JEE Main 2021 Paper 1 (B.E./B.Tech) Aug 27 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 4 (Aug-Sep), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 August 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The angle between the straight lines, whose direction cosines are given by the equations 2l + 2m − n = 0 and mn + nl + lm = 0, is : [4 marks]

(A) $\frac{\pi}{2}$

(B) $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

(C) $\cos^{-1}\left(\frac{8}{9}\right)$

(D) $\frac{\pi}{3}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:

Concept:

- Direction cosines satisfy $l^2 + m^2 + n^2 = 1$. A nonzero proportional triple gives direction ratios, so normalising the triples is optional when using the dot-product angle formula.
- For direction vectors $\mathbf{a}, \mathbf{b}$, $\cos\theta = |\mathbf{a} \cdot \mathbf{b}|/(|\mathbf{a}||\mathbf{b}|)$ gives the acute or right angle between the lines. Perpendicular lines have zero dot product.

Step 1: Eliminate the third direction cosine The linear condition gives $n = 2l + 2m$. Substituting this into the quadratic condition gives

$$m(2l + 2m) + l(2l + 2m) + lm = 0.$$

Collecting terms and factoring,

$$2l^2 + 5lm + 2m^2 = (2l + m)(l + 2m) = 0.$$

Thus the simultaneous conditions determine two directions. The all-zero triple is excluded because it cannot be a direction-cosine triple.

Step 2: Find and compare the two directions From $2l + m = 0$, choose $l = 1, m = -2$, giving $n = -2$. From $l + 2m = 0$, choose $l = -2, m = 1$, again giving $n = -2$. Hence direction vectors are $(1, -2, -2)$ and $(-2, 1, -2)$. Their dot product is

$$1(-2) + (-2)(1) + (-2)(-2) = -2 - 2 + 4 = 0.$$

Both vectors have length 3, so division by their lengths still gives cosine zero. In particular the cosine is neither $8/9$, nor $1/2$, nor $-4/9$, ruling out the other three listed angles.

Step 3: Conclusion The two straight lines are perpendicular, so their angle is $\pi/2$. Hence, the correct option is (A).

Quick Tip to Solve: Eliminate one cosine before normalising: the homogeneous quadratic often factors directly into the two required directions.

2. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval : [4 marks]

(A) [68, 69)

(B) [62, 63)

(C) [65, 66)

(D) [60, 61)

Correct Answer: (B) [62, 63)

Solution:

Concept:

- For every integer r, $\lfloor x+r \rfloor = \lfloor x \rfloor + r$. Thus all entries of this matrix depend on the single integer $n = \lfloor x \rfloor$.
- Adding a multiple of one row to another does not change a determinant. Once n is found, $\lfloor x \rfloor = n$ means $n \le x < n+1$.

Step 1: Reduce the greatest-integer entries Put $n = \lfloor x \rfloor$. Since every shift in the entries is integral, the matrix becomes

$$A = \begin{pmatrix} n+1 & n+2 & n+3 \\ n & n+3 & n+3 \\ n & n+2 & n+4 \end{pmatrix}.$$

This substitution is valid for negative as well as positive x, and requires no assumption that x itself is an integer. The determinant is constant throughout each interval $[n, n+1)$.

Step 2: Evaluate and solve the determinant condition Use $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - R_1$. Then

$$\det A = \begin{vmatrix} n+1 & n+2 & n+3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}.$$

Expansion in the first row gives

$$\det A = (n+1) + (n+2) + (n+3) = 3n + 6.$$

Consequently $3n + 6 = 192$, whence $n = 62$. The other proposed left endpoints, 68, 65 and 60, give determinants 210, 201 and 186 respectively, so none meets the required value. At the right endpoint 63 the floor changes, so that endpoint must be excluded.

Step 3: Conclusion We require $\lfloor x \rfloor = 62$, giving $x \in [62, 63)$. Hence, the correct option is (B).

Quick Tip to Solve: Replace every shifted floor by one integer plus its shift, then subtract nearly equal rows before expanding the determinant.

3. Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$, then the value of $\tan(M - m)$ is equal to : [4 marks]

(A) $2 + \sqrt{3}$

(B) $2 - \sqrt{3}$

(C) $3 + 2\sqrt{2}$

(D) $3 - 2\sqrt{2}$

Correct Answer: (D) $3 - 2\sqrt{2}$

Solution:

Concept:

- The principal function $\tan^{-1} t$ is strictly increasing for every real t, so its extrema occur at the extrema of its argument.
- Use $\sin x + \cos x = \sqrt{2}\sin(x + \pi/4)$ and $\tan(u - v) = (\tan u - \tan v)/(1 + \tan u \tan v)$.

Step 1: Determine the range of the argument For $0 \le x \le \pi/2$, both sine and cosine are nonnegative, and

$$(\sin x + \cos x)^2 = 1 + \sin 2x.$$

Since $0 \le \sin 2x \le 1$, the argument lies between 1 and $\sqrt{2}$. The value 1 occurs at either endpoint; $\sqrt{2}$ occurs at $x = \pi/4$. Monotonicity of the principal inverse tangent therefore gives

$$m = \tan^{-1} 1 = \pi/4, \qquad M = \tan^{-1}\sqrt{2}.$$

No additional branch adjustment is necessary because both inverse tangents are in the first quadrant.

Step 2: Apply the tangent subtraction identity Substitute the tangent values directly:

$$\tan(M - m) = \frac{\sqrt{2} - 1}{1 + \sqrt{2}}.$$

Rationalising the denominator gives

$$\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = \frac{(\sqrt{2} - 1)^2}{2 - 1} = 3 - 2\sqrt{2}.$$

This is about 0.172, as expected for the tangent of the small positive difference of the two angles. The reciprocal $3 + 2\sqrt{2}$ is greater than 1 and cannot be right here. The alternatives involving $\sqrt{3}$ are different from the value obtained from the actual argument range.

Step 3: Conclusion The requested value is $3 - 2\sqrt{2}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the range inside an increasing inverse function first; use its tangent values instead of trying to evaluate the inverse angles.

4. If two tangents drawn from a point P to the parabola $y^2 = 16(x - 3)$ are at right angles, then the locus of point P is : [4 marks]

(A) x + 3 = 0

(B) x + 1 = 0

(C) x + 2 = 0

(D) x + 4 = 0

Correct Answer: (B) x + 1 = 0

Solution:

Concept:

- After translating the vertex by $X = x - 3$, the parabola is $y^2 = 4aX$ with $a = 4$. Its tangent of nonzero slope m is $y = mX + a/m$.
- Two nonvertical lines of slopes m_1, m_2 are perpendicular exactly when $m_1 m_2 = -1$. The intersection of such tangents to a parabola lies on its directrix.

Step 1: Write the equation satisfied by the tangent slopes Let the point of intersection be $P = (h, k)$. Substitution into the tangent equation gives

$$k = m(h - 3) + \frac{4}{m}.$$

Multiplying through by the nonzero slope,

$$(h - 3)m^2 - km + 4 = 0.$$

Thus the two tangents from P have slopes equal to the two roots of this quadratic. For the eventual locus $h = -1$, its leading coefficient is nonzero, so the quadratic-root argument applies without a degeneracy.

Step 2: Impose perpendicularity and verify the locus By the product-of-roots formula,

$$m_1 m_2 = \frac{4}{h - 3}.$$

For perpendicular tangents this equals -1, giving $h - 3 = -4$ and hence $h = -1$. Conversely, when $h = -1$, the slope equation is $-4m^2 - km + 4 = 0$. Its discriminant is $k^2 + 64 > 0$, so there are two distinct real tangents, and their slopes have product -1. Therefore every point of the vertical line obtained meets the condition. The other listed vertical lines have the wrong displacement from the vertex and do not give this product.

Step 3: Conclusion The locus is the directrix $x = -1$, or $x + 1 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For perpendicular tangents to $y^2 = 4a(x - h)$, translate the directrix to $x = h - a$; remember to retain the vertex shift.

5. If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points (0, 1) and (2, β), then β is a root of the equation : [4 marks]

(A) $y^5 - 2y - 2 = 0$

(B) $2y^5 - 2y - 1 = 0$

(C) $2y^5 - y^2 - 2 = 0$

(D) $y^5 - y^2 - 1 = 0$

Correct Answer: (D) $y^5 - y^2 - 1 = 0$

Solution:

Concept:

- Treat x as a function of y: an equation that is nonlinear in $y(x)$ may become linear in $x(y)$.
- For $dx/dy + P(y)x = Q(y)$, multiply by the integrating factor $e^{\int P(y)\,dy}$, so the left side becomes the derivative of the integrating factor times x.

Step 1: Solve with y as the independent variable Near the given point $(0, 1)$, $y \neq 0$, so divide the differential equation by dy and then by y:

$$\frac{dx}{dy} + \frac{2}{y}x = 10y^2.$$

The integrating factor is $e^{\int 2/y\,dy} = y^2$. Multiplying by it gives

$$\frac{d}{dy}(xy^2) = 10y^4.$$

Integration yields

$$xy^2 = 2y^5 + C.$$

Differentiating this relation recovers $y^2dx + (2xy - 10y^4)dy = 0$, the original relation multiplied by y, confirming the solution on the relevant nonzero branch.

Step 2: Use the initial point and the second point At $(x, y) = (0, 1)$, the integrated equation gives $0 = 2 + C$, so $C = -2$. The solution curve through the point is therefore

$$xy^2 = 2y^5 - 2.$$

At $(2, \beta)$, this becomes

$$2\beta^2 = 2\beta^5 - 2, \qquad 2\beta^5 - 2\beta^2 - 2 = 0.$$

Dividing by 2 gives $\beta^5 - \beta^2 - 1 = 0$. Thus the coefficient of the squared term after normalisation is -1, while the fifth-power coefficient and constant are 1 and -1. These coefficients distinguish the correct polynomial from the other listed equations.

Step 3: Conclusion We obtain $\beta^5 - \beta^2 - 1 = 0$, which is the polynomial in option (D). Hence, the correct option is (D).

Quick Tip to Solve: When a differential equation contains x linearly but powers of y, first test whether solving for x as a function of y makes it linear.

6. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations

$x + y + z = 4$,

$3x + 2y + 5z = 3$,

$9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is : [4 marks]

(A) R

(B) $(-\infty, -9) \cup (-9, \infty)$

(C) $[-9, -8)$

(D) $(-\infty, -9) \cup [-8, \infty)$

Correct Answer: (A) R

Solution:

Concept:

- A square linear system has a unique solution whenever its coefficient determinant is nonzero. A zero determinant requires checking consistency separately; it does not by itself imply no solution.
- Write $n = \lfloor \lambda \rfloor$. Elimination must act on the right-hand side as well as on the coefficient rows.

Step 1: Eliminate x and y from the third equation The first two equations are $x + y + z = 4$ and $3x + 2y + 5z = 3$. Five times the second minus six times the first gives

$$9x + 4y + 19z = -9.$$

Subtract this result from the third printed equation, $9x + 4y + (28 + n)z = n$. The remaining condition is

$$(n + 9)z = n + 9.$$

This single relation separates the ordinary case from the potentially singular case without expanding a determinant.

Step 2: Check both parameter cases If $n \neq -9$, we get $z = 1$. The first two equations then give $x + y = 3$ and $3x + 2y = -2$, so $x = -8, y = 11$. If $n = -9$, elimination gives $0 = 0$, not a contradiction; the third equation is a linear combination of the first two. Indeed the same triple $(-8, 11, 1)$ satisfies every equation even in that case. More directly its third left side is $-72 + 44 + 28 + n = n$. Thus a solution exists for every integer floor value, including the entire interval $-9 \leq \lambda < -8$; excluding this interval or a single endpoint would be unjustified.

Step 3: Conclusion The system has a solution for every real λ, so the parameter set is $\mathbb{R}$. Hence, the correct option is (A).

Quick Tip to Solve: At a singular parameter, check the augmented equation: a cancellation to 0=0 preserves consistency and must not be discarded.

7. The set of all values of K > -1, for which the equation $(3x^2+4x+3)^2-(k+1)(3x^2+4x+3)(3x^2+4x+2)+k(3x^2+4x+2)^2=0$ has real roots, is : [4 marks]

(A) $\left(1, \frac{5}{2}\right]$

(B) [2, 3)

(C) $\left[-\frac{1}{2}, 1\right)$

(D) $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

Correct Answer: (A) $\left(1, \frac{5}{2}\right]$

Solution:

Concept:

- For expressions A, B, $A^2-(k+1)AB+kB^2=(A-B)(A-kB)$. Here the two quadratic expressions differ by exactly 1.
- A quadratic written as a square plus its minimum has a transparent real range. This range converts the existence of real roots into inequalities on the parameter.

Step 1: Factor the expression before expanding Put $B=3x^2+4x+2$, so that $A=3x^2+4x+3=B+1$. The given equation factors as

$$(A-B)(A-kB)=0.$$

Since $A-B=1$ identically, only $A-kB=0$ is possible. Consequently

$$(1-k)B+1=0.$$

For $k=1$, this is $1=0$, so that value is excluded. For every other parameter value, the condition becomes $B=1/(k-1)$. No division by B has been made here, so no possible real root is lost.

Step 2: Use the minimum value of the quadratic Completing the square gives

$$B=3\left(x+\frac{2}{3}\right)^2+\frac{2}{3}\geq\frac{2}{3}.$$

Its range is exactly $[2/3,\infty)$. Thus real roots exist exactly when

$$\frac{1}{k-1}\geq\frac{2}{3}.$$

The left side must be positive, hence $k>1$; multiplying by $3(k-1)>0$ then gives $3\geq 2(k-1)$, or $k\leq 5/2$. At $k=5/2$, the minimum is attained at $x=-2/3$, so the upper endpoint is included. Values below 1 cannot work because the required quadratic value is negative. The remaining intervals either omit valid values or include invalid ones.

Step 3: Conclusion The required parameter set is $(1,5/2]$, already satisfying $k>-1$. Hence, the correct option is (A).

Quick Tip to Solve: When two repeated quadratic expressions differ by a constant, factor in those expressions first and use the range of the simpler quadratic.

8. A box open from top is made from a rectangular sheet of dimension a × b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to : [4 marks]

(A) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

(B) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

(C) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

(D) $\frac{a+b+\sqrt{a^2+b^2+ab}}{6}$

Correct Answer: (C) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

Solution:

Concept:

- The cut size is the height of the box. Removing two corner squares along each edge leaves base dimensions $a-2x$ and $b-2x$.
- For a physical box, $0 < x < \min(a,b)/2$. An interior maximum of its differentiable volume satisfies $V\prime(x) = 0$.

Step 1: Express the volume The three dimensions must all be positive. Multiplying them gives

$$V(x) = x(a-2x)(b-2x) = abx - 2(a+b)x^2 + 4x^3.$$

At either endpoint of the allowed closed interval the volume vanishes; inside it the volume is positive. Therefore its greatest value occurs at an interior stationary point.

Step 2: Solve and select the maximum Differentiate and solve the quadratic:

$$V\prime = ab - 4(a+b)x + 12x^2 = 0.$$

Thus

$$x = \frac{a+b \pm \sqrt{a^2+b^2-ab}}{6}.$$

If $D = \sqrt{a^2+b^2-ab} > 0$, then $V\prime\prime = -4(a+b) + 24x$ equals $-4D$ at the smaller root and $4D$ at the larger root. The smaller root is the maximum. It lies in the physical interval: assuming $a \le b$, $D < a+b$, and $D > b-2a$ whenever the latter is positive, because $D^2 - (b-2a)^2 = 3a(b-a) \ge 0$. The square case also gives a strictly interior root.

Step 3: Check the alternatives Option (A) incorrectly halves the required cut size. Options (B) and (D) use a^2+b^2+ab, whereas the stationary quadratic gives the minus sign. The plus-root expression also does not describe the required maximum.

Step 4: Conclusion The maximizing cut size is $x = \frac{a+b-\sqrt{a^2+b^2-ab}}{6}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the three box dimensions before differentiating; then reject any stationary root that is not a physical maximum.

9. Let Z be the set of all integers,

$$A = \{(x, y) \in Z \times Z : (x-2)^2 + y^2 \leq 4\}$$

$$B = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$$

$$C = \{(x, y) \in Z \times Z : (x-2)^2 + (y-2)^2 \leq 4\}$$

If the total number of relation from A ∩ B to A ∩ C is 2^p, then the value of p is : [4 marks]

(A) 16

(B) 25

(C) 49

(D) 9

Correct Answer: (B) 25

Solution:

Concept:

- A relation from a finite set S to a finite set T is any subset of $S \times T$. Hence there are $2^{|S||T|}$ relations.
- The inequalities describe closed disks of radius 2, but only integer-coordinate points count. Boundary points are included by $\leq$.

Step 1: Enumerate the first intersection In $A \cap B$, the first disk restricts x to $0, 1, 2, 3, 4$, while the second restricts it to $-2, -1, 0, 1, 2$. Only $x = 0, 1, 2$ can occur. For $x = 0$, the first inequality forces $y = 0$. For $x = 1$, both inequalities give $y^2 \leq 3$, so $y = -1, 0, 1$. For $x = 2$, the second forces $y = 0$. Consequently

$$A \cap B = \{(0, 0), (1, -1), (1, 0), (1, 1), (2, 0)\},$$

and its size is 5.

Step 2: Enumerate the second intersection For $A \cap C$, the possible common ordinates are $y = 0, 1, 2$. At $y = 0$, the inequality for C forces $x = 2$. At $y = 1$, both give $(x-2)^2 \leq 3$, allowing $x = 1, 2, 3$. At $y = 2$, the inequality for A forces $x = 2$. Thus

$$A \cap C = \{(2, 0), (1, 1), (2, 1), (3, 1), (2, 2)\},$$

also of size 5.

Step 3: Count all relations There are $5 \cdot 5 = 25$ ordered pairs in the Cartesian product. Each can independently be included or excluded from a relation, giving 2^{25} relations. This is not the number of functions, so the formula 5^5 is inapplicable. The other exponents 16, 49 and 9 would correspond to incorrectly counted intersections of sizes 4, 7 and 3.

Step 4: Conclusion Comparing $2^p = 2^{25}$, we obtain $p = 25$. Hence, the correct option is (B).

Quick Tip to Solve: For relations, first count each set, multiply their sizes, and only then put 2 to that power.

10. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is : [4 marks]

(A) 9

(B) 10

(C) 4

(D) 6

Correct Answer: (A) 9

Solution:

Concept:

- Use horizontal strips when the parabola is already easy to express as x in terms of y. Area is $\int (x_{\text{right}} - x_{\text{left}})\, dy$.
- The tangent at a point has the same derivative as the curve there. The x-axis supplies the boundary $y = 0$.

Step 1: Find the tangent The parabola is $x = 1 + (y-2)^2$. At ordinate $y = 3$, its abscissa is $x = 2$, so the point of tangency is $(2, 3)$. Differentiating the original equation gives

$$2(y-2)\frac{dy}{dx} = 1.$$

The slope at that point is $1/2$. Therefore the tangent is

$$y - 3 = \frac{1}{2}(x-2), \qquad x = 2y - 4.$$

The tangent meets the x-axis at $(-4, 0)$, and the parabola meets it at $(5, 0)$. These intersections close the required region.

Step 2: Integrate the horizontal width For $0 \le y \le 3$, the right boundary is the parabola and the left boundary is the tangent. Their difference is

$$1 + (y-2)^2 - (2y-4) = (y-3)^2 \ge 0.$$

The region therefore has area

$$\mathcal{A} = \int_0^3 (y-3)^2\, dy = \left[\frac{(y-3)^3}{3}\right]_0^3 = 0 - (-9) = 9.$$

No second region needs adding: the tangent and parabola touch at $y = 3$, and the horizontal axis closes the region below.

Step 3: Compare with the options The positive width integral gives exactly 9 square units. The values 10, 4 and 6 do not agree with this integral. In particular, using the difference of boundary abscissae at just one height as an area would omit the changing width and is invalid.

Step 4: Conclusion The bounded area is 9 square units. Hence, the correct option is (A).

Quick Tip to Solve: For a sideways parabola and the x-axis, integrate with respect to y to avoid splitting the parabola into two branches.

11. If $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$**, then** $\frac{dy}{dx}$ **at** $x = \frac{5\pi}{6}$ **is :** [4 marks]

(A) $-\frac{1}{2}$

(B) -1

(C) $\frac{1}{2}$

(D) 0

Correct Answer: (A) $-\frac{1}{2}$

Solution:

Concept:

- Principal square roots are nonnegative, so $\sqrt{u^2} = |u|$. The specified interval fixes the signs of half-angle expressions.
- The principal value of $\cot^{-1} z$ lies in $(0, \pi)$, allowing a branch-correct simplification before differentiation.

Step 1: Resolve the square roots using the interval Put $t = x/2$. Since $\pi/2 < x < \pi$, we have $\pi/4 < t < \pi/2$. Thus $\sin t > \cos t > 0$. The half-angle identities give

$$1 + \sin x = (\sin t + \cos t)^2, \qquad 1 - \sin x = (\sin t - \cos t)^2.$$

Both displayed linear expressions are positive on this interval. Therefore their principal square roots are respectively $\sin t + \cos t$ and $\sin t - \cos t$, with no additional minus signs.

Step 2: Simplify the inverse cotangent The fraction inside the inverse function becomes

$$\frac{(\sin t + \cos t) + (\sin t - \cos t)}{(\sin t + \cos t) - (\sin t - \cos t)} = \frac{2\sin t}{2\cos t} = \tan t.$$

Since $\tan t = \cot(\pi/2 - t)$, and $0 < \pi/2 - t < \pi/4$ is within the principal range,

$$y = \frac{\pi}{2} - t = \frac{\pi}{2} - \frac{x}{2}.$$

This equality holds throughout the given interval, so differentiation is legitimate there.

Step 3: Differentiate and choose the value It follows that $dy/dx = -1/2$, independent of x in the interval. In particular $5\pi/6$ is interior and gives the same value. Option (C) has the wrong sign, often caused by ignoring the root signs. Option (B) misses the half-angle factor, while (D) would incorrectly treat the expression as constant.

Step 4: Conclusion At $x = 5\pi/6$, $\frac{dy}{dx} = -\frac{1}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Before simplifying square roots of half-angle squares, use the given interval to remove the absolute values correctly.

12. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is : [4 marks]

(A) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

(B) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

(C) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$

(D) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$

Correct Answer: (A) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

Solution:

Concept:

- The substitution $x = t^2$ removes $\sqrt{x}$, with $dx = 2t\,dt$ and limits still 0 and 1.
- Partial fractions reduce this rational integral to inverse tangent integrals: $\int dt/(1 + c^2t^2) = \tan^{-1}(ct)/c$.

Step 1: Remove the radical With $x = t^2$, the integral becomes

$$I = \int_0^1 \frac{2t^2\,dt}{(1+t^2)(1+3t^2)(3+t^2)}.$$

All denominators are positive throughout the interval, so this substitution introduces no singularity. Decompose the rational function as

$$\frac{2t^2}{(1+t^2)(1+3t^2)(3+t^2)} = \frac{1}{2(1+t^2)} - \frac{3}{8(1+3t^2)} - \frac{3}{8(3+t^2)}.$$

Multiplying by the common denominator verifies this identity; its constant and fourth-degree terms cancel and its quadratic term is $2t^2$.

Step 2: Evaluate the elementary integrals Integrating term by term gives

$$I = \left[\frac{1}{2}\tan^{-1}t - \frac{3}{8\sqrt{3}}\tan^{-1}(\sqrt{3}t) - \frac{3}{8\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right]_0^1.$$

The inverse tangent values at the upper endpoint are $\pi/4, \pi/3, \pi/6$, respectively, and all lower endpoint values vanish. Hence

$$I = \frac{\pi}{8} - \frac{3}{8\sqrt{3}}\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \frac{\pi}{8} - \frac{\sqrt{3}\pi}{16}.$$

Step 3: Match the exact expression Factoring $\pi/8$ gives $I = (\pi/8)(1 - \sqrt{3}/2)$, which is positive as required. Option (D) is twice this value. Options (B) and (C) have the incorrect radical coefficient and therefore fail the exact evaluation.

Step 4: Conclusion $I = \frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: After substituting x=t squared, decompose in the three quadratic factors; the two inverse tangent angles add to pi/2.

13. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is : [4 marks]

(A) $\left(1,\frac{1}{2}\right)$

(B) $\left(1,-\frac{1}{2}\right)$

(C) $\left(-1,\frac{1}{2}\right)$

(D) $\left(-1,-\frac{1}{2}\right)$

Correct Answer: (B) $\left(1,-\frac{1}{2}\right)$

Solution:

Concept:

- For $x\to+\infty$, $\sqrt{x^2-x+1}/x=\sqrt{1-1/x+1/x^2}\to 1$. A finite limit requires cancellation of the leading linear term.
- Rationalizing a difference of square roots exposes the finite constant remaining after the leading terms cancel.

