JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 5 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 5 September 2020, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the function

$$f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$$ **is**

twice differentiable, then the ordered pair (k_1, k_2) is equal to : [4 marks]

(A) $\left(\frac{1}{2}, -1\right)$

(B) (1, 1)

(C) (1, 0)

(D) $\left(\frac{1}{2}, 1\right)$

Correct Answer: (D) $\left(\frac{1}{2}, 1\right)$

Solution:

Concept:

- A piecewise function can be twice differentiable at its joining point only if the function and its first derivative join continuously there, and the derivative itself is differentiable.
- For the polynomial branch, $f\prime(x) = 2k_1(x-\pi)$ and $f\prime\prime(x) = 2k_1$. For the cosine branch, $f\prime(x) = -k_2 \sin x$ and $f\prime\prime(x) = -k_2 \cos x$.

Step 1: Enforce continuity at the joining point The left branch gives $f(\pi) = -1$, independently of k_1. The limiting value of the right branch is $k_2 \cos \pi = -k_2$. Differentiability implies continuity, so these values must coincide:

$$-1 = -k_2 \quad \Longrightarrow \quad k_2 = 1.$$

Thus option (A), with $k_2 = -1$, and option (C), with $k_2 = 0$, already fail continuity.

Step 2: Match the first and second derivatives With $k_2 = 1$, both one-sided first derivatives at π equal zero: the polynomial gives zero and the cosine gives $-\sin \pi = 0$. For the second derivative, differentiate these first-derivative branches and match their limits:

$$2k_1 = -\cos \pi = 1 \quad \Longrightarrow \quad k_1 = \frac{1}{2}.$$

These equalities also ensure existence of the second derivative at the joining point by the one-sided difference quotients of $f\prime$. Option (B) has $k_1 = 1$, which would give the unequal second derivatives 2 and 1.

Step 3: Conclusion The required ordered pair is $\boxed{(k_1, k_2) = (\frac{1}{2}, 1)}$. Hence, the correct option is (D).

Quick Tip to Solve: At a piecewise join, test continuity first, then match successive derivatives; each condition usually determines a different parameter.

2. If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$

are m and M respectively, then the ordered pair (m,M) is equal to : [4 marks]

(A) $\left(0,2\sqrt{2}\right)$

(B) (-4, 0)

(C) (-4, 4)

(D) (0, 4)

Correct Answer: (B) (-4, 0)

Solution:

Concept:

- Subtracting one row from another preserves a determinant. Expanding along a row with a zero entry reduces this determinant to two small minors.
- Write $a=\sin^2\theta$, $b=\cos^2\theta$. Then $b-a=\cos 2\theta$, whose range follows from the given interval for θ.

Step 1: Reduce the determinant Apply $R_1\leftarrow R_1-R_2$. The first row becomes $(b-a,b-a,0)$. Expanding along that row, with the alternating cofactor signs, gives

$$\begin{aligned}f(\theta)&=(b-a)\{2(1+b)-10\}\\&\quad-(b-a)\{2b-12\}\\&=4(b-a)=4\cos 2\theta.\end{aligned}$$

The third-column zero removes the third cofactor; keeping the minus sign of the second cofactor is essential.

Step 2: Find the full attained range The restriction $\pi/4\le\theta\le\pi/2$ gives $\pi/2\le 2\theta\le\pi$. On this interval cosine decreases continuously from 0 to -1. Therefore

$$-4\le f(\theta)\le 0.$$

Both endpoints are attained: $f(\pi/4)=0$ and $f(\pi/2)=-4$. Hence the minimum is -4 and the maximum is 0. Options (A) and (D) incorrectly make the determinant nonnegative, while option (C) includes the unattainable positive value 4.

Step 3: Conclusion The ordered pair is $\boxed{(m,M)=(-4,0)}$. Hence, the correct option is (B).

Quick Tip to Solve: When two trigonometric rows differ only through sine-squared and cosine-squared, subtract them before expanding the determinant.

3. If y = y(x) is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}+e^x=0$ satisfying y(0) = 1, then a value of $y(\log_e 13)$ is : [4 marks]

(A) -1

(B) 1

(C) 0

(D) 2

Correct Answer: (A) -1

Solution:

Concept:

- Rearrange a separable differential equation so that the terms involving y multiply dy, and those involving x multiply dx.
- The initial condition selects the integration constant. Here $5+e^x>0$ for every real x, so the logarithm of this expression is well defined.

Step 1: Separate and integrate The equation can be rearranged without changing its solutions on the initial-condition branch as

$$\frac{dy}{y+2}=-\frac{e^x\,dx}{5+e^x}.$$

Integrating the left side directly and using $u=5+e^x$, $du=e^x dx$, on the right yields

$$\log|y+2|=-\log(5+e^x)+C.$$

Consequently $(y+2)(5+e^x)=K$ for a nonzero constant K. The value $y=-2$ would make the original equation undefined and is excluded.

Step 2: Use the initial value and evaluate At $x=0$, the condition $y=1$ gives

$$K=(1+2)(5+1)=18.$$

Hence the solution through the prescribed point is

$$y(x)=\frac{18}{5+e^x}-2.$$

At $x=\log_e 13$, we have $e^x=13$, so

$$y(\log_e 13)=\frac{18}{18}-2=-1.$$

This solution has $y+2>0$ for finite x, consistent with the branch chosen by the initial value. The other listed values 1, 0 and 2 do not satisfy the constant-product relation at $e^x=13$.

Step 3: Conclusion The required value is $\boxed{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: After separating this equation, keep the integrated result as a constant product; it makes substitution of the initial value immediate.

4. Let $\lambda \in \mathbf{R}$. The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

is inconsistent for: [4 marks]

(A) exactly one positive value of λ

(B) exactly one negative value of λ

(C) exactly two values of λ

(D) every value of λ

Correct Answer: (B) exactly one negative value of λ

Solution:

Concept:

- A square linear system has a unique solution whenever its coefficient determinant is nonzero. Only zero-determinant parameter values can produce inconsistency.
- At a singular value, eliminate a variable and compare the remaining equations including their right-hand sides; proportional coefficients with unequal constants prove inconsistency.

Step 1: Find the possible exceptional values For the coefficient matrix, expansion along the first row gives

$$\begin{aligned}\Delta &= 2(-24+10) + 4(4-\lambda) \\ &\quad + \lambda(-10+6\lambda) \\ &= 6\lambda^2 - 14\lambda - 12 \\ &= 2(3\lambda+2)(\lambda-3).\end{aligned}$$

Thus all real values except 3 and $-2/3$ give a unique solution. Both exceptional values must be checked; a zero determinant alone does not establish inconsistency.

Step 2: Check each singular value Subtract twice equation 2 from equation 1, and subtract λ times equation 2 from equation 3. This yields

$$\begin{aligned}8x_2 + (\lambda-2)x_3 &= -3, \\ (6\lambda-10)x_2 + (4-\lambda)x_3 &= 3-2\lambda.\end{aligned}$$

At $\lambda = 3$, both reduce to $8x_2 + x_3 = -3$, so the system is consistent. At $\lambda = -2/3$, the second left side is $-7/4$ times the first left side. Consistency would require its constant to be $21/4$, but the actual constant is $13/3$. These are unequal, proving inconsistency. Thus there is one negative exceptional value and no positive inconsistent value; this excludes (A), (C) and (D).

Step 3: Conclusion The system is inconsistent only for $\boxed{\lambda = -\frac{2}{3}}$, exactly one negative value. Hence, the correct option is (B).

Quick Tip to Solve: Factor the coefficient determinant first, but always check the augmented equations at each root before declaring a system inconsistent.

5. If S is the sum of the first 10 terms of the series

$$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\tan^{-1}\left(\frac{1}{21}\right)+....$$

then tan(S) is equal to : [4 marks]

(A) $\frac{10}{11}$
(B) $\frac{5}{11}$
(C) $-\frac{6}{5}$
(D) $\frac{5}{6}$

Correct Answer: (D) $\frac{5}{6}$

Solution:

Concept:

- The denominators $3, 7, 13, 21, \ldots$ have the form n^2+n+1 for $n = 1, 2, 3, 4, \ldots$.
- The tangent subtraction identity gives $\tan^{-1}(n+1)-\tan^{-1} n = \tan^{-1}(1/(n^2+n+1))$. Both sides lie between 0 and $\pi/2$, so there is no branch adjustment.

Step 1: Write a typical term as a difference For positive integer n, the tangent of the difference is

$$\tan\{\tan^{-1}(n+1)-\tan^{-1} n\} = \frac{(n+1)-n}{1+n(n+1)} = \frac{1}{n^2+n+1}.$$

Since the difference is a positive acute angle, taking the principal inverse tangent is legitimate. Thus each term equals $\tan^{-1}(n+1)-\tan^{-1} n$. The first denominator corresponds to $n = 1$, so the tenth term corresponds to $n = 10$.

Step 2: Telescope and take the tangent In the sum from $n = 1$ to 10, every intermediate inverse tangent cancels:

$$\begin{aligned} S &= \sum_{n=1}^{10}\{\tan^{-1}(n+1)-\tan^{-1} n\} \\ &= \tan^{-1} 11-\tan^{-1} 1. \end{aligned}$$

Now use the subtraction formula once more:

$$\tan S = \frac{11-1}{1+11\cdot 1} = \frac{10}{12} = \frac{5}{6}.$$

The sum is positive and acute, so a negative value such as option (C) is impossible. Options (A) and (B) do not equal the endpoint expression; the numerator and denominator must both come from the tangent subtraction formula.

Step 3: Conclusion The required value is $\boxed{\tan S = \frac{5}{6}}$. Hence, the correct option is (D).

Quick Tip to Solve: For inverse-tangent sums with denominator n^2+n+1, try consecutive arguments $n+1$ and n; the resulting differences telescope.

6. If

$\int \left(e^{2x} + 2e^{x} - e^{-x} - 1\right) e^{\left(e^{x}+e^{-x}\right)} dx = g\left(x\right) e^{\left(e^{x}+e^{-x}\right)} + c$

where c is a constant of integration, then g(0) is equal to : [4 marks]

(A) 1

(B) 2

(C) e

(D) e^2

Correct Answer: (B) 2

Solution:

Concept:

- For an exponential product, $\frac{d}{dx}[g(x)e^{h(x)}] = [g\prime(x) + g(x)h\prime(x)]e^{h(x)}$. Match the factor outside the exponential.
- Here $h(x) = e^x + e^{-x}$, so $h\prime(x) = e^x - e^{-x}$. The term e^{2x} suggests trying $g(x) = e^x + a$ for a constant a.

Step 1: Match the product derivative Substitute $g = e^x + a$ into the product-rule factor:

$$\begin{aligned} g\prime + gh\prime &= e^x + (e^x + a)(e^x - e^{-x}) \\ &= e^{2x} + (a+1)e^x - ae^{-x} - 1. \end{aligned}$$

Comparing this with the given factor $e^{2x} + 2e^x - e^{-x} - 1$ gives $a + 1 = 2$ and $-a = -1$, both yielding $a = 1$. Therefore the natural prefactor of the antiderivative is $g(x) = e^x + 1$.

Step 2: Verify and evaluate the prefactor Differentiate the proposed antiderivative directly:

$$\begin{aligned} \frac{d}{dx}\{(e^x + 1)e^{e^x+e^{-x}}\} &= \{e^x + (e^x + 1)(e^x - e^{-x})\}e^{e^x+e^{-x}} \\ &= (e^{2x} + 2e^x - e^{-x} - 1)e^{e^x+e^{-x}}. \end{aligned}$$

This reproduces the integrand exactly, including the negative e^{-x} term. The additive integration constant is represented separately by c in the question. Evaluate the prefactor itself, rather than the whole antiderivative: $g(0) = e^0 + 1 = 2$. Option (A) misses the added 1; options (C) and (D) confuse the prefactor with an exponential factor.

Step 3: Conclusion The prefactor is $g(x) = e^x + 1$, and $\boxed{g(0) = 2}$. Hence, the correct option is (B).

Quick Tip to Solve: When an integrand is a polynomial in e^x times an exponential, differentiate a simple prefactor times that exponential before attempting substitution.

7. If α is positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim_{x\to\alpha^+} \frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) $\frac{3}{\sqrt{2}}$

(D) $\frac{3}{2}$

Correct Answer: (C) $\frac{3}{\sqrt{2}}$

Solution:

Concept:

- The factorisation $p(x) = x^2 - x - 2 = (x-2)(x+1)$ identifies the positive root and the small quantity entering the cosine.
- Use $1 - \cos u = 2\sin^2(u/2)$ and $\lim_{v\to 0} \sin v/v = 1$. Taking a square root produces an absolute value, whose sign must be decided from the direction of approach.

Step 1: Find the root and determine the sign The roots are 2 and -1, so $\alpha = 2$. The denominator becomes $x - 2$. As $x \to 2^+$, both $x - 2$ and $x + 1$ are positive, hence $p(x) > 0$ and tends to zero from above. Therefore $\sin(p(x)/2) > 0$ sufficiently near the limit point. This sign check is necessary: the square root is always nonnegative, regardless of the sign of its argument before squaring.

Step 2: Factor the expression into standard limits Writing $p = p(x)$, we have $\sqrt{1-\cos p} = \sqrt{2}\,|\sin(p/2)|$. On the stated right-hand approach this equals $\sqrt{2}\sin(p/2)$, so

$$\frac{\sqrt{1-\cos p(x)}}{x-2} = \frac{1}{\sqrt{2}} \frac{\sin(p(x)/2)}{p(x)/2} \frac{p(x)}{x-2} = \frac{x+1}{\sqrt{2}} \frac{\sin(p(x)/2)}{p(x)/2}.$$

The sine quotient tends to 1, while $x + 1 \to 3$. Thus the limit is $3/\sqrt{2}$. Option (A) omits the factor 3 coming from the polynomial; (B) and (D) use an incorrect factor of 2 in place of $\sqrt{2}$.

Step 3: Conclusion The required right-hand limit is $\boxed{3/\sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: After using the half-angle identity, keep the absolute value until the side of approach fixes its sign; then factor the polynomial.

8. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to : [4 marks]

(A) 8

(B) 9

(C) 16

(D) 6

Correct Answer: (A) 8

Solution:

Concept:

- An ellipse with equation $x^2/a^2 + y^2/b^2 = 1$, where $a > b > 0$, has foci $(\pm c, 0)$, with $c^2 = a^2 - b^2$.
- For every point on an ellipse, the sum of its distances from the two foci equals the major-axis length $2a$. The major semiaxis is the square root of the larger denominator.

Step 1: Put the conic in standard form Divide the given equation by 144:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1.$$

Both coefficients are positive, so the conic is an ellipse centred at the origin. Its major axis lies along the x-axis because $16 > 9$. Consequently $a^2 = 16$, $b^2 = 9$, and $a = 4$, $b = 3$. These are lengths, so their positive square roots are used.

Step 2: Identify the given points and use the distance sum The focal parameter is

$$c = \sqrt{a^2 - b^2} = \sqrt{16 - 9} = \sqrt{7}.$$

Thus the given points $A(\sqrt{7}, 0)$ and $B(-\sqrt{7}, 0)$ are precisely the foci. The ellipse definition now gives $PA + PB = 2a = 8$, independently of where P lies on the curve. As a direct check, at the vertex $P = (4, 0)$, the distances are $4 - \sqrt{7}$ and $4 + \sqrt{7}$, whose sum is 8. Option (B) confuses the denominator $b^2 = 9$ with a distance; (C) uses $a^2 = 16$ instead of $2a$; (D) uses the minor-axis length $2b = 6$.

Step 3: Conclusion For every allowed point P, $\boxed{PA + PB = 8}$. Hence, the correct option is (A).

Quick Tip to Solve: Before applying the constant focal-distance sum, verify that the given points really are the foci using $c^2 = a^2 - b^2$.

9. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$ is : [4 marks]

(A) $\frac{5}{9}$

(B) $\frac{5}{27}$

(C) $\frac{25}{81}$

(D) $\frac{25}{9}$

Correct Answer: (C) $\frac{25}{81}$

Solution:

Concept:

- For a real variable, $x^2 = |x|^2$. The substitution $t = |x| \geq 0$ converts the equation into an ordinary quadratic.
- Each positive root t yields two distinct real roots $x = t$ and $x = -t$. The product requested must include all roots of the original equation, not just the positive ones.

Step 1: Solve for the magnitude of each root Let $t = |x|$. Substitution gives

$$9t^2 - 18t + 5 = 0 = (3t - 1)(3t - 5).$$

Therefore $t = 1/3$ or $t = 5/3$. Both values meet $t \geq 0$, so neither is rejected. Notice that ordinary Vieta applied at this stage gives the product of the two magnitude values, not yet the product of all original x-roots.

Step 2: Recover all signed roots and multiply The first magnitude gives $x = \pm 1/3$, and the second gives $x = \pm 5/3$. Substitution confirms both signs work because the equation contains only x^2 and $|x|$. These four distinct real roots exhaust the possibilities. Their product is

$$\left(\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(-\frac{5}{3}\right) = \left(-\frac{1}{9}\right)\left(-\frac{25}{9}\right) = \frac{25}{81}.$$

Option (A), $5/9$, is only the product of the two positive magnitudes. Option (B) omits factors from the four-root product, and (D) does not contain the full denominator 3^4. In particular, two negative roots make the final product positive.

Step 3: Conclusion The product of all roots is $\boxed{25/81}$. Hence, the correct option is (C).

Quick Tip to Solve: For an equation quadratic in $|x|$, first solve for nonnegative magnitudes, then restore both signs before taking a product.

10. If $3^{2\sin 2\alpha-1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth terms of this A.P. is: [4 marks]

(A) 66

(B) 81

(C) 65

(D) 78

Correct Answer: (A) 66

Solution:

Concept:

- Three consecutive terms a, b, c form an arithmetic progression when $a + c = 2b$. Their common difference is $d = b - a$.
- For real α, $-1 \le \sin 2\alpha \le 1$. This bounds the possible exponential term and selects the valid root after solving the algebraic condition.

Step 1: Use the sum and product of the outer terms Write $u = 3^{2\sin 2\alpha-1}$ and $v = 3^{4-2\sin 2\alpha}$. Since the middle term is 14, the A.P. condition gives $u + v = 28$. Adding their exponents also gives

$$uv = 3^{(2\sin 2\alpha-1)+(4-2\sin 2\alpha)} = 3^3 = 27.$$

Hence u and v are roots of $z^2 - 28z + 27 = 0$, which factors as $(z - 1)(z - 27) = 0$. The two outer terms are therefore 1 and 27, but their order must still be fixed.

Step 2: Select the valid first term and find the sixth The exponent of u lies in $[-3, 1]$, so

$$\frac{1}{27} \le u \le 3.$$

Thus $u = 27$ is impossible and the first term is $u = 1$. This is attainable: $2\sin 2\alpha - 1 = 0$ requires $\sin 2\alpha = 1/2$, which has real solutions. Therefore $d = 14 - 1 = 13$, and

$$a_6 = a_1 + 5d = 1 + 5(13) = 66.$$

The initial terms $1, 14, 27$ check the required difference directly. Option (C), 65, counts only five differences and omits the first term; (B) and (D) are not obtained from the valid first term and difference.

Step 3: Conclusion The sixth term of the progression is $\boxed{66}$. Hence, the correct option is (A).

Quick Tip to Solve: When two exponential A.P. terms have complementary exponents, use their product as well as their sum; then enforce the sine range.

11. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, -4), then PQ^2 is equal to: [4 marks]

(A) 36

(B) 48

(C) 21

(D) 29

Correct Answer: (A) 36

Solution:

Concept:

- The squared distance from $P(x, y)$ to $Q(0, -4)$ is $PQ^2 = x^2 + (y + 4)^2$. Maximising the square also maximises the nonnegative distance.
- The ellipse constraint can eliminate x^2, leaving a function of y on a closed interval. A derivative sign test on that interval establishes the global maximum.

Step 1: Eliminate one coordinate using the ellipse From $4x^2 + 5y^2 = 20$, we obtain

$$x^2 = 5 - \frac{5}{4}y^2.$$

Since $x^2 \geq 0$, the possible values of y satisfy $-2 \leq y \leq 2$. Now substitute in the distance:

$$PQ^2 = 5 - \frac{5}{4}y^2 + (y + 4)^2 = 21 + 8y - \frac{1}{4}y^2.$$

This substitution respects the full ellipse: for each y in this interval, at least one corresponding real x exists.

Step 2: Maximise over the permitted interval Let $F(y) = 21 + 8y - y^2/4$. Then

$$F\prime(y) = 8 - \frac{y}{2} \geq 7 > 0 \qquad (-2 \leq y \leq 2).$$

Thus the squared distance is strictly increasing throughout the allowed interval and reaches its largest value at $y = 2$. The ellipse equation then gives $x = 0$, so the farthest point is $P = (0, 2)$, directly above Q. Consequently

$$PQ^2 = 21 + 16 - 1 = 36.$$

Option (C), 21, corresponds to $y = 0$, not the maximum. The value 29 is below the maximum, while 48 exceeds the proven upper bound. No interior stationary point lies in the allowed interval: $F\prime(y) = 0$ would require $y = 16$.

