JEE Main 2021 Paper 1 (B.E./B.Tech) Aug 26 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 4 (Aug-Sep), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 26 August 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. if α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is : [4 marks]

(A) (11, 26)

(B) (10.5, 25)

(C) (11, 25)

(D) (10.5, 26)

Correct Answer: (D) (10.5, 26)

Solution:

Concept:

- For n observations, the mean is $\bar{x} = \sum x_i/n$. Correct the sum by subtracting the wrong observation and adding the right one.
- The population variance is $\sigma^2 = \sum x_i^2/n - \bar{x}^2$. A correction affects both the sum of squares and the square of the mean.

Step 1: Recover the original sums The number of observations remains 20. The incorrectly calculated mean and standard deviation give the following totals:

$$S = 20(10) = 200.$$

$$2.5^2 = \frac{T}{20} - 10^2 \quad \Longrightarrow \quad T = 20(106.25) = 2125.$$

Here S is the original sum and T is the original sum of squares. We use the variance formula with denominator 20, since the question gives the standard deviation of these observations.

Step 2: Correct the totals and calculate the parameters Replace 25 by 35 exactly once. Thus

$$S_c = 200 - 25 + 35 = 210, \qquad \alpha = \frac{210}{20} = 10.5.$$

$$T_c = 2125 - 25^2 + 35^2 = 2725.$$

The corrected variance, rather than the corrected standard deviation, is the requested β:

$$\beta = \frac{2725}{20} - (10.5)^2 = 136.25 - 110.25 = 26.$$

The correction increases the mean by only $10/20 = 0.5$, so the options with mean 11 fail immediately. The option with variance 25 also fails the direct sum-of-squares calculation.

Step 3: Conclusion The corrected parameters are $(\alpha, \beta) = (10.5, 26)$, and the standard deviation is $\sqrt{26}$. Hence, the correct option is (D).

Quick Tip to Solve: Correct the sum and sum of squares separately; adding the change in an observation directly to the variance is invalid.

2. Let y = y(x) be a solution curve of the differential equation $(y+1)\tan^2 x\,dx + \tan x\,dy + y\,dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim\limits_{x\to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is : [4 marks]

(A) $-\frac{\pi}{4}$

(B) $\frac{\pi}{4} - 1$

(C) $\frac{\pi}{4} + 1$

(D) $\frac{\pi}{4}$

Correct Answer: (D) $\frac{\pi}{4}$

Solution:

Concept:

- A linear equation $y' + P(x)y = Q(x)$ is solved using the integrating factor $e^{\int P(x)\,dx}$.
- The condition at $x \to 0^+$ must be applied to the general solution; it is not a finite initial value for $y(0)$.

Step 1: Put the equation in linear form On $0 < x < \pi/2$, $\tan x$ is nonzero, so division by it is valid. Collect the terms containing y:

$$\tan x\, y' + (\tan^2 x + 1)y = -\tan^2 x.$$

$$y' + \frac{\sec^2 x}{\tan x} y = -\tan x.$$

Since $\frac{d}{dx}\log(\tan x) = \sec^2 x/\tan x$, the integrating factor is $\tan x$. Multiplying through gives

$$\frac{d}{dx}(y\tan x) = -\tan^2 x = 1 - \sec^2 x.$$

Step 2: Integrate and use the limiting condition Integration on the stated interval yields

$$y\tan x = x - \tan x + C, \qquad y = (x + C)\cot x - 1.$$

Therefore

$$\lim_{x\to 0^+} xy(x) = \lim_{x\to 0^+} \{x(x + C)\cot x - x\} = C,$$

because $x\cot x \to 1$. The given limit is 1, so $C = 1$. Substitute $x = \pi/4$, where $\cot x = 1$:

$$y(\pi/4) = (\pi/4 + 1) - 1 = \pi/4.$$

The negative and shifted values in the other three options do not satisfy this uniquely determined solution. The subtraction of 1 comes from integrating the $-\sec^2 x$ term and cannot be omitted.

Step 3: Conclusion $y(\pi/4) = \pi/4$. Hence, the correct option is (D).

Quick Tip to Solve: When the coefficient of y is the logarithmic derivative of tan x, multiply by tan x before integrating.

3. Let A and B be independent events such that P(A) = p, P(B) = 2p. The largest value of p, for which P (exactly one of A, B occurs) = $\frac{5}{9}$, is : [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{2}{9}$
(C) $\frac{4}{9}$
(D) $\frac{5}{12}$

Correct Answer: (D) $\frac{5}{12}$

Solution:

Concept:

- For independent events, $P(A \cap B) = P(A)P(B) = 2p^2$. Independence does not mean that the two events are disjoint.
- Exactly one means $(A \cap B^c) \cup (A^c \cap B)$, a disjoint union. Hence its probability is $P(A) + P(B) - 2P(A \cap B)$.

Step 1: Form the probability equation The two ways that exactly one event can occur have probabilities

$$P(A \cap B^c) = p(1-2p), \qquad P(A^c \cap B) = (1-p)2p.$$

Their sum must equal $5/9$, so

$$p(1-2p) + 2p(1-p) = \frac{5}{9}.$$

$$3p - 4p^2 = \frac{5}{9} \quad \Longrightarrow \quad 36p^2 - 27p + 5 = 0.$$

Subtracting the intersection only once would calculate at least one event, which is different from the demand here.

Step 2: Solve and compare the admissible roots The quadratic factors as

$$(3p-1)(12p-5) = 0.$$

Thus $p = 1/3$ or $p = 5/12$. The requirement $P(B) = 2p \leq 1$, together with nonnegative probabilities, gives $0 \leq p \leq 1/2$, so both roots are permitted. For $p = 5/12$, the exactly-one probabilities are $5/72$ and $35/72$, whose sum is $5/9$. For $p = 1/3$, the sum is also $5/9$, but this is the smaller root. The other displayed candidates, $2/9$ and $4/9$, are not roots of the quadratic. Since $5/12 > 1/3$, select the larger admissible root.

Step 3: Conclusion The largest possible value is $p = 5/12$. Hence, the correct option is (D).

Quick Tip to Solve: Exactly one requires subtracting twice the intersection from the sum of the two event probabilities.

4. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$$(1+\cos^2\theta)x + \sin^2\theta y + 4\sin 3\theta z = 0$$

$$\cos^2\theta x + (1+\sin^2\theta)y + 4\sin 3\theta z = 0$$

$$\cos^2\theta x + \sin^2\theta y + (1+4\sin 3\theta)z = 0$$

has a non-trivial solution, then the value of θ is : [4 marks]

(A) $\frac{4\pi}{9}$
(B) $\frac{7\pi}{18}$
(C) $\frac{\pi}{18}$
(D) $\frac{5\pi}{18}$

Correct Answer: (B) $\frac{7\pi}{18}$

Solution:

Concept:

- A homogeneous system has a nonzero solution precisely when its coefficient matrix is singular. Subtracting equations can reveal the singularity condition without expanding a determinant.
- Use $\cos^2\theta + \sin^2\theta = 1$, and enforce the given interval after solving the trigonometric condition.

Step 1: Compare the three equations Denote the equations in their printed order by E_1, E_2, E_3. Subtracting E_2 from E_1 cancels all trigonometric terms and gives

$$x - y = 0.$$

Similarly, subtracting E_3 from E_2 gives

$$y - z = 0.$$

Consequently any solution must have $x = y = z = t$. Substituting into the first equation yields

$$(1+\cos^2\theta+\sin^2\theta+4\sin 3\theta)t = 0,$$

$$(2+4\sin 3\theta)t = 0.$$

For a non-trivial solution $t \neq 0$; thus the coefficient must vanish.

Step 2: Solve within the prescribed interval The necessary and sufficient condition is

$$\sin 3\theta = -\frac{1}{2}.$$

As $0 < \theta < \pi/2$, we have $0 < 3\theta < 3\pi/2$. In this interval the only angle with sine $-1/2$ is $7\pi/6$. The angle $11\pi/6$ lies outside it. Therefore

$$3\theta = \frac{7\pi}{6} \quad \Longrightarrow \quad \theta = \frac{7\pi}{18}.$$

For this angle $x = y = z = 1$ is indeed a nonzero solution, verifying sufficiency. The other options make 3θ equal to $4\pi/3, \pi/6, 5\pi/6$; their sines are respectively $-\sqrt{3}/2, 1/2, 1/2$, so none works.

Step 3: Conclusion The required angle is $\theta = 7\pi/18$. Hence, the correct option is (B).

Quick Tip to Solve: In a system whose rows differ only in their diagonal 1, subtract adjacent equations to force all unknowns equal.

5. Let $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$, 0 < x < 1. Then : [4 marks]

(A) $(1-x)^2 f'(x) - 2(f(x))^2 = 0$

(B) $(1+x)^2 f'(x) + 2(f(x))^2 = 0$

(C) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$

(D) $(1+x)^2 f'(x) - 2(f(x))^2 = 0$

Correct Answer: (C) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$

Solution:

Concept:

- On $0 < x < 1$, let $t = \cot^{-1}\sqrt{(1-x)/x}$. Since the square root is positive, $0 < t < \pi/2$ and $\sin t = \sqrt{x}$.
- The identity $\cos(2\tan^{-1} u) = (1-u^2)/(1+u^2)$ turns the composite function into a rational function before differentiation.

Step 1: Simplify and differentiate the function From $\cot^2 t = (1-x)/x$, we have $\csc^2 t = 1/x$. The acute-angle restriction selects the positive square root, giving $\sin t = \sqrt{x}$. Hence

$$f(x) = \cos(2\tan^{-1}\sqrt{x}) = \frac{1-x}{1+x}.$$

Differentiate this quotient directly:

$$f'(x) = \frac{-(1+x)-(1-x)}{(1+x)^2} = -\frac{2}{(1+x)^2}.$$

Thus an identity that actually follows is $(1+x)^2 f'(x) + 2 = 0$; its final term is the constant 2, not $2(f(x))^2$.

Step 2: Test the four printed identities Set $x = 1/2$, which lies strictly inside the stated domain. Then

$$f(1/2) = \frac{1}{3}, \qquad f'(1/2) = -\frac{8}{9}.$$

Substitution into the left sides of the four options gives respectively

$$\text{(A)}: -\frac{2}{9}-\frac{2}{9}=-\frac{4}{9},$$
$$\text{(B)}: -2+\frac{2}{9}=-\frac{16}{9},$$
$$\text{(C)}: -\frac{2}{9}+\frac{2}{9}=0,$$
$$\text{(D)}: -2-\frac{2}{9}=-\frac{20}{9}.$$

Option (C) remains to be checked for the entire domain. Its left side simplifies to

$$(1-x)^2 f'(x)+2f(x)^2=0$$

for every $0<x<1$, since the two terms are $-2(1-x)^2/(1+x)^2$ and $+2(1-x)^2/(1+x)^2$.

Step 3: Conclusion $(1-x)^2 f'(x)+2(f(x))^2=0$ throughout the given domain. Hence, the correct option is (C).

Quick Tip to Solve: Simplify the nested inverse trigonometric expression first; direct differentiation makes the branch information harder to track.

6. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set : [4 marks]

(A) {80, 83, 86, 89}

(B) {84, 86, 88, 90}

(C) {79, 81, 83, 85}

(D) {84, 87, 90, 93}

Correct Answer: (C) {79, 81, 83, 85}

Solution:

Concept:

- For any sets of patients H, L, inclusion-exclusion gives $P(H\cup L)=P(H)+P(L)-P(H\cap L)$. No independence assumption is given.
- The intersection is contained in each set, while the union cannot exceed the entire population. These facts supply upper and lower bounds for the common percentage.

Step 1: Bound the common percentage Work with percentages of the same total population. Since the union cannot exceed 100 percent,

$$89+98-K\le 100 \quad\Longrightarrow\quad K\ge 87.$$

Also every patient with both ailments has the heart ailment, so $K\le 89$. The lungs condition gives the

weaker upper bound 98. Therefore all feasible values satisfy

$$87 \le K \le 89.$$

These bounds are attainable: the percentages with heart only, lungs only, both, and neither are $89 - K, 98 - K, K, K - 87$, respectively. They are all nonnegative precisely on this interval and sum to 100.

Step 2: Check each proposed set The question asks for the set to which K cannot belong, so a set must have no element in the feasible interval. Set (A), $\{80, 83, 86, 89\}$, contains the possible value 89. Set (B), $\{84, 86, 88, 90\}$, contains the possible value 88. Set (D), $\{84, 87, 90, 93\}$, contains the possible value 87. In contrast, every element of set (C), $\{79, 81, 83, 85\}$, is less than 87. None can be the percentage with both ailments. Multiplying 0.89 by 0.98 would assume independence and is not justified by the question.

Step 3: Conclusion K cannot belong to $\{79, 81, 83, 85\}$. Hence, the correct option is (C).

Quick Tip to Solve: For percentages a and b, the overlap lies between max(0,a+b-100) and min(a,b); compare whole sets with that interval.

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7. The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with : [4 marks]

(A) centre at (0, -1) and radius $\sqrt{2}$

(B) centre at (0, 1) and radius $\sqrt{2}$

(C) centre (0, 0) and radius $\sqrt{2}$

(D) centre at (0, 1) and radius 2

Correct Answer: (B) centre at (0, 1) and radius $\sqrt{2}$

Solution:

Concept:

- Write $z = x + iy$ and multiply a complex quotient by the conjugate of its denominator. Its real and imaginary parts determine its argument.
- Argument $\pi/4$ requires equal positive real and imaginary parts. Equality alone gives the supporting circle; the positivity selects the relevant arc.

Step 1: Resolve the complex quotient The denominator cannot vanish, so $z \neq -1$. Rationalisation gives

$$\frac{z-1}{z+1} = \frac{(x-1+iy)(x+1-iy)}{(x+1)^2+y^2}$$

$$= \frac{x^2+y^2-1+2iy}{(x+1)^2+y^2}.$$

Its real part is $(x^2 + y^2 - 1)/D$ and imaginary part is $2y/D$, where $D = (x + 1)^2 + y^2 > 0$. The numerator must also be nonzero because the argument of zero is undefined; hence $z \neq 1$.

Step 2: Identify the circle and the allowed arc For argument $\pi/4$, set the real and imaginary parts

equal and require them to be positive:

$$x^2 + y^2 - 1 = 2y > 0.$$

Complete the square:

$$x^2 + (y-1)^2 = 2, \qquad y > 0.$$

The supporting circle therefore has centre $(0, 1)$ and radius $\sqrt{2}$. Strictly, the argument equation describes only its portion with $y > 0$, with the endpoints $z = \pm 1$ excluded. The rest of the circle has the wrong argument or an undefined quotient. The centre $(0, -1)$ would reverse the sign of the linear y term; the origin-centred circle would omit it. Radius 2 would require the right side to be 4, which is not obtained.

Step 3: Conclusion The circle containing the locus has centre $(0, 1)$ and radius $\sqrt{2}$. The specified argument selects its arc with $y > 0$. Hence, the correct option is (B).

Quick Tip to Solve: After using tan of an argument, restore its quadrant condition; otherwise an extra arc can enter the locus.

8. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is : [4 marks]

(A) 5

(B) 7

(C) $5\sqrt{3}$

(D) $3\sqrt{5}$

Correct Answer: (D) $3\sqrt{5}$

Solution:

Concept:

- For a circle of radius R, a line at perpendicular distance $d < R$ from its centre cuts a chord of length $2\sqrt{R^2 - d^2}$. The perpendicular from the centre bisects the chord.
- The line $y = mx + c$ is tangent to $y^2 = 4ax$ when $c = a/m$, for nonzero slope m. The given parabola has $a = 30/4 = 15/2$.

Step 1: Find the circle centre and radius Divide the circle equation by 4 and complete the square:

$$x^2 + y^2 + 30x + \frac{675}{4} = 0, \qquad (x + 15)^2 + y^2 = \frac{225}{4}.$$

Thus the centre is $C = (-15, 0)$ and the radius is $R = 15/2$. The given point $(-30, 0)$ is outside this circle, so the line through it can cut a nonzero chord.

Step 2: Apply the tangency condition A nonvertical line through $(-30, 0)$ has equation $y = m(x + 30)$. Comparing its intercept $30m$ with a/m gives

$$30m = \frac{15}{2m} \quad \Longrightarrow \quad m^2 = \frac{1}{4}.$$

Both slopes yield the same distance from the circle centre. The vertical line through the given point is $x = -30$, which neither touches the parabola nor cuts the circle.

Step 3: Calculate the chord length The distance of $(-15, 0)$ from $mx - y + 30m = 0$ is

$$d = \frac{|15m|}{\sqrt{1+m^2}}, \qquad d^2 = \frac{225(1/4)}{5/4} = 45.$$

Therefore

$$L = 2\sqrt{\frac{225}{4} - 45} = 2\sqrt{\frac{45}{4}} = 3\sqrt{5}.$$

Squaring the proposed lengths gives 25, 49, 75 and 45 respectively; only the last agrees with $L^2 = 45$.

Step 4: Conclusion The chord length is $3\sqrt{5}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the tangent slope first, then use the centre-to-line distance; finding the two circle intersections separately is unnecessary.

9. The value of $\int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2 \right)^{\frac{1}{2}} dx$ **is :** [4 marks]

(A) $\log_e 4$

(B) $\log_e 16$

(C) $2\log_e 16$

(D) $4\log_e (3 + 2\sqrt{2})$

Correct Answer: (B) $\log_e 16$

Solution:

Concept:

- The identity $u^2 + u^{-2} - 2 = (u - u^{-1})^2$ simplifies the radical. For real v, the principal square root satisfies $\sqrt{v^2} = |v|$, not simply v.
- An even integrand on $[-a, a]$ has integral twice its integral on $[0, a]$. Here $|x| < 1$ throughout, so $1 - x^2$ is positive and the denominator never vanishes.

Step 1: Simplify the expression inside the radical Put $u = (x+1)/(x-1)$. Its reciprocal is $(x-1)/(x+1)$, since neither $x = 1$ nor $x = -1$ occurs on the interval. Subtracting the two ratios gives

$$u - u^{-1} = \frac{(x+1)^2 - (x-1)^2}{x^2 - 1} = \frac{4x}{x^2 - 1}.$$

Consequently the integrand is

$$\left| \frac{4x}{x^2 - 1} \right| = \frac{4|x|}{1 - x^2}.$$

Retaining the absolute value is essential: the original radical cannot be negative.

Step 2: Use symmetry and integrate The simplified integrand is even. With $a = 1/\sqrt{2}$, the required

integral is

$$I = 8\int_0^{1/\sqrt{2}} \frac{x}{1-x^2}\,dx.$$

Substitute $t = 1 - x^2$, so $dt = -2x\,dx$, or use its logarithmic antiderivative directly:

$$I = \left[-4\ln(1-x^2)\right]_0^{1/\sqrt{2}} = -4\ln\frac{1}{2} = 4\ln 2.$$

The logarithm arguments are positive over the entire interval, and the integral is finite.

Step 3: Compare the logarithmic forms Since $4\ln 2 = \ln(2^4) = \ln 16$, the second option agrees exactly. Option (A), $\ln 4$, is half the required result; option (C) doubles it. Option (D) involves a different logarithmic argument and does not arise from the endpoint $1 - x^2 = 1/2$. A zero result from oddness would incorrectly discard the absolute value introduced by the square root.

Step 4: Conclusion The integral equals $\ln 16$. Hence, the correct option is (B).

Quick Tip to Solve: Whenever a radical becomes a perfect square, insert an absolute value before testing symmetry; this often changes an apparently odd integrand into an even one.

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10. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, **and Q = $A^T BA$, then the inverse of the matrix A Q^{2021} A^T is equal to :** [4 marks]

(A) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Correct Answer: (B) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

Solution:

Concept:

- A real matrix A is orthogonal if $A^TA = AA^T = I$. Then conjugation preserves powers: $(A^TBA)^n = A^TB^nA$. Adjacent factors cancel by orthogonality.
- If $N^2 = 0$, the binomial theorem gives $(I + N)^n = I + nN$. Its inverse is $I - nN$, because their product is $I - n^2N^2 = I$.

Step 1: Check orthogonality of the given matrix Write

$$A = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}.$$

Its columns have squared length $(1+4)/5 = 1$, and their inner product is $(2-2)/5 = 0$. Hence $A^T A = I$, and because A is square, $AA^T = I$ too. In particular, the transpose used in the question is its inverse; complex conjugation is not needed for this real matrix.

Step 2: Cancel the conjugating matrices Using $Q = A^T BA$, multiply successive factors and cancel each adjacent AA^T:

$$Q^{2021} = A^T B^{2021} A.$$

Therefore

$$AQ^{2021}A^T = (AA^T)B^{2021}(AA^T) = B^{2021}.$$

Thus only the power of the much simpler triangular matrix B needs to be inverted, rather than the power of Q itself.

Step 3: Use the nilpotent part of the triangular matrix Let

$$N = B - I = \begin{pmatrix} 0 & 0 \\ i & 0 \end{pmatrix}.$$

Direct multiplication gives $N^2 = 0$. Thus

$$B^{2021} = I + 2021N, \qquad (B^{2021})^{-1} = I - 2021N = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}.$$

The positive lower entry in (C) describes the power, not its inverse. Option (D) places the nonzero entry in the wrong position, and (A) has the wrong diagonal entries.

Step 4: Conclusion The inverse is $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$. Hence, the correct option is (B).

Quick Tip to Solve: Before expanding a matrix power, look for an orthogonal similarity and a nilpotent remainder; they can reduce a large exponent to a linear coefficient.

11. If the sum of an infinite GP a, ar, ar^2, ar^3, ... is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, ... is : [4 marks]

(A) $\frac{5}{2}$

(B) $\frac{1}{2}$

(C) $\frac{25}{2}$

(D) $\frac{9}{2}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept:

- For a convergent real geometric progression, $|r| < 1$ and $a+ar+ar^2+\cdots = a/(1-r)$. Convergence must hold before using this infinite-sum formula.
- The squared terms form another GP with first term a^2 and ratio r^2. The requested subsequence $ar^2, ar^4, \ldots$ has first term ar^2 and ratio r^2, not first term a.

Step 1: Translate the two supplied sums The first sum gives

$$\frac{a}{1-r} = 15 \quad \Longrightarrow \quad a = 15(1-r).$$

The sum of the squared terms is

$$\frac{a^2}{1-r^2} = 150.$$

Since $|r| < 1$, the factors $1-r$ and $1+r$ are nonzero; cancelling them in the next step does not lose a permitted solution.

Step 2: Determine the common ratio and first term Substitute $a = 15(1-r)$ into the second equation:

$$\frac{225(1-r)^2}{(1-r)(1+r)} = 150.$$

Dividing by 75 and simplifying gives

$$3(1-r) = 2(1+r), \qquad 3-3r = 2+2r, \qquad r = \frac{1}{5}.$$

Hence $a = 15(4/5) = 12$. As a check, $12/(4/5) = 15$ and $144/(24/25) = 150$, so both original conditions are satisfied.

Step 3: Sum the required even-power terms The requested series starts with ar^2, so its sum is

$$S = \frac{ar^2}{1-r^2} = \frac{12(1/25)}{1-1/25} = \frac{12}{24} = \frac{1}{2}.$$

This is consistent with the first term $12/25 = 0.48$ and a small common ratio $1/25$: the sum should be only slightly larger than 0.48. The other proposed values are far too large, especially the sum $25/2$, which corresponds to including a different leading term.

Step 4: Conclusion The sum of $ar^2 + ar^4 + ar^6 + \cdots$ is $1/2$. Hence, the correct option is (B).

Quick Tip to Solve: Write the first term and common ratio of the requested subsequence explicitly; retaining the original first term is the common error in even-power GP sums.

12. The sum of solutions of the equation

$\frac{\cos x}{1+\sin x} = |\tan 2x|$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ **is :** [4 marks]

(A) $-\frac{11\pi}{30}$

(B) $\frac{\pi}{10}$

(C) $-\frac{7\pi}{30}$

(D) $-\frac{\pi}{15}$

Correct Answer: (A) $-\frac{11\pi}{30}$

Solution:

Concept:

- The half-angle identity gives $\cos x/(1+\sin x) = \tan(\pi/4 - x/2)$. On the specified open interval, its angle lies strictly between 0 and $\pi/2$, so the left side is positive.
- The equality $|\tan u| = \tan v$, when $0 < v < \pi/2$, is equivalent to $u = v + k\pi$ or $u = -v + k\pi$, with integer k. Check the given domain after solving both families.

Step 1: Replace the left side by a single tangent Put $t = \pi/4 - x/2$. Since $-\pi/2 < x < \pi/2$, we have $0 < t < \pi/2$. Also $1 + \sin x > 0$, so this replacement introduces no zero denominator. The equation becomes

$$|\tan 2x| = \tan t.$$

Therefore $\tan 2x = \tan t$ or $\tan 2x = -\tan t$. These two alternatives exhaust the absolute-value equation because $\tan t > 0$.

