JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 4 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 4 September 2020, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is : [4 marks]

(A) $\frac{1}{3\sqrt{3}}$
(B) $\frac{2}{3\sqrt{3}}$
(C) $\frac{4}{3\sqrt{3}}$
(D) $\frac{4}{3}$

Correct Answer: (C) $\frac{4}{3\sqrt{3}}$

Solution:

Concept:

- A rectangle with its upper side on the x-axis has horizontal lower side. The two lower vertices on $y = x^2 - 1$ therefore have coordinates $(-t, t^2 - 1)$ and $(t, t^2 - 1)$, with $0 < t < 1$.
- Area equals width times height. An interior maximum is found by differentiating this area with respect to t, and its global nature is checked using the endpoints.

Step 1: Express the area in one variable The equal ordinates of the lower vertices force their abscissae to be opposites. Thus the width is $2t$, whereas the vertical distance from the lower side to the x-axis is $1 - t^2$, not t^2. Consequently

$$A(t) = 2t(1 - t^2) = 2t - 2t^3, \qquad 0 \le t \le 1.$$

The endpoints represent degenerate rectangles and both have zero area.

Step 2: Locate and verify the maximum Differentiate and set the first derivative equal to zero:

$$A\prime(t) = 2 - 6t^2 = 0 \quad \Longrightarrow \quad t = \frac{1}{\sqrt{3}}.$$

Only the positive root lies in the chosen parameter interval. Also $A\prime\prime(t) = -12t < 0$ at this point, so it gives a strict local maximum. Since the area is continuous on $[0, 1]$, and is zero at both endpoints, this positive interior value is the global maximum.

Step 3: Evaluate the area and compare options At the maximizing position, the width is $2/\sqrt{3}$ and the height is $2/3$. Therefore

$$A_{\max} = \frac{2}{\sqrt{3}} \cdot \frac{2}{3} = \frac{4}{3\sqrt{3}}.$$

Options (A) and (B) underestimate this product; (D) drops the factor $\sqrt{3}$ in the width. The computed value agrees only with (C).

Step 4: Conclusion The largest area is $\boxed{\frac{4}{3\sqrt{3}}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Use the positive half-width as the variable: the full width is twice it, while the height is the distance below the x-axis.

2. If a and b are real numbers such that

$$(2+\alpha)^4 = a + b\alpha$$

where $\alpha = \frac{-1+i\sqrt{3}}{2}$ then a + b is equal to : [4 marks]

(A) 33

(B) 9

(C) 24

(D) 57

Correct Answer: (B) 9

Solution:

Concept:

- The number $\alpha = (-1 + i\sqrt{3})/2$ is a non-real cube root of unity. It satisfies $\alpha^2 + \alpha + 1 = 0$ and $\alpha^3 = 1$.
- Reduce all powers of α to the form $u + v\alpha$. Because α is non-real, its representation with real coefficients is unique.

Step 1: Reduce the first square Rather than expanding the fourth power into five terms, square once and use $\alpha^2 = -1 - \alpha$:

$$\begin{aligned}(2+\alpha)^2 &= 4 + 4\alpha + \alpha^2 \\ &= 4 + 4\alpha - 1 - \alpha \\ &= 3 + 3\alpha = 3(1+\alpha).\end{aligned}$$

This reduction is valid for the specified value of α; replacing it by the real cube root 1 would not satisfy the same quadratic relation.

Step 2: Square again and identify the coefficients Using the reduced square gives

$$\begin{aligned}(2+\alpha)^4 &= 9(1+\alpha)^2 \\ &= 9(1 + 2\alpha + \alpha^2) \\ &= 9(1 + 2\alpha - 1 - \alpha) = 9\alpha.\end{aligned}$$

Compare this expression with $a + b\alpha$. Taking imaginary parts first gives $b = 9$, since $\operatorname{Im}\alpha \neq 0$; the real parts then give $a = 0$. Thus there is no ambiguity in the real coefficients.

Step 3: Compute the requested sum The problem asks for the sum of the coefficients, not the modulus of the original complex number or a coefficient from an unreduced expansion. We obtain

$$a + b = 0 + 9 = 9.$$

The alternatives 33, 24 and 57 cannot occur because substituting any of those sums would contradict the unique reduced form $0 + 9\alpha$.

Step 4: Conclusion $\boxed{a + b = 9}$. Hence, the correct option is (B).

Quick Tip to Solve: For a non-real cube root of unity, reduce after each multiplication using $\alpha^2 = -1 - \alpha$, instead of retaining high powers.

3. Let a_1, a_2, ..., an be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ... + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to: [4 marks]

(A) (2480, 249)

(B) (2480, 248)

(C) (2490, 248)

(D) (2490, 249)

Correct Answer: (C) (2490, 248)

Solution:

Concept:

- For an arithmetic progression with first term a_1 and common difference d, $a_k = a_1 + (k-1)d$ and $S_k = k(a_1 + a_k)/2$.
- The integer condition on d, together with the allowed interval for n, restricts $n-1$ to an appropriate positive divisor of $a_n - a_1$.

Step 1: Determine the number of terms The last term is 300 and the first term is 1, so

$$300 = 1 + (n-1)d \quad \Longrightarrow \quad (n-1)d = 299 = 13 \cdot 23.$$

Since n is a number of terms, it is an integer. Also $15 \leq n \leq 50$ gives $14 \leq n - 1 \leq 49$. The positive divisors of 299 are 1, 13, 23 and 299; only 23 lies in this interval. Hence $n - 1 = 23$, $n = 24$, and $d = 13$. A negative difference is impossible because the final term exceeds the first.

Step 2: Find the requested term The index $n - 4$ is $24 - 4 = 20$, so the term is

$$a_{n-4} = a_{20} = 1 + 19 \cdot 13 = 248.$$

Equivalently, move back four common differences from the last term: $300 - 4 \cdot 13 = 248$. This checks the indexing and avoids confusing the number of steps with the term number.

Step 3: Calculate the corresponding partial sum There are 20 terms in the required partial sum. Using its actual first and last terms gives

$$S_{n-4} = S_{20} = \frac{20}{2}(1 + 248) = 10 \cdot 249 = 2490.$$

Thus the sum is 2490, whereas the last included term is 248. Options (A) and (B) use an incorrect sum of 2480, and (D) incorrectly uses 249 as the term instead of as the sum of the two end terms.

Step 4: Conclusion The ordered pair is $\boxed{(S_{n-4}, a_{n-4}) = (2490, 248)}$. Hence, the correct option is (C).

Quick Tip to Solve: Factor the difference between the last and first terms, then use the permitted range of $n - 1$ to choose its divisor.

4. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to: [4 marks]

(A) 36

(B) 9

(C) 27

(D) 18

Correct Answer: (D) 18

Solution:

Concept:

- For a quadratic $Ax^2 + Bx + C = 0$, the sum of roots is $-B/A$ and their product is C/A.
- A shared root satisfies both polynomial equations. Eliminating its square gives a simple relation between that root and the parameter.

Step 1: Eliminate the square of the common root As α is a root of both quadratics,

$$\alpha^2 - \alpha + 2\lambda = 0, \qquad 3\alpha^2 - 10\alpha + 27\lambda = 0.$$

Subtract three times the first equation from the second:

$$-7\alpha + 21\lambda = 0 \quad \Longrightarrow \quad \alpha = 3\lambda.$$

This step uses the equations of the common root directly; it does not assume the other roots coincide.

Step 2: Determine the nonzero parameter Substitution into the first equation yields

$$9\lambda^2 - 3\lambda + 2\lambda = 0 \quad \Longrightarrow \quad \lambda(9\lambda - 1) = 0.$$

The question explicitly excludes $\lambda = 0$, hence $\lambda = 1/9$ and $\alpha = 1/3$. Keeping the zero branch would also make the requested quotient undefined, so it cannot be used.

Step 3: Recover the other roots and form the quotient From the sums of roots,

$$\beta = 1 - \alpha = \frac{2}{3}, \qquad \gamma = \frac{10}{3} - \alpha = 3.$$

Their products provide an additional check: $\alpha\beta = 2/9 = 2\lambda$ and $\alpha\gamma = 1 = 9\lambda$. Therefore

$$\frac{\beta\gamma}{\lambda} = \frac{(2/3)\cdot 3}{1/9} = 18.$$

The values 36, 9 and 27 do not match this quotient. In particular, 9 is the product-of-roots coefficient in the second quadratic, rather than the requested ratio.

Step 4: Conclusion $\boxed{\beta\gamma/\lambda = 18}$. Hence, the correct option is (D).

Quick Tip to Solve: For two quadratics sharing a root, first cancel their squared terms; use the nonzero-parameter condition before dividing.

5. If the system of equations

x+y+z=2

2x+4y–z=6

3x+2y+λz=μ

has infinitely many solutions, then [4 marks]

(A) 2λ - μ = 5

(B) λ - 2μ = -5

(C) $2\lambda + \mu = 14$

(D) $\lambda + 2\mu = 14$

Correct Answer: (C) $2\lambda + \mu = 14$

Solution:

Concept:

- Infinitely many solutions require dependent equations that remain consistent, so the coefficient matrix and augmented matrix have the same rank below the number of unknowns.
- When two coefficient rows are independent, a third dependent equation must be their linear combination with exactly the same combination of the right-hand sides.

Step 1: Form the required linear combination The first two coefficient rows $(1, 1, 1)$ and $(2, 4, -1)$ are not proportional, so they are independent. Let the third equation be u times the first plus v times the second. Comparing the coefficients of x and y gives

$$u + 2v = 3, \qquad u + 4v = 2.$$

Subtracting yields $2v = -1$, hence $v = -1/2$ and $u = 4$. These values are forced by the two independent columns.

Step 2: Enforce consistency in the remaining entries The same linear combination must give both the coefficient of z and the constant term:

$$\lambda = 4 \cdot 1 + \left(-\frac{1}{2}\right)(-1) = \frac{9}{2},$$

$$\mu = 4 \cdot 2 + \left(-\frac{1}{2}\right) 6 = 5.$$

With these values, the third equation is exactly four times the first minus half the second, so it imposes no extra condition. The first two equations leave one free variable, confirming infinitely many solutions rather than no solution.

Step 3: Check the listed relations Substitute the determined values into the alternatives. They give

$$2\lambda - \mu = 4, \qquad \lambda - 2\mu = -\frac{11}{2},$$

$$2\lambda + \mu = 14, \qquad \lambda + 2\mu = \frac{29}{2}.$$

Thus the assertions in (A), (B) and (D) are false, while the relation in (C) holds. Checking only a zero

determinant would have missed the need for the correct value of μ.

Step 4: Conclusion The parameters are $\lambda = 9/2$, $\mu = 5$, giving $\boxed{2\lambda + \mu = 14}$. Hence, the correct option is (C).

Quick Tip to Solve: For infinitely many solutions, apply the same row dependence to the constants as to the coefficients; determinant zero alone is insufficient.

6. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. if

$x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix}$, $x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$b_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $b_2 = \begin{bmatrix}0\\2\\0\end{bmatrix}$ **and** $b_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$,

then the determinant of A is equal to : [4 marks]

(A) $\frac{3}{2}$

(B) 4

(C) 2

(D) $\frac{1}{2}$

Correct Answer: (C) 2

Solution:

Concept:

- Place the solution vectors as columns of a matrix $X = [x_1\ x_2\ x_3]$ and the corresponding right-hand sides as columns of $B = [b_1\ b_2\ b_3]$. Then $AX = B$.
- The determinant of a product is the product of determinants: $\det(A)\det(X) = \det(B)$. The determinant of a triangular matrix is the product of its diagonal entries.

Step 1: Assemble the column matrices The given equations $Ax_j = b_j$ have the same coefficient matrix for all three right-hand sides. Combining them without changing their order gives

$$A\begin{pmatrix}1 & 0 & 0\\ 1 & 2 & 0\\ 1 & 1 & 1\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{pmatrix}.$$

Thus X is lower triangular and B is diagonal. It is unnecessary to solve for all nine entries of A. The pairing of each solution with its right-hand side must be preserved when forming the columns.

Step 2: Evaluate the two known determinants Because both matrices are triangular, their determinants are immediate:

$$\det(X) = 1 \cdot 2 \cdot 1 = 2, \qquad \det(B) = 1 \cdot 2 \cdot 2 = 4.$$

In particular, $\det(X) \neq 0$; the three solution vectors are independent, so division by its determinant is legitimate. The number 4 is the determinant of the right-hand-side matrix, not yet the determinant asked for.

Step 3: Apply the determinant product identity Taking determinants of $AX = B$,

$$\det(A) \cdot 2 = 4 \quad \Longrightarrow \quad \det(A) = 2.$$

The alternative $1/2$ reverses the quotient $\det(B)/\det(X)$, and 4 omits the divisor altogether. The value $3/2$ has no basis in the triangular determinants. As an independent structural check, multiplication of a triangular inverse of X by B gives a matrix with diagonal $1, 1, 2$, whose determinant is again 2.

Step 4: Conclusion $\boxed{\det A = 2}$. Hence, the correct option is (C).

Quick Tip to Solve: When several systems share the same matrix, stack the solutions and right-hand sides as columns and take determinants of the resulting matrix equation.

7. The solution of the differential equation
$\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ **is:**
(where c is a constant of integration) [4 marks]

(A) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$
(B) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$
(C) $x - \log_e(y+3x) = C$
(D) $x - 2\log_e(y+3x) = C$
Correct Answer: (A) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

Solution:

Concept:

- A linear combination of the dependent and independent variables suggests the substitution $u = y + 3x$. Its derivative is $du/dx = dy/dx + 3$, exactly the combination present here.
- After substitution, separate the variables and integrate. For real logarithms the original equation requires $u > 0$ and $u \neq 1$, since $\ln u$ is a denominator.

Step 1: Combine the derivative terms Rearrange the given equation without changing its sign:

$$\frac{dy}{dx} + 3 = \frac{y+3x}{\ln(y+3x)}.$$

Put $u = y + 3x$. Then

$$\frac{du}{dx} = \frac{u}{\ln u}, \qquad dx = \frac{\ln u}{u}\, du.$$

This substitution removes the explicit appearance of both x and y from the right-hand side.

Step 2: Integrate and restore the original variables Use $v = \ln u$, so $dv = du/u$. Thus

$$x = \int \frac{\ln u}{u}\, du = \frac{1}{2}(\ln u)^2 + C_1.$$

Moving the logarithmic term to the left and renaming the arbitrary constant gives

$$x - \frac{1}{2}\left[\ln(y + 3x)\right]^2 = C.$$

As a verification, differentiating this relation gives $1 - (\ln u/u)u' = 0$, and hence $u' = u/\ln u$, exactly the transformed equation. Option B instead integrates a logarithm of x, which is not the separated variable. Options C and D have only a first power of the logarithm; differentiating them produces $u' = u$ or $u' = u/2$, respectively, so neither satisfies the required equation.

Step 3: Conclusion On intervals where $y+3x > 0$ and $y+3x \neq 1$, the solution is $x-\frac{1}{2}[\ln(y+3x)]^2 = C$. Hence, the correct option is (A).

Quick Tip to Solve: When $dy/dx + a$ occurs with $y + ax$, substitute the entire combination before separating variables.

8. The minimum value of $2^{\sin x} + 2^{\cos x}$ is : [4 marks]

(A) $2^{-1+\sqrt{2}}$

(B) $2^{1-\frac{1}{\sqrt{2}}}$

(C) $2^{1-\sqrt{2}}$

(D) $2^{-1+\frac{1}{\sqrt{2}}}$

Correct Answer: (B) $2^{1-\frac{1}{\sqrt{2}}}$

Solution:

Concept:

- For positive numbers a, b, the arithmetic-geometric mean inequality gives $a + b \geq 2\sqrt{ab}$, with equality exactly when $a = b$. Both exponential terms here are positive.
- The identity $\sin x + \cos x = \sqrt{2}\sin(x + \pi/4)$ gives the lower bound $-\sqrt{2}$. Since the base 2 exceeds 1, its exponential is increasing.

Step 1: Bound the sum using its product Apply AM-GM to $a = 2^{\sin x}$ and $b = 2^{\cos x}$:

$$2^{\sin x} + 2^{\cos x} \geq 2\sqrt{2^{\sin x + \cos x}} = 2^{1+(\sin x+\cos x)/2}.$$

Now $\sin x + \cos x \geq -\sqrt{2}$. Increasingness of the exponential therefore gives

$$2^{1+(\sin x+\cos x)/2} \geq 2^{1-\sqrt{2}/2} = 2^{1-1/\sqrt{2}}.$$

Both inequalities have the same direction; no independent minima of sine and cosine have been assumed.

Step 2: Check attainability and compare the options A lower bound is a minimum only when it can

be attained. Choose $x = 5\pi/4$, or add any integral multiple of 2π. Then

$$\sin x = \cos x = -\frac{1}{\sqrt{2}}.$$

The AM-GM equality condition and the lower bound for their sum hold simultaneously, and

$$2^{\sin x} + 2^{\cos x} = 2 \cdot 2^{-1/\sqrt{2}} = 2^{1-1/\sqrt{2}}.$$

Option A has exponent $\sqrt{2} - 1$, larger than $1 - 1/\sqrt{2}$, so it is not the attained minimum. Options C and D have negative exponents and lie below the proved lower bound. Only option B agrees with both the inequality and its equality case.

Step 3: Conclusion The minimum is $2^{1-1/\sqrt{2}}$, attained when $\sin x = \cos x = -1/\sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: After using AM-GM, verify that its equality condition and the trigonometric bound occur at the same angle.

9. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws total a of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is : [4 marks]

(A) $\frac{5}{6}$
(B) $\frac{5}{31}$
(C) $\frac{31}{61}$
(D) $\frac{30}{61}$

Correct Answer: (D) $\frac{30}{61}$

Solution:

Concept:

- Two fair dice have 36 equally likely ordered outcomes. A total of 6 has five favourable outcomes, while a total of 7 has six.
- If neither player wins in a complete pair of turns, the game returns to its initial state. Independence allows the probabilities for that pair to be multiplied.

Step 1: Find the success and restart probabilities Player A wins on a turn with probability

$$p = \frac{5}{36},$$

from $(1,5), (2,4), (3,3), (4,2), (5,1)$. Player B succeeds with probability

$$q = \frac{6}{36} = \frac{1}{6},$$

from the six ordered pairs whose sum is 7. A complete unsuccessful cycle consists first of A failing, then B failing. Consequently its probability is

$$r = (1-p)(1-q) = \frac{31}{36} \cdot \frac{5}{6} = \frac{155}{216}.$$

A success by A ends the game before B can throw, so B's failure is not an additional factor in A's immediate winning probability.

Step 2: Sum all ways A can eventually win A can win on the first turn, after one failed cycle, after two failed cycles, and so on. These events are disjoint, giving

$$P_A = p + rp + r^2p + \cdots = \frac{p}{1-r}.$$

Since $0 < r < 1$, the geometric series converges. Substituting the values,

$$P_A = \frac{5/36}{1 - 155/216} = \frac{5/36}{61/216} = \frac{30}{61}.$$

The complementary probability of B winning is $((31/36)(1/6))/(1-r) = 31/61$, so the two sum to 1 and the chance of indefinite play is zero. Thus option C represents B's probability, not A's. Option A is B's failure probability on a single turn; option B does not include the correct repeated-cycle factor.

Step 3: Conclusion The probability that A wins the game is $30/61$. Hence, the correct option is (D).

Quick Tip to Solve: For an alternating game, write one complete unsuccessful cycle first; then sum the geometric series for the starting player.

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10. If the perpendicular bisector of the line segment joining the points P(1 ,4) and Q(k, 3) has y-intercept equal to –4, then a value of k is : [4 marks]

(A) $\sqrt{14}$

(B) -4

(C) –2

(D) $\sqrt{15}$

Correct Answer: (B) -4

Solution:

Concept:

- Every point on the perpendicular bisector of a segment is equidistant from its two endpoints. This gives an equation using squared distances without finding a slope.
- A y-intercept of -4 means that $R = (0, -4)$ belongs to the bisector. Equate RP^2 and RQ^2 to determine the unknown coordinate.

Step 1: Use the known point on the bisector The line in the question crosses the y-axis at $R = (0, -4)$.

For $P = (1, 4)$, the squared distance is

$$RP^2 = (1-0)^2 + \big(4-(-4)\big)^2 = 1 + 64 = 65.$$

For $Q = (k, 3)$, it is

$$RQ^2 = (k-0)^2 + \big(3-(-4)\big)^2 = k^2 + 49.$$

Because R lies on the perpendicular bisector, these squared distances must be equal. Squaring creates no sign issue because distances are nonnegative.

Step 2: Solve for k and check the listed choices Equating the two expressions gives

$$k^2 + 49 = 65, \qquad k^2 = 16, \qquad k = \pm 4.$$

The question asks for a value, rather than all values, and only -4 occurs in the options. Check it geometrically: with $k = -4$, the segment slope is $(3-4)/(-4-1) = 1/5$, so the bisector slope is -5. Its midpoint is $(-3/2, 7/2)$, yielding

$$y - \frac{7}{2} = -5\left(x + \frac{3}{2}\right), \qquad y = -5x - 4.$$

Its y-intercept is indeed -4. The squared values of the other choices are 14, 4, and 15, respectively; none equals 16, so none makes the known intercept equidistant from the endpoints.

Step 3: Conclusion A permitted value is $k = -4$; the other mathematical value is $+4$, which is not listed. Hence, the correct option is (B).

Quick Tip to Solve: If a bisector passes through a specified point, equate that point's squared distances to the endpoints instead of calculating slopes.