Step 3: Conclusion The farthest point is $(0, 2)$, giving $\boxed{PQ^2 = 36}$. Hence, the correct option is (A).

Quick Tip to Solve: For distances to an ellipse, eliminate the squared coordinate using the constraint and check the resulting function only on its feasible interval.

12. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be : [4 marks]

(A) 63

(B) 36

(C) 54

(D) 38

Correct Answer: (B) 36

Solution:

Concept:

- For two sets, inclusion-exclusion gives $n(C \cup T) = n(C) + n(T) - n(C \cap T)$. Percentages may be used in the same identity when they refer to one population.
- A union cannot exceed the whole population, and an intersection cannot exceed either of its component sets. These conditions give lower and upper bounds for the overlap.

Step 1: Find the possible interval for the overlap Let C be the coffee-likers and T the tea-likers. The survey gives $n(C) = 73\%$, $n(T) = 65\%$, and $n(C \cap T) = x\%$. Hence

$$n(C \cup T) = 73 + 65 - x = 138 - x.$$

Because the union is at most 100%, we need $138 - x \leq 100$, or $x \geq 38$. Also the common group is contained in the smaller group, so $x \leq 65$. Thus $38 \leq x \leq 65$. The data do not require everybody to like at least one drink, so the overlap need not equal 38.

Step 2: Test all four proposed values The choices 63, 54, and 38 lie in the permitted interval. To confirm feasibility, split the population into coffee only, tea only, both, and neither; these percentages are

$$73 - x, \quad 65 - x, \quad x, \quad x - 38.$$

For each of those three choices all four numbers are nonnegative and sum to 100. But $x = 36$ gives a union of $138 - 36 = 102\%$, equivalently a negative -2% for neither. This is impossible in any population. Therefore only option (B) violates the survey constraints; options (A), (C), and (D) can all occur.

Step 3: Conclusion The overlap cannot be $\boxed{36\%}$. Hence, the correct option is (B).

Quick Tip to Solve: For two percentage groups, the overlap lies between $\max(0, a+b-100)$ and $\min(a, b)$; do not assume the union is exactly 100%.

13. If the four complex numbers $z, \overline{z}, \overline{z} - 2\mathrm{Re}(\overline{z})$ and $z - 2Re(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to : [4 marks]

(A) $4\sqrt{2}$

(B) 4

(C) 2

(D) $2\sqrt{2}$

Correct Answer: (D) $2\sqrt{2}$

Solution:

Concept:

- Write a complex number as $z = x + iy$, where x and y are its real and imaginary parts. Its modulus is the distance from the origin: $|z| = \sqrt{x^2 + y^2}$.
- Conjugation reflects a point in the real axis. Subtracting twice the real part reverses its real coordinate, so the four given points have coordinates $(\pm x, \pm y)$.

Step 1: Identify the four vertices The four complex numbers become

$$z = x + iy, \quad \overline{z} = x - iy, \quad \overline{z} - 2\,\mathrm{Re}(\overline{z}) = -x - iy, \quad z - 2\,\mathrm{Re}(z) = -x + iy.$$

Thus they are the corners of a rectangle centred at the origin. Distinct square vertices require both coordinates to be nonzero. Their horizontal and vertical side lengths are respectively $2|x|$ and $2|y|$.

Step 2: Use the stated side length For this rectangle to be a square of side 4, both side lengths must equal 4:

$$2|x| = 2|y| = 4, \qquad |x| = |y| = 2.$$

Therefore

$$|z| = \sqrt{x^2 + y^2} = \sqrt{4 + 4} = 2\sqrt{2}.$$

Equivalently, the origin is the centre and the modulus is half the square diagonal, not its full diagonal. The value $4\sqrt{2}$ is the full diagonal, 4 is the side, and 2 is half the side; none is the distance from the centre to a vertex.

Step 3: Conclusion The modulus is $\boxed{2\sqrt{2}}$ units. Hence, the correct option is (D).

Quick Tip to Solve: For vertices $(\pm x, \pm y)$, first read the side lengths as $2|x|$ and $2|y|$; the modulus is half the diagonal.

14. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is : [4 marks]

(A) 2

(B) 3

(C) 1

(D) 4

Correct Answer: (A) 2

Solution:

Concept:

- For n observations, the mean satisfies $\sum x_i = n\overline{x}$. Population variance satisfies $\sigma^2 = \frac{1}{n}\sum x_i^2 - \overline{x}^2$; the divisor here is n, not $n-1$.
- If the missing observations are u, v, their absolute difference follows from $(u-v)^2 = 2(u^2+v^2) - (u+v)^2$. This avoids solving separately for their order.

Step 1: Find the sum of the missing observations The sum of all seven observations is

$$7 \times 8 = 56.$$

The five listed values have sum

$$2 + 4 + 10 + 12 + 14 = 42.$$

Therefore the missing observations satisfy

$$u + v = 56 - 42 = 14.$$

The mean alone fixes their sum but does not fix their difference, so the variance is also needed.

Step 2: Recover the sum of squares and the difference Using the given variance,

$$\sum x_i^2 = 7(16 + 8^2) = 7 \times 80 = 560.$$

The known squares add to

$$2^2 + 4^2 + 10^2 + 12^2 + 14^2 = 4 + 16 + 100 + 144 + 196 = 460.$$

Hence $u^2 + v^2 = 100$, and

$$(u-v)^2 = 2(100) - 14^2 = 200 - 196 = 4.$$

Taking the nonnegative square root gives $|u - v| = 2$. As a check, the missing numbers are 6 and 8: their sum is 14 and their squares sum to 100. Differences 3, 1 and 4 would give squared differences 9, 1 and 16, contrary to the computed value 4.

Step 3: Conclusion The absolute difference of the remaining observations is $\boxed{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the mean to get the missing sum and the variance to get the missing sum of squares; then compute the difference without a quadratic.

15. If (a, b, c) is the image of the point (1, 2, -3) in the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a + b + c is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) -1

Correct Answer: (B) 2

Solution:

Concept:

- A point on the line can be written $H = (-1, 3, 0) + t(2, -2, -1)$. The vector $\mathbf{d} = (2, -2, -1)$ gives the direction of the reflecting line.
- For reflection in a line, the perpendicular foot H is the midpoint of the original point P and its image P_{im}. Thus $(P - H) \cdot \mathbf{d} = 0$ and $P_{\text{im}} = 2H - P$.

Step 1: Locate the perpendicular foot Let $P = (1, 2, -3)$, and choose $A = (-1, 3, 0)$ on the line. Then $P - A = (2, -1, -3)$. The projection parameter is

$$t = \frac{(P - A) \cdot \mathbf{d}}{\mathbf{d} \cdot \mathbf{d}} = \frac{2(2) + (-1)(-2) + (-3)(-1)}{2^2 + (-2)^2 + (-1)^2} = \frac{9}{9} = 1.$$

Consequently the foot is

$$H = (-1, 3, 0) + (2, -2, -1) = (1, 1, -1).$$

This projection uses the whole direction vector; reflecting separately in coordinate planes would not reflect in this line.

Step 2: Reflect through the foot and add the coordinates The image point is

$$P_{\text{im}} = 2(1, 1, -1) - (1, 2, -3) = (1, 0, 1).$$

Therefore

$$a + b + c = 1 + 0 + 1 = 2.$$

Check that the midpoint of $(1, 2, -3)$ and $(1, 0, 1)$ is $(1, 1, -1)$, which lies on the line. Also $(P - H) \cdot \mathbf{d} = (0, 1, -2) \cdot (2, -2, -1) = 0$, confirming perpendicularity. The alternatives 1, 3 and -1 fail this coordinate sum; the foot itself is not the reflected image.

Step 3: Conclusion The image is $(a, b, c) = (1, 0, 1)$, so $\boxed{a + b + c = 2}$. Hence, the correct option is (B).

Quick Tip to Solve: For reflection in a line, compute its perpendicular foot first and double that foot before subtracting the original point.

16. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}}dx$ is: [4 marks]

(A) π

(B) $\frac{3\pi}{2}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Concept:

- On a symmetric interval, replacing x by $-x$ preserves the value of an integral after reversing its limits. Here $\sin(-x) = -\sin x$.
- For any real t, $\frac{1}{1+e^t} + \frac{1}{1+e^{-t}} = 1$. Pairing the two integrands removes the exponential without needing an antiderivative.

Step 1: Write the reflected integral Denote the given integral by

$$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{1+e^{\sin x}}.$$

Substitute $x = -u$; then $dx = -du$, the limits interchange, and $\sin x = -\sin u$. Reversing the limits cancels the minus sign. Renaming the integration variable gives

$$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{1+e^{-\sin x}}.$$

The denominator is always positive, so both continuous integrands are defined over the entire interval.

Step 2: Add the two forms Add the two equal expressions for I. With $t = \sin x$, multiplying the second fraction by e^t/e^t yields

$$\frac{1}{1+e^t} + \frac{e^t}{1+e^t} = 1.$$

Therefore

$$2I = \int_{-\pi/2}^{\pi/2} 1\,dx = \pi, \qquad I = \frac{\pi}{2}.$$

The interval has length π, and the paired values average to $1/2$, providing a direct check. Options π and $3\pi/2$ are too large for an integrand strictly between 0 and 1; $\pi/4$ contradicts the exact pairing identity.

Step 3: Conclusion The integral is $\boxed{\pi/2}$. Hence, the correct option is (C).

Quick Tip to Solve: When an odd function occurs in $1/(1+e^{g(x)})$ on symmetric limits, pair the values at x and $-x$.

17. If the common tangent to the parabolas,
$y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$,
then c is equal to : [4 marks]

(A) $\frac{1}{\sqrt{2}}$
(B) $\frac{1}{2\sqrt{2}}$
(C) $\frac{1}{2}$
(D) $\frac{1}{4}$

Correct Answer: (A) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The tangent of slope $m \neq 0$ to $y^2 = 4ax$ is $y = mx + a/m$. For $x^2 = 4ay$, interchanging the coordinates gives $x = ny + a/n$. Here $a = 1$.
- A line tangent to a circle centred at the origin has distance from the origin equal to its radius. For $Ax + By + C = 0$, that distance is $|C|/\sqrt{A^2 + B^2}$.

Step 1: Find the common tangent Let the common tangent have equation $y = mx + k$. Tangency to $y^2 = 4x$ gives $k = 1/m$. For the other parabola, write its tangent as $x = ny + 1/n$. Since $n = 1/m$, this becomes

$$x = \frac{y}{m} + m, \qquad y = mx - m^2.$$

Thus the same line requires

$$\frac{1}{m} = -m^2, \qquad m^3 = -1.$$

The real slope is $m = -1$ and the intercept is $k = -1$. The common tangent is therefore $x + y + 1 = 0$. A vertical or horizontal line cannot supply a missed common tangent: the vertex tangents to the two parabolas are different axes.

Step 2: Use tangency to the circle The circle $x^2 + y^2 = c^2$ is centred at the origin with radius $|c|$. The distance from its centre to the common tangent is

$$r = \frac{|1|}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}.$$

Hence the positive value of c in the options is $1/\sqrt{2}$. As a check, the perpendicular foot is $(-1/2, -1/2)$, whose squared distance from the origin is $1/2$. The other listed values have squares $1/8, 1/4, 1/16$; none gives this tangency distance.

Step 3: Conclusion The positive parameter in the options is $\boxed{c = 1/\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a common tangent, equate the two slope-intercept forms first; only then apply the circle-centre distance formula.

18. If $2^{10} + 2^9.3^1 + 2^8 .3^2 + \ldots\ldots + 2.3^9 + 3^{10} = S - 2^{11}$, then S is equal to : [4 marks]

(A) $\frac{3^{11}}{2} + 2^{10}$

(B) $3^{11} — 2^{12}$

(C) 2.3^{11}

(D) 3^{11}

Correct Answer: (D) 3^{11}

Solution:

Concept:

- The sum $a^n + a^{n-1}b + \cdots + b^n$ equals $(b^{n+1} - a^{n+1})/(b - a)$ for $a \neq b$. It follows by subtracting two shifted copies of the sum.
- The displayed terms are products of powers of 2 and 3, not binomial-expansion terms: there are no binomial coefficients. Successive terms have the common ratio $3/2$.

Step 1: Sum the eleven terms Let

$$T = 2^{10} + 2^9 \cdot 3 + 2^8 \cdot 3^2 + \cdots + 2 \cdot 3^9 + 3^{10}.$$

There are eleven terms, corresponding to powers of 3 from 0 through 10. Multiplying by 3 and by 2 and subtracting cancels all intermediate terms:

$$3T - 2T = 3^{11} - 2^{11}.$$

Hence

$$T = 3^{11} - 2^{11}.$$

Equivalently the geometric-series formula gives $2^{10}((3/2)^{11} - 1)/((3/2) - 1)$, with exactly the same result.

Step 2: Use the relation defining S The question states $T = S - 2^{11}$, so rearrange before substituting:

$$S = T + 2^{11} = (3^{11} - 2^{11}) + 2^{11} = 3^{11}.$$

The subtracted power cancels, rather than producing another negative contribution. Option $3^{11} - 2^{12}$ results from subtracting 2^{11} twice. Option $2 \cdot 3^{11}$ introduces an extra factor of two, while $3^{11}/2 + 2^{10}$ is incompatible with the eleven-term geometric sum. Thus only the last option satisfies the stated relation.

Step 3: Conclusion The required value is $\boxed{S = 3^{11}}$. Hence, the correct option is (D).

Quick Tip to Solve: For mixed powers $a^{n-k}b^k$, subtract aT from bT; all interior terms cancel and only the endpoint powers remain.

19. If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu. units, then : [4 marks]

(A) n = 7

(B) $\vec{b} . \vec{c} = 10$

(C) $\vec{a} . \vec{c} = 17$

(D) n = 9

Correct Answer: (B) $\vec{b} . \vec{c} = 10$

Solution:

Concept:

- The volume of a parallelepiped with coterminous edge vectors is the absolute scalar triple product: $V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$. The determinant must therefore be considered with both signs.
- The scalar product is obtained by multiplying corresponding Cartesian components and adding. The restriction $n \geq 0$ removes negative parameter values.

Step 1: Evaluate the volume determinant Using the three given vectors as the rows of a determinant gives

$$D = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 12 + n^2 - (6 + n) + n(2n - 4) = 3n^2 - 5n + 6.$$

Thus the volume condition is $|3n^2 - 5n + 6| = 158$, rather than the determinant itself being assigned a sign without checking.

Step 2: Find the admissible parameter For $D = 158$,

$$3n^2 - 5n - 152 = (n - 8)(3n + 19) = 0.$$

Only $n = 8$ satisfies the nonnegative restriction. The alternative $D = -158$ gives $3n^2 - 5n + 164 = 0$, whose discriminant is $25 - 1968 < 0$; it supplies no real parameter.

Step 3: Test the listed statements At $n = 8$, the two scalar products are

$$\mathbf{b} \cdot \mathbf{c} = 2 + 4n - 3n = 10, \qquad \mathbf{a} \cdot \mathbf{c} = 1 + n + 3n = 33.$$

Consequently the proposed values $n = 7$ and $n = 9$ are both false, and the proposed value 17 for the second scalar product is also false. Exactly the scalar-product statement in option (B) holds.

Step 4: Conclusion The admissible value is $n = 8$, and $\mathbf{b} \cdot \mathbf{c} = 10$. Hence, the correct option is (B).

Quick Tip to Solve: Compute the determinant before checking the statements, and retain the absolute value until both determinant signs have been tested.

20. The negation of the Boolean expression x $\leftrightarrow \sim$ y is equivalent to : [4 marks]

(A) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(B) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

(C) $(x \wedge y) \wedge (\sim x \vee \sim y)$

(D) $(x \wedge \sim y) \vee (\sim x \wedge y)$

Correct Answer: (A) $(x \wedge y) \vee (\sim x \wedge \sim y)$

Solution:

Concept:

- A biconditional $p \leftrightarrow q$ is true when its two statements have the same truth value. Its negation is true when their truth values differ.
- Here the second statement is $\neg y$, so the negated biconditional is true when x and y have the same truth value, not when they differ.

Step 1: Expand the given biconditional The original statement has the disjunctive normal form

$$x \leftrightarrow \neg y = (x \wedge \neg y) \vee (\neg x \wedge y).$$

It is therefore true precisely in the cases $(x, y) = (T, F)$ and (F, T). Negating the entire expression reverses its value in each of the four possible cases.

Step 2: List the cases that survive negation The negated statement is true for (T, T) and (F, F), and false for the other two assignments. These two true cases are represented respectively by $x \wedge y$ and $\neg x \wedge \neg y$. Combining them with OR gives

$$\neg(x \leftrightarrow \neg y) = (x \wedge y) \vee (\neg x \wedge \neg y).$$

This is exactly the expression in option (A).

Step 3: Exclude the other expressions Option (B) factors as $\neg x \wedge (y \vee \neg y) = \neg x$; at (F, T) it is true whereas the desired negation is false. Option (C) requires both x and y true and also at least one false, so it is always false. Option (D) is the original exclusive-OR expression: at (T, T) it is false although the required negation is true. Thus none of these three expressions is equivalent.

Step 4: Conclusion The negation is $(x \wedge y) \vee (\neg x \wedge \neg y)$. Hence, the correct option is (A).

Quick Tip to Solve: For a biconditional involving a negated variable, test the equal truth-value cases first; this prevents confusing equivalence with exclusive OR.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. If the line, 2x - y + 3 = 0 is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines 4x - 2y + α = 0 and 6x - 3y + β = 0, respectively, then the sum of all possible values of α and β is : [4 marks]

Solution:

Concept:

- The perpendicular distance between parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $|c_1 - c_2|/\sqrt{a^2 + b^2}$. Their coefficients of both variables must first be identical.
- An absolute distance generally allows two positions, one on either side of the reference line. Both signs must be included when finding all possible constants.

Step 1: Normalize the first pair of lines Divide $4x - 2y + \alpha = 0$ by 2 to obtain $2x - y + \alpha/2 = 0$. It is parallel to $2x - y + 3 = 0$ with a common normal of length $\sqrt{5}$. The prescribed distance gives

$$\frac{|\alpha/2 - 3|}{\sqrt{5}} = \frac{1}{\sqrt{5}} \quad \Longrightarrow \quad \alpha/2 - 3 = \pm 1.$$

The complete set of values is therefore $\alpha = 4, 8$. Keeping just one sign would omit a valid parallel line.

Step 2: Normalize the second pair of lines Similarly, division of $6x - 3y + \beta = 0$ by 3 gives $2x - y + \beta/3 = 0$. Its prescribed distance gives

$$\frac{|\beta/3 - 3|}{\sqrt{5}} = \frac{2}{\sqrt{5}} \quad \Longrightarrow \quad \beta/3 - 3 = \pm 2.$$

Consequently $\beta = 3, 15$. Each value gives the same required distance because the two possible lines lie on opposite sides of the reference line.

Step 3: Add all possible constant values The question asks for the sum of all possible values of the constants themselves. Thus add the two values of α and the two values of β, each once:

$$(4 + 8) + (3 + 15) = 12 + 18 = 30.$$

No coordinate of a point needs to be selected: normalizing the parallel line equations already reduces the distance calculation to their constant terms.

Step 4: Conclusion The sum of all possible values of α and β is $\boxed{30}$. This is a dimensionless numerical answer.

Quick Tip to Solve: Normalize parallel line equations before comparing constants; then use both signs in each absolute-value equation.

22. Let $f(x) = x.\left[\frac{x}{2}\right]$, for -10< x < 10, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to _____. [4 marks]

Solution:

Concept:

- The greatest integer function $\lfloor t \rfloor$ is constant between successive integers and jumps at each integer. Thus $\lfloor x/2 \rfloor$ can jump only at even integer values of x.
- A multiplier can remove a jump when it vanishes at the jump point. Continuity must therefore be checked for the product $x\lfloor x/2 \rfloor$, especially at $x = 0$.

Step 1: Identify every candidate point Write a candidate as $x = 2k$, where k is an integer. The open domain $-10 < x < 10$ requires

$$k = -4, -3, -2, -1, 0, 1, 2, 3, 4.$$

There are nine candidate locations. The endpoints -10 and 10 do not belong to the given domain, so they are not counted. At any other point the floor factor is locally constant and multiplication by x preserves continuity.

Step 2: Compare the one-sided limits Immediately to the left of $2k$, the floor factor equals $k - 1$; at and immediately to its right it equals k. Therefore

$$\lim_{x \to (2k)^-} f(x) = 2k(k-1), \qquad \lim_{x \to (2k)^+} f(x) = f(2k) = 2k^2.$$

The right limit minus the left limit is $2k$. For every nonzero value of k in the list this difference is nonzero, so those points are discontinuities.

Step 3: Check the removable candidate at zero At $k = 0$, both limits in the preceding calculation are zero and $f(0) = 0$. More directly, for small negative x, $f(x) = -x$, whereas for small positive x, $f(x) = 0$; both approach zero. Thus zero is continuous despite the jump in the floor factor. Removing this one candidate leaves $9 - 1 = 8$ discontinuity points.

