JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 6 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 September 2020, Shift 2. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The integral $\int_1^2 e^x.x^x\,(2+\log_e x)dx$ equals : [4 marks]

(A) e(4e + 1)

(B) e(2e – 1)

(C) e(4e – 1)

(D) $4e^2 - 1$

Correct Answer: (C) e(4e – 1)

Solution:

Concept:

- The fundamental theorem of calculus gives $\int_a^b F\prime(x)\,dx = F(b)-F(a)$. Identify the whole integrand as a derivative before attempting integration by parts.
- For $x > 0$, logarithmic differentiation of $x^x = e^{x\ln x}$ gives $\frac{d}{dx}x^x = x^x(1+\ln x)$. The interval $[1, 2]$ lies in this domain.

Step 1: Differentiate the candidate product Take $F(x) = e^x x^x$. Both factors depend on x, so use the product rule rather than treating the exponent of x as a constant:

$$\begin{aligned} F\prime(x) &= e^x x^x + e^x x^x(1+\ln x) \\ &= e^x x^x(2+\ln x). \end{aligned}$$

The first copy of 1 in the bracket comes from differentiating e^x; the second comes from differentiating $x\ln x$. Thus this is exactly the printed integrand.

Step 2: Evaluate the endpoints and compare the options No further substitution is required. Substitute the upper limit first and subtract the lower limit:

$$\begin{aligned} I &= [e^x x^x]_1^2 \\ &= e^2\,2^2 - e\,1^1 \\ &= 4e^2 - e = e(4e-1). \end{aligned}$$

Option (A) adds the lower-limit value instead of subtracting it. Option (B) uses an incorrect upper-limit factor 2 instead of $2^2 = 4$. Option (D) subtracts 1, although the lower endpoint is e. Only option (C) agrees with both endpoint values.

Step 3: Conclusion The integral equals $\boxed{e(4e-1)}$. Hence, the correct option is (C).

Quick Tip to Solve: For products involving $e^x x^x$, differentiate the product mentally: its logarithmic derivative is $2+\ln x$.

2. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to : [4 marks]

(A) $\frac{8}{3}$

(B) $\frac{4}{3}$

(C) $\frac{7}{2}$

(D) $\frac{16}{3}$

Correct Answer: (A) $\frac{8}{3}$

Solution:

Concept:

- The enclosed area between two graphs is $A = \int_a^b (y_{\text{upper}} - y_{\text{lower}})\,dx$, with intersection coordinates providing the integration limits.
- When the difference is even, symmetry about the y-axis allows $\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$. Area is nonnegative and is measured in square units.

Step 1: Find the intersections and identify the upper curve Equate the two ordinates:

$$x^2 - 1 = 1 - x^2 \quad \Longrightarrow \quad 2x^2 = 2 \quad \Longrightarrow \quad x = \pm 1.$$

Both intersections have $y = 0$. For $-1 < x < 1$, the difference between the second graph and the first is

$$(1 - x^2) - (x^2 - 1) = 2 - 2x^2 > 0.$$

Thus $y = 1 - x^2$ is the upper boundary throughout the enclosed interval. Integrating either curve alone would measure area from the axis, rather than the full area between both boundaries.

Step 2: Integrate the vertical separation Use the full interval, or double its right-hand half by symmetry:

$$\begin{aligned} A &= \int_{-1}^{1} (2 - 2x^2)\,dx \\ &= 2\int_0^1 (2 - 2x^2)\,dx \\ &= 2\left[2x - \frac{2x^3}{3}\right]_0^1 \\ &= 2\left(2 - \frac{2}{3}\right) = \frac{8}{3}. \end{aligned}$$

Option (B), $4/3$, is only half of the enclosed area and results from omitting the symmetry factor. Option (D), $16/3$, doubles the entire region once more. The remaining value $7/2$ does not equal the integral of the actual separation. The computed positive area therefore selects (A).

Step 3: Conclusion The enclosed area is $\boxed{\frac{8}{3}}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: Find intersections before integrating; for these symmetric parabolas, integrate the vertical gap on $[0, 1]$ and double once.

3. The set of all real values of λ for which the function

$$f(x) = \left(1 - \cos^2 x\right)(\lambda + \sin x), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

has exactly one maxima and exactly one minima, is : [4 marks]

(A) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$
(B) $\left(-\frac{3}{2}, \frac{3}{2}\right)$
(C) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$
(D) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Correct Answer: (A) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

Solution:

Concept:

- On $(-\pi/2, \pi/2)$, the substitution $t = \sin x$ is strictly increasing and maps the domain onto $(-1, 1)$. It preserves the type and number of local extrema.
- Write $1 - \cos^2 x = \sin^2 x$. For a differentiable function an interior extremum requires a stationary point; a sign change of the first derivative establishes a strict maximum or minimum.

Step 1: Reduce the function to a cubic In terms of t, the given function becomes

$$g(t) = t^2(\lambda + t) = \lambda t^2 + t^3, \qquad -1 < t < 1.$$

Since $\cos x > 0$ in the original interval, $f\prime(x) = g\prime(t)\cos x$ has the same sign as $g\prime(t)$. Differentiate and factor:

$$g\prime(t) = 2\lambda t + 3t^2 = t(2\lambda + 3t).$$

The candidates are $t = 0$ and $t = -2\lambda/3$. To obtain two distinct interior stationary points, we need $\lambda \neq 0$ and $-1 < -2\lambda/3 < 1$, or $|\lambda| < 3/2$.

Step 2: Classify both candidates and check the exclusions The second derivative is $g\prime\prime(t) = 2\lambda + 6t$. At the two candidates it gives

$$g\prime\prime(0) = 2\lambda, \qquad g\prime\prime(-2\lambda/3) = -2\lambda.$$

For any nonzero λ these have opposite signs, so precisely one candidate is a maximum and the other is a minimum. There are no further roots of $g\prime$, hence no other interior extrema. At $\lambda = 0$, $g(t) = t^3$ is increasing and its stationary point is not an extremum. At $\lambda = \pm 3/2$, the second candidate is an excluded endpoint. Values outside that interval give only one interior stationary point. Options (B) and (D) wrongly admit zero, while (C) unnecessarily excludes valid values such as $\lambda = 1$.

Step 3: Conclusion The required set is $\boxed{(-3/2, 3/2) \setminus \{0\}}$. Hence, the correct option is (A).

Quick Tip to Solve: A monotone substitution preserves extrema: replace $\sin x$ by t, then count distinct derivative roots inside $(-1, 1)$.

4. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is : [4 marks]

(A) $\left(\frac{11}{5}, \frac{28}{5}\right)$
(B) $\left(\frac{29}{5}, \frac{11}{5}\right)$
(C) $\left(\frac{29}{5}, \frac{8}{5}\right)$
(D) $\left(\frac{8}{5}, \frac{29}{5}\right)$

Correct Answer: (A) $\left(\frac{11}{5}, \frac{28}{5}\right)$

Solution:

Concept:

- A line with nonzero intercepts p, q has equation $x/p+y/q=1$. Thus the stated intercepts determine the reflecting line uniquely.
- For $ax+by+c=0$, the reflection of (x_0, y_0) is $(x_0-2ad,\ y_0-2bd)$, where $d=(ax_0+by_0+c)/(a^2+b^2)$. The line bisects the segment joining a point to its image at right angles.

Step 1: Form the line and evaluate the signed factor The intercept form gives

$$\frac{x}{3}+y=1 \quad \Longrightarrow \quad x+3y-3=0.$$

Therefore $a=1$, $b=3$, $c=-3$. For the original point $P=(-1,-4)$,

$$d=\frac{1(-1)+3(-4)-3}{1^2+3^2}=\frac{-16}{10}=-\frac{8}{5}.$$

Keep the negative sign: the displacement toward and beyond the line must be opposite to this signed normal factor. Replacing it by an unsigned distance without tracking direction would send the point to the wrong side.

Step 2: Calculate and verify the image Apply the reflection formula to both coordinates:

$$x\prime=-1-2(1)(-8/5)=\frac{11}{5},$$
$$y\prime=-4-2(3)(-8/5)=\frac{28}{5}.$$

The midpoint of P and $P\prime$ is $(3/5, 4/5)$, and it satisfies $x+3y-3=0$. Also,

$$P\prime-P=(16/5, 48/5)=\frac{16}{5}(1,3),$$

which is parallel to the line's normal. These two checks establish the reflection. For options (B), (C) and (D), evaluate the midpoint test: the respective midpoints give $x+3y-3=-33/10, -21/5, 0$, so (B) and (C) fail. In (D) the displacement is not parallel to $(1,3)$. Thus even a midpoint on the line alone is insufficient for (D).

Step 3: Conclusion The image is $\boxed{(11/5, 28/5)}$. Hence, the correct option is (A).

Quick Tip to Solve: Check any reflected point by verifying both that its midpoint lies on the mirror line and that the joining segment follows the normal.

5. For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, (fof)(x) = x. Then $f\left(-\frac{1}{2}\right)$ is equal to : [4 marks]

(A) $\frac{1}{3}$

(B) –3

(C) $-\frac{1}{3}$

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- The condition $(f \circ f)(x) = x$ means that composing the same function twice returns the input wherever both applications are defined.
- After substituting a rational function into itself, simplify the numerator and denominator separately. An equality valid for every admissible real input yields a polynomial identity in x.

Step 1: Compute the second composition For admissible x, substitute $f(x) = (a-x)/(a+x)$ into the definition again and multiply numerator and denominator by $a + x$:

$$f(f(x)) = \frac{a(a+x) - (a-x)}{a(a+x) + (a-x)} = \frac{a(a-1) + (a+1)x}{a(a+1) + (a-1)x}.$$

The denominator is nonzero because the question explicitly requires $f(x) \neq -a$. Therefore the condition can be cross-multiplied without introducing an admissible zero denominator.

Step 2: Determine the parameter and substitute the input Setting the last fraction equal to x and collecting terms gives

$$(a-1)\left[a - (a+1)x - x^2\right] = 0.$$

The bracket has coefficient -1 for x^2, so it cannot vanish identically for all admissible inputs. Hence $a = 1$. For this value the formula indeed gives $f(f(x)) = x$ wherever defined. The degenerate value $a = 0$ does not satisfy the identity: it gives $f(x) = -1$, and a second application again gives -1, rather than every input. Now evaluate

$$f(-1/2) = \frac{1 + 1/2}{1 - 1/2} = \frac{3/2}{1/2} = 3.$$

Both applications are permitted at this input. Option (A) is the reciprocal of the required quotient, while (B) and (C) introduce an incorrect negative sign. Only (D) equals the direct substitution.

Step 3: Conclusion We obtain $a = 1$ and $\boxed{f(-1/2) = 3}$. Hence, the correct option is (D).

Quick Tip to Solve: For an involution, simplify $f(f(x))$ first and compare polynomial coefficients; do not guess the parameter from a single input.

6. For all twice differentiable functions f : R → R,
with f(0) = f(1) = f'(0) = 0 [4 marks]

(A) f"(x) $\neq$ 0, at every point x $\in$ (0, 1)

(B) f"(x) = 0, for some x $\in$ (0, 1)

(C) f"(0) = 0

(D) f"(x) = 0, at every point x $\in$ (0, 1)

Correct Answer: (B) f"(x) = 0, for some x $\in$ (0, 1)

Solution:

Concept:

- Rolle's theorem states that a function continuous on a closed interval, differentiable inside it, and equal at the endpoints has a zero derivative at an interior point.
- Twice differentiability of f on $\mathbb{R}$ ensures that f is differentiable and continuous, while $f\prime$ is itself differentiable and continuous. Thus Rolle's theorem can be applied successively.

Step 1: Apply Rolle to the original function The function is continuous on $[0,1]$, differentiable on $(0,1)$, and satisfies $f(0) = f(1) = 0$. Hence there is a point $c \in (0,1)$ such that

$$f\prime(c) = 0.$$

This is an existence statement about the first derivative. It does not assert that the derivative is zero at every point, nor does it yet say anything about the second derivative. The additional condition $f\prime(0) = 0$ supplies a second zero for the derivative.

Step 2: Apply Rolle to the derivative and reject stronger claims On $[0,c]$, the function $f\prime$ is continuous and is differentiable on $(0,c)$. Since $f\prime(0) = f\prime(c) = 0$, Rolle's theorem gives some $d \in (0,c)$ with

$$f\prime\prime(d) = 0.$$

Because $0 < d < c < 1$, this proves the interior-zero assertion (B) and contradicts (A). To see why neither (C) nor (D) is forced, consider the admissible polynomial

$$f(x) = x^2(x-1) = x^3 - x^2.$$

It has $f(0) = f(1) = f\prime(0) = 0$, but $f\prime\prime(x) = 6x - 2$. Thus $f\prime\prime(0) = -2$, rejecting (C), and the second derivative is not identically zero on $(0,1)$, rejecting (D). It does vanish at $x = 1/3$, in agreement with the theorem.

Step 3: Conclusion For every function satisfying the hypotheses, $\boxed{f\prime\prime(\xi) = 0 \text{ for some } \xi \in (0,1)}$. Hence, the correct option is (B).

Quick Tip to Solve: Equal endpoint values give one interior zero of $f\prime$; pair it with the given zero at 0 and apply Rolle a second time.

7. Let f : R → R be a function defined by
f(x) = max {x, x^2}. Let S denote the set of all points in R, where f is not differentiable. Then : [4 marks]

(A) {0, 1}

(B) {0}

(C) ϕ(an empty set)

(D) {1}

Correct Answer: (A) {0, 1}

Solution:

Concept:

- The maximum selects the larger of the two expressions. Determine the active expression by the sign of $x^2 - x = x(x-1)$.
- A function formed from differentiable pieces can fail to be differentiable where its active expression changes. At such a point compare the left-hand and right-hand derivatives.

Step 1: Locate the changes of expression The two expressions agree when $x = x^2$, so the only possible junctions are $x = 0$ and $x = 1$. For $x < 0$, both factors of $x(x-1)$ are negative, so their product is positive and $x^2 > x$. For $0 < x < 1$, the product is negative, so $x > x^2$. For $x > 1$, it is positive again. Thus

$$f(x) = \begin{cases} x^2, & x \le 0, \\ x, & 0 \le x \le 1, \\ x^2, & x \ge 1. \end{cases}$$

The formulas agree at the endpoints, so the function is continuous there. Continuity alone does not settle differentiability.

Step 2: Compare the one-sided slopes At zero, the derivative of the left piece is $2x$, which tends to zero, whereas the derivative of the right piece is one. Hence

$$f'_-(0) = 0 \neq 1 = f'_+(0).$$

At one, the slope of the left piece is one and that of the right piece is two:

$$f'_-(1) = 1 \neq 2 = f'_+(1).$$

Both points therefore fail the derivative test. At every other real number the function agrees locally with a single polynomial, so it is differentiable. Options (B) and (D) each omit one genuine corner, while option (C) incorrectly asserts there are no corners.

Step 3: Conclusion The complete set of points of non-differentiability is $S = \{0, 1\}$. Hence, the correct option is (A).

Quick Tip to Solve: For a maximum of two smooth expressions, first solve their equality, then test the one-sided slopes only at those switching points.

8. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

If B = A + A^4 , then det (B) : [4 marks]

(A) lies in (1, 2)

(B) lies in (2, 3).

(C) is zero.

(D) is one.

Correct Answer: (A) lies in (1, 2)

Solution:

Concept:

- Write $R(t) = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}$. Matrix multiplication and the angle-addition formulas give $R(s)R(t) = R(s+t)$.
- Consequently $A^n = R(n\theta)$. A matrix of the form $\begin{pmatrix} u & v \\ -v & u \end{pmatrix}$ has determinant $u^2 + v^2$.

Step 1: Express the entries of the sum Here $A = R(\theta)$ and $A^4 = R(4\theta)$, so

$$B = \begin{pmatrix} \cos\theta + \cos 4\theta & \sin\theta + \sin 4\theta \\ -\sin\theta - \sin 4\theta & \cos\theta + \cos 4\theta \end{pmatrix}.$$

The determinant must be taken after adding the matrices. In particular, the determinant of a sum is not generally the sum of the determinants, so simply adding $\det A$ and $\det A^4$ would give an unjustified answer.

Step 2: Simplify and locate the determinant Using the displayed form and then the cosine difference identity,

$$\begin{aligned} \det B &= (\cos\theta + \cos 4\theta)^2 + (\sin\theta + \sin 4\theta)^2 \\ &= 2 + 2(\cos\theta\cos 4\theta + \sin\theta\sin 4\theta) \\ &= 2 + 2\cos 3\theta. \end{aligned}$$

As $\theta = \pi/5$, we have $3\theta = 3\pi/5$, an angle between $\pi/2$ and $2\pi/3$. Cosine is strictly decreasing on this interval, hence

$$-\tfrac{1}{2} < \cos(3\pi/5) < 0.$$

It follows that $1 < \det B < 2$. Equivalently its exact value is $(5 - \sqrt{5})/2$. The value is positive and strictly above one, excluding zero and one, and it is below two, excluding the interval in option (B).

Step 3: Conclusion $\det B = (5 - \sqrt{5})/2$ lies in $(1, 2)$. Hence, the correct option is (A).

Quick Tip to Solve: For a sum of two planar rotation matrices, square and add its cosine-sum and sine-sum entries; the angle difference gives the determinant immediately.

9. The common difference of the A.P.
$b_1, b_2, \ldots, b_m$ is 2 more than the common
difference of A.P. $a_1, a_2, \ldots, a_n$. If
$a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to : [4 marks]

(A) 127
(B) 81
(C) –127
(D) -81

Correct Answer: (D) -81

Solution:

Concept:

- For an arithmetic progression with first term a_1 and common difference d, the nth term is $a_n = a_1 + (n-1)d$.
- Subtracting two terms eliminates the first term: $a_r - a_s = (r-s)d$. The second progression has common difference $d_b = d_a + 2$, retaining its algebraic sign.

Step 1: Determine the first progression The supplied terms give

$$\begin{aligned} a_{100} - a_{40} &= 60d_a, \\ -399 - (-159) &= 60d_a, \\ d_a &= -4. \end{aligned}$$

The sequence is decreasing, so its common difference is negative. Now use the fortieth term to find its first term:

$$a_1 = -159 - 39(-4) = -3.$$

Therefore

$$a_{70} = a_1 + 69d_a = -3 + 69(-4) = -279.$$

As a check, $a_1 + 99d_a = -3 - 396 = -399$, reproducing the other given term.

Step 2: Work backwards through the second progression Two more than -4 is -2, so $d_b = -2$, not -6 or $+2$. The condition $b_{100} = a_{70}$ gives

$$b_{100} = -279 = b_1 + 99(-2).$$

Solving this equation yields

$$b_1 = -279 + 198 = -81.$$

The index factor is 99 because the hundredth term lies 99 equal increments beyond the first. Substitution confirms $-81 + 99(-2) = -279$. A positive starting value such as 81 or 127 would not produce the required hundredth term. The alternative -127 gives -325 as the hundredth term and also fails the condition. Thus the signs and indexing select a unique printed option.

Step 3: Conclusion The first term of the second arithmetic progression is $b_1 = -81$. Hence, the correct option is (D).

Quick Tip to Solve: Find a common difference by subtracting two distant terms, and remember that recovering the first term from the hundredth uses 99 increments.

10. If the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then |k| equals : [4 marks]

(A) 3

(B) 9

(C) 1

(D) 2

Correct Answer: (A) 3

Solution:

Concept:

- The general term in $(a+b)^{10}$, choosing the second term r times, is $T_{r+1}=\binom{10}{r}a^{10-r}b^r$, where $0\le r\le 10$.
- A constant term has net power zero in x. Keep the coefficient, its sign and its power of k separate from that exponent condition.

Step 1: Identify which term is independent of x Take $a=\sqrt{x}=x^{1/2}$ and $b=-kx^{-2}$. Then

$$\begin{aligned} T_{r+1} &= \binom{10}{r}(x^{1/2})^{10-r}(-kx^{-2})^r \\ &= \binom{10}{r}(-k)^r x^{(10-r)/2-2r} \\ &= \binom{10}{r}(-k)^r x^{5-5r/2}. \end{aligned}$$

The required exponent vanishes only when

$$5-\frac{5r}{2}=0, \qquad r=2.$$

This is an allowed integer between zero and ten, so there is exactly one constant term. It is the third term, because the general term is indexed by $r+1$.

Step 2: Equate its coefficient to the given value For $r=2$, the negative sign disappears upon squaring, and the coefficient becomes

$$\binom{10}{2}(-k)^2=\frac{10\cdot 9}{2}k^2=45k^2.$$

By the condition in the question,

$$45k^2=405, \qquad k^2=9, \qquad k=\pm 3.$$

Both signs give the same constant coefficient; the question asks for the absolute value precisely to avoid distinguishing them. Thus $|k|=3$, not 9, which is the value of k^2. The choices 1 and 2 would give constant coefficients 45 and 180 respectively, so neither meets the stated value 405.

Step 3: Conclusion The required magnitude is $|k|=3$. Hence, the correct option is (A).

Quick Tip to Solve: In a constant-term problem, solve the exponent equation before simplifying coefficients; distinguish the selection count r from the term number r+1.

11. Let z = x + iy be a non-zero complex number such that $z^2 = i|z|^2$, where i = $\sqrt{-1}$, then z lies on the : [4 marks]

(A) line, y = –x

(B) real axis

(C) line, y = x

(D) imaginary axis

Correct Answer: (C) line, y = x

Solution:

Concept:

- For $z = x + iy$, with real x, y, we have $z^2 = (x^2 - y^2) + 2ixy$ and $|z|^2 = x^2 + y^2$.
- Two complex numbers are equal exactly when both their real parts and their imaginary parts are equal. The condition $z \neq 0$ excludes $(x, y) = (0, 0)$.

