JEE Main 2021 Paper 1 (B.E./B.Tech) Aug 31 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 4 (Aug-Sep), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 31 August 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has : [4 marks]

(A) no solution

(B) infinitely many solution

(C) exactly two solutions

(D) a unique solution

Correct Answer: (B) infinitely many solution

Solution:

Concept:

- A homogeneous system $A\mathbf{x} = \mathbf{0}$ is always consistent because the zero vector is a solution. Its solution is unique precisely when $\det A \neq 0$.
- When $\alpha + \beta + \gamma = 2\pi$, the cosine identity is $\cos^2\alpha + \cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma = 1$.

Step 1: Form the coefficient determinant Write $a = \cos\alpha$, $b = \cos\beta$, and $c = \cos\gamma$. The coefficient matrix is symmetric, and expansion along its first row gives

$$\Delta = \begin{vmatrix} 1 & c & b \\ c & 1 & a \\ b & a & 1 \end{vmatrix} = 1 - a^2 - b^2 - c^2 + 2abc.$$

This calculation uses the coefficients of the equations, rather than the values of the unknowns x, y, z.

Step 2: Use the angle condition Since $\gamma = 2\pi - (\alpha + \beta)$, we have $c = \cos(\alpha + \beta) = ab - \sin\alpha\sin\beta$. Squaring $c - ab = -\sin\alpha\sin\beta$ and using $\sin^2 t = 1 - \cos^2 t$ gives

$$a^2 + b^2 + c^2 - 2abc = 1.$$

Substitution in the determinant therefore yields $\Delta = 0$. This identity remains valid even if some sines vanish; no division by a sine was used.

Step 3: Classify the solution set The matrix is singular, so its null space has positive dimension. There is a nonzero vector $\mathbf{v}$ with $A\mathbf{v} = 0$, and every real multiple $t\mathbf{v}$ is another solution. Thus there cannot be exactly two solutions. The zero solution rules out option (A), while singularity rules out the unique-solution claim in (D).

Step 4: Conclusion The system has infinitely many solutions. Hence, the correct option is (B).

Quick Tip to Solve: For a homogeneous system, check the determinant only: a zero determinant gives infinitely many solutions, never no solution.

2. The domain of the function

$f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$ **is :** [4 marks]

(A) $\left[0, \frac{1}{4}\right]$

(B) $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

(C) $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

(D) $\left[0, \frac{1}{2}\right]$

Correct Answer: (C) $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

Solution:

Concept:

- For real principal values, both $\sin^{-1} u$ and $\cos^{-1} u$ require $-1 \le u \le 1$. Denominators must also be nonzero.
- The domain of a sum is the intersection of the domains of its two terms. Squaring $|v| \le 1$ is equivalent to $v^2 \le 1$ for real v.

Step 1: Constrain the inverse cosine First exclude $x = -1$. Its argument must satisfy

$$\left|\frac{x-1}{x+1}\right| \le 1 \iff (x-1)^2 \le (x+1)^2 \iff x \ge 0.$$

The excluded value -1 does not lie in this range, so this term has domain $[0, \infty)$.

Step 2: Constrain the inverse sine Exclude $x = 1$. Because $(x-1)^2 > 0$ for all other x, multiplication by it preserves the inequality directions. The upper bound gives

$$3x^2 + x - 1 \le (x-1)^2 \iff (2x-1)(x+2) \le 0,$$

so $-2 \le x \le \frac{1}{2}$. The lower bound gives

$$3x^2 + x - 1 \ge -(x-1)^2 \iff x(4x-1) \ge 0,$$

so $x \le 0$ or $x \ge \frac{1}{4}$. Their intersection is $[-2, 0] \cup [\frac{1}{4}, \frac{1}{2}]$; the forbidden value 1 is already absent.

Step 3: Intersect and compare the options Intersect this last set with $[0, \infty)$: only 0 survives from its negative interval, while the entire interval $[\frac{1}{4}, \frac{1}{2}]$ survives. Endpoints are included because arguments equal to 1 or -1 are allowed. Option (B) retains invalid negative inputs; (A) and (D) admit inputs between 0 and $\frac{1}{4}$, where the sine argument is below -1.

Step 4: Conclusion The domain is $\{0\} \cup [\frac{1}{4}, \frac{1}{2}]$. Hence, the correct option is (C).

Quick Tip to Solve: Find each inverse-trigonometric domain separately, and take their intersection only after solving both argument inequalities.

3. Let S = {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g(1) is : [4 marks]

(A) $\frac{1}{10}$
(B) $\frac{1}{15}$
(C) $\frac{1}{5}$
(D) $\frac{1}{30}$

Correct Answer: (A) $\frac{1}{10}$

Solution:

Concept:

- An onto function between two finite sets having the same number of elements is a bijection. Every output occurs exactly once.
- For equally likely bijections on a six-element set, probability equals the number satisfying the condition divided by $6!$. Fixing two distinct outputs leaves $4!$ bijections.

Step 1: Count all onto functions There are six domain elements and six codomain elements. Surjectivity requires all six codomain values to appear among the six images. Hence no two distinct inputs can share an image. An onto function here is therefore a permutation, with

$$N_{\text{total}} = 6! = 720.$$

It would be incorrect to use 6^6 as the sample-space size, since that includes functions which are not onto.

Step 2: List the allowed ordered image pairs The condition involves the images of the distinct inputs 1 and 3. The equation $g(3) = 2g(1)$ requires $g(1) \in \{1, 2, 3\}$, because the doubled value must remain in S. The possible ordered pairs are exactly

$$(g(1), g(3)) = (1, 2),\ (2, 4),\ (3, 6).$$

In each pair the images are distinct, as a bijection requires. There are no pairs beginning with 4, 5 or 6, since their doubles lie outside the codomain.

Step 3: Complete the remaining images For each of these three choices, the other four inputs can be bijected onto the four unused outputs in $4! = 24$ ways. These three cases are disjoint, so

$$N_{\text{favourable}} = 3 \cdot 4! = 72, \qquad P = \frac{72}{720} = \frac{1}{10}.$$

Equivalently, the ordered distinct images of inputs 1 and 3 have $6 \cdot 5 = 30$ equally likely possibilities, of which three work. This also rules out $1/15$, $1/5$ and $1/30$, which use the wrong favourable count or denominator.

Step 4: Conclusion The required probability is $\frac{1}{10}$. Hence, the correct option is (A).

Quick Tip to Solve: For a random permutation, count the ordered images of the specified inputs first; all completions then have the same size.

4. Let f : N $\to$ N be a function such that f(m + n) = f(m) + f(n) for every m, n$\in$N. If f(6) = 18, then f(2) . f(3) is equal to : [4 marks]

(A) 6

(B) 54

(C) 18

(D) 36

Correct Answer: (B) 54

Solution:

Concept:

- On the natural numbers, additivity $f(m+n) = f(m) + f(n)$ determines every positive-integer value from $f(1)$.
- Repeated addition gives $f(k) = kf(1)$ for each positive integer k; no continuity or other analytic assumption is needed on this domain.

Step 1: Determine the value at one Let $c = f(1)$. Using the given equation repeatedly,

$$f(2) = 2c, \qquad f(3) = 3c, \qquad f(6) = 6c.$$

For completeness, the induction step is $f(k+1) = f(k) + f(1) = kc + c = (k+1)c$. Thus this formula follows directly from additivity for all natural inputs, rather than being an assumed form for the function.

Step 2: Apply the supplied value The condition $f(6) = 18$ now becomes $6c = 18$, giving $c = 3$. Therefore

$$f(2) = 2 \cdot 3 = 6, \qquad f(3) = 3 \cdot 3 = 9.$$

One can check these values without finding $f(1)$: writing 6 as $3 + 3$ gives $2f(3) = 18$, while writing 6 as $2 + 2 + 2$ gives $3f(2) = 18$. Both approaches yield the same pair.

Step 3: Evaluate the requested product The dot in $f(2) \cdot f(3)$ means multiplication of the two function values, so

$$f(2)f(3) = 6 \times 9 = 54.$$

Option (A), 6, is only $f(2)$, not the product. Option (C), 18, is the supplied value $f(6)$; additivity applies to a sum of inputs and does not imply $f(2)f(3) = f(6)$. Option (D), 36, would incorrectly square $f(2)$ and ignore that $f(3) = 9$.

Step 4: Conclusion The product $f(2) \cdot f(3)$ equals 54. Hence, the correct option is (B).

Quick Tip to Solve: On natural numbers, expand an additive function by repeated addition; do not mistake additivity for multiplicativity.

5. If $\alpha = \lim\limits_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ and $\beta = \lim\limits_{x \to 0} (\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is : [4 marks]

(A) (1, −3)

(B) (−1, 3)

(C) (−1, −3)

(D) (1, 3)

Correct Answer: (D) (1, 3)

Solution:

Concept:

- For a $0/0$ limit, differentiating numerator and denominator is valid when the derivative ratio has a limit and the usual differentiability conditions hold.
- For a positive-base variable power $u(x)^{v(x)}$, take logarithms. For roots α, β of $ax^2 + bx - 4 = 0$, use $\alpha + \beta = -b/a$ and $\alpha\beta = -4/a$.

Step 1: Evaluate the first root At $x = \pi/4$, both $\tan^3 x - \tan x$ and $\cos(x + \pi/4)$ vanish. Applying the derivative ratio gives

$$\alpha = \lim_{x \to \pi/4} \frac{(3\tan^2 x - 1)\sec^2 x}{-\sin(x + \pi/4)} = \frac{(3-1)2}{-1} = -4.$$

In particular the minus sign from differentiating the cosine is essential; losing it would give the wrong root and coefficient signs.

Step 2: Evaluate the variable-power limit Near zero, $\cos x > 0$, so write the logarithm of the expression as $\cot x \log(\cos x)$. Since $\log(\cos x) = -x^2/2 + O(x^4)$ and $\cot x = 1/x + O(x)$,

$$\lim_{x \to 0} \cot x \log(\cos x) = \lim_{x \to 0} \left(-\frac{x}{2} + O(x^3)\right) = 0.$$

Exponentiating yields $\beta = e^0 = 1$. The same limit is obtained from both sides.

Step 3: Find and check the coefficients The root product is -4, so $-4/a = -4$, giving $a = 1$. The root sum is -3, so $-b/a = -3$, giving $b = 3$. As a direct check,

$$x^2 + 3x - 4 = (x + 4)(x - 1).$$

These factors have the calculated roots. The other choices change the sign of the sum or the product, so they cannot have both roots -4 and 1.

Step 4: Conclusion The ordered pair is $(a, b) = (1, 3)$. Hence, the correct option is (D).

Quick Tip to Solve: For a power limit, evaluate its logarithm first; then use both the root sum and product to fix coefficient signs.

6. The locus of mid-points of the line segments joining $(-3, -5)$ and the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is : [4 marks]

(A) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

(B) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

(C) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

(D) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

Correct Answer: (C) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

Solution:

Concept:

- If (x, y) is the midpoint of (x_1, y_1) and (X, Y), then $X = 2x - x_1$ and $Y = 2y - y_1$.
- A locus is obtained by eliminating the moving endpoint using its known curve equation. The affine midpoint map is invertible, so this substitution gives the full locus.

Step 1: Express the moving endpoint Let $P = (X, Y)$ be the point moving on the ellipse and $M = (x, y)$ the midpoint of the segment joining P to $A = (-3, -5)$. The midpoint formula gives

$$x = \frac{X - 3}{2}, \qquad y = \frac{Y - 5}{2}.$$

Solving for the coordinates of P, we get $X = 2x + 3$ and $Y = 2y + 5$. The signs are positive here because the fixed coordinates are negative.

Step 2: Substitute into the ellipse The moving endpoint satisfies $X^2/4 + Y^2/9 = 1$. Therefore the midpoint coordinates obey

$$\frac{(2x + 3)^2}{4} + \frac{(2y + 5)^2}{9} = 1.$$

Multiplication by 36 gives

$$9(2x + 3)^2 + 4(2y + 5)^2 = 36.$$

Expanding carefully yields $36x^2 + 108x + 81 + 16y^2 + 80y + 100 = 36$. Thus the constant term after moving 36 to the left is $81 + 100 - 36 = 145$.

Step 3: Check the geometry and options The resulting ellipse has centre $(-3/2, -5/2)$, the midpoint of the original centre $(0, 0)$ and the fixed point. Its semiaxes are 1 and $3/2$, half the original semiaxes. This confirms both the scale and translation. Option (A) has the wrong quadratic coefficients, while (B) and (D) have the wrong linear terms and hence the wrong centre. Every point obtained by the substitution corresponds back to an endpoint on the given ellipse.

Step 4: Conclusion The locus is $36x^2 + 16y^2 + 108x + 80y + 145 = 0$. Hence, the correct option is (C).

Quick Tip to Solve: For midpoint loci, solve the midpoint formulas for the original moving coordinates before substituting into the curve.

7. If $\frac{dy}{dx} = \frac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y}\log_e 2}$, y(0) = 0, then for y = 1, the value of x lies in the interval : [4 marks]

(A) (1, 2)

(B) $\left(\frac{1}{2}, 1\right]$

(C) (2, 3)

(D) $\left(0, \frac{1}{2}\right]$

Correct Answer: (A) (1, 2)

Solution:

Concept:

- Cancel a common nonzero factor before classifying a differential equation. Here $2^x > 0$ for every real x.
- The logarithmic derivative identity $\frac{d}{dy}\log h(y) = h\prime(y)/h(y)$ converts a separated rational expression into an elementary integral.

Step 1: Simplify and separate the variables Cancel the common factor 2^x in the numerator and denominator:

$$\frac{dy}{dx} = \frac{y + 2^y}{1 + 2^y \log 2}.$$

At the initial value $y = 0$, the numerator is 1 and the denominator is positive. Along the branch from $y = 0$ to $y = 1$, both remain positive. We can therefore invert and separate on this branch:

$$dx = \frac{1 + 2^y \log 2}{y + 2^y}\, dy.$$

Step 2: Integrate and apply the initial condition The numerator is exactly the derivative of $y + 2^y$. Thus integration gives

$$x = \log(y + 2^y) + C.$$

There is no absolute-value ambiguity on $0 \le y \le 1$, since $y + 2^y > 0$. Substituting $x = 0, y = 0$ yields $0 = \log 1 + C$, so $C = 0$. Differentiating $x = \log(y + 2^y)$ with respect to x recovers the simplified equation, which checks the integration.

Step 3: Locate the value when the ordinate is one Putting $y = 1$ in the implicit solution gives $x = \log(1 + 2) = \log 3$. Since $e < 3 < e^2$, taking natural logarithms gives $1 < \log 3 < 2$; numerically it is about 1.0986. Therefore it lies in $(1, 2)$, rather than in either interval ending at 1 or in $(2, 3)$. The logarithm here is natural, as required by the derivative of 2^y.

Step 4: Conclusion For $y = 1$, $x = \log 3 \in (1, 2)$. Hence, the correct option is (A).

Quick Tip to Solve: After cancelling the exponential factor, look for a numerator equal to the derivative of the denominator in the separated integral.

8. If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, x > 0, $\phi > 0$, and y(1) = -1, then $\phi\left(\frac{y^2}{4}\right)$ is equal to : [4 marks]

(A) 4 ϕ (2)

(B) 4ϕ (1)

(C) 2 ϕ (1)

(D) ϕ (1)

Correct Answer: (B) 4ϕ (1)

Solution:

Concept:

- The substitution $u = y^2/x^2$ converts the repeated ratio into a single dependent variable. Differentiating the quotient gives $u' = 2yy'/x^2 - 2y^2/x^3$.
- The chain rule gives $\frac{d}{dx}\log\phi(u) = \frac{\phi'(u)}{\phi(u)}u'$. Positivity of ϕ makes this logarithm valid.

Step 1: Reduce the differential equation Write $y^2 = ux^2$ and differentiate to obtain $yy' = xu + \frac{1}{2}x^2u'$. Substitution into the given equation cancels the common term xu:

$$\frac{x^2}{2}\frac{du}{dx} = x\frac{\phi(u)}{\phi'(u)}.$$

Since $x > 0$, rearrangement yields $\frac{\phi'(u)}{\phi(u)}\frac{du}{dx} = 2/x$.

Step 2: Integrate and use the initial condition Integrating the chain-rule form gives

$$\log\phi(u) = 2\log x + C, \qquad \phi(u) = Kx^2.$$

At $x = 1$, the initial value $y = -1$ gives $u = 1$, hence $K = \phi(1)$. Thus the relation along the solution is $\phi(y^2/x^2) = x^2\phi(1)$.

Step 3: Evaluate the requested expression The argument $y^2/4$ corresponds to $x = 2$ in the solution relation. Consequently,

$$\phi\left(\frac{y(2)^2}{4}\right) = 4\phi(1).$$

The coefficient is four, rather than two or one; the initial argument is 1, rather than 2. Therefore the other three expressions do not follow from the initial condition.

Step 4: Conclusion At $x = 2$, $\phi(y^2/4) = 4\phi(1)$. Hence, the correct option is (B).

Quick Tip to Solve: When a differential equation repeatedly contains y^2/x^2, differentiate that ratio first; its derivative often cancels the explicit homogeneous term.

9. The sum of the roots of the equation

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$**, is :** [4 marks]

(A) $\log_2 14$

(B) $\log_2 11$

(C) $\log_2 12$

(D) $\log_2 13$

Correct Answer: (B) $\log_2 11$

Solution:

Concept:

- Set $t = 2^x > 0$, so $x = \log_2 t$. A real logarithm requires a positive argument, giving the additional restriction $10 - 1/t > 0$, or $t > 1/10$.
- The base-change identity $2\log_4 A = \log_2 A$ allows the equation to be combined into one base-two logarithm. Vieta's formula then gives the product of the transformed roots.

Step 1: Combine the logarithms After the substitution, the equation becomes

$$\log_2 t + 1 - 2\log_2(3 + t) + \log_2(10 - 1/t) = 0.$$

Every argument is positive on $t > 1/10$. Therefore the logarithm laws give

$$\log_2\left(\frac{2t(10 - 1/t)}{(3 + t)^2}\right) = 0.$$

A base-two logarithm vanishes exactly when its argument equals one.

Step 2: Find and check the transformed roots Equating the argument to one and clearing the positive denominator yields

$$20t - 2 = (t + 3)^2, \qquad t^2 - 14t + 11 = 0.$$

The roots are $t_1 = 7 + \sqrt{38}$ and $t_2 = 7 - \sqrt{38}$. Since $\sqrt{38} < 6.2$, even the smaller root exceeds 0.8, so both satisfy the logarithmic domain. Each positive t gives exactly one real x.

Step 3: Add the original roots Using $t_1 t_2 = 11$, we obtain

$$x_1 + x_2 = \log_2 t_1 + \log_2 t_2 = \log_2(t_1 t_2) = \log_2 11.$$

The numbers 14, 12 and 13 in the other options are not the product of the transformed roots. In particular, 14 is their sum, which is not the quantity needed when adding logarithms.

Step 4: Conclusion The sum of the two real roots is $\log_2 11$. Hence, the correct option is (B).

Quick Tip to Solve: If the unknown occurs both as x and 2^x inside logarithms, put $t = 2^x$; the sum of the original roots is the logarithm of the product of the new roots.

10. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of | z − (3 + 3i) | is : [4 marks]

(A) $2\sqrt{2}-1$

(B) $3\sqrt{2}$

(C) $6\sqrt{2}$

(D) $2\sqrt{2}$

Correct Answer: (D) $2\sqrt{2}$

Solution:

Concept:

- A complex quotient is purely imaginary precisely when its real part is zero, provided its denominator is nonzero. Write $z = x + iy$ and impose $z \neq 1$.
- For a circle with centre C, radius r, and an exterior point P, the least distance from P to the circle is $PC - r$. Equality occurs on the radial line towards P.

Step 1: Obtain the locus of the complex number Multiply numerator and denominator by the conjugate of $z - 1$:

$$\frac{z-i}{z-1} = \frac{(x + i(y-1))((x-1) - iy)}{(x-1)^2 + y^2}.$$

The real part of the numerator is $x(x-1) + y(y-1)$. Setting it to zero gives

$$x^2 + y^2 - x - y = 0, \qquad (x - \tfrac{1}{2})^2 + (y - \tfrac{1}{2})^2 = \tfrac{1}{2}.$$

Thus the locus is a circle, with the undefined point $(1, 0)$ excluded.

Step 2: Minimise the required distance The centre is $C = (1/2, 1/2)$, the radius is $r = 1/\sqrt{2}$, and the fixed point is $P = (3, 3)$. Thus

$$PC = \sqrt{(5/2)^2 + (5/2)^2} = \frac{5}{\sqrt{2}}.$$

The nearest point lies on CP, at $(1, 1)$. It is allowed: for $z = 1 + i$, the quotient is $-i$, which is purely imaginary. Hence the excluded point causes no difficulty.

Step 3: Compare with the alternatives The least distance is

$$PC - r = \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 2\sqrt{2}.$$

The alternatives $2\sqrt{2} - 1$, $3\sqrt{2}$ and $6\sqrt{2}$ do not equal this centre-distance-minus-radius value. Directly, the distance from $(1, 1)$ to $(3, 3)$ is $\sqrt{4+4} = 2\sqrt{2}$, confirming the calculation.

Step 4: Conclusion The minimum value of $|z - (3 + 3i)|$ is $2\sqrt{2}$. Hence, the correct option is (D).

Quick Tip to Solve: Translate a purely imaginary complex ratio into a real-part condition, then complete squares; minimum-modulus questions often become distances from a point to a circle.

11. Let a_1, a_2, a_3, ... be an A.P. If $\frac{a_1+a_2+....+a_{10}}{a_1+a_2+....+a_p} = \frac{100}{p^2}$, p $\neq$ 10, then $\frac{a_{11}}{a_{10}}$ is equal to : [4 marks]

(A) $\frac{19}{21}$

(B) $\frac{100}{121}$

(C) $\frac{21}{19}$

(D) $\frac{121}{100}$

Correct Answer: (C) $\frac{21}{19}$

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $a_n = a + (n-1)d$ and $S_n = n[2a + (n-1)d]/2$.
- The given ratio compares sums, rather than individual terms. Cross-multiplication and the condition $p \neq 10$ determine the relation between a and d.

Step 1: Insert the arithmetic-sum formula Let $a = a_1$. The given equality becomes

$$\frac{10(2a+9d)}{p(2a+(p-1)d)} = \frac{100}{p^2}.$$

The sum S_p is nonzero because it is a denominator, and p is a positive integer. Cross-multiplying and cancelling $10p$ yields

$$p(2a+9d) = 10(2a+(p-1)d).$$

Step 2: Determine the first term Collect the terms containing a and d:

$$2a(p-10) + (10-p)d = 0, \qquad (p-10)(2a-d) = 0.$$

As $p \neq 10$, the first factor cannot vanish; therefore $2a = d$. The possibility $d = 0$ would force $a = 0$, making the given denominator sum zero, so $d \neq 0$.

Step 3: Form the requested term ratio Now the tenth and eleventh terms are

$$a_{10} = \frac{d}{2} + 9d = \frac{19d}{2}, \qquad a_{11} = \frac{d}{2} + 10d = \frac{21d}{2}.$$

Consequently $a_{11}/a_{10} = 21/19$. The reciprocal $19/21$ reverses the required order; $100/121$ and $121/100$ resemble ratios of squared indices and do not represent these individual terms.

Step 4: Conclusion The required ratio is $a_{11}/a_{10} = 21/19$. Hence, the correct option is (C).

Quick Tip to Solve: Keep sum ratios and term ratios separate: first substitute $S_n = n[2a + (n-1)d]/2$, find a/d, and only then calculate the requested individual terms.

12. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is : [4 marks]

(A) $\frac{4}{\sqrt{5}}$
(B) $\frac{16}{\sqrt{5}}$
(C) $\frac{8}{\sqrt{5}}$
(D) $\frac{12}{\sqrt{5}}$

Correct Answer: (C) $\frac{8}{\sqrt{5}}$

Solution:

Concept:

- The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x, y)$ is half the absolute value of the corresponding coordinate determinant.
- A fixed nonzero triangle area with a fixed base gives two parallel lines for the third vertex. The shortest distance from the origin to $Ax + By + C = 0$ is $|C|/\sqrt{A^2 + B^2}$.

Step 1: Express the fixed-area condition For the stated vertices, the determinant formula gives

$$12 = \frac{1}{2}\left|5(2-\beta) + 3(\beta - 6) + \alpha(6-2)\right|.$$

Simplifying the expression inside the absolute value yields

$$|4\alpha - 2\beta - 8| = 24, \qquad |2\alpha - \beta - 4| = 12.$$

The absolute value must be retained because either orientation of the triangle is allowed.

Step 2: Identify both possible lines The two signs give

$$2\alpha - \beta - 16 = 0 \quad \text{or} \quad 2\alpha - \beta + 8 = 0.$$

Thus the set A is the union of these parallel lines. Their distances from the origin are, respectively,

$$d_1 = \frac{16}{\sqrt{2^2 + (-1)^2}} = \frac{16}{\sqrt{5}}, \qquad d_2 = \frac{8}{\sqrt{5}}.$$

Step 3: Choose the smaller attainable distance On either line, the perpendicular foot from the origin is the closest point on that line, so both line distances are attained by admissible points. The smaller value is $8/\sqrt{5}$. The value $16/\sqrt{5}$ belongs to the more distant line, while $4/\sqrt{5}$ and $12/\sqrt{5}$ come from neither of the two area equations.

