JEE Main 2021 Paper 1 (B.E./B.Tech) Feb 24 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 1 (February), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 24 February 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)$ X = O, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × 1 null matrix, has : [4 marks]

(A) no solution

(B) exactly two solutions

(C) infinitely many solutions

(D) a unique solution

Correct Answer: (C) infinitely many solutions

Solution:

Concept:

- A real matrix is symmetric when $A^T = A$ and skew-symmetric when $B^T = -B$. Squaring a skew-symmetric matrix produces a symmetric matrix.
- A homogeneous system $CX = O$ always has its zero solution. It has a unique solution exactly when C is nonsingular; a nonzero null vector gives infinitely many scalar multiples.

Step 1: Determine the transpose of the coefficient matrix Put $C = A^2B^2 - B^2A^2$. Since $(A^2)^T = A^2$ and $(B^2)^T = (-B)^2 = B^2$, transposing reverses the order of multiplication:

$$C^T = B^2A^2 - A^2B^2 = -C.$$

Thus the coefficient matrix is skew-symmetric. This conclusion does not assume that A and B commute, so it remains valid for every pair permitted by the question.

Step 2: Use the odd order to test singularity For a matrix of order three,

$$\det C = \det C^T = \det(-C) = (-1)^3 \det C = -\det C.$$

Over the real numbers this gives $2 \det C = 0$, hence $\det C = 0$. Therefore its rank is less than three and the homogeneous system has at least one free variable. If $V \neq O$ is a null vector, then tV solves the system for every real t. The option claiming no solution is impossible because O always solves a homogeneous system. Exactly two solutions is impossible for a real linear homogeneous system, and uniqueness would require a nonzero determinant.

Step 3: Conclusion The system has infinitely many solutions. Hence, the correct option is (C).

Quick Tip to Solve: For an odd-order skew-symmetric coefficient matrix, use the determinant transpose identity before attempting elimination.

2. The value of the integral, $\int_1^3 [x^2 - 2x - 2]dx$, where [x] denotes the greatest integer less than or equal to x, is : [4 marks]

(A) -5

(B) $-\sqrt{2} - \sqrt{3} + 1$

(C) -4

(D) $-\sqrt{2} - \sqrt{3} - 1$

Correct Answer: (D) $-\sqrt{2} - \sqrt{3} - 1$

Solution:

Concept:

- The greatest integer function is constant between consecutive integer crossings. Isolated values at the crossing points do not affect a definite integral.
- Completing the square gives $x^2 - 2x - 2 = (x-1)^2 - 3$. A shift $u = x - 1$ changes the limits from $1, 3$ to $0, 2$.

Step 1: Locate the integer crossings With $u = x - 1$, write

$$I = \int_0^2 \lfloor u^2 - 3 \rfloor \, du.$$

On this interval $u^2 - 3$ increases from -3 to 1. The relevant crossings occur at $u = 1, \sqrt{2}, \sqrt{3}, 2$, where the expression equals $-2, -1, 0, 1$, respectively. Hence the integrand is -3 on $[0, 1)$, -2 on $[1, \sqrt{2})$, -1 on $[\sqrt{2}, \sqrt{3})$, and zero on $[\sqrt{3}, 2)$.

Step 2: Add the rectangular areas The signed areas of these four constant pieces give

$$\begin{aligned} I &= -3(1-0) - 2(\sqrt{2}-1) - (\sqrt{3}-\sqrt{2}) + 0(2-\sqrt{3}) \\ &= -3 - 2\sqrt{2} + 2 - \sqrt{3} + \sqrt{2} \\ &= -1 - \sqrt{2} - \sqrt{3}. \end{aligned}$$

The value at $u = 2$, where the floor becomes 1, contributes no area because it is a single point. Numerically the result is about -4.146, so neither integer value -5 nor -4 is correct. The remaining radical option has its constant term $+1$, whereas the exact area calculation gives -1.

Step 3: Conclusion The integral equals $\boxed{-\sqrt{2} - \sqrt{3} - 1}$. Hence, the correct option is (D).

Quick Tip to Solve: Complete the square first, then split a greatest-integer integral at the exact points where the inner expression reaches an integer.

3. Let a, b∈R. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is (20, b, −a−9), then | a + b |, is equal to : [4 marks]

(A) 88

(B) 90

(C) 86

(D) 84

Correct Answer: (A) 88

Solution:

Concept:

- For reflection of a point in a line, the midpoint of the original point and its image lies on the mirror line.
- The segment joining the point to its image is perpendicular to the mirror line. A parametric point on this line is $(3 + 7t, 2 + 5t, 1 - 9t)$.

Step 1: Put the midpoint on the line Let the image be $Q = (20, b, -a-9)$. The midpoint of $P = (a, 6, 9)$ and Q is

$$M = \left(\frac{a+20}{2}, \frac{b+6}{2}, -\frac{a}{2}\right).$$

Because M lies on the given line, its coordinates satisfy

$$\frac{a+20}{2} = 3 + 7t, \qquad \frac{b+6}{2} = 2 + 5t, \qquad -\frac{a}{2} = 1 - 9t.$$

The last relation gives $a = 18t - 2$, which can now be used in the first relation.

Step 2: Find the coordinates and check the reflection Substitution gives $9t + 9 = 3 + 7t$, so $t = -3$. Consequently

$$a = 18(-3) - 2 = -56, \qquad b = 10(-3) - 2 = -32.$$

To verify the geometric condition as well, $Q - P = (76, -38, 38)$ and the direction vector of the line is $(7, 5, -9)$. Their dot product is

$$76(7) - 38(5) + 38(-9) = 532 - 190 - 342 = 0.$$

Thus the joining segment is perpendicular to the line and the midpoint construction gives the required mirror image. Finally, $|a + b| = |-56 - 32| = 88$; the other three offered values do not equal this exact result.

Step 3: Conclusion $\boxed{|a + b| = 88}$. Hence, the correct option is (A).

Quick Tip to Solve: For reflection in a line, place the midpoint on the line and then check that the joining vector is perpendicular to its direction vector.

4. For the system of linear equations:

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$**,**

consider the following statements :

(A) The system has unique solution if $k \neq 2, k \neq -2$**.**

(B) The system has unique solution if k = −2

(C) The system has unique solution if k = 2

(D) The system has no solution if k = 2

(E) The system has infinite number of solutions if k $\neq -2$**.**

Which of the following statements are correct? [4 marks]

(A) (B) and (E) only

(B) (C) and (D) only

(C) (A) and (E) only

(D) (A) and (D) only

Correct Answer: (D) (A) and (D) only

Solution:

Concept:

- A square linear system has a unique solution when its coefficient determinant is nonzero. A zero determinant requires a separate consistency check.
- Eliminating x leaves two equations in y, z; proportional left sides with incompatible right sides prove that no solution exists.

Step 1: Eliminate the first variable From $x - 2y = 1$, obtain $x = 1 + 2y$. Substitute this into $x - y + kz = -2$:

$$y + kz = -3.$$

Together with the third equation, the reduced system is

$$y + kz = -3, \qquad ky + 4z = 6.$$

Its coefficient determinant is $4 - k^2$. It is nonzero exactly when $k \neq 2$ and $k \neq -2$, so statement (A) is true. In particular, all nonexceptional values have a unique solution, disproving the broad infinite-solution claim in statement (E).

Step 2: Examine both exceptional values At $k = 2$, the first reduced equation gives $2y + 4z = -6$ after multiplication by two, but the other equation demands $2y + 4z = 6$. This contradiction proves there is no solution: (D) is true and (C) is false. At $k = -2$, the reduced equations are $y - 2z = -3$ and $-2y + 4z = 6$; the latter is exactly -2 times the former. Taking $z = t$ gives $y = 2t - 3$, $x = 4t - 5$, hence infinitely many solutions. Statement (B), claiming uniqueness there, is false. Thus precisely statements (A) and (D) hold; every other offered pair contains a false statement.

Step 3: Conclusion The valid statements are A and D: uniqueness holds away from the exceptional parameters, and inconsistency holds at k = 2. Hence, the correct option is (D).

Quick Tip to Solve: A vanishing determinant does not distinguish inconsistency from infinitely many solutions; substitute each exceptional parameter into the reduced equations.

5. Let f be a twice differentiable function defined on R such that f(0) = 1, f'(0) = 2 and f'(x) $\neq$ 0 for all x $\in$ R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}$ = 0, for all x$\in$R, then the value of f(1) lies in the interval : [4 marks]

(A) (0, 3)

(B) (9, 12)

(C) (3, 6)

(D) (6, 9)

Correct Answer: (D) (6, 9)

Solution:

Concept:

- The given determinant is $f(x)f''(x) - [f'(x)]^2$. Setting it to zero supplies a differential relation.
- Since $f'(x) \neq 0$ everywhere, the quotient f/f' is well defined on all of $\mathbb{R}$. Its derivative can be evaluated by the quotient rule.

Step 1: Reduce the determinant equation Expanding the determinant gives

$$ff'' - (f')^2 = 0.$$

Rather than divide by f, which has not yet been shown nonzero, differentiate the permitted quotient:

$$\left(\frac{f}{f'}\right)' = \frac{(f')^2 - ff''}{(f')^2} = 0.$$

A differentiable function with zero derivative on $\mathbb{R}$ is constant. The initial values fix this constant as $f(0)/f'(0) = 1/2$. Therefore $f/f' = 1/2$, or $f' = 2f$, everywhere.

Step 2: Solve and locate the requested value Multiply the last equation by the integrating factor e^{-2x}:

$$\frac{d}{dx}\left(e^{-2x}f(x)\right) = e^{-2x}(f' - 2f) = 0.$$

Thus $e^{-2x}f(x) = f(0) = 1$, giving $f(x) = e^{2x}$. This function satisfies both initial conditions and has $f'(x) = 2e^{2x} \neq 0$, so it meets every hypothesis. At $x = 1$, $f(1) = e^2 \approx 7.389$. This lies strictly between 6 and 9, outside each of the other three disjoint intervals $(0, 3), (9, 12), (3, 6)$.

Step 3: Conclusion $f(1) = e^2 \in (6, 9)$. Hence, the correct option is (D).

Quick Tip to Solve: When a determinant gives $ff'' = (f')^2$ and $f' \neq 0$, differentiate f/f' to avoid prematurely dividing by f.

6. Let f(x) be a differentiable function defined on [0, 2] such that f'(x) = f'(2 − x) for all x∈ (0, 2), f(0) = 1 and f(2) = e^2. Then the value of $\int_0^2 f(x)dx$ is : [4 marks]

(A) $1 + e^2$

(B) $2(1 + e^2)$

(C) $1 - e^2$

(D) $2(1 - e^2)$

Correct Answer: (A) $1 + e^2$

Solution:

Concept:

- The substitution $u = 2 - x$ reflects the interval $[0, 2]$ into itself, so $\int_0^2 f(2-x)\,dx = \int_0^2 f(x)\,dx$.
- By the chain rule, the derivative of $f(x) + f(2 - x)$ is $f'(x) - f'(2 - x)$. The given derivative symmetry makes this sum constant.

Step 1: Find the constant reflected sum Define $g(x) = f(x) + f(2 - x)$ on $[0, 2]$. For every interior point the chain rule and the hypothesis give

$$g'(x) = f'(x) - f'(2 - x) = 0.$$

Hence g is constant. Evaluating at the endpoint, using continuity from differentiability, fixes this constant:

$$f(x) + f(2 - x) = f(0) + f(2) = 1 + e^2.$$

The reflected arguments must be added, since their derivatives cancel; subtracting the function values would not produce a zero derivative.

Step 2: Integrate the symmetry identity Let $I = \int_0^2 f(x)\,dx$. Integrating the identity over the whole interval gives

$$I + \int_0^2 f(2 - x)\,dx = \int_0^2 (1 + e^2)\,dx = 2(1 + e^2).$$

In the second integral put $u = 2 - x$; reversing its limits cancels the minus sign, leaving I. Therefore $2I = 2(1 + e^2)$, and $I = 1 + e^2$. The factor-two alternative forgets that the left side is twice the desired integral. Both alternatives involving $1 - e^2$ have the wrong endpoint sum and would even be negative, contrary to the exact calculation.

Step 3: Conclusion $\int_0^2 f(x)\,dx = \boxed{1 + e^2}$. Hence, the correct option is (A).

Quick Tip to Solve: If derivatives agree at reflected points, differentiate the sum of the original function at those points, then integrate the resulting constant identity.

7. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} -55x, & if\ x < -5 \\ 2x^3 - 3x^2 - 120x, & if\ -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & if\ x > 4, \end{cases}$$

Let A = {x ∈ R : f is increasing}. Then A is equal to : [4 marks]

(A) $(-5, \infty)$

(B) $(-\infty, -5) \cup (4, \infty)$

(C) $(-5, -4) \cup (4, \infty)$

(D) $(-\infty, -5) \cup (-4, \infty)$

Correct Answer: (C) $(-5, -4) \cup (4, \infty)$

Solution:

Concept:

- On an interval where a differentiable function has positive derivative, it is increasing. A piecewise definition requires a separate derivative calculation on each piece.
- The derivative sign is found by factoring the polynomial, while each factorization must be restricted to the interval where its formula actually applies.

Step 1: Differentiate the first two pieces For $x < -5$, $f'(x) = -55 < 0$, so the first piece is decreasing throughout its domain. On $-5 < x < 4$,

$$f'(x) = 6x^2 - 6x - 120 = 6(x - 5)(x + 4).$$

Both factors are negative on $(-5, -4)$, so the derivative is positive there. On $(-4, 4)$, $x - 5 < 0$ but $x + 4 > 0$, so the derivative is negative. The stationary point $x = -4$ separates the increasing and decreasing intervals.

Step 2: Check the last piece and combine For $x > 4$, the applicable formula is $2x^3 - 3x^2 - 36x - 336$, whose derivative is

$$f'(x) = 6x^2 - 6x - 36 = 6(x - 3)(x + 2) > 0.$$

Thus this entire last piece is increasing. The definitions agree at the joins: at -5 both give 275, and at 4 both give -400; there is no additional increasing stretch hidden by a jump. The standard open intervals of increase are therefore $(-5, -4)$ and $(4, \infty)$. Option (A) incorrectly includes the decreasing middle interval, and options (B) and (D) include $x < -5$, where the slope is negative.

Step 3: Conclusion $\boxed{A = (-5, -4) \cup (4, \infty)}$. Hence, the correct option is (C).

Quick Tip to Solve: For piecewise polynomials, factor each derivative separately and intersect its positive-sign intervals with that piece's own domain.

8. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are : [4 marks]

(A) $(-2, 8)$

(B) $(2, 8)$

(C) $(1, 5)$

(D) $(3, 13)$

Correct Answer: (B) $(2, 8)$

Solution:

Concept:

- The perpendicular distance from a point (x, y) to a line $Ax+By+C = 0$ is $|Ax+By+C|/\sqrt{A^2 + B^2}$. This distance, rather than a vertical separation, determines the closest point.
- A quadratic written as a square plus a positive constant attains its least value when the square vanishes. This gives a global minimum without restricting the parameter to the listed options.

Step 1: Parameterise the parabola and form the distance Every point on the parabola can be written as $P = (t, t^2 + 4)$, with t real. Write the line as $4x - y - 1 = 0$. The required distance is therefore

$$d(t) = \frac{|4t - (t^2 + 4) - 1|}{\sqrt{17}} = \frac{|-(t-2)^2 - 1|}{\sqrt{17}}.$$

The expression within the absolute value is always negative, so removing the absolute value changes its sign. Thus

$$d(t) = \frac{(t-2)^2 + 1}{\sqrt{17}}.$$

Step 2: Minimise and check the listed coordinates The square $(t - 2)^2$ is nonnegative and equals zero only at $t = 2$. Consequently the least distance is $1/\sqrt{17}$, attained at $P = (2, 8)$. For the other listed parameters $t = -2, 1, 3$, the distances are respectively $17/\sqrt{17}, 2/\sqrt{17}, 2/\sqrt{17}$, all larger. This comparison also confirms that choosing the vertex of the parabola would not answer the stated distance problem.

Step 3: Conclusion The unique closest point is $\boxed{P = (2, 8)}$. Hence, the correct option is (B).

Quick Tip to Solve: Substitute a parametric point into the line equation, then complete the square before differentiating; the fixed denominator cannot affect where the distance is least.

9. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is : [4 marks]

(A) $\frac{135}{2^9}$

(B) $\frac{65}{2^8}$

(C) $\frac{65}{2^7}$

(D) $\frac{35}{2^7}$

Correct Answer: (A) $\frac{135}{2^9}$

Solution:

Concept:

- A set with five elements has 2^5 subsets. Selecting each subset independently and uniformly gives $2^5 \cdot 2^5 = 4^5$ equally likely ordered pairs.
- For each element there are four membership patterns: in neither subset, in the first only, in the second only, or in both. Exactly two elements must have the last pattern.

Step 1: Choose the common elements Let the selected subsets be A and B. First choose the two elements that will belong to $A \cap B$. There are

$$\binom{5}{2} = 10$$

ways to choose them. Their membership in both subsets is now fixed. Choosing the intersection first avoids counting a pair with three or more common elements, which would not satisfy the question.

Step 2: Assign the remaining elements and divide by all pairs Each of the remaining three elements may belong to neither subset, to A alone, or to B alone. It cannot belong to both, since the intersection must contain exactly the chosen two elements. Thus the favourable count is

$$N = \binom{5}{2} 3^3 = 10 \cdot 27 = 270.$$

Since the full sample space contains $4^5 = 1024$ pairs, the probability is

$$\Pr(|A \cap B| = 2) = \frac{270}{1024} = \frac{135}{512} = \frac{135}{2^9}.$$

Equivalently, each element is common with probability $1/4$, giving the binomial probability $\binom{5}{2}(1/4)^2(3/4)^3$. Options (B), (C) and (D) do not equal this exact fraction; they do not count the three allowed patterns of a noncommon element correctly.

Step 3: Conclusion The required probability is $\boxed{135/2^9}$. Hence, the correct option is (A).

Quick Tip to Solve: For two independent random subsets, assign each element one of four membership patterns. To prescribe an intersection of size k, use $\binom{n}{k}3^{n-k}/4^n$.

10. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is : [4 marks]

(A) $\sqrt{7}-1$

(B) $\frac{1}{\sqrt{7}}$

(C) $2\sqrt{2}-1$

(D) $\frac{1}{2\sqrt{2}}$

Correct Answer: (B) $\frac{1}{\sqrt{7}}$

Solution:

Concept:

- For a principal inverse sine of a positive number, $\theta=\sin^{-1}u$ lies in $(0,\pi/2)$. Therefore the cosine and the half-angle square roots used here are positive.
- Two successive half-angle identities reduce the quarter-angle. First find the cosine of the half-angle; then use the positive square root for the tangent, since the quarter-angle is in the first quadrant.

Step 1: Determine the cosine of the original angle Put

$$\theta=\sin^{-1}\left(\frac{\sqrt{63}}{8}\right).$$

Since $0<\theta<\pi/2$, we have

$$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\frac{63}{64}}=\frac{1}{8}.$$

The positive sign is forced by the principal range; using a negative sign would introduce an angle outside that range. Now apply the half-angle formula once:

$$\cos\frac{\theta}{2}=\sqrt{\frac{1+1/8}{2}}=\sqrt{\frac{9}{16}}=\frac{3}{4}.$$

Step 2: Apply the half-angle formula again and compare options The angle $\theta/4$ is positive and smaller than $\pi/8$. Therefore

$$\tan\frac{\theta}{4}=\sqrt{\frac{1-3/4}{1+3/4}}=\sqrt{\frac{1}{7}}=\frac{1}{\sqrt{7}}.$$

Options (A) and (C) exceed one, whereas the tangent of this angle is less than $\tan(\pi/8)<1$. Option (D) has square $1/8$, but the exact double half-angle calculation requires square $1/7$. This distinguishes the two small positive candidates without relying on a decimal approximation.

Step 3: Conclusion A possible value, and the principal value, is $\boxed{1/\sqrt{7}}$. Hence, the correct option is (B).

Quick Tip to Solve: When an inverse sine contains $\sqrt{m^2-1}/m$, compute its cosine first; two half-angle formulas can then remove a quarter-angle neatly.

11. The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ **is :** [4 marks]

(A) $6\sqrt{3}$ square units

(B) $12\sqrt{3}$ square units

(C) $11\sqrt{3}$ square units

(D) $9\sqrt{3}$ square units

Correct Answer: (B) $12\sqrt{3}$ square units

Solution:

Concept:

- For vertical strips between two curves, area is the integral of the upper ordinate minus the lower ordinate over the interval where that difference is nonnegative.
- The given inequalities specify both bounding parabolas: the lower one is $y = 5x^2$, and the upper one is $y = 2x^2 + 9$. Their intersection points determine the integration limits.

Step 1: Find the interval occupied by the region For an admissible point the lower ordinate cannot exceed the upper ordinate. Therefore

$$5x^2 \le 2x^2 + 9 \quad \Longleftrightarrow \quad x^2 \le 3.$$

The curves meet at $x = \pm\sqrt{3}$, with $y = 15$. Thus the entire region lies between these two x-coordinates. On this interval its vertical thickness is

$$(2x^2 + 9) - 5x^2 = 9 - 3x^2 \ge 0.$$

This order of subtraction guarantees positive area and includes both the left and right halves of the region.

Step 2: Integrate the thickness and check the result The required area is

$$\begin{aligned} A &= \int_{-\sqrt{3}}^{\sqrt{3}} (9 - 3x^2)\, dx \\ &= 2\int_{0}^{\sqrt{3}} (9 - 3x^2)\, dx \\ &= 2[9x - x^3]_0^{\sqrt{3}} \\ &= 2(9\sqrt{3} - 3\sqrt{3}) = 12\sqrt{3}. \end{aligned}$$

The factor of two is justified because the thickness is even. Option (A) gives only the right half of the region and omits its equal left half. Options (C) and (D) disagree with the exact endpoint evaluation. The answer is expressed in square units because it is an area, not a length or an ordinate.

Step 3: Conclusion The area is $\boxed{12\sqrt{3} \text{ square units}}$. Hence, the correct option is (B).

Quick Tip to Solve: First solve lower curve $\le$ upper curve for the limits; when the thickness is even, integrate over the positive half and multiply by two.

12. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$ is : [4 marks]

(A) $\frac{n(2n+1)(3n+1)}{6}$

(B) $\frac{n(n+1)(2n+1)}{6}$

(C) $\frac{n(n+1)^2(n+2)}{12}$

(D) $\frac{n(n-1)(2n+1)}{6}$

Correct Answer: (B) $\frac{n(n+1)(2n+1)}{6}$

Solution:

Concept:

- The hockey-stick identity gives $\sum_{k=2}^{n}\binom{k}{2} = \binom{n+1}{3}$. It follows by summing Pascal's identity $\binom{k}{2} = \binom{k+1}{3} - \binom{k}{3}$.
- The general formulae $\binom{n+1}{2} = n(n+1)/2$ and $\binom{n+1}{3} = n(n+1)(n-1)/6$ reduce the binomial sum to a polynomial.

Step 1: Evaluate the sum inside the parentheses Write the requested expression as

$$S = \binom{n+1}{2} + 2\sum_{k=2}^{n}\binom{k}{2}.$$

By Pascal's identity,

$$\sum_{k=2}^{n}\binom{k}{2} = \sum_{k=2}^{n}\left(\binom{k+1}{3} - \binom{k}{3}\right) = \binom{n+1}{3}.$$

All intermediate terms cancel, and the initial $\binom{2}{3}$ is zero. This argument also applies at $n = 2$, so the lower endpoint of the stated domain is included.

Step 2: Simplify and rule out the other expressions Substitution gives

$$\begin{aligned} S &= \frac{n(n+1)}{2} + \frac{2n(n+1)(n-1)}{6} \\ &= \frac{n(n+1)}{6}[3 + 2(n-1)] \\ &= \frac{n(n+1)(2n+1)}{6}. \end{aligned}$$

As a quick independent check, at $n = 2$ the original series is $\binom{3}{2} + 2\binom{2}{2} = 5$. Options (A), (C), and (D) respectively give $35/3$, 6, and $5/3$, so none can be an identity for every allowed integer. Option (B) gives 5 and agrees with the general derivation. Recognising its familiar sum-of-squares form is a useful check, but the telescoping calculation establishes why that form arises.

