JEE Main 2021 Paper 1 (B.E./B.Tech) Aug 31 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 4 (Aug-Sep), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 31 August 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The number of real roots of the equation

$e^{4x} + 2e^{3x} - e^x - 6 = 0$ **is :** [4 marks]

(A) 2

(B) 4

(C) 1

(D) 0

Correct Answer: (C) 1

Solution:

Concept:

- The substitution $t = e^x$ gives $t > 0$. Since the exponential function is one-to-one from the real numbers onto the positive numbers, each positive root in t gives exactly one real root in x.
- A continuous strictly increasing function and a strictly decreasing function can meet at most once. A change of sign supplies existence of a root by the intermediate value theorem.

Step 1: Replace the exponential by a positive variable Put $t = e^x$. The equation becomes

$$t^4 + 2t^3 - t - 6 = 0.$$

We must count only positive roots: zero and negative values cannot equal e^x. Factoring the first two terms and rearranging gives

$$t^3(t+2) = t + 6, \qquad t^3 = \frac{t+6}{t+2}.$$

Division is valid because $t + 2 > 0$.

Step 2: Prove uniqueness and check existence For $t > 0$, the left side t^3 is strictly increasing, whereas

$$\frac{t+6}{t+2} = 1 + \frac{4}{t+2}$$

is strictly decreasing. Hence there can be at most one intersection. For $P(t) = t^4 + 2t^3 - t - 6$, we have

$$P(1) = -4, \qquad P(2) = 24.$$

Continuity therefore gives a root between 1 and 2. Together these facts prove exactly one positive root, so neither zero roots nor two or four roots is possible. The degree of the polynomial alone does not count the allowable exponential roots.

Step 3: Conclusion There is exactly 1 real root of the given equation. Hence, the correct option is (C).

Quick Tip to Solve: After substituting $e^x = t$, restrict to $t > 0$; compare an increasing and a decreasing side to count roots without finding them.

2. Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If

$$\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt, \qquad 0 \le x \le 1$$

and f(0) = 0, then

$$\lim_{x\to 0} \frac{1}{x^2}\int_0^x f(t)\,dt$$

: [4 marks]

(A) equals 0

(B) equals 1

(C) does not exist

(D) equals $\frac{1}{2}$

Correct Answer: (D) equals $\frac{1}{2}$

Solution:

Concept:

- Differentiating the integral identity at interior points gives $\sqrt{1-f\prime(x)^2} = f(x)$, hence $f(x)^2 + f\prime(x)^2 = 1$.
- Twice differentiability on $(0,1)$ does not imply right continuity at the endpoint 0. A prescribed isolated value $f(0)$ cannot determine an integral or an interior solution without endpoint regularity.

Step 1: Check what the literal hypotheses permit The printed problem does not state right continuity at zero. Consider

$$f(0) = 0, \qquad f(x) = 1 \quad (0 < x \le 1).$$

This function is non-negative and twice differentiable throughout $(0,1)$, with $f\prime(x) = 0$. Both integrals in the question equal x: a change at the single endpoint does not change an integral. The derivative integrand at that endpoint may likewise be assigned any value for integration. However,

$$\frac{1}{x^2}\int_0^x f(t)\,dt = \frac{1}{x} \longrightarrow +\infty.$$

Thus the initial value alone does not justify saying that the interior function starts continuously from zero.

Step 2: Derive the conventional answer with the required extra condition If right continuity at 0 is additionally imposed, $f(x) \to 0$. The interior identity then implies $|f\prime(x)| \to 1$. Near zero the derivative cannot vanish or change sign, and its sign must be positive because $f \ge 0$ starts continuously from zero. Separating variables therefore gives

$$\sin^{-1} f(x) = x, \qquad f(x) = \sin x$$

in a right neighbourhood of zero. Consequently

$$\lim_{x\to 0^+} \frac{1}{x^2}\int_0^x f(t)\,dt = \lim_{x\to 0^+} \frac{1-\cos x}{x^2} = \frac{1}{2}.$$

This is the conventional result and corresponds to D. The admissible discontinuous example above proves that this value is not forced by the literal hypotheses. Nor can C describe all admissible functions, since the continuous example $f(x) = \sin x$ has the finite limit $1/2$.

Step 3: Conclusion The literal hypotheses are underdetermined because right continuity at 0 is omitted. With right continuity at 0, the limit is $\frac{1}{2}$, the conventional answer (D).

Quick Tip to Solve: Before using an endpoint initial value in a differential equation, check that continuity at the endpoint connects it to the interior solution.

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3. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $\left|2\vec{a} + 3\vec{b}\right| = \left|3\vec{a} + \vec{b}\right|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $\left|\vec{b}\right|$ is equal to : [4 marks]

(A) 4

(B) 6

(C) 5

(D) 8

Correct Answer: (C) 5

Solution:

Concept:

- The squared magnitude expands by the dot product: $|p\vec{a} + q\vec{b}|^2 = p^2|\vec{a}|^2 + q^2|\vec{b}|^2 + 2pq\vec{a} \cdot \vec{b}$.
- For angle 60°, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|/2$. A vector of unit magnitude has norm one; scalar multiplication scales its norm by the absolute value of the scalar.

Step 1: Use the unit-vector condition Since $\vec{a}/8$ is a unit vector,

$$\left|\frac{1}{8}\vec{a}\right| = 1 \quad \Longrightarrow \quad |\vec{a}| = 8.$$

Let $b = |\vec{b}|$, which must be non-negative. Using the given angle,

$$\vec{a} \cdot \vec{b} = 8b\cos 60^\circ = 4b.$$

This dot product is positive; replacing it by zero would incorrectly treat the vectors as perpendicular.

Step 2: Expand the equal magnitudes and solve Square both sides of the given magnitude equality. Both sides are non-negative, so this operation is equivalent to the original condition:

$$4|\vec{a}|^2 + 9b^2 + 12\vec{a} \cdot \vec{b} = 9|\vec{a}|^2 + b^2 + 6\vec{a} \cdot \vec{b}.$$

Substitute $|\vec{a}| = 8$ and $\vec{a} \cdot \vec{b} = 4b$:

$$8b^2 + 24b - 320 = 0, \qquad b^2 + 3b - 40 = 0, \qquad (b+8)(b-5) = 0.$$

The algebraic roots are -8 and 5. A magnitude cannot be negative, so retain $b = 5$. The other offered positive magnitudes 4, 6 and 8 do not satisfy the quadratic. For example, the necessary relation $8b^2 + 24b = 320$ holds at 5 and fails at each of those values.

Step 3: Conclusion $\left|\vec{b}\right| = 5$. Hence, the correct option is (C).

Quick Tip to Solve: Expand squared vector norms before substituting the angle; reject negative roots when the unknown is a magnitude.

4. The function

$f(x) = \left|x^2 - 2x - 3\right| \cdot e^{\left|9x^2 - 12x + 4\right|}$ **is not differentiable at exactly :** [4 marks]

(A) four points

(B) three points

(C) two points

(D) one point

Correct Answer: (C) two points

Solution:

Concept:

- An absolute value of a smooth function can have a corner at a simple zero of its argument. A zero without a sign change need not create any failure of differentiability.
- The factor $e^{h(x)}$ is smooth whenever h is smooth and is always strictly positive. Multiplication by this nonzero factor cannot remove the corner of another factor.

Step 1: Simplify the exponential factor first The expression inside the exponential absolute value is a perfect square:

$$9x^2 - 12x + 4 = (3x - 2)^2 \geq 0.$$

Therefore $e^{|9x^2 - 12x + 4|} = e^{(3x-2)^2}$ is differentiable for every real x, including $x = 2/3$. Although the polynomial vanishes there, its absolute value equals the polynomial on both sides. Counting every zero of an expression inside absolute-value signs would give an incorrect extra point.

Step 2: Check the simple zeros of the remaining factor Write

$$g(x) = x^2 - 2x - 3 = (x - 3)(x + 1).$$

Its zeros are -1 and 3. At these points $g\prime(x) = 2x - 2$ equals -4 and 4, respectively, so both zeros are simple. The one-sided derivatives of $|g|$ are opposite nonzero numbers at each zero. If $h(x) = e^{(3x-2)^2}$, the one-sided derivatives of $h|g|$ at a zero are h times those slopes: the term involving $h\prime|g|$ vanishes there. Since $h > 0$, the slopes remain unequal. Everywhere else, g has constant sign locally and the product is smooth. Thus there are precisely two failures, rather than one, three or four.

Step 3: Conclusion The function is not differentiable exactly at $x = -1$ and $x = 3$, hence at two points. Hence, the correct option is (C).

Quick Tip to Solve: Factor the arguments of absolute values first: a perfect square stays smooth, while a simple zero produces a corner.

5. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is 3 r^2, then $r^2 - d$ is equal to : [4 marks]

(A) $7 - 7\sqrt{3}$

(B) $7 + \sqrt{3}$

(C) $7 - \sqrt{3}$

(D) $7 + 3\sqrt{3}$

Correct Answer: (B) $7 + \sqrt{3}$

Solution:

Concept:

- Three successive GP terms may be written a, ar, ar^2. Their fourth term is ar^3, so a prescribed fourth term connects the first term and common ratio.
- Three numbers form an AP precisely when twice their middle term equals the sum of the outer terms. The common difference is the second term minus the first.

Step 1: Translate the progression conditions The original terms are a, ar, ar^2, and after doubling the middle term they are $a, 2ar, ar^2$. The fourth-term condition gives

$$ar^3 = 3r^2 \quad \Longrightarrow \quad ar = 3.$$

Here $r \neq 0$, since an increasing GP cannot have both its second and third terms zero. The AP condition is

$$4ar = a + ar^2 \quad \Longrightarrow \quad r^2 - 4r + 1 = 0.$$

The first term is nonzero because $ar = 3$, so cancellation of a is also valid.

Step 2: Select the increasing ratio and calculate the difference The ratio roots are

$$r = 2 \pm \sqrt{3}.$$

Both are positive, and $a = 3/r > 0$. Increasing positive terms require $r > 1$, which selects $r = 2 + \sqrt{3}$. The smaller root would give decreasing terms. Next,

$$a = \frac{3}{r} = 3(2 - \sqrt{3}), \qquad d = 2ar - a = 6 - a = 3\sqrt{3}.$$

Consequently

$$r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3} = 7 + 4\sqrt{3} - 3\sqrt{3} = 7 + \sqrt{3}.$$

The coefficients in the other options do not result from this subtraction. In particular, d belongs to the modified AP, not to differences between the original GP terms; using the latter would change the result.

Step 3: Conclusion $r^2 - d = 7 + \sqrt{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the fourth GP term to get ar first, and apply the AP condition only after doubling the middle term.

6. Which of the following is not correct for relation R on the set of real numbers ? [4 marks]

(A) (x, y) $\in$ R $\Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.

(B) (x, y) $\in$ R $\Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.

(C) (x, y) $\in$ R $\Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.

(D) (x, y) $\in$ R $\Leftrightarrow |x - y| \leq 1$ is reflexive nd symmetric.

Correct Answer: (B) (x, y) $\in$ R $\Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.

Solution:

Concept:

- A relation is reflexive when every (x, x) belongs to it, symmetric when xRy implies yRx, and transitive when xRy and yRz imply xRz.
- A single counterexample disproves symmetry or transitivity. Absolute distance $|x - y|$ is unchanged by interchanging its arguments, whereas $|x| - |y|$ changes sign.

Step 1: Test the relation in option B For $0 < |x - y| \leq 1$, symmetry holds because $|x - y| = |y - x|$. Transitivity, however, fails. Take

$$x = 0, \qquad y = \frac{3}{4}, \qquad z = \frac{3}{2}.$$

Then $|x - y| = |y - z| = 3/4$, so both xRy and yRz hold, while $|x - z| = 3/2 > 1$, so xRz does not hold. Thus the assertion that this relation is symmetric and transitive is false.

Step 2: Verify that the other descriptions are correct In A, choose $x = 3/4, y = 0$: the forward difference of magnitudes lies in $(0, 1]$, but the reversed difference is negative, disproving symmetry. For transitivity take $x = 3/2, y = 3/4, z = 0$. Each successive magnitude difference is $3/4$, but the overall difference is $3/2$, so transitivity also fails. Hence A is a correct description.

In C, setting $y = x$ gives zero, so the relation is reflexive. It is not symmetric: $x = 0, y = 2$ satisfies $|x| - |y| = -2 \leq 1$, whereas reversing them gives $2 > 1$. Finally D is reflexive because $|x - x| = 0 \leq 1$, and symmetric by the symmetry of absolute distance. Therefore A, C and D state true properties; the requested incorrect statement is B.

Step 3: Conclusion Option (B) is not correct: its relation is symmetric but not transitive. Hence, the correct option is (B).

Quick Tip to Solve: For a distance bounded by 1, test three equally spaced points: two allowed short steps can combine into a disallowed longer step.

7. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}\,dx$ is equal to : (where C is a constant of integration) [4 marks]

(A) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$

(B) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$

(C) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

(D) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

Correct Answer: (C) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

Solution:

Concept:

- When the powers of two linear factors have total exponent two in the denominator, a substitution using their ratio often removes the remaining rational factor.
- For $u=(x-1)/(x+2)$, differentiation gives $du=3\,dx/(x+2)^2$. The fourth root requires a positive radicand, so the real domain here is $x>1$ or $x<-2$.

Step 1: Introduce the ratio of the linear factors On either interval of the real domain, set

$$u=\frac{x-1}{x+2}>0, \qquad du=\frac{3}{(x+2)^2}\,dx.$$

Since $x-1=u(x+2)$, the expression under the fourth root becomes

$$(x-1)^3(x+2)^5=u^3(x+2)^8.$$

Its positive fourth root is $u^{3/4}(x+2)^2$. The square is non-negative even when $x<-2$, so this simplification respects the real fourth-root convention on both intervals. The endpoints -2 and 1 are excluded because the denominator vanishes.

Step 2: Integrate the power and verify the result The integral reduces to

$$I=\int \frac{dx}{u^{3/4}(x+2)^2}=\frac{1}{3}\int u^{-3/4}\,du=\frac{4}{3}u^{1/4}+C.$$

Substitution back gives

$$I=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C.$$

As a direct check, differentiating this expression yields

$$\frac{4}{3}\cdot\frac{1}{4}u^{-3/4}\cdot\frac{3}{(x+2)^2}=\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}.$$

The inverse-ratio options A and B differentiate with the wrong sign as well as different powers, while D has power $5/4$ instead of $1/4$ and produces an extra factor of u. Thus only C matches the integrand. The arbitrary integration constant may be chosen separately on disconnected domain intervals.

Step 3: Conclusion The integral is $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$. Hence, the correct option is (C).

Quick Tip to Solve: Use $(x-1)/(x+2)$ as the substitution and rewrite the full fourth root before applying the power rule.

8. If p and q are the lengths of the perpendiculars from the origin on the lines,
x cosec α − y sec α = k cot 2α and
x sinα + y cosα = k sin2α
respectively, then k^2 is equal to : [4 marks]

(A) $4p^2 + q^2$
(B) $2p^2 + q^2$
(C) $p^2 + 2q^2$
(D) $p^2 + 4q^2$

Correct Answer: (A) $4p^2 + q^2$

Solution:

Concept:

- The perpendicular distance from the origin to $Ax+By=C$ is $|C|/\sqrt{A^2+B^2}$. Squaring removes the absolute value without changing the distance.
- Use $\sin^2\theta+\cos^2\theta=1$ to eliminate the angle after expressing the two squared distances.

Step 1: Find the first squared distance Here the coefficients of the first line are $\csc\alpha$ and $-\sec\alpha$. Thus

$$p^2=\frac{k^2\cot^2 2\alpha}{\csc^2\alpha+\sec^2\alpha}.$$

The denominator simplifies to

$$\csc^2\alpha+\sec^2\alpha=\frac{1}{\sin^2\alpha\cos^2\alpha}=\frac{4}{\sin^2 2\alpha}.$$

Consequently $4p^2=k^2\cos^2 2\alpha$. The stated lines require their trigonometric coefficients to be defined; within that domain all these simplifications are valid.

Step 2: Find the second distance and eliminate the angle For the second line the squared coefficient norm is $\sin^2\alpha+\cos^2\alpha=1$, giving

$$q^2=k^2\sin^2 2\alpha.$$

Add this relation to the first one:

$$4p^2+q^2=k^2(\cos^2 2\alpha+\sin^2 2\alpha)=k^2.$$

The other choices attach the wrong coefficient to one of the squared distances and do not eliminate the angle identically.

Step 3: Conclusion $\boxed{k^2 = 4p^2 + q^2}$. Hence, the correct option is (A).

Quick Tip to Solve: Square each distance first; the first normal has squared length $4/\sin^2 2\alpha$, while the second has length one.

9. cosec18° is a root of the equation : [4 marks]

(A) $x^2 + 2x - 4 = 0$

(B) $4x^2 + 2x - 1 = 0$

(C) $x^2 - 2x + 4 = 0$

(D) $x^2 - 2x - 4 = 0$

Correct Answer: (D) $x^2 - 2x - 4 = 0$

Solution:

Concept:

- The exact value $\sin 18^\circ = (\sqrt{5} - 1)/4$ gives the reciprocal directly. Rationalising a surd allows its quadratic equation to be formed.
- If $x = 1 + \sqrt{5}$, isolating the radical and squaring produces a polynomial satisfied by x; substituting verifies the chosen root.

Step 1: Evaluate the reciprocal Using the standard special-angle value,

$$x = \csc 18^\circ = \frac{4}{\sqrt{5} - 1} = \frac{4(\sqrt{5} + 1)}{5 - 1} = 1 + \sqrt{5}.$$

This is positive and greater than three, consistent with the reciprocal of a small positive sine. The sign is important: replacing the reciprocal with $\sin 18^\circ$ would instead produce the polynomial involving $4x^2$ seen among the distractors.

Step 2: Form and test the equation Isolate the radical and square the equality:

$$x - 1 = \sqrt{5}, \qquad (x - 1)^2 = 5,$$

$$x^2 - 2x - 4 = 0.$$

For a direct check, $x^2 = 6 + 2\sqrt{5}$, so $x^2 - 2x - 4 = 0$. Option A has value $4x$, which is positive. Option C has value 8, rather than zero. Option B is positive for this $x > 3$, so it cannot vanish. Squaring also allows the conjugate root $1 - \sqrt{5}$, but the question asks only for an equation having the specified number as a root, not for a unique real root.

Step 3: Conclusion $\csc 18^\circ$ satisfies $\boxed{x^2 - 2x - 4 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: Rationalise $4/(\sqrt{5} - 1)$ before forming the quadratic; its value is $1 + \sqrt{5}$, not $\sqrt{5} - 1$.

10. If the following system of linear equations

2x + y + z = 5

x − y + z = 3

x + y + az = b

has no solution, then : [4 marks]

(A) $a = -\frac{1}{3}, b \neq \frac{7}{3}$

(B) $a \neq \frac{1}{3}, b = \frac{7}{3}$

(C) $a \neq -\frac{1}{3}, b = \frac{7}{3}$

(D) $a = \frac{1}{3}, b \neq \frac{7}{3}$

Correct Answer: (D) $a = \frac{1}{3}, b \neq \frac{7}{3}$

Solution:

Concept:

- A system has no solution when elimination produces a false constant equation, such as $0 = c$ with $c \neq 0$. A zero determinant alone does not distinguish inconsistency from infinitely many solutions.
- Use the first two independent equations to express x, y in terms of z, and impose the third equation.

Step 1: Express the first two variables in terms of the third Adding the first two equations eliminates y:

$$3x + 2z = 8, \qquad x = \frac{8 - 2z}{3}.$$

From $x - y + z = 3$,

$$y = x + z - 3 = \frac{z - 1}{3}.$$

These expressions satisfy both initial equations for every real z, so they represent the whole common line of those two planes. No restriction on a or b has yet been used.

Step 2: Apply the final equation and identify inconsistency Substitute into $x + y + az = b$:

$$\frac{8 - 2z + z - 1}{3} + az = b,$$

$$\left(a - \frac{1}{3}\right) z = b - \frac{7}{3}.$$

If $a \neq 1/3$, this determines exactly one z and hence a solution. When $a = 1/3$, consistency requires $b = 7/3$; that case gives infinitely many solutions. Therefore no solution requires $a = 1/3$ and $b \neq 7/3$. In particular $a = -1/3$ gives a nonzero coefficient and cannot force inconsistency. This rules out A and the conditions in B and C.

Step 3: Conclusion The required condition is $\boxed{a = \frac{1}{3},\ b \neq \frac{7}{3}}$. Hence, the correct option is (D).

Quick Tip to Solve: After elimination, inspect both the remaining coefficient and the constant: a zero coefficient with a nonzero constant signals no solution.

11. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x-axis at a distance R and S (> R) respectively from the origin, is : [4 marks]

(A) 4(S + R)

(B) 2(S − R)

(C) 4(S − R)

(D) 2(S + R)

Correct Answer: (C) 4(S − R)

Solution:

Concept:

- For the horizontal parabola $(y-k)^2 = 4a(x-h)$, the vertex is (h,k), the focus is $(h+a,k)$, and the latus rectum has length $4|a|$.
- Translation changes the vertex position but preserves lengths. The focal distance is the separation of the focus and vertex, not either distance from the origin.

Step 1: Identify the focal parameter Both points lie on the positive x-axis, so the given vertex and focus are

$$V = (R, 0), \qquad F = (S, 0), \qquad S > R.$$

The axis is horizontal and the focus lies to the right of the vertex. Hence the parabola opens towards increasing x, and its positive focal parameter is

$$a = VF = S - R.$$

It is the difference of these coordinates that matters; adding them would use the origin as though it were the vertex.

Step 2: Find the latus rectum length The translated equation is

$$y^2 = 4(S-R)(x-R).$$

The latus rectum is the chord through the focus perpendicular to the axis, so put $x = S$. Then

$$y^2 = 4(S-R)^2, \qquad y = \pm 2(S-R).$$

Its endpoints are therefore separated by $2(S-R) - [-2(S-R)] = 4(S-R)$. Option B gives only half the full chord. Options A and D use the sum $S + R$, which incorrectly makes the answer depend on the position of the origin. The result is positive because $S > R$.

Step 3: Conclusion The length is $\boxed{4(S-R)}$. Hence, the correct option is (C).

Quick Tip to Solve: First subtract the vertex coordinate from the focus coordinate to obtain a; multiply that distance by four for the full latus rectum.

12. If the function

$$f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , \quad x<0\\ k & , \quad x=0\\ \frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , \quad x>0\end{cases}$$

is continuous at x = 0, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to : [4 marks]

(A) -5

(B) 5

(C) -4

(D) 4

Correct Answer: (A) -5

Solution:

Concept:

- Continuity at zero requires the left limit, the right limit and the assigned value k to be equal.
- Use $\lim_{t\to 0}\log(1+t)/t=1$, $\lim_{x\to 0}\sin x/x=1$, and rationalisation of the square-root denominator.

Step 1: Evaluate the right-hand limit Use $\cos^2 x-\sin^2 x-1=-2\sin^2 x$ and rationalise:

$$\frac{-2\sin^2 x}{\sqrt{1+x^2}-1}=-2\left(\frac{\sin x}{x}\right)^2(\sqrt{1+x^2}+1).$$

As $x\to 0^+$, the two factors tend to one and two respectively. Thus the right limit is -4. Continuity forces $k=-4$, which is nonzero, so the requested reciprocal of k is defined.

Step 2: Evaluate the left-hand limit and combine Near zero the logarithm arguments are positive. Split the logarithm into a difference:

$$\lim_{x\to 0^-}\frac{\log(1+x/a)-\log(1-x/b)}{x}=\frac{1}{a}+\frac{1}{b}.$$

Here a, b must be nonzero for the given expression to be defined. Continuity equates this limit to $k=-4$. Therefore

$$\frac{1}{a}+\frac{1}{b}+\frac{4}{k}=-4+\frac{4}{-4}=-5.$$

The values -4 and 4 omit or mishandle the reciprocal term, while 5 has the wrong sign. Keeping the negative numerator in the right-hand branch is essential.

Step 3: Conclusion The value of the expression is $\boxed{-5}$. Hence, the correct option is (A).

Quick Tip to Solve: Obtain k from the right-hand branch first; continuity immediately supplies the sum of reciprocals without finding a and b separately.