Step 4: Conclusion The least possible segment length is $8/\sqrt{5}$ units. Hence, the correct option is (C).

Quick Tip to Solve: A fixed triangle area with a fixed base gives two parallel loci. Compute the origin-to-line distance for both signs of the determinant before selecting the minimum.

13. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If f(0) = 0, f(1) = 1 and f(2) = 2, then [4 marks]

(A) f"(x) = 0 for all x ∈ (0, 2)

(B) f"(x) = 0 for some x ∈ (0, 2)

(C) f'(x) = 0 for some x ∈ [0, 2]

(D) f"(x) > 0 for all x ∈ (0, 2)

Correct Answer: (B) f"(x) = 0 for some x ∈ (0, 2)

Solution:

Concept:

- The mean value theorem gives an interior point where the derivative equals the secant slope, for a function continuous on a closed interval and differentiable inside it.
- Rolle's theorem applied to the first derivative gives a zero of the second derivative when the first derivative has equal values at two interior points. Twice differentiability ensures that the first derivative is continuous there.

Step 1: Find equal first-derivative values Apply the mean value theorem to f separately on $[0, 1]$ and $[1, 2]$. Its continuity and differentiability assumptions are satisfied on both intervals. There exist $c_1 \in (0, 1)$ and $c_2 \in (1, 2)$ such that

$$f'(c_1) = \frac{f(1) - f(0)}{1 - 0} = 1, \qquad f'(c_2) = \frac{f(2) - f(1)}{2 - 1} = 1.$$

Step 2: Apply Rolle theorem to the derivative The function f' is differentiable throughout $(0, 2)$, hence continuous on the interior closed interval $[c_1, c_2]$. Since it has equal endpoint values, Rolle's theorem supplies $c \in (c_1, c_2)$ with

$$f''(c) = 0.$$

Because $0 < c_1 < c < c_2 < 2$, this is exactly an interior zero of the second derivative.

Step 3: Check the stronger alternatives The example $f(x) = x + x(x - 1)(x - 2)$ has the three prescribed values but $f''(x) = 6x - 6$, which is not zero everywhere; this rules out (A). The simpler example $f(x) = x$ has $f' = 1$ and $f'' = 0$, so neither (C) nor (D) is compulsory. The theorem guarantees existence at some point, rather than a claim at every point.

Step 4: Conclusion The second derivative vanishes at some interior point of $(0, 2)$. Hence, the correct option is (B).

Quick Tip to Solve: When three points of a differentiable function lie on one straight line, apply the mean value theorem on the two adjacent intervals, then Rolle's theorem to its derivative.

14. If [x] is the greatest integer $\leq$ x, then

$\pi^2 \int_0^2 \left(\sin \frac{\pi x}{2}\right)(x-[x])^{[x]}dx$ **is equal to :** [4 marks]

(A) $2(\pi - 1)$

(B) $4(\pi - 1)$

(C) $4(\pi + 1)$

(D) $2(\pi + 1)$

Correct Answer: (B) $4(\pi - 1)$

Solution:

Concept:

- The greatest-integer function is constant between consecutive integers. Here $[x] = 0$ on $(0, 1)$ and $[x] = 1$ on $(1, 2)$; changing finitely many endpoint values does not change the integral.
- For a constant a, $\int \sin(ax)\,dx = -\cos(ax)/a$. The integral of a linear factor times $\sin(ax)$ follows from integration by parts.

Step 1: Split at the integer boundary Write the required value as I and take $a = \pi/2$. For $0 < x < 1$, the factor $(x - [x])^{[x]}$ is $x^0 = 1$. For $1 < x < 2$, it is $x - 1$. Therefore

$$\frac{I}{\pi^2} = \int_0^1 \sin(ax)\,dx + \int_1^2 (x-1)\sin(ax)\,dx.$$

Any isolated endpoint convention for the zeroth power has no effect on these definite integrals.

Step 2: Evaluate the two integrals The first integral equals

$$\left[-\frac{\cos(ax)}{a}\right]_0^1 = \frac{1}{a} = \frac{2}{\pi}.$$

For the second, integrate by parts with $u = x - 1$ and $dv = \sin(ax)dx$:

$$\int_1^2 (x-1)\sin(ax)\,dx = \left[-\frac{(x-1)\cos(ax)}{a} + \frac{\sin(ax)}{a^2}\right]_1^2 = \frac{1}{a} - \frac{1}{a^2}.$$

The sine term at the lower limit is essential for the negative $1/a^2$ contribution.

Step 3: Restore the outside factor Adding and multiplying by π^2 gives

$$I = \pi^2\left(\frac{4}{\pi} - \frac{4}{\pi^2}\right) = 4(\pi - 1).$$

The coefficient two in (A) and (D) misses one contribution of $2/\pi$. The plus sign in (C) and (D) conflicts with subtraction of the lower-limit sine term.

Step 4: Conclusion The integral equals $4(\pi - 1)$. Hence, the correct option is (B).

Quick Tip to Solve: Split a floor-function integral at every integer first. Only after replacing the floor by its constant value should you simplify the power and integrate.

15. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is : [4 marks]

(A) $\frac{92}{5}$

(B) $\frac{134}{5}$

(C) $\frac{536}{25}$

(D) $\frac{112}{5}$

Correct Answer: (C) $\frac{536}{25}$

Solution:

Concept:

- For a population of n observations, the mean is $\bar{x} = \sum x_i/n$, and the variance is $\sigma^2 = \sum x_i^2/n - \bar{x}^2$. The divisor is the number of observations, rather than the sample divisor $n - 1$.
- When observations are removed, subtract their values from the sum and their squares from the sum of squares. Recalculate the mean of the remaining observations before finding their variance.

Step 1: Recover the original sums The seven observations have mean 8, so their sum is $7 \times 8 = 56$. Their variance is 16. Rearranging the variance formula gives the second moment and then the sum of squares:

$$\frac{\sum x_i^2}{7} = 16 + 8^2 = 80, \qquad \sum x_i^2 = 560.$$

This uses all seven observations and does not require knowing them separately.

Step 2: Remove the specified observations The removed values are 6 and 8. Their sum is 14 and their sum of squares is $36 + 64 = 100$. Hence the five retained observations satisfy

$$S = 56 - 14 = 42, \qquad T = 560 - 100 = 460.$$

Their new mean is $42/5$, which is different from 8. Keeping the old mean would therefore give an incorrect variance.

Step 3: Calculate and compare the variance Apply the same population formula to the five observations:

$$\sigma_{\text{new}}^2 = \frac{460}{5} - \left(\frac{42}{5}\right)^2$$
$$= 92 - \frac{1764}{25} = \frac{536}{25}.$$

This is 21.44. The other listed values, $92/5$, $134/5$, and $112/5$, do not equal the second moment minus the square of the new mean.

Step 4: Conclusion The variance of the remaining five observations is $\boxed{536/25}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep two running totals when deleting data: the sum and the sum of squares. Only after the deletions should you calculate the new mean.

16. Let $\vec{a}, \vec{b}, \vec{c}$ three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies.

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$, then $\vec{r}$ is equal to : [4 marks]

(A) $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$

(B) $\frac{1}{3}(2\vec{a} + \vec{b} - \vec{c})$

(C) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$

(D) $\frac{1}{2}(\vec{a} + \vec{b} + 2\vec{c})$

Correct Answer: (C) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$

Solution:

Concept:

- The vector triple-product identity is $\vec{u} \times (\vec{v} \times \vec{w}) = \vec{v}(\vec{u} \cdot \vec{w}) - \vec{w}(\vec{u} \cdot \vec{v})$. The order of the vectors determines the signs.
- Let the common nonzero magnitude be m. Mutually perpendicular vectors satisfy $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$, and form an orthogonal basis in three dimensions.

Step 1: Expand the three products Apply the identity to each term, using perpendicularity to eliminate the mixed dot products. For the first term,

$$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} = m^2(\vec{r} - \vec{b}) - \vec{a}(\vec{a} \cdot \vec{r}).$$

The next terms are $m^2(\vec{r} - \vec{c}) - \vec{b}(\vec{b} \cdot \vec{r})$ and $m^2(\vec{r} - \vec{a}) - \vec{c}(\vec{c} \cdot \vec{r})$, respectively.

Step 2: Use the orthogonal decomposition The expansion of any vector in this equal-length orthogonal basis gives

$$\vec{a}(\vec{a} \cdot \vec{r}) + \vec{b}(\vec{b} \cdot \vec{r}) + \vec{c}(\vec{c} \cdot \vec{r}) = m^2\vec{r}.$$

Thus, on adding the three terms, the given equation reduces to

$$3m^2\vec{r} - m^2(\vec{a} + \vec{b} + \vec{c}) - m^2\vec{r} = \vec{0}.$$

Division by m^2 is allowed because the basis vectors have nonzero length.

Step 3: Solve and reject the other coefficients The remaining vector equation is $2\vec{r} = \vec{a} + \vec{b} + \vec{c}$. Consequently each of the three basis coefficients of $\vec{r}$ must be exactly $1/2$. Option (A) uses coefficients $1/3$; option (B) has unequal coefficients including a negative one; option (D) gives coefficient 1 for $\vec{c}$. Each fails the reduced equation.

Step 4: Conclusion $\boxed{\vec{r} = \frac{1}{2}(\vec{a} + \vec{b} + \vec{c})}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal-length perpendicular vectors, the sum of their three projection numerators equals the common squared length times the projected vector.

17. The distance of the point $(-1, 2, -2)$ from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{5}{2}$

(C) $\frac{\sqrt{42}}{2}$

(D) $\frac{\sqrt{34}}{2}$

Correct Answer: (D) $\frac{\sqrt{34}}{2}$

Solution:

Concept:

- The line common to two planes has direction perpendicular to both plane normals. Its direction can therefore be obtained from their cross product, or by solving their homogeneous equations.
- For a line through the origin with direction $\vec{d}$, the squared distance of a point with position vector $\vec{p}$ is $|\vec{p}|^2 - (\vec{p} \cdot \vec{d})^2/|\vec{d}|^2$. This removes the squared component parallel to the line.

Step 1: Find the intersection line Both planes pass through the origin. From the second plane, $x + z = 2y$. Substitute this into the first plane to obtain

$$2(x + z) + 3y = 4y + 3y = 7y = 0.$$

Thus $y = 0$ and $z = -x$. The entire common line is $(x, y, z) = (t, 0, -t)$, with direction $\vec{d} = (1, 0, -1)$. A normal to either single plane would not be the direction of this line.

Step 2: Calculate the perpendicular component The given point is $P = (-1, 2, -2)$, so $\vec{p} = (-1, 2, -2)$. Compute

$$|\vec{p}|^2 = 1 + 4 + 4 = 9, \qquad \vec{p} \cdot \vec{d} = -1 + 2 = 1, \qquad |\vec{d}|^2 = 2.$$

The squared perpendicular distance is therefore

$$D^2 = 9 - \frac{1^2}{2} = \frac{17}{2} = \frac{34}{4}.$$

We take the nonnegative square root, since distance cannot be negative.

Step 3: Check the nearest point and options The closest point occurs at $t = (\vec{p} \cdot \vec{d})/|\vec{d}|^2 = 1/2$, giving $H = (1/2, 0, -1/2)$. Then $\overrightarrow{HP} = (-3/2, 2, -3/2)$, perpendicular to $(1, 0, -1)$, and its squared length is $9/4 + 4 + 9/4 = 34/4$. Options (A), (B), and (C) have squared values $1/2$, $25/4$, and $42/4$, so none matches.

Step 4: Conclusion The required distance is $\boxed{\sqrt{34}/2}$ units. Hence, the correct option is (D).

Quick Tip to Solve: When both plane equations are homogeneous, their common line passes through the origin, so the projection formula needs no extra reference point.

18. Negation of the statement (p $\vee$ r) $\Rightarrow$ (q $\vee$ r) is : [4 marks]

(A) p $\wedge \sim$ q $\wedge \sim$ r

(B) $\sim$ p $\wedge$ q $\wedge \sim$ 4

(C) $\sim$ p $\wedge$ q $\wedge$ r

(D) p $\wedge$ q $\wedge$ r

Correct Answer: (A) p $\wedge \sim$ q $\wedge \sim$ r

Solution:

Concept:

- An implication $A \Rightarrow B$ is false only when its antecedent A is true and its consequent B is false. Thus $\neg(A \Rightarrow B) = A \wedge \neg B$.
- De Morgan's law changes the negation of a disjunction into a conjunction: $\neg(q \vee r) = (\neg q) \wedge (\neg r)$. Boolean simplification can then remove an impossible term.

Step 1: Negate the entire implication Use $A = p \vee r$ and $B = q \vee r$. Negating the complete statement, rather than separately reversing its two sides, gives

$$\neg\big[(p \vee r) \Rightarrow (q \vee r)\big] = (p \vee r) \wedge \neg(q \vee r).$$

The antecedent must continue to hold while the consequent must fail. An implication is not negated by merely changing its arrow direction.

Step 2: Apply De Morgan and distribute The consequent is false precisely when both q and r are false. Hence

$$\begin{aligned}(p \vee r) \wedge \neg q \wedge \neg r &= (p \wedge \neg q \wedge \neg r) \\ &\quad \vee (r \wedge \neg q \wedge \neg r) \\ &= p \wedge \neg q \wedge \neg r.\end{aligned}$$

The second term vanishes because it requires r and $\neg r$ to be true together.

Step 3: Check the truth assignment The resulting conjunction is true exactly at $p = \mathrm{T}$, $q = \mathrm{F}$, $r = \mathrm{F}$. At that assignment the original antecedent is true and consequent false, so the original implication is indeed false. The other options all include positive q, which makes $q \vee r$ true and the implication true. They therefore cannot express its negation, regardless of their remaining factors.

Step 4: Conclusion The negation is $\boxed{p \wedge \neg q \wedge \neg r}$. Hence, the correct option is (A).

Quick Tip to Solve: For the negation of an implication, first write antecedent AND NOT consequent. Apply De Morgan only after that step.

19. An angle of intersection of the curves, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab$, $a>b$, is : [4 marks]

(A) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$

(B) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

(C) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

(D) $\tan^{-1}\left(2\sqrt{ab}\right)$

Correct Answer: (C) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

Solution:

Concept:

- The angle between smooth curves at an intersection is the angle between their tangents. If their tangent slopes are m_1, m_2, the acute angle satisfies $\tan\theta = |(m_1-m_2)/(1+m_1m_2)|$.
- The ellipse has positive semiaxes $a > b > 0$. Implicit differentiation gives its tangent slope $-b^2x/(a^2y)$, whereas the circle has slope $-x/y$. The coordinates must satisfy both curves.

Step 1: Find the coordinate ratio at intersection Solve $x^2+y^2=ab$ together with $b^2x^2+a^2y^2=a^2b^2$. Substituting $x^2=ab-y^2$ gives

$$(a^2-b^2)y^2 = ab^2(a-b).$$

Since $a>b$, cancellation is valid, and

$$y^2=\frac{ab^2}{a+b}, \qquad x^2=\frac{a^2b}{a+b}, \qquad \left|\frac{x}{y}\right|=\sqrt{\frac{a}{b}}.$$

Neither coordinate is zero, so both tangent slopes are finite.

Step 2: Evaluate the slope-angle formula Write $u=x/y$; then $m_1=-b^2u/a^2$ and $m_2=-u$. Their product is $(b^2/a^2)u^2=b/a$. Therefore

$$\begin{aligned}\tan\theta &= \frac{|u|(1-b^2/a^2)}{1+b/a}\\ &= \sqrt{\frac{a}{b}}\left(1-\frac{b}{a}\right)=\frac{a-b}{\sqrt{ab}}.\end{aligned}$$

Absolute values give the same acute angle at intersections in all quadrants.

Step 3: Compare with the listed angles The acute angle is $\tan^{-1}((a-b)/\sqrt{ab})$. It tends to zero as the ellipse approaches the circle when a approaches b, consistent with tangency in that limit. Option (A) incorrectly uses $a+b$; option (B) adds a factor 2 in the denominator; option (D) lacks the necessary ratio and does not have this limiting behaviour.

Step 4: Conclusion An angle of intersection is $\boxed{\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the ratio of the squared coordinates before finding tangent slopes; the individual intersection coordinates are unnecessary.

20. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81,\ 0 \le x \le \frac{\pi}{4}$ is : [4 marks]

(A) 3

(B) 1

(C) 0

(D) 2

Correct Answer: (B) 1

Solution:

Concept:

- The identity $\sec^2 x = 1 + \tan^2 x$ makes the two exponential terms share a common factor. For a positive base, $a^{t+1} = a\,a^t$.
- On $[0, \pi/4]$, the function $\tan x$ increases continuously from 0 to 1. Its square therefore also increases continuously from 0 to 1, and an exponential with base greater than 1 preserves this strict increase.

Step 1: Reduce to one exponential Let $t = \tan^2 x$. The interval in the question gives $0 \le t \le 1$, and the identity gives $\sec^2 x = t + 1$. Taking the printed base as 32, factor the equation as

$$32^t + 32^{t+1} = 32^t(1 + 32) = 81.$$

Consequently

$$32^t = \frac{81}{33} = \frac{27}{11}.$$

This step does not treat the sum of powers as a power of a sum.

Step 2: Establish existence and uniqueness The number $27/11$ lies strictly between 1 and 32. Since 32^t increases continuously from 1 to 32 for t in $[0, 1]$, exactly one value

$$t = \frac{\ln(27/11)}{\ln 32}$$

lies in the required interval. Each allowed t corresponds to exactly one x, namely $x = \tan^{-1}\sqrt{t}$, because tangent is one-to-one and nonnegative on this interval.

Step 3: Check the number rather than unnecessary roots The left side of the original equation has value 33 at $x = 0$ and $32 + 1024 = 1056$ at $x = \pi/4$, and is strictly increasing between them. The value 81 is crossed once. Thus option (C), which asserts no solution, fails the endpoint check; options (A) and (D), which assert three and two, violate strict monotonicity. Only the count 1 remains.

Step 4: Conclusion There is exactly $\boxed{1}$ solution in the specified interval. Hence, the correct option is (B).

Quick Tip to Solve: Use the difference of one between secant-squared and tangent-squared exponents to factor first, then count roots through monotonicity.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K.2^{16}$, then K is equal to _____. [4 marks]

Solution:

Concept:

- In expanding $(u + v + w)^{10}$, selecting u, v, and w respectively r, s, t times contributes coefficient $10!/(r!s!t!)$, where $r + s + t = 10$.
- Here the three terms are a, $2b$, and $4ab$. Selecting the mixed term contributes to both the power of a and the power of b, so its count must occur in both exponent equations.

Step 1: Determine the selection counts Choose a from r factors, $2b$ from s factors, and $4ab$ from t factors. Their product is

$$a^r(2b)^s(4ab)^t = 2^s 4^t a^{r+t} b^{s+t}.$$

To obtain a^7b^8, the nonnegative integer counts must obey

$$r + t = 7, \qquad s + t = 8, \qquad r + s + t = 10.$$

Adding the first two equations and subtracting the third gives $t = 5$, after which $r = 2$ and $s = 3$. These counts are unique.

Step 2: Calculate the coefficient There are $10!/(2!3!5!)$ ways to position these selections among the ten factors. The numerical multiplier of each such selection is $2^3 4^5 = 2^{13}$. The complete coefficient is therefore

$$\begin{aligned} C &= \frac{10!}{2!3!5!}\, 2^{13} \\ &= 2520\, 2^{13}. \end{aligned}$$

All three selection counts must be retained in the factorial denominator, including the five mixed-term selections.

Step 3: Express the coefficient in the requested form The question defines $C = K\, 2^{16}$. Divide the coefficient by 2^{16}, rather than equating K directly to the multinomial factor:

$$K = 2520\, 2^{13-16} = \frac{2520}{8} = 315.$$

Equivalently, $2520 = 315 \times 2^3$, so $2520 \times 2^{13} = 315 \times 2^{16}$, verifying exactly the specified power of two. The answer is a dimensionless integer.

Step 4: Conclusion The required value is $\boxed{K = 315}$.

Quick Tip to Solve: For a mixed term such as ab, introduce its selection count in both exponent equations; solve the counts before calculating any factorials.

22. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is _____.

[4 marks]

Solution:

Concept:

- Four-digit positive integers form the inclusive interval $1000 \le n \le 9999$, containing $9999 - 1000 + 1 = 9000$ integers.
- The count of multiples of a positive integer k in this interval is $\lfloor 9999/k \rfloor - \lfloor 999/k \rfloor$. Inclusion and exclusion subtracts the union of the forbidden sets, counting their intersection only once.

Step 1: Count the forbidden multiples separately Let S_3 and S_7 be the sets of four-digit multiples of 3 and 7. Using the inclusive endpoints,

$$|S_3| = 3333 - 333 = 3000, \qquad |S_7| = 1428 - 142 = 1286.$$

For example, the first admissible multiple of 7 is 1001 and the last is 9996; multiples below 1000 must not be included.

Step 2: Correct for the common multiples An integer divisible by both 3 and 7 is divisible by their least common multiple, 21, since these two divisors are coprime. Hence

$$|S_3 \cap S_7| = 476 - 47 = 429.$$

Subtracting the two separate counts from 9000 removes every common multiple twice. Restore that overlap once to count the complement of the union:

$$N = 9000 - 3000 - 1286 + 429 = 5143.$$

This complement imposes both restrictions simultaneously; it does not count numbers divisible by exactly one of the two divisors.

Step 3: Conclusion There are $\boxed{5143}$ four-digit numbers that are multiples of neither 7 nor 3. The requested numerical entry is 5143.

Quick Tip to Solve: For neither-divisible counting, start with all integers, subtract each multiple count, and add back the multiples of the least common multiple.

23. If $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx =$
$\alpha \log_e|1 + \tan x| + \beta \log_e|1 - \tan x + \tan^2 x| + \gamma \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$, when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is _____.

[4 marks]

Solution:

Concept:

- The substitution $t = \tan x$ converts the trigonometric rational function into a rational function of t, on intervals where $\cos x \neq 0$.

- Factor $t^3+1=(t+1)(t^2-t+1)$. The quadratic satisfies $t^2-t+1=(t-\frac{1}{2})^2+\frac{3}{4}>0$, so its reciprocal integrates to an inverse tangent.

Step 1: Convert the integral and decompose the fraction Since $\sin x = t\cos x$ and $dt=\sec^2 x\,dx$, the powers of $\cos x$ cancel to give

$$I=\int \frac{t}{1+t^3}\,dt.$$

The partial fractions are

$$\frac{t}{1+t^3}=-\frac{1}{3(t+1)}+\frac{t+1}{3(t^2-t+1)}.$$

Multiplying this identity by $(t+1)(t^2-t+1)$ gives the numerator t, confirming the signs.

Step 2: Integrate and compare the coefficients Write $t+1=\frac{1}{2}(2t-1)+\frac{3}{2}$. The derivative of the quadratic is $2t-1$, so

$$I=-\frac{1}{3}\log|t+1|+\frac{1}{6}\log(t^2-t+1)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)+C.$$

Thus $\alpha=-1/3$, $\beta=1/6$, and $\gamma=1/\sqrt{3}$. The logarithm of the quadratic needs no absolute value because its argument is positive. Restoring $t=\tan x$ yields exactly the prescribed form, locally away from its singularities. Therefore

$$18(\alpha+\beta+\gamma^2)=18\left(-\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right)=3.$$

Step 3: Conclusion The required value is $\boxed{3}$, a dimensionless numerical entry.

Quick Tip to Solve: For a cubic sum in sine and cosine, try tangent substitution, then split the quadratic numerator into its denominator derivative and a constant.

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24. The number of elements in the set $\left\{A=\begin{pmatrix}a & b\\ 0 & d\end{pmatrix} : a,b,d\in\{-1,0,1\}\ and\ (I-A)^3=I-A^3\right\}$, where I is 2×2 identity matrix, is : [4 marks]

Solution:

Concept:

- The identity matrix commutes with every square matrix, so the binomial expansion $(I-A)^3=I-3A+3A^2-A^3$ is valid.
- An idempotent matrix satisfies $A^2=A$. For an upper triangular matrix, equating corresponding entries reduces this relation to scalar equations.

Step 1: Reduce the given matrix equation Expanding the left side and cancelling $I-A^3$ from both sides gives

$$-3A+3A^2=0 \quad\Longleftrightarrow\quad A^2=A.$$

For the stated matrix,

$$A^2 = \begin{pmatrix} a^2 & b(a+d) \\ 0 & d^2 \end{pmatrix}.$$

Consequently $a^2 = a$, $d^2 = d$, and $b(a + d - 1) = 0$. Although the original allowed values include -1, the diagonal equations force each of a, d to belong to $\{0, 1\}$.