Step 3: Conclusion The sum is $\boxed{n(n+1)(2n+1)/6}$. Hence, the correct option is (B).

Quick Tip to Solve: Use Pascal's identity to telescope a sum whose upper binomial index increases while the lower index stays fixed; test the smallest allowed n to eliminate false identities.

13. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is : [4 marks]

(A) $\frac{69}{256}$
(B) $\frac{71}{256}$
(C) $-\frac{71}{256}$
(D) $-\frac{69}{256}$

Correct Answer: (C) $-\frac{71}{256}$

Solution:

Concept:

- The centroid of a triangle is the componentwise average of its vertices. Hence the two supplied centroid coordinates give independent linear relations among a, b, c.
- An arithmetic progression satisfies $2b = a + c$. For roots of $ax^2 + bx + 1 = 0$, Vieta's formulae give $\alpha + \beta = -b/a$ and $\alpha\beta = 1/a$.

Step 1: Recover the coefficients from the centroid and progression The x-coordinate of the centroid gives

$$\frac{a + 2 + a}{3} = \frac{10}{3} \quad \Rightarrow \quad a = 4.$$

Its y-coordinate gives $c + 2b = 7$. Using the progression condition $c = 2b - a = 2b - 4$, we obtain

$$(2b - 4) + 2b = 7 \quad \Rightarrow \quad b = \frac{11}{4}, \qquad c = \frac{3}{2}.$$

These values indeed satisfy $2b = a + c = 11/2$, and their ordinates sum to 7, so both the geometric and progression requirements are checked.

Step 2: Express the required root combination using Vieta's formulae It is unnecessary to solve for the individual roots. Instead,

$$\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta.$$

Here

$$\alpha + \beta = -\frac{11}{16}, \qquad \alpha\beta = \frac{1}{4}.$$

Therefore

$$\alpha^2 + \beta^2 - \alpha\beta = \frac{121}{256} - \frac{3}{4} = \frac{121 - 192}{256} = -\frac{71}{256}.$$

The roots may be nonreal, but Vieta's algebraic identities remain valid. Option (B) has the wrong sign, while (A) and (D) have numerator 69 instead of the exact difference 71. The negative value is consistent with the subtraction of three times the positive root product.

Step 3: Conclusion The required value is $\boxed{-71/256}$. Hence, the correct option is (C).

Quick Tip to Solve: For symmetric expressions in two quadratic roots, rewrite in terms of their sum and product before using the quadratic formula; here the product is subtracted three times.

14. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1, 2) and the tangent line to this curve at origin is y = x, then the possible values of a, b, c are : [4 marks]

(A) a = -1, b = 1, c = 1

(B) a = 1, b = 1, c = 0

(C) a = $\frac{1}{2}$, b = $\frac{1}{2}$, c = 1

(D) a = 1, b = 0, c = 1

Correct Answer: (B) a = 1, b = 1, c = 0

Solution:

Concept:

- A tangent at a stated point requires the point itself to lie on the curve. Thus a tangent at the origin gives both a position condition and a slope condition.
- For $y = ax^2 + bx + c$, the derivative is $dy/dx = 2ax + b$. At $x = 0$, the tangent slope is b, and the curve's ordinate is c.

Step 1: Use the tangent at the origin Since the origin $(0, 0)$ lies on the curve,

$$0 = a(0)^2 + b(0) + c \quad \Rightarrow \quad c = 0.$$

The tangent line $y = x$ has slope 1. Matching its slope to the derivative at the point of tangency gives

$$\left.\frac{dy}{dx}\right|_{x=0} = b = 1.$$

Both conditions are essential: merely requiring a parallel tangent somewhere would not imply the same coefficients. The stated point of tangency fixes where to evaluate the derivative.

Step 2: Apply the second point and verify every condition The curve also passes through $(1, 2)$, so

$$2 = a + b + c = a + 1 \quad \Rightarrow \quad a = 1.$$

The resulting curve is $y = x^2 + x$. It gives $y(0) = 0$, $y(1) = 2$, and derivative $2x + 1$, whose value at zero is 1. Its tangent equation at the origin is consequently $y - 0 = 1(x - 0)$, exactly $y = x$. All of options (A), (C), and (D) have $c = 1$, so their curves do not even pass through the origin; option (D) additionally has zero slope there. Thus none of those alternatives can satisfy the given tangent condition.

Step 3: Conclusion The coefficients are $\boxed{a = 1,\ b = 1,\ c = 0}$. Hence, the correct option is (B).

Quick Tip to Solve: When a tangent is specified at the origin, first impose $f(0) = 0$, then match $f\prime(0)$ to its slope; use any remaining point only after these two conditions.

15. If a curve y = f(x) passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$? [4 marks]

(A) $\frac{31}{5}$

(B) 10

(C) 5

(D) $\frac{62}{5}$

Correct Answer: (B) 10

Solution:

Concept:

- The product rule gives $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$, so the differential equation can be integrated directly.
- The point $(1, 2)$ fixes the integration constant. The prescribed definite integral then determines the parameter b.

Step 1: Integrate the differential equation Use the product rule before attempting an integrating factor. The equation becomes

$$\frac{d}{dx}(xy) = bx^4.$$

Integrating with respect to x yields

$$xy = \frac{b}{5}x^5 + C, \qquad f(x) = \frac{b}{5}x^4 + \frac{C}{x}.$$

The integration constant must be retained until the point condition is applied; setting it to zero immediately would assume the answer.

Step 2: Apply the point and integral conditions At $x = 1$, the ordinate is 2, hence $C = 2 - b/5$. Therefore

$$\begin{aligned}\int_1^2 f(x)\,dx &= \frac{b}{5}\left[\frac{x^5}{5}\right]_1^2 + \left(2 - \frac{b}{5}\right)[\ln x]_1^2 \\ &= \frac{31b}{25} + \left(2 - \frac{b}{5}\right)\ln 2.\end{aligned}$$

Equate this to $62/5$. Rearranging around $b = 10$ gives

$$(b - 10)\left(\frac{31}{25} - \frac{\ln 2}{5}\right) = 0.$$

The second factor is positive, so $b = 10$ is the unique solution.

Step 3: Check the proposed value and reject the others For $b = 10$, the constant is $C = 0$ and $f(x) = 2x^4$. It satisfies both $f(1) = 2$ and

$$\int_1^2 2x^4\,dx = \frac{2}{5}(32 - 1) = \frac{62}{5}.$$

Since the coefficient of b in the integral equation is nonzero, the other three proposed parameter values cannot produce this integral.

Step 4: Conclusion The required value is $\boxed{b = 10}$. Hence, the correct option is (B).

Quick Tip to Solve: When the left side is $xy' + y$, integrate it as $(xy)'$; apply the point condition before evaluating the integral.

16. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?
[4 marks]

(A) $2x^2 - 18y^2 = 9$
(B) $y^2 = \frac{1}{6\sqrt{3}}x$
(C) $x^2 + 9y^2 = 9$
(D) $x^2 + y^2 = 7$

Correct Answer: (C) $x^2 + 9y^2 = 9$

Solution:

Concept:

- A tangent to a differentiable curve at a point has the derivative of the curve there as its slope.
- Both conditions are essential: the point must lie on the curve, and its derivative must equal the given line slope.

Step 1: Find the required tangent slope Write the proposed line in slope-intercept form:

$$y = 2 - \frac{x}{\sqrt{3}}.$$

Thus the required slope is $-1/\sqrt{3}$. The point $P = (3\sqrt{3}/2, 1/2)$ lies on this line since

$$\frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 2\sqrt{3}.$$

Merely checking that a curve contains P is insufficient: several options do, but their tangents need not coincide.

Step 2: Differentiate the ellipse in option C For $x^2 + 9y^2 = 9$, the point condition is

$$\left(\frac{3\sqrt{3}}{2}\right)^2 + 9\left(\frac{1}{2}\right)^2 = \frac{27}{4} + \frac{9}{4} = 9.$$

Differentiating implicitly gives

$$2x + 18y\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = -\frac{x}{9y}.$$

At P, this is

$$-\frac{3\sqrt{3}/2}{9/2} = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}.$$

The point and slope are both correct, so the tangent is the stated line.

Step 3: Eliminate the other three curves For option A, differentiation gives $y\prime = x/(9y)$, which is $+1/\sqrt{3}$ at P, the wrong sign. For option B, $2yy\prime = 1/(6\sqrt{3})$, so $y\prime = 1/(6\sqrt{3})$ at $y = 1/2$, also positive. For option D, $y\prime = -x/y = -3\sqrt{3}$, which has the wrong magnitude. Although these curves contain the point, none has the required tangent direction there.

Step 4: Conclusion The curve is $\boxed{x^2 + 9y^2 = 9}$. Hence, the correct option is (C).

Quick Tip to Solve: Check point membership first, then compare derivatives. A point lying on both a curve and a line does not prove tangency.

17. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \to q)) \to \sim p$

(b) $((p \vee q) \wedge \sim p) \to q$

Then which of the following statements is correct? [4 marks]

(A) (b) is a tautology but not (a).

(B) (a) and (b) both are not tautologies.

(C) (a) and (b) both are tautologies.

(D) (a) is a tautology but not (b).

Correct Answer: (C) (a) and (b) both are tautologies.

Solution:

Concept:

- An implication $A \to B$ is false only when A is true and B is false.
- A tautology is true for every assignment of truth values. If a true antecedent always forces a true consequent, the implication is a tautology.

Step 1: Test statement (a) through its antecedent Consider

$$(\neg q \wedge (p \to q)) \to \neg p.$$

If its antecedent is false, the implication is automatically true. If the antecedent is true, q is false and $p \to q$ is true. An implication with a false consequent can be true only when its antecedent p is false. Thus $\neg p$ is true, exactly as required. This proves (a) for all truth assignments without assuming that p or q is always false.

Step 2: Test statement (b) through its antecedent Now consider

$$((p \vee q) \wedge \neg p) \to q.$$

Again, a false antecedent makes the implication true. A true antecedent says that at least one of p, q is true while p itself is false. Therefore q must be true. Its consequent follows whenever its antecedent holds, so (b) is also a tautology. This is the familiar disjunctive syllogism written as a compound implication.

Step 3: Verify every truth assignment For an explicit check, let T and F mean true and false. The

resulting implication values are

p	q	(a)	(b)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	T	T

Each column consists entirely of T. Option A wrongly excludes (a); option D wrongly excludes (b); option B excludes both. The option asserting that both are tautologies is the only one consistent with these results.

Step 4: Conclusion Both compound statements are tautologies. Hence, the correct option is (C).

Quick Tip to Solve: To test an implication for tautology, try to make the antecedent true and consequent false. If that is impossible, the implication is always true.

18. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is : [4 marks]

(A) $3600\sqrt{3}$ m

(B) $1200\sqrt{3}$ m

(C) $1800\sqrt{3}$ m

(D) $2400\sqrt{3}$ m

Correct Answer: (B) $1200\sqrt{3}$ m

Solution:

Concept:

- For a horizontal distance d and constant height h, the angle of elevation obeys $\tan\theta = h/d$.
- The plane travels horizontally at its constant height; distance travelled equals speed multiplied by time. Convert the speed to metres per second first.

Step 1: Convert the speed and calculate the displacement The speed in SI units is

$$432\,\text{km}\,\text{h}^{-1} = 432 \times \frac{5}{18} = 120\,\text{m}\,\text{s}^{-1}.$$

Consequently the horizontal distance travelled in 20 seconds is

$$s = 120 \times 20 = 2400\,\text{m}.$$

Let the height be h, and let d_1, d_2 be the horizontal distances from A to the plane's vertical projection at the first and second observations. The falling elevation angle corresponds to the plane moving farther from A.

Step 2: Express the two horizontal distances Using the two right triangles gives

$$\tan 60^\circ = \frac{h}{d_1}, \qquad \tan 30^\circ = \frac{h}{d_2}.$$

Therefore

$$d_1 = \frac{h}{\sqrt{3}}, \qquad d_2 = \sqrt{3}h.$$

For the receding flight along the same vertical plane, $d_2 - d_1 = 2400$. Thus

$$\sqrt{3}h - \frac{h}{\sqrt{3}} = 2400, \qquad \frac{2h}{\sqrt{3}} = 2400.$$

Solving yields $h = 1200\sqrt{3}\,\text{m}$.

Step 3: Check the height against the observations At this height, the horizontal distances are $d_1 = 1200\,\text{m}$ and $d_2 = 3600\,\text{m}$. Their difference is $2400\,\text{m}$, exactly the distance flown, and their ratios with the height reproduce 60° and 30°. The heights in options A, C and D would instead require displacements of 7200, 3600 and $4800\,\text{m}$, respectively, so they fail the speed-time condition.

Step 4: Conclusion The height is $\boxed{1200\sqrt{3}\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the difference of the two horizontal distances for a receding plane: $s = h(\cot 30^\circ - \cot 60^\circ)$.

19. The negation of the statement

$\sim p \wedge (p \vee q)$ **is :** [4 marks]

(A) $p \vee \sim q$

(B) $\sim p \vee q$

(C) $\sim p \wedge q$

(D) $p \wedge \sim q$

Correct Answer: (A) $p \vee \sim q$

Solution:

Concept:

- De Morgan's laws change a conjunction into a disjunction when negating: $\neg(A \wedge B) = \neg A \vee \neg B$.
- Distributive and absorption laws can simplify the original statement before negating it. Double negation gives $\neg\neg p = p$.

Step 1: Simplify the original compound statement Distribute $\neg p$ across the bracket:

$$\begin{aligned} \neg p \wedge (p \vee q) &= (\neg p \wedge p) \vee (\neg p \wedge q) \\ &= \text{F} \vee (\neg p \wedge q) \\ &= \neg p \wedge q. \end{aligned}$$

The first term is always false, because a statement and its negation cannot both be true. Thus the

given statement is true precisely when p is false and q is true.

Step 2: Negate the simplified expression Apply De Morgan's law to the whole conjunction:

$$\neg(\neg p \wedge q) = (\neg\neg p) \vee \neg q = p \vee \neg q.$$

This says that either p is true or q is false. It is false only for $p = \mathrm{F}, q = \mathrm{T}$, exactly the truth assignment for which the original statement is true. Negating each symbol without changing the connective would give the wrong result.

Step 3: Check the alternatives Option A is the expression just obtained. For $p = \mathrm{T}, q = \mathrm{F}$, the required negation is true, whereas option B, $\neg p \vee q$, and option C, $\neg p \wedge q$, are false. Option D, $p \wedge \neg q$, is false when $p = q = \mathrm{T}$, but the required negation is true there. These checks eliminate all three incorrect alternatives and confirm the simplification.

Step 4: Conclusion The negation is $\boxed{p \vee \neg q}$. Hence, the correct option is (A).

Quick Tip to Solve: Simplify $\neg p \wedge (p \vee q)$ to $\neg p \wedge q$ before negating; this makes the De Morgan step immediate.

20. The vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right) = 1$ and $\vec{r}.\left(\hat{i}-2\hat{j}\right) = -2$, and the point (1, 0, 2) is : [4 marks]

(A) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right) = \frac{7}{3}$

(B) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right) = 7$

(C) $\vec{r}.\left(3\hat{i}+7\hat{j}+3\hat{k}\right) = 7$

(D) $\vec{r}.\left(\hat{i}-7\hat{j}+3\hat{k}\right) = \frac{7}{3}$

Correct Answer: (B) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right) = 7$

Solution:

Concept:

- If two planes are $P_1 = 0$ and $P_2 = 0$, every plane of the form $P_1 + \lambda P_2 = 0$ contains their intersection.
- Substitute the additional point to determine λ. A scalar multiple of a plane equation represents the same plane only when both sides scale together.

Step 1: Form the family through the intersection Write $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. The two given planes are

$$x + y + z - 1 = 0, \qquad x - 2y + 2 = 0.$$

Their common points satisfy every linear combination

$$(x + y + z - 1) + \lambda(x - 2y + 2) = 0.$$

This construction guarantees the entire intersection line is contained in the required plane, rather

than merely one point on that line.

Step 2: Use the extra point to determine the member Substitute $(1, 0, 2)$:

$$(1 + 0 + 2 - 1) + \lambda(1 - 0 + 2) = 0.$$

Hence $2 + 3\lambda = 0$, giving $\lambda = -2/3$. Insert this value into the family and multiply the complete equation by three:

$$3(x + y + z - 1) - 2(x - 2y + 2) = 0.$$

Expanding and collecting terms gives

$$x + 7y + 3z - 7 = 0.$$

Thus the normal vector can be chosen as $\hat{i} + 7\hat{j} + 3\hat{k}$.

Step 3: Verify the point and rule out the alternatives At the specified point, the left side $x + 7y + 3z$ equals $1 + 0 + 6 = 7$, confirming the constant. Options A and D have right side $7/3$, so the point does not satisfy either equation. In option C, the left side is $3 + 0 + 6 = 9$, not 7; it also fails the point condition. Option B has both the correct normal and constant, and the family construction proves it contains the intersection line.

Step 4: Conclusion The required plane is $\boxed{\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the constant inside each plane expression when forming $P_1 + \lambda P_2 = 0$; omitting it changes the family of planes.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (−5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _____ [4 marks]

Solution:

Concept:

- The distance formula converts a fixed ratio of distances from two fixed points into an algebraic locus.
- For a circle $(x - a)^2 + (y - b)^2 = r^2$, complete the square to read its centre and squared radius.

Step 1: Translate the distance ratio into coordinates Let $P = (x, y)$. Its distances from $(5, 0)$ and $(-5, 0)$ are $\sqrt{(x - 5)^2 + y^2}$ and $\sqrt{(x + 5)^2 + y^2}$, respectively. The first is three times the second, so

$$\sqrt{(x - 5)^2 + y^2} = 3\sqrt{(x + 5)^2 + y^2}.$$

Both sides are nonnegative; squaring therefore preserves the condition and introduces no sign ambiguity.

Step 2: Expand and complete the square Squaring gives

$$(x-5)^2+y^2=9\big((x+5)^2+y^2\big).$$

Expand the two sides carefully, including the signs of the linear terms:

$$x^2-10x+25+y^2=9x^2+90x+225+9y^2.$$

Bring all terms together and divide by eight:

$$x^2+y^2+\frac{25}{2}x+25=0.$$

Completing the square yields

$$\left(x+\frac{25}{4}\right)^2+y^2=\frac{625}{16}-25=\frac{225}{16}.$$

The centre is $(-25/4,0)$ and $r=15/4$.

Step 3: Evaluate the requested numerical entry The exact value requested is

$$4r^2=4\times\frac{225}{16}=\frac{225}{4}=56.25.$$

A numerical-value entry rounded to the nearest integer is consequently 56. As a check on the geometry, the circle lies nearer $(-5,0)$, which is consistent with the distance to $(5,0)$ being the larger distance. Reversing the ratio changes the sign of the centre but not the radius; keeping the unsquared factor three instead of nine would, however, give a wrong radius.

Step 4: Conclusion The exact value is $\boxed{4r^2=56.25}$, so the numerical answer to the nearest integer is $\boxed{56}$.

Quick Tip to Solve: Square the complete distance ratio, so a factor of three becomes nine. Complete the square before evaluating $4r^2$.

22. If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is _____. [4 marks]

Solution:

Concept:

- The population variance of N observations is $\sigma^2=\frac{1}{N}\sum x_i^2-\bar{x}^2$, where $\bar{x}$ is their arithmetic mean.
- There are nine observations equal to 1 and one equal to k. The strict variance inequality must be applied before choosing the largest natural number.

Step 1: Compute the mean and the mean of the squares The ten observations have total $9+k$, and

the sum of their squares is $9 + k^2$. Thus

$$\bar{x} = \frac{k+9}{10}, \qquad \frac{1}{10}\sum x_i^2 = \frac{k^2+9}{10}.$$

The denominator is ten because variance here refers to the entire given data set, rather than a sample estimate.

Step 2: Simplify the variance and impose the bound Subtract the square of the mean:

$$\begin{aligned}\sigma^2 &= \frac{k^2+9}{10} - \frac{(k+9)^2}{100} \\ &= \frac{10k^2 + 90 - k^2 - 18k - 81}{100} \\ &= \frac{9(k-1)^2}{100}.\end{aligned}$$

Consequently $\sigma^2 < 10$ gives

$$(k-1)^2 < \frac{1000}{9}, \qquad |k-1| < \frac{10\sqrt{10}}{3}.$$

Since k is natural, $k \geq 1$; hence $k < 1 + 10\sqrt{10}/3 \approx 11.54$.

Step 3: Check the two boundary integers For $k = 11$, the variance is $9(10)^2/100 = 9 < 10$. For $k = 12$, it is $9(11)^2/100 = 10.89 > 10$. The variance increases as k increases beyond one, so no larger natural value works.

Step 4: Conclusion The maximum possible value is $\boxed{k = 11}$. This is a dimensionless natural number.

Quick Tip to Solve: When all but one observation are identical, simplify the variance to a multiple of the squared deviation of the exceptional observation before testing integers.

23. Let λ be an integer. If the shortest distance between the lines $\mathbf{x - \lambda = 2y - 1 = -2z}$ and $\mathbf{x = y + 2\lambda = z - \lambda}$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____. [4 marks]

Solution:

Concept:

- For lines through points P, Q with direction vectors $\mathbf{u}, \mathbf{v}$, the shortest distance is $d = \frac{|(Q-P)\cdot(\mathbf{u}\times\mathbf{v})|}{|\mathbf{u}\times\mathbf{v}|}$.
- Introduce one common parameter in each chain of equalities to identify a point and direction vector. An integer restriction is imposed after solving the distance equation.

Step 1: Write the lines in parametric form Put $x - \lambda = 2y - 1 = -2z = t$ for the first line. Then

$$(x, y, z) = (\lambda, \tfrac{1}{2}, 0) + t(1, \tfrac{1}{2}, -\tfrac{1}{2}).$$

We may therefore use $P = (\lambda, 1/2, 0)$ and $\mathbf{u} = (2, 1, -1)$, since scaling a direction vector does not

change the line. For the second line put $x = y + 2\lambda = z - \lambda = s$. This gives

$$(x, y, z) = (0, -2\lambda, \lambda) + s(1, 1, 1).$$

Thus $Q = (0, -2\lambda, \lambda)$ and $\mathbf{v} = (1, 1, 1)$.

Step 2: Evaluate the scalar triple product The cross product and joining vector are

$$\mathbf{u} \times \mathbf{v} = (2, -3, 1), \qquad |\mathbf{u} \times \mathbf{v}| = \sqrt{14},$$

$$Q - P = (-\lambda, -2\lambda - \tfrac{1}{2}, \lambda).$$

Their dot product is $-2\lambda + 6\lambda + 3/2 + \lambda = 5\lambda + 3/2$. Hence

$$d = \frac{|10\lambda + 3|}{2\sqrt{14}}.$$

Step 3: Use the specified distance and integer condition The prescribed distance is $\sqrt{7}/(2\sqrt{2}) = 7/(2\sqrt{14})$. Therefore

$$|10\lambda + 3| = 7.$$

The two algebraic possibilities are $10\lambda + 3 = 7$, giving $\lambda = 2/5$, and $10\lambda + 3 = -7$, giving $\lambda = -1$. Only the latter is an integer, as required.

Step 4: Conclusion Thus $\lambda = -1$, and the requested value is $\boxed{|\lambda| = 1}$, a dimensionless number.

Quick Tip to Solve: Do not interpret the coefficients in a chain of equalities as direction ratios directly: parameterize the coordinates first, then apply the triple-product formula.

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24. If a + α = 1, b + β = 2 and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \neq 0$, then the value of the expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is _____. [4 marks]

Solution:

Concept:

- The substitution $x \mapsto 1/x$ is valid for every nonzero real x. It exchanges the two unknown values $f(x)$ and $f(1/x)$.
- Adding the resulting equations isolates their sum using $a + \alpha = 1$ and $b + \beta = 2$; finding the individual function values is unnecessary.