13. If $\frac{dy}{dx} = \frac{2^{x+y}-2^x}{2^y}$, y(0) = 1, then y(1) is equal to : [4 marks]

(A) $\log_2(2 + e)$

(B) $\log_2(1 + e)$

(C) $\log_2(2e)$

(D) $\log_2(1 + e^2)$

Correct Answer: (B) $\log_2(1 + e)$

Solution:

Concept:

- The substitution $u = 2^y - 1$ converts the equation into a separable equation because $du/dx = (\log 2)2^y\, dy/dx$.
- The initial condition fixes the integration constant. Since $u(0) = 1$, the positive exponential solution applies, and taking $\log_2$ recovers y.

Step 1: Transform and integrate the equation Rewrite the given derivative as

$$\frac{dy}{dx} = 2^x \frac{2^y - 1}{2^y}.$$

With $u = 2^y - 1$, multiplication by $(\log 2)2^y$ gives

$$\frac{du}{dx} = (\log 2)2^x u.$$

On the solution through the initial point, u is positive. Separate and integrate:

$$\frac{du}{u} = (\log 2)2^x\, dx, \qquad \log u = 2^x + C.$$

The factor $\log 2$ cancels when integrating 2^x; it must not remain as an extra factor in the exponent.

Step 2: Use the initial condition and evaluate Since $y(0) = 1$, $u(0) = 2^1 - 1 = 1$. Therefore $0 = 1 + C$, giving $C = -1$ and

$$2^y - 1 = e^{2^x - 1}, \qquad y(x) = \log_2(1 + e^{2^x - 1}).$$

At $x = 1$, the exponent is $2 - 1 = 1$, so $y(1) = \log_2(1 + e)$. Differentiating the relation $2^y - 1 = e^{2^x-1}$ reproduces the given derivative and confirms the initial condition. Option A adds an extra one inside the logarithm; C replaces the necessary sum by a product; D fails to subtract the integration constant in the exponent.

Step 3: Conclusion $\boxed{y(1) = \log_2(1 + e)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a denominator 2^y and numerator containing $2^y - 1$, try $u = 2^y - 1$; apply the initial condition before substituting $x = 1$.

14. $\lim\limits_{x\to 0} \frac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ **is equal to :** [4 marks]

(A) π^2

(B) $2\pi^2$

(C) $4\pi^2$

(D) 4π

Correct Answer: (C) $4\pi^2$

Solution:

Concept:

- When a sine argument approaches π, write it as $\pi - t$ with $t \to 0$; then $\sin(\pi - t) = \sin t$.
- The identity $1 - \cos^4 x = \sin^2 x(1 + \cos^2 x)$ reduces the limit to the standard ratio $\sin x/x$.

Step 1: Convert the argument to a small angle As $x \to 0$, $\pi \cos^4 x \to \pi$, rather than zero. Set

$$t = \pi(1 - \cos^4 x).$$

Then $t \to 0$, and

$$\sin(\pi \cos^4 x) = \sin(\pi - t) = \sin t.$$

This substitution prevents the incorrect use of $\sin u \sim u$ for an argument tending to π. It is the small difference from π that controls the leading term.

Step 2: Evaluate the two standard-limit factors For nonzero x sufficiently close to zero,

$$\frac{\sin^2(\pi \cos^4 x)}{x^4} = \left(\frac{\sin t}{t}\right)^2 \left(\frac{\pi(1 - \cos^4 x)}{x^2}\right)^2.$$

Now

$$\frac{1 - \cos^4 x}{x^2} = \left(\frac{\sin x}{x}\right)^2 (1 + \cos^2 x) \longrightarrow 2.$$

The first factor tends to one and the second tends to $(2\pi)^2$, so the result is $4\pi^2$. Options A and B lose the factor two before squaring, and D fails to square the factor π. The squared form also confirms that the limit cannot be negative.

Step 3: Conclusion The limit is $\boxed{4\pi^2}$. Hence, the correct option is (C).

Quick Tip to Solve: Near $x = 0$, first use $1 - \cos^4 x \sim 2x^2$; the outer squared sine then gives $(2\pi x^2)^2$.

15. If $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9}$, r = 1, 2, 3, ..., i = $\sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to : [4 marks]

(A) $a_2a_6 - a_4a_8$

(B) a_9

(C) $a_1a_9 - a_3a_7$

(D) a_5

Correct Answer: (C) $a_1a_9 - a_3a_7$

Solution:

Concept:

- De Moivre's formula gives $a_r = \omega^r$, where $\omega = \cos(2\pi/9) + i\sin(2\pi/9)$ and $\omega^9 = 1$. Multiplication therefore adds the subscripts: $a_r a_s = \omega^{r+s}$.
- A determinant vanishes when one row is a scalar multiple of another. Use this dependence before expanding the nine entries individually.

Step 1: Express the rows as powers of one root The three rows are $(\omega, \omega^2, \omega^3)$, $(\omega^4, \omega^5, \omega^6)$, and $(\omega^7, \omega^8, \omega^9)$. Every entry of the second row equals the corresponding entry of the first row multiplied by ω^3. Likewise, the third row is ω^6 times the first. Thus the rows are dependent. Applying $R_2 \leftarrow R_2 - \omega^3 R_1$ produces a zero row, so the given determinant is zero.

Step 2: Identify the option that also vanishes In option (C), both products have the same total exponent:

$$a_1a_9 - a_3a_7 = \omega^{10} - \omega^{10} = 0.$$

Option (B) is $a_9 = 1$, and option (D) is $a_5 = \omega^5 \neq 0$. In option (A), the difference is $\omega^8 - \omega^{12} = \omega^8(1 - \omega^4) \neq 0$, since ω is a primitive ninth root of unity and $\omega^4 \neq 1$. Thus only (C) agrees with the determinant.

Step 3: Conclusion The determinant is $0 = a_1a_9 - a_3a_7$. Hence, the correct option is (C).

Quick Tip to Solve: For consecutive powers arranged row-wise in a determinant, first test whether successive rows have a common scalar ratio.

16. Let *, $\square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

[4 marks]

(A) $* = \vee, \square = \vee$

(B) $* = \wedge, \square = \wedge$

(C) $* = \wedge, \square = \vee$

(D) $* = \vee, \square = \wedge$

Correct Answer: (C) $* = \wedge, \square = \vee$

Solution:

Concept:

- An implication $A \Rightarrow B$ is false only when its antecedent A is true and its consequent B is false. A tautology must avoid this combination for all truth values of p, q.
- The symbols $\wedge$ and $\vee$ mean conjunction (both true) and disjunction (at least one true), respectively. The negation $\sim q$ reverses the truth value of q.

Step 1: Test the conjunction-to-disjunction choice For option (C), the implication is $(p \wedge \sim q) \Rightarrow (p \vee q)$. If its antecedent is true, then necessarily p is true and q is false. Its consequent $p \vee q$ is therefore true. For every other assignment, the antecedent is false, making the implication true automatically. Hence the expression is true for all four assignments and is a tautology.

Step 2: Exclude the remaining pairs with counterexamples Take p false and q false. For option (A), $p \vee \sim q$ is true but $p \vee q$ is false, so the implication fails. The same assignment makes option (D) fail because its consequent $p \wedge q$ is false. For option (B), take p true and q false: its antecedent $p \wedge \sim q$ is true whereas $p \wedge q$ is false. Thus each of the other three choices has an explicit false instance and cannot be a tautology.

Step 3: Conclusion The required operations are $* = \wedge$ and $\square = \vee$. Hence, the correct option is (C).

Quick Tip to Solve: To reject an implication as a tautology, find just one assignment with a true antecedent and a false consequent.

17. The sum of 10 terms of the series

$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$. **is :**

[4 marks]

(A) 1

(B) $\frac{120}{121}$

(C) $\frac{99}{100}$

(D) $\frac{143}{144}$

Correct Answer: (B) $\frac{120}{121}$

Solution:

Concept:

- The numerator in the general term is $2r + 1 = (r + 1)^2 - r^2$. Dividing by $r^2(r + 1)^2$ turns the term into a difference of reciprocal squares.

- A telescoping sum cancels intermediate terms. For ten terms starting at $r = 1$, the last surviving negative term uses $r + 1 = 11$, rather than 10 or 12.

Step 1: Find the general term and split it The displayed numerators are $3, 5, 7, \ldots$, with denominators $1^2 \cdot 2^2, 2^2 \cdot 3^2, 3^2 \cdot 4^2, \ldots$. Thus the term indexed by r is

$$T_r = \frac{2r+1}{r^2(r+1)^2} = \frac{(r+1)^2 - r^2}{r^2(r+1)^2} = \frac{1}{r^2} - \frac{1}{(r+1)^2}.$$

This identity is exact for every positive integer r.

Step 2: Cancel the intermediate reciprocal squares Adding the ten terms gives

$$\begin{aligned} S_{10} &= \left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{9}\right) + \cdots + \left(\frac{1}{100} - \frac{1}{121}\right) \\ &= 1 - \frac{1}{121} = \frac{120}{121}. \end{aligned}$$

Every internal reciprocal square occurs once positively and once negatively. Option (A) incorrectly discards the final remainder; $99/100$ corresponds to nine terms and $143/144$ to eleven terms. Keeping the upper index at ten therefore distinguishes the correct choice.

Step 3: Conclusion The sum of the first ten terms is $\frac{120}{121}$. Hence, the correct option is (B).

Quick Tip to Solve: After telescoping, explicitly write the last original term to avoid shifting the number of terms by one.

18. Let the equation of the plane, that passes through the point (1, 4, −3) and contains the line of intersection of the

planes $3x - 2y + 4z - 7 = 0$

and $x + 5y - 2z + 9 = 0$, be

$\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) -23

(B) -15

(C) 23

(D) 15

Correct Answer: (A) -23

Solution:

Concept:

- Every plane of the form $F_1 + \lambda F_2 = 0$ contains the intersection of $F_1 = 0$ and $F_2 = 0$. Determine λ by substituting the additional point.
- Plane equations may be multiplied by any nonzero constant. Normalize the final constant to the printed value $+3$ before reading α, β, γ.

Step 1: Use the point to choose the required family member Write $F_1 = 3x - 2y + 4z - 7$ and $F_2 = x + 5y - 2z + 9$. At $(1, 4, -3)$, their values are

$$F_1 = 3 - 8 - 12 - 7 = -24, \qquad F_2 = 1 + 20 + 6 + 9 = 36.$$

The required plane therefore obeys $-24 + 36\lambda = 0$, giving $\lambda = 2/3$. Multiplying the family equation by three, use $3F_1 + 2F_2 = 0$.

Step 2: Expand and normalize the equation Collecting coefficients gives

$$3F_1 + 2F_2 = 11x + 4y + 8z - 3 = 0.$$

The question instead fixes the constant term as $+3$, so multiply the entire equation by -1:

$$-11x - 4y - 8z + 3 = 0.$$

Hence $\alpha = -11$, $\beta = -4$, $\gamma = -8$, and their sum is -23. Substitution of the point checks the result: $-11 - 16 + 24 + 3 = 0$. The positive value 23 would come from ignoring the normalization sign; neither 15 nor -15 matches the correctly collected coefficients.

Step 3: Conclusion $\alpha + \beta + \gamma = -23$. Hence, the correct option is (A).

Quick Tip to Solve: When a plane equation specifies its constant term, scale every coefficient to that constant before taking any requested sum.

19. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is : [4 marks]

(A) $12\sqrt{15}$

(B) $12\sqrt{10}$

(C) $8\sqrt{10}$

(D) $6\sqrt{10}$

Correct Answer: (B) $12\sqrt{10}$

Solution:

Concept:

- For a right triangle, the tangent of the elevation angle equals height divided by horizontal distance. Distinguish the angle to the mark from the angle to the top.
- If each segment subtends an angle θ at the observer, the top has elevation 2θ. The identity $\tan 2\theta = 2\tan\theta/(1 - \tan^2\theta)$ relates the two heights.

Step 1: Introduce heights using the given ratio Let the lower segment have length $3k$ metres and the upper segment have length $7k$ metres, with $k > 0$. The total height is $10k$. At the observation point, 18 m from the base, let the angle between the horizontal and the line to the mark be θ. Equality of

the two subtended angles makes the angle from the mark to the top another θ. Thus

$$\tan\theta = \frac{3k}{18} = \frac{k}{6}, \qquad \tan 2\theta = \frac{10k}{18} = \frac{5k}{9}.$$

Step 2: Solve the double-angle equation Substitute into the tangent identity and cancel the positive factor k:

$$\frac{5k}{9} = \frac{k/3}{1 - k^2/36} \quad \Longrightarrow \quad 1 - \frac{k^2}{36} = \frac{3}{5}.$$

Therefore $k^2 = 72/5$, and the positive height satisfies

$$H^2 = (10k)^2 = 100 \cdot \frac{72}{5} = 1440, \qquad H = 12\sqrt{10}\ \text{m}.$$

The total elevation is acute, consistent with the positive denominator $3/5$. Squaring the other proposed positive heights gives 2160, 640 and 360, rather than 1440, so they do not meet the equal-angle condition.

Step 3: Conclusion The pole is $12\sqrt{10}$ m high. Hence, the correct option is (B).

Quick Tip to Solve: Equal angles subtended by two vertical segments do not mean equal segment lengths; express the top elevation as twice the mark elevation.

20. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves? [4 marks]

(A) $x^2 + y^2 = 169$

(B) $144x^2 + 25y^2 = 3600$

(C) $25x^2 + 12y^2 = 3600$

(D) $x^2 + y^2 = 60$

Correct Answer: (B) $144x^2 + 25y^2 = 3600$

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, the point $(a\cos\theta, b\sin\theta)$ lies on the curve. Its tangent is $x\cos\theta/a + y\sin\theta/b = 1$.
- Match the coefficients only after putting the given line in the normalized form with right-hand side 1. The semiaxes are determined by the reciprocal coefficients.

Step 1: Normalize the given equation Divide $12x\cos\theta + 5y\sin\theta = 60$ by 60 to obtain

$$\frac{x\cos\theta}{5} + \frac{y\sin\theta}{12} = 1.$$

Comparing with the parametric tangent form gives $a = 5$ and $b = 12$. Hence the candidate curve is

$$\frac{x^2}{25} + \frac{y^2}{144} = 1 \quad \Longleftrightarrow \quad 144x^2 + 25y^2 = 3600.$$

Step 2: Confirm tangency and reject the other curves The contact point is $(5\cos\theta, 12\sin\theta)$. It lies on the ellipse since $\cos^2\theta + \sin^2\theta = 1$, and substituting it into the line gives $60(\cos^2\theta + \sin^2\theta) = 60$. The gradient of the ellipse at that point is proportional to $(12\cos\theta, 5\sin\theta)$, the normal to the line, which verifies tangency even at the axis endpoints. At $\theta = 0$, the line is $x = 5$; it is not tangent to either proposed circle, whose radii are 13 and $\sqrt{60}$. Nor is it tangent to option (C), whose x-intercepts are ± 12. Thus those curves cannot describe this tangent family.

Step 3: Conclusion The curve is $144x^2 + 25y^2 = 3600$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce a trigonometric line to right-hand side 1; its coefficients then reveal the reciprocals of the ellipse semiaxes.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let [t] denote the greatest integer $\leq$ t. Then the value of

8. $\int_{-\frac{1}{2}}^{1} ([2x] + |x|)dx$ **is _____.** [4 marks]

Solution:

Concept:

- The greatest-integer function $[2x]$ is constant between successive points where $2x$ is an integer. Break the interval at $x = 0$ and $x = 1/2$.
- The absolute value satisfies $|x| = -x$ for negative x and $|x| = x$ for nonnegative x. Values at isolated endpoints do not change a definite integral.

Step 1: Integrate the step-function part On $[-1/2, 0)$, $[2x] = -1$. On $[0, 1/2)$, it is zero, and on $[1/2, 1)$, it is one. The value two occurs only at $x = 1$, so it contributes no area. Thus

$$\int_{-1/2}^{1} [2x]\,dx = -1\left(\frac{1}{2}\right) + 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{2}\right) = 0.$$

In particular, the greatest integer of a negative noninteger is -1, rather than zero; truncation toward zero would give an incorrect result.

Step 2: Integrate the absolute-value part and apply the factor Split at zero to handle the change in sign:

$$\int_{-1/2}^{1} |x|\,dx = \int_{-1/2}^{0} (-x)\,dx + \int_{0}^{1} x\,dx$$
$$= \frac{1}{8} + \frac{1}{2} = \frac{5}{8}.$$

By linearity, the integral of $[2x] + |x|$ is $0 + 5/8$. The question asks for eight times this integral, so

$$8\int_{-1/2}^{1} ([2x] + |x|)\,dx = 8 \cdot \frac{5}{8} = 5.$$

The two absolute-value areas may also be checked as right triangles, with areas $\frac{1}{2}(\frac{1}{2})^2$ and $\frac{1}{2}(1)^2$, respectively.

Step 3: Conclusion The required numerical value is $\boxed{5}$.

Quick Tip to Solve: For a greatest-integer integral, mark every jump first and treat each constant strip separately; endpoint values have zero area.

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22. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For $z = x + iy$, multiply a complex quotient by the conjugate of its denominator to separate real and imaginary parts.
- The shortest distance from a point P outside a circle with centre C and radius r is $CP - r$, provided the nearest point belongs to the required locus.

Step 1: Convert the argument condition into a circle Writing $z = x + iy$, with $z \neq -2$, gives

$$\frac{z-2}{z+2} = \frac{x^2 + y^2 - 4 + 4iy}{(x+2)^2 + y^2}.$$

An argument of $\pi/4$ means equal positive real and imaginary parts. Hence $x^2 + y^2 - 4 = 4y$, with $y > 0$. Completing the square gives

$$x^2 + (y-2)^2 = 8.$$

Thus the locus is the portion of the circle with centre $C = (0, 2)$, radius $2\sqrt{2}$, and positive ordinate. Merely setting the tangent of the argument equal to one would also allow the wrong quadrant, so the positivity condition must be retained.

Step 2: Minimise the distance on the permitted locus The expression is the squared distance from z to $P = (9\sqrt{2}, 2)$. Since $CP = 9\sqrt{2} > 2\sqrt{2}$, the point is outside the circle. The triangle inequality gives $PZ \geq CP - CZ = 7\sqrt{2}$. Equality occurs on the ray from C towards P, at $Z = (2\sqrt{2}, 2)$. This point has $y = 2 > 0$, so it satisfies the required argument and is neither excluded endpoint. Therefore the lower bound is attained, rather than being only an infimum:

$$\min PZ^2 = (9\sqrt{2} - 2\sqrt{2})^2 = (7\sqrt{2})^2 = 98.$$

Step 3: Conclusion The minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is $\boxed{98}$.

Quick Tip to Solve: After finding an argument circle, retain its quadrant restriction and check that the nearest circle point lies on the allowed arc.

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23. If 'R' is the least value of 'a' such that the function f(x) = x^2 + ax + 1 is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function f(x) = x^2 + ax + 1 is decreasing on [1, 2], then the value of |R − S| is _____. [4 marks]

Solution:

Concept:

- For a differentiable function, a nonnegative derivative throughout an interval ensures increasing behaviour, and a nonpositive derivative ensures decreasing behaviour.
- For $f(x) = x^2 + ax + 1$, the derivative $f\prime(x) = 2x + a$ increases with x; its extreme values on $[1, 2]$ occur at the endpoints.

Step 1: Find the least parameter for increasing behaviour Differentiate with respect to x, treating a as a constant:

$$f\prime(x) = 2x + a.$$

Because this linear expression increases as x increases, its minimum on the interval is $2 + a$, at $x = 1$. Consequently $f\prime(x) \geq 0$ everywhere on $[1, 2]$ precisely when $a \geq -2$. A value below -2 makes the derivative negative on a nonempty interval immediately to the right of 1, so it cannot give an increasing function. At $a = -2$, the derivative is zero only at 1 and positive for $1 < x \leq 2$; hence this endpoint value is allowed even with strict increasing behaviour. Therefore $R = -2$.

Step 2: Find the greatest parameter for decreasing behaviour For the derivative to be nonpositive throughout the interval, its maximum must be nonpositive. That maximum is $4 + a$, attained at $x = 2$. Hence

$$4 + a \leq 0 \quad \Longleftrightarrow \quad a \leq -4.$$

If $a > -4$, the derivative is positive on a nonempty interval immediately to the left of 2, violating decreasing behaviour. At $a = -4$, it is negative for $1 \leq x < 2$, and zero only at 2. Thus $S = -4$. Taking the absolute difference, rather than changing either sign prematurely, gives

$$|R - S| = |-2 - (-4)| = 2.$$

Step 3: Conclusion The required value is $\boxed{2}$, with $R = -2$ and $S = -4$.

Quick Tip to Solve: For an affine derivative, test its minimum for increasing behaviour and its maximum for decreasing behaviour; endpoint zeros are allowed.

24. The mean of 10 numbers 7 × 8, 10 × 10, 13 × 12, 16 × 14, ... is _____. [4 marks]

Solution:

Concept:

- The mean of ten numbers is their sum divided by ten; here each number is a product, so the products must be summed.
- The identities $\sum_{r=1}^{n} r = n(n+1)/2$ and $\sum_{r=1}^{n} r^2 = n(n+1)(2n+1)/6$ evaluate a quadratic general term.

Step 1: Express the ten products using one index In the first factors, $7, 10, 13, 16, \ldots$, the common difference is 3. In the second factors, $8, 10, 12, 14, \ldots$, the common difference is 2. With $r = 1$ denoting the first product, their general terms are $3r + 4$ and $2r + 6$, respectively. Therefore the r-th number is

$$a_r = (3r + 4)(2r + 6) = 6r^2 + 26r + 24, \qquad 1 \le r \le 10.$$

For a quick indexing check, $a_1 = 6 + 26 + 24 = 56 = 7 \times 8$, while $a_2 = 100 = 10 \times 10$. The tenth product is $34 \times 26 = 884$; it is included exactly once in the required ten numbers.

Step 2: Sum the quadratic expression and divide by ten Using $n = 10$, the needed sums are

$$\sum_{r=1}^{10} r = 55, \qquad \sum_{r=1}^{10} r^2 = 385.$$

The constant term contributes 10×24, not just 24. Thus

$$\begin{aligned} \sum_{r=1}^{10} a_r &= 6(385) + 26(55) + 24(10) \\ &= 2310 + 1430 + 240 = 3980, \\ \overline{a} &= \frac{3980}{10} = 398. \end{aligned}$$

One must not multiply the separate means of the two factor sequences: the paired factors increase together, so averaging their products requires the quadratic sum above.

Step 3: Conclusion The mean of the ten given numbers is $\boxed{398}$.

Quick Tip to Solve: When a sequence consists of paired products of arithmetic progressions, expand its general term first; the sum reduces to sums of integers and squares.

25. If the variable line 3x + 4y = α lies between the two circles $(x - 1)^2 + (y - 1)^2 = 1$ and $(x - 9)^2 + (y - 1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is _____. [4 marks]

Solution:

Concept:

- The perpendicular distance from (h, k) to $3x + 4y = \alpha$ is $|3h + 4k - \alpha|/5$. A line cuts a chord only when this distance is less than the radius.
- Tangency is allowed by the condition of not intercepting a chord. The additional requirement that the line lie between the circles places their centres on opposite sides.

Step 1: Locate the interval between the centres The first circle has centre $C_1 = (1, 1)$ and radius $r_1 = 1$; the second has centre $C_2 = (9, 1)$ and radius $r_2 = 2$. The value of $3x + 4y$ at the centres is 7 and 31 respectively. Hence a separating line lying between the circles must have $7 < \alpha < 31$. On this

interval its distances from the centres simplify without ambiguity:

$$d_1 = \frac{\alpha - 7}{5}, \qquad d_2 = \frac{31 - \alpha}{5}.$$

Lines outside this interval might avoid both circles, but would fail the stated condition of lying between them. This excludes the unbounded exterior ranges that arise if only absolute-distance inequalities are solved.

Step 2: Impose the no-chord condition and sum integers Avoiding a chord means $d_1 \geq 1$ and $d_2 \geq 2$. Therefore

$$\alpha - 7 \geq 5, \qquad 31 - \alpha \geq 10,$$

which together give $12 \leq \alpha \leq 21$. Both boundary values are included: at 12 the line is tangent to the first circle, and at 21 it is tangent to the second. A tangent has only one common point and does not intercept a nonzero chord. The integral values are consequently $12, 13, \ldots, 21$, a list of $21 - 12 + 1 = 10$ numbers. Their sum is

$$\frac{10(12 + 21)}{2} = 165.$$

Step 3: Conclusion The sum of all admissible integral values of α is $\boxed{165}$.

