JEE Main 2020 Paper 1 (B.E./B.Tech) Sep 6 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 2 (September), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 6 September 2020, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to : [4 marks]

(A) (–3, –1)

(B) (–4, –1)

(C) (1, 3)

(D) (–3, 3)

Correct Answer: (A) (–3, –1)

Solution:

Concept:

- A determinant is a signed scalar, so vertical determinant bars do not mean that its value must be nonnegative.
- Adding or subtracting a multiple of another row leaves a determinant unchanged. Also, $\cos^2 x + \sin^2 x = 1$ and $-1 \le \sin 2x \le 1$.

Step 1: Simplify the first two rows Write $c = \cos^2 x$, $s = \sin^2 x$ and $u = \sin 2x$, so $c + s = 1$. Denote the displayed determinant by D. Subtract the third row from each of the first two rows. These operations preserve its value and isolate the constant entries:

$$D = \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ c & s & 1+u \end{vmatrix}.$$

The same original third row is used for both subtractions; it remains unchanged. This avoids expanding products of squared trigonometric functions.

Step 2: Expand and obtain the range Expansion along the first row gives

$$\begin{aligned} D &= -\{1(1+u) - (-1)c\} - \{1s - 0c\} \\ &= -1 - u - c - s = -2 - \sin 2x. \end{aligned}$$

Because $\sin 2x$ takes every value in $[-1, 1]$, the determinant ranges over $[-3, -1]$. In particular, $x = \pi/4$ gives $D = -3$, whereas $x = 3\pi/4$ gives $D = -1$, so both endpoints are attained. Option (B) incorrectly extends the lower endpoint to -4; options (C) and (D) include positive values even though $-2 - \sin 2x$ is always negative.

Step 3: Conclusion The minimum is $m = -3$ and the maximum is $M = -1$. Thus $(m, M) = (-3, -1)$. Hence, the correct option is (A).

Quick Tip to Solve: Use row differences before expanding when rows share trigonometric entries; bound the resulting sine expression only after preserving its sign.

2. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated? [4 marks]

(A) 2! 3! 4!

(B) $(3!)^3.(4!)$

(C) 3! $(4!)^3$

(D) $(3!)^2.(4!)$

Correct Answer: (B) $(3!)^3.(4!)$

Solution:

Concept:

- When specified people must sit consecutively, first treat each such group as a single block, then arrange the people inside each block.
- The multiplication principle combines independent choices; n distinct objects in a row have $n!$ arrangements.

Step 1: Arrange the three family blocks Call the two three-member families F_1, F_2 and the four-member family F_3. The condition that members of the same family are not separated forces each family to occupy one uninterrupted stretch of seats. Compress each stretch into one block. There are three distinct blocks, regardless of their unequal sizes, so their left-to-right order can be chosen in

$$3! = 6$$

ways. The two smaller families are still distinct: having the same number of members does not make the families interchangeable. There is therefore no division by $2!$.

Step 2: Arrange members within their blocks For any fixed block order, the members of F_1 can be seated in $3!$ ways and those of F_2 independently in $3!$ ways. The four distinct members of F_3 have $4!$ internal orders. Hence

$$N = 3! \times 3! \times 3! \times 4! = (3!)^3(4!) = 5184.$$

Each admissible seating determines a unique block order and unique internal orders, so this product neither misses nor repeats any seating. Option (A) omits necessary permutations; option (D) omits one $3!$ factor. Option (C) gives $4!$ internal permutations to the smaller families, which have only three members each.

Step 3: Conclusion The required number of seatings is $(3!)^3(4!) = 5184$. Hence, the correct option is (B).

Quick Tip to Solve: Count one factorial for arranging the family blocks, then one factorial for the members of each family; equal family sizes do not make families identical.

3. The area (in sq. units) of the region

$A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$ [4 marks]

(A) $\frac{1}{6}$

(B) $\frac{5}{6}$

(C) $\frac{1}{3}$

(D) $\frac{7}{6}$

Correct Answer: (B) $\frac{5}{6}$

Solution:

Concept:

- Both inequalities involve $|x|$ and either $|y|$ or y^2, so the region is symmetric about both coordinate axes.
- At a fixed height in the first quadrant, the permitted horizontal length is the smaller of the bounds supplied by the two inequalities. Integrate that length and multiply by four.

Step 1: Determine the horizontal boundary In the first quadrant, $x, y \geq 0$, and the conditions become

$$x \leq 1 - y, \qquad x \leq 2y^2, \qquad 0 \leq y \leq 1.$$

Thus $0 \leq x \leq \min(1 - y, 2y^2)$. The two right boundaries meet when

$$2y^2 = 1 - y \quad \Longrightarrow \quad (2y - 1)(y + 1) = 0.$$

Only $y = 1/2$ belongs to this quadrant. For $0 \leq y \leq 1/2$, the parabola provides the smaller bound. For $1/2 \leq y \leq 1$, the straight side of the diamond provides the smaller bound. The negative root -1 must not be used in this first-quadrant integral.

Step 2: Integrate and restore all four quadrants The area in one quadrant is

$$\begin{aligned} A_1 &= \int_0^{1/2} 2y^2 \, dy + \int_{1/2}^{1} (1 - y) \, dy \\ &= \left[\frac{2y^3}{3}\right]_0^{1/2} + \left[y - \frac{y^2}{2}\right]_{1/2}^{1} \\ &= \frac{1}{12} + \frac{1}{8} = \frac{5}{24}. \end{aligned}$$

The axes have zero area, so symmetry gives $A = 4A_1 = 5/6$. The alternatives $1/6$, $1/3$ and $7/6$ do not equal this sum. In particular, integrating only the parabola or only the diamond side over the full interval would ignore the change in the active boundary.

Step 3: Conclusion The required area is $\frac{5}{6}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For inequalities of the form $|x| \leq g(y)$, horizontal strips often avoid square roots; find where the competing bounds switch before integrating.

4. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point : [4 marks]

(A) $\frac{(t_1+t_2)}{2}$

(B) $\frac{(t_2-t_1)}{2}$

(C) $2a(t_1 + t_2) + b$

(D) $a(t_2 - t_1) + b$

Correct Answer: (A) $\frac{(t_1+t_2)}{2}$

Solution:

Concept:

- The instantaneous velocity is the derivative of position, $v(t) = f\prime(t)$. Since $a, b > 1$ and $t > 0$, this derivative is positive and speed equals velocity.
- For motion without a change of direction, average speed on $[t_1, t_2]$ is the difference quotient $\{f(t_2) - f(t_1)\}/(t_2 - t_1)$.

Step 1: Calculate the average speed Take $0 < t_1 < t_2$. The derivative of the given position function is $f\prime(t) = 2at + b > 0$, so the car moves in one direction throughout the interval. Its distance travelled therefore equals its positive displacement. The constant c cancels when positions are subtracted:

$$\begin{aligned} v_{\text{av}} &= \frac{at_2^2 + bt_2 + c - (at_1^2 + bt_1 + c)}{t_2 - t_1} \\ &= \frac{a(t_2 - t_1)(t_2 + t_1) + b(t_2 - t_1)}{t_2 - t_1} \\ &= a(t_1 + t_2) + b. \end{aligned}$$

The cancellation is legitimate because the interval has positive length.

Step 2: Find the time having this speed Let t_* be the requested time. Equate its instantaneous speed to the average speed:

$$2at_* + b = a(t_1 + t_2) + b \quad \Longrightarrow \quad t_* = \frac{t_1 + t_2}{2}.$$

Division by $2a$ is valid because $a > 1$. This midpoint lies strictly inside the interval and is unique because $v(t) = 2at + b$ increases strictly. Option (B) gives half the interval length rather than the absolute time; it generally lies outside the required location. Options (C) and (D) are expressions of speed involving a and b, whereas the question asks for a time. They also do not solve the equality above.

Step 3: Conclusion The average speed is attained at $t = (t_1 + t_2)/2$. Hence, the correct option is (A).

Quick Tip to Solve: For quadratic position, velocity is linear in time; its interval average equals its midpoint value, provided speed and velocity have the same sign throughout.

5. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is : [4 marks]

(A) $\frac{10}{99}$

(B) $\frac{5}{33}$

(C) $\frac{15}{101}$

(D) $\frac{5}{101}$

Correct Answer: (B) $\frac{5}{33}$

Solution:

Concept:

- A selection of three distinct numbers is an unordered set, so the total number of equally likely selections is $\binom{11}{3}$.
- Write an increasing three-term arithmetic progression as $a, a+d, a+2d$, where d is a positive integer. Translation of all eleven numbers preserves this property.

Step 1: Count the total selections Subtract the smallest natural number in the block and add one to every element. This bijection turns the block into $1, 2, \ldots, 11$ without changing any common difference, so the starting natural number does not affect the probability. Since the three numbers are selected without repetition, every three-element subset is equally likely. The sample-space size is

$$\binom{11}{3} = \frac{11 \times 10 \times 9}{3 \times 2 \times 1} = 165.$$

Positive common difference means that a favourable subset is written in increasing order; it does not create additional outcomes by permuting its entries.

Step 2: Count progressions by their common difference For a fixed d, the last term must satisfy $a + 2d \leq 11$. Therefore a can be any integer from 1 to $11 - 2d$. A positive number of choices exists precisely for $d = 1, 2, 3, 4, 5$, giving

$$N = (11-2) + (11-4) + (11-6) + (11-8) + (11-10) = 9 + 7 + 5 + 3 + 1 = 25.$$

Every increasing progression has exactly one first term and one common difference, so none is double counted. Thus

$$P = \frac{25}{165} = \frac{5}{33}.$$

Options (A), (C) and (D) disagree with this integer count over the sample space of 165 subsets. In particular, counting only consecutive triples would incorrectly restrict the difference to one.

Step 3: Conclusion The required probability is $\frac{5}{33}$. Hence, the correct option is (B).

Quick Tip to Solve: For three-term progressions among n consecutive integers, sum $n - 2d$ over $1 \leq d \leq \lfloor (n-1)/2 \rfloor$, then divide by $\binom{n}{3}$.

6. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively: [4 marks]

(A) 6 and 8

(B) 5 and 8

(C) 5 and 7

(D) 4 and 9

Correct Answer: (B) 5 and 8

Solution:

Concept:

- A system in three unknowns has infinitely many solutions when its coefficient and augmented matrices have equal rank smaller than three.
- Elementary elimination reveals whether the last equation is dependent on the earlier equations or imposes an additional independent constraint.

Step 1: Eliminate the first unknown Subtract the first equation from the second and third equations. This gives the equivalent conditions

$$y + 2z = 3, \qquad 2y + (\lambda - 1)z = \mu - 2,$$

along with $x + y + z = 2$. Now subtract twice the first of these new equations from the second:

$$(\lambda - 5)z = \mu - 8.$$

The first two original equations are independent: their coefficient rows $(1, 1, 1)$ and $(1, 2, 3)$ are not proportional. Consequently the coefficient rank is already at least two, and an infinite solution set requires the final equation to supply no new constraint.

Step 2: Require dependence and verify the free parameter If $\lambda \neq 5$, the last equation determines z, after which y and x are fixed, giving a unique solution. If $\lambda = 5$ but $\mu \neq 8$, it becomes $0 = \mu - 8$, which is inconsistent. Thus infinitely many solutions require

$$\lambda = 5, \qquad \mu = 8.$$

These values are sufficient: put $z = t$, where t is any real number. Then

$$y = 3 - 2t, \qquad x = t - 1.$$

Substitution in the third equation gives $(t - 1) + 3(3 - 2t) + 5t = 8$ for every t. Option (A) fixes z uniquely; option (C) is inconsistent; option (D) also gives a unique solution. Only option (B) leaves a free parameter.

Step 3: Conclusion The values are $\lambda = 5$ and $\mu = 8$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: After elimination, infinite solutions require a zero coefficient and a zero right-hand side in the dependent row; setting the determinant to zero alone is insufficient.

7. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is : [4 marks]

(A) 1

(B) 3

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Concept:

- For a monic quadratic $x^2 - Sx + P = 0$, the roots have sum S and product P. Here $\alpha + \beta = 64$ and $\alpha\beta = 256$.
- For positive real roots, principal eighth roots obey the usual laws of exponents. Replacing a ratio by a power divided by a power of the product avoids finding either root separately.

Step 1: Establish that both roots are positive The discriminant is $64^2 - 4(256) = 3072 > 0$, so the roots are real. Their product is positive and their sum is positive; hence both roots are positive. We can therefore take positive real eighth roots without any sign ambiguity. This check matters because an eighth root of an eighth power otherwise gives an absolute value.

Step 2: Express each term using the product Use $\alpha\beta = 256$ to put both terms over a common denominator:

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} = \left(\frac{\alpha^8}{(\alpha\beta)^5}\right)^{1/8} = \frac{\alpha}{256^{5/8}}.$$

$$\left(\frac{\beta^3}{\alpha^5}\right)^{1/8} = \frac{\beta}{256^{5/8}}.$$

Since $256 = 2^8$, the common denominator equals $2^5 = 32$.

Step 3: Add the two terms and compare the options Their sum is therefore

$$\frac{\alpha + \beta}{32} = \frac{64}{32} = 2.$$

The values 1, 3 and 4 in the other options do not equal this exact result. In particular, there is no need to approximate the irrational roots: their sum and product determine the expression directly.

Step 4: Conclusion The required value is $\boxed{2}$. Hence, the correct option is (C).

Quick Tip to Solve: When complementary powers occur in a ratio of roots, multiply through by a power of their product to turn the fractional power into a simple root sum.

8. Let a , b, c , d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then : [4 marks]

(A) a, c, p are in G.P.

(B) a, b, c, d are in G.P.

(C) a, b, c, d are in A.P.

(D) a, c, p are in A.P.

Correct Answer: (B) a, b, c, d are in G.P.

Solution:

Concept:

- A sum of squares of real quantities can equal zero only when every squared quantity is zero.
- Nonzero numbers form a geometric progression when each successive ratio is the same. Rearranging a quadratic expression as squared differences can reveal these ratios directly.

Step 1: Complete the three squares Collect the terms according to the adjacent pairs (a, b), (b, c) and (c, d). The given equation is exactly

$$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0.$$

Indeed, the squared terms supply $(a^2 + b^2 + c^2)p^2$, the cross terms supply $-2(ab + bc + cd)p$, and the remaining constants supply $b^2 + c^2 + d^2$. Thus no inequality or approximation has been introduced.

Step 2: Set every squared term equal to zero All five numbers are real. Consequently, each square is nonnegative, and none can cancel another. We must have

$$ap = b, \qquad bp = c, \qquad cp = d.$$

Since the numbers are nonzero, division gives

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p.$$

Therefore a, b, c, d have common ratio p, as required for a geometric progression.

Step 3: Check why the other conclusions are not necessary The distinctness excludes a constant progression. A nonconstant geometric progression is not generally an arithmetic progression. Neither claimed relation involving a, c, p is forced: for example, $a = 3, p = 2$ gives $(a, b, c, d) = (3, 6, 12, 24)$, all five numbers distinct and nonzero. But $3, 12, 2$ are neither in G.P. nor in A.P. This example rejects (A), (C) and (D) while satisfying the original equation.

Step 4: Conclusion The numbers a, b, c, d are in G.P. with common ratio p. Hence, the correct option is (B).

Quick Tip to Solve: For a real quadratic expression with adjacent products, look for a sum such as $(ap - b)^2 + (bp - c)^2 + (cp - d)^2$ before using the discriminant.

9. The region represented by

$\{z = x + iy \in C : |z| - Re(z) \leq 1\}$ is also given by the

inequality : $\{z = x + iy \in C : |z| - Re(z) \leq 1\}$ [4 marks]

(A) $y^2 \leq 2\left(x + \frac{1}{2}\right)$

(B) $y^2 \leq x + \frac{1}{2}$

(C) $y^2 \geq 2(x + 1)$

(D) $y^2 \geq x + 1$

Correct Answer: (A) $y^2 \leq 2\left(x + \frac{1}{2}\right)$

Solution:

Concept:

- For $z = x + iy$, its modulus is $|z| = \sqrt{x^2 + y^2}$ and its real part is $\mathrm{Re}(z) = x$.
- Squaring an inequality involving a square root is reversible only after establishing that its other side is nonnegative. A Cartesian description must retain every necessary domain condition.

Step 1: Rewrite the modulus inequality The given condition becomes

$$\sqrt{x^2 + y^2} - x \leq 1 \quad \Longleftrightarrow \quad \sqrt{x^2 + y^2} \leq x + 1.$$

The left side is nonnegative, so any permitted point must have $x + 1 \geq 0$. Under this condition both sides are nonnegative and can be squared without reversing the inequality.

Step 2: Square and simplify Squaring gives

$$x^2 + y^2 \leq (x + 1)^2 = x^2 + 2x + 1.$$

Cancel the common x^2 term to obtain

$$y^2 \leq 2x + 1 = 2\left(x + \frac{1}{2}\right).$$

Conversely, this final inequality implies $2x + 1 \geq 0$, since $y^2 \geq 0$. Thus $x \geq -1/2$, which guarantees $x + 1 > 0$; taking square roots recovers the original condition. No extra restriction needs to be appended.

Step 3: Identify the region and eliminate alternatives The boundary is the right-opening parabola $y^2 = 2x + 1$, and the inequality includes its boundary and the region on its right. Option (B) has the wrong coefficient: the point $(0, 1)$ satisfies the original inequality but fails (B). Options (C) and (D) have the wrong direction; both exclude $(0, 0)$, although that point plainly satisfies the original condition.

Step 4: Conclusion The region is $\boxed{y^2 \leq 2(x + 1/2)}$. Hence, the correct option is (A).

Quick Tip to Solve: After squaring a modulus inequality, verify the converse: here $y^2 \leq 2x + 1$ automatically makes the quantity $x + 1$ positive.

10. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$ (n, a > 1) then the standard deviation of n observations x_1 , x_2 , ..., x_n is : [4 marks]

(A) $a - 1$

(B) $n\sqrt{a-1}$

(C) $\sqrt{n(a-1)}$

(D) $\sqrt{a-1}$

Correct Answer: (D) $\sqrt{a-1}$

Solution:

Concept:

- Standard deviation is unchanged by subtracting a fixed number from every observation. For $u_i = x_i - a$, the variance is $\sigma^2 = \overline{u^2} - (\overline{u})^2$.
- The standard deviation is the nonnegative square root of the variance. Both first and second moments must be averages, so their sums are divided by the number n of observations.

Step 1: Find the mean of the shifted observations Put $u_i = x_i - a$. The first given relation states $\sum_{i=1}^{n} u_i = n$. Hence

$$\overline{u} = \frac{1}{n}\sum_{i=1}^{n} u_i = 1.$$

Equivalently, the mean of the original observations is $\overline{x} = a + 1$. The shift changes the mean but does not change any observation's deviation from its own mean.

Step 2: Compute the variance from the second moment The second relation gives

$$\overline{u^2} = \frac{1}{n}\sum_{i=1}^{n} u_i^2 = \frac{na}{n} = a.$$

Therefore

$$\sigma^2 = \overline{u^2} - (\overline{u})^2 = a - 1.$$

One may also expand $(x_i - \overline{x})^2 = (u_i - 1)^2$; summing yields $na - 2n + n = n(a-1)$, and dividing by n gives the same variance.

Step 3: Take the square root and compare Since $a > 1$, the variance is positive and

$$\sigma = \sqrt{a-1}.$$

Option (A) is the variance rather than the standard deviation. Options (B) and (C) retain an unwanted factor of n or $\sqrt{n}$: they result from failing to normalize one of the sums. The standard deviation here is independent of the number of observations after the supplied moments have been averaged.

Step 4: Conclusion The standard deviation is $\boxed{\sqrt{a-1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use a convenient origin: divide the two supplied shifted sums by n, subtract the square of the first average from the second, then take one square root.

11. The general solution of the differential equation

$\sqrt{1+x^2+y^2+x^2y^2}$ **+ xy**$\frac{dy}{dx}$ **= 0 is :**

(where C is a constant of integration) [4 marks]

(A) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(B) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(C) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(D) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

Correct Answer: (C) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

Solution:

Concept:

- The radical factors as $\sqrt{(1+x^2)(1+y^2)}=\sqrt{1+x^2}\sqrt{1+y^2}$ for real x, y. This makes the differential equation separable.
- Use $u=\sqrt{1+x^2}$ in the remaining integral. A solution is considered on an interval where $x\neq 0$; the original equation cannot hold at $x=0$ or $y=0$ for a finite derivative.

Step 1: Separate the variables Move the radical to the other side and divide by $x\sqrt{1+y^2}$:

$$\frac{y}{\sqrt{1+y^2}}\,dy=-\frac{\sqrt{1+x^2}}{x}\,dx.$$

The integral on the left is $\sqrt{1+y^2}$. We now evaluate the right-hand integral with its minus sign retained.

Step 2: Integrate the expression in x Let $u=\sqrt{1+x^2}$. Then $x^2=u^2-1$ and $x\,dx=u\,du$. Hence

$$\int\frac{\sqrt{1+x^2}}{x}\,dx=\int\frac{u^2}{u^2-1}\,du.$$

$$=u+\frac{1}{2}\log_e\left(\frac{u-1}{u+1}\right).$$

Here $u>1$ when $x\neq 0$, so the logarithm argument is positive. The identity follows from $u^2/(u^2-1)=1+1/(u^2-1)$.