Step 2: Solve both tangent families in the allowed interval The first family gives

$$2x = \frac{\pi}{4} - \frac{x}{2} + k\pi \quad \Longrightarrow \quad x = \frac{\pi}{10} + \frac{2k\pi}{5}.$$

The only permitted integers are $k = -1, 0$, yielding $-3\pi/10$ and $\pi/10$. The next value $k = 1$ gives the excluded endpoint $\pi/2$. The second family gives

$$2x = -\frac{\pi}{4} + \frac{x}{2} + k\pi \quad \Longrightarrow \quad x = -\frac{\pi}{6} + \frac{2k\pi}{3}.$$

Only $k = 0$ lies in the open interval, yielding $-\pi/6$. None of these three values equals $\pm\pi/4$, where $\tan 2x$ is undefined.

Step 3: Add the distinct solutions The three permitted solutions are distinct, so their sum is

$$-\frac{3\pi}{10} + \frac{\pi}{10} - \frac{\pi}{6} = -\frac{\pi}{5} - \frac{\pi}{6} = -\frac{11\pi}{30}.$$

The other options result neither from this complete set nor from a valid endpoint choice; including $\pi/2$, or omitting one sign branch of the absolute value, changes the required sum.

Step 4: Conclusion The sum of all solutions is $-11\pi/30$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the half-angle quotient first, then use both tangent sign families; check open endpoints and the excluded poles before summing roots.

13. On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line x + 2y = 0. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5-e^2)A$ is : [4 marks]

(A) 6

(B) 12

(C) 14

(D) 24

Correct Answer: (A) 6

Solution:

Concept:

- For $x^2/a^2+y^2/b^2=1$ with $a>b$, the foci are $(\pm c,0)$, where $c^2=a^2-b^2$, and the eccentricity obeys $e^2=1-b^2/a^2$.
- Implicit differentiation gives the tangent slope $-b^2x/(a^2y)$. The triangle with the two foci as its base has area $\frac{1}{2}(2c)|y_P|=c|y_P|$, since the focal axis is the x-axis.

Step 1: Use the perpendicularity condition The line $x+2y=0$ has slope $-1/2$. A perpendicular tangent must therefore have slope 2. Differentiating the ellipse equation yields

$$\frac{x}{4}+\frac{y}{2}\frac{dy}{dx}=0 \quad\Longrightarrow\quad \frac{dy}{dx}=-\frac{x}{2y}.$$

Thus at P, $-x/(2y)=2$, or $x=-4y$. This relation is compatible with the second-quadrant condition $x<0, y>0$.

Step 2: Determine the point and the focal data Substitute $x=-4y$ into the ellipse:

$$\frac{16y^2}{8}+\frac{y^2}{4}=1 \quad\Longrightarrow\quad \frac{9}{4}y^2=1.$$

The second quadrant selects $y=2/3$, giving $P=(-8/3,2/3)$. Here $a^2=8$, $b^2=4$, so

$$c=\sqrt{8-4}=2, \qquad e^2=1-\frac{4}{8}=\frac{1}{2}.$$

Accordingly the focal separation is $SS\prime=4$, not the major-axis length $2\sqrt{8}$.

Step 3: Calculate the triangle area and requested product The base $SS\prime$ lies on the x-axis, and its altitude from P is $2/3$. Therefore

$$A=\frac{1}{2}\times 4\times\frac{2}{3}=\frac{4}{3}.$$

Finally,

$$(5-e^2)A=\left(5-\frac{1}{2}\right)\frac{4}{3}=\frac{9}{2}\frac{4}{3}=6.$$

The other proposed numbers do not agree with this base-and-altitude calculation; doubling the triangle area would incorrectly produce 12.

Step 4: Conclusion The required value of $(5-e^2)A$ is 6. Hence, the correct option is (A).

Quick Tip to Solve: For a triangle formed by a point on an ellipse and its foci, use the focal separation as the base and the point's ordinate as the altitude.

14. The sum of the series

$\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+.....+\frac{2^{100}}{x^{2^{100}}+1}$ **when x = 2 is :** [4 marks]

(A) $1+\frac{2^{101}}{4^{101}-1}$

(B) $1+\frac{2^{100}}{4^{101}-1}$

(C) $1-\frac{2^{100}}{4^{100}-1}$

(D) $1-\frac{2^{101}}{2^{400}-1}$

Correct Answer: The exact sum is $1-\frac{2^{101}}{2^{2^{101}}-1}$. None of the printed options equals this expression, so the printed question has no correct listed option.

Solution:

Concept:

- The identity $1/(u+1)=1/(u-1)-2/(u^2-1)$ follows by bringing the right side over a common denominator. With $u=x^{2^k}$, its coefficients match successive terms of this series.
- For a finite telescoping sum, keep the final uncancelled term. Here the indices run from $k=0$ through $k=100$, so there are 101 terms; the final denominator involves $x^{2^{101}}$.

Step 1: Rewrite the general term as a difference Let

$$S=\sum_{k=0}^{100}\frac{2^k}{x^{2^k}+1}.$$

Applying the identity to $u=x^{2^k}$ gives

$$\frac{2^k}{x^{2^k}+1}=\frac{2^k}{x^{2^k}-1}-\frac{2^{k+1}}{x^{2^{k+1}}-1}.$$

The denominators are nonzero at $x=2$, so the identity is applicable to every term. In particular the second fraction of term k is the first fraction of term $k+1$.

Step 2: Cancel adjacent terms and substitute x On adding the 101 differences, all interior fractions cancel and only the two endpoint fractions remain:

$$S=\frac{1}{x-1}-\frac{2^{101}}{x^{2^{101}}-1}.$$

Therefore, at $x=2$,

$$S=1-\frac{2^{101}}{2^{2^{101}}-1}.$$

This is strictly less than 1 and greater than zero. The terminal exponent is 2^{101}, because the last step squares $x^{2^{100}}$; it is not the linear exponent 400.

Step 3: Compare with every printed option Options (A) and (B) exceed 1, whereas the exact sum is below 1. Option (C) subtracts $2^{100}/(4^{100}-1)=2^{100}/(2^{200}-1)$, and option (D) subtracts $2^{101}/(2^{400}-1)$. Neither denominator equals the endpoint denominator $2^{2^{101}}-1$. These are exact expressions, so their being numerically close to 1 does not make them equal. In particular, comparing the positive

correction in (C) to the correct correction would require $2^{2^{101}} - 1 = 2(2^{200} - 1)$, which is false; (D) would require $2^{101} = 400$, also false.

Step 4: Conclusion The exact sum is $1 - \dfrac{2^{101}}{2^{2^{101}} - 1}$. None of the printed options equals this expression, so the printed question has no correct listed option.

Quick Tip to Solve: In repeated-squaring telescoping sums, track the exponent symbolically through the last cancellation; very large finite powers cannot be replaced by arbitrary large exponents.

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15. If $^{20}C_r$ is the co-efficient of x^r in the expansion of $(1 + x)^{20}$, then the value of $\sum_{r=0}^{20} r^2.{}^{20}C_r$ is equal to : [4 marks]

(A) 420×2^{19}

(B) 380×2^{19}

(C) 380×2^{18}

(D) 420×2^{18}

Correct Answer: (D) 420×2^{18}

Solution:

Concept:

- The binomial theorem gives $F(x) = (1 + x)^{20} = \sum_{r=0}^{20} \binom{20}{r} x^r$. Differentiation introduces the index as a multiplier.
- For a square weight, use $r^2 = r(r - 1) + r$. The second derivative produces the first term, while the first derivative produces the second.

Step 1: Differentiate the binomial expansion Differentiate the polynomial once and twice. As the sum is finite, termwise differentiation is valid everywhere. We obtain

$$F\prime(x) = 20(1 + x)^{19} = \sum_{r=1}^{20} r \binom{20}{r} x^{r-1},$$

$$F\prime\prime(x) = 20 \cdot 19(1 + x)^{18} = \sum_{r=2}^{20} r(r - 1) \binom{20}{r} x^{r-2}.$$

Setting $x = 1$ makes every remaining power of x equal to one. The omitted initial terms contribute zero to their respective weighted sums.

Step 2: Combine the two weighted sums The required expression includes both the falling-factorial weight and the linear weight. Therefore

$$\begin{aligned} S &= F\prime\prime(1) + F\prime(1) \\ &= 380 \cdot 2^{18} + 20 \cdot 2^{19} \\ &= (380 + 40)2^{18} = 420 \cdot 2^{18}. \end{aligned}$$

The coefficient 380 alone would count only $r(r-1)$, so options (B) and (C) cannot represent the square weight. Option (A) is twice the value obtained after combining both contributions.

Step 3: Conclusion The sum is $\boxed{420 \times 2^{18}}$. Hence, the correct option is (D).

Quick Tip to Solve: For $\sum r^2 \binom{n}{r}$, split r^2 into $r(r-1)+r$, then evaluate the first two binomial derivatives at one.

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16. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$, then $\vec{a}.(\vec{b} \times \vec{c})$ is equal to : [4 marks]

(A) -2

(B) -6

(C) 6

(D) 2

Correct Answer: (A) -2

Solution:

Concept:

- A scalar triple product is unchanged by cyclic permutation: $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a})$. Interchanging two vectors reverses its sign.
- The cross product is antisymmetric, so $\vec{a} \times \vec{c} = \vec{b}$ implies $\vec{c} \times \vec{a} = -\vec{b}$. A vector dotted with itself is its squared magnitude.

Step 1: Express the triple product using the given cross product Write the quantity to be evaluated as $T = \vec{a} \cdot (\vec{b} \times \vec{c})$. A cyclic permutation puts the known cross product inside the dot product:

$$T = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{b} \cdot (-\vec{b}) = -|\vec{b}|^2.$$

The negative sign is essential: the given relation has $\vec{a} \times \vec{c}$, whereas the cyclic expression contains $\vec{c} \times \vec{a}$. This method avoids finding all three components of $\vec{c}$.

Step 2: Evaluate the magnitude and check the remaining condition Here $\vec{b} = (0, 1, -1)$, so

$$|\vec{b}|^2 = 0^2 + 1^2 + (-1)^2 = 2, \qquad T = -2.$$

The extra condition $\vec{a} \cdot \vec{c} = 3$ fixes the component of $\vec{c}$ parallel to $\vec{a}$, but cannot change this triple product. For a direct consistency check, $\vec{c} = (5/3, 2/3, 2/3)$ satisfies both given equations and produces $T = -2$. The positive options lose the antisymmetry sign; -6 introduces an unwarranted factor $|\vec{a}|^2 = 3$.

Step 3: Conclusion The required scalar triple product is $\boxed{-2}$. Hence, the correct option is (A).

Quick Tip to Solve: Cycle a scalar triple product until the supplied cross product appears, then check its order before substituting.

---

17. A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P? [4 marks]

(A) $(-1, 1, 2)$

(B) $(0, 1, 1)$

(C) $(1, 0, 1)$

(D) $(2, -1, 1)$

Correct Answer: (B) $(0, 1, 1)$

Solution:

Concept:

- If a line is the intersection of $L_1 = 0$ and $L_2 = 0$, a plane through that line has equation $L_1 + \lambda L_2 = 0$, with the second original plane checked separately if needed.
- Two planes are perpendicular when their normal vectors have zero dot product. A point lies on a plane exactly when its coordinates satisfy the plane equation.

Step 1: Determine the parameter from perpendicularity Take

$$P: \ x + 2y + 3z + 1 + \lambda(x - y - z - 6) = 0.$$

Its normal is $(1 + \lambda, 2 - \lambda, 3 - \lambda)$. The normal of the given perpendicular plane is $(-2, 1, 1)$. Thus

$$-2(1 + \lambda) + (2 - \lambda) + (3 - \lambda) = 0,$$

giving $3 - 4\lambda = 0$, or $\lambda = 3/4$. The omitted family member $x - y - z - 6 = 0$ has normal dot product -4, so it does not satisfy perpendicularity.

Step 2: Obtain the plane and test every point Multiply the family equation by four and substitute the parameter:

$$4(x + 2y + 3z + 1) + 3(x - y - z - 6) = 0.$$

Hence

$$P: \ 7x + 5y + 9z - 14 = 0.$$

The left sides at the four proposed points are respectively

$$\begin{aligned}(-1, 1, 2): \ &-7 + 5 + 18 - 14 = 2,\\ (0, 1, 1): \ &0 + 5 + 9 - 14 = 0,\\ (1, 0, 1): \ &7 + 0 + 9 - 14 = 2,\\ (2, -1, 1): \ &14 - 5 + 9 - 14 = 4.\end{aligned}$$

Only the second point gives zero; the other three are outside the required plane.

Step 3: Conclusion The plane is $7x + 5y + 9z - 14 = 0$, and $\boxed{(0, 1, 1)}$ lies on it. Hence, the correct option is (B).

Quick Tip to Solve: For a plane containing an intersection line, use a linear combination of the two defining equations and impose the normal-vector condition.

18. The value of

$\lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{n^2}{n^2+4r^2}$ **is :** [4 marks]

(A) $\frac{1}{2}\tan^{-1}(2)$

(B) $\frac{1}{2}\tan^{-1}(4)$

(C) $\tan^{-1}(4)$

(D) $\frac{1}{4}\tan^{-1}(4)$

Correct Answer: (B) $\frac{1}{2}\tan^{-1}(4)$

Solution:

Concept:

- A left-endpoint Riemann sum $\Delta x \sum f(x_r)$ converges to the definite integral on its interval when f is continuous.
- The antiderivative $\int dx/(1+4x^2) = \frac{1}{2}\tan^{-1}(2x) + C$ follows by substituting $u = 2x$.

Step 1: Identify the interval and the mesh width Divide numerator and denominator of the summand by n^2:

$$\frac{1}{n}\sum_{r=0}^{2n-1} \frac{1}{1+4(r/n)^2}.$$

Let $x_r = r/n$, $\Delta x = 1/n$. There are $2n$ subintervals, not n, so the interval has length $2n\Delta x = 2$. The sample points are the left endpoints of the equal subdivision of $[0, 2]$. The function $f(x) = 1/(1+4x^2)$ is continuous throughout this interval because its denominator is strictly positive.

Step 2: Integrate and distinguish the proposed values The limit is consequently

$$I = \int_0^2 \frac{dx}{1+4x^2}.$$

Set $u = 2x$, so $dx = du/2$; the endpoints become zero and four. Thus

$$I = \frac{1}{2}\int_0^4 \frac{du}{1+u^2} = \frac{1}{2}\left[\tan^{-1} u\right]_0^4 = \frac{1}{2}\tan^{-1}(4).$$

Option (A) uses the incorrect upper endpoint one for x. Option (C) omits the substitution factor $1/2$, and option (D) inserts an additional factor $1/2$ unsupported by the mesh width.

Step 3: Conclusion The limit equals $\boxed{\frac{1}{2}\tan^{-1}(4)}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the interval length as the number of terms times the mesh width before converting a sum into an integral.

19. Let ABC be a triangle with A(−3, 1) and ∠ACB = θ, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is 2x + y − 3 = 0 and the equation of angle bisector of C is 7x − 4y − 1 = 0, then tanθ is equal to : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{4}{3}$

(D) 2

Correct Answer: (C) $\frac{4}{3}$

Solution:

Concept:

- The median from B passes through the midpoint of AC. The angle bisector from C makes an angle $\theta/2$ with either side at C.
- For slopes m_1, m_2, the tangent of the acute angle between the lines is $|(m_1 - m_2)/(1 + m_1 m_2)|$. The double-angle identity is $\tan(2u) = 2\tan u/(1 - \tan^2 u)$.

Step 1: Locate vertex C using the midpoint condition Write $C = (h, k)$. Since $A = (-3, 1)$, the midpoint of AC is $M = ((h-3)/2, (k+1)/2)$. It belongs to the given median, so

$$2\frac{h-3}{2} + \frac{k+1}{2} - 3 = 0 \quad \Rightarrow \quad 2h + k - 11 = 0.$$

Also C lies on its angle bisector, giving $7h - 4k - 1 = 0$. Solving with $k = 11 - 2h$ gives $15h - 45 = 0$, hence $C = (3, 5)$.

Step 2: Calculate the half-angle and then the required angle The slope of CA is $(5-1)/(3+3) = 2/3$, while the bisector slope is $7/4$. Because $0 < \theta < \pi/2$, the acute angle between this side and the internal bisector is $\theta/2$. Therefore

$$\tan\frac{\theta}{2} = \left|\frac{7/4 - 2/3}{1 + (7/4)(2/3)}\right| = \frac{13/12}{13/6} = \frac{1}{2}.$$

Now

$$\tan\theta = \frac{2(1/2)}{1 - (1/2)^2} = \frac{4}{3}.$$

The value $1/2$ is the half-angle tangent, not the required tangent. The value $3/4$ is its reciprocal after doubling; neither it nor 2 satisfies the double-angle calculation.

Step 3: Conclusion We obtain $\boxed{\tan\theta = 4/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the median only to locate the midpoint of the opposite side; then compare the side with the bisector to obtain the half-angle.

20. If the truth value of the Boolean expression $((p \vee q) \wedge (q \to r) \wedge (\sim r)) \to (p \wedge q)$ is false, then the truth values of the statements p, q, r respectively can be : [4 marks]

(A) T F T

(B) F F T

(C) T F F

(D) F T F

Correct Answer: (C) T F F

Solution:

Concept:

- An implication $X \to Y$ is false only when its antecedent X is true and its consequent Y is false.
- A conjunction is true only when every constituent is true. The implication $q \to r$ is equivalent to $(\sim q) \vee r$.

Step 1: Force the antecedent to be true Let $X = (p \vee q) \wedge (q \to r) \wedge (\sim r)$ and $Y = p \wedge q$. Since the entire implication is false, X must be true. In particular $\sim r$ is true, so $r = \mathrm{F}$. Next $q \to r$ must be true. With a false consequent r, this is possible only if $q = \mathrm{F}$. Finally $p \vee q$ must be true; since q is false, $p = \mathrm{T}$. These requirements determine all three values uniquely.

Step 2: Check the consequent and the alternative assignments For $(p, q, r) = (\mathrm{T}, \mathrm{F}, \mathrm{F})$, the three factors of X are all true:

$$p \vee q = \mathrm{T}, \quad q \to r = \mathrm{T}, \quad \sim r = \mathrm{T}.$$

But $Y = p \wedge q = \mathrm{F}$, so the outer implication is indeed false. In options (A) and (B), r is true, making $\sim r$ false and therefore the antecedent false; the outer implication is then true. In option (D), q is true and r false, making $q \to r$ false. It also makes the antecedent false and cannot satisfy the question.

Step 3: Conclusion The truth values of p, q, r are respectively $\boxed{\mathrm{T}, \mathrm{F}, \mathrm{F}}$. Hence, the correct option is (C).

Quick Tip to Solve: To make a long implication false, first force its antecedent true and consequent false; propagate these constraints through the conjunction.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $z = \frac{1-i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + ... + \left(z^{21} + \frac{1}{z^{21}}\right)^3$ is _____. [4 marks]

Solution:

Concept:

- The complex number $z = (1 - i\sqrt{3})/2 = \cos(\pi/3) - i\sin(\pi/3)$ has modulus one and satisfies $z^6 = 1$. Its inverse is its conjugate.
- De Moivre's formula gives $z^k + z^{-k} = 2\cos(k\pi/3)$. Therefore the terms in the sum repeat after

every six indices.

Step 1: List one complete period of the cubed terms Define $a_k = (z^k + z^{-k})^3$. Since $z^6 = 1$, both powers and their inverses repeat with period six. For $k = 1, 2, 3, 4, 5, 6$, the values of $z^k + z^{-k}$ are respectively

$$1,\ -1,\ -2,\ -1,\ 1,\ 2.$$

Cubing them, with the negative signs retained, gives

$$a_1, a_2, a_3, a_4, a_5, a_6 = 1, -1, -8, -1, 1, 8.$$

Their sum is $1 - 1 - 8 - 1 + 1 + 8 = 0$. Thus every complete group of six consecutive terms starting at the first term contributes zero.

Step 2: Group the twenty-one terms and include the constant The requested expression is $21 + \sum_{k=1}^{21} a_k$, with the constant 21 outside the sum. The first eighteen terms form three complete periods and have total zero. The remaining terms have indices 19, 20 and 21, congruent to 1, 2 and 3 modulo six. Hence

$$\sum_{k=1}^{21} a_k = 3(0) + (1 - 1 - 8) = -8.$$

Adding the initial constant now gives

$$21 + (-8) = 13.$$

This grouping uses every term exactly once; replacing the cube of a sum by the sum of cubes would lose the cross terms and give an incorrect sequence.

Step 3: Conclusion The value of the complete expression is $\boxed{13}$.

Quick Tip to Solve: For powers of a root of unity, write one full period first and group the sum into complete periods plus its remainder.

22. The sum of all integral values of k ($k \neq 0$) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is _____. [4 marks]

Solution:

Concept:

- For a quadratic $ax^2 + bx + c = 0$, real roots exist exactly when $D = b^2 - 4ac \geq 0$. A negative discriminant gives no real root.
- Before clearing denominators, record the excluded inputs. Here $x \neq 1, 2$ and the parameter satisfies $k \neq 0$.

Step 1: Clear the denominators carefully Combine the fractions first:

$$\frac{2}{x-1} - \frac{1}{x-2} = \frac{x-3}{(x-1)(x-2)}.$$

Multiplying by the nonzero denominator and by k gives

$$k(x-3) = 2(x-1)(x-2),$$

so the equivalent polynomial equation on the allowed domain is

$$2x^2 - (k+6)x + (3k+4) = 0.$$

At $x = 1$ its value is $2k$, and at $x = 2$ it is k. Neither vanishes for an allowed parameter. Thus clearing denominators introduces no excluded root, and the discriminant test applies directly.

Step 2: Find and sum the allowed integers The discriminant is

$$D = (k+6)^2 - 8(3k+4) = k^2 - 12k + 4.$$

No real roots requires

$$(k-6)^2 < 32, \qquad 6 - 4\sqrt{2} < k < 6 + 4\sqrt{2}.$$

The bounds are approximately 0.343 and 11.657, so the integral values are precisely $1, 2, \ldots, 11$. All are nonzero. The inequalities are strict because a zero discriminant would give a repeated real root, not no real roots. Therefore

$$\sum_{k=1}^{11} k = \frac{11(12)}{2} = 66.$$

Step 3: Conclusion The required sum of all integral parameter values is $\boxed{66}$.

Quick Tip to Solve: When a rational equation becomes quadratic, test the excluded denominator roots before using its discriminant.

23. If ${}^1P_1 + 2.{}^2P_2 + 3.{}^3P_3 + ... + 15.{}^{15}P_{15} = {}^qP_r - s, 0 \le s \le 1$**, then** ${}^{q+s}C_{r-s}$ **is equal to _____.**

[4 marks]

Solution:

Concept:

- For a positive integer n, ${}^nP_n = n!$. The identity $n\,n! = (n+1)! - n!$ converts the weighted sum into a telescoping sum.
- For permutations ${}^qP_r = q!/(q-r)!$, while ${}^aC_b = a!/[b!(a-b)!]$. The factorial denominator can be either $0!$ or $1!$, both equal to one.

Step 1: Evaluate the given sum exactly Let S denote the left-hand side. Since each displayed permutation selects all its objects,

$$S = \sum_{n=1}^{15} n\,n! = \sum_{n=1}^{15} \big((n+1)! - n!\big).$$

Consecutive factorial terms cancel, leaving

$$S = 16! - 1.$$

The permutation value is an integer, and $0 \le s \le 1$; consequently s is either zero or one. The natural factorial representation is ${}^{16}P_{16} - 1$, which identifies the intended values as $q = 16, r = 16, s = 1$.

Step 2: Calculate the intended value and check uniqueness With those values, the requested expression becomes

$${}^{q+s}C_{r-s} = {}^{17}C_{15} = {}^{17}C_2 = \frac{17 \cdot 16}{2} = 136.$$

There is a parameter ambiguity in the literal statement: ${}^{16}P_{15} = 16!/1! = 16!$ also. Thus $q = 16, r = 15, s = 1$ satisfies the same displayed equation but gives

$${}^{17}C_{14} = {}^{17}C_3 = \frac{17 \cdot 16 \cdot 15}{6} = 680.$$

Both choices obey the usual permutation restrictions. Therefore the standard factorial identification gives the keyed value, but the supplied conditions alone do not make that value unique; an extra restriction such as $q = r$ is needed for uniqueness.

Step 3: Conclusion The intended answer is $\boxed{136}$, using $(q, r, s) = (16, 16, 1)$. Literally, the parameters are not unique: $(16, 15, 1)$ also satisfies the question and gives 680.

Quick Tip to Solve: For sums of weighted factorials, try writing each summand as a difference of consecutive factorials; also check whether parameter identification is unique.

24. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi} + 1\right) k$ is equal to _____. [4 marks]

Solution:

Concept:

- A circle with circumference k has radius $k/(2\pi)$ and area $k^2/(4\pi)$. A square with perimeter p has area $p^2/16$.
- The total wire length is fixed, so one variable determines both areas. A positive second derivative proves that a stationary point is the unique minimum.

Step 1: Express both areas using the circumference Let k metres of wire form the circle. The remaining $36 - k$ metres form the square, whose side is $(36 - k)/4$. The admissible range is $0 < k < 36$, since both pieces form figures. Hence the total area in square metres is

$$A(k) = \frac{k^2}{4\pi} + \frac{(36 - k)^2}{16}.$$

It is important to use circumference rather than radius as the variable: the sum of the wire lengths is a perimeter constraint, not a constraint on areas or diameters.

