JEE Main 2020 Paper 1 (B.E./B.Tech) Jan 9 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2020, Session 1 (January), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 9 January 2020, Shift 1. The total duration is 3 hours.

(ii) The paper has 75 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 25 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 5 numerical value questions. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, no negative marking and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to : [4 marks]

(A) 6

(B) 8

(C) 4

(D) 7

Correct Answer: (B) 8

Solution:

Concept:

- The multiplication principle counts a sequence of independent choices by multiplying the number of choices available at each stage.
- A five-digit number cannot begin with zero. Once a digit is used, it is excluded from every other position because all digits must be distinct.

Step 1: Fix the tens digit and count the leading choices The digit in the tens place is fixed as 2, so that place contributes exactly one choice. The leading digit must be one of 1 through 9, with 2 excluded. Hence there are eight choices for the ten-thousands place. Zero is excluded only from this leading position; it remains available for the other unfilled places.

Step 2: Fill the remaining three places without repetition After choosing the leading digit and fixing 2, two of the ten digits have been used. There are eight choices for the thousands place, seven for the hundreds place and six for the units place. These positions are distinct, so their order matters. The required count is therefore

$$N = 8 \times 8 \times 7 \times 6 = 2688.$$

This counts each admissible number once because its successive digit choices are unique.

Step 3: Compare the count with the stated multiple The question gives the count in the form $336k$. Dividing the calculated number by 336 gives

$$k = \frac{2688}{336} = 8.$$

The alternatives 6, 4 and 7 would produce counts 2016, 1344 and 2352 respectively, so none equals the count obtained from the positional restrictions.

Step 4: Conclusion Thus $k = 8$. Hence, the correct option is (B).

Quick Tip to Solve: Fix the specified digit first, then choose the leading nonzero digit before filling the remaining places. This keeps the special restriction on zero separate from the no-repetition rule.

2. The value of

$\cos^3\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)$

is : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2\sqrt{2}}$

Correct Answer: (D) $\frac{1}{2\sqrt{2}}$

Solution:

Concept:

- For complementary angles, $\cos(\pi/2-\theta)=\sin\theta$ and $\sin(\pi/2-\theta)=\cos\theta$.
- The identities $\sin^2\theta+\cos^2\theta=1$ and $2\sin\theta\cos\theta=\sin 2\theta$ simplify products without requiring separate radical values.

Step 1: Express both angles using a single variable Put $\theta=\pi/8$. The other angle is $3\pi/8=\pi/2-\theta$, so its cosine is $\sin\theta$ and its sine is $\cos\theta$. The given sum becomes

$$S=\cos^3\theta\sin\theta+\sin^3\theta\cos\theta.$$

Both terms are positive because the two original angles lie in the first quadrant.

Step 2: Factor the common trigonometric product Take $\sin\theta\cos\theta$ outside the sum, leaving a sum of squares:

$$\begin{aligned}S&=\sin\theta\cos\theta(\cos^2\theta+\sin^2\theta)\\&=\sin\theta\cos\theta.\end{aligned}$$

Thus the cubic powers disappear through the Pythagorean identity. Expanding separate half-angle radicals would give the same result but would introduce unnecessary algebra.

Step 3: Evaluate the remaining double angle Since $2\theta=\pi/4$,

$$S=\tfrac{1}{2}\sin(\pi/4)=\frac{1}{2\sqrt{2}}.$$

Option (A) is twice this value and forgets the factor one-half in the double-angle identity. Options (B) and (C) also fail the exact expression: they are respectively $1/2$ and $1/4$, whereas the calculated value is approximately 0.3536.

Step 4: Conclusion The required value is $\frac{1}{2\sqrt{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: When two angles add to $\pi/2$, replace the sine and cosine of one by the complementary functions of the other before expanding powers.

3. If for all real triplets (a, b, c), f(x) = a + bx + cx^2; then $\int_0^1 f(x)dx$ is equal to : [4 marks]

(A) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$
(B) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$
(C) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$
(D) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

Correct Answer: (A) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

Solution:

Concept:

- Integrating a polynomial term by term gives $\int_0^1 (a+bx+cx^2)\,dx = a + b/2 + c/3$.
- An identity valid for every real triple (a, b, c) must reproduce the coefficients of all three independent terms. Checking only one particular quadratic is not enough to prove it.

Step 1: Compute the integral directly Treat a, b, c as arbitrary constants and integrate each power of x:

$$I = \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1$$
$$= a + \frac{b}{2} + \frac{c}{3}.$$

The value at the lower limit vanishes in every term. This expression is the target that a proposed formula involving sampled values must match.

Step 2: Substitute the three sampled values into option A The values required by option (A) are

$$f(0) = a, \quad f(1) = a + b + c, \quad f(1/2) = a + b/2 + c/4.$$

Consequently,

$$\frac{f(0)+f(1)+4f(1/2)}{6} = \frac{6a+3b+2c}{6}$$
$$= a + \frac{b}{2} + \frac{c}{3} = I.$$

Thus the displayed combination gives the exact integral for every quadratic, including lower-degree polynomials.

Step 3: Exclude the other proposed formulas For a fast disproof, choose the permitted constant polynomial $f(x) = 1$, corresponding to $a = 1, b = c = 0$. Its integral is 1. Option (B) gives $2(3 + 2) = 10$, option (C) gives $2/3$, and option (D) gives $(1 + 3)/2 = 2$. None equals 1, so none can hold for all real coefficient triples.

Step 4: Conclusion Therefore $\int_0^1 f(x)\,dx = \frac{1}{6}\{f(0)+f(1)+4f(1/2)\}$. Hence, the correct option is (A).

Quick Tip to Solve: For a proposed integration identity, first test the constant polynomial to reject bad options, then match the coefficients of $1, x, x^2$ to prove the surviving formula.

4. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is : [4 marks]

(A) $\frac{1}{18\pi}$
(B) $\frac{1}{36\pi}$
(C) $\frac{1}{54\pi}$
(D) $\frac{5}{6\pi}$

Correct Answer: (A) $\frac{1}{18\pi}$

Solution:

Concept:

- The volume of a sphere of radius r is $4\pi r^3/3$. An ice shell has the difference between the outer spherical volume and the fixed iron-ball volume.
- The rate of decrease of a quantity is the positive magnitude of its negative derivative. The iron radius stays constant, so the outer radius and ice thickness have equal time derivatives.

Step 1: Write the ice volume in terms of thickness Let h cm denote the ice thickness after t minutes. The outer radius is $10 + h$ cm, while the iron ball retains radius 10 cm. Therefore the ice volume is

$$V = \frac{4\pi}{3}\left((10+h)^3 - 10^3\right).$$

Melting removes ice, so the given volume-loss rate means $dV/dt = -50\,\text{cm}^3\,\text{min}^{-1}$.

Step 2: Differentiate and substitute the current radius Differentiate with respect to time, applying the chain rule to the changing outer radius:

$$\frac{dV}{dt} = 4\pi(10+h)^2\frac{dh}{dt}.$$

At the specified instant $h = 5$ cm, the outer radius is 15 cm. Hence

$$-50 = 4\pi(15)^2\frac{dh}{dt}, \qquad \frac{dh}{dt} = -\frac{50}{900\pi} = -\frac{1}{18\pi}.$$

The fixed inner volume contributes no derivative.

Step 3: Interpret the sign and compare the choices The negative derivative confirms that the layer becomes thinner. The question asks the rate at which the thickness decreases, so report its positive magnitude in centimetres per minute. Using 10 cm in the surface-area factor would confuse the fixed iron radius with the changing outer radius. The values in (B), (C) and (D) do not satisfy $4\pi(15)^2|dh/dt| = 50$.

Step 4: Conclusion The thickness decreases at $\frac{1}{18\pi}\,\text{cm}\,\text{min}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a melting coating, differentiate using the current outer radius, which is the core radius plus the coating thickness; keep the core volume constant.

5. Let z be complex number such that
$\left|\frac{z-i}{z+2i}\right| = 1$ **and |z| =** $\frac{5}{2}$**.**
Then the value of |z + 3i| is : [4 marks]

(A) $2\sqrt{3}$
(B) $\sqrt{10}$
(C) $\frac{15}{4}$
(D) $\frac{7}{2}$

Correct Answer: (D) $\frac{7}{2}$

Solution:

Concept:

- For $z = x + iy$, its modulus satisfies $|z|^2 = x^2 + y^2$. A modulus of a difference is the distance between the corresponding points in the complex plane.
- The condition $|(z - i)/(z + 2i)| = 1$ implies $|z - i| = |z + 2i|$, provided the denominator is nonzero. Equal distances yield a linear equation after squaring.

Step 1: Use the ratio of distances to determine the imaginary part Write $z = x + iy$ with real x, y. Equality of the two moduli gives

$$x^2 + (y - 1)^2 = x^2 + (y + 2)^2.$$

The x^2 and y^2 terms cancel, leaving

$$-2y + 1 = 4y + 4, \qquad y = -\frac{1}{2}.$$

This is the horizontal perpendicular bisector of the points i and $-2i$. In particular, it does not include the forbidden point $-2i$.

Step 2: Use the prescribed modulus to find the real part squared We are given $|z| = 5/2$, so

$$x^2 + y^2 = \frac{25}{4}.$$

Substituting $y = -1/2$ gives $x^2 = 25/4 - 1/4 = 6$. Either sign $x = \pm\sqrt{6}$ is possible, but the required distance depends only on x^2, so both points give the same answer.

Step 3: Evaluate the requested shifted modulus The number $z+3i$ has imaginary part $-1/2+3 = 5/2$. Therefore

$$|z + 3i|^2 = x^2 + (y + 3)^2 = 6 + \frac{25}{4} = \frac{49}{4}.$$

Taking the nonnegative square root gives $7/2$. The squared values associated with (A), (B) and (C) are 12, 10 and $225/16$, respectively, and none equals $49/4$.

Step 4: Conclusion Thus $|z + 3i| = \frac{7}{2}$. Hence, the correct option is (D).

Quick Tip to Solve: When two shifted moduli are equal, square and subtract first: the quadratic terms cancel, giving one coordinate before you use the given modulus.

6. The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ **is equal to :** [4 marks]

(A) 4π

(B) 2π

(C) π^2

(D) $2\pi^2$

Correct Answer: (C) π^2

Solution:

Concept:

- For an integrable function g, reflection gives $\int_0^a xg(x)\,dx = \int_0^a (a-x)g(a-x)\,dx$.
- Here $g(x) = \sin^8 x/(\sin^8 x + \cos^8 x)$ is continuous and periodic with period π. Its denominator is positive because sine and cosine never vanish together.

Step 1: Remove the factor x by reflection Let $J = \int_0^{2\pi} g(x)\,dx$ and let I be the requested integral. Since even powers give $g(2\pi - x) = g(x)$, the substitution $u = 2\pi - x$ yields

$$I = \int_0^{2\pi} (2\pi - x)g(x)\,dx = 2\pi J - I.$$

Adding the original and reflected expressions therefore gives $I = \pi J$. This step uses symmetry of the whole interval; it does not replace x pointwise by a constant.

Step 2: Evaluate the unweighted integral by exchanging sine and cosine A quarter-period shift interchanges the numerator functions, so

$$g(x + \pi/2) = \frac{\cos^8 x}{\sin^8 x + \cos^8 x} = 1 - g(x).$$

The integral over an interval containing two full periods is unchanged by a shift of its endpoints. Thus

$$J = \int_0^{2\pi} (1 - g(x))\,dx = 2\pi - J, \qquad J = \pi.$$

This pairing avoids expanding either eighth power.

Step 3: Combine the two symmetry results Substituting the value of the unweighted integral into $I = \pi J$ gives

$$I = \pi \cdot \pi = \pi^2.$$

Option (D) is twice this value and results from missing the division by two in one of the paired integrals. Options (A) and (B), equal to 4π and 2π, also differ from the exact result obtained by the two identities.

Step 4: Conclusion The required integral equals π^2. Hence, the correct option is (C).

Quick Tip to Solve: For a weighted integral $xg(x)$, first reflect the full interval to remove x. Then pair the sine and cosine fractions so their numerators add to the denominator.

7. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to :

(where C is a constant of integration) [4 marks]

(A) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$

(B) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

(C) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{13}{7}} + C$

(D) $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}} + C$

Correct Answer: (B) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

Solution:

Concept:

- The ratio of the two linear factors is a useful substitution because its derivative contains the square of the denominator.
- For a real seventh root, fractional powers with denominator seven are defined for both signs of the base. Work on an interval excluding the singular points $x = -4, 3$.

Step 1: Choose the ratio as the new variable Put $t = (x-3)/(x+4)$. The quotient rule gives

$$\frac{dt}{dx} = \frac{(x+4)-(x-3)}{(x+4)^2} = \frac{7}{(x+4)^2}.$$

Thus $dx = (x+4)^2 dt/7$. Also $x - 3 = t(x+4)$, so the two powers in the denominator combine to a total power of two in $x+4$. This is why this substitution eliminates both linear factors together.

Step 2: Integrate the resulting power The integral becomes

$$I = \frac{1}{7}\int t^{-6/7}\,dt = \frac{1}{7}\frac{t^{1/7}}{1/7} + C = t^{1/7} + C.$$

Substituting the ratio back gives

$$I = \left(\frac{x-3}{x+4}\right)^{1/7} + C.$$

The integration constant can be chosen independently on each interval separated by a singular point.

Step 3: Differentiate to select the option The derivative of this answer is

$$\frac{1}{7}t^{-6/7}\frac{7}{(x+4)^2} = \frac{1}{(x+4)^{8/7}(x-3)^{6/7}},$$

exactly the printed integrand. Option (A) instead differentiates to $\frac{3}{2}t^{-4/7}/(x+4)^2$; option (C) gives $-t^{6/7}/(x+4)^2$; and option (D) gives $t^{-8/7}/(x+4)^2$. Their powers or coefficients therefore do not match.

Step 4: Conclusion The antiderivative is $\left(\frac{x-3}{x+4}\right)^{1/7} + C$. Hence, the correct option is (B).

Quick Tip to Solve: When the exponents of two linear factors add to two, try their ratio and compare its derivative with the integrand.

8. Let f be any function continuous on [a, b] and twice differentiable on (a, b). If for all x $\in$ (a, b), f'(x) $>$ 0 and f"(x) $<$ 0, then for any c $\in$ (a, b), $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than : [4 marks]

(A) 1

(B) $\frac{b-c}{c-a}$

(C) $\frac{b+a}{b-a}$

(D) $\frac{c-a}{b-c}$

Correct Answer: (D) $\frac{c-a}{b-c}$

Solution:

Concept:

- The Mean Value Theorem equates the slope of a chord with a derivative at an interior point.
- The conditions $f\prime > 0$ and $f\prime\prime < 0$ mean that the function is increasing while its derivative is strictly decreasing.

Step 1: Apply the theorem on both subintervals Continuity on $[a, b]$ and differentiability inside it allow the Mean Value Theorem on $[a, c]$ and $[c, b]$. There are points $u \in (a, c)$, $v \in (c, b)$ such that

$$\frac{f(c) - f(a)}{c - a} = f\prime(u), \qquad \frac{f(b) - f(c)}{b - c} = f\prime(v).$$

Since $u < c < v$ and the derivative is strictly decreasing, the first slope is strictly greater than the second.

Step 2: Rearrange using positive quantities The increasing-function condition ensures $f(c) - f(a) > 0$ and $f(b) - f(c) > 0$. The interval lengths are positive too. We may therefore divide and cross-multiply without reversing the inequality:

$$\frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c} \quad \Longrightarrow \quad \frac{f(c) - f(a)}{f(b) - f(c)} > \frac{c - a}{b - c}.$$

This proves the proposed bound for every interior choice of c.

Step 3: Check why the other bounds are not universal For example, take $f(x) = \sqrt{x}$, $a = 1$, $b = 4$, and c close to 1 from the right. Both derivative conditions hold, but the ratio of function increments approaches zero. It is then less than 1, less than $(b - c)/(c - a)$, and less than $(b + a)/(b - a) = 5/3$. Thus (A), (B), and (C) cannot hold in general; the ratio of interval lengths in (D) has the correct orientation.

Step 4: Conclusion For every $c \in (a, b)$, the ratio is greater than $(c - a)/(b - c)$. Hence, the correct option is (D).

Quick Tip to Solve: For a concave increasing function, the left chord is steeper than the right chord; compare slopes before comparing increments.

9. In a box, there are 20 cards, out of which 10 are lebelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is : [4 marks]

(A) $\frac{13}{16}$
(B) $\frac{11}{16}$
(C) $\frac{15}{16}$
(D) $\frac{9}{16}$

Correct Answer: (B) $\frac{11}{16}$

Solution:

Concept:

- Replacement makes successive draws independent, with $P(A) = P(B) = 1/2$.
- The second A must occur with zero, one, or two preceding B cards if it is to precede the third B. These stopping cases are disjoint.

Step 1: Count sequences ending at the second A Let k be the number of B cards drawn before the second A. Success means $k = 0, 1, 2$. The final card is A, while the preceding $k + 1$ positions contain exactly one A and k B cards. There are $\binom{k+1}{1} = k+1$ ways to place that earlier A. Every such complete sequence has probability $(1/2)^{k+2}$, because its length is $k + 2$.

Step 2: Add the mutually exclusive cases For $k = 0$, only AA occurs, giving $1/4$. For $k = 1$, ABA and BAA give $2/8 = 1/4$. For $k = 2$, ABBA, BABA, and BBAA give $3/16$. Consequently

$$P = \frac{1}{4} + \frac{1}{4} + \frac{3}{16} = \frac{4+4+3}{16} = \frac{11}{16}.$$

We stop at the second A, so appending arbitrary later draws would double-count outcomes.

Step 3: Verify by a fixed number of draws Imagine that four draws are made even if the stopping event has already happened. The second A precedes the third B exactly when these first four draws contain at least two A cards. Fewer than two A cards means at least three B cards, so failure has already occurred; two or more A cards means success. Hence

$$P = 1 - \frac{\binom{4}{0} + \binom{4}{1}}{16} = 1 - \frac{5}{16} = \frac{11}{16}.$$

This independently rules out the proposed values $13/16$, $15/16$, and $9/16$.

Step 4: Conclusion The required probability is $\boxed{11/16}$. Hence, the correct option is (B).

Quick Tip to Solve: For a race between the second A and third B, inspect the first four draws: at least two A cards is exactly the success event.

10. If $f(x) = \begin{cases} \frac{\sin(a+2)x+\sin x}{x}; & x<0 \\ b \quad\quad\quad\quad ; & x=0 \\ \frac{\left(x+3x^2\right)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}}; & x>0 \end{cases}$

is continuous at x = 0, then a + 2b is equal to : [4 marks]

(A) 0

(B) -1

(C) -2

(D) 1

Correct Answer: (A) 0

Solution:

Concept:

- Continuity at zero requires the left-hand limit, the value $f(0) = b$, and the right-hand limit to be equal.
- Use $\lim_{u\to 0} \sin u/u = 1$ and the difference-of-cubes identity to evaluate the two limits without dividing by zero.

Step 1: Find the left-hand limit For $x < 0$, split the numerator into two terms. If $a + 2 \neq 0$, write the first term in the standard sine-limit form; if $a + 2 = 0$, that term is identically zero and the same result follows. Thus

$$L = \lim_{x\to 0^-} \left(\frac{\sin((a+2)x)}{x} + \frac{\sin x}{x} \right) = (a+2) + 1 = a + 3.$$

The sine argument is $(a + 2)x$; the extra $\sin x$ contributes a separate 1.

Step 2: Find the right-hand limit For positive x, factor out $x^{1/3}$ from the numerator to obtain

$$R = \lim_{x\to 0^+} \frac{(1+3x)^{1/3} - 1}{x}.$$

Put $u = (1 + 3x)^{1/3}$. Since $(u - 1)(u^2 + u + 1) = u^3 - 1 = 3x$, the quotient equals

$$\frac{(1+3x)^{1/3} - 1}{x} = \frac{3}{(1+3x)^{2/3} + (1+3x)^{1/3} + 1}.$$

The denominator tends to 3, so $R = 1$.

Step 3: Impose continuity and evaluate the expression Both limits must equal the value at the joining point. Therefore

$$a + 3 = b = 1, \qquad a = -2, \quad b = 1.$$

Substituting these values into the requested combination gives

$$a + 2b = -2 + 2(1) = 0.$$

The proposed alternatives $-1, -2, 1$ do not equal this combination. In particular, choosing -2 would report the value of a alone rather than the expression asked for.

Step 4: Conclusion Continuity gives $a = -2$, $b = 1$, and $\boxed{a + 2b = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: Factor out the lowest power of x before evaluating a limit with cube roots, then rationalise using the difference of cubes.

11. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines x + 3y – 1 = 0 and 3x – y + 1 = 0. Then the line passing through the points C and P also passes through the point : [4 marks]

(A) (–9, –7)

(B) (9, 7)

(C) (7, 6)

(D) (–9, –6)

Correct Answer: (D) (–9, –6)

Solution:

Concept:

- The centroid is the coordinate-wise average of the three vertices of a triangle.
- A line through two points can be obtained from its slope or by eliminating the parameter in a two-point equation. Substitution then tests each candidate point.

Step 1: Find the centroid The vertices are $(3, -1)$, $(1, 3)$, and $(2, 4)$. Their centroid is

$$C = \left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) = (2, 2).$$

Averaging the negative ordinate with its sign is essential: the sum of the three ordinates is 6, not 8.

Step 2: Solve the two equations for P The given lines can be written as $x + 3y = 1$ and $3x - y = -1$. From the second equation, $y = 3x + 1$. Substituting into the first gives

$$x + 3(3x + 1) = 1, \qquad 10x = -2, \qquad x = -\frac{1}{5}.$$

Consequently $y = 3(-1/5) + 1 = 2/5$, so $P = (-1/5, 2/5)$. Substitution in both original line equations confirms the intersection.

Step 3: Form the required line and test the options The slope of CP is

$$m = \frac{2 - 2/5}{2 - (-1/5)} = \frac{8/5}{11/5} = \frac{8}{11}.$$

Hence

$$y - 2 = \frac{8}{11}(x - 2), \qquad 8x - 11y + 6 = 0.$$

The left side at $(-9, -7)$, $(9, 7)$, and $(7, 6)$ is respectively 11, 1, and -4, so none lies on CP. At $(-9, -6)$, it is $-72 + 66 + 6 = 0$. This last point lies on the same line as C and P.

Step 4: Conclusion The line is $8x - 11y + 6 = 0$, and it passes through $\boxed{(-9, -6)}$. Hence, the correct option is (D).