Step 1: Determine the coefficient a Write the expression as

$$x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}-a\right).$$

If $a\neq 1$, the bracket approaches the nonzero number $1-a$, and multiplying by $x\to+\infty$ makes the expression diverge. More explicitly, the radical differs from 1 by a quantity tending to zero, so the nonzero constant cannot cancel. Because the question specifies a finite value b, necessarily $a=1$. Options with $a=-1$ cannot satisfy the requirement; they add rather than cancel the two positive leading terms.

Step 2: Rationalize to determine b For $a=1$, multiply numerator and denominator by the conjugate:

$$\sqrt{x^2-x+1}-x=\frac{-x+1}{\sqrt{x^2-x+1}+x}=\frac{-1+1/x}{\sqrt{1-1/x+1/x^2}+1}.$$

Since x is positive near positive infinity, division by x has the indicated sign. The numerator tends to -1, and the denominator tends to 2, so $b=-1/2$. The denominator stays positive, making this limit operation valid.

Step 3: Check the sign and alternatives For $x>1$, the radicand x^2-x+1 is smaller than x^2. Thus the radical minus x is negative, confirming that $b=+1/2$ cannot be correct. Options (C) and (D) already fail the leading-term cancellation. Only option (B) has both required values.

Step 4: Conclusion The ordered pair is $\boxed{(a,b)=(1,-1/2)}$. Hence, the correct option is (B).

Quick Tip to Solve: First cancel the leading growth to find a; only then use the conjugate to extract the constant limit.

14. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is : [4 marks]

(A) $\frac{1}{8}$

(B) $\frac{5}{8}$

(C) $\frac{5}{16}$

(D) 1

Correct Answer: (C) $\frac{5}{16}$

Solution:

Concept:

- The head count in three fair tosses has binomial distribution: $P(H = k) = \binom{3}{k}/8$ for $k = 0, 1, 2, 3$.
- The two persons toss independently. Probabilities for a specified pair of head counts multiply, and mutually exclusive matching cases add.

Step 1: List the distribution of each head count Three fair coins have $2^3 = 8$ equally likely sequences. There is one sequence with no head, three with one head, three with two heads, and one with three heads. Thus either person has probabilities

$$P(H = 0) = \frac{1}{8}, \quad P(H = 1) = \frac{3}{8}, \quad P(H = 2) = \frac{3}{8}, \quad P(H = 3) = \frac{1}{8}.$$

The counts are not equally likely: the middle counts occur in three different sequences each. Treating four possible counts as equiprobable would therefore give an incorrect result.

Step 2: Add the four matching cases The event of equal counts is the disjoint union of $(0, 0), (1, 1), (2, 2), (3, 3)$. Independence gives

$$P(H_A = H_B) = \sum_{k=0}^{3} P(H_A = k)P(H_B = k).$$

Since the distributions are identical, this is the sum of the squared probabilities:

$$\left(\frac{1}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^2 + \left(\frac{1}{8}\right)^2 = \frac{1 + 9 + 9 + 1}{64} = \frac{5}{16}.$$

Step 3: Distinguish counts from identical sequences Option (A), $1/8$, is the probability that the two ordered sequences match exactly; equality of head counts permits more outcomes, such as HHT and HTH. Option (B) doubles the desired probability by incorrectly counting the matching cases twice. Option (D) would mean equal counts are certain, contradicted by one person obtaining HHH and the other TTT.

Step 4: Conclusion The required probability is $\frac{5}{16}$. Hence, the correct option is (C).

Quick Tip to Solve: For two independent identical discrete distributions, the probability of equal values is the sum of the squared individual probabilities.

15. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, −3) from the line 3x + 4y = 5, is given by : [4 marks]

(A) $10\frac{d^2y}{dx^2} = 11$

(B) $11\frac{d^2x}{dy^2} = 10$

(C) $10\frac{d^2x}{dy^2} = 11$

(D) $11\frac{d^2y}{dx^2} = 10$

Correct Answer: (D) $11\frac{d^2y}{dx^2} = 10$

Solution:

Concept:

- The perpendicular distance from (x_0, y_0) to $Ax + By + C = 0$ is $d = |Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$.
- A parabola with vertical axis can be written $(x - h)^2 = 4c(y - k)$. Its latus rectum has length $4c$ for the usual upward-opening form, and differentiating twice eliminates the vertex coordinates.

Step 1: Calculate the latus rectum The line is $3x + 4y - 5 = 0$ and the given point is $(2, -3)$. Thus

$$d = \frac{|3(2) + 4(-3) - 5|}{\sqrt{3^2 + 4^2}} = \frac{|-11|}{5} = \frac{11}{5}.$$

The absolute value makes this distance positive; using the signed numerator would incorrectly reverse the sign of the curvature.

Step 2: Differentiate the parabola family Write the family in the standard vertical-axis form

$$(x - h)^2 = \frac{11}{5}(y - k).$$

Here h and k specify its vertex and are the arbitrary constants to eliminate. Differentiate with respect to x:

$$2(x - h) = \frac{11}{5}\frac{dy}{dx}.$$

Differentiate once more:

$$2 = \frac{11}{5}\frac{d^2y}{dx^2}, \qquad 11\frac{d^2y}{dx^2} = 10.$$

Step 3: Compare the proposed differential equations The dependent variable is y, because the axis is parallel to the y-axis. Therefore the alternatives using d^2x/dy^2 describe the wrong orientation. Option (A) has curvature $11/10$, whereas the required curvature is $10/11$. The reciprocal must not be interchanged when rearranging the second derivative.

Step 4: Conclusion The required listed equation is $11\dfrac{d^2y}{dx^2} = 10$. Hence, the correct option is (D).

Quick Tip to Solve: For $(x - h)^2 = Ly - Lk$, two x-derivatives give $Ly'' = 2$: find the latus rectum first, then eliminate the translations.

16. The equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x-axis is : [4 marks]

(A) $\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$

(B) $\vec{r}.\left(\hat{i}+3\hat{k}\right)+6=0$

(C) $\vec{r}.\left(\hat{i}-3\hat{k}\right)+6=0$

(D) $\vec{r}.\left(\hat{j}-3\hat{k}\right)-6=0$

Correct Answer: (A) $\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$

Solution:

Concept:

- Every plane through the intersection of $P_1=0$ and $P_2=0$ has equation $P_1+\mu P_2=0$, with a real parameter μ.
- A plane parallel to the x-axis has a normal perpendicular to $(1,0,0)$; hence its x-coefficient is zero.

Step 1: Form the pencil of planes The given vector equations become

$$P_1=x+y+z-1=0, \qquad P_2=2x+3y-z+4=0.$$

A common point makes both expressions zero, so it also satisfies every linear combination. Therefore write

$$(x+y+z-1)+\mu(2x+3y-z+4)=0.$$

Collecting the coefficients gives

$$(1+2\mu)x+(1+3\mu)y+(1-\mu)z-1+4\mu=0.$$

Step 2: Impose parallelism to the x-axis The direction vector of the x-axis is $\hat{i}$. Its dot product with the plane normal must vanish. Consequently

$$1+2\mu=0, \qquad \mu=-\frac{1}{2}.$$

Substituting this value into the pencil gives

$$-\frac{1}{2}y+\frac{3}{2}z-3=0.$$

Multiplying by -2, which preserves the same plane, gives

$$y-3z+6=0.$$

Step 3: Check the position and the alternatives The resulting equation contains no x-term, so changing x while holding y and z fixed keeps a point in the plane: this confirms the required parallel direction. It equals $P_2-2P_1=0$, confirming that the entire common line is retained. Options (B) and (C) have

nonzero x-coefficients and fail parallelism. Option (D) has the correct normal but the wrong constant, so it is a distinct parallel plane rather than the required plane.

Step 4: Conclusion The equation is $\vec{r} \cdot (\hat{j} - 3\hat{k}) + 6 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: In a pencil of planes, eliminate the coefficient of the coordinate along the required parallel axis before simplifying the remaining equation.

17. Let A(a, 0), B(b, 2b + 1) and C(0, b), b $\neq$ 0, |b| $\neq$ 1, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is : [4 marks]

(A) $\frac{-2b}{b+1}$

(B) $\frac{2b}{b+1}$

(C) $\frac{2b^2}{b+1}$

(D) $\frac{-2b^2}{b+1}$

Correct Answer: (D) $\frac{-2b^2}{b+1}$

Solution:

Concept:

- The area of a triangle with coordinates (x_i, y_i) is half the absolute value of its coordinate determinant.
- An absolute-value equation $|F(a)| = 2$ gives two signed equations, $F(a) = 2$ and $F(a) = -2$; both are needed when summing all possible values.

Step 1: Write the determinant for the area For the three given points, twice the signed area is

$$\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = a((2b+1) - b) + b^2 = a(b+1) + b^2.$$

The determinant may be positive or negative depending on the orientation of the vertices. Since the geometric area is one square unit, its absolute value satisfies

$$|a(b+1) + b^2| = 2.$$

Step 2: Find both admissible values of a Splitting the absolute value into its positive and negative cases yields

$$a(b+1) + b^2 = 2 \quad \text{or} \quad a(b+1) + b^2 = -2.$$

Because the hypothesis $|b| \neq 1$ implies $b \neq -1$, division by $b + 1$ is valid. Thus

$$a_1 = \frac{2 - b^2}{b+1}, \qquad a_2 = \frac{-2 - b^2}{b+1}.$$

They are distinct, since their difference is $4/(b + 1)$, which is nonzero.

Step 3: Add the values and select the expression Adding cancels the constants coming from the two

orientations:

$$a_1 + a_2 = \frac{2 - b^2 - 2 - b^2}{b+1} = -\frac{2b^2}{b+1}.$$

Options (A) and (B) incorrectly have a numerator linear in b; the determinant produces b^2. Option (C) has the correct magnitude but the opposite sign. The negative sign follows directly from transferring b^2 across the equality in each case.

Step 4: Conclusion The sum of all possible values is $-\frac{2b^2}{b+1}$. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed nonzero area, solve both signs of the determinant. Their sum often cancels the area constant and saves calculation.

18. The Boolean expression (p ∧ q) ⇒ ((r ∧ q) ∧ p) is equivalent to : [4 marks]

(A) (p ∧ q) ⇒ (r ∧ q)

(B) (q ∧ r) ⇒ (p ∧ q)

(C) (p ∧ q) ⇒ (r ∨ q)

(D) (p ∧ r) ⇒ (p ∧ q)

Correct Answer: (A) (p ∧ q) ⇒ (r ∧ q)

Solution:

Concept:

- An implication $u \Rightarrow v$ is equivalent to $\neg u \vee v$; it is false only when u is true and v is false.
- Conjunction is associative and commutative. The absorption identities permit simplification after converting an implication to a disjunction.

Step 1: Simplify the given implication Let $s = p \wedge q$. Its consequent $(r \wedge q) \wedge p$ equals $s \wedge r$. Therefore the expression becomes

$$s \Rightarrow (s \wedge r) \equiv \neg s \vee (s \wedge r).$$

Distribute disjunction over conjunction:

$$\neg s \vee (s \wedge r) \equiv (\neg s \vee s) \wedge (\neg s \vee r) \equiv \neg s \vee r.$$

In terms of the original statements this is $\neg p \vee \neg q \vee r$.

Step 2: Reduce option A to the same expression For option (A), use the implication rule again:

$$(p \wedge q) \Rightarrow (r \wedge q) \equiv \neg p \vee \neg q \vee (r \wedge q).$$

Because $\neg q \vee (r \wedge q) = \neg q \vee r$, this also simplifies to

$$\neg p \vee \neg q \vee r.$$

Alternatively, when its antecedent is true, q is already true, so requiring $r \wedge q$ imposes exactly the additional condition r.

Step 3: Exclude the remaining alternatives Option (C) is always true: whenever $p \wedge q$ is true, q makes $r \vee q$ true. The given expression is false at $p = q =$ true, $r =$ false, so it is not that tautology. At the same truth assignment, (B) and (D) both have false antecedents and are true, while the original is false. This single counterexample rejects both of those alternatives without needing a full truth table.

Step 4: Conclusion The equivalent expression is $(p \wedge q) \Rightarrow (r \wedge q)$. Hence, the correct option is (A).

Quick Tip to Solve: When an implication has a conjunction as antecedent, its factors are already true in the only case where the consequent needs checking.

19. Two poles, AB of length a metres and CD of length a + b (b $\neq$ a) metres are erected at the same horizontal level with bases at B and D. If BD = x and tan$\angle$ACB = $\frac{1}{2}$, then : [4 marks]

(A) $x^2 + 2(a + 2b)x - b(a + b) = 0$

(B) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(C) $x^2 - 2ax + b(a + b) = 0$

(D) $x^2 - 2ax + a(a + b) = 0$

Correct Answer: (C) $x^2 - 2ax + b(a + b) = 0$

Solution:

Concept:

- For two downward lines of sight from a common point, their included angle is the difference of their depression angles measured from the horizontal.
- The identity $\tan(\alpha - \beta) = (\tan\alpha - \tan\beta)/(1 + \tan\alpha\tan\beta)$ relates the two right triangles to the given angle.

Step 1: Identify the two right triangles The tops A and C are vertically above B and D, respectively. Their heights are a and $a + b$, and the horizontal separation is $x > 0$. From C, the vertical fall to B is $a + b$, while the vertical fall to A is b. If their depression angles are α and β, then

$$\tan\alpha = \frac{a+b}{x}, \qquad \tan\beta = \frac{b}{x}.$$

Thus $\angle ACB = \alpha - \beta$. These differences use the heights relative to C; substituting a as the fall to A would misidentify the geometry.

Step 2: Apply the tangent difference formula The given tangent is $1/2$, so

$$\frac{1}{2} = \frac{(a+b)/x - b/x}{1 + ((a+b)/x)(b/x)}.$$

The numerator is a/x. Multiplying numerator and denominator by x^2 yields

$$\frac{1}{2} = \frac{ax}{x^2 + b(a+b)}.$$

Cross-multiplication is valid for the physical positive lengths and gives

$$x^2 + b(a+b) = 2ax.$$

Step 3: Rearrange and compare the options Bringing all terms to one side gives

$$x^2 - 2ax + b(a+b) = 0.$$

The cross term must be negative after moving $2ax$ to the left; options (A) and (B) have a positive linear term and also the wrong coefficient. Option (D) replaces the product of the two vertical falls $b(a+b)$ by $a(a+b)$, so it does not arise from the two slopes. The stated existence of the configuration selects positive real x satisfying this relation.

Step 4: Conclusion The relation is $x^2 - 2ax + b(a+b) = 0$. Hence, the correct option is (C).

Quick Tip to Solve: From the higher top, use the total height and the height difference as the two vertical falls, then apply the tangent subtraction identity.

20. If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + ...$, then the value of e^{1+y} at $x = \frac{1}{2}$ is : [4 marks]

(A) $\frac{1}{2}e^2$

(B) 2e

(C) $\frac{1}{2}\sqrt{e}$

(D) $2e^2$

Correct Answer: (A) $\frac{1}{2}e^2$

Solution:

Concept:

- For $|x| < 1$, $\sum_{n=1}^{\infty} x^n = x/(1-x)$ and $-\log(1-x) = \sum_{n=1}^{\infty} x^n/n$.
- The coefficient $(n-1)/n = 1 - 1/n$ splits this power series into a geometric series and a logarithmic series. The given domain ensures their convergence.

Step 1: Express the general term The displayed terms have powers 2, 3, 4 with coefficients $1/2, 2/3, 3/4$. Hence

$$y = \sum_{n=2}^{\infty} \frac{n-1}{n} x^n = \sum_{n=2}^{\infty} x^n - \sum_{n=2}^{\infty} \frac{x^n}{n}.$$

Both sums converge absolutely because $0 < x < 1$, so this subtraction is justified. The coefficient is not $n/(n+1)$ at the same power index; matching the first term avoids that indexing error.

Step 2: Sum the two standard series The first sum is $x^2/(1-x)$. Removing the first term from the logarithmic series gives

$$\sum_{n=2}^{\infty} \frac{x^n}{n} = -\log(1-x) - x.$$

Therefore

$$y = \frac{x^2}{1-x} + \log(1-x) + x = \frac{x}{1-x} + \log(1-x).$$

The logarithm is real on the stated interval, and its negative value is consistent with the subtraction of the second series.

Step 3: Substitute and exponentiate At $x = 1/2$,

$$y = 1 + \log\frac{1}{2} = 1 - \log 2, \qquad 1 + y = 2 - \log 2.$$

Consequently

$$e^{1+y} = e^{2-\log 2} = \frac{e^2}{2}.$$

Option (B) would require the exponent $1 + \log 2$, option (C) would require $1/2 - \log 2$, and option (D) would require $2 + \log 2$. None agrees with the computed exponent; in particular the logarithm enters with a negative value at one half.

Step 4: Conclusion The value is $\frac{1}{2}e^2$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever a coefficient is $(n-1)/n$, separate it as $1 - 1/n$ and combine the geometric sum with $\log(1-x)$.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The probability distribution of random variable X is given by :

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let p = P(1 < X < 4 | X < 3). If 5p = λK, then λ equal to _____. [4 marks]

Solution:

Concept:

- The probabilities of all mutually exclusive values of a discrete random variable add to one. This normalizes K before evaluating the requested parameter.
- Conditional probability is $P(A \mid B) = P(A \cap B)/P(B)$, provided $P(B) > 0$. Only values satisfying the strict inequalities belong to these events.

Step 1: Normalize the distribution The possible values are 1, 2, 3, 4 and 5, with probabilities $K, 2K, 2K, 3K, K$. Their sum must equal one:

$$K + 2K + 2K + 3K + K = 9K = 1, \qquad K = \frac{1}{9}.$$

All entries are then nonnegative, so this is a valid distribution. Retaining K for the conditional-probability calculation will also show how the common factor cancels.

Step 2: Find the conditional event Put $A = \{1 < X < 4\}$ and $B = \{X < 3\}$. Among the listed integer values,

$$A = \{2, 3\}, \qquad B = \{1, 2\}, \qquad A \cap B = \{2\}.$$

The strict inequality in B excludes 3 even though 3 lies in A. Thus

$$P(B) = K + 2K = 3K, \qquad P(A \cap B) = 2K.$$

Since $3K = 1/3 > 0$, conditioning is well-defined and

$$p = \frac{2K}{3K} = \frac{2}{3}.$$

Step 3: Use the defining relation for lambda Insert this p into the printed equation $5p = \lambda K$:

$$5\left(\frac{2}{3}\right) = \lambda\left(\frac{1}{9}\right), \qquad \lambda = \frac{10}{3} \times 9 = 30.$$

There is no physical unit: the probability and normalization constant are dimensionless. A common error is using $P(A) = 4K$ as the numerator; that counts the excluded outcome 3 and is not the intersection required for conditional probability. Another error is replacing the conditional denominator by the total probability one.

Step 4: Conclusion The required dimensionless numerical value is $\lambda = 30$.

Quick Tip to Solve: List the actual discrete outcomes in both events before conditioning: the strict inequalities here leave only X=2 in their intersection.

22. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to _____. [4 marks]

Solution:

Concept:

- Write a complex number as $z = x + iy$, so $\text{Re}\, z = x$ and $|z - 3| = \sqrt{(x-3)^2 + y^2}$.
- The argument of a nonzero difference determines the direction of the chord joining its two points. An argument of $\pi/4$ gives equal, positive real and imaginary differences.

Step 1: Convert the modulus condition to a locus For each of the two points the given condition is

$$\sqrt{(x-3)^2 + y^2} = x.$$

The right side is nonnegative, so squaring is legitimate for points on the locus. Expanding and cancelling x^2 gives

$$x^2 - 6x + 9 + y^2 = x^2, \qquad x = \frac{y^2 + 9}{6}.$$

This expression is positive for every real y, so it also satisfies the original unsquared equation. Thus no unwanted branch has been introduced.

Step 2: Use the direction of the difference Put $z_j = x_j + iy_j$. Since $\arg(z_1 - z_2) = \pi/4$, we have $x_1 - x_2 = y_1 - y_2 > 0$. In particular, $y_1 - y_2 \neq 0$. Subtract the two locus equations and factor:

$$x_1 - x_2 = \frac{y_1^2 - y_2^2}{6} = \frac{(y_1 - y_2)(y_1 + y_2)}{6}.$$

Replace the left side by $y_1 - y_2$, then divide by that nonzero difference. We obtain $y_1 + y_2 = 6$. This calculation uses the chord slope, rather than treating either point as fixed.

Step 3: Conclusion The imaginary part of $z_1 + z_2$ is $y_1 + y_2 = \boxed{6}$. This is a dimensionless numerical value.

Quick Tip to Solve: For two points on $x = (y^2 + 9)/6$, factor the difference of squares: the chord slope immediately determines the sum of their ordinates.

23. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set T = {A $\subseteq$ S : A $\neq \phi$ and the sum of all the elements of A is not a multiple of 3} is _____. [4 marks]

Solution:

Concept:

- A sum is divisible by three exactly when its residue modulo three is zero. Elements with residue zero never change the residue of a subset sum.
- The number of ways to select a objects from two distinct objects is $\binom{2}{a}$. Choices from disjoint residue classes multiply.

Step 1: Separate the three residue classes In the given set, the residue-zero elements are $3, 6, 9$, the residue-one elements are $1, 4$, and the residue-two elements are $2, 5$. First choose a subset of the four nonzero-residue elements. If it contains a residue-one elements and b residue-two elements, its sum has residue $a + 2b$, where $0 \leq a, b \leq 2$. There are $2^4 = 16$ choices in total. We count and remove those whose sum is divisible by three.

Step 2: Count the forbidden residue and restore zero-residue choices Within these bounds, $a + 2b \equiv 0 \pmod 3$ occurs precisely for

$$(a, b) = (0, 0),\ (1, 1),\ (2, 2).$$

Their respective numbers of subsets are

$$\binom{2}{0}\binom{2}{0} = 1, \qquad \binom{2}{1}\binom{2}{1} = 4, \qquad \binom{2}{2}\binom{2}{2} = 1.$$

Thus $16 - (1 + 4 + 1) = 10$ subsets have nonzero residue. Each can be combined independently with any subset of $\{3, 6, 9\}$, giving $2^3 = 8$ choices without changing its residue. Every resulting subset is nonempty, since its nonzero-residue part cannot be empty. There is therefore no further subtraction for the empty set.

Step 3: Conclusion The required number is $10 \times 8 = \boxed{80}$ subsets, a dimensionless count.

Quick Tip to Solve: Remove multiples of three from the residue calculation first; they contribute a factor of two each to every admissible subset.

24. $3 \times 7^{22} + 2 \times 10^{22} - 44$ **when divided by 18 leaves the remainder _____.** [4 marks]

Solution:

Concept:

- Congruences modulo 18 preserve addition and multiplication, so large powers can be replaced by their repeating residues.
- The remainder in Euclidean division by 18 is the unique representative in $\{0, 1, \ldots, 17\}$, even when an intermediate residue is negative.

Step 1: Reduce the two large powers For powers of seven, compute a short cycle:

$$7^2 = 49 \equiv 13 \pmod{18}, \qquad 7^3 = 343 \equiv 1 \pmod{18}.$$

Since $22 = 3 \cdot 7 + 1$, this yields

$$7^{22} = (7^3)^7 \cdot 7 \equiv 7 \pmod{18}.$$

For powers of ten, $10^2 = 100 \equiv 10 \pmod{18}$. Multiplying this congruence repeatedly by ten shows that every positive power of ten has residue ten. In particular, $10^{22} \equiv 10 \pmod{18}$. This is a direct induction, so it does not require ten to be relatively prime to eighteen.

Step 2: Combine the residues and choose the standard remainder Substitute the reduced powers into the entire expression, keeping the coefficients and the negative constant:

$$3 \cdot 7^{22} + 2 \cdot 10^{22} - 44 \equiv 3 \cdot 7 + 2 \cdot 10 - 44 = -3 \pmod{18}.$$

The value -3 is a useful congruent representative but cannot itself be a division remainder. Adding one divisor gives $-3 + 18 = 15$, which lies between zero and seventeen. Equivalently the whole expression minus fifteen is divisible by eighteen, so this representative is the required remainder.

Step 3: Conclusion The remainder on division by 18 is $\boxed{15}$, a dimensionless integer.

Quick Tip to Solve: Look for short power cycles before expanding: $7^3 \equiv 1$ and $10^2 \equiv 10 \pmod{18}$ reduce this expression immediately.

25. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The overall mean is the weighted mean of the group means: $\bar{x} = (n_1\bar{x}_1 + n_2\bar{x}_2)/(n_1 + n_2)$.
- The combined population variance is the weighted sum of each within-group variance plus the squared displacement of that group mean from the overall mean.