Step 2: Differentiate and verify the minimum Differentiating with respect to k gives

$$A'(k) = \frac{k}{2\pi} - \frac{36-k}{8}.$$

At a stationary point,

$$\frac{k}{2\pi} = \frac{36-k}{8}, \qquad 4k = \pi(36-k).$$

Therefore

$$\left(\frac{4}{\pi} + 1\right) k = 36.$$

The second derivative is

$$A''(k) = \frac{1}{2\pi} + \frac{1}{8} > 0.$$

Thus the area is strictly convex throughout its domain. Its stationary point $k = 36\pi/(\pi+4)$ lies strictly between zero and 36, so it is the unique global minimum, rather than an endpoint or a maximum.

Step 3: Conclusion At the minimum, the requested quantity is $\boxed{36}$.

Quick Tip to Solve: Use the perimeter assigned to one figure as the variable; then the other perimeter is the total wire length minus that variable.

25. The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is _____. [4 marks]

Solution:

Concept:

- The inequalities $3x^2 \le 4y \le 6x + 24$ put the parabola $y = 3x^2/4$ below the line $y = (6x + 24)/4$.
- The enclosed area is the integral of upper ordinate minus lower ordinate between the intersection abscissae.

Step 1: Find the horizontal extent of the region At an intersection, both boundary values of $4y$ coincide:

$$3x^2 = 6x + 24.$$

Dividing by 3 and factoring gives

$$x^2 - 2x - 8 = (x-4)(x+2) = 0.$$

Thus the intersections have abscissae -2 and 4. The vertical gap is

$$\frac{6x + 24 - 3x^2}{4} = \frac{3}{4}(4-x)(x+2).$$

It is nonnegative precisely on $[-2, 4]$. This checks both the integration interval and which curve is uppermost, avoiding an erroneous signed area.

Step 2: Integrate the vertical gap Using vertical strips, the required area is

$$A = \int_{-2}^{4} \left(\frac{6x + 24}{4} - \frac{3x^2}{4} \right) dx.$$

Take the common factor $1/4$ outside and integrate each term:

$$A = \frac{1}{4} \left[3x^2 + 24x - x^3 \right]_{-2}^{4}.$$

At the upper endpoint the bracket is $48 + 96 - 64 = 80$; at the lower endpoint it is $12 - 48 + 8 = -28$. Therefore

$$A = \frac{1}{4}\big(80 - (-28)\big) = \frac{108}{4} = 27.$$

The answer is positive as required for a geometric area. The factor of four in the original inequality has been retained in both boundary ordinates, not discarded during integration.

Step 3: Conclusion The area of S is $\boxed{27}$ square units.

Quick Tip to Solve: After finding the intersections, factor upper minus lower to check its sign on the interval before integrating.

26. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1), (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to _____.

[4 marks]

Solution:

Concept:

- For a moving point $P(x, y)$, the squared distance to (a, b) is $(x - a)^2 + (y - b)^2$. No square roots are needed when squared distances are summed.
- A circle in standard form is $(x - h)^2 + (y - k)^2 = R^2$. Its diameter satisfies $d^2 = 4R^2$.

Step 1: Translate the distance condition into coordinates Write the squared distances from $P(x, y)$ to the four fixed points in the stated order:

$$x^2 + y^2, \quad (x - 1)^2 + y^2, \quad x^2 + (y - 1)^2, \quad (x - 1)^2 + (y - 1)^2.$$

Their sum equals 18. Collecting terms gives

$$2x^2 + 2(x - 1)^2 + 2y^2 + 2(y - 1)^2 = 18,$$

or, after expansion,

$$4x^2 + 4y^2 - 4x - 4y + 4 = 18.$$

Each coordinate occurs twice with offset zero and twice with offset one. This symmetry also predicts that the centre will be the centre of the unit square.

Step 2: Complete squares and calculate the diameter Divide by four and rearrange:

$$x^2 - x + y^2 - y = \frac{7}{2}.$$

Completing both squares adds $1/4 + 1/4$ to the right-hand side:

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{7}{2} + \frac{1}{2} = 4.$$

Thus the centre is $(1/2, 1/2)$, the radius is $R = 2$, and the diameter is $d = 4$. The question requests the square of the diameter, rather than the radius squared. Accordingly,

$$d^2 = (2R)^2 = 4R^2 = 4 \cdot 4 = 16.$$

As a check, the sum of squared distances from the centre itself is 2, and moving distance 2 from the centre increases that sum by $4 \cdot 2^2 = 16$, giving the required 18.

Step 3: Conclusion The circle has diameter 4, and hence $\boxed{d^2 = 16}$.

Quick Tip to Solve: For a symmetric collection of fixed points, centre the coordinates at their average before completing squares; distinguish radius squared from diameter squared.

---

27. If y = y(x) is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at x = 0 is equal to _____. [4 marks]

Solution:

Concept:

- Implicit differentiation treats y as a function of x. Thus $d\log(x + y)/dx = (1 + y\prime)/(x + y)$, with $x + y > 0$.
- To evaluate a second derivative at a point, first determine the corresponding value of y, then its first derivative, and finally differentiate again.

Step 1: Determine the point and the first derivative At $x = 0$, the given relation becomes $\log_e y = 0$, so $y = 1$; this is consistent with the logarithm domain. Differentiating the complete equation gives

$$\frac{1 + y'}{x + y} = 4(y + xy').$$

At $(x, y) = (0, 1)$, this reduces to

$$1 + y' = 4, \qquad y' = 3.$$

The product xy must be differentiated as $y + xy'$, not simply as y'. The implicit function is locally regular here because the partial derivative of $\log(x + y) - 4xy$ with respect to y equals one at the point.

Step 2: Differentiate again before substituting Differentiating the first-derivative relation using the

quotient rule on the left and the product rule on the right yields

$$\frac{y''(x+y)-(1+y')^2}{(x+y)^2} = 4(2y' + xy'').$$

Now substitute $x = 0$, $y = 1$ and $y' = 3$. The left denominator is one and the term xy'' vanishes, so

$$y'' - (1+3)^2 = 4(2 \cdot 3).$$

Therefore

$$y'' - 16 = 24, \qquad y'' = 40.$$

Substituting $x = 0$ only after differentiating is essential: substituting it before taking the second derivative would discard the contribution from the derivative of xy'.

Step 3: Conclusion The required second derivative at $x = 0$ is $\boxed{40}$.

Quick Tip to Solve: For an implicit second derivative at a point, compute y and y-prime first, but substitute the point only after taking the next derivative.

28. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is _____. [4 marks]

Solution:

Concept:

- A three-digit number must have a nonzero hundreds digit. An even number has units digit 0, 4 or 6 from the supplied digit set.
- When repetition is forbidden, every occupied position removes one available digit. Separate cases by whether the units digit is zero, since that changes the leading-digit restriction.

Step 1: Count numbers ending in zero Fix the units digit as zero. The hundreds digit may then be any of the five nonzero digits $1, 3, 4, 6, 7$. Once it is selected, four of the unused digits remain available for the tens position. Zero is already used, so it cannot reappear. Thus this case gives

$$N_0 = 5 \cdot 4 = 20.$$

Every counted arrangement is a genuine three-digit number and is even. Fixing the final digit first automatically ensures parity and leaves only the no-repetition rule to enforce in the other positions.

Step 2: Count numbers ending in a nonzero even digit If the units digit is 4, the hundreds digit can be chosen from $1, 3, 6, 7$: four choices, since zero cannot lead and 4 is already used. After those two positions are fixed, four unused digits remain for the tens position; zero is allowed there. Hence there are $4 \cdot 4 = 16$ such numbers. The same reasoning applies to a final digit of 6, also giving 16 numbers.

$$N_{4,6} = 2 \cdot 4 \cdot 4 = 32.$$

The three possible units digits define disjoint cases, and exhaust all even possibilities in the set.

Therefore addition, rather than multiplication between these cases, gives

$$N = N_0 + N_{4,6} = 20 + 32 = 52.$$

Treating zero like the other even final digits would incorrectly apply the same number of leading choices to all three cases.

Step 3: Conclusion The number of three-digit even numbers without repeated digits is $\boxed{52}$.

Quick Tip to Solve: When zero is among the digits, fix the units digit first and separate zero from nonzero even endings to enforce the leading-digit rule.

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29. Let a, b ∈ R, b ∈ 0, Define a function

$$f(x) = \begin{cases} a\sin\frac{\pi}{2}(x-1), & for\, x \le 0 \\ \frac{\tan 2x - \sin 2x}{bx^3}, & for\, x > 0 \end{cases}.$$

If f is continuous at x = 0, then 10 − ab is equal to _____. [4 marks]

Solution:

Concept:

- Continuity at zero requires the right-hand limit to equal the value supplied by the branch containing zero: $\lim_{x\to 0^+} f(x) = f(0)$.
- The identities $\tan t - \sin t = \sin t(1-\cos t)/\cos t$, $\sin t/t \to 1$, and $(1-\cos t)/t^2 \to 1/2$ evaluate the cubic-order numerator without cancelling terms incorrectly.

Step 1: Evaluate the value at zero The first branch applies at zero because its condition includes equality. Substituting into its sine argument gives $\frac{\pi}{2}(0-1) = -\frac{\pi}{2}$. Therefore

$$f(0) = a\sin(-\pi/2) = -a.$$

The same expression is continuous from the left, so only the right-hand limit imposes a condition on the parameters. The minus sign in this value is essential.

Step 2: Evaluate the right-hand limit Rewrite the difference of tangent and sine before taking the limit:

$$\frac{\tan 2x - \sin 2x}{bx^3} = \frac{1}{b}\frac{\sin 2x}{x}\frac{1-\cos 2x}{x^2}\frac{1}{\cos 2x}.$$

As $x \to 0^+$, these three factors tend respectively to 2, 2, and 1. Thus the right-hand limit is $4/b$. In particular, the numerator is cubic near zero; replacing both sine and tangent just by their identical linear terms would lose the term that determines this limit.

Step 3: Impose continuity and calculate the expression Equate the two values at the joining point:

$$-a = \frac{4}{b} \quad \Longrightarrow \quad ab = -4.$$

The denominator requires nonzero b, so multiplying by b is legitimate for a defined right branch.

Hence

$$10 - ab = 10 - (-4) = 14.$$

The individual values of a and b are unnecessary because the requested expression depends only on their product.

Step 4: Conclusion The required numerical value is $\boxed{14}$.

Quick Tip to Solve: For a difference such as tangent minus sine, factor it into sine times one minus cosine; this exposes the cubic term and preserves its sign.

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30. Let the line L be the projection of the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ in the plane x − 2y − z = 3. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to _____. [4 marks]

Solution:

Concept:

- The orthogonal projection of a point P on $\mathbf{n} \cdot \mathbf{r} = c$ is $P - [(\mathbf{n} \cdot P - c)/|\mathbf{n}|^2]\mathbf{n}$.
- The projected direction is $\mathbf{v} - (\mathbf{v} \cdot \mathbf{n})\mathbf{n}/|\mathbf{n}|^2$. A vector from a point on a line perpendicular to its direction gives the shortest distance directly.

Step 1: Project one point of the original line The original line passes through $P = (1, 3, 4)$ with direction $\mathbf{v} = (2, 1, 2)$. The plane has normal $\mathbf{n} = (1, -2, -1)$, with $|\mathbf{n}|^2 = 6$ and constant $c = 3$. Since $\mathbf{n} \cdot P = 1 - 6 - 4 = -9$, the projected point is

$$P_0 = P - \frac{-9 - 3}{6}\mathbf{n} = P + 2\mathbf{n} = (3, -1, 2).$$

As a check, $3 - 2(-1) - 2 = 3$, so this point lies on the required plane.

Step 2: Project the direction vector The scalar product $\mathbf{v} \cdot \mathbf{n} = 2 - 2 - 2 = -2$. Remove the normal component to obtain

$$\mathbf{v}_0 = \mathbf{v} + \frac{1}{3}\mathbf{n} = \left(\frac{7}{3}, \frac{1}{3}, \frac{5}{3}\right).$$

A convenient proportional direction is $\mathbf{w} = (7, 1, 5)$. It satisfies $\mathbf{n} \cdot \mathbf{w} = 7 - 2 - 5 = 0$, as required for a line lying in the plane. Thus the projected line is $\mathbf{r} = (3, -1, 2) + t(7, 1, 5)$.

Step 3: Find the perpendicular distance For the given point $Q = (0, 0, 6)$, the displacement from P_0 is $\mathbf{u} = Q - P_0 = (-3, 1, 4)$. Its scalar product with the line direction is

$$\mathbf{u} \cdot \mathbf{w} = -21 + 1 + 20 = 0.$$

Therefore P_0 itself is the foot of the perpendicular from Q to the projected line. Consequently

$$d^2 = |\mathbf{u}|^2 = 9 + 1 + 16 = 26.$$

There is no need to minimize a quadratic or calculate a cross product once this perpendicularity is established.

Step 4: Conclusion The requested squared distance is $\boxed{d^2 = 26}$.

Quick Tip to Solve: Project both a point and the direction of the line, then test whether the displacement from the given point is already perpendicular to the projected direction.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The fractional change in the magnetic field intensity at a distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at the centre of the same coil is : (Take r $<$ a) [4 marks]

(A) $\frac{3}{2}\frac{a^2}{r^2}$
(B) $\frac{2}{3}\frac{a^2}{r^2}$
(C) $\frac{2}{3}\frac{r^2}{a^2}$
(D) $\frac{3}{2}\frac{r^2}{a^2}$

Correct Answer: (D) $\frac{3}{2}\frac{r^2}{a^2}$

Solution:

Concept:

- For a circular coil of radius a, the axial magnetic field is $B(r) = \mu_0 N I a^2/[2(a^2 + r^2)^{3/2}]$, where N is the number of turns and I the current.
- The fractional decrease from the central field is $[B(0) - B(r)]/B(0)$. For a near-centre point, the leading binomial term is obtained from $(1 + u)^{-3/2} = 1 - 3u/2 + O(u^2)$.

Step 1: Form the ratio to the central field At the centre, put $r = 0$ in the axial-field formula to get $B(0) = \mu_0 N I/(2a)$. Dividing the field at the specified point by this value cancels the current, number of turns and magnetic constant:

$$\frac{B(r)}{B(0)} = \frac{a^3}{(a^2 + r^2)^{3/2}} = \left(1 + \frac{r^2}{a^2}\right)^{-3/2}.$$

The ratio is less than one, consistent with the field decreasing away from the centre along the axis.

Step 2: Expand the fractional decrease near the centre The exact fractional decrease is

$$\delta = 1 - \left(1 + \frac{r^2}{a^2}\right)^{-3/2}.$$

Using the near-centre, leading-order approximation represented by the options, set $u = r^2/a^2$ and expand:

$$\delta = \frac{3}{2}\frac{r^2}{a^2} + O\left(\frac{r^4}{a^4}\right).$$

The condition $r < a$ makes the binomial series applicable; retaining only its first correction gives the usual small-displacement approximation. It is most accurate when r/a is much smaller than one.

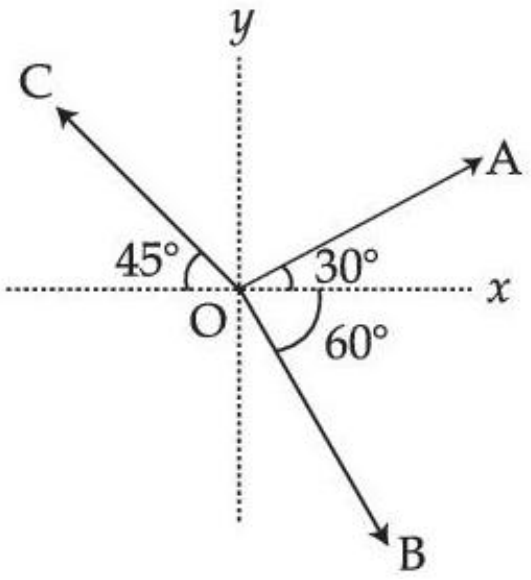
Step 3: Compare the alternatives The decrease must vanish at the centre. Options (A) and (B), which

are proportional to a^2/r^2, diverge there and therefore cannot be correct. Both remaining choices have the right quadratic dependence, but differentiating or expanding $(1+u)^{-3/2}$ fixes the coefficient at $3/2$, rather than $2/3$. If change is defined with a signed difference $B(r) - B(0)$, its sign is negative; the options give its magnitude.

Step 4: Conclusion To leading order, the magnitude of the fractional change is $\boxed{3r^2/(2a^2)}$. Hence, the correct option is (D).

Quick Tip to Solve: Divide by the central field first: the coil current and turn count cancel, leaving a single binomial expression in the small ratio squared.

32. The magnitude of vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with x-axis will be : [4 marks]



(A) $\tan^{-1}\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

(B) $\tan^{-1}\frac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$

(C) $\tan^{-1}\frac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$

(D) $\tan^{-1}\frac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$

Correct Answer: (A) $\tan^{-1}\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

Solution:

Concept:

- A vector of magnitude V at angle θ from the positive x-axis has components $V\cos\theta$ and $V\sin\theta$. Subtraction reverses both components of the subtracted vector.
- The direction of a resultant $\mathbf{R}$ follows from $\tan\phi = R_y/R_x$, with its quadrant checked using the signs of both components.

Step 1: Read the directions and resolve the vectors Let the common magnitude be V. In the figure, $\overrightarrow{OA}$ is at 30°, $\overrightarrow{OB}$ at -60°, and $\overrightarrow{OC}$ at 135°, all measured from the positive x-axis. Therefore

$$\overrightarrow{OA} = \tfrac{V}{2}(\sqrt{3}, 1),$$
$$\overrightarrow{OB} = \tfrac{V}{2}(1, -\sqrt{3}),$$
$$\overrightarrow{OC} = \tfrac{V}{2}(-\sqrt{2}, \sqrt{2}).$$

The negative horizontal component of the last vector is particularly important.

Step 2: Subtract the third vector component by component For $\mathbf{R} = \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$, subtract both components of $\overrightarrow{OC}$. This gives

$$R_x = \frac{V}{2}(\sqrt{3} + 1 + \sqrt{2}), \qquad R_y = \frac{V}{2}(1 - \sqrt{3} - \sqrt{2}).$$

Thus $R_x > 0$ and $R_y < 0$; the resultant lies in the fourth quadrant, below the positive x-axis. Reversing only the horizontal component of the subtracted vector would give an incorrect direction.

Step 3: Calculate the angle and identify the option The common factor $V/2$ cancels in the component ratio:

$$\tan\phi = \frac{1 - \sqrt{3} - \sqrt{2}}{1 + \sqrt{3} + \sqrt{2}}.$$

The principal inverse tangent here is negative and lies between -90° and zero, which is exactly the quadrant established above. Option (A) has both the required numerator and denominator. Options (B) and (C) give a positive tangent with the wrong signs, while (D) interchanges the required component expressions.

Step 4: Conclusion $\phi = \tan^{-1}\left(\dfrac{1 - \sqrt{3} - \sqrt{2}}{1 + \sqrt{3} + \sqrt{2}}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: Write the two signed components of the vector being subtracted before combining terms; then check the quadrant before choosing an inverse-tangent expression.

33. Car B overtakes another car A at a relative speed of 40 ms^{-1}. How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A? [4 marks]

(A) 4 ms^{-1}

(B) 0.2 ms^{-1}

(C) 40 ms^{-1}

(D) 0.1 ms^{-1}

Correct Answer: (D) 0.1 ms^{-1}

Solution:

Concept:

- A car rear-view mirror is convex. With Cartesian signs, a real object has negative object distance u, while a convex mirror has positive focal length f.
- The mirror equation is $1/v + 1/u = 1/f$. Differentiating it with respect to time relates image speed to object speed: $\dot{v} = -(v/u)^2\dot{u}$.

Step 1: Find the instantaneous image position Work in the frame of car A, because both its mirror and the observer move with that car. The relative speed of car B is $40\,\mathrm{m\,s^{-1}}$. At the specified instant, $u = -1.9\,\mathrm{m}$ and $f = +0.10\,\mathrm{m}$. Substitution in the mirror formula gives

$$\frac{1}{v} = \frac{1}{0.10} - \frac{1}{-1.9} = 10 + \frac{10}{19} = \frac{200}{19}.$$

Thus $v = 19/200 = 0.095\,\mathrm{m}$, behind the convex mirror.

Step 2: Differentiate the mirror equation The focal length is constant, so differentiating the two reciprocal distances yields

$$-\frac{\dot{v}}{v^2} - \frac{\dot{u}}{u^2} = 0.$$

It follows that the magnitude of the image velocity relative to the mirror is

$$|\dot{v}| = \left|\frac{v}{u}\right|^2 |\dot{u}|.$$

The speed ratio is the square of the linear magnification, not the magnification itself.

Step 3: Substitute and compare the speeds Here the magnitude of the instantaneous distance ratio is $0.095/1.9 = 0.05 = 1/20$. Therefore

$$|\dot{v}| = \frac{40}{20^2} = \frac{40}{400} = 0.10\,\mathrm{m\,s^{-1}}.$$

The virtual image is much slower than the object. The values 4 and $0.2\,\mathrm{m\,s^{-1}}$ do not follow from the squared ratio, and $40\,\mathrm{m\,s^{-1}}$ would wrongly equate image and object speeds.

Step 4: Conclusion The image appears to move at $\boxed{0.1\,\mathrm{m\,s^{-1}}}$ relative to car A. Hence, the correct option is (D).

Quick Tip to Solve: For image speed along a mirror axis, first find the instantaneous image distance and then use the square of the image-to-object distance ratio.

34. Inside a uniform spherical shell :

(1) the gravitational field is zero

(2) the gravitational potential is zero

(3) the gravitational field is same everywhere

(4) the gravitational potential is same everywhere

(5) all of the above

Choose the most appropriate answer from the options given below : [4 marks]

(A) (1), (3) and (4) only

(B) (5) only

(C) (1), (2) and (3) only

(D) (2), (3) and (4) only

Correct Answer: (A) (1), (3) and (4) only

Solution:

Concept:

- Newton's shell theorem gives zero gravitational field at every point strictly inside a uniform spherical shell, regardless of the point's distance from its centre.
- With gravitational potential chosen to be zero at infinity, a shell of mass M and radius R has constant interior potential $V = -GM/R$. A zero field implies spatially constant potential, not necessarily zero potential.

Step 1: Evaluate the field statements For a spherically symmetric shell, take a concentric Gaussian sphere with radius smaller than R. It encloses no mass. Spherical symmetry and the gravitational flux law then give zero field on that sphere; allowing its radius to vary covers the whole interior. Thus $\mathbf{g} = 0$ everywhere inside. Statement (1) is true, and statement (3) is also true: the field has the same value, namely zero, at all interior points. These statements are compatible rather than competing alternatives.

Step 2: Evaluate the potential statements Because $\mathbf{g} = -\nabla V = 0$ in the interior, potential has no spatial variation there. Matching continuously to the surface potential gives

$$V_{\text{inside}} = V(R) = -\frac{GM}{R}.$$

For a massive shell, this constant is negative and nonzero when the conventional reference at infinity is used. Hence statement (2), which says potential is zero, is false, while statement (4), which says potential is the same everywhere, is true. Potential measures work per unit mass relative to a reference; a vanishing force does not erase its value.

Step 3: Select the complete correct combination The true statements are exactly (1), (3), and (4). Since statement (2) is false, statement (5), asserting that all the preceding claims hold, is false as well. Option (B) therefore fails. Options (C) and (D) both include the false zero-potential claim. Only option (A) includes all three true statements while omitting that false claim.

Step 4: Conclusion Statements (1), (3), and (4) only are correct. Hence, the correct option is (A).

Quick Tip to Solve: Always separate a zero potential gradient from a zero potential value: zero field inside a shell means a flat potential at its nonzero surface value.

35. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube. [Take surface tension of water T = 7.3 $\times$ 10^{-2} Nm^{-1}, angle of contact = 0, g = 10 ms^2 and density of water = 1.0 $\times$ 10^3 kg m^{-3}] [4 marks]

(A) 3.62 mm

(B) 2.19 mm

(C) 5.34 mm

(D) 4.97 mm

Correct Answer: (B) 2.19 mm

Solution:

Concept:

- For a wetting liquid in a circular capillary of diameter D, the capillary rise is $h = 4T\cos\theta/(\rho g D)$, where T is surface tension and θ the contact angle.
- Both limbs belong to the same connected liquid, so pressures at a common horizontal level agree. Their different meniscus curvatures produce a level difference equal to the difference of their capillary rises.

Step 1: Express the level difference Let the narrower and wider diameters be $D_1 = 5.0 \times 10^{-3}\,\mathrm{m}$ and $D_2 = 8.0 \times 10^{-3}\,\mathrm{m}$. The narrower limb has the larger capillary rise because water wets the wall and $\cos 0 = 1$. Subtracting the rises eliminates the arbitrary common reference level:

$$\Delta h = \frac{4T}{\rho g}\left(\frac{1}{D_1} - \frac{1}{D_2}\right).$$

This is also obtained by equating hydrostatic pressure differences to the difference in the Laplace pressure jumps at the two menisci.

