a bare helium nucleus.

Step 3: Conclusion The final speed ratio is $\boxed{v_H : v_{He} = 2 : 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For ions crossing the same voltage, first compare their charge states; then use $v \propto \sqrt{q/m}$.

27. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? [4 marks]

(A) 6

(B) 7

(C) 5

(D) 3

Correct Answer: (A) 6

Solution:

Concept:

- For a fixed quantity of ideal gas, $Q_P = nC_P\Delta T$ and $Q_V = nC_V\Delta T$, where the heat capacities are molar and the same n is used for both processes.
- Equipartition gives $C_V = fR/2$ and Mayer's relation gives $C_P - C_V = R$. Consequently $\gamma = C_P/C_V = (f+2)/f$, with f the molecular degrees of freedom.

Step 1: Extract the heat capacities of the sample It is convenient to use the total heat capacities nC_P and nC_V, since neither the mass nor the number of moles is specified. For the constant-pressure heating,

$$nC_P = \frac{160}{50} = 3.2 \text{ cal K}^{-1}.$$

For constant-volume cooling, both heat and temperature change are negative. Their quotient is positive:

$$nC_V = \frac{-240}{-100} = 2.4 \text{ cal K}^{-1}.$$

A temperature interval in degrees Celsius equals the same interval in kelvin, so no offset of 273 is added.

Step 2: Use the ratio to find the degrees of freedom The factor n cancels from the heat-capacity ratio:

$$\gamma = \frac{3.2}{2.4} = \frac{4}{3}.$$

Using the ideal-gas equipartition relation,

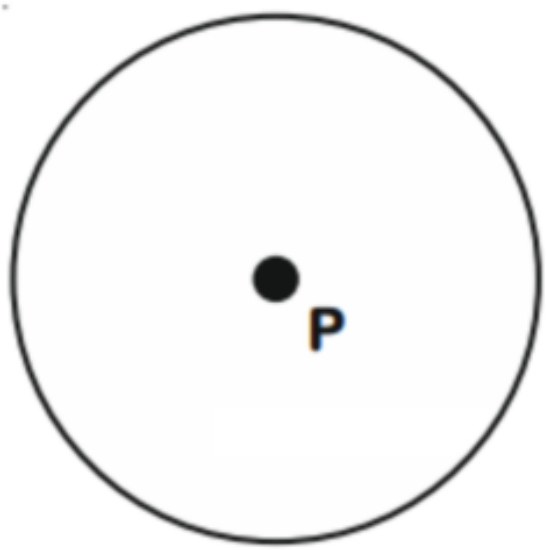
$$1 + \frac{2}{f} = \frac{4}{3}, \qquad \frac{2}{f} = \frac{1}{3}, \qquad f = 6.$$

The other offered values can be checked directly: $f = 7$ gives $\gamma = 9/7$, $f = 5$ gives $7/5$, and $f = 3$ gives $5/3$. None is the measured ratio $4/3$. A direct ratio of heats would be wrong because the two temperature intervals differ.

Step 3: Conclusion Each molecule has $\boxed{6}$ degrees of freedom. Hence, the correct option is (A).

Quick Tip to Solve: Compare $Q/|\Delta T|$, rather than the heats alone, before using $f = 2/(\gamma - 1)$.

28. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is : [4 marks]



(A) $\vec{B}$

(B) zero

(C) much large than $\left|\vec{B}\right|$ and parallel to $\vec{B}$

(D) much large than $\left|\vec{B}\right|$ but opposite to $\vec{B}$

Correct Answer: (B) zero

Solution:

Concept:

- A perfectly diamagnetic material expels magnetic induction from its bulk: its ideal relative permeability is zero and its induced response opposes the externally applied field.
- A completely enclosed cavity in such a magnetic shield, with no independent magnetic source inside it, has no applied magnetic induction. A paramagnetic material responds to a local field; it has no remanent magnetization after its exciting field is removed.

Step 1: Apply the shielding condition to the enclosed cavity The external uniform field is applied to the whole sphere, but the cavity is surrounded on every side by perfectly diamagnetic material. The field is excluded from that surrounding material, and magnetic flux cannot cross the inner spherical boundary. The enclosed cavity therefore receives no field from the external source. This is the ideal magnetic-shielding limit, not merely the small reduction that an ordinary weak diamagnet would produce. The central location of the small cavity preserves the stated symmetry.

Step 2: Check the effect of the paramagnetic filling For a linear paramagnet the induced magnetization satisfies $\mathbf{M} = \chi_m \mathbf{H}$, with positive susceptibility. In this shielded cavity there is no exciting field and no permanent internal source, so

$$\mathbf{H} = \mathbf{0}, \qquad \mathbf{M} = \mathbf{0}.$$

It follows that

$$\mathbf{B}_{\text{inside}} = \mu_0(\mathbf{H} + \mathbf{M}) = \mathbf{0}.$$

The paramagnetic filling cannot amplify a field which is excluded from its region. Option (A) ignores the surrounding shield. Options (C) and (D) both assign a large nonzero field without an internal source; neither follows from paramagnetism. Moreover a paramagnet normally aligns with its local applied field, so the opposite-field description is inappropriate.

Step 3: Conclusion The magnetic field inside the paramagnetic substance is $\boxed{\mathbf{0}}$. Hence, the correct option is (B).

Quick Tip to Solve: Check whether the local exciting field reaches a material before applying its paramagnetic or diamagnetic response.

29. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is : [4 marks]

(A) $\frac{3Q_2}{4\pi\varepsilon_0 R}$
(B) $\frac{3Q_1}{4\pi\varepsilon_0 R}$
(C) $\frac{3Q_1}{16\pi\varepsilon_0 R}$
(D) $\frac{Q_2}{4\pi\varepsilon_0 R}$

Correct Answer: (C) $\frac{3Q_1}{16\pi\varepsilon_0 R}$

Solution:

Concept:

- A spherical shell of charge Q and radius a produces potential kQ/a throughout its interior, and kQ/r at an external point of radius r, where $k = 1/(4\pi\varepsilon_0)$.
- Electric potentials add algebraically. The outer shell contributes the same potential at both radii here, so only the inner shell contributes to the potential difference.

Step 1: Write the potential on each sphere At radius R, the inner sphere contributes kQ_1/R and the outer sphere contributes its constant interior potential $kQ_2/(4R)$. Thus

$$V(R) = \frac{kQ_1}{R} + \frac{kQ_2}{4R}.$$

At radius $4R$, the contributions are

$$V(4R) = \frac{kQ_1}{4R} + \frac{kQ_2}{4R}.$$

These expressions already include the common outer-shell contribution. It must not be counted as a field-producing contribution between the spheres.

Step 2: Subtract and identify the matching option Subtract the second potential from the first:

$$V(R) - V(4R) = kQ_1\left(\frac{1}{R} - \frac{1}{4R}\right).$$

Therefore

$$V(R) - V(4R) = \frac{3kQ_1}{4R} = \frac{3Q_1}{16\pi\varepsilon_0 R}.$$

The stated equal surface charge densities also imply $Q_2/(4\pi(4R)^2) = Q_1/(4\pi R^2)$, hence $Q_2 = 16Q_1$; this does not change the subtraction. Option (B) is four times the result. Substituting the density relation into (A) or (D) also gives values inconsistent with the derived difference. The units are volts because kQ/R has the dimensions of electric potential.

Step 3: Conclusion The potential difference is $\boxed{3Q_1/(16\pi\varepsilon_0 R)}$. Hence, the correct option is (C).

Quick Tip to Solve: Between concentric shells, subtract potentials before using any charge-density relation: the outer shell's constant potential cancels.

30. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10 m/s^2 . Assume there is no rotational motion and loss of energy after the collision is negligable.] [4 marks]

(A) 23 J

(B) 21 J

(C) 20 J

(D) 19 J

Correct Answer: (B) 21 J

Solution:

Concept:

- During the short bullet-block collision, horizontal momentum is conserved. The bullet sticks, so the collision is perfectly inelastic and kinetic energy is not conserved across it.
- After the collision, mechanical energy is conserved during the fall. The decrease Mgh in gravitational potential energy becomes additional kinetic energy, where M is the combined mass.

Step 1: Find the speed just after impact The stationary block has no initial momentum. With bullet mass $m = 0.1\,\text{kg}$, block mass $1.9\,\text{kg}$, and bullet speed $u = 20\,\text{m s}^{-1}$, the total mass is $M = 2.0\,\text{kg}$. Conservation of horizontal momentum gives

$$mu = Mv, \qquad v = \frac{0.1 \times 20}{2.0} = 1\,\text{m s}^{-1}.$$

Immediately after the collision,

$$K_{\text{after}} = \frac{1}{2}Mv^2 = \frac{1}{2}(2.0)(1)^2 = 1\,\text{J}.$$

Using the bullet's initial kinetic energy directly after sticking would neglect the energy dissipated in this inelastic event.

Step 2: Add the kinetic energy gained during the fall The combined system drops through $h = 1\,\text{m}$, and the question excludes further energy loss and rotational motion. Its loss of potential energy is

$$Mgh = 2.0 \times 10 \times 1 = 20\,\text{J}.$$

Hence its translational kinetic energy just before reaching the floor is

$$K_{\text{floor}} = K_{\text{after}} + Mgh = 1 + 20 = 21\,\text{J}.$$

Option (C) includes only the vertical fall energy and omits the remaining horizontal motion. Options (A) and (D) give neither the momentum-conserved initial energy nor the correct gravitational increment. The collision and the subsequent fall must therefore be treated as separate stages.

Step 3: Conclusion The final kinetic energy is $\boxed{21\,\text{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a sticking collision followed by a fall, use momentum for the impact first, then energy conservation for the fall.

31. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixure is 31°C. The value of 'm' is close to :
(Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} $°C^{-1}$) [4 marks]

(A) 2.6

(B) 2

(C) 4

(D) 3.2

Correct Answer: (B) 2

Solution:

Concept:

- In an insulated mixture, heat released by the hot substance equals heat absorbed by the initially colder water and calorimeter. The calorimeter water equivalent adds directly to the mass of water receiving heat.
- Steam first condenses at 100°C, releasing latent heat mL, and the condensed water then cools to the common final temperature. Thus each gram releases $L + c(100 - T_f)$.

Step 1: Find the heat absorbed by the cold system The calorimeter behaves thermally like $20\,\text{g}$ of water. Hence the effective cold-water mass is $180 + 20 = 200\,\text{g}$. Its temperature rise is $31 - 25 = 6^\circ\text{C}$,

giving

$$Q_{\text{gained}} = (180 + 20) \times 1 \times (31 - 25) = 1200\,\text{cal}.$$

The calorimeter must be included because it is warmed along with its contents.

Step 2: Include condensation and subsequent cooling For m grams of steam, condensation releases $540m$ cal. The resulting liquid also cools from 100°C to 31°C, releasing $69m$ cal. Energy conservation therefore gives

$$m[540 + 1(100 - 31)] = 1200, \qquad m = \frac{1200}{609} = 1.97\,\text{g}.$$

The final temperature is below the boiling point, consistently allowing all the steam to condense. The alternatives 2.6, 4, and 3.2 g would release appreciably more heat than the required 1200 cal; none is as close as 2 g.

Step 3: Conclusion The required mass of steam is approximately 2 g. Hence, the correct option is (B).

Quick Tip to Solve: For steam mixed with cooler water, count both latent heat and cooling of the condensate; add the calorimeter water equivalent to the cold-water mass.

---

32. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2 . After this the phase difference between the two waves is : [4 marks]

(A) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$

(B) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$

(C) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$

(D) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$

Correct Answer: (A) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$

Solution:

Concept:

- The wavelength in a medium of refractive index n is λ/n, where λ is the vacuum wavelength. The frequency remains unchanged when light enters the medium.
- A path length L in that medium contains nL/λ wavelengths, giving a phase accumulation $2\pi nL/\lambda$. The product nL is the optical path length.

Step 1: Calculate the phase accumulation on each path The waves start with the same phase, so their initial phase terms cancel when we compare them. For the first path, the number of wavelengths traversed is $L_1/(\lambda/n_1) = n_1L_1/\lambda$. For the second path it is n_2L_2/λ. Thus

$$\phi_1 = \frac{2\pi n_1 L_1}{\lambda}, \qquad \phi_2 = \frac{2\pi n_2 L_2}{\lambda}.$$

Each factor is dimensionless, as a phase in radians must be.

Step 2: Subtract the optical path contributions Using the first-path-minus-second-path convention for

the phase difference gives

$$\Delta\phi = \phi_1 - \phi_2 = \frac{2\pi}{\lambda}(n_1L_1 - n_2L_2).$$

Reversing which wave is used as the reference reverses the sign; interference depends on the phase difference modulo 2π. Option (B) incorrectly pairs each refractive index with the other path. Options (C) and (D) divide path lengths by refractive indices, whereas a larger refractive index shortens the wavelength and increases the number of phase cycles accumulated. As an additional check, equal optical paths give zero relative phase, exactly as this expression predicts.

Step 3: Conclusion The phase difference is $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$, with the stated path-order convention. Hence, the correct option is (A).

Quick Tip to Solve: Convert geometrical lengths into optical lengths nL before multiplying their difference by the vacuum wave number $2\pi/\lambda$.

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33. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\overrightarrow{E} = E_0\hat{j}\cos(\omega t - kx)$.

The magnetic field $\overrightarrow{B}$, at the moment t = 0 is : [4 marks]

(A) $\overrightarrow{B} = \frac{E_0}{\sqrt{\mu_0\epsilon_0}}\cos(kx)\,\hat{j}$

(B) $\overrightarrow{B} = \frac{E_0}{\sqrt{\mu_0\epsilon_0}}\cos(kx)\,\hat{k}$

(C) $\overrightarrow{B} = E_0\sqrt{\mu_0\epsilon_0}\cos(kx)\,\hat{k}$

(D) $\overrightarrow{B} = E_0\sqrt{\mu_0\epsilon_0}\cos(kx)\,\hat{j}$

Correct Answer: (C) $\overrightarrow{B} = E_0\sqrt{\mu_0\epsilon_0}\cos(kx)\,\hat{k}$

Solution:

Concept:

- For an electromagnetic wave in vacuum, $E_0 = cB_0$ and $c = 1/\sqrt{\mu_0\varepsilon_0}$. The electric and magnetic fields oscillate in phase.
- The propagation direction is that of $\vec{E} \times \vec{B}$. Both fields are perpendicular to each other and to the direction of wave propagation.