Step 3: Rearrange and check the signs After integrating the separated equation,

$$\sqrt{1+y^2}+u=-\frac{1}{2}\log_e\left(\frac{u-1}{u+1}\right)+C.$$

Reverse the ratio inside the logarithm to absorb the minus sign. This is option (C). Differentiation verifies the choice: the derivative of the logarithmic term on its right is $-1/(xu)$, while $u'=x/u$. Thus $yy'/\sqrt{1+y^2}=-u/x$, exactly the separated equation. Options (A), (B) and (D) have incompatible signs.

Step 4: Conclusion The general solution is

$$\boxed{\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Factor the four-term radical first. In the x-integral, $u=\sqrt{1+x^2}$ turns the integrand into $1+1/(u^2-1)$; track the leading minus sign.

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12. If $I_1 = \int_0^1 (1-x^{50})^{100}dx$ and $I_2 = \int_0^1 (1-x^{50})^{101}dx$ such that $I_2 = \alpha I_1$ then α equals to : [4 marks]

(A) $\frac{5051}{5050}$
(B) $\frac{5050}{5051}$
(C) $\frac{5050}{5049}$
(D) $\frac{5049}{5050}$

Correct Answer: (B) $\frac{5050}{5051}$

Solution:

Concept:

- Two integrals whose exponents differ by one can be related by integration by parts, without expanding a large binomial power.
- The identity $(1-x^{50})^{100}-(1-x^{50})^{101}=x^{50}(1-x^{50})^{100}$ expresses the extra integral in terms of I_1-I_2.

Step 1: Apply integration by parts to the second integral Take $u=(1-x^{50})^{101}$ and $dv=dx$. Then $v=x$ and $du=-5050x^{49}(1-x^{50})^{100}\,dx$. Therefore

$$I_2=\left[x(1-x^{50})^{101}\right]_0^1+5050\int_0^1 x^{50}(1-x^{50})^{100}\,dx.$$

The boundary term vanishes at both limits: at zero the factor x vanishes, and at one the factor $1-x^{50}$ vanishes.

Step 2: Express the remaining integral as a difference Write $x^{50}=1-(1-x^{50})$ inside the last integral. This yields

$$\int_0^1 x^{50}(1-x^{50})^{100}\,dx=I_1-I_2.$$

Thus the integration-by-parts relation becomes

$$I_2 = 5050(I_1 - I_2), \qquad 5051I_2 = 5050I_1.$$

Since the integrand defining I_1 is positive for $0 \le x < 1$, its integral is positive and division by I_1 is valid.

Step 3: Find the ratio and check the options Comparison with $I_2 = \alpha I_1$ gives

$$\alpha = \frac{I_2}{I_1} = \frac{5050}{5051}.$$

On $(0, 1)$, raising $1 - x^{50}$ from power 100 to 101 makes it smaller. Hence $0 < I_2 < I_1$, so options (A) and (C), both greater than one, are impossible. Option (D) is below one but fails the exact recurrence derived above.

Step 4: Conclusion The multiplier is $\boxed{\alpha = 5050/5051}$. Hence, the correct option is (B).

Quick Tip to Solve: For $\int_0^1 (1-x^m)^n \, dx$, integrate the higher power by parts with $dv = dx$; rewrite the resulting x^m factor as $1 - (1 - x^m)$.

13. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to : [4 marks]

(A) $\frac{5}{8}$

(B) $\frac{7}{8}$

(C) $\frac{1}{8}$

(D) $\frac{3}{8}$

Correct Answer: (C) $\frac{1}{8}$

Solution:

Concept:

- The fractional part of a positive real number is what remains after subtracting its greatest integer: $\{z\} = z - \lfloor z \rfloor$.
- For an integer $N = 8k + r$, where $0 \le r < 8$, division by eight gives $N/8 = k + r/8$. Thus only the remainder modulo eight is needed.

Step 1: Group the power into even pairs The exponent 200 is even, so write the numerator as a power of nine rather than attempting to calculate its enormous decimal expansion:

$$3^{200} = (3^2)^{100} = 9^{100} = (8 + 1)^{100}.$$

Every term of the binomial expansion except the constant term contains a factor of eight. Indeed,

$$(8 + 1)^{100} = 1 + \binom{100}{1} 8 + \binom{100}{2} 8^2 + \cdots + 8^{100}.$$

Consequently the entire sum after the first term is an integer multiple of eight.

Step 2: Extract the fractional part Let that integer multiple be $8k$. Then

$$3^{200} = 8k + 1, \qquad \frac{3^{200}}{8} = k + \frac{1}{8}.$$

Because $0 < 1/8 < 1$, the integer part of the quotient is exactly k, and its fractional part is $1/8$. Options (A) and (B) would require remainders 5 and 7, which the expansion excludes. Option (D), with remainder 3, applies to odd positive powers of three, not to this even power.

Step 3: Conclusion Therefore $\{3^{200}/8\} = \boxed{1/8}$, a dimensionless number. Hence, the correct option is (C).

Quick Tip to Solve: For a fractional part of an integer divided by a small integer, find the remainder first. Here $3^2 \equiv 1 \pmod 8$ immediately settles every even exponent.

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14. A ray of light coming from the point (2, $2\sqrt{3}$) is incident at an angle 30° on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point : [4 marks]

(A) (3, -$\sqrt{3}$)

(B) (4, -$\sqrt{3}$)

(C) $\left(4, -\frac{\sqrt{3}}{2}\right)$

(D) $\left(3, -\frac{1}{\sqrt{3}}\right)$

Correct Answer: (A) (3, -$\sqrt{3}$)

Solution:

Concept:

- Reflection in a vertical line reverses the horizontal component of the direction while preserving its vertical component. Therefore the reflected line has the opposite of the slope of the incident line.
- The stated angle is with the reflecting line $x = 1$. A ray making $30°$ with a vertical line makes $60°$ with a horizontal line, so its slope has magnitude $\tan 60° = \sqrt{3}$.

Step 1: Locate the point of incidence Write $P = (2, 2\sqrt{3})$. The incident ray travels from P towards the mirror $x = 1$. For the downward incident direction its slope is $\sqrt{3}$, so its equation is

$$y - 2\sqrt{3} = \sqrt{3}(x - 2).$$

At the mirror, $x = 1$, which gives

$$y_A = 2\sqrt{3} - \sqrt{3} = \sqrt{3}.$$

Thus $A = (1, \sqrt{3})$. The reflected ray travels rightwards and downwards, with slope $-\sqrt{3}$.

Step 2: Find the reflected line and test the options The equation of the reflected line AB is therefore

$$y - \sqrt{3} = -\sqrt{3}(x - 1), \qquad y = \sqrt{3}(2 - x).$$

It meets the x-axis at $B = (2, 0)$. Substituting $x = 3$ gives $y = -\sqrt{3}$, so option (A) lies on this line. At $x = 4$, the required ordinate is $-2\sqrt{3}$, excluding both (B) and (C). At $x = 3$, the ordinate is not $-1/\sqrt{3}$, excluding (D). Choosing the upward incident orientation instead gives a reflected line with positive ordinate for all listed x-values, so none of the listed points is on that branch.

Step 3: Conclusion The reflected line is $y = \sqrt{3}(2 - x)$, which passes through $\boxed{(3, -\sqrt{3})}$. Hence, the correct option is (A).

Quick Tip to Solve: When the mirror is vertical, first convert an angle with the mirror into an angle with the x-axis; reflection then changes the sign of the slope.

15.

$$\lim_{x \to 1} \left(\frac{\int_0^{(x-1)^2} t \cos\left(t^2\right) dt}{(x-1)\sin(x-1)} \right)$$

[4 marks]

(A) is equal to 0

(B) is equal to $\frac{1}{2}$

(C) does not exist

(D) is equal to $-\frac{1}{2}$

Correct Answer: (A) is equal to 0

Solution:

Concept:

- The substitution $u = t^2$ gives $du = 2t\,dt$, so $\int t\cos(t^2)\,dt = \frac{1}{2}\sin(t^2) + C$.
- The standard small-angle limit is $\lim_{v \to 0} \sin v/v = 1$. Factoring the expression into such ratios makes its order of vanishing explicit.

Step 1: Evaluate the integral before taking the limit Put $h = x - 1$; then $h \to 0$ as $x \to 1$, and the upper integration limit is h^2. Applying the antiderivative at the two limits gives

$$\int_0^{h^2} t\cos(t^2)\,dt = \left[\frac{1}{2}\sin(t^2)\right]_0^{h^2} = \frac{1}{2}\sin(h^4).$$

Notice that the square in the integrand acts on the entire upper limit. Thus its argument is $(h^2)^2 = h^4$, not h^2. The lower endpoint contributes zero.

Step 2: Use the standard sine ratios For sufficiently small nonzero h, the quotient can be factored exactly as

$$\frac{\sin(h^4)}{2h\sin h} = \frac{1}{2}\,h^2\,\frac{\sin(h^4)}{h^4}\,\frac{h}{\sin h}.$$

The second and third factors after h^2 tend to one, whereas h^2 tends to zero from both sides. Therefore the limit exists and equals zero. The nonzero alternatives $1/2$ and $-1/2$ omit the remaining factor h^2;

the alternative claiming nonexistence is excluded by this common two-sided limit.

Step 3: Conclusion The given limit is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: Evaluate a simple integral before applying a limit. Track the power of its upper endpoint carefully: integrating a leading term t up to h^2 produces order h^4.

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16. If f(x + y) = f(x)f(y) and $\sum_{x=1}^{\infty} f(x) = 2$, x, y $\in$ N, where N is the set of all natural number, then the value of $\frac{f(4)}{f(2)}$ is : [4 marks]

(A) $\frac{2}{3}$

(B) $\frac{1}{9}$

(C) $\frac{1}{3}$

(D) $\frac{4}{9}$

Correct Answer: (D) $\frac{4}{9}$

Solution:

Concept:

- For natural-number arguments, the relation $f(x+y) = f(x)f(y)$ forces $f(n) = [f(1)]^n$, by repeated addition of one.
- A geometric series $r + r^2 + r^3 + \cdots$ converges for $|r| < 1$ and then has sum $r/(1-r)$.

Step 1: Express the function values using one number Let $r = f(1)$. Taking $x = n$, $y = 1$ gives

$$f(n+1) = f(n)r.$$

Starting with $f(1) = r$, this yields $f(2) = r^2$, $f(3) = r^3$, and in general $f(n) = r^n$. Hence the given infinite sum is precisely

$$\sum_{n=1}^{\infty} f(n) = r + r^2 + r^3 + \cdots = 2.$$

The finite stated sum requires convergence of this geometric series. The value $r = 0$ would give sum zero and therefore cannot satisfy the condition.

Step 2: Determine the ratio and evaluate the quotient Using the sum formula with the first term r, rather than with first term one, gives

$$\frac{r}{1-r} = 2 \quad \Longrightarrow \quad r = 2 - 2r \quad \Longrightarrow \quad r = \frac{2}{3}.$$

This value satisfies $|r| < 1$, as required. Therefore

$$\frac{f(4)}{f(2)} = \frac{r^4}{r^2} = r^2 = \frac{4}{9}.$$

Option (A) is r, not the quotient. Options (B) and (C) come from incorrect powers or an incorrect

first term in the series. As a check, using the functional equation directly gives $f(4) = f(2)^2$, so the requested quotient equals $f(2) = 4/9$.

Step 3: Conclusion The requested dimensionless ratio is $\boxed{4/9}$. Hence, the correct option is (D).

Quick Tip to Solve: In an additive-to-multiplicative functional equation on natural numbers, set $r = f(1)$. Check whether the geometric series starts at r or at one before using its sum.

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17. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line : [4 marks]

(A) $x + 3 = 0$

(B) $x + 2y = 0$

(C) $x + 2 = 0$

(D) $2x + 1 = 0$

Correct Answer: (A) $x + 3 = 0$

Solution:

Concept:

- A tangent of finite nonzero slope m to $y^2 = 4a(x + h)$ has equation $y = m(x + h) + a/m$. It follows by translating the standard tangent equation.
- Two nonvertical lines meet at right angles when the product of their slopes is -1. We can impose this before eliminating the slope parameter.

Step 1: Write the two perpendicular tangents For the first parabola, $a = 1$ and $h = 1$. Its tangent with slope m is

$$L_1 : \quad y = m(x + 1) + \frac{1}{m}.$$

The second parabola has $a = 2$ and $h = 2$. Its tangent must have slope $-1/m$, so

$$L_2 : \quad y = -\frac{x + 2}{m} - 2m.$$

Here $m \neq 0$. Neither right-opening parabola has a horizontal tangent at a finite point. A vertical vertex tangent would need a horizontal partner, so it does not add an exceptional perpendicular pair.

Step 2: Eliminate the slope at the intersection At the common point, equate the two values of y and multiply by m:

$$m^2(x + 1) + 1 = -(x + 2) - 2m^2.$$

Collecting terms gives

$$(m^2 + 1)x + 3m^2 + 3 = 0, \qquad (m^2 + 1)(x + 3) = 0.$$

Since m is real, $m^2 + 1 > 0$, and hence every intersection has $x = -3$, independent of the tangent

chosen. For example $m = 1$ gives the intersection $(-3, -1)$. This point violates $x + 2y = 0$, $x + 2 = 0$, and $2x + 1 = 0$, explicitly excluding (B), (C), and (D).

Step 3: Conclusion The perpendicular tangents intersect on the line $\boxed{x + 3 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: For perpendicular tangents to two translated parabolas, assign slopes m and $-1/m$ first. Equating their tangent equations often removes the parameter in one factorisation.

18. Which of the following points lies on the locus of the foot of perpedicular drawn upon any tangent to the ellipse,

$$\frac{x^2}{4} + \frac{y^2}{2} = 1$$

from any of its foci? [4 marks]

(A) $\left(-1, \sqrt{3}\right)$
(B) $\left(-2, \sqrt{3}\right)$
(C) $\left(-1, \sqrt{2}\right)$
(D) $(1, 2)$

Correct Answer: (A) $\left(-1, \sqrt{3}\right)$

Solution:

Concept:

- For $x^2/a^2 + y^2/b^2 = 1$, the foci are $(\pm c, 0)$, where $c^2 = a^2 - b^2$. Here $a^2 = 4$, $b^2 = 2$, and $c = \sqrt{2}$.
- A nonvertical tangent $y = mx + k$ to this ellipse satisfies $k^2 = a^2m^2 + b^2 = 4m^2 + 2$. The perpendicular foot can be obtained by projecting a focus onto this line.

Step 1: Find the coordinates of a perpendicular foot Use a focus $F = (f, 0)$, where $f = \sqrt{2}$ or $f = -\sqrt{2}$, so $f^2 = 2$. Write the tangent as $mx - y + k = 0$. The perpendicular foot $H = (X, Y)$ from F is

$$X = \frac{f - mk}{1 + m^2}, \qquad Y = \frac{mf + k}{1 + m^2}.$$

These coordinates satisfy the tangent equation, and the vector from F to H is a multiple of its normal $(m, -1)$, verifying both conditions defining the foot.

Step 2: Determine the locus and compare all four points Squaring and adding cancels the mixed terms:

$$X^2 + Y^2 = \frac{(f - mk)^2 + (mf + k)^2}{(1 + m^2)^2}$$
$$= \frac{f^2 + k^2}{1 + m^2} = \frac{2 + 4m^2 + 2}{1 + m^2} = 4.$$

Thus the feet lie on the auxiliary circle $X^2 + Y^2 = 4$. The vertical tangents $x = \pm 2$, omitted by slope notation, give feet $(\pm 2, 0)$, also on this circle. The four proposed points give squared distances from the origin equal to 4, 7, 3, and 5 respectively. Only $(-1, \sqrt{3})$ satisfies the circle equation. It is

an attainable foot: the line through it perpendicular to the segment joining it to $(\sqrt{2}, 0)$ has slope $(1+\sqrt{2})/\sqrt{3}$ and intercept $\sqrt{3}+m$, which obey $k^2 = 4m^2 + 2$.

Step 3: Conclusion The required point is $\boxed{(-1, \sqrt{3})}$. Hence, the correct option is (A).

Quick Tip to Solve: The feet of perpendiculars from a focus of an ellipse to its tangents lie on its auxiliary circle $x^2 + y^2 = a^2$. Test squared distances before calculating individual tangents.

19. The shortest distance between the lines

$$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$$

and x + y + z + 1 = 0, 2x – y + z + 3 = 0 is : [4 marks]

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (D) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- A line given as the intersection of two planes has a direction perpendicular to both plane normals, obtained from their cross product.
- For points A, B on two nonparallel lines with directions $\mathbf{u}, \mathbf{v}$, their shortest distance is $d = |(\overrightarrow{AB}) \cdot (\mathbf{u} \times \mathbf{v})|/|\mathbf{u} \times \mathbf{v}|$.

Step 1: Find a point and direction on each line The zero denominator in the first symmetric equation means that $x = 1$, not that the line is undefined. Its parametric form is $(x, y, z) = (1, -1 - t, t)$. Thus take $A = (1, -1, 0)$ and $\mathbf{u} = (0, -1, 1)$. The second line is the intersection of the two given planes. The point $B = (0, 1, -2)$ satisfies both: $0 + 1 - 2 + 1 = 0$ and $0 - 1 - 2 + 3 = 0$.

Step 2: Construct the common normal The plane normals are $\mathbf{n}_1 = (1, 1, 1)$ and $\mathbf{n}_2 = (2, -1, 1)$, so a direction of their intersection is

$$\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = (2, 1, -3).$$

The common normal to the two lines is therefore

$$\mathbf{u} \times \mathbf{v} = (2, 2, 2), \qquad |\mathbf{u} \times \mathbf{v}| = 2\sqrt{3}.$$

Step 3: Project the joining vector onto the normal Use $\overrightarrow{AB} = (-1, 2, -2)$. Its scalar product with the common normal is

$$(-1, 2, -2) \cdot (2, 2, 2) = -2 + 4 - 4 = -2.$$

Hence

$$d = \frac{|-2|}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

The absolute value makes the distance positive. The other proposed lengths $1, 1/2, 1/\sqrt{2}$ do not equal this normal projection; the full distance between the chosen points is not the required minimum.

Step 4: Conclusion The shortest distance is $\boxed{1/\sqrt{3}}$ unit. Hence, the correct option is (D).

Quick Tip to Solve: For a line given by two plane equations, cross their normals first; then use the scalar triple product for the shortest distance.

20. The negation of the Boolean expression p $\vee$ (~p $\wedge$ q) is equivalent to : [4 marks]

(A) $p \wedge \sim q$

(B) $\sim p \vee \sim q$

(C) $\sim p \wedge q$

(D) $\sim p \wedge \sim q$

Correct Answer: (D) $\sim p \wedge \sim q$

Solution:

Concept:

- De Morgan's laws state $\neg(r \vee s) = (\neg r) \wedge (\neg s)$ and $\neg(r \wedge s) = (\neg r) \vee (\neg s)$.
- Distributivity gives $p \vee (\neg p \wedge q) = (p \vee \neg p) \wedge (p \vee q)$, while $p \vee \neg p$ is always true.

Step 1: Simplify the original proposition Let $E = p \vee (\neg p \wedge q)$. Distribute the first disjunction over the conjunction:

$$E = (p \vee \neg p) \wedge (p \vee q).$$

Since $p \vee \neg p$ is true for either truth value of p, this reduces to $E = p \vee q$. This step also follows directly: if p is true, E is true; if p is false, E has precisely the truth value of q.

Step 2: Negate the simplified expression Apply De Morgan's law to the whole simplified expression, not merely to its second term:

$$\neg E = \neg(p \vee q) = (\neg p) \wedge (\neg q).$$

A direct check on the original expression gives the same result:

$$\begin{aligned} \neg E &= (\neg p) \wedge \neg(\neg p \wedge q) \\ &= (\neg p) \wedge (p \vee \neg q) \\ &= ((\neg p) \wedge p) \vee ((\neg p) \wedge \neg q) \\ &= (\neg p) \wedge \neg q. \end{aligned}$$

Step 3: Check the alternatives by truth values The negation must be true only when both p and q are false. Option (A), $p \wedge \neg q$, is true when p is true and q false, so it fails. Option (C), $\neg p \wedge q$, is true when q is true, so it also fails. Option (B), $\neg p \vee \neg q$, is already true when only one is false and therefore is

too broad. Only option (D) requires both to be false.

Step 4: Conclusion The required negation is $\boxed{\neg p \wedge \neg q}$. Hence, the correct option is (D).

Quick Tip to Solve: Simplify a compound proposition before negating it; negating an OR changes it to AND and negates both operands.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. Let f : R → R be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is _____. [4 marks]

Solution:

Concept:

- The derivative at zero is defined by $f\prime(0) = \lim_{h\to 0}[f(h) - f(0)]/h$, provided its one-sided limits agree.
- To test $f''(0)$, first find $f\prime(0)$, then take the one-sided limits of $[f\prime(h) - f\prime(0)]/h$. Bounded sine and cosine terms multiplied by positive powers of h vanish.

Step 1: Find the first derivative at zero Since $f(0) = 0$, the left difference quotient is $h^4 \sin(1/h) + 5h$ and the right one is $h^4 \cos(1/h) + \lambda h$. For every fixed real λ, both tend to zero because sine and cosine are bounded. Hence $f\prime(0) = 0$. Matching these first difference quotients alone places no restriction on λ; the question concerns the second derivative.