Step 1: Find the average of the girls There are twenty boys and thirty girls, with overall mean fifteen. Equating the total score computed by the two groups to that computed from all candidates gives

$$20 \cdot 12 + 30\mu = 50 \cdot 15.$$

Consequently $30\mu = 750 - 240 = 510$, so $\mu = 17$. This is larger than the overall mean, as expected: the boys have a mean below fifteen, so the girls must offset their deficit. The group weights in all subsequent calculations are $20/50$ and $30/50$, not equal weights.

Step 2: Combine the within-group and between-group variation Relative to the overall mean fifteen, the boys' mean differs by -3 and the girls' mean differs by 2. Each group has variance two about its own mean. When shifting the origin to the overall mean, the average cross term vanishes because deviations within a group sum to zero. Hence

$$\begin{aligned}\sigma^2 &= \frac{20}{50}\{2 + (12-15)^2\} + \frac{30}{50}\{2 + (17-15)^2\} \\ &= \frac{2}{5}(11) + \frac{3}{5}(6) = \frac{22+18}{5} = 8.\end{aligned}$$

Merely averaging the two variances would give two and miss the spread caused by the different group means. We now add the two numerical quantities requested in the question.

Step 3: Conclusion We obtain $\mu = 17$, $\sigma^2 = 8$, and therefore $\boxed{\mu + \sigma^2 = 25}$. The requested numerical entry is 25.

Quick Tip to Solve: In a pooled variance, include both the variance within each group and the squared distance between that group mean and the overall mean.

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26. If $\int \frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx = \frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) + C$, where C is a constant of integration, then u + v is equal to _____. [4 marks]

Solution:

Concept:

- For a positive differentiable function $D(x)$, $\int D\prime(x)/D(x)\,dx = \ln D(x) + C$.
- A numerator built from the same exponentials as its denominator can be expressed as a linear combination of the denominator and its derivative.

Step 1: Decompose the numerator Let $D = 4e^x + 7e^{-x}$. It is strictly positive for every real x, and its derivative is $D\prime = 4e^x - 7e^{-x}$. Seek constants A, B such that

$$2e^x + 3e^{-x} = A(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x}).$$

Matching coefficients gives $4(A + B) = 2$ and $7(A - B) = 3$. Thus

$$A + B = \frac{1}{2}, \qquad A - B = \frac{3}{7}, \qquad A = \frac{13}{28}, \quad B = \frac{1}{28}.$$

The two signs in the derivative are essential: differentiating e^{-x} produces a minus sign.

Step 2: Integrate and compare the requested coefficients Divide the decomposition by D and integrate term by term:

$$\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}}\,dx = \int \left(\frac{13}{28} + \frac{1}{28}\frac{D\prime}{D}\right) dx$$
$$= \frac{13x}{28} + \frac{1}{28}\ln(4e^x + 7e^{-x}) + C.$$

No absolute-value qualification is needed inside the logarithm because $D > 0$. Compare this with $(ux + v\ln D)/14 + C$: $u/14 = 13/28$ and $v/14 = 1/28$, so $u = 13/2$ and $v = 1/2$. As a check, differentiating the result recombines the numerator to $2e^x + 3e^{-x}$ exactly.

Step 3: Conclusion The dimensionless coefficient sum is $u + v = \frac{13}{2} + \frac{1}{2} = \boxed{7}$.

Quick Tip to Solve: For a ratio of two linear combinations of e^x and e^{-x}, write the numerator as $AD + BD\prime$ before making a substitution.

27. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in [0, 4π]. Then $\frac{8S}{\pi}$ is equal to _____. [4 marks]

Solution:

Concept:

- The identity $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$ reduces the fourth powers.
- The product $t = \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ satisfies $-\frac{1}{2} \le t \le \frac{1}{2}$. Use this range to reject extraneous algebraic roots.

Step 1: Reduce to a quadratic with a restricted variable Set $t = \sin\theta\cos\theta$. The given equation becomes

$$1 - 2t^2 - t = 0, \qquad 2t^2 + t - 1 = (2t - 1)(t + 1) = 0.$$

Its algebraic roots are $t = 1/2$ and $t = -1$. The second cannot occur for real θ, since the magnitude of $\sin\theta\cos\theta$ is at most one half. We therefore retain only $t = 1/2$, which is equivalent to $\sin 2\theta = 1$. The range check ensures that no spurious trigonometric solutions enter the sum.

Step 2: List every solution in the closed interval and add The complete real solution of $\sin 2\theta = 1$ is

$$2\theta = \frac{\pi}{2} + 2k\pi, \qquad \theta = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}.$$

Imposing $0 \le \theta \le 4\pi$ gives $k = 0, 1, 2, 3$, hence the four angles are

$$\frac{\pi}{4}, \quad \frac{5\pi}{4}, \quad \frac{9\pi}{4}, \quad \frac{13\pi}{4}.$$

Neither endpoint is a solution, and the next value $17\pi/4$ exceeds the upper bound. Their sum is $S = (1 + 5 + 9 + 13)\pi/4 = 7\pi$ radians. This enumeration uses the period π of the product, so no repeated or missed roots arise.

Step 3: Conclusion Thus $S = 7\pi$ radians and the requested dimensionless value is $\frac{8S}{\pi} = \boxed{56}$.

Quick Tip to Solve: After substituting $t = \sin\theta\cos\theta$, enforce $|t| \le 1/2$ before solving for angles; then list roots using the period π.

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28. Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane 2x − y + z + 3 = 0 and let R(3, 5, γ) be a point of this plane. Then the square of the length of the line segment SR is _____. [4 marks]

Solution:

Concept:

- For the plane $\mathbf{n}\cdot\mathbf{r} + d = 0$, reflection of $\mathbf{q}$ is $\mathbf{s} = \mathbf{q} - 2(\mathbf{n}\cdot\mathbf{q} + d)\mathbf{n}/|\mathbf{n}|^2$.
- A point stated to lie on a plane satisfies its equation. The squared distance between two points is the sum of the squares of their coordinate differences.

Step 1: Find the reflected point The plane has normal vector $\mathbf{n} = (2, -1, 1)$, with $|\mathbf{n}|^2 = 4 + 1 + 1 = 6$. At $Q = (1, 3, 4)$, the plane expression equals

$$2(1) - 3 + 4 + 3 = 6.$$

Thus the perpendicular foot is $H = Q - (6/6)(2, -1, 1) = (-1, 4, 3)$. Reflection places H midway between Q and S, so

$$S = 2H - Q = (-3, 5, 2).$$

The midpoint satisfies the plane equation: $2(-1) - 4 + 3 + 3 = 0$. Also $S - Q = (-4, 2, -2)$ is parallel to the plane normal. These checks establish that this is a reflection in the plane rather than a translation in some other direction.

Step 2: Determine the second point and compute the squared distance Since $R = (3, 5, \gamma)$ lies in the plane, substitution gives

$$2(3) - 5 + \gamma + 3 = 0, \qquad \gamma = -4.$$

Therefore $R = (3, 5, -4)$. Its displacement from the reflected point is

$$R - S = (3 - (-3), 5 - 5, -4 - 2) = (6, 0, -6).$$

By the Cartesian distance formula,

$$SR^2 = 6^2 + 0^2 + (-6)^2 = 36 + 36 = 72.$$

The question requests the square of the length, so we stop at seventy-two instead of taking its square root. Reflection preserves distances to points in the mirror plane, which also allows the result to be checked using QR^2.

Step 3: Conclusion The square of the segment length is $\boxed{SR^2 = 72}$ square coordinate units.

Quick Tip to Solve: If the other point lies in the mirror plane, reflection preserves its distance to the original point: $SR^2 = QR^2$ offers a quick independent check.

29. Let A ($\sec\theta$, $2\tan\theta$) and B ($\sec\phi$, $2\tan\phi$), where $\theta + \phi = \pi/2$, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- A point must satisfy the equation of a conic before a tangent or normal to that conic can be assigned at that point.
- The identity $\sec^2 t - \tan^2 t = 1$ tests the stated coordinates directly; both angles must also have finite secants and tangents.

Step 1: Test the coordinates of A The printed hyperbola is $2x^2 - y^2 = 2$, while the printed coordinates are $x = \sec\theta$, $y = 2\tan\theta$. Substitution gives

$$2\sec^2\theta - 4\tan^2\theta = 2 - 2\tan^2\theta.$$

For this to equal 2, we require $\tan^2\theta = 0$, hence $\theta = k\pi$ for an integer k. The coordinates do not parametrise the printed hyperbola for a general angle.

Step 2: Apply the complementary-angle condition The same calculation at B requires $\tan\phi = 0$, so $\phi = l\pi$ for an integer l. But then $\theta + \phi = (k + l)\pi$, which cannot equal $\pi/2$. Equivalently, choosing an allowed A forces $\phi = \pi/2 - k\pi$, where the coordinates of B are undefined. Thus there is no pair of finite real points satisfying all the printed conditions. In particular, the two normals in the question cannot be constructed as stated; there is no intersection coordinate β to square. Replacing the hyperbola or the factor 2 in the coordinates would create a different question and is not justified.

Step 3: Conclusion As printed, the conditions are inconsistent, so $(2\beta)^2$ has no defined numerical value. The official answer key dropped this question and awarded bonus marks to all candidates.

Quick Tip to Solve: Before using a conic parametrisation, substitute it into the printed conic; this immediately detects a mismatched coefficient.

30. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to _____. [4 marks]

Solution:

Concept:

- The radius to a point of tangency is perpendicular to the tangent line. A normal vector to $4x + 3y = 10$ is $(4, 3)$.
- The two equal circles touch externally, so their distinct centres lie on opposite sides of the tangent at the common point.

Step 1: Find the two possible centres The tangency point is $P = (1, 2)$, which lies on the given line since $4 + 6 = 10$. The vector $(4, 3)$ has length $\sqrt{4^2 + 3^2} = 5$, exactly the radius. Therefore moving

from P by this vector or its negative gives the only centres at distance 5 on the normal:

$$C_1 = (1,2) + (4,3) = (5,5), \qquad C_2 = (1,2) - (4,3) = (-3,-1).$$

The centre separation is 10, the sum of the radii, confirming external contact. Exchanging the names of the centres cannot change the required product.

Step 2: Evaluate the requested absolute product For the first centre the sum of coordinates is $\alpha + \beta = 5 + 5 = 10$. For the other it is $\gamma + \delta = -3 - 1 = -4$. Thus

$$|(\alpha + \beta)(\gamma + \delta)| = |10(-4)| = 40.$$

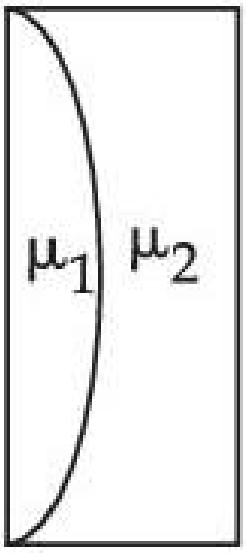
One can also obtain the same result without assigning centre names: the coordinate sums are $(1+2) \pm (4+3) = 3 \pm 7$, whose product is $3^2 - 7^2 = -40$. The absolute value in the question removes its negative sign. The result is a product of coordinate sums measured in the chosen coordinate units.

Step 3: Conclusion The required value is $\boxed{40}$ (in square coordinate units).

Quick Tip to Solve: When the normal vector to the tangent already has the given radius as its length, add and subtract that vector directly from the contact point.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses. [4 marks]



(A) $\frac{1}{\mu_2-\mu_1}$
(B) $\mu_1-\mu_2$
(C) $\frac{1}{\mu_1-\mu_2}$
(D) $\mu_2-\mu_1$

Correct Answer: (B) $\mu_1-\mu_2$

Solution:

Concept:

- Thin lenses in contact have additive powers: $1/f = 1/f_1 + 1/f_2$, with signed focal lengths.
- For the equal curvature magnitude $R > 0$, the plano-convex lens has power $(\mu_1 - 1)/R$, while the plano-concave lens has power $-(\mu_2 - 1)/R$.

Step 1: Assign the individual lens powers The figure shows a convex part of refractive index μ_1 and a concave part of refractive index μ_2, with matching curved surfaces and plane external faces. For the convex lens in air, the lens-maker formula gives

$$P_1 = \frac{\mu_1 - 1}{R}.$$

The concave lens has negative power of magnitude $(\mu_2 - 1)/R$, so

$$P_2 = -\frac{\mu_2 - 1}{R}.$$

The common radius is a positive magnitude; the opposite curvature effects are already included in the signs of the two powers.

Step 2: Add powers and form the specified ratio Treating the pair as thin lenses in contact,

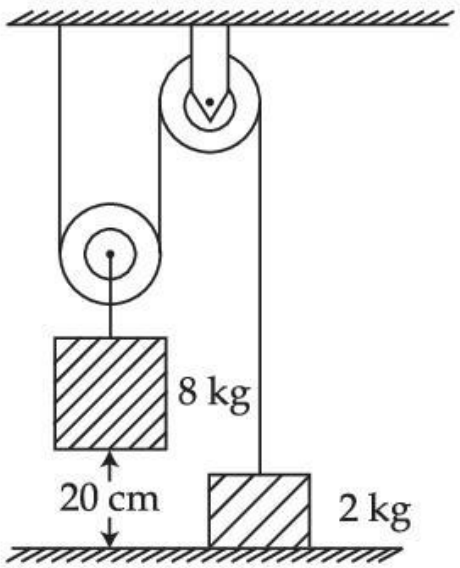
$$\frac{1}{f} = P_1 + P_2 = \frac{\mu_1 - 1 - \mu_2 + 1}{R} = \frac{\mu_1 - \mu_2}{R}.$$

Multiplying by R gives $R/f = \mu_1 - \mu_2$. The question asks for radius divided by focal length, so the reciprocal expressions in (A) and (C) are not the requested ratio. Option (D) reverses the sign. As a consistency check, when the refractive indices are equal the assembly has parallel external faces with no net focusing power: $1/f = 0$, and hence $R/f = 0$. For $\mu_1 > \mu_2$ the pair is converging, consistent with the positive answer.

Step 3: Conclusion $\boxed{R/f = \mu_1 - \mu_2}$. Hence, the correct option is (B).

Quick Tip to Solve: Add the signed powers first, then check whether the question asks for R/f or its reciprocal.

32. The boxes of masse 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest. (use g = 10 m/s^2) [4 marks]



(A) 0.34 s

(B) 0.2 s

(C) 0.25 s

(D) 0.4 s

Correct Answer: (D) 0.4 s

Solution:

Concept:

- A massless string over smooth pulleys has uniform tension T. The movable pulley supporting the 8 kg box is pulled up by two tensions.

- Conservation of string length makes the 2 kg box move twice as far, and accelerate twice as fast, as the movable pulley in the opposite direction.

Step 1: Write equations using the string constraint Let the 8 kg box accelerate downward at a. When its movable pulley descends by x, both supporting string segments lengthen by x. The free end must rise by $2x$; hence the 2 kg box accelerates upward at $2a$. Newton's second law for the two boxes gives

$$80 - 2T = 8a, \qquad T - 20 = 2(2a) = 4a.$$

Thus $T = 20 + 4a$. Substitution into the first equation yields

$$80 - 2(20 + 4a) = 8a \quad \Rightarrow \quad 40 = 16a,$$

so $a = 2.5\,\mathrm{m\,s^{-2}}$. The tension is 30 N, greater than the lighter box's weight; it therefore leaves the floor immediately.

Step 2: Use the fall distance shown in the figure The bottom of the 8 kg box is initially 20 cm above the ground, so it must descend $s = 0.20\,\mathrm{m}$. Starting from rest with constant acceleration,

$$s = \tfrac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2(0.20)}{2.5}} = \sqrt{0.16} = 0.40\,\mathrm{s}.$$

This equals option (D), not (A), (B) or (C). In particular, treating the system as a simple Atwood machine would miss both the doubled supporting tension and the acceleration constraint. The displacement used must be that of the heavy box, not the twice-larger displacement of the string's free end.

Step 3: Conclusion The 8 kg box strikes the ground after $\boxed{0.40\,\mathrm{s}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a movable pulley, write the string-length relation before the force equations; here the free end has twice the pulley acceleration.

33. Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor. [4 marks]

(A) 4.18 m

(B) 2.94 m

(C) 2.45 m

(D) 7.35 m

Correct Answer: (D) 7.35 m

Solution:

Concept:

- A drop released from rest falls a distance $s = \frac{1}{2}gt^2$, neglecting air resistance.

- Equal release intervals refer to time, not equal separations in distance; later drops have fallen for shorter times.

Step 1: Determine the elapsed times for the drops Let the common interval between releases be Δt. The first drop is released at time 0, the second at Δt, and the third at $2\Delta t$. The first strikes the floor exactly when the third starts falling, so the complete fall time is $T = 2\Delta t$. Since the nozzle is $H = 9.8\,\text{m}$ above the floor,

$$H = \tfrac{1}{2}gT^2.$$

At that same instant the second drop has been falling for $T - \Delta t = T/2$. The factor one-half applies to its elapsed time, not directly to its distance.

Step 2: Calculate the height above the floor The second drop has descended

$$s_2 = \tfrac{1}{2}g(T/2)^2 = \frac{1}{4}\left(\tfrac{1}{2}gT^2\right) = \frac{H}{4} = 2.45\,\text{m}.$$

This is the distance below the nozzle. Its position measured upward from the floor is therefore

$$h_2 = H - s_2 = 9.8 - 2.45 = 7.35\,\text{m}.$$

Option (C) is the tempting value 2.45 m, but it measures the fall distance rather than the height from the floor. The other two values, 4.18 m and 2.94 m, do not satisfy the squared-time relation for a half-duration fall. The calculation can be done without specifying g because the ratio s_2/H cancels it.

Step 3: Conclusion The second drop is $\boxed{7.35\,\text{m}}$ above the floor. Hence, the correct option is (D).

Quick Tip to Solve: If a drop has fallen for half the total fall time, it has descended one-quarter of the height and remains three-quarters above the floor.

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34. Two discs have moments of inertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by : [4 marks]

(A) $\frac{I_1 I_2}{(I_1+I_2)}(\omega_1 - \omega_2)^2$

(B) $\frac{(I_1-I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$

(C) $\frac{I_1 I_2}{2(I_1+I_2)}(\omega_1 - \omega_2)^2$

(D) $\frac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$

Correct Answer: (C) $\frac{I_1 I_2}{2(I_1+I_2)}(\omega_1 - \omega_2)^2$

Solution:

Concept:

- With no external torque about the common axis, the total angular momentum of the two discs is conserved.
- Friction brings both discs to a common angular velocity while converting part of their rotational

kinetic energy into heat.

Step 1: Find the common angular velocity Take the given angular velocities with the same chosen positive rotational sense. The initial angular momentum is $I_1\omega_1 + I_2\omega_2$. After slipping ceases, both discs rotate together with angular velocity ω, so

$$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2, \qquad \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}.$$

The frictional torques between the discs are internal and cancel in the total angular momentum balance. Energy, however, is not conserved as mechanical energy during the slipping process.

Step 2: Subtract the final kinetic energy The initial and final energies are

$$K_i = \tfrac{1}{2}I_1\omega_1^2 + \tfrac{1}{2}I_2\omega_2^2, \qquad K_f = \frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}.$$

Expanding the numerator of their difference gives

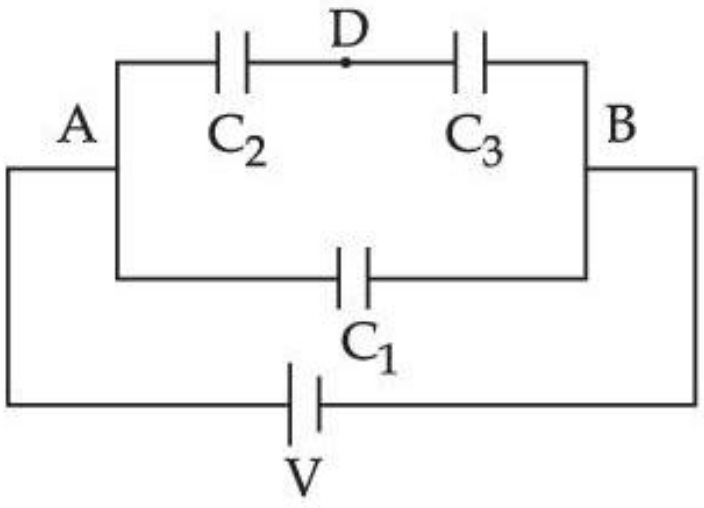
$$\Delta K = \frac{I_1 I_2(\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2)}{2(I_1 + I_2)} = \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2.$$

Option (A) is twice this loss. Option (B) does not vanish for equal initial angular velocities when the inertias differ, contradicting the absence of slipping then. Option (D) lacks the inertia factors required for energy dimensions. The derived loss is always nonnegative and vanishes when the speeds coincide.

Step 3: Conclusion $\boxed{\Delta K = \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2}$. Hence, the correct option is (C).

Quick Tip to Solve: For rotational sticking, conserve angular momentum to find the common speed, then subtract energies; do not conserve kinetic energy.

35. Three capacitors $C_1 = 2\mu F$, $C_2 = 6\mu F$ and $C_3 = 12\mu F$ are connected as shown in figure. Find the ratio of the charges on capacitors C_1, C_2 and C_3 respectively : [4 marks]



(A) 2 : 1 : 1

(B) 2 : 3 : 3

(C) 1 : 2 : 2

(D) 3 : 4 : 4

Correct Answer: (C) 1 : 2 : 2

Solution:

Concept:

- Capacitors in series carry equal charge, and their equivalent capacitance obeys $1/C_s = 1/C_2 + 1/C_3$.
- Parallel branches share the same potential difference. The charge on a capacitor or equivalent branch is $Q = CV$.

Step 1: Identify the branches from the connections In the figure, capacitor $C_1 = 2\,\mu$F is connected directly between A and B. The upper branch contains $C_2 = 6\,\mu$F and $C_3 = 12\,\mu$F in series, with their common isolated junction at D. Both branches are across the same battery voltage V. The upper branch equivalent capacitance is

$$C_s = \frac{C_2 C_3}{C_2 + C_3} = \frac{6 \times 12}{6 + 12}\,\mu\text{F} = 4\,\mu\text{F}.$$

The two series capacitors therefore have the same charge magnitude, even though their individual voltages differ.

Step 2: Calculate and compare the three charges The direct branch has charge $Q_1 = C_1 V = 2V$ in microcoulombs when V is in volts. The series branch carries $Q_s = C_s V = 4V$ microcoulombs, which is the magnitude of the charge on each capacitor:

$$Q_2 = Q_3 = 4V, \qquad Q_1 : Q_2 : Q_3 = 2V : 4V : 4V = 1 : 2 : 2.$$

Only option (C) has this ratio. Although all the alternatives show equal second and third entries, their ratio to the first charge differs; the branch capacitances fix that comparison. In particular, one must

not assign the whole battery voltage separately to both C_2 and C_3. Their voltages are $Q_s/C_2 = 2V/3$ and $Q_s/C_3 = V/3$, whose sum correctly equals V.

Step 3: Conclusion The charge ratio is $\boxed{Q_1 : Q_2 : Q_3 = 1 : 2 : 2}$. Hence, the correct option is (C).

Quick Tip to Solve: First replace a series branch by its equivalent capacitance; its branch charge is the charge on each member of that series pair.

36. if the rms speed of oxygen molecules at 0°C is 160 m/s, find the rms speed of hydrogen molecules at 0°C. [4 marks]

(A) 640 m/s

(B) 40 m/s

(C) 80 m/s

(D) 332 m/s

Correct Answer: (A) 640 m/s

Solution:

Concept:

- The root-mean-square molecular speed is $v_{\text{rms}} = \sqrt{3RT/M}$, where T is absolute temperature and M is molar mass.
- At the same temperature, speeds vary inversely as the square root of molar mass. Molecular oxygen is O_2 and molecular hydrogen is H_2.

Step 1: Compare the two gases at the same temperature Both gases are at $0°C$, or approximately $273\,K$. Their thermal factors $3RT$ are identical, so there is no need to substitute the gas constant or temperature separately. Divide the expressions for their rms speeds to cancel these common factors:

$$\frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}}.$$

This ratio must exceed one because hydrogen molecules are lighter.

