# JEE Main 2021 Paper 1 (B.E./B.Tech) Aug 26 Shift 2 Question Paper with Solutions PDF

## Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

**Time Allowed: 3 Hours** **Maximum Marks: 300**

### General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 4 (Aug-Sep), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 26 August 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

## Mathematics – Section A: Multiple Choice Questions (80 marks)

**1. Let [t] denote the greatest integer less than or equal to t. Let f(x) = x − [x], g(x) = 1 − x + [x], and h(x) = min{f(x), g(x)}, x ∈ [−2, 2]. Then h is :** [4 marks]

(A) continuous in $[-2, 2]$ but not differentiable at more than four points in $(-2, 2)$

(B) not continuous at exactly three points in $[-2, 2]$

(C) continuous in $[-2, 2]$ but not differentiable at exactly three points in $(-2, 2)$

(D) not continuous at exactly four points in $[-2, 2]$

**Correct Answer:** (A) continuous in $[-2, 2]$ but not differentiable at more than four points in $(-2, 2)$

**Solution:**

**Concept:**

- The fractional part is $\{x\} = x - \lfloor x \rfloor$, lying in $[0, 1)$. On each interval between successive integers it increases linearly with slope one.
- The minimum of $\{x\}$ and $1 - \{x\}$ forms two linear pieces. Continuity is checked with one-sided limits, and differentiability requires equal left and right derivatives.

**Step 1:** Write the two pieces on a unit interval Let $n$ be an integer and put $t = x - n$ for $n \le x < n+1$. Then $f(x) = t$, $g(x) = 1 - t$, and therefore

$$h(x) = \begin{cases} x - n, & n \le x \le n + \frac{1}{2}, \\ n + 1 - x, & n + \frac{1}{2} \le x < n + 1. \end{cases}$$

The two expressions agree at the midpoint, where the value is $1/2$. At every integer the actual value is zero.

**Step 2:** Check continuity at all joining points At an interior integer $n$, the value approached from the previous unit interval is $n - x \to 0$; from the next interval it is $x - n \to 0$. Both limits equal $h(n) = 0$. The midpoint limits also agree, and the appropriate one-sided limits at $-2$ and $2$ equal zero. Hence there is no discontinuity anywhere in the given closed interval.

**Step 3:** Count the corners in the open interval At each half-integer the slope changes from $+1$ to $-1$; at each interior integer it changes from $-1$ to $+1$. Thus the nondifferentiable points are

$$-\tfrac{3}{2},\ -1,\ -\tfrac{1}{2},\ 0,\ \tfrac{1}{2},\ 1,\ \tfrac{3}{2}.$$

There are seven such points. Options (B) and (D) wrongly assert discontinuities, while (C) counts only the three integer corners and misses the four midpoint corners.

**Step 4:** Conclusion The function is continuous on $[-2, 2]$ and is not differentiable at seven points of $(-2, 2)$, which is more than four. Hence, the correct option is (A).

**Quick Tip to Solve:** For the minimum of a fractional part and its complement, check corners at both integers and half-integers; the jumps of the fractional part itself disappear in the minimum.

**2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to :** [4 marks]

(A) $A^6 - A$

(B) $A^5$

(C) $A^5 - A$

(D) $A^6$

**Correct Answer:** (A) $A^6 - A$

**Solution:**

**Concept:**

- Obtain a low-power pattern by direct matrix multiplication and prove it by induction before using large exponents.
- Matrix subtraction is entrywise. In this problem only the entry in the second row and first column varies with a positive integer power.

**Step 1:** Calculate the first powers Direct multiplication gives

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad A^3 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

These suggest that, for every integer $n \geq 1$,

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

The expression also gives the original matrix when $n = 1$.

**Step 2:** Verify the pattern Assume the displayed formula holds for $n$. Multiplication of its second row by the first column of $A$ gives $(n-1)+1 = n$, while the other two entries of that row remain one. The first and third rows remain $(1, 0, 0)$. Thus multiplication by $A$ produces precisely the claimed formula with $n$ replaced by $n+1$, completing the induction.

**Step 3:** Subtract and compare the options The large powers therefore give

$$A^{2025} - A^{2020} = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Likewise, $A^6 - A$ has this same matrix, because its variable entry is $5 - 0 = 5$. Option (C), $A^5 - A$, has entry four instead. The matrices in (B) and (D) have diagonal entries one, whereas the required difference has every diagonal entry zero, so neither can be correct.

**Step 4:** Conclusion $A^{2025} - A^{2020} = A^6 - A$. Hence, the correct option is (A).

**Quick Tip to Solve:** When a matrix power depends linearly on the exponent, subtract the exponents after proving the pattern; also use diagonal entries to reject impossible options quickly.

**3. The local maximum value of the function $f(x) = \left(\frac{2}{x}\right)^{x^2}$, x > 0, is** [4 marks]

(A) $(2\sqrt{e})^{\frac{1}{e}}$

(B) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(C) $(e)^{\frac{2}{e}}$

(D) 1

**Correct Answer:** (C) $(e)^{\frac{2}{e}}$

**Solution:**

**Concept:**

- For a positive function, the natural logarithm is strictly increasing, so maximizing the function is equivalent to maximizing its logarithm.
- Logarithmic differentiation handles an expression with a variable base and variable exponent: set $L(x) = \log f(x)$ and use the sign of $L\prime(x)$.

**Step 1:** Take the logarithm and differentiate Since $x > 0$, both $f(x)$ and its logarithm are defined. Write

$$L(x) = x^2 \log\left(\frac{2}{x}\right).$$

Using the product rule and $\frac{d}{dx}\log(2/x) = -1/x$, we obtain

$$L\prime(x) = 2x\log(2/x) - x = x\big(2\log(2/x) - 1\big).$$

Because $x$ is positive, a stationary point requires $\log(2/x) = 1/2$, giving $x_0 = 2/\sqrt{e}$.

**Step 2:** Confirm that the stationary point is a maximum The quantity $\log(2/x)$ is strictly decreasing. It exceeds $1/2$ for $0 < x < x_0$, so $L\prime > 0$; it is below $1/2$ for $x > x_0$, so $L\prime < 0$. Thus $L$, and hence $f$, increases before $x_0$ and decreases after it. This sign change establishes the required local maximum without confusing the location with its value.

**Step 3:** Evaluate the maximum value At the stationary point, $x_0^2 = 4/e$ and $\log(2/x_0) = 1/2$. Therefore

$$\log f(x_0) = \frac{4}{e}\cdot\frac{1}{2} = \frac{2}{e}, \qquad f(x_0) = e^{2/e}.$$

For comparison, the logarithms of the proposed values in (A), (B), and (D) are $(\log 2 + 1/2)/e$, $(e/4)(\log 4 - 1/2)$, and zero respectively; none equals $2/e$.

**Step 4:** Conclusion The local maximum value is $e^{2/e}$, attained at $x = 2/\sqrt{e}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For $u(x)^{v(x)}$, maximize its logarithm first; remember to exponentiate the final logarithmic value to answer a question about the function itself.

**4. If the value of the integral**

$\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$, **where** $\alpha, \beta \in$ **R,** $5\alpha + 6\beta = 0$, **and [x] denotes the greatest integer less than or equal to x; then the value of** $(\alpha + \beta)^2$ **is equal to :** [4 marks]

(A) 100

(B) 25

(C) 16

(D) 36

**Correct Answer:** (B) 25

**Solution:**

**Concept:**

- Split the integration range at each integer, where $\lfloor x \rfloor$ changes. Values at individual endpoints do not affect a definite integral.
- On $[k, k+1)$, substitute $t = x - k$ so that the fractional part is $t$ and the same integration limits $0$ and $1$ apply to every piece.

**Step 1:** Reduce the five pieces to standard integrals Write the integral as $I$. For $k = 0, 1, 2, 3, 4$, we have $\lfloor x \rfloor = k$, $x + \lfloor x \rfloor = 2k + t$, and $x - \lfloor x \rfloor = t$. Consequently,

$$I = \sum_{k=0}^{4} \int_0^1 (2k + t)e^{-t}\, dt.$$

The two necessary integrals are

$$\int_0^1 e^{-t}\, dt = 1 - e^{-1}, \qquad \int_0^1 te^{-t}\, dt = 1 - 2e^{-1}.$$

The latter follows by integration by parts, with antiderivative $-(t+1)e^{-t}$.

**Step 2:** Sum the contributions Using $\sum_{k=0}^{4} k = 10$, we obtain

$$\begin{aligned} I &= 2(10)(1 - e^{-1}) + 5(1 - 2e^{-1}) \\ &= 25 - 30e^{-1}. \end{aligned}$$

Thus $\alpha = -30$ and $\beta = 25$ meet the integral equation. They also satisfy the additional printed condition, since $5(-30) + 6(25) = 0$.

**Step 3:** Use the additional relation and evaluate the demand The relation $5\alpha + 6\beta = 0$ gives $\alpha = -6\beta/5$. Substituting this into $I = \alpha/e + \beta$ yields $I = \beta(1 - 6/(5e))$. Since $e > 2$, the multiplier is nonzero, so this determines $\beta$ uniquely as 25 and then $\alpha = -30$. Finally,

$$(\alpha + \beta)^2 = (-30 + 25)^2 = 25.$$

The other options, 100, 16 and 36, do not equal the square of the required sum; in particular the negative sum must be squared only after combining the coefficients.

**Step 4:** Conclusion $\alpha = -30$, $\beta = 25$, and $(\alpha + \beta)^2 = 25$. Hence, the correct option is (B).

**Quick Tip to Solve:** For integrals containing both $x$ and $\lfloor x \rfloor$, a unit-interval substitution often leaves a simple sum over integers and two common integrals.

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**5. Let y(x) be the solution of the differential equation**
**$2x^2$ dy + ($e^y$ − 2x)dx = 0, x > 0. If y(e) = 1, then y(1) is equal to :** [4 marks]

(A) 0

(B) 2

(C) $\log_e 2$

(D) $\log_e$ (2e)

**Correct Answer:** (C) $\log_e 2$

**Solution:**

**Concept:**

- The substitution $z = e^{-y}$ converts the exponential term in this differential equation into a linear equation for $z$. Its derivative is $z\prime = -e^{-y}y\prime$.
- A first-order linear equation $z\prime + P(x)z = Q(x)$ is solved using integrating factor $e^{\int P(x)dx}$. Here $x > 0$, so $\log x$ is defined.

**Step 1:** Transform the differential equation Divide the given equation by $dx$ to obtain

$$2x^2y\prime + e^y - 2x = 0, \qquad y\prime = \frac{1}{x} - \frac{e^y}{2x^2}.$$

Multiplying by $-e^{-y}$ and setting $z = e^{-y}$ gives

$$z\prime = -\frac{z}{x} + \frac{1}{2x^2}, \qquad z\prime + \frac{1}{x}z = \frac{1}{2x^2}.$$

The sign from differentiating $e^{-y}$ is essential in this step.

**Step 2:** Integrate and apply the supplied value The integrating factor is $e^{\int dx/x} = x$, because $x > 0$. Hence

$$\frac{d}{dx}(xz) = \frac{1}{2x}, \qquad xe^{-y} = \frac{1}{2}\log x + C.$$

At $x = e$, the condition $y(e) = 1$ gives $e\,e^{-1} = 1 = 1/2 + C$, so $C = 1/2$. The solution through this point therefore satisfies

$$xe^{-y} = \frac{1 + \log x}{2}.$$

It is real on the connected interval $x > e^{-1}$, which contains both input points in the question.

**Step 3:** Evaluate the solution at one At $x = 1$, the last equation becomes $e^{-y(1)} = 1/2$, and therefore $y(1) = \log_e 2$. This value is neither zero nor two, ruling out (A) and (B). Option (D) is $1 + \log_e 2$, which would instead give $e^{-y(1)} = 1/(2e)$, contradicting the equation at one. As a check, the explicit solution $y = \log(2x/(1 + \log x))$ gives $y(e) = 1$ exactly.

**Step 4:** Conclusion $y(1) = \log_e 2$. Hence, the correct option is (C).

**Quick Tip to Solve:** When $e^y$ appears beside $y\prime$, try $e^{-y}$; its derivative can turn the nonlinear equation into a linear one. Carry the negative derivative sign carefully.

**6. The domain of the function $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :** [4 marks]

(A) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$
(B) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$
(C) $\left(-\frac{1}{2}, \infty\right) - \{0\}$
(D) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

**Correct Answer:** (D) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

**Solution:**

**Concept:**

- The real inverse cosecant $\text{cosec}^{-1} u$ requires $|u| \geq 1$; equality is allowed at both endpoints of its real input range.
- A rational input must also be defined. Squaring an absolute-value inequality with nonnegative sides is valid, and multiplication by $x^2 > 0$ preserves it when $x \neq 0$.

**Step 1:** Impose both domain conditions The rational expression $(1+x)/x$ requires $x \neq 0$. The inverse cosecant then imposes

$$\left|\frac{1+x}{x}\right| \geq 1.$$

Both sides are nonnegative, so we may square to get $(1+x)^2/x^2 \geq 1$. Since zero has already been excluded, $x^2$ is strictly positive, allowing multiplication without a reversal of inequality.

**Step 2:** Solve the inequality and retain the endpoint We have

$$(1+x)^2 \geq x^2 \quad \Longleftrightarrow \quad 1 + 2x \geq 0 \quad \Longleftrightarrow \quad x \geq -\frac{1}{2}.$$

At $x = -1/2$, the argument is $-1$, which is a permitted inverse-cosecant input. At $x = 0$, however, the argument does not exist, so zero must remain excluded even though it satisfies the simplified inequality.

**Step 3:** Check the alternatives The resulting set is $[-1/2, 0) \cup (0, \infty)$. Option (A) includes inputs such as $x = -3/4$, whose argument is $-1/3$ and is not permitted. Option (B) unnecessarily omits positive inputs below one; for example $x = 1/2$ gives the allowed argument three. Option (C) wrongly omits $-1/2$, where the inverse cosecant is defined. These checks also show why neither an open left endpoint nor an extra restriction at one is justified.

**Step 4:** Conclusion The domain is $\left[-\frac{1}{2}, \infty\right) \setminus \{0\}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For real inverse secant or inverse cosecant, require the input magnitude to be at least one, then separately exclude zeros of any denominator before simplifying.

**7. A fair die is tossed until six is obtained on it. Let x be the number of required tosses, then the conditional probability P(x $\geq$ 5 | x > 2) is :** [4 marks]

(A) $\frac{125}{216}$

(B) $\frac{11}{36}$

(C) $\frac{5}{6}$

(D) $\frac{25}{36}$

**Correct Answer:** (D) $\frac{25}{36}$

**Solution:**

**Concept:**

- If independent trials have success probability $p$, the number $X$ of trials up to the first success satisfies $P(X > m) = (1-p)^m$.
- Conditional probability is $P(A \mid B) = P(A \cap B)/P(B)$ when $P(B) > 0$. Here the event $X \geq 5$ is contained in $X > 2$.

**Step 1:** Translate the waiting-time events into failures A success is rolling six, with probability $1/6$. A failure is any of the other five faces, with probability $q = 5/6$. The event $X \geq 5$ means that each of the first four tosses fails; the fifth toss is not required to be a six, because later first successes are also included. Thus

$$P(X \geq 5) = q^4.$$

Similarly $X > 2$ means that the first two tosses fail, so $P(X > 2) = q^2$.

**Step 2:** Apply the conditional-probability formula Whenever $X \geq 5$, it follows automatically that $X > 2$. The intersection in the numerator is therefore simply $X \geq 5$. Since $q^2$ is positive,

$$P(X \geq 5 \mid X > 2) = \frac{q^4}{q^2} = q^2 = \left(\frac{5}{6}\right)^2 = \frac{25}{36}.$$

Equivalently, after the first two failures have been specified, only tosses three and four must additionally fail, independently of the preceding tosses.

**Step 3:** Distinguish the other probabilities Option (A), $125/216 = q^3$, counts three further failures rather than the required two. Option (C), $5/6 = q$, counts only one further failure. Option (B), $11/36 = 1 - q^2$, is the complementary conditional probability that a six occurs within the next two tosses. It therefore corresponds to $X < 5$, given $X > 2$, rather than the requested event. No factor $1/6$ is needed because the question asks for a tail probability, not for success on a particular toss.

**Step 4:** Conclusion $P(X \geq 5 \mid X > 2) = 25/36$. Hence, the correct option is (D).

**Quick Tip to Solve:** For first-success waiting times, translate $X \geq r$ into failure on the first $r - 1$ trials. Conditioning on earlier failures cancels those trials from the count.

**8. If $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$, then the value of tan p is :** [4 marks]

(A) $\frac{101}{102}$

(B) $\frac{50}{51}$

(C) 100

(D) $\frac{51}{50}$

**Correct Answer:** (B) $\frac{50}{51}$

**Solution:**

**Concept:**

- The subtraction identity is $\tan(\alpha-\beta) = (\tan\alpha - \tan\beta)/(1+\tan\alpha\tan\beta)$. It converts a suitable inverse-tangent term into a difference.
- A sum of consecutive differences telescopes: the negative term of one summand cancels the positive term of the preceding summand. Principal inverse tangents lie in $(-\pi/2, \pi/2)$.

**Step 1:** Express each summand as a difference For every positive integer $r$, consider the two positive numbers $2r+1$ and $2r-1$. Their inverse tangents have a positive difference smaller than $\pi/2$. Hence the principal-value identity applies without adding a multiple of $\pi$:

$$\tan^{-1}(2r+1) - \tan^{-1}(2r-1) = \tan^{-1}\left(\frac{2}{1+(2r+1)(2r-1)}\right) = \tan^{-1}\left(\frac{1}{2r^2}\right).$$

**Step 2:** Cancel the intermediate inverse tangents Substitute this identity into the given sum. Its first terms are $(\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}5 - \tan^{-1}3)$. The intermediate angles cancel in pairs, leaving only the last positive term and the first negative term:

$$p = \tan^{-1}101 - \tan^{-1}1.$$

The upper endpoint is $2(50)+1 = 101$, rather than 100.

**Step 3:** Take the tangent and compare the choices Use the subtraction formula once more:

$$\tan p = \frac{101-1}{1+101\cdot 1} = \frac{100}{102} = \frac{50}{51}.$$

Thus option (A) has the wrong numerator; option (C) omits the denominator from the tangent subtraction formula; and option (D) is the reciprocal of the required result.

**Step 4:** Conclusion The value is $\boxed{\tan p = 50/51}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Look for odd neighbours $2r-1$ and $2r+1$ when an inverse tangent contains $1/(2r^2)$; the resulting differences telescope.

**9. Two fair dice are thrown. The numbers on them are taken as $\lambda$ and $\mu$, and a system of linear equations**

**$x + y + z = 5$**

**$x + 2y + 3z = \mu$**

**$x + 3y + \lambda z = 1$**

**is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :** [4 marks]

(A) $p = \frac{1}{6}$ and $q = \frac{1}{36}$

(B) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(C) $p = \frac{5}{6}$ and $q = \frac{1}{36}$

(D) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

**Correct Answer:** (B) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

**Solution:**

**Concept:**

- A square linear system has a unique solution exactly when its coefficient determinant is nonzero. A zero determinant requires a separate consistency check.
- The two fair dice give 36 equally likely ordered pairs $(\lambda, \mu)$. Probabilities are obtained by counting these ordered pairs, not unordered outcomes.

**Step 1:** Find when the coefficient matrix is singular The coefficient determinant depends only on $\lambda$. Subtract the first row from the second and third rows, which preserves the determinant:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \lambda - 1 \end{vmatrix} = \lambda - 5.$$

Therefore all values of $\lambda$ except 5 give a unique solution, regardless of the second die.

**Step 2:** Check the singular case for consistency Subtracting the first original equation from the other two gives $y + 2z = \mu - 5$ and $2y + (\lambda - 1)z = -4$. When $\lambda = 5$, the left side of the second reduced equation is twice that of the first. Consistency consequently requires

$$2(\mu - 5) = -4, \qquad \mu = 3.$$

For the outcome $(5, 3)$, the remaining equations are dependent and there are infinitely many solutions. For the other five outcomes with $\lambda = 5$, they contradict each other and there is no solution.

**Step 3:** Count outcomes and reject the alternatives Unique solutions occur for five possible values of $\lambda$, each combined with six values of $\mu$. Hence

$$p = \frac{5 \cdot 6}{36} = \frac{5}{6}, \qquad q = \frac{5}{36}.$$

The remaining probability is $1/36$, belonging to infinitely many solutions, not no solution. This excludes (C); (A) and (D) also give the wrong value of $p$.

**Step 4:** Conclusion $\boxed{p = 5/6,\ q = 5/36}$. Hence, the correct option is (B).

**Quick Tip to Solve:** After finding the singular die value, compare the reduced right-hand sides; singularity alone does not imply that the system has no solution.

**10. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :** [4 marks]

(A) $y^3(x - 2) = x^2$

(B) $x^3(x - 2) = y^2$

(C) $y^2(x - 2) = x^3$

(D) $x^2(x - 2) = y^3$

**Correct Answer:** (C) $y^2(x - 2) = x^3$

**Solution:**

**Concept:**

- For $x^2 - y^2 = 4$, the chord whose midpoint is $(h, k)$ has equation $hx - ky = h^2 - k^2$, obtained from the midpoint-chord relation $T = S_1$.
- A nonvertical tangent of slope $m$ to $y^2 = 4ax$ is $y = mx + a/m$. Here $a = 2$, so its intercept is $2/m$.

**Step 1:** Write the chord using its midpoint Let the midpoint of a nondegenerate chord be $(h, k)$. Applying the midpoint formula gives

$$hx - ky = h^2 - k^2.$$

For $k \neq 0$, write this as

$$y = \frac{h}{k}x - \frac{h^2 - k^2}{k}.$$

Thus the chord has slope $m = h/k$ and vertical intercept $-(h^2 - k^2)/k$. The midpoint coordinates refer to the chord, and need not satisfy the hyperbola equation themselves.

**Step 2:** Impose tangency to the parabola Since the same line touches $y^2 = 8x$, its intercept must equal $2/m = 2k/h$. Equating intercepts and clearing denominators yields

$$-\frac{h^2 - k^2}{k} = \frac{2k}{h}, \qquad -h(h^2 - k^2) = 2k^2.$$

Rearrange carefully to obtain

$$h^3 = k^2(h - 2).$$

This condition is also obtained by setting the discriminant to zero after substituting the line into the parabola.

**Step 3:** Identify the locus equation Replace the variable midpoint coordinates $(h, k)$ by $(x, y)$. The required algebraic locus is

$$y^2(x - 2) = x^3.$$

As usual, only its points arising from real nondegenerate chords are relevant. The exceptional vertical tangent to the parabola is $x = 0$, which does not cut this hyperbola in a real chord. Options (A) and (D) incorrectly cube the ordinate, while (B) gives an incorrect extra power of the abscissa; none equals the condition derived from the tangent intercept.

**Step 4:** Conclusion The locus is $\boxed{y^2(x-2) = x^3}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Use $T = S_1$ for a midpoint chord, then compare its slope and intercept with $y = mx + a/m$ for the parabola.

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**11. The value of**

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ **is :** [4 marks]

(A) $\frac{1}{4\sqrt{2}}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{1}{8\sqrt{2}}$

**Correct Answer:** (C) $\frac{1}{8}$

**Solution:**

**Concept:**

- Supplementary angles have equal sines: $\sin(\pi - \theta) = \sin\theta$. This pairs the first and last factors of a symmetric sine product.
- The double-angle identity $2\sin\theta\cos\theta = \sin 2\theta$, together with $\sin(\pi/4) = 1/\sqrt{2}$, evaluates the remaining product exactly.

**Step 1:** Pair the supplementary factors Denote the required expression by $P$. The angles $7\pi/8$, $6\pi/8$, and $5\pi/8$ are supplementary to $\pi/8$, $2\pi/8$, and $3\pi/8$, respectively. All their sines are positive. Therefore

$$P = 2\left(\sin\frac{\pi}{8}\sin\frac{2\pi}{8}\sin\frac{3\pi}{8}\right)^2.$$

The factor 2 outside the original product must remain outside the square.

**Step 2:** Evaluate the product inside the square Use $3\pi/8 = \pi/2 - \pi/8$, so that $\sin(3\pi/8) = \cos(\pi/8)$. Then

$$\sin\frac{\pi}{8}\sin\frac{3\pi}{8} = \sin\frac{\pi}{8}\cos\frac{\pi}{8} = \frac{1}{2}\sin\frac{\pi}{4} = \frac{\sqrt{2}}{4}.$$

Multiplication by the middle factor $\sin(2\pi/8) = \sqrt{2}/2$ gives

$$\sin\frac{\pi}{8}\sin\frac{2\pi}{8}\sin\frac{3\pi}{8} = \frac{1}{4}.$$

**Step 3:** Restore the square and the prefactor Substitution now gives

$$P = 2\left(\frac{1}{4}\right)^2 = \frac{2}{16} = \frac{1}{8}.$$

There is no remaining square root: the two factors of $\sqrt{2}$ already multiplied to 2 in the inner product. Consequently the radical-valued choices (A) and (D) cannot be correct. Choice (B) is the inner three-sine product, before squaring and restoring the prefactor; it is not the full expression.

**Step 4:** Conclusion The given product equals $\boxed{1/8}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Pair sines of supplementary angles first; then turn $\sin(\pi/8)\sin(3\pi/8)$ into a double-angle expression.

**12. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation :** [4 marks]

(A) $x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

(B) $x^2 + \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$

(C) $x^2 + \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

(D) $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$

**Correct Answer:** (A) $x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

**Solution:**

**Concept:**

- Write a complex number as $r(\cos\theta + i\sin\theta)$. De Moivre's theorem raises its modulus to the power and multiplies its argument by that power.
- For real numbers $p, q$, the monic quadratic having them as roots is $x^2 - (p+q)x + pq = 0$. Both their sum and their product are needed.

**Step 1:** Convert the base to polar form The modulus of $\sqrt{3}+i$ is $\sqrt{3+1} = 2$. Both components are positive, so its argument lies in the first quadrant and equals $\pi/6$. Hence

$$(\sqrt{3}+i)^{100} = 2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right).$$

Dividing by the stated factor $2^{99}$ leaves a factor of 2; it does not leave a unit-modulus complex number.

**Step 2:** Reduce the argument and identify the components Because $50\pi/3 = 16\pi + 2\pi/3$, periodicity reduces the angle to $2\pi/3$. Thus

$$p + iq = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + i\sqrt{3}.$$

Equality of the real and imaginary parts gives $p = -1$ and $q = \sqrt{3}$. The positive imaginary part is essential: using an angle in the fourth quadrant would give an incorrect sign for $q$.

**Step 3:** Form the quadratic and compare coefficients The sum and product are

$$p+q=\sqrt{3}-1, \qquad pq=-\sqrt{3}.$$

Therefore the required monic equation is

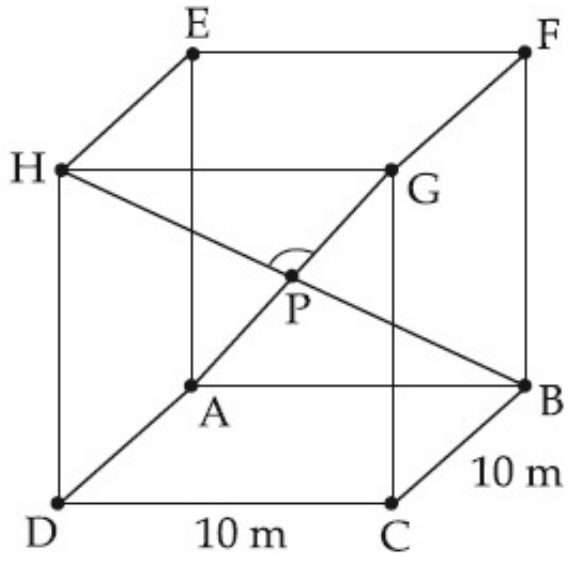
$$x^2-(\sqrt{3}-1)x-\sqrt{3}=0.$$

Its factorisation $(x+1)(x-\sqrt{3})=0$ checks both roots directly. Choice (C) reverses the sign of the sum term; choices (B) and (D) have positive constant term and hence cannot have roots of opposite signs as required here.