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11. The integral

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3 x . \sin^2 3x \left(2\sec^2 x . \sin^2 3x + 3\tan x . \sin 6x\right) dx$$

is equal to: [4 marks]

(A) $-\frac{1}{9}$
(B) $-\frac{1}{18}$
(C) $\frac{7}{18}$
(D) $\frac{9}{2}$

Correct Answer: (B) $-\frac{1}{18}$

Solution:

Concept:

- A product of powers of $\tan x$ and $\sin 3x$ suggests applying the product and chain rules to $\tan^4 x \sin^4(3x)$.
- Use $\sin 6x = 2\sin 3x \cos 3x$. This converts the mixed sine term into the derivative of a power of $\sin 3x$.

Step 1: Identify the exact derivative Define $F(x) = \tan^4 x \sin^4(3x)$. Differentiate both factors:

$$\begin{aligned} F'(x) &= 4\tan^3 x \sec^2 x \sin^4(3x) \\ &\quad + 12\tan^4 x \sin^3(3x)\cos 3x \\ &= 4\tan^3 x \sec^2 x \sin^4(3x) + 6\tan^4 x \sin^2(3x)\sin 6x. \end{aligned}$$

The integrand expands to $2\tan^3 x \sec^2 x \sin^4(3x) + 3\tan^4 x \sin^2(3x)\sin 6x$, so it equals $F'(x)/2$. The coefficient one-half multiplies both terms, an essential check before evaluating the endpoints.

Step 2: Apply the limits and compare the signs The function and its derivative are continuous throughout the integration interval, since $\cos x \neq 0$ there. The fundamental theorem gives

$$I = \frac{1}{2}\big[F(\pi/3) - F(\pi/6)\big].$$

At the upper limit, $\sin(3\pi/3) = \sin\pi = 0$, so $F(\pi/3) = 0$. At the lower limit,

$$F(\pi/6) = \left(\frac{1}{\sqrt{3}}\right)^4 \sin^4(\pi/2) = \frac{1}{9}.$$

Therefore $I = \frac{1}{2}(0 - 1/9) = -1/18$. Option A omits the one-half factor. Options C and D have the wrong positive sign as well as the wrong magnitude; the endpoint difference is negative, even though not every part of the original integrand has an immediately obvious sign.

Step 3: Conclusion The definite integral equals $-1/18$. Hence, the correct option is (B).

Quick Tip to Solve: For a product with two derivative-like terms, differentiate the product of the corresponding powers and compare every coefficient.

12. The function

$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1}x, & |x| \le 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$ **is :** [4 marks]

(A) continuous on R–{–1} and differentiable on R–{–1, 1}

(B) both continuous and differentiable on R–{1}

(C) both continuous and differentiable on R–{–1}

(D) continuous on R–{1} and differentiable on R–{–1, 1}

Correct Answer: (D) continuous on R–{1} and differentiable on R–{–1, 1}

Solution:

Concept:

- For a piecewise function, check continuity at each joining point before comparing one-sided derivatives. A jump discontinuity prevents differentiability.
- For $|x| > 1$, the outer formula has derivative $-1/2$ on $x < -1$ and $+1/2$ on $x > 1$. Inside, the derivative is $1/(1 + x^2)$.

Step 1: Check the function at the two joins At $x = -1$, the middle formula gives

$$f(-1) = \frac{\pi}{4} + \arctan(-1) = 0.$$

The left-hand limit from $(|x| - 1)/2$ is also 0, and the right-hand limit from the middle formula is 0. Thus the function is continuous at -1. At $x = 1$,

$$f(1) = \frac{\pi}{4} + \arctan 1 = \frac{\pi}{2}.$$

The left-hand limit is $\pi/2$, whereas the right-hand limit from the outer formula is 0. Hence there is a jump at 1. Away from these joins, the relevant formula is continuous; the absolute-value cusp at 0 is irrelevant because the outer formula is not used there.

Step 2: Compare derivatives at the continuous join At -1, use the continuous value 0 when forming the one-sided derivatives. From the outer branch the left derivative is $-1/2$. From the inner branch the right derivative is

$$\frac{1}{1 + (-1)^2} = \frac{1}{2}.$$

They differ, so f is not differentiable at -1. At 1 it is already discontinuous, so differentiability is impossible there. At every other real point each local branch is differentiable. Options A and C incorrectly claim continuity excludes -1, although that join is continuous and 1 is not. Option B wrongly asserts differentiability at -1. Only D gives the two exceptional sets correctly.

Step 3: Conclusion The function is continuous on $\mathbb{R} \setminus \{1\}$ and differentiable on $\mathbb{R} \setminus \{-1, 1\}$. Hence, the correct option is (D).

Quick Tip to Solve: At a piecewise join, match function values first; then check slopes only at joins that are continuous.

13. Let $f:(0,\infty)\to(0,\infty)$ be a differentiable function such that f(1) = e and $\lim\limits_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$. If f(x) = 1, then x is equal to : [4 marks]

(A) $\frac{1}{e}$

(B) e

(C) $\frac{1}{2e}$

(D) 2e

Correct Answer: (A) $\frac{1}{e}$

Solution:

Concept:

- The given limit is a derivative with respect to t; throughout that limit, x and $f(x)$ are constants.
- For a positive differentiable function, $f'(x)/f(x)$ is the derivative of $\ln f$. The initial value fixes the constant of integration.

Step 1: Evaluate the limiting difference quotient For fixed $x>0$, define $g(t)=t^2f^2(x)-x^2f^2(t)$. Since $g(x)=0$, the stated limit is $g'(x)$. Differentiating with respect to t gives

$$g'(t)=2tf^2(x)-2x^2f(t)f'(t).$$

Consequently the zero-limit condition becomes

$$2xf^2(x)-2x^2f(x)f'(x)=0.$$

Both x and $f(x)$ are positive, so division by $2xf(x)$ is legitimate. This gives $xf'(x)=f(x)$.

Step 2: Separate variables and impose the initial value The differential equation can be written

$$\frac{f'(x)}{f(x)}=\frac{1}{x}.$$

Integrating on the connected interval $(0,\infty)$ gives $\ln f(x)=\ln x+C$, or $f(x)=Ax$, with $A>0$. Since $f(1)=e$, we obtain $A=e$. Thus $f(x)=ex$, and the required condition $f(x)=1$ yields $ex=1$. As a check on the other options, $x=e,1/(2e),2e$ give $f(x)=e^2,1/2,2e^2$, respectively; none equals one.

Step 3: Conclusion Therefore $x=1/e$. Hence, the correct option is (A).

Quick Tip to Solve: When a limit contains $f(x)$ and $f(t)$, hold x fixed while differentiating with respect to t; only then set $t=x$.

14. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$ where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to : [4 marks]

(A) 30

(B) 50

(C) 15

(D) 45

Correct Answer: (A) 30

Solution:

Concept:

- Count incidences (u, X_i) with $u \in X_i$ in two ways: by the sets and by the elements of their union.
- If every element belongs to exactly r sets, then the sum of their cardinalities equals r times the cardinality of the union. Overlaps must be counted with their stated multiplicity.

Step 1: Count memberships in the first family Write $|T| = m$. There are 50 sets X_i, each containing 10 elements, so counting the pairs (u, i) with $u \in X_i$ by the index gives

$$\sum_{i=1}^{50} |X_i| = 50 \times 10 = 500.$$

The same count by elements of T gives $20m$, since each element occurs in exactly 20 of these sets. Therefore

$$20m = 500 \quad \Rightarrow \quad m = 25.$$

The union thus has 25 distinct elements; 500 is the number of memberships, not the number of distinct elements.

Step 2: Count memberships in the second family Each of the n sets Y_i contains five elements. Their total number of incidences is therefore $5n$. Every one of the same 25 elements belongs to exactly six of the Y_i, so the element-wise count is

$$5n = 6|T| = 6 \times 25 = 150.$$

Dividing by five gives $n = 30$. To test the alternatives, $n = 50, 15, 45$ would give total incidence counts $250, 75, 225$, respectively. Each differs from the required 150. No assumption that the sets are disjoint has been made; indeed their stipulated repeated memberships are essential to the count.

Step 3: Conclusion The required number of sets is $n = 30$. Hence, the correct option is (A).

Quick Tip to Solve: For a uniform membership problem, use (number of sets)(size per set) = (multiplicity)(union size) separately for each family.

15. Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If P(1, β), $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is : [4 marks]

(A) 4x – 3y = 2

(B) 8x – 2y = 5

(C) 7x – 4y = 1

(D) 4x – 2y = 1

Correct Answer: (D) 4x – 2y = 1

Solution:

Concept:

- For an ellipse centred at the origin with its major axis along the x-axis, the directrices are $x = \pm a/e$, and $b^2 = a^2(1 - e^2)$.
- Implicit differentiation of $x^2/a^2 + y^2/b^2 = 1$ gives tangent slope $-b^2x/(a^2y)$; the normal slope is its negative reciprocal when the tangent slope is nonzero.

Step 1: Determine the ellipse and the point The given directrix is vertical, so use the horizontal-major-axis form. From $a/e = 4$ and $e = 1/2$,

$$a = 2, \qquad a^2 = 4, \qquad b^2 = 4\left(1 - \frac{1}{4}\right) = 3.$$

The ellipse is therefore

$$\frac{x^2}{4} + \frac{y^2}{3} = 1.$$

Since $P = (1, \beta)$, substitution gives $1/4 + \beta^2/3 = 1$, hence $\beta^2 = 9/4$. The condition $\beta > 0$ selects $\beta = 3/2$, rather than the lower point on the ellipse.

Step 2: Find the normal and check the options Differentiate the ellipse:

$$\frac{x}{2} + \frac{2y}{3}\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = -\frac{3x}{4y}.$$

At $P = (1, 3/2)$, the tangent slope is $-1/2$; hence the normal slope is 2. Its equation is

$$y - \frac{3}{2} = 2(x - 1) \quad \Rightarrow \quad 4x - 2y = 1.$$

Option (A) has slope $4/3$, option (B) slope 4, and option (C) slope $7/4$; none is perpendicular to a tangent of slope $-1/2$. Option (D) has slope 2 and passes through the required point, satisfying both conditions.

Step 3: Conclusion The normal at P is $4x - 2y = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Use the sign condition on β before finding the slope: the upper and lower ellipse points have normals with different slopes.

16. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is : [4 marks]

(A) 330

(B) 792

(C) 252

(D) 462

Correct Answer: (D) 462

Solution:

Concept:

- For consecutive binomial coefficients, $\binom{m}{r+1}/\binom{m}{r} = (m-r)/(r+1)$. This determines the exponent from consecutive coefficient ratios.
- The coefficients of $(1+x)^m$ increase towards the middle and then decrease symmetrically. For odd m, the two central coefficients are equal maxima.

Step 1: Recover the exponent from the ratio Put $m = n + 5$, and let the three coefficients be $\binom{m}{r}, \binom{m}{r+1}, \binom{m}{r+2}$, in their printed order. The ratio $5 : 10 : 14$ gives

$$\frac{m-r}{r+1} = 2, \qquad \frac{m-r-1}{r+2} = \frac{7}{5}.$$

The first equality yields $m = 3r + 2$. Substitute this in the second equality:

$$5(2r+1) = 7(r+2), \qquad 10r + 5 = 7r + 14.$$

It follows that $r = 3$, $m = 11$, and $n = 6$, which is a positive integer as required. The check $\binom{11}{3} : \binom{11}{4} : \binom{11}{5} = 165 : 330 : 462 = 5 : 10 : 14$ verifies the chosen positions.

Step 2: Evaluate the central coefficient For exponent 11 the middle coefficients are $\binom{11}{5}$ and $\binom{11}{6}$, and they are equal. Their value is

$$\binom{11}{5} = \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} = 462.$$

For completeness, the successive ratio $(11-k)/(k+1)$ is greater than one before the middle, equals one at $k = 5$, and is less than one after it. Thus this is a maximum, not merely one possible coefficient. The value 330 is a noncentral coefficient; 792 and 252 are not the maximal coefficients for exponent 11 and fail the recovered-exponent condition.

Step 3: Conclusion The largest coefficient is 462. Hence, the correct option is (D).

Quick Tip to Solve: Use ratios of adjacent binomial coefficients before computing any factorials; two consecutive ratios usually determine both the exponent and the position.

17. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°,then PC (in m) is equal to : [4 marks]

(A) $200\sqrt{3}$

(B) 400

(C) 100

(D) $400\sqrt{3}$

Correct Answer: (B) 400

Solution:

Concept:

- A still lake forms a plane-mirror image: a cloud at height h above the surface has an image h below it, at the same horizontal position.
- The tangent of an elevation or depression angle is the vertical separation divided by the horizontal distance. The sine of the elevation angle connects the cloud-observer separation to the sight-line length.

Step 1: Express both angles using one horizontal distance Let the cloud be $h\,\mathrm{m}$ above the lake and let $d\,\mathrm{m}$ be its horizontal separation from P. Since P is 200 m above the lake, its vertical separation from the cloud is $h-200$, whereas its vertical separation from the cloud's image is $h+200$. The cloud is seen above the horizontal, so $h>200$. The two angles give

$$\frac{h-200}{d}=\tan 30^\circ=\frac{1}{\sqrt{3}},\qquad \frac{h+200}{d}=\tan 60^\circ=\sqrt{3}.$$

The different signs of 200 reflect the fact that the image lies below the water surface.

Step 2: Solve for the height and the sight-line length Dividing the second equation by the first eliminates d:

$$\frac{h+200}{h-200}=3,\qquad h+200=3h-600.$$

Thus $h=400\,\mathrm{m}$, so the cloud is 200 m above P. In the elevation triangle,

$$\sin 30^\circ=\frac{h-200}{PC}=\frac{200}{PC}\quad\Rightarrow\quad PC=400\,\mathrm{m}.$$

As an independent check, $d=200\sqrt{3}\,\mathrm{m}$ and $PC^2=d^2+200^2=160000\,\mathrm{m}^2$. Option (A) is the horizontal distance, not the slant distance. Options (C) and (D) disagree with this right-triangle calculation and therefore cannot be PC.

Step 3: Conclusion The distance is $PC=400\,\mathrm{m}$. Hence, the correct option is (B).

Quick Tip to Solve: For an observer above a reflecting lake, use $h-H$ for the real cloud and $h+H$ for its image; then distinguish horizontal distance from sight-line distance.

18. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point : [4 marks]

(A) (–3, 1)

(B) (1, –3)

(C) (–1, 3)

(D) (–3, 6)

Correct Answer: (D) (–3, 6)

Solution:

Concept:

- If two circles are $S_1 = 0$ and $S_2 = 0$, then $S_1 + \lambda S_2 = 0$ passes through their common points. Exclude a parameter value that removes the quadratic terms when a circle is required.
- For $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$. Substituting a point into the final equation tests whether it belongs to the circle.

Step 1: Write the family and locate its centre Using the two printed circles, write

$$(x^2 + y^2 - 6x) + \lambda(x^2 + y^2 - 4y) = 0.$$

For $\lambda \neq -1$, divide by $1 + \lambda$ to obtain a circle whose centre is

$$\left(\frac{3}{1+\lambda}, \frac{2\lambda}{1+\lambda}\right).$$

The excluded value gives a straight line, not a member of the required circle family. Inserting the centre into the given line yields

$$\frac{6}{1+\lambda} - \frac{6\lambda}{1+\lambda} + 12 = 0.$$

Multiplication by $1 + \lambda$ gives $6 - 6\lambda + 12 + 12\lambda = 0$, so $\lambda = -3$.

Step 2: Find the circle and test all proposed points At $\lambda = -3$, the family equation becomes

$$-2(x^2 + y^2) - 6x + 12y = 0, \qquad x^2 + y^2 + 3x - 6y = 0.$$

Its centre is $(-3/2, 3)$, which satisfies $2x - 3y + 12 = -3 - 9 + 12 = 0$. Evaluate the left side of the circle equation at each option:

$$(-3, 1): \ 10 - 9 - 6 = -5,$$
$$(1, -3): \ 10 + 3 + 18 = 31,$$
$$(-1, 3): \ 10 - 3 - 18 = -11,$$
$$(-3, 6): \ 45 - 9 - 36 = 0.$$

Only the last point gives zero, so it lies on the circle. This explicit substitution also rules out every competing option without relying on the position of the point in a sketch.

Step 3: Conclusion The circle passes through $(-3, 6)$. Hence, the correct option is (D).

Quick Tip to Solve: After using the centre condition to fix a circle family, substitute each candidate point into the normalized equation; only a zero residual indicates membership.

19. The distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is : [4 marks]

(A) 7

(B) 1

(C) $\frac{1}{7}$

(D) $\frac{7}{5}$

Correct Answer: (B) 1

Solution:

Concept:

- A line through a point parallel to a given line has the same direction vector. Here the direction ratios are $(2, 3, -6)$.
- To measure distance in that direction, first intersect this parallel line with the plane. The required distance is the length of the displacement, not the perpendicular distance to the plane.

Step 1: Write the line through the given point Let $P = (1, -2, 3)$ and let the line parameter be t. A general point on the required line is

$$R = (1 + 2t, -2 + 3t, 3 - 6t).$$

At $t = 0$ this is the given point; increasing t moves along the direction specified in the question. Both signs of the parameter are allowed because distance is a nonnegative length.

Step 2: Locate its intersection with the plane Substitute the coordinates of R into $x - y + z = 5$:

$$1 + 2t - (-2 + 3t) + 3 - 6t = 5.$$

Thus $6 - 7t = 5$, giving $t = 1/7$. The displacement from P to the intersection is therefore $(2/7, 3/7, -6/7)$.

Step 3: Calculate the length and compare the options The direction vector has magnitude $\sqrt{2^2 + 3^2 + (-6)^2} = 7$. Hence

$$PR = |t|\sqrt{49} = \frac{1}{7} \times 7 = 1.$$

Option (C), $1/7$, is only the parameter, not a distance. Option (A) is the length of the direction vector before multiplying by the parameter; option (D) is not obtained by the intersection calculation.

Step 4: Conclusion The distance measured parallel to the given line is $\boxed{1 \text{ unit}}$. Hence, the correct option is (B).

Quick Tip to Solve: After finding the line parameter, multiply its absolute value by the magnitude of the direction vector; the parameter alone is generally not the distance.

20. Contrapositive of the statement :

'If a function f is differentiable at a, then it is also continuous at a', is: [4 marks]

(A) If a function f is continuous at a, then it is not differentiable at a.

(B) If a function f is not continuous at a, then it is differentiable at a.

(C) If a function f is not continuous at a, then it is not differentiable at a.

(D) If a function f is continuous at a, then it is differentiable at a.

Correct Answer: (C) If a function f is not continuous at a, then it is not differentiable at a.

Solution:

Concept:

- For an implication $p \Rightarrow q$, the contrapositive is $\neg q \Rightarrow \neg p$: interchange the two clauses and negate both.
- A statement and its contrapositive are logically equivalent. The converse $q \Rightarrow p$ and inverse $\neg p \Rightarrow \neg q$ need not follow from the original statement.

Step 1: Identify the hypothesis and conclusion Let p mean that f is differentiable at a, and let q mean that f is continuous at a. The given sentence is precisely $p \Rightarrow q$. Its hypothesis concerns differentiability, while its conclusion concerns continuity, both at the same point.

Step 2: Negate and reverse the clauses The negation $\neg q$ says that f is not continuous at a. The negation $\neg p$ says that f is not differentiable at a. Therefore the required implication is: if a function is not continuous at a, then it is not differentiable at a. This agrees with the calculus fact that differentiability necessarily implies continuity.

Step 3: Check all the offered statements Option (C) has exactly the form $\neg q \Rightarrow \neg p$. Option (D), continuity implying differentiability, is the converse and can fail, for example for $f(x) = |x|$ at zero. Option (A) says $q \Rightarrow \neg p$, which incorrectly excludes functions that are both continuous and differentiable. Option (B) says $\neg q \Rightarrow p$, contradicting the necessity of continuity for differentiability.

Step 4: Conclusion The contrapositive is: If a function f is not continuous at a, then it is not differentiable at a. Hence, the correct option is (C).

Quick Tip to Solve: Label the two clauses before transforming the sentence: contrapositive means reverse their order and negate each one.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If the variance of the following frequency distribution :

Class : 10–20 20–30 30–40

Frequency : 2 x 2

is 50, then x is equal to____ [4 marks]

Solution:

Concept:

- For grouped data, represent each class by its midpoint. The midpoints of 10–20, 20–30 and 30–40 are $15, 25, 35$, respectively.
- Variance is the frequency-weighted mean square deviation from the mean: $\sigma^2 = \sum f_i(m_i - \bar{m})^2 / \sum f_i$. Frequencies, unlike the observations, belong in the weights.

Step 1: Find the total frequency and mean The frequencies are $2, x, 2$, so $N = x + 4$. The equal outer frequencies are placed symmetrically around the central midpoint 25. Directly,

$$\bar{m} = \frac{2(15) + 25x + 2(35)}{x + 4} = \frac{100 + 25x}{x + 4} = 25.$$

Thus the unknown central frequency does not change the mean. As a frequency, the answer must be nonnegative.

Step 2: Use the given variance The deviations of the midpoints from 25 are $-10, 0, 10$, with squared deviations $100, 0, 100$. Therefore

$$\sigma^2 = \frac{2(100) + x(0) + 2(100)}{x + 4} = \frac{400}{x + 4}.$$

The middle class increases the total frequency without contributing to the numerator. Equating this expression to the stated variance gives $400/(x + 4) = 50$.

