JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 4 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 September 2020, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is : [4 marks]

(A) 5

(B) 3

(C) 7

(D) 9

Correct Answer: (C) 7

Solution:

Concept:

- For n observations, the arithmetic mean is $\bar{x} = \sum x_i/n$, so it determines the sum of all observations.
- The population variance is $\sigma^2 = \sum x_i^2/n - \bar{x}^2$. For two unknown observations, their sum and sum of squares determine the square of their difference.

Step 1: Find the sum of the missing observations Let the remaining observations be u and v. The mean is taken over all eight observations, including these two. Hence the total sum is $8(10) = 80$. The six listed observations have sum $5 + 7 + 10 + 12 + 14 + 15 = 63$, giving

$$u + v = 80 - 63 = 17.$$

Step 2: Use the variance to find their squares The variance formula gives

$$\sum_{i=1}^{8} x_i^2 = 8(13.5 + 10^2) = 908.$$

The known squares add to $25 + 49 + 100 + 144 + 196 + 225 = 739$. Subtract these from the total, rather than subtracting their mean, to obtain

$$u^2 + v^2 = 908 - 739 = 169.$$

Step 3: Calculate the absolute difference Use the identity that eliminates the product:

$$(u - v)^2 = 2(u^2 + v^2) - (u + v)^2 = 2(169) - 17^2 = 49.$$

Therefore $|u - v| = 7$. As a consistency check, the two values are $(17 + 7)/2 = 12$ and $(17 - 7)/2 = 5$. Repeated observations are permitted. The proposed differences 5, 3 and 9 would give squared differences 25, 9 and 81, inconsistent with 49.

Step 4: Conclusion The absolute difference of the remaining observations is 7. Hence, the correct option is (C).

Quick Tip to Solve: Combine the mean and variance using $(u - v)^2 = 2(u^2 + v^2) - (u + v)^2$; finding the product first is unnecessary.

2. Let α and β be the roots of x^2 - 3x + p=0 and γ and δ be the roots of x^2 - 6x + q = 0. If $\alpha, \beta, \gamma, \delta$ form a geometric progression.Then ratio (2q + p) : (2q - p) is: [4 marks]

(A) 9 : 7

(B) 5 : 3

(C) 3 : 1

(D) 33 :31

Correct Answer: (A) 9 : 7

Solution:

Concept:

- For a monic quadratic $x^2 - sx + t = 0$, the sum and product of its roots are s and t.
- Four consecutive terms of a geometric progression may be written a, ar, ar^2, ar^3, with common ratio r. Pair sums then differ by a factor r^2.

Step 1: Express the roots as a progression Write $\alpha = a$, $\beta = ar$, $\gamma = ar^2$ and $\delta = ar^3$, in their stated order. From the coefficients of the two quadratics,

$$a(1 + r) = 3, \qquad ar^2(1 + r) = 6.$$

The first sum is nonzero, so division by $a(1 + r)$ is valid. Dividing the second equation by the first gives $r^2 = 2$. This conclusion does not require choosing a sign for r.

Step 2: Relate the constant coefficients The products of the roots are

$$p = \alpha\beta = a^2r, \qquad q = \gamma\delta = a^2r^5.$$

Consequently $q/p = r^4 = 4$, so $q = 4p$. Here $p \neq 0$: a zero root in a finite geometric progression would contradict the nonzero sum of the first pair. Thus cancellation of p in the requested ratio is justified.

Step 3: Evaluate and compare the ratios Substitute the product relation before attempting to compute individual roots:

$$(2q + p) : (2q - p) = (8p + p) : (8p - p) = 9p : 7p = 9 : 7.$$

The answer is the same for either possible common ratio $r = \sqrt{2}$ or $r = -\sqrt{2}$. Ratios $5 : 3$, $3 : 1$ and $33 : 31$ would require a different relation between the root products and therefore do not satisfy the given pair sums.

Step 4: Conclusion The required ratio is $9 : 7$. Hence, the correct option is (A).

Quick Tip to Solve: For four consecutive GP terms, divide the second pair sum by the first to obtain r^2; the ratio of their products is then r^4.

3. Let $f(x) = |x-2|$ and g(x) = f(f(x)), $x \in [0,4]$. Then $\int_0^3 (g(x) - f(x))dx$ is equal to: [4 marks]

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

Correct Answer: (A) 1

Solution:

Concept:

- An absolute value $|t|$ equals t for $t \geq 0$ and $-t$ for $t < 0$; locate sign changes before integration.
- The composition $g(x) = f(f(x))$ means replacing the input of f by its output. Linearity allows integration of $g - f$ directly.

Step 1: Simplify the composed absolute value On the whole stated domain $0 \leq x \leq 4$, the quantity $f(x) = |x-2|$ lies between 0 and 2. Therefore

$$g(x) = \big||x-2| - 2\big| = 2 - |x-2|.$$

The outer absolute value reverses the sign because its argument is nonpositive. It follows that $g(x) - f(x) = 2 - 2|x-2|$.

Step 2: Split the integral at the corner The integration interval ends at 3, not 4. Split at $x = 2$, where the inner absolute value changes formula:

$$\int_0^3 |x-2|\,dx = \int_0^2 (2-x)\,dx + \int_2^3 (x-2)\,dx.$$

The first contribution is $[2x - x^2/2]_0^2 = 2$, while the second is $[(x-2)^2/2]_2^3 = 1/2$. Thus the total is $5/2$.

Step 3: Evaluate the required difference Using the simplified composition,

$$\int_0^3 (g-f)\,dx = \int_0^3 2\,dx - 2\int_0^3 |x-2|\,dx = 6 - 2\left(\frac{5}{2}\right) = 1.$$

Alternatively, the integrand is $2x - 2$ on $[0,2]$ and $6 - 2x$ on $[2,3]$, whose integrals are 0 and 1. This independent check rules out 0, $1/2$ and $3/2$; those values arise from missing a contribution or a factor of 2.

Step 4: Conclusion The integral equals 1. Hence, the correct option is (A).

Quick Tip to Solve: Check the range of the inner function first: $0 \leq f \leq 2$ immediately makes the composition $g = 2 - f$.

4. The integral $\int \left(\frac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to (where C is a constant of integration): [4 marks]

(A) $\sec x - \frac{x\tan x}{x\sin x+\cos x} + C$

(B) $\sec x + \frac{x\tan x}{x\sin x+\cos x} + C$

(C) $\tan x - \frac{x\sec x}{x\sin x+\cos x} + C$

(D) $\tan x + \frac{x\sec x}{x\sin x+\cos x} + C$

Correct Answer: (C) $\tan x - \frac{x\sec x}{x\sin x+\cos x} + C$

Solution:

Concept:

- An antiderivative can be verified by differentiation: its derivative must equal the complete printed integrand.
- For $D = x\sin x + \cos x$, the derivative simplifies to $D\prime = x\cos x$, since the two sine terms cancel.

Step 1: Simplify the promising expression The integrand is x^2/D^2, where $D = x\sin x + \cos x$. Option (C) has the useful cancellation

$$\tan x - \frac{x\sec x}{D} = \frac{\sin x\, D - x}{\cos x\, D} = \frac{\sin x - x\cos x}{D}.$$

In the last equality, use $\sin^2 x - 1 = -\cos^2 x$. This quotient is convenient because its numerator has derivative $x\sin x$.

Step 2: Differentiate the simplified quotient Put $N = \sin x - x\cos x$. Then $N\prime = x\sin x$ and $D\prime = x\cos x$. The quotient rule gives

$$\frac{d}{dx}\left(\frac{N}{D}\right) = \frac{x\sin x\, D - x\cos x\, N}{D^2}.$$

Expanding only the numerator gives

$$x^2\sin^2 x + x\sin x\cos x - x\sin x\cos x + x^2\cos^2 x = x^2.$$

Thus the derivative is exactly x^2/D^2.

Step 3: Check the options and domain The plus sign in (D) adds rather than cancels the quotient derivative. Options (A) and (B) use $\sec x$ as the leading term and therefore do not give the required cancellation. For example, at $x = 0$, the target integrand is 0, whereas (D) has derivative 2. The verified identity holds on intervals where the printed expressions are defined; $D = 0$ is excluded from the original integrand. Add an arbitrary constant on each such interval.

Step 4: Conclusion

$$\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx = \tan x - \frac{x\sec x}{x\sin x + \cos x} + C.$$

Hence, the correct option is (C).

Quick Tip to Solve: For MCQ antiderivatives, simplify a candidate and differentiate it; here the quotient numerator and denominator give an immediate cancellation.

5. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx \, (x \geq 0)$. Then f(3) – f(1) is eqaul to : [4 marks]

(A) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(B) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(C) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(D) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Correct Answer: (B) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Solution:

Concept:

- The difference of two values of the same antiderivative is a definite integral, so its arbitrary constant cancels.
- For a square-root integrand, $t = \sqrt{x}$ removes the radical. The identity $t^2 = (1 + t^2) - 1$ then separates standard rational integrals.

Step 1: Convert the difference to a definite integral By the fundamental theorem of calculus,

$$f(3) - f(1) = \int_1^3 \frac{\sqrt{x}}{(1+x)^2} \, dx.$$

The integrand is continuous and positive on this interval. Put $t = \sqrt{x}$, so $x = t^2$ and $dx = 2t \, dt$. The lower and upper limits become 1 and $\sqrt{3}$, respectively. Thus the integral becomes $\int_1^{\sqrt{3}} 2t^2/(1+t^2)^2 \, dt$.

Step 2: Find a convenient antiderivative Using $t^2 = (1 + t^2) - 1$, or directly differentiating the expression below, obtain

$$\frac{d}{dt}\left(\arctan t - \frac{t}{1+t^2}\right) = \frac{1}{1+t^2} - \frac{1-t^2}{(1+t^2)^2} = \frac{2t^2}{(1+t^2)^2}.$$

This derivative check also protects against losing the factor of 2 introduced by dx.

Step 3: Apply the transformed limits Evaluate the antiderivative at both endpoints:

$$f(3) - f(1) = \left[\arctan t - \frac{t}{1+t^2}\right]_1^{\sqrt{3}} = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right).$$

Therefore the result is $\pi/12 + 1/2 - \sqrt{3}/4$. The inverse tangents lie in their principal first-quadrant range. The other alternatives have an incorrect angle difference, an incorrect endpoint sign, or both; the angle contribution must be $\pi/3 - \pi/4 = \pi/12$.

Step 4: Conclusion

$$f(3) - f(1) = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}.$$

Hence, the correct option is (B).

Quick Tip to Solve: After substituting $t = \sqrt{x}$, use $\arctan t - t/(1+t^2)$ as a derivative-checked primitive and transform both limits.

6. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is : [4 marks]

(A) 10/3

(B) 5

(C) 20/3

(D) 6

Correct Answer: (D) 6

Solution:

Concept:

- A straight line joining intercepts has a height that varies linearly with horizontal position.
- The two poles stand on the same horizontal baseline. Choosing convenient coordinates does not change the intersection height, and their unknown separation cancels.

Step 1: Choose coordinates along the ground Let the ground separation be $AC = d > 0$. Choose $A = (0, 0)$ and $C = (d, 0)$. Since both poles are vertical above the horizontal ground, their tops are $B = (0, 15)$ and $D = (d, 10)$. The height of any point is simply its positive y-coordinate. We need the intersection of the crossed segments BC and AD, rather than of the two vertical poles.

Step 2: Write the equations of the crossed segments The line through A and D rises by 10 over a horizontal distance d, so

$$AD: \quad y = \frac{10x}{d}.$$

The line from B to C falls from height 15 to 0 over the same distance, hence

$$BC: \quad y = 15 - \frac{15x}{d}.$$

Both equations use the same horizontal coordinate measured from A.

Step 3: Solve for the intersection height At P, the two expressions for y agree:

$$\frac{10x}{d} = 15 - \frac{15x}{d}, \qquad \frac{x}{d} = \frac{15}{25} = \frac{3}{5}.$$

Substituting into the rising line gives $y = 10(3/5) = 6$ metres. The ratio x/d lies strictly between 0 and 1, confirming that the intersection is on both segments. The general height formula is $h = h_1h_2/(h_1+h_2)$, which here gives $150/25 = 6$. None of $10/3$, 5 or $20/3$ satisfies both line equations at a common horizontal position.

Step 4: Conclusion The height of P above AC is $6\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: For crossed lines joining opposite pole tops and bases, use $h = h_1h_2/(h_1 + h_2)$; the horizontal separation cancels.

7. Let y = y(x) be the solution of the differential equation,
xy'- y = x^2(xcosx + sinx), x > 0. if y (π) = π then
$y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ **is equal to :** [4 marks]

(A) $1 + \frac{\pi}{2}$
(B) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$
(C) $2 + \frac{\pi}{2}$
(D) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$

Correct Answer: (C) $2 + \frac{\pi}{2}$

Solution:

Concept:

- For $x > 0$, the quotient rule gives $(y/x)' = (xy' - y)/x^2$. This converts the given differential equation into a direct integration.
- The product rule gives $(x \sin x)' = x \cos x + \sin x$. The condition at $x = \pi$ fixes the integration constant.

Step 1: Integrate the differential equation Divide the equation by x^2, which is permitted because $x > 0$. Then

$$\frac{d}{dx}\left(\frac{y}{x}\right) = x \cos x + \sin x = \frac{d}{dx}(x \sin x).$$

Consequently $y/x = x \sin x + C$, and hence $y = x^2 \sin x + Cx$. The constant term is multiplied by x; it is not an additive constant in y.

Step 2: Apply the initial condition At $x = \pi$, the sine term vanishes. Thus

$$\pi = \pi^2 \sin \pi + C\pi = C\pi, \qquad C = 1.$$

The required solution on the positive half-line is therefore $y = x^2 \sin x + x$. Substitution in the original first-order equation also verifies this expression.

Step 3: Differentiate and combine the requested quantities Differentiate twice using the product rule:

$$y' = 2x \sin x + x^2 \cos x + 1,$$

$$y'' = 2 \sin x + 4x \cos x - x^2 \sin x.$$

Adding y before substituting cancels the quadratic sine terms:

$$y'' + y = 2 \sin x + 4x \cos x + x.$$

At $x = \pi/2$, sine is one and cosine is zero. Thus the sum is $2 + \pi/2$. Options containing $\pi^2/4$ miss this cancellation; the option beginning with 1 misses the second derivative of the product.

Step 4: Conclusion

$$y''(\pi/2) + y(\pi/2) = 2 + \frac{\pi}{2}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For an equation containing $xy' - y$, test the derivative of y/x. Add the requested expressions before evaluating to expose cancellations.

8. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be: [4 marks]

(A) 37

(B) 65

(C) 29

(D) 55

Correct Answer: (D) 55

Solution:

Concept:

- For two sets, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Here probabilities can be represented by percentages of the same city population.
- An intersection cannot exceed either individual set, and a union cannot exceed the whole population. Both restrictions are needed.

Step 1: Use the upper bound on the union The total percentage reading at least one of the newspapers is

$$63 + 76 - x = 139 - x.$$

It must be no greater than 100, even if some residents read neither newspaper. Therefore

$$139 - x \leq 100 \quad \Longrightarrow \quad x \geq 39.$$

The common error of setting the union equal to 100 would force $x = 39$, but the question does not say that everyone reads a newspaper.

Step 2: Bound the intersection from above Every person who reads both newspapers must belong to the group reading A, which contains only 63% of the population. Similarly the intersection cannot exceed 76%. Thus

$$x \leq \min(63, 76) = 63.$$

Combining these necessary bounds gives $39 \leq x \leq 63$. They are also sufficient, since all four groups then have nonnegative percentages.

Step 3: Check the listed values and construct a distribution Of 37, 65, 29 and 55, only 55 belongs to the feasible interval. To verify existence directly, take 55% reading both. Then A only contains $63 - 55 = 8\%$, B only contains $76 - 55 = 21\%$, and neither contains $100 - (8 + 21 + 55) = 16\%$. These four percentages are nonnegative and add to 100. Values 37 and 29 would make the union exceed 100, whereas 65 exceeds the entire A readership.

Step 4: Conclusion A possible overlap is 55%. Hence, the correct option is (D).

Quick Tip to Solve: For two percentage sets, the overlap lies between $\max(0, a + b - 100)$ and $\min(a, b)$; test the options against both ends.

9. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$

and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, **where** $i = \sqrt{-1}$ **then which one of the following is not true?** [4 marks]

(A) a^2 - c^2 = 1

(B) $0 \le a^2 + b^2 \le 1$

(C) a^2 - d^2 = 0

(D) $a^2 - b^2 = \frac{1}{2}$

Correct Answer: (D) $a^2 - b^2 = \frac{1}{2}$

Solution:

Concept:

- For $S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $S^2 = I$. Thus $A = \cos\theta I + i\sin\theta S$ has powers $A^n = \cos(n\theta)I + i\sin(n\theta)S$.
- These are ordinary algebraic squares, not modulus squares: $(i\sin\alpha)^2 = -\sin^2\alpha$.

Step 1: Evaluate the fifth power The trigonometric addition formulas show that multiplying two matrices of the given form adds their angles. Therefore

$$A^5 = \begin{pmatrix} \cos\alpha & i\sin\alpha \\ i\sin\alpha & \cos\alpha \end{pmatrix}, \quad \alpha = \frac{5\pi}{24}.$$

Hence $a = d = \cos\alpha$ and $b = c = i\sin\alpha$.

Step 2: Verify the first three statements Using $i^2 = -1$,

$$a^2 - c^2 = \cos^2\alpha + \sin^2\alpha = 1, \qquad a^2 - d^2 = 0.$$

Thus (A) and (C) are true. For (B),

$$a^2 + b^2 = \cos^2\alpha - \sin^2\alpha = \cos(2\alpha) = \cos\frac{5\pi}{12}.$$

Since $5\pi/12$ lies strictly between 0 and $\pi/2$, this value lies strictly between 0 and 1. Therefore (B) is true as well.

Step 3: Identify the false statement The remaining difference has the same form as the determinant:

$$a^2 - b^2 = \cos^2\alpha - (i\sin\alpha)^2 = \cos^2\alpha + \sin^2\alpha = 1.$$

It is not $1/2$. The word "not" in the question means we must select this false assertion, rather than one of the three identities already verified. Confusing the square of the imaginary entry with its positive modulus square would reverse the relevant signs.

Step 4: Conclusion The assertion $a^2 - b^2 = 1/2$ is false because its left side is 1. Hence, the correct option is (D).

Quick Tip to Solve: For this matrix pattern, replace powers by angle multiples, then square the imaginary entries carefully: $i^2 = -1$.

10. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function,

$\phi(t)=\frac{5}{12}+t-t^2$, then $a^2 + b^2$ is equal to : [4 marks]

(A) 145

(B) 126

(C) 135

(D) 116

Correct Answer: (B) 126

Solution:

Concept:

- For an ellipse with semi-major axis a, semi-minor axis b and eccentricity e, $b^2=a^2(1-e^2)$. Its full latus rectum has length $2b^2/a$.
- A concave quadratic is maximized at its vertex. Completing the square finds the maximum without confusing the maximizing input with the maximum value.

Step 1: Find the eccentricity Complete the square in the given function:

$$\phi(t)=\frac{5}{12}+t-t^2=\frac{5}{12}+\frac{1}{4}-\left(t-\frac{1}{2}\right)^2=\frac{2}{3}-\left(t-\frac{1}{2}\right)^2.$$

The squared term is nonnegative and is zero when $t=1/2$. The maximum value is therefore $2/3$, which is the eccentricity e, not the argument $1/2$. The value satisfies $0<e<1$, as required for the given ellipse.

Step 2: Combine the axis and latus rectum relations Using $e=2/3$,

$$b^2=a^2\left(1-\frac{4}{9}\right)=\frac{5}{9}a^2.$$

The stated latus rectum length is the full length 10, so

$$\frac{2b^2}{a}=10, \qquad b^2=5a.$$

Combining these equations yields $5a^2/9=5a$. Since a semi-major axis is positive, divide by $5a$ to obtain $a=9$.

Step 3: Evaluate the requested sum and check consistency It follows that $a^2=81$ and $b^2=5a=45$. Therefore

$$a^2+b^2=81+45=126.$$

As a check, $2b^2/a=90/9=10$, and $1-b^2/a^2=1-45/81=4/9=e^2$. The condition $a>b$ also holds because $81>45$. Only 126 satisfies both original geometric conditions, so the other numerical options are excluded.

Step 4: Conclusion $a^2+b^2=126$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the full latus rectum $2b^2/a$ distinct from its half-length, and use the quadratic maximum as e, not its vertex coordinate.

11. Let [t] denote the greatest integer $\leq$ t. Then the equation in x,
$[x]^2 + 2[x+2] - 7 = 0$ has : [4 marks]

(A) no integral solution.

(B) exactly two solutions.

(C) exactly four integral solutions.

(D) infinitely many solutions.

Correct Answer: (D) infinitely many solutions.

Solution:

Concept:

- The floor function satisfies $\lfloor x+m \rfloor = \lfloor x \rfloor + m$ for every integer m. A floor value n corresponds to the entire interval $[n, n+1)$.
- The unknown x need not be an integer. Solving an equation in its floor first gives integer floor values, and then intervals of real solutions.