Step 1: Replace the argument by its reciprocal The given equation is

$$af(x) + \alpha f(1/x) = bx + \frac{\beta}{x}.$$

Since $x \neq 0$, its reciprocal is also an admissible argument. Replacing x by $1/x$ gives

$$af(1/x) + \alpha f(x) = \frac{b}{x} + \beta x.$$

This second relation has the same two function values in the opposite order.

Step 2: Add the equations and use the coefficient sums Adding corresponding sides produces

$$(a+\alpha)\big(f(x) + f(1/x)\big) = (b+\beta)\left(x + \frac{1}{x}\right).$$

Insert the two stated sums to obtain

$$f(x) + f(1/x) = 2\left(x + \frac{1}{x}\right).$$

There is no division by $a - \alpha$, so this reasoning also covers equal values of those coefficients. It uses only the combinations of coefficients supplied in the question.

Step 3: Evaluate the requested quotient For a nonzero real argument,

$$x + \frac{1}{x} = \frac{x^2+1}{x} \neq 0.$$

The denominator of the requested expression is therefore nonzero. Dividing the derived sum by this expression yields

$$\frac{f(x) + f(1/x)}{x + 1/x} = 2.$$

The result is constant and does not depend on the separate values of the four coefficients.

Step 4: Conclusion The requested dimensionless value is $\boxed{2}$.

Quick Tip to Solve: For an equation coupling $f(x)$ and $f(1/x)$, substitute the reciprocal and add when the target is their sum; subtract when the target is their difference.

25. Let $i = \sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}} + \frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}} = k$, and $n = [|k|]$ be the greatest integral part of | k |. Then $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$ is equal to _____. [4 marks]

Solution:

Concept:

- De Moivre's identity states $(\rho(\cos\theta + i\sin\theta))^m = \rho^m(\cos m\theta + i\sin m\theta)$ for integer m.
- The greatest integral part $n = \lfloor |k| \rfloor$ fixes the upper summation limit. The common limits allow subtraction term by term.

Step 1: Evaluate the numerator powers The complex numbers in the numerators have modulus two

and arguments $2\pi/3$ and $\pi/3$, respectively:

$$-1+i\sqrt{3}=2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right),$$

$$1+i\sqrt{3}=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).$$

Raising to the twenty-first power multiplies these arguments by 21, giving 14π and 7π. Thus the powers are 2^{21} and -2^{21}, respectively.

Step 2: Evaluate the denominators and determine the limit Each of $1-i$ and $1+i$ has modulus $\sqrt{2}$, with argument $-\pi/4$ and $\pi/4$. Their twenty-fourth powers have arguments -6π and 6π, so

$$(1-i)^{24}=(1+i)^{24}=2^{12}.$$

It follows that

$$k=\frac{2^{21}}{2^{12}}-\frac{2^{21}}{2^{12}}=512-512=0.$$

Therefore $n=\lfloor|0|\rfloor=0$, and both sums run from $j=0$ to $j=5$, inclusive.

Step 3: Subtract and add the six summands Combine the two sums and put $r=j+5$:

$$\sum_{j=0}^{5}\left((j+5)^2-(j+5)\right)=\sum_{r=5}^{10}r(r-1).$$

The six values are $20, 30, 42, 56, 72, 90$. Their total is

$$20+30+42+56+72+90=310.$$

Including both endpoints is essential: the index range contains six terms.

Step 4: Conclusion The required dimensionless sum is $\boxed{310}$.

Quick Tip to Solve: Reduce the arguments of complex powers before attempting arithmetic; here the two real numerator powers cancel because the denominator powers coincide.

26. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is _____. [4 marks]

Solution:

Concept:

- The expression $|x-5|$ equals $5-x$ for $x<5$, and $x-5$ for $x\geq 5$. Every root found in a case must satisfy its case restriction.
- A lower bound can exclude an entire interval without solving its quadratic: for $x\geq 5$, the square $(x+1)^2$ is already at least 36.

Step 1: Solve on the interval below five For $x<5$, substitute $|x-5|=5-x$ in the equation. This

gives

$$(x+1)^2+5-x=\frac{27}{4},$$

$$x^2+x+6=\frac{27}{4}, \qquad x^2+x-\frac{3}{4}=0.$$

Multiply by four and factor:

$$4x^2+4x-3=(2x-1)(2x+3)=0.$$

The two roots are $x=1/2$ and $x=-3/2$. Both lie below five, so both survive the case restriction.

Step 2: Exclude all arguments at least five When $x\geq 5$, we have $x+1\geq 6$, and $|x-5|\geq 0$. Hence

$$(x+1)^2+|x-5|\geq 36>\frac{27}{4}.$$

No value in this interval, including the boundary $x=5$, can satisfy the equation. This completes the real-domain case split.

Step 3: Check the two surviving roots At $x=1/2$, the two terms are $9/4$ and $9/2$, whose sum is $27/4$. At $x=-3/2$, they are $1/4$ and $13/2$, again summing to $27/4$. These are distinct real roots, and the preceding interval analysis excludes every other real value.

Step 4: Conclusion The equation has exactly $\boxed{2}$ distinct real roots.

Quick Tip to Solve: Before solving both modulus cases, compare the square with the target on each interval; a simple bound can eliminate a whole case and avoid extraneous roots.

27. The students S_1, S_2, ..., S_{10} are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____. [4 marks]

Solution:

Concept:

- The groups A, B, C are labelled, and the ten students are distinct. Choosing the r members of C can be done in $\binom{10}{r}$ ways.
- Each remaining student has two choices, A or B. Of the 2^{10-r} assignments, exactly two leave one of these two groups empty.

Step 1: Separate the possible sizes of group C Every group must be nonempty, while C contains at most three students. Its allowed sizes are therefore $r=1,2,3$. These three cases are disjoint, so their counts can be added. First choose the actual members of C; this contributes $\binom{10}{r}$ for a fixed size r.

Step 2: Assign the remaining students to A and B After choosing C, there are $10-r$ students left. Independent assignment to the two named groups gives 2^{10-r} possibilities. The assignment placing everyone in A leaves B empty; the assignment placing everyone in B leaves A empty. Remove these

two forbidden assignments. Thus the count for this size is

$$\binom{10}{r}(2^{10-r}-2).$$

There is no division by a factorial because exchanging the membership of the named groups changes the grouping.

Step 3: Evaluate and add the three cases The required total is

$$\begin{aligned} N &= \binom{10}{1}(2^9-2)+\binom{10}{2}(2^8-2) \\ &\quad +\binom{10}{3}(2^7-2) \\ &= 10(510)+45(254)+120(126) \\ &= 5100+11430+15120 \\ &= 31650. \end{aligned}$$

Each permissible grouping is counted exactly once, according to the unique set of students it puts in C.

Step 4: Conclusion The total number of possible groupings is $\boxed{31650}$.

Quick Tip to Solve: Fix the size and membership of the restricted group first, then use onto assignments for the remaining labelled groups; subtract the two empty-group assignments.

---

28. The sum of first four terms of a geometric progression (G. P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _____. [4 marks]

Solution:

Concept:

- Write the first four terms of a real geometric progression as a, ar, ar^2, ar^3. The specified nonzero product ensures $a \neq 0$ and $r \neq 0$.
- The sum of the reciprocal terms can be related to the original sum by reversing its order. The product of the first three terms is $(ar)^3$.

Step 1: Use the product of the first three terms The given product gives

$$a \cdot ar \cdot ar^2 = a^3r^3 = (ar)^3 = 1.$$

For real terms this implies $ar = 1$. Consequently $a = 1/r$, and the first four terms can be written

$$\frac{1}{r}, \quad 1, \quad r, \quad r^2.$$

The third term, denoted by α, is therefore r. We need the common ratio, rather than a separate

calculation of every term.

Step 2: Relate the two supplied sums Let the original sum be S, and the reciprocal sum be T. In terms of r,

$$S = \frac{1}{r} + 1 + r + r^2, \qquad T = r + 1 + \frac{1}{r} + \frac{1}{r^2}.$$

Multiplying the second equation by r gives $rT = r^2 + r + 1 + 1/r = S$. Since $T = 65/18 \neq 0$,

$$r = \frac{S}{T} = \frac{65/12}{65/18} = \frac{18}{12} = \frac{3}{2}.$$

This determines the sign as well as the magnitude of the ratio.

Step 3: Identify the third term and verify the data The third term is $\alpha = r = 3/2$. The four terms are $2/3, 1, 3/2, 9/4$, whose sum is $(8 + 12 + 18 + 27)/12 = 65/12$. Their reciprocals are $3/2, 1, 2/3, 4/9$, with sum $(27 + 18 + 12 + 8)/18 = 65/18$. The first three terms have product one, so all three supplied conditions hold simultaneously.

Step 4: Conclusion Thus $\alpha = 3/2$, and the requested dimensionless value is $\boxed{2\alpha = 3}$.

Quick Tip to Solve: For consecutive geometric terms, reverse the reciprocal sum and look for a common scaling factor; combine that relation with the product condition before solving polynomial equations.

---

29. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to _____. [4 marks]

Solution:

Concept:

- The normal to a circle at a point passes through its centre; the tangent is perpendicular to that radius.
- For a triangle with its base on the x-axis, the area is half the base length multiplied by the perpendicular height. The height of the vertex $(5, 7)$ above this axis is 7 units.

Step 1: Find the normal and tangent The centre is $C = (2, 3)$, and the given point is $P = (5, 7)$. Since $(5 - 2)^2 + (7 - 3)^2 = 9 + 16 = 25$, it lies on the circle. The radius has slope $(7 - 3)/(5 - 2) = 4/3$. Thus the normal and tangent respectively are

$$y - 7 = \frac{4}{3}(x - 5), \qquad y - 7 = -\frac{3}{4}(x - 5).$$

Writing these without fractions gives $4x - 3y + 1 = 0$ and $3x + 4y - 43 = 0$. The normal follows the radius, while the tangent uses its negative reciprocal slope.

Step 2: Calculate the intercepts and area Set $y = 0$ in each line. The normal meets the x-axis at $N = (-1/4, 0)$; the tangent meets it at $T = (43/3, 0)$. The base length is the difference between these

intercepts, including the negative sign of the normal intercept:

$$NT = \frac{43}{3} - \left(-\frac{1}{4}\right) = \frac{175}{12}.$$

The two lines meet at P, so the triangular area is

$$A = \frac{1}{2} \cdot \frac{175}{12} \cdot 7 = \frac{1225}{24}.$$

These intercepts describe the triangle bounded by the x-axis line and the tangent and normal. The normal intercept is slightly to the negative side of the origin; using its absolute coordinate as a positive intercept would give the wrong base.

Step 3: Conclusion Therefore $24A = 24(1225/24) = \boxed{1225}$. The area itself is $1225/24$ square units.

Quick Tip to Solve: Find both x-intercepts before calculating the base; subtract a negative intercept rather than subtracting its magnitude.

---

30. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & if\ n \geq r \geq 0 \\ 0, & otherwise \end{cases}$ **The maximum value of k for which the sum** $\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ **exists, is equal to _____.** [4 marks]

Solution:

Concept:

- Vandermonde's identity gives $\sum_i \binom{a}{i}\binom{b}{m-i} = \binom{a+b}{m}$, with out-of-range coefficients defined as zero.
- Pascal's identity is $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$. In an even-numbered row, the unique greatest binomial coefficient is the central coefficient.

Step 1: Reduce the two convolutions For nonnegative integer k, collect the coefficient of x^k in $(1+x)^{10}(1+x)^{15}$. This gives the first sum as $\binom{25}{k}$. The coefficient of x^{k+1} in $(1+x)^{12}(1+x)^{13}$ gives the second sum as $\binom{25}{k+1}$. Hence their sum simplifies to

$$S(k) = \binom{25}{k} + \binom{25}{k+1} = \binom{26}{k+1}.$$

The zero convention makes this identity valid even near or beyond the ends of the rows; terms do not become undefined there.

Step 2: Find the peak and examine the wording For $0 \leq j < 26$, consecutive coefficients have ratio

$$\frac{\binom{26}{j+1}}{\binom{26}{j}} = \frac{26-j}{j+1}.$$

This ratio exceeds 1 for $j \leq 12$ and is less than 1 for $j \geq 13$. Thus $\binom{26}{j}$ is uniquely greatest at $j = 13$, so the sum is greatest when $k+1 = 13$, or $k = 12$. However, the wording asks only when the sum

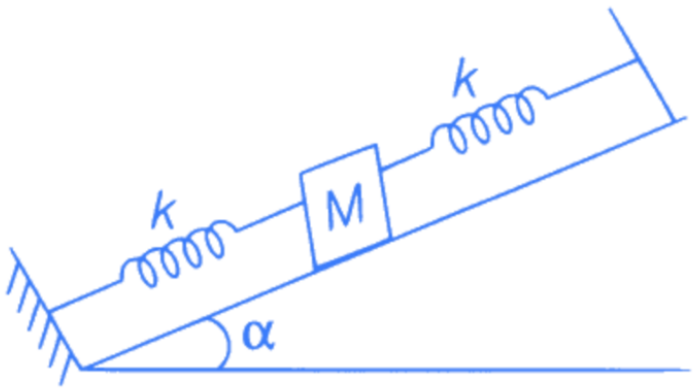
"exists. With the stated zero extension, every finite sum is defined for every nonnegative integer k; for $k > 25$ it is simply zero. Consequently existence alone imposes no largest k.

Step 3: Conclusion The answer-key value is $\boxed{12}$, obtained when the displayed sum is required to be maximum. Literally, "the maximum value of k for which the sum exists has no finite answer, since the given zero convention defines the sum for arbitrarily large nonnegative k.

Quick Tip to Solve: Convert binomial convolutions to a single coefficient, but distinguish a greatest value of a sum from mere existence of that sum.

Physics – Section A: Multiple Choice Questions (80 marks)

31. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is : [4 marks]



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

Correct Answer: (C) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

Solution:

Concept:

- For a displacement x from static equilibrium, simple harmonic motion obeys $M\ddot{x} = -k_{\text{eff}}x$; hence $\omega = \sqrt{k_{\text{eff}}/M}$ and $f = \omega/(2\pi)$.
- A constant force such as the component $Mg\sin\alpha$ shifts the equilibrium position, but does not change the derivative of the restoring force with respect to displacement.

Step 1: Displace the block from equilibrium Take x along the incline, measured from the position where the spring forces balance the component of weight. If the block is moved up the plane by x, the

lower spring changes its length by x, producing an extra force kx down the plane. The upper spring changes its length oppositely and also contributes an extra force kx down the plane. The constant forces at equilibrium already cancel, so they do not enter the incremental restoring force. The plane is smooth, so there is no frictional force or damping term.

Step 2: Use the effective restoring constant The two restoring contributions add:

$$F_{\text{net}} = -kx - kx = -2kx.$$

Thus

$$\ddot{x} + \frac{2k}{M}x = 0, \qquad \omega = \sqrt{\frac{2k}{M}}, \qquad f = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}.$$

Options (A) and (B) incorrectly put $g\sin\alpha$ inside the frequency; their square roots do not even have the dimensions of an angular frequency. Option (D) corresponds to an effective spring constant $k/2$, appropriate to two equal springs in series, whereas here each spring changes length by the full block displacement. The support arrangement therefore gives addition of stiffnesses.

Step 3: Conclusion The frequency is $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Measure displacement from equilibrium and add the changes in both spring forces; gravity then drops out of the oscillation frequency.

32. If the source of light used in a Young's double slit experiment is changed from red to violet :

[4 marks]

(A) the fringes will become brighter.

(B) the intensity of minima will increase.

(C) consecutive fringe lines will come closer.

(D) the central bright fringe will become a dark fringe.

Correct Answer: (C) consecutive fringe lines will come closer.

Solution:

Concept:

- Young's double-slit fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D the screen distance and d the slit separation.
- Constructive interference occurs at path difference $m\lambda$, and destructive interference at $(m + 1/2)\lambda$. At the geometrical centre the path difference is zero for every wavelength.

Step 1: Compare the two wavelengths Violet light has a shorter wavelength than red light. Changing only the source colour leaves the slit separation and the screen distance fixed. Therefore

$$\frac{\beta_{\text{violet}}}{\beta_{\text{red}}} = \frac{\lambda_{\text{violet}}}{\lambda_{\text{red}}} < 1.$$

The separation between neighbouring bright fringes, or neighbouring dark fringes, decreases. The

order positions $y_m = m\lambda D/d$ move towards the central fringe, so the visible lines become more closely spaced. This conclusion uses wavelength rather than the perceived colour brightness.

Step 2: Check the intensity claims At the centre, the two paths remain equal. Their phase difference is zero, so the central bright fringe remains bright and (D) is false. For coherent waves of intensities I_1 and I_2, the minimum intensity is

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2.$$

For equally illuminated identical slits it is zero for either colour. There is no necessary increase of minimum intensity, so (B) is not the wavelength effect. Brightness also depends on source power and the eye or detector response, none of which is specified. Thus (A) cannot be inferred from replacing red by violet. The unambiguous consequence is the decreased fringe width.

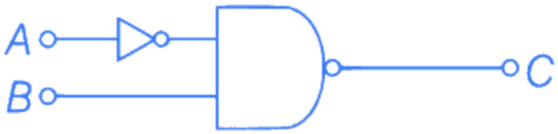
Step 3: Conclusion Consecutive fringe lines come closer because $\beta \propto \lambda$ and violet light has the smaller wavelength. Hence, the correct option is (C).

Quick Tip to Solve: For a colour change in a fixed double-slit arrangement, compare wavelengths first: shorter wavelength means narrower fringes.

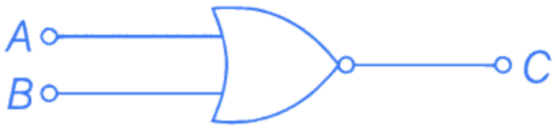
33. The logic circuit shown above is equivalent to : [4 marks]



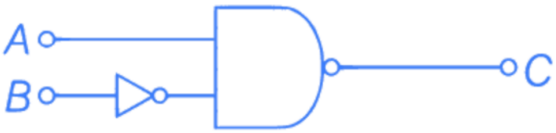
(A) $C = \overline{\overline{A}B}$



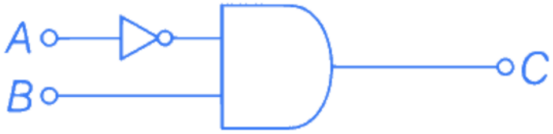
(B) $C = \overline{A + B}$



(C) $C = \overline{A}\overline{B}$

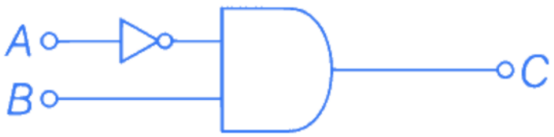


(D) $C = \overline{A}B$



Correct Answer: (D)

$C = \overline{A}B$



Solution:

Concept:

- An inverter changes X into $\overline{X}$; an OR gate gives $X + Y$, while an AND gate gives XY. A bubble on the output indicates complementation.
- De Morgan's identity $\overline{X + Y} = \overline{X}\,\overline{Y}$ converts an inverted OR output into an AND of the complemented inputs.

Step 1: Write the expression for the given circuit Input B first passes through a NOT gate, so the lower input of the OR gate is $\overline{B}$. Its upper input is A, without inversion. The OR output is therefore $A + \overline{B}$. The final inverter complements the whole expression:

$$C = \overline{A + \overline{B}} = \overline{A}\,B.$$

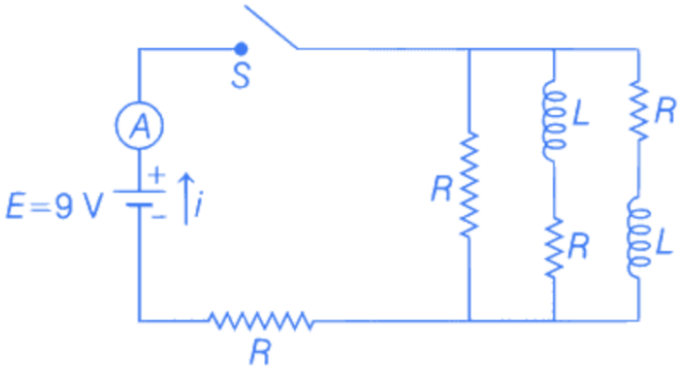
The final equality follows because complementing $\overline{B}$ restores B. Hence an equivalent circuit must invert only A and then AND that result with the unchanged input B.

Step 2: Compare all four graphical alternatives Option (D) has exactly a NOT gate on the A input followed by an ordinary AND gate, so it gives $\overline{A}B$. Option (A) has an output bubble on that AND gate and gives $\overline{\overline{A}B}$, the complement of the required output. Option (B) is a NOR gate on the unchanged inputs and gives $\overline{A + B} = \overline{A}\,\overline{B}$. Option (C) inverts B before a NAND gate, giving $\overline{A\overline{B}}$. As a check, the required output is 1 only when $A = 0, B = 1$, and is 0 for the other three input pairs. This truth condition agrees with option (D) and distinguishes it from the other drawings.

Step 3: Conclusion The circuit implements $\boxed{C = \overline{A}\,B}$, which is an AND gate with its A input inverted. Hence, the correct option is (D).

Quick Tip to Solve: Trace every inversion bubble before simplifying; an inverter after an OR gate complements the entire OR expression.

34. Figure shows a circuit that contains four identical resistors with resistance R = 2.0Ω, two identical inductors with inductance L = 2.0 mH and an ideal battery with emf E = 9V. The current 'i' just after the switch 'S' is closed will be : [4 marks]



(A) 3.0 A

(B) 3.37 A

(C) 9 A

(D) 2.25 A

Correct Answer: (D) 2.25 A

Solution:

Concept:

- An ideal inductor obeys $V = L\,di/dt$, so its current cannot jump instantaneously under a finite applied voltage.
- Immediately after a circuit is connected, an initially unenergised inductor has zero current and its branch behaves as an open circuit. Ohm's law then applies to the remaining resistive path.

Step 1: Identify the branches that conduct initially Before the switch closes, the battery is disconnected from the network and both inductors are unenergised. Their currents are initially zero. At $t = 0^+$, each inductor current remains zero by current continuity. Consequently, neither of the two vertical branches containing an inductor and a resistor carries current at that instant, regardless of the order of those two components within its branch. It is the initial transient, rather than the eventual steady state, that the question asks for. The given inductance sets the subsequent time variation but is not needed for this initial value.

Step 2: Reduce the initial network and calculate current The middle vertical branch containing only a resistor R remains connected between the top and bottom rails. The resistor in the lower return wire is in series with it. Thus the initial equivalent resistance seen by the ideal battery is

$$R_{\text{eq}}(0^+) = R + R = 2R = 4.0\,\Omega.$$

The ammeter reads the battery current, which is

$$i(0^+) = \frac{E}{2R} = \frac{9}{4}\,\text{A} = 2.25\,\text{A}.$$

The larger values 3.0 A, 3.37 A and 9 A cannot arise from this 4-ohm initial path under a 9-V supply. In particular, treating an inductor as a wire immediately after closing would incorrectly include branches whose currents have not yet risen from zero.

Step 3: Conclusion The initial current is $\boxed{2.25\ \text{A}}$, flowing in the indicated direction from the battery. Hence, the correct option is (D).

Quick Tip to Solve: At switching, preserve inductor current; for initially unenergised inductors remove their branches before reducing the resistor network.

35. A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin i.e. ma = $-\alpha x^2$. The distance at which the particle stops : [4 marks]

(A) $\left[\frac{3mv_0^2}{2\alpha}\right]^{\frac{1}{3}}$

(B) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$

(C) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$

(D) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$

Correct Answer: (A) $\left[\frac{3mv_0^2}{2\alpha}\right]^{\frac{1}{3}}$

Solution:

Concept:

- The work-energy theorem gives $K_f - K_i = \int F(x)\,dx$. It applies directly when the force is specified as a function of position.
- Here $F(x) = -\alpha x^2$, with $\alpha > 0$. During the initial motion along positive x the force opposes the displacement, removing kinetic energy until the first stopping point.