Step 3: Solve and verify the frequency Since $x + 4 > 0$, multiplication gives $400 = 50x + 200$. Hence $50x = 200$ and $x = 4$. With frequencies $2, 4, 2$, the total is 8, the mean is still 25, and the weighted sum of squared deviations is 400. The resulting variance $400/8 = 50$ exactly reproduces the supplied value.

Step 4: Conclusion The unknown frequency is $\boxed{x = 4}$. This is a count and therefore has no physical unit.

Quick Tip to Solve: When equal outer frequencies lie at equal distances from the middle midpoint, use that midpoint as the mean before forming the variance.

22. Let {x} and [x] denote the fractional part of x and the greatest integer $\leq$ x respectively of a real number x. If $\int_0^n \{x\}\,dx, \int_0^n [x]\,dx$ and 10(n^2 – n), $(n \in N, n > 1)$ are three consecutive terms of a G.P., then n is equal to_____. [4 marks]

Solution:

Concept:

- On every unit interval $[k, k+1)$, $[x] = k$ and $\{x\} = x - k$. Values at isolated integer endpoints do not change either definite integral.
- For three consecutive positive terms A, B, C of a geometric progression, the necessary and sufficient condition is $B^2 = AC$. Here n is a natural number greater than one.

Step 1: Integrate the fractional part over the unit intervals For each integer $k = 0, 1, \ldots, n-1$, substitute $u = x - k$:

$$\int_k^{k+1} \{x\}\,dx = \int_0^1 u\,du = \frac{1}{2}.$$

There are n such intervals, so the first term is $A = n/2$. This uses the repeated triangular area of the fractional-part function rather than treating that function as x across the whole interval.

Step 2: Integrate the greatest integer function On the same interval its constant value is k, so

$$B = \int_0^n [x]\,dx = \sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}.$$

The third term given in the question is $C = 10(n^2 - n) = 10n(n-1)$. All three terms are positive for the allowed n, so cancellation by n and $n-1$ is legitimate.

Step 3: Apply the progression condition and check Use $B^2 = AC$:

$$\frac{n^2(n-1)^2}{4} = \frac{n}{2}\,10n(n-1) = 5n^2(n-1).$$

Dividing by $n^2(n-1)/4$ yields $n - 1 = 20$, hence $n = 21$. The actual terms are then $21/2, 210, 4200$; their successive ratios are both 20. The algebraic roots 0 and 1 from an uncancelled equation do not satisfy the given domain.

Step 4: Conclusion The required natural number is $\boxed{n = 21}$, a dimensionless value.

Quick Tip to Solve: For integer upper limits, calculate fractional-part integrals one unit interval at a time; then use the square-of-the-middle-term test for a G.P.

23. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of
$\left|\hat{i} \times \left(\vec{a} \times \hat{i}\right)\right|^2 + \left|\hat{j} \times \left(\vec{a} \times \hat{j}\right)\right|^2 + \left|\hat{k} \times \left(\vec{a} \times \hat{k}\right)\right|^2$ **is equal to____** [4 marks]

Solution:

Concept:

- The vector triple-product identity is $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{w}(\mathbf{u} \cdot \mathbf{v})$. The order of the vectors matters.
- The Cartesian unit vectors form an orthonormal basis, so the squared magnitude of $a\hat{i} + b\hat{j} + c\hat{k}$ is $a^2 + b^2 + c^2$. Perpendicular components have no cross terms.

Step 1: Find the first double cross product Take $\mathbf{u} = \mathbf{w} = \hat{i}$ and $\mathbf{v} = \vec{a}$. The identity gives

$$\hat{i} \times (\vec{a} \times \hat{i}) = \vec{a} - 2\hat{i} = \hat{j} + 2\hat{k}.$$

Its squared magnitude is $1^2 + 2^2 = 5$. The result removes the component of $\vec{a}$ parallel to the unit vector and retains the two perpendicular components.

Step 2: Evaluate the other two terms For the $\hat{j}$ expression, $\vec{a} \cdot \hat{j} = 1$, giving

$$\hat{j} \times (\vec{a} \times \hat{j}) = \vec{a} - \hat{j} = 2\hat{i} + 2\hat{k}.$$

Its squared magnitude is $4 + 4 = 8$. Similarly,

$$\hat{k} \times (\vec{a} \times \hat{k}) = \vec{a} - 2\hat{k} = 2\hat{i} + \hat{j},$$

whose squared magnitude is $4 + 1 = 5$.

Step 3: Add and independently check the sum The required sum is therefore $5 + 8 + 5 = 18$. As a check, each squared Cartesian component of $\vec{a}$ occurs in exactly two of these three projections. Their sum must be $2|\vec{a}|^2 = 2(2^2 + 1^2 + 2^2) = 2(9) = 18$, agreeing with the direct computation. Squared magnitudes must be added here; squaring the sum of the three vectors would be a different expression.

Step 4: Conclusion The value of the stated expression is $\boxed{18}$, dimensionless for the vector as given.

Quick Tip to Solve: For a unit vector $\mathbf{e}$, replace $\mathbf{e} \times (\vec{a} \times \mathbf{e})$ by the part of $\vec{a}$ perpendicular to $\mathbf{e}$.

24. Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line,
$x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is _____. [4 marks]

Solution:

Concept:

- Because the circle is centred at the origin, endpoints of a diameter are opposite points: write $P = (u, v)$ and $Q = (-u, -v)$, with $u^2 + v^2 = 9$.
- The perpendicular distance from (x_0, y_0) to $x + y - 2 = 0$ is $|x_0 + y_0 - 2|/\sqrt{2}$. Distances are

nonnegative, so absolute values cannot be omitted.

Step 1: Express the product using one variable Put $s = u + v$. The two distances are

$$\alpha = \frac{|s-2|}{\sqrt{2}}, \qquad \beta = \frac{|-s-2|}{\sqrt{2}} = \frac{|s+2|}{\sqrt{2}}.$$

Consequently

$$\alpha\beta = \frac{|(s-2)(s+2)|}{2} = \frac{|s^2-4|}{2}.$$

This formula remains valid whether the two endpoints lie on the same side or opposite sides of the line.

Step 2: Determine the full allowed range By $(u+v)^2 \le 2(u^2+v^2)$, we have $0 \le s^2 \le 18$. The lower endpoint occurs when $u = -v$; the upper endpoint occurs when $u = v = \pm 3/\sqrt{2}$. Thus both endpoint values are actually attained on the given circle, rather than being unattainable bounds.

Step 3: Maximise and check the product Let $z = s^2$. On $0 \le z \le 4$, the product is $(4-z)/2$, decreasing from 2 to zero. On $4 \le z \le 18$, it is $(z-4)/2$, increasing up to $(18-4)/2 = 7$. Hence the larger endpoint value is 7. At $P = (3/\sqrt{2}, 3/\sqrt{2})$ and its opposite endpoint, $s = 3\sqrt{2}$, and direct substitution gives the same product.

Step 4: Conclusion The maximum product is $\boxed{\alpha\beta = 7 \text{ square units}}$.

Quick Tip to Solve: For opposite points on a circle, express both line distances through the same linear coordinate and retain the absolute value before maximising.

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25. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is _____. [4 marks]

Solution:

Concept:

- Choose the positions of the four correct responses using a combination: $\binom{n}{r} = n!/[r!(n-r)!]$. The questions are distinguishable, so different sets of correct positions are different answer patterns.
- Each question has one correct alternative and three incorrect alternatives. After the positions are fixed, apply the multiplication principle to the independent choices for the two incorrect responses.

Step 1: Choose the questions answered correctly Exactly four out of six questions must be correct. Choose their positions without ordering them, because the candidate submits one response at each fixed question position. The number of choices is

$$\binom{6}{4} = \frac{6!}{4!2!} = 15.$$

The four correct responses are then forced: each of these questions has only one correct alternative. Therefore no extra factor of four is attached to these selected positions.

Step 2: Count the incorrect responses at the other positions The remaining two questions must both be answered incorrectly. For either such question there are three wrong alternatives, and the choice at one question does not restrict the other. Consequently each chosen set of four correct positions gives

$$3 \times 3 = 3^2 = 9$$

complete response patterns. These patterns are disjoint for different choices of correct positions, because every completed answer sheet uniquely identifies its four correct positions.

Step 3: Multiply the independent counts The position choice and the wrong-answer choices jointly determine the entire answer sheet. Thus

$$N = \binom{6}{4} 1^4 3^2 = 15 \times 9 = 135.$$

Using 4^2 instead of 3^2 would permit a remaining answer to be correct, violating the word exactly. Using a permutation for the correct questions would count the same set repeatedly.

Step 4: Conclusion The required number of answer patterns is $\boxed{135}$. This is a dimensionless count, and it agrees with the numerical answer key.

Quick Tip to Solve: For exactly r correct answers among n questions with m options each, count $\binom{n}{r}(m-1)^{n-r}$, provided every question is answered.

Physics – Section A: Multiple Choice Questions (80 marks)

26. A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is : [4 marks]

(A) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$

(B) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$

(C) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$

(D) $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$

Correct Answer: (C) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$

Solution:

Concept:

- During ascent both gravity and air resistance act downward. With upward displacement y positive, Newton's law gives $dv/dt = -g - kv^2$, because the mass cancels.
- For motion described in terms of height, use $dv/dt = v\,dv/dy$. The launch condition is $v = u$ at $y = 0$, while at maximum height $v = 0$.

Step 1: Express the equation of motion in terms of height The resistive force has magnitude mkv^2

and is opposite to the upward velocity. Hence

$$m\frac{dv}{dt} = -mg - mkv^2, \qquad v\frac{dv}{dy} = -(g + kv^2).$$

Rearrange while preserving the sign of the decreasing speed:

$$dy = -\frac{v\,dv}{g + kv^2}.$$

The parameter k has dimension inverse length, so ku^2/g is dimensionless.

Step 2: Integrate between launch and the highest point The height gained is obtained by integrating the speed from u down to zero:

$$h = -\int_u^0 \frac{v\,dv}{g + kv^2} = \int_0^u \frac{v\,dv}{g + kv^2}.$$

Use $s = g + kv^2$, so that $ds = 2kv\,dv$. The limits for s are g and $g + ku^2$, giving

$$h = \frac{1}{2k}\left[\ln s\right]_g^{g+ku^2} = \frac{1}{2k}\ln\left(1 + \frac{ku^2}{g}\right).$$

Step 3: Check the form and reject the alternatives The integral contains $v\,dv$, which produces a logarithm after substitution, not an inverse tangent; options (A) and (B) therefore use the wrong integral. Option (D) has the wrong coefficient and argument. As a further check, $\ln(1 + x) \sim x$ for small x, so the derived height tends to $u^2/(2g)$ when drag vanishes. For positive drag, $\ln(1 + x) < x$, so the height is lower than in vacuum.

Step 4: Conclusion The maximum height is $\boxed{h = \frac{1}{2k}\ln(1 + ku^2/g)}$. Hence, the correct option is (C).

Quick Tip to Solve: For drag proportional to v^2, convert acceleration to $v\,dv/dy$ when the required quantity is a distance; this avoids solving for time.

27. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is: [4 marks]

(A) 2

(B) 1

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (D) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- In a circular orbit, gravity supplies the centripetal force. At orbital radius r, the resulting orbital speed is $v_o = \sqrt{GM/r}$, where G is the gravitational constant.
- Escape velocity is the minimum speed that gives zero total mechanical energy at infinity. From a planet's surface it is $v_e = \sqrt{2GM/R}$, neglecting atmospheric resistance and rotation.

Step 1: Calculate the speed in the low circular orbit Let the orbiting body have mass m. Its gravitational attraction to the planet balances the centripetal force:

$$\frac{GMm}{R^2} = \frac{mv_o^2}{R}.$$

Here the orbital radius is replaced by R as explicitly permitted in the question. Cancel m and one power of R to obtain

$$v_o^2 = \frac{GM}{R}, \qquad v_o = \sqrt{\frac{GM}{R}}.$$

The result does not depend on the mass of the orbiting body.

Step 2: Obtain the escape speed from energy conservation For minimum escape, the body reaches infinity with zero speed. Taking gravitational potential energy to be zero there, the initial total energy must also vanish:

$$\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0.$$

It follows that

$$v_e^2 = \frac{2GM}{R}, \qquad v_e = \sqrt{\frac{2GM}{R}}.$$

Escape requires more speed than a bound circular orbit, since the orbiting body still has negative total energy.

Step 3: Form the requested ratio and compare the choices The question asks for orbital speed divided by escape speed, so

$$\frac{v_o}{v_e} = \sqrt{\frac{GM/R}{2GM/R}} = \frac{1}{\sqrt{2}}.$$

Options (A), (B) and (C) give respectively 2, 1 and $\sqrt{2}$, all at least unity; none is consistent with the orbital speed being smaller than the escape speed. The reciprocal $\sqrt{2}$ would answer the reverse ratio,

not the one requested.

Step 4: Conclusion The ratio of orbital speed to escape velocity is $\boxed{1/\sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: At the same radius, escape speed is $\sqrt{2}$ times circular orbital speed. Read the order of the requested ratio before selecting its reciprocal.

28. The electric field of a plane electromagnetic wave is given by

$$\overrightarrow{E} = E_0 (\widehat{x} + \widehat{y}) \sin (kz - \omega t)$$

Its magnetic field will be given by : [4 marks]

(A) $\frac{E_0}{c} (\widehat{x} + \widehat{y}) \sin (kz - \omega t)$

(B) $\frac{E_0}{c} (\widehat{x} - \widehat{y}) \sin (kz - \omega t)$

(C) $\frac{E_0}{c} (\widehat{x} - \widehat{y}) \cos (kz - \omega t)$

(D) $\frac{E_0}{c} (-\widehat{x} + \widehat{y}) \sin (kz - \omega t)$

Correct Answer: (D) $\frac{E_0}{c} (-\widehat{x} + \widehat{y}) \sin (kz - \omega t)$

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the electric field, magnetic field and propagation direction form a right-handed system: $\mathbf{B} = (1/c)\hat{\mathbf{n}} \times \mathbf{E}$.
- The electric and magnetic fields oscillate in phase and have magnitudes related by $E = cB$. A phase $kz - \omega t$ represents propagation along positive z.

Step 1: Identify the direction of propagation Keep the phase constant: $kz - \omega t = \phi$. Then

$$z = \frac{\omega}{k} t + \frac{\phi}{k}.$$

As time increases, the constant-phase point moves towards larger z. Thus $\hat{\mathbf{n}} = \hat{\mathbf{z}}$, and the magnetic direction must be found by crossing $\hat{\mathbf{z}}$ into the electric field. Reversing this cross-product order would reverse the propagation direction.

Step 2: Evaluate the cross product Using the cyclic unit-vector rules,

$$\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}, \qquad \hat{\mathbf{z}} \times \hat{\mathbf{y}} = -\hat{\mathbf{x}}.$$

Therefore

$$\mathbf{B} = \frac{E_0}{c} \left[\hat{\mathbf{z}} \times (\hat{\mathbf{x}} + \hat{\mathbf{y}})\right] \sin(kz - \omega t)$$
$$= \frac{E_0}{c} (-\hat{\mathbf{x}} + \hat{\mathbf{y}}) \sin(kz - \omega t).$$

This vector is perpendicular to $\mathbf{E}$, and its magnitude is $\sqrt{2}E_0/c$, as required.

Step 3: Check direction and phase against the options Option (A) makes the fields parallel and

cannot represent the wave. Option (B) has the opposite magnetic direction, so its Poynting vector points along negative z. Option (C) also replaces the sine by cosine and puts the fields out of phase. For the derived field,

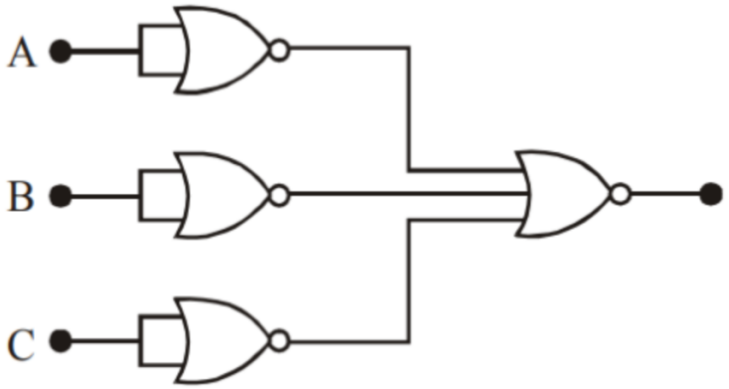
$$(\hat{\mathbf{x}} + \hat{\mathbf{y}}) \times (-\hat{\mathbf{x}} + \hat{\mathbf{y}}) = 2\hat{\mathbf{z}},$$

confirming energy propagation along the positive z-axis.

Step 4: Conclusion The magnetic field is $\boxed{\mathbf{B} = \frac{E_0}{c}(-\hat{\mathbf{x}} + \hat{\mathbf{y}})\sin(kz - \omega t)}$. Hence, the correct option is (D).

Quick Tip to Solve: First read propagation from the phase, then use $\mathbf{B} = \hat{\mathbf{n}} \times \mathbf{E}/c$. This fixes the sign without guessing from perpendicularity alone.

29. Identify the operation performed by the circuit given below : [4 marks]



(A) AND

(B) NAND

(C) OR

(D) NOT

Correct Answer: (A) AND

Solution:

Concept:

- A NOR gate gives the complement of the OR of its inputs: $Y = \overline{P + Q}$. Connecting its inputs together makes it an inverter, since $P + P = P$.
- De Morgan's theorem converts a complemented sum into a product of complements: $\overline{P + Q + R} = \bar{P}\,\bar{Q}\,\bar{R}$. Here a plus sign means logical OR and multiplication means logical AND.

Step 1: Reduce the three input gates In each of the three left-hand gates, the two inputs of a NOR symbol are tied together. The upper gate therefore receives A at both inputs, the middle gate receives B, and the lower receives C. Their outputs are

$$P = \overline{A + A} = \bar{A}, \qquad Q = \overline{B + B} = \bar{B}, \qquad R = \overline{C + C} = \bar{C}.$$

The small circles at the gate outputs signify inversion; ignoring them would incorrectly treat these gates as ordinary OR gates.

Step 2: Combine the signals in the final NOR gate The three intermediate outputs feed the final three-input NOR gate. Hence its output is

$$Y = \overline{P + Q + R} = \overline{\bar{A} + \bar{B} + \bar{C}}.$$

Apply De Morgan's theorem to the complete sum, keeping all three inversion signs:

$$Y = \overline{\bar{A}}\,\overline{\bar{B}}\,\overline{\bar{C}} = ABC.$$

Thus the output is one precisely when all three original inputs are one.

Step 3: Distinguish the possible gate operations For $A = B = C = 1$, the three first-stage outputs are zero and the final NOR output is one. If any original input is zero, its first-stage output is one, forcing the final output to zero. This is exactly the AND truth condition. NAND would give zero for all-one inputs; OR would give one even when only one input is one. NOT is a single-input inversion and cannot describe this three-input product. The circuit therefore implements an AND operation despite being built entirely from NOR gates.

Step 4: Conclusion The equivalent operation is the three-input AND function $\boxed{Y = ABC}$. Hence, the correct option is (A).

Quick Tip to Solve: Treat a tied-input NOR gate as an inverter, then apply De Morgan's theorem to the final NOR. Track each output bubble before simplifying.

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30. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2 . When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is: [4 marks]

(A) $gdS(x_2 + x_1)^2$

(B) $gdS\left(x_2^2 + x_1^2\right)$

(C) $\frac{1}{4}gdS(x_2 - x_1)^2$

(D) $\frac{3}{4}gdS(x_2 - x_1)^2$

Correct Answer: (C) $\frac{1}{4}gdS(x_2 - x_1)^2$

Solution:

Concept:

- A liquid column of uniform cross-section S, density d, and height h has mass dSh and centre of mass at $h/2$. Its gravitational potential energy is $U = dgSh^2/2$.
- At equilibrium the two open vessels have equal liquid height. Conservation of volume determines that height; mechanical energy is dissipated during the flow, so gravitational potential energy need not remain constant.

Step 1: Find the common equilibrium height The initial total volume is $Sx_1 + Sx_2$. The connecting pipe has negligible volume, so the final volume is $2Sh$, with h the common height. Therefore

$$2Sh = S(x_1 + x_2), \qquad h = \frac{x_1 + x_2}{2}.$$

Equal height follows from equal pressure at the connected bottom levels for the same liquid exposed to the same atmospheric pressure.

Step 2: Calculate the initial and final potential energies Measure potential energy from the ground. Initially, adding the energies of the two columns gives

$$U_i = \frac{dgS}{2}(x_1^2 + x_2^2).$$

Finally both columns have height h, so

$$U_f = 2\left(\frac{dgS}{2}h^2\right) = \frac{dgS}{4}(x_1 + x_2)^2.$$

The factor $1/2$ in a single-column energy is essential: the liquid is distributed from the ground to its top, rather than concentrated at the top surface.

Step 3: Find the energy decrease and identify its magnitude Subtract the final energy from the initial energy:

$$\begin{aligned} U_i - U_f &= \frac{dgS}{4}\{2x_1^2 + 2x_2^2 - (x_1 + x_2)^2\} \\ &= \frac{dgS}{4}(x_2 - x_1)^2. \end{aligned}$$

Thus the signed change $U_f - U_i$ is negative; the positive options describe the amount of energy lost. Options (A) and (B) do not vanish for equal initial heights, although no flow then occurs. Option (D) has the correct dependence on height difference but an incorrect coefficient. The lost gravitational energy is dissipated in the equilibration process.