Step 2: Count the permitted off-diagonal entries There are four ordered diagonal pairs. If $(a, d) = (0, 0)$, then $-b = 0$, so only $b = 0$ is allowed. If $(a, d) = (1, 1)$, then $b = 0$ again. For $(a, d) = (0, 1)$ or $(1, 0)$, the factor $a + d - 1$ is zero, so any of the three choices $-1, 0, 1$ is allowed for b. The number of matrices is therefore

$$1 + 1 + 3 + 3 = 8.$$

Every counted choice satisfies all three entry equations, and hence the original cubic identity. Distinct triples (a, b, d) give distinct matrices, so there is no duplication.

Step 3: Conclusion The set contains $\boxed{8}$ matrices. The requested numerical entry is 8.

Quick Tip to Solve: Expand the polynomial identity before enumerating matrices; idempotence immediately eliminates negative diagonal entries and leaves only four diagonal cases.

25. If the line y = mx bisects the area enclosed by the lines x = 0, y = 0, x = $\frac{3}{2}$ and the curve y = 1 + 4x − x², then 12 m is equal to _____. [4 marks]

Solution:

Concept:

- The area under a nonnegative curve $y = f(x)$ between vertical boundaries a, b is $\int_a^b f(x)\,dx$.
- If the line $y = mx$ remains inside the region throughout $0 \le x \le 3/2$, the portion below it is the triangle of area $\frac{1}{2}(3/2)(3m/2) = 9m/8$.

Step 1: Evaluate the enclosed area The parabola $f(x) = 1 + 4x - x^2$ is positive on the interval: it increases there since $f\prime(x) = 4 - 2x > 0$, and $f(0) = 1$. Thus the region lies between the curve and the x-axis. Its area is

$$S = \int_0^{3/2} (1 + 4x - x^2)\,dx = \left[x + 2x^2 - \frac{x^3}{3}\right]_0^{3/2} = \frac{39}{8}.$$

A bisector must leave area $39/16$ on each side.

Step 2: Find the slope and verify the line stays inside Equate the area beneath the line to half the total:

$$\frac{9m}{8} = \frac{39}{16} \quad \Longrightarrow \quad m = \frac{13}{6}.$$

To justify using the full triangular area, examine the vertical separation:

$$f(x) - mx = 1 + \frac{11}{6}x - x^2.$$

This is concave, so its minimum on the closed interval occurs at an endpoint. Its values at 0 and $3/2$ are 1 and $3/2$, both positive. Also $m > 0$; hence the line lies above the x-axis and below the curve throughout the interval. The two resulting subregions really have equal area, rather than merely equal signed integrals.

Step 3: Conclusion With $m = 13/6$, the requested value is $12m = \boxed{26}$, a dimensionless numerical entry.

Quick Tip to Solve: When an area-bisecting line passes through the origin, use its triangular area first, then verify it does not cross the upper curve inside the interval.

26. Let f(x) be a cubic polynomial with f(1) = −10, f(−1) = 6, and has a local minima at x = 1, and f'(x) has a local minima at x = −1. Then f(3) is equal to _____. [4 marks]

Solution:

Concept:

- For a differentiable polynomial, an interior local minimum at $x = c$ requires $f\prime(c) = 0$. A positive second derivative verifies a strict local minimum.
- If the quadratic $f\prime(x)$ has a local minimum at -1, its derivative $f\prime\prime(-1)$ is zero and its leading coefficient is positive.

Step 1: Translate the two minimum conditions Write $f(x) = ax^3 + bx^2 + cx + d$, with $a \neq 0$. Then

$$f\prime(x) = 3ax^2 + 2bx + c, \qquad f\prime\prime(x) = 6ax + 2b.$$

The minimum of $f\prime$ at -1 gives $-6a + 2b = 0$, hence $b = 3a$; it also requires $a > 0$. The minimum of f at 1 gives $3a + 2b + c = 0$, so $c = -9a$.

Step 2: Use the function values and evaluate the cubic The difference between the two supplied values eliminates b and d:

$$f(1) - f(-1) = 2a + 2c = -10 - 6 = -16.$$

Substituting $c = -9a$ gives $-16a = -16$, so $a = 1$, $b = 3$, and $c = -9$. Now $f(1) = 1 + 3 - 9 + d = -10$ yields $d = -5$. Therefore

$$f(x) = x^3 + 3x^2 - 9x - 5, \qquad f(3) = 27 + 27 - 27 - 5 = 22.$$

The minimum claims are consistent: $f\prime\prime(1) = 12 > 0$, while $f\prime(x) = 3(x + 1)^2 - 12$ has its strict minimum at -1. Also $f(-1) = -1 + 3 + 9 - 5 = 6$, checking the second supplied value.

Step 3: Conclusion The requested polynomial value is $f(3) = \boxed{22}$, a dimensionless numerical entry.

Quick Tip to Solve: For cubic constraints at 1 and -1, subtract the two values to isolate odd-power coefficients; translate extrema of the derivative using the second derivative.

27. Suppose, the line $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. Then $(\alpha + \beta)$ is equal to _____. [4 marks]

Solution:

Concept:

- A line lies entirely in a plane precisely when a point on it belongs to the plane and its direction vector is perpendicular to the plane normal.
- Equivalently, substitute the parametric equations of the line into the plane equation: both the constant term and the coefficient of the parameter must vanish.

Step 1: Parametrize the given line Let the common ratio be t. The line is

$$x = 2 + \alpha t, \qquad y = 2 - 5t, \qquad z = -2 + 2t.$$

It passes through $P = (2, 2, -2)$ and has direction vector $\boldsymbol{d} = (\alpha, -5, 2)$. The plane normal is $\boldsymbol{n} = (1, 3, -2)$. The negative coordinate in P follows from the numerator $z + 2$, and must be retained.

Step 2: Impose containment for every parameter value Substitution into the plane gives

$$(2 + \alpha t) + 3(2 - 5t) - 2(-2 + 2t) + \beta = 0,$$

or, after collecting terms,

$$(12 + \beta) + (\alpha - 19)t = 0.$$

Since the whole line lies on the plane, this identity must hold for every real t, rather than for just one point of intersection. Thus $\beta = -12$ and $\alpha = 19$. Independently, $\boldsymbol{d} \cdot \boldsymbol{n} = 19 - 15 - 4 = 0$ verifies parallelism to the plane, while $2 + 6 + 4 - 12 = 0$ verifies the base point. Both necessary conditions hold, so the line is contained in the plane.

Step 3: Conclusion The required sum is $\alpha + \beta = 19 - 12 = \boxed{7}$. This is a dimensionless numerical entry.

Quick Tip to Solve: For a symmetric line contained in a plane, separate the point condition from the direction condition; the latter determines the denominator containing the unknown.

28. A tangent line L is drawn at the point (2, −4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to _____. [4 marks]

Solution:

Concept:

- For the parabola $y^2 = 4px$, the tangent at (x_1, y_1) is $yy_1 = 2p(x + x_1)$. The point must first satisfy the parabola equation.
- A circle centered at the origin is tangent to a line when its radius equals the perpendicular distance from the origin to that line. In $x^2 + y^2 = a$, the parameter a is the squared radius.

Step 1: Obtain the tangent to the parabola Here $4p = 8$, so $p = 2$. The given point $(2, -4)$ lies on the

parabola because $(-4)^2 = 8(2) = 16$. Its tangent is therefore

$$-4y = 4(x+2) \quad \Longleftrightarrow \quad x + y + 2 = 0.$$

The same result follows by differentiation: $2y\,dy/dx = 8$, giving slope -1 at $y = -4$, so $y + 4 = -(x-2)$.

Step 2: Use the tangency distance condition The perpendicular distance of the origin from $x+y+2=0$ is

$$d = \frac{|2|}{\sqrt{1^2+1^2}} = \sqrt{2}.$$

The circle radius is $r = \sqrt{a}$, not a. Hence tangency requires $\sqrt{a} = \sqrt{2}$, which gives $a = 2$. For a direct check, substituting $y = -x - 2$ into the circle produces

$$x^2 + (x+2)^2 = 2 \quad \Longleftrightarrow \quad 2(x+1)^2 = 0.$$

There is exactly one intersection, $(-1, -1)$, confirming that this line is tangent to the circle as well. The two tangency points need not be the same.

Step 3: Conclusion The requested circle parameter is $a = \boxed{2}$, the square of its radius, in squared coordinate units.

Quick Tip to Solve: When a line is tangent to a circle centered at the origin, square its perpendicular distance to obtain the constant in the circle equation.

29. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$, then 160 S is equal to _____. [4 marks]

Solution:

Concept:

- The numerator sequence has successive differences $2, 4, 6, \ldots$. Its nth term is therefore $7 + 2(1 + 2 + \cdots + (n-1)) = n^2 - n + 7$, where the first term corresponds to $n = 1$.
- For $|x| < 1$, the geometric series and its derivatives give $\sum_{n=1}^{\infty} x^n = x/(1-x)$ and $\sum_{n=1}^{\infty} n(n-1)x^n = 2x^2/(1-x)^3$. Here $x = 1/5$, so the series converge absolutely.

Step 1: Identify and separate the numerator The successive numerators are $7, 9, 13, 19, \ldots$. Subtracting the constant 7 leaves $0, 2, 6, 12, \ldots = n(n-1)$. Thus we can split the original sum without changing its order:

$$S = \sum_{n=1}^{\infty} \frac{n(n-1)}{5^n} + 7\sum_{n=1}^{\infty} \frac{1}{5^n}.$$

The quadratic numerator grows much more slowly than the denominator 5^n, which also confirms convergence.

Step 2: Evaluate the two convergent sums Twice differentiating $\sum_{n=0}^{\infty} x^n = (1-x)^{-1}$, then multiplying

by x^2, produces the first required sum. Therefore

$$\begin{aligned} S &= \frac{2(1/5)^2}{(1-1/5)^3} + \frac{7(1/5)}{1-1/5} \\ &= \frac{2}{25}\frac{125}{64} + \frac{7}{4} \\ &= \frac{5}{32} + \frac{56}{32} = \frac{61}{32}. \end{aligned}$$

In particular, the denominator of the first term is 5^1, so no extra factor of 5 is introduced.

Step 3: Calculate the requested multiple The question asks for $160S$, rather than S alone. Multiplying the exact fraction gives

$$160S = 160 \times \frac{61}{32} = 5 \times 61 = 305.$$

Keeping fractions throughout avoids a rounding error in this final integer.

Step 4: Conclusion The requested numerical value is $\boxed{305}$. The series and its requested multiple are dimensionless.

Quick Tip to Solve: For numerators with constant second differences, express the nth numerator using $n(n-1)$; its infinite sum follows immediately from the twice-differentiated geometric series.

30. Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8.\left(\frac{area\,\Delta APQ}{area\,\Delta BPQ}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- Completing squares in a circle equation reveals its centre and radius. Tangents drawn from an external point are equal, so their chord of contact is perpendicular to the line joining that point to the centre.
- Let M be the intersection of the chord of contact PQ with AB. From the right triangle formed by a radius and a tangent, $BM \cdot BA = r^2$. Triangles with the same base have areas proportional to their perpendicular heights.

Step 1: Find the centre and its distance from A Write the equation in standard form:

$$x^2 - 2x + y^2 + 4y + 1 = 0 \quad \Longrightarrow \quad (x-1)^2 + (y+2)^2 = 4.$$

Hence $B = (1, -2)$ and $r = 2$. With $A = (3, 1)$, the distance between the external point and the centre is

$$BA = \sqrt{(3-1)^2 + (1+2)^2} = \sqrt{13}.$$

Since $\sqrt{13} > 2$, the two distinct tangents and their contact chord exist.

Step 2: Obtain the distances from the common base The perpendicular bisector of PQ passes through both A and B. Thus M lies on AB, and $PM \perp AB$. In right triangle BPA, $BP = r$ and $BP \perp AP$;

projecting on BA gives

$$BM = \frac{BP^2}{BA} = \frac{4}{\sqrt{13}}.$$

The contact chord lies between A and B, so

$$AM = AB - BM = \sqrt{13} - \frac{4}{\sqrt{13}} = \frac{9}{\sqrt{13}}.$$

These are the respective perpendicular heights to the same line PQ.

Step 3: Use the common-base area ratio Both triangles use the identical base PQ. Consequently its length and the factor $1/2$ cancel:

$$\frac{\text{area}\,\triangle APQ}{\text{area}\,\triangle BPQ} = \frac{AM}{BM} = \frac{9/\sqrt{13}}{4/\sqrt{13}} = \frac{9}{4}.$$

Multiplying this ratio by the printed factor 8 gives $8(9/4) = 18$; finding the individual tangent points is unnecessary.

Step 4: Conclusion The requested dimensionless value is $\boxed{18}$.

Quick Tip to Solve: For tangents from an external point, use $BM = r^2/BA$. The common-base area ratio then simplifies to $(BA^2 - r^2)/r^2$.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50 $\times$ 10^3 kg. The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column. [Use Y = 2.0 $\times$ 10^{11} Pa, g = 9.8 m/s^2] [4 marks]

(A) 3.60×10^{-8}

(B) 2.60×10^{-7}

(C) 1.87×10^{-3}

(D) 7.07×10^{-4}

Correct Answer: (B) 2.60×10^{-7}

Solution:

Concept:

- Young modulus relates longitudinal stress to longitudinal strain: $Y = \sigma/\varepsilon$, so $\varepsilon = F/(AY)$. Compression strain here means the positive magnitude of the fractional shortening.
- An annular column has load-bearing area $A = \pi(R^2 - r^2)$, not πR^2. Identical columns under a uniformly distributed load carry equal shares of the total weight.

Step 1: Distribute the weight among the columns The supported mass is $M = 50 \times 10^3\,\text{kg}$. Its

downward weight is

$$Mg = (50 \times 10^3)(9.8) = 4.90 \times 10^5\,\text{N}.$$

Four identical columns carry this load equally, giving a compressive force in each column of

$$F = \frac{Mg}{4} = 1.225 \times 10^5\,\text{N}.$$

Using the full weight for one column would overestimate its strain by a factor of four.

Step 2: Calculate the load-bearing annular area Convert both radii to metres before squaring: $R = 100\,\text{cm} = 1.00\,\text{m}$ and $r = 50\,\text{cm} = 0.50\,\text{m}$. Hence

$$A = \pi\left(1.00^2 - 0.50^2\right) = 0.75\pi\,\text{m}^2.$$

The hollow central region does not support axial force. The compressive stress is therefore $\sigma = (1.225 \times 10^5)/(0.75\pi)\,\text{Pa}$.

Step 3: Apply Young modulus and compare the options Substitution of $Y = 2.0 \times 10^{11}\,\text{Pa}$ gives

$$\varepsilon = \frac{1.225 \times 10^5}{(0.75\pi)(2.0 \times 10^{11})} = 2.60 \times 10^{-7}.$$

Strain has no units because the stress and modulus both have units of pascals. Option (A) has the wrong magnitude, while (C) and (D) are thousands of times too large for this stress in steel.

Step 4: Conclusion The compression strain of each column is $\boxed{2.60 \times 10^{-7}}$, a dimensionless magnitude. Hence, the correct option is (B).

Quick Tip to Solve: Divide the total weight by the number of identical supports first, and use the difference of squared radii for a hollow cross-section.

32. A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is :
(Assume that the current is flowing in the clockwise direction.) [4 marks]

(A) 3×10^{-7} T, outside the plane of triangle
(B) $2\sqrt{3} \times 10^{-7}$ T, outside the plane of triangle
(C) $2\sqrt{3} \times 10^{-5}$ T, inside the plane of triangle
(D) 3×10^{-5} T, inside the plane of triangle
Correct Answer: (D) 3×10^{-5} T, inside the plane of triangle

Solution:

Concept:

- A finite straight conductor produces field $B = \mu_0 I(\sin\alpha + \sin\beta)/(4\pi d)$ at perpendicular distance d, where the endpoint angles are measured from the perpendicular to the wire.
- At the centroid of an equilateral triangular loop, all three sides give equal magnetic fields normal

to the plane. Their directions agree by the right-hand rule, so their magnitudes add.

Step 1: Find the geometry relative to one side The triangle is equilateral with side $a = 9\,\text{cm} = 0.09\,\text{m}$. Its centroid is also its incentre, and the distance to a side is one third of its altitude:

$$d = \frac{1}{3}\left(\frac{\sqrt{3}a}{2}\right) = \frac{a}{2\sqrt{3}}.$$

The perpendicular meets the side at its midpoint. For either endpoint,

$$\tan\alpha = \frac{a/2}{d} = \sqrt{3}, \qquad \alpha = \beta = 60^\circ.$$

These are angles at the observation point, measured from that perpendicular.

Step 2: Add the fields from all three sides With $\mu_0/(4\pi) = 10^{-7}\,\text{T m A}^{-1}$ and $I = 1.5\,\text{A}$, the contribution of one side is

$$B_1 = 10^{-7}\frac{I}{d}\,2\sin 60^\circ = 10^{-7}\frac{I\sqrt{3}}{a/(2\sqrt{3})} = 10^{-7}\frac{6I}{a}.$$

Hence the field of the complete triangle is

$$B = 3B_1 = 10^{-7}\frac{18(1.5)}{0.09} = 3.0\times10^{-5}\,\text{T}.$$

Omitting the sum over the three sides would give an incorrect value.

Step 3: Determine the direction and reject the other values Curl the fingers of the right hand clockwise along the current as viewed on the page. The thumb then points into the page. Thus (A) and (B) have both the wrong direction and the wrong power of ten. Option (C) points inward but retains an incorrect factor $\sqrt{3}$ in the magnitude; that factor cancels when the inradius is substituted.

Step 4: Conclusion The magnetic field is $\boxed{3.0\times10^{-5}\,\text{T}}$, into the plane of the triangle. Hence, the correct option is (D).

Quick Tip to Solve: At the centre of an equilateral triangular loop, use inradius $a/(2\sqrt{3})$, endpoint angles 60°, and add the three equal inward fields.

33. A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the end of light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its midpoint? [4 marks]

(A) 18.75 kgm^2

(B) 1.905×10^5 kgm^2

(C) 19.05 kgm^2

(D) 1.875×10^5 kgm^2

Correct Answer: (C) 19.05 kgm^2

Solution:

Concept:

- The moment of inertia of a uniform solid sphere about any diameter is $I_{\text{cm}} = 2mr^2/5$. Its finite radius must be included, even when the connecting rod is light.
- The parallel-axis theorem gives $I = I_{\text{cm}} + md^2$, where d is the perpendicular distance between the sphere-centre axis and the required parallel axis. The rod contributes negligibly because it is light.

Step 1: Identify the distance of each centre from the axis The rotation axis passes through the midpoint of the rod and is perpendicular to it. The two equal sphere centres are separated by $5\,\text{m}$, so each centre is at distance

$$d = \frac{5}{2} = 2.5\,\text{m}$$

from the required axis. Each sphere has $m = 1.5\,\text{kg}$ and $r = 50\,\text{cm} = 0.50\,\text{m}$. The given separation is between centres; no radius needs to be added to it.

Step 2: Shift the moment of inertia of each sphere For one sphere, the moment about its own parallel diameter is

$$I_{\text{cm}} = \frac{2}{5}(1.5)(0.50)^2 = 0.15\,\text{kg}\,\text{m}^2.$$

The parallel-axis contribution is

$$md^2 = (1.5)(2.5)^2 = 9.375\,\text{kg}\,\text{m}^2.$$

Thus each sphere contributes $0.15 + 9.375 = 9.525\,\text{kg}\,\text{m}^2$. Both contributions are positive; moving an axis away from a centre increases the inertia.

Step 3: Add the identical contributions and compare The complete system has

$$I_{\text{total}} = 2\left(\frac{2}{5}mr^2 + md^2\right) = 2(9.525) = 19.05\,\text{kg}\,\text{m}^2.$$

Option (A), $18.75\,\text{kg}\,\text{m}^2$, is the point-mass approximation and omits the two centre-of-mass terms totalling $0.30\,\text{kg}\,\text{m}^2$. Options (B) and (D) have an erroneous factor of 10^4, associated with failing to convert lengths consistently to metres before squaring.

Step 4: Conclusion The moment of inertia is $\boxed{19.05\,\text{kg}\,\text{m}^2}$. Hence, the correct option is (C).

Quick Tip to Solve: For separated extended bodies, add each body's own centre-of-mass inertia as well as md^2; the point-mass expression alone loses the radius correction.

34. Statement I :

Two forces $\left(\vec{P}+\vec{Q}\right)$ and $\left(\vec{P}-\vec{Q}\right)$ where $\vec{P}\perp\vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement II :

In the situation given above.

θ_1 = 60° and θ_2 = 90°

In the light of the above statements, choose the most appropriate answer from the options given below :- [4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false.

Correct Answer: (B) Both Statement I and Statement II are true

Solution:

Concept:

- For two force magnitudes F_1, F_2 at angle θ, the resultant obeys $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$. Angles between forces lie in $[0, \pi]$.
- Because $\vec{P} \perp \vec{Q}$, the magnitudes of $\vec{P}+\vec{Q}$ and $\vec{P}-\vec{Q}$ are both $\sqrt{P^2+Q^2}$. The stated changes of angle concern the relative directions in which forces of these magnitudes act.

Step 1: Reduce the two force magnitudes to one symbol Put $F = \sqrt{P^2+Q^2}$. Orthogonality gives

$$|\vec{P}+\vec{Q}|^2 = |\vec{P}-\vec{Q}|^2 = P^2+Q^2 = F^2.$$

The resultant formula becomes

$$R^2 = 2F^2(1+\cos\theta).$$

For nonzero forces, a larger angle produces a smaller resultant because cosine decreases throughout the allowed angular range. This already indicates the ordering in Statement I.

Step 2: Calculate the angle for the first resultant For the first arrangement, $R_1 = \sqrt{3(P^2+Q^2)} = \sqrt{3}F$. Squaring and substituting gives

$$3F^2 = 2F^2(1+\cos\theta_1) \quad \Longrightarrow \quad \cos\theta_1 = \frac{1}{2}.$$

Therefore $\theta_1 = 60°$. The positive cosine is consistent with a resultant exceeding $\sqrt{2}F$.

Step 3: Calculate the second angle and evaluate both statements For the second arrangement, $R_2 = \sqrt{2(P^2+Q^2)} = \sqrt{2}F$. Hence

$$2F^2 = 2F^2(1+\cos\theta_2) \quad \Longrightarrow \quad \cos\theta_2 = 0, \qquad \theta_2 = 90°.$$

It follows that $\theta_1 < \theta_2$, proving Statement I, and the actual angles equal the values in Statement II.

Thus any option declaring either statement false is ruled out.

Step 4: Conclusion Both Statement I and Statement II are true: $\theta_1 = 60^\circ$ and $\theta_2 = 90^\circ$. Hence, the correct option is (B).

Quick Tip to Solve: When two forces have equal magnitudes F, write $R^2 = 2F^2(1+\cos\theta)$; squared resultants then reveal the angle directly.

35. A free electron of 2.6 eV energy collides with a H^+ ion. This results in the formation of a hydrogen atom in the first excited state and a photon is released. Find the frequency of the emitted photon. (h = 6.6 $\times$ 10^{-34} Js) [4 marks]

(A) 1.45 $\times$ 10^{16} MHz

(B) 0.19 $\times$ 10^{15} MHz

(C) 1.45 $\times$ 10^{9} MHz

(D) 9.0 $\times$ 10^{27} MHz

Correct Answer: (C) 1.45 $\times$ 10^{9} MHz

Solution:

Concept:

- The energy of hydrogen in level n relative to a free electron and proton at rest is $E_n = -13.6/n^2\,\text{eV}$. The first excited state corresponds to $n = 2$, not $n = 1$.
- Energy conservation in radiative capture gives photon energy equal to the electron's initial kinetic energy plus the magnitude of the final binding energy. The photon frequency is $\nu = E_\gamma/h$, with electron-volts converted to joules.

Step 1: Calculate the final bound-state energy The electron is initially free and has kinetic energy $2.6\,\text{eV}$, measured relative to the ionisation threshold. The final hydrogen atom is in its first excited state, for which

$$E_2 = -\frac{13.6}{2^2} = -3.4\,\text{eV}.$$

The atom releases binding energy on capture; this energy is added to, rather than subtracted from, the incoming kinetic energy when determining the emitted photon energy.

Step 2: Apply energy conservation and convert units Neglecting the very small atomic recoil in the usual hydrogen energy model,

$$2.6 = -3.4 + E_\gamma \qquad \Longrightarrow \qquad E_\gamma = 6.0\,\text{eV}.$$

Using $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$,

$$E_\gamma = 6.0(1.6 \times 10^{-19}) = 9.6 \times 10^{-19}\,\text{J}.$$

This positive energy also confirms that the capture process can emit a photon into the stated bound state.

Step 3: Find the frequency in the units used by the options With the supplied $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$,

$$\nu = \frac{9.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.45 \times 10^{15}\,\mathrm{Hz}.$$

Since $1\,\mathrm{MHz} = 10^{6}\,\mathrm{Hz}$, this is $1.45 \times 10^{9}\,\mathrm{MHz}$. Options (A) and (D) are vastly too large after this conversion, while (B) also has the wrong frequency. Only (C) matches both the energy and the stated MHz unit.