**Step 4:** Conclusion The equation is $\boxed{x^2-(\sqrt{3}-1)x-\sqrt{3}=0}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Before extracting real and imaginary parts, divide by the factor stated in the question; here $2^{100}/2^{99}=2$.

**13. A hall has a square floor of dimension 10 m $\times$ 10 m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :** [4 marks]




(A) 5

(B) $2\sqrt{10}$

(C) $5\sqrt{3}$

(D) $5\sqrt{2}$

**Correct Answer:** (D) $5\sqrt{2}$

**Solution:**

**Concept:**

- The cosine of the angle between vectors $\mathbf{u}$ and $\mathbf{v}$ is $(\mathbf{u}\cdot\mathbf{v})/(|\mathbf{u}||\mathbf{v}|)$. Use vectors pointing along the two rays of the specified angle.
- The space diagonals of a rectangular hall bisect each other. For a square floor of side 10 and height $h$, their intersection has half of every relevant coordinate.

**Step 1:** Assign coordinates consistent with the figure Take the floor vertices as $A=(0,0,0)$, $B=(10,0,0)$, $C=(10,10,0)$, and $D=(0,10,0)$. The roof vertices directly above $C$ and $D$ are $G=(10,10,h)$ and $H=(0,10,h)$. The diagonals $AG$ and $BH$ meet at their common midpoint $P=(5,5,h/2)$. This coordinate choice reproduces the square floor and the vertical walls shown.

**Step 2:** Use the rays forming the stated angle The vectors for $\angle GPH$ are

$$\overrightarrow{PG}=(5,5,h/2), \qquad \overrightarrow{PH}=(-5,5,h/2).$$

Their dot product is $-25+25+h^2/4=h^2/4$; each squared length is $50+h^2/4$. Consequently

$$\cos\angle GPH=\frac{h^2/4}{50+h^2/4}=\frac{h^2}{200+h^2}.$$

Taking one ray in the reverse direction would instead calculate the supplementary angle.

**Step 3:** Solve for the positive height The given cosine is $1/5$, so

$$\frac{h^2}{200+h^2}=\frac{1}{5}, \qquad 5h^2=200+h^2, \qquad h^2=50.$$

A physical height is positive; therefore $h=\sqrt{50}=5\sqrt{2}$ metres. The other options have squared heights 25, 40, and 75 respectively, none of which satisfies $4h^2=200$, so they are excluded by the angle condition.

**Step 4:** Conclusion The hall has height $\boxed{5\sqrt{2}\text{ m}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Construct vectors from the vertex of the specified angle. The horizontal terms cancel in their dot product for these two space-diagonal rays.

**14. The value of** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right) dx$ **is** [4 marks]

(A) $\frac{\pi}{2}$

(B) $\frac{5\pi}{4}$

(C) $\frac{3\pi}{4}$

(D) $\frac{3\pi}{2}$

**Correct Answer:** (C) $\frac{3\pi}{4}$

**Solution:**

**Concept:**

- On symmetric limits, replacing $x$ by $-x$ leaves the value of a definite integral unchanged. Here $\sin(-x) = -\sin x$, while $1 + \sin^2 x$ is even.
- For positive $a$, $1/(1+a) + 1/(1+a^{-1}) = 1$. Pairing these complementary fractions removes the exponential denominator.

**Step 1:** Reflect the integral across the origin Let the given integral be $I$. The substitution $x = -t$, with reversal of the limits cancelling the negative differential, gives

$$I = \int_{-\pi/2}^{\pi/2} \frac{1 + \sin^2 t}{1 + \pi^{-\sin t}}\, dt.$$

The factor $\pi^{\sin t}$ is always positive, so both the original and reflected denominators are nonzero throughout the interval. The integrands are continuous and may be added term by term.

**Step 2:** Add the two equal representations Rename the dummy variable $t$ as $x$ and add the original expression. With $a = \pi^{\sin x}$, the two reciprocal fractions have sum one. Therefore

$$2I = \int_{-\pi/2}^{\pi/2} (1 + \sin^2 x) \left(\frac{1}{1 + \pi^{\sin x}} + \frac{1}{1 + \pi^{-\sin x}}\right) dx = \int_{-\pi/2}^{\pi/2} (1 + \sin^2 x)\, dx.$$

This simplification uses reflection of the whole integral, rather than incorrectly treating its original denominator as an even function.

**Step 3:** Evaluate the elementary integral Using $\sin^2 x = (1 - \cos 2x)/2$, its integral over these limits is $\pi/2$. The integral of 1 is $\pi$, and hence

$$2I = \pi + \frac{\pi}{2} = \frac{3\pi}{2}, \qquad I = \frac{3\pi}{4}.$$

Choice (D) is $2I$, before division by two. Choice (A) omits the sine-squared contribution, and choice (B) is also inconsistent with the evaluated reflected sum.

**Step 4:** Conclusion The integral equals $\boxed{3\pi/4}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For a denominator $1 + a^{\sin x}$ on symmetric limits, pair the values at $x$ and $-x$ before attempting integration.

**15. A circle C touches the line x = 2y at the point (2, 1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of $C_1$. Then the diameter of C is :** [4 marks]

(A) $7\sqrt{5}$

(B) 15

(C) $\sqrt{285}$

(D) $4\sqrt{15}$

**Correct Answer:** (A) $7\sqrt{5}$

**Solution:**

**Concept:**

- The centre of a circle touching a given line lies on the normal to that line through the point of contact. Its radius is the distance from its centre to the contact point.
- The common chord of two circles is their radical axis. If it is a diameter of one circle, its centre lies on the radical axis and has equal powers with respect to both circles.

**Step 1:** Locate the unknown centre Write the known circle as $x^2 + (y+1)^2 = 6$. Thus its centre is $O_1 = (0, -1)$ and its squared radius is 6. The tangent line is $x - 2y = 0$, with normal vector $(1, -2)$. Therefore the centre of the required circle can be written as $O = (2 + t, 1 - 2t)$. Since the contact point is $T = (2, 1)$, its squared radius is

$$r^2 = OT^2 = t^2 + (-2t)^2 = 5t^2.$$

**Step 2:** Use the diameter condition The chord $PQ$ passes through $O_1$, because it is a diameter of the known circle. Hence $O_1$ is on the radical axis. Its power relative to the known circle is $-6$, rather than $+6$. Equal powers give

$$OO_1^2 - r^2 = -6.$$

Substitute the centre and radius expressions:

$$\begin{aligned}(2+t)^2 + (2-2t)^2 - 5t^2 &= -6,\\ 8 - 4t &= -6,\\ t &= \frac{7}{2}.\end{aligned}$$

**Step 3:** Calculate and check the diameter The radius is positive and equals $\sqrt{5}\,|t| = 7\sqrt{5}/2$. Thus the diameter is

$$2r = 7\sqrt{5}.$$

This agrees with the first option. The other three proposed diameters do not satisfy the equal-power equation with a centre on the specified normal; they are therefore excluded. Using a positive power of 6 would describe the wrong chord condition and cannot be used here.

**Step 4:** Conclusion The diameter of circle $C$ is $\boxed{7\sqrt{5}}$ units. Hence, the correct option is (A).

**Quick Tip to Solve:** When a common chord is a diameter of one circle, substitute that circle's centre into the equal-power equation; its power is the negative squared radius.

**16.** $\lim\limits_{x\to 2}\left(\sum\limits_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ **is equal to :** [4 marks]

(A) $\frac{9}{44}$

(B) $\frac{5}{24}$

(C) $\frac{1}{5}$

(D) $\frac{7}{36}$

**Correct Answer:** (A) $\frac{9}{44}$

**Solution:**

**Concept:**

- A finite sum of rational functions is continuous at a point where every denominator is nonzero. The limit can then be found by direct substitution term by term.
- The identity $1/[k(k+1)] = 1/k - 1/(k+1)$ turns consecutive rational terms into a telescoping sum; only its endpoint terms survive.

**Step 1:** Check continuity and substitute There are only nine summands, so no convergence or interchange of an infinite series is involved. At $x = 2$, the denominator of the term indexed by $n$ becomes

$$4n(n+1) + 4(2n+1) + 4 = 4(n^2 + 3n + 2) = 4(n+1)(n+2).$$

It is strictly positive for every $n = 1, \ldots, 9$. Thus substitution is valid and the required limit is

$$L = \frac{1}{2}\sum_{n=1}^{9}\frac{1}{(n+1)(n+2)}.$$

**Step 2:** Split the fractions and telescope For each term use

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}.$$

Hence

$$\begin{aligned} 2L &= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) \\ &\quad + \cdots + \left(\frac{1}{10} - \frac{1}{11}\right) \\ &= \frac{1}{2} - \frac{1}{11} = \frac{9}{22}. \end{aligned}$$

Therefore

$$L = \frac{1}{2}\cdot\frac{9}{22} = \frac{9}{44}.$$

The upper endpoint is $11$, because the final summand has $n = 9$.

**Step 3:** Compare the proposed values The exact fraction obtained is $9/44$, approximately $0.204545$. The other values $5/24$, $1/5$, and $7/36$ are approximately $0.208333$, $0.200000$, and $0.194444$, respectively, so none equals the telescoped result. In particular, replacing the final denominator $11$ by $10$ would be an endpoint error.

**Step 4:** Conclusion The limit is $\boxed{9/44}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Factor the denominator after substituting the limit point, then write out the first and last telescoping terms to avoid shifting an endpoint.

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**17. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to :** [4 marks]

(A) $4\sqrt{3}$

(B) 6

(C) $6\sqrt{3}$

(D) $3\sqrt{6}$

**Correct Answer:** (C) $6\sqrt{3}$

**Solution:**

**Concept:**

- For $x^2/a^2 - y^2/b^2 = 1$, the eccentricity satisfies $e^2 = 1 + b^2/a^2$. The conjugate axis is the $y$-axis, where $x = 0$.
- The tangent at $(x_1, y_1)$ is $xx_1/a^2 - yy_1/b^2 = 1$. Implicit differentiation gives its slope, and the normal has the negative reciprocal slope.

**Step 1:** Determine the hyperbola parameters From the eccentricity $e = \sqrt{5}/2$, obtain

$$\frac{b^2}{a^2} = e^2 - 1 = \frac{1}{4}, \qquad b^2 = \frac{a^2}{4}.$$

Substitute the given point $P = (-2\sqrt{6}, \sqrt{3})$ into the hyperbola:

$$\frac{24}{a^2} - \frac{3}{b^2} = 1.$$

Consequently $(24 - 12)/a^2 = 1$, giving $a^2 = 12$ and $b^2 = 3$.

**Step 2:** Find the tangent intercept The tangent equation is

$$\frac{-2\sqrt{6}\,x}{12} - \frac{\sqrt{3}\,y}{3} = 1.$$

Setting $x = 0$ gives $y = -\sqrt{3}$. Thus $Q = (0, -\sqrt{3})$. The negative sign matters: the point of tangency lies above the axis, but its tangent meets the conjugate axis below the origin.

**Step 3:** Find the normal intercept and distance Differentiate the hyperbola to get

$$\frac{dy}{dx} = \frac{b^2 x}{a^2 y}.$$

At $P$, the tangent slope is $-1/\sqrt{2}$; hence the normal slope is $\sqrt{2}$. The normal is

$$y - \sqrt{3} = \sqrt{2}(x + 2\sqrt{6}).$$

At $x = 0$, this yields $y = \sqrt{3} + 4\sqrt{3} = 5\sqrt{3}$, so $R = (0, 5\sqrt{3})$. Both intercepts have zero $x$-coordinate, and therefore

$$QR = |5\sqrt{3} - (-\sqrt{3})| = 6\sqrt{3}.$$

The alternatives $4\sqrt{3}$, $6$, and $3\sqrt{6}$ differ from this distance and are excluded.

**Step 4:** Conclusion The length $QR$ is $\boxed{6\sqrt{3}}$ units. Hence, the correct option is (C).

**Quick Tip to Solve:** For intercepts on the conjugate axis, set x equal to zero in the tangent and normal separately; subtract their signed y-coordinates only at the end.

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**18. Consider the two statements :**

**(S1) : (p $\rightarrow$ q) $\vee$ ($\sim$ q $\rightarrow$ p) is a tautology .**

**(S2) : (p $\wedge \sim$ q) $\wedge$ ($\sim$ p $\wedge$ q) is a fallacy.**

**Then :** [4 marks]

(A) only (S1) is true.

(B) both (S1) and (S2) are false.

(C) both (S1) and (S2) are true.

(D) only (S2) is true.

**Correct Answer:** (C) both (S1) and (S2) are true.

**Solution:**

**Concept:**

- The implication $p \rightarrow q$ is equivalent to $\neg p \vee q$. A tautology is true for every assignment of truth values, whereas a fallacy or contradiction is false for every assignment.
- Logical conjunction and disjunction are associative and commutative. The excluded-middle expression $p \vee \neg p$ is always true, and $p \wedge \neg p$ is always false.

**Step 1:** Simplify the expression in the first statement Translate each implication before combining the expressions. In particular, $\neg q \rightarrow p$ becomes $q \vee p$, because the negation of its antecedent is $q$. Thus

$$\begin{aligned}(p \rightarrow q) \vee (\neg q \rightarrow p) &= (\neg p \vee q) \vee (q \vee p) \\ &= (\neg p \vee p) \vee q \\ &= \text{True}.\end{aligned}$$

This holds whether $q$ is true or false. Consequently the claim in $S_1$ is true.

**Step 2:** Simplify the expression in the second statement Rearrange the conjunction without changing

any connective:

$$(p \wedge \neg q) \wedge (\neg p \wedge q) = (p \wedge \neg p) \wedge (q \wedge \neg q)$$
$$= \text{False}.$$

It demands that $p$ and its negation are both true, which is impossible. It also demands both $q$ and its negation. Therefore this compound expression is false under all four truth assignments. It is a fallacy, so the claim in $S_2$ is also true.

**Step 3:** Choose the combined verdict Both assertions about their respective logical expressions are correct. Notice the distinction between a false compound expression and a false statement about that expression: saying that an always-false expression is a fallacy is itself correct. Options asserting that only one of the statements is true therefore fail, and the option declaring both statements false fails as well.

**Step 4:** Conclusion Both $S_1$ and $S_2$ are true. Hence, the correct option is (C).

**Quick Tip to Solve:** Eliminate implication symbols first. Then search for p or its negation joined by disjunction, or joined by conjunction, to recognize a tautology or contradiction immediately.

**19. Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $\vec{r} \cdot \left(\hat{i} + \hat{j} + 4\hat{k}\right) = 16$ and $\vec{r} \cdot \left(-\hat{i} + \hat{j} + \hat{k}\right) = 6$. Then which of the following points does NOT lie on P?** [4 marks]

(A) (3, 3, 2)

(B) (6, −6, 2)

(C) (4, 2, 2)

(D) (−8, 8, 6)

**Correct Answer:** (C) (4, 2, 2)

**Solution:**

**Concept:**

- If two planes are $F_1 = 0$ and $F_2 = 0$, every plane of the family $F_1 + \lambda F_2 = 0$ contains their line of intersection.
- A point lies on a plane precisely when its coordinates satisfy the plane equation. To find the point that does not lie on a plane, test all proposed triples after determining the equation.

**Step 1:** Form the family through the common line The given vector equations correspond to

$$F_1 = x + y + 4z - 16 = 0, \qquad F_2 = -x + y + z - 6 = 0.$$

Hence write the required plane as

$$x + y + 4z - 16 + \lambda(-x + y + z - 6) = 0.$$

This construction preserves the entire intersection line, since both expressions vanish on that line, for every value of $\lambda$.

**Step 2:** Use the specified point The plane must contain $(1, 2, 3)$. Substituting gives

$$1 + 2 + 12 - 16 + \lambda(-1 + 2 + 3 - 6) = 0,$$

so $-1 - 2\lambda = 0$, or $\lambda = -1/2$. Multiplying the plane equation by two yields

$$2(x + y + 4z - 16) - (-x + y + z - 6) = 0,$$

and therefore

$$P: \quad 3x + y + 7z = 26.$$

As a check, the given point satisfies $3 + 2 + 21 = 26$.

**Step 3:** Test all four options Evaluate the left side $3x + y + 7z$ at each proposed point:

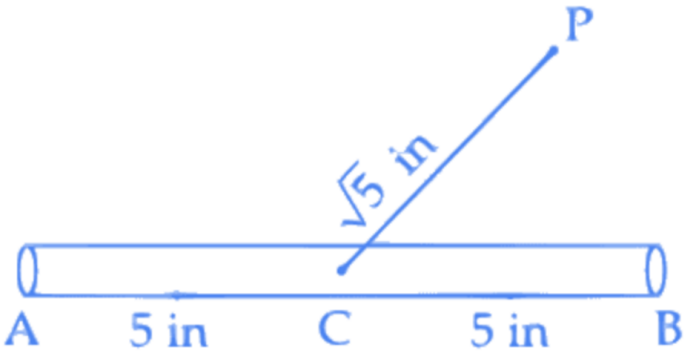
$$\begin{aligned} (3, 3, 2): &\quad 9 + 3 + 14 = 26, \\ (6, -6, 2): &\quad 18 - 6 + 14 = 26, \\ (4, 2, 2): &\quad 12 + 2 + 14 = 28, \\ (-8, 8, 6): &\quad -24 + 8 + 42 = 26. \end{aligned}$$

The first, second and fourth points satisfy the plane equation exactly. The third point gives $28$ instead of $26$, and is the only point outside the plane. The word NOT in the question therefore selects that point.

**Step 4:** Conclusion The point $\boxed{(4, 2, 2)}$ does not lie on $P$. Hence, the correct option is (C).

**Quick Tip to Solve:** Keep the plane in an integer-coefficient form before checking options; evaluating one linear expression at every point is faster and safer than solving extra equations.

**20. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that PC = $\sqrt{5}$ inches and $\angle$PCB = $\tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :** [4 marks]




(A) $\tan^{-1}\left(\frac{3}{4}\right)$
(B) $\tan^{-1}(1)$
(C) $\tan^{-1}\left(\frac{4}{3}\right)$
(D) $\tan^{-1}\left(\frac{1}{2}\right)$

**Correct Answer:** (A) $\tan^{-1}\left(\frac{3}{4}\right)$

**Solution:**

**Concept:**

- If a point is a distance r from the pivot of a line, its perpendicular distance from the line is $r|\sin\beta|$, where beta is the angle between the line and the segment joining the pivot to the point.
- The difference identity $\tan(\alpha-\beta) = (\tan\alpha - \tan\beta)/(1+\tan\alpha\tan\beta)$ determines the rotation once the original and required angles are known.

**Step 1:** Express the required distance Let the initial acute angle $\angle PCB$ be $\alpha = \tan^{-1}2$. Since $PC = \sqrt{5}$, its perpendicular components relative to the initial pencil direction have lengths 1 and 2 inches. If the final acute angle between $CP$ and the pencil is $\beta$, the requested distance requires

$$\sqrt{5}\sin\beta = 1, \qquad \sin\beta = \frac{1}{\sqrt{5}}.$$

Hence $\cos\beta = 2/\sqrt{5}$ and $\tan\beta = 1/2$.

**Step 2:** Find the acute rotation Rotate the pencil towards $CP$. The angle turned is $\theta = \alpha - \beta$, since both angles are acute and $\alpha > \beta$. Thus

$$\tan\theta = \frac{2 - 1/2}{1 + 2(1/2)} = \frac{3/2}{2} = \frac{3}{4}.$$

It follows that $\theta = \tan^{-1}(3/4)$. Rotating in the opposite direction reaches the same distance after a right-angle turn; that rotation is not acute. The perpendicular foot is within the pencil because its distance from $C$ is $PC\cos\beta = 2$ inches, less than its half-length of 5 inches.

**Step 3:** Check the other choices For the proposed acute rotations, $\sin(\alpha - \theta) = (2\cos\theta - \sin\theta)/\sqrt{5}$. Consequently the perpendicular distance is $|2\cos\theta - \sin\theta|$. The choices with tangents $1$, $4/3$, and $1/2$ give distances $1/\sqrt{2}$, $2/5$, and $3/\sqrt{5}$ inches, respectively, rather than $1$. For tangent $3/4$, the distance is $|2(4/5) - 3/5| = 1$ inch, as required.

**Step 4:** Conclusion The acute rotation is $\boxed{\tan^{-1}(3/4)}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Convert the point-to-line distance into r times the sine of the final angle; use the tangent subtraction identity instead of evaluating inverse tangents numerically.

## Mathematics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**21. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _____.** [4 marks]

**Solution:**

**Concept:**

- A three-digit number $100a + 10b + c$ is divisible by $11$ precisely when $a - b + c$ is a multiple of $11$. Digit restrictions apply independently to every position.
- One can organize qualifying numbers by their hundreds digit, then sum each finite list. A boundary such as 500 must be checked directly rather than treated as an entire fifth hundreds block.

**Step 1:** Restrict the hundreds digit A three-digit number at most $500$ has hundreds digit $1, 2, 3, 4$, or it is exactly $500$. The digit $1$ is forbidden, and $500$ is not divisible by $11$. Thus only the blocks beginning with $2$, $3$, and $4$ can contribute. The tens and units digits may include zero but may not include $1$.

**Step 2:** Enumerate with the divisibility rule For a number with digits $a, b, c$, the difference $a - b + c$ must be $0$ or $11$: in the relevant digit range it cannot be a negative multiple of $11$ or reach $22$. Checking the allowed tens and units digits gives the complete lists

$$a = 2 : 209, 220, 242, 253, 264, 275, 286, 297;$$
$$a = 3 : 308, 330, 352, 363, 374, 385, 396;$$
$$a = 4 : 407, 429, 440, 462, 473, 484, 495.$$

Every listed number contains no digit $1$, is divisible by $11$, and is below $500$. Numbers such as $231$, $319$, and $451$ are excluded because they contain the forbidden digit, despite being multiples of $11$.

**Step 3:** Sum the three disjoint lists The block totals are

$$S_2 = 209 + 220 + 242 + 253 + 264 + 275 + 286 + 297 = 2046,$$
$$S_3 = 308 + 330 + 352 + 363 + 374 + 385 + 396 = 2508,$$
$$S_4 = 407 + 429 + 440 + 462 + 473 + 484 + 495 = 3190.$$

These lists are disjoint and exhaust the possible hundreds digits. Therefore

$$S = S_2 + S_3 + S_4 = 2046 + 2508 + 3190 = 7744.$$

There are $8 + 7 + 7 = 22$ contributing integers. This is an integer sum, so no rounding or physical unit is involved.

**Step 4:** Conclusion The required sum is $\boxed{7744}$.

**Quick Tip to Solve:** Group allowed multiples by hundreds digit and apply the alternating-digit test; check the upper endpoint separately and exclude every occurrence of the forbidden digit.

**22. Let a and b respectively be the points of local maximum and local minimum of the function f(x) = $2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by y = f(x), the x-axis and the lines x = a and x = b, then 4A is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The first derivative locates stationary points; the sign of the second derivative distinguishes a local maximum from a local minimum.
- Total geometric area is the integral of the absolute value of the function. An interval must be split wherever the curve crosses the $x$-axis.

**Step 1:** Locate the two stationary points Differentiate the given cubic:

$$f'(x) = 6x^2 - 6x - 12 = 6(x + 1)(x - 2), \qquad f''(x) = 12x - 6.$$

Thus the stationary points have abscissae $-1$ and $2$. Since $f''(-1) = -18 < 0$, the local maximum occurs at $a = -1$. Since $f''(2) = 18 > 0$, the local minimum occurs at $b = 2$. These are the limits of the required region, rather than the roots of the cubic.

**Step 2:** Split the region at its intercept Factor the function as $f(x) = x(2x^2 - 3x - 12)$. Its other two zeros are $(3 \pm \sqrt{105})/4$, both outside $[-1, 2]$, so only $x = 0$ splits this interval. The function is positive on $[-1, 0]$ and negative on $[0, 2]$. Therefore

$$A = \int_{-1}^{0} f(x)\,dx - \int_{0}^{2} f(x)\,dx.$$

The minus sign makes the contribution below the axis positive.

**Step 3:** Evaluate both positive contributions An antiderivative is

$$F(x) = \frac{x^4}{2} - x^3 - 6x^2.$$

Here $F(-1) = -9/2$, $F(0) = 0$ and $F(2) = -24$. Consequently

$$A = \left(0 + \frac{9}{2}\right) - (-24 - 0) = \frac{57}{2}.$$

Adding signed integrals without this sign correction would give the net signed area, which is not what is asked.

**Step 4:** Conclusion The total area is $A = 57/2$ square units. Hence the required numerical value is $\boxed{4A = 114}$.

**Quick Tip to Solve:** Find every axis crossing between the stationary points before integrating; reverse the sign of each contribution below the axis.

---

**23. If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then $\lambda$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The scalar projection of $\mathbf{u}$ on a nonzero vector $\mathbf{v}$ is $\mathbf{u} \cdot \mathbf{v}/|\mathbf{v}|$. The denominator is the length of the direction vector, not its squared length.
- After squaring an equation involving a positive square root, check the candidate in the original equation to exclude a wrong sign.

**Step 1:** Form the direction of projection Add the two vectors on which the projection is taken:

$$\mathbf{v} = (2 - \lambda)\hat{i} + 6\hat{j} - 2\hat{k}.$$

The vector being projected is $\mathbf{u} = \hat{i} + 2\hat{j} + \hat{k}$. Thus

$$\mathbf{u} \cdot \mathbf{v} = (2 - \lambda) + 12 - 2 = 12 - \lambda, \qquad |\mathbf{v}| = \sqrt{(2 - \lambda)^2 + 40}.$$

Its length cannot vanish because its second and third components are fixed nonzero numbers.

**Step 2:** Use the specified unit projection The given scalar projection equals 1, so

$$\frac{12 - \lambda}{\sqrt{(2 - \lambda)^2 + 40}} = 1.$$

In particular $12 - \lambda > 0$. Squaring and expanding gives

$$\begin{aligned} (12 - \lambda)^2 &= (2 - \lambda)^2 + 40, \\ 144 - 24\lambda + \lambda^2 &= 44 - 4\lambda + \lambda^2, \\ 100 &= 20\lambda. \end{aligned}$$

The quadratic terms cancel, leaving the single candidate $\lambda = 5$.

**Step 3:** Check the sign and the original relation For $\lambda = 5$, the sum vector is $-3\hat{i} + 6\hat{j} - 2\hat{k}$, whose

length is $\sqrt{9+36+4} = 7$. Its dot product with $\mathbf{u}$ is $-3+12-2 = 7$. The projection is therefore $7/7 = 1$, with the required positive sign. This direct check also confirms that squaring introduced no extraneous value.

**Step 4:** Conclusion The parameter is $\boxed{\lambda = 5}$, a dimensionless numerical value.

**Quick Tip to Solve:** Add the direction vectors first, then use dot product divided by their sum vector's length. Check the sign after squaring.

---

**24. Let $a_1, a_2, \ldots\ldots, a_{10}$ be an AP with common difference $-3$ and $b_1, b_2, \ldots\ldots\ldots, b_{10}$ be a GP with common ratio 2. Let $c_k = a_k + b_k$, $k = 1, 2, \ldots\ldots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- An arithmetic progression with first term $A$ and common difference $-3$ has $a_k = A - 3(k-1)$.
- A geometric progression with first term $B$ and ratio 2 has $b_k = B2^{k-1}$. Sum the AP and GP separately, using linearity of finite sums.