Step 1: Compare real and imaginary parts Substitute the Cartesian form of z into the given equation:

$$(x^2 - y^2) + 2ixy = i(x^2 + y^2).$$

The right-hand side has zero real part. Equating the two components yields

$$x^2 - y^2 = 0, \qquad 2xy = x^2 + y^2.$$

The real equation alone gives the two possible lines $y = x$ and $y = -x$, so it cannot by itself determine the correct choice. The imaginary equation must be imposed at the same time.

Step 2: Enforce both conditions and exclude the origin Rearrange the imaginary equation into a nonnegative square:

$$x^2 - 2xy + y^2 = 0 \quad \Longrightarrow \quad (x - y)^2 = 0.$$

Since the coordinates are real, this forces $x = y$. It also satisfies the real equation automatically. For $x = y = t \neq 0$, direct substitution gives

$$z = t(1 + i), \quad z^2 = 2it^2 = i|z|^2,$$

so every nonzero point on this line works. On $y = -x$, the imaginary part of z^2 is $-2x^2$, opposite to the required $+2x^2$, unless $x = 0$, which is forbidden. On either coordinate axis the two component equations force both coordinates to vanish; thus no allowed point lies solely on the real or imaginary axis. These checks eliminate options (A), (B) and (D).

Step 3: Conclusion The number lies on $y = x$, with the origin excluded by $z \neq 0$. Hence, the correct option is (C).

Quick Tip to Solve: When a complex equation contains the modulus squared, expand into real coordinates and equate both components; one component may leave an extra, false locus.

12. If α and β are the roots of the equation
$2x(2x + 1) = 1$, then β is equal to : [4 marks]

(A) $-2\alpha\,(\alpha+1)$

(B) $2\alpha\,(\alpha+1)$

(C) $2\alpha^2$

(D) $2\alpha\,(\alpha-1)$

Correct Answer: (A) $-2\alpha\,(\alpha+1)$

Solution:

Concept:

- For a quadratic $ax^2+bx+c=0$ with roots α,β, their sum is $\alpha+\beta=-b/a$.
- Each root also satisfies the original equation. Use that equation to reduce quadratic expressions in α to linear ones before comparing the options.

Step 1: Obtain the sum of the roots First expand the equation without losing the coefficient outside the brackets:

$$2x(2x+1)=1 \quad\Longleftrightarrow\quad 4x^2+2x-1=0.$$

Vieta's relation therefore gives

$$\alpha+\beta=-\frac{2}{4}=-\frac{1}{2}, \qquad \beta=-\frac{1}{2}-\alpha.$$

There is no need to compute the two radical roots separately. Since the coefficient of x^2 is four, using a monic quadratic formula without first normalising would give an incorrect root sum.

Step 2: Reduce the candidate expressions As α is a root,

$$4\alpha^2+2\alpha-1=0 \quad\Longrightarrow\quad 2\alpha^2=\frac{1}{2}-\alpha.$$

Consequently the first candidate reduces to

$$\begin{aligned}-2\alpha(\alpha+1) &= -2\alpha^2-2\alpha\\ &= -\left(\frac{1}{2}-\alpha\right)-2\alpha\\ &= -\frac{1}{2}-\alpha=\beta.\end{aligned}$$

For comparison, option (B) reduces to $1/2+\alpha$, the negative of β, and cannot equal the nonzero root β. Option (C) is $1/2-\alpha$, which differs from β by one. Option (D) becomes $1/2-3\alpha$; equality with β would require $\alpha=1/2$, but substituting that value into $4\alpha^2+2\alpha-1$ gives one rather than zero. Thus only the first expression agrees for either possible assignment of the roots.

Step 3: Conclusion $\beta=-2\alpha(\alpha+1)$. Hence, the correct option is (A).

Quick Tip to Solve: For an expression in one root of a quadratic, eliminate its square with the quadratic equation and compare with the sum-of-roots formula.

13. The probabilities of three events A, B and C are given by

P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5.

If P(A∪B) = 0.8, P(A∩C) = 0.3, P(A∩B∩C) = 0.2, P(B∩C) = β

and P(A∪B∪C) = α, where 0.85 $\leq \alpha \leq$ 0.95, then β lies in the interval : [4 marks]

(A) [0.35, 0.36]

(B) [0.20, 0.25]

(C) [0.25, 0.35]

(D) [0.36, 0.40]

Correct Answer: (C) [0.25, 0.35]

Solution:

Concept:

- For two events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; the intersection is subtracted because it was counted twice.
- For three events, add individual probabilities, subtract the three pairwise intersections, and add back the triple intersection. Here α denotes the union probability and $\beta = P(B \cap C)$.

Step 1: Find the missing pairwise intersection The first union supplies the probability needed in the three-event formula. Rearranging gives

$$P(A \cap B) = 0.6 + 0.4 - 0.8 = 0.2.$$

This is the intersection of A and B, including any outcomes also in C; it must not be treated as the probability of A and B alone.

Step 2: Apply inclusion-exclusion to all three events Insert all the given probabilities and the intersection just obtained:

$$\begin{aligned} \alpha &= P(A) + P(B) + P(C) \\ &\quad - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &\quad + P(A \cap B \cap C) \\ &= 0.6 + 0.4 + 0.5 - 0.2 - 0.3 - \beta + 0.2 \\ &= 1.2 - \beta. \end{aligned}$$

The triple intersection is added back once after the three pairwise subtractions.

Step 3: Convert the bounds on the union Since $\beta = 1.2 - \alpha$, the largest union probability gives the smallest intersection probability:

$$1.2 - 0.95 \leq \beta \leq 1.2 - 0.85.$$

Thus $0.25 \leq \beta \leq 0.35$. Option B omits admissible values greater than 0.25, while options A and D lie mostly or entirely above the upper bound. Only C gives the full required interval.

Step 4: Conclusion The required interval is $\boxed{[0.25, 0.35]}$. Hence, the correct option is (C).

Quick Tip to Solve: Isolate the unknown probability before using bounds; subtracting a bounded variable reverses which endpoint gives the maximum.

14. If $y = \left(\frac{2}{\pi}x - 1\right) cosec\, x$ is the solution of the differential equation,
$\frac{dy}{dx} + p\,(x)\, y = \frac{2}{\pi} cosec\, x$,
$0 < x < \frac{\pi}{2}$, then the function p(x) is equal to : [4 marks]

(A) cot x

(B) sec x

(C) tan x

(D) cosec x

Correct Answer: (A) cot x

Solution:

Concept:

- A prescribed solution can be differentiated and substituted into $y\prime + p(x)y = q(x)$ to identify its coefficient $p(x)$.
- The product rule gives $(uv)\prime = u\prime v + uv\prime$, and $\frac{d}{dx}\csc x = -\csc x \cot x$.

Step 1: Differentiate the given product Write $u(x) = 2x/\pi - 1$, so that $y = u(x)\csc x$. Both factors depend on x; differentiating just the linear factor would miss the term that determines the unknown coefficient. Applying the product rule gives

$$\frac{dy}{dx} = \frac{2}{\pi}\csc x - \left(\frac{2x}{\pi} - 1\right)\csc x \cot x.$$

Recognise the product in the second term as y itself, obtaining

$$\frac{dy}{dx} = \frac{2}{\pi}\csc x - y\cot x.$$

Step 2: Compare with the differential equation Move the second term to the left to get

$$\frac{dy}{dx} + y\cot x = \frac{2}{\pi}\csc x.$$

This has exactly the stated right-hand side. Comparing with $dy/dx + p(x)y = (2/\pi)\csc x$, we obtain $p(x) = \cot x$. The identification holds throughout the given interval: $\csc x$ is finite there and $2x/\pi - 1 < 0$, so y never vanishes and division by y is legitimate.

Step 3: Check the remaining functions The coefficient must cancel precisely the product-rule term $-y\cot x$. The functions $\sec x$, $\tan x$, and $\csc x$ are not identically equal to $\cot x$ on the interval and leave a nonzero residual for general x. Equality at an isolated point, if any, cannot make a function satisfy the equation on its entire domain.

Step 4: Conclusion The function is $\boxed{p(x) = \cot x}$ for $0 < x < \pi/2$. Hence, the correct option is (A).

Quick Tip to Solve: When a linear differential equation supplies its solution, differentiate it and rewrite repeated factors as y before comparing coefficients.

15. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is : [4 marks]

(A) $\left(\frac{6}{5}, \frac{53}{10}\right)$
(B) $\left(\frac{3}{10}, \frac{16}{5}\right)$
(C) $\left(\frac{-53}{10}, \frac{16}{5}\right)$
(D) $\left(\frac{-16}{5}, \frac{53}{10}\right)$

Correct Answer: (D) $\left(\frac{-16}{5}, \frac{53}{10}\right)$

Solution:

Concept:

- At a point where two curves touch, they share a tangent. The radius of the circle at that point is perpendicular to this common tangent.
- A circle centre is equidistant from all points of the circle. For centre (h, k), compare squared distances to avoid introducing the unknown radius.

Step 1: Locate the centre on the normal For the parabola $y = x^2$, the derivative is $dy/dx = 2x$. At the contact point $T = (2, 4)$, the tangent has slope 4, so its perpendicular normal has slope $-1/4$. Since the circle radius through T lies on this normal, its centre (h, k) satisfies

$$k - 4 = -\frac{1}{4}(h - 2), \qquad h + 4k = 18.$$

Using the tangent line instead of this normal would impose the wrong geometric condition.

Step 2: Use equal radii to the two specified points The point $P = (0, 1)$ and the contact point T both lie on the circle. Equate their squared distances from the centre:

$$h^2 + (k - 1)^2 = (h - 2)^2 + (k - 4)^2.$$

Expand and cancel the common quadratic terms:

$$h^2 + k^2 - 2k + 1 = h^2 - 4h + 4 + k^2 - 8k + 16.$$

Consequently $4h + 6k = 19$, a second linear equation independent of the normal condition.

Step 3: Solve and verify the centre From the first equation $h = 18 - 4k$. Substitution in the second gives

$$72 - 16k + 6k = 19, \qquad k = \frac{53}{10}, \qquad h = -\frac{16}{5}.$$

This pair satisfies $h + 4k = 18$ and $4h + 6k = 19$, confirming tangency and equal radii. None of A, B or C satisfies both equations; therefore their proposed centres cannot meet both requirements.

Step 4: Conclusion The centre is $\boxed{\left(-\frac{16}{5}, \frac{53}{10}\right)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a circle touching a known curve, combine its normal at contact with the perpendicular-bisector condition for two known circle points.

16. If the tangent to the curve, y = f (x) = $x\log_e x$, ($x > 0$) at a point (c, f(c)) is parallel to the line-segment joining the points (1, 0) and (e, e), then c is equal to : [4 marks]

(A) $\frac{e-1}{e}$

(B) $e^{\left(\frac{1}{1-e}\right)}$

(C) $e^{\left(\frac{1}{e-1}\right)}$

(D) $\frac{1}{e-1}$

Correct Answer: (C) $e^{\left(\frac{1}{e-1}\right)}$

Solution:

Concept:

- The derivative $f\prime(c)$ gives the slope of the tangent at $(c, f(c))$. Parallel lines have equal slopes.
- For $f(x) = x\ln x$ on $x > 0$, the product rule yields $f\prime(x) = \ln x + 1$. The secant slope between (x_1, y_1) and (x_2, y_2) is $(y_2 - y_1)/(x_2 - x_1)$.

Step 1: Compute the reference line slope The line segment joins $(1, 0)$ and (e, e), so its rise is e and its horizontal change is $e - 1$. Its slope is therefore

$$m = \frac{e-0}{e-1} = \frac{e}{e-1}.$$

Both endpoints also lie on the curve, since $\ln 1 = 0$ and $\ln e = 1$, consistent with the specified coordinates.

Step 2: Equate the tangent and secant slopes Differentiate the function, retaining both product-rule terms:

$$f\prime(x) = 1 \cdot \ln x + x\frac{1}{x} = \ln x + 1.$$

At $x = c$, parallelism requires

$$\ln c + 1 = \frac{e}{e-1}.$$

Subtracting 1 gives

$$\ln c = \frac{e-(e-1)}{e-1} = \frac{1}{e-1}.$$

The numerator simplifies to positive 1; replacing it with negative 1 would select the wrong exponential.

Step 3: Exponentiate and distinguish the choices As c is positive, exponentiation is reversible and gives $c = e^{1/(e-1)}$. The derivative $\ln x + 1$ increases strictly on the positive axis, so this solution is unique. Option B has negative logarithm and cannot give the required slope greater than 1. Option A is below 1 and fails for the same reason. Option D omits the exponential needed to solve for c, rather than for its logarithm.

Step 4: Conclusion The required abscissa is $\boxed{c = e^{1/(e-1)}}$. Hence, the correct option is (C).

Quick Tip to Solve: In a tangent-parallel-to-chord question, equate f prime at c to the chord slope; remember to exponentiate after isolating the logarithm.

17. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies : [4 marks]

(A) $e^4 + 2e^2 - 1 = 0$

(B) $e^4 + e^2 - 1 = 0$

(C) $e^2 + 2e - 1 = 0$

(D) $e^2 + e - 1 = 0$

Correct Answer: (B) $e^4 + e^2 - 1 = 0$

Solution:

Concept:

- Write the ellipse as $x^2/a^2 + y^2/b^2 = 1$, where $a > b > 0$, with $b^2 = a^2(1 - e^2)$ and $0 < e < 1$.
- An upper end of its right latus rectum is $L = (ae, b^2/a)$. Implicit differentiation gives tangent slope $-b^2x/(a^2y)$, whose negative reciprocal is the normal slope.

Step 1: Find the normal at a latus-rectum end Choose the point $L = (ae, b^2/a)$; symmetry ensures that choosing another end gives the same relation for eccentricity. At this point the normal slope is

$$m_n = \frac{a^2y}{b^2x} = \frac{a^2(b^2/a)}{b^2(ae)} = \frac{1}{e}.$$

Thus the equation of the normal is

$$y - \frac{b^2}{a} = \frac{1}{e}(x - ae).$$

Both the coordinate and the normal slope use the semi-axes, not the full axis lengths.

Step 2: Impose passage through a minor-axis extremity An extremity of the minor axis has x-coordinate zero. At $x = 0$, the normal gives

$$y = \frac{b^2}{a} - a = \frac{b^2 - a^2}{a} = -ae^2.$$

This intercept is negative, so the extremity must be $(0, -b)$. Equating its ordinate gives $b = ae^2$, or $b/a = e^2$. Squaring and using the ellipse identity yields

$$1 - e^2 = e^4.$$

Step 3: Identify the admissible equation Rearrange to $e^4 + e^2 - 1 = 0$. The positive solution for e^2 is $(\sqrt{5} - 1)/2$, which lies between zero and one and therefore defines an ellipse. Options A, C and D impose different polynomials; none follows from $b/a = e^2$ together with $b^2/a^2 = 1 - e^2$. In particular, failing to square the eccentricity ratio leads to an incorrect quadratic in e.

Step 4: Conclusion The eccentricity satisfies $\boxed{e^4 + e^2 - 1 = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: At a latus-rectum end, evaluate the normal intercept on the minor axis first; then replace the semi-axis ratio using eccentricity.

18. Consider the statement :

"For an integer n, if $n^3 - 1$ is even, then n is odd."

The contrapositive statement of this statement is : [4 marks]

(A) For an integer n, if n is even, then $n^3 - 1$ is even.

(B) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(C) For an integer n, if n is odd, then $n^3 - 1$ is even.

(D) For an integer n, if n is even, then $n^3 - 1$ is odd.

Correct Answer: (D) For an integer n, if n is even, then $n^3 - 1$ is odd.

Solution:

Concept:

- For statements P and Q, the contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$: reverse the implication and negate both component statements.
- Every integer is either even or odd. Thus for an integer, not odd means even, and not even means odd; this equivalence depends on retaining the integer domain.

Step 1: Identify the hypothesis and conclusion Let P be the statement that $n^3 - 1$ is even, and let Q be the statement that n is odd. The given sentence has the form $P \Rightarrow Q$. The phrase for an integer n restricts the domain and stays in the transformed sentence. It is not itself the hypothesis that is to be negated.

Step 2: Reverse and negate the statements The contrapositive begins with the negation of Q, namely n is not odd, and concludes with the negation of P, namely $n^3 - 1$ is not even. Since n and $n^3 - 1$ are integers, these become

$$n \text{ is even} \quad \Rightarrow \quad n^3 - 1 \text{ is odd.}$$

For a check, write $n = 2k$; then $n^3 - 1 = 8k^3 - 1 = 2(4k^3 - 1) + 1$, which is indeed odd for every integer k.

Step 3: Separate the contrapositive from related forms Option A reverses the statements without correctly negating the conclusion, so it is not the required form. Option B has the form $\neg P \Rightarrow \neg Q$, the inverse. Option C has the form $Q \Rightarrow P$, the converse. Although the converse happens to be true for this parity example, its logical form is different from the requested contrapositive. Option D alone reverses the order and negates both original component statements.

Step 4: Conclusion The contrapositive is: for an integer n, if n is even, then $n^3 - 1$ is odd. Hence, the correct option is (D).

Quick Tip to Solve: Write P implies Q before transforming a sentence: the contrapositive starts with not Q and ends with not P, keeping the domain unchanged.

19. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is : [4 marks]

(A) $\frac{1}{\sqrt{3}-1}$

(B) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(C) $\frac{1}{\sqrt{3}+1}$

(D) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Correct Answer: (A) $\frac{1}{\sqrt{3}-1}$

Solution:

Concept:

- For an angle of elevation, the tangent is the vertical rise divided by the horizontal separation from the summit. Measure both distances relative to the observation point.
- A climb of length s at inclination θ has horizontal component $s\cos\theta$ and vertical component $s\sin\theta$. The length along the slope is not the horizontal displacement.

Step 1: Use the initial elevation Let the summit have height h km above the ground and let its foot be d km horizontally from the initial point. The initial angle gives

$$\tan 45^\circ = \frac{h}{d} = 1, \qquad d = h.$$

The climb is towards the summit, so the remaining horizontal separation decreases. The ground level is the reference for the required height.

Step 2: Resolve the climb into components The one-kilometre climb gives

$$\Delta x = \cos 30^\circ = \frac{\sqrt{3}}{2}\ \text{km}, \qquad \Delta y = \sin 30^\circ = \frac{1}{2}\ \text{km}.$$

Thus the summit is $h - \frac{1}{2}$ km above the new position and $h - \frac{\sqrt{3}}{2}$ km away horizontally. Use the new angle, rather than treating the entire climb as a horizontal walk.

Step 3: Apply the final elevation and compare options Using $\tan 60^\circ = \sqrt{3}$,

$$h - \frac{1}{2} = \sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right),$$

$$(\sqrt{3}-1)h = 1, \qquad h = \frac{1}{\sqrt{3}-1}.$$

This is approximately 1.366 km and leaves a positive horizontal separation of 0.500 km at the final point. Options B, C and D give different heights and do not satisfy this elevation equation with the prescribed climb.

Step 4: Conclusion The height of the summit is $\boxed{\frac{1}{\sqrt{3}-1}\ \text{km}}$. Hence, the correct option is (A).

Quick Tip to Solve: For an inclined climb, subtract its vertical component from the height and its horizontal component from the original horizontal distance.

20. A plane P meets the coordinate axes at A, B and C respectively. The centroid of $\triangle ABC$ is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is : [4 marks]

(A) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

(B) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

(C) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

(D) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

Correct Answer: (C) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

Solution:

Concept:

- A plane meeting the coordinate axes at $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ has intercept form $x/a+y/b+z/c = 1$. Its normal has direction ratios $1/a : 1/b : 1/c$.
- The centroid of a triangle is obtained by taking the arithmetic mean of the three vertex coordinates. A line perpendicular to a plane is parallel to the plane's normal.

Step 1: Find the three axis intercepts Write the intercept points as $A = (a, 0, 0)$, $B = (0, b, 0)$, $C = (0, 0, c)$. Their centroid is

$$G = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2).$$

Equating corresponding coordinates gives $a = 3$, $b = 3$, and $c = 6$. The intercepts are three times the centroid coordinates, because only one vertex contributes to each coordinate sum.

Step 2: Read the normal direction from the plane The required plane is therefore

$$\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1 \quad \Longleftrightarrow \quad 2x + 2y + z = 6.$$

For a plane $Ax + By + Cz = D$, the vector (A, B, C) is perpendicular to every direction in the plane. Thus the required line has direction ratios $2 : 2 : 1$, equivalently $1/3 : 1/3 : 1/6$. The intercept ratios themselves are not the normal ratios.

Step 3: Pass the normal line through the centroid Through $G = (1, 1, 2)$, the line is

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}.$$

In parametric form it is $(x, y, z) = (1, 1, 2)+t(2, 2, 1)$, so it passes through the centroid at $t = 0$. Options A, B and D have direction vectors $(1, 1, 2)$, $(2, 1, 1)$, and $(1, 2, 2)$, respectively; none is proportional to $(2, 2, 1)$.

Step 4: Conclusion The required normal line is $\boxed{\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For an intercept triangle, multiply centroid coordinates by three to get intercepts, then take their reciprocals to get the normal direction.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Consider the data on x taking the values
0, 2, 4, 8,....., 2^n with frequencies
${}^nC_0$, ${}^nC_1$, ${}^nC_2$,...., ${}^nC_n$ respectively. If the
mean of this data is $\frac{728}{2^n}$, then n is equal to _____ . [4 marks]

Solution:

Concept:

- The mean of a frequency distribution is $\bar{x} = \sum f_r x_r / \sum f_r$. The frequency must be multiplied by its corresponding data value before summation.
- The binomial theorem gives $\sum_{r=0}^{n} \binom{n}{r} t^r = (1+t)^n$. Here the first data value is zero, whereas the later values are 2^r for $r \geq 1$.

