JEE Main 2021 Paper 1 (B.E./B.Tech) Feb 24 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 1 (February), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 February 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If $\int \frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$, where c is a constant of integration, then the ordered pair (a, b) is equal to : [4 marks]

(A) (-1, 3)

(B) (1, 3)

(C) (1, -3)

(D) (3, 1)

Correct Answer: (B) (1, 3)

Solution:

Concept:

- The derivative of $\sin x + \cos x$ is $\cos x - \sin x$, exactly the numerator of the integrand.
- Use $(\sin x + \cos x)^2 = 1 + \sin 2x$ and the standard integral $\int du/\sqrt{a^2-u^2} = \sin^{-1}(u/a) + C$, for $a > 0$.

Step 1: Choose the substitution Let $u = \sin x + \cos x$. Then $du = (\cos x - \sin x)\,dx$. Rewrite the expression inside the radical before integrating:

$$8 - \sin 2x = 8 - (u^2 - 1) = 9 - u^2.$$

Thus the numerator and denominator transform together, with no extra multiplier. Notice that the constant under the radical becomes 9 rather than 8; using $\sqrt{8}$ as the inverse-sine denominator would therefore be incorrect.

Step 2: Integrate and compare the constants The required antiderivative is

$$\int \frac{du}{\sqrt{9-u^2}} = \sin^{-1}\left(\frac{u}{3}\right) + C = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + C.$$

Comparing with the given form gives $a = 1$ and $b = 3$. A direct derivative check yields

$$\frac{d}{dx}\sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) = \frac{\cos x - \sin x}{\sqrt{9-(\sin x+\cos x)^2}}.$$

This equals the original integrand. Options (A) and (C) reverse the sign of this derivative, while (D) has an incorrect radical and scale. The argument has magnitude at most $\sqrt{2}/3 < 1$, so the real inverse sine is defined for every real x.

Step 3: Conclusion The ordered pair is $\boxed{(a, b) = (1, 3)}$. Hence, the correct option is (B).

Quick Tip to Solve: When the numerator is the derivative of $\sin x + \cos x$, square that expression to rewrite any occurrence of $\sin 2x$ before integrating.

2. If f : R $\to$ R is a function defined by f(x)= [x - 1] $\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is : [4 marks]

(A) continuous for every real x

(B) discontinuous at all integral values of x except at x = 1

(C) discontinuous only at x = 1

(D) continuous only at x = 1

Correct Answer: (A) continuous for every real x

Solution:

Concept:

- The greatest-integer function $\lfloor x-1 \rfloor$ is constant between consecutive integers and can jump only at integers.
- A function is continuous at $x = n$ when its two one-sided limits equal its value there; a vanishing continuous factor can remove a finite jump.

Step 1: Simplify the trigonometric factor Interpret the printed cosine as $\cos((x-\frac{1}{2})\pi)$. The identity $\cos(\theta - \pi/2) = \sin\theta$ gives

$$f(x) = \lfloor x-1 \rfloor \sin(\pi x).$$

If x is not an integer, a small neighbourhood of x contains no jump of the greatest-integer factor. On that neighbourhood the factor is a fixed integer, so its product with the continuous sine function is continuous. It remains to inspect every integer, rather than judging continuity from the floor function alone.

Step 2: Check both sides at an arbitrary integer Let n be any integer. Immediately to its left, $\lfloor x-1 \rfloor = n-2$, and immediately to its right it equals $n-1$. Consequently,

$$\lim_{x\to n^-} f(x) = (n-2)\sin(n\pi) = 0, \qquad \lim_{x\to n^+} f(x) = (n-1)\sin(n\pi) = 0.$$

At the point itself,

$$f(n) = \lfloor n-1 \rfloor \sin(n\pi) = (n-1)\cdot 0 = 0.$$

Hence the jump of the greatest-integer factor contributes no discontinuity to the product. This argument includes $n = 1$ and all negative integers as well. Options (B), (C) and (D) each assert a discontinuity somewhere; the preceding limits rule out all three.

Step 3: Conclusion The function is continuous for every real x. Hence, the correct option is (A).

Quick Tip to Solve: For a floor-function product, inspect only its possible jump points and check whether the other factor vanishes at each one.

3. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt}$ = 0.5P – 450. If P(0) = 850, then the time at which population becomes zero is : [4 marks]

(A) $\log_e 18$

(B) $\frac{1}{2}\log_e 18$

(C) $2\log_e 18$

(D) $\log_e 9$

Correct Answer: (C) $2\log_e 18$

Solution:

Concept:

- The equation $P\prime = 0.5P - 450$ has equilibrium population 900, since its right-hand side vanishes there.
- Shifting by the equilibrium reduces the equation to exponential growth: if $Q = P - 900$, then $Q\prime = Q/2$.

Step 1: Solve using the equilibrium shift Set $Q(t) = P(t) - 900$. Because the derivative of a constant is zero,

$$\frac{dQ}{dt} = \frac{1}{2}Q.$$

Separating variables gives $dQ/Q = dt/2$, so $Q = Ce^{t/2}$. Therefore

$$P(t) = 900 + Ce^{t/2}.$$

Use the initial population $P(0) = 850$:

$$850 = 900 + C \quad \Longrightarrow \quad C = -50.$$

Thus the population law satisfying both the differential equation and the initial condition is $P(t) = 900 - 50e^{t/2}$. The negative coefficient is essential: the initial population lies below equilibrium and initially decreases.

Step 2: Find and verify the extinction time To find the first time at which the population is zero, solve

$$900 - 50e^{t/2} = 0 \quad \Longrightarrow \quad e^{t/2} = 18.$$

Taking natural logarithms,

$$\frac{t}{2} = \ln 18 \quad \Longrightarrow \quad t = 2\ln 18.$$

For $t \geq 0$, $P\prime(t) = -25e^{t/2} < 0$, so this is the unique zero reached from the initial population. The model is used only until this extinction time, since negative values afterward cannot represent a population. Options (A) and (B) omit the factor arising from the coefficient $1/2$; option (D) also uses the wrong exponential ratio. Substituting $t = 2\ln 18$ gives $900 - 50 \cdot 18 = 0$, as required.

Step 3: Conclusion The population becomes zero at $\boxed{t = 2\ln 18}$, in the time units used in the model. Hence, the correct option is (C).

Quick Tip to Solve: For $P\prime = kP - c$, first shift by the equilibrium c/k; this avoids sign errors when applying the initial condition.

4. Let f : R $\to$ R be defined as f (x) = 2x – 1 and g : R - {1} $\to$ R be defined as g(x) = $\frac{x-\frac{1}{2}}{x-1}$. Then the composition function f(g(x)) is : [4 marks]

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Correct Answer: (A) one-one but not onto

Solution:

Concept:

- Compute the composition on the domain of the inner function: here $x \in \mathbb{R} \setminus \{1\}$, and the stated codomain is $\mathbb{R}$.
- One-one means distinct inputs have distinct outputs. Onto means that every element of the stated codomain is attained.

Step 1: Simplify the composition Let $h = f \circ g$. Substitution into $f(u) = 2u - 1$ gives

$$h(x) = 2\frac{x - \frac{1}{2}}{x - 1} - 1 = \frac{2x - 1 - (x - 1)}{x - 1} = \frac{x}{x - 1}, \qquad x \neq 1.$$

Equivalently, $h(x) = 1 + 1/(x - 1)$. The denominator restriction remains part of the domain after simplification; it must not be forgotten when deciding the mapping properties.

Step 2: Prove the one-one property Suppose $h(u) = h(v)$ for two allowed inputs $u, v \neq 1$. Since both denominators are nonzero, cross-multiplication is valid:

$$\frac{u}{u - 1} = \frac{v}{v - 1} \quad \Longrightarrow \quad u(v - 1) = v(u - 1).$$

Cancelling uv gives $-u = -v$, hence $u = v$. Thus h is one-one, ruling out options (B) and (D).

Step 3: Determine which outputs are possible For an output y, the equation $y = x/(x - 1)$ becomes $(y - 1)x = y$. If $y \neq 1$, it has the allowed solution

$$x = \frac{y}{y - 1},$$

which cannot equal 1. If $y = 1$, the original equation would require $x = x - 1$, which is impossible. Therefore the range is exactly $\mathbb{R} \setminus \{1\}$. Because the codomain includes 1, the composition is not onto $\mathbb{R}$, so option (C) is false.

Step 4: Conclusion The composition is one-one but not onto $\mathbb{R}$. Hence, the correct option is (A).

Quick Tip to Solve: To test whether a rational map is onto, solve $y = h(x)$ for x and look for excluded output values, keeping the codomain in view.

5. The function

f(x) = $\frac{4x^3-3x^2}{6} - 2\sin x + (2x-1)\cos x$ **:** [4 marks]

(A) increases in $\left(-\infty, \frac{1}{2}\right]$

(B) decreases in $\left(-\infty, \frac{1}{2}\right]$

(C) increases in $\left[\frac{1}{2}, \infty\right)$

(D) decreases in $\left[\frac{1}{2}, \infty\right)$

Correct Answer: (C) increases in $\left[\frac{1}{2}, \infty\right)$

Solution:

Concept:

- A differentiable function is increasing where its derivative is positive and decreasing where its derivative is negative.
- The auxiliary function $h(x) = x - \sin x$ has derivative $1 - \cos x \geq 0$, with $h(0) = 0$; thus it has the sign of x away from zero.

Step 1: Differentiate and factor Apply the product rule to $(2x-1)\cos x$. The cosine terms from the two trigonometric summands cancel:

$$\begin{aligned} f\prime(x) &= 2x^2 - x - 2\cos x + 2\cos x - (2x-1)\sin x \\ &= 2x^2 - x - (2x-1)\sin x \\ &= (2x-1)(x - \sin x). \end{aligned}$$

Factoring the derivative is more useful than separately estimating its original polynomial and trigonometric terms.

Step 2: Establish the signs of both factors Since $x - \sin x = \int_0^x (1 - \cos t)\, dt$, it is strictly positive for $x > 0$ and strictly negative for $x < 0$. The integrand is nonnegative and cannot vanish identically on any nontrivial interval. Thus $f\prime(x) > 0$ for $x < 0$, $f\prime(x) < 0$ for $0 < x < 1/2$, and $f\prime(x) > 0$ for $x > 1/2$. At 0 and $1/2$ the derivative is zero.

Step 3: Compare the proposed intervals On $[1/2, \infty)$, the derivative is positive in the interior, so the function increases throughout the interval; a zero derivative at its endpoint causes no difficulty. Hence (C) holds and (D) fails. On $(-\infty, 1/2]$, the function first increases up to 0 and then decreases from 0 to $1/2$. Therefore it neither increases nor decreases on that whole interval. This rules out both (A) and (B), without incorrectly assuming that $x - \sin x$ is positive for negative x.

Step 4: Conclusion The function increases on $\boxed{[1/2, \infty)}$. Hence, the correct option is (C).

Quick Tip to Solve: After factoring a derivative, make a sign chart for every factor; the inequality $\sin x \leq x$ applies for $x \geq 0$, not for all real x.

6. The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$ is : [4 marks]

(A) $2^{13} - 13$

(B) $2^{16} - 1$

(C) 2^{14}

(D) $2^{13} - 14$

Correct Answer: (D) $2^{13} - 14$

Solution:

Concept:

- Differentiating $(1+z)^n = \sum_{r=0}^{n} \binom{n}{r} z^r$ produces weighted binomial sums involving $r\binom{n}{r}$.
- The sum of all odd-index binomial coefficients of order $n \geq 1$ is 2^{n-1}, obtained by subtracting the expansions at 1 and -1.

Step 1: Evaluate the alternating weighted sum Write the first part as

$$S_1 = \sum_{r=1}^{15} (-1)^r r \binom{15}{r}.$$

Differentiate the binomial expansion and multiply by z:

$$15z(1+z)^{14} = \sum_{r=1}^{15} r \binom{15}{r} z^r.$$

Substituting $z = -1$ gives

$$S_1 = 15(-1)(1-1)^{14} = 0.$$

The alternating signs begin with a negative term, exactly as in the printed expression. The weighted terms are not the ordinary alternating binomial sum, so the differentiation step is needed.

Step 2: Evaluate the truncated odd-index sum For order 14, subtraction of the two binomial expansions gives

$$\sum_{\substack{r=1 \\ r \text{ odd}}}^{13} \binom{14}{r} = \frac{(1+1)^{14} - (1-1)^{14}}{2} = 2^{13}.$$

The question includes the odd indices only through 11. Therefore the sole missing odd coefficient is $\binom{14}{13} = 14$, and

$$S_2 = 2^{13} - 14.$$

Combining the independent parts gives $S_1 + S_2 = 0 + 2^{13} - 14$. Subtracting 13 instead of 14 would confuse the index with the value of the coefficient, leading to option (A). Options (B) and (C) use the wrong complete binomial total and do not account for the omitted term.

Step 3: Conclusion The required value is $\boxed{2^{13} - 14}$. Hence, the correct option is (D).

Quick Tip to Solve: For a truncated parity sum, evaluate the complete odd or even binomial sum first, then subtract the actual omitted coefficients rather than their indices.

7. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is : [4 marks]

(A) 1050

(B) 575

(C) 560

(D) 1625

Correct Answer: (D) 1625

Solution:

Concept:

- A committee is an unordered selection, so choosing r people from n gives $\binom{n}{r}$ possibilities.
- For a fixed composition, multiply the independent choices from the two groups; add the counts for mutually exclusive compositions.

Step 1: List every possible committee composition Let i be the number of Indians in the committee. The required number of foreigners is $2i$, and the condition of at least two Indians gives $i \geq 2$. Availability imposes $i \leq 6$ and $2i \leq 8$, so $i \leq 4$. Thus the only allowable pairs (Indians, foreigners) are

$$(2, 4), \qquad (3, 6), \qquad (4, 8).$$

These cases exhaust the possibilities. In particular, the committee size is not fixed in advance: it can be 6, 9 or 12. Restricting attention to just one size would miss valid committees.

Step 2: Count and add the disjoint cases For each pair choose the Indians and foreigners independently:

$$\begin{aligned} N &= \binom{6}{2}\binom{8}{4} + \binom{6}{3}\binom{8}{6} + \binom{6}{4}\binom{8}{8} \\ &= 15 \cdot 70 + 20 \cdot 28 + 15 \cdot 1 \\ &= 1050 + 560 + 15 \\ &= 1625. \end{aligned}$$

No ordering factor is needed, since a scientific committee is a selection rather than a sequence of appointed positions. The three cases cannot overlap because their numbers of Indians differ. Option (A) counts only the first case, and option (C) counts only the second. Option (B) gives $560 + 15 = 575$, omitting the valid committees with two Indians and four foreigners. Adding every allowed composition selects (D).

Step 3: Conclusion The committee can be formed in $\boxed{1625}$ ways. Hence, the correct option is (D).

Quick Tip to Solve: When one group must be twice another, let the smaller count be a variable, use availability to bound it, and sum products of combinations over all allowed values.

8. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then, which of these stones is / are on the path of the man? [4 marks]

(A) A only

(B) All the three

(C) C only

(D) B only

Correct Answer: (D) B only

Solution:

Concept:

- The intercept form of a line is $x/a+y/b=1$, where a, b are its nonzero intercepts on the coordinate axes.
- A point lies on a line precisely when its coordinates satisfy the equation. The given arithmetic mean fixes $1/a+1/b$, even though it need not fix the individual intercepts.

Step 1: Translate the mean condition The two numbers whose arithmetic mean is given are the reciprocals of the intercepts, not the intercepts themselves. Therefore

$$\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{1}{4}, \qquad \frac{1}{a}+\frac{1}{b}=\frac{1}{2}.$$

We do not need to determine either intercept separately: all three candidate points have equal coordinates, so only this sum is relevant.

Step 2: Test the three stones At a point (s, s), the left side of the line equation becomes

$$\frac{s}{a}+\frac{s}{b}=s\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{s}{2}.$$

For A, B and C the values of this expression are respectively $1/2, 1, 2$. Only the value 1 satisfies the line equation. In particular, A and C do not lie on the path, so neither the option naming A only, the option naming C only, nor the option naming all three is possible.

Step 3: Conclusion Only stone B at $(2, 2)$ lies on the path. Hence, the correct option is (D).

Quick Tip to Solve: For candidate points on $y=x$, substitute (s, s) into intercept form and use the sum of reciprocal intercepts directly.

9. Let p and q be two positive numbers such that p + q = 2 and p^4+q^4 = 272. Then p and q are roots of the equation : [4 marks]

(A) $x^2 - 2x + 8 = 0$

(B) $x^2 - 2x + 136=0$

(C) $x^2 - 2x + 16 = 0$

(D) $x^2 - 2x + 2 = 0$

Correct Answer: As printed, the conditions are inconsistent: no such positive numbers exist, and none of the four options is a valid answer.

Solution:

Concept:

- For real roots p, q, their sum and product determine the monic quadratic $x^2 - (p+q)x + pq = 0$.
- The requirement that p, q are positive must be checked before selecting a polynomial from algebraic identities.

Step 1: Check whether the hypotheses can hold If $p, q > 0$ and $p + q = 2$, then both numbers are less than 2. More sharply,

$$p^4 + q^4 < (p+q)^4 = 16,$$

because the omitted mixed terms $4p^3q + 6p^2q^2 + 4pq^3$ are strictly positive. The printed condition $p^4 + q^4 = 272$ therefore contradicts positivity. No pair of positive real numbers satisfies the question as stated.

Step 2: Check the algebraic products and the listed options Writing $r = pq$, the sum-of-fourth-powers identity gives

$$p^4 + q^4 = ((p+q)^2 - 2r)^2 - 2r^2 = 16 - 16r + 2r^2.$$

Consequently

$$r^2 - 8r - 128 = 0, \qquad (r-16)(r+8) = 0.$$

The product -8 would correspond to two real roots of opposite signs, while product 16 with sum 2 gives a negative quadratic discriminant. Neither possibility describes two positive numbers. Indeed, each listed polynomial has product greater than 1, whereas positive numbers of sum 2 satisfy $pq \leq 1$. The presence of the algebraic product 16 in option C does not make that option valid under the printed hypotheses.

Step 3: Conclusion As printed, the conditions are inconsistent: no such positive numbers exist, and none of the four options is a valid answer.

Quick Tip to Solve: Always apply positivity and the bound $pq \leq (p+q)^2/4$ before accepting a product obtained from a higher-power identity.

10. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is : [4 marks]

(A) $12\pi - 3\sqrt{3}$

(B) $24\pi + 3\sqrt{3}$

(C) $24\pi - 3\sqrt{3}$

(D) $12\pi + 3\sqrt{3}$

Correct Answer: (C) $24\pi - 3\sqrt{3}$

Solution:

Concept:

- The circle has radius 6 and area 36π. The inside of the right-opening parabola is $x \geq y^2/9$.
- Subtract the common region from the disk. Horizontal strips and symmetry about the x-axis avoid splitting the circular boundary into several pieces.

Step 1: Find the intersection ordinates Substituting $x = y^2/9$ into the circle, or substituting $y^2 = 9x$, gives

$$x^2 + 9x - 36 = 0, \qquad (x-3)(x+12) = 0.$$

The root -12 is inadmissible because $x = y^2/9 \geq 0$. Thus the intersections are $(3, \pm 3\sqrt{3})$. For $0 \leq y \leq 3\sqrt{3}$, the common region extends horizontally from $x = y^2/9$ to $x = \sqrt{36 - y^2}$.

Step 2: Integrate the common region and subtract Its area is

$$A_{\text{common}} = 2\int_0^{3\sqrt{3}} \left(\sqrt{36 - y^2} - \frac{y^2}{9}\right) dy.$$

An antiderivative of the first term is $\frac{y}{2}\sqrt{36 - y^2} + 18\sin^{-1}(y/6)$. At the upper limit it equals $9\sqrt{3}/2 + 6\pi$; the integral of $y^2/9$ equals $3\sqrt{3}$. Hence

$$A_{\text{common}} = 12\pi + 3\sqrt{3}, \qquad A_{\text{outside}} = 36\pi - A_{\text{common}} = 24\pi - 3\sqrt{3}.$$

The sign is fixed by subtraction: adding the common region or retaining its area would select a different option.

Step 3: Conclusion The required area is $24\pi - 3\sqrt{3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a right-opening parabola inside a disk, horizontal strips give right boundary minus parabola; subtract that overlap from the full disk.

11. If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is : [4 marks]

(A) $\sqrt{3}$

(B) $\frac{3}{2}$

(C) $2\sqrt{3}$

(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- For $0 < x < \pi/2$, the geometric series with first term and ratio $\cos^2 x$ converges.
- The sum is $\cos^2 x/(1 - \cos^2 x) = \cot^2 x$, and $e^{u \log_e 2} = 2^u$.

Step 1: Sum the series and identify the admissible root Let the exponential expression be t. Since $0 < \cos^2 x < 1$,

$$\cos^2 x + \cos^4 x + \cdots = \cot^2 x, \qquad t = 2^{\cot^2 x}.$$

The equation factors as

$$(t - 1)(t - 8) = 0.$$

Here $\cot^2 x > 0$ throughout the given open interval, so $t > 1$. Thus $t = 1$ is impossible; it would require the excluded endpoint $x = \pi/2$. We must use $t = 8$.

Step 2: Evaluate the requested ratio From $2^{\cot^2 x} = 8 = 2^3$, injectivity of the exponential yields $\cot^2 x = 3$. Because x is in the first quadrant, $\cot x = \sqrt{3}$, rather than the negative square root. Dividing numerator and denominator of the desired expression by the positive number $\sin x$ gives

$$\frac{2\sin x}{\sin x + \sqrt{3}\cos x} = \frac{2}{1 + \sqrt{3}\cot x} = \frac{2}{1+3} = \frac{1}{2}.$$

This direct simplification excludes the other three numerical options without requiring separate angle calculations.

Step 3: Conclusion The expression equals $1/2$. Hence, the correct option is (D).

Quick Tip to Solve: Convert a geometric series in even powers of cosine to $\cot^2 x$, then use the interval to discard inadmissible exponential roots.

12. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is : [4 marks]

(A) x = 0

(B) x = - $\frac{a}{2}$

(C) x = a

(D) x = $\frac{a}{2}$

Correct Answer: (A) x = 0

Solution:

Concept:

- The focus of $y^2 = 4ax$ is $(a, 0)$. The midpoint transformation expresses the moving endpoint in terms of the midpoint.
- A horizontal parabola $(Y - k)^2 = 4A(X - h)$ has vertex (h, k) and directrix $X = h - A$.

Step 1: Express the endpoint through the midpoint Let (X, Y) be the midpoint and (x, y) the moving point on the original parabola. The midpoint relations are

$$X = \frac{x + a}{2}, \qquad Y = \frac{y}{2},$$

and hence $x = 2X - a$, $y = 2Y$. Substituting these relations into the original curve eliminates the moving endpoint:

$$(2Y)^2 = 4a(2X - a), \qquad Y^2 = 2aX - a^2.$$

This is the entire midpoint locus because each point on it reverses to a point on the original parabola by the same linear transformation.

Step 2: Read the directrix from standard form Complete the horizontal shift:

$$Y^2 = 2a\left(X - \frac{a}{2}\right) = 4\left(\frac{a}{2}\right)\left(X - \frac{a}{2}\right).$$

The new vertex is $(a/2, 0)$, and its focal parameter is $A = a/2$. Therefore the directrix is

$$X = \frac{a}{2} - \frac{a}{2} = 0.$$

The line $X = a/2$ is the vertex line, not the directrix; $X = a$ passes through the new focus. The alternative $X = -a/2$ would omit the horizontal translation. The usual nondegenerate-parabola assumption is $a \neq 0$.