Step 4: Conclusion The energy decreases by $\boxed{\frac{1}{4}gdS(x_2 - x_1)^2}$; equivalently $\Delta U = -\frac{1}{4}gdS(x_2 - x_1)^2$. Hence, the correct option is (C), giving the magnitude of the decrease.

Quick Tip to Solve: For liquid columns, use each centre of mass at half its height. Check the final loss by setting the initial heights equal: the answer must then vanish.

31. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is : [4 marks]

(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- For a uniform disc rotating about its central axis perpendicular to its plane, $I = \frac{1}{2}MR^2$, where M is its mass and R its radius.
- The mass of a uniform disc of density ρ and thickness t is $M = \rho\pi R^2 t$. Equal material and thickness imply equal ρ and t, not equal masses.

Step 1: Express the mass ratio using the radii The larger disc contains more material because its area is larger. With $R_1 = R$ and $R_2 = \alpha R$, the common density and thickness cancel in the ratio:

$$\frac{M_2}{M_1} = \frac{\rho\pi(\alpha R)^2 t}{\rho\pi R^2 t} = \alpha^2.$$

Thus the change in radius changes both the mass and the squared distance factor in the moment of inertia.

Step 2: Use the fourth-power dependence Substituting the mass expression into the disc formula gives $I = \frac{1}{2}\rho\pi t R^4$. Hence the given inertia ratio requires

$$\frac{I_2}{I_1} = \frac{M_2 R_2^2}{M_1 R_1^2} = \alpha^2\alpha^2 = \alpha^4 = 16.$$

Radii are positive, so $\alpha = 16^{1/4} = 2$. The values $\sqrt{2}$, $2\sqrt{2}$, and 4 would give inertia ratios 4, 64, and 256, respectively, rather than 16. Treating the two masses as equal would incorrectly produce the last value.

Step 3: Conclusion The radius ratio is $\alpha = 2$, a dimensionless number. Hence, the correct option is (B).

Quick Tip to Solve: For discs of equal material and thickness, combine $M \propto R^2$ with $I \propto MR^2$ before taking the inertia ratio: $I \propto R^4$.

32. Find the Binding energy per neucleon for $^{120}_{50}Sn$. Mass of proton m_p = 1.00783 U, mass of neutron m_n = 1.00867 U and mass of tin nucleus m_{Sn} = 119.902199 U. (take 1U = 931 MeV) [4 marks]

(A) 9.0 MeV

(B) 8.5 MeV

(C) 8.0 MeV

(D) 7.5 MeV

Correct Answer: (B) 8.5 MeV

Solution:

Concept:

- The mass defect is $\Delta m = Zm_p + Nm_n - m_{\text{nucleus}}$, where Z counts protons and $N = A - Z$ counts neutrons.
- The total binding energy is $E_b = \Delta mc^2$. Using the supplied conversion, each unit of mass defect corresponds to $931\,\text{MeV}$; divide by A to obtain energy per nucleon.

Step 1: Count the nucleons and add their free masses For tin with mass number 120 and atomic number 50, there are 50 protons and $120 - 50 = 70$ neutrons. Using exactly the masses supplied in the question,

$$Zm_p = 50(1.00783) = 50.39150\,\text{u},$$
$$Nm_n = 70(1.00867) = 70.60690\,\text{u},$$
$$m_{\text{free}} = 120.99840\,\text{u}.$$

The free-nucleon mass exceeds the bound nuclear mass; the positive difference represents the energy released on assembly.

Step 2: Convert the mass defect and divide by the nucleon count The defect and its associated total binding energy are

$$\Delta m = 120.99840 - 119.902199$$
$$= 1.096201\,\text{u},$$
$$E_b = 1.096201(931) = 1020.563131\,\text{MeV}.$$

Therefore $E_b/A = 1020.563131/120 = 8.50469\,\text{MeV}$. This is closest to $8.5\,\text{MeV}$. Neither 9.0, 8.0, nor $7.5\,\text{MeV}$ agrees with the result at the precision of the options. The division by 120, rather than by 50 or 70, is essential because every nucleon is counted.

Step 3: Conclusion The binding energy per nucleon is approximately $8.5\,\text{MeV}$. Hence, the correct option is (B).

Quick Tip to Solve: First compute the total mass defect with both proton and neutron counts; convert to energy only once, then divide by the mass number.

33. A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be: [4 marks]

(A) 4 A/m

(B) 1 A/m

(C) 0.75 A/m

(D) 2.25 A/m

Correct Answer: (C) 0.75 A/m

Solution:

Concept:

- For a paramagnetic specimen obeying Curie law, $\chi = C/T$, where χ is susceptibility, T is absolute temperature and C is the Curie constant.
- Magnetisation satisfies $M = \chi H$. For the same specimen in an applied field B, $H \simeq B/\mu_0$, so $M \propto B/T$. All temperatures must be in kelvin.

Step 1: Form a ratio that cancels the specimen constant Let subscripts 1 and 2 describe the two stated conditions. Combining the field dependence with the temperature dependence gives

$$\frac{M_2}{M_1} = \frac{B_2}{B_1}\frac{T_1}{T_2}.$$

This ratio avoids needing the Curie constant or the sample volume. The new field is smaller and the new temperature is higher, so both changes reduce the magnetisation; the result must be appreciably below $6\,\mathrm{A\,m^{-1}}$.

Step 2: Substitute both field and temperature changes Insert $B_1 = 0.4\,\mathrm{T}$, $B_2 = 0.3\,\mathrm{T}$, $T_1 = 4\,\mathrm{K}$, and $T_2 = 24\,\mathrm{K}$:

$$M_2 = 6\left(\frac{0.3}{0.4}\right)\left(\frac{4}{24}\right) = 6\left(\frac{3}{4}\right)\left(\frac{1}{6}\right) = 0.75\,\mathrm{A\,m^{-1}}.$$

The field factor is $3/4$, while thermal agitation contributes an additional factor $1/6$. Using temperature alone would give $1\,\mathrm{A\,m^{-1}}$, option B, and neglects the field reduction. The values 4 and $2.25\,\mathrm{A\,m^{-1}}$ likewise fail the combined ratio $M_2/M_1 = 1/8$.

Step 3: Conclusion The magnetisation under the second condition is $0.75\,\mathrm{A\,m^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write $M_2/M_1 = (B_2/B_1)(T_1/T_2)$ before substituting: field goes directly into the ratio, while absolute temperature goes inversely.

34. A particle of charge q and mass m is subjected to an electric field $E = E_0 (1 - ax^2)$ in the x-direction, where a and E_0 are constants. Initially the particle was at rest at x = 0. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is : [4 marks]

(A) a

(B) $\sqrt{\frac{2}{a}}$

(C) $\sqrt{\frac{3}{a}}$

(D) $\sqrt{\frac{1}{a}}$

Correct Answer: (C) $\sqrt{\frac{3}{a}}$

Solution:

Concept:

- The work-energy theorem states that the change in kinetic energy is $\Delta K = \int F_x \, dx$. Here the electric force is position dependent: $F_x = qE_0(1 - ax^2)$.
- With initial kinetic energy zero, a later turning position is found by setting the accumulated work to zero. Zero force identifies an extremum of kinetic energy, not generally a turning point.

Step 1: Integrate the electric force from the initial point At $x = 0$ the particle starts from rest, so $K(0) = 0$. At position x,

$$K(x) = qE_0 \int_0^x (1 - a\xi^2) \, d\xi$$
$$= qE_0 \left(x - \frac{ax^3}{3} \right).$$

The integration variable ξ keeps the upper limit distinct from the variable inside the integral. The mass affects the speed corresponding to this energy, but it does not affect the position where that energy vanishes.

Step 2: Exclude the initial root and identify the distance For nonzero applied force scale qE_0, the condition $K(x) = 0$ becomes

$$x \left(1 - \frac{ax^2}{3} \right) = 0.$$

The root $x = 0$ is explicitly excluded. The remaining root has $x^2 = 3/a$, giving distance $|x| = \sqrt{3/a}$ for the positive a implicit in the alternatives. Option D, $\sqrt{1/a}$, instead solves $E = 0$, where energy is maximal along the outward motion. At $x^2 = 2/a$, option B, the bracket is $1/3$, not zero. Option A also has the wrong dimensions: a has units of inverse length squared.

Step 3: Conclusion The non-initial zero of kinetic energy occurs at a distance $\sqrt{3/a}$ from the origin. Hence, the correct option is (C).

Quick Tip to Solve: For a varying electric field, integrate $qE(x)$ to find a turning point. Setting $E = 0$ only locates zero acceleration and gives a different position.

35. A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to :

(Given bulk modulus of metal, B = 8×10^{10} Pa) [4 marks]

(A) 0.6

(B) 20

(C) 1.67

(D) 5

Correct Answer: (C) 1.67

Solution:

Concept:

- The bulk modulus is $B = -\Delta P/(\Delta V/V)$. A positive applied hydrostatic pressure produces negative volumetric strain, indicating compression.
- For isotropic compression of a cube of side l, $V = l^3$, and small strains satisfy $\Delta V/V \simeq 3\Delta l/l$. Thus the magnitude of linear strain is one third of the volumetric strain.

Step 1: Calculate the fractional volume reduction Convert the pressure to pascals before combining it with the modulus: $P = 4\,\text{GPa} = 4 \times 10^9\,\text{Pa}$. The compressive volume strain is

$$\frac{\Delta V}{V} = -\frac{P}{B} = -\frac{4 \times 10^9}{8 \times 10^{10}} = -0.05.$$

Thus the volume decreases by 5%. This percentage concerns the whole volume, and cannot directly be assigned to one side of the cube.

Step 2: Convert volumetric strain to linear strain Pressure acts equally in every direction, so the three perpendicular sides have the same fractional contraction. Expanding $(l + \Delta l)^3$ to first order gives

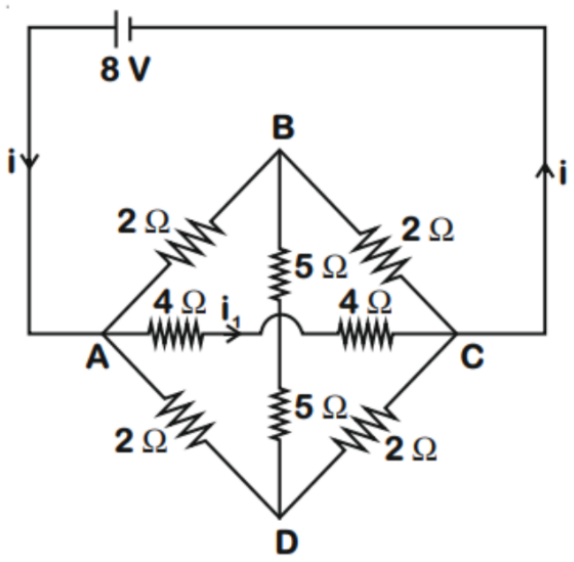
$$\frac{\Delta V}{V} \simeq 3\frac{\Delta l}{l}, \qquad \frac{\Delta l}{l} \simeq -\frac{0.05}{3} = -0.01667.$$

The percentage change in the side length therefore has magnitude 1.667%, closest to 1.67%. Option D, 5, confuses volume strain with linear strain. Options A and B do not follow the pressure-to-modulus ratio and the factor of three. Even using the finite-volume check, $l_f/l_i = (0.95)^{1/3}$, gives a contraction near 1.70%, consistent with the same closest option.

Step 3: Conclusion The side length decreases by approximately 1.67%. Hence, the correct option is (C).

Quick Tip to Solve: Under hydrostatic compression, calculate P/B for volume strain first; divide it by three to get the small fractional change in each side.

36. The value of current i_1 flowing from A to C in the circuit diagram is : [4 marks]



(A) 1 A

(B) 4 A

(C) 5 A

(D) 2 A

Correct Answer: (A) 1 A

Solution:

Concept:

- In a balanced Wheatstone bridge, equal resistance ratios in the two outer paths make the intermediate junctions equipotential, so no current flows in the connecting bridge branch.
- An ideal battery fixes the potential difference across every parallel path. Ohm law for a particular branch is $I = V/R_{\text{branch}}$, using that branch resistance rather than the equivalent resistance of the entire circuit.

Step 1: Identify the balanced outer bridge and the crossing The upper path is $A \to B \to C$, with two $2\,\Omega$ resistors. The lower path $A \to D \to C$ has the same pair. Hence

$$\frac{R_{AB}}{R_{BC}} = \frac{2}{2} = \frac{R_{AD}}{R_{DC}}.$$

Both B and D are at the midpoint potential between A and C; the vertical branch containing the two $5\,\Omega$ resistors carries no current. The little bridge at the central crossing shows that the horizontal and vertical wires are not connected there.

Step 2: Apply the battery voltage to the horizontal branch The horizontal A-to-C branch contains two series resistors of $4\,\Omega$ each, so

$$R_{AC,\text{horizontal}} = 4 + 4 = 8\,\Omega, \qquad i_1 = \frac{8\,\text{V}}{8\,\Omega} = 1\,\text{A}.$$

The battery positive terminal is on the A side, so this current flows from A to C as requested. Each outer path carries $8/4 = 2$ A; adding all three branch currents gives total battery current $2+2+1 = 5$ A. Thus option C gives the total, while D is the current of an outer path. Option B is their sum, not the indicated horizontal current.

Step 3: Conclusion The current i_1 is 1 A, directed from A to C. Hence, the correct option is (A).

Quick Tip to Solve: Read the crossing symbol first. Then use bridge balance to eliminate the vertical branch and apply the full battery voltage to the two horizontal series resistors.

37. A quantity x is given by $\left(\frac{IFv^2}{WL^4}\right)$ in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of : [4 marks]

(A) Coefficient of viscosity

(B) Force constant

(C) Energy density

(D) Planck's constant

Correct Answer: (C) Energy density

Solution:

Concept:

- Dimensions multiply and divide by adding and subtracting exponents. The fundamental dimensions used here are mass M, length L and time T.
- Moment of inertia has dimension $[I] = ML^2$, force has $[F] = MLT^{-2}$, speed has $[v] = LT^{-1}$, and work has $[W] = ML^2T^{-2}$.

Step 1: Form the numerator dimensions The given expression contains the square of velocity, not velocity itself. Thus the numerator contains two factors of mass, five factors of length and time to the power minus four:

$$[IFv^2] = (ML^2)(MLT^{-2})(L^2T^{-2}) = M^2L^5T^{-4}.$$

The symbol I denotes moment of inertia; it must not be mistaken for electric current.

Step 2: Divide by the denominator dimensions The additional length factor in the denominator is L^4, so

$$[WL^4] = (ML^2T^{-2})L^4 = ML^6T^{-2}.$$

Hence

$$[x] = \frac{M^2L^5T^{-4}}{ML^6T^{-2}} = ML^{-1}T^{-2}.$$

This is also the dimension of pressure.

Step 3: Compare all four quantities Energy density is energy per unit volume, giving $[W/L^3] = ML^{-1}T^{-2}$, exactly the required expression. The coefficient of dynamic viscosity has dimension $ML^{-1}T^{-1}$; its time exponent is different. A force constant is force per displacement and has dimension MT^{-2}, with no length factor. Planck's constant is energy multiplied by time, with dimension ML^2T^{-1}. These three dimensions therefore cannot match x.

Step 4: Conclusion The dimensional formula is $\boxed{ML^{-1}T^{-2}}$, the same as energy density. Hence, the correct option is (C).

Quick Tip to Solve: After simplifying the exponents, compare the result with energy per unit volume or force per unit area; both have pressure dimensions.

38. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is : [4 marks]

(A) $\frac{1}{2}CV_0^2$
(B) $\frac{1}{4}CV_0^2$
(C) $\frac{1}{3}CV_0^2$
(D) $\frac{1}{6}CV_0^2$

Correct Answer: (D) $\frac{1}{6}CV_0^2$

Solution:

Concept:

- When isolated capacitors are connected in parallel, their final potential difference is common and their total charge is conserved. The battery has already been disconnected here.
- The energy stored in a capacitor is $U = \frac{1}{2}CV^2$. Electrical energy need not be conserved during redistribution: part becomes heat and electromagnetic radiation in the connecting circuit.

Step 1: Conserve the initial charge Initially only the capacitor C is charged, so the total charge available is $Q_0 = CV_0$. The second capacitor, of capacitance $C/2$, contributes no initial charge. In parallel the equivalent capacitance is

$$C_{\text{eq}} = C + \frac{C}{2} = \frac{3C}{2}.$$

The common final potential is therefore

$$V_f = \frac{Q_0}{C_{\text{eq}}} = \frac{CV_0}{3C/2} = \frac{2V_0}{3}.$$

Step 2: Subtract the final stored energy The initial energy is $U_i = \frac{1}{2}CV_0^2$. Both capacitors contribute to the final energy; using their combined capacitance gives

$$U_f = \frac{1}{2}\left(\frac{3C}{2}\right)\left(\frac{2V_0}{3}\right)^2 = \frac{1}{3}CV_0^2.$$

Thus the energy lost from the electric field is

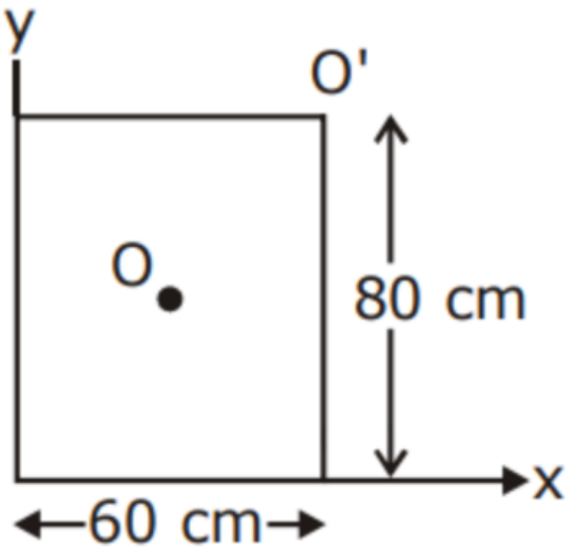
$$U_i - U_f = \left(\frac{1}{2} - \frac{1}{3}\right)CV_0^2 = \frac{1}{6}CV_0^2.$$

Step 3: Check what each option represents Option (A) is the entire initial energy and would incorrectly imply no energy remains. Option (C) is the final stored energy rather than the loss. Option (B) is the familiar loss for sharing charge between two equal capacitors, whereas the second capacitance here is only $C/2$. The loss is positive and smaller than the initial energy, as required for the redistribution process.

Step 4: Conclusion The energy loss is $\boxed{\frac{1}{6}CV_0^2}$. Hence, the correct option is (D).

Quick Tip to Solve: Conserve charge to find the common voltage first; calculate energy afterwards. For unequal capacitors the equal-capacitance shortcut gives the wrong loss.

39. For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is : [4 marks]



(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{1}{8}$
(D) $\frac{2}{3}$

Correct Answer: (B) $\frac{1}{4}$

Solution:

Concept:

- For a uniform rectangular sheet of mass m, sides a, b, and an axis through its centre perpendicular to its plane, $I_O = \frac{m}{12}(a^2 + b^2)$.
- The parallel-axis theorem gives $I_{O'} = I_O + md^2$, where d is the distance between the central axis and the parallel axis through the corner.

Step 1: Find the centre-to-corner distance The figure gives the sides as $60\,\text{cm}$ and $80\,\text{cm}$. The centre

lies halfway along both sides, so the displacement from O to $O\prime$ has perpendicular components $30\,\mathrm{cm}$ and $40\,\mathrm{cm}$. Consequently,

$$d^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 = \frac{a^2+b^2}{4}.$$

Numerically, $d = 50\,\mathrm{cm} = 0.50\,\mathrm{m}$.

Step 2: Apply the parallel-axis theorem Both specified axes are perpendicular to the same sheet, so they are parallel. The central moment is

$$I_O = \frac{m(a^2+b^2)}{12}.$$

Shifting to the corner adds $m(a^2+b^2)/4$, giving

$$I_{O\prime} = \frac{m(a^2+b^2)}{12} + \frac{m(a^2+b^2)}{4} = \frac{m(a^2+b^2)}{3}.$$

The corner value is larger because all mass is, on average, farther from that axis.

Step 3: Take the ratio in the requested order The question asks for the central moment divided by the corner moment, not the inverse. Hence

$$\frac{I_O}{I_{O\prime}} = \frac{1/12}{1/3} = \frac{1}{4}.$$

The mass and side lengths cancel, so the answer holds for any uniform rectangle. Options (A), (C) and (D) give different ratios and do not satisfy the exact shift term $md^2 = 3I_O$.

Step 4: Conclusion The requested ratio is $\boxed{I_O/I_{O\prime} = 1/4}$. Hence, the correct option is (B).

Quick Tip to Solve: For a rectangle the centre-to-corner distance squared is one quarter of the sum of the squared sides; the parallel-axis addition is three times the central moment.

40. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is : [4 marks]

(A) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$

(B) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$

(C) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$

(D) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

Correct Answer: (B) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$

Solution:

Concept:

- After an initially unenergised series L-R circuit is connected to a constant emf V, its current grows as $i(t) = \frac{V}{R}(1 - e^{-Rt/L})$. The time constant is $\tau = L/R$.
- The magnetic energy is $U = \frac{1}{2}Li^2$, so the energy fraction is the square of the current fraction, not the current fraction itself.

Step 1: Relate the target energy to the current The eventual current is $i_{\max} = V/R$, giving $U_{\max} = \frac{1}{2}L(V/R)^2$. At the required instant,

$$\frac{U(t)}{U_{\max}} = \left(\frac{i(t)}{i_{\max}}\right)^2 = \frac{1}{n}.$$

The current is positive during growth, so its fraction must be $1/\sqrt{n}$. For a finite positive time the specified fraction requires $n > 1$.

