JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 5 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 5 September 2020, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If

$\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = \mathbf{A}\log_e|B(\theta)| + C$,

where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be : [4 marks]

(A) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$

(B) $\frac{2\sin\theta+1}{\sin\theta+3}$

(C) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$

(D) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$

Correct Answer: (C) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$

Solution:

Concept:

- When the numerator contains $\cos\theta$ and the denominator is a polynomial in $\sin\theta$, use $t = \sin\theta$, so that $dt = \cos\theta\, d\theta$.
- Factor the quadratic denominator and resolve it into partial fractions. The identity $\int du/u = \log_e|u| + C$ then gives a logarithm of a quotient.

Step 1: Transform and factor the denominator Using $\cos^2\theta = 1 - \sin^2\theta$, the denominator becomes

$$5 + 7\sin\theta - 2\cos^2\theta = 3 + 7\sin\theta + 2\sin^2\theta.$$

Consequently, on any interval where the original denominator is nonzero,

$$I = \int \frac{dt}{2t^2+7t+3} = \int \frac{dt}{(2t+1)(t+3)}.$$

The factorisation follows by expansion: the middle terms are $6t + t = 7t$.

Step 2: Integrate the partial fractions Write

$$\frac{1}{(2t+1)(t+3)} = \frac{2/5}{2t+1} - \frac{1/5}{t+3}.$$

The numerator on combining these fractions is $(2t + 6 - 2t - 1)/5 = 1$. Therefore,

$$I = \frac{1}{5}\log_e|2t+1| - \frac{1}{5}\log_e|t+3| + C.$$

The factor $1/2$ from integrating $1/(2t+1)$ is essential.

Step 3: Identify a suitable logarithmic form Restoring $t = \sin\theta$, take

$$A = \frac{1}{5}, \qquad B(\theta) = \frac{2\sin\theta+1}{\sin\theta+3}.$$

This is a valid representation of the given antiderivative. Thus dividing this $B(\theta)$ by A multiplies the

quotient by 5. Options (A) and (B) instead divide by 5 or leave it unchanged; option (D) reverses the quotient and would require a corresponding sign change in A.

Step 4: Conclusion One valid choice gives $\frac{B(\theta)}{A} = \frac{5(2\sin\theta + 1)}{\sin\theta + 3}$. Hence, the correct option is (C).

Quick Tip to Solve: After the sine substitution, factor before integrating, and remember that the question asks for B/A, rather than the logarithm argument alone.

2. If α and β are the roots of the equation,
$7x^2 - 3x - 2 = 0$, then the value of
$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ **is equal to :** [4 marks]

(A) $\frac{1}{24}$
(B) $\frac{27}{32}$
(C) $\frac{27}{16}$
(D) $\frac{3}{8}$

Correct Answer: (C) $\frac{27}{16}$

Solution:

Concept:

- For $ax^2 + bx + c = 0$, the sum and product of its roots are $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. Symmetric rational expressions can be evaluated without finding either root.
- Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ to express the common denominator in terms of the sum and product.

Step 1: Find the sum and product of the roots Set $s = \alpha + \beta$ and $p = \alpha\beta$. From $7x^2 - 3x - 2 = 0$,

$$s = \frac{3}{7}, \qquad p = -\frac{2}{7}.$$

Neither root equals 1 or -1, because substituting those values into the polynomial gives 2 and 8 respectively. The requested expression is therefore defined.

Step 2: Combine the fractions symmetrically The numerator is

$$\alpha(1 - \beta^2) + \beta(1 - \alpha^2) = s - ps = s(1 - p).$$

The denominator is

$$\begin{aligned}(1 - \alpha^2)(1 - \beta^2) &= 1 - (\alpha^2 + \beta^2) + \alpha^2\beta^2 \\ &= 1 - s^2 + 2p + p^2 = (1 + p)^2 - s^2.\end{aligned}$$

This route avoids radicals and preserves the negative sign of the product.

Step 3: Substitute and simplify Substitution gives

$$s(1-p) = \frac{3}{7}\left(1+\frac{2}{7}\right) = \frac{27}{49},$$

$$(1+p)^2 - s^2 = \left(1-\frac{2}{7}\right)^2 - \left(\frac{3}{7}\right)^2 = \frac{16}{49}.$$

Hence the required sum is $(27/49)/(16/49) = 27/16$. Option (B) introduces an extra factor of 2 in the denominator. Options (A) and (D) do not equal the numerator-to-denominator ratio obtained from the root identities.

Step 4: Conclusion The value is $\frac{27}{16}$. Hence, the correct option is (C).

Quick Tip to Solve: For a symmetric expression in two quadratic roots, combine fractions first and then replace every occurrence of the sum and product using Vieta's relations.

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3. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is : [4 marks]

(A) -2^{15}i

(B) -2^{15}

(C) 2^{15}i

(D) 6^5

Correct Answer: (A) -2^{15}i

Solution:

Concept:

- For a nonzero complex number, write $z = r(\cos\phi + i\sin\phi)$. In a quotient, divide moduli and subtract arguments.
- De Moivre's theorem gives $z^n = r^n(\cos n\phi + i\sin n\phi)$. Reduce the resulting angle modulo 2π only after multiplying it by the power.

Step 1: Express the numerator and denominator in polar form The numerator $-1+i\sqrt{3}$ lies in the second quadrant, has modulus 2 and argument $2\pi/3$. The denominator $1-i$ has modulus $\sqrt{2}$ and argument $-\pi/4$. Thus

$$\frac{-1+i\sqrt{3}}{1-i} = \sqrt{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right),$$

because $2\pi/3 - (-\pi/4) = 11\pi/12$. A wrong quadrant at this stage would change the final sign.

Step 2: Raise to the thirtieth power Applying De Moivre's theorem,

$$\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} = 2^{15}\left(\cos\frac{55\pi}{2} + i\sin\frac{55\pi}{2}\right).$$

Here $(\sqrt{2})^{30} = 2^{15}$ and $30(11\pi/12) = 55\pi/2$. The angle is $26\pi + 3\pi/2$, so its cosine is 0 and its sine is

-1.

Step 3: Check the magnitude and direction The resulting number is purely imaginary and lies on the negative imaginary axis. This rules out the real values in options (B) and (D); in addition, 6^5 does not have the required modulus 2^{15}. Option (C) has the right modulus but points along the positive imaginary axis. Only option (A) has both the required magnitude and the required sign.

Step 4: Conclusion The value is $-2^{15}i$. Hence, the correct option is (A).

Quick Tip to Solve: For a large complex power, track the modulus and argument separately; use the original quadrants before reducing the multiplied angle.

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4. If L = $\sin^2\left(\frac{\pi}{16}\right)$ - $\sin^2\left(\frac{\pi}{8}\right)$ and
M = $\cos^2\left(\frac{\pi}{16}\right)$ - $\sin^2\left(\frac{\pi}{8}\right)$, then : [4 marks]

(A) L = $-\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(B) M = $\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(C) M = $\frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

(D) L = $\frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

Correct Answer: (B) M = $\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

Solution:

Concept:

- The half-angle identities are $\sin^2 u = (1 - \cos 2u)/2$ and $\cos^2 u = (1 + \cos 2u)/2$. They replace squares by functions of twice the angle.
- For the angles here, $\cos(\pi/4) = 1/\sqrt{2}$. Evaluating both L and M allows every proposed relation to be checked explicitly.

Step 1: Evaluate the common subtracted term Use the sine-square identity with $u = \pi/8$:

$$\sin^2\frac{\pi}{8} = \frac{1 - \cos(\pi/4)}{2} = \frac{1}{2} - \frac{1}{2\sqrt{2}}.$$

For the other angle the doubled angle is $\pi/8$, rather than $\pi/4$. Therefore,

$$\sin^2\frac{\pi}{16} = \frac{1}{2} - \frac{1}{2}\cos\frac{\pi}{8}, \qquad \cos^2\frac{\pi}{16} = \frac{1}{2} + \frac{1}{2}\cos\frac{\pi}{8}.$$

Step 2: Subtract to determine L and M Subtracting the common sine-square term gives

$$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8},$$

$$M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}.$$

The constant halves cancel in both subtractions. As a sign check, $L < 0$, since sine increases between

$\pi/16$ and $\pi/8$, whereas $M > 0$.

Step 3: Compare all four statements Option (B) agrees exactly with the evaluated expression for M. Option (A) equals $-L$, so it has the wrong sign for L. Option (C) equals $M/2$ and option (D) equals $L/2$; each incorrectly halves the expression. These are unequal to M and L, since neither quantity is zero.

Step 4: Conclusion $M = \dfrac{1}{2\sqrt{2}} + \dfrac{1}{2}\cos\dfrac{\pi}{8}$. Hence, the correct option is (B).

Quick Tip to Solve: When subtracting two squared trigonometric functions, apply the power-reduction identities first; the constant halves often cancel immediately.

5. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is : [4 marks]

(A) $\frac{2}{13}\left(3^{50}-1\right)$
(B) $\frac{1}{13}\left(3^{50}-1\right)$
(C) $\frac{1}{26}\left(3^{49}-1\right)$
(D) $\frac{1}{26}\left(3^{50}-1\right)$

Correct Answer: (D) $\frac{1}{26}\left(3^{50}-1\right)$

Solution:

Concept:

- In a geometric progression with first term a and common ratio r, its n-th term is ar^{n-1}. Moving each term four places forward multiplies it by r^4.
- For $r \neq 1$, the sum of the first n terms is $S_n = a(r^n - 1)/(r - 1)$. Positivity of every term implies both $a > 0$ and $r > 0$.

Step 1: Use the two blocks of three consecutive terms The first given block has sum

$$ar + ar^2 + ar^3 = ar(1 + r + r^2) = 3.$$

The later block is

$$ar^5 + ar^6 + ar^7 = ar^5(1 + r + r^2) = 243.$$

Each term in the second block is four places beyond the corresponding term of the first block. Dividing the equations cancels the same nonzero factor and gives

$$r^4 = \frac{243}{3} = 81.$$

Step 2: Find the positive common ratio and first term Since the progression consists of positive terms, r cannot be negative. Hence $r = 3$. Substituting in the first block,

$$a \cdot 3(1 + 3 + 9) = 3, \qquad 39a = 3, \qquad a = \frac{1}{13}.$$

As a check, the sixth-through-eighth block is the earlier sum multiplied by 3^4, giving $3 \cdot 81 = 243$ as required.

Step 3: Calculate the sum and reject the distractors The finite geometric sum now gives

$$S_{50} = \frac{1}{13}\frac{3^{50}-1}{3-1} = \frac{3^{50}-1}{26}.$$

Option (B) omits division by $r - 1 = 2$, and option (A) is four times the required sum. Option (C) uses the power 49 instead of 50, which corresponds to one fewer term in this sum formula. Thus none of those expressions equals the required total.

Step 4: Conclusion The sum of the first 50 terms is $\frac{1}{26}(3^{50} - 1)$. Hence, the correct option is (D).

Quick Tip to Solve: Divide sums of equal-length blocks of a geometric progression before finding the first term; their ratio gives a power of the common ratio directly.

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6. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to : [4 marks]

(A) 9

(B) 3

(C) -9

(D) -3

Correct Answer: (D) -3

Solution:

Concept:

- A homogeneous system always has the zero solution. A nonzero solution requires dependence among its equations; subtracting equations can reveal that dependence without expanding a determinant.
- Ratios such as y/z require $z \neq 0$. This must be justified from the equations and the given nonzero-solution condition before dividing.

Step 1: Subtract the first equation from the third The equations are $x + y + 3z = 0$, $x + 3y + k^2z = 0$, and $3x + y + 3z = 0$. Subtracting the first from the third gives

$$2x = 0, \qquad x = 0.$$

Returning to the first equation then yields

$$y + 3z = 0, \qquad y = -3z.$$

These two relations hold for every solution, irrespective of the value of k.

Step 2: Use the nonzero condition and check consistency If $z = 0$, the relation $y = -3z$ gives $y = 0$, and we already have $x = 0$. That would be the excluded zero solution. Therefore $z \neq 0$ and $y/z = -3$. The second equation becomes

$$3(-3z) + k^2 z = 0, \qquad (k^2 - 9)z = 0.$$

Thus $k^2 = 9$, or $k = \pm 3$, both real as required. Conversely, either value permits solutions $(0, -3z, z)$ for any nonzero real z.

Step 3: Evaluate the requested expression Now substitute the forced values:

$$x + \frac{y}{z} = 0 - 3 = -3.$$

The value 9 in option (A) is k^2, which is not the requested expression. Option (B) loses the negative sign in $y = -3z$, and option (C) confuses $3y/z = -9$ with y/z. The expression is independent of the scale chosen for the nonzero solution.

Step 4: Conclusion $x + \frac{y}{z} = -3$. Hence, the correct option is (D).

Quick Tip to Solve: Before computing a determinant, subtract equations with matching terms; then prove the denominator is nonzero before evaluating a solution ratio.

7. If x = 1 is a critical point of the function
$f(x) = (3x^2 + ax - 2 - a)e^x$, then : [4 marks]

(A) x = 1 is a local maxima and x = $-\frac{2}{3}$ is a local minima of f.
(B) x = 1 and x = $-\frac{2}{3}$ are local maxima of f.
(C) x = 1 and x = $-\frac{2}{3}$ are local minima of f.
(D) x = 1 is a local minima and x = $-\frac{2}{3}$ is a local maxima of f.

Correct Answer: (D) x = 1 is a local minima and x = $-\frac{2}{3}$ is a local maxima of f.

Solution:

Concept:

- A critical point of a differentiable function satisfies $f\prime(x) = 0$. Apply the product rule to the polynomial multiplied by e^x.
- Since $e^x > 0$ for every real x, the sign of the derivative is determined entirely by its polynomial factor. A change from negative to positive gives a local minimum; the reverse gives a local maximum.

Step 1: Use the given critical point Differentiating the given function gives

$$f\prime(x) = e^x\{(6x + a) + (3x^2 + ax - 2 - a)\} = e^x\{3x^2 + (a + 6)x - 2\}.$$

The condition at $x = 1$ is therefore $e(3 + a + 6 - 2) = 0$. Because $e \neq 0$, this gives $a = -7$. The

constant terms involving a cancel in the derivative; keeping both is essential before substitution.

Step 2: Classify both stationary points With this value of the parameter,

$$f\prime(x) = e^x(3x^2 - x - 2) = e^x(3x + 2)(x - 1).$$

The two zeros are $-2/3$ and 1. For $x < -2/3$, both linear factors are negative, so the derivative is positive. Between $-2/3$ and 1, their signs are opposite, so the derivative is negative. For $x > 1$, both are positive. Thus the function first increases, then decreases, then increases again. It has a maximum at $-2/3$ and a minimum at 1. Options (B) and (C) incorrectly assign the same kind of extremum to both points, while (A) reverses the actual changes of sign.

Step 3: Conclusion The point $x = 1$ is a local minimum and $x = -2/3$ is a local maximum of f. Hence, the correct option is (D).

Quick Tip to Solve: For a polynomial times an exponential, factor the derivative and discard the positive exponential when constructing its sign chart.

8. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is : [4 marks]

(A) 2250

(B) 2255

(C) 3000

(D) 1500

Correct Answer: (A) 2250

Solution:

Concept:

- A selection is unordered, so choosing k questions from a section of five can be done in $\binom{5}{k}$ ways.
- The three sections are distinct. Because all must contribute at least one question and the total is five, their positive contribution counts can only be permutations of $(3, 1, 1)$ or $(2, 2, 1)$.

Step 1: Count the selections with a three-question section Choose which of the three sections supplies three questions. Choose its questions in $\binom{5}{3} = 10$ ways, then choose one question from each of the other sections in 5 ways each. Thus this distribution contributes

$$3\binom{5}{3}\binom{5}{1}\binom{5}{1} = 3(10)(5)(5) = 750.$$

There is no additional ordering factor for the questions chosen inside a section: rearranging an already chosen set does not produce a new selection.

Step 2: Count the selections with a one-question section For the other possible distribution, choose the section that supplies one question in three ways. Choose that question in five ways, and choose

two questions from each remaining section in $\binom{5}{2} = 10$ ways. This gives

$$3\binom{5}{1}\binom{5}{2}^2 = 3(5)(10)(10) = 1500.$$

The cases are disjoint, since their section counts differ, and exhaustive, since no positive count can reach four while leaving at least one for both other sections. Adding gives $750 + 1500 = 2250$. The value 1500 counts only the second distribution; 2255 and 3000 do not equal the complete case sum.

Step 3: Conclusion The number of allowed selections is 2250. Hence, the correct option is (A).

Quick Tip to Solve: List the positive section-count partitions of the required total first; multiply choices within a case and add only disjoint cases.

9. If the length of the chord of the circle,
$x^2 + y^2 = r^2$ ($r > 0$) along the line, $y - 2x = 3$ is r,
then r^2 is equal to : [4 marks]

(A) $\frac{9}{5}$
(B) $\frac{24}{5}$
(C) $\frac{12}{5}$
(D) 12

Correct Answer: (C) $\frac{12}{5}$

Solution:

Concept:

- The circle $x^2 + y^2 = r^2$ has centre $O = (0, 0)$ and radius r. The perpendicular from the centre to a chord bisects that chord.
- For a line $ax + by + c = 0$, its distance from the origin is $|c|/\sqrt{a^2 + b^2}$. A chord of length L at distance d satisfies $(L/2)^2 + d^2 = r^2$.

Step 1: Find the distance to the chord Write the line as $2x - y + 3 = 0$, which is equivalent to the given equation. Its perpendicular distance from the circle centre is

$$d = \frac{|3|}{\sqrt{2^2 + (-1)^2}} = \frac{3}{\sqrt{5}}, \qquad d^2 = \frac{9}{5}.$$

The line need not pass through the centre. Using its intercept as the distance would be incorrect because the perpendicular direction is determined by the coefficients of both coordinates.

Step 2: Insert the stated chord length Here the full chord length is $L = r$; consequently its half-length is $r/2$. Applying Pythagoras to a radius, the perpendicular and the half-chord gives

$$\frac{r^2}{4} + \frac{9}{5} = r^2, \qquad \frac{3r^2}{4} = \frac{9}{5}, \qquad r^2 = \frac{12}{5}.$$

This is consistent with an actual chord because $r^2 = 12/5 > 9/5 = d^2$. Option (A) gives only the squared perpendicular distance and would make the line tangent, with zero chord length. Option (B) doubles the required squared radius, while (D) omits the coefficient contribution in the distance formula; neither satisfies the right-triangle relation above.

Step 3: Conclusion The required squared radius is $r^2 = 12/5$. Hence, the correct option is (C).

Quick Tip to Solve: When a chord equals the radius, its half-chord is $r/2$, so immediately use $d^2 = 3r^2/4$ after finding the line distance.

10. The area (in sq. units) of the region
$A = \{(x, y) : (x - 1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where [t]
denotes the greatest integer function, is : [4 marks]

(A) $\frac{8}{3}\sqrt{2} - 1$
(B) $\frac{4}{3}\sqrt{2} + 1$
(C) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$
(D) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$
Correct Answer: (C) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

Solution:

Concept:

- For vertical slices between lower and upper curves, the area is the integral of upper minus lower height wherever the upper boundary lies above the lower boundary.
- The greatest-integer function is constant between consecutive integers: $[x] = 0$ for $0 \le x < 1$, and $[x] = 1$ for $1 \le x < 2$. Values at isolated endpoints do not affect area.

Step 1: Separate the intervals of the lower curve On the first interval the lower curve $(x - 1)[x]$ is zero; on the second it is $x - 1$. The upper curve remains $2\sqrt{x}$ throughout. The upper curve is nonnegative on the first interval. On the second, $x - 1 \le 1$ while $2\sqrt{x} \ge 2$, so the required slices all have positive height. Thus there is no additional intersection or excluded interval to account for.

$$\mathcal{A} = \int_0^1 2\sqrt{x}\,dx + \int_1^2 \left(2\sqrt{x} - (x - 1)\right) dx.$$

Step 2: Evaluate the two contributions Combine the upper-curve terms and subtract the lower-curve contribution only over the interval where it is nonzero:

$$\begin{aligned}\mathcal{A} &= \int_0^2 2\sqrt{x}\,dx - \int_1^2 (x - 1)\,dx \\ &= \left[\frac{4}{3}x^{3/2}\right]_0^2 - \left[\frac{(x-1)^2}{2}\right]_1^2 \\ &= \frac{8\sqrt{2}}{3} - \frac{1}{2}.\end{aligned}$$

The lower-bound area is a triangle of base one and height one, hence one-half, providing a check on the subtraction. Option (A) subtracts one instead of one-half. Options (B) and (D) use an incorrect upper-curve contribution, since the integral of $2\sqrt{x}$ from zero to two is $8\sqrt{2}/3$.

Step 3: Conclusion The area is $\frac{8\sqrt{2}}{3} - \frac{1}{2}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Break a greatest-integer integral at each integer, then disregard isolated jump values: they change a boundary point, not the area.

11.

$$\lim_{x\to 0} \frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$$

[4 marks]

(A) is equal to 0.

(B) is equal to $\sqrt{e}$.

(C) is equal to 1.

(D) does not exist.

Correct Answer: (C) is equal to 1.

Solution:

Concept:

- The standard real exponential limit is $\lim_{u\to 0}(e^u-1)/u = 1$. A composite expression can use this limit only after its inner variable is shown to tend to zero.
- Rationalisation removes cancellation in $\sqrt{1+x^2+x^4}-1$ and reveals its order of smallness without requiring a power-series expansion.

Step 1: Introduce and estimate the exponent Put

$$u = \frac{\sqrt{1+x^2+x^4}-1}{x} \quad (x \neq 0).$$

Multiply the numerator and denominator of the radical difference by its conjugate to get

$$u = \frac{x^2+x^4}{x(\sqrt{1+x^2+x^4}+1)} = \frac{x(1+x^2)}{\sqrt{1+x^2+x^4}+1} \longrightarrow 0.$$

The denominator tends to two, so this conclusion holds from both sides of zero. Also the radical difference is strictly positive for nonzero real x, hence division by it in a punctured neighbourhood is legitimate.