Step 4: Conclusion The photon frequency is $\boxed{1.45 \times 10^{9}\,\mathrm{MHz}}$, equivalently $1.45 \times 10^{15}\,\mathrm{Hz}$. Hence, the correct option is (C).

Quick Tip to Solve: For radiative capture, add the incoming kinetic energy to the final binding energy magnitude, then convert Hz to MHz only after dividing by Planck's constant.

36. Two thin metallic spherical shells of radii r_1 and r_2 ($r_1 < r_2$) are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature $\theta_2(\theta_1 < \theta_2)$. The rate at which heat flows radially through the material is :- [4 marks]

(A) $\frac{4\pi K r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$

(B) $\frac{\pi r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$

(C) $\frac{K(\theta_2-\theta_1)}{r_2-r_1}$

(D) $\frac{K(\theta_2-\theta_1)(r_2-r_1)}{4\pi r_1 r_2}$

Correct Answer: (A) $\frac{4\pi K r_1 r_2(\theta_2-\theta_1)}{r_2-r_1}$

Solution:

Concept:

- In steady radial conduction the same heat per second crosses every concentric spherical surface. The area at radius r is $A(r) = 4\pi r^2$, so a plane-slab formula with one fixed area is not applicable.
- Fourier's law relates heat current to the temperature gradient. Since the outer boundary is hotter, heat flows inward; we calculate its positive magnitude H, with conductivity K.

Step 1: Write the differential thermal resistance For a thin layer of thickness dr, its resistance is $dR = dr/(4\pi K r^2)$. These layers are traversed successively by the same heat current and hence their resistances add. Integrating between the two radii gives

$$R = \int_{r_1}^{r_2} \frac{dr}{4\pi K r^2} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

The limits have this order because $r_2 > r_1$, making the resistance positive.

Step 2: Use the boundary temperature difference The temperature drop along the direction of heat

flow is $\theta_2 - \theta_1 > 0$. Therefore

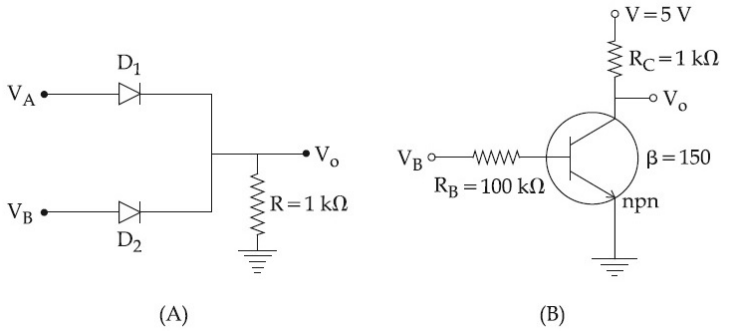
$$H = \frac{\theta_2 - \theta_1}{R} = \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}.$$

The factor $r_1 r_2$ comes from the changing spherical area, rather than choosing either boundary area. Option B omits the conductivity and the correct geometric factor; C omits the area dependence; D reverses the radius factor and has incorrect dimensions for heat per second. The result also increases with conductivity and temperature difference, as it should.

Step 3: Conclusion The heat flows radially inward at a rate $H = \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$. Hence, the correct option is (A).

Quick Tip to Solve: For concentric spherical layers, integrate $dr/(4\pi K r^2)$; using a single fixed area gives the wrong radius dependence.

37. If V_A and V_B are the input voltages (either 5V or 0V) and V_0 is the output voltage then the two gates represented in the following circuit (A) and (B) are :- [4 marks]



(A) AND and OR Gate

(B) OR and NOT Gate

(C) NAND and NOR Gate

(D) AND and NOT Gate

Correct Answer: (B) OR and NOT Gate

Solution:

Concept:

- For positive logic, an input near $5\,\mathrm{V}$ represents 1 and an input near $0\,\mathrm{V}$ represents 0. A diode conducts when its anode is sufficiently above its cathode.
- An npn transistor with its emitter grounded acts as an inverter when the output is taken from the collector through a pull-up resistor: cutoff gives a high output and saturation gives a low output.

Step 1: Identify the diode circuit A The two diode anodes connect to the inputs, while their cathodes join the output. The resistor pulls the output down to ground when neither diode conducts. With both inputs low, the output is low. If either input is high, its diode conducts and raises the output to a

high logic level; with both inputs high the output remains high. Thus

$$00 \longmapsto 0, \quad 01 \longmapsto 1, \quad 10 \longmapsto 1, \quad 11 \longmapsto 1.$$

This is the OR truth table. The small forward diode drop does not change these logic assignments.

Step 2: Identify the transistor circuit B When the base input is zero, the transistor is cut off and the collector resistor pulls the output up to $5\,\mathrm{V}$. When the base input is high, the transistor conducts. With a typical base-emitter drop of $0.7\,\mathrm{V}$, the base current is about $43\,\mu\mathrm{A}$, so $\beta I_B \approx 6.45\,\mathrm{mA}$, exceeding the approximately $5\,\mathrm{mA}$ allowed by the collector resistor. The transistor therefore saturates and its collector output is low. Circuit B is a NOT gate. Options A and D incorrectly identify A as AND; C requires inverted two-input functions that these circuits do not provide.

Step 3: Conclusion Circuit A is an OR gate and circuit B is a NOT gate. Hence, the correct option is (B).

Quick Tip to Solve: For diode logic inspect the diode direction and pull resistor; for a common-emitter switch, a collector output reverses the input.

38. Consider two separate ideal gases of electrons and protons having same number of particles. The temperature of both the gases are same. The ratio of the uncertainty in determining the position of an electron to that of a proton is proportional to :- [4 marks]

(A) $\left(\frac{m_p}{m_e}\right)^{3/2}$

(B) $\sqrt{\frac{m_e}{m_p}}$

(C) $\sqrt{\frac{m_p}{m_e}}$

(D) $\frac{m_p}{m_e}$

Correct Answer: (C) $\sqrt{\frac{m_p}{m_e}}$

Solution:

Concept:

- At a common temperature, the mean translational kinetic energy of a classical ideal-gas particle is $\frac{3}{2}k_BT$, independent of its mass. Equal kinetic energies do not mean equal momenta.
- The uncertainty relation is $\Delta x\,\Delta p_x \geq \hbar/2$. The characteristic minimum position uncertainty therefore varies inversely with the thermal momentum spread.

Step 1: Find the mass dependence of the momentum spread For an ideal gas with no net drift, the mean momentum component is zero. Equipartition in one direction gives

$$\frac{\langle p_x^2\rangle}{2m} = \frac{1}{2}k_BT, \qquad \Delta p_x = \sqrt{mk_BT}.$$

Equivalently, using three dimensions changes only a common numerical factor and leaves the same square-root mass dependence. Thus the proton, although moving more slowly at this temperature,

has the larger typical momentum because its mass is much greater.

Step 2: Compare the position uncertainties Using the same uncertainty scale for each gas,

$$\Delta x \propto \frac{1}{\Delta p_x} \propto \frac{1}{\sqrt{mT}}.$$

Because the temperatures are equal,

$$\frac{\Delta x_e}{\Delta x_p} \propto \sqrt{\frac{m_p}{m_e}}.$$

The equal particle counts introduce no further factor: this comparison concerns the position and momentum of a particle of each species. Option B gives the inverse ratio, which would incorrectly predict a smaller electron position uncertainty. Options A and D have mass exponents inconsistent with $p^2 \propto mT$. The heavier proton has the larger momentum spread and hence the smaller characteristic position uncertainty.

Step 3: Conclusion The required proportionality is $\Delta x_e/\Delta x_p \propto \sqrt{m_p/m_e}$. Hence, the correct option is (C).

Quick Tip to Solve: At equal temperature first use $p \propto \sqrt{m}$, then invert this dependence for the minimum position uncertainty.

39. A bob of mass 'm' suspended by a thread of length l undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by 1/3rd of the original length, then the time period of the simple harmonic oscillations will be :- [4 marks]

(A) T

(B) $\frac{3}{2}$T

(C) $\frac{3}{4}$T

(D) $\frac{4}{3}$T

Correct Answer: (D) $\frac{4}{3}$T

Solution:

Concept:

- For small oscillations of a simple pendulum, $T = 2\pi\sqrt{l/g}$. A buoyant force reduces the effective restoring weight of a completely immersed bob.
- If the bob has density ρ_b and the liquid has density ρ_l, the effective downward force is $mg(1-\rho_l/\rho_b)$. The inertial mass remains m in the ideal pendulum model.

Step 1: Calculate the effective restoring acceleration Let the volume of the bob be V, so that $m = \rho_b V$. The buoyant force is $\rho_l V g$ and acts upward. Its tangential component opposes the tangential component of gravity. Hence

$$g_{\text{eff}} = g\left(1 - \frac{\rho_l}{\rho_b}\right) = g\left(1 - \frac{1}{4}\right) = \frac{3}{4}g.$$

For a small angular displacement θ, the restoring equation is $\ddot{\theta} + (g_{\text{eff}}/l\prime)\theta = 0$. Viscous damping and hydrodynamic added mass are not part of the stated simple-harmonic model.

Step 2: Include the increased length and compare periods The increase is one-third of the original length, not one-third of the final length. Therefore

$$l\prime = l + \frac{l}{3} = \frac{4}{3}l.$$

The new-to-old period ratio is

$$\frac{T\prime}{T} = \sqrt{\frac{l\prime}{l}\frac{g}{g_{\text{eff}}}} = \sqrt{\frac{4}{3}\cdot\frac{4}{3}} = \frac{4}{3}.$$

Both effects increase the period: the longer thread lowers the angular frequency and buoyancy weakens the restoring force. The unchanged period and the smaller period in A and C are therefore excluded immediately. B does increase it, but its factor does not follow from the two exact changes.

Step 3: Conclusion The new period is $T\prime = \frac{4}{3}T$. Hence, the correct option is (D).

Quick Tip to Solve: In a pendulum-period ratio, multiply the length ratio by the inverse effective-gravity ratio before taking the square root.

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40. Statement : 1

If three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are represented by three sides of a triangle and $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement : 2

A triangle made up of three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement - I is false but Statement - II is true

(B) Statement - I is true but Statement - II is false

(C) Both Statement-I and Statement-II are false

(D) Both Statement-I and Statement-II are true

Correct Answer: (A) Statement - I is false but Statement - II is true

Solution:

Concept:

- A force triangle is a vector diagram. Its closure gives $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$, which specifies magnitudes and directions but does not specify where the forces act.
- Translatory equilibrium requires zero resultant force. A claim about concurrence or complete equilibrium of a rigid body additionally concerns lines of action and torque; it cannot follow from the vector sum alone.

Step 1: Test Statement I with a counterexample Consider an equilateral triangle of side a, with vertices $A = (0, 0)$, $B = (a, 0)$, $C = (a/2, \sqrt{3}a/2)$. Let three equal forces of magnitude F act along its physical sides, directed from A to B, B to C, and C to A. Their vectors are

$$\vec{F}_1 = F(1, 0), \quad \vec{F}_2 = F(-1/2, \sqrt{3}/2), \quad \vec{F}_3 = F(-1/2, -\sqrt{3}/2).$$

Their sum is zero, so they satisfy exactly $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$. Yet the three side-lines do not pass through a common point. Moreover, about A, the first and third forces have zero moment, while the force along BC gives a moment $\sqrt{3}Fa/2$. The body therefore has a nonzero resultant torque. This disproves the assertion that the stated vector condition necessarily makes the forces concurrent and in equilibrium. Statement I is false.

Step 2: Evaluate Statement II without adding concurrence Three directed sides taken cyclically around a triangle form a closed vector chain. Thus

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \quad \Longrightarrow \quad \vec{a}_{\text{CM}} = 0.$$

Statement II claims only translatory equilibrium, so it is true even when a torque remains. A is consequently the only choice matching the literal assertions. B reverses the two truth values, C incorrectly rejects vector closure as a condition for translation, and D incorrectly treats concurrence as a consequence of closure.

Step 3: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (A). This differs from the question-sheet key, which accepts both statements: the explicit equilateral-triangle counterexample satisfies the printed vector equation while having nonconcurrent lines of action and a nonzero torque, so Statement I does not follow as printed.

Quick Tip to Solve: A closed force triangle establishes zero resultant, not concurrence. Check lines of action and moments separately before claiming full rigid-body equilibrium.

41. If velocity [V], time [T] and force [F] are chosen as the base quantities, the dimensions of the mass will be : [4 marks]

(A) [$FT^{-1}\ V^{-1}$]

(B) [FTV^{-1}]

(C) [$FT^2\ V$]

(D) [FVT^{-1}]

Correct Answer: (B) [FTV^{-1}]

Solution:

Concept:

- Force, velocity and time may serve as independent base quantities. A mass dimension must then be written as a product $[M] = [F]^a[T]^b[V]^c$.
- Newton's second law gives $F = ma$, while acceleration has the dimension of velocity divided by time. Thus $[a] = [V][T]^{-1}$.

Step 1: Express mass through force and acceleration Rearranging Newton's second law at the dimensional level gives

$$[M] = \frac{[F]}{[a]} = \frac{[F]}{[V][T]^{-1}} = [F][T][V]^{-1}.$$

The time factor moves to the numerator because it was in the denominator of acceleration. This result can also be recognized from impulse: force times time has momentum dimensions, and dividing momentum by velocity gives mass. No numerical value of acceleration or velocity is required.

Step 2: Verify the exponents and reject the alternatives In the conventional mass-length-time system, $[F] = \mathrm{MLT}^{-2}$ and $[V] = \mathrm{LT}^{-1}$. Substitution confirms

$$[FTV^{-1}] = (\mathrm{MLT}^{-2})(\mathrm{T})(\mathrm{L}^{-1}\mathrm{T}) = \mathrm{M}.$$

Option A gives MT^{-2}, rather than mass. Option C gives $\mathrm{ML}^2\mathrm{T}^{-1}$, and option D gives $\mathrm{ML}^2\mathrm{T}^{-4}$; their unwanted time or length factors do not cancel. Alternatively, matching exponents in $F^aT^bV^c$ requires $a = 1$, $a + c = 0$, and $-2a + b - c = 0$, yielding $b = 1, c = -1$.

Step 3: Conclusion The dimensions of mass are $[FTV^{-1}]$. Hence, the correct option is (B).

Quick Tip to Solve: Use $m = F/(V/T) = FT/V$; the impulse-over-velocity form is a quick check on the time exponent.

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42. The magnetic field vector of an electromagnetic wave is given by $B = B_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz - \omega t)$; where $\hat{i}, \hat{j}$ represents unit vector along x and y-axis respectively. At t = 0s, two electric charges q_1 of 4π coulomb and q_2 of 2π coulomb located at $\left(0, 0, \frac{\pi}{k}\right)$ and $\left(0, 0, \frac{3\pi}{k}\right)$, respectively, have the same velocity of 0.5 c $\hat{i}$, (where c is the velocity of light). The ratio of the force acting on charge q_1 to q_2 is :- [4 marks]

(A) $2\sqrt{2} : 1$

(B) $1 : \sqrt{2}$

(C) 2 : 1

(D) $\sqrt{2} : 1$

Correct Answer: (C) 2 : 1

Solution:

Concept:

- The instantaneous force on a moving charge is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. Charges with the same velocity in identical local fields have forces proportional to their charges.
- For a plane wave, the phase $kz - \omega t$ determines both local fields. Points separated by an integer wavelength have the same electric and magnetic fields at the same instant.

Step 1: Compare the wave phases at the charge positions At $t = 0$, the two phases are

$$\phi_1 = k\left(\frac{\pi}{k}\right) = \pi, \qquad \phi_2 = k\left(\frac{3\pi}{k}\right) = 3\pi.$$

Since $\cos\pi = \cos 3\pi = -1$, the magnetic fields at the charges are equal. Their separation is $2\pi/k$,

exactly one wavelength, so the corresponding electric fields are equal too. There is no relative field-magnitude factor of $\sqrt{2}$ between these two locations.

Step 2: Factor the common Lorentz-force term Both velocities are $\vec{v} = (c/2)\hat{\imath}$. Consequently the vector $\vec{E} + \vec{v} \times \vec{B}$ is identical for both charges. Its common value is nonzero: the magnetic contribution has magnitude no greater than $vB = cB/2$, whereas the plane-wave electric field has magnitude cB. Therefore

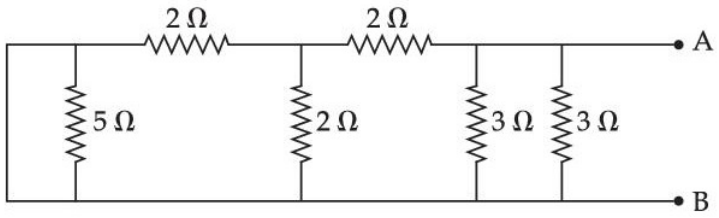
$$\frac{F_1}{F_2} = \frac{q_1}{q_2} = \frac{4\pi}{2\pi} = 2.$$

The charges are both positive, so their force directions also agree. It is unnecessary to resolve every component because those components cancel in the ratio. A and D introduce unjustified polarization factors; B even reverses the expected ordering despite the first charge being twice the second.

Step 3: Conclusion The ratio of the force magnitudes is $F_1 : F_2 = 2 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: Before resolving electric and magnetic forces, compare the phases. Equal phase and equal velocity leave only the charge ratio.

43. The equivalent resistance of the given circuit between the terminals A and B is : [4 marks]



(A) 0Ω

(B) 3Ω

(C) $\frac{9}{2}\Omega$

(D) 1Ω

Correct Answer: (D) 1Ω

Solution:

Concept:

- An ideal wire has zero resistance, so every point on a continuous wire is at the same potential. A resistor connected across such a wire is short-circuited.
- Series resistances add, whereas parallel branches obey $1/R_{\text{eq}} = \sum_i 1/R_i$. Identify circuit nodes before combining resistors.

Step 1: Remove the shorted resistor The wire on the extreme left directly connects the upper-left junction to the lower rail, which is terminal B. The two ends of the $5\,\Omega$ resistor are therefore at the same potential. Its voltage and current are zero, so it does not affect the equivalent resistance. This wire does not directly join A to B: the upper rail still contains the two $2\,\Omega$ resistors.

Step 2: Reduce the left-hand network Call the junction between the two upper $2\,\Omega$ resistors C. From

C to B there are two separate $2\,\Omega$ paths: the vertical resistor and the left upper resistor followed by the left wire. They are parallel, not series. Thus

$$R_{CB} = \frac{2 \times 2}{2 + 2} = 1\,\Omega.$$

The remaining upper resistor connects A to C in series with this equivalent, giving

$$R_{\text{left}} = 2 + 1 = 3\,\Omega.$$

Step 3: Combine the three branches across the terminals The reduced left branch and the two right vertical resistors are each $3\,\Omega$, all connected directly between A and B. Consequently

$$\frac{1}{R_{AB}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\,\Omega^{-1}, \qquad R_{AB} = 1\,\Omega.$$

The value $3\,\Omega$ omits the two parallel branches; $9/2\,\Omega$ cannot be the resistance of three equal parallel branches. A zero value would require a continuous wire directly from A to B, which is absent.

Step 4: Conclusion The equivalent resistance is $1\,\Omega$. Hence, the correct option is (D).

Quick Tip to Solve: Mark the entire lower rail and the upper-left corner as one node first; this immediately exposes the shorted $5\,\Omega$ resistor.

44. Choose the incorrect statement :

(1) The electric lines of force entering into a Gaussian surface provide negative flux.

(2) A charge 'q' is placed at the centre of a cube. The flux through all the faces will be the same.

(3) In a uniform electric field net flux through a closed Gaussian surface containing no net charge, is zero.

(4) When electric field is parallel to a Gaussian surface, it provides a finite non-zero flux.

Choose the most appropriate answer from the options given below [4 marks]

(A) (3) and (4) only

(B) (2) and (4) only

(C) (4) only

(D) (1) and (3) only

Correct Answer: (C) (4) only

Solution:

Concept:

- Electric flux through a surface is $\Phi = \int \vec{E} \cdot d\vec{A}$, with the area vector normal to the surface. For a closed surface this normal points outward.
- Gausśś law gives $\oint \vec{E} \cdot d\vec{A} = q_{\text{enclosed}}/\varepsilon_0$; symmetry can distribute this flux equally among equivalent faces.

Step 1: Check the sign and the symmetry statements For statement (1), a field line entering a closed

Gaussian surface has a component opposite to the outward normal. Therefore $\vec{E} \cdot d\vec{A} < 0$, giving negative flux; the statement is true. For statement (2), a point charge at the centre of a cube has identical geometry relative to every face. The total flux q/ε_0 is shared equally, so each face has flux $q/(6\varepsilon_0)$. Statement (2) is also true.

Step 2: Apply Gauss's law to the third statement The closed surface in statement (3) encloses no net charge. Gausśś law therefore gives

$$\Phi_{\text{net}} = \frac{0}{\varepsilon_0} = 0.$$

A uniform field can enter some portions and leave others, but their signed contributions cancel. Zero net flux does not mean that the field itself vanishes. Statement (3) is true.

Step 3: Test the parallel-field statement For statement (4), a field parallel to the surface is perpendicular to its area vector. Hence

$$d\Phi = E\, dA \cos 90^\circ = 0.$$

A tangent field has no component crossing the surface, so the claimed finite non-zero flux is incorrect. The other three statements are correct; options (A), (B), and (D) each wrongly include a correct statement among the incorrect ones.

Step 4: Conclusion Only statement (4) is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: In a flux formula the angle is measured from the surface normal, not from the surface itself; a parallel field gives zero flux.

45. A mixture of hydrogen and oxygen has volume 500 cm^3, temperature 300 K, pressure 400 kPa and mass 0.76 g. The ratio of masses of oxygen to hydrogen will be :- [4 marks]

(A) 3 : 8

(B) 3 : 16

(C) 16 : 3

(D) 8 : 3

Correct Answer: (C) 16 : 3

Solution:

Concept:

- For an ideal-gas mixture, $PV = (n_{\text{H}} + n_{\text{O}})RT$, where the subscripts denote molecular hydrogen and molecular oxygen.
- The molar masses of H_2 and O_2 are 2 and $32\,\text{g mol}^{-1}$, respectively. Mass equals mole number multiplied by molar mass.

Step 1: Find the total number of moles Convert all quantities to SI units before applying the gas law:

$$P = 4.00 \times 10^5\,\text{Pa}, \quad V = 5.00 \times 10^{-4}\,\text{m}^3, \quad T = 300\,\text{K}.$$

Thus $PV = 200\,\text{J}$. Using $R = 8.314\,\text{J mol}^{-1}\text{K}^{-1}$,

$$n = \frac{200}{8.314 \times 300} \simeq 0.0802\,\text{mol} \simeq 0.080\,\text{mol}.$$

The rounded mole value is sufficient to distinguish the widely separated answer choices.

Step 2: Resolve the two gas masses Let the hydrogen and oxygen masses in grams be h and o. The total mass and total mole number give

$$h + o = 0.76, \qquad \frac{h}{2} + \frac{o}{32} \simeq 0.080.$$

Multiplying the second equation by 32 gives $16h + o \simeq 2.56$. Subtracting the mass equation yields

$$15h \simeq 1.80, \qquad h \simeq 0.12\,\text{g}, \qquad o \simeq 0.64\,\text{g}.$$

Keeping more digits in the gas constant gives nearly the same masses and the same selected ratio.

Step 3: Take the masses in the requested order The problem asks for oxygen to hydrogen, not hydrogen to oxygen. Therefore

$$\frac{o}{h} \simeq \frac{0.64}{0.12} = \frac{16}{3}.$$

As a check, these masses correspond to 0.020 mol of oxygen and 0.060 mol of hydrogen, whose total is 0.080 mol. Option (B) reverses the requested ratio. Options (A) and (D) do not satisfy both the total mass and gas-law constraint.

Step 4: Conclusion The ratio of the mass of oxygen to that of hydrogen is approximately $16 : 3$. Hence, the correct option is (C).

Quick Tip to Solve: Use the gas law for total moles and the mass equation separately; a mole ratio is not a mass ratio when molar masses differ.

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46. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1 : 2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is :- [4 marks]

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{1}{8}$
(D) $\frac{1}{4}$

Correct Answer: (C) $\frac{1}{8}$

Solution:

Concept:

- In the absence of an external horizontal impulse, total horizontal momentum is conserved during fragmentation. Internal forces can still change the kinetic energy.

- Kinetic energy is $K = \frac{1}{2}mv^2$. The fractional change means $(K_f - K_i)/K_i$, not K_f/K_i.

Step 1: Use momentum conservation to find the other speed Let the smaller fragment have mass m and the larger fragment mass $2m$; the original block then has mass $3m$. Take its initial direction as positive. If the larger fragment has speed v, conservation of momentum gives

$$(3m)(40) = m(60) + (2m)v.$$

Cancelling m and solving,

$$120 = 60 + 2v, \qquad v = 30\,\mathrm{m\,s^{-1}}.$$

The positive sign shows that both fragments move in the original direction.