Step 3: Conclusion The directrix of the midpoint locus is $x = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Under a midpoint map, substitute the inverse coordinate relations first; then identify both the vertex shift and the new focal parameter.

13. The system of linear equations

3x - 2y - kz = 10

2x - 4y - 2z = 6

x+2y - z = 5m

is inconsistent if : [4 marks]

(A) k $\neq$ 3, m $\in$ **R**

(B) k = 3, m $\neq \frac{4}{5}$

(C) k = 3, m = $\frac{4}{5}$

(D) k $\neq$ 3, m $\neq \frac{4}{5}$

Correct Answer: (B) k = 3, m $\neq \frac{4}{5}$

Solution:

Concept:

- A linear system has a unique solution whenever its coefficient determinant is nonzero.
- When the rows become dependent, consistency also requires the same dependence among the right-hand sides; a contradiction makes the system inconsistent.

Step 1: Locate the singular coefficient matrix The coefficient matrix is

$$A = \begin{pmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{pmatrix}.$$

Expanding along the first row yields

$$\det A = 3(8) + 2(0) - k(8) = 8(3 - k).$$

Thus for every $k \neq 3$ the determinant is nonzero and there is exactly one solution for every real m. Inconsistency can therefore occur only at $k = 3$, ruling out both options with $k \neq 3$.

Step 2: Check the compatibility of the constants At $k = 3$, subtract the second equation from the first. Their left sides give

$$(3x - 2y - 3z) - (2x - 4y - 2z) = x + 2y - z,$$

while their right sides give $10 - 6 = 4$. The third equation has precisely this left side but right side $5m$. Consequently it is compatible if and only if $5m = 4$. The first two coefficient rows are independent, so this equality really does give a consistent system with one free variable; it is not another inconsistent case. If $5m \neq 4$, the same quantity would have two unequal values and no solution is possible.

Step 3: Conclusion The system is inconsistent exactly when $k = 3$ and $m \neq 4/5$. Hence, the correct option is (B).

Quick Tip to Solve: After setting the determinant to zero, seek a simple row relation and apply the same relation to the constants to test consistency.

14. $\lim\limits_{x\to 0} \dfrac{\int_0^{x^2} (\sin\sqrt{t})\,dt}{x^3}$ **is equal to :** [4 marks]

(A) $\frac{1}{15}$

(B) 0

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

Correct Answer: The two-sided limit as printed does not exist. The value $2/3$, option (C), is the right-hand limit only; no listed option gives the stated two-sided limit.

Solution:

Concept:

- For nonnegative t close to zero, $\sin\sqrt{t} = \sqrt{t} + O(t^{3/2})$.
- The upper bound x^2 is nonnegative on both sides of zero, and $(x^2)^{3/2} = |x|^3$, not x^3.

Step 1: Find the leading term of the integral Use the small-angle expansion and integrate over the nonnegative interval $[0, x^2]$:

$$\int_0^{x^2} \sin\sqrt{t}\,dt = \int_0^{x^2} \left(\sqrt{t} + O(t^{3/2})\right) dt = \frac{2}{3}|x|^3 + O(|x|^5).$$

The absolute value is essential because the square root is the nonnegative real root. An equivalent exact check uses $u = \sqrt{t}$, giving $2\int_0^{|x|} u\sin u\,du = 2(\sin|x| - |x|\cos|x|)$. This confirms both the sign and the leading coefficient.

Step 2: Compare the one-sided limits Dividing by the printed denominator, for $x \neq 0$, gives

$$\frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3} = \frac{2}{3}\frac{|x|^3}{x^3} + O(x^2).$$

Therefore the right-hand limit is $2/3$, whereas the left-hand limit is $-2/3$. These are different, so the two-sided real limit written in the question does not exist. Option C would be correct for $x \to 0^+$, but that restriction is not present in the printed text. Neither zero nor either of the other positive values can be the two-sided limit. Replacing $\sqrt{x^2}$ by x in the derivative-based working would wrongly conceal this distinction.

Step 3: Conclusion The two-sided limit as printed does not exist. The value $2/3$, option (C), is the right-hand limit only; no listed option gives the stated two-sided limit.

Quick Tip to Solve: Whenever an integral ends at x^2 and contains a square root, keep $\sqrt{x^2} = |x|$ and check both approaches to zero.

15. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is : [4 marks]

(A) 0

(B) $2t^3$

(C) $-2t^3$

(D) $-t^3$

Correct Answer: (C) $-2t^3$

Solution:

Concept:

- The slope of the tangent to $y = x^3$ is $dy/dx = 3x^2$. A second intersection must satisfy both the tangent equation and the curve.
- If a point divides PQ internally in the ratio $PR : RQ = m : n$, its ordinate is $(ny_P + my_Q)/(m+n)$.

Step 1: Write the tangent at P At $P(t, t^3)$, the slope is $3t^2$. Hence the tangent is

$$y - t^3 = 3t^2(x - t), \qquad y = 3t^2x - 2t^3.$$

The parameter is kept symbolic, since the question asks for an ordinate in terms of t.

Step 2: Find the other intersection Substitute the tangent equation into the cubic:

$$x^3 - 3t^2x + 2t^3 = 0.$$

The point of tangency gives a repeated root. Factorising,

$$(x - t)^2(x + 2t) = 0.$$

Thus the other point is $Q(-2t, -8t^3)$. The repeated root $x = t$ corresponds to P, not to the second intersection.

Step 3: Apply the internal division formula For $PR : RQ = 1 : 2$, the ordinate is

$$y_R = \frac{2t^3 + 1(-8t^3)}{1 + 2} = -2t^3.$$

The weight of P is 2 and that of Q is 1; reversing these weights would give the wrong point. The ordinate is neither zero nor $2t^3$ nor $-t^3$, so the other listed expressions do not satisfy this section formula. The wording presumes a distinct second intersection; at $t = 0$ the intersections coalesce, and the resulting formula still has the limiting value zero.

Step 4: Conclusion The required ordinate is $\boxed{-2t^3}$. Hence, the correct option is (C).

Quick Tip to Solve: For a cubic tangent-intersection problem, factor out the double root at the point of contact before applying the section formula.

16. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is : [4 marks]

(A) 30

(B) 25

(C) $20\sqrt{3}$

(D) $25\sqrt{3}$

Correct Answer: (D) $25\sqrt{3}$

Solution:

Concept:

- In a right triangle, $\tan\theta$ is the vertical height divided by the horizontal distance.
- Complementary acute angles satisfy $\tan\theta\tan(90^\circ-\theta)=1$.

Step 1: Express the two tangents Let the shorter pole have height $h>0$ metres. The taller pole then has height $3h$. The observer stands at the midpoint between their feet, so the horizontal distance to each foot is $150/2=75$ metres. If the elevation angle of the shorter top is θ, that of the taller top is $90^\circ-\theta$. Therefore,

$$\tan\theta=\frac{h}{75},\qquad \tan(90^\circ-\theta)=\frac{3h}{75}.$$

Step 2: Use the complementary-angle condition Multiply the two equations rather than attempting to find either angle separately:

$$\frac{h}{75}\frac{3h}{75}=1.$$

It follows that

$$3h^2=75^2=5625,\qquad h^2=1875.$$

Since a height is positive,

$$h=\sqrt{1875}=25\sqrt{3}\text{ m}.$$

Step 3: Check the value against the geometry With this value, the two tangents are $1/\sqrt{3}$ and $\sqrt{3}$. They correspond to 30° and 60°, confirming that the angles are complementary and that the taller pole is three times as high. For any proposed shorter height, its squared value must satisfy $3h^2=5625$; 30, 25 and $20\sqrt{3}$ all fail this equation. Thus only the fourth option meets both the distance and height-ratio conditions.

Step 4: Conclusion The shorter pole has height $\boxed{25\sqrt{3}\text{ m}}$. Hence, the correct option is (D).

Quick Tip to Solve: When two elevation angles from a common horizontal distance are complementary, the product of the two heights equals the square of that distance.

17. The equation of the plane passing through the point (1, 2, -3) and perpendicular to the planes 3x + y - 2z = 5 and 2x - 5y - z = 7, is : [4 marks]

(A) 6x - 5y + 2z + 10 =0

(B) 3x - 10y - 2z + 11 = 0

(C) 6x - 5y - 2z - 2 = 0

(D) 11x + y + 17z + 38 = 0

Correct Answer: (D) 11x + y + 17z + 38 = 0

Solution:

Concept:

- The coefficients of x, y, z in a plane equation form its normal vector.
- Two planes are perpendicular exactly when their normal vectors have zero dot product. A cross product gives a vector perpendicular to two given nonparallel vectors.

Step 1: Find a normal to the required plane The given planes have normals

$$\mathbf{n}_1 = (3, 1, -2), \qquad \mathbf{n}_2 = (2, -5, -1).$$

The required plane must be perpendicular to both, so its normal must be perpendicular to both of these normals. Compute

$$\mathbf{n}_1 \times \mathbf{n}_2 = (-11, -1, -17).$$

Changing all three signs gives the equally valid normal $(11, 1, 17)$.

Step 2: Use the specified point The plane through $(1, 2, -3)$ with this normal has point-normal equation

$$11(x - 1) + (y - 2) + 17(z + 3) = 0.$$

On collecting constants,

$$11x + y + 17z + 38 = 0.$$

This step fixes the constant term; orthogonality alone determines only the orientation, not which parallel plane is required.

Step 3: Verify all the conditions The dot products with the original normals are

$$(11, 1, 17) \cdot (3, 1, -2) = 33 + 1 - 34 = 0,$$

$$(11, 1, 17) \cdot (2, -5, -1) = 22 - 5 - 17 = 0.$$

Also, substituting the specified point gives $11+2-51+38 = 0$. Thus both perpendicularity requirements and the incidence condition hold. The normals in the other options are not proportional to $(11, 1, 17)$; since the given normals are independent, none can define a plane perpendicular to both.

Step 4: Conclusion The plane is $\boxed{11x + y + 17z + 38 = 0}$. Hence, the correct option is (D).

Quick Tip to Solve: For a plane perpendicular to two given planes, cross their normals and then use the required point to determine the constant.

18. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 is : [4 marks]

(A) $19\sqrt{2}$

(B) $2\sqrt{19}$

(C) 38

(D) $\sqrt{38}$

Correct Answer: (D) $\sqrt{38}$

Solution:

Concept:

- A symmetric line equation can be parametrised by setting all three ratios equal to a single parameter.
- The distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Step 1: Parametrise the line Set the common ratio equal to λ. Then

$$x = 3 + \lambda, \qquad y = 4 + 2\lambda, \qquad z = 5 + 2\lambda.$$

These coordinates describe every point of the line. In particular, the direction vector is $(1, 2, 2)$, and $(3, 4, 5)$ is a point on it.

Step 2: Locate its intersection with the plane Substitute these coordinates into $x + y + z = 17$:

$$(3 + \lambda) + (4 + 2\lambda) + (5 + 2\lambda) = 17.$$

Hence $12 + 5\lambda = 17$, giving $\lambda = 1$. Therefore the intersection point is $S = (4, 6, 7)$. The coefficient 5 is nonzero, so this is a unique intersection. Its coordinates sum to 17, which checks the plane equation.

Step 3: Measure the distance to the given point The question asks for the distance from $A = (1, 1, 9)$ to S, rather than the perpendicular distance from A to the plane. The coordinate differences are $3, 5, -2$. Thus

$$AS = \sqrt{3^2 + 5^2 + (-2)^2} = \sqrt{9 + 25 + 4} = \sqrt{38}.$$

The value 38 is the squared distance and must not be reported as the distance. The other radical choices have different squares, so they do not agree with the coordinate-distance calculation.

Step 4: Conclusion The required distance is $\boxed{\sqrt{38}}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the actual line-plane intersection first; a point-to-plane distance formula answers a different question here.

19. The statement among the following that is a tautology is : [4 marks]

(A) $B \rightarrow [A \wedge (A \rightarrow B)]$

(B) $[A \wedge (A \rightarrow B)] \rightarrow B$

(C) $[A \wedge (A \vee B)]$

(D) $[A \vee (A \wedge B)]$

Correct Answer: (B) $[A \wedge (A \rightarrow B)] \rightarrow B$

Solution:

Concept:

- A tautology is true for every possible assignment of truth values to its variables.
- The implication $X \rightarrow Y$ is equivalent to $\neg X \vee Y$. It is false only when X is true and Y is false.

Step 1: Simplify the antecedent in option B Use $A \rightarrow B \equiv \neg A \vee B$ and distribute:

$$A \wedge (A \rightarrow B) \equiv A \wedge (\neg A \vee B)$$

$$\equiv (A \wedge \neg A) \vee (A \wedge B) \equiv A \wedge B.$$

Thus option B is equivalent to $(A \wedge B) \rightarrow B$.

Step 2: Show that option B is always true If $A \wedge B$ is true, B must be true, so the implication is true. If $A \wedge B$ is false, the implication is true by the truth rule for implication. Equivalently,

$$\neg(A \wedge B) \vee B \equiv \neg A \vee \neg B \vee B \equiv \text{True}.$$

This covers all four assignments of A and B, rather than just an example.

Step 3: Reject each of the remaining statements For option A, choose A false and B true. The consequent $A \wedge (A \rightarrow B)$ is false while the antecedent B is true, so the implication is false. Option C simplifies by absorption to $A \wedge (A \vee B) \equiv A$, and option D similarly becomes $A \vee (A \wedge B) \equiv A$. Both are false whenever A is false. Consequently none of A, C or D is true for every assignment, whereas B is.

Step 4: Conclusion The tautology is $\boxed{[A \wedge (A \rightarrow B)] \rightarrow B}$. Hence, the correct option is (B).

Quick Tip to Solve: Recognise the modus ponens form: if A holds and A implies B, then B must hold; use counterexamples to eliminate other choices.

20. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is : [4 marks]

(A) $\frac{5}{36}$
(B) $\frac{3}{16}$
(C) $\frac{1}{2}$
(D) $\frac{1}{32}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept:

- For a fair die, the probabilities of an odd result and an even result are both $1/2$. Independent rolls therefore give a binomial count.
- For n rolls, exactly r odd results have probability $\binom{n}{r}2^{-n}$; exactly r even results have the same formula.

Step 1: Determine the number of rolls Let n be the number of independent rolls. The stated equality is

$$\binom{n}{2}\left(\frac{1}{2}\right)^n = \binom{n}{3}\left(\frac{1}{2}\right)^n.$$

For the meaningful situation in which both counts are possible, $n \geq 3$. Cancel the common positive factor and use

$$\frac{\binom{n}{3}}{\binom{n}{2}} = \frac{n-2}{3} = 1.$$

Thus $n = 5$.

Step 2: Add the probabilities for odd counts Let X denote the number of odd outcomes in these five rolls. An odd number of odd outcomes means $X = 1, 3$ or 5, which are disjoint events. Hence

$$P(X \text{ odd}) = \frac{\binom{5}{1} + \binom{5}{3} + \binom{5}{5}}{2^5}$$

$$= \frac{5 + 10 + 1}{32} = \frac{16}{32} = \frac{1}{2}.$$

The count refers to how many rolls show odd faces, not to the numerical sum of all the faces.

Step 3: Check using symmetry Every sequence of five odd/even results has probability $1/32$. Flipping the parity of the first roll pairs each sequence with an even count with exactly one sequence having an odd count. Thus precisely half the total probability belongs to odd counts, confirming the direct sum. The alternatives $5/36$, $3/16$, and $1/32$ are not this total; in particular $1/32$ accounts only for all five outcomes being odd.

Step 4: Conclusion The required probability is $\boxed{1/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For independent fair odd/even trials, flipping one fixed trial pairs odd-count and even-count sequences, so each class has probability one-half.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let P = $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$**, where** $\alpha \in$ **R. Suppose Q = [** q_{ij}**] is a matrix satisfying PQ =** kI_3 **for some non-zero k** $\in$ **R.**

If $q_{23} = -\frac{k}{8}$ **and** $|Q| = \frac{k^2}{2}$**, then** $a^2 + k^2$ **is equal to _____.** [4 marks]

Solution:

Concept:

- From $PQ = kI_3$ with $k \neq 0$, both matrices are invertible and $Q = kP^{-1}$.
- For a 3×3 matrix, $\det(kP^{-1}) = k^3/\det P$. The inverse entry $(P^{-1})_{ij}$ uses cofactor C_{ji} divided by $\det P$.

Step 1: Compute the relevant determinant and cofactor Expanding the determinant of P along the first row gives

$$|P| = 15\alpha - 3\alpha + 20 = 12\alpha + 20.$$

To obtain entry (2,3) of the inverse, use cofactor (3,2):

$$C_{32} = -\begin{vmatrix} 3 & -2 \\ 2 & \alpha \end{vmatrix} = -(3\alpha + 4).$$

The transpose of the cofactor indices is essential when forming the adjugate.

Step 2: Use the specified entry of Q Since $Q = kP^{-1}$, the supplied condition becomes

$$-\frac{k(3\alpha + 4)}{12\alpha + 20} = -\frac{k}{8}.$$

Cancel the nonzero k and cross-multiply:

$$8(3\alpha + 4) = 12\alpha + 20.$$

Therefore $12\alpha = -12$, or $\alpha = -1$. It follows that $|P| = 8$, consistent with the required invertibility. The determinant denominator has therefore not vanished during the calculation.

Step 3: Use the determinant of Q to find k Now apply the determinant scaling law:

$$|Q| = \frac{k^3}{|P|} = \frac{k^3}{8} = \frac{k^2}{2}.$$

Since $k \neq 0$, division by k^2 gives $k/8 = 1/2$, so $k = 4$. Finally,

$$\alpha^2 + k^2 = (-1)^2 + 4^2 = 1 + 16 = 17.$$

As a consistency check, the inverse entry is $-1/8$ when $\alpha = -1$, and multiplying it by k gives exactly

the stated $q_{23} = -k/8$.

Step 4: Conclusion The requested value is $\boxed{17}$.

Quick Tip to Solve: Transpose the cofactor indices for an inverse entry, and remember that multiplying a three-by-three matrix by k multiplies its determinant by k cubed.

22. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is ..... [4 marks]

Solution:

Concept:

- Put $t = \sin x$. Since $0 < x < \pi/2$, the transformed variable takes every value in $0 < t < 1$.
- The least admissible right-hand side is the minimum of $f(t) = 4/t + 1/(1-t)$, provided equality occurs inside the interval.

Step 1: Separate the minimum from a nonnegative term Using a common denominator gives the exact identity

$$\frac{4}{t} + \frac{1}{1-t} - 9 = \frac{4(1-t) + t - 9t(1-t)}{t(1-t)} = \frac{(3t-2)^2}{t(1-t)}.$$

The denominator is positive throughout the allowed interval, while the squared numerator is nonnegative. Therefore every possible value of the left-hand side is at least 9. This argument treats the open endpoints correctly: neither endpoint needs to be included to establish the bound.

Step 2: Check that equality is attained Equality requires $3t - 2 = 0$, hence $t = 2/3$. This lies strictly between 0 and 1, so $x = \sin^{-1}(2/3)$ lies in the required interval and is an actual solution. Direct substitution gives

$$\frac{4}{2/3} + \frac{1}{1-2/3} = 6 + 3 = 9.$$

Thus the bound is a minimum, rather than merely an unattained infimum. For any smaller value of α, the nonnegative-square identity rules out a solution.

Step 3: Conclusion The minimum value of α is $\boxed{9}$.

Quick Tip to Solve: For expressions of the form $a/t + b/(1-t)$, subtract $(\sqrt{a} + \sqrt{b})^2$ and look for a squared numerator.

23. If the least and the largest real values of a, for which the equation z + α|z – 1| + 2i = 0 (z $\in$ C and i = $\sqrt{-1}$) has a solution, are p and q respectively; then 4(p^2 + q^2) is equal to _____. [4 marks]

Solution:

Concept:

- Write $z = x + iy$, with x, y real. The modulus term is real because α is real.

- The parameter must satisfy the unsquared real equation. Squaring can introduce roots whose signs are incompatible with that equation.

Step 1: Reduce the complex equation to a real range problem The imaginary part of $z+\alpha|z-1|+2i=0$ gives $y=-2$. Its real part is consequently

$$x+\alpha\sqrt{(x-1)^2+4}=0, \qquad \alpha=h(x)=-\frac{x}{\sqrt{(x-1)^2+4}}.$$

The square root is strictly positive for every real x. Thus each real x gives a genuine solution with this value of α, and the exact admissible set is the range of h.

Step 2: Find the range and test the claimed largest value Differentiating gives

$$h'(x)=\frac{x-5}{\left((x-1)^2+4\right)^{3/2}}.$$

Hence h decreases up to $x=5$ and increases thereafter. Its minimum is

$$h(5)=-\frac{5}{\sqrt{20}}=-\frac{\sqrt{5}}{2}.$$

The limiting values are $\lim_{x\to-\infty}h(x)=1$ and $\lim_{x\to\infty}h(x)=-1$. The value 1 is not attained: it would require $x<0$ and $x^2=(x-1)^2+4$, giving $x=5/2>0$, a contradiction. Continuity therefore gives exactly

$$\alpha\in\left[-\frac{\sqrt{5}}{2},1\right).$$

There are admissible positive values arbitrarily close to 1, but none equal to 1. Thus no largest admissible real value exists.

Step 3: Conclusion As printed, $p=-\sqrt{5}/2$, but q does not exist, so $4(p^2+q^2)$ is undefined. The supplied answer key gives 10; that value does not follow from the printed equation.

Quick Tip to Solve: After squaring a modulus equation, retain the sign condition from the original equation; a discriminant alone need not describe the parameter range.

24. $\lim\limits_{n\to\infty}\tan\left\{\sum\limits_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ **is equal to _____.** [4 marks]

Solution:

Concept:

- For positive r, both $\tan^{-1}(r+1)$ and $\tan^{-1}r$ lie in $(0,\pi/2)$. Their positive difference is also in that interval.
- The identity $\tan(A-B)=(\tan A-\tan B)/(1+\tan A\tan B)$ turns each summand into a difference of successive inverse tangents.

Step 1: Identify a difference of consecutive terms Take $A = \tan^{-1}(r+1)$ and $B = \tan^{-1} r$. Then

$$\tan(A-B) = \frac{r+1-r}{1+r(r+1)} = \frac{1}{1+r+r^2}.$$

Since $0 < A - B < \pi/2$, the principal inverse tangent gives the exact identity

$$\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}(r+1) - \tan^{-1} r.$$

The branch check ensures that no multiple of π has been lost.

Step 2: Sum first and then take the limit The adjacent terms cancel in the finite sum:

$$S_n = \sum_{r=1}^{n} \left(\tan^{-1}(r+1) - \tan^{-1} r\right) = \tan^{-1}(n+1) - \frac{\pi}{4}.$$

One can evaluate its tangent before passing to the limit:

$$\tan S_n = \frac{(n+1)-1}{1+(n+1)} = \frac{n}{n+2}.$$

This tends to 1 as n tends to infinity. Equivalently, $S_n \to \pi/4$, where tangent is continuous. Both approaches avoid incorrectly replacing a sum of inverse tangents by the inverse tangent of a sum.

Step 3: Conclusion The required limit is $\boxed{1}$.

Quick Tip to Solve: When the denominator is r^2+r+1, try the tangent difference for the consecutive numbers $r+1$ and r.