Quick Tip to Solve: After finding the centroid and intersection, write the line in integer-coefficient form; testing the four options then needs only substitution.

12. The number of real roots of the equation,
$e^{4x} + e^{3x} - 4e^{2x} + e^{x} + 1 = 0$ is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

Concept:

- Set $t = e^x$. Then $t > 0$, and the one-to-one exponential function associates exactly one real x with each positive solution t.
- The polynomial in t is reciprocal: its coefficients read the same in reverse order. Dividing by t^2 allows the substitution $u = t + 1/t \geq 2$.

Step 1: Convert to a reciprocal polynomial With $t = e^x > 0$, the equation becomes

$$t^4 + t^3 - 4t^2 + t + 1 = 0.$$

Division by t^2 is valid because an exponential is never zero. It gives

$$t^2 + \frac{1}{t^2} + t + \frac{1}{t} - 4 = 0.$$

Since $t^2 + t^{-2} = (t + t^{-1})^2 - 2$, putting $u = t + t^{-1}$ changes the equation to

$$u^2 + u - 6 = (u - 2)(u + 3) = 0.$$

Step 2: Enforce positivity before counting roots For positive t, the inequality $t + 1/t \geq 2$ follows from $(\sqrt{t} - 1/\sqrt{t})^2 \geq 0$. Thus the candidate $u = -3$ is inadmissible. The remaining value $u = 2$ gives

$$t + \frac{1}{t} = 2, \qquad t^2 - 2t + 1 = 0, \qquad (t - 1)^2 = 0.$$

Hence the only positive value of t is 1, and its corresponding real value is $x = \ln 1 = 0$.

Step 3: Distinguish multiplicity from distinct real roots An equivalent factorisation is

$$t^4 + t^3 - 4t^2 + t + 1 = (t - 1)^2(t^2 + 3t + 1).$$

The quadratic roots $(-3 \pm \sqrt{5})/2$ are both negative, so neither can equal e^x. The repeated factor $(t-1)^2$ still gives only one distinct real x. Thus the degree of the quartic does not imply four real roots of the exponential equation, and the proposed counts 2, 3, and 4 are excluded.

Step 4: Conclusion The equation has exactly one real root, $\boxed{x = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: For a reciprocal polynomial in e to the x, divide by its middle power and use t plus 1 over t, remembering that t must be positive.

13. Let the observations x_i $(1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_1 - 5) = 10$ and $\sum_{i=1}^{10}(x_1 - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3$, $x_2 - 3$, ..., $x_{10} - 3$, then the ordered pair (μ, λ) is equal to : [4 marks]

(A) (6, 6)

(B) (3, 3)

(C) (3, 6)

(D) (6, 3)

Correct Answer: The printed fixed subscript x_1 makes the two sums contradictory; the usual x_i interpretation gives option (B).

Solution:

Concept:

- For observations x_i, the mean is $\bar{x} = \frac{1}{n}\sum_i x_i$, and the population variance is $\frac{1}{n}\sum_i (x_i - a)^2 - (\bar{x} - a)^2$, where a is any fixed reference value.
- Subtracting a constant from every observation reduces the mean by that constant and leaves the variance unchanged. A summation must, however, distinguish a varying subscript from a fixed subscript.

Step 1: Check the summands as written The two displayed sums contain the fixed subscript x_1, although their summation index is i. Taken literally, the first equation gives

$$10(x_1 - 5) = 10 \quad \Longrightarrow \quad x_1 = 6.$$

The second would then give $10(6-5)^2 = 10$, contradicting the stated value 40. Thus no observations satisfy both equations in their literal form; none of the four pairs can be inferred literally.

Step 2: Evaluate the usual indexed interpretation If the summands are read as the indexed deviations $x_i - 5$, the first equation gives $\sum_i x_i = 10 + 50 = 60$, hence $\bar{x} = 6$. The shifted observations $y_i = x_i - 3$ have mean

$$\mu = \bar{x} - 3 = 3.$$

Their variance equals that of the original observations:

$$\lambda = \frac{40}{10} - (6-5)^2 = 4 - 1 = 3.$$

The pair is therefore $(3, 3)$ under that interpretation. The alternatives with mean 6 forget the translation; $(3, 6)$ uses an incorrect variance.

Step 3: Conclusion The literal fixed-subscript equations are inconsistent, so they have no valid option. Under the standard indexed-summand interpretation, $(\mu, \lambda) = (3, 3)$, corresponding to option (B).

Quick Tip to Solve: For shifted data, calculate variance from deviations about a convenient reference, then change only the mean. Check whether the summation subscript actually varies.

14. If f'(x) = $\tan^{-1}$(secx + tanx), $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and f(0) = 0, then f(1) is equal to : [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{\pi-1}{4}$
(C) $\frac{\pi+1}{4}$
(D) $\frac{\pi+2}{4}$

Correct Answer: (C) $\frac{\pi+1}{4}$

Solution:

Concept:

- The identity $\sec x + \tan x = (1+\sin x)/\cos x = \tan(\pi/4 + x/2)$ converts the derivative into a simple angle.
- The principal inverse tangent takes values in $(-\pi/2, \pi/2)$. The given interval determines whether $\tan^{-1}(\tan u) = u$ can be used without an added multiple of π.

Step 1: Check the angle and simplify the derivative Because $-\pi/2 < x < \pi/2$, the angle $u = \pi/4 + x/2$ lies strictly between 0 and $\pi/2$. It is inside the principal interval of inverse tangent. Therefore

$$f\prime(x) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}.$$

In particular the plus sign is essential: replacing it by a minus sign would give the wrong linear term in the derivative.

Step 2: Integrate and impose the initial condition An antiderivative of this linear function is

$$f(x) = \frac{\pi x}{4} + \frac{x^2}{4} + C.$$

Substitution of $x = 0$ gives $C = f(0) = 0$. Hence

$$f(1) = \frac{\pi}{4} + \frac{1}{4} = \frac{\pi + 1}{4}.$$

Equivalently, integrate the derivative from 0 to 1, which automatically includes the initial condition. Differentiating the displayed expression for f recovers $\pi/4 + x/2$, so both its coefficient and sign are checked.

Step 3: Compare the alternatives Option (A) omits the contribution $\pi/4$. Option (B) gives the wrong sign for the quadratic contribution at 1. Option (D) would require a quadratic contribution $1/2$, whereas integrating $x/2$ gives $x^2/4$. Only option (C) agrees with both the derivative and the initial value.

Step 4: Conclusion

$$\boxed{f(1) = \frac{\pi + 1}{4}}$$

Hence, the correct option is (C).

Quick Tip to Solve: Before cancelling an inverse tangent with a tangent, put the inner angle in its principal interval; then integrate the resulting linear expression.

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15. If for some α and β in R, the intersection of the following three places

$x + 4y - 2z = 1$

$x + 7y - 5z = b$

$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to : [4 marks]

(A) -10

(B) 0

(C) 10

(D) 2

Correct Answer: (C) 10

Solution:

Concept:

- Two nonparallel planes intersect in a line. A third plane contains that entire line when its equation is a linear combination of the first two equations.
- The coefficients on both sides must use the same linear combination; making only the coefficient determinant zero does not establish consistency.

Step 1: Express the third plane using the first two Denote the right-hand-side parameter written b in the second equation by β, the parameter named in the question. Let $P_3 = uP_1 + vP_2$, including their right-hand sides. Matching the coefficients of x and y gives

$$u + v = 1, \qquad 4u + 7v = 5.$$

Subtracting four times the first relation from the second gives $3v = 1$. Therefore $v = 1/3$ and $u = 2/3$. The first two normal vectors are not proportional, so those two planes already define a genuine line.

Step 2: Match the remaining coefficient and constant The coefficient of z in this same combination must equal α:

$$\alpha = -2\left(\frac{2}{3}\right) - 5\left(\frac{1}{3}\right) = -3.$$

The constant must likewise equal 5:

$$\frac{2}{3} + \frac{\beta}{3} = 5 \quad \Longrightarrow \quad \beta = 13.$$

Thus $\alpha + \beta = -3 + 13 = 10$. As a direct check, twice the first equation plus the second becomes $3x + 15y - 9z = 15$; division by 3 gives exactly $x + 5y - 3z = 5$, the required third plane.

Step 3: Check the option and the rank condition The third equation is dependent on the first two, while the first two are independent. Hence the coefficient and augmented ranks are both 2, so the intersection has one free parameter and is a line in three-dimensional space. The other proposed sums, -10, 0 and 2, disagree with the uniquely determined coefficient and constant.

Step 4: Conclusion

$$\boxed{\alpha = -3, \quad \beta = 13, \quad \alpha + \beta = 10}$$

Hence, the correct option is (C).

Quick Tip to Solve: For a common line, form a linear combination of whole plane equations, including their constants; dependence of the normals alone can also describe an inconsistent system.

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16. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to : [4 marks]

(A) 17

(B) 16

(C) 15

(D) 14

Correct Answer: (B) 16

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$ with $a > b$, the squared eccentricity is $e^2 = 1 - b^2/a^2$.
- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, the squared eccentricity is $e^2 = 1 + b^2/a^2$. The sign changes because its focal distance satisfies $c^2 = a^2 + b^2$.

Step 1: Find the squared eccentricities The larger denominator of the ellipse is 18, so its semi-major axis has $a^2 = 18$, while $b^2 = 4$. Therefore

$$e_1^2 = 1 - \frac{4}{18} = \frac{7}{9}.$$

For the hyperbola, the positive x^2 term gives $a^2 = 9$, and the magnitude of the other denominator is

$b^2 = 4$. Hence

$$e_2^2 = 1 + \frac{4}{9} = \frac{13}{9}.$$

These values meet the necessary checks: $0 < e_1 < 1$, whereas $e_2 > 1$. Using the ellipse formula for both conics would violate the latter check.

Step 2: Substitute the point into the final ellipse The point (e_1, e_2) has coordinates $x = e_1$ and $y = e_2$. Consequently

$$k = 15e_1^2 + 3e_2^2 = 15\left(\frac{7}{9}\right) + 3\left(\frac{13}{9}\right).$$

Combining with a common denominator,

$$k = \frac{105 + 39}{9} = \frac{144}{9} = 16.$$

The final ellipse uses the squared coordinates, so there is no reason to calculate the square roots e_1 and e_2 separately. This also avoids unnecessary rounding.

Step 3: Check the alternatives The exact substitution yields 16, corresponding to option (B). The proposed values 17, 15 and 14 do not satisfy the equation at the stated point. Keeping the rational squared eccentricities throughout checks the result without any approximation to the radicals.

Step 4: Conclusion

$$\boxed{k = 16}$$

Hence, the correct option is (B).

Quick Tip to Solve: When the final expression contains only squared eccentricities, retain e squared throughout and use the minus sign for an ellipse but the plus sign for a hyperbola.

17. A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle? [4 marks]

(A) 3x – 4y – 24 = 0

(B) 4x + 3y – 8 = 0

(C) 3x + 4y – 6 = 0

(D) 4x – 3y + 17 = 0

Correct Answer: (B) 4x + 3y – 8 = 0

Solution:

Concept:

- A circle tangent to the y-axis at $(0, 4)$ has centre $(h, 4)$ and radius $|h|$: the radius at contact is perpendicular to the y-axis.
- A line $ax + by + c = 0$ is tangent precisely when its perpendicular distance $|ah + bk + c|/\sqrt{a^2 + b^2}$ from the centre (h, k) equals the radius.

Step 1: Determine the centre and radius Use the point $(2,0)$ on the circle with centre $(h,4)$:

$$(2-h)^2+(0-4)^2=h^2.$$

Expanding gives $4-4h+h^2+16=h^2$, so $h=5$. Thus the centre is $(5,4)$, the radius is 5 and the equation is

$$(x-5)^2+(y-4)^2=25.$$

At $(0,4)$, the radius points horizontally, as required for contact with the vertical y-axis.

Step 2: Test every proposed line by its distance Each line has coefficient norm $\sqrt{3^2+4^2}=5$, so compare the following distances with the radius 5:

$$\begin{aligned} d_A &= \frac{|15-16-24|}{5}=5, \\ d_B &= \frac{|20+12-8|}{5}=\frac{24}{5}, \\ d_C &= \frac{|15+16-6|}{5}=5, \\ d_D &= \frac{|20-12+17|}{5}=5. \end{aligned}$$

Hence A, C and D are tangent, while B has distance less than 5 and cuts the circle at two points. The question asks for the line that is not tangent, so the inequality for B is decisive.

Step 3: Identify the requested exception The line $4x+3y-8=0$ passes 4.8 units from the centre, leaving a nonzero chord inside the circle. By contrast all three other listed lines meet the distance-equals-radius condition exactly. Selecting a line at distance 5 would answer the opposite question.

Step 4: Conclusion

$$\boxed{4x+3y-8=0}$$

is not a tangent. Hence, the correct option is (B).

Quick Tip to Solve: After fixing the centre from the contact point, evaluate each line at the centre; divide its absolute value by the normal-vector length and compare with the radius.

18. If the matrices A = $\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, B = adjA and C = 3A, then $\frac{|adjB|}{|C|}$ is equal to : [4 marks]

(A) 8

(B) 2

(C) 72

(D) 16

Correct Answer: (A) 8

Solution:

Concept:

- For an n by n matrix M, $\det(\text{adj}\, M) = (\det M)^{n-1}$. Here all matrices are 3 by 3, so the exponent is 2.
- Multiplying every entry of an n by n matrix by a scalar k multiplies its determinant by k^n. Thus $\det(3A) = 27 \det A$, not merely $3 \det A$.

Step 1: Compute the determinant of A Apply the determinant-preserving row operations $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - R_1$:

$$\det A = \begin{vmatrix}1 & 1 & 2\\ 0 & 2 & 2\\ 0 & -2 & 1\end{vmatrix}.$$

Expanding down the first column gives

$$\det A = \begin{vmatrix}2 & 2\\ -2 & 1\end{vmatrix} = 2 \cdot 1 - 2(-2) = 6.$$

Subtracting rows rather than expanding the original determinant helps keep track of the negative entry. The nonzero value also confirms that A and its adjugate are nonsingular.

Step 2: Apply the adjugate identity twice Since $B = \text{adj}\, A$, its determinant is

$$\det B = (\det A)^2 = 6^2 = 36.$$

Applying the same identity to B gives

$$\det(\text{adj}\, B) = (\det B)^2 = 36^2 = 1296.$$

Meanwhile $C = 3A$, so

$$\det C = 3^3 \det A = 27 \cdot 6 = 162.$$

The required ratio is therefore $1296/162 = 8$. One may also simplify symbolically first: the ratio is $(\det A)^4/(27 \det A) = (\det A)^3/27$.

Step 3: Check the listed values Substituting $\det A = 6$ in the simplified ratio gives $216/27 = 8$, agreeing with the direct computation. Options 2, 72 and 16 cannot result from the correct powers:

the adjugate contributes a square at each stage, while scaling a 3 by 3 matrix contributes a cube.

Step 4: Conclusion

$$\boxed{\frac{|\operatorname{adj} B|}{|C|} = 8}$$

Hence, the correct option is (A).

Quick Tip to Solve: Write the matrix order above the determinant rules: an adjugate uses exponent n minus 1, whereas scaling the entire matrix uses exponent n.

19. Negation of the statement :

$\sqrt{5}$ is an integer or 5 is an irrational is : [4 marks]

(A) $\sqrt{5}$ is not an integer and 5 is not irrational.

(B) $\sqrt{5}$ is irrational or 5 is an integer.

(C) $\sqrt{5}$ is an integer and 5 is irrational.

(D) $\sqrt{5}$ is not an integer or 5 is not irrational.

Correct Answer: (A) $\sqrt{5}$ is not an integer and 5 is not irrational.

Solution:

Concept:

- A compound statement formed with inclusive "or" is written $p \vee q$; it is false exactly when both component statements are false.
- De Morgan's law gives $\neg(p \vee q) = (\neg p) \wedge (\neg q)$. Negating the components and changing the connective are both necessary.

Step 1: Identify the component statements Let p denote "$\sqrt{5}$ is an integer" and let q denote "5 is irrational". The given sentence therefore has the logical form $p \vee q$. We seek its formal negation, so we preserve the subjects and properties in each component rather than replacing them by a different true statement.

Step 2: Negate both components and combine them The negation of p is "$\sqrt{5}$ is not an integer", and the negation of q is "5 is not irrational". A disjunction fails only when neither alternative holds. Thus these two negated components must be joined by "and":

$$\neg(p \vee q) = (\neg p) \wedge (\neg q).$$

Step 3: Check the listed alternatives Option (A) has exactly this form. Option (D) negates the components but incorrectly retains "or". Option (C) changes the connective without negating either component. Option (B) instead gives a different disjunction; the fact that its components are true for these numbers does not make it the formal negation required by the question. The original components are both false, which is consistent with their conjunction of negations being true.

Step 4: Conclusion The negation is "$\sqrt{5}$ is not an integer and 5 is not irrational". Hence, the correct option is (A).

Quick Tip to Solve: For the negation of an "or" sentence, negate each complete component and replace "or" by "and"; do not merely write another true sentence.

20. The product $2^{\frac{1}{4}}.4^{\frac{1}{16}}.8^{\frac{1}{48}}.16^{\frac{1}{128}}$ ... to ∞ is equal to : [4 marks]

(A) $2^{\frac{1}{4}}$

(B) $2^{\frac{1}{2}}$

(C) 1

(D) 2

Correct Answer: (B) $2^{\frac{1}{2}}$

Solution:

Concept:

- For positive factors, an infinite product can be evaluated from its finite partial products by writing all factors as powers of a common base.
- The exponent laws give $(a^m)^r = a^{mr}$. A convergent geometric series satisfies $\sum_{n=1}^{\infty} ar^{n-1} = a/(1-r)$ for $|r| < 1$.

Step 1: Write each factor as a power of 2 The bases are $2, 4, 8, 16, \ldots = 2^1, 2^2, 2^3, 2^4, \ldots$. The denominators of their exponents are $4, 16, 48, 128, \ldots$, following the pattern $n2^{n+1}$. Hence the nth factor is

$$(2^n)^{1/(n2^{n+1})} = 2^{1/2^{n+1}}.$$

The exponent n from the base cancels the factor n in the denominator; omitting this cancellation would give a different series.

Step 2: Add the exponents of a partial product For the first N factors let the product be P_N. Multiplication of powers with the same base gives

$$P_N = 2^{S_N}, \qquad S_N = \sum_{n=1}^{N} \frac{1}{2^{n+1}}.$$

The first exponents after simplification are $1/4, 1/8, 1/16, 1/32$, so this is a geometric series with first term $1/4$ and common ratio $1/2$. Consequently

$$S_N = \frac{1/4}{1-1/2}(1 - 2^{-N}) = \frac{1}{2}(1 - 2^{-N}).$$

Step 3: Take the limit and compare the options As $N \to \infty$, $2^{-N} \to 0$, giving $S_N \to 1/2$. Continuity of the exponential function yields $P_N \to 2^{1/2}$. Option (A) keeps only the first exponent, option (C) would require zero total exponent, and option (D) would require total exponent one. None agrees with the sum $1/2$.

Step 4: Conclusion The infinite product equals $2^{1/2} = \sqrt{2}$. Hence, the correct option is (B).

Quick Tip to Solve: Before summing exponents in an infinite product, convert every base to the same base; here that cancels the factor n in each denominator.

Mathematics – Section B: Numerical Value Questions (20 marks)

21. The number of distinct solutions of the equation

$\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x|$ **in the interval [0, 2π], is ____.** [4 marks]

Solution:

Concept:

- For a positive base unequal to one, $\log_b u + \log_b v = \log_b(uv)$, provided both arguments are positive.
- The double-angle identity $\sin 2x = 2 \sin x \cos x$ converts a product equation into one trigonometric equation. Domain restrictions must be retained.

Step 1: Impose the logarithmic domain and combine terms Both logarithms must exist, so $\sin x \neq 0$ and $\cos x \neq 0$. Moving the cosine logarithm to the left gives

$$\log_{1/2} |\sin x| + \log_{1/2} |\cos x| = 2.$$

Since both absolute values are positive on the domain, the product rule applies:

$$\log_{1/2} |\sin x \cos x| = 2.$$

Step 2: Convert to a double-angle equation The definition of logarithm now gives

$$|\sin x \cos x| = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Multiplying by two and using the double-angle identity yields

$$|\sin 2x| = \frac{1}{2}.$$

There is no reversal of an inequality here: this is an equality, so a base below one changes no sign. The nonzero product also guarantees that any solution of this transformed equation satisfies both original logarithmic domain restrictions.

Step 3: Count all distinct solutions in the stated interval Put $t = 2x$, so $t \in [0, 4\pi]$. In the first full period $[0, 2\pi]$, the four solutions of $|\sin t| = 1/2$ are

$$t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}.$$

Adding 2π gives another four solutions in the second period. No interval endpoint is a solution, since its sine is zero. Each distinct t gives one distinct $x = t/2$, with no overlap between the two groups.

Step 4: Conclusion There are $4 + 4 = \boxed{8}$ distinct solutions. This is a dimensionless count.

Quick Tip to Solve: After replacing $\sin x \cos x$ by $\sin 2x/2$, double the interval as well; here the new variable covers two full periods.

22. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is _____. [4 marks]

Solution:

Concept:

- The vector along a segment is obtained by subtracting the coordinates of its initial point from those of its final point.
- The scalar projection of $\mathbf{a}$ on a line of direction $\mathbf{b}$ is $\mathbf{a} \cdot \mathbf{b}/|\mathbf{b}|$. The length of the projection is its absolute value.

Step 1: Form the two direction vectors Let $A = (1, -1, 3)$, $B = (2, -4, 11)$, $C = (-1, 2, 3)$, and $D = (3, -2, 10)$. The segment to be projected has vector

$$\mathbf{a} = \overrightarrow{AB} = (2 - 1, -4 + 1, 11 - 3) = (1, -3, 8).$$

The direction of the receiving line is

$$\mathbf{b} = \overrightarrow{CD} = (3 + 1, -2 - 2, 10 - 3) = (4, -4, 7).$$

The projection uses this direction, not the position vector of either endpoint of the line.

Step 2: Compute the dot product and the direction magnitude The dot product is

$$\mathbf{a} \cdot \mathbf{b} = 1(4) + (-3)(-4) + 8(7) = 4 + 12 + 56 = 72.$$

The magnitude of the line direction is

$$|\mathbf{b}| = \sqrt{4^2 + (-4)^2 + 7^2} = \sqrt{16 + 16 + 49} = 9.$$

The unit vector along the line is therefore $\hat{\mathbf{b}} = (4, -4, 7)/9$. Dividing by nine is necessary because the direction vector itself does not have unit length.