Step 1: Reduce to an integer quadratic Let $n = \lfloor x \rfloor$, so n is an integer. Because 2 is an integer, $\lfloor x+2 \rfloor = n+2$. Substitute in the equation:

$$n^2 + 2(n+2) - 7 = 0 \quad \Longrightarrow \quad n^2 + 2n - 3 = 0.$$

Factorization gives $(n+3)(n-1) = 0$, hence $n = -3$ or $n = 1$. These are two possible floor values, not merely two possible real values of x.

Step 2: Recover all real solutions The definition of floor now gives

$$\lfloor x \rfloor = -3 \iff -3 \leq x < -2, \qquad \lfloor x \rfloor = 1 \iff 1 \leq x < 2.$$

Thus the complete real solution set is $[-3, -2) \cup [1, 2)$. Each interval includes its left endpoint and excludes its right endpoint, because the floor jumps to the next integer at the right endpoint.

Step 3: Compare the number of solutions with the options Every real number in either interval satisfies the equation, so there are infinitely many solutions. For instance, in the second interval $\lfloor x \rfloor = 1$ and $\lfloor x+2 \rfloor = 3$, yielding $1 + 6 - 7 = 0$ throughout. There are exactly two integral solutions, namely -3 and 1, which rules out both the claim of no integral solution and the claim of four integral solutions. The statement of exactly two solutions confuses those integer members with all real solutions.

Step 4: Conclusion The equation has infinitely many real solutions. Hence, the correct option is (D).

Quick Tip to Solve: After solving for $\lfloor x \rfloor$, always replace each allowed integer n by the interval $[n, n+1)$ before counting real solutions.

12. Let $u = \frac{2z+i}{z-ki}$, z = x + iy and k > 0. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where PQ = 5, then the value of k is : [4 marks]

(A) 2

(B) 4

(C) 1/2

(D) 3/2

Correct Answer: No value satisfies the literal two-intercept premise: the pole $z = ki$ is excluded. For the intended circle closure, $k = 2$, option (A).

Solution:

Concept:

- For $z = x + iy$, multiply the numerator and denominator by the complex conjugate of the denominator to separate real and imaginary parts.
- The algebraic locus obtained here is a circle. Its intercept separation follows by setting $x = 0$; the original rational expression also has the domain restriction $z \neq ki$.

Step 1: Separate real and imaginary parts The denominator is $x + i(y - k)$. Multiplying by its conjugate, with $D = x^2 + (y - k)^2$, gives

$$\operatorname{Re} u = \frac{2x^2 + (2y + 1)(y - k)}{D}, \qquad \operatorname{Im} u = \frac{(2k + 1)x}{D}.$$

For $D \neq 0$, the condition that their sum is 1 therefore becomes

$$x^2 + y^2 + (2k + 1)x + y - k^2 - k = 0.$$

This is the circle equation associated with the given rational locus.

Step 2: Find the intercepts of the algebraic circle Put $x = 0$ in this circle equation:

$$y^2 + y - k^2 - k = 0 = (y - k)(y + k + 1).$$

The circle has y-axis intercept ordinates k and $-k - 1$, whose separation is $|k - (-k - 1)| = 2k + 1$, since $k > 0$. Interpreting the stated curve as this circle, $PQ = 5$ gives $2k + 1 = 5$, so $k = 2$. The other options give separations 9, 2 and 4 respectively.

Step 3: Check the rational-domain qualification There is a subtle domain issue in the printed wording: the intercept $(0, k)$ is exactly where $D = 0$, so the original expression for u is undefined there. Strictly, its locus is the circle with that point deleted and has only the other y-axis intercept. Thus two actual intercepts of the rational locus do not exist. The listed answer 2 corresponds to the intended circle, including its missing point, and this interpretation is necessary for the stated distance to make sense.

Step 4: Conclusion The literal rational locus has no admissible k satisfying the two-intercept premise. For its intended circle closure, $k = 2$, corresponding to option (A).

Quick Tip to Solve: After clearing a complex denominator, check whether the resulting circle includes an excluded pole; here the conventional intercept calculation uses the circle closure.

13. If $\left(a+\sqrt{2}b\cos x\right)\left(a-\sqrt{2}b\cos y\right)=a^2-b^2$
where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ **is :** [4 marks]

(A) $\frac{a-2b}{a+2b}$
(B) $\frac{a-b}{a+b}$
(C) $\frac{a+b}{a-b}$
(D) $\frac{2a+b}{2a-b}$

Correct Answer: (C) $\frac{a+b}{a-b}$

Solution:

Concept:

- Treat x as a differentiable function of y, and differentiate both factors using the product rule.
- The derivative of $\cos x$ with respect to y is $-\sin x\,dx/dy$. The constants a, b do not change during differentiation.

Step 1: Differentiate the given product The right side is constant, so its derivative vanishes. Keeping the undifferentiated factor beside each differentiated factor gives

$$-\sqrt{2}b\sin x\,\frac{dx}{dy}(a-\sqrt{2}b\cos y)+(a+\sqrt{2}b\cos x)\sqrt{2}b\sin y=0.$$

Notice that the derivative of the second factor is positive: differentiating its minus cosine produces a second minus sign.

Step 2: Isolate the required derivative Since $b>0$, cancel $\sqrt{2}b$, move the first term to the other side, and divide by its coefficient:

$$\frac{dx}{dy}=\frac{(a+\sqrt{2}b\cos x)\sin y}{(a-\sqrt{2}b\cos y)\sin x}.$$

This is the derivative with x dependent on y; interchanging the variables here would give its reciprocal.

Step 3: Evaluate at the specified point At $x=y=\pi/4$, both sine values are $1/\sqrt{2}$, so their ratio is one. Also $\sqrt{2}b\cos(\pi/4)=b$. Hence

$$\left.\frac{dx}{dy}\right|_{(\pi/4,\pi/4)}=\frac{a+b}{a-b}.$$

The denominator is positive because $a>b$, and the ratio exceeds one. Option (B) is the reciprocal; (A) and (D) use incorrect coefficients of the constants.

Step 4: Conclusion The required derivative is $\frac{a+b}{a-b}$. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate before substituting the point; check that two minus signs make the second product-rule term positive.

14. Let f be a twice differentiable function on (1, 6). If f(2) = 8, f'(2) = 5, f'(x) $\geq$ 1 and f"(x) $\geq$ 4, for all x $\in$ (1, 6), then : [4 marks]

(A) f(5) $\leq$ 10

(B) f(5) + f'(5) $\geq$ 28

(C) f(5) + f'(5) $\leq$ 26

(D) f'(5) + f"(5) $\leq$ 20

Correct Answer: (B) f(5) + f'(5) $\geq$ 28

Solution:

Concept:

- A lower bound on f'' gives a lower bound on the increase of f'; a lower bound on f' similarly bounds the increase of f.
- If a differentiable function has nonnegative derivative on an interval, it is nondecreasing there. Apply this fact to functions obtained by subtracting suitable polynomials.

Step 1: Bound the first derivative at five Define $g(x) = f'(x) - 4x$. Its derivative satisfies $g'(x) = f''(x) - 4 \geq 0$, so $g(5) \geq g(2)$. Therefore

$$f'(5) - 20 \geq 5 - 8 = -3, \qquad f'(5) \geq 17.$$

More generally, for $2 \leq x \leq 5$, the same argument gives $f'(x) \geq 5 + 4(x - 2)$.

Step 2: Bound the function value at five Subtract an antiderivative of the last lower bound. The function

$$h(x) = f(x) - 8 - 5(x - 2) - 2(x - 2)^2$$

has $h(2) = 0$ and $h'(x) = f'(x) - 5 - 4(x - 2) \geq 0$ on $[2, 5]$. Thus $h(5) \geq 0$, which yields

$$f(5) \geq 8 + 5(3) + 2(3)^2 = 41.$$

Using monotonicity avoids requiring any unstated continuity of the second derivative.

Step 3: Compare the four proposed statements Adding the established bounds gives $f(5) + f'(5) \geq 41 + 17 = 58$, and consequently it is certainly at least 28. Options (A) and (C) contradict these lower bounds. Although $f''(5) \geq 4$, together with $f'(5) \geq 17$ this gives a sum at least 21, so option (D) also fails. The required claim need not be the strongest available bound.

Step 4: Conclusion It follows that $f(5) + f'(5) \geq 28$. Hence, the correct option is (B).

Quick Tip to Solve: Propagate the second-derivative bound to the first derivative, then to the function; a weaker proposed bound can still be true.

15. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+.....+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to: [4 marks]

(A) (11, 103)

(B) (10, 103)

(C) (10, 97)

(D) (11, 97)

Correct Answer: (A) (11, 103)

Solution:

Concept:

- The product in the k-th parenthesis is $(2k)^2(2k-1)$, for $k = 1, \ldots, 10$. The initial standalone 1 must also be counted.
- Use $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$ and $\sum_{k=1}^{n} k^3 = [n(n+1)/2]^2$.

Step 1: Express the displayed pattern as a sum The factors $2, 4, 6, \ldots, 20$ are $2k$, while $1, 3, 5, \ldots, 19$ are $2k-1$. Thus the required quantity is

$$S = 1 + \sum_{k=1}^{10} \{1 - (2k)^2(2k-1)\}.$$

There are ten parentheses in addition to the leading one, giving eleven constant terms. Expand the product as

$$(2k)^2(2k-1) = 8k^3 - 4k^2.$$

The minus sign before the product therefore makes the square-sum contribution positive.

Step 2: Evaluate the two standard power sums For $n = 10$,

$$\sum_{k=1}^{10} k^3 = 55^2 = 3025, \qquad \sum_{k=1}^{10} k^2 = \frac{10 \cdot 11 \cdot 21}{6} = 385.$$

Substituting these separately keeps the coefficients and signs visible:

$$S = 11 - 8(3025) + 4(385) = 11 - 24200 + 1540 = 11 - 22660.$$

As a check on the indexing, the final product is $20^2 \cdot 19$, exactly the last one in the question.

Step 3: Match the form in the question The large subtracted term has the prescribed factor 220:

$$22660 = 220 \cdot 103.$$

Consequently $S = 11 - 220 \cdot 103$, which matches the offered pair $(11, 103)$. Pairs beginning with ten omit the separate leading constant; using 97 produces a different numerical sum. The task is to select an offered pair representing the expression, rather than to claim a unique representation for unrestricted real α, β.

Step 4: Conclusion An offered ordered pair is $(\alpha, \beta) = (11, 103)$. Hence, the correct option is (A).

Quick Tip to Solve: Count the standalone term before summing the parentheses, and expand the even-square times odd-factor pattern into cube and square sums.

16. Let x_0 be the point of Local maxima of $f(x) = \vec{a}.\left(\vec{b} \times \vec{c}\right)$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$, $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ at x = x_0 is : [4 marks]

(A) 14

(B) -30

(C) -4

(D) -22

Correct Answer: (D) -22

Solution:

Concept:

- The scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is evaluated as a determinant, with the three vector components as its rows.
- At a stationary point, $f'(x) = 0$; a negative second derivative identifies a strict local maximum.

Step 1: Compute the scalar triple product First form the cross product with its signs in the usual component order:

$$\vec{b} \times \vec{c} = (x^2 - 2)\hat{i} + (2x - 7)\hat{j} + (4 - 7x)\hat{k}.$$

Taking its dot product with $(x, -2, 3)$ gives

$$f(x) = x(x^2 - 2) - 2(2x - 7) + 3(4 - 7x) = x^3 - 27x + 26.$$

In particular, the middle cofactor of the cross product includes its negative sign; overlooking it changes the stationary points.

Step 2: Identify the local maximum Differentiate the polynomial:

$$f'(x) = 3x^2 - 27 = 3(x - 3)(x + 3), \qquad f''(x) = 6x.$$

The stationary points are -3 and 3. At -3, the second derivative is $-18 < 0$, whereas at 3 it is $18 > 0$. Therefore the specified point of local maximum is $x_0 = -3$; the positive root gives a local minimum and must not be substituted instead.

Step 3: Evaluate the requested sum of dot products Compute each ordinary dot product independently:

$$\vec{a} \cdot \vec{b} = -4x - 3, \qquad \vec{b} \cdot \vec{c} = -3x - 14, \qquad \vec{c} \cdot \vec{a} = 10x + 4.$$

Their sum simplifies to $3x - 13$. At $x_0 = -3$, it is

$$3(-3) - 13 = -22.$$

This calculation distinguishes the requested sum from the scalar triple product itself. The other three proposed numbers do not equal the sum evaluated at the maximum point.

Step 4: Conclusion The value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is -22. Hence, the correct option is (D).

Quick Tip to Solve: Use the second derivative to choose between the two stationary roots before evaluating the separate dot-product expression.

17. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90^o$ and area$(\Delta ABC) = 5\sqrt{5}$ s units, then the abscissa of the vertex C is : [4 marks]

(A) $1 + 2\sqrt{5}$

(B) $2\sqrt{5} - 1$

(C) $1 + \sqrt{5}$

(D) $2 + \sqrt{5}$

Correct Answer: (A) $1 + 2\sqrt{5}$

Solution:

Concept:

- Because $\angle BAC = 90°$, the side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have zero dot product.
- The area of a triangle with adjacent side vectors (u_1, u_2) and (v_1, v_2) is $\frac{1}{2}|u_1v_2 - u_2v_1|$.

Step 1: Parametrize the perpendicular side From $A(1, 2)$ and $B(3, 1)$, the vector along AB is $(2, -1)$. A perpendicular vector is $(1, 2)$, since $2(1) - 1(2) = 0$. Hence write

$$\overrightarrow{AC} = t(1, 2), \qquad C = (1 + t, 2 + 2t).$$

The real parameter t allows both possible directions along the line through A, rather than prematurely choosing one of the two triangles.

Step 2: Use the prescribed area The determinant of the adjacent side vectors is $2(2t) - (-1)t = 5t$. Thus

$$\text{Area}(\triangle ABC) = \frac{1}{2}|5t| = \frac{5}{2}|t|.$$

Equating this to $5\sqrt{5}$ gives $|t| = 2\sqrt{5}$. The same result follows from $|AB| = \sqrt{5}$, $|AC| = \sqrt{5}|t|$, and the right-triangle area formula. The absolute value is essential because either orientation has positive area.

Step 3: Apply the first-quadrant condition If $t = -2\sqrt{5}$, the proposed vertex has coordinates $(1 - 2\sqrt{5}, 2 - 4\sqrt{5})$, both negative, contradicting the stated quadrant. Therefore $t = 2\sqrt{5}$, and

$$C = (1 + 2\sqrt{5}, 2 + 4\sqrt{5}).$$

Its abscissa is $1 + 2\sqrt{5}$. The remaining options do not give the required horizontal displacement from A while meeting the area and perpendicularity conditions. Both coordinates of the selected vertex are positive, as required.

Step 4: Conclusion The abscissa of C is $1 + 2\sqrt{5}$. Hence, the correct option is (A).

Quick Tip to Solve: Choose a perpendicular direction vector first, use the area to find its scale, and use the quadrant only to choose the sign.

18. Given the following two statements:

$(S_1) : (q \vee p) \rightarrow (p \leftrightarrow \sim q)$ **is a tautology**

$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ **is a fallacy. Then:** [4 marks]

(A) both (S_1) and (S_2) are not correct

(B) only (S_1) is correct

(C) only (S_2) is correct

(D) both (S_1) and (S_2) are correct

Correct Answer: (A) both (S_1) and (S_2) are not correct

Solution:

Concept:

- A tautology is true for every assignment of its variables; a fallacy (contradiction) is false for every assignment.
- An implication is false only when its antecedent is true and consequent false. A biconditional is true exactly when both sides have equal truth values.

Step 1: Test the proposed tautology To disprove a tautology, one counterexample suffices. Take $p = T$ and $q = T$. Then $q \vee p = T$, but $\sim q = F$, so $p \leftrightarrow \sim q = T \leftrightarrow F = F$. Therefore

$$(q \vee p) \rightarrow (p \leftrightarrow \sim q) = T \rightarrow F = F.$$

Thus the formula in S_1 is not a tautology, making statement S_1 incorrect. There is no need to check the remaining assignments to establish this failure.

Step 2: Test the proposed fallacy To disprove a fallacy, find a single assignment making the formula true. Set $q = F$ and $p = T$. Then $\sim q = T$, and $\sim p = F$ agrees with $q = F$. Hence

$$\sim p \leftrightarrow q = F \leftrightarrow F = T, \qquad \sim q \wedge (\sim p \leftrightarrow q) = T \wedge T = T.$$

So the second displayed formula is not always false. Statement S_2, which calls it a fallacy, is also incorrect.

Step 3: Select the statement about the two claims The truth of a displayed formula for one particular assignment must be distinguished from the truth of the claim that it is a tautology or fallacy. We found a false assignment for the first formula and a true assignment for the second. Therefore neither universal claim holds. Option (B) incorrectly accepts the first claim, option (C) incorrectly accepts the second, and option (D) accepts both. Only option (A) agrees with both counterexamples.

Step 4: Conclusion Both S_1 and S_2 are not correct. Hence, the correct option is (A).

Quick Tip to Solve: Disprove a tautology with one false row and disprove a fallacy with one true row; a full truth table is then unnecessary.

19. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to: [4 marks]

(A) ${}^{50}C_6 - {}^{30}C_6$

(B) ${}^{51}C_7 - {}^{30}C_7$

(C) ${}^{50}C_7 - {}^{30}C_7$

(D) ${}^{51}C_7 + {}^{30}C_7$

Correct Answer: (B) ${}^{51}C_7 - {}^{30}C_7$

Solution:

Concept:

- Pascal's identity gives $\binom{n}{k} = \binom{n+1}{k+1} - \binom{n}{k+1}$. Consecutive terms therefore telescope when the lower index is fixed.
- In a finite telescoping sum, every intermediate contribution cancels. The surviving terms are determined by the first lower endpoint and the last upper endpoint, including both endpoints.

Step 1: Rewrite the summation index Put $n = 50 - r$. When $r = 0$, $n = 50$; when $r = 20$, $n = 30$. Reversing this finite list does not change its sum. Thus the question contains twenty-one terms, not twenty:

$$S = \binom{30}{6} + \binom{31}{6} + \cdots + \binom{50}{6}.$$

This endpoint check is essential because removing the term at 30 changes the lower boundary of the answer.

Step 2: Apply Pascal's identity term by term For each integer n from 30 through 50, write $\binom{n}{6} = \binom{n+1}{7} - \binom{n}{7}$. Hence

$$S = \left(\binom{31}{7} - \binom{30}{7}\right) + \left(\binom{32}{7} - \binom{31}{7}\right) + \cdots + \left(\binom{51}{7} - \binom{50}{7}\right).$$

Every coefficient from $\binom{31}{7}$ to $\binom{50}{7}$ occurs once positively and once negatively.

Step 3: Check the remaining boundaries The uncancelled upper contribution is $\binom{51}{7}$, while the lower contribution is $-\binom{30}{7}$. Option A keeps the wrong lower index; option C ends the upper boundary one term early; option D adds the lower boundary instead of subtracting it. None represents this telescoping sum.

Step 4: Conclusion

$$S = \binom{51}{7} - \binom{30}{7}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For a consecutive sum of binomial coefficients, increase the lower index by one and subtract the boundary just before the first term.

20. Let P(3, 3) be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to : [4 marks]

(A) $\left(\frac{9}{2}, 2\right)$

(B) $\left(\frac{3}{2}, 2\right)$

(C) (9,3)

(D) $\left(\frac{9}{2}, 3\right)$

Correct Answer: (D) $\left(\frac{9}{2}, 3\right)$

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, implicit differentiation gives the tangent slope $dy/dx = b^2x/(a^2y)$.
- The normal is perpendicular to the tangent, so their slopes multiply to -1. For this horizontal hyperbola its eccentricity satisfies $e^2 = 1 + b^2/a^2$.

Step 1: Find the slope of the given normal The normal passes through $P(3, 3)$ and $(9, 0)$. Its slope is therefore

$$m_N = \frac{0-3}{9-3} = -\frac{1}{2}.$$

Both coordinate differences must be taken in the same order. The tangent slope at P is consequently $m_T = -1/m_N = 2$; the line joining the two given points is the normal, so using its slope as the tangent would reverse the required ratio.

Step 2: Use the derivative at P Differentiating the hyperbola gives

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = \frac{b^2x}{a^2y}.$$

At $x = y = 3$, this is b^2/a^2. Equating it to 2 yields $b^2 = 2a^2$. The positive ratio also agrees with the positive tangent slope in the first quadrant.

Step 3: Substitute P and compute eccentricity Since P lies on the curve,

$$\frac{9}{a^2} - \frac{9}{2a^2} = 1 \quad \Longrightarrow \quad a^2 = \frac{9}{2}.$$

Therefore $b^2 = 9$, and $e^2 = 1 + 2 = 3$. The two options having $e^2 = 2$ violate the slope ratio; the pair $(9, 3)$ has the right eccentricity but fails the point equation. Substitution verifies $9/(9/2) - 9/9 = 2 - 1 = 1$.

Step 4: Conclusion The ordered pair is $\boxed{(a^2, e^2) = (9/2, 3)}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the normal to get the tangent slope first; at a point with equal coordinates that tangent slope is simply the ratio of the squared axes.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If the system of equations

x - 2y + 3z = 9

2x + y + z = b

x - 7y + az = 24,

has infinitely many solutions, then a - b is equal to..... [4 marks]

Solution:

Concept:

- Infinitely many solutions require a consistent linear system with rank less than the number of unknowns. A third equation dependent on two independent equations supplies no new restriction.
- Row operations must be applied to the constants as well as the coefficients. A zero coefficient row with a nonzero constant would make the system inconsistent.