Step 1: Calculate the total frequency The frequencies are all the binomial coefficients in the expansion of $(1+1)^n$. Therefore

$$N = \sum_{r=0}^{n} \binom{n}{r} = 2^n.$$

This is the denominator in the weighted mean. There are $n+1$ listed data categories, but the number of categories is not the sum of their frequencies.

Step 2: Calculate the weighted sum carefully The data attached to $\binom{n}{0}$ is 0, not $2^0 = 1$. Hence

$$\sum f_r x_r = 0\binom{n}{0} + \sum_{r=1}^{n} \binom{n}{r} 2^r.$$

Including and then removing the $r = 0$ binomial term yields

$$\sum f_r x_r = (1+2)^n - 1 = 3^n - 1.$$

The subtraction of one is essential: using 3^n would count a contribution absent from the printed data.

Step 3: Equate the means and determine the integer The mean calculated from the data must equal the stated mean, so

$$\frac{3^n - 1}{2^n} = \frac{728}{2^n}.$$

Since 2^n is positive, cancel the equal denominators:

$$3^n - 1 = 728, \qquad 3^n = 729 = 3^6.$$

Powers of three are strictly increasing for integer n, so this determines the unique value. As a check, the total frequency is 64, the weighted sum is 728, and their ratio is precisely the given mean.

Step 4: Conclusion The required value is $\boxed{n = 6}$, a dimensionless integer.

Quick Tip to Solve: When the data begin with zero followed by powers, subtract the missing constant term from the binomial sum before forming the mean.

22. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}|=|\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is _____ . [4 marks]

Solution:

Concept:

- For vectors $\vec{a},\vec{b}$, $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2$. Squaring a norm equality produces a useful scalar relation.
- Two perpendicular vectors have zero scalar product. Because $\vec{y}\neq\vec{0}$, its squared norm is positive, allowing division by $|\vec{y}|^2$.

Step 1: Square the equality of magnitudes The given relation compares lengths, not the vectors themselves. Both sides are nonnegative, so squaring is valid:

$$|\vec{x}+\vec{y}|^2=|\vec{x}|^2.$$

Expanding the left side and cancelling $|\vec{x}|^2$ gives

$$2\vec{x}\cdot\vec{y}+|\vec{y}|^2=0.$$

Thus $2\vec{x}\cdot\vec{y}=-|\vec{y}|^2$. In particular, the scalar product is negative; the original vectors need not be perpendicular.

Step 2: Translate the second condition into a scalar equation The condition $(2\vec{x}+\lambda\vec{y})\perp\vec{y}$ means

$$(2\vec{x}+\lambda\vec{y})\cdot\vec{y}=0.$$

By distributivity of the dot product, this becomes

$$2\vec{x}\cdot\vec{y}+\lambda|\vec{y}|^2=0.$$

This equation uses exactly the same scalar product found from the magnitude condition, so there is no need to choose components or introduce an angle.

Step 3: Eliminate the scalar product Substitute $2\vec{x}\cdot\vec{y}=-|\vec{y}|^2$:

$$-|\vec{y}|^2+\lambda|\vec{y}|^2=0,\qquad(\lambda-1)|\vec{y}|^2=0.$$

The nonzero-vector assumption rules out $|\vec{y}|^2=0$; therefore $\lambda-1=0$. For a direct check, the first derived equation already says $(2\vec{x}+\vec{y})\cdot\vec{y}=0$, agreeing with the required perpendicularity.

Step 4: Conclusion The required parameter is $\boxed{\lambda=1}$, which is dimensionless.

Quick Tip to Solve: Square a norm equality first, then compare the resulting dot-product relation directly with the perpendicularity equation.

23. The sum of distinct values of λ for which the system of equations

$$(\lambda - 1)\, x + (3\lambda + 1)\, y + 2\lambda z = 0$$

$$(\lambda - 1)\, x + (4\lambda - 2)\, y + (\lambda + 3)\, z = 0$$

$$2x + (3\lambda + 1)\, y + 3\,(\lambda - 1)\, z = 0$$

has non-zero solutions, is _____ . [4 marks]

Solution:

Concept:

- A homogeneous system of three linear equations has a nonzero solution exactly when its coefficient matrix is singular, that is, when its determinant is zero.
- Adding a multiple of one row to another leaves a determinant unchanged. Factoring a common scalar from a row factors the same scalar from the determinant; repeated roots count only once when distinct parameter values are requested.

Step 1: Form the determinant and simplify one row Let D be the coefficient determinant. Subtract the first row from the second row. The new second row is

$$\big(0, \lambda - 3, 3 - \lambda\big) = (\lambda - 3)(0, 1, -1).$$

Consequently, without dividing by a factor that might vanish,

$$D = (\lambda - 3)\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ 0 & 1 & -1 \\ 2 & 3\lambda + 1 & 3\lambda - 3 \end{vmatrix}.$$

Keeping the factor explicitly ensures that $\lambda = 3$ is retained as a candidate.

Step 2: Evaluate and factor the remaining determinant Expand the smaller expression along its first row:

$$\begin{aligned} D &= (\lambda - 3)\big[(\lambda - 1)(6\lambda - 2) \\ &\qquad - 2(3\lambda + 1) - 4\lambda\big] \\ &= (\lambda - 3)(6\lambda^2 - 18\lambda) \\ &= 6\lambda(\lambda - 3)^2. \end{aligned}$$

Here the first minor is $(3\lambda - 3) + (3\lambda + 1) = 6\lambda - 2$; retaining its sign avoids introducing false roots.

Step 3: Identify distinct admissible values Nonzero solutions exist when

$$6\lambda(\lambda - 3)^2 = 0,$$

so the distinct values are 0 and 3. A zero determinant is sufficient as well as necessary for a homogeneous square system, because the null space then has positive dimension. Although 3 is a

repeated root of the determinant polynomial, it is only one value of the parameter. Their sum is therefore $0 + 3 = 3$, rather than a sum counting multiplicity.

Step 4: Conclusion The sum of the distinct values is $\boxed{3}$, a dimensionless number.

Quick Tip to Solve: Factor a parameter-dependent row instead of dividing by it; this preserves singular cases. Count a repeated determinant root once for distinct values.

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24. Suppose that a function f : R $\to$ R satisfies
f(x + y) = f(x)f(y) for all x, y $\in$ R and f(1) = 3.
If $\sum_{i=1}^{n} f(i) = 363$ **then n is equal to _____ .** [4 marks]

Solution:

Concept:

- The relation $f(x + y) = f(x)f(y)$, together with $f(1) = 3$, determines positive-integer values recursively: $f(k + 1) = 3f(k)$. No continuity assumption is needed for these integer arguments.
- The sum of a geometric progression with first term a, common ratio $r \neq 1$, and n terms is $a(r^n - 1)/(r - 1)$. Here the first term is $f(1)$, not $f(0)$.

Step 1: Determine the integer values of the function Use the functional equation with $x = k$ and $y = 1$:

$$f(k + 1) = f(k)f(1) = 3f(k).$$

Starting with $f(1) = 3$, this gives $f(2) = 9$, $f(3) = 27$, and, by induction,

$$f(k) = 3^k \quad \text{for every positive integer } k.$$

Only these arguments occur in the required sum. It is unnecessary to make any claim about the form of the function at arbitrary real arguments.

Step 2: Evaluate the stated sum The sum from $i = 1$ to n is

$$\sum_{i=1}^{n} f(i) = 3 + 3^2 + \cdots + 3^n.$$

This is a geometric progression with first term 3 and ratio 3, so

$$\frac{3(3^n - 1)}{3 - 1} = 363.$$

The initial factor of three comes from starting at $i = 1$. Starting at zero would add an extra term and produce the wrong equation.

Step 3: Solve for the number of terms and verify Multiply by two and divide by three:

$$3(3^n - 1) = 726, \qquad 3^n - 1 = 242, \qquad 3^n = 243 = 3^5.$$

Thus $n = 5$. Directly checking the five terms gives

$$3 + 9 + 27 + 81 + 243 = 363.$$

Every term is positive, so the partial sums strictly increase; therefore no other positive integer n can give the same sum. This also confirms that the number sought is the count of terms, rather than the value of the last term.

Step 4: Conclusion The required value is $\boxed{n = 5}$, a dimensionless integer.

Quick Tip to Solve: Apply a multiplicative functional equation at consecutive integers to get a geometric progression, and check whether the summation starts at zero or one.

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25. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____ . [4 marks]

Solution:

Concept:

- The letters of LETTER are L, E, T, T, E, R. The two vowels are identical E letters; the consonants contain two identical T letters.
- To keep vowels apart, first arrange all consonants and then put the vowels in distinct gaps. For repeated objects divide the factorial count by the factorial of each repetition.

Step 1: Arrange the four consonants Remove the two E letters temporarily. The remaining letters L, T, T, R can occupy four positions. Permuting four distinguishable objects would give $4!$ orders, but interchanging the two T letters produces no new word. Therefore the number of different consonant arrangements is

$$N_c = \frac{4!}{2!} = 12.$$

Do not divide by a second factor of $2!$ here: the E letters have not yet been inserted.

Step 2: Choose separate gaps for the vowels Every consonant order has five possible gaps: one before the first consonant, three between neighbouring consonants and one after the last consonant. For example, an order can be represented as

$$_L_T_T_R_.$$

Place one E in each of two different gaps. This guarantees that at least one consonant separates the vowels. Since both E letters are identical, we select an unordered pair of gaps rather than arrange two labelled vowels:

$$N_v = \binom{5}{2} = 10.$$

Step 3: Multiply the independent counts Each of the 12 consonant arrangements allows the same 10 gap choices, so the multiplication rule gives

$$N = 12 \times 10 = 120.$$

Every admissible word is counted exactly once: removing its vowels recovers one consonant arrangement and the two occupied gaps. As a check, all unrestricted arrangements number $6!/(2!2!) = 180$. Treating EE as one block gives $5!/2! = 60$ adjacent-vowel words, leaving $180 - 60 = 120$.

Step 4: Conclusion The required number of words is $\boxed{120}$. This is a dimensionless count.

Quick Tip to Solve: With identical vowels that must stay apart, arrange consonants first and choose distinct gaps; do not multiply by a vowel factorial.

Physics – Section A: Multiple Choice Questions (80 marks)

26. Two planets have masses M and 16 M and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is : [4 marks]

(A) $2\sqrt{\frac{GM}{a}}$

(B) $\sqrt{\frac{GM^2}{ma}}$

(C) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

(D) $4\sqrt{\frac{GM}{a}}$

Correct Answer: (C) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

Solution:

Concept:

- The gravitational potential energies due to the two planets add: $U = -GMm/r_1 - 16GMm/r_2$. Mechanical energy is conserved during the flight.
- The minimum launch energy must reach the maximum potential energy between the planets. Comparing only the two surfaces misses this intervening barrier.

Step 1: Locate the point of zero net force Let x be the distance from the smaller planet's centre, measured towards the larger planet. Between the surfaces, $a \le x \le 8a$. The two gravitational pulls oppose one another. Their magnitudes are equal when

$$\frac{GMm}{x^2} = \frac{16GMm}{(10a - x)^2}.$$

Taking positive square roots gives $10a - x = 4x$, hence $x = 2a$. The potential energy increases from the larger planet's facing surface to this point, and decreases afterwards towards the smaller planet; thus this is the barrier.

Step 2: Calculate the initial and barrier potentials At the facing surface of the larger planet the distances from the two centres are $8a$ and $2a$:

$$U_i = -\frac{GMm}{8a} - \frac{16GMm}{2a} = -\frac{65GMm}{8a}.$$

At the zero-force point the distances are $2a$ and $8a$:

$$U_{\max} = -\frac{GMm}{2a} - \frac{16GMm}{8a} = -\frac{5GMm}{2a}.$$

Step 3: Apply the limiting energy condition At threshold the body has vanishing speed at the top of the potential barrier. Therefore

$$\frac{1}{2}mv_{\min}^2 = U_{\max} - U_i = \frac{45GMm}{8a},$$

$$v_{\min}^2 = \frac{45GM}{4a}, \qquad v_{\min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}.$$

The mass m cancels, ruling out option (B). Options (A) and (D) give different squared speeds, $4GM/a$ and $16GM/a$, instead of $45GM/(4a)$. Strictly, crossing requires an arbitrarily small excess above this limiting threshold; this is the usual minimum-speed convention.

Step 4: Conclusion The minimum firing speed is $\boxed{\frac{3}{2}\sqrt{5GM/a}}$, in units of speed. Hence, the correct option is (C).

Quick Tip to Solve: For transfer between two attracting bodies, locate the zero-force point first: the highest potential on the route determines the launch threshold.

27. Given the masses of various atomic particles

m_p = 1.0072 u, m_n = 1.0087 u, m_e = 0.000548 u,

$m_{\overline{v}}$ = 0, m_d = 2.0141 u, where p ≡ proton, n ≡ neutron,

e ≡ electron, $\overline{v}$ ≡ antineutrino and d ≡ deuteron. Which of the following process is allowed by momentum and energy conservation? [4 marks]

(A) n + n → deuterium atom

(electron bound to the nucleus)

(B) n + p → d + γ

(C) p → n + e^+ + $\overline{v}$

(D) e^+ + e^- → γ

Correct Answer: (B) n + p → d + γ

Solution:

Concept:

- An isolated reaction must conserve both total energy and vector momentum. A positive rest-mass release is necessary for an initially slow system to release energy, but it is not sufficient by itself.
- In the centre-of-momentum frame the total momentum is zero. A single massive final particle must then be at rest, while a single photon can never have zero momentum at positive energy.

Step 1: Check neutron capture by a proton For option (B), the initial rest mass exceeds the deuteron

mass:

$$\Delta m = m_n + m_p - m_d = 1.0087 + 1.0072 - 2.0141 = 0.0018\,\text{u} > 0.$$

This corresponds to approximately $0.0018 \times 931.5 = 1.68\,\text{MeV}$ of released energy. A photon can carry most of this energy, while the deuteron recoils with equal and opposite momentum. Thus the two final particles allow energy and momentum to be conserved simultaneously.

Step 2: Reject the single-particle final states In option (A), two neutrons produce only one deuterium atom. In their centre-of-momentum frame that atom would have to be at rest. Its approximate rest mass is $m_d + m_e = 2.014648\,\text{u}$, apart from the tiny electronic binding correction, whereas the initial rest mass is $2m_n = 2.0174\,\text{u}$. The excess energy has no other particle or radiation to carry it away. Initial kinetic energy only increases this mismatch.

In option (D), an electron and positron cannot annihilate into a single photon in isolation. In the centre-of-momentum frame the initial momentum is zero, whereas a photon with energy $E > 0$ has momentum magnitude $E/c > 0$. At least two photons or another recoiling object are needed.

Step 3: Test the proposed proton decay For option (C), the final rest mass is larger than the initial rest mass, even with the antineutrino mass given as zero:

$$m_p - (m_n + m_e) = 1.0072 - 1.0087 - 0.000548 = -0.002048\,\text{u}.$$

Thus a free proton cannot spontaneously supply the rest energy needed for the neutron and positron. The permitted choice among the listed isolated processes is consequently radiative neutron-proton capture.

Step 4: Conclusion $\boxed{n + p \longrightarrow d + \gamma}$ is allowed by energy and momentum conservation. Hence, the correct option is (B).

Quick Tip to Solve: After checking the mass defect, inspect the number of final particles: a lone product usually cannot absorb both released energy and momentum.

28. A square loop of side $2a$ and carrying current I is kept in xz plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point (0, b, 0), (b $>>$ a). The magnitude of torque on the loop about z-axis will be : [4 marks]

(A) $\frac{2\mu_0 I^2 a^2}{\pi b}$

(B) $\frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$

(C) $\frac{\mu_0 I^2 a^2 b}{2\pi(a^2+b^2)}$

(D) $\frac{\mu_0 I^2 a^2}{2\pi b}$

Correct Answer: (A) $\frac{2\mu_0 I^2 a^2}{\pi b}$

Solution:

Concept:

- A small current loop has magnetic moment $\boldsymbol{\mu} = IA\,\hat{n}$ and torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ in a nearly uniform external magnetic field.

- The long straight wire produces $B = \mu_0 I/(2\pi r)$. Since $b \gg a$, the field across the square is approximately its value at the centre.

Step 1: Find the field direction and loop moment The square lies in the xz plane, so its area vector is parallel to the y-axis. Its side is $2a$, giving

$$A = (2a)^2 = 4a^2, \qquad |\boldsymbol{\mu}| = 4Ia^2.$$

The wire runs parallel to the z-axis at $(0, b, 0)$. At the origin, the perpendicular displacement from the wire is along the negative y-axis, so the right-hand rule puts its field along the x-axis, with sign depending on the wire's current direction. Thus the moment and field are perpendicular and their torque is along the z-axis.

Step 2: Use the distant-wire approximation The origin is a distance b from the wire. In the stated limit $b \gg a$, replace the field over the loop by

$$B_0 = \frac{\mu_0 I}{2\pi b}.$$

The torque magnitude about z is therefore

$$\tau_z = \mu B_0 = 4Ia^2 \frac{\mu_0 I}{2\pi b} = \frac{2\mu_0 I^2 a^2}{\pi b}.$$

Changing either current direction reverses the torque direction but does not change this magnitude.

Step 3: Distinguish the approximation from finite-size corrections An exact force calculation on the two z-directed sides, located at $x = \pm a$, gives

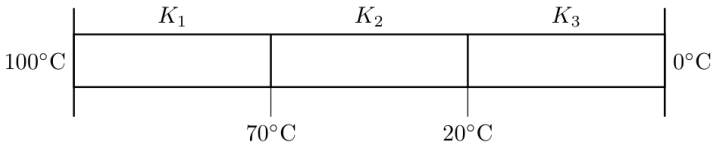
$$\tau_z^{\text{exact}} = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}.$$

The other two sides have forces parallel to z and contribute no torque about z. The exact expression in (B) reduces to (A) when a^2/b^2 is neglected, as the specified distant-wire condition requires. Options (C) and (D) have a coefficient four times too small, arising from using a^2 instead of the actual area $4a^2$.

Step 4: Conclusion For the given limit $b \gg a$, $\boxed{\tau_z \simeq 2\mu_0 I^2 a^2/(\pi b)}$. Hence, the correct option is (A).

Quick Tip to Solve: Read the side length carefully: a square of side 2a has area 4a squared. Use the field at its centre only when the wire is far away.

29. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1 , K_2 and K_3 , respecrtively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1 , K_2 and K_3 is : [4 marks]



(A) $K_1 : K_3 = 2 : 3$,
$K_2 : K_3 = 2 : 5$
(B) $K_1 < K_2 < K_3$
(C) $K_1 : K_2 = 5 : 2$,
$K_1 : K_3 = 3 : 5$
(D) $K_1 > K_2 > K_3$
Correct Answer: (A) $K_1 : K_3 = 2 : 3$,
$K_2 : K_3 = 2 : 5$

Solution:

Concept:

- For a uniform rod in steady conduction, the heat current is $H = KA\Delta T/L$, where K is thermal conductivity, A cross-sectional area and L length.
- Rods joined in series without heat loss carry the same heat current. With equal lengths and areas, conductivity is inversely proportional to the temperature drop across each rod.

Step 1: Read the temperature drops in order The figure places the rods in the order K_1, K_2, K_3 from the hot end to the cold end. Their boundary temperatures are $100°\text{C}$, $70°\text{C}$, $20°\text{C}$ and $0°\text{C}$. Therefore the respective drops are

$$\Delta T_1 = 100 - 70 = 30\,\text{K},$$

$$\Delta T_2 = 70 - 20 = 50\,\text{K}, \qquad \Delta T_3 = 20 - 0 = 20\,\text{K}.$$

Temperature differences have the same numerical value in kelvin and degrees Celsius; no offset is added when taking these differences.

Step 2: Equate the heat currents There is no storage of energy at either joint in the steady state and no heat escapes through the lateral surfaces. Thus the energy entering each rod per second equals the energy leaving it:

$$\frac{K_1 A(30)}{L} = \frac{K_2 A(50)}{L} = \frac{K_3 A(20)}{L}.$$

Cancel the identical geometrical factor A/L:

$$30K_1 = 50K_2 = 20K_3.$$

Consequently

$$K_1 : K_2 : K_3 = \frac{1}{30} : \frac{1}{50} : \frac{1}{20} = 10 : 6 : 15.$$

Step 3: Compare the proposed relationships From this ratio,

$$K_1 : K_3 = 10 : 15 = 2 : 3, \qquad K_2 : K_3 = 6 : 15 = 2 : 5.$$

Both statements in option (A) hold. The correct ordering is $K_2 < K_1 < K_3$, excluding both (B) and (D). Option (C) claims $K_1 : K_2 = 5 : 2$, whereas the actual value is $5 : 3$; its proposed ratio $K_1 : K_3 = 3 : 5$ also differs from $2 : 3$. The largest drop occurs across the least conducting rod, as expected physically.

Step 4: Conclusion $\boxed{K_1 : K_3 = 2 : 3, \quad K_2 : K_3 = 2 : 5}$. Hence, the correct option is (A).

Quick Tip to Solve: In equal rods in series, the largest temperature drop identifies the smallest thermal conductivity; take reciprocal drops to obtain the ratios.