Step 3: Evaluate and check the projection length The scalar projection is

$$\mathbf{a} \cdot \hat{\mathbf{b}} = \frac{72}{9} = 8.$$

It is positive for the direction C to D; reversing the direction would change its sign but leave the projected length unchanged. As a magnitude check, $|\mathbf{a}| = \sqrt{74}$, which exceeds 8, as required because a projection cannot be longer than the original segment.

Step 4: Conclusion The projection of the segment on the given line has length $\boxed{8}$ coordinate units.

Quick Tip to Solve: Use the difference of each pair of endpoints first, then divide their dot product by the magnitude of the receiving line direction.

23. The coefficient of x^4 is the expansion of $(1 + x + x^2)^{10}$ is _____. [4 marks]

Solution:

Concept:

- In expanding $(1+x+x^2)^{10}$, choose one term from each of ten factors; the exponents of the chosen terms add.
- If a factors contribute x, b factors contribute x^2, and the rest contribute 1, their contribution has exponent $a+2b$ and coefficient $10!/[a!b!(10-a-b)!]$.

Step 1: List every way to obtain the exponent four To obtain x^4, let a count selections of x and b count selections of x^2. We require

$$a + 2b = 4, \qquad a, b \geq 0.$$

The only possibilities are $(a, b) = (4, 0), (2, 1), (0, 2)$. In all three cases the remaining selections are 1. This list is exhaustive because b cannot exceed two, and each permitted value of b fixes a.

Step 2: Count the selections for each case For four x selections and no x^2, choose four of ten factors:

$$\binom{10}{4} = 210.$$

For two x selections and one x^2, first choose the factor contributing x^2, then two of the other nine:

$$10\binom{9}{2} = 10(36) = 360.$$

For two x^2 selections and no x, choose two factors:

$$\binom{10}{2} = 45.$$

No additional ordering multiplier is needed, because the choices of positions already distinguish all selections.

Step 3: Add the disjoint contributions Every selection belonging to one case has a different number of x^2 terms from a selection in either of the other cases. Thus the three counts are disjoint and can be added without double counting:

$$[x^4](1 + x + x^2)^{10} = 210 + 360 + 45 = 615.$$

Each original term has coefficient one, so there are no further numerical factors in these counts.

Step 4: Conclusion The coefficient of x^4 is $\boxed{615}$, a dimensionless number.

Quick Tip to Solve: For a low power in $(1 + x + x^2)^n$, list the nonnegative solutions of $a + 2b = r$ and count factor positions for each solution.

24. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation $(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$, then $y(3)$ is equal to _____. [4 marks]

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ has integrating factor $\mu(x) = e^{\int P(x)\,dx}$, making the left side a product derivative.
- The initial value fixes the integration constant; a direct substitution into both the equation and the initial condition checks the result.

Step 1: Put the equation in linear form Because $x \geq 0$, $x + 1 > 0$ and division by $x + 1$ is valid throughout the stated domain. Rearranging the printed equation gives

$$\frac{dy}{dx} - \frac{y}{x+1} = (x+1) - \frac{3}{x+1}.$$

Here $P(x) = -1/(x+1)$, so an integrating factor is

$$\mu(x) = e^{-\log(x+1)} = \frac{1}{x+1}.$$

The domain ensures there is no absolute-value ambiguity in this logarithm.

Step 2: Integrate the product derivative Multiplying the linear equation by the integrating factor gives

$$\frac{d}{dx}\left(\frac{y}{x+1}\right) = 1 - \frac{3}{(x+1)^2}.$$

Integration yields

$$\frac{y}{x+1} = x + \frac{3}{x+1} + C.$$

The sign of the rational term is positive: differentiating $3/(x+1)$ gives $-3/(x+1)^2$. Multiplying back by $x + 1$ gives $y = x(x+1) + 3 + C(x+1)$.

Step 3: Apply the initial condition and evaluate At $x = 2$, the condition $y(2) = 0$ gives $0 = 6 + 3 + 3C$, hence $C = -3$. Therefore

$$y = x(x+1) + 3 - 3(x+1) = x^2 - 2x.$$

It follows that $y(3) = 9 - 6 = 3$. As a check, $y(2) = 4 - 4 = 0$, and the differential equation becomes

$$(x+1)(2x-2) = (x+1)^2 + x^2 - 2x - 3 = 2x^2 - 2,$$

so the obtained function satisfies the original relation identically.

Step 4: Conclusion The required value is $y(3) = \boxed{3}$. No physical unit is specified.

Quick Tip to Solve: After finding the integrating factor $1/(x+1)$, integrate $1 - 3/(x+1)^2$ carefully: the rational antiderivative has a positive sign.

25. If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$,
$\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ **and**

$$\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}\,(a \in R)$$

are coplanar and $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$**, then the value of** λ **is _____.** [4 marks]

Solution:

Concept:

- Three vectors are coplanar precisely when their scalar triple product, the determinant formed from their components, is zero.
- For vectors $\mathbf{u}, \mathbf{v}$, $|\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2|\mathbf{v}|^2 - (\mathbf{u} \cdot \mathbf{v})^2$. This avoids expanding the cross product component by component.

Step 1: Use coplanarity to determine the parameter The component matrix has diagonal entries $a+1$ and off-diagonal entries a. Subtracting its first row from its second and third rows gives the determinant directly:

$$\det\begin{pmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{pmatrix} = 3a + 1 = 0.$$

Thus $a = -1/3$. Equivalently, the matrix is the identity plus a times the all-ones matrix; its eigenvalues are $1, 1, 1+3a$.

Step 2: Evaluate the dot product and cross-product magnitude At this value, $\mathbf{p} = (2, -1, -1)/3$, $\mathbf{q} = (-1, 2, -1)/3$, and $\mathbf{r} = (-1, -1, 2)/3$. Therefore

$$\mathbf{p} \cdot \mathbf{q} = \frac{-2-2+1}{9} = -\frac{1}{3}, \qquad |\mathbf{q}|^2 = |\mathbf{r}|^2 = \frac{2}{3}.$$

Also $\mathbf{r} \cdot \mathbf{q} = -1/3$, so

$$|\mathbf{r} \times \mathbf{q}|^2 = \frac{2}{3}\frac{2}{3} - \left(-\frac{1}{3}\right)^2 = \frac{1}{3}.$$

The cross-product magnitude is nonzero, so division by its square is legitimate.

Step 3: Substitute into the given relation The equation now reads

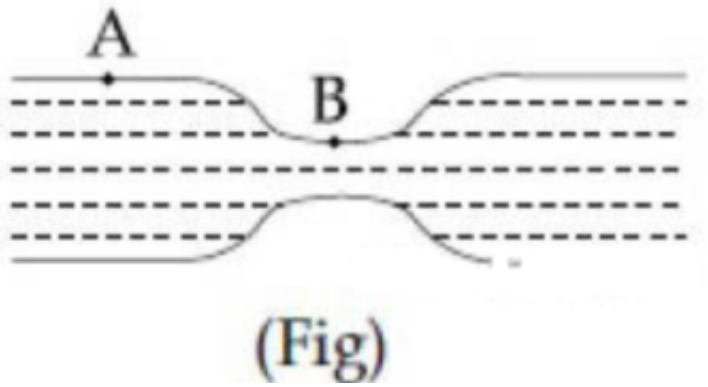
$$3\left(-\frac{1}{3}\right)^2 - \lambda\left(\frac{1}{3}\right) = 0 \quad \Longrightarrow \quad \frac{1}{3} - \frac{\lambda}{3} = 0.$$

The square on the dot product is essential: the negative dot product contributes a positive term.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{\lambda = 1}$.

Quick Tip to Solve: For three vectors with a common component pattern, write the determinant as $I + aJ$; two eigenvalues are immediately 1.

26. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm^{-3}) [4 marks]



(Fig)

(A) 1810 cm^3/s

(B) 2420 cm^3/s

(C) 3020 cm^3/s

(D) 2720 cm^3/s

Correct Answer: (D) 2720 cm^3/s

Solution:

Concept:

- For steady incompressible flow, the volume flow rate $Q = Av$ is constant along the tube.
- For a horizontal tube, Bernoulli's equation is $P + \frac{1}{2}\rho v^2 =$ constant; the gravitational terms at the two points cancel.

Step 1: Relate the two water speeds The cross-sectional area at A is twice that at B. Continuity therefore gives

$$A_A v_A = A_B v_B, \qquad 40 v_A = 20 v_B, \qquad v_B = 2 v_A.$$

The narrower portion has the larger speed, so its pressure is lower. Hence the stated pressure change is $P_A - P_B = 700\,\text{Pa}$.

Step 2: Use the pressure difference to obtain the speed Apply Bernoulli between A and B and insert $\rho = 1000\,\text{kg m}^{-3}$:

$$700 = \frac{1}{2}(1000)(v_B^2 - v_A^2) = 500(4v_A^2 - v_A^2).$$

Thus

$$v_A^2 = \frac{7}{15}\,\text{m}^2\,\text{s}^{-2}, \qquad v_A \simeq 0.683\,\text{m s}^{-1}.$$

Notice that the difference of the squares, rather than the square of the difference of speeds, occurs in Bernoulli's equation.

Step 3: Convert the flow rate and compare the options Convert the area before multiplication:

$$A_A = 40 \times 10^{-4} = 4.0 \times 10^{-3}\,\mathrm{m}^2.$$

Therefore

$$Q = A_A v_A \simeq 2.7325 \times 10^{-3}\,\mathrm{m}^3\,\mathrm{s}^{-1} = 2732.5\,\mathrm{cm}^3\,\mathrm{s}^{-1}.$$

The closest listed value is $2720\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. Options A, B and C give 1810, 2420 and 3020 respectively and do not agree with this result; their deviations are much larger than the rounding in D.

Step 4: Conclusion The calculated flow rate is approximately $2.73 \times 10^3\,\mathrm{cm}^3\,\mathrm{s}^{-1}$, represented by $2720\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Use continuity first: an area ratio of 2 makes the speed ratio 1:2, so the kinetic-pressure difference becomes $3\rho v_A^2/2$.

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27. An electric dipole of moment $\vec{p} = \left(-\hat{i} - 3\hat{j} + 2\hat{k}\right) \times 10^{-29}$ C.m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}.\vec{p} = 0$) is parallel to : [4 marks]

(A) $\left(+\hat{i} + 3\hat{j} - 2\hat{k}\right)$

(B) $\left(+\hat{i} - 3\hat{j} - 2\hat{k}\right)$

(C) $\left(-\hat{i} + 3\hat{j} - 2\hat{k}\right)$

(D) $\left(-\hat{i} - 3\hat{j} + 2\hat{k}\right)$

Correct Answer: (A) $\left(+\hat{i} + 3\hat{j} - 2\hat{k}\right)$

Solution:

Concept:

- The field of an ideal dipole at a point away from the origin is $\mathbf{E} = (4\pi\varepsilon_0 r^3)^{-1}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}]$.
- When the position vector is perpendicular to the dipole moment, the point is in the equatorial plane. There the field is opposite to the dipole moment.

Step 1: Check the equatorial condition from the components The dipole moment has components proportional to $(-1, -3, 2)$, while the position vector has components $(1, 3, 5)$. Their dot product is

$$\mathbf{p}\cdot\mathbf{r} = (-1 - 9 + 10) \times 10^{-29} = 0.$$

Both vectors are nonzero, so this equality confirms that their directions are perpendicular. The common factor $10^{-29}\,\mathrm{C\,m}$ changes the field magnitude but cannot change its direction.

Step 2: Simplify the dipole-field expression Since $\mathbf{p}\cdot\hat{\mathbf{r}} = 0$, the radial projection term vanishes

exactly:

$$\mathbf{E} = -\frac{\mathbf{p}}{4\pi\varepsilon_0 r^3}.$$

The denominator is positive because $r = \sqrt{35}$ is a positive distance. Thus the electric field has the direction of $-\mathbf{p}$, namely

$$\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}.$$

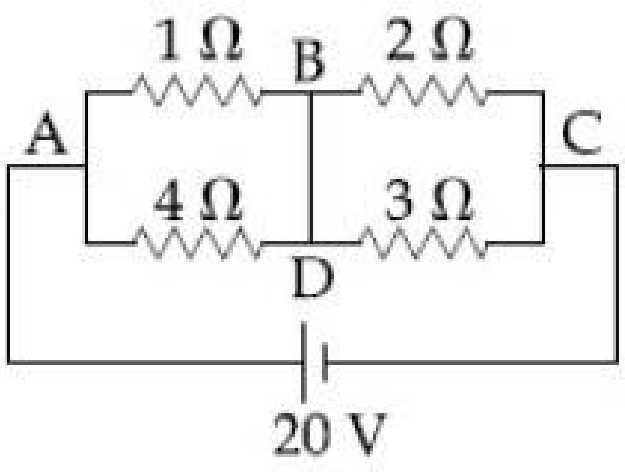
No separate evaluation of the numerical magnitude is needed when only a parallel direction is asked for.

Step 3: Compare all the proposed directions Option A has the required component ratio $1 : 3 : -2$. Option B changes the sign of the second component, and option C changes the sign of the first component, so neither is a scalar multiple in the field direction. Option D is the original dipole-moment vector and points opposite to the equatorial field. The minus sign in the equatorial-field formula therefore decides the choice.

Step 4: Conclusion The field is directed along $\boxed{\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}$. Hence, the correct option is (A).

Quick Tip to Solve: First calculate $\mathbf{p} \cdot \mathbf{r}$. If it vanishes, the dipole field points along $-\mathbf{p}$ without further vector algebra.

28. In the given circuit diagram, a wire is joining points B and D. The current in this wire is : [4 marks]



(A) 4 A

(B) 2 A

(C) zero

(D) 0.4 A

Correct Answer: (B) 2 A

Solution:

Concept:

- An ideal wire has zero resistance, so its ends B and D have the same electric potential. Equal potential does not imply zero current through that wire.
- Combine resistors sharing the same two nodes in parallel, then apply Kirchhoff's junction law at B to find the current in the joining wire.

Step 1: Reduce the resistors using the common node With B and D shorted, call their common potential node X. The $1\,\Omega$ and $4\,\Omega$ resistors are in parallel between A and X; the $2\,\Omega$ and $3\,\Omega$ resistors are in parallel between X and C. Therefore

$$R_{AX} = \frac{1 \times 4}{1+4} = 0.8\,\Omega, \qquad R_{XC} = \frac{2 \times 3}{2+3} = 1.2\,\Omega.$$

These combinations are in series across the 20 V source, giving total resistance $2\,\Omega$ and total current $10\,\text{A}$.

Step 2: Calculate the currents entering and leaving B The voltage drops across the two combinations are

$$V_{AX} = 10(0.8) = 8\,\text{V}, \qquad V_{XC} = 10(1.2) = 12\,\text{V}.$$

Thus the current from A to B through the $1\,\Omega$ resistor is $8/1 = 8\,\text{A}$, while the current from B to C through the $2\,\Omega$ resistor is $12/2 = 6\,\text{A}$. The positive terminal is at the left of the source as drawn, so these directions are consistent with the figure.

Step 3: Apply current conservation to the joining wire At B, the 8 A arriving from A divides into 6 A towards C and the remainder towards D:

$$I_{BD} = 8 - 6 = 2\,\text{A}.$$

At D, the independent check is $8/4 + 2 = 12/3$, or $2 + 2 = 4\,\text{A}$. Hence the junction laws agree. The bridge is not balanced because $1/2 \neq 4/3$; zero current is therefore incorrect. The values 4 A and 0.4 A also fail the B-node current balance.

Step 4: Conclusion The current in the joining wire is $\boxed{2\,\text{A}}$, flowing from B to D. Hence, the correct option is (B).

Quick Tip to Solve: When a bridge is shorted, merge the joined nodes, find the four resistor currents, and subtract the two currents at either joining point.

29. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$.

They collide completely inelastically. The energy lost in the process is : [4 marks]

(A) $\frac{1}{3}mu^2$

(B) $\frac{1}{8}mu^2$

(C) $\frac{3}{4}mu^2$

(D) $\sqrt{\frac{2}{3}}mu^2$

Correct Answer: (B) $\frac{1}{8}mu^2$

Solution:

Concept:

- In a completely inelastic collision the particles stick together. Total vector momentum is conserved, but kinetic energy is generally reduced.
- For equal masses, the final velocity is the arithmetic mean of the initial velocity vectors. Kinetic energy uses the squared magnitude of velocity, including all perpendicular components.

Step 1: Find the common final velocity The velocities before collision are $\mathbf{u}_1 = u\hat{\mathbf{i}}$ and $\mathbf{u}_2 = (u/2)\hat{\mathbf{i}} + (u/2)\hat{\mathbf{j}}$. The combined mass after sticking is $2m$. Momentum conservation gives

$$2m\mathbf{v} = m\mathbf{u}_1 + m\mathbf{u}_2, \qquad \mathbf{v} = \frac{3u}{4}\hat{\mathbf{i}} + \frac{u}{4}\hat{\mathbf{j}}.$$

Both the horizontal and vertical momentum equations are satisfied by this velocity.

Step 2: Evaluate the kinetic energies before and after collision The second particle has speed squared $u^2/4 + u^2/4 = u^2/2$, rather than u^2. Consequently

$$K_i = \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u^2}{2}\right) = \frac{3}{4}mu^2.$$

The common final speed obeys $v^2 = (9+1)u^2/16 = 5u^2/8$. Therefore

$$K_f = \frac{1}{2}(2m)\frac{5u^2}{8} = \frac{5}{8}mu^2.$$

The reduction is physically expected since relative motion is eliminated when the particles stick.

Step 3: Take the difference and test the choices The energy converted into internal energy is

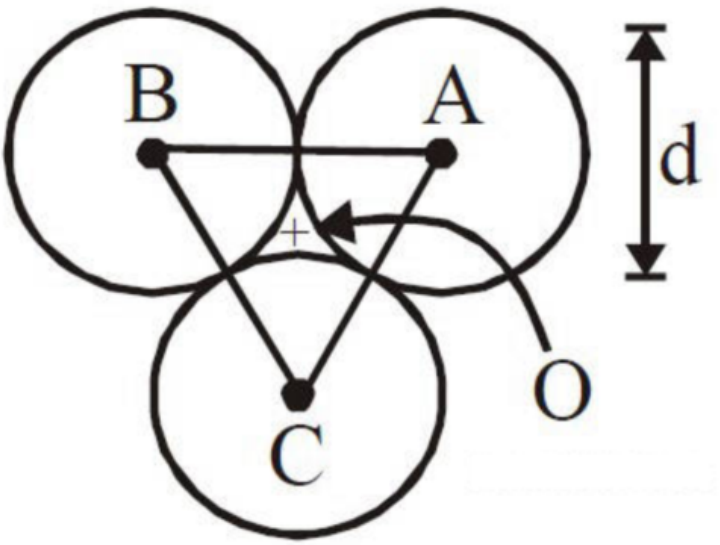
$$\Delta K = K_i - K_f = \left(\frac{3}{4} - \frac{5}{8}\right)mu^2 = \frac{1}{8}mu^2.$$

Option A gives $mu^2/3$, which is too large. Option C equals the entire initial kinetic energy and would wrongly require the final system to be at rest despite its nonzero momentum. Option D exceeds the initial kinetic energy and is impossible for this isolated sticking collision.

Step 4: Conclusion The energy lost is $\boxed{\frac{1}{8}mu^2}$. Hence, the correct option is (B).

Quick Tip to Solve: For two equal masses that stick, the energy loss is $m|\mathbf{u}_1 - \mathbf{u}_2|^2/4$; use the vector relative velocity.

30. Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is : [4 marks]



(A) $\frac{13}{23}$
(B) $\frac{23}{13}$
(C) $\frac{15}{13}$
(D) $\frac{13}{15}$

Correct Answer: (A) $\frac{13}{23}$

Solution:

Concept:

- A solid sphere of radius $R = d/2$ has moment of inertia $I_{\text{cm}} = 2mR^2/5 = md^2/10$ about any diameter.
- The parallel-axis theorem adds $m\ell^2$ when an axis is displaced by perpendicular distance ℓ from a sphere's centre. Add the contributions of all three spheres.

Step 1: Locate the centroid axis relative to the centres The sphere centres form an equilateral triangle of side d. Its altitude is $\sqrt{3}d/2$, and the centroid lies two-thirds of the way from a vertex to the opposite side. Each centre is therefore at distance

$$\ell = \frac{2}{3}\frac{\sqrt{3}d}{2} = \frac{d}{\sqrt{3}}$$

from the perpendicular centroid axis. The physical spheres have radius $d/2$; this radius must not be confused with the side d between their centres.

Step 2: Add the three moments about the centroid Each sphere contributes its own diameter moment

plus the displacement term. Thus

$$I_0 = 3\left(\frac{md^2}{10} + m\frac{d^2}{3}\right) = \left(\frac{3}{10} + 1\right) md^2 = \frac{13}{10} md^2.$$

Keeping the diameter moments is necessary; treating the spheres as point masses would miss $3md^2/10$ and give a different ratio.

Step 3: Shift to the perpendicular axis through a sphere centre The system has total mass $3m$, and its centre of mass is the triangle centroid by symmetry. The new parallel axis through A is a distance $d/\sqrt{3}$ away. Hence

$$I_A = I_0 + (3m)\left(\frac{d}{\sqrt{3}}\right)^2 = \frac{23}{10} md^2, \qquad \frac{I_0}{I_A} = \frac{13}{23}.$$

Option B reverses the ratio and exceeds one, although the centroid axis must have the smaller inertia. Options C and D do not include the correct parallel-axis increase of md^2.

Step 4: Conclusion The required ratio is $\boxed{I_0/I_A = 13/23}$. Hence, the correct option is (A).

Quick Tip to Solve: Calculate the centroid moment once, then shift the whole assembly using its total mass $3m$ and the centroid-to-vertex distance $d/\sqrt{3}$.

31. Three harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to : [4 marks]

(A) 5.8 I_0

(B) 3 I_0

(C) 0.2 I_0

(D) I_0

Correct Answer: (A) 5.8 I_0

Solution:

Concept:

- For coherent waves of the same frequency, add their displacements or complex amplitudes before calculating intensity. The intensity is proportional to the square of the resultant amplitude.
- If each wave has amplitude a, its intensity is $I_0 = Ka^2$, with the same positive proportionality constant K for all three waves.

Step 1: Add the two symmetric phase contributions Take the phase of the first wave as the reference. The amplitude phasors of the other two waves make angles $+\pi/4$ and $-\pi/4$. Their perpendicular components cancel, while their components along the reference direction add. Thus their combined amplitude is $2a\cos(\pi/4) = \sqrt{2}a$, in phase with the first wave. This cancellation concerns the transverse phasor components, not the waves themselves.