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25. Let A = {n ∈ N: n is a 3-digit number}
B = {9k + 2: k ∈ N}
and C = {9k + l: k ∈ N} for some $l(0 < l < 9)$
If the sum of all the elements of the set A ∩ (B ∪ C) is 274 × 400, then l is equal to _____. [4 marks]

Solution:

Concept:

- Three-digit integers in a fixed residue class modulo 9 form an arithmetic progression with common difference 9.
- A set union counts common elements once. For different residues 2 and l modulo 9, the two progressions are disjoint.

Step 1: Sum the elements belonging to the first residue For B, the first three-digit number is $9(11)+2 = 101$, and the last is $9(110)+2 = 992$. There are 100 terms. Hence

$$\sum_{n \in A \cap B} n = \frac{100}{2}(101+992) = 54650.$$

If $l = 2$, then $B = C$, so the union has this same sum, which is not $274 \times 400 = 109600$. Thus $l \neq 2$, and the residue classes may be added without any double counting.

Step 2: Express the second sum and solve for the residue For every integer l with $1 \leq l \leq 8$, the first and last three-digit members of C are $99 + l$ and $990 + l$. They again correspond to $k = 11, \ldots, 110$, so there are 100 terms. Therefore

$$\sum_{n \in A \cap C} n = 50(1089 + 2l) = 54450 + 100l.$$

The stated union sum becomes

$$54650 + 54450 + 100l = 109600, \qquad 100l = 500, \qquad l = 5.$$

This is an allowed residue and differs from 2. As a direct check, the second progression runs from 104 to 995, with sum $50(104 + 995) = 54950$; adding 54650 indeed gives 109600.

Step 3: Conclusion The required value is $\boxed{l = 5}$.

Quick Tip to Solve: Find the first and last three-digit representatives of each residue class; check equal residues separately before adding union sums.

26. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is _____. [4 marks]

Solution:

Concept:

- Completing squares identifies the centre and radius of a circle. A diameter has its midpoint at that centre and length twice the radius.
- The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord; the radius, half-chord and centre-to-chord distance form a right triangle.

Step 1: Find the fixed chord length and midpoint Rewrite the given circle as

$$(x - 1)^2 + (y - 3)^2 = 4.$$

Its centre is $O = (1, 3)$, and its radius is 2. Let AB be the diameter which is also a chord of the other circle. Then its midpoint is O and $OA = OB = 2$, irrespective of the direction of the diameter. This midpoint observation fixes the perpendicular distance needed for the second circle; the actual endpoints need not be calculated.

Step 2: Use the right triangle in the second circle Let the centre of circle C be $P = (2, 1)$. Because O is the midpoint of chord AB, $PO \perp AB$. The distance between the centres is

$$PO = \sqrt{(2 - 1)^2 + (1 - 3)^2} = \sqrt{5}.$$

The radius $R = PA$ is therefore obtained from the right triangle POA:

$$R^2 = PO^2 + OA^2 = 5 + 4 = 9, \qquad R = 3.$$

The positive square root is required because a radius is a length. The perpendicularity also explains why the specified diameter must have a particular direction, although its orientation has no effect on the numerical radius.

Step 3: Conclusion The radius of circle C is $\boxed{3}$ units.

Quick Tip to Solve: If a diameter of one circle is a chord of another, use the first centre as the chord midpoint and add the two perpendicular squared lengths.

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27. If $\int_{-a}^{a} (|x| + |x-2|)dx = 22$, (a > 2) and [x] denotes the greatest integer $\leq$ x, then $\int_{-a}^{a} (x + [x])dx$ is equal to _____. [4 marks]

Solution:

Concept:

- The signs in $|x| + |x-2|$ change only at 0 and 2. Since $a > 2$, both breakpoints lie inside the integration interval.
- For noninteger x, $\lfloor x \rfloor + \lfloor -x \rfloor = -1$. The exceptional integer points do not affect a definite integral.

Step 1: Determine the integration limit from the absolute values On $[-a, 0]$, the integrand is $2 - 2x$; on $[0, 2]$, it is 2; and on $[2, a]$, it is $2x - 2$. Thus

$$\begin{aligned} 22 &= \int_{-a}^{0} (2 - 2x)\, dx + \int_{0}^{2} 2\, dx + \int_{2}^{a} (2x - 2)\, dx \\ &= (a^2 + 2a) + 4 + (a^2 - 2a) = 2a^2 + 4. \end{aligned}$$

Hence $a^2 = 9$, and the given condition $a > 2$ selects $a = 3$. There is no freedom to choose the negative root or to reverse the limits.

Step 2: Pair the floor values across the origin The function x is odd, so $\int_{-3}^{3} x\, dx = 0$. For the remaining term, substitute $x = -t$ in the negative half and pair it with the positive half:

$$\begin{aligned} \int_{-3}^{3} \lfloor x \rfloor\, dx &= \int_{0}^{3} \left(\lfloor t \rfloor + \lfloor -t \rfloor \right) dt \\ &= \int_{0}^{3} (-1)\, dt = -3. \end{aligned}$$

For a direct check, the floor values on the six unit intervals are $-3, -2, -1, 0, 1, 2$; their sum is -3. Values at the finitely many integer endpoints do not change the integral. Therefore adding the zero integral of x leaves -3.

Step 3: Conclusion The integral as printed is $\boxed{-3}$. The supplied answer key gives 3, but the floor-function identity and direct unit-interval sum both give the negative value.

Quick Tip to Solve: Do not treat the floor function as odd: away from integers its values at opposite arguments add to -1, rather than zero.

28. Let M be any 3 $\times$ 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is _____. [4 marks]

Solution:

Concept:

- The trace of M^TM equals the sum of the squares of all nine entries of M. An entry 0, 1 or 2 contributes 0, 1 or 4 respectively.
- Count matrices by first choosing the positions occupied by 2 and then the positions occupied by 1. The remaining entries are forced to be zero.

Step 1: List all possible compositions of the entries If a entries equal 2 and b entries equal 1, the trace condition is

$$4a + b = 7, \qquad a, b \geq 0, \qquad a + b \leq 9.$$

Because two entries equal to 2 would already contribute 8, only $a = 0$ and $a = 1$ are possible. They give the two disjoint cases $(a, b) = (0, 7)$ and $(1, 3)$. This uses all entries of M, including those off the diagonal: it is the diagonal of the product M^TM, not just the diagonal of M, that is being summed.

Step 2: Count the arrangements in each case and add In the first case, choose seven of the nine positions for the 1s. The other two are 0s, giving

$$N_1 = \binom{9}{7} = 36.$$

In the second case, choose one position for the 2 and three of the remaining eight positions for the 1s:

$$N_2 = \binom{9}{1}\binom{8}{3} = 9 \times 56 = 504.$$

All unchosen positions are 0s. Entries of the same value are indistinguishable, so there is no extra permutation factor. The cases do not overlap, and every admissible matrix has exactly one of these compositions. Consequently the complete count is

$$N = N_1 + N_2 = 36 + 504 = 540.$$

Step 3: Conclusion The number of such matrices is $\boxed{540}$.

Quick Tip to Solve: For a condition on $\mathrm{tr}(M^TM)$, count squared entries over the entire matrix; it is the sum of all entry squares.

29. Let B_i (i = 1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)).

Then $\frac{P(B_1)}{P(B_3)}$ is equal to _____. [4 marks]

Solution:

Concept:

- For independent events, write $r_i = P(B_i)$, where $0 < r_i < 1$. The probability that none occurs is $p = (1 - r_1)(1 - r_2)(1 - r_3)$.
- The probability that only one event occurs divided by p is its odds: $x = \alpha/p = r_1/(1 - r_1)$, $y = \beta/p$, and $z = \gamma/p$. Thus $r_1 = x/(1 + x)$ and $r_3 = z/(1 + z)$.

Step 1: Convert the given equations to odds Since $p > 0$, divide each equation by p^2. This removes the common factors from the independent-event probabilities and gives

$$x - 2y = xy, \qquad y - 3z = 2yz.$$

Rearrange these equations before substituting:

$$x(1 - y) = 2y, \qquad y(1 - 2z) = 3z.$$

All three odds are positive. Hence these relations also ensure that the denominators used next are nonzero and positive.

Step 2: Express the first odds in terms of the third The second equation gives $y = 3z/(1 - 2z)$. Substituting this into the first equation yields

$$x = \frac{2y}{1 - y} = \frac{6z}{1 - 5z}.$$

The positivity of x implies $0 < z < 1/5$, consistent with the divisions. Convert these odds back to the probabilities requested in the question:

$$r_1 = \frac{x}{1 + x} = \frac{6z}{1 + z}, \qquad r_3 = \frac{z}{1 + z}.$$

Step 3: Take the probability ratio The common factor $z/(1 + z)$ is positive, so it cancels without losing a possible case:

$$\frac{P(B_1)}{P(B_3)} = \frac{6z/(1 + z)}{z/(1 + z)} = 6.$$

The individual event probabilities need not be determined; the two equations fix their ratio.

Step 4: Conclusion The required numerical value is $\boxed{6}$.

Quick Tip to Solve: When probabilities of only one event and of no event are given, divide by the no-event probability to introduce independent-event odds.

30. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is ____. [4 marks]

Solution:

Concept:

- Coplanarity with two nonparallel vectors means $\vec{c} = u\vec{a} + v\vec{b}$ for scalars u, v. Perpendicularity is expressed by a zero scalar product.
- Use $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}+\vec{b}|^2 + 2(\vec{a}+\vec{b})\cdot\vec{c} + |\vec{c}|^2$. This avoids unnecessary component expansion.

Step 1: Compute the scalar products of the known vectors From $\vec{a} = (-1, 1, 1)$ and $\vec{b} = (2, 0, 1)$,

$$\vec{a}\cdot\vec{a} = 3, \qquad \vec{b}\cdot\vec{b} = 5, \qquad \vec{a}\cdot\vec{b} = -1.$$

Their scalar products are not those of parallel vectors, so they span a plane and the given coplanarity condition can be used directly.

Step 2: Determine the coplanar vector Write $\vec{c} = u\vec{a} + v\vec{b}$. The two conditions give

$$3u - v = 7, \qquad -u + 5v = 0.$$

Thus $u = 5v$, so $14v = 7$, $v = 1/2$, and $u = 5/2$. Consequently

$$\vec{c} = \tfrac{5}{2}\vec{a} + \tfrac{1}{2}\vec{b} = (-\tfrac{3}{2}, \tfrac{5}{2}, 3).$$

This satisfies $\vec{a}\cdot\vec{c} = 7$ and $\vec{b}\cdot\vec{c} = 0$, providing a direct check of both constraints.

Step 3: Evaluate the requested squared magnitude We have $|\vec{a}+\vec{b}|^2 = 3 + 5 - 2 = 6$. Also,

$$|\vec{c}|^2 = \vec{c}\cdot(\tfrac{5}{2}\vec{a} + \tfrac{1}{2}\vec{b}) = \tfrac{5}{2}(7) + \tfrac{1}{2}(0) = \tfrac{35}{2}.$$

Therefore

$$2|\vec{a}+\vec{b}+\vec{c}|^2 = 2\left(6 + 2(7+0) + \tfrac{35}{2}\right) = 75.$$

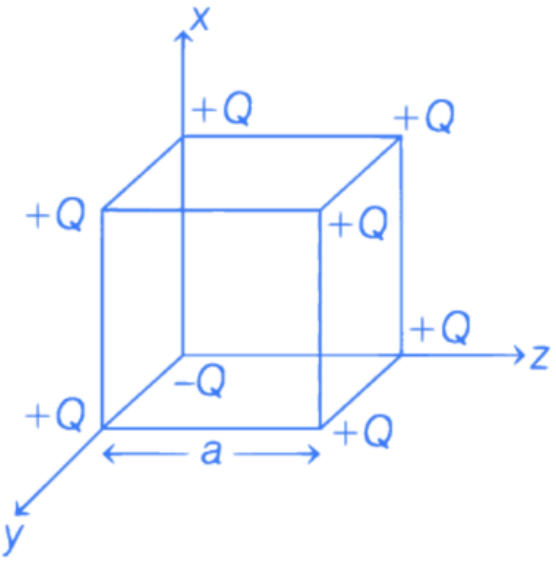
Alternatively the sum vector is $(-1/2, 7/2, 5)$, whose squared magnitude is $75/2$.

Step 4: Conclusion The required value is $\boxed{75}$.

Quick Tip to Solve: For a coplanar unknown vector, express it as a linear combination of the two given vectors and impose the dot-product constraints.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is −Q. The electric field at the centre of cube is : [4 marks]



(A) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(B) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(C) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(D) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

Correct Answer: (D) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

Solution:

Concept:

- The electric field is a vector sum. Eight equal charges at the vertices of a cube produce zero field at its centre by symmetry.
- Replacing the charge $+Q$ at the origin by $-Q$ is equivalent to adding an extra charge $-2Q$ there. Its field alone gives the net result.

Step 1: Replace the asymmetric arrangement by a symmetric one plus a correction Imagine first that the origin also carries $+Q$. For every vertex there is an opposite vertex at the same distance from the centre, so their fields cancel in pairs. Changing just the origin charge from $+Q$ to $-Q$ changes it by $-2Q$. All other charges remain unchanged. Superposition therefore reduces the calculation to the field of this extra negative charge, not merely that of $-Q$.

Step 2: Find the distance and direction The centre is at $(a/2, a/2, a/2)$. Its position from the origin and its magnitude are

$$\vec{r} = \frac{a}{2}(\hat{x}+\hat{y}+\hat{z}), \qquad r = \frac{\sqrt{3}a}{2}.$$

The field of a negative charge points toward the origin, opposite to $\vec{r}$. Using $\vec{E} = (4\pi\varepsilon_0)^{-1} q\vec{r}/r^3$,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{(-2Q)(a/2)}{(\sqrt{3}a/2)^3} (\hat{x} + \hat{y} + \hat{z}).$$

Step 3: Simplify and compare the options Since $(\sqrt{3}a/2)^3 = 3\sqrt{3}a^3/8$, the coefficient becomes

$$-\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}.$$

The positive coefficients in (A) and (C) point away from the negative vertex and are incorrect. Option (B) has the right direction but half the required coefficient because it omits the removal of the original positive charge.

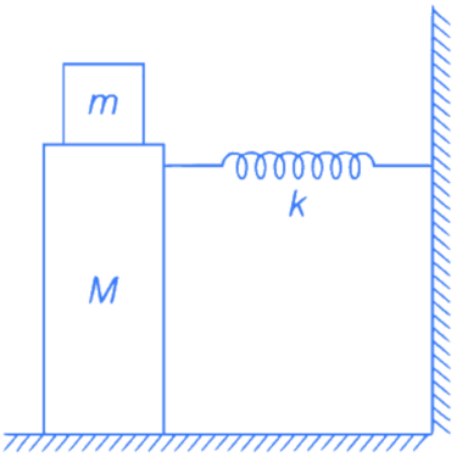
Step 4: Conclusion

$$\boxed{\vec{E} = -\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For a single altered vertex charge in a symmetric arrangement, calculate only the change in charge at that vertex.

32. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillations will be : [4 marks]



(A) $A\sqrt{\frac{M}{M-m}}$

(B) $A\sqrt{\frac{M-m}{M}}$

(C) $A\sqrt{\frac{M+m}{M}}$

(D) $A\sqrt{\frac{M}{M+m}}$

Correct Answer: (D) $A\sqrt{\frac{M}{M+m}}$

Solution:

Concept:

- For a spring oscillator, $\omega = \sqrt{k/M}$ and the speed at equilibrium is its maximum value $v = A\omega$.
- During the brief sticking event horizontal momentum is conserved. Mechanical energy need not be conserved during attachment, but is conserved in the subsequent oscillation.

Step 1: Find the speed just before attachment The block is at equilibrium, so its displacement and the spring force are zero at the instant the new mass is fixed. All its oscillator energy is kinetic, giving

$$v = A\sqrt{\frac{k}{M}}.$$

The added mass is gently placed and has no initial horizontal velocity. The equilibrium position of a horizontal spring is unchanged by adding a mass because the vertical weight is balanced by the surface.

Step 2: Conserve horizontal momentum through sticking During the very short attachment interval, the spring supplies negligible horizontal impulse. If $v\prime$ is the common speed just afterward,

$$Mv = (M+m)v\prime, \qquad v\prime = \frac{M}{M+m}v.$$

Some kinetic energy is lost in this inelastic event. Assuming energy conservation across it would therefore give an incorrect amplitude.

Step 3: Relate the new maximum speed to the new amplitude Afterward the mass of the oscillator is $M+m$, so its new angular frequency is $\omega\prime = \sqrt{k/(M+m)}$. It still starts from equilibrium, hence $v\prime = A\prime\omega\prime$. Thus

$$A\prime = \frac{M}{M+m}\frac{A\sqrt{k/M}}{\sqrt{k/(M+m)}} = A\sqrt{\frac{M}{M+m}}.$$

For positive added mass this is real and less than A. Options (A) and (C) predict an increase; (B) even becomes imaginary when $m > M$, unlike this physical process.

Step 4: Conclusion The new amplitude is $\boxed{A\sqrt{M/(M+m)}}$. Hence, the correct option is (D).

Quick Tip to Solve: At equilibrium attachment, first conserve momentum to find the reduced speed, then divide by the new angular frequency.

33. A current through a wire depends on time as

$i = \alpha_0 t + \beta t^2$

where α_0 = 20 A/s and β = 8 As^{-2}. Find the charge crossed through a section of the wire in 15 s. [4 marks]

(A) 2250 C

(B) 2100 C

(C) 260 C

(D) 11250 C

Correct Answer: (D) 11250 C

Solution:

Concept:

- Current is the rate of flow of charge: $i = dq/dt$. The total charge passing in an interval is the area under the current-time curve, $q = \int i\, dt$.
- Both terms of $i = \alpha_0 t + \beta t^2$ contribute. Since the current varies with time, multiplying its final value by the interval does not give the transferred charge.

Step 1: Integrate the current over the specified interval Take the interval to run from $t = 0$ to $t = 15\,\text{s}$. Integrating each term separately gives

$$q = \int_0^{15} (\alpha_0 t + \beta t^2)\, dt = \left[\frac{\alpha_0 t^2}{2} + \frac{\beta t^3}{3}\right]_0^{15}.$$

The lower limit contributes zero for both terms. The powers increase by one on integration, and their new exponents divide the coefficients.

Step 2: Substitute the given coefficients Using $\alpha_0 = 20\,\text{A}\,\text{s}^{-1}$ and $\beta = 8\,\text{A}\,\text{s}^{-2}$,

$$q = \frac{20(15)^2}{2} + \frac{8(15)^3}{3}.$$

The first contribution is $10 \times 225 = 2250\,\text{C}$. The second is $(8/3) \times 3375 = 9000\,\text{C}$. Each has units of ampere-second, which is coulomb.

Step 3: Add both contributions and test the choices The total is

$$q = 2250 + 9000 = 11250\,\text{C}.$$

Option (A) includes only the linear-current contribution and misses the quadratic term entirely. Options (B) and (C) do not equal the integral of either the full current or its two contributions. As a check, the average current is $11250/15 = 750\,\text{A}$, below the final current $20(15) + 8(15)^2 = 2100\,\text{A}$, as expected for an increasing current.

Step 4: Conclusion The charge transferred is $\boxed{11250\,\text{C}}$. Hence, the correct option is (D).

Quick Tip to Solve: For polynomial current, integrate each power of time separately and include every term before substituting the end time.

34. Match List I with List II.

	List I		List II
(a)	Isothermal	(i)	Pressure constant
(b)	Isochoric	(ii)	Temperature constant
(c)	Adiabatic	(iii)	Volume constant
(d)	Isobaric	(iv)	Heat content is constant

Choose the correct answer from the options given below : [4 marks]

(A) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

(B) (a) - (ii), (b) - (iv), (c) - (iii), (d) - (i)

(C) (a) - (iii), (b) - (ii), (c) - (i), (d) - (iv)

(D) (a) - (i), (b) - (iii), (c) - (ii), (d) - (iv)

Correct Answer: (A) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

Solution:

Concept:

- An isothermal process keeps temperature constant, an isochoric process keeps volume constant, and an isobaric process keeps pressure constant.
- An adiabatic process has no heat exchange, $\delta Q = 0$. Heat is energy transferred during a process, rather than a state variable or stored heat content.

Step 1: Match the three direct constant-variable definitions The prefix iso denotes an unchanged quantity. Therefore the isothermal process (a) corresponds to constant temperature (ii). An isochoric process (b) takes place at fixed volume and corresponds to (iii). An isobaric process (d) takes place at fixed pressure and corresponds to (i). These three definitions already select one complete option unambiguously.

Step 2: Interpret the adiabatic entry carefully An adiabatic wall prevents heat transfer, so the correct thermodynamic statement is $Q = 0$ during the process. Entry (iv), phrased as heat content being constant, is the intended no-heat-exchange entry in this matching list. It should not be interpreted as constant internal energy: the first law permits internal energy to change when an adiabatic gas does work. Thus the intended remaining match is (c)-(iv).

Step 3: Compare all four proposed matchings The complete intended correspondence is

$$(a)-(ii), \quad (b)-(iii), \quad (c)-(iv), \quad (d)-(i).$$

Option (A) gives exactly this sequence. Option (B) exchanges the isochoric and adiabatic entries. Option (C) incorrectly calls an isothermal process constant-volume and an adiabatic process constant-pressure. Option (D) interchanges the constant-pressure and constant-temperature processes and also mismatches the adiabatic entry.

Step 4: Conclusion The intended sequence is constant temperature, constant volume, no heat exchange, and constant pressure, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Match temperature, volume and pressure first; remember that adiabatic means no heat transfer, not necessarily no temperature change.

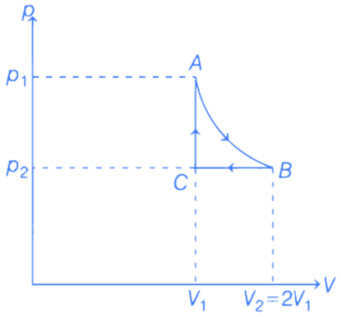
35. n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes.

A $\rightarrow$ B : Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure charges from P_1 to P_2

B $\rightarrow$ C : Isobaric compression at pressure P_2 to initial volume V_1.

C $\rightarrow$ A : Isochoric change leading to change of pressure from P_2 to P_1.

Total workdone in the complete cycle ABCA is : [4 marks]



(A) nRTln 2

(B) 0

(C) $nRT\left(\ln 2 - \frac{1}{2}\right)$

(D) $nRT\left(\ln 2 + \frac{1}{2}\right)$

Correct Answer: (C) $nRT\left(\ln 2 - \frac{1}{2}\right)$

Solution:

Concept:

- Work done by a gas is $W = \int P\,dV$. Expansion contributes positive work, compression negative work, and an isochoric path contributes zero.
- For an ideal gas on an isotherm, $PV = nRT$, so $W = nRT\ln(V_f/V_i)$. The total cyclic work is the signed area enclosed on the pressure-volume diagram.

Step 1: Calculate the isothermal expansion work Along $A \rightarrow B$, temperature is T and volume doubles. Hence

$$W_{AB} = nRT\ln\frac{2V_1}{V_1} = nRT\ln 2.$$

At the endpoint B, the ideal-gas equation gives

$$P_2(2V_1) = nRT, \qquad P_2V_1 = \frac{nRT}{2}.$$

This endpoint relation must be used for the following constant-pressure compression.

Step 2: Find work on the two returning paths Along $B \rightarrow C$, the pressure is P_2 and volume decreases from $2V_1$ to V_1. Thus

$$W_{BC} = P_2(V_1 - 2V_1) = -P_2V_1 = -\frac{nRT}{2}.$$

The minus sign represents work done on the gas. Along $C \rightarrow A$, volume stays at V_1, so $dV = 0$ and

$W_{CA} = 0$, even though pressure and temperature rise.