**Step 1:** Determine the first terms Write $c_k = A - 3(k-1) + B2^{k-1}$. The specified second and third terms give

$$A - 3 + 2B = 12 \quad \Rightarrow \quad A + 2B = 15,$$
$$A - 6 + 4B = 13 \quad \Rightarrow \quad A + 4B = 19.$$

Subtracting the first equation from the second yields $2B = 4$. Hence $B = 2$ and $A = 11$. This step treats the two constituent sequences separately; the sequence $c_k$ itself need not be either an AP or a GP.

**Step 2:** Sum the arithmetic part The ten arithmetic terms start with 11 and end with

$$a_{10} = 11 + 9(-3) = -16.$$

Their sum is

$$\sum_{k=1}^{10} a_k = \frac{10}{2}(11 - 16) = -25.$$

A negative AP sum is consistent here: the later terms have become negative because the common difference is negative.

**Step 3:** Sum the geometric part and combine There are ten geometric terms with first term 2 and ratio 2. Applying the finite geometric-series formula gives

$$\sum_{k=1}^{10} b_k = 2\frac{2^{10} - 1}{2 - 1} = 2(1024 - 1) = 2046.$$

Therefore, by $c_k = a_k + b_k$,

$$\sum_{k=1}^{10} c_k = \sum a_k + \sum b_k = -25 + 2046 = 2021.$$

As a check on the recovered first terms, $c_2 = 8 + 4 = 12$ and $c_3 = 5 + 8 = 13$, exactly as stated.

**Step 4:** Conclusion The required dimensionless sum is $\boxed{2021}$.

**Quick Tip to Solve:** Use the two consecutive mixed terms to find the AP and GP first terms, then apply their separate sum formulae.

**25. Let $\lambda \neq 0$ be in R. If $\alpha$ and $\beta$ are the roots of the equation $x^2 - x + 2\lambda = 0$, and $\alpha$ and $\gamma$ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- A common root satisfies both quadratic equations. Eliminating its squared term produces a simple relation between that root and the parameter.
- For a quadratic $ax^2 + bx + c = 0$, the sum of its roots is $-b/a$ and their product is $c/a$. The given condition $\lambda \neq 0$ must be retained during division.

**Step 1:** Exploit the common root Because $\alpha$ is a root of both equations,

$$\alpha^2 - \alpha + 2\lambda = 0, \qquad 3\alpha^2 - 10\alpha + 27\lambda = 0.$$

Subtract three times the first equation from the second. This eliminates $\alpha^2$ and gives

$$-7\alpha + 21\lambda = 0 \quad \Rightarrow \quad \alpha = 3\lambda.$$

No assertion about the other two roots being equal is needed; only $\alpha$ is specified to be common.

**Step 2:** Find the allowed parameter and root Substitute this relation in the first quadratic:

$$9\lambda^2 - 3\lambda + 2\lambda = 0 \quad \Rightarrow \quad \lambda(9\lambda - 1) = 0.$$

The root $\lambda = 0$ is excluded by the question. Therefore

$$\lambda = \frac{1}{9}, \qquad \alpha = \frac{1}{3}.$$

Keeping the nonzero condition prevents including a spurious parameter for which the requested quotient would be undefined.

**Step 3:** Recover the other roots and evaluate The root sums for the two equations are $1$ and $10/3$, respectively. Consequently

$$\beta = 1 - \alpha = \frac{2}{3}, \qquad \gamma = \frac{10}{3} - \alpha = 3.$$

These values also satisfy the product relations: $\alpha\beta = 2/9 = 2\lambda$, and $\alpha\gamma = 1 = 9\lambda$. Finally,

$$\frac{\beta\gamma}{\lambda} = \frac{(2/3)\,3}{1/9} = 18.$$

The checks establish consistency with both original quadratics.

**Step 4:** Conclusion The required dimensionless value is $\boxed{18}$.

**Quick Tip to Solve:** For two quadratics sharing one root, eliminate the squared term first. Then use root sums to obtain the remaining roots.

---

**26. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x − y is _____.** [4 marks]

**Solution:**

**Concept:**

- For four observations with mean $\mu$, their sum is $4\mu$, and their variance is $\frac{1}{4}\sum z_i^2 - \mu^2$.
- Knowing $x + y$ and $x^2 + y^2$ determines $xy$. The inequality $x > y$ then fixes the order of the two roots.

**Step 1:** Use the original mean The four original observations have mean 5, so

$$\frac{3 + 7 + x + y}{4} = 5 \quad \Rightarrow \quad x + y = 10.$$

This is a condition on their sum only. It does not determine $x$ and $y$ individually, so the variance is also needed before substituting into the new list.

**Step 2:** Use the variance to determine the observations The given variance 10 means

$$10 = \frac{3^2 + 7^2 + x^2 + y^2}{4} - 5^2.$$

Hence

$$x^2 + y^2 = 4(10 + 25) - 58 = 82.$$

Using $(x + y)^2 = x^2 + y^2 + 2xy$, we obtain $100 = 82 + 2xy$, or $xy = 9$. Therefore the values of $x, y$ are the roots of

$$t^2 - 10t + 9 = (t - 9)(t - 1) = 0.$$

Since $x > y$, the required assignment is $x = 9$, $y = 1$. Interchanging these values would change the last observation $x - y$.

**Step 3:** Compute the new mean The new four observations, in their printed order, become

$$3 + 2x = 21, \qquad 7 + 2y = 9, \qquad x + y = 10, \qquad x - y = 8.$$

Their sum is $21 + 9 + 10 + 8 = 48$; thus their mean is $48/4 = 12$. As a check on the original data, the deviations of $3, 7, 9, 1$ from 5 are $-2, 2, 4, -4$, and the average of their squares is $(4+4+16+16)/4 = 10$.

Both original statistical conditions are satisfied.

**Step 4:** Conclusion The mean of the new four numbers is $\boxed{12}$.

**Quick Tip to Solve:** Convert variance to the sum of squares, then combine it with the known sum. Use the ordering condition before evaluating an asymmetric expression.

---

**27. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$ is a positive integer, is _____.**

[4 marks]

**Solution:**

**Concept:**

- In polar form, multiplying complex numbers adds arguments and dividing subtracts them. An integer is positive real only when its argument is an integer multiple of $2\pi$.
- The positive-real condition alone must be followed by an integer-magnitude check. Here the resulting congruence also forces an even exponent.

**Step 1:** Express the quotient in polar form Use

$$2i = 2e^{i\pi/2}, \qquad 1 - i = \sqrt{2}\, e^{-i\pi/4}.$$

For a positive integer $n$, the quotient is

$$\frac{(2i)^n}{(1-i)^{n-2}} = 2^{(n+2)/2} \exp\left(i\frac{(3n-2)\pi}{4}\right).$$

The same expression remains valid for $n = 1$, when the denominator has a negative exponent, since $1 - i \neq 0$. Its magnitude is always strictly positive.

**Step 2:** Impose positivity on the argument For the quotient to be positive real, its argument must equal $2m\pi$ for some integer $m$. Thus

$$\frac{(3n-2)\pi}{4} = 2m\pi \quad \Rightarrow \quad 3n - 2 = 8m.$$

Equivalently $3n \equiv 2 \pmod 8$. Multiplying by 3, the inverse of 3 modulo 8, gives

$$n \equiv 6 \pmod 8.$$

The least positive integer in this residue class is $n = 6$. All smaller positive exponents fail the positive-real argument condition, even if their magnitude might be integral.

**Step 3:** Check the integer value directly At $n = 6$, the polar magnitude is $2^{(6+2)/2} = 16$, an integer. A Cartesian check is

$$(2i)^6 = -64, \qquad (1-i)^2 = -2i, \qquad (1-i)^4 = -4.$$

The quotient is therefore $(-64)/(-4) = 16 > 0$. This verifies both required features: it is positive and it is an integer, rather than merely real.

**Step 4:** Conclusion The least positive exponent is $\boxed{n = 6}$; the corresponding quotient equals 16.

**Quick Tip to Solve:** Check the argument modulo $2\pi$, rather than modulo $\pi$: the latter also admits negative real values.

---

**28. Let Q be the foot of the perpendicular from the point P(7, −2, 13) on the plane containing the lines $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$. Then $(PQ)^2$, is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The cross product of two independent direction vectors in a plane gives a normal vector to that plane. A point on either line fixes the plane constant.
- For a point $(x_0, y_0, z_0)$, its perpendicular distance from $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$.

**Step 1:** Find a normal vector From the symmetric line equations, take direction vectors $\mathbf{u} = (6, 7, 8)$ and $\mathbf{v} = (3, 5, 7)$. Their cross product is

$$\mathbf{u} \times \mathbf{v} = (49 - 40,\ 24 - 42,\ 30 - 21) = (9, -18, 9).$$

It is nonzero, so the direction vectors are independent. Dividing this normal by 9, use the simpler vector $(1, -2, 1)$. Scaling a plane normal does not change either the plane or the resulting distance.

**Step 2:** Write and verify the common plane The first line passes through $(-1, 1, 3)$. Hence the plane with this normal is

$$(x + 1) - 2(y - 1) + (z - 3) = 0 \quad \Rightarrow \quad x - 2y + z = 0.$$

The base point $(1, 2, 3)$ of the second line satisfies $1 - 4 + 3 = 0$. Its direction satisfies $3 - 2(5) + 7 = 0$, while the first direction satisfies $6 - 2(7) + 8 = 0$. Thus both complete lines lie in this plane, confirming that the computed plane is the one specified.

**Step 3:** Calculate the squared perpendicular distance Since $Q$ is the perpendicular foot, $PQ$ is exactly the point-to-plane distance from $P = (7, -2, 13)$. Therefore

$$PQ = \frac{|7 - 2(-2) + 13|}{\sqrt{1 + (-2)^2 + 1}} = \frac{24}{\sqrt{6}}.$$

Squaring avoids an unnecessary radical simplification:

$$(PQ)^2 = \frac{24^2}{6} = 96.$$

The absolute value guarantees a nonnegative distance; the question asks for its square, not the distance itself.

**Step 4:** Conclusion The requested squared distance is $\boxed{(PQ)^2 = 96}$ square units.

**Quick Tip to Solve:** Reduce the cross-product normal to small integers, verify a point of the other line, and square the distance formula directly.

**29. Let $\binom{n}{k}$ denotes ${}^nC_k$ and $\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k}, & if\, 0 \le k \le n \\ 0, & otherwise \end{cases}$**

**If $A_k = \sum_{i=0}^{9} \binom{9}{i} \begin{bmatrix} 12 \\ 12-k+i \end{bmatrix} + \sum_{i=0}^{8} \binom{8}{i} \begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$ and $A_4 - A_3 = 190$ p, then p is equal to :** [4 marks]

**Solution:**

**Concept:**

- The zero extension of a binomial coefficient permits sums to include every integer index without adding unwanted terms: $\binom{n}{r} = 0$ when $r < 0$ or $r > n$.
- Vandermonde's identity is $\sum_i \binom{a}{i}\binom{b}{k-i} = \binom{a+b}{k}$. It follows by equating the coefficient of $x^k$ in $(1+x)^a(1+x)^b$.

**Step 1:** Rewrite the second factors By symmetry of binomial coefficients, $\binom{n}{n-r} = \binom{n}{r}$, with both sides understood to be zero outside the permitted range. Therefore the bracketed coefficients in the two sums can be written with lower index $k-i$:

$$A_k = \sum_{i=0}^{9} \binom{9}{i}\binom{12}{k-i} + \sum_{i=0}^{8} \binom{8}{i}\binom{13}{k-i}.$$

This step is useful because both products now have complementary lower indices whose sum is exactly $k$.

**Step 2:** Apply the coefficient identity The first sum is the coefficient of $x^k$ in $(1+x)^9(1+x)^{12}$, and the second is the same coefficient in $(1+x)^8(1+x)^{13}$. Both products equal $(1+x)^{21}$. Hence

$$A_k = \binom{21}{k} + \binom{21}{k} = 2\binom{21}{k}.$$

The finite summation limits already include all nonzero terms, so no additional end terms have been omitted.

**Step 3:** Evaluate the required difference Use $\binom{21}{3} = 21\cdot 20\cdot 19/6 = 1330$ and $\binom{21}{4} = 21\cdot 20\cdot 19\cdot 18/24 = 5985$. Then

$$A_4 - A_3 = 2(5985 - 1330) = 9310 = 190p.$$

Dividing by the given factor gives $p = 9310/190 = 49$. The parameter is a pure number, so no physical unit is involved.

**Step 4:** Conclusion The required value is $\boxed{p = 49}$.

**Quick Tip to Solve:** Convert $\binom{n}{n-k+i}$ to $\binom{n}{k-i}$ before applying Vandermonde; the upper indices then simply add.

**30. Let A be a 3 × 3 real matrix. If det(2Adj(2 Adj(Adj(2A)))) = $2^{41}$, then the value of det($A^2$) equal _____.**

[4 marks]

**Solution:**

**Concept:**

- For a $3 \times 3$ matrix $M$, $\det(cM) = c^3 \det M$ and $\det(\text{Adj } M) = (\det M)^2$.
- Also $\det(A^2) = (\det A)^2$. Tracking determinants from the innermost matrix outward avoids having to compute any matrix entries.

**Step 1:** Name the unknown determinant Put $d = \det A$. The given determinant is nonzero, so $d \neq 0$. Begin at the innermost matrix $2A$, whose determinant is $2^3 d$. Taking the first adjugate squares this entire determinant:

$$\det(\text{Adj}(2A)) = (2^3 d)^2 = 2^6 d^2.$$

The scalar multiplying a matrix must contribute its cube, rather than just a factor of two, because the order is three.

**Step 2:** Move outward through the remaining operations The next adjugate again squares the determinant. The scalar 2 immediately outside it then contributes $2^3$:

$$\det(\text{Adj}(\text{Adj}(2A))) = 2^{12} d^4,$$
$$\det(2\,\text{Adj}(\text{Adj}(2A))) = 2^{15} d^4.$$

Taking the outer adjugate produces $2^{30} d^8$. Finally, the outermost scalar 2 contributes another factor $2^3$. Thus the determinant stated in the question is $2^{33} d^8$.

**Step 3:** Solve for the requested squared determinant Equating this result to the given value gives

$$2^{33} d^8 = 2^{41}, \qquad d^8 = 2^8 = 256.$$

As $A$ is real, $d^2 \geq 0$. Setting $u = d^2$ gives $u^4 = 256 = 4^4$, so $u = 4$. Although $d$ itself could be either 2 or $-2$, both give the same determinant of $A^2$.

**Step 4:** Conclusion Therefore $\boxed{\det(A^2) = 4}$.

**Quick Tip to Solve:** For a matrix of order three, each adjugate doubles the determinant exponent; each external scalar 2 adds three to its power of two.

## Physics – Section A: Multiple Choice Questions (80 marks)

**31. The temperature of equal masses of three different liquids x, y and z are 10°C, 20°C and 30°C respectively. The temperature of mixture when x is mixed with y is 16°C and that when y is mixed with z is 26°C. The temperature of mixture when x and z are mixed will be :** [4 marks]

(A) 28.32°C

(B) 25.62°C

(C) 23.84°C

(D) 20.28°C

**Correct Answer:** (C) 23.84°C

**Solution:**

**Concept:**

- For an insulated mixture with no phase change, heat lost equals heat gained: $mc_h(T_h - T) = mc_c(T - T_c)$.
- For equal masses, the equilibrium temperature is the specific-heat-weighted average, not generally the arithmetic mean. Let $c_x, c_y, c_z$ denote the three specific heats.

**Step 1:** Use the first mixture to compare specific heats When $x$, initially at $10^\circ\text{C}$, is mixed with $y$, initially at $20^\circ\text{C}$, the final temperature is $16^\circ\text{C}$. Conservation of heat gives

$$mc_x(16 - 10) = mc_y(20 - 16), \qquad 6c_x = 4c_y.$$

Thus $c_x/c_y = 2/3$. Equal masses cancel; equal specific heats must not be assumed.

**Step 2:** Obtain a common ratio for all three liquids The mixture of $y$ and $z$ reaches $26^\circ\text{C}$, so

$$mc_y(26 - 20) = mc_z(30 - 26), \qquad 6c_y = 4c_z.$$

Hence $c_y/c_z = 2/3$. Combining the two ratios gives $c_x : c_y : c_z = 4 : 6 : 9$. In particular, $c_z/c_x = 9/4$, so the final mixture of $x$ and $z$ lies closer to the initially hotter liquid.

**Step 3:** Calculate the final temperature and compare the options Let the final temperature be $T$ in degrees Celsius. Applying heat balance to $x$ and $z$ yields

$$4(T - 10) = 9(30 - T), \qquad 13T = 310.$$

Therefore $T = 310/13 = 23.846\ldots\ ^\circ\text{C}$, corresponding to the listed $23.84^\circ\text{C}$. The other three temperatures do not satisfy this heat balance; the simple mean $20^\circ\text{C}$ would require equal specific heats.

**Step 4:** Conclusion The equilibrium temperature is approximately $23.85^\circ\text{C}$, represented by the listed $23.84^\circ\text{C}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Infer the two specific-heat ratios from the known mixtures before calculating the third mixture; multiplying the ratios gives $c_x/c_z$.

**32. The de-Broglie wavelength of a particle having kinetic energy E is $\lambda$. How much extra energy must be given to this particle so that the de-Broglie wavelength reduces to 75% of the initial value?**

[4 marks]

(A) $\frac{1}{9}$E

(B) $\frac{7}{9}$E

(C) E

(D) $\frac{16}{9}$E

**Correct Answer:** (B) $\frac{7}{9}$E

**Solution:**

**Concept:**

- The de Broglie relation is $\lambda = h/p$, where $p$ is momentum and $h$ is Planck's constant.
- For the same nonrelativistic particle, $K = p^2/(2m) = h^2/(2m\lambda^2)$. Thus kinetic energy varies inversely as the square of wavelength.

**Step 1:** Translate the wavelength reduction into a momentum ratio The new wavelength is $\lambda_f = 0.75\lambda = 3\lambda/4$. The mass and Planck constant do not change. Consequently,

$$\frac{p_f}{p_i} = \frac{\lambda}{\lambda_f} = \frac{4}{3}.$$

The momentum increases when the wavelength decreases. A direct proportionality between energy and wavelength would therefore give the wrong trend.

**Step 2:** Find the final kinetic energy Because kinetic energy is proportional to the square of momentum for a fixed mass,

$$\frac{E_f}{E} = \left(\frac{p_f}{p_i}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}.$$

Thus $E_f = 16E/9$. This value is the total energy after acceleration, whereas the question asks how much additional energy must be supplied.

**Step 3:** Subtract the original energy and reject the distractors The extra energy is

$$\Delta E = E_f - E = \left(\frac{16}{9} - 1\right) E = \frac{7}{9}E.$$

Option (D) gives the final energy instead of the increment. An extra energy $E$, option (C), would double the energy and reduce the wavelength to $\lambda/\sqrt{2}$, not $3\lambda/4$. Option (A) would make the final energy $10E/9$, which is likewise insufficient. All energy ratios are dimensionless, so $\Delta E$ has the same unit as $E$.

**Step 4:** Conclusion The required extra energy is $\boxed{7E/9}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For a wavelength ratio $f$, first find the final energy $E/f^2$, then subtract $E$ to obtain the energy supplied.

**33. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle will be :** [4 marks]

(A) $\sqrt{rg}$

(B) $\sqrt{2rg}$

(C) $2\sqrt{rg}$

(D) $\sqrt{\frac{rg}{2}}$

**Correct Answer:** (A) $\sqrt{rg}$

**Solution:**

**Concept:**

- A conical pendulum has a constant height, so the vertical component of string tension balances weight, while its horizontal component provides centripetal force.
- If the string makes angle $\theta$ with the vertical, $r = L\sin\theta$, $T\cos\theta = mg$, and $T\sin\theta = mv^2/r$.

**Step 1:** Determine the angle made by the string The radius is the horizontal projection of the string length. Hence

$$\sin\theta = \frac{r}{L} = \frac{1}{\sqrt{2}}.$$

Since the suspended particle has its string inclined between the vertical and horizontal, $\theta = 45^\circ$, giving $\tan\theta = 1$. This angle is measured from the vertical, the direction used in resolving the weight balance.

**Step 2:** Eliminate the unknown tension There is no vertical acceleration because the circular path lies in a horizontal plane. Resolve the tension and divide its horizontal balance by the vertical balance:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2/r}{mg}, \qquad \tan\theta = \frac{v^2}{rg}.$$

Thus $v^2 = rg\tan\theta$. Both tension and mass cancel, as expected for this ideal conical-pendulum speed.

**Step 3:** Substitute and compare the proposed speeds For the given geometry, $\tan\theta = 1$, so

$$v^2 = rg, \qquad v = \sqrt{rg}.$$

The positive square root is required because speed is a magnitude. The other choices would give $v^2/(rg) = 2, 4, 1/2$, respectively, instead of the required value 1. They therefore contradict the ratio of horizontal to vertical tension components. Since $rg$ has units $\mathrm{m^2\,s^{-2}}$, the result has the required speed units $\mathrm{m\,s^{-1}}$.

**Step 4:** Conclusion The speed is $\boxed{\sqrt{rg}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Divide the horizontal and vertical force equations to remove tension immediately; then use $r/L$ to determine $\tan\theta$.

**34. A cylindrical container of volume $4.0 \times 10^{-3}$ m$^3$ contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400 K. The pressure of the mixture of gases is :**

**[Take gas constant as 8.3 J mol$^{-1}$ K$^{-1}$]** [4 marks]

(A) $249 \times 10^1$ Pa

(B) $24.9 \times 10^3$ Pa

(C) $24.9 \times 10^5$ Pa

(D) 24.9 Pa

**Correct Answer:** (C) $24.9 \times 10^5$ Pa

**Solution:**

**Concept:**

- For ideal gases in the same container, Dalton's law gives total pressure as the sum of the partial pressures.
- Each partial pressure is $p_i = n_i RT/V$. Hence $P = (n_1 + n_2)RT/V$, where $n_i$ is the amount in moles, $T$ is absolute temperature and $V$ is the common volume.

**Step 1:** Count the total amount of gas The container holds one mole of hydrogen and two moles of carbon dioxide. Under the ideal-gas assumption the different molecular masses do not change the pressure contributed by one mole at the same temperature and volume. Therefore

$$n_{\text{total}} = 1 + 2 = 3 \text{ mol}.$$

The volume occupied by each component for calculating its partial pressure is the full container volume, not a separate fraction of it.

**Step 2:** Substitute the common temperature and volume Use $T = 400\,\text{K}$, $R = 8.3\,\text{J mol}^{-1}\,\text{K}^{-1}$ and $V = 4.0 \times 10^{-3}\,\text{m}^3$:

$$P = \frac{3 \times 8.3 \times 400}{4.0 \times 10^{-3}}\,\text{Pa} = \frac{9960}{0.004}\,\text{Pa}.$$

The SI unit conversion is already included because one joule per cubic metre is one pascal. No temperature conversion is required as the given temperature is in kelvin.

**Step 3:** Simplify and check the power of ten Evaluating the quotient gives

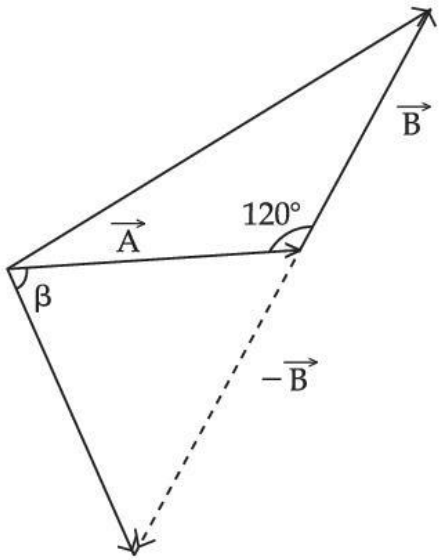
$$P = 2.49 \times 10^6\,\text{Pa} = 24.9 \times 10^5\,\text{Pa}.$$

Only option (C) has this value. Options (A) and (B) are smaller by factors of 1000 and 100, respectively; option (D) loses the volume scale almost entirely. Equivalently, adding the hydrogen partial pressure $8.3 \times 10^5$ Pa to the carbon dioxide partial pressure $16.6 \times 10^5$ Pa gives the same result.

**Step 4:** Conclusion The mixture pressure is $\boxed{24.9 \times 10^5\,\text{Pa}}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For an ideal-gas mixture at a common temperature, add the moles first and apply $PV = nRT$ once; carefully retain the $10^{-3}$ volume.

**35. The angle between vector $\left(\vec{A}\right)$ and $\left(\vec{A}-\vec{B}\right)$ is :** [4 marks]




(A) $\tan^{-1}\left(\frac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

(B) $\tan^{-1}\left(\frac{A}{0.7B}\right)$

(C) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$

(D) $\tan^{-1}\left(\frac{B\cos\theta}{A-B\sin\theta}\right)$

**Correct Answer:** (C) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$

**Solution:**

**Concept:**

- Vector subtraction means adding the reverse vector: $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$. Resolve components parallel and perpendicular to $\vec{A}$.
- The marked $120^\circ$ is between $-\vec{A}$ and $\vec{B}$ at their junction. Thus the angle between the positive directions of $\vec{A}$ and $\vec{B}$ is $60^\circ$.

**Step 1:** Choose axes along the first vector Take the positive horizontal direction along $\vec{A}$, and let the direction above it be positive perpendicular. Translating vectors without rotating them, the figure gives

$$\vec{A}=(A,0), \qquad \vec{B}=(B\cos 60^\circ, B\sin 60^\circ)=\left(\frac{B}{2},\frac{\sqrt{3}B}{2}\right).$$

The supplement of the marked angle must be used here; using $120^\circ$ directly between the vectors would reverse the sign of the parallel component.

**Step 2:** Subtract the two sets of components The vector from the first starting point to the bottom

vertex of the figure represents $\vec{A}-\vec{B}$. Its components are

$$\vec{A}-\vec{B}=\left(A-\frac{B}{2},-\frac{\sqrt{3}B}{2}\right).$$

The negative perpendicular component places it below $\vec{A}$, in agreement with the marked angle $\beta$. For the acute angle drawn, the parallel component is positive.

**Step 3:** Form the tangent of the required angle The angle with $\vec{A}$ is obtained from the magnitude of the perpendicular component divided by the parallel component:

$$\tan\beta=\frac{\sqrt{3}B/2}{A-B/2}=\frac{\sqrt{3}B}{2A-B}.$$

Consequently $\beta=\tan^{-1}(\sqrt{3}B/(2A-B))$, corresponding to option (C). Option (A) uses the wrong trigonometric components and a signed downward angle. Option (B) omits the parallel subtraction entirely; option (D) does not give the fixed-angle components shown in the diagram.

**Step 4:** Conclusion The marked angle is $\boxed{\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)}$. Hence, the correct option is (C).

**Quick Tip to Solve:** When vectors are drawn head to tail, first convert the interior junction angle to the angle between their positive directions, then resolve components.

**36. A light beam is described by $E=800\sin\omega\left(t-\frac{x}{c}\right)$. An electron is allowed to move normal to the propagation of light beam with a speed of $3\times10^7$ ms$^{-1}$. What is the maximum magnetic force exerted on the electron?** [4 marks]

(A) $1.28\times10^{-18}$ N

(B) $1.28\times10^{-21}$ N

(C) $12.8\times10^{-17}$ N

(D) $12.8\times10^{-18}$ N

**Correct Answer:** (D) $12.8\times10^{-18}$ N

**Solution:**

**Concept:**

- In an electromagnetic wave in vacuum, the field amplitudes satisfy $E_0=cB_0$, where $c=3.0\times10^8\,\mathrm{m\,s^{-1}}$. The coefficient of the sine is the electric-field amplitude.
- The magnetic force magnitude is $F_B=|q|vB\sin\theta$. Its maximum occurs when the magnetic field is at its amplitude and the velocity is perpendicular to that field; use $e=1.6\times10^{-19}\,\mathrm{C}$.