Step 2: Solve the exponential equation Substitute the current-growth law and rearrange:

$$1 - e^{-Rt/L} = \frac{1}{\sqrt{n}}, \qquad e^{-Rt/L} = \frac{\sqrt{n}-1}{\sqrt{n}}.$$

Taking logarithms gives

$$-\frac{Rt}{L} = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right).$$

Therefore

$$t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right).$$

The emf cancels because the target was expressed relative to the maximum energy.

Step 3: Check the sign and alternatives For $n > 1$, the logarithm in the result is positive, so the time is positive. Options (A) and (D) have logarithm arguments below one and would give negative times. Option (C) does not solve the exponential equation: substituting it would give a current fraction $2/(\sqrt{n}+1)$, not $1/\sqrt{n}$. As $n \to 1^+$, the answer tends to infinity, consistent with reaching the steady value only asymptotically.

Step 4: Conclusion The required time is $\boxed{\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)}$. Hence, the correct option is (B).

Quick Tip to Solve: Take the square root of the energy fraction before using the LR current law, then check that the logarithm gives a positive time.

41. A person pushes a box on a rough horizontal plateform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box? [4 marks]

(A) 5690 J

(B) 5250 J

(C) 2780 J

(D) 3280 J

Correct Answer: (B) 5250 J

Solution:

Concept:

- The work done by the person is the integral of the applied force along the displacement, $W = \int F\,dx$. Its graphical value is the area under the force-distance graph.
- When force varies linearly with distance, its average over that interval is the arithmetic mean of its endpoint values. Friction affects net work but does not change the work attributed to the applied force.

Step 1: Calculate the first interval The force is constant at $200\,\mathrm{N}$ for the first $15\,\mathrm{m}$. Since the push acts along the motion, its work is

$$W_1 = Fd = (200)(15) = 3000\,\mathrm{J}.$$

This is the rectangular area under the first part of the force-distance graph. No coefficient of friction is needed to calculate this particular work.

Step 2: Calculate the decreasing-force interval The total displacement is $30\,\mathrm{m}$, so the remaining distance is $30 - 15 = 15\,\mathrm{m}$. Over this interval the force falls linearly from $200\,\mathrm{N}$ to $100\,\mathrm{N}$. Its mean value is

$$\overline{F} = \frac{200 + 100}{2} = 150\,\mathrm{N}.$$

Thus the trapezoidal area is

$$W_2 = \overline{F}\,\Delta x = (150)(15) = 2250\,\mathrm{J}.$$

Step 3: Add the two contributions and check Both pushes have the same direction as the displacement, so both contributions are positive:

$$W = W_1 + W_2 = 3000 + 2250 = 5250\,\mathrm{J}.$$

The overall average applied force is $5250/30 = 175\,\mathrm{N}$, lying between the endpoint forces. The other listed values, 5690, 2780 and $3280\,\mathrm{J}$, do not equal the sum of the required rectangle and trapezium. In particular, using a single final force or treating the force as decreasing over the entire path would ignore the first constant interval.

Step 4: Conclusion The work done by the person is $\boxed{5250\,\mathrm{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: Split the path where the force law changes: use a rectangle for the constant part and a trapezium for the linearly decreasing part.

42. Match the thermodynamic processes taking place in a system with the correct conditions. In the table : ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy of the system.

Process	Condition
(I) Adiabatic	(1) $\Delta W = 0$
(II) Isothermal	(2) $\Delta Q = 0$
(III) Isochoric	(3) $\Delta U \neq 0$, $\Delta W \neq 0$, $\Delta Q \neq 0$
(IV) Isobaric	(4) $\Delta U = 0$

[4 marks]

(A) (I) - (1), (II) - (1), (III) - (2), (IV) - (3)

(B) (I) - (2), (II) - (4), (III) - (1), (IV) - (3)

(C) (I) - (1), (II) - (2), (III) - (4), (IV) - (4)

(D) (I) - (2), (II) - (1), (III) - (4), (IV) - (3)

Correct Answer: (B) (I) - (2), (II) - (4), (III) - (1), (IV) - (3)

Solution:

Concept:

- Using work done by the system as positive, the first law is $\Delta Q = \Delta U + \Delta W$. Each named process imposes its own constraint.
- For an ideal gas, internal energy depends only on temperature. Thus an isothermal change has $\Delta U = 0$; constant volume instead forces boundary work to vanish.

Step 1: Match adiabatic and isothermal changes An adiabatic process permits no transfer of heat across the boundary. Its defining condition is $\Delta Q = 0$, so (I) matches condition (2). Work can still be done, at the expense of internal energy. In an isothermal ideal-gas process the temperature remains constant. Therefore the internal energy does not change, $\Delta U = 0$, so (II) matches condition (4). Heat and work can each be nonzero and equal in this case.

Step 2: Match constant volume and constant pressure In an isochoric process the volume is fixed. Consequently the boundary work $\Delta W = \int P\, dV$ is zero, giving (III)-(1); any supplied heat changes internal energy. In the nontrivial isobaric process represented by the table, pressure remains fixed while volume and temperature change. Thus work, heat and internal-energy change are all nonzero, giving (IV)-(3). Constant pressure does not mean that work or internal energy is zero.

Step 3: Select the complete matching code Collecting the four results yields

$$(\mathrm{I}) - (2), \quad (\mathrm{II}) - (4), \quad (\mathrm{III}) - (1), \quad (\mathrm{IV}) - (3).$$

Option (B) has this exact sequence. Option (A) wrongly assigns zero work to an adiabatic change and zero heat to an isochoric change. Option (C) similarly misidentifies adiabatic and isothermal conditions. Option (D) interchanges the isothermal and isochoric conditions. These errors are exposed directly by the defining constraints, without numerical calculation.

Step 4: Conclusion The correct matching is (I)-(2), (II)-(4), (III)-(1), (IV)-(3). Hence, the correct option is (B).

Quick Tip to Solve: Remember the zero quantity for each process: adiabatic means no heat, isochoric means no boundary work, and isothermal ideal-gas means no internal-energy change.

43. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2 . The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the vertical,it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be: [4 marks]

(A) 10 π rad s^{-1}

(B) 20 π rad s^{-1}

(C) $10(3)^{1/4}$ rad s^{-1}

(D) 20 rad s^{-1}

Correct Answer: (C) $10(3)^{1/4}$ rad s^{-1}

Solution:

Concept:

- The magnetic potential energy of a current loop is $U = -\mu B\cos\theta$, where θ is the angle between the magnetic moment (normal to the coil) and the magnetic field.
- For free rotation about the given diameter, the decrease in magnetic potential energy becomes rotational kinetic energy: $U_i - U_f = \frac{1}{2}I\omega^2$. The coil starts from rest.

Step 1: Find the initial and final orientations The plane of the coil is initially vertical, so its magnetic moment is horizontal. The applied field is vertical; hence initially $\theta_i = 90°$, giving $U_i = 0$. The magnetic torque turns the moment toward the field. After the coil rotates through $60°$, the angle between its moment and the field is $\theta_f = 30°$, not $60°$. Thus the energy released is

$$\Delta U = \mu B\cos 30° = 20(4)\frac{\sqrt{3}}{2} = 40\sqrt{3}\,\text{J}.$$

Step 2: Apply conservation of energy The centre of mass lies on the horizontal diameter, so gravity produces no change in potential energy during this rotation. Using the stated moment of inertia about this same axis,

$$\frac{1}{2}(0.8)\omega^2 = 40\sqrt{3}, \qquad \omega^2 = 100\sqrt{3}.$$

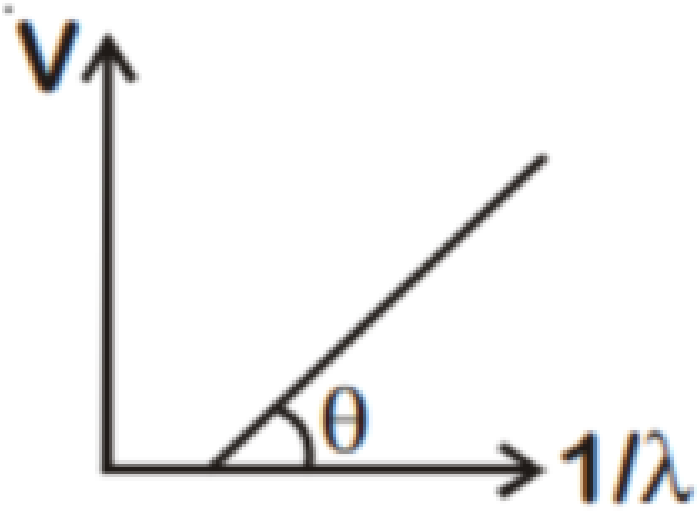
Taking the positive square root for angular speed gives $\omega = 10\,3^{1/4}\,\text{rad}\,\text{s}^{-1}$. Options (A) and (B) incorrectly introduce a factor of π; no period or full-revolution conversion occurs here. Option (D) would require kinetic energy 160 J, exceeding even the available $\mu B = 80$ J.

Step 3: Conclusion The angular speed is $\boxed{10\,3^{1/4}\,\text{rad}\,\text{s}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Always measure the angle in $-\mu B\cos\theta$ from the normal to the coil, not from its plane.

44. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased :

[4 marks]



(A) Slope of the straight line get more steep

(B) Graph does not change

(C) Straight line shifts to left

(D) Straight line shifts to right

Correct Answer: (B) Graph does not change

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where ϕ is the work function of the emitting material and e is the magnitude of electronic charge.
- Intensity measures incident energy per unit area per unit time. At a fixed wavelength, increasing intensity increases the photon arrival rate, while each photon still has energy hc/λ.

Step 1: Express the graph as a straight line With $x = 1/\lambda$, the equation for the stopping potential becomes

$$V_s = \frac{hc}{e}x - \frac{\phi}{e}.$$

Its slope is hc/e, containing only physical constants. Its vertical intercept is $-\phi/e$, and its horizontal intercept is $\phi/(hc) = 1/\lambda_0$, where λ_0 is the threshold wavelength. Neither intercept depends on the incident intensity for the same metal.

Step 2: Identify the effect of increased intensity A larger photon flux can eject more electrons per second, increasing the photocurrent when the frequency is above threshold. It does not increase the maximum kinetic energy of an individual emitted electron, since that energy is fixed by the energy of one incident photon minus the work function. Consequently the reverse potential needed to stop the fastest electrons is unchanged at each wavelength. The slope cannot become steeper, excluding (A); neither a left nor a right shift can occur because the threshold wavelength and work function remain

fixed, excluding (C) and (D). All points on the original stopping-potential graph remain at the same positions.

Step 3: Conclusion The stopping-potential graph remains unchanged when only the intensity is increased. Hence, the correct option is (B).

Quick Tip to Solve: Use intensity to predict photocurrent, and photon frequency or wavelength to predict stopping potential.

45. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1}. [4 marks]

(A) 91 kmh^{-1}

(B) 81 kmh^{-1}

(C) 61 kmh^{-1}

(D) 71 kmh^{-1}

Correct Answer: (A) 91 kmh^{-1}

Solution:

Concept:

- For sound of speed v, a source approaching a stationary observer at speed u produces frequency $f_1 = f_0 v/(v-u)$. The wave is compressed in front of the moving source.
- When the reflected sound returns, the stationary wall acts as a source and the driver is an approaching observer. Reflection from a stationary wall preserves frequency in the air frame.

Step 1: Include both Doppler shifts Let the speed of the bus be u and take the air and wall to be stationary. The driver and horn move together, so the directly heard horn frequency is $f_0 = 420\,\text{Hz}$. The wall receives the frequency

$$f_1 = 420\frac{330}{330-u}.$$

The reflected wave has this same frequency in the air, but the driver moves toward it. Therefore the driver receives

$$f_2 = f_1\frac{330+u}{330} = 420\frac{330+u}{330-u}.$$

Step 2: Solve and convert the speed Substitute the measured reflected frequency $490\,\text{Hz}$ and cross-multiply:

$$\frac{490}{420} = \frac{330+u}{330-u} = \frac{7}{6},$$

$$7(330-u) = 6(330+u), \qquad 330 = 13u.$$

Thus $u = 330/13 = 25.3846\,\text{m s}^{-1}$. Multiplication by 3.6 converts to $91.3846\,\text{km h}^{-1}$, matching the rounded option $91\,\text{km h}^{-1}$. The speeds in (B), (C) and (D), when substituted in the double-shift ratio, give smaller reflected frequencies than $490\,\text{Hz}$; they cannot match the observation.

Step 3: Conclusion The bus speed is approximately $\boxed{91\,\text{km h}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: For an echo heard by a moving source approaching a fixed wall, use the combined factor $(v+u)/(v-u)$.

Physics – Section B: Numerical Value Questions (20 marks)

46. Orange light of wavelength 6000×10^{-10} m illuminates a single slit of width 0.6×10^{-4} m. The maximum possible number of diffraction minima produced on both sides of the central maximum is _____. [4 marks]

Solution:

Concept:

- For a slit of width a, diffraction minima satisfy $a \sin\theta = m\lambda$, with nonzero integer order m. Positive and negative orders occur symmetrically about the central maximum.
- The largest geometrically permitted order is limited by $|\sin\theta| \leq 1$. Count the orders on both sides, without counting the central maximum as a minimum.

Step 1: Convert the dimensions and find the order limit The wavelength and slit width in SI units are

$$\lambda = 6000 \times 10^{-10} = 6.0 \times 10^{-7}\,\text{m},$$

$$a = 0.6 \times 10^{-4} = 6.0 \times 10^{-5}\,\text{m}.$$

Their ratio is dimensionless:

$$\frac{a}{\lambda} = \frac{6.0 \times 10^{-5}}{6.0 \times 10^{-7}} = 100.$$

The condition $m\lambda/a \leq 1$ therefore allows orders through $m = 100$. The order $m = 101$ would require $\sin\theta = 1.01$, which is impossible.

Step 2: Count the orders on both sides The positive side has the formal orders $1, 2, \ldots, 100$, and the negative side has $-1, -2, \ldots, -100$. The maximum possible count, including the limiting grazing directions at $\theta = \pm 90^\circ$, is

$$N = 2m_{\max} = 2(100) = 200.$$

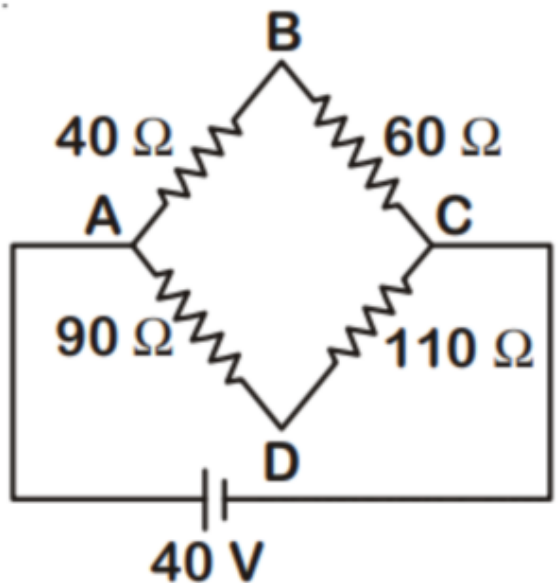
The order zero represents the central bright maximum, so it is not added. The endpoint qualification matters: the hundredth orders occur at grazing angles and would not be intercepted at finite position on a plane screen perpendicular to the incident beam. The question asks the maximum possible count, for which these limiting orders are included.

Step 3: Conclusion The maximum possible number of diffraction minima on both sides, including the limiting grazing orders, is $\boxed{200}$ (a dimensionless count).

Quick Tip to Solve: First calculate a/λ; then double the permitted nonzero orders, checking whether an integer endpoint is at grazing incidence.

47. Four resistances 40 Ω, 60 Ω, 90 Ω and 110 Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40 V and internal resistance negligible.The potential difference across BD in V is _____.

[4 marks]



Solution:

Concept:

- With an ideal battery across A and C, both paths ABC and ADC have the full battery voltage. The resistors within each path are in series.
- The potential at a junction follows the voltage-divider rule. Since no conducting branch joins B and D, no current flows between them while their potential difference is measured ideally.

Step 1: Find the two branch currents The diagram gives $R_{AB} = 40\,\Omega$, $R_{BC} = 60\,\Omega$, $R_{AD} = 90\,\Omega$, and $R_{DC} = 110\,\Omega$. The positive terminal is connected to A. Choose $V_C = 0$, so $V_A = 40\,\text{V}$. The upper and lower branch currents, both flowing from A to C, are

$$I_{ABC} = \frac{40}{40 + 60} = 0.4\,\text{A},$$

$$I_{ADC} = \frac{40}{90 + 110} = 0.2\,\text{A}.$$

It would be incorrect to use a single current through all four resistors: the battery junctions divide the circuit into parallel paths.

Step 2: Compare the junction potentials Along the upper path, the drop from A to B is $0.4(40) = 16\,\text{V}$; hence

$$V_B = 40 - 16 = 24\,\text{V}.$$

Along the lower path, the drop from A to D is $0.2(90) = 18\,\text{V}$; hence

$$V_D = 40 - 18 = 22\,\text{V}.$$

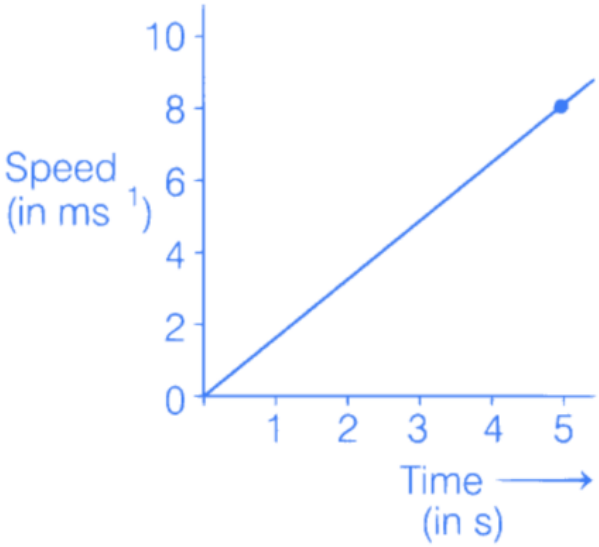
Thus $V_B - V_D = 24 - 22 = 2\,\text{V}$. Checking from C gives the same potentials: $0.4(60) = 24\,\text{V}$ and

$0.2(110) = 22\,\text{V}$. The bridge is not balanced because $40/60 \neq 90/110$, so the voltage cannot be zero.

Step 3: Conclusion The potential difference across BD has magnitude $\boxed{2\,\text{V}}$, with B at the higher potential.

Quick Tip to Solve: For an open bridge diagonal, calculate the two junction voltages as separate potential dividers across the supply.

48. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5 s will be_____. [4 marks]



Solution:

Concept:

- Distance travelled equals the area under a speed-time graph: $s = \int v(t)\,dt$. Because speed is nonnegative, the entire area contributes positively to distance.
- A straight line on a speed-time plot represents constant acceleration. For a line beginning at zero speed, the area over a time interval is a triangle of area $\frac{1}{2} \times$ base $\times$ height.

Step 1: Read the relevant coordinates The graph starts at the origin, so the initial speed is $v(0) = 0$. The marked point at $t = 5\,\text{s}$ has speed $8\,\text{m}\,\text{s}^{-1}$; it is not at the tick labelled 10. The portion of the line beyond this point is outside the requested interval. From 0 to $5\,\text{s}$, the enclosed region is a triangle whose horizontal base is $5\,\text{s}$ and whose vertical height is $8\,\text{m}\,\text{s}^{-1}$.

Step 2: Calculate the triangular area The distance during the stated interval is therefore

$$s = \frac{1}{2}(5\,\text{s})(8\,\text{m}\,\text{s}^{-1}) = 20\,\text{m}.$$

As a check, uniform acceleration implies that the average speed is the arithmetic mean of the endpoint

speeds:

$$v_{\text{avg}} = \frac{0+8}{2} = 4\,\text{m}\,\text{s}^{-1}, \qquad s = v_{\text{avg}}(5\,\text{s}) = 20\,\text{m}.$$

Multiplying the final speed by the full time would give the rectangular area and incorrectly count the whole interval as motion at the final speed. The slope $8/5 = 1.6\,\text{m}\,\text{s}^{-2}$ is acceleration, not the requested distance.

Step 3: Conclusion The distance travelled from $t = 0$ to $t = 5\,\text{s}$ is $\boxed{20\,\text{m}}$.

Quick Tip to Solve: On a speed-time graph, use area for distance and slope for acceleration; read the plotted endpoint rather than the top axis tick.

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49. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively.

If $|\Delta T| = C|\Delta P|$ then value of C in (K/atm.) is _____. [4 marks]

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$, where P, V, T are its pressure, volume and absolute temperature.
- For small changes, the isothermal volume contraction is $V|\Delta P|/P$; the isobaric volume contraction is $V|\Delta T|/T$. These must be evaluated from the same initial state.

Step 1: Find the volume change at constant temperature Let the initial volume be V. With temperature fixed, differentiate Boyle's relation $PV =$ constant:

$$P\,dV + V\,dP = 0, \qquad dV = -\frac{V}{P}dP.$$

An additional positive pressure therefore reduces the volume. To first order in the small pressure change, its magnitude is

$$|\Delta V|_T = \frac{V}{P}|\Delta P|.$$

The minus sign indicates contraction, and is removed when comparing magnitudes.