Step 2: Apply the exponential limit Because $\sqrt{1+x^2+x^4}-1 = xu$, the entire given quotient is exactly

$$\frac{x(e^u-1)}{xu} = \frac{e^u-1}{u} \quad (x \neq 0).$$

Taking the limit now gives one by the standard exponential limit. Cancelling only after forming this

exact identity avoids treating the original zero-over-zero expression as a ratio of its separate limits. Option (A) wrongly retains a vanishing factor after cancellation; option (B) would require a nonzero exponent limit. Option (D) is excluded because the same finite standard limit is reached from positive and negative x.

Step 3: Conclusion The two-sided limit equals 1. Hence, the correct option is (C).

Quick Tip to Solve: When a radical difference occurs both outside and inside an exponential, set the exponent equal to u and look for $(e^u - 1)/u$.

12. Let y = y(x) be the solution of the differential equation
cosx$\frac{dy}{dx}$ + 2ysinx = sin2x, x $\in \left(0, \frac{\pi}{2}\right)$.
If y$\left(\frac{\pi}{3}\right)$ = 0, then y$\left(\frac{\pi}{4}\right)$ is equal to : [4 marks]

(A) $\frac{1}{\sqrt{2}} - 1$
(B) $\sqrt{2} - 2$
(C) $2 - \sqrt{2}$
(D) $2 + \sqrt{2}$

Correct Answer: (B) $\sqrt{2} - 2$

Solution:

Concept:

- A linear differential equation $y' + P(x)y = Q(x)$ is solved by multiplying by the integrating factor $e^{\int P(x)\,dx}$. The left side then becomes a product derivative.
- On $0 < x < \pi/2$, $\cos x > 0$. Thus division by $\cos x$ is valid throughout the specified interval, and $\int 2\tan x\,dx = -2\log(\cos x)$.

Step 1: Put the equation in linear form Using $\sin 2x = 2\sin x\cos x$, divide the equation by $\cos x$:

$$\frac{dy}{dx} + 2\tan x\, y = 2\sin x.$$

Its integrating factor is $\sec^2 x$. Multiplication gives

$$\frac{d}{dx}(y\sec^2 x) = 2\sin x\sec^2 x.$$

Since $d(\sec x)/dx = \sin x/\cos^2 x$, integration yields $y\sec^2 x = 2\sec x + C$. Equivalently, the general solution on the stated interval is $y = 2\cos x + C\cos^2 x$.

Step 2: Apply the condition and evaluate the requested value At $x = \pi/3$, the condition $y = 0$ gives

$$0 = 2\left(\frac{1}{2}\right) + C\left(\frac{1}{4}\right), \qquad C = -4.$$

Therefore, substituting $x = \pi/4$,

$$y\left(\frac{\pi}{4}\right) = 2\left(\frac{1}{\sqrt{2}}\right) - 4\left(\frac{1}{2}\right) = \sqrt{2} - 2.$$

As a check, $2\cos x - 4\cos^2 x$ vanishes at $\pi/3$ and differentiation restores the original equation. Option (A) is half the obtained value; (C) reverses its sign, and (D) changes the sign of the constant contribution. Thus none of those alternatives satisfies the solved initial-value problem.

Step 3: Conclusion The required value is $y(\pi/4) = \sqrt{2} - 2$. Hence, the correct option is (B).

Quick Tip to Solve: After finding an integrating factor, keep $y\sec^2 x = 2\sec x + C$ until the initial condition is applied; it makes the constant easy to determine.

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13. If a + x = b + y = c + z + 1, where a, b, c, x, y, z are non-zero distinct real numbers, then

$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ **is equal to :** [4 marks]

(A) y(b – a)

(B) y(a – b)

(C) y(a – c)

(D) 0

Correct Answer: (B) y(a – b)

Solution:

Concept:

- Adding a multiple of another column to a column does not change the determinant. This can expose a column containing two zeros.
- Introduce the common value $t = a + x = b + y = c + z + 1$. The last equality gives $c + z = t - 1$, which differs from the first two sums by one.

Step 1: Use the common value in the third column Denote the required determinant by D. Its third column has entries $t, t, t - 1$. Hence the column operation $C_3 \leftarrow C_3 - C_1 - C_2$ gives

$$D = \begin{vmatrix} x & a+y & -y \\ y & b+y & -y \\ z & c+y & -y \end{vmatrix} = -y \begin{vmatrix} x & a+y & 1 \\ y & b+y & 1 \\ z & c+y & 1 \end{vmatrix}.$$

The factor is negative because every entry of the transformed third column is $-y$, not y.

Step 2: Reduce the second column and expand In the remaining determinant, use $C_2 \leftarrow C_2 + C_1 -$

$(t+y)C_3$. The first two entries become zero, while the third is -1, because $z+c=t-1$. Therefore

$$D=-y\begin{vmatrix} x & 0 & 1 \\ y & 0 & 1 \\ z & -1 & 1 \end{vmatrix}=-y(x-y).$$

Expansion along the second column has cofactor sign $(-1)^{3+2}=-1$, so the inner determinant is $x-y$. Finally, $x=t-a$ and $y=t-b$, giving $x-y=b-a$.

Step 3: Check the alternatives Thus $D=-y(b-a)=y(a-b)$. Option (A) has the opposite sign. Option (C) incorrectly replaces b by c; the reduction shows the last row disappears from the final minor. Option (D) is impossible here because $y\neq 0$ and $a\neq b$.

Step 4: Conclusion The determinant equals $\boxed{y(a-b)}$. Hence, the correct option is (B).

Quick Tip to Solve: When several sums share a common value, replace them by one symbol before applying column operations; retain any exceptional constant such as the final $+1$.

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14. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation : [4 marks]

(A) $x^2 - 20x + 18 = 0$

(B) $2x^2 - 20x + 19 = 0$

(C) $x^2 - 10x + 18 = 0$

(D) $x^2 - 10x + 19 = 0$

Correct Answer: (D) $x^2 - 10x + 19 = 0$

Solution:

Concept:

- The arithmetic mean of five observations is their sum divided by five. It directly determines $a+b$.
- For ungrouped data, the population variance is $\sigma^2=\frac{\sum x_i^2}{n}-\bar{x}^2$. A monic quadratic with roots a, b is $x^2-(a+b)x+ab=0$.

Step 1: Find the sum of the unknown observations There are five observations, and their mean is given as 5. Thus

$$\frac{3+5+7+a+b}{5}=5 \quad\Longrightarrow\quad a+b=25-15=10.$$

The coefficient of x in the required monic quadratic must consequently be -10. This immediately rules out option (A), whose roots sum to 20.

Step 2: Obtain the product from the variance The standard deviation is 2, so the variance is 4. Substitute the mean and known observations into the variance formula:

$$4=\frac{3^2+5^2+7^2+a^2+b^2}{5}-25.$$

Consequently,

$$a^2 + b^2 = 5(4 + 25) - (9 + 25 + 49) = 145 - 83 = 62.$$

Using $(a + b)^2 = a^2 + b^2 + 2ab$, we obtain

$$100 = 62 + 2ab \quad \Longrightarrow \quad ab = 19.$$

The divisor here is 5, as appropriate to the standard deviation of the complete data, rather than the sample-estimation divisor 4.

Step 3: Form and verify the quadratic Substitution of the sum and product gives $x^2 - 10x + 19 = 0$. Its roots are $5 \pm \sqrt{6}$, so their deviations from the mean contribute $6 + 6$ to the squared deviations. Together with $4 + 0 + 4$ from the known observations, the variance is $20/5 = 4$, as required. Option (B) has product $19/2$, while option (C) has product 18; neither agrees with the data.

Step 4: Conclusion The required equation is $\boxed{x^2 - 10x + 19 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the mean to find the sum of the unknown roots and the variance to find their squared sum; the product then follows without solving for each observation.

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15. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x = $\frac{1}{2}$ is : [4 marks]

(A) $\frac{2\sqrt{3}}{3}$

(B) $\frac{2\sqrt{3}}{5}$

(C) $\frac{\sqrt{3}}{10}$

(D) $\frac{\sqrt{3}}{12}$

Correct Answer: (C) $\frac{\sqrt{3}}{10}$

Solution:

Concept:

- If u and v depend differentiably on x, then $\frac{du}{dv} = \frac{du/dx}{dv/dx}$, provided $dv/dx \neq 0$.
- Half-angle and double-angle identities simplify the inverse tangents. Principal-value branches must be checked locally at the specified point before differentiating the simplified expressions.

Step 1: Simplify the first inverse tangent Let $u = \tan^{-1}((\sqrt{1 + x^2} - 1)/x)$. Rationalise its argument:

$$\frac{\sqrt{1 + x^2} - 1}{x} = \frac{x}{\sqrt{1 + x^2} + 1}.$$

Set $\theta = \tan^{-1} x$. Near $x = 1/2$, θ is positive and less than $\pi/2$, so the argument equals $\tan(\theta/2)$. Since

$\theta/2$ lies in the principal inverse-tangent interval,

$$u = \frac{1}{2}\tan^{-1} x, \qquad \left.\frac{du}{dx}\right|_{x=1/2} = \frac{1}{2(1+1/4)} = \frac{2}{5}.$$

Step 2: Simplify the second inverse tangent locally Write $v = \tan^{-1}(2x\sqrt{1-x^2}/(1-2x^2))$, and put $\phi = \sin^{-1} x$. The argument becomes $\tan(2\phi)$. At $x = 1/2$, $\phi = \pi/6$, and hence $2\phi = \pi/3$ is strictly inside the principal inverse-tangent interval. The equality $v = 2\sin^{-1} x$ therefore holds throughout a neighbourhood of this point. Thus

$$\left.\frac{dv}{dx}\right|_{x=1/2} = \frac{2}{\sqrt{1-1/4}} = \frac{4}{\sqrt{3}}.$$

Step 3: Divide the two derivatives The requested derivative is the derivative of the first expression with respect to the second, so preserve that order:

$$\left.\frac{du}{dv}\right|_{x=1/2} = \frac{2/5}{4/\sqrt{3}} = \frac{\sqrt{3}}{10}.$$

Options (A) and (B) are too large because they do not retain both the half-angle factor in u and the double-angle factor in v. Option (D) also fails the direct quotient of the explicitly evaluated derivatives above.

Step 4: Conclusion The derivative is $\boxed{\sqrt{3}/10}$. Hence, the correct option is (C).

Quick Tip to Solve: Recognise the half-angle expression after rationalising, then check the inverse-tangent branch only near the evaluation point before taking the derivative quotient.

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16. The statement

$(p \to (q \to p)) \to (p \to (p \vee q))$ **is :** [4 marks]

(A) a tautology

(B) a contradiction

(C) equivalent to (p $\vee$ q) $\wedge$ ($\sim$ p)

(D) equivalent to (p $\wedge$ q) $\vee$ ($\sim$ q)

Correct Answer: (A) a tautology

Solution:

Concept:

- An implication $r \to s$ is equivalent to $\neg r \vee s$. It is false only when its antecedent is true and its consequent is false.
- A tautology is true for every assignment of truth values. The complementary pair $p \vee \neg p$ is always true, and a disjunction containing it is always true.

Step 1: Simplify the antecedent Write the given statement as $A \to B$, where $A = p \to (q \to p)$ and

$B = p \to (p \vee q)$. Replace each implication in A by a disjunction:

$$A \equiv \neg p \vee (\neg q \vee p) \equiv (\neg p \vee p) \vee \neg q \equiv \mathrm{T}.$$

Here T denotes a statement that is always true. Intuitively, whenever p is true, the implication $q \to p$ must be true; if p is false, the outer implication is true automatically.

Step 2: Simplify the consequent and the full implication Apply the same rule to B:

$$B \equiv \neg p \vee (p \vee q) \equiv (\neg p \vee p) \vee q \equiv \mathrm{T}.$$

Therefore the entire statement is equivalent to $\mathrm{T} \to \mathrm{T}$, which is true for all choices of p and q. There is no truth assignment under which it becomes false.

Step 3: Eliminate the remaining descriptions A contradiction would be false for all assignments, so option (B) is excluded. The expression in option (C), $(p \vee q) \wedge \neg p$, is false when p is true, and hence cannot equal our always-true statement. For option (D), choose p false and q true: both $p \wedge q$ and $\neg q$ are false. Thus option (D) is not a tautology either. Only option (A) describes the full statement correctly.

Step 4: Conclusion The given compound statement is a $\boxed{\text{tautology}}$. Hence, the correct option is (A).

Quick Tip to Solve: Replace each arrow by ¬(antecedent) ∨ (consequent), then look for $p \vee \neg p$ before constructing a full truth table.

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17. If the line y = mx + c is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true? [4 marks]

(A) 5m = 4

(B) 8m + 5 = 0

(C) $c^2 = 369$

(D) $4c^2 = 369$

Correct Answer: (D) $4c^2 = 369$

Solution:

Concept:

- The line $y = mx + c$ is tangent to $x^2/a^2 - y^2/b^2 = 1$ when $c^2 = a^2m^2 - b^2$. This follows by requiring a repeated intersection after substituting the line in the conic.
- For a circle centred at the origin, tangency means that the perpendicular distance from the origin to the line equals the radius. Thus $|c|/\sqrt{1+m^2} = r$.

Step 1: Apply tangency to the hyperbola Here $a^2 = 100$ and $b^2 = 64$, so the hyperbola tangency condition is

$$c^2 = 100m^2 - 64.$$

For completeness, substitution of $y = mx + c$ into the hyperbola and multiplication by 6400 gives

$$(64 - 100m^2)x^2 - 200mcx - 100c^2 - 6400 = 0.$$

Setting this quadratic's discriminant equal to zero reduces to the stated relation. The minus sign in -64 is characteristic of the hyperbola; the ellipse tangency formula would have the other sign.

Step 2: Apply tangency to the circle and solve The circle $x^2 + y^2 = 36$ has radius 6. Write the line as $mx - y + c = 0$. Its distance from the origin must be 6, giving

$$\frac{|c|}{\sqrt{m^2+1}} = 6 \quad \Longrightarrow \quad c^2 = 36(m^2+1).$$

Equating the two expressions for c^2,

$$100m^2 - 64 = 36m^2 + 36, \qquad 64m^2 = 100, \qquad m^2 = \frac{25}{16}.$$

Hence $m = \pm 5/4$, and

$$c^2 = 36\left(1 + \frac{25}{16}\right) = 36 \cdot \frac{41}{16} = \frac{369}{4}.$$

Step 3: Check which relation always holds Neither $m = 4/5$ in option (A) nor $m = -5/8$ in option (B) agrees with $m^2 = 25/16$. Option (C) misses the factor of four in the intercept calculation. Option (D), $4c^2 = 369$, follows for either allowed sign of the slope or intercept.

Step 4: Conclusion The true relation is $\boxed{4c^2 = 369}$. Hence, the correct option is (D).

Quick Tip to Solve: For a common tangent, equate the two formulas for the same c^2; using squared quantities avoids selecting a slope or intercept sign prematurely.

18. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0)? [4 marks]

(A) (2, 2)

(B) (–2, 4)

(C) (2, 6)

(D) (–2, 6)

Correct Answer: (D) (–2, 6)

Solution:

Concept:

- The slope of the tangent to an implicitly defined curve is obtained by differentiating with respect to x, treating y as a function of x.
- At a regular point (x_0, y_0), a tangent of slope m has equation $y - y_0 = m(x - x_0)$. A candidate point must satisfy this line equation, although it need not lie on the original curve.

Step 1: Differentiate the curve implicitly Start with $x^4e^y + 2\sqrt{y+1} = 3$. The first term needs the product rule and the chain rule; the square-root term also contains $y(x)$. Therefore

$$4x^3e^y + x^4e^y\frac{dy}{dx} + \frac{1}{\sqrt{y+1}}\frac{dy}{dx} = 0.$$

Collecting the derivative terms gives

$$\frac{dy}{dx} = -\frac{4x^3e^y}{x^4e^y + 1/\sqrt{y+1}}.$$

The factor 2 multiplying the square root cancels the $1/2$ from differentiating the square root. Omitting that cancellation would give an incorrect tangent slope.

Step 2: Evaluate the slope at the given point First check that $(1, 0)$ belongs to the curve: $1^4e^0 + 2\sqrt{1} = 1 + 2 = 3$. At this point the denominator in the derivative is $1 + 1 = 2$, which is nonzero. Hence

$$m = -\frac{4 \cdot 1^3 \cdot e^0}{1^4e^0 + 1/\sqrt{1}} = -\frac{4}{2} = -2.$$

The tangent equation is consequently

$$y - 0 = -2(x - 1), \qquad \text{or} \qquad 2x + y = 2.$$

Step 3: Substitute the proposed points into the tangent For option (A), $2x + y = 2(2) + 2 = 6$. For option (B), it is $2(-2) + 4 = 0$. For option (C), it is $2(2) + 6 = 10$. None equals 2. In contrast, option (D) gives $2(-2) + 6 = 2$, so $(-2, 6)$ lies on the tangent. These substitutions test the required line directly without unnecessarily substituting points into the nonlinear curve.

Step 4: Conclusion The tangent is $2x + y = 2$, and the point on it is $\boxed{(-2, 6)}$. Hence, the correct option is (D).

Quick Tip to Solve: After implicit differentiation, reduce the tangent to a simple linear equation and test the offered points there; the question asks about the tangent, not the original curve.

19. If the sum of the first 20 terms of the series

$\log_{(7^{1/2})}x + \log_{(7^{1/3})}x + \log_{(7^{1/4})}x + \ldots$ **is 460,**

then x is equal to : [4 marks]

(A) e^2

(B) $7^{1/2}$

(C) 7^2

(D) $7^{46/21}$

Correct Answer: (C) 7^2

Solution:

Concept:

- Change of base gives $\log_{7^{1/k}} x = k \log_7 x$, since $\log_7(7^{1/k}) = 1/k$. The argument must satisfy $x > 0$.
- The multipliers of $\log_7 x$ form an arithmetic progression. Its sum is the number of terms multiplied by the average of the first and last terms.

Step 1: Convert every term to base seven Set $t = \log_7 x$. The successive bases are $7^{1/2}, 7^{1/3}, 7^{1/4}, \ldots$, so the corresponding terms are $2t, 3t, 4t, \ldots$. In particular, the first term has multiplier 2, not 1. The twentieth multiplier is therefore $2 + 19 = 21$. Thus the sum specified in the question is

$$S_{20} = t(2 + 3 + 4 + \cdots + 21).$$

All bases are positive and different from one, so applying the change-of-base identity is valid throughout the series.

Step 2: Sum the multipliers and determine the argument There are twenty consecutive integers from 2 to 21. Their sum and the resulting equation for t are

$$2 + 3 + \cdots + 21 = \frac{20(2 + 21)}{2} = 230,$$

$$230t = 460 \quad \Longrightarrow \quad t = 2.$$

Returning to the definition of t gives $\log_7 x = 2$, hence $x = 7^2$. For a direct check, this value makes the original terms $4, 6, 8, \ldots, 42$, whose sum is $20(4 + 42)/2 = 460$. Option (B) would give only 115, while option (D) would give $230(46/21)$, not 460. Option (A) also fails because $\log_7(e^2) = 2/\ln 7$, which is not 2.

Step 3: Conclusion The required positive argument is $x = 7^2 = 49$. Hence, the correct option is (C).

Quick Tip to Solve: When logarithm bases have reciprocal exponents, change to their common base first; count the twenty multipliers as 2 through 21.

20. If for some $\alpha \in$ R, the lines

L_1 : $\frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and

L_2 : $\frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar,

then the line L_2 passes through the point : [4 marks]

(A) (10, 2, 2)

(B) (2, –10, –2)

(C) (10, –2, –2)

(D) (–2, 10, 2)

Correct Answer: (B) (2, –10, –2)

Solution:

Concept:

- Lines through position vectors $\mathbf{p}, \mathbf{q}$ with directions $\mathbf{u}, \mathbf{v}$ are coplanar when $(\mathbf{q}-\mathbf{p}) \cdot (\mathbf{u} \times \mathbf{v}) = 0$.
- A point belongs to a line only when all three parametric coordinate equations hold for one common parameter. Matching a single coordinate is insufficient.

Step 1: Apply the coplanarity condition From the symmetric equations take $\mathbf{p} = (-1, 2, 1)$, $\mathbf{q} = (-2, -1, -1)$, $\mathbf{u} = (2, -1, 1)$ and $\mathbf{v} = (\alpha, 5-\alpha, 1)$. The displacement between the chosen points is $\mathbf{q}-\mathbf{p} = (-1, -3, -2)$. Compute

$$\mathbf{u} \times \mathbf{v} = (\alpha - 6, \alpha - 2, 10 - \alpha).$$

Therefore coplanarity requires

$$-(\alpha - 6) - 3(\alpha - 2) - 2(10 - \alpha) = 0,$$

$$-2\alpha - 8 = 0 \quad \Longrightarrow \quad \alpha = -4.$$

The two directions are not parallel at this value, and the scalar triple product condition consequently places both intersecting lines in one plane.

Step 2: Test the listed points on the second line Substituting $\alpha = -4$, write L_2 as

$$x = -2 - 4t, \qquad y = -1 + 9t, \qquad z = -1 + t.$$

Option (B) has $z = -2$, which fixes $t = -1$. This gives $x = 2$ and $y = -10$, so all coordinates agree. Option (A) has $z = 2$, requiring $t = 3$, but this predicts $(-14, 26, 2)$, not $(10, 2, 2)$. Option (C), with $z = -2$, must have $(x, y) = (2, -10)$, not $(10, -2)$. Option (D) again has $z = 2$, so its first two coordinates also fail the same parameter test.

Step 3: Conclusion The line L_2 passes through $(2, -10, -2)$. Hence, the correct option is (B).

Quick Tip to Solve: Use coplanarity to find the unknown direction first, then use the coordinate with denominator 1 to fix the parameter for each candidate point.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ be such that $|\vec{a}| = 2$, $\left|\vec{b}\right| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $\left|\vec{a} + \vec{b} - \vec{c}\right|$ is _____. [4 marks]

Solution:

Concept:

- The scalar projection of $\mathbf{b}$ on a nonzero vector $\mathbf{a}$ is $(\mathbf{a} \cdot \mathbf{b})/|\mathbf{a}|$. Equal projections on the same vector therefore give equal dot products.
- Expand a squared magnitude by dot products: $|\mathbf{v}|^2 = \mathbf{v} \cdot \mathbf{v}$. Perpendicular vectors have zero dot product, so their cross term disappears.