Step 2: Differentiate the two nonzero branches Use the product and chain rules, keeping the factor $-1/x^2$ from differentiating $1/x$. For $x < 0$,

$$f\prime(x) = 5x^4 \sin(1/x) - x^3 \cos(1/x) + 10x.$$

For $x > 0$,

$$f\prime(x) = 5x^4 \cos(1/x) + x^3 \sin(1/x) + 2\lambda x.$$

The signs of the two oscillating chain-rule terms differ because the derivative of cosine introduces another minus sign.

Step 3: Match the second difference quotients Dividing these derivatives by h, since $f\prime(0) = 0$, yields

$$\lim_{h\to 0^-} \frac{f\prime(h)}{h} = \lim_{h\to 0^-} \{5h^3 \sin(1/h) - h^2 \cos(1/h) + 10\} = 10,$$
$$\lim_{h\to 0^+} \frac{f\prime(h)}{h} = \lim_{h\to 0^+} \{5h^3 \cos(1/h) + h^2 \sin(1/h) + 2\lambda\} = 2\lambda.$$

Every oscillating contribution vanishes by its power bound. The second derivative exists exactly when

$10 = 2\lambda$.

Step 4: Conclusion Therefore $\boxed{\lambda = 5}$, and for this value $f''(0) = 10$.

Quick Tip to Solve: For a second derivative at a joining point, compare the difference quotients of the first derivative, after calculating its value at the join.

22. Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _____. [4 marks]

Solution:

Concept:

- A finite set with k elements has exactly 2^k subsets: each element is either included or excluded independently.
- Every positive integer has a unique factorization into a power of two times an odd integer. This separates the unknown exponents in a difference of powers of two.

Step 1: Translate the condition into powers of two Set A has 2^m subsets and set B has 2^n subsets. The statement that the former exceeds the latter by 112 gives

$$2^m - 2^n = 112.$$

The right side is positive, so $m > n$. In particular $m - n$ is a positive integer, which is needed when deciding the parity of the factor below.

Step 2: Separate the power of two from the odd factor Factor out the smaller power:

$$2^n\left(2^{m-n} - 1\right) = 112 = 2^4 \cdot 7.$$

Because $m - n \geq 1$, the number 2^{m-n} is even and $2^{m-n} - 1$ is odd. Thus all factors of two on the left occur in 2^n. Comparing the exact powers of two on both sides forces $n = 4$, rather than merely saying that 2^n divides 112.

Step 3: Determine the remaining exponent and check After cancelling 2^4, the remaining equation is

$$2^{m-4} - 1 = 7, \qquad 2^{m-4} = 8 = 2^3.$$

Therefore $m - 4 = 3$, giving $m = 7$. The counts of subsets are then $2^7 = 128$ and $2^4 = 16$. Their difference is $128 - 16 = 112$, exactly as required. Notice that the requested quantity is the product of the numbers of elements, not the difference of the subset counts or the product of those counts.

Step 4: Conclusion The required product is $mn = 7 \times 4 = \boxed{28}$. This is a dimensionless integer.

Quick Tip to Solve: Factor a difference of powers of two using the smaller power; the remaining factor is odd, so its power-of-two exponent is fixed immediately.

23. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\left|\vec{a}+\vec{b}\right|+\left|\vec{a}-\vec{b}\right|$ is_____. [4 marks]

Solution:

Concept:

- For unit vectors $\mathbf{a}, \mathbf{b}$, the parallelogram identity gives $|\mathbf{a}+\mathbf{b}|^2+|\mathbf{a}-\mathbf{b}|^2=4$.
- Cauchy–Schwarz gives $(\alpha u+\beta v)^2 \le (\alpha^2+\beta^2)(u^2+v^2)$, with equality when the two component pairs are proportional.

Step 1: Relate the two magnitudes Put $u=|\mathbf{a}+\mathbf{b}|$ and $v=|\mathbf{a}-\mathbf{b}|$, both nonnegative. Expanding their squares gives

$$u^2=2+2\mathbf{a}\cdot\mathbf{b}, \qquad v^2=2-2\mathbf{a}\cdot\mathbf{b}.$$

Their sum is therefore $u^2+v^2=4$, independently of the angle between the unit vectors. The expression to maximize is $S=\sqrt{3}\,u+v$.

Step 2: Apply a sharp upper bound Apply Cauchy–Schwarz to the real vectors $(\sqrt{3},1)$ and (u,v):

$$S^2 \le (3+1)(u^2+v^2)=4\cdot 4=16.$$

Since $S \ge 0$, this proves $S \le 4$. Merely bounding each magnitude by 2 separately would give a larger bound that cannot be attained simultaneously; their fixed sum of squares must be used.

Step 3: Show that equality can actually occur Equality requires $u : v = \sqrt{3} : 1$. Together with $u^2+v^2=4$, this gives $u=\sqrt{3}$ and $v=1$. The first expansion then requires

$$3=2+2\mathbf{a}\cdot\mathbf{b}, \qquad \mathbf{a}\cdot\mathbf{b}=\frac{1}{2}.$$

Two unit vectors at an angle 60° satisfy this condition, so the bound is attainable. For them $S=\sqrt{3}\sqrt{3}+1=4$. Thus the bound is a maximum, not just an estimate.

Step 4: Conclusion The greatest value is $\boxed{4}$, attained when the angle between the unit vectors is 60°. The value is dimensionless.

Quick Tip to Solve: For sums and differences of two unit vectors, use their fixed sum of squared lengths, then match Cauchy–Schwarz equality conditions.

24. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is _____. [4 marks]

Solution:

Concept:

- A vertical pole and a segment on horizontal ground form a right triangle, so the squared slant distance equals the squared horizontal distance plus the squared pole height.
- A quadratic with positive leading coefficient has a unique minimum at its vertex; completing the square also reveals the minimum and its location directly.

Step 1: Express both squared distances using one variable Let $AM = x$ metres. Since M lies on the segment AB of length 10 metres, $0 \le x \le 10$ and $BM = 10 - x$. The poles have heights $AD = 8$ metres and $BC = 11$ metres. By the Pythagorean theorem applied separately to triangles MAD and MBC,

$$MD^2 = x^2 + 8^2, \qquad MC^2 = (10 - x)^2 + 11^2.$$

The different heights contribute constant terms when the point moves along the ground.

Step 2: Complete the square in the total Add the two expressions, with all numerical squared distances measured in square metres:

$$\begin{aligned} S(x) &= MD^2 + MC^2 \\ &= x^2 + 64 + (10 - x)^2 + 121 \\ &= 2x^2 - 20x + 285 \\ &= 2(x - 5)^2 + 235. \end{aligned}$$

The square is nonnegative and its coefficient is positive. Therefore the least possible total occurs precisely when $x - 5 = 0$. The value $x = 5$ is allowed because it lies in $[0, 10]$.

Step 3: Verify the minimizing location At $x = 5$, the horizontal distances from the two feet are equal. The squared slant distances are $MD^2 = 25 + 64 = 89$ and $MC^2 = 25 + 121 = 146$, giving the sum 235. Moving a distance t metres from this midpoint increases the sum by $2t^2$, proving it is a strict minimum. The question asks for the location AM, not for the minimum squared-distance sum 235, nor for the distance to either pole top.

Step 4: Conclusion The required distance of M from A is $AM = \boxed{5\,\text{m}}$.

Quick Tip to Solve: When minimizing a sum of squared distances to two vertical pole tops, the fixed height squares do not affect the minimizing ground point.

25. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is _____. [4 marks]

Solution:

Concept:

- The tangent of an angle of elevation equals the vertical separation divided by the horizontal separation. Both separations must be measured from the observer's current position.
- A displacement of length s up an incline of angle θ has horizontal component $s\cos\theta$ and vertical component $s\sin\theta$.

Step 1: Express the original horizontal distance Let the height of the hill above the horizontal plane be h, and let the initial horizontal distance from its foot be x. The first observation gives

$$\tan 45^\circ = \frac{h}{x} = 1, \qquad x = h.$$

Thus the initial horizontal separation happens to equal the height, but it decreases by only the horizontal component of the walk.

Step 2: Resolve the walk along the slope The observer walks 80 m up a slope at 30°, toward the top. The horizontal advance and rise are

$$80\cos 30^\circ = 40\sqrt{3}\ \text{m}, \qquad 80\sin 30^\circ = 40\ \text{m}.$$

Hence the new horizontal separation is $h - 40\sqrt{3}$, and the top is now $h - 40$ metres above the observer.

Step 3: Use the second elevation and solve Using $\tan 75^\circ = 2 + \sqrt{3}$,

$$\frac{h - 40}{h - 40\sqrt{3}} = 2 + \sqrt{3}.$$

Rearrangement gives

$$h - 40 = (2 + \sqrt{3})h - 80\sqrt{3} - 120,$$

$$(1 + \sqrt{3})h = 80 + 80\sqrt{3}, \qquad h = 80\ \text{m}.$$

The remaining horizontal distance $80 - 40\sqrt{3}$ is positive, so the position is geometrically consistent.

Step 4: Conclusion The height of the hill is $\boxed{80\ \text{m}}$. The numerical value to enter is $\boxed{80}$.

Quick Tip to Solve: Resolve a walk on a slope into horizontal advance and vertical rise before using the new angle of elevation; subtract both components from the original triangle.

26. You are given that Mass of $^{7}_{3}Li$ = 7.0160u,
Mass of $^{4}_{2}He$ = 4.0026u
and Mass of $^{1}_{1}H$ = 1.0079u.
When 20 g of $^{7}_{3}Li$ is converted into $^{4}_{2}He$ by proton capture, the energy liberated, (in kWh), is :
[Mass of nucleon = 1 GeV/c^2] [4 marks]

(A) 6.82×10^5
(B) 4.5×10^5
(C) 8×10^6
(D) 1.33×10^6

Correct Answer: (D) 1.33×10^6

Solution:

Concept:

- Energy released by a nuclear reaction is $Q = \Delta m c^2$, where Δm is the initial mass minus the final mass.
- Conserve nucleon number and charge to write $^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \rightarrow 2\,{}^{4}_{2}\mathrm{He}$. With the supplied approximation, $1\,\mathrm{u}\,c^2 \simeq 1\,\mathrm{GeV}$.

Step 1: Find the energy released per capture The lithium nucleus and captured proton have total mass $7.0160 + 1.0079 = 8.0239\,\mathrm{u}$. The two helium products have mass $2(4.0026) = 8.0052\,\mathrm{u}$. Therefore

$$\Delta m = 8.0239 - 8.0052 = 0.0187\,\mathrm{u},$$

$$Q = 0.0187\,\mathrm{GeV} = 18.7\,\mathrm{MeV}.$$

Atomic electron masses cancel as well, because the reactants and products each contain four electrons when neutral atomic masses are used.

Step 2: Count the reactions in the sample Each capture consumes one lithium nucleus. Taking the molar mass as approximately $7\,\mathrm{g\,mol^{-1}}$, the number of captures is

$$N = \frac{20}{7}(6.022 \times 10^{23}) \simeq 1.72 \times 10^{24}.$$

The energy conversion is $1\,\mathrm{GeV} = 1.602 \times 10^{-10}\,\mathrm{J}$, so

$$E = N(0.0187)(1.602 \times 10^{-10}) \simeq 5.15 \times 10^{12}\,\mathrm{J}.$$

Step 3: Convert the total to kilowatt-hours Since $1\,\mathrm{kWh} = 3.6 \times 10^6\,\mathrm{J}$,

$$E \simeq \frac{5.15 \times 10^{12}}{3.6 \times 10^6} = 1.43 \times 10^6\,\mathrm{kWh}.$$

The listed value $1.33 \times 10^6\,\mathrm{kWh}$ is the intended closest choice; it corresponds to the usual $931.5\,\mathrm{MeV}$

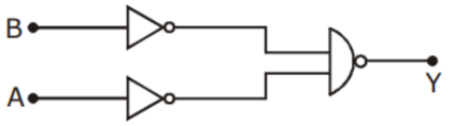
per atomic mass unit instead of the supplied rounded $1000\,\text{MeV}$. The other values differ far more and cannot result from this mass defect.

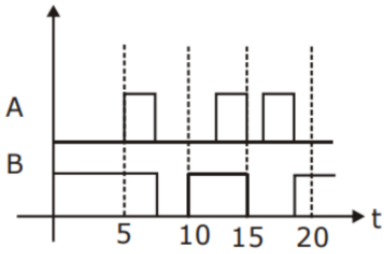
Step 4: Conclusion The intended listed result is $\boxed{1.33 \times 10^6\,\text{kWh}}$. Hence, the correct option is (D). Strictly, the stated $1\,\text{GeV}/c^2$ approximation gives about $1.43 \times 10^6\,\text{kWh}$; the small mismatch reflects the mass-energy conversion used in the options.

Quick Tip to Solve: Balance the nuclear reaction first: proton capture by lithium produces two helium nuclei, so the final mass is twice the stated helium mass.

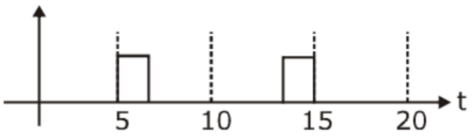
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27. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B. [4 marks]

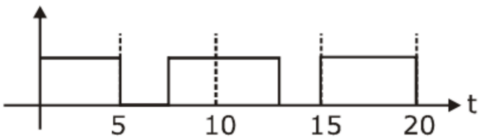




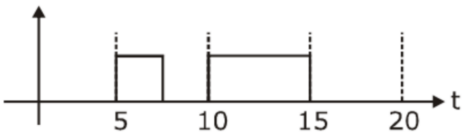
(A)



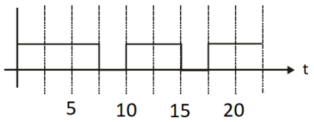
(B)



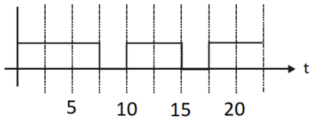
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- A NOT gate reverses a binary input, and a NAND gate gives the complement of the product of its inputs.
- De Morgan's theorem gives $\overline{\bar{A}\,\bar{B}} = A + B$, where the plus sign denotes logical OR rather than numerical addition.

Step 1: Identify the signal entering the final gate The input labelled A first passes through an inverter, so its signal at the lower input of the final gate is $\bar{A}$. Similarly B passes through its own inverter and reaches the upper input as $\bar{B}$. The last gate is AND-shaped with a bubble at its output, identifying it as a NAND gate. Therefore the complete expression is

$$Y = \overline{\bar{A}\,\bar{B}} = A + B.$$

Step 2: Apply the OR truth table to the input traces The output is high whenever either input is high, and it is low only when both input traces are on their low baseline. Explicitly,

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Initially B is high, so Y must start high even though A starts low. Across the displayed middle interval from 10 to 15, B is high throughout; any extra A pulse there cannot introduce a low gap in Y. Later, the last A pulse joins the final high portion of B, keeping Y high across that transition.

Step 3: Match the complete output waveform Option D has exactly the union of the high portions of A and B, with low intervals only where neither input is high. Option A wrongly starts low and retains only short pulses. Option C also starts low, contrary to the initial high B input. Option B falls at the start of the first A pulse even though B remains high there, so it is not the required OR trace. The output bubbles must be included: ignoring either inversion gives a different logic function.

Step 4: Conclusion The combination acts as an OR gate: $\boxed{Y = A + B}$. Hence, the correct option is (D).

Quick Tip to Solve: Push the output inversion through the NAND gate using De Morgan's law: the two input inversions cancel, leaving an OR operation on the original signals.

28. A particle of charge q and mass m is moving with a velocity $-v\hat{i}$ (v $\neq$ 0) towards a large screen placed in the Y - Z plane at a distance d. If there is a magnetic field $\overrightarrow{B} = B_0\hat{k}$, the maximum value of v for which the particle will not hit the screen is : [4 marks]

(A) $\frac{2qdB_0}{m}$
(B) $\frac{qdB_0}{3m}$
(C) $\frac{qdB_0}{2m}$
(D) $\frac{qdB_0}{m}$

Correct Answer: (D) $\frac{qdB_0}{m}$

Solution:

Concept:

- A magnetic field perpendicular to the velocity bends a charged particle into a circle of radius $r = mv/(|q|B_0)$, without changing its speed.
- Whether a plane is reached depends on the maximum displacement toward that plane, not the full diameter of the circular orbit.

Step 1: Locate the circle relative to the initial position Take the screen to be $x = 0$, with the particle initially at $x = d$ and velocity along $-\hat{\mathbf{i}}$. Since $\mathbf{B} = B_0\hat{\mathbf{k}}$, the Lorentz force is perpendicular to the initial velocity and lies along the y direction. Thus the circle centre has the same initial x coordinate, namely d. The circle extends leftward by one radius from its centre; its smallest x coordinate is

$$x_{\min} = d - r.$$

Step 2: Find the limiting radius and speed The particle turns back before crossing the screen if $r < d$. At the threshold it just grazes the screen, so the limiting value is obtained by setting

$$r = d, \qquad \frac{mv_{\lim}}{qB_0} = d,$$

$$v_{\lim} = \frac{qdB_0}{m}.$$

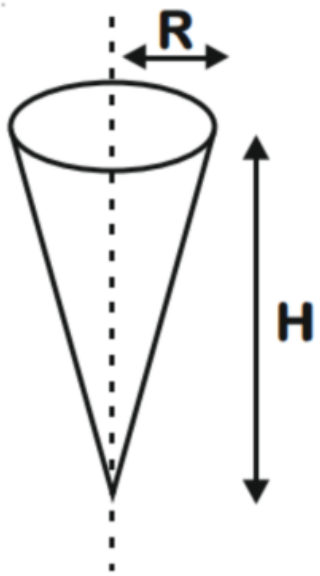
Here the positive q in the options is the charge magnitude; reversing its sign reverses the bend in y but leaves the closest x coordinate unchanged.

Step 3: Check the proposed alternatives Option A would give $r = 2d$, so the orbit intersects the screen. Options B and C give radii $d/3$ and $d/2$; these speeds avoid the screen but are smaller than the limiting value and hence are not maximal. Option D gives exactly the limiting radius. If touching the screen counts as hitting it, the strictly safe speeds obey $v < qdB_0/m$; the question uses the conventional threshold value as its maximum.

Step 4: Conclusion The limiting speed is $\boxed{v_{\max} = qdB_0/m}$, with strict non-contact speeds below this threshold. Hence, the correct option is (D).

Quick Tip to Solve: At the initial point the magnetic force is sideways, so the circle centre is sideways too: its approach toward the screen is one radius, not two.

29. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is : [4 marks]



(A) $\frac{M\left(R^2+H^2\right)}{3}$

(B) $\frac{MR^2}{2}$

(C) $\frac{MR^2}{3}$

(D) $\frac{MH^2}{3}$

Correct Answer: (B) $\frac{MR^2}{2}$

Solution:

Concept:

- The axial moment of inertia is $I = \int r^2\, dm$, with r the perpendicular distance from the symmetry axis.
- A uniform thin conical shell can be divided into narrow circular rings; their masses are proportional to ring circumference times slant width.

Step 1: Parameterise the conical surface Let the slant height of the cone be $L = \sqrt{R^2 + H^2}$, and let σ be its uniform surface mass density. Its lateral area is πRL, so

$$\sigma = \frac{M}{\pi RL}.$$

At radius r measured from the axis, similarity of cones gives slant distance $s = Lr/R$, hence $ds = (L/R)dr$. No circular lid is included because the cone is hollow and open at the top.

Step 2: Obtain the mass of a thin ring The lateral strip of slant width ds and circumference $2\pi r$ has mass

$$dm = \sigma(2\pi r)\, ds = \frac{M}{\pi RL}(2\pi r)\frac{L}{R}\, dr = \frac{2M}{R^2} r\, dr.$$

Integrating this expression from zero to R gives M, confirming that all of the conical shell mass is included. Each ring lies entirely at perpendicular distance r from the axis, so its contribution is $dI = r^2 dm$.

Step 3: Integrate and compare the answers The total moment is

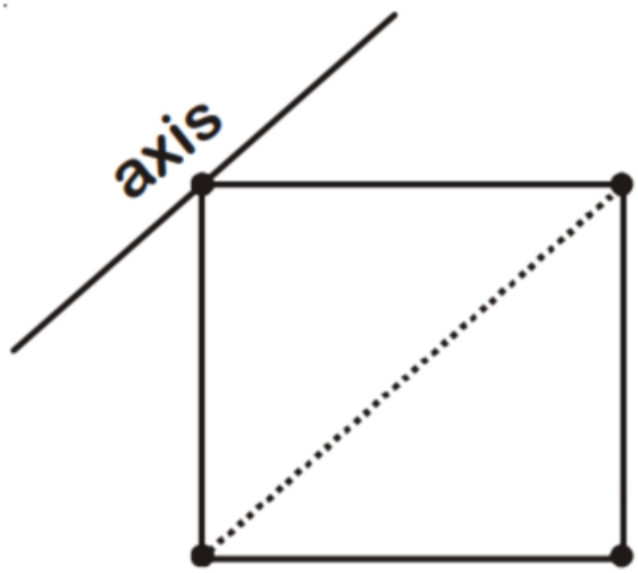
$$I = \frac{2M}{R^2}\int_0^R r^3\,dr = \frac{2M}{R^2}\frac{R^4}{4} = \frac{MR^2}{2}.$$

The slant height cancels at fixed M and R, so terms involving H in options A and D cannot describe this axial moment. Option C has the incorrect radial mass weighting; the shell has more mass in rings near its broad end than a uniform distribution in radius would have.