Step 4: Conclusion The discontinuities are at $x = -8, -6, -4, -2, 2, 4, 6, 8$. Their number is $\boxed{8}$, a dimensionless count.

Quick Tip to Solve: For a floor function multiplied by another factor, test zeros of the multiplier separately; they can cancel an otherwise genuine jump.

23. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is _____. [4 marks]

Solution:

Concept:

- For one fair die, showing either a three or a five is a success with probability $p = 2/6 = 1/3$. Independent dice therefore produce a binomial count of successes.
- For repeated independent trials of an event of probability P, the expected number of occurrences

in N trials is NP. This expectation need not be interpreted as a guaranteed count.

Step 1: Define the count for one throw Let X be the number of the four dice that show one of the two specified faces. Each die either succeeds or fails independently, so

$$X \sim \text{Bin}(4, 1/3), \qquad P(X = r) = \binom{4}{r}(1/3)^r(2/3)^{4-r}.$$

The desired event is $X \geq 2$. It allows two, three or four successes; counting exactly two alone would leave out valid outcomes.

Step 2: Use the complementary event The complement consists only of zero successes or exactly one success. Consequently

$$\begin{aligned} P(X \geq 2) &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - \left(\frac{2}{3}\right)^4 - 4\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 \\ &= 1 - \frac{16}{81} - \frac{32}{81} = \frac{33}{81} = \frac{11}{27}. \end{aligned}$$

The factor 4 in the one-success term selects which die shows a three or a five.

Step 3: Apply expectation over the repeated throws Let Y count the throws, among the 27 repetitions, in which at least two dice succeed. Its success probability is $11/27$, obtained above, so

$$E(Y) = 27 \times \frac{11}{27} = 11.$$

Equivalently, each throw contributes an indicator with mean $11/27$, and adding the 27 indicator means gives the same result. Independence of the dice is used inside each throw, while the repetitions give the stated repeated-trial setting.

Step 4: Conclusion The expected number of occurrences is $\boxed{11}$ throws.

Quick Tip to Solve: For an at-least-two event, subtract zero and one from unity; this usually needs fewer binomial terms than direct addition.

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24. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is : [4 marks]

Solution:

Concept:

- The word SYLLABUS contains two copies each of S and L, and one copy each of Y, A, B and U. There are six distinct letter types altogether.
- The requested pattern is exactly $2 + 1 + 1$: one identical pair and two singleton letters, with the singleton types different from each other and from the paired type. For fixed letters, the arrangement count is $4!/2!$.

Step 1: Choose the type of the repeated pair Only S and L occur at least twice in the original word.

Thus there are two choices for the letter used as the identical pair. A singleton such as Y cannot be chosen for that role, since doing so would require a second copy absent from SYLLABUS. Treating the two copies of S or L as distinguishable would also count the same resulting word more than once.

Step 2: Choose the two distinct singleton types After choosing the pair, five other letter types remain. For example, when S is paired, the available types are L, Y, A, B and U. Choose two different types without regard to order:

$$\binom{5}{2} = 10.$$

The other repeated type is allowed as a singleton: selecting one L with a pair of S still has exactly the required pattern. Taking two Ls would instead produce two pairs and is excluded by the distinct-singleton choice.

Step 3: Arrange each selected multiset For each selection, the four letters consist of two identical copies of the pair type and two different singletons. Their distinct arrangements number

$$\frac{4!}{2!} = 12.$$

Multiplying the independent choices gives

$$2\binom{5}{2}\frac{4!}{2!} = 2 \times 10 \times 12 = 240.$$

Each word determines its unique pair type, its two singleton types and its order, so no word is counted twice. The question allows meaningless words, so no dictionary restriction is imposed.

Step 4: Conclusion The number of admissible four-letter words is $\boxed{240}$, a dimensionless count.

Quick Tip to Solve: First classify the multiplicity pattern, then choose letter types and arrange the multiset; repeated printed copies are never distinct objects.

25. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ..... [4 marks]

Solution:

Concept:

- The general term in $(a+b)^n$ is $T_{r+1} = \binom{n}{r}a^{n-r}b^r$, where r is an integer from 0 to n. Its numerical coefficient and its power of x must both satisfy the question.
- The binomial coefficients obey $\binom{n}{r} = \binom{n}{n-r}$. A coefficient can therefore correspond to two different term indices; both must be checked against the required natural-number exponent.

Step 1: Write the general term Choose r factors of x^{-2} and $22-r$ factors of x^m. Their contribution is

$$T_{r+1} = \binom{22}{r}(x^m)^{22-r}(x^{-2})^r = \binom{22}{r}x^{m(22-r)-2r}.$$

For this to be the term in x, its exponent must be exactly 1, rather than 0 or the numerical coefficient.

Step 2: Locate the coefficient 1540 The first coefficients are $1, 22, 231, 1540$, since

$$\binom{22}{3} = \frac{22 \cdot 21 \cdot 20}{3 \cdot 2 \cdot 1} = 1540.$$

The coefficients increase up to the middle and then repeat symmetrically. Thus the only possible indices are $r = 3$ and $r = 19$. No other term has the specified coefficient.

Step 3: Impose the power condition For $r = 3$, the exponent condition gives $19m - 6 = 1$, so $m = 7/19$, which is not a natural number. For $r = 19$,

$$3m - 38 = 1 \quad \Rightarrow \quad m = 13.$$

Checking directly, the term is $\binom{22}{19} x^{39-38} = 1540x$, so both the coefficient and exponent are correct.

Step 4: Conclusion The required natural number is $\boxed{m = 13}$. It is dimensionless.

Quick Tip to Solve: When a binomial coefficient is specified, test both symmetric term indices before imposing the requested power of the variable.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to?
(Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, g = 980 cm/s^2, density of mercury = 13.6 g/cm^3) [4 marks]

(A) 5.8×10^{18}
(B) 4.0×10^{16}
(C) 5.8×10^{16}
(D) 4.0×10^{18}

Correct Answer: (D) 4.0×10^{18}

Solution:

Concept:

- For an ideal gas, $PV = Nk_BT$, while the mean translational kinetic energy per molecule is $\bar{K} = \frac{3}{2}k_BT$. Eliminating temperature gives $PV = \frac{2}{3}N\bar{K}$.
- The pressure of a mercury column is $P = \rho gh$. Use one consistent unit system: in cgs, pressure times volume is measured in erg, matching the kinetic energy supplied.

Step 1: Find the gas pressure The given pressure is a mercury column of height $h = 2\,\text{cm}$, not 2 units of absolute pressure. With $\rho = 13.6\,\text{g cm}^{-3}$ and $g = 980\,\text{cm s}^{-2}$,

$$P = 13.6 \times 980 \times 2 = 26656\,\text{dyn cm}^{-2}.$$

The volume is already in cgs units, $V = 4\,\mathrm{cm}^3$.

Step 2: Eliminate the unknown temperature There is no need to determine T or to insert a separate value of Boltzmann's constant. The kinetic-theory relation immediately gives

$$N = \frac{3PV}{2\bar{K}} = \frac{3(26656)(4)}{2(4 \times 10^{-14})}.$$

Here $PV = 106624\,\mathrm{erg}$, so dividing by the energy per molecule leaves a dimensionless count.

Step 3: Evaluate and compare the choices The numerator is $319872\,\mathrm{erg}$, and hence

$$N = 3.9984 \times 10^{18} \simeq 4.0 \times 10^{18}.$$

Options (B) and (C) have an exponent two orders too small; option (A) has the right order but an incorrect numerical factor. Option (D) matches both the magnitude and the coefficient.

Step 4: Conclusion The number of molecules is approximately $\boxed{4.0 \times 10^{18}}$. Hence, the correct option is (D).

Quick Tip to Solve: If molecular kinetic energy is given, use $N = 3PV/(2\bar{K})$ directly and keep all quantities in the same energy unit.

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27. An electron is constrained to move along the y-axis with a speed of 0.1 c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is
$\vec{E} = 30\hat{j}\sin\left(1.5 \times 10^7 t - 5 \times 10^{-2} x\right)$ **V/m.**
The maximum magnetic force experienced by the electron will be :
(given c = 3 $\times$ 10^8 ms^{-1} and electron charge = 1.6 $\times$ 10^{-19} C) [4 marks]

(A) 4.8 $\times$ 10^{-19} N
(B) 2.4 $\times$ 10^{-18} N
(C) 3.2 $\times$ 10^{-18} N
(D) 1.6 $\times$ 10^{-18} N
Correct Answer: (A) 4.8 $\times$ 10^{-19} N

Solution:

Concept:

- In a plane electromagnetic wave, the electric and magnetic fields are mutually perpendicular and transverse to the propagation direction. Their amplitudes satisfy $E_0 = cB_0$.
- The magnetic part of the Lorentz force has magnitude $F_B = |q|vB\sin\theta$, where θ is the angle between the particle velocity and the magnetic field. It is distinct from the electric force.

Step 1: Identify the field directions The phase has the form $\omega t - kx$, so the wave propagates along the positive x-axis. Its electric field is along $\hat{j}$, the y-direction. The magnetic field is therefore along the z-axis, with $\vec{E} \times \vec{B}$ pointing along positive x. Since the electron moves along y, its velocity is perpendicular to $\vec{B}$ and $\sin\theta = 1$.

Step 2: Calculate the magnetic amplitude The sine factor has maximum magnitude 1, so $E_0 = 30\,\mathrm{V\,m^{-1}}$. The magnetic amplitude is

$$B_0 = \frac{30}{3 \times 10^8} = 1.0 \times 10^{-7}\,\mathrm{T}.$$

The speed of the electron is $v = 0.1c = 3.0 \times 10^7\,\mathrm{m\,s^{-1}}$. The field varies in time, but its maximum magnitude is sufficient for the requested maximum force.

Step 3: Evaluate the maximum magnetic force Using the magnitude of the electron charge,

$$F_{B,\max} = (1.6 \times 10^{-19})(3.0 \times 10^7)(1.0 \times 10^{-7}) = 4.8 \times 10^{-19}\,\mathrm{N}.$$

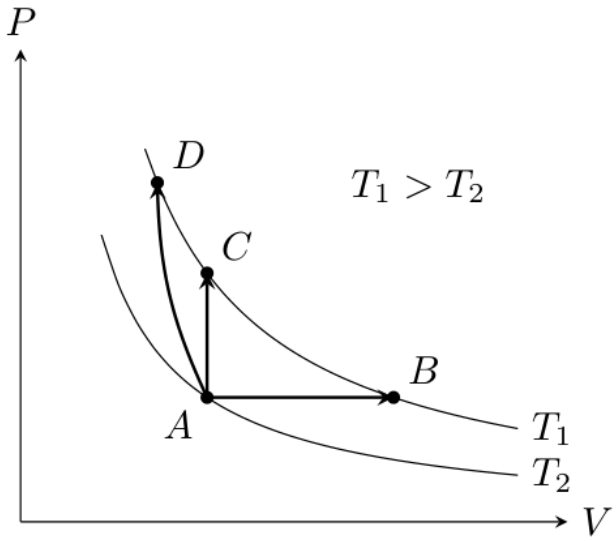
The negative charge reverses the force direction, not its magnitude. The other three options exceed this value and do not equal evE_0/c. Including the electric force would answer a different question.

Step 4: Conclusion The maximum magnetic force is $\boxed{4.8 \times 10^{-19}\,\mathrm{N}}$. Hence, the correct option is (A).

Quick Tip to Solve: For transverse motion in an electromagnetic wave, substitute $B_0 = E_0/c$ first; then $F_{B,\max} = e(v/c)E_0$.

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28. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A → B, A → C and A → D. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the work done as W_{AB}, W_{AC} and W_{AD}.
The correct relation between these parameters are : [4 marks]



(A) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(B) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(C) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$
(D) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
Correct Answer: (B) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$

Solution:

Concept:

- For a fixed amount of ideal monoatomic gas, $U = \frac{3}{2}nRT$. Internal energy is a state function: its change depends only on the initial and final temperatures, not on the path.
- Work done by the gas is $W = \int P\,dV$. Expansion gives positive work, a constant-volume process gives zero work, and compression gives negative work.

Step 1: Read the endpoint temperatures The lower isotherm in the diagram is labelled T_2, and the upper one is labelled T_1, with $T_1 > T_2$. Point A lies on the lower isotherm, while points B, C and D all lie on the same upper isotherm. Their pressures and volumes differ, but their common product $PV = nRT_1$ gives the same final temperature.

Step 2: Compare the internal-energy changes Every process begins at A and ends at temperature T_1. Consequently,

$$E_{AB} = E_{AC} = E_{AD} = \frac{3}{2}nR(T_1 - T_2) > 0.$$

A steeper path on a pressure-volume plot does not imply a larger internal-energy change. This equality rules out options (A), (C) and (D), each of which assigns unequal changes to endpoints at the same temperature.

Step 3: Determine the work signs Along A to B the volume increases, so $W_{AB} > 0$. Along A to C the path is vertical, so $dV = 0$ throughout and $W_{AC} = 0$, even though pressure and temperature rise. Along A to D the endpoint is to the left of A and the path is a compression, so $W_{AD} < 0$. These signs refer to work done by the gas; using work done on it would reverse the nonzero signs.

Step 4: Conclusion $\boxed{E_{AB} = E_{AC} = E_{AD}, \quad W_{AB} > 0, \quad W_{AC} = 0, \quad W_{AD} < 0}$. Hence, the correct option is (B).

Quick Tip to Solve: On a pressure-volume diagram, read endpoint isotherms for internal energy and read left/right volume changes for the sign of work.

29. With increasing biasing voltage of a photodiode, the photocurrent magnitude : [4 marks]

(A) Increases initially and after attaining certain value, it decreases

(B) Increases linearly

(C) Increases initially and saturates finally

(D) Remains constant

Correct Answer: (C) Increases initially and saturates finally

Solution:

Concept:

- A photodiode is normally operated in reverse bias. Absorbed photons create electron-hole pairs, and the junction electric field separates and collects these carriers to produce photocurrent.
- At fixed illumination the generation rate of photocarriers is fixed. Increasing the collection field can improve carrier collection initially, but the current saturates once essentially all generated

carriers are collected.

Step 1: Describe the initial collection process At a small reverse bias, some photogenerated carriers can recombine before reaching the contacts. Increasing the reverse-bias magnitude strengthens the electric field and widens the depletion region. Carrier separation and collection become more effective, so the measured photocurrent initially increases. This is a collection effect, not an unlimited increase in the rate of incident photons.

Step 2: Apply the fixed-generation limit For constant incident intensity and frequency, the number of usable electron-hole pairs generated per second is approximately fixed. If that rate is G and the collection efficiency is η, the photocurrent magnitude can be written $I_{\text{ph}} = e\eta G$. Increasing reverse bias can raise η towards its collection limit, but cannot make it grow without bound. Once collection is effectively complete, the photocurrent approaches a saturation value.

Step 3: Distinguish the alternatives Option (C) describes both stages: an initial rise followed by saturation. A linear rise without a limiting value, as in (B), ignores the fixed carrier-generation rate. A subsequent decrease, as in (A), is not the normal photodiode characteristic. Option (D) describes only the approximately flat saturation region, not the initial approach to it. This comparison concerns normal reverse-bias operation below breakdown.

Step 4: Conclusion The photocurrent magnitude increases initially and finally saturates. Hence, the correct option is (C).

Quick Tip to Solve: At fixed light intensity, separate carrier generation from carrier collection: bias improves collection initially, while illumination sets the saturation photocurrent.

30. Assume that the displacement(s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If Δp ~ 10 Pa, v ~ 300 m/s, ρ ~ 1 kg/m^3 and f ~ 1000 Hz, then s will be of the order of (take the multiplicative constant to be 1) : [4 marks]

(A) 1 mm

(B) $\frac{3}{100}$ mm

(C) 10 mm

(D) $\frac{1}{10}$ mm

Correct Answer: (B) $\frac{3}{100}$ mm

Solution:

Concept:

- Write the displacement as $s = C\Delta p\, v^a \rho^b f^d$, because the question states direct proportionality to the pressure difference. The multiplicative constant C is set to 1.
- The relevant dimensions are $[s] = L$, $[\Delta p] = ML^{-1}T^{-2}$, $[v] = LT^{-1}$, $[\rho] = ML^{-3}$ and $[f] = T^{-1}$. Equate mass, length and time powers independently.

Step 1: Determine the density and speed powers In the proposed product the mass exponent is $1 + b$. Displacement has no mass dimension, so $b = -1$. The length exponent is $-1 + a - 3b$; setting this

equal to 1 gives

$$-1 + a + 3 = 1 \quad \Rightarrow \quad a = -1.$$

Both density and sound speed therefore occur in the denominator, not the numerator.

Step 2: Determine the frequency power The time exponent of the product is $-2 - a - d$, which must vanish. Substituting $a = -1$ gives $-1 - d = 0$, so $d = -1$. Hence

$$s = \frac{\Delta p}{\rho v f}.$$

Indeed, $[\rho v f] = ML^{-2}T^{-2}$, and pressure divided by this quantity has dimension length. The assumption of direct proportionality to pressure is essential to fix these exponents.

Step 3: Substitute and convert the length Use the supplied approximate SI values and the instructed constant 1:

$$s \sim \frac{10}{(1)(300)(1000)} = 3.33 \times 10^{-5}\,\text{m} = 3.33 \times 10^{-2}\,\text{mm}.$$

This is of order $0.03\,\text{mm}$, matching option (B). Options (A), (C) and (D) give 1, 10 and 0.1 mm respectively, rather than the scale obtained from the specified unit constant. No additional factor such as 2π is required in this dimensional estimate.

Step 4: Conclusion The displacement is of order $\boxed{\frac{3}{100}\,\text{mm}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a sound-wave dimensional estimate, $\Delta p/(\rho v)$ has units of velocity; divide once more by frequency to obtain displacement.

31. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is : [4 marks]

(A) 0

(B) $\frac{5}{6}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

Correct Answer: (D) $\frac{3}{4}$

Solution:

Concept:

- With no external torque about the shaft, total angular momentum is conserved during coupling: $L_i = L_f$. Internal friction can dissipate mechanical energy without changing this total.
- The rotational kinetic energy of a rigid body is $K = \frac{1}{2}I\omega^2$, where I is its moment of inertia about the rotation axis. Coupled wheels have a common final angular speed.

Step 1: Find the common angular speed Initially only the wheel of inertia I rotates, while the second wheel has zero angular momentum. The shaft contributes negligibly to the inertia. After coupling the

inertias add, so conservation gives

$$I\omega + 3I(0) = (I + 3I)\Omega.$$

Thus $\Omega = \omega/4$. Conserving kinetic energy at this stage would be incorrect because sudden coupling involves slipping and friction, which convert some rotational energy into heat.

Step 2: Compare the initial and final kinetic energies The energy before coupling is $K_i = \frac{1}{2}I\omega^2$. Using the common final speed and the total inertia gives

$$K_f = \frac{1}{2}(4I)\left(\frac{\omega}{4}\right)^2 = \frac{1}{8}I\omega^2 = \frac{1}{4}K_i.$$

The requested fraction is the energy lost divided by the initial energy, rather than the fraction remaining:

$$\frac{K_i - K_f}{K_i} = 1 - \frac{1}{4} = \frac{3}{4}.$$

Option (C) is the remaining fraction, not the loss. Option (A) would require energy conservation despite the dissipative coupling. Option (B) does not follow from either the conserved angular momentum or the energy ratio.

Step 3: Conclusion The fractional loss of kinetic energy is $\boxed{3/4}$. Hence, the correct option is (D).

Quick Tip to Solve: For a rotating wheel coupled to a stationary wheel on the same free shaft, the energy remaining fraction is $I_1/(I_1 + I_2)$; subtract it from one for the loss.

32. A galvanometer of resistance G is converted into a voltmeter of range 0 – 1 V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2 V will be : [4 marks]

(A) G

(B) R_1

(C) R_1 + G

(D) R_1 - G

Correct Answer: (C) R_1 + G

Solution:

Concept:

- A voltmeter is made by adding a high resistance in series with a galvanometer. Its full-scale voltage satisfies $V = I_g R_{\text{total}}$, where I_g is the galvanometer full-scale current.
- The same galvanometer is used for both ranges, so its full-scale current and internal resistance G remain unchanged. The series multiplier resistance controls the range.

Step 1: Write the original full-scale condition At the original upper limit of $1\,\text{V}$, the current through the galvanometer and the series resistance is I_g. The original total resistance includes both components,

not just the added multiplier:

$$I_g(G + R_1) = 1\,\text{V}.$$

Therefore $I_g = (1\,\text{V})/(G + R_1)$. The resistance of the galvanometer cannot be neglected because no approximation about its size has been supplied.

Step 2: Determine the extra series resistance Let X be the additional resistance connected in series with the existing R_1. At the new full-scale voltage, the same current must give full deflection:

$$I_g(G + R_1 + X) = 2\,\text{V}.$$

Dividing this equation by the original condition eliminates the unknown full-scale current:

$$\frac{G + R_1 + X}{G + R_1} = 2 \quad \Rightarrow \quad X = G + R_1.$$

Thus doubling the voltage range requires doubling the entire original resistance. Adding only R_1, as in (B), fails to duplicate the original internal resistance contribution. Adding only G, as in (A), does not generally double the total either. The difference $R_1 - G$ in (D) gives a new total $2R_1$, which also misses $2G$.