Step 1: Translate the projection information into dot products Since $|\mathbf{a}| = 2$, the given equality of projections reads

$$\frac{\mathbf{a} \cdot \mathbf{b}}{2} = \frac{\mathbf{a} \cdot \mathbf{c}}{2}.$$

Thus $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, or equivalently

$$\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = 0.$$

This shows that $\mathbf{a}$ is perpendicular to the difference $\mathbf{b} - \mathbf{c}$, even though its angle with either vector separately need not be known. Also, the stated perpendicularity of $\mathbf{b}$ and $\mathbf{c}$ gives $\mathbf{b} \cdot \mathbf{c} = 0$.

Step 2: Expand the required squared magnitude Keep the minus sign on $\mathbf{c}$ while expanding:

$$\begin{aligned} |\mathbf{a} + \mathbf{b} - \mathbf{c}|^2 &= |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 \\ &\quad + 2\mathbf{a} \cdot \mathbf{b} - 2\mathbf{a} \cdot \mathbf{c} - 2\mathbf{b} \cdot \mathbf{c} \\ &= 2^2 + 4^2 + 4^2 = 36. \end{aligned}$$

The two terms involving $\mathbf{a}$ cancel because the projections are equal, while the final cross term vanishes because $\mathbf{b} \perp \mathbf{c}$. A magnitude is nonnegative, so take the positive square root rather than a signed root. No coordinate components or individual angles are needed.

Step 3: Conclusion The required magnitude is $|\mathbf{a} + \mathbf{b} - \mathbf{c}| = \sqrt{36} = 6$, in the same magnitude units as the given vectors. The numerical entry is 6.

Quick Tip to Solve: Equal projections on a common vector cancel its two mixed dot products in the square of a sum containing one plus and one minus sign.

22. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set C = {f : A → B | 2 ∈ f(A) and f is not one-one} is _____. [4 marks]

Solution:

Concept:

- A function from a three-element labelled domain to a four-element codomain has 4^3 possibilities, because each domain element independently receives one image.
- To impose that 2 appears in the image, subtract functions avoiding 2. Then subtract the injective functions that still satisfy the same image condition; these are exactly the unwanted one-one cases.

Step 1: Count all functions whose image contains two There are $4^3 = 64$ functions from A to B. If the value 2 is avoided, each of a, b, c has only the three choices $1, 3, 4$, giving $3^3 = 27$ functions. Hence the number with at least one domain element mapped to 2 is

$$N_{2 \text{ present}} = 4^3 - 3^3 = 64 - 27 = 37.$$

This subtraction correctly includes cases in which two or all three elements map to 2; there is no restriction on multiplicity at this stage. Directly choosing one preimage of 2 without controlling others would count some functions more than once.

Step 2: Remove the one-one functions with this property The total number of injective functions from a three-element set into a four-element set is $4 \cdot 3 \cdot 2 = 24$. Among them, those avoiding 2 use all of $1, 3, 4$ once and number $3! = 6$. Therefore the injective functions whose image does contain 2 number

$$N_{\text{injective},2 \text{ present}} = 24 - 6 = 18.$$

Subtract these from 37:

$$|C| = 37 - 18 = 19.$$

As a separate check, image size one contributes just the constant function with value 2. Image size two contributes three choices of the other image value, each with $2^3 - 2 = 6$ functions using both values. Thus $1 + 3 \cdot 6 = 19$ agrees with the subtraction method.

Step 3: Conclusion There are 19 functions with $2 \in f(A)$ that are not one-one. Thus the number of elements of C, a dimensionless count, is 19.

Quick Tip to Solve: Apply the compulsory-image condition to both the total functions and the injective functions before subtracting the two counts.

23. If the lines x + y = a and x – y = b touch the curve y = x^2 – 3x + 2 at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to _____.

[4 marks]

Solution:

Concept:

- Intersections with the x-axis satisfy $y = 0$. The derivative dy/dx gives the slope of the tangent at each such point.
- A tangent through (x_0, y_0) with slope m has equation $y - y_0 = m(x - x_0)$. Rewrite the given lines in slope-intercept form to identify which intercept belongs to each line.

Step 1: Locate the intercepts and find the tangent slopes For the curve to meet the x-axis, solve

$$x^2 - 3x + 2 = (x - 1)(x - 2) = 0.$$

The two points are therefore $(1, 0)$ and $(2, 0)$. Differentiating the polynomial gives

$$\frac{dy}{dx} = 2x - 3.$$

At $x = 1$, the slope is -1; at $x = 2$, the slope is $+1$. The line $x + y = a$ is $y = -x + a$, with slope -1, so it touches at the first point. The line $x - y = b$ is $y = x - b$, with slope $+1$, so it touches at the second point.

Step 2: Determine the two constants from their contact points The tangent through $(1, 0)$ with slope -1 is

$$y = -(x - 1) \quad \Longrightarrow \quad x + y = 1.$$

Comparing with $x + y = a$ gives $a = 1$. Similarly, the tangent through $(2, 0)$ with slope $+1$ is

$$y = x - 2 \quad \Longrightarrow \quad x - y = 2,$$

giving $b = 2$. Consequently

$$\frac{a}{b} = \frac{1}{2} = 0.50.$$

The assignment of contact points is fixed by their different slopes; reversing the two constants would conflict with the given line equations. Both tangent lines pass through their respective intercepts and have the derivative there, which verifies contact with the stated curve.

Step 3: Conclusion The ratio is $a/b = 0.50$. This is a dimensionless number; the required numerical entry is 0.50.

Quick Tip to Solve: Match the slopes of the given tangent lines with the derivative at each root before substituting the contact points to find their constants.

24. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is _____. [4 marks]

Solution:

Concept:

- The finite geometric sum gives $1 + x + x^2 + x^3 = (1 - x^4)/(1 - x)$. This identity simplifies the sixth power into two factors with easily tracked low-degree coefficients.
- In $(1 - x)^{-6}$, the coefficient of x^r is $\binom{r+5}{5}$. Equivalently, it counts nonnegative solutions of a sum of six exponents equal to r.

Step 1: Rewrite the expression to isolate the restriction on exponents Work with coefficients around $x = 0$, using the geometric-sum identity:

$$(1 + x + x^2 + x^3)^6 = (1 - x^4)^6(1 - x)^{-6}.$$

The numerator begins as

$$(1 - x^4)^6 = 1 - 6x^4 + \text{terms of degree at least } 8.$$

Terms of degree eight or more cannot contribute to the coefficient of x^4, because the other factor contains only nonnegative powers. Thus there are only two contributions to track: the degree-four term of the denominator expansion and the product of $-6x^4$ with its constant term.

Step 2: Extract the coefficient and check it by counting By the negative-binomial expansion,

$$[x^4](1 - x)^{-6} = \binom{9}{5} = 126, \qquad [x^0](1 - x)^{-6} = 1.$$

Hence the required coefficient is

$$126 - 6(1) = 120.$$

A counting check gives the same result. Choosing a monomial from each of six factors gives exponent choices $r_1, \ldots, r_6$ with sum 4 and $0 \le r_i \le 3$. Without the upper bound there are $\binom{9}{5} = 126$ choices. The invalid choices have one exponent equal to 4 and all others zero, giving exactly six cases. Two exponents cannot both reach 4 because their sum would exceed the required total. Removing those six leaves 120 valid selections.

Step 3: Conclusion The coefficient of x^4 is 120. It is a dimensionless integer, so the numerical entry is 120.

Quick Tip to Solve: For a small target degree, replace a finite geometric sum by a quotient and retain only numerator terms capable of contributing to that degree.

25. In a bombing attack, there is 50% chance that a bomb will hit the target. Atleast two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _____.

[4 marks]

Solution:

Concept:

- Independent trials with the same success probability follow a binomial distribution. Here a hit is a success and $p = 1/2$.
- Destruction needs at least two hits, so its complement consists of exactly zero hits or exactly one hit. Counting these two cases avoids summing every successful case separately.

Step 1: Express the probability of failure Let X count hits among n bombs. Independence gives $X \sim \text{Bin}(n, 1/2)$. Thus

$$P(X = 0) = \binom{n}{0}\left(\frac{1}{2}\right)^n = \frac{1}{2^n}, \qquad P(X = 1) = \binom{n}{1}\left(\frac{1}{2}\right)^n = \frac{n}{2^n}.$$

These outcomes are mutually exclusive. The target avoids complete destruction precisely when fewer than two bombs hit, so

$$P(X \geq 2) = 1 - \frac{n+1}{2^n}.$$

Step 2: Impose the required probability The required chance of complete destruction is at least 0.99. Therefore

$$1 - \frac{n+1}{2^n} \geq 0.99 \quad \Longleftrightarrow \quad \frac{n+1}{2^n} \leq 0.01.$$

Check the consecutive candidate integers:

$$n = 10: \quad \frac{11}{1024} = 0.0107421875 > 0.01,$$
$$n = 11: \quad \frac{12}{2048} = 0.005859375 < 0.01.$$

Thus ten bombs give only 98.92578125%, whereas eleven give 99.4140625%.

Step 3: Verify minimality The failure probability decreases with the number of bombs, since

$$\frac{(n+2)/2^{n+1}}{(n+1)/2^n} = \frac{n+2}{2n+2} < 1 \qquad (n > 0).$$

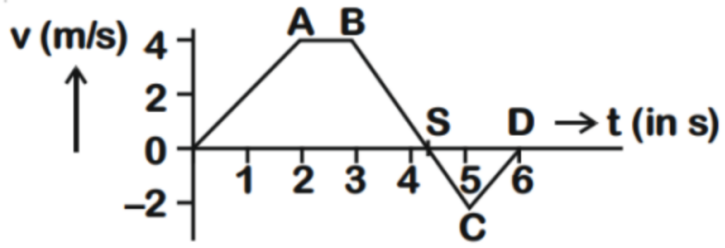
Since ten fails, all smaller positive numbers fail too; eleven is therefore the first admissible integer, rather than merely one possible choice.

Step 4: Conclusion The minimum required number is $\boxed{11}$ bombs. This count is dimensionless and gives a probability of complete destruction above 99%.

Quick Tip to Solve: For at least two binomial successes, subtract the zero-success and one-success cases from one; then check consecutive integers to establish the minimum.

Physics – Section A: Multiple Choice Questions (80 marks)

26. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is : [4 marks]



(A) 12 m

(B) 11 m

(C) $\frac{49}{4}$ m

(D) $\frac{37}{3}$ m

Correct Answer: (D) $\frac{37}{3}$ m

Solution:

Concept:

- Displacement is the signed area under a velocity-time graph. Total distance is the area under its speed-time graph, so every area is counted positively even when velocity is negative.
- A triangular region has area $\frac{1}{2} \times$ base $\times$ height, and a rectangular region has area base times height. With seconds and metres per second on the axes, these areas are lengths in metres.

Step 1: Read the vertices and zero crossing The graph rises from $(0,0)$ to $A = (2,4)$, remains at $4\,\mathrm{m\,s^{-1}}$ until $B = (3,4)$, then falls linearly to $C = (5,-2)$, and rises to $D = (6,0)$. The slope of BC is

$$\frac{-2-4}{5-3} = -3\,\mathrm{m\,s^{-2}}.$$

Hence its zero crossing satisfies $0 = 4 - 3(t-3)$, giving $t_S = 13/3\,\mathrm{s}$, consistent with the stated 4.333 seconds.

Step 2: Add the positive-velocity areas From zero to two seconds the region is triangular; from two to three seconds it is rectangular. Between three seconds and S, it is another triangle. Their combined area is

$$d_+ = \frac{1}{2}(2)(4) + (1)(4) + \frac{1}{2}\left(\frac{13}{3} - 3\right)(4) = 4 + 4 + \frac{8}{3} = \frac{32}{3}\,\mathrm{m}.$$

Each term is positive because the body moves in the positive direction during these intervals.

Step 3: Include the reverse motion as positive distance Below the time axis, the two triangles have height $2\,\mathrm{m\,s^{-1}}$. Their total magnitude is

$$d_- = \frac{1}{2}\left(5 - \frac{13}{3}\right)(2) + \frac{1}{2}(6-5)(2) = \frac{2}{3} + 1 = \frac{5}{3}\,\mathrm{m}.$$

Therefore the total distance is $d_+ + d_- = 37/3\,\mathrm{m}$. Subtracting these regions would give displacement instead. The alternatives 12, 11 and $49/4$ do not equal the sum of the graph's absolute areas; in particular rounding S too early is unnecessary.

Step 4: Conclusion The total distance covered in six seconds is $\boxed{\frac{37}{3}\,\mathrm{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: Split a velocity-time graph at every zero crossing and add the magnitudes of all areas; negative velocity still contributes positive distance.

27. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6 mA it produces a deflection of 2°, its figure of merit is close to : [4 marks]

(A) 6×10^{-3} A/div

(B) 666° A/div

(C) 3×10^{-3} A/div

(D) 333° A/div

Correct Answer: (C) 3×10^{-3} A/div

Solution:

Concept:

- The figure of merit of a galvanometer is the current required to produce one scale division of deflection. If current I gives deflection θ on its uniformly calibrated scale, the figure of merit is $k = I/\theta$.
- Current sensitivity is the reciprocal quantity, $S_I = \theta/I$. Distinguishing these definitions is essential: a sensitive galvanometer has a large deflection per unit current and a small current per division.

Step 1: Identify the measured current and deflection The stated current is $6\,\mathrm{mA}$, and the corresponding deflection is 2°. On the given angular scale this is a reading of two divisions. Convert milliamperes to amperes before evaluating the current per division:

$$I = 6 \times 10^{-3}\,\mathrm{A}, \qquad \theta = 2.$$

The deflection is a scale reading here; converting the angle to radians would change the scale convention and would not give the requested amperes per division.

Step 2: Calculate the current for one division In the linear range the current and deflection are proportional. Thus half the current producing two divisions produces one division:

$$k = \frac{I}{\theta} = \frac{6 \times 10^{-3}}{2} = 3 \times 10^{-3}\,\mathrm{A/div}.$$

Equivalently, the instrument requires $3\,\mathrm{mA}$ for each unit division. Its use as a null detector identifies when the current vanishes; it does not alter the definition of figure of merit.

Step 3: Compare the alternatives Option (A) gives the entire current for two divisions, so it misses

the division by two. The numbers in (B) and (D) are far too large to describe a current of only a few milliamperes per division. In particular, taking a reciprocal would calculate sensitivity rather than figure of merit, and would have units of divisions per ampere. Only (C) has the required value and the stated current-per-division convention.

Step 4: Conclusion The figure of merit is $\boxed{3 \times 10^{-3}\,\mathrm{A/div}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the units first: figure of merit is A/div, so divide current by deflection; sensitivity has the reciprocal units and must be calculated oppositely.

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28. The quantities x = $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$, y = $\frac{E}{B}$ and z = $\frac{l}{CR}$ are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and ε_0, μ_0, - free space permittivity and permeability respectively. Then : [4 marks]

(A) Only y and z have the same dimension

(B) x, y and z have the same dimension

(C) Only x and y have the same dimension

(D) Only x and z have the same dimension

Correct Answer: (B) x, y and z have the same dimension

Solution:

Concept:

- The quantity $1/\sqrt{\mu_0\varepsilon_0}$ is the speed of electromagnetic waves in vacuum, so it has the dimensions of velocity, $[LT^{-1}]$.
- Dimensions can also be found using physical equations: electric force is qE, magnetic force for perpendicular motion is qvB, and the resistance-capacitance product RC is a time constant.

Step 1: Find the dimensions of the first quantity Maxwell's relation for the speed of light is

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = x.$$

Consequently $[x] = [c] = LT^{-1}$. This identifies the dimension without separately carrying the dimensions of permeability and permittivity, whose product is the inverse square of a speed.

Step 2: Compare the electric and magnetic fields A charge experiences electric force $F_E = qE$ and magnetic force $F_B = qvB$ when its velocity is perpendicular to the magnetic field. Since both are forces, their dimensional equations imply

$$[E] = \frac{[F]}{[q]}, \qquad [B] = \frac{[F]}{[q][v]}.$$

Taking the quotient gives

$$[y] = \frac{[E]}{[B]} = [v] = LT^{-1}.$$

This argument compares dimensions only; it does not require the two forces in the question to be

numerically equal.

Step 3: Reduce the resistance-capacitance product Capacitance is charge divided by potential difference and resistance is potential difference divided by current. Therefore

$$[CR] = \left[\frac{Q}{V}\frac{V}{I}\right] = \left[\frac{Q}{I}\right] = T, \qquad [z] = \frac{[l]}{[CR]} = LT^{-1}.$$

All three expressions thus have the dimensions of speed. The alternatives saying that only a particular pair has the same dimension fail because the excluded third expression also reduces to LT^{-1}. Equal dimensions do not, by themselves, require equal numerical magnitudes.

Step 4: Conclusion $\boxed{[x] = [y] = [z] = LT^{-1}}$: all three quantities have the same dimensions. Hence, the correct option is (B).

Quick Tip to Solve: Recognise RC as a time and E/B as a speed before expanding electromagnetic quantities into base dimensions.

29. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to :
(ignore viscosity of air) [4 marks]

(A) r
(B) r^4
(C) r^3
(D) r^2

Correct Answer: (B) r^4

Solution:

Concept:

- For a small sphere moving slowly through a viscous liquid, Stokes' drag is $6\pi\eta rv$, where η is dynamic viscosity, r is radius and v is speed relative to the liquid.
- Terminal speed is reached when effective weight equals viscous drag. A free fall from rest through height h, with negligible air resistance, gives $v^2 = 2gh$.

Step 1: Balance the forces in water Write the density of water as ρ_w to distinguish it from the sphere's density ρ. The sphere's volume is $4\pi r^3/3$, so its weight and buoyancy are respectively $(4\pi r^3/3)\rho g$ and $(4\pi r^3/3)\rho_w g$. At downward terminal speed v_t,

$$\frac{4}{3}\pi r^3(\rho - \rho_w)g = 6\pi\eta r v_t.$$

The buoyant force must be subtracted, since it acts upward; omitting it changes the coefficient even though the radius dependence would remain the same.

Step 2: Determine the radius dependence of terminal speed Divide the force balance by $6\pi\eta r$:

$$v_t = \frac{2(\rho - \rho_w)g}{9\eta} r^2.$$

For fixed sphere material and water conditions, ρ, ρ_w, g and η do not depend on r. Therefore $v_t \propto r^2$, rather than being proportional to the radius itself.

Step 3: Match the entry speed to the terminal speed The ball falls from rest in air, and the stated neglect of air viscosity permits $v_{\text{entry}} = \sqrt{2gh}$. Equating this to the water terminal speed gives

$$2gh = v_t^2 = \frac{4(\rho - \rho_w)^2 g^2}{81\eta^2} r^4, \qquad h = \frac{2(\rho - \rho_w)^2 g}{81\eta^2} r^4.$$

Thus the fourth power results from squaring the quadratic dependence of terminal speed. Options (A) and (C) have the wrong powers, while (D) mistakes the dependence of v_t for the dependence of the fall height.

Step 4: Conclusion The required height satisfies $\boxed{h \propto r^4}$. Hence, the correct option is (B).

Quick Tip to Solve: First obtain $v_t \propto r^2$ from Stokes' law, then use $h = v_t^2/(2g)$; the second squaring changes the power to four.

30. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is : [4 marks]

(A) $2\sqrt{\alpha_1\alpha_2}$

(B) $4\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2L_1}{(L_2+L_1)^2}$

(C) $\frac{\alpha_1+\alpha_2}{2}$

(D) $\frac{\alpha_1L_1+\alpha_2L_2}{L_1+L_2}$

Correct Answer: (D) $\frac{\alpha_1L_1+\alpha_2L_2}{L_1+L_2}$

Solution:

Concept:

- For a uniform temperature rise ΔT, a freely expanding wire of original length L and linear expansion coefficient α extends by $\Delta L = \alpha L \Delta T$, to first order in the temperature change.
- Wires joined end-to-end have additive lengths and additive extensions. The effective coefficient is defined by giving the complete combination the same total expansion as a single wire of its total initial length.

Step 1: Write the expansion of each wire Both wires experience the same temperature change. Their individual increases in length are

$$\Delta L_1 = \alpha_1 L_1 \Delta T, \qquad \Delta L_2 = \alpha_2 L_2 \Delta T.$$

Since the wires lie in sequence, one wire's extension does not replace the other's. The combination's initial length is $L_1 + L_2$, and its final length is the sum of the two final lengths. Thus

$$\Delta L_{\text{total}} = (\alpha_1 L_1 + \alpha_2 L_2)\Delta T.$$

Step 2: Define the equivalent single-wire coefficient Let α_{eff} denote the effective coefficient. Applying its definition to the total initial length gives

$$\Delta L_{\text{total}} = \alpha_{\text{eff}}(L_1 + L_2)\Delta T.$$

Equating the two expressions and cancelling the common temperature change yields

$$\alpha_{\text{eff}} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}.$$

It is a length-weighted mean: a longer segment contributes more to the total extension under the same temperature rise.

Step 3: Check the limiting cases and options For $L_1 = L_2$, the expression reduces to $(\alpha_1 + \alpha_2)/2$, but option (C) incorrectly applies this special case to arbitrary lengths. If L_2 becomes negligible, the coefficient must approach α_1; option (B) instead tends to zero and option (A) ignores the lengths entirely. Also, setting $\alpha_1 = \alpha_2 = \alpha$ gives $\alpha_{\text{eff}} = \alpha$, as required for two pieces of the same material. These checks support the weighted mean in (D).

Step 4: Conclusion The effective coefficient is $\boxed{\alpha_{\text{eff}} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}}$, with units K^{-1}. Hence, the correct option is (D).

Quick Tip to Solve: For end-to-end thermal expansion, add actual length increases first; divide by the total original length and the common temperature rise afterwards.

31. A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant k = 4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored? [4 marks]

(A) $\frac{l}{4}$
(B) $\frac{l}{2}$
(C) $\frac{2l}{3}$
(D) $\frac{l}{3}$

Correct Answer: (D) $\frac{l}{3}$

Solution:

Concept:

- With the battery connected, the potential difference remains V; the stored energy is $U = \frac{1}{2}CV^2$. Thus doubling the energy requires doubling the capacitance.