Step 2: Include the reference wave and square the amplitude The first wave contributes another

amplitude a in the same direction. Consequently,

$$a_{\text{res}} = a + \sqrt{2}a = (1+\sqrt{2})a.$$

The resultant intensity is therefore

$$I = Ka_{\text{res}}^2 = (1+\sqrt{2})^2 I_0 = (3+2\sqrt{2})I_0.$$

Using $\sqrt{2} \simeq 1.414$, we obtain $I \simeq 5.828 I_0$. Simply adding the three individual intensities would give $3I_0$, but that neglects interference. The values $0.2I_0$ and I_0 would require a much stronger cancellation than the given phases allow.

Step 3: Conclusion The intensity is $(3+2\sqrt{2})I_0 \simeq 5.8I_0$. Hence, the correct option is (A).

Quick Tip to Solve: For equal amplitudes at phases $0, +\theta, -\theta$, use $I/I_0 = (1+2\cos\theta)^2$; symmetric phasors cancel only perpendicular components.

32. A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$ follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, $v\hat{i}$ and $-2v\hat{j}$. Then which of the following statements (A, B, C, D) are the correct ?

(Trajectory shown is schematic and not to scale) :

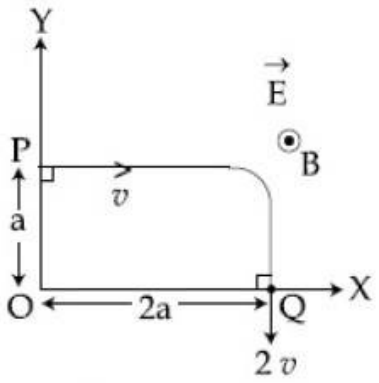
(A) E = $\frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(B) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(C) Rate of work done by both the fields at Q is zero

(D) The difference between the magnitude of angular momentum of the particle at P and Q is 2mav.

[4 marks]



(A) (A), (B), (C), (D)

(B) (A), (B), (C)

(C) (A), (C), (D)

(D) (B), (C), (D)

Correct Answer: (B) (A), (B), (C)

Solution:

Concept:

- The magnetic force $q\mathbf{v} \times \mathbf{B}$ is perpendicular to velocity and does no work. Only the electric force changes the kinetic energy.
- Instantaneous electric power is $q\mathbf{E} \cdot \mathbf{v}$. Angular momentum about the shown origin is $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$.

Step 1: Use the displacement to test statement A The figure gives $P = (0, a)$ and $Q = (2a, 0)$. Since the electric field points along $+x$, its work between these points is $qE(2a)$. The speeds are v and $2v$, so the work-energy theorem gives

$$2qaE = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{3}{2}mv^2.$$

Thus $E = 3mv^2/(4qa)$, exactly as stated in (A). No magnetic-field contribution is included in this energy equation.

Step 2: Evaluate the instantaneous powers At P, velocity is parallel to the electric field, giving

$$\mathcal{P}_P = qEv = \frac{3mv^3}{4a}.$$

Statement (B) is therefore true. At Q, velocity points along $-y$, perpendicular to the electric field, so electric power is zero. Magnetic power is zero at every point. Hence the rate of work done by the two fields together at Q is zero, and (C) is true.

Step 3: Compare the angular momentum magnitudes Using the origin O shown in the diagram,

$$\mathbf{L}_P = (a\hat{\mathbf{j}}) \times (mv\hat{\mathbf{i}}) = -mav\hat{\mathbf{k}},$$

$$\mathbf{L}_Q = (2a\hat{\mathbf{i}}) \times (-2mv\hat{\mathbf{j}}) = -4mav\hat{\mathbf{k}}.$$

The difference of their magnitudes is $4mav - mav = 3mav$, rather than $2mav$. Therefore (D) is false. Every option containing statement (D) is excluded, leaving the combination (A), (B), (C).

Step 4: Conclusion The first three statements are correct, whereas the fourth statement is false. Hence, the correct option is (B).

Quick Tip to Solve: Use energy to find E, the dot product to find power, and the perpendicular distance from O to the momentum line to find each angular momentum.

33. A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be : [4 marks]

(A) $\frac{h}{\sqrt{2}}$

(B) $\frac{h}{3\sqrt{2}}$

(C) $\frac{3}{4}h\sqrt{2}$

(D) $\frac{h}{2(\sqrt{2}+1)}$

Correct Answer: (C) $\frac{3}{4}h\sqrt{2}$

Solution:

Concept:

- For near-normal viewing from air through parallel layers, a layer of real thickness t and refractive index μ contributes an apparent thickness t/μ.
- The apparent depth below the upper surface is the sum of these reduced thicknesses. Refraction must be accounted for in each liquid separately.

Step 1: Identify the two equal real thicknesses The total vessel depth is $2h$, and each liquid occupies half of it. Thus each layer has thickness h. The lower liquid has index $2\sqrt{2}$, whereas the upper liquid has index $\sqrt{2}$. The observer views the bottom through both layers from air, so neither layer can be omitted. The liquids being immiscible ensures that their interface is well defined.

Step 2: Add the apparent thickness contributions For a paraxial ray emerging into air at a small angle θ, Snell's law gives the ray angle θ/μ inside a layer of index μ. Its lateral travel across that layer is approximately $h\theta/\mu$. Summing the lateral travels and extending the emerging ray backwards gives

$$d_{\text{app}} = \frac{h}{2\sqrt{2}} + \frac{h}{\sqrt{2}} = \frac{3h}{2\sqrt{2}}.$$

Rationalising the denominator,

$$d_{\text{app}} = \frac{3\sqrt{2}}{4}h.$$

This is smaller than the real depth $2h$, as expected when looking from air into optically denser liquids.

Step 3: Check the alternatives against both layers The expression $h/\sqrt{2}$ accounts only for the upper layer and therefore misses a positive contribution from the lower layer. The values $h/(3\sqrt{2})$ and $h/[2(\sqrt{2}+1)]$ are even smaller than the apparent thickness of the upper layer alone. Neither can represent the combined apparent depth. Option (C) is the sum obtained from the two interfaces.

Step 4: Conclusion The apparent depth is $\frac{3\sqrt{2}}{4}h$. Hence, the correct option is (C).

Quick Tip to Solve: For several transparent parallel layers viewed from air, add t_i/μ_i; do not average the refractive indices first.

34. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B, respectively is : [4 marks]

(A) 7 : 9

(B) 5 : 7

(C) 3 : 5

(D) 5 : 9

Correct Answer: (B) 5 : 7

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom, the internal energy per mole is $U_m = fRT/2$, giving molar heat capacity $C_V = fR/2$.
- A rigid diatomic molecule has three translational and two rotational degrees of freedom. One active vibrational mode adds two quadratic energy terms: kinetic and potential.

Step 1: Count the active terms for gas A Gas A is rigid, so its bond does not vibrate. It has three independent translational directions and two rotations perpendicular to the molecular axis. Rotation about the bond axis is not counted in the usual diatomic ideal-gas model. Therefore

$$f_A = 3 + 2 = 5, \qquad U_m^A = \frac{5}{2}RT, \qquad C_V^A = \frac{5}{2}R.$$

These heat capacities are the molar constant-volume quantities denoted by C_V in the options.

Step 2: Include vibration in gas B Gas B has the same translational and rotational contributions, plus one active vibration of the bond. A vibrational mode contains both kinetic and potential energy; each receives $RT/2$ per mole. Therefore that mode contributes R to the molar heat capacity, rather than only $R/2$. Hence

$$f_B = 3 + 2 + 2 = 7, \qquad C_V^B = \frac{7}{2}R.$$

The different molecular mass affects speeds and mass-specific heat capacities, but it does not enter the equipartition result per mole.

Step 3: Form the required ratio and compare options Taking the ratio in the stated order,

$$\frac{C_V^A}{C_V^B} = \frac{5R/2}{7R/2} = \frac{5}{7}.$$

The ratio $3 : 5$ would correspond to monatomic versus rigid diatomic heat capacities, which is not this case. The ratios $7 : 9$ and $5 : 9$ require more active energy terms than the single additional vibrational mode provides. The mass $m/4$ is not a reason to change the number of active degrees of freedom.

Step 4: Conclusion The molar heat capacities are in the ratio $C_V^A : C_V^B = 5 : 7$. Hence, the correct option is (B).

Quick Tip to Solve: Count one kinetic and one potential term for each active vibrational mode; for a diatomic gas, one such mode changes C_V from $5R/2$ to $7R/2$.

35. The electric fields of two plane electromagnetic plane waves in vacuum are given by $\overrightarrow{E_1} = E_0\hat{j}\cos(\omega t - kx)$ and

$$\overrightarrow{E_2} = E_0\hat{k}\cos(\omega t - ky)$$

At t = 0, a particle of charge q is at origin with a velocity $\overrightarrow{v} = 0.8c\hat{j}$ (c is the speed of light in vacuum). The instantaneous force experienced by the particle is : [4 marks]

(A) $E_0q\left(0.8\hat{i} - \hat{j} + 0.4\hat{k}\right)$

(B) $E_0q\left(-0.8\hat{i} + \hat{j} + \hat{k}\right)$

(C) $E_0q\left(0.8\hat{i} + \hat{j} + 0.2\hat{k}\right)$

(D) $E_0q\left(0.4\hat{i} - 3\hat{j} + 0.8\hat{k}\right)$

Correct Answer: (C) $E_0q\left(0.8\hat{i} + \hat{j} + 0.2\hat{k}\right)$

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, $\mathbf{B} = (\hat{\mathbf{n}} \times \mathbf{E})/c$, where $\hat{\mathbf{n}}$ is its propagation direction.
- The instantaneous force on a charge is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Add the electric and magnetic fields of both waves before applying this expression.

Step 1: Find the field vectors at the origin At $t = 0$ and the origin, both cosine factors are 1. Thus

$$\mathbf{E} = E_0(\hat{\mathbf{j}} + \hat{\mathbf{k}}).$$

The phase $\omega t - kx$ identifies propagation along $+x$; the other wave propagates along $+y$. Their magnetic fields are

$$\mathbf{B}_1 = \frac{E_0}{c}(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = \frac{E_0}{c}\hat{\mathbf{k}},$$

$$\mathbf{B}_2 = \frac{E_0}{c}(\hat{\mathbf{j}} \times \hat{\mathbf{k}}) = \frac{E_0}{c}\hat{\mathbf{i}}.$$

These signs ensure that $\mathbf{E} \times \mathbf{B}$ points along each wave's propagation direction.

Step 2: Calculate the magnetic part of the force Using the given instantaneous velocity $\mathbf{v} = 0.8c\hat{\mathbf{j}}$,

$$\mathbf{v} \times \mathbf{B} = 0.8c\hat{\mathbf{j}} \times \frac{E_0}{c}(\hat{\mathbf{k}} + \hat{\mathbf{i}})$$
$$= 0.8E_0(\hat{\mathbf{i}} - \hat{\mathbf{k}}).$$

Here $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$. The speed of light cancels, leaving the same field units as $\mathbf{E}$.

Step 3: Combine electric and magnetic forces Adding the electric contribution gives

$$\mathbf{F} = qE_0\left[\hat{\mathbf{j}} + \hat{\mathbf{k}} + 0.8\hat{\mathbf{i}} - 0.8\hat{\mathbf{k}}\right] = qE_0(0.8\hat{\mathbf{i}} + \hat{\mathbf{j}} + 0.2\hat{\mathbf{k}}).$$

Options (A) and (D) have a negative y component, although the magnetic force has no y component

and the electric y component is positive. Option (B) has the wrong x sign and fails to include the reduction of the z component.

Step 4: Conclusion The instantaneous force is $qE_0(0.8\hat{\mathbf{i}} + \hat{\mathbf{j}} + 0.2\hat{\mathbf{k}})$. Hence, the correct option is (C).

Quick Tip to Solve: Determine propagation from the phase first; then use $\mathbf{B} = \hat{\mathbf{n}} \times \mathbf{E}/c$ to avoid reversing the magnetic field.

---

36. Radiation, with wavelength 6561 $\overset{o}{A}$ falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to : [4 marks]

(A) 1.8eV

(B) 0.8eV

(C) 1.1eV

(D) 1.6eV

Correct Answer: (C) 1.1eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the metal's work function.
- For an electron moving perpendicular to a uniform magnetic field, $p = eBr$ and $K = p^2/(2m_e)$. The largest orbit radius corresponds to the largest kinetic energy.

Step 1: Convert the data and find the photon energy The wavelength is $6561\,\text{Å} = 656.1\,\text{nm}$. The magnetic field and largest radius are $B = 3 \times 10^{-4}\,\text{T}$ and $r = 10\,\text{mm} = 1.0 \times 10^{-2}\,\text{m}$. Using $hc \simeq 1240\,\text{eV nm}$,

$$E_\gamma = \frac{1240}{656.1}\,\text{eV} \simeq 1.890\,\text{eV}.$$

The wavelength must be expressed in nanometres for this numerical form of hc.

Step 2: Obtain the maximum electron kinetic energy The magnetic force provides centripetal acceleration, so $evB = m_e v^2/r$. Thus the electron momentum is $p = eBr$, and

$$K_{\max} = \frac{e^2 B^2 r^2}{2m_e}.$$

Substituting $e = 1.602 \times 10^{-19}\,\text{C}$ and $m_e = 9.11 \times 10^{-31}\,\text{kg}$,

$$K_{\max} = \frac{(1.602 \times 10^{-19})^2 (3 \times 10^{-4})^2 (10^{-2})^2}{2(9.11 \times 10^{-31})} \simeq 1.268 \times 10^{-19}\,\text{J}.$$

Dividing by $1.602 \times 10^{-19}\,\text{J/eV}$ gives $K_{\max} \simeq 0.792\,\text{eV}$. This energy is far below the electron rest energy, so the nonrelativistic expression is appropriate.

Step 3: Subtract the kinetic energy from the photon energy The incident photon supplies both the

escape energy and the outgoing kinetic energy. Therefore

$$\phi = E_\gamma - K_{\max} \simeq 1.890 - 0.792 = 1.098\,\text{eV}.$$

The option $0.8\,\text{eV}$ is close to the electron kinetic energy, not the work function. The values $1.8\,\text{eV}$ and $1.6\,\text{eV}$ would leave too little kinetic energy to produce the stated largest radius.

Step 4: Conclusion The work function is approximately $1.1\,\text{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate $e^2B^2r^2/(2m_e)$ in joules, convert it to electron-volts, and subtract it from hc/λ; the orbit energy is not the work function.

---

37. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is added to its energy, the wavelength become $\lambda/2$. Value of ΔE, is : [4 marks]

(A) E

(B) 3E

(C) 2E

(D) 4E

Correct Answer: (B) 3E

Solution:

Concept:

- The de Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is momentum.
- For a nonrelativistic particle of fixed mass m, its kinetic energy is $K = p^2/(2m)$. Hence wavelength varies inversely as the square root of kinetic energy.

Step 1: Relate the two momenta The initial momentum is $p = h/\lambda$. The final wavelength is half the initial value, so the final momentum must be twice as large:

$$p_f = \frac{h}{\lambda/2} = \frac{2h}{\lambda} = 2p.$$

The mass is unchanged when energy is supplied, so the same kinetic-energy relation applies before and after the addition.

Step 2: Calculate the final kinetic energy Squaring the momentum ratio gives the energy ratio:

$$\frac{K_f}{E} = \frac{p_f^2/(2m)}{p^2/(2m)} = \left(\frac{p_f}{p}\right)^2 = 4.$$

Thus the final kinetic energy is $4E$. This is the total final energy of motion, not the extra energy that was added.

Step 3: Subtract the original energy By the meaning of the added energy, $K_f = E + \Delta E$. Therefore

$$\Delta E = 4E - E = 3E.$$

Adding only E or $2E$ would give final energies $2E$ or $3E$, so neither halves the wavelength. Adding $4E$ would instead give $5E$; that option confuses the final energy with the increment.

Step 4: Conclusion The required added energy is $\boxed{\Delta E = 3E}$. Hence, the correct option is (B).

Quick Tip to Solve: First find the final kinetic energy from $K \propto 1/\lambda^2$, then subtract the initial energy to obtain the energy added.

38. The aperture diameter of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 $\overset{o}{A}$, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to : [4 marks]

(A) 20 m

(B) 200 m

(C) 600 m

(D) 60 m

Correct Answer: (D) 60 m

Solution:

Concept:

- For a circular aperture of diameter D, the Rayleigh criterion gives the minimum angular separation $\theta_{\min} = 1.22\lambda/D$, in radians.
- For objects at the same distant surface, a small angular separation corresponds to a transverse distance $s = L\theta$, where L is their distance from the telescope.

Step 1: Convert every length to SI units The aperture diameter is $D = 5\,\text{m}$. The wavelength and Earth–Moon distance become

$$\lambda = 5500 \times 10^{-10}\,\text{m} = 5.5 \times 10^{-7}\,\text{m},$$

$$L = 4 \times 10^5\,\text{km} = 4 \times 10^8\,\text{m}.$$

The aperture is specified by its diameter; using half of it would incorrectly double the answer.

Step 2: Find the limiting angular separation Substitute the wavelength and aperture diameter into the circular-aperture criterion:

$$\theta_{\min} = \frac{1.22(5.5 \times 10^{-7})}{5} = 1.342 \times 10^{-7}\,\text{rad}.$$

This very small angle justifies the small-angle approximation $s/L \simeq \theta$.

Step 3: Convert the angle to a lunar separation The corresponding surface separation is

$$s = (4 \times 10^8)(1.342 \times 10^{-7}) = 53.68\,\text{m}.$$

Among the given approximate choices, $60\,\text{m}$ is closest. The alternatives $20\,\text{m}$, $200\,\text{m}$ and $600\,\text{m}$ differ substantially from this diffraction-limited value and are rejected.

Step 4: Conclusion The minimum separation is about $\boxed{54\,\mathrm{m}}$, closest to $60\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $s = 1.22\lambda L/D$ directly after converting angstroms and kilometres into metres; the aperture quantity here is the diameter.

---

39. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of : [4 marks]

(A) Energy

(B) Momentum

(C) Area

(D) Volume

Correct Answer: (A) Energy

Solution:

Concept:

- Planck's constant has dimensions of action: $[h] = ML^2T^{-1}$. The speed of light has dimensions $[c] = LT^{-1}$.
- From the gravitational force law $F = Gm_1m_2/r^2$, the gravitational constant has dimensions $[G] = M^{-1}L^3T^{-2}$.

Step 1: Find the dimension inside the radical The expression contains the fifth power of speed, not the fifth power of length alone. Thus

$$[c^5] = L^5T^{-5}, \qquad [hc^5] = (ML^2T^{-1})(L^5T^{-5}) = ML^7T^{-6}.$$

Multiplication adds the powers of each independent base dimension.

Step 2: Divide by the dimension of the gravitational constant Division subtracts the denominator exponents. In particular the inverse mass power of G increases the numerator mass power:

$$\left[\frac{hc^5}{G}\right] = \frac{ML^7T^{-6}}{M^{-1}L^3T^{-2}} = M^2L^4T^{-4}.$$

This is the dimension of the full radicand, before taking the square root.

Step 3: Take the square root and compare The square root halves every exponent, giving

$$[f] = (M^2L^4T^{-4})^{1/2} = ML^2T^{-2}.$$

Energy has precisely these dimensions, as follows from work being force times distance. Momentum instead has dimension MLT^{-1}, area has L^2, and volume has L^3; none matches the calculated mass, length and time powers together.

Step 4: Conclusion The quantity f has the dimensions of $\boxed{\text{energy}}$. Hence, the correct option is (A).

Quick Tip to Solve: Carry the square root through every dimensional exponent only after completing the product and quotient inside it.

---

40. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is : [4 marks]

(A) 0.001 mm

(B) 0.01 cm

(C) 0.02 mm

(D) 0.001 cm

Correct Answer: (D) 0.001 cm

Solution:

Concept:

- The pitch of a screw gauge is the axial distance moved by the screw in one complete rotation.
- The least count is the pitch divided by the number of equal divisions on the circular scale. It is the displacement corresponding to one circular-scale division.

Step 1: Determine the pitch from the rotations Six complete rotations advance the screw through $3\,\text{mm}$. Since each rotation produces the same axial displacement,

$$\text{pitch} = \frac{3\,\text{mm}}{6} = 0.5\,\text{mm}.$$

The given $3\,\text{mm}$ is the motion for all six turns, so it must not be treated as the pitch of one turn.

Step 2: Divide the pitch by the scale divisions The circular scale has 50 divisions for one complete revolution. The axial motion associated with advancing by one division is therefore

$$\text{LC} = \frac{0.5\,\text{mm}}{50} = 0.01\,\text{mm}.$$

As a direct check, 50 such least-count intervals total $0.5\,\text{mm}$, and six revolutions total the specified $3\,\text{mm}$.

Step 3: Express the result in the offered unit Because $1\,\text{cm} = 10\,\text{mm}$, the value in centimetres is

$$0.01\,\text{mm} = \frac{0.01}{10}\,\text{cm} = 0.001\,\text{cm}.$$

Option (A) is ten times smaller than the calculated value. Option (B), $0.01\,\text{cm} = 0.1\,\text{mm}$, is ten times larger; option (C), $0.02\,\text{mm}$, is twice the correct least count.

Step 4: Conclusion The least count is $\boxed{0.01\,\text{mm} = 0.001\,\text{cm}}$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate pitch per single revolution first, then divide by circular divisions; convert units only when comparing with the choices.

41. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body : [4 marks]

(A) starts moving in an elliptical orbit around the planet.

(B) Falls vertically downwards towards the planet

(C) Escapes from the Planet's Gravitational field.

(D) continues to move in a circular orbit

Correct Answer: (A) starts moving in an elliptical orbit around the planet.

Solution:

Concept:

- The initial circular orbital speed at radius R is $v = \sqrt{GM/R}$, with M the planet's mass.
- During the short completely inelastic collision, conserve the combined bodies' linear momentum; the bodies stick together. Afterward, the planet's central gravitational force conserves orbital energy and angular momentum.

Step 1: Conserve momentum along the tangent The second body has mass $m/2$ and velocity $\vec{v}/2$, in the same direction as A. If the combined velocity is $\vec{u}$, momentum conservation gives

$$m\vec{v} + \frac{m}{2}\frac{\vec{v}}{2} = \left(m + \frac{m}{2}\right)\vec{u}.$$

Consequently $\vec{u} = (5/6)\vec{v}$. The new velocity remains tangential but is smaller than the circular speed.