30. A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5 P is : [4 marks]

(A) $\frac{R}{3}$

(B) $\frac{3R}{2}$

(C) $\frac{R}{2}$

(D) 2R

Correct Answer: (A) $\frac{R}{3}$

Solution:

Concept:

- For a thin lens in air, the lens maker formula is $P = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index of its material.
- The two lenses have the same material, so the factor $\mu - 1$ is common. A plane surface has infinite radius and contributes zero to the curvature sum.

Step 1: Write the power of the double-convex lens Take the light to travel from left to right. For a double-convex lens with equal radius magnitudes, the Cartesian signs are $R_1 = +R$ and $R_2 = -R$. The powers of the two curved surfaces therefore add:

$$P = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R}.$$

Using the same positive sign for both radii would incorrectly make the double-convex lens have zero power. Here R denotes the common positive magnitude stated in the question.

Step 2: Write the power of the plano-convex lens Let r be the positive radius magnitude of its curved surface. One surface is plane, so its reciprocal radius is zero. Either orientation of the lens gives the

same positive power:

$$P' = \frac{\mu - 1}{r}.$$

The question requires $P' = 1.5P$. The plano-convex lens has only one curved surface, so increasing its power beyond that of the double-convex lens requires a substantially smaller radius, rather than a larger one.

Step 3: Solve the power ratio and check the options Cancel the material factor in the ratio:

$$\frac{P'}{P} = \frac{(\mu - 1)/r}{2(\mu - 1)/R} = \frac{R}{2r} = \frac{3}{2}.$$

Hence $R = 3r$, giving

$$r = \frac{R}{3}.$$

For comparison, $r = 3R/2$ would give $P'/P = 1/3$; $r = R/2$ would give 1; and $r = 2R$ would give $1/4$. These respectively reject (B), (C) and (D). The result has units of length and is smaller than the original radius, consistent with the required stronger curvature.

Step 4: Conclusion The curved surface of the plano-convex lens must have radius $\boxed{R/3}$. Hence, the correct option is (A).

Quick Tip to Solve: Compare curvature sums rather than powers alone: two equal convex surfaces contribute 2/R, whereas one curved surface and one plane contribute 1/r.

31. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by

$y(t) = y_0 \sin^2 \omega t$, where 'y' is measured from the lower end of unstretched spring. Then ω is: [4 marks]

(A) $\sqrt{\frac{g}{y_0}}$

(B) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$

(C) $\sqrt{\frac{2g}{y_0}}$

(D) $\sqrt{\frac{g}{2y_0}}$

Correct Answer: (D) $\sqrt{\frac{g}{2y_0}}$

Solution:

Concept:

- Take downward extension from the unstretched end as positive. Gravity stretches the spring, while the restoring force is upward: $m\ddot{y} = mg - ky$.
- The angular frequency about the equilibrium extension $y_{\text{eq}} = mg/k$ is $\Omega = \sqrt{k/m}$. The frequency parameter inside a squared sine is half the oscillation frequency.

Step 1: Rewrite the given displacement Use the double-angle identity before comparing frequencies:

$$y(t) = y_0 \sin^2(\omega t) = \frac{y_0}{2}\big[1 - \cos(2\omega t)\big].$$

Thus the mean extension and oscillation amplitude are both $y_0/2$. The mass starts from zero extension with zero velocity, as required for release from the unstretched position. It oscillates between 0 and y_0, so the equilibrium point is halfway between them.

Step 2: Compare the equilibrium and acceleration Equating the mean extension to the static equilibrium extension gives

$$\frac{y_0}{2} = \frac{mg}{k}, \qquad \frac{k}{m} = \frac{2g}{y_0}.$$

The cosine term oscillates with angular frequency 2ω. Consequently,

$$2\omega = \sqrt{\frac{k}{m}}, \qquad \omega^2 = \frac{k}{4m} = \frac{g}{2y_0}.$$

This also follows by differentiating twice: at $t = 0$, $\ddot{y} = 2y_0\omega^2 = g$. Option (C) is the actual spring oscillation frequency Ω, rather than the parameter ω in the squared sine. Options (A) and (B) also give the wrong initial acceleration.

Step 3: Conclusion Therefore $\omega = \sqrt{\dfrac{g}{2y_0}}$, in $\mathrm{rad\,s^{-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a displacement proportional to $\sin^2 \omega t$, first rewrite it using $1 - \cos 2\omega t$; the physical oscillation frequency is 2ω.

32. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit : [4 marks]

(A) Ammeter is always connected in series and voltmeter in parallel

(B) Both ammeter and voltmeter must be connected in series

(C) Both ammeter and voltmeter must be connected in parallel

(D) Ammeter is always used in parallel and voltmeter is series

Correct Answer: (A) Ammeter is always connected in series and voltmeter in parallel

Solution:

Concept:

- Ohm's law relates the potential difference across a resistor to the current through it: $V = IR$, provided its temperature and other physical conditions remain constant.
- An ammeter measures branch current and has very low internal resistance. A voltmeter measures the potential difference between two points and has very high internal resistance.

Step 1: Connect the current-measuring instrument Place the ammeter in the same conducting path as the resistor, so that the current to be measured flows through the ammeter. This is a series connection. Its small internal resistance makes the extra voltage drop across it small and therefore limits the

disturbance of the original current. Connecting it directly in parallel with the resistor would create a low-resistance bypass and could produce a dangerously large current. It would not provide the intended measurement of resistor current.

Step 2: Connect the potential-measuring instrument Join the voltmeter terminals to the two ends of the resistor. The instrument must compare the potentials of these two points, so it is connected in parallel with the resistor. Its large internal resistance keeps the diverted current small. A voltmeter put in series would greatly increase circuit resistance and would measure the drop across itself rather than the required drop across the resistor. With the proper connections, pairs of measured values (V, I) can be plotted to verify a straight-line relation with constant ratio V/I. Option (B) incorrectly puts the voltmeter in series; option (C) incorrectly puts the ammeter in parallel; option (D) reverses both required connections.

Step 3: Conclusion The ammeter is connected in series with the resistor and the voltmeter in parallel across it. Hence, the correct option is (A).

Quick Tip to Solve: Current is measured by inserting the meter into the current path; voltage is measured by placing the meter across the two points of interest.

33. Two identical electric point dipoles have dipole moments $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along the x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm', their speed when they are infinitely far apart is : [4 marks]

(A) $\frac{p}{a}\sqrt{\frac{3}{2\pi\epsilon_0 ma}}$

(B) $\frac{p}{a}\sqrt{\frac{1}{\pi\epsilon_0 ma}}$

(C) $\frac{p}{a}\sqrt{\frac{1}{2\pi\epsilon_0 ma}}$

(D) $\frac{p}{a}\sqrt{\frac{2}{\pi\epsilon_0 ma}}$

Correct Answer: (C) $\frac{p}{a}\sqrt{\frac{1}{2\pi\epsilon_0 ma}}$

Solution:

Concept:

- The interaction energy of fixed point dipoles separated by $\mathbf{r} = r\hat{\mathbf{r}}$ is $U = [\mathbf{p}_1 \cdot \mathbf{p}_2 - 3(\mathbf{p}_1 \cdot \hat{\mathbf{r}})(\mathbf{p}_2 \cdot \hat{\mathbf{r}})]/(4\pi\varepsilon_0 r^3)$.
- With no external force, the initially stationary pair retains zero total momentum. Equal masses therefore move with equal speeds in opposite directions, and mechanical energy is conserved.

Step 1: Evaluate the initial potential energy Choose the unit separation vector along the positive x-axis. The dipole moments are antiparallel and remain fixed:

$$\mathbf{p}_1 \cdot \mathbf{p}_2 = -p^2, \qquad (\mathbf{p}_1 \cdot \hat{\mathbf{r}})(\mathbf{p}_2 \cdot \hat{\mathbf{r}}) = -p^2.$$

At separation a, their interaction energy is therefore

$$U_i = \frac{-p^2 + 3p^2}{4\pi\varepsilon_0 a^3} = \frac{p^2}{2\pi\varepsilon_0 a^3}.$$

It is positive and decreases towards zero as the separation increases, consistent with their repulsion. This sign is essential: the initial stored energy becomes translational kinetic energy.

Step 2: Share the released energy between both dipoles At infinite separation, $U_f = 0$. If each dipole has speed v, the total final kinetic energy includes both masses:

$$K_f = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2.$$

Energy conservation then gives

$$mv^2 = \frac{p^2}{2\pi\varepsilon_0 a^3}, \qquad v = \frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}.$$

Compared with this result, options (A), (B) and (D) would give total kinetic energies respectively three, two and four times the available initial interaction energy. They therefore violate energy conservation.

Step 3: Conclusion Each dipole reaches speed $\frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$. Hence, the correct option is (C).

Quick Tip to Solve: For two identical freely moving bodies, remember to include both kinetic energies: their total is mv^2, not $mv^2/2$.

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34. Particle A of mass m_1 moving with velocity $\left(\sqrt{3}\hat{i} + \hat{j}\right) ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V_1}$ and $\vec{V_2}$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V_1} = \left(\hat{i} + \sqrt{3}\hat{j}\right)$, the angle between $\vec{V_1}$ and $\vec{V_2}$ is : [4 marks]

(A) 105°

(B) 15°

(C) -45°

(D) 60°

Correct Answer: (A) 105°

Solution:

Concept:

- The total linear momentum of an isolated colliding pair is conserved separately in each Cartesian direction. No assumption about kinetic-energy conservation is needed here.
- The angle between two nonzero vectors can be obtained from their directions or from $\cos\theta = (\mathbf{V}_1 \cdot \mathbf{V}_2)/(|\mathbf{V}_1||\mathbf{V}_2|)$, with $0 \le \theta \le 180°$.

Step 1: Find the second particle velocity The initial velocity of A is $\mathbf{u} = \sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}}$, while B is initially at rest. Write momentum conservation using $m_1 = 2m_2$:

$$2m_2\mathbf{u} = 2m_2\mathbf{V}_1 + m_2\mathbf{V}_2.$$

Thus

$$\begin{aligned}\mathbf{V}_2 &= 2(\mathbf{u} - \mathbf{V}_1)\\ &= 2(\sqrt{3}-1)\hat{\mathbf{i}} + 2(1-\sqrt{3})\hat{\mathbf{j}}\\ &= 2(\sqrt{3}-1)(\hat{\mathbf{i}} - \hat{\mathbf{j}})\ \mathrm{m\,s^{-1}}.\end{aligned}$$

Its x-component is positive and y-component negative, so its direction is -45° from the positive x-axis.

Step 2: Compare the two velocity directions The given final velocity of A is $\mathbf{V}_1 = \hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}}$. Both components are positive and

$$\tan\alpha = \frac{\sqrt{3}}{1}, \qquad \alpha = 60^\circ.$$

The smaller angle between the two velocity vectors is consequently

$$\theta = 60^\circ - (-45^\circ) = 105^\circ.$$

Option (C) gives the signed orientation of B, not the angle between the velocities. Option (D) gives the orientation of A alone. Option (B) would result from treating B's direction as $+45^\circ$, contrary to its negative y-component. The result uses the vector directions, not merely their speed magnitudes.

Step 3: Conclusion The angle between $\mathbf{V}_1$ and $\mathbf{V}_2$ is 105°. Hence, the correct option is (A).

Quick Tip to Solve: Find the unknown velocity by vector momentum conservation, then locate both vectors in their correct quadrants before subtracting their angles.

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35. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1 + \frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is : [4 marks]

(A) $\frac{2}{5}ML^2$
(B) $\frac{5}{12}ML^2$
(C) $\frac{7}{18}ML^2$
(D) $\frac{3}{7}ML^2$

Correct Answer: (C) $\frac{7}{18}ML^2$

Solution:

Concept:

- For a thin slice of rod of length dx, its mass is $dm = \lambda(x)dx$. The total mass is found by integrating the stated density over the rod.
- The moment of inertia about a perpendicular axis through A is $I_A = \int_0^L x^2\,dm$, since each slice is at perpendicular distance x from that axis.

Step 1: Express the density constant in terms of total mass Measure x from A as specified; the density rises linearly from λ_0 at A to $2\lambda_0$ at B. Integrating gives

$$M = \int_0^L \lambda_0 \left(1 + \frac{x}{L}\right) dx$$
$$= \lambda_0 \left[L + \frac{L^2}{2L}\right] = \frac{3}{2}\lambda_0 L.$$

Therefore $\lambda_0 = 2M/(3L)$. Using M/L for λ_0 would incorrectly treat the density at A as the average density and would give an incorrect inertia.

Step 2: Integrate the inertia contribution of each slice Weight each slice mass by the square of its distance from A:

$$I_A = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right) dx$$
$$= \lambda_0 \left(\frac{L^3}{3} + \frac{L^4}{4L}\right) = \frac{7}{12}\lambda_0 L^3$$
$$= \frac{7}{12}\frac{2M}{3L}L^3 = \frac{7}{18}ML^2.$$

This exceeds the uniform-rod value $ML^2/3$, as expected because more mass lies towards the distant end B. Direct integration fixes the coefficient at $7/18$; the coefficients $2/5$, $5/12$ and $3/7$ in the other options do not follow from the stated density and mass normalization.

Step 3: Conclusion The required moment of inertia is $I_A = \frac{7}{18}ML^2$, in $\mathrm{kg\,m^2}$ for SI data. Hence, the correct option is (C).

Quick Tip to Solve: For a variable-density rod, calculate $M = \int dm$ first and only then substitute the density constant into $I = \int x^2 dm$.

36. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T as : [4 marks]

(A) $\sqrt{T}$

(B) T

(C) $\frac{1}{T}$

(D) $\frac{1}{\sqrt{T}}$

Correct Answer: (A) $\sqrt{T}$

Solution:

Concept:

- For a dilute gas with fixed molecular collision diameter d, the mean free path is $\ell = 1/(\sqrt{2}\pi d^2 n)$, where n is the number of molecules per unit volume.
- The ideal-gas relation gives $n = P/(k_B T)$, and the characteristic molecular speed varies as $\sqrt{T}$. The mean collision time scales as free path divided by speed.

Step 1: Find the temperature dependence of the mean free path The comparison is made at the stated pressure P, for the same dilute gas, so the molecular mass and collision diameter are constant. Substitute its number density into the free-path formula:

$$\ell = \frac{k_{\mathrm{B}}T}{\sqrt{2}\pi d^2 P}.$$

Consequently $\ell \propto T$ at constant pressure. As temperature rises, the gas expands and its number density decreases; this increases the distance a molecule typically travels before a collision. Holding number density fixed instead would describe a different constraint and would lead to a different dependence.

Step 2: Divide by the molecular speed For example, the mean molecular speed is

$$\bar{v} = \sqrt{\frac{8k_{\mathrm{B}}T}{\pi m}},$$

where m is the molecular mass. The mean interval between collisions therefore has the dependence

$$\tau = \frac{\ell}{\bar{v}} \propto \frac{T}{\sqrt{T}} = \sqrt{T}.$$

Although molecules move faster at higher temperature, the reduction of collision partners per unit volume more than offsets that effect at constant pressure. Option (B) accounts only for the free path and omits the changing speed. Option (D) would apply to fixed number density rather than fixed pressure. Option (C) also fails to include both the density and speed dependence consistently.

Step 3: Conclusion At fixed pressure, the mean time between successive collisions varies as $\sqrt{T}$. Hence, the correct option is (A). The supplied key selects the fourth option, with inverse-square-root dependence on temperature; that dependence holds at fixed number density. For the stated pressure P, the ideal-gas relation gives $n \propto T^{-1}$, so $\tau \propto 1/(n\bar{v}) \propto \sqrt{T}$, proving (A) for this question.

Quick Tip to Solve: Always identify the quantity held fixed: at constant pressure $\ell \propto T$, whereas at constant density ℓ is temperature independent.

37. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to (1 + β)v, this angle changes to 45°. The value of β is close to : [4 marks]

(A) 0.50

(B) 0.73

(C) 0.37

(D) 0.41

Correct Answer: (B) 0.73

Solution:

Concept:

- The velocity seen by the driver is the rain velocity relative to the car, $\vec{u}_{\text{rel}} = \vec{u}_{\text{rain}} - \vec{v}_{\text{car}}$.
- Because the rain falls vertically when the car is stationary, let its downward ground speed be u. For a horizontal car speed v_c, the observed inclination θ to the horizontal satisfies $\tan\theta = u/v_c$.

Step 1: Resolve the first relative velocity The car moves horizontally and changes only the horizontal component of the observed rain velocity. Its vertical component remains the downward rain speed u, while the magnitude of its horizontal component is the car speed v. The angle is measured from the horizontal, so the vertical component belongs in the numerator. Thus

$$\tan 60^\circ = \frac{u}{v}, \qquad u = \sqrt{3}\, v.$$

Step 2: Use the second inclination When the car speed becomes $(1+\beta)v$, the downward component is still u. The new inclination gives

$$\tan 45^\circ = \frac{u}{(1+\beta)v} = 1.$$

Consequently, $(1+\beta)v = u = \sqrt{3}\, v$, and cancelling the nonzero speed yields

$$\beta = \sqrt{3} - 1 = 0.732\ldots.$$

Of the stated values, 0.73 is the close match. The values 0.50, 0.37 and 0.41 do not satisfy the required speed ratio $1 + \beta = \sqrt{3}$; using an angle measured from the vertical would incorrectly invert this ratio.

Step 3: Conclusion The required fractional increase is $\boxed{\beta \simeq 0.73}$, a dimensionless number. Hence, the correct option is (B).

Quick Tip to Solve: For rain angles measured from the horizontal, use vertical speed divided by car speed in the tangent; increasing car speed makes this angle smaller.

38. Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell? [4 marks]

(A) $\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^2} > F > 0$ for r < R

(B) $F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$ for r > R

(C) $F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$ for all r

(D) $F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^2}$ for r < R

Correct Answer: (B) $F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$ for r > R

Solution:

Concept:

- For a uniformly charged spherical shell, spherical symmetry makes its electric field radial and constant in magnitude on a concentric Gaussian sphere.
- Gauss's law is $\oint \vec{E}\cdot d\vec{A} = Q_{\text{enc}}/\varepsilon_0$. The force on a test charge is $\vec{F} = q\vec{E}$.

Step 1: Evaluate the field outside the shell Choose a concentric Gaussian sphere of radius $r > R$. It encloses the entire charge Q, so symmetry and Gauss's law give

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}, \qquad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$

The charge distribution therefore acts externally like a point charge at its centre. Multiplying by q gives the stated Coulomb expression for the radial force. For like charges it is outward; for unlike charges its direction reverses.

Step 2: Check the interior and eliminate the other statements A Gaussian sphere with $r < R$ encloses none of the shell's charge. Its flux is zero and spherical symmetry then implies

$$E(4\pi r^2) = 0, \qquad E = 0, \qquad F = 0.$$

Thus option (A), which requires a strictly positive interior force, is false. Option (D) wrongly assigns the nonzero surface-field value throughout the interior. Option (C) extends the exterior inverse-square expression into the empty interior, where it does not apply. Only option (B) restricts that expression to its correct region. The shell radius sets the boundary between the two regions, whereas the exterior field depends on the observation distance r.

Step 3: Conclusion For $r > R$, $\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}}$, with the radial direction determined by the signs of the charges. Hence, the correct option is (B).

Quick Tip to Solve: For a uniformly charged thin spherical shell, remember the two regions separately: zero field inside and a central point-charge field outside.

39. Assuming the nitrogen molecule is moving with r.m.s. velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to :

(Given : nitrogen molecule weight : 4.64 $\times$ 10^{-26} kg,

Boltzman constant: 1.38 $\times$ 10^{-23} J/K,

Planck constant : 6.63 $\times$ 10^{-34} J.s) [4 marks]

(A) 0.44 $\overset{o}{A}$

(B) 0.34 $\overset{o}{A}$

(C) 0.20 $\overset{o}{A}$

(D) 0.24 $\overset{o}{A}$

Correct Answer: (D) 0.24 $\overset{o}{A}$

Solution:

Concept:

- The translational rms speed of a molecule of mass m at temperature T is $v_{\text{rms}} = \sqrt{3k_BT/m}$. This uses the mass of one molecule, not the molar mass.
- The de Broglie wavelength is $\lambda = h/p = h/(mv)$. Combining the relations gives $\lambda = h/\sqrt{3mk_BT}$.

Step 1: Find the momentum at the specified speed Use the given molecular mass $m = 4.64 \times 10^{-26}\,\text{kg}$, temperature $T = 400\,\text{K}$ and Boltzmann constant $k_B = 1.38 \times 10^{-23}\,\text{J K}^{-1}$. Since the problem specifies rms speed, its translational kinetic energy is $3k_BT/2$. Therefore

$$p_{\text{rms}} = \sqrt{3mk_BT}.$$

Substituting carefully,

$$3mk_BT = 3(4.64 \times 10^{-26})(1.38 \times 10^{-23})(400)$$
$$= 7.68384 \times 10^{-46}\,\text{kg}^2\,\text{m}^2\,\text{s}^{-2}.$$

Taking the square root gives $p_{\text{rms}} \simeq 2.772 \times 10^{-23}\,\text{kg m s}^{-1}$.

Step 2: Calculate the wavelength and convert the unit Using the given Planck constant,

$$\lambda = \frac{6.63 \times 10^{-34}}{2.772 \times 10^{-23}}\,\text{m} \simeq 2.39 \times 10^{-11}\,\text{m}.$$

One angstrom is $10^{-10}\,\text{m}$, so

$$\lambda \simeq 0.239\,\text{Å} \simeq 0.24\,\text{Å}.$$

The alternatives 0.44 and 0.34 angstrom are too large, while 0.20 angstrom is too small for the computed momentum. Using the mean or most probable molecular speed instead of the explicitly specified rms speed would change the numerical coefficient and is inappropriate here.