- The dielectric-filled and air-filled parts have the same plate separation and voltage. They act as parallel capacitors, so their capacitances add.

Step 1: Express the two parallel capacitances Let the inserted length be x, measured along the plate length l. The dielectric occupies area wx, while the uncovered area is $w(l-x)$. The slab fills the complete separation d, so there is no air layer in series with it. Consequently,

$$C_{\text{dielectric}} = \frac{k\varepsilon_0 wx}{d}, \qquad C_{\text{air}} = \frac{\varepsilon_0 w(l-x)}{d}.$$

Before insertion the entire area is air-filled and $C_0 = \varepsilon_0 wl/d$.

Step 2: Apply the required energy ratio Adding the two contributions and using $k = 4$,

$$C = \frac{\varepsilon_0 w}{d}\big[l + (k-1)x\big] = C_0\left(1 + \frac{3x}{l}\right).$$

Since the battery maintains the same voltage,

$$\frac{U}{U_0} = \frac{C}{C_0} = 2 \quad \Longrightarrow \quad 1 + \frac{3x}{l} = 2 \quad \Longrightarrow \quad x = \frac{l}{3}.$$

The proposed lengths $l/4, l/2, 2l/3$ would give energy ratios $7/4, 5/2, 3$, respectively, and hence do not satisfy the requirement. The needed length lies between zero and l, as expected physically.

Step 3: Conclusion The slab must be inserted through a length $\boxed{l/3}$. Hence, the correct option is (D).

Quick Tip to Solve: First identify whether the battery stays connected: fixed voltage makes energy proportional to capacitance. A slab inserted sideways creates parallel area regions.

32. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is : [4 marks]

(A) 128

(B) 32

(C) 326

(D) $\frac{1}{32}$

Correct Answer: (A) 128

Solution:

Concept:

- For a fixed mass of an ideal gas in a reversible adiabatic process, $PV^{\gamma} =$ constant, where $\gamma = C_P/C_V$.
- A diatomic gas with translational and rotational degrees of freedom has $C_V = 5R/2$, $C_P = 7R/2$, and therefore $\gamma = 7/5$. Density is $\rho = M/V$ for fixed mass M.

Step 1: Rewrite the adiabatic law in terms of density The mass of the gas remains constant during

compression. Therefore its density increase fixes the inverse volume change:

$$\frac{V_i}{V_f} = \frac{\rho_f}{\rho_i} = 32.$$

The adiabatic relation gives

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} \quad \Longrightarrow \quad \frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma} = \left(\frac{\rho_f}{\rho_i}\right)^{\gamma}.$$

Pressure rises faster than density because compression also increases the temperature when heat cannot escape.

Step 2: Evaluate the pressure multiplier With the diatomic value $\gamma = 7/5$,

$$n = 32^{7/5} = (2^5)^{7/5} = 2^7 = 128.$$

The answer 32 would correspond to constant temperature, for which pressure is simply proportional to density, rather than to an adiabatic compression. The fraction $1/32$ predicts a pressure decrease during compression and has the wrong direction. The value 326 is not the result of raising 32 to the required adiabatic exponent. The dimensionless multiplier is greater than 32, consistent with the accompanying rise in temperature.

Step 3: Conclusion The final pressure is $\boxed{128P_i}$, so $\boxed{n = 128}$. Hence, the correct option is (A).

Quick Tip to Solve: For a fixed gas mass, use $P \propto \rho^{\gamma}$. Writing $32 = 2^5$ makes the diatomic exponent $7/5$ cancel immediately.

33. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is (h << R, where R is the radius of the earth) [4 marks]

(A) $\frac{R^2\omega^2}{2g}$

(B) $\frac{R^2\omega^2}{g}$

(C) $\frac{R^2\omega^2}{8g}$

(D) $\frac{R^2\omega^2}{4g}$

Correct Answer: (A) $\frac{R^2\omega^2}{2g}$

Solution:

Concept:

- A spring balance measures apparent weight. At the equator, rotation reduces the balance reading to $m(g - \omega^2 R)$, neglecting the Earth's oblateness.
- Above a pole the distance from the rotation axis is zero, so there is no centrifugal reduction. Gravity at height h is $g_h = gR^2/(R+h)^2$.

Step 1: Write the two balance readings Let the object have mass m. At the equator its circular motion requires centripetal acceleration $\omega^2 R$ towards the rotation axis. The gravitational force supplies this after subtracting the outward support force from the balance, so

$$W_{\text{eq}} = mg - m\omega^2 R.$$

At a height h directly above a pole the object remains on the axis and has no circular acceleration due to rotation. Its reading is therefore

$$W_{\text{pole},h} = mg\left(1 + \frac{h}{R}\right)^{-2}.$$

Step 2: Expand for small height and equate The stated condition $h \ll R$ permits the first-order binomial approximation

$$\left(1 + \frac{h}{R}\right)^{-2} \simeq 1 - \frac{2h}{R}.$$

Equal apparent weights then give

$$g - \omega^2 R = g\left(1 - \frac{2h}{R}\right) \quad \Longrightarrow \quad \omega^2 R = \frac{2gh}{R}.$$

Thus $h = R^2\omega^2/(2g)$. The other expressions miss the coefficient 2 in the inverse-square expansion and do not give equal readings at this order. The expression has dimensions of length and vanishes with ω, as it should when rotation is absent.

Step 3: Conclusion The required height is $\boxed{h \simeq R^2\omega^2/(2g)}$. Hence, the correct option is (A).

Quick Tip to Solve: At the equator subtract $\omega^2 R$; above the pole subtract the height correction $2gh/R$. Equal readings mean these two reductions are equal.

34. A ring is hung on a nail. It can oscillate, without slipping or sliding
(i) in its plane with a time period T_1 and,
(ii) back and forth in a direction perpendicular to its plane,
with a period T_2. The ratio $\frac{T_1}{T_2}$ will be : [4 marks]

(A) $\frac{\sqrt{2}}{3}$
(B) $\frac{2}{\sqrt{3}}$
(C) $\frac{2}{3}$
(D) $\frac{3}{\sqrt{2}}$

Correct Answer: (B) $\frac{2}{\sqrt{3}}$

Solution:

Concept:

- For a physical pendulum undergoing small oscillations, $T = 2\pi\sqrt{I_P/(Mga)}$, where I_P is the moment of inertia about the pivot axis and a is the pivot-to-centre-of-mass distance.
- For a thin ring of mass M and radius r, the central moment of inertia is Mr^2 normal to its plane and $Mr^2/2$ about a diameter in its plane. The parallel-axis theorem adds Mr^2 at a tangent pivot.

Step 1: Find the inertia for oscillation within the plane The nail supports the ring at a point on its rim without slipping, so the centre of mass is a distance r below the pivot. When the ring swings in its own plane, its rotation axis passes through the nail and is perpendicular to the plane of the ring. Hence

$$I_1 = Mr^2 + Mr^2 = 2Mr^2, \qquad T_1 = 2\pi\sqrt{\frac{2Mr^2}{Mgr}} = 2\pi\sqrt{\frac{2r}{g}}.$$

The restoring torque for either mode is $-Mgr\sin\theta$, which becomes $-Mgr\theta$ for small angular displacement.

Step 2: Find the inertia for the perpendicular swing For a back-and-forth swing normal to the ring's plane, the pivot axis is tangent to the ring and lies in its plane. The parallel central axis is a diameter, giving

$$I_2 = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2, \qquad T_2 = 2\pi\sqrt{\frac{3r}{2g}}.$$

Taking the ratio removes all common factors:

$$\frac{T_1}{T_2} = \sqrt{\frac{2}{3/2}} = \frac{2}{\sqrt{3}}.$$

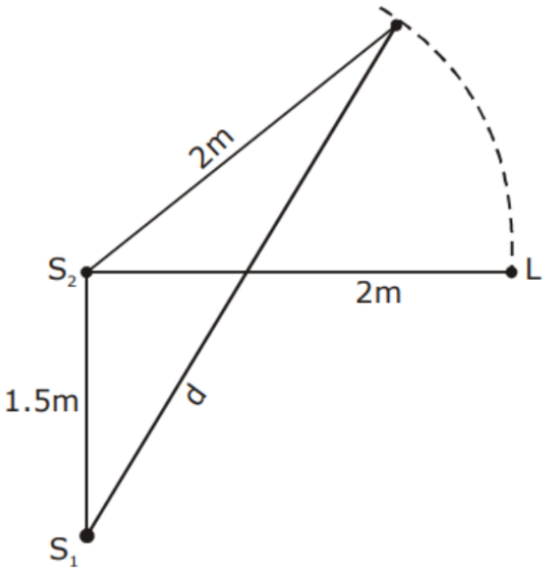
The first inertia is larger, so the ratio must exceed unity; this rules out (A) and (C). Option (D) does not equal the square root of the inertia ratio.

Step 3: Conclusion The required ratio is $\boxed{T_1/T_2 = 2/\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: Draw the rotation axis mentally for each swing. Both pivots are on the rim, but the relevant central axes are a normal and a diameter.

35. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, λ = 1 m, in phase. S_1 and S_2 are placed 1.5 m apart (see fig). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum

when he is 2 m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is : [4 marks]



(A) 12 m

(B) 2 m

(C) 3 m

(D) 5 m

Correct Answer: (C) 3 m

Solution:

Concept:

- For two in-phase coherent sources, intensity maxima occur at path difference $\Delta = n\lambda$, and minima at $\Delta = (n + \frac{1}{2})\lambda$.
- Keeping the distance from S_2 fixed means the listener moves on a circle of radius $2\,\mathrm{m}$ centred at S_2. Moving away from S_1 increases the other path and hence the path difference.

Step 1: Calculate the initial path difference The original listener position is directly in front of S_2, and the figure shows this line perpendicular to S_1S_2. The source separation is $1.5\,\mathrm{m}$, while $S_2L = 2\,\mathrm{m}$. Pythagoras therefore gives

$$S_1L = \sqrt{(1.5)^2 + 2^2} = \sqrt{6.25} = 2.5\,\mathrm{m}.$$

The initial path difference is

$$\Delta_i = S_1L - S_2L = 2.5 - 2 = 0.5\,\mathrm{m} = \frac{\lambda}{2},$$

which agrees with the stated initial minimum.

Step 2: Locate the adjacent maximum in the direction of motion As the listener moves away from S_1, the distance S_2L stays $2\,\mathrm{m}$, whereas the distance from S_1 increases. The first maximum encountered thus has path difference λ, not zero:

$$d - 2 = 1 \quad \Longrightarrow \quad d = 3\,\mathrm{m}.$$

This is geometrically accessible because the distances from S_1 on that circle range from 0.5 to $3.5\,\mathrm{m}$. Option (B) would give zero path difference but lies in the opposite direction of change. Options (A) and (D) exceed the largest possible distance $3.5\,\mathrm{m}$ and cannot describe a point on the specified circle.

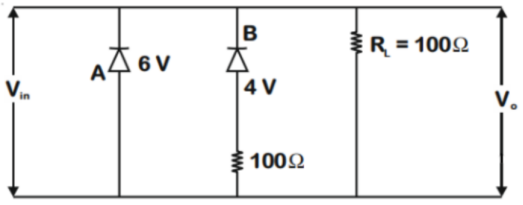
Step 3: Conclusion The adjacent maximum occurs at $\boxed{d = 3\,\mathrm{m}}$ from S_1. Hence, the correct option is (C).

Quick Tip to Solve: Find the initial path difference first, then follow whether it increases or decreases. Adjacent maxima on opposite sides of a minimum correspond to different directions of motion.

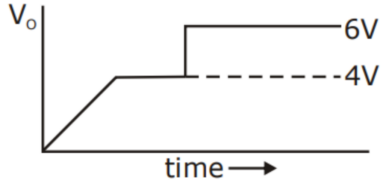
36. Two Zener diodes (A and B) having breakdown voltages of 6 V and 4 V respectively, are connected as shown in the circuit below. The output voltage V_0 variation with input voltage linearly increasing with time, is given by :

(V_{input} = 0 V at t = 0)

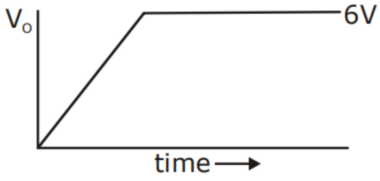
(figures are qualitative) [4 marks]



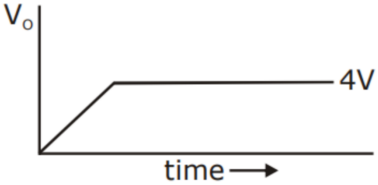
(A) Output rises linearly to $4\,\mathrm{V}$, remains flat briefly, jumps to $6\,\mathrm{V}$, and then remains constant.



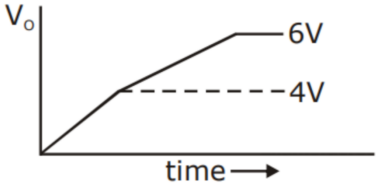
(B) Output rises linearly to $6\,\mathrm{V}$ and then remains constant.



(C) Output rises linearly to $4\,\mathrm{V}$ and then remains constant.

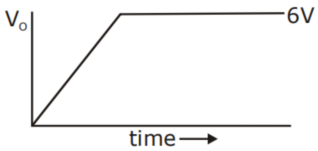


(D) Output rises linearly to $4\,\mathrm{V}$, continues to rise with a smaller slope to $6\,\mathrm{V}$, and then remains constant.



Correct Answer: (B)

Output rises linearly to $6\,\mathrm{V}$ and then remains constant.



Solution:

Concept:

- In the ideal Zener clipping model, a reverse-biased diode conducts strongly at its breakdown voltage and holds the voltage directly across it approximately constant.
- A Zener diode in series with a resistor does not fix the voltage across the whole branch at the Zener voltage: any additional branch voltage is taken up by the resistor.

Step 1: Examine the lower-breakdown branch The output is measured between the common upper and lower rails. Diode B has breakdown voltage $4\,\mathrm{V}$, but its branch contains a series resistance $100\,\Omega$. Below breakdown it carries negligible reverse current. Above $4\,\mathrm{V}$, its ideal branch current is

$$I_B = \frac{V_0 - 4\,\mathrm{V}}{100\,\Omega}.$$

Thus V_0 can exceed $4\,\mathrm{V}$: the excess is dropped across the resistor. A permanent plateau at $4\,\mathrm{V}$, as in (C), is therefore not the intended clipping behaviour. There is also no mechanism for the flat segment and abrupt jump shown in (A).

Step 2: Identify the diode directly across the output Diode A is connected directly between the output rails and has breakdown voltage $6\,\mathrm{V}$. In the qualitative Zener-regulator model used by the graphs, the output follows the rising input until A reaches breakdown and then is clamped near $6\,\mathrm{V}$. Diode B changes its branch current at $4\,\mathrm{V}$, rather than introducing a separate output plateau. The selected characteristic is consequently a straight rising segment followed by a horizontal segment at $6\,\mathrm{V}$, as in (B), rather than the intermediate change of slope in (D). At the final level, $I_B = (6-4)/100 = 0.02\,\mathrm{A}$, so the resistor accommodates the difference between the two breakdown voltages.

Step 3: Conclusion For the qualitative Zener-clipping characteristic, the output rises to $\boxed{6\,\mathrm{V}}$ and then remains approximately constant. Hence, the correct option is (B).

Quick Tip to Solve: Locate the Zener connected directly across the output. A lower-voltage Zener with a series resistor can conduct earlier without clamping the entire output to its own breakdown voltage.

37. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt}$ = bv^2(t), where v(t) is its instantaneous velocity. The instantaneous acceleration of the satellite is : [4 marks]

(A) -bv^3(t)

(B) $-\frac{2bv^3}{M(t)}$

(C) $-\frac{bv^3}{M(t)}$

(D) $-\frac{bv^3}{2M(t)}$

Correct Answer: (C) $-\frac{bv^3}{M(t)}$

Solution:

Concept:

- When stationary dust sticks to a spaceship, conserve the momentum of the spaceship and the captured dust together. No external thrust is specified.
- The captured dust initially has zero momentum in the frame in which its velocity is zero. The spaceship must give it momentum, reducing its own speed.

Step 1: Choose a short capture interval At time t, let the spaceship have mass M and speed v. In time dt, it captures stationary dust of mass dM, and its speed changes by dv. The initial momentum of the combined system is $Mv + dM(0)$. After capture, its mass is $M + dM$, so conservation gives

$$Mv = (M + dM)(v + dv).$$

Step 2: Expand to first order Expanding the product and omitting the second-order small term $dM\,dv$, obtain

$$Mv = Mv + M\,dv + v\,dM.$$

Hence

$$M\frac{dv}{dt} = -v\frac{dM}{dt}.$$

The sign is negative because stationary material must be accelerated at the expense of the original spaceship's momentum.

Step 3: Insert the given rate and compare Using $dM/dt = bv^2(t)$,

$$a(t) = \frac{dv}{dt} = -\frac{v(t)}{M(t)}bv^2(t) = -\frac{bv^3(t)}{M(t)}.$$

Option (A) omits the instantaneous mass, while (B) and (D) insert incorrect factors of two. There is no such factor in the momentum balance; it is momentum rather than kinetic energy that is conserved during capture.

Step 4: Conclusion The instantaneous acceleration is $-\frac{bv^3(t)}{M(t)}$, opposite to the direction of motion. Hence, the correct option is (C).

Quick Tip to Solve: For stationary material added to a freely moving body, start from $M\,dv = -v\,dM$; conserving its kinetic energy would give the wrong coefficient.

38. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be : [4 marks]

(A) 5×10^2 Am^2

(B) 0.5×10^2 Am^2

(C) 500×10^2 Am^2

(D) 50×10^2 Am^2

Correct Answer: (A) 5×10^2 Am^2

Solution:

Concept:

- A long solenoid carrying current I, with n turns per metre, produces magnetic field intensity $H = nI$. The field intensity is measured in $\mathrm{A\,m^{-1}}$.
- For a linear magnetic material, $\chi_m = \mu_r - 1$, magnetisation $\mathcal{M} = \chi_m H$, and magnetic dipole moment $m = \mathcal{M}V$, where V is its volume.

Step 1: Convert the turn density The given turn density is per centimetre. Since one metre contains 100 centimetres,

$$n = 10 \times 100 = 1000\,\mathrm{m^{-1}}.$$

Therefore the magnetising field intensity due to the solenoid current is

$$H = nI = 1000(0.5) = 500\,\mathrm{A\,m^{-1}}.$$

Using 10 directly as turns per metre would introduce a factor-of-100 error.

Step 2: Compute the induced magnetic moment The susceptibility of the iron core is

$$\chi_m = 1000 - 1 = 999.$$

Consequently,

$$\mathcal{M} = 999(500) = 4.995 \times 10^5\,\mathrm{A\,m^{-1}},$$

$$m = \mathcal{M}V = (4.995 \times 10^5)(10^{-3}) = 499.5\,\mathrm{A\,m^2}.$$

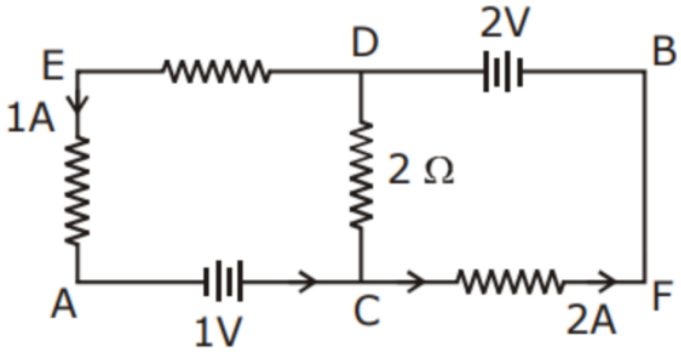
The volume multiplies magnetisation, which is moment per unit volume. It must not be confused with magnetic flux or flux density.

Step 3: Match the precision of the choices Here $\mu_r \gg 1$, so the usual approximation $\chi_m \simeq \mu_r$ gives $m \simeq 500\,\mathrm{A\,m^2}$. This is $5 \times 10^2\,\mathrm{A\,m^2}$. The alternatives are 50, 50000 and 5000 in the same unit, all incompatible with the substituted values. The exact 499.5 value rounds to the listed 500.

Step 4: Conclusion The magnetic moment is approximately $5 \times 10^2\,\mathrm{A\,m^2}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert turns/cm to turns/m before finding H, and use magnetic moment = susceptibility $\times H \times$ volume.

39. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is : [4 marks]



(A) +2 V

(B) -2 V

(C) +1 V

(D) -1 V

Correct Answer: (C) +1 V

Solution:

Concept:

- Kirchhoff's junction rule states that the total current entering a junction equals the total leaving it. In an ideal battery, potential rises from the short plate to the long plate.
- Along the direction of current through a resistor, potential falls by IR. Evaluate $V_B - V_A$ along a path whose voltage changes are known.

Step 1: Find the middle-branch current The left branch carries $1\,\text{A}$ downwards from E to A. Since there is no branching at A, the same current flows from A to C through the 1 V cell. At C, $2\,\text{A}$ leaves towards F. The additional $1\,\text{A}$ must therefore enter C from D:

$$I_{D\to C} = 2 - 1 = 1\,\text{A}.$$

This direction is essential: taking it upwards would reverse the resistor voltage.

Step 2: Read the cell polarities and resistor drop The long plate of the 1 V cell is on the C side, so

$$V_C - V_A = +1\,\text{V}.$$

The middle $2\,\Omega$ resistor carries 1 A from D to C. Hence

$$V_D - V_C = IR = 1(2) = 2\,\text{V}.$$

The long plate of the upper 2 V cell is on the D side, giving

$$V_B - V_D = -2\,\text{V}.$$

Step 3: Add the changes along A-C-D-B The desired potential of B relative to A is the algebraic sum

along this path:

$$V_B - V_A = (V_C - V_A) + (V_D - V_C) + (V_B - V_D)$$
$$= 1 + 2 - 2 = +1\,\text{V}.$$

Thus B is at a higher potential than A. The negative alternatives reverse the requested reference, while +2 V fails to include all three changes. The unlabelled outer resistors are unnecessary because the middle-branch current and its resistance are known.

Step 4: Conclusion The potential at B with respect to A is $+1$ V. Hence, the correct option is (C).