Step 1: Eliminate x from the last two equations Write the first equation as $E_1 : x - 2y + 3z = 9$. Subtract twice E_1 from the second equation and subtract E_1 from the third. The equivalent equations are

$$5y - 5z = b - 18, \qquad -5y + (a-3)z = 15.$$

The first original equation and its second row are independent because the latter has a nonzero y coefficient but no x coefficient.

Step 2: Impose the condition for dependence Add the two reduced equations to eliminate y. This gives

$$(a-8)z = b - 3.$$

If $a \neq 8$, this equation fixes z uniquely; the preceding equation then fixes y, and the first fixes x, giving a unique solution. For infinitely many solutions the last relation must instead be an identity, so its coefficient and constant must both vanish: $a = 8$ and $b = 3$. If only the coefficient vanished, a nonzero right-hand side would give no solution.

Step 3: Verify an actual free parameter For these values the reduced equation is $5y - 5z = -15$, or $y = z - 3$. Let $z = t$, where t is any real number. The first equation gives $x = 3 - t$. Substituting $(x, y, z) = (3-t, t-3, t)$ into the second gives 3 and into the third gives 24 for every t. Thus consistency and infinitely many distinct solutions are both verified, rather than merely assuming that a zero determinant is sufficient.

Step 4: Conclusion

$$\boxed{a - b = 8 - 3 = 5}.$$

Quick Tip to Solve: After eliminating one variable, a dependent row must reduce to 0=0; zero determinant alone does not guarantee infinitely many solutions.

22. Suppose a differentiable function f(x) satisfies the identity

$f(x+y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y.

$\lim_{x\to 0} \frac{f(x)}{x} = 1$, then f'(3) is equal to _____. [4 marks]

Solution:

Concept:

- The derivative at zero is $f\prime(0) = \lim_{h\to 0}[f(h) - f(0)]/h$. The supplied limit determines it once $f(0)$ is known.
- In the functional identity, hold x fixed and let the increment y tend to zero. This extracts the derivative at any x directly, without guessing a formula for f.

Step 1: Find the value and derivative at zero Set $x = y = 0$ in the identity. It gives $f(0) = 2f(0)$, hence $f(0) = 0$. The stated limit therefore equals the derivative at zero:

$$f\prime(0) = \lim_{h\to 0} \frac{f(h) - f(0)}{h} = 1.$$

There is no extra constant term to include in the difference quotient.

Step 2: Form the difference quotient at x Replace y by h in the identity and subtract $f(x)$ from both sides. For nonzero h, division by h gives

$$\frac{f(x+h) - f(x)}{h} = \frac{f(h)}{h} + xh + x^2.$$

Keep x fixed during this limit. The first term tends to 1 by the given condition, the second tends to zero, and the third is independent of h. Differentiability makes the left side tend to $f\prime(x)$. Thus $f\prime(x) = 1 + x^2$ for every real x.

Step 3: Evaluate and check the identity At $x = 3$, $f\prime(3) = 1 + 3^2 = 10$. As a consistency check, integrating the derivative and using $f(0) = 0$ gives $f(x) = x + x^3/3$. Its additive difference is

$$((x+y)^3 - x^3 - y^3)/3 = x^2y + xy^2,$$

exactly the two extra terms in the question. This confirms the coefficient of x^2 in the derivative and avoids confusing the cubic term in f with its derivative.

Step 4: Conclusion

$$\boxed{f\prime(3) = 10}.$$

Quick Tip to Solve: For a differentiable addition identity, subtract f(x), divide by the increment, and let the increment approach zero before trying to find the whole function.

23. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is _____. [4 marks]

Solution:

Concept:

- For independent shots with constant hit probability p, the probability of missing every one of n shots is $(1-p)^n$.
- The event of at least one hit is the complement of no hits. Its probability is therefore $1-(1-p)^n$, and it increases with n when $0 < p < 1$.

Step 1: Translate the probability condition Each shot has hit probability $1/10$, so its miss probability is $9/10$. Using the repeated independent-trial model, the probability of n consecutive misses is $(9/10)^n$. Hence the required strict inequality becomes

$$1-\left(\frac{9}{10}\right)^n > \frac{1}{4} \quad \Longleftrightarrow \quad \left(\frac{9}{10}\right)^n < \frac{3}{4}.$$

Adding individual hit probabilities would count outcomes with two or more hits repeatedly and would not give the probability requested.

Step 2: Test the smallest candidate integers With one shot the hit probability is 0.1, below 0.25. With two shots,

$$P(\text{at least one hit}) = 1-0.9^2 = 1-0.81 = 0.19 < 0.25.$$

With three shots,

$$P(\text{at least one hit}) = 1-0.9^3 = 1-0.729 = 0.271 > 0.25.$$

These are exact finite decimals, so no rounding near the strict boundary is involved.

Step 3: Establish that the candidate is minimal The miss probability is multiplied by $0.9 < 1$ whenever another shot is added. It strictly decreases, so the complementary probability strictly increases. Since two shots fail while three shots succeed, every smaller positive number of shots fails and three is the least admissible integer. A logarithmic calculation is unnecessary here; checking adjacent integers directly both answers the question and proves minimality.

Step 4: Conclusion The least required number of shots is $\boxed{3}$.

Quick Tip to Solve: For at least one success, calculate one minus the probability of all failures; then check the integer immediately below your proposed minimum.

24. Let $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20} a_r x^r$

Then $\frac{a_7}{a_{13}}$ is equal to _____. [4 marks]

Solution:

Concept:

- A quadratic with different constant and leading coefficients can possess a scaled reciprocal symmetry. Replacing x by a suitable constant divided by x reverses its coefficient sequence.
- Equality of polynomials allows coefficients of the same power to be compared. Multiplying a reciprocal expression by the total degree power of x removes negative exponents.

Step 1: Find the reciprocal substitution Let $P(x)=(2x^2+3x+4)^{10}$. The leading and constant coefficients of the quadratic suggest replacing x by $2/x$. For nonzero x,

$$2(2/x)^2+3(2/x)+4=\frac{8}{x^2}+\frac{6}{x}+4=\frac{2}{x^2}(2x^2+3x+4).$$

The middle term also matches this scaling, so the relation holds for the entire quadratic, not merely for its endpoints.

Step 2: Raise to the tenth power and compare coefficients The preceding identity yields

$$x^{20}P(2/x)=2^{10}P(x).$$

On substituting $P(x)=\sum_{r=0}^{20} a_r x^r$, it becomes

$$\sum_{r=0}^{20} 2^r a_r x^{20-r}=2^{10}\sum_{r=0}^{20} a_r x^r.$$

Both sides are polynomials after multiplication by x^{20}, so their coefficients agree. The coefficient of x^{13} on the left comes from $r=7$; on the right it is $2^{10}a_{13}$.

Step 3: Obtain the required ratio Therefore $2^7 a_7=2^{10}a_{13}$, which gives $a_7/a_{13}=2^3=8$. Division is valid: all coefficients of the quadratic are positive, and the expansion contains terms of degree 13, so $a_{13}>0$. A full multinomial expansion would involve several contributions for each coefficient; the reciprocal identity pairs all of them at once and avoids omitting any combination.

Step 4: Conclusion

$$\boxed{\frac{a_7}{a_{13}}=8}.$$

Quick Tip to Solve: When coefficient indices add to the polynomial degree, look for a scaled reciprocal substitution that reverses those indices.

25. If the equation of a plane P, passing through the intersection of the planes, x + 4y - z + 7 = 0 and 3x + y + 5z = 8 is ax + by + 6z = 15 for some a, b $\in$ R, then the distance of the point (3, 2, -1) from the plane P is..... [4 marks]

Solution:

Concept:

- A plane through the line of intersection of two nonparallel planes is a linear combination of their equations. Its coefficients must be compared only after allowing an overall nonzero multiplier.
- The perpendicular distance of (x_0, y_0, z_0) from $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$.

Step 1: Write a general member of the family Use the given equations with all terms on the left. Every required plane has an equation of the form

$$\lambda(x + 4y - z + 7) + \mu(3x + y + 5z - 8) = 0.$$

The desired equation is $ax + by + 6z - 15 = 0$, so we may choose the multiplier normalization to make the coefficients of z and the constant exactly 6 and -15. Thus

$$-\lambda + 5\mu = 6, \qquad 7\lambda - 8\mu = -15.$$

This avoids treating the two original equations as though their constants were both on the same side.

Step 2: Determine the plane coefficients From the first relation, $\lambda = 5\mu - 6$. Substituting in the second gives

$$35\mu - 42 - 8\mu = -15 \quad \Rightarrow \quad \mu = 1, \quad \lambda = -1.$$

Hence the family member is

$$-(x + 4y - z + 7) + (3x + y + 5z - 8) = 0,$$

or

$$2x - 3y + 6z - 15 = 0.$$

Therefore $a = 2$, $b = -3$. As a check, this is the difference of the two plane equations, and consequently vanishes at every point of their intersection.

Step 3: Evaluate the perpendicular distance For the specified point $(3, 2, -1)$, the signed numerator is

$$2(3) - 3(2) + 6(-1) - 15 = -21.$$

The length of the normal vector is

$$\sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{49} = 7.$$

Distance is nonnegative, so use the absolute value:

$$d = \frac{|-21|}{7} = 3.$$

The point is not on the plane; the negative numerator merely identifies its side of the plane.

Step 4: Conclusion The distance of $(3, 2, -1)$ from plane P is $\boxed{3}$ units.

Quick Tip to Solve: Normalize the two fixed coefficients first: the z coefficient and constant determine the plane family parameters without needing a and b in advance.

Physics – Section A: Multiple Choice Questions (80 marks)

26. A air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is (g = 980 cm/s^2). [4 marks]

(A) 1.52 gm

(B) 4.51 gm

(C) 3.15 gm

(D) 4.15 gm

Correct Answer: (D) 4.15 gm

Solution:

Concept:

- Archimedes' buoyant force equals the weight of displaced water: $F_B = \rho V g$. The bubble displaces the volume of a sphere, $V = 4\pi r^3/3$.
- With upward positive and negligible drag, the only external vertical forces are buoyancy upward and the bubble's weight downward. Therefore $F_B - mg = ma$.

Step 1: Calculate the displaced volume Keep all quantities in the centimetre-gram-second system because the radius, density, acceleration and answer choices use those units. With $r = 1\,\text{cm}$,

$$V = \frac{4\pi}{3}(1)^3 = \frac{4\pi}{3}\,\text{cm}^3.$$

The mass of displaced water is therefore

$$\rho V = (1)\frac{4\pi}{3} = 4.18879\,\text{g}.$$

This is not quite the bubble mass: the upward acceleration tells us the bubble is lighter than the displaced water.

Step 2: Apply the upward force balance Newton's second law gives

$$\rho V g - mg = ma \quad \Rightarrow \quad m = \frac{\rho V g}{g + a}.$$

The stated upward acceleration is $9.8\,\text{cm}\,\text{s}^{-2}$, whereas $g = 980\,\text{cm}\,\text{s}^{-2}$. Thus

$$m = \frac{(4\pi/3)(980)}{980 + 9.8} = \frac{4.18879}{1.01} = 4.1473\,\text{g}.$$

Retaining the small acceleration term is necessary to distinguish the answer from simply the displaced mass.

Step 3: Compare the choices physically The computed mass rounds to $4.15\,\mathrm{g}$. Option (B), $4.51\,\mathrm{g}$, exceeds the displaced water mass and would accelerate downward. Options (A) and (C) are much smaller and would give considerably larger upward acceleration. Only option (D) reproduces the given small upward acceleration. No surface-tension force is added to the net external force on the complete spherical bubble.

Step 4: Conclusion The bubble has mass $\boxed{4.15\,\mathrm{g}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a freely accelerating submerged body with negligible drag, write m = displaced mass divided by (1 + a/g); an upward acceleration requires m below the displaced mass.

27. On the x-axis and at a distance x from the origin, the gravitational field due a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is: [4 marks]

(A) $A\left(x^2+a^2\right)^{3/2}$

(B) $A\left(x^2+a^2\right)^{1/2}$

(C) $\frac{A}{(x^2+a^2)^{1/2}}$

(D) $\frac{A}{(x^2+a^2)^{3/2}}$

Correct Answer: (C) $\frac{A}{(x^2+a^2)^{1/2}}$

Solution:

Concept:

- Field and potential satisfy $g_x = -d\Phi/dx$. The additive constant is fixed by the condition $\Phi(\infty) = 0$, rather than by the field alone.
- The useful derivative is $d(x^2+a^2)^{-1/2}/dx = -x(x^2+a^2)^{-3/2}$. It matches the given field apart from the factor and minus sign.

Step 1: Integrate the stated field The field in the question is directed along the positive x-axis and has component

$$g_x = \frac{Ax}{(x^2+a^2)^{3/2}}.$$

Therefore

$$\frac{d\Phi}{dx} = -\frac{Ax}{(x^2+a^2)^{3/2}}.$$

Substitute $u = x^2 + a^2$, so $du = 2x\,dx$, to obtain

$$\Phi = -\frac{A}{2}\int u^{-3/2}\,du = Au^{-1/2} + C = \frac{A}{\sqrt{x^2+a^2}} + C.$$

The sign follows the field as printed; one should not insert an inward direction that is not stated.

Step 2: Use the reference at infinity For large positive x, $1/\sqrt{x^2+a^2}$ tends to zero. The prescribed reference $\Phi(\infty) = 0$ consequently gives $C = 0$. Thus

$$\Phi(x) = \frac{A}{\sqrt{x^2+a^2}}.$$

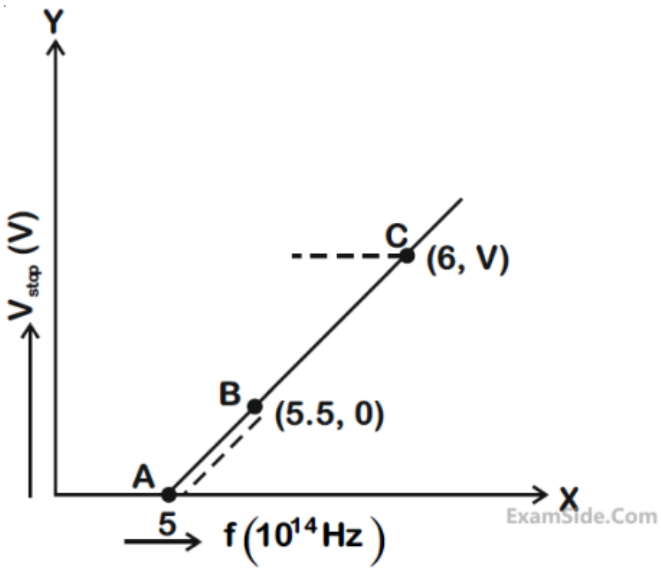
The question asks for the magnitude, so with the positive constant represented in the options this is the required expression. Differentiating it gives $-Ax/(x^2+a^2)^{3/2}$; taking its negative recovers exactly the field component.

Step 3: Reject the other powers Options (A) and (B) grow without bound at infinity, violating the specified zero reference. Option (D) does decay, but its derivative gives a denominator with power $5/2$ and a factor 3, not the given field. Option (C) alone has both the correct derivative and the correct limiting value. Integration reduces the denominator power because the factor $x\,dx$ supplies the differential of x^2+a^2.

Step 4: Conclusion The magnitude of gravitational potential is $\boxed{A/\sqrt{x^2+a^2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Check a proposed potential by both tests: its negative derivative must give the field, and its value must satisfy the chosen reference at infinity.

28. Given figure shows few data points in a phot electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is:
(Plancks constant h = 6.62 $\times$ 10^{-34} J.s) [4 marks]



(A) 2.10 eV
(B) 2.27 eV
(C) 2.59 eV
(D) 1.93 eV
Correct Answer: (B) 2.27 eV

Solution:

Concept:

- Einstein's photoelectric equation is $eV_{\text{stop}} = hf - \phi$, where ϕ is the work function and f is incident frequency.
- At threshold, the maximum electron kinetic energy and stopping potential are zero. The frequency-axis intercept therefore gives $f_0 = \phi/h$.

Step 1: Read the threshold from the graph There are two conflicting threshold indications in the supplied figure. The drawn line reaches the frequency axis at A, beside the tick 5. Reading that intercept gives $f_0 = 5 \times 10^{14}\,\text{Hz}$. However, B is explicitly labelled $(5.5, 0)$, although it is drawn above the axis. Its printed zero ordinate instead means $V_{\text{stop}} = 0$, giving $f_0 = 5.5 \times 10^{14}\,\text{Hz}$. Thus the drawing and the numerical coordinate are internally inconsistent. We retain both as printed and first calculate from the explicitly labelled zero-potential coordinate.

Step 2: Convert the threshold energy into electron volts The least energy needed to eject an electron is the work function:

$$\phi = hf_0 = (6.62 \times 10^{-34})(5.5 \times 10^{14}) = 3.641 \times 10^{-19}\,\text{J}.$$

Convert to electron volts using $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$:

$$\phi = \frac{3.641 \times 10^{-19}}{1.602 \times 10^{-19}}\,\text{eV} \simeq 2.273\,\text{eV}.$$

This rounds to $2.27\,\text{eV}$. The units cancel correctly because dividing an energy in joules by the joules per electron volt yields the energy in electron volts.

Step 3: Compare with the remaining options If the drawn intercept A at $5 \times 10^{14}\,\text{Hz}$ is used instead, Einstein's equation gives

$$\phi = (6.62 \times 10^{-34})(5 \times 10^{14}) = 3.31 \times 10^{-19}\,\text{J} \simeq 2.07\,\text{eV},$$

which is near option (A), $2.10\,\text{eV}$. Hence the figure does not uniquely support one numerical threshold without choosing between its conflicting indications. The printed coordinate B leads to option (B), $2.27\,\text{eV}$, which is also the supplied answer key. Options (C) and (D) follow neither indication. No corrected ordinate or shifted intercept is assumed.

Step 4: Conclusion The labelled coordinate B gives $\boxed{2.27\,\text{eV}}$, option (B), matching the supplied answer key. The drawn intercept instead gives about $2.07\,\text{eV}$, near option (A); the printed graph therefore contains an internal inconsistency.

Quick Tip to Solve: Identify threshold by zero stopping potential, but cross-check printed coordinates against the drawn intercept. If they disagree, state both readings rather than silently changing the graph.

29. A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is : [4 marks]

(A) 6.4×10^{-2} J

(B) 9.2×10^{-3} J

(C) 7.2×10^{-2} J

(D) 11.7×10^{-3} J

Correct Answer: (C) 7.2×10^{-2} J

Solution:

Concept:

- A magnetic dipole of moment m in a uniform magnetic field experiences torque $\tau = mB\sin\theta$, where θ is the angle between its moment and the field.
- Its potential energy is $U = -mB\cos\theta$. Stable equilibrium is parallel to the field and unstable equilibrium is antiparallel. Minimum external work equals the increase in potential energy.

Step 1: Find the product of magnetic moment and field The measured torque refers to $\theta = 30°$, not to a right angle. Hence

$$0.018 = m(0.06)\sin 30° = m(0.06)\frac{1}{2}.$$

This gives

$$m = \frac{0.018}{0.03} = 0.60\,\text{A m}^2, \qquad mB = 0.60(0.06) = 0.036\,\text{J}.$$

One can find mB directly from $\tau/\sin 30°$; computing m separately is simply a useful dimensional check.

Step 2: Compare the two equilibrium energies At stable equilibrium $\theta = 0$, so

$$U_{\text{stable}} = -mB = -0.036\,\text{J}.$$

At unstable equilibrium $\theta = \pi$, so

$$U_{\text{unstable}} = +mB = +0.036\,\text{J}.$$

Rotating slowly from the first orientation to the second requires positive external work. The minimum is achieved without leaving extra kinetic energy in the magnet:

$$W_{\min} = \Delta U = 0.036 - (-0.036) = 0.072\,\text{J}.$$

The external work is opposite in sign to the work performed by the magnetic field.

Step 3: Match the energy rather than multiplying by a fixed torque The torque changes as the magnet rotates, vanishing at both equilibrium orientations. It is therefore incorrect to multiply the measured $0.018\,\text{N m}$ by the full angular displacement as if it were constant. The potential-energy calculation

includes its angular variation automatically. Written in scientific notation, $0.072 = 7.2\times10^{-2}$, matching (C); none of the other numerical choices equals the energy increase $2mB$.

Step 4: Conclusion The minimum work required is $\boxed{7.2 \times 10^{-2}\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: For stable-to-unstable rotation of a magnetic dipole, use W = 2mB; infer mB from the given torque after dividing by the sine of its angle.

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30. Starting from the origin at time t = 0, with initial velocity $5\hat{j}$ ms^{-1} , a particle moves in the x-y plane with a constant acceleration of $\left(10\hat{i}+4\hat{j}\right)$ ms^{-2}. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively: [4 marks]

(A) 5s and 25 m

(B) 2s and 18 m

(C) 2s and 24 m

(D) 4s and 52 m

Correct Answer: (B) 2s and 18 m

Solution:

Concept:

- For constant acceleration, each Cartesian component obeys $s_i = u_i t + \frac{1}{2}a_i t^2$. The x and y motions share the same elapsed time.
- The initial velocity $5\hat{j}\,\text{m s}^{-1}$ means $u_x = 0$, $u_y = 5\,\text{m s}^{-1}$. The acceleration components are $a_x = 10$ and $a_y = 4\,\text{m s}^{-2}$.