Step 3: Add the signed work contributions Adding the three legs yields

$$W_{\text{cycle}} = W_{AB} + W_{BC} + W_{CA} = nRT\left(\ln 2 - \frac{1}{2}\right).$$

The value is positive because $\ln 2 > 1/2$, consistent with the clockwise cycle drawn. Option (A) ignores compression, (B) incorrectly assumes all cyclic work vanishes, and (D) assigns a positive sign to compression. A cycle has zero change in internal energy, but can still have nonzero work and net heat transfer.

Step 4: Conclusion

$$\boxed{W_{\text{cycle}} = nRT\left(\ln 2 - \frac{1}{2}\right).}$$

Hence, the correct option is (C).

Quick Tip to Solve: For a cycle, compute work leg by leg with expansion positive and compression negative; zero cyclic internal-energy change does not mean zero work.

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36. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temperature is T + ΔT. Calculate the increase in the volume of the metal box. [4 marks]

(A) $3a^3\alpha\Delta T$

(B) $4\pi a^3\alpha\Delta T$

(C) $\frac{4}{3}\pi a^3\alpha\Delta T$

(D) $4a^3\alpha\Delta T$

Correct Answer: (A) $3a^3\alpha\Delta T$

Solution:

Concept:

- For a small uniform temperature rise, each linear dimension changes according to $a_f = a(1+\alpha\Delta T)$. Here α is the coefficient of linear expansion and $\alpha\Delta T$ is dimensionless.
- The volume of a cube is the cube of its side. To first order, the coefficient of volume expansion of an isotropic solid is 3α.

Step 1: Expand the three dimensions The initial enclosed volume is $V_i = a^3$. Heating the sheet uniformly increases all three perpendicular sides by the same fractional amount; the empty interior expands with the boundaries of the box. Thus

$$V_f = [a(1+\alpha\Delta T)]^3 = a^3(1+\alpha\Delta T)^3.$$

Step 2: Retain the first-order change Using the binomial expansion,

$$V_f = a^3 \left[1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3\right].$$

For the stated small temperature rise, neglect the squared and cubed fractional changes. Subtracting V_i gives

$$\Delta V = 3a^3\alpha\Delta T.$$

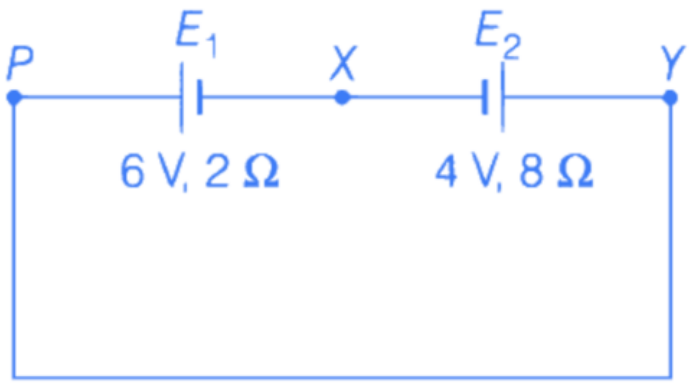
The factor 3 comes from the three expanding dimensions. A factor involving π would correspond to different geometry, while 4 is not the linearized cubic factor.

Step 3: Conclusion The increase in the volume is $\boxed{3a^3\alpha\Delta T}$. Hence, the correct option is (A).

Quick Tip to Solve: For a cube, expand $(1 + \alpha\Delta T)^3$ only to first order; do not confuse the side increment with the volume increment.

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37. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is : [4 marks]



(A) 10.0 V

(B) 2.0 V

(C) 5.6 V

(D) 3.6 V

Correct Answer: (C) 5.6 V

Solution:

Concept:

- A real cell has emf E and internal resistance r. Its terminal voltage is $E - Ir$ when it supplies current and $E + Ir$ when it is being charged.
- Kirchhoff's loop law sets the algebraic sum of emfs equal to the resistive voltage drop. The two internal resistances are in the same closed loop.

Step 1: Find the circulating current The long plate of the 6 V cell is at P, while the long plate of the 4

V cell is at Y. Since P and Y are joined by the external wire, the cells oppose one another around the loop. The 6 V source drives current through the 4 V source in its charging direction. Hence

$$I = \frac{E_1 - E_2}{r_1 + r_2} = \frac{6-4}{2+8} = 0.2\,\text{A}.$$

Step 2: Calculate the common terminal voltage P and Y have the same potential because their connecting wire has negligible resistance. Across the supplying cell,

$$V_P - V_X = 6 - (0.2)(2) = 5.6\,\text{V}.$$

Independently, the charging cell gives

$$V_Y - V_X = 4 + (0.2)(8) = 5.6\,\text{V}.$$

The agreement checks the polarity and the current. Adding the emfs gives 10 V and subtracting them gives 2 V, but neither includes the internal drops needed for the terminal voltage. The 3.6 V value would incorrectly treat the 4 V cell as discharging.

Step 3: Conclusion The magnitude of the potential difference across X and Y is $\boxed{5.6\,\text{V}}$, with Y at the higher potential. Hence, the correct option is (C).

Quick Tip to Solve: For unequal cells joined with like terminals together, the stronger cell supplies current and the weaker cell charges; check both terminal-voltage formulas.

38. The focal length f is related to the radius of curvature r of the spherical convex mirror by :

[4 marks]

(A) f = r
(B) f = − r
(C) f = $+\frac{1}{2}$ r
(D) f = $-\frac{1}{2}$ r

Correct Answer: (C) f = $+\frac{1}{2}$ r

Solution:

Concept:

- In the paraxial approximation, a spherical mirror has focal length equal to half its signed radius of curvature: $f = R/2$. This relation follows from the law of reflection near the principal axis.
- In the Cartesian convention, distances are measured from the pole in the direction of incident light. A convex mirror has both its virtual focus and centre of curvature behind the mirror.

Step 1: Locate the focus and centre A parallel incident ray is reflected so that its backward extension meets the principal axis at the virtual principal focus. For a convex spherical mirror, that focus lies between the pole and the centre of curvature. Both points lie on the same side of the pole, so their signed distances have the same sign.

Step 2: Apply the spherical-mirror relation Writing the printed radius as r, the relation is

$$r = 2f \quad \Longrightarrow \quad f = +\frac{r}{2}.$$

The positive relative sign is essential: $f = -r/2$ would place the focus and centre on opposite sides. The alternatives $f = r$ and $f = -r$ also miss the factor of two. If r is treated as a positive magnitude for the convex mirror, its focus is behind the mirror and f is positive in the usual convention.

Step 3: Conclusion The required relation is $\boxed{f = +\frac{1}{2}r}$. Hence, the correct option is (C).

Quick Tip to Solve: For any spherical mirror use the signed relation $R = 2f$; the focus and centre of curvature are on the same side of the pole.

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39. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern. [4 marks]

(A) 4 : 1

(B) 2 : 1

(C) 1 : 4

(D) 3 : 1

Correct Answer: (A) 4 : 1

Solution:

Concept:

- The question specifies that light amplitude is proportional to slit width. Therefore a width ratio $3:1$ gives an amplitude ratio $a_1 : a_2 = 3:1$, not an intensity ratio of $3:1$.
- For coherent beams the maximum and minimum intensities are proportional to $(a_1 + a_2)^2$ and $(a_1 - a_2)^2$, respectively.

Step 1: Use the stated amplitude ratio Let the amplitude from the narrow slit be a. The amplitude from the wide slit is then $3a$. At a bright fringe, the waves arrive in phase and their displacements add. At a dark fringe, their phase difference is π, so the resultant amplitude is their difference rather than zero.

Step 2: Compare the intensity extremes With the same intensity proportionality constant C for both positions,

$$I_{\max} = C(3a + a)^2 = 16Ca^2,$$

$$I_{\min} = C(3a - a)^2 = 4Ca^2.$$

Consequently,

$$\frac{I_{\max}}{I_{\min}} = \frac{16}{4} = 4.$$

The ratio 2 is the ratio of resultant amplitudes, before squaring. The ratio $1:4$ reverses maximum and minimum, and $3:1$ merely repeats the slit-width ratio. Unequal amplitudes explain why the

minimum remains nonzero.

Step 3: Conclusion The maximum-to-minimum intensity ratio is $\boxed{4:1}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert widths to amplitudes using the given rule, add or subtract the amplitudes, and only then square to obtain intensities.

40. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be : [4 marks]

(A) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

(B) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

(C) $\sqrt{G(1+2\sqrt{2})}$

(D) $\frac{1}{2}\sqrt{G(1+2\sqrt{2})}$

Correct Answer: (D) $\frac{1}{2}\sqrt{G(1+2\sqrt{2})}$

Solution:

Concept:

- Four identical particles in the symmetric rotating configuration occupy the vertices of a square inscribed in the circle. The net gravitational force on each particle points towards the centre.
- The force between two masses is Gm^2/d^2. For uniform circular motion, the inward resultant equals mv^2/R, with R the circle radius.

Step 1: Resolve the three attractions For a particle at one vertex, the opposite particle is $2R$ away. It exerts a radial force

$$F_o = \frac{Gm^2}{4R^2}.$$

Each of the two adjacent particles is $\sqrt{2}R$ away, so its attraction has magnitude $Gm^2/(2R^2)$. Its line makes 45° with the inward radius. The tangential components cancel by symmetry, while the inward components add.

Step 2: Equate the radial force to the centripetal force The resultant is

$$F = \frac{Gm^2}{4R^2} + 2\frac{Gm^2}{2R^2}\cos 45^\circ = \frac{Gm^2}{4R^2}(1+2\sqrt{2}).$$

Thus

$$v^2 = \frac{Gm}{4R}(1+2\sqrt{2}).$$

For $m = 1\,\text{kg}$ and $R = 1\,\text{m}$,

$$v = \frac{1}{2}\sqrt{G(1+2\sqrt{2})}.$$

All three forces are attractive, so a negative opposite-particle contribution is impossible. The alterna-

tives with coefficients $1/2$ or 1 in v^2 overestimate the inward force.

Step 3: Conclusion The speed of each particle is $\boxed{\frac{1}{2}\sqrt{G(1+2\sqrt{2})}}$ in SI units. Hence, the correct option is (D).

Quick Tip to Solve: Use the circle radius for centripetal acceleration, but the square side $\sqrt{2}R$ and diagonal $2R$ for the gravitational separations.

41. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is : [4 marks]

(A) 1 : 4

(B) 8 : 1

(C) 2 : 1

(D) 1 : 8

Correct Answer: (B) 8 : 1

Solution:

Concept:

- The angular speed of a satellite in a circular orbit is $\omega = 2\pi/T$, because one revolution sweeps an angle of 2π radians in the period T.
- Angular speed is inversely proportional to period. Both periods must be written in the same time unit before forming a ratio.

Step 1: Write the angular speeds separately For the first satellite, $T_1 = 1\,\text{h}$, while for the second, $T_2 = 8\,\text{h}$. Therefore

$$\omega_1 = \frac{2\pi}{1\,\text{h}}, \qquad \omega_2 = \frac{2\pi}{8\,\text{h}}.$$

The common factor and time unit cancel in the comparison; conversion to seconds would produce exactly the same ratio.

Step 2: Invert the period ratio Dividing the two expressions gives

$$\frac{\omega_1}{\omega_2} = \frac{2\pi/T_1}{2\pi/T_2} = \frac{T_2}{T_1} = \frac{8}{1}.$$

The first satellite completes eight revolutions while the second completes one, so it must have the larger angular speed. The alternative $1 : 8$ confuses angular speed with period, while $1 : 4$ and $2 : 1$ do not follow the reciprocal dependence. Kepler's third law is unnecessary because neither orbital radius nor linear speed is requested.

Step 3: Conclusion The required angular-velocity ratio is $\boxed{\omega_1 : \omega_2 = 8 : 1}$. Hence, the correct option is (B).

Quick Tip to Solve: For angular-speed ratios, invert the period ratio directly; use Kepler's law only if radii or orbital linear speeds are needed.

42. If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters. [4 marks]

(A) $Y = \frac{9K\eta}{3K-\eta} N/m^2$

(B) $Y = \frac{9K\eta}{2\eta+3K} N/m^2$

(C) $\eta = \frac{3YK}{9K+Y} N/m^2$

(D) $K = \frac{Y\eta}{9\eta-3Y} N/m^2$

Correct Answer: (D) $K = \frac{Y\eta}{9\eta-3Y} N/m^2$

Solution:

Concept:

- For an isotropic linearly elastic material, Young's modulus Y, shear modulus η, bulk modulus K, and Poisson's ratio ν satisfy $Y = 2\eta(1+\nu)$ and $Y = 3K(1-2\nu)$.
- Eliminating Poisson's ratio gives the standard identity $Y = 9K\eta/(3K+\eta)$. All three moduli have the unit $\mathrm{N\,m^{-2}}$.

Step 1: Eliminate Poisson's ratio From the shear relation,

$$\nu = \frac{Y}{2\eta} - 1.$$

Substituting this in the bulk relation yields

$$Y = 3K\left[1 - 2\left(\frac{Y}{2\eta} - 1\right)\right] = 3K\left(3 - \frac{Y}{\eta}\right).$$

Multiplication by η and collection of terms give

$$Y\eta = 9K\eta - 3KY, \qquad Y(3K+\eta) = 9K\eta.$$

Step 2: Rearrange for the printed alternative Collecting the terms containing K,

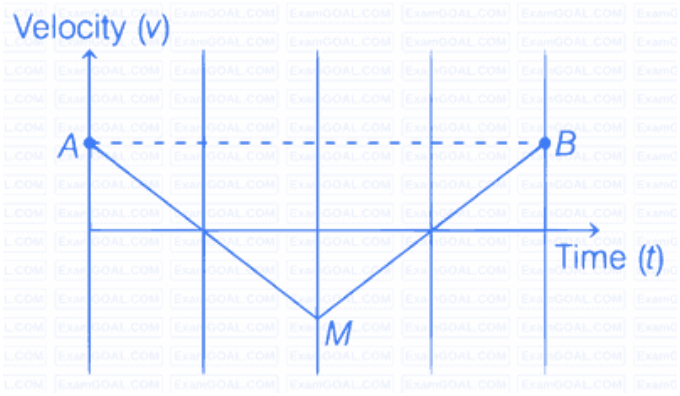
$$Y\eta = K(9\eta - 3Y) \quad \Longrightarrow \quad K = \frac{Y\eta}{9\eta - 3Y}.$$

This matches the fourth relation. The first relation uses a minus sign instead of a plus sign in the denominator of Y. The second puts an incorrect factor 2 in front of η. Solving instead for rigidity gives $\eta = 3YK/(9K - Y)$, so the plus sign in the third relation is also wrong. The printed unit annotation identifies the SI unit of the resulting modulus.

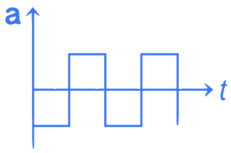
Step 3: Conclusion The correct relation is $\boxed{K = \frac{Y\eta}{9\eta - 3Y}}$. Hence, the correct option is (D).

Quick Tip to Solve: Memorize $Y = 9K\eta/(3K+\eta)$, then rearrange it algebraically rather than guessing the signs in equivalent expressions.

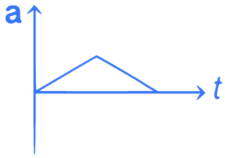
43. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph? [4 marks]



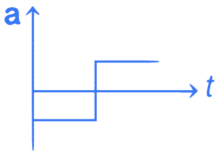
(A)



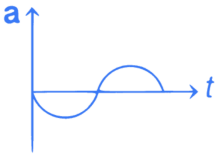
(B)



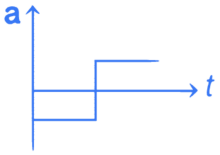
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- Instantaneous acceleration is the slope of the velocity-time graph: $a = dv/dt$. A straight segment therefore represents a constant acceleration.
- The sign of acceleration is determined by the direction of the slope, not by whether velocity itself lies above or below zero.

Step 1: Read the slope on AM From A to M the velocity decreases linearly with time. The line has the same negative slope on both sides of its crossing of the time axis, so acceleration remains a constant negative value throughout AM. The particle changes its direction when velocity crosses zero, but its acceleration does not change there. Thus the acceleration graph must be a horizontal segment below the time axis until the instant corresponding to M.

Step 2: Read the slope on MB From M to B the velocity increases linearly, giving a constant positive acceleration. The equal horizontal and vertical changes in the two halves show equal magnitudes of the negative and positive slopes. The acceleration therefore jumps from a negative constant to a positive constant at M. At the ideal sharp corner the derivative is not defined; the horizontal pieces describe the intervals on either side.

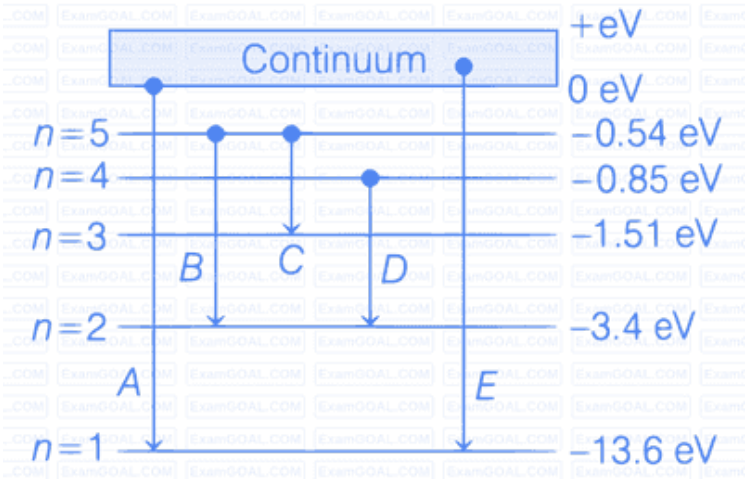
Step 3: Compare the proposed shapes Option (C) has exactly one switch, from a negative horizontal segment to a positive horizontal segment. Option (A) incorrectly changes sign at every velocity-axis crossing as well. Options (B) and (D) make the acceleration vary continuously with time even though each velocity segment is straight. Neither gives the required constant slopes.

Step 4: Conclusion The acceleration is constant and negative on AM, then constant and positive on MB. Hence, the correct option is (C).

Quick Tip to Solve: For a velocity-time graph, differentiate its slope segment by segment; a zero velocity is not necessarily a zero acceleration.

44. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E.

The transitions A, B and C respectively represent : [4 marks]



(A) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

(B) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.

(C) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

(D) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.

Correct Answer: (C) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

Solution:

Concept:

- A hydrogen emission series is named by its final level: Lyman ends at $n = 1$, Balmer at $n = 2$, and Paschen at $n = 3$.
- For a fixed final level n_f, the first member starts at $n_f + 1$; the series limit starts at $n = \infty$, the zero-energy threshold.

Step 1: Identify transition A The diagram shows A falling from the zero-energy boundary of the continuum to the ground level at $-13.6\,\text{eV}$. This is the transition $\infty \to 1$, which is the series limit of the Lyman series. It is not the first Lyman member, since that member would begin at $n = 2$. An ionization transition has the reverse direction, from a bound level into the continuum.

Step 2: Count the Balmer member B B starts at $n = 5$ and ends at $n = 2$. Balmer members are counted as $3 \to 2$ first, $4 \to 2$ second, and $5 \to 2$ third. Therefore B is the third member of the Balmer series. The intermediate energy levels crossed by the arrow do not represent separate transitions; the arrow specifies one direct jump between its endpoints.

Step 3: Count the Paschen member C and choose C falls from $n = 5$ to $n = 3$. The first Paschen member is $4 \to 3$, so $5 \to 3$ is the second member. Together the three identifications are Lyman limit, third Balmer member, and second Paschen member. Options (A) and (B) incorrectly call B the second Balmer member, while (D) incorrectly calls A the first Lyman member.

Step 4: Conclusion A, B and C are respectively the Lyman series limit, the third Balmer member and the second Paschen member. Hence, the correct option is (C).

Quick Tip to Solve: For a transition $n_i \to n_f$, its member number within the series is $n_i - n_f$; use the final level to name the series.

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45. The work done by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. α and β are constants. Then the dimensions of β will be : [4 marks]

(A) $[M^0LT^0]$

(B) $[ML^2T^{-2}]$

(C) $[MLT^{-2}]$

(D) $[M^2LT^2]$

Correct Answer: (C) $[MLT^{-2}]$

Solution:

Concept:

- The argument of an exponential must be dimensionless. Thus the numerator and denominator of $x^2/(\alpha kT)$ must have identical dimensions.
- Boltzmann constant times absolute temperature has energy dimensions: $[kT] = [ML^2T^{-2}]$. Work has these same dimensions.

Step 1: Determine the dimensions of alpha Write temperature as Θ when tracking dimensions, so it is not confused with the time dimension T. Since displacement has dimension L, dimensionlessness of the exponent gives

$$[\alpha][kT] = L^2.$$

Consequently,

$$[\alpha] = \frac{L^2}{ML^2T^{-2}} = M^{-1}T^2.$$

The minus sign in the exponent and the exponential function itself introduce no additional dimension.

Step 2: Use the prefactor to determine beta The exponential is dimensionless, so the prefactor must supply all the dimensions of work. Therefore

$$[W] = [\alpha][\beta]^2.$$

Substituting the value just obtained,

$$[\beta]^2 = \frac{ML^2T^{-2}}{M^{-1}T^2} = M^2L^2T^{-4}.$$

Taking the square root of the powers yields

$$[\beta] = MLT^{-2}.$$

These are the dimensions of force, not of energy or displacement.

Step 3: Check the choice against both conditions With this value, $[\alpha\beta^2] = [M^{-1}T^2][M^2L^2T^{-4}] = [ML^2T^{-2}]$, as required for work. Option (A) is length alone, option (B) is energy, and option (D) has different mass and time powers; none makes the prefactor an energy with the required dimensions of alpha.

Step 4: Conclusion The dimensions of β are $[MLT^{-2}]$. Hence, the correct option is (C).

Quick Tip to Solve: In an expression containing an exponential, use its dimensionless argument first, then apply dimensional homogeneity to the prefactor.

46. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is : [4 marks]

(A) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

(B) $2\pi\sqrt{\frac{3Gm}{d^3}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$

(D) $2\pi\sqrt{\frac{d^3}{3Gm}}$

Correct Answer: (D) $2\pi\sqrt{\frac{d^3}{3Gm}}$

Solution:

Concept:

- Both stars revolve with the same angular velocity about their common centre of mass; their orbital radii add to the separation d.
- For circular motion the mutual gravitational force supplies each star's centripetal force: $F = Gm_1m_2/d^2 = m_i\omega^2 r_i$.

Step 1: Locate the centre of mass Let the distances of the stars of mass m and $2m$ from the centre of mass be r_1 and r_2. The centre-of-mass condition and the fixed separation give

$$mr_1 = 2mr_2, \qquad r_1 + r_2 = d.$$

Thus

$$r_1 = \frac{2d}{3}, \qquad r_2 = \frac{d}{3}.$$

The lighter star follows the larger orbit, so it would be incorrect to use d as either star's orbital radius.

Step 2: Apply the force equation The force between the stars depends on their full separation, not their individual distances from the centre of mass:

$$F = \frac{G(m)(2m)}{d^2} = \frac{2Gm^2}{d^2}.$$

For the lighter star,

$$\frac{2Gm^2}{d^2} = m\omega^2\frac{2d}{3}, \qquad \omega^2 = \frac{3Gm}{d^3}.$$

Using the heavier star with radius $d/3$ gives the same angular velocity, as required.