Quick Tip to Solve: First use the junction rule to find the central current; then follow A-C-D-B, assigning each battery rise and resistor change its sign.

40. The correct match between the entries in column I and column II are :

I	II
Radiation	**Wavelength**
(a) Microwave	**(i) 100 m**
(b) Gamma rays	**(ii) 10^{-15} m**
(c) A.M. radio waves	**(iii) 10^{-10} m**
(d) X-rays	**(iv) 10^{-3} m**

[4 marks]

(A) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(B) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

(C) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(D) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

Correct Answer: (B) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

Solution:

Concept:

- Electromagnetic waves obey $c = f\lambda$. A shorter wavelength therefore corresponds to a larger frequency and photon energy $E = hc/\lambda$.
- The order from longer to shorter wavelength among the four given types is AM radio waves, microwaves, X-rays and gamma rays. Match the numerical scales before checking the option codes.

Step 1: Match the two long-wavelength entries AM broadcasting uses radio waves with wavelengths of order hundreds of metres; $100\,\text{m}$ is its appropriate listed scale. Thus (c) matches (i). Microwaves have much shorter wavelengths, extending down to the millimetre scale, so $10^{-3}\,\text{m}$ matches (a). This gives

$$(a) \longleftrightarrow (iv), \qquad (c) \longleftrightarrow (i).$$

In particular, $10^{-3}\,\text{m}$ is one millimetre, not an atomic or nuclear distance.

Step 2: Match the ionising radiation entries X-rays have wavelengths comparable with atomic dimensions. The listed $10^{-10}\,\text{m}$, one angstrom, is therefore assigned to X-rays. Gamma rays can have

nuclear-scale wavelengths; the listed $10^{-15}\,\mathrm{m}$ is assigned to gamma rays. Hence

$$(b) \longleftrightarrow (ii), \qquad (d) \longleftrightarrow (iii).$$

The gamma-ray entry has the highest frequency and energy of these four representative wavelengths.

Step 3: Check the complete matching Together the matches are (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii). Only option (B) has all four. Option (A) assigns nuclear-scale wavelength to microwaves and 100 m to gamma rays; (C) interchanges microwave and X-ray scales; (D) assigns 100 m to microwaves and millimetres to AM waves. Each conflicts with the spectrum order.

Step 4: Conclusion Microwaves match millimetres, gamma rays nuclear wavelengths, AM waves 100 m, and X-rays atomic wavelengths. Hence, the correct option is (B).

Quick Tip to Solve: Remember the scale sequence: radio hundreds of metres, microwave millimetres, X-ray atomic sizes, gamma-ray nuclear sizes; then read off the matching code.

41. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge (+q) each, while 2, 4, 6, 8, 10 have charge (–q) each. The potential V and the electric field E at the centre of the circle are respectively.

(Take V = 0 at infinity) [4 marks]

(A) V = 0; E = 0

(B) $V = \frac{10q}{4\pi\varepsilon_0 R}$; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

(C) $V = \frac{10q}{4\pi\varepsilon_0 R}$; E = 0

(D) V = 0; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

Correct Answer: (A) V = 0; E = 0

Solution:

Concept:

- Electric potential is a scalar: $V = k\sum_i q_i/r_i$, where $k = 1/(4\pi\varepsilon_0)$. Charge signs enter this sum directly.
- Electric field is a vector. Equal charges uniformly distributed on a regular polygon produce zero field at its centre because their equal radial contributions sum symmetrically.

Step 1: Add the signed scalar potentials Every charge is at the same distance R from the centre. There are five charges $+q$ and five charges $-q$, so

$$V = \frac{k}{R}(5q - 5q) = 0.$$

The specified zero of potential at infinity fixes the reference consistently with this point-charge formula. Using the sum of the absolute values, $10q$, would incorrectly ignore the negative charges.

Step 2: Separate the two regular pentagons Successive charges are separated by $360^\circ/10 = 36^\circ$. The five positive charges are therefore separated from each other by 72°, forming a regular pentagon.

Their five field vectors at the centre have equal magnitudes kq/R^2, separated by 72°, and add to zero. The five negative charges form another regular pentagon, rotated by 36°; their field contributions, directed towards the negative vertices, also sum to zero.

Step 3: Superpose the two field sums Adding the two separate vector results gives

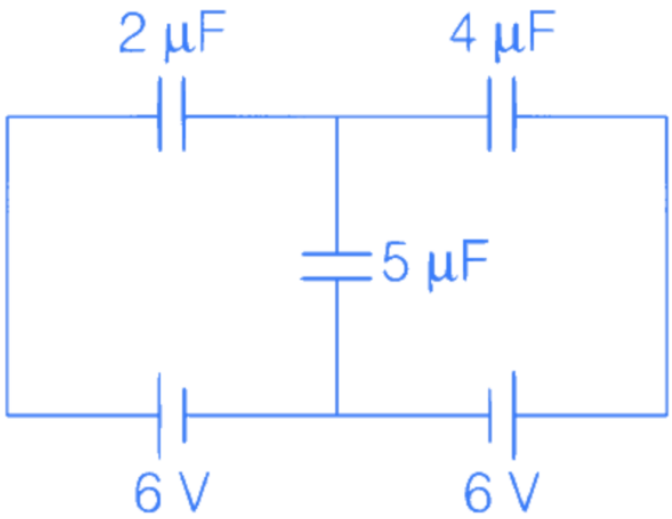
$$\vec{E} = \vec{E}_+ + \vec{E}_- = \vec{0} + \vec{0} = \vec{0}.$$

One must not cancel diametrically opposite pairs: opposite vertices here carry opposite signs, so their fields at the centre point in the same direction. The valid cancellation is the full fivefold vector symmetry within each sign group. Options (B) and (C) have a nonzero scalar potential, and (D) treats field magnitudes as if they were all parallel.

Step 4: Conclusion At the centre, $V = 0$ and $\vec{E} = \vec{0}$. Hence, the correct option is (A).

Quick Tip to Solve: For alternating charges on a regular decagon, split the arrangement into two regular pentagons; each sign group has zero resultant central field.

42. In the circuit shown, charge on the 5 μF capacitor is : [4 marks]



(A) 5.45 μC

(B) 18.00 μC

(C) 10.90 μC

(D) 16.36 μC

Correct Answer: (D) 16.36 μC

Solution:

Concept:

- For a capacitor between nodes at potentials V_i and V_j, the charge on its plate at node i is $C(V_i - V_j)$.
- The top central junction is isolated from the batteries by capacitors. Its initially neutral connected

plates must retain zero total charge; apply this node-charge equation rather than treating all three capacitors as simple series elements.

Step 1: Assign the battery-controlled potentials Take the lower central node as zero potential. The 6 V cell on the left has its positive long plate at the outer left node, so the left node is at $+6\,\mathrm{V}$. The right cell likewise has its positive plate at the outer right node, so the right node is also at $+6\,\mathrm{V}$. Denote the top central potential by x volts. Thus both outer capacitors connect this top junction to nodes of the same potential.

Step 2: Conserve charge on the isolated junction The plates connected to the top central junction belong to the 2, 4 and 5 microfarad capacitors. Measuring charge in microcoulombs, their total is

$$2(x-6)+4(x-6)+5(x-0)=0.$$

Simplifying,

$$11x-36=0, \qquad x=\frac{36}{11}\,\mathrm{V}.$$

The 2 and 4 microfarad capacitors are effectively parallel between equal-potential outer nodes and the top node. Their combined 6 microfarad value is in series with the 5 microfarad capacitor across 6 V.

Step 3: Find the charge on the central capacitor The voltage across the $5\,\mu\mathrm{F}$ capacitor is $x-0$. Therefore its charge magnitude is

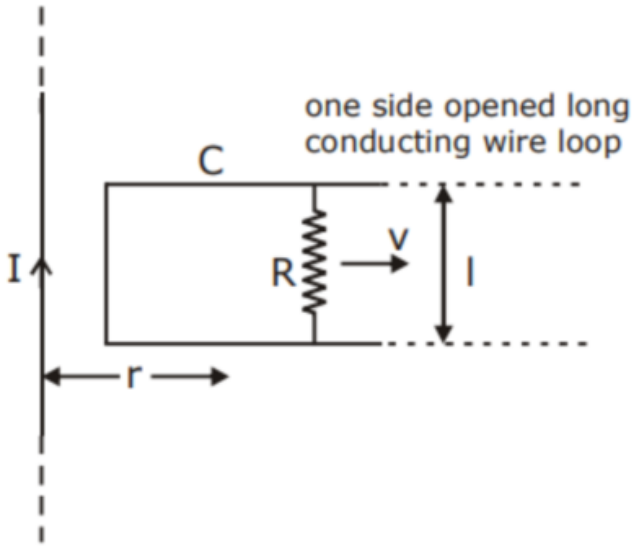
$$Q_5=C_5x=5\left(\frac{36}{11}\right)=\frac{180}{11}\,\mu\mathrm{C}=16.36\,\mu\mathrm{C}.$$

As a check, the equivalent series capacitance is $30/11\,\mu\mathrm{F}$, which draws $(30/11)6=180/11\,\mu\mathrm{C}$. The other listed values do not satisfy the junction charge balance with the shown cell polarities.

Step 4: Conclusion The charge on the $5\,\mu\mathrm{F}$ capacitor is $16.36\,\mu\mathrm{C}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the cell polarities to identify equal-potential outer nodes first; combine the 2 and 4 microfarad capacitors in parallel before using the series relation.

43. An infinitely long, straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown, in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is : [4 marks]



(A) $\frac{\mu_0}{4\pi}\frac{Ivl}{Rr}$
(B) $\frac{\mu_0}{\pi}\frac{Ivl}{Rr}$
(C) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$
(D) $\frac{2\mu_0}{\pi}\frac{Ivl}{Rr}$

Correct Answer: (C) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$

Solution:

Concept:

- An infinitely long straight wire carrying current I produces a magnetic field $B(x) = \mu_0 I/(2\pi x)$ at perpendicular distance x. Here μ_0 is the permeability of free space.
- A conducting rod of length l moving with speed v perpendicular to its length and the magnetic field develops motional emf $\mathcal{E} = Blv$. The induced current is the emf divided by the total circuit resistance.

Step 1: Evaluate the field at the connector The straight wire and the sliding connector are parallel. Every point of the vertical connector is at the same distance r from the wire, so the magnetic field has the same magnitude all along its length. The right-hand rule gives a field into the plane of the figure at the connector. Thus,

$$B(r) = \frac{\mu_0 I}{2\pi r}.$$

Although the field varies across the width of the whole loop, only the connector moves; the stationary conductor contributes no motional emf.

Step 2: Calculate the emf and apply Ohm's law The connector moves rightwards, perpendicular to

this field, and its length is perpendicular to its velocity. The magnitude of its emf is therefore

$$\mathcal{E} = B(r)lv = \frac{\mu_0 Ivl}{2\pi r}.$$

Only the connector has appreciable resistance, namely R. With negligible self-inductance, there is no inductive delay, and the instantaneous induced current is

$$i(r) = \frac{\mathcal{E}}{R} = \frac{\mu_0}{2\pi}\frac{Ivl}{Rr}.$$

Options (A), (B) and (D) have respectively one-half, twice and four times this magnitude. The factor 2π follows from the field of an infinite wire; the loop does not introduce any additional multiplying factor.

Step 3: Conclusion The induced current has magnitude $i(r) = \frac{\mu_0 Ivl}{2\pi Rr}$, decreasing inversely with separation. It flows upwards through the connector, counterclockwise around the loop. Hence, the correct option is (C).

Quick Tip to Solve: For a sliding bar parallel to a long wire, use the field at the bar itself in Blv; do not replace the varying loop field by an average field.

44. A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100 s. The effective half life of the nucleus is close to : [4 marks]

(A) 12 sec

(B) 9 sec

(C) 55 sec

(D) 6 sec

Correct Answer: (B) 9 sec

Solution:

Concept:

- For a radioactive sample, the decay constant λ is related to its half-life T by $\lambda = \ln 2/T$. The number of undecayed parent nuclei follows $N(t) = N_0 e^{-\lambda t}$.
- When the same parent nucleus can decay through two independent competing channels, either channel removes it from the parent population. The effective decay constant is the sum $\lambda_{\text{eff}} = \lambda_1 + \lambda_2$, rather than an average of the half-lives.

Step 1: Add the two rates of disappearance Let N be the number of parent nuclei still present. In a short time interval the two processes remove nuclei at rates $\lambda_1 N$ and $\lambda_2 N$. Consequently,

$$-\frac{dN}{dt} = (\lambda_1 + \lambda_2)N.$$

This is simultaneous decay of one population through alternative channels, not two consecutive stages

of a decay chain. With the stated individual half-lives,

$$\lambda_1 = \frac{\ln 2}{10}\,\mathrm{s}^{-1}, \qquad \lambda_2 = \frac{\ln 2}{100}\,\mathrm{s}^{-1}.$$

Step 2: Convert the combined decay constant to a half-life Adding the constants and using the half-life relation gives

$$\lambda_{\text{eff}} = \ln 2\left(\frac{1}{10} + \frac{1}{100}\right) = \frac{11\ln 2}{100}\,\mathrm{s}^{-1},$$
$$T_{\text{eff}} = \frac{\ln 2}{\lambda_{\text{eff}}} = \frac{100}{11}\,\mathrm{s} \approx 9.09\,\mathrm{s}.$$

The additional channel speeds up disappearance, so the effective half-life must be less than $10\,\mathrm{s}$. Thus $12\,\mathrm{s}$ and $55\,\mathrm{s}$ cannot be correct. Although $6\,\mathrm{s}$ is smaller, it is inconsistent with the calculated rate sum; the slow second channel causes only a modest reduction.

Step 3: Conclusion The effective half-life is $100/11\,\mathrm{s} \approx 9.09\,\mathrm{s}$, closest to $9\,\mathrm{s}$. Hence, the correct option is (B).

Quick Tip to Solve: For competing radioactive channels, add reciprocal half-lives: $1/T_{\text{eff}} = 1/T_1 + 1/T_2$. The result must be smaller than either individual half-life.

45. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is : [4 marks]

(A) 36 km/hr

(B) 54 km/hr

(C) 24 km/hr

(D) 18 km/hr

Correct Answer: (B) 54 km/hr

Solution:

Concept:

- For sound of speed c in still air, a source approaching a stationary observer with speed u produces observed frequency $f_1 = fc/(c-u)$, where f is its emitted frequency.
- Reflection from a stationary wall preserves frequency in the wall's frame. The approaching driver then acts as a moving observer receiving the reflected wave, giving a second Doppler factor $(c+u)/c$.

Step 1: Apply the Doppler effect on the outward and return journeys Write the horn frequency as $f = 440\,\mathrm{Hz}$ and the frequency heard after reflection as $f_r = 480\,\mathrm{Hz}$. Let the car's speed towards the wall be u. On the outward journey the wall receives

$$f_1 = f\frac{c}{c-u}.$$

The wall is stationary and reflects a wave with frequency f_1. The driver moves towards this returning wave, so the frequency he hears is

$$f_r = f_1\frac{c+u}{c} = f\frac{c+u}{c-u}.$$

The observed increase therefore contains two Doppler shifts; applying only a moving-source formula would omit the return journey.

Step 2: Solve for the car's speed and convert the units Substituting the two frequencies and the speed of sound $c = 345\,\mathrm{m\,s^{-1}}$,

$$\frac{480}{440} = \frac{345+u}{345-u},$$
$$480(345-u) = 440(345+u),$$
$$(480-440)345 = (480+440)u,$$
$$u = \frac{40\times 345}{920} = 15\,\mathrm{m\,s^{-1}}.$$

Hence $u = 15\times 3.6 = 54\,\mathrm{km\,h^{-1}}$. Inserting the speeds 36, 24 and $18\,\mathrm{km\,h^{-1}}$ in the two-shift ratio would give a reflected frequency below 480 Hz, since each is less than the required speed.

Step 3: Conclusion The car approaches the wall at $15\,\mathrm{m\,s^{-1}} = 54\,\mathrm{km\,h^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: For an echo heard by a source moving towards a fixed wall, use $u = c(f_r - f)/(f_r + f)$; the frequency change includes both journeys.

Physics – Section B: Numerical Value Questions (20 marks)

46. A prism of angle A = 1° has a refractive index μ = 1.5. A good estimate for the minimum angle of deviation (in degrees) is close to $\frac{N}{10}$.

Value of N is ____. [4 marks]

Solution:

Concept:

- At minimum deviation a prism has a symmetric ray path: its internal refraction angles are both $A/2$, and its incidence and emergence angles are equal. The refractive index in air obeys $\mu = \sin[(A+\delta_{\min})/2]/\sin(A/2)$.
- For a thin prism, all these angles are small and $\sin\theta \approx \theta$ when angles are expressed in radians. This gives $\delta_{\min} \approx (\mu-1)A$, with deviation and prism angle in the same angular units.

Step 1: Obtain the thin-prism approximation The prism angle is only $A = 1^\circ$, so the small-angle approximation is appropriate for the requested good estimate. Starting with the exact minimum-deviation relation,

$$\mu = \frac{\sin[(A+\delta_{\min})/2]}{\sin(A/2)} \approx \frac{A+\delta_{\min}}{A}.$$

Solving this expression gives $\delta_{\min} \approx (\mu-1)A$. Although the approximation is derived using radians,

its final linear relation also holds in degrees because the conversion factor multiplies both angles equally.

Step 2: Substitute the refractive index and identify the requested number With $\mu = 1.5$ and $A = 1^\circ$, the estimated deviation is

$$\delta_{\min} \approx (1.5 - 1) \times 1^\circ = 0.5^\circ.$$

The question writes the numerical value in degrees as $N/10$. Therefore,

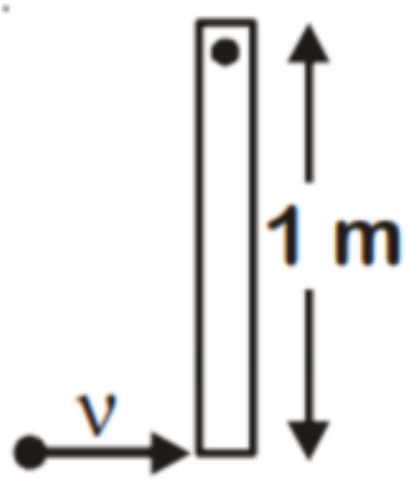
$$\frac{N}{10} = 0.5 \quad \Longrightarrow \quad N = 5.$$

As a check, the exact formula is $\delta_{\min} = 2\sin^{-1}[1.5\sin(0.5^\circ)] - 1^\circ$, which is approximately 0.50002°. This is very close to the thin-prism result; the small difference has no effect on the requested estimate. The deviation is positive because the prism's refractive index exceeds that of air.

Step 3: Conclusion The minimum deviation is approximately 0.5°. The requested dimensionless number is $\boxed{N = 5}$.

Quick Tip to Solve: For a prism angle of only one degree, first use $\delta \approx (\mu - 1)A$; then convert that deviation into the scaled number requested in the question.

47. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be _____. [4 marks]



Solution:

Concept:

- During a brief collision, angular momentum about a fixed hinge is conserved when the external angular impulse about it is negligible. The hinge force can provide a large impulse, but its moment about the hinge is zero.
- A uniform thin rod of mass M and length L has moment of inertia $ML^2/3$ about one end. A point particle of mass m stuck at its other end adds mL^2, and both rotate with a common angular speed ω.

Step 1: Evaluate the angular momentum before impact Take moments about the upper suspension point. The rod is initially at rest, and the particle moves horizontally towards its lower end, as the figure shows. Its momentum is perpendicular to the radius from the hinge, so its initial angular momentum magnitude is

$$L_{\text{initial}} = mvL = (0.1)(80)(1) = 8\,\text{kg}\,\text{m}^2\,\text{s}^{-1}.$$

Over the very short impact, the angular impulse from gravity is negligible. The hinge's impulsive force has zero lever arm about the hinge, justifying conservation about this point even though the system's linear momentum need not be conserved.

Step 2: Include both contributions to the final moment of inertia The particle sticks to the rod, so this is a perfectly inelastic collision. Both objects rotate together immediately afterwards. The combined moment of inertia about the hinge is

$$I_{\text{total}} = \frac{ML^2}{3} + mL^2 = \frac{0.9 \times 1^2}{3} + 0.1 \times 1^2 = 0.4\,\text{kg}\,\text{m}^2.$$

Applying angular momentum conservation,

$$I_{\text{total}}\omega = mvL \quad \Longrightarrow \quad \omega = \frac{8}{0.4} = 20\,\text{rad}\,\text{s}^{-1}.$$

The particle's contribution cannot be omitted. Kinetic energy is not conserved during sticking; some of the initial energy becomes deformation, sound and heat. That is why the energy-conservation method would be inappropriate here.

Step 3: Conclusion The angular speed of the rod immediately after the collision is $\boxed{20\,\text{rad}\,\text{s}^{-1}}$, so the numerical entry is 20.

Quick Tip to Solve: In a sticking collision with a hinged rod, conserve angular momentum about the hinge and include the stuck particle in the final moment of inertia.

48. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) _____.

[4 marks]

Solution:

Concept:

- Power is the rate of doing work. For the body driven by the stated engine, the work supplied increases its kinetic energy, so $P = dK/dt$. Constant power therefore gives $K(t) = Pt$ when the body starts from rest.
- Since $K = mv^2/2$, the speed is $v(t) = \sqrt{2Pt/m}$. Distance is the time integral of speed. Constant power does not imply constant force or constant acceleration.

Step 1: Find the speed as a function of time Let t be measured from the instant the body starts moving. The engine supplies energy Pt by time t, and the initial kinetic energy is zero. Hence,

$$\frac{1}{2}mv(t)^2 = Pt \quad \Longrightarrow \quad v(t) = \sqrt{\frac{2P}{m}}\, t^{1/2}.$$

The positive square root describes the motion in its forward direction. Here $P = 1\,\mathrm{J\,s^{-1}} = 1\,\mathrm{W}$ and $m = 2\,\mathrm{kg}$, so the speed grows in proportion to $\sqrt{t}$, rather than in proportion to t.