Step 3: Conclusion The wavelength is $\boxed{\lambda \simeq 2.39 \times 10^{-11}\,\text{m} = 0.24\,\text{Å}}$. Hence, the correct option is (D).

Quick Tip to Solve: Combine rms momentum directly as $p = \sqrt{3mk_BT}$, then convert metres to angstroms only at the end to avoid powers-of-ten errors.

40. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field $\vec{B}$. The magnetic moment of this moving particle: [4 marks]

(A) $\frac{mv^2\vec{B}}{2B^2}$

(B) $-\frac{mv^2\vec{B}}{2B^2}$

(C) $-\frac{mv^2\vec{B}}{B^2}$

(D) $-\frac{mv^2\vec{B}}{2\pi B^2}$

Correct Answer: (B) $-\frac{mv^2\vec{B}}{2B^2}$

Solution:

Concept:

- A charge in a circular orbit is equivalent to a current of magnitude $I = |q|/T$, where $T = 2\pi r/v$ is its period. The loop moment has magnitude $\mu = I\pi r^2$.
- For motion perpendicular to a uniform magnetic field, the Lorentz force provides the centripetal force: $|q|vB = mv^2/r$. The moment direction follows conventional current, which reverses for a negative charge.

Step 1: Obtain the magnitude of the loop moment The magnetic force bends the particle into a circular path without changing its speed. Its orbit radius is

$$r = \frac{mv}{|q|B}.$$

The current magnitude and area are $I = |q|v/(2\pi r)$ and $A = \pi r^2$. Hence

$$\mu = IA = \frac{|q|vr}{2} = \frac{mv^2}{2B}.$$

The cancellation of π comes from multiplying the current by the circular area, while cancellation of $|q|$ follows after inserting the magnetic orbit radius. Neither a residual π nor an extra factor of two remains.

Step 2: Determine the vector direction and compare options Take $\vec{B}$ along the positive normal to the orbital plane. A positive charge circulates clockwise when viewed from that normal, so the right-hand rule for its conventional current gives a moment along the negative normal. A negative charge orbits in the opposite direction, but its conventional current reverses relative to its motion, again giving a moment opposite to $\vec{B}$. Therefore

$$\vec{\mu} = -\mu\frac{\vec{B}}{B} = -\frac{mv^2}{2B^2}\vec{B}.$$

Option (A) has the wrong direction, option (C) has twice the correct magnitude, and option (D) incorrectly retains a factor π in the denominator. The moment is diamagnetically opposed to the field regardless of the sign of the particle's charge.

Step 3: Conclusion The magnetic moment is $\boxed{\vec{\mu} = -\frac{mv^2}{2B^2}\vec{B}}$, opposite to the applied magnetic field. Hence, the correct option is (B).

Quick Tip to Solve: Compute $\mu = |q|vr/2$ for the magnitude, then use conventional current to obtain its direction; the orbital moment opposes the magnetic field.

41. A particle moving in the xy plane experiences a velocity dependent force $\vec{F} = k\left(v_y\hat{i} + v_x\hat{j}\right)$, where v_x and v_y are the x and y components of its velocity $\vec{v}$. If $\vec{a}$ is the acceleration of the particle, then which of the following statements is true for the particle? [4 marks]

(A) kinetic energy of particle is constant in time

(B) quantity $\vec{v} \times \vec{a}$ is constant in time

(C) quantity $\vec{v}.\vec{a}$ is constant in time

(D) $\vec{F}$ arises due to a magnetic field

Correct Answer: (B) quantity $\vec{v} \times \vec{a}$ is constant in time

Solution:

Concept:

- Newton's second law gives $a_x = (k/m)v_y$ and $a_y = (k/m)v_x$. Write $c = k/m$, a constant, to simplify the component equations.
- For motion in the xy plane, $\vec{v} \times \vec{a} = (v_x a_y - v_y a_x)\hat{k}$, while $\vec{v} \cdot \vec{a} = v_x a_x + v_y a_y$. A quantity is conserved if its time derivative vanishes.

Step 1: Test the cross product Substituting the acceleration components yields

$$\vec{v} \times \vec{a} = c(v_x^2 - v_y^2)\hat{k}.$$

Its derivative is

$$\frac{d}{dt}(v_x^2 - v_y^2) = 2v_x\dot{v}_x - 2v_y\dot{v}_y$$
$$= 2cv_xv_y - 2cv_yv_x = 0.$$

Thus $v_x^2 - v_y^2$ is conserved, and so the full cross-product vector, including its fixed normal direction, remains constant throughout the motion.

Step 2: Check the other statements The instantaneous power is

$$\frac{dK}{dt} = \vec{F} \cdot \vec{v} = 2kv_xv_y,$$

which is generally nonzero, ruling out (A). Also $\vec{v} \cdot \vec{a} = 2cv_xv_y$, whose derivative is

$$\frac{d}{dt}(\vec{v} \cdot \vec{a}) = 2c^2(v_x^2 + v_y^2).$$

It is generally nonzero, so (C) is false. A purely magnetic force always has zero dot product with velocity and does no work. The present force generally does work and its two components have the same exchange sign, unlike the antisymmetric components of a planar magnetic force. Therefore (D) is false as a general explanation.

Step 3: Conclusion Thus $\vec{v} \times \vec{a} = \frac{k}{m}(v_x^2 - v_y^2)\hat{k}$ is constant. Hence, the correct option is (B).

Quick Tip to Solve: For coupled equations $\dot{v}_x = cv_y$ and $\dot{v}_y = cv_x$, differentiate the difference of squares; its two terms cancel exactly.

42. A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P pascal. At another point where pressure is $\frac{P}{2}$ Pascal its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to : [4 marks]

(A) $\sqrt{\frac{P}{2\rho}+v^2}$

(B) $\sqrt{\frac{P}{\rho}+v^2}$

(C) $\sqrt{\frac{2P}{\rho}+v^2}$

(D) $\sqrt{\frac{P}{\rho}+v}$

Correct Answer: (B) $\sqrt{\frac{P}{\rho}+v^2}$

Solution:

Concept:

- For steady streamline flow of an ideal incompressible fluid, Bernoulli's equation is $P+\frac{1}{2}\rho u^2+\rho gh=$ constant along a streamline.
- The pipe is horizontal, so its two points have equal height and their gravitational terms cancel. A decrease of pressure therefore corresponds to an increase of kinetic energy per unit volume.

Step 1: Apply Bernoulli between the two points At the first point the pressure and speed are P and v. At the second they are $P/2$ and V. Substituting these values at equal elevation gives

$$P+\frac{1}{2}\rho v^2=\frac{P}{2}+\frac{1}{2}\rho V^2.$$

The pressure decrease $P/2$ supplies the kinetic energy increase: $\frac{1}{2}\rho(V^2-v^2)=P/2$. No height difference enters, and continuity need not be used because both pressures are already specified.

Step 2: Solve for the speed and inspect the alternatives Multiply the equation by $2/\rho$ to obtain

$$V^2-v^2=\frac{P}{\rho},\qquad V^2=v^2+\frac{P}{\rho}.$$

Since V is speed, take the positive root: $V=\sqrt{v^2+P/\rho}$. The ratio P/ρ has units $\mathrm{m^2\,s^{-2}}$, the same as v^2, so the dimensions agree. Option (A) underestimates the increase in squared speed by a factor of two, while (C) overestimates it by a factor of two. Option (D) adds a speed to a squared speed, which is dimensionally invalid. The resulting $V>v$ also agrees with the reduction in pressure and the physical energy balance.

Step 3: Conclusion The speed at the lower-pressure point is $\boxed{V=\sqrt{\frac{P}{\rho}+v^2}}\,\mathrm{m\,s^{-1}}$ when SI values are substituted. Hence, the correct option is (B).

Quick Tip to Solve: In a horizontal pipe, write the pressure drop as $\frac{1}{2}\rho\Delta(v^2)$; retain the one-half on both sides before simplifying.

43. For a plane electromagnetic wave, the magnetic field at a point x and time t is

$\vec{B}(x,t) = \left[1.2 \times 10^{-7} \sin\left(0.5 \times 10^3 x + 1.5 \times 10^{11} t\right) \hat{k}\right]$ **T**

The instantaneous electric field $\vec{E}$ corresponding to $\vec{B}$ is :

(speed of light c = 3 × 10^8 ms^{-1}) [4 marks]

(A) $\vec{E}(x,t) = \left[36 \sin\left(1 \times 10^3 x + 1.5 \times 10^{11} t\right) \hat{i}\right] \frac{V}{m}$

(B) $\vec{E}(x,t) = \left[36 \sin\left(0.5 \times 10^3 x + 1.5 \times 10^{11} t\right) \hat{k}\right] \frac{V}{m}$

(C) $\vec{E}(x,t) = \left[36 \sin\left(1 \times 10^3 x + 0.5 \times 10^{11} t\right) \hat{j}\right] \frac{V}{m}$

(D) $\vec{E}(x,t) = \left[-36 \sin\left(0.5 \times 10^3 x + 1.5 \times 10^{11} t\right) \hat{j}\right] \frac{V}{m}$

Correct Answer: (D) $\vec{E}(x,t) = \left[-36 \sin\left(0.5 \times 10^3 x + 1.5 \times 10^{11} t\right) \hat{j}\right] \frac{V}{m}$

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, the electric and magnetic fields have the same phase and satisfy $E_0 = cB_0$. Both fields are transverse to the propagation direction.
- The direction of energy propagation is the direction of $\vec{E} \times \vec{B}$. A phase of the form $kx + \omega t$ travels along the negative x-axis.

Step 1: Find the amplitude and retain the phase The magnetic amplitude is $B_0 = 1.2 \times 10^{-7}\,\mathrm{T}$. Therefore the electric amplitude is

$$E_0 = (3 \times 10^8)(1.2 \times 10^{-7}) = 36\,\mathrm{V\,m^{-1}}.$$

The two fields belong to the same travelling wave, so neither its wave number $0.5 \times 10^3\,\mathrm{m^{-1}}$ nor its angular frequency $1.5 \times 10^{11}\,\mathrm{s^{-1}}$ changes. Their ratio is also $\omega/k = 3 \times 10^8\,\mathrm{m\,s^{-1}}$, consistent with the given light speed.

Step 2: Determine the electric-field direction For a fixed phase, $kx + \omega t = \text{constant}$ gives $dx/dt = -\omega/k$, so propagation is along $-\hat{i}$. Since the magnetic field is along $\hat{k}$, the electric field must point along $-\hat{j}$ when the sine is positive:

$$(-\hat{j}) \times \hat{k} = -\hat{i}.$$

This determines the sign as well as the transverse axis. Option (A) has a longitudinal field and a different wave number. Option (B) makes the electric field parallel to the magnetic field. Option (C) changes both wave parameters and has the wrong direction. Only (D) keeps the required phase and direction.

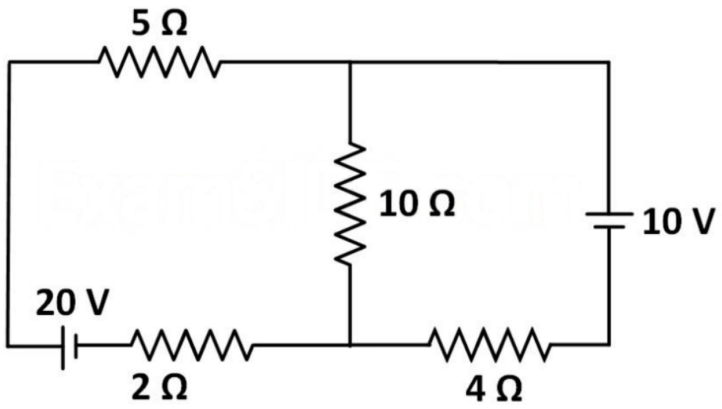
Step 3: Conclusion The corresponding field is

$$\vec{E}(x,t) = -36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\,\hat{j}\ \mathrm{V\,m^{-1}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For a phase $kx + \omega t$, first mark propagation as $-x$; then use $\vec{E} \times \vec{B}$ to select the sign of the transverse electric field.

44. In the figure shown, the current in the 10 V battery is close to : [4 marks]



(A) 0.36 A from negative to positive terminal

(B) 0.42 A from positive to negative terminal

(C) 0.71 A from positive to negative terminal

(D) 0.21 A from positive to negative terminal

Correct Answer: (D) 0.21 A from positive to negative terminal

Solution:

Concept:

- Kirchhoff's junction law requires the algebraic sum of the branch currents at a node to vanish. A branch containing a source and a resistor can be treated by its net voltage drop.
- Across an ideal cell the longer plate is the positive terminal. Current entering that terminal flows through the cell from positive to negative and charges it.

Step 1: Express the three branch currents Take the lower central junction as zero potential and let the upper junction be at potential V. Assign positive branch currents from the upper junction to the lower junction. The middle branch has resistance $10\,\Omega$, giving $I_m = V/10$. The left branch has $5 + 2 = 7\,\Omega$ and the 20 V cell is crossed from its positive to its negative plate on that route. The right branch similarly crosses the 10 V cell from positive to negative and then the $4\,\Omega$ resistor. Thus

$$I_l = \frac{V - 20}{7}, \qquad I_r = \frac{V - 10}{4}.$$

Negative values will indicate that the actual current is opposite to our chosen branch direction.

Step 2: Apply the junction law and identify the direction All three currents have been defined outward from the upper node, so their signed sum is zero:

$$\frac{V - 20}{7} + \frac{V}{10} + \frac{V - 10}{4} = 0.$$

Multiplication by 140 gives

$$20V - 400 + 14V + 35V - 350 = 0, \qquad V = \frac{750}{69} = \frac{250}{23}\,\text{V}.$$

The current in the 10 V cell is therefore

$$I_r = \frac{250/23 - 10}{4} = \frac{5}{23}\,\text{A} \simeq 0.217\,\text{A}.$$

It is positive in the assigned downward direction, which is from the positive plate to the negative plate within that battery. The magnitudes 0.36 A, 0.42 A and 0.71 A do not obey this node equation; option (A) also specifies the opposite direction.

Step 3: Conclusion The battery current is approximately $0.21\,\text{A}$, flowing from its positive to its negative terminal. Hence, the correct option is (D).

Quick Tip to Solve: For parallel branches containing cells, use one node voltage and write each current as $(V - \mathcal{E})/R$ with its actual plate polarity; the current sign then supplies the direction.

45. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm, 5.45 mm, 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter
of the pencil should therefore be recorded as : [4 marks]

(A) (5.54 ± 0.07) mm

(B) (5.5375 ± 0.0740) mm

(C) (5.5375 ± 0.0739) mm

(D) (5.538 ± 0.074) mm

Correct Answer: (A) (5.54 ± 0.07) mm

Solution:

Concept:

- A measured result is reported as a central value together with its uncertainty. The uncertainty ordinarily has one significant figure, except when retaining an extra digit is specifically justified.
- The reported central value is rounded to the same decimal place as the reported absolute uncertainty. Extra digits in a calculated average do not create additional experimental precision.

Step 1: Round the quoted uncertainty The problem supplies the standard deviation $0.07395\,\text{mm}$ as the measure of spread to use with the average. Its first significant digit is 7 in the hundredths place. Keeping one significant figure, the following digit is 3, so the 7 does not increase:

$$0.07395\,\text{mm} \longrightarrow 0.07\,\text{mm}.$$

The zeros before 7 merely locate the decimal point; they are not significant figures. We are reporting the supplied spread, rather than replacing it by a standard error of the mean or a least count that the question has not specified.

Step 2: Round the mean to the matching place The reported uncertainty is in hundredths of a millimetre, so the mean must also end at hundredths. The given mean is $5.5375\,\text{mm}$, and the next

digit after the hundredths digit 3 is 7. Therefore

$$5.5375\,\text{mm} \longrightarrow 5.54\,\text{mm}.$$

Combining the rounded values gives $(5.54 \pm 0.07)\,\text{mm}$. Options (B) and (C) retain several unjustified significant digits in both uncertainty and mean. Option (D) retains two significant figures in the uncertainty and a thousandths digit in the mean, whereas the usual uncertainty convention here requires only hundredths. The unrounded average remains useful during calculation but is not the appropriate final measurement notation.

Step 3: Conclusion The diameter should be recorded as

$$d = (5.54 \pm 0.07)\,\text{mm}.$$

Hence, the correct option is (A).

Quick Tip to Solve: Round the uncertainty first, then round the mean to the uncertainty's last decimal place; do not round the mean independently by counting its significant figures.

Physics – Section B: Numerical Value Questions (20 marks)

46. A Young's double-slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is _____.

[4 marks]

Solution:

Concept:

- For two equal-intensity coherent slits, the intensity at path difference Δ is $I = 4I_s \cos^2(\pi\Delta/\lambda)$, where I_s is the intensity contributed by either slit alone.
- A path difference equal to an integral multiple of the wavelength gives constructive interference. The phase difference is $\phi = 2\pi\Delta/\lambda$, so intensity is periodic in path difference.

Step 1: Use the known bright-fringe intensity At the first point, the path difference is $\Delta = \lambda$. Its phase difference is 2π, so the waves are in phase, even though their paths are not equal. Substitution into the interference expression gives

$$K = 4I_s \cos^2 \pi = 4I_s.$$

Consequently the stated intensity K is the maximum intensity of this two-slit pattern, and $I_s = K/4$. This relation fixes the normalization without requiring the source power, slit separation or screen distance. Using K as the intensity of a single slit instead would overcount the intensity by a factor of four.

Step 2: Calculate the new intensity and compare coefficients At the second point, $\Delta = \lambda/6$, hence

$$\phi = \frac{2\pi}{\lambda}\frac{\lambda}{6} = \frac{\pi}{3}.$$

The half-phase entering the cosine-squared formula is $\phi/2 = \pi/6$. Thus

$$I = 4I_s \cos^2\frac{\pi}{6} = K\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3K}{4}.$$

The question writes this same intensity as $nK/12$. Comparing the two forms,

$$\frac{nK}{12} = \frac{3K}{4} \quad \Longrightarrow \quad n = 12 \times \frac{3}{4} = 9.$$

The intensity is three quarters of its maximum, as expected for a point whose phase difference is smaller than the dark-fringe value π.

Step 3: Conclusion The intensity is $3K/4 = 9K/12$, so the required dimensionless integer is $\boxed{n = 9}$.

Quick Tip to Solve: Normalize by the known maximum intensity before substituting the new path difference, and remember that $\cos^2$ uses half the phase difference.

47. The centre of mass of solid hemisphere of radius 8 cm is x from the centre of the flat surface. Then value of x is _____. [4 marks]

Solution:

Concept:

- For a body of uniform density ρ, the centre-of-mass coordinate is $z_{\text{cm}} = \int z\, dm/M$. Rotational symmetry places the centre of mass on the axis perpendicular to the flat face.
- A solid hemisphere of radius R can be divided into thin circular discs at height z, measured from the centre of its flat surface. Each disc has area $\pi(R^2 - z^2)$.

Step 1: Set up the mass and first moment Choose the flat circular face as $z = 0$ and the curved pole as $z = R$. At height z, the spherical relation gives the disc radius squared as $R^2 - z^2$. A disc of thickness dz therefore has mass

$$dm = \rho\pi(R^2 - z^2)\, dz.$$

The hemisphere has total mass

$$M = \rho\,\frac{2\pi R^3}{3}.$$

Equal masses on opposite sides of the axis cancel all lateral first moments, leaving only the positive axial coordinate to calculate. The reference point is the centre of the flat surface, not the curved surface or a point on the circular rim.

Step 2: Evaluate the axial coordinate and substitute the radius The mass-weighted first moment

about the flat-face plane is

$$\int_0^R z\,dm = \rho\pi \int_0^R (R^2 z - z^3)\,dz.$$

Evaluating both terms gives

$$\rho\pi \left[\frac{R^2 z^2}{2} - \frac{z^4}{4}\right]_0^R = \frac{\rho\pi R^4}{4}.$$

Dividing by the total mass,

$$z_{\text{cm}} = \frac{\rho\pi R^4/4}{2\rho\pi R^3/3} = \frac{3R}{8}.$$

With $R = 8\,\text{cm}$, this becomes

$$x = \frac{3}{8}(8\,\text{cm}) = 3\,\text{cm}.$$

The value lies inside the hemisphere and is below its half-height because more of its volume is concentrated near the broad flat face. The expression $R/2$ would apply to a uniform thin hemispherical shell, not the solid body specified here.

Step 3: Conclusion The centre of mass is $\boxed{x = 3\,\text{cm}}$ from the centre of the flat surface, along the symmetry axis toward the curved surface. The numerical entry in centimetres is 3.

Quick Tip to Solve: Distinguish a solid hemisphere, with centre at $3R/8$, from a thin hemispherical shell, with centre at $R/2$; both distances start at the flat-face centre.

48. In a series LR circuit, power of 400W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\frac{n}{3\pi}\right)$ μF, then value of n is _____. [4 marks]

Solution:

Concept:

- The average power in an AC circuit is $P = VI\cos\phi = I^2R$, where V and I are rms values. In a series LR circuit, $\cos\phi = R/Z$ and $\sin\phi = X_L/Z$.
- Adding a capacitor in series makes the power factor unity when the net reactance vanishes: $X_C = X_L$. Capacitive reactance is $X_C = 1/(2\pi fC)$.