Step 1: Determine the magnetic-field direction The phase $\omega t - kx$ represents propagation in the positive x direction. Since the electric field is directed along $\hat{j}$, the magnetic field must point along $\hat{k}$ when the electric scalar amplitude is positive:

$$\hat{j} \times \hat{k} = \hat{\imath}.$$

The field directions reverse together during the oscillation, preserving positive energy flow. Options (A) and (D), which put both fields along $\hat{j}$, violate their mutual perpendicularity.

Step 2: Find the amplitude and set the time to zero Using the vacuum amplitude relation gives

$$B_0 = \frac{E_0}{c} = E_0\sqrt{\mu_0\varepsilon_0}.$$

Therefore the full travelling magnetic field is

$$\vec{B}(x,t) = E_0\sqrt{\mu_0\varepsilon_0}\,\hat{k}\cos(\omega t - kx).$$

At $t = 0$, the cosine is even, so $\cos(-kx) = \cos(kx)$. Option (B) has the correct direction but multiplies the electric amplitude by c, rather than dividing by it; this also gives the wrong dimensions for a magnetic field. Option (C) alone satisfies the amplitude relation, direction condition and common phase of the two fields.

Step 3: Conclusion At $t = 0$, $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(kx)\hat{k}$. Hence, the correct option is (C).

Quick Tip to Solve: Check electromagnetic-wave options twice: use $\vec{E} \times \vec{B}$ for direction, then $B_0 = E_0/c$ for magnitude.

34. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate $\frac{dB}{dt}$ = 0.032 Ts^{-1}. The induced current in the loop is close to (Resistivity of the metal wire is 1.23 $\times$ 10^{-8} Ωm) [4 marks]

(A) 0.53 A

(B) 0.43 A

(C) 0.34 A

(D) 0.61 A

Correct Answer: (D) 0.61 A

Solution:

Concept:

- Faraday's law gives the induced-emf magnitude $\mathcal{E} = A_{\text{loop}}|dB/dt|$ for one fixed loop in a perpendicular, spatially uniform field.
- The resistance is $R_{\text{wire}} = \rho\ell/A_{\text{wire}}$, where ℓ is the total wire length and A_{wire} is its cross-sectional area. Ohm's law then gives $I = \mathcal{E}/R_{\text{wire}}$.

Step 1: Calculate the loop area and induced emf The wire length is the perimeter of the square, so its side is

$$a = \frac{0.30}{4} = 0.075\,\text{m}.$$

The area linked by the magnetic flux is therefore $a^2 = 0.005625\,\text{m}^2$, rather than the wire cross-sectional area. The induced emf is

$$\mathcal{E} = (0.005625)(0.032) = 1.80 \times 10^{-4}\,\text{V}.$$

Only its magnitude is required; Lenz's law would determine the circulation direction if the field orientation were specified.

Step 2: Find the wire resistance and divide The stated wire diameter is $4\,\text{mm}$, giving a radius of $2 \times 10^{-3}\,\text{m}$. Thus

$$A_{\text{wire}} = \pi(2 \times 10^{-3})^2 = 4\pi \times 10^{-6}\,\text{m}^2,$$

$$R_{\text{wire}} = \frac{(1.23 \times 10^{-8})(0.30)}{4\pi \times 10^{-6}} = 2.94 \times 10^{-4}\,\Omega.$$

Consequently

$$I = \frac{1.80 \times 10^{-4}}{2.94 \times 10^{-4}} \simeq 0.613\,\text{A}.$$

The result is closest to 0.61 A. The other proposed currents, 0.53, 0.43 and 0.34 A, do not follow from the specified flux-change rate and wire resistance.

Step 3: Conclusion The induced current is approximately 0.61 A. Hence, the correct option is (D).

Quick Tip to Solve: Keep the two areas distinct: the square area determines flux, while the circular wire cross section determines resistance.

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35. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is :

Planck's constant h = 6.63 $\times$ 10^{-34} J.s. Speed of light c = 3 $\times$ 10^{8} m/s [4 marks]

(A) 1.5 eV

(B) 2.0 eV

(C) 3.1 eV

(D) 1.1 eV

Correct Answer: (C) 3.1 eV

Solution:

Concept:

- For photodetection across a semiconductor band gap, a photon must supply at least the energy E_g needed to promote an electron from the valence band to the conduction band.
- The photon energy is $E = hc/\lambda$. The longest detectable wavelength corresponds to the minimum accepted photon energy, so $E_g = hc/\lambda_{\max}$.

Step 1: Apply the cutoff-wavelength condition The maximum detectable wavelength is $400\,\text{nm} = 4.00 \times 10^{-7}\,\text{m}$. At this threshold, photon energy just equals the gap. Photons with longer wavelengths have less energy and cannot bridge the band gap, whereas shorter wavelengths supply enough energy. Substituting the constants given in the question,

$$E_g = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{4.00 \times 10^{-7}}\,\text{J}.$$

The numerator is $1.989 \times 10^{-25}\,\text{J m}$, which cancels the wavelength unit correctly.

Step 2: Convert the energy into electronvolts and compare The numerical energy is

$$E_g = 4.9725 \times 10^{-19}\,\text{J}.$$

Using $1\,\text{eV} \simeq 1.60 \times 10^{-19}\,\text{J}$,

$$E_g = \frac{4.9725 \times 10^{-19}}{1.60 \times 10^{-19}}\,\text{eV} \simeq 3.11\,\text{eV}.$$

The small variation from using a more precise elementary charge does not affect the selected option. Band gaps of 1.5, 2.0, or $1.1\,\text{eV}$ would have cutoff wavelengths longer than $400\,\text{nm}$, so none matches the stated maximum wavelength. The threshold refers to energy per photon, not the total energy delivered by a beam.

Step 3: Conclusion The band gap is approximately $3.1\,\text{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: At a photodiode wavelength cutoff, use $E_g(\text{eV}) \simeq 1240/\lambda_{\max}(\text{nm})$, giving $1240/400 = 3.1$.

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36. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(\mathbf{r}) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$.

Then the gravitational field is maximum at : [4 marks]

(A) $r = \frac{1}{\sqrt{3}}R$

(B) r = R

(C) $r = \sqrt{\frac{3}{4}}R$

(D) $r = \sqrt{\frac{5}{9}}R$

Correct Answer: (D) $r = \sqrt{\frac{5}{9}}R$

Solution:

Concept:

- For spherical symmetry, only the mass enclosed within radius r contributes to the field there. A spherical shell of radius s has mass $dm = 4\pi s^2 \rho(s)\,ds$.
- The internal gravitational-field magnitude is $g(r) = GM(r)/r^2$. Integrate the variable density to obtain $M(r)$ before differentiating the field; a uniform-density formula does not apply.

Step 1: Integrate the mass enclosed within radius r Use s as the integration variable to distinguish shell radius from the observation radius. With the stated density,

$$\begin{aligned} M(r) &= 4\pi\rho_0 \int_0^r \left(s^2 - \frac{s^4}{R^2}\right) ds \\ &= 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right). \end{aligned}$$

Hence, for $0 < r \le R$,

$$g(r) = 4\pi G\rho_0 \left(\frac{r}{3} - \frac{r^3}{5R^2}\right).$$

This tends to zero at the centre, as required by symmetry.

Step 2: Maximise the field and check both regions Differentiating the internal field gives

$$\frac{dg}{dr} = 4\pi G\rho_0 \left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0.$$

Thus $r^2 = 5R^2/9$, or $r = R\sqrt{5/9}$. The second derivative is $-24\pi G\rho_0 r/(5R^2) < 0$ at this positive radius, so the stationary point is a maximum. The field rises before this point and falls afterwards up to the surface. Outside the planet the total enclosed mass is constant and $g = GM(R)/r^2$ decreases with radius, so no larger external maximum exists. Neither $R/\sqrt{3}$ nor $R\sqrt{3/4}$ makes the derivative zero, and the surface lies on the decreasing part of the field curve.

Step 3: Conclusion The gravitational field is maximum at $r = R\sqrt{5/9} = \sqrt{5}\,R/3$. Hence, the correct option is (D).

Quick Tip to Solve: For a radially varying density, first integrate $4\pi r^2\rho(r)$; maximising density or enclosed mass alone does not maximise the gravitational field.

37. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is : [4 marks]

(A) MLT^{-2}

(B) ML^0T^{-3}

(C) $M^2L^0T^{-1}$

(D) ML^2T^{-2}

Correct Answer: (B) ML^0T^{-3}

Solution:

Concept:

- The solar constant is an energy flux: energy received per unit area per unit time. Its dimensions must therefore be obtained from the definition, rather than from the word constant.
- Energy has dimensions $[E] = ML^2T^{-2}$, area has dimensions L^2, and time has dimensions T. Numerical values and the orientation of the receiving surface do not affect these dimensions.

Step 1: Express the defined quantity as a quotient If energy E falls on an area S during a time interval Δt, the solar constant is $I = E/(S\Delta t)$. Equivalently, it is power per unit area. Its SI unit is therefore $\mathrm{W\,m^{-2}}$, not joule or watt alone. Both the area and time in the definition must be included in the denominator.

Step 2: Cancel the length dimensions Substitution of the base dimensions gives

$$[I] = \frac{ML^2T^{-2}}{L^2T} = ML^0T^{-3}.$$

The two powers of length cancel completely, while division by time decreases the time exponent from -2 to -3. Option (A) has the dimensions of force and option (D) has those of energy, so neither is an energy flux. Option (C) incorrectly contains the square of mass and the time exponent -1. Only

option (B) has one power of mass, zero powers of length, and time to the power -3.

Step 3: Conclusion The dimensions of the solar constant are $\boxed{ML^0T^{-3}}$, equivalent to power per unit area. Hence, the correct option is (B).

Quick Tip to Solve: For radiation flux, start from $\mathrm{W\,m^{-2}}$; the area cancels the length squared in the dimensions of power.

38. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is : [4 marks]

(A) $\frac{1}{500}$

(B) 500

(C) 250

(D) $\frac{1}{250}$

Correct Answer: (A) $\frac{1}{500}$

Solution:

Concept:

- Each photon carries energy $\varepsilon = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is its wavelength. A shorter wavelength means a more energetic photon.
- A source of power P emits $\dot{N} = P/\varepsilon = P\lambda/(hc)$ photons per second. For equal powers the photon emission rate is directly proportional to wavelength, rather than inversely proportional to it.

Step 1: Relate photon number to the fixed power In an interval Δt, both sources emit the same energy $P\Delta t$. The numbers of photons are

$$N_X = \frac{P\Delta t\lambda_X}{hc}, \qquad N_v = \frac{P\Delta t\lambda_v}{hc}.$$

Thus the common power $200\,\mathrm{W}$, observation time and constants cancel in the ratio. In the comparison of photon densities under the same propagation conditions, both radiations travel at c, so this is also the photon-number-density ratio asked for.

Step 2: Substitute the two wavelengths Use the same wavelength unit for numerator and denominator:

$$\frac{n_X}{n_v} = \frac{N_X}{N_v} = \frac{\lambda_X}{\lambda_v} = \frac{1\,\mathrm{nm}}{500\,\mathrm{nm}} = \frac{1}{500}.$$

Each X-ray photon is 500 times more energetic, so the same energy contains 500 times fewer such photons. Option (B) reverses this ratio. Options (C) and (D) introduce an unwarranted factor of two; neither the common source power nor photon energy supplies such a factor.

Step 3: Conclusion The ratio of X-ray photon number density to visible-light photon number density is $\boxed{1/500}$, a dimensionless ratio. Hence, the correct option is (A).

Quick Tip to Solve: At equal power, longer-wavelength light contains more photons: photon number rate is proportional to wavelength.

39. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed? [4 marks]

(A) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.

(B) Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.

(C) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.

(D) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

Correct Answer: The key selects (D), but the literal wording is ambiguous: (A) is also false for a conducting wire, and (D) ignores the capacitor's charging transient.

Solution:

Concept:

- In resistance mode, a multimeter applies a small voltage from its internal battery and detects the resulting current. For an analogue meter, a conducting low-resistance path gives a deflection; an open circuit gives no steady deflection.
- A metal wire and an ordinary resistor conduct for either polarity. An LED conducts appreciably only in forward bias, while a capacitor carries a transient charging current and then blocks steady direct current.

Step 1: Test the statement about a metal wire A metal wire provides a small but finite resistance between the probes. With either orientation of the probes, the meter's battery drives current through it. The wire therefore gives a deflection, normally close to the low-resistance end of the scale, in both directions. Reversing the probes does not turn this conducting connection into an open circuit. Consequently the claim in option (A), that a metal wire gives no deflection for either polarity, is physically false in the stated resistance-measuring mode. This remains true whether the comparison is immediate or after the reading settles.

Step 2: Check the remaining component responses An LED can glow when the meter forward-biases it and supplies sufficient voltage, but reverse bias suppresses its current; this is the polarity dependence described in option (B). An ordinary resistor has the same resistance for either polarity, so option (C) correctly gives equal deflections. For a capacitor, the initial charging current can cause a momentary kick, especially on reversing the probes. Once charged, however, the capacitor blocks the meter's direct current and there is no sustained deflection in either connection. The words "just reversed" include this charging transient, so option (D), with its unqualified assertion of no deflection, is also false on that reading. If only settled readings are compared, (D) is true while (A) remains false. Thus the two time interpretations cannot justify a unique selection of (D) from the wording as printed.

Step 3: Conclusion The answer key selects (D), reflecting the capacitor's transient deflection immediately after reversal. However, (A) also contradicts the finite current through a metal wire. As printed, the question has no unique defensible answer: (A) is false, and the immediate-response interpretation also makes (D) false. The key choice (D) therefore requires this qualification.

Quick Tip to Solve: Separate a capacitor's momentary charging kick from its final DC reading; a metal wire always provides a conducting test path.

40. The radius R of a nucleus of mass number A can be estimated by the formula

$R = (1.3 \times 10^{-15})A^{1/3}$ m.

It follows that the mass density of a nucleus is of the order of :

($M_{prot.} \cong M_{neut} \simeq 1.67 \times 10^{-27}$ kg) [4 marks]

(A) 10^{24} kg m^{-3}

(B) 10^{10} kg m^{-3}

(C) 10^{17} kg m^{-3}

(D) 10^{3} kg m^{-3}

Correct Answer: (C) 10^{17} kg m^{-3}

Solution:

Concept:

- The nuclear mass is approximately $M = Am_N$, where A is the number of nucleons and $m_N = 1.67 \times 10^{-27}\,\text{kg}$ is the stated proton or neutron mass. Binding-energy corrections do not change the required order of magnitude.
- Treating the nucleus as a sphere, its volume is $V = 4\pi R^3/3$. The radius law $R = r_0 A^{1/3}$, with $r_0 = 1.3 \times 10^{-15}\,\text{m}$, makes the volume proportional to A.