Step 1: Use the x coordinate to find the time The particle starts at the origin, so there is no initial-position term. The horizontal motion is

$$x = u_x t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(10)t^2 = 5t^2.$$

At the event described, $x = 20\,\text{m}$, hence

$$20 = 5t^2 \quad \Rightarrow \quad t^2 = 4.$$

The algebraic roots are $t = \pm 2\,\text{s}$. Since motion is considered after the stated start at $t = 0$, select $t = 2\,\text{s}$. The horizontal acceleration, not the initial vertical speed, determines when the required x coordinate is reached.

Step 2: Evaluate the y coordinate at that same time The vertical displacement includes both the initial vertical velocity and the acceleration contribution:

$$y_0 = u_y t + \frac{1}{2}a_y t^2 = 5t + 2t^2.$$

Substituting $t = 2\,\text{s}$ gives

$$y_0 = 5(2) + 2(2)^2 = 10 + 8 = 18\,\text{m}.$$

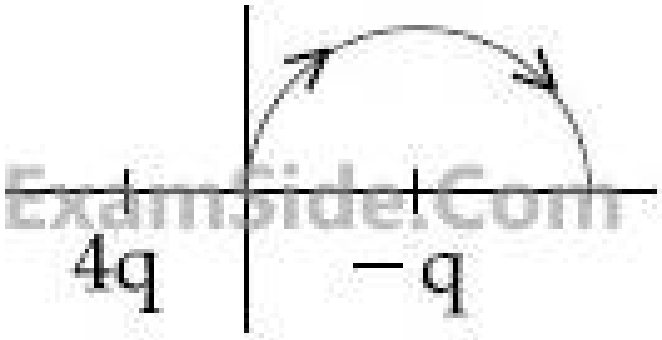
The two contributions have the same sign, so the particle is above the x-axis. Omitting either term would lose part of the displacement; using the full acceleration magnitude would incorrectly mix the independent coordinate equations.

Step 3: Check the proposed pairs For $t = 5\,\text{s}$, the horizontal coordinate would be $125\,\text{m}$, excluding (A). At $t = 4\,\text{s}$, it would be $80\,\text{m}$, excluding (D). Both (B) and (C) give the right time, but only (B) gives the vertical coordinate $18\,\text{m}$. Substituting the resulting pair directly into both component equations confirms the requested event.

Step 4: Conclusion The values are $\boxed{t = 2\,\text{s},\ y_0 = 18\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Find time from the coordinate with simpler initial conditions, then use that identical time in the other component; do not combine the accelerations into a scalar magnitude.

31. A two point charges 4q and -q are fixed on the x-axis at x = $-\frac{d}{2}$ and x = $\frac{d}{2}$ respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will : [4 marks]



(A) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

(B) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$

(C) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$

(D) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

Correct Answer: (D) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

Solution:

Concept:

- The potential due to fixed point charges adds algebraically: $V = k\sum Q_i/r_i$, where $k = 1/(4\pi\varepsilon_0)$. Potential energy of the moving charge is $U = qV$.
- Electrostatic forces are conservative. Thus $\Delta U = q(V_f - V_i)$ depends only on the endpoints, not on whether the charge travels on a semicircle or a straight line.

Step 1: Find the initial potential At the origin both fixed charges are at distance $d/2$. Include the negative sign of the second charge while adding potentials; potential is a scalar, so no directional

components are required.

$$V_i = k\left(\frac{4q}{d/2} - \frac{q}{d/2}\right) = \frac{6kq}{d}.$$

The initial interaction energy of the moving charge with the fixed pair is therefore $U_i = 6kq^2/d$. The energy of interaction between the two fixed charges is unchanged and cancels from the energy difference.

Step 2: Evaluate the final potential and subtract At $x = d$, the distances from $4q$ and $-q$ are respectively $3d/2$ and $d/2$. Hence

$$V_f = k\left(\frac{4q}{3d/2} - \frac{q}{d/2}\right) = \frac{2kq}{3d}.$$

Consequently,

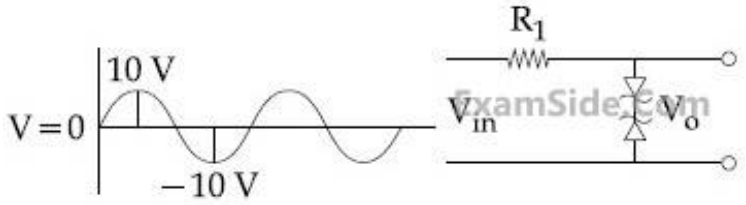
$$\Delta U = q(V_f - V_i) = \frac{kq^2}{d}\left(\frac{2}{3} - 6\right) = -\frac{16kq^2}{3d} = -\frac{4q^2}{3\pi\varepsilon_0 d}.$$

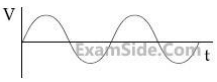
The negative sign means a decrease. Options (A) and (B) predict the opposite sign, while (C) has the wrong magnitude.

Step 3: Conclusion The energy decreases by $4q^2/(3\pi\varepsilon_0 d)$. Hence, the correct option is (D).

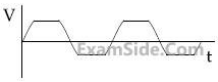
Quick Tip to Solve: Calculate endpoint potentials first. The semicircular route changes neither the potential difference nor the electrostatic energy change.

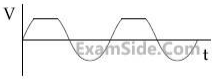
32. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is: (Graphs drawn are schematic and not to scale) [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- A reverse-biased Zener diode conducts strongly when its reverse voltage reaches the breakdown value V_Z. The series resistor limits this current and takes up the excess input voltage.
- The two oppositely oriented Zener diodes form a bidirectional shunt limiter. In either polarity, one diode is forward biased while the other is reverse biased.

Step 1: Examine the region below breakdown The input is an alternating waveform of peak magnitude $10\,\mathrm{V}$, whereas the specified breakdown voltage is $6\,\mathrm{V}$. Near each zero crossing, the magnitude is too small to bring the reverse-biased member into breakdown. The shunt branch then carries negligible current, so the voltage drop across R_1 is negligible and the output follows the input. Thus the rising and falling portions near zero remain sinusoidal; the output is neither a rectified waveform nor a constant voltage.

Step 2: Apply clipping to both half-cycles On the positive half-cycle, one diode becomes forward biased and the other reaches reverse breakdown. Further increase in input is mainly dropped across R_1, leaving a nearly constant positive output. On the negative half-cycle the two diodes exchange roles, giving the corresponding negative limiting level. With the ideal forward-diode approximation the limiting levels are approximately $\pm 6\,\mathrm{V}$; including a small forward drop changes their magnitudes slightly but not the shape. Both the positive and negative peaks must therefore be flattened. Option (A) is an unclipped sine wave, (B) removes a half-cycle, and (D) clips only one polarity. Only (C) has symmetric flattened extrema while retaining both half-cycles.

Step 3: Conclusion The output follows the input near zero and is limited at both positive and negative peaks. Hence, the correct option is (C).

Quick Tip to Solve: For two back-to-back Zener diodes, interchange their forward and reverse roles on the next half-cycle: both polarities are clipped.

33. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is : [4 marks]

(A) 0.50 W

(B) 0.072 W

(C) 0.10 W

(D) 0.125 W

Correct Answer: (C) 0.10 W

Solution:

Concept:

- The power delivered to an external resistor is $P_{\text{ext}} = VI$, where V is the terminal voltage under load, not the battery emf $\mathcal{E}$.
- For a discharging battery, $\mathcal{E} = V + Ir$. Its internal heating is $P_{\text{int}} = I^2 r = I(\mathcal{E} - V)$, with r the internal resistance.

Step 1: Determine the common circuit current The external resistor dissipates $0.5\,\mathrm{W}$ while the terminal voltage is $2.5\,\mathrm{V}$. The current through the resistor is also the current through the battery internal resistance because they belong to the same series circuit. Thus

$$I = \frac{P_{\text{ext}}}{V} = \frac{0.5}{2.5} = 0.20\,\mathrm{A}.$$

Using the emf $3.0\,\mathrm{V}$ in this division would be incorrect: only the terminal voltage acts across the external resistor. As a check, the external resistance is $R = V/I = 12.5\,\Omega$.

Step 2: Calculate the loss inside the battery The internal voltage drop equals the difference between the emf and the loaded terminal voltage:

$$Ir = \mathcal{E} - V = 3.0 - 2.5 = 0.50\,\mathrm{V}.$$

Therefore

$$P_{\text{int}} = I(Ir) = 0.20 \times 0.50 = 0.10\,\mathrm{W}.$$

Equivalently, $r = 0.50/0.20 = 2.5\,\Omega$ and $I^2 r = (0.20)^2(2.5) = 0.10\,\mathrm{W}$. Energy conservation gives the same result: the source supplies $\mathcal{E}I = 0.60\,\mathrm{W}$, of which $0.50\,\mathrm{W}$ reaches the resistor. The other listed powers do not equal this difference.

Step 3: Conclusion The power dissipated in the internal resistance is $0.10\,\mathrm{W}$. Hence, the correct option is (C).

Quick Tip to Solve: Use terminal voltage to find the load current, then multiply that current by the lost voltage $\mathcal{E} - V$ to find internal heating.

34. A beam of plane polarised light of large cross-sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to : [4 marks]

(A) 1.0×10^{-5} J

(B) 1.0×10^{-4} J

(C) 1.5×10^{-4} J

(D) 5.0×10^{-4} J

Correct Answer: (B) 1.0×10^{-4} J

Solution:

Concept:

- Malus law for plane-polarised incident light is $I(t) = I_0 \cos^2\theta(t)$, where θ is the angle between the incident polarisation and the transmission axis.
- Transmitted power is intensity times illuminated polariser area: $P(t) = I_0 A \cos^2\theta(t)$. Over a full revolution the average of $\cos^2\theta$ is $1/2$.

Step 1: Set the illumination area and rotation period The incident beam is larger than the polariser, so the entire polariser area $A = 3 \times 10^{-4}\,\mathrm{m}^2$ is illuminated. It is this area, rather than the unspecified cross-section of the incident beam, that enters the transmitted power. With constant angular speed $\omega = 31.4\,\mathrm{rad\,s^{-1}}$, one revolution takes

$$T = \frac{2\pi}{\omega} \simeq \frac{6.28}{31.4} = 0.20\,\mathrm{s}.$$

The initial direction of the transmission axis cannot affect energy accumulated over a complete turn.

Step 2: Integrate the transmitted power The incident intensity is already that of plane-polarised light, so apply Malus law directly and average only once:

$$E = \int_0^T I_0 A \cos^2(\omega t + \theta_0)\, dt = \frac{I_0 A T}{2}.$$

Substitution gives

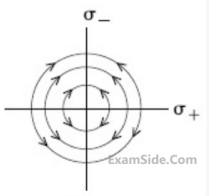
$$E = \frac{3.3 \times 3 \times 10^{-4} \times 0.20}{2} = 9.9 \times 10^{-5}\,\mathrm{J}.$$

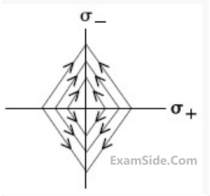
This is close to 1.0×10^{-4} J. Option (A) is too small by roughly a factor of ten, and the larger values in (C) and (D) do not follow from the half-intensity average. Applying an additional factor of one-half for unpolarised light would also be unjustified.

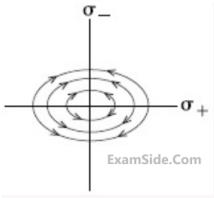
Step 3: Conclusion The energy transmitted in one revolution is approximately 1.0×10^{-4} J. Hence, the correct option is (B).

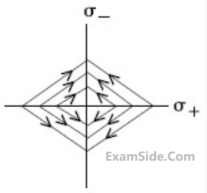
Quick Tip to Solve: For a complete turn of an analyser, replace $\cos^2\theta$ by $1/2$, then multiply average intensity by area and the full rotation period.

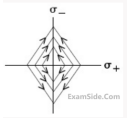
35. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- An infinite plane sheet produces a uniform field of magnitude $|\sigma|/(2\varepsilon_0)$, perpendicular to the sheet. The field points away from a positive sheet and towards a negative sheet.
- Superposition adds these two uniform, perpendicular fields. Within any one quadrant both components are constant, so the field lines there must be straight, not curved.

Step 1: Determine the component directions In the option diagrams the positive sheet is horizontal and the negative sheet is vertical. Above the horizontal sheet its contribution points upward, and below it the contribution points downward. To the right of the negative sheet its contribution points left, while to its left the contribution points right. Thus lines leave the horizontal positive sheet and head towards the vertical negative sheet. For example, the field in the upper-right quadrant points upward and to the left; in the lower-right quadrant it points downward and to the left.

Step 2: Use the unequal charge densities to select the slopes Choose horizontal x and vertical y coordinates in the plane of the drawings. In every quadrant,

$$|E_y| = \frac{|\sigma_+|}{2\varepsilon_0}, \qquad |E_x| = \frac{|\sigma_-|}{2\varepsilon_0}.$$

The stated inequality gives

$$\left|\frac{dy}{dx}\right| = \frac{|E_y|}{|E_x|} = \frac{|\sigma_+|}{|\sigma_-|} > 1.$$

The straight segments must therefore be steeper than $45°$, producing the vertically extended arrangement shown in (B), with arrows directed from the positive sheet towards the negative sheet. The curved lines of (A) and (C) contradict constant field components within a quadrant. The horizontally

extended straight pattern of (D) would require the opposite inequality of magnitudes.

Step 3: Conclusion The field lines are straight within each quadrant and have the steeper vertical component shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: First reject curved lines for uniform infinite-sheet fields. Then compare the perpendicular component magnitudes to decide whether the straight pattern is tall or wide.

36. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the are : [4 marks]

(A) 1, 3, 5, ...

(B) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$, ...

(C) 1, 2, 3, ...

(D) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$,

Correct Answer: (B) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$, ...

Solution:

Concept:

- Any two crests of a periodic transverse wave are separated by an integer multiple of the wavelength: $\Delta x_{cc} = n\lambda$, where n is a positive integer.
- A crest and a trough are separated by an odd number of half-wavelengths: $\Delta x_{ct} = (2m+1)\lambda/2$, with m a nonnegative integer. Neither separation is stated to be between adjacent extrema.

Step 1: Write both integer restrictions Let the measured crest-to-crest distance $5\,\mathrm{m}$ contain n full wavelengths, and let the measured crest-to-trough distance $1.5\,\mathrm{m}$ contain an odd number of half-wavelengths. The conditions are

$$5 = n\lambda, \qquad 1.5 = \frac{(2m+1)\lambda}{2}.$$

Eliminating λ gives

$$3n = 5(2m+1).$$

Since 3 and 5 are coprime, n must be a multiple of 5. Write $n = 5k$, where k is a positive integer.

Step 2: Find all allowed wavelengths and test them Substituting $n = 5k$ into the first condition gives $\lambda = 1/k\,\mathrm{m}$. The second condition becomes $2m + 1 = 3k$. Its left side is odd, so k must also be odd. Conversely, every positive odd k makes $n = 5k$ an integer and $m = (3k-1)/2$ a nonnegative integer, so all such wavelengths really work. Hence

$$\lambda = 1,\ \frac{1}{3},\ \frac{1}{5}, \ldots\ \mathrm{m}.$$

Option (A) incorrectly includes 3 and $5\,\mathrm{m}$, which cannot satisfy both distances; (C) similarly includes invalid integer wavelengths. Option (D) makes $3/\lambda$ even rather than odd. In particular, assuming

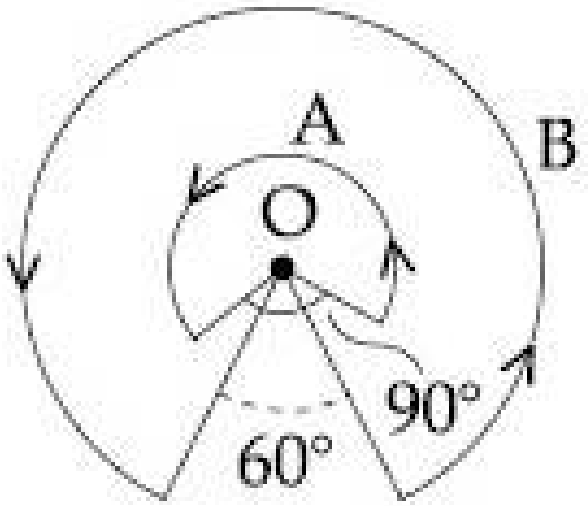
adjacent crests would give $5\,\mathrm{m}$, inconsistent with the other measurement.

Step 3: Conclusion The possible wavelengths are $1, 1/3, 1/5, \ldots\ \mathrm{m}$. Hence, the correct option is (B).

Quick Tip to Solve: Do not assume measured extrema are adjacent. Use integer wavelengths for crest-to-crest distances and odd half-wavelengths for crest-to-trough distances.

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37. A wire A, bent in the shape of an arc of a circle, carrying a current of 2 A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is : [4 marks]



(A) 4 : 6

(B) 6 : 4

(C) 2 : 5

(D) 6 : 5

Correct Answer: (D) 6 : 5

Solution:

Concept:

- Biot–Savart law gives the field at the centre of a circular arc: $B = \mu_0 I\theta/(4\pi R)$, where θ is the angle occupied by the conducting arc in radians.
- For two arcs in the same medium, the common factor $\mu_0/(4\pi)$ cancels. Thus the ratio depends on current, angular extent and inverse radius, not merely on current.

Step 1: Read the conducting angles The inner arc A has a missing angle of $90°$, so its conducting angle is $\theta_A = 360° - 90° = 270°$. The outer arc B has a missing angle of $60°$, so $\theta_B = 300°$. These are the long arcs occupied by the wires; using the small empty sectors would give the wrong field.

Step 2: Form and simplify the field ratio Use $I_A = 2\,\mathrm{A}$, $R_A = 2\,\mathrm{cm}$, $I_B = 3\,\mathrm{A}$ and $R_B = 4\,\mathrm{cm}$. The

same angular and length units may be used in this ratio:

$$\frac{B_A}{B_B} = \frac{I_A\theta_A R_B}{I_B\theta_B R_A} = \frac{2 \times 270 \times 4}{3 \times 300 \times 2} = \frac{6}{5}.$$

The right-hand rule determines each field direction; the requested ratio is their magnitudes. Ratios $4:6$ and $6:4$ neglect the unequal angular spans, while $2:5$ does not satisfy the arc-field expression.

Step 3: Conclusion The magnetic-field magnitude ratio is $B_A : B_B = 6:5$. Hence, the correct option is (D).

Quick Tip to Solve: For an incomplete circular wire, subtract the empty sector from a full turn before using $B \propto I\theta/R$.

38. Match the $\frac{C_P}{C_V}$ ratio for ideal gases with different type of molecules :

Molecule Type	C_P/C_V
(A) Monatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

[4 marks]

(A) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(B) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(C) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(D) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Correct Answer: (C) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom per molecule, $C_V = fR/2$ and $C_P = C_V + R$. Here these heat capacities are molar.
- The ratio is $\gamma = C_P/C_V = 1 + 2/f$. Rigid molecules have no vibrational contribution; non-rigid diatomic molecules include the kinetic and potential terms of one vibration.

Step 1: Count the active degrees of freedom A monatomic molecule has only three translations, so $f = 3$. A rigid diatomic molecule has three translations and two rotations, so $f = 5$. Allowing its one vibrational mode adds two quadratic terms, giving $f = 7$ for the non-rigid diatomic molecule. The non-linear triatomic rigid molecule represented in the listed matching has three translations and three rotations, hence $f = 6$.

Step 2: Match each ratio with the printed code Substitution gives

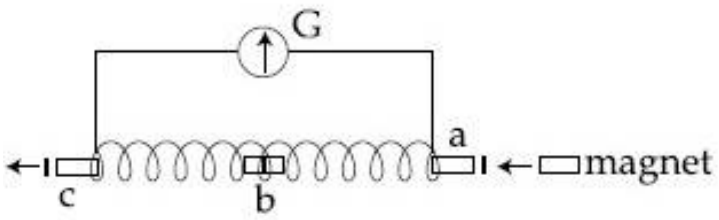
$$\gamma_{\text{mono}} = 1 + \frac{2}{3} = \frac{5}{3} \quad \longrightarrow \text{IV},$$
$$\gamma_{\text{diatomic,rigid}} = 1 + \frac{2}{5} = \frac{7}{5} \quad \longrightarrow \text{I},$$
$$\gamma_{\text{diatomic,nonrigid}} = 1 + \frac{2}{7} = \frac{9}{7} \quad \longrightarrow \text{II},$$
$$\gamma_{\text{triatomic,rigid}} = 1 + \frac{2}{6} = \frac{4}{3} \quad \longrightarrow \text{III}.$$

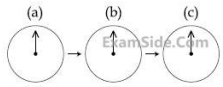
Only the third option contains all four matches. The first and fourth options already give an incorrect monatomic ratio; the second interchanges the rigid and non-rigid diatomic ratios.

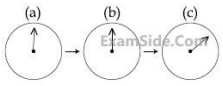
Step 3: Conclusion The matching is monatomic–IV, rigid diatomic–I, non-rigid diatomic–II, rigid triatomic–III. Hence, the correct option is (C).

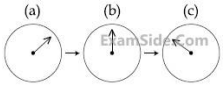
Quick Tip to Solve: Use $\gamma = 1 + 2/f$: more active degrees of freedom increase heat capacity and bring γ closer to one.