Step 1: Find the current, impedance and inductive reactance The initial circuit dissipates $400\,\text{W}$ at $250\,\text{V}$, with power factor 0.8. Its rms current follows from real power:

$$I = \frac{P}{V\cos\phi} = \frac{400}{250(0.8)} = 2\,\text{A}.$$

Therefore

$$Z = \frac{V}{I} = \frac{250}{2} = 125\,\Omega, \qquad R = Z\cos\phi = 100\,\Omega.$$

Since the circuit is inductive, $\sin\phi = \sqrt{1 - 0.8^2} = 0.6$. The inductor has reactance

$$X_L = Z\sin\phi = 125(0.6) = 75\,\Omega.$$

As a cross-check, $I^2R = 4(100) = 400\,\text{W}$, which recovers the stated dissipated power.

Step 2: Choose the capacitor for zero net reactance The frequency remains $f = 50\,\text{Hz}$ when the capacitor is added. Unity power factor in the series RLC circuit requires

$$2\pi fL - \frac{1}{2\pi fC} = 0, \qquad \frac{1}{2\pi fC} = 75\,\Omega.$$

Thus

$$C = \frac{1}{2\pi(50)(75)}\,\text{F} = \frac{1}{7500\pi}\,\text{F}.$$

Converting farads to microfarads,

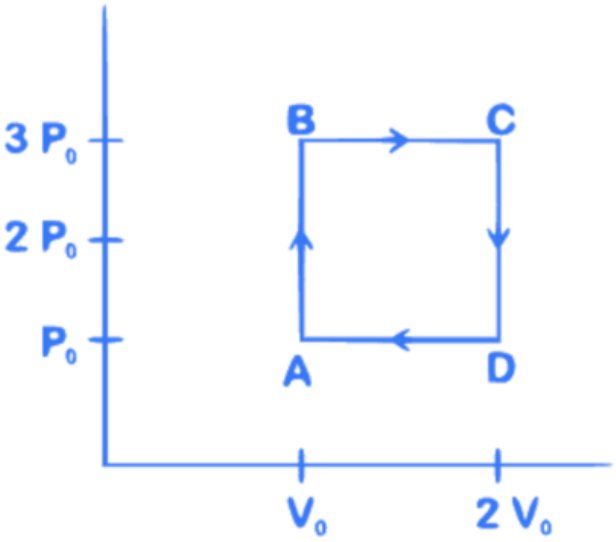
$$C = \frac{10^6}{7500\pi}\,\mu\text{F} = \frac{400}{3\pi}\,\mu\text{F}.$$

Comparing this with $C = n/(3\pi)\,\mu\text{F}$ gives $n = 400$. The initial 2 A current was used only to determine the original circuit parameters; after reactance cancellation the current changes, so it must not be imposed on the final resonant circuit.

Step 3: Conclusion The required capacitance is $C = \frac{400}{3\pi}\,\mu\text{F}$, and the requested dimensionless number is $\boxed{n = 400}$.

Quick Tip to Solve: Find I from real power first, then calculate X_L; series power-factor correction requires cancelling that reactance at the same source frequency.

49. An engine operates by taking a monoatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to _____. [4 marks]



Solution:

Concept:

- For a monoatomic ideal gas, internal energy is $U = \frac{3}{2}nRT = \frac{3}{2}PV$. Heat supplied on a process is $Q = \Delta U + W$, taking work done by the gas as positive.
- The efficiency of a heat engine is $\eta = W_{\text{net}}/Q_{\text{in}}$. Only positive heat transfers enter the heat-input denominator.

Step 1: Read the states and find cyclic work The corners are $A = (V_0, P_0)$, $B = (V_0, 3P_0)$, $C = (2V_0, 3P_0)$ and $D = (2V_0, P_0)$. The clockwise rectangle gives positive net work, equal to its area:

$$W_{\text{net}} = (3P_0 - P_0)(2V_0 - V_0) = 2P_0V_0.$$

The vertical processes do no work; the positive expansion work exceeds the negative compression work.

Step 2: Calculate the positive heat transfers On the constant-volume heating $A \to B$,

$$Q_{AB} = \tfrac{3}{2}(3P_0V_0 - P_0V_0) = 3P_0V_0.$$

On the constant-pressure expansion $B \to C$,

$$\Delta U_{BC} = \tfrac{3}{2}(6P_0V_0 - 3P_0V_0) = \tfrac{9}{2}P_0V_0, \qquad W_{BC} = 3P_0V_0.$$

Thus $Q_{BC} = \frac{15}{2}P_0V_0$. Both remaining legs cool the gas and reject heat, so

$$Q_{\text{in}} = 3P_0V_0 + \tfrac{15}{2}P_0V_0 = \tfrac{21}{2}P_0V_0.$$

Step 3: Evaluate the efficiency The common pressure-volume scale cancels:

$$\eta = \frac{2}{21/2} = \frac{4}{21}, \qquad 100\eta = 19.0476\ldots\%.$$

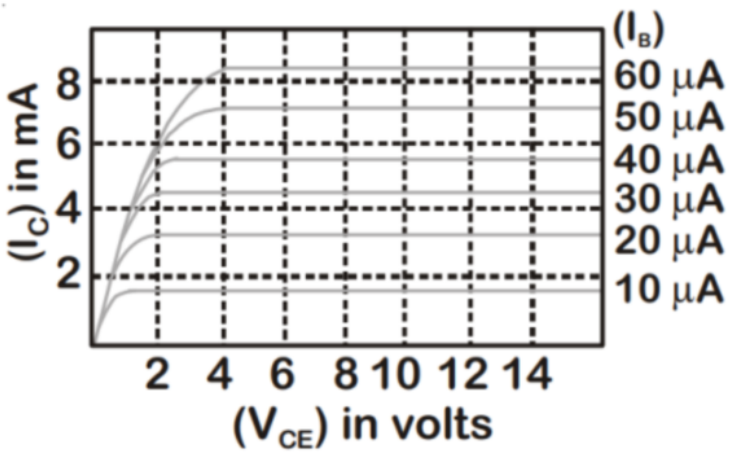
Counting the rejected heat as supplied heat would give an incorrect denominator.

Step 4: Conclusion The percentage efficiency is approximately $\boxed{19\%}$. The numerical entry is $\boxed{19}$.

Quick Tip to Solve: For a rectangular ideal-gas cycle, use the enclosed area for net work and add heat only along the two heating legs.

50. The output characteristics of a transistor is shown in the figure.

When V_{CE} is 10V and I_C = 4.0 mA, then value of β_{ac} is _____. [4 marks]



Solution:

Concept:

- The small-signal common-emitter current gain is $\beta_{\text{ac}} = (\Delta I_C/\Delta I_B)_{V_{CE}}$. It is a ratio of current changes at a fixed collector-emitter voltage.
- An output-characteristic family labels each curve by its base current. Read the vertical separation of nearby curves at the specified voltage rather than using the single operating-point ratio I_C/I_B.

Step 1: Read nearby output curves Draw a vertical reading line at $V_{CE} = 10\,\text{V}$, where the curves are almost horizontal. Around $I_C = 4.0\,\text{mA}$, the curves for $I_B = 30\,\mu\text{A}$ and $50\,\mu\text{A}$ have collector currents approximately $4.0\,\text{mA}$ and $7.0\,\text{mA}$, respectively. These lie at and near the operating region and give the local incremental slope.

Step 2: Form the ratio with consistent units The corresponding changes are

$$\Delta I_C = 7.0 - 4.0 = 3.0\,\text{mA}, \qquad \Delta I_B = 50 - 30 = 20\,\mu\text{A}.$$

Therefore

$$\beta_{\text{ac}} = \frac{3.0 \times 10^{-3}}{20 \times 10^{-6}} = 150.$$

The milliampere-to-microampere conversion contributes a factor of one thousand. Using the absolute collector current of $4.0\,\text{mA}$ in the numerator would calculate a different quantity, rather than the required change in current.

Step 3: Conclusion The alternating-current gain is $\boxed{\beta_{\text{ac}} = 150}$, a dimensionless current ratio.

Quick Tip to Solve: For ac gain, keep collector voltage fixed and divide the spacing of nearby collector currents by the spacing of their labelled base currents.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. For a reaction,

$4M(s) + nO_2(g) \rightarrow 2M_2O_n(s)$

the free energy change is plotted as a function of temperature. The temperature below which the oxide is stable could be inferred from the plot as the point at which : [4 marks]

(A) the free energy change shows a change from negative to positive value

(B) the slope changes from positive to negative

(C) the slope changes from negative to positive

(D) the slope changes from positive to zero

Correct Answer: (A) the free energy change shows a change from negative to positive value

Solution:

Concept:

- The formation of an oxide from its elements is thermodynamically favourable when its reaction Gibbs energy satisfies $\Delta G < 0$. The limiting temperature for equilibrium has $\Delta G = 0$.
- For approximately constant enthalpy and entropy changes, $\Delta G = \Delta H - T\Delta S$, so a Gibbs-energy plot has slope $-\Delta S$. Its sign of height and its slope describe different properties.

Step 1: Apply the stability criterion The reaction $4\,\mathrm{M(s)} + \mathrm{nO_2(g)} \longrightarrow 2\,\mathrm{M_2O}_n\mathrm{(s)}$ consumes gaseous oxygen and produces a solid oxide. Below the stability limit, oxide formation has negative free-energy change. At the limit, metal and oxide are in equilibrium. On heating past it, the formation free-energy change becomes positive, and formation is no longer spontaneous under the stated conditions.

Step 2: Distinguish a zero crossing from a slope change The required temperature is therefore where the plotted ΔG crosses from negative to positive values as temperature increases. A change from positive to negative slope, from negative to positive slope, or from positive to zero slope says how the reaction entropy changes. None of those slope conditions by itself establishes $\Delta G = 0$. Thus the three slope-based alternatives do not identify the oxide stability boundary.

Step 3: Conclusion The boundary is the change of free energy from negative to positive. Hence, the correct option is (A).

Quick Tip to Solve: On an Ellingham-style plot, locate the zero of formation Gibbs energy for a stability boundary; a kink alone is not that boundary.

52. For the given cell :

$Cu(s) \mid Cu^{2+}(C_1M) \mid\mid Cu^{2+}(C_2M) \mid Cu(s)$

change in Gibbs energy (ΔG) is negative, if : [4 marks]

(A) $C_2 = \sqrt{2}C_1$

(B) $C_2 = \frac{C_1}{\sqrt{2}}$

(C) $C_1 = 2C_2$

(D) $C_1 = C_2$

Correct Answer: (A) $C_2 = \sqrt{2}C_1$

Solution:

Concept:

- The copper electrodes form a concentration cell, so their standard electrode potentials cancel: $E^\circ_{\text{cell}} = 0$. The reduction potential of Cu^{2+}/Cu increases with copper-ion concentration.
- For a cell written with the left electrode as anode and right electrode as cathode, $E_{\text{cell}} = E_{\text{right}} - E_{\text{left}}$ and $\Delta G = -nFE_{\text{cell}}$, with $n = 2$.

Step 1: Write the concentration-cell emf The Nernst equation for either copper electrode is

$$E = E^\circ + \frac{RT}{2F} \ln[Cu^{2+}].$$

Subtracting left from right gives

$$E_{\text{cell}} = \frac{RT}{2F} \ln \frac{C_2}{C_1}.$$

Consequently,

$$\Delta G = -RT \ln \frac{C_2}{C_1}.$$

Since $R, T > 0$, negative Gibbs-energy change requires $C_2/C_1 > 1$. This means the right-hand copper-ion solution must be more concentrated.

Step 2: Check all concentration relations For $C_2 = \sqrt{2}C_1$, the ratio is $\sqrt{2} > 1$, so the emf is positive and ΔG is negative. For $C_2 = C_1/\sqrt{2}$, the ratio is less than one, reversing both signs. For $C_1 = 2C_2$, the ratio is $1/2$, again giving positive ΔG. Equal concentrations give zero emf and zero Gibbs-energy change, rather than a negative value.

Step 3: Conclusion The required condition is $\boxed{C_2 = \sqrt{2}C_1}$. Hence, the correct option is (A).

Quick Tip to Solve: For identical metal electrodes, the more concentrated metal-ion solution is the cathode; check the cell direction before assigning the sign of Gibbs energy.

53. Which one of the following statements is not true? [4 marks]

(A) Lactose contains α-glycosidic linkage between C_1 of galactose and C_4 of glucose

(B) Lactose is a reducing sugar and it gives Fehling's test

(C) On acid hydrolysis, lactose gives one molecule of D(+)-glucose and one molecule of D(+)-galactose

(D) Lactose ($C_{11}H_{22}O_{11}$) is a disaccharide and it contains 8 hydroxyl groups

Correct Answer: Both (A) and (D) are false as printed: beta linkage is required, and the molecular formula must be C12H22O11.

Solution:

Concept:

- Lactose is a disaccharide formed from galactose and glucose with a $\beta(1 \rightarrow 4)$ glycosidic linkage. The anomeric carbon of galactose is joined to carbon 4 of glucose.
- The glucose anomeric carbon remains a free hemiacetal, so lactose can open to an aldehyde form and acts as a reducing sugar. Its molecular formula is $C_{12}H_{22}O_{11}$.

Step 1: Test the linkage and reducing-sugar statements The linkage in lactose is beta, not alpha. The carbon positions stated in the first option are correct, but its alpha designation makes that statement false. The free anomeric centre on the glucose unit explains why lactose reduces Fehling's reagent; the second statement is therefore true. Acid hydrolysis breaks the glycosidic bond and gives one molecule each of D-glucose and D-galactose, so the third statement is true as well.

Step 2: Check the last statement literally Two hexose units combine with loss of one water molecule:

$$C_6H_{12}O_6 + C_6H_{12}O_6 \longrightarrow C_{12}H_{22}O_{11} + H_2O.$$

Lactose contains eight free hydroxyl groups: two hydroxyl groups of the separate monosaccharides participate in forming the glycosidic bond. However, the last option prints $C_{11}H_{22}O_{11}$, which has the wrong carbon count. Thus that statement also contains a false assertion, even though its disaccharide and hydroxyl-group descriptions are correct.

Step 3: Conclusion The intended linkage answer is (A): lactose has a beta linkage. Taken literally, (D) is also false because its printed formula has 11 rather than 12 carbons. Therefore the question as printed does not have a unique false statement.

Quick Tip to Solve: Check the anomeric configuration and the molecular formula separately: two six-carbon sugar units retain all twelve carbons after losing water.

54. The correct match between Item-I (starting material) and Item-II (reagent) for the preparation of benzaldehyde is :

Item-I	Item-II
(I) Benzene	(P) HCl and $SnCl_2$, H_3O^+
(II) Benzonitrile	(Q) H_2, Pd-$BaSO_4$, S and quinoline
(III) Benzoyl Chloride	(R) CO, HCl and $AlCl_3$

[4 marks]

(A) (I) - (R), (II) - (P) and (III) - (Q)

(B) (I) - (P), (II) - (Q) and (III) - (R)

(C) (I) - (Q), (II) - (R) and (III) - (P)

(D) (I) - (R), (II) - (Q) and (III) - (P)

Correct Answer: (A) (I) - (R), (II) - (P) and (III) - (Q)

Solution:

Concept:

- The correct reagent must transform the listed starting material into C_6H_5CHO, without reducing the aldehyde further. Identify the functional-group change before matching the reagent letters.
- Benzene undergoes formylation, benzonitrile undergoes controlled reduction followed by hydrolysis, and benzoyl chloride undergoes selective catalytic hydrogenation.

Step 1: Match benzene and benzonitrile Benzene reacts with carbon monoxide and hydrogen chloride in the presence of a Lewis acid such as $AlCl_3$ in the Gattermann–Koch formylation. Thus starting material (I) matches reagent (R). Benzonitrile is converted by $SnCl_2/HCl$ to an iminium salt, which on acidic hydrolysis gives benzaldehyde. This is the Stephen reduction sequence, so (II) matches (P).

Step 2: Match the acid chloride and eliminate other pairings Benzoyl chloride is selectively hydrogenated with hydrogen over poisoned palladium on barium sulfate in the Rosenmund reduction. Sulfur and quinoline suppress further reduction of the aldehyde. Therefore (III) matches (Q). These assignments give

$$(\mathrm{I}) - (\mathrm{R}), \qquad (\mathrm{II}) - (\mathrm{P}), \qquad (\mathrm{III}) - (\mathrm{Q}).$$

The second and third alternatives incorrectly assign benzene to reagents for other functional groups. The fourth retains the benzene match but swaps the specific nitrile and acid-chloride reductions.

Step 3: Conclusion The match is (I)-(R), (II)-(P), (III)-(Q). Hence, the correct option is (A).

Quick Tip to Solve: Remember the starting groups: Gattermann–Koch uses an aromatic ring, Stephen uses a nitrile, and Rosenmund uses an acid chloride.

55. For a d^4 metal ion in an octahedral field, the correct electronic configuration is : [4 marks]

(A) $t_{2g}^4 e_g^0$ when $\Delta_0 < P$

(B) $t_{2g}^3 e_g^1$ when $\Delta_0 > P$

(C) $e_g^2 t_{2g}^2$ when $\Delta_0 < P$

(D) $t_{2g}^3 e_g^1$ when $\Delta_0 < P$

Correct Answer: (D) $t_{2g}^3 e_g^1$ when $\Delta_0 < P$

Solution:

Concept:

- An octahedral ligand field splits the five d orbitals into the lower-energy t_{2g} set and the higher-energy e_g set. The separation is the crystal-field splitting energy Δ_o.
- The fourth d electron either pairs in a lower orbital at an energy cost P, the pairing energy, or enters a higher orbital at an energy cost Δ_o. Comparing these two costs decides the spin state.

Step 1: Fill the three lower orbitals first For a d^4 ion, the first three electrons occupy the three different t_{2g} orbitals singly, following Hund's rule. At this stage the distribution is t_{2g}^3, with each lower orbital occupied once. The position of the fourth electron is therefore the only decision needed to distinguish the two possible ground-state configurations.

Step 2: Compare promotion with pairing When $\Delta_o < P$, occupying an e_g orbital costs less energy than pairing in t_{2g}. The fourth electron consequently enters e_g, giving the high-spin configuration

$$t_{2g}^3 e_g^1.$$

When $\Delta_o > P$, pairing is cheaper and the low-spin configuration is $t_{2g}^4 e_g^0$. Thus (A) and (B) attach their configurations to the wrong inequality. Option (C) puts two electrons in the higher set before the lower set is singly filled, so it is not the ground-state distribution for the stated case.

Step 3: Conclusion The correct weak-field configuration is $t_{2g}^3 e_g^1$ when $\Delta_o < P$. Hence, the correct option is (D).

Quick Tip to Solve: For the fourth octahedral d electron, compare the price of promotion with the price of pairing: the cheaper process determines high spin or low spin.

56. The value of K_C is 64 at 800 K for the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The value of K_C for the following reaction is :

$NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{3}{2}H_2(g)$ [4 marks]

(A) 8

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{64}$

Correct Answer: (B) $\frac{1}{8}$

Solution:

Concept:

- Reversing a balanced reaction replaces its equilibrium constant K by $1/K$, because the product and reactant concentration factors exchange places.
- Multiplying every stoichiometric coefficient by a factor r raises the equilibrium constant to the power r. All constants here refer to the same temperature, $800\,\mathrm{K}$.

Step 1: Reverse the given reaction The supplied equilibrium is

$$\mathrm{N_2(g)} + 3\,\mathrm{H_2(g)} \rightleftharpoons 2\,\mathrm{NH_3(g)}, \qquad K_c = 64.$$

On reversing it, we obtain

$$2\,\mathrm{NH_3(g)} \rightleftharpoons \mathrm{N_2(g)} + 3\,\mathrm{H_2(g)}, \qquad K_{\text{reverse}} = \frac{1}{64}.$$

This reversal is essential: the target reaction consumes ammonia, whereas the given reaction forms it.

Step 2: Halve every coefficient The target reaction has one mole of ammonia on its left, so it is one-half of the reversed reaction. Therefore

$$K_{\text{target}} = \left(\frac{1}{64}\right)^{1/2} = \frac{1}{8}.$$

Equivalently, its concentration quotient is $[\mathrm{N_2}]^{1/2}[\mathrm{H_2}]^{3/2}/[\mathrm{NH_3}]$, the reciprocal square root of the original quotient. Option (A), 8, would correspond to halving without reversing. Option (D), $1/64$, accounts for reversal but omits halving. Option (C), $1/4$, is neither the required reciprocal square root nor the result of the stated coefficient change.

Step 3: Conclusion For the reaction as written, $K_c = 1/8$ at $800\,\mathrm{K}$. Hence, the correct option is (B).

Quick Tip to Solve: Express the new reaction as a multiple of the old one first; a multiplier of $-1/2$ means take the reciprocal square root of its equilibrium constant.

57. The average molar mass of chlorine is 35.5 g mol^{-1}. The ratio of ^{35}Cl to ^{37}Cl in naturally occuring chlorine is close to : [4 marks]

(A) 1 : 1

(B) 2 : 1

(C) 3 : 1

(D) 4 : 1

Correct Answer: (C) 3 : 1

Solution:

Concept:

- The mean molar mass of an isotopic mixture is its abundance-weighted mean. Isotopes have the same atomic number but different mass numbers.
- If x is the atomic fraction of $^{35}\mathrm{Cl}$, then $1-x$ is that of $^{37}\mathrm{Cl}$; their approximate molar masses are 35 and $37\,\mathrm{g\,mol^{-1}}$.

Step 1: Write the weighted-average equation The average is much closer to 35 than to 37, so the lighter isotope must be more abundant. Using the two isotope fractions gives

$$35x + 37(1-x) = 35.5.$$

The fractions must add to one; simply averaging 35 and 37 would assume equal abundance and would produce 36 instead of the stated 35.5.