Step 4: Conclusion The moment of inertia about the cone's symmetry axis is $\boxed{I = MR^2/2}$. Hence, the correct option is (B).

Quick Tip to Solve: For a thin cone use surface rings, with $dm \propto r\,dr$. A solid cone requires volume slices and has a different coefficient.

30. Four point masses, each of mass m, are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is : [4 marks]



(A) $3ml^2\omega$

(B) $4ml^2\omega$

(C) $ml^2\omega$

(D) $2ml^2\omega$

Correct Answer: (A) $3ml^2\omega$

Solution:

Concept:

- For rotation about a fixed axis, the component of angular momentum about that axis is $L_{\text{axis}} = I_{\text{axis}}\omega$.
- For point masses, $I = \sum mr_{\perp}^2$; distances must be measured perpendicularly to the actual axis, not along square edges.

Step 1: Determine the four perpendicular distances Put the corner on the axis at $(0, 0)$, and take the other three vertices as $(l, 0)$, $(0, -l)$, and $(l, -l)$. The indicated axis is parallel to the other diagonal and has equation $y = x$. The perpendicular distance of (x, y) from this line is $|x-y|/\sqrt{2}$. Consequently the four distances are

$$0, \qquad \frac{l}{\sqrt{2}}, \qquad \frac{l}{\sqrt{2}}, \qquad \sqrt{2}\,l.$$

The farthest corner is across the square from the corner on the axis.

Step 2: Sum the moment of inertia All four particles have the same mass m. The corner on the axis contributes zero, the two neighbouring corners contribute $ml^2/2$ each, and the opposite corner contributes $2ml^2$. Thus

$$I = m\left[0 + \frac{l^2}{2} + \frac{l^2}{2} + 2l^2\right] = 3ml^2.$$

As a check, the central parallel diagonal has moment ml^2; shifting it outward by $l/\sqrt{2}$ adds $(4m)(l^2/2) = 2ml^2$, giving the same total.

Step 3: Calculate angular momentum and eliminate distractors Multiply the moment of inertia by the angular frequency:

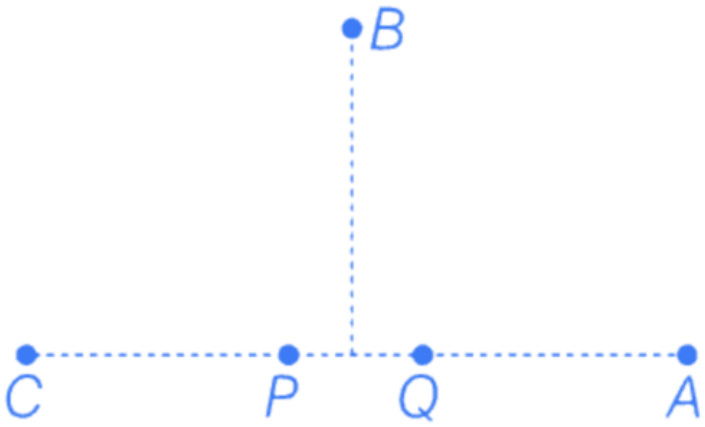
$$L_{\text{axis}} = I\omega = 3ml^2\omega.$$

Option C corresponds to the parallel axis through the centre, not through the corner. Option D retains only the farthest particle's contribution and misses the two adjacent masses. Option B would require all four masses to be a distance l from the axis, which is not the geometry shown.

Step 4: Conclusion The angular momentum about the indicated axis is $\boxed{3ml^2\omega}$. Hence, the correct option is (A).

Quick Tip to Solve: List the squared perpendicular distances first: $0, l^2/2, l^2/2, 2l^2$. Their sum immediately gives the moment of inertia.

31. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio : [4 marks]



(A) 4 : 1 : 0

(B) 2 : 1 : 0

(C) 0 : 1 : 2

(D) 0 : 1 : 4

Correct Answer: (B) 2 : 1 : 0

Solution:

Concept:

- For two coherent sources each contributing intensity I_0, the resultant intensity is $I = 2I_0(1+\cos\delta)$, where δ is their phase difference at the observation point.
- Propagation over distance r subtracts phase kr, with $k = 2\pi/\lambda$. The initial phase lead and the path-induced phase difference must therefore be combined with their signs.

Step 1: Find the relative phase at each point Take the phase of P minus that of Q. Since P initially leads Q by $\pi/2$, the phase difference after propagation is

$$\delta = \frac{\pi}{2} - \frac{2\pi}{20}(r_P - r_Q).$$

At A, P is farther away by 5 m, so $r_P - r_Q = 5$ m and $\delta_A = 0$. At B, the paths are equal, giving $\delta_B = \pi/2$. At C, P is closer by 5 m, so $\delta_C = \pi$.

Step 2: Calculate and compare the three intensities Substitute these phases into the interference formula:

$$I_A = 4I_0, \qquad I_B = 2I_0, \qquad I_C = 0.$$

Thus A is a constructive-interference point, B has the sum of the individual intensities, and C is a destructive-interference point. Dividing by $2I_0$ gives $2:1:0$. Option A incorrectly doubles only the first entry; options C and D interchange the bright and dark sides by reversing the initial phase lead.

Step 3: Conclusion The intensities are $I_A : I_B : I_C = 2:1:0$. Hence, the correct option is (B).

Quick Tip to Solve: Write the arriving phase as initial phase minus kr; this prevents reversing the bright and dark sides when the sources already have a phase difference.

32. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of : [4 marks]

(A) 10^{-3}

(B) 10^{-1}

(C) 10^{-2}

(D) 10^{-4}

Correct Answer: (A) 10^{-3}

Solution:

Concept:

- The second hand completes one revolution in $T = 60\,\text{s}$, so its angular speed is $\omega = 2\pi/T$. Constant speed does not mean constant velocity because the direction changes.
- The characteristic acceleration of a point at radius r is the centripetal value $a = \omega^2 r$. A vector average over a specified finite interval instead equals $\Delta \boldsymbol{v}/\Delta t$.

Step 1: Compute the acceleration scale The tip is at $r = 0.1\,\text{m}$. Its speed is

$$v = \frac{2\pi(0.1)}{60} = 1.047 \times 10^{-2}\,\text{m s}^{-1}.$$

Therefore,

$$a = \frac{v^2}{r} = \left(\frac{2\pi}{60}\right)^2 (0.1) = 1.097 \times 10^{-3}\,\text{m s}^{-2}.$$

This is of order 10^{-3}, not 10^{-2}, 10^{-1} or 10^{-4}.

Step 2: Interpret the wording about the average For a time interval Δt, the magnitude of the vector average is

$$|\overline{\boldsymbol{a}}| = \frac{2v|\sin(\omega\Delta t/2)|}{\Delta t}.$$

For a short interval this tends to $v\omega = \omega^2 r$, giving the scale calculated above. The question does not specify an averaging interval; its listed alternatives indicate this characteristic circular-motion acceleration. A vector average over exactly one complete revolution would be zero, since the initial and final velocities would coincide. It must not be confused with the nonzero centripetal acceleration.

Step 3: Conclusion The intended acceleration scale is approximately $1.1 \times 10^{-3}\,\text{m s}^{-2}$. Hence, the correct option is (A).

Quick Tip to Solve: Use 60 s for a second hand and square $2\pi/60$. For a finite vector average, check the stated interval before using the centripetal value.

33. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5mm is noticed on the pitch scale. The nature of zero error involved and the least count of the screw gauge, are respectively : [4 marks]

(A) Positive, 0.1 mm

(B) Positive, 0.1 μm

(C) Positive, 10 μm

(D) Negative, 2 μm

Correct Answer: (C) Positive, 10 μm

Solution:

Concept:

- The pitch is the axial distance moved in one complete rotation. The least count is the pitch divided by the number of circular-scale divisions.
- A positive reading when the measuring faces are closed is a positive zero error. The zero correction is the negative of that error; neither changes the instrument least count.

Step 1: Calculate the least count One complete turn moves the screw through $0.5\,\text{mm}$, so the pitch is $0.5\,\text{mm}$. There are 50 equal divisions on the circular scale. Hence one division corresponds to

$$\text{LC} = \frac{0.5}{50}\,\text{mm} = 0.01\,\text{mm} = 10\,\mu\text{m}.$$

The conversion uses $1\,\text{mm} = 1000\,\mu\text{m}$. The four-division initial offset is not the least count; it describes the zero error.

Step 2: Determine the sign and check the options The circular scale is four divisions ahead of the pitch-scale reference before use. Thus it records a positive reading even at the zero setting:

$$e_0 = 4 \times 0.01 = +0.04\,\text{mm}.$$

The correction to a measured reading would consequently be $-0.04\,\text{mm}$. The required pair is positive zero error and $10\,\mu\text{m}$ least count. Option A has a least count ten times too large; option B has a least count one hundred times too small. Option D has both the wrong sign and an incorrect division value.

Step 3: Conclusion The zero error is positive and the least count is $10\,\mu\text{m}$. Hence, the correct option is (C).

Quick Tip to Solve: Find pitch divided by divisions first. Then use the closed-jaw reading to decide the error sign; do not divide the pitch by the zero-offset divisions.

34. An object of mass m is suspended at the end of a massless wire of length L and area of crosssection A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is : [4 marks]

(A) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

(B) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

(C) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

(D) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$

Correct Answer: (A) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

Solution:

Concept:

- Young modulus is stress divided by longitudinal strain: $Y = (F/A)/(\Delta L/L)$. In the elastic range this gives an effective spring constant $k = YA/L$.
- A mass undergoing small oscillations under a linear restoring force has angular frequency $\omega = \sqrt{k/m}$ and ordinary frequency $f = \omega/(2\pi)$.

Step 1: Find the incremental restoring force Let the equilibrium extension produced by the weight be x_0. Then $kx_0 = mg$, where $k = YA/L$. Now displace the mass downward by a small additional distance x. The tension increases to $k(x_0 + x)$, while its weight remains mg. Taking downward as positive,

$$m\ddot{x} = mg - k(x_0 + x) = -kx = -\frac{YA}{L}x.$$

Thus the net force is proportional and opposite to displacement measured from equilibrium.

Step 2: Read the frequency from the motion equation Comparing $\ddot{x} + (YA/mL)x = 0$ with the standard equation $\ddot{x} + \omega^2 x = 0$,

$$\omega = \sqrt{\frac{YA}{mL}}, \qquad f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}.$$

The weight changes the equilibrium extension, but cancels from the incremental motion equation. A thicker or stiffer wire has a greater frequency, whereas a longer wire or a larger mass has a smaller frequency. Option B inverts the required ratio. Option C gives the wrong dependence on area and length, and option D also gives an increasing frequency with mass. These fail the physical stiffness check.

Step 3: Conclusion The oscillation frequency is $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$. Hence, the correct option is (A).

Quick Tip to Solve: Replace an elastic wire by a spring with $k = YA/L$, and measure displacement from the stretched equilibrium position so that the constant weight cancels.

35. An electron is moving along +x direction with a velocity of 6×10^6 ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along +y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be : [4 marks]

(A) 3×10^{-4} T, along –z direction

(B) 5×10^{-3} T, along –z direction

(C) 5×10^{-3} T, along +z direction

(D) 3×10^{-4} T, along +z direction

Correct Answer: (C) 5×10^{-3} T, along +z direction

Solution:

Concept:

- A charge in electric and magnetic fields experiences $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$. Straight motion here requires the transverse electric and magnetic forces to cancel.
- When velocity and magnetic field are perpendicular, cancellation requires $E = vB$. The cross-product direction must be checked independently of this magnitude condition.

Step 1: Convert the electric field and find the magnitude The field is $300\,\mathrm{V\,cm^{-1}}$, and one metre contains 100 centimetres. Therefore

$$E = 300 \times 100 = 3.0 \times 10^4\,\mathrm{V\,m^{-1}}.$$

With $v = 6.0 \times 10^6\,\mathrm{m\,s^{-1}}$,

$$B = \frac{E}{v} = \frac{3.0 \times 10^4}{6.0 \times 10^6} = 5.0 \times 10^{-3}\,\mathrm{T}.$$

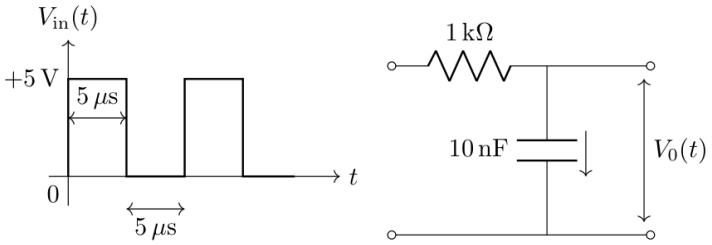
The common magnitude of the charge cancels; using the electron mass is unnecessary.

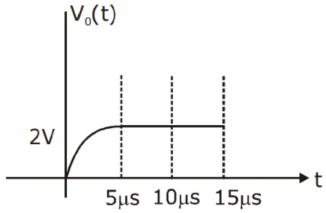
Step 2: Determine the field direction The electric field points along $+y$, so zero net force demands $\boldsymbol{v} \times \boldsymbol{B}$ along $-y$. Since $\hat{\boldsymbol{x}} \times \hat{\boldsymbol{z}} = -\hat{\boldsymbol{y}}$, the magnetic field must point along $+z$. For an electron the electric force is along $-y$, while the magnetic force is along $+y$, confirming cancellation. Option B has the correct magnitude but the opposite direction, making the forces add. Options A and D have the wrong magnitude; A additionally has the wrong direction. The negative charge reverses both forces, not just the magnetic force.

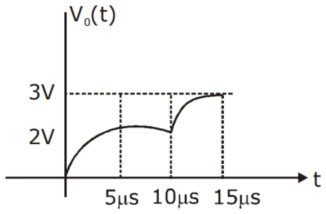
Step 3: Conclusion The required field is $5.0 \times 10^{-3}\,\mathrm{T}$ along $+z$. Hence, the correct option is (C).

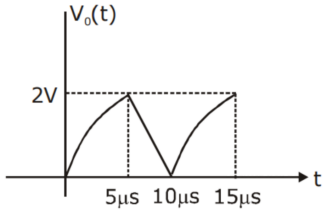
Quick Tip to Solve: Cancel the charge in $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = 0$ first. Convert V/cm to V/m before dividing by the speed.

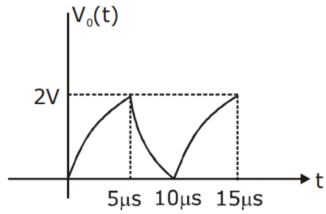
36. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_0(t)$, across the capacitor is correctly depicted by : [4 marks]



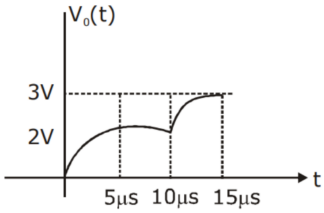
(A)


(B)


(C)


(D)


Correct Answer: (B)



Solution:

Concept:

- For a resistor feeding a capacitor, $RC\,dV_0/dt + V_0 = V_{in}$. The capacitor voltage is continuous and approaches the input level exponentially during each constant-input interval.

- The charging solution is $V_0 = V_s + (V_i - V_s)e^{-t/RC}$; during a zero-input interval it is $V_0 = V_i e^{-t/RC}$. The starting value for a new interval is the ending value of the preceding one.

Step 1: Find the charging time and first maximum The circuit has $R = 1\,\text{k}\Omega$ and $C = 10\,\text{nF}$, so

$$\tau = RC = 10^3 \times 10^{-8}\,\text{s} = 10\,\mu\text{s}.$$

During the first $5\,\mu$s, the input is 5 V. Starting from zero capacitor voltage,

$$V_0(5) = 5(1 - e^{-5/10}) \approx 1.97\,\text{V}.$$

Thus the first segment is a concave-down exponential rise approaching 2 V at the switching instant, rather than a jump to 5 V.

Step 2: Follow the discharge and the next pulse From 5 to $10\,\mu$s, the input is zero and the capacitor discharges through the same resistor. Its voltage falls exponentially but cannot reach zero in this finite interval:

$$V_0(10) = 1.97e^{-1/2} \approx 1.19\,\text{V}.$$

The next 5 V pulse raises the voltage again, giving

$$V_0(15) = 5 + (1.19 - 5)e^{-1/2} \approx 2.69\,\text{V}.$$

The second peak is higher than the first because residual charge remains. Only graph B shows this rise, partial discharge and subsequent rise toward about 3 V. Graph A omits discharge; C and D incorrectly return to zero after each low-input interval. The plotted voltages are schematic, so exponential behaviour and the nonzero trough decide the answer.

Step 3: Conclusion The capacitor voltage rises, decreases only partly, then rises to a higher peak, as in graph B. Hence, the correct option is (B).

Quick Tip to Solve: Compare each pulse duration with RC. A capacitor retains a factor $e^{-\Delta t/RC}$ of its voltage during a zero-input interval, so a finite low pulse cannot empty it completely.

37. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is: [4 marks]

(A) 1 : 2

(B) 1 : 3

(C) 1 : 6

(D) 3 : 4

Correct Answer: (C) 1 : 6

Solution:

Concept:

- Gravity is a central force directed towards the planet, so its torque about the planet vanishes and the satellite's angular momentum is conserved.
- At the closest and farthest points of an elliptical orbit the radial speed is zero. The velocity is perpendicular to the radius there, giving angular momentum magnitude $L = mrv$.

Step 1: Identify the two orbital positions Let r_c, v_c denote distance and speed at closest approach, and r_f, v_f those at the farthest point. The given speed relation means $v_f = v_c/6$. The orbit need not be circular: the changing speed is precisely why the radius factors must be retained. The planet is at a focus of the ellipse, so both distances must be measured from the planet, not from the centre of the ellipse.

Step 2: Apply angular momentum conservation The same angular momentum applies at both apsides. Since the radius and velocity are perpendicular at each,

$$mr_c v_c = mr_f v_f.$$

Cancel the satellite mass and divide by $r_f v_c$:

$$\frac{r_c}{r_f} = \frac{v_f}{v_c} = \frac{1}{6}.$$

This inverse relation also agrees with Kepler's equal-area law: near the planet the radius is shorter and the satellite moves faster.

Step 3: Compare the proposed ratios The required order is closest distance followed by farthest distance, hence $r_c : r_f = 1 : 6$. Ratios $1 : 2$, $1 : 3$ and $3 : 4$ would require closest-to-farthest speed ratios 2, 3 and $4/3$, respectively, rather than the stated factor of six. Conservation of speed or of kinetic energy alone would be inappropriate because gravitational potential energy changes around the orbit.

Step 4: Conclusion The ratio of closest to farthest distances is $\boxed{1 : 6}$. Hence, the correct option is (C).

Quick Tip to Solve: At the two ends of an elliptical orbit use $r_c v_c = r_f v_f$; the distance ratio is the inverse of the speed ratio.

38. If the potential energy between two molecules is given by

U = $-\frac{A}{r^6}+\frac{B}{r^{12}}$,

then at equilibrium, separation between molecules, and the potential energy are : [4 marks]

(A) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$

(B) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

(C) $\left(\frac{B}{A}\right)^{1/6}, 0$

(D) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$

Correct Answer: (A) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$

Solution:

Concept:

- The radial force is $F_r = -\mathrm{d}U/\mathrm{d}r$. A finite equilibrium separation therefore satisfies $\mathrm{d}U/\mathrm{d}r = 0$.
- A stable equilibrium is a minimum of potential energy, verified by $\mathrm{d}^2U/\mathrm{d}r^2 > 0$. The constants A and B in the attractive and repulsive terms are positive.

Step 1: Differentiate the potential For $U = -Ar^{-6} + Br^{-12}$, differentiate with respect to the separation:

$$\frac{\mathrm{d}U}{\mathrm{d}r} = 6Ar^{-7} - 12Br^{-13}.$$

Setting this derivative to zero and multiplying by $r^{13}/6$ gives

$$Ar^6 - 2B = 0, \qquad r_0^6 = \frac{2B}{A}.$$

Thus the positive physical separation is $r_0 = (2B/A)^{1/6}$.

Step 2: Evaluate the energy and check stability At this separation, $r_0^{-6} = A/(2B)$ and $r_0^{-12} = A^2/(4B^2)$. Substituting both terms,

$$U(r_0) = -\frac{A^2}{2B} + \frac{A^2}{4B} = -\frac{A^2}{4B}.$$

The second derivative is

$$U''(r_0) = -\frac{42A}{r_0^8} + \frac{156B}{r_0^{14}} = \frac{36A}{r_0^8} > 0,$$

where $B = Ar_0^6/2$ was used. Hence this stationary point is indeed stable.

Step 3: Eliminate the alternatives Option (B) has the right separation but omits the positive repulsive contribution when stating the energy. At the separation in (C), $Ar^6 = B$, the potential is zero but its derivative is not zero, so this is not equilibrium. Option (D) also fails $Ar^6 = 2B$. Both the distance and the energy must agree simultaneously.