Step 2: Check whether the orbit is bound The specific mechanical energy after impact is

$$\varepsilon = \frac{u^2}{2} - \frac{GM}{R} = \frac{25}{72}\frac{GM}{R} - \frac{GM}{R} = -\frac{47GM}{72R} < 0.$$

Negative mechanical energy rules out escape. The tangential speed is nonzero, so the specific angular momentum Ru is also nonzero; the body does not begin a purely radial fall.

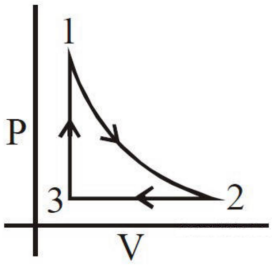
Step 3: Identify the new orbit A circular orbit at the collision radius would require speed $\sqrt{GM/R} = v$, independent of the satellite mass. Since $u = 5v/6 \neq v$, the orbit cannot remain circular. A bound Kepler orbit with nonzero angular momentum and a speed different from the local circular value is elliptical. The impact point is its apocentre because the radial speed is zero and the tangential speed is sub-circular.

Step 4: Conclusion The combined body starts moving in an $\boxed{\text{elliptical orbit}}$ around the planet. Hence, the correct option is (A).

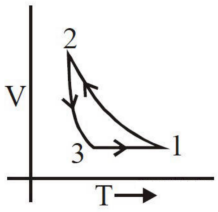
Quick Tip to Solve: After an orbital collision, compare the new tangential speed with the local circular and escape speeds; the satellite's mass does not set those speeds.

42. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure ? where, $1 \rightarrow 2$ is adiabatic.

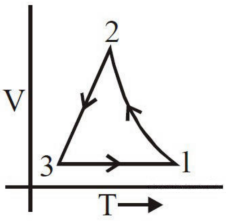
(Graphs are schematic and are not to scale) [4 marks]



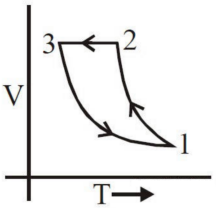
(A)



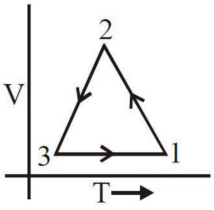
(B)



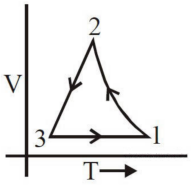
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nRT$. An isobar therefore obeys $V \propto T$, while an isochore has constant volume.
- For an adiabatic process, $PV^{\gamma} =$ constant, equivalently $TV^{\gamma-1} =$ constant, where $\gamma > 1$. During adiabatic expansion, volume increases and temperature decreases.

Step 1: Translate the adiabatic expansion In the supplied P-V cycle, the path $1 \to 2$ is an adiabatic expansion: the volume rises while the pressure falls. In the V-T plane, $TV^{\gamma-1} =$ constant is a curved path with negative slope. Thus point 2 must lie above and to the left of point 1, with the arrow directed from 1 to 2.

Step 2: Translate the constant-pressure compression The horizontal path $2 \to 3$ on the pressure–volume diagram has constant pressure. Applying the ideal-gas equation gives

$$V = \frac{nR}{P}T.$$

In a plot with volume vertical and temperature horizontal, this is a straight line of positive slope, whose extension passes through the origin. Compression reduces both volume and temperature, so the arrow goes down and left from 2 to 3.

Step 3: Translate the constant-volume heating and compare The vertical path $3 \to 1$ on the original graph is at constant volume. It becomes a horizontal rightward line on the V-T graph as temperature rises. Option (B) alone has all three features together: curved adiabatic $1 \to 2$, straight rising isobar $2 \to 3$, and horizontal isochore $3 \to 1$. Option (A) curves the isobar, (C) assigns equal volumes to 2 and 3, and (D) makes the adiabatic path straight.

Step 4: Conclusion The equivalent cycle is the V-T diagram in option (B). Hence, the correct option is (B).

Quick Tip to Solve: Translate each process separately: constant volume is horizontal on a V-T plot, constant pressure is a straight line through the origin, and an adiabat is curved.

43. A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and $2a$, respectively from the axis of the wire is : [4 marks]

(A) 2

(B) $\frac{1}{2}$

(C) $\frac{3}{2}$

(D) $\frac{2}{3}$

Correct Answer: (D) $\frac{2}{3}$

Solution:

Concept:

- A circular Amperian loop concentric with the wire gives $B(2\pi r) = \mu_0 I_{\text{enclosed}}$, because cylindrical symmetry makes the magnetic field tangential and constant on that loop.
- For uniform current density in a wire of radius a, the current inside radius $r < a$ is Ir^2/a^2. Outside the wire the enclosed current is the whole current I.

Step 1: Find the field inside the wire The point at $r = a/3$ lies inside the conducting material. It is therefore incorrect to insert this distance in the external line-current formula. The enclosed current is proportional to the circular cross-sectional area, giving

$$I_{\text{enclosed}} = I\frac{(a/3)^2}{a^2} = \frac{I}{9}.$$

Using Ampere's law on the loop of circumference $2\pi a/3$,

$$B_{a/3} = \frac{\mu_0(I/9)}{2\pi(a/3)} = \frac{\mu_0 I}{6\pi a}.$$

Step 2: Find the external field and form the ratio The point at $r = 2a$ is outside the wire, so its Amperian loop encloses all of I. Thus

$$B_{2a} = \frac{\mu_0 I}{2\pi(2a)} = \frac{\mu_0 I}{4\pi a}.$$

The order requested is the inner field divided by the outer field:

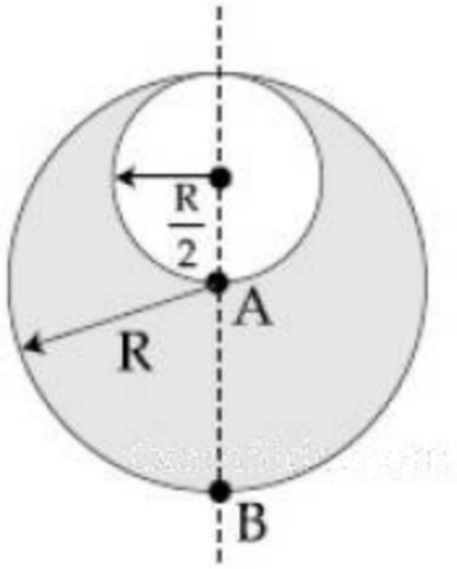
$$\frac{B_{a/3}}{B_{2a}} = \frac{\mu_0 I/(6\pi a)}{\mu_0 I/(4\pi a)} = \frac{4}{6} = \frac{2}{3}.$$

The common current, permeability and wire radius cancel. The value $3/2$ reverses the requested order; 2 and $1/2$ do not follow from the correctly enclosed currents.

Step 3: Conclusion The magnetic-field ratio is $\boxed{2/3}$, a dimensionless quantity. Hence, the correct option is (D).

Quick Tip to Solve: Use the inside-wire formula $B \propto r$ before the surface and the outside formula $B \propto 1/r$ after it; the enclosed current is not the same in both regions.

44. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\overrightarrow{E_A}|}{|\overrightarrow{E_B}|}$ of magnitude of electric field $\overrightarrow{E_A}$ and $\overrightarrow{E_B}$, respectively, at points A and B due to the remaining portion is : [4 marks]



(A) $\frac{17}{54}$
(B) $\frac{18}{54}$
(C) $\frac{18}{34}$
(D) $\frac{21}{34}$

Correct Answer: (C) $\frac{18}{34}$

Solution:

Concept:

- A uniformly charged solid sphere produces an internal field $\vec{E} = \rho\vec{r}/(3\varepsilon_0)$, where $\vec{r}$ is measured from its centre; outside it acts as its total charge concentrated at the centre.
- A cavity can be represented by superposing the complete sphere of density ρ with a smaller sphere of density $-\rho$ occupying the removed volume. Add fields as vectors before taking magnitudes.

Step 1: Read the geometry and evaluate the field at A In the figure A is the centre of the original sphere. Let C be the centre of the cavity, above A by $R/2$. The smaller sphere has radius $R/2$, touches the top surface internally and has A on its lower surface. The complete sphere gives zero field at A. The removed positive sphere would give a downward field there, so subtracting it leaves an upward field of magnitude

$$|E_A| = \frac{\rho(R/2)}{3\varepsilon_0} = \frac{\rho R}{6\varepsilon_0}.$$

For negative charge density both directions reverse together and the magnitude ratio is unchanged.

Step 2: Subtract the fields at B and simplify B is the bottom surface point of the original sphere, so $AB = R$ and $CB = 3R/2$. The full sphere gives a downward field of magnitude $\rho R/(3\varepsilon_0)$. The

positive charge removed is

$$Q_c = \frac{4\pi}{3}\rho\left(\frac{R}{2}\right)^3 = \frac{\pi\rho R^3}{6}.$$

Its field at B is downward with magnitude

$$E_c(B) = \frac{Q_c}{4\pi\varepsilon_0(3R/2)^2} = \frac{\rho R}{54\varepsilon_0}.$$

Therefore the remaining material gives

$$|E_B| = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17\rho R}{54\varepsilon_0}.$$

Consequently,

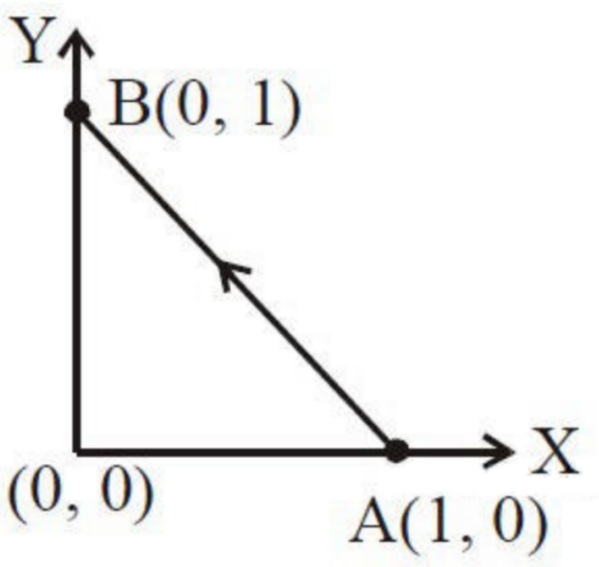
$$\frac{|E_A|}{|E_B|} = \frac{1/6}{17/54} = \frac{9}{17} = \frac{18}{34}.$$

Neither $17/54$, $18/54$, nor $21/34$ equals this dimensionless ratio; the first is merely the coefficient in the field at B.

Step 3: Conclusion The required ratio is $\boxed{18/34}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep the two centres separate: B is a distance R from the large-sphere centre but $3R/2$ from the cavity centre. This distinction controls the subtracted field.

45. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is : (all quantities are in SI units) [4 marks]



(A) 2

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

Correct Answer: (C) 1

Solution:

Concept:

- The work done by a force along a specified path is the line integral $W = \int_A^B \vec{F} \cdot d\vec{r}$, where $d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$.
- For $\vec{F} = -x\hat{i} + y\hat{j}$, the work element is $-x\,dx + y\,dy$. Its two terms are exact differentials, providing an independent endpoint check on the path integral.

Step 1: Parameterise the directed line segment The line from A(1,0) to B(0,1) satisfies $x + y = 1$. Choose x as the parameter, with x decreasing from 1 to 0. Then

$$y = 1 - x, \qquad dy = -dx.$$

Substitution in the scalar product gives

$$\vec{F} \cdot d\vec{r} = -x\,dx + (1 - x)(-dx) = -dx.$$

Both components do positive work on this motion: the x component of force and the x displacement are both negative, while the y component and the y displacement are both positive. Thus the answer must have positive sign.

Step 2: Integrate and verify from the endpoints The directed bounds must correspond to A followed by B, rather than the reverse:

$$W = \int_1^0 (-1)\,dx = -[x]_1^0 = 1\,\text{J}.$$

A second way is to integrate the two exact differential terms separately:

$$W = \left[-\frac{x^2}{2} + \frac{y^2}{2}\right]_{(1,0)}^{(0,1)} = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1\,\text{J}.$$

Each Cartesian contribution is $1/2$ J, so option (B) includes only one contribution. Options (A) and (D), giving 2 and $3/2$, respectively, disagree with both integrations. The exact differential also confirms that the stated straight path has been handled consistently.

Step 3: Conclusion The work done from A to B is $\boxed{1\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write $\vec{F} \cdot d\vec{r}$ before integrating. On $x + y = 1$, substitute both $y = 1 - x$ and $dy = -dx$, and preserve the directed bounds.

Physics – Section B: Numerical Value Questions (20 marks)

46. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be _____. [4 marks]

Solution:

Concept:

- An inductor opposes a change of current with induced emf $\mathcal{E} = -L\,dI/dt$. Here L is the self-inductance and the minus sign states Lenz's law.
- When the current changes uniformly, its rate is constant, so the emf magnitude is $|\mathcal{E}| = L|\Delta I|/\Delta t$. Use seconds before calculating an inductance in henry.

Step 1: Convert the time and identify the current change The reverse voltage has magnitude $100\,\mathrm{V}$; it opposes the decrease of current rather than implying a negative self-inductance. The current falls from $0.25\,\mathrm{A}$ to zero, giving

$$\Delta I = 0 - 0.25 = -0.25\,\mathrm{A}, \qquad |\Delta I| = 0.25\,\mathrm{A}.$$

The short duration is given in milliseconds, not seconds:

$$\Delta t = 0.025\,\mathrm{ms} = 0.025 \times 10^{-3}\,\mathrm{s} = 2.5 \times 10^{-5}\,\mathrm{s}.$$

The uniform rate permits the finite current change divided by its duration to be used directly, without any approximation to a varying slope.

Step 2: Apply the emf law and express the result in mH The magnitude of the current-change rate is

$$\left|\frac{\Delta I}{\Delta t}\right| = \frac{0.25}{2.5 \times 10^{-5}} = 1.0 \times 10^{4}\,\mathrm{A\,s^{-1}}.$$

Rearranging the induction relation gives

$$L = \frac{|\mathcal{E}|\Delta t}{|\Delta I|} = \frac{100(2.5 \times 10^{-5})}{0.25} = 1.0 \times 10^{-2}\,\mathrm{H}.$$

Since $1\,\mathrm{H} = 10^{3}\,\mathrm{mH}$,

$$L = 10\,\mathrm{mH}.$$

As a dimensional check, voltage divided by current-change rate has units $\mathrm{V\,s/A}$, which are henry. Substitution back gives $(0.010)(10^{4}) = 100\,\mathrm{V}$, reproducing the stated voltage.

Step 3: Conclusion The self-inductance of the choke is $\boxed{10\,\mathrm{mH}}$, so the required numerical entry is $\boxed{10}$.

Quick Tip to Solve: Convert milliseconds to seconds first. Use the magnitude of the current change to find positive L; the negative sign in the emf law describes opposition to that change.

47. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is :
(Breaking stress of wire = 4.8×10^7 Nm^{-2} and
area of cross-section of the wire = 10^{-2} cm^2) is: [4 marks]

Solution:

Concept:

- Breaking stress σ_b is the largest tensile force per cross-sectional area before a wire breaks. Thus the maximum permitted tension is $T_{\max} = \sigma_b A$.
- For a mass m rotating at angular speed ω on radius r, the required centripetal force is $m\omega^2 r$. In the space-station setting the wire tension supplies this force without an added mg term.

Step 1: Find the largest safe tension The cross-sectional area is stated in square centimetres, whereas the breaking stress is in $\mathrm{N\,m^{-2}}$. Convert the area using $1\,\mathrm{cm^2} = 10^{-4}\,\mathrm{m^2}$:

$$A = 10^{-2}\,\mathrm{cm^2} = 10^{-2} \times 10^{-4}\,\mathrm{m^2} = 10^{-6}\,\mathrm{m^2}.$$

The limiting wire tension is therefore

$$T_{\max} = (4.8 \times 10^7)(10^{-6}) = 48\,\mathrm{N}.$$

The wire length is the radius of the mass's circular motion, $r = 0.3\,\mathrm{m}$. The limiting rotation occurs when the tensile stress just reaches the stated breaking value.

Step 2: Equate tension with the centripetal requirement At the limiting angular speed,

$$m\omega_{\max}^2 r = T_{\max}.$$

Substitute the mass $10\,\mathrm{kg}$, radius $0.3\,\mathrm{m}$ and maximum tension $48\,\mathrm{N}$:

$$\omega_{\max}^2 = \frac{48}{10(0.3)} = \frac{48}{3} = 16\,\mathrm{s^{-2}}.$$

Choose the positive square root because angular speed is a magnitude:

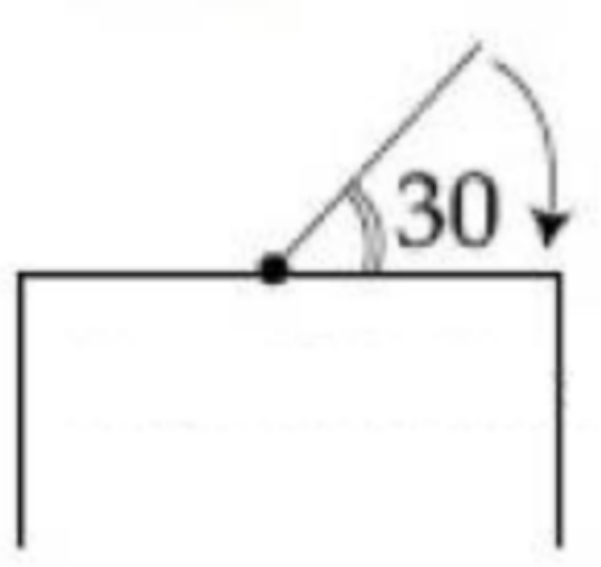
$$\omega_{\max} = 4\,\mathrm{rad\,s^{-1}}.$$

A check using this speed gives $10 \times 4^2 \times 0.3 = 48\,\mathrm{N}$, exactly the breaking tension. Any greater angular speed would demand a greater tension because the required force varies as ω^2, so the wire would exceed its breaking stress.

Step 3: Conclusion The maximum angular speed is $\boxed{4\,\mathrm{rad\,s^{-1}}}$. The required numerical entry is $\boxed{4}$.

Quick Tip to Solve: For a rotating mass on a wire use $\omega_{\max} = \sqrt{\sigma_b A/(mr)}$. Converting an area from $\mathrm{cm^2}$ to $\mathrm{m^2}$ contributes a factor 10^{-4}.

48. One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}$ s^{-1}, where n is an integer. The value of n is _____. [4 marks]



Solution:

Concept:

- For a rigid uniform rod of mass M and length l rotating about a fixed end, the moment of inertia is $I = \frac{1}{3}Ml^2$ and its rotational kinetic energy is $\frac{1}{2}I\omega^2$.
- The centre of mass lies at the midpoint of the rod. During the fall, loss of gravitational potential energy of that centre becomes rotational kinetic energy, because the fixed pivot does no work.

Step 1: Find the vertical fall of the centre of mass Initially the rod makes $30°$ with the horizontal. Its free end is elevated, but the gravitational potential energy must be calculated from the midpoint rather than from that free end. Relative to the final horizontal position, the centre of mass falls by

$$h = \frac{l}{2}\sin 30° = \frac{l}{4}.$$

For $l = 1\,\text{m}$, this is $h = 0.25\,\text{m}$. The rod is released from rest, so its initial kinetic energy is zero. Just before it touches the table, the decrease of gravitational potential energy is $Mgl/4$.

Step 2: Convert the lost potential energy to rotation For rotation about the fixed end, energy conservation up to the instant immediately before impact gives

$$Mg\frac{l}{4} = \frac{1}{2}\left(\frac{1}{3}Ml^2\right)\omega^2.$$

Cancel the mass and one factor of length to obtain

$$\omega^2 = \frac{3g}{2l}.$$

Using the conventional JEE estimate $g = 10\,\mathrm{m\,s^{-2}}$ with $l = 1\,\mathrm{m}$,

$$\omega^2 = \frac{3(10)}{2(1)} = 15\,\mathrm{s^{-2}}, \qquad \omega = \sqrt{15}\,\mathrm{s^{-1}}.$$

The angular-speed magnitude is positive. Comparing with the form $\sqrt{n}\,\mathrm{s^{-1}}$, the coefficient inside the square root is 15. Using the end's height instead of the midpoint height would double the available energy; using the moment of inertia about the midpoint would also be incorrect because the pivot is at an end.

Step 3: Conclusion The bar hits the table with angular speed $\sqrt{15}\,\mathrm{rad\,s^{-1}}$. Hence the dimensionless integer is $\boxed{n = 15}$.

Quick Tip to Solve: Use the centre-of-mass drop $(l/2)\sin\theta$, and take the moment of inertia about the actual fixed end. For a uniform rod this gives $\omega^2 = 3g\sin\theta/l$.

---

49. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is _____ [4 marks]

Solution:

Concept:

- Velocity is the first time derivative, $v = dx/dt$, and acceleration is $a_x = dv/dt$. The symbol a in the given displacement equation is a constant, not the instantaneous acceleration.
- The constants a, b, c do not depend on time. Differentiating the given squared displacement twice exposes the dependence of acceleration on position.

Step 1: Differentiate the displacement relation Start with the printed relation and differentiate both sides with respect to time:

$$x^2 = at^2 + 2bt + c, \qquad 2x\frac{dx}{dt} = 2at + 2b.$$

Thus $xv = at + b$, or $v = (at + b)/x$. This expression includes the changing denominator, so treating the velocity as a linear function of time would be incorrect.

Step 2: Eliminate time from the acceleration Differentiating $xv = at + b$ again gives

$$v^2 + xa_x = a.$$

Also, multiply the original relation by a and compare with $(at + b)^2$:

$$(at + b)^2 = ax^2 + b^2 - ac.$$

Therefore

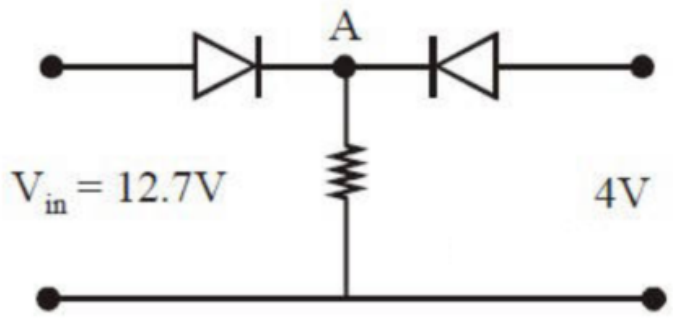
$$v^2 = a + \frac{b^2 - ac}{x^2}, \qquad a_x = \frac{a - v^2}{x} = \frac{ac - b^2}{x^3}.$$

The numerator is constant. Hence the position dependence is an inverse cube, on any interval where the displacement and derivatives are defined. The sign of $ac - b^2$ determines the acceleration's direction, but it does not change the exponent. For the stated nonzero position-dependent acceleration, the degenerate case $ac = b^2$ is excluded.