**Step 1:** Find the magnetic-field amplitude The given wave has $E_0=800\,\mathrm{V\,m^{-1}}$, so its associated magnetic-field amplitude is

$$B_0=\frac{E_0}{c}=\frac{800}{3.0\times10^8}=\frac{8}{3}\times10^{-6}\,\mathrm{T}.$$

The magnetic field is transverse to the propagation direction. For the maximum force, the electron can move in the transverse direction perpendicular to this magnetic field.

**Step 2:** Evaluate the maximum magnetic force With $v = 3 \times 10^7\,\mathrm{m\,s^{-1}}$, substitute the magnitudes into the magnetic part of the Lorentz force:

$$F_{B,\max} = evB_0 = (1.6 \times 10^{-19})(3 \times 10^7)\left(\frac{8}{3} \times 10^{-6}\right) = 1.28 \times 10^{-17}\,\mathrm{N}.$$

This is the magnetic force only; the electric force $eE_0$ must not be added when the question asks specifically for the magnetic force. Options with $10^{-18}$, $10^{-21}$, or $12.8 \times 10^{-17}$ do not equal this product.

**Step 3:** Conclusion The maximum magnetic force is $1.28 \times 10^{-17}\,\mathrm{N} = 12.8 \times 10^{-18}\,\mathrm{N}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Use $F_{B,\max} = evE_0/c$ directly, and normalize powers of ten before comparing the options.

**37. The two thin coaxial rings, each of radius 'a' and having charges +Q and −Q respectively are separated by a distance of 's'. The potential difference between the centres of the two rings is :**

[4 marks]

(A) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{\sqrt{s^2+a^2}}\right]$

(B) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{\sqrt{s^2+a^2}}\right]$

(C) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right]$

(D) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right]$

**Correct Answer:** (D) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right]$

**Solution:**

**Concept:**

- The potential on the axis of a uniformly charged thin ring is $V(x) = kq/\sqrt{a^2 + x^2}$, where $k = 1/(4\pi\varepsilon_0)$, $a$ is its radius and $x$ is the axial distance from its centre.
- Potential is a signed scalar, so the contributions of the positive and negative rings add algebraically. At its own centre each ring contributes $kq/a$.

**Step 1:** Calculate the potentials at both centres Let $O_+$ and $O_-$ be the centres of the rings carrying $+Q$ and $-Q$, respectively. Their separation is $s$, hence

$$V_+ = \frac{kQ}{a} - \frac{kQ}{\sqrt{a^2 + s^2}}.$$

At the other centre the roles of the rings interchange:

$$V_- = \frac{kQ}{\sqrt{a^2 + s^2}} - \frac{kQ}{a} = -V_+.$$

Both the self-ring and the other-ring contributions are essential.

**Step 2:** Subtract and compare the options The potential difference, positive-ring centre minus negative-ring centre, is therefore

$$V_+ - V_- = 2kQ\left(\frac{1}{a} - \frac{1}{\sqrt{a^2+s^2}}\right).$$

Since $2k = 1/(2\pi\varepsilon_0)$, the required expression has both the factor of two and the minus sign. The sum expressions fail because the other ring has opposite charge, while the expression with $1/(4\pi\varepsilon_0)$ gives only one centre potential. As a check, the difference vanishes for $s = 0$.

**Step 3:** Conclusion The potential difference is $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{\sqrt{s^2+a^2}}\right]$. Hence, the correct option is (D).

**Quick Tip to Solve:** For equal and opposite rings, the two centre potentials are opposite, so their difference is twice either positive centre potential.

**38. If you are provided a set of resistances 2Ω, 4Ω, 6Ω and 8Ω. Connect these resistances so as to obtain an equivalent resistance of $\frac{46}{3}\Omega$.** [4 marks]

(A) 4Ω and 6Ω are in parallel with 2Ω and 8Ω in series

(B) 6Ω and 8Ω are in parallel with 2Ω and 4Ω in series

(C) 2Ω and 6Ω are in parallel with 4Ω and 8Ω in series

(D) 2Ω and 4Ω are in parallel with 6Ω and 8Ω in series

**Correct Answer:** (D) 2Ω and 4Ω are in parallel with 6Ω and 8Ω in series

**Solution:**

**Concept:**

- Resistors in series add: $R_s = R_1 + R_2$. Two resistors in parallel have equivalent resistance $R_p = R_1R_2/(R_1+R_2)$.
- In each proposed connection, first reduce the named parallel pair, then add the two remaining series resistances. The target is $46/3\,\Omega$, which is about $15.33\,\Omega$.

**Step 1:** Test the pair of smaller resistances in parallel Connect the $2\,\Omega$ and $4\,\Omega$ resistors across the same two nodes. This gives

$$R_p = \frac{2\times 4}{2+4} = \frac{4}{3}\,\Omega.$$

Place this parallel combination in series with the $6\,\Omega$ and $8\,\Omega$ resistors. The resulting total is

$$R_{\text{eq}} = 6 + 8 + \frac{4}{3} = 14 + \frac{4}{3} = \frac{46}{3}\,\Omega.$$

The parallel branch carries split current, while the two series resistors carry the total current.

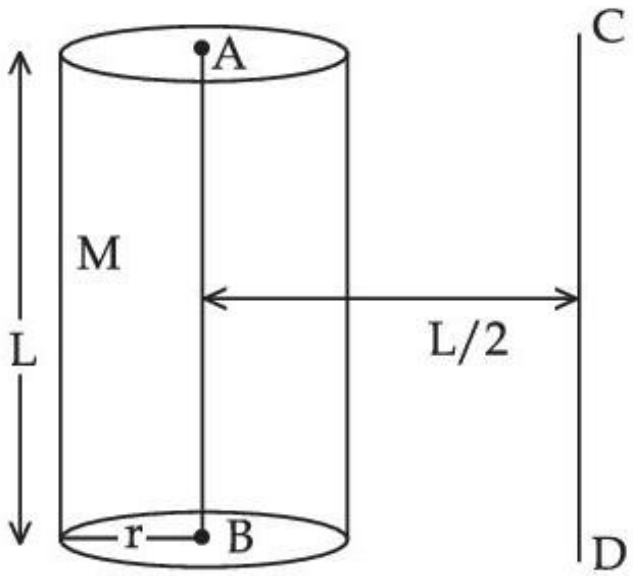
**Step 2:** Check the other proposed arrangements For $4\,\Omega \parallel 6\,\Omega$ with $2\,\Omega$ and $8\,\Omega$ in series, the total is $10 + 24/10 = 62/5\,\Omega$. For $6\,\Omega \parallel 8\,\Omega$ with $2\,\Omega$ and $4\,\Omega$ in series, it is $6 + 48/14 = 66/7\,\Omega$. For $2\,\Omega \parallel 6\,\Omega$

with $4\,\Omega$ and $8\,\Omega$ in series, it is $12 + 12/8 = 27/2\,\Omega$. None of these equals $46/3\,\Omega$.

**Step 3:** Conclusion Use $2\,\Omega$ and $4\,\Omega$ in parallel, with $6\,\Omega$ and $8\,\Omega$ in series with that pair. Hence, the correct option is (D).

**Quick Tip to Solve:** The target is close to $14\,\Omega$: retain $6 + 8$ in series and obtain the remaining $4/3\,\Omega$ from $2 \parallel 4$.

**39. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is 2.7 kg m$^2$.** [4 marks]




(A) 14.9 kg/m$^3$

(B) $7.5 \times 10^1$ kg/m$^3$

(C) $7.5 \times 10^2$ kg/m$^3$

(D) $1.49 \times 10^2$ kg/m$^3$

**Correct Answer:** (D) $1.49 \times 10^2$ kg/m$^3$

**Solution:**

**Concept:**

- A solid cylinder has moment of inertia $I_{AB} = \frac{1}{2}Mr^2$ about its central longitudinal axis. For a parallel axis at distance $h$, the parallel-axis theorem gives $I_{CD} = I_{AB} + Mh^2$.
- Density is $\rho = M/V$, with cylinder volume $V = \pi r^2 L$. Convert centimetres to metres before using the specified SI moment of inertia.

**Step 1:** Use the axis separation shown in the figure The figure labels the distance between axes $AB$ and $CD$ as $L/2$, rather than the radius. With $L = 0.80\,\mathrm{m}$ and $r = 0.20\,\mathrm{m}$, the offset is $h = 0.40\,\mathrm{m}$.

Therefore

$$2.7 = M\left(\frac{0.20^2}{2} + 0.40^2\right) = M(0.02 + 0.16) = 0.18M.$$

Solving gives $M = 15\,\text{kg}$. Using the transverse-axis formula would be incorrect because both axes in the picture are parallel to the cylinder length.

**Step 2:** Divide the mass by the cylinder volume The volume and density are

$$V = \pi(0.20)^2(0.80) = 0.032\pi\,\text{m}^3,$$

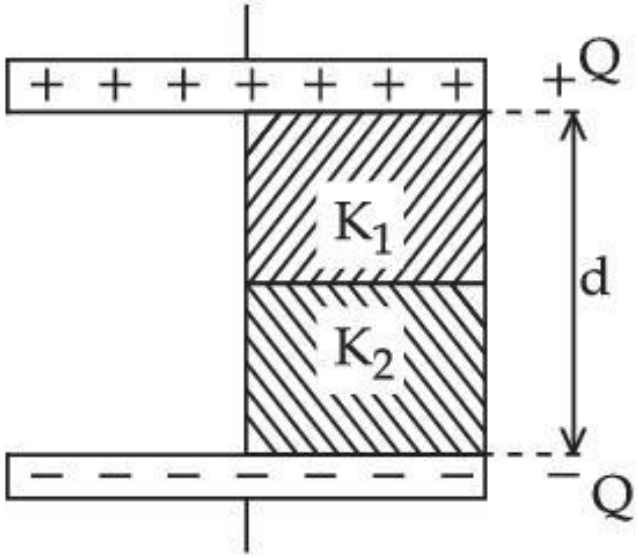
$$\rho = \frac{15}{0.032\pi} \simeq 149.2\,\text{kg m}^{-3}.$$

Thus the density is approximately $1.49 \times 10^2\,\text{kg m}^{-3}$. The other values, 14.9, 75, and $750\,\text{kg m}^{-3}$, fail to give the required mass when multiplied by the cylinder volume.

**Step 3:** Conclusion The material density is approximately $1.49 \times 10^2\,\text{kg m}^{-3}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Read the offset from the drawing first: here it is $L/2$, so the added term is $M(L/2)^2$, not $Mr^2$.

**40. A parallel plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant $K_1$ and $K_2$ of same area A/2 and thickness d/2 are inserted in the space between the plates. The capacitance of the capacitor will be given by:** [4 marks]




(A) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{K_1K_2}{K_1+K_2}\right)$

(B) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{K_1K_2}{2(K_1+K_2)}\right)$

(C) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{K_1+K_2}{K_1K_2}\right)$

(D) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{2(K_1+K_2)}{K_1K_2}\right)$

**Correct Answer:** (A) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{K_1K_2}{K_1+K_2}\right)$

**Solution:**

**Concept:**

- A parallel-plate region of area $S$, separation $\ell$ and dielectric constant $K$ has capacitance $C = K\varepsilon_0 S/\ell$, neglecting edge effects.
- Side-by-side regions share the same plate voltage and combine in parallel. Layers stacked along the electric field combine in series because they carry the same free charge.

**Step 1:** Separate the empty and filled halves In the figure the left half of area $A/2$ is empty across the full separation $d$. Its capacitance is

$$C_{\text{air}} = \frac{\varepsilon_0 A}{2d}.$$

The right half contains two successive layers, each with area $A/2$ and thickness $d/2$. Their individual capacitances are

$$C_1 = \frac{K_1\varepsilon_0(A/2)}{d/2} = \frac{K_1\varepsilon_0 A}{d}, \qquad C_2 = \frac{K_2\varepsilon_0 A}{d}.$$

These two layers are in series, rather than parallel.

**Step 2:** Combine the layers and add the empty region The filled-half equivalent is

$$C_{\text{filled}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d}\frac{K_1 K_2}{K_1 + K_2}.$$

Adding the parallel empty half gives

$$C = C_{\text{air}} + C_{\text{filled}} = \frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2}\right).$$

Setting $K_1 = K_2 = 1$ correctly recovers $\varepsilon_0 A/d$. The extra factor of two in the second expression and the inverse dielectric factors in the other expressions fail this empty-capacitor check.

**Step 3:** Conclusion The capacitance is $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2}\right)$. Hence, the correct option is (A).

**Quick Tip to Solve:** First divide the plate into side-by-side areas; within each area, combine dielectric layers along the field in series.

**41. A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a :** [4 marks]

(A) hyperbola

(B) parabola in the direction of motion of plane

(C) straight line vertically down the plane

(D) parabola in a direction opposite to the motion of plane

**Correct Answer:** (C) straight line vertically down the plane

**Solution:**

**Concept:**

- When released, the bomb retains the horizontal velocity of the aircraft. Neglecting air resistance, gravity accelerates it only vertically.
- For a plane continuing at constant horizontal velocity, the relative horizontal coordinate is the bomb coordinate minus the plane coordinate. The plane frame then has no horizontal relative acceleration.

**Step 1:** Compare horizontal motion in the ground frame Let the plane move with constant horizontal speed $u$, and take the release point as the origin at $t = 0$. The bomb has no horizontal force after release, so its horizontal position is $x_b = ut$. The plane is at $x_p = ut$ at the same time. Consequently,

$$x_{\text{relative}} = x_b - x_p = ut - ut = 0.$$

The bomb therefore remains directly below the plane, even though both continue to move horizontally relative to the ground.

**Step 2:** Find the vertical path in the plane frame The initial vertical speed is zero. Taking upward as positive, the bomb has

$$y_{\text{relative}} = -\frac{1}{2}gt^2.$$

Because its relative horizontal coordinate is fixed, these positions form a vertical straight line down from the plane. A ground observer sees a parabola because the horizontal coordinate changes in that frame. Neither a forward nor a backward parabola describes the plane-frame path, and a hyperbola is inconsistent with constant gravitational acceleration and zero relative horizontal motion.

**Step 3:** Conclusion To an observer moving with the plane, the bomb travels in a straight line vertically downward. Hence, the correct option is (C).

**Quick Tip to Solve:** For a dropped projectile, subtract the carrier velocity before deciding the path seen by an observer on that carrier.

**42. An electric bulb of 500 watt at 100 volt is used in a circuit having a 200 V supply. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500 W.** [4 marks]

(A) 20 Ω

(B) 30 Ω

(C) 5 Ω

(D) 10 Ω

**Correct Answer:** (A) 20 Ω

**Solution:**

**Concept:**

- The rated bulb must operate at its rated voltage to deliver the rated power. Use $P = VI$ to obtain the required current and $R_b = V^2/P$ for its operating resistance.
- A series resistor carries the same current as the bulb and drops the excess supply voltage. Its resistance is $R = V_R/I$; the full supply voltage is divided between bulb and resistor.

**Step 1:** Determine the required bulb current For the bulb to deliver $500\,\text{W}$ at its rated $100\,\text{V}$, the current must be

$$I = \frac{P}{V_b} = \frac{500}{100} = 5\,\text{A}.$$

Its operating resistance is

$$R_b = \frac{V_b^2}{P} = \frac{100^2}{500} = 20\,\Omega.$$

Connecting it directly to the $200\,\text{V}$ supply would not preserve the rated operating condition, so an external series voltage drop is required.

**Step 2:** Choose the resistance that drops the excess voltage The series resistor must drop

$$V_R = V_s - V_b = 200 - 100 = 100\,\text{V}.$$

Since it carries the same $5\,\text{A}$,

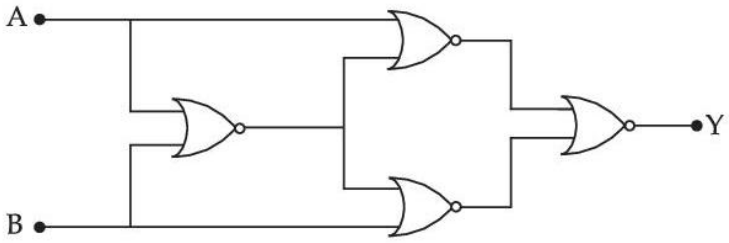
$$R = \frac{V_R}{I} = \frac{100}{5} = 20\,\Omega.$$

As a check, the circuit resistance is $20 + 20 = 40\,\Omega$, giving $I = 200/40 = 5\,\text{A}$ and bulb power $I^2 R_b = 25 \times 20 = 500\,\text{W}$. A $30\,\Omega$ resistor gives only $4\,\text{A}$, while $5\,\Omega$ and $10\,\Omega$ give currents greater than $5\,\text{A}$; none maintains the required bulb power.

**Step 3:** Conclusion The required series resistance is $R = 20\,\Omega$. Hence, the correct option is (A).

**Quick Tip to Solve:** Calculate the rated current first, then divide the excess supply voltage by that same current to size the series resistor.

**43. Four NOR gates are connected as shown in figure. The truth table for the given figure is :** [4 marks]




(A)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(B)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(C)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(D)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Correct Answer:** (D)

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Solution:**

**Concept:**

- A NOR gate produces the complement of the logical OR: $\text{NOR}(P, Q) = \overline{P + Q}$. The symbol $+$ denotes OR, and juxtaposition denotes AND.
- The intermediate outputs must be calculated before evaluating the final gate. A four-NOR arrangement of this form implements equality of the two inputs, also called XNOR.

**Step 1:** Label the intermediate gate outputs Let the output of the leftmost NOR gate be $X$, and the

upper and lower middle outputs be $U$ and $V$. Reading the connections gives

$$X = \overline{A + B}, \qquad U = \overline{A + X}, \qquad V = \overline{B + X}.$$

The rightmost gate receives $U, V$, so $Y = \overline{U + V}$. In particular, the first gate is shared by both middle gates; treating the network as four gates in series would give the wrong result.

**Step 2:** Evaluate each input combination For $(A, B) = (0, 0)$, we have $X = 1$, hence $U = V = 0$, giving $Y = 1$. For $(0, 1)$, $X = 0$, $U = 1, V = 0$, giving $Y = 0$. For $(1, 0)$, $X = 0$, $U = 0, V = 1$, again giving $Y = 0$. Finally, for $(1, 1)$, $X = 0$, $U = V = 0$, giving $Y = 1$.

| $A$ | $B$ | $Y$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Step 3:** Identify the matching table The output is high precisely when the inputs are equal: $Y = AB + \overline{A}\,\overline{B}$. Option (B) instead gives XOR, which is high for unequal inputs. Option (A) follows $\overline{B}$, and option (C) follows $B$; both ignore the required dependence on the other input. Thus only the fourth table matches all four rows.

**Step 4:** Conclusion The output sequence for inputs $00, 01, 10, 11$ is $1, 0, 0, 1$: the circuit is an XNOR gate. Hence, the correct option is (D).

**Quick Tip to Solve:** For a NOR network, write the output of each internal node first; test equal and unequal inputs to distinguish XOR from XNOR.

**44. Match List - I with List - II**

**Choose the most appropriate answer from the options given below :** [4 marks]

| | **List-I** | | **List-II** |
|---|---|---|---|
| (a) | Magnetic Induction | (i) | $ML^2T^{-2}A^{-1}$ |
| (b) | Magnetic Flux | (ii) | $M^0L^{-1}A$ |
| (c) | Magnetic Permeability | (iii) | $MT^{-2}A^{-1}$ |
| (d) | Magnetization | (iv) | $MLT^{-2}A^{-2}$ |

(A) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(B) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(C) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(D) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

**Correct Answer:** (D) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

**Solution:**

**Concept:**

- Dimensional formulae follow from defining equations. Use $[F] = MLT^{-2}$, $[v] = LT^{-1}$, and electric current dimension $A$.
- Magnetic flux is field times area; permeability relates magnetic induction to magnetic field intensity. Magnetization is magnetic dipole moment per unit volume.

**Step 1:** Find magnetic induction and flux The magnetic force on a moving charge is $F = qvB$ when velocity is perpendicular to the field. Since $[q] = AT$,

$$[B] = \frac{MLT^{-2}}{(AT)(LT^{-1})} = MT^{-2}A^{-1}.$$

Thus magnetic induction, entry (a), matches (iii). Flux through a perpendicular plane is $\Phi = BS$, where $S$ is area, giving

$$[\Phi] = [B]L^2 = ML^2T^{-2}A^{-1}.$$

Therefore entry (b) matches (i). This area factor is essential: magnetic flux and magnetic induction have different dimensions.

**Step 2:** Find magnetization and permeability A current loop has magnetic dipole moment $m = IS$, with dimensions $AL^2$. Magnetization is the dipole moment per unit volume, so

$$[\mathcal{M}] = \frac{AL^2}{L^3} = AL^{-1} = M^0L^{-1}A.$$

Hence entry (d) matches (ii). Magnetic field intensity $H$ has the same dimensions $AL^{-1}$, and $B = \mu H$ implies

$$[\mu] = \frac{MT^{-2}A^{-1}}{AL^{-1}} = MLT^{-2}A^{-2}.$$

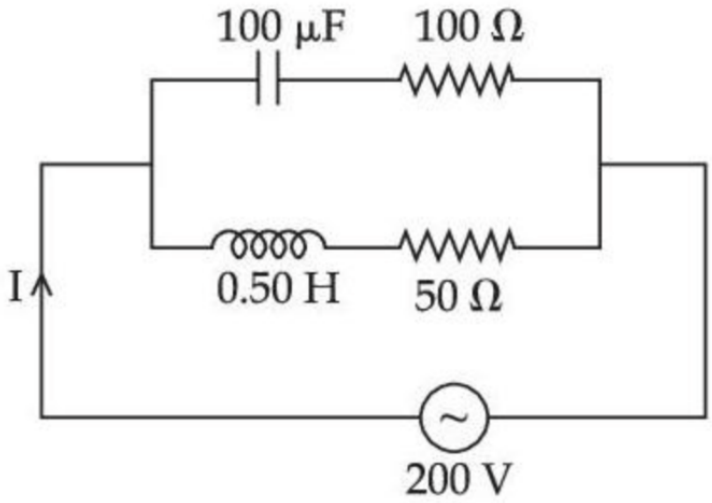
Thus entry (c) matches (iv).

**Step 3:** Compare all four associations The complete ordered matching is (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii). Option (A) assigns induction the dimension of magnetization; option (B) makes the same incorrect induction match. Option (C) gets induction and permeability right but interchanges the dimensions needed for flux and magnetization. Only option (D) satisfies all four defining relations.

**Step 4:** Conclusion All four dimensional formulae match the fourth listed combination. Hence, the correct option is (D).

**Quick Tip to Solve:** Remember $[B] = MT^{-2}A^{-1}$, then obtain flux by multiplying by area and permeability by dividing by $[H] = AL^{-1}$.

**45. In the given circuit the AC source has $\omega$ = 100 rad s$^{-1}$. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?** [4 marks]




(A) 5.9 A

(B) 3.16 A

(C) 0.94 A

(D) 6 A

**Correct Answer:** (B) 3.16 A

**Solution:**

**Concept:**

- In a parallel AC circuit, every branch has the same applied voltage, but branch currents add as phasors, not as ordinary magnitudes.
- For angular frequency $\omega$, inductive reactance is $X_L = \omega L$ and capacitive reactance is $X_C = 1/(\omega C)$. With the voltage as reference, impedances are $R + jX_L$ and $R - jX_C$.

**Step 1:** Calculate the two branch impedances The upper branch contains $100\,\Omega$ in series with $100\,\mu$F; the lower has $50\,\Omega$ in series with $0.50\,$H. At the given frequency,

$$X_C = \frac{1}{100(100 \times 10^{-6})} = 100\,\Omega, \qquad X_L = 100(0.50) = 50\,\Omega.$$

Thus $Z_C = 100 - j100\,\Omega$ and $Z_L = 50 + j50\,\Omega$. The signs differ because the capacitor current leads the voltage whereas the inductor current lags.

**Step 2:** Add the branch currents as complex quantities Using the applied $200\,$V across each branch,

$$I_C = \frac{200}{100 - j100} = 1 + j1\,\text{A},$$

$$I_L = \frac{200}{50 + j50} = 2 - j2\,\text{A}.$$

Therefore the source-current phasor is

$$I = I_C + I_L = 3 - j1\,\text{A}, \qquad |I| = \sqrt{3^2 + 1^2} = \sqrt{10}\,\text{A}.$$

The net reactive component is inductive, with magnitude $1\,\text{A}$, while the in-phase components add to $3\,\text{A}$.

**Step 3:** Select the magnitude The magnitude is approximately $3.16\,\text{A}$, which matches option (B). Adding branch magnitudes would incorrectly ignore the opposite reactive components. Options (A) and (D), near $6\,\text{A}$, are too large, while $0.94\,\text{A}$ is smaller even than the net in-phase current and cannot be the source-current magnitude.

**Step 4:** Conclusion The current flowing from the source has magnitude $I = \sqrt{10}\,\text{A} \approx 3.16\,\text{A}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For parallel RC and RL branches, resolve currents into in-phase and reactive components before adding them.

**46. If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is :** [4 marks]

(A) 86.4 s

(B) 4.32 s

(C) 43.2 s

(D) 8.64 s

**Correct Answer:** (C) 43.2 s

**Solution:**

**Concept:**

- For small oscillations of a simple pendulum, $T = 2\pi\sqrt{\ell/g}$. At fixed gravitational acceleration the period varies as the square root of length.
- A clock counts oscillations using its original calibration. If its actual period increases, each indicated second takes longer and the clock loses time.

**Step 1:** Relate the fractional period and length changes Logarithmic differentiation of the period relation at fixed $g$ gives

$$\frac{\Delta T}{T} \simeq \frac{1}{2}\frac{\Delta \ell}{\ell}.$$

The stated $0.1\%$ increase in length means

$$\frac{\Delta \ell}{\ell} = \frac{0.1}{100} = 0.001, \qquad \frac{\Delta T}{T} = 0.0005.$$

Thus the new period is approximately $1.0005T$. The positive change concerns period, rather than oscillation frequency; the frequency decreases.

**Step 2:** Find the daily error One day has $24 \times 60 \times 60 = 86400\,\text{s}$. To first order, the loss of indicated

time over a real day is the fractional period increase multiplied by the elapsed time:

$$\Delta t = 86400(0.0005) = 43.2\,\text{s}.$$

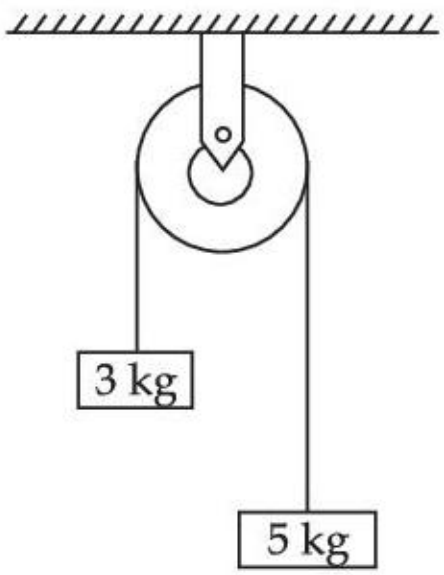
More precisely, the indicated interval is $86400/\sqrt{1.001}$, so the exact loss is $86400[1-(1.001)^{-1/2}]$, approximately $43.17\,\text{s}$. The usual small-change approximation gives the displayed answer.

**Step 3:** Check the direction and choices The longer pendulum completes fewer oscillations in a day, so its reading falls behind the correct time. The value $86.4\,\text{s}$ would follow from using the full fractional length change instead of half. Values $4.32\,\text{s}$ and $8.64\,\text{s}$ involve an extra factor-of-ten error. The period law and seconds-per-day conversion jointly select $43.2\,\text{s}$.