Step 1: Integrate the retarding work Let the particle start at the origin with speed v_0, and let its first stopping distance be $s > 0$. Its initial kinetic energy is $mv_0^2/2$, and at the stop its kinetic energy is zero. The force is not constant, so multiplying the final force by the full displacement would not give the work. Integrating gives

$$W = \int_0^s -\alpha x^2\,dx = -\frac{\alpha s^3}{3}.$$

The negative sign reflects the loss of kinetic energy during outward motion. No assumption of a constant acceleration has been made.

Step 2: Equate the work to the kinetic-energy change The work-energy equation becomes

$$0 - \frac{1}{2}mv_0^2 = -\frac{\alpha s^3}{3}.$$

Solving for the positive distance,

$$s^3 = \frac{3mv_0^2}{2\alpha}, \qquad s = \left(\frac{3mv_0^2}{2\alpha}\right)^{1/3}.$$

Equivalently, substituting $a = v\,dv/dx$ into $ma = -\alpha x^2$ and integrating gives the same result. Since $[\alpha] = [F]/[x]^2 = \mathrm{kg\,m^{-1}s^{-2}}$, the quantity mv_0^2/α has dimensions of length cubed. Option (B) has the wrong speed dependence and omits the mass, while the square roots in (C) and (D) do not have the required length dimensions. The positive cube root also increases with the initial kinetic energy, as expected.

Step 3: Conclusion The first stopping distance is $\boxed{\left(\frac{3mv_0^2}{2\alpha}\right)^{1/3}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a force proportional to x^2, integrate to get a work proportional to x^3; the stopping distance must involve a cube root.

36. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be : [4 marks]

(A) 10^{-2} nm

(B) 10^{-1} nm

(C) 10^{-3} nm

(D) 10^{-4} nm

Correct Answer: (C) 10^{-3} nm

Solution:

Concept:

- An electron accelerated through a potential difference V gains kinetic energy eV, where e is the elementary charge.
- The shortest X-ray wavelength corresponds to the largest possible photon energy: $hc/\lambda_{\min} = eV$. The useful conversion is $hc = 1240\,\mathrm{eV\,nm}$.

Step 1: Convert the accelerating voltage The tube voltage is 1.24 million volts, so $V = 1.24 \times 10^6\,\mathrm{V}$. An electron gains one electron-volt of energy for each volt through which it is accelerated. Therefore its maximum available energy is

$$E_{\max} = 1.24 \times 10^6\,\mathrm{eV}.$$

This is an upper limit: ordinarily a photon may carry only part of the electron energy, giving a longer wavelength.

Step 2: Apply the cutoff condition At the short-wavelength limit the entire kinetic energy of one electron is converted into a single photon. Thus

$$\lambda_{\min} = \frac{hc}{eV} = \frac{1240}{1.24 \times 10^6}\,\mathrm{nm} = 10^{-3}\,\mathrm{nm}.$$

The wavelength varies inversely with accelerating voltage, rather than directly with it. The tube target material affects characteristic spectral lines, but does not alter this energy-conservation cutoff.

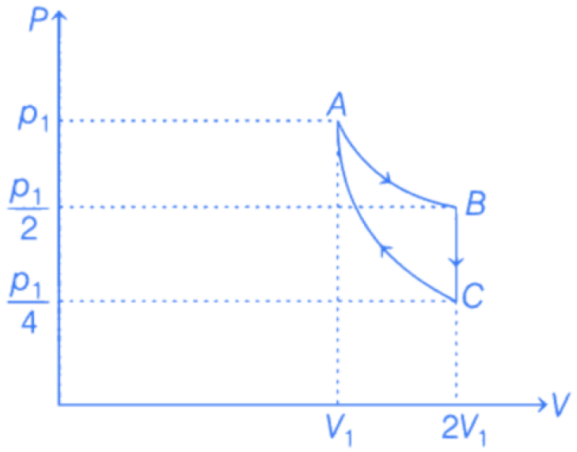
Step 3: Check the alternatives Options (A) and (B) give wavelengths ten and one hundred times too large, respectively; they cannot be the shortest wavelength at the stated voltage. Option (D) would require ten times the available photon energy. Only option (C) satisfies $hc/\lambda = eV$.

Step 4: Conclusion The shortest wavelength is $\boxed{10^{-3}\,\text{nm}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\lambda_{\min}(\text{nm}) = 1240/V(\text{V})$ to avoid converting electron-volts into joules.

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37. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B $\rightarrow$ C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to : [4 marks]



(A) $-\frac{RT}{2(\gamma-1)}$

(B) $RT\left(\ln 2 - \frac{1}{2(\gamma-1)}\right)$

(C) $RT\ln 2$

(D) 0

Correct Answer: (B) $RT\left(\ln 2 - \frac{1}{2(\gamma-1)}\right)$

Solution:

Concept:

- Work done by the gas is positive during expansion and negative during compression. For an isothermal ideal-gas process, $W = nRT\ln(V_f/V_i)$.
- For an adiabatic ideal-gas process, $Q = 0$ and $W = -\Delta U = nR(T_i - T_f)/(\gamma-1)$, where $\gamma = C_P/C_V$.

Step 1: Find the temperatures and volumes Let the initial temperature at A be T; for one mole, $P_1V_1 = RT$. The expansion A to B is isothermal and halves the pressure, so $V_B = 2V_1$ and $T_B = T$.

Cooling B to C occurs at fixed volume $2V_1$, while $P_C = P_1/4$. Consequently

$$T_C = \frac{P_C V_C}{R} = \frac{(P_1/4)(2V_1)}{R} = \frac{T}{2}.$$

Step 2: Calculate the work on each leg The isothermal expansion gives

$$W_{AB} = RT \ln \frac{2V_1}{V_1} = RT \ln 2.$$

There is no volume change on B to C, hence $W_{BC} = 0$. The stated adiabatic compression raises the temperature from $T/2$ to T, giving

$$W_{CA} = \frac{R(T/2 - T)}{\gamma - 1} = -\frac{RT}{2(\gamma - 1)}.$$

The negative sign is essential because work is done on the gas during this compression.

Step 3: Add the signed contributions The cyclic work is the sum over all three processes:

$$W_{\text{net}} = W_{AB} + W_{BC} + W_{CA} = RT \left(\ln 2 - \frac{1}{2(\gamma - 1)} \right).$$

Option (A) includes only the compression contribution, whereas option (C) includes only the expansion. Returning to the initial state makes the internal-energy change zero, but it does not make the net work zero, so option (D) does not follow.

Step 4: Conclusion $\boxed{W_{\text{net}} = RT \left(\ln 2 - \frac{1}{2(\gamma - 1)} \right)}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the work for each leg separately; a closed cycle has zero net internal-energy change, not necessarily zero work.

38. A body weights 49N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?
[Use $g = \frac{GM}{R^2}$ = 9.8 ms^{-2} and radius of earth, R = 6400 km.] [4 marks]

(A) 49 N

(B) 49.83 N

(C) 48.83 N

(D) 49.17 N

Correct Answer: (C) 48.83 N

Solution:

Concept:

- A spring balance reads the normal support force. At the pole there is no centripetal acceleration due to the Earth's rotation, so its reading is mg.

- At the equator the support force is $N = m(g - \omega^2 R)$, with $\omega = 2\pi/T_{\mathrm{E}}$ and one day $T_{\mathrm{E}} = 86400\,\mathrm{s}$.

Step 1: Find the mass from the polar reading The gravitational acceleration specified in the question is $g = 9.8\,\mathrm{m\,s^{-2}}$. At the north pole the rotation does not require centripetal acceleration, because the body lies on the rotation axis. Hence

$$m = \frac{49}{9.8} = 5\,\mathrm{kg}.$$

The mass remains the same when the body is moved; only the balance reading changes.

Step 2: Calculate the rotational correction Taking the Earth's rotation period as one day and converting the radius gives

$$\omega = \frac{2\pi}{86400}\,\mathrm{s^{-1}}, \qquad R = 6.4 \times 10^6\,\mathrm{m}.$$

Thus

$$m\omega^2 R = 5\left(\frac{2\pi}{86400}\right)^2 (6.4 \times 10^6) \simeq 0.169\,\mathrm{N}.$$

At the equator gravity supplies the inward force needed for circular motion, so the upward normal reaction is smaller than mg by this amount.

Step 3: Find the equatorial balance reading Using the inward radial equation $mg - N = m\omega^2 R$, we obtain

$$N = 49 - 0.169 \simeq 48.83\,\mathrm{N}.$$

Option (A) neglects rotation. Options (B) and (D) increase the weight, contrary to the required inward centripetal acceleration. The given model uses one specified radius and gravitational acceleration, so no additional correction for the Earth's shape is required.

Step 4: Conclusion The balance reads $\boxed{48.83\,\mathrm{N}}$ at the equator. Hence, the correct option is (C).

Quick Tip to Solve: At the equator subtract $m\omega^2 R$ from the polar reading; at the pole the distance from the rotation axis is zero.

39. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is : [4 marks]

(A) circular

(B) straight line

(C) parabolic

(D) elliptical

Correct Answer: (D) elliptical

Solution:

Concept:

- For simple harmonic motion, $x = A\sin(\omega t + \phi)$, where A is amplitude and ω is angular frequency.
- The velocity is $v = A\omega\cos(\omega t + \phi)$. Eliminating time gives $v^2 = \omega^2(A^2 - x^2)$.

Step 1: Eliminate the phase Write $\theta = \omega t + \phi$. Then $x/A = \sin\theta$ and $v/(A\omega) = \cos\theta$. The identity

$\sin^2\theta + \cos^2\theta = 1$ gives

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1.$$

This equation relates instantaneous velocity and displacement without requiring the time or the initial phase.

Step 2: Identify the curve and its intercepts The relation is the standard equation of an ellipse centred at the origin in the x-v plane. Its displacement intercepts are $x = \pm A$, where the particle turns and $v = 0$; its velocity intercepts are $v = \pm A\omega$ at the mean position $x = 0$. Both signs of velocity occur, since a given interior displacement is crossed in opposite directions during a full oscillation.

Step 3: Rule out the other shapes A straight line would require a linear relation between v and x, which SHM does not have. The relation between v^2 and x^2 is linear, but this does not make the velocity-displacement curve parabolic. A circle can appear only after a particular normalization of the two axes, such as plotting v/ω against x; the general velocity-displacement relation has different scale factors and is elliptical.

Step 4: Conclusion The velocity-displacement graph is $\boxed{\text{elliptical}}$. Hence, the correct option is (D).

Quick Tip to Solve: Eliminate the SHM phase with $\sin^2\theta + \cos^2\theta = 1$; remember that the full curve includes positive and negative velocity.

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40. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be : [4 marks]

(A) 1.30%

(B) 1.33%

(C) 1.13%

(D) 1.03%

Correct Answer: (C) 1.13%

Solution:

Concept:

- Rearranging the pendulum relation gives $g = 4\pi^2 L/T^2$. The constant $4\pi^2$ introduces no measurement uncertainty.
- For the maximum fractional error in a product or quotient, add the absolute fractional errors multiplied by their powers: $\Delta g/g = \Delta L/L + 2\Delta T/T$.

Step 1: Express the instrument uncertainties The meter scale has a minimum division of $1\,\text{mm}$, so use $\Delta L = 0.001\,\text{m}$ for $L = 1.0\,\text{m}$. The stopwatch resolution is $0.01\,\text{s}$, giving $\Delta T = 0.01\,\text{s}$ for the measured period $T = 1.95\,\text{s}$. These are absolute uncertainties and must be divided by their corresponding measured quantities before combining them.

Step 2: Apply the powers in the formula Since $g = 4\pi^2 LT^{-2}$, the magnitude of the time exponent is

two. Thus the maximum percentage uncertainty is

$$\frac{\Delta g}{g} \times 100 = \left(\frac{0.001}{1.0} + 2\frac{0.01}{1.95}\right) 100$$
$$= 0.100 + 1.02564$$
$$= 1.12564\%.$$

The two error contributions add even though the time appears in the denominator: we are finding the maximum error magnitude, not a signed algebraic change.

Step 3: Round and compare with the options Rounding the percentage error to two decimal places gives 1.13%. Option (D), 1.03%, is approximately the time contribution alone and misses the length error. Options (A) and (B) do not result from the stated fractional uncertainties. Combining errors in quadrature would describe a different statistical prescription; the standard maximum-error rule is appropriate here.

Step 4: Conclusion The percentage error in g is $\boxed{1.13\%}$. Hence, the correct option is (C).

Quick Tip to Solve: First write $g \propto LT^{-2}$; the time percentage error is doubled and then added to the length percentage error.

41. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains :

[4 marks]

(A) may increase or decrease in size and change its orientation.

(B) increase in size but no change in orientation.

(C) decrease in size and changes orientation.

(D) have no relation with external magnetic field.

Correct Answer: (A) may increase or decrease in size and change its orientation.

Solution:

Concept:

- A ferromagnetic domain is a region whose atomic magnetic moments have a common spontaneous orientation. Different domains can cancel one another in an unmagnetized specimen.
- An applied magnetic field favours domain magnetization parallel to itself. Magnetization proceeds through domain-wall motion and rotation of magnetization within domains.

Step 1: Consider the domain energy in the field The magnetic interaction energy decreases when a domain's magnetic moment points more nearly along the applied field. Domains already favourably oriented therefore tend to occupy more of the specimen. Their boundaries move, so these domains grow at the expense of domains pointing in unfavourable directions. The latter domains shrink; it is not correct to assert that every domain increases in size.

Step 2: Include changes in orientation In addition to wall motion, the magnetization direction of domains can rotate toward the external field, especially as the field increases. Thus both a redistribution of domain volumes and a change in orientation can contribute to the resultant magnetization. A soft

ferromagnetic material has relatively mobile domain walls and is easily magnetized, making these changes possible under a comparatively small external field.

Step 3: Select the complete statement Option (A) allows domains to increase or decrease in size and to change orientation, covering the two physical mechanisms. Option (B) incorrectly excludes rotation and assumes only growth. Option (C) allows rotation but describes only shrinking, whereas favourable domains grow. Option (D) ignores the coupling of domain magnetization with the external magnetic field and cannot explain the strong magnetization of a ferromagnet.

Step 4: Conclusion The domains may grow or shrink and may change their orientation. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish domain-wall motion, which changes domain sizes, from magnetization rotation, which changes their orientation.

42. Which of the following equations represents a travelling wave? [4 marks]

(A) $y = Ae^{x}\cos(\omega t - \theta)$

(B) $y = Ae^{-x^2}(vt + \theta)$

(C) $y = A \sin(15x - 2t)$

(D) $y = A \sin x \cos\omega t$

Correct Answer: (C) $y = A \sin(15x - 2t)$

Solution:

Concept:

- A travelling disturbance has a fixed profile of the form $y = f(x - ut)$ or $y = f(x + ut)$, where u is its propagation speed.
- For a sinusoidal travelling wave, space and time occur in one phase: $y = A\sin(kx - \omega t + \phi)$. Its speed is ω/k.

Step 1: Test the sinusoidal phase For option (C),

$$y = A\sin(15x - 2t) = A\sin\left[15\left(x - \frac{2}{15}t\right)\right].$$

This is precisely a function of $x - ut$, with $u = 2/15$ in the units consistent with x and t. A point of constant phase satisfies $15x - 2t =$ constant, so its position advances in the positive x direction at constant speed.

Step 2: Examine the separated expressions Option (A), $Ae^{x}\cos(\omega t - \theta)$, has a fixed position-dependent amplitude and the same time phase everywhere, rather than a profile translated through space. Option (B), $Ae^{-x^2}(vt + \theta)$, changes the overall size of a fixed Gaussian profile with time. The symbol v appearing there does not by itself make a wave travel; the coordinate must occur in a translating combination such as $x - vt$.

Step 3: Recognize the standing-wave option Option (D), $A\sin x\cos\omega t$, is a product of a spatial factor and a time factor. Its nodes remain at the fixed positions $x = n\pi$, identifying a stationary-wave pattern.

In contrast, option (C) moves its phase pattern without changing its amplitude. Therefore only option (C) is the required travelling wave among the listed expressions.

Step 4: Conclusion The travelling wave is $\boxed{y = A\sin(15x - 2t)}$. Hence, the correct option is (C).

Quick Tip to Solve: Look for a single argument combining position and time, such as $kx - \omega t$; a product of separate spatial and temporal factors usually describes a standing pattern.

43. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x (x << d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency : (m = mass of charged particle) [4 marks]

(A) $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

(B) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

(C) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$

(D) $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$

Correct Answer: (B) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

Solution:

Concept:

- Coulomb attraction between a proton of charge $+q$ and an electron of charge $-q$, separated by r, has magnitude kq^2/r^2, where $k = 1/(4\pi\varepsilon_0)$.
- Symmetric forces have cancelling components along the line of the fixed electrons, but their perpendicular components add toward the midpoint.
- Simple harmonic motion follows when the restoring force is $F_x = -m\omega^2 x$. Linearize the force using $x \ll d$; m is the mass of the moving proton.

Step 1: Find the distance and force components Put the fixed electrons at $(d, 0)$ and $(-d, 0)$, and the displaced proton at $(0, x)$. Its distance from either electron is $r = \sqrt{d^2 + x^2}$. The perpendicular projection of either attractive force is negative for positive x, because the force points back toward the electron line. Consequently, the combined force is

$$F_x = -2\frac{kq^2}{r^2}\frac{x}{r} = -\frac{2kq^2x}{(d^2 + x^2)^{3/2}}.$$

Step 2: Linearize for a small displacement Because $x^2/d^2 \ll 1$, replace $(d^2 + x^2)^{3/2}$ by d^3 to first order. Then

$$m\ddot{x} = -\frac{2kq^2}{d^3}x, \qquad \omega^2 = \frac{2kq^2}{md^3} = \frac{q^2}{2\pi\varepsilon_0 md^3}.$$

The negative force coefficient establishes stability for this perpendicular displacement; the result does

not assert stability for displacements along the electron line.

Step 3: Compare the options Taking the positive square root gives an angular frequency proportional to q, inversely proportional to $\sqrt{m}$, and inversely proportional to $d^{3/2}$. Option (A) has four times the required value of ω^2, so its frequency is twice the correct frequency. Options (C) and (D) invert the dimensional combination and have units of time rather than inverse time.

Step 4: Conclusion

$$\boxed{\omega = \sqrt{\frac{q^2}{2\pi\varepsilon_0 m d^3}}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Add the two restoring projections before linearizing: each projection supplies a factor x/r, making the small-displacement force proportional to x/d^3.

44. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules : [4 marks]

(A) continuously lose their energy till it reaches wall.

(B) are attracted by the walls of container.

(C) suffer change in momentum when impinge on the walls of container.

(D) continuously stick to the walls of container.

Correct Answer: (C) suffer change in momentum when impinge on the walls of container.

Solution:

Concept:

- Pressure is normal force per unit area. Gas pressure arises from the momentum transferred by molecules to a container wall during repeated collisions.
- Newton's second law in momentum form gives average force as total momentum transfer divided by the elapsed time: $F = \Delta p/\Delta t$.
- In the ideal-gas model wall collisions are elastic: molecular kinetic energy is conserved while the velocity component normal to a wall reverses.

Step 1: Find the momentum change in one collision Consider a molecule of mass m striking a wall normally with velocity component v_x. Before impact its normal momentum is $+mv_x$; after elastic reflection it is $-mv_x$. Thus the molecule undergoes a change $-2mv_x$, and the wall receives the equal and opposite impulse $+2mv_x$. A momentum change therefore occurs even though the molecule has the same kinetic energy after reflection.

Step 2: Relate repeated impacts to pressure A large number of molecules strike each wall every second. Their impulses add, producing a nearly steady average force over macroscopic time intervals. Dividing this normal force by wall area gives gas pressure. For N molecules occupying volume V, the calculation gives

$$P = \frac{Nm}{V}\langle v_x^2\rangle = \frac{Nm}{3V}\langle v^2\rangle.$$

This positive pressure depends on the rate of momentum transfer rather than on any continuous loss

of molecular energy.

Step 3: Eliminate the alternative statements Option (A) incorrectly requires continuous energy loss; ideal-gas molecules move freely between collisions. Option (B) attributes pressure to attraction by the wall, whereas pressure is an outward push from impacts and exists without such attraction. Option (D) says molecules continuously stick to the wall, contrary to the reflecting collisions used in kinetic theory. Only option (C) identifies the necessary momentum exchange.

Step 4: Conclusion Gas molecules exert pressure because they suffer a change in momentum when they impinge on the container walls. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish momentum from kinetic energy: elastic reflection reverses a velocity component, so it changes momentum without requiring any loss of speed or energy.

45. Match List - I with List - II.

	List I		List II
(a)	Source of microwave frequency	(i)	Radioactive decay of nucleus
(b)	Source of infrared frequency	(ii)	Magnetron
(c)	Source of Gamma Rays	(iii)	Inner shell electrons
(d)	Source of X-rays	(iv)	Vibration of atoms and molecules
		(v)	LASER
		(vi)	RC circuit

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(vi), (b)-(v), (c)-(i), (d)-(iv)

(B) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(C) (a)-(ii), (b)-(iv), (c)-(vi), (d)-(iii)

(D) (a)-(vi), (b)-(iv), (c)-(i), (d)-(v)

Correct Answer: (B) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Solution:

Concept:

- Electromagnetic radiation is classified by frequency, and characteristic production mechanisms identify the spectral region of a source.
- Microwaves can be generated by a magnetron, whereas infrared radiation is associated with atomic and molecular vibrations in matter.
- Nuclear energy-level transitions produce gamma radiation; transitions involving inner-shell atomic electrons produce characteristic X-rays.

Step 1: Match the microwave and infrared sources A magnetron is a vacuum device in which electrons interact with electric and magnetic fields to sustain microwave oscillations. Therefore (a) matches (ii). Infrared frequencies correspond to relatively low-energy motions of atoms and molecules, including molecular vibrational transitions, so (b) matches (iv). An RC circuit is not the listed microwave generator, and LASER is not the specific vibrational source identified for the infrared entry in this

matching exercise.

Step 2: Match gamma rays and X-rays A radioactive nucleus may release excess nuclear energy as a gamma photon. Thus (c) matches (i), radioactive decay of the nucleus. Characteristic X-rays result when an inner-shell vacancy is filled by an electron from a higher atomic shell, releasing the large energy difference as a photon. Thus (d) matches (iii), inner-shell electrons. Although both are high-frequency radiation, their origins distinguish the two entries.

Step 3: Check the complete sequence against each option The four assignments together are

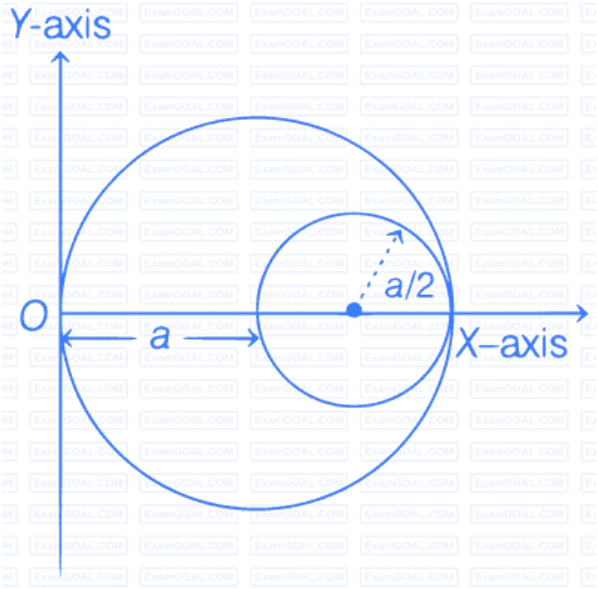
$$(a)\to(ii),\quad (b)\to(iv),\quad (c)\to(i),\quad (d)\to(iii).$$

Option (B) contains all four. Option (A) incorrectly pairs microwaves with an RC circuit and X-rays with vibrations. Option (C) has the first, second and fourth matches right but wrongly assigns gamma rays to an RC circuit. Option (D) likewise has incorrect microwave and X-ray sources. Matching only one pair would not reliably distinguish all choices.