Step 3: Convert angular velocity into period One complete revolution corresponds to angle 2π, hence

$$T_{\text{rev}} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}.$$

The square root has the dimensions of time. Options (B) and (C) instead have inverse-time dimensions, and option (A) uses the wrong numerical conversion between angular velocity and period.

Step 4: Conclusion The revolution period is $2\pi\sqrt{d^3/(3Gm)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a circular binary system, the relative orbit obeys $T^2 = 4\pi^2 d^3/[G(m_1 + m_2)]$; use the sum of the two masses.

47. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be : [4 marks]

(A) 4 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Correct Answer: (D) 1 : 4

Solution:

Concept:

- In series, capacitors carry equal charge and their potential differences add, giving $1/C_s = 1/C_1 + 1/C_2$.
- In parallel, each capacitor has the same potential difference and the charges add, giving $C_p = C_1 + C_2$.

Step 1: Find the equivalent series capacitance Let the capacitance of each identical capacitor be C. In a series combination the same charge Q appears on each capacitor. Each has potential difference Q/C, so the total potential difference is $2Q/C$. Therefore

$$C_s = \frac{Q}{2Q/C} = \frac{C}{2}.$$

Equivalently, the reciprocal rule gives $1/C_s = 2/C$. The equivalent series capacitance is smaller than either individual capacitance.

Step 2: Find the equivalent parallel capacitance In a parallel combination each capacitor is connected across the same potential difference V. Each stores charge CV, making the total supplied charge $2CV$. Thus

$$C_p = \frac{2CV}{V} = 2C.$$

The parallel capacitance is larger than either individual capacitance because the combination stores twice the charge at the same potential difference.

Step 3: Take the ratio in the requested order The question asks for series first and parallel second. Consequently,

$$\frac{C_s}{C_p} = \frac{C/2}{2C} = \frac{1}{4}, \qquad C_s : C_p = 1 : 4.$$

The value of C cancels, so no numerical capacitance is needed. Option (A) reverses the requested order. Options (B) and (C) compare one equivalent capacitance to a single capacitor instead of comparing the two equivalent combinations.

Step 4: Conclusion The equivalent capacitances in series and parallel are in the ratio $1 : 4$. Hence, the correct option is (D).

Quick Tip to Solve: For n equal capacitors, series gives C/n and parallel gives nC, so the series-to-parallel ratio is $1 : n^2$.

48. Moment of inertia (M. I.) of four bodies, having same mass and radius, are reported as;

I_1 = M.I. of thin circular ring about its diameter,

I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

I_4 = M.I. of solid sphere about its diameter.

Then : [4 marks]

(A) $I_1 = I_2 = I_3 > I_4$

(B) $I_1 + I_3 < I_2 + I_4$

(C) $I_1 = I_2 = I_3 < I_4$

(D) $I_1 + I_2 = I_3 + \frac{5}{2} I_4$

Correct Answer: (A) $I_1 = I_2 = I_3 > I_4$

Solution:

Concept:

- The moment of inertia depends on the specified rotation axis as well as mass and size: $I = \int r_\perp^2 \, dm$.
- For a planar circular ring, the perpendicular-axis theorem relates its central normal axis and any two perpendicular diameters.

Step 1: Evaluate the ring and disc moments All four bodies have the same mass M and radius R. A thin ring has moment MR^2 about its central axis perpendicular to its plane. Symmetry makes the moments about two perpendicular diameters equal. By the perpendicular-axis theorem,

$$I_1 = \frac{1}{2} MR^2.$$

A uniform circular disc about its central normal axis has

$$I_2 = \frac{1}{2} MR^2.$$

The ring axis specified in the question is a diameter, not the normal axis.

Step 2: Evaluate the cylinder and sphere moments A uniform solid cylinder about its symmetry axis has the same radial mass dependence as a disc, giving

$$I_3 = \frac{1}{2} MR^2.$$

For a uniform solid sphere about any diameter,

$$I_4 = \frac{2}{5} MR^2.$$

Since $1/2 > 2/5$, the first three moments are equal and all are larger than the fourth. The common factors M and R^2 are positive and do not affect this comparison.

Step 3: Check the other proposed relations In units of MR^2, the four coefficients are $0.5, 0.5, 0.5, 0.4$. Option (B) compares 1.0 with 0.9, so its less-than sign is false. Option (C) reverses the sphere comparison. In option (D), the left side is 1.0, while the right side is $0.5 + (5/2)(0.4) = 1.5$, so that

equality is also false.

Step 4: Conclusion $I_1 = I_2 = I_3 > I_4$. Hence, the correct option is (A).

Quick Tip to Solve: Before using a standard moment-of-inertia formula, underline the axis: a ring about a diameter has half its moment about the central normal.

49. Given below are two statements :

Statement I : Two photons having equal linear momenta have equal wavelengths.

Statement II : If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (D) Statement I is true but Statement II is false

Solution:

Concept:

- For a photon, momentum magnitude and wavelength are related by $p = h/\lambda$, with Planck constant h fixed.
- Photon energy is $E = h\nu = hc/\lambda = pc$. Both momentum magnitude and energy are inversely proportional to wavelength.

Step 1: Evaluate Statement I Consider two photons with equal momentum magnitudes, $p_1 = p_2$. Applying the photon momentum relation to each,

$$\lambda_1 = \frac{h}{p_1}, \qquad \lambda_2 = \frac{h}{p_2}.$$

The denominators are equal, so $\lambda_1 = \lambda_2$. Equal momentum vectors also necessarily have equal magnitudes and lead to the same result. Statement I is therefore true; no assumption about equal intensity or the number of photons is needed.

Step 2: Evaluate Statement II If wavelength is decreased from λ to a smaller positive value λ', then

$$\frac{p'}{p} = \frac{\lambda}{\lambda'} > 1, \qquad \frac{E'}{E} = \frac{\lambda}{\lambda'} > 1.$$

Both momentum and energy increase rather than decrease. For example, halving the wavelength doubles each quantity. Statement II has the inverse dependence backwards and is false.

Step 3: Match the pair of truth values The required combination is Statement I true and Statement II false, which is option (D). Option (A) reverses both truth values, option (B) wrongly rejects the

equality of wavelengths for equal photon momenta, and option (C) wrongly accepts the claimed decrease in energy and momentum. This conclusion concerns individual photons rather than the total energy in a beam.

Step 4: Conclusion Statement I is true, while Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: Remember the linked photon relations $E = pc = hc/\lambda$: a shorter wavelength means a more energetic photon with greater momentum.

50. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be : [4 marks]

(A) 0.875

(B) 0.5

(C) 3.5

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- The emitter current splits into collector and base currents: $I_E = I_C + I_B$. The corresponding small changes obey the same relation.
- The common-emitter current gain is $\beta = \Delta I_C/\Delta I_B$, whereas the common-base gain is $\alpha = \Delta I_C/\Delta I_E$. Both gains are dimensionless.

Step 1: Find the base-current change The given changes are $\Delta I_E = 4\,\text{mA}$ and $\Delta I_C = 3.5\,\text{mA}$. Applying current conservation at the transistor gives

$$\Delta I_B = \Delta I_E - \Delta I_C = 4 - 3.5 = 0.5\,\text{mA}.$$

The base receives the part of the emitter-current change that does not pass to the collector. We must use this smaller current change in the denominator for common-emitter gain.

Step 2: Calculate and compare the gains Substitute the two changes expressed in the same unit:

$$\beta = \frac{3.5}{0.5} = 7.$$

As a cross-check, $\alpha = 3.5/4 = 0.875$ and $\beta = \alpha/(1-\alpha) = 0.875/0.125 = 7$. Thus option (A) gives the common-base gain, rather than the requested common-emitter gain. The number 0.5 in option (B) is the base-current change in mA, and 3.5 in option (C) is the collector-current change in mA; neither is the current ratio requested.

Step 3: Conclusion The common-emitter current gain is $\boxed{\beta = 7}$. Hence, the correct option is (D).

Quick Tip to Solve: When emitter and collector current changes are given, first subtract them to obtain the base-current change; do not mistake α for β.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. An unpolarised light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be _____ Lumens. [4 marks]

Solution:

Concept:

- Malus law gives the transmitted intensity of an ideal analyzer as $I = I_p \cos^2\theta$, where I_p is the intensity after the polarizer and θ is the angle between the transmission axes.
- For initially parallel polarizer and analyzer axes, the initial transmitted intensity is the maximum, I_p. Rotating the analyzer changes the relative angle, without changing the light incident on it.

Step 1: Identify the intensity incident on the analyzer Use the initially parallel-axis configuration of the polarization experiment. At the initial position $\theta = 0$, so

$$I_{\text{initial}} = I_p \cos^2 0 = I_p = 100\,\text{lumens}.$$

The incident beam is unpolarised before the polarizer, but the stated 100 lumens is already measured after the analyzer. Therefore we must not halve this measured value again. The first polarizer has already produced plane-polarised light.

Step 2: Apply the analyzer rotation The new angle between the transmission axes is $30°$. Clockwise versus anticlockwise rotation makes no difference here because the square of the cosine is even. Hence

$$I_{\text{new}} = 100\cos^2 30° = 100\left(\frac{\sqrt{3}}{2}\right)^2 = 75\,\text{lumens}.$$

This value is three quarters of the initial maximum. Strictly, if the initial relative angle were an unspecified θ_0, the ratio would be $\cos^2(\theta_0 + 30°)/\cos^2\theta_0$; the numerical result uses the initial parallel setting.

Step 3: Conclusion For the initial parallel setting, the emerging intensity is $\boxed{75\,\text{lumens}}$, so the required entry is $\boxed{75}$.

Quick Tip to Solve: Apply Malus law to the intensity just before the analyzer. A measured initial maximum must not be divided by two again.

52. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x : y, where x is _____. [4 marks]

Solution:

Concept:

- In the absence of an external impulse, total linear momentum is conserved along both the original direction and the perpendicular direction.
- The balls are identical, so their equal masses cancel from the momentum equations. The equal deflection angles are on opposite sides of the original line of motion.

Step 1: Conserve transverse momentum Take the original direction of the moving ball as the positive horizontal axis. Initially the transverse momentum is zero because one ball moves horizontally and the other is at rest. Let the final speeds be v_1 and v_2, with directions $+30^\circ$ and -30°. Then

$$0 = mv_1 \sin 30^\circ - mv_2 \sin 30^\circ.$$

Since $m \neq 0$ and $\sin 30^\circ \neq 0$, this immediately gives $v_1 = v_2$.

Step 2: Check the speed and state the ratio Conservation along the original direction gives

$$9m = mv_1 \cos 30^\circ + mv_2 \cos 30^\circ.$$

Putting $v_1 = v_2 = v$, we obtain

$$v = \frac{9}{2\cos 30^\circ} = 3\sqrt{3}\,\mathrm{m\,s^{-1}}.$$

The ratio of the final speeds is therefore $v_1 : v_2 = 1 : 1$. No assumption of an elastic collision is needed. Indeed, the final kinetic energy is smaller than the initial kinetic energy for these speeds, so imposing elastic-collision energy conservation would be inappropriate.

Step 3: Conclusion The velocity-magnitude ratio is $\boxed{1:1}$. Thus, in the reduced ratio $x : y$, the required entry is $\boxed{x = 1}$.

Quick Tip to Solve: For equal masses deflected through equal angles on opposite sides, transverse momentum alone gives equal final speeds; use horizontal momentum only as a check.

53. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is _____ $\times$ 10^7 m/s. [4 marks]

Solution:

Concept:

- For a homogeneous medium, the electromagnetic wave speed is $v = 1/\sqrt{\mu\varepsilon}$, where μ and ε are its permeability and permittivity.
- Using $\mu = \mu_r\mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$ and $c = 1/\sqrt{\mu_0\varepsilon_0}$, the speed becomes $v = c/\sqrt{\mu_r\varepsilon_r}$.

Step 1: Form the material factor The two relative material constants are separately specified:

$$\varepsilon_r = 2, \qquad \mu_r = 2.$$

Their product enters the wave-speed formula. Hence

$$\sqrt{\mu_r \varepsilon_r} = \sqrt{2 \times 2} = 2.$$

One must not use only the electric permittivity, since this medium also has a relative magnetic permeability different from unity.

Step 2: Calculate and express the requested coefficient Taking the vacuum light speed as $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$v = \frac{3.0 \times 10^8}{2} = 1.5 \times 10^8\,\mathrm{m\,s^{-1}}.$$

To match the power of ten printed in the blank, rewrite this as

$$v = 15 \times 10^7\,\mathrm{m\,s^{-1}}.$$

The given frequency is $5\,\mathrm{GHz}$. It is not an additional multiplicative factor in the speed formula; it would instead determine the wavelength through $\lambda = v/f = 0.030\,\mathrm{m}$. The speed is lower than the vacuum speed, as expected for the stated material factor.

Step 3: Conclusion The wave speed is $\boxed{15 \times 10^7\,\mathrm{m\,s^{-1}}}$. The required numerical entry is $\boxed{15}$.

Quick Tip to Solve: Multiply the two relative material constants before taking the square root, then rewrite the speed using the power of ten specified beside the blank.

54. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _____ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston. [4 marks]

Solution:

Concept:

- Pascal law equates the pressure transmitted through an ideal hydraulic fluid: $F_s/A_s = F_L/A_L$. Thus the supported load is proportional to A_L/A_s for a fixed input force.
- For a circular piston, $A = \pi d^2/4$; changing a diameter by a factor changes the area by the square of that factor.

Step 1: Write the initial and final load relations Let the original smaller and larger piston areas be A_s and A_L. A mass m supplies force mg at the smaller piston and supports a mass of 100 kg on the larger piston. Therefore

$$\frac{mg}{A_s} = \frac{100g}{A_L}, \qquad 100 = m\frac{A_L}{A_s}.$$

The input mass remains unchanged after the piston diameters are altered. Gravity cancels, so the supported mass changes exactly as the area ratio changes.

Step 2: Square the diameter changes The larger diameter becomes four times its original value, so $A'_L = 16A_L$. The smaller diameter becomes one fourth of its original value, so $A'_s = A_s/16$. The new supported mass is

$$M' = m\frac{16A_L}{A_s/16} = 256m\frac{A_L}{A_s} = 256 \times 100.$$

$$M' = 25600\,\text{kg}.$$

The two factors of 16 reinforce each other: one increases the output area and the other increases the input pressure at the same applied weight. Using diameter ratios directly would miss this square-law amplification.

Step 3: Conclusion The press can lift $\boxed{25600\,\text{kg}}$, so the required entry is $\boxed{25600}$.

Quick Tip to Solve: Convert each diameter change into an area change first. A fourfold increase at one piston and a fourfold decrease at the other multiply the load capacity by 4^4.

55. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction μ = 0.5, the maximum height in cm at which a stationary block will not slip downward is _____ cm. [4 marks]

Solution:

Concept:

- At a point on a curved surface, resolve the weight along the local tangent and normal. Static equilibrium requires $mg\sin\theta \le \mu mg\cos\theta$, or $\tan\theta \le \mu$.
- The local inclination is set by the derivative: $\tan\theta = |dy/dx|$. Use the greatest permitted slope to determine the highest point of equilibrium.

Step 1: Determine the local slope The surface profile is

$$y = \frac{x^2}{4}.$$

Differentiating with respect to the horizontal coordinate gives

$$\frac{dy}{dx} = \frac{x}{2}.$$

On either side of the bottom, the magnitude of the tangent slope is $|x|/2$. The normal reaction is $N = mg\cos\theta$, while the friction needed to prevent downward sliding is $mg\sin\theta$. Its maximum available value is μN.

Step 2: Apply limiting equilibrium and calculate the height The greatest height occurs when the limiting friction is just sufficient:

$$\tan\theta = \mu = 0.5, \qquad \frac{|x|}{2} = 0.5.$$

Thus $|x| = 1$. Substitution into the profile, with the coordinates in the usual SI length unit, gives

$$y_{\max} = \frac{1^2}{4} = 0.25\,\text{m} = 25\,\text{cm}.$$

At larger values of $|x|$, the tangent is steeper and the required friction exceeds μN; at smaller values it does not. Both sides of the parabola give the same maximum height because the height depends on x^2.

Step 3: Conclusion The maximum height at which the block can remain stationary is $\boxed{25\,\text{cm}}$. The numerical entry is $\boxed{25}$.

Quick Tip to Solve: For a curved rough track, apply the incline condition locally: set $|dy/dx| = \mu$ at the limiting point, then substitute its coordinate into the curve.

56. A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _____. [4 marks]

Solution:

Concept:

- For an ideal transformer, the voltage ratio equals the turns ratio: $V_p/V_s = N_p/N_s$. A step-down transformer has fewer secondary turns than primary turns.
- The rectifier converts the secondary alternating supply into a direct supply. In the ideal voltage-ratio model used here, the specified output requirement sets the secondary voltage at 12 V.

Step 1: Identify primary and secondary quantities The primary winding is connected to the domestic alternating supply, so $V_p = 220\,\text{V}$. The radio requires a 12 V supply, and the secondary has $N_s = 24$ turns. Thus use $V_s = 12\,\text{V}$ in the ideal transformer calculation. The transformer itself operates with alternating current; the rectifier is on its secondary side and provides the required current direction for the radio. It does not change the turns ratio relation.

Step 2: Calculate the primary turns Rearrange the ideal transformer equation to isolate the unknown:

$$N_p = N_s \frac{V_p}{V_s}.$$

Substitute the given values:

$$N_p = 24 \times \frac{220}{12} = 2 \times 220 = 440.$$

As a check, the secondary has $24/12 = 2$ turns per volt, so 220 V requires $2 \times 220 = 440$ primary turns. The primary-to-secondary turns ratio is $440 : 24 = 220 : 12$, exactly the required voltage ratio. This is a step-down arrangement, consistent with $N_p > N_s$. No diode drops, filtering details or a separate peak-voltage specification are given for a more detailed rectifier model.

Step 3: Conclusion The number of turns in the primary winding is $\boxed{440}$.

Quick Tip to Solve: Keep the high-voltage side paired with the primary turns: $N_p = N_s V_p/V_s$. A step-down supply must give $N_p > N_s$.

57. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be _____ N. [g = 10 ms^{-2}] [4 marks]

Solution:

Concept:

- The horizontal applied force is balanced by the normal reaction of the wall: $N = F$. There is no horizontal acceleration while the block remains at rest.
- Static friction acts along the wall and adjusts up to its limiting magnitude $f_{s,\max} = \mu_s N$. The weight is mg, vertically downward.

Step 1: Identify the vertical forces The block tends to slide downward under gravity, so the friction exerted by the wall acts upward. Since the applied force is horizontal, it has no vertical component. Vertical equilibrium therefore requires $f_s = mg$. With the given mass and gravitational acceleration,

$$mg = 0.5 \times 10 = 5\,\text{N}.$$

It is the friction, rather than the horizontal push itself, that supports this weight.

Step 2: Apply the limiting-friction condition The available friction must be at least the required 5 N. Thus

$$mg \le \mu_s F, \qquad F \ge \frac{mg}{\mu_s}.$$

At the threshold for just preventing downward slipping, equality holds. Substituting the supplied coefficient gives

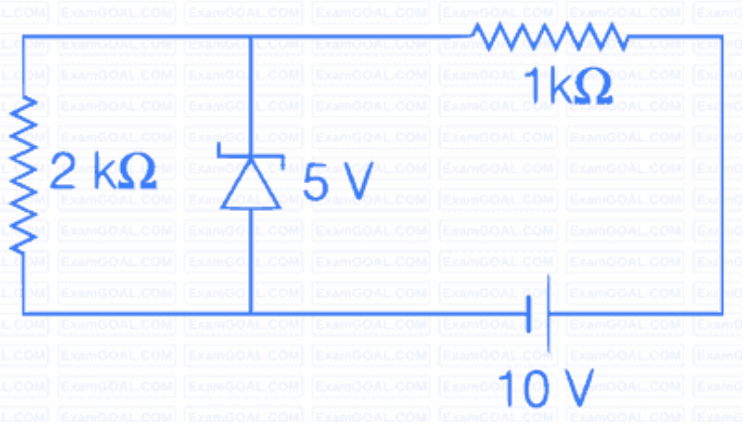
$$F_{\min} = \frac{0.5 \times 10}{0.2} = 25\,\text{N}.$$

For a larger horizontal push the actual static friction is still 5 N; only its maximum possible value increases. Treating static friction as always equal to its limiting value would incorrectly exclude these larger forces.

Step 3: Conclusion The minimum horizontal force required is $\boxed{25\,\text{N}}$, so the numerical entry is 25. Any force at least this large can keep the block at rest in the ideal static-friction model.

Quick Tip to Solve: For a block pressed against a vertical wall, first use horizontal equilibrium to obtain the normal reaction, then impose $\mu_s N \ge mg$.

58. In connection with the circuit drawn below, the value of current flowing through 2 kΩ resistor is _____ $\times 10^{-4}$ A. [4 marks]



Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains its rated voltage across the parallel load, provided that the series resistor supplies sufficient current.
- Components connected across the same two nodes have the same potential difference. The load current follows Ohm's law, $I_L = V_L/R_L$.

Step 1: Check the Zener breakdown condition The circuit places the 5 V Zener diode in parallel with the $2\,\text{k}\Omega$ load, while a $1\,\text{k}\Omega$ resistor connects the combination to the 10 V supply. If the Zener were absent, the divider would give

$$V_L = 10\frac{2}{1+2} = \frac{20}{3}\,\text{V} > 5\,\text{V}.$$

Therefore the reverse voltage is sufficient to bring the Zener into breakdown. In regulation, the load voltage is clamped at 5 V, rather than being the unregulated divider voltage.

Step 2: Find the load and Zener currents The current specifically requested is the current in the $2\,\text{k}\Omega$ branch. Hence

$$I_L = \frac{5}{2\times 10^3} = 2.5\times 10^{-3}\,\text{A} = 25\times 10^{-4}\,\text{A}.$$

As a consistency check, the series resistor carries

$$I_s = \frac{10-5}{1\times 10^3} = 5\,\text{mA}.$$

The remaining current is $I_Z = I_s - I_L = 2.5\,\text{mA}$, which is positive. Thus the assumed conducting Zener state is self-consistent; assigning all the series current to the load would overlook the parallel diode branch.

Step 3: Conclusion The load current is $\boxed{25\times 10^{-4}\,\text{A}}$, so the required numerical coefficient is $\boxed{25}$.

Quick Tip to Solve: Check that the series current exceeds the load current before using the Zener voltage as the load voltage; their difference must flow through the diode.

59. An audio signal v_m = 20 sin 2π(1500t) amplitude modulates a carrier v_c = 80 sin 2π(100,000t). The value of percent modulation is _____. [4 marks]

Solution:

Concept:

- For sinusoidal amplitude modulation, the modulation index is the ratio of the peak message amplitude to the peak unmodulated carrier amplitude: $\mu = V_m/V_c$.
- Percentage modulation is 100μ. The frequencies determine the carrier and sideband positions, but do not replace the amplitude ratio in this definition.

Step 1: Read the peak amplitudes The message is $v_m = 20\sin[2\pi(1500t)]$, so its peak voltage is $V_m = 20$. The unmodulated carrier is $v_c = 80\sin[2\pi(100000t)]$, so its peak voltage is $V_c = 80$. These are the coefficients multiplying the sine functions. The values 1500 and 100000 are the corresponding frequencies in hertz; using their ratio would calculate a frequency ratio, which is not the modulation index.