**Step 4:** Conclusion The pendulum clock loses approximately $43.2\,\text{s}$ per day. Hence, the correct option is (C).

**Quick Tip to Solve:** For a small pendulum-length change, take half its fractional value and multiply by 86400 to find the daily time error in seconds.

**47. Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2$ Nm$^{-2}$. What is the minimum radius of the wire ? (Take g = 10 ms$^{-2}$)** [4 marks]




(A) 125 cm

(B) 1250 cm

(C) 12.5 cm

(D) 1.25 cm

**Correct Answer:** (C) 12.5 cm

**Solution:**

**Concept:**

- Tensile stress is force divided by cross-sectional area, $\sigma = T/(\pi r^2)$. The smallest safe radius occurs when the working stress equals the breaking stress.
- For an ideal Atwood machine with masses $m_1, m_2$, the wire tension is common on both sides, but is unequal to either individual weight because both masses accelerate.

**Step 1:** Determine the acceleration and tension The $5\,\text{kg}$ block moves downward and the $3\,\text{kg}$ block upward. Newton's second law gives

$$5g - T = 5a, \qquad T - 3g = 3a.$$

Adding and substituting $g = 10\,\text{m}\,\text{s}^{-2}$,

$$a = \frac{(5-3)g}{5+3} = 2.5\,\text{m}\,\text{s}^{-2}.$$

Using the lighter block,

$$T = 3(g + a) = 3(10 + 2.5) = 37.5\,\text{N}.$$

The heavier-block equation gives $T = 5(10 - 2.5) = 37.5\,\text{N}$, independently checking the result.

**Step 2:** Apply the limiting stress condition For the circular wire, the area is $\pi r^2$. With the given breaking stress $\sigma_b = (24/\pi) \times 10^2\,\text{N}\,\text{m}^{-2}$, the limiting condition is

$$\frac{37.5}{\pi r_{\text{min}}^2} = \frac{24}{\pi} \times 10^2.$$

The factors of $\pi$ cancel, giving

$$r_{\text{min}}^2 = \frac{37.5}{2400} = 0.015625\,\text{m}^2, \qquad r_{\text{min}} = 0.125\,\text{m} = 12.5\,\text{cm}.$$

A larger radius reduces stress below the limit; a smaller radius increases it beyond the breaking value.

**Step 3:** Check units and options The stress was specified in $\text{N}\,\text{m}^{-2}$, so the radius obtained by substitution is in metres. It must be multiplied by 100 to express it in centimetres. Thus neither $1.25\,\text{cm}$ nor $125\,\text{cm}$ nor $1250\,\text{cm}$ is the minimum radius; these differ from the calculated value by powers of ten.

**Step 4:** Conclusion The minimum radius is $0.125\,\text{m} = 12.5\,\text{cm}$. Hence, the correct option is (C).

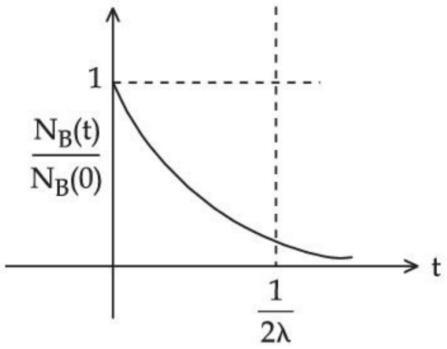
**Quick Tip to Solve:** Find the accelerating-system tension first; then use $r_{\text{min}} = \sqrt{T/(\pi\sigma_b)}$ and convert metres to centimetres only at the end.

**48. At time t = 0, a material is composed of two radioactive atoms A and B, where $N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is $\lambda$. However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t)$ / $N_B(0)$ with respect to time t?**

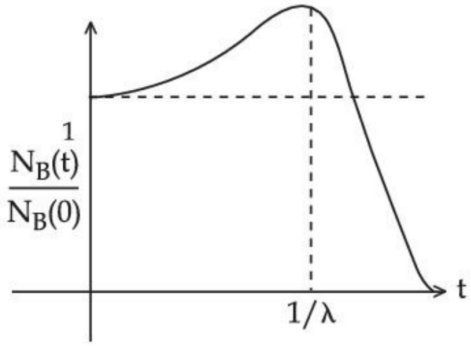
**[$N_A$ (0) = No. of A atoms at t = 0**

**$N_B$ (0) = No. of B atoms at t = 0]** [4 marks]

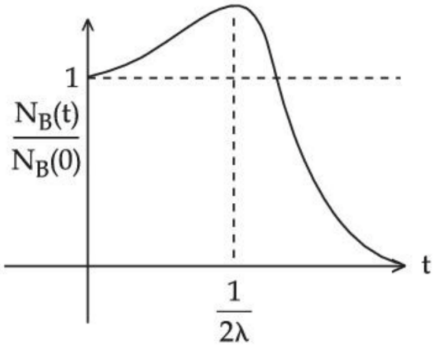
(A)



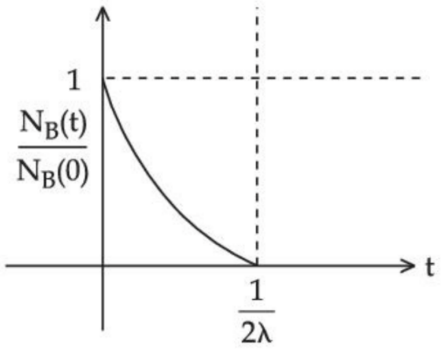

(B)



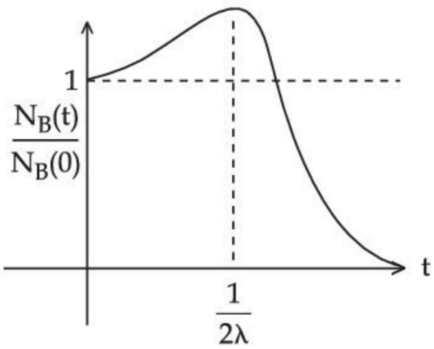

(C)




(D)




**Correct Answer:** (C)



**Solution:**

**Concept:**

- In a decay chain $A \to B \to C$, the number of B nuclei changes through production from A and loss by decay of B. The two contributions must both be retained.
- For equal decay constants $\lambda$, $N_A(t) = N_A(0)e^{-\lambda t}$ and $dN_B/dt = \lambda N_A - \lambda N_B$. Here $N_A(0) = 2N_B(0)$.

**Step 1:** Solve the production-decay equation Substitute the A population into the equation for B:

$$\frac{dN_B}{dt} + \lambda N_B = 2\lambda N_B(0)e^{-\lambda t}.$$

Multiplication by the integrating factor $e^{\lambda t}$ gives

$$\frac{d}{dt}\left(N_B e^{\lambda t}\right) = 2\lambda N_B(0).$$

Integrating from zero to $t$ and applying the stated initial B population yields

$$N_B(t)e^{\lambda t} = N_B(0)(1 + 2\lambda t), \qquad \frac{N_B(t)}{N_B(0)} = (1 + 2\lambda t)e^{-\lambda t}.$$

**Step 2:** Locate the maximum and long-time behaviour Let $f(t) = N_B(t)/N_B(0)$. Then

$$f(0) = 1, \qquad f'(t) = \lambda(1 - 2\lambda t)e^{-\lambda t}.$$

The initial slope is positive, so the B population initially rises. Its slope vanishes at $t = 1/(2\lambda)$, changing from positive to negative there. The peak value is $2e^{-1/2} > 1$. At long times the exponential dominates the linear factor, and $f(t) \to 0$ asymptotically.

**Step 3:** Compare the four curves Option (C) starts at one, rises to a peak at $1/(2\lambda)$, and then falls toward zero. Option (B) places the peak at $1/\lambda$, which contradicts the derivative. Options (A) and (D) decrease immediately, whereas production from A initially exceeds loss from B. Furthermore, the exact positive expression never reaches zero at a finite time, as the fourth curve suggests.

**Step 4:** Conclusion The population ratio is $(1 + 2\lambda t)e^{-\lambda t}$, with its maximum at $t = 1/(2\lambda)$. Hence, the correct option is (C).

**Quick Tip to Solve:** In a decay chain, check the initial production-minus-loss rate and the time of the maximum before selecting a graph.

**49. A transmitting antenna at top of a tower has a height of 50 m and the height of receiving antenna is 80 m. What is range of communication for Line of Sight (LoS) mode?**

**[use radius of earth = 6400 km]** [4 marks]

(A) 45.5 km

(B) 80.2 km

(C) 144.1 km

(D) 57.28 km

**Correct Answer:** (D) 57.28 km

**Solution:**

**Concept:**

- The horizon distance for an antenna of height $h \ll R$ above a spherical Earth is $d \simeq \sqrt{2Rh}$, where $R$ is the Earth's radius.
- For two elevated antennas the limiting line of sight consists of two horizon distances, so $d_{\max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$. Use the same length unit in every factor.

**Step 1:** Convert the radius and identify both heights The transmitter height is $h_t = 50\,\text{m}$, the receiver height is $h_r = 80\,\text{m}$, and

$$R = 6400\,\text{km} = 6.4 \times 10^6\,\text{m}.$$

A ray just clearing the Earth is tangent to its surface. The right triangle formed with the Earth's centre gives $d^2 = (R+h)^2 - R^2 = 2Rh + h^2$. Since both antenna heights are tiny compared with $R$, the $h^2$ contribution can be neglected consistently.

**Step 2:** Calculate the two horizon distances For the transmitter,

$$d_t = \sqrt{2(6.4 \times 10^6)(50)} = \sqrt{6.4 \times 10^8}\,\text{m} \approx 25.30\,\text{km}.$$

For the receiver,

$$d_r = \sqrt{2(6.4 \times 10^6)(80)} = \sqrt{1.024 \times 10^9}\,\text{m} = 32.00\,\text{km}.$$

Both antennas extend the available direct path, so their horizon distances add rather than subtract.

**Step 3:** Find the communication range Therefore,

$$d_{\max} = 25.30 + 32.00 = 57.30\,\text{km}.$$

This is the approximately $57.28\,\text{km}$ result listed in option (D), with the small difference arising from square-root rounding. The other choices $45.5$, $80.2$ and $144.1\,\text{km}$ do not follow from the sum of the two horizon distances. Using only one antenna would omit a substantial part of the range, while replacing the two square roots by one square root of the summed heights would also be incorrect.

**Step 4:** Conclusion The line-of-sight communication range is about $57.3\,\text{km}$, corresponding to $57.28\,\text{km}$ in the options. Hence, the correct option is (D).

**Quick Tip to Solve:** Compute the horizon distance separately for each antenna, keeping height and Earth radius in matching units, then add the two distances.

**50. A refrigerator consumes an average 35W power to operate between temperature $-10°C$ to $25°C$. If there is no loss of energy then how much average heat per second does it transfer?** [4 marks]

(A) 263 J/s

(B) 298 J/s

(C) 350 J/s

(D) 35 J/s

**Correct Answer:** (A) 263 J/s

**Solution:**

**Concept:**

- The coefficient of performance of an ideal reversible refrigerator is $\beta = Q_c/W = T_c/(T_h - T_c)$. Here $Q_c$ is the heat removed from the cold chamber and $W$ is the work supplied in the same interval.
- Temperature ratios in thermodynamics use absolute temperature. Energy conservation also gives $Q_h = Q_c + W$, where $Q_h$ is the heat rejected to the surroundings.

**Step 1:** Convert the two temperatures to kelvin The cold chamber is at $-10°\mathrm{C}$ and the surroundings are at $25°\mathrm{C}$. Using the standard examination conversion gives

$$T_c = 263\,\mathrm{K}, \qquad T_h = 298\,\mathrm{K}.$$

The temperature difference is $35\,\mathrm{K}$. A Celsius difference equals the corresponding kelvin difference, but a Celsius temperature itself cannot be used in the numerator of the performance formula.

**Step 2:** Calculate the heat removed in one second The ideal, lossless refrigerator has

$$\beta = \frac{263}{298 - 263} = \frac{263}{35}.$$

The electrical power supplied is $\dot{W} = 35\,\mathrm{J\,s^{-1}}$, so

$$\dot{Q}_c = \beta \dot{W} = \frac{263}{35} \times 35 = 263\,\mathrm{J\,s^{-1}}.$$

This is the rate at which heat is transferred out of the refrigerated space, the cooling rate asked for here.

**Step 3:** Distinguish cooling from heat rejection The hot surroundings receive both the extracted heat and the input work: $\dot{Q}_h = 263 + 35 = 298\,\mathrm{J\,s^{-1}}$. Thus option (B) describes the heat rejected at the hot end, whereas option (D) gives only the work input. Option (C) agrees with neither the ideal cooling rate nor the energy balance.

**Step 4:** Conclusion The refrigerator removes heat from its cold chamber at $263\,\mathrm{J\,s^{-1}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Label the cold-side heat and hot-side heat separately: a refrigerator removes $Q_c$, while its surroundings receive $Q_c + W$.

## Physics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**51. Two waves are simultaneously passing through a string and their equations are : $y_1 = A_1 \sin k(x - vt)$, $y_2 = A_2 \sin k(x - vt + x_0)$. Given amplitudes $A_1 = 12$ mm and $A_2 = 5$ mm, $x_0 = 3.5$ cm and wave number $k = 6.28$ cm$^{-1}$. The amplitude of resulting wave will be ..... mm.** [4 marks]

**Solution:**

**Concept:**

- Two waves of the same frequency combine by superposition. Their resultant amplitude is $R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$, where $\delta$ is their phase difference.
- A displacement of the argument by a length $x_0$ produces phase difference $\delta = kx_0$. The product is dimensionless when the units of $k$ and $x_0$ are reciprocal.

**Step 1:** Find the relative phase The two waves have the same travelling argument $k(x - vt)$; the second has the extra term $kx_0$. Therefore their phase difference is constant:

$$\delta = kx_0 = (6.28\,\text{cm}^{-1})(3.5\,\text{cm}) = 21.98\,\text{rad}.$$

Since $6.28$ represents $2\pi$ to the stated precision, this is approximately $7\pi$. Hence $\cos\delta \simeq -1$: the waves interfere destructively.

**Step 2:** Combine the unequal amplitudes Substitute $A_1 = 12\,\text{mm}$ and $A_2 = 5\,\text{mm}$ into the amplitude formula:

$$R = \sqrt{12^2 + 5^2 + 2(12)(5)(-1)}\,\text{mm}.$$

Consequently

$$R = \sqrt{144 + 25 - 120}\,\text{mm} = \sqrt{49}\,\text{mm} = 7\,\text{mm}.$$

Using 21.98 radians directly gives a value extremely close to the same result; the small departure comes from rounding $2\pi$ to 6.28.

**Step 3:** Check the physical range The allowed resultant amplitudes lie between the amplitude difference and the amplitude sum: $7\,\text{mm} \le R \le 17\,\text{mm}$. The almost opposite phases put the resultant at the lower limit. The cancellation is incomplete because the amplitudes are unequal; destructive interference does not imply zero displacement amplitude in this case.

**Step 4:** Conclusion The resultant amplitude is $\boxed{7\,\text{mm}}$; the numerical entry is 7.

**Quick Tip to Solve:** Reduce $kx_0$ modulo $2\pi$ before using the amplitude formula; an odd multiple of $\pi$ gives $|A_1 - A_2|$.

**52. A source of light is placed in front of a screen. Intensity of light on the screen is I. Two Polaroids $P_1$ and $P_2$ are so placed in between the source of light and screen that the intensity of light on screen is I/2. $P_2$ should be rotated by an angle of (degrees) so that the intensity of light on the screen becomes $\frac{3I}{8}$.** [4 marks]

**Solution:**

**Concept:**

- An ideal Polaroid transmits half the intensity of incident unpolarized light. The transmitted beam is plane polarized along its transmission axis.
- Malus law for a second Polaroid is $I_{\text{out}} = I_{\text{in}} \cos^2\theta$, where $\theta$ is the angle between the two transmission axes.

**Step 1:** Identify the initial relative orientation The source produces intensity $I$ at the screen before the Polaroids are inserted. After the first ideal Polaroid the available intensity is $I/2$. Since the measured intensity after both Polaroids is also $I/2$, the second initially transmits all the polarized light from the first. Thus $\cos^2\theta_0 = 1$, so the axes are initially parallel, with equivalent orientations differing by $180°$.

**Step 2:** Apply Malus law after rotating the second Polaroid Let the smallest rotation from parallel axes be $\theta$. The first Polaroid remains fixed, so its transmitted intensity is still $I/2$. The final screen intensity is therefore

$$\frac{I}{2}\cos^2\theta = \frac{3I}{8}.$$

Cancelling the common source intensity gives

$$\cos^2\theta = \frac{3}{4}, \qquad \cos\theta = \frac{\sqrt{3}}{2}$$

for the smallest positive rotation between $0°$ and $90°$.

**Step 3:** Select the required angle The angle with cosine $\sqrt{3}/2$ is $30°$. Rotating clockwise or anticlockwise by this magnitude has the same effect because the transmitted intensity depends on the square of the cosine. A $60°$ rotation would instead give $I/8$, while $90°$ would extinguish the beam; neither produces the prescribed intensity.

**Step 4:** Conclusion The smallest required rotation of $P_2$ is $\boxed{30°}$. The numerical entry in degrees is 30.

**Quick Tip to Solve:** Normalize the final intensity to the light reaching the analyser, $I/2$, rather than directly to the original unpolarized intensity $I$.

**53. If the maximum value of accelerating potential provided by a ratio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is .....**

**[$m_p$ = 1.67 $\times$ $10^{-27}$ kg, e = 1.6 $\times$ $10^{-19}$C, Speed of light = 3 $\times$ $10^8$ m/s]** [4 marks]

**Solution:**

**Concept:**

- A proton gains energy $eV$ each time it crosses the accelerating gap. It crosses the gap twice per complete revolution, so the energy gained per revolution is $2eV$.
- In the nonrelativistic cyclotron model the kinetic energy at speed $v$ is $K = \frac{1}{2}m_p v^2$. The revolution count is obtained by dividing the required kinetic energy by the gain per revolution.

**Step 1:** Calculate the required kinetic energy The target speed is one sixth of the speed of light:

$$v = \frac{3 \times 10^8}{6} = 5 \times 10^7 \,\mathrm{m\,s^{-1}}.$$

Neglecting the initial injection energy, as in the standard cyclotron calculation, the proton must gain

$$K = \frac{1}{2}(1.67 \times 10^{-27})(5 \times 10^7)^2 = 2.0875 \times 10^{-12}\,\mathrm{J}.$$

The magnetic field bends the trajectory but does no work; the increase in kinetic energy comes from the alternating gap voltage.

**Step 2:** Find the work supplied in one complete turn The accelerating potential is $V = 12\,\mathrm{kV} = 1.2 \times 10^4\,\mathrm{V}$. Each crossing supplies

$$eV = (1.6 \times 10^{-19})(1.2 \times 10^4) = 1.92 \times 10^{-15}\,\mathrm{J}.$$

There are two crossings per turn, so

$$\Delta K_{\mathrm{turn}} = 2eV = 3.84 \times 10^{-15}\,\mathrm{J}.$$

Using $eV$ as the energy per turn would count half-turns instead of revolutions.

**Step 3:** Divide the energy by the gain per revolution The continuous revolution estimate is

$$N = \frac{2.0875 \times 10^{-12}}{3.84 \times 10^{-15}} = 543.62 \text{ revolutions}.$$

Thus the proton makes 543 complete revolutions and reaches the target during the following revolution in this ideal energy-counting model. The integer-part numerical entry used for this question is 543; the computed continuous estimate is retained here to make that convention explicit.

**Step 4:** Conclusion The calculated count is $N \simeq 543.62$ revolutions, corresponding to 543 complete revolutions and part of the next. The recorded numerical entry is $\boxed{543}$.

**Quick Tip to Solve:** Count two accelerating-gap crossings per revolution. Distinguish the continuous turn estimate from the integer count of completed turns.

**54. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum to known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as .....%**

[4 marks]

**Solution:**

**Concept:**

- For a simple pendulum, $T = 2\pi\sqrt{l/g}$, where $l$ is its length. Hence $l = gT^2/(4\pi^2)$.
- At a specified angular amplitude $\theta_0$, the energy is $E = mgl(1 - \cos\theta_0)$. The known mass and fixed angular amplitude carry no stated measurement uncertainty.
- For a product of powers, the maximum fractional uncertainty is the sum of the absolute powers times their respective fractional uncertainties.

**Step 1:** Express the energy in terms of the measured quantities Take the energy relative to the lowest point of the pendulum. At the extreme angular displacement the speed is zero, so all its oscillation energy is gravitational potential energy. Substituting the length inferred from the time period gives

$$E = mg\left(\frac{gT^2}{4\pi^2}\right)(1 - \cos\theta_0) = \frac{m(1 - \cos\theta_0)}{4\pi^2}g^2T^2.$$

For the specified oscillation amplitude this establishes $E \propto g^2T^2$. The amplitude would be needed for a numerical energy, but it cancels from the requested fractional uncertainty.

**Step 2:** Propagate the two stated uncertainties The powers of both uncertain quantities are two. Their maximum percentage uncertainties therefore contribute twice each:

$$\frac{\Delta E}{E} = 2\frac{\Delta g}{g} + 2\frac{\Delta T}{T}.$$

With the given accuracies,

$$100\frac{\Delta E}{E} = 2(4\%) + 2(3\%) = 8\% + 6\% = 14\%.$$

The uncertainties add in this maximum-error convention rather than cancelling because one cannot rely on their signs being opposite.

**Step 3:** Check the scope of the estimate The known mass contributes no uncertainty. No uncertainty in the chosen angular amplitude is specified, so none is included. This result concerns the standard pendulum energy estimate at fixed angular excursion; it uses the indirect dependence of length on both $g$ and $T$, rather than treating the inferred length as an independent exact quantity.

**Step 4:** Conclusion The estimated energy is known to a maximum percentage uncertainty of $\boxed{14\%}$. The numerical entry is 14.

**Quick Tip to Solve:** Eliminate an indirectly measured length before propagating errors; its dependence on $g$ introduces an additional power of $g$.

**55. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad $s^{-1}$ in a uniform horizontal magnetic field of 3.0 $\times$ $10^{-2}$ T. The maximum emf induced the coil will be ..... $\times$ $10^{-2}$ volt (rounded off to the nearest integer)** [4 marks]

**Solution:**

**Concept:**

- The magnetic flux linkage of a coil with $N$ turns is $N\Phi = NBA\cos\theta$, where $A$ is its area and $\theta$ is the angle between its normal and the magnetic field.
- For uniform rotation with angular speed $\omega$, Faraday law gives a sinusoidal induced emf whose maximum magnitude is $\varepsilon_{\max} = NBA\omega$.

**Step 1:** Convert the radius and calculate the coil area The radius is $r = 8.0\,\text{cm} = 0.080\,\text{m}$. The area enclosed by each circular turn is therefore

$$A = \pi r^2 = \pi(0.080)^2 = 0.0064\pi\,\text{m}^2.$$

The radius must be converted before squaring. The 20 turns multiply the flux linkage; they do not change the geometrical area of an individual turn.

**Step 2:** Use the maximum rate of change of flux The vertical rotation diameter is perpendicular to the horizontal magnetic field, so the flux through the coil varies from $+BA$ to $-BA$ during a turn. Differentiating $NBA\cos\omega t$ gives

$$\varepsilon(t) = NBA\omega\sin\omega t.$$

Its peak magnitude is obtained when $|\sin\omega t| = 1$:

$$\varepsilon_{\max} = 20(3.0\times10^{-2})(0.0064\pi)(50) = 0.192\pi\,\text{V}.$$

Numerically this is $0.6032\,\text{V}$.

**Step 3:** Express the result in the required numerical form The blank multiplies $10^{-2}\,\text{V}$, so write

$$0.6032\,\text{V} = 60.32\times10^{-2}\,\text{V}.$$

The requested coefficient, rounded to the nearest integer, is 60. This is a peak emf, not the root-mean-square value, so no division by $\sqrt{2}$ is appropriate. Likewise $50\,\text{rad}\,\text{s}^{-1}$ is already an angular speed; it must not be multiplied by another $2\pi$.

**Step 4:** Conclusion The maximum induced emf is approximately $0.603\,\text{V} = 60.32\times10^{-2}\,\text{V}$. The required nearest-integer entry is $\boxed{60}$.

**Quick Tip to Solve:** Use $NBA\omega$ directly when the angular speed is in radians per second, then round the coefficient of the power of ten specified in the blank.

**56. Two simple harmonic motions are represented by the equations $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of second motion is ..... times the amplitude in first motion.** [4 marks]

**Solution:**

**Concept:**

- The amplitude of $x = A\sin(\omega t + \phi)$ is $|A|$; changing the phase $\phi$ does not change the maximum displacement.
- For $x = a\sin\omega t + b\cos\omega t$, the amplitude is $\sqrt{a^2 + b^2}$. The sine and cosine contributions differ in phase by $\pi/2$, so their coefficients cannot simply be added.

**Step 1:** Read the amplitude of the first motion The first displacement is already in the standard single-sinusoid form:

$$x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right).$$

Its amplitude is $A_1 = 5$ in the displacement units of the question. Its angular frequency is $2\pi$, and its initial phase is $\pi/4$. Neither the angular frequency nor the phase changes the magnitude of the largest displacement.

**Step 2:** Reduce the second motion to a single sinusoid Use the identity

$$\sin\theta + \cos\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right).$$

Substitution into the given second motion gives

$$x_2 = 5\sqrt{2}\left[\sqrt{2}\sin\left(2\pi t + \frac{\pi}{4}\right)\right] = 10\sin\left(2\pi t + \frac{\pi}{4}\right).$$

Thus its amplitude is $A_2 = 10$. Equivalently, its sine and cosine coefficients are both $5\sqrt{2}$, and $\sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2} = 10$.

**Step 3:** Form the dimensionless amplitude ratio The two amplitudes are expressed in the same displacement units, so the units cancel:

$$\frac{A_2}{A_1} = \frac{10}{5} = 2.$$

Both motions even have the same frequency and phase after simplification, so $x_2 = 2x_1$ at every instant. The factor $\sqrt{2}$ printed outside the brackets is only part of the amplitude; the trigonometric sum supplies a further factor of $\sqrt{2}$.

**Step 4:** Conclusion The second amplitude is $\boxed{2}$ times the first amplitude. The ratio is dimensionless.

**Quick Tip to Solve:** Convert $a\sin\omega t + b\cos\omega t$ to amplitude $\sqrt{a^2 + b^2}$ before comparing motions; do not add the coefficients directly.

**57. A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20 mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x} \times 10^{-5}$ Nm. The value of x is .....** [4 marks]

**Solution:**

**Concept:**

- The magnetic moment of a single current loop is $m = IA$, where $I$ is its current and $A$ its enclosed area.
- The torque is $\tau = IAB\sin\theta$, with $\theta$ measured between the area normal and the magnetic field. An equilateral triangle of side $a$ has area $\sqrt{3}a^2/4$.

**Step 1:** Convert the data and find the enclosed area The side length is $a = 10\,\text{cm} = 0.10\,\text{m}$, the field is $B = 20\,\text{mT} = 0.020\,\text{T}$, and the current is $I = 0.2\,\text{A}$. Thus the area enclosed by the triangular wire is

$$A = \frac{\sqrt{3}}{4}(0.10)^2 = 2.5\sqrt{3} \times 10^{-3}\,\text{m}^2.$$

The area of the loop, rather than its perimeter, enters its magnetic moment.