Step 3: Conclusion The acceleration varies as x^{-3}. Comparing with x^{-n} gives $\boxed{n = 3}$, a dimensionless integer.

Quick Tip to Solve: Differentiate the squared-displacement equation implicitly; the product rule gives $v^2 + xa_x = a$ without taking a square root.

50. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is _____. [4 marks]



Solution:

Concept:

- A forward-biased conducting diode has the stated built-in drop $0.7\,\text{V}$, while its additional resistance is negligible. A reverse-biased diode is an open circuit.
- The common cathode node is driven by the higher input through its conducting diode. The resistor provides the return path to the lower reference line.

Step 1: Test the diode connected to the higher input In the circuit, the left diode has its anode at $12.7\,\text{V}$ and its cathode at A. With the pull-down resistor alone, A would be at the reference potential, so this diode is forward biased. In its conducting state, the anode-to-cathode drop is the given $0.7\,\text{V}$. Hence

$$12.7 - V_A = 0.7 \quad \Rightarrow \quad V_A = 12.0\,\text{V}.$$

Negligible forward resistance does not mean we may discard the separately specified built-in potential. It means there is no extra current-dependent voltage drop to include.

Step 2: Check the other diode for consistency The right diode also has its cathode at A, while its

anode is connected to the $4\,\mathrm{V}$ input. For the proposed node voltage,

$$V_{\text{anode}} - V_{\text{cathode}} = 4 - 12 = -8\,\mathrm{V}.$$

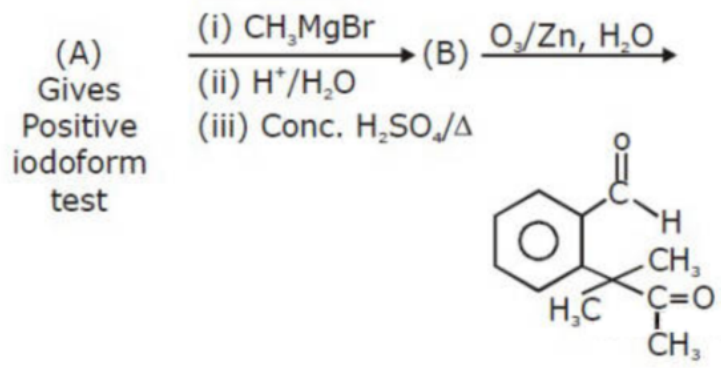
It is reverse biased, so it cannot pull the node down to the lower input. The left input therefore supplies current through the left diode and resistor to the reference line, with no conducting path into the right input. Both diode states agree with the calculated voltage. Choosing $3.3\,\mathrm{V}$ from the lower input would forward bias the left diode strongly, so that alternative state is inconsistent.

Step 3: Conclusion The voltage at A, measured relative to the lower line, is $\boxed{12\,\mathrm{V}}$.

Quick Tip to Solve: For common-cathode diode selectors, first try the largest input minus the stated diode drop, then check that every other diode is reverse biased.

Chemistry – Section A: Multiple Choice Questions (80 marks)

51. Identify (A) in the following reaction sequence : [4 marks]



(A) $C_6H_5CH{=}C(CH_3)COCH_3$

(B) $C_6H_5C(CH_3){=}C(CH_3)COCH_3$

(C) $C_6H_5CH{=}CHCOCH_3$

(D) $C_6H_5C(CH_3){=}CHCOCH_3$

Correct Answer: (A) $C_6H_5CH{=}C(CH_3)COCH_3$

Solution:

Concept:

- A methyl ketone, $RCOCH_3$, gives a positive iodoform test. Methylmagnesium bromide adds a methyl group to the carbonyl carbon; acidic work-up produces an alcohol.
- An allylic tertiary alcohol can form a carbocation in hot concentrated H_2SO_4. Intramolecular aromatic attack can close a five-membered ring, and reductive ozonolysis then cleaves its alkene.

Step 1: Follow the candidate in option A Option A is $C_6H_5CH{=}C(CH_3)COCH_3$. It contains the required methyl ketone. Carbonyl addition of CH_3MgBr, followed by H^+/H_2O, gives the allylic tertiary alcohol

$$C_6H_5CH{=}C(CH_3)C(OH)(CH_3)_2.$$

On protonation and loss of water, the carbon bearing the two methyl groups becomes electrophilic. The ortho position of the benzene ring attacks this centre intramolecularly; loss of an aromatic proton restores the benzene ring and closes a five-membered ring. The resulting indene has a gem-dimethyl saturated carbon and a methyl substituent on one alkene carbon.

Step 2: Trace both carbonyls in the final product Reductive ozonolysis breaks the double bond of that five-membered ring. The alkene carbon directly attached to the aromatic ring carries hydrogen, so it becomes the ortho aldehyde group $-\mathrm{CHO}$. The other alkene carbon carries methyl, so it becomes $-\mathrm{COCH_3}$, still bonded to the gem-dimethyl carbon. This gives exactly the pictured adjacent substituents: $-\mathrm{CHO}$ and $-\mathrm{C(CH_3)_2COCH_3}$.

Option B has a methyl instead of hydrogen on the aromatic-side alkene carbon, so it would yield an acetyl group there rather than an aldehyde. Option C lacks the methyl on the other alkene carbon and would form an aldehyde there. Option D has both of those mismatches. Thus the substitution pattern, as well as the iodoform test, selects only A.

Step 3: Conclusion The required starting compound is $\mathrm{C_6H_5CH{=}C(CH_3)COCH_3}$. Hence, the correct option is (A).

Quick Tip to Solve: Work backward from ozonolysis: an aldehyde requires a hydrogen on its original alkene carbon, whereas a methyl ketone requires a methyl substituent.

52. B has a smaller first ionization enthalpy than Be. Consider the following statements :

(I) It is easier to remove 2p electron than 2s electron

(II) 2p electron of B is more shielded from the nucleus by the inner core of electrons than the 2s electrons of Be.

(III) 2s electron has more penetration power than 2p electron.

(IV) atomic radius of B is more than Be (Atomic number B = 5, Be = 4)

The correct statements are : [4 marks]

(A) (I), (III) and (IV)

(B) (II), (III) and (IV)

(C) (I), (II) and (IV)

(D) (I), (II) and (III)

Correct Answer: (D) (I), (II) and (III)

Solution:

Concept:

- First ionization enthalpy is the energy needed to remove the outermost electron from an isolated gaseous atom. Its value depends on nuclear attraction, shielding, subshell energy and electronic configuration.
- For orbitals of the same principal shell, an s electron penetrates nearer the nucleus more effectively than a p electron. Greater penetration reduces shielding and strengthens attraction to the nucleus.

Step 1: Compare the electrons removed from Be and B The ground-state configurations are

$$\text{Be} : 1s^2\,2s^2, \qquad \text{B} : 1s^2\,2s^2\,2p^1.$$

The first electron removed from beryllium is a $2s$ electron, whereas boron loses its single $2p$ electron. The $2p$ orbital has less penetration than the $2s$ orbital and is more effectively shielded from nuclear attraction. In addition, Be has a filled $2s$ subshell. These features make boron's outer electron easier to remove despite its larger nuclear charge. Therefore statement I is true.

Step 2: Evaluate shielding, penetration and atomic size Statement II is true: the less penetrating $2p$ electron experiences stronger shielding by the inner core than the more penetrating $2s$ electron. Statement III is precisely the penetration ordering $2s > 2p$, so it is also true. These are linked explanations of the observed ionization-enthalpy exception.

Statement IV is false. Across this part of Period 2, the nuclear charge increases while electrons enter the same principal shell; the effective nuclear attraction increases and the atomic radius decreases. Boron therefore has a smaller atomic radius than beryllium, not a larger one. Each of options A, B and C includes statement IV, so all three are eliminated. Only option D contains the three true statements without IV.

Step 3: Conclusion The correct statements are (I), (II) and (III). Hence, the correct option is (D).

Quick Tip to Solve: For the Be–B ionization exception, compare the subshells being ionized: removing a higher-energy, less penetrating $2p$ electron is easier than removing a $2s$ electron.

53. Which of these will produce the highest yield in Friedel Crafts reaction? [4 marks]

(A) Chlorobenzene, C_6H_5Cl

(B) Aniline, $C_6H_5NH_2$

(C) Phenol, C_6H_5OH

(D) Benzamide, $C_6H_5CONH_2$

Correct Answer: (C) Phenol, C_6H_5OH

Solution:

Concept:

- Friedel–Crafts reactions are electrophilic aromatic substitutions using a Lewis-acid catalyst such as $AlCl_3$. Ring activation favours substitution, but strong coordination to the catalyst can suppress it.
- Electron-donating groups increase aromatic electron density by resonance, while electron-withdrawing groups reduce it. A substituent must be assessed together with its interaction with the reaction conditions.

Step 1: Identify the substituents and their effects The four substrates are chlorobenzene (A), aniline (B), phenol (C) and benzamide (D). Chlorine has a strong electron-withdrawing inductive effect. Although it can donate by resonance and direct substitution to ortho and para positions, chlorobenzene is deactivated overall and is a poor substrate for a Friedel–Crafts reaction.

The $-CONH_2$ group of benzamide is attached to the ring through its carbonyl carbon. It withdraws

electron density by resonance and induction, deactivating the aromatic ring strongly. It therefore does not give the highest yield.

Step 2: Compare the two nominally activating groups The $-NH_2$ group of aniline is normally a strong resonance donor. Under Friedel–Crafts conditions, however, its nitrogen lone pair coordinates strongly to the Lewis acid, for example $AlCl_3$. This prevents normal lone-pair donation and makes aniline unsuitable for ordinary Friedel–Crafts substitution. It is therefore incorrect to choose B merely by ranking free amino and hydroxyl groups as ring activators.

The hydroxyl group of phenol supplies electron density to the aromatic ring by resonance, especially at the ortho and para positions. Among these four listed substrates it offers the favourable activated ring for electrophilic substitution and gives the highest yield. Thus the deactivated chloro and amide substrates and the catalyst-complexing amino substrate are all excluded in the comparison.

Step 3: Conclusion Phenol, bearing $-OH$, gives the highest yield among the listed substrates. Hence, the correct option is (C).

Quick Tip to Solve: In Friedel–Crafts comparisons, check catalyst complexation before selecting aniline solely because $-NH_2$ is a strong ring activator.

54. For the following reactions

$$A \xrightarrow{700K} \text{Product}$$

$$A \xrightarrow[catalyst]{500K} \text{Product}$$

it was found that E_a is decreased by 30 kJ/mol in the presence of catalyst.

If the rate remains unchanged, the activation energy for catalysed reaction is (Assume pre exponential factor is same): [4 marks]

(A) 198 kJ/mol

(B) 135 kJ/mol

(C) 105 kJ/mol

(D) 75 kJ/mol

Correct Answer: (D) 75 kJ/mol

Solution:

Concept:

- The Arrhenius equation is $k = A_0 \exp[-E_a/(RT)]$, where A_0 is the pre-exponential factor, E_a the activation energy, R the gas constant and T the absolute temperature.
- For equal rate constants and the same pre-exponential factor, the dimensionless exponential factors are equal. Thus the ratio E_a/T is the same in both cases.

Step 1: Express the two activation energies Let E_c be the activation energy of the catalysed reaction, expressed in kJ mol^{-1}. Since the catalyst decreases the barrier by $30\,\text{kJ mol}^{-1}$, the uncatalysed barrier

is $E_c + 30$. The temperatures must be paired with the appropriate barriers: $700\,\mathrm{K}$ for the uncatalysed route and $500\,\mathrm{K}$ for the catalysed route.

With the same initial reaction conditions and unchanged rate, equate the rate constants. The common pre-exponential factor cancels:

$$\exp\left[-\frac{E_c + 30}{R(700)}\right] = \exp\left[-\frac{E_c}{R(500)}\right].$$

Use consistent energy units for R and the barriers; the gas constant cancels in the next step.

Step 2: Solve and check the barrier Taking logarithms gives

$$\frac{E_c + 30}{700} = \frac{E_c}{500}.$$

Consequently,

$$500E_c + 15000 = 700E_c, \qquad E_c = 75\,\mathrm{kJ\,mol^{-1}}.$$

The uncatalysed barrier is therefore $105\,\mathrm{kJ\,mol^{-1}}$. As a consistency check, $105/700 = 75/500 = 0.15$ in the same energy units, so the Arrhenius exponent is unchanged. Option C, 105, is the uncatalysed value rather than the requested one. Options A and B do not satisfy the equal-ratio equation and are ruled out.

Step 3: Conclusion The activation energy for the catalysed reaction is $\boxed{75\,\mathrm{kJ\,mol^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: With equal Arrhenius prefactors and equal rates, equate E_a/T; first label the catalysed and uncatalysed barriers to avoid reporting the wrong one.

55. The K_{sp} for the following dissociation is 1.6×10^{-5}

$$PbCl_{2(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2Cl^{-}_{(aq)}$$

Which of the following choices is correct for a mixture of 300 mL 0.134 M $Pb(NO_3)_2$ and 100 mL 0.4 M NaCl ? [4 marks]

(A) $Q > K_{sp}$

(B) Not enough data provided

(C) $Q < K_{sp}$

(D) $Q = K_{sp}$

Correct Answer: (A) $Q > K_{sp}$

Solution:

Concept:

- The ionic product of lead(II) chloride is $Q = [\mathrm{Pb^{2+}}][\mathrm{Cl^{-}}]^2$, calculated using the concentrations immediately after mixing, before precipitation.

- Precipitation is favourable when $Q > K_{sp}$; dilution on mixing must be included for both ions. Dissolved nitrate and sodium ions do not enter this expression.

Step 1: Calculate the diluted ion concentrations The total volume is $300 + 100 = 400\,\text{mL} = 0.400\,\text{L}$. The lead salt supplies one lead ion per formula unit, while sodium chloride supplies one chloride ion. Thus

$$[\text{Pb}^{2+}] = \frac{0.300(0.134)}{0.400} = 0.1005\,\text{mol L}^{-1}, \qquad [\text{Cl}^{-}] = \frac{0.100(0.4)}{0.400} = 0.100\,\text{mol L}^{-1}.$$

These are the initial concentrations in the mixture, rather than the stock-solution concentrations.

Step 2: Compare the ionic product with the solubility product Substitute both concentrations, remembering the square on chloride:

$$Q = 0.1005(0.100)^2 = 1.005 \times 10^{-3}.$$

Consequently $Q/K_{sp} = (1.005 \times 10^{-3})/(1.6 \times 10^{-5}) \approx 62.8 > 1$. The mixture is supersaturated and lead(II) chloride precipitates. The supplied concentrations, volumes and solubility product are sufficient, so option (B) is excluded. Options (C) and (D) would describe an initially unsaturated or exactly saturated mixture, respectively; neither fits this calculation. After equilibrium is established with solid present the ionic product becomes equal to the solubility product, but that is not the initial mixing test.

Step 3: Conclusion The initial ionic product is $1.005 \times 10^{-3} > 1.6 \times 10^{-5}$, so $Q > K_{sp}$. Hence, the correct option is (A).

Quick Tip to Solve: First dilute each ion to the combined volume, then apply its stoichiometric exponent in the ionic product.

56. The de Broglie wavelength of an electron in the 4th Bohr orbit is : [4 marks]

(A) $2\pi a_0$

(B) $6\pi a_0$

(C) $8\pi a_0$

(D) $4\pi a_0$

Correct Answer: (C) $8\pi a_0$

Solution:

Concept:

- An electron forms a standing matter wave in an allowed Bohr orbit: $2\pi r_n = n\lambda_n$, where n is the principal quantum number.
- For the hydrogen Bohr model, $r_n = n^2 a_0$, where a_0 is the first-orbit radius. Combining these relations gives $\lambda_n = 2\pi n a_0$.

Step 1: Relate wavelength to orbital circumference The matter wavelength is $\lambda = h/(m_e v)$. Bohr quantisation gives $m_e v r_n = n\hbar = nh/(2\pi)$. Dividing the latter equation by the momentum therefore

gives

$$\lambda_n = \frac{h}{m_e v} = \frac{2\pi r_n}{n}.$$

The entire circumference contains n wavelengths; the wavelength does not equal the circumference except in the first orbit. This distinction matters because the radius itself increases quadratically with the orbit number.

Step 2: Insert the fourth-orbit radius and compare the options In the fourth orbit, $n = 4$ and $r_4 = 4^2 a_0 = 16a_0$. Therefore

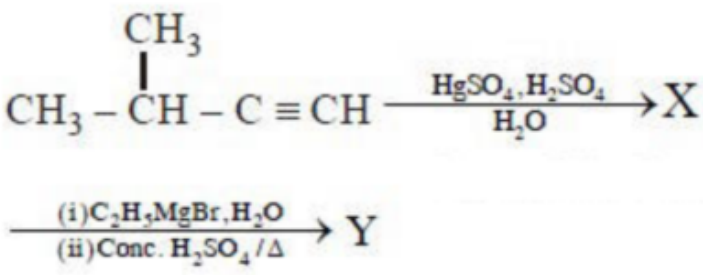
$$\lambda_4 = \frac{2\pi(16a_0)}{4} = 8\pi a_0.$$

The same result follows immediately from $\lambda_n = 2\pi n a_0$. The first option, $2\pi a_0$, is the first-orbit wavelength; $6\pi a_0$ corresponds to the third orbit, and $4\pi a_0$ to the second. Thus all three fail the required fourth-orbit standing-wave condition. Since a_0 has dimensions of length, the final expression also has the correct dimensions. The electron wavelength increases with n, consistently with the decreasing Bohr orbital speed.

Step 3: Conclusion The de Broglie wavelength in the fourth Bohr orbit is $\boxed{8\pi a_0}$. Hence, the correct option is (C).

Quick Tip to Solve: For hydrogen, use $\lambda_n = 2\pi n a_0$; do not confuse the wavelength with the circumference $2\pi n^2 a_0$.

57. The major product (Y) in the following reactions is : [4 marks]



(A) $CH_2{=}C(CH_3)CH(C_2H_5)CH_3$

(B) $CH_3CH(CH_3)C(CH_3){=}CHCH_3$

(C) $CH_3CH(CH_3)C(C_2H_5){=}CH_2$

(D) $(CH_3)_2C{=}C(CH_3)CH_2CH_3$

Correct Answer: (D) $(CH_3)_2C{=}C(CH_3)CH_2CH_3$

Solution:

Concept:

- Mercuric-ion-catalysed hydration of a terminal alkyne follows Markovnikov addition. The initially formed enol tautomerises to the methyl ketone $RCOCH_3$.
- A Grignard reagent adds its alkyl group to a carbonyl carbon. Acidic dehydration of the resulting

alcohol favours the more substituted, more stable alkene.

Step 1: Identify the ketone formed by hydration The starting alkyne is $(CH_3)_2CH{-}C{\equiv}CH$. With $HgSO_4/H_2SO_4/H_2O$, the oxygen ultimately occupies the internal alkyne carbon. Enol-to-ketone tautomerisation gives

$$(CH_3)_2CH{-}C{\equiv}CH \longrightarrow (CH_3)_2CH{-}CO{-}CH_3.$$

Thus X is 3-methylbutan-2-one. This hydration retains the original five-carbon skeleton and converts the terminal carbon into the methyl group next to the carbonyl.

Step 2: Add ethyl and eliminate water Ethylmagnesium bromide supplies an ethyl group to the carbonyl carbon. Hydrolysis of its alkoxide produces the tertiary alcohol

$$(CH_3)_2CH{-}C(OH)(CH_3){-}CH_2CH_3.$$

On heating with concentrated sulfuric acid, elimination from the neighbouring isopropyl methine carbon gives

$$(CH_3)_2C{=}C(CH_3)CH_2CH_3.$$

This alkene has four carbon substituents on its double bond and is the most substituted available product. Option (B) is an internal but only trisubstituted alkene; option (C) has a terminal methylene and is less substituted. Option (A) also contains a terminal methylene and a different placement of the carbon branches, so it is not the favoured dehydration product. The carbon count remains seven after ethyl addition, as required.

Step 3: Conclusion The major product is $(CH_3)_2C{=}C(CH_3)CH_2CH_3$, the tetrasubstituted alkene (2,3-dimethylpent-2-ene). Hence, the correct option is (D).

Quick Tip to Solve: Carry the carbon skeleton through hydration and Grignard addition, then compare the substitution of each possible dehydration double bond.

58. If the magnetic moment of a dioxygen species is 1.73 B.M, it may be : [4 marks]

(A) O_2^- or O_2^+

(B) O_2, O_2^- or O_2^+

(C) O_2 or O_2^+

(D) O_2 or O_2^-

Correct Answer: (A) O_2^- or O_2^+

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons.
- The frontier orbitals of dioxygen are the two degenerate antibonding π^* orbitals. Adding or removing one electron changes the unpaired-electron count.

Step 1: Infer the number of unpaired electrons For one unpaired electron the spin-only expression

gives

$$\mu = \sqrt{1(1+2)} = \sqrt{3} \approx 1.73\,\text{B.M.}$$

Thus the required species must have exactly one unpaired electron. Two unpaired electrons would give $\sqrt{8} \approx 2.83\,\text{B.M.}$, and a closed-shell species would have zero spin-only moment. It is therefore necessary to compare the molecular-orbital occupancies rather than simply classify all oxygen species as paramagnetic.

Step 2: Compare neutral oxygen with its singly charged ions Neutral O_2 has two electrons in the pair of degenerate π^* orbitals, one in each orbital by Hund's rule. It therefore has two unpaired electrons and cannot yield the stated moment. In O_2^-, the added electron pairs with one of these electrons, leaving one unpaired. In O_2^+, removal of one antibonding electron also leaves exactly one unpaired electron. Both ions therefore have $\mu = \sqrt{3}\,\text{B.M.}$. Options (B), (C) and (D) all include neutral oxygen, whose predicted moment is too large, whereas option (A) contains exactly the two eligible singly charged species. The opposite ionic charges do not imply opposite magnetic behaviour; unpaired spins determine the moment.

Step 3: Conclusion The species can be either O_2^- or O_2^+. Hence, the correct option is (A).

Quick Tip to Solve: For dioxygen questions, fill the two π^* orbitals first; both adding and removing one electron leave a single unpaired spin.