Step 2: Use molecular masses and calculate the speed Take $M_{O_2} = 32\,g\,mol^{-1}$ and $M_{H_2} = 2\,g\,mol^{-1}$. The units cancel in the ratio. Thus

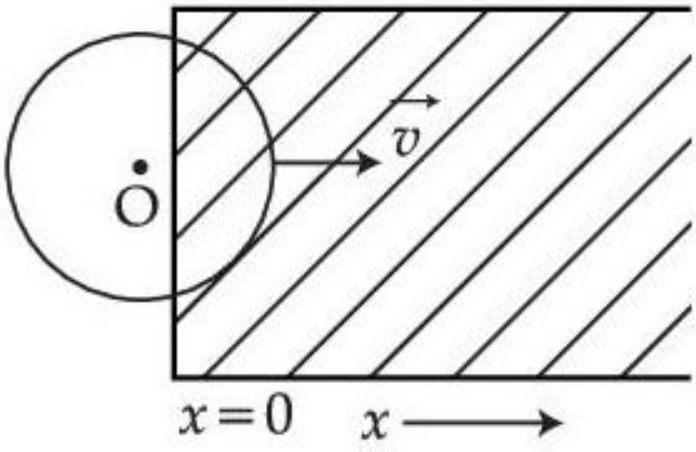
$$v_{H_2} = 160\sqrt{\frac{32}{2}} = 160 \times 4 = 640\,m\,s^{-1}.$$

The choices 40 and 80 m/s incorrectly make the lighter gas slower. The value 332 m/s does not have the required factor of four relative to the given oxygen speed. We use the supplied oxygen speed as the reference rather than replace it by another value.

Step 3: Conclusion The rms speed of hydrogen molecules is $640\,m\,s^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For gases at equal temperature, form the speed ratio first; remember to use the molecular masses 32 and 2, not atomic masses.

37. A constant magnetic field of 1T is applied in the x > 0 region. A metallic circular ring of radius 1m is moving with a constant velocity of 1 m/s along the x-axis. At t = 0s, the centre of O of the ring is at x = −1m. What will be the value of the induced emf in the ring at t = 1s? (Assume the velocity of the ring does not change.) [4 marks]



(A) 1V

(B) 2πV

(C) 2V

(D) 0V

Correct Answer: (C) 2V

Solution:

Concept:

- Faraday's law gives the magnitude of induced emf as $|\mathcal{E}| = |d\Phi/dt|$. For a uniform field perpendicular to the ring, $\Phi = BA$, with A the area inside the field.
- When a boundary sweeps across a loop, the area entering per second equals the chord length at the boundary multiplied by the speed normal to it.

Step 1: Locate the ring at the specified instant The centre moves according to $x_c = -1 + vt$, with all lengths in metres and time in seconds. At $v = 1\,\mathrm{m\,s^{-1}}$ and $t = 1\,\mathrm{s}$,

$$x_c = -1 + 1 \times 1 = 0.$$

The field boundary therefore passes through the centre of the ring. The chord cut by this boundary is the full diameter, $\ell = 2r = 2\,\mathrm{m}$, rather than the circumference.

Step 2: Calculate the instantaneous flux-change rate In a small time dt, a strip of area $dA = \ell v\,dt$ enters the field to first order. Hence

$$|\mathcal{E}| = B\frac{dA}{dt} = B\ell v = 1 \times 2 \times 1 = 2\,\mathrm{V}.$$

Although the field and velocity are constant, the part of the ring enclosed by the field is changing, so zero emf is incorrect. The 1 V choice uses only one radius as the chord. The 2π V choice uses the circumference, which is not the length sweeping across the straight boundary. Lenz's law determines

polarity; the options ask for the magnitude.

Step 3: Conclusion The induced emf at $t = 1\,\mathrm{s}$ has magnitude $2\,\mathrm{V}$. Hence, the correct option is (C).

Quick Tip to Solve: For a loop entering a straight field boundary, use Bv times the intercepted chord; it becomes $2Brv$ when the centre reaches the boundary.

38. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is : [4 marks]

(A) -60 G

(B) $+2$ G

(C) -20 G

(D) -4 G

Correct Answer: (D) -4 G

Solution:

Concept:

- Gravitational potential is a scalar and contributions from different masses add algebraically, with zero potential chosen at infinity.
- A spherical shell of mass M and radius R has constant potential $-GM/R$ everywhere inside it. A central point mass m contributes $-Gm/r$.

Step 1: Find the potential due to the shell The observation point is $25\,\mathrm{m}$ from the centre, which is inside the shell of radius $50\,\mathrm{m}$. Therefore the shell theorem gives

$$V_{\text{shell}} = -\frac{G(100)}{50} = -2G.$$

The radius in this expression is the shell radius, not the distance of the interior point from its centre. A vanishing gravitational field inside a uniform shell means that its potential is constant, not that its potential is zero.

Step 2: Add the contribution of the central mass The separate $50\,\mathrm{kg}$ mass is at a distance of $25\,\mathrm{m}$ from the point. Its potential is

$$V_{\text{centre}} = -\frac{G(50)}{25} = -2G.$$

Superposition now gives

$$V_{\text{total}} = V_{\text{shell}} + V_{\text{centre}} = -2G - 2G = -4G.$$

Both contributions are negative because gravity is attractive and the reference is infinity. The positive choice has the wrong sign, while -20G and -60G do not follow from either mass-to-distance ratio. The potential has the physical unit $\mathrm{J\,kg^{-1}} = \mathrm{m^2\,s^{-2}}$; the numerical coefficient multiplying G carries $\mathrm{kg\,m^{-1}}$.

Step 3: Conclusion The total potential is $-4G$, in SI potential units when G is substituted in SI.

Hence, the correct option is (D).

Quick Tip to Solve: Inside a shell use its radius for the shell potential, but use the actual separation for an additional point mass; then add the scalar potentials.

39. For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be : [4 marks]

(A) 1Ω

(B) 5Ω

(C) 4Ω

(D) 2Ω

Correct Answer: (D) 2Ω

Solution:

Concept:

- Current sensitivity is deflection per unit current: $S_I = \theta/I$. It must be used with the same deflection unit as the stated scale reading.
- Ohm's law for the galvanometer coil is $V = IR_g$, where R_g is its resistance.

Step 1: Convert the full-scale deflection to current The current sensitivity is $2\,\text{div}\,\text{mA}^{-1}$. Thus each milliampere produces two divisions of deflection, and reaching fifty divisions requires

$$I_g = \frac{\theta_{\text{fs}}}{S_I} = \frac{50\,\text{div}}{2\,\text{div}\,\text{mA}^{-1}} = 25\,\text{mA}.$$

The divisions cancel, leaving a current. Multiplying 50 by 2 instead would reverse the definition of sensitivity and give the wrong full-scale current.

Step 2: Apply the voltage-current relation The full-scale voltage is $50\,\text{mV}$, applied across the same galvanometer coil. Therefore

$$R_g = \frac{V_g}{I_g} = \frac{50 \times 10^{-3}}{25 \times 10^{-3}} = 2\,\Omega.$$

The milli prefixes cancel directly, so dividing numerical millivolts by numerical milliamperes already gives ohms. As a check, $25\,\text{mA} \times 2\,\Omega = 50\,\text{mV}$, reproducing the supplied voltage. Resistances of 1, 5 and 4 ohms would instead require voltages of 25, 125 and 100 mV at that current, so none fits both the sensitivity and the full-scale reading.

Step 3: Conclusion The galvanometer resistance is $2\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: First obtain full-scale current by dividing scale divisions by current sensitivity; only then divide the full-scale voltage by that current.

40. A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm? [4 marks]

(A) 0.96 V

(B) 1.25 V

(C) 0.24 V

(D) 1.5 V

Correct Answer: (B) 1.25 V

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where ϕ is the work function of the material.
- For the same photosensitive material, subtracting equations at two wavelengths eliminates the unknown work function. The useful conversion is $hc/e \simeq 1240\,\text{V nm}$.

Step 1: Eliminate the work function Let $V_1 = 0.48\,\text{V}$, $\lambda_1 = 670.5\,\text{nm}$, and $\lambda_2 = 474.6\,\text{nm}$. The material is unchanged, so the same work function appears in both experiments. Subtracting the first equation from the second gives

$$V_2 - V_1 = \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right).$$

The new wavelength is shorter, so its photons have higher energy and the new stopping voltage must exceed 0.48 V.

Step 2: Evaluate the increase in stopping voltage Using wavelengths in nanometres consistently with the conversion factor,

$$V_2 = 0.48 + 1240\left(\frac{1}{474.6} - \frac{1}{670.5}\right)\text{ V}.$$

The photon energies are approximately 2.613 eV and 1.849 eV, giving an increase of 0.763 V. Thus

$$V_2 \simeq 1.243\,\text{V} \simeq 1.25\,\text{V}$$

at the precision of the options. The 0.24 V choice incorrectly decreases the stopping voltage. The 0.96 V and 1.5 V choices have increases inconsistent with the photon-energy difference; stopping voltage does not simply scale with inverse wavelength because the work function must first be subtracted.

Step 3: Conclusion The stopping voltage is approximately $1.25\,\text{V}$. Hence, the correct option is (B).

Quick Tip to Solve: When the metal stays the same, subtract the photoelectric equations; the stopping-voltage change equals the photon-energy change expressed in electronvolts.

41. Match List - I with List - II.

	List - I		List - II
(a)	R_H **(Rydberg constant)**	**(i)**	$kg\,m^{-1}s^{-1}$
(b)	**h (Planck's constant)**	**(ii)**	$kg\,m^{2}s^{-1}$
(c)	μ_B **(Magnetic field energy density)**	**(iii)**	m^{-1}
(d)	η **(coefficient of viscocity)**	**(iv)**	$kg\,m^{-1}s^{-2}$

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(B) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(C) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

(D) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

Correct Answer: (B) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

Solution:

Concept:

- Units follow from defining equations: a wave number has units of inverse length, and an action has units of energy multiplied by time.
- Energy density has units of energy per volume, while dynamic viscosity follows from shear stress equal to viscosity times velocity gradient.

Step 1: Match the Rydberg and Planck constants The Rydberg formula is $1/\lambda = R_H(1/n_1^2 - 1/n_2^2)$. The bracket is dimensionless, so R_H has unit m^{-1}, giving (a)-(iii). For Planck's constant, $E = h\nu$ gives

$$[h] = \mathrm{J\,s} = \mathrm{kg\,m^2\,s^{-1}},$$

so (b)-(ii). The use of the symbol μ_B in the question does not change the explicitly named quantity, magnetic field energy density.

Step 2: Match energy density and viscosity Energy per volume has unit

$$\frac{\mathrm{J}}{\mathrm{m^3}} = \mathrm{kg\,m^{-1}\,s^{-2}},$$

which gives (c)-(iv). For viscosity, $\tau = \eta\, dv/dy$, where stress τ has unit $\mathrm{kg\,m^{-1}\,s^{-2}}$ and velocity gradient has unit $\mathrm{s^{-1}}$. Thus $[\eta] = \mathrm{kg\,m^{-1}\,s^{-1}}$, giving (d)-(i). The resulting sequence is (iii), (ii), (iv), (i). Option A interchanges the first two matches, option C has several incorrect assignments, and option D interchanges energy density with viscosity. Only option B preserves all four defining relations.

Step 3: Conclusion The matching is a to iii, b to ii, c to iv, and d to i. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish energy density from viscosity by the time exponent: energy density has s^{-2}, while dynamic viscosity has s^{-1}.

42. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density : [4 marks]

(A) $[FL^{-4}T^2]$

(B) $[FL^{-3}T^2]$

(C) $[FL^{-5}T^2]$

(D) $[FL^{-3}T^3]$

Correct Answer: (A) $[FL^{-4}T^2]$

Solution:

Concept:

- Mass density is mass per unit volume: $[\rho] = ML^{-3}$ in the usual mass-length-time system.
- When force replaces mass as a fundamental quantity, Newton's second law $F = ma$ expresses mass in terms of force, length and time.

Step 1: Express mass through the new fundamental quantities Acceleration has dimension LT^{-2}. From Newton's second law,

$$F = MLT^{-2} \quad \Rightarrow \quad M = FL^{-1}T^2.$$

This relation replaces only the old mass dimension. The symbols F, L and T are now independent fundamental dimensions; they should not be interpreted as numerical values or SI unit names.

Step 2: Substitute into the density formula Volume still has dimension L^3, so divide the derived mass dimension by L^3:

$$[\rho] = \frac{FL^{-1}T^2}{L^3} = FL^{-4}T^2.$$

The powers of length add as $-1 - 3 = -4$, while the time exponent remains $+2$. A useful check is to substitute $F = MLT^{-2}$ back into this result:

$$(MLT^{-2})L^{-4}T^2 = ML^{-3},$$

which is the usual density dimension. Choices B and C have the wrong length powers, and choice D also has an extra time factor. In particular, retaining L^{-3} alone would overlook the inverse length introduced when mass is eliminated.

Step 3: Conclusion The dimension of density in the force-length-time system is $[FL^{-4}T^2]$. Hence, the correct option is (A).

Quick Tip to Solve: When force is fundamental, replace every mass factor by $FL^{-1}T^2$ before collecting the powers of length and time.

43. A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current i_0, which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) x < a and (ii) a < x < b ? [4 marks]

(A) $\frac{x^2}{a^2}$

(B) $\frac{a^2}{x^2}$

(C) $\frac{x^2}{b^2-a^2}$

(D) $\frac{b^2-a^2}{x^2}$

Correct Answer: (A) $\frac{x^2}{a^2}$

Solution:

Concept:

- Ampere's circuital law for a circular path concentric with a long cable is $B(2\pi x) = \mu_0 I_{\text{enc}}$. Cylindrical symmetry makes the field constant along that path.
- For uniform current density, the fraction of the inner-wire current enclosed is the fraction of its cross-sectional area. Current outside the circular path contributes no net field inside this ideal coaxial geometry.

Step 1: Find the field inside the inner conductor The inner wire has current density $J = i_0/(\pi a^2)$. A circular path of radius $x < a$ encloses area πx^2, hence current $i_0 x^2/a^2$. Applying Ampere's law gives

$$B_{\text{inside}} = \frac{\mu_0 i_0 x}{2\pi a^2}.$$

Thus the field grows linearly from zero at the axis; it does not follow the inverse-distance law inside a uniformly conducting wire.

Step 2: Find the field between the conductors and form the ratio In the gap $a < x < b$, the complete inner current is enclosed and none of the outer-shell current is enclosed. Therefore

$$B_{\text{gap}} = \frac{\mu_0 i_0}{2\pi x}, \qquad \frac{B_{\text{inside}}}{B_{\text{gap}}} = \frac{x^2}{a^2}.$$

The expressions are compared using the same radial variable, as intended by the options. They agree at the boundary $x = a$. Option (B) reverses the requested ratio, while (C) and (D) incorrectly introduce an annular area that is not involved in either enclosed current.

Step 3: Conclusion The requested ratio is $\boxed{x^2/a^2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a coaxial cable, first identify which currents lie inside the Amperian circle; only then substitute the enclosed area fraction.

44. The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall?

[Given 1 cal = 4.2 J and specific heat of water = 1 cal g^{-1} °0C^{-1}] [4 marks]

(A) 0.147° C

(B) 14.76° C

(C) 1.476° C

(D) 0.014° C

Correct Answer: (A) 0.147° C

Solution:

Concept:

- When falling water loses gravitational potential energy, the ideal temperature rise follows from converting that energy into internal energy: $mgh = mc\Delta T$.
- The specific heat must be expressed consistently with joules and kilograms. Here $1\,\text{cal}\,\text{g}^{-1}\,^{\circ}\text{C}^{-1} = 4200\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$.

Step 1: Convert the given specific heat The supplied conversion is $1\,\text{cal} = 4.2\,\text{J}$, and one kilogram contains $1000\,\text{g}$. Consequently

$$c = 1 \times 4.2 \times 1000 = 4200\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}.$$

A temperature interval of one kelvin is the same size as an interval of one degree Celsius. The factor of one thousand is essential: using the gram value with a mass in kilograms would give an incorrect temperature rise.

Step 2: Equate the lost potential energy to heat For a mass m falling through $h = 63\,\text{m}$, the available energy is $mg(63)$. In the ideal conversion used for this question,

$$mg(63) = m(4200)\Delta T, \qquad \Delta T = \frac{9.8 \times 63}{4200} = 0.147\,^{\circ}\text{C}.$$

The mass cancels because both the potential-energy loss and heat capacity scale with the amount of water. The lower water is warmer. Options (B) and (C) are much too large, and (D) is roughly a factor of ten too small; none equals the energy balance. This calculation is the maximum rise for complete conversion into heat of the water.

Step 3: Conclusion The bottom water is hotter by $\boxed{0.147\,^{\circ}\text{C}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $\Delta T = gh/c$ for a waterfall and convert the specific heat to $\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ before evaluating.

45. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion ? (Take g = 10 ms^{-2}) [4 marks]

(A) h_{max} = 10 m

T = 2.5 s

(B) h_{max} = 15.625 m

T = 3.54 s

(C) h_{max} = 15.625 m

T = 1.77 s

(D) h_{max} = 3.54 m

T = 0.125 s

Correct Answer: (C) h_{max} = 15.625 m

T = 1.77 s

Solution:

Concept:

- The horizontal and vertical components of the launch velocity are independent. Only the vertical component $u_y = u\sin\theta$ determines the maximum height and the ascent time.
- At the highest point the vertical velocity is zero. With downward acceleration of magnitude g, $t_{\text{up}} = u_y/g$ and $H = u_y^2/(2g)$.

Step 1: Resolve the initial velocity and calculate the ascent time The given speed is $25\,\text{m}\,\text{s}^{-1}$, the launch angle is $45°$, and $g = 10\,\text{m}\,\text{s}^{-2}$. Therefore

$$u_y = 25\sin 45° = \frac{25}{\sqrt{2}}\,\text{m}\,\text{s}^{-1}.$$

Using $v_y = u_y - gt$ with $v_y = 0$ at the top gives

$$t_{\text{up}} = \frac{25}{10\sqrt{2}} = 1.7678\,\text{s} \approx 1.77\,\text{s}.$$

The football still has horizontal velocity at the top; it is only the vertical component that vanishes.

Step 2: Calculate the maximum height and compare the pairs The vertical kinematic equation $v_y^2 = u_y^2 - 2gH$ gives

$$H = \frac{(25/\sqrt{2})^2}{2(10)} = \frac{625}{40} = 15.625\,\text{m}.$$

Both quantities must match the same option. Option (B) has the correct height but gives approximately the full flight time $2t_{\text{up}} = 3.54\,\text{s}$, rather than the time to the top. Option (A) does not use the proper vertical component, and both values in (D) are inconsistent with the vertical equations. Option (C) alone supplies the correct height and ascent time.

Step 3: Conclusion $\boxed{H = 15.625\,\text{m}, \quad t_{\text{up}} = 1.77\,\text{s}}$. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish ascent time from total flight time: for equal launch and landing heights, the latter is twice the former.

46. The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light at maxima? [4 marks]

(A) I_0

(B) $2\ I_0$

(C) $5\ I_0$

(D) $4\ I_0$

Correct Answer: (D) $4\ I_0$

Solution:

Concept:

- For coherent waves with phase difference δ, the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$. The interference term cannot be omitted.
- Intensity is proportional to the square of wave amplitude. Equal waves in phase have twice the individual amplitude and therefore four times the individual intensity.

Step 1: Check the stated minimum Both waves have individual intensity I_0. Substituting $I_1 = I_2 = I_0$ into the interference equation yields

$$I = 2I_0(1 + \cos\delta) = 4I_0\cos^2(\delta/2).$$

At destructive interference, $\delta = (2n+1)\pi$, so $\cos\delta = -1$ and $I_{\min} = 0$. This agrees with the condition in the question and confirms complete destructive interference for the equal amplitudes.

Step 2: Apply the constructive-interference condition At a maximum, $\delta = 2n\pi$, and therefore

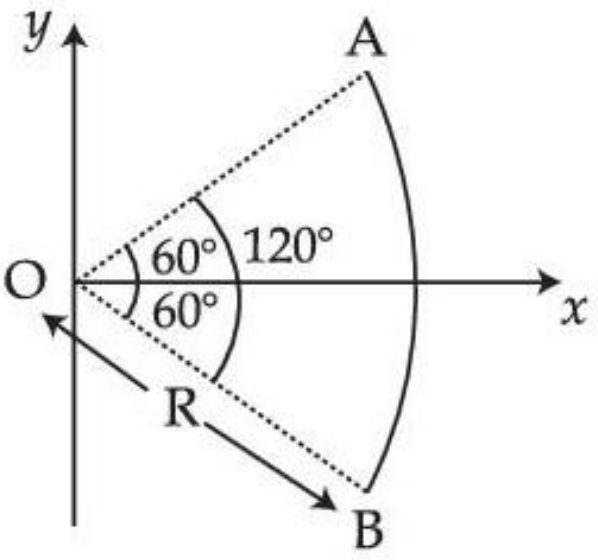
$$I_{\max} = I_0 + I_0 + 2\sqrt{I_0 I_0} = 4I_0.$$

Option (A) is merely the intensity of one source. Option (B), $2I_0$, is the incoherent sum or the spatial average of the fringes, not their peak intensity. Option (C), $5I_0$, exceeds the largest possible value since $\cos\delta$ cannot exceed one. There is no conflict with energy conservation: bright fringes gain intensity while dark fringes lose it, and the pattern redistributes energy. The maximum is four times either individual intensity but twice their noninterfering sum.

Step 3: Conclusion The intensity at a bright fringe is $\boxed{4I_0}$. Hence, the correct option is (D).

Quick Tip to Solve: Add amplitudes before squaring at an interference maximum; adding the individual intensities alone misses the interference term.

47. Figure shows a rod AB, which is bent in a 120° circular arc of radius R. A charge (−Q) is uniformly distributed over rod AB. What is the electric field $\vec{E}$ at the centre of curvature O ? [4 marks]



(A) $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}(\hat{i})$

(B) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$

(C) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$

(D) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(-\hat{i})$

Correct Answer: (B) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$

Solution:

Concept:

- For a uniformly charged arc of angular span 2θ, its length is $2R\theta$, with angles measured in radians. Its charge-density magnitude is $|\lambda| = Q/(2R\theta)$.
- Symmetry cancels the transverse field components at the centre. The field due to negative charge points from the observation point towards the charged arc.

Step 1: Express the charge density and direction The arc spans $120° = 2\pi/3$ radians and lies symmetrically about the positive x-axis, from $-\pi/3$ to $+\pi/3$. Hence

$$|\lambda| = \frac{Q}{R(2\pi/3)} = \frac{3Q}{2\pi R}.$$

For every element above the axis there is an equal element below it. Their vertical fields cancel, while their horizontal fields both point towards the negative arc, along $+\hat{\mathbf{i}}$. Thus a negative sign in the final direction would be incorrect.

Step 2: Integrate the horizontal field components An element at angular position ϕ has charge magnitude $|dq| = |\lambda| R\, d\phi$. Its field magnitude at the centre is $k|dq|/R^2$, where $k = 1/(4\pi\varepsilon_0)$.

Therefore

$$E_x = \frac{k|\lambda|}{R}\int_{-\pi/3}^{\pi/3}\cos\phi\, d\phi = \frac{k|\lambda|}{R}\sqrt{3} = \frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}.$$

Option (A) loses the extra factor of π from the arc length, (C) gives half the field, and (D) has the opposite direction. Both the angular normalization and the sign of charge are needed to choose the answer.

Step 3: Conclusion $\boxed{\vec{E} = \frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}\hat{\mathbf{i}}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a charged arc, use radians in its length and resolve along its symmetry axis; negative charge reverses the direction relative to a positive arc.

48. For a transistor α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between α and β will be : [4 marks]

(A) $\alpha = \frac{1-\beta}{\beta}$

(B) $\beta = \frac{\alpha}{1-\alpha}$

(C) $\alpha\beta = 1$

(D) $\alpha = \frac{\beta}{1-\beta}$

Correct Answer: (B) $\beta = \frac{\alpha}{1-\alpha}$

Solution:

Concept:

- For the transistor-current magnitudes used in current-gain definitions, Kirchhoff's current law gives $I_E = I_C + I_B$.
- The common-base current gain is $\alpha = I_C/I_E$, while the common-emitter current gain is $\beta = I_C/I_B$. Their different denominators are essential.

Step 1: Express the base current using the common-base gain From $\alpha = I_C/I_E$, write $I_C = \alpha I_E$. Substitution into current conservation yields

$$I_B = I_E - I_C = I_E(1-\alpha).$$

For normal forward-active operation the collector current is slightly smaller than the emitter current, so $0 < \alpha < 1$. The base current is the small difference between these two currents, which explains why β can be much greater than one.