**Step 2:** Use the angle between the normal and the field When the plane of the coil is parallel to the magnetic field, its area normal is perpendicular to the field. Consequently $\theta = 90^\circ$ and $\sin\theta = 1$; the torque is maximum, not zero. For the single loop described,

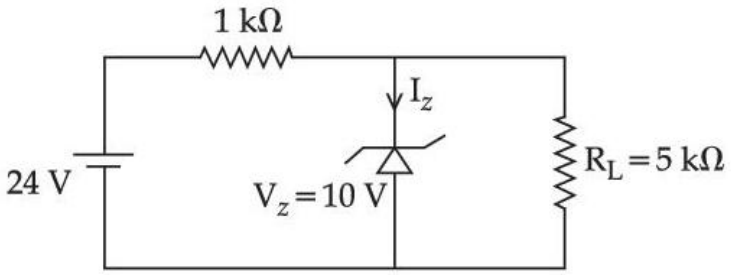
$$\begin{aligned}\tau &= IAB \\ &= (0.2)(2.5\sqrt{3} \times 10^{-3})(0.020) \\ &= \sqrt{3} \times 10^{-5}\,\text{N m}.\end{aligned}$$

**Step 3:** Compare with the stated form The question expresses the same torque as $\sqrt{x} \times 10^{-5}\,\text{N m}$. Matching its positive coefficient gives $\sqrt{x} = \sqrt{3}$, so squaring yields $x = 3$. The number $x$ is dimensionless; the unit belongs to the torque.

**Step 4:** Conclusion The torque is $\sqrt{3} \times 10^{-5}\,\text{N m}$, so the required value is $\boxed{x = 3}$.

**Quick Tip to Solve:** In $\tau = IAB\sin\theta$, measure the angle from the loop normal: a plane parallel to the field gives maximum torque.

**58. For the given circuit, the power across Zener diode is ..... mW.** [4 marks]




**Solution:**

**Concept:**

- A reverse-biased Zener diode in breakdown holds the parallel load at its Zener voltage $V_Z$, provided a positive current is available through the diode.
- The series-resistor current splits as $I_S = I_L + I_Z$. The diode dissipates $P_Z = V_Z I_Z$, not the total power delivered by the source.

**Step 1:** Calculate the series current at the regulated voltage The figure gives a $24\,\text{V}$ supply, a $1\,\text{k}\Omega$ series resistor, a $10\,\text{V}$ Zener diode and a $5\,\text{k}\Omega$ load in parallel with it. In breakdown the load and diode each have a potential difference of $10\,\text{V}$. The series resistor therefore has a drop of $24 - 10 = 14\,\text{V}$, giving

$$I_S = \frac{24 - 10}{1000} = 0.014\,\text{A} = 14\,\text{mA}.$$

**Step 2:** Subtract the current drawn by the load The load is connected across the regulated output, so its current must be found using $10\,\text{V}$, rather than the full supply voltage:

$$I_L = \frac{10}{5000} = 0.002\,\text{A} = 2\,\text{mA}.$$

By Kirchhoff's junction rule,

$$I_Z = I_S - I_L = 14 - 2 = 12\,\text{mA}.$$

This is positive, confirming that the assumed Zener conduction is consistent with the circuit.

**Step 3:** Find the diode power in the requested unit Only the $12\,\text{mA}$ branch current flows through the Zener. Hence

$$P_Z = V_Z I_Z = 10(12 \times 10^{-3}) = 0.120\,\text{W} = 120\,\text{mW}.$$

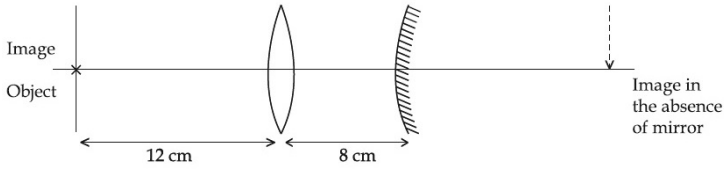
Using the entire $14\,\text{mA}$ would incorrectly include the load branch in the diode power.

**Step 4:** Conclusion The Zener diode dissipates $\boxed{120\,\text{mW}}$; the numerical entry is $\boxed{120}$.

**Quick Tip to Solve:** For a shunt Zener regulator, first find the series current, then subtract the load current before using $P_Z = V_Z I_Z$.

**59. An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.**

**When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be ..... (cm)** [4 marks]




**Solution:**

**Concept:**

- A convex mirror has its centre of curvature behind the mirror at distance $R = 2f_m$. A ray directed towards that centre strikes the mirror normally and retraces its path.
- For the returned image to coincide with the object through the same lens, the converging rays from the lens must be directed towards the mirror's centre of curvature. Optical reversibility then returns them to the object.

**Step 1:** Locate the point towards which the lens converges the rays The convex mirror has focal length $f_m = 15\,\text{cm}$, so its centre of curvature is $R = 30\,\text{cm}$ behind its pole. The mirror is $8\,\text{cm}$ to the right of the lens. Its centre therefore lies

$$v = 8 + 30 = 38\,\text{cm}$$

to the right of the lens. Rays emerging from the lens towards this centre reflect back along themselves, and the second passage through the lens recreates the object.

**Step 2:** Interpret the image position when the mirror is removed Removing the mirror leaves the original convergence produced by the lens unchanged. The rays can now reach the point $38\,\text{cm}$ to the right of the lens, forming the stated real, inverted image. As a consistency check, the object distance is $u = -12\,\text{cm}$, so the thin-lens formula gives

$$\frac{1}{f} = \frac{1}{38} - \frac{1}{-12} = \frac{25}{228}, \qquad f = 9.12\,\text{cm}.$$

This positive focal length is appropriate for a convex lens and is less than the object distance, as required for a real image.

**Step 3:** Add the distances on opposite sides of the lens The object is $12\,\text{cm}$ to the left of the lens and the final image without the mirror is $38\,\text{cm}$ to the right. Their separation is therefore the sum,

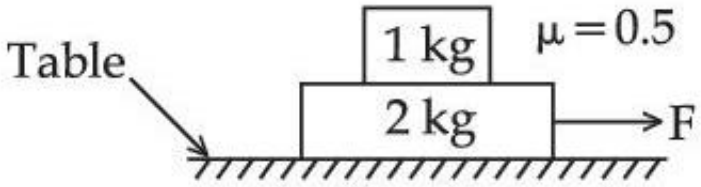
$$D = 12 + 38 = 50\,\text{cm}.$$

Subtracting the distances would be incorrect because the two points are on opposite sides of the lens.

**Step 4:** Conclusion The image is $38\,\text{cm}$ beyond the lens, and its distance from the object is $\boxed{50\,\text{cm}}$.

**Quick Tip to Solve:** When a lens-mirror combination sends rays back to the same object, locate the mirror centre first; here the lens alone focuses at that centre.

**60. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is ..... N. (take g = 10 $ms^{-2}$)** [4 marks]




**Solution:**

**Concept:**

- The largest static friction between two surfaces is $f_{\max} = \mu_s N$. Static friction takes the value needed to prevent slipping, up to this limit.
- On the smooth table, the only external horizontal force on the two-block system is $F$. If the blocks move together, $a = F/(m + M)$.

**Step 1:** Identify the friction required by the upper block The diagram shows a $1\,\text{kg}$ upper block on a $2\,\text{kg}$ lower block, with $F$ applied to the lower one. The upper block has no separately applied horizontal force: friction from the lower block must accelerate it to the right. If their common acceleration is $a$, Newton's second law for the upper block gives

$$f = ma = (1)a.$$

The equal and opposite friction on the lower block is internal to the combined system.

**Step 2:** Apply the limiting condition for no relative slipping For the upper block there is no vertical acceleration, so the contact normal force is its own weight,

$$N = mg = 1 \times 10 = 10\,\text{N}.$$

Therefore

$$f_{\max} = \mu_s N = 0.5 \times 10 = 5\,\text{N}, \qquad a_{\max} = \frac{f_{\max}}{m} = 5\,\text{m s}^{-2}.$$

A larger acceleration would require more friction than the contact can supply, and the upper block would slip backwards relative to the lower block.

**Step 3:** Use the total mass to obtain the applied force The total moving mass is $1 + 2 = 3\,\text{kg}$. Since the table is smooth, no external friction opposes the system, and

$$F_{\max} = (m + M)a_{\max} = 3 \times 5 = 15\,\text{N}.$$

The lower-block equation provides a check: $F - f = Ma$ gives $15 - 5 = 2 \times 5$, as required.

**Step 4:** Conclusion The maximum force for motion without relative slipping is $\boxed{15\,\text{N}}$; the required numerical value is $\boxed{15}$.

**Quick Tip to Solve:** Find the acceleration limit from the upper block alone, then multiply by the combined mass to find the force applied to the lower block.

## Chemistry – Section A: Multiple Choice Questions (80 marks)

**61. Which one of the following phenols does not give colour when condensed with phthalic anhydride in presence of conc. $H_2SO_4$ ?** [4 marks]

(A) Phenol

(B) 4-Methylphenol (p-cresol)

(C) Benzene-1,3-diol (resorcinol)

(D) Benzene-1,2-diol (catechol)

**Correct Answer:** (B) 4-Methylphenol (p-cresol)

**Solution:**

**Concept:**

- Condensation of suitable phenols with phthalic anhydride in concentrated $H_2SO_4$ produces phthalein-type dyes; their conjugated forms account for the characteristic colour test.
- The usual phthalein condensation uses a reactive position para to the phenolic OH. A substituent occupying that position prevents the normal dye-forming condensation of that phenol.

**Step 1:** Locate the para position in each structure Number the carbon carrying OH as carbon 1. The opposite carbon is carbon 4, the para position. Option (A) is phenol and has hydrogen at carbon 4. Option (B) is $p$-cresol, or 4-methylphenol, and has a $CH_3$ group at carbon 4. Option (C) is resorcinol, benzene-1,3-diol; option (D) is catechol, benzene-1,2-diol. Both dihydroxybenzenes retain available para-reactive sites relative to an activating hydroxyl group.

**Step 2:** Identify the structure that fails the colour test In $p$-cresol, the para site needed for the usual condensation is occupied by a methyl group. It therefore does not form the normal coloured phthalein product under the test conditions. This obstruction is absent in phenol, which gives phenolphthalein, so (A) is rejected. Resorcinol forms the fluorescein dye system and hence (C) is rejected. Catechol likewise gives a coloured phthalein condensation product, so (D) is rejected. The deciding feature is the blocked para position, rather than simply the number of hydroxyl groups.

**Step 3:** Conclusion The phenol that does not give the colour test is $p$-cresol (4-methylphenol). Hence, the correct option is (B).

**Quick Tip to Solve:** Mark the carbon para to each phenolic hydroxyl group: in $p$-cresol that position is already occupied by $CH_3$.

**62. The bond order and magnetic behaviour of $O_2^-$ ion are respectively :** [4 marks]

(A) 1.5 and paramagnetic

(B) 1.5 and diamagnetic

(C) 2 and diamagnetic

(D) 1 and paramagnetic

**Correct Answer:** (A) 1.5 and paramagnetic

**Solution:**

**Concept:**

- Bond order is $(N_b - N_a)/2$, where $N_b$ and $N_a$ are the numbers of electrons in bonding and antibonding molecular orbitals.
- An atom, molecule or ion with an unpaired electron is paramagnetic. Adding an electron to an antibonding orbital lowers bond order by one half.

**Step 1:** Fill the valence molecular orbitals Neutral $O_2$ has 12 valence electrons and bond order 2. The superoxide ion $O_2^-$ has one additional electron, hence 13 valence electrons. Using the orbital ordering appropriate to oxygen, its valence configuration is

$$(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2 \\ (\pi 2p_x)^2(\pi 2p_y)^2(\pi^* 2p_x)^2(\pi^* 2p_y)^1.$$

The two degenerate antibonding $\pi^*$ orbitals therefore contain three electrons in total.

**Step 2:** Count bonding electrons and inspect the pairing The bonding orbitals contain $N_b = 2+2+4 = 8$ electrons, while the antibonding orbitals contain $N_a = 2+3 = 5$. Thus

$$\text{Bond order} = \frac{8-5}{2} = 1.5.$$

The last $\pi^*$ orbital has one unpaired electron, so the ion is paramagnetic. Option (B) has the correct bond order but incorrectly claims diamagnetism. Option (C) retains the neutral oxygen bond order and is also wrong about magnetism. Option (D) has the correct magnetic behaviour but its bond order 1 corresponds to the further reduced peroxide ion, not superoxide.

**Step 3:** Conclusion The bond order of $O_2^-$ is 1.5, and it is paramagnetic. Hence, the correct option is (A).

**Quick Tip to Solve:** Going from $O_2$ to $O_2^-$, add one electron to $\pi^*$: bond order falls from 2 to 1.5 and one electron remains unpaired.

**63. Given below are two statements : one is labelled as Assertion (A) and other is labelled as Reason (R).**

**Assertion (A) : Sucrose is a disaccharide and a non-reducing sugar.**

**Reason (R) : Sucrose involves glycosidic linkage between $C_1$ of $\beta$-glucose and $C_2$ of $\alpha$-fructose.**

**Choose the most appropriate answer from the options given below :** [4 marks]

(A) Both (A) and (R) are true but (R) is not the true explanation of (A).

(B) (A) is false but (R) is true.

(C) (A) is true but (R) is false.

(D) Both (A) and (R) are true and (R) is the true explanation of (A).

**Correct Answer:** (C) (A) is true but (R) is false.

**Solution:**

**Concept:**

- A disaccharide yields two monosaccharide units on hydrolysis. Sucrose yields glucose and fructose.
- A reducing sugar needs an available anomeric hemiacetal or hemiketal group that can open to a carbonyl form. In sucrose, both anomeric centres participate in the glycosidic bond.

**Step 1:** Evaluate the assertion Sucrose consists of one glucose unit and one fructose unit joined by a glycosidic linkage, so it is a disaccharide. Its glycosidic bond joins both anomeric carbons: carbon 1 of glucose and carbon 2 of fructose. Neither anomeric hydroxyl remains free to generate the corresponding open-chain carbonyl form without hydrolysing the glycosidic bond. Thus sucrose does not reduce Fehling's or Tollens' reagent in the usual reducing-sugar tests. Both parts of the assertion are therefore true.

**Step 2:** Check the anomeric configurations in the reason The actual linkage is between carbon 1 of $\alpha$-D-glucose and carbon 2 of $\beta$-D-fructose, commonly written as an $\alpha(1\rightarrow 2)\beta$ linkage. The reason instead states $\beta$-glucose and $\alpha$-fructose. Although it names the correct carbon numbers, it reverses the two anomeric configurations and is therefore false as a complete statement. It cannot be accepted merely because both anomeric centres are involved.

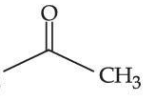
**Step 3:** Select the truth combination The assertion is true and the reason is false, which is exactly option (C). Options (A) and (D) require both statements to be true, so both fail because of the incorrect configurations in the reason. Option (B) requires a false assertion and a true reason; both truth values are reversed relative to the actual structure of sucrose.

**Step 4:** Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

**Quick Tip to Solve:** For sucrose remember both the carbon numbers and the configurations: $\alpha$-glucose C1 joins $\beta$-fructose C2.

**64. Match List - I with List - II :**

**Choose the most appropriate match :** [4 marks]

| List - I (Chemical Reaction) | | List - II (Reagent used) | |
|---|---|---|---|
| (a) | $CH_3COOCH_2CH_3 \rightarrow CH_3CH_2OH$ | (i) | $CH_3MgBr/H_3O^+$ (1.equivalent) |
| (b) | $CH_3COOCH_3 \rightarrow CH_3CHO$ | (ii) | $H_2SO_4/H_2O$ |
| (c) | $CH_3C \equiv N \rightarrow CH_3CHO$ | (iii) | DIBAL-H/$H_2O$ |
| (d) | $CH_3C \equiv N \rightarrow CH_3$  $CH_3$ | (iv) | $SnCl_2$, $HCl/H_2O$ |

(A) a-ii, b-iv, c-iii, d-i

(B) a-iv, b-ii, c-iii, d-i

(C) a-ii, b-iii, c-iv, d-i

(D) a-iii, b-ii, c-i, d-iv

**Correct Answer:** (C) a-ii, b-iii, c-iv, d-i

**Solution:**

**Concept:**

- Acid-catalysed ester hydrolysis gives a carboxylic acid and an alcohol. Controlled hydride reduction of an ester can stop at the aldehyde.
- A nitrile gives an aldehyde on Stephen reduction, whereas addition of one equivalent of a Grignard reagent followed by hydrolysis gives a ketone.

**Step 1:** Identify the ester reactions In (a), ethyl ethanoate supplies ethanol by hydrolysis. Water in the presence of sulphuric acid cleaves the ester into ethanoic acid and ethanol, so (a) matches (ii).

$$CH_3COOCH_2CH_3 + H_2O \xrightarrow{H^+} CH_3COOH + CH_3CH_2OH$$

In (b), methyl ethanoate must be reduced to ethanal, rather than hydrolysed or completely reduced to ethanol. DIBAL-H under controlled conditions, followed by water, gives this partial reduction. Thus (b) matches (iii).

**Step 2:** Identify the nitrile reactions In (c), ethanenitrile is converted into ethanal by the Stephen reaction. Stannous chloride and hydrochloric acid first form the iminium salt, and hydrolysis releases the aldehyde. Thus (c) matches (iv). In (d), methylmagnesium bromide attacks the electrophilic carbon of the nitrile. Hydrolysis of the resulting imine salt gives acetone:

$$CH_3CN \xrightarrow[H_3O^+]{CH_3MgBr} CH_3COCH_3.$$

Thus (d) matches (i).

**Step 3:** Compare the complete match Combining the four independent transformations gives (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i). Option (A) incorrectly interchanges the ester-reduction and nitrile-reduction reagents. Option (B) assigns Stephen reagent to ester hydrolysis. Option (D) assigns acid hydrolysis to the ester-to-aldehyde change and assigns the Grignard reagent to aldehyde formation. Each of

these fails at least one required functional-group conversion.

**Step 4:** Conclusion The matching is a-ii, b-iii, c-iv, d-i. Hence, the correct option is (C).

**Quick Tip to Solve:** First distinguish ester hydrolysis from reduction; then remember that a nitrile plus a Grignard reagent gives a ketone after hydrolysis.

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**65. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**

**Assertion (A) : Barium carbonate is insoluble in water and is highly stable.**

**Reason (R) : The thermal stability of the carbonates increases with increasing cationic size.** [4 marks]

(A) Both (A) and (R) are true and (R) is the true explanation of (A)

(B) (A) is true but (R) is false

(C) Both (A) and (R) are true but (R) is not the true explanation of (A)

(D) (A) is false but (R) is true.

**Correct Answer:** (A) Both (A) and (R) are true and (R) is the true explanation of (A)

**Solution:**

**Concept:**

- The carbonates of alkaline-earth metals are sparingly soluble in water. Their resistance to thermal decomposition generally increases down Group 2.
- A larger divalent cation has lower polarising power, so it distorts the carbonate ion less strongly and makes decomposition to the oxide more difficult.

**Step 1:** Evaluate the assertion Barium carbonate, $BaCO_3$, is insoluble for ordinary qualitative purposes: only a very small amount dissolves in water. It is also among the most thermally stable Group 2 carbonates. On strong heating, its decomposition is represented by

$$BaCO_3 \longrightarrow BaO + CO_2.$$

Its high decomposition temperature supports the stability part of the assertion. The assertion is therefore true. Insoluble here means sparingly soluble, rather than an absolute absence of dissolved ions.

**Step 2:** Evaluate the reason Down the group, the radius of the doubly charged metal ion increases. Since its charge remains the same, the larger cation has less ability to polarise the electron cloud of $CO_3^{2-}$. The carbonate ion is consequently less destabilised, and decomposition requires stronger heating. Therefore the thermal stability increases in the order

$$MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3.$$

The stated trend in the reason is true.

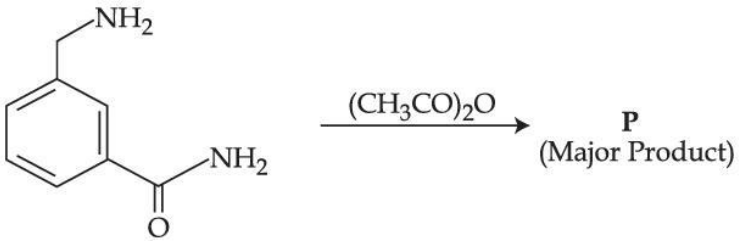
**Step 3:** Relate the reason to the assertion Barium lies near the bottom of Group 2 and has a large cation. The trend given in the reason accounts directly for the high thermal stability stated in the

assertion. The insolubility is also an established property of barium carbonate; the reason addresses the stability comparison. In the intended assertion-reason classification this is the explanatory pair. Options (B) and (D) incorrectly make one statement false, and option (C) denies the stated stability explanation.

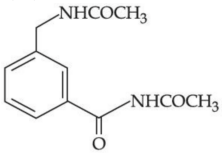
**Step 4:** Conclusion Both statements are true, and the increase in cationic size explains the high thermal stability of barium carbonate. Hence, the correct option is (A).

**Quick Tip to Solve:** For Group 2 carbonates, connect a larger cation with weaker polarisation and therefore greater resistance to thermal decomposition.

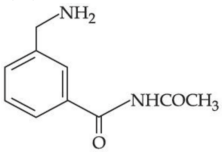
**66. The major product in the above reaction is :** [4 marks]



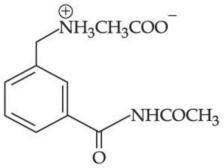

(A) $m$-$CH_3CONHCH_2C_6H_4CONHCOCH_3$



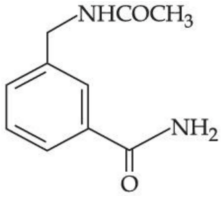

(B) $m$-$H_2NCH_2C_6H_4CONHCOCH_3$



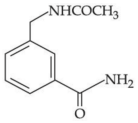

(C) $m$-$[H_3NCH_2C_6H_4CONHCOCH_3]^+ CH_3COO^-$




(D) $m$-$CH_3CONHCH_2C_6H_4CONH_2$




**Correct Answer:** (D) $m$-$CH_3CONHCH_2C_6H_4CONH_2$




**Solution:**

**Concept:**

- Acetic anhydride acetylates a primary amine by nucleophilic acyl substitution, replacing one N-H hydrogen with an acetyl group $CH_3CO^-$.
- An amide nitrogen is much less nucleophilic than an amine nitrogen because its lone pair is delocalised into the adjacent carbonyl group.

**Step 1:** Identify the two nitrogen environments The substrate contains a benzylamine side chain, $-CH_2NH_2$, and a carboxamide group, $-CONH_2$, attached at meta positions on the benzene ring. The first nitrogen behaves as a primary aliphatic amine because the methylene group separates it from the ring. The second nitrogen belongs to an amide and participates in carbonyl resonance. These two

amino-looking groups therefore do not have equal reactivity.

**Step 2:** Apply selective acetylation The available lone pair on the benzylamine nitrogen attacks a carbonyl carbon of acetic anhydride. Loss of acetate and proton transfer form the N-acetyl derivative:

$$\mathrm{ArCH_2NH_2 + (CH_3CO)_2O \longrightarrow ArCH_2NHCOCH_3 + CH_3COOH.}$$

Here Ar is the benzene ring bearing the unchanged carboxamide substituent. Ordinary acetylation therefore converts $-\mathrm{CH_2NH_2}$ into $-\mathrm{CH_2NHCOCH_3}$, while the ring-bound $-\mathrm{CONH_2}$ remains unchanged.

**Step 3:** Select the matching structure The product in (D) has precisely this combination: an acetylated benzylamine and an unmodified carboxamide. Structure (A) additionally acetylates the already deactivated amide nitrogen. Structure (B) acetylates only the less nucleophilic nitrogen and leaves the reactive amine unchanged. Structure (C) shows an ammonium acetate salt together with amide acetylation, rather than the neutral amide produced by nucleophilic acyl substitution at the free amine.

**Step 4:** Conclusion The major product is $m$-$\mathrm{CH_3CONHCH_2C_6H_4CONH_2}$, with acetylation at the free amine nitrogen. Hence, the correct option is (D).

**Quick Tip to Solve:** When an amine and an amide occur together, acetylate the amine first: the amide lone pair is already engaged in carbonyl resonance.

**67. Indicate the complex/complex ion which did not show any geometrical isomerism :** [4 marks]

(A) $[CoCl_2(en)_2]$

(B) $[Co(CN)_5(NC)]^{3-}$

(C) $[Co(NH_3)_3(NO_2)_3]$

(D) $[Co(NH_3)_4Cl_2]^+$

**Correct Answer:** (B) $[Co(CN)_5(NC)]^{3-}$

**Solution:**

**Concept:**

- Geometrical isomers have the same coordination bonds but different relative positions of ligands around the central metal.
- In an octahedral $MA_5B$ complex, all six positions for the single B ligand are equivalent; there is no distinct cis/trans arrangement.

**Step 1:** Classify the ligand patterns Complex (B), $[\mathrm{Co(CN)_5(NC)}]^{3-}$, has five carbon-bound cyanide ligands and one nitrogen-bound isocyanide ligand. For geometrical classification it has the octahedral pattern $\mathrm{MA_5B}$. Moving the single distinct ligand to any other vertex gives a structure superposable by rotation. There is therefore only one geometrical arrangement for this pattern.

**Step 2:** Check the other complexes In (A), the two chloride ligands of $[\mathrm{CoCl_2(en)_2}]$ can be adjacent or opposite, giving cis and trans forms; the two bidentate ethylenediamine ligands occupy the remaining positions. In (C), the octahedral $\mathrm{MA_3B_3}$ arrangement has facial and meridional forms: the three

identical ligands occupy one face or a meridian. In (D), $[\mathrm{Co(NH_3)_4Cl_2}]^+$ has the pattern $\mathrm{MA_4B_2}$, again allowing adjacent chlorides (cis) or opposite chlorides (trans).

**Step 3:** Distinguish linkage from geometrical changes The notation CN versus NC specifies the atom through which the ambidentate cyanide ligand binds. Changing a ligand from carbon bonding to nitrogen bonding concerns linkage isomerism. It does not create a second geometrical placement for an octahedral complex containing just one ligand of the distinct linkage type. Thus the presence of an ambidentate ligand in (B) does not overturn the $\mathrm{MA_5B}$ positional argument. Each other option has a genuine pair of different spatial arrangements.

**Step 4:** Conclusion $[\mathrm{Co(CN)_5(NC)}]^{3-}$ does not show geometrical isomerism. Hence, the correct option is (B).

**Quick Tip to Solve:** Count ligand types around the octahedron: a single distinct ligand in $\mathrm{MA_5B}$ cannot generate a cis/trans pair.

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**68. Arrange the following Cobalt complexes in the order of increasing Crystal Field Stabilization Energy (CFSE) value.**

**Complexes :** $\underset{A}{[CoF_6]^{3-}}, \underset{B}{[Co(H_2O)_6]^{2+}}, \underset{C}{[Co(NH_3)_6]^{3+}}$ *and* $\underset{D}{[Co(en)_3]^{3+}}$

**Choose the correct option :** [4 marks]

(A) A < B < C < D

(B) B < A < C < D

(C) B < C < D < A

(D) C < D < B < A

**Correct Answer:** (B) B < A < C < D

**Solution:**

**Concept:**

- Octahedral CFSE is calculated from the populations of $t_{2g}$ and $e_g$: $E = (-0.4n_t + 0.6n_e)\Delta_o$. The stabilisation value is its positive magnitude.
- The splitting $\Delta_o$ generally increases with metal oxidation state and ligand field strength. Relevant ligand strength increases from fluoride and water to ammonia and ethylenediamine.