Step 3: Conclusion The additional series resistance is $\boxed{R_1 + G}$. Hence, the correct option is (C).

Quick Tip to Solve: For a range multiplied by n, add $(n - 1)$ times the original total voltmeter resistance, including the galvanometer resistance.

33. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is : [4 marks]

(A) 2200 Hz

(B) 3300 Hz

(C) 1100 Hz

(D) 550 Hz

Correct Answer: (A) 2200 Hz

Solution:

Concept:

- The air column above water behaves as a pipe closed at the water surface and open at the top. Its effective resonant lengths are odd multiples of $\lambda/4$.
- Two successive resonant lengths differ by $\lambda/2$. A constant end correction cancels when the difference is taken, and the wave speed obeys $v = f\lambda$.

Step 1: Use the change in water height If H is the fixed total tube height and y the water height from the bottom, the air length is $L = H - y$. Raising the water therefore decreases the air-column length.

Between the two stated successive resonances its magnitude of change is

$$|\Delta L| = 24.5 - 17.0 = 7.5\,\text{cm} = 0.075\,\text{m}.$$

The unknown total tube height is not needed. Nor should either quoted water height itself be treated as a resonant air length; each is measured from the bottom rather than from the open end.

Step 2: Find the wavelength and frequency For a closed-open tube the resonance condition is $L + e = (2n-1)\lambda/4$, where e is the open-end correction. Subtracting the conditions for adjacent resonances eliminates e and gives

$$\frac{\lambda}{2} = 0.075\,\text{m}, \qquad \lambda = 0.150\,\text{m}.$$

Now use the given air-wave speed:

$$f = \frac{v}{\lambda} = \frac{330}{0.150} = 2200\,\text{Hz}.$$

The value $1100\,\text{Hz}$ in (C) would require twice this wavelength, and $550\,\text{Hz}$ in (D) requires four times it. The value $3300\,\text{Hz}$ in (B) corresponds to $0.100\,\text{m}$, inconsistent with the measured resonance separation.

Step 3: Conclusion The tuning-fork frequency is $\boxed{2200\,\text{Hz}}$. Hence, the correct option is (A).

Quick Tip to Solve: For successive resonances use $f = v/(2\Delta L)$; the water-height change has the same magnitude as the air-length change.

34. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/(g – °C) (1 cal = 4.2 × 10^7 ergs) close to : [4 marks]

(A) 87.5 °C

(B) 83.3 °C

(C) 38.4 °C

(D) 119.2 °C

Correct Answer: (A) 87.5 °C

Solution:

Concept:

- The kinetic energy of a body of mass m and speed v is $K = \frac{1}{2}mv^2$. Only the stated fraction of this energy heats the bullet.
- Heating without a change of state obeys $Q = mc\Delta T$, with specific heat capacity c. All energy and mass units must be consistent before substitution.

Step 1: Calculate the heat retained by the bullet The bullet mass is $5\,\text{g} = 0.005\,\text{kg}$, and its speed

before impact is $210\,\mathrm{m\,s^{-1}}$. Thus

$$K = \frac{1}{2}(0.005)(210)^2 = 110.25\,\mathrm{J}.$$

The statement assigns only one half to the bullet, the rest to the wood. Therefore

$$Q_{\text{bullet}} = \frac{K}{2} = 55.125\,\mathrm{J}.$$

The target is fixed, and the specified energy partition is enough to calculate the bullet's heating without knowing the wood's mass or specific heat.

Step 2: Convert the specific heat and compute the rise Use $1\,\mathrm{erg} = 10^{-7}\,\mathrm{J}$, so the supplied conversion gives $1\,\mathrm{cal} = 4.2\,\mathrm{J}$. Consequently,

$$c = 0.030 \times 4.2 = 0.126\,\mathrm{J\,g^{-1}\,^{\circ}C^{-1}}.$$

Keeping the mass in grams to match this form of c, apply calorimetry:

$$\Delta T = \frac{55.125}{5(0.126)} = 87.5\,^{\circ}\mathrm{C}.$$

Equivalently, mass cancels in $\Delta T = v^2/(4c)$ if c is in $\mathrm{J\,kg^{-1}\,K^{-1}}$. Options (B), (C) and (D) do not equal the stated energy divided by the bullet's heat capacity. Assigning all the kinetic energy to the bullet would incorrectly double the rise.

Step 3: Conclusion The bullet temperature rises by $\boxed{87.5\,^{\circ}\mathrm{C}}$. Hence, the correct option is (A).

Quick Tip to Solve: When a fraction η of kinetic energy heats the moving object itself, mass cancels: $\Delta T = \eta v^2/(2c)$ in consistent SI units.

35. The value of the acceleration due to gravity is g_1 at a height h = $\frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals : [4 marks]

(A) $\frac{5}{9}$

(B) $\frac{1}{9}$

(C) $\frac{7}{9}$

(D) $\frac{4}{9}$

Correct Answer: (A) $\frac{5}{9}$

Solution:

Concept:

- Outside a spherical Earth, gravitational acceleration at radius r is GM/r^2. If surface gravity is g, then $g_h = g[R/(R+h)]^2$.
- In the standard uniform-density Earth model, gravity at depth d is $g_d = g(1 - d/R)$. Only the mass

enclosed within radius $R - d$ contributes to the net field there.

Step 1: Find the acceleration at the stated height Height is measured from the surface, so the distance from the Earth's centre is $R + h = 3R/2$, not $R/2$. Using the inverse-square dependence gives

$$g_1 = g\left(\frac{R}{3R/2}\right)^2 = \frac{4}{9}g.$$

The reduction factor is dimensionless. It is the square of the radius ratio, because the field outside the Earth is that of its full mass concentrated at the centre.

Step 2: Equate the gravity at depth to the same value For the usual uniform-density model, the enclosed mass at an interior radius r is $M(r/R)^3$. Hence $GM(r/R)^3/r^2 = gr/R$, giving the linear depth law with $r = R - d$. Set that acceleration equal to g_1:

$$g\left(1 - \frac{d}{R}\right) = \frac{4}{9}g.$$

Cancelling the nonzero surface gravity and rearranging,

$$\frac{d}{R} = 1 - \frac{4}{9} = \frac{5}{9}.$$

Option (D), $4/9$, is the fractional radius remaining inside the Earth, not the fractional depth. Options (B) and (C) would give interior gravity $8g/9$ and $2g/9$, respectively, which do not match the height value. The result lies between zero and one, as a physical depth below the surface must.

Step 3: Conclusion The required ratio is $\boxed{d/R = 5/9}$. Hence, the correct option is (A).

Quick Tip to Solve: Use inverse-square gravity outside and linear gravity inside; convert the interior radius fraction to depth by subtracting it from one.

36. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to :

[g is the acceleration due to gravity] [4 marks]

(A) t = $3.4\sqrt{\left(\frac{h}{g}\right)}$

(B) t = $1.8\sqrt{\left(\frac{h}{g}\right)}$

(C) t = $\sqrt{\frac{2h}{3g}}$

(D) t = $\frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

Correct Answer: (A) t = $3.4\sqrt{\left(\frac{h}{g}\right)}$

Solution:

Concept:

- A released packet retains the instantaneous velocity of the helicopter. It subsequently accelerates downward at g, independently of the helicopter motion.
- For constant acceleration, $v^2 = u^2 + 2as$ determines the release speed; $y = y_0 + ut + \frac{1}{2}at^2$ determines the subsequent position, using one consistent sign convention.

Step 1: Find the packet velocity at release The helicopter starts from rest and accelerates upward at g through height h. Therefore its upward speed, and hence the upward initial speed of the packet just after release, is

$$u^2 = 0 + 2gh, \qquad u = \sqrt{2gh}.$$

Dropping the packet means releasing it, not bringing its ground-frame velocity to zero. It initially continues upward, slows to rest, and only then falls to the ground.

Step 2: Solve for the positive time to reach the ground Take the ground as $y = 0$, upward as positive, and set $t = 0$ at release. The initial height is h, so

$$0 = h + ut - \frac{1}{2}gt^2.$$

The quadratic formula gives

$$t = \frac{u \pm \sqrt{u^2 + 2gh}}{g}.$$

Since $u^2 = 2gh$, the positive root is

$$t = \frac{\sqrt{2gh} + 2\sqrt{gh}}{g} = (\sqrt{2} + 2)\sqrt{\frac{h}{g}} \simeq 3.414\sqrt{\frac{h}{g}}.$$

The other root is negative and describes an extrapolated position before release. Of the choices, only (A) has the correct coefficient. The smaller values (B), (C) and (D) omit or underestimate the extra time due to the upward release velocity; the packet takes longer than a body released from rest at

height h.

Step 3: Conclusion The elapsed time after release is $\boxed{t \simeq 3.4\sqrt{h/g}}$. Hence, the correct option is (A).

Quick Tip to Solve: Before using a free-fall formula, find the carrier velocity at release. For upward release use the positive root $t = (u + \sqrt{u^2 + 2gh})/g$.

37. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is : [4 marks]

(A) $\frac{2}{3}$R
(B) $\frac{4}{9}$R
(C) $\frac{1}{3}$R
(D) $\frac{8}{9}$R

Correct Answer: With the printed specific gravity $27/8$, the result is $\boxed{r = (\sqrt[3]{19}/3)R}$. None of the printed options is exactly correct; a unique listed answer cannot be justified from the stated data.

Solution:

Concept:

- Archimedes' principle states that the upward buoyant force equals the weight of displaced water. When the sphere is just submerged, the displaced volume is its full outer volume, not the volume occupied by the shell material.
- If the water density is ρ_w, the shell material has density $\rho_s = (27/8)\rho_w$. The material volume is the difference of the outer and inner spherical volumes.

Step 1: Write the floating equilibrium The hollow cavity does not contain shell material. Neglecting the weight of air, the downward weight and upward thrust are therefore

$$W = \rho_s g \frac{4\pi}{3}(R^3 - r^3), \qquad F_B = \rho_w g \frac{4\pi}{3} R^3.$$

Floating at rest requires $W = F_B$. The same radius R must be used for the displaced water because the sphere is fully submerged.

Step 2: Solve for the inner radius Cancel the common factors and substitute the stated density ratio:

$$\frac{27}{8}(R^3 - r^3) = R^3, \qquad r^3 = \frac{19}{27}R^3.$$

Thus

$$r = \frac{\sqrt[3]{19}}{3}R \simeq 0.88947R.$$

This retains the cube root required by the volume dependence; taking a linear ratio directly would be incorrect.

Step 3: Check the printed choices For option (A), $r = 2R/3$ would require specific gravity $27/19$,

rather than the stated $27/8$. Options (B) and (C) likewise fail the volume balance. Option (D), $8R/9$, is numerically close but is not exact: its required specific gravity is $729/217$, not $27/8$. No rounding instruction permits replacing the exact cube root by that fraction.

Step 4: Conclusion With the printed specific gravity $27/8$, the result is $\boxed{r = (\sqrt[3]{19}/3)R}$. None of the printed options is exactly correct; a unique listed answer cannot be justified from the stated data.

Quick Tip to Solve: For a hollow floating sphere, balance density times material volume against water density times outer volume before taking any cube root.

38. A square loop of side $2a$, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b >> a). The magnitude of the torque on the loop about zaxis is given by : [4 marks]

(A) $\frac{2\mu_0 I^2 a^2}{\pi b}$

(B) $\frac{\mu_0 I^2 a^2}{2\pi b}$

(C) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$

(D) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$

Correct Answer: (A) $\frac{2\mu_0 I^2 a^2}{\pi b}$

Solution:

Concept:

- The magnetic field of a long straight wire has magnitude $\mu_0 I/(2\pi s)$, where s is the perpendicular distance from that wire. Its direction is tangent to circles about the wire.
- Torque about the z-axis is $\tau_z = xF_y - yF_x$. The loop lies at $y = 0$, so only the transverse force component F_y contributes to this torque.

Step 1: Find the relevant magnetic field component Take the long-wire current along positive z; reversing either current reverses the torque but not its magnitude. At the loop position $(x, 0, z)$, the vector from the wire is $x\hat{x} - b\hat{y}$. Consequently

$$\mathbf{B} = \frac{\mu_0 I}{2\pi(x^2 + b^2)}(b\hat{x} + x\hat{y}).$$

The two loop sides parallel to z are at $x = \pm a$, each of length $2a$.

Step 2: Calculate the couple of forces On either vertical side, the magnitude of the y-directed force is

$$F_y = I(2a)B_x = \frac{\mu_0 I^2 ab}{\pi(a^2 + b^2)}.$$

The currents on those opposite sides are opposite, giving opposite forces separated by $2a$. The horizontal sides have force along z, so they do not contribute to τ_z. Thus

$$|\tau_z| = 2aF_y = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}.$$

Step 3: Use the large-separation approximation Since $b \gg a$, replace $a^2 + b^2$ by b^2:

$$|\tau_z| \simeq \frac{2\mu_0 I^2 a^2}{\pi b}.$$

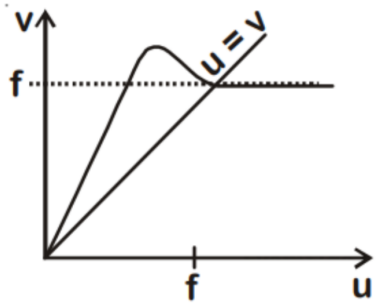
Option (B) has a factor four too small. Options (C) and (D) have a different dependence on loop size and distance, characteristic of retaining an irrelevant field-gradient contribution rather than the leading magnetic couple.

Step 4: Conclusion The torque magnitude is $\boxed{2\mu_0 I^2 a^2/(\pi b)}$. Hence, the correct option is (A).

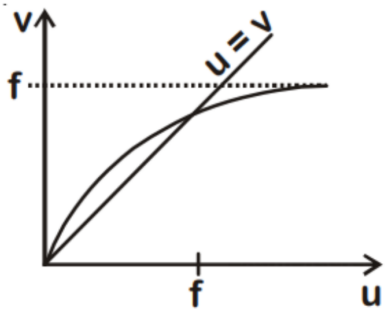
Quick Tip to Solve: For a small loop far from a wire, check the leading answer with $\tau = mB$: its area is $4a^2$, not a^2.

39. For a concave lens of focal length f, the relation between object and image distances u and v, respectively, from its pole can best be represented by (u = v is the reference line) : [4 marks]

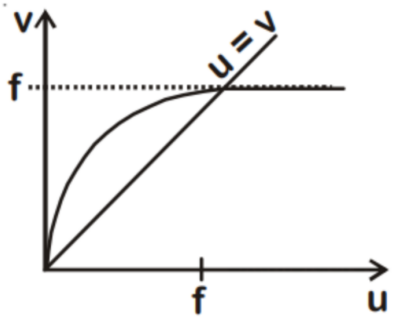
(A)



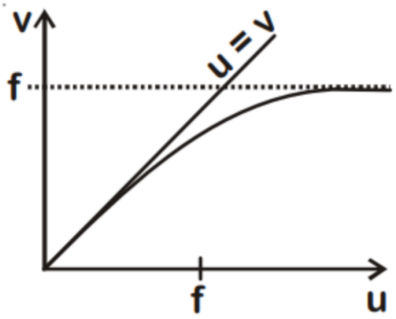
(B)



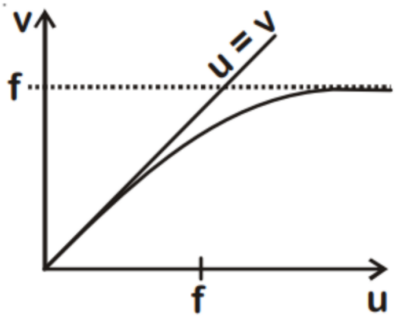
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- A concave lens forms a virtual, erect and diminished image of a real object. The object and image are on the same side of the lens, with the image lying between the lens and its principal focus.
- For a graph of positive distance magnitudes, write $U = |u|$, $V = |v|$ and $F = |f|$. The Cartesian lens formula then gives $1/V = 1/F + 1/U$.

Step 1: Obtain the distance relation The displayed graphs use positive axes for the distances. In the Cartesian convention for a concave lens, $u = -U$, $v = -V$, and the signed focal length is $-F$. Substitution into $1/v - 1/u = 1/f$ yields

$$-\frac{1}{V} + \frac{1}{U} = -\frac{1}{F}, \qquad V = \frac{FU}{F+U}.$$

This is the curve to compare with the reference line $V = U$.

Step 2: Check the slope and limiting values For all $U > 0$, $V/U = F/(F+U) < 1$, so the curve must stay below the reference line. Also

$$\frac{dV}{dU} = \frac{F^2}{(F+U)^2} > 0, \qquad \frac{d^2V}{dU^2} = -\frac{2F^2}{(F+U)^3} < 0.$$

The curve starts at the origin with slope one, then bends downward while increasing. As $U \to \infty$, $V \to F$ from below; it cannot rise above the horizontal focal-distance asymptote.

Step 3: Compare all four graphs Option (A) overshoots the focal-distance level and later turns downward, contradicting monotonic increase. Options (B) and (C) show portions above the line

$u = v$, which would require an image farther from the lens than the object. Option (D) alone begins tangent to the reference line, remains below it, and approaches the horizontal line $v = f$ from below.

Step 4: Conclusion The concave-lens relation is $V = FU/(F + U)$, so the fourth graph has the required behaviour. Hence, the correct option is (D).

Quick Tip to Solve: For a concave lens, the image-distance magnitude is smaller than both the object distance and focal length; use both bounds to reject graph options.

---

40. A physical quantity z depends on four observables a, b, c and d, as z = $\frac{a^2 b^{\frac{2}{3}}}{\sqrt{c} d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is : [4 marks]

(A) 13.5 %

(B) 14.5%

(C) 16.5%

(D) 12.25%

Correct Answer: (B) 14.5%

Solution:

Concept:

- For a product of measured quantities raised to powers, the maximum fractional error is the sum of the absolute powers multiplied by the respective fractional errors. Denominator factors therefore add to the maximum error.
- Write the quantity as $z = a^2 b^{2/3} c^{-1/2} d^{-3}$. Its exponents are 2, 2/3, $-1/2$, and -3, respectively.

Step 1: Form the fractional-error equation Logarithmic differentiation identifies the sensitivity to each observable:

$$\frac{dz}{z} = 2\frac{da}{a} + \frac{2}{3}\frac{db}{b} - \frac{1}{2}\frac{dc}{c} - 3\frac{dd}{d}.$$

For a maximum uncertainty, the signs of the individual deviations are not known. We add their possible magnitudes rather than subtracting the denominator contributions. The same expression applies directly to percentage uncertainties.

Step 2: Calculate each contribution The given errors contribute

$$a : 2(2\%) = 4\%,$$
$$b : \frac{2}{3}(1.5\%) = 1\%,$$
$$c : \frac{1}{2}(4\%) = 2\%,$$
$$d : 3(2.5\%) = 7.5\%.$$

The square root belongs to c only, so its error multiplier is one half; the exponent three on d multiplies the full error in that observable.

Step 3: Add the maximum errors and compare Adding the four independent maximum contributions

gives

$$\frac{\Delta z}{z}\,100\% = 4\% + 1\% + 2\% + 7.5\% = 14.5\%.$$

Option (A) misses one percentage point, while (C) overestimates the square-root contribution. Option (D) also fails the exponent-weighted sum. A root-sum-square rule would require a statistical uncertainty model, which is not the maximum-error rule tested here.

Step 4: Conclusion The percentage error in z is $\boxed{14.5\%}$. Hence, the correct option is (B).

Quick Tip to Solve: Write all powers explicitly, take their absolute values, and multiply each percentage error before adding; denominator errors never cancel in a maximum-error estimate.

41. An electrical power line, having a total resistance of 2 Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately : [4 marks]

(A) 85%

(B) 96%

(C) 72%

(D) 91%

Correct Answer: (B) 96%

Solution:

Concept:

- The power delivered to a load is $P_{\text{out}} = V_{\text{load}} I$. The current in the transmission line is the same as the load current, so its resistive heating is $P_{\text{loss}} = I^2 R$.
- Efficiency is the ratio of useful output power to supplied input power: $\eta = P_{\text{out}}/(P_{\text{out}} + P_{\text{loss}})$. The stated voltage is the receiving-end voltage because the line delivers the stated power at that voltage.

Step 1: Find the line current Convert the delivered power to watts and use the load voltage:

$$P_{\text{out}} = 1000\,\text{W}, \qquad I = \frac{1000}{220} = \frac{50}{11}\,\text{A} \simeq 4.545\,\text{A}.$$

Using the output power divided by 220 V is essential; treating 220 V as a voltage drop across the 2-ohm line would describe a different circuit.