Step 1: Write the density and cancel the mass number Density means mass divided by volume. Cube the entire radius expression, including its mass-number factor:

$$\rho = \frac{Am_N}{(4\pi/3)r_0^3 A} = \frac{3m_N}{4\pi r_0^3}.$$

Thus A cancels. The approximate equality of nuclear densities is a consequence of the cube-root dependence of radius on mass number; no particular nucleus needs to be chosen.

Step 2: Evaluate the power of ten and coefficient First calculate $r_0^3 = (1.3)^3 \times 10^{-45} = 2.197 \times 10^{-45}\,\text{m}^3$. Therefore

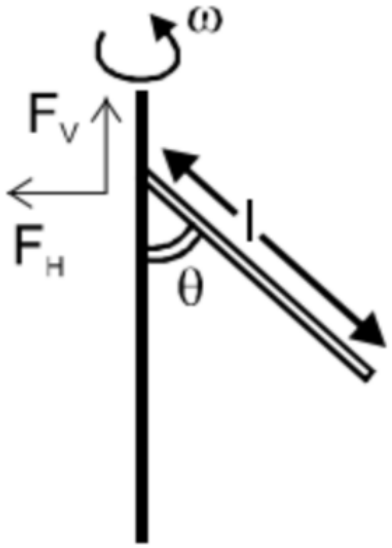
$$\rho = \frac{3(1.67 \times 10^{-27})}{4\pi(2.197 \times 10^{-45})}$$
$$\simeq 1.81 \times 10^{17}\,\text{kg m}^{-3}.$$

The exponent $-27 - (-45) = 18$ is multiplied by a coefficient about 0.181, giving the displayed 10^{17} result. Option (A) is seven orders of magnitude too large; option (B) is seven orders too small, and option (D) resembles ordinary material density rather than nuclear density.

Step 3: Conclusion The nuclear mass density is approximately $1.8 \times 10^{17}\,\text{kg m}^{-3}$, and is of order $\boxed{10^{17}\,\text{kg m}^{-3}}$. Hence, the correct option is (C).

Quick Tip to Solve: Cube $R = r_0 A^{1/3}$ before substituting: the factor A cancels the nuclear mass, leaving density independent of nucleus size.

41. A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that : [4 marks]



(A) $\cos\theta = \frac{2g}{3l\omega^2}$

(B) $\cos\theta = \frac{3g}{2l\omega^2}$

(C) $\cos\theta = \frac{g}{2l\omega^2}$

(D) $\cos\theta = \frac{g}{l\omega^2}$

Correct Answer: (B) $\cos\theta = \frac{3g}{2l\omega^2}$

Solution:

Concept:

- The centre of mass of a uniform rod is at its midpoint. It moves in a horizontal circle of radius $r = (l/2)\sin\theta$, so the horizontal pivot force supplies its centripetal acceleration.
- The centre of mass has no vertical acceleration, giving $F_V = mg$. About the centre of mass, gravity has zero moment, while the two pivot-force components provide the torque that changes the direction of angular momentum.

Step 1: Determine the pivot forces For a constant angle, the centre of mass travels with angular speed ω. Thus

$$F_H = m\omega^2 \frac{l}{2}\sin\theta, \qquad F_V = mg.$$

The horizontal force points towards the shaft. Its moment about the centre of mass opposes the moment of the upward vertical force; the corresponding perpendicular lever arms are $(l/2)\cos\theta$ and $(l/2)\sin\theta$, respectively.

Step 2: Apply the angular-momentum equation about the midpoint Taking torque into the paper as

positive, equate the net moment to the rate given in the question:

$$F_V \frac{l}{2}\sin\theta - F_H \frac{l}{2}\cos\theta = \frac{ml^2}{12}\omega^2 \sin\theta\cos\theta.$$

Substitute the forces and move the horizontal-force term to the right:

$$mg\frac{l}{2}\sin\theta = m\omega^2 l^2\left(\frac{1}{4}+\frac{1}{12}\right)\sin\theta\cos\theta$$
$$= \frac{m\omega^2 l^2}{3}\sin\theta\cos\theta.$$

For the inclined configuration shown, $\sin\theta \neq 0$, so cancellation gives

$$\cos\theta = \frac{3g}{2l\omega^2}.$$

The factor comes from adding the orbital coefficient $1/4$ and the midpoint rotational coefficient $1/12$. Options (A), (C) and (D) have different coefficients and fail this torque balance. A nonzero inclination is possible when $\omega^2 > 3g/(2l)$; equality is the vertical limiting configuration.

Step 3: Conclusion The rod's inclination satisfies $\boxed{\cos\theta = 3g/(2l\omega^2)}$. Hence, the correct option is (B).

Quick Tip to Solve: About the midpoint, the vertical and horizontal pivot forces have opposing moments. Keep the supplied angular-momentum rate on the correct side before combining the coefficients $1/4$ and $1/12$.

42. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is : [4 marks]

(A) 1
(B) $\frac{1}{2}$
(C) $\sqrt{2}$
(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (D) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- For a mass m on a spring of force constant k, the angular frequency is $\omega = \sqrt{k/m}$. In simple harmonic motion the speed at equilibrium is its maximum value, $v_{\max} = \omega A$.
- When part of a moving block breaks off without an additional impulse, the remaining part retains its instantaneous velocity. Its new frequency changes with its mass, while the spring constant and horizontal equilibrium position remain unchanged.

Step 1: Determine the velocity at the instant of separation The separation occurs at equilibrium,

where the spring extension is zero. Just before the break the entire block has speed

$$v_0 = A\sqrt{\frac{k}{m}}.$$

Immediately after the break, both pieces still have this instantaneous speed; there is no impulsive spring force at zero extension. In particular, the speed of the mass remaining attached to the spring does not double. The detached piece carries away its own share of the original kinetic energy.

Step 2: Use the new frequency to find the new amplitude The attached mass is now $m' = m/2$, so

$$\omega' = \sqrt{\frac{k}{m/2}} = \sqrt{2}\,\omega.$$

Its displacement is still zero when its speed is v_0; hence this is the maximum speed of its new oscillation. Writing $A' = fA$,

$$A' = \frac{v_0}{\omega'} = \frac{A\omega}{\sqrt{2}\omega} = \frac{A}{\sqrt{2}}.$$

Equivalently, the remaining kinetic energy is half its original value, and spring energy is proportional to amplitude squared. Option (A) ignores the frequency change; option (B) scales amplitude directly with energy instead of its square root; option (C) reverses the dependence of amplitude on frequency at fixed speed.

Step 3: Conclusion The remaining system oscillates with amplitude $A/\sqrt{2}$, so $\boxed{f = 1/\sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: At an equilibrium-point mass change, keep the instantaneous velocity fixed and use $A' = v/\omega'$; do not conserve the energy of the attached piece alone across separation.

43. Two resistors 400Ω and 800Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of 10 kΩ across 400 Ω resistor is close to : [4 marks]

(A) 2.05 V

(B) 1.95 V

(C) 2 V

(D) 1.8 V

Correct Answer: (B) 1.95 V

Solution:

Concept:

- A voltmeter has a finite resistance and is connected in parallel with the resistor whose voltage is measured. Thus connecting it changes the circuit resistance and the voltage being measured.
- For parallel resistances, $R_p^{-1} = R^{-1} + R_V^{-1}$. The voltage divider then gives $V_p = V R_p/(R_p + R_s)$, where R_s is the remaining series resistance.

Step 1: Replace the measured branch by its equivalent resistance The meter resistance is $R_V =$

$10\,\text{k}\Omega = 10000\,\Omega$. It is across the $400\,\Omega$ resistor, so their parallel combination is smaller than $400\,\Omega$:

$$R_p = \frac{400 \times 10000}{400 + 10000} = \frac{5000}{13}\,\Omega \approx 384.615\,\Omega.$$

The $800\,\Omega$ resistor stays in series with this combined branch; the meter is not inserted in series.

Step 2: Apply the divider and compare the choices Both the meter and the $400\,\Omega$ resistor have the same potential difference. Therefore,

$$V_V = 6\frac{5000/13}{800 + 5000/13} = \frac{150}{77}\,\text{V} \approx 1.948\,\text{V}.$$

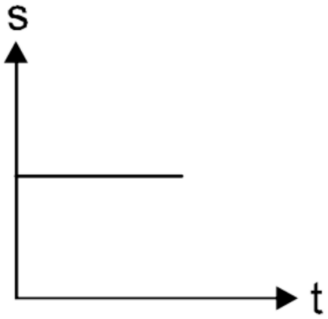
Without the meter the drop would be $6(400/1200) = 2\,\text{V}$, but the finite meter resistance lowers the branch resistance and its voltage. Thus $2.05\,\text{V}$ has the wrong direction of change, $2\,\text{V}$ ignores loading, and $1.8\,\text{V}$ overestimates the loading effect.

Step 3: Conclusion The meter reads approximately $1.95\,\text{V}$. Hence, the correct option is (B).

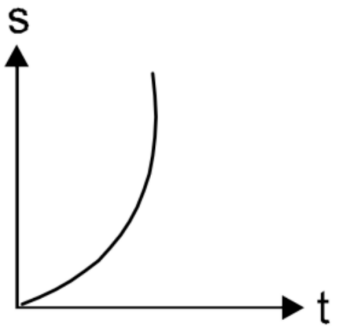
Quick Tip to Solve: For a finite-resistance voltmeter, first replace the measured resistor and meter by their parallel equivalent; only then use the voltage divider.

44. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) : [4 marks]

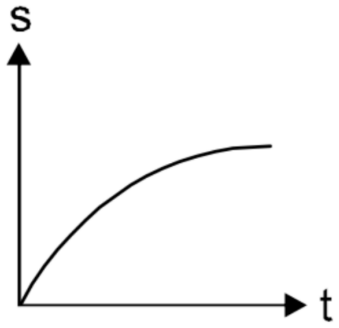
(A)



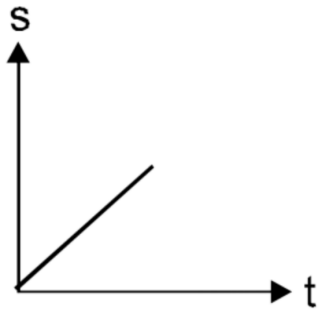
(B)



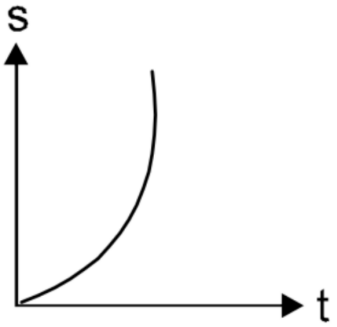
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Power is the time rate of doing work. For a particle of fixed mass m, constant positive power P increases kinetic energy uniformly: $P = d(\frac{1}{2}mv^2)/dt$.
- The slope of a displacement-time graph is velocity, $v = ds/dt$. An increasing velocity therefore requires an increasing graph slope, regardless of whether the acceleration itself is decreasing.

Step 1: Find the time dependence of speed Integrating the energy equation from the starting instant gives

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = Pt, \qquad v = \sqrt{v_0^2 + \frac{2P}{m}t}.$$

The positive square root applies because the motion is unidirectional along increasing displacement. For $P > 0$, speed increases with time. Equivalently $P = Fv = mav$, so $a = P/(mv) > 0$ for positive speed.

Step 2: Relate increasing speed to graph curvature For the schematic case starting from rest, integrate $ds/dt = \sqrt{2P/m}\, t^{1/2}$:

$$s - s_0 = \frac{2}{3}\sqrt{\frac{2P}{m}}\, t^{3/2}.$$

Thus displacement grows faster than linearly and the curve bends upwards. A nonzero initial speed also gives positive curvature. Graph (B) has this increasing slope. Graph (A) has zero slope and describes rest; graph (C) has decreasing slope and describes slowing motion; graph (D) has constant slope and constant speed. None of these three can represent a particle gaining kinetic energy continuously at constant positive power.

Step 3: Conclusion The displacement-time graph must rise with increasing slope, as in graph (B). Hence, the correct option is (B).

Quick Tip to Solve: On an s-t plot inspect the slope first: constant positive power makes speed increase, so the graph must be concave upwards, even though acceleration decreases.

45. A metallic sphere cools from 50°C to 40° in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to : [4 marks]

(A) 35°C

(B) 31°C

(C) 33°C

(D) 28°C

Correct Answer: (C) 33°C

Solution:

Concept:

- Newton's law of cooling states that the rate of decrease of temperature is proportional to the excess temperature above constant surroundings: $d\theta/dt = -k\theta$, where $\theta = T - T_a$.
- The solution is $\theta(t) = \theta(0)e^{-kt}$. Consequently, equal time intervals multiply the excess temperature by the same factor; they do not subtract equal temperature drops.

Step 1: Use the first interval to obtain the cooling factor The atmospheric temperature is $T_a = 20°\text{C}$. Initially the excess temperature is $50 - 20 = 30°\text{C}$, and after $300\,\text{s}$ it is $40 - 20 = 20°\text{C}$. Thus the factor for any $300\,\text{s}$ interval is

$$e^{-300k} = \frac{20}{30} = \frac{2}{3}.$$

This uses temperature differences, for which degrees Celsius and kelvin have equal numerical increments.

Step 2: Apply the same factor for the next five minutes The next five minutes is another $300\,\text{s}$. Therefore the new excess over the atmosphere is

$$\theta_2 = 20\left(\frac{2}{3}\right) = \frac{40}{3}^{\circ}\text{C}.$$

Adding the atmospheric temperature gives

$$T_2 = 20 + \frac{40}{3} = \frac{100^\circ}{3}\,\mathrm{C} \approx 33.33^\circ\mathrm{C}.$$

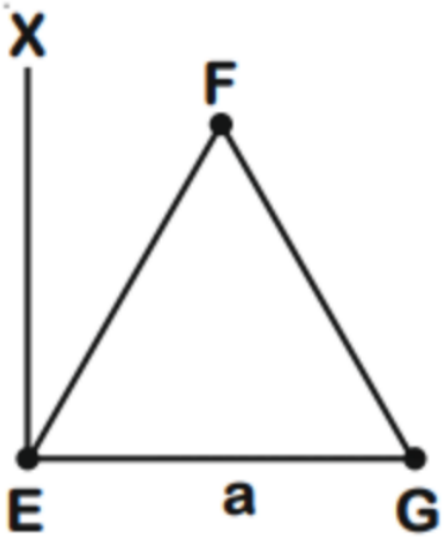
Of the listed values, $33^\circ\mathrm{C}$ is closest. Values $35^\circ\mathrm{C}$, $31^\circ\mathrm{C}$ and $28^\circ\mathrm{C}$ are farther from the exponential-cooling result. Subtracting another $10^\circ\mathrm{C}$ would incorrectly treat cooling as uniform, although its rate decreases as the sphere approaches its surroundings.