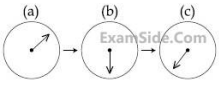
39. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil?
Three positions shown describe :
(a) the magnet's entry
(b) magnet is completely inside and
(c) magnet's exit. [4 marks]

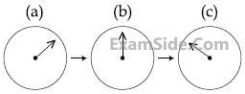


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Faraday's law gives $\mathcal{E} = -d\Phi_B/dt$, where Φ_B is the magnetic flux linked with the coil. Deflection indicates induced current, rather than the presence of a magnetic field alone.

- Lenz's law makes the induced current oppose the change of flux. Reversing the sign of that change reverses the current and therefore the galvanometer deflection.

Step 1: Follow the flux as the magnet enters As the magnet approaches and enters the coil, the flux linked with it increases in magnitude. Its rate of change is nonzero, so a current is induced and the galvanometer deflects to one side. In the option drawings this first deflection is shown toward the right. A stationary vertical needle at entry would incorrectly imply no changing flux.

Step 2: Compare the central and exit positions At the central position, the magnet is completely inside and the symmetric flux linkage is momentarily maximum. The slope of flux against position is then zero; since the speed is constant, $d\Phi_B/dt = v\, d\Phi_B/dx = 0$. Thus the needle returns to its central vertical position. During exit the flux linkage decreases, so $d\Phi_B/dt$ changes sign and the needle deflects to the opposite side. Constant speed does not mean constant flux change: the coil's linkage varies with position. The required sequence is right deflection, zero deflection, then left deflection. Option A shows no induction at all; B omits the entry deflection, and D incorrectly shows a nonzero central reading.

Step 3: Conclusion The galvanometer deflects oppositely on entry and exit and is undeflected at the central inside position. Hence, the correct option is (C).

Quick Tip to Solve: In magnet–coil questions, track whether flux is increasing, stationary or decreasing; the needle responds to its rate of change.

40. The specific heat of water = 4200 J kg^{-1}K^{-1} and the latent heat of ice = 3.4 $\times$ 10^5 J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams) : [4 marks]

(A) 63.8

(B) 61.7

(C) 69.3

(D) 64.6

Correct Answer: (B) 61.7

Solution:

Concept:

- When warm water cools without a change of phase, its released heat is $Q = mc\Delta T$, where c is its specific heat capacity.
- Ice already at its melting point absorbs $Q = m_{\text{melt}} L_f$ while melting, without a temperature rise. Heat conservation relates these quantities when losses are neglected.

Step 1: Calculate the heat released by the water Convert the initial water mass to $0.200\,\text{kg}$. Its

temperature falls from $25^\circ\mathrm{C}$ to $0^\circ\mathrm{C}$, a change of $25\,\mathrm{K}$. Therefore

$$Q = (0.200)(4200)(25) = 21000\,\mathrm{J}.$$

The ice begins at $0^\circ\mathrm{C}$, so no heat is needed to warm the solid before melting begins.

Step 2: Use the latent heat to find the melted mass The heat needed to melt all $0.100\,\mathrm{kg}$ of ice would be $0.100(3.4 \times 10^5) = 34000\,\mathrm{J}$, greater than the available heat. Consequently only some ice melts and the final mixture can remain at $0^\circ\mathrm{C}$. Setting the heat absorbed equal to that released gives

$$m_{\mathrm{melt}} = \frac{21000}{3.4 \times 10^5} = 0.0617647\,\mathrm{kg} = 61.7647\,\mathrm{g}.$$

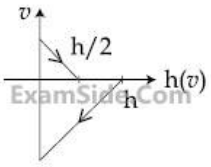
This is below the initial ice mass, as required. Among the listed values, $61.7\,\mathrm{g}$ is closest; the other three values would require more than the available $21000\,\mathrm{J}$ of latent heat.

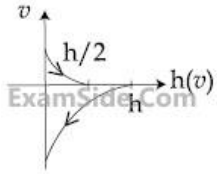
Step 3: Conclusion Approximately $61.8\,\mathrm{g}$ of ice melts; the closest listed amount is $61.7\,\mathrm{g}$. Hence, the correct option is (B).

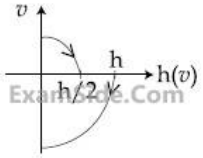
Quick Tip to Solve: First test whether the available heat can melt all the ice. If not, divide the heat released by the latent heat and keep the final temperature at the melting point.

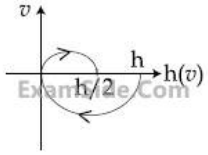
41. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :

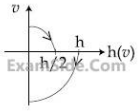
(graph are drawn schematically and on not to scale) [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- With height y measured upward from the floor and upward velocity positive, constant gravitational acceleration gives $v^2 - u^2 = -2g(y - y_0)$.
- The downward and upward branches must have opposite velocity signs. The bounce changes velocity rapidly at the same height $y = 0$, and a smaller rebound height means a smaller speed

immediately after impact.

Step 1: Find the downward branch The ball starts from rest at $y = h$. During the fall it has negative velocity, so

$$v = -\sqrt{2g(h-y)}, \qquad 0 \le y \le h.$$

It begins at $(h, 0)$ and moves toward $(0, -\sqrt{2gh})$. This is a curved branch, not a straight line: v^2, rather than v, is linear in height. Its slope is positive and becomes increasingly steep as the ball approaches its release height.

Step 2: Find the rebound branch and choose the graph On the rebound, the ball rises to $h/2$, where its velocity becomes zero. Thus

$$v = +\sqrt{2g(h/2-y)} = +\sqrt{gh-2gy}, \qquad 0 \le y \le h/2.$$

Immediately after the collision its speed is $\sqrt{gh}$, smaller than the incident speed by a factor $1/\sqrt{2}$. The upper branch therefore ends on the height axis at $h/2$, while the lower branch ends at h. At the floor the graph jumps from negative to positive velocity. The third graph has these curved branches and the correct height intercepts. Straight branches, a reversed time direction on the falling branch, or exchanging the intercepts cannot represent this motion.

Step 3: Conclusion The correct velocity–height plot is the third graph, with the downward branch below the axis ending at h and the upward branch above it ending at $h/2$. Hence, the correct option is (C).

Quick Tip to Solve: For a velocity–height graph, start with $v^2 = 2g \times$ height difference, then select the sign of v separately for falling and rising.

42. Choose the correct option relating wave lengths of different parts of electromagnetic wave spectrum: [4 marks]

(A) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$

(B) $\lambda_{\text{visible}} > \lambda_{\text{x-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$

(C) $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{x-rays}}$

(D) $\lambda_{\text{x-rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

Correct Answer: (A) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$

Solution:

Concept:

- All electromagnetic waves have speed c in vacuum, with $c = \nu\lambda$. Hence wavelength decreases when frequency increases.
- The spectrum in increasing frequency is radio, microwave, infrared, visible, ultraviolet, X-ray and gamma ray. The wavelength order is the reverse of this sequence.

Step 1: Place the four bands in spectral order Among the bands listed, radio waves have the lowest frequencies and the longest wavelengths. Microwaves have shorter wavelengths than radio waves

but much longer wavelengths than visible light. X-rays have higher frequencies than visible light and therefore shorter wavelengths. Qualitatively, radio wavelengths are macroscopic, visible wavelengths are a fraction of a micrometre and X-ray wavelengths are smaller still.

Step 2: Compare the printed inequalities The four selected bands consequently satisfy

$$\lambda_{\text{radio}} > \lambda_{\text{microwave}} > \lambda_{\text{visible}} > \lambda_{\text{X-ray}}.$$

This is exactly the first option. The second option places visible wavelengths above radio wavelengths, contradicting the spectral sequence. The third makes X-rays longest, although they have the highest frequency among these four bands. The fourth correctly starts with short X-rays but puts visible light beyond radio waves, which is again inconsistent. Since the question concerns wavelength rather than frequency, reversing the direction of the complete order would be a mistake.

Step 3: Conclusion The correct decreasing wavelength order is radio waves, microwaves, visible light and X-rays. Hence, the correct option is (A).

Quick Tip to Solve: Remember the frequency order radio–microwave–infrared–visible–ultraviolet–X-ray–gamma; reverse it whenever wavelengths are compared.

---

43. Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If both particles move along the x-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavlength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is : [4 marks]

(A) $\Delta\lambda = \frac{5}{2}\lambda_0$

(B) $\Delta\lambda = \frac{3}{2}\lambda_0$

(C) $\Delta\lambda = 2\lambda_0$

(D) $\Delta\lambda = 4\lambda_0$

Correct Answer: (D) $\Delta\lambda = 4\lambda_0$

Solution:

Concept:

- The de Broglie wavelength of a nonrelativistic particle is $\lambda = h/p = h/(mv)$; the mass of particle A does not change in the collision.
- In a one-dimensional elastic collision with a stationary target, conservation of momentum and kinetic energy gives $v_A = (m_A - m_B)v_0/(m_A + m_B)$.

Step 1: Find the velocity after collision Take the initial direction as positive. Substitute the two given masses into the elastic-collision formula:

$$v_A = \frac{m/2 - m/3}{m/2 + m/3}v_0 = \frac{1/6}{5/6}v_0 = \frac{v_0}{5}.$$

The positive sign shows that A continues in its initial direction. Its momentum magnitude therefore becomes one fifth of its initial momentum. As a check, the target velocity is $v_B = 2(m/2)v_0/(5m/6) =$

$6v_0/5$, giving the same total momentum and kinetic energy as initially.

Step 2: Calculate the change in wavelength Initially, $\lambda_0 = h/[(m/2)v_0]$. After collision,

$$\lambda_f = \frac{h}{(m/2)(v_0/5)} = 5\lambda_0.$$

The question asks for the increase, not the final wavelength, so

$$\Delta\lambda = \lambda_f - \lambda_0 = (5-1)\lambda_0 = 4\lambda_0.$$

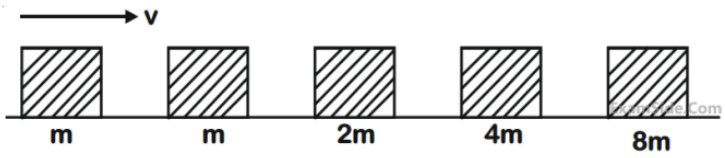
Options (A), (B) and (C) do not equal this difference; using a change in speed directly as a change in wavelength would miss their inverse relationship.

Step 3: Conclusion The wavelength increases by $\boxed{4\lambda_0}$. Hence, the correct option is (D).

Quick Tip to Solve: First obtain the final speed from the collision law, then invert its ratio to find the wavelength ratio; subtract one only at the end.

---

44. Blocks of masses m, 2m, 4m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to : [4 marks]



(A) 37

(B) 77

(C) 87

(D) 94

Correct Answer: (D) 94

Solution:

Concept:

- In every perfectly inelastic collision the colliding blocks stick together; horizontal momentum is conserved on the frictionless floor.
- Kinetic energy is not conserved during sticking. For fixed total momentum P, the kinetic energy of the combined body is $P^2/(2M)$.

Step 1: Combine all the masses The moving block has mass m and initial momentum mv. The stationary blocks have masses $m, 2m, 4m, 8m$. Thus, once the last collision has occurred,

$$M = m + m + 2m + 4m + 8m = 16m.$$

There is no external horizontal impulse, so the final momentum remains mv. Conservation of momentum gives

$$16mV = mv, \qquad V = \frac{v}{16}.$$

The phrase that the last block starts moving refers to its inelastic impact, after which all the blocks move together.

Step 2: Compare final and initial energies The original kinetic energy is carried only by the incident block:

$$K_i = \frac{1}{2}mv^2.$$

The final kinetic energy is

$$K_f = \frac{1}{2}(16m)\left(\frac{v}{16}\right)^2 = \frac{mv^2}{32} = \frac{K_i}{16}.$$

Hence the fraction converted into deformation, heat and sound is

$$\frac{K_i - K_f}{K_i} = 1 - \frac{1}{16} = \frac{15}{16}.$$

The percentage loss is $100(15/16) = 93.75\%$. Values 37, 77 and 87 are not close to the total loss through all four impacts; the closest listed integer is 94.

Step 3: Conclusion $p = 93.75$, which is close to $\boxed{94}$. Hence, the correct option is (D).

Quick Tip to Solve: For a chain of sticking collisions with initially stationary targets, use the initial momentum and the final combined mass directly.

45. Dimensional formula for thermal conductivity is (here K denotes the temperature): [4 marks]

(A) $MLT^{-3}K^{-1}$

(B) $MLT^{-2}K^{-2}$

(C) $MLT^{-2}K$

(D) $MLT^{-3}K$

Correct Answer: (A) $MLT^{-3}K^{-1}$

Solution:

Concept:

- Fourier's law for steady one-dimensional conduction is $\dot{Q} = kA\Delta\theta/L$, where k is thermal conductivity.
- Heat has the dimensions of energy, $[Q] = ML^2T^{-2}$, and heat transfer rate has dimensions ML^2T^{-3}.

Step 1: Isolate the conductivity For a slab of area A, thickness L, and temperature difference $\Delta\theta$, the magnitude of the conducted heat per unit time is proportional to the area and temperature difference, and inversely proportional to thickness. Rearranging Fourier's law gives

$$k = \frac{\dot{Q}L}{A\Delta\theta} = \frac{QL}{tA\Delta\theta}.$$

The temperature difference has dimension K, the same fundamental dimension as absolute temperature; it is not an energy or a dimensionless ratio.

Step 2: Substitute the fundamental dimensions Use $[A] = \mathrm{L}^2$, $[L] = \mathrm{L}$, $[t] = \mathrm{T}$, and $[\Delta\theta] = \mathrm{K}$. Therefore

$$[k] = \frac{\mathrm{ML^2T^{-2}\,L}}{\mathrm{T\,L^2\,K}} = \mathrm{MLT^{-3}K^{-1}}.$$

Equivalently, the SI unit is $\mathrm{W\,m^{-1}\,K^{-1}}$. Option (B) has the wrong time and temperature powers; (C) has the wrong time power and puts temperature in the numerator; (D) also has temperature in the numerator. Only (A) matches both the conduction law and the SI unit.

Step 3: Conclusion The dimensional formula is $\boxed{\mathrm{MLT^{-3}K^{-1}}}$. Hence, the correct option is (A).

Quick Tip to Solve: Remember conductivity as heat flow rate times thickness divided by area and temperature difference; this also reconstructs its SI unit.

Physics – Section B: Numerical Value Questions (20 marks)

46. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is ____. [4 marks]

Solution:

Concept:

- For geometrically similar uniform laminas, mass scales as the square of the linear scale and centroidal moment of inertia scales as its fourth power.
- The parallel-axis theorem is $I = I_C + Md^2$. An equilateral triangular lamina of mass M and altitude H has polar centroidal moment $I_0 = MH^2/9$.

Step 1: Locate the removed triangle Let the original triangle have mass M, altitude H, and vertex A above the base BC. Since D and E are midpoints, ADE is similar to ABC with half the linear size. Its mass is $M/4$, altitude is $H/2$, and centroidal polar moment is $I_0/16$. Measured downwards from A, G lies at $2H/3$, whereas the centroid of ADE lies at $2(H/2)/3 = H/3$. Their separation is therefore $d = H/3$, not $H/2$.

Step 2: Subtract about the specified axis The removed triangle's moment about the original axis through G is found by the parallel-axis theorem:

$$I_{ADE,G} = \frac{I_0}{16} + \frac{M}{4}\left(\frac{H}{3}\right)^2 = \frac{I_0}{16} + \frac{MH^2}{36}.$$

Since $I_0 = MH^2/9$, the second term equals $I_0/4$. Thus

$$I_{ADE,G} = \frac{5I_0}{16}, \qquad I_{\text{remaining,G}} = I_0 - \frac{5I_0}{16} = \frac{11I_0}{16}.$$

Both terms must refer to the same original axis. The remaining body has a new centroid, but the question does not ask for its centroidal moment, so no further axis shift is required.

Step 3: Conclusion Comparing with $NI_0/16$ gives $\boxed{N = 11}$.

Quick Tip to Solve: Scale the small triangle about its own centroid first, then shift to the original centroid before subtracting its inertia.

---

47. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1 . If another stationary disc having radius $\frac{R}{2}$ and same mass M is droped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2 the energy lost in the process is p% of the initial energy. Value of p is _____. [4 marks]

Solution:

Concept:

- A uniform disc has moment of inertia $I = MR^2/2$ about its symmetry axis. Its radius enters quadratically.
- With no external torque about the common axis, total angular momentum is conserved. Friction transfers angular momentum between the discs but dissipates mechanical energy.

Step 1: Find the shared angular speed The initial rotating disc has

$$I_1 = \frac{1}{2}MR^2.$$

The second disc has the same mass but half the radius, hence

$$I_2 = \frac{1}{2}M\left(\frac{R}{2}\right)^2 = \frac{1}{8}MR^2 = \frac{I_1}{4}.$$

Initially only the first disc has angular momentum. After slipping ceases, the common angular speed follows from

$$I_1\omega_1 = (I_1 + I_2)\omega_2, \qquad \omega_2 = \frac{4}{5}\omega_1.$$

Equal masses therefore do not mean equal moments of inertia.

Step 2: Evaluate the energy decrease The initial rotational energy is $K_i = I_1\omega_1^2/2$. When both discs rotate together their energy is

$$K_f = \frac{1}{2}\left(\frac{5}{4}I_1\right)\left(\frac{4}{5}\omega_1\right)^2 = \frac{4}{5}\left(\frac{1}{2}I_1\omega_1^2\right) = \frac{4}{5}K_i.$$

Consequently,

$$\frac{K_i - K_f}{K_i} = 1 - \frac{4}{5} = \frac{1}{5}, \qquad p = 100 \times \frac{1}{5} = 20.$$

The lost rotational energy becomes internal energy through friction between the surfaces. Conserving kinetic energy during the contact would incorrectly omit this dissipation; it is angular momentum that remains constant about the axis.

Step 3: Conclusion The energy lost is 20% of the initial energy, so $\boxed{p = 20}$.

Quick Tip to Solve: For a stationary coaxial disc added to a rotating one, the retained energy fraction is $I_1/(I_1 + I_2)$.

48. A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to _____. [4 marks]

Solution:

Concept:

- The internal energy of a monatomic ideal gas is $U = (3/2)nRT$; it depends on temperature and amount, not directly on pressure or volume.
- For thermal mixing with no specified external heat transfer or mechanical work, total initial internal energy equals the final equilibrium internal energy. The same gas has the same molar heat capacity in both portions.

Step 1: Write the energy balance Denote the two initial amounts by $n_1 = 0.1\,\text{mol}$ and $n_2 = 0.05\,\text{mol}$, with temperatures $T_1 = 200\,\text{K}$ and $T_2 = 400\,\text{K}$. The total amount after addition is $n_f = 0.15\,\text{mol}$. The sum of the initial internal energies is

$$U_i = \frac{3}{2}R(n_1T_1 + n_2T_2).$$

At equilibrium both portions have the same temperature T_f, so

$$U_f = \frac{3}{2}R(n_1 + n_2)T_f.$$

There is no need to know the vessel's volume because it cancels from the ideal-gas internal-energy calculation.

Step 2: Calculate the weighted temperature Equating these energies cancels the common factor $3R/2$:

$$T_f = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} = \frac{(0.1)(200) + (0.05)(400)}{0.15} = \frac{40}{0.15} = 266.67\,\text{K}.$$

The colder portion contains twice as many moles as the hotter portion, so the answer is closer to 200 K than to 400 K. An unweighted average of 300 K would wrongly treat the two amounts as equal. The exact result is $800/3\,\text{K}$, which rounds to 267 K at the nearest integer. The value 266 K is a truncation, rather than the exact equilibrium value.

Step 3: Conclusion The equilibrium temperature is $\boxed{T_f = 800/3\,\text{K} \simeq 266.67\,\text{K}}$. To the nearest integer it is 267 K; the listed key value 266 K approximates it by truncation.

Quick Tip to Solve: For mixing the same ideal gas, use a mole-weighted temperature average; a simple arithmetic mean works only for equal amounts.

49. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is _____. [4 marks]

Solution:

Concept:

- The objective forms a magnified real intermediate image, and the eyepiece acts as a simple microscope viewing it.
- For the standard compound-microscope approximation, the magnitude of angular magnification for a final image at the near point is $M = (L/f_o)(1 + D/f_e)$. Here L is the tube length, f_o the objective focal length, f_e the eyepiece focal length and D the near-point distance.

Step 1: Choose the near-point formula The final image is virtual and lies 25 cm from the eyepiece. Thus the specified viewing distance is $D = 25\,\text{cm}$, and the eyepiece factor is $1 + D/f_e$. It is not the relaxed-eye factor D/f_e, which applies when the final image is at infinity. We use magnitudes of magnification; inversion by the objective does not change the stated magnification of 100.

Step 2: Find the objective magnification In the usual microscope approximation the objective magnification is the tube length divided by its focal length. With both lengths in centimetres,

$$m_o \simeq \frac{L}{f_o} = \frac{20}{1} = 20.$$

The total magnification is the product of the objective and eyepiece magnifications, so the eyepiece must provide $100/20 = 5$.