Step 2: Calculate the heating loss and input power The quoted resistance is the total line resistance, so no additional factor of two is needed:

$$P_{\text{loss}} = \left(\frac{50}{11}\right)^2 (2) = \frac{5000}{121}\,\text{W} \simeq 41.32\,\text{W}.$$

Conservation of energy then gives

$$P_{\text{in}} = 1000 + 41.32 = 1041.32\,\text{W}.$$

The source must supply both the useful load power and the energy dissipated in the wire.

Step 3: Calculate and compare efficiencies Therefore

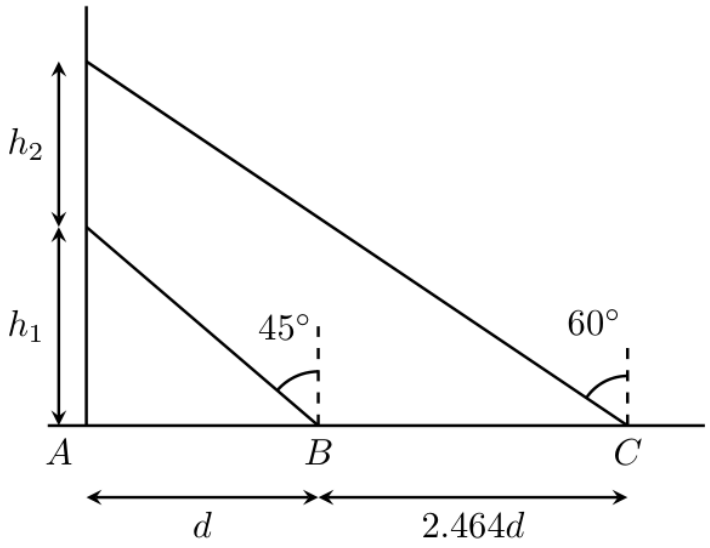
$$\eta = \frac{1000}{1041.32} \times 100\% \simeq 96.03\%.$$

The loss is only about four percent of the input, so the alternatives 85 percent, 72 percent and 91 percent all imply losses much larger than the calculated line heating. The result also lies below 100 percent, as required for a resistive transmission line.

Step 4: Conclusion The efficiency is approximately $\boxed{96\%}$. Hence, the correct option is (B).

Quick Tip to Solve: When a line delivers power at a specified receiving voltage, find the current from those two values, then include I^2R in the input power.

42. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distanced (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d(point C). Then the height h_2 is (given tan 30° = 0.5774) [4 marks]



(A) 0.464d

(B) d

(C) 0.732d

(D) 1.464d

Correct Answer: (B) d

Solution:

Concept:

- The balloon moves on the vertical line through A. Its first altitude is h_1, while its second altitude is the total $h_1 + h_2$, because h_2 is the additional rise.
- The angles are measured from the vertical. For such an angle θ, horizontal distance divided by altitude is $\tan\theta$; equivalently, altitude equals horizontal distance times $\cot\theta$.

Step 1: Use the first observation Initially the girl is at B, with horizontal separation $AB = d$. The sightline makes 45° with the vertical, so

$$\tan 45^\circ = \frac{d}{h_1} = 1, \qquad h_1 = d.$$

This is the altitude at the first observation, and it must be subtracted from the later total altitude to obtain the rise requested.

Step 2: Find the second altitude The girl moves farther from A, by $BC = 2.464d$. Thus

$$AC = AB + BC = 3.464d.$$

At the second observation the sightline makes 60° with the vertical, or 30° with the horizontal. Hence

$$h_1 + h_2 = AC\tan 30^\circ = (3.464d)(0.5774) \simeq 2.0001d.$$

The slight departure from $2d$ reflects the rounded tangent and distance data.

Step 3: Extract the additional rise and compare Subtracting the first altitude gives

$$h_2 \simeq 2.0001d - d \simeq d.$$

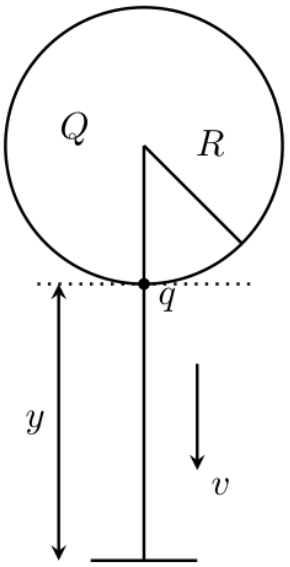
Option (A) incorrectly combines the displacement with a simple subtraction, and (C) or (D) would not reproduce the second sightline with horizontal separation $3.464d$. Using 60 degrees as an elevation angle would also be incorrect, since the problem explicitly measures it from the vertical.

Step 4: Conclusion The further rise is $\boxed{h_2 \simeq d}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert an angle from the vertical to its complementary elevation angle, and distinguish the final altitude from the additional rise.

43. A solid sphere of radius R carries a charge Q + q distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then :

(assume the remaining portion to be spherical). [4 marks]



(A) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(B) $v^2 = 2y\left[\frac{qQR}{4\pi\varepsilon_0 (R+y)^3 m} + g\right]$

(C) $v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(D) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

Correct Answer: (C) $v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

Solution:

Concept:

- Outside a spherically symmetric charge distribution, the potential equals that of a point charge at its centre: $V(r) = kQ/r$, with $k = 1/(4\pi\varepsilon_0)$.
- The detached piece starts from rest. Its gain in kinetic energy equals the loss of gravitational and electrostatic potential energies; the remaining sphere carries charge Q, not $Q + q$.

Step 1: Identify the initial and final separations The piece is initially on the bottom surface, so its distance from the centre is R. After falling vertically down by y, its distance is $R + y$. Thus the initial and final interaction energies are kqQ/R and $kqQ/(R + y)$. The sphere is treated as stationary, consistently with the stated approximation that the remaining portion stays spherical.

Step 2: Apply conservation of mechanical energy Take the final gravitational potential as zero. The initial gravitational potential then exceeds the final value by mgy. Therefore

$$\frac{1}{2}mv^2 = mgy + kqQ\left(\frac{1}{R} - \frac{1}{R+y}\right) = mgy + \frac{kqQy}{R(R+y)}.$$

Both gravity and repulsion do positive work for like charges; for opposite signs the electrostatic term automatically subtracts from the gravitational contribution.

Step 3: Solve and compare the alternatives Multiply by $2/m$ and factor out $2y$:

$$v^2 = 2y\left[g + \frac{qQ}{4\pi\varepsilon_0 mR(R+y)}\right].$$

Option (A) omits the factor two from kinetic energy. Option (B) has an incorrect radial dependence, whereas (D) does not even give zero speed as the drop tends to zero. Setting the charge interaction to zero correctly recovers free fall, $v^2 = 2gy$.

Step 4: Conclusion

$$\boxed{v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For a charged fragment falling away from a sphere, use the change in kqQ/r; the electric force is not constant over a finite fall.

---

44. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is : [4 marks]

(A) Zero

(B) $\frac{3}{2}CV^2$

(C) $\frac{9}{2}CV^2$

(D) $\frac{25}{6}CV^2$

Correct Answer: (B) $\frac{3}{2}CV^2$

Solution:

Concept:

- The algebraic charge on each isolated connected node is conserved. Charges on plates joined with opposite polarities must be subtracted.
- Parallel capacitors have a common final voltage and equivalent capacitance $C_{\text{eq}} = C + 2C = 3C$. Their stored energy is $U = \frac{1}{2}C_{\text{eq}}V_f^2$.

Step 1: Calculate the signed charges before connection The first capacitor initially stores a charge of magnitude $Q_1 = CV$. The second has both twice the capacitance and twice the voltage, so its plate charge is $Q_2 = (2C)(2V) = 4CV$. Consider the node formed by joining the positive plate of the first capacitor to the negative plate of the second. Its total charge is

$$Q_{\text{node}} = CV - 4CV = -3CV.$$

The other connected node consequently has charge $+3CV$.

Step 2: Find the common final voltage The two capacitors share the same pair of nodes. With the chosen node ordering,

$$(C + 2C)V_f = -3CV \quad \Rightarrow \quad V_f = -V.$$

The negative sign indicates that the larger initially charged capacitor determines the final polarity. The voltage magnitude is V, not zero; the initial plate charges were unequal. Each capacitor finally stores energy according to this same voltage magnitude.

Step 3: Add the final energies and reject the alternatives The total energy is

$$U_f = \frac{1}{2}CV^2 + \frac{1}{2}(2C)V^2 = \frac{3}{2}CV^2.$$

Initially the stored energy was $\frac{1}{2}CV^2 + \frac{1}{2}(2C)(2V)^2 = \frac{9}{2}CV^2$, which is option (C), not the final energy. The difference is dissipated during charge redistribution. Zero would require equal and opposite initial charges. Option (D) results from adding the initial charge magnitudes as if like polarities were joined.

Step 4: Conclusion

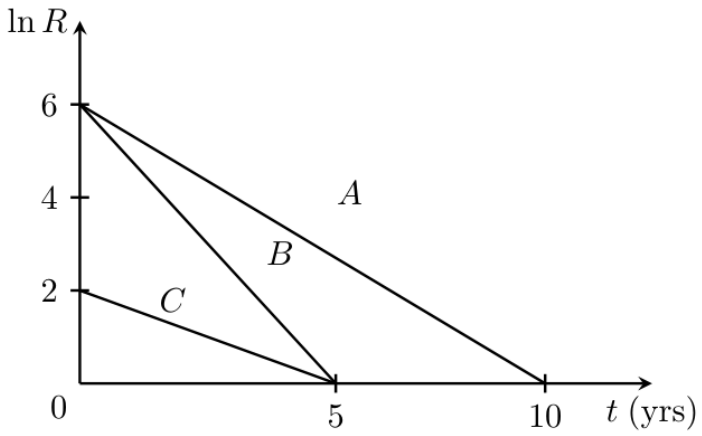
$$\boxed{U_f = \frac{3}{2}CV^2}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Label one connected node and add its signed plate charges first. Use charge conservation to find the voltage before calculating energy.

45. Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives

$T_{\frac{1}{2}}(A)$: $T_{\frac{1}{2}}(B)$: $T_{\frac{1}{2}}(C)$ are in the ratio : [4 marks]



(A) 3 : 2 : 1

(B) 2 : 1 : 1

(C) 4 : 3 : 1

(D) 2 : 1 : 3

Correct Answer: (D) 2 : 1 : 3

Solution:

Concept:

- Activity follows $R(t) = R_0 e^{-\lambda t}$. Hence a plot of $\ln R$ against time is a straight line with slope $-\lambda$.
- The half-life is $T_{1/2} = \ln 2/\lambda$. Half-lives are inversely proportional to slope magnitudes, not to vertical intercepts.

Step 1: Read the endpoints of each straight line For line A, the plotted activity logarithm decreases from 6 at $t = 0$ to 0 at $t = 10$ years. For B it decreases from 6 to 0 in 5 years. For C it decreases from 2 to 0 in 5 years. The graph is logarithmic: reaching zero on its vertical axis means $\ln R = 0$, not that the radioactive activity itself vanishes.

Step 2: Calculate the three decay constants Use the magnitude of the slope $\Delta(\ln R)/\Delta t$:

$$\lambda_A = \frac{6}{10} = \frac{3}{5}\ \text{yr}^{-1}, \qquad \lambda_B = \frac{6}{5}\ \text{yr}^{-1}, \qquad \lambda_C = \frac{2}{5}\ \text{yr}^{-1}.$$

Although A and B have equal initial activities, B decays faster. C has a lower initial activity, but its shallow slope indicates the smallest decay constant and therefore the longest half-life.

Step 3: Invert the constants to obtain the ratio The common factor $\ln 2$ cancels from the half-life ratio:

$$T_A : T_B : T_C = \frac{5}{3} : \frac{5}{6} : \frac{5}{2} = 2 : 1 : 3.$$

Multiplying all three fractions by $6/5$ produces the integer ratio. Options (A), (B) and (C) all make C no longer lived than A; that contradicts its smaller slope magnitude. Equal horizontal intercepts for B and C do not imply equal half-lives because their initial logarithmic activities differ.

Step 4: Conclusion

$$\boxed{T_A : T_B : T_C = 2 : 1 : 3}.$$

Hence, the correct option is (D).

Quick Tip to Solve: On a $\ln R$-versus-time graph, first find each slope. Invert the slope magnitudes for half-life ratios; do not compare intercept times alone.

Physics – Section B: Numerical Value Questions (20 marks)

46. A compound microscope consists of an objective lens of focal length 1 cm and an eyepiece of focal length 5 cm with a separation of 10 cm.

The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is _____. [4 marks]

Solution:

Concept:

- Minimum eye strain occurs when the final image is at infinity, so the intermediate image must lie in the first focal plane of the eyepiece.
- For the objective, use the Cartesian thin-lens equation $1/f_o = 1/v_o - 1/u_o$. A real object to its left

has $u_o < 0$.

Step 1: Locate the intermediate image The separation between objective and eyepiece is $L = 10\,\text{cm}$. To produce parallel emergent rays, the intermediate image must be $f_e = 5\,\text{cm}$ before the eyepiece. Its distance from the objective is consequently

$$v_o = L - f_e = 10 - 5 = 5\,\text{cm}.$$

This is a real image formed by the objective between the two lenses. Normal adjustment is the relaxed-eye condition; it is different from placing the final image at the near point.

Step 2: Apply the objective lens equation With $f_o = 1\,\text{cm}$, substitute into the lens formula:

$$\frac{1}{1} = \frac{1}{5} - \frac{1}{u_o}.$$

All distances in this equation are measured in centimetres. Rearranging gives

$$\frac{1}{u_o} = \frac{1}{5} - 1 = -\frac{4}{5} \quad \Rightarrow \quad u_o = -\frac{5}{4}\,\text{cm}.$$

The negative sign confirms that the object lies on the incident-light side of the objective. Its requested distance is the positive magnitude $5/4\,\text{cm}$, slightly beyond the objective focal point, as a microscope requires.

Step 3: Match the numerical parameter The distance is written in the question as $n/40\,\text{cm}$. Equate positive distances:

$$\frac{n}{40} = \frac{5}{4} \quad \Rightarrow \quad n = 40 \times \frac{5}{4} = 50.$$

As a check, the objective magnification magnitude is $5/(5/4) = 4$, and the intermediate image really lies $10-5 = 5\,\text{cm}$ from the eyepiece. Thus the same image satisfies both lens conditions simultaneously.

Step 4: Conclusion The object is $\boxed{1.25\,\text{cm}}$ from the objective, so $\boxed{n = 50}$ (dimensionless).

Quick Tip to Solve: For minimum eye strain, subtract the eyepiece focal length from the lens separation to obtain the objective image distance before using its lens equation.

47. Two concentric circular coils, C_1 and C_2 are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A where t is in s. The emf induced in C_1 (in mV), at the instant t = 1 s is $\frac{4}{x}$. The value of x is ___ . [4 marks]

Solution:

Concept:

- The magnetic field at the centre of an N_2-turn circular coil is $B = \mu_0 N_2 I/(2r_2)$. Over a much smaller concentric coil, this field is taken as uniform.
- The induced emf magnitude is $|\mathcal{E}_1| = N_1 \pi r_1^2 |dB/dt|$. The turns of both coils enter the flux linkage.

Step 1: Differentiate the current and convert the radii The smaller coil has $N_1 = 500$, $r_1 = 0.01\,\text{m}$; the larger has $N_2 = 200$, $r_2 = 0.20\,\text{m}$. Their radius ratio is $1/20$, so the usual central-field approximation applies. The current is $I(t) = 5t^2 - 2t + 3$ in amperes, giving

$$\left.\frac{dI}{dt}\right|_{t=1} = 10(1) - 2 = 8\,\text{A s}^{-1}.$$

The constant term has zero derivative.

Step 2: Calculate the changing flux linkage The mutual inductance in this approximation is

$$M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{2r_2}.$$

Substitute $\mu_0 = 4\pi \times 10^{-7}\,\text{H m}^{-1}$:

$$M = \frac{(4\pi \times 10^{-7})(500)(200)\pi(0.01)^2}{2(0.20)} = \pi^2 \times 10^{-5}\,\text{H}.$$

Use the receiving coil area and the field-producing coil radius.

Step 3: Find the emf and extract the requested value Thus

$$|\mathcal{E}_1| = M\left|\frac{dI}{dt}\right| = 8\pi^2 \times 10^{-5}\,\text{V} = 0.08\pi^2\,\text{mV}.$$

This is approximately $0.79\,\text{mV}$, or $0.8\,\text{mV}$ with the standard numerical approximation $\pi^2 \simeq 10$. Since the stated emf is $4/x$ in millivolts,

$$x \simeq \frac{4}{0.8} = 5.$$

Keeping more digits would give $x = 50/\pi^2 \simeq 5.07$; the requested numerical entry is therefore 5. We use the emf magnitude.

Step 4: Conclusion The emf magnitude is approximately $\boxed{0.8\,\text{mV}}$, giving $\boxed{x = 5}$ (dimensionless numerical entry).

Quick Tip to Solve: Use the large coil radius in the magnetic-field denominator and the small coil area in the flux. Differentiate the current before substituting the time.

48. A particle of mass 200 MeV/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is :
(Given the mass of the hydrogen atom to be 1 GeV/c^2) _____ . [4 marks]

Solution:

Concept:

- Linear momentum is conserved in the collision. The particle stops, so its entire initial momentum becomes the recoil momentum of the atom.
- Hydrogen energies are $E_n = -13.6/n^2\,\text{eV}$. Excitation from $n = 1$ to $n = 2$ requires $10.2\,\text{eV}$, in addition to recoil energy.

Step 1: Find the excitation energy and mass ratio The first excited state is $n = 2$. Its energy is $-3.4\,\text{eV}$, whereas the ground state has energy $-13.6\,\text{eV}$. The internal energy increase is therefore

$$\Delta E = (-3.4) - (-13.6) = 10.2\,\text{eV}.$$

Write the incoming particle mass as m and the atom mass as M. The supplied values give

$$\frac{m}{M} = \frac{200\,\text{MeV}/c^2}{1000\,\text{MeV}/c^2} = \frac{1}{5}.$$

Only this ratio is needed, so no conversion to kilograms is necessary.

Step 2: Express the recoil energy using momentum conservation Let p be the incident momentum. The hydrogen atom was initially at rest, and the incident particle is finally at rest. Hence the atom recoils with momentum p. In the nonrelativistic regime,

$$K_i = \frac{p^2}{2m}, \qquad K_H = \frac{p^2}{2M} = \frac{m}{M}K_i = \frac{1}{5}K_i.$$

The kinetic energies are only of order electronvolts, vastly smaller than the rest energies in megaelectronvolts, so this kinetic-energy expression is appropriate.

Step 3: Apply energy conservation and evaluate the entry The initial kinetic energy supplies recoil and internal excitation:

$$K_i = \frac{1}{5}K_i + 10.2 \quad \Rightarrow \quad \frac{4}{5}K_i = 10.2 \quad \Rightarrow \quad K_i = 12.75\,\text{eV}.$$

The recoil energy is $2.55\,\text{eV}$, and $2.55 + 10.2 = 12.75$, checking the energy balance. The question writes $K_i = N/4$ in electronvolts, so

$$N = 4(12.75) = 51.$$

Using only the excitation energy would incorrectly neglect the momentum carried by the recoiling atom.

Step 4: Conclusion The initial kinetic energy is $\boxed{12.75\,\text{eV}}$. Therefore $\boxed{N = 51}$ (dimensionless).

Quick Tip to Solve: When the projectile stops after exciting an atom, use its conserved momentum to express the recoil energy as $(m/M)K_i$, then add the excitation energy.

49. A force $\vec{F} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ N acts at a point $\left(4\hat{i} + 3\hat{j} - \hat{k}\right)$ m. Then the magnitude of torque about the point $\left(\hat{i} + 2\hat{j} + \hat{k}\right)$ m will be $\sqrt{x}$ N m. The value of x is _____. [4 marks]

Solution:

Concept:

- The torque of a force about a specified point is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is measured from that point to the point of application.
- For perpendicular Cartesian components, $|\boldsymbol{\tau}|^2 = \tau_x^2 + \tau_y^2 + \tau_z^2$. The squared magnitude is independent of the choice of axis labels.

Step 1: Find the relative position vector The coordinates of the point where the force acts cannot be used directly: the moment is required about another point, rather than the origin. Subtract the reference-point position from the application-point position:

$$\mathbf{r} = (4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j} - 2\hat{k}\ \text{m}.$$

Thus the displacement components are $(3, 1, -2)$, while the force components are $(1, 2, 3)$ in newtons.

Step 2: Evaluate the cross product Use the determinant in the order position crossed with force. Reversing this order reverses the torque direction, although its magnitude is unchanged.

$$\boldsymbol{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}.$$

$$\boldsymbol{\tau} = \big(1 \cdot 3 - (-2) \cdot 2\big)\hat{i} - \big(3 \cdot 3 - (-2) \cdot 1\big)\hat{j} + \big(3 \cdot 2 - 1 \cdot 1\big)\hat{k}.$$

Therefore $\boldsymbol{\tau} = 7\hat{i} - 11\hat{j} + 5\hat{k}\ \text{N m}$.