Step 3: Conclusion After the next five minutes the temperature is approximately $33.3^\circ\mathrm{C}$, closest to $33^\circ\mathrm{C}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal cooling intervals, multiply the excess above room temperature by the same ratio, then add room temperature back.

Physics – Section B: Numerical Value Questions (20 marks)

46. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}$ ma^2 where N is an integer. The value of N is _____. [4 marks]



Solution:

Concept:

- The moment of inertia of point particles about a specified axis is $I = \sum_j m_j r_j^2$, where r_j is the perpendicular distance of particle j from that axis.
- The triangular frame is massless, so only the three vertex particles contribute. An axis in the plane must be treated using distances within that plane, rather than distances from the centre of the triangle.

Step 1: Find the perpendicular distances from EX Choose $E = (0,0)$, $G = (a,0)$, and $F =$

$(a/2, \sqrt{3}a/2)$, as follows from the equilateral triangle geometry. Since EX passes through E and is perpendicular to EG in the triangle's plane, it is the vertical axis $x = 0$. The perpendicular distance to it is therefore the horizontal coordinate magnitude. Hence

$$r_E = 0, \qquad r_F = \frac{a}{2}, \qquad r_G = a.$$

In particular, the height $\sqrt{3}a/2$ of F above EG is not F's distance from EX.

Step 2: Sum the particle contributions and identify N The particle at E lies directly on the rotation axis and contributes zero. Using the other two distances,

$$I = m(0)^2 + m\left(\frac{a}{2}\right)^2 + m(a)^2 = \frac{1}{4}ma^2 + ma^2 = \frac{5}{4}ma^2.$$

The question expresses this same inertia as $(N/20)ma^2$. Comparing coefficients of the nonzero common factor ma^2,

$$\frac{N}{20} = \frac{5}{4}, \qquad N = 20 \times \frac{5}{4} = 25.$$

The dimensions are mass times length squared, as required for inertia. The requested N is a dimensionless integer, rather than the physical moment of inertia itself.

Step 3: Conclusion The moment of inertia is $I = \frac{5}{4}ma^2 = \frac{25}{20}ma^2$; hence the required integer is $\boxed{N = 25}$.

Quick Tip to Solve: For an in-plane axis through a triangle vertex, project each vertex perpendicularly onto the axis; do not substitute the triangle height unless it is that perpendicular distance.

47. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is ____. [4 marks]

Solution:

Concept:

- For a fixed spherical mirror, $1/f = 1/u + 1/v$, with signed object and image distances u and v. The focal length is constant as the object moves.
- Differentiating the mirror equation relates instantaneous velocities. Taking magnitudes gives $|dv/dt| = (v^2/u^2)|du/dt|$, so image speed depends on the square of the distance ratio.

Step 1: Differentiate the fixed-focus mirror relation Use one consistent Cartesian axis for both distances. Since f does not change, differentiation with respect to time gives

$$0 = -\frac{1}{u^2}\frac{du}{dt} - \frac{1}{v^2}\frac{dv}{dt}.$$

Rearranging,

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}.$$

The minus sign describes opposite signs of the two signed velocities; it is not a negative speed. The problem asks only for the magnitude of the image's velocity.

Step 2: Substitute the distances at the stated instant The distance magnitudes at this instant are $|u| = 30\,\text{cm}$ and $|v| = 10\,\text{cm}$, while the object speed is $9\,\text{cm}\,\text{s}^{-1}$. Therefore

$$\left|\frac{dv}{dt}\right| = \left(\frac{10}{30}\right)^2 \times 9 = \frac{1}{9} \times 9 = 1\,\text{cm}\,\text{s}^{-1}.$$

The distance signs disappear upon squaring. The image is moving more slowly because the image distance is smaller than the object distance. Multiplying object speed by $10/30$ alone would use linear magnification incorrectly; longitudinal displacement and speed require its square. No assumption of a constant image speed over a finite interval is needed.

Step 3: Conclusion The instantaneous image speed is $\boxed{1\,\text{cm}\,\text{s}^{-1}}$. The numerical value to enter is 1.

Quick Tip to Solve: For mirror image speed, differentiate the mirror formula: the speed ratio is the square of the image-to-object distance ratio, not the linear magnification.

48. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m^2 . If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _____. [4 marks]

Solution:

Concept:

- The magnetic moment magnitude of an N-turn planar coil is $\mu = NIA$, where I is its current and A is the area of each turn.
- The magnetic torque magnitude is $\tau = \mu B \sin\theta = NIAB\sin\theta$, where θ is the angle between the coil's area normal and the magnetic field, rather than between its plane and the field.

Step 1: Translate the stated orientation into the torque angle The coil is held with its plane parallel to the magnetic field. Its normal is consequently perpendicular to the field, so $\theta = 90^\circ$ and $\sin\theta = 1$. The magnetic torque therefore has its maximum magnitude,

$$\tau = NIAB.$$

To keep the coil at rest, an applied holding torque must balance this magnetic torque in the opposite direction. Thus the stated $1.5\,\text{N}\,\text{m}$ is the magnitude used in the magnetic torque equation.

Step 2: Calculate the magnetic moment and field The given turn count is $N = 500$, current is $I = 0.5\,\text{A}$, and area per turn is $A = 3 \times 10^{-4}\,\text{m}^2$. Thus

$$NIA = 500 \times 0.5 \times 3 \times 10^{-4} = 0.075\,\text{A}\,\text{m}^2.$$

The required field strength is

$$B = \frac{\tau}{NIA} = \frac{1.5}{0.075} = 20\,\text{T}.$$

The area is already in square metres and the 500 turns are counted once. Using the angle of the plane directly would incorrectly give zero torque; the correct angle belongs to the normal. Finally $\mathrm{N\,m/(A\,m^2)} = \mathrm{T}$, confirming the unit.

Step 3: Conclusion The field strength is $\boxed{20\ \mathrm{T}}$, so the numerical value to enter is 20.

Quick Tip to Solve: In coil-torque questions, identify the angle of the normal to the field: a plane parallel to the field gives maximum torque, not zero torque.

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49. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0 . It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$. The nearest integer to I is____. [4 marks]

Solution:

Concept:

- Kinetic friction has magnitude $\mu_k mg\cos\theta$ and always opposes sliding. Gravity contributes $mg\sin\theta$ down the incline, where $\theta = 30^\circ$.
- The work-energy theorem relates the change in kinetic energy to the net work. The outward and return distances are the same, but the gravitational work reverses sign while frictional work remains negative.

Step 1: Write the energy balance during ascent Let s be the distance travelled before the block stops. Both gravity and friction oppose its upward velocity. Therefore the initial kinetic energy pays for the gain in potential energy and for frictional dissipation:

$$\frac{1}{2}mv_0^2 = mgs(\sin\theta + \mu_k\cos\theta).$$

The mass and the unknown distance need not be calculated separately.

Step 2: Write the return balance and eliminate the distance On descent gravity does positive work while friction still does negative work. The given return speed is $v_0/2$, so

$$\frac{1}{2}m(v_0/2)^2 = mgs(\sin\theta - \mu_k\cos\theta).$$

Dividing by the ascent equation gives

$$\frac{1}{4} = \frac{\sin\theta - \mu_k\cos\theta}{\sin\theta + \mu_k\cos\theta}.$$

Consequently $5\mu_k\cos\theta = 3\sin\theta$, and

$$\mu_k = \frac{3}{5}\tan 30^\circ = \frac{\sqrt{3}}{5} = 0.346410\ldots.$$

The return is physically possible because this is less than $\tan 30^\circ$.

Step 3: Conclusion Thus $I = 1000\mu_k = 346.410\ldots$, and the nearest integer is $\boxed{346}$. Both I and the

friction coefficient are dimensionless.

Quick Tip to Solve: For the same incline distance, divide the return energy equation by the ascent equation; the ratio uses the square of the speed ratio, not the speed ratio itself.

50. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer ? [4 marks]

Solution:

Concept:

- A refrigerator removes heat Q_c from a cold reservoir and rejects $Q_h = Q_c + W$ to the surroundings. Minimum work for prescribed reservoir temperatures is achieved reversibly.
- For a reversible cyclic refrigerator the entropy balance is $Q_h/T_h = Q_c/T_c$. Temperatures in this relation must be absolute temperatures, even though their differences can be expressed in degrees Celsius.

Step 1: Calculate the heat extracted while water freezes The water begins at its freezing point and becomes ice at the same temperature. Only latent heat must be removed; no separate sensible cooling term is needed. With mass $m = 100\,\mathrm{g}$ and latent heat $L = 80\,\mathrm{cal\,g^{-1}}$,

$$Q_c = mL = 100 \times 80 = 8000\,\mathrm{cal}.$$

The cold reservoir is at $T_c = 273\,\mathrm{K}$, and the surroundings are at $T_h = 27 + 273 = 300\,\mathrm{K}$.

Step 2: Use reversible heat transfer to find the rejected heat The phrase minimum possible work requires the reversible limit. Hence

$$Q_h = Q_c \frac{T_h}{T_c} = 8000 \frac{300}{273} = 8791.20879\ldots\ \mathrm{cal}.$$

As a check, the required work is

$$W_{\min} = Q_h - Q_c = 791.20879\ldots\ \mathrm{cal} > 0.$$

The rejected heat exceeds the latent heat extracted because the surroundings receive both that heat and the work supplied to the refrigerator. Using Celsius temperatures in the ratio, or identifying Q_h with Q_c, would violate this balance.

Step 3: Conclusion The heat released to the surroundings, to the nearest integer, is $\boxed{8791\ \mathrm{cal}}$.

Quick Tip to Solve: For minimum refrigerator work, first find the latent heat removed, then multiply it by the hot-to-cold absolute-temperature ratio to obtain the heat rejected.

Chemistry – Section A: Multiple Choice Questions (80 marks)

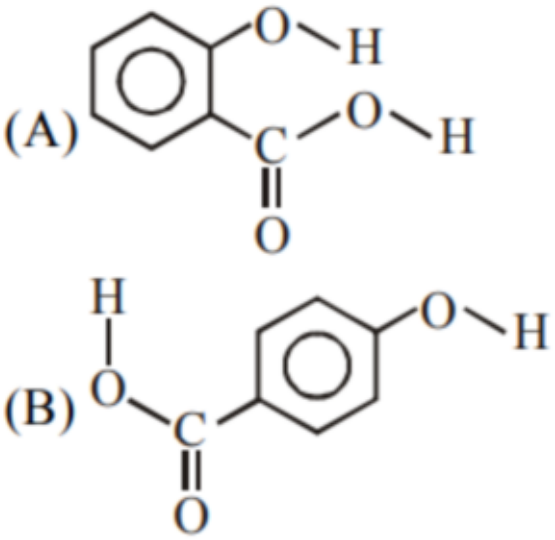
51. Consider the following molecules and statements related to them :

(a) (B) is more likely to be crystalline than (A)

(b) (B) has higher boiling point than (A)

(c) (B) dissolves more readily than (A) in water

Identify the correct option from below : [4 marks]



(A) only (a) is true

(B) (a), (b) and (c) are true

(C) (b) and (c) are true

(D) (a) and (b) are true

Correct Answer: (B) (a), (b) and (c) are true

Solution:

Concept:

- Molecule (A) is ortho-hydroxybenzoic acid and molecule (B) is para-hydroxybenzoic acid. Their identical functional groups occupy different relative positions.
- Intramolecular hydrogen bonding occurs within one molecule and reduces its availability for association with neighbours. Intermolecular hydrogen bonding links different molecules and can strengthen bulk association.

Step 1: Compare hydrogen bonding and crystal packing In (A), the adjacent phenolic hydroxyl and carboxyl groups can form a favourable six-membered intramolecular hydrogen bond. In (B), the para groups are too far apart for that internal association, so hydrogen bonding primarily connects different molecules. The para arrangement is also more symmetric and supports more regular packing in a crystal. Therefore (B) is more likely to be crystalline than (A), making statement (a) true.

Step 2: Evaluate boiling behaviour and hydration The stronger intermolecular association in (B)

requires more energy to separate molecules in the usual comparison of these isomers; it is therefore assigned the higher boiling point in statement (b). In (A), internal hydrogen bonding masks part of the polar functionality. The hydroxyl and carboxyl groups of (B) remain available to hydrogen-bond with water, making its interaction with water more favourable and its dissolution more ready in the comparison asked. Thus (c) is also true. The options that omit (a), (b), or (c) each discard one of these consequences of the ortho-versus-para arrangement.

Step 3: Conclusion All three statements are true. Hence, the correct option is (B).

Quick Tip to Solve: In an ortho-versus-para hydrogen-bonding comparison, check whether an internal six-membered hydrogen bond ties up the polar groups before comparing intermolecular association.

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52. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is : [4 marks]

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

Concept:

- Successive ionization enthalpies are the energies needed to remove electrons one at a time from an isolated gaseous atom and its increasingly positive ions.
- The major jump occurs after all valence electrons have been removed: the next electron belongs to a compact inner shell and is much more strongly bound. A gradual rise within a shell should not be mistaken for this shell-change discontinuity.

Step 1: Locate the discontinuity in the data The enthalpies are $800,\ 2427,\ 3658,\ 25024,\ 32824\,\text{kJ mol}^{-1}$. The consecutive increases are

$$2427 - 800 = 1627,$$
$$3658 - 2427 = 1231,$$
$$25024 - 3658 = 21366,$$
$$32824 - 25024 = 7800$$

in kJ mol^{-1}. The striking jump is from the third ionization to the fourth: the fourth value is nearly seven times the third.

Step 2: Identify the number of outer-shell electrons The first three removals take electrons from the valence shell. After the third removal, a stable inner-shell configuration is exposed. Removing the fourth electron breaks into this core and demands the exceptionally large energy. Therefore there were three valence electrons in the neutral atom. Two valence electrons would place the large jump at the third removal, not the fourth. Four or five valence electrons would put it at the fifth or sixth removal, respectively. These alternatives are inconsistent with the location of the observed discontinuity.

Step 3: Conclusion The number of valence electrons is $\boxed{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Count the electrons removed before the first major shell-change jump; do not count the high-energy core electron that produces the jump.