59. Complex X of composition $Cr(H_2O)_6Cl_n$ has a spin only magnetic moment of 3.83 BM. It reacts with $AgNO_3$ and shows geometrical isomerism. The IUPAC nomenclature of X is : [4 marks]

(A) Hexaaqua chromium (III) chloride

(B) Tetraaquadichlorido chromium(IV) chloride dihydrate

(C) Tetraaquadichlorido chromium (III) chloride dihydrate

(D) Dichloridotetraaqua chromium (IV) chloride dihydrate

Correct Answer: (C) Tetraaquadichlorido chromium (III) chloride dihydrate

Solution:

Concept:

- The spin-only moment $\mu = \sqrt{n(n+2)}\,\text{B.M.}$ identifies the unpaired-electron count. Chromium(III) is $3d^3$ and has three unpaired electrons in an octahedral complex.
- Only chloride ions outside the coordination sphere react directly with silver nitrate. An octahedral $[MA_4B_2]$ complex exhibits cis-trans geometrical isomerism.

Step 1: Determine chromium oxidation state and the coordination sphere The reported $3.83\,\text{B.M.}$ is approximately the three-unpaired-electron value $\sqrt{3(3+2)} = \sqrt{15} \approx 3.87\,\text{B.M.}$. This supports chromium(III), with a d^3 configuration, rather than chromium(IV). Since water is neutral and chloride is mononegative, the composition has three chlorides overall. The reaction with $AgNO_3$ shows that at least one chloride remains outside the coordination sphere. A hexaaqua cation contains only identical ligands and would not exhibit geometrical isomerism.

Step 2: Use geometrical isomerism and name the hydrate Two coordinated chlorides and four

water ligands give the octahedral cation $[Cr(H_2O)_4Cl_2]^+$, which has cis and trans arrangements. The remaining chloride is the counter-ion, and the two remaining water molecules are water of crystallisation:

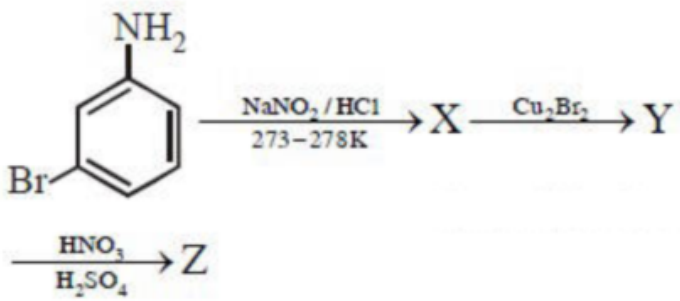
$$[Cr(H_2O)_4Cl_2]Cl \cdot 2\,H_2O.$$

The formula contains six waters and three chlorides in total, matching the composition. Its name is tetraaquadichloridochromium(III) chloride dihydrate. Option (A) fails the geometrical-isomerism condition; options (B) and (D) assign chromium(IV), inconsistent with the magnetic evidence. In naming mixed ligands, aqua is alphabetised before chlorido, with multiplicative prefixes ignored.

Step 3: Conclusion $X = [Cr(H_2O)_4Cl_2]Cl \cdot 2\,H_2O$: tetraaquadichloridochromium(III) chloride dihydrate. Hence, the correct option is (C).

Quick Tip to Solve: Combine all three clues: magnetic moment fixes oxidation state, silver nitrate detects outer chloride, and cis-trans isomerism fixes coordinated chlorides.

60. The major product Z obtained in the following reaction scheme is : [4 marks]



(A) 1,3-Dibromo-4-nitrobenzene

(B) 1,3-Dibromo-5-nitrobenzene

(C) 1,3-Dibromo-2-nitrobenzene

(D) 1,3-Dibromo-2,4-dinitrobenzene

Correct Answer: (A) 1,3-Dibromo-4-nitrobenzene

Solution:

Concept:

- An aromatic primary amine undergoes diazotisation with $NaNO_2/HCl$ at 273–278 K; the diazonium group can then be replaced by bromine using a cuprous bromide reagent.
- Halogens deactivate a benzene ring by induction but direct electrophilic substitution to ortho and para positions by resonance. Steric hindrance helps distinguish activated positions.

Step 1: Follow the first two functional-group changes The starting material is meta-bromoaniline: place its amino group at carbon 1 and its existing bromine at carbon 3. Diazotisation converts $-NH_2$ to $-N_2{}^+Cl^-$, giving diazonium intermediate X. The cuprous bromide step replaces that diazonium group with bromine, expelling nitrogen. Thus Y is 1,3-dibromobenzene. The original bromine

substituent stays at carbon 3 throughout; neither of these transformations moves the substituents around the ring.

Step 2: Locate the major nitration position In 1,3-dibromobenzene, positions 2, 4 and 6 are ortho or para to both bromines and are electronically favoured for nitration. Position 2 is crowded between two bromine substituents. Positions 4 and 6 are equivalent by symmetry, each being ortho to one bromine and para to the other; substitution there is less crowded and gives the major mononitration product. Accordingly Z is 1,3-dibromo-4-nitrobenzene, shown in option (A). Option (B) places nitro meta to both bromines and lacks their resonance direction. Option (C) places it at the sterically hindered position between the bromines. Option (D) shows two nitro groups, rather than the major mononitration product of the indicated step.

Step 3: Conclusion The major product is 1,3-dibromo-4-nitrobenzene (the symmetry-equivalent 6-nitro placement represents the same product). Hence, the correct option is (A).

Quick Tip to Solve: Complete the diazonium replacement before applying directing effects: nitration acts on 1,3-dibromobenzene, not on the original amino compound.

61. If enthalpy of atomisation for $Br_{2(l)}$ is x kJ/mol and bond enthalpy for Br_2 is y kJ/mol, the relation between them : [4 marks]

(A) does not exist

(B) is x $<$ y

(C) is x $>$ y

(D) is x $=$ y

Correct Answer: (C) is x $>$ y

Solution:

Concept:

- Bond enthalpy refers to bond cleavage in the gaseous state. Breaking the bromine bond produces gaseous atoms: $\mathrm{Br_2(g)} \longrightarrow 2\,\mathrm{Br\,(g)}$.
- For the comparison in this question, atomising liquid bromine involves both vaporising the liquid and breaking the bond. Enthalpy is a state function, so the two energy requirements add by Hess's law.

Step 1: Write the gaseous bond-cleavage process The quantity y corresponds to dissociation of gaseous bromine molecules. Thus it accounts for the energy required to break the covalent bond, but does not include energy needed to remove molecules from the liquid.

$$\mathrm{Br_2(g)} \longrightarrow 2\,\mathrm{Br\,(g)}, \qquad \Delta H = y.$$

The phase label is essential: using a gaseous starting material and using a liquid starting material are different thermochemical processes.

Step 2: Include vaporisation before dissociation Use the path liquid molecules to gaseous molecules to gaseous atoms. The first step is endothermic because intermolecular attractions in liquid bromine

must be overcome.

$$\mathrm{Br_2(l)} \longrightarrow \mathrm{Br_2(g)}, \qquad \Delta H = \Delta_{\mathrm{vap}} H > 0.$$

For the dissociation comparison intended in the question, Hess's law therefore gives

$$x = \Delta_{\mathrm{vap}} H + y, \qquad x - y = \Delta_{\mathrm{vap}} H > 0.$$

Equality would omit vaporisation. The reverse inequality would incorrectly treat vaporisation as exothermic. The processes are connected by a definite thermochemical cycle, so saying that no relation exists is also inappropriate.

Step 3: State the normalisation convention explicitly The keyed comparison takes both energies for dissociation of one mole of bromine molecules. Under the standard thermochemical definition, atomisation enthalpy is instead quoted per mole of gaseous atoms. If that convention is used, write x_{atom} to avoid confusing the two quantities:

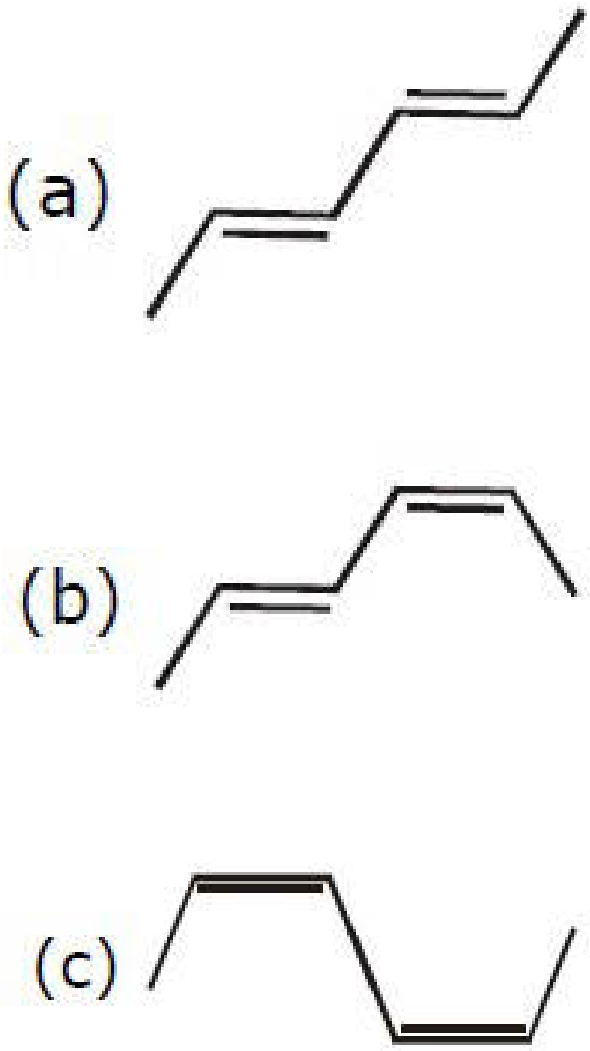
$$x_{\mathrm{atom}} = \frac{\Delta_{\mathrm{vap}} H + y}{2}.$$

Positivity of vaporisation alone does not imply that this per-atom quantity exceeds y. Thus the wording has a normalisation ambiguity: the answer key uses the full-molecule dissociation convention. The comparison $x > y$ applies to that convention, not to a silent replacement of a per-mole-atoms definition.

Step 4: Conclusion The keyed answer is (C), $x > y$, when both energies refer to dissociation of one mole of $\mathrm{Br_2}$ molecules. With the standard per-mole-gaseous-atoms definition of atomisation, the factor of one-half must instead be retained as shown above; the wording does not specify that normalisation clearly.

Quick Tip to Solve: Check the starting phase before comparing dissociation energies: a liquid-to-atom path includes vaporisation before bond cleavage.

62. **The correct order of heat of combustion for following alkadienes is :** [4 marks]



(A) (b) < (c) < (a)

(B) (a) < (c) < (b)

(C) (a) < (b) < (c)

(D) (c) < (b) < (a)

Correct Answer: (C) (a) < (b) < (c)

Solution:

Concept:

- For isomers giving identical complete-combustion products, the less stable reactant releases more heat. Heat of combustion here means the positive magnitude of heat liberated.
- Trans double bonds generally reduce steric repulsion relative to cis double bonds. Compare the geometry at each double bond rather than the bend about the intervening single bond.

Step 1: Identify the double-bond configurations In structure (a), the carbon-chain substituents lie

on opposite sides at both double bonds, giving the trans,trans isomer. Structure (b) has one trans and one cis double bond. In structure (c), the chain substituents are on the same side at both double bonds, giving the cis,cis isomer. All three have the same carbon skeleton and two conjugated double bonds; therefore differences in formula or the number of double bonds cannot decide the order.

Step 2: Compare stability and reverse the order The cis arrangements bring substituents closer and increase steric crowding. The trans,trans isomer is consequently the most stable, the cis,trans isomer is intermediate, and the cis,cis isomer is the least stable.

$$\text{Stability: } (a) > (b) > (c).$$

Combustion leads to the same amounts of carbon dioxide and water for all three. A higher-energy reactant must lose more energy to reach those common products, so the magnitude of combustion heat follows the reverse order.

$$Q_{\text{comb}}(a) < Q_{\text{comb}}(b) < Q_{\text{comb}}(c).$$

Options placing (a) last or (c) first invert this stability argument; the order (a), (c), (b) incorrectly makes the doubly cis isomer more stable than the mixed isomer.

Step 3: Conclusion The trans,trans isomer releases the least heat, the mixed cis,trans isomer is intermediate, and the cis,cis isomer releases the most. Hence, the correct option is (C).

Quick Tip to Solve: For geometrical isomers with identical combustion products, count the cis arrangements first, then reverse the stability order to obtain heat released.

63. The compound that cannot act both as oxidising and reducing agent is : [4 marks]

(A) H_3PO_4

(B) H_2SO_3

(C) H_2O_2

(D) HNO_2

Correct Answer: (A) H_3PO_4

Solution:

Concept:

- An oxidising agent accepts electrons and is reduced; a reducing agent donates electrons and is oxidised. An element in an intermediate oxidation state can often undergo either change.
- Use $+1$ for hydrogen and normally -2 for oxygen, but oxygen in a peroxide has oxidation state -1. The sum of oxidation states in a neutral molecule is zero.

Step 1: Find the oxidation state in phosphoric acid Let the oxidation state of phosphorus in H_3PO_4 be p.

$$3(+1) + p + 4(-2) = 0 \quad \Rightarrow \quad p = +5.$$

This is phosphorus in its highest usual oxidation state. It cannot be oxidised further under the ordinary

redox conditions considered here. Therefore phosphoric acid cannot serve as both an oxidising and a reducing agent. This conclusion does not require claiming that it is a strong oxidising agent; its inability to undergo both directions is what the question asks.

Step 2: Check why the other compounds can undergo both changes In H_2SO_3, sulphur is $+4$, intermediate between lower states and $+6$: it can be oxidised to sulphate or reduced to sulphur. In HNO_2, nitrogen is $+3$, and can be oxidised to $+5$ or reduced to lower states. In H_2O_2, oxygen is -1. It may be reduced to -2 in water or oxidised to 0 in oxygen. For example, the two peroxide half-reactions in acid are

$$H_2O_2 + 2\,H^+ + 2\,e^- \longrightarrow 2\,H_2O,$$

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

Thus options (B), (C) and (D) all allow both electron acceptance and electron donation, unlike phosphoric acid.

Step 3: Conclusion The compound that cannot act as both types of redox agent is H_3PO_4. Hence, the correct option is (A).

Quick Tip to Solve: Check for an intermediate oxidation state, remembering the peroxide exception: oxygen is minus one in hydrogen peroxide.

64. The acidic, basic and amphoteric oxides, respectively, are : [4 marks]

(A) Cl_2O, CaO, P_4O_{10}

(B) N_2O_3, Li_2O, Al_2O_3

(C) MgO, Cl_2O, Al_2O_3

(D) Na_2O, SO_3, Al_2O_3

Correct Answer: (B) N_2O_3, Li_2O, Al_2O_3

Solution:

Concept:

- Acidic oxides react with bases and often form acids with water. Basic oxides react with acids and often form hydroxides with water.
- An amphoteric oxide reacts with both acids and strong bases. Aluminium oxide is the standard amphoteric example among the compounds listed.

Step 1: Identify the acidic and basic members Dinitrogen trioxide is the anhydride of nitrous acid and is acidic. Lithium oxide is an ionic oxide of an alkali metal and is basic. Their reactions with water illustrate the distinction:

$$N_2O_3 + H_2O \longrightarrow 2\,HNO_2,$$

$$Li_2O + H_2O \longrightarrow 2\,LiOH.$$

Thus the first two entries in option (B) occur in exactly the acidic-then-basic order required. Merely choosing a familiar metal oxide is insufficient; each position in the ordered list must satisfy its assigned behaviour.

Step 2: Confirm amphoterism and eliminate the alternatives Aluminium oxide dissolves in acids to give aluminium salts and reacts with strong aqueous base to form aluminate:

$$Al_2O_3 + 6\,HCl \longrightarrow 2\,AlCl_3 + 3\,H_2O,$$

$$Al_2O_3 + 2\,NaOH + 3\,H_2O \longrightarrow 2\,Na[Al(OH)_4].$$

Therefore it correctly occupies the third position in option (B). Option (A) has acidic P_4O_{10} in the amphoteric position. Option (C) reverses the first two classes: MgO is basic and Cl_2O is acidic. Option (D) similarly starts with basic Na_2O followed by acidic SO_3. None of these other ordered triples meets all three requirements.

Step 3: Conclusion The acidic, basic and amphoteric oxides are respectively N_2O_3, Li_2O and Al_2O_3. Hence, the correct option is (B).

Quick Tip to Solve: Test an oxide triple position by position; aluminium oxide reacts with both acid and base, while alkali-metal oxides are basic.

65. The electronic configurations of bivalent europium and trivalent cerium are (atomic number : Xe = 54, Ce = 58, Eu = 63) [4 marks]

(A) [Xe] $4f^7$ $6s^2$ and [Xe] $4f^2$ $6s^2$

(B) [Xe] $4f^2$ and [Xe] $4f^7$

(C) [Xe] $4f^4$ and [Xe] $4f^9$

(D) [Xe] $4f^7$ and [Xe] $4f^1$

Correct Answer: (D) [Xe] $4f^7$ and [Xe] $4f^1$

Solution:

Concept:

- A cation has fewer electrons than its neutral atom by the magnitude of its positive charge. The closed xenon core contains 54 electrons.
- When lanthanoid cations form, the outer $6s$ electrons are removed before the inner $4f$ electrons; a $5d$ electron, where present, is also removed before a $4f$ electron.

Step 1: Form the bivalent europium ion Neutral europium has atomic number 63, hence nine electrons beyond xenon. Its ground-state configuration is $[Xe]\,4f^7 6s^2$. Bivalent europium is Eu^{2+}, so removal of the two outer $6s$ electrons leaves

$$Eu^{2+}: \quad [Xe]\,4f^7.$$

The count is $54 + 7 = 61 = 63 - 2$, confirming the charge. The half-filled $4f^7$ subshell also accounts for the relative stability of the divalent state. Retaining $6s^2$, as in option (A), would retain the neutral-atom electron count rather than form this cation.

Step 2: Form the trivalent cerium ion and compare options Neutral cerium is represented by

$[\mathrm{Xe}]\,4f^1 5d^1 6s^2$. To form Ce^{3+}, remove the two $6s$ electrons and the $5d$ electron:

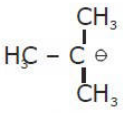
$$\mathrm{Ce}^{3+}: \quad [\mathrm{Xe}]\,4f^1.$$

The independent electron-count check gives $58 - 3 = 55$, exactly one electron beyond the xenon core. Consequently the pair must be $4f^7$ followed by $4f^1$. Option (B) exchanges inappropriate populations, and option (C) supplies neither required electron count. Option (A) is not a pair of the specified ionic configurations. Only option (D) has the correct populations in the requested Eu-then-Ce order.

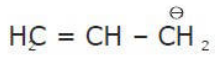
Step 3: Conclusion $\mathrm{Eu}^{2+} = [\mathrm{Xe}]4f^7$ and $\mathrm{Ce}^{3+} = [\mathrm{Xe}]4f^1$. Hence, the correct option is (D).

Quick Tip to Solve: Check a lanthanoid cation twice: remove outer electrons first, then verify that core plus remaining electrons equals atomic number minus charge.

66. The increasing order of basicity for the following intermediates is (from weak to strong) [4 marks]



(i)



(ii)



(iii)



(iv)



(v)

(A) (iii) < (i) < (ii) < (iv) < (v)
(B) (iii) < (iv) < (ii) < (i) < (v)
(C) (v) < (i) < (iv) < (ii) < (iii)
(D) (v) < (iii) < (ii) < (iv) < (i)
Correct Answer: (D) (v) < (iii) < (ii) < (iv) < (i)

Solution:

Concept:

- A more stabilised anion is generally a weaker base. Basicity may be compared using the acidity of the conjugate acid: the stronger the conjugate acid, the weaker its conjugate base.
- Resonance and greater s-character stabilise negative charge, whereas electron-releasing alkyl groups destabilise a carbanion. Cyanide is especially weakly basic compared with the hydrocarbon-derived anions shown.

Step 1: Identify the stabilised anions The five species are tert-butyl carbanion (i), allyl carbanion (ii), acetylide (iii), methyl carbanion (iv), and cyanide (v). Cyanide is the conjugate base of HCN, which is far more acidic than the hydrocarbons corresponding to the other anions. It is therefore the weakest base in this group. In acetylide, the negatively charged carbon is sp hybridised, with 50 percent s-character. Its electron pair is held more strongly than in an sp3 carbanion. This places acetylide above cyanide in basicity but below the allyl and alkyl carbanions.

Step 2: Compare resonance and alkyl-group effects The allyl anion spreads its negative charge over two terminal carbon atoms through resonance:

$$CH_2{=}CH{-}CH_2{}^- \longleftrightarrow {-}CH_2{-}CH{=}CH_2.$$

It is thus more stable, and less basic, than the localised methyl carbanion. Comparing methyl with tert-butyl, the three methyl groups in the latter donate electron density by the positive inductive effect. This destabilises the already negatively charged centre and makes tert-butyl carbanion the stronger proton acceptor. Combining these comparisons gives

$$(v) < (iii) < (ii) < (iv) < (i).$$

Options (A) and (B) wrongly put cyanide at the strongly basic end. Option (C) also makes acetylide the strongest base and places the alkyl carbanions too low. Only option (D) respects all the stabilisation comparisons.

Step 3: Conclusion The increasing basicity is $(v) < (iii) < (ii) < (iv) < (i)$. Hence, the correct option is (D).

Quick Tip to Solve: Rank carbanion stability first using conjugate-acid acidity, resonance and s-character; reverse that order to rank proton-accepting strength.

67. $[Pd(F)(Cl)(Br)(I)]^{2-}$ has n number of geometrical isomers. Then, the spin-only magnetic moment and crystal field stabilisation energy [CFSE] of $[Fe(CN)_6]^{n-6}$, respectively, are:

[Note : Ignore the pairing energy] [4 marks]

(A) 1.73 BM and –2.0 Δ_0

(B) 5.92 BM and 0

(C) 2.84 BM and –1.6 Δ_0

(D) 0 BM and –2.4 Δ_0

Correct Answer: (A) 1.73 BM and –2.0 Δ_0

Solution:

Concept:

- A square-planar complex with four different monodentate ligands has three geometrical arrangements, distinguished by which ligand lies trans to a fixed ligand.
- For an octahedral complex, each electron in t_{2g} contributes $-0.4\Delta_o$ and each electron in e_g contributes $+0.6\Delta_o$. The spin-only moment is $\sqrt{u(u+2)}$ BM, where u is the number of unpaired electrons.