Step 2: Convert the diameters and substitute Use all quantities in SI units: $T = 7.3 \times 10^{-2}\,\mathrm{N\,m^{-1}}$, $\rho = 1.0 \times 10^{3}\,\mathrm{kg\,m^{-3}}$, and the given gravitational acceleration magnitude $g = 10\,\mathrm{m\,s^{-2}}$. Then

$$\frac{4T}{\rho g} = \frac{4(7.3 \times 10^{-2})}{(10^3)(10)} = 2.92 \times 10^{-5}\,\mathrm{m^2}.$$

The reciprocal-diameter difference is $200 - 125 = 75\,\mathrm{m^{-1}}$. Keeping diameters in metres avoids a factor-of-one-thousand error.

Step 3: Calculate and compare the alternatives Multiply the preceding factors:

$$\Delta h = (2.92 \times 10^{-5})(75)\,\mathrm{m} = 2.19 \times 10^{-3}\,\mathrm{m} = 2.19\,\mathrm{mm}.$$

As a check, the separate rises are $5.84\,\mathrm{mm}$ and $3.65\,\mathrm{mm}$, whose difference is $2.19\,\mathrm{mm}$. The other options do not equal this difference. Adding the rises would be inappropriate because both water menisci rise relative to the same hydrostatic reference.

Step 4: Conclusion The water level in the narrower limb is higher by $\boxed{2.19\,\mathrm{mm}}$. Hence, the correct option is (B).

Quick Tip to Solve: When diameters are given, use the factor four in the capillary formula and subtract reciprocal diameters; do not add the two capillary rises.

36. An electric appliance supplies 6000 J/min heat to the system. If the system delivers a power of 90W. How long it would take to increase the internal energy by 2.5×10^3 J ? [4 marks]

(A) 2.5×10^2 s

(B) 4.1×10^1 s

(C) 2.4×10^3 s

(D) 2.5×10^1 s

Correct Answer: (A) 2.5×10^2 s

Solution:

Concept:

- The first law is $\Delta U = Q - W$, with W positive when the system does work on its surroundings.
- For constant input and output powers, the rate of storage of internal energy is $dU/dt = P_{\text{in}} - P_{\text{out}}$. Convert both powers to the same unit before subtracting.

Step 1: Convert the heat input to power The appliance supplies heat at a rate of $6000\,\text{J/min}$. Since one minute is $60\,\text{s}$, its input power is

$$P_{\text{in}} = \frac{6000}{60} = 100\,\text{J}\,\text{s}^{-1} = 100\,\text{W}.$$

The stated $90\,\text{W}$ is work delivered by the system, so it removes energy rather than increasing the stored internal energy.

Step 2: Calculate the net energy stored each second Apply the first law to a time interval t. The heat received is $100t$ joules and the work delivered is $90t$ joules. Thus

$$\Delta U = (100 - 90)t = 10t.$$

Only $10\,\text{J}$ of the supplied energy remains in the system each second; the rest leaves as mechanical or electrical work.

Step 3: Find the required duration The required increase is $2.5 \times 10^3\,\text{J}$. Therefore

$$t = \frac{2.5 \times 10^3}{10} = 250\,\text{s}.$$

Using the full heat input alone would ignore the work output. Adding the two powers would also reverse the physical role of work delivered. The other listed durations do not satisfy the energy balance.

Step 4: Conclusion The time required is $250\,\text{s} = 2.5 \times 10^2\,\text{s}$. Hence, the correct option is (A).

Quick Tip to Solve: For constant heating with simultaneous work output, divide the required energy increase by input power minus output power, after converting J/min to watts.

37. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds : [4 marks]

(A) 0.4

(B) 0.8

(C) 0.125

(D) 0.2

Correct Answer: (D) 0.2

Solution:

Concept:

- The magnetic energy stored in a coil carrying current I is $U = \frac{1}{2}LI^2$, where L is its inductance.
- The winding resistance dissipates power $P = I^2R$. When this coil is connected to an ideal battery, its RL time constant is $\tau = L/R$; the ideal source adds no resistance.

Step 1: Determine the inductance from stored energy The current is $8\,\text{A}$, so $I^2 = 64\,\text{A}^2$. Substituting the magnetic energy gives

$$64 = \frac{1}{2}L(8)^2 = 32L, \qquad L = 2\,\text{H}.$$

The factor of one-half is essential: magnetic energy is not simply LI^2.

Step 2: Determine the resistance from heating The same current dissipates $640\,\text{W}$ in the winding. Joule heating therefore gives

$$640 = (8)^2R = 64R, \qquad R = 10\,\Omega.$$

This resistance is a property of the actual coil even though the battery is ideal; ideality of the source does not make the coil resistance vanish.

Step 3: Form the time constant For the growth of current in a series RL circuit, the exponential factor is $e^{-tR/L}$. Its characteristic time is consequently

$$\tau = \frac{L}{R} = \frac{2}{10} = 0.2\,\text{s}.$$

One may independently eliminate the current between the two given equations: $2U/P = L/R = 128/640\,\text{s}$. Options 0.4, 0.8, and 0.125 do not equal this ratio; using U/P misses the energy factor of two.

Step 4: Conclusion The circuit time constant is $\boxed{0.2\,\text{s}}$. Hence, the correct option is (D).

Quick Tip to Solve: When both stored magnetic energy and resistive power are known at the same current, use the shortcut $\tau = 2U/P$; the current cancels.

38. A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance X_L = 250 $\pi\Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be : (Take π^2 = 10) [4 marks]

(A) 4 μF

(B) 25 μF

(C) 400 μF

(D) 40 μF

Correct Answer: (A) 4 μF

Solution:

Concept:

- The average power in a driven series LCR circuit is $P = V_{\text{rms}}^2 R/[R^2 + (X_L - X_C)^2]$, with $X_C = 1/(\omega C)$.
- For fixed source voltage, frequency, resistance and inductance, changing the capacitor maximizes the power at resonance, where $X_C = X_L$.

Step 1: Identify the condition for maximum power The resistance $R = 3\,\text{k}\Omega$ and source voltage are fixed. The denominator of the power expression is smallest when its nonnegative reactance term vanishes:

$$X_L - X_C = 0.$$

This makes the impedance purely resistive, the current largest, and voltage and current in phase. The source voltage does not affect which capacitance gives this maximum.

Step 2: Calculate the resonant capacitance The angular frequency is

$$\omega = 2\pi f = 100\pi\,\text{rad}\,\text{s}^{-1}.$$

Since $X_L = 250\pi\,\Omega$, impose $1/(\omega C) = 250\pi$, obtaining

$$C = \frac{1}{(100\pi)(250\pi)} = \frac{1}{25000\pi^2}\,\text{F}.$$

Step 3: Convert to microfarads and test the options Use the approximation explicitly supplied in the question, $\pi^2 = 10$:

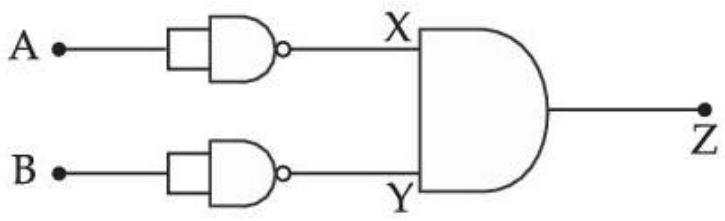
$$C = \frac{1}{250000}\,\text{F} = 4 \times 10^{-6}\,\text{F} = 4\,\mu\text{F}.$$

For this capacitance, $X_C = 250\pi\,\Omega$ exactly within the stated approximation. Larger choices 25, 40, or 400 μF reduce X_C below X_L, leaving an inductive mismatch and hence a smaller average power.

Step 4: Conclusion The capacitance for maximum average power is $\boxed{4\,\mu\text{F}}$. Hence, the correct option is (A).

Quick Tip to Solve: For maximum power in a series LCR circuit with a fixed resistance, first set $X_C = X_L$; the given voltage and resistance are unnecessary for finding the resonant capacitor.

39. Identify the logic operation carried out by the given circuit :- [4 marks]



(A) OR

(B) AND

(C) NOR

(D) NAND

Correct Answer: (C) NOR

Solution:

Concept:

- A NAND gate gives the complement of the product of its inputs. With both inputs tied to the same signal, it acts as an inverter: $\overline{A \cdot A} = \overline{A}$.
- The final gate in the diagram is an AND gate, so its output is the product of its two input signals. De Morgan law gives $\overline{A}\,\overline{B} = \overline{A + B}$.

Step 1: Read the two input gates The upper small gate has the shape of an AND gate with a bubble at its output, identifying it as NAND. Its inputs are joined to A, so its output is

$$X = \overline{A \cdot A} = \overline{A}.$$

The lower small gate has the same configuration for input B, giving $Y = \overline{B}$. The bubbles must be included; omitting them would give the wrong overall operation.

Step 2: Combine the inverted signals The larger gate at the right has no output bubble and is an AND gate. Therefore

$$Z = XY = \overline{A}\,\overline{B} = \overline{A + B}.$$

This expression is the complement of the logical OR of the original inputs, which is precisely the NOR operation.

Step 3: Check the truth values For $(A, B) = (0, 0)$, both inverter outputs equal one and the final output is one. For $(0, 1)$, $(1, 0)$, or $(1, 1)$, at least one final AND input is zero, so $Z = 0$. Thus the output sequence is $1, 0, 0, 0$. OR would instead give $0, 1, 1, 1$, AND gives $0, 0, 0, 1$, and NAND gives $1, 1, 1, 0$, ruling out the other choices.

Step 4: Conclusion The circuit implements $Z = \overline{A + B}$, the NOR operation. Hence, the correct option is (C).

Quick Tip to Solve: Replace every NAND gate with joined inputs by a NOT operation, then apply De Morgan law to the expression at the final gate.

40. A particular hydrogen like ion emits radiation of frequency 2.92×10^{15} Hz when it makes transition from n = 3 to n = 1. The frequency in Hz of radiation emitted in transition from n = 2 to n = 1 will be : [4 marks]

(A) 0.44×10^{15}

(B) 6.57×10^{15}

(C) 4.38×10^{15}

(D) 2.46×10^{15}

Correct Answer: (D) 2.46×10^{15}

Solution:

Concept:

- For a hydrogen-like ion, the transition frequency is $\nu = cRZ^2(1/n_f^2 - 1/n_i^2)$, where Z is the nuclear charge.
- Ratios of frequencies for the same ion cancel the common factor cRZ^2. Frequency is proportional to the energy difference, not simply to a principal quantum number.

Step 1: Express the two transition frequencies For the given transition from $n = 3$ to $n = 1$,

$$\nu_{31} = cRZ^2\left(1 - \frac{1}{9}\right) = cRZ^2\frac{8}{9}.$$

For the required transition from $n = 2$ to $n = 1$,

$$\nu_{21} = cRZ^2\left(1 - \frac{1}{4}\right) = cRZ^2\frac{3}{4}.$$

Both are emissions because the final energy level lies below the initial level.

Step 2: Take the ratio and substitute Dividing eliminates the unknown ionic charge as well as the constants:

$$\frac{\nu_{21}}{\nu_{31}} = \frac{3/4}{8/9} = \frac{27}{32}.$$

Hence

$$\nu_{21} = 2.92 \times 10^{15} \times \frac{27}{32} = 2.46375 \times 10^{15}\,\text{Hz}.$$

To the precision used in the options this is 2.46×10^{15} Hz.

Step 3: Check the physical scale The $2 \to 1$ energy gap is smaller than the $3 \to 1$ gap, so its frequency must be below 2.92×10^{15} Hz. This immediately excludes the two larger options. The ratio $27/32$ is about 0.844, rather than about 0.151, so the much smaller 0.44×10^{15} option is also inconsistent with the level spacings.

Step 4: Conclusion The required frequency is $\boxed{2.46 \times 10^{15}\,\text{Hz}}$. Hence, the correct option is (D).

Quick Tip to Solve: For two spectral transitions of the same hydrogen-like ion, take the ratio of the reciprocal-square level differences; the nuclear charge cancels.

41. In a photoelectric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work function ϕ = 2.5 eV. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. (h = 6.63 $\times$ 10^{-34} Js, c = 3 $\times$ 10^{8} ms^{-1}) [4 marks]

(A) 1.3 V

(B) 1.1 V

(C) 1.9 V

(D) 0.6 V

Correct Answer: (A) 1.3 V

Solution:

Concept:

- Einstein photoelectric equation is $eV_s = hc/\lambda - \phi$, where V_s is the stopping potential and ϕ is the work function.
- For the same cathode, subtraction cancels ϕ: the change in stopping potential depends only on the change in photon energy, provided both wavelengths produce emission.

Step 1: Check that both lights produce photoelectrons The photon energy in electronvolts is approximately $1243/\lambda$ when the wavelength is in nanometres, using the supplied values of h and c and $e = 1.60 \times 10^{-19}\,\mathrm{C}$. Thus the two photon energies are about $4.44\,\mathrm{eV}$ and $3.11\,\mathrm{eV}$. Both exceed the work function $2.5\,\mathrm{eV}$, so a stopping potential exists in each case.

Step 2: Subtract the two photoelectric equations Let the first and second stopping potentials be V_1 and V_2. Then

$$V_1 - V_2 = \frac{hc}{e}\left(\frac{1}{280 \times 10^{-9}} - \frac{1}{400 \times 10^{-9}}\right).$$

The work function cancels because the metal is unchanged. Inserting the constants yields

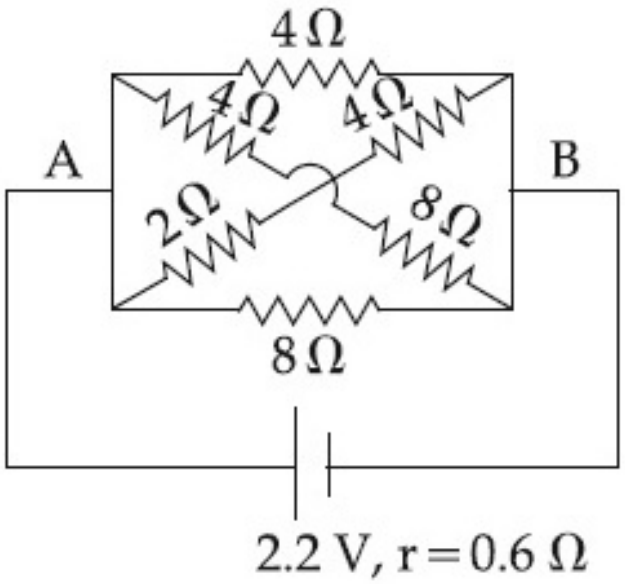
$$V_1 - V_2 = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.60 \times 10^{-19}} \times 1.07143 \times 10^{6} \approx 1.33\,\mathrm{V}.$$

Step 3: Interpret the change Increasing the wavelength lowers the photon energy and therefore the maximum kinetic energy. The stopping potential consequently decreases: $V_2 - V_1 \approx -1.33\,\mathrm{V}$. The question lists positive magnitudes, so the matching value is $1.3\,\mathrm{V}$. The 1.9 and $0.6\,\mathrm{V}$ choices are close to individual stopping potentials, whereas the requested quantity is their difference; $1.1\,\mathrm{V}$ is not obtained from the energy difference.

Step 4: Conclusion The stopping potential decreases by approximately $\boxed{1.3\,\mathrm{V}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a wavelength change on the same metal, subtract the two stopping-potential equations before substituting; the work function disappears.

42. In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6Ω. Calculate the power dissipated in the whole circuit : [4 marks]



(A) 1.32 W

(B) 0.65 W

(C) 2.2 W

(D) 4.4 W

Correct Answer: (C) 2.2 W

Solution:

Concept:

- Resistors in series add, whereas parallel branches satisfy $1/R_{\text{ext}} = \sum 1/R_i$. A wire crossing with a bridge is not a junction.
- Include the internal resistance r when calculating the current: $I = \mathcal{E}/(R_{\text{ext}} + r)$. Power dissipated in the whole circuit is $I^2(R_{\text{ext}} + r) = \mathcal{E}I$.

Step 1: Read the four branches between A and B The left vertical wire makes the two left corners a common node A, and the right vertical wire similarly makes both right corners node B. The diagonals cross without electrical contact, as indicated by the bridge in the drawing. Thus the top and bottom branches have resistances $4\,\Omega$ and $8\,\Omega$. The diagonal branches have resistances $4 + 8 = 12\,\Omega$ and $2 + 4 = 6\,\Omega$.

Step 2: Reduce the external network and find the current These four branches are parallel:

$$\frac{1}{R_{\text{ext}}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{6} = \frac{15}{24} = \frac{5}{8}.$$

Hence $R_{\text{ext}} = 1.6\,\Omega$. Including the cell resistance gives

$$R_{\text{total}} = 1.6 + 0.6 = 2.2\,\Omega, \qquad I = \frac{2.2}{2.2} = 1\,\text{A}.$$

Step 3: Calculate the total dissipation The phrase whole circuit includes heat in the internal resistance as well as the external resistors. Therefore

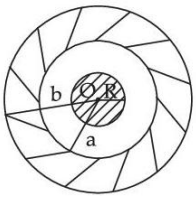
$$P = I^2 R_{\text{total}} = 1^2(2.2) = 2.2\,\text{W}.$$

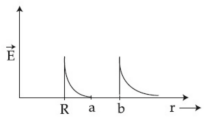
As an energy check, the external network dissipates $1.6\,\text{W}$ and the cell internally dissipates $0.6\,\text{W}$, summing to the same result. The other listed powers do not equal the source power $\mathcal{E}I$ for the current found from the full circuit.

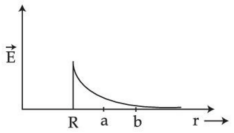
Step 4: Conclusion The power dissipated in the whole circuit is $\boxed{2.2\,\text{W}}$. Hence, the correct option is (C).

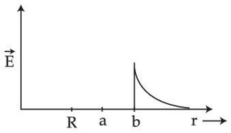
Quick Tip to Solve: First collapse wires into common nodes and distinguish a bridged crossing from a junction. For whole-circuit power include internal resistance and check with $P = \mathcal{E}I$.

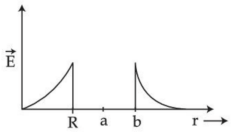
43. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in the figure. The approximate variation electric field $\vec{E}$ as a function of distance r from centre O is given by [4 marks]



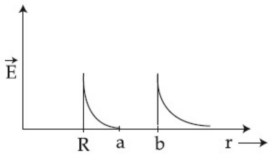
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- In electrostatic equilibrium the electric field inside the material of a conductor is zero; excess charge lies on its surfaces.
- For a spherical Gaussian surface, Gauss's law gives $E(4\pi r^2) = q_{\text{enclosed}}/\varepsilon_0$. Spherical symmetry makes the field radial and constant in magnitude on that surface.

Step 1: Identify the conductor regions The solid metal sphere occupies $0 < r < R$, while the metal shell occupies $a < r < b$. Thus the field vanishes throughout both these intervals. The field cannot rise linearly or quadratically inside the solid sphere, as it might for a nonconducting volume charge distribution. This immediately excludes graph (D).

Step 2: Apply Gauss's law in the cavity For $R < r < a$, a Gaussian sphere encloses the central charge q. Therefore

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2}.$$

The field jumps from zero to $q/(4\pi\varepsilon_0 R^2)$ just outside the inner sphere, then decreases inversely with the square of distance until the shell is reached. It becomes zero in the shell material. Graph (C) omits the nonzero cavity field; graph (B) incorrectly continues the field through the shell.

Step 3: Find the exterior behaviour The initially uncharged shell acquires $-q$ on its inner surface and $+q$ on its outer surface. Outside the shell the net enclosed charge is still q, giving

$$E(r) = \frac{q}{4\pi\varepsilon_0 r^2}, \qquad r > b.$$

Consequently the field jumps up again at b and then approaches zero as r increases. Among the approximate sketches, only (A) has both nonzero inverse-square regions separated by the zero-field metal shell.

Step 4: Conclusion The field is zero for $r < R$ and $a < r < b$, and varies as $1/r^2$ in the cavity and exterior. Hence, the correct option is (A).

Quick Tip to Solve: First mark the metal regions on the distance axis: the field must be zero there. Then use enclosed charge to fill the remaining intervals.

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44. The rms speeds of the molecules of Hydrogen, Oxygen and Carbon dioxide at the same temperature are V_H, V_O and V_C respectively then : [4 marks]

(A) $V_H > V_O > V_C$

(B) $V_C > V_O > V_H$

(C) $V_H = V_O > V_C$

(D) $V_H = V_O = V_C$

Correct Answer: (A) $V_H > V_O > V_C$

Solution:

Concept:

- For an ideal gas, $v_{\text{rms}} = \sqrt{3RT/M}$, where T is absolute temperature and M is molar mass in consistent units.
- At equal temperature all gases have the same mean translational kinetic energy per molecule, but their speeds depend on molecular mass.

Step 1: List the molecular masses Hydrogen is molecular H_2, oxygen is molecular O_2, and carbon dioxide is CO_2. Their molar masses are respectively 2, 32, and $44\,\text{g mol}^{-1}$. The labels H, O and C in

the question identify the gases; they do not mean isolated hydrogen, oxygen or carbon atoms. Thus the mass order is

$$M_H < M_O < M_C.$$

Step 2: Compare their rms speeds Since $3RT$ is common, take the inverse square roots of these masses:

$$V_H : V_O : V_C = \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{32}} : \frac{1}{\sqrt{44}}.$$

For example,

$$\frac{V_H}{V_O} = \sqrt{\frac{32}{2}} = 4, \qquad \frac{V_O}{V_C} = \sqrt{\frac{44}{32}} > 1.$$

Both comparisons are strict, so hydrogen is fastest, followed by oxygen and then carbon dioxide.

Step 3: Check the alternative statements Option (B) reverses the inverse-mass dependence and would make the heaviest gas fastest. Option (C) incorrectly equates the hydrogen and oxygen speeds despite a factor of sixteen between their molecular masses. Option (D) confuses equal average kinetic energies with equal speeds. From $\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_BT$, a lighter molecule must move faster to have the same mean kinetic energy.

Step 4: Conclusion The order is $V_H > V_O > V_C$. Hence, the correct option is (A).

Quick Tip to Solve: At a common temperature, rank rms speeds in the reverse order of molar masses; use square-root mass ratios for a quantitative check.

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45. In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20th division of the circular scale coincides with reference line. Calculate the true reading. [4 marks]

(A) 5.00 mm

(B) 5.25 mm

(C) 5.15 mm

(D) 5.20 mm

Correct Answer: (C) 5.15 mm

Solution:

Concept:

- The least count of a screw gauge is the pitch divided by the number of circular-scale divisions.
- The true reading is the observed reading minus the zero error. A positive reading with the jaws closed is a positive zero error.

Step 1: Calculate the least count The main scale advances $0.5\,\text{mm}$ in one complete rotation, so the pitch is $0.5\,\text{mm}$. There are 50 equal circular divisions. Thus

$$\text{LC} = \frac{0.5}{50} = 0.01\,\text{mm}.$$

Each circular division contributes one hundredth of a millimetre, not half a millimetre; the pitch refers to the entire revolution.

Step 2: Determine the observed reading and zero error For the measurement, the main-scale reading is $5\,\text{mm}$ and the circular-scale reading is 20 divisions. Therefore

$$x_{\text{obs}} = 5 + 20(0.01) = 5.20\,\text{mm}.$$

When the ratchet is closed, the fifth division coincides with the reference line instead of zero. The instrument therefore reads $+5$ divisions for zero separation:

$$e = +5(0.01) = +0.05\,\text{mm}.$$

This offset is already included in every observed reading.

Step 3: Apply the correction with the proper sign The instrument overestimates the length, so subtract the offset:

$$x_{\text{true}} = x_{\text{obs}} - e = 5.20 - 0.05 = 5.15\,\text{mm}.$$

Option (D) leaves the zero error uncorrected. Option (B) adds the positive error instead of subtracting it. Option (A) discards the circular-scale contribution entirely. Equivalently, the effective circular reading is $20 - 5 = 15$ divisions, giving $5 + 15(0.01)\,\text{mm}$.

Step 4: Conclusion The corrected measurement is $5.15\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: Write observed reading and zero error separately, with signs, before applying true reading = observed reading minus zero error.

46. What equal length of an iron wire and a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3Ω ?

(Given resistivities of iron and copper-nickel alloy wire are 12 $\mu\Omega$ and 51 $\mu\Omega$ cm respectively)

[4 marks]

(A) 82 m

(B) 97 m

(C) 110 m

(D) 90 m

Correct Answer: (B) 97 m

Solution:

Concept:

- A uniform wire has resistance $R = \rho l/A$, where ρ is resistivity, l is length and A is cross-sectional area.
- Parallel resistors have additive conductances: $1/R_{\text{eq}} = 1/R_1 + 1/R_2$. Both wires here have the same length and area.