53. The strengths of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mass percentage and molarity (M), respectively, are: (Take molar mass of hydrogen peroxide as 34 g/mol) [4 marks]

(A) 0.85 and 0.5

(B) 0.85 and 0.25

(C) 1.7 and 0.25

(D) 1.7 and 0.5

Correct Answer: (D) 1.7 and 0.5

Solution:

Concept:

- Volume strength is the volume of oxygen at STP liberated by one volume of hydrogen peroxide solution on complete decomposition.
- The decomposition is $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. Thus one mole of peroxide gives half a mole of oxygen, or $11.2\,L$ at STP.

Step 1: Convert volume strength to molarity Take $1\,L$ of the given solution. A strength of 5.6 volumes means that it produces $5.6\,L$ of oxygen at STP. Therefore

$$n(O_2) = \frac{5.6}{22.4} = 0.25\,\text{mol}.$$

The balanced equation requires two moles of peroxide per mole of oxygen, so

$$n(H_2O_2) = 2(0.25) = 0.50\,\text{mol}.$$

Since this amount was in one litre, the molarity is $0.50\,\text{mol}\,L^{-1}$, not $0.25\,\text{mol}\,L^{-1}$.

Step 2: Convert the solute mass to a mass percentage The peroxide mass in that litre is

$$m(H_2O_2) = 0.50 \times 34 = 17\,g.$$

The solution density is $1\,g\,mL^{-1}$, so its total mass is $1000\,g$. Consequently

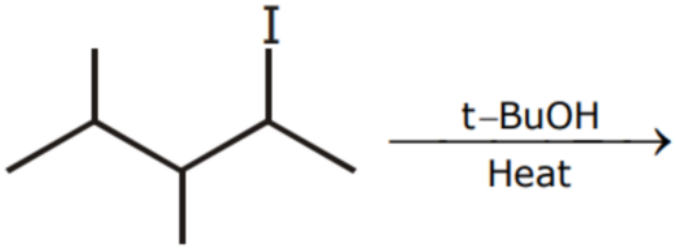
$$\text{mass percentage} = \frac{17}{1000} \times 100 = 1.7\%.$$

The 0.85 percent alternatives miss the factor of two in the peroxide-to-oxygen stoichiometry, while the 0.25 M alternatives count oxygen moles as peroxide moles. Only one option contains both correct values in the requested order.

Step 3: Conclusion The mass percentage and molarity are $\boxed{1.7\%\text{ and }0.5\,M}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Use volume strength $= 11.2M$ for peroxide at STP, then use the actual solution density to convert grams per litre to mass percent.

54. The major product in the following reaction is : [4 marks]



(A) $(CH_3)_2C{=}C(CH_3)CH_2CH_3$

(B) $CH_3CH[OC(CH_3)_3]CH(CH_3)CH(CH_3)CH_3$

(C) $CH_2{=}CHCH(CH_3)CH(CH_3)CH_3$

(D) (E)-3,4-dimethylpent-2-ene, $CH_3CH{=}C(CH_3)CH(CH_3)CH_3$

Correct Answer: (A) $(CH_3)_2C{=}C(CH_3)CH_2CH_3$

Solution:

Concept:

- The substrate is a secondary alkyl iodide heated in tert-butanol, a protic solvent and weak, bulky nucleophile. These conditions favour ionization followed by elimination.
- In an E1 pathway, carbocation rearrangement can precede proton loss. A hydride shift is favoured when it converts a secondary carbocation into a more stable tertiary carbocation; elimination then favours the more substituted alkene.

Step 1: Form and rearrange the carbocation Write the substrate from the iodide end as

$$CH_3CH(I)CH(CH_3)CH(CH_3)CH_3.$$

. Iodide departs from the secondary carbon, producing

$$CH_3CH^+CH(CH_3)CH(CH_3)CH_3.$$

. The adjacent carbon bears a hydrogen and is bonded to three carbon groups. Migration of that hydrogen with its bonding electron pair to the positively charged carbon gives a tertiary carbocation:

$$CH_3CH_2C^+(CH_3)CH(CH_3)CH_3.$$

This 1,2-hydride shift preserves the carbon skeleton and increases carbocation stability.

Step 2: Select the most stable elimination product Loss of the beta hydrogen from the neighbouring $CH(CH_3)_2$ group now gives

$$CH_3CH_2C(CH_3){=}C(CH_3)_2.$$

This is the tetrasubstituted alkene shown in option (A), 2,3-dimethylpent-2-ene. The unrearranged internal alkene (D) is only trisubstituted, and the terminal alkene (C) is still less substituted. Option (B) is a substitution product, which is disfavoured relative to elimination on heating under these conditions. The reagent shown is tert-butanol, not potassium tert-butoxide; treating it as a strong

bulky E2 base would incorrectly favour the terminal alkene.

Step 3: Conclusion The major product is $(CH_3)_2C{=}C(CH_3)CH_2CH_3$, the rearranged tetrasubstituted alkene. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish tert-butanol from tert-butoxide: a weak alcohol solvent allows carbocation rearrangement, so check for a hydride shift before choosing the alkene.

55. Among the statements (I – IV), the correct ones are:

(I) Be has smaller atomic radius compared to Mg.

(II) Be has higher ionization enthalpy than Al.

(III) Charge/radius ratio of Be is greater than that of Al.

(IV) Both Be and Al form mainly covalent compounds. [4 marks]

(A) (I), (II) and (IV)

(B) (II), (III) and (IV)

(C) (I), (III) and (IV)

(D) (I), (II) and (III)

Correct Answer: (A) (I), (II) and (IV)

Solution:

Concept:

- Atomic radius generally increases down a group because a new electron shell is added; first ionization enthalpy depends on effective nuclear attraction and the occupied subshell.
- The charge/radius comparison uses charge and radius consistently; aluminium has a much larger nuclear charge. The Be–Al diagonal relationship also accounts for their polarizing power and covalent bonding.

Step 1: Check the size and ionization statements Be and Mg belong to group 2, with Be above Mg. Be has fewer occupied shells, so its atomic radius is smaller: statement (I) is true. Be has the stable $2s^2$ valence configuration and a compact atom. Aluminium loses a more weakly held $3p$ electron. Thus the first ionization enthalpy of Be is higher than that of Al, making (II) true.

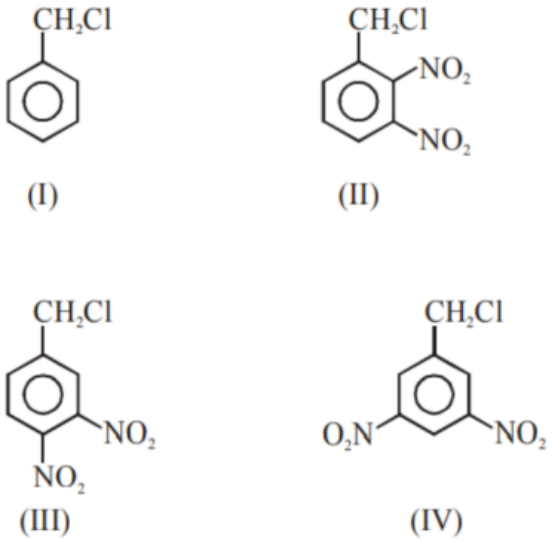
Step 2: Compare charge density and bonding For the atomic charge/radius comparison, the nuclear charges are $+4e$ for Be and $+13e$ for Al. Aluminium has a larger atomic radius, but this increase does not compensate for its much larger nuclear charge. Thus Be does not have the greater atomic charge/radius ratio asserted in (III). This comparison should not be confused with the approximate similarity of the ionic potentials of their common cations. Both small, strongly polarizing cations produce appreciable covalent character in their compounds; this is a standard Be–Al diagonal resemblance. Therefore (IV) is true.

Step 3: Eliminate the incorrect combinations The correct set is (I), (II) and (IV). Option (B) wrongly includes (III) and omits the true size statement. Option (C) includes (III) and omits the true ionization statement. Option (D) also includes (III), while omitting the true bonding statement.

Step 4: Conclusion The correct statements are (I), (II) and (IV). Hence, the correct option is (A).

Quick Tip to Solve: For a charge/radius comparison, identify whether atomic nuclear charge or ionic charge is meant, and use the corresponding radius consistently.

56. The decreasing order of reactivity of the following compounds towards nucleophilic substitution (S_N2) is : [4 marks]



(A) (IV) > (II) > (III) > (I)

(B) (II) > (III) > (IV) > (I)

(C) (II) > (III) > (I) > (IV)

(D) (III) > (II) > (IV) > (I)

Correct Answer: (D) (III) > (II) > (IV) > (I)

Solution:

Concept:

- In an S_N2 substitution, the nucleophile attacks the carbon bearing the leaving group as the carbon–halogen bond breaks in a single concerted step.
- For the substituted benzyl chlorides shown, electron-withdrawing nitro groups favour nucleophilic attack. Their positions matter: a para nitro group transmits a resonance effect, while an ortho substituent also introduces steric hindrance.

Step 1: Identify the four benzyl chlorides All four compounds have the same benzylic CH_2Cl reacting group, so their differences arise from ring substituents. Compound (I) is unsubstituted benzyl chloride. Compound (II) has ortho and meta nitro groups relative to CH_2Cl; (III) has meta and para nitro groups; (IV) has two meta nitro groups. The leaving group and benzylic carbon type are otherwise unchanged.

Step 2: Compare the positional effects Compound (III) benefits from the strong withdrawing influence of the para nitro group together with the additional meta nitro group, without an ortho group close to the benzylic reaction centre. Compound (II) also has strong nitro-group withdrawal, but its ortho substituent hinders approach and makes it less reactive than (III). Compound (IV), with both nitro groups meta, has predominantly inductive withdrawal and is slower than (II), but is still activated relative to unsubstituted (I).

Step 3: Match the complete sequence These comparisons give

$$(\text{III}) > (\text{II}) > (\text{IV}) > (\text{I}).$$

Option (A) incorrectly puts the dimeta compound first. Options (B) and (C) reverse the order of (II) and (III); option (C) also places the unsubstituted compound ahead of the activated dimeta compound. Only (D) satisfies every comparison.

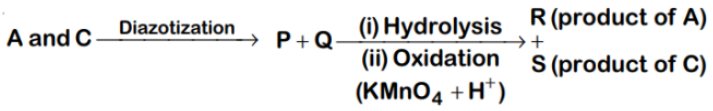
Step 4: Conclusion The decreasing reactivity is $(\text{III}) > (\text{II}) > (\text{IV}) > (\text{I})$. Hence, the correct option is (D).

Quick Tip to Solve: For substituted benzyl halides, identify each substituent position before comparing withdrawal; include the steric cost of an ortho substituent.

57. Three isomers A, B and C (mol. formula $C_8H_{11}N$) give the following results :

R has lower boiling point than S

A, B and C, respectively are : [4 marks]





(A) A: 2-ethylaniline; B: 3-methylbenzylamine; C: 3-ethylaniline.

(B) A: 3-ethylaniline; B: N-methylbenzylamine; C: 2-ethylaniline.

(C) A: 4-ethylaniline; B: N-ethylaniline; C: 2-ethylaniline.

(D) A: 2-ethylaniline; B: N-methylbenzylamine; C: 4-ethylaniline.

Correct Answer: (D) A: 2-ethylaniline; B: N-methylbenzylamine; C: 4-ethylaniline.

Solution:

Concept:

- A primary aromatic amine forms a diazonium salt, which on hydrolysis replaces NH_2 by OH. Acidified permanganate oxidizes an alkyl side chain having a benzylic hydrogen to COOH.
- A secondary amine reacts with benzenesulfonyl chloride to give a sulfonamide without an N–H bond; this product does not dissolve in alkali. Intramolecular hydrogen bonding can reduce intermolecular association and lower boiling point.

Step 1: Use the test for B The alkali-insoluble Hinsberg product identifies B as a secondary amine. A primary amine gives a sulfonamide retaining an acidic N–H proton and hence an alkali-soluble salt; a tertiary amine does not give this sulfonamide. The benzylamine in option (A) is primary, so that option is excluded. The secondary amines in the remaining alternatives require the boiling-point comparison to distinguish them.

Step 2: Identify the products from A and C Diazotization followed by hydrolysis converts each ethylaniline into the corresponding ethylphenol. Subsequent oxidation of the ethyl side chain converts it to a carboxyl group, preserving its position relative to the phenolic group. Thus 2-ethylaniline produces 2-hydroxybenzoic acid, whereas 4-ethylaniline produces 4-hydroxybenzoic acid. The ortho acid forms an intramolecular hydrogen bond between neighbouring hydroxy and carboxyl groups. The para acid cannot form this intramolecular bond and associates more strongly between molecules.

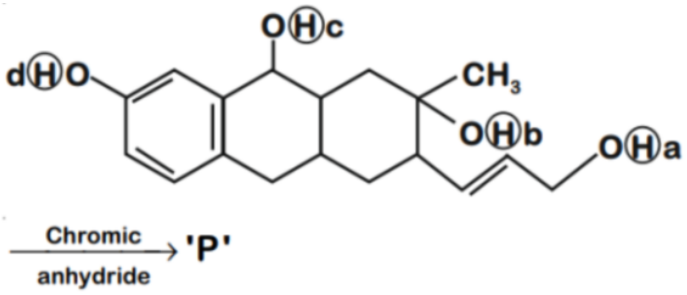
Step 3: Apply the lower-boiling-point condition Since R, the product of A, has the lower boiling point, A must give the ortho acid and C the para acid. Therefore A is 2-ethylaniline, B is N-methylbenzylamine and C is 4-ethylaniline. Option (B) puts a meta isomer at A; option (C) puts the para isomer at A and the ortho isomer at C, reversing the required association trend. Option (D) alone has the correct assignment.

Step 4: Conclusion A, B and C are respectively 2-ethylaniline, N-methylbenzylamine and 4-ethylaniline. Hence, the correct option is (D).

Quick Tip to Solve: Use the Hinsberg test to classify B first, then compare the positions of OH and COOH in the oxidation products rather than in the starting amines.

58. Consider the following reaction :

The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these –OH group(s)? [4 marks]



(A) (b) only

(B) (b) and (d)

(C) (c) and (d)

(D) (d) only

Correct Answer: (A) (b) only

Solution:

Concept:

- Chromic anhydride oxidizes primary and secondary alcohols because the carbon carrying OH has a hydrogen that can be removed during carbonyl formation.
- A tertiary alcohol has no hydrogen on its OH-bearing carbon and resists this ordinary oxidation. Ceric ammonium nitrate is used as a qualitative test for an alcoholic OH group; a phenolic OH is distinguished from an alcoholic OH.