Step 4: Conclusion The equilibrium separation and potential energy are $\boxed{(2B/A)^{1/6},\ -A^2/(4B)}$. Hence, the correct option is (A).

Quick Tip to Solve: First find r_0^6, then substitute its reciprocal into both potential terms; do not confuse zero potential with zero force.

39. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e, $\lambda_{He^{++}}$ and λ_p is : [4 marks]

(A) $\lambda_e > \lambda_{He^{++}} > \lambda_p$

(B) $\lambda_e < \lambda_p < \lambda_{He^{++}}$

(C) $\lambda_e > \lambda_p > \lambda_{He^{++}}$

(D) $\lambda_e < \lambda_{He^{++}} = \lambda_p$

Correct Answer: (C) $\lambda_e > \lambda_p > \lambda_{He^{++}}$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, with h Planck's constant and p the particle momentum.
- For a nonrelativistic particle of mass m and kinetic energy K, $p = \sqrt{2mK}$. At equal K, wavelength varies inversely as $\sqrt{m}$.

Step 1: Express the wavelength at fixed energy The question gives the same kinetic energy to all three particles, not the same speed and not acceleration through the same voltage. Combining $K = p^2/(2m)$ with the de Broglie relation gives

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

Here h and K are common positive factors, so the ordering is decided entirely by the mass. A smaller mass gives a smaller momentum at fixed energy, and therefore a longer wavelength.

Step 2: Order the masses and wavelengths An electron is much lighter than a proton, while a doubly ionized helium atom is essentially a helium nucleus with mass about four proton masses:

$$m_e \ll m_p < m_{He^{++}} \simeq 4m_p.$$

Reversing this mass order in the inverse-square-root law yields

$$\lambda_e > \lambda_p > \lambda_{He^{++}}.$$

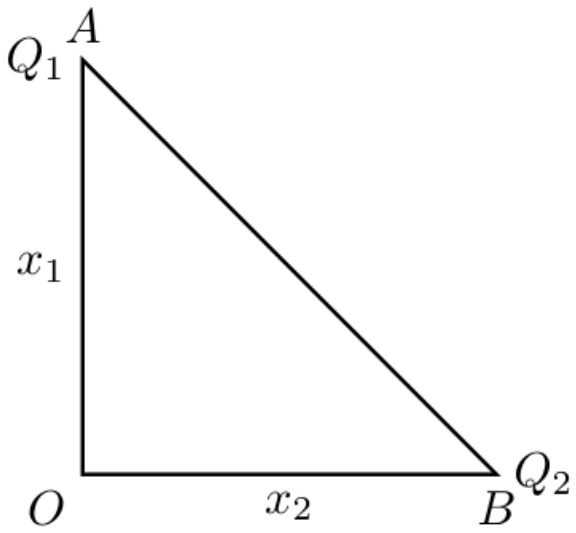
In particular, $\lambda_{He^{++}} \simeq \lambda_p/2$, so these two wavelengths cannot be equal.

Step 3: Check each competing ordering Option (A) incorrectly puts the heavier helium ion ahead of the proton in wavelength. Options (B) and (D) incorrectly give the very light electron the shortest wavelength; (D) also equates wavelengths for unequal nuclear masses. The double positive charge does not enter the relation for already specified kinetic energy. It would matter if a common accelerating potential, rather than common energy, had been specified.

Step 4: Conclusion The wavelengths satisfy $\boxed{\lambda_e > \lambda_p > \lambda_{He^{++}}}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal kinetic energies compare $1/\sqrt{m}$; do not introduce the charge factor used for particles accelerated through the same voltage.

40. Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then $\frac{Q_1}{Q_2}$ is proportional to : [4 marks]



(A) $\frac{x_1^3}{x_2^3}$

(B) $\frac{x_2^2}{x_1^2}$

(C) $\frac{x_1}{x_2}$

(D) $\frac{x_2}{x_1}$

Correct Answer: (C) $\frac{x_1}{x_2}$

Solution:

Concept:

- The electric field of a point charge is $\mathbf{E} = kQ\mathbf{r}/r^3$, where $\mathbf{r}$ is directed from the charge to the field point and $k = 1/(4\pi\varepsilon_0)$.
- Two perpendicular vectors have zero scalar product. The condition that the resultant field is normal to AB can therefore be imposed directly without introducing an angle.

Step 1: Write the field components at the origin Choose $O = (0, 0)$, $A = (0, x_1)$ and $B = (x_2, 0)$, consistent with the figure. The charges at A and B contribute fields along the vertical and horizontal axes, respectively. Retaining the algebraic signs of the charges,

$$\mathbf{E}(O) = -\frac{kQ_2}{x_2^2}\,\hat{\mathbf{i}} - \frac{kQ_1}{x_1^2}\,\hat{\mathbf{j}}.$$

The hypotenuse has direction vector $\overrightarrow{AB} = x_2\hat{\mathbf{i}} - x_1\hat{\mathbf{j}}$.

Step 2: Impose perpendicularity Take the scalar product with the direction of the hypotenuse:

$$\mathbf{E} \cdot \overrightarrow{AB} = -\frac{kQ_2}{x_2} + \frac{kQ_1}{x_1} = 0,$$
$$\frac{Q_1}{x_1} = \frac{Q_2}{x_2},$$
$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}.$$

This is an exact ratio for the pictured geometry. In particular, the two charges have the same sign; opposite signs would not satisfy the derived zero scalar product.

Step 3: Compare the functional dependence The required dependence is linear in x_1/x_2. The inverse-square field law alone does not give the charge ratio, because the required direction supplies an additional length ratio. The squared inverse ratio in (B), and the inverted linear ratio in (D), do not obey the scalar-product equation for general unequal lengths. The cubic ratio in (A) also fails that equation; it would arise from using an incorrect geometrical slope.

Step 4: Conclusion The result is $\boxed{Q_1/Q_2 = x_1/x_2}$, proportional to x_1/x_2. Hence, the correct option is (C).

Quick Tip to Solve: For a resultant field perpendicular to a line, take its scalar product with a vector along that line; this avoids inverting a slope.

41. A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is : [4 marks]

(A) 2.6 m from the mirror, real

(B) 1 m from the mirror, real

(C) 2.6 m from the mirror, virtual

(D) 1 m from the mirror, virtual

Correct Answer: (A) 2.6 m from the mirror, real

Solution:

Concept:

- For a thin lens in air, $1/v - 1/u = 1/f$ in the Cartesian convention. A convex lens has positive focal length.
- A plane mirror places the image the same distance behind it as the object is in front. On the return journey, treat that mirror image as the object for the lens.

Step 1: Find the first image formed by the lens Take the lens at coordinate $x = 0$, the object at $x = -1\,\mathrm{m}$, and the mirror at $x = 2\,\mathrm{m}$. Initially light travels to the right. With $u_1 = -1\,\mathrm{m}$ and $f = 0.5\,\mathrm{m}$,

$$\frac{1}{v_1} = \frac{1}{f} + \frac{1}{u_1} = 2 - 1 = 1\,\mathrm{m}^{-1}.$$

The lens forms its first real image at $x = 1\,\mathrm{m}$, before the rays reach the mirror.

Step 2: Locate the object seen by the lens on return This first image lies one metre in front of the mirror. Reflection therefore makes the returning rays appear to originate from a point one metre behind the mirror, at $x = 3\,\mathrm{m}$. This point is a virtual image of the mirror alone, but it supplies a real object for the returning lens traversal: the incident reflected rays diverge from that position. Its distance from the lens is three metres.

Step 3: Apply the lens formula on the return path Choose positive distances in the new direction of propagation, towards the left. Then $u_2 = -3\,\mathrm{m}$ and

$$\frac{1}{v_2} = 2 - \frac{1}{3} = \frac{5}{3}\,\mathrm{m}^{-1}, \qquad v_2 = 0.6\,\mathrm{m}.$$

The emergent rays actually meet $0.6\,\mathrm{m}$ to the left of the lens, at original coordinate $x = -0.6\,\mathrm{m}$. Their distance from the mirror is $2 + 0.6 = 2.6\,\mathrm{m}$. The one-metre alternatives refer to an intermediate image, while the virtual-image alternatives misidentify the final converging rays.

Step 4: Conclusion The final image is real and lies $\boxed{2.6\,\mathrm{m}}$ from the mirror, on the object side of the lens. Hence, the correct option is (A).

Quick Tip to Solve: Complete all three imaging stages: lens, mirror, lens again. Reverse the propagation direction before applying the lens formula on return.

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42. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom.The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_p}{C_v}\right)$ are given, respectively by: [4 marks]

(A) U = $\frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

(B) U = 5RT and $\gamma = \frac{6}{5}$

(C) U = 5RT and $\gamma = \frac{7}{5}$

(D) U = $\frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

Correct Answer: (A) U = $\frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

Solution:

Concept:

- Equipartition assigns mean energy $k_BT/2$ to each active quadratic degree of freedom of a molecule. For one mole this contributes $RT/2$ per degree of freedom.
- For an ideal gas, $C_V = \mathrm{d}U/\mathrm{d}T$ per mole and Mayer's relation is $C_P - C_V = R$. The heat-capacity ratio is $\gamma = C_P/C_V$.

Step 1: Count the active degrees of freedom The specified three translational and two rotational degrees give

$$f = 3 + 2 = 5.$$

No vibrational contribution is stated, so none is added. For one mole, $N_Ak_B = R$, and summing the

energy of the five active degrees gives

$$U = N_A \frac{5}{2} k_B T = \frac{5}{2} RT.$$

The factor of one half is essential: each degree of freedom contributes half of RT, not all of RT.

Step 2: Determine the heat capacities Differentiate the internal energy with respect to temperature at fixed volume:

$$C_V = \frac{5}{2} R.$$

At constant pressure the gas also performs expansion work, so the ideal-gas relation supplies

$$C_P = C_V + R = \frac{7}{2} R.$$

Consequently,

$$\gamma = \frac{C_P}{C_V} = \frac{(7/2)R}{(5/2)R} = \frac{7}{5}.$$

Both heat capacities are molar quantities, so their units cancel in the ratio.

Step 3: Check both entries in each option The pair must contain both $U = (5/2)RT$ and $\gamma = 7/5$. Options (B) and (C) double the internal energy by overlooking the equipartition factor of one half. Options (B) and (D) give $6/5$, whereas adding R to $5R/2$ requires the numerator seven. Equivalently $\gamma = 1 + 2/f = 1 + 2/5$, confirming the result independently of the numerical value of temperature.

Step 4: Conclusion For one mole, $\boxed{U = \frac{5}{2}RT, \quad \gamma = \frac{7}{5}}$. Hence, the correct option is (A).

Quick Tip to Solve: With f active degrees use $U = fRT/2$ and $\gamma = 1 + 2/f$; count only the modes specified as active.

43. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is :

(g = 10 ms^{-2}) [4 marks]

(A) 0.45 m

(B) 0.60 m

(C) 0.20 m

(D) 0.80 m

Correct Answer: (C) 0.20 m

Solution:

Concept:

- At the point where slipping begins, static friction reaches its limiting magnitude: $f_{\max} = \mu N$, where N is the normal reaction.
- For slow crawling on a surface inclined at angle θ to the horizontal, gravity has tangential

component $mg\sin\theta$ and normal component $mg\cos\theta$.

- On a hemisphere of radius R, the height above the bottom is $h = R(1-\cos\theta)$.

Step 1: Resolve the forces at the limiting point The insect crawls slowly, so we use limiting static equilibrium rather than introduce a centripetal acceleration. The normal reaction balances the inward normal component of its weight, while friction acts uphill against the impending downward slip. Thus

$$N = mg\cos\theta, \qquad mg\sin\theta = \mu N.$$

Dividing the second equation by the first cancels both the insect's mass and gravitational acceleration: $\tan\theta = \mu = 0.75 = 3/4$.

Step 2: Convert the angle into height A right triangle with opposite and adjacent sides in the ratio $3:4$ has hypotenuse in the ratio 5. Therefore $\cos\theta = 4/5 = 0.8$. The radius to the insect has vertical projection $R\cos\theta$ below the centre; the bottom is R below the centre. Hence

$$h = R - R\cos\theta = 1(1-0.8) = 0.20\,\text{m}.$$

This is the vertical height, not the distance crawled along the curved surface.

Step 3: Check the alternatives The values $0.45\,\text{m}$, $0.60\,\text{m}$, and $0.80\,\text{m}$ do not satisfy $\cos\theta = 0.8$ together with $h/R = 1-\cos\theta$. In particular, 0.60 is $\sin\theta$, not the height ratio, and $0.80\,\text{m}$ is the vertical projection of the radius, not the rise above the bottom.

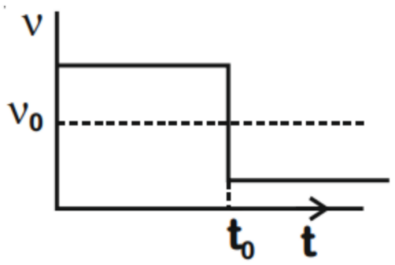
Step 4: Conclusion The insect starts slipping at $h = 0.20\,\text{m}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the limiting slope from $\tan\theta = \mu$, then use the vertical rise $R(1-\cos\theta)$; do not confuse it with the radius projection.

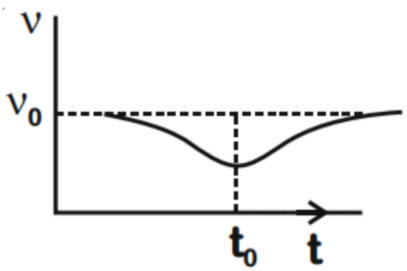
44. A sound source S is moving along a straight track with speed v, and is emitting, sound of frequency v_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by:

(t_0 represents the instant when the distance between the source and observer is minimum) [4 marks]

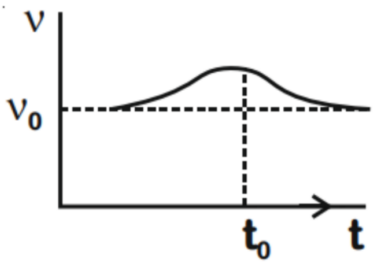
(A)



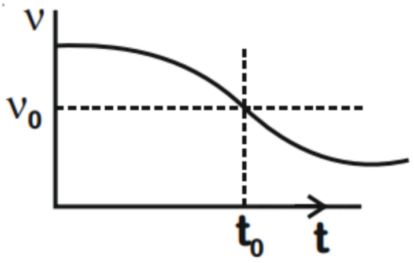
(B)



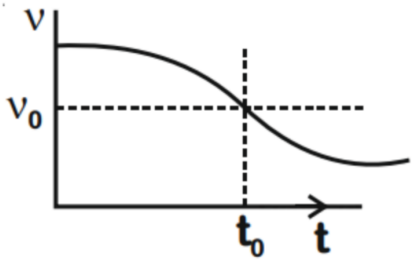
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- The Doppler shift of a stationary observer depends on the component of source velocity along the source-observer line, rather than on the total source speed alone.
- With sound speed c, source frequency ν_0, and angle θ between source velocity and the line directed from source to observer, the frequency is $\nu = \nu_0 c/(c - v\cos\theta)$.

Step 1: Identify the shift before and after closest approach As the source approaches the observer, its velocity has a component towards the observer, so $v\cos\theta > 0$. Successive wavefronts are compressed and the observed frequency is above ν_0. After passing the point of closest approach, the radial component reverses sign: the source recedes and the observed frequency is below ν_0. Thus the correct curve must pass from a higher to a lower frequency, not stay entirely above or entirely below the emitted frequency.

Step 2: Use the finite separation from the track The observer is a finite perpendicular distance from the track. Consequently the direction of the source-observer line changes continuously as the source travels, and the radial velocity changes continuously through zero. In the closest-approach construction at t_0, the source velocity is perpendicular to that line, so $\cos\theta = 0$ and the plotted

frequency passes through ν_0. Far before and far after the passage, the radial velocity approaches its positive and negative limiting values, so the curve tends towards corresponding upper and lower levels.

Step 3: Compare all four graphical choices Graph (A) shows a discontinuous jump, appropriate to a collinear idealisation rather than the stated finite lateral separation. Graph (B) remains below ν_0 on both sides and graph (C) remains above it, so each misses the reversal from approach to recession. Graph (D) is the smooth decreasing transition through ν_0 required by the changing radial velocity.

Step 4: Conclusion The observed frequency decreases smoothly from above ν_0 to below ν_0, as shown in graph (D). Hence, the correct option is (D).

Quick Tip to Solve: For a source passing beside an observer, track the radial velocity: approach raises the pitch, recession lowers it, and finite lateral separation makes the transition smooth.

45. An AC circuit has R= 100 Ω, C = 2 μF and L = 80 mH, connected in series. The quality factor of the circuit is : [4 marks]

(A) 20

(B) 2

(C) 0.5

(D) 400

Correct Answer: (B) 2

Solution:

Concept:

- For a series RLC circuit, the resonant angular frequency is $\omega_0 = 1/\sqrt{LC}$, where L and C are in SI units.
- The quality factor is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$. It compares resonant reactance with resistance and is dimensionless.

Step 1: Convert the component values into SI units The resistance is already in ohms. Convert the inductance and capacitance before substituting, because milli and micro denote different powers of ten:

$$R = 100\,\Omega, \qquad L = 80 \times 10^{-3}\,\mathrm{H} = 0.080\,\mathrm{H},$$

$$C = 2 \times 10^{-6}\,\mathrm{F}.$$

At resonance, inductive and capacitive reactances cancel in the total impedance, but their individual magnitudes determine the quality factor.

Step 2: Calculate the resonant reactance and quality factor Using the series-circuit expression gives

$$\sqrt{\frac{L}{C}} = \sqrt{\frac{0.080}{2 \times 10^{-6}}} = \sqrt{4.0 \times 10^{4}} = 200\,\Omega.$$

Therefore

$$Q = \frac{200}{100} = 2.$$

As a cross-check, $\omega_0 = 1/\sqrt{0.080(2 \times 10^{-6})} = 2500\,\text{rad}\,\text{s}^{-1}$, so $\omega_0 L = 2500(0.080) = 200\,\Omega$, giving the same ratio.

Step 3: Reject the other numerical choices Option (C), 0.5, is the reciprocal $R/\sqrt{L/C}$, whereas the series quality factor uses reactance divided by resistance. Option (A), 20, has an extra factor of ten and does not follow the SI substitution. Option (D), 400, is also inconsistent with the independently calculated reactance of $200\,\Omega$ and resistance of $100\,\Omega$. The ratio must have no unit.

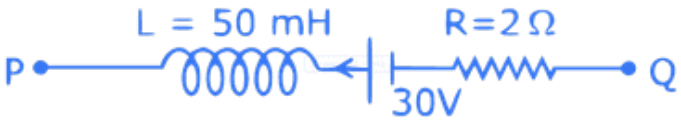
Step 4: Conclusion The quality factor of the circuit is $Q = 2$, a dimensionless quantity. Hence, the correct option is (B).

Quick Tip to Solve: For a series circuit, use $Q = \sqrt{L/C}/R$ and convert mH and microfarads separately before taking the square root.

Physics – Section B: Numerical Value Questions (20 marks)

46. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is _____.

[4 marks]



Solution:

Concept:

- Across a resistor the potential falls by IR when traversed in the direction of current. A cell produces a rise from its negative terminal to its positive terminal.
- For an ideal inductor in the passive sign convention, the potential drop along the current direction is $L\,dI/dt$. A decreasing current makes this drop negative: the inductor supplies energy to sustain the current.

Step 1: Read the direction and assign the signed rate The arrow in the figure points left, so the current flows from the Q side towards P. The longer plate of the 30 V cell is on the left, making its left terminal positive. The values are

$$I = 1\,\text{A}, \quad R = 2\,\Omega, \quad L = 50\,\text{mH} = 0.050\,\text{H}.$$

Since the current magnitude is decreasing, its derivative in the chosen current direction is $dI/dt = -10^2 = -100\,\text{A}\,\text{s}^{-1}$, rather than positive 100.

Step 2: Add the potential changes from Q to P Traversing from Q to P follows the current through

the resistor, crosses the cell from negative to positive, and then crosses the inductor in the current direction. The potential changes are therefore

$$\Delta V_R = -IR = -2\,\text{V}, \qquad \Delta V_{\text{cell}} = +30\,\text{V},$$

$$\Delta V_L = -L\frac{dI}{dt} = -(0.050)(-100) = +5\,\text{V}.$$

The positive inductor contribution is consistent with Lenz's law: the induced emf supports the existing leftward current as that current decreases.

Step 3: Obtain the requested potential difference The sum of all changes along the path beginning at Q and ending at P is exactly $V_P - V_Q$. Thus

$$V_P - V_Q = -2 + 30 + 5 = 33\,\text{V}.$$

No assumption about the remainder of the complete circuit is needed, because every element on this P-Q segment and the instantaneous current derivative are specified.

Step 4: Conclusion The potential difference is $\boxed{V_P - V_Q = 33\,\text{V}}$. The numerical entry is 33.

Quick Tip to Solve: Walk from the subtracted point Q to the first point P, recording every rise and fall. Use a negative derivative when the given current is decreasing.

47. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)$ %. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is_____. [4 marks]

Solution:

Concept:

- The density of a sphere of mass m and diameter d is $\rho = 6m/(\pi d^3)$. The numerical factors 6 and π are exact.
- For maximum possible relative error in a product or quotient, add the absolute fractional errors multiplied by the magnitudes of their powers: $\Delta\rho/\rho = \Delta m/m + 3\Delta d/d$.

Step 1: Express density in terms of the measured quantities The volume of a sphere is $4\pi r^3/3$, and its radius is $r = d/2$. Therefore

$$V = \frac{4\pi}{3}\left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6}, \qquad \rho = \frac{m}{V} = \frac{6m}{\pi d^3}.$$

The diameter is cubed, so its relative measurement uncertainty contributes three times to the density uncertainty. The fact that diameter rather than radius is measured does not introduce an extra factor of two in the relative error.

Step 2: Calculate the maximum percentage error For the largest possible error, the effects of the mass and diameter uncertainties reinforce one another, so their magnitudes are added. They are not

subtracted, even though diameter appears in the denominator. Using the given percentage errors,

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} \times 100\% = 6.0\% + 3(1.5\%) = 10.5\%.$$

This is the standard first-order propagation of limiting measurement errors; adding errors in quadrature would estimate a different statistical quantity, not the requested maximum.

Step 3: Convert the percentage to the requested parameter The question writes the maximum percentage error as $(x/100)\%$. Equate this complete expression to the calculated percentage, keeping the percent sign on both sides:

$$\frac{x}{100} = 10.5 \quad \Longrightarrow \quad x = 1050.$$

Thus 10.5 is the percentage error itself, whereas 1050 is the numerical parameter that the question asks us to enter.

Step 4: Conclusion The maximum error in density is 10.5%, so $\boxed{x = 1050}$. The parameter x is dimensionless.

Quick Tip to Solve: For $\rho \propto md^{-3}$, add the mass percentage error to three times the diameter percentage error, then convert to the exact form requested.

48. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is _____. [4 marks]

Solution:

Concept:

- Linear momentum is conserved during the collision when external impulse is negligible. In a completely inelastic collision the bodies stick, so the final moving mass is $2m$.
- For two vectors of equal magnitude mu, separated by angle θ, the resultant magnitude satisfies $p^2 = 2m^2u^2(1 + \cos\theta)$. Kinetic energy need not be conserved.

Step 1: Write the initial and final momenta Let each body have mass m, initial speed u, and let θ be the angle between the initial velocity vectors. The initial total momentum is the vector sum $m\vec{u}_1 + m\vec{u}_2$; its magnitude is not generally $2mu$, because the directions differ. After sticking, the common speed is $u/2$, so the magnitude of final momentum is

$$p_f = (2m)\frac{u}{2} = mu.$$

Momentum conservation requires the magnitude of the initial vector sum to equal this value.

Step 2: Use the angle between the two initial velocities Apply the dot product to the initial momentum vector:

$$p_i^2 = (m\vec{u}_1 + m\vec{u}_2)^2 = 2m^2u^2 + 2m^2u^2\cos\theta.$$

Substituting $p_i = p_f = mu$ gives

$$m^2u^2 = 2m^2u^2(1+\cos\theta),$$

$$\cos\theta = -\frac{1}{2}.$$

The angle between two velocity vectors lies between 0° and 180°, hence $\theta = 120^\circ$.

Step 3: Check the speed and energy behaviour The equivalent equal-vector result is $v_f = u\cos(\theta/2)$. At 120°, this gives $u\cos 60^\circ = u/2$, as required. The initial kinetic energy is mu^2, while the final kinetic energy is $\frac{1}{2}(2m)(u/2)^2 = mu^2/4$. The loss is permitted because the collision is completely inelastic; imposing kinetic-energy conservation would contradict the stated sticking process.

Step 4: Conclusion The angle between the initial velocities is $\boxed{120^\circ}$. The numerical entry in degrees is 120.

Quick Tip to Solve: For equal masses sticking after collision, use $v_f = u\cos(\theta/2)$. Always use the vector momentum sum, not the scalar sum of the initial speeds.

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49. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2 . The ratio $\frac{P_2}{P_1}$ is _____. [4 marks]

Solution:

Concept:

- For an ideal gas, $PV = nRT$, where n counts independently moving gas particles, whether molecules or atoms.
- Dissociation of one diatomic molecule produces two atoms, so a dissociation fraction α increases the total amount from n to $n(1+\alpha)$.

Step 1: Count the particles after dissociation Let the initial amount of diatomic gas be n_1 moles. The undissociated molecules contribute $0.75n_1$ moles, while the dissociated molecules produce $2(0.25n_1) = 0.50n_1$ moles of atoms. Hence the final amount of gaseous particles is

$$n_2 = 0.75n_1 + 0.50n_1 = 1.25n_1.$$

The mass is unchanged, but the number of particles that exert pressure has increased.

Step 2: Compare the two equilibrium states Apply the ideal gas law separately before and after heating. The gas constant cancels on division:

$$\frac{P_2}{P_1} = \frac{n_2}{n_1}\frac{T_2}{T_1}\frac{V_1}{V_2}.$$

Using the absolute temperatures and the doubled volume gives

$$\frac{P_2}{P_1} = 1.25 \times \frac{2000}{250} \times \frac{1}{2} = 1.25 \times 8 \times \frac{1}{2} = 5.$$

The temperature increase alone would give a factor of four after allowing for expansion; the extra particles multiply this by 1.25.

Step 3: Conclusion The required pressure ratio is $\boxed{P_2/P_1 = 5}$. This is a dimensionless ratio.

Quick Tip to Solve: For partial dissociation of a diatomic gas into atoms, first multiply the original mole count by $1 + \alpha$, then compare nT/V.

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50. Suppose that intensity of a laser is $\frac{315}{\pi}$ W/m^2.
The rms electric field, in units of V/m associated with this source is close to the nearest integer is _____.
($\in_0$ = 8.86 $\times$ 10^{-12} C^2 Nm^{-2}; c = 3 $\times$ 10^8 ms^{-1}) [4 marks]

Solution:

Concept:

- The time-averaged intensity of an electromagnetic wave is $I = \varepsilon_0 c E_{\text{rms}}^2$. The rms field already includes the time averaging.
- For a sinusoidal wave, $E_{\text{rms}} = E_0/\sqrt{2}$, so the equivalent peak-field formula is $I = \frac{1}{2}\varepsilon_0 c E_0^2$.

Step 1: Choose the rms intensity relation The question asks for the rms electric field, not the maximum field. Rearrange the average-intensity relation directly:

$$E_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0 c}}.$$

There is no additional factor of two inside this square root. Such a factor would instead calculate the peak electric field. Here the given intensity is already an average energy flux in W m^{-2}.

Step 2: Substitute the constants and round at the end Use $I = 315/\pi$, $\varepsilon_0 = 8.86 \times 10^{-12}$, and $c = 3 \times 10^8$, in SI units:

$$\varepsilon_0 c = 2.658 \times 10^{-3}, \qquad I \simeq 100.268\,\text{W m}^{-2}.$$

$$E_{\text{rms}} = \sqrt{\frac{100.268}{0.002658}} \simeq \sqrt{37723} \simeq 194.22\,\text{V m}^{-1}.$$

Since the fractional part is less than one half, rounding to the nearest integer gives 194. Retaining the square root until the last line avoids mistaking the squared field for the field itself.

Step 3: Conclusion The rms electric field, to the nearest integer, is $\boxed{194\,\text{V m}^{-1}}$.

Quick Tip to Solve: Use $I = \varepsilon_0 c E_{\text{rms}}^2$ for rms fields; reserve the factor $1/2$ for a formula written in terms of the peak field.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. The correct statement with respect to dinitrogen is : [4 marks]

(A) N_2 is paramagnetic in nature

(B) it can be used as an inert diluent for reactive chemicals

(C) it can combine with dioxygen at 25°C

(D) liquid dinitrogen is not used in cryosurgery

Correct Answer: (B) it can be used as an inert diluent for reactive chemicals

Solution:

Concept:

- Dinitrogen, N_2, contains a strong $N \equiv N$ triple bond. Its high bond dissociation enthalpy makes it relatively unreactive under ordinary conditions.
- All electrons in the ground-state molecular orbitals of N_2 are paired, making it diamagnetic. Liquid nitrogen provides very low temperatures and is a cryogenic refrigerant.

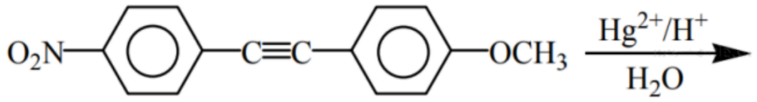
Step 1: Identify the correct chemical use Because the nitrogen molecule is kinetically resistant to many ordinary reactions, nitrogen gas can dilute reactive chemicals without itself reacting readily. It can also displace air and reduce unwanted contact with oxygen. This is the sense in which it is an inert diluent; it does not mean that nitrogen is incapable of reacting under every possible condition. Therefore the statement in option (B) is correct.

Step 2: Check each remaining statement Option (A) is false: paired molecular-orbital electrons make N_2 diamagnetic rather than paramagnetic. Option (C) is false: appreciable direct combination with dioxygen requires high temperatures, as in an electric arc or lightning; the reaction is not an ordinary room-temperature process at $25^\circ C$. Option (D) is false: liquid nitrogen is indeed used in cryosurgery, where its very low temperature freezes and destroys selected tissue. Thus none of the other statements describes dinitrogen correctly.

Step 3: Conclusion Dinitrogen can be used as an inert diluent for reactive chemicals. Hence, the correct option is (B).

Quick Tip to Solve: Link the strong nitrogen triple bond with low room-temperature reactivity, and remember that liquid nitrogen is used for freezing and cryosurgery.

52. The major product obtained from the following reaction is : [4 marks]



(A) p-$NO_2-C_6H_4-CO-CH_2-C_6H_4-OCH_3$-$p$

(B) p-$NO_2-C_6H_4-CH_2-CO-C_6H_4-OH$-$p$

(C) p-$NO_2-C_6H_4-CH_2-CO-C_6H_4-OCH_3$-$p$

(D) p-$NO_2-C_6H_4-CO-CH_2-C_6H_4-OH$-$p$

Correct Answer: (C) p-$NO_2-C_6H_4-CH_2-CO-C_6H_4-OCH_3$-$p$

Solution:

Concept:

- Mercuric-ion-catalysed acidic hydration of an alkyne adds water across the triple bond and gives an enol, which tautomerises to a ketone.
- In an unsymmetrical diaryl alkyne, electron-donating and electron-withdrawing substituents influence the orientation of hydration by stabilising or destabilising developing positive charge.

Step 1: Choose the favoured orientation Write the starting alkyne as $Ar_N - C \equiv C - Ar_M$, where Ar_N is the para-nitrophenyl group and Ar_M the para-methoxyphenyl group. The methoxy group donates electron density through resonance, whereas the nitro group withdraws it. The developing positive character is therefore better supported at the alkyne carbon beside Ar_M. Water addition places the enolic hydroxyl group on this carbon.

Step 2: Tautomerise and compare the products The resulting enol has the connectivity $Ar_N - CH = C(OH) - Ar_M$. Keto–enol tautomerism gives

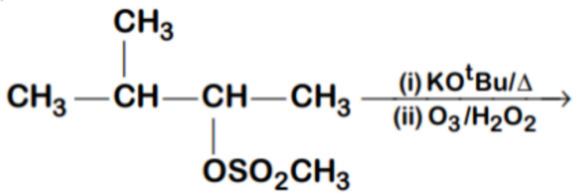
$$Ar_N - CH_2 - CO - Ar_M.$$

Thus the carbonyl is adjacent to the methoxy-substituted ring, while the nitro-substituted ring is bonded to the methylene carbon. This is option (C). Option (A) puts the carbonyl at the opposite end of the original triple bond. Options (B) and (D) also replace the methoxy group by a phenolic hydroxyl group, a cleavage not required by the stated alkyne-hydration transformation.

Step 3: Conclusion The major ketone is p-$NO_2-C_6H_4-CH_2-CO-C_6H_4-OCH_3$-$p$. Hence, the correct option is (C).

Quick Tip to Solve: For this diaryl alkyne, locate the electron-donating methoxy ring: hydration places the eventual carbonyl next to that ring, without removing its methyl group.

53. The major products of the following reaction are : [4 marks]



(A) $CH_3COCH_3 + CH_3COOH$

(B) $CH_3COCH_3 + CH_3CHO$

(C) $(CH_3)_2CHCHO + HCHO$

(D) $(CH_3)_2CHCOOH + HCOOH$

Correct Answer: (D) $(CH_3)_2CHCOOH + HCOOH$

Solution:

Concept:

- Potassium tert-butoxide is a bulky base. In an E2 elimination it preferentially removes an accessible beta hydrogen, favouring the less substituted Hofmann alkene.
- Ozonolysis cleaves the carbon–carbon double bond. With an oxidative H_2O_2 work-up, aldehyde fragments are oxidised to carboxylic acids, while ketones remain ketones.

Step 1: Form the major alkene The substrate is $CH_3-CH(CH_3)-CH(OSO_2CH_3)-CH_3$. The mesylate group is the leaving group. Removing a beta hydrogen from the terminal methyl group beside the leaving-group carbon produces

$$CH_2=CH-CH(CH_3)-CH_3.$$

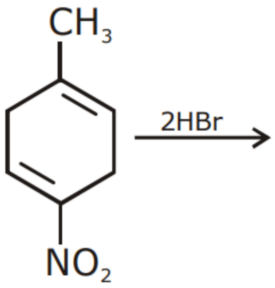
The alternative beta site gives the more substituted alkene, but is more crowded. The bulky tert-butoxide base therefore favours the terminal alkene, 3-methylbut-1-ene, under the heated elimination conditions.

Step 2: Cleave the double bond and oxidise the fragments Cleavage of $CH_2=CH-CH(CH_3)-CH_3$ first corresponds to the carbonyl fragments HCHO and $(CH_3)_2CHCHO$. Both are aldehydes, so the oxidative work-up gives HCOOH and $(CH_3)_2CHCOOH$, the pair in (D). Option (C) stops at the aldehydes and would describe a reductive work-up instead. Options (A) and (B) use the fragments from the more substituted alkene; (B) additionally leaves an aldehyde unoxidised. Neither matches both the bulky-base selectivity and the stated peroxide work-up.

Step 3: Conclusion The products are 2-methylpropanoic acid, $(CH_3)_2CHCOOH$, and formic acid, HCOOH. Hence, the correct option is (D).

Quick Tip to Solve: Apply the reagents in order: bulky base chooses the less substituted alkene, then peroxide work-up converts each aldehyde cleavage fragment into an acid.

54. The major product of the following reaction is : [4 marks]



(A) 1,3-Dibromo-1-methyl-4-nitrocyclohexane

(B) 2,5-Dibromo-1-methyl-4-nitrocyclohexane

(C) 1,4-Dibromo-1-methyl-4-nitrocyclohexane

(D) 2,4-Dibromo-1-methyl-4-nitrocyclohexane

Correct Answer: (A) 1,3-Dibromo-1-methyl-4-nitrocyclohexane

Solution:

Concept:

- In ionic addition of HBr to an alkene, protonation follows the pathway giving the more stable carbocation, after which bromide captures that carbon.
- An alkyl group stabilises adjacent positive charge through hyperconjugation and inductive donation. A nitro group strongly withdraws electron density and destabilises a carbocation on the carbon bearing it.

Step 1: Add hydrogen bromide to the methyl-substituted bond Number the methyl-bearing ring carbon as C1 and the nitro-bearing ring carbon as C4. The double bonds are C1=C2 and C4=C5. For C1=C2, protonation at C2 leaves the positive charge on C1, where the methyl substituent stabilises the tertiary carbocation. Bromide therefore attaches at C1. The opposite orientation would place positive charge on the less substituted neighbouring carbon.

Step 2: Add the second equivalent and identify the option At C4=C5, placing positive charge on the nitro-bearing carbon C4 is unfavourable because NO_2 is strongly electron withdrawing. Protonation therefore places hydrogen on C4 and positive charge on C5; bromide subsequently attaches at C5. After saturation of both double bonds, the bromines occupy C1 and C5 relative to the original numbering. Reflecting the ring about C1–C4 expresses the identical connectivity as C1 and C3, as drawn in option (A). Option (B) has the wrong orientation next to methyl; (C) has the wrong orientation next to nitro; (D) has both orientations wrong. No stereochemical selection is required by the drawings.

Step 3: Conclusion The product has bromine on the methyl-bearing carbon and on the carbon adjacent to the nitro-bearing carbon, as in 1,3-dibromo-1-methyl-4-nitrocyclohexane. Hence, the correct option is (A).

Quick Tip to Solve: Assess each double bond separately: methyl favours positive charge at its own carbon, whereas nitro strongly disfavors positive charge on the carbon bearing it.

55. The lanthanoid that does NOT show +4 oxidation state is : [4 marks]

(A) Tb

(B) Dy

(C) Ce

(D) Eu

Correct Answer: (D) Eu

Solution:

Concept:

- Lanthanoids predominantly form the +3 oxidation state by losing the two outer 6s electrons and one additional electron.
- An empty, half-filled or completely filled 4f subshell gives extra stability. Thus accessible +2 and +4 states depend on the electronic arrangement, not merely on membership of the series.

Step 1: Identify the especially stable europium ion Europium has the configuration $[\mathrm{Xe}]4f^7 6s^2$. Removing its two 6s electrons produces Eu^{2+}, with the stable half-filled $4f^7$ subshell. Its familiar oxidation states are +2 and +3. Forming +4 would require removing further strongly bound 4f electrons, with no compensating empty, half-filled or filled subshell, so europium is the required exception.

Step 2: Compare the remaining candidates Cerium can form Ce^{4+}, which has the empty $4f^0$ subshell. Terbium can form Tb^{4+}, with the half-filled $4f^7$ arrangement. Dysprosium also exhibits +4 in suitable compounds, although +3 is its normal and much more common state. Consequently Tb, Dy and Ce do not satisfy the negative wording of the question. The question asks whether +4 occurs, rather than which state is most stable or most abundant.

Step 3: Conclusion Europium is the lanthanoid among the choices that does not show the +4 oxidation state in this classification. Hence, the correct option is (D).

Quick Tip to Solve: For lanthanoid exceptions, write the 4f occupancy after ionisation; Eu(II), Ce(IV) and Tb(IV) are useful stability anchors.

56. The variation of equilibrium constant with temperature is given below :

Temperature	Equilibrium Constant
$T_1 = 25^oC$	$K_1 = 10$
$T_2 = 100^oC$	$K_2 = 100$

The values of ΔH^o, ΔG^o at
T_1 and ΔG^o at T_2 (in kJ mol^{-1}) respectively, are close to :
[Use R = 8.314 J K^{-1} mol^{-1}] [4 marks]

(A) 28.4, –5.71 and –14.29

(B) 0.64, –7.14 and –5.71

(C) 28.4, –7.14 and –5.71

(D) 0.64, –5.71 and –14.29

Correct Answer: (A) 28.4, –5.71 and –14.29

Solution:

Concept:

- The integrated van't Hoff equation is $\ln(K_2/K_1) = \frac{\Delta H^\circ}{R}(1/T_1 - 1/T_2)$, treating the reaction enthalpy as constant over the interval.
- The standard Gibbs energy at each temperature is $\Delta G^\circ = -RT\ln K$. Temperatures must be in kelvin and joules must be converted to kilojoules.

Step 1: Calculate the enthalpy change Convert the temperatures: $T_1 = 298\,\text{K}$ and $T_2 = 373\,\text{K}$. The equilibrium constant increases from 10 to 100, so the forward reaction is endothermic. Substitution gives

$$\Delta H^\circ = \frac{8.314\ln(100/10)}{1/298 - 1/373} = 2.84 \times 10^4\,\text{J mol}^{-1} = 28.4\,\text{kJ mol}^{-1}.$$

The positive sign also checks against the observed increase in equilibrium constant on heating.

Step 2: Calculate the two Gibbs energy changes Use the appropriate equilibrium constant separately at each temperature.

$$\Delta G_1^\circ = -8.314(298)\ln 10\,\text{J mol}^{-1} = -5.71\,\text{kJ mol}^{-1}.$$

$$\Delta G_2^\circ = -8.314(373)\ln 100\,\text{J mol}^{-1} = -14.29\,\text{kJ mol}^{-1}.$$

Both are negative because both equilibrium constants exceed unity. Options (B) and (D) give the wrong enthalpy magnitude; (C) assigns incorrect Gibbs energies and does not reproduce the specified temperatures and constants.

Step 3: Conclusion The three values are $28.4, -5.71, -14.29\,\text{kJ mol}^{-1}$, respectively. Hence, the correct option is (A).