Step 2: Form the common-emitter gain and check the alternatives Taking the ratio of collector current to base current gives

$$\beta = \frac{I_C}{I_B} = \frac{\alpha I_E}{(1-\alpha)I_E} = \frac{\alpha}{1-\alpha}.$$

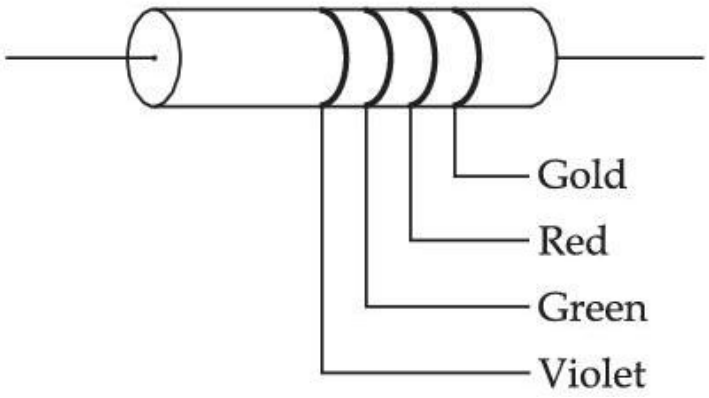
Equivalently, solving for α gives $\alpha = \beta/(1+\beta)$. This plus sign rules out option (D). The gains are not reciprocals, ruling out (C), and option (A) contradicts this algebra. For an explicit check, choose $\alpha = 0.98$; the relation gives $\beta = 49$, and indeed $49/(1+49) = 0.98$. The emitter current, rather than

the base current, is the larger denominator in α, so its value remains below unity while β can be large.

Step 3: Conclusion The correct relation is $\boxed{\beta = \alpha/(1-\alpha)}$. Hence, the correct option is (B).

Quick Tip to Solve: Start from $I_E = I_C + I_B$ instead of memorizing signs; rearrangement immediately gives both current-gain conversion formulas.

49. The Colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is : [4 marks]



(A) (5700 $\pm$ 285) Ω

(B) (7500 $\pm$ 750) Ω

(C) (5700 $\pm$ 375) Ω

(D) (7500 $\pm$ 375) Ω

Correct Answer: (D) (7500 $\pm$ 375) Ω

Solution:

Concept:

- In a four-band resistor code, the first two bands give significant digits, the third gives a power-of-ten multiplier, and the fourth gives the tolerance.
- The relevant colour values are violet 7, green 5, red multiplier 10^2, and gold tolerance $\pm 5\%$. The gold band identifies the tolerance end.

Step 1: Read the significant digits and multiplier The labelled bands in the figure, read from the end opposite the gold band, are violet, green, red, and gold. The first two digits therefore form 75, not 57. The red third band multiplies this number by 10^2:

$$R_{\text{nominal}} = 75 \times 10^2 = 7500\,\Omega.$$

The colour labels in the monochrome figure determine the values, so there is no need to infer colours from shading. Reading from the wrong end or reversing the first two digits would produce an incorrect nominal resistance.

Step 2: Calculate the tolerance in ohms and compare A gold tolerance band permits a deviation of

five per cent on either side of the nominal resistance. Its absolute value is

$$\Delta R = \frac{5}{100}(7500) = 375\,\Omega.$$

Thus the permitted interval runs from $7500 - 375 = 7125\,\Omega$ to $7500 + 375 = 7875\,\Omega$. Options (A) and (C) start with $5700\,\Omega$, which reverses the significant digits. Option (B) uses $750\,\Omega$, a ten-per-cent tolerance that corresponds to silver rather than gold. Only option (D) combines the correct ordered digits, multiplier, and tolerance. The tolerance must be calculated from the nominal value, not appended as a separate colour-code digit.

Step 3: Conclusion The resistance is $\boxed{(7500 \pm 375)\,\Omega}$. Hence, the correct option is (D).

Quick Tip to Solve: Locate the gold or silver tolerance band first, read the other bands from the opposite end, and calculate the percentage of the nominal resistance.

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50. An antenna is mounted on a 400 m tall building. What will be the wavelength of signal that can be radiated effectively by the transmission tower upto a range of 44 km? [4 marks]

(A) 37.8 m

(B) 605 m

(C) 75.6 m

(D) 302 m

Correct Answer: The stated building height and range do not uniquely determine wavelength; 605 m is the conventional keyed result only when the building height is ignored.

Solution:

Concept:

- The geometrical radio horizon is $d \simeq \sqrt{2R_E H}$, where H is the total antenna height above the ground and $R_E \simeq 6.4 \times 10^6\,\text{m}$.
- Efficient radiation is commonly associated with an antenna length $l = \lambda/4$. Antenna length and the elevation of the antenna above ground are different quantities.

Step 1: Calculate the height associated with the stated range Using the usual single-tower horizon model, a range of $44\,\text{km}$ corresponds to

$$H = \frac{d^2}{2R_E} = \frac{(44 \times 10^3)^2}{2(6.4 \times 10^6)} = 151.25\,\text{m}.$$

This is the total height required in that geometrical calculation, rather than an additional height above the building.

Step 2: Check the building height and the offered answer The printed building alone is $400\,\text{m}$ tall. Its geometrical horizon is already

$$d_0 = \sqrt{2(6.4 \times 10^6)(400)} \simeq 71.6\,\text{km}.$$

Thus equating the full horizon to $44\,\mathrm{km}$ would require a negative extra height, $151.25 - 400 = -248.75\,\mathrm{m}$. If instead the range only means coverage up to that distance, the building satisfies it but it does not determine the antenna length or wavelength.

Step 3: Identify the conventional option calculation Disregarding the building and identifying l with $151.25\,\mathrm{m}$ gives $\lambda = 4l = 605\,\mathrm{m}$, which is option (B), the keyed response. The other lengths do not follow from that quarter-wave calculation. However, this calculation cannot simultaneously incorporate the explicitly printed building height; the range and building data do not establish a unique wavelength.

Step 4: Conclusion The conventional keyed answer is (B), $605\,\mathrm{m}$, obtained by using the range-derived height as a quarter-wave antenna length. Taken literally with the $400\,\mathrm{m}$ building, the data do not uniquely determine a wavelength, so no unique option is physically established.

Quick Tip to Solve: Keep the antenna length used in $l = \lambda/4$ distinct from its total elevation used in the radio-horizon formula.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Two simple harmonic motion, are represented by the equations $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$ $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ Ratio of amplitude of y_1 to y_2 = x : 1. The value of x is _____. [4 marks]

Solution:

Concept:

- For $y = A\sin(\omega t + \phi)$, the amplitude is $|A|$; a constant phase changes the starting position but not the maximum displacement.
- For equal-frequency terms $a\sin\omega t + b\cos\omega t$, the resultant amplitude is $\sqrt{a^2 + b^2}$. The quadrature components must be combined by squares.

Step 1: Read the first amplitude The first displacement is already in standard harmonic form:

$$y_1 = 10\sin(3\pi t + \pi/3).$$

Therefore $A_1 = 10$, in the displacement units used in the question. The factor 3π fixes the angular frequency; neither it nor the phase $\pi/3$ multiplies the amplitude.

Step 2: Combine the second displacement components First distribute the factor outside the bracket:

$$y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t.$$

The sine and cosine components are a quarter cycle apart, so their coefficients cannot simply be added. Their amplitude is

$$A_2 = \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{25 + 75} = 10.$$

Equivalently, $\sin u + \sqrt{3}\cos u = 2\sin(u + \pi/3)$, so $y_2 = 10\sin(3\pi t + \pi/3)$, identical to y_1.

Step 3: Form the requested ratio Both amplitudes are measured in the same units, so the units cancel in the ratio:

$$A_1 : A_2 = 10 : 10 = 1 : 1 = x : 1.$$

The numerical factor outside the second bracket is only 5, but the bracket itself has amplitude 2. Ignoring that factor of 2 would give an incorrect ratio.

Step 4: Conclusion The amplitude ratio is $1 : 1$, hence $\boxed{x = 1}$ (dimensionless).

Quick Tip to Solve: For a sine plus cosine at the same frequency, take the square root of the sum of squared coefficients before comparing amplitudes.

---

52. X different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are exited to states with principal quantum number n = 6 ? The value of X is _____. [4 marks]

Solution:

Concept:

- An emission wavelength corresponds to a downward transition between two distinct hydrogen energy levels, with $h\nu = E_i - E_f$.
- In a sample excited to level n, cascades populate intermediate levels. All unordered pairs of levels from 1 to n can therefore contribute spectral lines.

Step 1: Count all accessible transitions The atoms initially reach $n = 6$, and can decay directly to any lower level or in several stages. For example, an atom may emit on $6 \to 4$ and then on $4 \to 1$; other atoms in the sample can follow different paths. Hence count transitions among all six accessible levels, not just the five transitions starting directly at level 6.

Step 2: Sum the numbers from each upper level Level 6 has five lower destinations, level 5 has four, and so on down to level 2, which has one. Thus

$$X = 5 + 4 + 3 + 2 + 1 = \frac{6(6-1)}{2} = 15.$$

This is the combination count ${}^6C_2$, since choosing two levels fixes which is the upper and which is the lower level. Upward absorption transitions are not counted a second time.

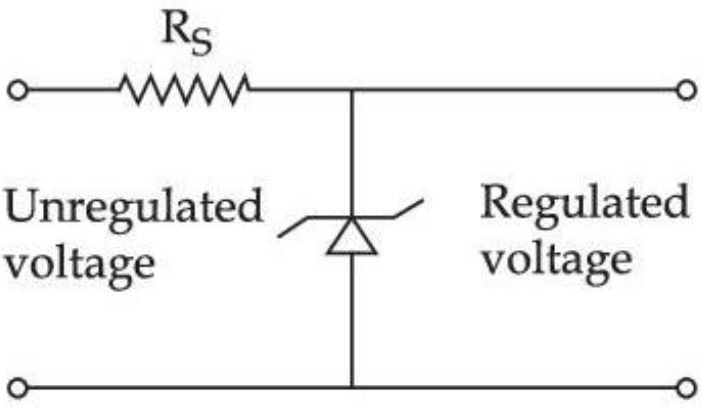
Step 3: Interpret the spectral count The hydrogen energy formula $E_n = -13.6/n^2\,\text{eV}$ assigns a photon energy to each downward pair. For these six levels the resulting energy differences give the fifteen possible emission wavelengths. A single atom need not emit all fifteen photons: the question concerns the spectrum of a sample containing many atoms and all possible cascades.

Step 4: Conclusion The number of possible emission wavelengths is $\boxed{X = 15}$ (a dimensionless count).

Quick Tip to Solve: For a sample excited to level n, count all level pairs using $n(n-1)/2$; use $n - 1$ only for direct transitions from the top level.

---

53. A zener diode of power rating 2W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_s for safe operation should be _____ Ω. [4 marks]



Solution:

Concept:

- In reverse breakdown the Zener voltage is approximately $V_Z = 10\,\text{V}$, and its dissipated power is $P_Z = V_Z I_Z$.
- The series resistor limits the current: $I_S = (V_{\text{in}} - V_Z)/R_S$ while breakdown regulation is active. The largest input voltage gives the largest possible Zener current.

Step 1: Find the maximum safe Zener current The power rating limits dissipation inside the diode, rather than in the resistor. Consequently

$$I_{Z,\max} = \frac{P_{Z,\max}}{V_Z} = \frac{2}{10} = 0.20\,\text{A}.$$

With no specified load current, the safe design uses the worst case in which all current through the series resistor can pass through the Zener.

Step 2: Use the highest supply voltage At the largest stated input, $V_{\text{in}} = 14\,\text{V}$, a conducting Zener holds approximately $10\,\text{V}$ across itself. The resistor therefore drops

$$V_R = 14 - 10 = 4\,\text{V}.$$

Limiting current to $0.20\,\text{A}$ requires

$$R_S \geq \frac{4}{0.20} = 20\,\Omega.$$

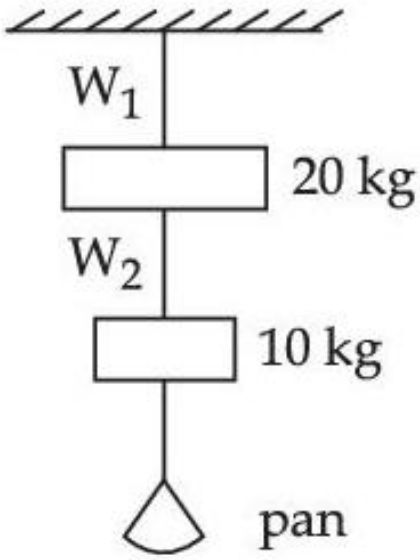
At the limiting value the resistor dissipates $I^2R = 0.8\,\text{W}$, while the diode dissipates its rated $2\,\text{W}$.

Step 3: Check the low-input limitation For an input below $10\,\text{V}$, including the printed $6\,\text{V}$ endpoint, the diode cannot remain in its 10 V reverse-breakdown region. This simple regulator cannot maintain a 10 V output there. The computed resistance is the minimum for safe operation at the maximum input; it does not make voltage regulation possible below the Zener voltage.

Step 4: Conclusion The limiting safe series resistance is $\boxed{R_S = 20\,\Omega}$; larger values also respect the power limit. Regulation at 10 V occurs only when the supply provides sufficient voltage and current.

Quick Tip to Solve: For Zener power safety, use the maximum input and minimum load current; a supply below the breakdown voltage cannot sustain regulation.

54. Wires W_1 and W_2 are made of same material having the breaking stress of 1.25×10^9 N/m^2. W_1 and W_2 have cross-sectional area of 8×10^{-7} m^2 and 4×10^{-7} m^2, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is _____ kg. (Use g = 10 m/s^2) [4 marks]



Solution:

Concept:

- Breaking stress is maximum tensile force per unit area: $T_{\max} = \sigma_b A$. Both wires have the same stress limit but different force limits.
- In static equilibrium each wire supports every mass hanging below it. The upper wire carries the 20 kg block as well as the lower block and added pan load.

Step 1: Calculate the two limiting tensions Using $\sigma_b = 1.25 \times 10^9\,\mathrm{N\,m^{-2}}$, the upper wire has capacity

$$T_{1,\max} = (1.25 \times 10^9)(8 \times 10^{-7}) = 1000\,\mathrm{N}.$$

The lower wire has half the cross-sectional area, so

$$T_{2,\max} = (1.25 \times 10^9)(4 \times 10^{-7}) = 500\,\mathrm{N}.$$

The area must remain in square metres for the stress multiplication.

Step 2: Write equilibrium for each supported load Let m kilograms be added to the pan, with the pan mass neglected as no value is given. The lower wire carries the 10 kg block and the added mass:

$$T_2 = (10 + m)g = 10(10 + m).$$

The upper wire carries both blocks and the pan load:

$$T_1 = (20 + 10 + m)g = 10(30 + m).$$

The two wire tensions are different because the 20 kg block lies between them.

Step 3: Apply both strength conditions The upper-wire condition is

$$10(30+m) \le 1000 \quad \Rightarrow \quad m \le 70\,\text{kg}.$$

The lower-wire condition is

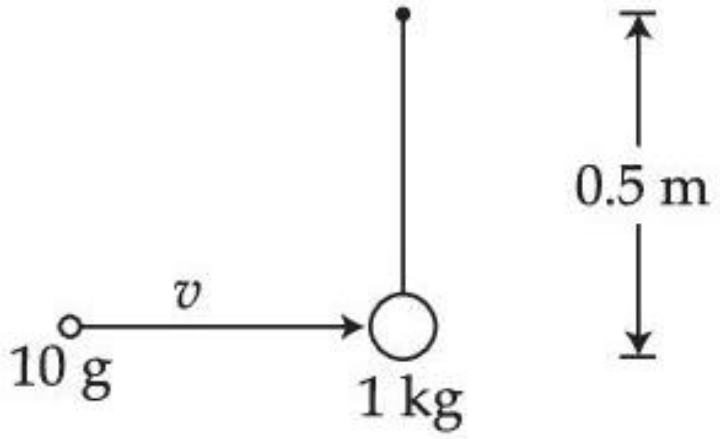
$$10(10+m) \le 500 \quad \Rightarrow \quad m \le 40\,\text{kg}.$$

Both must hold, so the smaller limit controls. At $m = 40\,\text{kg}$, $T_2 = 500\,\text{N}$ and $T_1 = 700\,\text{N} < 1000\,\text{N}$.

Step 4: Conclusion The maximum additional mass is $\boxed{40\,\text{kg}}$, limited by wire W_2.

Quick Tip to Solve: For vertically connected loaded wires, draw a separate supported-mass list for each wire before comparing its tension with breaking stress times area.

55. A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of v = _____ m/s so that the pendulum describes a circle. (Assume the string to be inextensible and g = 10 m/s^2) [4 marks]



Solution:

Concept:

- For a bob on a flexible string to just complete a vertical circle, the top tension may fall to zero: $u_{\text{top}}^2 = gL$.
- Mechanical energy is conserved during the subsequent swing, while horizontal momentum is conserved during the brief collision. A recoiling bullet has velocity opposite to its incident velocity.

Step 1: Find the minimum bottom speed of the bob From bottom to top the bob rises through $2L$. If U is its speed just after impact, conservation of mechanical energy gives

$$\frac{1}{2}MU^2 = \frac{1}{2}Mu_{\text{top}}^2 + Mg(2L).$$

At the limiting condition $u_{\text{top}}^2 = gL$, this becomes

$$U_{\min} = \sqrt{5gL} = \sqrt{5(10)(0.5)} = 5\,\text{m s}^{-1}.$$

Merely reaching the top with zero speed would leave the string slack before the top.

Step 2: Conserve momentum through the impact Take the initial bullet direction as positive. The bullet mass is $10\,\text{g} = 0.010\,\text{kg}$; after recoil its velocity is $-100\,\text{m s}^{-1}$. At the bottom the string is vertical, so its impulsive force does not affect horizontal momentum. Thus

$$0.010v = 1 \times U + 0.010(-100) = U - 1.$$

Substituting the minimum $U = 5\,\text{m s}^{-1}$ gives

$$0.010v = 4 \quad \Rightarrow \quad v = 400\,\text{m s}^{-1}.$$

Step 3: Check the collision interpretation The bob receives momentum $0.010(v + 100)$, because the bullet reverses direction. Treating the recoil speed as positive would instead subtract its contribution incorrectly. Kinetic energy need not be conserved in the collision; only the later ideal pendulum swing uses energy conservation. A larger incident speed gives the bob more than the minimum required bottom speed.

Step 4: Conclusion The minimum incident bullet speed is $\boxed{v = 400\,\text{m s}^{-1}}$.

Quick Tip to Solve: Find the bob speed from $\sqrt{5gL}$ first, then apply collision momentum with a negative velocity for the recoiling bullet.

56. An ac circuit has an inductor and a resistor resistance R in series, such that X_L = 3R. Now, a capacitor is added in series such that X_C = 2R. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$. The value of x is _____. [4 marks]

Solution:

Concept:

- The power factor of a series circuit is $\cos\phi = R/Z$, with R the resistance and Z the magnitude of impedance.
- For series inductive and capacitive reactances, $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The reactances subtract because their voltage contributions oppose in phase.

Step 1: Calculate the original power factor Before adding the capacitor the circuit has resistance R and inductive reactance $3R$. Its impedance is

$$Z_{\text{old}} = \sqrt{R^2 + (3R)^2} = \sqrt{10}\,R.$$

Therefore

$$\cos\phi_{\text{old}} = \frac{R}{\sqrt{10}R} = \frac{1}{\sqrt{10}}.$$

The circuit is inductive, so the current lags the applied voltage.

Step 2: Calculate the new impedance and power factor The added series capacitor has reactance $X_C = 2R$, while the inductor retains $X_L = 3R$. Hence the remaining net reactance is

$$X_{\text{new}} = 3R - 2R = R.$$

It is still inductive, but less strongly so. The new impedance and power factor are

$$Z_{\text{new}} = \sqrt{R^2 + R^2} = \sqrt{2}R, \qquad \cos\phi_{\text{new}} = \frac{1}{\sqrt{2}}.$$

Adding the magnitudes of the reactances would miss their opposing phase effects.

Step 3: Compare the factors in the requested order The question asks for new divided by old, rather than the reverse:

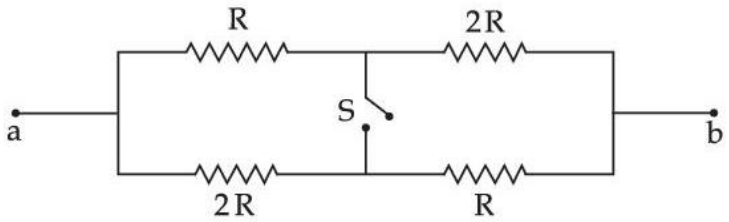
$$\frac{\cos\phi_{\text{new}}}{\cos\phi_{\text{old}}} = \frac{1/\sqrt{2}}{1/\sqrt{10}} = \sqrt{5}.$$

Thus the ratio is $\sqrt{5} : 1$. This improvement is sensible: partial capacitive compensation reduces the phase difference, though it does not reach resonance because $X_L \neq X_C$.

Step 4: Conclusion The power-factor ratio is $\sqrt{5} : 1$, so $\boxed{x = 1}$ (dimensionless).

Quick Tip to Solve: Subtract capacitive from inductive reactance before forming impedance, then keep the requested new-to-old ratio in the right order.

57. The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is x : 8. The value of x is _____. [4 marks]



Solution:

Concept:

- In series, resistances add because the same current passes through them. In parallel, the potential difference is common and conductances add: $R_{\text{eq}}^{-1} = R_1^{-1} + R_2^{-1}$.
- Closing an ideal switch makes its two terminals equipotential. The circuit must then be regrouped according to common nodes, rather than by the visual positions of the resistors.

Step 1: Find the resistance with the switch open With the central switch open, current cannot cross between the midpoints. Each complete path from a to b has one resistor R and one resistor $2R$ in series, so each branch has resistance $3R$. These two branches are connected in parallel between the

same terminals. Therefore,

$$R_{\text{open}} = \frac{(3R)(3R)}{3R+3R} = \frac{3R}{2}.$$

The order of the two resistors within a series branch does not change its total.

Step 2: Regroup the network when the switch closes The two central junctions become a single node. Between a and that node, R and $2R$ are in parallel; the same pair is in parallel between that node and b. These two equivalent groups are in series:

$$R_{\text{closed}} = \frac{R(2R)}{R+2R} + \frac{(2R)R}{2R+R} = \frac{2R}{3} + \frac{2R}{3} = \frac{4R}{3}.$$

Thus the switch reduces the equivalent resistance, as expected when an extra conducting path is supplied.

Step 3: Compare the two equivalent resistances Taking the ratio cancels the unknown positive resistance R:

$$\frac{R_{\text{open}}}{R_{\text{closed}}} = \frac{3R/2}{4R/3} = \frac{9}{8}.$$

Comparing this result with the specified ratio $x : 8$ directly gives the required dimensionless numerator.

Step 4: Conclusion The ratio is $9 : 8$, so $\boxed{x = 9}$ (dimensionless).

Quick Tip to Solve: After closing a switch, merge its endpoints into one node before deciding which resistors are parallel.

58. A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be x $\times$ 10^{-8} T. The value of x is _____. [4 marks]

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, the electric and magnetic fields are in phase, perpendicular to one another and related at each event by $E = cB$.
- The vacuum wave speed is $c = 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}$. The ratio E/B is this speed regardless of the frequency of the wave.

Step 1: Apply the instantaneous field relation The question supplies the electric field at a particular point and instant, rather than an amplitude or an rms value. Because the two fields have the same phase in a travelling plane wave, their instantaneous magnitudes obey the same ratio as their amplitudes. Consequently,

$$B = \frac{E}{c}.$$

No factor of $\sqrt{2}$ is required: such a conversion would only be relevant if an rms value were being compared with an amplitude.

Step 2: Substitute the electric field Using the stated field magnitude $E = 6\,\mathrm{V\,m^{-1}}$,

$$B = \frac{6}{3.0 \times 10^{8}}\,\mathrm{T} = 2.0 \times 10^{-8}\,\mathrm{T}.$$

The units are consistent because electric field divided by speed has the unit tesla. The given frequency $30\,\mathrm{MHz}$ would determine the wavelength, but it is not necessary for the field ratio in free space.

Step 3: Read the requested coefficient The field has already been expressed in the requested form $x \times 10^{-8}\,\mathrm{T}$. Matching the coefficients gives $x = 2$. Dividing by the speed, rather than multiplying by it, also gives the physically appropriate very small magnetic field associated with a field of a few volts per metre.

Step 4: Conclusion The magnetic field magnitude is $\boxed{2 \times 10^{-8}\,\mathrm{T}}$; hence $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: For a vacuum plane wave, use the same type of electric and magnetic field value in $E = cB$; frequency is irrelevant to this ratio.

59. A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _____ cm. (Take speed of sound in air as 340 ms^{-1}) [4 marks]

Solution:

Concept:

- An organ pipe closed at one end has a displacement node at the closed end and a displacement antinode at the open end.
- Its resonant lengths are odd quarter wavelengths: $L = (2n-1)\lambda/4$, where $n = 1, 2, \ldots$. The shortest resonating pipe operates in the fundamental mode.