Step 1: Classify the labelled hydroxy groups Group (a) is the terminal primary alcohol: its OH-bearing carbon is bonded to one carbon. Group (c) is secondary because its carbon is attached to two other carbon atoms. Group (b) is tertiary: its carbon is attached to two ring carbons and a methyl group, leaving no C–H bond there. Group (d) is phenolic because its oxygen is directly bonded to an aromatic ring carbon.

Step 2: Determine which alcoholic group survives On treatment with chromic anhydride, the primary group (a) and secondary group (c) are oxidized to carbonyl-containing functionality, so they no longer provide the relevant alcohol test in P. Group (b) remains an alcohol because its tertiary carbon cannot undergo the usual alcohol-to-carbonyl oxidation without carbon–carbon bond cleavage. The phenolic site (d), whether considered separately in oxidation chemistry or in identification, is not the surviving alcoholic OH responsible for the stated test.

Step 3: Eliminate the alternative assignments The positive ceric ammonium nitrate test of P is therefore assigned to (b) only. Option (B) improperly includes phenolic (d) with the alcohol. Option (C) includes the oxidized secondary alcohol (c) as well as phenolic (d). Option (D) attributes the alcohol test solely to the phenolic group and overlooks the resistant tertiary alcohol.

Step 4: Conclusion The positive test is due to the surviving tertiary alcoholic OH group labelled b only. Hence, the correct option is (A).

Quick Tip to Solve: For oxidation followed by an alcohol test, classify the OH-bearing carbon first: tertiary alcohols lack the C–H bond needed for ordinary carbonyl formation.

59. For the reaction

2A + 3B + $\frac{3}{2}$C $\rightarrow$ 3P, which statement is correct ? [4 marks]

(A) $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$

(B) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

(C) $\frac{dn_A}{dt} = \frac{3}{2}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

(D) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$

Correct Answer: (D) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$

Solution:

Concept:

- Introduce the extent of reaction ξ. The mole changes equal each signed stoichiometric coefficient times $d\xi$; reactant amounts decrease together in the coefficient ratio.
- For $2\mathrm{A}+3\mathrm{B}+\frac{3}{2}\mathrm{C} \rightarrow 3\mathrm{P}$, the common positive extent rate is $-\frac{1}{2}\,dn_A/dt = -\frac{1}{3}\,dn_B/dt = -\frac{2}{3}\,dn_C/dt$.

Step 1: Write the mole-change relations Let $\dot{\xi} = d\xi/dt$. Consumption of A, B and C gives

$$\frac{dn_A}{dt} = -2\dot{\xi}, \qquad \frac{dn_B}{dt} = -3\dot{\xi}, \qquad \frac{dn_C}{dt} = -\frac{3}{2}\dot{\xi}.$$

All three derivatives are negative during the forward reaction. Their signs are the same, but their magnitudes are not equal because different mole amounts are consumed per unit extent.

Step 2: Express each derivative in terms of A Eliminate the common extent rate between A and B:

$$\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt}.$$

Similarly, the coefficient ratio for A and C is $2/(3/2) = 4/3$, giving

$$\frac{dn_A}{dt} = \frac{4}{3}\frac{dn_C}{dt}.$$

No extra minus sign is inserted between reactant derivatives because all are consumption derivatives.

Step 3: Test the printed options Option (A) assumes equal consumption rates despite unequal coefficients. Option (B) has the correct A-to-B factor but uses the reciprocal of the required A-to-C factor. Option (C) uses incorrect factors for both comparisons. Option (D) has the coefficient ratios $2/3$ and $4/3$, and substituting the three derivatives above verifies that every term equals $-2\dot{\xi}$.

Step 4: Conclusion

$$\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Check a rate relation by replacing each mole derivative with its signed coefficient times the same extent rate; all terms must become identical.

60. Complex A has a composition of $H_{12}O_6Cl_3Cr$. If the complex on treatment with conc.H_2SO_4 loses 13.5% of its original mass, the correct molecular formula of A is :

[Given: atomic mass of Cr = 52 amu and Cl = 35 amu] [4 marks]

(A) $[Cr(H_2O)_5Cl]Cl_2.H_2O$

(B) $[Cr(H_2O)_4Cl_2]Cl.2H_2O$

(C) $[Cr(H_2O)_3Cl_3].3H_2O$

(D) $[Cr(H_2O)_6]Cl_3$

Correct Answer: (B) $[Cr(H_2O)_4Cl_2]Cl.2H_2O$

Solution:

Concept:

- Water inside the coordination sphere is bound to the metal; water written outside the brackets is water of crystallization and is removed by a dehydrating agent.
- The total composition fixes the molar mass. The mass fraction removed therefore determines the number of external water molecules per formula unit without changing the total number of waters originally present.

Step 1: Calculate the molar mass from the given composition The empirical composition contains one Cr, three Cl and six water equivalents: $H_{12}O_6Cl_3Cr = CrCl_3 \cdot 6\,H_2O$ as an overall composition.

Using the supplied chlorine mass,

$$M = 52 + 3(35) + 6(18) = 265 \text{ g mol}^{-1}.$$

The approximate percentage in the question must be evaluated with these given masses rather than substituting a different chlorine atomic mass.

Step 2: Find the number of removable waters Concentrated sulfuric acid removes water of crystallization. The mass lost per mole is

$$0.135(265) = 35.775 \text{ g mol}^{-1} \approx 36 \text{ g mol}^{-1}.$$

Since each water molecule contributes 18 g mol^{-1}, the number outside the coordination sphere is

$$n = \frac{35.775}{18} \approx 2.$$

The slight difference from exactly 2 is the rounding implicit in the stated 13.5 percent.

Step 3: Locate the matching hydrate isomer Two external waters leave four water ligands inside the sphere. The six-coordinate chromium(III) complex therefore contains four aqua ligands and two coordinated chlorides, with one chloride counterion: $[Cr(H_2O)_4Cl_2]Cl \cdot 2\,H_2O$. Option (A) has only one external water and would lose about 6.8 percent; option (C) has three and would lose about 20.4 percent. Option (D) has no external water and cannot account for the observed dehydration.

Step 4: Conclusion The molecular formula is $[Cr(H_2O)_4Cl_2]Cl \cdot 2\,H_2O$. Hence, the correct option is (B).

Quick Tip to Solve: Convert dehydration percentage to grams per mole, divide by 18, and place that many water molecules outside the coordination brackets.

61. The d-electron configuration of $[Ru(en)_3]Cl_2$ and $[Fe(H_2O)_6]Cl_2$, respectively are : [4 marks]

(A) $t_{2g}^4 e_g^2$ and $t_{2g}^6 e_g^0$
(B) $t_{2g}^6 e_g^0$ and $t_{2g}^6 e_g^0$
(C) $t_{2g}^6 e_g^0$ and $t_{2g}^4 e_g^2$
(D) $t_{2g}^4 e_g^2$ and $t_{2g}^4 e_g^2$

Correct Answer: (C) $t_{2g}^6 e_g^0$ and $t_{2g}^4 e_g^2$

Solution:

Concept:

- In an octahedral complex the five metal d orbitals split into lower-energy t_{2g} and higher-energy e_g sets. The competition between splitting energy Δ_o and pairing energy P determines the spin state.
- Neutral ligands do not change the oxidation-state calculation. Both ethylenediamine (en) and water are neutral, so two external chloride ions imply a metal oxidation state of $+2$.

Step 1: Count the metal d electrons For $[Ru(en)_3]Cl_2$, the complex cation is $[Ru(en)_3]^{2+}$. Ruthenium is a group-8 metal, so Ru^{2+} has a $4d^6$ configuration. Similarly, $[Fe(H_2O)_6]^{2+}$ contains Fe^{2+}, whose configuration is $3d^6$. Thus both complexes contain six d electrons; their different occupancies arise from their different ligand-field splittings.

Step 2: Fill the octahedral levels The 4d metal ruthenium has relatively large octahedral splitting, strengthened further by the en ligands. Pairing in the lower set is favoured, giving low-spin $t_{2g}^6 e_g^0$, with no unpaired electrons. In contrast, water is a weak-field ligand for the 3d ion Fe^{2+}. Its smaller splitting favours the high-spin arrangement $t_{2g}^4 e_g^2$, containing four unpaired electrons.

Step 3: Compare both entries in each option Option (C) gives the low-spin ruthenium configuration first and the high-spin iron configuration second. Option (A) reverses these two occupancies. Option (B) incorrectly makes the aqua complex low spin, while option (D) incorrectly makes the ruthenium complex high spin. Equal d-electron counts therefore do not imply equal spin states.

Step 4: Conclusion The respective configurations are $t_{2g}^6 e_g^0$ and $t_{2g}^4 e_g^2$. Hence, the correct option is (C).

Quick Tip to Solve: For an octahedral d-six pair, first identify whether each metal is 3d or 4d and then compare ligand strength; do not assign both ions the same spin state.

62. The incorrect statement is : [4 marks]

(A) In manganate and permanganate ions, the p-bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese

(B) Manganate ion is green in colour and permanganate ion in purple in colour

(C) Manganate and permanganate ions are paramagnetic

(D) Manganate and permanganate ions are tetrahedral

Correct Answer: (C) Manganate and permanganate ions are paramagnetic

Solution:

Concept:

- Manganese has atomic number 25 and a group-7 valence count. For these oxoanions, the formal d count is 7 − oxidation state. An unpaired electron makes an ion paramagnetic.
- Manganate, MnO_4^{2-}, and permanganate, MnO_4^-, both have tetrahedral oxygen arrangements. Their characteristic colours must be distinguished from the origin of magnetism.

Step 1: Calculate the oxidation states Taking each oxygen as -2, write $x+4(-2) = -2$ for manganate, which gives $x = +6$. For permanganate, $x + 4(-2) = -1$, so $x = +7$. Consequently their metal-ion configurations are $d^{7-6} = d^1$ and $d^{7-7} = d^0$, respectively. This comparison immediately distinguishes their magnetic behaviour.

Step 2: Test the magnetic statement Manganate has one d electron, which remains unpaired, and is therefore paramagnetic. Permanganate has no d electrons and no unpaired electrons, so it is diamagnetic. The statement that both ions are paramagnetic is false. The intense purple colour of permanganate does not imply an unpaired d electron; its colour is associated with charge-transfer

absorption.

Step 3: Check the remaining statements Manganate is green and permanganate is purple, so (B) is correct. Both anions are tetrahedral, so (D) is correct. In the conventional bonding description used for these oxoanions, oxygen p orbitals overlap with manganese d orbitals in the additional metal–oxygen bonding, corresponding to (A). Thus the false magnetic claim in (C), rather than either colour or geometry, is the requested incorrect statement.

Step 4: Conclusion MnO_4^{2-} is paramagnetic but MnO_4^{-} is diamagnetic. Hence, the correct option is (C).

Quick Tip to Solve: Find manganese oxidation states before discussing magnetism: manganate is d-one, whereas permanganate is d-zero despite its intense colour.

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63. The increasing order of the reactivity of the following compound in nucleophilic addition reaction is :

Propanal, Benzaldehyde, Propanone, Butanone [4 marks]

(A) Butanone $<$ Propanone $<$ Benzaldehyde $<$ Propanal

(B) Benzaldehyde $<$ Butanone $<$ Propanone $<$ Propanal

(C) Propanal $<$ Propanone $<$ Butanone $<$ Benzaldehyde

(D) Benzaldehyde $<$ Propanal $<$ Propanone $<$ Butanone

Correct Answer: (A) Butanone $<$ Propanone $<$ Benzaldehyde $<$ Propanal

Solution:

Concept:

- A nucleophile attacks the electrophilic carbon atom of a carbonyl group. Alkyl groups reduce that electrophilicity by electron donation and also hinder approach to the carbonyl carbon.
- Aldehydes generally undergo nucleophilic addition more readily than ketones because an aldehyde has only one carbon substituent and one small hydrogen atom at its carbonyl carbon.

Step 1: Compare the two ketones Propanone is CH_3COCH_3, whereas butanone is $CH_3COCH_2CH_3$. Replacing a methyl group by the larger, more electron-donating ethyl group makes nucleophilic attack less favourable. Both the increased crowding around the carbonyl group and the greater positive inductive effect lower butanone reactivity. Thus butanone $<$ propanone.

Step 2: Compare the two aldehydes Propanal, CH_3CH_2CHO, is an aliphatic aldehyde. Benzaldehyde, C_6H_5CHO, has its carbonyl group conjugated with the aromatic ring. Resonance reduces the effective positive character of the carbonyl carbon and stabilises the starting carbonyl compound. Consequently benzaldehyde is less reactive than propanal, while its aldehydic carbonyl remains more accessible than the ketonic carbonyls in this comparison.

Step 3: Assemble and test the increasing order Combining the steric and electronic trends gives

$$\text{butanone} < \text{propanone} < \text{benzaldehyde} < \text{propanal}.$$

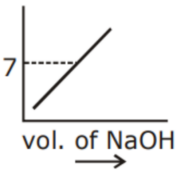
This is option (A). Option (B) wrongly places benzaldehyde below both ketones. Option (C) reverses

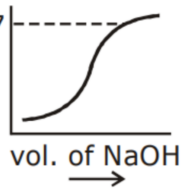
the aldehyde–ketone trend by placing propanal first. Option (D) likewise puts the aliphatic aldehyde below both ketones and also reverses the order within the ketone pair.

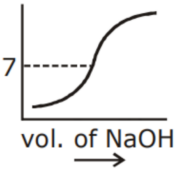
Step 4: Conclusion The increasing reactivity order is butanone $<$ propanone $<$ benzaldehyde $<$ propanal. Hence, the correct option is (A).

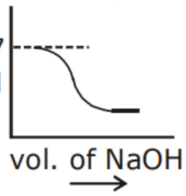
Quick Tip to Solve: Build the order in pairs: compare the ketones by alkyl crowding, compare the aldehydes by conjugation, and finally use the aldehyde–ketone trend.

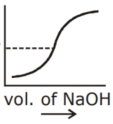
64. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- HCl and NaOH are strong electrolytes. Their net neutralisation reaction is $\mathrm{H^+ + OH^- \longrightarrow H_2O}$, with equal reacting amounts of hydrogen and hydroxide ions.
- The pH scale is logarithmic: $\mathrm{pH} = -\log_{10}[\mathrm{H^+}]$. A strong acid–strong base titration has a steep pH rise near equivalence, which is neutral at ordinary room temperature.

Step 1: Locate the equivalence volume The original acid amount is

$$n(\mathrm{HCl}) = 0.100 \times 0.1 = 0.0100 \text{ mol}.$$

Each 2 mL addition supplies $0.002 \times 0.1 = 0.0002$ mol of hydroxide. Thus 50 such additions, totalling 100 mL, exactly neutralise the acid. The original pH is 1; at equivalence only neutral sodium chloride remains and the pH is approximately 7.