Step 2: Integrate the speed over the first nine seconds The motion is along one straight line without reversal, so the distance equals the integral of speed. Denoting the final time by T,

$$\begin{aligned} s(T) &= \int_0^T \sqrt{\frac{2P}{m}}\, t^{1/2}\, dt \\ &= \frac{2}{3}\sqrt{\frac{2P}{m}}\, T^{3/2}. \end{aligned}$$

Substituting the SI values gives

$$s(9\,\mathrm{s}) = \frac{2}{3}\sqrt{\frac{2\times 1}{2}}\,(9)^{3/2}\,\mathrm{m} = \frac{2}{3}\times 27\,\mathrm{m} = 18\,\mathrm{m}.$$

As a consistency check, the final speed is $3\,\mathrm{m\,s^{-1}}$, whereas the average speed is $18/9 = 2\,\mathrm{m\,s^{-1}}$. Their ratio is $2/3$, as expected for speed proportional to $t^{1/2}$. A constant-acceleration average-speed formula would give the wrong result because the acceleration decreases with time.

Step 3: Conclusion The body travels $\boxed{18\,\mathrm{m}}$ in the first $9\,\mathrm{s}$, so the requested numerical entry is 18.

Quick Tip to Solve: For motion from rest under constant power, use $s = (2/3)v_{\text{final}}T$; this follows from $v \propto \sqrt{t}$, not constant acceleration.

49. The surface of a metal is illuminated alternately with photons of energies E_1 = 4 eV and E_2 = 2.5 eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is _____. [4 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E - \phi$, where E is the photon energy and ϕ is the metal's work function. Both energies may be expressed in electron-volts.
- The maximum kinetic energy is $\frac{1}{2}m_e v_{\max}^2$. Therefore a speed ratio of 2 corresponds to a kinetic-energy ratio of 4, since the emitted particles have the same electron mass.

Step 1: Relate the two maximum kinetic energies Write the first speed as v_1 and the second as v_2. The higher photon energy gives the higher maximum speed, so $v_1/v_2 = 2$. Using the same work function in both experiments gives

$$K_1 = (4 - \phi)\,\text{eV}, \qquad K_2 = (2.5 - \phi)\,\text{eV}.$$

Taking their ratio cancels the factor converting electron-volts to joules and the electron mass:

$$\frac{4 - \phi}{2.5 - \phi} = \frac{v_1^2}{v_2^2} = 4.$$

Step 2: Solve for the work function and check emission Rearrange the energy-ratio equation without replacing the kinetic-energy ratio by the speed ratio:

$$4 - \phi = 10 - 4\phi \quad \Rightarrow \quad 3\phi = 6 \quad \Rightarrow \quad \phi = 2.$$

With this value, the maximum kinetic energies are $2\,\text{eV}$ and $0.5\,\text{eV}$, respectively. Both are positive, as required because photoelectrons are emitted in both cases. Their ratio is 4, so their speeds indeed have ratio $\sqrt{4} = 2$.

Step 3: Conclusion The work function of the metal is $\boxed{2\,\text{eV}}$. The numerical entry in the blank is 2.

Quick Tip to Solve: Square a photoelectron speed ratio before substituting it into an Einstein-equation energy ratio; the work function is subtracted from each photon energy first.

50. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is _____.
(Molar mass of N_2 gas 28 g). [4 marks]

Solution:

Concept:

- For an ideal gas, the root mean square molecular speed is $v_{\text{rms}} = \sqrt{3RT/M}$, where R is the gas constant, T is absolute temperature and M is molar mass in consistent units.
- Equal rms speeds for different gases require equal values of T/M. A lighter gas therefore needs a

lower absolute temperature to have the same rms speed as a heavier gas.

Step 1: Convert the nitrogen temperature to kelvin The temperature in the rms-speed equation must be measured from absolute zero, so the given Celsius value cannot be inserted directly:

$$T_{N_2} = 300 + 273 = 573\,\text{K}.$$

The molar masses are $M_{N_2} = 28\,\text{g mol}^{-1}$ and $M_{H_2} = 2\,\text{g mol}^{-1}$. Since only their ratio is needed, using grams consistently for both gives the same result as converting both to kilograms.

Step 2: Equate the rms speeds and calculate the temperature Let the required hydrogen temperature be T_H. Squaring the equality of the two rms speeds and cancelling $3R$ gives

$$\frac{3RT_H}{M_{H_2}} = \frac{3RT_{N_2}}{M_{N_2}} \quad \Rightarrow \quad \frac{T_H}{2} = \frac{573}{28}.$$

Hence

$$T_H = 573\left(\frac{2}{28}\right) = \frac{573}{14} = 40.93\,\text{K}.$$

Hydrogen is fourteen times lighter per molecule, so its temperature must be fourteen times smaller. This confirms the direction of the temperature change and excludes an inverted mass ratio.

Step 3: Conclusion The required temperature is $\boxed{40.93\,\text{K}}$, approximately $\boxed{41\,\text{K}}$ to the nearest kelvin.

Quick Tip to Solve: For equal rms speeds, write $T_1/T_2 = M_1/M_2$ after converting Celsius to kelvin. The lighter gas must have the lower temperature.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The correct statement about probability density (except at infinite distance from nucleus) is :

[4 marks]

(A) It can be zero for 1s orbital

(B) It can be zero for 3p orbital

(C) It can never be zero for 2s orbital

(D) It can negative for 2p orbital

Correct Answer: (B) It can be zero for 3p orbital

Solution:

Concept:

- The probability density of an electron is $|\psi|^2$, where ψ is its orbital wavefunction. It is non-negative everywhere and vanishes at a node, where the wavefunction is zero.
- For a hydrogen-like orbital, the number of radial nodes is $n - l - 1$, and the number of angular nodes is l. Here n is the principal quantum number and l is the azimuthal quantum number.

Step 1: Check which orbitals possess finite-distance nodes For the $1s$ orbital, $n = 1$ and $l = 0$, so it has neither radial nor angular nodes. Its probability density is positive at every finite distance and

tends to zero only asymptotically. Thus statement (A) is false. For $3p$, $n = 3$ and $l = 1$, giving

$$n - l - 1 = 3 - 1 - 1 = 1 \text{ radial node}, \qquad l = 1 \text{ angular node}.$$

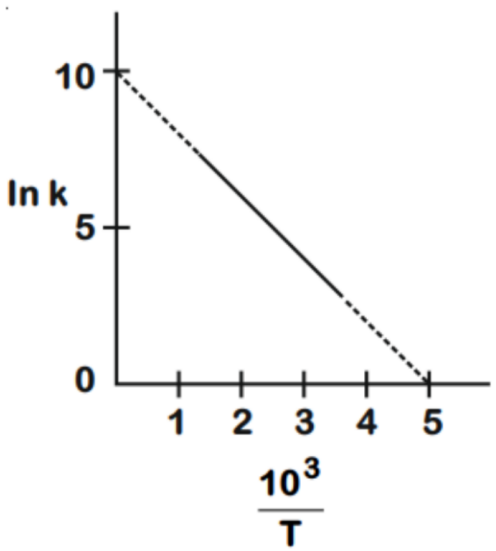
The probability density of a $3p$ orbital can therefore be zero at a finite position, establishing statement (B).

Step 2: Eliminate the remaining statements The $2s$ orbital has $2 - 0 - 1 = 1$ radial node, so its probability density does vanish on a finite spherical nodal surface. Statement (C), which says it can never be zero, is false. A $2p$ wavefunction may have positive and negative lobes, but the probability density is its modulus squared. Consequently, $|\psi|^2 \geq 0$, and it cannot be negative. Statement (D) confuses the sign of the wavefunction with a probability density and is false.

Step 3: Conclusion The probability density can be zero for the $3p$ orbital at its nodes. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the sign of ψ from the non-negative density $|\psi|^2$; then count radial and angular nodes to test whether the density can vanish.

52. The rate constant (k) of a reaction is measured at differenct temperatures (T), and the data are plotted in the given figure. The activation energy of the reaction in kJ mol^{-1} is :
(R is gas constant) [4 marks]



(A) R
(B) 2R
(C) $\frac{1}{R}$
(D) $\frac{1}{2R}$
Correct Answer: (B) 2R

Solution:

Concept:

- The Arrhenius equation $k = Ae^{-E_a/(RT)}$ becomes $\ln k = \ln A - E_a/(RT)$. Here E_a is activation energy, A is the pre-exponential factor and R is the gas constant.
- The gradient depends on the horizontal-axis variable. For a plot against $x = 10^3/T$, the slope is $-E_a/(10^3 R)$, rather than $-E_a/R$, when E_a is expressed in joules per mole.

Step 1: Read the slope from the plotted intercepts The graph has vertical intercept $\ln k = 10$ at $x = 0$ and horizontal intercept $x = 5$ at $\ln k = 0$, where $x = 10^3/T$. Therefore its gradient is

$$m = \frac{0 - 10}{5 - 0} = -2.$$

The negative slope agrees with positive activation energy: increasing $1/T$ means decreasing temperature, and the rate constant consequently decreases.

Step 2: Include the axis scale and convert the energy unit Since $1/T = x/10^3$, the Arrhenius equation in terms of the printed horizontal coordinate is

$$\ln k = \ln A - \frac{E_a}{10^3 R} x.$$

Equating its slope with the measured value gives

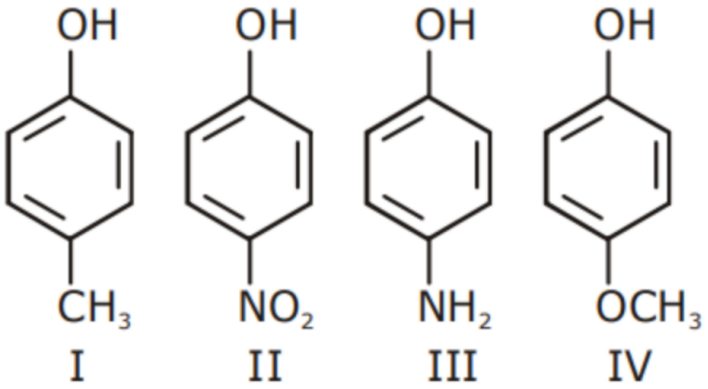
$$-\frac{E_a}{10^3 R} = -2 \quad \Rightarrow \quad E_a = 2000R\,\text{J mol}^{-1} = 2R\,\text{kJ mol}^{-1}.$$

Thus option (A) misses the factor 2 obtained from the slope. Options (C) and (D) place R in the denominator, whereas isolating E_a requires multiplying by R. The axis factor of 10^3 cancels when joules are converted to kilojoules.

Step 3: Conclusion The activation energy is $\boxed{2R\,\text{kJ mol}^{-1}}$, using the numerical value of R in $\text{J mol}^{-1}\,\text{K}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Read the horizontal-axis multiplier before using an Arrhenius slope. For $\ln k$ versus $10^3/T$, multiply the slope magnitude by $1000R$ to obtain joules per mole.

53. **The increasing order of boiling points of the following compounds is :** [4 marks]



(A) I < III < IV < II

(B) IV < I < II < III

(C) I < IV < III < II

(D) III < I < II < IV

Correct Answer: (C) I < IV < III < II

Solution:

Concept:

- Boiling separates molecules against intermolecular attractions. For related aromatic compounds, hydrogen bonding, dipole interactions and dispersion forces together determine the boiling-point order.
- All four compounds are para-substituted phenols. Their phenolic OH groups form intermolecular hydrogen bonds; the para substituent changes the number and strength of intermolecular interactions without permitting an ortho-type intramolecular hydrogen bond.

Step 1: Identify the four para-substituted phenols The structures are I: p-cresol with CH_3, II: p-nitrophenol with NO_2, III: p-aminophenol with NH_2, and IV: p-methoxyphenol with OCH_3. The methyl group in I supplies no additional hydrogen-bond donor or acceptor. The ether oxygen in IV supplies an extra acceptor and increases polar interactions, giving stronger overall molecular association than in I. Hence $I < IV$.

Step 2: Compare the amino and nitro derivatives The amino group in III provides additional hydrogen-bonding interactions through its nitrogen and its $N-H$ bonds, so association is stronger than in the methoxy derivative IV. The nitro derivative II has a strongly polar NO_2 group with two oxygen acceptors. Its electron-withdrawing effect also makes the phenolic hydrogen a stronger hydrogen-bond donor. The resulting overall intermolecular attractions place II above III in this comparison. Combining the comparisons gives

$$I < IV < III < II.$$

Option (A) reverses the IV/III pair. Option (B) wrongly puts IV below I and II below III. Option (D) wrongly makes the amino compound the lowest and the methoxy compound the highest.

Step 3: Conclusion The increasing boiling-point order is $\boxed{I < IV < III < II}$. Hence, the correct option is (C).

Quick Tip to Solve: First identify every substituent and its hydrogen-bond donor or acceptor sites. For these para phenols, compare intermolecular association; do not invoke ortho intramolecular hydrogen bonding.

54. The correct order of the ionic radii of O^{2-}, N^{3-}, F^-, Mg^{2+}, Na^+ and Al^{3+} is : [4 marks]

(A) $N^{3-} < O^{2-} < F^- < Na^+ < Mg^{2+} < Al^{3+}$

(B) $N^{3-} < F^- < O^{2-} < Mg^{2+} < Na^+ < Al^{3+}$

(C) $Al^{3+} < Na^+ < Mg^{2+} < O^{2-} < F^- < N^{3-}$

(D) $Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$

Correct Answer: (D) $Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$

Solution:

Concept:

- Isoelectronic ions have the same number of electrons and the same occupied-shell pattern. In such a series, increasing nuclear charge pulls the common electron cloud inward, decreasing ionic radius.
- Determine electron counts before comparing charges: an anion gains electrons and a cation loses electrons. The nuclear charge is still set by the atomic number Z, not by the ion's net charge alone.

Step 1: Verify that all six ions are isoelectronic Nitrogen, oxygen and fluorine have atomic numbers 7, 8 and 9, respectively. Their listed anions contain $7+3=10$, $8+2=10$ and $9+1=10$ electrons. Sodium, magnesium and aluminium have atomic numbers 11, 12 and 13, and their listed cations contain $11-1=10$, $12-2=10$ and $13-3=10$ electrons. Thus every ion has the neon-like configuration $1s^2 2s^2 2p^6$, so nuclear charge provides the required ordering criterion.

Step 2: Reverse the nuclear-charge order to obtain the radii The nuclear charges increase as

$$N^{3-} < O^{2-} < F^- < Na^+ < Mg^{2+} < Al^{3+}.$$

Because all these ions have ten electrons, the ion with 13 protons holds its electron cloud most tightly, while the ion with 7 protons holds it least tightly. Therefore the radii increase in the reverse order:

$$r(Al^{3+}) < r(Mg^{2+}) < r(Na^+) < r(F^-) < r(O^{2-}) < r(N^{3-}).$$

Option (A) gives the reverse trend. Option (B) also puts the largest anion below smaller ions. Option (C) reverses both the sodium/magnesium pair and the oxygen/fluorine pair. Only (D) consistently follows increasing radius with decreasing nuclear charge.

Step 3: Conclusion The order is $\boxed{Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}}$, where each comparison denotes ionic radius. Hence, the correct option is (D).

Quick Tip to Solve: Count electrons first. Once an isoelectronic series is established, arrange atomic numbers in descending order to obtain ionic radii in ascending order.

55. Consider the complex ions,

trans-$[Co(en)_2Cl_2]^+$ (A) and

cis-$[Co(en)_2Cl_2]^+$ (B).

The correct statement regarding them is : [4 marks]

(A) both (A) and (B) can be optically active.

(B) both (A) and (B) can not be optically active.

(C) (A) can not be optically active, but (B) can be optically active.

(D) (A) can be optically active, but (B) can not be optically active.

Correct Answer: (C) (A) can not be optically active, but (B) can be optically active.

Solution:

Concept:

- An octahedral complex is optically active when its three-dimensional arrangement is chiral: its mirror image cannot be superimposed on it by rotation.
- Ethylenediamine, en, is a neutral bidentate ligand. Two en ligands occupy four coordination sites around cobalt; the two chloride ligands occupy the remaining sites.

Step 1: Examine the trans arrangement In complex (A), the two chloride ligands are opposite each other in the octahedron. The four nitrogen donor atoms of the two en ligands occupy the remaining equatorial sites. This arrangement possesses a plane of symmetry. Reflection therefore gives a configuration that can be superimposed on the original one, so the trans complex is achiral and cannot show optical isomerism.

Step 2: Examine the cis arrangement In complex (B), the chloride ligands occupy adjacent positions. The two chelate rings can wrap around the cobalt centre in two opposite senses. These configurations are the right-handed and left-handed forms, conventionally denoted Δ and Λ. They are non-superimposable mirror images, so the cis complex has a pair of optical isomers. An individual enantiomer can rotate plane-polarised light; an equal mixture would be optically inactive because the rotations cancel.

Step 3: Compare the statements Option (A) incorrectly makes the trans form optically active. Option (B) incorrectly excludes the optical activity of the cis form. Option (D) reverses their behaviour. Only option (C) assigns an achiral trans form and a potentially optically active cis form. This conclusion depends on the complete arrangement of the chelate rings, rather than on the presence of cobalt or chloride alone.

Step 4: Conclusion The trans complex cannot be optically active, whereas the cis complex can exist as optically active enantiomers. Hence, the correct option is (C).

Quick Tip to Solve: For octahedral complexes of the type $[M(en)_2X_2]$, test the chelate arrangement for symmetry: the trans form is achiral and the cis form has an enantiomeric pair.

56. Lattice enthalpy and enthalpy of solution of NaCl are 788 kJ mol^{-1}, and 4 kJ mol^{-1}, respectively. The hydration enthalpy of NaCl is : [4 marks]

(A) –780 kJ mol^{-1}

(B) –784 kJ mol^{-1}

(C) 780 kJ mol^{-1}

(D) 784 kJ mol^{-1}

Correct Answer: (B) –784 kJ mol^{-1}

Solution:

Concept:

- Hess's law states that the enthalpy change between fixed initial and final states is independent of the path. Dissolution can therefore be split into lattice dissociation and hydration.
- Lattice dissociation requires energy to separate crystal ions into gaseous ions. Hydration releases energy when water stabilises those gaseous ions; its enthalpy is negative.

Step 1: Write the two stages of dissolution The positive lattice enthalpy given here is the energy required for $\mathrm{NaCl(s)} \longrightarrow \mathrm{Na^+(g) + Cl^-(g)}$, namely $+788\,\mathrm{kJ\,mol^{-1}}$. Hydration then converts the separated gaseous ions to aqueous ions. The required hydration enthalpy is the sum for one mole of sodium ions and one mole of chloride ions, rather than an individual ionic value.

Step 2: Apply the enthalpy balance Adding the two stages must give $\mathrm{NaCl(s)} \longrightarrow \mathrm{Na^+(aq) + Cl^-(aq)}$. Therefore,

$$\Delta H_{\mathrm{sol}} = \Delta H_{\mathrm{latt,diss}} + \Delta H_{\mathrm{hyd}}.$$

Substitute the signed quantities, keeping all values in the same molar units:

$$4 = 788 + \Delta H_{\mathrm{hyd}} \quad \Longrightarrow \quad \Delta H_{\mathrm{hyd}} = -784\,\mathrm{kJ\,mol^{-1}}.$$

Step 3: Check the sign and the alternatives The hydration energy released is slightly smaller in magnitude than the energy absorbed in breaking the lattice. Consequently the overall dissolution is weakly endothermic, consistent with the given $+4\,\mathrm{kJ\,mol^{-1}}$. A value of -780 would give $+8$ for the solution enthalpy, so option (A) fails. The positive values in (C) and (D) would make hydration endothermic and cannot reproduce the stated balance.

Step 4: Conclusion The hydration enthalpy of NaCl is $-784\,\mathrm{kJ\,mol^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Write lattice dissociation as positive and hydration as negative before substituting: the small solution enthalpy is the difference between two large opposing contributions.

57. The major product formed in the following reaction is :

$CH_3CH = CHCH(CH_3)_2 \xrightarrow{HBr}$ [4 marks]

(A) $Br(CH_2)_3CH(CH_3)_2$

(B) $CH_3CH(Br)CH_2CH(CH_3)_2$

(C) $CH_3CH_2CH(Br)CH(CH_3)_2$

(D) $CH_3CH_2CH_2C(Br)(CH_3)_2$

Correct Answer: (D) $CH_3CH_2CH_2C(Br)(CH_3)_2$

Solution:

Concept:

- In the absence of a peroxide reagent, HBr adds to an alkene by an ionic electrophilic mechanism: protonation generates a carbocation and bromide then attacks it.
- A neighbouring hydride can migrate with its bonding electron pair when this converts a secondary carbocation into a more stable tertiary carbocation.

Step 1: Generate the carbocation next to the branch Number the chain in the written direction as $CH_3{-}CH{=}CH{-}CH(CH_3)_2$. Proton addition at the second carbon places the positive charge on the third carbon:

$$CH_3{-}CH{=}CH{-}CH(CH_3)_2 + H^+ \longrightarrow CH_3{-}CH_2{-}CH^+{-}CH(CH_3)_2.$$

The initially formed cation is secondary. Importantly, the adjacent fourth carbon bears a hydrogen and is bonded to three carbon groups; it can supply a hydride to the electron-deficient centre.

Step 2: Rearrange and capture bromide A 1,2-hydride shift transfers the hydrogen from the fourth carbon to the third carbon and moves the positive charge to the fourth carbon:

$$CH_3{-}CH_2{-}CH^+{-}CH(CH_3)_2 \longrightarrow CH_3{-}CH_2{-}CH_2{-}C^+(CH_3)_2.$$

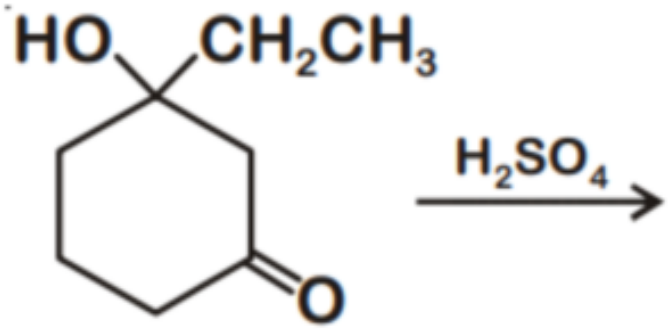
The new carbocation is tertiary and is stabilised by greater alkyl electron donation and hyperconjugation. Attack by Br^- then produces $CH_3CH_2CH_2C(Br)(CH_3)_2$, the rearranged major product.