Step 4: Conclusion The sources are magnetron for microwaves, atomic and molecular vibrations for infrared, radioactive nuclear decay for gamma rays, and inner-shell electrons for X-rays. Hence, the correct option is (B).

Quick Tip to Solve: Use the origin to distinguish the highest-frequency pair: gamma rays come from the nucleus, while characteristic X-rays come from inner electron shells.

46. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be : [4 marks]



(A) $\frac{1}{6}a$
(B) $\frac{2}{3}a$
(C) $\frac{5}{6}a$
(D) $\frac{10}{11}a$

Correct Answer: (C) $\frac{5}{6}a$

Solution:

Concept:

- For a uniform lamina, the centroid is its centre of mass, and mass is proportional to area. A removed region can be treated as a negative mass.
- The centroid formula for a disc of mass M minus a removed disc of mass m is $x_c = (Mx_1 - mx_2)/(M - m)$. Coordinates must use the same origin.
- The figure is symmetric about the horizontal axis, so the remaining lamina has $y_c = 0$. The required coordinate is measured from O, not from the original disc centre.

Step 1: Read the two centre positions The large disc has radius a, with its leftmost point at O. Its centre therefore lies at $x_1 = a$. The hole has radius $a/2$ and touches the large circle internally at its rightmost point $x = 2a$. Hence its centre is

$$x_2 = 2a - \frac{a}{2} = \frac{3a}{2}.$$

Both centres lie on the horizontal axis in the given figure.

Step 2: Subtract the removed area and its moment Let M be the mass of the complete large disc before cutting. The hole has one quarter of its area, so its removed mass is $m = M/4$, and the remaining mass is $3M/4$. Its first moment about O is

$$Ma - \frac{M}{4}\frac{3a}{2} = Ma - \frac{3Ma}{8} = \frac{5Ma}{8}.$$

Consequently,

$$x_c = \frac{5Ma/8}{3M/4} = \frac{5a}{6}, \qquad y_c = 0.$$

Removing material to the right shifts the centroid left of the original centre, as this result requires.

Step 3: Distinguish the origin from the centre The shift from the original centre is $a - 5a/6 = a/6$, explaining why option (A) is tempting but incorrect: it gives the displacement rather than the coordinate from O. Option (B) does not follow from the removed area and its lever arm. Option (D) gives a different first moment and also fails the negative-mass calculation. The coordinate from the stated origin is the value in option (C).

Step 4: Conclusion The centroid is on the positive horizontal axis at $(5a/6, 0)$ relative to O. Hence, the correct option is (C).

Quick Tip to Solve: Locate both centres from the marked origin first, then use negative area for the hole. A shift of $a/6$ from the large-disc centre is a coordinate of $5a/6$ from O.

47. Zener breakdown occurs in a p − n junction having p and n both : [4 marks]

(A) heavily doped and have wide depletion layer.

(B) lightly doped and have narrow depletion layer.

(C) heavily doped and have narrow depletion layer.

(D) lightly doped and have wide depletion layer.

Correct Answer: (C) heavily doped and have narrow depletion layer.

Solution:

Concept:

- A reverse-biased junction has a depletion region in which the electric field is large. The field strength depends on the potential difference divided by the depletion width.
- Heavy doping on both sides of a p–n junction makes the depletion layer very narrow, since a small width contains enough fixed ions to establish the junction field.
- Zener breakdown occurs through field-induced electron tunnelling across this narrow region at a comparatively low reverse breakdown voltage.

Step 1: Relate doping to depletion width When the p and n regions are brought together, electrons and holes diffuse and recombine near the junction, leaving fixed ionized dopants. The depletion region must provide the space charge that balances the built-in potential. A high dopant density provides that charge within a short distance; therefore, heavy doping on both sides produces a narrow depletion layer. Lightly doped junctions instead need a wider depleted region.

Step 2: Identify the Zener mechanism Under reverse bias the potential drop is concentrated across the depletion layer. Since the layer is narrow, even a modest reverse voltage creates a strong electric field. This field permits electrons to tunnel from occupied valence-band states on the p side into available conduction-band states on the n side. The reverse current then rises sharply: this is Zener breakdown. It is the large field across a small width, rather than a wide region for repeated collisions, that identifies this mechanism.

Step 3: Eliminate inconsistent combinations Option (A) combines heavy doping with a wide layer, contradicting the doping-width relation. Option (B) pairs light doping with a narrow layer and fails for the same reason. Option (D), light doping and a wide layer, describes the kind of junction in which avalanche multiplication is favoured: carriers gain energy and create additional carriers through collisions. Only option (C) simultaneously gives the doping and depletion width appropriate to Zener tunnelling.

Step 4: Conclusion Zener breakdown occurs when both p and n regions are heavily doped and the depletion layer is narrow. Hence, the correct option is (C).

Quick Tip to Solve: Associate Zener breakdown with a strong field across a very thin junction: heavy doping makes the layer narrow, so a low reverse voltage can cause tunnelling.

48. The de-Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is : [4 marks]

(A) 4 : 2

(B) 4 : 3

(C) 4 : 1

(D) 1 : 4

Correct Answer: (C) 4 : 1

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of a particle's momentum.
- Equal de Broglie wavelengths imply equal momentum magnitudes. In the nonrelativistic regime relevant here, $p = mv$, so velocities are inversely proportional to masses.
- An alpha particle contains two protons and two neutrons, so its mass is approximately four times the proton mass: $m_\alpha \simeq 4m_p$.

Step 1: Use the equality of wavelengths For a proton and an alpha particle, respectively, write

$$\lambda_p = \frac{h}{m_p v_p}, \qquad \lambda_\alpha = \frac{h}{m_\alpha v_\alpha}.$$

The question gives $\lambda_p = \lambda_\alpha$. Cancelling the common constant h and cross-multiplying therefore gives

$$m_p v_p = m_\alpha v_\alpha.$$

Equal wavelength fixes momentum, not speed, kinetic energy, or charge-to-mass ratio.

Step 2: Insert the particle mass ratio The alpha particle is a helium nucleus with four nucleons. Neglecting the small proton-neutron mass difference and nuclear binding correction, as in this question, put $m_\alpha = 4m_p$. Thus

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = 4, \qquad v_p : v_\alpha = 4 : 1.$$

The lighter proton must move faster to carry the same momentum as the heavier alpha particle; this provides a direct physical check on the direction of the ratio.

Step 3: Check the other proposed ratios Option (A), $4 : 2$, would give only a factor of two and can result from incorrectly using the alpha particle's charge instead of its mass. Charge does not enter the de Broglie relation here. Option (B), $4 : 3$, is inconsistent with the four-nucleon mass. Option (D), $1 : 4$, reverses the order and would make the heavier alpha particle faster despite equal momenta. Option (C) alone has the right magnitude and ordering.

Step 4: Conclusion The required proton-to-alpha velocity ratio is $\boxed{v_p : v_\alpha = 4 : 1}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal wavelengths, first write equal momenta. Then reverse the mass ratio to obtain the velocity ratio; do not use the particles'charges.

49. According to Bohr atomic model, in which of the following transitions will the frequency be maximum? [4 marks]

(A) n = 2 to n = 1

(B) n = 3 to n = 2

(C) n = 4 to n = 3

(D) n = 5 to n = 4

Correct Answer: (A) n = 2 to n = 1

Solution:

Concept:

- For a hydrogen-like atom of fixed atomic number Z, Bohr energy levels are $E_n = -13.6Z^2/n^2\,\text{eV}$. Lower levels are more widely separated.
- A downward transition emits a photon with $h\nu = E_i - E_f$. Therefore the largest positive energy gap gives the largest emitted frequency.
- All listed transitions are between adjacent levels of the same atom, so the common factor $13.6Z^2/h$ cancels when comparing their frequencies.

Step 1: Write the frequency for a downward transition For an electron dropping from n_i to $n_f < n_i$, use the difference of the negative level energies:

$$h\nu = 13.6Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\text{eV}.$$

The bracket is positive, and it is this energy difference, rather than just the change in the integer n, that must be compared. Each option changes n by one but the level spacings are unequal.

Step 2: Compare all four energy gaps The dimensionless factors for the four options are

$$\begin{aligned} 2 \to 1 &: 1 - \tfrac{1}{4} = \tfrac{3}{4}, \\ 3 \to 2 &: \tfrac{1}{4} - \tfrac{1}{9} = \tfrac{5}{36}, \\ 4 \to 3 &: \tfrac{1}{9} - \tfrac{1}{16} = \tfrac{7}{144}, \\ 5 \to 4 &: \tfrac{1}{16} - \tfrac{1}{25} = \tfrac{9}{400}. \end{aligned}$$

Their approximate values are 0.750, 0.139, 0.0486, and 0.0225, respectively. The first is plainly the largest, and multiplication by the same positive energy scale preserves this ordering.

Step 3: Interpret and select the transition The $2 \to 1$ transition terminates in the ground state and spans the widest of these adjacent-level gaps. Its photon therefore has the greatest energy and highest frequency; equivalently it has the shortest wavelength. Options (B), (C), and (D) describe progressively higher adjacent levels, where the Bohr levels crowd closer together, and their explicitly calculated energy differences are all smaller. No numerical value of Planck's constant is needed for this comparison.

Step 4: Conclusion The maximum frequency is emitted in the transition $\boxed{n = 2 \to n = 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For adjacent Bohr transitions, the lower pair of levels has the larger gap. Compare $1/n_f^2 - 1/n_i^2$ if there is any doubt; equal changes in n do not mean equal photon energies.

50. Given below are two statements :

Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (B) Statement I is false but Statement II is true

Solution:

Concept:

- A transistor has two junctions within one continuous semiconductor crystal. Its thin, lightly doped base allows carriers injected by the emitter to reach the collector.
- For a common-emitter transistor, the current gain is $\beta = I_C/I_B$ for dc currents; for small changes at a fixed operating point, $\beta_{ac} = \Delta I_C/\Delta I_B$.

Step 1: Evaluate Statement I Two separate PN diodes joined back to back do not form a transistor. Their intervening connection is not the very thin shared semiconductor base of a transistor. Carriers injected across one separate diode cannot be efficiently collected across the other junction as they are in a single transistor structure. Hence Statement I is false.

Step 2: Evaluate Statement II and select the combination In the common-emitter configuration, the base current controls the much larger collector current. Their ratio is precisely the amplification factor β, so Statement II is true. The combination false/true excludes the both-false and both-true choices; the true/false choice reverses both judgements.

Step 3: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (B).

Quick Tip to Solve: Remember that the two junctions must share a thin semiconductor base; joining two finished diodes does not reproduce carrier transport through that base.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be _____ cm. [4 marks]

Solution:

Concept:

- Young's modulus is $Y = (F/A)/(\Delta L/L)$, so the extension of a uniform wire is $\Delta L = FL/(AY)$.
- For a circular cross-section, $A = \pi d^2/4$. For the same material and force, extension is therefore proportional to L/d^2, not to length alone.

Step 1: Compare the new geometry Let the original length and diameter be L and d. The new values are $L\prime = 2L$ and $d\prime = 2d$, so $A\prime = \pi(2d)^2/4 = 4A$. The increased length tends to double the extension, while the fourfold area tends to divide it by four.

Step 2: Calculate and convert the extension The material retains its Young's modulus and the applied force is unchanged. Taking the ratio avoids needing either unknown quantity:

$$\frac{\Delta L\prime}{\Delta L} = \frac{L\prime}{L}\frac{A}{A\prime} = \frac{2}{4} = \frac{1}{2}.$$

$$\Delta L\prime = \frac{0.04}{2}\,\mathrm{m} = 0.02\,\mathrm{m} = 2\,\mathrm{cm}.$$

The answer blank requests centimetres, so the numerical entry is 2.

Step 3: Conclusion The new elongation is $\boxed{2\,\mathrm{cm}}$; enter $\boxed{2}$.

Quick Tip to Solve: When a wire diameter changes, square its scale factor before applying the extension ratio $\Delta L \propto L/d^2$.

52. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be _____ $\times 10^{-2}$ cm. [4 marks]

Solution:

Concept:

- For a nonmagnetic dielectric, the wave speed is $v = c/\sqrt{\varepsilon_r}$, with $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$.
- The frequency is set by the source and remains unchanged at an interface. Thus the wavelength in the medium is $\lambda = v/f$.

Step 1: Find the speed in the dielectric The relative electric permittivity is $\varepsilon_r = 2.25$, giving refractive index $n = \sqrt{2.25} = 1.5$. Hence

$$v = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\,\mathrm{m\,s^{-1}}.$$

The wave slows on entering the dielectric; it is the wavelength, rather than the frequency, that

decreases.

Step 2: Express the wavelength in the requested units Convert the frequency before division: $3\,\text{GHz} = 3 \times 10^9\,\text{Hz}$. Therefore

$$\lambda = \frac{2 \times 10^8}{3 \times 10^9}\,\text{m} = \frac{1}{15}\,\text{m} = \frac{20}{3}\,\text{cm}.$$

Writing this in the specified form gives

$$\lambda = \frac{2000}{3} \times 10^{-2}\,\text{cm} = 666.666\ldots \times 10^{-2}\,\text{cm}.$$

The integer entry is obtained by rounding this coefficient.

Step 3: Conclusion The wavelength is approximately $6.667\,\text{cm}$, or $667 \times 10^{-2}\,\text{cm}$ to the requested integer coefficient. Enter $\boxed{667}$.

Quick Tip to Solve: Calculate the wavelength in metres first, convert to centimetres, then divide by 10^{-2} to find the requested coefficient.

53. A cylindrical wire of radius 0.5 mm and conductivity 5 $\times$ 10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is _____. [4 marks]

Solution:

Concept:

- The microscopic form of Ohm's law is $J = \sigma E$, where J is current density and σ is conductivity.
- For a uniform cylindrical wire, $I = JA = \sigma E \pi r^2$. Radius, electric field and the final current must all be expressed in compatible units.

Step 1: Convert the radius and electric field The stated radius is $r = 0.5\,\text{mm} = 5 \times 10^{-4}\,\text{m}$, and the field is $E = 10\,\text{mV m}^{-1} = 10^{-2}\,\text{V m}^{-1}$. Thus

$$A = \pi(5 \times 10^{-4})^2 = 25\pi \times 10^{-8}\,\text{m}^2, \qquad J = (5 \times 10^7)(10^{-2}) = 5 \times 10^5\,\text{A m}^{-2}.$$

Step 2: Compute the current and compare coefficients Multiplying the current density by the cross-sectional area gives

$$I = (5 \times 10^5)(25\pi \times 10^{-8}) = 125\pi \times 10^{-3}\,\text{A} = 125\pi\,\text{mA}.$$

The question writes this same current as $x^3\pi\,\text{mA}$. Cancel the common π and the common current unit:

$$x^3 = 125 \quad \Longrightarrow \quad x = 5.$$

Taking a cube root is essential because the requested quantity is not the coefficient itself.

Step 3: Conclusion The current is $125\pi\,\text{mA}$, giving the dimensionless value $\boxed{x = 5}$.

Quick Tip to Solve: Keep π symbolic in $I = \sigma E \pi r^2$; it cancels immediately when comparing with $x^3\pi\,\text{mA}$.

54. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(K.E.)_A : (K.E.)_B$ will be $\frac{A}{1}$, so the value of A will be _____.

[4 marks]

Solution:

Concept:

- For a nonrelativistic body, $p = mv$ and $K = \frac{1}{2}mv^2$. Eliminating the speed gives $K = p^2/(2m)$.
- When two bodies have equal linear momentum, their kinetic energies vary inversely with their masses. Equal momentum therefore does not imply equal kinetic energy.

Step 1: Express both energies using their common momentum Denote the common momentum magnitude by p. For the masses $m_A = 1\,\text{kg}$ and $m_B = 2\,\text{kg}$,

$$K_A = \frac{p^2}{2m_A}, \qquad K_B = \frac{p^2}{2m_B}.$$

These expressions compare the two kinetic energies without requiring the unspecified momentum value.

Step 2: Form the ratio and identify the required number Divide the expressions and cancel the common factor $p^2/2$:

$$\frac{K_A}{K_B} = \frac{m_B}{m_A} = \frac{2}{1}.$$

Physically, the lighter body must move twice as fast to carry the same momentum, so its kinetic energy is larger. This check also prevents inadvertently using the direct mass ratio $1/2$, which would apply to equal speeds instead. The problem denotes the numerator of the ratio $A/1$ by the same letter as the first body; that requested numerator is 2.

Step 3: Conclusion The kinetic-energy ratio is $K_A : K_B = 2 : 1$, so the required dimensionless value is $\boxed{A = 2}$.

Quick Tip to Solve: Choose the useful energy form from the condition: use $K = p^2/(2m)$ when momentum is equal, and $K = mv^2/2$ when speed is equal.

55. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be _____.

[4 marks]

Solution:

Concept:

- The root mean square molecular speed is $v_{\text{rms}} = \sqrt{3RT/M}$, where M is the gas molar mass and T is absolute temperature.
- For the same gas, $v_{\text{rms}} \propto \sqrt{T}$. Pressure is not an independent factor in this relation; it affects the density consistently with the ideal-gas equation.

Step 1: Convert both temperatures to kelvin Using the conventional exam conversion,

$$T_1 = 27 + 273 = 300\,\text{K}, \qquad T_2 = 127 + 273 = 400\,\text{K}.$$

A ratio formed directly from Celsius temperatures would be incorrect, since molecular kinetic energy is proportional to the absolute temperature.

Step 2: Scale the speed and match the stated form The gas species remains unchanged, so the constants and molar mass cancel:

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{400}{300}} = \frac{2}{\sqrt{3}}.$$

$$v_2 = 200\frac{2}{\sqrt{3}} = \frac{400}{\sqrt{3}}\,\text{m}\,\text{s}^{-1}.$$

The change from one to two atmospheres requires no extra multiplier. Equivalently, in $v_{\text{rms}} = \sqrt{3P/\rho}$, the density also changes and $P/\rho = RT/M$. Comparing with the requested $x/\sqrt{3}$ form gives the coefficient directly.

Step 3: Conclusion The final rms speed is $400/\sqrt{3}\,\text{m}\,\text{s}^{-1}$, so $\boxed{x = 400}$.

Quick Tip to Solve: For the same gas, form $v_2/v_1 = \sqrt{T_2/T_1}$ using kelvin temperatures; a separate pressure factor would count the state change incorrectly.

56. A series L-C-R circuit is designed to resonate at an angular frequency $\omega_0 = 10^5$ rad/s. The circuit draws 16W power from 120V source at resonance. The value of resistance 'R' in the circuit is _____ Ω. [4 marks]

Solution:

Concept:

- In a series LCR circuit at resonance, $X_L = X_C$. The reactive contributions cancel and the total impedance is the resistance: $Z = R$.
- The phase angle is zero at resonance, so the power factor is unity. With rms source voltage V, average power is $P = VI = V^2/R$.

Step 1: Apply the resonance condition The circuit resonates at $\omega_0 = 10^5\,\text{rad}\,\text{s}^{-1}$. At that frequency, $\omega_0 L = 1/(\omega_0 C)$, so the net reactance vanishes. Although the given frequency constrains the product LC, individual values of L and C are unnecessary for finding the resistance from the measured power.

Step 2: Calculate the resistance and check the power The source voltage is $120\,\text{V}$ and the average power drawn is $16\,\text{W}$. Rearranging the resonant power expression gives

$$R = \frac{V^2}{P} = \frac{(120)^2}{16} = \frac{14400}{16} = 900\,\Omega.$$

The current provides an independent consistency check:

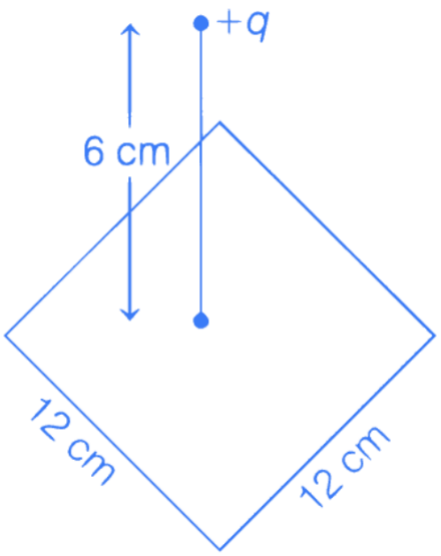
$$I = \frac{P}{V} = \frac{16}{120} = \frac{2}{15}\,\text{A}, \qquad \frac{V}{I} = \frac{120}{2/15} = 900\,\Omega.$$

The voltage in the ac power formula is its rms value, as usual for an unspecified source rating.

Step 3: Conclusion The resistance required in the resonant circuit is $\boxed{R = 900\,\Omega}$. Enter $\boxed{900}$.

Quick Tip to Solve: At series resonance, replace the entire impedance by R and use $P = V_{\text{rms}}^2/R$; the resonant frequency is not needed for this calculation.

57. A point charge of $+12\mu$C is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be _____ $\times$ 10^3 Nm^2/C. [4 marks]



Solution:

Concept:

- Gauss's law gives the total outward electric flux through a closed surface: $\Phi_{\text{closed}} = q/\varepsilon_0$, where q is its enclosed charge.
- For a point charge at the centre of a cube, symmetry divides the total flux equally between its six congruent faces. The result depends on the enclosed charge, not the side length.

Step 1: Complete the square into a cube The square has side $12\,\text{cm}$, and the charge is $6\,\text{cm}$ above its centre. Imagine the square as the bottom face of a cube extending towards the charge. Half the cube side is $6\,\text{cm}$, so the charge is exactly at the cube centre. Every face subtends the same solid angle at the charge. Although the square is an open surface, this construction lets us use a symmetric closed Gaussian surface without integrating the varying field.

Step 2: Divide the total flux between the faces The enclosed charge is $q = 12 \times 10^{-6}\,\text{C}$. Therefore,

the magnitude through the square is

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{12 \times 10^{-6}}{6(8.854 \times 10^{-12})} = 2.2599 \times 10^5 \,\mathrm{N\,m^2/C}.$$

This is $225.99 \times 10^3 \,\mathrm{N\,m^2/C}$. The flux is outward from the cube on that face; only its magnitude is requested. Taking the full q/ε_0 would count all six faces, rather than this one square.

Step 3: Conclusion The electric-flux magnitude is approximately $226 \times 10^3 \,\mathrm{N\,m^2/C}$. Hence, the required numerical entry is $\boxed{226}$.

Quick Tip to Solve: When a charge is half a square side above its centre, complete a cube: the square receives one-sixth of the total Gauss-law flux.

58. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is _____ $\times 10^{-1}$ kg m^2. [4 marks]

Solution:

Concept:

- For each straight side of a uniform wire frame, the moment of inertia about a perpendicular axis through its midpoint is $I_{\text{mid}} = ma^2/12$, where m and a are that side's mass and length.
- The parallel-axis theorem gives $I = I_{\text{mid}} + mr^2$. For a regular hexagon of side a, the perpendicular distance from the centre to a side midpoint is the apothem $r = \sqrt{3}a/2$.

Step 1: Find the dimensions of each side The uniform bar forms six equal sides, so its mass and length are divided equally:

$$a = \frac{2.4}{6} = 0.4\,\mathrm{m}, \qquad m = \frac{6}{6} = 1\,\mathrm{kg}.$$

The centre of mass of the symmetric hexagonal frame is its geometric centre. Joining this centre to adjacent vertices produces equilateral triangles; the altitude to a side bisects that side and has length $\sqrt{3}a/2$. Thus $r^2 = 3a^2/4 = 0.12\,\mathrm{m}^2$.