Step 1: Convert the material and size data The common diameter is $2\,\text{mm}$, so the radius is $1 \times 10^{-3}\,\text{m}$

and

$$A = \pi(10^{-3})^2 = \pi \times 10^{-6}\,\mathrm{m}^2.$$

Using the stated resistivities in micro-ohm centimetres,

$$\rho_1 = 12 \times 10^{-8}\,\Omega\mathrm{m}, \qquad \rho_2 = 51 \times 10^{-8}\,\Omega\mathrm{m}.$$

The factor 10^{-8} combines 10^{-6} for micro and 10^{-2} for centimetres.

Step 2: Express the parallel combination Let the common length be l. Substitution into the parallel formula gives

$$\frac{1}{R_{\mathrm{eq}}} = \frac{A}{l}\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right),$$

$$R_{\mathrm{eq}} = \frac{l}{A}\frac{\rho_1\rho_2}{\rho_1 + \rho_2}.$$

The effective resistivity factor is

$$\frac{12 \times 51}{12 + 51} \times 10^{-8} = 9.7143 \times 10^{-8}\,\Omega\mathrm{m}.$$

It is below both individual resistivities, as expected from a parallel combination.

Step 3: Solve for the common length With $R_{\mathrm{eq}} = 3\,\Omega$,

$$l = \frac{3\pi \times 10^{-6}}{9.7143 \times 10^{-8}} \approx 97.0\,\mathrm{m}.$$

At this length the separate resistances are approximately $3.71\,\Omega$ and $15.75\,\Omega$, which combine to about $3\,\Omega$. Since equivalent resistance is proportional to length, the other proposed lengths do not give the required value; using the diameter as the radius would also give an erroneous factor of four.

Step 4: Conclusion Each wire must have length approximately $97\,\mathrm{m}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal lengths and cross sections, replace the two resistivities by their parallel factor $\rho_1\rho_2/(\rho_1 + \rho_2)$ before solving for length.

47. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2}. The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket : [Use g = 10 m/s^2] [4 marks]

(A) 6.0×10^2 kg s^{-1}

(B) 500 kg s^{-1}

(C) 10 kg s^{-1}

(D) 60 kg s^{-1}

Correct Answer: (D) 60 kg s^{-1}

Solution:

Concept:

- For exhaust speed u relative to the rocket, the thrust magnitude is $T = u\dot{m}_f$, where $\dot{m}_f$ is the positive fuel consumption rate.
- For upward motion against gravity, Newton's law at an instant is $T - mg = ma$. The initial mass is used for the initial acceleration.

Step 1: Identify the required net force The rocket has initial mass $m = 1000\,\text{kg}$ and must accelerate upward at $a = 20\,\text{m}\,\text{s}^{-2}$. Thus the required upward resultant force is

$$ma = 1000(20) = 2.0 \times 10^4\,\text{N}.$$

The engines must also overcome the weight; the thrust itself is not merely ma. With the given $g = 10\,\text{m}\,\text{s}^{-2}$, the weight is $1.0 \times 10^4\,\text{N}$.

Step 2: Find the exhaust thrust Taking upward as positive gives

$$T - mg = ma \quad \Rightarrow \quad T = m(a + g).$$

Consequently

$$T = 1000(20 + 10) = 3.0 \times 10^4\,\text{N}.$$

This balance is instantaneous: as fuel burns, the rocket mass changes, but the question specifies its initial mass, so the requested initial burning rate is evaluated with that mass.

Step 3: Convert thrust to fuel consumption rate The exhaust gases have relative speed $u = 500\,\text{m}\,\text{s}^{-1}$. Their momentum carried away per second gives

$$\dot{m}_f = \frac{T}{u} = \frac{3.0 \times 10^4}{500} = 60\,\text{kg}\,\text{s}^{-1}.$$

For a check, $500(60) = 30000\,\text{N}$; subtracting the weight leaves $20000\,\text{N}$, giving the required acceleration. Option (A) gives ten times too much thrust, (B) is also too large, and (C) cannot even supply thrust equal to the initial weight.

Step 4: Conclusion The initial fuel-burning rate must be $60\,\text{kg}\,\text{s}^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: For a vertical rocket, compute thrust from $m(a + g)$, then divide by the exhaust speed relative to the rocket.

48. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are : [4 marks]

(A) $[M^0\ L^1\ T^0]$

(B) $[M^{-1}\ L^{-1}\ T^2]$

(C) $[M^1\ L^1\ T^{-2}]$

(D) $[M^0\ L^0\ T^0]$

Correct Answer: (D) $[M^0\ L^0\ T^0]$

Solution:

Concept:

- Energy has dimensions $[E] = \mathrm{ML^2T^{-2}}$; angular momentum has dimensions $[L_{\text{ang}}] = \mathrm{ML^2T^{-1}}$.
- The gravitational constant has dimensions $[G] = \mathrm{M^{-1}L^3T^{-2}}$, obtained from $F = Gm_1m_2/r^2$. Powers of quantities multiply their dimensional exponents.

Step 1: Distinguish the symbol from a base dimension The question uses the letter L for angular momentum, whereas the base-dimension symbol L denotes length. Angular momentum is position times linear momentum, giving $\mathrm{ML^2T^{-1}}$, not simply length. The factor denoted M in the formula is the physical mass and contributes dimension M. Keeping these meanings separate prevents a misleading substitution.

Step 2: Insert every dimensional factor For $P = EL^2M^{-5}G^{-2}$, substitute

$$[P] = (\mathrm{ML^2T^{-2}})(\mathrm{ML^2T^{-1}})^2 \times \mathrm{M^{-5}}(\mathrm{M^{-1}L^3T^{-2}})^{-2}.$$

The squared angular momentum contributes $\mathrm{M^2L^4T^{-2}}$. The inverse square of G contributes $\mathrm{M^2L^{-6}T^4}$; the negative power reverses each exponent.

Step 3: Add exponents of each base quantity The mass exponent is $1 + 2 - 5 + 2 = 0$. The length exponent is $2 + 4 - 6 = 0$, and the time exponent is $-2 - 2 + 4 = 0$. Hence

$$[P] = \mathrm{M^0L^0T^0}.$$

All dimensions cancel, so the expression is dimensionless. Options (A), (B) and (C) retain nonzero powers of one or more base dimensions and cannot match the result. This cancellation does not imply that the numerical value of P is necessarily one.

Step 4: Conclusion P is dimensionless, with dimensions $[\mathrm{M^0L^0T^0}]$. Hence, the correct option is (D).

Quick Tip to Solve: For a product with negative powers, make a separate exponent sum for mass, length and time; do not confuse angular momentum with the length dimension.

49. The material filled between the plates of a parallel plate capacitor has resistivity 200 Ωm. The value of capacitance of the capacitor is 2 pF. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is : (given the value of relative permittivity of material is 50) [4 marks]

(A) 9.0 μA

(B) 9.0 mA

(C) 0.9 mA

(D) 0.9 μA

Correct Answer: (C) 0.9 mA

Solution:

Concept:

- For a parallel plate capacitor filled by a material of relative permittivity K, $C = K\varepsilon_0 A/d$.
- The same material provides a leakage resistance $R = \rho d/A$; steady leakage current obeys $I = V/R$. The leakage is conduction through the dielectric.

Step 1: Eliminate the unknown geometry The plate area A and separation d are not given, but they occur reciprocally in capacitance and resistance. Multiplying gives

$$RC = \left(\rho\frac{d}{A}\right)\left(K\varepsilon_0\frac{A}{d}\right) = \rho K\varepsilon_0.$$

Therefore the current can be found without determining either geometric dimension:

$$I = \frac{VC}{\rho K\varepsilon_0}.$$

The capacitance must be expressed in farads and resistivity in ohm metres.

Step 2: Calculate the leakage resistance Use $\rho = 200\,\Omega\text{m}$, $K = 50$, $C = 2\times10^{-12}\,\text{F}$, and $\varepsilon_0 = 8.85\times10^{-12}\,\text{F m}^{-1}$. Then

$$R = \frac{200(50)(8.85\times10^{-12})}{2\times10^{-12}} = 4.425\times10^4\,\Omega.$$

The unit check is $(\Omega\text{m})(\text{F/m})/\text{F} = \Omega$, so the result has the required resistance unit.

Step 3: Apply the potential difference With $V = 40\,\text{V}$,

$$I = \frac{40}{4.425\times10^4}\,\text{A} \approx 9.04\times10^{-4}\,\text{A} = 0.904\,\text{mA}.$$

This rounds to $0.9\,\text{mA}$. Option (B) is larger by a factor of ten. Options (A) and (D) incorrectly put the result in the microampere range. The current calculated here is the leakage current through the finite-resistivity filling, not the transient charging current immediately after switching on the source.

Step 4: Conclusion The leakage current is approximately $0.9\,\text{mA}$. Hence, the correct option is (C).

Quick Tip to Solve: When the same dielectric sets capacitance and leakage resistance, use $RC = \rho K\varepsilon_0$ to cancel the unknown plate geometry.

50. Statement I : By doping silicon semiconductor with pentavalent material, the electrons density increases.

Statement II : The n-type semiconductor has net negative charge.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement - I is true but Statement - II is false.

(B) Statement - I is false but Statement - II is true.

(C) Both Statement I and Statement II are true.

(D) Both Statement I and Statement II are false.

Correct Answer: (A) Statement - I is true but Statement - II is false.

Solution:

Concept:

- A pentavalent donor atom has five valence electrons. Four form covalent bonds in the silicon lattice, while the fifth can supply a mobile electron.
- Carrier type and total charge are different properties: an n-type sample has electrons as majority carriers but remains electrically neutral in bulk.

Step 1: Examine the effect of donor doping Pure silicon has equal electron and hole concentrations. Replacing a small fraction of silicon atoms by pentavalent atoms introduces donor levels close to the conduction band. At ordinary temperatures, their extra electrons can enter that band. Consequently the free-electron concentration increases compared with intrinsic silicon. Statement I is therefore true. The letter n describes the negative sign of the majority mobile carriers; it does not specify a net charge on the specimen.

Step 2: Balance the charges in the sample An electron released by a donor leaves behind an immobile positively ionised donor. Thus the additional negative mobile charge is balanced by positive donor-ion charge. In a uniformly doped, unbiased bulk sample the neutrality condition is $n = p + N_D^+$ when acceptors are absent. Here n and p denote electron and hole number densities, and N_D^+ is the ionised donor density. The crystal has no net negative charge merely because electrons conduct most of its current. Statement II is false.

Step 3: Compare the four combinations Option (A) has precisely the required true-false combination. Options (B) and (D) incorrectly deny the donor-induced increase of electron concentration, while option (C) incorrectly assigns a net negative charge to the n-type material.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish majority-carrier sign from net charge: each donor electron is accompanied by a positive donor ion.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter lying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is ..... . (Take g = 10 m/s^2) [4 marks]

Solution:

Concept:

- For a smooth table, gravity supplies the gain in kinetic energy; the normal reaction does no work and friction is absent.
- A uniform chain has linear mass density $\lambda = M/L$. A hanging length x has its centre of mass $x/2$ below the tabletop.

Step 1: Choose the energy reference Take the tabletop as zero gravitational potential energy, with upward height positive. The chain has total length $L = 3\,\text{m}$ and mass $M = 3\,\text{kg}$, so $\lambda = 1\,\text{kg}\,\text{m}^{-1}$. Initially $2\,\text{m}$ lies at tabletop height and $1\,\text{m}$ hangs vertically. The hanging part has mass $1\,\text{kg}$ and its centre is $0.5\,\text{m}$ below the tabletop. Therefore

$$U_i = -(1)(10)(0.5) = -5\,\text{J}.$$

The horizontal portion makes no contribution to this chosen potential energy.

Step 2: Find the final potential energy Just as the chain completely leaves the table, its full length hangs below the edge. Its centre of mass is $L/2 = 1.5\,\text{m}$ below the tabletop, giving

$$U_f = -(3)(10)(1.5) = -45\,\text{J}.$$

Equivalently, for a hanging length x, $U(x) = -\lambda g x^2/2$; using $x = 1$ and $x = 3$ yields the same energies.

Step 3: Apply conservation of mechanical energy The chain begins to slip from rest. With no dissipative work, its final kinetic energy is the decrease in gravitational potential energy:

$$k = U_i - U_f = -5 - (-45) = 40\,\text{J}.$$

The positive result is consistent with the chain losing height as it accelerates. Using only the initially hanging mass would miss the gravitational work on the rest of the chain as it leaves the tabletop.

Step 4: Conclusion The kinetic energy is $40\,\text{J}$, so the required numerical value is $\boxed{k = 40}$.

Quick Tip to Solve: For a uniform chain, write $U = -\lambda g x^2/2$ for the hanging length; subtract the initial and final values.

52. The electric field in a plane electromagnetic wave is given by $\vec{E} = 200\cos\left[\left(\frac{0.5\times10^3}{m}\right)x - \left(1.5\times10^{11}\frac{rad}{s}\times t\right)\right]\frac{V}{m}\hat{j}$. If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2. If the radiation pressure exerted by the E.M. wave on the surface during a 10 minute exposure is $\frac{x}{10^9}\frac{N}{m^2}$. Find the value of x . [4 marks]

Solution:

Concept:

- For a sinusoidal plane electromagnetic wave, $I = \frac{1}{2}\varepsilon_0 c E_0^2$, where E_0 is the electric-field amplitude.
- At normal incidence on a perfectly reflecting surface, radiation pressure is $P = 2I/c$, because reflection reverses the normal photon momentum.

Step 1: Read the field amplitude The coefficient of the cosine gives $E_0 = 200\,\mathrm{V\,m^{-1}}$. The angular frequency and wave number give $c = \omega/k = (1.5\times10^{11})/(0.5\times10^3) = 3\times10^8\,\mathrm{m\,s^{-1}}$, consistent with propagation in vacuum. The time average of the squared cosine is $1/2$; this is why the intensity uses the peak field with a factor of one-half, rather than the peak energy flux directly.

Step 2: Calculate the average pressure Substitute the average intensity into the reflection formula:

$$P = \frac{2}{c}\left(\frac{1}{2}\varepsilon_0 c E_0^2\right) = \varepsilon_0 E_0^2.$$

Taking $\varepsilon_0 = 8.85\times10^{-12}\,\mathrm{F\,m^{-1}}$,

$$P = (8.85\times10^{-12})(200)^2 = 3.54\times10^{-7}\,\mathrm{N\,m^{-2}}.$$

The factor of two from reflection exactly cancels the sinusoidal averaging factor.

Step 3: Identify the requested coefficient Pressure is force per unit area and is already a rate of momentum transfer per area. The stated area would be used to find force, $F = PA$, and the ten-minute duration would be used to find impulse, $J = PAt$; neither changes the steady average pressure. Expressing the result in the requested form gives

$$P = \frac{354}{10^9}\,\mathrm{N\,m^{-2}}.$$

Step 4: Conclusion The radiation pressure is $3.54\times10^{-7}\,\mathrm{N\,m^{-2}}$; hence $\boxed{x = 354}$.

Quick Tip to Solve: For perfect reflection of a sinusoidal wave, use $P = \varepsilon_0 E_0^2$; area and exposure time affect impulse, not pressure.

53. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of ..... m. (Take g = 10 m/s^2) [4 marks]

Solution:

Concept:

- For upward projection from a common level, the centre height is $y = ut - \frac{1}{2}gt^2$ while the ball is in free flight.
- After the second launch both balls have the same downward acceleration, so their relative acceleration vanishes. Their relative velocity remains constant until contact.

Step 1: Write the two trajectories Let t be the time in seconds measured from the first launch. The first ball has $u = 35\,\mathrm{m\,s^{-1}}$, so

$$y_1 = 35t - 5t^2.$$

The second starts after $3\,\mathrm{s}$, giving

$$y_2 = 35(t-3) - 5(t-3)^2, \qquad t \geq 3.$$

Subtracting gives the vertical separation of the centres before contact:

$$y_1 - y_2 = 150 - 30t.$$

The separation decreases at $30\,\mathrm{m\,s^{-1}}$, as expected from the three-second difference in gravitational deceleration.

Step 2: Locate the meeting height Treating the balls as particles, set $y_1 = y_2$. This gives $t = 5\,\mathrm{s}$, or two seconds after the second launch. The common trajectory height is then

$$y = 35(5) - 5(5)^2 = 175 - 125 = 50\,\mathrm{m}.$$

At this instant the first ball is descending and the second is ascending, so a collision is physically possible.

Step 3: Check the stated finite radii Each radius is $0.05\,\mathrm{m}$, so actual first contact occurs when the centre separation is $0.10\,\mathrm{m}$. Thus $150 - 30t_c = 0.10$, giving $t_c = 4.99667\,\mathrm{s}$. At that instant the lower surface of the upper ball, equivalently the contact point, is about $50.000\,\mathrm{m}$ above the launch level (the centre heights are about 50.050 and $49.950\,\mathrm{m}$). The finite size therefore leaves the requested whole-metre answer unchanged.

Step 4: Conclusion The balls collide at approximately $\boxed{50\,\mathrm{m}}$; the required numerical value is 50.

Quick Tip to Solve: Use the three-second delay explicitly in the second trajectory; subtracting the two heights removes the quadratic term.

54. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is ..... cm. [4 marks]

Solution:

Concept:

- A soap bubble has two liquid-air surfaces, so its pressure excess is $\Delta P = 4T/r$, where T is surface tension and r is its radius.
- For nested bubbles, pressure jumps add in sequence from the atmosphere through the outer bubble into the inner bubble.

Step 1: Label the three pressure regions Let P_0 be atmospheric pressure, P_1 the gas pressure inside the larger bubble but outside the smaller one, and P_2 the pressure inside the smaller bubble. The two radii are $r_1 = 6\,\text{cm}$ for the outer bubble and $r_2 = 3\,\text{cm}$ for the inner one. Both films are soap films with the same surface tension T. Across each film the pressure rises towards the enclosed gas, so

$$P_1 - P_0 = \frac{4T}{r_1}, \qquad P_2 - P_1 = \frac{4T}{r_2}.$$

Step 2: Add the excess pressures The desired pressure is measured inside the smaller bubble relative to the atmosphere, rather than merely relative to the gas immediately outside it. Adding the two equations gives

$$P_2 - P_0 = 4T\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

This addition is essential: using only $4T/r_2$ would omit the pressure already produced by the outer bubble.

Step 3: Find the equivalent single radius For a single equivalent soap bubble of radius R, the same atmospheric excess is $4T/R$. Hence

$$\frac{4T}{R} = 4T\left(\frac{1}{6} + \frac{1}{3}\right)\text{cm}^{-1}.$$

Cancelling the common $4T$ gives

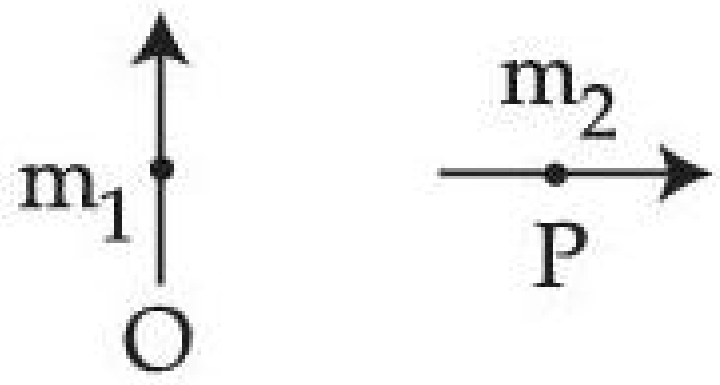
$$\frac{1}{R} = \frac{1}{2}\,\text{cm}^{-1}, \qquad R = 2\,\text{cm}.$$

This radius is smaller than either original radius, as required for a larger excess pressure. The problem equates pressure, so no volume-conservation relation is involved.

Step 4: Conclusion The equivalent radius is $\boxed{2\,\text{cm}}$, and the required numerical value is 2.

Quick Tip to Solve: Nested pressure jumps add: for equal surface tension, the equivalent bubble satisfies $1/R = 1/r_1 + 1/r_2$.

55. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is ..... $\times 10^{-7}$ Nm. [4 marks]



Solution:

Concept:

- The magnetic field of a short dipole at an equatorial point is opposite its magnetic moment and has magnitude $B_{\text{eq}} = \mu_0 m_1/(4\pi r^3)$.
- The torque on a second magnetic dipole is $\boldsymbol{\tau} = \mathbf{m}_2 \times \mathbf{B}_1$, with magnitude $m_2 B_1 \sin\theta$.

Step 1: Identify the geometry from the figure The first moment $\mathbf{m}_1$ points vertically upward at O, whereas P lies horizontally to its right. Thus OP is perpendicular to $\mathbf{m}_1$: P is on the equatorial line of the first dipole. The second moment $\mathbf{m}_2$ points rightward at P. The field of the first dipole at P therefore points downward, opposite $\mathbf{m}_1$, and is perpendicular to $\mathbf{m}_2$. The axial field formula, which contains an extra factor of two, does not apply here.

Step 2: Calculate the field at P Using the short-dipole approximation stated in the question, with $m_1 = 1\,\text{A}\,\text{m}^2$, $r = OP = 1\,\text{m}$, and $\mu_0/(4\pi) = 10^{-7}\,\text{T}\,\text{m}\,\text{A}^{-1}$,

$$B_1 = \frac{\mu_0}{4\pi}\frac{m_1}{r^3} = 10^{-7}\frac{1}{1^3} = 10^{-7}\,\text{T}.$$

The cube of the separation is important for a dipole field.

Step 3: Evaluate the torque magnitude Here $\theta = 90^\circ$, so the sine factor is unity. With $m_2 = 1\,\text{A}\,\text{m}^2$,

$$\tau = m_2 B_1 \sin 90^\circ = (1)(10^{-7}) = 1 \times 10^{-7}\,\text{N}\,\text{m}.$$

The torque direction is into the plane of the figure, since rightward $\mathbf{m}_2$ crossed with downward $\mathbf{B}_1$ points inward. The question requests the magnitude coefficient.

Step 4: Conclusion The torque magnitude is $1 \times 10^{-7}\,\text{N}\,\text{m}$, so the required numerical value is $\boxed{1}$.

Quick Tip to Solve: First classify the observation point as axial or equatorial; then use the angle between the second moment and the local field.

56. Two travelling waves produces a standing wave represented by equation,

y = 1.0 mm cos(1.57 cm^{-1}) x sin(78.5 s^{-1})t.

The node closest to the origin in the region x > 0 will be at x = ... cm. [4 marks]

Solution:

Concept:

- In a standing wave $y = A\cos(kx)\sin(\omega t)$, the displacement amplitude at position x is $|A\cos(kx)|$.
- A node is a point that remains at zero displacement for all times; consequently its spatial amplitude factor must vanish.

Step 1: Separate the space and time factors The given standing wave has amplitude scale $A = 1.0\,\text{mm}$, wave number $k = 1.57\,\text{cm}^{-1}$, and angular frequency $\omega = 78.5\,\text{s}^{-1}$. Its spatial factor is $\cos(kx)$, while $\sin(\omega t)$ controls the oscillation in time. A zero of the time factor makes the entire string pass through equilibrium at one instant; it does not locate nodes. To find a point that is always at rest, set the spatial factor to zero.

Step 2: Solve the node condition The cosine vanishes at odd multiples of $\pi/2$, so

$$kx = \frac{(2n+1)\pi}{2}, \qquad n \in \mathbb{Z}.$$

Hence the node positions are

$$x_n = \frac{(2n+1)\pi}{2k}.$$

For $x > 0$, the smallest distance from the origin corresponds to $n = 0$. The origin itself has $\cos 0 = 1$, and is an antinode, so it cannot be included as a node.

Step 3: Calculate the first positive node Because the wave number is in reciprocal centimetres, the resulting position is directly in centimetres:

$$x_{\min} = \frac{\pi}{2(1.57)}\,\text{cm} \simeq 1.00\,\text{cm}.$$

Equivalently, the wavelength is $2\pi/k \simeq 4\,\text{cm}$, and the first node is one quarter wavelength from the antinode at the origin. Neither the amplitude in millimetres nor the temporal frequency changes this position.

Step 4: Conclusion The nearest node in $x > 0$ is at $\boxed{x = 1\,\text{cm}}$; the required numerical value is 1.

Quick Tip to Solve: For a cosine standing-wave profile, the origin is an antinode and the first positive node is at $\lambda/4$.

57. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. the difference in wavelengths of red and violet light is ..... nm. [4 marks]

Solution:

Concept:

- For a bright fringe of order n, the path difference is $n\lambda$. In the small-angle arrangement its displacement from the centre is $y_n = n\lambda D/d$, where D is screen distance and d is slit separation.
- The first bright fringes of both colours have the same order $n = 1$. Their separation therefore measures the difference of their wavelengths directly; the central fringe has zero path difference for every colour.

Step 1: Convert the dimensions to SI units The slit separation is $d = 0.3 \times 10^{-3}\,\text{m}$, the screen distance is $D = 1.5\,\text{m}$, and the first-fringe positions are $y_v = 2.0 \times 10^{-3}\,\text{m}$ and $y_r = 3.5 \times 10^{-3}\,\text{m}$. Thus the red fringe is farther from the centre, consistent with red light having a larger wavelength.