Step 2: Compute kinetic energy before and after the split Before fragmentation the block has a single speed, so

$$K_i = \frac{1}{2}(3m)(40)^2 = 2400m.$$

After fragmentation, add the energies of the two fragments rather than averaging their speeds:

$$K_f = \frac{1}{2}m(60)^2 + \frac{1}{2}(2m)(30)^2 = 1800m + 900m = 2700m.$$

If m is expressed in kilograms, these energies are in joules. The increase comes from internal energy released during the split; a smooth surface does not require kinetic-energy conservation.

Step 3: Form the fractional increase The change is

$$\Delta K = 2700m - 2400m = 300m.$$

Divide by the initial kinetic energy:

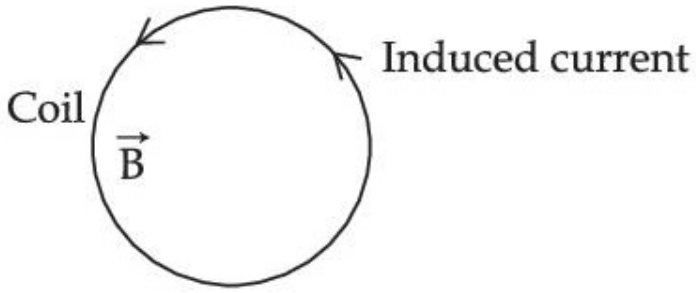
$$\frac{\Delta K}{K_i} = \frac{300m}{2400m} = \frac{1}{8}.$$

The mass scale cancels, as a fractional change must be dimensionless. The final-to-initial ratio is $9/8$, while the requested change is $1/8$. None of $1/3$, $2/3$, or $1/4$ agrees with the momentum-conserving fragment speeds.

Step 4: Conclusion The kinetic energy increases by a fraction $1/8$ of its initial value. Hence, the correct option is (C).

Quick Tip to Solve: For a splitting block, conserve momentum first and calculate kinetic energies afterward; internal energy can increase the total kinetic energy.

47. A coil is placed in a magnetic field $\vec{B}$ as shown below :
A current is induced in the coil because $\vec{B}$ is : [4 marks]



(A) Outward and decreasing with time
(B) Parallel to the plane of coil and decreasing with time
(C) Outward and increasing with time
(D) Parallel to the plane of coil and increasing with time
Correct Answer: (A) Outward and decreasing with time

Solution:

Concept:

- Faradayś law is $\mathcal{E} = -d\Phi_B/dt$, where $\Phi_B = \int \vec{B} \cdot d\vec{A}$. Only the magnetic field component normal to the coil contributes to its flux.
- Lenzś law says that the field generated by the induced current opposes the change of magnetic flux. The right-hand rule relates current circulation to its induced magnetic field.

Step 1: Read the current direction in the figure The arrow on the upper-left portion of the circular coil points down and left along the circumference. The shown induced current therefore circulates anticlockwise as viewed on the page. Curling the fingers of the right hand in that direction makes the thumb point out of the page. Thus the induced magnetic field is outward, toward the observer.

Step 2: Infer the required change of applied flux An outward induced field tries to maintain outward flux if that external flux is decreasing. Choose the outward normal as positive. An outward applied field that decreases has $d\Phi_B/dt < 0$. Faradayś law then gives an emf in the positive, anticlockwise sense:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} > 0.$$

This agrees with the figure. It is the change that the induced field opposes, not necessarily the direction of the applied field itself.

Step 3: Eliminate the other proposed fields An outward field increasing with time would require an inward induced field to oppose its growth. It would drive clockwise current, contrary to the arrows, so option (C) is wrong. In options (B) and (D), the applied field is parallel to the coil plane. Its normal component is zero, and for this fixed planar coil its flux remains zero even when the field magnitude changes:

$$\Phi_B = BA\cos 90^\circ = 0.$$

Those choices cannot account for the shown induced current.

Step 4: Conclusion The applied magnetic field is outward and decreasing with time. Hence, the correct option is (A).

Quick Tip to Solve: First infer the induced field from the current arrows, then ask which proposed flux change that field would oppose.

48. For a body executing S.H.M. :

(1) Potential energy is always equal to its K.E.

(2) Average potential and kinetic energy over any given time interval are always equal.

(3) Sum of the kinetic and potential energy at any point of time is constant.

(4) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below : [4 marks]

(A) (3) and (4)

(B) only (3)

(C) (2) and (3)

(D) only (2)

Correct Answer: (A) (3) and (4)

Solution:

Concept:

- For undamped simple harmonic motion, $x = A\cos(\omega t + \phi)$, where A is amplitude and ω angular frequency. The restoring-force constant satisfies $k = m\omega^2$.
- The energies are $U = \frac{1}{2}kx^2$ and $K = \frac{1}{2}mv^2$. Their sum is the constant $E = \frac{1}{2}kA^2$.

Step 1: Express the instantaneous energies Differentiating the displacement gives $v = -A\omega\sin(\omega t+\phi)$. Therefore

$$U = E\cos^2(\omega t + \phi), \qquad K = E\sin^2(\omega t + \phi).$$

At an extreme position, $U = E$ and $K = 0$; at equilibrium, $K = E$ and $U = 0$. Hence the two energies are not always equal, and statement (1) is false. The identity $\sin^2\theta + \cos^2\theta = 1$ gives $K + U = E$ at every instant, so statement (3) is true.

Step 2: Distinguish arbitrary intervals from a complete period Over one full period T, the averages of both squared sine and squared cosine are $1/2$. Thus

$$\langle K\rangle_T = \langle U\rangle_T = \frac{E}{2}.$$

Statement (4) is true. This equality does not hold over every arbitrarily chosen interval. For example, take a short interval near an extreme position: the potential energy stays near E, while the kinetic energy stays near zero. Their averages over that interval cannot be equal, making statement (2) false.

Step 3: Select the correct pair The true statements are precisely (3) and (4). Option (B) omits the true full-period average relation. Option (C) includes the false claim about every given interval, and option (D) contains only that false claim. It is essential to distinguish an instantaneous equality, a

finite-interval average, and an average over a complete oscillation.

Step 4: Conclusion Statements (3) and (4) are correct. Hence, the correct option is (A).

Quick Tip to Solve: Use the extreme position to disprove claims of equal instantaneous energies, and use full-period averages only when the interval is specified.

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49. Statement - I :

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect a capacitor across the output parallel to the load R_L.

Statement - II :

To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect an inductor in series with R_L.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- A full-wave rectifier produces a unidirectional but pulsating output, containing a dc component and alternating ripple components. A filter suppresses the ripple delivered to the load.
- A capacitor opposes changes of voltage and has reactance $X_C = 1/(\omega C)$; an inductor opposes changes of current and has reactance $X_L = \omega L$.

Step 1: Examine the parallel capacitor A capacitor placed across the load charges when the rectified voltage rises near a peak. When the rectifier voltage falls, the capacitor releases stored charge through the load, sustaining its voltage between successive peaks. With a suitable large capacitance, the discharge is slow compared with the interval between peaks:

$$\tau = R_L C.$$

Equivalently, its small reactance to ripple gives the alternating component a bypass path across the load. It does not short-circuit the dc component. Thus statement I describes a valid smoothing arrangement.

Step 2: Examine the series inductor An inductor in series with the load develops an emf opposing rapid changes of current:

$$\mathcal{E}_L = -L\frac{di}{dt}.$$

It stores magnetic energy as the current rises and releases it as the current falls, thereby reducing

current ripple. Its reactance to the alternating ripple is $X_L = \omega L$, whereas an ideal inductor offers no reactance to steady dc. A suitable choke therefore passes the dc component while impeding ripple, so statement II is also true.

Step 3: Interpret steady output and compare the alternatives Both arrangements are standard filters for making rectifier output approximately steady. Finite practical components leave some residual ripple; the word steady here refers to this smoothing action, not to a mathematical demand for exactly zero ripple with any component value. Options (A) and (B) incorrectly reject one of the two valid methods, and option (C) rejects both. The capacitor belongs in parallel to maintain voltage; the inductor belongs in series to smooth current.

Step 4: Conclusion Both statement I and statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: Remember the filter placements by their conserved quantity: a capacitor smooths load voltage in parallel, while an inductor smooths load current in series.

50. If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is : (Given : $r < R_E$) [4 marks]

(A) $1 - \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$

(B) $1 + \frac{r}{R_E} + \frac{r^2}{R_E^2} + \frac{r^3}{R_E^3}$

(C) $1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} + \frac{r^3}{R_E^3}$

(D) $1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$

Correct Answer: (D) $1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$

Solution:

Concept:

- For the standard uniform-density Earth model, the gravitational acceleration at depth r is $g_d = g_0(1 - r/R_E)$, where $g_0 = GM/R_E^2$ is the surface value.
- Outside a spherical Earth, the inverse-square law gives $g_h = g_0[R_E/(R_E + r)]^2$ at height r. The requested ratio is depth divided by height.

Step 1: Write the two accelerations At depth r, the distance from the centre is $R_E - r$; the mass enclosed decreases as the cube of that distance, giving the linear depth formula. At height r, the full Earth mass acts as a central mass. Thus

$$g_d = g_0\left(1 - \frac{r}{R_E}\right), \qquad g_h = \frac{g_0}{(1 + r/R_E)^2}.$$

Step 2: Expand the ratio exactly Put $x = r/R_E$, with $0 < x < 1$. Cancelling the common surface acceleration gives

$$\frac{g_d}{g_h} = (1 - x)(1 + x)^2.$$

Multiplication, without discarding powers of x, yields

$$(1-x)(1+2x+x^2) = 1+x-x^2-x^3.$$

Option (A) has the wrong linear sign. Option (B) has the wrong quadratic and cubic signs, and option (C) has the wrong cubic sign. All three therefore fail the exact product; a first-order approximation alone would not distinguish the last three options.

Step 3: Conclusion

$$\boxed{\frac{g_d}{g_h} = 1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Form the exact product $(1-x)(1+x)^2$ before expanding; retaining the cubic term separates the closely matching options.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A parallel plate capacitor of capacitance 200 μF is connected to a battery of 200 V. A dielectric slab of dielectric constant 2 is now inserted into the space between plates of capacitor while the battery remain connected. The change in the electrostatic energy in the capacitor will be _____J.

[4 marks]

Solution:

Concept:

- The electrostatic energy stored in a capacitor is $U = \frac{1}{2}CV^2$. Here C is capacitance and V is the potential difference across the plates.
- When a dielectric completely occupies the plate gap, the capacitance changes from C_0 to KC_0. The connected ideal battery maintains constant voltage, rather than constant charge.

Step 1: Identify the fixed quantity and initial energy The battery remains connected throughout insertion, so the voltage stays at $200\,\text{V}$. The initial capacitance in SI units is

$$C_0 = 200\,\mu\text{F} = 200 \times 10^{-6}\,\text{F}.$$

The initial field energy is therefore

$$U_i = \frac{1}{2}(200 \times 10^{-6})(200)^2 = 4\,\text{J}.$$

Using the isolated-capacitor formula with fixed charge would give an energy decrease and would be inappropriate here.

Step 2: Calculate the final stored energy and change For dielectric constant $K = 2$, the final

capacitance is

$$C_f = 2C_0 = 400 \times 10^{-6}\,\text{F}.$$

At the unchanged potential difference,

$$U_f = \frac{1}{2}(400 \times 10^{-6})(200)^2 = 8\,\text{J}.$$

Hence the change in the capacitor's stored electrostatic energy is

$$\Delta U = U_f - U_i = 8 - 4 = 4\,\text{J}.$$

The battery supplies additional charge as capacitance grows. Its energy transfer is not identical to the change of stored field energy, because the dielectric also experiences an electric force.

Step 3: Conclusion The electrostatic energy increases by $\boxed{4\,\text{J}}$. The numerical entry is $\boxed{4}$.

Quick Tip to Solve: Check whether the battery is connected first: fixed voltage makes stored energy proportional to capacitance and hence to the dielectric constant.

52. A long solenoid with 1000 turns/m has a core material with relative permeability 500 and volume 10^3 cm^3. If the core material is replaced by another material having relative permeability of 750 with same volume maintaining same current of 0.75 A in the solenoid, the fractional change in the magnetic moment of the core would be approximately $\left(\frac{x}{499}\right)$. Find the value of x. [4 marks]

Solution:

Concept:

- For a linear magnetic material, $M = \chi_m H$, where M is magnetisation, H is magnetising field and $\chi_m = \mu_r - 1$ is magnetic susceptibility.
- A uniformly magnetised core of volume $\mathcal{V}$ has magnetic moment $m = M\mathcal{V}$. In a long solenoid, $H = nI$, so unchanged turn density, current and core volume make m proportional to $\mu_r - 1$.

Step 1: Calculate the initial and final magnetic moments The applied magnetising field is the same for both cores:

$$H = nI = 1000(0.75) = 750\,\text{A m}^{-1}.$$

The volume conversion is

$$\mathcal{V} = 10^3\,\text{cm}^3 = 10^{-3}\,\text{m}^3.$$

The two susceptibilities are $\chi_1 = 500 - 1 = 499$ and $\chi_2 = 750 - 1 = 749$. Thus

$$m_1 = 499(750)(10^{-3}) = 374.25\,\text{A m}^2,$$

$$m_2 = 749(750)(10^{-3}) = 561.75\,\text{A m}^2.$$

Step 2: Form the fractional increase relative to the first core The fractional change means the increase

divided by the original magnetic moment, not by the final moment. Therefore

$$\frac{\Delta m}{m_1} = \frac{m_2 - m_1}{m_1} = \frac{749 - 499}{499} = \frac{250}{499}.$$

The common factors $nI\mathcal{V}$ cancel, so the supplied current and volume need not be calculated to obtain the ratio. Comparing with the prescribed form $x/499$ identifies the numerator. Using μ_r directly in place of susceptibility would lose the exact denominator specified in the question.

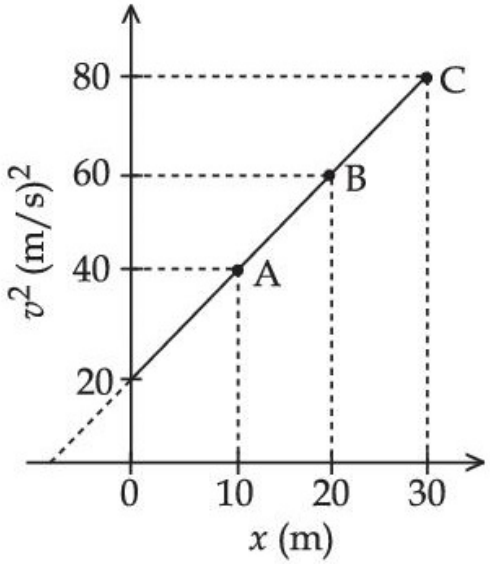
Step 3: Conclusion

$$\frac{\Delta m}{m_1} = \frac{250}{499}, \qquad \boxed{x = 250}.$$

The requested number x is dimensionless.

Quick Tip to Solve: For the magnetic moment of the material itself, use susceptibility $\mu_r - 1$, and divide its change by the initial susceptibility.

53. A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x(displacement) plot. The acceleration of the particle is _____ m/s^2. [4 marks]



Solution:

Concept:

- For constant acceleration, the displacement equation is $v^2 = u^2 + 2ax$. Therefore a graph of v^2 against displacement x is a straight line with slope $2a$, not a.
- The vertical intercept gives u^2, while the slope must be evaluated from changes in the coordinates of two points. The graph uses $\mathrm{m^2\,s^{-2}}$ vertically and metres horizontally.

Step 1: Read two clear points and calculate the slope The labelled graph gives point A at $x = 10\,\mathrm{m}$,

$v^2 = 40\,\mathrm{m^2\,s^{-2}}$, and point C at $x = 30\,\mathrm{m}$, $v^2 = 80\,\mathrm{m^2\,s^{-2}}$. Hence

$$\text{slope} = \frac{80-40}{30-10} = \frac{40}{20} = 2\,\mathrm{m\,s^{-2}}.$$

Point B, $(20, 60)$, lies on the same straight line and confirms the same gradient. The positive slope shows that the acceleration is positive for increasing displacement.

Step 2: Relate the gradient to acceleration Comparing the straight-line form $v^2 = u^2 + (2a)x$ with the graph shows

$$2a = 2\,\mathrm{m\,s^{-2}}, \qquad a = 1\,\mathrm{m\,s^{-2}}.$$

The intercept is $20\,\mathrm{m^2\,s^{-2}}$, which represents u^2. As a consistency check, the graph equation is $v^2 = 20 + 2x$; it produces 40, 60, and 80 at the three marked displacements. Dividing one ordinate by its displacement would incorrectly include the nonzero intercept, and treating the slope as acceleration would give twice the required result.

Step 3: Conclusion The particle has acceleration $\boxed{1\,\mathrm{m\,s^{-2}}}$. The numerical entry is $\boxed{1}$.

Quick Tip to Solve: On a v^2-versus-x graph, subtract two ordinate values to find the slope, then divide that slope by two.

54. In a Young's double slit experiment, the slits are separated by 0.3 mm and the screen is 1.5 m away from the plane of slits. Distance between fourth bright fringes on both sides of central bright is 2.4 cm. The frequency of light used is _____ $\times$ 10^{14} Hz. [4 marks]

Solution:

Concept:

- In Young's double-slit experiment, the bright fringe at order m occurs at $y_m = m\lambda D/d$ from the central maximum, where d is slit separation and D is screen distance.
- The fringe width is $\beta = \lambda D/d$. Light in air satisfies $f = c/\lambda$, with $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. Positions on opposite sides have opposite signs.

Step 1: Convert the separation into fringe width The fourth bright fringes are at $+4\beta$ and -4β, so their separation contains eight fringe widths:

$$8\beta = 2.4\,\mathrm{cm} = 2.4 \times 10^{-2}\,\mathrm{m}.$$

Thus

$$\beta = \frac{2.4 \times 10^{-2}}{8} = 3.0 \times 10^{-3}\,\mathrm{m}.$$

Using four fringe widths would treat the given distance as measured from the central bright fringe, whereas the question measures between two symmetric fringes.

Step 2: Find the wavelength and then the frequency The slit separation is $d = 0.3\,\mathrm{mm} = 3.0 \times 10^{-4}\,\mathrm{m}$,

and $D = 1.5\,\text{m}$. Therefore

$$\lambda = \frac{\beta d}{D} = \frac{(3.0 \times 10^{-3})(3.0 \times 10^{-4})}{1.5} = 6.0 \times 10^{-7}\,\text{m}.$$

This is a visible-light wavelength of $600\,\text{nm}$, consistent with an optical interference experiment. The frequency is

$$f = \frac{3.0 \times 10^{8}}{6.0 \times 10^{-7}} = 5.0 \times 10^{14}\,\text{Hz}.$$

The blank asks for the coefficient multiplying $10^{14}\,\text{Hz}$, rather than the full SI number.

Step 3: Conclusion The light frequency is $\boxed{5.0 \times 10^{14}\,\text{Hz}}$. Hence the requested entry is $\boxed{5}$.

Quick Tip to Solve: The separation between the mth bright fringes on opposite sides is $2m\beta$; count both sides before extracting wavelength.

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55. The diameter of a spherical bob is measured using a vernier callipers. 9 divisions of the main scale, in the vernier callipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8^{th} division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.04 cm, then the radius of the bob is _____ $\times$ 10^{-2} cm. [4 marks]

Solution:

Concept:

- For a direct vernier, the least count is one main-scale division minus one vernier-scale division. The observed length equals the main-scale reading plus the coinciding vernier division number times the least count.
- A positive zero error means the instrument reads too large, so subtract it from the observed reading. The callipers measure the diameter of the sphere; its radius is half the corrected diameter.

Step 1: Calculate the least count and observed diameter One main-scale division is $1\,\text{mm}$. Since ten vernier divisions equal nine main-scale divisions,

$$1\,\text{VSD} = \frac{9}{10}(1\,\text{mm}) = 0.9\,\text{mm}.$$

The least count is therefore

$$LC = 1.0 - 0.9 = 0.1\,\text{mm}.$$

The eighth vernier division coincides, giving an added reading of $8LC = 0.8\,\text{mm}$. Thus the observed diameter is

$$D_{\text{obs}} = 10 + 0.8 = 10.8\,\text{mm} = 1.08\,\text{cm}.$$

Step 2: Correct the diameter and halve it The positive zero error is $0.04\,\text{cm}$; its correction is negative. Subtracting it in the same unit gives

$$D = 1.08 - 0.04 = 1.04\,\text{cm}.$$

The required radius is

$$r = \frac{D}{2} = \frac{1.04}{2} = 0.52\,\text{cm} = 52 \times 10^{-2}\,\text{cm}.$$

This is smaller than half the observed diameter, as a positive zero error requires. Adding the zero error would produce a length too large; reporting the corrected diameter itself would also miss the final factor of two.

Step 3: Conclusion The radius is $\boxed{52 \times 10^{-2}\,\text{cm}}$. The requested coefficient is $\boxed{52}$.

Quick Tip to Solve: Keep the zero error and scale reading in the same unit, subtract a positive zero error, and only then convert the measured diameter to radius.

---

56. A sample of gas with γ = 1.5 is taken through an adiabatic process in which the volume is compressed from 1200 cm^3 to 300 cm^3. If the initial pressure is 200 kPa. The absolute value of the workdone by the gas in the process = _____ J. [4 marks]

Solution:

Concept:

- For a reversible adiabatic process of an ideal gas, PV^γ is constant, where $\gamma = C_P/C_V$. Thus $P_f/P_i = (V_i/V_f)^\gamma$.
- The work done by the gas in an adiabatic change is $W = (P_iV_i - P_fV_f)/(\gamma - 1)$. Compression gives negative work by the gas, while the question asks for its absolute value.

Step 1: Calculate the final pressure using the compression ratio The volume ratio is

$$\frac{V_i}{V_f} = \frac{1200}{300} = 4.$$

With $\gamma = 1.5 = 3/2$, the adiabatic relation gives

$$P_f = 200\,\text{kPa}\,(4)^{3/2} = 200(8)\,\text{kPa} = 1600\,\text{kPa}.$$

For calculating work in joules, use $P_i = 2.00 \times 10^5\,\text{Pa}$, $P_f = 1.60 \times 10^6\,\text{Pa}$, $V_i = 1.20 \times 10^{-3}\,\text{m}^3$, and $V_f = 3.00 \times 10^{-4}\,\text{m}^3$.

Step 2: Evaluate the signed work and take its magnitude The pressure-volume products are

$$P_iV_i = (2.00 \times 10^5)(1.20 \times 10^{-3}) = 240\,\text{J},$$

$$P_fV_f = (1.60 \times 10^6)(3.00 \times 10^{-4}) = 480\,\text{J}.$$

Therefore the work done by the gas is

$$W = \frac{240 - 480}{1.5 - 1} = \frac{-240}{0.5} = -480\,\text{J}.$$

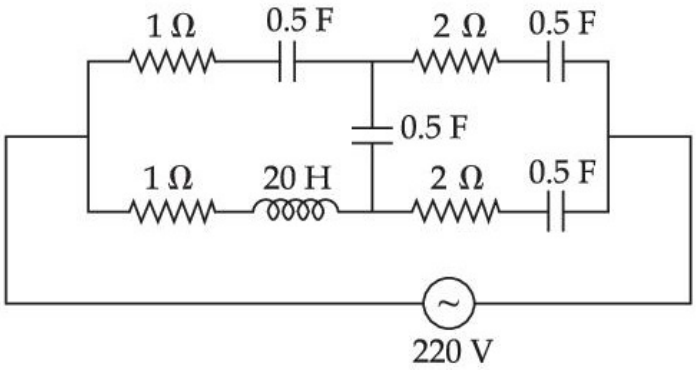
Its negative sign agrees with compression: the surroundings perform positive work on the gas. Since the change is adiabatic, no heat enters, and the positive work on the gas increases its internal energy.

Taking the absolute value removes only the sign, not the physical distinction.

Step 3: Conclusion The absolute value of the work done by the gas is $\boxed{480\,\text{J}}$. The numerical entry is $\boxed{480}$.

Quick Tip to Solve: Use the adiabatic pressure ratio before applying the work formula, and check that compression makes work done by the gas negative.

57. At very high frequencies, the effective impendence of the given circuit will be _____ Ω. [4 marks]



Solution:

Concept:

- The capacitor impedance is $Z_C = 1/(j\omega C)$, whereas the inductor impedance is $Z_L = j\omega L$. Here $\omega = 2\pi f$ is the angular frequency.
- In the high-frequency limit a finite capacitor behaves as a short circuit and a finite inductor behaves as an open circuit. Combine resistances only after identifying which circuit nodes become connected.

Step 1: Replace the reactive components by their limits For every $0.5\,\text{F}$ capacitor, $|Z_C| = 1/(0.5\omega) \to 0$ as frequency becomes very large. The $20\,\text{H}$ inductor instead has $|Z_L| = 20\omega \to \infty$. Consequently no current flows through the lower left branch containing the inductor and its series $1\,\Omega$ resistor. That resistor cannot contribute to the conducting network.