Step 2: Add the six shifted side contributions All six side contributions are equal. Each side has a finite length, so its own midpoint inertia must be included in addition to the displacement term:

$$I_{\text{one}} = \frac{ma^2}{12} + m\frac{3a^2}{4} = \frac{5ma^2}{6}.$$

Consequently,

$$I = 6I_{\text{one}} = 5ma^2 = 5(1)(0.4)^2 = 0.80\,\mathrm{kg\,m^2}.$$

The axis is normal to the frame plane, so this is the required polar moment. Treating all the wire mass as concentrated only at the side midpoints would omit $ma^2/12$ for each side.

Step 3: Conclusion The moment of inertia is $0.80\,\mathrm{kg\,m^2} = 8 \times 10^{-1}\,\mathrm{kg\,m^2}$. The required numerical entry is $\boxed{8}$.

Quick Tip to Solve: For a regular hexagonal wire, use each side as a small rod and shift its midpoint axis by the apothem, not by the circumradius.

59. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is −5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W. The value of x is _____.
[Gain in $dB = 10\log_{10}\left(\frac{P_o}{P_i}\right)$] [4 marks]

Solution:

Concept:

- Power gain in decibels is $G = 10\log_{10}(P_o/P_i)$. The factor is 10 for a power ratio; it is not the amplitude-ratio factor 20.
- Uniform attenuation adds in decibels along successive cable lengths. A negative gain means that the output power is smaller than the input power.

Step 1: Calculate the total attenuation The cable contributes $-5\,\text{dB}$ per kilometre over $20\,\text{km}$. Hence its total power gain is

$$G = (-5)(20) = -100\,\text{dB}.$$

The input signal power must be expressed in watts before comparing it with the stated output form:

$$P_i = 0.1\,\text{kW} = 0.1(1000) = 100\,\text{W} = 10^2\,\text{W}.$$

The negative sign must be retained: inserting a positive attenuation into the gain formula would incorrectly predict amplification.

Step 2: Convert decibels back to a power ratio Substitute the total gain into the given definition and divide by 10:

$$-100 = 10\log_{10}\left(\frac{P_o}{P_i}\right), \qquad \log_{10}\left(\frac{P_o}{P_i}\right) = -10.$$

Taking the base-10 antilogarithm gives $P_o/P_i = 10^{-10}$. Therefore,

$$P_o = 10^2 \times 10^{-10} = 10^{-8}\,\text{W}.$$

Comparing the exponent with $P_o = 10^{-x}\,\text{W}$ gives $-x = -8$. The ratio is dimensionless, while the received power itself retains watts.

Step 3: Conclusion The receiver gets $10^{-8}\,\text{W}$, so $\boxed{x = 8}$. The exponent x is dimensionless.

Quick Tip to Solve: Add cable losses in dB first, divide the total by 10, and only then convert back to a power ratio; convert kW to W before matching exponents.

60. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be _____ Hz. [Velocity of sound in air is 340 m/s.] [4 marks]

Solution:

Concept:

- For a source and observer approaching through stationary air, $f_{\text{heard}} = f(v + v_o)/(v - v_s)$, where the speeds are measured relative to the air.
- The beat frequency is the absolute difference of the two frequencies heard together. A driver hears their own horn at its emitted frequency because source and listener move together.

Step 1: Identify the two frequencies at one car Convert the speed of either car:

$$u = 7.2 \times \frac{5}{18} = 2\,\text{m}\,\text{s}^{-1}.$$

Each driver hears their own horn at $676\,\text{Hz}$. For the other horn, the observer moves towards the source at $v_o = 2\,\text{m}\,\text{s}^{-1}$, and that source also moves towards the observer at $v_s = 2\,\text{m}\,\text{s}^{-1}$. Both effects raise the received frequency; using only the relative speed in either the numerator or denominator would miss one of these contributions.

Step 2: Calculate the shifted frequency and its difference With sound speed $v = 340\,\text{m}\,\text{s}^{-1}$, the approaching horn is heard at

$$f_{\text{heard}} = 676\frac{340 + 2}{340 - 2} = 676\frac{342}{338} = 684\,\text{Hz}.$$

Here $676 = 2(338)$, so the simplification is exact. Its beats with the driver's own horn have frequency

$$f_{\text{beat}} = |684 - 676| = 8\,\text{Hz}.$$

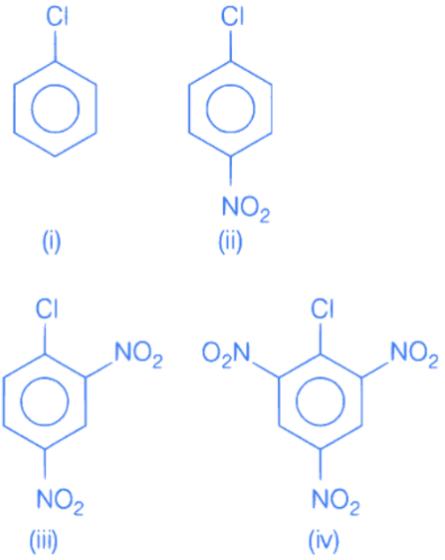
Interchanging the two cars leaves all the speeds and emitted frequencies unchanged. Thus the second driver hears the same pair of frequencies and the same beat rate, rather than a different Doppler shift.

Step 3: Conclusion The beat frequency heard by each driver is $\boxed{8\,\text{Hz}}$.

Quick Tip to Solve: At each car compare the Doppler-shifted horn of the other car with the unchanged frequency of its own horn; the two identical cars give the same beat rate.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is : [4 marks]



(A) (iv) < (i) < (ii) < (iii)

(B) (iv) < (iii) < (ii) < (i)

(C) (iv) < (i) < (iii) < (ii)

(D) (i) < (ii) < (iii) < (iv)

Correct Answer: (D) (i) < (ii) < (iii) < (iv)

Solution:

Concept:

- In nucleophilic aromatic substitution, an electron-withdrawing nitro group at an ortho or para position relative to the leaving group stabilizes the anionic addition intermediate by resonance.
- More suitably positioned NO_2 groups increase the electrophilic character of the carbon bearing chlorine and lower the energy of the intermediate. This is different from the directing effects in electrophilic substitution.

Step 1: Compare the four aryl chlorides Compound (i) is chlorobenzene, with no nitro group to stabilize the intermediate. Compound (ii) is para-nitrochlorobenzene and has one activating NO_2 group para to chlorine. Compound (iii) has two such groups, one ortho and one para. Compound (iv) has three nitro groups at both ortho positions and the para position. All four retain the same leaving group, chlorine, so the key difference is stabilization by the nitro substituents.

Step 2: Arrange the increasing reactivity An incoming nucleophile first adds at the carbon bearing chlorine. The negative charge of the resulting sigma complex can delocalize to ortho and para positions and then into the nitro groups. Thus each additional nitro group in the indicated positions

favours addition and the overall substitution:

$$\text{(i)} < \text{(ii)} < \text{(iii)} < \text{(iv)}.$$

Option (D) matches this order. Options (A), (B) and (C) all place the trinitro compound (iv) first, as the least reactive, contradicting the strong activation from its three nitro groups. In particular, the aryl carbon-chlorine bond is not analysed using an ordinary alkyl-halide carbocation-stability order.

Step 3: Conclusion The increasing tendency towards nucleophilic substitution is

$$\boxed{\text{(i)} < \text{(ii)} < \text{(iii)} < \text{(iv)}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For nucleophilic substitution of an aryl chloride, count nitro groups ortho and para to chlorine: they stabilize the addition intermediate and increase reactivity.

62. The incorrect statement among the following is : [4 marks]

(A) RuO_4 is an oxidizing agent

(B) Cr_2O_3 is an amphoteric oxide.

(C) $VOSO_4$ is a reducing agent

(D) Red colour of ruby is due to the presence of Co^{3+}

Correct Answer: (D) Red colour of ruby is due to the presence of Co^{3+}

Solution:

Concept:

- Transition-metal ions show characteristic oxidation states, redox behaviour and colours. A metal in a very high oxidation state often acts as an oxidizing agent by accepting electrons.
- Ruby is corundum, Al_2O_3, containing a small amount of Cr^{3+} substituting for Al^{3+}. Chromium(III), rather than cobalt(III), produces its red colour.

Step 1: Verify the three chemical-property statements In RuO_4, ruthenium has oxidation state $+8$. It can be reduced to lower oxidation states while oxidizing another species, so statement (A) is correct. Chromium(III) oxide, Cr_2O_3, is amphoteric: it reacts with acids and with strong bases under suitable conditions. Hence statement (B) is correct as well. In $VOSO_4$, the sulfate ion has charge -2, so the vanadyl ion is VO^{2+}; vanadium is consequently $+4$, since oxygen contributes -2.

Step 2: Check the vanadium redox role and ruby colour Vanadium(IV) can be oxidized to vanadium(V), donating an electron and therefore acting as a reducing agent. For example, its acidic-medium oxidation half-reaction can be written

$$VO^{2+} + H_2O \longrightarrow {VO_2}^{+} + 2\,H^{+} + e^{-}.$$

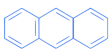
This makes statement (C) correct. The red colour of ruby is produced by Cr^{3+} impurity ions in the aluminium oxide lattice. Statement (D) instead attributes it to Co^{3+}, which identifies the wrong ion.

Since the question asks for the incorrect statement, the three correct chemical-property statements must be rejected as answers.

Step 3: Conclusion The incorrect statement is the claim that ruby's red colour is due to Co^{3+}; the correct ion is Cr^{3+}. Hence, the correct option is (D).

Quick Tip to Solve: Remember ruby as chromium(III)-doped corundum; then check oxidation states separately when evaluating the other transition-metal statements.

63. Which one of the following compounds is non-aromatic? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- An aromatic molecule requires a planar cyclic network of continuously overlapping p orbitals containing $4n + 2$ pi electrons. Meeting an electron count without a continuous cyclic p system is insufficient.
- A tetrahedral sp^3 carbon in a ring interrupts conjugation. Such a ring is non-aromatic; it is not classified as antiaromatic simply by counting its double bonds.

Step 1: Check the first three cyclic pi systems Option (A) is anthracene, a fused planar aromatic system with 14 pi electrons, consistent with $4n + 2$ for $n = 3$. Option (B) is the cyclobutadiene dication shown with one double bond and two positively charged carbons. Its two empty p orbitals complete the cyclic conjugation, and the remaining double bond supplies two pi electrons, satisfying the aromatic count for $n = 0$. In option (C), furan, two double bonds provide four pi electrons and one oxygen lone pair supplies two more. The other oxygen lone pair is not counted in the cyclic pi system. Furan therefore has six pi electrons and is aromatic.

Step 2: Identify the interrupted conjugation Option (D) is cyclohepta-1,3,5-triene. Its three double bonds contain six pi electrons, but the remaining ring carbon is a neutral CH_2 group. That carbon is sp^3-hybridized and has no p orbital available for continuous overlap around the whole seven-membered ring. Consequently, the three double bonds do not form an uninterrupted cyclic conjugated system. This distinguishes the neutral molecule from the tropylium cation, where every ring atom has a p orbital. Options (A), (B) and (C) have the required continuous p framework, whereas (D) fails that first structural test.

Step 3: Conclusion The cycloheptatriene structure is non-aromatic because its sp^3 carbon interrupts cyclic conjugation. Hence, the correct option is (D).

Quick Tip to Solve: Before applying the $4n+2$ rule, check every atom around the ring for a p orbital; a neutral CH_2 vertex breaks the aromatic circuit.

64. According to Bohr's atomic theory :

(A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$.

(B) The product of velocity (v) of electron and principal quantum number (n), $'vn' \propto Z^2$.

(C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$.

(D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$.

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A), (C) and (D) only

(B) (A) only

(C) (C) only

(D) (A) and (D) only

Correct Answer: (D) (A) and (D) only

Solution:

Concept:

- For a hydrogen-like atom, the Bohr radius and electron speed scale as $r_n \propto n^2/Z$ and $v_n \propto Z/n$, where Z is nuclear charge and n is the orbit number.
- Kinetic energy is $K = mv^2/2$, orbital frequency is $f = v/(2\pi r)$, and the Coulomb force magnitude is $F = kZe^2/r^2$.

Step 1: Test the energy and velocity statements Using the speed scaling gives

$$K \propto v^2 \propto \frac{Z^2}{n^2}.$$

Thus statement (A) is correct. On multiplying the speed by the orbit number,

$$vn \propto \frac{Z}{n} n = Z.$$

The dependence is on the first power of nuclear charge, rather than its square, so statement (B) is incorrect. The electron mass and fundamental constants are the same in all these proportionalities.

Step 2: Calculate the frequency dependence An orbital revolution covers the circumference $2\pi r$. Consequently,

$$f = \frac{v}{2\pi r} \propto \frac{Z/n}{n^2/Z} = \frac{Z^2}{n^3}.$$

Statement (C) gives an extra power of Z, and is therefore false. The frequency is inversely proportional to the cube of n, but checking only that part would miss the incorrect nuclear-charge dependence.

Step 3: Calculate the force and select the pair The nucleus has charge $+Ze$, so the numerator of Coulomb force contains one additional factor of Z:

$$F = \frac{kZe^2}{r^2} \propto \frac{Z}{n^4/Z^2} = \frac{Z^3}{n^4}.$$

Statement (D) is correct. Thus (A) and (D) hold, whereas (B) and (C) fail. The first and third options include the false frequency statement, and the second omits the true force statement.

Step 4: Conclusion The true statements concern kinetic energy and Coulomb force. Hence, the correct option is (D).

Quick Tip to Solve: Start with $r \propto n^2/Z$ and $v \propto Z/n$; derive all other powers from these two relations rather than memorising separate lists.

65. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4? [4 marks]

(A) $CH_3COCH_2CH_3$ (butan-2-one)

(B) $C_6H_{11}COCH_3$ (cyclohexyl methyl ketone)

(C) CH_3CH_2CHO (propanal)

(D) CH_3CHO (ethanal)

Correct Answer: (C) CH_3CH_2CHO (propanal)

Solution:

Concept:

- Acidic hydration of an alkyne with $HgSO_4/H_2SO_4$ forms an enol, which undergoes keto-enol tautomerism to a carbonyl compound.
- A terminal alkyne $R{-}C{\equiv}CH$ gives the methyl ketone $RCOCH_3$ by Markovnikov hydration. Ethyne is the exception and gives ethanal.

Step 1: Apply the hydration rule Water adds across the triple bond with hydroxyl attached to the more substituted carbon of a terminal alkyne. The initial enol rearranges by proton transfer and double-bond migration:

$$R{-}C{\equiv}CH \xrightarrow[HgSO_4/H_2SO_4]{H_2O} R{-}C(OH){=}CH_2 \longrightarrow RCOCH_3.$$

The product retains the original carbon skeleton. This method therefore makes methyl ketones from terminal alkynes other than ethyne; it does not generally make aldehydes from such alkynes.

Step 2: Find precursors for the obtainable options But-1-yne, $CH_3CH_2C{\equiv}CH$, gives $CH_3CH_2COCH_3$, which is the butan-2-one in option (A). Similarly, $C_6H_{11}C{\equiv}CH$, with C_6H_{11} denoting the cyclohexyl group, gives cyclohexyl methyl ketone, option (B). Hydration of ethyne produces vinyl alcohol, $CH_2{=}CHOH$, whose tautomer is CH_3CHO. Thus the ethanal shown in option (D) is also obtainable.

Step 3: Exclude propanal For a three-carbon alkyne the only open-chain possibility is propyne, $CH_3C{\equiv}CH$. Under the stated mercuric-ion acidic conditions it produces acetone, CH_3COCH_3, rather than propanal, CH_3CH_2CHO. Internal alkynes give ketones, so choosing an internal alkyne cannot supply this aldehyde either. Propanal can instead be obtained using suitable anti-Markovnikov hydration conditions, but those are different from the reagents in this question.

Step 4: Conclusion Propanal, CH_3CH_2CHO, cannot be prepared by the stated alkyne hydration. Hence, the correct option is (C).

Quick Tip to Solve: For $HgSO_4/H_2SO_4$ hydration, look for a methyl ketone; allow ethanal only as the special product from ethyne.

66. The correct set from the following in which both pairs are in correct order of melting point is : [4 marks]

(A) LiCl $>$ LiF ; NaCl $>$ MgO

(B) LiCl $>$ LiF ; MgO $>$ NaCl

(C) LiF $>$ LiCl ; NaCl $>$ MgO

(D) LiF $>$ LiCl ; MgO $>$ NaCl

Correct Answer: (D) LiF $>$ LiCl ; MgO $>$ NaCl

Solution:

Concept:

- The melting point of an ionic solid generally increases with the strength of electrostatic attraction in its lattice. Lattice attraction increases with ionic charge and decreases with interionic distance.
- For similar structures, the magnitude of lattice energy roughly follows $U \propto |z_+ z_-|/(r_+ + r_-)$. Polarisation can introduce covalent character and modify a simple ionic comparison.

Step 1: Compare lithium fluoride and lithium chloride Both salts contain Li^+, and both anions have charge -1. The fluoride ion is smaller than chloride, so the distance between the ion centres is smaller in LiF. Electrostatic attraction is consequently stronger. In addition, the larger chloride ion is more readily polarised by the small lithium cation, so $LiCl$ has greater covalent character. Both considerations support

$$T_m(\text{LiF}) > T_m(\text{LiCl}).$$

Hence options (A) and (B), which reverse this first pair, are excluded.

Step 2: Compare magnesium oxide and sodium chloride In MgO, the ions are Mg^{2+} and O^{2-}, giving a charge-product magnitude of four. In $NaCl$, the corresponding charge-product magnitude is one. The doubly charged ions in magnesium oxide produce substantially stronger lattice attraction, also assisted by their relatively small sizes. More energy is needed to disrupt that lattice during melting:

$$T_m(\text{MgO}) > T_m(\text{NaCl}).$$

Option (C) has the first pair right but this second pair wrong.

Step 3: Combine the two comparisons The requested option must satisfy both inequalities simultaneously. The fluoride-chloride comparison selects the options beginning with $LiF > LiCl$, and the oxide-chloride comparison then selects $MgO > NaCl$. Comparing only one pair would leave a distractor that is half correct.

Step 4: Conclusion The correct pair of orders is $LiF > LiCl$ and $MgO > NaCl$. Hence, the correct option is (D).

Quick Tip to Solve: Compare charge products before ionic sizes. For equal charges, check size and polarisation; for MgO versus $NaCl$, the doubled ionic charges dominate.

67. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene?

[4 marks]



(A) $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{H^+}$

(B) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$

(C) $\xrightarrow{BB_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$

(D) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

Correct Answer: (C) $\xrightarrow{BB_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$

Solution:

Concept:

- The nitro group is a deactivating meta-directing group, while the amino group activates a benzene ring and directs substitution to ortho and para positions.
- An aromatic primary amine gives a diazonium salt with $NaNO_2/HCl$ at 273–278 K. Sandmeyer substitution with $CuBr/HBr$ replaces the diazonium group by bromine.

Step 1: Install the first bromine before reducing the nitro group The target has its two bromines meta to one another. We must first exploit the meta-directing effect of $-NO_2$. Bromination in the presence of iron gives mainly meta-bromonitrobenzene:

$$C_6H_5NO_2 \xrightarrow{Br_2/Fe} \text{m-}BrC_6H_4NO_2.$$

Reducing first would generate aniline, whose amino group directs bromination to ortho and para sites; that route does not establish the required meta relationship.

Step 2: Turn the nitro group into a diazonium leaving group Reduce the nitro group with Sn/HCl, followed by the usual work-up, to obtain meta-bromoaniline without changing the position of the first bromine. Then diazotise the amino group using nitrous acid generated in situ:

$$\text{m-}BrC_6H_4NO_2 \xrightarrow{Sn/HCl} \text{m-}BrC_6H_4NH_2,$$

$$\text{m-}BrC_6H_4NH_2 \xrightarrow{NaNO_2/HCl} \text{m-}BrC_6H_4N_2{}^+Cl^-.$$

The low temperature stabilises the diazonium salt until the substitution step.

Step 3: Replace the diazonium group by bromine Treatment with cuprous bromide in hydrobromic

acid replaces $-N_2^+$ at the original nitro-group position:

$$\text{m-BrC}_6\text{H}_4\text{N}_2^+\text{Cl}^- \xrightarrow{\text{CuBr/HBr}} \text{m-C}_6\text{H}_4\text{Br}_2 + \text{N}_2.$$

This is the sequence represented by option (C), whose first reagent denotes bromination with iron. Option (A) never reduces the nitro group into an amine suitable for diazotisation. Options (B) and (D) reduce before bromination and also lack the complete correct diazotisation/Sandmeyer sequence.

Step 4: Conclusion The required sequence is bromination, Sn/HCl reduction, NaNO_2/HCl diazotisation, then CuBr/HBr. Hence, the correct option is (C).

Quick Tip to Solve: Use a directing group before replacing it: let $-NO_2$ place the first bromine meta, then convert that nitro group into the second bromine through a diazonium salt.

68. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are : [4 marks]

(A) 5.92, 4.90 and 0 BM

(B) 4.90, 0 and 1.73 BM

(C) 5.82, 0 and 0 BM

(D) 4.90, 0 and 2.83 BM

Correct Answer: (B) 4.90, 0 and 1.73 BM

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons.
- Determine oxidation state and d-electron count first, then use the coordination geometry and ligand field to decide whether the electrons pair.

Step 1: Find the iron electron arrangement In $[\text{FeCl}_4]^{2-}$, four chloride ligands contribute -4, so iron is in oxidation state $+2$, with configuration $3d^6$. This tetrahedral chloride complex is high spin: the tetrahedral splitting is small and chloride is a weak-field ligand. Four electrons remain unpaired. Thus

$$\mu_{\text{Fe}} = \sqrt{4(4+2)} = \sqrt{24} = 4.90\,\text{BM}.$$

Options (A) and (C) already fail this first value; five unpaired electrons would correspond to a different electron count.

Step 2: Find the cobalt electron arrangement Each oxalate ion has charge -2, so three ligands contribute -6. The overall charge -3 gives cobalt oxidation state $+3$, hence $3d^6$. The octahedral cobalt(III) oxalate complex has a sufficiently large splitting for a low-spin t_{2g}^6 arrangement. All electrons are paired:

$$n = 0, \qquad \mu_{\text{Co}} = 0\,\text{BM}.$$

It is incorrect to treat this cobalt(III) complex as the same high-spin case as tetrahedral iron(II)

chloride.

Step 3: Find the manganese electron count For manganate, MnO_4^{2-}, oxygen contributes a total oxidation number -8, giving manganese oxidation state $+6$. Manganese is a group-7 element, so Mn^{6+} is d^1. Its one d electron is necessarily unpaired:

$$\mu_{Mn} = \sqrt{1(1+2)} = \sqrt{3} = 1.73\,BM.$$

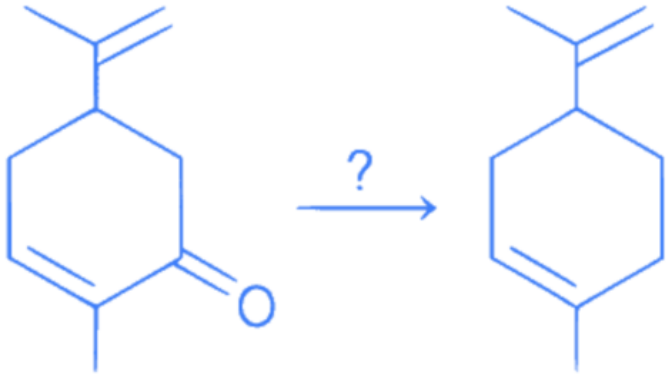
Option (D) gives the value for two unpaired electrons and is excluded; manganate must not be confused with the d^0 permanganate ion.

Step 4: Conclusion The moments are $4.90,\ 0,\ 1.73\,BM$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Check the ion charge carefully: manganate MnO_4^{2-} is d^1, whereas permanganate MnO_4^- is d^0.