Step 1: Find the wavelength of the sound Resonance requires the natural frequency of the air column to equal the tuning-fork frequency, $f = 250\,\mathrm{Hz}$. With the specified speed $v = 340\,\mathrm{m\,s^{-1}}$, the wavelength in air is

$$\lambda = \frac{v}{f} = \frac{340}{250}\,\mathrm{m} = 1.36\,\mathrm{m}.$$

This uses the speed of sound in the air column; the mechanical dimensions of the tuning fork do not enter the calculation.

Step 2: Choose the shortest permitted air column The least separation between a displacement node and the adjacent antinode is one quarter of a wavelength. Therefore the shortest closed-pipe length is

$$L_{\min} = \frac{\lambda}{4} = \frac{1.36}{4}\,\mathrm{m} = 0.34\,\mathrm{m}.$$

The half-wavelength relation belongs to a pipe open at both ends and would be inappropriate here. Higher odd modes, such as $3\lambda/4$, give longer pipes and cannot answer the demand for the shortest length.

Step 3: Convert to the requested unit Since $1\,\mathrm{m} = 100\,\mathrm{cm}$,

$$L_{\min} = 0.34 \times 100\,\mathrm{cm} = 34\,\mathrm{cm}.$$

No end correction is specified, so the ideal closed-pipe resonance condition is used. Substitution into $f = v/(4L)$ returns $340/(4 \times 0.34) = 250\,\text{Hz}$, checking the resonance.

Step 4: Conclusion The shortest resonating closed organ pipe has length $\boxed{34\,\text{cm}}$.

Quick Tip to Solve: For the shortest closed pipe use $L = v/(4f)$; reserve $v/(2f)$ for an open pipe.

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60. A heat engine operates between a cold reservoir at temperature T_2 = 400 K and a hot reservoir at temperature T_1. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be _____ K. [4 marks]

Solution:

Concept:

- For a cyclic heat engine, the net work is $W = Q_H - Q_C$, and its efficiency is $\eta = W/Q_H = 1 - Q_C/Q_H$.
- No engine between absolute temperatures T_H and T_C can exceed the reversible Carnot efficiency: $\eta \le 1 - T_C/T_H$. Equality determines the smallest permitted hot-reservoir temperature.

Step 1: Calculate the actual efficiency The engine absorbs $Q_H = 300\,\text{J}$ and rejects $Q_C = 240\,\text{J}$ during each cycle. Since the internal energy returns to its starting value after one cycle,

$$W = 300 - 240 = 60\,\text{J}.$$

Its efficiency is consequently

$$\eta = \frac{60}{300} = 0.20.$$

The rejected heat itself is not work; confusing these quantities would incorrectly produce an efficiency of 0.80.

Step 2: Impose the second-law restriction With the cold reservoir at $T_C = 400\,\text{K}$, the Carnot bound requires

$$0.20 \le 1 - \frac{400}{T_H}.$$

Rearranging, with the positive absolute temperature T_H, gives

$$\frac{400}{T_H} \le 0.80, \qquad T_H \ge \frac{400}{0.80} = 500\,\text{K}.$$

A reservoir cooler than this would have a Carnot efficiency below the observed engine efficiency, violating the second law.

Step 3: Identify the limiting engine The minimum corresponds to equality in the Carnot bound, so the engine must be reversible in that limiting case. The entropy check gives the same result:

$$\Delta S_{\text{reservoirs}} = \frac{240}{400} - \frac{300}{T_H} \ge 0.$$

At $T_H = 500\,\text{K}$, this expression is zero. Irreversibility would require a larger hot-reservoir temperature for the same heat exchanges.

Step 4: Conclusion The minimum hot-reservoir temperature is $\boxed{500\,\text{K}}$.

Quick Tip to Solve: When heat exchanges are known, obtain the efficiency first and then use the Carnot inequality to find the minimum hot temperature.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The addition of dilute NaOH to Cr^{3+} salt solution will give : [4 marks]

(A) a solution of $[Cr(OH)_4]^-$

(B) precipitate of $Cr_2O_3(H_2O)_n$

(C) precipitate of $[Cr(OH)_6]^{3-}$

(D) precipitate of $Cr(OH)_3$

Correct Answer: (B) precipitate of $Cr_2O_3(H_2O)_n$

Solution:

Concept:

- Chromium(III) salts give a grey-green hydrous chromium(III) oxide precipitate when dilute alkali is added; its composition is written $Cr_2O_3(H_2O)_n$.
- A hydrous metal oxide has variable water content. A simple hydroxide formula may be used as an idealised representation, but the hydrated oxide identifies the actual precipitate more accurately in this question.

Step 1: Identify the precipitate formed by dilute alkali Adding dilute $NaOH$ to an aqueous Cr^{3+} salt supplies hydroxide ions. Chromium(III) is precipitated as a grey-green gelatinous hydrous oxide, conventionally represented in the qualitative description by $Cr_2O_3(H_2O)_n$. The subscript n allows for the variable amount of water retained by the hydrous precipitate; sodium and the original salt anions remain in the solution.

Step 2: Distinguish the hydrated oxide from idealised hydroxide The familiar ionic equation $Cr^{3+} + 3\,OH^- \longrightarrow Cr(OH)_3$ describes the stoichiometric precipitation in an idealised hydroxide form. However, chromium(III) hydroxide precipitates are hydrous oxides rather than a well-defined simple hydroxide phase. The printed alternatives explicitly distinguish these descriptions, and the appropriate description of the actual gelatinous solid is the hydrated oxide in option (B), consistent with the answer key.

Step 3: Reject the other product descriptions Option (A) describes a soluble hydroxo complex, associated with dissolution of the amphoteric precipitate in excess alkali, rather than its initial formation with dilute alkali. Option (C) also gives an anionic hydroxo complex and incorrectly calls it the precipitate. Option (D) is the idealised hydroxide formula, whereas the more precise hydrous-oxide composition is explicitly offered in option (B). No separate precipitation of either soluble complex is required here.

Step 4: Conclusion The product is a precipitate of hydrous chromium(III) oxide, $Cr_2O_3(H_2O)_n$. Hence, the correct option is (B).

Quick Tip to Solve: When both a simple hydroxide and a hydrous oxide are offered for chromium(III), identify the actual gelatinous precipitate as the hydrated oxide.

62. Given below are two statements :

Statement I : Ethyl pent-4-yn-oate on reaction with CH_3MgBr gives a 3°-alcohol.

Statement II : In this reaction one mole of ethyl pent-4-yn-oate utilizes two moles of CH_3MgBr.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are false.

(B) Statement I is false but Statement II is true.

(C) Statement I is true but Statement II is false.

(D) Both Statement I and Statement II are true.

Correct Answer: (C) Statement I is true but Statement II is false.

Solution:

Concept:

- An ester normally consumes two equivalents of a Grignard reagent through carbonyl addition and, after aqueous work-up, gives a tertiary alcohol.
- A terminal alkyne contains an acidic hydrogen. A Grignard reagent is also a strong base and consumes an additional equivalent by deprotonating that hydrogen.

Step 1: Locate both reactive sites in the ester Ethyl pent-4-ynoate has the condensed structure $HC{\equiv}C{-}CH_2{-}CH_2{-}COOC_2H_5$. It contains both an ester carbonyl and a terminal $C{\equiv}C{-}H$ bond. Therefore treating it simply as an ester would miss an acid-base reaction. Methylmagnesium bromide removes the terminal hydrogen, producing methane and the corresponding alkynyl magnesium species.

Step 2: Count the carbonyl-addition equivalents At the ester group, the first methyl addition is followed by elimination of the ethoxy group, producing a ketone intermediate. A second methyl addition converts that ketone into a tertiary alkoxide. Aqueous work-up protonates the alkoxide to give the tertiary alcohol $HC{\equiv}C{-}CH_2{-}CH_2{-}C(OH)(CH_3)_2$. The carbon carrying the hydroxyl group is bonded to three carbon groups, so Statement I is true.

Step 3: Add the acid-base consumption and judge the statements The terminal alkyne consumes one equivalent in deprotonation, while conversion of the ester carbonyl to the tertiary alcohol consumes two more. Thus the total requirement is

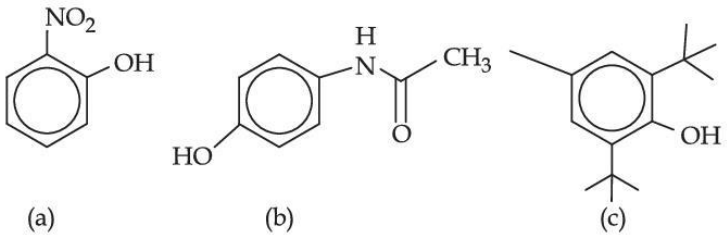
$$1 + 2 = 3 \text{ mol of } CH_3MgBr \text{ per mol of ester.}$$

Statement II claims only two moles and is false. Options (A) and (B) wrongly reject formation of the tertiary alcohol; option (D) wrongly accepts the incomplete reagent count.

Step 4: Conclusion Statement I is true but Statement II is false; three equivalents of CH_3MgBr are required. Hence, the correct option is (C).

Quick Tip to Solve: Before counting Grignard additions, count acidic hydrogens such as terminal alkyne hydrogens; each consumes an extra equivalent.

63. The compound/s which will show significant intermolecular H-bonding is/are : [4 marks]



(A) (b) only

(B) (c) only

(C) (a) and (b) only

(D) (a), (b) and (c)

Correct Answer: (A) (b) only

Solution:

Concept:

- Significant intermolecular hydrogen bonding requires an accessible hydrogen bonded to oxygen, nitrogen or fluorine and a suitable lone-pair acceptor on another molecule.
- Intramolecular hydrogen bonding can use up a donor within one molecule, while bulky groups adjacent to a donor can sterically suppress close approach by other molecules.

Step 1: Examine compound (a) Compound (a) is ortho-nitrophenol: the nitro group lies next to the phenolic hydroxyl group. The hydroxyl hydrogen can bond to a nitro oxygen within the same molecule, producing a favourable six-membered intramolecular arrangement. This internal association predominates and makes compound (a) much less available for significant intermolecular hydrogen bonding of the kind asked for.

Step 2: Examine compound (b) Compound (b) contains a phenolic $O-H$ group and an amide $N-H$ group, together with a carbonyl oxygen that acts as a hydrogen-bond acceptor. The functional groups are not arranged to lock the phenolic hydrogen into the ortho intramolecular interaction seen in (a). Other molecules can approach these sites, so appreciable intermolecular hydrogen bonding occurs.

Step 3: Check steric shielding in compound (c) In compound (c), the phenolic hydroxyl group is flanked by bulky tert-butyl substituents. These groups hinder the close approach required to form a strong intermolecular hydrogen-bonded network. Thus (c) is not included among the compounds showing significant intermolecular association. Only (b) qualifies: option (B) includes only the hindered phenol, option (C) wrongly includes the intramolecularly bonded compound, and option (D) includes both exclusions.

Step 4: Conclusion Only the middle compound, with accessible phenolic and amide groups, shows significant intermolecular hydrogen bonding. Hence, the correct option is (A).

Quick Tip to Solve: For substituted phenols, check ortho intramolecular bonding and steric shielding before assuming that every hydroxyl group forms an intermolecular network.

64. Which one of the following reactions will not yield propionic acid? [4 marks]

(A) $CH_3CH_2COCH_3$ + OI^-/H_3O^+

(B) $CH_3CH_2CH_3$ + $KMnO_4$(Heat), OH^-/H_3O^+

(C) $CH_3CH_2CCl_3$ + OH^-/H_3O^+

(D) $CH_3CH_2CH_2Br$ + Mg, CO_2 dry ether/H_3O^+

Correct Answer: The key selects (D), but the literal propane in (B) also fails as a standard permanganate preparation of propionic acid.

Solution:

Concept:

- A haloform reaction converts a methyl ketone $RCOCH_3$ into $RCOO^-$; acidification gives $RCOOH$.
- Hydrolysis of $RCCl_3$ gives a carboxylic acid with the same carbon skeleton, whereas carboxylation of $RMgBr$ adds one carbon.
- A saturated alkane is not a standard substrate for oxidation by hot alkaline permanganate to a single, same-carbon carboxylic acid.

Step 1: Check the routes that give propionic acid In (A), the ethyl group is the group retained in the haloform reaction. Thus $CH_3CH_2COCH_3$ gives propionate and then CH_3CH_2COOH. In (C), the carbon bearing the three chlorine atoms becomes the carboxyl carbon upon hydrolysis. The product after acidification is again propionic acid. Neither route adds a carbon to the ethyl fragment.

Step 2: Count carbons in the Grignard route In (D), magnesium first gives $CH_3CH_2CH_2MgBr$. Its reaction with carbon dioxide forms a new carbon–carbon bond, so the product contains four carbons:

$$CH_3CH_2CH_2MgBr \xrightarrow[H_3O^+]{CO_2} CH_3CH_2CH_2COOH.$$

The acid is butanoic acid, not propionic acid. This is the definite carbon-counting distinction tested by the keyed choice.

Step 3: Account for the literal alkane in option (B) Option (B) actually prints $CH_3CH_2CH_3$, propane, rather than propan-1-ol. Propane does not undergo the usual primary-alcohol oxidation with alkaline permanganate. A clean conversion of this printed alkane into propionic acid cannot be justified by the normal reaction rules. Therefore (B) also does not provide the stated standard preparation; silently inserting an oxygen atom would change the question.

Step 4: Conclusion Option (D) is the keyed answer: it yields butanoic acid. With propane literally printed in (B), the question does not have a unique defensible answer under the standard reactions: both (B) and (D) fail as standard preparations of propionic acid.

Quick Tip to Solve: For a Grignard reagent followed by carbon dioxide, count the carbon in carbon dioxide as one additional carbon in the acid.

65. The correct order of ionic radii for the ions, P^{3-}, S^{2-}, Ca^{2+}, K^{+}, Cl^{-} is : [4 marks]

(A) $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

(B) $Cl^{-} > S^{2-} > P^{3-} > Ca^{2+} > K^{+}$

(C) $P^{3-} > S^{2-} > Cl^{-} > Ca^{2+} > K^{+}$

(D) $K^{+} > Ca^{2+} > P^{3-} > S^{2-} > Cl^{-}$

Correct Answer: (A) $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

Solution:

Concept:

- Isoelectronic species contain the same number of electrons and have the same occupied principal shells. In this series the outermost occupied shell is the third shell.
- Within an isoelectronic series, increasing nuclear charge attracts the same electron cloud more strongly, so ionic radius decreases.

Step 1: Count the electrons in every ion Phosphorus has atomic number 15, sulfur 16, chlorine 17, potassium 19 and calcium 20. Adding electrons for anions and subtracting them for cations gives

$$N_e(P^{3-}) = 15 + 3 = 18, \quad N_e(S^{2-}) = 16 + 2 = 18,$$
$$N_e(Cl^{-}) = 17 + 1 = 18, \quad N_e(K^{+}) = 19 - 1 = 18,$$
$$N_e(Ca^{2+}) = 20 - 2 = 18.$$

All five ions therefore have the argon electron configuration. Comparing neutral-atom radii would be inappropriate because the species here are ions.

Step 2: Order the nuclear attractions The electron count and shell pattern are fixed, while the nuclear charges increase in the order $15 < 16 < 17 < 19 < 20$. The phosphorus ion has the weakest nuclear attraction for the 18 electrons and is largest. Calcium has the largest nuclear charge and is smallest. Hence

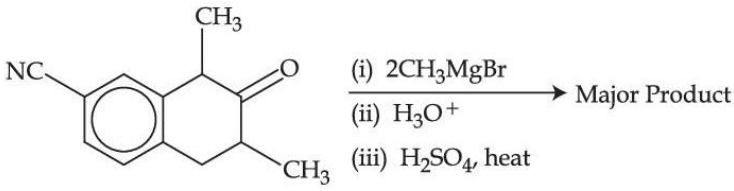
$$r(P^{3-}) > r(S^{2-}) > r(Cl^{-}) > r(K^{+}) > r(Ca^{2+}).$$

Option (B) reverses the anion order. Option (C) incorrectly places calcium above potassium. Option (D) incorrectly makes the cations larger than the anions despite their larger nuclear charges.

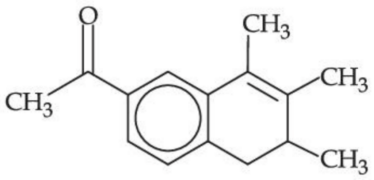
Step 3: Conclusion The order is $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$. Hence, the correct option is (A).

Quick Tip to Solve: First check whether all ions have the same electron count; then reverse the order of their atomic numbers to obtain the radius order.

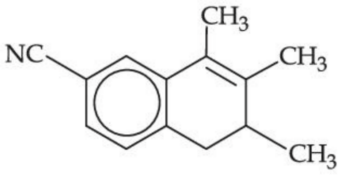
66. Which one of the following is the major product of the given reaction? [4 marks]



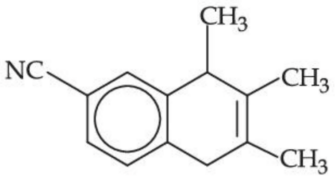
(A) Acetyl substituent on the aromatic ring; the upper, conjugated double bond in the non-aromatic ring.



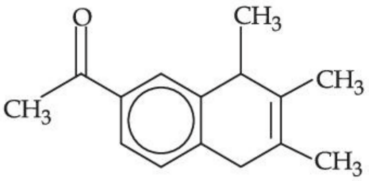
(B) Nitrile substituent on the aromatic ring; the upper, conjugated double bond in the non-aromatic ring.



(C) Nitrile substituent on the aromatic ring; the right-hand, non-conjugated double bond in the non-aromatic ring.

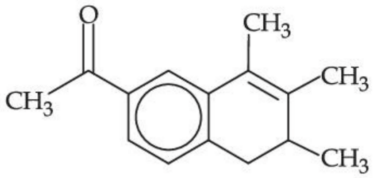


(D) Acetyl substituent on the aromatic ring; the right-hand, non-conjugated double bond in the non-aromatic ring.



Correct Answer: (A)

Acetyl substituent on the aromatic ring; the upper, conjugated double bond in the non-aromatic ring.



Solution:

Concept:

- A Grignard reagent adds to a ketone to form a tertiary alkoxide; acid work-up gives the corresponding tertiary alcohol.
- Addition of one equivalent of CH_3MgBr to a nitrile gives an imine magnesium salt, which hydrolyses to a methyl ketone.

- Acid-catalysed dehydration favours a stable, substituted alkene; conjugation with an aromatic ring provides additional stabilisation. An acetyl group is the group $-COCH_3$.

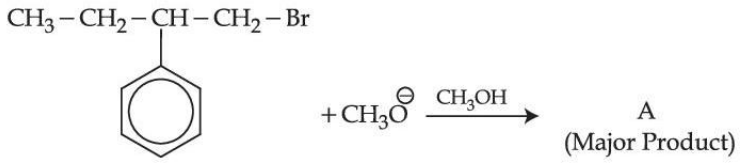
Step 1: Use both equivalents of methylmagnesium bromide The starting structure contains two electrophilic groups: the cyclic ketone and the aromatic nitrile. One methyl group adds to the cyclic carbonyl carbon, producing an alkoxide at that position. A second methyl group adds to the nitrile carbon, giving an imine salt. Only after these additions does the aqueous acid work-up occur. The nitrile-derived imine then hydrolyses to an aromatic acetyl group, $ArCOCH_3$. It cannot retain the nitrile group in the final product. The ketone formed on this work-up is not exposed to an additional equivalent of unquenched Grignard reagent.

Step 2: Dehydrate the tertiary alcohol selectively The former cyclic carbonyl carbon now bears an OH group and the newly introduced methyl group. Sulfuric acid and heat remove water and a neighbouring hydrogen. Of the depicted ring alkenes, the upper double bond is conjugated with the benzene ring. The right-hand double bond is separated from the aromatic ring by a saturated carbon and lacks this stabilisation. Thus the major product has both the aromatic acetyl substituent and the upper conjugated double bond. Options (B) and (C) incorrectly retain the nitrile. Option (D) has the acetyl group but the less favourable, non-conjugated double bond.

Step 3: Conclusion The major product is the acetyl-substituted conjugated alkene shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: When a molecule contains a nitrile and a ketone, track one Grignard addition at each group before applying hydrolysis and dehydration.

67. The major product (A) formed in the reaction given below is : [4 marks]



(A) $CH_3CH_2CH(C_6H_4OCH_3-p)CH_2Br$

(B) $CH_3CH_2C(C_6H_5)=CH_2$

(C) $CH_3CH_2CH(C_6H_5)CH_2OH$

(D) $CH_3CH_2CH(C_6H_5)CH_2OCH_3$

Correct Answer: (D) $CH_3CH_2CH(C_6H_5)CH_2OCH_3$

Solution:

Concept:

- A primary alkyl bromide undergoes bimolecular nucleophilic substitution with an alkoxide ion to form an ether.
- Methoxide, CH_3O^-, contributes its oxygen atom to the new carbon–oxygen bond; the alkyl skeleton is retained in an S_N2 reaction.

Step 1: Locate the carbon bearing bromine The bromine is attached to the terminal CH_2 carbon in $CH_3CH_2CH(C_6H_5)CH_2Br$. This is a primary carbon, even though the neighbouring carbon carries a phenyl group. Methoxide is a small, effective nucleophile. It attacks the carbon bearing the leaving group and expels bromide in a single substitution step. No benzylic carbocation or carbon-skeleton rearrangement is required.

Step 2: Replace bromide by methoxy The product of the substitution is

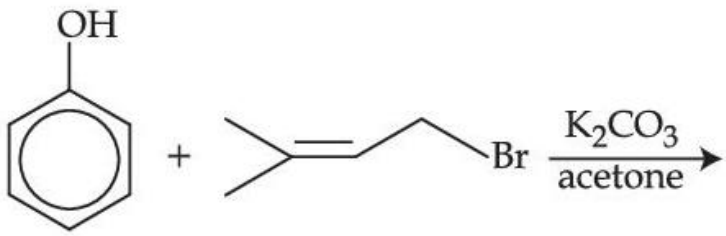
$$CH_3CH_2CH(C_6H_5)CH_2Br + CH_3O^- \longrightarrow CH_3CH_2CH(C_6H_5)CH_2OCH_3 + Br^-.$$

This is the methyl ether in option (D). Methanol is the solvent; it does not convert the attacking methoxide ion into hydroxide. Consequently the alcohol in (C) is not the substitution product. The alkene in (B) would require elimination, which is less favoured than substitution for this primary bromide with a small nucleophile under the stated conditions. Option (A) would require aromatic substitution at a ring carbon that has no leaving group; the terminal bromine is not even displaced in that option. It therefore cannot follow the normal nucleophilic substitution route.

Step 3: Conclusion The product is $CH_3CH_2CH(C_6H_5)CH_2OCH_3$. Hence, the correct option is (D).

Quick Tip to Solve: Classify the carbon bonded directly to the halogen, rather than its neighbour, before comparing substitution and elimination.

68. The major product of the following reaction, if it occurs by S_N2 mechanism is : [4 marks]



(A) $PhOCH_2CH_2C(CH_3)_2OPh$

(B) $PhOC(CH_3)_2CH{=}CH_2$

(C) $PhOCH_2CH_2C(CH_3){=}CH_2$

(D) $PhOCH_2CH{=}C(CH_3)_2$

Correct Answer: (D) $PhOCH_2CH{=}C(CH_3)_2$

Solution:

Concept:

- Potassium carbonate generates phenoxide from phenol; phenoxide is the oxygen nucleophile in a Williamson ether synthesis.
- An S_N2 substitution replaces the leaving group at the carbon to which it is bonded without carbocation rearrangement.
- Acetone is a polar aprotic solvent that favours nucleophilic substitution by anionic nucleophiles.

Step 1: Identify the nucleophile and electrophile The reacting oxygen belongs to phenol. In the presence of K_2CO_3, phenoxide $C_6H_5O^-$ is available to attack the primary allylic carbon bearing bromine. The alkyl bromide is $BrCH_2CH{=}C(CH_3)_2$. The question explicitly specifies an S_N2 mechanism, so we replace bromide at that same CH_2 position rather than choosing a rearranged allylic product.

Step 2: Form the ether without moving the double bond The carbon–oxygen bond forms at the bromomethyl end:

$$C_6H_5O^- + BrCH_2CH{=}C(CH_3)_2 \longrightarrow C_6H_5OCH_2CH{=}C(CH_3)_2 + Br^-.$$

The original double bond and both methyl groups retain their positions. This connectivity is displayed in (D). Option (B) puts phenoxy at a different allylic carbon and moves the double bond; that is not the direct substitution requested. Option (C) also changes the alkene position and carbon substitution pattern. Option (A) adds two phenoxy groups and removes the alkene, whereas the stated substitution consumes one phenoxide and expels one bromide. It is therefore not a product of this single displacement.