Step 2: Identify the remaining connections The upper left capacitor connects the upper $1\,\Omega$ resistor to the central node. The vertical capacitor shorts the upper and lower central nodes together. Both right-hand capacitors also become wires, so the two $2\,\Omega$ resistors connect between the same central node and the right supply terminal. They therefore form a parallel pair, rather than a series pair.

Step 3: Combine the conducting resistances The equivalent resistance of the right-hand pair is

$$R_p = \frac{2 \times 2}{2 + 2} = 1\,\Omega.$$

It lies in series with the upper left $1\,\Omega$ resistor, giving

$$Z_\infty = 1 + R_p = 1 + 1 = 2\,\Omega.$$

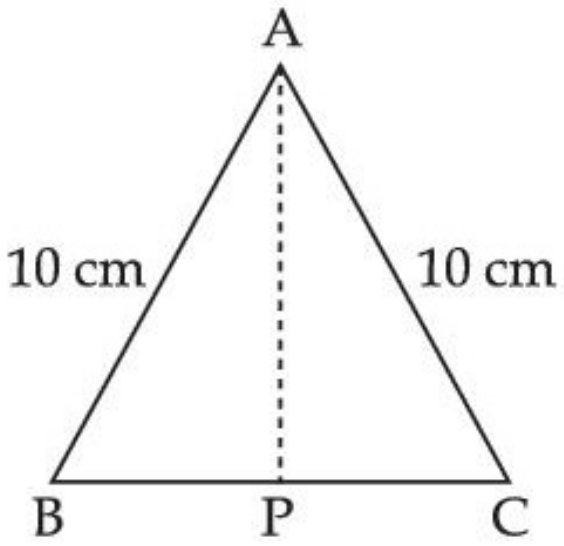
The reactive impedances vanish or disconnect their branches in this limit, so the remaining impedance is purely resistive; the supply voltage does not affect its value.

Step 4: Conclusion The effective impedance at very high frequencies is $\boxed{2\,\Omega}$. The numerical entry is 2.

Quick Tip to Solve: First short the capacitors and open the inductor; the vertical short makes the two right-hand resistors parallel.

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58. Cross-section view of a prism is the equilateral triangle ABC in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (midpoint of BC) to A is _____ $\times 10^{-10}$ s. (Given, speed of light in vacuum = 3 $\times$ 10^8 m/s and cos30° = $\frac{\sqrt{3}}{2}$) [4 marks]



Solution:

Concept:

- At minimum deviation the path through a prism is symmetric: $i = e$ and $r_1 = r_2 = A/2$, where A is its prism angle.
- For refraction from air into the prism, Snell's law gives $\mu = \sin i / \sin r$. Light travels inside the medium at $v = c/\mu$, so its transit time over length ℓ is $t = \mu\ell/c$.

Step 1: Obtain the refractive index The prism cross-section is equilateral, so its apex angle is $A = 60°$. The stated incidence angle equals the prism angle, hence $i = 60°$. Minimum deviation then requires

$$r_1 = r_2 = \frac{60°}{2} = 30°.$$

Applying Snell's law at the first face gives

$$\mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

This determines the material's speed of light without requiring the deviation angle itself.

Step 2: Calculate the distance from P to A In an equilateral triangle the line from the apex to the midpoint of the opposite side is perpendicular to that side. With $AB = AC = 10\,\text{cm}$, the altitude is

$$AP = 10\cos 30^\circ = 5\sqrt{3}\,\text{cm} = 0.05\sqrt{3}\,\text{m}.$$

Use the altitude, not the side length of $10\,\text{cm}$, because the requested journey is specifically from P to A.

Step 3: Use the speed inside the prism The propagation speed is

$$v = \frac{3 \times 10^8}{\sqrt{3}}\,\text{m s}^{-1}.$$

Therefore

$$t = \frac{AP}{v} = \frac{(0.05\sqrt{3})(\sqrt{3})}{3 \times 10^8} = \frac{0.15}{3 \times 10^8} = 5 \times 10^{-10}\,\text{s}.$$

The two factors of $\sqrt{3}$, one from geometry and one from refraction, multiply to 3.

Step 4: Conclusion The time is $\boxed{5 \times 10^{-10}\,\text{s}}$, so the required numerical coefficient is 5.

Quick Tip to Solve: At minimum deviation put each internal refraction angle equal to half the prism angle before applying Snell's law.

59. A resistor dissipates 192 J of energy in 1s when a current of 4A is passed through it. Now, when the current is doubled, the amount of thermal energy dissipated in 5s in _____ J. [4 marks]

Solution:

Concept:

- For a constant current through a resistor, Joule's law is $H = I^2Rt$, where H is heat energy, R is resistance and t is the elapsed time.
- For the same resistor with unchanged resistance, heating is proportional to the square of current and to time. The rate of heating is the electrical power $P = I^2R$.

Step 1: Find the resistance from the first observation The initial energy is $H_1 = 192\,\text{J}$, the current is $I_1 = 4\,\text{A}$, and the interval is $t_1 = 1\,\text{s}$. Substitution in Joule's law gives

$$192 = 4^2R(1), \qquad R = \frac{192}{16} = 12\,\Omega.$$

Thus the initial power is $192\,\text{W}$. The given energy is associated with one second; it is not the energy that must remain fixed when the current changes.

Step 2: Calculate the power at the doubled current The new current is

$$I_2 = 2I_1 = 8\,\text{A}.$$

Keeping the resistance at $12\,\Omega$, the new power is

$$P_2 = I_2^2 R = 8^2 \times 12 = 768\,\text{W}.$$

This is four times the original power, because the current occurs squared. Merely doubling the original energy rate would miss this quadratic dependence.

Step 3: Calculate the energy in the new time interval The new current flows for $t_2 = 5\,\text{s}$, so

$$H_2 = P_2 t_2 = 768 \times 5 = 3840\,\text{J}.$$

Equivalently, the ratio method gives

$$\frac{H_2}{H_1} = \left(\frac{I_2}{I_1}\right)^2 \frac{t_2}{t_1} = 2^2 \times 5 = 20,$$

and $20 \times 192 = 3840$. Both calculations agree and include the change in time as well as current.

Step 4: Conclusion The thermal energy dissipated in five seconds is $\boxed{3840\,\text{J}}$. The numerical entry is 3840.

Quick Tip to Solve: For the same resistor, multiply the original heat by the square of the current ratio and then by the time ratio.

60. A bandwidth of 6 MHz is available for A.M. transmission. If the maximum audio signal frequency used for modulating the carrier wave is not to exceed 6 kHz. The number of stations that can be broadcasted within this band simultaneously without interfering with each other will be _____.

[4 marks]

Solution:

Concept:

- A conventional amplitude-modulated transmission contains an upper sideband and a lower sideband around its carrier. A signal of maximum frequency f_m therefore requires channel bandwidth $B_{\text{AM}} = 2f_m$.
- The maximum number of independent channels is the available bandwidth divided by the bandwidth per channel, with all frequencies expressed in consistent units. No additional guard-band allowance is specified here.

Step 1: Find the bandwidth of one station The highest audio frequency is $f_m = 6\,\text{kHz}$. For a carrier at frequency f_c, the furthest sideband frequencies are $f_c - f_m$ and $f_c + f_m$. The total occupied interval is consequently

$$B_{\text{station}} = (f_c + f_m) - (f_c - f_m) = 2f_m = 12\,\text{kHz}.$$

The carrier frequency itself sets the location of this interval, not its width.

Step 2: Express the available band in the same units The available bandwidth is $6\,\text{MHz}$. Since one megahertz is one thousand kilohertz,

$$B_{\text{available}} = 6 \times 1000 = 6000\,\text{kHz}.$$

Comparing 6 MHz directly with 6 kHz without this conversion would give an incorrect channel count. Also, using only 6 kHz per station would omit one of the two AM sidebands.

Step 3: Count the independent station channels Place each station in its own nonoverlapping $12\,\text{kHz}$ interval. The number of complete intervals is

$$N = \frac{B_{\text{available}}}{B_{\text{station}}} = \frac{6000}{12} = 500.$$

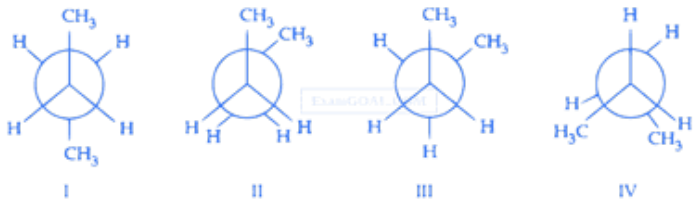
This ratio is dimensionless, as expected for a count. The result assumes ideal adjacent channels with the bandwidth specified in the question; no extra separation has been requested.

Step 4: Conclusion The band can accommodate $\boxed{500}$ stations simultaneously. The numerical entry is 500.

Quick Tip to Solve: For ordinary AM, double the highest audio frequency to get each station's bandwidth, then divide using matching units.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Arrange the following conformational isomers of n-butane in order of their increasing potential energy : [4 marks]



(A) II < III < IV < I

(B) I < IV < III < II

(C) II < IV < III < I

(D) I < III < IV < II

Correct Answer: (D) I < III < IV < II

Solution:

Concept:

- Rotation around the central carbon–carbon bond in n-butane produces staggered and eclipsed Newman conformations. Staggered conformations have lower torsional strain than eclipsed conformations.

- Among staggered forms, the anti arrangement places the two methyl groups 180° apart and is more stable than gauche. Among eclipsed forms, methyl–methyl eclipsing is more destabilising than methyl–hydrogen eclipsing.

Step 1: Identify the two staggered conformations In structure I, the two CH_3 groups are opposite each other when viewed down the central bond. This is the anti staggered arrangement and has the lowest energy. In structure III, the methyl groups are separated by a 60° dihedral angle. It is the gauche staggered form: torsional eclipsing is absent, but the methyl groups are closer and their steric repulsion raises its energy above I.

Step 2: Compare the eclipsed conformations Structure IV is eclipsed with methyl–hydrogen overlaps. Its torsional strain makes it higher in energy than the gauche staggered form III. Structure II has the two methyl groups eclipsing one another. That fully eclipsed arrangement combines torsional strain with the strongest methyl–methyl repulsion and has the greatest potential energy of the four. Thus the eclipsed forms must be ordered IV before II.

Step 3: Assemble the order and test the choices The energy therefore rises in the sequence

$$E_{\text{anti}} < E_{\text{gauche}} < E_{\text{eclipsed (CH}_3\text{/H)}} < E_{\text{fully eclipsed (CH}_3\text{/CH}_3\text{)}}.$$

In the labels of the figure this is I, III, IV, II. Options (A) and (C) put the fully eclipsed form II first, contrary to its high strain. Option (B) puts eclipsed IV below staggered III, reversing their torsional stability. Only option (D) has both comparisons correct.

Step 4: Conclusion The increasing potential-energy order is $\boxed{\text{I} < \text{III} < \text{IV} < \text{II}}$. Hence, the correct option is (D).

Quick Tip to Solve: Rank anti below gauche first; then rank methyl–hydrogen eclipsing below methyl–methyl eclipsing.

62. The Eu^{2+} ion is a strong reducing agent in spite of its ground state electronic configuration (outermost) : [Atomic number of Eu = 63] [4 marks]

(A) $4f^7 6s^2$

(B) $4f^6$

(C) $4f^7$

(D) $4f^6 6s^2$

Correct Answer: (C) $4f^7$

Solution:

Concept:

- Europium has atomic number 63 and neutral configuration $\text{Eu} : [\text{Xe}]\, 4f^7 6s^2$. During formation of its common cations, the two outer $6s$ electrons are removed before the $4f$ electrons.
- A reducing agent donates an electron and is itself oxidised. A half-filled subshell gives extra stability but does not alone determine the preferred oxidation state in chemical reactions; the overall energetics must also be considered.

Step 1: Count the electrons in the divalent ion A neutral europium atom has 63 electrons. Forming Eu^{2+} removes two electrons, leaving 61. Both lost electrons come from $6s^2$, so

$$Eu \longrightarrow Eu^{2+} + 2\,e^-, \qquad [Xe]\,4f^7 6s^2 \longrightarrow [Xe]\,4f^7.$$

The xenon core contains 54 electrons; adding the seven $4f$ electrons gives the required total of 61.

Step 2: Relate the configuration to reducing behaviour The seven $4f$ orbitals are singly occupied in the divalent ion, giving a half-filled $4f^7$ subshell. Nevertheless Eu^{2+} can lose a further electron and form the common trivalent state,

$$Eu^{2+} \longrightarrow Eu^{3+} + e^-.$$

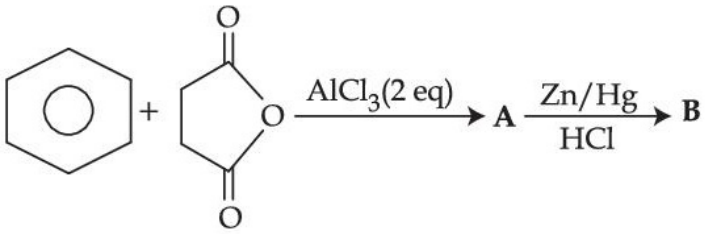
This electron donation explains its reducing action. The phrase "in spite of" signals that the half-filled configuration does not prevent oxidation; it does not ask us to replace that configuration by the trivalent one.

Step 3: Eliminate the other configurations Option (A), $4f^7 6s^2$, is the neutral atom's outer configuration and retains the electrons that must be removed. Option (B), $4f^6$, corresponds to Eu^{3+}, not the stated divalent ion. Option (D), $4f^6 6s^2$, both retains $6s$ electrons and has the wrong electron count for Eu^{2+}. Option (C), $4f^7$, is the only configuration consistent with removing two outer electrons.

Step 4: Conclusion The ground-state outer configuration of Eu^{2+} is $\boxed{4f^7}$. Hence, the correct option is (C).

Quick Tip to Solve: Write neutral Eu as xenon core plus $4f^7 6s^2$, then remove the two $6s$ electrons to form Eu(II).

63. The structures A and B formed in the following reaction are : [Ph = $-C_6H_5$] [4 marks]



(A) $A = C_6H_5COCH_2CH_3$; $B = C_6H_5CH(OH)CH_2CH_3$

(B) $A = C_6H_5COCH_2CH_2COOH$; $B = C_6H_5CH_2CH_2CH_2CH_2OH$

(C) $A = C_6H_5COCH_2CH_3$; $B = C_6H_5CH_2CH_2CH_3$

(D) $A = C_6H_5COCH_2CH_2COOH$; $B = C_6H_5CH_2CH_2CH_2COOH$

Correct Answer: (D) $A = C_6H_5COCH_2CH_2COOH$; $B = C_6H_5CH_2CH_2CH_2COOH$

Solution:

Concept:

- An acid anhydride reacts with benzene in the presence of $AlCl_3$ by Friedel–Crafts acylation. A cyclic anhydride opens to give an aromatic ketone bearing a carboxylic-acid group after work-up.

- Clemmensen reduction, $Zn(Hg)/HCl$, converts the ketone carbonyl $C{=}O$ into CH_2. Under these conditions the carboxylic-acid group is retained.

Step 1: Identify the acylating reagent The cyclic reagent in the scheme is succinic anhydride. Its ring contains the two carbonyl groups joined through oxygen and a two-carbon CH_2CH_2 chain on the opposite side. Activation by $AlCl_3$ allows benzene to attack an anhydride carbonyl. Ring opening and work-up preserve all four carbons of the succinic unit, giving

$$A = C_6H_5COCH_2CH_2COOH.$$

The product is a keto acid, not the shorter ketone $C_6H_5COCH_2CH_3$.

Step 2: Apply the second reagent selectively The next reagent is zinc amalgam with hydrochloric acid. It reduces the ketone group immediately adjacent to the phenyl ring to a methylene group:

$$C_6H_5COCH_2CH_2COOH \xrightarrow{Zn(Hg)/HCl} C_6H_5CH_2CH_2CH_2COOH.$$

The terminal $COOH$ remains a carboxylic acid; it is not reduced to CH_2OH. Thus B is 4-phenylbutanoic acid, while A is 4-oxo-4-phenylbutanoic acid.

Step 3: Match both products to the options Option (D) shows the required keto acid as A and the corresponding acid with three methylene groups between the phenyl ring and the carboxyl carbon as B. Options (A) and (C) omit the terminal carboxyl group already at the acylation stage and lose part of the anhydride skeleton. Option (B) gives a terminal alcohol after reduction, which is not the chemoselectivity of the stated Clemmensen reagent. Both stages therefore select (D).

Step 4: Conclusion $A = C_6H_5COCH_2CH_2COOH$ and $B = C_6H_5CH_2CH_2CH_2COOH$. Hence, the correct option is (D).

Quick Tip to Solve: Open the cyclic anhydride without losing carbon atoms, then reduce only the ketone carbonyl with zinc amalgam and acid.

64. In which one of the following sets all species show disproportionation reaction? [4 marks]

(A) ClO_2^-, F_2, MnO_4^- and $Cr_2O_7^{2-}$

(B) $Cr_2O_7^{2-}$, MnO_4^-, ClO_2^- and Cl_2

(C) MnO_4^-, ClO_2^-, Cl_2 and Mn^{3+}

(D) ClO_4^-, MnO_4^-, ClO_2^- and F_2

Correct Answer: No correct option as printed: every set contains permanganate, whose Mn(VII) cannot disproportionate.

Solution:

Concept:

- Disproportionation is a redox reaction in which the same element in one oxidation state is both oxidised and reduced.
- An element already in its highest ordinary oxidation state cannot undergo disproportionation, since

the oxidation branch would require a still higher state.

Step 1: Check the ion that occurs in every set The printed species is MnO_4^-, not MnO_4^{2-}. With oxygen at -2, its manganese oxidation state satisfies $x - 8 = -1$, giving $x = +7$. This is manganese's maximum ordinary oxidation state, so permanganate can be reduced but cannot supply the oxidation half of a disproportionation. Every offered set contains this ion; therefore every set fails the stated requirement.

Step 2: Identify why option C appears intended The other three species in (C) can disproportionate. Chlorine gives chloride and hypochlorite in cold alkali:

$$Cl_2 + 2\,OH^- \longrightarrow Cl^- + ClO^- + H_2O.$$

Chlorite has chlorine at $+3$, allowing both higher and lower states, while Mn^{3+} can form Mn^{2+} and MnO_2. Manganate, unlike the printed permanganate, does disproportionate:

$$3\,MnO_4^{2-} + 4\,H^+ \longrightarrow 2\,MnO_4^- + MnO_2 + 2\,H_2O.$$

Options (A) and (B) also contain dichromate, with chromium at its highest usual state $+6$; (A) and (D) contain fluorine, which cannot be oxidised to a positive state in this context. These provide additional exclusions.

Step 3: Conclusion No option is chemically correct with MnO_4^- as printed in every set. The answer key selects (C), which would be correct if that species were MnO_4^{2-}; the two ions must not be interchanged in solving the literal question.

Quick Tip to Solve: Calculate the oxidation state before applying a remembered reaction: manganate has Mn(VI), whereas permanganate has Mn(VII).

65. Match List - I with List - II

	List - I(Parmeter)		List - II(Unit)
(a)	**Cell constant**	**(i)**	$S\,cm^2mol^{-1}$
(b)	**Molar conductivity**	**(ii)**	**Dimensionless**
(c)	**Conductivity**	**(iii)**	m^{-1}
(d)	**Degree of dissociation of electrolyte**	**(iv)**	$\Omega^{-1}m^{-1}$

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(B) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(C) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(D) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Correct Answer: (A) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Solution:

Concept:

- Cell constant is electrode separation divided by electrode area: $G^* = l/A$. Conductivity relates conductance to geometry through $\kappa = G(l/A)$.
- Molar conductivity is conductivity divided by molar concentration in consistent volume units. Degree of dissociation is a ratio of numbers of particles or moles.

Step 1: Match the geometry and conductivity units Because l has unit m and area has unit m^2,

$$[l/A] = \mathrm{m}^{-1}.$$

Thus cell constant (a) matches (iii). Conductance is the reciprocal of resistance, so it has unit Ω^{-1}, also called the siemens (S). Multiplying by cell constant gives

$$[\kappa] = \Omega^{-1}\mathrm{m}^{-1}.$$

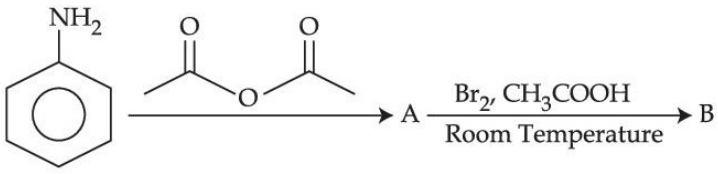
Thus conductivity (c) matches (iv); it cannot be dimensionless.

Step 2: Match the remaining quantities and check the options Molar conductivity is the conductance contribution of one mole after the appropriate geometric normalisation. Its SI unit is $\mathrm{S\,m^2\,mol^{-1}}$, and the commonly used centimetre-based unit appearing in the list is $\mathrm{S\,cm^2\,mol^{-1}}$. Therefore (b) matches (i). The degree of dissociation $\alpha = n_{\text{dissociated}}/n_{\text{initial}}$ is a ratio of like quantities and has no unit, so (d) matches (ii). Option (B) wrongly makes conductivity dimensionless and assigns a conductivity unit to a fraction. Option (C) interchanges the geometric and transport quantities. Option (D) makes the cell constant dimensionless, contradicting its definition.

Step 3: Conclusion The matching is a–iii, b–i, c–iv, d–ii. Hence, the correct option is (A).

Quick Tip to Solve: Begin with the two definitions l/A and $\kappa = G(l/A)$; their units determine the matching without memorising the full table.

66. The major products A and B formed in the following reaction sequence are : [4 marks]



(A) A = $C_6H_5NHCOCH_3$; B = m-$BrC_6H_4NHCOCH_3$

(B) A = p-$H_2NC_6H_4COCH_3$; B = 2-Br$-$4-$COCH_3-C_6H_3NH_2$

(C) A = $C_6H_5NHCOCH_3$; B = p-$BrC_6H_4NHCOCH_3$

(D) A = p-$H_2NC_6H_4COCH_3$; B = 2,6$-Br_2-$4-$COCH_3-C_6H_2NH_2$

Correct Answer: (C) A = $C_6H_5NHCOCH_3$; B = p-$BrC_6H_4NHCOCH_3$

Solution:

Concept:

- Aniline reacts with acetic anhydride by acylation at nitrogen, producing acetanilide, $C_6H_5NHCOCH_3$.
- The acetamido group remains ortho/para directing but is less activating than an amino group, allowing controlled bromination of the aromatic ring.

Step 1: Identify the first product A The lone pair on aniline nitrogen attacks an acyl group of acetic anhydride. Replacement of one N–H hydrogen by $COCH_3$ gives

$$C_6H_5NH_2 + (CH_3CO)_2O \longrightarrow C_6H_5NHCOCH_3 + CH_3COOH.$$

Thus A is acetanilide. This reaction protects the strongly activating amino group. It is not a Friedel–Crafts acylation of the benzene ring: no Lewis-acid catalyst is supplied and the amine nitrogen is the reactive nucleophilic site. Options (B) and (D), which show a ring acetyl group with an unchanged amino group, already have an incorrect A.

Step 2: Locate the bromine atom in B Bromine in acetic acid at room temperature substitutes on the ring of acetanilide. Resonance donation by $-NHCOCH_3$ favours the ortho and para positions. The para site is less sterically crowded than either adjacent ortho site, so para bromination gives the major product:

$$C_6H_5NHCOCH_3 + Br_2 \longrightarrow p\text{-}BrC_6H_4NHCOCH_3 + HBr.$$

The N-acetyl protecting group remains attached because the sequence has no hydrolysis step. Option (A) places bromine meta to the acetamido group and is inconsistent with its directing effect. Option (C) alone shows acetanilide followed by its para-bromo derivative.

Step 3: Conclusion A is acetanilide and B is p-bromoacetanilide. Hence, the correct option is (C).

Quick Tip to Solve: Acetylate aniline before bromination to moderate its activation; then choose the less crowded para product and retain the protecting group until hydrolysis is specified.

67. Which of the following is not an example of fibrous protein? [4 marks]

(A) Keratin

(B) Albumin

(C) Collagen

(D) Myosin

Correct Answer: (B) Albumin

Solution:

Concept:

- Fibrous proteins have elongated, thread-like molecular arrangements and generally perform structural or mechanical functions.
- Globular proteins fold into compact shapes; albumin is a standard example of a globular protein rather than a fibrous one.

Step 1: Classify the structural proteins Keratin is a fibrous protein of hair, nails and related protective tissues. Its polypeptide chains form extended structural assemblies rather than compact soluble globules. Collagen forms strong fibres in connective tissues such as tendons and skin. Its characteristic triple-helical units assemble into fibrils and provide tensile strength. Both therefore meet the fibrous-protein description, so neither (A) nor (C) answers the negative wording of the question.