Step 2: Calculate and check the percentage Using the peak values in the modulation-index definition gives

$$\mu = \frac{V_m}{V_c} = \frac{20}{80} = \frac{1}{4} = 0.25.$$

Therefore

$$\text{Percentage modulation} = 100\mu = 100 \times 0.25 = 25\%.$$

The result is below 100 percent, so the envelope does not undergo overmodulation. A further envelope check gives maximum and minimum amplitudes $80 + 20 = 100$ and $80 - 20 = 60$. Their ratio formula yields $(100 - 60)/(100 + 60) = 40/160 = 0.25$, confirming the same modulation index.

Step 3: Conclusion The percent modulation is $\boxed{25\%}$, and the requested numerical entry is $\boxed{25}$.

Quick Tip to Solve: Read amplitudes from the coefficients outside the sine functions. Convert the amplitude ratio into a percentage only after calculating the dimensionless modulation index.

60. A resonance circuit having inductance and resistance 2 $\times$ 10^{-4} H and 6.28Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is _____. [π = 3.14] [4 marks]

Solution:

Concept:

- For a series resonant RLC circuit, the quality factor is $Q = \omega_0 L/R$, where ω_0 is the resonant angular frequency, L the inductance and R the resistance.
- Ordinary frequency and angular frequency are related by $\omega_0 = 2\pi f_0$. The quality factor is dimensionless and measures the sharpness of resonance.

Step 1: Convert the frequency and write the formula The specified resonant frequency is $f_0 = 10\,\text{MHz} = 10 \times 10^6\,\text{Hz} = 10^7\,\text{Hz}$. The other data are $L = 2 \times 10^{-4}\,\text{H}$, $R = 6.28\,\Omega$, and $\pi = 3.14$. The

angular frequency must be used in the quality-factor expression, so

$$Q = \frac{2\pi f_0 L}{R}.$$

Using $f_0 L/R$ alone would omit the factor 2π, while leaving the frequency in megahertz would produce an error of a million.

Step 2: Substitute and simplify First calculate the inductive reactance at resonance:

$$X_L = 2(3.14)(10^7)(2 \times 10^{-4}) = 12560\,\Omega.$$

Then divide by the resistance:

$$Q = \frac{12560}{6.28} = 2000.$$

Equivalently, $2\pi = 6.28$ cancels the numerical resistance in the direct substitution, leaving $10^7 \times 2 \times 10^{-4} = 2000$. The units cancel because both $\omega_0 L$ and R are measured in ohms. The large value indicates a narrow resonance for this ideal circuit model.

Step 3: Conclusion The quality factor is $\boxed{Q = 2000}$. It is dimensionless, so the required numerical entry is 2000.

Quick Tip to Solve: At resonance compute X_L/R; convert MHz to Hz first and include 2π when turning frequency into angular frequency.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The electrode potential of M^{2+}/M of 3d-series elements shows positive value for : [4 marks]

(A) Zn

(B) Fe

(C) Cu

(D) Co

Correct Answer: (C) Cu

Solution:

Concept:

- The quoted M^{2+}/M electrode potential refers to the reduction half-reaction $M^{2+} + 2\,e^- \longrightarrow M$, measured relative to the standard hydrogen electrode.
- The sign depends on the overall balance of atomisation, ionisation and hydration energies; it does not follow from atomic number alone.

Step 1: Compare the reduction-potential signs For the listed 3d metals, the standard reduction

potentials of the divalent-metal couples are approximately

$$E^\circ(\mathrm{Zn^{2+}/Zn}) = -0.76\,\mathrm{V},$$
$$E^\circ(\mathrm{Fe^{2+}/Fe}) = -0.44\,\mathrm{V},$$
$$E^\circ(\mathrm{Cu^{2+}/Cu}) = +0.34\,\mathrm{V},$$
$$E^\circ(\mathrm{Co^{2+}/Co}) = -0.28\,\mathrm{V}.$$

Only copper has a positive value among these four candidates. The other three couples lie below the hydrogen reference on the standard reduction-potential scale.

Step 2: Explain why copper is the exception Converting copper metal into hydrated $\mathrm{Cu^{2+}}$ requires substantial energy for atomisation and ionisation. In particular, removing the second electron disrupts the stable filled $3d^{10}$ configuration of $\mathrm{Cu^+}$. Hydration does not compensate fully for the overall energy cost, so oxidation of copper by standard hydrogen ions is unfavourable. Accordingly, the reverse reduction of $\mathrm{Cu^{2+}}$ has a positive potential. Zinc, iron and cobalt all have negative reduction potentials for the divalent couples, so none satisfies the requested positive sign.

Step 3: Conclusion Copper has the positive $\mathrm{M^{2+}/M}$ reduction potential. Hence, the correct option is (C).

Quick Tip to Solve: Keep reduction and oxidation conventions separate: reversing the half-reaction reverses the potential sign. Copper is the positive divalent couple among these choices.

62. Which of the following reagent is used for the following reaction ?

$$\mathrm{CH_3CH_2CH_3} \xrightarrow{?} \mathrm{CH_3CH_2CHO}$$

[4 marks]

(A) Molybdenum oxide

(B) Manganese acetate

(C) Copper at high temperature and pressure

(D) Potassium permanganate

Correct Answer: (A) Molybdenum oxide

Solution:

Concept:

- Controlled catalytic oxidation of an alkane can convert a terminal methyl group into an aldehyde while retaining the carbon skeleton.
- The catalyst and reaction conditions determine how far oxidation proceeds. Strong oxidants need not stop at the aldehyde stage.

Step 1: Identify the required transformation The reactant is propane, $\mathrm{CH_3CH_2CH_3}$, and the product is propanal, $\mathrm{CH_3CH_2CHO}$. Both contain three carbon atoms. A terminal $-\mathrm{CH_3}$ group has become $-\mathrm{CHO}$, so the desired process is controlled oxidation rather than carbon-chain cleavage. The net

oxygen balance can be represented as

$$CH_3CH_2CH_3 + O_2 \longrightarrow CH_3CH_2CHO + H_2O.$$

The catalyst must permit oxidation to the aldehyde without continuing extensively to the carboxylic acid or complete combustion.

Step 2: Choose the catalyst and reject the alternatives Molybdenum oxide is the catalyst associated with this controlled oxidation of an alkane to an aldehyde. Manganese acetate is associated with other controlled oxidation conditions, notably oxidation of ethane to ethanoic acid, rather than the stated propane-to-propanal transformation. Copper under high-temperature, high-pressure oxidation conditions is classically associated with methane-to-methanol conversion; copper also catalyses alcohol dehydrogenation, but the reactant here is an alkane, not an alcohol. Potassium permanganate is a strong oxidising agent and is not the selective catalyst specified for stopping this alkane oxidation at propanal.

Step 3: Conclusion The required catalyst is molybdenum oxide. Hence, the correct option is (A).

Quick Tip to Solve: Identify whether the starting material is an alkane or an alcohol before selecting an oxidation reagent; propane-to-propanal is a controlled catalytic oxidation.

63. Which of the following are isostructural pairs ?

A. SO_4^{2-} **and** CrO_4^{2-}

B. $SiCl_4$, and $TiCl_4$

C. NH_3 and NO_3^-

D. BCl_3 and $BrCl_3$ [4 marks]

(A) B and C only

(B) C and D only

(C) A and B only

(D) A and C only

Correct Answer: (C) A and B only

Solution:

Concept:

- Isostructural species have the same molecular geometry. Count the bonding regions and lone pairs at the central atom before comparing shapes.
- Four bonding regions without a central lone pair give a tetrahedral shape; three bonding regions without a central lone pair give a trigonal planar shape.

Step 1: Check pairs A and B In pair A, both SO_4^{2-} and CrO_4^{2-} have four oxygen atoms arranged around the central atom in a tetrahedral geometry. The resonance representations do not change this four-coordinate arrangement, so A is an isostructural pair. In pair B, both molecular $SiCl_4$ and $TiCl_4$ have four chlorine atoms surrounding the central atom tetrahedrally. Neither has a central lone pair that changes the molecular shape. Thus B is also isostructural; their different central elements do not

prevent the same geometry.

Step 2: Check pairs C and D In pair C, NH_3 has three bonds and one lone pair on nitrogen, giving a trigonal pyramidal shape. The nitrate ion, NO_3^-, has three bonding regions with no central lone pair and is trigonal planar. Hence C is not isostructural. In pair D, BCl_3 is trigonal planar, whereas $BrCl_3$ has three bonds and two lone pairs on bromine. Its electron-pair arrangement is trigonal bipyramidal, with lone pairs occupying equatorial sites, and its molecular shape is T-shaped. Hence D is not isostructural. Choices containing C or D are therefore excluded.

Step 3: Conclusion Only pairs A and B are isostructural. Hence, the correct option is (C).

Quick Tip to Solve: Compare molecular shapes rather than merely the number of attached atoms: central lone pairs make NH_3 pyramidal and $BrCl_3$ T-shaped.

64. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is : [4 marks]

(A) Al $<$ Mg $<$ Si $<$ S $<$ P

(B) Al $<$ Mg $<$ S $<$ Si $<$ P

(C) Mg $<$ Al $<$ Si $<$ S $<$ P

(D) Mg $<$ Al $<$ Si $<$ P $<$ S

Correct Answer: (A) Al $<$ Mg $<$ Si $<$ S $<$ P

Solution:

Concept:

- First ionization enthalpy is the energy required to remove the most loosely bound electron from an isolated gaseous atom: $M(g) \longrightarrow M^+(g) + e^-$.
- Across period 3 the effective nuclear attraction generally increases, but filled and half-filled subshells produce the important exceptions at Mg/Al and P/S.

Step 1: Compare magnesium and aluminium Magnesium has configuration $[Ne]3s^2$, whereas aluminium has $[Ne]3s^2 3p^1$. The first electron removed from aluminium is a higher-energy, less penetrating $3p$ electron. It is easier to remove than an electron from the filled $3s$ subshell of magnesium. Thus $I_1(Al) < I_1(Mg)$, despite the general left-to-right increase.

Step 2: Place silicon, sulfur and phosphorus Silicon lies beyond magnesium and aluminium and has a larger effective nuclear attraction, giving a higher first ionization enthalpy than either. Phosphorus has the relatively stable half-filled configuration $3p^3$, with one electron in each p orbital. Sulfur has $3p^4$, so one orbital contains an electron pair. Repulsion within that pair makes electron removal easier in sulfur than in phosphorus. Therefore the last three entries are Si $<$ S $<$ P. Option (B) incorrectly places sulfur below silicon; (C) and (D) incorrectly place magnesium below aluminium. Option (D) additionally reverses the phosphorus-sulfur exception.

Step 3: Conclusion The increasing order is $\boxed{Al < Mg < Si < S < P}$. Hence, the correct option is (A).

Quick Tip to Solve: Apply the period trend first, then check the filled s^2 and half-filled p^3 exceptions before choosing an order.

65. Out of the following, which type of interaction is reponsible for the stabilisation of α-helix structure of proteins ? [4 marks]

(A) Covalent bonding

(B) vander Waals forces

(C) Ionic bonding

(D) Hydrogen bonding

Correct Answer: (D) Hydrogen bonding

Solution:

Concept:

- The α-helix is a secondary structure in which a polypeptide backbone coils into a regular helix.
- Its characteristic stabilising interactions are intramolecular hydrogen bonds between peptide carbonyl oxygen atoms and peptide $\mathrm{N-H}$ groups.

Step 1: Locate the hydrogen-bond donors and acceptors Each peptide linkage contains a carbonyl group and usually an $\mathrm{N-H}$ group. Oxygen carries a partial negative charge, while hydrogen bonded to nitrogen carries a partial positive charge. In the helical arrangement, the carbonyl oxygen of residue i accepts a hydrogen bond from the $\mathrm{N-H}$ group of residue $i+4$. This repeated pattern links nearby turns of the same polypeptide chain and favours the regular coil.

Step 2: Distinguish the other interactions Covalent peptide bonds join amino-acid residues and maintain the primary sequence; they do not identify the interaction responsible for stabilising this particular secondary structure. Van der Waals forces can contribute to molecular packing but are not the characteristic repeated links of an α-helix. Ionic interactions involve charged groups and can influence tertiary structure, but are not the regular backbone interaction asked for here. Therefore options (A), (B) and (C) do not describe the defining stabilising mechanism. The side chains project outward from the helix while the hydrogen bonds occur within its backbone framework.

Step 3: Conclusion The α-helix is stabilised by intramolecular hydrogen bonding between peptide groups. Hence, the correct option is (D).

Quick Tip to Solve: For protein structure questions, associate peptide bonds with primary structure and the regular backbone hydrogen-bond pattern with the α-helix.

66. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH ? [4 marks]

(A) 4-Methyl-5-propylbenzene-1,2-diol

(B) 6-Methyl-4-propylbenzene-1,3-diol

(C) 2-Propylphenol

(D) 4-Propylphenol

Correct Answer: (C) 2-Propylphenol

Solution:

Concept:

- Phenol condenses with phthalic anhydride in concentrated H_2SO_4 to form a phthalein dye; its alkaline form can be pink because of an extended conjugated quinonoid structure.
- The usual phenolphthalein-type condensation uses the para position of a phenolic ring. Both the substitution pattern and the number of hydroxyl groups matter when distinguishing the colour tests.

Step 1: Inspect the substituted phenols Option (C) is 2-propylphenol: the propyl group occupies an ortho position relative to OH, leaving the para carbon available for condensation. Two such phenolic rings can react with phthalic anhydride under acid catalysis to form the corresponding substituted phenolphthalein framework. The propyl group does not block the required para position.

Step 2: Identify the alkaline colour and exclude the distractors On treatment with NaOH, deprotonation and conversion to a conjugated quinonoid form give the pink colour of the phthalein dye. Option (D), 4-propylphenol, already has its para position occupied by propyl and therefore cannot undergo the same ordinary para condensation. Option (A) is an ortho-dihydroxybenzene derivative, and option (B) is a resorcinol-type, meta-dihydroxybenzene derivative. These do not give the characteristic simple phenolphthalein test sought here; resorcinol-type condensation is associated with fluorescein-type dyes rather than this pink phenolphthalein response. Thus choosing a compound merely because it contains phenolic hydroxyl groups is insufficient.

Step 3: Conclusion 2-Propylphenol has the required free para position and gives the pink phthalein colour in alkaline medium. Hence, the correct option is (C).

Quick Tip to Solve: Before applying the phenolphthalein test, mark the para position relative to OH; a para alkyl substituent blocks the usual condensation site.

67. Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Statement I is true but Statement II is false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (C) Both Statement I and Statement II are false

Solution:

Concept:

- On heating, borax gives boric anhydride, which reacts with metallic oxides to form characteristic metaborate beads.
- A non-luminous oxidising flame favours cupric copper, Cu^{2+}; a luminous reducing flame can reduce it to cuprous copper, Cu^{+}, or metallic copper.

Step 1: Evaluate Statement I Cupric metaborate is $Cu(BO_2)_2$, and it produces a coloured copper bead, characteristically blue or blue-green depending on temperature and conditions. It is not colourless. In a reducing flame copper(II) may be reduced to copper(I), giving colourless cuprous metaborate, or further to red metallic copper. Although the reduction direction named in the statement is plausible, the statement explicitly calls the cupric compound colourless, making the complete statement false.

Step 2: Evaluate Statement II and choose the truth pair Heating copper sulphate supplies copper(II) oxide, which reacts with boric anhydride in the oxidising, non-luminous flame. The relevant bead-forming reaction is

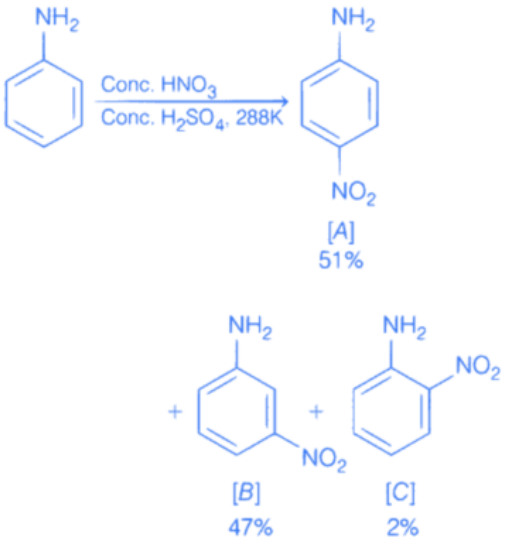
$$CuO + B_2O_3 \longrightarrow Cu(BO_2)_2.$$

The product is cupric metaborate, not cuprous metaborate. Formation of the cuprous compound requires reducing conditions. Statement II therefore incorrectly identifies the oxidation state and is also false. Option (A) wrongly makes Statement I true, option (B) wrongly makes Statement II true, and option (D) wrongly makes both true. The word describing the colour must be checked as carefully as the flame condition.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (C).

Quick Tip to Solve: Keep the copper oxidation state separate from the flame colour: oxidising flame gives coloured cupric metaborate; reducing conditions can give colourless cuprous material.

68. In the following reaction the reason why meta-nitro product also formed is : [4 marks]



(A) $-NH_2$ group is highly meta-directive

(B) $-NO_2$ substitution always takes place at meta-position

(C) Formation of anilinium ion

(D) low temperature

Correct Answer: (C) Formation of anilinium ion

Solution:

Concept:

- The free amino group $-NH_2$ donates its lone pair to the benzene ring by resonance, activating ortho and para positions.
- In strongly acidic nitrating mixture, aniline is protonated to the anilinium ion $C_6H_5NH_3^+$. This positively charged substituent is deactivating and meta-directing.

Step 1: Account for protonation in the reaction medium The reaction uses concentrated HNO_3 and concentrated H_2SO_4, so the amino nitrogen is exposed to a strongly acidic medium. The acid-base reaction is

$$C_6H_5NH_2 + H^+ \rightleftharpoons C_6H_5NH_3^+.$$

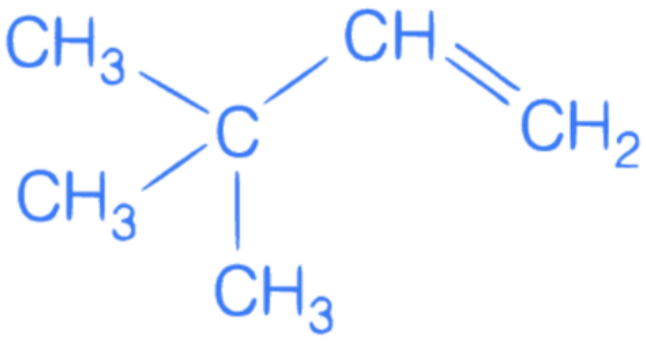
Protonation uses the nitrogen lone pair to form the new $N-H$ bond. Consequently that lone pair is no longer available for the resonance donation characteristic of free aniline.

Step 2: Relate the ionic form to the meta product The positively charged $-NH_3^+$ group withdraws electron density and particularly destabilises the intermediates for ortho or para attack. Relative to those pathways, meta substitution is favoured for the protonated species. The mixture can therefore contain a substantial meta-nitro product even though unprotonated aniline directs predominantly ortho and para. Option (A) is false because free $-NH_2$ is not meta-directing. Option (B) is false because the incoming nitro group does not always enter a meta position; orientation depends on the group already on the ring. Low temperature alone, option (D), does not explain the reversal of directing effect.

Step 3: Conclusion The meta-nitro product forms because aniline is protonated to the meta-directing anilinium ion in the acidic medium. Hence, the correct option is (C).

Quick Tip to Solve: For aniline in concentrated acid, first replace the directing group mentally by $-NH_3^+$, then apply electrophilic-substitution orientation rules.

69. What is the major product formed by HI on reaction with [4 marks]



(A) $(CH_3)_3C-CH(I)-CH_3$

(B) $CH_3-CH(CH_3)-CH(I)-CH_2-CH_3$

(C) $(CH_3)_3C-CH_2-CH_2I$

(D) $CH_3-C(I)(CH_3)-CH(CH_3)-CH_3$

Correct Answer: (D) $CH_3-C(I)(CH_3)-CH(CH_3)-CH_3$

Solution:

Concept:

- Addition of HI to an unsymmetrical alkene proceeds through protonation followed by attack of iodide on a carbocation.
- A secondary carbocation adjacent to a quaternary carbon can rearrange by a 1,2-methyl shift if that produces a more stable tertiary carbocation.

Step 1: Form the initial Markovnikov carbocation The alkene is $(CH_3)_3C-CH=CH_2$. Proton addition at its terminal CH_2 carbon gives the secondary carbocation

$$(CH_3)_3C-CH^+-CH_3.$$

The alternative protonation would generate a primary terminal carbocation and is less favourable. Direct attack of iodide on this initial secondary centre would give the unrearranged product represented by option (A), so stopping at Markovnikov orientation alone is not enough.

Step 2: Shift a methyl group and capture iodide The adjacent quaternary carbon has no hydrogen available for a hydride shift, but it has methyl groups that can migrate with their bonding electron pair. A 1,2-methyl shift transfers one methyl group to the positively charged carbon and moves the positive charge to the neighbouring carbon. The resulting ion is $(CH_3)_2C^+-CH(CH_3)_2$, a tertiary

carbocation. Iodide attack gives $(CH_3)_2C(I)-CH(CH_3)_2$. Option (B) has a different carbon skeleton and does not represent this single favourable rearrangement. Option (C) places iodine at the terminal carbon, contrary to the preferred ionic pathway. Option (D) is the rearranged tertiary iodide.

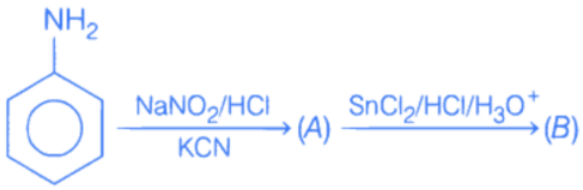
Step 3: Conclusion The major product is 2-iodo-2,3-dimethylbutane, given by

$$CH_3-C(I)(CH_3)-CH(CH_3)-CH_3.$$

Hence, the correct option is (D).

Quick Tip to Solve: After Markovnikov protonation, inspect the adjacent carbon for a hydride or alkyl shift before adding the nucleophile.

70. 'A' and 'B' in the following reactions are : [4 marks]



(A) A: benzenediazonium chloride; B: chlorobenzene

(B) A: benzenediazonium chloride; B: benzaldehyde

(C) A: benzonitrile; B: chlorobenzene

(D) A: benzonitrile; B: benzaldehyde

Correct Answer: (D) A: benzonitrile; B: benzaldehyde

Solution:

Concept:

- Aniline reacts with $NaNO_2/HCl$ to form benzenediazonium chloride; replacement of the diazonium group by cyanide gives benzonitrile.
- Stephen reduction converts a nitrile to an aldehyde using $SnCl_2/HCl$, followed by hydrolysis of the intermediate iminium salt.

Step 1: Identify intermediate A The first reagent combination diazotises aniline: nitrous acid generated from $NaNO_2/HCl$ converts $-NH_2$ into the diazonium group. The subsequent cyanide substitution introduces a carbon atom as $-CN$, with loss of nitrogen. Thus the product labelled A after the full first sequence is benzonitrile, C_6H_5CN, rather than the benzenediazonium chloride intermediate. This rules out options (A) and (B), both of which stop before the cyanide substitution.

Step 2: Apply the reduction and hydrolysis sequence The reagent $SnCl_2/HCl$ partially reduces the nitrile to its iminium salt. Acidic hydrolysis, represented by H_3O^+, then releases the aldehyde. In condensed form the conversion is

$$C_6H_5CN \xrightarrow{SnCl_2/HCl} C_6H_5CH{=}NH \xrightarrow{H_3O^+} C_6H_5CHO.$$

The imine is shown schematically; it is protonated as its salt in the acidic reaction medium. Product B is therefore benzaldehyde. Chlorobenzene would require replacement by chlorine, not the nitrile reduction and hydrolysis indicated here, so option (C) is also excluded. The nitrile carbon becomes the aldehyde carbon and remains attached to the benzene ring.