Step 3: Solve for the eyepiece focal length Substitute the given total magnification into the near-point expression:

$$100 = 20\left(1 + \frac{25}{f_e}\right).$$

Consequently,

$$1 + \frac{25}{f_e} = 5, \qquad \frac{25}{f_e} = 4, \qquad f_e = \frac{25}{4} = 6.25\,\text{cm}.$$

Checking gives an eyepiece factor of $1 + 25/6.25 = 5$, and hence total magnification $20 \times 5 = 100$.

Step 4: Conclusion The focal length of the eyepiece lens is $\boxed{6.25\,\text{cm}}$. The numerical entry is $\boxed{6.25}$.

Quick Tip to Solve: First identify whether the final image is at infinity or at the near point; the extra 1 in the eyepiece factor is essential here.

50. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304 Å. The corresponding difference for the Paschen series (in Å) is ____.

[4 marks]

Solution:

Concept:

- The hydrogen Rydberg formula is $1/\lambda = R(1/n^2 - 1/m^2)$, where n is the fixed lower level and $m > n$ the upper level.
- Within a series the longest wavelength comes from $m = n + 1$, and the shortest wavelength is the series limit $m \to \infty$. Lyman has $n = 1$; Paschen has $n = 3$.

Step 1: Express the wavelength spread of a series The smallest energy gap in a series is its first transition, so

$$\lambda_{\max} = \frac{n^2(n+1)^2}{R(2n+1)}.$$

At the limit the photon energy is largest, giving

$$\lambda_{\min} = \frac{n^2}{R}.$$

Subtract wavelengths themselves, rather than wavenumbers, because the question asks for their difference. Simplifying,

$$\Delta\lambda_n = \frac{n^2}{R}\left(\frac{(n+1)^2}{2n+1} - 1\right) = \frac{n^4}{R(2n+1)}.$$

Step 2: Use the Lyman spread to eliminate the constant For the Lyman series $n = 1$, so

$$\Delta\lambda_L = \frac{1}{3R} = 304\,\text{Å}.$$

For the Paschen series $n = 3$,

$$\Delta\lambda_P = \frac{81}{7R}.$$

Taking the ratio avoids inserting an independently rounded value of the Rydberg constant and preserves the numerical information supplied in the problem.

Step 3: Calculate the Paschen spread The ratio of the two wavelength differences is

$$\frac{\Delta\lambda_P}{\Delta\lambda_L} = \frac{81/(7R)}{1/(3R)} = \frac{243}{7}.$$

Therefore,

$$\Delta\lambda_P = 304 \times \frac{243}{7} = \frac{73872}{7} = 10553.142857\ldots\ \text{Å}.$$

The larger wavelength spread is consistent with the infrared wavelengths of the Paschen series, whereas Lyman lines lie in the ultraviolet.

Step 4: Conclusion The corresponding difference for the Paschen series is $\boxed{10553.14\,\text{Å}}$, so the numerical entry is $\boxed{10553.14}$.

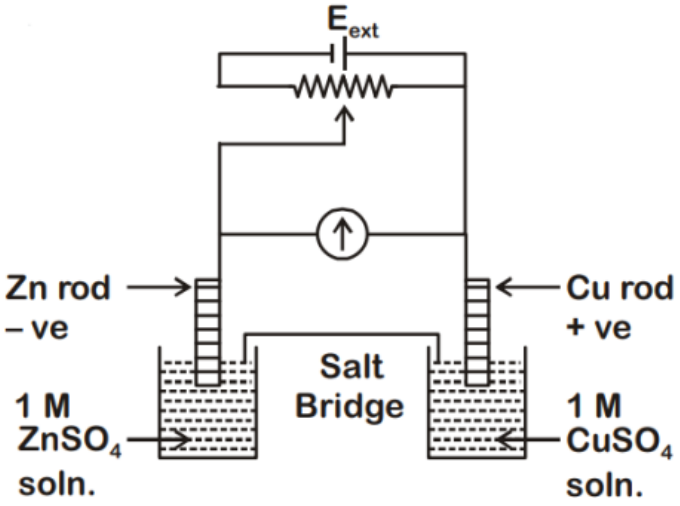
Quick Tip to Solve: For any hydrogen series, derive the spread as $n^4/[R(2n+1)]$; ratios then remove the Rydberg constant.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. $E^0_{Cu^{2+}|Cu}$ = **+0.34 V**

$E^0_{Zn^{2+}|Zn}$ = **-0.76 V**

Identify the incorrect statement from the option below for the above cell : [4 marks]



(A) If $E_{ext} < 1.1$ V, Zn dissolves at anode and Cu deposits at cathode

(B) If $E_{ext} = 1.1$ V, no flow of e^- or current occurs

(C) If $E_{ext} > 1.1$ V, e^- flows from Cu to Zn

(D) If $E_{ext} > 1.1$ V, Zn dissolves at Zn electrode and Cu deposits at Cu electrode

Correct Answer: (D) If $E_{ext} > 1.1$ V, Zn dissolves at Zn electrode and Cu deposits at Cu electrode

Solution:

Concept:

- The standard Daniell-cell potential is the cathode reduction potential minus the anode reduction potential: $E^\circ_{\text{cell}} = 0.34 - (-0.76) = 1.10\,\text{V}$.
- An external voltage opposing the spontaneous cell reaction can slow it, stop it, or reverse it. Reversal exchanges the oxidation and reduction processes at the two electrodes.

Step 1: Identify the spontaneous reaction Without a sufficiently large opposing external voltage, zinc is oxidised and copper ions are reduced:

$$\text{Zn} \longrightarrow \text{Zn}^{2+} + 2\,\text{e}^-, \qquad \text{Cu}^{2+} + 2\,\text{e}^- \longrightarrow \text{Cu}.$$

Thus zinc dissolves at the anode, copper deposits at the cathode, and electrons travel through the external circuit from zinc to copper. The driving voltage of this reaction is $1.10\,\text{V}$.

Step 2: Compare the external voltage with the cell emf If $E_{\text{ext}} < 1.10\,\text{V}$, the spontaneous reaction still wins, so statement (A) is correct. If the opposing voltage is exactly $1.10\,\text{V}$, the net driving emf is zero and no net current flows, as in statement (B). If the opposing voltage exceeds $1.10\,\text{V}$, the reaction is driven in reverse and the electron flow is from copper to zinc, making statement (C) correct.

Step 3: Check what happens after reversal In the reversed cell, copper undergoes oxidation and zinc ions undergo reduction:

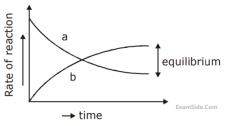
$$\mathrm{Cu} \longrightarrow \mathrm{Cu}^{2+} + 2\,\mathrm{e}^-, \qquad \mathrm{Zn}^{2+} + 2\,\mathrm{e}^- \longrightarrow \mathrm{Zn}.$$

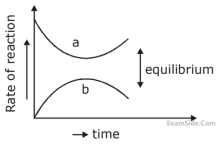
Copper therefore dissolves and zinc is deposited. Statement (D) instead asserts the same electrode changes as in the spontaneous direction even though the external voltage has reversed the current. That is the incorrect statement sought.

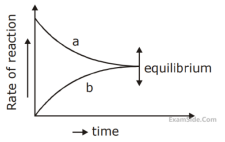
Step 4: Conclusion For $E_{\text{ext}} > 1.10\,\text{V}$, copper dissolves and zinc deposits. Hence, the correct option is (D).

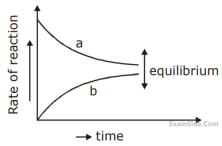
Quick Tip to Solve: When an opposing voltage exceeds the cell emf, reverse both half-reactions together; reversing only electron flow gives inconsistent electrode changes.

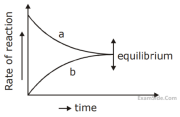
52. For the equilibrium A $\rightleftharpoons$ B , the variation of the rate of the forward (a) and reverse (b) reaction with time is given by : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- At chemical equilibrium the forward and reverse reaction rates are equal and nonzero: $r_f = r_b$. Equal rates keep macroscopic concentrations constant.
- For an initially reactant-rich mixture, the forward rate decreases as reactant is consumed, while the reverse rate rises as product accumulates. At fixed conditions both approach the same constant rate.

Step 1: Follow the rates from the start The graphs begin with a substantial forward rate, labelled a, and zero reverse rate, labelled b. This represents starting with A and essentially no B. Initially the forward reaction converts A into B. As A is depleted, the forward rate falls; as B is formed, the reverse reaction becomes faster. These trends explain the descending and ascending curves before equilibrium.

Step 2: Apply the equality condition at equilibrium For the elementary representation one may write

$r_f = k_f[A]$ and $r_b = k_b[B]$. The balance condition is

$$\frac{d[B]}{dt} = r_f - r_b = 0.$$

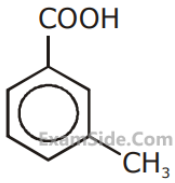
The two rates must therefore converge to one common horizontal value. They do not become zero: molecules continue to react in both directions, with no net conversion. The graph in option (C) displays precisely this approach to the same limiting rate.

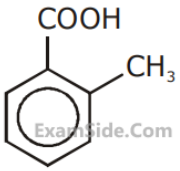
Step 3: Reject the other rate plots In (A), the curves cross and subsequently approach unequal values, so product concentration would continue changing. In (B), the rates turn in opposite directions and remain unequal rather than settling at a common constant value. In (D), both curves level off but at different heights; constant individual rates do not establish equilibrium unless those rates are equal. Hence none of these three plots can depict the stated equilibrium under unchanged conditions.

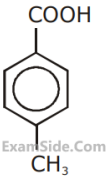
Step 4: Conclusion The forward and reverse rates approach an equal, constant, nonzero value in graph (C). Hence, the correct option is (C).

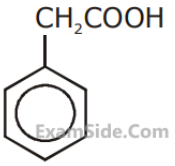
Quick Tip to Solve: Judge an equilibrium rate graph by the final heights of the two curves: they must coincide, even when the initial rates differ.

53. [P] on treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2Br$ while heating [P] with sodalime gave toluene. The compound [P] is [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- Soda-lime decarboxylation replaces the carboxyl group of a carboxylate by hydrogen: $\mathrm{RCOONa} + \mathrm{NaOH} \xrightarrow{\mathrm{CaO},\ \Delta} \mathrm{RH} + \mathrm{Na_2CO_3}$.
- In electrophilic aromatic substitution, $-\mathrm{CH_3}$ directs ortho and para, while $-\mathrm{COOH}$ directs meta. Molecular symmetry determines whether two allowed substitution sites give the same product.

Step 1: Use the decarboxylation result Toluene must remain after the carbon of the carboxyl group

is removed. The methylbenzoic-acid structures in (A), (B), and (C) satisfy this condition: removing the ring carboxyl group leaves a methyl group attached to benzene. Option (D) is phenylacetic acid, $C_6H_5CH_2COOH$; its sodium salt also gives toluene on decarboxylation. Thus this test alone is insufficient to select an option.

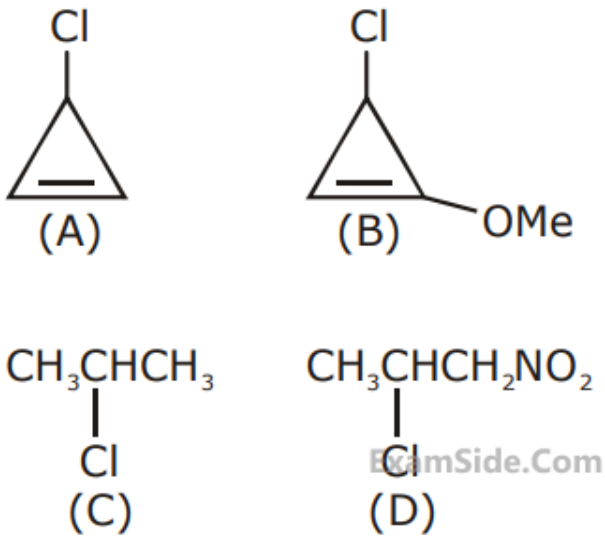
Step 2: Locate the preferred bromination sites Consider option (C), para-methylbenzoic acid, numbering the carboxyl carbon attachment as ring position 1 and the methyl group as position 4. The carboxyl group directs substitution to positions 3 and 5. These same positions are ortho to the methyl group, so both substituents favour them. A symmetry plane exchanges positions 3 and 5; replacing hydrogen at either produces the same brominated constitutional isomer.

Step 3: Compare the remaining structures In the meta acid (A), the ring lacks the symmetry that makes all available favoured positions equivalent, and the directing influences do not isolate one equivalent pair. In the ortho acid (B), the favoured positions relative to both substituents are inequivalent, allowing distinct isomers. In phenylacetic acid (D), the $-CH_2COOH$ side chain permits distinct ortho and para ring-substitution products. Only the para acid has the required equivalent favoured positions. Monobromination changes $C_8H_8O_2$ to $C_8H_7O_2Br$, agreeing with the stated formula.

Step 4: Conclusion Compound [P] is para-methylbenzoic acid (p-toluic acid), the structure shown in (C). Hence, the correct option is (C).

Quick Tip to Solve: Combine the decarboxylation carbon skeleton with directing effects, then use symmetry to count distinct bromination products.

54. The decreasing order of reactivity of the following organic molecules towards $AgNO_3$ solution is : [4 marks]



(A) (B) > (A) > (C) > (D)

(B) (A) > (B) > (C) > (D)

(C) (A) > (B) > (D) > (C)

(D) (C) > (D) > (A) > (B)

Correct Answer: (A) (B) > (A) > (C) > (D)

Solution:

Concept:

- Silver nitrate promotes ionisation of the carbon–chlorine bond: Ag^+ removes chloride as insoluble $AgCl$. Easier carbocation formation gives faster reaction.
- A cyclic, planar, conjugated carbocation with two pi electrons is aromatic by the $4n + 2$ rule with $n = 0$. Resonance donation stabilises a positive charge, whereas an electron-withdrawing inductive effect destabilises it.

Step 1: Compare the two cyclopropenyl systems Loss of chloride from compound (A) gives the cyclopropenyl cation. The positively charged carbon has an empty p orbital that overlaps with the p orbitals of the other two ring carbons. The two pi electrons are delocalised around the three-membered ring, producing an aromatic cation. This strong stabilisation makes (A) ionise more readily than an ordinary secondary alkyl chloride. Compound (B) forms the corresponding methoxy-substituted cyclopropenyl cation. It retains the aromatic two-pi-electron ring and gains additional stabilisation from donation of an oxygen lone pair through the methoxy group. The positive charge can be delocalised towards oxygen. Thus, the cation from (B) is more stabilised than that from (A), giving $(B) > (A)$ in reactivity.

Step 2: Compare the acyclic chlorides and assemble the order Compound (C), $CH_3CH(Cl)CH_3$, gives an ordinary secondary carbocation, stabilised by the electron-releasing effects and hyperconjugation of its methyl groups. It lacks the aromatic stabilisation of the cyclic cations, so it reacts more slowly than (A) and (B). Compound (D), $CH_3CH(Cl)CH_2NO_2$, would also develop positive charge at the carbon bearing chlorine. Its nitro group withdraws electron density through sigma bonds and opposes formation of that positive charge. The intervening saturated carbon prevents direct resonance donation to the cation. Therefore (D) is less reactive than (C). Option (B) incorrectly puts (A) ahead of the methoxy-stabilised system (B). Option (C) also reverses the acyclic pair, while option (D) puts the less stabilised acyclic cations ahead of the aromatic systems. Only option (A) satisfies both comparisons.

Step 3: Conclusion The methoxy-substituted cyclopropenyl chloride reacts fastest, followed by the unsubstituted cyclopropenyl chloride, isopropyl chloride and finally the nitro-substituted acyclic chloride. This is the order established by aromatic stabilisation, methoxy donation and the nitro inductive effect. Hence, the correct option is (A).

Quick Tip to Solve: For silver nitrate tests, compare the cations formed after chloride leaves. Check cyclic conjugation and aromaticity before applying the usual alkyl-carbocation order, then account for donating and withdrawing substituents.

55. What are the functional groups present in the structure of maltose? [4 marks]

(A) One acetal and one ketal

(B) One ketal and one hemiketal

(C) Two acetals

(D) One acetal and one hemiacetal

Correct Answer: (D) One acetal and one hemiacetal

Solution:

Concept:

- A hemiacetal carbon is bonded to an alkoxy group and a hydroxyl group. An acetal carbon instead has two alkoxy groups and no hydroxyl group.
- Maltose consists of two glucose units joined through an $\alpha(1 \rightarrow 4)$ glycosidic linkage. Glucose is an aldose, so its cyclic anomeric group is a hemiacetal, rather than a hemiketal.

Step 1: Identify the glycosidic anomeric centre The anomeric carbon C1 of the first glucose unit forms the glycosidic bond to the oxygen on C4 of the second glucose unit. At this C1, the original anomeric hydroxyl group has been replaced by an alkoxy linkage. This carbon is now bonded to the ring oxygen and the glycosidic oxygen, so it has two C–O–C connections and is an acetal centre.

Step 2: Examine the remaining anomeric centre The C1 atom of the second glucose unit does not take part in the glycosidic bond. It retains its hydroxyl group along with its bond to the ring oxygen. It is therefore a hemiacetal centre and can undergo ring opening, which accounts for the reducing character of maltose. There is only one acetal centre, so the option listing two acetals is excluded. Neither glucose unit supplies a ketose-derived anomeric centre; consequently the options containing a ketal or a hemiketal do not describe maltose.

Step 3: Conclusion Maltose contains one acetal group and one hemiacetal group. Hence, the correct option is (D).

Quick Tip to Solve: Count the anomeric hydroxyl groups: a free anomeric OH identifies the reducing hemiacetal end, while an anomeric glycosidic linkage gives an acetal.

56. The number of isomers possible for
$[Pt(en)(NO_2)_2]$ is : [4 marks]

(A) 3

(B) 1

(C) 4

(D) 2

Correct Answer: (A) 3

Solution:

Concept:

- In $[\mathrm{Pt(en)(NO_2)_2}]$, en is a neutral bidentate ligand and the two monodentate ligands each have charge -1. Platinum is therefore Pt(II), a d^8 metal with a square-planar arrangement.
- The ambidentate ligand $\mathrm{NO_2}^-$ can bind through nitrogen (nitro) or through oxygen (nitrito). These alternative donor atoms produce linkage isomerism.

Step 1: Fix the positions occupied by ethylenediamine The two donor nitrogen atoms of en belong to the same chelating ligand and occupy adjacent positions of the square plane. The two remaining sites are also adjacent. There is thus no independent cis/trans choice for the two monodentate ligands: the chelate has already fixed their relative positions. Counting an additional trans form would require the short en ligand to span opposite coordination sites.

Step 2: Count the distinct donor-atom combinations Each of the two remaining ligands can be nitrogen-bound or oxygen-bound. The distinct possibilities are (i) both nitrogen-bound, (ii) both oxygen-bound and (iii) one nitrogen-bound and one oxygen-bound. Interchanging the two remaining sites in the mixed case does not produce a new isomer, since the two ends of en are equivalent. Thus the apparent fourth assignment is identical to the third. The square-planar complex supplies no additional optical pair. The numbers 1 and 2 omit linkage possibilities, whereas 4 double-counts the mixed arrangement.

Step 3: Conclusion There are three distinct linkage isomers: nitro–nitro, nitrito–nitrito and nitro–nitrito. Hence, the correct option is (A).

Quick Tip to Solve: For a square-planar complex with one symmetric bidentate ligand, first fix the chelate and then count the distinct linkage choices at the two remaining sites.

57. The elements with atomic numbers 101 and 104 belong to, respectively : [4 marks]

(A) Group 6 and Actinoids

(B) Actinoids and Group 4

(C) Group 11 and Group 4

(D) Actinoids and Group 6

Correct Answer: (B) Actinoids and Group 4

Solution:

Concept:

- The actinoid series extends from actinium, $Z = 89$, to lawrencium, $Z = 103$. These elements belong to the inner-transition series associated with filling of the $5f$ subshell.
- After the actinoid series, the seventh-period transition series continues with rutherfordium, $Z = 104$. Its group is determined by its transition-series position and valence-shell pattern.

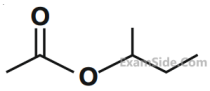
Step 1: Locate the element with atomic number 101 Atomic number 101 identifies mendelevium, Md. Since 101 lies between 89 and 103, mendelevium is an actinoid. Its representative ground-state configuration is $[\mathrm{Rn}]\,5f^{13}7s^2$, which places it in the late part of the actinoid sequence. It is not a group 6 transition metal and is not assigned to group 11. This immediately rules out the alternatives that put the first element in either of those groups.

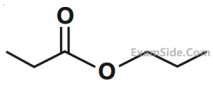
Step 2: Locate the element with atomic number 104 Atomic number 104 is rutherfordium, Rf. It follows lawrencium at the end of the actinoid series and belongs to group 4, below titanium, zirconium and hafnium. Its representative valence configuration $6d^27s^2$ is consistent with that classification. It does not belong to the actinoid series, which ends at 103, and it is not in group 6. Group 6 lies two columns later in the transition series; confusing the period number or the presence of d electrons with the group number gives the wrong assignment.

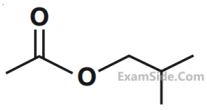
Step 3: Conclusion The elements with atomic numbers 101 and 104 belong to the actinoids and group 4, respectively. Hence, the correct option is (B).

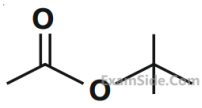
Quick Tip to Solve: Remember the end of the actinoid series: Lr is 103, and the next element Rf, 104, starts the group-4 position of the seventh-period transition series.