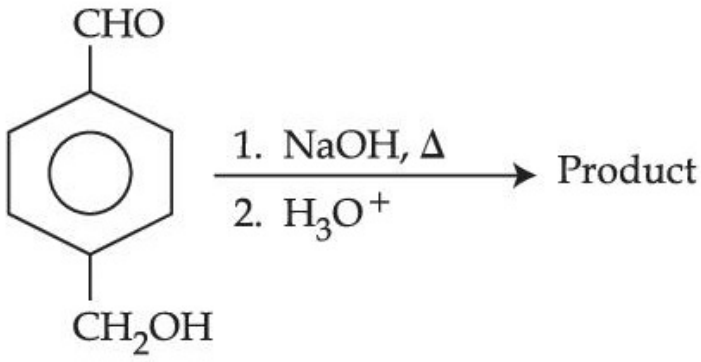
Step 2: Compare albumin with the muscle protein Myosin is a muscle protein with an extended coiled-coil tail that assembles into thick filaments; it is placed among fibrous proteins in this classification. Albumin, by contrast, has a compact folded molecular shape and belongs to the globular class. Serum albumin participates in transport and the maintenance of colloid osmotic pressure, functions that suit a soluble globular protein rather than an insoluble structural fibre. The classification concerns overall protein architecture and typical function, not merely whether a protein contains alpha-helical secondary structure. A globular protein may also contain helices, so the presence of a helix alone cannot make albumin fibrous. Thus (D) is excluded along with keratin and collagen, leaving only albumin.

Step 3: Conclusion Albumin is a globular protein and is not an example of a fibrous protein. Hence, the correct option is (B).

Quick Tip to Solve: Use the standard contrast: keratin and collagen form structural fibres, whereas albumin is a compact, soluble globular protein.

68. For the reaction given below :-

The compound which is not formed as a product in the reaction is a : [4 marks]



(A) compound with both alcohol and acid functional groups

(B) monocarboxylic acid

(C) dicarboxylic acid

(D) diol

Correct Answer: (C) dicarboxylic acid

Solution:

Concept:

- Aldehydes without an alpha-hydrogen undergo the Cannizzaro reaction in concentrated alkali: one aldehyde molecule is oxidised to carboxylate while another is reduced to alcohol.
- Acid work-up converts the carboxylate to a carboxylic acid; a pre-existing benzyl alcohol substituent is retained under these conditions.

Step 1: Identify the reacting and spectator groups The substrate is p-(hydroxymethyl)benzaldehyde, $HOCH_2-C_6H_4-CHO$. The aldehyde carbon is bonded directly to an aromatic ring and has no enolisable alpha-hydrogen. Hence base promotes disproportionation rather than an ordinary aldol condensation. The para $-CH_2OH$ group is already an alcohol; sodium hydroxide followed by acidification is not an oxidising reagent that converts this group into a second carboxyl group.

Step 2: Write both Cannizzaro products and test the descriptions The aldehyde groups in two molecules change in opposite directions. After acidification the organic products are

$$2\,HOCH_2-C_6H_4-CHO + H_2O \longrightarrow HOCH_2-C_6H_4-COOH + HOCH_2-C_6H_4-CH_2OH.$$

This balanced net equation includes water; the alkali and acid work-up are supplied in the question. The oxidised molecule contains one alcohol and one acid group, so it satisfies (A); it is also a monocarboxylic acid, satisfying (B). The reduced molecule contains two hydroxymethyl groups and is a diol, satisfying (D). Neither product contains two carboxyl groups. Making a dicarboxylic acid would require oxidation of the original hydroxymethyl group as well, which this reaction sequence does not provide.

Step 3: Conclusion A dicarboxylic acid is not formed in the stated Cannizzaro reaction. Hence, the correct option is (C).

Quick Tip to Solve: Track each functional group separately: Cannizzaro changes the aldehyde into acid or alcohol, while an alcohol already on the ring remains an alcohol.

69. Spin only magnetic moment in BM of $[Fe(CO)_4(C_2O_4)]^+$ is : [4 marks]

(A) 5.92

(B) 0

(C) 1

(D) 1.73

Correct Answer: (D) 1.73

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.
- Carbon monoxide is a neutral strong-field ligand; oxalate is a bidentate ligand of charge -2. Strong ligand-field splitting favours the low-spin arrangement here.

Step 1: Find the oxidation state and d-electron count Let the oxidation state of iron be x. Four carbonyl ligands contribute zero charge and one oxalate ligand contributes -2. The total complex charge is $+1$, so

$$x + 4(0) - 2 = +1, \qquad x = +3.$$

Neutral iron has configuration $[\text{Ar}]3d^6 4s^2$; removal of two 4s electrons and one 3d electron gives Fe^{3+} with a $3d^5$ configuration. The four monodentate carbonyl ligands and bidentate oxalate supply six donor atoms.

Step 2: Fill the split orbitals and calculate the moment For the strong-field, six-coordinate complex the five d electrons occupy the lower t_{2g} orbitals before the higher e_g orbitals. The low-spin arrangement is $t_{2g}^5 e_g^0$: two lower orbitals are doubly occupied and the third contains one electron. Therefore $n = 1$, giving

$$\mu = \sqrt{1(1+2)} = \sqrt{3} \simeq 1.73\,\text{BM}.$$

A value of 5.92 BM in (A) would require five unpaired electrons and the high-spin d^5 state. Zero in (B) would require all electrons paired, impossible for this odd d-electron count. The value 1 in (C) confuses the number of unpaired electrons with the magnetic moment.

Step 3: Conclusion The complex has one unpaired electron and a spin-only magnetic moment of $1.73\,\text{BM}$. Hence, the correct option is (D).

Quick Tip to Solve: First use ligand charges to obtain Fe(III), then distinguish the low-spin d^5 count $n = 1$ from the high-spin count $n = 5$.

70. The incorrect expression among the following is : [4 marks]

(A) $\frac{\Delta G_{System}}{\Delta S_{Total}} = -T$ (at constant P)

(B) $K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

(C) $K = e^{-\Delta G^\circ/RT}$

(D) For isothermal process $w_{reversible} = -nRT \ln \frac{V_f}{V_i}$

Correct Answer: (B) $K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

Solution:

Concept:

- The standard Gibbs-energy change satisfies $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -RT \ln K$, with K the dimensionless equilibrium constant.
- At fixed temperature and pressure, $\Delta G_{\text{system}} = -T\Delta S_{\text{total}}$. Reversible isothermal ideal-gas work follows by integrating $-P\,dV$.

Step 1: Derive the relation that must contain K Substituting $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ into the equilibrium relation gives

$$\ln K = -\frac{\Delta H^\circ - T\Delta S^\circ}{RT}.$$

Consequently,

$$K = \exp\left[-\frac{\Delta H^\circ - T\Delta S^\circ}{RT}\right].$$

Option (B) omits both the logarithm or exponential and the necessary minus sign. The quotient written there equals $-\ln K$, not K. This error remains even though the quotient is dimensionless.

Step 2: Verify the other three expressions Option (C) is precisely the exponential form $K = e^{-\Delta G^\circ/(RT)}$ and is correct. For a system exchanging heat with a reservoir at temperature T, $\Delta S_{\text{surroundings}} = -\Delta H_{\text{system}}/T$ at constant pressure; adding the system entropy gives $\Delta G = -T\Delta S_{\text{total}}$, the relation intended in (A). For reversible isothermal expansion of an ideal gas,

$$w = -\int_{V_i}^{V_f} \frac{nRT}{V}\,dV = -nRT \ln \frac{V_f}{V_i},$$

agreeing with (D) under the usual chemical sign convention. The negative sign gives negative work for expansion, as expected for work done by the gas.

Step 3: Conclusion The incorrect expression is $K = (\Delta H^\circ - T\Delta S^\circ)/(RT)$. The correct relation is $\ln K = -(\Delta H^\circ - T\Delta S^\circ)/(RT)$. Hence, the correct option is (B).

Quick Tip to Solve: Remember $\Delta G^\circ = -RT \ln K$: dividing Gibbs energy by RT yields a logarithm with a minus sign, never the equilibrium constant directly.

71. Match List - I with List - II :

	List - I(Metal Ion)		List - II(Group of Qualitative analysis)
(a)	Mn^{2+}	**(i)**	**Group - III**
(b)	As^{3+}	**(ii)**	**Group - IIA**
(c)	Cu^{2+}	**(iii)**	**Group - IV**
(d)	Al^{3+}	**(iv)**	**Group - IIB**

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(B) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(C) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

(D) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

Correct Answer: (B) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Solution:

Concept:

- Classical qualitative analysis separates cations by selective precipitation. The group depends on the reagent and medium, not merely on the charge of the ion.
- Group II sulphides precipitate with hydrogen sulphide in acidic solution. Group III hydroxides precipitate with ammonium hydroxide in the presence of ammonium chloride; Group IV sulphides need an alkaline medium.

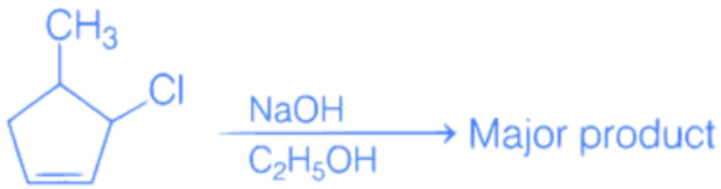
Step 1: Classify manganese and aluminium Mn^{2+} belongs to Group IV. Its sulphide is precipitated in an ammoniacal medium, where the sulphide-ion concentration is sufficiently high. Therefore (a) matches (iii). Aluminium belongs to Group III: Al^{3+} gives $\mathrm{Al(OH)_3}$ with the group reagent $\mathrm{NH_4OH}$ in the presence of $\mathrm{NH_4Cl}$. Thus (d) matches (i). The buffer controls the hydroxide concentration and prevents later-group ions from precipitating at this stage.

Step 2: Separate the two Group II subgroups Copper and arsenic both belong to Group II, but they lie in different subgroups. Copper belongs to Group IIA, the copper subgroup, whose sulphides are insoluble in yellow ammonium sulphide. Hence (c) matches (ii). Arsenic belongs to Group IIB, whose sulphides dissolve in yellow ammonium sulphide to form thio salts, so (b) matches (iv). Options (A) and (C) wrongly put manganese in Group III; option (D) incorrectly puts it in Group IIB. Only option (B) contains all four correct pairings.

Step 3: Conclusion Manganese is in Group IV, arsenic in IIB, copper in IIA and aluminium in III. Hence, the correct option is (B).

Quick Tip to Solve: For Group II, remember that arsenic belongs to the sulphide-soluble subgroup IIB, while copper belongs to IIA.

72. The major product of the following reaction is : [4 marks]



(A) 2-Methylcyclopentan-1-ol

(B) 1-Methylcyclopent-3-en-1-ol

(C) 4-Methylcyclopent-2-en-1-ol

(D) 2-Methylcyclopenta-1,3-diene

Correct Answer: (D) 2-Methylcyclopenta-1,3-diene

Solution:

Concept:

- Alcoholic NaOH promotes elimination of hydrogen halide from a halo compound. The new double bond forms between the carbon bearing chlorine and an adjacent carbon bearing hydrogen.
- A conjugated diene has alternating double and single bonds. Delocalisation of its pi electrons stabilises it relative to a cumulated diene.

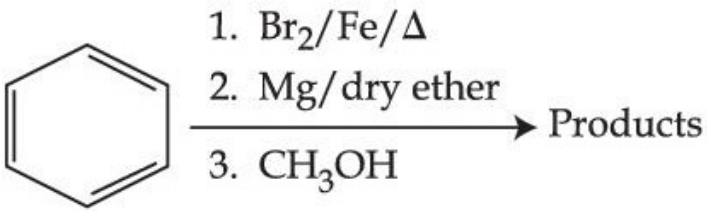
Step 1: Identify the carbon pair for elimination In the given five-membered ring, chlorine is on an allylic carbon next to the methyl-bearing saturated carbon. The reagent is NaOH in ethanol, so consider removal of a beta hydrogen together with chloride rather than replacement of chlorine by hydroxyl. Remove the hydrogen on the adjacent methyl-bearing carbon. The electron pair from its carbon–hydrogen bond forms the new double bond to the carbon from which chloride leaves. The original double bond of the ring remains unchanged.

Step 2: Compare the possible products This elimination produces a five-membered ring containing two conjugated double bonds, with methyl attached to one of the unsaturated carbons: 2-methylcyclopenta-1,3-diene. Removal of the vinylic hydrogen on the other side would instead place adjacent double bonds at one carbon in a small ring, a highly unfavourable cumulated arrangement. Option (A) even removes the original unsaturation and would require reduction as well as substitution. Options (B) and (C) are alcohols, whereas the alcoholic base favours dehydrohalogenation. Option (D) alone represents the conjugated elimination product.

Step 3: Conclusion The major product is 2-methylcyclopenta-1,3-diene, the conjugated diene shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: With alcoholic base, first look for loss of hydrogen halide; when an alkene is already present, test whether elimination extends conjugation.

73. For the following sequence of reactions, the correct products are : [4 marks]



(A) $C_6H_6 + CH_3OMgBr$

(B) $C_6H_5Br + HMgOCH_3$

(C) $C_6H_5CH_3 + Mg(OH)Br$

(D) $C_6H_5OCH_3 + HMgBr$

Correct Answer: (A) $C_6H_6 + CH_3OMgBr$

Solution:

Concept:

- Bromination of benzene with Br_2 and iron proceeds by electrophilic aromatic substitution, giving bromobenzene.
- Magnesium in dry ether converts an aryl bromide into a Grignard reagent. Such reagents are strong bases and react with the acidic hydrogen of an alcohol.

Step 1: Form the organomagnesium intermediate Iron generates the Lewis-acid bromination catalyst in the reaction mixture. A hydrogen on benzene is substituted by bromine without destroying aromaticity. Next, magnesium inserts into the carbon–bromine bond in dry ether:

$$C_6H_6 + Br_2 \longrightarrow C_6H_5Br + HBr$$

$$C_6H_5Br + Mg \longrightarrow C_6H_5MgBr$$

Dry conditions are essential because any water would protonate and destroy phenylmagnesium bromide before the final reagent is added.

Step 2: Protonate with methanol and compare options The carbon bonded to magnesium behaves as a strong base. Methanol supplies its hydroxyl proton, not a methyl carbocation. Proton transfer restores benzene and leaves the methoxide group bound to magnesium:

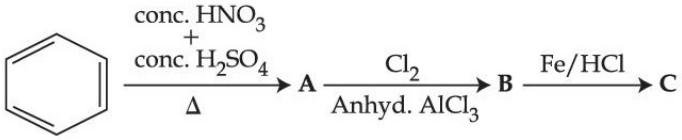
$$C_6H_5MgBr + CH_3OH \longrightarrow C_6H_6 + CH_3OMgBr$$

Thus the organic product is the starting aromatic hydrocarbon, together with magnesium bromide methoxide. Option (B) incorrectly retains bromine on the aromatic carbon after Grignard formation. Option (C) would require transfer of a methyl group from methanol, which does not occur. Option (D) incorrectly predicts ether formation rather than the rapid acid–base reaction.

Step 3: Conclusion The products are benzene and CH_3OMgBr. Hence, the correct option is (A).

Quick Tip to Solve: When a Grignard reagent encounters an alcohol, replace its carbon–magnesium bond by carbon–hydrogen before considering any carbon–carbon bond formation.

74. Identify correct A, B and C in the reaction sequence given below : [4 marks]



(A) A: nitrobenzene; B: 1-chloro-4-nitrobenzene; C: 4-chloroaniline

(B) A: nitrobenzene; B: 1-chloro-3-nitrobenzene; C: 3-chloroaniline

(C) A: nitrobenzene; B: 1-chloro-3-nitrobenzene; C: 3-aminophenol

(D) A: nitrobenzene; B: 1-chloro-3-nitrobenzene; C: 3-chlorophenol

Correct Answer: (B) A: nitrobenzene; B: 1-chloro-3-nitrobenzene; C: 3-chloroaniline

Solution:

Concept:

- Concentrated nitric acid with concentrated sulphuric acid nitrates benzene by electrophilic aromatic substitution, producing nitrobenzene.
- The nitro group withdraws electron density by inductive and resonance effects and directs further electrophilic substitution mainly to the meta position. Iron and hydrochloric acid reduce a nitro group to an amino group.

Step 1: Identify A and apply the directing effect The nitrating mixture generates the nitronium ion, $NO_2{}^+$, which substitutes for one ring hydrogen. Therefore A is nitrobenzene, $C_6H_5NO_2$. During the following chlorination with Cl_2 and anhydrous $AlCl_3$, the group already present controls the preferred site. The strongly electron-withdrawing nitro group destabilises the ortho and para substitution intermediates more than the meta intermediate. Consequently B is 1-chloro-3-nitrobenzene, not the para isomer.

Step 2: Reduce the nitro group without replacing chlorine Treatment with Fe/HCl reduces $-NO_2$ to $-NH_2$; the aryl carbon–chlorine bond is retained. The substituents remain meta to one another, so C is 3-chloroaniline. Option (A) fails because it shows para chlorination. Option (C) replaces the chlorine substituent by hydroxyl during a reduction that does not cause such substitution. Option (D) gives a chlorophenol, requiring replacement of the nitro functionality by hydroxyl rather than its reduction to an amine. Only option (B) follows both the directing effect and the final functional-group transformation.

Step 3: Conclusion A is nitrobenzene, B is 1-chloro-3-nitrobenzene and C is 3-chloroaniline. Hence, the correct option is (B).

Quick Tip to Solve: Follow each functional group separately: nitro directs chlorine meta, and the final reduction changes only nitro into amino.

75. The deposition of X and Y on ground surfaces is referred as wet and dry depositions, respectively. X and Y are : [4 marks]

(A) X = Ammonium salts, Y = CO_2

(B) X = SO_2, Y = Ammonium salts

(C) X = Ammonium salts, Y = SO_2

(D) X = CO_2, Y = SO_2

Correct Answer: (C) X = Ammonium salts, Y = SO_2

Solution:

Concept:

- Wet deposition removes atmospheric pollutants through precipitation such as rain, snow or fog. Dissolved ions and salts are carried to the ground in water.
- Dry deposition transfers gases and particles directly to ground surfaces without precipitation. Sulphur dioxide is an important gaseous pollutant deposited by this route.

Step 1: Identify the material in wet deposition Ammonia can react with atmospheric acids, forming ammonium salts such as ammonium sulphate and ammonium nitrate. These water-soluble species can enter droplets and reach the ground with rain or other precipitation. Thus ammonium salts are the intended example X of wet deposition. This description concerns the route of removal rather than claiming that every ammonium particle is always removed only by rain. The presence of a water carrier is what makes the deposition wet.

Step 2: Identify the dry-deposited pollutant and eliminate alternatives Sulphur dioxide can be transferred directly from air to soil, vegetation and other surfaces, so Y is SO_2. It may also dissolve and contribute to acid rain, but direct deposition of the gas is the dry route asked for here. Option (A) gives the wet-deposition species correctly but substitutes carbon dioxide for the intended dry pollutant. Option (B) reverses the stated pairing. Option (D) assigns carbon dioxide as the wet-deposition example, missing the ammonium salts carried by precipitation. Therefore only option (C) gives both intended examples in the required order.

Step 3: Conclusion X is ammonium salts and Y is SO_2. Hence, the correct option is (C).

Quick Tip to Solve: Classify deposition by its carrier: precipitation carries dissolved salts in wet deposition, while gases can contact surfaces directly in dry deposition.

76. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Lithium salts are hydrated.

Reason (R) : Lithium has higher polarising power than other alkali metal group members.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(B) (A) is correct but (R) is not correct.

(C) (A) is not correct but (R) is correct.

(D) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Correct Answer: (A) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Solution:

Concept:

- The small Li^+ ion has a high charge density and a strong electric field. Its interactions with polar water molecules give it a particularly high hydration enthalpy among alkali-metal cations.
- Polarising power increases as cation size decreases at the same charge. Lithium therefore has the greatest polarising power within the alkali-metal group.

Step 1: Evaluate the assertion The assertion says that lithium salts are hydrated. This is the characteristic textbook behaviour of lithium salts; for example, lithium chloride can crystallise as $LiCl \cdot 2\,H_2O$. The very small lithium ion strongly attracts the oxygen ends of surrounding water molecules, making retention of water of crystallisation favourable. Its hydration is stronger than that of the larger alkali-metal ions. Accordingly, Assertion (A) is correct.

Step 2: Evaluate the reason The ions of the alkali metals have the same positive charge, but ionic size increases down the group. The smallest ion, Li^+, produces the strongest local electric field and has the highest polarising power. The Reason (R) is therefore correct. This comparison is between lithium and the other members of its own group, not an assertion that it exceeds every other cation.

Step 3: Connect the two statements Hydration is explained by the strong ion–dipole attraction between the small lithium ion and water, expressed by its high hydration enthalpy. Polarising power, in the usual Fajans-rule sense, describes the distortion of an anion's electron cloud and explains the increased covalent character of lithium salts. Although both properties have small ionic size as a common origin, this statement about polarising power does not itself explain water of crystallisation. Thus the two statements are true, but the given reason is not the correct explanation. Options (B) and (C) incorrectly reject a true statement, while option (D) incorrectly identifies the specified reason as the explanation.

Step 4: Conclusion Both statements are correct, but the reason is not the correct explanation of the assertion. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish high hydration enthalpy, which explains hydration, from high polarising power, which describes distortion of an anion and covalent character.

77. Which one of the following statements is incorrect? [4 marks]

(A) Atomic hydrogen is produced when H_2 molecules at a high temperature are irradiated with UV radiation.

(B) At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%

(C) Bond dissociation enthalpy of H_2 is highest among diatomic gaseous molecules which contain a single bond.

(D) Dihydrogen is produced on reacting zinc with HCl as well as $NaOH_{(aq)}$.

Correct Answer: (B) At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%

Solution:

Concept:

- The hydrogen molecule has a strong single covalent bond. Heating or supplying sufficient photon energy can dissociate H_2 into hydrogen atoms.
- The NCERT dissociation figure of approximately 8.1% applies near $5000\,\mathrm{K}$, rather than $2000\,\mathrm{K}$. Dissociation increases strongly with temperature.

Step 1: Test the numerical temperature statement Option (B) claims that dissociation is nearly 8.1% at about $2000\,\mathrm{K}$. The stated percentage belongs to about $5000\,\mathrm{K}$. At the lower temperature, a much smaller fraction has enough energy to overcome the strong hydrogen–hydrogen bond. The numerical pairing in (B) is therefore incorrect; the question asks us to select this incorrect statement rather than a true property.

Step 2: Check the remaining statements Option (A) describes production of atomic hydrogen by irradiation of hot dihydrogen with ultraviolet light; the supplied energy can break the bond, so it is correct. Option (C) is also correct: among diatomic gaseous molecules containing a single bond, the bond dissociation enthalpy of H_2, about $435.8\,\mathrm{kJ\,mol^{-1}}$, is the largest. The restriction to single bonds excludes strongly bound multiply bonded molecules such as nitrogen. Option (D) correctly states that zinc liberates hydrogen with both acid and aqueous alkali. For example,

$$\mathrm{Zn} + 2\,\mathrm{HCl} \longrightarrow \mathrm{ZnCl_2} + \mathrm{H_2}$$

Zinc is amphoteric and also gives hydrogen while forming zincate in aqueous sodium hydroxide.

Step 3: Conclusion The incorrect statement is the claim of 8.1% dissociation at $2000\,\mathrm{K}$; the corresponding temperature is about $5000\,\mathrm{K}$. Hence, the correct option is (B).

Quick Tip to Solve: In numerical factual statements, verify the temperature and percentage together; a correct percentage paired with the wrong temperature is still false.

78. Which among the following is not a polyester? [4 marks]

(A) Novolac

(B) PHBV

(C) Dacron

(D) Glyptal

Correct Answer: (A) Novolac

Solution:

Concept:

- A polyester contains recurring ester linkages, $-\mathrm{COO}-$, in its polymer backbone. Classify the polymer by this linkage, rather than merely by whether its preparation is a condensation reaction.
- Phenol reacts with formaldehyde to form phenol-formaldehyde resins. A methylene bridge, $-\mathrm{CH_2}-$, between aromatic rings is different from an ester linkage.

Step 1: Identify the linkage in Novolac Novolac is the linear phenol-formaldehyde polymer obtained under acidic conditions with excess phenol. Its phenolic rings are connected mainly by methylene bridges. The oxygen of a phenolic hydroxyl group remains in an $-\mathrm{OH}$ group; it is not part of the repeating $-\mathrm{COO}-$ linkage of a polyester. Thus Novolac is the required exception.

Step 2: Check the three polyester choices PHBV is poly(β-hydroxybutyrate-co-β-hydroxyvalerate), whose hydroxy-acid units are joined by ester bonds. Dacron is polyethylene terephthalate, formed from ethylene glycol and terephthalic acid, again giving ester linkages. Glyptal is made from ethylene glycol and phthalic acid or its anhydride, and also contains ester linkages. These structures exclude options (B), (C) and (D). Condensation polymerisation alone therefore cannot distinguish Novolac from all the others: the backbone functional group must be examined.

Step 3: Conclusion Novolac is a phenol-formaldehyde resin and is not a polyester. Hence, the correct option is (A).