Step 3: Conclusion $A = C_6H_5CN$ (benzonitrile) and $B = C_6H_5CHO$ (benzaldehyde). Hence, the correct option is (D).

Quick Tip to Solve: Read every reagent below an arrow: diazotisation followed by cyanide gives a nitrile, and $SnCl_2/HCl$ followed by water signals Stephen aldehyde synthesis.

71. Identify products A and B. [4 marks]



(A) A: 2-methylcyclopentan-1-ol; B: 2-methylcyclopentan-1-one.

(B) A: $OHC(CH_2)_3COCH_3$; B: $HOOC(CH_2)_3COCH_3$.

(C) A: 1-methylcyclopentane-1,2-diol; B: 2-hydroxy-2-methylcyclopentan-1-one.

(D) A: 1-methylcyclopentane-1,2-diol; B: 2-methylcyclopent-2-en-1-one.

Correct Answer: (C) A: 1-methylcyclopentane-1,2-diol; B: 2-hydroxy-2-methylcyclopentan-1-one.

Solution:

Concept:

- Cold dilute $KMnO_4$ hydroxylates an alkene: two hydroxyl groups add across its carbon–carbon double bond, producing a vicinal diol.
- Chromium(VI) oxide oxidises a secondary alcohol to a ketone. A tertiary alcohol lacks a hydrogen on its hydroxyl-bearing carbon and does not undergo this ordinary alcohol oxidation.

Step 1: Identify the two alkene carbons The reactant is 1-methylcyclopentene. One double-bond carbon bears a methyl group and is bonded to two other carbons of the ring; the neighbouring double-bond carbon bears hydrogen. The cold dilute reagent at 273 K favours addition of hydroxyl groups rather than oxidative cleavage of the ring.

Step 2: Form the vicinal diol A Add one hydroxyl group to each of the two formerly double-bonded carbons. This gives 1-methylcyclopentane-1,2-diol. The hydroxyl group on the methyl-bearing carbon is tertiary, whereas the adjacent hydroxyl group is secondary. Thus a product with only one hydroxyl group cannot be A, and the open-chain aldehyde/ketone is also inconsistent with these mild conditions.

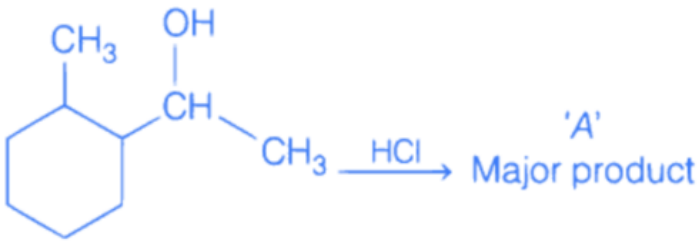
Step 3: Oxidise the secondary alcohol selectively On treatment with CrO_3, the secondary hydroxyl-bearing carbon becomes a carbonyl carbon. The neighbouring tertiary hydroxyl group remains. Numbering the ketone carbon as carbon 1, B is 2-hydroxy-2-methylcyclopentan-1-one. Option D

incorrectly removes the tertiary hydroxyl group and introduces another double bond; that is not the stated oxidation sequence.

Step 4: Conclusion A is 1-methylcyclopentane-1,2-diol and B is 2-hydroxy-2-methylcyclopentan-1-one. Hence, the correct option is (C).

Quick Tip to Solve: After alkene hydroxylation, classify each alcohol carbon separately: only the secondary centre here can become a ketone without breaking carbon–carbon bonds.

72. What is the final product (major) 'A' in the given reaction ? [4 marks]



(A) 1-Chloro-1-ethyl-2-methylcyclohexane.

(B) 1-Methyl-2-vinylcyclohexane.

(C) 1-Ethylidene-2-methylcyclohexane.

(D) 1-(1-Chloroethyl)-2-methylcyclohexane.

Correct Answer: (A) 1-Chloro-1-ethyl-2-methylcyclohexane.

Solution:

Concept:

- In acid, an alcohol is protonated so that water becomes the leaving group. Ionisation of a secondary alcohol can form a secondary carbocation.
- A neighbouring hydride can migrate with its bonding pair when the resulting carbocation is more stable. Chloride then captures the rearranged carbocation.

Step 1: Generate the initial carbocation The hydroxyl-bearing side-chain carbon is attached to a methyl group and a cyclohexyl ring carbon. Protonation by HCl gives an oxonium ion; loss of water leaves a secondary carbocation on that side-chain carbon. Direct replacement at this stage would give the unrearranged chloride in option D, but the neighbouring ring carbon provides a stabilising rearrangement.

Step 2: Apply a hydride shift The ring carbon bonded to the side chain bears a hydrogen. Migration of this hydride to the adjacent carbocation changes the side chain from $-CH^{+}CH_3$ to $-CH_2CH_3$. The positive charge moves onto the ring carbon. That ring carbon is now attached to its two ring neighbours and the ethyl substituent, so the new carbocation is tertiary rather than secondary.

Step 3: Capture the rearranged cation Chloride attacks the tertiary ring carbocation. The chlorine and ethyl group therefore occupy the same ring carbon, and the original methyl substituent remains on the adjacent ring carbon. This is 1-chloro-1-ethyl-2-methylcyclohexane. The vinyl and ethylidene

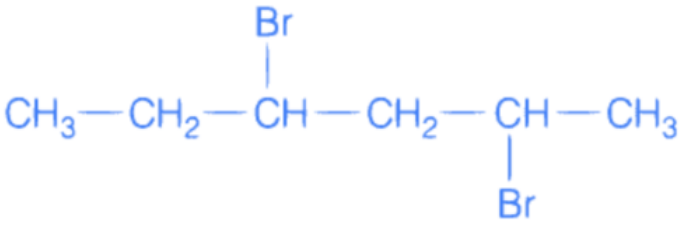
choices are elimination products and do not represent the major rearranged chloride formed with the given reagent.

Step 4: Conclusion The major product is 1-chloro-1-ethyl-2-methylcyclohexane. Hence, the correct option is (A).

Quick Tip to Solve: Before attaching chloride to an alcohol-derived carbocation, check whether one neighbouring hydride shift changes a secondary cation into a tertiary cation.

73. The product formed in the first step of the reaction of

with excess Mg/Et_2O ($Et = C_2H_5$) is [4 marks]



(A) $CH_3CH_2CH(MgBr)CH_2CH(MgBr)CH_3$.

(B) A cyclohexane dimer: join C3 and C5 of the upper chain to C5 and C3, respectively, of a reversed lower chain, with both chains numbered as the original substrate.

(C) A cyclohexane dimer: join C3 and C5 of the upper chain to C3 and C5, respectively, of a lower chain in the same orientation, with both chains numbered as the original substrate.

(D) 1,2-Dimethylcyclopropane.

Correct Answer: (A) $CH_3CH_2CH(MgBr)CH_2CH(MgBr)CH_3$.

Solution:

Concept:

- Magnesium inserts into a carbon–halogen bond in dry ether to form a Grignard reagent: $RBr + Mg \longrightarrow RMgBr$.
- With excess magnesium, both carbon–bromine bonds of a dibromide can undergo insertion. The first product retains the original carbon skeleton.

Step 1: Locate the two reactive bonds The displayed substrate is $CH_3CH_2CH(Br)CH_2CH(Br)CH_3$, which has two secondary carbon–bromine bonds. Dry diethyl ether is the standard coordinating solvent for formation of organomagnesium compounds. It stabilises the magnesium centres without supplying an acidic proton that would immediately destroy the carbon–magnesium bonds.

Step 2: Insert magnesium at both sites Because magnesium is in excess, replace each C–Br bond by a C–MgBr bond. No carbon atom is removed and no new carbon–carbon bond is required for this first step. The stoichiometry is

$$CH_3CH_2CH(Br)CH_2CH(Br)CH_3 + 2\,Mg \longrightarrow CH_3CH_2CH(MgBr)CH_2CH(MgBr)CH_3.$$

The product is therefore a di-Grignard reagent rather than a hydrocarbon.

Step 3: Reject later coupling products The two dimeric cyclohexane choices require intermolecular carbon–carbon coupling and a change in connectivity. The cyclopropane choice also requires new carbon–carbon bond formation and is not the direct magnesium-insertion product asked for in the first step. No water or other proton source is supplied, so replacing bromine by hydrogen is not the specified reaction either.

Step 4: Conclusion The first product is $CH_3CH_2CH(MgBr)CH_2CH(MgBr)CH_3$, the di-Grignard reagent. Hence, the correct option is (A).

Quick Tip to Solve: For the first step with excess magnesium in dry ether, insert Mg into each carbon–halogen bond before considering any subsequent coupling or hydrolysis.

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74. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are : [4 marks]

(A) X = $Na[Al(OH)_4]$, Y = CO_2, Z = $Al_2O_3.xH_2O$

(B) X = $Al(OH)_3$, Y = CO_2, Z = Al_2O_3

(C) X = $Al(OH)_3$, Y = SO_2, Z = $Al_2O_3.xH_2O$

(D) X = $Na[Al(OH)_4]$, Y = SO_2, Z = Al_2O_3

Correct Answer: (A) X = $Na[Al(OH)_4]$, Y = CO_2, Z = $Al_2O_3.xH_2O$

Solution:

Concept:

- Alumina is amphoteric and dissolves in aqueous alkali as soluble sodium tetrahydroxoaluminate, $Na[Al(OH)_4]$.
- Carbon dioxide neutralises the alkaline aluminate solution and precipitates hydrated alumina; heating this precipitate later gives anhydrous alumina.

Step 1: Identify the soluble leaching product X Treatment of Al_2O_3 with hot concentrated sodium hydroxide transfers aluminium into the soluble aluminate complex. In its aqueous representation the reaction is

$$Al_2O_3 + 2\,NaOH + 3\,H_2O \longrightarrow 2\,Na[Al(OH)_4].$$

Thus X must be $Na[Al(OH)_4]$, not insoluble $Al(OH)_3$. This eliminates the alternatives that put aluminium hydroxide at the leaching stage.

Step 2: Identify the gas and precipitate Passing CO_2 lowers the alkalinity and permits precipitation of hydrated aluminium oxide. The common hydroxide representation is

$$2\,Na[Al(OH)_4] + CO_2 \longrightarrow 2\,Al(OH)_3\downarrow + Na_2CO_3 + H_2O.$$

Aluminium hydroxide is a hydrated-alumina form, conventionally written $Al_2O_3.xH_2O$. The question uses that general hydrated-oxide notation for Z.

Step 3: Distinguish precipitation from calcination The precipitate at this stage is hydrated, so an

option assigning anhydrous Al_2O_3 directly to Z skips the subsequent heating step. Calcination removes its water and produces pure alumina for electrolysis. The standard gas in this purification sequence is carbon dioxide, not sulfur dioxide; the combination of soluble aluminate, carbon dioxide and hydrated alumina uniquely matches option A.

Step 4: Conclusion $X = Na[Al(OH)_4]$, $Y = CO_2$, and $Z = Al_2O_3.xH_2O$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the Bayer-process stages distinct: alkali gives soluble aluminate, carbon dioxide gives hydrated alumina, and only heating gives anhydrous alumina.

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75. (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option. [4 marks]

(A) H_2O_2 acts as reducing and oxidising agent respectively in equations (A) and (B).

(B) H_2O_2 act as oxidizing and reducing agent respectively in equations (A) and (B).

(C) H_2O_2 acts as oxidising agent in equations (A) and (B).

(D) H_2O_2 acts as reducing agent in equations (A) and (B).

Correct Answer: (D) H_2O_2 acts as reducing agent in equations (A) and (B).

Solution:

Concept:

- Oxygen has oxidation number -1 in H_2O_2 and 0 in O_2. Conversion of peroxide into oxygen is oxidation.
- A reducing agent supplies electrons and is itself oxidised. An oxidising agent accepts electrons and is itself reduced.

Step 1: Analyse reaction A In HOCl, hydrogen is $+1$ and oxygen is -2, so chlorine is $+1$. Chloride in the products has chlorine at -1, showing that chlorine is reduced by two oxidation-number units. Meanwhile the peroxide oxygen goes from -1 to zero in molecular oxygen. Peroxide therefore loses electrons and acts as the reducing agent.

Step 2: Analyse reaction B Molecular iodine has iodine at oxidation number zero, whereas iodide has iodine at -1. Iodine is again reduced. The peroxide still forms O_2, so its oxygen is oxidised from -1 to zero. The alkaline oxidation half-reaction is

$$H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-.$$

Combining it with $I_2 + 2\,e^- \longrightarrow 2\,I^-$ gives the stated second reaction.

Step 3: Compare the descriptions in the options Peroxide is not becoming water in either listed reaction; it is becoming oxygen. Consequently its ability to act as an oxidising agent in other reactions is irrelevant here. Both mixed-role choices assign the wrong role to one reaction, and the choice claiming oxidation of both other reactants contradicts their decreasing oxidation numbers. The same reducing role applies to both equations.

Step 4: Conclusion H_2O_2 acts as a reducing agent in both of the given reactions. Hence, the correct option is (D).

Quick Tip to Solve: Track peroxide oxygen first: -1 to 0 means peroxide is oxidised and hence is the reducing agent, whatever the medium.

76. Which of the following ore is concentrated using group 1 cyanide salt ? [4 marks]

(A) Sphalerite

(B) Malachite

(C) Siderite

(D) Calamine

Correct Answer: (A) Sphalerite

Solution:

Concept:

- Sulfide ores are commonly concentrated by froth flotation. A depressant prevents a chosen sulfide from joining the froth, allowing selective separation.
- An alkali-metal cyanide such as NaCN acts as a depressant for zinc sulfide, sphalerite, in separation of mixed sulfide ores.

Step 1: Recognise the cyanide salt A group 1 cyanide salt refers to an alkali-metal cyanide, most commonly sodium cyanide, NaCN. In this context it is used as a selective reagent in flotation, not as a statement that every listed ore is dissolved by cyanide. Cyanide depresses the flotation of ZnS by forming soluble zinc cyanide complexes at its surface.

Step 2: Match the ore to its composition Sphalerite is zinc blende, ZnS, and is the sulfide ore to which this cyanide-depressant treatment applies. A representative complex-formation equation is

$$\mathrm{ZnS} + 4\,\mathrm{NaCN} \longrightarrow \mathrm{Na_2[Zn(CN)_4]} + \mathrm{Na_2S}.$$

When ZnS and PbS occur together, depression of sphalerite permits lead sulfide to enter the froth preferentially. The zinc-bearing material can subsequently be recovered by suitable adjustment of flotation conditions.

Step 3: Eliminate the other ore names Malachite is a basic copper carbonate, $\mathrm{CuCO_3 \cdot Cu(OH)_2}$; siderite is iron carbonate, $\mathrm{FeCO_3}$; and calamine is zinc carbonate, $\mathrm{ZnCO_3}$. None is the zinc sulfide selectively depressed by alkali cyanide in the standard mixed-sulfide flotation process. The relevant ore among the four is therefore sphalerite; cyanide need not itself carry that ore into the froth to be used in its concentration procedure.

Step 4: Conclusion The ore is sphalerite, ZnS. Hence, the correct option is (A).

Quick Tip to Solve: In flotation questions, pair NaCN with depression of ZnS; distinguish zinc blende (sulfide) from calamine (carbonate).

77. The major components in "Gun Metal" are : [4 marks]

(A) Al, Cu, Mg and Mn

(B) Cu, Ni and Fe

(C) Cu, Sn and Zn

(D) Cu, Zn and Ni

Correct Answer: (C) Cu, Sn and Zn

Solution:

Concept:

- Gun metal is a copper-based alloy whose principal constituents are copper, tin and zinc.
- Alloy questions are distinguished by the complete set of constituents, rather than by the presence of copper alone.

Step 1: Recall the gun-metal composition Traditional gun metal contains copper as the major component with smaller amounts of tin and zinc. A common nominal formulation is approximately 88% copper, 10% tin and 2% zinc, although commercial compositions can vary. The question asks for the major component elements rather than a fixed numerical composition, so the decisive set is Cu, Sn and Zn.

Step 2: Compare the listed alloy families The aluminium–copper–magnesium–manganese combination in option A describes the familiar duralumin family, in which aluminium is the base metal. Copper–nickel–iron compositions belong to nickel-containing copper alloys rather than the usual tin-containing gun metal. Copper–zinc–nickel in option D is the characteristic constituent set of German silver or nickel silver, despite that alloy containing no silver.

Step 3: Select the matching set Only option C includes both tin and zinc together with copper. Tin is essential to distinguish the gun-metal or bronze family from the copper–zinc brass family and the copper–nickel alloy families. Therefore neither a copper-containing alternative lacking tin nor the aluminium-based alternative answers the question. No additional constituent or processing step is needed to identify the requested alloy from these four sets.

Step 4: Conclusion The major components of gun metal are copper, tin and zinc, Cu, Sn, and Zn. Hence, the correct option is (C).

Quick Tip to Solve: Memorise alloy constituent sets together: gun metal is Cu–Sn–Zn, while German silver is Cu–Zn–Ni and duralumin is aluminium based.

78. The gas released during anaerobic degradation of vegetation may lead to : [4 marks]

(A) Ozone hole

(B) Corrosion of metals

(C) Acid rain

(D) Global warming and cancer

Correct Answer: The key intends (D), but methane establishes global warming, not the additional cancer claim; no compound option is wholly correct.

Solution:

Concept:

- Anaerobic decomposition is microbial breakdown of organic matter in the absence of oxygen. Waterlogged vegetation can release marsh gas, whose principal combustible component is CH_4.
- Methane is a greenhouse gas: absorption of outgoing infrared radiation increases atmospheric heat retention. Different environmental effects must be distinguished by their chemical causes.

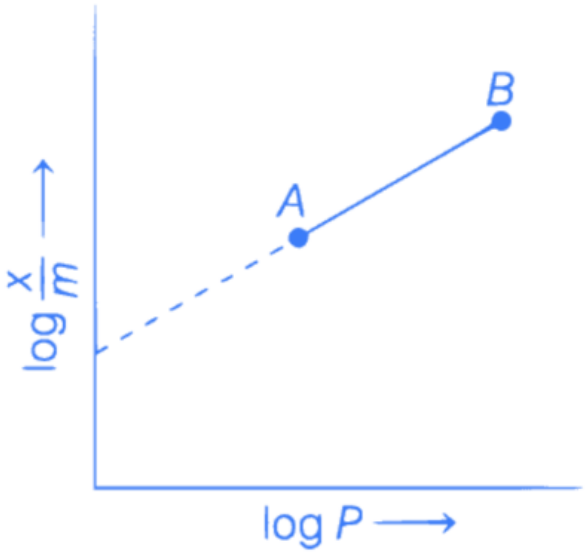
Step 1: Identify the decomposition product Vegetation contains organic carbon. In oxygen-poor sediments, anaerobic microorganisms convert organic material into a mixture containing methane and carbon dioxide. Methane released from wetlands is therefore relevant to the greenhouse effect. This is different from aerobic decay, in which oxygen is consumed and carbon dioxide and water are the main products.

Step 2: Compare the listed environmental effects Ozone depletion is chiefly associated with ozone-destroying radicals from substances such as chlorofluorocarbons. Acid rain is associated mainly with sulfur and nitrogen oxides, and corrosion involves chemical or electrochemical attack on metals. These do not identify the characteristic methane-related effect. The only listed choice containing global warming is (D). Its additional reference to cancer should not be interpreted as establishing that methane itself is a human carcinogen; that is not the chemical basis for selecting this choice.

Step 3: Conclusion The characteristic effect is global warming due to methane. Among the supplied choices, the intended answer is (D), Global warming and cancer, with the qualification that methane itself does not establish the cancer claim. The official key intends (D), but no listed option is wholly correct as worded.

Quick Tip to Solve: First identify marsh gas as methane, then select the environmental effect linked to greenhouse absorption rather than confusing it with acid rain or ozone depletion.

79. In Freundlich adsorption isotherm, slope of AB line is : [4 marks]



(A) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \, to \, 1\right)$
(B) n with (n, 0.1 to 0.5)
(C) $\log \frac{1}{n}$ with $(n < 1)$
(D) log n with (n > 1)
Correct Answer: (A) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \, to \, 1\right)$

Solution:

Concept:

- The Freundlich adsorption isotherm relates the mass of gas adsorbed per unit mass of adsorbent to its equilibrium pressure at a fixed temperature: $x/m = kP^{1/n}$, where k and n are empirical constants.
- Taking logarithms converts a power law into a straight-line relation. In $y = c + st$, the coefficient s of the horizontal-axis variable is the slope, while c is the vertical intercept.

Step 1: Write the logarithmic form The graph has $\log(x/m)$ on the vertical axis and $\log P$ on the horizontal axis. Taking the logarithm of both sides of the adsorption equation gives

$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P.$$

The same logarithm base must be used on both axes. Changing that common base does not change the coefficient $1/n$.

Step 2: Identify the slope of AB Compare the equation with $y = c + st$, using $y = \log(x/m)$ and $t = \log P$. Therefore

$$\text{slope of AB} = \frac{\Delta \log(x/m)}{\Delta \log P} = \frac{1}{n}, \qquad \text{intercept} = \log k.$$

For the usual Freundlich range the exponent lies between zero and one, $0 < 1/n < 1$. The limiting

cases zero and one describe pressure-independent adsorption and directly proportional adsorption, respectively, and explain the range written in option (A).

Step 3: Eliminate the other expressions Option (B) gives n, which would be the reciprocal slope if the two axes were exchanged; they are not exchanged here. Options (C) and (D) take logarithms of constants, whereas the slope is the coefficient multiplying $\log P$. Neither $\log(1/n)$ nor $\log n$ appears as that coefficient. The upward straight line is consistent with a positive adsorption exponent.

Step 4: Conclusion The slope of AB is $1/n$, with the Freundlich exponent conventionally between zero and one. Hence, the correct option is (A).

Quick Tip to Solve: For an adsorption graph, read both axes before naming the slope: the log-log form of $x/m = kP^{1/n}$ has slope $1/n$ and intercept $\log k$.

80. Match List I with List II.

List I(Monomer Unit)	List II(Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1, 3-butadiene	(ii) Buna-N
(c) Isoprene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below: [4 marks]

(A) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

(B) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

(C) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

(D) (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)

Correct Answer: (B) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

Solution:

Concept:

- A polymer is identified by the monomer units that form its repeating chain. Ring-opening polymerisation of caprolactam produces a polyamide, whereas substituted dienes form rubber-like addition polymers.
- A copolymer contains repeating units derived from more than one monomer. Buna-N is formed from buta-1,3-diene and acrylonitrile, so acrylonitrile in the list is matched with Buna-N rather than treated as its only monomer.

Step 1: Match the polyamide precursor Caprolactam, written as caprolactum in List I, is the cyclic amide used to make nylon 6 by ring-opening polymerisation. The six-carbon monomer gives a polyamide chain containing the repeating unit

$$[-\mathrm{NH}-(\mathrm{CH_2})_5-\mathrm{CO}-]_n.$$

Thus (a) matches (iii). This immediately rules out options (A), (C) and (D), but the remaining

matches should also be checked independently.

Step 2: Match the two diene monomers 2-Chloro-1,3-butadiene is chloroprene, $CH_2{=}CCl{-}CH{=}CH_2$. Its addition polymer is neoprene, hence (b) matches (iv). Isoprene is 2-methylbuta-1,3-diene, $CH_2{=}C(CH_3){-}CH{=}CH_2$; natural rubber is predominantly its cis-1,4-polymer. Therefore (c) matches (i). The chlorine substituent identifies neoprene, while the methyl substituent identifies the isoprene units of natural rubber.