Quick Tip to Solve: Use signed centre values to select the separating interval before solving distance inequalities, and include tangency when only chords are forbidden.

26. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _____. [4 marks]

Solution:

Concept:

- The six letters of VOWELS are distinct, so unrestricted six-letter arrangements number $6!$. The vowels are O and E; the consonants are V, W, L and S.
- To count arrangements in which all four consonants are consecutive, treat them as one block and then multiply by the internal arrangements of that block.

Step 1: Count the unrestricted arrangements All six letters must be used exactly once, and words with or without meaning are permitted. There is therefore no dictionary restriction or repetition factor. The total number of arrangements is

$$6! = 720.$$

The phrase that all consonants never come together excludes the arrangements in which the entire group of four consonants occupies consecutive positions. It does not forbid a pair or a triple of consonants from being adjacent. Counting via the complementary event avoids accidentally imposing the stronger, different requirement that every consonant be isolated by vowels. Indeed, two vowels could not separate four consonants into four isolated positions.

Step 2: Subtract arrangements with one complete consonant block Combine V, W, L and S into a single block. Together with O and E, there are now three distinct units. Their relative positions can be

chosen in $3!$ ways. Within the block, the four distinct consonants can be ordered in $4!$ ways. Thus the forbidden number is

$$3!\,4! = 6 \times 24 = 144.$$

Each forbidden word has exactly one maximal block containing all four consonants, so this product counts every excluded arrangement once. The vowels are already distinguished in the $3!$ factor; an additional factor of $2!$ would double-count them. Subtracting the complement yields

$$720 - 144 = 576.$$

All remaining words contain at least one vowel separating some of the consonants, exactly as required.

Step 3: Conclusion The number of required six-letter words is $\boxed{576}$.

Quick Tip to Solve: For a condition that all specified letters must not occur together, subtract the single-block count from the unrestricted permutation count.

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27. If $x\phi(x) = \int_5^x (3t^2 - 2\phi'(t))dt$, x > -2, and $\phi(0) = 4$, then $\phi(2)$ is _____. [4 marks]

Solution:

Concept:

- Differentiating an integral with variable upper limit uses the fundamental theorem of calculus. Every solution of an integral identity must satisfy both the differentiated equation and the original identity.
- An integral from 5 to 5 is zero, so the printed identity imposes $5\phi(5) = 0$, independently of any differentiation.

Step 1: Derive the necessary differential equation Differentiate the printed equality with respect to x. The product rule on its left and the fundamental theorem on its right give

$$\phi(x) + x\phi\prime(x) = 3x^2 - 2\phi\prime(x).$$

Consequently

$$\frac{d}{dx}\big((x+2)\phi(x)\big) = 3x^2, \qquad (x+2)\phi(x) = x^3 + C.$$

Since $\phi(0) = 4$, this constant must be $C = 8$. The only possible candidate on $x > -2$ is therefore $\phi(x) = (x^3 + 8)/(x + 2) = x^2 - 2x + 4$. It would give $\phi(2) = 4$, agreeing with the stated answer key, but this is only a necessary consequence until the original integral has been checked.

Step 2: Check the original lower limit and expose the contradiction The printed lower limit is 5. Putting $x = 5$ in the original equation gives $5\phi(5) = 0$, hence $\phi(5) = 0$. In contrast, the necessary candidate gives

$$\phi(5) = 25 - 10 + 4 = 19.$$

The contradiction can also be checked at every point: for this candidate, $\phi\prime(t) = 2t - 2$, and

$$\int_5^x (3t^2 - 2\phi\prime(t))\, dt = x^3 - 2x^2 + 4x - 95,$$

whereas $x\phi(x) = x^3 - 2x^2 + 4x$. The two sides differ by 95, so the integration constant lost by differentiation cannot be ignored. Thus no differentiable function satisfies all the printed conditions on $x > -2$.

Step 3: Conclusion As printed, the conditions are inconsistent and $\phi(2)$ is not defined by any admissible function. The keyed value $\boxed{4}$ follows from the differentiated equation and $\phi(0) = 4$, but does not satisfy the original integral identity.

Quick Tip to Solve: After differentiating an integral equation, substitute the candidate back into the original equality; differentiation loses a constant constraint.

---

28. If $\left(\frac{3^6}{4^4}\right) k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$, then k is equal to _____.

[4 marks]

Solution:

Concept:

- The general term in $(a+b)^{12}$ is $T_{r+1} = \binom{12}{r} a^{12-r} b^r$, where r is an integer from 0 to 12.
- A term independent of x has total exponent zero. Its coefficient includes the binomial coefficient, the powers of all numerical factors, and the sign.

Step 1: Find the index of the constant term Use $a = x/4$ and $b = -12/x^2$. The general term is

$$\begin{aligned} T_{r+1} &= \binom{12}{r} \left(\frac{x}{4}\right)^{12-r} \left(-\frac{12}{x^2}\right)^r \\ &= \binom{12}{r} \frac{(-12)^r}{4^{12-r}} x^{12-3r}. \end{aligned}$$

The exponent combines $12 - r$ from the first factor and $-2r$ from the second. For independence of x, set $12 - 3r = 0$, giving $r = 4$. This index is in the permitted range, so a constant term exists and is unique. It is the fifth term, because the index is $r + 1$, and its sign is positive since 4 is even.

Step 2: Evaluate the coefficient and compare with the required form Substituting $r = 4$ gives

$$\begin{aligned} T_5 &= \binom{12}{4} \frac{12^4}{4^8} = 495 \frac{(3 \cdot 4)^4}{4^8} \\ &= \frac{495 \cdot 3^4}{4^4}. \end{aligned}$$

The question expresses this constant term as $(3^6/4^4)k$. Divide the coefficient just found by the

displayed factor, keeping the common denominator unchanged:

$$k = \frac{495 \cdot 3^4}{3^6} = \frac{495}{9} = 55.$$

The binomial coefficient is essential; using only $12^4/4^8$ would omit the number of ways to choose the four second-factor contributions. No residual power of x remains.

Step 3: Conclusion The required value of k is $\boxed{55}$.

Quick Tip to Solve: Combine powers of the variable before computing coefficients, then factor the numerical answer into the exact form requested.

29. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to _____. [4 marks]

Solution:

Concept:

- For independent units, probabilities of joint states multiply. Let A mean that the first unit works and B that the second unit works.
- The instrument operates only on $A \cap B$. Given an observed failure $F = (A \cap B)^c$, use $P(E \mid F) = P(E \cap F)/P(F)$, where $E = A^c \cap B$.

Step 1: Calculate the probability of instrument failure The working probabilities are $P(A) = 0.9$ and $P(B) = 0.8$. Since the units function independently, the probability that both work is their product. Therefore

$$P(A \cap B) = 0.9(0.8) = 0.72, \qquad P(F) = 1 - 0.72 = 0.28.$$

This includes three mutually exclusive states: only the first fails, only the second fails, and both fail. Failure of the instrument does not imply failure of both units.

Step 2: Condition on the failure that has occurred The event that only the first unit fails is $E = A^c \cap B$. Its unconditional probability is

$$P(E) = (1 - 0.9)(0.8) = 0.08.$$

Every outcome in E is an instrument failure, so $E \subset F$. The question tells us the instrument has failed; hence the required probability is conditional:

$$p = P(E \mid F) = \frac{0.08}{0.28} = \frac{2}{7}.$$

Using 0.08 alone would ignore this information. As a check, the other failure states have probabilities 0.18 and 0.02, which sum with 0.08 to 0.28.

Step 3: Evaluate the requested multiple Multiply the conditional probability, rather than the uncondi-

tional joint probability, by the stated factor:

$$98p = 98\left(\frac{2}{7}\right) = 14(2) = 28.$$

The result is dimensionless, as it is a multiple of a probability.

Step 4: Conclusion The required probability is $p = 2/7$, and the numerical answer is $\boxed{98p = 28}$.

Quick Tip to Solve: When an instrument is known to have failed, divide the relevant failure-state probability by the probability of any instrument failure.

30. The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point $(-1, -1, 2)$ is _____.

[4 marks]

Solution:

Concept:

- A point on the symmetric line is obtained by setting all three ratios equal to a parameter t. Its coordinates are $x = 1 + 2t$, $y = 2 + 3t$, $z = -1 + 6t$.
- The line meets a plane where these coordinates satisfy its equation. For points $P(x, y, z)$ and $Q(x_0, y_0, z_0)$, the squared distance is $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$.

Step 1: Substitute the line into the plane Use the same parameter for all three coordinates; the denominators are direction ratios, not the coordinates of the required point. Substitute into the plane $2x - y + z = 6$:

$$2(1 + 2t) - (2 + 3t) + (-1 + 6t) = 6.$$

Collect the constant terms and the coefficients of t separately to obtain

$$-1 + 7t = 6 \quad \Longrightarrow \quad t = 1.$$

The nonzero coefficient 7 confirms there is a unique intersection.

Step 2: Find and check the intersection Putting $t = 1$ into the coordinate expressions gives

$$P = (1 + 2,\ 2 + 3,\ -1 + 6) = (3, 5, 5).$$

Check the result directly: $2(3) - 5 + 5 = 6$, so it is on the plane. Also, the three line ratios are $(3 - 1)/2 = 1$, $(5 - 2)/3 = 1$, and $(5 + 1)/6 = 1$; hence it is on the given line as well.

Step 3: Calculate the squared distance The reference point is $Q = (-1, -1, 2)$. Subtract its coordinates carefully, including both negative signs:

$$\begin{aligned} PQ^2 &= (3 - (-1))^2 + (5 - (-1))^2 + (5 - 2)^2 \\ &= 4^2 + 6^2 + 3^2 \\ &= 16 + 36 + 9 = 61. \end{aligned}$$

The question asks for the square of the distance, so taking a square root would answer a different quantity.

Step 4: Conclusion The intersection is $(3, 5, 5)$, and its squared distance from $(-1, -1, 2)$ is $\boxed{61}$ square units.

Quick Tip to Solve: Parameterise the line first, impose the plane equation, and keep the distance squared when that is the requested quantity.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped? [4 marks]

(A) $\sqrt{\frac{2ghv^2+1}{h^2}}$

(B) $\sqrt{2ghv^2 + h^2}$

(C) $\sqrt{\frac{2v^2h}{g} + h^2}$

(D) $\sqrt{\frac{2gh}{v^2} + h^2}$

Correct Answer: (C) $\sqrt{\frac{2v^2h}{g} + h^2}$

Solution:

Concept:

- On release, a food packet retains the helicopter's horizontal velocity v, while its initial vertical velocity is zero. Air resistance is neglected.
- Free fall through height h gives $h = gt^2/2$. Horizontal travel is $x = vt$; the direct helicopter-to-man distance at release is $R = \sqrt{x^2 + h^2}$.

Step 1: Find the time needed to reach the ground Take downward displacement as positive. Gravity acts vertically, so the vertical motion is independent of the inherited horizontal motion. With zero initial vertical speed,

$$h = \frac{1}{2}gt^2 \quad \Longrightarrow \quad t = \sqrt{\frac{2h}{g}}.$$

This is the positive flight time. The horizontal speed does not enter this equation because it does not contribute to vertical acceleration or vertical displacement.

Step 2: Calculate the separation at release To reach the man, the packet must travel the horizontal separation between the helicopter and the man during its flight. Thus

$$x = vt = v\sqrt{\frac{2h}{g}}.$$

The question asks for the distance of the helicopter from the man, whose vertical separation is also h.

Applying Pythagoras gives

$$R = \sqrt{x^2 + h^2} = \sqrt{\frac{2v^2h}{g} + h^2}.$$

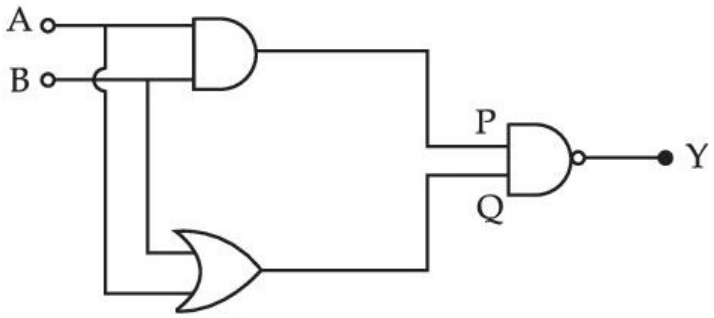
The horizontal lead distance x alone is not the direct separation.

Step 3: Check the choices Both terms inside the correct square root have dimensions of length squared. Options (A), (B) and (D) do not reproduce the free-fall time and the resulting horizontal distance; in particular, the term must contain v^2/g, not g/v^2 or gv^2. The result also tends to h as $v \to 0$, as expected for a release directly above the man.

Step 4: Conclusion The required distance is $\boxed{\sqrt{2v^2h/g + h^2}}$. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish the horizontal release lead $v\sqrt{2h/g}$ from the direct helicopter-to-target separation, which also includes height.

32. In the following logic circuit the sequence of the inputs A, B are (0, 0), (0, 1), (1, 0) and (1, 1). The output Y for this sequence will be : [4 marks]



(A) 1, 0, 1, 0

(B) 0, 1, 0, 1

(C) 1, 1, 1, 0

(D) 0, 0, 1, 1

Correct Answer: (C) 1, 1, 1, 0

Solution:

Concept:

- An AND gate has output AB, and an OR gate has output $A + B$, where $+$ denotes Boolean OR rather than arithmetic addition.
- The last gate is NAND: its output is the complement of the AND of its inputs. The small circle at its output denotes inversion.

Step 1: Identify the two intermediate signals The upper gate receives both A and B and is an AND gate, giving $P = AB$. The lower gate receives the same two inputs and is an OR gate, giving $Q = A + B$. The curved bridge where wires cross indicates that the crossing wires are not joined at that point. Thus the diagram does not short the two input variables together.

Step 2: Simplify the output expression The final gate has an AND-shaped body and an output inversion circle, so

$$Y = \overline{PQ} = \overline{AB(A+B)}.$$

In Boolean algebra, $A^2 = A$ and $B^2 = B$. Consequently,

$$AB(A+B) = AAB + ABB = AB + AB = AB,$$

and therefore $Y = \overline{AB}$. One can also see this directly: whenever $AB = 1$, the OR output is necessarily 1, so the second factor adds no further restriction.

Step 3: Evaluate the ordered input pairs For $(A, B) = (0,0), (0,1), (1,0), (1,1)$, the products AB are respectively $0, 0, 0, 1$. Inverting them gives

$$Y : 1,\ 1,\ 1,\ 0.$$

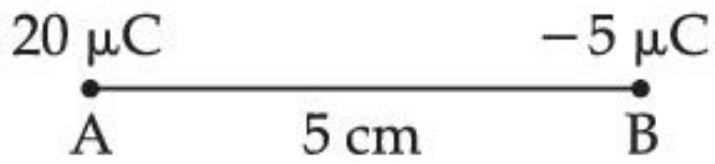
Thus only the last input pair produces zero. Options (A) and (B) incorrectly make the output depend only on B; option (D) incorrectly makes it depend only on A. None describes the NAND combination in the circuit.

Step 4: Conclusion The output sequence is $\boxed{1, 1, 1, 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Simplify $AB(A+B)$ to AB before constructing the truth table; the output bubble then converts the result to NAND.

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33. Two particles A and B having charges 20μC and -5μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force? [4 marks]



(A) At 5 cm from 20 μC on the left side of system

(B) At 5 cm from -5 μC on the right side

(C) At 1.25 cm from -5 μC between two charges

(D) At midpoint between two charges

Correct Answer: (B) At 5 cm from -5 μC on the right side

Solution:

Concept:

- A nonzero test charge experiences zero resultant electric force exactly where the resultant electric field is zero. The field magnitude of a point charge is $E = k|q|/r^2$.
- Fields point away from a positive charge and toward a negative charge. Cancellation requires

opposite field directions as well as equal magnitudes.

Step 1: Locate the possible zero-field region Put $+20\,\mu\text{C}$ at $x = 0$ and $-5\,\mu\text{C}$ at $x = 5\,\text{cm}$. Between them, both fields point from the positive charge toward the negative charge, so they cannot cancel. A zero can occur only outside the interval. It must be on the side of the smaller-magnitude charge: the point must be closer to $-5\,\mu\text{C}$ to compensate for its weaker charge.

Step 2: Equate the two field magnitudes Let the point be at distance $r > 0$ to the right of the negative charge. Its distances from the positive and negative charges are respectively $r + 5$ and r, measured in centimetres. Equating magnitudes, with common factors cancelled, gives

$$\frac{20}{(r+5)^2} = \frac{5}{r^2}.$$

Taking the positive square root yields

$$\frac{2}{r+5} = \frac{1}{r} \quad \Longrightarrow \quad 2r = r + 5 \quad \Longrightarrow \quad r = 5\,\text{cm}.$$

The fields there have equal magnitudes and opposite directions.

Step 3: Exclude the remaining positions On the left side of the positive charge, the larger charge is also the nearer one, so its field always dominates; option (A) fails. Options (C) and (D) lie between unlike charges, where their fields add rather than cancel. At the required position the distances are 10 cm and 5 cm, and $20/10^2 = 5/5^2$, providing a direct check.

Step 4: Conclusion The third charged particle should be placed $\boxed{5\,\text{cm}}$ to the right of the $-5\,\mu\text{C}$ charge. Hence, the correct option is (B).

Quick Tip to Solve: For two unlike unequal charges, the zero-field point lies outside them on the side of the smaller charge magnitude.

34. An object is placed at the focus of concave lens having focal length f. What is the magnification and distance of the image from the optical centre of the lens? [4 marks]

(A) 1, ∞

(B) Very high, ∞

(C) $\frac{1}{2}$, $\frac{f}{2}$

(D) $\frac{1}{4}$, $\frac{f}{4}$

Correct Answer: (C) $\frac{1}{2}$, $\frac{f}{2}$

Solution:

Concept:

- The Cartesian lens formula is $1/v - 1/u = 1/F$, where F is the signed focal length. A concave lens has $F = -f$ when $f > 0$ denotes its focal-length magnitude.
- For an object on the incident side, $u < 0$. Linear magnification is $m = v/u$; positive magnification signifies an upright image.

Step 1: Assign the correct signs The object is at the focus on the incident side, a distance f from the lens. Thus $u = -f$, while the concave lens has focal length $F = -f$. Keeping these as signed quantities avoids applying the familiar converging-lens rule about an image at infinity. A concave lens diverges the incident rays and produces a virtual image for this real object.

Step 2: Apply the lens equation Substitute into the thin-lens formula:

$$\frac{1}{v} - \frac{1}{-f} = \frac{1}{-f}.$$

It follows that

$$\frac{1}{v} + \frac{1}{f} = -\frac{1}{f} \quad \Longrightarrow \quad \frac{1}{v} = -\frac{2}{f} \quad \Longrightarrow \quad v = -\frac{f}{2}.$$

The negative sign puts the image on the same side of the lens as the object. The distance from the optical centre, a positive magnitude, is $f/2$.

Step 3: Calculate and interpret the magnification The image magnification is

$$m = \frac{v}{u} = \frac{-f/2}{-f} = \frac{1}{2}.$$

It is positive and less than unity, so the image is upright and diminished, as a concave lens should produce. Options (A) and (B), which put the image at infinity, describe neither this signed lens calculation nor the expected virtual image. Option (D) gives excessive reduction and does not satisfy the lens equation.

Step 4: Conclusion The magnification is $\boxed{1/2}$, and the image is $\boxed{f/2}$ from the optical centre on the object side. Hence, the correct option is (C).

Quick Tip to Solve: Use a negative signed focal length for a concave lens; an object at its focus still forms a finite virtual image.

35. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil: [4 marks]

(A) $\frac{\mu_0 IN}{2(b-a)} \log_e \left(\frac{b}{a}\right)$

(B) $\frac{\mu_0 I}{8} \left[\frac{a+b}{a-b}\right]$

(C) $\frac{\mu_0 I}{4(a-b)} \left[\frac{1}{a} - \frac{1}{b}\right]$

(D) $\frac{\mu_0 I}{8} \left(\frac{a-b}{a+b}\right)$

Correct Answer: (A) $\frac{\mu_0 IN}{2(b-a)} \log_e \left(\frac{b}{a}\right)$

Solution:

Concept:

- A circular turn of radius r carrying current I produces field $B = \mu_0 I/(2r)$ at its centre, normal to the plane according to the right-hand rule.
- For a tightly wound spiral, approximate the turns as concentric circles. Uniform radial winding

gives $dN = N\,dr/(b-a)$; contributions from all turns have the same direction and add.

Step 1: Express the number of turns in a radial strip The coil contains N turns spread over radial width $b-a$, with $b > a > 0$. For tight, uniform winding, the turn density per unit radius is

$$n_r = \frac{N}{b-a}.$$

A narrow strip from radius r to $r+dr$ contains $dN = n_r\,dr$. Because the winding is tight, the small radial advance per turn can be neglected in using the circular-turn centre-field formula.

Step 2: Integrate the field contributions Each turn in the strip contributes approximately $\mu_0 I/(2r)$. Therefore

$$dB = \frac{\mu_0 I}{2r}\,dN = \frac{\mu_0 IN}{2(b-a)}\frac{dr}{r}.$$

Integrate from the innermost to the outermost turn:

$$B = \frac{\mu_0 IN}{2(b-a)}\int_a^b \frac{dr}{r} = \frac{\mu_0 IN}{2(b-a)}\ln\left(\frac{b}{a}\right).$$

The logarithm is dimensionless and positive, so this expression gives the positive field magnitude.

Step 3: Check the form and limiting behaviour The field must be proportional to both I and N, so options (B), (C) and (D), which omit N, cannot represent this coil. As an additional check, for a very narrow radial winding, $\ln(b/a) \approx (b-a)/a$, and the answer tends to $\mu_0 IN/(2a)$, the centre field of N nearly equal circular turns. Its direction is perpendicular to the spiral plane, fixed by the current sense.

Step 4: Conclusion The magnetic field magnitude is $\boxed{\frac{\mu_0 IN}{2(b-a)}\ln(b/a)}$. Hence, the correct option is (A).

Quick Tip to Solve: Count turns per unit radial width and integrate dr/r; the logarithm signals that inner turns contribute more strongly.

36. A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is : [4 marks]

(A) 4

(B) 1

(C) 3

(D) 2

Correct Answer: (C) 3

Solution:

Concept:

- For an elastic collision, both vector linear momentum and total kinetic energy are conserved. Initially only the body of mass M moves, with speed V_0.
- Equal scattering angles mean that the outgoing transverse momenta are equal and opposite. A possible angle must satisfy $0 \le \cos^2\theta \le 1$.

Step 1: Apply the two momentum components Let the final speeds of masses M and m be u and v, respectively, on opposite sides of the initial direction. For a nonzero common angle θ, transverse momentum conservation gives $Mu = mv$. The longitudinal equation is therefore

$$MV_0 = (Mu + mv)\cos\theta = 2Mu\cos\theta.$$

Thus $u = V_0/(2\cos\theta)$ and $v = (M/m)u$. The cancellation of the transverse sine is appropriate for the limiting oblique scattering configuration.

Step 2: Use elastic energy conservation to find the bound Put $k = M/m$. Conservation of kinetic energy gives

$$MV_0^2 = Mu^2 + mv^2 = M(1+k)u^2.$$

Substitute the longitudinal result to obtain

$$4\cos^2\theta = 1 + k, \qquad k = 4\cos^2\theta - 1 \le 3.$$

Hence ratios greater than 3 would demand $\cos^2\theta > 1$, which is impossible. The value 3 is the limiting value as the common scattering angle tends to zero; ratios below it allow finite equal angles. Option 4 exceeds the bound, while 1 and 2 are possible ratios but are not the largest limiting value.

Step 3: Conclusion The maximum limiting mass ratio is $M/m = 3$. Hence, the correct option is (C).

Quick Tip to Solve: When the two scattering angles are equal, use transverse momentum first; it turns the longitudinal equation into twice one outgoing momentum.

37. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses : [4 marks]

(A) $V = \frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$

(B) $V = \sqrt{\frac{4G(M_1+M_2)}{r}}$

(C) $V = \frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$

(D) $V = \frac{\sqrt{2G}(M_1+M_2)}{r}$

Correct Answer: (B) $V = \sqrt{\frac{4G(M_1+M_2)}{r}}$

Solution:

Concept:

- Gravitational potential energy is additive: $U = -GM_1m/r_1 - GM_2m/r_2$, taking zero potential energy at infinity.
- The threshold for escape has zero final speed at infinity, so the initial kinetic energy equals the magnitude of the initial negative potential energy.

Step 1: Evaluate the potential at the midpoint The stated point is halfway between the two centres. Its distance from each centre is $r/2$; it is not a point halfway between their surfaces. Therefore

$$U_i = -\frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = -\frac{2Gm(M_1+M_2)}{r}.$$

The radii do not enter this expression because the point is outside both bodies and each spherical body acts gravitationally as a point mass at its centre. The test particle contributes a factor m to both its potential and kinetic energies.

Step 2: Set the total energy to the escape threshold For escape with no residual kinetic energy at infinity, conservation of energy gives

$$\frac{1}{2}mV^2 - \frac{2Gm(M_1+M_2)}{r} = 0.$$

Cancel the nonzero test mass and rearrange:

$$V^2 = \frac{4G(M_1+M_2)}{r}.$$

The positive square root is the speed. Adding the two potentials before taking the root is essential: adding separate escape speeds would be incorrect. Options (A) and (C) supply too little kinetic energy because of their extra factors. Option (D) does not even have the dimensions of speed, whereas the square root of GM/r does.

Step 3: Conclusion The minimum escape speed is $V = \sqrt{4G(M_1+M_2)/r}$. Hence, the correct option is (B).

Quick Tip to Solve: For escape in the field of several fixed bodies, add their negative potentials at the launch point and then use $V = \sqrt{-2\phi}$.

38. A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b >> a). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is : [4 marks]

(A) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$

(B) $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{a}$

(C) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{b^2}{a}$

(D) $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{b}$

Correct Answer: (A) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$

Solution:

Concept:

- Mutual inductance is the flux through one loop per unit current in the other: $\mathcal{M} = \Phi/I$ for a single turn.
- The field of a finite straight wire at perpendicular distance d is $B = \mu_0 I(\sin\alpha + \sin\beta)/(4\pi d)$. Since $a \ll b$, the field is approximately uniform over the small loop.

Step 1: Calculate the field at the centre of the large square For each side of the square of side b, the perpendicular distance from the centre is $d = b/2$. The two endpoints subtend angles 45° with the perpendicular. Thus the field due to one side is

$$B_1 = \frac{\mu_0 I}{4\pi(b/2)}(2\sin 45^\circ) = \frac{\mu_0 I}{4\pi}\frac{2\sqrt{2}}{b}.$$

By the right-hand rule, all four sides produce a field in the same direction normal to the common plane. Their contributions add rather than cancel.

Step 2: Find the flux through the small loop Multiplying by four gives

$$B = \frac{\mu_0 I}{4\pi}\frac{8\sqrt{2}}{b}.$$

The small square has area a^2. Its normal is parallel to the field, and the approximation $b \gg a$ allows the central field to be used throughout that area:

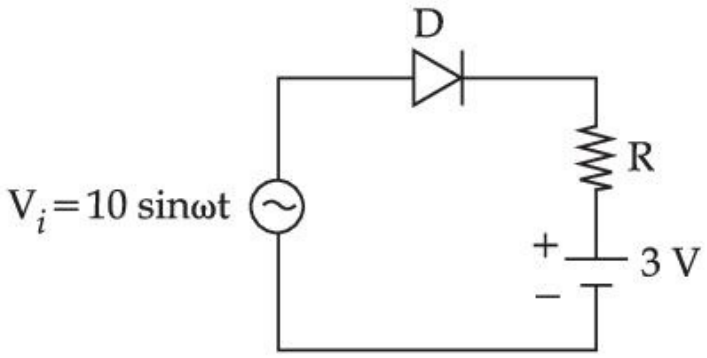
$$\Phi \simeq Ba^2, \qquad \mathcal{M} = \frac{\Phi}{I} = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}\,a^2}{b}.$$

This has dimensions of inductance because μ_0 is inductance per length and a^2/b is a length. The alternatives without the area factor have incorrect dimensions; the expression proportional to b^2/a also uses the wrong loop area and separation scale.

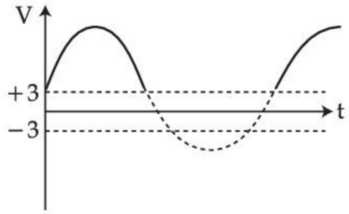
Step 3: Conclusion The mutual inductance is $\mathcal{M} = \frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$. Hence, the correct option is (A).

Quick Tip to Solve: For a tiny loop inside a much larger loop, calculate the large loop's central field and multiply by the small loop's area, not the large area.

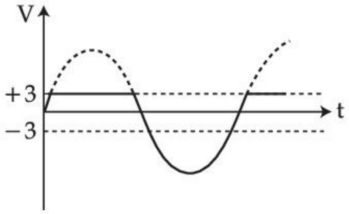
39. Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode is ideal one : [4 marks]



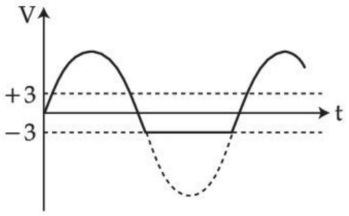
(A)



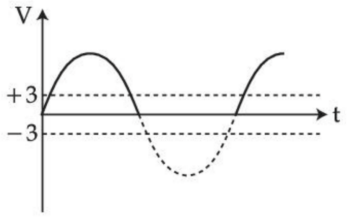
(B)



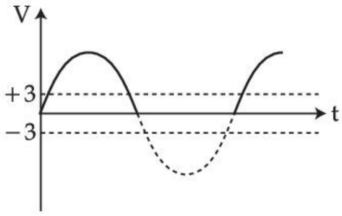
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- An ideal diode conducts only when its anode is at a higher potential than its cathode; its forward voltage drop is zero. In reverse bias its current is zero.
- The battery fixes the lower terminal of R at $+3\,\mathrm{V}$ relative to the bottom wire. The resistor voltage is $V_R = IR$, not the potential of its upper terminal alone.

Step 1: Determine when the diode conducts The anode is connected to the source terminal at $V_i = 10\sin\omega t$. If the diode is off, no current flows through R, so both resistor terminals are at $+3\,\mathrm{V}$. The diode therefore starts conducting only when $10\sin\omega t > 3$. When it conducts, the upper terminal of R follows the input because the diode has no forward drop. Kirchhoff's voltage law then gives

$$V_R = 10\sin\omega t - 3 \quad (10\sin\omega t > 3).$$

Step 2: Identify the rectified output waveform During the rest of the cycle, including the entire negative half-cycle, the diode is reverse biased and the resistor current vanishes. Thus

$$V_R = \max\{0,\ 10\sin\omega t - 3\}\ \mathrm{V}.$$

The output contains positive pulses separated by intervals at zero, and has no negative part. Its maximum is $10 - 3 = 7\,\mathrm{V}$. Among the qualitative sketches, (D) represents the positive rectified waveform returning to the zero baseline. Sketch (A) clamps the output at a positive level and would refer to a different pair of terminals; (B) passes negative voltage and (C) includes a negative conducting segment. The battery shifts the conduction threshold, so the exact pulse begins at $\omega t = \sin^{-1}(0.3)$ and ends at $\pi - \sin^{-1}(0.3)$, not at the input zero crossings.

Step 3: Conclusion The resistor voltage is $V_R = \max\{0, 10\sin\omega t - 3\}\,\mathrm{V}$, represented qualitatively by the positive pulses in (D). Hence, the correct option is (D).

Quick Tip to Solve: Always distinguish voltage across the resistor from the diode cathode potential: subtract the series battery voltage only while the diode conducts.

40. A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is 2 $\times$ 10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight. [4 marks]

(A) 2×10^{-9} m

(B) 5×10^{-8} m

(C) 4×10^{-8} m

(D) 5×10^{-10} m

Correct Answer: (D) 5×10^{-10} m

Solution:

Concept:

- Young's modulus relates tensile stress to longitudinal strain: $Y = (F/A)/(\Delta L/L)$. For a nonuniform tensile force use $d\ell = T(x)\,dx/(AY)$.
- A freely hanging uniform rod supports different weights at different heights. Its tension rises linearly from zero at the bottom to its full weight W at the top.

Step 1: Express the tension in a small element Take x upwards from the free lower end of a rod of total length L. The portion below that section has fraction x/L of the rod's weight. Therefore

$$T(x) = \frac{Wx}{L}, \qquad d\ell = \frac{Wx}{LAY}\,dx.$$

The lower end is not subject to the full weight: treating the rod as though a point load W acts at its bottom would overestimate its extension by a factor of two. Lateral contraction is neglected as the question specifies, so A is constant.

Step 2: Integrate and substitute the SI values Adding the extensions over the whole rod gives

$$\Delta L = \frac{W}{LAY}\int_0^L x\,dx = \frac{WL}{2AY}.$$

The printed weight $10\,\mathrm{kg\,m\,s^{-2}}$ is $10\,\mathrm{N}$, not a mass of $10\,\mathrm{kg}$. Also, $A = 100\,\mathrm{cm^2} = 10^{-2}\,\mathrm{m^2}$ and $L = 20\,\mathrm{cm} = 0.20\,\mathrm{m}$. Consequently

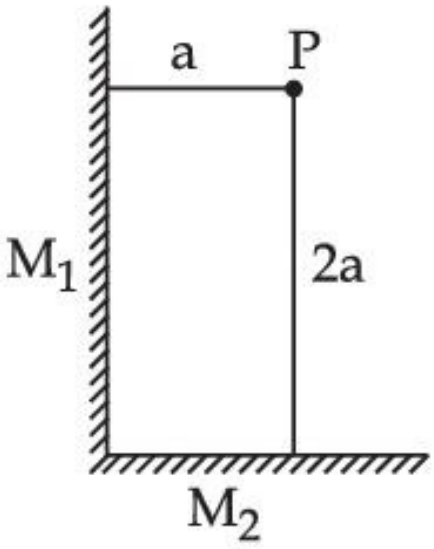
$$\Delta L = \frac{10(0.20)}{2(10^{-2})(2\times 10^{11})} = 5\times 10^{-10}\,\mathrm{m}.$$

The unit is a length since AY is a force. The other offered values result from incorrect conversion, an incorrect force, or a missing self-weight factor; none equals the integrated result.

Step 3: Conclusion The elongation due to the rod's own weight is $5 \times 10^{-10}\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: For a uniform hanging rod, use the average tension $W/2$; first check whether the given quantity is weight in newtons or mass in kilograms.

41. Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is : (Take $\sqrt{5}$ = 2.3) [4 marks]



(A) 3a

(B) 4.6a

(C) 2.3a

(D) $2\sqrt{10}$a

Correct Answer: No listed option gives the literal shortest separation, 2a; (B) is the keyed distance between first-order images.

Solution:

Concept:

- A plane mirror forms an image at the same perpendicular distance behind the mirror as the object is in front. Two perpendicular reflecting mirrors produce three images.
- Use Cartesian coordinates with the mirrors as the axes; reflection changes the sign of the coordinate normal to the reflecting mirror. Distances between images follow from the distance formula.

Step 1: Locate all the images Put the mirror intersection at the origin, the vertical mirror M_1 at $x = 0$, and the horizontal mirror M_2 at $y = 0$. The source is at $P = (a, 2a)$. Reflection in M_1 gives $I_1 = (-a, 2a)$, and reflection in M_2 gives $I_2 = (a, -2a)$. Successive reflections in both mirrors give the third image $I_3 = (-a, -2a)$. Both reflection orders produce this same third image, so it is counted only once.

Step 2: Compare the image separations and the printed choices The three pairwise distances are

$$I_1I_2 = \sqrt{(2a)^2 + (4a)^2} = 2\sqrt{5}\,a,$$

$$I_1I_3 = 4a, \qquad I_2I_3 = 2a.$$

Thus the literal shortest distance between images is $2a$, absent from the offered options. If only

the two single-reflection images are intended, their separation is $2\sqrt{5}\,a \simeq 4.6a$, using the supplied $\sqrt{5} = 2.3$, which explains option (B) and the supplied key. However, that restricted pair is not the shortest pair when all images formed by the perpendicular mirrors are included. Values $3a$, $2.3a$ and $2\sqrt{10}a$ are not the literal minimum or the separation of the two first-order images.

Step 3: Conclusion The literal shortest image separation is $2a$, so no listed option is correct as worded. The supplied key selects (B), $4.6a$, which is the separation of the two first-order images only.

Quick Tip to Solve: For two perpendicular mirrors, include the image formed by two reflections before deciding which pair of images has the smallest separation.

42. Match List - I with List - II.

	List - I		List - II
(a)	**Torque**	**(i)**	**MLT^{-1}**
(b)	**Impulse**	**(ii)**	**MT^{-2}**
(c)	**Tension**	**(iii)**	**ML^2T^{-2}**
(d)	**Surface Tension**	**(iv)**	**MLT^{-2}**

Choose the most appropriate answer from the option given below : [4 marks]

(A) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(B) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(C) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(D) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Correct Answer: (A) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Solution:

Concept:

- Force has dimensions $[F] = \mathrm{MLT}^{-2}$, since force equals mass times acceleration. Tension is a force, irrespective of the origin of the force.
- Multiply or divide the dimensions of the defining quantities. Numerical constants and angles are dimensionless, so they do not affect a dimensional formula.

Step 1: Match torque and impulse Torque is the moment of a force about an axis. Its magnitude is force times the perpendicular lever arm, which has length dimensions. Thus

$$[\tau] = [F][L] = \mathrm{ML}^2\mathrm{T}^{-2},$$

so (a) matches (iii). Impulse is the time integral of force, and is also the change in momentum:

$$[J] = [F][T] = \mathrm{MLT}^{-1}.$$

Therefore (b) matches (i). Torque and impulse are not interchangeable: torque contains two powers of length and impulse has only one inverse power of time.

Step 2: Match tension and surface tension The tension transmitted by a stretched string is a force, so

$$[\text{tension}] = \text{MLT}^{-2},$$

which matches (c) with (iv). Surface tension is force per unit length of an imaginary line drawn on the liquid surface:

$$[\gamma] = \frac{[F]}{[L]} = \text{MT}^{-2}.$$

Hence (d) matches (ii). The same surface-tension dimensions follow from surface energy per unit area. Combining all four results gives (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii), exactly option (A). Option (B) assigns surface-tension dimensions to torque; (C) swaps torque with impulse; (D) swaps impulse and force. Each alternative fails at least one defining dimensional relation.

Step 3: Conclusion Torque matches (iii), impulse (i), tension (iv), and surface tension (ii). Hence, the correct option is (A).

Quick Tip to Solve: Begin with force MLT^{-2}: multiply by length for torque, multiply by time for impulse, and divide by length for surface tension.

---

43. For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If p = p_0 at v =0 is the given boundary condition, then the maximum temperature one mole of gas can attain is : (Here R is the gas constant) [4 marks]

(A) $\frac{p_0}{aeR}$

(B) $\frac{ap_0}{eR}$

(C) infinity

(D) 0°C

Correct Answer: (A) $\frac{p_0}{aeR}$

Solution:

Concept:

- For one mole of an ideal gas, $pv = RT$, where p is pressure, v is volume and T is absolute temperature.
- The differential relation determines pressure as a function of volume. A maximum of temperature occurs where its derivative with respect to volume changes from positive to negative.

Step 1: Integrate the pressure relation Separate the variables in the given equation, retaining the negative sign:

$$\frac{dp}{p} = -a\,dv.$$

Integration gives $\ln p = -av + C$. The boundary condition $p = p_0$ at $v = 0$ fixes $C = \ln p_0$, so

$$p(v) = p_0 e^{-av}.$$

Thus pressure decreases exponentially with increasing volume; it is not constant along this process.

Step 2: Maximise the absolute temperature Substitute the pressure into the ideal-gas equation for one mole:

$$T(v) = \frac{p_0}{R} v e^{-av}.$$

Differentiating by the product rule yields

$$\frac{dT}{dv} = \frac{p_0}{R} e^{-av}(1 - av).$$

For the positive process parameter a, this derivative is positive for $v < 1/a$ and negative for $v > 1/a$. Therefore $v = 1/a$ is the maximum, rather than a minimum. At this volume the pressure is p_0/e, giving

$$T_{\max} = \frac{p_0}{aeR}.$$

The factor a has dimensions of inverse volume, so this expression has temperature units. Option (B) places this factor in the wrong position. The product ve^{-av} tends to zero at large volume, which rules out an infinite maximum. A fixed Celsius temperature is not implied by the given variable parameters.

Step 3: Conclusion The maximum temperature is $\boxed{p_0/(aeR)}$, attained at $v = 1/a$. Hence, the correct option is (A).

Quick Tip to Solve: First obtain $p(v)$; then maximise pv, since for a fixed amount of ideal gas its temperature is proportional to that product.

44. Which of the following equations is dimensionally incorrect?

Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume, p = pressure, W = work done, T = torque, $\in$ = permittivity, E = electric field, J = current density, L = length. [4 marks]

(A) $v = \frac{\pi p a^4}{8\eta L}$

(B) $h = \frac{2s\cos\theta}{\rho r g}$

(C) $J = \in \frac{\partial E}{\partial t}$

(D) $W = \Gamma\theta$

Correct Answer: (A) $v = \frac{\pi p a^4}{8\eta L}$

Solution:

Concept:

- An equation is dimensionally valid only when both sides have identical powers of mass M, length L, time T and electric current I. Numerical factors and trigonometric functions are dimensionless.
- Dynamic viscosity has dimensions $[\eta] = ML^{-1}T^{-1}$, pressure has $[p] = ML^{-1}T^{-2}$, and the question defines v as volume, not volume flow rate.

Step 1: Check the volume equation For option (A), a and L are lengths. Therefore

$$\left[\frac{\pi p a^4}{8\eta L}\right] = \frac{(ML^{-1}T^{-2})L^4}{(ML^{-1}T^{-1})L} = L^3T^{-1}.$$

The left side is a volume and must have dimensions L^3. The right side is volume per unit time, so the equation as written is dimensionally incorrect. It resembles Poiseuille's expression for volume flow rate; a time factor or a rate symbol would be needed to equate the two sides.

Step 2: Confirm that the remaining equations are consistent Surface tension is force per unit length, with dimensions MT^{-2}. The denominator $\rho r g$ has dimensions $(ML^{-3})L(LT^{-2}) = ML^{-1}T^{-2}$. Hence $2s\cos\theta/(\rho r g)$ has dimensions L, matching height in (B). For (C), $[\epsilon] = M^{-1}L^{-3}T^4I^2$, while $[E] = MLT^{-3}I^{-1}$. Consequently

$$[\epsilon\,\partial E/\partial t] = IL^{-2},$$

which is current per unit area, as required for J. Finally, torque has dimensions ML^2T^{-2} and angle is dimensionless in radians. Thus $\Gamma\theta$ has the dimensions of work, so (D) is also consistent. Only the volume equation fails the required comparison.

Step 3: Conclusion Option (A) equates a volume to a volume flow rate. Hence, the correct option is (A).

Quick Tip to Solve: When a familiar formula looks correct, check the definition of its left-hand symbol: volume and volume per second differ by a factor of time.

45. Angular momentum of a single particle moving with constant speed along circular path : [4 marks]

(A) changes in magnitude but remains same in the direction

(B) remains same in magnitude and direction

(C) remains same in magnitude but changes in the direction

(D) is zero

Correct Answer: (B) remains same in magnitude and direction

Solution:

Concept:

- Angular momentum about the centre of a circular path is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{p} = m\mathbf{v}$. The radius vector is perpendicular to the instantaneous tangential velocity.
- For uniform circular motion, the radius and speed are constant. The right-hand rule fixes the angular-momentum direction perpendicular to the fixed plane of motion.

Step 1: Calculate the angular-momentum magnitude Take the origin at the centre of the circular path, as in the usual statement of this result. If the circle has radius r, the angle between $\mathbf{r}$ and the tangential momentum is 90°. Thus

$$L = rp\sin 90^\circ = mrv.$$

The particle has a fixed mass, its path has a fixed radius, and its speed is constant by hypothesis. All three factors in this product are constant. Although the linear-momentum vector turns continuously, that does not make the angular-momentum magnitude change.

Step 2: Determine the direction and eliminate the alternatives The vectors $\mathbf{r}$ and $\mathbf{p}$ rotate together

in the same circular plane. Their cross product always points along the same normal to that plane: outwards for one sense of revolution and inwards for the other. The sense of revolution does not change during uniform circular motion, so the direction of $\mathbf{L}$ is fixed. The same conclusion follows from torque about the centre. The centripetal force lies along the radius and therefore

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{0}, \qquad \frac{d\mathbf{L}}{dt} = \mathbf{0}.$$

Option (A) incorrectly changes the magnitude, while (C) incorrectly transfers the changing direction of linear momentum to angular momentum. Option (D) is false because the nonzero radius and tangential momentum have a nonzero cross product.

Step 3: Conclusion About the centre of the circular path, angular momentum remains the same in magnitude and direction. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish linear momentum from angular momentum: the former turns in uniform circular motion, while the latter stays normal to the orbital plane.

46. In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is : [4 marks]

(A) $2R^2$

(B) Zero

(C) R

(D) $R\sqrt{2}$

Correct Answer: (C) R

Solution:

Concept:

- The complex impedance of a series LCR circuit is $\mathcal{Z} = R + j(X_L - X_C)$, where inductive and capacitive reactances have opposite signs.
- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Reactive voltages cancel when $X_L = X_C$, but the resistor still opposes the current.

Step 1: Form the net reactance The inductor voltage leads the current by 90°, while the capacitor voltage lags it by 90°. Since the same current flows through all three series components, their reactive voltage phasors are oppositely directed. The circuit's net reactance is consequently their difference:

$$X = X_L - X_C.$$

The given equality is $X_L = R = X_C$. Substitution gives $X = R - R = 0$, not $2R$. Equal reactance magnitudes therefore cancel; they must not be added as positive scalar resistances.

Step 2: Evaluate the remaining impedance Insert the net reactance into the impedance expression:

$$Z = \sqrt{R^2 + 0^2} = R.$$

Equivalently, $\mathcal{Z} = R$, so the applied voltage and the current are in phase. This is the series-resonance condition at the specified frequency. Energy continues to be exchanged between the inductor and capacitor, but their combined reactive contribution to the source voltage is zero. Option (B) would require the actual resistance also to vanish; cancellation of reactance does not remove dissipation in the resistor. Option (D), $R\sqrt{2}$, would arise if the magnitude of the net reactance were R, which it is not. Option (A), $2R^2$, even has units of resistance squared and cannot represent an impedance. Thus only the remaining resistance gives the correct dimension and value.

Step 3: Conclusion The circuit impedance is $\boxed{Z = R}$. Hence, the correct option is (C).

Quick Tip to Solve: In a series LCR circuit subtract the two reactances first, then combine the result with resistance in quadrature.

47. A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option : [4 marks]

(A) $K_p < K_e$ and $P_p = P_e$

(B) $K_p = K_e$ and $P_p = P_e$

(C) $K_p < K_e$ an $P_p < P_e$

(D) $K_p > K_e$ and $P_p = P_e$

Correct Answer: (A) $K_p < K_e$ and $P_p = P_e$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/P$, where P is the magnitude of particle momentum. Equal wavelengths therefore imply equal momentum magnitudes.
- For nonrelativistic motion, kinetic energy is $K = P^2/(2m)$. At fixed momentum it decreases when mass increases; a proton is much heavier than an electron.

Step 1: Compare the momenta Apply the same de Broglie relation to each particle:

$$P_p = \frac{h}{\lambda_p}, \qquad P_e = \frac{h}{\lambda_e}.$$

Since the two wavelengths are equal, their denominators are equal and hence $P_p = P_e = P$. This compares momentum magnitudes; it does not require the two particles to travel in the same spatial direction. The equality already excludes option (C), which assigns unequal momenta.