Step 3: Compare the magnitude with the given form Square the three torque components and add them; do not add their signed values.

$$|\boldsymbol{\tau}|^2 = 7^2 + (-11)^2 + 5^2 = 49 + 121 + 25 = 195\ (\text{N m})^2.$$

Hence the magnitude is $\sqrt{195}$ N m, exactly of the stated form $\sqrt{x}$ N m.

Step 4: Conclusion The torque magnitude is $\sqrt{195}$ N m, so $\boxed{x = 195}$. The requested parameter x is dimensionless.

Quick Tip to Solve: For torque about a displaced point, subtract that point first. The check $|\mathbf{r} \times \mathbf{F}|^2 = r^2F^2 - (\mathbf{r} \cdot \mathbf{F})^2$ gives $14 \cdot 14 - 1 = 195$.

50. A beam of electrons of energy E scatters from a target having atomic spacing of 1 $\overset{o}{A}$. The first maximum intensity occurs at θ = 60°. Then E (in eV) is _____.
(Planck constant h = 6.64 $\times$ 10^{-34} Js,
1 eV = 1.6 $\times$ 10^{-19} J, electron
mass m = 9.1 $\times$ 10^{-31} kg) [4 marks]

Solution:

Concept:

- Bragg reflection from planes separated by d obeys $2d\sin\theta = n\lambda$. For the first maximum, the order is $n = 1$.
- For a nonrelativistic electron, $\lambda = h/p$ and $E = p^2/(2m) = h^2/(2m\lambda^2)$. Convert joules to electron volts using the value printed in the question.

Step 1: Find the wavelength for the first maximum The plane spacing is $d = 1$ angstrom $= 10^{-10}$ m, and the angle in Bragg's condition is 60°. With first-order diffraction,

$$\lambda = 2d\sin 60^\circ = 2(10^{-10})\frac{\sqrt{3}}{2} = \sqrt{3}\times 10^{-10}\ \text{m}.$$

Thus $\lambda^2 = 3\times 10^{-20}\ \text{m}^2$. The factor of two comes from the path difference between rays reflected from neighbouring planes; omitting it changes the energy by a factor of four.

Step 2: Substitute into the kinetic-energy relation Eliminate the momentum using the de Broglie relation and substitute the supplied constants, rather than a rounded wavelength formula:

$$E = \frac{(6.64\times 10^{-34})^2}{2(9.1\times 10^{-31})(3\times 10^{-20})}\ \text{J}.$$

$$E = \frac{44.0896\times 10^{-68}}{54.6\times 10^{-51}} = 8.0750\times 10^{-18}\ \text{J}.$$

This energy is far below the electron rest energy, so the nonrelativistic kinetic-energy expression is consistent.

Step 3: Convert the energy into electron volts Divide by the energy represented by one electron volt:

$$E = \frac{8.0750\times 10^{-18}}{1.6\times 10^{-19}}\ \text{eV} = 50.4689\ \text{eV}.$$

The numerical conversion multiplies the coefficient by ten because the powers differ by one; it must not be treated as an additional physical approximation. Rounding to two decimal places gives the requested numerical value.

Step 4: Conclusion $\boxed{E \approx 50.47\ \text{eV}}$.

Quick Tip to Solve: Square the Bragg wavelength before substituting: here $\lambda^2 = 3\times 10^{-20}\ \text{m}^2$, which avoids rounding $\sqrt{3}$ too early.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Which of the following derivatives of alcohols is unstable in an aqueous base? [4 marks]

(A) $\mathrm{RO{-}C(CH_3)_3}$ (tert-butyl ether)

(B) $\mathrm{RO{-}C(=O){-}CH_3}$ (acetate ester)

(C) 2-(OR)-tetrahydropyran (tetrahydropyranyl ether)

(D) $\mathrm{RO{-}CH_2{-}C_6H_5}$ (benzyl ether)

Correct Answer: (B) $\mathrm{RO{-}C(=O){-}CH_3}$ (acetate ester)

Solution:

Concept:

- An alcohol may be protected as an ether, an acetal-type ether, or an ester. The functional group determines whether aqueous base removes the protecting group.
- Esters undergo base-promoted nucleophilic acyl substitution (saponification); ordinary ethers and acetals are normally stable to aqueous base.

Step 1: Identify the functional group in each option Option (A), $\mathrm{RO{-}C(CH_3)_3}$, is a tert-butyl ether. Option (B), $\mathrm{RO{-}C(=O){-}CH_3}$, is an acetate ester: the oxygen attached to R is bonded directly to a carbonyl carbon. Option (C) is a tetrahydropyranyl ether, with an acetal centre next to the ring oxygen. Option (D), $\mathrm{RO{-}CH_2{-}C_6H_5}$, is a benzyl ether. The presence of a carbonyl in (B) is the distinguishing feature for hydrolysis by hydroxide.

Step 2: Apply base-promoted ester hydrolysis Hydroxide attacks the electrophilic carbonyl carbon of the acetate group, generating a tetrahedral intermediate. Its collapse replaces the alkoxy group, and proton transfer gives the alcohol and the resonance-stabilised acetate ion. The overall reaction is

$$\mathrm{CH_3COOR + OH^- \longrightarrow CH_3COO^- + ROH.}$$

Formation of carboxylate makes the base hydrolysis effectively irreversible under these conditions. Thus the alcohol derivative in (B) is consumed rather than remaining protected.

Step 3: Rule out the other derivatives The tert-butyl ether in (A) is commonly cleaved by acid, which enables loss of the tert-butyl group; aqueous base does not provide that activation. The tetrahydropyranyl derivative in (C) is an acetal and is acid-labile but base-stable. The benzyl ether in (D) is usually removed by hydrogenolysis, not ordinary aqueous base. None of these three contains the ester carbonyl needed for the saponification pathway.

Step 4: Conclusion The acetate ester $\mathrm{ROCOCH_3}$ is unstable in aqueous base because it hydrolyses to ROH and acetate. Hence, the correct option is (B).

Quick Tip to Solve: Look for $-\mathrm{O{-}C(=O){-}}$: an alcohol protected as an ester is base-labile, unlike common ether protecting groups.

52. A flask contains a mixture of compounds A and B. Both compounds decompose by first-order kinetics. The half-lives for A and B are 300 s and 180 s, respectively. If the concentrations of A and B are equal initially, the time required for the concentration of A to be four times that of B(in s) :
(Use ln 2 = 0.693) [4 marks]

(A) 180

(B) 120

(C) 300

(D) 900

Correct Answer: (D) 900

Solution:

Concept:

- For independent first-order decay, $[A]_t = [A]_0 e^{-k_A t}$ and $[B]_t = [B]_0 e^{-k_B t}$. The rate constant is $k = \ln 2/t_{1/2}$.
- When initial concentrations are equal, their ratio depends on the difference of rate constants: $[A]_t/[B]_t = e^{(k_B - k_A)t}$.

Step 1: Compute the difference of rate constants A has the longer half-life, so it decomposes more slowly and its concentration eventually exceeds that of B. Using the printed half-lives,

$$k_A = \frac{0.693}{300}\ \text{s}^{-1}, \qquad k_B = \frac{0.693}{180}\ \text{s}^{-1}.$$

$$k_B - k_A = 0.693\left(\frac{1}{180} - \frac{1}{300}\right) = \frac{0.693}{450}\ \text{s}^{-1}.$$

The positive sign is essential: using the reverse difference would predict a negative elapsed time for a ratio greater than one.

Step 2: Set up the required concentration ratio Write the integrated rate laws before taking their ratio. Since $[A]_0 = [B]_0$, the initial concentration cancels:

$$\frac{[A]_t}{[B]_t} = \frac{e^{-k_A t}}{e^{-k_B t}} = e^{(k_B - k_A)t} = 4.$$

Taking natural logarithms gives

$$t = \frac{\ln 4}{k_B - k_A} = \frac{2(0.693)}{0.693/450} = 900\ \text{s}.$$

Thus the supplied approximation for $\ln 2$ cancels exactly in the ratio.

Step 3: Check the result and reject the other times After 900 s, A has experienced three half-lives and retains $1/8$ of its starting concentration. B has experienced five half-lives and retains $1/32$. Their ratio is $(1/8)/(1/32) = 4$, as required. At 180 s, 120 s and 300 s, the ratio is respectively $2^{0.4}$, $2^{4/15}$ and $2^{2/3}$; each is less than four. Hence options (A), (B) and (C) do not satisfy the condition.

Step 4: Conclusion The required time is $\boxed{900\ \text{s}}$. Hence, the correct option is (D).

Quick Tip to Solve: For equal initial amounts undergoing first-order decay, compare the difference in the number of half-lives: a ratio of four requires two more halvings of B than A.

53. The correct electronic configuration and spin-only magnetic moment (BM) of Gd^{3+} (Z = 64), respectively, are : [4 marks]

(A) [Xe]$5f^7$ and 8.9

(B) [Xe]$4f^7$ and 7.9

(C) [Xe]$5f^7$ and 7.9

(D) [Xe]$4f^7$ and 8.9

Correct Answer: (B) [Xe]$4f^7$ and 7.9

Solution:

Concept:

- Neutral gadolinium has configuration $[\mathrm{Xe}]4f^7 5d^1 6s^2$. In forming a cation, the outer $6s$ electrons and then the $5d$ electron are removed.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons. Hund's rule gives one electron in each of the seven f orbitals for f^7.

Step 1: Form the tripositive ion Gadolinium has 64 electrons in the neutral atom. The xenon core accounts for 54, leaving ten electrons distributed as $4f^7 5d^1 6s^2$. Removal of three electrons gives

$$\mathrm{Gd} \longrightarrow \mathrm{Gd}^{3+} + 3\,\mathrm{e}^-, \qquad \mathrm{Gd}^{3+} : [\mathrm{Xe}]4f^7.$$

The electron count is $54 + 7 = 61 = 64 - 3$, a useful independent check. The $4f$ shell, rather than the $5f$ shell, belongs to the lanthanoid series containing gadolinium.

Step 2: Count the unpaired electrons and calculate the moment There are seven orbitals in an f subshell. In the half-filled $4f^7$ configuration, Hund's rule places one electron in each orbital with parallel spins, so $n = 7$. The spin-only moment is therefore

$$\mu = \sqrt{7(7+2)}\ \mathrm{BM} = \sqrt{63}\ \mathrm{BM} = 7.94\ \mathrm{BM}.$$

Rounded to one decimal place, this is 7.9 BM. The formula uses the number of unpaired electrons, not the total number of electrons in the ion.

Step 3: Match both entries to the options Option (B) gives both the correct shell, $4f^7$, and the correct rounded moment, 7.9 BM. Options (A) and (C) place the seven electrons in $5f$, which is the wrong subshell even though (C) has the right moment. Option (D) has the right configuration but gives 8.9 BM, inconsistent with $\sqrt{63}$. Both pieces of information must agree; matching only one is insufficient.

Step 4: Conclusion The configuration is $[\mathrm{Xe}]4f^7$ and the spin-only moment is 7.9 BM. Hence, the correct option is (B).

Quick Tip to Solve: For Gd^{3+}, remember the half-filled $4f^7$ shell: seven unpaired electrons immediately give $\sqrt{63} \approx 7.9$ BM.

54. The most appropriate reagent for conversion of C_2H_5CN into $CH_3CH_2CH_2NH_2$ is [4 marks]

(A) $NaBH_4$

(B) CaH_2

(C) $Na(CN)BH_3$

(D) $LiAlH_4$

Correct Answer: (D) $LiAlH_4$

Solution:

Concept:

- A nitrile $R-C\equiv N$ is reduced to the primary amine $R-CH_2-NH_2$. The nitrile carbon is retained as the methylene carbon next to nitrogen.
- Lithium aluminium hydride, $LiAlH_4$, is a strong hydride reducing agent capable of reducing nitriles; hydrolysis of the aluminium-bound intermediate releases the amine.

Step 1: Recognise the functional-group transformation The starting material C_2H_5CN is propanenitrile, $CH_3CH_2-C\equiv N$. The product $CH_3CH_2CH_2NH_2$ is propan-1-amine. There is no change in the number of carbon atoms: the carbon of the nitrile group becomes the terminal CH_2NH_2 group. Thus the required operation is complete reduction of the carbon-nitrogen triple bond, rather than substitution or removal of the nitrile carbon.

Step 2: Choose the reagent and state the work-up $LiAlH_4$ supplies hydride to the nitrile carbon and reduces the multiple bond through an imine-type intermediate. A subsequent aqueous work-up protonates the nitrogen-containing intermediate to give the free primary amine. In condensed form,

$$CH_3CH_2CN + 4\,[H] \longrightarrow CH_3CH_2CH_2NH_2.$$

The usual reagent sequence is $LiAlH_4$ in dry ether, followed by water. Water is added only after the hydride reduction because it would otherwise consume the reducing agent.

Step 3: Reject the weaker or unsuitable alternatives $NaBH_4$ in option (A) normally reduces aldehydes and ketones and is not the standard reagent for direct nitrile-to-amine reduction. CaH_2 in option (B) is mainly used as a drying agent and does not supply this selective organic transformation under ordinary conditions. Sodium cyanoborohydride, $NaBH_3CN$, in option (C) is a mild reagent associated with reductive amination of imines or iminium ions; it is not the appropriate direct nitrile reducing agent here. The strong hydride in (D) meets the requirement.

Step 4: Conclusion Use $LiAlH_4$, followed by aqueous work-up, to obtain propan-1-amine. Hence, the correct option is (D).

Quick Tip to Solve: In nitrile reduction, retain the nitrile carbon and replace $-C\equiv N$ by $-CH_2NH_2$; choose the strong hydride $LiAlH_4$.

55. Which of the following is not an essential amino acid : [4 marks]

(A) Valine

(B) Leucine

(C) Lysine

(D) Tyrosine

Correct Answer: (D) Tyrosine

Solution:

Concept:

- Essential amino acids are those that the human body cannot synthesise in sufficient amounts. They must therefore be supplied by dietary proteins; essentiality concerns nutritional requirement, not whether an amino acid occurs in proteins.
- Non-essential amino acids can normally be made by the body. Tyrosine is formed from the essential amino acid phenylalanine by hydroxylation of its aromatic ring.

Step 1: Classify the listed amino acids Valine and leucine are branched-chain essential amino acids. Lysine is also an essential amino acid. The human body does not normally produce enough of any of these three to meet its needs, so all three belong in the dietary essential group. Thus options (A), (B) and (C) cannot answer a question asking for the amino acid that is not essential.

Step 2: Identify the biosynthetically available member Tyrosine is a hydroxylated aromatic amino acid. Normal metabolism converts phenylalanine into tyrosine through the enzyme phenylalanine hydroxylase. Accordingly, tyrosine need not ordinarily be supplied independently in the diet when adequate phenylalanine is available. It is classified as non-essential in the standard nutritional classification used here. This does not mean that tyrosine is biologically unimportant; it is a protein constituent and precursor for several substances.

Step 3: Conclusion Tyrosine is not an essential amino acid in the usual classification. Hence, the correct option is (D).

Quick Tip to Solve: For this list, remember that valine, leucine and lysine are essential; tyrosine can be made from phenylalanine.

56. The structure of PCl_5 in the solid state is : [4 marks]

(A) square pyramidal

(B) tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$

(C) square planar $[PCl_4]^+$ and octahedral $[PCl_6]^-$

(D) trigonal bipyramidal

Correct Answer: (B) tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$

Solution:

Concept:

- The physical state matters when assigning the structure of phosphorus pentachloride. A discrete gaseous molecule of PCl_5 is trigonal bipyramidal, whereas the solid is an ionic lattice.
- In the solid, chloride transfer between two formula units produces tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$ ions. The lattice stabilises these oppositely charged species.

Step 1: Write the solid-state ion formation The solid-state formulation is represented by

$$2\,PCl_5 \longrightarrow [PCl_4]^+ + [PCl_6]^-.$$

There are ten chlorine atoms and two phosphorus atoms on each side, and the net charge remains zero. The cation and anion occur in a one-to-one ratio. Therefore, treating the solid as a collection of isolated five-coordinate molecules would miss the ionic structure.

Step 2: Assign the coordination geometries In $[PCl_4]^+$, phosphorus has four bonding pairs and no lone pair, giving a tetrahedral arrangement. In $[PCl_6]^-$, six chlorine atoms surround phosphorus, giving an octahedral arrangement. Thus the tetrahedral/octahedral pair in (B) is correct. Option (C) incorrectly makes the four-coordinate ion square planar. Option (A) gives a square-pyramidal molecular geometry, which is not the solid structure, and (D) describes the molecular gas rather than the ionic solid.

Step 3: Conclusion Solid PCl_5 consists of tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$ ions. Hence, the correct option is (B).

Quick Tip to Solve: First check the physical state: gaseous PCl5 is trigonal bipyramidal, but solid PCl5 contains a tetrahedral cation and an octahedral anion.

57. Consider the following reaction:

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$; $\Delta H^o = +58$ kJ

For each of the following cases (a, b), the direction in which the equilibrium shifts is :

(a) Temperature is decreased.

(b) Pressure is increased by adding N_2 at constant T. [4 marks]

(A) (a) towards reactant, (b) towards product

(B) (a) towards reactant, (b) no change

(C) (a) towards product, (b) towards reactant

(D) (a) towards product, (b) no change

Correct Answer: (B) (a) towards reactant, (b) no change

Solution:

Concept:

- Le Chatelier's principle predicts that cooling favours the exothermic direction. Here the forward dissociation has positive enthalpy, so heat acts like a reactant in the forward process.
- Adding a non-reacting gas raises total pressure without changing the reacting gases' partial pressures when temperature and container volume are unchanged. Equilibrium depends on partial pressures, not total pressure alone.

Step 1: Apply the temperature change For the reaction

$$\mathrm{N_2O_4(g)} \rightleftharpoons 2\,\mathrm{NO_2(g)}, \qquad \Delta H^\circ = +58\,\mathrm{kJ},$$

the forward reaction absorbs heat. Decreasing temperature favours the reverse, heat-releasing association. Consequently, the equilibrium shifts towards the reactant N_2O_4, excluding options (C) and (D), both of which predict a shift towards product.

Step 2: Distinguish inert-gas addition from compression Nitrogen does not participate in this equilibrium. Increasing pressure by adding N_2 to the same container is different from reducing the volume: at fixed temperature and volume, each reacting gas still has partial pressure $p_i = n_iRT/V$. Thus neither p_{NO_2} nor $p_{N_2O_4}$ changes, and

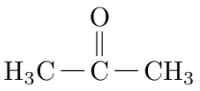
$$Q_p = \frac{p_{\mathrm{NO_2}}^2}{p_{\mathrm{N_2O_4}}}$$

remains equal to its equilibrium value. There is no shift in case (b). Option (A) confuses total pressure with the reacting partial pressures. If the vessel were expanded at constant total pressure, a different result could follow, but that is not the stated pressure-increase operation.

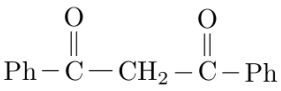
Step 3: Conclusion Cooling shifts equilibrium towards reactant, while increasing total pressure by adding nitrogen produces no change in the reacting equilibrium. Hence, the correct option is (B).

Quick Tip to Solve: When an inert gas is added, inspect partial pressures first; the fewer-moles rule applies to compression, not simply to an increase in total pressure.

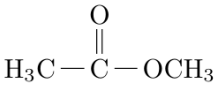
58. The increasing order of the acidity of the α-hydrogen of the following compounds is : [4 marks]



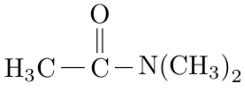
(A)



(B)



(C)



(D)

(A) $(C) < (A) < (B) < (D)$

(B) $(B) < (C) < (A) < (D)$

(C) $(A) < (C) < (D) < (B)$

(D) $(D) < (C) < (A) < (B)$

Correct Answer: (D) $(D) < (C) < (A) < (B)$

Solution:

Concept:

- An alpha hydrogen is removed from the carbon adjacent to a carbonyl group. Its acidity increases as the resulting conjugate base becomes more stable by resonance and electron withdrawal.
- The compounds are (A) acetone, (B) dibenzoylmethane, (C) methyl acetate and (D) N,N-dimethylacetamide. Compare the carbonyl groups and their substituents before ranking their alpha-hydrogen acidity.

Step 1: Compare the single-carbonyl compounds In acetone (A), the negative charge of the enolate can be delocalised from the alpha carbon onto the carbonyl oxygen. In ester (C), the methoxy group donates a lone pair into the carbonyl, reducing its electron-withdrawing effect and making alpha deprotonation less favourable than in the ketone. The amide nitrogen in (D) is an even stronger resonance donor than the ester oxygen, so its alpha hydrogen is the least acidic of these three. Their increasing acidity is therefore $(D) < (C) < (A)$.

Step 2: Include the compound with two carbonyl groups In (B), the central methylene lies between two benzoyl carbonyl groups. Removal of one of its hydrogens yields a conjugate base whose charge can be delocalised onto either carbonyl oxygen. This additional stabilisation makes (B) much more acidic than any single-carbonyl member. Combining both comparisons gives

$$(D) < (C) < (A) < (B).$$

Option (A) wrongly places the amide last, option (B) wrongly places the doubly activated methylene first, and option (C) wrongly places acetone below both ester and amide. Only option (D) agrees with conjugate-base stability.