Quick Tip to Solve: For polymer classification, locate the repeating backbone bond: an oxygen-containing polymer is a polyester only if the linkage is an ester.

79. Which one of the following correctly represents the order of stability of oxides, X_2O; (X = halogen)? [4 marks]

(A) $Br > Cl > I$

(B) $Br > I > Cl$

(C) $Cl > I > Br$

(D) $I > Cl > Br$

Correct Answer: (D) $I > Cl > Br$

Solution:

Concept:

- The stability of halogen oxides depends on bonding and the properties of the particular halogen. It cannot be predicted by applying the oxidising-power order of the free halogens.
- For the oxides considered here, the standard stability order is iodine oxides greater than chlorine oxides greater than bromine oxides. Larger iodine can support stronger stabilising interactions in its oxides.

Step 1: Apply the halogen-oxide stability trend The question asks about the relative stability of the compounds X_2O, rather than the relative reactivity of X_2. Iodine oxides are the most stable in this comparison, whereas bromine oxides are the least stable. The stability of iodine oxides is associated with strong interactions involving the large, polarizable iodine atom; chlorine oxides lie between iodine and bromine oxides. Thus the required descending sequence is

$$I_2O > Cl_2O > Br_2O.$$

Step 2: Compare the printed sequences Expressing the order by the halogen symbol gives $I > Cl > Br$. Option (A) puts bromine first and iodine last, reversing the essential endpoints. Option (B) also incorrectly assigns bromine the greatest stability. Option (C) correctly leaves bromine last but incorrectly places chlorine above iodine. Only option (D) has both the correct endpoints and the correct middle member. A monotonic electronegativity or atomic-size argument by itself would miss this particular empirical trend.

Step 3: Conclusion The order of stability is $I > Cl > Br$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the oxide-stability sequence separate from halogen oxidising power: for these oxides remember iodine first, chlorine next, bromine last.

80. The number of S = O bonds present in sulphurous acid, peroxodisulphuric acid and pyrosulphuric acid, respectively are : [4 marks]

(A) 2, 3 and 4

(B) 1, 4 and 3

(C) 2, 4 and 3

(D) 1, 4 and 4

Correct Answer: (D) 1, 4 and 4

Solution:

Concept:

- Count terminal $\mathrm{S{=}O}$ bonds from each acid structure. An $\mathrm{S{-}OH}$ bond, a bridging $\mathrm{S{-}O{-}S}$ bond and a peroxide $\mathrm{O{-}O}$ bond are all single bonds and must not be included.
- Sulphurous acid is $\mathrm{H_2SO_3}$, peroxodisulphuric acid is $\mathrm{H_2S_2O_8}$, and pyrosulphuric acid is $\mathrm{H_2S_2O_7}$. The prefixes identify different bridges, not different numbers of terminal double bonds in the last two acids.

Step 1: Count the bonds in sulphurous acid The conventional oxoacid structure of sulphurous acid is $\mathrm{HO{-}S(=O){-}OH}$. Of its three oxygen atoms, two are in hydroxyl groups and one is doubly bonded to sulphur. Therefore the number of $\mathrm{S{=}O}$ bonds in this acid is one. This immediately rules out options (A) and (C), which start with two.

Step 2: Count the bonds in the two disulphur acids Peroxodisulphuric acid has the connectivity $\mathrm{HO{-}S(=O)_2{-}O{-}O{-}S(=O)_2{-}OH}$. Each sulphur has two terminal double-bonded oxygen atoms, giving $2 + 2 = 4$ $\mathrm{S{=}O}$ bonds overall. Pyrosulphuric acid instead has $\mathrm{HO{-}S(=O)_2{-}O{-}S(=O)_2{-}OH}$. Replacing the peroxide bridge by a single oxygen bridge does not remove a terminal double bond from either sulphur, so it too has four. Option (B) incorrectly counts only three in pyrosulphuric acid; the three requested counts are one, four and four.

Step 3: Conclusion The numbers of $\mathrm{S{=}O}$ bonds are $1, 4, 4$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Draw the bridge first, then count terminal oxygen double bonds. Never count the oxygen atoms in an OH group or an O–O bridge as S=O bonds.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 1.22 g of an organic acid is separately dissolved in 100 g of benzene (K_b = 2.6 K kg mol^{-1}) and 100 g of acetone (K_b = 1.7 K kg mol^{-1}). The acid is known to dimerize in benzene but remain as a monomer in acetone. The boiling point of the solution in acetone increases by 0.17°C. The increase in boiling point of solution in benzene in °C is x × 10^{-2}. The value of x is _____. (Nearest integer) [Atomic mass : C = 12.0, H = 1.0, O =16.0] [4 marks]

Solution:

Concept:

- Boiling-point elevation obeys $\Delta T_b = iK_b m$, where i is the van't Hoff factor and m is the molality based on the original solute formula units.
- A monomeric solute has $i = 1$. Complete dimerisation joins every two acid molecules into one particle, so $i = 1/2$; equal acid masses in equal solvent masses have the same original molality.

Step 1: Obtain the acid molality from acetone The acid remains monomeric in acetone, so the observed rise measures the original molality directly. Substitution gives

$$0.17 = 1 \times 1.7 \times m, \qquad m = 0.100\,\text{mol kg}^{-1}.$$

With $0.100\,\text{kg}$ of solvent, this corresponds to $0.0100\,\text{mol}$ of acid formula units. Its molar mass would be $1.22/0.0100 = 122\,\text{g mol}^{-1}$, although identifying the acid is unnecessary.

Step 2: Include dimerisation in benzene The same mass of acid is dissolved in the same mass of benzene. Before association its molality is therefore also $0.100\,\text{mol kg}^{-1}$. Dimerisation halves the number of independently moving solute particles; the particle molality is $0.0500\,\text{mol kg}^{-1}$. Hence

$$\Delta T_b = \frac{1}{2} \times 2.6 \times 0.100 = 0.130\,\text{K}.$$

A temperature interval has the same numerical value in kelvin and degrees Celsius. Comparing $0.130 = 13 \times 10^{-2}$ with the requested form gives the required integer. Using $i = 1$ in benzene would incorrectly double the result.

Step 3: Conclusion The boiling-point rise in benzene is $0.13\,^\circ\text{C}$, so $x = 13$ (nearest integer).

Quick Tip to Solve: For equal solute and solvent masses, divide the two elevation equations: the ratio is simply the ratio of i times Kb; the molar mass cancels.

82. The value of magnetic quantum number of the outermost electron of Zn^+ ion is _____.

[4 marks]

Solution:

Concept:

- The magnetic quantum number m_l specifies an orbital orientation for a given azimuthal quantum number l, taking integer values from $-l$ to $+l$. For an s orbital, $l = 0$.
- To form a transition-metal cation, remove the outer ns electron before removing an electron from the filled or partially filled inner d subshell. Outermost means the largest occupied principal quantum number.

Step 1: Write the configuration of the zinc ion Zinc has atomic number 30 and neutral ground-state configuration

$$\mathrm{Zn}: \ [\mathrm{Ar}]\, 3d^{10}4s^2.$$

Removing one electron gives the singly charged ion, not the usual doubly charged zinc ion:

$$\mathrm{Zn}^+: \ [\mathrm{Ar}]\, 3d^{10}4s^1.$$

The electron removed comes from 4s because that is the outer occupied shell. The remaining 4s electron is therefore the outermost electron of the ion; selecting a 3d electron instead would answer a different question.

Step 2: Assign its magnetic quantum number For the outermost electron, $n = 4$ and the orbital is s, so $l = 0$. The allowed magnetic quantum numbers obey

$$m_l = -l, -l+1, \ldots, +l.$$

On substituting $l = 0$, there is just one allowed value, $m_l = 0$. The electron spin quantum number may be $+1/2$ or $-1/2$, but spin is a separate label and does not alter this answer. A closed 3d subshell contains several orbital orientations, yet none of those are the outermost 4s orbital asked for here.

Step 3: Conclusion The outermost electron is in the 4s orbital, whose magnetic quantum number is $m_l = 0$. The required numerical value is 0 (dimensionless).

Quick Tip to Solve: For a metal ion, write the ionic configuration first; then find the subshell with largest n. Every s electron has magnetic quantum number zero.

83. The pH of a solution obtained by mixing 50 mL of 1 M HCl and 30 mL of 1 M NaOH is x $\times$ 10^{-4}. The value of x is _____. (Nearest integer) [log 2.5 = 0.3979]

[4 marks]

Solution:

Concept:

- Strong HCl and NaOH dissociate completely and neutralise in the ratio $H^+ + OH^- \longrightarrow H_2O$. Compare their mole amounts before calculating the final concentration.
- For excess strong acid, $\mathrm{pH} = -\log_{10}[H^+]$, using the molar concentration after mixing. The final

liquid volume is the sum of the two supplied volumes.

Step 1: Find the excess acid after neutralisation Convert each volume to litres and use $n = CV$:

$$n(\mathrm{HCl}) = 1 \times 0.050 = 0.050\,\mathrm{mol},$$

$$n(\mathrm{NaOH}) = 1 \times 0.030 = 0.030\,\mathrm{mol}.$$

The base consumes an equal amount of acid, leaving $0.050 - 0.030 = 0.020\,\mathrm{mol}$ of excess hydrogen ions. The resulting solution is acidic, so calculating the pH from excess hydroxide would have the wrong physical starting point.

Step 2: Calculate the pH and the scaled integer The total volume is $50 + 30 = 80\,\mathrm{mL} = 0.080\,\mathrm{L}$, not the difference between the volumes. Thus

$$[\mathrm{H}^+] = \frac{0.020}{0.080} = 0.25\,\mathrm{mol\,L^{-1}}.$$

Using the logarithm provided in the question,

$$\mathrm{pH} = -\log_{10}(2.5 \times 10^{-1}) = 1 - 0.3979 = 0.6021.$$

The question asks for the coefficient of 10^{-4}, rather than for pH itself. Therefore

$$x = \frac{0.6021}{10^{-4}} = 6021.$$

No buffer equation is needed, since neither a weak acid nor its conjugate base is present.

Step 3: Conclusion The pH is $0.6021 = 6021 \times 10^{-4}$; hence $x = 6021$ (nearest integer, dimensionless).

Quick Tip to Solve: Neutralise using moles first, divide the excess by the total mixed volume, and only then take the logarithm and apply the requested power-of-ten scaling.

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84. For the reaction A → B, the rate constant k(in s^{-1}) is given by

$$\log_{10} k = 20.35 - \frac{(2.47 \times 10^3)}{T}$$

The energy of activation in kJ mol^{-1} is _____. (Nearest integer) [Given : R = 8.314 J K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- The Arrhenius relation is $k = Ae^{-E_a/(RT)}$. In common logarithms it becomes $\log_{10} k = \log_{10} A - E_a/(2.303RT)$, with E_a the activation energy.
- The coefficient of $1/T$ has magnitude $E_a/(2.303R)$. With R in $\mathrm{J\,K^{-1}\,mol^{-1}}$, the calculated activation energy is in $\mathrm{J\,mol^{-1}}$, requiring division by 1000 for kilojoules.

Step 1: Match the temperature-dependent coefficients The given equation is

$$\log_{10} k = 20.35 - \frac{2.47 \times 10^3}{T}.$$

Comparing this with the logarithmic Arrhenius equation identifies $20.35 = \log_{10} A$. It is the coefficient 2.47×10^3, not the intercept, that determines the activation energy:

$$\frac{E_a}{2.303R} = 2.47 \times 10^3.$$

The negative sign agrees with a positive activation energy: increasing temperature makes the negative term smaller in magnitude, so the rate constant increases.

Step 2: Substitute R and convert the energy unit Rearrange before inserting the supplied gas constant:

$$E_a = 2.303(8.314)(2.47 \times 10^3)\,\mathrm{J\,mol^{-1}}.$$

The product is approximately $4.729 \times 10^4\,\mathrm{J\,mol^{-1}}$, or

$$E_a \approx 47.29\,\mathrm{kJ\,mol^{-1}}.$$

Rounding to the nearest integer yields 47. Omitting the factor 2.303 would treat the given common logarithm as a natural logarithm and underestimate the energy. Leaving the answer in joules would also fail to match the requested unit.

Step 3: Conclusion The activation energy is approximately $47.29\,\mathrm{kJ\,mol^{-1}}$, so the required nearest-integer value is $47\,\mathrm{kJ\,mol^{-1}}$.

Quick Tip to Solve: Check the logarithm base before reading an Arrhenius slope: log base ten introduces 2.303, while a natural logarithm does not.

85. Sodium oxide reacts with water to produce sodium hydroxide. 20.0 g of sodium oxide is dissolved in 500 mL of water. Neglecting the change in volume, the concentration of the resulting NaOH solution is _____ $\times$ 10^{-1} M. (Nearest integer) [Atomic mass : Na = 23.0, O = 16.0, H = 1.0] [4 marks]

Solution:

Concept:

- Molarity is the amount of solute in moles divided by the final solution volume in litres: $M = n/V$. The specified negligible volume change lets us use $V = 0.500\,\mathrm{L}$.
- The balanced reaction fixes the mole ratio. One mole of sodium oxide produces two moles of sodium hydroxide, so the mass of oxide cannot be treated directly as the mass of hydroxide.

Step 1: Balance the reaction and calculate the oxide amount The reaction with water is

$$\mathrm{Na_2O + H_2O \longrightarrow 2\,NaOH}.$$

Using the given atomic masses, the molar mass of sodium oxide is

$$M_{Na_2O} = 2(23.0) + 16.0 = 62.0\,\text{g mol}^{-1}.$$

Consequently, the amount initially dissolved is

$$n(Na_2O) = \frac{20.0}{62.0} = 0.32258\,\text{mol}.$$

Water is the solvent and is present in excess; sodium oxide determines the amount of hydroxide produced.

Step 2: Convert to hydroxide molarity and the requested coefficient The stoichiometric coefficient of sodium hydroxide is two, giving

$$n(NaOH) = 2\left(\frac{20.0}{62.0}\right) = 0.64516\,\text{mol}.$$

Neglecting the volume change as directed,

$$M = \frac{0.64516}{0.500} = 1.29032\,\text{mol L}^{-1}.$$

To express this in the form $a \times 10^{-1}$ M, divide by 10^{-1}:

$$a = \frac{1.29032}{0.1} = 12.9032.$$

Round this coefficient, rather than the molarity itself, to the nearest integer.

Step 3: Conclusion The concentration is approximately $1.2903\,\text{M} = 12.903 \times 10^{-1}\,\text{M}$. The requested nearest-integer entry is $\boxed{13}$.

Quick Tip to Solve: First apply the 1:2 oxide-to-hydroxide mole ratio, then convert 500 mL to litres. Round only the coefficient of the power of ten requested in the blank.

86. According to molecular orbital theory, the number of unpaired electron(s) in O_2^{2-} is : [4 marks]

Solution:

Concept:

- The peroxide ion $O_2{}^{2-}$ has two more electrons than neutral dioxygen. Its total electron count is $2(8) + 2 = 18$, including the inner-shell electrons.
- Electrons fill molecular orbitals in order of energy. Degenerate orbitals are filled according to Hund's rule, and each orbital can finally hold two electrons with opposite spins.

Step 1: Count electrons and fill the bonding orbitals For oxygen and its ions, the relevant ordering of the valence molecular orbitals is

$$\sigma_{2s} < \sigma^*_{2s} < \sigma_{2p_z} < \pi_{2p_x} = \pi_{2p_y} < \pi^*_{2p_x} = \pi^*_{2p_y}.$$

The ion has fourteen valence electrons, since each oxygen contributes six and the negative charge adds two. The first ten valence electrons fill the bonding and the lower antibonding orbitals in pairs:

$$(\sigma_{2s})^2(\sigma^*_{2s})^2(\sigma_{2p_z})^2(\pi_{2p_x})^2(\pi_{2p_y})^2.$$

The inner $1s$ molecular orbitals are also completely paired and do not affect the unpaired-electron count.

Step 2: Fill the antibonding pi orbitals and count spins Four valence electrons remain after the first ten have been placed. They fill the two degenerate antibonding pi orbitals as

$$(\pi^*_{2p_x})^2(\pi^*_{2p_y})^2.$$

Each now contains an opposite-spin pair. In neutral O_2, these orbitals each contain one electron; the two additional electrons in the peroxide ion pair those electrons. No electron occupies the higher $\sigma^*_{2p_z}$ orbital. Thus every occupied orbital is doubly occupied, and there is no singly occupied molecular orbital.

Step 3: Conclusion The peroxide ion O_2^{2-} is diamagnetic and contains $\boxed{0}$ unpaired electrons. The requested integer is 0.

Quick Tip to Solve: Distinguish neutral oxygen, superoxide and peroxide by adding electrons to the antibonding pi orbitals: their unpaired-electron counts are two, one and zero, respectively.

87. The transformation occurring in Duma's method is given below :

$$C_2H_7N + \left(2x + \frac{y}{2}\right) CuO \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right) Cu$$

The value of y is _____. (Integer answer) [4 marks]

Solution:

Concept:

- In Dumas' method, an organic nitrogen compound is oxidised by hot copper(II) oxide. Its carbon gives carbon dioxide, its hydrogen gives water, and its nitrogen is measured as nitrogen gas.
- Atom conservation determines the unknown coefficients. If the general composition is $C_xH_yN_z$, the product amounts are x of CO_2, $y/2$ of H_2O, and $z/2$ of N_2.

Step 1: Read the hydrogen balance directly The molecular formula printed on the left is C_2H_7N. It contains seven hydrogen atoms per molecule. The water coefficient on the right is $y/2$, and each water molecule contains two hydrogen atoms. Therefore the number of hydrogen atoms on the right, per molecule of organic compound, is

$$2\left(\frac{y}{2}\right) = y.$$

Equating the left and right hydrogen counts immediately gives

$$y = 7.$$

A fractional coefficient for water is permitted in a balanced equation; it does not require the parameter y to be even.

Step 2: Check the entire equation for consistency The carbon and nitrogen balances similarly give $x = 2$ and $z = 1$. Hence the coefficient of copper(II) oxide is

$$2x + \frac{y}{2} = 2(2) + \frac{7}{2} = \frac{15}{2}.$$

Multiplying the whole reaction by two gives the integer-coefficient equation

$$2\,C_2H_7N + 15\,CuO \longrightarrow 4\,CO_2 + 7\,H_2O + N_2 + 15\,Cu.$$

Carbon, hydrogen, nitrogen, copper and oxygen all balance. In particular, the seven water molecules contain fourteen hydrogen atoms, exactly those in two molecules of the starting compound.

Step 3: Conclusion The parameter equals the hydrogen subscript of C_2H_7N, so the required integer is $\boxed{y = 7}$.

Quick Tip to Solve: For this Dumas template, compare the molecular formula with $C_xH_yN_z$. The denominator 2 belongs to the water coefficient, not to the hydrogen subscript being requested.

88. CH_4 is absorbed on 1g charcoal at 0°C following the Freundlich adsorpiton isotherm 10.0 mL of CH_4 is adsorbed at 100 mm of Hg, whereas 15.0 mL is adsorbed at 200 mm of Hg. The volume of CH_4 adsorbed at 300 mm of Hg is 10^x mL. The value of x is _____ $\times 10^{-2}$.

(Nearest integer) [Use $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$] [4 marks]

Solution:

Concept:

- At fixed temperature the Freundlich isotherm is $V = kP^{1/n}$ for a fixed mass of adsorbent, where V is adsorbed gas volume and P is pressure. Taking logarithms makes the relation linear.
- Ratios remove the constant k, so two measured volumes determine the exponent $1/n$. The logarithm used here is base ten, matching the data supplied in the question.

Step 1: Determine the Freundlich exponent Write the two observations in the form

$$10.0 = k(100)^{1/n}, \qquad 15.0 = k(200)^{1/n}.$$

Dividing the second equation by the first gives

$$1.5 = 2^{1/n}, \qquad \frac{1}{n} = \frac{\log_{10}(1.5)}{\log_{10} 2}.$$

The supplied logarithms yield

$$\log_{10}(1.5) = \log_{10} 3 - \log_{10} 2 = 0.4771 - 0.3010 = 0.1761.$$

Thus

$$\frac{1}{n} = \frac{0.1761}{0.3010} \simeq 0.58505.$$

The exponent is less than one, so adsorption increases more slowly than pressure in this range.

Step 2: Find the logarithm of the volume at the new pressure Use the observation at 100 mm of Hg as a reference. At 300 mm of Hg,

$$\frac{V_{300}}{10.0} = \left(\frac{300}{100}\right)^{1/n} = 3^{1/n}.$$

Therefore

$$\log_{10} V_{300} = 1 + \frac{0.1761}{0.3010}(0.4771) \simeq 1.27912.$$

Since the required volume is 10^x mL, this logarithm is precisely x. The blank asks for the coefficient when $x = a \times 10^{-2}$, so

$$a = 100x \simeq 127.912.$$

The nearest integer is obtained only after this scaling.

Step 3: Conclusion The exponent is $x \simeq 1.27912 = 127.912 \times 10^{-2}$, and the adsorbed volume is about $19.0\,\mathrm{mL}$. The required integer entry is $\boxed{128}$.

Quick Tip to Solve: Do not assume volume is proportional to pressure. First obtain the Freundlich exponent from the ratio of the two observations, then calculate the logarithm requested by 10^x.

89. The empirical formula for a compound with a cubic close packed arrangement of anions and with cations occupying all the octahedral sites in A_xB. The value of x is _____. (Integer answer) [4 marks]

Solution:

Concept:

- In a cubic close packed arrangement, the number of octahedral voids equals the number of close packed particles. The number of tetrahedral voids, which is twice as large, is irrelevant here.
- An empirical formula records the simplest whole-number ratio of cations to anions. Full occupation of octahedral sites gives one cation for every anion, independently of the size of the crystal.

Step 1: Count the anions in a conventional cubic cell A cubic close packed lattice is represented by a face-centred cubic unit cell. Anions at the eight corners are each shared by eight cells, and anions at the six faces are each shared by two cells. The number belonging to one unit cell is therefore

$$N_{\text{anion}} = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) = 1 + 3 = 4.$$

Shared particles must be counted fractionally; counting all fourteen drawn spheres as whole anions would give the wrong stoichiometry.

Step 2: Count the occupied octahedral sites and simplify the ratio There is one octahedral site at

the body centre, wholly inside the unit cell. There are also twelve edge-centre octahedral sites, each shared by four neighbouring cells. Their effective total is

$$N_{\text{oct}} = 1 + 12\left(\frac{1}{4}\right) = 4.$$

All these sites are occupied, so the cell contains four cations and four anions. Taking A as the cation and B as the anion in the stated formula, the ratio is

$$A : B = 4 : 4 = 1 : 1.$$

Dividing both subscripts by four gives the empirical formula AB, rather than retaining the unit-cell contents A_4B_4.

Step 3: Conclusion The empirical formula is $AB = A_1B$. Consequently the required dimensionless integer is $\boxed{x = 1}$.

Quick Tip to Solve: Remember the void counts per close packed particle: one octahedral void and two tetrahedral voids. Full octahedral occupation therefore gives a 1:1 empirical formula.

90. In the electrolytic refining of blister copper, the total number of main impurities, from the following, removed as anode mud is _____

Pb, Sb, Se, Te, Ru, Ag, Au and Pt [4 marks]

Solution:

Concept:

- In electrolytic refining, impure copper is the anode, pure copper is the cathode, and a copper salt solution is the electrolyte. Copper dissolves at the anode and is deposited selectively at the cathode.
- The standard classification separates the more readily oxidised impurities, which enter the electrolyte, from less readily oxidised impurities, which are left behind and collect below the anode as anode mud.

Step 1: Identify the impurities excluded from the mud count The desired anode process is

$$\text{Cu} \longrightarrow \text{Cu}^{2+} + 2\,\text{e}^{-}.$$

Impurities more electropositive than copper are more readily oxidised. In the supplied list, lead and antimony fall in this category, so they are excluded from the less readily oxidised impurities counted as the main anode-mud constituents in this question. Their presence does not mean they are deposited as pure metals on the copper cathode: dissolution at the anode and selective deposition at the cathode are separate processes.

Step 2: List and count the retained impurities The remaining listed impurities are selenium, tellurium, ruthenium, silver, gold and platinum. These are retained with the insoluble residue rather than following copper through its anode-dissolution and cathode-deposition cycle. Count each named

element once:

$$\underbrace{\mathrm{Se},\ \mathrm{Te},\ \mathrm{Ru},\ \mathrm{Ag},\ \mathrm{Au},\ \mathrm{Pt}}_{6\ \text{elements}}.$$

Thus six of the eight listed impurities contribute to the requested main anode-mud count. The count is of elements in the supplied list; it is not a count of chemical species or of atoms in a reaction equation.

Step 3: Conclusion The six listed anode-mud impurities are $\mathrm{Se}, \mathrm{Te}, \mathrm{Ru}, \mathrm{Ag}, \mathrm{Au}$ and Pt. The required integer is $\boxed{6}$.

Quick Tip to Solve: Classify each impurity by its tendency to oxidise relative to copper before counting. Do not count every impurity in blister copper as anode mud.