Step 1: Count the square-planar isomers In $[\mathrm{PdFClBrI}]^{2-}$, palladium is in the $+2$ oxidation state and has the usual square-planar d^8 arrangement. Fix the fluoride ligand at one position. Its trans partner can be chloride, bromide or iodide. For each choice, the two remaining ligands occupy the remaining adjacent positions; interchanging those two gives a rotated equivalent arrangement, not a new geometrical isomer. Thus $n = 3$.

Step 2: Find the iron oxidation state and electron arrangement The charge on the iron complex is $n - 6 = 3 - 6 = -3$. Six cyanide ligands contribute a total charge of -6, so $x - 6 = -3$ gives iron in the $+3$ oxidation state. Consequently iron is d^5. Cyanide is a strong-field ligand, and the octahedral complex is low spin:

$$(t_{2g})^5(e_g)^0.$$

The five lower-level electrons form two pairs and leave one unpaired electron.

Step 3: Evaluate both quantities and compare the options With one unpaired electron,

$$\mu = \sqrt{1(1+2)} = \sqrt{3} \simeq 1.73 \text{ BM}.$$

Ignoring pairing energy as directed,

$$\text{CFSE} = 5(-0.4\Delta_o) + 0(0.6\Delta_o) = -2.0\Delta_o.$$

The 5.92 BM value would require five unpaired electrons, contrary to the strong-field low-spin arrangement. The 2.84 BM and zero-moment alternatives likewise have the wrong unpaired-electron counts.

Step 4: Conclusion There are three geometrical isomers, and the iron complex has $\mu = 1.73$ BM and $\text{CFSE} = -2.0\Delta_o$. Hence, the correct option is (A).

Quick Tip to Solve: First use the isomer count to determine the complex charge; only then assign the iron oxidation state and its low-spin electron configuration.

68. 'X' melts at low temperature and is a bad conductor of electricity in both liquid and solid state. X is : [4 marks]

(A) Mercury

(B) Carbon tetrachloride

(C) Zinc sulphide

(D) Silicon carbide

Correct Answer: (B) Carbon tetrachloride

Solution:

Concept:

- A low melting point generally indicates weak attractions between discrete particles. A nonpolar molecular solid is held together mainly by London dispersion forces.
- Electrical conduction requires mobile charged particles. Neutral molecules supply neither mobile ions nor delocalised electrons in the solid or liquid state.

Step 1: Combine the melting and conductivity clues The substance must satisfy both observations at the same time. Weak intermolecular forces permit melting at a low temperature, whereas neutral molecules account for poor electrical conduction even after melting. Melting changes the arrangement and mobility of the molecules; it does not, by itself, produce ions or free electrons. A molecular substance therefore fits the two clues more directly than an ionic lattice or a metal.

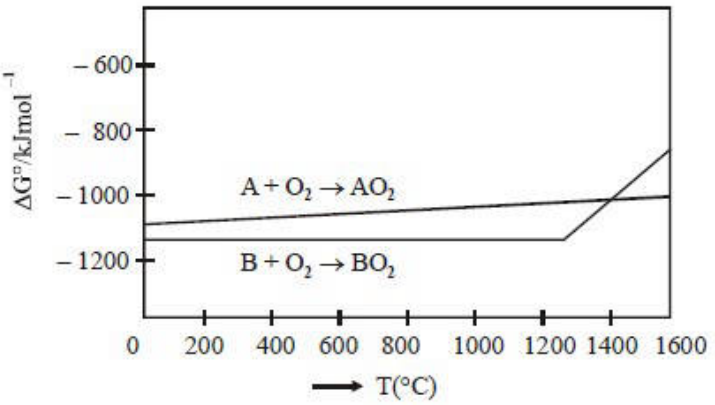
Step 2: Identify the molecular candidate Carbon tetrachloride, CCl_4, consists of discrete covalent molecules. Its tetrahedral symmetry makes the molecule nonpolar overall, even though individual carbon–chlorine bonds are polar. The molecules interact mainly through comparatively weak dispersion forces. It consequently has a low melting point and remains a poor electrical conductor in both its solid and liquid forms.

Step 3: Eliminate the other three substances Mercury has a low melting point, but it is a metal and conducts electricity through mobile electrons, so it fails the conductivity condition. Zinc sulphide has a strongly bound crystalline lattice and is not a low-melting molecular substance; the weak-force explanation does not apply to it. Silicon carbide has an extended covalent network with strong bonds throughout the solid and a very high melting or decomposition temperature. Thus neither zinc sulphide nor silicon carbide satisfies the low-temperature melting clue. The description cannot be decided from conductivity alone; the thermal behaviour is essential.

Step 4: Conclusion Carbon tetrachloride has weak intermolecular attractions and no mobile charge carriers in either state. Hence, the correct option is (B).

Quick Tip to Solve: Use both clues together: a low melting point plus poor conduction in the melt is characteristic of a neutral molecular substance.

69. According to the following diagram, A reduces BO_2 when the temperature is : [4 marks]



(A) $>$ 1400 °C

(B) $>$ 1200 °C but $<$ 1400 °C

(C) $<$ 1400 °C

(D) $<$ 1200 °C

Correct Answer: (A) $>$ 1400 °C

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy of oxide formation against temperature. The lower formation line corresponds to the more thermodynamically stable oxide at that temperature.
- For $A + BO_2 \longrightarrow AO_2 + B$, $\Delta G^\circ_{\text{reaction}} = \Delta G^\circ_f(AO_2) - \Delta G^\circ_f(BO_2)$. Reduction is thermodynamically favoured when this difference is negative.

Step 1: Construct the oxygen-transfer reaction The plotted formation reactions are $A + O_2 \longrightarrow AO_2$ and $B + O_2 \longrightarrow BO_2$. Reverse the second and add it to the first. Oxygen cancels, leaving

$$A + BO_2 \longrightarrow AO_2 + B.$$

Both formation reactions use the same amount of oxygen, so their Gibbs-energy ordinates can be subtracted directly without an additional stoichiometric factor.

Step 2: Locate the temperature at which the line order reverses At lower temperatures, the BO_2 formation line lies below the AO_2 line. Therefore the difference $\Delta G^\circ_f(AO_2) - \Delta G^\circ_f(BO_2)$ is positive and A cannot favourably take oxygen from BO_2. The rising portion of the B-oxide line crosses the A-oxide line near $1400\,^\circ C$. Above that intersection, the A-oxide line lies below the B-oxide line.

Step 3: Apply the negative-Gibbs-energy criterion For temperatures above $1400\,^\circ C$, formation of AO_2 releases more Gibbs energy than is needed to decompose BO_2; thus the oxygen-transfer reaction has negative standard Gibbs energy. The interval between 1200 and 1400 degrees still has the wrong line order, so (B) fails. Options (C) and (D) also select temperatures below the crossing. At the crossing itself, the standard Gibbs-energy difference is zero, which is the boundary rather than a negative driving force.

Step 4: Conclusion A reduces BO_2 when $T > 1400\,^\circ C$. Hence, the correct option is (A).

Quick Tip to Solve: For reduction of one oxide by another element, subtract the oxide-formation ordinates: the reducing element must have its oxide line lower on the graph.

70. A chemist has 4 samples of artificial sweetener A, B, C and D. To identify these samples, he performed certain experiments and noted the following observations :

(i) A and D both form blue-violet colour with ninhydrin.

(ii) Lassaigne extract of C gives positive $AgNO_3$ test and negative $Fe_4[Fe(CN)_6]_3$ test.

(iii) Lassaigne extract of B and D gives positive sodium nitroprusside test

Based on these observations which option is correct ? [4 marks]

(A) A : Aspartame ; B : Alitame ; C : Saccharin ; D ; Sucralose

(B) A : Alitame ; B : Saccharin ; C : Aspartame ; D ; Sucralose

(C) A : Aspartame ; B : Saccharin ; C : Sucralose ; D ; Alitame

(D) A : Saccharin ; B : Alitame ; C : Sucralose ; D ; Aspartame

Correct Answer: (C) A : Aspartame ; B : Saccharin ; C : Sucralose ; D ; Alitame

Solution:

Concept:

- Ninhydrin gives the characteristic blue-violet colour with compounds having a suitable free amino group. Aspartame and alitame are peptide-derived sweeteners with such groups.
- After sodium fusion, a positive silver nitrate halogen test indicates halogen, a negative Prussian-blue nitrogen test indicates absence of nitrogen, and sodium nitroprusside detects sulphur as sulphide.

Step 1: Use the halogen and nitrogen tests to identify C Sample C gives a positive silver nitrate test but a negative nitrogen test in its Lassaigne extract. Sucralose contains chlorine atoms and contains no nitrogen, matching both observations. Aspartame and alitame contain nitrogen and are not chlorinated sweeteners. Saccharin contains nitrogen and sulphur rather than chlorine. Therefore $C =$ sucralose, restricting attention to options (C) and (D).

Step 2: Combine the amino-group and sulphur observations Both A and D give the ninhydrin colour, so they must be the two amino-containing peptide-derived sweeteners: aspartame and alitame. Samples B and D give the sulphur test. Among the four sweeteners listed, alitame and saccharin contain sulphur. Since D belongs to both sets, D must be alitame: it gives the amino-group response and contains sulphur. The other ninhydrin-positive sample A is therefore aspartame.

Step 3: Complete the assignments and check all options The remaining sulphur-containing sweetener B is saccharin. The full assignment is

$$A = \text{aspartame}, \quad B = \text{saccharin},$$

$$C = \text{sucralose}, \quad D = \text{alitame}.$$

Option (A) incorrectly assigns saccharin to the halogen-positive, nitrogen-negative sample and sucralose to a ninhydrin-positive sample. Option (B) has the same ninhydrin problem and assigns nitrogen-containing aspartame to C. Option (D) puts saccharin at A and sulphur-free aspartame at D, contradicting two tests. Only option (C) satisfies all three observations simultaneously.

Step 4: Conclusion A is aspartame, B is saccharin, C is sucralose and D is alitame. Hence, the correct option is (C).

Quick Tip to Solve: Identify C from chlorine without nitrogen first, then find D as the intersection of the ninhydrin-positive and sulphur-positive sets.

Chemistry – Section B: Numerical Value Questions (20 marks)

71. How much amount of NaCl should be added to 600 g of water (ρ = 1.00 g/mL) to decrease the freezing point of water to – 0.2 °C ? _____.
(The freezing point depression constant for water = 2K kg mol^{-1}) [4 marks]

Solution:

Concept:

- For a dilute solution, freezing-point depression is $\Delta T_f = iK_f m$, where m is moles of solute per kilogram of solvent and i counts dissolved particles per formula unit.
- Sodium chloride dissociates as $\mathrm{NaCl} \longrightarrow \mathrm{Na^+} + \mathrm{Cl^-}$. In the dilute, ideal complete-dissociation treatment, $i = 2$, and the molar mass of NaCl is $58.5\,\mathrm{g\,mol^{-1}}$.

Step 1: Calculate the required depression and solvent mass Pure water freezes at $0\,^\circ\mathrm{C}$, while the target temperature is $-0.2\,^\circ\mathrm{C}$. The magnitude of the depression is therefore

$$\Delta T_f = 0 - (-0.2) = 0.2\,\mathrm{K}.$$

A Celsius temperature difference has the same numerical value as a kelvin difference. The given water mass is $600\,\mathrm{g} = 0.600\,\mathrm{kg}$. Molality uses the mass of solvent, so the density is unnecessary here; it would only be needed to convert a volume of water to mass.

Step 2: Determine the required molality and amount of salt Using complete dissociation of the dilute sodium chloride solution, substitute $i = 2$ and $K_f = 2\,\mathrm{K\,kg\,mol^{-1}}$:

$$m = \frac{\Delta T_f}{iK_f} = \frac{0.2}{2 \times 2} = 0.050\,\mathrm{mol\,kg^{-1}}.$$

Multiplying by the solvent mass gives

$$n(\mathrm{NaCl}) = 0.050 \times 0.600 = 0.0300\,\mathrm{mol}.$$

This is the amount of sodium chloride formula units, not the sum of the sodium-ion and chloride-ion amounts; the factor of two has already been included.

Step 3: Convert the amount to mass and verify The mass required is

$$w = nM = 0.0300 \times 58.5 = 1.755\,\mathrm{g}.$$

As a check, dissolving this amount produces approximately 0.0600 mol of ions, or 0.100 mol of dissolved particles per kilogram of water. Multiplying that particle molality by $K_f = 2$ gives $0.200\,\mathrm{K}$, exactly the requested depression. Omitting dissociation would incorrectly double the salt mass.

Step 4: Conclusion The required mass of NaCl is $1.755\,\mathrm{g}$, reported as approximately $1.75\,\mathrm{g}$ (numerical entry 1.75).

Quick Tip to Solve: Convert the solvent mass to kilograms and include the two ions from each sodium chloride formula unit before converting moles to grams.

72. 108 g of silver (molar mass 108 g mol^{-1}) is deposited at cathode from $AgNO_3$(aq) solution by a certain quantity of electricity. The volume (in L) of oxygen gas produced at 273 K and 1 bar pressure from water by the same quantity of electricity is _____. [4 marks]

Solution:

Concept:

- Faraday law relates transferred charge to the amount of electrons. Depositing one mole of silver by $\mathrm{Ag^+ + e^- \longrightarrow Ag}$ requires one mole of electrons.
- Water oxidation is $\mathrm{2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-}$, so four moles of electrons correspond to one mole of oxygen. For an ideal gas, $PV = nRT$, using consistent pressure and gas-constant units.

Step 1: Find the electron amount from the silver deposit The deposited silver has mass $108\,\mathrm{g}$ and molar mass $108\,\mathrm{g\,mol^{-1}}$, giving

$$n(\mathrm{Ag}) = \frac{108}{108} = 1.00\,\mathrm{mol}.$$

Each silver ion accepts one electron to become a silver atom. Thus the quantity of electricity corresponds to 1.00 mol of electrons, or one faraday of charge. The exact numerical value of the Faraday constant is not required because the same charge is used in the second electrolysis.

Step 2: Use the electron stoichiometry for oxygen The balanced water-oxidation half-reaction releases four electrons per oxygen molecule:

$$\mathrm{2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-}.$$

Therefore the same one mole of electrons corresponds to

$$n(\mathrm{O_2}) = \frac{1.00}{4} = 0.250\,\mathrm{mol}.$$

A factor of two would be appropriate to hydrogen formation from two protons, not to oxygen formation from water. The four-electron oxygen stoichiometry is the decisive distinction.

Step 3: Calculate the gas volume at the stated pressure At $273\,\mathrm{K}$ and $1\,\mathrm{bar}$, take $R = 0.08314\,\mathrm{L\,bar\,mol^{-1}\,K^{-1}}$. The ideal-gas volume is

$$V = \frac{nRT}{P} = \frac{0.250 \times 0.08314 \times 273}{1} = 5.6743\,\mathrm{L}.$$

The familiar molar volume of approximately $22.4\,\mathrm{L}$ applies at one atmosphere, whereas this question specifies one bar. At one bar the molar volume here is about $22.70\,\mathrm{L}$, and a quarter of it gives the result above. Keeping the pressure unit consistent avoids a small but significant numerical error.

Step 4: Conclusion The oxygen volume is $5.67\,\mathrm{L}$ at $273\,\mathrm{K}$ and $1\,\mathrm{bar}$ (numerical entry 5.67).

Quick Tip to Solve: Convert the deposited metal to moles of electrons, divide by four for oxygen, and use the gas constant in litre-bar units at one bar.

73. The molarity of HNO_3 in a sample which has density 1.4 g/mL and mass percentage of 63% is _____.

(Molecular Weight of HNO_3 = 63) [4 marks]

Solution:

Concept:

- Molarity is the amount of solute in moles per litre of solution: $M = n/V$, with V in litres. The relevant volume is the volume of solution, not the volume of solvent alone.
- A mass percentage of 63% means $63\,\mathrm{g}$ of $\mathrm{HNO_3}$ per $100\,\mathrm{g}$ of solution. Density converts solution volume to solution mass through $m = \rho V$.

Step 1: Choose one litre of solution Take $1.000\,\mathrm{L} = 1000\,\mathrm{mL}$ of the sample. This choice makes the number of moles numerically equal to the molarity once the solute mass has been calculated. The mass of the entire sample is

$$m_{\text{solution}} = (1.4\,\mathrm{g\,mL^{-1}})(1000\,\mathrm{mL}) = 1400\,\mathrm{g}.$$

The given density is that of the solution; this mass includes both the nitric acid and the water.

Step 2: Calculate the amount of nitric acid The solute mass fraction is $63/100 = 0.63$. Hence,

$$m_{\mathrm{HNO_3}} = 0.63(1400) = 882\,\mathrm{g}.$$

The given molecular weight corresponds to a molar mass of $63\,\mathrm{g\,mol^{-1}}$. The amount of nitric acid is therefore

$$n_{\mathrm{HNO_3}} = \frac{882\,\mathrm{g}}{63\,\mathrm{g\,mol^{-1}}} = 14\,\mathrm{mol}.$$

Moles count the compound, so no additional factor for the atoms in one molecule is required.

Step 3: Divide by the solution volume The $14\,\mathrm{mol}$ belongs to the $1.000\,\mathrm{L}$ sample chosen at the start. Thus,

$$M = \frac{14\,\mathrm{mol}}{1.000\,\mathrm{L}} = 14\,\mathrm{mol\,L^{-1}}.$$

Interpreting the percentage as grams per $100\,\mathrm{mL}$ would incorrectly replace a mass percentage with a mass-by-volume percentage and omit the density conversion.

Step 4: Conclusion The molarity is $\boxed{14\,\mathrm{mol\,L^{-1}}}$, or $14\,\mathrm{M}$. The numerical value to enter is $\boxed{14}$.

Quick Tip to Solve: For density ρ in $\mathrm{g\,mL^{-1}}$, mass percentage w, and molar mass M_r, use $M = 10\rho w/M_r$; here it gives $10(1.4)(63)/63 = 14$.

74. The mass percentage of nitrogen in histamine is _____. [4 marks]

Solution:

Concept:

- The mass percentage of an element is its mass contribution to one mole of a compound divided by the molar mass of the compound, multiplied by 100.
- Histamine has molecular formula $C_5H_9N_3$. Its three nitrogen atoms consist of two in the imidazole ring and one in the amino group of the two-carbon side chain.

Step 1: Identify the molecular formula Histamine is the imidazole derivative with an aminoethyl side chain. The substituted imidazole ring contributes $C_3H_3N_2$, while the side chain $-CH_2-CH_2-NH_2$ contributes C_2H_6N. Combining these contributions gives

$$C_3H_3N_2 + C_2H_6N \quad \Longrightarrow \quad C_5H_9N_3.$$

The ring and the terminal amino group must both be counted: ignoring either ring nitrogen would underestimate the nitrogen mass.

Step 2: Calculate the molar mass Using the standard atomic masses $C = 12$, $H = 1$, and $N = 14$, the molar mass is

$$M(\text{histamine}) = 5(12) + 9(1) + 3(14) = 111\,\text{g mol}^{-1}.$$

The nitrogen contribution in one mole of histamine is

$$m_N = 3(14) = 42\,\text{g}.$$

All three nitrogen atoms are included irrespective of their different bonding environments.

Step 3: Convert the nitrogen fraction into a percentage Divide the nitrogen mass by the mass of the whole compound, rather than by the mass of carbon or by the number of atoms:

$$\%N = \frac{42}{111} \times 100 = 37.8378\ldots\%.$$

Thus the mass percentage is about 37.84%, or 38% to the nearest integer. The atom-count fraction $3/(5+9+3)$ is not appropriate because the atoms have unequal masses. The percentage is dimensionless; the gram units cancel in the ratio.

Step 4: Conclusion The mass percentage of nitrogen in histamine is $\boxed{37.84\%}$, approximately $\boxed{38\%}$.

Quick Tip to Solve: First write the complete formula $C_5H_9N_3$; then use $3(14)/111$, not the fraction of atoms that are nitrogen.

75. The hardness of a water sample containing 10^{-3} M $MgSO_4$ expressed as $CaCO_3$ equivalents (in ppm) is _____.
(molar mass of $MgSO_4$ is 120.37 g/mol) [4 marks]

Solution:

Concept:

- Water hardness due to Mg^{2+} is expressed as the mass concentration of an equivalent amount of $CaCO_3$. Both magnesium and calcium are divalent, so one mole of dissolved $MgSO_4$ corresponds to one mole of $CaCO_3$ in this convention.
- For a dilute water sample, $1\,\text{mg}\,\text{L}^{-1}$ is expressed as approximately $1\,\text{ppm}$. The equivalent weight of a divalent salt is its molar mass divided by two.

Step 1: Find the magnesium concentration Each formula unit of $MgSO_4$ supplies one magnesium ion:

$$MgSO_4 \longrightarrow Mg^{2+} + {SO_4}^{2-}.$$

Thus a $10^{-3}\,\text{mol}\,\text{L}^{-1}$ solution contains $10^{-3}\,\text{mol}\,\text{L}^{-1}$ of the hardness-causing magnesium ions. The sulfate ion does not produce an additional independent contribution to hardness.

Step 2: Convert to calcium carbonate equivalents The molar mass of $CaCO_3$ is $40 + 12 + 3(16) = 100\,\text{g}\,\text{mol}^{-1}$. Because both cations have charge $+2$, their mole-for-mole equivalence gives

$$c_{CaCO_3} = (10^{-3}\,\text{mol}\,\text{L}^{-1})(100\,\text{g}\,\text{mol}^{-1}) = 0.100\,\text{g}\,\text{L}^{-1}.$$

The charge factor cancels; multiplying the final result by two would count the divalency twice.

Step 3: Check with the supplied magnesium sulfate molar mass The actual concentration of magnesium sulfate is $10^{-3}(120.37) = 0.12037\,\text{g}\,\text{L}^{-1}$, or $120.37\,\text{mg}\,\text{L}^{-1}$. Converting by the equivalent-weight ratio gives

$$120.37 \times \frac{100/2}{120.37/2} = 100\,\text{mg}\,\text{L}^{-1}.$$

This agrees with the direct molar method. The 120.37 value describes the actual dissolved salt; the requested hardness is reported on the calcium carbonate scale. For dilute water this concentration is $100\,\text{ppm}$.

Step 4: Conclusion The hardness of the sample, expressed as calcium carbonate equivalents, is $\boxed{100\,\text{ppm}}$.

Quick Tip to Solve: For a salt supplying one divalent hardness ion, multiply its molarity directly by 100×1000 to obtain $\text{mg}\,\text{L}^{-1}$ as $CaCO_3$.