JEE Main 2021 Paper 1 (B.E./B.Tech) Aug 27 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 4 (Aug-Sep), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 August 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is : [4 marks]

(A) $3x^2 - 2y - 6 = 0$

(B) $3x^2 + 2y - 6 = 0$

(C) $2x^2 + 3y - 9 = 0$

(D) $2x^2 - 3y + 9 = 0$

Correct Answer: (C) $2x^2 + 3y - 9 = 0$

Solution:

Concept:

- The midpoint of points (x_1, y_1) and (x_2, y_2) is $((x_1 + x_2)/2, (y_1 + y_2)/2)$. Introduce the parameter first and eliminate it only after both midpoint conditions have been used.
- Every point on the perpendicular bisector of a segment is equidistant from its endpoints. Thus $CA^2 = CB^2$ locates its intersection with the y-axis without dividing by a slope.

Step 1: Find the first midpoint For $A = (0, 6)$ and $B = (2t, 0)$, the midpoint is $M = (t, 3)$. Write $C = (0, c)$, since it lies on the y-axis. The point C must satisfy the perpendicular-bisector condition even when $t = 0$; the distance method therefore covers that case too.

Step 2: Locate the perpendicular bisector on the axis The squared distances from C to the two endpoints give

$$(c - 6)^2 = 4t^2 + c^2.$$

Cancelling c^2 and collecting terms,

$$36 - 12c = 4t^2, \qquad c = 3 - \frac{t^2}{3}.$$

Only the intersection point is needed; no equation for the entire perpendicular bisector is necessary.

Step 3: Eliminate the parameter If $P = (x, y)$ is the midpoint of M and C, then

$$x = \frac{t}{2}, \qquad y = \frac{3 + c}{2} = 3 - \frac{t^2}{6}.$$

Substitute $t = 2x$:

$$y = 3 - \frac{2x^2}{3}, \qquad 2x^2 + 3y - 9 = 0.$$

At $t = 0$ the point is $(0, 3)$, and the curve opens downward. Option (A) has the wrong opening direction, (B) has a different coefficient ratio, and (D) also opens upward.

Step 4: Conclusion The locus is $\boxed{2x^2 + 3y - 9 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: For a perpendicular-bisector intersection with an axis, equate squared distances instead of finding a slope; this also avoids exceptional vertical or horizontal cases.

2. If $(\sin^{-1}x)^2 - (\cos^{-1}x)^2 = a$; $0 < x < 1$, $a \neq 0$, then the value of $2x^2 - 1$ is : [4 marks]

(A) $\cos\left(\frac{4a}{\pi}\right)$

(B) $\sin\left(\frac{2a}{\pi}\right)$

(C) $\cos\left(\frac{2a}{\pi}\right)$

(D) $\sin\left(\frac{4a}{\pi}\right)$

Correct Answer: (B) $\sin\left(\frac{2a}{\pi}\right)$

Solution:

Concept:

- For $-1 \le x \le 1$, the principal inverse functions satisfy $\sin^{-1} x + \cos^{-1} x = \pi/2$. For $0 < x < 1$, both angles lie between zero and $\pi/2$.
- Factor a difference of squares before using the double-angle identity $2\sin^2\theta - 1 = -\cos 2\theta$. All inverse-trigonometric angles here are measured in radians.

Step 1: Recover the inverse sine angle Put $\theta = \sin^{-1} x$ and $\phi = \cos^{-1} x$. The given equation is $\theta^2 - \phi^2 = a$. Because $\theta + \phi = \pi/2$,

$$(\theta - \phi)\frac{\pi}{2} = a, \qquad \theta - \phi = \frac{2a}{\pi}.$$

Adding this to the equation for their sum gives

$$2\theta = \frac{\pi}{2} + \frac{2a}{\pi}.$$

The angle ranges imply $-\pi^2/4 < a < \pi^2/4$, so this manipulation stays within the principal branches.

Step 2: Express the requested quantity Since $x = \sin\theta$,

$$2x^2 - 1 = -\cos 2\theta = -\cos\left(\frac{\pi}{2} + \frac{2a}{\pi}\right).$$

Using $\cos(\pi/2 + u) = -\sin u$, this becomes $\sin(2a/\pi)$. The negative sign in the double-angle identity is essential; omitting it would reverse the result.

Step 3: Check the alternative expressions The argument is $2a/\pi$, not $4a/\pi$, since the factor from the sum of the inverse angles is $\pi/2$. This rules out (A) and (D). Option (C) has cosine rather than sine and approaches 1 as a approaches zero, whereas the required quantity approaches 0. Thus it is not the required identity.

Step 4: Conclusion $\boxed{2x^2 - 1 = \sin(2a/\pi)}$. Hence, the correct option is (B).

Quick Tip to Solve: A difference of squared inverse sine and inverse cosine becomes linear after using their constant sum; apply the double-angle identity only afterwards.

3. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is : [4 marks]

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) -1

(D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- The characteristic polynomial of $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ is $\lambda^2 + \lambda - 2K$. Cayley-Hamilton therefore gives $A^2 + A - 2KI = 0$.
- To enforce a matrix polynomial equation, reduce higher powers to a linear combination of A and I. A nonzero off-diagonal entry then determines its coefficient.

Step 1: Reduce the higher powers From $A^2 = -A + 2KI$, multiplication by A gives

$$A^3 = (1 + 2K)A - 2KI.$$

Multiplying again and using the same quadratic relation,

$$A^4 = -(1 + 4K)A + (2K + 4K^2)I.$$

This reduction avoids expanding four complete matrix products and keeps every occurrence of the unknown parameter explicit.

Step 2: Apply the given equation entry by entry The condition $A(A^3 + 3I) = 2I$ is $A^4 + 3A - 2I = 0$. Substitute the reduced fourth power:

$$(2 - 4K)A + (4K^2 + 2K - 2)I = 0.$$

The upper-right entry of A is 2, while that of I is zero. Consequently,

$$2(2 - 4K) = 0, \qquad K = \frac{1}{2}.$$

The fixed nonzero entry ensures this is necessary, rather than just one possible simplification.

Step 3: Verify the remaining coefficient For $K = 1/2$, the identity coefficient is $4(1/4) + 2(1/2) - 2 = 0$, so every entry of the required matrix equation vanishes. Equivalently, the reduced formula gives $A^4 = -3A + 2I$, exactly as required. Values $-1/2$, -1 and 1 leave the upper-right coefficient nonzero and therefore fail the original condition.

Step 4: Conclusion $\boxed{K = \frac{1}{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: After reducing a matrix polynomial to $uA + vI$, inspect a known nonzero off-diagonal entry first; it forces $u = 0$ immediately.

4. If $S = \left\{z \in C : \frac{z-i}{z+2i} \in R\right\}$, then : [4 marks]

(A) S contains exactly two elements

(B) S contains only one element

(C) S is a circle in the complex plane

(D) S is a straight line in the complex plane

Correct Answer: (D) S is a straight line in the complex plane

Solution:

Concept:

- A complex number is real precisely when its imaginary part is zero. A rational expression in z must also have a nonzero denominator.
- Write $z = x + iy$ and multiply numerator and denominator by the conjugate of the denominator. This produces a real denominator and makes the imaginary-part condition explicit.

Step 1: Separate the real and imaginary parts For $z = x + iy$, the expression becomes

$$\frac{z-i}{z+2i} = \frac{x + i(y-1)}{x + i(y+2)}.$$

The point $z = -2i$, or $(x, y) = (0, -2)$, is excluded because the original denominator is zero there. For all other points, multiply by $x - i(y+2)$ to obtain

$$\frac{z-i}{z+2i} = \frac{x^2 + (y-1)(y+2) - 3ix}{x^2 + (y+2)^2}.$$

The sign of the imaginary term follows from $x(y-1) - x(y+2) = -3x$.

Step 2: Impose reality and classify the locus The denominator is strictly positive on the domain, so the quotient is real if and only if $-3x = 0$, that is, $x = 0$. Conversely, if $x = 0$ and $y \neq -2$, the quotient reduces to the real number $(y-1)/(y+2)$. Thus every point of the imaginary axis except the pole satisfies the condition; there is no additional restriction on y.

Step 3: Compare the listed geometric descriptions The set contains infinitely many points, so neither the one-element nor the two-element description is possible. Its Cartesian condition is linear, rather than a circle equation. Among the offered geometric descriptions, (D) identifies this straight-line locus, with the understood exclusion of the point where the quotient is undefined.

Step 4: Conclusion The locus lies on $\boxed{\operatorname{Re} z = 0}$, with $z = -2i$ excluded. Thus the straight-line description is the intended one. Hence, the correct option is (D).

Quick Tip to Solve: Always remove the pole before classifying a complex quotient locus; a real quotient condition often reduces to collinearity, with an excluded endpoint.

5. Let y = y(x) be the solution of the differential equation

$\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ **such that y(0) = 7. Then y(π) is equal to :** [4 marks]

(A) $2e^{\pi^2} + 5$

(B) $e^{\pi^2} + 5$

(C) $3e^{\pi^2} + 5$

(D) $7e^{\pi^2} + 5$

Correct Answer: (A) $2e^{\pi^2} + 5$

Solution:

Concept:

- An equation containing $y + g(x)$ together with $-g\prime(x)$ suggests substituting $u = y + g(x)$. The derivative then combines the troublesome nonhomogeneous terms.
- The separable equation $u\prime = 2xu$ has solution $u = Ce^{x^2}$. The initial value fixes C, and substituting back is needed before evaluating y.

Step 1: Choose a substitution that cancels the cosine Define

$$u = y + 2\sin x - 5.$$

Then $u\prime = y\prime + 2\cos x$. Substitution of the stated differential equation gives

$$u\prime = 2x(y + 2\sin x - 5) = 2xu.$$

The shift by 5 and the added sine both matter: together they turn the full bracket on the right into exactly u.

Step 2: Solve and use the initial condition The differential equation has integrating factor e^{-x^2}, since

$$\frac{d}{dx}\left(ue^{-x^2}\right) = 0.$$

Hence $u = Ce^{x^2}$. At $x = 0$, the given value $y(0) = 7$ yields $u(0) = 7 + 0 - 5 = 2$, so $C = 2$. This method also includes the possible zero solution of the transformed equation without dividing by u.

Step 3: Return to the original function and evaluate Undo the substitution:

$$y(x) = 2e^{x^2} - 2\sin x + 5.$$

Differentiating gives $y\prime = 4xe^{x^2} - 2\cos x$, which agrees with the given equation because its bracket equals $2e^{x^2}$. Since $\sin\pi = 0$,

$$y(\pi) = 2e^{\pi^2} + 5.$$

The other options use constants 1, 3 or 7 in place of 2; each would violate $y(0) = 7$.

Step 4: Conclusion $\boxed{y(\pi) = 2e^{\pi^2} + 5}$. Hence, the correct option is (A).

Quick Tip to Solve: When a bracket contains $y + 2\sin x - 5$ and the equation has $-2\cos x$, differentiate that entire bracket to reveal a homogeneous equation.

6. Let us consider a curve, y = f(x) passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point (x, f(x)) is given by f(x) + xf'(x) = x^2. Then : [4 marks]

(A) $x^2 + 2xf(x) - 12 = 0$

(B) $x^3 + xf(x) + 12 = 0$

(C) $x^3 - 3xf(x) - 4 = 0$

(D) $x^2 + 2xf(x) + 4 = 0$

Correct Answer: (C) $x^3 - 3xf(x) - 4 = 0$

Solution:

Concept:

- The product rule gives $\frac{d}{dx}[xf(x)] = f(x) + xf\prime(x)$. Recognising this exact derivative converts the given differential relation into one direct integration.
- The integration constant is determined by the stated point on the curve. Verify both the differential relation and the point rather than using the point alone to select an option.

Step 1: Integrate the product derivative The given relation is

$$\frac{d}{dx}[xf(x)] = x^2.$$

Integrating with respect to x gives

$$xf(x) = \frac{x^3}{3} + C.$$

There is no need to divide by x at this stage. Working with the product is shorter and avoids introducing an apparent difficulty at zero before the initial value is used.

Step 2: Apply the point on the curve The curve passes through $(-2, 2)$, so $f(-2) = 2$. Therefore

$$-4 = -\frac{8}{3} + C, \qquad C = -\frac{4}{3}.$$

Substituting this constant and multiplying by 3 yields

$$3xf(x) = x^3 - 4, \qquad x^3 - 3xf(x) - 4 = 0.$$

On the interval containing the initial point, $f(x) = x^2/3 - 4/(3x)$, which is defined for negative x.

Step 3: Verify the relation and reject the distractors Differentiating $xf(x) = (x^3 - 4)/3$ returns $f + xf\prime = x^2$, and substituting $x = -2$ returns $f = 2$. At the given point, the left sides of options (A) and (B) are respectively -16 and 0 for (B), so (B) needs the differential check. For (B), differentiating $xf = -x^3 - 12$ gives $f + xf\prime = -3x^2$; (D) likewise gives $-x$, not x^2.

Step 4: Conclusion $\boxed{x^3 - 3xf(x) - 4 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Recognise $f + xf\prime$ as the derivative of xf; an option passing through the specified point must still satisfy the differential equation.

7. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim_{x\to\beta} \frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to : [4 marks]

(A) $b^2 + 4c$

(B) $2(b^2 + 4c)$

(C) $2(b^2 - 4c)$

(D) $b^2 - 4c$

Correct Answer: (C) $2(b^2 - 4c)$

Solution:

Concept:

- Near $u = 0$, the exponential expansion gives $e^{2u} = 1 + 2u + 2u^2 + O(u^3)$. Thus $(e^{2u} - 1 - 2u)/u^2$ tends to 2.
- For the distinct roots α, β of $x^2 + bx + c$, factorisation gives $x^2 + bx + c = (x - \alpha)(x - \beta)$. Vieta's formulae relate the squared root difference to the discriminant.

Step 1: Factor the quantity that vanishes Put $u = x^2 + bx + c = (x - \alpha)(x - \beta)$. As x approaches β, u approaches zero, while

$$\frac{u}{x - \beta} = x - \alpha \longrightarrow \beta - \alpha.$$

Because the roots are distinct, this limiting factor is nonzero. Factoring isolates the precise order of vanishing in the original denominator.

Step 2: Use the exponential limit For $x \neq \beta$ sufficiently close to β, rewrite the expression as

$$\frac{e^{2u} - 1 - 2u}{u^2} (x - \alpha)^2.$$

The first factor tends to 2: the constant and linear terms cancel, leaving $2u^2$ as the leading term of the numerator. The second factor tends to $(\beta - \alpha)^2$. The requested limit is therefore $2(\beta - \alpha)^2$.

Step 3: Express the answer in the coefficients Using $\alpha + \beta = -b$ and $\alpha\beta = c$,

$$(\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta = b^2 - 4c.$$

Hence the limit equals $2(b^2 - 4c)$. Options (A) and (B) have the wrong sign of $4c$; option (D) omits the factor 2 arising from the quadratic term of e^{2u}.

Step 4: Conclusion The limit is $\boxed{2(b^2 - 4c)}$. Hence, the correct option is (C).

Quick Tip to Solve: After subtracting the constant and linear terms from an exponential, retain its quadratic term; then use the root factorisation to match the denominator.

8. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6} - x$ and its opposite face occurs with probability $\frac{1}{6} + x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is : [4 marks]

(A) $\frac{1}{16}$
(B) $\frac{1}{8}$
(C) $\frac{1}{9}$
(D) $\frac{1}{12}$

Correct Answer: (B) $\frac{1}{8}$

Solution:

Concept:

- For two independent rolls, the probability of an ordered pair of faces is the product of their individual probabilities.
- A total of seven occurs precisely when the two faces are opposite. The six ordered outcomes form three opposite-face pairs, with two orders for each pair.

Step 1: Identify the favourable outcomes Write the probabilities of the biased opposite faces as $p = \frac{1}{6} - x$ and $q = \frac{1}{6} + x$. Whether these faces are 1 and 6, 2 and 5, or 3 and 4 does not affect the calculation: they always sum to seven. Their contribution is $pq + qp = 2pq$. Each of the remaining two opposite-face pairs has probability $1/6$ on each face and contributes $2(1/6)^2$. The rolls are independent, so the bias does not change between rolls.

Step 2: Form and solve the probability equation Adding the three disjoint pairs gives

$$P(\text{sum } 7) = 2\left(\frac{1}{6} - x\right)\left(\frac{1}{6} + x\right) + 4\left(\frac{1}{6}\right)^2 = \frac{1}{6} - 2x^2.$$

Equate this to the specified value:

$$2x^2 = \frac{1}{6} - \frac{13}{96} = \frac{3}{96} = \frac{1}{32}, \qquad x^2 = \frac{1}{64}.$$

Thus $x = \pm 1/8$. The restriction $x > 0$ selects $x = 1/8$, which also satisfies $x < 1/6$.

Step 3: Check the alternatives For $x = 1/8$, the biased face probabilities are $1/24$ and $7/24$, both positive. The probability becomes $1/6 - 2/64 = 13/96$, as required. Options (A), (C) and (D) give different positive squares from $1/64$, so none satisfies the same equation.

Step 4: Conclusion The required value is $x = \frac{1}{8}$. Hence, the correct option is (B).

Quick Tip to Solve: Count both orders of every opposite-face pair; the bias changes the sum-seven probability by $-2x^2$.

9. If $x^2 + 9y^2 - 4x + 3 = 0$, x, y $\in$ R, then x and y respectively lie in the intervals : [4 marks]

(A) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(B) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and [1, 3]

(C) [1, 3] and [1, 3]

(D) [1, 3] and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Correct Answer: (D) [1, 3] and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Solution:

Concept:

- Completing the square converts a quadratic relation into the standard form of an ellipse.
- In a sum of nonnegative squared terms equal to one, each term lies between zero and one. Equality cases show whether the resulting bounds are attained.

Step 1: Complete the square in x Group the terms involving x, while retaining the constant exactly as printed:

$$x^2 - 4x + 9y^2 + 3 = 0 \quad \Longrightarrow \quad (x-2)^2 + 9y^2 = 1.$$

This is an ellipse centred at $(2, 0)$, with horizontal semi-axis 1 and vertical semi-axis $1/3$. The coefficient 9 multiplies the square of y; it does not change the centre of the x-coordinate.

Step 2: Find the range of each coordinate Since $9y^2 \geq 0$, we have $(x-2)^2 \leq 1$. Therefore

$$-1 \leq x - 2 \leq 1 \quad \Longrightarrow \quad 1 \leq x \leq 3.$$

Likewise $(x-2)^2 \geq 0$ implies $9y^2 \leq 1$, giving

$$-\frac{1}{3} \leq y \leq \frac{1}{3}.$$

The points $(1, 0)$ and $(3, 0)$ attain the two horizontal bounds. The points $(2, -1/3)$ and $(2, 1/3)$ attain the vertical bounds, so both intervals are closed.

Step 3: Compare the ordered intervals The question asks for the intervals for x and y, respectively; their order matters. Options (A) and (B) incorrectly restrict x to an interval around zero, even though the ellipse is centred at $x = 2$. Option (C) assigns the horizontal range to y, whereas every point on the ellipse has $|y| \leq 1/3$. Only option (D) gives both ranges in their correct order. These are coordinate projections; not every arbitrary pair drawn from the two intervals lies on the ellipse.

Step 4: Conclusion We obtain $x \in [1, 3]$ and $y \in [-1/3, 1/3]$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Complete the square first: the centre shifts the coordinate interval, while the semi-axis gives its half-width.

10. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)}\, dx$ **is equal to :** [4 marks]

(A) 6

(B) 8

(C) 5

(D) 10

Correct Answer: (C) 5

Solution:

Concept:

- The reflection identity is $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$.
- If reflected integrands sum to one, adding their equal integrals gives the interval length; each integral is half that length.

Step 1: Expose the reflected logarithm Factor the quadratic exactly:

$$x^2 - 44x + 484 = (22 - x)^2.$$

The endpoints satisfy $6 + 16 = 22$. Set

$$f(x) = \frac{\ln x^2}{\ln x^2 + \ln(22 - x)^2}.$$

For $6 \le x \le 16$, both x and $22 - x$ lie between 6 and 16. Their squares are positive and exceed one, so both logarithms are defined and positive. In particular, the denominator never vanishes on the interval.

Step 2: Reflect the integral about the midpoint With $u = 22 - x$, the bounds 6 and 16 interchange and $dx = -du$. Reversing the bounds cancels the minus sign. Thus the same integral I is

$$I = \int_6^{16} \frac{\ln(22 - x)^2}{\ln(22 - x)^2 + \ln x^2}\, dx.$$

Add this expression to the original one. The numerators add to the common denominator, so

$$2I = \int_6^{16} 1\, dx = 16 - 6 = 10, \qquad I = 5.$$

No separate antiderivative of the logarithmic quotient is required.

Step 3: Check the value against the choices The reflection exchanges the two logarithms without changing the denominator. This is the reason the average value of the integrand over the symmetric interval is $1/2$, even though the integrand is not constant. Multiplying that average by the interval length 10 again gives 5. Options (A) and (B) disagree with this exact symmetry, while (D) gives the whole interval length rather than half of it.

Step 4: Conclusion The definite integral is 5. Hence, the correct option is (C).

Quick Tip to Solve: When a quadratic becomes $(a + b - x)^2$, test reflection $x \mapsto a + b - x$ before attempting integration.

11. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is : [4 marks]

(A) $\frac{5}{2+\sqrt{3}}$
(B) $\frac{10}{2+3\sqrt{3}}$
(C) $\frac{5}{3+\sqrt{3}}$
(D) $\frac{10}{3+2\sqrt{3}}$

Correct Answer: (D) $\frac{10}{3+2\sqrt{3}}$

Solution:

Concept:

- A square of side s has perimeter $4s$ and area s^2. A regular hexagon of side h consists of six equilateral triangles and has area $\frac{3\sqrt{3}}{2}h^2$.
- The fixed wire length provides the constraint $4s + 6h = 20$. A positive second derivative confirms a stationary point is a strict minimum.

Step 1: Express the combined area using one side Let h metres be the side of the regular hexagon and s metres be the side of the square. Both shapes use all of their respective pieces, so

$$4s + 6h = 20, \qquad s = 5 - \frac{3}{2}h.$$

The feasible interval is $0 < h < 10/3$. The combined area, in square metres, is

$$A(h) = \left(5 - \frac{3}{2}h\right)^2 + \frac{3\sqrt{3}}{2}h^2.$$

Step 2: Differentiate and locate the minimum Differentiate the area function:

$$A'(h) = -3\left(5 - \frac{3}{2}h\right) + 3\sqrt{3}h.$$

The equation $A'(h) = 0$ becomes

$$15 = \left(\frac{9}{2} + 3\sqrt{3}\right)h, \qquad h = \frac{10}{3 + 2\sqrt{3}}.$$

Also $A''(h) = \frac{9}{2} + 3\sqrt{3} > 0$, so the quadratic is strictly convex and this stationary value gives its global minimum. The value lies strictly between zero and $10/3$, so both pieces have positive length.

Step 3: Check the perimeter and options The stationary condition can also be written $s = \sqrt{3}h$. Substituting it directly into $4s + 6h = 20$ gives $(4\sqrt{3} + 6)h = 20$, reproducing the same result. Each of options (A), (B) and (C) gives a different positive side and therefore fails the stationary equation.

Step 4: Conclusion The minimizing hexagon side is $\frac{10}{3+2\sqrt{3}}$ m. Hence, the correct option is (D).

Quick Tip to Solve: Apply the wire constraint to side lengths first; minimizing the area then reduces to a convex quadratic in one variable.

12. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...$, is equal to : [4 marks]

(A) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(B) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

(C) $\frac{1-x}{1+x} + \log_e(1-x)$

(D) $\frac{1+x}{1-x} + \log_e(1-x)$

Correct Answer: (A) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

Solution:

Concept:

- For $|x| < 1$, the geometric series gives $\sum_{n=2}^{\infty} x^n = x^2/(1-x)$.
- The logarithmic expansion is $-\ln(1-x) = \sum_{n=1}^{\infty} x^n/n$. The given condition $0 < x < 1$ ensures both series converge.

Step 1: Rewrite the general coefficient The displayed coefficients are $3/2, 5/3, 7/4, \ldots$. For the term with power x^n, starting at $n = 2$, the coefficient is

$$\frac{2n-1}{n} = 2 - \frac{1}{n}.$$

Consequently the required series S separates as

$$S = 2\sum_{n=2}^{\infty} x^n - \sum_{n=2}^{\infty} \frac{x^n}{n}.$$

Both sums begin at 2, so the missing first-power term must be accounted for when applying standard expansions.