Step 2: Determine the shape near equivalence Before equivalence, for V litres of added base,

$$[\mathrm{H^+}] = \frac{0.0100 - 0.1V}{0.100 + V}.$$

At 50 mL, the pH is about 1.48; at 98 mL, the remaining acid concentration is $0.0002/0.198$, giving pH about 3.00. The final small addition therefore produces a sharp rise towards 7. If base addition continues past equivalence, excess hydroxide makes the solution alkaline. This is the usual rising sigmoidal strong acid–strong base curve, with its steep central region passing through pH 7.

Step 3: Compare the plotted alternatives Graph (C) places pH 7 in the steep transition of a rising titration curve. Graph (A) incorrectly makes pH linear in base volume. Graph (B) puts 7 near the upper flattened part instead of the neutral equivalence region of the standard complete titration curve. Graph (D) decreases even though base is being added. The stated 100 mL addition reaches the neutral point on graph (C); the alkaline continuation depicts what happens upon adding further base.

Step 4: Conclusion The strong acid–strong base titration curve is graph (C), whose sharp rise passes through pH 7. Hence, the correct option is (C).

Quick Tip to Solve: Find the neutralisation volume first, then identify the rising curve whose steep equivalence region is at pH 7; equal pH increments do not follow equal base additions.

65. Consider the hypothetical situation where the azimuthal quantum number, l, takes values 0, 1, 2, ..., n + 1, where n is the principal quantum number. Then, the element with atomic number :

[4 marks]

(A) 13 has a half-filled valence subshell

(B) 9 is the first alkali metal

(C) 8 is the first noble gas

(D) 6 has a 2p-valence subshell

Correct Answer: (A) 13 has a half-filled valence subshell

Solution:

Concept:

- The question changes the allowed azimuthal quantum numbers. A subshell with quantum number l contains $2l + 1$ orbitals and can hold $2(2l + 1)$ electrons, with at most two per orbital.
- For $n = 1$, the hypothetical allowed values are $l = 0, 1, 2$, corresponding to 1s, 1p and 1d subshells. Their capacities are respectively 2, 6 and 10; a d subshell is half filled when it contains five electrons.

Step 1: Fill the hypothetical first-shell subshells Using the first-shell filling sequence intended in this hypothetical model, fill the 1s and 1p subshells before 1d. The first two subshells accommodate $2 + 6 = 8$ electrons. An atom with $Z = 13$ then has $13 - 8 = 5$ electrons in 1d:

$$\text{configuration}: \quad 1s^2\, 1p^6\, 1d^5.$$

By Hund's rule, these five electrons singly occupy the five d orbitals. The valence subshell is therefore half filled.

Step 2: Check the alkali and noble-gas claims The ordinary configurations of real elements must not be substituted into this altered model. With 1d now allowed in the first shell, $Z = 9$ has $1s^2 1p^6 1d^1$

in the intended sequence and is not the first alkali metal. Also, $Z = 8$ fills only 1s and 1p, leaving the allowed 1d subshell empty; it does not close this hypothetical first shell. A noble-gas conclusion cannot be drawn merely because the p subshell is full while another subshell of the same shell is allowed.

Step 3: Reject the remaining configuration For $Z = 6$, the intended configuration is $1s^2 1p^4$, so its valence electrons occupy a hypothetical 1p subshell rather than 2p. Thus (D) is false. Among the statements supplied, the half-filled 1d subshell at $Z = 13$ is the one consistent with the hypothetical first-shell scheme. The familiar rule $l \leq n - 1$ cannot be restored while solving a question that expressly replaces it.

Step 4: Conclusion Atomic number 13 gives $1s^2 1p^6 1d^5$, with a half-filled valence d subshell in the stipulated model. Hence, the correct option is (A).

Quick Tip to Solve: Recalculate the allowed subshells before using familiar electronic configurations: the altered rule permits 1p and 1d, with capacities six and ten.

66. The compound A in the following reaction is :

$$\mathrm{A} \xrightarrow[\text{(ii) conc. } \mathrm{H_2SO_4},\ \Delta]{\text{(i) } \mathrm{CH_3MgBr/H_2O}} \mathrm{B}$$

$$\mathrm{B} \xrightarrow[\text{(ii) } \mathrm{Zn/H_2O}]{\text{(i) } \mathrm{O_3}} \mathrm{C + D}$$

$$\mathrm{C} \xrightarrow[\text{(ii) } \Delta]{\text{(i) conc. KOH}} \mathrm{C_6H_5COO^-K^+ + C_6H_5CH_2OH}$$

$$\mathrm{D} \xrightarrow[\Delta]{\mathrm{Ba(OH)_2}} \mathrm{(CH_3)_2C{=}CHCOCH_3}$$

[4 marks]

(A) $\mathrm{C_6H_5COCH_2CH_3}$

(B) $\mathrm{C_6H_5COCH(CH_3)_2}$

(C) $\mathrm{C_6H_5COCH_3}$

(D) $\mathrm{C_6H_5CH_2COCH_3}$

Correct Answer: (D) $\mathrm{C_6H_5CH_2COCH_3}$

Solution:

Concept:

- Reductive ozonolysis cleaves an alkene double bond into carbonyl compounds. Each alkene carbon becomes a carbonyl carbon while retaining its other substituents.
- Benzaldehyde has no alpha hydrogen and undergoes Cannizzaro disproportionation with concentrated alkali. Acetone has alpha hydrogens and undergoes aldol condensation to mesityl oxide on heating.

Step 1: Identify products C and D The pair $\mathrm{C_6H_5COO^-K^+}$ and $\mathrm{C_6H_5CH_2OH}$ identifies C as benzaldehyde, $\mathrm{C_6H_5CHO}$: two benzaldehyde molecules disproportionate into benzoate and benzyl alcohol.

The product from D is $(CH_3)_2C{=}CHCOCH_3$, or mesityl oxide. It arises by self-aldol condensation and dehydration of acetone, so $D = CH_3COCH_3$.

Step 2: Reconstruct alkene B To reverse ozonolysis, join the two carbonyl carbons by a double bond and remove their oxygens. Benzaldehyde contributes a carbon bearing phenyl and hydrogen; acetone contributes one bearing two methyl groups. Therefore

$$B = C_6H_5CH{=}C(CH_3)_2.$$

Cleaving this bond gives precisely C_6H_5CHO and $(CH_3)_2CO$, as required by both subsequent reactions.

Step 3: Trace the Grignard addition and dehydration Option (D), $C_6H_5CH_2COCH_3$, accepts a methyl group from CH_3MgBr; aqueous work-up gives $C_6H_5CH_2C(OH)(CH_3)_2$. Acid-catalysed dehydration forms the more substituted, phenyl-conjugated alkene $C_6H_5CH{=}C(CH_3)_2$. Options (A), (B) and (C) have phenyl directly attached to the original carbonyl carbon. After methyl addition and dehydration, that phenyl-bearing alkene carbon also bears a methyl substituent and cannot yield benzaldehyde on cleavage. They fail the observed Cannizzaro-product test.

Step 4: Conclusion Compound A is $C_6H_5CH_2COCH_3$, 1-phenylpropan-2-one. Hence, the correct option is (D).

Quick Tip to Solve: Work backwards from the diagnostic products: Cannizzaro products identify benzaldehyde, and mesityl oxide identifies acetone; then reconstruct the alkene.

67. A mixture of one mole each of H_2 , He and O_2 each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of H_2 is 2 atm, the total pressure of the gases in the cylinder is : [4 marks]

(A) 14 atm

(B) 38 atm

(C) 6 atm

(D) 22 atm

Correct Answer: (C) 6 atm

Solution:

Concept:

- For a non-reacting ideal-gas mixture, Dalton's law states that the total pressure is the sum of the component partial pressures: $P = p_{H_2} + p_{He} + p_{O_2}$.
- The partial pressure of component i is $p_i = n_iRT/V$. At a common temperature and volume, partial pressure depends on mole number rather than molecular mass.

Step 1: Compare the amounts of the three gases The cylinder contains one mole each of hydrogen, helium and oxygen. All three occupy the same available volume V, and all are at temperature T. Hence $n_{H_2} = n_{He} = n_{O_2} = 1$, so their values of n_iRT/V are equal. Hydrogen has partial pressure $2\,\text{atm}$; therefore helium and oxygen each also have partial pressure $2\,\text{atm}$. The different molecular masses do not alter this conclusion.

Step 2: Add the partial pressures and check the options Apply Dalton's law to all three components:

$$P = 2 + 2 + 2 = 6\,\text{atm}.$$

Equivalently, hydrogen contributes one-third of the total mole number, so its mole fraction is $x_{H_2} = 1/3$. The relation $p_{H_2} = x_{H_2} P$ gives $P = 2/(1/3) = 6\,\text{atm}$, confirming the sum. Options (A), (B) and (D) give 14, 38 and 22 atm; none equals three times the pressure of one equal-mole component. They would incorrectly assign unequal pressure contributions to equal amounts of gas.

Step 3: Conclusion The total pressure is $\boxed{6\,\text{atm}}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal mole amounts of k ideal gases at a common temperature and volume, multiply any one partial pressure by k; do not weight by molar mass.

68. The incorrect statement(s) among (a) – (d) regarding acid rain is (are) :

(a) It can corrode water pipes.

(b) It can damage structures made up of stone.

(c) It cannot cause respiratory ailments in animals.

(d) It is not harmful for trees [4 marks]

(A) (c) and (d)

(B) (a), (b) and (d)

(C) (c) only

(D) (a), (c) and (d)

Correct Answer: (A) (c) and (d)

Solution:

Concept:

- Acid rain contains acids formed from atmospheric sulfur and nitrogen oxides. Its acidity can attack construction materials and metals and can damage vegetation.
- The question asks for incorrect statements: first decide the truth of each claim, paying particular attention to the negative words "cannot" and "not".

Step 1: Check the claims about pipes and stone Statement (a) is correct. Acidic water can attack metal water pipes, increase corrosion and dissolve metal ions. Thus corrosion of pipes is a possible harmful effect, and (a) must not be selected as incorrect. Statement (b) is also correct. Many stone structures contain carbonate minerals, which react with acids. For limestone or marble, the essential reaction is

$$CaCO_3 + 2\,H^+ \longrightarrow Ca^{2+} + CO_2 + H_2O.$$

This chemical attack damages the surface of the stone. Therefore both (a) and (b) describe harmful effects correctly.

Step 2: Check the biological effects and eliminate options Statement (c) is incorrect: the assertion that acid rain cannot cause respiratory ailments denies its adverse biological effects. Acidic pollution

and the associated sulfur and nitrogen oxide pollutants can harm respiratory systems. Statement (d) is incorrect as well. Acid rain can injure foliage, reduce growth and change soil chemistry by leaching nutrients and increasing the availability of toxic metal ions. It is therefore harmful to trees. The complete incorrect set is (c) and (d). Option (C) misses (d), while options (B) and (D) wrongly include the true statement (a); (B) also includes the true statement (b).

Step 3: Conclusion The third and fourth statements are incorrect. Hence, the correct option is (A).

Quick Tip to Solve: For an "incorrect statements" question, label each claim true or false before matching the option; a negative word can reverse an otherwise familiar fact.

69. Match the following drugs with their therapeutic actions:

(i) Ranitidine	**(a) Antidepressant**
(ii) Nardil (Phenelzine)	**(b) Antibiotic**
(iii) Chloramphenicol	**(c) Antihistamine**
(iv) Dimetane (Brompheniramine)	**(d) Antacid**
	(e) Analgesic

[4 marks]

(A) (i)-(a); (ii)-(c); (iii)-(b); (iv)-(e)

(B) (i)-(e); (ii)-(a); (iii)-(c); (iv)-(d)

(C) (i)-(d); (ii)-(a); (iii)-(b); (iv)-(c)

(D) (i)-(d); (ii)-(c); (iii)-(a); (iv)-(e)

Correct Answer: (C) (i)-(d); (ii)-(a); (iii)-(b); (iv)-(c)

Solution:

Concept:

- Drug classes are matched to their principal therapeutic action: antacids reduce gastric acidity, antidepressants treat depressive disorders, antibiotics act against bacteria, and antihistamines counter histamine-mediated allergy.
- The commercial names in the table identify specific drugs. Match all four entries separately before choosing the combined code; a single correct pair does not establish an option.

Step 1: Match the gastric and psychiatric drugs Ranitidine reduces gastric acid secretion by blocking histamine H_2 receptors in the stomach. In the classification used in this question it belongs to the antacid group, so (i) matches (d), rather than antidepressant or analgesic. Nardil is phenelzine, a monoamine oxidase inhibitor used as an antidepressant. It acts by limiting the breakdown of certain neurotransmitters; thus (ii) matches (a), not antihistamine. These two pairs already exclude (A) and (B), since neither assigns both of these actions correctly.

Step 2: Match the antibiotic and allergy drug Chloramphenicol is an antibiotic that inhibits bacterial protein synthesis. It is not an antidepressant or an antihistamine, so (iii) matches (b). Dimetane is brompheniramine, an H_1-receptor antihistamine used against allergic symptoms. Therefore (iv)

matches (c), not analgesic. Collecting the pairs gives

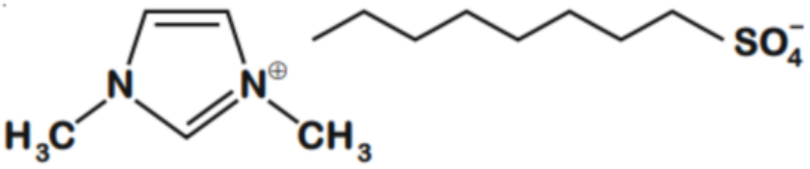
(i)–(d), (ii)–(a), (iii)–(b), (iv)–(c).

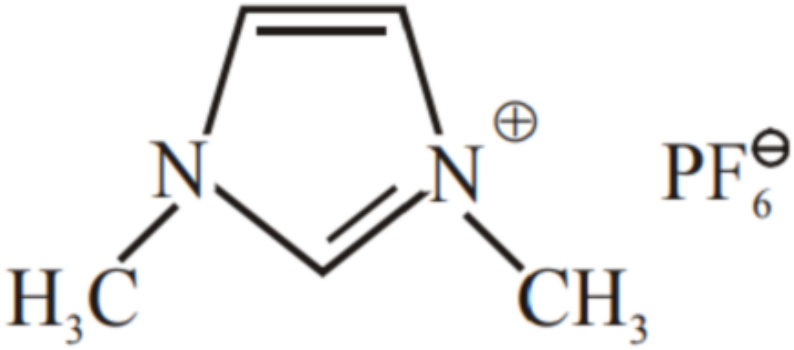
Option (D) has the first pair correct but assigns the remaining drugs inappropriate actions. Option (C) alone has all four correct pairs. The unused action (e), analgesic, need not be assigned to any drug.