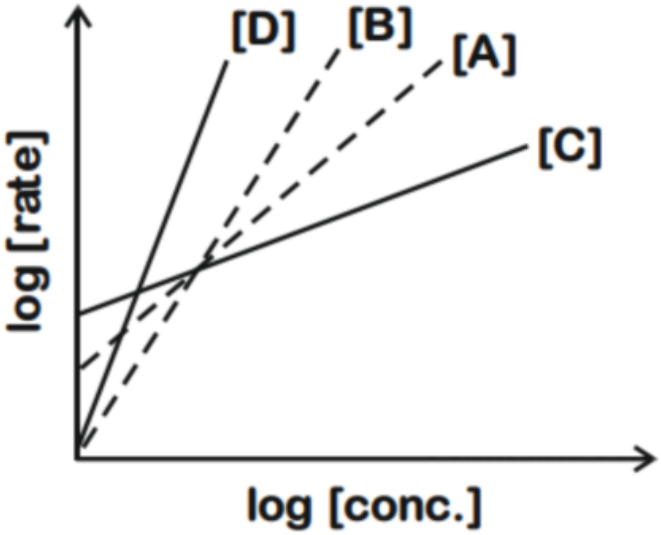
Quick Tip to Solve: First check the sign from how K changes with temperature; then use kelvin in both the van't Hoff and Gibbs equations.

57. Consider the following reactions

$A \rightarrow P_1$; $B \rightarrow P_2$; $C \rightarrow P_3$; $D \rightarrow P_4$,

The order of the above reactions are a, b, c, and d, respectively. The following graph is obtained when log[rate] vs. log[conc.] are plotted

Among the following, the correct sequence for the order of the reactions is : [4 marks]



(A) d > b > a > c

(B) d > a > b > c

(C) a > b > c > d

(D) c > a > b > d

Correct Answer: (A) d > b > a > c

Solution:

Concept:

- For a rate law $r = k[X]^n$, taking logarithms gives $\log r = \log k + n \log[X]$.
- On a plot of log(rate) against log(concentration), the slope equals the reaction order n; the vertical intercept equals log(k).

Step 1: Read slopes rather than heights The different lines represent A, B, C and D, whose reaction orders are a, b, c and d. Because both graph axes are logarithmic variables, the straight-line slope directly measures the exponent of concentration in each rate law. A line with a larger upward inclination therefore represents a larger order. Its position above another line at one chosen concentration is not sufficient: different rate constants shift the intercepts without changing the order.

Step 2: Rank the four inclinations In the supplied graph D is the steepest rising line. B is less steep than D but steeper than A; C has the shallowest slope. Thus

$$\text{slope}(D) > \text{slope}(B) > \text{slope}(A) > \text{slope}(C),$$

so $d > b > a > c$. Option (B) interchanges a and b, even though B is steeper than A. Option (C) puts d

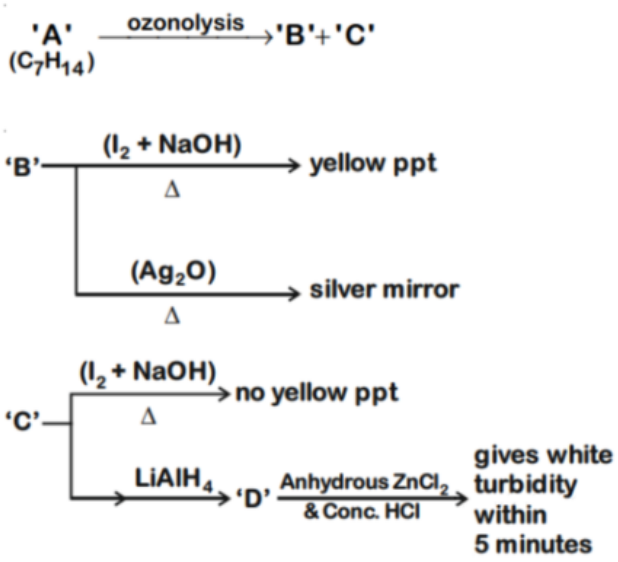
last despite its largest slope. Option (D) puts c first despite its smallest slope. None of those rankings agrees with the plotted inclinations.

Step 3: Conclusion The orders follow $d > b > a > c$. Hence, the correct option is (A).

Quick Tip to Solve: In a log(rate)-log(concentration) graph, compare slopes only; differing intercepts tell you about rate constants, not reaction orders.

58. Consider the following reactions

'A' is : [4 marks]



(A) $CH_3CH{=}CHCH_2CH_2CH_2CH_3$

(B) $CH_3CH{=}C(C_2H_5)_2$

(C) $(CH_3)_2C{=}CHCH_2CH_2CH_3$

(D) $CH_3CH{=}C(CH_3)CH_2CH_2CH_3$

Correct Answer: (B) $CH_3CH{=}C(C_2H_5)_2$

Solution:

Concept:

- Ozonolysis cleaves an alkene double bond and converts its two carbon atoms into carbonyl carbons, preserving the groups originally attached to each.
- Ethanal gives both the iodoform test and Tollens test. A ketone is reduced by $LiAlH_4$ to a secondary alcohol, which gives Lucas-test turbidity within about five minutes.

Step 1: Identify the two carbonyl products Product B gives a yellow precipitate with $I_2/NaOH$ and a silver mirror with silver oxide. The first identifies the iodoform reaction and the second an aldehyde; their common aldehyde is ethanal, CH_3CHO. Since the alkene contains seven carbon atoms and B contains two, product C contains five. C gives no iodoform precipitate, so it is not a methyl ketone. Reduction of C gives D, whose Lucas-test response indicates a secondary alcohol. Hence C is the

five-carbon non-methyl ketone pentan-3-one, $C_2H_5COC_2H_5$.

Step 2: Reconstruct the alkene and reject the alternatives Join the two carbonyl carbon atoms with a double bond, retaining their substituents. The ethanal side carries H and methyl; the pentan-3-one side carries two ethyl groups. This gives

$$CH_3CH{=}C(C_2H_5)_2.$$

It is 3-ethylpent-2-ene, shown in option (B). Option (A) gives two aldehydes, making the reduced five-carbon product primary. Option (C) produces acetone as one fragment and fails the required pair of tests for B. Option (D) produces a methyl ketone as C, which would give the yellow iodoform precipitate contrary to the observation.

Step 3: Conclusion A is $CH_3CH{=}C(C_2H_5)_2$, 3-ethylpent-2-ene. Hence, the correct option is (B).

Quick Tip to Solve: Identify carbonyl fragments from their tests before joining their carbonyl carbons back into the original alkene double bond.

59. The set that contains atomic numbers of only transition elements, is : [4 marks]

(A) 21, 32, 53, 64

(B) 9, 17, 34, 38

(C) 37, 42, 50, 64

(D) 21, 25, 42, 72

Correct Answer: (D) 21, 25, 42, 72

Solution:

Concept:

- A transition element has an incomplete d subshell in its atom or in at least one of its common oxidation states.
- Atomic numbers identify positions in the periodic table. Main-group s- and p-block elements cannot form an all-transition-element set.

Step 1: Identify every member of the candidate set The atomic numbers in option (D) correspond to scandium, manganese, molybdenum and hafnium: $21 = \mathrm{Sc}$, $25 = \mathrm{Mn}$, $42 = \mathrm{Mo}$ and $72 = \mathrm{Hf}$. These belong to the d block and meet the transition-element criterion. For example, Sc has a $3d^1$ atom, Mn has an incompletely filled $3d$ subshell, Mo has an incompletely filled $4d$ subshell and Hf has a $5d^2$ atom. Thus every entry in this set qualifies.

Step 2: Find a decisive exclusion in every other set Option (A) includes germanium $(Z = 32)$ and iodine $(Z = 53)$, which are p-block elements. Option (B) includes fluorine (9), chlorine (17) and selenium (34), all p-block elements, together with the s-block element strontium (38). Option (C) includes rubidium (37), an alkali metal, and tin (50), a p-block element. These exclusions suffice regardless of the remaining members of those sets. They cannot be sets containing only transition elements.

Step 3: Conclusion The required atomic-number set is $21, 25, 42, 72$. Hence, the correct option is (D).

Quick Tip to Solve: To eliminate a proposed all-transition set quickly, look first for a familiar alkali metal, halogen or other unmistakable main-group atomic number.

60. Which of the following compounds shows geometrical isomerism? [4 marks]

(A) 2-methylpent-1-ene

(B) 4-methylpent-1-ene

(C) 2-methylpent-2-ene

(D) 4-methylpent-2-ene

Correct Answer: (D) 4-methylpent-2-ene

Solution:

Concept:

- An alkene can exhibit geometrical isomerism when each carbon of its double bond carries two different substituents.
- Restricted rotation around the double bond preserves distinct spatial arrangements. If either double-bond carbon has two identical groups, those arrangements are not distinct isomers.

Step 1: Eliminate terminal alkenes In 2-methylpent-1-ene and 4-methylpent-1-ene, the terminal double-bond carbon is $CH_2=$. Its two substituents are both hydrogen, so the necessary condition fails at that carbon. Moving a methyl branch along the saturated chain does not remove the pair of identical hydrogens at the terminal carbon. Thus neither option (A) nor option (B) can show geometrical isomerism.

Step 2: Compare the two internal alkenes For 2-methylpent-2-ene, write its structure on a separate line:

$$(CH_3)_2C{=}CHCH_2CH_3.$$

Carbon 2 has two methyl groups, so option (C) also fails. For 4-methylpent-2-ene the structure is

$$CH_3CH{=}CHCH(CH_3)CH_3.$$

Here carbon 2 carries H and methyl, while carbon 3 carries H and the $CH(CH_3)_2$ group. The two groups on each individual double-bond carbon are different. This permits two arrangements, distinguished by the relative positions of the higher-priority substituents, and hence a geometrical-isomer pair.

Step 3: Conclusion Only 4-methylpent-2-ene satisfies the condition at both double-bond carbons. Hence, the correct option is (D).

Quick Tip to Solve: Inspect the two substituents on each double-bond carbon separately; a single identical pair rules out geometrical isomerism.

61. Arrange the following solutions in the decreasing order of pOH :

(A) 0.01 M HCl

(B) 0.01 M NaOH

(C) 0.01 M CH_3COONa

(D) 0.01 M NaCl [4 marks]

(A) $\mathrm{B} > \mathrm{C} > \mathrm{D} > \mathrm{A}$

(B) $\mathrm{A} > \mathrm{D} > \mathrm{C} > \mathrm{B}$

(C) $\mathrm{A} > \mathrm{C} > \mathrm{D} > \mathrm{B}$

(D) $\mathrm{B} > \mathrm{D} > \mathrm{C} > \mathrm{A}$

Correct Answer: (B) $\mathrm{A} > \mathrm{D} > \mathrm{C} > \mathrm{B}$

Solution:

Concept:

- At room temperature, $\mathrm{pH} + \mathrm{pOH} = 14$; an acidic solution has high pOH and a basic solution has low pOH. Thus arranging decreasing pOH places the most acidic solution first.
- Hydrochloric acid and sodium hydroxide ionize essentially completely at this concentration. Sodium acetate is a salt of a weak acid and strong base, whereas sodium chloride is neutral.

Step 1: Evaluate the strong acid and strong base For solution A, complete ionization of HCl gives $[\mathrm{H}^+] = 10^{-2}\,\mathrm{mol\,L^{-1}}$. Hence $\mathrm{pH} = 2$ and $\mathrm{pOH} = 12$. For solution B, NaOH supplies $[\mathrm{OH}^-] = 10^{-2}\,\mathrm{mol\,L^{-1}}$, giving $\mathrm{pOH} = 2$. The identical analytical concentrations do not make their pOH values equal, since they supply opposite ions.

Step 2: Place the two salts between the extremes Solution D contains NaCl, whose ions do not appreciably hydrolyse: it is neutral and has $\mathrm{pOH} = 7$. In solution C, acetate accepts a proton from water:

$$\mathrm{CH_3COO^-} + \mathrm{H_2O} \rightleftharpoons \mathrm{CH_3COOH} + \mathrm{OH^-}.$$

Thus sodium acetate is basic and its pOH is below 7. Its hydrolysis is partial, so at the same concentration it produces less hydroxide than fully dissociated sodium hydroxide. Consequently $2 < \mathrm{pOH}(C) < 7$.

Step 3: Compare with each listed order The acid must come first and the strong base last. This rejects options (A) and (D), which put B first and A last. Neutral sodium chloride has a higher pOH than basic sodium acetate, so D must precede C; option (C) reverses those two. Only option (B) satisfies both comparisons.

Step 4: Conclusion The decreasing order is $\boxed{\mathrm{A} > \mathrm{D} > \mathrm{C} > \mathrm{B}}$. Hence, the correct option is (B).

Quick Tip to Solve: Rank acid, neutral salt, hydrolysing basic salt and strong base first; then remember that a stronger base has a smaller pOH.

62. The species that has a spin-only magnetic moment of 5.9 BM, is :

(T_d = tetrahedral) [4 marks]

(A) $[MnBr_4]^{2-}$ (T_d)

(B) $[NiCl_4]^{2-}$ (T_d)

(C) $Ni(CO)_4$ (T_d)

(D) $[Ni(CN)_4]^{2-}$ (square planar)

Correct Answer: (A) $[MnBr_4]^{2-}$ (T_d)

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons; ligand-field geometry determines their pairing.
- Oxidation state must be determined before counting d electrons. Bromide and chloride are weak-field ligands, while cyanide favours a paired square-planar nickel(II) complex.

Step 1: Find the required unpaired-electron count The stated moment is close to $\sqrt{35} = 5.92\,\text{BM}$. Setting $n(n+2) = 35$ gives $n = 5$. Thus the required species must contain five unpaired electrons, rather than merely an odd number of d electrons. The small difference between 5.9 and 5.92 is the rounding in the question.

Step 2: Count electrons in the manganese complex In $[MnBr_4]^{2-}$, let the oxidation state of manganese be x. Then $x - 4 = -2$, so $x = +2$. Manganese has atomic number 25; removing its two 4s electrons gives Mn^{2+} with configuration $3d^5$. Tetrahedral bromide is weak field: the five d electrons remain singly occupied, distributed as $e^2t_2^3$. Therefore $n = 5$, exactly as required.

Step 3: Eliminate the nickel alternatives For tetrahedral $[NiCl_4]^{2-}$, nickel is $+2$, hence d^8, and there are two unpaired electrons: $\mu = \sqrt{8} \approx 2.83\,\text{BM}$. Neutral $Ni(CO)_4$ has nickel in oxidation state zero and a d^{10} configuration, so all electrons are paired. Square-planar $[Ni(CN)_4]^{2-}$ is a strong-field d^8 complex with no unpaired electrons. The last two are diamagnetic and cannot have moment 5.9 BM.

Step 4: Conclusion $\boxed{[MnBr_4]^{2-}}$ has five unpaired electrons and $\mu \approx 5.9\,\text{BM}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert 5.9 BM to five unpaired electrons before examining the choices, then check oxidation state and geometry together.

63. A solution of two components containing n_1 moles of the 1st component and n_2 moles of the 2nd component is prepared. M_1 and M_2 are the molecular weights of component 1 and 2 respectively. If d is the density of the solution in g mL^{-1}, C_2 is the molarity and x_2 is the mole fraction of the 2nd component, then C_2 can be expressed as : [4 marks]

(A) $C_2 = \frac{1000x_2}{M_1+x_2(M_2-M_1)}$

(B) $C_2 = \frac{1000dx_2}{M_1+x_2(M_2-M_1)}$

(C) $C_2 = \frac{dx_2}{M_1+x_2(M_2-M_1)}$

(D) $C_2 = \frac{dx_1}{M_1+x_2(M_2-M_1)}$

Correct Answer: (B) $C_2 = \frac{1000dx_2}{M_1+x_2(M_2-M_1)}$

Solution:

Concept:

- Molarity of component 2 is $C_2 = n_2/V$, with V in litres; mole fraction is $x_2 = n_2/(n_1 + n_2)$.
- The total solution mass is $n_1M_1 + n_2M_2$. Dividing this mass in grams by density d in $\mathrm{g\,mL^{-1}}$ gives volume in millilitres.

Step 1: Express the solution volume using its mass The two components contribute masses n_1M_1 and n_2M_2, so the total mass is their sum. No additive-volume approximation is needed because the measured density already describes the final solution. Thus

$$V_{\mathrm{mL}} = \frac{n_1M_1 + n_2M_2}{d}, \qquad V_{\mathrm{L}} = \frac{n_1M_1 + n_2M_2}{1000d}.$$

The factor 1000 converts millilitres to litres.

Step 2: Rewrite the molarity in terms of mole fraction Substitute the final volume into the definition of molarity:

$$C_2 = \frac{1000dn_2}{n_1M_1 + n_2M_2}.$$

Divide numerator and denominator by $n_1 + n_2$. Since $x_1 = 1 - x_2$, this gives

$$C_2 = \frac{1000dx_2}{(1 - x_2)M_1 + x_2M_2} = \frac{1000dx_2}{M_1 + x_2(M_2 - M_1)}.$$

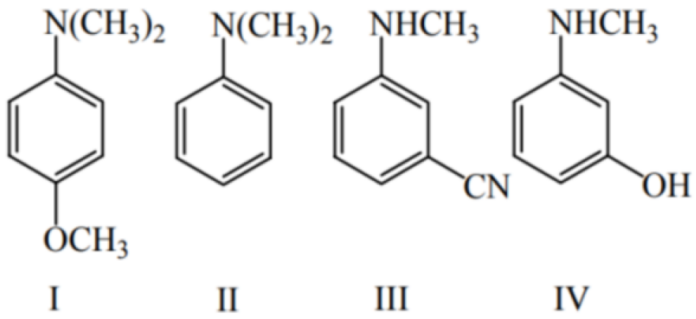
This expression retains density because mole fraction alone does not specify the volume occupied by the mixture.

Step 3: Check the dimensions and reject the other expressions Option (A) drops density and therefore cannot give a concentration in moles per litre. Option (C) omits the factor 1000 needed with the stated millilitre density units; it would give moles per millilitre. Option (D) also uses x_1 in the numerator, so it cannot represent the amount of component 2 and would fail the limit $x_2 \to 0$. Option (B) has the required dependence and conversion.

Step 4: Conclusion $\boxed{C_2 = \frac{1000dx_2}{M_1 + x_2(M_2 - M_1)}}$, in $\mathrm{mol\,L^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Take one mole of mixture: its mass is the mole-fraction-weighted molar mass, its volume follows from density, and component 2 contributes exactly x_2 moles.

64. The increasing order of pK_b values of the following compounds is : [4 marks]



(A) I < II < IV < III

(B) I < II < III < IV

(C) II < I < III < IV

(D) II < IV < III < I

Correct Answer: (A) I < II < IV < III

Solution:

Concept:

- Basicity depends on the availability of the nitrogen lone pair for protonation. $pK_b = -\log K_b$, so stronger bases have smaller pK_b.
- For substituted anilines, electron donation raises basicity whereas electron withdrawal lowers it. The position of a substituent decides whether its resonance effect reaches the amino group directly.

Step 1: Compare compounds I and II Compound II is N,N-dimethylaniline. Compound I has the same dimethylamino group but additionally carries a methoxy group para to it. The methoxy group donates electron density by resonance through the aromatic ring. Para resonance donation outweighs inductive withdrawal, making I more basic than II. Hence

$$K_b(\mathrm{I}) > K_b(\mathrm{II}), \qquad pK_b(\mathrm{I}) < pK_b(\mathrm{II}).$$

Step 2: Compare the meta-substituted secondary amines Compounds III and IV are meta-substituted N-methylanilines, with cyano and hydroxy groups respectively. At the meta position, direct resonance donation from hydroxy does not operate at the carbon bearing the amino group; its electron-withdrawing inductive effect reduces basicity relative to the unsubstituted reference. Cyano withdraws electron density much more strongly than hydroxy, leaving III the weaker base. Therefore IV is more basic than III and $pK_b(\mathrm{IV}) < pK_b(\mathrm{III})$.

Step 3: Assemble the order and test the alternatives Compound II is more basic than meta-hydroxy compound IV; para methoxy donation makes I strongest. The basicity order is therefore I > II > IV > III. Stronger basicity means smaller pK_b, giving increasing values in that sequence. Options (B) and (C) wrongly put the cyano compound below IV in pK_b; option (D) wrongly makes the para-methoxy compound I the weakest base.

Step 4: Conclusion The increasing order of pK_b is $\boxed{\mathrm{I} < \mathrm{II} < \mathrm{IV} < \mathrm{III}}$. Hence, the correct option is (A).

Quick Tip to Solve: First rank lone-pair availability using both substituent position and electron effects; only then convert stronger basicity into lower pK_b.

65. For the reaction

$Fe_2N(s) + \frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s) + NH_3(g)$ [4 marks]

(A) $K_C = K_p(RT)^{1/2}$

(B) $K_C = K_p(RT)^{-1/2}$

(C) $K_C = K_p(RT)$

(D) $K_C = K_p(RT)^{3/2}$

Correct Answer: (A) $K_C = K_p(RT)^{1/2}$

Solution:

Concept:

- For an ideal gaseous equilibrium, $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is gaseous product stoichiometry minus gaseous reactant stoichiometry.
- Pure solids have unit activity and are omitted from both equilibrium-constant expressions. Their coefficients must not enter Δn_g.

Step 1: Count only the gaseous species The reaction is

$$\mathrm{Fe_2N\,(s)} + \tfrac{3}{2}\mathrm{H_2(g)} \rightleftharpoons 2\,\mathrm{Fe\,(s)} + \mathrm{NH_3(g)}.$$

On the product side there is one mole of gaseous ammonia; on the reactant side there are $3/2$ moles of gaseous hydrogen per stoichiometric reaction. Both iron-containing solids are excluded. Thus

$$\Delta n_g = 1 - \frac{3}{2} = -\frac{1}{2}.$$

Including the two moles of solid iron would give an erroneous exponent.