Step 2: Evaluate the two convergent sums Using the geometric series and removing the $n = 1$ term from the logarithmic series gives

$$\sum_{n=2}^{\infty} x^n = \frac{x^2}{1-x}, \qquad \sum_{n=2}^{\infty} \frac{x^n}{n} = -\ln(1-x) - x.$$

Substitute these into S, carefully retaining the minus sign before the second sum:

$$S = \frac{2x^2}{1-x} + \ln(1-x) + x = x\frac{1+x}{1-x} + \ln(1-x).$$

Since $0 < x < 1$, the logarithm is real and all rearrangements of these absolutely convergent series are valid.

Step 3: Check the leading terms and reject other forms Expanding the result near zero cancels the linear term and gives $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \cdots$, as required. Option (B) has the wrong sign for its leading quadratic coefficient. Options (C) and (D) have a nonzero constant term as $x \to 0$, whereas the original series tends to zero. These checks leave only option (A), consistent with the exact summation.

Step 4: Conclusion The sum is $x\frac{1+x}{1-x} + \ln(1-x)$. Hence, the correct option is (A).

Quick Tip to Solve: For coefficients such as $(2n-1)/n$, split them into a constant and $1/n$ to combine geometric and logarithmic series.

13. If for x, y $\in$ R, x $>$ 0, y = $\log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9}$ + ... upto ∞ terms and $\frac{2+4+6+....+2y}{3+6+9+.....+3y} = \frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to : [4 marks]

(A) $(10^6, 6)$

(B) $(10^4, 6)$

(C) $(10^2, 3)$

(D) $(10^6, 9)$

Correct Answer: (D) $(10^6, 9)$

Solution:

Concept:

- For $x > 0$, the power rule gives $\log_{10}(x^a) = a\log_{10} x$.
- The geometric sum $1 + 1/3 + 1/9 + \cdots$ equals $3/2$. The arithmetic sums in the ratio have identical term counts and proportional terms.

Step 1: Sum the logarithmic expression for y Put $L = \log_{10} x$. The successive terms are $L, L/3, L/9, \ldots$, forming a convergent geometric series with common ratio $1/3$. Hence

$$y = L\left(1 + \frac{1}{3} + \frac{1}{9} + \cdots\right) = \frac{L}{1 - 1/3} = \frac{3}{2}L.$$

The second displayed condition contains $4/L$, so $L \neq 0$. In particular $x = 1$, which would make all logarithms zero, cannot satisfy that condition.

Step 2: Evaluate the arithmetic-sum ratio For the indicated finite sums, factor the common sequence of positive integers:

$$\frac{2 + 4 + 6 + \cdots + 2y}{3 + 6 + 9 + \cdots + 3y} = \frac{2(1 + 2 + \cdots + y)}{3(1 + 2 + \cdots + y)} = \frac{2}{3}.$$

Equivalently, each sum is its multiplier times $y(y + 1)/2$. The question's notation asks for a positive integer term count; the value obtained below indeed has that property. Therefore

$$\frac{2}{3} = \frac{4}{L} \quad \Longrightarrow \quad L = 6.$$

It follows that $x = 10^6$ and $y = \frac{3}{2} \cdot 6 = 9$.

Step 3: Verify both equations and compare the pairs For $x = 10^6$, the logarithmic series is $6 + 2 + 2/3 + \cdots = 9$. With nine terms in each arithmetic sum, the ratio is still $2/3 = 4/6$, so the two conditions hold simultaneously. Option (A) has the right value of x but the wrong value of y. Options (B) and (C) satisfy the logarithmic relation but make $4/\log_{10} x$ equal to 1 and 2, respectively, rather than $2/3$.

Step 4: Conclusion The ordered pair is $(x, y) = (10^6, 9)$. Hence, the correct option is (D).

Quick Tip to Solve: First factor the logarithm series; in the ratio of proportional arithmetic sums, cancel the common sum before solving for the logarithm.

14. The distance of the point (1, −2, 3) from the plane x − y + z = 5 measured parallel to a line, whose direction ratios are 2, 3, −6 is : [4 marks]

(A) 3

(B) 5

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- A line through $P = (x_0, y_0, z_0)$ with direction vector $\mathbf{d}$ has equation $\mathbf{r} = P + t\mathbf{d}$. Its displacement length is $|t|\,|\mathbf{d}|$.
- To measure distance to a plane along that line, find its intersection with the plane. The perpendicular-distance formula applies only when the prescribed direction is normal to the plane.

Step 1: Write the line through the given point Take $P = (1, -2, 3)$ and direction vector $\mathbf{d} = (2, 3, -6)$. The line parallel to this direction and passing through P is

$$x = 1 + 2t, \qquad y = -2 + 3t, \qquad z = 3 - 6t.$$

The vector has length $\sqrt{2^2 + 3^2 + (-6)^2} = 7$. The parameter is not itself the distance because this direction vector is not a unit vector.

Step 2: Intersect the line with the plane Insert the three coordinates into $x - y + z = 5$:

$$(1 + 2t) - (-2 + 3t) + (3 - 6t) = 5.$$

Collecting the constant terms gives 6, and collecting parameter terms gives $-7t$. Thus

$$6 - 7t = 5, \qquad t = \frac{1}{7}.$$

The intersection is $Q = (9/7, -11/7, 15/7)$. As a check, its coordinates satisfy $9/7 + 11/7 + 15/7 = 5$, confirming that it lies on the required plane.

Step 3: Calculate the displacement length The displacement from the original point to the intersection is

$$\overrightarrow{PQ} = \frac{1}{7}(2, 3, -6), \qquad PQ = \frac{1}{7}\sqrt{4 + 9 + 36} = 1.$$

This is the distance along the specified direction. The shortest perpendicular distance would instead be $|1 + 2 + 3 - 5|/\sqrt{3} = 1/\sqrt{3}$; it answers a different measurement. Options (A), (B) and (C) do not equal the intersection displacement length and therefore cannot be selected.

Step 4: Conclusion The requested distance is 1 unit. Hence, the correct option is (D).

Quick Tip to Solve: Find the line-plane parameter first, then multiply its absolute value by the norm of the direction-ratio vector.

15. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes x − y − z − 1 = 0 and 2x + y − 3z + 4 = 0, is : [4 marks]

(A) $3x - y - 5z + 2 = 0$

(B) $3x - 4z + 3 = 0$

(C) $-x + 2y + 2z - 3 = 0$

(D) $4x - y - 5z + 2 = 0$

Correct Answer: (D) $4x - y - 5z + 2 = 0$

Solution:

Concept:

- A plane containing the intersection of two distinct planes has equation $P_1 + \lambda P_2 = 0$, apart from the second plane itself.
- The perpendicular distance of $Ax + By + Cz + D = 0$ from the origin is $|D|/\sqrt{A^2 + B^2 + C^2}$. Both containment and distance must be checked.

Step 1: Form the pencil of planes The planes supplied in the question are $P_1 = x - y - z - 1$ and $P_2 = 2x + y - 3z + 4$. Thus every point on their common line satisfies the following plane for each real λ:

$$(1 + 2\lambda)x + (\lambda - 1)y - (1 + 3\lambda)z + 4\lambda - 1 = 0.$$

This construction enforces the line condition before we test the distance.

Step 2: Apply the prescribed distance Squaring the positive distance condition gives

$$\frac{(4\lambda - 1)^2}{(1 + 2\lambda)^2 + (\lambda - 1)^2 + (1 + 3\lambda)^2} = \frac{2}{21}.$$

On expanding,

$$308\lambda^2 - 184\lambda + 15 = 0, \qquad \lambda = \frac{1}{2} \text{ or } \frac{15}{154}.$$

Taking $\lambda = 1/2$ and multiplying the equation by 2 yields $4x - y - 5z + 2 = 0$.

Step 3: Check the listed alternatives For this plane the distance is $2/\sqrt{16 + 1 + 25} = \sqrt{2/21}$, as required. Options A, B and C instead have distances $2/\sqrt{35}$, $3/5$ and 1, respectively, so none can meet the stated distance. The other pencil parameter produces a plane that is not among the options.

Step 4: Conclusion The required listed plane is $\boxed{4x - y - 5z + 2 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: For a plane through an intersection line, use a pencil first; then square the distance equation and check the resulting listed plane directly.

16. **If** $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \ldots\ldots \left(1+\frac{n^2}{n^2}\right)^n$, **then** $\lim\limits_{n\to\infty} (U_n)^{\frac{-4}{n^2}}$ **is equal to :** [4 marks]

(A) $\frac{e^2}{16}$

(B) $\frac{4}{e}$

(C) $\frac{16}{e^2}$

(D) $\frac{4}{e^2}$

Correct Answer: (A) $\frac{e^2}{16}$

Solution:

Concept:

- Taking logarithms converts a product with powers into a weighted sum: $\log \prod a_k^k = \sum k \log a_k$. All factors here are positive.
- A sum $\frac{1}{n}\sum_{k=1}^{n} f(k/n)$ tends to $\int_0^1 f(x)\,dx$ for continuous f. Exponentiation can be applied after finding the logarithmic limit.

Step 1: Take the logarithm of the required expression Write the product as $U_n = \prod_{k=1}^{n}(1+k^2/n^2)^k$. If the required limit is L, consider its logarithm:

$$\log\left(U_n^{-4/n^2}\right) = -\frac{4}{n^2}\sum_{k=1}^{n} k\log(1+k^2/n^2) = -\frac{4}{n}\sum_{k=1}^{n}\frac{k}{n}\log\left(1+\left(\frac{k}{n}\right)^2\right).$$

The weight k is essential; dropping it would give a different integral.

Step 2: Evaluate the limiting integral The function $x\log(1+x^2)$ is continuous on $[0,1]$, so

$$\log L = -4\int_0^1 x\log(1+x^2)\,dx.$$

Put $t = 1+x^2$, so $dt = 2x\,dx$. The endpoints become 1 and 2, giving

$$\int_0^1 x\log(1+x^2)\,dx = \frac{1}{2}[t\log t - t]_1^2 = \log 2 - \frac{1}{2}.$$

Hence $\log L = 2 - 4\log 2$.

Step 3: Return from the logarithm Continuity of the exponential gives $L = e^{2-4\log 2} = e^2/16$. Also $U_n > 1$ and its exponent is negative, so the required value must be below 1; this immediately rules out B and C. Option D has logarithm $2\log 2 - 2$, which differs from the value just computed.

Step 4: Conclusion The limit is $\boxed{e^2/16}$. Hence, the correct option is (A).

Quick Tip to Solve: In weighted product limits, take logs and rewrite each weight as k/n; the remaining factor $1/n$ reveals the Riemann sum.

17. The statement $(p \wedge (p \to q) \wedge (q \to r)) \to r$ is : [4 marks]

(A) a tautology

(B) equivalent to $p \to \sim r$

(C) a fallacy

(D) equivalent to $q \to \sim r$

Correct Answer: (A) a tautology

Solution:

Concept:

- An implication $X \to Y$ is false only when X is true and Y is false. If its antecedent is false, the implication is true.
- A tautology is true for every truth assignment. Modus ponens says that p and $p \to q$ together imply q.

Step 1: Name the antecedent and test its true case Let $H = p \wedge (p \to q) \wedge (q \to r)$. The statement to classify is $H \to r$. If H is true, each of its three factors must be true. In particular, p is true and $p \to q$ is true, forcing q to be true. Now $q \to r$ is true with q true, so r must also be true. Therefore the implication is true whenever its antecedent is true.

Step 2: Include every remaining truth assignment For all assignments where H is false, $H \to r$ is true by the definition of implication, irrespective of the value of r. This covers every possible assignment: either H is true, when the preceding deduction forces r, or H is false. Alternatively, a false implication would require H true and r false, which contradicts the two successive applications of modus ponens.

Step 3: Rule out the other classifications The statement cannot be a fallacy because it is never false. It is also not equivalent to either $p \to \sim r$ or $q \to \sim r$: take $p = q = r$ true. The given statement is true, while each of these proposed equivalent implications is false. Thus B, C and D are excluded by the definition and this explicit counterexample.

Step 4: Conclusion The statement is true for every truth assignment, so it is $\boxed{\text{a tautology}}$. Hence, the correct option is (A).

Quick Tip to Solve: To test a chained implication, assume its entire antecedent is true and propagate the truth through the chain; also remember the false-antecedent case.

18. $\sum_{k=0}^{20} \left({}^{20}C_k\right)^2$ **is equal to :** [4 marks]

(A) ${}^{40}C_{21}$

(B) ${}^{40}C_{19}$

(C) ${}^{40}C_{20}$

(D) ${}^{41}C_{20}$

Correct Answer: (C) ${}^{40}C_{20}$

Solution:

Concept:

- The symmetry identity is $\binom{n}{k} = \binom{n}{n-k}$. It allows a square of a binomial coefficient to become a convolution.
- The coefficient of x^m in a product is obtained by pairing terms whose exponents add to m. Here $(1+x)^{20}(1+x)^{20} = (1+x)^{40}$.

Step 1: Express the sum as a coefficient Use binomial symmetry on one factor in every summand:

$$S = \sum_{k=0}^{20} \binom{20}{k}^2 = \sum_{k=0}^{20} \binom{20}{k}\binom{20}{20-k}.$$

In the product of the two binomial expansions, the term of degree k from the first factor combines with the term of degree $20-k$ from the second. Every pair contributing to degree 20 appears exactly once in this sum.

Step 2: Combine the two binomial expansions The product is

$$(1+x)^{20}(1+x)^{20} = (1+x)^{40}.$$

By the binomial theorem, its coefficient of x^{20} is $\binom{40}{20}$. Equating the same coefficient in the product form and in the combined power proves $S = \binom{40}{20}$. This is an instance of Vandermonde's identity and requires no expansion of the 21 individual squares.

Step 3: Distinguish the neighboring options Options A and B are equal to each other by symmetry, but each is smaller than $\binom{40}{20}$:

$$\binom{40}{19} = \binom{40}{21} = \frac{20}{21}\binom{40}{20}.$$

Pascal's rule gives $\binom{41}{20} = \binom{40}{20} + \binom{40}{19}$, which is larger. Thus D is also excluded, leaving only C.

Step 4: Conclusion $\sum_{k=0}^{20} \binom{20}{k}^2 = \boxed{\binom{40}{20}}$. Hence, the correct option is (C).

Quick Tip to Solve: For $\sum_k \binom{n}{k}^2$, replace one factor by $\binom{n}{n-k}$ and read the coefficient of x^n in $(1+x)^{2n}$.

19. A tangent and a normal are drawn at the point P(2, −4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q(a, b) is a point such that AQBP is a square, then 2a + b is equal to : [4 marks]

(A) −16

(B) −18

(C) −12

(D) −20

Correct Answer: (A) −16

Solution:

Concept:

- For $y^2 = 4px$, the directrix is $x = -p$. Implicit differentiation gives the tangent slope $dy/dx = 2p/y$.
- In a square, opposite vertices have the same midpoint. For the cyclic order A, Q, B, P, the diagonals are AB and QP, so $Q = A + B - P$.

Step 1: Find the tangent and normal at P Here $4p = 8$, so $p = 2$ and the directrix is $x = -2$. At $P = (2, -4)$, the tangent slope is $4/y = -1$. The normal slope is its negative reciprocal, 1. Their equations are therefore

$$y + 4 = -(x - 2), \qquad y + 4 = x - 2.$$

In simplified form the tangent is $y = -x - 2$, while the normal is $y = x - 6$.

Step 2: Intersect both lines with the directrix Set $x = -2$. On the tangent, $y = 0$, hence $A = (-2, 0)$. On the normal, $y = -8$, hence $B = (-2, -8)$. The vectors from P to these points are $(-4, 4)$ and $(-4, -4)$. Both have length $4\sqrt{2}$ and their dot product is zero, confirming that they are equal perpendicular sides, as required for the stated square.

Step 3: Locate the fourth vertex and evaluate The diagonals AB and QP bisect each other. Thus

$$Q = A + B - P = (-2, 0) + (-2, -8) - (2, -4) = (-6, -4).$$

Consequently $a = -6$, $b = -4$, and $2a + b = 2(-6) - 4 = -16$. The other choices do not agree with these uniquely determined coordinates; choosing Q on the right of the directrix would repeat P, rather than complete the square.

Step 4: Conclusion $Q = (-6, -4)$, and $\boxed{2a + b = -16}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the square's diagonal midpoint relation $Q = A + B - P$ after finding the tangent and normal intersections; it avoids extra distance equations.

20. Let $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then : [4 marks]

(A) $b^2 - a^2 = a^2 + c^2$

(B) b^2, c^2, a^2 are in A.P.

(C) c^2, a^2, b^2 are in A.P.

(D) a^2, b^2, c^2 are in A.P.

Correct Answer: (B) b^2, c^2, a^2 are in A.P.

Solution:

Concept:

- The sine rule gives $\sin A = a/(2R)$, $\sin B = b/(2R)$, $\sin C = c/(2R)$, where R is the circumradius.
- Use $\sin(u - v) = \sin u \cos v - \cos u \sin v$ and $A + B = \pi - C$. The cosine rule gives $2ab\cos C = a^2 + b^2 - c^2$.

Step 1: Cross-multiply and expand the sine differences The given ratio implies

$$\sin A \sin(C - B) = \sin B \sin(A - C).$$

Expanding both differences and collecting terms gives

$$\sin C(\sin A \cos B + \sin B \cos A) = 2 \cos C \sin A \sin B.$$

The bracket is $\sin(A + B) = \sin C$, because the angles belong to a triangle. Therefore

$$\sin^2 C = 2 \cos C \sin A \sin B.$$

No division by $\cos C$ has been used, so no extra case is lost.

Step 2: Convert the relation to side lengths Substitute the sine-rule expressions into this identity and multiply by $4R^2$:

$$c^2 = 2ab \cos C.$$

Now apply the cosine rule, $2ab\cos C = a^2 + b^2 - c^2$. Hence

$$2c^2 = a^2 + b^2, \qquad c^2 = \frac{a^2 + b^2}{2}.$$

This is precisely the condition that b^2, c^2, a^2, in that order, are in arithmetic progression.

Step 3: Check the order of the progression Option B places c^2 in the middle and therefore matches the derived equation. C and D put a^2 and b^2, respectively, in the middle; those are different conditions. Option A states $b^2 = 2a^2 + c^2$, which also does not follow. It is essential to preserve the middle term when identifying the arithmetic progression.

Step 4: Conclusion $\boxed{b^2, c^2, a^2 \text{ are in A.P.}}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand the sine differences before introducing side lengths. The sum $\sin A \cos B + \sin B \cos A$ collapses to $\sin C$.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\vec{a} = \hat{i}+5\hat{j}+\alpha\hat{k}$, $\vec{b} = \hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c} = -\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $\left|\vec{b} \times \vec{c}\right| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _____. [4 marks]

Solution:

Concept:

- For perpendicular vectors, the scalar product vanishes: $\vec{a} \cdot \vec{b} = 0$. Here this relates the unknown components α and β.
- The square of a vector's magnitude is the sum of the squares of its Cartesian components. Squaring the given cross-product magnitude gives 75.

Step 1: Compute the cross product Using the determinant formula with $\vec{b} = (1, 3, \beta)$ and $\vec{c} = (-1, 2, -3)$,

$$\vec{b} \times \vec{c} = (-9 - 2\beta,\ 3 - \beta,\ 5).$$

Therefore the prescribed magnitude yields

$$(-9 - 2\beta)^2 + (3 - \beta)^2 + 25 = 75.$$

Expanding gives $5\beta^2 + 30\beta + 115 = 75$, or

$$\beta^2 + 6\beta + 8 = 0.$$

Thus the two possible values are $\beta = -2$ and $\beta = -4$.

Step 2: Apply perpendicularity to find alpha The zero dot product condition is

$$\vec{a} \cdot \vec{b} = 1 + 5(3) + \alpha\beta = 16 + \alpha\beta = 0.$$

Hence $\alpha = -16/\beta$. For $\beta = -2$, this gives $\alpha = 8$; for $\beta = -4$, it gives $\alpha = 4$. Both roots are nonzero, so this division is valid. Both pairs satisfy the dot-product equation, and each cross product has square magnitude 75.

Step 3: Compare the two possible squared magnitudes For $\vec{a} = (1, 5, \alpha)$,

$$|\vec{a}|^2 = 1^2 + 5^2 + \alpha^2 = 26 + \alpha^2.$$

The values are $26 + 64 = 90$ when $\alpha = 8$, and $26 + 16 = 42$ when $\alpha = 4$. The question asks for the greatest value, so select 90 rather than taking just the first root of the quadratic without comparing. This is a pure numerical squared magnitude; no physical unit is specified.

Step 4: Conclusion The possible values of $|\vec{a}|^2$ are 90 and 42. The greatest is $\boxed{90}$.

Quick Tip to Solve: Keep both roots from a cross-product magnitude equation, use the dot product to find the matching components, then compare the requested magnitudes.

22. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _____.

[4 marks]

Solution:

Concept:

- For a polynomial f, the sign of f at its stationary points and its end behaviour determine how often its graph crosses the horizontal axis.
- The intermediate value theorem gives a root between two points with opposite signs. Strict monotonicity on that interval then makes the root unique.

Step 1: Locate the stationary points Put $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$. Differentiating and factorising gives

$$f'(x) = 12x^3 + 12x^2 - 24x = 12x(x+2)(x-1).$$

Thus the only stationary points are $x = -2, 0, 1$. These split the real line into four intervals on which the derivative has a fixed nonzero sign.

Step 2: Evaluate the polynomial and identify crossings The required values are

$$f(-2) = 48 - 32 - 48 + 4 = -28, \qquad f(0) = 4, \qquad f(1) = 3 + 4 - 12 + 4 = -1.$$

Since the leading term is $3x^4$, the polynomial tends to positive infinity as x tends to either positive or negative infinity. Consequently its signs alternate from positive at the far left to negative at -2, positive at 0, negative at 1, and positive at the far right. Continuity gives at least one root in each of $(-\infty, -2), (-2, 0), (0, 1), (1, \infty)$.

Step 3: Check that the roots are distinct and exhaustive The derivative signs on these four intervals are respectively negative, positive, negative and positive. Therefore the polynomial is strictly monotone on each interval and has exactly one root there. None of the separating stationary points is a root because their polynomial values are nonzero. A quartic cannot have more than four distinct real roots, so these four crossings account for every possible real root.

Step 4: Conclusion The equation has exactly $\boxed{4}$ distinct real roots.

Quick Tip to Solve: For a quartic with three real stationary points, test their heights and the end signs; alternating signs establish four distinct crossings without solving the quartic.

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23. If $A = \{x \in R : |x - 2| > 1\}$,
$B = \{x \in R : \sqrt{x^2 - 3} > 1\}$,
$C = \{x \in R : |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _____.

[4 marks]

Solution:

Concept:

- Translate each inequality into intervals on the real line, taking care over strict and non-strict endpoints. The complement is relative to $\mathbb{R}$.

- A finite set containing k distinct elements has 2^k subsets, including the empty set and the set itself.

Step 1: Find the three sets The first inequality gives

$$A = (-\infty, 1) \cup (3, \infty).$$

For B, the square root is nonnegative, so squaring the inequality $\sqrt{x^2-3} > 1$ is reversible. It gives $x^2 - 3 > 1$, or $x^2 > 4$. This condition also ensures the radicand is nonnegative. Hence

$$B = (-\infty, -2) \cup (2, \infty).$$

Finally, $|x - 4| \geq 2$ means $x - 4 \leq -2$ or $x - 4 \geq 2$, giving

$$C = (-\infty, 2] \cup [6, \infty).$$

Step 2: Intersect the sets and take the complement Intersecting A with B, the negative branch is $x < -2$, while the positive branch requires $x > 3$. The condition from C keeps the entire negative branch but restricts the positive branch to $x \geq 6$. Thus

$$A \cap B \cap C = (-\infty, -2) \cup [6, \infty).$$

Its complement is $[-2, 6)$. Notice that -2 is included in the complement because it is excluded from B, while 6 is excluded from the complement because it belongs to all three sets.

Step 3: Count the integer elements and their subsets The integer set requested is

$$(A \cap B \cap C)^c \cap \mathbb{Z} = \{-2, -1, 0, 1, 2, 3, 4, 5\}.$$

There are $5 - (-2) + 1 = 8$ distinct elements. Each element can independently be either selected or omitted from a subset, giving $2^8 = 256$ possibilities.

Step 4: Conclusion The number of subsets is $\boxed{256}$.

Quick Tip to Solve: Draw the interval endpoints mentally before counting integers: a strict inequality can change the subset count by a factor of two.

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24. If $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$, x > 0 where C is the constant of integration, then the value of $9\left(\sqrt{3}a + b\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- An identity between antiderivatives can be differentiated to give an identity between their derivatives; the arbitrary constant disappears.
- After putting the derivatives over a common denominator, compare coefficients of equal powers of x. This avoids evaluating the original integral separately.