Step 2: Use the equal fringe orders For the first bright fringe, rearrange the fringe-position relation to $\lambda = yd/D$. Subtract the violet relation from the red relation before substituting:

$$\lambda_r - \lambda_v = \frac{d}{D}(y_r - y_v) = \frac{0.3 \times 10^{-3}}{1.5}(1.5 \times 10^{-3}) = 3.0 \times 10^{-7}\,\text{m}.$$

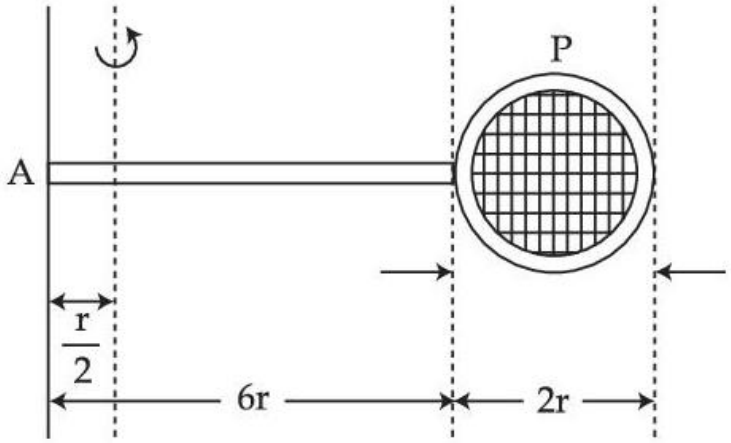
The individual wavelengths provide a useful check: $\lambda_v = 400\,\text{nm}$ and $\lambda_r = 700\,\text{nm}$. The small-angle condition is well satisfied since both displacements are much smaller than the screen distance.

Step 3: Conclusion Since $1\,\text{nm} = 10^{-9}\,\text{m}$, the wavelength difference is $\boxed{300\,\text{nm}}$. The numerical entry is $\boxed{300}$.

Quick Tip to Solve: For equal-order fringes, subtract the measured positions first: $\Delta\lambda = d\,\Delta y/(nD)$. This avoids carrying two separate wavelength calculations.

58. Consider a badminton racket with length scales as shown in the figure.

If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be ..... Mr^2. [4 marks]



Solution:

Concept:

- The racket is a uniform straight rod of length $6r$ and mass M, joined to a thin circular ring of radius r and mass M. Negligible thread mass contributes no moment of inertia.
- For a rod, the central transverse moment is $ML^2/12$. A thin ring has moment $Mr^2/2$ about an in-plane diameter, using symmetry and the perpendicular-axis theorem. A parallel displacement a adds Ma^2.

Step 1: Locate the component centres relative to the axis Measure distance along the handle from A. The required rotation axis passes through $x = r/2$ and lies in the ring plane. The rod centre is at $x = 3r$, so its displacement is $5r/2$. The near edge of the circular portion is at $6r$; hence its centre is at $7r$, a distance $13r/2$ from the rotation axis. These distances use the given ring diameter $2r$, not the radius as a diameter.

Step 2: Add the two parallel-axis moments For the rod,

$$I_{\text{rod}} = \frac{M(6r)^2}{12} + M\left(\frac{5r}{2}\right)^2 = \frac{37}{4}Mr^2.$$

For the ring, the axis through its centre parallel to the required axis is a diameter, rather than the perpendicular axis through its face. Therefore

$$I_{\text{ring}} = \frac{Mr^2}{2} + M\left(\frac{13r}{2}\right)^2 = \frac{171}{4}Mr^2.$$

Add these component moments about the specified line.

Step 3: Conclusion The total is $I = (37 + 171)Mr^2/4 = \boxed{52Mr^2}$. Enter $\boxed{52}$.

Quick Tip to Solve: Determine whether the ring axis is in its plane before selecting $Mr^2/2$ or Mr^2; then shift each component from its own centre.

59. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be ..... Hz. [4 marks]

Solution:

Concept:

- Sound speed is measured relative to the medium. With no wind the air is at rest relative to the ground, so $v = 340\,\mathrm{m\,s^{-1}}$ can be used directly with the given source and detector speeds.
- When the source and detector recede, the moving source increases the wavelength to $(v + v_s)/f_s$, while the receding detector meets successive wavefronts at relative speed $v - v_d$. Thus $f_d = f_s(v - v_d)/(v + v_s)$.

Step 1: Identify the two recession effects The phrase that the source and detector move away from each other with speed $20\,\mathrm{m\,s^{-1}}$ with respect to the ground gives $v_s = v_d = 20\,\mathrm{m\,s^{-1}}$. Each is measured relative to the stationary air. The source recession and detector recession both lower the detected frequency, so the original frequency must exceed $1800\,\mathrm{Hz}$. The relative separation speed alone is not the speed to put in either individual factor.

Step 2: Invert the Doppler relation Substitute the signed recession factors into the frequency ratio:

$$1800 = f_s\frac{340 - 20}{340 + 20} = f_s\frac{320}{360} = f_s\frac{8}{9}.$$

Hence

$$f_s = 1800\frac{9}{8} = 2025\,\mathrm{Hz}.$$

Equivalently, the increased wavelength is $360/2025\,\mathrm{m}$, and a detector meeting wavefronts at $320\,\mathrm{m\,s^{-1}}$ counts $320/(360/2025) = 1800$ wavefronts per second, independently checking the result.

Step 3: Conclusion The original source frequency is $\boxed{2025\,\mathrm{Hz}}$, so the numerical entry is $\boxed{2025}$.

Quick Tip to Solve: For mutual recession, use a minus sign for detector speed in the numerator and a plus sign for source speed in the denominator; check that the observed frequency decreases.

60. An amplitude modulated wave is represented by
$C_m(t) = 10(1 + 0.2\cos 12560t)\sin(111 \times 10^4 t)$ volts. The modulating frequency in kHz will be ..... . [4 marks]

Solution:

Concept:

- A sinusoidally amplitude-modulated signal has form $A_c[1+\mu\cos(\omega_m t)]\sin(\omega_c t)$. The slowly varying bracket is its amplitude envelope, and ω_m is the modulating angular frequency.
- Angular frequency and ordinary frequency differ by $\omega = 2\pi f$. The coefficient of time in a trigonometric argument is an angular frequency in radians per second, not a frequency in hertz.

Step 1: Match the given wave to the standard form Compare the wave with the standard form,

writing its expression as

$$10[1 + 0.2\cos(12560t)]\sin(111 \times 10^4 t).$$

The constant carrier amplitude is $10\,\text{V}$, the modulation index is 0.2, and the modulating angular frequency is $\omega_m = 12560\,\text{rad s}^{-1}$. The coefficient 111×10^4 belongs to the carrier oscillation; it must not be used to find the frequency of the message.

Step 2: Convert the envelope frequency to kilohertz Divide the modulating angular frequency by 2π:

$$f_m = \frac{\omega_m}{2\pi} = \frac{12560}{2\pi}\,\text{Hz}.$$

With $\pi \simeq 3.14$, this gives $f_m = 2000\,\text{Hz}$. Since one kilohertz is $1000\,\text{Hz}$,

$$f_m = \frac{2000}{1000} = 2\,\text{kHz}.$$

Using a more precise value of π gives about $1.999\,\text{kHz}$, which has the same intended numerical value at the precision of the supplied coefficient.

Step 3: Conclusion The modulating frequency is $\boxed{2\,\text{kHz}}$. Enter $\boxed{2}$.

Quick Tip to Solve: Read the frequency of the cosine inside the amplitude bracket, then divide its angular coefficient by 2π and convert hertz to kilohertz.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which one of the following complexes is violet in colour? [4 marks]

(A) $[Fe(CN)_6]^{4-}$

(B) $[Fe(SCN)_6]^{4-}$

(C) $Fe_4[Fe(CN_6)]_3 \cdot H_2O$

(D) $[Fe(CN)_5NOS]^{4-}$

Correct Answer: (D) $[Fe(CN)_5NOS]^{4-}$

Solution:

Concept:

- The sodium nitroprusside test detects sulphide through formation of a violet thionitroprusside complex. The relevant reagent anion is $[Fe(CN)_5NO]^{2-}$.
- Characteristic qualitative-analysis colours identify the complex formed, rather than merely the metal present. Sulphide addition produces $[Fe(CN)_5NOS]^{4-}$, the violet species in this list.

Step 1: Identify the violet sulphide-test product In the alkaline sulphur test, sulphide ions react with nitroprusside to give the violet-coloured thionitroprusside ion:

$$[Fe(CN)_5NO]^{2-} + S^{2-} \longrightarrow [Fe(CN)_5NOS]^{4-}.$$

The five cyanide ligands and the sulphur-containing nitrosyl group distinguish this formula from the

six-cyanide and thiocyanate complexes. The charge is consistent with addition of a dianionic sulphide ion to the reagent dianion.

Step 2: Eliminate the other colour assignments Option (A), the ferrocyanide ion $[Fe(CN)_6]^{4-}$, is associated with pale yellow ferrocyanide salts, rather than the violet sulphide test. Option (B) is an iron-thiocyanate formula; the familiar diagnostic iron-thiocyanate colour is blood red, not this violet product. Option (C) represents the hydrated ferric ferrocyanide material known as Prussian blue, whose identifying colour is blue. These alternatives must not be confused with thionitroprusside merely because they all contain iron and nitrogen ligands.

Step 3: Conclusion The violet complex is $\boxed{[Fe(CN)_5NOS]^{4-}}$, the thionitroprusside ion. Hence, the correct option is (D).

Quick Tip to Solve: In sulphur detection, pair nitroprusside with violet and lead sulphide with black; the violet complex contains the added sulphur in its formula.

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62. Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere?

[4 marks]

(A) $CuCl_2$

(B) AgCl

(C) $ZnCl_2$

(D) Cu_2Cl_2

Correct Answer: (A) $CuCl_2$

Solution:

Concept:

- Aqueous transition-metal ions with partly occupied d orbitals can absorb visible light in ligand-field transitions. The hydrated Cu^{2+} ion has a $3d^9$ configuration and gives a coloured aqueous solution.
- Cu^+, Ag^+ and Zn^{2+} each have a filled d^{10} subshell. They do not give the ordinary d–d absorption of hydrated copper(II). Nitrogen prevents atmospheric oxygen from changing the copper(I) species.

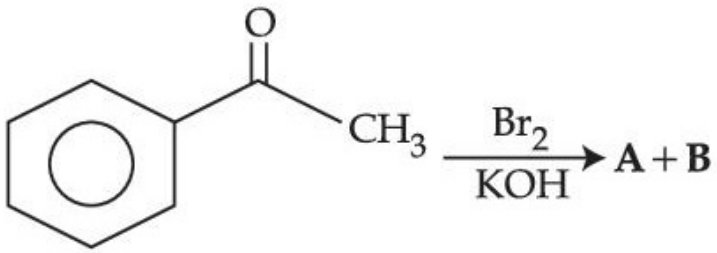
Step 1: Examine the copper(II) salt $CuCl_2$ dissolves in water to give copper(II) ions surrounded by water ligands. Their partly filled d levels are split by the ligand environment, allowing absorption of visible radiation. The solution is coloured, typically blue to blue-green depending on chloride concentration and hydration. This colour does not require oxygen to be present, so replacing air with nitrogen does not remove the underlying copper(II) electronic configuration.

Step 2: Check the remaining salts in nitrogen AgCl is sparingly soluble; its Ag^+ ion has $4d^{10}$, so it does not produce the stated coloured solution. $ZnCl_2$ dissolves but Zn^{2+} has $3d^{10}$, yielding a colourless solution. In Cu_2Cl_2, copper is in the +1 oxidation state with $3d^{10}$; it is not the coloured copper(II) salt. The nitrogen atmosphere is relevant because one must not attribute an oxygen-driven oxidation colour to the copper(I) alternative. Thus only one choice directly supplies the required coloured hydrated ion.

Step 3: Conclusion $\boxed{CuCl_2}$ gives the coloured aqueous solution in nitrogen. Hence, the correct option is (A).

Quick Tip to Solve: Find the oxidation state before counting d electrons: copper(II) is d^9, whereas copper(I), silver(I) and zinc(II) are d^{10}.

63. The major products formed in the following reaction sequence A and B are : [4 marks]



(A) A = C_6H_5COOK; B = $CHBr_3$.

(B) A = 1-(3,5-dibromophenyl)ethan-1-one; B = 1-(3,5-dihydroxyphenyl)ethan-1-one.

(C) A = 2-bromo-1-(3-bromophenyl)ethan-1-one; B = 1-(3-bromophenyl)-2-hydroxyethan-1-one.

(D) A = $C_6H_5COCBr_3$; B = C_6H_5CHO.

Correct Answer: (A) A = C_6H_5COOK; B = $CHBr_3$.

Solution:

Concept:

- Acetophenone, $C_6H_5COCH_3$, contains a methyl group directly bonded to a carbonyl group. Methyl ketones undergo the haloform reaction with halogen and alkali.
- With Br_2/KOH, all three methyl hydrogens are replaced by bromine before hydroxide-induced carbon–carbon cleavage. The carbonyl-containing fragment becomes a carboxylate salt and the methyl carbon becomes bromoform.

Step 1: Apply exhaustive alpha bromination The starting structure is acetophenone, so the reactive methyl ketone group is $-COCH_3$. Base removes an alpha hydrogen and bromine substitutes at that carbon. Repetition replaces the other two alpha hydrogens, forming the trihalomethyl ketone $C_6H_5COCBr_3$. This intermediate is especially susceptible to attack by hydroxide at the carbonyl carbon. Stopping at a singly brominated ketone would miss the characteristic exhaustive substitution under haloform conditions.

Step 2: Identify the cleavage products and option Cleavage gives benzoate and the tribromomethyl fragment, which is protonated to bromoform. With potassium hydroxide the isolated carboxylate is potassium benzoate:

$$C_6H_5COCH_3 + 3\,Br_2 + 4\,KOH \longrightarrow C_6H_5COOK + CHBr_3 + 3\,KBr + 3\,H_2O.$$

Thus A is C_6H_5COOK and B is $CHBr_3$. Option (B) instead changes substituents on the aromatic ring, and option (C) leaves the methyl-ketone skeleton intact; neither describes haloform cleavage. Option

(D) pairs a trihalogenated intermediate with benzaldehyde, which is not the cleavage product. No acid work-up is shown, so benzoic acid should not replace its potassium salt.

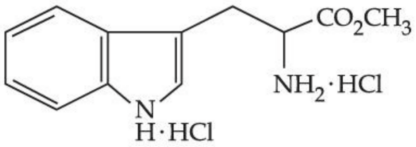
Step 3: Conclusion The products are $\boxed{C_6H_5COOK}$ and $\boxed{CHBr_3}$. Hence, the correct option is (A).

Quick Tip to Solve: For $RCOCH_3$ with bromine and alkali, split the bond beside the carbonyl: $RCOOK$ and $CHBr_3$ are the final products.

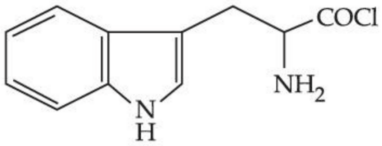
64. The major product formed in the following reaction is : [4 marks]



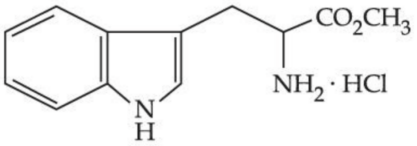
(A) Methyl tryptophan ester with hydrochloride shown at both the side-chain amino nitrogen and the indole nitrogen.



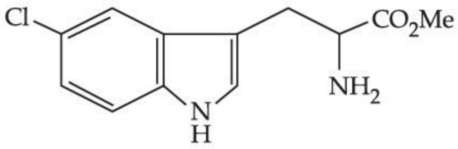
(B) Tryptophan acid chloride, with the side-chain NH_2 and indole NH unchanged.



(C) Methyl tryptophan ester hydrochloride, with the side-chain amino group protonated and the indole NH unchanged.

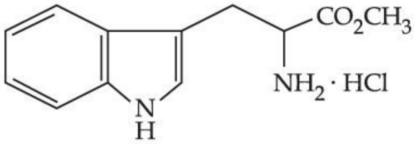


(D) Methyl tryptophan ester with chlorine on the benzene ring and an unprotonated side-chain amino group.



Correct Answer: (C)

Methyl tryptophan ester hydrochloride, with the side-chain amino group protonated and the indole NH unchanged.



Solution:

Concept:

- Thionyl chloride in methanol converts the carboxylic acid group of an amino acid to its methyl ester. The acidic medium also forms the hydrochloride of a basic amino group.
- The side-chain amino nitrogen has an available lone pair, whereas the indole nitrogen contributes its lone pair to the aromatic system and is much less basic.

Step 1: Identify the groups that react The starting molecule is tryptophan, containing a carboxylic acid, a primary amino group and an indole ring. With $SOCl_2$ and CH_3OH, the net carboxyl transformation is

$$\mathrm{RCH(NH_2)COOH} \longrightarrow \mathrm{RCH(NH_2)COOCH_3}.$$

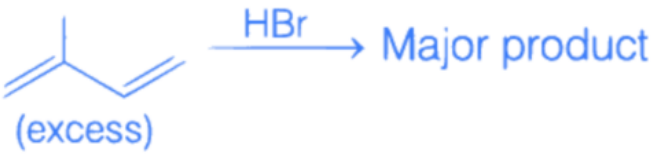
Here R denotes the indol-3-ylmethyl group. Methanol is the nucleophile that supplies the methyl ester group; the acid chloride is not the final product in this medium.

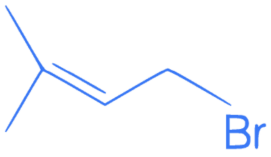
Step 2: Locate salt formation and compare the structures Hydrogen chloride generated in the reaction protonates the ordinary amino group, giving $-\mathrm{NH_3{}^+Cl}-$, also represented as $-\mathrm{NH_2 \cdot HCl}$. The pyrrolic nitrogen of indole remains as NH: its lone pair sustains aromaticity and does not behave like the primary amino lone pair. Thus the product has CO_2CH_3, one side-chain amino hydrochloride and an unchanged indole ring. Option (A) incorrectly shows a hydrochloride at the indole nitrogen as well. Option (B) stops at the acid chloride despite methanol being present. Option (D) introduces ring chlorine without an electrophilic aromatic chlorination reagent.

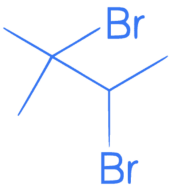
Step 3: Conclusion The major product is the methyl ester hydrochloride of tryptophan with only the side-chain amino group protonated. Hence, the correct option is (C).

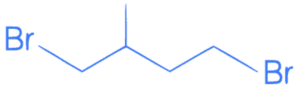
Quick Tip to Solve: With an amino acid and thionyl chloride in methanol, check for both methyl ester formation and an amino hydrochloride; distinguish an amine nitrogen from a pyrrolic nitrogen.

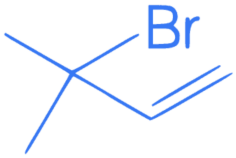
65. The major product formed in the following reaction is : [4 marks]



(A) $(CH_3)_2C{=}CHCH_2Br$ 

(B) $(CH_3)_2C(Br)CH(Br)CH_3$ 

(C) $BrCH_2CH(CH_3)CH_2CH_2Br$ 

(D) $(CH_3)_2C(Br)CH{=}CH_2$ 

Correct Answer: (A)

$(CH_3)_2C{=}CHCH_2Br$ 

Solution:

Concept:

- A conjugated diene reacts with hydrogen bromide through a resonance-stabilised allylic carbocation. Capture can give 1,2- and 1,4-addition products.
- The diene is present in excess, so the relevant products contain one equivalent of added HBr. The more substituted surviving double bond stabilises the 1,4-addition product.

Step 1: Form the allylic intermediate The substrate is isoprene, $CH_2{=}C(CH_3){-}CH{=}CH_2$. Proton addition to its first terminal carbon gives an allylic cation, which can be represented by

$$CH_3{-}C^+(CH_3){-}CH{=}CH_2 \longleftrightarrow CH_3{-}C(CH_3){=}CH{-}CH_2{}^+.$$

The positive charge is delocalised, so bromide need not attach only to the carbon adjacent to the entering hydrogen. This is the essential difference from addition to an isolated alkene.

Step 2: Compare the allowed addition products Bromide capture at the far end gives the 1,4-addition product:

$$CH_3-C(CH_3)=CH-CH_2Br.$$

Its internal double bond is trisubstituted and therefore more stable than the terminal double bond of the 1,2-addition product $(CH_3)_2C(Br)-CH=CH_2$. The former is the major product represented by (A); (D) is the competing 1,2-addition structure. The diene is explicitly in excess, rather than hydrogen bromide being in excess, so double addition to make a dibromide is not selected. Thus (B) and (C), each containing two bromine atoms and no remaining double bond, do not describe the required principal monoaddition product.

Step 3: Conclusion The major product is $(CH_3)_2C=CHCH_2Br$, the more stable 1,4-addition product. Hence, the correct option is (A).

Quick Tip to Solve: Read which reactant is in excess, then draw both resonance positions of an allylic cation before choosing between monoaddition products.

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66. Given below are two statements :

Statement I : The limiting molar conductivity of KCl (strong electrolyte) is higher compared to that of CH_3COOH (weak electrolyte).

Statement II : Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is true but Statement II is false.

(B) Statement I is false but Statement II is true.

(C) Both Statement I and Statement II are true.

(D) Both Statement I and Statement II are false.

Correct Answer: (D) Both Statement I and Statement II are false.

Solution:

Concept:

- Limiting molar conductivity, Λ_m°, is the molar conductivity at infinite dilution and equals the sum of the limiting ionic contributions.
- Molar conductivity generally increases upon dilution. A weak electrolyte also gains a higher degree of ionisation as its concentration decreases.

Step 1: Evaluate Statement I using ionic contributions For the two electrolytes, Kohlrausch's law gives

$$\Lambda_m^\circ(KCl) = \lambda^\circ(K^+) + \lambda^\circ(Cl^-),$$
$$\Lambda_m^\circ(CH_3COOH) = \lambda^\circ(H^+) + \lambda^\circ(CH_3COO^-).$$

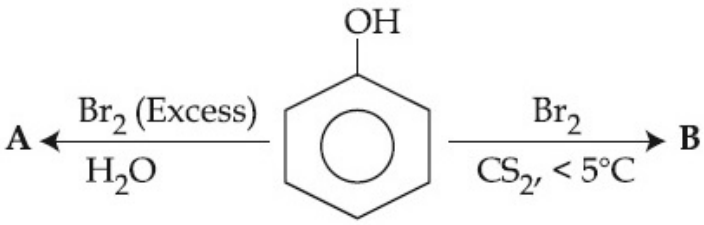
Hydrogen ions have exceptionally high ionic mobility in water. Typical limiting values near room temperature are about 150 for KCl and 391 for acetic acid, in $\mathrm{S\,cm^2\,mol^{-1}}$. Consequently, limiting conductivity of the weak acid is higher, and Statement I is false. Calling an electrolyte strong does not establish a larger limiting value.

Step 2: Evaluate Statement II and select the truth pair Upon dilution, interionic attractions weaken for strong electrolytes, enabling the ions to move more freely. For weak electrolytes such as acetic acid, dilution additionally increases ionisation and hence the number of ions per mole of dissolved electrolyte. Therefore Λ_m increases as concentration decreases, approaching its limiting value. Statement II asserts the opposite trend, so it is false. Options (A), (B) and (C) each require at least one true statement and are excluded. This trend concerns molar conductivity, not the conductivity per unit volume, which normally decreases on dilution.

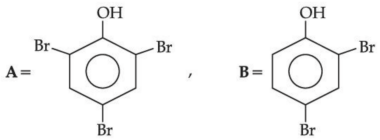
Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (D).

Quick Tip to Solve: Keep conductivity and molar conductivity distinct; at infinite dilution compare the actual ionic mobilities, especially the unusually mobile hydrogen ion.

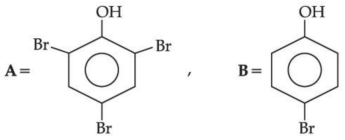
67. The correct options for the products A and B of the following reactions are : [4 marks]



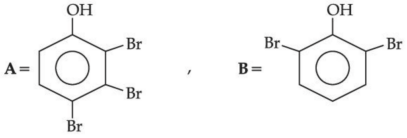
(A) A: 2,4,6-tribromophenol; B: 2,4-dibromophenol.



(B) A: 2,4,6-tribromophenol; B: 4-bromophenol.



(C) A: 2,3,4-tribromophenol; B: 2,6-dibromophenol.

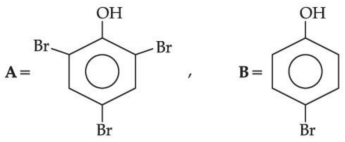


(D) A: 4-bromophenol; B: 4-bromophenol.



Correct Answer: (B)

A: 2,4,6-tribromophenol; B: 4-bromophenol.



Solution:

Concept:

- The hydroxyl group strongly activates the phenol ring and directs electrophilic substitution to the ortho and para positions.
- Bromine water with excess bromine gives 2,4,6-tribromophenol; a nonpolar solvent and low temperature favour controlled monobromination, mainly at para.