Step 2: Find the volume change at constant pressure At fixed pressure the ideal-gas equation gives $V/T = nR/P$, a constant. Thus

$$dV = \frac{V}{T}dT, \qquad |\Delta V|_P = \frac{V}{T}|\Delta T|.$$

Cooling means $\Delta T < 0$, so this process also contracts the gas. The same amount of gas and the same initial volume occur in both expressions; neither needs to be calculated separately.

Step 3: Equate the stated magnitudes The question sets these two volume reductions equal. Hence

$$\frac{V}{P}|\Delta P| = \frac{V}{T}|\Delta T| \quad \Rightarrow \quad |\Delta T| = \frac{T}{P}|\Delta P|.$$

Comparing with $|\Delta T| = C|\Delta P|$, substitute the initial values:

$$C = \frac{300\,\text{K}}{2\,\text{atm}} = 150\,\text{K atm}^{-1}.$$

Because the requested coefficient uses atmospheres, no conversion of the pressure to pascals is necessary.

Step 4: Conclusion The coefficient is $\boxed{C = 150\,\text{K atm}^{-1}}$. The numerical entry is $\boxed{150}$.

Quick Tip to Solve: For equal small fractional volume changes of an ideal gas, equate $|\Delta P|/P$ with $|\Delta T|/T$; keep pressure in the unit requested for the coefficient.

50. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right) D$ where N is an integer, the value of N is _____. [4 marks]

Solution:

Concept:

- A converging thin lens forming a real image obeys $1/f = 1/u + 1/v$ when u and v denote positive object and image distance magnitudes.
- With object and screen fixed a distance D apart, $u + v = D$. The two lens positions interchange u and v; their separation is $d = |v - u|$.

Step 1: Determine the conjugate distances The fixed object–screen distance is $D = 100\,\text{cm}$ and the displacement between the two sharp-image positions is $d = 40\,\text{cm}$. Choose the position with $v > u$. Then

$$u + v = 100, \qquad v - u = 40.$$

Adding and subtracting gives

$$v = \frac{100 + 40}{2} = 70\,\text{cm}, \qquad u = \frac{100 - 40}{2} = 30\,\text{cm}.$$

At the second position these two distances are exchanged, which explains why the same lens gives a sharp image twice.

Step 2: Calculate the focal length Apply the real-image thin-lens relation using these positive magnitudes:

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{70} = \frac{70 + 30}{2100} = \frac{1}{21}\ \text{cm}^{-1}.$$

Therefore

$$f = 21\,\text{cm} = 0.21\,\text{m}.$$

Equivalently, $f = (D^2 - d^2)/(4D)$ gives $(10000 - 1600)/400 = 21\,\text{cm}$. The positive focal length is consistent with a converging lens and a real image on a screen.

Step 3: Convert to power and find the integer Power in dioptres is the reciprocal of the focal length in metres:

$$P = \frac{1}{0.21}\,\text{D} = \frac{100}{21}\,\text{D} \simeq 4.76190\,\text{D}.$$

The printed form is $P \simeq (N/100)\,\text{D}$, so

$$N \simeq 100(4.76190) = 476.190.$$

The closest integer is 476; indeed $476/100 = 4.76$, close to the calculated positive power.

Step 4: Conclusion The lens power is approximately $+4.76\,\text{D}$, and the required integer is $\boxed{N = 476}$.

Quick Tip to Solve: In the lens displacement method use $f = (D^2 - d^2)/(4D)$, then convert centimetres to metres before taking the reciprocal for dioptres.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The process that is NOT endothermic in nature is : [4 marks]

(A) $Ar(g) + e^- \rightarrow Ar^-(g)$

(B) $H(g) + e^- \rightarrow H^-(g)$

(C) $Na(g) \rightarrow Na^+(g) + e^-$

(D) $O^-(g) + e^- \rightarrow O^{2-}(g)$

Correct Answer: (B) $H(g) + e^- \rightarrow H^-(g)$

Solution:

Concept:

- An endothermic process absorbs energy, so its enthalpy change is positive; an exothermic electron-attachment process releases energy.
- Electron gain enthalpy depends on the electronic configuration and charge of the species. First electron attachment and attachment to an existing anion need not have the same sign.

Step 1: Check electron attachment to hydrogen A hydrogen atom has configuration $1s^1$. Accepting an electron completes its $1s$ shell:

$$\text{H}\,(\text{g}) + \text{e}^- \longrightarrow \text{H}^-(\text{g}).$$

The electron is bound to the atom and energy is released (the first electron gain enthalpy is negative). Thus the process in option (B) is exothermic and satisfies the demand for a process that is NOT endothermic. The word NOT is essential: we seek the exception to energy absorption.

Step 2: Eliminate the other three processes Argon already has the closed-shell configuration $[\text{Ne}]3s^2 3p^6$. An additional electron must enter a higher-energy shell, so forming the isolated gaseous

negative ion in option (A) requires energy. In option (C), removing an electron from neutral gaseous sodium is first ionisation:

$$\mathrm{Na(g)} \longrightarrow \mathrm{Na^{+}(g)} + \mathrm{e^{-}}.$$

Ionisation must overcome attraction to the nucleus and is endothermic. Finally, option (D) adds an electron to an already negative oxygen ion:

$$\mathrm{O^{-}(g)} + \mathrm{e^{-}} \longrightarrow \mathrm{O^{2-}(g)}.$$

Electrostatic repulsion must be overcome, so this second electron gain is also endothermic. Stability of oxide ions in an ionic crystal does not change the positive enthalpy of this isolated gaseous-ion step.

Step 3: Conclusion Only $\mathrm{H(g)} + \mathrm{e^{-}} \longrightarrow \mathrm{H^{-}(g)}$ releases energy among the listed processes. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish first electron gain by a neutral atom from second electron gain by an anion; adding an electron to a negative gaseous ion requires energy.

52. The incorrect statement(s) among (a) - (c) is (are)

(a) W(VI) is more stable than Cr(VI).

(b) In the presence of HCl, permanganate titrations provide satisfactory results.

(c) Some lanthanoid oxides can be used as phosphors. [4 marks]

(A) (a) and (b) only

(B) (a) only

(C) (b) only

(D) (b) and (c) only

Correct Answer: (C) (b) only

Solution:

Concept:

- The stability of a high oxidation state generally increases on descending a transition-metal group; oxidation-state trends must be compared within the same group.
- An acid used in a redox titration must supply protons without itself being oxidised by the titrant. Lanthanoid ions also have characteristic electronic transitions useful in luminescent materials.

Step 1: Assess the oxidation-state and phosphor statements Statement (a) compares chromium and tungsten in Group 6. The highest oxidation state, $+6$, is more stable for the heavier member tungsten than for chromium. Chromium(VI) compounds are strong oxidants and readily reduce to chromium(III), whereas tungsten(VI) is especially stable. Thus (a) is correct. Statement (c) is also correct: certain lanthanoid oxides are used in phosphors, where excitation is followed by characteristic light emission. Such applications are associated with the electronic energy levels of lanthanoid ions. The statement says some oxides, and does not assert that every lanthanoid oxide behaves identically.

Step 2: Identify the interference caused by hydrochloric acid Acidic permanganate is a strong

oxidising agent. Hydrochloric acid introduces chloride ions, which it can oxidise to chlorine:

$$2\,MnO_4^- + 16\,H^+ + 10\,Cl^- \longrightarrow 2\,Mn^{2+} + 8\,H_2O + 5\,Cl_2.$$

This unwanted reaction consumes additional permanganate, so the measured titre no longer represents only the intended analyte. Therefore the claim in (b) that HCl permits satisfactory permanganate titrations is incorrect; dilute sulfuric acid is normally used instead. Options (A) and (B) wrongly reject (a), and option (D) wrongly rejects (c). Only (b) is incorrect.

Step 3: Conclusion The sole incorrect statement is statement b, owing to oxidation of chloride by acidic permanganate. Hence, the correct option is (C).

Quick Tip to Solve: For permanganate titrations check whether the acid anion can itself be oxidised; chloride causes an unwanted redox reaction and an erroneous titre.

53. Which of the following compounds will form the precipitate with aq. $AgNO_3$ solution most readily? [4 marks]

(A) $CH_3CH_2OCH_2Br$

(B) 1-(Bromomethyl)piperidine

(C) $C_6H_5N(C_2H_5)CH_2Br$

(D) 4-$CH_3OC_6H_4N(C_2H_5)CH_2CH_2Br$

Correct Answer: (B) 1-(Bromomethyl)piperidine

Solution:

Concept:

- Aqueous silver nitrate detects bromide released by ionisation of an organic bromide: $Ag^+ + Br^- \longrightarrow AgBr\,(s)$. Faster carbon–bromine bond ionisation produces precipitate more readily.
- A lone pair on an adjacent nitrogen or oxygen can stabilise the developing positive carbon through resonance. An aliphatic amine nitrogen donates its lone pair more effectively than an ether oxygen or an aniline nitrogen.

Step 1: Identify the most stabilised ionisation product In option (B), the bromomethyl carbon is bonded directly to the nitrogen of a saturated piperidine ring. When bromide leaves, the available nitrogen lone pair delocalises into the vacant carbon orbital:

$$R_2N{-}CH_2{-}Br \longrightarrow R_2N{-}CH_2{}^+ + Br^-, \qquad R_2N{-}CH_2{}^+ \longleftrightarrow R_2N^+{=}CH_2.$$

Here the two R groups are the two ends of the piperidine carbon chain. The iminium contributor gives carbon a complete octet and strongly stabilises the ionisation product. The nitrogen lone pair is not tied up by conjugation with an aromatic ring, so this assistance is particularly effective.

Step 2: Compare the remaining structures Option (A), $C_2H_5OCH_2Br$, has an adjacent ether oxygen and can form an oxonium-stabilised cation, but oxygen is more electronegative and is a weaker lone-pair donor than the aliphatic nitrogen in (B). In option (C), the nitrogen is attached to a phenyl ring; its lone pair participates in aromatic conjugation, making it less available to stabilise the adjacent

bromomethyl-derived cation. In option (D), bromine is on the terminal carbon of a two-carbon substituent, $\mathrm{N{-}CH_2{-}CH_2Br}$. The nitrogen is not directly adjacent to the carbon losing bromide, so the same direct iminium resonance stabilisation is absent; it is also an arylamine. These alternatives do not ionise as readily as (B).

Step 3: Connect ionisation with the observed precipitate The bromide released from the fastest-ionising compound is removed from solution as sparingly soluble silver bromide:

$$\mathrm{Ag^+(aq) + Br^-(aq) \longrightarrow AgBr(s).}$$

Thus the comparison concerns ease of bromide release, rather than the number of bromine atoms (each option has one) or simply the length of the alkyl chain.

Step 4: Conclusion The bromomethyl-substituted piperidine in (B) gives the precipitate most readily because its ionisation is strongly assisted by the adjacent aliphatic nitrogen lone pair. Hence, the correct option is (B).

Quick Tip to Solve: For silver nitrate tests on heteroatom-substituted halides, first draw the cation after halide loss and look for direct lone-pair donation into its vacant orbital.

54. If the equilibrium constant for
A $\rightleftharpoons$ B + C is $K_{eq}^{(1)}$ and that of
B + C $\rightleftharpoons$ P is $K_{eq}^{(2)}$, the equilibrium
constant for A $\rightleftharpoons$ P is : [4 marks]

(A) $\frac{K_{eq}^{(1)}}{K_{eq}^{(2)}}$
(B) $K_{eq}^{(1)} + K_{eq}^{(2)}$
(C) $K_{eq}^{(2)} - K_{eq}^{(1)}$
(D) $K_{eq}^{(1)}\ K_{eq}^{(2)}$

Correct Answer: (D) $K_{eq}^{(1)}\ K_{eq}^{(2)}$

Solution:

Concept:

- The equilibrium constant is written as the product of product activities divided by the product of reactant activities, each raised to its stoichiometric coefficient.
- When two reactions are added in their forward directions, their equilibrium constants multiply; intermediate species cancel from the net reaction and from the multiplied expressions.

Step 1: Write both equilibrium expressions For the first reaction, $\mathrm{A \rightleftharpoons B + C}$, the constant is

$$K_{eq}^{(1)} = \frac{a_B a_C}{a_A},$$

where a_X is the activity of species X. For the second reaction, $\mathrm{B} + \mathrm{C} \rightleftharpoons \mathrm{P}$,

$$K_{eq}^{(2)} = \frac{a_P}{a_B a_C}.$$

In the usual concentration notation the same cancellation follows with concentrations in square brackets. Both reactions are used in the directions stated: the products of the first provide exactly the reactants of the second.

Step 2: Add the reactions and cancel the intermediate factors Adding the two equations cancels one B and one C from opposite sides and leaves the required net reaction $\mathrm{A} \rightleftharpoons \mathrm{P}$. Multiply the equilibrium expressions:

$$K_{eq}^{(1)} K_{eq}^{(2)} = \frac{a_B a_C}{a_A} \frac{a_P}{a_B a_C} = \frac{a_P}{a_A} = K_{eq}.$$

Thus option (D) is obtained directly. The quotient in (A) would require reversal of one of the component reactions, which is not what produces the target equation. Adding or subtracting numerical equilibrium constants, as in (B) and (C), does not cancel intermediate activity factors and has no basis in the mass-action expressions. Equivalently, standard Gibbs energies add, and $\Delta G^\circ = -RT \ln K$ turns their addition into multiplication of constants.

Step 3: Conclusion The equilibrium constant for $\mathrm{A} \rightleftharpoons \mathrm{P}$ is $\boxed{K_{eq}^{(1)} K_{eq}^{(2)}}$. Hence, the correct option is (D).

Quick Tip to Solve: Add the reaction equations first. Every forward reaction contributes its constant as a factor; a reversed reaction contributes the reciprocal.

55. The shortest wavelength of H atom in the Lyman series is λ_1. The longest wavelength in the Balmar series of He^+ is : [4 marks]

(A) $\frac{5\lambda_1}{9}$

(B) $\frac{36\lambda_1}{5}$

(C) $\frac{27\lambda_1}{5}$

(D) $\frac{9\lambda_1}{5}$

Correct Answer: (D) $\frac{9\lambda_1}{5}$

Solution:

Concept:

- The Rydberg formula for a one-electron species is $1/\lambda = RZ^2(1/n_f^2 - 1/n_i^2)$, where Z is nuclear charge and $n_i > n_f$.
- Lyman lines terminate at $n_f = 1$, whereas Balmer lines terminate at $n_f = 2$. Wavelength is inversely proportional to the transition energy.

Step 1: Identify the Lyman series limit The shortest wavelength in a fixed series corresponds to its

largest energy gap. For hydrogen, $Z = 1$, and this is the transition from $n_i \to \infty$ to $n_f = 1$. Thus

$$\frac{1}{\lambda_1} = R\left(1 - \frac{1}{\infty^2}\right) = R.$$

Hence $R = 1/\lambda_1$. A transition from level 2 would instead give the longest Lyman wavelength, so it cannot define the given series limit.

Step 2: Choose the longest Balmer line of the helium ion The helium ion He^+ has one electron and $Z = 2$, so the hydrogen-like formula applies with a factor $Z^2 = 4$. The longest Balmer wavelength has the smallest possible energy gap ending at level 2, namely $3 \to 2$. Substitution gives

$$\frac{1}{\lambda_{He}} = 4R\left(\frac{1}{4} - \frac{1}{9}\right) = 4R\frac{5}{36} = \frac{5R}{9}.$$

Invert this expression and use the hydrogen limit:

$$\lambda_{He} = \frac{9}{5R} = \frac{9\lambda_1}{5}.$$

The factor $36/5$ would omit the helium nuclear-charge factor. The ratios $5/9$ and $27/5$ do not satisfy the Rydberg expression for this transition.

Step 3: Conclusion The longest wavelength is $\boxed{9\lambda_1/5}$. Hence, the correct option is (D).

Quick Tip to Solve: First choose the transition using shortest versus longest wavelength, then insert the nuclear-charge factor Z^2; these are separate decisions.

56. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is : [4 marks]

(A) Zero

(B) -RT $\ln \frac{V_2}{V_1}$

(C) C_V ($T_2 - T_1$)

(D) – RT ($V_2 - V_1$)

Correct Answer: (A) Zero

Solution:

Concept:

- Pressure-volume work in the chemistry sign convention is $w = -\int_{V_1}^{V_2} p_{ext}\, dV$; the relevant pressure is the pressure resisting expansion.
- Expansion into a vacuum is free expansion: $p_{ext} = 0$, even when the gas itself initially has a nonzero pressure.

Step 1: Identify the resisting pressure The gas begins at 1 bar and 298 K, but the space into which it expands is a vacuum. Therefore the surroundings exert no opposing pressure on the expanding boundary. The initial internal gas pressure is not the external pressure in the work integral. Doubling

the volume specifies $V_2 = 2V_1$, so there is an actual volume change, but displacement alone does not imply work: an opposing force must act through that displacement.

Step 2: Evaluate the work and compare the formulas Throughout the expansion into the evacuated region, $p_{\text{ext}} = 0$. Therefore

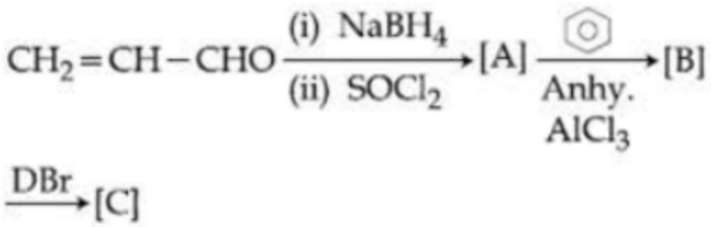
$$w = -\int_{V_1}^{2V_1} 0\, dV = 0 \text{ J}.$$

This result is independent of the number of moles and of the initial pressure and temperature. If work done by the gas is used as the sign convention, it is also zero. The logarithmic expression in option (B) describes reversible isothermal expansion, where a nonzero external pressure is continuously matched to the gas pressure; it does not describe free expansion. Option (C), involving heat capacity and temperature change, is related to the internal-energy change rather than the defining expression for expansion work. The question does not impose the extra conditions needed to identify it with work. Option (D) is not the pressure-volume work formula and is dimensionally inappropriate when R is the gas constant. None of these expressions replaces the zero resisting pressure in this process.

Step 3: Conclusion The work done during this free expansion is $\boxed{0\text{ J}}$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever a gas expands into a vacuum, set the external pressure to zero before using any reversible-expansion formula.

57. The major product [C] of the following reaction sequence will be : [4 marks]



(A) $C_6H_5CH_2CH(Br)CH_2D$

(B) $C_6H_5CH(Br)CH_2CH_2D$

(C) $C_6H_5CH_2CH(D)CH_2Br$

(D) $C_6H_5CH(Br)CH(D)CH_3$

Correct Answer: (B) $C_6H_5CH(Br)CH_2CH_2D$

Solution:

Concept:

- $NaBH_4$ reduces an aldehyde to a primary alcohol without normally reducing an isolated carbon-carbon double bond; $SOCl_2$ replaces the hydroxyl group by chlorine.
- Friedel–Crafts alkylation joins the allyl group to benzene. Ionic addition of DBr can be followed by a hydride shift to a resonance-stabilised benzylic carbocation.

Step 1: Find the intermediates before deuterium addition The starting compound is acrolein,

$CH_2{=}CHCHO$. Reduction changes only its aldehyde group:

$$CH_2{=}CHCHO \xrightarrow{NaBH_4} CH_2{=}CHCH_2OH.$$

Thionyl chloride then forms allyl chloride, $[A] = CH_2{=}CHCH_2Cl$. In the presence of anhydrous $AlCl_3$, benzene undergoes Friedel–Crafts alkylation to give allylbenzene:

$$[B] = C_6H_5CH_2CH{=}CH_2.$$

The carbon skeleton therefore contains three side-chain carbon atoms attached to the benzene ring.

Step 2: Add DBr and stabilise the carbocation With no peroxide specified, addition proceeds by the ionic route. Deuterium adds to the terminal carbon so that the initially formed positive charge is on the internal carbon:

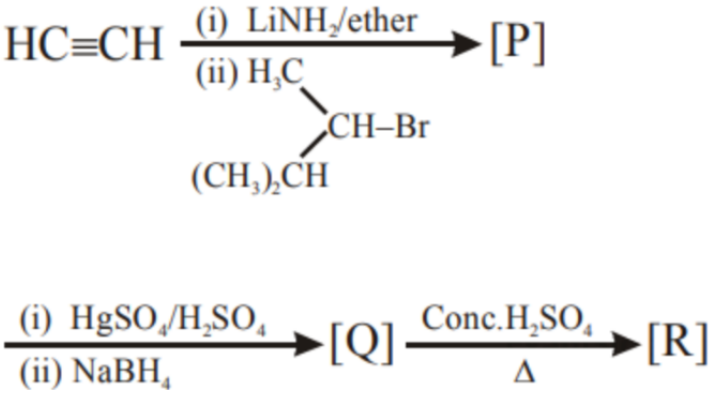
$$C_6H_5CH_2CH{=}CH_2 + D^+ \longrightarrow C_6H_5CH_2CH^+CH_2D.$$

A neighbouring benzylic hydrogen migrates with its bonding pair to this cationic centre. This 1,2-hydride shift gives the resonance-stabilised benzylic cation $C_6H_5CH^+CH_2CH_2D$. Bromide attacks that centre to form $C_6H_5CH(Br)CH_2CH_2D$. Option (A) traps the unrearranged secondary cation. Option (C) places bromine at the terminal position, contrary to this ionic pathway. Option (D) puts deuterium at the middle carbon, whereas it was added to the terminal carbon and stays there during the hydride shift.

Step 3: Conclusion The major product is $\boxed{C_6H_5CH(Br)CH_2CH_2D}$. Hence, the correct option is (B).