Step 1: Differentiate the proposed antiderivative Write $D = x^2 + x + 1$, which is positive for every real x. The chain rule gives

$$\frac{d}{dx}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = \frac{\sqrt{3}}{2D}.$$

Indeed, the denominator in the chain-rule expression is $1 + (2x+1)^2/3 = 4D/3$. For the rational term, the quotient rule yields

$$\frac{d}{dx}\left(\frac{2x+1}{D}\right) = \frac{2D - (2x+1)^2}{D^2} = \frac{-2x^2 - 2x + 1}{D^2}.$$

Step 2: Compare the polynomial coefficients Differentiating the stated identity and multiplying by D^2, we obtain

$$1 = \frac{a\sqrt{3}}{2}(x^2 + x + 1) + b(-2x^2 - 2x + 1).$$

This holds throughout $x > 0$, so the two polynomials have identical coefficients. The coefficient of x^2 (equivalently that of x) and the constant term give

$$\frac{a\sqrt{3}}{2} - 2b = 0, \qquad \frac{a\sqrt{3}}{2} + b = 1.$$

Subtracting the first equation from the second gives $3b = 1$. Therefore $b = 1/3$ and $a\sqrt{3} = 4/3$.

Step 3: Evaluate the requested combination Substitute these values directly into the combination in the question:

$$9(\sqrt{3}a + b) = 9\left(\frac{4}{3} + \frac{1}{3}\right) = 9 \cdot \frac{5}{3} = 15.$$

As a check, the quadratic and linear coefficients vanish and the constant coefficient becomes $2/3 + 1/3 = 1$, so the differentiated expression is exactly the original integrand.

Step 4: Conclusion The requested numerical value is $\boxed{15}$.

Quick Tip to Solve: When an integral is already expressed using unknown constants, differentiate that expression and compare coefficients instead of integrating from scratch.

25. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to _____. [4 marks]

Solution:

Concept:

- A system of three linear equations in three unknowns has infinitely many solutions when the coefficient and augmented matrices have the same rank, less than three.

- If two coefficient rows are independent, the third must be a linear combination of them, with exactly the same combination applying to the right-hand sides.

Step 1: Express the third coefficient row using the first two The first two coefficient rows are $(2, 1, -1)$ and $(1, -1, -1)$. They are independent, since the minor from their first two columns is $2(-1) - 1(1) = -3 \neq 0$. Let the third row be u times the first plus v times the second. Comparing the coefficients of x and y gives

$$2u + v = 3, \qquad u - v = 3.$$

Adding the two equations gives $3u = 6$, hence $u = 2$ and $v = -1$.

Step 2: Impose the dependence on the remaining terms The coefficient of z in this same combination is

$$\beta = 2(-1) - (-1) = -1.$$

For consistency, the constant on the third equation must also be twice the first constant minus the second constant:

$$3 = 2 \cdot 3 - \alpha.$$

Therefore $\alpha = 3$. With these values, the entire third equation is twice the first equation minus the second. There is no additional independent restriction.

Step 3: Confirm infinitely many solutions and calculate the value The two independent equations leave one free parameter. More explicitly, using $\alpha = 3$, subtraction of the second equation from the first gives $x + 2y = 0$. Hence $x = -2y$, and the second equation gives $z = -3y - 3$. Any real y produces a solution, confirming infinitely many solutions rather than merely a zero determinant. Finally,

$$\alpha + \beta - \alpha\beta = 3 - 1 - 3(-1) = 5.$$

Step 4: Conclusion The required value is $\boxed{5}$.

Quick Tip to Solve: A zero coefficient determinant is necessary but not sufficient for infinitely many solutions: check the identical row dependence in the constants as well.

26. Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to _____. [4 marks]

Solution:

Concept:

- For equally weighted observations, the variance is the mean of their squares minus the square of their mean: $\sigma^2 = \overline{x^2} - (\bar{x})^2$.
- The standard sums are $\sum_{k=1}^{n} k = n(n+1)/2$ and $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$. The divisor for the variance here is n, not $n - 1$.

Step 1: Calculate the two required means The observations are the n consecutive natural numbers

$1, 2, \ldots, n$, all with equal weight. Their arithmetic mean is

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} k = \frac{n+1}{2}.$$

The mean of their squares is obtained separately:

$$\overline{x^2} = \frac{1}{n}\sum_{k=1}^{n} k^2 = \frac{(n+1)(2n+1)}{6}.$$

The mean of the squares must not be confused with the square of the mean; their difference measures the spread of the numbers.

Step 2: Simplify the variance and use its given value Substituting in the variance formula gives

$$\begin{aligned}\sigma^2 &= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} \\ &= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12} \\ &= \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}.\end{aligned}$$

The specified variance is 14, so $n^2 - 1 = 168$ and $n^2 = 169$. The algebraic possibilities are $n = 13$ and $n = -13$, but the latter cannot count observations.

Step 3: Check the natural-number and oddness conditions The admissible value is $n = 13$, which is both positive and odd as required. For a direct numerical check, the mean of $1, \ldots, 13$ is 7, and the mean of their squares is $14 \cdot 27/6 = 63$. The variance is therefore $63 - 7^2 = 14$, in agreement with the given condition.

Step 4: Conclusion The required odd natural number is $\boxed{13}$.

Quick Tip to Solve: Remember $\mathrm{Var}(1, 2, \ldots, n) = (n^2 - 1)/12$; use the population-variance divisor and discard a negative root for the number of observations.

27. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _____.

[4 marks]

Solution:

Concept:

- A six-digit palindrome has the form $abccba$, where the leading digit a is from 1 to 9, while b, c are from 0 to 9.
- Divisibility by $55 = 5 \cdot 11$ is equivalent to divisibility by both 5 and 11, since these factors are coprime.

Step 1: Apply divisibility by eleven For a six-digit palindrome $abccba$, the alternating sums of digits

are

$$a+c+b \quad \text{and} \quad b+c+a.$$

They are equal, so their difference is zero, a multiple of 11. Thus every six-digit palindrome is already divisible by 11; this condition places no further restriction on a, b, c. The even number of digits is crucial to this cancellation.

Step 2: Apply divisibility by five A decimal integer is divisible by 5 exactly when its last digit is 0 or 5. The last digit of the palindrome equals its first digit a. A six-digit number cannot start with zero, so $a = 0$ is inadmissible. Consequently $a = 5$, and every allowed palindrome must have the form

$$5bccb5.$$

Conversely, a number of this form has six digits, is palindromic, is divisible by 5, and is also divisible by 11 by the preceding argument. It is therefore divisible by 55.

Step 3: Count the independent remaining digits There are ten possibilities for b, including zero, and independently ten possibilities for c, also including zero. Once these two digits are fixed, all six positions are determined. Different ordered choices of (b, c) give different numbers, so no duplication occurs. The multiplication principle therefore gives

$$1 \cdot 10 \cdot 10 = 100.$$

For example, the endpoint choice $b = c = 0$ gives 500005, which is still a valid six-digit number and must be included.

Step 4: Conclusion The number of six-digit palindromes divisible by 55 is $\boxed{100}$.

Quick Tip to Solve: Every even-length palindrome is divisible by eleven; for divisibility by fifty-five, only its final digit then needs to be tested.

28. Let the equation $x^2 + y^2 + px + (1 - p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer$\}$ is _____. [4 marks]

Solution:

Concept:

- For $x^2 + y^2 + 2gx + 2fy + c = 0$, the squared radius is $r^2 = g^2 + f^2 - c$. A genuine circle requires a strictly positive squared radius.
- First find the allowed intervals of the real parameter p. Then map both intervals under $q = p^2$ and take their union before counting integer values.

Step 1: Translate the radius restriction into a quadratic inequality Here $2g = p$, $2f = 1 - p$, and $c = 5$. Completing the squares, or using the radius formula, gives

$$r^2 = \frac{p^2 + (1-p)^2}{4} - 5 = \frac{2p^2 - 2p - 19}{4}.$$

Since $0 < r \leq 5$, we need

$$0 < 2p^2 - 2p - 19 \leq 100, \qquad 19 < 2p^2 - 2p \leq 119.$$

Solving these two quadratic inequalities together yields

$$p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right].$$

Step 2: Find the squared parameter values from each branch On the negative interval, squaring reverses the order, whereas it preserves order on the positive interval. Hence the negative branch gives

$$q \in \left(10 - \frac{\sqrt{39}}{2},\ 60 - \frac{\sqrt{239}}{2}\right],$$

and the positive branch gives

$$q \in \left(10 + \frac{\sqrt{39}}{2},\ 60 + \frac{\sqrt{239}}{2}\right].$$

The endpoints are approximately $(6.878, 52.270]$ and $(13.122, 67.730]$, respectively. The strict lower bounds come from excluding radius zero.

Step 3: Take the union and count distinct integers The first interval supplies integer values $7, 8, \ldots, 52$, and the second supplies $14, 15, \ldots, 67$. Their overlap must be counted only once: the set S contains squared values, not the different parameters producing them. The union is therefore exactly all integers from 7 through 67, inclusive. Its cardinality is

$$|S| = 67 - 7 + 1 = 61.$$

Step 4: Conclusion The number of elements of S is $\boxed{61}$.

Quick Tip to Solve: When a parameter is squared, map its positive and negative intervals separately and combine the resulting sets before counting integers.

29. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to _____. [4 marks]

Solution:

Concept:

- The point $(b\cos\theta, 2a\sin\theta)$ lies on the ellipse with positive semi-axes b and $2a$. Its tangent is $x\cos\theta/b + y\sin\theta/(2a) = 1$.
- A triangle formed with the coordinate axes has area one-half the absolute product of its two intercepts. A finite triangle requires both sine and cosine to be nonzero.

Step 1: Find the tangent intercepts Setting $y = 0$ gives the x-intercept $b/\cos\theta$; setting $x = 0$ gives

the y-intercept $2a/\sin\theta$. Thus the area, irrespective of the quadrant of contact, is

$$A = \frac{1}{2}\left|\frac{b}{\cos\theta}\frac{2a}{\sin\theta}\right| = \frac{ab}{|\sin\theta\cos\theta|} = \frac{2ab}{|\sin 2\theta|}.$$

Tangents at the endpoints of the axes do not form finite intercept triangles and are excluded.

Step 2: Minimise the positive area Since $|\sin 2\theta| \le 1$, the denominator cannot exceed one, so $A \ge 2ab$. Equality is attainable: take $\theta = \pi/4$, which gives intercepts $\sqrt{2}b$ and $2\sqrt{2}a$. Their half-product is exactly $2ab$, proving that the bound is a minimum rather than only an infimum.

Step 3: Conclusion The minimum area is $2ab$ square units. Comparing it with kab, the dimensionless value is $\boxed{k = 2}$.

Quick Tip to Solve: Express an intercept triangle area using the tangent parameter; the bound on $|\sin 2\theta|$ gives the minimum immediately.

30. If $y^{1/4} + y^{-1/4} = 2x$, and
$(x^2-1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\ \alpha - \beta\ |$ is equal to _____. [4 marks]

Solution:

Concept:

- Write $u = y^{1/4} > 0$. Then $u + u^{-1} = 2x$, so the real branches satisfy $x \ge 1$ and $u = x \pm \sqrt{x^2-1}$.
- For $x > 1$, the two reciprocal branches obey opposite logarithmic derivatives. Keeping the branch sign explicit avoids choosing only one of the possible functions.

Step 1: Differentiate each branch Put $s = \sqrt{x^2-1}$ and $\varepsilon = +1$ or -1. Then $u = x + \varepsilon s$ and $u'/u = \varepsilon/s$. Since $y = u^4$, logarithmic differentiation gives

$$y' = \frac{4\varepsilon y}{s}.$$

This relation is valid on either smooth branch for $x > 1$; the branch point itself need not be differentiated.

Step 2: Eliminate the branch sign Differentiate once more, using $s' = x/s$:

$$y'' = \frac{4\varepsilon y'}{s} - \frac{4\varepsilon x y}{s^3} = \frac{16y}{s^2} - \frac{xy'}{s^2}.$$

Multiplication by $s^2 = x^2 - 1$ yields

$$(x^2-1)y'' + xy' - 16y = 0.$$

Both signs lead to the same equation, so the coefficient comparison applies to both real branches.

Step 3: Conclusion Comparing coefficients gives $\alpha = 1$ and $\beta = -16$. Thus the required dimensionless

value is

$$|\alpha - \beta| = |1 - (-16)| = \boxed{17}.$$

Quick Tip to Solve: For $u + u^{-1} = 2x$, use $u = x \pm \sqrt{x^2 - 1}$ and logarithmic differentiation; the branch sign disappears in the second-order equation.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin :- [4 marks]

(A) $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

(B) $E = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

(C) $E = \frac{2\varepsilon_0}{\sigma}\left(\frac{1}{(Z^2+R^2)^{1/2}} + Z\right)$

(D) $E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{(Z^2+R^2)} + \frac{1}{Z^2}\right)$

Correct Answer: (A) $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{(Z^2+R^2)^{1/2}}\right)$

Solution:

Concept:

- By rotational symmetry, the transverse electric-field components from a uniformly charged disc cancel on its axis. Only the component normal to the disc remains.
- A thin annular ring of radius r and width dr carries charge $dq = 2\pi\sigma r\, dr$. Its axial field at positive distance Z is $dE_z = (4\pi\varepsilon_0)^{-1} Z\, dq/(Z^2 + r^2)^{3/2}$.

Step 1: Integrate over concentric rings For the point on the positive z-axis, sum the ring contributions from $r = 0$ to R:

$$E_z = \frac{\sigma Z}{2\varepsilon_0}\int_0^R \frac{r\, dr}{(Z^2 + r^2)^{3/2}}.$$

Since the antiderivative is $-(Z^2 + r^2)^{-1/2}$,

$$E_z = \frac{\sigma Z}{2\varepsilon_0}\left(\frac{1}{Z} - \frac{1}{\sqrt{Z^2 + R^2}}\right).$$

Step 2: Check the physical limits and options This becomes $\sigma[1 - Z/\sqrt{Z^2 + R^2}]/(2\varepsilon_0)$. It tends to zero far from the finite disc, whereas the plus sign in the second expression would give a nonzero limit. The third expression inverts the field prefactor and adds quantities of unlike dimensions; the fourth has an extra inverse-length-squared factor. They cannot describe this electric field.

Step 3: Conclusion For $Z > 0$, $\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{\sqrt{Z^2 + R^2}}\right)\hat{\mathbf{z}}$, measured in $\mathrm{N\,C^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use rings for an axial disc field, and reject a plus sign by checking that a finite disc must have vanishing field at infinity.

32. Which of the following is not a dimensionless quantity? [4 marks]

(A) Relative magnetic permeability (μ_r)

(B) Power factor

(C) Permeability of free space (μ_0)

(D) Quality factor

Correct Answer: (C) Permeability of free space (μ_0)

Solution:

Concept:

- A dimensionless quantity is a pure ratio of quantities with the same dimensions, or a function of a dimensionless argument such as a phase angle.
- The magnetic force per unit length between parallel currents satisfies $F/L = \mu_0 I_1 I_2/(2\pi r)$, allowing the dimensions of permeability to be determined directly.

Step 1: Find the dimensions of free-space permeability Rearranging the parallel-wire force equation gives $\mu_0 = 2\pi r(F/L)/(I_1 I_2)$. Since r/L is dimensionless,

$$[\mu_0] = \frac{[F]}{[I]^2} = \mathrm{M\,L\,T^{-2}\,A^{-2}}.$$

Its SI unit is $\mathrm{N\,A^{-2}}$, equivalently $\mathrm{H\,m^{-1}}$. Thus it is a dimensional material constant, even though its numerical value in a chosen unit system can be written as a number.

Step 2: Check the remaining three quantities Relative permeability is $\mu_r = \mu/\mu_0$, so the dimensions cancel. Power factor is $\cos\phi$, also equal to real power divided by apparent power. Quality factor can be written $Q = 2\pi \times$ energy stored/energy lost per cycle; its numerator and denominator both have energy units. These three quantities are consequently dimensionless, leaving only free-space permeability.

Step 3: Conclusion Permeability of free space μ_0 has dimensions $\mathrm{M\,L\,T^{-2}\,A^{-2}}$, so it is not dimensionless. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish absolute permeability μ_0 from relative permeability μ/μ_0; only the ratio is dimensionless.

33. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be : [4 marks]

(A) ohm

(B) mho

(C) joule

(D) newton

Correct Answer: (A) ohm

Solution:

Concept:

- The electric-field intensity E has SI unit volt per metre, because E is the potential gradient in an electrostatic field.
- The magnetising-field intensity H has SI unit ampere per metre. It differs from magnetic flux density B, whose unit is tesla.

Step 1: Divide the SI units Write the units explicitly before cancelling the common length factor:

$$\left[\frac{E}{H}\right]_{\mathrm{SI}} = \frac{\mathrm{V\,m^{-1}}}{\mathrm{A\,m^{-1}}} = \mathrm{V\,A^{-1}}.$$

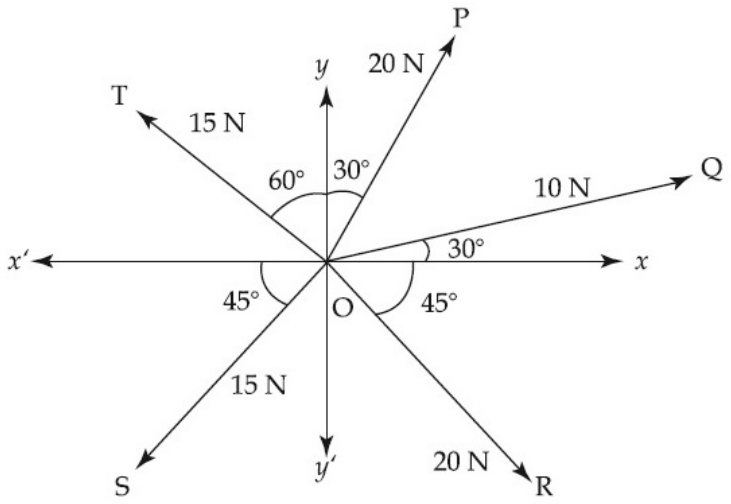
From Ohm's law $V = IR$, a volt divided by an ampere is an ohm, the unit of resistance or impedance. The cancellation involves metres only; no assumption about the numerical field strengths is required.

Step 2: Compare with the candidate units A mho, also called a siemens, is $\mathrm{A\,V^{-1}}$, the reciprocal unit, so it would describe H/E rather than E/H. A joule measures energy and has dimensions $\mathrm{M\,L^2T^{-2}}$. A newton measures force and has dimensions $\mathrm{M\,L\,T^{-2}}$. Neither includes the electric-current dependence of $\mathrm{V\,A^{-1}}$. Thus only the resistance unit is consistent.

Step 3: Conclusion The unit of E/H is $\mathrm{V/A} = \Omega$, or ohm. Hence, the correct option is (A).

Quick Tip to Solve: Remember the pair $E : \mathrm{V/m}$ and $H : \mathrm{A/m}$; their ratio has impedance units, while the reverse ratio has conductance units.

34. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately ..... N.
[Take $\sqrt{3} = 1.7$, $\sqrt{2} = 1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis] [4 marks]



(A) $9.25\hat{i} + 5\hat{j}$
(B) $3\hat{i} + 15\hat{j}$
(C) $2.5\hat{i} - 14.5\hat{j}$
(D) $-1.5\hat{i} - 15.5\hat{j}$
Correct Answer: (A) $9.25\hat{i} + 5\hat{j}$

Solution:

Concept:

- The resultant force is found by adding Cartesian components: $R_x = \sum F\cos\theta$ and $R_y = \sum F\sin\theta$, with θ measured from the positive x-axis.
- The diagram places OP, OQ, OR, OS and OT at $60°, 30°, -45°, 225°$ and $150°$, respectively. Use the supplied approximations $\sqrt{3} = 1.7$ and $\sqrt{2} = 1.4$.

Step 1: Resolve each force with the correct signs Using $\sin 45° = \cos 45° = \sqrt{2}/2 = 0.7$, the component pairs in newtons are

$$\vec{OP} = (10, 17), \qquad \vec{OQ} = (8.5, 5),$$
$$\vec{OR} = (14, -14), \qquad \vec{OS} = (-10.5, -10.5),$$
$$\vec{OT} = (-12.75, 7.5).$$

For OT, the ray is $60°$ to the left of the positive y-axis, hence $150°$ from positive x. Its x-component is negative and y-component positive.

Step 2: Add horizontal and vertical components separately The horizontal sum is

$$R_x = 10 + 8.5 + 14 - 10.5 - 12.75 = 9.25 \text{ N}.$$

The vertical sum is

$$R_y = 17 + 5 - 14 - 10.5 + 7.5 = 5 \text{ N}.$$

Both components are positive. The other candidates either give the wrong component magnitudes or place the resultant below the x-axis, contrary to the component sum; adding force magnitudes without directions would miss these cancellations.

Step 3: Conclusion The resultant is $\boxed{\vec{R} = (9.25\hat{\mathbf{i}} + 5\hat{\mathbf{j}})\ \mathrm{N}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert every marked angle to a signed direction from positive x before adding components; check each component sign from its quadrant.

---

35. A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is −7°C. Assuming the volume constant? [4 marks]

(A) 181.46 kg

(B) 214.15 kg

(C) 219.07 kg

(D) 123.54 kg

Correct Answer: (D) 123.54 kg

Solution:

Concept:

- At fixed volume, the mass of displaced air is proportional to air density. For air treated as an ideal gas of fixed composition, $\rho = PM/(R_{\mathrm{g}}T)$, so $\rho \propto P/T$.
- The load supported by the balloon in the stated idealised comparison scales with the displaced-air mass. Use absolute temperatures and the normal barometric pressure $76\ \mathrm{cm\ of\ Hg}$.

Step 1: Convert temperatures and form the density ratio At ground level, $T_1 = 27 + 273 = 300\ \mathrm{K}$ and $P_1 = 76\ \mathrm{cm\ of\ Hg}$. At the stated height, $T_2 = -7 + 273 = 266\ \mathrm{K}$ and $P_2 = 45\ \mathrm{cm\ of\ Hg}$. Therefore

$$\frac{\rho_2}{\rho_1} = \frac{P_2 T_1}{P_1 T_2} = \frac{45 \times 300}{76 \times 266}.$$

Pressure can remain in the same mercury-column units because it enters only as a ratio.

Step 2: Scale the supported total load The balloon volume is constant, so the corresponding supported load is

$$m_2 = 185 \frac{45 \times 300}{76 \times 266} = 123.541\ldots\ \mathrm{kg}.$$

The reduction in pressure outweighs the density increase caused by the lower temperature, so the load decreases. The values above the initial load are inconsistent with this density ratio; the other smaller candidate does not reproduce the pressure-temperature scaling. Using Celsius directly would give an unphysical negative result.

Step 3: Conclusion The balloon carries approximately $\boxed{123.54\ \mathrm{kg}}$ at the given height. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed-volume balloon, compare P/T using kelvin temperatures; the same pressure unit may be used in numerator and denominator.

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36. An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C, the radius of curvature of this mirror is : [4 marks]

(A) $\frac{2d_1d_2}{d_1-d_2}$
(B) $\frac{2d_1d_2}{d_1+d_2}$
(C) $\frac{d_1d_2}{d_1+d_2}$
(D) $\frac{d_1d_2}{d_1-d_2}$

Correct Answer: (A) $\frac{2d_1d_2}{d_1-d_2}$

Solution:

Concept:

- The mirror formula relates signed distances from the pole: $1/f = 1/u + 1/v$. For a concave mirror, the magnitude of the focal length is half its radius: $|f| = R/2$.
- An object beyond the centre of curvature forms a real image between the focus and centre. The two stated distances are measured from the centre, so they must first be converted to pole distances.

Step 1: Express the pole distances Let $R > 0$ be the radius of curvature. The object lies a further distance d_1 beyond the centre, whereas the image lies d_2 towards the pole from it. Hence the magnitudes are $U = R + d_1$ and $V = R - d_2$. Both object and real image lie on the reflecting side; using Cartesian signs gives $u = -U$, $v = -V$, and $f = -R/2$.

Step 2: Apply the mirror formula and simplify The negative signs cancel to give

$$\frac{2}{R} = \frac{1}{R+d_1} + \frac{1}{R-d_2}.$$

Combining the fractions and cross-multiplying,

$$2(R+d_1)(R-d_2) = R(2R+d_1-d_2).$$

After expansion, the $2R^2$ terms cancel, leaving

$$R(d_1-d_2) = 2d_1d_2.$$

Thus the denominator is a difference, not a sum. The factor 2 follows from $R = 2|f|$; omitting it would give the focal length rather than the requested radius. Options B and C use the wrong sum, while D misses this factor.

Step 3: Conclusion The radius of curvature is $\boxed{R = \frac{2d_1d_2}{d_1-d_2}}$. Hence, the correct option is (A).