Step 3: Conclusion The increasing order of alpha-hydrogen acidity is amide < ester < ketone < beta-diketone. Hence, the correct option is (D).

Quick Tip to Solve: For alpha-hydrogen acidity, stabilisation by two carbonyls dominates; among single carbonyls, amide < ester < ketone.

59. The difference between the radii of 3rd and 4th orbits of Li^{2+} is R_1 . The difference between the radii of 3rd and 4th orbits of He^+ is ΔR_2 .

Ratio $\Delta R_1 : \Delta R_2$ is : [4 marks]

(A) 8 : 3

(B) 3 : 2

(C) 2 : 3

(D) 3 : 8

Correct Answer: (C) 2 : 3

Solution:

Concept:

- For a hydrogen-like ion containing a single electron, the Bohr radius of the nth orbit is $r_n = a_0 n^2/Z$, where Z is nuclear charge and a_0 is the hydrogen ground-state radius.
- Both Li^{2+} and He^+ have one electron. Thus the same Bohr formula applies, but their nuclear charges are 3 and 2 respectively.

Step 1: Calculate the common orbit factor The required difference is the fourth-orbit radius minus the third-orbit radius, which is positive:

$$\Delta r = r_4 - r_3 = \frac{a_0}{Z}(4^2 - 3^2) = \frac{7a_0}{Z}.$$

The squared-principal-quantum-number difference is identical for both ions, so their ratio depends only on the inverse nuclear charges. There is no need to substitute a numerical value of the Bohr radius.

Step 2: Form the ratio in the specified order For lithium and helium respectively,

$$\Delta R_1 = \frac{7a_0}{3}, \qquad \Delta R_2 = \frac{7a_0}{2}.$$

Therefore,

$$\frac{\Delta R_1}{\Delta R_2} = \frac{7a_0/3}{7a_0/2} = \frac{2}{3}.$$

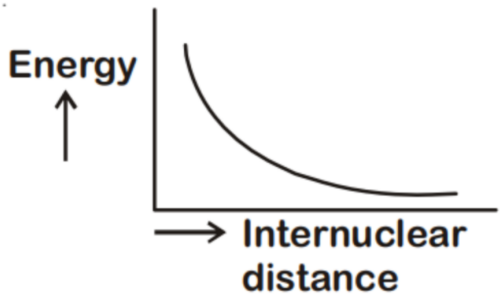
The ratio is dimensionless because the length unit cancels. The stronger attraction of the lithium nucleus also predicts its orbital gap to be smaller, confirming that the ratio must be below one. Option (B) reverses the order; options (A) and (D) introduce a factor unrelated to the common transition between the same two orbit numbers.

Step 3: Conclusion The ratio $\Delta R_1 : \Delta R_2 = 2 : 3$. Hence, the correct option is (C).

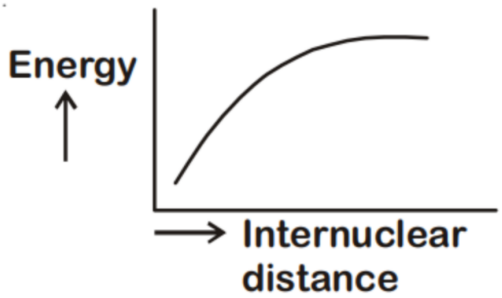
Quick Tip to Solve: For the same pair of orbits in one-electron ions, radius differences vary as 1/Z; write the nuclear charges in reverse order for the ratio.

60. **The potential energy curve for the H_2 molecule as a function of internuclear distance is :** [4 marks]

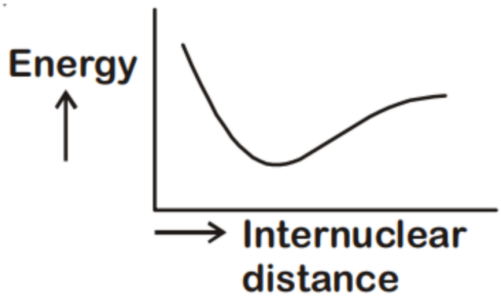
(A)



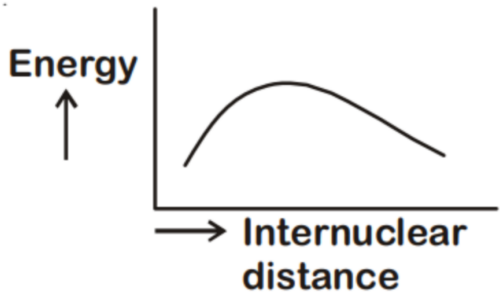
(B)



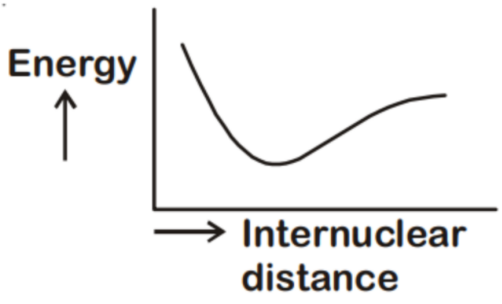
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- The potential energy of two hydrogen atoms is conventionally measured relative to two separated atoms. A stable bond requires a minimum of potential energy at a finite internuclear separation.
- At long distances the interaction approaches zero; at very short distances nuclear and electron repulsions raise the energy sharply. At intermediate distances attractive interactions lower the energy.

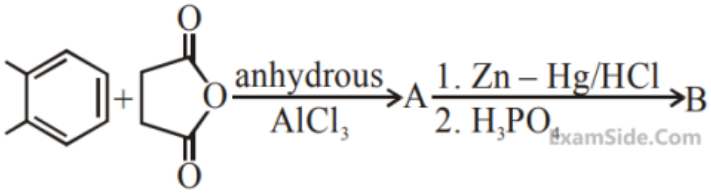
Step 1: Follow the energy as the atoms approach Starting with two well-separated hydrogen atoms, attractive interactions between each nucleus and the other atom's electron lower the potential energy as separation decreases. The energy reaches a minimum at the equilibrium bond length, approximately $74\,\mathrm{pm}$. At this minimum, the net force is zero because $F = -dU/dr$; small displacements increase the energy, which is the condition for a stable molecular bond.

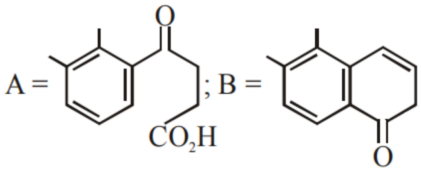
Step 2: Check both sides of the minimum Bringing the nuclei even closer makes repulsive interactions dominate, so the energy rises steeply on the short-distance side. Increasing the separation from equilibrium breaks the bond and raises the energy towards the separated-atom limit. Consequently the curve must have a well-shaped minimum, as in option (C). The absolute vertical placement of a plotted energy curve depends on the energy reference, but the minimum and the two limiting trends do not. Option (A) decreases without a stable finite-distance minimum; option (B) rises monotonically and has no such minimum; option (D) has a maximum, corresponding to unstable rather than stable equilibrium.

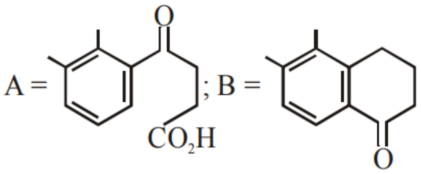
Step 3: Conclusion The potential-energy curve has a minimum at the equilibrium bond length and rises on either side. Hence, the correct option is (C).

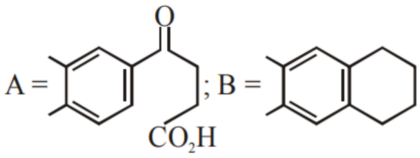
Quick Tip to Solve: Look for a potential-energy well: a stable bond sits at a minimum, with steep repulsion to the left and dissociation to the right.

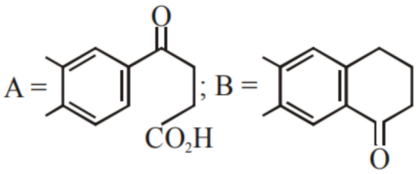
61. In the following reaction sequence the major products A and B are : [4 marks]



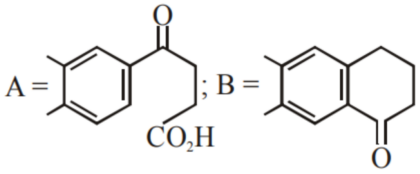
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- Friedel-Crafts acylation with succinic anhydride gives an aryl keto acid, $\mathrm{ArCOCH_2CH_2COOH}$. The two methyl groups of the starting o-xylene activate the ring and favour the less hindered substitution site.
- Clemmensen reduction, $\mathrm{Zn(Hg)/HCl}$, converts the ketonic carbonyl to a methylene group while retaining the carboxylic acid. Subsequent acid-promoted intramolecular acylation closes a six-membered ring.

Step 1: Locate the first acylation site The two methyl groups of o-xylene are ortho to one another. A position para to one methyl group and meta to the other is less crowded than a position ortho to a

methyl group. Acylation there gives A = 4-(3,4-dimethylphenyl)-4-oxobutanoic acid, with the side chain $-COCH_2CH_2COOH$. The A structures of options (A) and (B), having the side chain next to a methyl group, therefore do not represent the major regioisomer.

Step 2: Apply reduction before cyclisation The $Zn(Hg)/HCl$ step changes $ArCOCH_2CH_2COOH$ into $ArCH_2CH_2CH_2COOH$. It removes the original ketone oxygen, but it does not reduce the acid group under the stated conditions. Thus the carbonyl in the final cyclic ketone must originate from the retained terminal carboxylic acid, rather than from the original aryl ketone.

Step 3: Close the six-membered ring H_3PO_4 promotes intramolecular Friedel-Crafts acylation of this acid at an available position ortho to the side chain. The chain contains three methylene groups, so closure gives a saturated six-membered ring fused to benzene, retaining one ketone group: 6,7-dimethyl-1-tetralone. Option (C) wrongly removes this final carbonyl, and option (A) also shows an extra double bond in the newly formed ring.

Step 4: Conclusion A is 4-(3,4-dimethylphenyl)-4-oxobutanoic acid and B is 6,7-dimethyl-1-tetralone, the pair shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: Track the two carbonyl groups separately: Clemmensen removes the aryl ketone, while the terminal acid supplies the carbonyl formed during ring closure.

62. The values of the crystal field stabilization energies for a high spin d^6 metal ion in octahedral and tetrahedral fields, respectively, are : [4 marks]

(A) $-0.4\Delta_0$ and $-0.27\Delta_t$

(B) $-1.6\Delta_0$ and $-0.4\Delta_t$

(C) $-0.4\Delta_0$ and $-0.6\Delta_t$

(D) $-2.4\Delta_0$ and $-0.27\Delta_t$

Correct Answer: (C) $-0.4\Delta_0$ and $-0.6\Delta_t$

Solution:

Concept:

- CFSE is the sum of orbital energies relative to the spherical-field barycentre; it excludes pairing energy unless that extra term is explicitly requested.
- In an octahedral field, each t_{2g} electron contributes $-0.4\Delta_o$ and each e_g electron contributes $+0.6\Delta_o$. In a tetrahedral field the lower e and upper t_2 contributions are $-0.6\Delta_t$ and $+0.4\Delta_t$.

Step 1: Fill the high-spin octahedral orbitals A high-spin d^6 ion occupies all five orbitals singly before the sixth electron pairs in a lower orbital. Its distribution is $t_{2g}^4 e_g^2$, not the low-spin t_{2g}^6 arrangement. Hence

$$\mathrm{CFSE_{oct}} = 4(-0.4\Delta_o) + 2(0.6\Delta_o) = -0.4\Delta_o.$$

This immediately excludes options (B) and (D), whose octahedral values correspond to different occupation counts or spin assumptions.

Step 2: Fill the tetrahedral orbitals The tetrahedral lower set contains two orbitals and the upper set contains three. Distributing six electrons in the high-spin arrangement gives $e^3 t_2^3$: one pair in the

lower set and four unpaired electrons overall. Consequently

$$\mathrm{CFSE}_{\mathrm{tet}} = 3(-0.6\Delta_t) + 3(0.4\Delta_t) = -0.6\Delta_t.$$

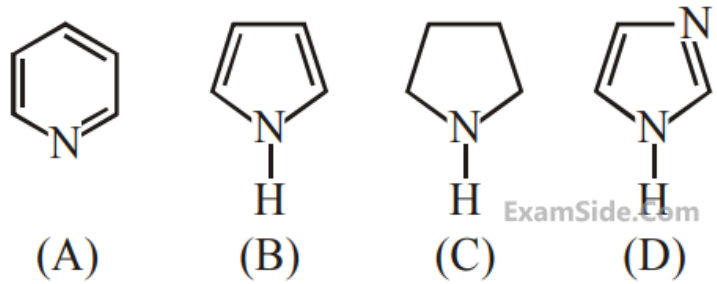
The splitting is expressed in units of Δ_t, as the question specifies; there is no reason to insert a conversion factor to Δ_o.

Step 3: Compare the ordered pair The required order is octahedral first and tetrahedral second. The ordered pair is therefore $(-0.4\Delta_o, -0.6\Delta_t)$. Option (A) has the correct first entry but an incorrect second entry. Option (C) contains both calculated values, whereas (B) and (D) already fail the octahedral calculation.

Step 4: Conclusion The stabilization energies are $-0.4\Delta_o$ and $-0.6\Delta_t$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Write the orbital populations before calculating CFSE; the coefficients reverse between octahedral and tetrahedral lower and upper sets.

63. The increasing order of basicity of the following compounds is : [4 marks]



(A) (B) (C) (D)

(A) $(A) < (B) < (C) < (D)$
(B) $(B) < (A) < (C) < (D)$
(C) $(D) < (A) < (B) < (C)$
(D) $(B) < (A) < (D) < (C)$

Correct Answer: (D) $(B) < (A) < (D) < (C)$

Solution:

Concept:

- Basicity depends on the availability of the nitrogen lone pair for protonation and on the stability of the resulting conjugate acid.
- A lone pair forming part of an aromatic sextet is much less available than a lone pair outside that sextet. Resonance stabilization of a conjugate acid can increase basicity.

Step 1: Identify the four nitrogen environments The structures are (A) pyridine, (B) pyrrole, (C) pyrrolidine and (D) imidazole. Pyrrole is aromatic because its nitrogen lone pair contributes two electrons to the six-electron aromatic system. Protonating this pair disrupts that stabilization, making pyrrole the weakest base. In pyridine, the lone pair is in an orbital outside the aromatic pi system, so

it can accept a proton without destroying aromaticity.

Step 2: Compare pyridine and imidazole Imidazole contains both a pyrrole-like nitrogen and a pyridine-like nitrogen. Protonation occurs chiefly at the pyridine-like nitrogen, whose lone pair is available. The positive charge in the resulting imidazolium ion is delocalized over the two nitrogens through resonance. This stabilizes its conjugate acid more than pyridinium and places imidazole above pyridine in basicity. Thus the order so far is $(B) < (A) < (D)$.

Step 3: Place the saturated amine and test the choices Pyrrolidine is a saturated secondary cyclic amine. Its nitrogen lone pair is localized on an sp^3-hybridized nitrogen, and the alkyl groups donate electron density, making it more basic than the aromatic heterocycles here. It is therefore last in an increasing order. Option (A) incorrectly puts pyridine below pyrrole; option (B) puts imidazole above pyrrolidine; option (C) puts imidazole below pyrrole. Only option (D) has all comparisons correct.

Step 4: Conclusion The increasing basicity is pyrrole < pyridine < imidazole < pyrrolidine. Hence, the correct option is (D).

Quick Tip to Solve: For heterocycles, first ask whether the lone pair belongs to the aromatic sextet, then compare resonance stabilization after protonation.

64. In the sixth period, the orbitals that are filled are : [4 marks]

(A) 6s, 5d, 5f, 6p

(B) 6s, 4f, 5d, 6p

(C) 6s, 6p, 6d, 6f

(D) 6s, 5f, 6d, 6p

Correct Answer: (B) 6s, 4f, 5d, 6p

Solution:

Concept:

- A period begins with filling of its outer ns subshell and ends when the corresponding np subshell is complete, apart from the first period.
- The sixth period includes the lanthanoids, so the inner $4f$ subshell is filled along with $6s$, $5d$ and $6p$. Subshell filling follows orbital energies, rather than simply increasing the principal quantum number.

Step 1: Locate the start and end of the period The sixth period starts after xenon. Caesium and barium occupy the $6s$ subshell, so the first subshell required is $6s$. The period ends at radon, whose outer configuration is $6s^2 6p^6$. Thus $6p$ is the final subshell in the period. An answer must account for the intervening inner-transition and transition elements as well as these two main-group blocks.

Step 2: Identify the intervening f and d blocks The lanthanoid series belongs to period six and progressively fills $4f$, while the period-six transition series fills $5d$. At the level of subshells represented across the period, the sequence is

$$6s, \quad 4f, \quad 5d, \quad 6p.$$

Individual ground-state configurations near the start of the f block may share electrons between

closely spaced orbitals; that does not change which four subshells are involved.

Step 3: Reject the other lists Option (A) substitutes $5f$ for $4f$; the $5f$ actinoid series belongs to the seventh period. Option (C) incorrectly lists $6d$ and $6f$ as sixth-period subshells. Option (D) likewise contains $5f$ and $6d$, associated with the later period. Only option (B) includes the sixth-period inner-transition subshell $4f$ and transition subshell $5d$, together with the correct outer s and p subshells.

Step 4: Conclusion The orbitals filled across period six are $6s, 4f, 5d, 6p$. Hence, the correct option is (B).

Quick Tip to Solve: For period n containing an f block, remember the pattern ns, (n-2)f, (n-1)d, np; for n = 6 this gives 6s, 4f, 5d, 6p.

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65. If a person is suffering from the deficiency of nor-adrenaline, what kind of drug can be suggested?

[4 marks]

(A) Anti-inflammatory

(B) Antihistamine

(C) Analgesic

(D) Antidepressant

Correct Answer: (D) Antidepressant

Solution:

Concept:

- Noradrenaline is a neurotransmitter involved in signal transmission and mood regulation. Low levels are associated with depressive symptoms in the textbook classification used here.
- Antidepressants can increase the availability or action of monoamine neurotransmitters at synapses, for example by inhibiting their reuptake or enzymatic breakdown.

Step 1: Relate the deficiency to neurotransmission Noradrenaline carries signals between nerve cells. After release into a synapse, its concentration is controlled by reuptake and metabolism. A deficiency reduces the neurotransmitter available for signalling. The class sought in this question should therefore address impaired neurotransmitter availability and mood regulation, rather than inflammation, allergy or pain.

Step 2: Identify the relevant class of drug In the standard chemistry classification, antidepressants are used for depressive conditions associated with insufficient monoamine neurotransmitter activity. Certain antidepressants inhibit enzymes that degrade neurotransmitters such as noradrenaline, while others inhibit their reuptake. Either mechanism can increase the amount remaining available at the synapse and improve transmission. Thus the pharmacological class corresponding to the stated deficiency is an antidepressant.

Step 3: Distinguish the distractor classes Anti-inflammatory drugs reduce inflammatory responses such as swelling and inflammation-associated discomfort; that is a different target. Antihistamines block histamine action and are classified chiefly with allergy treatment, so the similarity between the

words amine and histamine is not a reason to choose them. Analgesics relieve pain without specifically correcting the stated neurotransmitter deficiency. These functions exclude options (A), (B) and (C), leaving option (D).

Step 4: Conclusion The relevant drug class is antidepressants. Hence, the correct option is (D).

Quick Tip to Solve: Match each drug class to its target: antidepressants to mood-related monoamine signalling, antihistamines to histamine, and analgesics to pain.

66. An Ellingham diagram provides information about : [4 marks]

(A) the temperature dependence of the standard Gibbs energies of formation of some metal oxides

(B) the pressure dependence of the standard electrode potentials of reduction reactions involved in the extraction of metals

(C) the conditions of pH and potential under which a species is thermodynamically stable

(D) the kinetics of the reduction process

Correct Answer: (A) the temperature dependence of the standard Gibbs energies of formation of some metal oxides

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change for oxidation, ΔG°, against temperature. Its curves commonly describe the formation of metal oxides.
- The relation $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ explains why many portions of the plotted curves are approximately straight lines, with slope $-\Delta S^\circ$.

Step 1: Read the two plotted variables The horizontal coordinate of an Ellingham diagram is temperature and the vertical coordinate is the standard Gibbs energy change for an oxidation reaction, commonly normalized per mole of oxygen. A point on a metal-oxide line therefore tells us the thermodynamic tendency of that metal to combine with oxygen at the selected temperature. It directly provides temperature dependence of oxide formation energies, as stated in option (A).

Step 2: Explain its metallurgical use Comparing an oxide-formation line with the oxidation line of a possible reducing agent helps decide whether reduction is thermodynamically favourable. If the oxidation of the reducing agent has a sufficiently more negative Gibbs energy, adding it to the reversed metal-oxide formation reaction can give a negative net ΔG°. This comparison concerns feasibility at a given temperature, rather than the speed at which the reduction actually occurs.