Quick Tip to Solve: After Markovnikov addition generates a carbocation beside a benzylic carbon, check for a hydride shift before attaching the halide.

58. The major product [R] in the following sequence of reactions as : [4 marks]



(A) $CH_2{=}C(CH_2CH_3)CH(CH_3)_2$

(B) $CH_2{=}CHCH(CH_3)CH(CH_3)_2$

(C) $CH_3CH_2C(CH_3){=}C(CH_3)_2$

(D) $CH_3CH{=}C(CH_3)CH(CH_3)_2$

Correct Answer: (C) $CH_3CH_2C(CH_3){=}C(CH_3)_2$

Solution:

Concept:

- A terminal alkyne forms an acetylide with $LiNH_2$; alkylation installs the alkyl group at the terminal carbon. Acidic mercuric-salt hydration gives a methyl ketone.
- $NaBH_4$ reduces the ketone to a secondary alcohol. Acid-catalysed dehydration can rearrange a secondary carbocation to a tertiary one before forming the most substituted alkene.

Step 1: Follow the alkyne through hydration and reduction Acetylene loses a terminal proton on treatment with lithium amide. Alkylation with the displayed bromide, $CH_3CH(Br)CH(CH_3)_2$, gives

$$[P] = HC{\equiv}CCH(CH_3)CH(CH_3)_2.$$

Hydration with $HgSO_4/H_2SO_4$ places the carbonyl group at the substituted alkyne carbon after enol-keto tautomerism. Thus the ketone is $CH_3COCH(CH_3)CH(CH_3)_2$. Reduction gives the alcohol

$$[Q] = CH_3CH(OH)CH(CH_3)CH(CH_3)_2.$$

No carbon atoms are lost in these transformations; the final alkene must retain all seven carbons.

Step 2: Rearrange the dehydration intermediate Concentrated sulphuric acid protonates the hydroxyl group, allowing water to leave. The initially formed secondary carbocation is $CH_3CH^+CH(CH_3)CH(CH_3)_2$. A hydride shift from the adjacent branched carbon generates the more stable tertiary cation $CH_3CH_2C^+(CH_3)CH(CH_3)_2$. Removal of the hydrogen on the neighbouring isopropyl carbon now gives

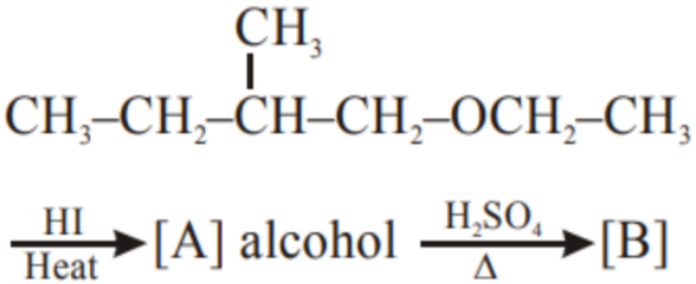
$$[R] = CH_3CH_2C(CH_3){=}C(CH_3)_2.$$

This double bond is tetrasubstituted and is the major alkene. Option (A) is a less substituted terminal alkene. Option (B) retains a terminal double bond and does not represent the major rearranged dehydration product. Option (D) corresponds to a less substituted internal alkene obtainable without the favoured rearrangement. The tertiary carbocation and the more substituted final double bond favour option (C).

Step 3: Conclusion The major product is $\boxed{CH_3CH_2C(CH_3){=}C(CH_3)_2}$, 2,3-dimethylpent-2-ene. Hence, the correct option is (C).

Quick Tip to Solve: Do not stop at the first alkene predicted by alcohol dehydration: look for an adjacent hydride shift that permits a more substituted double bond.

59. The major product [B] in the following reactions is : [4 marks]



(A) $CH_3CH_2CH{=}CHCH_3$

(B) $CH_2{=}CH_2$

(C) $CH_3CH{=}C(CH_3)CH_3$

(D) $CH_3CH_2C(CH_3){=}CH_2$

Correct Answer: (C) $CH_3CH{=}C(CH_3)CH_3$

Solution:

Concept:

- Hot hydrogen iodide protonates an ether oxygen. Iodide cleaves the less hindered primary alkyl group by an S_N2 reaction, leaving the other group as an alcohol.
- Acid-catalysed dehydration gives an alkene. Carbocation rearrangement and the greater stability of a more substituted double bond determine the major product.

Step 1: Identify the alcohol produced by ether cleavage The ether is $CH_3CH_2CH(CH_3)CH_2OCH_2CH_3$. Both carbons directly attached to oxygen are primary, but the ethyl side is less sterically hindered than the branched pentyl side. After protonation, iodide preferentially attacks the ethyl carbon. The products are ethyl iodide and the alcohol

$$[A] = CH_3CH_2CH(CH_3)CH_2OH.$$

This is 2-methylbutan-1-ol. It contains five carbon atoms; ethanol is not the alcohol selected by the dominant cleavage pathway.

Step 2: Determine the major dehydration product On heating with sulphuric acid the alcohol loses water to generate an alkene. The initially accessible terminal alkene is $CH_3CH_2C(CH_3){=}CH_2$. Under the acidic conditions, protonation at its terminal carbon gives the stable tertiary 2-methylbutan-2-yl carbocation. Loss of a proton from the adjacent ethyl carbon then produces the more substituted alkene

$$CH_3CH{=}C(CH_3)CH_3.$$

The same rearranged product can be represented through hydride migration during acid-catalysed elimination. This trisubstituted alkene is favoured over the less substituted terminal alkene in option (D). Option (A) has an unbranched pentene skeleton and does not retain the methyl branching of the alcohol. Option (B), ethene, would arise from ethanol rather than the alcohol isolated in the dominant ether-cleavage route. Thus the product in option (C) is the major alkene.

Step 3: Conclusion The major product is $\boxed{CH_3CH{=}C(CH_3)CH_3}$, 2-methylbut-2-ene. Hence, the

correct option is (C).

Quick Tip to Solve: For an unsymmetrical primary ether, identify which side iodide attacks before dehydrating the alcohol; then compare alkene substitution.

60. 250 mL of a waste solution obtained from the workshop of a goldsmith contains 0.1 M $AgNO_3$ and 0.1 M AuCl. The solution was electrolyzed at 2V by passing a current of 1A for 15 minutes. The metal/metals electrodeposited will be

[$E^0_{Ag^+/Ag}$ = 0.80 V, $E^0_{Au^+/Au}$ = 1.69 V] [4 marks]

(A) Silver and gold in equal mass proportion

(B) Silver and gold in proportion to their atomic weights

(C) Only gold

(D) Only silver

Correct Answer: (C) Only gold

Solution:

Concept:

- The ion with the higher reduction potential is preferentially reduced at the cathode. Here $\mathrm{Au}^+ + \mathrm{e}^- \longrightarrow \mathrm{Au}$ is favoured over $\mathrm{Ag}^+ + \mathrm{e}^- \longrightarrow \mathrm{Ag}$.
- Faraday law gives the amount reduced as $n = It/(zF)$, where z is the number of electrons per ion and $F \simeq 96500\ \mathrm{C\,mol^{-1}}$.

Step 1: Compare the reduction tendencies and ion inventory The supplied standard reduction potentials are $E^\circ_{\mathrm{Au}^+/\mathrm{Au}} = 1.69$ V and $E^\circ_{\mathrm{Ag}^+/\mathrm{Ag}} = 0.80$ V. Gold(I) has the larger value and is preferentially discharged. Initially both concentrations are 0.1 M, so their comparable concentration corrections do not overturn this large potential difference. The amount of gold(I) in 250 mL is

$$n_{\mathrm{Au}^+} = 0.1 \times 0.250 = 0.025\ \mathrm{mol}.$$

Since each gold(I) ion needs one electron, complete removal would need

$$Q_{\mathrm{Au}} = 0.025F \simeq 2412.5\ \mathrm{C}.$$

Step 2: Compare the charge passed with the gold requirement The electrolysis lasts $15 \times 60 = 900$ s, so

$$Q = It = 1 \times 900 = 900\ \mathrm{C}.$$

The amount of gold deposited is consequently

$$n_{\mathrm{Au}} = \frac{900}{96500} \simeq 0.00933\ \mathrm{mol}.$$

This is less than the available 0.025 mol, so gold(I) is not exhausted. Its remaining concentration is approximately $(0.025 - 0.00933)/0.250 = 0.0627$ M, which still leaves gold strongly preferred for

cathodic reduction. Thus the supplied charge is consumed in gold deposition before silver deposition is required. The two mixed-metal choices (A) and (B) incorrectly assume simultaneous discharge in fixed proportions. Option (D) reverses the reduction-potential order. The applied cell voltage does not by itself impose equal deposition of the two metals.

Step 3: Conclusion Only gold is electrodeposited for the stated charge and time. Hence, the correct option is (C).

Quick Tip to Solve: For selective electrolysis, compare reduction potentials first and then check whether the passed charge is enough to exhaust the preferred ion.

61. The molecule in which hybrid MOs involve only one d-orbital of the central atom is : [4 marks]

(A) XeF_4

(B) $[Ni(CN)_4]^{2-}$

(C) $[CrF_6]^{3-}$

(D) BrF_5

Correct Answer: (B) $[Ni(CN)_4]^{2-}$

Solution:

Concept:

- In the valence-bond hybridisation description used here, the superscript on d counts the participating d orbitals. Thus dsp^2 uses one d orbital, whereas d^2sp^3 and sp^3d^2 use two.
- The central-ion electronic configuration and ligand strength determine the hybridisation of a metal complex. For main-group species, count both bond pairs and lone pairs to obtain the steric number.

Step 1: Identify the square-planar complex In $[Ni(CN)_4]^{2-}$, four cyanide ligands contribute a total charge of -4, so nickel is $+2$. Its configuration is $3d^8$. Cyanide is a strong-field ligand and pairs the d electrons, making one inner d orbital available. The four bonds use one d, one s and two p orbitals: dsp^2. The resulting square-planar complex therefore meets the requirement of exactly one d orbital.

Step 2: Eliminate the other geometries For XeF_4, xenon has four bond pairs and two lone pairs. Its steric number is six, giving sp^3d^2 in the conventional hybridisation model. In $[CrF_6]^{3-}$, chromium is $+3$, or d^3, and the octahedral complex uses d^2sp^3. For BrF_5, five bond pairs and one lone pair again give steric number six and sp^3d^2. Each of A, C and D therefore involves two d orbitals rather than one.

Step 3: Conclusion Only $[Ni(CN)_4]^{2-}$ has dsp^2 hybridisation involving one d orbital. Hence, the correct option is (B).

Quick Tip to Solve: For this type of question, write the hybridisation first and then read the exponent on d; do not count only the visible bonds in xenon or bromine compounds.

62. The one that can exhibit highest paramagnetic behaviour among the following is :

gly = glycinato; bpy = 2, 2'-bipyridine [4 marks]

(A) $[Fe(en)(bpy)(NH_3)_2]^{2+}$

(B) $[Pd(gly)_2]$

(C) $[Co(OX)_2(OH)_2]^-$ ($\Delta_0 > P$)

(D) $[Ti(NH_3)_6]^{3+}$

Correct Answer: (C) $[Co(OX)_2(OH)_2]^-$ ($\Delta_0 > P$)

Solution:

Concept:

- Paramagnetism increases with the number n of unpaired electrons. The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, so compare unpaired-electron counts rather than merely atomic numbers.
- Find the metal oxidation state before finding its d-electron count. Neutral ligands such as en, bpy and ammonia do not change the charge balance; glycinato and hydroxido are singly negative and oxalato is doubly negative.

Step 1: Count unpaired electrons in the cobalt option In $[Co(ox)_2(OH)_2]^-$, the two oxalato ligands supply -4 and the hydroxido ligands supply -2. Charge balance gives $x - 6 = -1$, hence cobalt is $+5$ and has a d^4 configuration. The condition $\Delta_o > P$ specifies low spin. The octahedral occupancy is therefore $t_{2g}^4 e_g^0$: the fourth electron pairs in one lower orbital, leaving two unpaired electrons. Its moment is $\sqrt{8}\,\mu_B$.

Step 2: Compare the remaining three complexes In A all ligands are neutral, giving Fe^{2+}, a d^6 ion. With the strong-field en and bpy ligands this is the low-spin t_{2g}^6 arrangement, with no unpaired electrons. In B two glycinato ligands give Pd^{2+}, a $4d^8$ ion: its square-planar complex is diamagnetic. In D ammonia is neutral, so Ti^{3+} is d^1 and has one unpaired electron, giving $\sqrt{3}\,\mu_B$. Thus their unpaired-electron counts are respectively 0, 0 and 1, all below the cobalt complex's count of 2.

Step 3: Conclusion $[Co(ox)_2(OH)_2]^-$ has the largest number of unpaired electrons, two, and hence the highest paramagnetic behaviour. Hence, the correct option is (C).

Quick Tip to Solve: Do the ligand-charge sum carefully: the cobalt ion here is +5, not the more familiar +3. That changes the d count and the magnetic prediction.

63. The reaction in which the hybridisation of the underlined atom is affected is : [4 marks]

(A) $\underline{Xe}F_4 + SbF_5 \longrightarrow$

(B) $H_2\underline{S}O_4 + NaCl \xrightarrow{420\,K}$

(C) $H_3\underline{P}O_2 \xrightarrow{\text{Disproportionation}}$

(D) $\underline{N}H_3 \xrightarrow{H^+}$

Correct Answer: (A) $\underline{Xe}F_4 + SbF_5 \longrightarrow$

Solution:

Concept:

- Hybridisation in the conventional electron-pair model follows the steric number: count sigma bonds plus lone pairs at the specified atom. Six domains correspond to sp^3d^2, five to sp^3d, and four to sp^3.
- A change in oxidation state or in the number of lone pairs does not by itself prove a hybridisation change. Compare the total number of electron domains before and after reaction.

Step 1: Follow fluoride abstraction from xenon tetrafluoride SbF_5 is a Lewis acid and accepts fluoride from XeF_4:

$$XeF_4 + SbF_5 \longrightarrow [XeF_3]^+ + [SbF_6]^-.$$

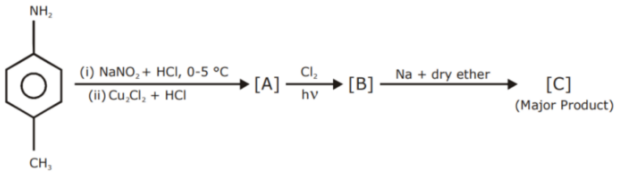
The underlined xenon initially has four Xe–F bonds and two lone pairs, hence six domains. In $XeF_3{}^+$, xenon has three sigma bonds and two lone pairs, hence five domains. Its conventional hybridisation changes from sp^3d^2 to sp^3d.

Step 2: Check why the other reactions do not qualify At $420\,K$, H_2SO_4 reacts with $NaCl$ to give $NaHSO_4$ and HCl. Sulfur remains surrounded by four oxygen atoms and remains sp^3. Disproportionation is $2\,H_3PO_2 \longrightarrow H_3PO_4 + PH_3$: phosphorus has four domains in H_3PO_2 and H_3PO_4, while three bonds and one lone pair in PH_3 also give four. Finally $NH_3 + H^+ \longrightarrow NH_4{}^+$ replaces the nitrogen lone pair with a bond; three bonds plus a lone pair become four bonds, so nitrogen stays sp^3. Options B, C and D do not change the hybridisation.

Step 3: Conclusion Only fluoride abstraction from XeF_4 changes the underlined atom's hybridisation, from sp^3d^2 to sp^3d. Hence, the correct option is (A).

Quick Tip to Solve: Protonating ammonia changes its shape from pyramidal to tetrahedral, but not its steric number. Count electron domains before deciding that hybridisation changed.

64. In the following reaction sequence, [C] is : [4 marks]



(A) p-$CH_3-C_6H_4-C_6H_4-CH_3$-p (4,4'-dimethylbiphenyl)

(B) p-$ClCH_2-C_6H_4-C_6H_4-CH_2Cl$-$p$ (4,4'-bis(chloromethyl)biphenyl)

(C) p-$ClC_6H_4-CH_2-CH_2-C_6H_4Cl-p$

(D) p-$ClC_6H_4-CH_2-C_6H_4-CH_2Cl-p$

Correct Answer: (C) p-$ClC_6H_4-CH_2-CH_2-C_6H_4Cl-p$

Solution:

Concept:

- Diazotisation converts a primary aromatic amino group into a diazonium group at 0–$5\,^\circ C$. Cuprous chloride then replaces that group by chlorine in the Sandmeyer reaction.
- Chlorination under light proceeds by a free-radical route at the benzylic side chain. Sodium in dry ether couples benzylic halides by the Wurtz reaction, joining the two carbons that bore chlorine.

Step 1: Identify the two intermediates The starting compound is p-toluidine, p-$CH_3C_6H_4NH_2$. Treatment with $NaNO_2/HCl$ forms the aryl diazonium salt, and Cu_2Cl_2/HCl replaces the diazonium group by Cl. Therefore $[A]$ is p-chlorotoluene, p-$ClC_6H_4CH_3$. The next reagents are Cl_2 and light, not a Lewis-acid catalyst for aromatic chlorination. Benzylic substitution gives $[B]$ = p-$ClC_6H_4CH_2Cl$, with the ring chlorine retained.

Step 2: Couple the benzyl chloride units and compare options The benzylic C–Cl bond reacts readily with sodium in dry ether, so two benzyl units couple:

$$2\,\text{p-}ClC_6H_4CH_2Cl + 2\,Na \longrightarrow \text{p-}ClC_6H_4CH_2CH_2C_6H_4Cl\text{-p} + 2\,NaCl.$$

The product is 1,2-bis(4-chlorophenyl)ethane, containing a $-CH_2-CH_2-$ bridge and one para chlorine on each ring. A joins the rings directly and retains methyl groups. B also joins the rings directly and retains chloromethyl groups. D gives a one-carbon bridge with a terminal chloromethyl group, so it is not the symmetrical coupling of two benzylic sites. Only C has both the correct bridge and the retained aryl chlorines.

Step 3: Conclusion $[C]$ is p-$ClC_6H_4-CH_2-CH_2-C_6H_4Cl-p$, formed by coupling the benzylic carbons. Hence, the correct option is (C).

Quick Tip to Solve: In a sequence containing both aryl and benzylic chlorine, mark the chlorine introduced under light: sodium preferentially couples the reactive benzylic sites here.

65. The Crystal Field Stabilization Energy (CFSE) of $[CoF_3(H_2O)_3]$ ($\Delta_0 < P$) is : [4 marks]

(A) -0.8 Δ_0

(B) -0.4 Δ_0

(C) -0.8 Δ_0 + 2P

(D) -0.4 Δ_0 + P

Correct Answer: (B) -0.4 Δ_0

Solution:

Concept:

- For an octahedral field, each t_{2g} electron contributes $-0.4\Delta_o$, while each e_g electron contributes $+0.6\Delta_o$, measured relative to the unsplit d-orbital barycentre.
- The condition $\Delta_o < P$ means that pairing costs more energy than promotion into the upper set. The complex is therefore high spin, and the electrons first occupy all five d orbitals singly.

Step 1: Determine the oxidation state and occupancy The neutral complex $[\mathrm{CoF_3(H_2O)_3}]$ contains three singly negative fluorido ligands and three neutral water ligands. Consequently cobalt is in the $+3$ state. Cobalt is group 9, so $\mathrm{Co^{3+}}$ has d^6 configuration. In the weak octahedral field specified, five electrons first occupy the five orbitals singly; the sixth pairs in a lower t_{2g} orbital. Thus the high-spin occupancy is $t_{2g}^4 e_g^2$, with four unpaired electrons.

Step 2: Evaluate the orbital stabilisation Substitute the occupancies into the octahedral expression:

$$\begin{aligned}\mathrm{CFSE} &= 4(-0.4\Delta_o) + 2(+0.6\Delta_o) \\ &= -1.6\Delta_o + 1.2\Delta_o = -0.4\Delta_o.\end{aligned}$$

The negative sign indicates net orbital stabilisation. This high-spin d^6 arrangement has the same one electron pair as the free-ion high-spin reference, so no additional pairing-energy cost is added to the CFSE quoted here. A uses an incorrect orbital total. C has both an incorrect coefficient and extra pairing terms; D incorrectly adds P to this orbital CFSE. The required stabilisation is the expression in B.

Step 3: Conclusion The high-spin configuration is $t_{2g}^4 e_g^2$, giving $\mathrm{CFSE} = -0.4\Delta_o$. Hence, the correct option is (B).

Quick Tip to Solve: Write the actual high-spin occupancy before calculating CFSE. For d6 it is four lower-set electrons and two upper-set electrons, not five and one.

66. Among the following compounds, which one has the shortest C – Cl bond? [4 marks]

(A) CH_3-Cl

(B) $(CH_3)_3C-Cl$

(C) $CH_2=CH-CH_2-Cl$

(D) $CH_2=CH-Cl$

Correct Answer: (D) $CH_2=CH-Cl$

Solution:

Concept:

- Bond length decreases as bond order and the s character of the carbon bonding orbital increase. A bond from an sp^2 carbon is generally shorter than the corresponding bond from an sp^3 carbon.
- In a vinyl halide, a chlorine lone pair can conjugate directly with the adjacent carbon–carbon pi system. This gives the C–Cl bond partial double-bond character, strengthening and shortening it.