Quick Tip to Solve: When distances are given from C, write the object and image distances from the pole as $R + d_1$ and $R - d_2$ before using the mirror formula.

37. A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second?

Given : 1 ly = 9.46 $\times$ 10^{15} m

1 AU = 1.5 $\times$ 10^{11} m [4 marks]

(A) 4.1 $\times$ 10^{8} s

(B) 4.5 $\times$ 10^{10} s

(C) 3.5 $\times$ 10^{6} s

(D) 7.2 $\times$ 10^{8} s

Correct Answer: (B) 4.5 $\times$ 10^{10} s

Solution:

Concept:

- Arc length obeys $s = r\theta$ only when θ is in radians. One degree contains 3600 seconds of arc, so 1 arcsecond $= \pi/(180 \times 3600)$ radians.
- Time equals total path length divided by speed. Four complete revolutions cover $4(2\pi r) = 8\pi r$; a light-year is a distance, not a time in this calculation.

Step 1: Convert the angle and find the radius The angle $4s$ denotes four seconds of arc. Therefore

$$\theta = \frac{4\pi}{180 \times 3600} = \frac{\pi}{162000} \text{ rad.}$$

The arc length in metres is

$$s = 4.4(9.46 \times 10^{15}) = 4.1624 \times 10^{16} \text{ m.}$$

Consequently, $r = s/\theta = 4.1624 \times 10^{16}(162000/\pi)$ m. Keeping π symbolic avoids an unnecessary rounding error.

Step 2: Calculate the time for four turns The speed is

$$v = 8(1.5 \times 10^{11}) = 1.2 \times 10^{12} \text{ m s}^{-1}.$$

Hence

$$t = \frac{8\pi r}{v} = \frac{8(162000)(4.1624 \times 10^{16})}{1.2 \times 10^{12}} = 4.495392 \times 10^{10} \text{ s.}$$

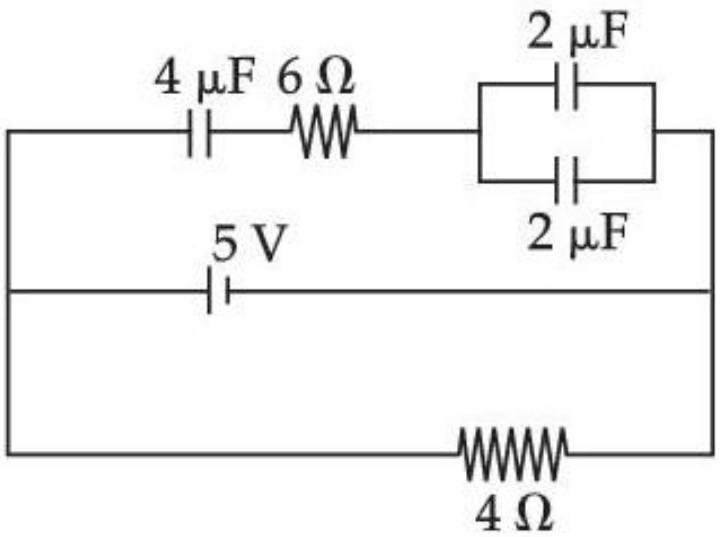
This rounds to 4.5×10^{10} s. The arc subtends an extremely small angle, so the circle is far larger than the given arc: using the arc itself as the circumference would grossly underestimate the time. Options A, C and D do not match the converted angle and four-revolution distance.

Step 3: Conclusion The required time is $\boxed{4.5 \times 10^{10} \text{ s}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert seconds of arc to radians first. In this problem the π in $8\pi r$ cancels the π in the angle exactly.

38. Calculate the amount of charge on capacitor of 4μF. The internal resistance of battery is 1Ω :

[4 marks]



(A) 8 μC

(B) zero

(C) 16 μC

(D) 4 μC

Correct Answer: (A) 8 μC

Solution:

Concept:

- After a DC circuit reaches steady state, a capacitor branch carries no conduction current. A resistor in that branch then has zero potential drop, although the capacitors can retain charge.
- The battery terminal voltage under load is $V = \mathcal{E} - Ir$. Parallel capacitances add; series capacitors carry equal charge and obey $1/C_{\text{eq}} = 1/C_1 + 1/C_2$.

Step 1: Find the loaded terminal voltage The upper branch is blocked by capacitors at steady state, so the only continuous current passes through the external $4\,\Omega$ resistor and the battery internal resistance $1\,\Omega$. Thus

$$I = \frac{5}{4+1} = 1\ \text{A}, \qquad V = I(4) = 4\ \text{V}.$$

Equivalently, $V = 5 - (1)(1) = 4$ V. The $6\,\Omega$ resistor is in the blocked upper branch and has no steady-state voltage drop; it must not be added to the conducting load resistance.

Step 2: Reduce the capacitors and calculate their charge The two $2\,\mu$F capacitors are parallel, giving

$$C_p = 2 + 2 = 4\ \mu\text{F}.$$

This combination is in series with the specified $4\,\mu$F capacitor. Therefore

$$C_{\text{eq}} = \frac{4 \times 4}{4+4} = 2\ \mu\text{F}, \qquad Q = C_{\text{eq}}V = 2(4) = 8\ \mu\text{C}.$$

The same series charge resides on the given capacitor. Zero current does not imply zero capacitor charge, excluding B. Multiplying $4\,\mu\text{F}$ by the full terminal voltage gives the incorrect $16\,\mu\text{C}$ in C; the two equal series capacitances each receive only half that voltage. Option D is also inconsistent with this division.

Step 3: Conclusion The charge on the $4\,\mu\text{F}$ capacitor is $\boxed{8\,\mu\text{C}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a steady DC capacitor network, first find the battery terminal voltage using only conducting branches, then reduce the capacitors across those terminals.

39. Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by : [4 marks]

(A) $\frac{Ml^2}{6}$

(B) Ml^2

(C) $\frac{Ml^2}{12}$

(D) $\frac{2}{3}Ml^2$

Correct Answer: (D) $\frac{2}{3}Ml^2$

Solution:

Concept:

- For a uniform rectangular lamina, the central perpendicular moment of inertia is $I_{\text{cm}} = M(a^2 + b^2)/12$. A square has $a = b = l$.
- The parallel-axis theorem gives $I = I_{\text{cm}} + Md^2$, where d is the perpendicular separation between two parallel axes and the first passes through the centre of mass.

Step 1: Calculate the central perpendicular inertia Treat the plate as a uniform thin square lamina. For axes through its centre along the two sides, symmetry gives $I_x = I_y = Ml^2/12$. The perpendicular-axis theorem then yields

$$I_{\text{cm}} = I_x + I_y = \frac{Ml^2}{6}.$$

This is the inertia about the centre, not the corner. Therefore option A stops before the required shift, and option C is the inertia about one in-plane central axis rather than the perpendicular one.

Step 2: Shift the axis from the centre to the corner The centre-to-corner distance is half the diagonal of the square:

$$d = \sqrt{(l/2)^2 + (l/2)^2} = \frac{l}{\sqrt{2}}, \qquad d^2 = \frac{l^2}{2}.$$

Both specified axes are perpendicular to the plate, so they are parallel and the parallel-axis theorem applies directly:

$$I_{\text{corner}} = \frac{Ml^2}{6} + M\frac{l^2}{2} = \frac{4Ml^2}{6} = \frac{2}{3}Ml^2.$$

The added term is positive, as expected because moving an axis away from the centre increases the mean squared distance of the mass from it. Option B would require an excessive shift contribution and does not follow from the actual half-diagonal distance.

Step 3: Conclusion The moment of inertia is $\boxed{I = \frac{2}{3}Ml^2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a corner axis of a square, use half the diagonal for the shift: $d^2 = l^2/2$, not $l^2/4$.

40. An ideal gas is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is : [4 marks]

(A) $\frac{1}{T}$
(B) $\frac{2}{T}$
(C) $\frac{4}{T}$
(D) $\frac{3}{T}$

Correct Answer: (C) $\frac{4}{T}$

Solution:

Concept:

- For a fixed amount of ideal gas, $PV = nR_gT$, where R_g is the universal gas constant. The pressure and temperature must follow the stated process relation.
- The fractional rate of volume change along the given expansion is $\beta = (1/V)(dV/dT) = d(\ln V)/dT$. Temperature is measured on the absolute Kelvin scale.

Step 1: Express the volume in terms of temperature Write the process condition as $PT^3 = K$, where K is constant. Therefore

$$P = \frac{K}{T^3}.$$

Substituting this pressure into the ideal-gas equation gives

$$V = \frac{nR_gT}{P} = \frac{nR_g}{K}T^4.$$

Thus the volume varies as the fourth power of temperature. The pressure decreases as the gas warms; ignoring that decrease would incorrectly treat the expansion as isobaric.

Step 2: Differentiate the process volume Let $A = nR_g/K$, so $V = AT^4$. At a point on this process,

$$\frac{dV}{dT} = 4AT^3, \qquad \beta = \frac{1}{AT^4}(4AT^3) = \frac{4}{T}.$$

The unit is inverse kelvin. Option A, $1/T$, applies to a fixed-pressure ideal-gas expansion and does not use the specified path. Option D retains only the magnitude of the pressure exponent and misses the extra power of T from $PV = nR_gT$. Option B likewise does not represent the power-law dependence derived above. The derivative here is along $PT^3 = K$, rather than a constant-pressure partial derivative.

Step 3: Conclusion Along the stated expansion, $\boxed{\beta = \frac{4}{T}}$, with SI unit K^{-1}. Hence, the correct option is (C).

Quick Tip to Solve: For an ideal-gas path PT^m = constant, first obtain $V \propto T^{m+1}$; its fractional volume derivative is $(m+1)/T$.

41. In a photoelectric experiment, increasing the intensity of incident light : [4 marks]

(A) increases the number of photons incident and also increases the K.E. of the ejected electrons

(B) increases the frequency of photons incident and increases the K.E. of the ejected electrons

(C) increases the frequency of photons incident and the K.E. of the ejected electrons remains unchanged

(D) increases the number of photons incident and the K.E. of the ejected electrons remains unchanged

Correct Answer: (D) increases the number of photons incident and the K.E. of the ejected electrons remains unchanged

Solution:

Concept:

- Each photon has energy $E = h\nu$. At a fixed frequency, increasing the intensity increases the photon flux, meaning the number incident per unit area per unit time.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the work function of the emitting surface. The maximum kinetic energy depends on frequency, not photon number.

Step 1: Identify what changes with intensity For monochromatic incident light, intensity is the energy delivered per unit area per unit time. If N photons arrive per unit area per unit time, then

$$\mathcal{I} = Nh\nu.$$

When the frequency remains the same, $h\nu$ remains the same. Increasing $\mathcal{I}$ therefore increases N. It does not make an individual photon more energetic or change its frequency. This immediately excludes B and C, both of which claim that frequency increases.

Step 2: Apply the energy balance to an emitted electron A single photon transfers its energy to an electron. For the most energetic emitted electron,

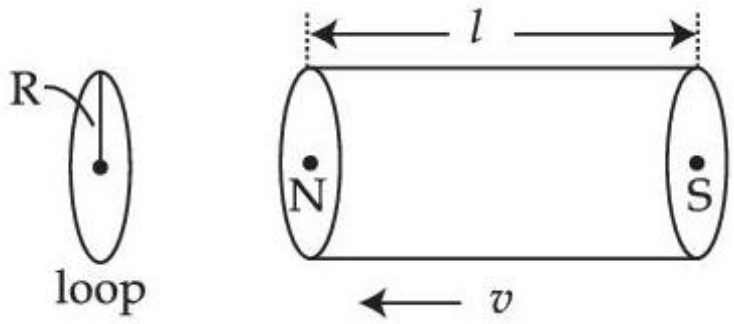
$$K_{\max} = h\nu - \phi.$$

Neither ν nor the material work function changes when only intensity increases. Hence the maximum kinetic energy and stopping potential are unchanged. More photons can release more electrons per unit time, increasing photoelectric current when emission and collection conditions allow it, but the kinetic-energy limit stays fixed. Option A incorrectly associates a larger photon arrival rate with a larger energy per emitted electron; an electron does not accumulate the energy of many ordinary incident photons in this experiment. These statements presume frequency above the threshold, as in a working photoelectric experiment.

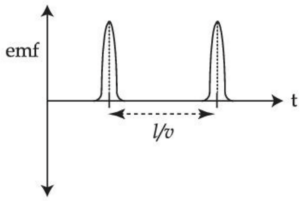
Step 3: Conclusion Increasing intensity increases the number of incident photons, while the kinetic-energy limit of the ejected electrons remains unchanged. Hence, the correct option is (D).

Quick Tip to Solve: Separate photon count from photon energy: intensity controls the count at fixed frequency, whereas ν controls $h\nu$ and the stopping potential.

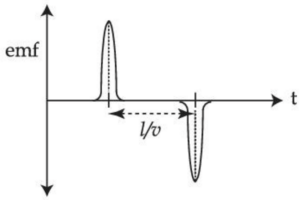
42. A bar magnet is passing through a conducting loop of radius R with velocity v. The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve : [4 marks]



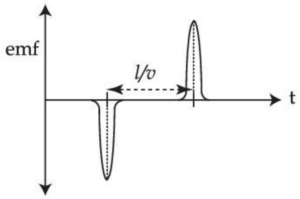
(A) Two positive pulses separated by l/v.



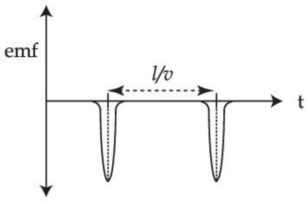
(B) A positive pulse followed by a negative pulse, separated by l/v.



(C) A negative pulse followed by a positive pulse, separated by l/v.



(D) Two negative pulses separated by l/v.



Correct Answer: Both B and C represent the same opposite-pulse emf under opposite choices of loop polarity; the question does not define that polarity.

Solution:

Concept:

- Faraday's law gives the oriented loop emf as $\mathcal{E} = -d\Phi_B/dt$. The sign depends on the chosen positive circulation and its corresponding area normal.

- A close-fitting long magnet changes the linked flux rapidly when either end crosses the loop. Entry and exit produce pulses of opposite signs, with their peak times approximately separated by l/v.

Step 1: Determine the pulse sequence independently of polarity As the leading north pole approaches and enters the loop, the linked flux grows in magnitude. While the loop surrounds the nearly uniform central portion of the magnet, the flux changes comparatively little and the emf is close to zero. As the trailing south pole leaves, the flux magnitude decreases back towards zero. The flux changes at entry and exit have opposite derivatives, so the induced emfs must have opposite signs. The pole separation is l, making the transit time between these events approximately $\Delta t = l/v$. Options A and D have pulses of the same sign and therefore fail this test.

Step 2: Check whether the sign convention is specified Choose the positive area normal in the direction of the field linked during entry. Then Φ_B first increases and later decreases, so $\mathcal{E}$ is first negative and then positive, as in curve C. Reversing the chosen normal reverses the positive loop circulation and multiplies both flux and emf by -1; the same physical induction is then drawn as curve B. The question labels neither a positive loop normal nor an ordered pair of voltage terminals. Consequently there is no stated polarity convention that distinguishes B from C. This is a sign ambiguity only: the opposite-pulse shape and the spacing l/v are fixed by Faraday's law.

Step 3: Conclusion The emf consists of opposite pulses separated by approximately l/v. Curve (C) follows when positive flux is chosen along the entering field; curve (B) is the same physical result with reversed polarity. Without a specified polarity, both (B) and (C) are defensible; there is no unique signed option.

Quick Tip to Solve: First determine whether entry and exit reverse $d\Phi_B/dt$. Then check the defined loop normal or voltage-terminal order before assigning a sign to an emf graph.

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43. Two ions of masses 4 amu and 16 amu have charges +2e and +3e respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same. Then : [4 marks]

(A) lighter ion will be deflected less than heavier ion

(B) lighter ion will be deflected more than heavier ion

(C) both ions will be deflected equally

(D) no ion will be deflected

Correct Answer: (B) lighter ion will be deflected more than heavier ion

Solution:

Concept:

- A perpendicular magnetic field supplies the centripetal force: $qvB = mv^2/r$, so the circular radius is $r = mv/(qB)$. Here m, q, v, B denote mass, charge magnitude, speed and field strength.
- For equal kinetic energy $K = mv^2/2$, momentum is $p = \sqrt{2mK}$. Therefore $r = \sqrt{2mK}/(qB)$, and the ion with the smaller radius bends more strongly.

Step 1: Express the radii at the common energy Label the lighter ion 1 and the heavier ion 2. The

magnetic force is always perpendicular to velocity, so it changes direction without changing kinetic energy. Because both ions enter perpendicular to the field, both move on curved paths. Their masses are $m_1 = 4\,\text{amu}$, $m_2 = 16\,\text{amu}$, and their charges are $q_1 = 2e$, $q_2 = 3e$. The common factors $\sqrt{2K}$ and B cancel when comparing radii.

Step 2: Compare the curvature and test the alternatives Using the mass and charge ratios gives

$$\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}\frac{q_2}{q_1}} = \sqrt{\frac{4}{16}\frac{3}{2}} = \frac{3}{4}.$$

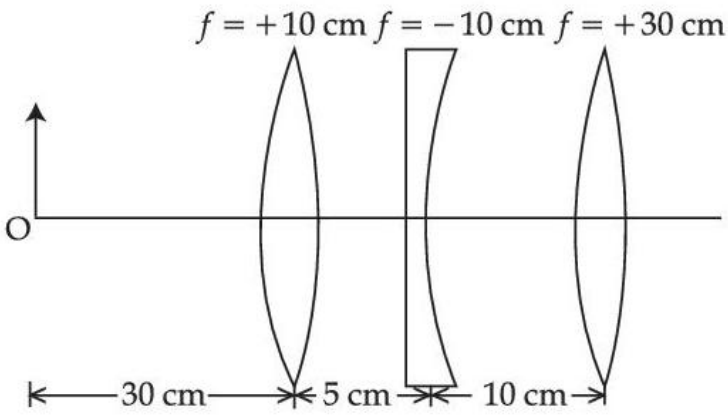
Thus the lighter ion has a radius only three-quarters that of the heavier ion. Its curvature $1/r$ is larger by a factor $4/3$, so it is deflected more. Option (A) reverses this comparison, and (C) would require equal radii. Option (D) is impossible because both charges are nonzero and their velocities are perpendicular to the magnetic field.

Step 3: Conclusion The lighter ion follows the more tightly curved path and is deflected more than the heavier ion. Hence, the correct option is (B).

Quick Tip to Solve: At equal kinetic energy compare $\sqrt{m}/q$, rather than m/q; equal energy does not mean equal speed.

44. Find the distance of the image from object O, formed by the combination of lenses in the figure :

[4 marks]



(A) 75 cm

(B) 10 cm

(C) 20 cm

(D) infinity

Correct Answer: (A) 75 cm

Solution:

Concept:

- For a thin lens, use the Cartesian convention $1/v - 1/u = 1/f$: distances to the right of each lens are positive for light travelling from left to right.

- The image formed by one lens is the object for the next. A converging beam has a virtual object on the right of the next lens, so its object distance is positive.

Step 1: Locate the image from the first lens Take the first lens as the coordinate origin. The object is at $x = -30\,\text{cm}$, the second lens at $x = 5\,\text{cm}$, and the third at $x = 15\,\text{cm}$. For the first convex lens, $f_1 = 10\,\text{cm}$ and $u_1 = -30\,\text{cm}$. Hence

$$\frac{1}{v_1} = \frac{1}{10} + \frac{1}{-30} = \frac{1}{15}, \qquad v_1 = 15\,\text{cm}.$$

Without the second lens, the first image would lie at the position of the third lens.

Step 2: Pass the converging beam through the second lens The second lens intercepts the beam before it reaches that first image. Its virtual object is $15 - 5 = 10\,\text{cm}$ to its right, so $u_2 = +10\,\text{cm}$. With $f_2 = -10\,\text{cm}$,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} = 0.$$

The rays emerging from the second lens are therefore parallel.

Step 3: Find the final image and measure from the object The third lens receives parallel rays and focuses them $f_3 = 30\,\text{cm}$ to its right. Thus its final image is at $x = 15 + 30 = 45\,\text{cm}$. The distance from the original object is

$$45 - (-30) = 75\,\text{cm}.$$

The final image is finite, so (D) refers only to the intermediate image after lens 2. Options (B) and (C) do not include the complete object-to-final-image separation.

Step 4: Conclusion The image is $75\,\text{cm}$ from the original object O. Hence, the correct option is (A).

Quick Tip to Solve: Keep one common position axis, but reset u at every lens; a converging incident beam gives a positive virtual-object distance.

45. In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^{3} kgm^{-3} ? Take viscosity of liquid = 1.8×10^{-5} Nsm^{-2}. (Neglect buoyancy due to air). [4 marks]

(A) 3.8×10^{-11} N

(B) 3.9×10^{-10} N

(C) 1.8×10^{-10} N

(D) 5.8×10^{-10} N

Correct Answer: (B) 3.9×10^{-10} N

Solution:

Concept:

- In the terminal-fall measurement of Millikan's experiment, an uncharged drop has no electric force. Neglecting air buoyancy, its upward viscous drag balances its downward weight.
- For a spherical drop, $m = \rho(4\pi r^3/3)$. At terminal speed v_t, Stokes'law gives $F_v = 6\pi\eta r v_t = mg$.

Step 1: Write the terminal-force balance An uncharged drop falls under gravity until the viscous resistance brings the acceleration to zero. The requested force in this terminal-fall experiment is therefore $F_{\text{v}} = mg$, directed upward. The quoted viscosity determines the terminal speed, but not the force required to balance the weight. Without the terminal condition, the drag would depend on the instantaneous speed; here the standard terminal-fall measurement is used.

Step 2: Evaluate the volume, mass and weight Insert $r = 2.0 \times 10^{-5}\,\text{m}$ and $\rho = 1.2 \times 10^{3}\,\text{kg m}^{-3}$:

$$V = \frac{4\pi}{3}(2.0 \times 10^{-5})^3 = 3.35 \times 10^{-14}\,\text{m}^3.$$

$$m = \rho V = 4.02 \times 10^{-11}\,\text{kg}.$$

$$F_{\text{v}} = mg = (4.02 \times 10^{-11})(9.8) \simeq 3.94 \times 10^{-10}\,\text{N}.$$

Rounding to the precision of the alternatives gives $3.9 \times 10^{-10}\,\text{N}$.

Step 3: Check the supplied viscosity and alternatives For consistency, $v_t = mg/(6\pi\eta r) \simeq 0.058\,\text{m s}^{-1}$; inserting this in Stokes'law recovers the same drag. Option (A) is about an order of magnitude too small. The values in (C) and (D) do not balance the calculated weight, so neither can represent the terminal drag.

Step 4: Conclusion The viscous force is approximately $3.9 \times 10^{-10}\,\text{N}$, upward. Hence, the correct option is (B).

Quick Tip to Solve: For an uncharged drop at terminal speed with buoyancy neglected, compute mg directly; viscosity is needed only if the speed is requested.

46. Electric field in a plane electromagnetic wave is given by E = 50 sin(500x − 10 × 10^{10} t) V/m The velocity of electromagnetic wave in this medium is :

(Given C = speed of light in vacuum) [4 marks]

(A) $\frac{3}{2}$C

(B) C

(C) $\frac{2}{3}$C

(D) $\frac{C}{2}$

Correct Answer: (C) $\frac{2}{3}$C

Solution:

Concept:

- A travelling plane wave has the form $E = E_0 \sin(kx - \omega t)$. Its phase speed is $v = \omega/k$, where k is the angular wave number and ω is angular frequency.
- The amplitude E_0 fixes the maximum electric field, not the propagation speed. The minus sign between kx and ωt indicates propagation along positive x.

Step 1: Read the coefficients of space and time Compare the supplied expression directly with

$E_0 \sin(kx - \omega t)$. The field amplitude is $50\,\mathrm{V\,m^{-1}}$, while

$$k = 500\,\mathrm{rad\,m^{-1}}, \qquad \omega = 10 \times 10^{10} = 10^{11}\,\mathrm{rad\,s^{-1}}.$$

The factor 10 multiplying 10^{10} must be retained. Both coefficients are angular quantities, so their ratio needs no extra factor of 2π.

Step 2: Calculate the speed from constant phase A crest satisfies $kx - \omega t =$ constant. Differentiating gives $k\,dx/dt = \omega$, hence

$$v = \frac{\omega}{k} = \frac{10^{11}}{500} = 2.0 \times 10^{8}\,\mathrm{m\,s^{-1}}.$$

The positive value of dx/dt also confirms that the wave travels in the positive x direction.