Step 3: Conclusion The major product by the stipulated mechanism is $C_6H_5OCH_2CH{=}C(CH_3)_2$. Hence, the correct option is (D).

Quick Tip to Solve: For a question explicitly specifying direct SN2, mark the carbon attached to the leaving group and replace only that bond.

69. Potassium permanganate on heating at 513 K gives a product which is : [4 marks]

(A) paramagnetic and colourless

(B) diamagnetic and green

(C) diamagnetic and colourless

(D) paramagnetic and green

Correct Answer: The key selects (D), describing green paramagnetic manganate; oxygen is also a colourless paramagnetic product, so (A) also fits the literal wording.

Solution:

Concept:

- On heating, potassium permanganate decomposes into potassium manganate, manganese dioxide and oxygen.
- The colour and magnetism of the manganate ion follow its electronic structure: manganese(VI) has a d^1 configuration and one unpaired electron.

Step 1: Write the thermal decomposition The decomposition on heating at the stated temperature is

$$2\,KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2.$$

The potassium manganate product K_2MnO_4 is green. It must be distinguished from purple permanganate, $KMnO_4$, and from the brown or black manganese dioxide formed alongside it. The green product appearing among the choices is therefore the manganate salt.

Step 2: Determine the number of unpaired electrons For MnO_4^{2-}, let the oxidation state of manganese be x. Charge balance gives $x + 4(-2) = -2$, hence $x = +6$. Neutral manganese has configuration $[Ar]3d^5 4s^2$. Removal of six electrons leaves one d electron. A single electron cannot be paired, so the manganate ion is paramagnetic. For comparison, manganese(VII) in permanganate is d^0, but using that oxidation state here would describe the reactant rather than the green decomposition product. Option (B) gets the green colour right but incorrectly calls a d^1 species diamagnetic. The green salt is not colourless, so (A) and (C) do not describe manganate. However, the balanced equation also produces oxygen. Gaseous oxygen is colourless and paramagnetic because its molecular orbitals contain two unpaired electrons. Therefore (A) does describe this other product. The wording does not explicitly restrict the product to the green manganese salt, so the literal wording admits both (A) and (D).

Step 3: Conclusion The keyed answer is (D), green paramagnetic potassium manganate. As written, (A) is also true of the oxygen formed, so the unrestricted phrase "a product" does not uniquely distinguish these two alternatives.

Quick Tip to Solve: Identify the manganese compound formed before counting d electrons: manganate is Mn(VI), while permanganate is Mn(VII).

70. Which one of the following tests used for the identification of functional groups in organic compounds does not use copper reagent? [4 marks]

(A) Barfoed's test

(B) Seliwanoff's test

(C) Benedict's test

(D) Biuret test for peptide bond

Correct Answer: (B) Seliwanoff's test

Solution:

Concept:

- Copper-based tests use Cu^{2+} either as an oxidising reagent for reducing sugars or as a complex-forming ion for peptide bonds.
- Seliwanoff's test distinguishes ketoses from aldoses using resorcinol and concentrated hydrochloric acid; copper salts are not its reagent.

Step 1: Check the tests containing copper Barfoed's reagent contains copper(II) acetate in dilute acetic acid. Reducing monosaccharides reduce the copper(II) ion to a red precipitate of copper(I) oxide on heating. Benedict's reagent contains copper(II) sulfate with sodium carbonate and sodium citrate; reducing sugars again reduce copper(II) to copper(I) oxide. In the Biuret test, alkaline copper(II) ions form a violet coordination complex with peptide bonds. Thus (A), (C) and (D) all explicitly rely on a copper reagent, although their conditions and the role of the copper ion differ.

Step 2: Identify the copper-free reaction Seliwanoff's reagent contains resorcinol in strongly acidic hydrochloric acid. Ketoses dehydrate rapidly under the acidic conditions to form hydroxymethylfurfural, which condenses with resorcinol to give a characteristic cherry-red colour. Aldoses respond

more slowly, so the rate of colour formation distinguishes the two classes of sugars. The colour is produced by this organic condensation chemistry, rather than by reduction or coordination of a copper salt. The presence of a coloured result therefore does not by itself imply a copper reagent. Among the four listed tests, only Seliwanoff's test uses this copper-free reagent combination.

Step 3: Conclusion Seliwanoff's test uses resorcinol and hydrochloric acid, without a copper reagent. Hence, the correct option is (B).

Quick Tip to Solve: Remember the pair resorcinol plus hydrochloric acid for Seliwanoff; Barfoed, Benedict and Biuret all contain copper.

71. Hydrolysis of sucrose gives : [4 marks]

(A) α-D-(−)-Glucose and β-D-(−)-Fructose

(B) α-D-(+)-Glucose and α-D-(−)-Fructose

(C) α-D-(−)-Glucose and α-D-(+)-Fructose

(D) α-D-(+)-Glucose and β-D-(−)-Fructose

Correct Answer: (D) α-D-(+)-Glucose and β-D-(−)-Fructose

Solution:

Concept:

- Sucrose is a disaccharide formed by linking the anomeric carbon of glucose to the anomeric carbon of fructose through an oxygen atom.
- The symbols α and β specify anomeric configuration, whereas (+) and (−) specify the direction of optical rotation; these are distinct properties.

Step 1: Identify the two monosaccharide units Sucrose contains an α-D-glucopyranose unit and a β-D-fructofuranose unit. Its glycosidic linkage joins carbon 1 of glucose to carbon 2 of fructose. Acid-catalysed hydrolysis, or hydrolysis by invertase, breaks this linkage by adding water. It produces one molecule of D-glucose and one molecule of D-fructose per sucrose molecule; it does not change their D-series configurations.

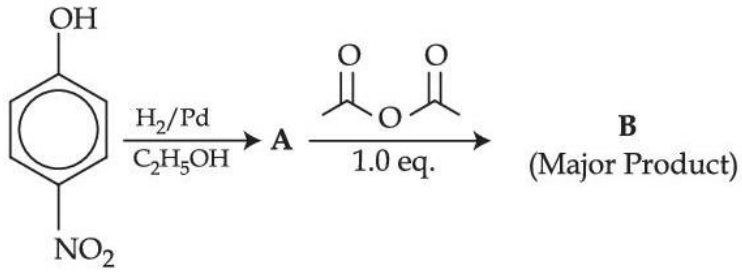
Step 2: Assign their optical rotations D-glucose is dextrorotatory, so its name carries (+). D-fructose is levorotatory, so its name carries (−). The combination represented in the options is therefore α-D-(+)-glucose and β-D-(−)-fructose. In aqueous solution the free sugars undergo mutarotation; the option describes the constituent anomeric units conventionally associated with sucrose.

Step 3: Eliminate the other pairs Option (A) incorrectly makes glucose levorotatory. Option (B) gives the incorrect fructose anomer for the sucrose unit. Option (C) reverses both optical signs and also gives the wrong fructose anomer. The hydrolysis mixture is called invert sugar because the negative rotation of fructose outweighs the positive rotation of glucose, reversing the original rotation of sucrose.

Step 4: Conclusion Hydrolysis gives α-D-(+)-glucose and β-D-(−)-fructose in the conventional representation. Hence, the correct option is (D).

Quick Tip to Solve: Remember sucrose as glucose α-1 to fructose β-2; glucose is (+), whereas fructose is (−).

72. The correct structures of A and B formed in the following reactions are : [4 marks]



(A) A: p-$HOC_6H_4NH_2$; B: 2-CH_3CO−4-NH_2−C_6H_3OH.

(B) A: p-$CH_3COOC_6H_4NH_2$; B: p-$CH_3COOC_6H_4NHCOCH_3$.

(C) A: p-$HOC_6H_4NH_2$; B: p-$CH_3COOC_6H_4NH_2$.

(D) A: p-$HOC_6H_4NH_2$; B: p-$HOC_6H_4NHCOCH_3$.

Correct Answer: (D) A: p-$HOC_6H_4NH_2$; B: p-$HOC_6H_4NHCOCH_3$.

Solution:

Concept:

- Catalytic hydrogenation with H_2/Pd reduces an aromatic nitro group to a primary amino group under the stated conditions.
- Acetic anhydride acetylates an amino group to an amide. In an aminophenol, the amino nitrogen is more nucleophilic than the phenolic oxygen, so one equivalent mainly gives N-acetylation.

Step 1: Reduce the nitro substituent The starting compound is para-nitrophenol: its hydroxyl and nitro groups occupy opposite positions on the benzene ring. Hydrogen in the presence of palladium reduces $-NO_2$ to $-NH_2$, leaving the phenolic hydroxyl group intact. Thus A is para-aminophenol, p-$HOC_6H_4NH_2$. There is no source of an acetyl group in this first stage, so A cannot already be an acetate ester.

Step 2: Apply the one-equivalent acetylation A has both an amino group and a hydroxyl group, but their reactivities toward acetic anhydride differ. The amino group attacks the anhydride more readily and forms $-NHCOCH_3$. With one equivalent the major product B is para-hydroxyacetanilide (paracetamol), p-$HOC_6H_4NHCOCH_3$. The hydroxyl group remains free; a second acetylation would require additional reagent and suitable conditions.

Step 3: Compare the four proposed products The first pair proposes carbon acetylation of the aromatic ring, which is not the normal result of these conditions. The second pair incorrectly makes A an O-acetylated compound before acetic anhydride is added. The third pair places the acetyl group on oxygen rather than on the more reactive amino nitrogen. Only the fourth pair shows para-aminophenol followed by its N-acetyl derivative, with the phenolic hydroxyl retained.

Step 4: Conclusion A is para-aminophenol and B is para-hydroxyacetanilide, p-$HOC_6H_4NHCOCH_3$. Hence, the correct option is (D).

Quick Tip to Solve: For one-equivalent acetylation of an aminophenol, compare nucleophilicity: the amino group normally reacts before the phenolic hydroxyl group.

73. Choose the correct statement from the following : [4 marks]

(A) The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group.

(B) The low solubility of CsI in water is due to its high lattice enthalpy.

(C) Among the alkali metal halides, LiF is least soluble in water.

(D) LiF has least negative standard enthalpy of formation among alkali metal fluorides.

Correct Answer: (C) Among the alkali metal halides, LiF is least soluble in water.

Solution:

Concept:

- Dissolution depends on the balance between the energy needed to separate a crystal lattice and the energy released on hydration of its ions.
- Small ions can form a strongly bound ionic lattice. A salt need not be readily soluble merely because its ions can be hydrated.

Step 1: Identify the least-soluble salt Among the alkali-metal halides, lithium fluoride is exceptionally sparingly soluble in water. Both Li^+ and F^- are small, so the attraction between oppositely charged ions in its lattice is strong. Although these ions are hydrated strongly, the overall balance of lattice disruption, hydration and entropy does not favour extensive dissolution. Thus the statement that LiF is the least soluble is correct.

Step 2: Check the bromide and iodide statements For alkali-metal bromides the standard enthalpy of formation does not become progressively less negative down the group as option (A) claims; the formation energetics reflect several contributions, including atomisation, ionisation and lattice formation. The relatively low solubility associated with CsI is connected with weak hydration of its large ions, not an unusually high lattice enthalpy. Hence option (B) gives the wrong explanation.

Step 3: Check the fluoride formation statement The very strong lattice of LiF stabilises the solid, giving it a strongly negative standard enthalpy of formation. Calling its formation enthalpy the least negative among alkali-metal fluorides reverses the relevant trend, so option (D) is incorrect. It is important to distinguish the enthalpy of forming a solid from its constituent elements from the enthalpy or free energy of dissolving that solid in water.

Step 4: Conclusion Among the alkali-metal halides, LiF is least soluble in water. Hence, the correct option is (C).

Quick Tip to Solve: For a solubility explanation, consider hydration as well as lattice strength; large ions such as those in CsI are hydrated weakly.

74. In stratosphere most of the ozone formation is assisted by : [4 marks]

(A) cosmic rays

(B) γ-rays

(C) ultraviolet radiation

(D) visible radiations

Correct Answer: (C) ultraviolet radiation

Solution:

Concept:

- Ozone formation in the stratosphere begins with photodissociation of molecular oxygen by sufficiently energetic ultraviolet radiation.
- The oxygen atoms produced combine with oxygen molecules to form ozone; a third body carries away excess energy and stabilises the product.

Step 1: Generate atomic oxygen An oxygen molecule must first absorb a photon with enough energy to break its oxygen–oxygen bond. Solar ultraviolet radiation provides the relevant energy in the stratosphere. The photochemical step is represented as

$$O_2 \xrightarrow{UV} O + O.$$

This is the essential initiating stage: ordinary oxygen molecules do not simply combine with one another to give ozone without an energy source and the intermediate atomic oxygen.

Step 2: Form and stabilise ozone An oxygen atom then combines with an oxygen molecule:

$$O + O_2 + M \longrightarrow O_3 + M.$$

Here M is a third body, such as another atmospheric molecule, that removes the excess energy. This ozone formation is part of the natural stratospheric oxygen–ozone cycle. Ozone can itself absorb ultraviolet light and dissociate, so its concentration reflects a dynamic balance rather than unlimited accumulation.

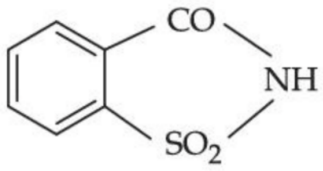
Step 3: Exclude the other radiation choices Visible photons do not have sufficient energy to drive the principal oxygen photodissociation process responsible for this ozone layer. Cosmic rays and gamma rays may cause high-energy interactions, but they are not the main solar radiation responsible for normal stratospheric ozone production. The question asks what assists most formation, so identifying the dominant photochemical source is decisive.

Step 4: Conclusion Most stratospheric ozone formation is assisted by ultraviolet radiation. Hence, the correct option is (C).

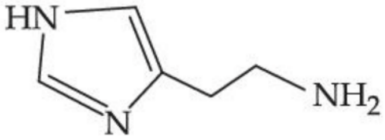
Quick Tip to Solve: Associate the ozone layer with ultraviolet light twice: UV helps make ozone from oxygen, and ozone then absorbs harmful UV radiation.

75. Which one of the following chemicals is responsible for the production of HCl in the stomach leading to irritation and pain? [4 marks]

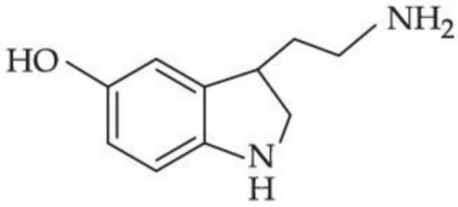
(A)



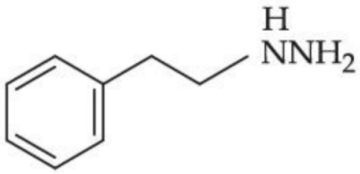
(B)



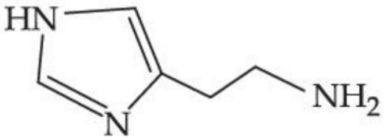
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- Histamine is a chemical messenger that stimulates acid secretion by acting on histamine receptors in the stomach.
- Histamine contains an imidazole ring attached to a two-carbon chain ending in a primary amino group; this structural pattern identifies it among the options.

Step 1: Identify the histamine structure Look for a five-membered aromatic ring containing two nitrogen atoms, one of which bears hydrogen. The ring carries a $-CH_2CH_2NH_2$ side chain. This is the imidazole-ethylamine framework of histamine, and it is the structure shown in option (B). Its ring differs from a benzene ring and from the fused bicyclic framework shown in option (C).

Step 2: Connect histamine with acid production Histamine stimulates gastric parietal cells through their H_2 receptors, promoting the secretion of hydrochloric acid into the stomach. Excess acid can produce irritation and pain. This is why histamine receptor antagonists such as cimetidine and ranitidine have been used to reduce gastric acid secretion: they block the action of histamine rather

than merely neutralising acid already present.

Step 3: Reject the remaining structures Option (A) has the cyclic sulfonimide framework of saccharin, an artificial sweetener, and is not the gastric acid messenger. Option (C) has a hydroxyl-substituted fused nitrogen-containing ring system rather than an imidazole ring. Option (D) is a phenethylhydrazine framework, recognised by its benzene ring and nitrogen–nitrogen linkage. Neither has the defining histamine structure required for this physiological action.

Step 4: Conclusion The compound is histamine, the imidazole-containing structure with an aminoethyl side chain. Hence, the correct option is (B).

Quick Tip to Solve: Identify histamine by its imidazole ring plus $-CH_2CH_2NH_2$ chain, then link it with gastric H_2 receptors.

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76. The oxide that gives H_2O_2 most readily on treatment with H_2O is : [4 marks]

(A) PbO_2

(B) Na_2O_2

(C) SnO_2

(D) $BaO_2 \cdot 8H_2O$

Correct Answer: (B) Na_2O_2

Solution:

Concept:

- A peroxide contains the peroxide ion O_2^{2-}, whose oxygen atoms have oxidation state -1. Protonation of this ion gives hydrogen peroxide.
- The ability to react readily with water depends on the nature of the compound and its accessibility in water; an ordinary metal dioxide need not contain a peroxide ion.

Step 1: Identify the actual peroxide compounds In sodium peroxide, Na_2O_2, the two sodium ions balance one peroxide ion. Hydrated barium peroxide also contains peroxide oxygen. In contrast, PbO_2 and SnO_2 are ordinary dioxides of metals in oxidation state $+4$, with oxygen in oxidation state -2. Their formulae do not signify an oxygen–oxygen peroxide group, so adding water does not simply release hydrogen peroxide.

Step 2: Write the ready hydrolysis reaction Sodium peroxide reacts readily with water and produces sodium hydroxide together with hydrogen peroxide:

$$Na_2O_2 + 2\,H_2O \longrightarrow 2\,NaOH + H_2O_2.$$

Equivalently, the peroxide ion takes up protons from water:

$$O_2^{2-} + 2\,H_2O \longrightarrow H_2O_2 + 2\,OH^-.$$

This explicitly shows why the peroxide unit can appear as hydrogen peroxide in the products without requiring a redox change of its oxygen atoms.

Step 3: Compare readiness with the remaining choice Barium peroxide octahydrate is much less soluble and does not undergo the same ready reaction with water as sodium peroxide. It is familiar as a reagent for preparing hydrogen peroxide by treatment with a dilute acid, but that is a different condition from adding water alone. The strongly alkaline sodium peroxide hydrolysis mixture can subsequently promote decomposition of hydrogen peroxide, so conditions matter when collecting the product; this does not change the requested identity of the oxide that hydrolyses most readily.

Step 4: Conclusion Sodium peroxide, Na_2O_2, gives hydrogen peroxide most readily on treatment with water. Hence, the correct option is (B).

Quick Tip to Solve: Use the stated reagent carefully: sodium peroxide reacts readily with water, while barium peroxide is commonly used with a dilute acid.

77. Which one of the following is used to remove most of plutonium from spent nuclear fuel? [4 marks]

(A) ClF_3

(B) O_2F_2

(C) I_2O_5

(D) BrO_3

Correct Answer: (B) O_2F_2

Solution:

Concept:

- Dioxygen difluoride, O_2F_2, is a powerful fluorinating and oxidising agent that can convert plutonium into its hexafluoride.
- Formation of a volatile fluoride provides a way to separate plutonium from material in spent nuclear fuel; the chemical transformation and volatility are both relevant.

Step 1: Identify the required chemical action To remove plutonium by a fluoride-volatility process, the plutonium is converted into a fluoride that can be separated from nonvolatile material. Dioxygen difluoride is a very strong fluorinating agent and oxidises plutonium to PuF_6, plutonium hexafluoride. The plutonium is thus removed through the formation of a separable volatile compound rather than by simple dissolution or precipitation.

Step 2: Match the reagent to the named use The particular oxygen fluoride used for this purpose is O_2F_2, which appears in option (B). Its formula is important: it is dioxygen difluoride, not oxygen difluoride. Oxygen fluorides show unusual bonding and strong oxidising behaviour because fluorine attracts electrons very strongly. The relevant application here is the powerful fluorination of plutonium to its higher fluoride.

Step 3: Eliminate the alternatives Chlorine trifluoride in option (A) is itself a reactive fluorinating compound, but the specific reagent associated with the stated removal of plutonium is dioxygen difluoride. Iodine pentoxide in option (C) is an oxide and does not supply fluorine to make plutonium hexafluoride; it is commonly associated with oxidation of carbon monoxide. The bromine oxide formula in option (D) likewise provides no fluorine and is not the reagent used for this purpose.

Step 4: Conclusion Dioxygen difluoride, O_2F_2, is used to remove plutonium from spent nuclear fuel by forming PuF_6. Hence, the correct option is (B).

Quick Tip to Solve: For the nuclear-fuel application, remember the pair O_2F_2 as reagent and volatile PuF_6 as product.

78. Lyophilic sols are more stable than lyophobic sols because : [4 marks]

(A) there is a strong electrostatic repulsion between the negatively charged colloidal particles.

(B) the colloidal particles have positive charge.

(C) the colloidal particles have no charge.

(D) the colloidal particles are solvated.

Correct Answer: (D) the colloidal particles are solvated.

Solution:

Concept:

- Lyophilic colloidal particles have a strong affinity for the dispersion medium. Solvation means that solvent molecules associate with their surfaces; in water this is called hydration.
- A solvation layer provides a protective barrier against aggregation. Electrical charge can also stabilize a sol, but charge alone does not explain the greater stability of lyophilic sols.

Step 1: Identify the extra stabilising factor In a lyophilic sol, the dispersed substance interacts favourably with the solvent. Solvent molecules surround each colloidal particle and form a solvated envelope. For particles to join into a larger aggregate, these favourable particle–solvent interactions must be disrupted and the protective layers must be displaced. This makes coagulation more difficult than in a lyophobic sol, whose particles have little affinity for the dispersion medium.

Step 2: Compare the alternatives Option (D) states this distinguishing factor directly. Option (A) describes electrostatic repulsion, which can stabilize lyophobic sols as well; it also unnecessarily specifies negative charge. Option (B) merely assigns positive charge, whose sign does not by itself produce greater stability. Option (C) is incorrect because lyophilic particles need not be uncharged, and absence of charge is not the protective mechanism. Thus the solvent envelope, rather than a particular sign of charge, explains the comparison.

Step 3: Conclusion Lyophilic sols are more stable because their colloidal particles are solvated. Hence, the correct option is (D).

Quick Tip to Solve: When comparing lyophilic with lyophobic sols, look for solvent affinity and the protective solvation layer, rather than the sign of charge.

79. Match List-I with List-II :

	List - I(Name of ore/mineral)		List - II(Chemical formula)
(a)	**Calmine**	**(i)**	Zns
(b)	**Malachite**	**(ii)**	$FeCO_3$
(c)	**Siderite**	**(iii)**	$ZnCO_3$
(d)	**Sphalerite**	**(iv)**	$CuCO_3.Cu(OH)_2$

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(B) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(C) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(D) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

Correct Answer: (A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Solution:

Concept:

- An ore or mineral is identified by its characteristic chemical composition. Calamine is zinc carbonate, whereas sphalerite (zinc blende) is zinc sulphide.
- Malachite is basic copper carbonate, $\mathrm{CuCO_3 \cdot Cu(OH)_2}$, and siderite is iron(II) carbonate, $\mathrm{FeCO_3}$.

Step 1: Assign each mineral its formula The entry labelled Calmine refers to calamine, the carbonate mineral of zinc. Its formula is $\mathrm{ZnCO_3}$, so (a) matches (iii). Malachite contains copper carbonate together with copper hydroxide and is written $\mathrm{CuCO_3 \cdot Cu(OH)_2}$; hence (b) matches (iv). Siderite is ferrous carbonate, $\mathrm{FeCO_3}$, giving (c)–(ii). Sphalerite is zinc sulphide, ZnS, giving (d)–(i). In the printed table, the sulphide entry is the one denoted by (i).

Step 2: Check the complete matching against the options The four matches together are

$$(a) \to (iii), \quad (b) \to (iv), \quad (c) \to (ii), \quad (d) \to (i).$$

Option (A) contains all four. Option (B) interchanges the identities of siderite and sphalerite. Option (C) additionally interchanges calamine and malachite. Option (D) assigns iron carbonate to malachite and basic copper carbonate to siderite. These alternatives contradict the metal and anion present in the named minerals.

Step 3: Conclusion Calamine matches zinc carbonate, malachite matches basic copper carbonate, siderite matches iron carbonate, and sphalerite matches zinc sulphide. Hence, the correct option is (A).

Quick Tip to Solve: Remember the zinc pair together: calamine is the carbonate and sphalerite is the sulphide; siderite contains iron.