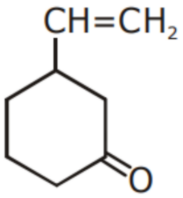
Step 3: Exclude the other structures Option (A) places bromine on a terminal carbon and does not follow this ionic addition pathway. Options (B) and (C) put bromine on secondary carbons; they correspond to capture before obtaining the more stable tertiary cation. Neither describes the major rearranged product. The carbon skeleton and all six carbon atoms are retained during the hydride shift; only the position of hydrogen and the positive charge changes.

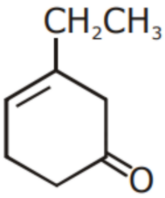
Step 4: Conclusion The major product is $CH_3CH_2CH_2C(Br)(CH_3)_2$, formed after a hydride shift to a tertiary carbocation. Hence, the correct option is (D).

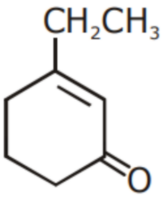
Quick Tip to Solve: Before trapping an alkene-addition carbocation with bromide, inspect its neighbouring carbon for a hydride shift that produces a tertiary centre.

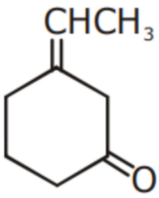
58. **The major product of the following reaction is :** [4 marks]

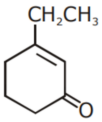


(A) 3-Vinylcyclohexan-1-one 

(B) 3-Ethylcyclohex-3-en-1-one 

(C) 3-Ethylcyclohex-2-en-1-one 

(D) 3-Ethylidenecyclohexan-1-one 

Correct Answer: (C) 3-Ethylcyclohex-2-en-1-one 

Solution:

Concept:

- Acid-catalysed dehydration of an alcohol proceeds by protonating the hydroxyl group, losing water and eliminating a neighbouring proton to form an alkene.
- An alkene conjugated with a carbonyl group is stabilised by delocalisation of its π electrons. This favours an α,β-unsaturated ketone over an otherwise comparable non-conjugated alkene.

Step 1: Locate the alcohol and carbonyl groups Number the ring from its ketone carbon as C1. The hydroxyl-bearing carbon is C3, and it also bears an ethyl group. Thus the reactant is 3-ethyl-3-hydroxycyclohexan-1-one, a β-hydroxy ketone. Concentrated H_2SO_4 protonates the alcohol oxygen so that the initially poor leaving group OH becomes water. Loss of water creates an electron-deficient centre at C3.

Step 2: Choose the favourable elimination Removal of a proton from C2 forms a double bond between C2 and C3. The product then has the sequence $O{=}C{-}C{=}C$, so the alkene is conjugated with the ketone. Its name is 3-ethylcyclohex-2-en-1-one. The ethyl substituent remains attached to C3, and the original carbonyl remains at C1; the reaction removes the alcohol as water rather than reducing or moving the ketone.

Step 3: Compare all four products Option (C) shows exactly this conjugated enone. Option (B) instead places the double bond between C3 and C4, separated from the carbonyl by a saturated

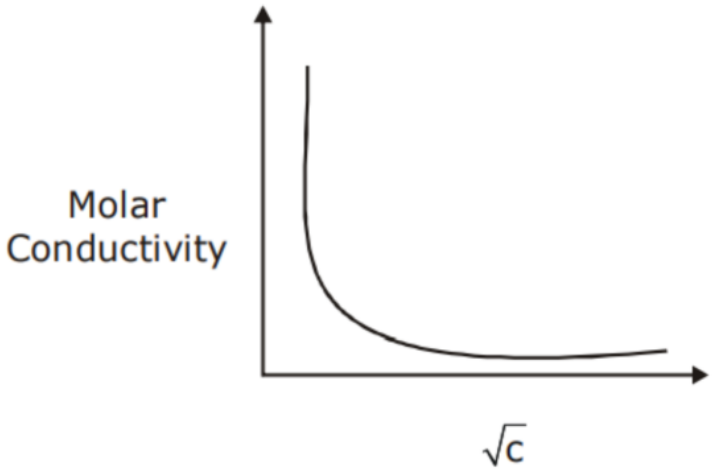
carbon, so it lacks conjugation. Option (D) has an exocyclic ethylidene double bond and is also not conjugated with the carbonyl. Option (A) has a vinyl side chain attached to a saturated ring carbon and likewise lacks the stabilised enone arrangement. These alternatives do not gain the conjugative stabilisation obtained by elimination toward C2.

Step 4: Conclusion The major product is 3-ethylcyclohex-2-en-1-one, the conjugated α, β-unsaturated ketone shown in option (C). Hence, the correct option is (C).

Quick Tip to Solve: For dehydration of a β-hydroxy carbonyl compound, first test elimination toward the carbonyl: it gives the stabilised conjugated enone.

59. The variation of molar conductivity with concentration of an electrolyte (X) in aqueous solution is shown in the given figure.

The electrolyte X is : [4 marks]



(A) HCl

(B) CH_3COOH

(C) NaCl

(D) KNO_3

Correct Answer: (B) CH_3COOH

Solution:

Concept:

- Molar conductivity Λ_m measures the conductivity contribution per mole of electrolyte. It increases on dilution as interionic effects diminish and, for a weak electrolyte, ionisation increases.
- A strong electrolyte approximately follows $\Lambda_m = \Lambda_m^\circ - A\sqrt{c}$ at low concentration. A weak electrolyte instead shows a strongly curved Λ_m-versus-$\sqrt{c}$ plot.

Step 1: Interpret the displayed curve The horizontal coordinate is $\sqrt{c}$, so moving left corresponds to dilution. The curve rises very sharply as this coordinate approaches zero rather than following an approximately straight line. Such a large change indicates that the number of ions supplied per

mole changes substantially with dilution. It is characteristic of a weak electrolyte whose degree of ionisation is small at ordinary concentration.

Step 2: Connect dilution with ionisation For ethanoic acid the equilibrium is $CH_3COOH + H_2O \rightleftharpoons H_3O^+ + CH_3COO^-$. Dilution favours ionisation, producing a much larger fraction of conducting ions. Writing its degree of ionisation as α, the approximate relation is $\Lambda_m = \alpha\Lambda_m^\circ$. The strong rise in α on dilution explains the sharp rise in molar conductivity. A curved plot also means that a simple straight-line extrapolation cannot reliably provide its limiting molar conductivity.

Step 3: Classify the options CH_3COOH is the only weak electrolyte listed and therefore matches the curve. HCl is a strong acid and is extensively ionised in water. $NaCl$ and KNO_3 are soluble salts and are strong electrolytes as well. Their low-concentration molar conductivity plots against $\sqrt{c}$ are approximately linear, so options (A), (C) and (D) do not explain the pronounced curvature.

Step 4: Conclusion The electrolyte X is ethanoic acid, CH_3COOH. Hence, the correct option is (B).

Quick Tip to Solve: On a Λ_m-versus-$\sqrt{c}$ plot, a sharp nonlinear rise near zero concentration identifies increased ionisation of a weak electrolyte.

60. The compound that has the largest H–M–H bond angle (M = N, O, S, C) is : [4 marks]

(A) CH_4

(B) H_2S

(C) NH_3

(D) H_2O

Correct Answer: (A) CH_4

Solution:

Concept:

- Valence-shell electron-pair repulsion determines molecular geometry. Lone pairs repel bonding pairs more strongly than bonding pairs repel each other and usually compress the bond angle.
- For the second-period central atoms considered here, the four electron-pair directions are approximately tetrahedral, with an ideal angle of 109.5° before lone-pair compression.

Step 1: Compare methane, ammonia and water In CH_4, carbon has four bonding pairs and no lone pair, giving a tetrahedral molecule with $\angle HCH \approx 109.5^\circ$. In NH_3, nitrogen has three bonding pairs and one lone pair. The extra lone-pair repulsion pushes the N–H bonds closer together, giving $\angle HNH \approx 107^\circ$. In H_2O, oxygen has two bonding pairs and two lone pairs; the stronger compression lowers $\angle HOH$ to about 104.5°.

Step 2: Include hydrogen sulfide H_2S is bent and has a bond angle of about 92°. Its bonding is predominantly associated with nearly perpendicular sulfur p orbitals, so its angle is much nearer 90° than a tetrahedral angle. Therefore the second-period comparison must not be used to assign H_2S the same angle as water simply because both central atoms have two lone pairs.

Step 3: Arrange the angles and choose The resulting order is

$$\angle HCH\ (CH_4) > \angle HNH\ (NH_3) > \angle HOH\ (H_2O) > \angle HSH\ (H_2S).$$

Thus option (A) has the largest angle. Option (B) is actually the smallest of the four. Options (C) and (D) are both smaller than methane because lone pairs compress their bond directions. The comparison is of angles at the central atom, not bond lengths or the number of hydrogen atoms.

Step 4: Conclusion CH_4 has the largest H–M–H bond angle, approximately $109.5°$. Hence, the correct option is (A).

Quick Tip to Solve: For CH_4, NH_3 and H_2O, count central-atom lone pairs: zero, one and two give progressively smaller bond angles.

61. Boron and silicon of very high purity can be obtained through : [4 marks]

(A) Liquation

(B) Electrolytic refining

(C) Zone refining

(D) Vapour phase refining

Correct Answer: (C) Zone refining

Solution:

Concept:

- Zone refining separates a crystalline solid from its impurities because the impurities are more soluble in the molten material than in the solid phase.
- Semiconductors require exceptionally high chemical purity; silicon and boron can be purified by repeated passage of a narrow molten zone.

Step 1: Identify the required level of purity Boron and silicon are used where even small impurity concentrations alter electrical properties. Ordinary separation of bulk gangue or removal of only a few metallic impurities is insufficient. The method must progressively concentrate trace impurities into a small region while leaving a large fraction of purified solid.

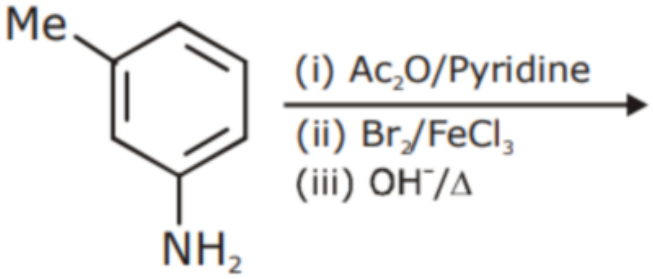
Step 2: Follow the moving molten zone A circular heater produces a narrow molten band in a rod of the impure solid. As the heater moves, material behind it crystallises. Impurities preferentially remain in the liquid and travel with the molten zone towards one end. Repeating the process increases purification; the impurity-rich end is finally cut off.

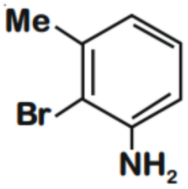
Step 3: Eliminate the other refining methods Liquation exploits a low melting point of a metal relative to its impurities and is not the method for obtaining these ultrapure semiconductors. Electrolytic refining requires a suitable electrochemical dissolution and deposition process, commonly used for metals such as copper. Vapour phase refining relies on formation and decomposition of a volatile compound, as in the Mond process for nickel. The stated pair specifically corresponds to zone refining.

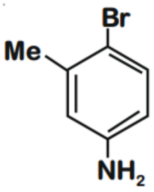
Step 4: Conclusion Very high purity boron and silicon are obtained by zone refining. Hence, the correct option is (C).

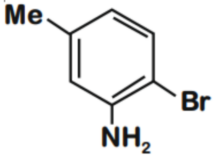
Quick Tip to Solve: Associate ultrapure semiconductor materials with a travelling molten zone: impurities remain preferentially in the melt.

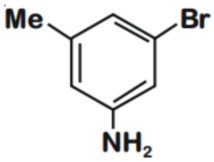
62. The final major product of the following reaction is : [4 marks]

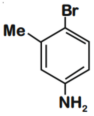


(A) 2-Bromo-3-methylaniline 

(B) 4-Bromo-3-methylaniline 

(C) 2-Bromo-5-methylaniline 

(D) 3-Bromo-5-methylaniline 

Correct Answer: (B) 4-Bromo-3-methylaniline 

Solution:

Concept:

- Acetylation converts the strongly activating amino group, $-NH_2$, into the less activating but still ortho/para-directing acetamido group, $-NHCOCH_3$.
- In electrophilic aromatic bromination the major substitution site reflects both directing effects and steric hindrance. Alkaline hydrolysis subsequently removes the acetyl protecting group.

Step 1: Protect the amino group The starting compound is 3-methylaniline: take the carbon bearing $-NH_2$ as C1 and that bearing the methyl group as C3. Acetic anhydride in pyridine acetylates nitrogen, producing the corresponding 3-methylacetanilide. The methyl group remains on the ring, and the reaction does not introduce a diazonium group.

Step 2: Locate the preferred bromination site The protected amino group directs bromine to C2, C4 and C6. The methyl group at C3 also directs to these same positions. C2 lies between the two existing substituents and is strongly crowded. C4 is para to the acetamido group and ortho to methyl; C6 is ortho to the larger acetamido group and para to methyl. Substitution predominantly occurs at C4, avoiding an ortho approach to the bulky acetamido group.

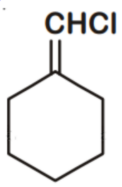
Step 3: Remove the protecting group and compare structures Heating with hydroxide hydrolyses the amide back to $-NH_2$ without shifting the ring substituents. The final product therefore has bromine para to the amino group and ortho to methyl, as shown in (B). Option (A) puts bromine between methyl and amino, option (C) makes bromine ortho to amino at the other site, and (D) makes bromine

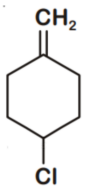
meta to both directing groups; none is the major product.

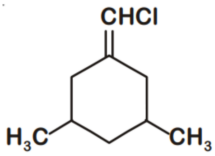
Step 4: Conclusion The major product is 4-bromo-3-methylaniline, with bromine para to $-NH_2$. Hence, the correct option is (B).

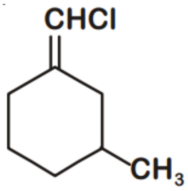
Quick Tip to Solve: For protected anilines, number the amino carbon first, select the least hindered directed site, then hydrolyse the protecting group without moving substituents.

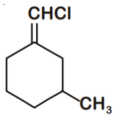
63. Among the following compounds, geometrical isomerism is exhibited by : [4 marks]

(A) Cyclohexane with an exocyclic =CHCl group 

(B) 4-Chlorocyclohexane with an exocyclic $=CH_2$ group 

(C) 3,5-Dimethylcyclohexane with an exocyclic =CHCl group 

(D) 3-Methylcyclohexane with an exocyclic =CHCl group 

Correct Answer: (D) 3-Methylcyclohexane with an exocyclic =CHCl group 

Solution:

Concept:

- Geometrical isomerism requires restricted motion and distinguishable spatial arrangements. For an alkene, each double-bonded carbon must carry two different substituent groups for ordinary E/Z isomerism.
- A substituted ring can also show cis/trans isomerism: two groups may lie on the same face or opposite faces. A chair flip changes axial/equatorial positions but does not interchange cis and trans.

Step 1: Reject the symmetric and terminal cases In (A), the external alkene carbon bears H and Cl, but the two paths around the unsubstituted cyclohexane ring from the other alkene carbon are identical. Thus two different groups are absent on that carbon. In (B), the external group is $=CH_2$, so that alkene carbon has two identical hydrogen atoms. A single chlorine substituent on the ring does not supply a pair of ring substituents capable of cis/trans isomerism.

Step 2: Check the two methyl groups in option C In (C), the two methyl groups occupy two saturated ring carbons. They can be cis, on the same ring face, or trans, on opposite faces. These arrangements cannot interconvert merely by a ring flip. Thus (C) exhibits geometrical isomerism arising from the ring substituents, even though a simple symmetry check on the exocyclic double bond alone would miss this possibility.

Step 3: Compare the unequal ring paths in option D In (D), one saturated ring carbon bears methyl. From the ring carbon of the exocyclic double bond, the two paths encounter that methyl-substituted carbon at different distances, so the paths are distinguishable. The external carbon bears H and Cl. Both alkene carbons consequently have unlike groups, allowing two geometrical arrangements about the exocyclic double bond. Option (D) is therefore a valid selection as well.

Step 4: Conclusion Both (C) and (D) exhibit geometrical isomerism, by the ring cis/trans and exocyclic alkene criteria respectively. The official final key accepts options (C) and (D); selecting (D) is correct.

Quick Tip to Solve: Check ring cis/trans possibilities separately from the double-bond test; a symmetric exocyclic alkene can still belong to a molecule with ring geometrical isomers.

64. Which one of the following polymers is not obtained by condensation polymerisation? [4 marks]

(A) Bakelite

(B) Nylon 6

(C) Buna-N

(D) Nylon 6, 6

Correct Answer: (C) Buna-N

Solution:

Concept:

- Addition polymerisation joins unsaturated monomers through their multiple bonds without eliminating a small molecule. Copolymerisation uses two different monomers.
- Condensation polymers are associated with reactions of functional groups, often accompanied by elimination of water or another small molecule.

Step 1: Identify the monomers of Buna-N Buna-N is the copolymer of 1,3-butadiene and acrylonitrile. Their carbon-carbon double bonds participate in chain growth, producing a rubber containing nitrile side groups. The monomer units add into the polymer backbone; water, ammonia or hydrogen chloride is not eliminated as the chain grows. It is an addition copolymer rather than a condensation polymer.

Step 2: Check the phenol-formaldehyde and diamine-diacid polymers Bakelite is obtained from phenol and formaldehyde through condensation, giving a cross-linked network. Nylon 6,6 is produced from hexamethylenediamine and adipic acid; amide linkages form with elimination of water. Therefore (A) and (D) are examples associated with condensation polymerisation, and cannot answer the negative question.

Step 3: Distinguish nylon 6 from the addition rubber Nylon 6 is obtained from caprolactam by ring opening under the heated, water-containing conditions described in the standard syllabus. Its formation is conventionally grouped with condensation polyamides; the equivalent amino-acid representation is condensation of 6-aminohexanoic acid to amide linkages. Although ring-opening polymerisation is a more specific mechanistic description for caprolactam, it does not make nylon 6 an addition copolymer of unsaturated monomers like Buna-N.

Step 4: Conclusion Buna-N is the addition copolymer among the listed choices and is not obtained by condensation polymerisation. Hence, the correct option is (C).

Quick Tip to Solve: For Buna-N, recall butadiene plus acrylonitrile: two unsaturated monomers add into a rubber chain, unlike the polyamides and phenolic resin.

65. Hydrogen peroxide, in the pure state, is : [4 marks]

(A) Linear and blue in color

(B) Linear and almost colorless

(C) Non-planar and almost colorless

(D) Planar and bluein color

Correct Answer: (C) Non-planar and almost colorless

Solution:

Concept:

- In H_2O_2, each oxygen forms one O–H bond and one O–O bond and retains two lone pairs. Its local electron-pair geometry does not produce a linear molecular chain.
- The stable peroxide molecule has a non-planar, open-book arrangement. The two O–H bonds are not all in the same plane as a flat H–O–O–H chain.

Step 1: Determine the shape about the oxygen atoms Both oxygen atoms have two bond pairs and two lone pairs. Lone-pair repulsions make the H–O–O and O–O–H bond angles bent rather than straight. Rotation about the O–O bond leads to a skew arrangement of the two O–H bonds. Consequently the complete four-atom molecule is non-planar; describing it simply as linear ignores the lone pairs.

Step 2: Relate the structure to the observable colour Pure hydrogen peroxide is a very pale blue, almost colourless liquid. The phrase almost colourless in the options is the appropriate description of this weak colour. It should not be interpreted as an intense blue liquid. The description of its geometry is decisive because only one option specifies a non-planar structure.

Step 3: Eliminate the alternatives Options (A) and (B) are rejected because both call the molecule linear. Changing the colour description cannot correct that structural error. Option (D) is also rejected because it calls the molecule planar, whereas peroxide has the characteristic skew, open-book conformation. Option (C) combines the correct non-planar structure with the stated almost colourless appearance.

Step 4: Conclusion Pure hydrogen peroxide is non-planar and almost colourless (very pale blue). Hence, the correct option is (C).

Quick Tip to Solve: Remember peroxide as an open book about its O–O bond; the formula H–O–O–H written on paper does not imply a planar or linear molecule.

66. An element crystallises in a face-centred cubic (fcc) unit cell with cell edge a. The distance between the centres of two nearest octahedral voids in the crystal lattice is : [4 marks]

(A) a

(B) $\frac{a}{2}$

(C) $\frac{a}{\sqrt{2}}$

(D) $\sqrt{2}a$

Correct Answer: (C) $\frac{a}{\sqrt{2}}$

Solution:

Concept:

- In an fcc unit cell, octahedral void centres occur at the body centre and at the centres of all twelve edges. Coordinates are measured relative to a cube of edge a.
- The distance between two positions is obtained from the Euclidean norm of their coordinate difference. A void-to-void separation must not be confused with an atom-to-void separation.

Step 1: Choose neighbouring octahedral centres Take the body-centred octahedral site at $P = (a/2, a/2, a/2)$. An edge-centred octahedral site is $Q = (a/2, 0, 0)$. Both points are void centres, rather than occupied fcc atom positions. Their coordinate difference is zero along one axis and $a/2$ along each of the other two.

Step 2: Calculate their separation Using the three-dimensional distance formula gives

$$\begin{aligned} PQ &= \sqrt{(a/2 - a/2)^2 + (a/2 - 0)^2 + (a/2 - 0)^2} \\ &= \sqrt{a^2/4 + a^2/4} = \frac{a}{\sqrt{2}}. \end{aligned}$$

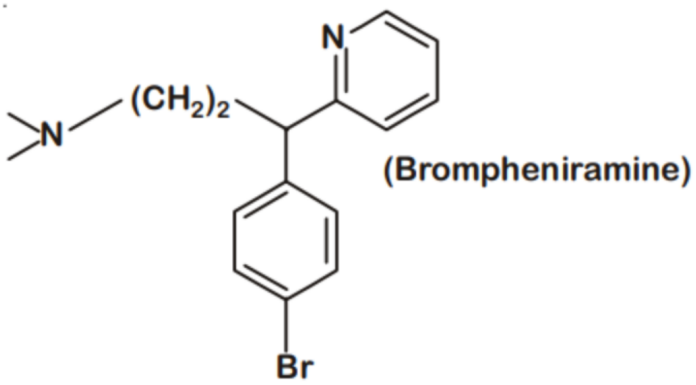
The same separation occurs between two adjacent edge-centred octahedral sites, for example $(a/2, 0, 0)$ and $(0, a/2, 0)$.