Step 3: Complete and verify the copolymer match Acrylonitrile, $CH_2{=}CH{-}CN$, combines with buta-1,3-diene to produce Buna-N, a nitrile rubber. Therefore (d) matches (ii). The complete correspondence is

$$(a) \to (iii), \qquad (b) \to (iv), \qquad (c) \to (i), \qquad (d) \to (ii).$$

Every pair appears together only in option (B). In particular, natural rubber does not come from caprolactam or chloroprene, and nylon 6 does not come from acrylonitrile; these facts also reject the incorrect pairings in the other options.

Step 4: Conclusion Caprolactam gives nylon 6; chloroprene gives neoprene; isoprene gives natural rubber; acrylonitrile is a comonomer of Buna-N. Hence, the correct option is (B).

Quick Tip to Solve: Use the monomer's identifying group: caprolactam means nylon 6, chlorine on the diene means neoprene, and the nitrile group points to Buna-N.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Number of amphoteric compounds among the following is _____.

(A) BeO

(B) BaO

(C) $Be(OH)_2$

(D) $Sr(OH)_2$

[4 marks]

Solution:

Concept:

- An amphoteric oxide or hydroxide reacts with both acids and strong bases. Reacting only with acid identifies an ordinary basic oxide or hydroxide, not an amphoteric one.
- Within group 2, the very small beryllium ion has high polarising power. Its oxide and hydroxide show amphoteric behaviour, while the heavier members give predominantly basic compounds.

Step 1: Test the two beryllium compounds Beryllium oxide reacts with acid and also dissolves in strong aqueous alkali, forming a beryllate complex:

$$BeO + 2\,H^+ \longrightarrow Be^{2+} + H_2O$$

$$BeO + 2\,OH^- + H_2O \longrightarrow [Be(OH)_4]^{2-}.$$

Beryllium hydroxide similarly exhibits both reactions:

$$Be(OH)_2 + 2\,H^+ \longrightarrow Be^{2+} + 2\,H_2O$$

$$Be(OH)_2 + 2\,OH^- \longrightarrow [Be(OH)_4]^{2-}.$$

Both listed beryllium compounds therefore qualify.

Step 2: Exclude the heavier group-2 compounds Barium oxide is a basic oxide; its reaction with water gives barium hydroxide, and with acid it gives a barium salt and water. Strontium hydroxide is a basic hydroxide and neutralises acids. Neither is counted as amphoteric in the usual aqueous acid-base classification. The selected entries are consequently (A) and (C), whereas (B) and (D) are excluded. The count is of compounds, not of the number of different elements present.

Step 3: Conclusion The amphoteric compounds are BeO and $Be(OH)_2$. Their number is $\boxed{2}$.

Quick Tip to Solve: In group 2, flag both the oxide and hydroxide of beryllium as amphoteric; the heavier alkaline-earth compounds are predominantly basic.

82. The stepwise formation of $[Cu(NH_3)_4]^{2+}$ is given below:
The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58 x 10^3, 5 x 10^2 and 10^2 respectively.
The overall equilibrium constants for dissociation of $[Cu(NH_3)_4]^{2+}$ is x $\times$ 10^{-12}.
The value of x is _____. (Rounded off to the nearest integer) [4 marks]

$$Cu^{2+} + NH_3 \overset{K_1}{\rightleftharpoons} [Cu(NH_3)]^{2+}$$

$$[Cu(NH_3)]^{2+} + NH_3 \overset{K_2}{\rightleftharpoons} [Cu(NH_3)_2]^{2+}$$

$$[Cu(NH_3)_2]^{2+} + NH_3 \overset{K_3}{\rightleftharpoons} [Cu(NH_3)_3]^{2+}$$

$$[Cu(NH_3)_3]^{2+} + NH_3 \overset{K_4}{\rightleftharpoons} [Cu(NH_3)_4]^{2+}$$

Solution:

Concept:

- When consecutive complex-formation equilibria are added, their equilibrium constants multiply. For four ligand additions, the overall formation constant is $\beta_4 = K_1K_2K_3K_4$.
- Reversing a chemical equilibrium replaces its constant by its reciprocal. The overall dissociation constant is therefore $K_d = 1/\beta_4$, not a sum of the individual constants.

Step 1: Combine the four ligand additions The intermediate monoammine, diammine and triammine

complexes cancel when all four formation reactions are added. The net equilibrium is

$$\mathrm{Cu}^{2+} + 4\,\mathrm{NH_3} \rightleftharpoons [\mathrm{Cu(NH_3)_4}]^{2+}.$$

Its constant is

$$\begin{aligned}\beta_4 &= (10^4)(1.58 \times 10^3)(5 \times 10^2)(10^2)\\ &= 7.90 \times 10^{11}.\end{aligned}$$

Multiplying the numerical coefficients and adding the powers separately prevents an exponent error.

Step 2: Reverse the equilibrium and round the coefficient The required process dissociates the tetraammine complex back into the copper ion and four ammonia ligands. Thus

$$K_\mathrm{d} = \frac{1}{7.90 \times 10^{11}} = 1.26582 \times 10^{-12}.$$

Comparison with $x \times 10^{-12}$ gives $x = 1.26582\ldots$. Rounding this coefficient to the nearest integer gives 1. The small constant is physically consistent with a large formation constant: the overall complex strongly favours association at standard equilibrium conditions.

Step 3: Conclusion $K_\mathrm{d} = 1.26582 \times 10^{-12}$, so the requested rounded coefficient is $\boxed{x = 1}$.

Quick Tip to Solve: Multiply the stepwise formation constants first, take a single reciprocal for dissociation, and round only the coefficient requested in the question.

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83. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_p}$ is x $\times$ 10^{-1}.
The value of x is _____. (Rounded off to the nearest integer)
[Mass of Li^{3+} = 8.3 mass of proton] [4 marks]

Solution:

Concept:

- For a particle initially at rest accelerated through potential difference V, the kinetic energy is $qV = p^2/(2m)$ in the nonrelativistic regime.
- The de Broglie wavelength is $\lambda = h/p = h/\sqrt{2mqV}$. At the same accelerating potential, both mass and charge must be included in comparing two ions.

Step 1: Form the wavelength ratio The proton has mass m_p and charge e. The lithium nucleus has mass $8.3m_p$ and charge $3e$. Using the same potential for both particles gives

$$\begin{aligned}\frac{\lambda_\mathrm{Li}}{\lambda_p} &= \frac{h/\sqrt{2(8.3m_p)(3e)V}}{h/\sqrt{2m_peV}}\\ &= \sqrt{\frac{m_pe}{(8.3m_p)(3e)}} = \frac{1}{\sqrt{24.9}}.\end{aligned}$$

The common factors h, 2 and V cancel. Equal potential does not imply equal kinetic energy because the charges differ.

Step 2: Evaluate and identify the requested coefficient Numerically, $\sqrt{24.9} \simeq 4.98999$, so

$$\frac{\lambda_{\mathrm{Li}}}{\lambda_p} \simeq 0.200401 = 2.00401 \times 10^{-1}.$$

Hence the unrounded coefficient is $x \simeq 2.00401$, which rounds to 2. The ratio is dimensionless, as expected for two wavelengths. Its value below one also agrees with the larger momentum of the heavier, triply charged lithium nucleus at this potential.

Step 3: Conclusion $\lambda_{\mathrm{Li}}/\lambda_p \simeq 2.00401 \times 10^{-1}$. The requested nearest integer is $\boxed{x = 2}$.

Quick Tip to Solve: For equal accelerating voltage use $\lambda \propto 1/\sqrt{mq}$; ignoring the triply charged lithium ion would give the wrong wavelength ratio.

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84. At 1990 K and 1 atm pressure, there are equal number of Cl_2, molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $Cl_{2\,(g)} \rightleftharpoons 2Cl_{(g)}$ under the above conditions is x $\times$ 10^{-1}.

The value of x is _____. (Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- For an ideal-gas mixture, the partial pressure is $p_i = y_i P$, where y_i is the mole fraction. Equal numbers of particles imply equal mole fractions.
- For $\mathrm{Cl_2(g)} \rightleftharpoons 2\,\mathrm{Cl(g)}$, the pressure expression is $K_p = p_{\mathrm{Cl}}^2/p_{\mathrm{Cl_2}}$, with partial pressures expressed in the stated pressure convention.

Step 1: Determine the equilibrium partial pressures The mixture contains equal numbers of chlorine molecules and chlorine atoms. Writing their amounts as n and n, the total is $2n$. Each species therefore has mole fraction $1/2$. At a total pressure of 1 atm,

$$p_{\mathrm{Cl_2}} = \tfrac{1}{2}(1) = 0.5 \text{ atm}, \qquad p_{\mathrm{Cl}} = 0.5 \text{ atm}.$$

The stoichiometric coefficient 2 does not change this given composition; it enters the equilibrium expression as an exponent.

Step 2: Substitute into the pressure expression The reaction has one gaseous reactant particle and two gaseous product particles, so

$$K_p = \frac{p_{\mathrm{Cl}}^2}{p_{\mathrm{Cl_2}}} = \frac{(0.5)^2}{0.5} = 0.5 = 5 \times 10^{-1}.$$

In the textbook convention using pressures numerically in atm this is the required value; with standard-state pressure factors the equilibrium constant is dimensionless and has the same numerical value for a 1 atm reference. Comparing directly with the question's $x \times 10^{-1}$ gives the exact coefficient 5, requiring no further numerical approximation.

Step 3: Conclusion $K_p = 5 \times 10^{-1}$, so the requested rounded coefficient is $\boxed{x = 5}$.

Quick Tip to Solve: Use the actual equilibrium mole fractions to find partial pressures first; stoichiometric coefficients determine exponents, not the given composition.

85. When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is x $\times$ 10^{-3}.

The value of x is _____. (Rounded off to the nearest integer)

[$K_{f(H_20)}$ = 1.86 K kg mol^{-1}] [4 marks]

Solution:

Concept:

- For a dilute solution, freezing-point depression measures the total number of dissolved particles: $\Delta T_f = iK_f m$, where m is the analytical molality before dissociation.
- Chloroacetic acid is monoprotic: $ClCH_2COOH \rightleftharpoons H^+ + ClCH_2COO^-$. If its degree of dissociation is α, each dissociated molecule adds one particle, so $i = 1 + \alpha$.
- For a weak monoprotic acid of analytical concentration c, the concentration-form dissociation constant is $K_a = c\alpha^2/(1-\alpha)$. Use the dilute aqueous approximation that 500 mL of water has mass 0.500 kg and solution volume approximately 0.500 L.

Step 1: Calculate the analytical concentration The molar mass of $ClCH_2COOH$ is $2(12) + 3(1) + 35.5 + 2(16) = 94.5\,\mathrm{g\,mol^{-1}}$. Thus

$$n = \frac{9.45}{94.5} = 0.100\,\mathrm{mol}, \qquad m = \frac{0.100}{0.500} = 0.200\,\mathrm{mol\,kg^{-1}}.$$

The corresponding dilute-solution molarity is $c \simeq 0.200\,\mathrm{mol\,L^{-1}}$. The mass of solvent, rather than the mass of solute or solution, belongs in molality.

Step 2: Find the dissociated fraction Insert the observed depression before rounding:

$$i = \frac{0.500}{1.86(0.200)} = 1.344086, \qquad \alpha = i - 1 = 0.344086.$$

This is a physically acceptable fraction between zero and one. Treating the acid as completely dissociated would give $i = 2$, inconsistent with the measured depression.

Step 3: Evaluate the equilibrium constant At equilibrium the acid concentration is $c(1-\alpha)$, while each ion concentration is $c\alpha$. Hence

$$K_a = \frac{(c\alpha)^2}{c(1-\alpha)} = \frac{0.200(0.344086)^2}{1 - 0.344086} = 0.0361008.$$

$$K_a = 36.1008 \times 10^{-3}.$$

Retaining digits until this final stage avoids changing the required rounded integer.

Step 4: Conclusion The stated values give $x = \boxed{36}$, rounded to the nearest integer. The supplied key value 34 does not follow from these numerical data under the usual dilute aqueous approximation.

Quick Tip to Solve: First obtain i from the observed freezing depression, then use $\alpha = i - 1$; do not replace $1 - \alpha$ by 1 when dissociation is this appreciable.

86. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is x $\times$ 10^{-1}.

The value of x is _____. (Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- Molarity is the amount of solute in moles divided by the final volume of solution in litres: $M = n/V$. It counts formula units of the stated compound per litre.
- The amount of solute follows from $n = m/M_{\text{mol}}$, with mass and molar mass expressed in compatible units. Millilitres must be converted to litres before substitution.
- The required answer is the coefficient x in $M = x \times 10^{-1}\,\text{mol}\,\text{L}^{-1}$, rather than the decimal molarity itself.

Step 1: Convert the given mass into moles The compound has molar mass $90\,\text{g}\,\text{mol}^{-1}$, and the sample mass is $4.5\,\text{g}$. Therefore

$$n = \frac{4.5\,\text{g}}{90\,\text{g}\,\text{mol}^{-1}} = 0.050\,\text{mol}.$$

No information about dissociation is needed because the question asks the analytical molarity of compound A. Its chemical identity does not enter this mass-to-amount conversion.

Step 2: Divide by the final solution volume The phrase making 250 mL of solution specifies the final volume, not the volume of solvent added. Convert it as

$$V = 250 \times 10^{-3} = 0.250\,\text{L}.$$

Now

$$M = \frac{0.050}{0.250} = 0.200\,\text{mol}\,\text{L}^{-1}.$$

Using 250 directly would produce moles per millilitre and an answer smaller by a factor of one thousand.

Step 3: Extract the requested coefficient Express the concentration with the exact power of ten used in the question:

$$0.200 = 2.00 \times 10^{-1}.$$

Comparison with $x \times 10^{-1}$ gives $x = 2.00$. The final instruction asks for the nearest integer, so the numerical entry is 2. As a check, $0.200\,\text{mol}\,\text{L}^{-1} \times 0.250\,\text{L} = 0.050\,\text{mol}$, recovering the original amount.

Step 4: Conclusion The molarity is $0.200\,\text{M} = 2.00 \times 10^{-1}\,\text{M}$. Hence the requested integer is $\boxed{x = 2}$.

Quick Tip to Solve: For a volume in mL, use $M = 1000m/(M_{\text{mol}}V_{\text{mL}})$, then read off the coefficient of the power of ten requested.

87. The reaction of sulphur in alkaline medium is given below:

The values of 'a' is _____. (Integer answer) [4 marks]

$$S_{8(s)} + a\ OH^-_{(aq)} \longrightarrow b\ S^{2-}_{(aq)} + c\ S_2O_3{}^{2-}_{(aq)} + d\ H_2O_{(l)}$$

Solution:

Concept:

- Sulphur reacts with alkali by disproportionation: elemental sulphur is simultaneously reduced to sulphide and oxidised to thiosulphate. All atoms and the net ionic charge must balance.
- The displayed equation is $S_8 + aOH^- \longrightarrow bS^{2-} + cS_2O_3{}^{2-} + dH_2O$. Its sulphur coefficient is fixed at one molecule of S_8, which determines the scale of every other coefficient.
- Water supplies the hydrogen and oxygen balance in alkaline medium. In the balanced net ionic equation, the total charge on either side must be the same.

Step 1: Write the element-balance equations From the displayed reactants and products, sulphur balance gives

$$8 = b + 2c.$$

Each hydroxide contains one hydrogen and each water molecule contains two, so

$$a = 2d, \qquad d = \frac{a}{2}.$$

Oxygen occurs in hydroxide, thiosulphate and water. Consequently

$$a = 3c + d.$$

Substituting $d = a/2$ gives $a = 6c$ and $d = 3c$. These relations use only element conservation; charge balance supplies the remaining condition.

Step 2: Enforce charge conservation The left side has net charge $-a$. Sulphide and thiosulphate each have charge -2, so the right side has charge $-2b - 2c$. Thus

$$a = 2b + 2c.$$

Combining this with $a = 6c$ gives $b = 2c$. The sulphur equation then becomes

$$8 = 2c + 2c = 4c, \qquad c = 2.$$

Therefore $b = 4$, $a = 12$ and $d = 6$.

Step 3: Check the complete equation The balanced reaction is

$$S_8 + 12\,OH^- \longrightarrow 4\,S^{2-} + 2\,S_2O_3{}^{2-} + 6\,H_2O.$$

It has eight sulphur atoms, twelve oxygen atoms and twelve hydrogen atoms on each side. The net charge is -12 on the left and $-8 - 4 = -12$ on the right. These independent checks confirm the coefficient is scaled to the stated S_8 reactant.

Step 4: Conclusion The coefficient of hydroxide in the given equation is $\boxed{a = 12}$.

Quick Tip to Solve: Keep the printed S_8 coefficient fixed at one; balance H and O first, then use charge to determine the sulphide-to-thiosulphate ratio.

88. For the reaction $A_{(g)} \rightarrow B_{(g)}$ the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is – xR, where x is _____. (Rounded off to the nearest integer)

[R = 8.31 J mol^{-1}K^{-1} and ln 10 = 2.3) [4 marks]

Solution:

Concept:

- The equilibrium constant determines the standard reaction Gibbs energy through $\Delta_r G^\circ = -RT \ln K$. The logarithm is natural, and K is dimensionless.
- A value $K > 1$ gives negative standard Gibbs energy, showing products are thermodynamically favoured in the standard-state comparison. At equilibrium the actual reaction Gibbs energy is zero.
- The requested coefficient multiplies R in $-xR$, so compare with $-RT \ln K$ and read $x = T \ln K$.

Step 1: Interpret the Gibbs-energy quantity The use of a supplied equilibrium constant indicates the standard reaction Gibbs-energy change, conventionally written $\Delta_r G^\circ$. This distinction matters: the actual value obeys $\Delta_r G = \Delta_r G^\circ + RT \ln Q$, and is zero when $Q = K$. The requested nonzero coefficient is obtained from the standard-state relation, with the gas standard pressure taken as the stated 1 atm.

Step 2: Evaluate the logarithm with the given value At 300 K, the equilibrium constant is $100.0 = 10^2$. Using the supplied $\ln 10 = 2.3$,

$$\ln K = \ln(10^2) = 2 \ln 10 = 4.6.$$

Therefore

$$\Delta_r G^\circ = -R(300)(4.6) = -1380R.$$

The minus sign is consistent with K exceeding one; changing it would incorrectly imply that the reactants were favoured in the standard-state comparison.

Step 3: Identify the coefficient and check units Comparison with the required form $-xR$ immediately gives $x = 1380$. If the energy itself is evaluated,

$$\Delta_r G^\circ = -1380(8.31) = -11467.8 \text{ J mol}^{-1}.$$

The coefficient 1380 represents $T \ln K$; multiplying by R, whose units are J mol^{-1} K^{-1}, supplies the energy units. The question asks for the coefficient, not this numerical energy.

Step 4: Conclusion Using the equilibrium-constant relation for the standard reaction Gibbs energy, $\Delta_r G^\circ = -1380R$. The requested rounded integer is $\boxed{x = 1380}$.

Quick Tip to Solve: When Gibbs energy is requested as $-xR$, calculate $T \ln K$ directly; multiplying by the numerical value of R is unnecessary.

89. Gaseous cyclobutene isomerises to butadiene in a first order process which has a 'k' value of 3.3 $\times$ 10^{-4} s^{-1} at 153°C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is _____.

(Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- For a first-order reaction, the integrated rate law is $kt = \ln([A]_0/[A]_t)$, where $[A]_0$ and $[A]_t$ are the initial and remaining reactant concentrations.
- Forty percent completion means 40 percent has reacted and 60 percent remains. The rate law uses the amount remaining, not the fraction converted.
- Because the supplied rate constant is in s^{-1}, direct substitution gives time in seconds. Divide by 60 to obtain minutes before rounding.

Step 1: Express the remaining fraction Let the initial amount of cyclobutene be n_0. At 40 percent completion, the amount converted to butadiene is $0.40n_0$, leaving

$$n_t = (1 - 0.40)n_0 = 0.60n_0.$$

The isomerisation is a one-to-one conversion and follows the stated first-order law. Thus the initial amount cancels from the concentration ratio:

$$\frac{[A]_0}{[A]_t} = \frac{1}{0.60} = \frac{5}{3}.$$

There is no need to assign an arbitrary starting concentration.

Step 2: Apply the integrated rate law At the given temperature, use the supplied $k = 3.3 \times 10^{-4}\,\mathrm{s}^{-1}$ directly. The temperature is already accounted for in this rate constant; no Arrhenius correction is required. Then

$$t = \frac{\ln(5/3)}{3.3 \times 10^{-4}} = \frac{0.510826}{3.3 \times 10^{-4}} = 1547.96\,\mathrm{s}.$$

A half-life calculation would correspond to 50 percent conversion and therefore would not answer this question.

Step 3: Convert the time to minutes Convert the unrounded time to the requested unit:

$$t = \frac{1547.96}{60} = 25.7993\,\mathrm{min}.$$

The nearest integer is 26. A substitution check gives $kt = 0.510826$, and $e^{-kt} = 0.6000$, which reproduces the required fraction of reactant remaining. Rounding seconds too early is avoidable, so the rounding is performed only after conversion to minutes.

Step 4: Conclusion The time needed for 40 percent isomerisation is $25.7993\,\mathrm{min}$, giving the requested integer $\boxed{26}$.

Quick Tip to Solve: Convert percentage completion into the fraction left first: for 40 percent conversion, put 0.60, rather than 0.40, in the first-order logarithm.

90. The coordination number of an atom in a body-centered cubic structure is _____.
[Assume that the lattice is made up of atoms.] [4 marks]

Solution:

Concept:

- The coordination number is the number of nearest neighbouring atoms surrounding a chosen atom. It depends on geometry and nearest-neighbour distance, rather than the number of atoms counted per unit cell.
- A body-centred cubic unit cell has atoms at its eight corners and one atom at the cube centre. The centre atom is equally close to all eight corner atoms.
- For cube edge length a, a centre-to-corner separation is $\sqrt{3}a/2$, while centre-to-centre and edge-neighbour separations are a. Thus the eight corner atoms form the nearest shell.

Step 1: Choose the atom at the body centre Take a cube with a body-centre atom at $(a/2, a/2, a/2)$. Its eight corners have coordinates formed by choosing each coordinate as either 0 or a. The displacement from the centre to any corner therefore has three components, each of magnitude $a/2$. Consequently every corner is at distance

$$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}a}{2}.$$

There are eight such equally distant atoms.

Step 2: Compare with the next neighbours The next body-centre atom in a neighbouring cell lies a distance a away along a cube-edge direction. Since

$$\frac{\sqrt{3}a}{2} \simeq 0.866a < a,$$

the corner atoms are closer and constitute the nearest-neighbour shell. Conversely, a corner atom is surrounded by the body-centre atoms of the eight cubes meeting at that corner, so choosing a different atom gives the same coordination number.

Step 3: Distinguish coordination from cell occupancy The number of atoms attributed to one conventional bcc unit cell is

$$8\left(\frac{1}{8}\right) + 1 = 2.$$

This sharing calculation counts atoms per cell; it does not reduce the number of physically present nearest neighbours around an atom. Likewise, the six atoms at distance a are next-nearest neighbours and are excluded from the coordination number. Counting the closest shell therefore gives eight neighbours.

Step 4: Conclusion Each atom in a body-centred cubic lattice has eight nearest neighbours. Its coordination number is $\boxed{8}$.

Quick Tip to Solve: For bcc, remember the distinct geometric counts: 2 atoms belong to a conventional cell, while 8 atoms surround any atom at the nearest distance.