Step 2: Substitute the exponent and rearrange The equilibrium constants satisfy

$$K_p = K_c(RT)^{-1/2}.$$

The question asks for K_c in terms of K_p, so multiply both sides by $(RT)^{1/2}$:

$$K_c = K_p(RT)^{1/2}.$$

The exponent changes sign on rearranging; retaining the negative sign would describe the relation for K_p, not the requested expression for K_c.

Step 3: Verify directly and eliminate distractors With solids omitted, $K_p = p_{\mathrm{NH_3}}/p_{\mathrm{H_2}}^{3/2}$ and $K_c = [\mathrm{NH_3}]/[\mathrm{H_2}]^{3/2}$. Replacing each partial pressure by concentration times RT introduces a net factor $(RT)^{1-3/2}$, confirming the result. Option (B) has the unrearranged exponent. Options (C) and (D) use exponents 1 and $3/2$, neither of which follows from the difference of gaseous coefficients.

Step 4: Conclusion $\boxed{K_c = K_p(RT)^{1/2}}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the signed gas-mole difference first, omit all pure solids, and only then rearrange $K_p = K_c(RT)^{\Delta n_g}$.

66. The presence of soluble fluoride ion upto 1 ppm concentration in drinking water, is : [4 marks]

(A) safe for teeth

(B) harmful to skin

(C) harmful for teeth

(D) harmful to bones

Correct Answer: (A) safe for teeth

Solution:

Concept:

- The effect of fluoride in water is concentration dependent. Small amounts of soluble fluoride help protect dental enamel, while excessive intake can cause fluorosis.
- For dilute aqueous solutions, $1\,\text{ppm}$ is approximately $1\,\text{mg}\,\text{L}^{-1}$. The question specifies the low-concentration range up to this level.

Step 1: Identify the role of low fluoride levels Tooth enamel contains hydroxyapatite, conventionally represented as $\text{Ca}_{10}(\text{PO}_4)_6(\text{OH})_2$. Fluoride can replace hydroxide sites and favour formation of the more acid-resistant fluoride-containing apatite. This reduces the susceptibility of enamel to attack by acids produced in the mouth and helps prevent dental caries. Thus the stated small concentration is associated with protection of teeth rather than tooth damage.

Step 2: Distinguish the specified level from excess exposure Fluoride is not harmless at every concentration. At excessive levels, prolonged exposure may lead to dental fluorosis, including mottling of enamel; much greater chronic exposure can affect bones. Those harmful effects do not describe the dilute level specified in this examination question. It is therefore essential to read the concentration qualifier rather than choose an answer merely because fluoride can be a pollutant at higher levels.

Step 3: Compare all four choices Option (A), safe for teeth, is the statement consistent with fluoride up to about $1\,\text{ppm}$ in the textbook context. Option (C) confuses this level with an excessive fluoride concentration responsible for dental fluorosis. Option (D) similarly refers to harmful bone effects from substantially excessive sustained exposure. Option (B), harmful to skin, is neither the relevant low-level role nor the characteristic effect being tested. The deciding feature is the concentration and the protective action on enamel.

Step 4: Conclusion At the stated concentration up to $1\,\text{ppm}$, soluble fluoride is $\boxed{\text{safe for teeth}}$. Hence, the correct option is (A).

Quick Tip to Solve: In fluoride questions, read the concentration before selecting an effect: a small amount protects enamel, while excess causes fluorosis.

67. Consider the Assertion and Reason given below.

Assertion (A) : Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.

Reason (R) : High density polymers are closely packed and are chemically inert.

Choose the correct answer from the following: [4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(B) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

(C) (A) is correct but (R) is wrong.

(D) (A) and (R) both are wrong.

Correct Answer: (A) Both (A) and (R) are correct and (R) is the correct explanation of (A).

Solution:

Concept:

- Ziegler–Natta catalysts promote coordination polymerisation of ethene, producing predominantly linear high-density polyethene (HDPE).
- Linear chains pack closely, increasing density, intermolecular attraction and mechanical strength. Chemical inertness makes the material useful for containers.

Step 1: Evaluate the assertion Ethene gives high-density polyethene in the presence of a Ziegler–Natta catalyst. Its polymerisation is carried out at an elevated temperature, typically about 333–$343\,\mathrm{K}$, and a pressure of about 6–$7\,\mathrm{atm}$. These conditions are elevated relative to room conditions; they should not be confused with the much more severe conditions used for low-density polyethene. HDPE is used for buckets, dustbins, bottles and pipes. Thus the assertion identifies the correct catalyst, polymer and applications.

Step 2: Evaluate the reason HDPE has comparatively little branching, so neighbouring chains can approach one another and pack closely. This produces a tough, rigid material. Polyethene is also chemically inert towards many common substances because its chains consist mainly of strong carbon–carbon and carbon–hydrogen bonds. The reason correctly describes the properties relevant to durable containers.

Step 3: Link the properties to the use Buckets and dustbins need shape retention, toughness and resistance to chemical attack. Close packing and chemical inertness supply these properties, so the reason explains why the polymer in the assertion is used for these articles. Option (B) incorrectly denies this explanatory link; (C) incorrectly rejects the reason; (D) incorrectly rejects both statements.

Step 4: Conclusion Both statements are correct in the intended HDPE context, and the reason explains the stated application. Hence, the correct option is (A).

Quick Tip to Solve: Identify the polymer from the catalyst first: Ziegler–Natta gives linear HDPE; the very high-pressure radical route gives branched LDPE.

68. Among the sulphates of alkaline earth metals, the solubilities of $BeSO_4$ and $MgSO_4$ in water, respectively, are : [4 marks]

(A) poor and poor

(B) poor and high

(C) high and poor

(D) high and high

Correct Answer: (D) high and high

Solution:

Concept:

- The solubility of an ionic salt depends on the competition between lattice enthalpy, hydration enthalpy and the entropy change on dissolution.
- For Group 2 sulphates, hydration becomes less favourable down the group, and solubility generally decreases from the lighter to the heavier members.

Step 1: Compare hydration and lattice separation Dissolution separates the ions in the sulphate crystal and hydrates them in water. Separating the lattice requires energy, whereas hydration releases energy. Small divalent cations have high charge density and interact strongly with water molecules. Consequently, the hydration enthalpies of Be^{2+} and Mg^{2+} are particularly favourable. The complete balance, rather than lattice energy alone, determines whether a salt dissolves appreciably.

Step 2: Apply the Group 2 trend Both $BeSO_4$ and $MgSO_4$ are soluble in water and have high solubility in the qualitative comparison asked here. Down the group, increasing cation size makes hydration progressively less favourable. The sulphates of the heavier alkaline earth metals therefore become less soluble, with $BaSO_4$ being extremely sparingly soluble. One must not extend the low solubility of barium sulphate to every Group 2 sulphate.

Step 3: Eliminate the other pairs The pair must therefore read high for beryllium sulphate and high for magnesium sulphate. Options (A) and (B) incorrectly call beryllium sulphate poorly soluble, while option (C) incorrectly calls magnesium sulphate poorly soluble. Only the last pair is consistent with both salts.

Step 4: Conclusion The respective solubilities are high and high. Hence, the correct option is (D).

Quick Tip to Solve: For alkaline earth sulphates remember the downward decrease in solubility; the light members Be and Mg are the soluble end of the series.

69. Kraft temperature is the temperature : [4 marks]

(A) below which the aqueous solution of detergents starts freezing.

(B) above which the formation of micelles takes place.

(C) below which the formation of micelles takes place.

(D) above which the aqueous solution of detergents starts boiling.

Correct Answer: (B) above which the formation of micelles takes place.

Solution:

Concept:

- Surfactant molecules contain a hydrophilic head and a hydrophobic tail; in water they can aggregate into micelles.
- Micelle formation requires a sufficiently high surfactant concentration, above the critical micelle concentration (CMC), and a temperature above the Kraft or Krafft temperature.

Step 1: Interpret the temperature threshold At sufficiently low temperature, the solubility of an ionic surfactant may be too small for the concentration of dissolved monomers to reach the critical micelle concentration. As temperature rises, surfactant solubility increases. The Kraft temperature marks the threshold beyond which micelles can form when the concentration is also sufficiently high. It is a property of the surfactant–solvent system, rather than the ordinary freezing or boiling point of water.

Step 2: Choose the correct side of the threshold The question asks what happens above or below this characteristic temperature. Micelle formation occurs above it, subject to the concentration condition. Therefore the statement in option (B) gives the correct definition. Option (C) reverses the temperature requirement: being below the Kraft temperature does not enable the usual micellar state.

Step 3: Exclude phase-change alternatives Options (A) and (D) describe freezing and boiling of the detergent solution. Those are bulk liquid phase changes and are not the definition of the Kraft temperature. A surfactant solution can remain liquid while crossing the micelle-formation threshold. Also, exceeding the Kraft temperature alone does not mean that a dilute solution below the CMC must contain micelles.

Step 4: Conclusion Kraft temperature is the temperature above which micelle formation becomes possible at an adequate surfactant concentration. Hence, the correct option is (B).

Quick Tip to Solve: Use two checks for micelles: temperature above the Kraft temperature and concentration above the CMC. Neither is a boiling or freezing condition.

70. The incorrect statement is : [4 marks]

(A) bronze is an alloy of copper and tin

(B) cast iron is used to manufacture wrought iron

(C) german silver is an alloy of zinc, copper and nickel

(D) brass is an alloy of copper and nickel

Correct Answer: (D) brass is an alloy of copper and nickel

Solution:

Concept:

- An alloy is a metallic mixture whose composition determines its name and useful properties. Brass, bronze and German silver are distinct copper alloys.
- Wrought iron is nearly pure iron obtained by removing most carbon and other impurities from cast iron.

Step 1: Check the first two statements Bronze is principally an alloy of copper and tin, so option (A) is a correct statement. Cast iron contains substantially more carbon than wrought iron. In producing wrought iron, cast iron is refined so that carbon and other impurities are oxidised and removed. Thus cast iron can indeed be used to manufacture wrought iron, and option (B) is also correct. Neither statement answers a question asking for the incorrect statement.

Step 2: Identify German silver German silver is an alloy of copper, zinc and nickel. Despite its name, silver is not an essential constituent of this alloy. The three metals listed in option (C) are therefore the correct ones. Their order in the wording does not change the alloy composition. This eliminates option (C) as the requested incorrect statement.

Step 3: Correct the composition of brass Brass is principally an alloy of copper and zinc, not copper and nickel. Copper–nickel alloys belong to a different alloy family, often called cupronickel. Option (D) replaces zinc with nickel and is therefore false. The distinction is compositional: bronze uses tin with copper, brass uses zinc with copper, and German silver contains copper together with both zinc and nickel.

Step 4: Conclusion The incorrect statement is that brass is an alloy of copper and nickel; brass is principally copper and zinc. Hence, the correct option is (D).

Quick Tip to Solve: Keep the copper alloys in a three-entry list: bronze = Cu–Sn, brass = Cu–Zn, German silver = Cu–Zn–Ni; then check the word incorrect.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. The number of Cl = O bonds in perchloric acid is, "_____". [4 marks]

Solution:

Concept:

- Perchloric acid has molecular formula $HClO_4$, with chlorine bonded to four oxygen atoms.
- In the conventional Lewis representation used for counting oxo bonds, its connectivity is

$HO-Cl(=O)_3$: one hydroxyl oxygen and three terminal oxo oxygens.

Step 1: Place the acidic hydrogen The hydrogen of an oxoacid is attached to oxygen, not directly to chlorine. Thus one of the four oxygen atoms belongs to an $-OH$ group. Its two bonds are an O–H single bond and a Cl–O single bond. Counting this hydroxyl oxygen as a chlorine–oxygen double bond would give the wrong structure and would not represent the acid group correctly.

Step 2: Count the remaining terminal oxygens After assigning one oxygen to the hydroxyl group, three oxygen atoms remain. Each is conventionally drawn as a terminal oxygen joined to chlorine by a double bond. The familiar structure is therefore $HO-Cl(=O)_3$. It contains three $Cl=O$ bonds and one $Cl-OH$ bond. These are bond types in the structural formula, so the O–H bond does not enter the requested count.

Step 3: Check the formula and interpretation The structure has one hydrogen, one chlorine and four oxygens, consistent with $HClO_4$. Taking hydrogen as +1 and each oxygen as -2 gives chlorine oxidation state +7, as expected for perchloric acid. This is a check on the composition, not a rule that the oxidation state equals the number of double bonds. Resonance and the detailed description of bonding do not change the conventional oxo-bond count asked in this examination question.

Step 4: Conclusion The number of $Cl=O$ bonds in perchloric acid is $\boxed{3}$. This is a dimensionless bond count.

Quick Tip to Solve: Write the oxoacid as an OH group plus terminal oxo groups before counting: four oxygens in perchloric acid leave three oxo oxygens after one OH oxygen.

72. The elevation of boiling point of 0.10 m aqueous $CrCl_3.xNH_3$ solution is two times that of 0.05 m aqueous $CaCl_2$ solution. The value of x is _____.

[Assume 100% ionisation of the complex and $CaCl_2$, coordination number of Cr as 6, and that all NH_3 molecules are present inside the coordination sphere] [4 marks]

Solution:

Concept:

- Boiling-point elevation obeys $\Delta T_b = iK_b m$, where i is the van't Hoff factor and m is the molality.
- Complete dissociation makes i equal to the number of dissolved ions per formula unit. Neutral ammine ligands inside a coordination sphere do not dissociate as separate particles.

Step 1: Find the particle factor of the complex Let i denote the factor for $CrCl_3 \cdot xNH_3$. Fully ionised $CaCl_2$ gives one calcium ion and two chloride ions, hence its factor is 3. Both solutions are aqueous, so the same solvent constant K_b applies. The given elevation ratio gives

$$iK_b(0.10) = 2 \times 3K_b(0.05).$$

Cancelling K_b and dividing by 0.10 yields $i = 3$. Thus each formula unit of the chromium complex produces three ions.

Step 2: Determine chloride placement The complex gives one chromium-containing complex cation. To obtain three ions altogether, two chloride ions must be outside the coordination sphere. Of the

three chlorides in the formula, the remaining one is coordinated directly to chromium. The ionic formula must therefore have the form $[Cr(NH_3)_xCl]Cl_2$, dissociating into one complex cation and two free chloride ions.

Step 3: Use the coordination number The coordination number of chromium is six. Each ammonia molecule is monodentate and occupies one coordination site, and the single inner-sphere chloride occupies one more. Since all ammonia molecules are inside the sphere,

$$x + 1 = 6 \quad \Longrightarrow \quad x = 5.$$

The complex is consequently $[Cr(NH_3)_5Cl]Cl_2$, whose three-ion dissociation agrees with the particle factor obtained from the boiling-point data.

Step 4: Conclusion The required value is $\boxed{x = 5}$, a dimensionless number of ammonia ligands per formula unit.

Quick Tip to Solve: First infer the number of outer-sphere chlorides from the colligative factor; then subtract inner-sphere chloride ligands from coordination number six.

73. Potassium chlorate is prepared by the electrolysis of KCl in basic solution

$6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10 g of $KClO_3$ using a current of 2 A is_____.

(Given : F = 96,500 C mol^{-1}; molar mass of $KClO_3$ = 122 g mol^{-1}) [4 marks]

Solution:

Concept:

- Faraday's law relates the amount of product to the useful charge: $Q_{\text{useful}} = nzF$, where n is the amount of product in moles, z is the number of electrons transferred per formula unit, and F is the Faraday constant.
- The given half-reaction releases six electrons for each chlorate ion formed, so $z = 6$. With current efficiency $\eta = 0.60$, only ηIt of the supplied charge forms the desired product.

Step 1: Calculate the amount of potassium chlorate Use the molar mass stated in the question. One mole of $KClO_3$ contains one mole of chlorate ions, so the amount needed is

$$n(KClO_3) = \frac{10}{122} = 0.081967\,\text{mol}.$$

The potassium ion is a spectator in the stated oxidation; it does not introduce an additional electron requirement.

Step 2: Find the charge required for the desired reaction Six moles of electrons are associated with one mole of chlorate. Therefore the charge actually used to form the specified mass is

$$Q_{\text{useful}} = \frac{10}{122} \times 6 \times 96{,}500 = 47{,}459.0\,\text{C}.$$

This is the productive charge, rather than the total charge supplied by the source.

Step 3: Account for current efficiency and convert the time The useful current is $0.60 \times 2 = 1.2\,\text{A}$. Since one ampere supplies one coulomb per second,

$$t = \frac{47{,}459.0}{1.2} = 39{,}549.2\,\text{s} = \frac{39{,}549.2}{3600}\,\text{h} = 10.986\,\text{h}.$$

The efficiency must increase the required time relative to the ideal case, because some current is consumed in other processes. Round only after converting seconds to hours.

Step 4: Conclusion The required time, rounded to the nearest hour, is $\boxed{11\,\text{h}}$. The numerical entry is $\boxed{11}$.

Quick Tip to Solve: Use $t = mzF/(M\eta I)$ directly for an electrolysis time, and place the fractional current efficiency in the denominator.

74. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is _____.
(Atomic mass, Ag = 108, Br = 80 g mol^{-1}) [4 marks]

Solution:

Concept:

- In the Carius method, the halogen present in a known mass of an organic compound is converted to the corresponding silver halide. Each formula unit of AgBr contains exactly one bromine atom.
- The mass fraction of bromine in pure silver bromide is $M(\text{Br})/M(\text{AgBr})$. The percentage in the original compound is the recovered bromine mass divided by the original sample mass, multiplied by 100.

Step 1: Calculate the molar mass of silver bromide Add the supplied atomic masses, retaining the one-to-one ratio of silver and bromine in the precipitate:

$$M(\text{AgBr}) = 108 + 80 = 188\,\text{g mol}^{-1}.$$

Thus every $188\,\text{g}$ of the precipitate contains $80\,\text{g}$ of bromine. The silver added during the estimation is not part of the original compound.

Step 2: Determine the mass of bromine in the precipitate The $1.88\,\text{g}$ precipitate corresponds to

$$n(\text{AgBr}) = \frac{1.88}{188} = 0.0100\,\text{mol}.$$

Because each mole of AgBr contains one mole of bromine atoms, the recovered bromine mass is

$$m(\text{Br}) = 0.0100 \times 80 = 0.800\,\text{g}.$$

Using the whole precipitate mass as the bromine mass would incorrectly include the mass of silver.

Step 3: Refer the bromine mass to the original sample The denominator is the $1.6\,\text{g}$ organic sample,

since the question asks for its composition, rather than the composition of the precipitate. Therefore

$$\text{Mass percentage of Br} = \frac{0.800}{1.6} \times 100 = 50\%.$$

The calculated bromine mass is half of the sample mass and less than the precipitate mass, providing a direct consistency check on the result.

Step 4: Conclusion The organic compound contains $\boxed{50\%}$ bromine by mass. The numerical entry is $\boxed{50}$.

Quick Tip to Solve: First multiply the silver-halide mass by the halogen's molar-mass fraction; then divide by the organic sample mass to find its percentage.

75. A spherical balloon of radius 3 cm containing helium gas has a pressure of 48 $\times$ 10^{-3} bar. At the same temperature, the pressure, of a spherical balloon of radius 12 cm containing the same amount of gas will be _____ $\times$ 10^{-6} bar. [4 marks]

Solution:

Concept:

- For a fixed amount of gas at constant temperature, Boyle's law gives $P_1V_1 = P_2V_2$. The pressure is inversely proportional to the gas volume under these conditions.
- A spherical balloon has volume $V = (4/3)\pi r^3$. Consequently, when both balloons are spherical, their volume ratio is the cube of their radius ratio, and the common geometrical factor cancels.

Step 1: Identify the constant quantities The question specifies the same amount of helium and the same temperature in the two states. Applying the ideal gas relation $PV = nRT$ to both therefore gives

$$P_1V_1 = P_2V_2.$$

The required pressure can be obtained from the relative volumes without calculating the number of moles or converting bar into pascal.

Step 2: Find the increase in spherical volume The radius changes from $3\,\text{cm}$ to $12\,\text{cm}$, a factor of four. Hence

$$\frac{V_2}{V_1} = \frac{(4/3)\pi(12)^3}{(4/3)\pi(3)^3} = \left(\frac{12}{3}\right)^3 = 64.$$

The units of the radii cancel in this ratio. Using a factor of four alone would treat the volume as proportional to radius and would miss the cubic dependence.

Step 3: Calculate the pressure in the requested form As the volume increases by a factor of 64, the pressure decreases by the same factor:

$$P_2 = P_1\frac{V_1}{V_2} = \frac{48 \times 10^{-3}}{64}\,\text{bar} = 0.75 \times 10^{-3}\,\text{bar}.$$

Rewrite the result with the power of ten specified in the blank:

$$0.75 \times 10^{-3} = 750 \times 10^{-6}.$$

The new pressure is smaller than the initial pressure, as required for an isothermal expansion with a fixed amount of gas.

Step 4: Conclusion The final pressure is $\boxed{750 \times 10^{-6}\,\text{bar}}$, so the required numerical entry is $\boxed{750}$.

Quick Tip to Solve: For an isothermal spherical balloon with a fixed amount of gas, use $P_2/P_1 = (r_1/r_2)^3$, then match the power of ten in the blank.