Step 3: Verify that the separation is the nearest one The octahedral-site array has coordinate differences in multiples of $a/2$, with the shortest nonzero permitted displacement having two such half-edge components. A single half-edge displacement from an octahedral centre would reach an occupied fcc atom rather than another octahedral void. Hence $a/2$ is not a void-to-void distance. The values a and $\sqrt{2}a$ correspond to larger separations and are not nearest neighbours.

Step 4: Conclusion The nearest octahedral void centres are separated by $a/\sqrt{2}$, with the same unit of length as a. Hence, the correct option is (C).

Quick Tip to Solve: Mark body and edge centres first; subtract their coordinates instead of using the radius formula for an octahedral hole.

67. The following molecule acts as an : [4 marks]



(A) Anti-depressant

(B) Antiseptic

(C) Anti-bacterial

(D) Anti-histamine

Correct Answer: (D) Anti-histamine

Solution:

Concept:

- Drugs are classified by their therapeutic action, so identifying the named molecule is more decisive here than merely noticing that it contains nitrogen or bromine.
- Histamine is involved in allergic responses. Antihistamines oppose its action at histamine receptors and thereby relieve symptoms such as sneezing and a runny nose.

Step 1: Identify the molecule shown The structure is explicitly labelled brompheniramine. Its substituted aromatic groups and tertiary amino side chain belong to this known antihistamine drug. The presence of a bromine substituent does not make the molecule an antiseptic; therapeutic action depends on the whole molecule and its interaction with the biological target.

Step 2: Connect its name with its action Brompheniramine is a first-generation antihistamine that blocks the action of histamine at H1 receptors. Thus it is used to counter allergic symptoms. The question asks for the class of the displayed drug, rather than for a functional group or for the general activity of all compounds containing an amine group.

Step 3: Eliminate the other therapeutic classes Antidepressants act on pathways associated with mood disorders, so option (A) does not describe brompheniramine. Antiseptics inhibit microorganisms on living tissue, whereas antibacterial drugs act against bacteria; neither (B) nor (C) is the therapeutic classification of this molecule. Its characteristic action is against histamine-mediated allergy.

Step 4: Conclusion Brompheniramine acts as an antihistamine. Hence, the correct option is (D).

Quick Tip to Solve: When a drug structure carries its name, first recall its therapeutic class: brompheniramine and chlorpheniramine are familiar antihistamines.

68. Reaction of ammonia with excess Cl_2 gives : [4 marks]

(A) NH_4Cl and N_2

(B) NH_4Cl and HCl

(C) NCl_3 and HCl

(D) NCl_3 and NH_4Cl

Correct Answer: (C) NCl_3 and HCl

Solution:

Concept:

- The products of the reaction between ammonia and chlorine depend on which reactant is in excess. Excess chlorine favours nitrogen trichloride, while excess ammonia gives nitrogen and ammonium chloride.
- In chlorination of ammonia, the three hydrogen atoms can be successively replaced by chlorine atoms, producing hydrogen chloride along with nitrogen trichloride.

Step 1: Apply the stated excess condition Here chlorine, not ammonia, is in excess. There is therefore enough chlorine to replace all three hydrogen atoms attached to nitrogen. The relevant overall reaction is

$$NH_3 + 3\,Cl_2 \longrightarrow NCl_3 + 3\,HCl.$$

This equation conserves one nitrogen atom, three hydrogen atoms and six chlorine atoms on each side. Nitrogen trichloride is the characteristic nitrogen-containing product under this condition.

Step 2: Distinguish the excess-ammonia reaction With excess ammonia the familiar equation is

$$8\,NH_3 + 3\,Cl_2 \longrightarrow N_2 + 6\,NH_4Cl.$$

That equation corresponds to a different reagent ratio and explains why option (A), although it lists products of an ammonia-chlorine reaction, is unsuitable for the condition printed in this question.

Step 3: Compare all product pairs The excess-chlorine equation gives nitrogen trichloride and hydrogen chloride together, which is option (C). Option (B) omits the nitrogen trichloride product. Option (D) replaces hydrogen chloride with ammonium chloride and mixes the product patterns for different excess conditions. Neither pair describes the stated overall reaction.

Step 4: Conclusion Ammonia reacts with excess chlorine to give NCl_3 and HCl. Hence, the correct option is (C).

Quick Tip to Solve: Underline the reactant in excess: excess chlorine gives nitrogen trichloride, whereas excess ammonia gives nitrogen and ammonium chloride.

69. The one that is NOT suitable for the removal of permanent hardness of water is : [4 marks]

(A) Ion-exchange method

(B) Calgon's method

(C) Treatment with sodium carbonate

(D) Clark's method

Correct Answer: (D) Clark's method

Solution:

Concept:

- Permanent hardness is caused mainly by dissolved chlorides and sulphates of calcium and magnesium. Their removal requires precipitation, sequestration or replacement of the hardness-producing ions.
- Temporary hardness arises from bicarbonates. Clark's method adds a calculated amount of slaked lime to remove bicarbonate hardness by forming insoluble precipitates.

Step 1: Identify the scope of Clark's method In Clark's method, lime reacts with bicarbonates. A representative reaction is

$$\mathrm{Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2\,CaCO_3\downarrow + 2\,H_2O.}$$

The calcium carbonate can be removed as a precipitate. This treatment addresses temporary bicarbonate hardness; it is not the method for removing the dissolved calcium and magnesium chlorides or sulphates responsible for permanent hardness.

Step 2: Check the methods that treat permanent hardness Ion-exchange materials replace the calcium and magnesium ions with other ions, thereby removing the cause of hardness. Calgon, sodium hexametaphosphate, sequesters the hardness-producing cations in soluble complexes. Sodium carbonate precipitates them as insoluble carbonates; for example,

$$\mathrm{CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3\downarrow + 2\,NaCl.}$$

Each of these methods can therefore address permanent hardness.

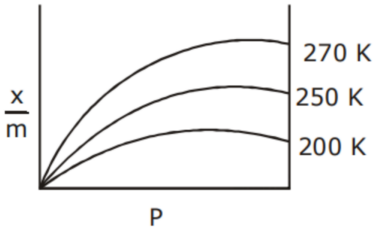
Step 3: Apply the negative wording The question asks which method is NOT suitable. Options (A), (B) and (C) are suitable because they remove or bind the calcium and magnesium ions without requiring those ions to be present as bicarbonates. Clark's method in option (D) specifically targets bicarbonates, so it is the required exception.

Step 4: Conclusion Clark's method is not suitable for removal of permanent hardness. Hence, the correct option is (D).

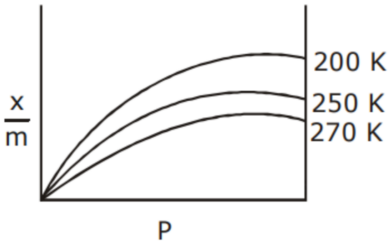
Quick Tip to Solve: Link Clark's lime treatment with bicarbonates and temporary hardness; use carbonate, Calgon or ion exchange for permanent hardness.

70. Adsorption of a gas follows Freundlich adsorption isotherm. If x is the mass of the gas adsorbed on mass m of the adsorbent, the correct plot of $\frac{x}{m}$ versus p is : [4 marks]

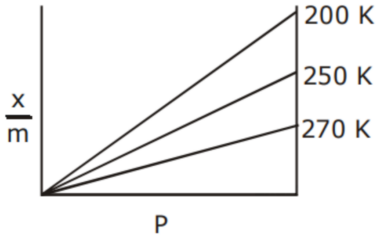
(A)



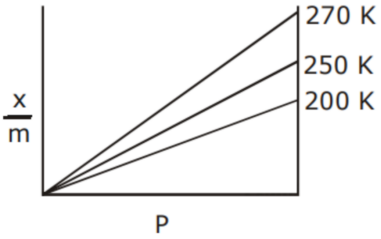
(B)



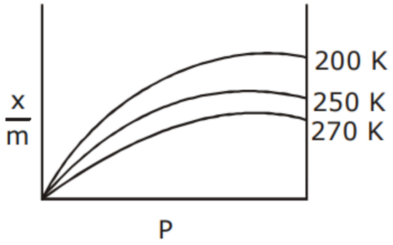
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- At a fixed temperature the Freundlich relation is $x/m = kp^{1/n}$, where k and n are constants for the gas-adsorbent system and $0 < 1/n < 1$ in the usual applicable pressure range.

- Physical adsorption is exothermic. At the same pressure, an increase in temperature reduces the amount adsorbed, so lower-temperature isotherms lie above higher-temperature ones.

Step 1: Determine the shape of each isotherm Put $a = 1/n$, with $0 < a < 1$. The relation becomes $x/m = kp^a$. Its first derivative is positive, while its second derivative is negative:

$$\frac{d(x/m)}{dp} = kap^{a-1} > 0, \qquad \frac{d^2(x/m)}{dp^2} = ka(a-1)p^{a-2} < 0.$$

Hence adsorption increases with pressure, but at a decreasing rate. The required curves bend downwards rather than remaining straight lines of constant slope. A logarithmic plot would be straight, but the printed axes are not logarithmic.

Step 2: Place the temperatures in order Adsorption releases heat. Raising the temperature opposes adsorption and favours desorption, so at any chosen pressure in the range shown,

$$(x/m)_{200\,\mathrm{K}} > (x/m)_{250\,\mathrm{K}} > (x/m)_{270\,\mathrm{K}}.$$

Thus the top curve must carry 200 K, the middle curve 250 K and the bottom curve 270 K. Option (B) has this order and the appropriate curved shape.

Step 3: Exclude the remaining graphs Option (A) has the curved shape but reverses the temperature dependence. Option (C) has the correct temperature order but shows straight lines for the printed linear axes. Option (D) has both the wrong temperature order and the wrong straight-line shape for the stated Freundlich regime.

Step 4: Conclusion The concave-down curves with 200 K highest and 270 K lowest give the required plot. Hence, the correct option is (B).

Quick Tip to Solve: Check both features before choosing: a Freundlich plot on linear axes is curved, and exothermic adsorption gives more adsorption at lower temperature.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The volume, in mL, of 0.02 M $K_2Cr_2O_7$ solution required to react with 0.288 g of ferrous oxalate in acidic medium is _____.

(Molar mass of Fe = 56 g mol^{-1}) [4 marks]

Solution:

Concept:

- In acidic medium dichromate accepts six electrons per mole: $\mathrm{Cr_2O_7^{2-}} + 14\,\mathrm{H^+} + 6\,\mathrm{e^-} \longrightarrow 2\,\mathrm{Cr^{3+}} + 7\,\mathrm{H_2O}$.
- Ferrous oxalate, $\mathrm{FeC_2O_4}$, contains two reducing components. Iron(II) loses one electron on oxidation to iron(III), and one oxalate ion loses two electrons on oxidation to carbon dioxide.

Step 1: Find the amount of ferrous oxalate Use the formula $\mathrm{FeC_2O_4}$ for the ferrous oxalate named in

the question. Its molar mass is

$$M = 56 + 2(12) + 4(16) = 144\,\text{g mol}^{-1}.$$

Therefore the sample contains

$$n(\text{FeC}_2\text{O}_4) = \frac{0.288}{144} = 0.002\,\text{mol}.$$

No waters of crystallisation are included because the question does not specify a hydrated formula.

Step 2: Count all electrons released Both the cation and anion of the salt are oxidised by acidified dichromate:

$$\text{Fe}^{2+} \longrightarrow \text{Fe}^{3+} + \text{e}^-, \qquad \text{C}_2\text{O}_4{}^{2-} \longrightarrow 2\,\text{CO}_2 + 2\,\text{e}^-.$$

Thus one mole of ferrous oxalate releases $1 + 2 = 3$ moles of electrons. The sample releases

$$n(e^-) = 3(0.002) = 0.006\,\text{mol}.$$

Counting only the iron oxidation would underestimate the required oxidant, since it would omit the reducing power of oxalate.

Step 3: Convert the oxidant amount to volume Each mole of dichromate accepts six moles of electrons, so electron balance requires

$$n(\text{K}_2\text{Cr}_2\text{O}_7) = \frac{0.006}{6} = 0.001\,\text{mol}.$$

For a solution of molarity $0.02\,\text{mol L}^{-1}$,

$$V = \frac{n}{c} = \frac{0.001}{0.02} = 0.050\,\text{L} = 50\,\text{mL}.$$

The electron balance also shows that two moles of ferrous oxalate react with one mole of dichromate.

Step 4: Conclusion The required volume of the $0.02\,\text{M}$ potassium dichromate solution is $\boxed{50\,\text{mL}}$. The numerical entry is 50.

Quick Tip to Solve: For ferrous oxalate, count three electrons per formula unit: one from iron(II) and two from oxalate. Dichromate accepts six in acidic medium.

72. Considering that $\Delta_0 > P$, the magnetic moment (in BM) of $[Ru(H_2O)_6]^{2+}$ would be _____. [4 marks]

Solution:

Concept:

- In an octahedral complex, the five metal d orbitals split into a lower t_{2g} set of three orbitals and an upper e_g set of two orbitals, separated by Δ_o.
- When $\Delta_o > P$, where P is the electron-pairing energy, pairing in the lower set is energetically preferable to occupation of the upper set. The complex is low spin.

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{BM}$, where n is the number of unpaired electrons, not the total number of d electrons.

Step 1: Determine the oxidation state and d count Water is a neutral ligand. Therefore the charge of $[\mathrm{Ru(H_2O)_6}]^{2+}$ places ruthenium in the +2 oxidation state. Ruthenium is a group 8 transition metal, so its +2 ion has

$$d\text{-electron count} = 8 - 2 = 6.$$

Thus the metal centre is a $4d^6$ ion. The electron count must be found for the ion in the complex rather than for the isolated neutral atom.

Step 2: Fill the octahedral levels using the given inequality Because $\Delta_o > P$, use the low-spin arrangement. The six electrons first occupy and then pair within the three lower-energy orbitals, giving

$$t_{2g}^6 e_g^0.$$

Each of the three t_{2g} orbitals contains an electron pair; the two e_g orbitals remain empty. Consequently all six d electrons are paired and the unpaired-electron count is $n = 0$. The stated inequality controls the filling, even though water is often discussed as a weak-field ligand in elementary examples involving 3d metals.

Step 3: Evaluate the magnetic moment Substituting the unpaired-electron count into the magnetic-moment expression gives

$$\mu = \sqrt{0(0+2)} = 0\,\mathrm{BM}.$$

Using six as n would confuse the total d-electron population with the number of unpaired electrons. The completely filled lower set has no net spin contribution and is diamagnetic.

Step 4: Conclusion The low-spin d^6 complex is diamagnetic and has magnetic moment $\boxed{0\,\mathrm{BM}}$. The numerical entry is 0.

Quick Tip to Solve: For an octahedral low-spin d-six ion, the lower three orbitals are fully paired: $t_{2g}^6 e_g^0$, so the magnetic moment is zero.

73. The number of chiral carbons present in sucrose is _____. [4 marks]

Solution:

Concept:

- A chiral carbon is a tetrahedral carbon attached to four different groups. In a sugar ring, the two paths around the ring must be compared as complete groups, rather than treating them as identical merely because both begin with carbon.
- Sucrose is formed from an α-D-glucopyranose unit and a β-D-fructofuranose unit. Their anomeric carbons, glucose C1 and fructose C2, are joined through the glycosidic oxygen.

Step 1: Count the centres in the glucose unit Number the glucose carbons from the anomeric carbon C1. In the cyclic glucose unit, C1, C2, C3, C4 and C5 each have four different substituent groups and are stereogenic. C1 is included because its hydrogen, ring oxygen, glycosidic oxygen substituent

and C2-containing chain are different groups. The terminal C6 carbon is a CH_2OH group, with two identical hydrogen atoms, so it is not chiral. Thus the glucose unit contributes five chiral carbons.

Step 2: Count the centres in the fructose unit In the cyclic fructose unit, C2, C3, C4 and C5 are stereogenic. In particular, anomeric C2 has a ring-oxygen group, a glycosidic-oxygen group and two different carbon-containing groups, so the absence of a hydrogen on it does not prevent chirality. Fructose C1 and C6 are both terminal CH_2OH carbons and therefore are not stereogenic. This unit contributes four chiral carbons.

Step 3: Add the two contributions The glycosidic linkage fixes the configurations of both anomeric centres; it does not turn either into a planar carbon or make two substituents identical. Consequently both counts remain valid in the disaccharide:

$$N_{\text{chiral carbon}} = 5 + 4 = 9.$$

Step 4: Conclusion Sucrose contains $\boxed{9}$ chiral carbon atoms. The required numerical entry is 9; a count of atoms has no physical unit.

Quick Tip to Solve: Count the cyclic units separately: glucose contributes five centres and fructose four. Include both anomeric carbons, and exclude every terminal CH_2OH carbon.

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74. For a reaction X + Y $\rightleftharpoons$ 2Z , 1.0 mol of X, 1.5 mol of Y and 0.5 mol of Z were taken in a 1 L vessel and allowed to react. At equilibrium, the concentration of Z was 1.0 mol L^{-1}. The equilibrium constant of reaction is $\frac{x}{15}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For $X + Y \rightleftharpoons 2\,Z$, the concentration equilibrium constant is $K_c = [Z]^2/([X][Y])$. The coefficients in the balanced reaction become powers in this expression.
- The reaction consumes equal amounts of X and Y, while forming twice that amount of Z. Initial concentrations can be converted directly from moles because the vessel volume is $1\,L$.

Step 1: Write the initial concentrations Divide each initial amount by the vessel volume. All concentrations below are in $mol\,L^{-1}$:

$$[X]_0 = 1.00, \qquad [Y]_0 = 1.50, \qquad [Z]_0 = 0.50.$$

The final Z concentration exceeds its initial value, so the net change is in the forward direction. It is essential to use the increase in Z, rather than its whole final concentration, to find how much reaction has occurred.

Step 2: Determine the equilibrium reactant concentrations Let a be the concentration of X consumed.

Stoichiometry gives changes $-a, -a, +2a$ for X, Y and Z respectively. Therefore

$$0.50 + 2a = 1.00 \quad \Rightarrow \quad a = 0.25\,\text{mol}\,\text{L}^{-1}.$$

The equilibrium concentrations of the reactants are consequently

$$[\text{X}] = 1.00 - 0.25 = 0.75, \qquad [\text{Y}] = 1.50 - 0.25 = 1.25.$$

Both remain positive, as required by the stated equilibrium mixture.

Step 3: Substitute into the equilibrium expression Use the equilibrium concentrations of all three substances:

$$K_c = \frac{(1.00)^2}{(0.75)(1.25)} = \frac{1}{(3/4)(5/4)} = \frac{16}{15}.$$

Comparison with the specified form $K_c = x/15$ immediately gives $x = 16$. The concentration powers cancel here because the total stoichiometric coefficients are equal on both sides.

Step 4: Conclusion The equilibrium constant is $K_c = 16/15$, so $\boxed{x = 16}$. The requested numerical entry is the dimensionless number 16.

Quick Tip to Solve: Z increases by 0.50, but its coefficient is two: subtract only 0.25 from each reactant before calculating K_c.

75. For a dimerization reaction,

2A(g) $\rightarrow$ A_2(g)

at 298 K, ΔU^o = –20 kJ mol^{-1}, ΔS^o = –30 JK^{-1} mol^{-1},

then the ΔG^o will be _____ J. [4 marks]

Solution:

Concept:

- For an ideal-gas reaction at a specified temperature, standard enthalpy and internal energy changes satisfy $\Delta H^\circ = \Delta U^\circ + \Delta n_g RT$, where Δn_g is the total gaseous product coefficient minus the total gaseous reactant coefficient.
- Standard Gibbs energy is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. All energy terms must use consistent units before substitution; here joules are convenient because the answer is requested in J.

Step 1: Calculate the change in gaseous amount The balanced reaction is $2\,\text{A}\,(\text{g}) \longrightarrow \text{A}_2(\text{g})$. There is one mole of gaseous product for every two moles of gaseous reactant in the reaction as written, so

$$\Delta n_g = 1 - 2 = -1.$$

The standard internal energy change is $-20\,\text{kJ}\,\text{mol}^{-1} = -20000\,\text{J}\,\text{mol}^{-1}$. Do not divide this value by two: it already refers to the reaction as written.

Step 2: Convert internal energy into enthalpy Substitute $T = 298\,\mathrm{K}$ and $R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$:

$$\begin{aligned}\Delta H^\circ &= -20000 + (-1)(8.314)(298)\\ &= -20000 - 2477.572\\ &= -22477.572\,\mathrm{J\,mol^{-1}}.\end{aligned}$$

The negative Δn_g makes the enthalpy change more negative than the internal energy change.

Step 3: Apply the entropy correction Since $\Delta S^\circ = -30\,\mathrm{J\,K^{-1}\,mol^{-1}}$, its temperature product is

$$T\Delta S^\circ = 298(-30) = -8940\,\mathrm{J\,mol^{-1}}.$$

Hence

$$\begin{aligned}\Delta G^\circ &= -22477.572 - (-8940)\\ &= -13537.572\,\mathrm{J\,mol^{-1}}.\end{aligned}$$

The entropy decreases in dimerization, so the entropy contribution makes the Gibbs energy less negative than the enthalpy. This also checks the sign of the last subtraction.

Step 4: Conclusion For the reaction as written, the required Gibbs energy change is approximately $\boxed{-13538\,\mathrm{J}}$ (equivalently $-13.538\,\mathrm{kJ\,mol^{-1}}$ as a molar reaction quantity). The numerical entry is -13538.

Quick Tip to Solve: When ΔU° is supplied, first add $\Delta n_g RT$, then subtract $T\Delta S^\circ$. Dimerization has $\Delta n_g = -1$.