Step 1: Determine product A in bromine water With excess Br_2 in water, substitution occurs at all three strongly activated ring positions. Taking the hydroxyl-bearing carbon as position 1, bromine enters positions 2, 4 and 6:

$$C_6H_5OH + 3\,Br_2 \longrightarrow C_6H_2Br_3OH + 3\,HBr.$$

Product A is therefore 2,4,6-tribromophenol. The hydroxyl group remains attached to the ring; the process replaces aromatic hydrogens rather than replacing the hydroxyl group. This is the familiar white-precipitate reaction of phenol with bromine water.

Step 2: Determine product B under controlled conditions For Br_2 in CS_2 below $5\,^\circ C$, the less polar medium and low temperature control the reaction to monobromination. Both ortho and para substitution are electronically favoured, but the para product has less steric crowding adjacent to the hydroxyl group and is the major product. Hence B is 4-bromophenol. Only option (B) pairs this with 2,4,6-tribromophenol as A. Option (A) incorrectly gives a dibrominated B, option (C) includes a meta bromine in A and a dibrominated B, while option (D) fails to account for exhaustive bromination in water.

Step 3: Conclusion A is 2,4,6-tribromophenol and B is predominantly 4-bromophenol. Hence, the correct option is (B).

Quick Tip to Solve: For phenol bromination, inspect the solvent before counting bromines: excess bromine water gives three substitutions, while cold nonpolar solvent mainly gives the para monobromo product.

68. Given below are two statements.

Statement I : In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement II : For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- An indicator must change colour within the steep pH change near the equivalence point. The pH at equivalence depends on the acid-base strength of the salt formed.
- Methyl orange changes in an acidic range, approximately pH 3.1–4.4; phenolphthalein changes in an alkaline range, approximately pH 8.2–10.0.

Step 1: Test Statement I for strong acid and weak base Titration of a strong acid with a weak base produces a salt whose cation is the conjugate acid of the weak base. For example, at equivalence in HCl-NH_3 titration the solution contains NH_4Cl, and NH_4^+ hydrolyses to release hydrogen ions. Thus the equivalence solution is acidic. Methyl orange has an acidic transition range and is a suitable indicator for this type of titration. Statement I is therefore true; an alkaline-range indicator would be a poor general choice here.

Step 2: Test Statement II for acetic acid and sodium hydroxide Acetic acid is weak whereas NaOH is strong. Their equivalence solution contains sodium acetate. Acetate hydrolyses according to

$$CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-,$$

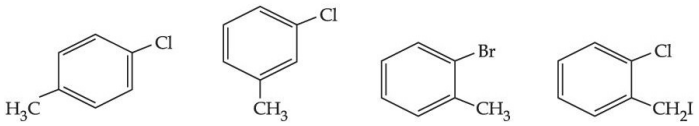
so the equivalence solution is alkaline. Phenolphthalein changes colour in the relevant alkaline region and is suitable for this titration. Statement II says it is not suitable, and is therefore false. The truth combination is true/false. Options (A), (C) and (D) assign the wrong truth value to at least one of the statements and are rejected.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: First decide whether the equivalence salt makes the solution acidic or alkaline, then match the indicator transition range to that region.

69. Among the following compounds I-IV, which one forms a yellow precipitate on reacting sequentially with (i) NaOH (ii) dil. HNO_3 (iii) $AgNO_3$?

[4 marks]



I II III IV

(A) II

(B) IV

(C) I

(D) III

Correct Answer: (B) IV

Solution:

Concept:

- An alkyl or benzylic halide can liberate halide ions on hydrolysis with aqueous alkali, whereas an unactivated aryl halide resists ordinary nucleophilic substitution.
- Silver nitrate precipitates halide ions: AgCl is white, AgBr is pale yellow, and AgI is yellow. Nitric acid removes interference from residual hydroxide.

Step 1: Identify the hydrolysable carbon-halogen bond In compounds I and II, chlorine is directly bonded to an aromatic ring carbon; in III, bromine is directly bonded to an aromatic ring carbon. These are aryl halides, whose carbon-halogen bonds have partial double-bond character through resonance and do not readily hydrolyse under the stated test. Compound IV has an additional $-CH_2I$ side chain next to a ring chlorine. Its iodine is attached to a benzylic, saturated carbon, so the benzylic carbon-iodine bond undergoes substitution readily.

Step 2: Follow the sequence of reagents For IV, alkali liberates iodide while replacing iodine by a hydroxyl group:

$$\text{o-ClC}_6\text{H}_4\text{CH}_2\text{I} + \text{NaOH} \longrightarrow \text{o-ClC}_6\text{H}_4\text{CH}_2\text{OH} + \text{NaI}.$$

Dilute HNO_3 neutralises excess alkali before adding $AgNO_3$; otherwise silver oxide or other interfering precipitates can form. The liberated iodide then gives

$$\text{Ag}^+ + \text{I}^- \longrightarrow \text{AgI}\downarrow.$$

Silver iodide is yellow. The chlorine still attached to the ring does not determine this result. Compounds I and II contain no iodine, and III contains aryl bromine, so none gives the required yellow iodide precipitate in this sequence.

Step 3: Conclusion Compound IV releases iodide and gives yellow AgI. Hence, the correct option is (B).

Quick Tip to Solve: In a silver-nitrate halogen test, inspect the carbon bonded to the halogen as well as the halogen itself; benzylic iodide reacts readily, while aryl halides resist hydrolysis.

70. Given below are two statements.

Statement I : According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement II : According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are false

(B) Both Statement I and Statement II are true

(C) Statement I is false but Statement II is true

(D) Statement I is true but Statement II is false

Correct Answer: (C) Statement I is false but Statement II is true

Solution:

Concept:

- For a hydrogen-like atom or ion, Bohr's model gives electron speed $v_n = Ze^2/(2\varepsilon_0 hn)$, where Z is nuclear charge number and n is principal quantum number.
- Consequently, electron speed is directly proportional to Z at fixed n, and inversely proportional to n at fixed Z.

Step 1: Derive the dependence on nuclear charge The electrostatic force supplies the centripetal force, and Bohr quantisation fixes angular momentum:

$$\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}, \qquad m_e v r = \frac{nh}{2\pi}.$$

Eliminating r gives

$$v_n = \frac{Ze^2}{2\varepsilon_0 hn}.$$

Thus reducing positive nuclear charge reduces the electron speed for the same orbit number. A weaker attraction does not accelerate the electron to a higher orbital speed. Statement I reverses the actual dependence and is false.

Step 2: Test the principal quantum number dependence For a fixed nucleus, the same formula gives $v_n \propto 1/n$. When n decreases, the orbit becomes smaller and the electron speed increases. For example, within a given hydrogen-like species, $v_1 = 2v_2$. This comparison holds between allowed stationary orbits; it is not a claim that an electron radiates continuously while circling in one orbit. Statement II is therefore true. Options (A) and (D) incorrectly make Statement II false, while option (B) incorrectly makes Statement I true. The required truth combination is false/true.

Step 3: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (C).

Quick Tip to Solve: Remember the compact proportionality $v_n \propto Z/n$: increasing nuclear charge speeds the electron up, while increasing the orbit number slows it down.

71. The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is : [4 marks]

(A) $Br_2/AlBr_3$, HNO_3/H_2SO_4, Mg/ether, CO_2, H_3O^+

(B) $Br_2/AlBr^3$, NaCN, H_3O^+, HNO_3/H_2SO_4

(C) $Br_2/AlBr_3$, HNO_3/H_2SO_4, NaCN, H_3O^+

(D) HNO_3/H_2SO_4, $Br_2/AlBr_3$, Mg/ether, CO_2, H_3O^+

Correct Answer: No listed sequence is fully valid as a practical synthesis; (D) is the intended formal/keyed answer, but a nitro group is incompatible with the Grignard step.

Solution:

Concept:

- The substituent already present on benzene controls the position of the next electrophilic substitution. The nitro group is strongly deactivating and meta-directing, whereas bromine is ortho/para-directing.
- In the formal Grignard carboxylation sequence, an aryl halide is converted into an arylmagnesium halide, treated with CO_2, and acidified to replace the halogen-bearing position by COOH.

Step 1: Establish the required relative positions The target has NO_2 and COOH meta to one another. Introduce the nitro group first by nitrating benzene with concentrated HNO_3/H_2SO_4. Bromination of nitrobenzene then gives the meta-bromo derivative because the existing nitro group directs the incoming electrophile to a meta position.

Step 2: Match the formal carbon-chain extension Option (D) next lists Mg in dry ether, followed by CO_2 and H_3O^+. In the intended reaction-sequence convention, carboxylation occurs at the carbon bearing bromine, so the meta relationship established during bromination is retained. Options (A) and (C) brominate first and then nitrate an ortho/para-directing bromobenzene; they do not establish the required meta relationship. Option (B) also assumes easy replacement of unactivated aryl bromide by NaCN, which is not an ordinary nucleophilic substitution.

Step 3: Distinguish the examination sequence from practical compatibility The positional logic identifies the listed answer, but there is a chemical limitation: a nitro group is incompatible with the usual preparation and use of a Grignard reagent. Thus this formal sequence should not be treated as an experimentally reliable direct synthesis without additional synthetic modifications. The question is using the standard reagent-sequence/regioselectivity convention, under which (D) is its intended answer.

Step 4: Conclusion Option (D) is the intended formal and keyed sequence because nitration followed by bromination establishes the meta relationship. However, the nitro group is incompatible with the subsequent Grignard step, so no listed option is a fully valid practical synthesis as written. The intended choice is (D), subject to this chemical defect.

Quick Tip to Solve: Choose the group that must direct the second substitution before tracing the reagent list; nitro establishes a meta relationship, while bromine does not.

72. Excess of isobutane on reaction with Br_2 in presence of light at 125°C gives which one of the following, as the major product? [4 marks]

(A) $CH_3CH(CH_2Br)CH_2Br$

(B) $CH_3C(Br)(CH_3)CH_2Br$

(C) $CH_3CH(CH_3)CH_2Br$

(D) $(CH_3)_3CBr$

Correct Answer: (D) $(CH_3)_3CBr$

Solution:

Concept:

- Photochemical bromination proceeds by a free-radical chain mechanism: light produces bromine radicals, which abstract hydrogen and lead to substitution by bromine.
- Bromination is highly selective: formation of a tertiary carbon radical is favoured over formation of a primary carbon radical. Excess alkane favours monosubstitution over repeated replacement.

Step 1: Identify the different hydrogen environments Isobutane is $(CH_3)_3CH$, or 2-methylpropane. It contains one tertiary hydrogen at the central carbon and nine primary hydrogens on the three equivalent methyl groups. Abstraction of the central hydrogen produces the tertiary tert-butyl radical, $(CH_3)_3C\cdot$; abstraction from a methyl group produces a less stable primary radical.

Step 2: Apply the selectivity of bromination The hydrogen-abstraction step in bromination is strongly sensitive to radical stability. The tertiary radical is stabilized by the three adjacent methyl groups through hyperconjugation and their electron-releasing effect. Its formation is sufficiently favoured to outweigh the larger number of primary hydrogen atoms. Reaction of that radical with Br_2 gives $(CH_3)_3CBr$, regenerating a bromine radical and continuing the chain.

Step 3: Compare the printed products Option (D) is tert-butyl bromide, or 2-bromo-2-methylpropane, the tertiary monobromination product. Option (C) is the primary monobromination product and is minor. Options (A) and (B) each contain two bromine atoms and require further substitution after the first bromination. Since isobutane is in excess, a bromine radical is much more likely to encounter fresh alkane than an already brominated molecule, suppressing these polybrominated products.

Step 4: Conclusion The major product is $(CH_3)_3CBr$, 2-bromo-2-methylpropane. Hence, the correct option is (D).

Quick Tip to Solve: For bromination, locate the most stable radical first; then use excess alkane as the clue that the main product is monobrominated.

73. Which one of the following is correct for the adsorption of a gas at a given temperature on a solid surface? [4 marks]

(A) $\Delta H > 0, \Delta S > 0$

(B) $\Delta H > 0, \Delta S < 0$

(C) $\Delta H < 0, \Delta S < 0$

(D) $\Delta H < 0, \Delta S > 0$

Correct Answer: (C) $\Delta H < 0, \Delta S < 0$

Solution:

Concept:

- Adsorption transfers gas molecules from the bulk gas to a solid surface. Their translational freedom decreases, so the entropy change of adsorption is negative.
- At a fixed temperature, spontaneity requires $\Delta G = \Delta H - T\Delta S < 0$. Binding of the adsorbate to the surface releases energy and makes ordinary gas adsorption exothermic.

Step 1: Determine the entropy change Before adsorption, gas molecules can move through the volume above the solid and occupy many spatial arrangements. After adsorption, they are localized at surface sites, with much less freedom of motion. The number of accessible configurations therefore decreases. For the adsorption process as written, this gives $\Delta S < 0$; the reverse process, desorption, has the opposite tendency.

Step 2: Use the free-energy condition Because temperature is positive and $\Delta S < 0$, the term $-T\Delta S$ is positive and opposes adsorption. Thus an energetically favourable enthalpy change is needed for spontaneous adsorption:

$$\Delta H < 0, \qquad |\Delta H| > T|\Delta S|$$

in the range where adsorption proceeds spontaneously. Formation of attractive surface–adsorbate interactions provides this negative enthalpy change. These signs also explain why physical adsorption generally decreases on raising temperature at fixed pressure.

Step 3: Eliminate the other sign combinations Options (A) and (B) assign a positive enthalpy change and therefore contradict the usual exothermic character of adsorption. Option (D) has negative enthalpy but incorrectly predicts greater entropy when the gas becomes surface-bound. Only option (C) includes both the release of heat and the loss of molecular freedom. The sign choice concerns adsorption, not the energy needed to remove molecules from the surface.

Step 4: Conclusion For adsorption of a gas on a solid, $\Delta H < 0$ and $\Delta S < 0$. Hence, the correct option is (C).

Quick Tip to Solve: Imagine a freely moving gas molecule becoming attached to a surface: it loses freedom and releases binding energy, giving two negative signs.

74. The polymer formed on heating Novolac with formaldehyde is : [4 marks]

(A) Bakelite

(B) Polyester

(C) Melamine

(D) Nylon 6, 6

Correct Answer: (A) Bakelite

Solution:

Concept:

- Novolac is the initial, largely linear phenol–formaldehyde condensation polymer. Phenolic rings are linked through methylene bridges at available ortho and para positions.
- Further heating with formaldehyde produces additional cross-links between chains, giving the rigid three-dimensional thermosetting phenol–formaldehyde resin Bakelite.

Step 1: Identify the starting polymer Novolac belongs to the phenol–formaldehyde family. Its phenolic rings retain reactive sites, so the initial polymer need not remain as independent linear chains. Formaldehyde supplies the carbon for additional methylene bridges, $-CH_2-$, between aromatic units. These bridges connect different parts of the polymer rather than merely increasing the length of a single chain.

Step 2: Follow the effect of heating with formaldehyde On further condensation, the available ortho and para positions on phenolic rings participate in cross-linking. The resulting network is Bakelite. The extensive three-dimensional structure restricts chain motion, producing a hard, rigid material. Once cured, it cannot be softened and remoulded by ordinary heating; this is the defining thermosetting behaviour associated with Bakelite.

Step 3: Distinguish the alternative polymer families A polyester contains ester linkages and is generally obtained from suitable polyfunctional acids and alcohols; the given phenol–formaldehyde reaction does not establish that polymer family. Melamine resin requires melamine as a nitrogen-containing starting material, which is absent here. Nylon 6,6 is a polyamide made from hexamethylenediamine and adipic acid, not from novolac and formaldehyde. Therefore none of those names describes the cross-linked continuation of the novolac synthesis.

Step 4: Conclusion Heating novolac with formaldehyde forms Bakelite, a cross-linked phenol–formaldehyde resin. Hence, the correct option is (A).

Quick Tip to Solve: Remember the phenol–formaldehyde sequence: initial linear novolac, followed by further cross-linking to thermosetting Bakelite.

75. The conversion of hydroxyapatite occurs due to presence of F^- ions in water. The correct formula of hydroxyapatite is : [4 marks]

(A) $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$

(B) $[3Ca(OH)_2 \cdot CaF_2]$

(C) $[Ca_3(PO_4)_2 \cdot CaF_2]$

(D) $[3Ca_3(PO_4)_2 \cdot CaF_2]$

Correct Answer: (A) $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$

Solution:

Concept:

- Hydroxyapatite is a calcium phosphate mineral containing hydroxide: its formula can be written as $Ca_{10}(PO_4)_6(OH)_2$.
- The equivalent component notation is $3\,Ca_3(PO_4)_2 \cdot Ca(OH)_2$. Replacement of hydroxide by fluoride produces fluorapatite, a different composition.

Step 1: Expand the hydroxyapatite composition Three formula units of $Ca_3(PO_4)_2$ supply nine calcium atoms and six phosphate groups. One unit of $Ca(OH)_2$ adds one calcium atom and two hydroxide groups. Combining these counts gives

$$3\,Ca_3(PO_4)_2 \cdot Ca(OH)_2 = Ca_{10}(PO_4)_6(OH)_2.$$

The ten calcium ions supply total charge $+20$; six phosphate ions supply -18, and the two hydroxide ions supply -2, so the formula is electrically neutral.

Step 2: Separate the starting mineral from its fluoride product The stem mentions fluoride because fluoride can replace hydroxide in the mineral. The corresponding fluorapatite composition is $Ca_{10}(PO_4)_6F_2$, commonly written $3\,Ca_3(PO_4)_2 \cdot CaF_2$. That is the product of substitution, not the hydroxyapatite formula being requested. In particular, the hydroxyl part of the name is the clue to the presence of OH^- in the original mineral.

Step 3: Check each option against the atom counts Option (A) contains the required three calcium phosphate units and one calcium hydroxide unit. Option (B) contains no phosphate at all and cannot be an apatite calcium phosphate mineral. Option (C) has fluoride instead of hydroxide and also lacks the required factor of three before calcium phosphate. Option (D) is the standard component representation of fluorapatite, so selecting it would confuse the converted material with hydroxyapatite.

Step 4: Conclusion Hydroxyapatite is $[3\,Ca_3(PO_4)_2 \cdot Ca(OH)_2]$. Hence, the correct option is (A).

Quick Tip to Solve: Use the name hydroxyapatite to identify hydroxide; a formula ending in calcium fluoride represents fluorapatite instead.

76. Which one of the following methods is most suitable for preparing deionized water? [4 marks]

(A) Synthetic resin method

(B) Clark's method

(C) Calgon's method

(D) Permutit method

Correct Answer: (A) Synthetic resin method

Solution:

Concept:

- Deionized water requires removal of dissolved cations and anions, rather than merely removal or masking of the ions responsible for hardness.
- A cation-exchange resin in the hydrogen form replaces cations with H^+, and an anion-exchange resin in the hydroxide form replaces anions with OH^-. The released ions combine to form water.

Step 1: Remove dissolved cations Water is first passed through a suitable synthetic cation-exchange resin in its hydrogen form. Dissolved metal cations are retained by the resin and hydrogen ions enter the water in their place. For example, writing an acidic resin site as $R{-}H$, the exchange for calcium is

$$2\,R{-}H + Ca^{2+} \longrightarrow R_2Ca + 2\,H^+.$$

Sodium and other cations are also exchanged; the process is not limited to calcium and magnesium.

Step 2: Remove dissolved anions An anion-exchange resin in its hydroxide form then retains anions such as chloride and sulfate, releasing an equivalent amount of hydroxide into the water. The hydrogen ions from the first stage and hydroxide ions from the second stage combine:

$$H^+ + OH^- \longrightarrow H_2O.$$

Sequential or mixed-bed use of the two resin types therefore removes both classes of dissolved ions and yields deionized water.

Step 3: Compare the other treatment methods Clark's method uses lime to remove temporary hardness and does not remove every dissolved electrolyte. Calgon treatment complexes hardness-producing ions, leaving soluble ionic species in the water. Permutit, a sodium zeolite, exchanges calcium and magnesium for sodium; this softens the water but leaves sodium salts present. These approaches therefore do not achieve the comprehensive cation and anion removal required by the word deionized.

Step 4: Conclusion The synthetic resin method is most suitable for preparing deionized water. Hence, the correct option is (A).

Quick Tip to Solve: Softening removes hardness; deionization removes both cations and anions. Look for a pair of hydrogen-form and hydroxide-form exchange resins.

77. Given below are two statements :

Statement I : The choice of reducing agents for metals extraction can be made by using Ellingham diagram, a plot of ΔG vs temperature.

Statement II : The value of ΔS increases from left to right in Ellingham diagram.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are false

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change for oxide formation against temperature, usually using a common amount of oxygen so reaction energies can be compared.
- For a reaction over a range without phase changes, $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$, giving the slope $d(\Delta G^\circ)/dT = -\Delta S^\circ$.

Step 1: Evaluate Statement I A reducing agent removes oxygen from a metal oxide by itself being oxidized. Subtracting the metal-oxide formation reaction from the reducing-agent oxidation reaction gives the required reduction reaction. If the reducing agent's oxidation line lies below the metal-oxide line at the selected temperature, the combined reaction has a negative standard Gibbs energy change. Thus the diagram gives a thermodynamic criterion for choosing a reducing agent and a suitable temperature. Statement I is true.

Step 2: Evaluate Statement II Moving left to right means increasing temperature on the horizontal axis; it does not mean the entropy change continually increases. Within an approximately straight segment, the reaction entropy is represented by the negative of the slope and is often treated as approximately constant. Most oxide-formation reactions consume gaseous oxygen and have negative entropy changes, so their lines slope upward. Different reactions, and changes of phase, can give different slopes; there is no universal left-to-right increase in ΔS. Statement II is false.

Step 3: Select the combined verdict An upward trend in the plotted Gibbs energy must not be confused with an upward trend in entropy: the vertical axis is ΔG°, while entropy controls the slope with a minus sign. Options (A) and (B) wrongly accept Statement II, and option (C) wrongly rejects the valid use of the diagram for reducing-agent selection. Only option (D) retains Statement I and rejects Statement II.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: On an Ellingham plot, read the vertical axis as Gibbs energy and the negative slope as reaction entropy; increasing temperature is not an entropy ranking.

78. What are the products formed in sequence when excess of CO_2 is passed in slaked lime? [4 marks]

(A) $Ca(HCO_3)_2$, $CaCO_3$

(B) $CaCO_3$, $Ca(HCO_3)_2$

(C) CaO, $Ca(HCO_3)_2$

(D) CaO, $CaCO_3$

Correct Answer: (B) $CaCO_3$, $Ca(HCO_3)_2$

Solution:

Concept:

- Slaked lime is calcium hydroxide, $Ca(OH)_2$. Carbon dioxide is an acidic oxide and first neutralises the hydroxide to give calcium carbonate.
- Calcium carbonate is sparingly soluble and produces milkiness. In excess carbon dioxide and water it forms soluble calcium hydrogen carbonate, $Ca(HCO_3)_2$.

Step 1: Identify the first reaction Passing carbon dioxide initially into slaked lime produces a white precipitate of calcium carbonate. The balanced equation is

$$Ca(OH)_2 + CO_2 \longrightarrow CaCO_3\downarrow + H_2O.$$

Thus the first product in the requested sequence must be carbonate, not hydrogen carbonate. Milkiness is the visible evidence for the suspended carbonate particles; it is not evidence for calcium oxide.

Step 2: Continue passing carbon dioxide The word excess means that the reaction does not stop at the first precipitate. Additional carbon dioxide dissolves in water and converts the carbonate into hydrogen carbonate:

$$CaCO_3 + CO_2 + H_2O \longrightarrow Ca(HCO_3)_2.$$

The resulting salt is soluble, so the milkiness disappears. Combining the stages would consume two carbon dioxide molecules per calcium hydroxide unit, but the question asks for the intermediate followed by the later product.

Step 3: Compare the four sequences Option (A) reverses the order of the two salts. Options (C) and (D) start with calcium oxide, which is not formed by passing carbon dioxide through slaked lime; calcium oxide instead gives slaked lime on addition of water. Only the carbonate followed by hydrogen carbonate matches both balanced reactions.

Step 4: Conclusion The products are $CaCO_3$ followed by $Ca(HCO_3)_2$. Hence, the correct option is (B).

Quick Tip to Solve: For the limewater test remember the sequence: carbon dioxide first causes milkiness, and excess carbon dioxide clears it by forming soluble hydrogen carbonate.

79. Given below are two statements.

Statement I : Frenkel defects are vacancy as well as interstitial defects.

Statement II : Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the most appropriate answer for the statements from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- A Frenkel defect forms when a small ion leaves its normal lattice site and occupies an interstitial site. One vacancy and one interstitial defect therefore appear together.
- An F-centre is an anion vacancy occupied by an electron. Absorption of visible light by these trapped electrons gives colour in a metal-excess, non-stoichiometric crystal.

Step 1: Evaluate Statement I Consider a cation initially at its proper position in an ionic lattice. When it shifts into a normally empty interstitial position, its original position becomes vacant. Since the same ion remains inside the crystal, no ion is lost overall. The result is a vacancy-interstitial pair, so Statement I is true. The numbers of cations and anions remain unchanged, preserving stoichiometry and electrical neutrality; the mass and density are not reduced by loss of ions.