69. Which of the following reagent is suitable for the preparation of the product in the above reaction?

[4 marks]



(A) Red P + Cl_2

(B) $NH_2 - NH_2/C_2H_5\overset{\ominus}{O}\overset{\oplus}{N}a$

(C) $NaBH_4$

(D) Ni/H_2

Correct Answer: (B) $NH_2 - NH_2/C_2H_5\overset{\ominus}{O}\overset{\oplus}{N}a$

Solution:

Concept:

- Wolff–Kishner reduction converts an aldehyde or ketone carbonyl group into a methylene group using hydrazine and a strong base under heating.
- The required reagent must remove carbonyl oxygen while preserving the carbon skeleton and the carbon-carbon double bonds shown in the reactant.

Step 1: Identify the functional-group change Compare the two ring structures in the reaction. The

starting molecule contains a cyclic ketone group, but the corresponding carbon in the product has no oxygen and is a methylene carbon. The alkene within the ring and the external isopropenyl double bond are retained. Hence the transformation is

$$R_2C = O \longrightarrow R_2CH_2,$$

rather than reduction of the ketone to an alcohol or general hydrogenation of all unsaturated groups.

Step 2: Choose the carbonyl deoxygenation reagent Hydrazine first condenses with the ketone to give a hydrazone. Under strongly basic heated conditions this intermediate loses nitrogen and gives the hydrocarbon at the former carbonyl carbon. The hydrazine/sodium ethoxide combination in option (B) supplies the Wolff–Kishner type conditions required here. Symbolically, the net conversion is

$$R_2C{=}O + H_2NNH_2 \longrightarrow R_2CH_2 + N_2 + H_2O.$$

The carbon-carbon double bonds need not be hydrogenated in this process, matching their survival in the printed product.

Step 3: Exclude the other reagents Red phosphorus with chlorine is not the hydrazine-mediated reduction that removes oxygen as required. Sodium borohydride, option (C), reduces the ketone to a secondary alcohol, leaving a $C{-}OH$ group instead of the required methylene. Nickel with hydrogen, option (D), readily hydrogenates carbon-carbon double bonds and would therefore change unsaturation that is explicitly retained in the product. None of these options provides the selective carbonyl-to-methylene conversion shown.

Step 4: Conclusion Hydrazine with sodium ethoxide is suitable for the required carbonyl-to-methylene reduction. Hence, the correct option is (B).

Quick Tip to Solve: When a carbonyl oxygen disappears but alkene groups remain, look for Wolff–Kishner conditions; $NaBH_4$ stops at the alcohol.

70. What is the correct order of the following elements with respect to their density? [4 marks]

(A) Cr < Zn < Co < Cu < Fe

(B) Cr < Fe < Co < Cu < Zn

(C) Zn < Cr < Fe < Co < Cu

(D) Zn < Cu < Co < Fe < Cr

Correct Answer: (C) Zn < Cr < Fe < Co < Cu

Solution:

Concept:

- Density is mass per unit volume, $\rho = M/V_m$, where M is molar mass and V_m is molar volume. It depends on both atomic mass and metallic packing.
- Density across the first transition series is not a strictly increasing function of atomic number; zinc is appreciably less dense than iron, cobalt and copper.

Step 1: Recall the approximate densities Useful room-temperature values, in $\mathrm{g\,cm^{-3}}$, are zinc about 7.1, chromium about 7.2, iron about 7.9, cobalt about 8.9, and copper about 9.0. These approximate values distinguish the required ordering without relying on unnecessary decimal places. In particular, zinc is slightly less dense than chromium even though its atomic mass is greater: its molar volume is also larger.

Step 2: Arrange the elements in increasing density Starting with the lowest of these values gives zinc, followed by chromium and then iron. Cobalt and copper are the densest two of the listed elements, with copper slightly denser than cobalt. Hence

$$\rho(\mathrm{Zn}) < \rho(\mathrm{Cr}) < \rho(\mathrm{Fe}) < \rho(\mathrm{Co}) < \rho(\mathrm{Cu}).$$

A trend based only on increasing atomic number would put zinc last and would miss the actual density drop at the end of this series.

Step 3: Check the alternatives against the ordering Option (A) wrongly places chromium below zinc and puts iron above copper. Option (B) incorrectly makes zinc the densest member. Option (D) starts with zinc correctly but then reverses the relative positions of copper, cobalt, iron and chromium. Option (C) alone agrees with every adjacent comparison, including the two close pairs zinc/chromium and cobalt/copper. Because the question concerns density, no comparison of reactivity or oxidation states is needed.

Step 4: Conclusion The increasing density order is $\mathrm{Zn} < \mathrm{Cr} < \mathrm{Fe} < \mathrm{Co} < \mathrm{Cu}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember the low density of zinc relative to the late transition metals; do not turn the increasing atomic-mass trend into an automatic density order.

71. Match List - I and List - II.

	List I		List II
(a)	Valium	(i)	Antifertility drug
(b)	Morphine	(ii)	Pernicious anaemia
(c)	Norethindrone	(iii)	Analgesic
(d)	Vitamin B_{12}	(iv)	Tranquilizer

[4 marks]

(A) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(B) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(C) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(D) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Correct Answer: (D) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Solution:

Concept:

- Therapeutic classification groups drugs by their principal medicinal action. Tranquilizers reduce anxiety, whereas analgesics relieve pain.
- An antifertility drug controls reproduction. Vitamin deficiency diseases are treated by supplying the deficient vitamin; pernicious anaemia is associated with vitamin B_{12} deficiency.

Step 1: Match the nervous-system drugs Valium is the trade name for diazepam, a tranquilizer used to reduce anxiety and tension. It therefore matches (iv), not an antifertility drug or a treatment for pernicious anaemia. Morphine is a powerful narcotic analgesic: it relieves severe pain by acting on the central nervous system. Thus morphine matches (iii). The first two assignments are consequently (a)-(iv) and (b)-(iii).

Step 2: Match the hormone and vitamin Norethindrone is a synthetic progestogen used in oral contraceptive formulations. Its relevant action here is antifertility, giving (c)-(i). Vitamin B_{12} is required for normal red-blood-cell formation; its deficiency or defective absorption is associated with pernicious anaemia. Hence (d)-(ii). These matches distinguish the contraceptive role of norethindrone from the nutritional role of the vitamin.

Step 3: Check all four option sets The complete sequence is (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii). Option (A) wrongly makes Valium an antifertility drug and norethindrone a tranquilizer. Option (B) interchanges the vitamin and contraceptive matches. Option (C) misclassifies all four entries, including morphine as a tranquilizer. Only option (D) preserves every assignment.

Step 4: Conclusion Valium is a tranquilizer, morphine an analgesic, norethindrone an antifertility drug, and vitamin B_{12} treats pernicious anaemia. Hence, the correct option is (D).

Quick Tip to Solve: In a drug-matching problem, first lock the familiar pairs morphine–analgesic and vitamin B_{12}–pernicious anaemia, then check the contraceptive and tranquilizer entries.

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72. The correct shape and $I-I-I$ bond angles respectively in I_3^- ion are : [4 marks]

(A) Linear; 180°

(B) T-shaped; 180° and 90°

(C) Trigonal planar; 120°

(D) Distorted trigonal planar; 135° and 90°

Correct Answer: (A) Linear; 180°

Solution:

Concept:

- VSEPR theory determines molecular shape from the number of bond pairs and lone pairs on the central atom. Molecular shape describes the atom positions, whereas electron-pair geometry also includes lone pairs.
- Five electron domains have a trigonal-bipyramidal electron-pair arrangement. Lone pairs prefer equatorial positions because these have fewer 90° interactions.

Step 1: Count the central iodine electron pairs The triiodide ion has $3(7) + 1 = 22$ valence electrons. Two single bonds use four electrons, and the terminal iodine atoms take twelve electrons as their three lone pairs each. The remaining six electrons form three lone pairs on the central iodine. The central atom therefore has two bonding pairs and three lone pairs, conventionally written AX_2E_3. Its steric number is five.

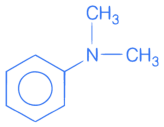
Step 2: Place the lone pairs and identify the shape The five electron domains adopt a trigonal-bipyramidal arrangement. Each equatorial position experiences two $90°$ interactions, while an axial position experiences three, so the three lone pairs occupy all three equatorial positions. The two I–I bonds then point along the two opposite axial directions. The three nuclei lie on one straight line, and the I–I–I bond angle is $180°$.

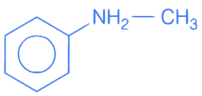
Step 3: Reject the alternative geometries A T-shaped molecule corresponds to three bonds and two lone pairs in this five-domain arrangement, rather than the two bonds present here. Trigonal planar geometry would require three bond directions in a plane. A distorted trigonal planar description with $135°$ and $90°$ angles likewise does not describe the two opposite bonds of I_3^-. Thus options (B), (C) and (D) fail the electron-domain count.

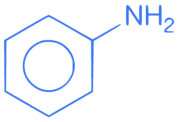
Step 4: Conclusion I_3^- is linear, with an I–I–I bond angle of $180°$. Hence, the correct option is (A).

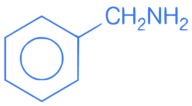
Quick Tip to Solve: For AX_2E_3, put all three lone pairs in the equatorial plane; the two surviving axial bonds immediately give a linear molecule.

73. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH? [4 marks]

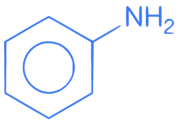
(A) $C_6H_5N(CH_3)_2$ 

(B) N-methyl-substituted aniline structure shown 

(C) $C_6H_5NH_2$ 

(D) $C_6H_5CH_2NH_2$ 

Correct Answer: (C)

$C_6H_5NH_2$ 

Solution:

Concept:

- A primary aromatic amine has an $-NH_2$ group directly bonded to an aromatic ring. On diazotization at low temperature it gives an arenediazonium salt.
- Arenediazonium ions couple with activated aromatic compounds such as β-naphthol in alkaline solution, producing an extended azo chromophore $-N{=}N-$ and hence a coloured dye.

Step 1: Identify the amine that can be diazotized Option (C) shows aniline, $C_6H_5NH_2$. Its amino group is primary and is attached directly to the benzene ring. It undergoes diazotization with nitrous acid, generated from sodium nitrite and hydrochloric acid, at 273–278 K:

$$C_6H_5NH_2 + NaNO_2 + 2\,HCl \longrightarrow C_6H_5N_2Cl + NaCl + 2\,H_2O.$$

The aromatic diazonium ion is sufficiently stable at this low temperature for the coupling reaction.

Step 2: Explain formation of the coloured product In sodium hydroxide solution, β-naphthol forms the more strongly activating naphthoxide ion. Its activated aromatic ring couples with the benzenediazonium ion to form an aryl azo linkage. The conjugation extends through the benzene ring, the $-N{=}N-$ group and the naphthol ring system, allowing absorption in the visible region and giving a coloured azo dye.

Step 3: Exclude the remaining structures Option (A) has two methyl groups attached to nitrogen and is a tertiary aromatic amine; it cannot supply the primary amino group needed for this diazonium salt. The nitrogen in the N-methyl-substituted structure shown in (B) is also substituted and is not the required primary aniline group. Option (D) is benzylamine, $C_6H_5CH_2NH_2$: its amino group is on a side chain, so its aliphatic diazonium intermediate loses nitrogen rather than behaving as a stable arenediazonium coupling reagent.

Step 4: Conclusion Aniline in option (C) forms benzenediazonium salt, which couples with alkaline β-naphthol to give a coloured azo dye. Hence, the correct option is (C).

Quick Tip to Solve: Look for $Ar{-}NH_2$: the amino nitrogen must be primary and directly attached to the aromatic ring. A $-CH_2-$ spacer changes the diazonium chemistry.

74. Match List - I and List - II.

	List-I		List-II
(a)	$RCOCl \longrightarrow RCHO$	**(i)**	$Br_2/NaOH$
(b)	$RCH_2COOH \longrightarrow RCHClCOOH$	**(ii)**	$H_2/Pd{-}BaSO_4$
(c)	$RCONH_2 \longrightarrow RNH_2$	**(iii)**	$Zn(Hg)$, **conc.** HCl
(d)	$RCOCH_3 \longrightarrow RCH_2CH_3$	**(iv)**	Cl_2, **red P,** H_2O

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(B) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(C) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(D) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Correct Answer: (A) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Solution:

Concept:

- Named-reaction matching is decided by the change in the functional group, carbon skeleton and oxidation level, rather than by a reagent name alone.
- Rosenmund reduction converts an acid chloride to an aldehyde; Clemmensen reduction converts a carbonyl group to methylene. Hofmann bromamide degradation removes the amide carbonyl carbon, and the Hell–Volhard–Zelinsky reaction halogenates a carboxylic acid at its alpha carbon.

Step 1: Identify the acid chloride and acid transformations In (a), $RCOCl \longrightarrow RCHO$, the acid chloride is selectively reduced to an aldehyde. Hydrogen over palladium poisoned with barium sulfate is the Rosenmund reagent, so (a)-(ii). In (b), $RCH_2COOH \longrightarrow RCHClCOOH$, chlorine replaces an alpha hydrogen while the acid group remains. Chlorine with red phosphorus, followed by water, gives this Hell–Volhard–Zelinsky transformation, so (b)-(iv).

Step 2: Identify the amide and ketone transformations In (c), $RCONH_2 \longrightarrow RNH_2$, the product has one carbon atom fewer than the amide. This is Hofmann bromamide degradation using $Br_{2/}NaOH$, giving (c)-(i). In (d), $RCOCH_3 \longrightarrow RCH_2CH_3$, the carbonyl oxygen is replaced by two hydrogen atoms without loss of carbon. Zinc amalgam and concentrated hydrochloric acid perform Clemmensen reduction, so (d)-(iii).

Step 3: Compare the proposed matches Together the assignments are (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii), exactly option (A). Option (B) incorrectly uses Clemmensen reagent to stop an acid chloride at an aldehyde and places Rosenmund reagent on the ketone. Options (C) and (D) put bromine/base on the alpha chlorination or chlorine/red phosphorus on the amide degradation. Those reagent changes do not produce the specified products.

Step 4: Conclusion Rosenmund reduction, alpha chlorination, Hofmann degradation and Clemmensen reduction use reagents (ii), (iv), (i) and (iii), respectively. Hence, the correct option is (A).

Quick Tip to Solve: Check carbon count first: loss of the amide carbonyl carbon signals Hofmann degradation; retention of carbon with $C{=}O \longrightarrow CH_2$ signals Clemmensen reduction.

75. Math List - I with List - II.

	List I (Metal)		List II (Ores)
(a)	Aluminium	(i)	Siderite
(b)	Iron	(ii)	Calamine
(c)	Copper	(iii)	Kaolinite
(d)	Zinc	(iv)	Malachite

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(B) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(C) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(D) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Correct Answer: (C) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Solution:

Concept:

- An ore is a naturally occurring mineral from which a metal can be extracted economically under suitable conditions. In a metal–mineral matching question, identify which metal occurs in each mineral formula.
- Siderite is iron carbonate, calamine is zinc carbonate, malachite is a basic copper carbonate, and kaolinite is a hydrated aluminium silicate. These compositions determine the requested matches.

Step 1: Match iron and zinc to their carbonates Siderite has formula $FeCO_3$, so the metal associated with siderite is iron. Thus (b)-(i). Calamine, in the standard school-level mineral list used here, is $ZnCO_3$, giving zinc as its metal and therefore (d)-(ii). The metal symbols Fe and Zn fix these two assignments without relying on the similarity of the mineral names.

Step 2: Match aluminium and copper Kaolinite is a hydrated aluminium silicate, commonly written $Al_2Si_2O_5(OH)_4$, equivalently $Al_2O_3 \cdot 2\,SiO_2 \cdot 2\,H_2O$. It contains aluminium, so (a)-(iii). Malachite is the green basic copper carbonate $CuCO_3 \cdot Cu(OH)_2$, and therefore matches copper: (c)-(iv). This matching concerns the metal present in the listed mineral; it does not imply that every listed mineral is the principal modern industrial feedstock.

Step 3: Select the only consistent option The complete sequence is aluminium–kaolinite, iron–siderite, copper–malachite and zinc–calamine, or (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii). Option (A) incorrectly assigns malachite to aluminium and kaolinite to iron. Option (B) assigns siderite to aluminium, and option (D) assigns calamine to aluminium. Their first matches alone rule them out; only option (C) has all four metal compositions right.

Step 4: Conclusion Aluminium matches kaolinite, iron siderite, copper malachite, and zinc calamine. Hence, the correct option is (C).

Quick Tip to Solve: Remember the carbonate trio by metal symbols: siderite $FeCO_3$, calamine $ZnCO_3$, and malachite, a basic carbonate of copper.

76. Given below are two statements :

Statement I : The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- Biochemical oxygen demand (BOD) is the amount of dissolved oxygen consumed by microorganisms while decomposing biodegradable organic matter in a water sample under specified test conditions.
- High BOD indicates a greater oxygen demand and often more organic pollution. This can deplete the oxygen available to fish and other aquatic organisms, so BOD must be distinguished from the dissolved-oxygen concentration itself.

Step 1: Evaluate Statement I Aquatic organisms depend on dissolved oxygen for respiration. When biodegradable organic waste enters water, aerobic microorganisms consume oxygen as they break the waste down. A large BOD therefore warns that oxygen may be removed faster than it is replenished, threatening aquatic life. BOD is consequently an important water-quality parameter for the survival of aquatic life, so Statement I is true. It measures oxygen demand rather than the amount of oxygen that the animals directly receive.

Step 2: Evaluate Statement II There is no general optimum BOD value of $6.5\,\text{ppm}$ for aquatic life. Clean water has low BOD, generally below about $5\,\text{ppm}$, while greater demand indicates increasing organic pollution. A value such as $6.5\,\text{ppm}$ must not be treated as a universal desirable BOD merely because a substantial dissolved-oxygen concentration is beneficial to fish. Oxygen demand and oxygen actually dissolved in water describe different aspects of water quality. Statement II is false.

Step 3: Choose the statement combination The logical combination obtained is Statement I true and Statement II false. Option (A) reverses these truth values, option (C) incorrectly accepts the asserted optimum, and option (D) rejects the valid connection between BOD and aquatic survival. Hence only option (B) represents both assessments correctly. Lower oxygen demand generally leaves more of the available oxygen for aquatic respiration, provided other water conditions are suitable.

Step 4: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: Keep BOD and dissolved oxygen separate: high dissolved oxygen supports aquatic life, whereas high BOD tends to consume that oxygen and signals organic pollution.

77. In polymer Buna-S : 'S' stands for : [4 marks]

(A) Sulphur

(B) Styrene

(C) Strength

(D) Sulphonation

Correct Answer: (B) Styrene

Solution:

Concept:

- Buna-S is a synthetic rubber produced by copolymerization of 1,3-butadiene and styrene. A copolymer contains repeat units derived from more than one kind of monomer.
- The name Buna reflects butadiene and the historical sodium-based polymerization process; the final letter identifies the comonomer. In Buna-S, S denotes styrene, whereas in Buna-N the N denotes acrylonitrile.

Step 1: Identify the monomers in Buna-S The two monomers used for Buna-S are 1,3-butadiene, $CH_2{=}CH{-}CH{=}CH_2$, and styrene, $C_6H_5CH{=}CH_2$. During addition copolymerization, their carbon–carbon double bonds participate in forming the polymer chains, which contain both butadiene-derived and styrene-derived units. The styrene component is responsible for the letter S in the name. The question asks for that naming component, rather than an additive used after polymer formation.

Step 2: Distinguish the monomer from vulcanization Sulphur can be used to create cross-links during vulcanization of rubber. That processing role makes sulphur a tempting choice, but sulphur is not what the S in Buna-S stands for. Styrene is incorporated as a comonomer during formation of the copolymer itself. Thus the source of the letter is a chemical starting material, not the later cross-linking agent or a description of a desirable property of the finished rubber.

Step 3: Eliminate the other proposed expansions Option (A), sulphur, confuses vulcanization with the name of the copolymer. Option (C), strength, is a physical property and does not name either monomer. Option (D), sulphonation, is a chemical modification process and is not the defining polymerization used to make Buna-S. Option (B), styrene, alone identifies the aromatic vinyl monomer that is copolymerized with butadiene.

Step 4: Conclusion In Buna-S, S stands for styrene. Hence, the correct option is (B).

Quick Tip to Solve: Pair the suffix with the comonomer: Buna-S contains styrene, while Buna-N contains acrylonitrile; sulphur belongs to the separate idea of vulcanization.

78. Most suitable salt which can be used for efficient clotting of blood will be : [4 marks]

(A) $Mg(HCO_3)_2$

(B) $NaHCO_3$

(C) $FeCl_3$

(D) $FeSO_4$

Correct Answer: (C) $FeCl_3$

Solution:

Concept:

- Blood is treated as a negatively charged colloidal system in the coagulation model. An electrolyte supplies counter-ions that neutralise the charge and promote aggregation.
- The Hardy–Schulze rule states that, for a given charged sol, coagulating power rises strongly with the magnitude of the charge on the oppositely charged ion. The cation is therefore decisive here.

Step 1: Identify the active ions The salts furnish the cations Mg^{2+}, Na^{+}, Fe^{3+} and Fe^{2+}, respectively. Since the colloidal particles are negatively charged, these positive ions act as the coagulating ions. Comparing anions such as chloride and sulfate alone would miss the counter-ion responsible for charge neutralisation.

Step 2: Apply the valency rule The charge of ferric ion is $+3$, greater than the $+2$ charges of magnesium and ferrous ions and the $+1$ charge of sodium ion. Hence ferric chloride has the greatest coagulating power among the listed electrolytes. This is the colloidal basis of the use of ferric salts as styptics: destabilisation and aggregation assist the formation of a clot.

Step 3: Eliminate the remaining salts $Mg(HCO_3)_2$ and $FeSO_4$ supply divalent cations and are weaker coagulants in this comparison; $NaHCO_3$ supplies only a monovalent cation and is weaker still. Thus neither bicarbonate choice nor ferrous sulfate gives the strongest counter-ion available. The comparison concerns their coagulating efficiency, rather than the number of atoms in a formula unit.

Step 4: Conclusion $FeCl_3$ is the most suitable salt, because its Fe^{3+} ions have the greatest charge and coagulating power. Hence, the correct option is (C).

Quick Tip to Solve: For coagulation questions, identify the sign of the sol first and compare the valencies of its counter-ions, not the charges of the co-ions.

79. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R : Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both A and R are true but R is NOT the correct explanation of A

(B) Both A and R are true and R is the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true.

Correct Answer: (B) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Hydrogen is the most abundant element in the Universe. Cosmic abundance and the composition of the Earth's troposphere refer to different systems and need not have the same ordering.
- Hydrogen is the lightest element; molecular hydrogen has a much smaller molar mass than the main atmospheric gases. At a fixed temperature, molecular speed varies inversely with the square root of molar mass.

Step 1: Evaluate the assertion The Universe contains predominantly hydrogen, including the hydrogen present in stars. In contrast, the major gas in the Earth's troposphere is nitrogen, followed by oxygen. Free hydrogen is only a trace constituent of this layer. Thus both parts of the assertion are true: a large cosmic abundance does not imply a large abundance in the lower atmosphere.

Step 2: Evaluate the reason Hydrogen has atomic number 1 and the smallest atomic mass of the elements. Consequently H_2 is an exceptionally light gas. The reason is therefore true. The dependence $v_{\text{rms}} = \sqrt{3RT/M}$ shows that hydrogen molecules have high thermal speeds compared with nitrogen and oxygen at the same temperature.

Step 3: Connect the statements in their intended context The contrast in the assertion is explained by the lightness of hydrogen: the Earth does not retain free hydrogen as efficiently as heavier gases, and hydrogen reaching the upper atmosphere can escape. This accounts for its scarcity in the troposphere despite its very large cosmic abundance. The reason is the accepted explanation of this contrast; it is not a claim that tropospheric nitrogen is the most abundant element throughout the Universe. Options (C) and (D) wrongly make one of the true statements false, and option (A) denies the relevant connection.

Step 4: Conclusion Both A and R are true, and R correctly explains the atmospheric contrast in A through the lightness and escape of hydrogen. Hence, the correct option is (B).