80. Which one of the following is formed (mainly) when red phosphorus is heated in a sealed tube at 803 K? [4 marks]

(A) White phosphorus

(B) Yellow phosphorus

(C) β-Black phosphorus

(D) α-Black phosphorus

Correct Answer: (D) α-Black phosphorus

Solution:

Concept:

- Phosphorus has several allotropes with different structures. The preparation conditions distinguish the alpha and beta forms of black phosphorus.
- Heating red phosphorus in a sealed tube at $803\,\mathrm{K}$ produces α-black phosphorus. The beta form is associated with heating white phosphorus under high pressure.

Step 1: Use the stated starting material and conditions The question specifies red phosphorus, a sealed tube and a temperature of $803\,\mathrm{K}$. These are the characteristic preparation conditions for the alpha form of black phosphorus. This is an allotropy conversion: the element remains phosphorus, while its bonding arrangement changes. A sealed tube retains the phosphorus during heating; it does not by itself imply the high-pressure preparation used for the beta form.

Step 2: Distinguish the proposed products Option (D) names α-black phosphorus and therefore agrees with the stated preparation. Option (C), β-black phosphorus, corresponds to a different preparation route involving white phosphorus and high pressure, so the two black forms should not be interchanged. White phosphorus and yellow phosphorus in options (A) and (B) refer to the white/yellow molecular form, rather than the black allotrope obtained by this stated treatment of red phosphorus. Selecting merely a colour without using the temperature and starting allotrope would miss the decisive information.

Step 3: Conclusion The main product is α-black phosphorus. Hence, the correct option is (D).

Quick Tip to Solve: Associate red phosphorus at 803 K in a sealed tube with alpha-black; distinguish it from the high-pressure route to beta-black.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The first order rate constant for the decomposition of $CaCO_3$ at 700 K is $6.36 \times 10^{-3} s^{-1}$ and activation energy is 209 kJ mol^{-1}. Its rate constant (in s^{-1}) at 600 K is $x \times 10^{-6}$. The value of x is _____. (Nearest integer)

[Given R = 8.31 J K^{-} mol^{-1}; log $6.36 \times 10^{-3} = -2.19$, $10^{-4.79} = 1.62 \times 10^{-5}$] [4 marks]

Solution:

Concept:

- The two-temperature Arrhenius equation is $\log_{10}(k_2/k_1) = -E_a(1/T_2 - 1/T_1)/(2.303R)$. Temperatures must be in kelvin and the energy units must agree with those of R.
- A positive activation energy means that cooling decreases the rate constant. For a first-order reaction the rate constant has units s^{-1}.

Step 1: Substitute the temperature change Take $T_1 = 700\,K$, $T_2 = 600\,K$, $k_1 = 6.36 \times 10^{-3}\,s^{-1}$, and $E_a = 209000\,J\,mol^{-1}$. Then

$$\frac{1}{600} - \frac{1}{700} = \frac{1}{4200}\,K^{-1}.$$

Hence

$$\log_{10}\frac{k_2}{k_1} = -\frac{209000}{2.303 \times 8.31 \times 4200} \simeq -2.60.$$

The negative sign is consistent with the lower temperature and prevents the erroneous prediction of a faster decomposition on cooling.

Step 2: Calculate the requested coefficient Using the logarithm supplied in the question,

$$\log_{10} k_2 = -2.19 - 2.60 = -4.79.$$

Thus

$$k_2 = 1.62 \times 10^{-5}\,s^{-1} = 16.2 \times 10^{-6}\,s^{-1}.$$

The question asks for the coefficient x, not for the rate constant itself. Therefore $x = 16.2$, which becomes 16 on rounding to the nearest integer. The rate-constant unit remains inverse seconds throughout; x is a dimensionless numerical coefficient.

Step 3: Conclusion The rate constant is approximately $1.62 \times 10^{-5}\,s^{-1}$, so the nearest-integer value of x is $\boxed{16}$.

Quick Tip to Solve: First determine the sign of the temperature effect, then rewrite the rate constant using exactly the power of ten requested.

82. The number of optical isomers possible for $[Cr(C_2O_4)_3]^{3-}$ is _____. [4 marks]

Solution:

Concept:

- Oxalate, $C_2O_4{}^{2-}$, is a bidentate ligand: each oxalate ion binds through two donor oxygen atoms.
- Three bidentate oxalate ligands give octahedral coordination around chromium. Their chelate arrangement has two non-superimposable mirror-image configurations, denoted Δ and Λ.

Step 1: Determine the coordination arrangement In $[Cr(C_2O_4)_3]^{3-}$, the three oxalate ligands occupy six donor positions around the metal. The coordination number is therefore six, giving an octahedral tris-chelate complex. Since all three ligands are identical, there is no separate cis/trans pair to multiply the count: the relevant difference is the handedness with which the three chelate rings wrap around the metal centre.

Step 2: Count the distinct mirror-image forms One chelate arrangement has a right-handed twist and is labelled Δ; reflection gives the left-handed Λ arrangement. Rotation of the whole complex cannot superimpose one configuration on the other. They therefore constitute a pair of optical isomers, or enantiomers. Different drawings obtained just by rotating either form are not additional isomers. Likewise, a racemic mixture is a mixture of these two forms, not a third molecular configuration. Thus the complete optical-isomer count is two.

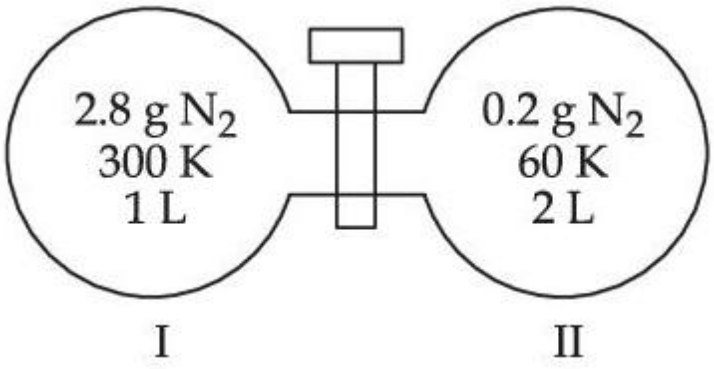
Step 3: Conclusion The complex has the two optical isomers Δ and Λ. The required dimensionless integer is $\boxed{2}$.

Quick Tip to Solve: For an octahedral complex with three identical symmetrical bidentate ligands, count the Delta and Lambda pair, not rotated versions of either form.

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83. Two flasks I and II shown below are connected by a valve of negligible volume.
When the valve is opened, the final pressure of the system in bar is x $\times$ 10^{-2}. The value of x is _____. (Integer answer)
[Assume - Ideal gas; 1 bar = 10^5 Pa; Molar mass of N_2 = 28.0 g mol^{-1}; R = 8.31 J mol^{-1} K^{-1}]

[4 marks]



Solution:

Concept:

- For an ideal gas, $PV = nRT$. The amount of nitrogen in each flask is its mass divided by the molar mass $28.0\,\mathrm{g\,mol^{-1}}$.
- For the combined rigid system with no external heat exchange, conservation of internal energy of the same ideal gas gives $n_f T_f = n_1 T_1 + n_2 T_2$. The final available volume is the sum of the two flask volumes.

Step 1: Find the amounts and the temperature-weighted total The diagram gives $2.8\,\mathrm{g}$ of nitrogen at $300\,\mathrm{K}$ in $1\,\mathrm{L}$, and $0.2\,\mathrm{g}$ at $60\,\mathrm{K}$ in $2\,\mathrm{L}$. Thus

$$n_1 = \frac{2.8}{28} = 0.100\,\mathrm{mol}, \qquad n_2 = \frac{0.2}{28} = \frac{1}{140}\,\mathrm{mol}.$$

Since the gas is the same in both flasks, its molar heat capacity cancels in the energy balance:

$$n_f T_f = 0.100(300) + \frac{60}{140} = \frac{213}{7}\,\mathrm{mol\,K}.$$

This corresponds to an equilibrium temperature of $284\,\mathrm{K}$.

Step 2: Apply the gas equation to the total volume The valve has negligible volume, so

$$V_f = (1+2)\,\mathrm{L} = 3.00 \times 10^{-3}\,\mathrm{m^3}.$$

Therefore

$$P_f = \frac{(n_f T_f)R}{V_f} = \frac{(213/7)(8.31)}{3.00 \times 10^{-3}} = 8.43 \times 10^4\,\mathrm{Pa}.$$

Converting with $1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$,

$$P_f = 0.843\,\mathrm{bar} = 84.3 \times 10^{-2}\,\mathrm{bar}.$$

Reporting the requested integer coefficient gives 84. One must combine amounts and energy, rather than average the two initial pressures without accounting for their volumes.

Step 3: Conclusion The final pressure is approximately $0.843\,\mathrm{bar}$. The requested integer coefficient is $\boxed{x = 84}$.

Quick Tip to Solve: For the same ideal gas in connected rigid flasks, use the energy-weighted sum of nT and divide by the combined volume; convert litres before using SI R.

84. 100 g of propane is completely reacted with 1000 g of oxygen. The mole fraction of carbon dioxide in the resulting mixture is x $\times$ 10^{-2}. The value of x is _____. (Nearest integer)
[Atomic weight : H = 1.008; C = 12.00; O = 16.00] [4 marks]

Solution:

Concept:

- Complete combustion of propane follows $C_3H_8 + 5\,O_2 \longrightarrow 3\,CO_2 + 4\,H_2O$. Mole fractions use the amount of a component divided by the total amount of all species in the resulting mixture.
- The limiting reactant determines the products. Excess oxygen must remain in the denominator together with the carbon dioxide and water formed.

Step 1: Determine the limiting reactant Using the supplied atomic weights,

$$M(C_3H_8) = 3(12.00) + 8(1.008) = 44.064\,\text{g mol}^{-1}.$$

Let $a = 100/44.064 = 2.2694\,\text{mol}$ be the propane amount. Initially $n(O_2) = 1000/32 = 31.25\,\text{mol}$. Complete reaction needs $5a = 11.347\,\text{mol}$ oxygen, less than the available amount. Propane is therefore limiting, and all of it reacts. The products contain $3a$ moles of carbon dioxide and $4a$ moles of water, along with $31.25 - 5a$ moles of unused oxygen.

Step 2: Calculate the product-mixture mole fraction The total amount in the resulting mixture is

$$n_{\text{total}} = 3a + 4a + (31.25 - 5a) = 31.25 + 2a.$$

Thus

$$y_{CO_2} = \frac{3a}{31.25 + 2a} = \frac{6.8083}{35.7889} \simeq 0.1902.$$

Water formed by combustion is included in the stated resulting mixture; neither the unused oxygen nor the water may be silently omitted from this calculation. Rewriting the result as 19.02×10^{-2} gives the requested coefficient $x \simeq 19.02$, whose nearest integer is 19. A mole fraction and its coefficient are dimensionless.

Step 3: Conclusion The carbon dioxide mole fraction is approximately 0.1902; therefore the nearest-integer coefficient is $\boxed{x = 19}$.

Quick Tip to Solve: After finding the limiting reactant, list every species left after reaction; excess oxygen contributes to the total moles just as the products do.

85. 40 g of glucose (Molar mass = 180) is mixed with 200 mL of water. The freezing point of solution is _____ K. (Nearest integer) [Given : K_f = 1.86 K kg mol^{-1}; Density of water = 1.00 g cm^{-3}; Freezing point of water = 273.15 K] [4 marks]

Solution:

Concept:

- For a nonvolatile solute, the freezing-point depression is $\Delta T_f = iK_f m$, where m is moles of solute per kilogram of solvent.

- Glucose is a molecular nonelectrolyte, so it does not dissociate in water and the van't Hoff factor is $i = 1$. The depression is subtracted from the freezing point of pure water.

Step 1: Calculate the amount of glucose The molar mass converts the given solute mass into moles. The amount of glucose is

$$n = \frac{40\,\text{g}}{180\,\text{g mol}^{-1}} = \frac{2}{9}\,\text{mol}.$$

This is the solute amount; the mass of the entire solution must not be used as the solvent mass.

Step 2: Convert the water volume to solvent mass Using the supplied water density, $200\,\text{mL} = 200\,\text{cm}^3$ has mass $200\,\text{g} = 0.200\,\text{kg}$. Therefore,

$$m = \frac{2/9}{0.200} = \frac{10}{9}\,\text{mol kg}^{-1}.$$

Molality uses the mass of water, so adding the glucose mass to the denominator would underestimate the depression.

Step 3: Find the lowered freezing point Substitute the molality and the given cryoscopic constant:

$$\Delta T_f = 1 \times 1.86 \times \frac{10}{9} = 2.0667\,\text{K}.$$

A temperature difference has the same numerical magnitude in kelvin and degrees Celsius. The absolute solution freezing point is

$$T_f = 273.15 - 2.0667 = 271.0833\,\text{K}.$$

Round the final absolute temperature, rather than prematurely rounding the molality or the depression.

Step 4: Conclusion The freezing point is $271.0833\,\text{K}$, giving the nearest-integer answer $\boxed{271\,\text{K}}$.

Quick Tip to Solve: For freezing-point problems, convert only the solvent mass to kilograms and subtract the depression before rounding the temperature.

86. The resistance of a conductivity cell with cell constant 1.14 cm^{-1}, containing 0.001 M KCl at 298 K is 1500 Ω. The molar conductivity of 0.001 M KCl solution at 298 K in S cm^2 mol^{-1} is _____. (Integer answer) [4 marks]

Solution:

Concept:

- The conductance of the cell is $G = 1/R$. Its conductivity is $\kappa = G(l/A)$, with the cell constant l/A measured in cm^{-1}.
- For concentration C in mol L^{-1}, molar conductivity is $\Lambda_m = 1000\kappa/C$, giving units $\text{S cm}^2\,\text{mol}^{-1}$.

Step 1: Convert resistance into conductance The measured resistance belongs to the solution-filled

conductivity cell. Taking its reciprocal gives

$$G = \frac{1}{1500\,\Omega} = 6.6667 \times 10^{-4}\,\mathrm{S}.$$

Conductance is not yet conductivity because it depends on the cell dimensions. The given cell constant removes this geometrical dependence; it is multiplied by conductance, not by resistance.

Step 2: Calculate the conductivity Use the cell constant in centimetre-based units so the requested final units can be obtained directly:

$$\kappa = \frac{1.14\,\mathrm{cm}^{-1}}{1500\,\Omega} = 7.60 \times 10^{-4}\,\mathrm{S\,cm}^{-1}.$$

The temperature is already specified as $298\,\mathrm{K}$; no temperature correction or limiting-conductivity extrapolation is needed.

Step 3: Calculate conductivity per mole The concentration is $0.001\,\mathrm{mol\,L}^{-1} = 10^{-6}\,\mathrm{mol\,cm}^{-3}$, since one litre contains $1000\,\mathrm{cm}^3$. Hence

$$\Lambda_m = \frac{7.60 \times 10^{-4}}{10^{-6}} = 760\,\mathrm{S\,cm^2\,mol^{-1}}.$$

Equivalently, $\Lambda_m = 1000(7.60 \times 10^{-4})/0.001$. Dividing conductivity by concentration in moles per cubic centimetre leaves square centimetres per mole, as required.

Step 4: Conclusion The molar conductivity is $\boxed{760\,\mathrm{S\,cm^2\,mol^{-1}}}$; the integer entry is $\boxed{760}$.

Quick Tip to Solve: With conductivity in S per centimetre and molarity in moles per litre, retain the factor 1000 in the molar-conductivity formula.

87. The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1 mW and wavelength of 1000 nm, in 0.1 second is x $\times$ 10^{13}. The value of x is _____. (Nearest integer)

(h = 6.63 $\times$ 10^{-34} Js, c = 3.00 $\times$ 10^{8} ms^{-1}) [4 marks]

Solution:

Concept:

- Each monochromatic photon has energy $\varepsilon = hc/\lambda$, where h is Planck's constant, c is light speed and λ is wavelength.
- The energy delivered by constant power P in time t is $E = Pt$. Thus the photon number is $N = Pt\lambda/(hc)$, and the requested coefficient is $x = N/10^{13}$.

Step 1: Express all quantities in SI units The wavelength and power must be converted before substitution:

$$P = 1\,\mathrm{mW} = 10^{-3}\,\mathrm{W}, \qquad \lambda = 1000\,\mathrm{nm} = 10^{-6}\,\mathrm{m}.$$

The given time is $t = 0.1\,\mathrm{s}$. Since one watt is one joule per second, the energy emitted during this

interval is

$$E = (10^{-3})(0.1) = 10^{-4}\,\text{J}.$$

This is the total pulse energy, not the energy of one photon.

Step 2: Calculate the energy of a single photon Using the supplied constants without changing their precision,

$$\varepsilon = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{10^{-6}} = 1.989 \times 10^{-19}\,\text{J}.$$

A longer wavelength gives a smaller photon energy, so a millijoule-scale source can emit a very large number of infrared photons.

Step 3: Divide the total energy by photon energy All photons have the same frequency, allowing the total energy to be divided by the energy per photon:

$$N = \frac{10^{-4}}{1.989 \times 10^{-19}} = 5.02765 \times 10^{14}.$$

Match the exact power of ten requested in the question:

$$N = 50.2765 \times 10^{13}, \qquad x = 50.2765.$$

The coefficient must be rounded only after writing the count with 10^{13}, rather than rounding the coefficient of 10^{14}.

Step 4: Conclusion The nearest integer is $\boxed{x = 50}$. The photon count is approximately 5.03×10^{14} photons; x itself is dimensionless.

Quick Tip to Solve: After computing the photon count, rewrite it using the exponent printed in the question before rounding its coefficient.

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88. When 5.1 g of solid NH_4HS is introduced into a two litre evacuated flask at 27°C, 20% of the solid decomposes into gaseous ammonia and hydrogen sulphide. The K_p for the reaction at 27°C is x $\times$ 10^{-2}. The value of x is _____. (Integer answer) [Given R = 0.082 L atm K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- For $\text{NH}_4\text{HS}\,(\text{s}) \rightleftharpoons \text{NH}_3(\text{g}) + \text{H}_2\text{S}\,(\text{g})$, the activity of the pure solid is one, so $K_p = p_{\text{NH}_3} p_{\text{H}_2\text{S}}$.
- Each mole of decomposed solid produces one mole of each gas. The partial pressure of either gas is $p = nRT/V$; total gas moles must not be used for each partial pressure.

Step 1: Find the amount that decomposes The molar mass of NH_4HS is $14 + 5(1) + 32 = 51\,\text{g mol}^{-1}$. Thus the initial amount and decomposed amount are

$$n_0 = \frac{5.1}{51} = 0.100\,\text{mol}, \qquad \xi = 0.20(0.100) = 0.0200\,\text{mol}.$$

The remaining solid is $0.0800\,\text{mol}$, so a pure solid phase is still present at equilibrium. Its amount is

not inserted into the equilibrium-constant expression.

Step 2: Calculate each gas partial pressure The flask initially contains no gas. Reaction stoichiometry gives $0.0200\,\text{mol}$ each of ammonia and hydrogen sulphide at equilibrium. With $T = 27 + 273 = 300\,\text{K}$ and $V = 2.00\,\text{L}$,

$$p_{NH_3} = p_{H_2S} = \frac{(0.0200)(0.082)(300)}{2.00} = 0.246\,\text{atm}.$$

Their sum is $0.492\,\text{atm}$, but this total pressure is not the partial pressure of either component.

Step 3: Form the equilibrium constant Multiply the two equilibrium partial pressures:

$$K_p = (0.246)(0.246) = 0.060516 = 6.0516 \times 10^{-2}.$$

Using pressures relative to the standard pressure makes the thermodynamic equilibrium constant dimensionless. In the conventional pressure-product notation the numerical value here comes from pressures in atmospheres. The requested integer coefficient is therefore approximately six.

Step 4: Conclusion $K_p \approx 6.05 \times 10^{-2}$, so the integer answer is $\boxed{x = 6}$ (dimensionless coefficient).

Quick Tip to Solve: For this 1:1 gas-producing decomposition, each partial pressure is half the total pressure, so use its square rather than the square of total pressure.

89. The number of species having non-pyramidal shape among the following is _____.

(A) SO_3

(B) NO_3^-

(C) PCl_3

(D) CO_3^{2-}

[4 marks]

Solution:

Concept:

- VSEPR theory counts electron-density regions around the central atom. A multiple bond counts as one bonding region when determining the arrangement of the bonded atoms.
- Three bonding regions with no central lone pair give a trigonal planar shape. Three bonds plus one lone pair give a tetrahedral electron-pair arrangement but a trigonal pyramidal molecular shape.

Step 1: Identify the geometry of the three oxygen species In SO_3, sulphur has three bonding directions to oxygen and no central lone pair in its usual molecular description. The molecule is trigonal planar. In NO_3^-, nitrogen also has three bonding regions and no lone pair; resonance distributes the double-bond character over the three N–O bonds. It is trigonal planar. In CO_3^{2-}, carbon has three bonding regions and no lone pair, again giving a trigonal planar ion. Resonance changes bond orders and lengths but does not introduce a fourth direction or turn these ions into pyramids.

Step 2: Check the phosphorus compound separately Phosphorus has five valence electrons. In PCl_3, it forms three single bonds to chlorine, retaining one lone pair. There are therefore four electron regions, denoted AX_3E. Their electron-pair arrangement is tetrahedral, while the positions of the three chlorine atoms give a trigonal pyramidal molecular shape. Calling it tetrahedral without

distinguishing electron geometry from molecular shape would misclassify the species requested here.

Step 3: Count only non-pyramidal molecular shapes The first, second and fourth species are planar and therefore non-pyramidal. The third species, PCl_3, is pyramidal and is excluded. Thus

$$N_{\text{non-pyramidal}} = 1 + 1 + 0 + 1 = 3.$$

The negative charge on nitrate or carbonate does not by itself imply a lone pair on the central atom; the electron distribution and bonded directions decide the shape.

Step 4: Conclusion The non-pyramidal species are SO_3, NO_3^- and CO_3^{2-}. Their number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Distinguish the electron-pair arrangement from molecular shape: an AX3E centre is pyramidal, whereas AX3 centres without a lone pair are planar.

90. Data given for the following reaction is as follows :

$FeO_{(s)} + C_{(graphite)} \rightarrow Fe_{(s)} + CO_{(g)}$

Substance	ΔH°(kJ mol^{-1})	ΔS°(J mol^{-1} K^{-1})
$FeO_{(s)}$	-266.3	**57.49**
$C_{(graphite)}$	**0**	**5.74**
$Fe_{(s)}$	**0**	**27.28**
$CO_{(g)}$	-110.5	**197.6**

The minimum temperature in K at which the reaction becomes spontaneous is _____. (Integer answer) [4 marks]

Solution:

Concept:

- For a reaction, $\Delta_r H^\circ$ and $\Delta_r S^\circ$ equal the stoichiometric sums for products minus those for reactants.
- The standard Gibbs energy is $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$. For positive enthalpy and entropy changes, spontaneity begins above $T_0 = \Delta_r H^\circ / \Delta_r S^\circ$.

Step 1: Calculate the reaction enthalpy For $\mathrm{FeO(s) + C(graphite) \longrightarrow Fe(s) + CO(g)}$, every stoichiometric coefficient is one. Using the supplied enthalpy data,

$$\Delta_r H^\circ = [0 + (-110.5)] - [(-266.3) + 0] = 155.8\,\mathrm{kJ\,mol^{-1}}.$$

The positive value shows that the forward reaction is endothermic. Elemental iron and graphite have zero standard formation enthalpy, as listed.

Step 2: Calculate the reaction entropy Apply the same products-minus-reactants rule to the entropy

column:

$$\begin{aligned}\Delta_r S^\circ &= (27.28 + 197.6) - (57.49 + 5.74)\\ &= 224.88 - 63.23\\ &= 161.65\,\mathrm{J\,mol^{-1}\,K^{-1}}.\end{aligned}$$

The increase is consistent with forming a gaseous product from solid reactants. The elemental entropies are not zero: their actual values in the table must be retained.

Step 3: Find the threshold for negative Gibbs energy Convert enthalpy to joules to match the entropy units. At the boundary of spontaneity,

$$0 = 155800 - T_0(161.65), \qquad T_0 = \frac{155800}{161.65} = 963.81\,\mathrm{K}.$$

Because the entropy change is positive, $\Delta_r G^\circ < 0$ for $T > 963.81\,\mathrm{K}$, within the constant-data approximation used here. Exactly at the threshold the reaction is at equilibrium, not spontaneously forward. The smallest integer temperature above this boundary is $964\,\mathrm{K}$.

Step 4: Conclusion The reaction becomes spontaneous above approximately $963.81\,\mathrm{K}$; the requested integer temperature is $\boxed{964\,\mathrm{K}}$.

Quick Tip to Solve: Use products minus reactants for both columns, then express enthalpy and entropy in consistent energy units before taking their ratio.