Step 2: Compare the kinetic energies at fixed momentum Use the kinetic-energy expression separately for the proton and electron:

$$K_p = \frac{P^2}{2m_p}, \qquad K_e = \frac{P^2}{2m_e}.$$

Their ratio is

$$\frac{K_p}{K_e} = \frac{m_e}{m_p} < 1,$$

since m_p is about 1836 times m_e. The heavier particle therefore has the smaller kinetic energy when

both carry the same momentum. Equivalently, the proton has the smaller speed, because $v = P/m$. Option (B) incorrectly assumes that equal momentum also means equal kinetic energy. Option (D) reverses the inverse dependence on mass, confusing fixed momentum with fixed speed. Option (A) satisfies both required comparisons. The familiar nonrelativistic expression is sufficient here; the qualitative energy ordering at fixed momentum also persists in the relativistic expression.

Step 3: Conclusion $\boxed{K_p < K_e \text{ and } P_p = P_e}$. Hence, the correct option is (A).

Quick Tip to Solve: For equal de Broglie wavelengths hold momentum fixed, then use $K = P^2/(2m)$; do not compare energies as if the speeds were equal.

48. Consider a galvanometer shunted with 5Ω resistance and 2% of current passes through it. What is the resistance of the given galvanometer ? [4 marks]

(A) 300 Ω

(B) 344 Ω

(C) 245 Ω

(D) 226 Ω

Correct Answer: (C) 245 Ω

Solution:

Concept:

- A shunt is connected in parallel with the galvanometer. Both branches have the same potential difference, so $I_g G = I_s S$, where G and S are the galvanometer and shunt resistances.
- The total current divides as $I = I_g + I_s$. The given two percent is the galvanometer current fraction, so the shunt carries the remaining ninety-eight percent.

Step 1: Express both branch currents Let the total current entering the parallel combination be I. The current through the galvanometer is

$$I_g = \frac{2}{100} I = 0.02I.$$

By conservation of current at the junction, the current through the shunt is

$$I_s = I - I_g = 0.98I.$$

The currents must be compared as branch currents. In particular, the ratio needed below is $0.98/0.02$, rather than $1/0.02$, because the total current does not flow through the shunt alone.

Step 2: Use equal voltage across the parallel branches The shunt resistance is $S = 5\,\Omega$. Equating the two potential differences and solving for G gives

$$I_g G = I_s S, \qquad G = \frac{I_s}{I_g} S = \frac{0.98I}{0.02I} \times 5\,\Omega = 49 \times 5\,\Omega = 245\,\Omega.$$

The unknown total current cancels, as expected for a current-division question expressed entirely

in percentages. The galvanometer resistance is much higher than the shunt resistance, consistent with most of the current bypassing the delicate instrument. A direct check is $I_g/I = S/(G+S) = 5/(245+5) = 0.02$. The other proposed resistances do not reproduce the required two-percent fraction: 300 and 344 ohms give smaller fractions, whereas 226 ohms gives a larger fraction. The result is therefore uniquely consistent with both the parallel-voltage condition and the current split.

Step 3: Conclusion The galvanometer resistance is $\boxed{245\,\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: When a fraction f of total current enters a galvanometer, its resistance is $G = S(1-f)/f$; use the complementary fraction for the shunt.

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49. A reversible engine has an efficiency of $\frac{1}{4}$. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink : [4 marks]

(A) 174°C

(B) 280°C

(C) 180.4°C

(D) 382°C

Correct Answer: No option is correct as printed: the sink is at 174 K, whereas option (A) states 174 degrees Celsius.

Solution:

Concept:

- A reversible engine operating between a hot reservoir at T_h and a cold reservoir at T_c has efficiency $\eta = 1 - T_c/T_h$. Reservoir temperatures in this ratio must be in kelvin.
- A decrease of $58°\mathrm{C}$ is a decrease of $58\,\mathrm{K}$. Doubling the stated efficiency changes it from $1/4$ to $1/2$, with the hot-reservoir temperature unchanged.

Step 1: Write the two efficiency conditions The original efficiency implies

$$\frac{1}{4} = 1 - \frac{T_c}{T_h}, \qquad T_c = \frac{3}{4}T_h.$$

After the sink is cooled, its temperature is $T_c - 58$ in kelvin. The doubled efficiency gives

$$\frac{1}{2} = 1 - \frac{T_c - 58}{T_h}, \qquad T_c - 58 = \frac{1}{2}T_h.$$

Subtract the latter relation from the former. This yields $58 = T_h/4$, so $T_h = 232\,\mathrm{K}$. The original sink temperature is therefore $T_c = (3/4)232 = 174\,\mathrm{K}$.

Step 2: Check the temperatures and the printed choices The cooled sink is at $174 - 58 = 116\,\mathrm{K}$. Substitution verifies both efficiencies exactly:

$$1 - \frac{174}{232} = \frac{1}{4}, \qquad 1 - \frac{116}{232} = \frac{1}{2}.$$

Convert the original temperature to Celsius only after using the absolute-temperature equations:

$$t_c = 174 - 273.15 = -99.15^\circ\text{C}.$$

Every printed option is a positive Celsius temperature. In particular, (A) prints 174°C, whereas the calculated number 174 is a kelvin temperature, not a Celsius temperature. The supplied key selects (A), but that unit label does not agree with the literal calculation. The other three printed Celsius values also fail the two efficiency equations. Thus there is no correct option as printed; treating 174 kelvin as 174 degrees Celsius would change the physical temperature.

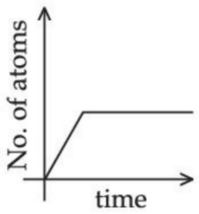
Step 3: Conclusion The original sink temperature is $\boxed{174\,\text{K} = -99.15^\circ\text{C}}$. No printed option has that temperature. The key selects (A), but its printed Celsius unit is inconsistent with the result; (A) would agree only if its unit were kelvin.

Quick Tip to Solve: Always evaluate heat-engine temperature ratios in kelvin. A Celsius temperature difference equals a kelvin difference, but their absolute temperatures are different.

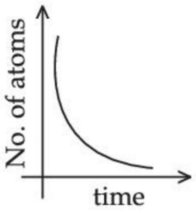
50. A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B versus time is :

(Assume that at t = 0, there are no B atoms in the sample) [4 marks]

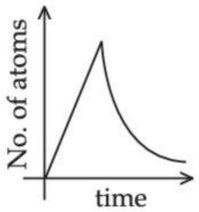
(A)



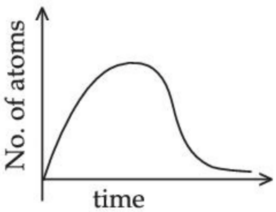
(B)



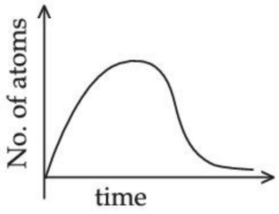
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- In a successive decay chain $A \to B \to C$, the intermediate species B is both produced by A and removed by its own decay. Its population is governed by the difference between these two rates.
- For decay constants $\lambda_A, \lambda_B > 0$, $N_A = N_0 e^{-\lambda_A t}$ and $dN_B/dt = \lambda_A N_A - \lambda_B N_B$. The condition $N_B(0) = 0$ fixes the initial population.

Step 1: Check the beginning of the graph Initially B is absent, so its own decay rate is zero, whereas A is present and produces B. Consequently

$$N_B(0) = 0, \qquad \left.\frac{dN_B}{dt}\right|_0 = \lambda_A N_0 > 0.$$

The correct curve must therefore start at the origin and rise. Option B starts with a nonzero B population and decreases, contradicting the stated initial condition.

Step 2: Follow production and loss at later times As A decays its production rate decreases. Meanwhile the increasing B population increases the loss rate. At the maximum, $\lambda_A N_A = \lambda_B N_B$; afterwards loss exceeds production. For unequal decay constants the exact result is

$$N_B = \frac{\lambda_A N_0}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right).$$

Both exponentials tend to zero, so B ultimately disappears into stable C. If the constants are equal, the result becomes $N_B = \lambda N_0 t e^{-\lambda t}$, with the same qualitative behaviour.

Step 3: Select the smooth rise and fall The rate equation gives a continuous slope, including zero slope at the maximum. Option A incorrectly leaves a permanent nonzero population. Option C draws a sharp corner at its peak, inconsistent with continuous decay rates. Option D shows a smooth rise from zero, a rounded maximum and an eventual fall towards zero, as required.

Step 4: Conclusion The number of B nuclei first increases, reaches a smooth maximum and then approaches zero. Hence, the correct option is (D).

Quick Tip to Solve: For an intermediate radioactive nucleus, compare its production rate with its decay rate; equality identifies the smooth maximum.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. When a rubber ball is taken to a depth of _____m in deep sea, its volume decreases by 0.5%. (The bulk modulus of rubber = 9.8 × 10^8 Nm^{-2}, Density of sea water = 10^3 kgm^{-3}, g = 9.8 m/s^2)

[4 marks]

Solution:

Concept:

- Bulk modulus measures the pressure increase needed for a fractional decrease in volume: $K = -\Delta P/(\Delta V/V)$. The minus sign expresses that compression reduces the volume.
- At depth h, the increase in water pressure relative to the surface is $\Delta P = \rho g h$, for approximately constant water density ρ.

Step 1: Express the compression as a fraction The volume decreases by 0.5%, so the magnitude of the volumetric strain is

$$\frac{|\Delta V|}{V} = \frac{0.5}{100} = 0.005.$$

This is a dimensionless fraction; inserting 0.5 directly would overestimate the pressure and depth by a factor of one hundred. Use the given bulk modulus of rubber, not an elastic modulus for a change in length.

Step 2: Calculate the required pressure increase Using the magnitude form of the bulk-modulus relation gives

$$\Delta P = K\frac{|\Delta V|}{V} = (9.8 \times 10^8)(0.005) = 4.9 \times 10^6\ \text{Pa}.$$

This is the additional pressure acting on the ball when it is lowered below the surface. Atmospheric pressure already acts on the ball at its initial position and therefore cancels when we take the pressure difference.

Step 3: Convert the pressure increase into depth Equate the calculated increase to the hydrostatic pressure:

$$h = \frac{\Delta P}{\rho g} = \frac{4.9 \times 10^6}{(10^3)(9.8)} = 500\,\text{m}.$$

The denominator has units of pressure per metre, so the quotient is a length. Substitution checks the result: a depth of 500 m supplies 4.9×10^6 Pa, which is 0.005 times the bulk modulus and produces the stated compression.

Step 4: Conclusion The required depth is $\boxed{500\,\text{m}}$. The numerical entry is 500.

Quick Tip to Solve: Convert percentage volume compression into a decimal first, then use the single relation $h = K(|\Delta V|/V)/(\rho g)$.

52. A particle of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1}. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For oscillations about the vertical spring equilibrium, displacement y obeys $y = A\cos\omega t$ when the particle is released from an extreme position. Here $\omega = \sqrt{k/m} = 2\pi/T$.
- The oscillation energy is $E = \frac{1}{2}kA^2$. Relative to equilibrium, $U = \frac{1}{2}ky^2$ and $K = E - U$; gravitational and spring potential changes together give this effective potential.

Step 1: Set the release position and energy expressions The mass is pulled slightly downward and released from rest. Take that instant as $t = 0$, with downward displacement amplitude A measured from the equilibrium position. Therefore

$$y = A\cos\omega t, \qquad U = E\cos^2\omega t, \qquad K = E\sin^2\omega t.$$

The equilibrium extension caused by gravity only shifts the reference position; it does not alter the small-oscillation angular frequency or these energy fractions.

Step 2: Find the first equality after release For kinetic and potential energies to be equal, each must be half the total energy. Thus

$$\sin^2\omega t = \cos^2\omega t, \qquad \cos^2\omega t = \frac{1}{2}.$$

Starting from $\omega t = 0$, the first positive phase satisfying this condition is $\omega t = \pi/4$. This corresponds to $|y| = A/\sqrt{2}$, not to $y = A/2$, because potential energy depends on the square of displacement.

Step 3: Compare the time with the given form Use $\omega = 2\pi/T$:

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\frac{T}{2\pi} = \frac{T}{8}.$$

Since the question writes this first time as T/x, comparison gives $x = 8$. As a check, the given mass and force constant give $\omega = 10\,s^{-1}$, so $T = \pi/5\,s$ and the equality occurs at $\pi/40\,s$. Later equalities recur during the oscillation; the requested value uses the first one.

Step 4: Conclusion The first equality occurs at $t = T/8$, so $\boxed{x = 8}$.

Quick Tip to Solve: When release is from an extreme position, equal SHM energies occur first after a phase change of $\pi/4$, or one-eighth of a period.

53. A square shaped wire with resistance of each side 3Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be _____.

[4 marks]

Solution:

Concept:

- For a uniform wire, $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area. Bending the same wire without stretching preserves its total resistance.
- Two diametrically opposite connection points divide a uniform circular wire into equal semicircles. These paths connect the same two terminals and therefore act in parallel.

Step 1: Find the resistance of the complete wire Before bending, the wire forms a square with four equal sides. Each side has resistance $3\,\Omega$. The total resistance along the full length is therefore

$$R_{\text{wire}} = 4 \times 3 = 12\,\Omega.$$

This is the resistance of the entire material length, before choosing the terminals on the new circle. Changing its shape alone does not create a different resistivity or change the amount of wire.

Step 2: Identify the two conducting paths Diametrically opposite points are separated by half the circumference in either direction. Each semicircle contains half the original wire length and hence has resistance

$$R_1 = R_2 = \frac{12}{2} = 6\,\Omega.$$

A current entering one terminal splits between the two semicircular arcs and recombines at the other terminal. The arcs are not in series, because current does not have to traverse one arc and then the other to get between the connection points.

Step 3: Combine the semicircles in parallel The reciprocal rule for parallel resistances gives

$$\frac{1}{R_{\text{eq}}} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}, \qquad R_{\text{eq}} = 3\,\Omega.$$

Equivalently, two equal resistances in parallel have half the value of either branch. The resulting resistance is smaller than the resistance of one semicircle, as a parallel combination must be. Treating the full 12-ohm circle as one series path would ignore the second available conducting route.

Step 4: Conclusion The resistance between diametrically opposite points is $\boxed{3\,\Omega}$. The numerical entry is 3.

Quick Tip to Solve: For any uniform closed wire of total resistance R, terminals dividing it into equal halves give $(R/2) \parallel (R/2) = R/4$.

54. A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is _____ m. [4 marks]

Solution:

Concept:

- A string fixed at both ends supports standing waves with $L = n\lambda_n/2$. Its normal-mode frequencies are $f_n = nv/(2L)$, where n is a positive integer.
- The transverse wave speed is $v = \sqrt{F/\mu}$, where F is tension and μ is mass per unit length. Adjacent allowed frequencies differ by the fundamental frequency $v/(2L)$.

Step 1: Extract the fundamental frequency from the spacing The two stated resonances are consecutive because 550 Hz is the next higher resonance above 500 Hz. Thus

$$f_1 = f_{n+1} - f_n = 550 - 500 = 50\,\text{Hz}.$$

The 500 Hz frequency is not the fundamental. Indeed, it equals ten times 50 Hz, so these are the tenth and eleventh harmonics. The rigid supports impose the same fixed-end boundary condition at both ends, allowing all integer harmonics.

Step 2: Calculate the speed of waves on the wire Insert the supplied tension and linear density:

$$v = \sqrt{\frac{900}{9.0 \times 10^{-4}}} = \sqrt{10^6} = 1000\,\text{m}\,\text{s}^{-1}.$$

The exponent in the density is negative; using 9.0×10^4 would reverse the scale of the speed. Dimensionally, tension divided by linear density has units of squared speed, as required.

Step 3: Determine and check the wire length For the fundamental mode, $f_1 = v/(2L)$. Therefore

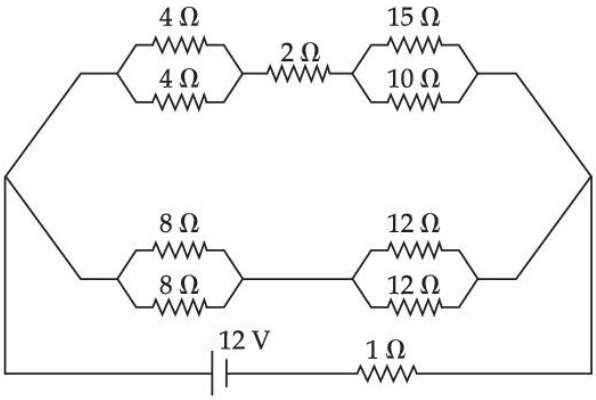
$$L = \frac{v}{2f_1} = \frac{1000}{2 \times 50} = 10\,\text{m}.$$

Checking against the observed resonances gives $f_{10} = 10(50) = 500\,\text{Hz}$ and $f_{11} = 11(50) = 550\,\text{Hz}$. Both are reproduced by the same length, so the interpretation of the frequency spacing is self-consistent.

Step 4: Conclusion The length of the wire is $\boxed{10\,\text{m}}$. The numerical entry is 10.

Quick Tip to Solve: For two consecutive resonances of a fixed-end string, subtract their frequencies to get the fundamental before calculating the length.

55. The voltage drop across 15Ω resistance in the given figure will be _____ V. [4 marks]



Solution:

Concept:

- Parallel resistors have the same potential difference. Their equivalent resistance satisfies $1/R_p = 1/R_1 + 1/R_2$, whereas series resistances add.
- Ohm's law gives $V = IR$. First reduce each branch between the two common junctions, then include the external $1\,\Omega$ series resistor when finding the source current.

Step 1: Reduce the upper and lower branches In the upper branch, the pair of 4-ohm resistors gives $4 \parallel 4 = 2\,\Omega$. The 15-ohm and 10-ohm pair gives

$$15 \parallel 10 = \frac{15 \times 10}{15 + 10} = 6\,\Omega.$$

Including the central 2-ohm resistor, the upper branch totals $2 + 2 + 6 = 10\,\Omega$. In the lower branch, $8 \parallel 8 = 4\,\Omega$ and $12 \parallel 12 = 6\,\Omega$, so this branch also totals $10\,\Omega$.

Step 2: Find the voltage across the branch network The upper and lower 10-ohm branches connect the same left and right junctions. Their equivalent is $10 \parallel 10 = 5\,\Omega$. The 1-ohm resistor beside the 12 V source is in series with this equivalent, hence

$$R_{\text{total}} = 5 + 1 = 6\,\Omega, \qquad I_{\text{source}} = \frac{12}{6} = 2\,\text{A}.$$

The potential difference across the two-branch network is therefore $2 \times 5 = 10\,\text{V}$. It is not 12 V because the series resistor also has a voltage drop.

Step 3: Obtain the drop across the 15-ohm resistor The upper branch current is $I_u = 10/10 = 1\,\text{A}$. Its right-hand parallel pair has equivalent resistance $6\,\Omega$, so its common voltage is

$$V_{15} = V_{10} = I_u(6) = 6\,\text{V}.$$

For a check, the separate currents are $6/15 = 0.4\,\text{A}$ and $6/10 = 0.6\,\text{A}$, which sum to the 1 A upper-branch current. The other two series blocks drop 2 V each, giving the correct branch total of 10 V.

Step 4: Conclusion The voltage drop across the $15\,\Omega$ resistor is $\boxed{6\,\text{V}}$. The numerical entry is 6.

Quick Tip to Solve: Reduce the parallel pairs first; remember that the 1-ohm resistor reduces the voltage available across the entire two-branch network.

56. A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be x : 4 where x = _____. [4 marks]

Solution:

Concept:

- In the absence of an external horizontal impulse, total linear momentum is conserved during splitting. Internal forces may change kinetic energy even though total momentum stays constant.
- The kinetic energy of a mass M moving at speed v is $Mv^2/2$. The fractional change is $(K_f - K_i)/K_i$, so its sign identifies an increase or decrease.

Step 1: Find the velocity of the other half Let the original mass be M and choose the initial direction as positive. After splitting, each piece has mass $M/2$. Conservation of horizontal momentum gives

$$40M = \frac{M}{2}(60) + \frac{M}{2}v_2.$$

Cancel M and multiply by two to obtain $80 = 60 + v_2$, hence $v_2 = 20\,\mathrm{m\,s^{-1}}$. Both pieces therefore move in the original direction. The smooth surface ensures that no horizontal frictional impulse needs to be included.

Step 2: Calculate initial and final kinetic energies The original kinetic energy is

$$K_i = \frac{1}{2}M(40)^2 = 800M.$$

The final energy must include both fragments, each with half the original mass:

$$K_f = \frac{1}{2}\frac{M}{2}(60)^2 + \frac{1}{2}\frac{M}{2}(20)^2 = 900M + 100M = 1000M.$$

Thus the kinetic energy has increased by $200M$. This does not contradict momentum conservation: energy stored internally can be released during the splitting process.

Step 3: Express the increase in the required ratio Divide the change by the original energy, rather than by the final energy:

$$\frac{\Delta K}{K_i} = \frac{1000M - 800M}{800M} = \frac{1}{4}.$$

The unknown mass cancels, as it should for a fractional change when equal pieces are specified. The given form is $x : 4$, meaning $x/4$. Comparing with $1/4$ yields $x = 1$. The positive sign confirms an increase, corresponding to 25 percent of the original kinetic energy.

Step 4: Conclusion The fractional kinetic-energy increase is $1/4$, so $\boxed{x = 1}$.

Quick Tip to Solve: Use momentum conservation to find the unknown fragment velocity first; kinetic energy need not be conserved when a body splits.

57. The electric field in an electromagnetic wave is given by E = (50 NC^{-1}) sinω (t − x/c) The energy contained in a cylinder of volume V is 5.5 × 10^{-12} J. The value of V is _____ cm^3. (given $\in_0$ = 8.8 × $10^{-12}C^2N^{-1}m^{-2}$) [4 marks]

Solution:

Concept:

- In a plane electromagnetic wave, electric and magnetic fields store equal amounts of energy. The total instantaneous energy density is $u = \varepsilon_0 E^2$, not just the electric contribution $\frac{1}{2}\varepsilon_0 E^2$.
- For a sinusoidal field of amplitude E_0, averaging over a cycle gives $\langle E^2 \rangle = E_0^2/2$. Therefore the mean total energy in a volume V is $U = \frac{1}{2}\varepsilon_0 E_0^2 V$.

Step 1: Identify the field amplitude The coefficient of the sine function is the amplitude, so $E_0 = 50\,\mathrm{N\,C^{-1}}$. The argument of the sine specifies the travelling-wave phase; it does not change this amplitude. The quoted wave energy is evaluated using the cycle-averaged energy density of the electromagnetic wave.

Step 2: Calculate the mean total energy density Include both the electric and magnetic energies before averaging. With the given permittivity,

$$\langle u \rangle = \frac{1}{2}(8.8 \times 10^{-12})(50)^2 = 1.1 \times 10^{-8}\,\mathrm{J\,m^{-3}}.$$

Using the maximum energy density instead would miss the sinusoidal averaging factor.

Step 3: Find the volume and convert its unit Energy equals mean energy density times volume. Thus

$$V = \frac{5.5 \times 10^{-12}}{1.1 \times 10^{-8}} = 5.0 \times 10^{-4}\,\mathrm{m^3}.$$

Since $1\,\mathrm{m} = 100\,\mathrm{cm}$, a cubic metre contains $10^6\,\mathrm{cm^3}$. Therefore

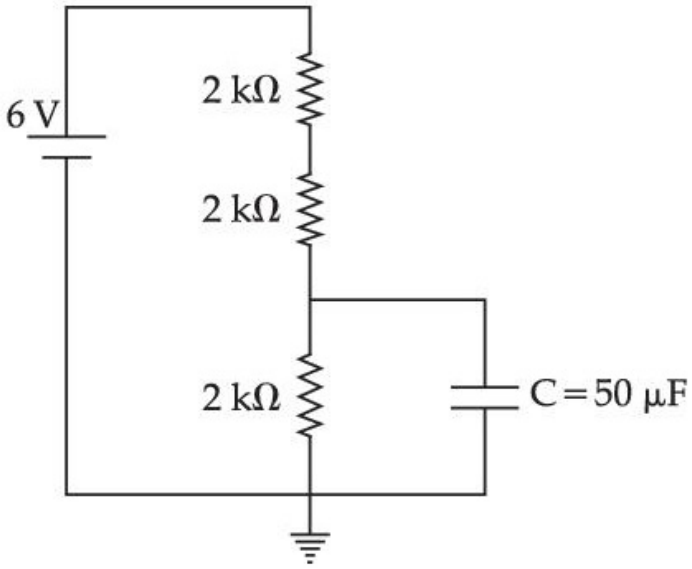
$$V = (5.0 \times 10^{-4})(10^6) = 500\,\mathrm{cm^3}.$$

The cylinder shape is irrelevant because the energy calculation depends on its volume.