Quick Tip to Solve: In abundance assertions, separate cosmic abundance from atmospheric retention; light gases have high thermal speeds and escape more readily.

80. Match List - I with List - II.

	List I (Salt)		List II (Flame colour wavelength)
(a)	LiCl	(i)	455.5 nm
(b)	NaCl	(ii)	670.8 nm
(c)	RbCl	(iii)	780.0 nm
(d)	CsCl	(iv)	589.2 nm

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(B) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(C) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

(D) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

Correct Answer: (D) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

Solution:

Concept:

- Heating an alkali-metal salt excites the metal atoms. Light is emitted when excited electrons return to lower energy levels, with $\Delta E = hc/\lambda$.
- The characteristic flame wavelengths given in the question are matched to the metal cations; changing chloride to another suitable salt does not change the identity of that metal.

Step 1: Match lithium and sodium Lithium salts produce a crimson-red flame, with the characteristic red line at $670.8\,\text{nm}$. Thus (a) matches (ii). Sodium salts produce the familiar intense yellow flame, represented here by $589.2\,\text{nm}$, so (b) matches (iv). These two well-known observations already distinguish most of the proposed combinations.

Step 2: Match rubidium and caesium The rubidium wavelength listed is $780.0\,\text{nm}$, giving (c)–(iii). Caesium has the characteristic blue line listed at $455.5\,\text{nm}$, giving (d)–(i). A metal can have several emission lines; the task is to assign the particular characteristic values supplied in List II, rather than to assume that each metal emits a single wavelength.

Step 3: Compare the full code The resulting sequence for (a), (b), (c), (d) is (ii), (iv), (iii), (i). Option (A) misassigns sodium and the heavier metals. Option (B) interchanges lithium and sodium. Option (C) assigns the blue caesium line to lithium and the red lithium line to rubidium. Only option (D) preserves all four required matches, including the sodium yellow line.

Step 4: Conclusion Lithium chloride matches (ii), sodium chloride matches (iv), rubidium chloride matches (iii), and caesium chloride matches (i). Hence, the correct option is (D).

Quick Tip to Solve: Anchor a flame-test matching problem with sodium at about 589 nm and lithium at about 671 nm, then assign the remaining listed lines.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is _____.

[4 marks]

Solution:

Concept:

- At the same temperature and pressure, gas volumes are proportional to amounts in moles. The stated volume ratios therefore give the stoichiometric coefficients relative to one mole of hydrocarbon.
- Complete combustion of C_xH_y is described by $C_xH_y + (x^+y/_4)O_2 \longrightarrow xCO_2 + (y/_2)H_2O$. Carbon balance fixes x, while oxygen balance then fixes y.

Step 1: Use the carbon dioxide ratio Take one mole, or one equal-volume unit, of the gaseous hydrocarbon. It produces four moles, or four volume units, of CO_2. Every carbon atom in the fuel appears in one molecule of carbon dioxide on complete oxidation. Hence

$$x = 4.$$

The water need not remain gaseous for this carbon balance; only the hydrocarbon, oxygen and carbon dioxide volume ratios are used.

Step 2: Use the oxygen requirement The required oxygen volume is six times the fuel volume, so

$$x + \frac{y}{4} = 6.$$

Substituting $x = 4$ gives

$$4 + \frac{y}{4} = 6, \qquad \frac{y}{4} = 2, \qquad y = 8.$$

The oxygen used to make carbon dioxide accounts for four of the six oxygen molecules; the remaining two oxygen molecules provide the four oxygen atoms needed in four water molecules.

Step 3: Verify the formula by balancing The molecular formula obtained is C_4H_8, and its combustion equation is

$$C_4H_8 + 6\,O_2 \longrightarrow 4\,CO_2 + 4\,H_2O.$$

Both sides contain four carbon atoms, eight hydrogen atoms and twelve oxygen atoms. Its coefficients reproduce both stated gas-volume ratios, which checks the hydrogen count independently of any structural identification of the hydrocarbon.

Step 4: Conclusion The hydrocarbon is C_4H_8, so the requested dimensionless value is $\boxed{y = 8}$.

Quick Tip to Solve: For a gaseous hydrocarbon, read the carbon count directly from the carbon dioxide volume, then use oxygen consumption to obtain the hydrogen count.

82. The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$. mol/L. The value of x is _____. (Rounded off to the nearest integer) [Given $\sqrt{2}$ = 1.41] [4 marks]

Solution:

Concept:

- For $\mathrm{PbI_2(s)} \rightleftharpoons \mathrm{Pb^{2+}(aq) + 2\,I^-(aq)}$, the solubility product is $K_{sp} = [\mathrm{Pb^{2+}}][\mathrm{I^-}]^2$. The pure solid is excluded.
- If the molar solubility is s, dissolution contributes s molar lead ions and $2s$ molar iodide ions. Lead nitrate supplies a common lead ion and suppresses dissolution.

Step 1: Write equilibrium concentrations Lead nitrate dissociates to supply an initial $0.1\ \mathrm{mol\,L^{-1}}$ concentration of $\mathrm{Pb^{2+}}$. If $s\ \mathrm{mol\,L^{-1}}$ of lead iodide dissolves, the concentrations are

$$[\mathrm{Pb^{2+}}] = 0.1 + s, \qquad [\mathrm{I^-}] = 2s.$$

Nitrate is a spectator ion; its concentration is not substituted into the solubility-product expression.

Step 2: Calculate the suppressed solubility Because K_{sp} is small, take $0.1 + s \simeq 0.1$. Then

$$8.0 \times 10^{-9} = 0.1(2s)^2 = 0.4s^2,$$

so

$$s^2 = 2.0 \times 10^{-8}, \qquad s = \sqrt{2} \times 10^{-4}\ \mathrm{mol\,L^{-1}}.$$

Using the supplied value $\sqrt{2} = 1.41$ gives $s = 1.41 \times 10^{-4}\ \mathrm{mol\,L^{-1}}$.

Step 3: Convert the requested coefficient and check Rewrite the solubility in the form requested:

$$s = 141 \times 10^{-6}\ \mathrm{mol\,L^{-1}}.$$

The neglected solubility is only about 0.14% of the initial lead-ion concentration, so the common-ion approximation is self-consistent. Keeping the small added lead concentration would not alter the nearest-integer coefficient. Notice that the factor of two for iodide is squared in K_{sp}; omitting it would give a different result.

Step 4: Conclusion The solubility is $141 \times 10^{-6}\ \mathrm{mol\,L^{-1}}$, so the requested nearest-integer value is $\boxed{x = 141}$.

Quick Tip to Solve: With a common lead ion, use its imposed concentration in the first factor, but retain the stoichiometric factor $2s$ for iodide before squaring.

83. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is _____ $\times 10^{-2}$. [4 marks]

Solution:

Concept:

- Acetylation converts aniline, $C_6H_5NH_2$, into acetanilide, $C_6H_5NHCOCH_3$, in a one-to-one mole ratio.
- Theoretical product mass is obtained from reactant moles and product molar mass. A purification loss is applied to this product mass after the chemical conversion is calculated.

Step 1: Calculate aniline moles Aniline has molecular formula C_6H_7N. Using $C = 12$, $H = 1$ and $N = 14$, its molar mass is

$$M_{\text{aniline}} = 6(12) + 7(1) + 14 = 93\,\text{g mol}^{-1}.$$

Therefore

$$n_{\text{aniline}} = \frac{1.86}{93} = 0.0200\,\text{mol}.$$

Complete reaction produces the same amount of acetanilide, since each aniline molecule receives one acetyl group.

Step 2: Find the theoretical product mass Acetanilide has molecular formula C_8H_9NO. Its molar mass is

$$M_{\text{acetanilide}} = 8(12) + 9(1) + 14 + 16 = 135\,\text{g mol}^{-1}.$$

The mass formed before purification is consequently

$$m_{\text{theoretical}} = 0.0200 \times 135 = 2.70\,\text{g}.$$

The product mass can exceed the original aniline mass because the acetylating reagent contributes atoms to the product.

Step 3: Apply the purification recovery A loss of 10% means that 90% of the product is retained. Thus

$$m_{\text{obtained}} = 2.70\left(\frac{90}{100}\right) = 2.43\,\text{g}.$$

Expressing this in the units multiplying the blank gives

$$2.43\,\text{g} = 243 \times 10^{-2}\,\text{g}.$$

The percentage is applied to product formed, not subtracted as 0.10 g and not applied as a change in the stoichiometric coefficient.

Step 4: Conclusion The purified acetanilide mass is $2.43\,\text{g} = \boxed{243} \times 10^{-2}\,\text{g}$. The requested numerical entry is 243.

Quick Tip to Solve: First convert the limiting reactant to moles and then to theoretical product mass; multiply by the recovery fraction only at the end.

84. Assuming ideal behaviour, the magnitude of log K for the following reaction at 25°C is x $\times$ 10^{-1}. The value of x is _____. (Integer answer)

$$3HC \equiv CH_{(g)} \rightleftharpoons C_6H_{6(l)}$$

[Given : $\Delta_f G^o(HC \equiv CH) = -2.04 \times 10^5$ J mol^{-1} ; $\Delta_f G^o(C_6H_6) = -1.24 \times 10^5$ J mol^{-1} ; R = 8.314 J K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- The standard reaction Gibbs energy is the sum of product formation Gibbs energies minus the corresponding reactant sum, each multiplied by its stoichiometric coefficient.
- The relation $\Delta_r G^\circ = -RT \ln K = -2.303RT \log_{10} K$ connects reaction Gibbs energy to the dimensionless equilibrium constant. Temperature must be in kelvin.

Step 1: Calculate the reaction Gibbs energy For the reaction $3\,C_2H_2(g) \rightleftharpoons C_6H_6(l)$, use the signed values supplied, even though the requested quantity is a magnitude. The result is

$$\begin{aligned}\Delta_r G^\circ &= (-1.24 \times 10^5) - 3(-2.04 \times 10^5) \\ &= 4.88 \times 10^5\,\text{J mol}^{-1}.\end{aligned}$$

The factor of three belongs to acetylene. Subtracting a negative reactant contribution produces a positive reaction Gibbs energy.

Step 2: Find the common logarithm At 25°C, use $T = 298$ K. Hence

$$\log_{10} K = -\frac{4.88 \times 10^5}{2.303(8.314)(298)} \simeq -85.5.$$

Since $\Delta_r G^\circ$ is positive, $K < 1$, consistent with the negative logarithm. The question asks for its magnitude, so $|\log_{10} K| \simeq 85.5$, not -85.5.

Step 3: Extract the numerical entry The required form is $|\log_{10} K| = x \times 10^{-1}$. Thus

$$x = 10|\log_{10} K| \simeq 855.$$

With the precision of the supplied formation energies, the integer answer is 855. The coefficient is dimensionless because both the logarithm and the thermodynamic equilibrium constant are dimensionless. Using $\ln K$ directly in place of $\log_{10} K$ would miss the factor 2.303 and would not yield the requested coefficient.

Step 4: Conclusion $\log_{10} K \simeq -85.5$, so its magnitude is 855×10^{-1}. The requested integer is $\boxed{x = 855}$.

Quick Tip to Solve: Keep the signs in the formation-energy subtraction, convert to a base-10 logarithm with 2.303, and take the magnitude only after calculating the logarithm.

85. **C_6H_6 freezes at 5.5°C. The temperature at which a solution of 10g of C_4H_{10} in 200g of C_6H_6 freeze is _____ °C. (The molal freezing point depression constant of C_6H_6 is 5.12°C/m.)** [4 marks]

Solution:

Concept:

- A non-electrolyte lowers the freezing point of a solvent by $\Delta T_f = iK_f m$, where m is the molality and i is the van't Hoff factor. Butane does not ionise in benzene, so $i = 1$.
- Molality uses the mass of solvent in kilograms, not the total solution mass. The solution freezes at $T_f = T_f^\circ - \Delta T_f$. A temperature difference in degrees Celsius has the same numerical value as in kelvin.

Step 1: Calculate the solute amount and molality For butane, the molar mass is $4(12) + 10(1) = 58\,\text{g mol}^{-1}$. Therefore, the amount dissolved is

$$n = \frac{10}{58} = 0.172414\,\text{mol}.$$

The benzene solvent has mass $200\,\text{g} = 0.200\,\text{kg}$. Thus

$$m = \frac{0.172414}{0.200} = 0.862069\,\text{mol kg}^{-1}.$$

Using 210 g here would incorrectly include the solute in the solvent mass.

Step 2: Find the freezing-point depression and subtract it The given constant is $K_f = 5.12\,\text{K kg mol}^{-1}$, so

$$\Delta T_f = 1(5.12)(0.862069) = 4.41379\,\text{K}.$$

Freezing-point depression is a positive magnitude, but the solution's freezing temperature is lower than that of pure benzene:

$$T_f = 5.5 - 4.41379 = 1.08621\,^\circ\text{C}.$$

The result is positive because the calculated depression is smaller than the pure-solvent freezing temperature above zero. It is not $5.5 + 4.41379$, which would describe an elevation rather than a depression.

Step 3: Conclusion The solution freezes at approximately $1.09\,^\circ\text{C}$. The whole-number numerical entry is $\boxed{1}$, in degrees Celsius.

Quick Tip to Solve: Convert only the solvent mass to kilograms before forming molality, then subtract the positive depression from the pure solvent freezing point.

86. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is f. The value of $\log_{10}\left(\frac{1}{f}\right)$ is _____ $\times 10^{-2}$. (Rounded off to the nearest integer)

[Assume : ln 10 = 2.303, ln 2 = 0.693] [4 marks]

Solution:

Concept:

- For a first-order reaction, the fraction remaining after time t is $f = e^{-kt}$, with rate constant $k = \ln 2/t_{1/2}$. The half-life is independent of the initial concentration.
- The requested common logarithm is $\log_{10}(1/f) = kt/\ln 10$. Both the reaction time and the half-life must use the same time unit; their ratio is dimensionless.

Step 1: Obtain the rate constant from the half-life The half-life of the hydrolysis at the stated temperature is $t_{1/2} = 3.33\,\text{h}$. Using the supplied value of $\ln 2$,

$$k = \frac{0.693}{3.33} = 0.208108\,\text{h}^{-1}.$$

The reaction time is $9\,\text{h}$, so there is no need to convert hours into seconds. The exponent controlling the remaining fraction is

$$kt = \frac{0.693 \times 9}{3.33} = 1.872973.$$

In particular, 9 h is not exactly three half-lives; rounding 3.33 h prematurely would distort the requested coefficient.

Step 2: Convert the natural logarithm and extract the coefficient Since $f = e^{-kt}$, its reciprocal is e^{kt}. Thus

$$\log_{10}\left(\frac{1}{f}\right) = \frac{kt}{\ln 10} = \frac{0.693 \times 9}{3.33 \times 2.303} = 0.81328.$$

If the blank is N in $N \times 10^{-2}$, then

$$N = 100(0.81328) = 81.328.$$

Rounding this coefficient, rather than rounding the logarithm to an integer first, gives 81. As a check, $f \approx e^{-1.873} \approx 0.154$, so its reciprocal is about 6.5 and its common logarithm should be close to 0.81.

Step 3: Conclusion $\log_{10}(1/f) \approx 81.33 \times 10^{-2}$, so the required nearest-integer, dimensionless coefficient is $\boxed{81}$.

Quick Tip to Solve: Combine the equations directly as $\log_{10}(1/f) = (t/t_{1/2})(\ln 2/\ln 10)$, and round only after applying the requested power-of-ten scale.

87. The magnitude of the change in oxidising power of the MnO_4^-/Mn^{2+} couple is x $\times$ 10^{-4} V, if the H^+ concentration is decreased from 1M to 10^{-4} M at 25°C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____. $\left[Given : \frac{2.303RT}{F} = 0.059\right]$

[4 marks]

Solution:

Concept:

- The acidic permanganate reduction is $\mathrm{MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O}$. Its electron number is $n = 5$, and its reaction quotient contains $[\mathrm{H^+}]^8$ in the denominator.
- At 25°C, the Nernst equation is $E = E^\circ - (0.059/5)\log_{10} Q$. A fall in reduction potential means that the species becomes a weaker oxidising agent.

Step 1: Write the part of the Nernst equation that changes For this reduction, with water activity taken as unity,

$$Q = \frac{[\mathrm{Mn^{2+}}]}{[\mathrm{MnO_4^-}][\mathrm{H^+}]^8}.$$

The permanganate and manganese(II) concentrations are unchanged, so their contributions and E° cancel when two potentials are subtracted. Expanding the proton term gives

$$E = \text{constant} + \frac{8(0.059)}{5}\log_{10}[\mathrm{H^+}].$$

The proton exponent is eight because eight hydrogen ions appear in the balanced half-reaction; it must not be replaced by the electron count five.

Step 2: Calculate the potential change and its magnitude Initially $[\mathrm{H^+}]_1 = 1\,\mathrm{M}$, while finally $[\mathrm{H^+}]_2 = 10^{-4}\,\mathrm{M}$. Hence

$$E_2 - E_1 = \frac{8(0.059)}{5}\log_{10}\left(\frac{10^{-4}}{1}\right)$$
$$= \frac{8(0.059)}{5}(-4) = -0.3776\,\mathrm{V}.$$

The negative sign confirms the expected decrease in oxidising power as the solution becomes less acidic. The question asks for the magnitude, so the reported change is positive. Expressing it in the specified scale,

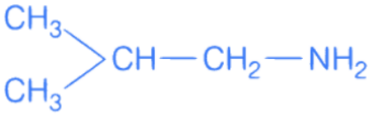
$$|\Delta E| = 0.3776\,\mathrm{V} = 3776 \times 10^{-4}\,\mathrm{V}.$$

Step 3: Conclusion The oxidising potential decreases by $0.3776\,\mathrm{V}$. Its magnitude is $3776 \times 10^{-4}\,\mathrm{V}$, giving $\boxed{x = 3776}$.

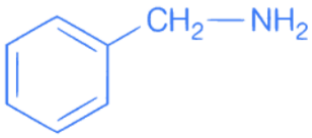
Quick Tip to Solve: Balance the half-reaction first: the proton coefficient gives the pH multiplier, while the electron coefficient belongs in the Nernst denominator.

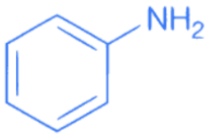
88. The total number of amines among the following which can be synthesized by Gabriel synthesis is _____.

[4 marks]

a. 

b. 

c. 

d. 

Solution:

Concept:

- Gabriel synthesis converts potassium phthalimide into an N-alkylphthalimide by nucleophilic substitution with a suitable alkyl halide. Hydrolysis then releases a primary amine, RNH_2.
- The substitution occurs at an alkyl or benzylic carbon. Ordinary aryl halides do not undergo the required substitution with phthalimide ion, so an aromatic amine with nitrogen directly bonded to the ring is excluded.

Step 1: Test the aliphatic and benzylic primary amines In structure (a), the amine is $(CH_3)_2CHCH_2NH_2$. It can be obtained by using $(CH_3)_2CHCH_2X$, where X is a suitable leaving group. Although the carbon chain is branched, the carbon bearing X is primary, so the required substitution is possible. Structure (b), $CH_3CH_2NH_2$, similarly follows from an ethyl halide CH_3CH_2X. It is a primary aliphatic amine. Structure (c), benzylamine $C_6H_5CH_2NH_2$, follows from a benzyl halide $C_6H_5CH_2X$. Its nitrogen is bonded to the side-chain CH_2, not directly to an aromatic carbon. Benzylic substitution is therefore suitable for Gabriel synthesis.

Step 2: Exclude the aryl amine and count the successful cases Structure (d) is aniline, $C_6H_5NH_2$. Making it by the Gabriel route would require substitution of a halogen directly attached to the benzene ring in C_6H_5X. The ordinary Gabriel conditions do not bring about that aryl substitution, so (d) is not included. All four species are primary amines in terms of substitution at nitrogen, but this fact alone is not sufficient. The nature of the carbon bonded to nitrogen must also be checked. The successful structures are exactly (a), (b) and (c); there is no fourth eligible structure in the list.

Step 3: Conclusion The amines obtainable by Gabriel synthesis are (a), (b) and (c). The total number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: Distinguish benzylamine from aniline: a ring separated from nitrogen by CH_2 permits the benzyl-halide route; a direct ring–nitrogen bond does not.

89. The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mmHg pressure is _____ L. (Rounded off to the nearest integer)

[Given R = 0.0826 L atm K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- The ideal gas equation is $PV = nRT$, so the volume is $V = nRT/P$. The number of moles is found by dividing the gas mass by its molar mass.
- The supplied gas constant uses litres, atmospheres and kelvin. Consequently the temperature must be absolute and the mercury pressure must be converted using $760\,\text{mmHg} = 1\,\text{atm}$.

Step 1: Convert the amount, temperature and pressure Acetylene is C_2H_2, with molar mass

$$M = 2(12) + 2(1) = 26\,\text{g mol}^{-1}.$$

The sample therefore contains

$$n = \frac{4.75}{26} = 0.182692\,\text{mol}.$$

For use with the given value of R, the other quantities are

$$T = 50 + 273 = 323\,\text{K}, \qquad P = \frac{740}{760} = 0.973684\,\text{atm}.$$

The gas is warmer than the usual standard temperature and its pressure is slightly below one atmosphere. Both changes tend to increase its volume relative to the corresponding standard-condition value.

Step 2: Substitute in the gas equation and round once Rearranging the ideal gas law, and retaining the given value of the gas constant,

$$\begin{aligned} V &= \frac{nRT}{P} \\ &= \frac{(4.75/26)(0.0826)(323)}{740/760} \\ &\approx 5.006\,\text{L}. \end{aligned}$$

The units reduce to litres because the mole, atmosphere and kelvin factors cancel. Using $323.15\,\text{K}$ would change only a small decimal amount and would give the same nearest integer. The requested rounding is applied to the final volume, not separately to n or P; setting either of those to a coarse integer would introduce a large error.

Step 3: Conclusion The acetylene occupies approximately $5.01\,\text{L}$, or $\boxed{5\,\text{L}}$ to the nearest integer. The numerical entry is 5.

Quick Tip to Solve: Before using $PV = nRT$, match pressure and volume units to R and convert Celsius to kelvin; keep the pressure fraction $740/760$ until the last step.

90. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is _____.

(A) α-sulphur

(B) β-sulphur

(C) S_2-form [4 marks]

Solution:

Concept:

- Paramagnetism results from one or more unpaired electrons. A substance with all electrons paired is diamagnetic; changing a crystal packing arrangement alone need not create unpaired electrons.
- Both rhombic α-sulphur and monoclinic β-sulphur consist of puckered S_8 rings. Diatomic sulphur, S_2, has an oxygen-like molecular-orbital arrangement with two unpaired electrons.

Step 1: Classify the two ring allotropes In α-sulphur, each sulphur atom is covalently bonded to two neighbouring sulphur atoms in an S_8 ring. The bonding and lone-pair electrons are paired, so the allotrope is diamagnetic. The β-sulphur crystal also contains S_8 rings. Its different crystal form changes the arrangement of those molecules rather than creating a pair of singly occupied molecular orbitals. It too is diamagnetic. Thus neither (A) nor (B) contributes to the required count. The different Greek letters label solid allotropes, not different electronic spin states of isolated sulphur atoms.

Step 2: Identify the unpaired electrons in the diatomic form The diatomic S_2 molecule has twelve valence electrons, like O_2. In its molecular-orbital description, the two degenerate antibonding π^* orbitals each contain one electron according to Hund's rule. These two electrons remain unpaired, and hence the molecule is paramagnetic. This distinguishes (C) from the closed-shell ring molecules. We count the number of listed allotropic forms that show paramagnetism, rather than the number of unpaired electrons. Although a single S_2 molecule has two unpaired electrons, it is only one eligible form in the three-member list.

Step 3: Conclusion Only the S_2 form, (C), is paramagnetic. The required number of allotropic forms is $\boxed{1}$, a dimensionless count.

Quick Tip to Solve: Use the oxygen analogy for S_2: two singly occupied π^* orbitals give paramagnetism. The two solid S_8 allotropes have paired electrons.