Step 3: Separate thermodynamics from the distractors Option (B) describes pressure dependence of standard electrode potentials, which is not what the axes of an Ellingham diagram show. Option (C), involving pH and electrode potential, describes a potential-pH or Pourbaix diagram. Option (D) concerns kinetics; reaction rates depend on activation barriers and other factors that a Gibbs-energy-versus-temperature plot does not supply. Only option (A) names both the correct energy quantity and the correct independent variable.

Step 4: Conclusion An Ellingham diagram shows the temperature dependence of standard Gibbs

energies of formation of metal oxides. Hence, the correct option is (A).

Quick Tip to Solve: Identify the axes: Gibbs energy versus temperature means Ellingham; potential versus pH means Pourbaix. Neither graph by itself gives a reaction rate.

67. A diatomic molecule X_2 has a body-centred cubic (bcc) structure with a cell edge of 300 pm. The density of the molecule is 6.17 g cm^{-3}. The number of molecules present in 200 g of X_2 is : (Avogadro constant (N_A) = 6 $\times$ 10^{23} mol^{-1}) [4 marks]

(A) 8 N_A

(B) 40 N_A

(C) 4 N_A

(D) 2 N_A

Correct Answer: (C) 4 N_A

Solution:

Concept:

- A body-centred cubic unit cell contains $Z = 2$ molecules: the eight corners contribute one molecule in total and the body centre contributes another.
- For a molecular crystal, density is $\rho = ZM/(N_A a^3)$, where M is the molar mass of the molecule and a is the cell edge. The number of molecules in mass m is $(m/M)N_A$.

Step 1: Convert the edge and calculate the cell volume The density is given in grams per cubic centimetre, so express the edge in centimetres before cubing. Since $1\,\text{pm} = 10^{-10}\,\text{cm}$,

$$a = 300 \times 10^{-10} = 3.00 \times 10^{-8}\,\text{cm}, \qquad a^3 = 2.70 \times 10^{-23}\,\text{cm}^3.$$

The cell volume is the cube of the edge, rather than the edge itself or the volume of a sphere around a molecule.

Step 2: Find the molecular molar mass Each bcc cell contains two X_2 molecules. Consequently,

$$M = \frac{\rho a^3 N_A}{Z} = \frac{6.17(2.70 \times 10^{-23})(6.00 \times 10^{23})}{2} = 49.977\,\text{g mol}^{-1} \simeq 50.0\,\text{g mol}^{-1}.$$

This is already the molar mass of the diatomic molecule; multiplying or dividing it by two again would count atoms rather than the specified molecular entities.

Step 3: Convert the sample mass into molecules The amount of X_2 in the sample is

$$n = \frac{200}{49.977} \simeq 4.00\,\text{mol}, \qquad N = nN_A \simeq 4N_A.$$

Thus $8N_A$ would double the required molecular count, $2N_A$ would halve it, and $40N_A$ would introduce a factor-of-ten error.

Step 4: Conclusion The 200 g sample contains $4N_A = 2.4 \times 10^{24}$ molecules of X_2, to the precision of the data. Hence, the correct option is (C).

Quick Tip to Solve: Use $Z = 2$ for a bcc molecular crystal and convert pm to cm before cubing; the molar mass then refers to the whole molecule.

68. The equation that represents the water-gas shift reaction is : [4 marks]

(A) $CO(g) + H_2O(g) \xrightarrow[Catalyst]{673K} CO_2(g) + H_2(g)$

(B) $CH_4(g) + H_2O(g) \xrightarrow[Ni]{1270K} CO(g) + 3H_2(g)$

(C) $C(s) + H_2O(g) \xrightarrow{1270K} CO(g) + H_2(g)$

(D) $2C(s) + O_2(g) + 4N_2(g) \xrightarrow{1273K} 2CO(g) + 4N_2(g)$

Correct Answer: (A) $CO(g) + H_2O(g) \xrightarrow[Catalyst]{673K} CO_2(g) + H_2(g)$

Solution:

Concept:

- Water gas is a mixture containing carbon monoxide and hydrogen. The water-gas shift reaction converts carbon monoxide with steam into carbon dioxide and additional hydrogen.
- The defining stoichiometry is $\mathrm{CO(g) + H_2O(g) \longrightarrow CO_2(g) + H_2(g)}$. It is distinct from making water gas by passing steam over hot carbon or by reforming methane.

Step 1: Identify the reactants and products of the shift In the shift process, carbon monoxide is oxidised to carbon dioxide while steam supplies the hydrogen that is liberated. The balanced equation is

$$\mathrm{CO(g) + H_2O(g) \xrightarrow[673\ K]{catalyst} CO_2(g) + H_2(g).}$$

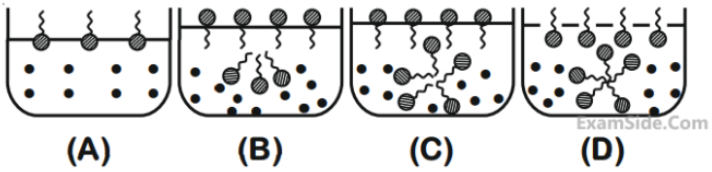
There is one carbon atom, two oxygen atoms and two hydrogen atoms on each side. This reaction increases the hydrogen yield from the carbon monoxide already present in a synthesis-gas mixture and is precisely the equation in option (A).

Step 2: Distinguish the other industrial gas reactions Option (B), $\mathrm{CH_4 + H_2O \longrightarrow CO + 3\,H_2}$, is steam reforming of methane over nickel: it creates carbon monoxide instead of converting it into carbon dioxide. Option (C), $\mathrm{C + H_2O \longrightarrow CO + H_2}$, is formation of water gas by the reaction of steam with hot coke. Option (D), $\mathrm{2\,C + O_2 + 4\,N_2 \longrightarrow 2\,CO + 4\,N_2}$, gives producer gas from carbon and air; nitrogen remains as a diluent. None of these equations represents the conversion of carbon monoxide by steam that defines the shift reaction.

Step 3: Conclusion The water-gas shift reaction is $\mathrm{CO(g) + H_2O(g) \longrightarrow CO_2(g) + H_2(g)}$, shown with a catalyst at 673 K. Hence, the correct option is (A).

Quick Tip to Solve: Look for CO and steam on the left and carbon dioxide plus extra hydrogen on the right; making CO is not the shift reaction.

69. Identify the correct molecular picture showing that happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant (◎ polar head; ∼ non-polar tail; • water). [4 marks]



(A) (C)

(B) (A)

(C) (D)

(D) (B)

Correct Answer: (C) (D)

Solution:

Concept:

- A surfactant has a hydrophilic polar head and a hydrophobic non-polar tail. In water, favourable orientations expose the heads to water while keeping the tails away from it.
- At the critical micellar concentration, surfactant molecules aggregate into micelles. For an aqueous micelle, the polar heads face outward into water and the non-polar tails cluster inward in the hydrophobic core.

Step 1: Check the orientation at the water surface An isolated surfactant at an air-water boundary places its polar head on the water side and its non-polar tail toward air. Picture (A) has this surface orientation but shows only isolated surface molecules, so it does not depict the onset of micellar aggregation. Pictures (B) and (C) put the surface heads toward air with tails immersed in water. That reverses the preferred orientation for a surfactant in an aqueous solution.

Step 2: Check the aggregate inside the solution At CMC, a group of molecules forms a micelle within the water. Picture (D) simultaneously shows the proper surface orientation and an aggregate whose polar heads lie around its exterior in contact with the surrounding water. Its wavy non-polar tails point inward, away from water. Picture (B) instead has an inward cluster of polar heads with tails exposed outward; picture (C) cannot be accepted because its surface monolayer is reversed even though its internal grouping resembles a micelle. The complete physically consistent picture is therefore (D).

Step 3: Match the picture label to the option letter The four drawings carry picture labels (A), (B), (C) and (D), while the answer options list those labels in the different order (C), (A), (D), (B). The required drawing is picture (D), which is listed as the third answer option. Distinguishing the drawing label from the option letter avoids selecting option (D), which actually names picture (B).

Step 4: Conclusion The fourth molecular picture correctly represents micellisation in water at CMC; its label is listed under the third answer option. Hence, the correct option is (C).

Quick Tip to Solve: For an aqueous micelle, put tails in the core and heads in water; then map the drawing label back to the separately ordered answer options.

70. The condition that indicates a polluted environment is : [4 marks]

(A) BOD value of 5 ppm

(B) eutrophication

(C) 0.03% of CO_2 in the atmosphere

(D) pH of rain water to be 5.6

Correct Answer: (B) eutrophication

Solution:

Concept:

- Eutrophication is nutrient enrichment of a water body, commonly from excess nitrates and phosphates. It promotes excessive algal growth and subsequent oxygen depletion.
- Biochemical oxygen demand (BOD) measures the oxygen required by microorganisms to decompose organic matter. High BOD indicates heavy organic pollution; normal atmospheric carbon dioxide and mildly acidic natural rain are different indicators.

Step 1: Recognise the consequence of eutrophication When nutrients from fertilisers or sewage enter a lake, algal populations can grow rapidly. The eventual death and microbial decomposition of that organic material consume dissolved oxygen. This can suffocate fish and disturb the aquatic ecosystem. Eutrophication therefore directly identifies an environmentally harmful enrichment process and is the definite pollution condition among the alternatives.

Step 2: Compare the numerical alternatives with their context A BOD of 5 ppm is around the usual upper boundary quoted for clean water, whereas heavily polluted water is associated with a much larger BOD, commonly 17 ppm or more. Thus the stated value alone is not the definite pollution indicator sought here. The 0.03% atmospheric CO_2 figure is the conventional background value used in this syllabus context, not a statement of excess carbon dioxide pollution. Natural rain dissolves CO_2, making weak carbonic acid; a pH near 5.6 is therefore its normal mildly acidic value. Acid rain is identified by pH below that reference. These comparisons eliminate (A), (C) and (D).

Step 3: Conclusion Eutrophication indicates pollution and deterioration of the aquatic environment. Hence, the correct option is (B).

Quick Tip to Solve: Use eutrophication as a nutrient-pollution signal; do not confuse normal rain pH near 5.6 with the lower pH characteristic of acid rain.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The minimum number of moles of O_2 required for complete combustion of 1 mole of propane and 2 moles of butane is _____. [4 marks]

Solution:

Concept:

- Complete combustion of a hydrocarbon converts every carbon atom into CO_2 and every pair of hydrogen atoms into H_2O.

- For C_xH_y, one mole requires $x + y/4$ moles of O_2. The oxygen demand of a mixture is the sum of the demands of its components.

Step 1: Balance combustion of propane Propane is C_3H_8. Its three carbon atoms give three molecules of carbon dioxide and its eight hydrogen atoms give four molecules of water. Hence,

$$C_3H_8 + 5\,O_2 \longrightarrow 3\,CO_2 + 4\,H_2O.$$

The products contain ten oxygen atoms in total, requiring five molecules of O_2. Therefore 1 mol of propane needs $5\,\text{mol}$ of oxygen.

Step 2: Balance combustion of butane Butane is C_4H_{10}. One mole produces 4 mol of CO_2 and 5 mol of H_2O, which together contain 13 mol of oxygen atoms. In molecular-oxygen form,

$$2\,C_4H_{10} + 13\,O_2 \longrightarrow 8\,CO_2 + 10\,H_2O.$$

Thus 1 mol of butane requires $13/2$ mol of O_2, and the stated 2 mol of butane require 13 mol of O_2.

Step 3: Add the stoichiometric requirements The minimum supply corresponds to exact complete combustion without excess oxygen left over. Both fuels must be burned, so add their independent requirements:

$$n(O_2) = 1(5) + 2\left(\frac{13}{2}\right) = 5 + 13 = 18\,\text{mol}.$$

There is no need to average the two combustion ratios; the amount of each fuel has already been specified.

Step 4: Conclusion The minimum amount of O_2 required is $18\,\text{mol}$. The numerical entry is 18.

Quick Tip to Solve: For each hydrocarbon use $x + y/4$ moles of oxygen per mole, multiply by its amount, and add the oxygen demands.

72. An oxidation-reduction reaction in which 3 electrons are transferred has a ΔG° of 17.37 kJ mol^{-1} at 25 °C. The value of E^o_{cell} (in V) is _____ $\times$ 10^{-2}. (1 F = 96,500 C mol^{-1}) [4 marks]

Solution:

Concept:

- The standard Gibbs energy change and standard cell potential are related by $\Delta G^\circ = -nFE^\circ_{\text{cell}}$, where n is the number of electrons transferred and F is the Faraday constant.
- When F is in coulombs per mole, use Gibbs energy in joules per mole. Since $1\,\text{V} = 1\,\text{J}\,\text{C}^{-1}$, the resulting cell potential is in volts.

Step 1: Convert the energy and identify the electron count The reaction transfers three electrons, so

$n = 3$. The stated energy is positive:

$$\Delta G^\circ = 17.37\,\text{kJ mol}^{-1} = 17370\,\text{J mol}^{-1}.$$

The temperature is 25 degrees Celsius, but no Nernst-equation correction is needed because both the Gibbs energy and the requested potential are standard-state quantities. Retain the positive sign of the given energy.

Step 2: Substitute in the thermodynamic relation Rearranging the energy-potential relation gives

$$E^\circ_{\text{cell}} = -\frac{\Delta G^\circ}{nF} = -\frac{17370}{3(96500)}\,\text{V}.$$

The denominator is 289500, and $289500 \times 0.06 = 17370$. Therefore

$$E^\circ_{\text{cell}} = -0.0600\,\text{V}.$$

The negative potential is consistent with the positive Gibbs energy: the reaction as written is not spontaneous under standard conditions.

Step 3: Express the result in the requested form The blank is the coefficient multiplying 10^{-2}, rather than the potential written directly in decimal form. Rewrite the calculated result as

$$-0.0600\,\text{V} = (-6.00) \times 10^{-2}\,\text{V}.$$

Thus the number entered in the blank must be negative six; omitting the conversion factor would incorrectly give an entry of negative 0.06.

Step 4: Conclusion $E^\circ_{\text{cell}} = -6 \times 10^{-2}\,\text{V}$. The coefficient required in the blank is -6.

Quick Tip to Solve: Convert kJ to J before dividing by nF, keep the minus sign in $\Delta G^\circ = -nFE^\circ$, and read the power of ten printed after the blank.

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73. A soft drink was bottled with a partial pressure of CO_2 of 3 bar over the liquid at room temperature. The partial pressure of CO_2 over the solution approaches a value of 30 bar when 44 g of CO_2 is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is _____ $\times 10^{-1}$.

(First dissociation constant of H_2CO_3 = 4.0 $\times$ 10^{-7}; log 2 = 0.3; density of the soft drink = 1 g mL^{-1}) . [4 marks]

Solution:

Concept:

- At a fixed temperature, Henry's law makes the dissolved gas concentration approximately proportional to its partial pressure when the solution is dilute.
- Dissolved CO_2 forms carbonic acid. For a weak monoprotic acid of concentration C, $[H^+] \simeq \sqrt{K_a C}$,

provided its ionization is small.

Step 1: Find the concentration at the bottling pressure The reference solution contains $44\,\mathrm{g}$ of CO_2, or one mole, in $1\,\mathrm{kg}$ of water at a pressure of $30\,\mathrm{bar}$. At $3\,\mathrm{bar}$, the pressure is one-tenth as large. Thus the approximate dissolved amount in the same quantity of water is

$$n(CO_2) \simeq 1 \times \frac{3}{30} = 0.10\,\mathrm{mol}.$$

This corresponds to $4.4\,\mathrm{g}$ of dissolved gas. With density $1\,\mathrm{g\,mL^{-1}}$, the solution volume is approximately $1\,\mathrm{L}$, so the acid concentration is $C \simeq 0.10\,\mathrm{mol\,L^{-1}}$. Retaining the small dissolved mass changes the volume only to about $1.0044\,\mathrm{L}$, which does not affect the requested approximate result.

Step 2: Use the first acid dissociation Write the relevant equilibrium as $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$. If x is the hydrogen-ion concentration,

$$K_a = \frac{x^2}{C - x} \simeq \frac{x^2}{C}.$$

Consequently,

$$[H^+] \simeq \sqrt{(4.0 \times 10^{-7})(0.10)} = 2.0 \times 10^{-4}\,\mathrm{mol\,L^{-1}}.$$

The ionized fraction is only 2.0×10^{-3}, justifying $C - x \simeq C$. Now use the given logarithm:

$$\mathrm{pH} = -\log(2 \times 10^{-4}) = 4 - 0.3 = 3.7.$$

The answer field asks for the coefficient multiplying 10^{-1}, rather than pH directly.

Step 3: Conclusion The approximate pH is $3.7 = 37 \times 10^{-1}$. Therefore, enter $\boxed{37}$; pH and its requested coefficient are dimensionless.

Quick Tip to Solve: First scale the dissolved gas concentration by the pressure ratio, then take the square root for the weak-acid hydrogen-ion concentration. Convert pH to the coefficient requested in the blank.

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74. The total number of coordination sites in ethylenediaminetetraacetate ($EDTA^{4-}$) is _____. [4 marks]

Solution:

Concept:

- A coordination site of a ligand is a donor atom that supplies a lone pair to form a coordinate bond with a metal ion. Denticity counts donor atoms bound by one ligand, rather than the ligand's charge.
- Ethylenediaminetetraacetate, $EDTA^{4-}$, has two amine nitrogen donor atoms and four carboxylate groups, each normally contributing one oxygen donor atom.

Step 1: Count the nitrogen donor atoms The ethylenediamine backbone contains two nitrogen atoms joined through a $-CH_2-CH_2-$ linkage. Each nitrogen retains a lone pair available for coordination with the central metal ion. Thus this part supplies two coordination sites. The carbon atoms in the

ethylene link do not donate lone pairs and are not additional sites. The nitrogens are separate donors even though they belong to a single ligand molecule.

Step 2: Count the oxygen donors and add the contributions Four acetate arms are attached to the two nitrogen atoms, giving four $-CH_2COO-$ groups in the fully deprotonated ligand. In its usual chelating mode, each carboxylate group attaches to the metal through one oxygen atom. Therefore the four arms contribute four more donor sites. Although a carboxylate contains two oxygen atoms, they are not both counted as separately binding the same metal in the conventional hexadentate EDTA complex. Adding the independent donors gives

$$\text{coordination sites} = 2 \text{ nitrogen donors} + 4 \text{ oxygen donors} = 6.$$

The superscript $4-$ describes the net ionic charge from the four deprotonated carboxyl groups; it does not specify the number of donor atoms. One EDTA ligand can therefore occupy all six sites of a typical octahedral complex.

Step 3: Conclusion $EDTA^{4-}$ is a hexadentate ligand with $\boxed{6}$ coordination sites. The requested number is dimensionless.

Quick Tip to Solve: For EDTA, count two nitrogen lone-pair donors plus one oxygen donor from each of four carboxylate arms; do not use the ionic charge as the denticity.

75. The number of chiral carbon(s) present in peptide, Ile-Arg-Pro, is _____. [4 marks]

Solution:

Concept:

- A chiral carbon is an approximately tetrahedral carbon attached to four different substituents. Count carbon stereocentres in each amino-acid residue separately.
- Peptide formation joins an amino group and a carboxyl group without removing the amino-acid carbon stereocentres. Carbonyl carbons are trigonal planar and are not chiral carbons.

Step 1: Count the stereocentres in isoleucine Isoleucine has the side chain $-CH(CH_3)CH_2CH_3$. Its alpha carbon has four different attachments: hydrogen, the peptide nitrogen, the carbonyl-containing chain and the side chain. It is therefore chiral. The first side-chain carbon also has four different groups: hydrogen, methyl, ethyl and the alpha-carbon-containing chain. This beta carbon is a second stereocentre. Thus the Ile residue contributes two chiral carbons, not just the one alpha carbon common to most amino acids.

Step 2: Count the remaining residues and total the result Arginine has the following side chain:

$$-(CH_2)_3-NH-C(=NH)-NH_2.$$

Its alpha carbon is chiral, while the side-chain methylene carbons each carry two identical hydrogen atoms and are achiral. The guanidino carbon is planar, so Arg contributes one chiral carbon. In proline, the alpha carbon is bonded to hydrogen, the carbonyl-containing group, nitrogen and the

carbon path around the ring. These are four different attachments, so it is chiral. The remaining ring carbons are methylene groups and introduce no further chiral carbons. Pro therefore contributes one. Peptide bonds do not create additional tetrahedral carbon centres: their carbonyl carbons remain planar. Consequently,

$$N_{\text{chiral C}} = 2\ (\text{Ile}) + 1\ (\text{Arg}) + 1\ (\text{Pro}) = 4.$$

Step 3: Conclusion The peptide Ile–Arg–Pro contains $\boxed{4}$ chiral carbon atoms. This is a dimensionless atom count.

Quick Tip to Solve: Start with one alpha-carbon stereocentre for each of these residues, then add the extra beta-carbon stereocentre of isoleucine. Peptide carbonyls are planar and do not add to the count.