Step 4: Conclusion The required volume is $\boxed{500\,\mathrm{cm^3}}$; enter 500.

Quick Tip to Solve: Use the mean total wave energy density, $\varepsilon_0 E_0^2/2$, and convert cubic metres to cubic centimetres with a factor of 10^6.

58. A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is _____μC. [4 marks]



Solution:

Concept:

- After a capacitor connected to a direct-current source has reached steady state, no conduction current passes through its branch. Its voltage is nevertheless fixed by the potentials of its two connection points.
- The charge on a specified capacitor plate is $Q = C(V_{\text{plate}} - V_{\text{other}})$. Three identical resistors in series divide the source voltage equally when the capacitor branch is open at steady state.

Step 1: Reduce the steady-state circuit The capacitor is connected across the lowest $2\,\text{k}\Omega$ resistor. At steady state its branch carries no current, so the source drives current through all three $2\,\text{k}\Omega$ resistors in series. The capacitor does not short out that resistor. The total resistance seen by the $6\,\text{V}$ battery is

$$R_{\text{total}} = 2 + 2 + 2 = 6\,\text{k}\Omega.$$

Step 2: Find the potential of the upper plate The series current is

$$I = \frac{6\,\text{V}}{6\,\text{k}\Omega} = 1\,\text{mA}.$$

The voltage across the bottom resistor is therefore

$$V_{\text{bottom}} = I(2\,\text{k}\Omega) = 2\,\text{V}.$$

The battery has its positive terminal at the upper node. Hence the upper capacitor plate, connected to the top of the bottom resistor, is at $+2\,\text{V}$ relative to the grounded lower plate.

Step 3: Calculate the signed plate charge The two capacitor plates have a potential difference of $2\,\text{V}$. Thus

$$Q_{\text{upper}} = (50\,\mu\text{F})(2\,\text{V}) = +100\,\mu\text{C}.$$

The positive sign follows from the higher potential of the upper plate. The lower plate carries equal negative charge; the numerical response asks specifically for the upper one.

Step 4: Conclusion The charge on the upper plate is $\boxed{+100\,\mu\text{C}}$; enter 100.

Quick Tip to Solve: Open the capacitor branch for steady DC, then find the voltage between its actual connection nodes before applying $Q = CV$.

---

59. A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is _____. (Take g = 10 ms^{-2}) [4 marks]

Solution:

Concept:

- In the accelerating frame of the car, the bob experiences a pseudo force $-m\mathbf{a}$, opposite to the acceleration of the car. The bob rests relative to the car when tension balances gravity and this pseudo force.
- The equilibrium string points along the effective gravitational field $\mathbf{g}_{\text{eff}} = \mathbf{g} - \mathbf{a}$. Its angle with the true vertical follows from the ratio of the horizontal and downward components of this field.

Step 1: Resolve the acceleration of the car Take the horizontal direction uphill as positive x, and the vertically upward direction as positive y. The acceleration has magnitude $a = 10\,\text{m}\,\text{s}^{-2}$ at 30° above the horizontal. Therefore

$$a_x = 10\cos 30^\circ = 5\sqrt{3}\,\text{m}\,\text{s}^{-2}, \qquad a_y = 10\sin 30^\circ = 5\,\text{m}\,\text{s}^{-2}.$$

The acceleration is along the incline, rather than vertically upward.

Step 2: Form the effective force components The pseudo acceleration has a backward horizontal component $5\sqrt{3}$ and a downward vertical component 5, in SI units. Gravity adds another downward component $g = 10$. Hence

$$|g_{\text{eff},x}| = 5\sqrt{3}, \qquad |g_{\text{eff},y}| = 10 + 5 = 15.$$

The string tilts backward from the vertical because the bob hangs opposite to the forward acceleration.

Step 3: Calculate the angle with the vertical If θ is the acute angle between the string and the downward vertical, equilibrium gives

$$\tan\theta = \frac{|g_{\text{eff},x}|}{|g_{\text{eff},y}|} = \frac{5\sqrt{3}}{15} = \frac{1}{\sqrt{3}}.$$

Thus $\theta = 30^\circ$. The mass cancels, as expected: the same tilt occurs for every bob mass in this accelerating frame.

Step 4: Conclusion The string makes an angle $\boxed{30^\circ}$ with the vertical; enter 30.

Quick Tip to Solve: For uphill acceleration, add $a\sin\alpha$ to g in the denominator: $\tan\theta = a\cos\alpha/(g + a\sin\alpha)$.

60. If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is _____ km. (Take radius of Earth = 6400 km) [4 marks]

Solution:

Concept:

- For an antenna of height $h \ll R$, the distance to its horizon is approximately $\sqrt{2Rh}$, where R is the radius of Earth. Two antennas can communicate along a line of sight up to the sum of their horizon distances.
- With a fixed total height $h_t + h_r$, the quantity $\sqrt{h_t} + \sqrt{h_r}$ is largest when the two heights are equal. Consistent length units must be used under every square root.

Step 1: Write the range for two antennas The maximum separation allowing an unobstructed line of sight is

$$d = \sqrt{2Rh_t} + \sqrt{2Rh_r} = \sqrt{2R}(\sqrt{h_t} + \sqrt{h_r}).$$

Here h_t and h_r denote transmitting and receiving antenna heights. The given constraint is $h_t + h_r = 160\,\text{m}$. Increasing only one height while reducing the other does not necessarily increase the range.

Step 2: Find the optimal division of the height Squaring the height-dependent factor gives

$$(\sqrt{h_t} + \sqrt{h_r})^2 = h_t + h_r + 2\sqrt{h_t h_r}.$$

The sum is fixed, while $h_t h_r \le [(h_t + h_r)/2]^2$, with equality only when $h_t = h_r$. The maximum therefore occurs for $h_t = h_r = 80\,\text{m}$, rather than putting all $160\,\text{m}$ into one antenna.

Step 3: Evaluate the maximum range Convert the optimal heights to kilometres: $80\,\text{m} = 0.080\,\text{km}$. Using $R = 6400\,\text{km}$,

$$d_{\max} = 2\sqrt{2(6400)(0.080)}\,\text{km} = 2\sqrt{1024}\,\text{km} = 64\,\text{km}.$$

Each antenna contributes a horizon distance of $32\,\text{km}$, giving the stated total separation.

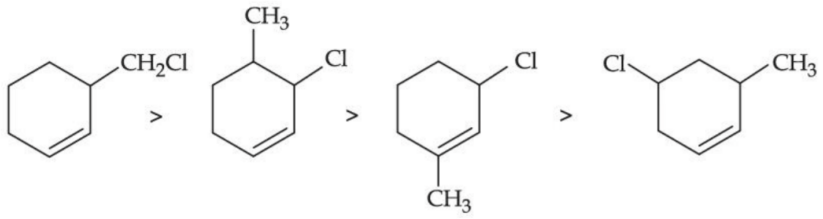
Step 4: Conclusion The maximum LOS communication range is $\boxed{64\,\text{km}}$; enter 64.

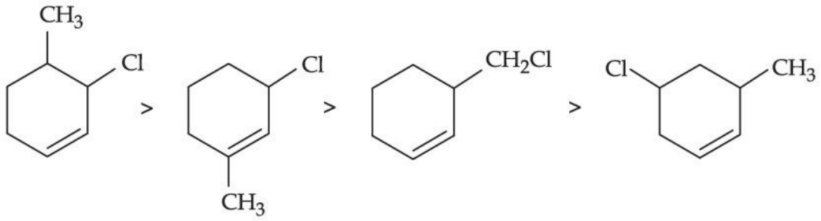
Quick Tip to Solve: For a fixed sum of antenna heights, split the height equally before using the sum of the two horizon distances.

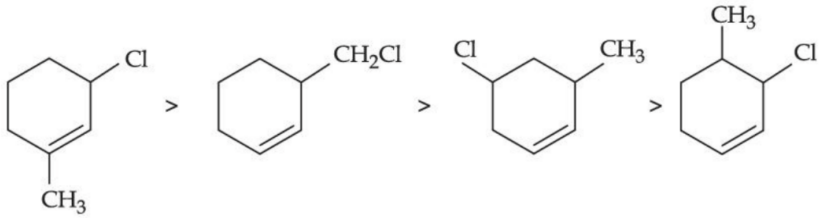
Chemistry – Section A: Multiple Choice Questions (80 marks)

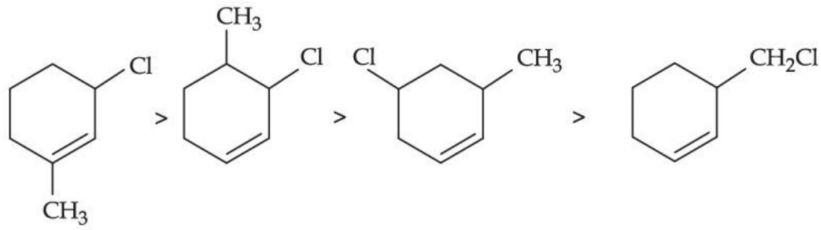
61. The correct order of reactivity of the given chlorides with acetate in acetic acid is : [4 marks]

(A)

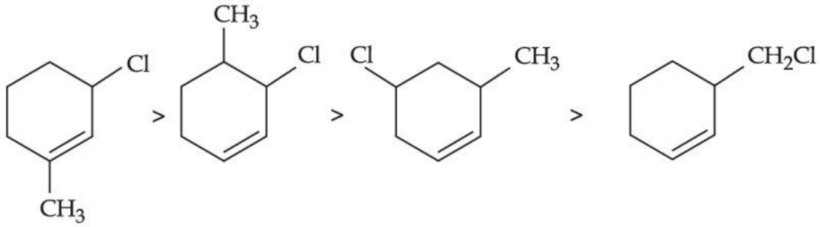


(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- Acetate in acetic acid is a relatively weak nucleophile in a polar protic medium. The relative substitution rates of these chlorides are governed chiefly by the ease of ionisation and the stability of the developing carbocation.
- An allylic carbocation is resonance-stabilised. A methyl substituent at a carbon bearing positive charge in a resonance contributor provides additional hyperconjugative stabilisation; a primary non-allylic chloride ionises least readily.

Step 1: Compare the two allylic chlorides Identify the chloride bonded directly to an allylic ring carbon, with a methyl group on the adjacent double-bond carbon. Loss of chloride gives a secondary allylic carbocation whose other resonance contributor is tertiary. This is the fastest substrate. The other allylic ring chloride has its methyl group on a saturated ring carbon; its allylic resonance contributors do not gain the same tertiary stabilisation. It therefore reacts next.

Step 2: Rank the non-allylic chlorides The ring chloride with the remote methyl group has its carbon bearing chlorine separated from the double bond by a saturated carbon. It is a non-allylic secondary chloride. The chloromethyl-substituted cyclohexene has chlorine on an exocyclic primary carbon;

its incipient positive charge is also not directly conjugated with the double bond. In this ionisation-favouring medium, the secondary non-allylic chloride reacts faster than this primary chloride.

Step 3: Match the complete sequence to the options The decreasing order is thus: allylic chloride giving a tertiary resonance contributor, then the other secondary allylic chloride, then the secondary non-allylic ring chloride, and finally the primary chloromethyl compound. Option (D) shows exactly this sequence. Option (A) incorrectly puts the primary chloride first; (B) places it ahead of a secondary non-allylic chloride and reverses the allylic comparison; (C) also puts it too high.

Step 4: Conclusion The reactivity sequence is the one shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: Before comparing primary and secondary labels, check whether the carbon losing chloride is allylic and draw both carbocation resonance contributors.

62. In the structure of the dichromate ion, there is a : [4 marks]

(A) linear symmetrical Cr-O-Cr bond.

(B) non-linear symmetrical Cr-O-Cr bond.

(C) linear unsymmetrical Cr-O-Cr bond.

(D) non-linear unsymmetrical Cr-O-Cr bond.

Correct Answer: (B) non-linear symmetrical Cr-O-Cr bond.

Solution:

Concept:

- The dichromate ion, $Cr_2O_7^{2-}$, consists of two tetrahedral chromium–oxygen units sharing one oxygen atom. Six oxygen atoms are terminal, while the seventh oxygen bridges the chromium centres.
- The chromium centres are equivalent and both have oxidation state $+6$. The bridging $Cr-O-Cr$ unit has equivalent chromium–oxygen links but a bent, rather than linear, arrangement.

Step 1: Identify the bridging oxygen atom Count the atoms in two CrO_4 tetrahedra. Sharing one oxygen gives $2 \times 4 - 1 = 7$ distinct oxygen atoms, as required for $Cr_2O_7^{2-}$. This shared oxygen makes a $Cr-O-Cr$ bridge between the two chromium centres. It should not be confused with a direct chromium–chromium bond or with two independent chromate ions.

Step 2: Determine the shape and equivalence of the bridge The angle at the bridging oxygen is about 126°, so the chromium centres and the bridging oxygen are not collinear. Thus the bridge is non-linear. Both sides of the bridge join equivalent $Cr(VI)$ centres in equivalent tetrahedral environments, making the two bridging $Cr-O$ bonds equivalent. This is what symmetrical means in the given alternatives; it does not mean that the bridge must be straight.

Step 3: Eliminate the incompatible descriptions Options (A) and (C) describe a linear bridge and therefore conflict with the bent dichromate structure. Option (D) correctly says non-linear but incorrectly makes the two sides of the bridge inequivalent. Option (B) is the only description that combines the bent geometry with the equivalent chromium–oxygen links.

Step 4: Conclusion The dichromate ion has a non-linear symmetrical $\mathrm{Cr-O-Cr}$ bridge. Hence, the correct option is (B).

Quick Tip to Solve: Remember dichromate as two tetrahedra sharing one oxygen: its bridge is bent, but the two chromium centres are equivalent.

63. Which one of the following compounds contains β-C_1-C_4 glycosidic linkage? [4 marks]

(A) Lactose

(B) Sucrose

(C) Maltose

(D) Amylose

Correct Answer: (A) Lactose

Solution:

Concept:

- A glycosidic linkage joins sugar units through an oxygen atom. Its numbers specify the participating carbon atoms, while α or β specifies the configuration at the anomeric carbon forming that bond.
- Lactose consists of galactose and glucose joined by a $\beta(1 \rightarrow 4)$ linkage. Maltose and amylose instead have $\alpha(1 \rightarrow 4)$ linkages, while sucrose links the anomeric carbons of glucose and fructose.

Step 1: Identify the linkage in lactose Lactose is composed of a galactose unit and a glucose unit. The anomeric carbon, C-1, of the galactose residue is in the beta configuration and is joined through oxygen to C-4 of glucose. Consequently, its glycosidic bond is described as β-C-1–C-4, or equivalently $\beta(1 \rightarrow 4)$. This directly satisfies both the configuration and the carbon positions asked for in the question.

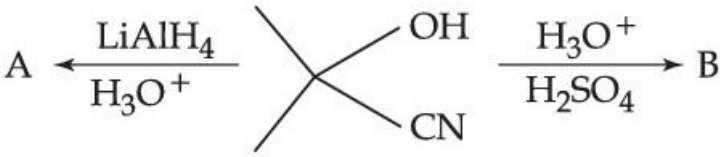
Step 2: Check sucrose against the carbon positions Sucrose contains glucose and fructose. Its glycosidic bond joins C-1 of alpha-glucose to C-2 of beta-fructose, using both anomeric centres. Thus it has a $(1 \rightarrow 2)$ connection, not a C-1–C-4 connection. The presence of a beta-fructose unit by itself does not make sucrose the requested carbohydrate.

Step 3: Distinguish maltose and amylose by configuration Maltose contains two glucose units linked through an $\alpha(1 \rightarrow 4)$ bond. Amylose is an unbranched chain of glucose residues with the same alpha linkage repeated along the chain. Although both have the required carbon positions, their alpha configuration is different from the beta configuration specified. Lactose is therefore the sole match among all four options.

Step 4: Conclusion Lactose contains the β-C-1–C-4 glycosidic linkage. Hence, the correct option is (A).

Quick Tip to Solve: Check both pieces of a linkage name: lactose is beta-1,4; maltose and amylose are alpha-1,4; sucrose joins carbons 1 and 2.

64. The major products A and B in the following set of reactions are : [4 marks]



(A) $A = (CH_3)_2C(OH)CHO, \quad B = (CH_3)_2C(OH)CO_2H$

(B) $A = (CH_3)_2C(OH)CH_2NH_2, \quad B = CH_2{=}C(CH_3)COOH$

(C) $A = (CH_3)_2C(OH)CH_2OH, \quad B = (CH_3)_2C(OH)CHO$

(D) $A = (CH_3)_2C(OH)CH_2NH_2, \quad B = (CH_3)_2C(OH)CHO$

Correct Answer: (B) $A = (CH_3)_2C(OH)CH_2NH_2, \quad B = CH_2{=}C(CH_3)COOH$

Solution:

Concept:

- A nitrile group, $-CN$, gives a primary amine $-CH_2NH_2$ on reduction with $LiAlH_4$ followed by aqueous work-up; its carbon remains in the chain.
- Acidic hydrolysis converts a nitrile into a carboxylic acid. Concentrated H_2SO_4 can also dehydrate a tertiary alcohol to a stable alkene.

Step 1: Identify the functional groups The starting cyanohydrin is $(CH_3)_2C(OH)CN$. Its central carbon bears two methyl groups, a hydroxyl group and the nitrile carbon. Treat the two arrows as independent reactions of this same substrate; B is not obtained from A.

Step 2: Apply the reducing reagent Lithium aluminium hydride adds hydrogen across the nitrile functionality. After protonation in the work-up the product is

$$A = (CH_3)_2C(OH)CH_2NH_2.$$

The alcohol survives the overall reduction and work-up. An aldehyde would require a different, selective partial reduction, so the proposed aldehyde A and the diol A are excluded.

Step 3: Hydrolyse and dehydrate on the acid pathway Acidic hydrolysis first replaces the nitrile with a carboxyl group, giving $(CH_3)_2C(OH)COOH$. Under the stated sulphuric-acid conditions this hydroxy acid loses water to give

$$B = CH_2{=}C(CH_3)COOH.$$

Loss of a proton from either equivalent methyl group gives the same conjugated unsaturated acid. An aldehyde is not the product of nitrile hydrolysis, which also rules out the final alternative.

Step 4: Conclusion $A = (CH_3)_2C(OH)CH_2NH_2$ and $B = CH_2{=}C(CH_3)COOH$. Hence, the correct option is (B).

Quick Tip to Solve: For a cyanohydrin, follow the nitrile carbon separately: reduction gives CH_2NH_2, while hydrolysis gives $COOH$; then check acid-induced dehydration.

65. Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature? (Given Z for Nd = 60, Yb = 70, La = 57, Ce = 58) [4 marks]

(A) Nd

(B) Yb

(C) La

(D) Ce

Correct Answer: (B) Yb

Solution:

Concept:

- A species is diamagnetic when every occupied orbital contains paired electrons. A filled $4f^{14}$ subshell has seven electron pairs and no unpaired electron.
- In forming lanthanoid cations, the outer $6s$ electrons are removed before the inner $4f$ electrons. The configuration must be assessed for the stated oxidation state, +2.

Step 1: Form the divalent ytterbium ion Ytterbium has atomic number 70. Its ground-state configuration is $\mathrm{Yb} : [\mathrm{Xe}]4f^{14}6s^2$. Removing the two outer s electrons gives

$$\mathrm{Yb}^{2+} : [\mathrm{Xe}]4f^{14}.$$

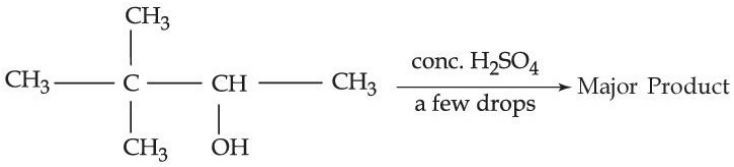
Each of the seven f orbitals contains a pair. Thus the divalent ion has no unpaired electron and is diamagnetic. The completely filled f shell also helps stabilise this oxidation state.

Step 2: Check the other listed elements Neodymium has an incompletely occupied f subshell; its divalent configuration is approximately $[\mathrm{Xe}]4f^4$, which has unpaired electrons. Divalent lanthanum retains an electron outside the xenon core, conventionally represented as $[\mathrm{Xe}]5d^1$, so it is paramagnetic. Divalent cerium also has an incompletely filled set of f/d orbitals and unpaired electrons. A neutral atom or a trivalent ion is not the requested species. Only divalent ytterbium supplies the filled-shell configuration required here.

Step 3: Conclusion Ytterbium, Yb^{2+}, has the diamagnetic configuration $[\mathrm{Xe}]4f^{14}$. Hence, the correct option is (B).

Quick Tip to Solve: For diamagnetic lanthanoid ions, look for $4f^0$ or $4f^{14}$, but remove electrons for the oxidation state actually asked.

66. The major product formed in the following reaction is : [4 marks]



(A) $(CH_3)_2C{=}C(CH_3)_2$

(B) $(CH_3)_2C{=}CHCH_3$

(C) $(CH_3)_3CCH{=}CH_2$

(D) $(CH_3)_2C{=}CHCH_2CH_3$

Correct Answer: (A) $(CH_3)_2C{=}C(CH_3)_2$

Solution:

Concept:

- Concentrated H_2SO_4 protonates an alcohol and allows water to leave. A carbocation formed under these conditions can rearrange before elimination.
- A 1,2-alkyl shift converts a less stable carbocation into a more stable one. Elimination preferentially forms the more highly substituted alkene when accessible.

Step 1: Generate the initial carbocation The substrate is $(CH_3)_3CCH(OH)CH_3$, or 3,3-dimethylbutan-2-ol. Protonation changes the poor leaving group hydroxide into water. Loss of water gives $(CH_3)_3C{-}CH^{+}{-}CH_3$, a secondary carbocation immediately adjacent to a quaternary carbon. Direct elimination toward the terminal methyl could give the less substituted alkene shown in option C, but it does not account for rearrangement.

Step 2: Rearrange by a methyl shift The adjacent quaternary carbon has no hydrogen available for a hydride shift. One methyl group migrates with its bonding electron pair to the cationic carbon. The positive charge then resides on the formerly quaternary carbon:

$$(CH_3)_2C^{+}{-}CH(CH_3)_2.$$

This is a tertiary carbocation, and its formation preserves all six carbon atoms.

Step 3: Eliminate to the most substituted alkene Removal of the hydrogen from the neighbouring CH group forms

$$(CH_3)_2C{=}C(CH_3)_2.$$

This tetrasubstituted alkene is 2,3-dimethylbut-2-ene. Option B contains only five carbon atoms and cannot be formed by simple dehydration. Option D has a different carbon skeleton and a less substituted double bond. Option A matches the rearranged, most stable alkene.

Step 4: Conclusion The major product is 2,3-dimethylbut-2-ene, $(CH_3)_2C{=}C(CH_3)_2$. Hence, the correct option is (A).

Quick Tip to Solve: Before applying the substitution rule for dehydration, inspect the first carbocation for a favourable 1,2-hydride or methyl shift.

67. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Treatment of bromine water with propene yields 1-bromopropan-2-ol.

Reason (R) : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(B) (A) is false but (R) is true

(C) Both (A) and (R) are true and (R) is the correct explanation of (A)

(D) (A) is true but (R) is false

Correct Answer: (C) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

Concept:

- An alkene reacts with bromine in water through a bridged bromonium ion. Water acts as the nucleophile and ultimately introduces the hydroxyl group.
- In an unsymmetrical bromonium ion, the more substituted carbon has greater positive character. Water attacks there, giving the Markovnikov orientation of the hydroxyl group.

Step 1: Evaluate the assertion Propene is $CH_3CH{=}CH_2$. Its reaction with aqueous bromine gives the bromohydrin $CH_3CH(OH)CH_2Br$. Numbering from the brominated end places bromine at carbon 1 and the hydroxyl group at carbon 2, so the product is 1-bromopropan-2-ol. The assertion is therefore true. A vicinal dibromide would be characteristic of bromine addition in a non-nucleophilic solvent rather than the major aqueous pathway.