Step 2: Evaluate Statement II The interstitial ion in a Frenkel defect is an ion displaced from a normal position; it is not an electron trapped in a missing-anion site. F-centres instead arise when an anion vacancy is occupied by an electron, as in an alkali halide heated in the vapour of its metal. These trapped electrons can absorb visible radiation and cause colour. Assigning this specific F-centre mechanism to a Frenkel vacancy-interstitial pair is therefore incorrect, making Statement II false.

Step 3: Select the matching combination The required truth values are true for Statement I and false for Statement II. This combination occurs only in option (C). Options (A) and (D) wrongly reject the vacancy-interstitial description, while option (B) wrongly accepts the F-centre explanation of colour as a Frenkel defect.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the occupant: a displaced ion in an interstitial site indicates Frenkel disorder; an electron in an anion vacancy indicates an F-centre.

80. The incorrect statement is : [4 marks]

(A) Cl_2 is more reactive than ClF.

(B) F_2 is more reactive than ClF.

(C) On hydrolysis ClF forms HOCl and HF.

(D) F_2 is a stronger oxidizing agent than Cl_2 in aqueous solution.

Correct Answer: (A) Cl_2 is more reactive than ClF.

Solution:

Concept:

- Interhalogen compounds are generally more reactive than the constituent halogens other than fluorine. Their heteronuclear bonds are comparatively weak; fluorine remains exceptionally reactive.
- Hydrolysis of chlorine monofluoride gives hypochlorous acid and hydrogen fluoride. In aqueous solution fluorine has a greater tendency to undergo reduction than chlorine.

Step 1: Compare chlorine monofluoride with the halogens The interhalogen ClF is more reactive than Cl_2, whereas F_2 is more reactive than ClF. The comparison with chlorine follows the usual greater reactivity of interhalogens, while fluorine is the standard exception. Consequently the statement that chlorine is more reactive than chlorine monofluoride reverses the correct order. This already identifies option (A) as incorrect, but the remaining statements should also be checked.

Step 2: Check the hydrolysis reaction Water hydrolyses chlorine monofluoride according to

$$ClF + H_2O \longrightarrow HOCl + HF.$$

Both atoms and charge balance in this equation. Fluorine occurs as fluoride in hydrogen fluoride, while chlorine forms hypochlorous acid. Thus option (C) gives the correct products, and option (B) also correctly places fluorine above chlorine monofluoride in reactivity.

Step 3: Check the aqueous oxidising strength An oxidising agent accepts electrons and is reduced. The aqueous reduction potential for the fluorine/fluoride couple is more positive than that for the chlorine/chloride couple. Fluorine therefore oxidises other species more strongly in aqueous solution, confirming option (D). Since the instruction asks for the incorrect statement, the three correct comparisons and reaction must be excluded.

Step 4: Conclusion The incorrect statement is that Cl_2 is more reactive than ClF. Hence, the correct option is (A).

Quick Tip to Solve: For interhalogen reactivity remember the exception explicitly: interhalogens exceed their constituent halogens except fluorine. Then check whether the question asks for the correct or incorrect statement.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _____. (Integer answer) [4 marks]

Solution:

Concept:

- An interhalogen of formula AB_3 contains a central halogen A bonded to three monovalent halogen atoms B. A neutral halogen contributes seven valence electrons.
- The VSEPR description of a T-shaped interhalogen is AX_3E_2: three bonding electron domains and two lone-pair domains, arranged around a trigonal-bipyramidal electron-domain framework.

Step 1: Count the electrons remaining on A The central halogen A starts with seven valence electrons. Each of the three single A–B bonds uses one electron supplied by A and one supplied by the corresponding outer halogen. After A contributes one electron to each bond, it retains

$$7 - 3 = 4$$

non-bonding electrons. Pairing these electrons gives

$$\text{number of lone pairs on A} = \frac{4}{2} = 2.$$

The three bonded atoms therefore do not imply that all five electron domains contain atoms.

Step 2: Check the stated molecular shape Three bond pairs together with two lone pairs produce five electron domains. Their electron-domain arrangement is trigonal bipyramidal. A lone pair has stronger repulsion than a bond pair and prefers an equatorial position, where it has fewer right-angle interactions. The two lone pairs occupy two equatorial sites. The remaining bonds occupy one equatorial site and the two axial sites, giving the three B atoms their T-shaped arrangement. This is the familiar arrangement in chlorine trifluoride.

Step 3: Separate molecular shape from electron geometry The molecular shape describes only the positions of the bonded atoms, while electron geometry includes both bonds and lone pairs. Thus a T-shaped molecule is compatible with five electron domains and is not a trigonal-planar molecule with only three domains. Counting just the visible arms of the T would miss the two non-bonding pairs.

Step 4: Conclusion The number of lone pairs of electrons on A is $\boxed{2}$.

Quick Tip to Solve: For neutral halogen AB_3, subtract the three electrons used in bonds from seven, then divide the remaining four electrons into pairs.

82. These are physical properties of an element

(A) Sublimation enthalpy

(B) Ionisation enthalpy

(C) Hydration enthalpy

(D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is _____ (Integer answer) [4 marks]

Solution:

Concept:

- The reduction potential of a metal ion/metal couple measures the tendency of $\mathrm{M}^{n+}(\mathrm{aq}) + n\,\mathrm{e}^- \longrightarrow \mathrm{M}(\mathrm{s})$. It depends on the free-energy change, not on gas-phase ionisation alone.
- The reverse metal-to-aqueous-ion process can be analysed as sublimation (atomisation), ionisation and hydration. Its standard free energy is related to the standard cell potential through $\Delta G^\circ = -nFE^\circ_{\text{cell}}$.

Step 1: Write a thermochemical path For a metallic element, the conversion from solid metal to aqueous cation can be separated into

$$\mathrm{M}(\mathrm{s}) \longrightarrow \mathrm{M}(\mathrm{g}),$$

$$\mathrm{M}(\mathrm{g}) \longrightarrow \mathrm{M}^{n+}(\mathrm{g}) + n\,\mathrm{e}^-,$$

and

$$\mathrm{M}^{n+}(\mathrm{g}) \longrightarrow \mathrm{M}^{n+}(\mathrm{aq}).$$

These are respectively sublimation, ionisation and hydration. Reversing the overall transformation gives deposition of metal from its aqueous ion. Hence all three energetic contributions influence the reduction tendency of the ion.

Step 2: Match the listed quantities to the path Quantity (A), sublimation enthalpy, is needed to separate metal atoms from the solid. Quantity (B), ionisation enthalpy, describes removal of electrons from gaseous atoms, including successive ionisations where needed. Quantity (C), hydration enthalpy, describes stabilisation of the resulting ion by water. A favourable hydration contribution can compensate for costly atomisation and ionisation, which is why ionisation energy by itself does not determine an aqueous electrode potential.

Step 3: Exclude electron gain enthalpy and count Electron gain enthalpy, quantity (D), refers to adding an electron to a gaseous neutral atom to produce an anion. That step is absent from the metal/aqueous-cation cycle considered here. It must not be substituted for the reverse of ionisation, because those processes involve different starting species. The relevant entries are therefore (A), (B) and (C), giving a total of three. These enthalpy terms explain trends; a complete quantitative electrode potential also requires the corresponding free-energy balance.

Step 4: Conclusion The total number of the listed properties is $\boxed{3}$: sublimation, ionisation and hydration enthalpies.

Quick Tip to Solve: For an aqueous metal-ion/metal potential, use the path solid metal → gaseous atom → gaseous cation → hydrated cation.

83. Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of 0.10 M C_2H_5OH is _____ (Integer answer)

(i) 0.10 M $Ba_3(PO_4)_2$

(ii) 0.10 M Na_2SO_4

(iii) 0.10 M KCl

(iv) 0.10 M Li_3PO_4 [4 marks]

Solution:

Concept:

- Freezing-point depression is a colligative property: $\Delta T_f = iK_f m$, where i is the van't Hoff factor, K_f is the solvent constant and m is molality.
- For the dilute aqueous solutions at the same stated concentration, compare the numbers of solute particles produced per formula unit. Ethanol remains molecular, so its reference factor is $i = 1$.

Step 1: Set the reference comparison The freezing point is lower when the depression ΔT_f is greater. With water common to every solution and the same dilute concentration specified, the intended comparison is controlled by dissociation. The molecular solute ethanol does not dissociate into ions in water. It gives one solute particle per molecule, providing the reference factor one. Each listed electrolyte must therefore be checked for a factor greater than one, rather than compared by its formula mass.

Step 2: Count the ions for each electrolyte Using the usual ideal-dissociation treatment of the stated aqueous solutions,

$$\mathrm{Ba_3(PO_4)_2} \longrightarrow 3\,\mathrm{Ba^{2+}} + 2\,\mathrm{PO_4{}^{3-}},$$

gives five ions per formula unit. Similarly,

$$\mathrm{Na_2SO_4} \longrightarrow 2\,\mathrm{Na^{+}} + \mathrm{SO_4{}^{2-}},$$

gives three;

$$\mathrm{KCl} \longrightarrow \mathrm{K^{+}} + \mathrm{Cl^{-}},$$

gives two; and

$$\mathrm{Li_3PO_4} \longrightarrow 3\,\mathrm{Li^{+}} + \mathrm{PO_4{}^{3-}},$$

gives four. Thus the respective ideal van't Hoff factors are 5, 3, 2 and 4.

Step 3: Count the solutions satisfying the inequality Every factor obtained exceeds ethanol's factor of one. Hence every listed solution has a greater freezing-point depression and therefore a lower freezing point in the stipulated dilute-solution comparison. The question asks how many solutions satisfy the condition, not which gives the greatest depression. Consequently all four entries must be counted. This comparison treats the listed concentrations as the concentrations of the stated aqueous solutions, as required by the problem.

Step 4: Conclusion All four listed solutions satisfy the comparison. The required integer is $\boxed{4}$.

Quick Tip to Solve: Count dissolved particles per formula unit before comparing freezing points: greater i gives greater depression at the same dilute concentration.

84. The OH^- concentration in a mixture of 5.0 mL of 0.0504 M NH_4Cl and 2 mL of 0.0210 M NH_3 solution is x $\times$ 10^{-6} M. The value of x is _____. (Nearest integer)

[Given K_w = 1 $\times$ 10^{-14} and K_b = 1.8 $\times$ 10^{-5}] [4 marks]

Solution:

Concept:

- A weak base and its conjugate acid form a basic buffer. Here NH_3 is the base and NH_4^+, supplied by NH_4Cl, is its conjugate acid.
- For $NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$, $K_b = [NH_4^+][OH^-]/[NH_3]$. The buffer approximation gives $[OH^-] = K_b n(NH_3)/n(NH_4^+)$, since both share the final volume.

Step 1: Calculate the amounts before mixing The salt dissociates to give ammonium ions. Converting millilitres to litres,

$$n(NH_4^+) = 0.0504 \times 5.0 \times 10^{-3} = 2.52 \times 10^{-4}\,\text{mol}.$$

The ammonia amount is

$$n(NH_3) = 0.0210 \times 2.0 \times 10^{-3} = 4.20 \times 10^{-5}\,\text{mol}.$$

There is no strong acid or strong base added to consume one component stoichiometrically. The mixture directly contains a conjugate acid-base pair.

Step 2: Use the buffer concentration ratio The final volume is $7.0\,\text{mL}$, but it cancels in the ratio of the two solute concentrations. Rearranging the base-dissociation expression gives

$$\begin{aligned}[OH^-] &= K_b \frac{[NH_3]}{[NH_4^+]} \\ &= 1.8 \times 10^{-5} \frac{4.20 \times 10^{-5}}{2.52 \times 10^{-4}} \\ &= 1.8 \times 10^{-5} \times \frac{1}{6} \\ &= 3.0 \times 10^{-6}\,\text{M}.\end{aligned}$$

The ammonium ion suppresses further ionisation of ammonia, so the isolated-weak-base square-root expression is inapplicable.

Step 3: Check the approximation and report x The initial mixed concentrations are $0.0060\,\text{M}$ for ammonia and $0.0360\,\text{M}$ for ammonium. The calculated hydroxide concentration is much smaller than both, so neglecting their small equilibrium changes is justified. The given water constant would permit conversion to hydrogen-ion concentration, but the question directly requests hydroxide. Comparing $3.0 \times 10^{-6}\,\text{M}$ with $x \times 10^{-6}\,\text{M}$ gives $x = 3.0$, whose nearest integer is three.

Step 4: Conclusion $[OH^-] = 3.0 \times 10^{-6}\,\text{M}$; therefore the required nearest-integer value is $\boxed{x = 3}$.

Quick Tip to Solve: In a weak-base/conjugate-acid buffer, calculate the mole ratio base/salt. The common final volume cancels, but the separate starting volumes must be included in the mole calculation.

85. The number of 4f electrons in the ground statement electronic configuration of Gd^{2+} is _____. [Atomic number of Gd = 64] [4 marks]

Solution:

Concept:

- The atomic number gives the number of electrons in the neutral atom. Gadolinium has atomic number 64, and its ground-state configuration is $[\mathrm{Xe}]\,4f^7 5d^1 6s^2$.
- When a metal forms a positive ion, electrons are removed first from the outermost shell. The two $6s$ electrons are removed before the inner $4f$ electrons; a half-filled $4f^7$ subshell is particularly stable.

Step 1: Write the neutral configuration The xenon core contains 54 electrons, leaving ten electrons to distribute beyond that core. For gadolinium the stable arrangement has seven electrons in the seven different $4f$ orbitals, one electron in $5d$, and two in $6s$. Thus the electron count is

$$54 + 7 + 1 + 2 = 64.$$

The $5d^1$ electron must not be incorrectly placed in $4f$ to give $4f^8$.

Step 2: Remove the two outer electrons The charge $+2$ means that the atom loses two electrons; it does not mean that every partially occupied subshell loses electrons. Removing both outer $6s$ electrons gives

$$\mathrm{Gd} \longrightarrow \mathrm{Gd}^{2+} + 2\,\mathrm{e}^-, \qquad \mathrm{Gd}^{2+} :\ [\mathrm{Xe}]\,4f^7 5d^1.$$

The ion has $54 + 7 + 1 = 62$ electrons, consistent with $64 - 2 = 62$. The $4f$ population has therefore remained unchanged.

Step 3: Conclusion The ground-state configuration of Gd^{2+} contains $\boxed{7}$ electrons in its $4f$ subshell. The integer answer is 7.

Quick Tip to Solve: Write neutral Gd as $4f^7 5d^1 6s^2$, then remove $6s$ electrons first; do not confuse the number of $4f$ electrons with all electrons outside xenon.

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86. The ratio of number of water molecules in Mohr's salt and potash alum is _____ $\times 10^{-1}$. (Integer answer) [4 marks]

Solution:

Concept:

- Water of crystallisation is the fixed number of water molecules associated with one formula unit of a crystalline hydrated salt. The hydration number is read from the coefficient after the dot in its formula.
- Mohr's salt is $(\mathrm{NH_4})_2\mathrm{Fe(SO_4)_2}\cdot 6\,\mathrm{H_2O}$, while potash alum is $\mathrm{KAl(SO_4)_2}\cdot 12\,\mathrm{H_2O}$. Compare equal numbers of formula units when taking the ratio.

Step 1: Read the hydration numbers One formula unit of Mohr's salt contains six water molecules. One formula unit of potash alum contains twelve water molecules. The ammonium groups in Mohr's

salt are not water, and the oxygen atoms within sulphate ions must not be counted as water molecules. Hence the relevant counts are

$$N_{\text{Mohr}} = 6, \qquad N_{\text{alum}} = 12.$$

These are stoichiometric counts, so no molar mass or density is needed.

Step 2: Express the ratio in the required form The order in the question is Mohr's salt divided by potash alum. Therefore

$$\frac{N_{\text{Mohr}}}{N_{\text{alum}}} = \frac{6}{12} = \frac{1}{2} = 0.5.$$

If the blank is denoted by x, the printed answer format requires

$$x \times 10^{-1} = 0.5, \qquad x = \frac{0.5}{0.1} = 5.$$

The ratio itself is one-half, but the integer placed before 10^{-1} is five.

Step 3: Conclusion The water-molecule ratio is $6 : 12 = 1 : 2 = 5 \times 10^{-1}$. Thus the required integer is $\boxed{5}$.

Quick Tip to Solve: Read the hydration suffix first, form the ratio in the stated order, and only then convert it into the mantissa multiplying the printed power of ten.

87. The following data was obtained for chemical reaction given below at 975 K.

$2NO_{(g)} + 2H_{2(g)} \rightarrow N_{2(g)} + 2H_2O_{(g)}$

The order of the reaction with respect to NO is _____. [Integer answer] [4 marks]

	[NO]	[H_2]	Rate
	mol L^{-1}	mol L^{-1}	mol $L^{-1}s^{-1}$
(A)	8×10^{-5}	8×10^{-5}	7×10^{-9}
(B)	24×10^{-5}	8×10^{-5}	2.1×10^{-8}
(C)	24×10^{-5}	32×10^{-5}	8.4×10^{-8}

Solution:

Concept:

- For an experimental rate law $v = k[\text{NO}]^a[\text{H}_2]^b$, the order with respect to NO is the exponent a. Reaction orders are determined by rate data, not directly by coefficients in the balanced overall equation.
- Compare two experiments with the same hydrogen concentration. Taking their rate ratio cancels both the rate constant at the common temperature and the unchanged hydrogen factor.

Step 1: Choose experiments A and B At 975 K, experiment A has $[\text{NO}] = 8 \times 10^{-5}\,\text{mol}\,\text{L}^{-1}$ and rate $7 \times 10^{-9}\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$. In B, NO becomes $24 \times 10^{-5}\,\text{mol}\,\text{L}^{-1}$, while the rate becomes $2.1 \times 10^{-8}\,\text{mol}\,\text{L}^{-1}\,\text{s}^{-1}$. Both have $[\text{H}_2] = 8 \times 10^{-5}\,\text{mol}\,\text{L}^{-1}$. Thus this comparison isolates the influence of NO.

Step 2: Calculate the NO exponent Divide the rate law for B by that for A:

$$\frac{v_B}{v_A} = \left(\frac{[\mathrm{NO}]_B}{[\mathrm{NO}]_A}\right)^a.$$

Substitution gives

$$\frac{2.1 \times 10^{-8}}{7 \times 10^{-9}} = \left(\frac{24 \times 10^{-5}}{8 \times 10^{-5}}\right)^a, \qquad 3 = 3^a.$$

Consequently $a = 1$. As a separate check, B to C leaves NO constant and quadruples hydrogen and the rate, giving $b = 1$; this does not alter the requested NO order.

Step 3: Conclusion The reaction is first order with respect to NO. The required integer is $\boxed{1}$.

Quick Tip to Solve: For the order of one reactant, use a pair of experiments in which the other reactant is unchanged; do not copy the coefficient 2 from the chemical equation.

88. The Born-Haber cycle for KCl is evaluated with the following data :

$\Delta_f H^\ominus$ **for KCl = −436.7 kJ mol^{-1} ;**

$\Delta_{sub} H^\ominus$ **for K = 89.2 kJ mol^{-1} ;**

$\Delta_{ionization} H^\ominus$ **for K = 419.0 kJ mol^{-1} ;**

$\Delta_{electron\ gain} H^\ominus$ **for Cl$_{(g)}$ = −348.6 kJ mol^{-1} ;**

$\Delta_{bond} H^\ominus$ **for Cl$_2$ = 243.0 kJ mol^{-1}**

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is _____ (Nearest integer) [4 marks]

Solution:

Concept:

- Hess's law states that the overall enthalpy change equals the sum of the enthalpy changes of the component steps. A Born-Haber cycle connects formation of an ionic solid with formation of its gaseous ions.
- For one mole of KCl, only half a mole of $\mathrm{Cl_2}$ is dissociated. Lattice formation enthalpy is negative, whereas the corresponding lattice dissociation enthalpy is positive; their magnitudes are equal.

Step 1: List the gaseous-ion preparation steps Start from $\mathrm{K(s)} + \frac{1}{2}\mathrm{Cl_2(g)}$. Subliming potassium costs $89.2\,\mathrm{kJ\,mol^{-1}}$, ionising the gaseous atoms costs $419.0\,\mathrm{kJ\,mol^{-1}}$, and producing one mole of chlorine atoms costs $243.0/2 = 121.5\,\mathrm{kJ\,mol^{-1}}$. Electron gain by chlorine releases $348.6\,\mathrm{kJ\,mol^{-1}}$, so its signed contribution is negative.

Step 2: Apply Hess's law with consistent signs Let L_f be the lattice enthalpy for $\mathrm{K^+(g) + Cl^-(g) \longrightarrow KCl(s)}$. Then

$$-436.7 = 89.2 + 419.0 + 121.5 - 348.6 + L_f,$$
$$-436.7 = 281.1 + L_f,$$
$$L_f = -717.8\,\mathrm{kJ\,mol^{-1}}.$$

The negative sign agrees with attraction between opposite gaseous ions when the crystal forms. The question asks for the magnitude, so take the absolute value and then round to the nearest integer.

Step 3: Conclusion The lattice enthalpy magnitude is $717.8\,\mathrm{kJ\,mol^{-1}}$, or $\boxed{718\,\mathrm{kJ\,mol^{-1}}}$ to the nearest integer. Enter 718.

Quick Tip to Solve: Use half the chlorine bond enthalpy for one mole of KCl, keep electron gain negative, and take the magnitude only after completing the signed Hess-law sum.

89. The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be _____. (Integer answer) [4 marks]

Solution:

Concept:

- In a peptide, the amino and carboxyl groups forming peptide bonds are amide linkages rather than free amino-acid ends. Count the remaining ionisable terminal and side-chain groups.
- At pH 12.5, carboxyl groups are deprotonated to $\mathrm{COO^-}$, the phenolic group of tyrosine is predominantly $\mathrm{O^-}$, and the terminal amino group is predominantly neutral $\mathrm{NH_2}$.

Step 1: Identify the groups that remain free In Gly-Glu-Asp-Tyr, the N-terminal glycine provides the free amino end and the C-terminal tyrosine provides the free carboxyl end. Glutamate contributes one additional side-chain carboxyl group, and aspartate contributes another. Tyrosine has a phenolic hydroxyl group on its aromatic side chain. The three peptide bonds consume the internal alpha-amino and alpha-carboxyl groups, so those groups must not be counted again.

Step 2: Count negative and positive charges At the stated strongly alkaline pH, the C-terminal carboxyl group contributes -1, the Glu side-chain carboxyl contributes -1, and the Asp side-chain carboxyl contributes -1. Since the pH is also above the usual phenolic ionisation range of Tyr, its phenolate contributes a fourth -1. The N-terminal amino group is unprotonated and contributes zero. Thus

$$Q = (-1) + (-1) + (-1) + (-1) + 0 = -4.$$

There are four negative charge units and no compensating positive charge in the usual predominant-form treatment.

Step 3: Conclusion The tetrapeptide carries a predominant net charge of -4, corresponding to $\boxed{4}$ negative charge units. The integer answer is 4.

Quick Tip to Solve: At high pH, count the free C-terminus, acidic side chains and Tyr phenolate; exclude peptide-bond groups and remember that the amino terminus loses its positive charge.

90. An aqueous KCl solution of density 1.20 g mL^{-1} has a molality of 3.30 mol kg^{-1}. The molarity of the solution in mol L^{-1} is _____ (Nearest integer) [Molar mass of KCl = 74.5] [4 marks]

Solution:

Concept:

- Molality is moles of solute per kilogram of solvent, while molarity is moles of solute per litre of solution. The solution volume must be obtained from its total mass and density.
- Choose a basis of $1.000\,\text{kg}$ of water. The given molality then directly supplies the amount of KCl, and its molar mass supplies the additional solute mass.

Step 1: Calculate the mass of the solution For the chosen basis, the amount of dissolved KCl is

$$n = 3.30\,\text{mol kg}^{-1} \times 1.000\,\text{kg} = 3.30\,\text{mol}.$$

Its mass is

$$m_{\text{KCl}} = 3.30 \times 74.5 = 245.85\,\text{g}.$$

Add this to the solvent mass, not to its volume:

$$m_{\text{solution}} = 1000 + 245.85 = 1245.85\,\text{g}.$$

Molality does not specify one litre of solution, so treating 3.30 directly as an exact molarity is not justified.

Step 2: Use density to obtain volume and concentration The density is $1.20\,\text{g mL}^{-1}$. Hence

$$V = \frac{1245.85}{1.20} = 1038.2083\,\text{mL} = 1.0382083\,\text{L}.$$

Divide the amount of solute by this solution volume:

$$M = \frac{3.30}{1.0382083} = 3.1786\,\text{mol L}^{-1}.$$

Carrying the unrounded volume prevents premature rounding. The nearest-integer instruction is applied only to the final molarity.

Step 3: Conclusion The molarity is approximately $3.18\,\text{mol L}^{-1}$, which rounds to $\boxed{3\,\text{mol L}^{-1}}$. The required integer is 3.

Quick Tip to Solve: Take one kilogram of solvent, add the solute mass, and divide the total mass by density. Convert millilitres to litres before calculating molarity.