Step 3: Conclusion The drugs match antacid, antidepressant, antibiotic and antihistamine, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Anchor this matching with phenelzine as an antidepressant and chloramphenicol as an antibiotic; then distinguish gastric H_2 blockade from allergy H_1 blockade.

70. An ionic micelle is formed on the addition of : [4 marks]

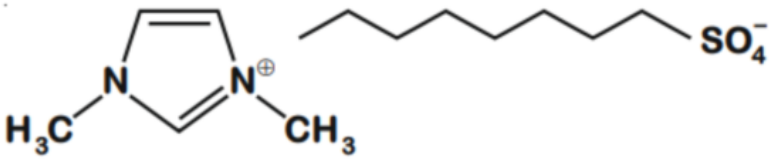
(A) excess water to liquid 

(B) excess water to liquid 

(C) liquid diethyl ether to aqueous NaCl solution

(D) sodium stearate to pure toluene

Correct Answer: (A)

excess water to liquid 

Solution:

Concept:

- An ionic micelle is an aggregate of surfactant ions in water. Its hydrophobic hydrocarbon chains cluster in the interior, while charged hydrophilic groups remain in contact with water.
- A suitable amphiphile has both a substantial nonpolar tail and an ionic head. Ionic character alone

is insufficient: small ions lacking a hydrophobic chain do not form ordinary surfactant micelles.

Step 1: Identify the amphiphile in the first liquid The first structure contains an imidazolium cation together with a long-chain alkyl sulfate anion. The alkyl sulfate ion has a hydrophobic hydrocarbon chain and a negatively charged sulfate head group. On adding sufficient water, the ionic liquid supplies these amphiphilic ions to an aqueous medium. When the aqueous surfactant concentration exceeds its critical micelle concentration (CMC), the alkyl chains aggregate away from water while the charged sulfate head groups face outward. Here the amphiphilic species is the alkyl sulfate anion. This is the structural requirement for an ionic micelle and identifies option (A). The important feature is the long-chain sulfate ion, rather than simply the presence of a positively charged nitrogen atom.

Step 2: Compare the remaining additions The second ionic liquid has the same small imidazolium-type cation but a hexafluorophosphate counterion, PF_6^-, instead of the long-chain alkyl sulfate. These ions do not provide the long hydrophobic tail needed for the aqueous micelles considered here, so option (B) fails. Diethyl ether is a small neutral molecule: adding it to aqueous sodium chloride does not create a long-chain ionic surfactant, excluding (C). Sodium stearate is a soap, but pure toluene is nonpolar and does not provide the aqueous environment in which ordinary ionic soap micelles form. Thus (D) is also excluded. Only the first addition combines the displayed amphiphilic ion with water.

Step 3: Conclusion An ionic micelle is formed on adding excess water to the long-chain alkyl sulfate ionic liquid shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: Inspect both ions in an ionic liquid: the surfactant tail may belong to the anion. For aqueous ionic micelles, locate a long hydrocarbon chain joined to a charged hydrophilic group.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. 6.023 $\times$ 10^{22} molecules are present in 10 g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is _____ $\times$ 10^{-3} [4 marks]

Solution:

Concept:

- Avogadro's constant relates molecules and moles: $n = N/N_A$, with $N_A = 6.023 \times 10^{23}\,\text{mol}^{-1}$ at the precision used here.
- Molarity is the amount of solute divided by the total solution volume in litres: $c = n/V$. For one pure substance, the number of moles is proportional to its mass.

Step 1: Determine the amount in the given sample The sample of mass $10\,\text{g}$ contains 6.023×10^{22} molecules. Convert this count into moles:

$$n_{10} = \frac{6.023 \times 10^{22}}{6.023 \times 10^{23}} = 0.100\,\text{mol}.$$

Thus its molar mass is

$$M_x = \frac{10\,\text{g}}{0.100\,\text{mol}} = 100\,\text{g mol}^{-1}.$$

This molecular-count calculation supplies the molar mass without needing the chemical identity of the substance. The exponent difference is $22 - 23 = -1$, so the sample contains one tenth of a mole, not one mole.

Step 2: Calculate the concentration and the requested coefficient For the new sample, the amount of solute is

$$n_5 = \frac{5\,\text{g}}{100\,\text{g mol}^{-1}} = 0.050\,\text{mol}.$$

The stated $2\,\text{L}$ is the volume of the whole solution and is already in the unit required for molarity. Therefore

$$c = \frac{0.050}{2} = 0.025\,\text{mol L}^{-1} = 25 \times 10^{-3}\,\text{mol L}^{-1}.$$

The blank requests the coefficient multiplying 10^{-3}, rather than the molarity written as an ordinary decimal. Consequently the numerical entry is 25. The same result follows immediately by halving the original amount of $0.100\,\text{mol}$, then dividing by two litres.

Step 3: Conclusion The molarity is $\boxed{25 \times 10^{-3}\,\text{mol L}^{-1}}$, so the required numerical entry is $\boxed{25}$.

Quick Tip to Solve: First convert the molecule count to moles, scale the amount with the sample mass, and divide by litres of solution. Check whether the blank asks for a scaled coefficient.

72. If 250 cm^3 of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular masses of A and B is _____ $\times$ 10^{-2} (to the nearest integer). [4 marks]

Solution:

Concept:

- Two dilute solutions are isotonic at a common temperature when they have equal osmotic pressures. For a non-electrolyte, $\Pi = cRT = (w/MV)RT$.
- Here w is the solute mass, M its molar mass, and V the solution volume. Both solutions contain proteins, so equality of osmotic pressure means equality of their molar concentrations in this treatment.

Step 1: Set the two osmotic pressures equal Both solutions are at $298\,\text{K}$, so their common factors R and T cancel when the pressures are equated. Convert the first solution volume using $1000\,\text{cm}^3 = 1\,\text{L}$:

$$V_A = 250\,\text{cm}^3 = 0.250\,\text{L}, \qquad V_B = 1.00\,\text{L}.$$

Using the given masses, the isotonic condition is

$$\frac{0.73}{M_A(0.250)} = \frac{1.65}{M_B(1.00)}.$$

The masses alone cannot be compared, because the same number of moles must be expressed per equal solution volume. Protein A is present in only one quarter of a litre.

Step 2: Solve the ratio and express it in the requested form Rearrange without inverting the required

ratio:

$$\frac{M_A}{M_B} = \frac{0.73(1.00)}{1.65(0.250)} = \frac{0.73}{0.4125} = 1.769696\ldots.$$

The question writes the ratio as a coefficient times 10^{-2}. Hence

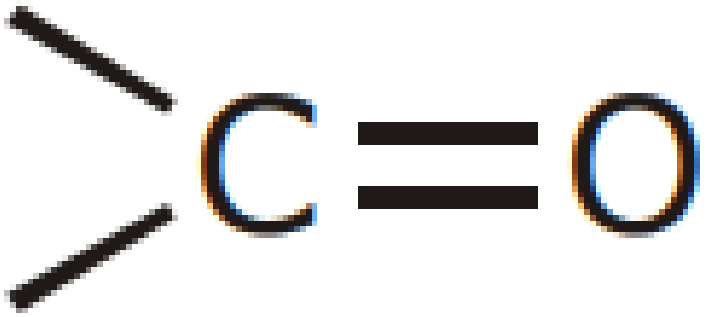
$$1.769696\ldots = 176.969696\ldots \times 10^{-2}.$$

Rounding this coefficient to the nearest integer gives 177. As a direction check, the mass concentration of A is $0.73/0.250 = 2.92\,\mathrm{g\,L^{-1}}$, which is greater than B's $1.65\,\mathrm{g\,L^{-1}}$. Equal molar concentrations therefore require A to have the larger molar mass, consistent with a ratio greater than one. A molecular-mass ratio is dimensionless.

Step 3: Conclusion The ratio is $M_A/M_B \approx 1.77 = \boxed{177 \times 10^{-2}}$. The required dimensionless numerical entry is $\boxed{177}$.

Quick Tip to Solve: For isotonic non-electrolytes at equal temperature, the molar-mass ratio equals the mass-concentration ratio. Convert both volumes to the same unit before forming that ratio.

73. The number of groups present in a tripeptide Asp–Glu–Lys is ____. [4 marks]



Solution:

Concept:

- The displayed group is a carbonyl group, C=O. Count a carbonyl whenever carbon is double-bonded to oxygen, including both carboxylic-acid and amide groups.
- A peptide bond forms when the amino group of one amino acid reacts with the carboxyl group of another, eliminating water. Its $-\mathrm{CO}-\mathrm{NH}-$ unit retains the original carboxyl carbonyl.

Step 1: Count carbonyl groups in the three amino acids Aspartic acid (Asp) has its alpha-carboxyl group and one side-chain carboxyl group, so it contributes two carbonyl groups. Glutamic acid (Glu) likewise has an alpha-carboxyl group and a side-chain carboxyl group, contributing two. Lysine (Lys) has one alpha-carboxyl group and an amino-containing side chain; the side-chain amino group

contributes no carbonyl. Thus the three separate residues supply

$$N_{C=O} = 2 + 2 + 1 = 5.$$

Step 2: Check what changes on forming the tripeptide In the sequence Asp–Glu–Lys, the alpha-carboxyl group of Asp forms a peptide bond to Glu, and that of Glu forms a peptide bond to Lys. These two condensations remove hydroxyl groups and amino hydrogens as water, but neither removes a carbonyl group. The final molecule therefore contains two peptide-amide carbonyls and three remaining carboxyl carbonyls: those on the Asp and Glu side chains and the terminal Lys carboxyl group.

$$N_{C=O} = 2 \text{ (amide)} + 3 \text{ (carboxyl)} = 5.$$

Counting only free carboxyl groups would miss the carbonyls present in the peptide backbone.

Step 3: Conclusion The tripeptide Asp–Glu–Lys contains $\boxed{5}$ carbonyl groups. The required numerical answer is 5.

Quick Tip to Solve: Peptide formation changes a carboxyl group into an amide without changing the total carbonyl count. Add the carbonyls of the constituent amino acids first.

74. The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is _____. [4 marks]

Solution:

Concept:

- Normality is the number of reacting equivalents per litre of solution. At complete acid-base neutralisation, the equivalents of acid equal the equivalents of base, so $N_aV_a = N_bV_b$ when the volumes use the same unit.
- Phosphinic acid is H_3PO_2, conventionally written $HOP(=O)H_2$. Only its hydrogen bonded to oxygen is ionisable in neutralisation; its two phosphorus-bonded hydrogens do not make it tribasic.

Step 1: Identify the neutralisation stoichiometry Phosphinic acid is monobasic, and sodium hydroxide provides one hydroxide ion per formula unit. Their reaction is

$$H_3PO_2 + NaOH \longrightarrow NaH_2PO_2 + H_2O.$$

The acid and base react in a 1:1 molar ratio. More directly, because both concentrations are already supplied as normalities, their reactive equivalents can be compared without any additional basicity factor. Multiplying the acid normality by three would incorrectly count all three hydrogen atoms as ionisable.

Step 2: Equate the acid and base equivalents The acid volume is $10\,\text{mL} = 0.010\,\text{L}$, so its amount in equivalents is

$$n_{\text{eq,acid}} = 0.1 \times 0.010 = 0.001\,\text{eq}.$$

The required sodium hydroxide solution is also $0.1\,\text{eq}\,\text{L}^{-1}$. Consequently,

$$V_b = \frac{0.001}{0.1} = 0.010\,\text{L} = 10\,\text{mL}.$$

Equivalently, retaining millilitres throughout gives $V_b = (0.1 \times 10)/0.1 = 10\,\text{mL}$. Equal normalities require equal volumes at the equivalence point, which checks the result.

Step 3: Conclusion The volume of $0.1\,\text{N}$ NaOH required is $\boxed{10\,\text{mL}}$. Enter 10 as the numerical answer.

Quick Tip to Solve: With normality already given, use $N_aV_a = N_bV_b$ directly; do not multiply by the number of hydrogen atoms in the acid formula.

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75. An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

The amount of Cr^{3+} obtained was 0.104 g. The efficiency of the process(in%) is (Take : F = 96000 C, At. mass of chromium = 52) _____. [4 marks]

Solution:

Concept:

- Faraday's law gives the theoretical amount produced from the charge passed: $Q = It$ and $n(e^-) = Q/F$, where I is current, t is time in seconds and F is the charge per mole of electrons.
- Current efficiency is the actual amount of the specified product divided by its theoretical amount for the total charge, multiplied by 100. The electron requirement must be read per mole of the product, not merely per written half-reaction.

Step 1: Calculate the charge and electron requirement Convert the electrolysis time to seconds before using the current:

$$t = 8 \times 60 = 480\,\text{s}, \qquad Q = 2 \times 480 = 960\,\text{C}.$$

With the supplied value $F = 96000\,\text{C}\,\text{mol}^{-1}$, the charge corresponds to

$$n(e^-) = \frac{960}{96000} = 0.010\,\text{mol}.$$

The given reduction is

$$\text{Cr}_2\text{O}_7^{2-} + 14\,\text{H}^+ + 6\,\text{e}^- \longrightarrow 2\,\text{Cr}^{3+} + 7\,\text{H}_2\text{O}.$$

Six moles of electrons produce two moles of chromium(III) ions; hence each mole of Cr^{3+} requires three moles of electrons.

Step 2: Compare the actual and theoretical chromium masses If every electron contributed to this reduction, the theoretical amount and mass of chromium(III) would be

$$n(\text{Cr}^{3+}) = \frac{0.010}{3} = \frac{1}{300}\,\text{mol},$$

$$m_{\text{theoretical}} = \frac{52}{300} = 0.17333\ldots\ \text{g}.$$

The measured amount is only $0.104\,\text{g}$, so the efficiency is

$$\eta = \frac{0.104}{52/300} \times 100 = 60\%.$$

As a cross-check, $0.104/52 = 0.002\,\text{mol}$ of chromium(III) requires $0.006\,\text{mol}$ of electrons. This is $0.006/0.010 = 0.60$ of the total electron supply.

Step 3: Conclusion The efficiency of the electrolytic process is $\boxed{60\%}$. The required numerical entry is 60.

Quick Tip to Solve: Divide the six electrons by the two chromium ions first: use three electrons per chromium ion, and convert the eight minutes to seconds before finding charge.