Step 2: Evaluate the reason The double bond forms a cyclic bromonium intermediate. Water attacks the secondary carbon rather than the terminal primary carbon because the secondary carbon better stabilises the developing positive character. Ring opening places OH_2^+ on carbon 2 and bromine on carbon 1; loss of a proton gives the neutral bromohydrin. Thus the reason is true.

Step 3: Relate the two statements The reason directly explains which carbon receives hydroxyl and therefore why the named product in the assertion is obtained. Both the chemical outcome and its mechanistic explanation agree. This eliminates the options calling either statement false and also the option claiming the true reason does not explain the assertion.

Step 4: Conclusion Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (C).

Quick Tip to Solve: In bromine water addition, place OH on the more substituted alkene carbon and Br on the other carbon; then name the bromohydrin.

68. The denticity of an organic ligand, biuret is : [4 marks]

(A) 2

(B) 4

(C) 3

(D) 6

Correct Answer: (A) 2

Solution:

Concept:

- Denticity is the number of donor atoms of one ligand that bind simultaneously to a single central metal ion. It is not simply the total number of heteroatoms in its formula.
- Biuret, $NH_2CONHCONH_2$, behaves as a bidentate ligand in the usual coordination-chemistry treatment of this question, forming two metal–donor links per ligand.

Step 1: Identify the ligand framework Biuret contains two urea-derived units joined through nitrogen. Its condensed structure is $H_2N-C(=O)-NH-C(=O)-NH_2$. Although this formula contains several nitrogen and oxygen atoms, the number of atoms with lone pairs is not itself the denticity. Resonance, protonation state and the geometry of binding determine which donor sites can coordinate to the same metal centre.

Step 2: Count the coordinating sites In its characteristic chelating mode, biuret uses two donor sites to bind a given metal ion. These provide two coordinate bonds from the same ligand and make biuret bidentate. In the usual alkaline biuret complex, deprotonation facilitates coordination through the appropriate nitrogen donor sites. The metal may bind more than one biuret ligand, but that does not change the denticity of each individual ligand.

Step 3: Compare the offered numbers The value 2 counts the donor sites of one chelating biuret ligand. A count of 4 may confuse several bonds around the metal with the contribution of one ligand. The values 3 and 6 likewise do not represent biuret's standard binding mode. Coordination number belongs to the metal centre, whereas denticity belongs to the ligand.

Step 4: Conclusion Biuret is a bidentate ligand, so its denticity is 2. Hence, the correct option is (A).

Quick Tip to Solve: Count the donor atoms of one ligand actually attached to one metal; do not equate denticity with the total N and O atoms in its formula.

69. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

Reason (R) : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is false but (R) is true.

(B) (A) is true but (R) is false

(C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(D) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Correct Answer: (C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:

Concept:

- Metallic character measures the tendency to lose electrons; non-metallic character is associated with attracting or accepting electrons.
- Across a period, increasing effective nuclear charge generally raises ionisation enthalpy and makes electron gain enthalpy more negative, with familiar subshell and noble-gas exceptions.

Step 1: Evaluate the assertion as a periodic trend From left to right in a period the nuclear charge increases while additional electrons enter the same principal shell. Shielding does not fully offset the extra nuclear charge, so the effective attraction on valence electrons rises and atomic size generally decreases. Electron loss becomes less favourable. Thus metallic character decreases and non-metallic character increases, making the assertion correct as a general trend.

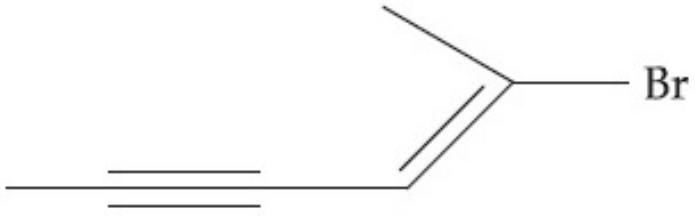
Step 2: Interpret the enthalpy statements Ionisation enthalpy generally increases because removing a valence electron requires more energy. Electron gain enthalpy generally decreases in its signed numerical value, meaning it becomes more negative: adding an electron is increasingly favourable toward the halogens. Here "decrease" must not be mistaken for a decrease in the magnitude of energy released. These trends have known exceptions but the statement describes the general progression across a period, so the reason is correct.

Step 3: Check the explanatory connection A higher ionisation enthalpy suppresses the loss of electrons and hence metallic behaviour. A more negative electron gain enthalpy favours electron acceptance and hence non-metallic behaviour. The two enthalpy trends therefore explain the assertion directly. The options declaring a statement false, or denying the explanatory link, do not match this relationship.

Step 4: Conclusion Both the assertion and the reason are correct, and the reason directly explains the assertion. Electron gain enthalpy decreases in its signed value, meaning it becomes more negative. Thus the supplied key, which calls the reason false, is not adopted for the question as printed. Hence, the correct option is (C).

Quick Tip to Solve: Keep the sign of electron gain enthalpy in mind: a decrease usually means a more negative value and a greater tendency to accept an electron.

70. Choose the correct name for compound given below : [4 marks]



(A) (4E)-5-Bromo-hex-4-en-2-yne

(B) (2E)-2-Bromo-hex-4-yn-2-ene

(C) (2E)-2-Bromo-hex-2-en-4-yne

(D) (4E)-5-Bromo-hex-2-en-4-yne

Correct Answer: (C) (2E)-2-Bromo-hex-2-en-4-yne

Solution:

Concept:

- The parent chain must include both multiple bonds. Number it to give the lowest set of multiple-bond locants; if the sets tie, the double bond receives the lower number.
- In a compound containing both a double and triple bond, the suffix is written en before yne. E/Z geometry follows the higher-priority substituent at each double-bond carbon.

Step 1: Choose and number the parent chain The structure has a six-carbon chain containing one double bond and one triple bond. Numbering from either end gives the set of multiple-bond locants $\{2, 4\}$. Resolve this tie by assigning the double bond locant 2, which puts the triple bond at 4. The bromine substituent then lies on carbon 2. The parent is therefore hex-2-en-4-yne, with a 2-bromo prefix.

Step 2: Assign the alkene configuration At carbon 2 of the double bond, bromine has higher priority than the methyl carbon because its atomic number is larger. At carbon 3, the carbon chain leading toward the triple bond has higher priority than hydrogen. In the displayed geometry these two higher-priority groups lie on opposite sides of the double bond. The configuration is therefore $2E$.

Step 3: Reject the alternative names Option A numbers the triple bond lower in a tie and places the alkene at 4, contrary to the enyne numbering rule. Option B writes the unsaturation suffixes in the wrong sequence. Option D combines locants inconsistent with the displayed brominated alkene. Only option C gives the correct numbering, suffix order and stereochemical descriptor together.

Step 4: Conclusion The name is (2E)-2-bromohex-2-en-4-yne, corresponding to the printed (2E)-2-Bromo-hex-2-en-4-yne. Hence, the correct option is (C).

Quick Tip to Solve: For tied enyne locants such as 2,4 from either end, give the double bond the lower number; write en before yne in the name.

71. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R) :

Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.

Reason (R) : Two liquids with a difference of more than 20° in their boiling points can be separated by simple distillations.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is false but (R) is true.

(B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) Both (A) and (R) are correct and (R) is the correct explanation of (A).

Correct Answer: (D) Both (A) and (R) are correct and (R) is the correct explanation of (A).

Solution:

Concept:

- Distillation separates volatile liquids using a difference in volatility, and therefore in their boiling points. The vapour is richer in the lower-boiling component.
- For liquids with a sufficiently large boiling-point difference, simple distillation is suitable; close-boiling mixtures need repeated vaporisation and condensation in a fractionating column.

Step 1: Evaluate the assertion Propanone, CH_3COCH_3, boils at approximately $56°C$. Propanol has a substantially higher boiling point: propan-1-ol boils near $97°C$, and even propan-2-ol boils near $82°C$. Thus the stated pair has a sufficiently large difference for simple distillation. On heating, propanone preferentially vaporises and is collected by condensation, while the less volatile alcohol remains enriched in the flask. The assertion is correct.

Step 2: Evaluate the reason The reason states the usual working rule used here: liquids whose boiling points differ by more than about $20°C$ can be separated by simple distillation. The separation relies on unequal vapour pressures and the resulting enrichment of the vapour in the more volatile liquid. The reason is correct in the context of this question.

Step 3: Connect the two statements The boiling-point difference for propanone and propanol exceeds the stated threshold. Therefore the reason directly explains why simple distillation is appropriate for this pair. Options describing either statement as false are excluded; the option saying the reason does not explain the assertion also misses this direct application.

Step 4: Conclusion Both the assertion and reason are correct, and the reason correctly explains the assertion. Hence, the correct option is (D).

Quick Tip to Solve: For a distillation assertion, first compare the actual boiling points, then check whether the stated separation rule explains that difference.

72. Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression? [4 marks]

(A) hydrazine

(B) glucose

(C) glycine

(D) $KHSO_4$

Correct Answer: (D) $KHSO_4$

Solution:

Concept:

- Freezing-point depression depends on the concentration of independently moving solute particles: $\Delta T_f = iK_f m$, where i is the van't Hoff factor.
- For dilute aqueous solutions of the same stated molarity, particle production through dissociation is the deciding comparison in this question. A molecule bearing internal charges is not automatically two separate solute particles.

Step 1: Compare the molecular solutes Glucose remains as neutral molecules in water, so its particle factor is approximately one. Glycine is largely present as a zwitterion, $H_3N^+CH_2COO^-$; its positive and negative sites belong to the same molecule and do not constitute two independent particles. Hydrazine, N_2H_4, is a weak base and ionises only partially in water. It consequently does not generate the large ionic particle concentration obtained from a soluble ionic salt.

Step 2: Identify the strongly dissociating solute Potassium hydrogen sulfate is an ionic solute and dissociates into potassium and hydrogen sulfate ions:

$$KHSO_4 \longrightarrow K^+ + HSO_4{}^-.$$

This produces about two particles per formula unit before considering further acid dissociation. Hydrogen sulfate can additionally ionise to a limited extent, $HSO_4{}^- \rightleftharpoons H^+ + SO_4{}^{2-}$, increasing the particle count further.

Step 3: Apply the colligative-property criterion The solvent and dilute analytical concentration are common, so the solution containing the greatest effective number of particles gives the largest depression. The ionic dissociation of potassium hydrogen sulfate dominates over the weak ionisation of hydrazine and the essentially one-particle behaviour of glucose and glycine. Thus choices (A), (B), and (C) give smaller depressions.

Step 4: Conclusion The $0.10\,M$ aqueous solution of $KHSO_4$ exhibits the largest freezing-point depression. Hence, the correct option is (D).

Quick Tip to Solve: Count separate dissolved particles, not charged sites: a zwitterion remains one particle, whereas a soluble ionic salt dissociates into separate ions.

73. The structure of product C, formed by the following sequence of reactions is : [4 marks]

$$CH_3COOH + SOCl_2 \longrightarrow A \xrightarrow[AlCl_3]{\text{Benzene}} B \xrightarrow[^-OH]{KCN} C$$

(A) $C_6H_5C(OH)(CN)CH_3$

(B) o-$CNC_6H_4COCH_3$

(C) $C_6H_5CH(CH_3)COOH$

(D) $C_6H_5CH_2CH_2CN$

Correct Answer: (A) $C_6H_5C(OH)(CN)CH_3$

Solution:

Concept:

- Thionyl chloride converts a carboxylic acid into its acid chloride, with gaseous sulfur dioxide and hydrogen chloride as by-products.
- An acid chloride acylates benzene in the presence of anhydrous aluminium chloride. Cyanide then adds nucleophilically to the carbonyl carbon of the ketone to form a cyanohydrin.

Step 1: Identify intermediate A Ethanoic acid reacts with thionyl chloride by replacement of its hydroxyl group with chlorine:

$$CH_3COOH + SOCl_2 \longrightarrow CH_3COCl + SO_2 + HCl.$$

Hence A is ethanoyl chloride, also called acetyl chloride. The methyl group and carbonyl carbon of the original acid are both retained.

Step 2: Identify intermediate B With benzene and $AlCl_3$, ethanoyl chloride undergoes Friedel–Crafts acylation. The acyl group attaches to the benzene ring, giving acetophenone:

$$C_6H_6 + CH_3COCl \xrightarrow{AlCl_3} C_6H_5COCH_3 + HCl.$$

B is therefore a ketone whose carbonyl carbon is bonded to phenyl and methyl groups.

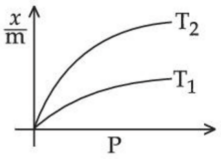
Step 3: Add cyanide to the carbonyl group Cyanide attacks the electrophilic carbonyl carbon of acetophenone. The carbonyl oxygen becomes an alkoxide, and proton transfer gives the cyanohydrin. Thus C has the connectivity $C_6H_5C(OH)(CN)CH_3$: the same central carbon bears phenyl, methyl, hydroxyl, and cyano groups. Option (A) shows exactly this tetrahedral centre. Option (B) instead substitutes the aromatic ring and retains the ketone; (C) contains a carboxylic acid; and (D) lacks the hydroxyl group and has a different carbon skeleton. None follows the stated carbonyl addition.

Step 4: Conclusion C is acetophenone cyanohydrin, $C_6H_5C(OH)(CN)CH_3$. Hence, the correct option is (A).

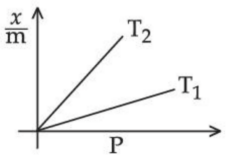
Quick Tip to Solve: Track the original carbonyl carbon: acylation attaches phenyl to it, and cyanohydrin formation adds CN and converts its oxygen into OH.

74. Select the graph that correctly describes the adsorption isotherms at two temperatures T_1, and T_2 ($T_1 > T_2$) for a gas :
(x - mass of the gas absorbed ; m - mass of adsorbent ; P - pressure) [4 marks]

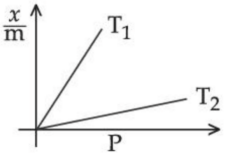
(A)



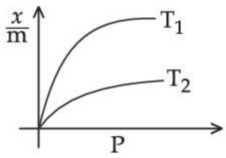
(B)



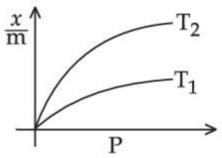
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- An adsorption isotherm plots the adsorbed mass per unit mass of adsorbent, x/m, against gas pressure at a fixed temperature.
- Physical adsorption is exothermic. At a given pressure, lowering temperature favours adsorption; at increasing pressure the surface gradually approaches saturation.

Step 1: Fix the relative positions of the curves The question specifies $T_1 > T_2$, so T_2 is the lower temperature. Because adsorption releases heat, a temperature increase favours desorption. Consequently, at the same pressure the amount adsorbed is greater at T_2 than at T_1. The curve labelled T_2 must lie above the curve labelled T_1. This already rules out the third and fourth graphs, where the hotter curve is higher.

Step 2: Check the pressure dependence At low pressure, raising pressure increases the frequency of gas molecules reaching the surface, and adsorption rises. As more adsorption sites become occupied, further increases in pressure cause smaller increases in x/m. The curves therefore bend towards a plateau rather than continuing as straight lines over the whole pressure range. This saturation

behaviour is represented by the first graph.

Step 3: Compare all four alternatives The first graph combines both required features: increasing adsorption with pressure followed by flattening, and a higher adsorption curve at the lower temperature T_2. The second graph has the right temperature ordering but retains an indefinitely linear pressure dependence. The third has both a linear form and the wrong ordering. The fourth has the right curved form but places T_1 above T_2, contradicting the exothermic temperature trend.

Step 4: Conclusion The first graph correctly places the lower-temperature curve T_2 above T_1 and shows approach to saturation. Hence, the correct option is (A).

Quick Tip to Solve: Check adsorption graphs in two passes: lower temperature means the higher curve, and increasing pressure should eventually give surface saturation.

75. The major component/ingredient of Portland Cement is : [4 marks]

(A) tricalcium aluminate

(B) tricalcium silicate

(C) dicalcium aluminate

(D) dicalcium silicate

Correct Answer: (B) tricalcium silicate

Solution:

Concept:

- Portland cement consists mainly of calcium silicates together with smaller proportions of calcium aluminates and ferrites.
- Tricalcium silicate is written as $3\,CaO \cdot SiO_2$, or Ca_3SiO_5; dicalcium silicate is $2\,CaO \cdot SiO_2$, or Ca_2SiO_4. The prefix refers to the lime-to-silica ratio.

Step 1: Identify the principal clinker constituent The dominant constituent in ordinary Portland cement is tricalcium silicate, commonly called alite. A typical textbook composition assigns it roughly half the cement mass, often about 50% to 60%, although exact proportions depend on manufacture. Its comparatively rapid hydration contributes strongly to the early strength of set cement. The question asks for the major ingredient, rather than merely any compound present.

Step 2: Compare the other listed compounds Dicalcium silicate is also an important cement constituent, but is present in a smaller proportion than tricalcium silicate and hydrates more slowly, contributing to later strength. Tricalcium aluminate, $3\,CaO \cdot Al_2O_3$, is a minor constituent and is associated with rapid hydration and setting behaviour. Dicalcium aluminate is not the major component of Portland cement. Thus the presence of calcium and oxygen in a formula alone does not identify the principal ingredient.

Step 3: Select by abundance rather than reactivity The standard composition distinguishes the two silicates by their relative proportions. Tricalcium aluminate may be highly reactive, but reactivity is not the same as mass fraction. Dicalcium silicate is significant but not the largest component. Among the four supplied names, only tricalcium silicate matches the requested major constituent.

Step 4: Conclusion The major component of Portland cement is tricalcium silicate, $3\,CaO \cdot SiO_2$. Hence, the correct option is (B).

Quick Tip to Solve: For Portland cement, associate tricalcium silicate with the largest fraction and early strength; dicalcium silicate contributes more slowly to later strength.

76. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite.

Reason (R) : The oxidation state of Al in cryolite is +3.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) (A) is true but (R) is false.

(B) (A) is false but (R) is true.

(C) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(D) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Correct Answer: (D) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Solution:

Concept:

- Aluminium is obtained by electrolytic reduction of purified alumina dissolved in molten cryolite. Cryolite lowers the operating temperature and improves the conductivity of the electrolyte.
- In a neutral compound the sum of oxidation states is zero. Cryolite has the formula Na_3AlF_6, with sodium at $+1$ and fluorine at -1.

Step 1: Evaluate the extraction assertion Bauxite is the principal ore of aluminium. After purification to alumina, Al_2O_3, the alumina is dissolved in a molten cryolite-containing bath and electrolysed in the Hall–Heroult process. Aluminium ions are reduced at the cathode:

$$Al^{3+} + 3\,e^- \longrightarrow Al.$$

Thus the assertion correctly describes the extraction route; it does not mean that unpurified bauxite is put directly into the electrolytic cell.

Step 2: Calculate the oxidation state in cryolite Let the oxidation state of aluminium in Na_3AlF_6 be z. Charge neutrality gives

$$3(+1) + z + 6(-1) = 0, \qquad z = +3.$$

Therefore the reason is also true. The aluminium in the complex fluoride anion has oxidation state $+3$, even though the entire anion carries a negative charge.

Step 3: Test whether the reason explains the assertion The role of cryolite in the extraction bath is to lower the fusion temperature and increase electrical conductivity, allowing economical electrolysis of

alumina. Merely knowing that aluminium in cryolite is in the $+3$ oxidation state does not establish these solvent and transport properties. Thus two true statements are present, but the second is not the explanation of the first. Options declaring either statement false are excluded, and option (C) incorrectly claims an explanatory relation.

Step 4: Conclusion Both the assertion and reason are correct, but the reason does not correctly explain the assertion. Hence, the correct option is (D).

Quick Tip to Solve: In assertion–reason questions, check truth first and causation separately: an oxidation state need not explain why an electrolyte additive is useful.

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77. Monomer of Novolac is : [4 marks]

(A) 3-Hydroxybutanoic acid

(B) phenol and melamine

(C) o-Hydroxymethylphenol

(D) 1, 3-Butadiene and styrene

Correct Answer: (C) o-Hydroxymethylphenol

Solution:

Concept:

- Novolac is a linear phenol–formaldehyde condensation resin formed under acidic conditions with phenol in excess.
- Phenol first undergoes hydroxymethylation at an ortho or para position. Hydroxymethylphenols then condense to form methylene bridges joining aromatic units.

Step 1: Identify the reactive phenolic intermediate Formaldehyde introduces a $-CH_2OH$ group at an activated position of phenol. The ortho product is o-hydroxymethylphenol, with hydroxyl and hydroxymethyl substituents adjacent on the ring. This intermediate contains the functionality needed to generate the methylene-linked phenolic structure of novolac through further condensation. It is the appropriate monomeric species among the given choices.

Step 2: Explain formation of the polymer linkage During condensation, the hydroxymethyl group reacts with a suitable position on another phenolic unit and water is eliminated. The resulting $-CH_2-$ bridge links the aromatic rings. Repetition yields the linear novolac resin. Subsequent curing can introduce more cross-links to give a harder thermosetting phenolic material; the question here concerns the monomeric precursor of novolac.

Step 3: Eliminate the unrelated monomer choices 3-Hydroxybutanoic acid is associated with hydroxyalkanoate polyester chemistry, not a methylene-linked phenolic resin. Phenol and melamine do not represent the phenol–formaldehyde precursor specified here; melamine is used with formaldehyde for a different resin family. 1,3-Butadiene and styrene form styrene–butadiene rubber by addition polymerisation and lack the phenolic hydroxymethyl functionality. Hence only the hydroxymethylphenol choice belongs to the novolac condensation sequence.

Step 4: Conclusion The monomeric precursor represented among the options is o-hydroxymethylphenol.

Hence, the correct option is (C).

Quick Tip to Solve: Recognise novolac from phenolic rings joined by methylene bridges; the corresponding reactive precursor is a hydroxymethylphenol, not a vinyl monomer pair.

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78. Given below are two statements :

Statement I : The process of producing syn-gas is called gasification of coal.

Statement II : The composition of syn-gas is CO + CO_2 + H_2 (1 : 1 : 1)

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- Synthesis gas, or syn-gas, is a mixture chiefly of carbon monoxide and hydrogen. These gases provide carbon and hydrogen for industrial synthesis.
- Passing steam over red-hot carbon produces water gas, which can be used as synthesis gas: $C(s) + H_2O(g) \longrightarrow CO(g) + H_2(g)$. Producing synthesis gas from coal is called coal gasification.

Step 1: Evaluate Statement I Coal contains carbon, and its conversion into a useful gaseous mixture by reaction with steam is gasification. The carbon is no longer retained as a solid fuel: it enters the gas phase mainly as carbon monoxide. The hydrogen comes from steam. Thus the stated name for obtaining syn-gas from coal is the standard one, and Statement I is true.

Step 2: Evaluate Statement II The characteristic mixture denoted by syn-gas is $CO + H_2$, not an equimolar ternary mixture of CO, CO_2 and H_2. Carbon dioxide may occur as an impurity or be formed by a water-gas shift reaction, but its inclusion in the fixed ratio $1 : 1 : 1$ does not define syn-gas. Hence Statement II is false.

Step 3: Compare the offered combinations Option (A) reverses the two truth values. Option (C) incorrectly rejects the gasification statement, while option (D) incorrectly accepts the asserted ternary composition. Only option (B) assigns true to Statement I and false to Statement II.

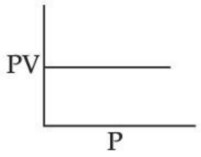
Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: Remember syn-gas as $CO + H_2$; distinguish its defining components from gases that may be present as impurities.

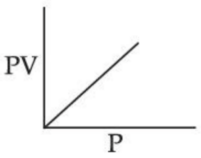
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79. Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas? (P and V stand for pressure and volume of the gas respectively) [4 marks]

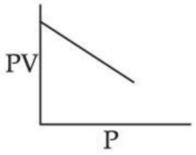
(A)



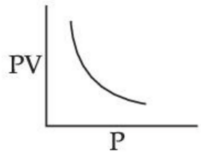
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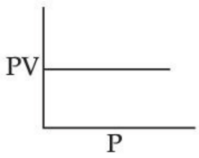
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- For a fixed amount n of ideal gas, the equation of state is $PV = nRT$, where R is the gas constant and T is absolute temperature.
- In a plot of PV against P, the vertical coordinate is the product of pressure and volume, not volume alone. At constant n and T, this product is constant.