Step 3: Express the result in terms of the vacuum speed With $C = 3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$,

$$\frac{v}{C} = \frac{2.0 \times 10^{8}}{3.0 \times 10^{8}} = \frac{2}{3}.$$

Thus (C) gives the calculated speed. Option (A) would imply $4.5 \times 10^{8}\,\mathrm{m\,s^{-1}}$, (B) would imply 3.0×10^{8}, and (D) would imply 1.5×10^{8}; none equals the ratio of the printed coefficients.

Step 4: Conclusion The wave speed is $2.0 \times 10^{8}\,\mathrm{m\,s^{-1}} = 2C/3$, along positive x. Hence, the correct option is (C).

Quick Tip to Solve: Read k and ω from the sine argument, then divide the time coefficient by the space coefficient; ignore the field amplitude.

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47. Five identical cells each of internal resistance 1Ω and emf 5V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same? [4 marks]

(A) 1 Ω

(B) 25 Ω

(C) 5 Ω

(D) 10 Ω

Correct Answer: (A) 1 Ω

Solution:

Concept:

- For n identical cells aiding in series, the equivalent emf is nE and internal resistance is nr. The external current is $I_s = nE/(R + nr)$.
- For identical cells connected with like terminals in parallel, the emf remains E, while internal resistance becomes r/n. The current is $I_p = E/(R + r/n)$.

Step 1: Write the two current expressions Here $n = 5$, $E = 5\,\mathrm{V}$, and $r = 1\,\Omega$. The series combination

has emf $25\,\mathrm{V}$ and internal resistance $5\,\Omega$, so

$$I_s = \frac{25}{R+5}.$$

The parallel combination has emf $5\,\mathrm{V}$ and internal resistance $0.2\,\Omega$, so

$$I_p = \frac{5}{R+0.2}.$$

Both expressions give amperes when resistances are inserted in ohms.

Step 2: Equate the currents and solve for the load Require the same current through the same external resistance in the two separate arrangements:

$$\frac{25}{R+5} = \frac{5}{R+0.2}.$$

Cross-multiplication yields

$$25R + 5 = 5R + 25, \qquad 20R = 20, \qquad R = 1\,\Omega.$$

This is also the general result $R = r$ for any $n > 1$ identical cells when the two load currents are equal.

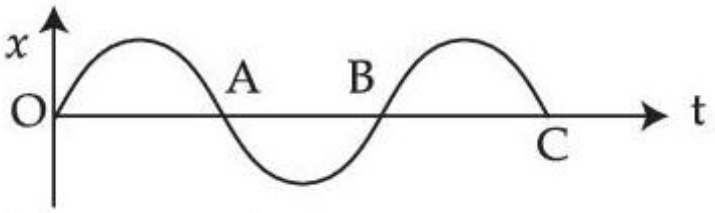
Step 3: Verify the result against the alternatives At $R = 1\,\Omega$, the two currents are $25/6\,\mathrm{A}$ and $5/1.2 = 25/6\,\mathrm{A}$, as required. At $R = 5\,\Omega$, they are $2.5\,\mathrm{A}$ and about $0.96\,\mathrm{A}$, so (C) fails. Options (B) and (D) also fail the derived equality $20R = 20$. The internal resistance of the cells must be included even though the external load is the same.

Step 4: Conclusion The required external resistance is $R = 1\,\Omega$. Hence, the correct option is (A).

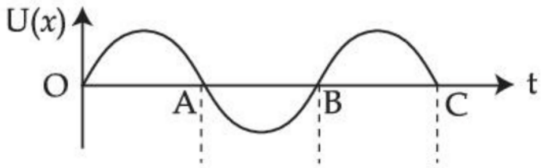
Quick Tip to Solve: For identical cells, equal load current in all-series and all-parallel arrangements occurs at $R = r$, independent of the number of cells.

48. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.

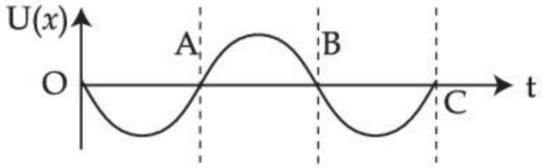
The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure : [4 marks]



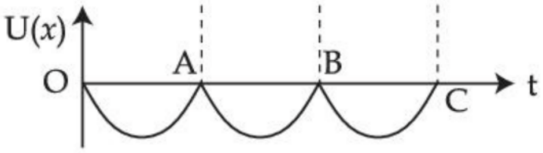
(A)



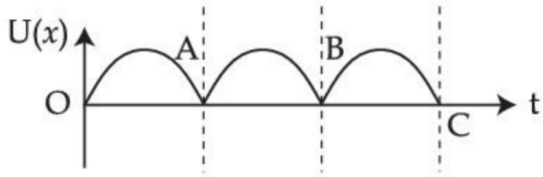
(B)



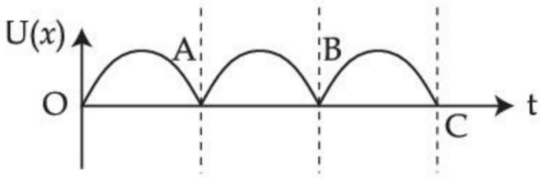
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- For an oscillator with effective spring constant $k > 0$, the potential energy relative to equilibrium is $U = \frac{1}{2}kx^2$. It vanishes at equilibrium and is largest at either extreme displacement.
- If $x = a\sin\omega t$, then $U = \frac{1}{2}ka^2\sin^2\omega t$. The energy repeats after $T/2$, half the displacement period $T = 2\pi/\omega$.

Step 1: Identify the zero-displacement instants The displacement graph begins at the origin O, then crosses the time axis at A, B and C. These instants correspond to $x = 0$, regardless of whether the particle is moving toward positive or negative displacement. Consequently the potential-energy graph must meet the time axis at O, A, B and C. Between every consecutive pair of these zero crossings, the particle reaches one extreme displacement.

Step 2: Use the square of displacement to locate energy maxima Both a positive extreme $x = +a$ and a negative extreme $x = -a$ have the same energy $U_{\max} = ka^2/2$. Thus a positive energy hump occurs in each of the intervals OA, AB and BC. In particular, the negative displacement during AB cannot produce negative potential energy, because the displacement is squared. The energy graph must stay on or above the time axis throughout the motion.

Step 3: Compare all four plotted alternatives Options (A) and (B) oscillate above and below zero, so neither represents $kx^2/2$. Option (C) gives negative humps and would make the equilibrium points energy maxima instead of minima. Option (D) alone has nonnegative humps and zeros at the required instants O, A, B and C. It also has twice as many energy oscillations as displacement oscillations, consistent with $\sin^2 \omega t = (1 - \cos 2\omega t)/2$.

Step 4: Conclusion The correct potential-energy plot is the nonnegative curve with zeros at O, A, B and C, shown in (D). Hence, the correct option is (D).

Quick Tip to Solve: To identify an SHM potential-energy graph, mark every zero of x, keep U nonnegative and double the oscillation frequency.

49. There are 10^{10} radioactive nuclei in a given radioactive element, its half-life time is 1 minute. How many nuclei will remain after 30 seconds? $(\sqrt{2} = 1.414)$ [4 marks]

(A) 2×10^{10}

(B) 7×10^{9}

(C) 10^{5}

(D) 4×10^{10}

Correct Answer: (B) 7×10^{9}

Solution:

Concept:

- Radioactive decay obeys $N = N_0 e^{-\lambda t} = N_0 (1/2)^{t/T_{1/2}}$. Here N_0 is the initial number, N the remaining number and $T_{1/2}$ the half-life.
- The fraction remaining is determined by the dimensionless ratio $t/T_{1/2}$. A fraction of a half-life must be used in the exponential law; decay is not linear in time.

Step 1: Put the elapsed time and half-life in the same unit The given initial count is $N_0 = 10^{10}$, and the half-life is $1\,\text{minute} = 60\,\text{s}$. The elapsed time is only $30\,\text{s}$, so

$$\frac{t}{T_{1/2}} = \frac{30}{60} = \frac{1}{2}.$$

Thus only half of one half-life has elapsed. The number has decreased, but it has not yet fallen to $N_0/2$.

Step 2: Calculate the surviving number Apply the decay formula with the fractional exponent:

$$N = 10^{10}\left(\frac{1}{2}\right)^{1/2} = \frac{10^{10}}{\sqrt{2}}.$$

Using the supplied $\sqrt{2} = 1.414$,

$$N = \frac{10^{10}}{1.414} \simeq 7.07 \times 10^{9}.$$

To the precision of the choices this is 7×10^9 nuclei. The complementary decayed number would instead be approximately 2.93×10^9, so it must not be confused with the requested surviving number.

Step 3: Check the magnitude and reject the other options Because $0 < t < T_{1/2}$, the surviving count must be between 5×10^9 and 10^{10}. Only option (B) lies in this interval. Options (A) and (D) exceed the initial count and cannot describe radioactive decay with no production. Option (C) is far below half the initial count, although less than one half-life has elapsed.

Step 4: Conclusion Approximately 7×10^9 radioactive nuclei remain after $30\,\text{s}$. Hence, the correct option is (B).

Quick Tip to Solve: For half of one half-life, divide the initial count by $\sqrt{2}$; do not subtract a fixed fraction linearly with time.

50. For a transistor in CE mode to be used as an amplifier, it must be operated in : [4 marks]

(A) Both cut-off and Saturation

(B) Saturation region only

(C) Cut-off region only

(D) The active region only

Correct Answer: (D) The active region only

Solution:

Concept:

- A common-emitter amplifier must reproduce small changes of the input signal as larger changes of collector current. In the forward-active region, the base-emitter junction is forward biased and the collector-base junction is reverse biased.
- The active-region current relation is approximately $I_C = \beta I_B$, where β is the common-emitter current gain. A quiescent operating point allows signal variations on either side without entering cut-off or saturation.

Step 1: Identify the region with controlled current gain For amplification, a small increase in base current must produce a corresponding increase in collector current. With the transistor biased in the active region, $\Delta I_C \approx \beta \Delta I_B$. The collector load converts this changing collector current into a changing output voltage. Thus both positive and negative small excursions of the input can be

represented in the output.

Step 2: Exclude the switching regions In cut-off, the base-emitter junction does not conduct appreciably, so the collector current is nearly zero. In saturation, both junctions are forward biased and the collector current is primarily limited by the external circuit rather than by the small-signal current-gain relation. These are useful OFF and ON states for switching, but neither alone provides the required linear amplification. Alternating between cut-off and saturation clips the signal rather than faithfully amplifying it. Therefore (A), (B) and (C) are excluded.

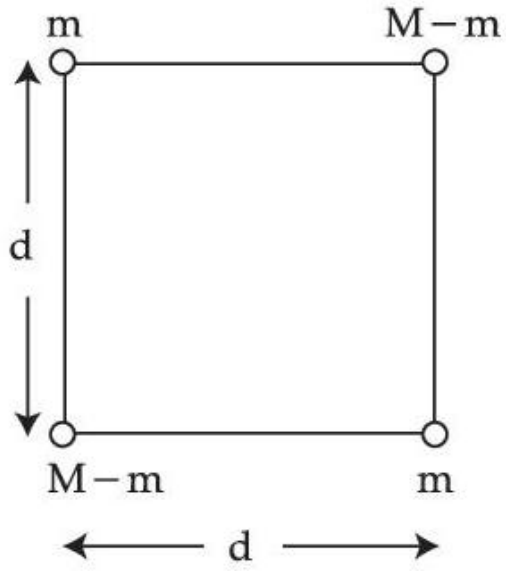
Step 3: Conclusion The transistor must be operated in the active region only, with a suitable bias point and a sufficiently small input signal. Hence, the correct option is (D).

Quick Tip to Solve: For a transistor amplifier, check for forward bias at the input junction and reverse bias at the collector junction; cut-off and saturation describe switching limits.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A body of mass (2M) splits into four masses (m, M − m, m, M − m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x : 1. The value of x is ..... . [4 marks]



Solution:

Concept:

- The gravitational potential energy of a pair of point masses is $-Gm_1m_2/r$, with zero energy at infinite separation. Add the contributions of all six distinct pairs, retaining the negative sign.
- For the square shown, unlike masses are adjacent and equal masses lie across diagonals. The four side distances are d, while each diagonal has length $\sqrt{2}d$. The total mass $2M$ and side d are fixed.

Step 1: Add the side and diagonal energies The side-pair mass product is $m(M-m)$ for each of the

four sides. The two diagonal products are m^2 and $(M-m)^2$. Thus

$$U = -\frac{G}{d}\left[4m(M-m) + \frac{m^2+(M-m)^2}{\sqrt{2}}\right].$$

Using $m^2+(M-m)^2 = M^2 - 2m(M-m)$, this becomes

$$U = -\frac{G}{d}\left[\frac{M^2}{\sqrt{2}} + (4-\sqrt{2})m(M-m)\right].$$

Step 2: Determine the nature of the stationary value Complete the square in the mass product:

$$m(M-m) = \frac{M^2}{4} - \left(m - \frac{M}{2}\right)^2.$$

Consequently

$$U = U_{\min} + \frac{G}{d}(4-\sqrt{2})\left(m - \frac{M}{2}\right)^2.$$

The coefficient of the square is positive, so $m = M/2$, or $M/m = 2$, is a minimum of the signed potential energy. Equivalently, $d^2U/dm^2 = 2G(4-\sqrt{2})/d > 0$. It maximises the magnitude of the negative potential energy, not the potential energy itself.

Step 3: Interpret the printed request literally For four positive masses, $0 < m < M$. The signed potential energy increases as m approaches either endpoint, and its supremum is $-GM^2/(\sqrt{2}d)$; this is not attained with all four masses positive. If zero masses are allowed, the maximum occurs at $m = 0$ or $m = M$; the ratio M/m is respectively undefined or 1. Neither interpretation makes the interior value 2 a maximum.

Step 4: Conclusion The supplied key value is $x = 2$, a dimensionless ratio, but the printed formula proves that this gives minimum gravitational potential energy (maximum binding-energy magnitude). As literally worded with four positive masses, the requested maximum has no attained value and no such finite x.

Quick Tip to Solve: With gravitational energy, distinguish maximum signed energy from maximum magnitude: the minus sign reverses the extremum of the positive pair sum.

52. The alternating current is given by $i = \left\{\sqrt{42}\sin\left(\frac{2\pi}{T}t\right) + 10\right\}$ A

The r.m.s. value of of this current is ..... A. [4 marks]

Solution:

Concept:

- The root-mean-square current is defined by $I_{\text{rms}} = \sqrt{\langle i^2\rangle}$, where the average is taken over a complete period. It determines the equivalent steady heating current in a resistor.
- Over one complete cycle, $\langle \sin\omega t\rangle = 0$ and $\langle \sin^2\omega t\rangle = 1/2$. A constant offset survives squaring; the cross term between that offset and a sinusoidal current has zero cycle average.

Step 1: Square the complete instantaneous current Write $\omega = 2\pi/T$ and retain the DC part while squaring:

$$i(t) = \sqrt{42}\sin\omega t + 10.$$

$$i^2(t) = 42\sin^2\omega t + 20\sqrt{42}\sin\omega t + 100.$$

Every term has units of A^2. Squaring only the sinusoidal term would discard the substantial heating contribution of the 10 A steady current.

Step 2: Average first, then take the square root Use the complete-period averages in the definition:

$$I_{\text{rms}}^2 = 42\left(\frac{1}{2}\right) + 20\sqrt{42}(0) + 100$$
$$= 21 + 100 = 121\ \mathrm{A}^2.$$

$$I_{\text{rms}} = \sqrt{121} = 11\ \mathrm{A}.$$

As a check, the AC part alone has RMS value $\sqrt{21}$ A. Its mean-square contribution adds to the DC mean-square contribution: $I_{\text{rms}}^2 = I_{\text{ac,rms}}^2 + I_{\text{dc}}^2$. The RMS current must therefore exceed 10 A, which it does. RMS values of these components are not added directly.

Step 3: Conclusion The root-mean-square current is $\boxed{11\ \mathrm{A}}$. The numerical entry is 11.

Quick Tip to Solve: For a sinusoid plus a constant offset, use $I_{\text{rms}} = \sqrt{I_{\text{dc}}^2 + I_0^2/2}$, not the sum of the two RMS values.

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53. A uniform conducting wire of length is 24a, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0. The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is 1 : $\sqrt{y}$ where y is ..... . [4 marks]

Solution:

Concept:

- The magnetic moment of a planar coil is $\mu = NIA$, where N is its number of turns, I is the current in each turn and A is the area enclosed by one turn.
- The same wire of length $24a$ and resistance R is reshaped without changing its total resistance. Connected across the same voltage V_0, both coils carry the same current $I = V_0/R$. The number of turns depends on the perimeter of one turn.

Step 1: Calculate the triangular coil moment Each equilateral triangular turn has perimeter $3a$, so

$$N_\triangle = \frac{24a}{3a} = 8.$$

The area of one turn is $A_\triangle = \sqrt{3}a^2/4$. Therefore

$$\mu_\triangle = 8\frac{V_0}{R}\frac{\sqrt{3}a^2}{4} = 2\sqrt{3}\frac{V_0 a^2}{R}.$$

The complete wire forms the eight-turn coil; treating it as a single triangle would neglect its multiple turns.

Step 2: Calculate the square moment and compare Each square turn has perimeter $4a$, so $N_{\square} = 24a/(4a) = 6$. Its area per turn is a^2, giving

$$\mu_{\square} = 6\frac{V_0 a^2}{R}.$$

The common factors of current and squared side cancel:

$$\frac{\mu_{\triangle}}{\mu_{\square}} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}}.$$

The ratio is therefore $1 : \sqrt{3}$, rather than the single-turn area ratio $\sqrt{3} : 4$. Comparing this with the requested form $1 : \sqrt{y}$ gives $\sqrt{y} = \sqrt{3}$.

Step 3: Conclusion The magnetic-moment ratio is $\mu_{\triangle} : \mu_{\square} = 1 : \sqrt{3}$, so $\boxed{y = 3}$. The parameter y is dimensionless.

Quick Tip to Solve: For coils made from the same fixed wire, compare NA: determine the turn count from the perimeter before comparing enclosed areas.

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54. First, a set of n equal resistors of 10 Ω each are connected in series to a battery of emf 20V and internal resistance 10Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is ..... . [4 marks]

Solution:

Concept:

- A battery of emf E and internal resistance r supplies current $I = E/(r + R_{\text{ext}})$. The internal resistance remains in series with the external network, even when that network contains parallel branches.
- For n identical resistors each of resistance R_0, the external equivalent resistance is nR_0 in series and R_0/n in parallel. Here $R_0 = r = 10\,\Omega$ and $E = 20\,\text{V}$.

Step 1: Write the current in each arrangement For the series arrangement the total resistance is $10n + 10 = 10(n + 1)\,\Omega$. Thus

$$I_s = \frac{20}{10(n+1)} = \frac{2}{n+1}\,\text{A}.$$

For the parallel arrangement, the external network has resistance $10/n\,\Omega$, while the battery still contributes $10\,\Omega$. Hence

$$I_p = \frac{20}{10 + 10/n} = \frac{2n}{n+1}\,\text{A}.$$

These are the total currents through the battery, not the current in a single parallel branch.

Step 2: Apply the measured current ratio The given increase means $I_p = 20I_s$. Divide the two

expressions before inserting a trial value:

$$\frac{I_p}{I_s} = \frac{2n/(n+1)}{2/(n+1)} = n.$$

Therefore $n = 20$. As a numerical check, $I_s = 2/21\,\text{A}$ and $I_p = 40/21\,\text{A}$; their ratio is exactly 20. The latter current is slightly less than the battery short-circuit limit $E/r = 2\,\text{A}$, which is physically consistent. Omitting internal resistance would instead give the incorrect ratio n^2.

Step 3: Conclusion The required number of equal resistors is $\boxed{n = 20}$. This is a dimensionless count.

Quick Tip to Solve: Include the battery internal resistance in both arrangements, then form the current ratio so that the common emf cancels.

55. If the velocity of a body related to displacement x is given by $v = \sqrt{5000 + 24x}$ m/s, then the acceleration of the body is ..... m/s^2. [4 marks]

Solution:

Concept:

- Acceleration is $a = dv/dt$. If velocity is specified as a function of position, apply the chain rule: $a = (dv/dx)(dx/dt) = v\,dv/dx$.
- Equivalently, $a = \frac{1}{2}\,d(v^2)/dx$. This form is particularly convenient when the square of velocity is a linear function of displacement, because no radical differentiation remains.

Step 1: Square the given position-dependent velocity The relation is $v = \sqrt{5000 + 24x}\,\text{m s}^{-1}$, with x measured in metres and the constants expressed in the corresponding SI units. Squaring gives

$$v^2 = 5000 + 24x.$$

The value 5000 determines the initial speed squared at $x = 0$; the coefficient of x controls how rapidly the kinetic energy per unit mass changes with displacement. It is the latter coefficient that determines the acceleration.

Step 2: Use the chain rule to find acceleration Differentiate with respect to position:

$$2v\frac{dv}{dx} = 24.$$

Since $a = v\,dv/dx$,

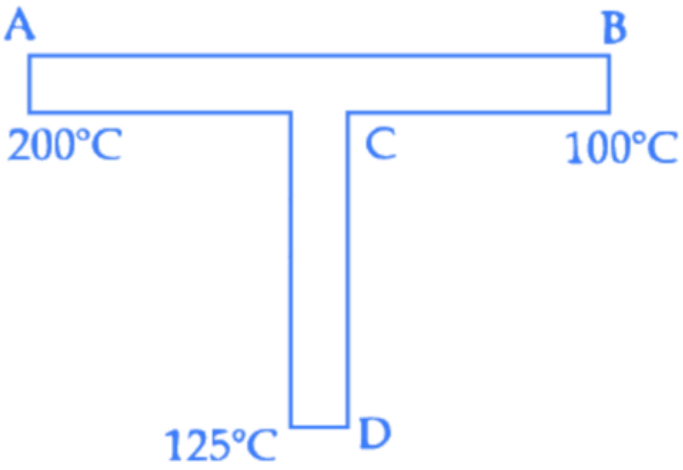
$$a = \frac{1}{2}\frac{d(v^2)}{dx} = \frac{24}{2} = 12\,\text{m s}^{-2}.$$

The result is constant over the domain of the stated motion. It can also be checked against $v^2 = u^2 + 2ax$: comparison gives $u^2 = 5000$ and $2a = 24$. Merely differentiating v with respect to x would give $12/\sqrt{5000 + 24x}$, which has units of inverse time and is not the acceleration; multiplication by velocity is essential.

Step 3: Conclusion The acceleration of the body is $\boxed{12\,\text{m s}^{-2}}$. The requested numerical entry is 12.

Quick Tip to Solve: When v is given as a square root of a linear expression in x, differentiate v^2 and halve its slope to obtain acceleration.

56. A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure. The end A, B and D are maintained at 200°C, 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is ..... . [4 marks]



Solution:

Concept:

- Thermal resistance relates heat current to temperature difference by $H = \Delta T / R_{\text{th}}$. For a uniform rod, resistance is proportional to length; half of a rod has half its total thermal resistance.
- At a steady junction with no heat source or storage, total heat current entering equals total heat current leaving. Rod CD and the full rod AB are identical, so each full rod has thermal resistance $10\,\text{K}\,\text{W}^{-1}$.

Step 1: Assign the branch resistances and junction temperature Let the common junction at the midpoint of AB have temperature T_C in degrees Celsius. Since AC and CB are equal halves of AB,

$$R_{AC} = R_{CB} = 5\,\text{K}\,\text{W}^{-1}, \qquad R_{CD} = 10\,\text{K}\,\text{W}^{-1}.$$

Temperature differences measured in degrees Celsius have the same numerical values as those measured in kelvin, so the given temperatures can be used directly in the conduction equations.

Step 2: Balance heat currents at C Take the heat arriving from A equal to that leaving towards B and D:

$$\frac{200 - T_C}{5} = \frac{T_C - 100}{5} + \frac{T_C - 125}{10}.$$

Multiplying by 10 and collecting terms yields

$$400 - 2T_C = 2T_C - 200 + T_C - 125,$$

$$5T_C = 725, \qquad T_C = 145^\circ\text{C}.$$

The heat current through CD is consequently

$$P = \frac{145 - 125}{10} = 2\,\text{W},$$

directed from C towards D. Check conservation: AC brings $55/5 = 11\,\text{W}$, while CB carries $45/5 = 9\,\text{W}$ and CD carries $2\,\text{W}$; the two outgoing currents sum to 11 W.

Step 3: Conclusion The heat current through rod CD is $\boxed{P = 2\,\text{W}}$, flowing from C to D. The numerical entry is 2.

Quick Tip to Solve: At a midpoint junction, halve the horizontal rod resistance before writing the heat-current balance; keep the full resistance for the vertical rod.

57. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ..... . [4 marks]

Solution:

Concept:

- For a constant force, work is the scalar product $W = \vec{F} \cdot \vec{d} = Fd\cos\theta$, where θ is the angle between force and displacement.
- Equal work over equal displacement means equal components of force along the displacement; the complete force magnitudes need not be equal.

Step 1: Write the work equations Let the common work be W, and let the force magnitudes be F_A and F_B. The displacement has the same magnitude d in the two cases. Only the component parallel to displacement contributes, so

$$W = F_A d\cos 45^\circ = F_B d\cos 60^\circ.$$

The components perpendicular to the displacement do no work. Thus comparing the full force magnitudes directly would ignore the different angles.

Step 2: Determine the force ratio Cancel the common nonzero displacement and rearrange the equality:

$$\frac{F_A}{F_B} = \frac{\cos 60^\circ}{\cos 45^\circ} = \frac{1/2}{1/\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

The larger angle for person B requires a larger force to supply the same forward component, consistent with this ratio being less than one.

Step 3: Compare with the specified expression The question writes this ratio as $1/\sqrt{x}$, with a positive square root. Therefore

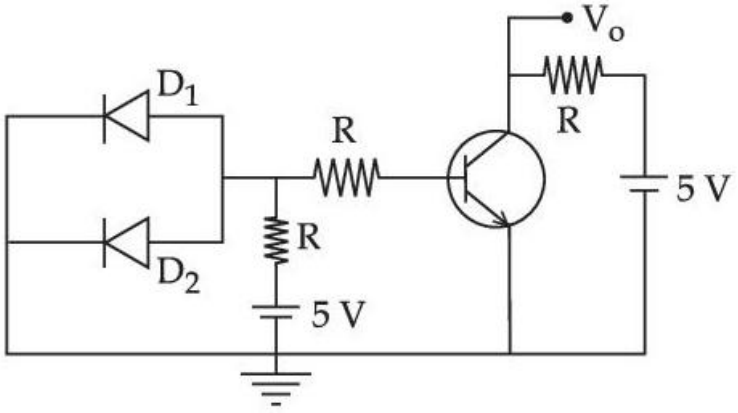
$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}} \quad \Longrightarrow \quad \sqrt{x} = \sqrt{2} \quad \Longrightarrow \quad x = 2.$$

Here x is dimensionless, since it is obtained from a ratio of forces.

Step 4: Conclusion The required value is $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: For equal work and displacement, equate $F\cos\theta$; the force ratio is the inverse ratio of the cosines.

58. A circuit is arranged as shown in figure. The output voltage V_0 is equal to ..... V. [4 marks]



Solution:

Concept:

- In the ideal switching model, a forward-biased diode conducts and clamps its connected node to the potential at its other terminal.
- An NPN transistor in cutoff has negligible base and collector current. With zero collector current, the collector resistor has no voltage drop.

Step 1: Identify the diode clamp Both diodes have their cathodes connected to the grounded left-hand wire. Their anodes meet at the node supplied from the left $5\,\mathrm{V}$ battery through a resistor. The positive supply tends to raise this node above ground, so the diodes become forward biased. In the ideal-diode model the conducting path holds the common-anode node at ground potential. The supply current is then diverted through the diode path to ground rather than providing a positive base drive.

Step 2: Determine the transistor state The transistor emitter is also grounded. Its base is connected through a resistor to the clamped node. Thus the base-emitter voltage is insufficient to turn the NPN transistor on:

$$V_{BE} = 0, \qquad I_B = 0.$$

The transistor is therefore in cutoff, and its collector current is $I_C = 0$. In particular, the circuit does not act as a saturated switch connecting the collector to the emitter.

Step 3: Evaluate the collector output The output V_0 is taken at the collector. The right-hand supply connects its positive $5\,\mathrm{V}$ terminal to the collector through the load resistance R. Applying the voltage-drop relation gives

$$V_0 = 5 - I_C R = 5 - 0 = 5\,\mathrm{V}.$$

No numerical value of R is needed because the collector branch carries no current in cutoff.

Step 4: Conclusion The output voltage is $\boxed{V_0 = 5\,\mathrm{V}}$.

Quick Tip to Solve: First decide whether the diode network supplies base drive. A cutoff NPN transistor leaves its collector output at the positive supply through the load resistor.

59. Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is ..... Hz. [4 marks]

Solution:

Concept:

- For a source and observer approaching each other through stationary air, the observed frequency is $f_{\text{obs}} = f(v + v_o)/(v - v_s)$.
- Here v is the speed of sound, v_o is the observer speed towards the source, and v_s is the source speed towards the observer. All speeds must use the same units.

Step 1: Identify the source and observer The whistle is produced in car X, so X is the source. Its speed is

$$v_s = 36 \times \frac{5}{18} = 10\,\text{m}\,\text{s}^{-1}.$$

The passenger in car Y is the observer, whose speed towards the source is

$$v_o = 72 \times \frac{5}{18} = 20\,\text{m}\,\text{s}^{-1}.$$

The given $1320\,\text{Hz}$ is the frequency received in Y, not the emitted frequency.

Step 2: Apply the approaching-motion formula The moving source reduces the wavelength in front of it, giving the denominator $v - v_s$; the approaching observer meets wavefronts faster, giving the numerator $v + v_o$. Thus

$$1320 = f\frac{340 + 20}{340 - 10} = f\frac{360}{330}.$$

Both effects raise the received frequency. Consequently, the actual frequency must be smaller than $1320\,\text{Hz}$.

Step 3: Solve for the actual frequency Rearranging before substitution avoids reversing the Doppler factor:

$$f = 1320\frac{330}{360} = 1320\frac{11}{12} = 1210\,\text{Hz}.$$

As a check, $1210 \times 360/330 = 1320$, which reproduces the observed frequency with the stated source and observer velocities.

Step 4: Conclusion The actual frequency of the whistle is $\boxed{1210\,\text{Hz}}$.

Quick Tip to Solve: Label the source and observer before substituting: approaching observer adds in the numerator; approaching source subtracts in the denominator.

60. A transmitting antenna has a height of 320 m and that of receiving antenna is 2000 m. The maximum distance between them for satisfactory communication in line of sight mode is 'd'. The value of 'd' is ..... km. [4 marks]

Solution:

Concept:

- For an antenna of height $h \ll R$ above a spherical Earth of radius R, its horizon distance is approximately $\sqrt{2Rh}$.
- The maximum line-of-sight separation of two antennas is the sum of their horizon distances: $d = \sqrt{2Rh_t} + \sqrt{2Rh_r}$. Use the standard Earth radius $R = 6.4 \times 10^6\,\text{m}$.

Step 1: Use both antenna horizons At the limiting separation, the direct ray just grazes the Earth. Each antenna contributes the distance from its top to the common tangent point. For either antenna, the right-triangle relation is

$$\ell^2 = (R+h)^2 - R^2 = 2Rh + h^2 \simeq 2Rh.$$

The heights $320\,\text{m}$ and $2000\,\text{m}$ are small compared with Earth's radius, so neglecting h^2 is justified. Add the two horizon distances rather than using only the taller antenna.

Step 2: Calculate the transmitting horizon For $h_t = 320\,\text{m}$,

$$d_t = \sqrt{2(6.4 \times 10^6)(320)} = \sqrt{4.096 \times 10^9} = 64000\,\text{m} = 64\,\text{km}.$$

All quantities inside the square root have first been expressed in metres; its result is also in metres.

Step 3: Calculate and add the receiving horizon For $h_r = 2000\,\text{m}$,

$$d_r = \sqrt{2(6.4 \times 10^6)(2000)} = \sqrt{2.56 \times 10^{10}} = 160000\,\text{m} = 160\,\text{km}.$$

Therefore

$$d = d_t + d_r = 64 + 160 = 224\,\text{km}.$$

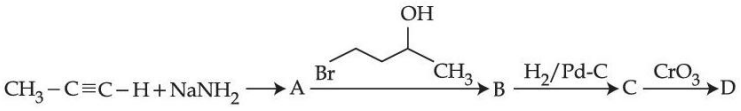
This is the limiting geometrical range for direct communication; no sky-wave reflection is being used.

Step 4: Conclusion The maximum line-of-sight distance is $\boxed{224\,\text{km}}$.

Quick Tip to Solve: Compute each antenna's horizon distance separately and add them; do not combine the two heights inside a single square root.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. In the following sequence of reactions, the final product D is : [4 marks]



(A) $CH_3CH_2CH_2CH_2CH_2CHO$

(B) $CH_3CH{=}CHCH_2CH_2CH_2COOH$

(C) $CH_3CH{=}CHCH(OH)CH_2CH_2CH_3$

(D) $CH_3CH_2CH_2CH_2CH_2COCH_3$

Correct Answer: (D) $CH_3CH_2CH_2CH_2CH_2COCH_3$

Solution:

Concept:

- A terminal alkyne reacts with $NaNH_2$ to form an acetylide nucleophile, which makes a carbon-carbon bond by substitution at a primary alkyl bromide.
- $H_2/Pd{-}C$ completely hydrogenates the triple bond. Subsequent oxidation of a secondary alcohol with CrO_3 gives a ketone without changing the carbon skeleton.

Step 1: Form and alkylate the acetylide Propyne contains an acidic terminal hydrogen. Its removal gives A:

$$CH_3{-}C{\equiv}C{-}H + NaNH_2 \longrightarrow CH_3{-}C{\equiv}C^-\,Na^+ + NH_3.$$

The bromide reagent is $BrCH_2CH_2CH(OH)CH_3$. Displacement of bromide by the acetylide joins the terminal alkyne carbon to the bromomethyl carbon, producing the seven-carbon intermediate B,

$$CH_3{-}C{\equiv}C{-}CH_2{-}CH_2{-}CH(OH){-}CH_3.$$

The hydroxyl group remains on its original secondary carbon.

Step 2: Hydrogenate and oxidise in sequence Catalytic hydrogenation over palladium on carbon reduces the triple bond fully, rather than stopping at the alkene. Thus C is

$$CH_3CH_2CH_2CH_2CH_2CH(OH)CH_3.$$

Oxidation converts the secondary $-CH(OH)-$ centre into $-CO-$, giving

$$CH_3CH_2CH_2CH_2CH_2COCH_3.$$

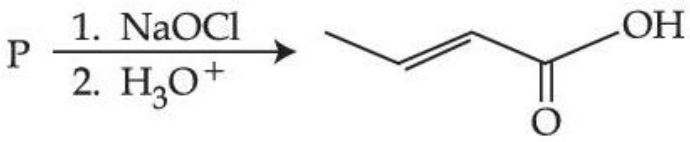
Numbering from the carbonyl end identifies this product as heptan-2-one.

Step 3: Compare all product options Option D is the seven-carbon methyl ketone just obtained. Option A is an aldehyde and has the wrong terminal functional group and carbon count. Option B retains a carbon-carbon double bond and is a carboxylic acid, although the alcohol was secondary. Option C retains both an alkene and an alcohol, contradicting the complete hydrogenation and subsequent oxidation. Thus only D follows every reagent in the stated order.

Step 4: Conclusion The final product D is heptan-2-one, $CH_3CH_2CH_2CH_2CH_2COCH_3$. Hence, the correct option is (D).

Quick Tip to Solve: Track the carbon skeleton during acetylide alkylation, then change only the functional groups affected by hydrogenation and oxidation.

62. The structure of the starting compound P used in the reaction given below is : [4 marks]



(A) $CH_3CH{=}CHCOCH_3$

(B) $CH_3CH_2CH{=}CHCHO$

(C) Six-membered cyclic enol ether: ring sequence $O{-}CH{=}CH{-}CH_2{-}CH_2{-}CH_2$, closed back to O.

(D) Cyclopentanone (a five-membered ring containing $C{=}O$).

Correct Answer: (A) $CH_3CH{=}CHCOCH_3$

Solution:

Concept:

- A methyl ketone $RCOCH_3$ undergoes haloform cleavage with hypochlorite, forming $RCOO^-$; acidification then gives $RCOOH$.
- The methyl carbon next to the carbonyl is lost in haloform formation, while the group R remains attached to the carbonyl carbon in the carboxylic acid.

Step 1: Read the product skeleton The product drawn is $CH_3CH{=}CHCOOH$, but-2-enoic acid. Thus the group attached to its carboxyl carbon is $CH_3CH{=}CH^-$. To obtain this acid by the methyl-ketone haloform reaction, the starting material must carry this same group next to $COCH_3$. Working backwards therefore gives

$$P{=}CH_3CH{=}CHCOCH_3.$$

This is pent-3-en-2-one, with five carbons before cleavage.

Step 2: Apply the reagents in the given order In the first stage, $NaOCl$ successively chlorinates the methyl group adjacent to the ketone carbonyl. Cleavage then produces the carboxylate and chloroform. For this substrate the retained carboxylate is $CH_3CH{=}CHCOO^-$. In the second stage, H_3O^+ protonates it:

$$CH_3CH{=}CHCOO^- + H_3O^+ \longrightarrow CH_3CH{=}CHCOOH + H_2O.$$

The carbon-carbon double bond in the R group is retained in the transformation shown.

Step 3: Eliminate the other structures Option A is exactly $CH_3CH{=}CHCOCH_3$, the required methyl ketone. Option B is an aldehyde with a longer carbon chain and cannot give the displayed four-carbon acid by this methyl-ketone cleavage. Option C is a cyclic enol ether and does not possess $-COCH_3$.

Option D is cyclopentanone, whose carbonyl is inside a ring and likewise has no methyl-ketone group. These structures do not match the indicated haloform route.

Step 4: Conclusion The starting compound is $CH_3CH{=}CHCOCH_3$, pent-3-en-2-one. Hence, the correct option is (A).

Quick Tip to Solve: For a haloform synthesis, replace the product's $-COOH$ by $-COCH_3$ while preserving the rest of its carbon skeleton.

63. Match List - I with List - II :

Choose the most appropriate answer from the options given below : [4 marks]

	List - I (Species)		List - II (No. of lone pairs of electrons on the central atom)
(a)	XeF_2	(i)	0
(b)	XeO_2F_2	(ii)	1
(c)	XeO_3F_2	(iii)	2
(d)	XeF_4	(iv)	3

(A) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(B) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(C) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(D) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

Correct Answer: (D) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

Solution:

Concept:

- Xenon contributes eight valence electrons. In the usual Lewis structures, each Xe-F single bond uses one xenon electron and each Xe=O double bond uses two.
- The xenon electrons left after bond formation form lone pairs. A double bond is one VSEPR domain, but it still requires two xenon electrons for electron counting.

Step 1: Count lone pairs in the first two species In XeF_2, two single bonds use two of xenon's eight valence electrons. The remaining six electrons form three lone pairs, so (a) matches (iv). In XeO_2F_2, the two Xe=O bonds use four xenon electrons and the two Xe-F bonds use two more. Two electrons remain, forming one lone pair. Hence (b) matches (ii). The oxygen double bonds must not be counted as single bonds in this electron bookkeeping.

Step 2: Count lone pairs in the remaining species For XeO_3F_2, three Xe=O bonds and two Xe-F bonds use all eight xenon electrons:

$$N_{LP} = \frac{8 - 3(2) - 2}{2} = 0.$$

Thus (c) matches (i). For XeF_4, four single bonds leave four xenon electrons:

$$N_{LP} = \frac{8-4}{2} = 2.$$

Thus (d) matches (iii). These counts also agree with the familiar linear XeF_2 and square-planar XeF_4 structures.

Step 3: Choose the complete matching The required sequence is

$$(a):(iv), \quad (b):(ii), \quad (c):(i), \quad (d):(iii).$$

Option D reproduces all four correspondences. Option A assigns zero lone pairs to XeO_2F_2 and one to XeO_3F_2, reversing their correct counts. Options B and C already fail for XeF_2, assigning two rather than three lone pairs, and they also fail for XeF_4.

Step 4: Conclusion The lone-pair counts in the listed order are three, one, zero and two. Hence, the correct option is (D).

Quick Tip to Solve: For these xenon species use $N_{LP} = (8 - N_F - 2N_O)/2$, with oxygen represented by Xe=O bonds.

64. Match items of List-I with those of List-II :

	List - I(Property)		List - II(Example)
(a)	**Diamagnetism**	**(i)**	**MnO**
(b)	**Ferrimagnetism**	**(ii)**	O_2
(c)	**Paramagnetism**	**(iii)**	**NaCl**
(d)	**Antiferromagnetism**	**(iv)**	Fe_3O_4

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(B) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(C) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(D) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

Correct Answer: (C) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Solution:

Concept:

- Diamagnetic substances have paired electrons and are weakly repelled by an applied magnetic field. Paramagnetic substances have unpaired electrons and are attracted by it.
- Antiferromagnets have equal antiparallel magnetic moments, giving zero resultant moment. Ferrimagnets have unequal antiparallel moments and therefore retain a net moment.

Step 1: Identify the non-ordered examples In NaCl, both Na^+ and Cl^- have closed-shell electron configurations. It is diamagnetic, so (a) matches (iii). Molecular oxygen has two unpaired electrons in its antibonding molecular orbitals and is paramagnetic, so (c) matches (ii). The presence of oxygen

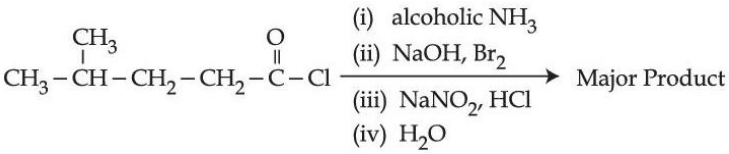
atoms in an oxide alone does not make that solid paramagnetic in the same way as the oxygen molecule.

Step 2: Identify the ordered magnetic solids In MnO, the magnetic moments of neighbouring manganese ions are arranged in equal and opposite directions below its ordering temperature. It is the antiferromagnetic example: (d)-(i). Magnetite, Fe_3O_4, has opposing sublattice moments that do not cancel completely. It is ferrimagnetic: (b)-(iv). Thus the full matching is (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i). Options (A), (B) and (D) each put the closed-shell salt or oxygen under an incompatible magnetic class.

Step 3: Conclusion Diamagnetism matches NaCl, ferrimagnetism matches magnetite, paramagnetism matches oxygen, and antiferromagnetism matches MnO. Hence, the correct option is (C).

Quick Tip to Solve: First identify closed-shell NaCl and the two unpaired electrons of oxygen; these two matches already isolate the correct row.

65. The major product of the following reaction is : [4 marks]



(A) $CH_3CH(CH_3)CHBrCH_2OH$

(B) $CH_3CH(CH_3)CH_2CH_2CH_2OH$

(C) $CH_3CH(CH_3)CH_2CH_2OH$

(D) $CH_3CH(CH_3)CH_2CH_2Cl$

Correct Answer: (C) $CH_3CH(CH_3)CH_2CH_2OH$

Solution:

Concept:

- An acid chloride reacts with ammonia to give a primary amide. Hofmann bromamide degradation converts $RCONH_2$ into RNH_2, removing the carbonyl carbon.
- A primary aliphatic amine gives the corresponding alcohol on treatment with nitrous acid, generated from $NaNO_2$ and HCl, followed by water.

Step 1: Form the amide and shorten the chain The starting acid chloride is $CH_3CH(CH_3)CH_2CH_2COCl$. Alcoholic ammonia replaces chlorine at the acyl group to give $CH_3CH(CH_3)CH_2CH_2CONH_2$. Bromine and sodium hydroxide then bring about Hofmann degradation. The carbonyl carbon is lost, while the alkyl group migrates intact. Thus the amine obtained is $CH_3CH(CH_3)CH_2CH_2NH_2$, with five carbon atoms rather than the six in the starting acid chloride.

Step 2: Replace the amino group by hydroxyl In the next step nitrous acid converts the primary aliphatic amino group into a highly unstable diazonium group. Loss of nitrogen and reaction with

water give the alcohol represented in option (C):

$$CH_3CH(CH_3)CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2} CH_3CH(CH_3)CH_2CH_2OH.$$

Option (B) incorrectly retains the carbonyl carbon as an extra methylene group. Option (A) introduces a carbon-bromine bond that Hofmann degradation does not leave in the alkyl chain. Option (D) is an alkyl chloride rather than the product of the final aqueous nitrous-acid treatment.

Step 3: Conclusion The major product is $CH_3CH(CH_3)CH_2CH_2OH$, 3-methylbutan-1-ol. Hence, the correct option is (C).

Quick Tip to Solve: Mark the acid chloride carbonyl carbon before starting: Hofmann degradation removes that carbon, and the final step replaces the terminal amino group by hydroxyl.

66. Which of the following is not a correct statement for primary aliphatic amines? [4 marks]

(A) The intermolecular association in primary amines is less than the intermolecular association in secondary amines.

(B) Primary amines on treating with nitrous acid solution from corresponding alcohols except mythyl amine.

(C) Primary amines are less basic than the secondary amines.

(D) Primary amines can be prepared by the Gabriel phthalimide synthesis.

Correct Answer: As printed, both (A) and (B) are incorrect; the supplied answer key selects (A).

Solution:

Concept:

- Primary amines contain two N–H bonds per molecule and form intermolecular hydrogen bonds. A comparable secondary amine contains only one N–H bond.
- Basicity in aqueous solution depends on electron donation as well as solvation; secondary alkylamines are generally more basic than corresponding primary alkylamines.

Step 1: Check the intermolecular association statement The nitrogen lone pair accepts hydrogen bonds, while the N–H bonds act as donors. Primary amines have two N–H bonds and can build a more extensive intermolecular hydrogen-bond network than comparable secondary amines. Hence the claim in option (A), that primary amines associate less than secondary amines, reverses the usual order. For isomeric amines the hydrogen-bond contribution to boiling point generally decreases from primary to secondary to tertiary.

Step 2: Compare the remaining chemical properties Option (B) states an exception for methylamine. Taken literally, this is also incorrect: methylamine reacts with nitrous acid to give methanol, with nitrogen evolution, just as other primary aliphatic amines give alcohols. In particular,

$$CH_3NH_2 + HNO_2 \longrightarrow CH_3OH + N_2 + H_2O.$$

Thus methylamine is not an exception to alcohol formation. The wordings "from" and "mythyl" do not change this chemical issue. Secondary alkylamines generally show greater aqueous basicity

than comparable primary alkylamines, so (C) states the usual trend. Gabriel phthalimide synthesis produces primary aliphatic amines without successive alkylation, making (D) correct.

Step 3: Conclusion The supplied key selects (A), whose hydrogen-bonding comparison is reversed. However, the printed methylamine exception also makes (B) incorrect. Consequently the literal question has no unique incorrect statement: both (A) and (B) qualify.

Quick Tip to Solve: Compare N–H bond counts for hydrogen bonding, then test any claimed exception explicitly: methylamine with nitrous acid gives methanol and nitrogen.

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67. Acidic ferric chloride solution on treatment with excess of potassium ferrocyanide gives a Prussian blue coloured colloidal species. It is : [4 marks]

(A) $Fe_4[Fe(CN)_6]_3$

(B) $K_5Fe[Fe(CN)_6]_2$

(C) $HFe[Fe(CN)_6]$

(D) $KFe[Fe(CN)_6]$

Correct Answer: (D) $KFe[Fe(CN)_6]$

Solution:

Concept:

- Ferrocyanide is $[Fe(CN)_6]^{4-}$, containing iron in the +2 oxidation state. Ferric chloride supplies Fe^{3+} ions.
- Prussian blue can occur as potassium-containing soluble/colloidal Prussian blue or as the insoluble neutral ferric ferrocyanide. The proportions and excess reagent distinguish these forms.

Step 1: Use the specified excess ferrocyanide The question explicitly treats ferric chloride with excess potassium ferrocyanide. Under these conditions the potassium-containing Prussian blue species is $KFe[Fe(CN)_6]$. Its formation is represented by

$$FeCl_3 + K_4[Fe(CN)_6] \longrightarrow KFe[Fe(CN)_6] + 3\,KCl.$$

One potassium ion is retained per ferric ion and ferrocyanide unit. The charge check is

$$(+1) + (+3) + (-4) = 0,$$

so the proposed formula is electrically neutral.

Step 2: Distinguish the alternative Prussian blue formula Option (A), $Fe_4[Fe(CN)_6]_3$, is the familiar insoluble ferric ferrocyanide composition, favoured with excess ferric salt. It balances four ferric cations against three ferrocyanide anions but does not express the potassium-containing colloidal species specified with excess ferrocyanide here. Option (B) is also charge-balanced with ferric iron and two ferrocyanide anions: $5 + 3 - 8 = 0$ is neutral, but its two ferrocyanide units do not give the 1:1 ferric-ferrocyanide composition of this Prussian blue species. Option (C) replaces the retained potassium by hydrogen; acid is present, but the specified ferrocyanide reagent supplies potassium and

gives the salt in (D).