Step 2: Find the fraction and convert it to a ratio Expanding and rearranging the equation,

$$37 - 2x = 35.5, \qquad 2x = 1.5, \qquad x = 0.75.$$

The heavier isotope therefore has fraction 0.25. Hence the number ratio is

$$^{35}\mathrm{Cl} : {}^{37}\mathrm{Cl} = 0.75 : 0.25 = 3 : 1.$$

As a check, three atoms of mass 35 together with one of mass 37 have mean $(3\times 35+37)/4 = 35.5$. The other proposed ratios give means 36 for 1:1, approximately 35.67 for 2:1, and 35.4 for 4:1. The ratio 3:1 reproduces the specified average using the given integer isotope masses.

Step 3: Conclusion The natural abundance ratio of $^{35}\mathrm{Cl}$ to $^{37}\mathrm{Cl}$ is close to $3:1$. Hence, the correct option is (C).

Quick Tip to Solve: For two isotopes, their abundance ratio equals the inverse ratio of the distances of their masses from the mean: here $(37-35.5) : (35.5-35) = 3:1$.

58. A set of solutions is prepared using 180 g of water as a solvent and 10 g of different nonvolatile solutes A, B and C. The relative lowering of vapour pressure in the presence of these solutes are in the order :

[Given, molar mass of A = 100 g mol^{-1}; B = 200 g mol^{-1}; C = 10,000 g mol^{-1}] [4 marks]

(A) A > C > B

(B) C > B > A

(C) A > B > C

(D) B > C > A

Correct Answer: (C) A > B > C

Solution:

Concept:

- For an ideal solution containing a nonvolatile molecular solute, Raoult's law gives relative lowering of vapour pressure as the solute mole fraction: $(p^0 - p)/p^0 = x_s$.
- For identical solvent amounts, $x_s = n_s/(n_w + n_s)$ increases with solute moles n_s. At fixed solute mass, the number of moles is inversely proportional to molar mass.

Step 1: Calculate the mole amounts The solvent amount is common to the three solutions:

$$n_w = \frac{180}{18} = 10\,\text{mol}.$$

For the equal 10 g solute portions,

$$n_A = \frac{10}{100} = 0.10\,\text{mol}, \qquad n_B = \frac{10}{200} = 0.050\,\text{mol},$$

$$n_C = \frac{10}{10000} = 0.0010\,\text{mol}.$$

Thus the highest-molar-mass solute contributes the fewest particles for the same mass.

Step 2: Compare the relative lowerings The respective solute mole fractions are

$$x_A = \frac{0.10}{10.10} \approx 0.009901, \qquad x_B = \frac{0.050}{10.050} \approx 0.004975,$$

$$x_C = \frac{0.0010}{10.0010} \approx 0.0001000.$$

These exact mole-fraction comparisons establish $x_A > x_B > x_C$; using the dilute approximation would give the same order. Option (A) incorrectly places C above B. Option (B) reverses the effect of molar mass, and option (D) incorrectly puts B above A. It is the count of solute particles, rather than the equal solute masses alone, that controls the colligative effect.

Step 3: Conclusion The relative lowering of vapour pressure follows $A > B > C$. Hence, the correct option is (C).

Quick Tip to Solve: When solvent mass and nonvolatile solute mass are fixed, rank this colligative effect by inverse molar mass: the lightest solute gives the greatest lowering.

59. Mischmetal is an alloy consisting mainly of : [4 marks]

(A) lanthanoid and actinoid metals

(B) actinoid and transition metals

(C) lanthanoid metals

(D) actinoid metals

Correct Answer: (C) lanthanoid metals

Solution:

Concept:

- Mischmetal is a lanthanoid-rich alloy, prepared as a mixture of rare-earth metals rather than as a single pure element. Its defining constituent family is the lanthanoids.
- The standard textbook composition is about 95 percent lanthanoid metals and about 5 percent iron, with small amounts of other elements such as carbon, sulphur, calcium and aluminium.

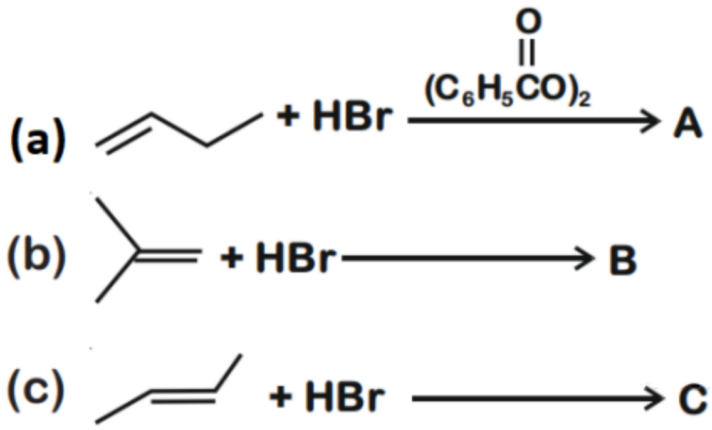
Step 1: Identify the predominant metallic family The word mainly asks for the majority constituents of the alloy. The lanthanoid portion includes metals such as cerium, lanthanum, neodymium and praseodymium; cerium is typically a major component. Since this rare-earth fraction makes up nearly the whole alloy, the appropriate description among the choices is lanthanoid metals. The presence of a smaller iron fraction does not change which family predominates.

Step 2: Check the alternatives against the composition Option (C) names the lanthanoid metals and therefore matches the principal constituents. Option (A) additionally makes actinoid metals a principal family, which is not the textbook composition of mischmetal. Option (B) combines actinoids with transition metals and leaves out the dominant lanthanoid fraction. Option (D) names only actinoids and is also inconsistent with the alloy. Lanthanoids and actinoids are both inner-transition series, but they are not interchangeable names: the lanthanoid series belongs to 4f filling, whereas the actinoid series belongs to 5f filling. This distinction is what separates the otherwise similar-sounding alternatives.

Step 3: Conclusion Mischmetal consists mainly of lanthanoid metals. Hence, the correct option is (C).

Quick Tip to Solve: Remember mischmetal as the lanthanoid-rich rare-earth alloy; minor iron content does not make actinoids part of its principal composition.

60. The increasing order of the boiling points of the major products A, B and C of the following reactions will be : [4 marks]



(A) A < C < B

(B) C < A < B

(C) B < C < A

(D) A < B < C

Correct Answer: (C) B < C < A

Solution:

Concept:

- Addition of HBr to an unsymmetrical alkene follows Markovnikov orientation without peroxide. In the presence of peroxide, the radical mechanism reverses the orientation for HBr.
- For isomeric alkyl bromides of the same molecular formula, a more compact carbon framework generally has a lower boiling point because reduced surface contact weakens intermolecular dispersion attractions.

Step 1: Identify each addition product In reaction (a), but-1-ene reacts with HBr in the presence of benzoyl peroxide. Anti-Markovnikov addition puts bromine on the terminal carbon, giving $A = CH_3CH_2CH_2CH_2Br$, 1-bromobutane. Reaction (b) starts with 2-methylpropene and has no peroxide. Markovnikov addition gives $B = (CH_3)_3CBr$, 2-bromo-2-methylpropane. Reaction (c) adds HBr to but-2-ene, giving $C = CH_3CHBrCH_2CH_3$, 2-bromobutane; the two double-bond carbons are equivalent for constitutional product identification.

Step 2: Rank the isomeric bromides All three products have formula C_4H_9Br, so their molar masses are equal. The tertiary, most compact bromide B has the lowest boiling point. The secondary bromide C boils below the less compact primary bromide A. The increasing order is therefore

$$B < C < A.$$

This agrees with the familiar boiling-point pattern of these butyl bromide isomers. Options (A) and (B) wrongly place B last, despite its compact structure. Option (D) puts the primary bromide A first and reverses the comparison of A with the other two products.

Step 3: Conclusion The increasing order of boiling points is $B < C < A$, where B is tert-butyl

bromide, C is sec-butyl bromide and A is n-butyl bromide. Hence, the correct option is (C).

Quick Tip to Solve: Identify the peroxide-dependent product before ranking boiling points: here peroxide makes A the primary bromide, while the most compact tertiary bromide B boils first.

61. Match the following :

Test/Method	Reagent
(i) Lucas Test	(a) $C_6H_5SO_2Cl$/ aq. KOH
(ii) Dumas method	(b) HNO_3/ $AgNO_3$
(iii) Kjeldahl's method	(c) CuO/CO_2
(iv) Hinsberg test	(d) Conc. HCl and $ZnCl_2$
	(e) H_2SO_4

[4 marks]

(A) (i)-(b), (ii)-(d), (iii)-(e), (iv)-(a)

(B) (i)-(d), (ii)-(c), (iii)-(e), (iv)-(a)

(C) (i)-(b), (ii)-(a), (iii)-(c), (iv)-(d)

(D) (i)-(d), (ii)-(c), (iii)-(b), (iv)-(e)

Correct Answer: (B) (i)-(d), (ii)-(c), (iii)-(e), (iv)-(a)

Solution:

Concept:

- The Lucas test distinguishes alcohols through the rate of substitution of the hydroxyl group using concentrated hydrochloric acid and anhydrous zinc chloride.
- Dumas and Kjeldahl methods determine nitrogen by different transformations. The Hinsberg test distinguishes classes of amines using benzenesulphonyl chloride in aqueous alkali.

Step 1: Match the alcohol and amine tests Lucas reagent is concentrated HCl with $ZnCl_2$, so (i) matches (d). Zinc chloride acts as a Lewis acid and facilitates replacement of the hydroxyl group by chloride. The Hinsberg reagent is $C_6H_5SO_2Cl$ in aqueous KOH, hence (iv) matches (a). Its sulphonamide products differ in their behaviour toward alkali, which allows the amine classes to be distinguished.

Step 2: Match the nitrogen estimation methods In the Dumas method the organic compound is heated with CuO in an atmosphere of CO_2; nitrogen is ultimately collected as N_2. Thus (ii) matches (c). In Kjeldahl's method concentrated H_2SO_4 digests the sample and converts suitable organic nitrogen into ammonium sulphate. Therefore (iii) matches (e). The pair $HNO_3/AgNO_3$ instead belongs to the test for halogens after sodium fusion and acidification.

Step 3: Select the complete matching sequence The required sequence is (i)-(d), (ii)-(c), (iii)-(e), (iv)-(a). Option (A) incorrectly assigns the Lucas test to silver nitrate, while option (C) also exchanges the elemental-analysis methods. Option (D) gets Lucas and Dumas right but gives the halogen reagent to Kjeldahl and sulphuric acid to Hinsberg, so it fails the last two matches.

Step 4: Conclusion Lucas uses hydrochloric acid/zinc chloride, Dumas uses copper(II) oxide/carbon

dioxide, Kjeldahl uses sulphuric acid, and Hinsberg uses benzenesulphonyl chloride in alkali. Hence, the correct option is (B).

Quick Tip to Solve: Anchor Lucas to concentrated $HCl/ZnCl_2$ and Hinsberg to $C_6H_5SO_2Cl$; then distinguish Dumas combustion from Kjeldahl acid digestion.

62. Which of the following compounds can be prepared in good yield by Gabriel phthalimide synthesis?

[4 marks]

(A) $C_6H_5NH_2$

(B) $C_6H_5CH_2NH_2$

(C) $C_6H_5CH_2CONH_2$

(D) $CH_3CH_2NHCH_3$

Correct Answer: (B) $C_6H_5CH_2NH_2$

Solution:

Concept:

- Gabriel synthesis uses the phthalimide anion as a nitrogen nucleophile. Alkylation followed by cleavage releases a primary amine, RNH_2.
- The substitution step requires a suitable alkyl halide. A primary benzylic halide reacts readily, whereas an ordinary aryl halide does not undergo the required S_N2 displacement.

Step 1: Identify the amine accessible by alkylation Benzylamine, $C_6H_5CH_2NH_2$, can be made from benzyl chloride or benzyl bromide. The electrophilic carbon is the saturated CH_2 carbon outside the aromatic ring. Potassium phthalimide displaces the halide at this primary benzylic carbon to give N-benzylphthalimide. Hydrolysis or hydrazinolysis then cleaves the protecting group and liberates benzylamine in good yield. The benzene ring remains unchanged throughout.

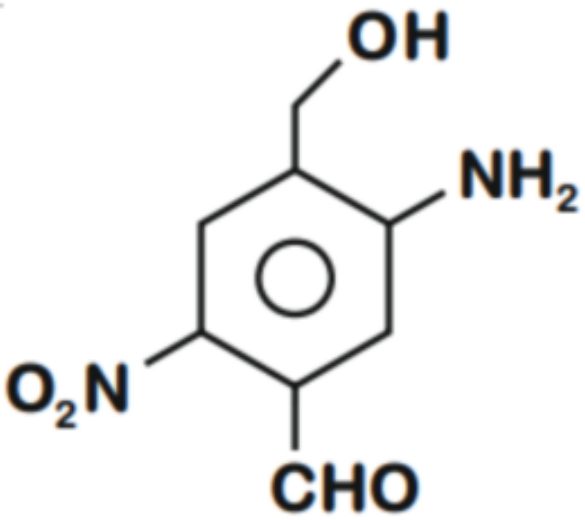
Step 2: Reject aniline and the amide Option (A), aniline, has nitrogen directly bonded to an aromatic ring carbon. Its proposed preparation by the usual Gabriel route would require displacement of halide from an unactivated aryl halide, which is unsuitable for the normal substitution step. Option (C), $C_6H_5CH_2CONH_2$, is phenylacetamide: its nitrogen is attached to a carbonyl carbon. Gabriel cleavage gives an amine and does not introduce the carbonyl group needed for this amide.

Step 3: Reject the secondary amine Option (D), $CH_3CH_2NHCH_3$, is a secondary amine containing two carbon groups on nitrogen. The nitrogen in the alkylated phthalimide acquires only one external alkyl group; cleavage supplies two N-H bonds and therefore gives a primary amine. It does not furnish this secondary amine as the intended product. Only option (B) satisfies both the primary-amine product requirement and the feasible benzylic substitution step.

Step 4: Conclusion Benzylamine, $C_6H_5CH_2NH_2$, can be prepared in good yield by Gabriel phthalimide synthesis. Hence, the correct option is (B).

Quick Tip to Solve: Check where nitrogen is attached: a benzylic CH_2NH_2 group works in Gabriel synthesis, but a directly attached aryl NH_2 group does not.

63. The IUPAC name of the following compound is : [4 marks]



(A) 2-nitro-4-hydroxymethyl-5-amino benzaldehyde

(B) 3-amino-4-hydroxymethyl-5-nitro benzaldehyde

(C) 4-amino-2-formyl-5-hydroxymehtyl nitrobenzene

(D) 5-amino-4-hydroxymethyl-2-nitro benzaldehyde

Correct Answer: (D) 5-amino-4-hydroxymethyl-2-nitro benzaldehyde

Solution:

Concept:

- Among the groups shown, the aldehyde group has higher naming priority than alcohol and amine groups. The ring is consequently named as a substituted benzaldehyde.
- The ring carbon carrying CHO is carbon 1. Choose the direction giving the lowest set of substituent locants, then list substituent prefixes alphabetically.

Step 1: Choose the parent and read the substituents The figure contains a benzene ring bearing CHO, NO_2, NH_2 and CH_2OH. The aldehyde supplies the parent name benzaldehyde. The alcohol is in a side chain, so that entire substituent is hydroxymethyl; it is not a phenolic hydroxy group. The remaining prefixes are amino and nitro. Number the ring carbon directly attached to the formyl group as position 1.

Step 2: Compare both directions of numbering Numbering toward the nitro group gives nitro at 2, hydroxymethyl at 4 and amino at 5. The ordered locant set is therefore $(2, 4, 5)$. Numbering in the opposite direction gives amino at 3, hydroxymethyl at 4 and nitro at 6, with locant set $(3, 4, 6)$. At the first point of difference, 2 is smaller than 3, so the former direction is mandatory. Alphabetical order of prefixes does not override this lowest-locant decision.

Step 3: Arrange the prefixes and compare the options The prefixes in alphabetical order are amino, hydroxymethyl and nitro. Keeping their selected locants attached gives 5-amino-4-hydroxymethyl-2-nitrobenzaldehyde. Option (A) has the right positions but does not arrange the prefixes alphabetically. Option (B) assigns the nitro group to an incorrect position and does not reproduce the diagram.

Option (C) uses nitrobenzene as the parent and demotes the aldehyde to formyl despite the aldehyde's higher priority.

Step 4: Conclusion The name is 5-amino-4-hydroxymethyl-2-nitrobenzaldehyde. Hence, the correct option is (D).

Quick Tip to Solve: First choose the principal functional group, then minimize the locant set; alphabetize prefixes only after the numbering is fixed.

64. The correct match between Item - I and Item - II is :

Item - I	Item - II
(a) Natural rubber	(I) 1, 3-butadiene + styrene
(b) Neoprene	(II) 1, 3-butadiene + acrylonitrile
(c) Buna-N	(III) Chloroprene
(d) Buna-S	(IV) Isoprene

[4 marks]

(A) (a) - (IV), (b) - (III), (c) - (II), (d) - (I)

(B) (a) - (III), (b) - (IV), (c) - (I), (d) - (II)

(C) (a) - (III), (b) - (IV), (c) - (II), (d) - (I)

(D) (a) - (IV), (b) - (III), (c) - (I), (d) - (II)

Correct Answer: (A) (a) - (IV), (b) - (III), (c) - (II), (d) - (I)

Solution:

Concept:

- Natural rubber is cis-1,4-polyisoprene. Its monomer is isoprene, or 2-methylbuta-1,3-diene.
- Neoprene is polychloroprene. Buna-S and Buna-N are copolymers of buta-1,3-diene with styrene and acrylonitrile, respectively.

Step 1: Match the two single-monomer rubbers Natural rubber forms by addition polymerisation of isoprene, $CH_2{=}C(CH_3){-}CH{=}CH_2$, giving the cis-1,4 arrangement characteristic of natural rubber. Thus (a) matches (IV). Neoprene is formed from chloroprene, $CH_2{=}CCl{-}CH{=}CH_2$, which is 2-chlorobuta-1,3-diene. Therefore (b) matches (III). The methyl group of isoprene and chlorine substituent of chloroprene identify these two monomers distinctly.

Step 2: Match the two Buna copolymers Buna-N is obtained by copolymerisation of buta-1,3-diene with acrylonitrile, $CH_2{=}CHCN$. The nitrile-containing comonomer accounts for the N designation and gives the match (c)-(II). Buna-S is the copolymer of buta-1,3-diene and styrene, $C_6H_5CH{=}CH_2$. Its S refers to styrene, hence (d)-(I). Both contain butadiene, so identifying only that common component cannot distinguish them.

Step 3: Check every offered sequence Combining the four independent assignments gives (a)-(IV), (b)-(III), (c)-(II), (d)-(I), exactly option (A). Options (B) and (C) exchange natural rubber and neoprene, assigning chloroprene to natural rubber and isoprene to neoprene. Option (D) correctly matches those first two rubbers but exchanges the Buna comonomers, incorrectly giving styrene to

Buna-N and acrylonitrile to Buna-S. It therefore cannot be selected.

Step 4: Conclusion Natural rubber corresponds to isoprene, neoprene to chloroprene, Buna-N to butadiene/acrylonitrile, and Buna-S to butadiene/styrene. Hence, the correct option is (A).

Quick Tip to Solve: Use N for the nitrile comonomer and S for styrene; natural rubber has isoprene and neoprene has chloroprene.

65. The element that can be refined by distillation is : [4 marks]

(A) gallium

(B) nickel

(C) zinc

(D) tin

Correct Answer: (C) zinc

Solution:

Concept:

- Distillation refines a metal that vaporises readily at a practical temperature while its impurities remain less volatile. The metal vapour is cooled to recover the purified metal.
- The criterion is a relatively low boiling point, rather than merely a low melting point. Zinc and mercury are standard examples of metals purified by this method.

Step 1: Apply the volatility criterion On heating impure zinc sufficiently, zinc passes into the vapour phase because it is relatively volatile. Nonvolatile impurities remain in the distillation vessel. The zinc vapour is led into a cooler receiver, where it condenses as purified zinc. This is a physical separation: the desired metal is recovered through vaporisation and condensation, without needing to dissolve it or selectively deposit it at an electrode.

Step 2: Distinguish the tempting low-melting metal Gallium melts close to room temperature, which can make it appear an attractive choice. However, easy melting does not mean easy boiling. Gallium has a very high boiling point and is associated with zone refining when very high purity is required. Distillation depends on the liquid-to-vapour separation and relative volatility, so a low melting point alone is not the relevant property. Zinc, with a much lower boiling point, is the standard suitable metal among these choices.

Step 3: Eliminate nickel and tin Nickel is classically purified by the Mond process: it forms volatile $Ni(CO)_4$, which is then decomposed to deposit pure nickel. That is chemical vapour-phase refining through a compound, not direct distillation of nickel metal. Tin can be purified by liquation when the impurities remain solid as the tin melts; its high boiling point makes direct distillation an unsuitable standard choice here. Thus the other listed metals do not replace zinc for this method.

Step 4: Conclusion Zinc can be refined by distillation. Hence, the correct option is (C).

Quick Tip to Solve: For distillation ask which metal vaporises easily; do not confuse gallium's low melting point with a low boiling point.

66. The reaction of NO with N_2O_4 at 250 K gives : [4 marks]

(A) N_2O_3

(B) N_2O_5

(C) N_2O

(D) NO_2

Correct Answer: (A) N_2O_3

Solution:

Concept:

- At low temperature nitric oxide and dinitrogen tetroxide combine to form dinitrogen trioxide, the anhydride corresponding to nitrous acid.
- The product can be identified by balancing nitrogen and oxygen atoms. In N_2O_3, nitrogen has the mean oxidation state $+3$.