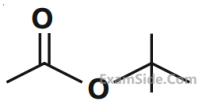
58. An organic compound (A) (molecular formula $C_6H_{12}O_2$) was hydrolysed with dil. H_2SO_4 to give a carboxylic acid (B) and an alcohol (C). 'C' gives white turbidity immediately when treated with anhydrous $ZnCl_2$ and conc. HCl. The organic compound (A) is : [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- Acid-catalysed hydrolysis of an ester gives a carboxylic acid and the alcohol corresponding to the ester alkoxy group: $\mathrm{RCOOR}\prime \xrightarrow{+\,H_2O} \mathrm{RCOOH} + \mathrm{R}\prime\mathrm{OH}$.
- Lucas reagent is concentrated HCl with anhydrous $\mathrm{ZnCl_2}$. An ordinary tertiary alcohol gives immediate turbidity because the insoluble alkyl chloride forms rapidly.

Step 1: Use the observation to identify the alcohol The immediate appearance of white turbidity means that C is a tertiary alcohol. For the structures shown, the tertiary alcohol is tert-butyl alcohol, $(\mathrm{CH_3})_3\mathrm{COH}$. Its carbon bearing the hydroxyl group is attached to three other carbon atoms. The ester must therefore contain the alkoxy fragment $-\mathrm{OC(CH_3)_3}$, since hydrolysis breaks the ester into the acid and this alcohol without changing its carbon skeleton.

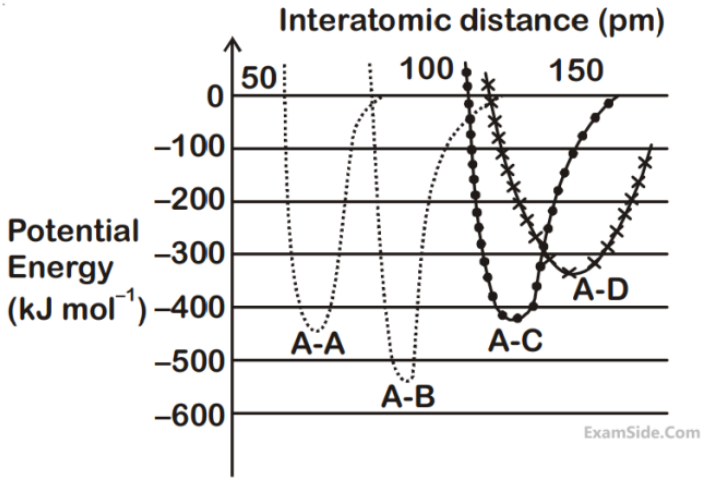
Step 2: Check the ester and eliminate the alternatives Option (D) is tert-butyl acetate, $\mathrm{CH_3COOC(CH_3)_3}$. It contains six carbon atoms and has formula $\mathrm{C_6H_{12}O_2}$, as required. Hydrolysis gives acetic acid and tert-butyl alcohol. In contrast, option (A) gives butan-2-ol, a secondary alcohol; option (B) gives propan-1-ol, a primary alcohol; and option (C) gives 2-methylpropan-1-ol, also a primary alcohol. These do not produce immediate turbidity in the Lucas test under the stated conditions. Thus matching only the molecular formula is insufficient; the alcohol-producing side of the ester determines the answer.

Step 3: Conclusion The ester is tert-butyl acetate, $\mathrm{CH_3COOC(CH_3)_3}$, whose hydrolysis supplies the tertiary alcohol required by the Lucas test. Hence, the correct option is (D).

Quick Tip to Solve: For an ester hydrolysis followed by an alcohol test, inspect the group attached to the ester oxygen: it becomes the alcohol directly.

59. The intermolecular potential energy for the molecules A, B, C and D given below suggests that :

[4 marks]



(A) A-B has the stiffest bond

(B) A-D has the shortest bond length

(C) A-A has the largest bond enthalpy

(D) D is more electronegative than other atoms

Correct Answer: (A) A-B has the stiffest bond

Solution:

Concept:

- The minimum of a potential-energy curve occurs at the equilibrium separation r_e. The well depth measures the energy needed to separate the bonded atoms to infinite distance.
- Near a stable minimum, $U(r) \simeq U(r_e) + \frac{1}{2}k(r - r_e)^2$, where $k = \mathrm{d}^2U/\mathrm{d}r^2\big|_{r_e}$ is the force constant. A narrow, sharply curved well indicates a stiff bond.

Step 1: Compare the curvature of the wells The A–B curve has the narrowest and most sharply curved well around its minimum. A small displacement from its equilibrium separation produces a comparatively large rise in potential energy and hence a strong restoring force. It therefore has the largest force constant among the curves shown and is the stiffest bond. The depth of the A–B well also exceeds those of the other curves, so this identification is consistent with a strongly bound pair.

Step 2: Check the other proposed conclusions The equilibrium distance is read from the horizontal coordinate of the minimum, not from the depth of the curve. The leftmost minimum belongs to A–A, whereas the A–D minimum lies farthest to the right. Hence A–D does not have the shortest bond length. A–A also does not have the greatest bond enthalpy: its well is less deep than the A–B well. Finally, these potential-energy curves do not by themselves establish an electronegativity ordering for the atoms. Their direct information concerns equilibrium distances, binding energies and the response to displacement, so the proposed statement about D being most electronegative is unsupported.

Step 3: Conclusion The curve for A–B indicates the stiffest bond. Hence, the correct option is (A).

Quick Tip to Solve: Read different features separately: the minimum position gives bond length, well depth gives dissociation energy, and curvature near the minimum gives stiffness.

60. The pair in which both the species have the same magnetic moment (spin only) is : [4 marks]

(A) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(B) $[Co(OH)_4]^{2-}$ and $[Fe(NH_3)_6]^{2+}$

(C) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)]^{2+}$

(D) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

Correct Answer: (D) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons. Equal spin-only moments require equal values of n.
- Determine the metal oxidation state and d-electron count before assigning spins. Water, chloride and hydroxide produce the high-spin arrangements relevant to the complexes listed here.

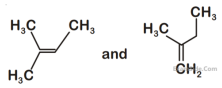
Step 1: Identify the matching high-spin pair In $[Cr(H_2O)_6]^{2+}$, chromium is Cr(II), with a d^4 configuration. The weak-field octahedral aqua complex is high spin: $t_{2g}^3 e_g^1$, giving four unpaired electrons. In $[Fe(H_2O)_6]^{2+}$, iron is Fe(II), with d^6. Its high-spin configuration is $t_{2g}^4 e_g^2$, also giving four unpaired electrons. Both therefore have $\mu = \sqrt{4(4+2)}\,\mu_B = \sqrt{24}\,\mu_B$.

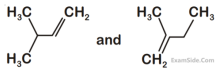
Step 2: Eliminate the remaining pairs In option (A), tetrahedral $[CoCl_4]^{2-}$ contains Co(II), d^7, with three unpaired electrons, whereas the Cr(II) aqua complex has four. In option (B), tetrahedral $[Co(OH)_4]^{2-}$ likewise has three unpaired electrons. Octahedral Fe(II), d^6, cannot have three unpaired electrons: its usual high-spin and low-spin cases have four and zero, respectively. Thus it cannot match that cobalt species. In option (C), Mn(II) is d^5; its aqua complex is high spin with five unpaired electrons, which differs from the four of the Cr(II) aqua species. Only option (D) pairs equal unpaired-electron counts.

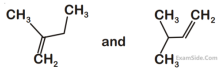
Step 3: Conclusion $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ each have four unpaired electrons and the same spin-only moment, $\sqrt{24}\,\mu_B$. Hence, the correct option is (D).

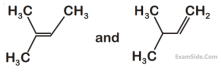
Quick Tip to Solve: Compare unpaired-electron counts rather than d-electron totals: high-spin octahedral d4 and d6 both have four unpaired electrons.

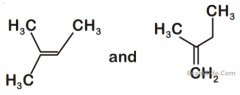
61. When neopentyl alcohol is heated with an acid, it slowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes? [4 marks]

(A)  and

(B) 

(C) 

(D) 

Correct Answer: (A) 

Solution:

Concept:

- Acid-catalysed dehydration converts an alcohol to an alkene by loss of water. Protonation changes the poor leaving group $-OH$ into the good leaving group H_2O.
- A carbocation rearranges by a 1,2-alkyl shift when this gives a more stable cation. Elimination from the rearranged cation preferentially gives the more substituted alkene (Zaitsev product).

Step 1: Identify the obstacle to direct elimination Neopentyl alcohol is $(CH_3)_3C-CH_2OH$. The carbon next to the alcohol-bearing carbon is quaternary: it has no hydrogen available for an ordinary elimination producing a double bond between these two carbons. Thus merely removing water without considering a skeletal rearrangement cannot explain the observed products.

Step 2: Form the rearranged carbon skeleton After protonation of oxygen, departure of water is accompanied by migration of a methyl group from the neighbouring quaternary carbon. This avoids treating a free primary neopentyl cation as a stable intermediate. The resulting tertiary carbocation has the connectivity $(CH_3)_2C^+-CH_2CH_3$. The migration preserves all five carbon atoms while putting the positive charge on a carbon attached to three carbon groups.

Step 3: Eliminate from the two inequivalent beta positions Removing a proton from the ethyl-side methylene group gives $(CH_3)_2C=CHCH_3$, 2-methylbut-2-ene. Removing a proton from either equivalent methyl group gives $CH_2=C(CH_3)CH_2CH_3$, 2-methylbut-1-ene. The first alkene is trisubstituted whereas the second is disubstituted, so the first is the major 85 percent product. Options containing 3-methylbut-1-ene fail to represent the two eliminations of the rearranged tertiary cation; reversing the major and minor products also fails.

Step 4: Conclusion A is 2-methylbut-2-ene (85 percent), and B is 2-methylbut-1-ene (15 percent). Hence, the correct option is (A).

Quick Tip to Solve: For neopentyl alcohol, first check the adjacent carbon for a beta hydrogen; its absence signals the need to consider a methyl shift before assigning the alkenes.

62. The region in the electromagnetic spectrum where the Balmar series lines appear is : [4 marks]

(A) Microwave

(B) Ultraviolet

(C) Visible

(D) Infrared

Correct Answer: (C) Visible

Solution:

Concept:

- Hydrogen emission lines arise when an electron falls from an upper level n_i to a lower level n_f, releasing a photon with energy hc/λ.
- The Balmer series consists of transitions ending at $n_f = 2$, with $n_i = 3, 4, 5, \ldots$. Its characteristic prominent lines lie in the visible region.

Step 1: Identify the final energy level The name of a hydrogen spectral series identifies its common lower level. Lyman transitions end at the ground state, $n_f = 1$, whereas Balmer transitions end at the second level. Thus the Balmer lines are found by setting $n_f = 2$ in the Rydberg relation, rather than by using the ionisation energy from the ground state.

Step 2: Check a characteristic wavelength The longest-wavelength Balmer line is the transition from $n_i = 3$ to $n_f = 2$. With the Rydberg constant $R_H \simeq 1.097 \times 10^7\,\mathrm{m}^{-1}$,

$$\frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36}, \qquad \lambda \simeq 656\,\mathrm{nm}.$$

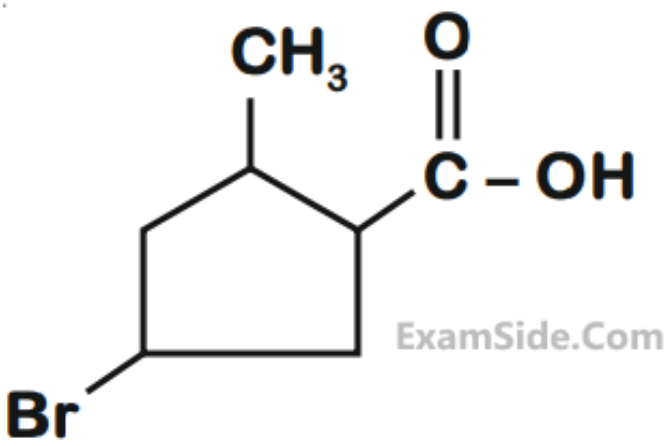
This is the red visible line. The next prominent lines occur near 486 nm, 434 nm and 410 nm, also in the visible part of the spectrum.

Step 3: Distinguish the alternative regions Ultraviolet is the standard region associated with the Lyman series, whose energy differences are larger. Infrared is associated with series such as Paschen, which ends at the third level, and the still smaller spacings involved in higher-level series. Microwave radiation has much longer wavelengths than these characteristic Balmer lines. Although the high-order Balmer lines converge to a near-ultraviolet limit around 365 nm, the conventional classification asked for here is the visible Balmer series.

Step 4: Conclusion The Balmer series is characteristically observed in the visible region. Hence, the correct option is (C).

Quick Tip to Solve: Remember the lower levels in order: Lyman ends at 1 (ultraviolet), Balmer at 2 (visible), and Paschen at 3 (infrared).

63. The IUPAC name of the following compound is : [4 marks]



(A) 3-Bromo-5-methylcyclopentane carboxylic acid

(B) 3-Bromo-5-methylcyclopentanoic acid

(C) 4-Bromo-2-methylcyclopentane carboxylic acid

(D) 5-Bromo-3-methylcyclopentanoic acid

Correct Answer: (C) 4-Bromo-2-methylcyclopentane carboxylic acid

Solution:

Concept:

- The principal functional group is $-\mathrm{COOH}$. When its carbon lies outside a ring, the ring name takes the suffix carboxylic acid.
- The ring carbon bearing the carboxyl group is carbon 1. Choose the direction giving substituents the lowest set of locants, then list substituent prefixes alphabetically.

Step 1: Choose the parent and suffix The figure shows a five-membered saturated carbon ring with an external $-\mathrm{C}(=\mathrm{O})\mathrm{OH}$ group. The carbonyl carbon is not one of the five ring atoms. The appropriate parent is therefore cyclopentanecarboxylic acid, also written as cyclopentane carboxylic acid in the options. The suffix cyclopentanoic acid would not correctly describe this external carboxyl carbon.

Step 2: Compare both directions of ring numbering Assign carbon 1 to the ring vertex attached to the carboxyl group. Numbering towards the methyl-bearing neighbouring vertex places methyl at carbon 2 and bromine at carbon 4. The opposite direction places bromine at carbon 3 and methyl at carbon 5. Compare the ascending locant sets $(2, 4)$ and $(3, 5)$: the first difference is 2 versus 3, so $(2, 4)$ is required. A lower number for bromo alone does not override the lowest-set-of-locants rule.

Step 3: Arrange the prefixes and reject mismatches Bromo precedes methyl alphabetically in the completed name even though its locant is larger. Thus the prefixes are 4-bromo and 2-methyl. Option (A) has the correct kind of parent suffix but uses the higher locant set. Options (B) and (D) use cyclopentanoic acid and also fail the required ring numbering. No stereochemical descriptor is needed because the drawing does not specify wedge or dash bonds or a relative configuration.

Step 4: Conclusion The compound is 4-bromo-2-methylcyclopentanecarboxylic acid. Hence, the correct option is (C).

Quick Tip to Solve: For a ring carrying an external carboxyl group, fix its attachment at carbon 1 before comparing the two complete locant sets; alphabetise only after numbering.

64. The ionic radii of O^{2-}, F^-, Na^+ and Mg^{2+} are in the order : [4 marks]

(A) $F^- > O^{2-} > Na^+ > Mg^{2+}$

(B) $Mg^{2+} > Na^+ > F^- > O^{2-}$

(C) $O^{2-} > F^- > Mg^{2+} > Na^+$

(D) $O^{2-} > F^- > Na^+ > Mg^{2+}$

Correct Answer: (D) $O^{2-} > F^- > Na^+ > Mg^{2+}$

Solution:

Concept:

- Isoelectronic species have the same number of electrons. For a fixed electron population, increasing nuclear charge draws the electron cloud closer and reduces ionic radius.
- The number of electrons in an ion is the atomic number minus its positive charge, or plus the magnitude of its negative charge.

Step 1: Count the electrons in each ion Oxygen has atomic number 8 and gains two electrons in O^{2-}, giving 10 electrons. Fluorine has atomic number 9 and gains one in F^-, again giving 10. Sodium has atomic number 11 and loses one in Na^+, while magnesium has atomic number 12 and loses two in Mg^{2+}. Both cations also have 10 electrons. All four ions therefore have the neon-like configuration $1s^2\,2s^2\,2p^6$.

Step 2: Order the attraction by nuclear charge The nuclei contain respectively 8, 9, 11 and 12 protons. Since the occupied electron shells and total electron counts are the same, the decisive trend is the increasing attraction of these electrons to the nucleus. The oxide ion has the weakest nuclear attraction in this series and the largest radius; the magnesium ion has the strongest attraction and the smallest radius. Hence radius decreases as $8 < 9 < 11 < 12$ increases.

Step 3: Compare the complete sequence with the options The resulting order is

$$r(O^{2-}) > r(F^-) > r(Na^+) > r(Mg^{2+}).$$

Option (A) reverses the oxide and fluoride pair despite their common electron count. Option (B) reverses the overall nuclear-charge trend. Option (C) incorrectly makes the magnesium ion larger than the sodium ion, even though magnesium has an extra proton attracting the same ten electrons. Only option (D) preserves both the anion pair and the cation pair.

Step 4: Conclusion The order is $O^{2-} > F^- > Na^+ > Mg^{2+}$. Hence, the correct option is (D).

Quick Tip to Solve: For isoelectronic ions, count protons rather than comparing neutral-atom sizes: more protons acting on the same electron cloud means a smaller ion.

65. For one mole of an ideal gas, which of these statements must be true?

(a) U and H each depends only on temperature

(b) Compressibility factor z is not equal to 1

(c) $C_{P,m} - C_{V,m} = R$

(d) $dU = C_V dT$ for any process [4 marks]

(A) (a), (c) and (d)

(B) (a) and (c)

(C) (c) and (d)

(D) (b), (c) and (d)

Correct Answer: (A) (a), (c) and (d)

Solution:

Concept:

- For a fixed amount of an ideal gas, internal energy U depends only on temperature. Since $H = U + pV$ and $pV = nRT$, enthalpy also depends only on temperature.
- The compressibility factor is $Z = pV/(nRT)$. For an ideal gas, $Z = 1$, and its molar heat capacities satisfy Mayer's relation $C_{P,m} - C_{V,m} = R$.

Step 1: Test statements (a) and (b) For one mole, the ideal-gas equation is $pV = RT$. The internal energy is a function $U(T)$, and therefore $H(T) = U(T) + RT$ is also a function only of temperature. Statement (a) is true. Substituting the same equation of state into $Z = pV/(RT)$ gives exactly 1. Statement (b), which says that the factor differs from 1, is false; it would describe nonideal behaviour rather than an ideal gas.

Step 2: Derive the heat-capacity difference For one mole, differentiate $H = U + RT$ with respect to temperature. Because both energies depend only on temperature, their derivatives equal the molar heat capacities:

$$\frac{dH}{dT} = \frac{dU}{dT} + R, \qquad C_{P,m} = C_{V,m} + R.$$

Thus statement (c) is true. This relation does not require either heat capacity to be constant over a finite temperature interval.

Step 3: Distinguish a state-function differential from heat transfer The definition at constant volume is $C_V = (\partial U/\partial T)_V$. For an ideal gas the absence of volume dependence makes this derivative the total derivative, so $dU = C_V\, dT$ for any path between neighbouring equilibrium states. Statement (d) is therefore true even if the actual process is not isochoric. It is the equality between heat supplied and dU, not this energy-temperature relation, that requires a suitable constraint. Options (B) and (C) omit a true statement; option (D) includes false statement (b).

Step 4: Conclusion The first, third and fourth statements are true. Hence, the correct option is (A).

Quick Tip to Solve: Do not confuse $dU = C_V dT$, valid for every ideal-gas path, with $\delta q = C_V dT$, which requires constant volume when only pressure-volume work occurs.

66. Which of the following will react with $CHCl_3$ + alc. KOH? [4 marks]

(A) Adenine and thymine

(B) Thymine and proline

(C) Adenine and lysine

(D) Adenine and proline

Correct Answer: (C) Adenine and lysine

Solution:

Concept:

- The carbylamine test uses chloroform and alcoholic potassium hydroxide to convert primary amines to isocyanides. Secondary and tertiary amines do not give this test.
- The diagnostic group is an amino nitrogen bonded to one carbon group and bearing two hydrogens, $R-NH_2$; a nitrogen atom in a molecule alone is not enough.

Step 1: Identify the reagent combination Chloroform with alcoholic potassium hydroxide is the characteristic carbylamine reagent. The overall reaction is

$$RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O.$$

Thus we must inspect the nitrogen functional groups in each candidate, rather than classify all nitrogen-containing biomolecules as reactive. The product is an isocyanide, with connectivity $R-N\equiv C$, and not a nitrile.

Step 2: Examine adenine and lysine Adenine contains an exocyclic primary amino group attached to its heteroaromatic ring. That $-NH_2$ group satisfies the primary-amine requirement, so adenine reacts. Lysine has the structure $HOOC-CH(NH_2)-(CH_2)_4-NH_2$. It possesses primary amino groups, including the terminal side-chain group, and also responds to the reagent. The presence of a carboxyl group in the amino acid does not convert these amino nitrogens into secondary amines.