Step 3: Conclusion With excess potassium ferrocyanide, the colloidal Prussian blue species is $KFe[Fe(CN)_6]$. The supplied key indicates the first option, the insoluble ferric ferrocyanide formula; the stated excess-ferrocyanide condition instead selects the potassium-containing form. Hence, the correct option is (D).

Quick Tip to Solve: Read which reagent is in excess: excess ferric salt gives the neutral insoluble ferric ferrocyanide formula, whereas excess potassium ferrocyanide gives the potassium-containing blue colloidal form.

68. The nature of oxides V_2O_3 and CrO is indexed as 'X' and 'Y' type respectively. The correct set of X and Y is : [4 marks]

(A)

X = basic		Y = amphoteric

(B)

X = amphoteric		Y = basic

(C)

X = acidic		Y = acidic

(D)

X = basic		Y = basic

Correct Answer: (D)

X = basic		Y = basic

Solution:

Concept:

- For a transition metal, oxides in lower oxidation states are generally basic; at high oxidation states the oxides become more covalent and acidic.
- The oxidation state is obtained by assigning oxygen -2 and balancing the total charge of the neutral oxide.

Step 1: Find the oxidation states For V_2O_3, let the oxidation state of vanadium be x. Charge balance gives

$$2x + 3(-2) = 0, \qquad x = +3.$$

For CrO, let the chromium oxidation state be y. Then

$$y + (-2) = 0, \qquad y = +2.$$

Both oxides place their metals in relatively low oxidation states. In particular, chromium(II) oxide must not be confused with chromium(III) oxide, which has different acid-base behaviour.

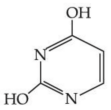
Step 2: Assign the acid-base character and eliminate alternatives Vanadium(III) oxide is basic, whereas the higher vanadium(V) oxide is amphoteric with a strong acidic tendency. Chromium(II) oxide is also basic; chromium(III) oxide is the familiar amphoteric chromium oxide and chromium(VI) oxide is acidic. Thus X = basic and Y = basic. Options (A) and (B) each incorrectly assign amphoterism to one of these lower oxides. Option (C) incorrectly describes both as acidic, a behaviour more characteristic of sufficiently high oxidation states.

Step 3: Conclusion Both V_2O_3 and CrO are basic oxides: X = basic, Y = basic. Hence, the correct option is (D).

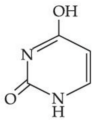
Quick Tip to Solve: Determine oxidation states before recalling oxide character; CrO contains chromium(II), not the chromium(III) of amphoteric chromium(III) oxide.

69. Out of following isomeric forms of uracil, which one is present in RNA? [4 marks]

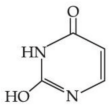
(A) 2,4-Dihydroxypyrimidine (the dienol form)



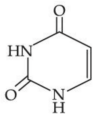
(B) 4-Hydroxypyrimidin-2(1H)-one (a mono-enol form)



(C) 2-Hydroxypyrimidin-4(3H)-one (a mono-enol form)

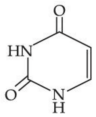


(D) Pyrimidine-2,4(1H,3H)-dione (the diketo form)



Correct Answer: (D)

Pyrimidine-2,4(1H,3H)-dione (the diketo form)



Solution:

Concept:

- Uracil is a pyrimidine base found in RNA. Its usual biological tautomer is the lactam, or diketo, form with carbonyl groups at positions 2 and 4.
- Keto-enol tautomerism moves a proton and changes adjacent bond orders without changing the molecular formula; it does not make all tautomers equally abundant.

Step 1: Locate the two oxygen substituents Inspect the oxygen atoms on the six-membered pyrimidine ring. In option (A), both oxygens are present as hydroxyl groups, giving a dienol structure. Options

(B) and (C) each show one hydroxyl group and one carbonyl group, so they are mono-enol tautomers. Option (D) has two carbonyl groups and neither oxygen is shown as hydroxyl. Its two ring nitrogen atoms bear hydrogen in the free-base structure.

Step 2: Choose the normal RNA base structure The predominant uracil structure used to represent the RNA base is pyrimidine-2,4(1H,3H)-dione. In a nucleotide, ribose is attached through N1, while the base retains the two carbonyl groups characteristic of this diketo form. These oxygen atoms provide the usual hydrogen-bond acceptor sites in adenine-uracil pairing. Therefore the dienol drawing in (A) and the mono-enol drawings in (B) and (C) are not the standard tautomer present in RNA. Their same elemental composition does not make them the preferred biological structure.

Step 3: Conclusion The diketo (lactam) structure of uracil is the form shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: For normal uracil, look for two carbonyl groups on the pyrimidine ring; the alternatives containing O–H groups are enol tautomers.

70. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Synthesis of ethyl phenyl ether may be achieved by Williamson synthesis.

Reason (R) : Reaction of bromobenzene with sodium ethoxide yields ethyl phenyl ether.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) (A) is correct but (R) is not correct

(C) (A) is not correct but (R) is correct

(D) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

Correct Answer: (B) (A) is correct but (R) is not correct

Solution:

Concept:

- Williamson synthesis forms an ether by an S_N2 reaction of an alkoxide or phenoxide with a suitable alkyl halide.
- Aryl halides do not undergo ordinary S_N2 substitution at the carbon bonded directly to the aromatic ring.

Step 1: Test the assertion with a valid synthesis Ethyl phenyl ether can be obtained from sodium phenoxide and an ethyl halide. For example,

$$C_6H_5ONa + C_2H_5Br \longrightarrow C_6H_5OC_2H_5 + NaBr.$$

The phenoxide oxygen attacks the primary ethyl carbon and bromide leaves. This is a valid Williamson synthesis, so assertion (A) is correct. Choosing the alkyl fragment to carry the leaving group is essential.

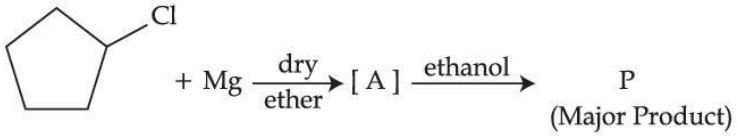
Step 2: Test the stated reason The reason instead places bromine directly on benzene and uses sodium ethoxide as the nucleophile. The carbon-bromine bond of bromobenzene has partial double-bond character through resonance, and backside attack at its trigonal aromatic carbon is unsuitable for the ordinary Williamson substitution. Consequently bromobenzene and sodium ethoxide do not give ethyl phenyl ether by the proposed normal reaction. Reason (R) is false.

Step 3: Select the assertion-reason combination Because the assertion is true and the reason is false, option (B) is selected. Options (A) and (D) both require a true reason and are excluded. Option (C) reverses the truth values by calling the feasible Williamson synthesis false and the unsuitable aryl-halide route true.

Step 4: Conclusion The assertion is correct, but the reason is not correct. Hence, the correct option is (B).

Quick Tip to Solve: For an aryl alkyl ether, use phenoxide with a primary alkyl halide; do not put the leaving group directly on the aromatic carbon.

71. In the following sequence of reactions the P is : [4 marks]



(A) Cyclopentane

(B) Ethoxycyclopentane

(C) Ethylcyclopentane

(D) Cyclopentene

Correct Answer: (A) Cyclopentane

Solution:

Concept:

- A Grignard reagent is an organomagnesium halide, RMgX, formed from an organic halide and magnesium in dry ether.
- The carbon bonded to magnesium behaves as a strong base. A compound containing an acidic hydrogen, such as ethanol, protonates this carbon to give RH.

Step 1: Form the organomagnesium intermediate The starting compound is chlorocyclopentane. Magnesium inserts into its carbon–chlorine bond in dry ether, giving cyclopentylmagnesium chloride. Let R denote the cyclopentyl group, $\mathrm{C_5H_9}$. The first stage is

$$\mathrm{RCl + Mg \xrightarrow{dry\ ether} RMgCl.}$$

The ether must be dry because water would destroy this strongly basic reagent before the intended second stage.

Step 2: React the intermediate with ethanol Ethanol supplies the proton on its oxygen atom. Proton

transfer is much faster than any proposed carbon-chain substitution. Thus

$$\mathrm{RMgCl} + \mathrm{C_2H_5OH} \longrightarrow \mathrm{RH} + \mathrm{C_2H_5OMgCl}.$$

Replacing the carbon–magnesium bond by a carbon–hydrogen bond restores the saturated five-membered ring, so the organic product is cyclopentane. The ring skeleton is retained and no carbon atom from ethanol is incorporated into it.

Step 3: Eliminate the other products Ethoxycyclopentane would require formation of a carbon–oxygen bond, whereas this reaction is acid–base protonolysis. Ethylcyclopentane would require a new carbon–carbon bond to an ethyl group, which ethanol does not supply here. Cyclopentene would require elimination to create a double bond; protonation of a Grignard reagent instead gives the alkane.

Step 4: Conclusion P is cyclopentane, $\mathrm{C_5H_{10}}$. Hence, the correct option is (A).

Quick Tip to Solve: For RMgX followed by water or an alcohol, replace $-\mathrm{MgX}$ by hydrogen before considering any substitution product.

72. In polythionic acid, $H_2S_xO_6$ (x = 3 to 5) the oxidation state(s) of sulphur is/are : [4 marks]

(A) + 5 only

(B) + 6 only

(C) + 3 and + 5 only

(D) 0 and + 5 only

Correct Answer: (D) 0 and + 5 only

Solution:

Concept:

- Oxidation states must be assigned to individual atoms from bonding; an average oxidation state need not describe any particular sulphur atom.
- In an $\mathrm{S-O}$ bond the bonding electrons are assigned to oxygen. A bond between two sulphur atoms contributes zero to either atom.

Step 1: Identify the sulphur-chain structure Polythionic acids have the arrangement

$$\mathrm{HO-S(=O)_2-S_{\mathit{x}-2}-S(=O)_2-OH},$$

with $x = 3, 4, 5$. There are two terminal sulphur atoms attached to oxygen, while each of the remaining $x - 2$ sulphur atoms is internal to the sulphur chain. The internal atoms have only sulphur–sulphur bonds, so none of their bonding electrons is assigned preferentially to a different element.

Step 2: Assign oxidation states to the two kinds of sulphur Each terminal sulphur has two double bonds to oxygen and one single bond to the hydroxyl oxygen. These contribute $+4$ and $+1$, respectively, giving $+5$. Its remaining sulphur–sulphur bond contributes zero. Every internal sulphur therefore has oxidation state zero. This assignment holds for all three stated chain lengths, not just for one chosen

value of x.

Step 3: Check the charge balance and options For the neutral acid, the total is

$$2(+1) + 2(+5) + (x-2)(0) + 6(-2) = 0.$$

The average value $10/x$ is not a substitute for the individual oxidation states. Option (A) omits the internal sulphur atoms, option (B) assigns the wrong terminal value, and option (C) incorrectly gives an internal atom the value $+3$.

Step 4: Conclusion The sulphur atoms have oxidation states 0 and $+5$ only. Hence, the correct option is (D).

Quick Tip to Solve: In sulphur oxoacid chains, separate oxygen-bearing terminal sulphur atoms from internal atoms before doing oxidation-number arithmetic.

73. In which one of the following molecules strongest back donation of an electron pair from halide to boron is expected? [4 marks]

(A) BCl_3

(B) BF_3

(C) BBr_3

(D) BI_3

Correct Answer: (B) BF_3

Solution:

Concept:

- Trigonal planar boron trihalides have an empty boron $2p$ orbital perpendicular to the molecular plane.
- A filled halogen p orbital can donate an electron pair into this orbital. The strength of this $p\pi$-$p\pi$ interaction depends strongly on orbital overlap.

Step 1: Compare the orbitals that overlap Boron uses a compact $2p$ orbital. Fluorine also donates from a $2p$ orbital, giving the best size and spatial match. Chlorine, bromine and iodine donate from $3p$, $4p$ and $5p$ orbitals, respectively. These are increasingly diffuse relative to the boron orbital and overlap less effectively sideways with it.

Step 2: Apply the back-donation trend The standard comparison of the boron halides is

$$BF_3 > BCl_3 > BBr_3 > BI_3$$

for the effectiveness of this halogen-to-boron back donation. Fluorine is highly electronegative, but choosing solely on electronegativity misses the orbital-matching factor tested here. The strong $2p$-$2p$ overlap in BF_3 gives its boron–fluorine bonds appreciable partial double-bond character.

Step 3: Select the molecule and reject the alternatives Option (B) contains fluorine and therefore gives the strongest back donation. Option (A), BCl_3, has the poorer $3p$-$2p$ overlap. The still larger

donor orbitals of bromine and iodine make options (C) and (D) weaker. This question concerns donation within the molecule, rather than a direct comparison of halogen electronegativities or ordinary sigma-bond polarity.

Step 4: Conclusion The strongest halide-to-boron back donation is expected in BF_3. Hence, the correct option is (B).

Quick Tip to Solve: For boron trihalides, check the halogen donor orbital against boron $2p$: the $2p$-$2p$ pair gives the best back-bonding overlap.

74. Deuterium resembles hydrogen in properties but : [4 marks]

(A) reacts slower than hydrogen

(B) reacts vigorously than hydrogen

(C) reacts just as hydrogen

(D) emits β^+ particles

Correct Answer: (A) reacts slower than hydrogen

Solution:

Concept:

- Protium and deuterium are hydrogen isotopes with the same atomic number and electronic configuration; this explains their broadly similar chemical behaviour.
- Deuterium has greater nuclear mass. Its bonds have lower vibrational zero-point energy and are generally harder to break than the corresponding protium bonds.

Step 1: Explain the shared chemistry Ordinary hydrogen is predominantly protium, ^1H, while deuterium is ^2H. Both neutral atoms contain one electron and have electronic configuration $1s^1$. They therefore form corresponding compounds and show similar valency. Similar electronic structure, however, does not imply identical reaction rates because reaction dynamics also depend on nuclear mass.

Step 2: Apply the isotope effect The greater mass of deuterium lowers the vibrational frequency of bonds involving it. Consequently their zero-point vibrational energy is lower, and the effective energy needed to reach a bond-breaking transition state is commonly greater. Thus deuterium generally reacts more slowly than hydrogen in the comparison intended here. This is a kinetic isotope effect: the same type of chemical change can occur, but its rate differs.

Step 3: Check the other statements Option (B) incorrectly predicts a greater vigour of reaction instead of the usual slower rate. Option (C) ignores the mass-dependent isotope effect and would imply identical behaviour in the very respect being tested. Option (D) is false because deuterium is stable and does not emit positrons. Radioactivity must not be inferred merely because an isotope contains an additional neutron; tritium is the radioactive hydrogen isotope.

Step 4: Conclusion Deuterium generally reacts more slowly than hydrogen. Hence, the correct option is (A).

Quick Tip to Solve: Use electronic configuration to compare isotope chemistry, then nuclear mass to compare physical properties and reaction rates.

75. Which refining process is generally used in the purification of low melting metals? [4 marks]

(A) Chromatographic method

(B) Liquation

(C) Electrolysis

(D) Zone refining

Correct Answer: (B) Liquation

Solution:

Concept:

- Liquation separates a readily fusible metal from solid impurities by exploiting a difference in melting points.
- The crude metal is heated enough to melt the desired metal while the higher-melting impurities remain solid. The liquid metal is then drained away.

Step 1: Relate the method to the stated property The question specifically identifies a low melting metal. Such a metal can be melted by moderate heating without melting many of the impurities mixed with it. In liquation, the impure metal is placed on a sloping hearth and heated above its melting point. The molten metal flows down the slope into a receiver, while the less fusible impurities are retained on the hearth.

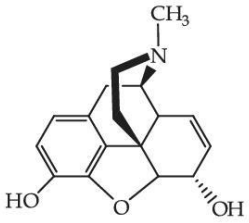
Step 2: State why liquation fits The useful separation is physical rather than a change in the chemical identity of the metal: one component flows as a liquid while the others remain as solid residue. Low melting metals such as tin can be refined by this method when the accompanying impurities are less fusible. The decisive condition is a sufficient melting-point difference; simply heating every mixture until all its constituents melt would not accomplish the required separation.

Step 3: Distinguish the alternative processes Chromatographic separation relies on different adsorption or distribution behaviour and is not the usual bulk refining method indicated by a low melting point. Electrolytic refining uses selective anodic dissolution and deposition at the cathode, so it is not identified by the property given. Zone refining relies on impurities being more soluble in a moving molten zone than in the solid metal and is especially useful for very high purity semiconductor materials.

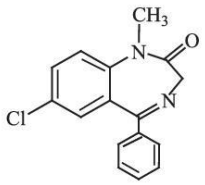
Step 4: Conclusion The process generally used here is liquation. Hence, the correct option is (B).

Quick Tip to Solve: Associate a low melting metal mixed with less fusible impurities with liquation; associate a moving molten zone and extreme purity with zone refining.

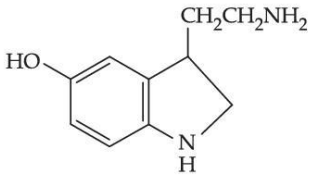
76. The correct statement about (A), (B), (C) and (D) is : [4 marks]



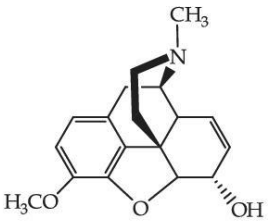
(A)



(B)



(C)



(D)

(A) (A), (B) and (C) are narcotic analgesics

(B) (B), (C) and (D) are tranquillizers

(C) (A) and (D) are tranquillizers

(D) (B) and (C) are tranquillizers

Correct Answer: (D) (B) and (C) are tranquillizers

Solution:

Concept:

- Narcotic analgesics act on the central nervous system to relieve pain; morphine and codeine are members of the opioid family.
- Tranquillizers reduce anxiety and mental tension. The standard classification includes diazepam and the serotonin-related example used in the question.

Step 1: Identify the four displayed compounds Compound (A) has the morphine skeleton with a phenolic hydroxyl group and is morphine. Compound (D) has the related skeleton with that phenolic group methylated to a methoxy group and is codeine. The substituted benzodiazepine structure (B) is diazepam, whereas the hydroxyindole-related structure (C) is the serotonin structure represented in the question. The structural difference between (A) and (D) does not change their placement in the opioid analgesic family.

Step 2: Apply the therapeutic classifications Morphine and codeine, (A) and (D), belong to the narcotic analgesics, so neither is the tranquillizer sought in the pair. Diazepam is a benzodiazepine tranquillizer. In the textbook classification used by this question, serotonin is associated with mood regulation and is listed in the tranquillizer discussion. Thus the intended tranquillizer pair is (B) and (C), while the narcotic analgesic pair is (A) and (D).

Step 3: Test all four statements The first statement is false because (B) and (C) are not the narcotic analgesics in this set. The second is false because it includes codeine, (D), among tranquillizers. The third is false because (A) and (D) are the analgesic pair. Only the fourth statement correctly selects

(B) and (C). Distinguish the letters attached to molecular structures from the separate letters that label answer options.

Step 4: Conclusion Diazepam and the serotonin-related compound form the tranquillizer pair in the classification tested. Hence, the correct option is (D).

Quick Tip to Solve: Identify morphine and its methyl ether codeine together first; their analgesic classification immediately eliminates any tranquillizer option containing either.

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77. The gas 'A' is having very low reactivity reaches to stratosphere. It is non-toxic and non-flammable but dissociated by UV-radiations in stratosphere. The intermediates formed initially from the gas 'A' are : [4 marks]

(A) $Cl\overset{\bullet}{O} + \overset{\bullet}{C}F_2Cl$

(B) $Cl\overset{\bullet}{O} + C\overset{\bullet}{H}_3$

(C) $\overset{\bullet}{C}H_3 + C\overset{\bullet}{F}_2Cl$

(D) $\overset{\bullet}{C}l + C\overset{\bullet}{F}_2Cl$

Correct Answer: (D) $\overset{\bullet}{C}l + C\overset{\bullet}{F}_2Cl$

Solution:

Concept:

- Chlorofluorocarbons are stable, non-flammable compounds that can reach the stratosphere, where short-wavelength ultraviolet radiation initiates photolysis.
- Homolytic cleavage of a carbon–chlorine bond gives a chlorine radical and a carbon-centred radical; subsequent ozone reactions are separate steps.

Step 1: Identify the gas and the bond broken The stated properties point to a chlorofluorocarbon such as dichlorodifluoromethane, CF_2Cl_2, commonly called CFC-12. Its low reactivity allows it to survive transport to the stratosphere. There ultraviolet radiation can supply the energy needed to break a carbon–chlorine bond homolytically. Each of the two fragments receives one electron from the broken bond.

Step 2: Write the initiating photochemical reaction The initial step is

$$CF_2Cl_2 \xrightarrow{h\nu} Cl^{\bullet} + {}^{\bullet}CF_2Cl.$$

Both products are radicals: the chlorine atom carries an unpaired electron, as does the carbon of the remaining chlorodifluoromethyl fragment. The atoms balance, with one carbon, two fluorines and two chlorines on each side. No oxygen or hydrogen atom occurs in this starting molecule.

Step 3: Separate initiation from the later ozone reaction The chlorine radical may subsequently attack ozone to form a chlorine monoxide radical, but $ClO^{\bullet}$ is not produced by the initial dissociation of the CFC. This rejects option (A); oxygen cannot appear from the stated photolysis alone. Options (B) and (C) contain methyl radicals even though the CFC has no hydrogen atoms, so they are also inconsistent. Option (D) gives exactly the chlorine and chlorodifluoromethyl radicals produced initially.

Step 4: Conclusion The initial intermediates are $Cl^{\bullet}$ and $^{\bullet}CF_2Cl$. Hence, the correct option is (D).

Quick Tip to Solve: For the initial photolysis step, break one C–Cl bond homolytically and check atom balance; do not replace the chlorine radical by its later ozone-reaction product.

78. The number of water molecules in gypsum, dead burnt plaster and plaster of Paris, respectively are : [4 marks]

(A) 2, 0 and 1

(B) 0.5, 0 and 2

(C) 5, 0 and 0.5

(D) 2, 0 and 0.5

Correct Answer: (D) 2, 0 and 0.5

Solution:

Concept:

- Water of crystallisation is water present in a definite stoichiometric ratio in a crystalline salt. Its coefficient is stated per formula unit of the anhydrous salt.
- Gypsum is calcium sulphate dihydrate, while plaster of Paris is calcium sulphate hemihydrate. Strong heating removes the water completely and gives anhydrous calcium sulphate.

Step 1: Write the formula of gypsum Gypsum contains two water molecules for each calcium sulphate formula unit. Its composition is $CaSO_4 \cdot 2\,H_2O$. Therefore the first entry must be 2. This immediately rules out option (B), which starts with 0.5, and option (C), which starts with 5. The water is not a reactant added during a calculation; it is the fixed hydration number of the crystalline solid.

Step 2: Identify the two heated forms Controlled heating of gypsum gives plaster of Paris, whose composition is $CaSO_4 \cdot \frac{1}{2}H_2O$, equivalently $2\,CaSO_4 \cdot H_2O$. Thus it has 0.5 water molecule per calcium sulphate formula unit, not one. More severe heating gives dead burnt plaster, $CaSO_4$, with no water of crystallisation. In the requested order the second entry is consequently 0 and the third is 0.5. Option (A) assigns one water molecule to plaster of Paris and is therefore incorrect; the remaining complete matching sequence is option (D).

Step 3: Conclusion The hydration numbers for gypsum, dead burnt plaster and plaster of Paris are respectively $\boxed{2,\ 0,\ 0.5}$. Hence, the correct option is (D).

Quick Tip to Solve: Attach the hydration number to one $CaSO_4$ unit: gypsum 2, plaster of Paris one-half, and dead burnt plaster zero.

79. The unit of the van der Waals gas equation parameter 'a' in $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$ is : [4 marks]

(A) kg m s^{-2}

(B) dm^3 mol^{-1}

(C) kg m s^{-1}

(D) atm dm^6 mol^{-2}

Correct Answer: (D) atm dm^6 mol^{-2}

Solution:

Concept:

- Every term added to pressure must itself have the dimensions of pressure. Thus the attraction correction an^2/V^2 has the same unit as P.
- The symbol n is the amount of gas in moles, and V is volume. The parameter a describes intermolecular attractions, whereas b has units of molar volume.

Step 1: Isolate the dimensions of the attraction parameter Apply dimensional consistency to the first bracket of the gas equation. Since the two summands must be comparable,

$$[a]\frac{[n]^2}{[V]^2}=[P].$$

Rearranging gives

$$[a]=[P][V]^2[n]^{-2}.$$

The second power applies to the whole volume unit; it is not a single power of length. This distinction separates the unit of a from the molar-volume unit associated with b.