Step 1: Find the dependence of the plotted ordinate Apply the ideal-gas equation at two positive pressures, P_1 and P_2, at the same temperature. The corresponding volumes obey

$$P_1V_1 = nRT = P_2V_2.$$

Although the volume decreases when pressure rises, the product used on the vertical axis does not change. Therefore the slope of the required plot is

$$\left(\frac{d(PV)}{dP}\right)_{n,T} = 0.$$

Step 2: Identify the graph and reject the distractors Graph (A) is a horizontal straight line at a positive value of PV, as required by $PV = nRT$. Graph (B) would make the product increase with pressure; graph (C) would make it decrease linearly; and graph (D) would make it decrease nonlinearly. None of these three behaviours is possible for the fixed quantity of ideal gas in an isothermal process. The familiar descending rectangular hyperbola describes V versus P, whereas this question explicitly plots PV versus P.

Step 3: Conclusion The product $PV = nRT$ remains constant, so its plot against pressure is horizontal. Hence, the correct option is (A).

Quick Tip to Solve: Check the vertical-axis label first: an ideal-gas PV-versus-P isotherm is horizontal, while V-versus-P is hyperbolic.

80. BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively :

[4 marks]

(A) A $>$ 50, B $<$ 27

(B) A $>$ 25, B $<$ 17

(C) A $<$ 5, B $>$ 17

(D) A $>$ 15, B $>$ 47

Correct Answer: (C) A $<$ 5, B $>$ 17

Solution:

Concept:

- Biochemical oxygen demand (BOD) measures the amount of dissolved oxygen required by microorganisms to decompose biodegradable organic matter in a water sample under specified conditions.
- A higher BOD indicates a greater load of biodegradable pollution. Clean water has a low BOD, conventionally below $5\,\text{ppm}$, whereas heavily polluted water may have BOD of $17\,\text{ppm}$ or more.

Step 1: Assign the expected demand to clean water Clean water contains relatively little biodegradable organic material. Microbial decomposition therefore consumes only a small amount of oxygen. Its BOD must fall in the low range; for sample A the appropriate relation is

$$\text{BOD}(A) < 5\,\text{ppm}.$$

A large value such as greater than 15, 25 or 50 ppm cannot represent the clean-water category in the given comparison.

Step 2: Assign the expected demand to polluted water Polluted water contains more biodegradable organic wastes, so bacteria require more oxygen while decomposing those wastes. For sample B the expected high BOD is

$$\text{BOD}(B) > 17\,\text{ppm}.$$

This does not mean that polluted water contains more dissolved oxygen: BOD is oxygen demand, and a high demand tends to deplete the available dissolved oxygen.

Step 3: Select the matching pair Only option (C) combines a low BOD for clean sample A with a high BOD for polluted sample B. Options (A), (B) and (D) all assign a high lower bound to the clean-water sample, so each fails the defining clean-water criterion.

Step 4: Conclusion The expected pair is $A < 5\,\text{ppm}$ and $B > 17\,\text{ppm}$. Hence, the correct option is (C).

Quick Tip to Solve: Read BOD as oxygen required by decomposers, not oxygen already dissolved: more biodegradable pollution means higher BOD.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Consider the following cell reaction :

$Cd_{(s)} + Hg_2SO_{4(s)} + \frac{9}{5}H_2O_{(l)} \rightleftharpoons CdSO_4.\frac{9}{5}H_2O_{(s)} + 2Hg_{(l)}$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H^\circ = -825.2$ kJ mol^{-1}, the standard entropy change ΔS° in J K^{-1} is _____. (Nearest integer) [Given : Faraday constant = 96487 C mol^{-1}] [4 marks]

Solution:

Concept:

- The standard Gibbs energy change of a cell reaction is $\Delta G^\circ = -nFE^\circ_{\text{cell}}$, where n is the number of electrons transferred and F is the Faraday constant.
- The thermodynamic relation $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ gives $\Delta S^\circ = (\Delta H^\circ - \Delta G^\circ)/T$. Enthalpy and Gibbs energy must use the same energy units.

Step 1: Determine the electron transfer Cadmium changes from oxidation state zero in the metal to $+2$ in cadmium sulfate, so its oxidation releases two electrons:

$$\text{Cd} \longrightarrow \text{Cd}^{2+} + 2\,\text{e}^-.$$

The mercury(I) dimer in Hg_2SO_4 accepts these two electrons to form two mercury atoms. Thus $n = 2$, not one, for the reaction as written.

Step 2: Calculate the standard Gibbs energy Using the supplied cell potential and Faraday constant,

$$\begin{aligned}\Delta G^\circ &= -2(96487)(4.315)\\ &= -832682.81\,\text{J mol}^{-1}.\end{aligned}$$

The negative sign follows from the positive cell emf. Convert the enthalpy to joules before subtracting:

$$\Delta H^\circ = -825.2 \times 1000 = -825200\,\text{J mol}^{-1}.$$

Step 3: Obtain and round the entropy change At $25^\circ\mathrm{C}$, use $T = 298\,\mathrm{K}$. Therefore

$$\Delta S^\circ = \frac{-825200 - (-832682.81)}{298}$$
$$= 25.1101\,\mathrm{J\,mol^{-1}\,K^{-1}}.$$

The numerator is positive because the Gibbs energy is more negative than the enthalpy. Using 298.15 K would give the same nearest integer. Round only this final entropy value.

Step 4: Conclusion The standard entropy change is approximately $+25\,\mathrm{J\,mol^{-1}\,K^{-1}}$; the required nearest-integer entry is $\boxed{25}$.

Quick Tip to Solve: First count electrons from the metal oxidation states, then convert kJ to J before using $\Delta S^\circ = (\Delta H^\circ + nFE^\circ)/T$.

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82. The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ($H_2C_2O_4.2H_2O$) in 250 mL of water in mol L^{-1} is x $\times$ 10^{-2}. The value of x is _____. (Nearest integer) [Atomic mass : H : 1.0, C : 12.0, O : 16.0] [4 marks]

Solution:

Concept:

- Molarity is the number of moles of solute per litre of solution: $M = n/V$. The specified volume is converted from millilitres to litres before substitution.
- For a crystalline hydrate, the weighed formula includes its water of crystallisation. Thus the molar mass of $\mathrm{H_2C_2O_4 \cdot 2\,H_2O}$ includes two water molecules per formula unit.

Step 1: Calculate the hydrate molar mass Using the given atomic masses, the anhydrous oxalic acid portion has molar mass

$$2(1) + 2(12) + 4(16) = 90\,\mathrm{g\,mol^{-1}}.$$

The two waters contribute

$$2\{2(1) + 16\} = 36\,\mathrm{g\,mol^{-1}}.$$

Hence the mass of one mole of the material actually weighed is

$$M_{\text{hydrate}} = 90 + 36 = 126\,\mathrm{g\,mol^{-1}}.$$

Leaving out the waters would overestimate the amount of oxalic acid.

Step 2: Find the molarity The number of moles in 6.3 g is

$$n = \frac{6.3}{126} = 0.0500\,\mathrm{mol}.$$

Using the stated 250 mL preparation volume,

$$V = 0.250\,\mathrm{L}, \qquad M = \frac{0.0500}{0.250} = 0.200\,\mathrm{mol\,L^{-1}}.$$

Each mole of the hydrate gives one mole of oxalic acid formula units in solution; dissociation does not multiply its analytical molarity.

Step 3: Match the coefficient requested The question expresses this concentration as $x \times 10^{-2}\,\mathrm{mol\,L^{-1}}$, rather than asking directly for the decimal molarity. Therefore

$$0.200 = x \times 0.01 \quad \Rightarrow \quad x = 20.$$

The coefficient is already an integer, so the requested rounding leaves it unchanged.

Step 4: Conclusion The molarity is $0.200\,\mathrm{mol\,L^{-1}} = 20 \times 10^{-2}\,\mathrm{mol\,L^{-1}}$. Thus $\boxed{x = 20}$.

Quick Tip to Solve: Include water of crystallisation in the weighed molar mass, and distinguish the concentration from the coefficient of its stated power of ten.

83. The total number of reagents from those given blow, that can convert nitrobenzene into aniline is _____. (Integer answer)

I. Sn – HCl

II. Sn – NH_4OH

III. Fe – HCl

IV. Zn – HCl

V. H_2 – Pd

VI. H_2 – Raney Nickel [4 marks]

Solution:

Concept:

- Conversion of nitrobenzene into aniline is reduction of the nitro group while retaining the benzene ring. In acidic notation, $\mathrm{C_6H_5NO_2 + 6\,H^+ + 6\,e^- \longrightarrow C_6H_5NH_2 + 2\,H_2O}$.
- Metal-acid pairs supply reducing equivalents, while catalytic hydrogenation uses molecular hydrogen with a metal catalyst. The reaction medium matters: a metal with a weakly basic reagent is not automatically equivalent to the same metal in hydrochloric acid.

Step 1: Check the metal-based reducing systems Reagents I, III and IV are respectively $\mathrm{Sn/HCl}$, $\mathrm{Fe/HCl}$ and $\mathrm{Zn/HCl}$. Each is a suitable reducing system for the nitro group and converts nitrobenzene into the amino compound. In the acidic reaction mixture, aniline is present chiefly as its anilinium salt; the usual work-up releases free aniline. Reagent II, $\mathrm{Sn/NH_4OH}$, lacks the strongly acidic metal-acid conditions and is not counted as a reagent for this conversion. It must not be treated as equivalent to $\mathrm{Sn/HCl}$.

Step 2: Check catalytic hydrogenation Reagents V and VI are hydrogen with palladium and hydrogen with Raney nickel. Both catalyse reduction of the nitro group by hydrogen:

$$\mathrm{C_6H_5NO_2 + 3\,H_2 \longrightarrow C_6H_5NH_2 + 2\,H_2O}.$$

Thus both systems are included. The three hydrogen molecules supply the six hydrogen atoms

required overall; the oxygen atoms leave as water.

Step 3: Count the successful entries The accepted entries are I, III, IV, V and VI. There are three metal-acid systems and two catalytic hydrogenation systems:

$$N = 3 + 2 = 5.$$

Only II is excluded, so the count is five rather than the total of six listed systems.

Step 4: Conclusion Five of the listed reagents, I, III, IV, V and VI, convert nitrobenzene into aniline. The integer answer is $\boxed{5}$.

Quick Tip to Solve: For nitro-to-amino reduction, count metal-acid pairs and catalytic hydrogenation separately; do not overlook a change from acidic to basic medium.

84. For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is _____. (Integer answer) [4 marks]

Solution:

Concept:

- For a first-order reaction with initial concentration $[A]_0$ and concentration $[A]_t$, the integrated law is $kt = \ln([A]_0/[A]_t)$. Here k is the rate constant.
- Percentage completion specifies the amount consumed. The integrated law instead uses the fraction remaining: 75 percent completion leaves one quarter, while 50 percent completion leaves one half.

Step 1: Write the time for 75 percent completion If the initial amount is a, after 75 percent has reacted the unreacted amount is $a/4$. Substitution into the first-order law gives

$$kt_{75} = \ln\left(\frac{a}{a/4}\right) = \ln 4, \qquad t_{75} = \frac{\ln 4}{k}.$$

Using the remaining concentration avoids the mistake of putting the consumed fraction, 0.75, in the denominator.

Step 2: Write the time for 50 percent completion At 50 percent completion, the remaining amount is $a/2$. Thus

$$kt_{50} = \ln\left(\frac{a}{a/2}\right) = \ln 2, \qquad t_{50} = \frac{\ln 2}{k}.$$

This is the first-order half-life, which is independent of the initial concentration. The same rate constant applies to both requested times.

Step 3: Form the ratio and interpret it Dividing eliminates both the rate constant and the initial amount:

$$\frac{t_{75}}{t_{50}} = \frac{\ln 4}{\ln 2} = \frac{\ln(2^2)}{\ln 2} = 2.$$

Equivalently, the first half-life reduces the amount from a to $a/2$, and the second from $a/2$ to $a/4$. Hence 75 percent completion requires two half-lives, rather than one and a half half-lives.

Step 4: Conclusion The requested ratio is $t_{75} : t_{50} = 2 : 1$, so the integer entry is $\boxed{2}$.

Quick Tip to Solve: Convert completion to fraction remaining: 75 percent complete means one quarter remains, which takes exactly two first-order half-lives.

85. The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4.5H_2O$ is _____. [4 marks]

Solution:

Concept:

- The water in a crystalline hydrate need not all have the same structural role. Coordinated water donates an oxygen lone pair to the metal ion; lattice water is held within the crystal chiefly by hydrogen bonding.
- Copper(II) sulphate pentahydrate is conventionally represented as $[\mathrm{Cu(H_2O)_4}]\mathrm{SO_4 \cdot H_2O}$. This form distinguishes four coordinated water molecules from the remaining water of crystallisation.

Step 1: Interpret the hydrate formula The composition $\mathrm{CuSO_4 \cdot 5\,H_2O}$ means that each formula unit contains five water molecules in total. It does not mean that all five water molecules form direct copper–oxygen coordinate bonds. To count the hydrogen-bonded water asked for here, use the structural distinction between the inner coordination sphere and the lattice.

Step 2: Separate coordinated water from lattice water Four water molecules are directly coordinated to $\mathrm{Cu^{2+}}$. The fifth water molecule is outside that set and is hydrogen bonded in the lattice, with sulphate and surrounding coordinated water participating in the network. Thus the number in the requested category is

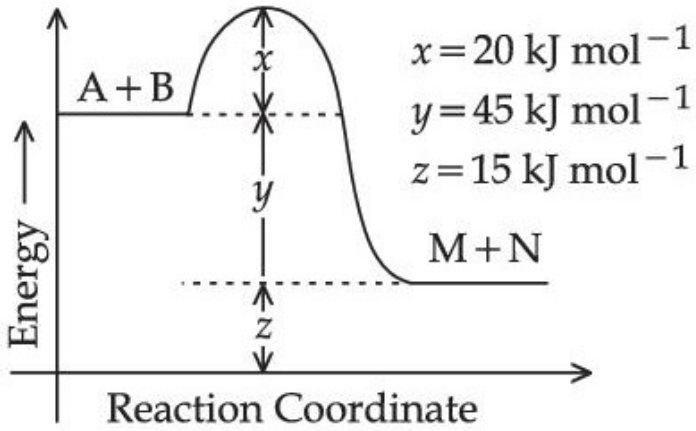
$$n(\text{hydrogen-bonded lattice water}) = 5 - 4 = 1.$$

The count concerns the number of water molecules of this structural type, not the number of individual hydrogen bonds. Coordinated water can also participate in hydrogen bonding, but it is classified as coordinated water in this standard hydrate description.

Step 3: Conclusion There is $\boxed{1}$ hydrogen-bonded lattice water molecule per formula unit of $\mathrm{CuSO_4 \cdot 5\,H_2O}$.

Quick Tip to Solve: Write the hydrate as $[\mathrm{Cu(H_2O)_4}]\mathrm{SO_4 \cdot H_2O}$ before counting; the water outside the coordination brackets is the required lattice water.

86. According to the following figure, the magnitude of the enthalpy change of the reaction $\mathrm{A}+\mathrm{B} \longrightarrow \mathrm{M}+\mathrm{N}$ in $\mathrm{kJ\,mol^{-1}}$ is equal to _____. (Integer answer) [4 marks]



Solution:

Concept:

- The enthalpy change of a reaction is the difference between product and reactant energy levels: $\Delta H = H_{\text{products}} - H_{\text{reactants}}$. Its sign is negative when products lie lower than reactants.
- The height of the activated complex above the reactants is the forward activation energy. It is different from the overall enthalpy change, which depends only on the initial and final levels.

Step 1: Read the labelled energy differences In the energy-profile diagram, the reactants $\mathrm{A}+\mathrm{B}$ lie above the products $\mathrm{M}+\mathrm{N}$. The vertical interval marked y joins their two energy levels, and its given value is $45\,\mathrm{kJ\,mol^{-1}}$. The interval $x = 20\,\mathrm{kJ\,mol^{-1}}$ extends from the reactant level to the peak, while $z = 15\,\mathrm{kJ\,mol^{-1}}$ locates the product level above the chosen zero.

Step 2: Calculate the signed change and its magnitude Using the lower horizontal line as the energy reference gives

$$H_{\text{products}} = z = 15\,\mathrm{kJ\,mol^{-1}}, \qquad H_{\text{reactants}} = z + y = 60\,\mathrm{kJ\,mol^{-1}}.$$

Therefore,

$$\Delta H = 15 - 60 = -45\,\mathrm{kJ\,mol^{-1}}.$$

This negative sign identifies an exothermic reaction. The question asks for the magnitude, so take the absolute value. Neither the peak height nor the arbitrary position of the zero changes the endpoint difference.

Step 3: Conclusion The magnitude of the enthalpy change is $\boxed{45\,\mathrm{kJ\,mol^{-1}}}$. The integer to enter is $\boxed{45}$.

Quick Tip to Solve: For reaction enthalpy, compare only the two horizontal endpoint levels. Use the peak only when an activation energy is requested.

87. Ge(Z = 32) in its ground state electronic configuration has x completely filled orbitals with m_l = 0. The value of x is _____. [4 marks]

Solution:

Concept:

- For a subshell with angular momentum quantum number l, the allowed magnetic quantum numbers are $-l, -l+1, \ldots, +l$. Thus every occupied subshell contains exactly one orbital with $m_l = 0$.
- A completely filled orbital contains two electrons with opposite spins. Hund's rule keeps electrons unpaired in separate degenerate orbitals until pairing is necessary; an occupied subshell need not have a completely filled $m_l = 0$ orbital.

Step 1: Write the ground-state electronic configuration Germanium has $Z = 32$, so the neutral atom has 32 electrons. Filling the subshells in the ground state gives

$$\mathrm{Ge} : 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^2\,4p^2.$$

The electron count is $2+2+6+2+6+10+2+2 = 32$, confirming that no subshell has been omitted.

Step 2: Count only paired orbitals with zero magnetic quantum number The fully occupied $1s, 2s, 3s, 4s$ subshells each contribute their sole $m_l = 0$ orbital, giving four. Each fully occupied $2p$ and $3p$ subshell contributes one such orbital, giving two more. The filled $3d$ subshell contributes one. In $4p^2$, the two electrons occupy different p orbitals with parallel spins; no 4p orbital is completely filled. It therefore adds zero irrespective of the labelling of the degenerate orbitals. Hence

$$x = 4 + 2 + 1 = 7.$$

We count spatial orbitals, not the fourteen electrons occupying these seven orbitals.

Step 3: Conclusion The number of completely filled orbitals with $m_l = 0$ is $\boxed{7}$.

Quick Tip to Solve: Count one zero-m_l orbital per filled subshell, then check partially filled subshells separately using Hund's rule.

88. A_3B_2 is a sparingly soluble salt of molar mass M (g mol^{-1}) and solubility x g L^{-1}. The solubility product satisfies $K_{sp} = a\left(\frac{x}{M}\right)^5$. The value of a is _____. (Integer answer) [4 marks]

Solution:

Concept:

- Molar solubility is the number of moles of salt dissolved per litre. If mass solubility is $x\,\mathrm{g\,L^{-1}}$ and molar mass is $M\,\mathrm{g\,mol^{-1}}$, then $s = x/M$ in $\mathrm{mol\,L^{-1}}$.
- The solubility product is the product of equilibrium ionic concentrations, each raised to its stoichiometric coefficient. The pure solid has unit activity and is absent from the expression.

Step 1: Convert the given solubility into ion concentrations The formula A_3B_2 gives three A ions and two B ions for each formula unit that dissolves. With charges chosen to make the formula neutral, write

$$\mathrm{A_3B_2(s)} \rightleftharpoons 3\,\mathrm{A^{2+}(aq)} + 2\,\mathrm{B^{3-}(aq)}.$$

If the molar solubility is s, the saturated solution therefore has $[A^{2+}] = 3s$ and $[B^{3-}] = 2s$, assuming no other source of either ion.

Step 2: Form the product and collect the numerical factor Apply the dissolution coefficients as the powers in the equilibrium expression:

$$\begin{aligned} K_{sp} &= [A^{2+}]^3[B^{3-}]^2 \\ &= (3s)^3(2s)^2 \\ &= 3^3 \times 2^2 \times s^{3+2} \\ &= 108s^5 = 108\left(\frac{x}{M}\right)^5. \end{aligned}$$

The total power five comes from the total of the two stoichiometric coefficients. The coefficient is not simply 3×2: each multiplicity is itself raised to the power of the number of corresponding ions. Comparing with the given expression identifies the required constant.

Step 3: Conclusion The required numerical coefficient is $a = 3^3 \times 2^2 = \boxed{108}$.

Quick Tip to Solve: For A_pB_q, substitute ps and qs before simplifying: the numerical coefficient is $p^p q^q$.

89. Consider the sulphides HgS, PbS, CuS, Sb_2S_3, As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is _____. [4 marks]

Solution:

Concept:

- Nitric acid is an oxidising acid, so the behaviour of a metal sulphide in it depends on oxidation as well as ordinary acid–base dissolution. The group-II sulphides must be distinguished individually.
- In the specified 50 percent nitric acid treatment, PbS, CuS, CdS and As_2S_3 are soluble, whereas HgS and Sb_2S_3 are not counted as soluble.

Step 1: Classify the six sulphides under the stated conditions Lead(II), copper(II) and cadmium sulphides undergo oxidative dissolution in nitric acid; their metal ions can remain in the acid solution as soluble nitrate species. Arsenic(III) sulphide is also dissolved by oxidative treatment, producing a soluble oxygen-containing arsenic species. Thus PbS, CuS, CdS and As_2S_3 belong in the soluble set. Their behaviour cannot be inferred merely from the small water solubility of the starting sulphides.

Step 2: Exclude the resistant sulphides and count HgS is resistant to this nitric acid treatment and is not included. Antimony sulphide also does not give a clear soluble product under the specified treatment; oxidation can leave insoluble hydrated antimony oxide. Therefore Sb_2S_3 is excluded as well. Of the six compounds listed, four satisfy the requested solubility condition:

$$N = 6 - 2 = 4.$$

Do not apply a blanket rule that all arsenic-group sulphides behave identically: arsenic sulphide and antimony sulphide differ in this test. Similarly, resistance to dilute non-oxidising acid does not by itself imply resistance to nitric acid.

Step 3: Conclusion The four soluble sulphides are PbS, CuS, As_2S_3 and CdS. The required number is $\boxed{4}$.

Quick Tip to Solve: For this nitric acid test, exclude HgS and Sb_2S_3 from the given list, then count the four remaining sulphides.

90. The number of halogen/(s) forming halic (V) acid is _____. [4 marks]

Solution:

Concept:

- A halic(V) acid is a halogen oxoacid in which the halogen has oxidation number $+5$. The familiar formula for these acids is HXO_3, where X represents the halogen.
- Fluorine differs from the other common halogens because it is the most electronegative element and does not show a positive oxidation state in its compounds. Chlorine, bromine and iodine form the corresponding $+5$ oxoacids.

Step 1: Identify the oxidation number in the acid formula Assign hydrogen the oxidation number $+1$ and each oxygen -2. For the neutral molecule HXO_3, let the oxidation number of X be n. Charge balance gives

$$+1 + n + 3(-2) = 0 \quad \Rightarrow \quad n = +5.$$

Thus the question refers to halogen oxoacids with the halogen in this oxidation state, rather than to every possible acid containing a halogen.

Step 2: List the halogens that form these acids Chlorine forms chloric(V) acid, $HClO_3$; bromine forms bromic(V) acid, $HBrO_3$; and iodine forms iodic(V) acid, HIO_3. These are three distinct halogens meeting the condition. Fluorine cannot occupy the $+5$ state required here, so a fluorine analogue is not added. In the standard halogen oxoacid chemistry considered in this question, the count is therefore

$$N = 1\ (\mathrm{Cl}) + 1\ (\mathrm{Br}) + 1\ (\mathrm{I}) = 3.$$

The Roman numeral V specifies the oxidation state; it does not ask for five oxygen atoms or for the total number of oxoacids of a halogen.

Step 3: Conclusion The halogens are chlorine, bromine and iodine. The required number is $\boxed{3}$.

Quick Tip to Solve: First translate halic(V) into oxidation state $+5$, then recall $HClO_3$, $HBrO_3$ and HIO_3; fluorine cannot take a positive oxidation state.