Step 1: Identify the carbon directly bonded to chlorine Option A is methyl chloride, CH_3Cl, with chlorine attached to an sp^3 carbon. Option B is tert-butyl chloride, $(CH_3)_3CCl$, again an sp^3 C–Cl bond. Option C is allyl chloride, $CH_2=CHCH_2Cl$: although it contains a double bond, the carbon directly carrying chlorine is the saturated CH_2 carbon and is sp^3. Option D is vinyl chloride, $CH_2=CHCl$, where chlorine is attached directly to an sp^2 carbon.

Step 2: Compare overlap and conjugation The sp^2 orbital in vinyl chloride has greater s character than the sp^3 orbitals in A–C. Its electron density lies closer to the carbon nucleus, shortening the sigma bond. In addition, donation of a chlorine lone pair into the adjoining pi system produces a resonance contributor with a C=Cl bond and charge separation. Consequently the real C–Cl bond has partial double-bond character. Allyl chloride does not receive this direct conjugative shortening, because its saturated CH_2 carbon interrupts the pi connection to chlorine. Neither methyl chloride nor tert-butyl chloride has the vinyl conjugation. Thus all three alternatives have a longer C–Cl bond than D.

Step 3: Conclusion Vinyl chloride, $CH_2=CHCl$, has the shortest C–Cl bond owing to the sp^2 carbon and partial double-bond character. Hence, the correct option is (D).

Quick Tip to Solve: Locate the carbon bonded to Cl, not just any double bond in the molecule: allyl chloride has an sp3 C–Cl bond, whereas vinyl chloride has an sp2 C–Cl bond.

67. The processes of calcination and roasting in metallurgical industries, respectively, can Iead to : [4 marks]

(A) Global warming and photochemical smog

(B) Photochemical smog and ozone layer depletion

(C) Photochemical smog and global warming

(D) Global warming and acid rain

Correct Answer: (D) Global warming and acid rain

Solution:

Concept:

- Calcination heats an ore in the absence or limited supply of air; carbonate ores decompose to oxides and release CO_2.
- Roasting heats sulphide ores in excess air, producing the metal oxide and SO_2. Carbon dioxide is a greenhouse gas, whereas sulphur dioxide is an acid-rain precursor.

Step 1: Identify the gas evolved during calcination A typical carbonate ore undergoes the thermal decomposition

$$ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2.$$

The carbon dioxide released absorbs outgoing infrared radiation from the Earth. Its increased atmospheric concentration strengthens the greenhouse effect and can therefore contribute to global warming. This identifies the first member of the required pair; the heating process itself is not the atmospheric phenomenon.

Step 2: Identify the gas evolved during roasting A representative sulphide roasting reaction is

$$2\,ZnS + 3\,O_2 \xrightarrow{\Delta} 2\,ZnO + 2\,SO_2.$$

Atmospheric oxidation of sulphur dioxide, followed by reaction with water, produces sulphuric acid. Deposition of these acidic products contributes to acid rain. Thus the second member must be acid rain, rather than global warming, photochemical smog or ozone-layer depletion.

Step 3: Eliminate the remaining pairs Options (B) and (C) assign photochemical smog to calcination, although the characteristic carbonate decomposition product is CO_2. Option (A) gives photochemical smog for roasting; that phenomenon mainly involves sunlight-driven reactions of nitrogen oxides and hydrocarbons, and does not describe the characteristic sulphur-dioxide effect identified here.

Step 4: Conclusion Calcination can contribute to global warming, while roasting of sulphide ores can contribute to acid rain. Hence, the correct option is (D).

Quick Tip to Solve: Write the gas evolved first: carbonate calcination gives CO_2, while sulphide roasting gives SO_2; then match each gas to its environmental effect.

68. An alkaline earth metal 'M' readily forms water soluble sulphate and water insoluble hydroxide. Its oxide MO is very stable to heat and does not have rock-salt structure. M is : [4 marks]

(A) Ca

(B) Mg

(C) Sr

(D) Be

Correct Answer: (D) Be

Solution:

Concept:

- In Group 2, sulphate solubility generally decreases down the group, while hydroxide solubility generally increases.
- The small, strongly polarising Be^{2+} ion gives beryllium compounds appreciable covalent character; BeO has tetrahedral coordination rather than the rock-salt arrangement.

Step 1: Use the solubility clues The required metal has a water-soluble sulphate but an insoluble hydroxide. Beryllium satisfies both: $BeSO_4$ is soluble in water, while $Be(OH)_2$ is insoluble in water and amphoteric. Its dissolution in acids or strong alkalis does not contradict the water-insolubility clue. These observations point towards the small metal ion at the top of Group 2.

Step 2: Apply the decisive oxide-structure clue Beryllium oxide is a thermally very stable solid with the wurtzite-type structure, in which the ions have tetrahedral coordination. It therefore does not possess the six-coordinate rock-salt structure. The strong bonding in BeO is consistent with its high thermal stability. The oxide clue distinguishes beryllium from magnesium even though magnesium sulphate is also soluble and magnesium hydroxide is sparingly soluble.

Step 3: Compare all the proposed metals For calcium and strontium, sulphates are much less soluble as one proceeds down the group, and their oxides have the rock-salt arrangement. Magnesium oxide also has the rock-salt structure and thus fails the explicit structural condition. Beryllium alone combines the stated solubility behaviour with an oxide lacking that arrangement; satisfying only the sulphate clue would not be sufficient.

Step 4: Conclusion The metal is beryllium, Be. Its oxide has the wurtzite-type structure. Hence, the correct option is (D).

Quick Tip to Solve: When Group 2 solubility clues leave Mg and Be plausible, use the oxide: BeO is tetrahedrally coordinated, while MgO has rock-salt coordination.

69. A sample of red ink (a colloidal suspension) is prepared by mixing eosin dye, egg white, HCHO and water. The component which ensures stability of the ink sample is : [4 marks]

(A) HCHO

(B) Water

(C) Eosin dye

(D) Egg white

Correct Answer: (D) Egg white

Solution:

Concept:

- A lyophobic colloid is relatively susceptible to coagulation when its particles aggregate; a protective lyophilic colloid improves its stability.
- Proteins in egg white are hydrophilic macromolecules. Their adsorption and hydration provide a protective layer around dispersed particles, hindering their approach and aggregation.

Step 1: Recognise the role required by the question The question asks which ingredient stabilises a colloidal ink, rather than which gives it its red colour or supplies its liquid medium. Stability requires the dispersed particles to remain separated instead of forming larger aggregates that can settle. A component that coats the particles and maintains their interaction with water can provide this protection.

Step 2: Identify the protective component Egg white contains proteins such as albumin. These behave as lyophilic colloids in water and can adsorb around the dispersed dye particles. The hydrated protein coating provides a barrier to close approach of neighbouring particles and reduces their tendency to coagulate. Egg white therefore acts as the protective colloid in this preparation, allowing the red colloidal dispersion to remain stable.

Step 3: Distinguish the other ingredients Eosin is the dye responsible for the red colour; identifying the colour-producing component does not answer the question about colloidal protection. Water is the dispersion medium in which the components are mixed, not the protective macromolecule. Formaldehyde, HCHO, serves as a preservative in the preparation rather than the lyophilic protective colloid. Consequently, options (A), (B) and (C) do not identify the stabilising component requested.

Step 4: Conclusion Egg white supplies the protective protein colloid that stabilises the ink suspension. Hence, the correct option is (D).

Quick Tip to Solve: Separate the roles in a colloidal formulation: dye gives colour, water is the medium, preservative prevents spoilage, and a hydrated protein supplies colloidal protection.

70. The mechanism of action of "Terfenadine" (Seldane) is : [4 marks]

(A) Activates the histamine receptor

(B) Helps in the secretion of histamine

(C) Inhibits the secretion of histamine

(D) Inhibits the action of histamine receptor

Correct Answer: (D) Inhibits the action of histamine receptor

Solution:

Concept:

- Histamine produces its effects by binding to histamine receptors. Blocking receptor action can reduce the response without preventing histamine production or secretion.
- Terfenadine is an antihistamine acting at peripheral H_1 receptors; in the terminology of this question, it inhibits the action of the histamine receptor.

Step 1: Locate the stage at which the drug acts A signalling molecule such as histamine must interact with a receptor before the associated biological response occurs. A drug can interfere either with production or release of the signalling molecule, or with its action at the target receptor. These mechanisms are distinct: a reduction in the response does not by itself imply that less histamine has been secreted.

Step 2: Apply the antihistamine mechanism Terfenadine belongs to the antihistamine class. It blocks histamine-mediated action at peripheral H_1 receptors, reducing effects associated with allergic responses. Thus its relevant site of action is the receptor, rather than the cellular machinery that produces or secretes histamine. The wording "inhibits the action of histamine receptor" corresponds to this receptor-blocking mechanism.

Step 3: Eliminate activation and secretion mechanisms Option (A) would activate the receptor and promote, rather than oppose, its histamine-like effects. Option (B) proposes increased secretion of histamine, which also does not describe an antihistamine action. Option (C) shifts the mechanism to inhibition of histamine secretion; terfenadine is identified here by inhibition at the receptor itself. Only option (D) matches the specified drug class and site of action.

Step 4: Conclusion Terfenadine inhibits histamine-receptor action, thereby opposing the effects of histamine. Hence, the correct option is (D).

Quick Tip to Solve: For an antihistamine question, distinguish receptor blocking from suppression of histamine secretion: preventing the response need not prevent release of the messenger.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The number of chiral centres present in threonine is _____. [4 marks]

Solution:

Concept:

- A tetrahedral carbon atom is a chiral centre when its four attached substituents are different. Each complete substituent must be compared, not merely its first atom.
- Threonine is 2-amino-3-hydroxybutanoic acid, with condensed structure:

$$\mathrm{HOOC{-}CH(NH_2){-}CH(OH){-}CH_3}.$$

Carbon atoms are numbered from the carboxyl carbon.

Step 1: Test the alpha carbon At carbon 2, the tetrahedral carbon bearing the amino group is attached to H, $\mathrm{NH_2}$, COOH, and $\mathrm{CH(OH)CH_3}$. These are four distinct groups. In particular, the two carbon-containing substituents differ because one is a carboxyl group and the other is a hydroxyethyl group. Hence carbon 2 is a chiral centre.

Step 2: Test the beta carbon At carbon 3, the carbon bearing the hydroxyl group is attached to H, OH, $\mathrm{CH_3}$, and $\mathrm{CH(NH_2)COOH}$. These four substituents are also different, so carbon 3 is a second chiral centre. The presence of the hydroxyl substituent in threonine therefore creates an additional stereogenic carbon beyond the usual alpha carbon of many amino acids.

Step 3: Exclude the remaining carbon atoms The carboxyl carbon is trigonal planar, not tetrahedral, so it is not counted as a carbon chiral centre. The terminal methyl carbon has three identical hydrogen substituents and therefore cannot be chiral. There are no other carbon atoms to inspect in the four-carbon chain. Counting only the two tetrahedral carbons with four unequal groups gives the requested number; the question asks for centres rather than the number of possible stereoisomers.

Step 4: Conclusion Threonine has $\boxed{2}$ chiral centres, at carbon 2 and carbon 3. The requested count is dimensionless.

Quick Tip to Solve: Write threonine as $\mathrm{HOOC{-}CH(NH_2){-}CH(OH){-}CH_3}$, then test the two CH carbons individually for four different substituents.

72. The number of molecules with energy greater than the threshold energy for a reaction increases five fold by a rise of temperature from 27°C to 42°C. Its energy of activation in J/mol is _____.
(Take ln 5 = 1.6094; R = 8.314 J mol^{-1} K^{-1}) [4 marks]

Solution:

Concept:

- The fraction of molecules exceeding an activation threshold varies as $e^{-E_a/(RT)}$, where E_a is the molar activation energy and R is the gas constant.
- For the same sample, the ratio of the numbers above the threshold at two temperatures is $N_2/N_1 =$

$\exp[(E_a/R)(1/T_1 - 1/T_2)]$. Temperatures in this expression must be in kelvin.

Step 1: Convert the temperatures and form the ratio The rise from $27^\circ\mathrm{C}$ to $42^\circ\mathrm{C}$ corresponds to $T_1 = 300\,\mathrm{K}$ and $T_2 = 315\,\mathrm{K}$, using the usual conversion for the given data. The number of molecules with sufficient energy increases fivefold, so $N_2/N_1 = 5$. The total population cancels when the two exponential fractions are divided, leaving only the temperature-dependent activation factor.

Step 2: Take logarithms and rearrange Taking natural logarithms gives

$$\ln 5 = \frac{E_a}{R}\left(\frac{1}{300} - \frac{1}{315}\right).$$

The reciprocal-temperature difference is positive because the higher-temperature sample contains more molecules above the threshold. Simplify it before substitution:

$$\frac{1}{300} - \frac{1}{315} = \frac{15}{300 \times 315} = \frac{1}{6300}\,\mathrm{K}^{-1}.$$

Therefore

$$E_a = R(\ln 5)(6300\,\mathrm{K}).$$

Step 3: Substitute the supplied constants Using the printed values $R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$ and $\ln 5 = 1.6094$,

$$E_a = 8.314 \times 1.6094 \times 6300 = 84297.47412\,\mathrm{J\,mol^{-1}}.$$

The kelvin factor cancels the inverse-kelvin unit in the gas constant. The result is positive, as expected for an activation barrier whose high-energy population increases with temperature.

Step 4: Conclusion The activation energy is $\boxed{84297.47\,\mathrm{J\,mol^{-1}}}$ (approximately $84.30\,\mathrm{kJ\,mol^{-1}}$).

Quick Tip to Solve: In the two-temperature equation, compute $1/T_1 - 1/T_2 = (T_2 - T_1)/(T_1T_2)$ first; this avoids subtracting rounded reciprocal temperatures.

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73. The osmotic pressure of a solution of NaCl is 0.10 atm and that of a glucose solution is 0.20 atm. The osmotic pressure of a solution formed by mixing 1 L of the sodium chloride solution with 2 L of the glucose solution is x $\times$ 10^{-3} atm. x is _____. (nearest integer) [4 marks]

Solution:

Concept:

- For a dilute solution, osmotic pressure obeys $\pi = iCRT$, where i accounts for the number of solute particles and C is the molar concentration.
- At the same temperature, $\pi V = n_{\text{particles}}RT$. On mixing nonreacting solutions, their effective particle amounts add, while dilution occurs into the total volume.

Step 1: Express the particle contributions before mixing Let the common temperature be T. The sodium chloride solution contributes an effective particle amount proportional to $\pi_1 V_1$, and the glucose solution contributes one proportional to $\pi_2 V_2$. The measured pressure of the sodium chloride

solution already includes its dissociation into ions, so an additional factor of two must not be inserted.

$$n_{1,\text{particles}}RT = (0.10)(1), \qquad n_{2,\text{particles}}RT = (0.20)(2).$$

Here pressures are in atmospheres and volumes are in litres; the same units are used in both terms.

Step 2: Calculate the pressure after dilution There is no reaction removing the dissolved particles. With additive volumes, the final volume is $1 + 2 = 3\,\text{L}$. Therefore,

$$\pi_{\text{mix}} = \frac{\pi_1 V_1 + \pi_2 V_2}{V_1 + V_2} = \frac{0.10 \times 1 + 0.20 \times 2}{3} = \frac{1}{6}\,\text{atm}.$$

The result lies between the two original pressures and is closer to $0.20\,\text{atm}$, as expected because twice as much glucose solution is mixed.

Step 3: Convert to the requested numerical coefficient The question writes the pressure as $x \times 10^{-3}\,\text{atm}$. Thus,

$$x = \frac{1/6}{10^{-3}} = 166.666\ldots.$$

Rounding this coefficient, rather than the pressure in atmospheres, to the nearest integer gives the requested entry.

Step 4: Conclusion The mixture has osmotic pressure $0.166666\ldots\,\text{atm}$, so the required nearest-integer coefficient is $\boxed{x = 167}$ (dimensionless).

Quick Tip to Solve: Use a volume-weighted average of the measured osmotic pressures at a common temperature; dissociation has already been included in those measured pressures.

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74. Consider the following equations :

$2Fe^{2+} + H_2O_2 \rightarrow xA + yB$

(in basic medium)

$2MnO_4^- + 6H^+ + 5H_2O_2 \rightarrow x'C + y'D + z'E$

(in acidic medium)

The sum of the stoichiometric coefficients x, y, x', y', and z' for products A, B, C, D and E, respectively, is _____. [4 marks]

Solution:

Concept:

- A balanced ionic redox equation conserves both atoms and net charge. Oxidation and reduction half-reactions are multiplied so that their transferred electrons cancel.
- Hydrogen peroxide is reduced to hydroxide ions when it oxidises ferrous ions in basic medium. In acidic permanganate solution, it acts as a reducing agent and is oxidised to oxygen.

Step 1: Balance the reaction in basic medium Each ferrous ion loses one electron, whereas one

peroxide molecule accepts two electrons. The half-reactions are

$$2\,Fe^{2+} \longrightarrow 2\,Fe^{3+} + 2\,e^-, \qquad H_2O_2 + 2\,e^- \longrightarrow 2\,OH^-.$$

Adding gives the ionic equation in the form requested:

$$2\,Fe^{2+} + H_2O_2 \longrightarrow 2\,Fe^{3+} + 2\,OH^-.$$

Thus the product coefficients are $x = 2$ and $y = 2$. The charge on each side is $+4$: on the right it is $+6 - 2$. Oxygen and hydrogen atoms also balance. The equation is written in ionic form as in the question, rather than adding an extra hydroxide precipitation reaction.

Step 2: Balance the acidic permanganate reaction Permanganate is reduced from manganese in oxidation state $+7$ to $+2$, accepting five electrons per manganese. Peroxide loses two electrons per molecule:

$$2\,MnO_4^- + 16\,H^+ + 10\,e^- \longrightarrow 2\,Mn^{2+} + 8\,H_2O,$$
$$5\,H_2O_2 \longrightarrow 5\,O_2 + 10\,H^+ + 10\,e^-.$$

Cancel the electrons and ten hydrogen ions to obtain

$$2\,MnO_4^- + 6\,H^+ + 5\,H_2O_2 \longrightarrow 2\,Mn^{2+} + 8\,H_2O + 5\,O_2.$$

Its product coefficients are $x' = 2$, $y' = 8$ and $z' = 5$, with their sum unchanged if the product letters are assigned in another order. Both sides contain two manganese atoms, eighteen oxygen atoms and sixteen hydrogen atoms; both sides have net charge $+4$.

Step 3: Add only the requested product coefficients The coefficients of the reactants are not included. Using the balanced product coefficients from both equations,

$$x + y + x' + y' + z' = 2 + 2 + 2 + 8 + 5 = 19.$$

Step 4: Conclusion The sum of the five product stoichiometric coefficients is $\boxed{19}$, a dimensionless integer.

Quick Tip to Solve: First cancel electrons and any common hydrogen ions, then count only the product coefficients; counting uncancelled half-reactions gives the wrong sum.

75. A 100 mL solution was made by adding 1.43 g of $Na_2CO_3.xH_2O$. The normality of the solution is 0.1 N. The value of x is _____.

(The atomic mass of Na is 23 g/mol) [4 marks]

Solution:

Concept:

- Normality is the number of equivalents of solute per litre of solution: $N = w/(EV)$, where w is mass in grams, E is equivalent mass in grams per equivalent, and V is volume in litres.
- In complete acid neutralisation, each carbonate ion consumes two hydrogen ions, so the n-factor

of sodium carbonate is two. Water of crystallisation increases the molar mass but does not change this neutralising capacity.

Step 1: Find the equivalents present in the solution Convert the final solution volume to litres before using its normality. The given $100\,\text{mL}$ is $0.100\,\text{L}$, so

$$\text{equivalents} = NV = (0.1\,\text{eq}\,\text{L}^{-1})(0.100\,\text{L}) = 0.0100\,\text{eq}.$$

The mass $1.43\,\text{g}$ refers to the whole hydrated salt, including its bound water, rather than to anhydrous sodium carbonate alone.

Step 2: Determine equivalent mass and molar mass The equivalent mass of the hydrate is therefore

$$E = \frac{1.43\,\text{g}}{0.0100\,\text{eq}} = 143\,\text{g}\,\text{eq}^{-1}.$$

Complete neutralisation is represented by

$$\text{CO}_3{}^{2-} + 2\,\text{H}^+ \longrightarrow \text{CO}_2 + \text{H}_2\text{O}.$$

Consequently one mole of the hydrate supplies two equivalents. Its molar mass must be

$$M = 2E = 286\,\text{g}\,\text{mol}^{-1}.$$

Using a factor of one would confuse molarity with normality and would give an inconsistent hydration number.

Step 3: Calculate the number of water molecules Using the given sodium atomic mass and the usual atomic masses $\text{C} = 12$, $\text{O} = 16$, $\text{H} = 1$, the anhydrous part has molar mass

$$M(\text{Na}_2\text{CO}_3) = 2(23) + 12 + 3(16) = 106\,\text{g}\,\text{mol}^{-1}.$$

Each mole of water contributes $18\,\text{g}\,\text{mol}^{-1}$, giving

$$106 + 18x = 286, \qquad x = \frac{180}{18} = 10.$$

As a check, $1.43/286 = 0.005$ mole in 0.100 litre has molarity $0.05\,\text{mol}\,\text{L}^{-1}$; multiplying by two recovers the stated normality of $0.1\,\text{N}$.

Step 4: Conclusion The hydration number is $\boxed{x = 10}$, a dimensionless count, and the salt is $\text{Na}_2\text{CO}_3 \cdot 10\,\text{H}_2\text{O}$.

Quick Tip to Solve: For a carbonate hydrate under complete neutralisation, obtain molar mass as twice the equivalent mass, then subtract 106 and divide the remaining mass by 18.