**Step 1:** Determine oxidation states and occupancies In A, $[\mathrm{CoF_6}]^{3-}$, cobalt is +3 and has a $d^6$ configuration. Fluoride is weak-field, giving high-spin $t_{2g}^4e_g^2$, with stabilisation $0.4\Delta_A$. In B, $[\mathrm{Co(H_2O)_6}]^{2+}$, cobalt is +2 and $d^7$. Its high-spin arrangement is $t_{2g}^5e_g^2$, with stabilisation $0.8\Delta_B$. In C and D, cobalt is +3 and the stronger ligands give low-spin $d^6$, namely $t_{2g}^6e_g^0$, with stabilisation $2.4\Delta_o$.

**Step 2:** Compare the cobalt complexes For the low-spin pair C and D, both stabilisation magnitudes equal $2.4\Delta_o$. Since ethylenediamine is stronger-field than ammonia, $\Delta_D > \Delta_C$, hence D has greater stabilisation than C. For A and B, the actual occupancy-based energies require more care:

$$S_A = 0.4\Delta_A, \qquad S_B = 0.8\Delta_B.$$

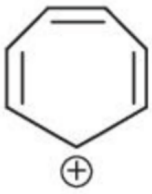
Consequently $S_B < S_A$ would require $\Delta_A > 2\Delta_B$. A higher cobalt oxidation state tends to increase splitting, but by itself does not prove this factor-of-two inequality. The given qualitative ligand and charge trends are insufficient to establish this literal CFSE comparison.

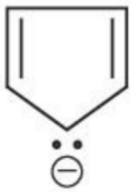
**Step 3:** Assemble and test the order The final answer key selects B, whose order is $B < A < C < D$. This is the conventional splitting-strength order used in this question: the increase from divalent to trivalent cobalt puts A after B, and the stronger ligands then put C and D after A. Under that convention the order matches option (B), while (A) reverses A/B and (C) and (D) misplace the strong-field complexes. However, crystal-field splitting $\Delta_o$ and occupancy-based CFSE are distinct quantities. Because A and B have different electron populations, the conventional order cannot be presented as a derivation of their actual stabilisation energies. No numerical splitting parameters are provided to settle that stricter interpretation.

**Step 4:** Conclusion The official-key answer is (B), $B < A < C < D$, using the intended crystal-field splitting-strength convention. Read literally as occupancy-based CFSE magnitudes, the A/B comparison is underdetermined by the supplied data; it requires $\Delta_A/\Delta_B$, with B below A only if that ratio exceeds 2.

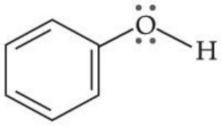
**Quick Tip to Solve:** Keep $\Delta_o$ separate from CFSE: first calculate the occupancy coefficient, then compare coefficient times splitting. A larger splitting alone need not mean larger CFSE.

**69. Which one of the following compounds is not aromatic?** [4 marks]

(A) Tropylium cation 

(B) Cyclopentadienyl anion 

(C) Cyclooctatetraene 

(D) Phenol 


**Correct Answer:** (C) Cyclooctatetraene 

**Solution:**

**Concept:**

- An aromatic ring must be cyclic, planar and continuously conjugated, with $4n + 2$ pi electrons in the conjugated loop.
- A cyclic structure with $4n$ pi electrons avoids antiaromatic destabilisation when it can adopt a nonplanar conformation, thereby becoming nonaromatic.

**Step 1:** Test the charged rings Option (A) is the tropylium cation. Its three double bonds contribute six pi electrons; the positively charged carbon provides an empty p orbital so that conjugation continues

around the seven-membered ring. The six-electron loop satisfies $4n+2$ with $n=1$, making it aromatic. Option (B) is the cyclopentadienyl anion. Its two double bonds provide four pi electrons and the negatively charged carbon supplies a lone pair in a p orbital. The total is six, so this planar, conjugated ring is also aromatic.

**Step 2:** Examine the eight-membered ring Option (C) is cyclooctatetraene. Its four double bonds supply eight pi electrons, which have the form $4n$, not $4n+2$. A planar fully conjugated eight-electron ring would be antiaromatic. Cyclooctatetraene instead adopts a nonplanar tub conformation, reducing continuous cyclic p-orbital overlap. Consequently the actual compound is nonaromatic, rather than an aromatic system satisfying the Huckel rule.

**Step 3:** Check the phenol option Option (D) is phenol. The benzene ring retains its planar, cyclic six-pi-electron conjugated system. Oxygen's lone pairs influence the electron density through resonance but do not destroy the aromatic benzene sextet. It therefore remains aromatic. Thus (A), (B), and (D) each possess an aromatic six-electron ring, while only the eight-membered ring in (C) fails the required combination of planarity and electron count.

**Step 4:** Conclusion Cyclooctatetraene is nonaromatic because its eight-pi-electron ring adopts a nonplanar tub shape. Hence, the correct option is (C).

**Quick Tip to Solve:** Before applying $4n+2$, check planarity: cyclooctatetraene avoids antiaromaticity by bending into a nonplanar tub conformation.

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**70. The number of stereoisomers possible for 1, 2-dimethyl cyclopropane is :** [4 marks]

(A) One

(B) Four

(C) Two

(D) Three

**Correct Answer:** (D) Three

**Solution:**

**Concept:**

- A disubstituted cyclopropane has distinct cis and trans arrangements because rotation about the ring bonds cannot interchange the faces of the ring.
- Two stereogenic centres can give at most four configurations, but internal symmetry can make two of those configurations identical as a meso form.

**Step 1:** Identify the stereogenic ring carbons In 1,2-dimethylcyclopropane, carbon 1 and carbon 2 each carry a methyl group and a hydrogen. At either carbon, the two directions around the ring are different: one leads directly to the other substituted carbon, while the other leads to the unsubstituted methylene carbon. Each of these two carbons is therefore stereogenic. Starting with $2^2=4$ configurations gives an upper limit, but symmetry must be checked before counting distinct molecules.

**Step 2:** Count the cis form When both methyl groups lie on the same face of the cyclopropane ring,

the molecule is cis. Since the two substituents are identical, a plane of symmetry interchanges the two substituted ring carbons. The opposite local configurations, conventionally represented by $(1R, 2S)$ and $(1S, 2R)$, describe the same meso stereoisomer. This cis arrangement is achiral and contributes just one distinct stereoisomer.

**Step 3:** Count the trans forms and compare options With the methyl groups on opposite ring faces, the trans arrangement lacks the internal mirror plane of the cis form. It occurs as a pair of nonsuperposable mirror images, represented by $(1R, 2R)$ and $(1S, 2S)$. These count separately as two enantiomers. The total is therefore one meso cis form plus two trans enantiomers:

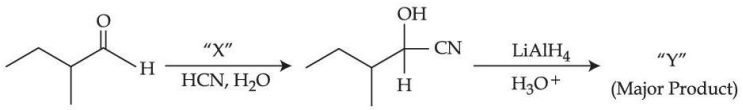
$$N = 1 + 2 = 3.$$

One ignores the trans forms; two counts only geometrical classes while overlooking chirality; four fails to identify the identical meso configurations.

**Step 4:** Conclusion There are three stereoisomers: one cis meso form and a trans enantiomeric pair. Hence, the correct option is (D).

**Quick Tip to Solve:** For identical substituents on adjacent cyclopropane carbons, count the cis meso form once and the two trans enantiomers separately.

**71. Consider the given reaction, identify 'X' and 'Y' :** [4 marks]




(A) $X = \text{NaOH}; \quad Y = \text{CH}_3\text{CH}_2\text{CH}(\text{CH}_3)\text{CH}(\text{OH})\text{NH}_2$

(B) $X = \text{HNO}_3; \quad Y = \text{CH}_3\text{CH}_2\text{CH}(\text{CH}_3)\text{CH}(\text{OH})\text{CH}_2\text{NH}_2$

(C) $X = \text{NaOH}; \quad Y = \text{CH}_3\text{CH}_2\text{CH}(\text{CH}_3)\text{CH}(\text{OH})\text{CH}_2\text{NH}_2$

(D) $X = \text{HNO}_3; \quad Y = \text{CH}_3\text{CH}_2\text{CH}(\text{CH}_3)\text{CH}(\text{OH})\text{NH}_2$

**Correct Answer:** (C) $X = \text{NaOH}; \quad Y = \text{CH}_3\text{CH}_2\text{CH}(\text{CH}_3)\text{CH}(\text{OH})\text{CH}_2\text{NH}_2$

**Solution:**

**Concept:**

- The carbonyl compound is 2-methylbutanal, $\text{CH}_3\text{CH}_2\text{CH}(\text{CH}_3)\text{CHO}$. Hydrogen cyanide adds across its carbonyl group to form a cyanohydrin.
- A small amount of base generates the nucleophile $\text{CN}^-$. Lithium aluminium hydride reduces a nitrile group $-\text{CN}$ to $-\text{CH}_2\text{NH}_2$, retaining the nitrile carbon in the product.

**Step 1:** Identify the catalyst X Hydrogen cyanide is a weak acid and by itself supplies only a small concentration of cyanide ions. The base $\text{NaOH}$ helps generate $\text{CN}^-$, which attacks the electrophilic carbonyl carbon. Protonation of the resulting alkoxide gives the displayed cyanohydrin. Thus $X = \text{NaOH}$; the alternatives containing $\text{HNO}_3$ are unsuitable because acid suppresses the concentration of the cyanide nucleophile.

**Step 2:** Track the nitrile carbon during reduction Put $R = \text{CH}_3\text{CH}_2\text{CH}(\text{CH}_3)^-$. The intermediate is

$RCH(OH)CN$. Hydride reduction followed by acidic work-up converts its nitrile to a primary amine:

$$RCH(OH)CN \xrightarrow[H_3O^+]{LiAlH_4} RCH(OH)CH_2NH_2.$$

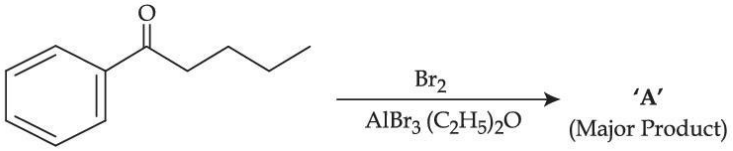
The alcohol group is regenerated during work-up. Crucially, the nitrogen is attached to the newly formed methylene carbon, rather than directly to the carbon bearing the hydroxyl group.

**Step 3:** Compare the two structural possibilities The product is $CH_3CH_2CH(CH_3)CH(OH)CH_2NH_2$, named 1-amino-3-methylpentan-2-ol. Options A and D omit the extra carbon between the hydroxyl-bearing carbon and nitrogen; they would incorrectly discard the carbon introduced by hydrogen cyanide. Option B has the appropriate amino-alcohol skeleton but gives the wrong catalyst. Option C has both the base catalyst and the retained nitrile carbon.

**Step 4:** Conclusion $X = NaOH$ and $Y = CH_3CH_2CH(CH_3)CH(OH)CH_2NH_2$. Hence, the correct option is (C).

**Quick Tip to Solve:** Count the carbon of $-CN$: reduction changes it to $-CH_2NH_2$; it never disappears.

**72. Consider the given reaction, the product A is :** [4 marks]




(A) $C_6H_5COCH_2CH_2CH_2CH_2Br$

(B) $p$-$BrC_6H_4COCH_2CH_2CH_2CH_3$

(C) $m$-$BrC_6H_4COCH_2CH_2CH_2CH_3$

(D) $C_6H_5COCH_2CH_2CH(Br)CH_3$

**Correct Answer:** (C) $m$-$BrC_6H_4COCH_2CH_2CH_2CH_3$

**Solution:**

**Concept:**

- $Br_2/AlBr_3$ supplies an electrophile for aromatic bromination. The ketone substituent attached to the ring is $-COCH_2CH_2CH_2CH_3$.
- An acyl group withdraws electron density by both inductive and resonance effects and is deactivating and meta-directing in electrophilic aromatic substitution.

**Step 1:** Identify the reaction conditions The substrate is an aryl alkyl ketone: the benzene ring is bonded directly to its carbonyl carbon. Bromine in the presence of the Lewis acid $AlBr_3$, in ether, is the condition shown for electrophilic substitution on the aromatic ring. There is no light or radical initiator to select a remote alkyl-chain carbon. Therefore the side-chain bromination structures A and D do not describe the intended major reaction.

**Step 2:** Apply the directing effect of the carbonyl group The carbonyl group withdraws electron density from the aromatic ring. Attack at an ortho or para position gives a resonance contrib-

utor of the arenium ion in which the positive charge is especially destabilised by the adjacent electron-withdrawing carbonyl group. The corresponding meta intermediate avoids this particularly unfavourable contributor. Hence substitution occurs preferentially at a ring carbon meta to the acyl group.

**Step 3:** Locate bromine in the listed products The alkyl chain and the carbonyl group remain intact, and one ring hydrogen is replaced by bromine. The product is $BrC_6H_4COCH_2CH_2CH_2CH_3$ with a meta relationship between bromine and the ketone substituent. The para isomer in B conflicts with the meta-directing effect. C shows the required meta isomer, whereas A and D place bromine on the alkyl chain.

**Step 4:** Conclusion The major product is the meta-brominated aryl ketone:

$$m\text{-}BrC_6H_4COCH_2CH_2CH_2CH_3.$$

Hence, the correct option is (C).

**Quick Tip to Solve:** For aromatic substitution, classify the group directly bonded to the ring: $-COR$ is meta-directing even though R is an alkyl group.

**73. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**

**Assertion (A) : Photochemical smog causes cracking of rubber.**

**Reason (R) : Presence of ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate in photochemical smog makes it oxidizing.**

**Choose the most appropriate answer from the options given below :** [4 marks]

(A) Both (A) and (R) are true but (R) is not the true explanation of (A)

(B) (A) is false but (R) is true.

(C) (A) is true but (R) is false.

(D) Both (A) and (R) are true and (R) is the true explanation of (A)

**Correct Answer:** (D) Both (A) and (R) are true and (R) is the true explanation of (A)

**Solution:**

**Concept:**

- Photochemical smog forms in sunlight from nitrogen oxides and hydrocarbons. It contains secondary pollutants such as ozone, peroxyacetyl nitrate, aldehydes and acrolein.
- It is called oxidizing smog because its reactive oxidants attack exposed materials and living tissues. Rubber contains carbon-carbon double bonds that are susceptible to ozone attack.

**Step 1:** Evaluate the assertion Rubber consists of long polymer chains containing unsaturated sites. Ozone and other oxidizing components of photochemical smog attack these sites, breaking or degrading the polymer chains. The material loses elasticity and develops cracks, especially under mechanical stress. Thus the assertion that photochemical smog causes cracking of rubber is true.

**Step 2:** Evaluate the reason The listed mixture includes ozone and peroxyacetyl nitrate, together with nitrogen oxides and organic pollutants such as acrolein and formaldehyde. These are characteristic constituents of photochemical smog, and the mixture has a strongly oxidizing character. The statement describes the collective character of the smog; it need not mean that every individual listed species is an equally strong oxidant. In this standard description, the reason is true.

**Step 3:** Connect the chemical property to the observed damage The oxidizing nature is precisely why the smog attacks unsaturated rubber and leads to deterioration and cracking. The reason therefore supplies the chemical explanation for the assertion, rather than merely listing an unrelated property. Option A incorrectly denies this explanatory connection. Options B and C incorrectly reject one of the two true statements, so neither fits the chemistry described.

**Step 4:** Conclusion Both the assertion and reason are true, and the reason correctly explains the assertion. Hence, the correct option is (D).

**Quick Tip to Solve:** Connect the oxidizing character of photochemical smog to ozone attack on the double bonds of rubber before choosing the assertion-reason code.

**74. The interaction energy of London forces between two particles is proportional to $r^x$, where r is the distance between the particles. The value of x is :** [4 marks]

(A) 3

(B) $-3$

(C) $-6$

(D) 6

**Correct Answer:** (C) $-6$

**Solution:**

**Concept:**

- London dispersion attraction arises from instantaneous dipoles inducing correlated dipoles in neighbouring particles. It occurs even between non-polar atoms or molecules.
- For two separated particles in the usual non-retarded molecular regime, the attractive interaction energy is $U(r) = -C_6/r^6$, where $C_6 > 0$ depends on their polarizabilities and excitation energies.

**Step 1:** Write the distance dependence of the energy The question asks for interaction energy, so use the inverse-sixth-power form for the London contribution. Expressing the denominator as a power gives

$$U(r) = -\frac{C_6}{r^6} = -C_6 r^{-6}.$$

The negative sign indicates that the interaction is attractive relative to infinitely separated particles; it does not alter the exponent of the distance.

**Step 2:** Compare with the required power The stated dependence is proportional to $r^x$. Comparing the factor $r^{-6}$ with $r^x$ gives $x = -6$. As a physical check, doubling the separation makes the magnitude of the attractive energy $1/2^6 = 1/64$ of its former value. The rapid decrease is characteristic of London dispersion attraction.

**Step 3:** Eliminate the other exponents The positive exponents 3 and 6 would make the magnitude increase with separation, contrary to a short-range attraction. The exponent $-3$ is associated with a different dipole interaction dependence and does not describe the London dispersion energy between two particles. Also distinguish energy from force: differentiating this energy introduces an inverse-seventh-power force, which is not what the question asks.

**Step 4:** Conclusion The interaction energy varies as $r^{-6}$, so $x = -6$. Hence, the correct option is (C).

**Quick Tip to Solve:** Memorise the London energy as $-C_6/r^6$, and check whether a question asks for energy or its distance derivative, the force.

**75. The number of non-ionisable hydrogen atoms present in the final product obtained from the hydrolysis of $PCl_5$ is :** [4 marks]

(A) 0

(B) 2

(C) 1

(D) 3

**Correct Answer:** (A) 0

**Solution:**

**Concept:**

- Complete hydrolysis of $PCl_5$ gives orthophosphoric acid, $H_3PO_4$, and hydrogen chloride. The intermediate oxychloride is $POCl_3$.
- The ionisability of hydrogen in a phosphorus oxyacid depends on bonding: hydrogen bonded through oxygen may ionise; a hydrogen bonded directly to phosphorus does not ionise as an acidic proton.

**Step 1:** Follow hydrolysis to the final phosphorus product Initial hydrolysis replaces chlorine by oxygen-containing groups and forms phosphorus oxychloride. Further hydrolysis gives orthophosphoric acid:

$$PCl_5 + H_2O \longrightarrow POCl_3 + 2\,HCl,$$

$$POCl_3 + 3\,H_2O \longrightarrow H_3PO_4 + 3\,HCl.$$

Hence the overall complete hydrolysis is $PCl_5 + 4\,H_2O \longrightarrow H_3PO_4 + 5\,HCl$. The word final excludes stopping at the oxychloride stage.

**Step 2:** Inspect the hydrogen bonding in the acid Orthophosphoric acid has the structure $O{=}P(OH)_3$. All three hydrogen atoms belong to hydroxyl groups, so each is bonded to oxygen. There is no $P{-}H$ bond. Its acidic dissociations occur stepwise and become progressively less favourable, but this does not make any of those hydroxyl hydrogens structurally non-ionisable.

**Step 3:** Count and compare the options The number of non-ionisable hydrogens in the phosphorus-containing final product is consequently zero. The alternatives 1 and 2 would correspond to counting direct phosphorus-hydrogen bonds found in other oxyacids, such as phosphorous or hypophosphorous acid, rather than in phosphoric acid. The value 3 incorrectly treats all hydroxyl hydrogens as

non-ionisable. The hydrogen chloride coproduct also supplies no non-ionisable hydrogen.

**Step 4:** Conclusion There are $0$ non-ionisable hydrogen atoms in the final product $H_3PO_4$. Hence, the correct option is (A).

**Quick Tip to Solve:** Draw the oxyacid as $O{=}P(OH)_3$ and count $P{-}H$ bonds, rather than subtracting from the formula mechanically.

**76. The sol given below with negatively charged colloidal particles is :** [4 marks]

(A) $FeCl_3$ added to hot water

(B) KI added to $AgNO_3$ solution

(C) $AgNO_3$ added to KI solution

(D) $Al_2O_3.xH_2O$ in water

**Correct Answer:** (C) $AgNO_3$ added to KI solution

**Solution:**

**Concept:**

- A precipitated sol preferentially adsorbs the lattice ion present in excess in its surrounding solution. That adsorbed ion determines the sign of the colloidal particle charge.
- For silver iodide, excess $Ag^+$ gives positively charged particles, whereas excess $I^-$ gives negatively charged particles. The order of mixing identifies the ion initially present in excess.

**Step 1:** Analyse silver nitrate added to potassium iodide When $AgNO_3$ is added to a solution of KI, silver iodide forms in an iodide-rich medium:

$$Ag^+ + I^- \longrightarrow AgI.$$

The newly formed $AgI$ particles preferentially adsorb the excess iodide ions from the surrounding solution. Their surface is therefore represented by $AgI \cdot I^-$, and the sol particles carry a negative charge. This corresponds to option C.

**Step 2:** Reverse the addition and check the sign When KI is instead added to $AgNO_3$ solution, the medium initially contains excess silver ions. The silver iodide particles adsorb $Ag^+$ and become positively charged. Thus option B forms the same sparingly soluble compound but the opposite surface charge; merely identifying $AgI$ is not enough to decide the answer.

**Step 3:** Check the remaining sols Adding $FeCl_3$ to hot water produces the familiar positively charged hydrated ferric oxide or ferric hydroxide sol. Hydrated aluminium oxide in water is also a standard positively charged sol. Neither A nor D supplies the negatively charged colloidal particles requested. The distinction is the adsorbed surface species, rather than the electrical neutrality of the starting salts or the precipitate formula.

**Step 4:** Conclusion $AgNO_3$ added to KI solution produces negatively charged silver iodide sol. Hence, the correct option is (C).

**Quick Tip to Solve:** Look at the solution receiving the added reagent: its excess lattice ion usually fixes the sign of the freshly formed sol.

77. **Given below are two statements :**

**Statement I : Sphalerite is a sulphide or of zinc and copper glance is a sulphide ore of copper.**

**Statement II : It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants' in a forth flotation method.**

**Choose the most appropriate answer from the options given below :** [4 marks]

(A) Statement I is true but Statement II is false.

(B) Both Statement I and Statement II are true.

(C) Statement I is false but Statement II is true.

(D) Both Statement I and Statement II are false.

**Correct Answer:** (B) Both Statement I and Statement II are true.

**Solution:**

**Concept:**

- Sphalerite or zinc blende is $ZnS$, a sulphide ore of zinc. Copper glance or chalcocite is $Cu_2S$, a sulphide ore of copper.
- Froth flotation exploits differences in surface wettability: oil preferentially wets suitable sulphide particles, while water wets the gangue. Selectivity can be improved using depressants and adjusted flotation conditions.

**Step 1:** Test the mineral identities in Statement I The mineral sphalerite contains zinc sulphide, $ZnS$, and is an important zinc ore. Copper glance contains copper(I) sulphide, $Cu_2S$, and is an ore of copper. Both substances are sulphides, as the first statement asserts. Its intended mineral classifications are therefore correct; Statement I is true.

**Step 2:** Test the separation principle in Statement II In a mixture containing different sulphide ores, their flotation behaviour can be made different by adjusting the oil-to-water proportion or adding a suitable depressant. The depressant prevents one sulphide from entering the froth while allowing another to be carried up with the air bubbles. This is a selective concentration step, not a requirement that all sulphide minerals float together under every condition.

**Step 3:** Use a standard example and choose the code For example, in separating $ZnS$ from $PbS$, sodium cyanide can depress zinc sulphide by forming a soluble zinc complex; lead sulphide can then be collected in the froth. This illustrates the method claimed in Statement II, so it is true. Options A and C incorrectly make one of the correct statements false, and D incorrectly rejects both. The option accepting both statements is B.

**Step 4:** Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

**Quick Tip to Solve:** Keep ore identities and flotation selectivity as separate checks: a depressant stops one sulphide from joining the froth.

**78. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).**

**Assertion (A) : Heavy water is used for the study of reaction mechanism.**

**Reason (R) : The rate of reaction for the cleavage of O - H bond is slower than that of O - D bond.**

**Choose the most appropriate answer from the options given below :** [4 marks]

(A) Both (A) and (R) are true but (R) is not the true explanation of (A).

(B) Both (A) and (R) are true and (R) is the true explanation of (A).

(C) (A) is false but (R) is true.

(D) (A) is true but (R) is false.

**Correct Answer:** (D) (A) is true but (R) is false.

**Solution:**

**Concept:**

- Heavy water, $D_2O$, contains deuterium, the heavier isotope of hydrogen. Isotopic substitution changes molecular vibration and can change a reaction rate without changing the basic chemical identity of the reactants.
- The primary kinetic isotope effect compares cleavage of a bond to hydrogen with cleavage of the corresponding bond to deuterium. For otherwise comparable reactions, cleavage of an $O-H$ bond is generally faster than cleavage of an $O-D$ bond.

**Step 1:** Judge the assertion Heavy water is used to investigate reaction mechanisms by replacing exchangeable hydrogen with deuterium and measuring the rate change. A substantial isotope effect gives evidence that cleavage of a bond involving that hydrogen contributes to the rate-controlling process. Consequently, the assertion that heavy water is used for studying reaction mechanisms is true.

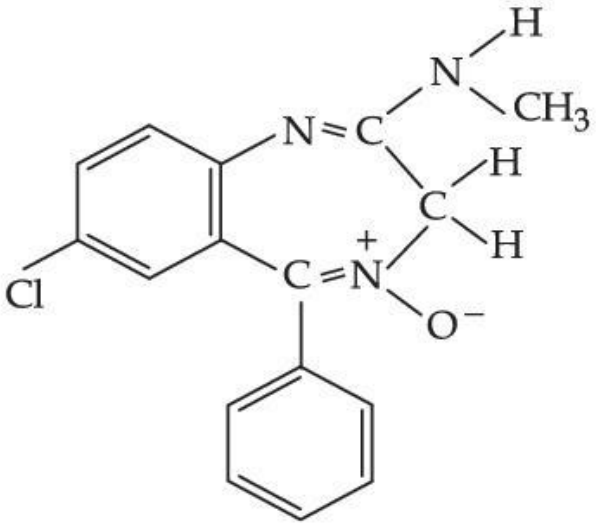
**Step 2:** Judge the reason Deuterium has the larger mass, so an $O-D$ stretching vibration has a lower frequency and a lower zero-point energy than the corresponding $O-H$ vibration. The deuterated bond therefore generally requires a greater effective activation energy for cleavage. The usual comparison is $k_H > k_D$, rather than the reverse stated in the reason. Thus the reason is false.

**Step 3:** Match the truth values The isotope effect supports the use of heavy water in mechanistic studies, but the printed reason reverses the direction of the effect. Options (A) and (B) incorrectly call both statements true, while option (C) incorrectly calls the assertion false. The required combination is a true assertion and a false reason.

**Step 4:** Conclusion The assertion is true, but the reason is false. Hence, the correct option is (D).

**Quick Tip to Solve:** For the usual primary isotope effect, remember that the heavier isotope forms the bond that is slower to cleave: compare $k_H$ with $k_D$ before judging the reason.

**79. The class of drug to which chlordiazepoxide with above structure belongs is :** [4 marks]




Chlordiazepoxide

(A) Antacid

(B) Analgesic

(C) Tranquilizer

(D) Antibiotic

**Correct Answer:** (C) Tranquilizer

**Solution:**

**Concept:**

- Drugs are classified by their therapeutic action. Tranquilizers act on the central nervous system and are used to reduce anxiety, tension and related mental disturbances.
- Chlordiazepoxide belongs to the benzodiazepine family. In the standard classification of drugs, it is an example of a tranquilizer rather than an antacid, analgesic or antibiotic.

**Step 1:** Identify the named drug The displayed structure is explicitly identified as chlordiazepoxide. Its therapeutic classification follows the pharmacological action of the compound; the presence of nitrogen, chlorine or aromatic rings alone is not sufficient to place an unfamiliar compound into a drug class. Chlordiazepoxide is a benzodiazepine with an anxiety-reducing effect and is listed among tranquilizers.