Step 3: Exclude thymine and proline Thymine has carbonyl groups and ring nitrogens but no exocyclic primary amino group. Its ring nitrogen atoms are not primary amines, so it does not give the carbylamine reaction. Proline has its amino nitrogen within a saturated five-membered ring: the nitrogen is bonded to two carbon atoms and is a secondary amine. It consequently fails this test. Options (A) and (B) incorrectly include thymine, and options (B) and (D) incorrectly include proline. The pair in which both members react is adenine and lysine.

Step 4: Conclusion Adenine and lysine react with chloroform and alcoholic potassium hydroxide. Hence, the correct option is (C).

Quick Tip to Solve: For a carbylamine question, explicitly locate a primary $-NH_2$ group; ring nitrogen and a secondary amino group do not qualify.

67. On combustion of Li, Na and K in excess of air, the major oxides formed, respectively, are :

[4 marks]

(A) Li_2O, Na_2O_2 and K_2O

(B) Li_2O_2, Na_2O_2 and K_2O_2

(C) Li_2O, Na_2O_2 and KO_2

(D) Li_2O, Na_2O and K_2O_2

Correct Answer: (C) Li_2O, Na_2O_2 and KO_2

Solution:

Concept:

- Alkali metals form normal oxides containing O^{2-}, peroxides containing ${O_2}^{2-}$, or superoxides containing ${O_2}^{-}$. The product depends on the metal and oxygen supply.
- Down Group 1, increasing cation size stabilises the larger peroxide and superoxide anions. Thus the major combustion product in excess air changes from oxide to peroxide to superoxide.

Step 1: Identify the lithium product Lithium has the smallest cation in this series and forms mainly the normal oxide. Its combustion is represented by

$$4\,Li + O_2 \longrightarrow 2\,Li_2O.$$

The oxide ion has charge -2, so two Li^+ ions are required per oxide ion. Lithium peroxide is therefore not the major product demanded here.

Step 2: Identify the sodium and potassium products Sodium burns predominantly to sodium peroxide, whereas the larger potassium cation stabilises potassium superoxide in excess air:

$$2\,Na + O_2 \longrightarrow Na_2O_2, \qquad K + O_2 \longrightarrow KO_2.$$

In the peroxide each oxygen has oxidation number -1; in the superoxide its average oxidation number is $-\frac{1}{2}$. These different oxygen species explain why simply writing the normal oxide for every metal gives the wrong sequence.

Step 3: Compare the options The required sequence is Li_2O, Na_2O_2, KO_2. Option (A) incorrectly gives potassium oxide; option (B) treats all three products as peroxides; option (D) gives sodium oxide and potassium peroxide. Only option (C) retains all three major products in the stated order.

Step 4: Conclusion The major oxides are $\boxed{Li_2O,\ Na_2O_2,\ KO_2}$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Remember the excess-oxygen sequence Li: oxide, Na: peroxide, K: superoxide; do not apply a single oxide formula to the whole group.

68. On heating, lead (II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is : [4 marks]

(A) +3

(B) +4

(C) +5

(D) +2

Correct Answer: (A) +3

Solution:

Concept:

- Heating a heavy-metal nitrate gives the metal oxide, nitrogen dioxide and oxygen. Brown NO_2 dimerises on cooling to colourless N_2O_4.
- The blue oxide N_2O_3 is nitrogen(III) oxide. With oxygen assigned oxidation number -2, charge balance determines nitrogen's oxidation state.

Step 1: Identify the brown gas and its dimer Lead(II) nitrate thermally decomposes as

$$2\,Pb(NO_3)_2 \longrightarrow 2\,PbO + 4\,NO_2 + O_2.$$

Therefore the brown gas (A) is NO_2, not nitric oxide, which is colourless. On cooling the dimerisation equilibrium shifts towards colourless dinitrogen tetroxide:

$$2\,NO_2 \rightleftharpoons N_2O_4.$$

Hence (B) is N_2O_4.

Step 2: Identify the blue nitrogen oxide Combination of nitrogen dioxide with nitric oxide gives dinitrogen trioxide, whose condensed form is blue. In terms of (B), the stoichiometric relationship is

$$N_2O_4 + 2\,NO \longrightarrow 2\,N_2O_3.$$

Thus the blue solid (C) is N_2O_3. This uses both the given reagent and the characteristic colour; stopping at the colourless dimer would identify the wrong species.

Step 3: Calculate the oxidation number Let the oxidation number of nitrogen in N_2O_3 be x. The molecule is neutral, so

$$2x + 3(-2) = 0 \quad \Longrightarrow \quad x = +3.$$

The value $+4$ belongs to NO_2 or N_2O_4, $+2$ to NO, and $+5$ to nitrate nitrogen. None of those is the oxidation state in the final blue oxide.

Step 4: Conclusion The blue solid is N_2O_3, in which nitrogen has oxidation number $\boxed{+3}$. Hence, the correct option is (A).

Quick Tip to Solve: Identify the final oxide before assigning oxidation state: brown NO_2, colourless N_2O_4 and blue N_2O_3 are different species.

69. Among statements (a) - (d), the correct ones are

(a) Lime stone is decomposed to CaO during the extraction of iron from its oxides.

(b) In the extraction of silver, silver is extracted as an anionic complex.

(c) Nickel is purified by Mond's process.

(d) Zr and Ti are purified by Van Arkel method. [4 marks]

(A) (a), (c) and (d) only

(B) (c) and (d) only

(C) (b), (c) and (d) only

(D) (a), (b), (c) and (d)

Correct Answer: (D) (a), (b), (c) and (d)

Solution:

Concept:

- In iron extraction limestone supplies the basic flux CaO, which removes silica as slag. In cyanide leaching silver enters solution as a negatively charged coordination complex.
- Mond refining uses volatile nickel tetracarbonyl. The van Arkel iodide process converts titanium or zirconium into volatile iodides and decomposes them on a hot filament.

Step 1: Check statements (a) and (b) Limestone decomposes in the blast furnace:

$$\mathrm{CaCO_3} \longrightarrow \mathrm{CaO} + \mathrm{CO_2}.$$

The resulting lime reacts with acidic silica to form calcium silicate slag, $\mathrm{CaSiO_3}$. Statement (a) is therefore correct. Silver undergoes cyanide leaching to give the soluble dicyanoargentate(I) ion $[\mathrm{Ag(CN)_2}]^-$. Its negative charge makes it an anionic complex, exactly as statement (b) says; silver is subsequently recovered from this solution.

Step 2: Check statement (c) In Mond's process impure nickel reacts with carbon monoxide to form volatile nickel tetracarbonyl:

$$\mathrm{Ni} + 4\,\mathrm{CO} \longrightarrow \mathrm{Ni(CO)_4}.$$

The carbonyl is separated and decomposed at a higher temperature to deposit pure nickel while carbon monoxide is released. This selective formation and decomposition of a volatile compound makes statement (c) correct.

Step 3: Check statement (d) and select the combination Titanium and zirconium are purified by the van Arkel method. A volatile tetraiodide is first formed, then decomposed on a hot tungsten filament; for zirconium,

$$\mathrm{Zr} + 2\,\mathrm{I_2} \longrightarrow \mathrm{ZrI_4}, \qquad \mathrm{ZrI_4} \longrightarrow \mathrm{Zr} + 2\,\mathrm{I_2}.$$

Titanium behaves similarly. Thus (d) is also correct. Every statement (a)–(d) is true. Options (A), (B) and (C) each omit at least one true statement, so none is the complete combination.

Step 4: Conclusion Every listed statement is correct, including limestone decomposition, anionic silver complex formation, Mond refining and van Arkel refining. Hence, the correct option is (D).

Quick Tip to Solve: Associate Mond with nickel carbonyl and van Arkel with titanium/zirconium iodides; check extraction statements separately from refining methods.

70. Match the following:

(i) Foam	**(a) smoke**
(ii) Gel	**(b) cell fluid**
(iii) Aerosol	**(c) jellies**
(iv) Emulsion	**(d) rubber**
	(e) froth
	(f) Milk

[4 marks]

(A) (i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)

(B) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(e)

(C) (i)-(d), (ii)-(b), (iii)-(e), (iv)-(f)

(D) (i)-(b), (ii)-(c), (iii)-(e), (iv)-(d)

Correct Answer: (A) (i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)

Solution:

Concept:

- Colloids are classified by the physical states of the dispersed phase and dispersion medium. A foam has gas dispersed in liquid; a gel has liquid dispersed in solid.
- An aerosol has a gaseous dispersion medium, while an emulsion contains one liquid dispersed in another liquid. Match the named examples to these phase combinations.

Step 1: Match foam and gel Froth contains bubbles of gas dispersed throughout a liquid, so it is the example of foam: $(i)-(e)$. Jellies contain liquid held in a solid-like network and are standard examples of gels: $(ii)-(c)$. A gel is therefore not identified merely by the fact that some material looks fluid; the continuous supporting network distinguishes it from a liquid dispersion.

Step 2: Match aerosol and emulsion Smoke consists of small solid particles dispersed in air, so its dispersion medium is gas and it is an aerosol: $(iii)-(a)$. Milk contains tiny liquid fat droplets dispersed in an aqueous liquid medium, so it is an emulsion: $(iv)-(f)$. The word aerosol covers both solid-in-gas and liquid-in-gas dispersions, but smoke supplies the solid-in-gas example here.

Step 3: Compare the complete codes The four matches in order are $(i)-(e)$, $(ii)-(c)$, $(iii)-(a)$, $(iv)-(f)$. Only option (A) contains all of them. Option (B) incorrectly calls froth an emulsion, option (C) calls froth an aerosol, and option (D) also assigns the examples to incompatible phase combinations. Rubber and cell fluid are distractors for the four examples selected in the correct code.

Step 4: Conclusion Froth is foam, jellies are gel, smoke is aerosol and milk is emulsion. Hence, the correct option is (A).

Quick Tip to Solve: Write dispersed phase/medium beside each example: froth gas/liquid, jelly liquid/solid, smoke solid/gas, milk liquid/liquid.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. At 300 K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state _____? [4 marks]

Solution:

Concept:

- For an ideal binary liquid solution, Raoult's law gives total pressure $P = x_A P_A^\circ + x_B P_B^\circ$, where x_i is mole fraction and P_i° is pure-component vapour pressure at the same temperature.
- Adding one component changes both mole fractions. Pure-component vapour pressures remain constant at fixed temperature, so the two compositions give two linear equations.

Step 1: Form the equation for the initial composition Let H denote the pure vapour pressure of n-hexane and J that of n-heptane, both in $\mathrm{mm\,Hg}$ at $300\,\mathrm{K}$. Initially there are $1 + 3 = 4$ moles, giving mole fractions $1/4$ and $3/4$. Hence

$$\frac{H + 3J}{4} = 550, \qquad H + 3J = 2200.$$

The coefficients use mole fractions, not the raw mole numbers; this division by total moles is essential.

Step 2: Form the equation after adding heptane After adding one mole of n-heptane the amounts become one mole of hexane and four moles of heptane. The total is now five moles. The pressure increases by $10\,\mathrm{mm\,Hg}$, so the new pressure is $560\,\mathrm{mm\,Hg}$, not 540. Therefore

$$\frac{H + 4J}{5} = 560, \qquad H + 4J = 2800.$$

Both pure-liquid pressures refer to the same temperature, so the same unknowns occur in both equations.

Step 3: Eliminate hexane pressure and verify Subtract the first equation from the second:

$$J = (H + 4J) - (H + 3J) = 2800 - 2200 = 600.$$

Then $H = 2200 - 3(600) = 400\,\mathrm{mm\,Hg}$. Substitution gives $(400 + 1800)/4 = 550$ initially and $(400 + 2400)/5 = 560$ finally. Thus the increase is reproduced exactly and both pure-component pressures are positive.

Step 4: Conclusion The vapour pressure of pure n-heptane at $300\,\mathrm{K}$ is $\boxed{600\,\mathrm{mm\,Hg}}$.

Quick Tip to Solve: For two compositions at the same temperature, multiply each total pressure by its total mole count before subtracting to isolate the added component.

72. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) _____.

(Take : log 2 = 0.30; log 2.5 = 0.40) [4 marks]

Solution:

Concept:

- For a first-order reaction, $kt = 2.303 \log_{10}([A]_0/[A]_t)$, where k is the rate constant and $[A]_t$ is the amount or concentration remaining.
- At a fixed temperature the rate constant is unchanged. Ratios of reaction times therefore equal ratios of the corresponding logarithms; percentages reacted must first be converted to fractions remaining.

Step 1: Use the 75 percent completion data If 75% has reacted, the remaining fraction is 0.25. Thus the initial-to-remaining ratio is $1/0.25 = 4$, not 0.75. The integrated first-order equation at 90 minutes is

$$k(90) = 2.303 \log 4.$$

Using the supplied approximation $\log 2 = 0.30$, we get $\log 4 = 2 \log 2 = 0.60$. This fixes the kinetic rate for the same reaction under the stated conditions.

Step 2: Express the time for 60 percent completion When 60% has reacted, 40% remains, giving $[A]_0/[A]_t = 1/0.40 = 2.5$. If the required time is t,

$$kt = 2.303 \log 2.5.$$

Divide by the earlier equation to cancel both k and 2.303:

$$\frac{t}{90} = \frac{\log 2.5}{\log 4} = \frac{0.40}{0.60} = \frac{2}{3}.$$

This avoids unnecessarily calculating and rounding the rate constant.

Step 3: Calculate the time and check the trend The required time is

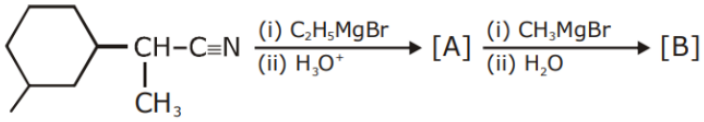
$$t = 90 \times \frac{2}{3} = 60 \,\text{min}.$$

Since 60% conversion is less than 75%, its completion time must be less than 90 minutes, which this value satisfies. A direct proportionality between percentage reacted and time would be inappropriate: first-order decay is exponential, and the logarithm applies to the unreacted fraction.

Step 4: Conclusion Approximately $\boxed{60 \,\text{minutes}}$ are needed for 60% completion using the supplied logarithms.

Quick Tip to Solve: Convert conversion into percentage remaining before taking logs; for the same first-order reaction use a ratio of logarithms to cancel the rate constant.

73. The number of chiral centres present in [B] is _____. [4 marks]



Solution:

Concept:

- A tetrahedral carbon is a chiral centre when its four attached groups are different. In a substituted ring, compare the two paths around the ring rather than treating both ring bonds as identical.
- A nitrile reacts with one Grignard reagent followed by acidic hydrolysis to give a ketone. A second Grignard reagent adds to that ketone and hydrolysis gives an alcohol.

Step 1: Identify the ketone intermediate Write the substituted cyclohexyl group as R. The starting nitrile is $\mathrm{R{-}CH(CH_3){-}C{\equiv}N}$. Ethylmagnesium bromide adds an ethyl group to the nitrile carbon; acidic hydrolysis then converts the intermediate into the ketone. Thus $[A]$ is $\mathrm{R{-}CH(CH_3){-}CO{-}C_2H_5}$. The carbonyl carbon is trigonal planar and is not a chiral centre at this stage.

Step 2: Find the final alcohol Methylmagnesium bromide adds a methyl group to the carbonyl carbon of $[A]$, and water protonates the alkoxide. Therefore $[B]$ has the connectivity $\mathrm{R{-}CH(CH_3){-}C(OH)(CH_3)(C_2H_5)}$. The alcohol carbon is bonded to hydroxy, methyl, ethyl and $\mathrm{CH(CH_3)R}$ groups. These are all distinct, so this newly tetrahedral carbon is a chiral centre.

Step 3: Count the other tetrahedral centres The adjacent side-chain CH is attached to H, methyl, the substituted cyclohexyl group and the alcohol-containing group; it contributes one more centre. The cyclohexane ring is substituted at two positions, one bearing methyl and the other bearing this side chain. Each substituted ring carbon bears H, its external substituent and two inequivalent ring paths: one direction reaches the other substituted position sooner than the other. Both therefore qualify. The remaining ring carbons are $\mathrm{CH_2}$, and the remaining alkyl carbons are $\mathrm{CH_2}$ or $\mathrm{CH_3}$, so none adds a centre.

Step 4: Conclusion The total is two substituted ring carbons, one side-chain methine carbon and one alcohol carbon: $\boxed{4}$ chiral centres.

Quick Tip to Solve: For ring stereocentres, follow both directions until the first point of difference; identical first atoms do not make the two ring paths identical.

74. The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is _____. [4 marks]

Solution:

Concept:

- The balanced synthesis is $\mathrm{N_2 + 3\,H_2 \longrightarrow 2\,NH_3}$. Stoichiometric coefficients compare mole amounts, not masses.
- The limiting reagent is the reactant with the smaller value of moles divided by its coefficient.

Quantitative reaction means the limiting reagent is consumed completely, with no yield correction.

Step 1: Convert the reactant masses to moles Use $M(\mathrm{N_2}) = 28\,\mathrm{g\,mol^{-1}}$, $M(\mathrm{H_2}) = 2\,\mathrm{g\,mol^{-1}}$, and $M(\mathrm{NH_3}) = 17\,\mathrm{g\,mol^{-1}}$. Convert kilograms to grams before dividing by these molar masses. The available amounts are

$$n(\mathrm{N_2}) = \frac{2800}{28} = 100\,\mathrm{mol}, \qquad n(\mathrm{H_2}) = \frac{1000}{2} = 500\,\mathrm{mol}.$$

Step 2: Identify the limiting reagent For every mole of dinitrogen, three moles of dihydrogen are required. The $100\,\mathrm{mol}$ of dinitrogen therefore needs $300\,\mathrm{mol}$ of dihydrogen. Since $500\,\mathrm{mol}$ is supplied, hydrogen is in excess and dinitrogen is limiting. Equivalently, the reaction extents allowed by the two reactants are $100/1 = 100$ and $500/3 \approx 166.7\,\mathrm{mol}$; the smaller controls product formation.

Step 3: Calculate the product mass The balanced equation gives two moles of ammonia for each mole of dinitrogen consumed. Hence

$$n(\mathrm{NH_3}) = 2(100) = 200\,\mathrm{mol}, \qquad m(\mathrm{NH_3}) = 200(17) = 3400\,\mathrm{g}.$$

As a mass check, only $300(2) = 600\,\mathrm{g}$ hydrogen reacts. The product mass is therefore $2800 + 600 = 3400\,\mathrm{g}$, while $400\,\mathrm{g}$ hydrogen remains. Using all the supplied hydrogen would incorrectly exceed the available nitrogen.

Step 4: Conclusion The mass of ammonia produced is $\boxed{3400\,\mathrm{g}}$. The numerical entry is $\boxed{3400}$.

Quick Tip to Solve: Always check the limiting reactant before converting the total supplied mass into product; excess hydrogen does not become ammonia without nitrogen.

75. A 20.0 mL solution containing 0.2 g impure H_2O_2 reacts completely with 0.316 g of $KMnO_4$ in acid solution. The purity of H_2O_2 (in %) is _____
(mol. wt. of H_2O_2 = 34; mol. wt. of $KMnO_4$ = 158) [4 marks]

Solution:

Concept:

- In acidic medium, permanganate is reduced to $\mathrm{Mn^{2+}}$, while hydrogen peroxide is oxidised to oxygen. The balanced redox equation determines their mole ratio.
- Purity is the mass of pure reacting substance divided by the mass of the impure sample, multiplied by 100. The sample volume is unnecessary when the complete sample reacts with a known reagent mass.

Step 1: Balance the redox reaction The reduction half-reaction is $\mathrm{MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O}$, and the oxidation half-reaction is $\mathrm{H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-}$. Equalising the electron transfer and adding gives

$$\mathrm{2\,MnO_4^- + 5\,H_2O_2 + 6\,H^+ \longrightarrow 2\,Mn^{2+} + 5\,O_2 + 8\,H_2O}.$$

Thus two moles of permanganate consume five moles of hydrogen peroxide. Potassium ions are

spectators and do not affect this ratio.

Step 2: Find the reacting peroxide amount The given molar mass of potassium permanganate is $158\,\mathrm{g\,mol^{-1}}$. Consequently

$$n(\mathrm{KMnO_4}) = \frac{0.316}{158} = 0.00200\,\mathrm{mol}.$$

Each formula unit supplies one permanganate ion. Complete reaction therefore requires

$$n(\mathrm{H_2O_2}) = \frac{5}{2}(0.00200) = 0.00500\,\mathrm{mol}.$$

The acidic medium is essential: using a different manganese reduction product would give a different, inapplicable stoichiometric ratio.

Step 3: Calculate the percentage purity Using the stated peroxide molar mass, the mass of pure hydrogen peroxide present is

$$m_{\text{pure}} = 0.00500(34) = 0.170\,\mathrm{g}.$$

This is the active peroxide contained in the $0.200\,\mathrm{g}$ impure sample. Hence

$$\text{Purity} = \frac{0.170}{0.200} \times 100 = 85\%.$$

No division by $20.0\,\mathrm{mL}$ is needed, because a concentration has not been requested and the supplied reagent reacts with the entire sample.

Step 4: Conclusion The hydrogen peroxide sample has purity $\boxed{85\%}$. The numerical entry is $\boxed{85}$.

Quick Tip to Solve: For acidic permanganate and peroxide, remember the mole ratio $2:5$; then compare the reacting peroxide mass with the impure sample mass.