Step 1: Write and balance the low-temperature reaction At the stated temperature, $250\,K$, the reaction is

$$2\,NO + N_2O_4 \longrightarrow 2\,N_2O_3.$$

The left side contains four nitrogen atoms and six oxygen atoms, and the two molecules of dinitrogen trioxide on the right contain the same totals. The printed conditions favour the low-temperature nitrogen(III) oxide product rather than requiring a higher oxide such as dinitrogen pentoxide. The formula of that product is N_2O_3.

Step 2: Check the oxidation states In nitric oxide nitrogen is at $+2$, while in N_2O_4 each nitrogen is at $+4$. Combining equal numbers of these nitrogen atoms gives a mean oxidation state of $+3$. For the balanced amounts, two $+2$ nitrogen atoms and two $+4$ nitrogen atoms give a total oxidation-state sum of $+12$ distributed over four nitrogen atoms. This agrees with two N_2O_3 molecules, in which nitrogen has a mean oxidation state of $+3$. The reaction is a comproportionation.

Step 3: Rule out the alternative oxides N_2O_5 has nitrogen at $+5$ and would require additional oxidation rather than this combination. N_2O has nitrogen at mean $+1$ and would require reduction beyond the given reactants. NO_2 has nitrogen at $+4$ and is linked to dissociation of N_2O_4; it is not the specified combination product of NO with N_2O_4 at 250 K. These distinctions leave the nitrogen(III) oxide as the consistent answer.

Step 4: Conclusion The reaction gives dinitrogen trioxide, N_2O_3. Hence, the correct option is (A).

Quick Tip to Solve: For NO $(+2)$ combining with N_2O_4 $(+4)$, check the intermediate nitrogen oxidation state $+3$ and balance N_2O_3.

67. A crystal is made up of metal ions 'M_1' and 'M_2' and oxide ions. Oxide ions form a ccp lattice structure. The cation 'M_1' occupies 50% of octahedral voids and the cation 'M_2' occupies 12.5% of tetrahedral voids of oxide lattice. The oxidation numbers of 'M_1' and 'M_2' are, respectively : [4 marks]

(A) + 3, + 1

(B) + 4, + 2

(C) + 1, + 3

(D) + 2, + 4

Correct Answer: (D) + 2, + 4

Solution:

Concept:

- For N oxide ions in a close-packed lattice, the numbers of octahedral and tetrahedral voids are N and $2N$, respectively. The cations occupy only the stated fractions of these sites.
- An ionic crystal is electrically neutral. Each oxide ion has charge -2, so the total positive charge of the metal ions must equal $2N$.

Step 1: Count the occupied sites The number of ions of the first metal is $0.50N = N/2$, since it occupies half of the octahedral voids. The second metal occupies $12.5\% = 1/8$ of the tetrahedral voids, so its population is $(1/8)(2N) = N/4$. Thus the ratio of the first metal, second metal and oxide ions is $2 : 1 : 4$. The number of tetrahedral sites must be doubled before applying the percentage.

Step 2: Apply charge neutrality Let the oxidation numbers of the first and second metals be x and y. Their positive charges balance the oxide charge:

$$\frac{N}{2}x + \frac{N}{4}y = 2N.$$

$$2x + y = 8.$$

This relation follows from the formula unit containing two ions of the first metal, one of the second metal and four oxide ions.

Step 3: Test the four ordered pairs For options (A), (B), (C) and (D), the values of $2x + y$ are 7, 10, 5 and 8, respectively. Only the last pair gives the required total positive charge. In particular, interchanging the two oxidation numbers does not preserve charge neutrality because their populations differ.

Step 4: Conclusion The oxidation numbers of M_1 and M_2 are $+2$ and $+4$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Convert site percentages into ion counts first: tetrahedral voids are twice as numerous as the oxide ions, while octahedral voids are equally numerous.

68. Dihydrogen of high purity ($>$ 99.95%) is obtained through : [4 marks]

(A) the electrolysis of acidified water using Pt electrodes

(B) the electrolysis of warm $Ba(OH)_2$ solution using Ni electrodes

(C) the electrolysis of brine solution

(D) the reaction of Zn with dilute HCl

Correct Answer: (B) the electrolysis of warm $Ba(OH)_2$ solution using Ni electrodes

Solution:

Concept:

- Electrolysis separates water into hydrogen at the cathode and oxygen at the anode. A suitable electrolyte provides ionic conductivity without being the intended source of the hydrogen.
- The prescribed method for dihydrogen of purity above 99.95% is electrolysis of warm aqueous barium hydroxide with nickel electrodes.

Step 1: Identify the specified preparation The combination of warm $Ba(OH)_2$ solution and nickel electrodes is the standard preparation associated with very high-purity dihydrogen. The electrolyte permits current to flow through the solution. Hydrogen is collected separately from the cathode, preventing its deliberate mixing with the oxygen formed at the anode.

Step 2: Write the electrode changes In the alkaline electrolyte, the cathodic reduction and anodic oxidation may be written as

$$2\,H_2O + 2\,e^- \longrightarrow H_2 + 2\,OH^-,$$

$$4\,OH^- \longrightarrow O_2 + 2\,H_2O + 4\,e^-.$$

Doubling the first equation and adding gives the net decomposition $2\,H_2O \longrightarrow 2\,H_2 + O_2$. Barium hydroxide supplies conductivity and is not consumed in this overall equation.

Step 3: Compare the alternatives Electrolysis of acidified water using platinum also generates hydrogen, but it is not the specified textbook procedure for the stated purity threshold. Brine electrolysis is primarily the chlor-alkali process and yields chlorine as well as hydrogen and sodium hydroxide; the option does not specify the required purification. Zinc with dilute hydrochloric acid is a laboratory preparation whose gas may contain impurities from the zinc and reagents, so it does not establish the quoted high purity.

Step 4: Conclusion Use electrolysis of warm $Ba(OH)_2$ solution with nickel electrodes. Hence, the correct option is (B).

Quick Tip to Solve: For the stated hydrogen purity above 99.95%, remember the complete combination: warm barium hydroxide electrolyte and nickel electrodes.

69. Match the following compounds (Column-I) with their uses (Column-II) :

S.No.	Column-I	S.No.	Column-II
(I)	$Ca(OH)_2$	(A)	casts of statues
(II)	NaCl	(B)	white wash
(III)	$CaSO_4$, $\frac{1}{2}H_2O$	(C)	antacid
(IV)	$CaCO_3$	(D)	washing soda preparation

[4 marks]

(A) (I)-(B), (II)-(C), (III)-(D), (IV)-(A)

(B) (I)-(B), (II)-(D), (III)-(A), (IV)-(C)

(C) (I)-(C), (II)-(D), (III)-(B), (IV)-(A)

(D) (I)-(D), (II)-(A), (III)-(C), (IV)-(B)

Correct Answer: (B) (I)-(B), (II)-(D), (III)-(A), (IV)-(C)

Solution:

Concept:

- The practical use of an inorganic compound follows from its chemical properties: basic hydroxides neutralise acids, while plaster of Paris sets on hydration.
- Washing soda is sodium carbonate decahydrate, $Na_2CO_3 \cdot 10\,H_2O$. Sodium chloride is an important starting material for manufacturing sodium carbonate.

Step 1: Match the calcium hydroxide and sodium chloride Calcium hydroxide, $Ca(OH)_2$, is slaked lime and is used in whitewashing, so (I) matches (B). The white layer hardens as it absorbs atmospheric carbon dioxide:

$$Ca(OH)_2 + CO_2 \longrightarrow CaCO_3 + H_2O.$$

Sodium chloride supplies sodium in the industrial preparation of sodium carbonate, from which washing soda is obtained by crystallisation. Therefore (II) matches (D), rather than the antacid use.

Step 2: Match the two remaining calcium compounds Calcium sulphate hemihydrate, $CaSO_4 \cdot \frac{1}{2}H_2O$, is plaster of Paris. It forms a workable paste and sets by hydration to gypsum, making it suitable for casts of statues. Thus (III) matches (A). Calcium carbonate, $CaCO_3$, neutralises excess stomach acid and is used as an antacid, so (IV) matches (C):

$$CaCO_3 + 2\,HCl \longrightarrow CaCl_2 + CO_2 + H_2O.$$

Step 3: Select the complete matching sequence The resulting sequence is (I)-(B), (II)-(D), (III)-(A), (IV)-(C). Option (A) incorrectly assigns sodium chloride to the antacid use and plaster of Paris to washing soda preparation. Option (C) assigns plaster of Paris to whitewashing, while option (D) assigns calcium hydroxide to washing soda preparation. Those contradictions rule out all three alternatives.

Step 4: Conclusion The matching sequence established above pairs each compound with its correct use. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the matching with plaster of Paris for casts and slaked lime for whitewash; then use sodium chloride for washing soda preparation and calcium carbonate as the antacid.

70. Reaction of an inorganic sulphite X with dilute H_2SO_4 generates compound Y. Reaction of Y with NaOH gives X. Further, the reaction of X with Y and water affords compound Z. Y and Z, respectively, are : [4 marks]

(A) S and Na_2SO_3

(B) SO_2 and $NaHSO_3$

(C) SO_2 and Na_2SO_3

(D) SO_3 and $NaHSO_3$

Correct Answer: (B) SO_2 and $NaHSO_3$

Solution:

Concept:

- Acidification of a sulphite releases sulphur dioxide because sulphurous acid is unstable and decomposes into water and SO_2.
- Sulphur dioxide is an acidic oxide. It forms sulphite with sufficient alkali and hydrogen sulphite when it reacts with sulphite in the presence of water.

Step 1: Identify the gas released on acidification Because X is an inorganic sulphite, treatment with dilute sulphuric acid releases SO_2, not sulphur or sulphur trioxide. Since the same substance X is regenerated using sodium hydroxide, identify it as sodium sulphite:

$$Na_2SO_3 + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + H_2O.$$

This establishes $Y = SO_2$. The sulphur oxidation state remains $+4$; acidification here is not an oxidation to sulphur trioxide.

Step 2: Check regeneration of the sulphite With sufficient sodium hydroxide, sulphur dioxide gives sodium sulphite:

$$SO_2 + 2\,NaOH \longrightarrow Na_2SO_3 + H_2O.$$

This agrees with the stated regeneration of X. The amount of alkali matters: full neutralisation gives the normal sulphite, while the hydrogen sulphite contains one sodium ion per sulphur-containing anion.

Step 3: Identify the product formed with additional sulphur dioxide The reaction of the sulphite with sulphur dioxide and water is

$$Na_2SO_3 + SO_2 + H_2O \longrightarrow 2\,NaHSO_3.$$

Thus Z is sodium hydrogen sulphite. Option (A) gives the wrong gas. Option (C) repeats sodium sulphite as the final product although it is converted to hydrogen sulphite on addition of sulphur dioxide. Option (D) incorrectly identifies the initial gas as sulphur trioxide.

Step 4: Conclusion Y is SO_2 and Z is $NaHSO_3$. Hence, the correct option is (B).

Quick Tip to Solve: Track the alkali-to-sulphur dioxide ratio: two hydroxide equivalents give sulphite, while additional sulphur dioxide converts sulphite into hydrogen sulphite.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. A solution of phenol in chloroform when treated with aqueous NaOH gives compound P as a major product. The mass percentage of carbon in P is _____. (to the nearest integer)
(Atomic mass : C = 12; H = 1; O = 16) [4 marks]

Solution:

Concept:

- Phenol treated with chloroform and aqueous sodium hydroxide undergoes the Reimer–Tiemann reaction. The major product is ortho-hydroxybenzaldehyde, also called salicylaldehyde.
- The mass percentage of an element is its mass contribution in one mole divided by the molar mass of the complete compound, multiplied by 100.

Step 1: Identify the major organic product Chloroform with aqueous sodium hydroxide supplies the reactive dichlorocarbene intermediate in the Reimer–Tiemann reaction. The phenoxide ring directs substitution principally to the ortho position; hydrolysis produces an aldehyde group next to the hydroxyl group. Therefore P is salicylaldehyde, $C_6H_4(OH)CHO$. The group introduced is an aldehyde group, not a carboxylic acid group.

Step 2: Determine its molecular formula and molar mass Count all atoms in $C_6H_4(OH)CHO$: the aromatic ring contributes six carbons, the aldehyde group contributes one more, and the total hydrogen and oxygen counts are six and two. Hence the formula is $C_7H_6O_2$. Using only the supplied atomic masses,

$$M(P) = 7(12) + 6(1) + 2(16) = 84 + 6 + 32 = 122\,\text{g mol}^{-1}.$$

The mass of carbon in one mole of the product is $84\,\text{g}$.

Step 3: Calculate and round the percentage The required mass percentage is

$$\%\ \text{carbon} = \frac{84}{122} \times 100 = 68.852459\ldots\%.$$

The question asks for the nearest integer, so round this value to 69, rather than truncating it to 68. The percentage calculation depends on the product formula, and both ortho and para hydroxybenzaldehyde have the same formula, although the ortho compound is the stated major product.

Step 4: Conclusion The mass percentage of carbon in P, to the nearest integer, is $\boxed{69\%}$. The numerical entry is 69.

Quick Tip to Solve: After identifying salicylaldehyde, count its formula as $C_7H_6O_2$; include the aldehyde carbon before calculating $84/122 \times 100$.

72. The atomic number of Unnilunium is _____. [4 marks]

Solution:

Concept:

- The temporary systematic names of elements are formed from roots representing the individual digits of the atomic number, read from left to right.
- The relevant digit roots are un for 1 and nil for 0. The suffix -ium completes the element name and contributes no additional digit.

Step 1: Separate the numerical roots Divide the given name Unnilunium as un-nil-un-ium. The repeated un root is significant: it appears once before nil and once after nil. Treating the entire ending unium as an unrelated suffix would lose the last digit, whereas the suffix itself is only ium.

Step 2: Translate the roots into digits The first un gives the hundreds digit 1, nil gives the tens digit 0, and the second un gives the units digit 1. Reading in that order gives

$$Z = \underbrace{1}_{\text{un}}\,\underbrace{0}_{\text{nil}}\,\underbrace{1}_{\text{un}} = 101.$$

This is concatenation of digits, not addition of the root values: adding $1 + 0 + 1$ would not represent an atomic number encoded by this nomenclature.

Step 3: Interpret the resulting atomic number The element with atomic number 101 is now known as mendelevium, but the question uses its systematic temporary name. The decoded number refers to its number of protons, so a nucleus of this element contains 101 protons. It does not specify a mass number or an isotope, and no neutron count is needed. Atomic number is a pure count and therefore has no physical unit. This also distinguishes the requested quantity from the relative atomic mass, which cannot be obtained from the name roots.

Step 4: Conclusion The atomic number of Unnilunium is $\boxed{101}$ (dimensionless). The required numerical entry is 101.

Quick Tip to Solve: Split a systematic element name into digit roots before the suffix -ium; un-nil-un directly encodes the digits 1, 0, 1.

73. The rate of a reaction decreased by 3.555 times when the temperature was changed from 40°C to 30°C. The activation energy (in kJ mol^{-1}) of the reaction is _____.

Take; R = 8.314 J mol^{-1} K^{-1} ln 3.555 = 1.268 [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = Ae^{-E_a/(RT)}$, where E_a is the activation energy, R is the gas constant and T is the absolute temperature. For the same reaction mixture, the change in rate on changing temperature gives the corresponding ratio of rate constants.
- Eliminating the pre-exponential factor between two temperatures gives $\ln(k_2/k_1) = (E_a/R)(1/T_1 - 1/T_2)$. Temperatures must be expressed in kelvin, and the logarithm must be natural when this

form is used.

Step 1: Identify the hotter and colder rate constants Take $T_1 = 30+273 = 303\,\text{K}$ and $T_2 = 40+273 = 313\,\text{K}$. Cooling decreases the rate by the stated factor, so the hotter rate is the larger one:

$$\frac{k_2}{k_1} = 3.555, \qquad \ln\frac{k_2}{k_1} = 1.268.$$

Writing the ratio in this order makes both the logarithm and the reciprocal-temperature difference positive, as required for a positive activation energy.

Step 2: Substitute in the two-temperature equation Use the supplied value of R, retaining joules until the final conversion:

$$1.268 = \frac{E_a}{8.314}\left(\frac{1}{303} - \frac{1}{313}\right).$$

The temperature factor is

$$\frac{1}{303} - \frac{1}{313} = \frac{10}{303\times 313}\,\text{K}^{-1}.$$

Hence

$$E_a = \frac{1.268\times 8.314\times 303\times 313}{10} = 99980.7\,\text{J mol}^{-1}.$$

Multiplying the reciprocal-temperature difference by E_a/R is dimensionless, consistent with the logarithm on the left.

Step 3: Conclusion The activation energy is $99.98\,\text{kJ mol}^{-1}$, or approximately $100\,\text{kJ mol}^{-1}$. The requested numerical entry is $\boxed{100}$.

Quick Tip to Solve: Put the hotter rate in the numerator and subtract its reciprocal temperature from the colder reciprocal temperature; this keeps both sides positive.

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74. If the solubility product of AB_2 is 3.20×10^{-11} M^3, then the solubility of AB_2 in pure water is _____ $\times 10^{-4}$ mol L^{-1}.

[Assuming that neither kind of ion reacts with water] [4 marks]

Solution:

Concept:

- Molar solubility s is the number of moles of the solid that dissolve per litre of saturated solution. The ion concentrations follow from the stoichiometric coefficients of its dissolution equation, rather than all being equal to s.
- For $\text{AB}_2(\text{s}) \rightleftharpoons \text{A}^{2+}(\text{aq}) + 2\,\text{B}^-(\text{aq})$, the solubility product is $K_{sp} = [\text{A}^{2+}][\text{B}^-]^2$. The pure solid is omitted from the equilibrium expression because its activity is constant.

Step 1: Express the equilibrium ion concentrations Let the molar solubility in pure water be $s\,\text{mol L}^{-1}$. Each mole of dissolved AB_2 supplies one mole of A ions and two moles of B ions. Since neither ion reacts with water, no hydrolysis correction is needed. There is also no added common ion in pure water. Therefore

$$[\text{A}^{2+}] = s, \qquad [\text{B}^-] = 2s.$$

These concentrations refer to the ions after the solution has reached saturation, not to the amount of undissolved solid.

Step 2: Evaluate the solubility from the product Substitute both stoichiometric concentrations in the equilibrium expression:

$$K_{sp} = s(2s)^2 = 4s^3 = 3.20 \times 10^{-11}.$$

Thus

$$s^3 = 8.00 \times 10^{-12}, \qquad s = \sqrt[3]{8.00 \times 10^{-12}} = 2.00 \times 10^{-4}\,\text{mol}\,\text{L}^{-1}.$$

A direct check gives $(2.00\times10^{-4})(4.00\times10^{-4})^2 = 3.20\times10^{-11}$, exactly the supplied solubility product. Its concentration power is three, matching the stated M^3 unit.

Step 3: Conclusion The solubility of AB_2 is $\boxed{2.00 \times 10^{-4}\,\text{mol}\,\text{L}^{-1}}$. The coefficient to enter in the blank is $\boxed{2}$.

Quick Tip to Solve: For a salt of formula AB_2, first write the ion concentrations as s and $2s$; the resulting factor of four in $K_{sp} = 4s^3$ is essential.

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75. For Freundlich adsorption isotherm, a plot of log (x/m) (y-axis) and log p (x-axis) gives a straight line. The intercept and slope for the line is 0.4771 and 2, respectively. The mass of gas, adsorbed per gram of adsorbent if the initial pressure is 0.04 atm, is _____ $\times$ 10^{-4} g.
(log 3 = 0.4771) [4 marks]

Solution:

Concept:

- The Freundlich adsorption isotherm is $x/m = kp^{1/n}$, where x is the mass of gas adsorbed, m is the mass of adsorbent and p is the gas pressure. The constants are determined at the temperature of the experiment.
- Taking common logarithms gives $\log(x/m) = \log k + (1/n)\log p$. On a graph with these axes, the intercept is $\log k$ and the slope is $1/n$. Use the slope stated in the question as the exponent of pressure.

Step 1: Read the constants from the straight line Compare the given intercept and slope with the logarithmic equation:

$$\log k = 0.4771 = \log 3, \qquad \frac{1}{n} = 2.$$

Therefore $k = 3$, while $n = 1/2$. The exponent in the original isotherm is the slope itself, namely 2; it is not n. Consequently the particular line supplied by this question gives

$$\frac{x}{m} = 3p^2.$$

Although the familiar empirical range often has a slope below unity, the calculation here follows the explicitly supplied straight-line slope of 2.

Step 2: Calculate the uptake for one gram of adsorbent Insert the given pressure as $p = 0.04$ in

atmospheres, the pressure convention of the supplied relation:

$$\frac{x}{m} = 3(0.04)^2 = 3(0.0016) = 0.0048.$$

This is the mass of adsorbed gas per unit mass of adsorbent. For $m = 1\,\text{g}$, the corresponding gas mass is

$$x = 0.0048\,\text{g} = 4.8 \times 10^{-3}\,\text{g} = 48 \times 10^{-4}\,\text{g}.$$

Converting to the power of ten printed beside the blank is necessary: the blank asks for the coefficient of 10^{-4}, rather than the mass expressed as a plain decimal.

Step 3: Conclusion The mass of gas adsorbed per gram of adsorbent is $\boxed{48 \times 10^{-4}\,\text{g}}$. Hence the required numerical entry is $\boxed{48}$.

Quick Tip to Solve: In a logarithmic Freundlich plot, take the antilogarithm of the intercept to obtain k, and use the slope directly as the pressure exponent.