**Step 2:** Compare the alternatives An antacid reduces excess stomach acidity, so option (A) does not describe the relevant therapeutic use. An analgesic relieves pain and is a different functional class, eliminating option (B). Antibiotics inhibit or destroy susceptible microorganisms and are used against microbial infections; this is not the action being classified here, eliminating option (D). A tranquilizer acts on the nervous system to relieve anxiety and tension, which matches chlordiazepoxide.

**Step 3:** Select the therapeutic class The characteristic classification needed in this question is therefore the tranquilizer class. This identifies the drug by its established therapeutic action and does not require a chemical reaction or a structural drawing in the answer. The name of the drug and its benzodiazepine family lead to the same classification.

**Step 4:** Conclusion Chlordiazepoxide is a tranquilizer. Hence, the correct option is (C).

**Quick Tip to Solve:** Associate chlordiazepoxide and diazepam with tranquilizers; classify by therapeutic action rather than by a single atom or functional group in the structure.

**80. Chalcogen group elements are :** [4 marks]

(A) Se, Tb and Pu.

(B) Se, Te and Po.

(C) S, Te and Pm.

(D) O, Ti and Po.

**Correct Answer:** (B) Se, Te and Po.

**Solution:**

**Concept:**

- The chalcogens are the elements of Group 16 of the periodic table. The familiar members are oxygen, sulphur, selenium, tellurium and polonium.
- For the main-group chalcogens, the characteristic outer-shell configuration is $ns^2np^4$. A proposed list is correct only when every element in that list belongs to Group 16.

**Step 1:** Write the Group 16 sequence The relevant sequence down the group is $\mathrm{O, S, Se, Te, Po}$. Oxygen and sulphur are the most familiar members, while selenium, tellurium and polonium continue the same vertical group. The question asks for a list composed entirely of members, so one incorrect symbol is enough to reject an option.

**Step 2:** Test each proposed list In option (A), selenium is a chalcogen, but Tb is terbium, a lanthanoid, and Pu is plutonium, an actinoid. In option (B), Se, Te and Po are selenium, tellurium and polonium, respectively, and all three belong to Group 16. In option (C), S and Te belong to the group, but Pm is promethium, a lanthanoid. In option (D), O and Po are chalcogens, but Ti is titanium, a transition element in Group 4.

**Step 3:** Choose the complete match Only the second list contains three chalcogen symbols without including an element from another group or block. In particular, the visually similar symbols Te and Ti must be distinguished: tellurium belongs to Group 16, whereas titanium does not. The same check rejects the lists containing rare-earth or actinoid elements.

**Step 4:** Conclusion The chalcogen group elements in the options are Se, Te and Po. Hence, the correct option is (B).

**Quick Tip to Solve:** Recall the sequence O, S, Se, Te, Po, and check every symbol in a proposed list; Te is tellurium, whereas Ti is titanium.

## Chemistry – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**81. In the sulphur estimation, 0.471 g of an organic compound gave 1.44 g of barium sulphate. The percentage of sulphur in the compound is _____%. (Nearest integer)**
**(Atomic Mass of Ba = 137 u)** [4 marks]

**Solution:**

**Concept:**

- In sulphur estimation, the sulphur in the organic sample is converted quantitatively into sulphate and precipitated as $BaSO_4$. Each mole of $BaSO_4$ contains one mole of sulphur atoms.
- The sulphur mass is obtained from the precipitate mass using $m_S = m_{BaSO_4}\, M_S/M_{BaSO_4}$, and its mass percentage is $100 m_S/m_{sample}$.

**Step 1:** Find the molar mass of the precipitate Using the given atomic mass of barium and the usual atomic masses of sulphur and oxygen, the molar mass is

$$M_{BaSO_4} = 137 + 32 + 4(16) = 233\,\text{g mol}^{-1}.$$

Thus 233 g of barium sulphate contains 32 g of sulphur. The full precipitate mass cannot be treated as sulphur mass because it also includes barium and oxygen.

**Step 2:** Calculate the mass of sulphur The 1.44 g precipitate therefore contains

$$m_S = 1.44\left(\frac{32}{233}\right) = 0.19777\,\text{g}.$$

This is the sulphur that was present in the 0.471 g organic sample. The one-to-one relationship between sulphur atoms and sulphate formula units supplies the stoichiometric conversion.

**Step 3:** Express the result as a percentage Divide by the original sample mass, rather than by the precipitate mass, and multiply by 100:

$$\%S = \frac{0.19777}{0.471} \times 100 = 41.988\%.$$

Rounding only at the final step gives 42 as the nearest integer. The intermediate mass also gives a useful check: about 0.198 g out of a 0.471 g sample is a little over two-fifths of the sample.

**Step 4:** Conclusion The sulphur content is approximately $41.988\%$, so the required nearest-integer percentage is $\boxed{42\%}$.

**Quick Tip to Solve:** For a barium sulphate precipitate, multiply its mass by $32/233$ first; then divide by the original organic sample mass to obtain the sulphur percentage.

**82. The equilibrium constant $K_c$ at 298 K for the reaction A + B $\rightleftharpoons$ C + D is 100. Starting with an equimolar solution with concentrations of A, B, C and D all equal to 1M, the equilibrium concentration of D is _____ $\times 10^{-2}$ M. (Nearest integer)** [4 marks]

**Solution:**

**Concept:**

- For $\mathrm{A} + \mathrm{B} \rightleftharpoons \mathrm{C} + \mathrm{D}$, the equilibrium constant is $K_c = [\mathrm{C}][\mathrm{D}]/([\mathrm{A}][\mathrm{B}])$. Each concentration is evaluated at equilibrium.
- The reaction quotient has the same expression for the initial mixture. If $Q_c < K_c$, the reaction proceeds forward; equal stoichiometric coefficients make the concentration changes equal in magnitude.

**Step 1:** Determine the direction of change Initially all four concentrations are 1 M, giving

$$Q_c = \frac{1 \times 1}{1 \times 1} = 1.$$

Since $1 < 100$, the mixture must form more products to reach equilibrium. Let $x$ M of each reactant be consumed. The equilibrium concentrations are then $[\mathrm{A}] = [\mathrm{B}] = (1 - x)$ M and $[\mathrm{C}] = [\mathrm{D}] = (1 + x)$ M. Physical concentrations require $0 < x < 1$.

**Step 2:** Solve the equilibrium equation Substitution into the constant gives

$$100 = \frac{(1 + x)^2}{(1 - x)^2}.$$

Both concentrations are positive, so the physically appropriate square root is

$$10 = \frac{1 + x}{1 - x}.$$

Hence $10 - 10x = 1 + x$, which gives $11x = 9$ and $x = 9/11$. A negative square-root ratio would contradict the positivity of the concentrations.

**Step 3:** Convert the concentration to the requested form The equilibrium concentration of D is

$$[\mathrm{D}] = 1 + \frac{9}{11} = \frac{20}{11} = 1.81818\ldots \text{ M}.$$

Writing this as $n \times 10^{-2}$ M gives $n = 181.81818\ldots$. Its nearest integer is 182. As a check, the product-to-reactant concentration ratio is 10, whose square reproduces the given equilibrium constant 100.

**Step 4:** Conclusion $[\mathrm{D}]_{\mathrm{eq}} = 1.81818\ldots$ M, so the required entry is $\boxed{182}$ in units of $10^{-2}$ M.

**Quick Tip to Solve:** With equal initial concentrations and 1:1:1:1 stoichiometry, use $\sqrt{K_c} = (1 + x)/(1 - x)$; convert to the stated power of ten only after solving.

**83. For water $\Delta_{vap}$ H = 41 kJ mol$^{-1}$ at 373 K and 1 bar pressure. Assuming that water vapour is an ideal gas that occupies a much larger volume than liquid water, the internal energy change during evaporation of water is _____ kJ mol$^{-1}$**

**[Use : R = 8.3 J mol$^{-1}$ K$^{-1}$]** [4 marks]

**Solution:**

**Concept:**

- Enthalpy is defined by $H = U + pV$. At constant pressure, the molar changes obey $\Delta H = \Delta U + p\Delta V$.
- For one mole of ideal water vapour, $pV_g = RT$. When the liquid volume is negligible compared with the vapour volume, $p\Delta V \simeq RT$, so $\Delta U_{vap} \simeq \Delta H_{vap} - RT$.

**Step 1:** Identify the expansion contribution Evaporation takes one mole of liquid water to one mole of vapour at 373 K. Because the vapour volume is much larger than the liquid volume, the increase in $pV$ is approximately the ideal-gas value for one mole. Thus

$$p\Delta V = p(V_g - V_l) \simeq pV_g = RT.$$

The enthalpy supplied includes this expansion contribution as well as the increase in internal energy.

**Step 2:** Calculate the correction in consistent units Use the stated value of the gas constant and the evaporation temperature:

$$RT = 8.3 \times 373 = 3095.9\,\text{J mol}^{-1} = 3.0959\,\text{kJ mol}^{-1}.$$

Converting joules to kilojoules before subtraction is essential because the quoted enthalpy of vaporisation is 41 kJ per mole. The pressure need not be inserted separately after applying the ideal-gas equation.

**Step 3:** Obtain the internal energy change Rearranging the enthalpy relation gives

$$\Delta U_{vap} = 41 - 3.0959 = 37.9041\,\text{kJ mol}^{-1}.$$

The positive sign is consistent with energy being absorbed during vaporisation. The internal energy increase is smaller than the enthalpy increase, since part of the energy is associated with the increase in $pV$. At integer precision, the result is 38 kJ per mole.

**Step 4:** Conclusion The internal energy change is $\boxed{37.9041\,\text{kJ mol}^{-1} \approx 38\,\text{kJ mol}^{-1}}$; the integer numerical entry is 38.

**Quick Tip to Solve:** For one mole of liquid forming an ideal vapour with negligible initial volume, subtract $RT$ from $\Delta H$, after expressing both terms in the same energy units.

**84. A metal surface is exposed to 500 nm radiation. The threshold frequency of the metal for photoelectric current is 4.3 $\times$ $10^{14}$ Hz. The velocity of ejected electron is _____ $\times$ $10^5$ $ms^{-1}$ (Nearest integer)**

**[Use : h = 6.63 $\times$ $10^{-34}$ Js, $m_e$ = 9.0 $\times$ $10^{-31}$ kg]** [4 marks]

**Solution:**

**Concept:**

- Einstein's photoelectric equation is $K_{\max} = h(\nu - \nu_0)$, where $\nu = c/\lambda$ is the incident frequency and $\nu_0$ is the threshold frequency.
- The maximum kinetic energy is also $K_{\max} = \frac{1}{2}m_e v^2$. Photoemission occurs only if the photon frequency exceeds the threshold; the threshold energy is the work function $h\nu_0$.

**Step 1:** Calculate the incident photon frequency Convert the wavelength into metres: $500\,\text{nm} = 5.00 \times 10^{-7}\,\text{m}$. Taking the speed of light as $3.00 \times 10^8\,\text{m s}^{-1}$,

$$\nu = \frac{3.00 \times 10^8}{5.00 \times 10^{-7}} = 6.00 \times 10^{14}\,\text{Hz}.$$

This exceeds $\nu_0 = 4.3 \times 10^{14}\,\text{Hz}$, so electrons can be emitted. Only the frequency excess supplies kinetic energy.

**Step 2:** Find the maximum kinetic energy Subtract the threshold frequency before multiplying by Planck's constant:

$$K_{\max} = 6.63 \times 10^{-34}(6.00 - 4.30) \times 10^{14} = 1.1271 \times 10^{-19}\,\text{J}.$$

Using the full photon energy here would overestimate the speed because the work function must first be supplied to liberate the electron.

**Step 3:** Calculate and round the speed Using the stated electron mass,

$$v = \sqrt{\frac{2K_{\max}}{m_e}} = \sqrt{\frac{2(1.1271 \times 10^{-19})}{9.0 \times 10^{-31}}} = \sqrt{2.50467 \times 10^{11}} = 5.0047 \times 10^5\,\text{m s}^{-1}.$$

The coefficient of $10^5\,\text{m s}^{-1}$ is therefore approximately 5.0047. Rounding that coefficient, rather than the speed in SI units, gives the numerical entry 5.

**Step 4:** Conclusion The emitted electron speed is approximately $5.0047 \times 10^5\,\text{m s}^{-1}$, so the required nearest-integer entry is $\boxed{5}$.

**Quick Tip to Solve:** First subtract the threshold energy from the photon energy; then use $v = \sqrt{2K/m_e}$ and round only the coefficient in the units requested.

**85. For the galvanic cell,**

**$Zn(s) + Cu^{2+}$ (0.02 M) $\rightarrow$ $Zn^{2+}$ (0.04 M) + Cu(s),**

**$E_{cell}$ = _____ $\times 10^{-2}$ V. (Nearest integer)**

**[Use : $E^0_{Cu/Cu^{2+}} = -0.34$ V, $E^0_{Zn/Zn^{2+}} = +0.76$ V, $\frac{2.303RT}{F} = 0.059\,V$]** [4 marks]

**Solution:**

**Concept:**

- The stated potentials refer to oxidation of the metals. The standard cell emf is the oxidation potential of the anode minus the oxidation potential of the cathode.
- For a reaction transferring $n$ electrons, $E = E^\circ - (0.059/n)\log_{10} Q$, where $Q$ is the reaction quotient; pure solids have unit activity.

**Step 1:** Identify the electrodes and standard emf Zinc is oxidised at the anode: $\mathrm{Zn} \longrightarrow \mathrm{Zn}^{2+} + 2\,\mathrm{e}^-$. Copper ions are reduced at the cathode: $\mathrm{Cu}^{2+} + 2\,\mathrm{e}^- \longrightarrow \mathrm{Cu}$. Thus two electrons are transferred. The given copper oxidation potential is $-0.34\,\mathrm{V}$, so its reduction potential is $+0.34\,\mathrm{V}$. Hence

$$E^\circ_{\text{cell}} = 0.76 + 0.34 = 1.10\,\mathrm{V}.$$

**Step 2:** Calculate the concentration correction Only the dissolved ions enter the quotient for the reaction as written. Therefore

$$Q = \frac{[\mathrm{Zn}^{2+}]}{[\mathrm{Cu}^{2+}]} = \frac{0.04}{0.02} = 2.$$

Using $n = 2$ and $\log_{10} 2 = 0.3010$,

$$\begin{aligned} E_{\text{cell}} &= 1.10 - \frac{0.059}{2}(0.3010) \\ &= 1.09112\,\mathrm{V}. \end{aligned}$$

The correction is negative because the product-ion concentration exceeds the reactant-ion concentration. Treating the two printed values as reduction potentials would reverse their stated meaning.

**Step 3:** Express the emf in the requested scale The blank multiplies $10^{-2}\,\mathrm{V}$, so convert the voltage before rounding:

$$\frac{1.09112}{10^{-2}} = 109.112.$$

Rounding this coefficient to the nearest integer gives 109; rounding the voltage itself to an integer would answer a different question.

**Step 4:** Conclusion $E_{\text{cell}} \simeq 109 \times 10^{-2}\,\mathrm{V}$. The required nearest-integer entry is $\boxed{109}$.

**Quick Tip to Solve:** Check whether each listed electrode potential is for oxidation or reduction, then form the reaction quotient in the direction of the cell reaction.

**86. 100 mL of $Na_3PO_4$ solution contains 3.45 g of sodium. The molarity of the solution is _____ $\times 10^{-2}$ mol $L^{-1}$. (Nearest integer)**

**[Atomic Masses - Na : 23.0 u, O : 16.0 u, P : 31.0 u]** [4 marks]

**Solution:**

**Concept:**

- Molarity is moles of dissolved solute divided by the solution volume in litres: $M = n_{\text{solute}}/V$.
- Each formula unit of $\mathrm{Na_3PO_4}$ contains three sodium atoms; therefore one mole of this salt contains three moles of sodium.

**Step 1:** Convert the mass of sodium into moles The quoted 3.45 g is the mass of sodium contained in the solution, rather than the mass of sodium phosphate. Use the given atomic mass of sodium:

$$n(\mathrm{Na}) = \frac{3.45\,\mathrm{g}}{23.0\,\mathrm{g\,mol^{-1}}} = 0.150\,\mathrm{mol}.$$

The oxygen and phosphorus atomic masses are not needed because the elemental sodium mass directly determines the number of salt formula units.

**Step 2:** Find the amount of sodium phosphate The stoichiometric relation between sodium atoms and salt formula units is 3:1. Consequently,

$$n(\mathrm{Na_3PO_4}) = \frac{n(\mathrm{Na})}{3} = \frac{0.150}{3} = 0.0500\,\mathrm{mol}.$$

Dissociation in water does not change the formal molarity of the dissolved salt. The sodium-ion concentration would be three times the salt concentration and is not what is asked.

**Step 3:** Calculate molarity and the requested coefficient The solution volume is $100\,\mathrm{mL} = 0.100\,\mathrm{L}$. Thus

$$M = \frac{0.0500}{0.100} = 0.500\,\mathrm{mol\,L^{-1}}.$$

Writing this in the specified form gives

$$0.500\,\mathrm{mol\,L^{-1}} = 50.0 \times 10^{-2}\,\mathrm{mol\,L^{-1}}.$$

Therefore the coefficient rounds to 50. Using 0.150 mol directly would incorrectly report the sodium concentration instead of the sodium phosphate molarity.

**Step 4:** Conclusion The sodium phosphate molarity is $0.500\,\mathrm{mol\,L^{-1}} = 50 \times 10^{-2}\,\mathrm{mol\,L^{-1}}$. The entry is $\boxed{50}$.

**Quick Tip to Solve:** When an elemental mass is given, first find moles of that element and divide by its subscript in the solute formula before using the solution volume.

**87. The overall stability constant of the complex ion $[Cu(NH_3)_4]^{2+}$ is $2.1 \times 10^{13}$. The overall dissociations constant is $y \times 10^{-14}$. Then y is _____. (Nearest integer)** [4 marks]

**Solution:**

**Concept:**

- The overall stability constant $\beta_4$ describes formation of $[Cu(NH_3)_4]^{2+}$ from $Cu^{2+}$ and four ammonia molecules.
- Reversing an equilibrium reciprocates its equilibrium constant. Thus the overall dissociation constant is $K_d = 1/\beta_4$, rather than the reciprocal of an individual stepwise formation constant.

**Step 1:** Write the overall formation and reverse equilibria The formation equilibrium is

$$Cu^{2+} + 4\,NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+}.$$

Its constant is

$$\beta_4 = \frac{[[Cu(NH_3)_4]^{2+}]}{[Cu^{2+}][NH_3]^4}.$$

The dissociation process has the complex as reactant and the free metal ion and ligands as products, so the numerator and denominator are interchanged.

**Step 2:** Calculate the reciprocal of the given constant Using the supplied overall stability constant,

$$\begin{aligned} K_d &= \frac{1}{2.1 \times 10^{13}} \\ &= \frac{1}{2.1} \times 10^{-13} \\ &= 4.7619 \times 10^{-14}. \end{aligned}$$

The small dissociation constant is consistent with the very large formation constant: the complex is strongly favoured relative to fully separated metal and ligand under the corresponding standard-state convention.

**Step 3:** Extract and round the coefficient The required representation is $K_d = y \times 10^{-14}$, so it is the coefficient 4.7619 that must be rounded:

$$y = 4.7619\ldots \longrightarrow 5.$$

Do not report zero by rounding the very small constant itself. Nor should the ligand subscript 4 be used as an extra exponent on the already overall constant; its effect is included in $\beta_4$.

**Step 4:** Conclusion $K_d = 4.7619 \times 10^{-14}$, so the requested nearest-integer coefficient is $\boxed{y = 5}$ (dimensionless equilibrium-constant convention).

**Quick Tip to Solve:** For the reverse of a complex-formation equilibrium, take one reciprocal, then adjust the power of ten before rounding its coefficient.

**88. 83 g of ethylene glycol dissolved in 625 g of water. The freezing point of the solution is _____ K. (Nearest integer)**

**[Use : Molal Freezing point depression constant of water = 1.86 K kg mol$^{-1}$]**

**Freezing Point of water = 273 K**

**Atomic masses : C : 12.0 u, O : 16.0 u, H : 1.0 u]** [4 marks]

**Solution:**

**Concept:**

- For a dilute solution, freezing-point depression obeys $\Delta T_f = iK_f m$, where $m$ is molality in moles of solute per kilogram of solvent.
- Ethylene glycol, $C_2H_6O_2$, is a molecular nonelectrolyte, so $i = 1$. The solution freezing temperature is $T_f = T_f^\circ - \Delta T_f$.

**Step 1:** Calculate the moles of ethylene glycol From the stated atomic masses, its molar mass is

$$M(C_2H_6O_2) = 2(12) + 6(1) + 2(16) = 62\,\text{g mol}^{-1}.$$

Hence the 83 g sample contains

$$n = \frac{83}{62} = 1.33871\,\text{mol}.$$

Ethylene glycol neither ionises into several particles nor changes its molecular mass on dissolving in this calculation.

**Step 2:** Find the molality and temperature depression The mass of solvent is $625\,\text{g} = 0.625\,\text{kg}$. Molality uses this water mass, not the combined mass of water and solute:

$$m = \frac{83/62}{0.625} = 2.14194\,\text{mol kg}^{-1}.$$

Therefore

$$\Delta T_f = 1(1.86)(2.14194) = 3.984\,\text{K}.$$

The factors of kilograms and moles cancel with the units of $K_f$, leaving a temperature difference in kelvin.

**Step 3:** Subtract the depression and round the temperature Use the stipulated freezing point of pure water, 273 K, consistently rather than replacing it by a more precise external value:

$$T_f = 273 - 3.984 = 269.016\,\text{K}.$$

The decrease has the expected sign for a nonvolatile solute dissolved in water. Round the final temperature, retaining the intermediate precision; premature rounding of molality is unnecessary.

**Step 4:** Conclusion The solution freezes at $269.016\,\text{K}$, or $\boxed{269\,\text{K}}$ to the nearest integer.

**Quick Tip to Solve:** For freezing-point problems, divide solute moles by kilograms of solvent, then subtract the calculated depression from the given pure-solvent freezing point.

**89. The reaction rate for the reaction**

**$[PtCl_4]^{2-} + H_2O \rightleftharpoons [Pt(H_2O)Cl_3]^- + Cl^-$**

**was measured as a function of concentrations of different species. It was observed that $\frac{-d[[PtCl_4]^{2-}]}{dt} = 4.8 \times 10^{-5}\left[[PtCl_4]^{2-}\right] - 2.4 \times 10^{-3}\left[[Pt(H_2O)Cl_3]^-\right][Cl^-]$.**

**where square brackets are used to denote molar concentrations. The equilibrium constant $K_c$ = _____ . (Nearest integer)** [4 marks]

**Solution:**

**Concept:**

- At equilibrium the forward and reverse reaction rates are equal, so the net disappearance rate of a reactant is zero.
- For this aqueous substitution, water is the solvent and its activity is incorporated into the constants. Thus $K_c = [[\mathrm{Pt(H_2O)Cl_3}]^-][\mathrm{Cl}^-]/[[\mathrm{PtCl_4}]^{2-}]$.

**Step 1:** Identify the two contributions to the net rate Let $A$ denote $[\mathrm{PtCl_4}]^{2-}$, $B$ denote $[\mathrm{Pt(H_2O)Cl_3}]^-$, and $C$ denote $\mathrm{Cl}^-$. The printed law becomes

$$-\frac{d[A]}{dt} = 4.8 \times 10^{-5}[A] - 2.4 \times 10^{-3}[B][C].$$

The positive term represents forward consumption of A. The negative term represents its regeneration by the reverse reaction; it must not be ignored when imposing equilibrium.

**Step 2:** Set the net rate to zero and isolate the ratio At equilibrium the concentrations remain constant even though both microscopic processes continue. Hence

$$4.8 \times 10^{-5}[A]_{\mathrm{eq}} = 2.4 \times 10^{-3}[B]_{\mathrm{eq}}[C]_{\mathrm{eq}}.$$

Dividing by the reverse coefficient and by $[A]_{\mathrm{eq}}$ gives

$$K_c = \frac{[B]_{\mathrm{eq}}[C]_{\mathrm{eq}}}{[A]_{\mathrm{eq}}} = \frac{4.8 \times 10^{-5}}{2.4 \times 10^{-3}} = 2.0 \times 10^{-2}.$$

**Step 3:** Apply the required numerical rounding The unrounded equilibrium constant is 0.020. Since the question asks for its nearest integer, without a multiplying power of ten outside the blank,

$$0.020 \longrightarrow 0.$$

This rounding does not mean that the true equilibrium constant vanishes or that products are completely absent. In the concentration-based dimensional convention the displayed ratio has units of $\mathrm{mol\,L^{-1}}$; standard-state-normalised thermodynamic constants are dimensionless.

**Step 4:** Conclusion The unrounded value is $K_c = 0.020$ ($0.020\,\mathrm{mol\,L^{-1}}$ in the concentration-unit convention). The requested nearest-integer entry is $\boxed{0}$.

**Quick Tip to Solve:** For a reversible net rate law, set the entire expression to zero; identify the equilibrium quotient before dividing the forward coefficient by the reverse one.

**90. A chloro compound "A".**

**(i) forms aldehydes on ozonolysis followed by the hydrolysis.**

**(ii) when vaporized completely 1.53 g of A, gives 448 mL of vapour at STP.**

**The number of carbon atoms in a molecule of compound A is _____.** [4 marks]

**Solution:**

**Concept:**

- At STP, one mole of an ideal gas occupies $22.4\,\mathrm{L}$. The molar mass follows from $M = mV_m/V$.
- Ozonolysis cleaves a carbon-carbon double bond into carbonyl compounds. Formation of aldehydes indicates alkene carbon atoms carrying hydrogen; replacing one H of an acyclic monoalkene by Cl gives $\mathrm{C}_n\mathrm{H}_{2n-1}\mathrm{Cl}$.

**Step 1:** Calculate the molecular mass from the vapour volume The vapour volume is $448\,\mathrm{mL} = 0.448\,\mathrm{L}$, so

$$n(A) = \frac{0.448}{22.4} = 0.0200\,\mathrm{mol}.$$

Thus

$$M(A) = \frac{1.53}{0.0200} = 76.5\,\mathrm{g\,mol^{-1}}.$$

Using 448 directly with a molar volume in litres would introduce a factor of one thousand. The molar mass is a molecular quantity and is not an empirical-formula mass.

**Step 2:** Use the chlorine content and alkene formula Use conventional atomic masses C = 12, H = 1 and Cl = 35.5. The compound contains chlorine. Two chlorine atoms would already contribute 71, leaving only 5.5 for carbon and hydrogen, which cannot accommodate even one carbon atom. Thus it has one chlorine atom. Ozonolysis identifies a carbon-carbon double bond. For the compatible chloroalkene formula,

$$M = 12n + (2n - 1) + 35.5 = 14n + 34.5.$$

Equating this to the measured molar mass gives

$$14n + 34.5 = 76.5, \qquad n = 3.$$

**Step 3:** Check the result against the chemical clue The resulting formula is $\mathrm{C_3H_5Cl}$, whose molar mass is $36 + 5 + 35.5 = 76.5$. It has one degree of unsaturation, consistent with an alkene. For example, $\mathrm{CH_2{=}CHCH_2Cl}$ yields HCHO and $\mathrm{ClCH_2CHO}$ after cleavage and workup, both aldehydes. This confirms compatibility; a unique positional isomer is not required to count the carbon atoms.

**Step 4:** Conclusion The molecular formula is $\mathrm{C_3H_5Cl}$, and the number of carbon atoms per molecule is $\boxed{3}$, a dimensionless count.

**Quick Tip to Solve:** Calculate the molar mass from the STP volume first, then use the unsaturation clue and chlorine mass to identify the molecular formula.