Step 2: Insert the specified pressure and volume units Take pressure in atmospheres, volume in cubic decimetres and amount in moles. Then

$$[a]=\text{atm}\,(\text{dm}^3)^2\,\text{mol}^{-2}=\text{atm}\,\text{dm}^6\,\text{mol}^{-2}.$$

As a check, multiplying this by n^2/V^2 cancels both mol^{-2} and dm^6, leaving atmospheres as required. Option (A) is a force unit and option (C) is a momentum unit, so neither represents pressure times squared volume per squared amount. Option (B) is molar volume, the appropriate dimension for b, not a.

Step 3: Conclusion The required unit is $\boxed{\text{atm}\,\text{dm}^6\,\text{mol}^{-2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Derive $[a]=[P][V]^2/[n]^2$ from the pressure bracket; do not confuse a with the excluded-volume parameter b.

80. Tyndall effect is more effectively shown by : [4 marks]

(A) true solution

(B) lyophilic colloid

(C) lyophobic colloid

(D) suspension

Correct Answer: (C) lyophobic colloid

Solution:

Concept:

- The Tyndall effect is scattering of a light beam by dispersed particles, making its path visible when viewed from the side. It depends on particle size and the optical contrast with the dispersion medium.
- Lyophobic colloids generally have a more pronounced difference in refractive index between the dispersed phase and medium than lyophilic colloids, and consequently show a stronger Tyndall effect.

Step 1: Compare true solutions and colloidal dispersions In a true solution the solute particles are molecular or ionic in size and do not produce the readily visible Tyndall cone characteristic of a colloid. Thus option (A) is unsuitable. In a colloidal dispersion, particles are large enough to scatter incident light appreciably while remaining dispersed. Both lyophilic and lyophobic systems can scatter light, so their relative effectiveness must be judged from the contrast between the particles and the surrounding medium.

Step 2: Choose the colloid with stronger optical contrast Lyophilic particles are strongly solvated and have an affinity for the dispersion medium. This reduces the effective optical contrast and makes their Tyndall effect comparatively weak. Lyophobic particles have little affinity for the medium and typically a greater refractive-index difference, so their scattering is more conspicuous. Hence option (C) is preferred over (B). A suspension, option (D), contains much coarser particles and can scatter or block light strongly, but it does not represent the characteristic stable colloidal Tyndall cone used in this comparison. The question concerns the most effective colloidal demonstration, for which a lyophobic colloid is the standard choice.

Step 3: Conclusion The Tyndall effect is more effectively shown by a lyophobic colloid. Hence, the correct option is (C).

Quick Tip to Solve: For lyophilic versus lyophobic sols, compare the refractive-index contrast: stronger solvation usually makes the lyophilic sol scatter less conspicuously.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. In Carius method for estimation of halogens, 0.2 g of an organic compound gave 0.188 g of AgBr. The percentage of bromine in the compound is _____. (Nearest integer)

[Atomic mass : Ag = 108, Br = 80] [4 marks]

Solution:

Concept:

- In the Carius method, the halogen in an organic compound is converted into a silver halide precipitate. Each mole of AgBr contains one mole of bromine atoms.
- The bromine mass is the precipitate mass multiplied by its bromine mass fraction, and percentage composition is that mass divided by the original sample mass, multiplied by 100.

Step 1: Calculate the bromine fraction of silver bromide Using the given atomic masses, the molar mass of silver bromide is

$$M(\mathrm{AgBr}) = 108 + 80 = 188\ \mathrm{g\,mol^{-1}}.$$

Therefore 188 g of the precipitate corresponds to 80 g of bromine. The measured precipitate is 0.188 g, so its amount is

$$n(\mathrm{AgBr}) = \frac{0.188}{188} = 0.00100\ \mathrm{mol}.$$

The one-to-one atom ratio means the sample contained the same amount of bromine atoms.

Step 2: Convert the bromine mass into a percentage The mass of bromine present in the original compound is

$$m(\mathrm{Br}) = 0.00100 \times 80 = 0.0800\ \mathrm{g}.$$

Divide by the mass of the organic sample, not by the mass of the precipitate:

$$\%\mathrm{Br} = \frac{0.0800}{0.200} \times 100 = 40.0\%.$$

The remainder of the precipitate mass is silver introduced by the analytical reagent; it must not be counted as bromine belonging to the compound. As a direct check, the same calculation is $100(80/188)(0.188/0.2) = 40$. The result is already an integer, so rounding to the nearest integer does not change it.

Step 3: Conclusion The bromine content is $\boxed{40\%}$; the required numerical entry is $\boxed{40}$.

Quick Tip to Solve: For a silver-halide precipitate, multiply its mass by halogen atomic mass divided by silver-halide molar mass before dividing by the original sample mass.

82. The reaction that occurs in a breath analyser, a device used to determine the alcohol level in a person's blood stream is

$$2K_2Cr_2O_7 + 8H_2SO_4 + 3C_2H_6O \rightarrow 2Cr_2(SO_4)_3 + 3C_2H_4O_2 + 2K_2SO_4 + 11H_2O$$

If the rate of appearance of $Cr_2(SO_4)_3$ is 2.67 mol min^{-1} at a particular time, the rate of disappearance of C_2H_6O at the same time is _____ mol min^{-1}. (Nearest integer) [4 marks]

Solution:

Concept:

- For a balanced reaction, the rate of consumption or formation of each species is proportional to its stoichiometric coefficient. Divide individual species rates by those coefficients to obtain a common reaction rate.
- Appearance and disappearance rates are both reported as positive magnitudes. A derivative for a consumed reactant is negative, but its disappearance rate is the negative of that derivative.

Step 1: Relate the two species using their coefficients The balanced equation consumes 3 mol of ethanol, C_2H_6O, while producing 2 mol of chromium(III) sulphate, $Cr_2(SO_4)_3$. Let N_E and N_C denote their amounts in moles. The common rate relation is

$$-\frac{1}{3}\frac{dN_E}{dt} = \frac{1}{2}\frac{dN_C}{dt}.$$

Thus the ethanol disappearance rate is $3/2$ times the chromium sulphate appearance rate. The ratio is determined only by the balanced equation, without needing a rate law or a reaction order.

Step 2: Substitute the appearance rate and round at the end With the given appearance rate of $2.67\ \text{mol}\,\text{min}^{-1}$,

$$-\frac{dN_E}{dt} = \frac{3}{2}(2.67) = 4.005\ \text{mol}\,\text{min}^{-1}.$$

This is larger than the product formation rate, as expected because three ethanol molecules are consumed for every two formula units of chromium sulphate formed. The question asks for the nearest integer, so 4.005 rounds to 4. Reversing the coefficient ratio would incorrectly predict a smaller ethanol consumption rate and would violate the stoichiometric amount balance.

Step 3: Conclusion The rate of disappearance of ethanol is $4.005\ \text{mol}\,\text{min}^{-1}$, or $\boxed{4\ \text{mol}\,\text{min}^{-1}}$ to the nearest integer. The required entry is $\boxed{4}$.

Quick Tip to Solve: Write each species rate over its balanced coefficient first; this prevents reversing the reactant-to-product rate ratio.

83. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is equal to $\frac{h^2}{xma_0^2}$. The value of 10x is _____. (a_0 is radius of Bohr's orbit) (Nearest integer) [Given : π = 3.14]

[4 marks]

Solution:

Concept:

- For a hydrogen atom, the radius of the orbit with principal quantum number n is $r_n = n^2 a_0$, where a_0 is the first Bohr radius.
- Bohr angular-momentum quantisation gives $mv_n r_n = nh/(2\pi)$. Combining this with $K_n = mv_n^2/2$ expresses kinetic energy in terms of h, m, n and a_0.

Step 1: Express kinetic energy through the orbit radius From the angular-momentum condition,

$$v_n = \frac{nh}{2\pi m r_n}.$$

Substitution in the kinetic-energy expression gives

$$K_n = \frac{1}{2}m\left(\frac{nh}{2\pi m r_n}\right)^2 = \frac{n^2h^2}{8\pi^2 m r_n^2}.$$

Because $r_n = n^2 a_0$, its square is $n^4 a_0^2$. Hence

$$K_n = \frac{h^2}{8\pi^2 m n^2 a_0^2}.$$

The resulting inverse-square dependence on n is consistent with the Bohr energy-level scaling.

Step 2: Set the principal quantum number to two For the second orbit $n = 2$, so

$$K_2 = \frac{h^2}{32\pi^2 m a_0^2}.$$

Comparing with the given expression $h^2/(xma_0^2)$ yields $x = 32\pi^2$. The requested quantity is $10x$, not x:

$$10x = 320\pi^2 = 320(3.14)^2 = 320(9.8596) = 3155.072.$$

Use the value of π supplied in the question before rounding. Rounding the final value to the nearest integer gives 3155. Both x and $10x$ are dimensionless because the dimensional factors have already been included in the kinetic-energy expression.

Step 3: Conclusion $x = 32\pi^2$, and the requested nearest-integer value is $\boxed{10x = 3155}$ (dimensionless).

Quick Tip to Solve: Use $r_n = n^2 a_0$ before squaring the radius; the final kinetic energy scales as $1/n^2$, not $1/n^4$.

84. 1 kg of 0.75 molal aqueous solution of sucrose can be cooled up to -4°C before freezing. The amount of ice (in g) that will be separated out is _____. (Nearest integer)

[Given : $K_f(H_2O)$ = 1.86 K kg mol^{-1}] [4 marks]

Solution:

Concept:

- For non-electrolyte sucrose, $\Delta T_f = K_f m$, with molality m defined as moles of solute per kilogram of liquid solvent. Sucrose is excluded from the ice.
- As pure water freezes, the sucrose amount stays fixed while the mass of liquid water decreases. The final liquid must have a freezing-point depression of 4 K to remain at -4°C.

Step 1: Find the initial masses of solvent and solute The molar mass of sucrose, $C_{12}H_{22}O_{11}$, is $342\ \text{g mol}^{-1}$. Let the initial water mass be w kg. A 0.75 molal solution contains $0.75w$ mol of sucrose, of mass $0.342(0.75w) = 0.2565w$ kg. Since the total solution mass is 1 kg,

$$w + 0.2565w = 1, \qquad w = \frac{1}{1.2565} = 0.7958615\ \text{kg}.$$

The conserved sucrose amount is consequently $n_s = 0.75w = 0.5968961$ mol.

Step 2: Calculate the water remaining liquid at the stated temperature At -4°C, the equilibrium molality of the unfrozen solution is

$$m_f = \frac{4}{1.86} = 2.1505376\ \text{mol kg}^{-1}.$$

Hence the remaining liquid water has mass

$$w_f = \frac{n_s}{m_f} = \frac{0.5968961 \times 1.86}{4} = 0.2775567\ \text{kg}.$$

The difference between initial and remaining water is the pure ice separated:

$$m_{\text{ice}} = 1000(w - w_f) = 518.3048\ \text{g}.$$

The original 1 kg is solution mass, so treating it as 1 kg of water would overestimate both the solute amount and the ice. Round only the final ice mass.

Step 3: Conclusion The ice separated is approximately 518.3 g, giving the required nearest-integer value $\boxed{518\ \text{g}}$.

Quick Tip to Solve: First separate solution mass into solute and solvent; then conserve solute moles while applying the final freezing-point depression to the remaining liquid water.

85. 1 mol of an octahedral metal complex with formula MCl_3 . 2L on reaction with excess of $AgNO_3$ gives 1 mol of AgCl. The denticity of Ligand L is _____. (Integer answer) [4 marks]

Solution:

Concept:

- An octahedral metal complex has coordination number six: six donor atoms are directly bonded to the central metal ion.
- Excess $AgNO_3$ precipitates ionisable chloride outside the coordination sphere as $AgCl$. A coordinated chloride occupies one donor position, whereas the denticity of a ligand counts the donor atoms it contributes.

Step 1: Identify the ionisable chloride One mole of the complex gives one mole of $AgCl$, so each formula unit has one chloride ion outside its coordination sphere. The precipitation is $Ag^+ + Cl^- \longrightarrow AgCl(s)$. There are three chloride ions in the overall formula. Consequently, the other two chlorides are coordinated to the metal; they are not counted as freely ionisable chloride in this test.

Step 2: Count the positions supplied by the two ligands Let the denticity of each identical ligand L be d. The two coordinated chloride ions contribute two donor atoms in total. The two ligands contribute $2d$ donor atoms. Thus the six positions of the octahedron require

$$2 + 2d = 6.$$

The coordination entity can therefore be represented as $[MCl_2L_2]^+$, with one chloride counter-ion outside.

Step 3: Solve for the denticity Rearranging gives $2d = 4$, hence $d = 2$. Each ligand binds through two donor atoms, so it is bidentate. A monodentate ligand would give only four coordinated donor atoms in all, whereas a tridentate ligand would give eight; neither agrees with the stated octahedral geometry.

Step 4: Conclusion The denticity of ligand L is $\boxed{2}$, a dimensionless integer.

Quick Tip to Solve: First subtract the precipitated chloride from the total chloride; use only the remaining coordinated chloride in the coordination-number equation.

86. The number of moles of CuO, that will be utilized in Dumas method for estimation nitrogen in a sample of 57.5 g of N, N-dimethylaminopentane is _____ $\times 10^{-2}$. (Nearest integer) [4 marks]

Solution:

Concept:

- In Dumas analysis the organic compound is heated with copper(II) oxide. Carbon becomes CO_2, hydrogen becomes H_2O, and nitrogen is ultimately collected as N_2.
- Each mole of CuO supplies one mole of oxygen atoms. For an oxygen-free compound $C_xH_yN_z$, complete oxidation of carbon and hydrogen requires $2x + y/2$ moles of CuO per mole of compound.

Step 1: Find the molecular formula and sample amount The pentyl group is C_5H_{11}, and the two

methyl groups attached to nitrogen supply C_2H_6. Thus N,N-dimethylaminopentane has formula $C_7H_{17}N$. Its molar mass is

$$M = 7(12) + 17(1) + 14 = 115\,\text{g mol}^{-1}.$$

The amount in the sample is therefore

$$n = \frac{57.5}{115} = 0.500\,\text{mol}.$$

Step 2: Balance the oxygen requirement For one mole of $C_7H_{17}N$, seven carbon atoms require fourteen oxygen atoms to produce seven CO_2 molecules. Seventeen hydrogen atoms require $17/2$ oxygen atoms to produce $17/2$ water molecules. Nitrogen is collected as the element, so no net oxygen is assigned to it. A balanced overall equation is

$$2\,C_7H_{17}N + 45\,CuO \longrightarrow 14\,CO_2 + 17\,H_2O + N_2 + 45\,Cu.$$

Step 3: Calculate the requested numerical coefficient The equation requires $45/2 = 22.5$ moles of CuO per mole of amine. Hence

$$n(CuO) = 0.500(22.5) = 11.25\,\text{mol}.$$

Writing this in the required form gives $11.25 = 1125 \times 10^{-2}$; the coefficient is already an integer.

Step 4: Conclusion The amount of CuO is $11.25\,\text{mol} = \boxed{1125} \times 10^{-2}\,\text{mol}$. The requested nearest integer is 1125.

Quick Tip to Solve: Count oxygen atoms, rather than oxygen molecules: one CuO supplies one oxygen atom, so the factor is $2x + y/2$.

87. The number of f electrons in the ground state electronic configuration of Np (Z = 93) is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- The ground-state electronic configuration of neptunium is $[\text{Rn}]\,5f^4\,6d^1\,7s^2$. The radon core contains 86 electrons, leaving seven electrons outside it for $Z = 93$.
- The phrase number of f electrons includes electrons in every occupied f subshell, including those inside the noble-gas core. A filled f subshell contains fourteen electrons.

Step 1: Write the ground-state configuration Neptunium is an actinoid. Its seven electrons beyond the radon core occupy $5f$, $6d$, and $7s$ as

$$\text{Np} : [\text{Rn}]\,5f^4\,6d^1\,7s^2.$$

The outer occupancies add to $4 + 1 + 2 = 7$, and $86 + 7 = 93$, which checks the atomic number. The

neutral ground state is required; configurations of neptunium ions must not be substituted.

Step 2: Include the filled f subshell in the core Expand the relevant part of the radon core:

$$[\mathrm{Rn}] = [\mathrm{Xe}]\, 4f^{14}\, 5d^{10}\, 6s^2\, 6p^6.$$

The xenon core has no occupied f subshell. Thus radon contributes fourteen $4f$ electrons. There are also four $5f$ electrons outside the core in neptunium. The $6d^1$ electron is a d electron and the $7s^2$ electrons are s electrons, so neither contributes to the f count.

Step 3: Add all f electrons The total is

$$N_f = N_{4f} + N_{5f} = 14 + 4 = 18.$$

Counting only four would answer a different question, namely the occupancy of the $5f$ subshell. Here the wording asks for the number of f electrons in the complete ground-state configuration, so the filled core subshell must be retained.

Step 4: Conclusion Neptunium contains $\boxed{18}$ f electrons in total: fourteen in $4f$ and four in $5f$. The requested integer is dimensionless.

Quick Tip to Solve: When a question asks for all electrons of a subshell type, expand the noble-gas core far enough to count filled inner subshells too.

88. 200 mL of 0.2 M HCl is mixed with 300 mL of 0.1 M NaOH. The molar heat of neutralization of this reaction is -57.1 kJ. The increase in temperature in °C of the system on mixing is x $\times$ 10^{-2}. The value of x is _____. (Nearest integer)

[Given : Specific heat of water = 4.18 J g^{-1} K^{-1}, Density of water = 1.00 g cm^{-3}]

[Assume no volume change on mixing) [4 marks]

Solution:

Concept:

- For a strong acid and strong base, $\mathrm{H^+ + OH^-} \longrightarrow \mathrm{H_2O}$; the amount neutralised equals the amount of the limiting reagent.
- The solution absorbs the released reaction heat: $q = mc\Delta T$. The molar neutralisation enthalpy is negative, but the temperature rise is positive because the surrounding solution gains heat.

Step 1: Find the limiting reagent The acid and base react in a one-to-one mole ratio. Their amounts are

$$n(\mathrm{HCl}) = 0.200(0.2) = 0.0400\,\mathrm{mol},$$

$$n(\mathrm{NaOH}) = 0.300(0.1) = 0.0300\,\mathrm{mol}.$$

Thus NaOH is limiting, and $0.0300\,\mathrm{mol}$ of water forms. Acid remains in excess; using all $0.0400\,\mathrm{mol}$ of acid in the heat calculation would overestimate the temperature rise.

Step 2: Calculate the released heat and solution mass The heat released has magnitude

$$q = 0.0300(57.1 \times 10^3) = 1713\,\text{J}.$$

With no volume change on mixing, the combined solution volume is $200 + 300 = 500\,\text{mL}$. At the given density, its mass is $500\,\text{g}$. Both solutions absorb the heat, so the full combined mass is used, rather than the mass of just the limiting reagent solution.

Step 3: Find the temperature rise and scale it Using the supplied specific heat,

$$\Delta T = \frac{1713}{500(4.18)} = 0.8196\,\text{K}.$$

A temperature difference has the same numerical value in kelvin and degrees Celsius. Since $\Delta T = x \times 10^{-2}\,^\circ\text{C}$,

$$x = \frac{0.8196}{0.01} = 81.96.$$

Rounding only at the end gives the nearest integer.

Step 4: Conclusion The temperature rises by approximately $0.820\,^\circ\text{C}$, and the requested value is $\boxed{x = 82}$.

Quick Tip to Solve: Use the limiting reagent for the heat released, but the total solution mass for the heat absorbed; keep the negative reaction enthalpy separate from the positive rise.

89. The number of moles of NH_3, that must be added to 2L of 0.80 M $AgNO_3$ in order to reduce the concentration of Ag^+ ions to 5.0×10^{-8} M ($K_{formation}$ for $[Ag(NH_3)_2]^+ = 1.0 \times 10^8$) is _____. (Nearest integer)

[Assume no volume change on adding NH_3] [4 marks]

Solution:

Concept:

- Silver(I) forms the diamminesilver(I) complex through $Ag^+ + 2\,NH_3 \rightleftharpoons [Ag(NH_3)_2]^+$. Its formation constant uses the equilibrium concentration of free ammonia.
- The total ammonia added is the sum of free ammonia and ammonia bound in the complex. Each mole of $[Ag(NH_3)_2]^+$ contains two moles of ammonia ligands.

Step 1: Use the silver mass balance As the volume remains $2\,\text{L}$, the total silver concentration is $0.80\,\text{M}$. At the required free silver concentration, the complex concentration is

$$[[Ag(NH_3)_2]^+] = 0.80 - 5.0 \times 10^{-8} \approx 0.80\,\text{M}.$$

Almost all the silver is complexed, but the specified tiny free silver concentration must still be used in the formation-constant expression.

Step 2: Calculate the required free ammonia The equilibrium expression is

$$K_f = \frac{[[\mathrm{Ag(NH_3)_2}]^+]}{[\mathrm{Ag^+}][\mathrm{NH_3}]^2}.$$

Consequently,

$$[\mathrm{NH_3}]^2_{\text{free}} = \frac{0.80 - 5.0 \times 10^{-8}}{(1.0 \times 10^8)(5.0 \times 10^{-8})} \approx 0.160.$$

Taking the positive square root gives $[\mathrm{NH_3}]_{\text{free}} \approx 0.400\,\mathrm{M}$. This is not the total added concentration, because much ammonia has become bound.

Step 3: Add bound and free ammonia The concentration bound as ligands is

$$[\mathrm{NH_3}]_{\text{bound}} = 2(0.80 - 5.0 \times 10^{-8}) \approx 1.60\,\mathrm{M}.$$

The total concentration added is therefore $1.60 + 0.400 = 2.00\,\mathrm{M}$. Multiplication by the unchanged solution volume gives

$$n_{\text{added}} = 2.00(2.00) = 4.00\,\mathrm{mol}.$$

Keeping the small free-silver correction gives the same nearest integer.

Step 4: Conclusion The ammonia required is approximately $\boxed{4\,\mathrm{mol}}$. The requested nearest integer is 4.

Quick Tip to Solve: Formation constants give free ligand concentration; add the ligand tied up in the complex before converting the total concentration to moles.

90. When 10 mL of an aqueous solution of $KMnO_4$ was titrated in acidic medium, equal volume of 0.1 M of an aqueous solution of ferrous sulphate was required for complete discharge of colour. The strength of $KMnO_4$ in grams per litre is _____ $\times 10^{-2}$. (Nearest integer)

[Atomic mass of K = 39, Mn = 55, O = 16] [4 marks]

Solution:

Concept:

- In acidic solution permanganate is reduced from manganese(VII) to manganese(II), gaining five electrons per mole. Each Fe^{2+} ion loses one electron to form Fe^{3+}.
- The balanced redox equation fixes the ratio $n(\mathrm{MnO_4^-}) : n(\mathrm{Fe^{2+}}) = 1 : 5$. Mass concentration, or strength, is molarity multiplied by molar mass.

Step 1: Write the acidic redox stoichiometry The net ionic equation is

$$\mathrm{MnO_4^-} + 8\,\mathrm{H^+} + 5\,\mathrm{Fe^{2+}} \longrightarrow \mathrm{Mn^{2+}} + 5\,\mathrm{Fe^{3+}} + 4\,\mathrm{H_2O}.$$

One mole of permanganate therefore consumes five moles of ferrous ions. The acidic medium is essential: using the reduction product for neutral or alkaline solution would produce a different electron ratio. Ferrous sulphate supplies one ferrous ion per formula unit.

Step 2: Find the permanganate concentration The ferrous sulphate volume equals the given $10\,\mathrm{mL}$ permanganate volume. Its amount is

$$n(\mathrm{Fe}^{2+}) = 0.100(0.0100) = 1.00 \times 10^{-3}\,\mathrm{mol}.$$

Thus the amount of permanganate in the aliquot is

$$n(\mathrm{KMnO_4}) = \frac{1.00 \times 10^{-3}}{5} = 2.00 \times 10^{-4}\,\mathrm{mol}.$$

Dividing by the $0.0100\,\mathrm{L}$ aliquot gives $M(\mathrm{KMnO_4}) = 0.0200\,\mathrm{mol\,L^{-1}}$.

Step 3: Convert molarity to strength The supplied atomic masses give

$$M_r(\mathrm{KMnO_4}) = 39 + 55 + 4(16) = 158.$$

Its mass concentration is therefore

$$s = 0.0200(158) = 3.16\,\mathrm{g\,L^{-1}}.$$

In the form requested, $3.16 = 316 \times 10^{-2}$, so the numerical coefficient is already an integer. The aliquot volume determines the titration amount, whereas strength refers to one litre of solution.

Step 4: Conclusion The strength is $3.16\,\mathrm{g\,L^{-1}} = \boxed{316} \times 10^{-2}\,\mathrm{g\,L^{-1}}$. The requested nearest integer is 316.

Quick Tip to Solve: For equal titrant volumes, use $5M_{\mathrm{KMnO4}} = M_{\mathrm{FeSO4}}$ in acidic medium, then multiply by 158 to obtain grams per litre.