JEE Main 2021 Paper 1 (B.E./B.Tech) Feb 25 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 1 (February), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 February 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is : [4 marks]

(A) $\frac{3}{4}$
(B) $\frac{3}{8}$
(C) $\frac{1}{27}$
(D) $\frac{1}{8}$

Correct Answer: (D) $\frac{1}{8}$

Solution:

Concept:

- For events H (hitting the target) and I (interception), the multiplication rule gives $P(H \cap I^c) = P(I^c)P(H \mid I^c)$. A missile must escape interception before it can hit.
- Independence of the three firings means their individual hit probabilities multiply. The conditional hit probability alone is not the overall hit probability.

Step 1: Find the hit probability for one missile The probability of avoiding interception is the complement of the given interception probability. Thus

$$P(I^c) = 1 - \frac{1}{3} = \frac{2}{3}, \qquad P(H) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}.$$

This accounts for both required stages: surviving interception and then striking the target. Intercepted missiles do not contribute to a successful hit.

Step 2: Multiply for three independent hits Each firing has success probability $1/2$. Because the firings are independent, the event that all three succeed has probability

$$P(H_1 \cap H_2 \cap H_3) = P(H_1)P(H_2)P(H_3) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

The value $3/4$ refers only to the conditional event for a single missile. The value $1/27$ would cube the interception probability, and $3/8$ does not represent the product of three hit probabilities.

Step 3: Conclusion The probability that all three missiles hit the target is $\boxed{1/8}$. Hence, the correct option is (D).

Quick Tip to Solve: First multiply survival probability by conditional hit probability; only then raise the resulting single-missile success probability to the number of independent firings.

2. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is : [4 marks]

(A) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

(B) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

(C) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

(D) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

Correct Answer: (C) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

Solution:

Concept:

- A line through (x_0, y_0, z_0) with direction ratios (a, b, c) has symmetric equation $(x - x_0)/a = (y - y_0)/b = (z - z_0)/c$.
- Two directions are perpendicular exactly when their scalar product is zero. Read the direction ratios directly from the denominators of their symmetric equations.

Step 1: Read the given direction and point The given line has direction vector $\mathbf{d} = (2, 3, -2)$. Every proposed line passes through $P = (0, 1, 2)$, as its numerators are $x, y - 1, z - 2$. Consequently, the direction test decides which of these candidates is perpendicular. The point through which the original line passes is not its direction vector.

Step 2: Test the candidate directions The respective direction vectors in options (A), (B), (C), and (D) are $(3, -4, 3)$, $(3, 4, -3)$, $(-3, 4, 3)$, and $(3, 4, 3)$. Their scalar products with the original direction are

$$\begin{aligned}(2, 3, -2) \cdot (3, -4, 3) &= 6 - 12 - 6 = -12,\\ (2, 3, -2) \cdot (3, 4, -3) &= 6 + 12 + 6 = 24,\\ (2, 3, -2) \cdot (-3, 4, 3) &= -6 + 12 - 6 = 0,\\ (2, 3, -2) \cdot (3, 4, 3) &= 6 + 12 - 6 = 12.\end{aligned}$$

Only the third candidate has a right-angle direction. In parametric form it is $(x, y, z) = (0, 1, 2) + t(-3, 4, 3)$, which also verifies the required point at $t = 0$.

Step 3: Conclusion The required candidate is $\boxed{\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: When every option already passes through the stated point, test only the denominator vectors by a dot product; retain the signs of all three ratios.

3. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\leq$ t, is : [4 marks]

(A) $\frac{e+1}{3}$

(B) $\frac{e-1}{3e}$

(C) $\frac{1}{3e}$

(D) $\frac{e+1}{3e}$

Correct Answer: (D) $\frac{e+1}{3e}$

Solution:

Concept:

- The greatest integer function $[u]$ is constant between successive integers. Here the sign of x determines whether x^3 lies in $(-1, 0)$ or $(0, 1)$.
- Changing a bounded integrand at finitely many isolated points does not change its definite integral. Split the integral at each interior jump of the greatest integer factor.

Step 1: Identify the two constant exponential factors For $-1 < x < 0$, we have $-1 < x^3 < 0$, so $[x^3] = -1$ and $e^{[x^3]} = e^{-1}$. For $0 < x < 1$, we have $0 < x^3 < 1$, so $[x^3] = 0$ and $e^{[x^3]} = 1$. At $x = 1$, the floor is 1, but that single endpoint contributes no area. Thus no extra interval carrying the value e is present.

Step 2: Integrate on the two intervals Let the requested integral be I. Use the constants just found and integrate x^2 directly:

$$\begin{aligned} I &= \frac{1}{e}\int_{-1}^{0} x^2\,dx + \int_{0}^{1} x^2\,dx \\ &= \frac{1}{e}\left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1} \\ &= \frac{1}{3e} + \frac{1}{3} = \frac{e+1}{3e}. \end{aligned}$$

Both contributions are positive. Option (B) incorrectly subtracts them, while option (C) omits the positive half of the interval. Option (A) uses an incorrect exponential factor on the negative half.

Step 3: Conclusion The integral equals $\boxed{\frac{e+1}{3e}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a floor-function integral, list the constant values on open intervals first; isolated endpoint jumps never add a separate area.

4. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is : [4 marks]

(A) 4

(B) 2

(C) 3

(D) 0

Correct Answer: (C) 3

Solution:

Concept:

- A real quadratic with positive leading coefficient is strictly positive for every real argument exactly when its minimum value is strictly positive; equivalently its discriminant is negative.
- Completing the square locates the minimum without testing sample values of x. The word strictly excludes a quadratic that merely touches zero.

Step 1: Complete the square in the variable x Write the quadratic as $Q(x) = x^2 - 2(3k-1)x + 8k^2 - 7$. Its leading coefficient is 1, so it opens upwards. Completing the square gives

$$\begin{aligned} Q(x) &= (x - (3k-1))^2 + 8k^2 - 7 - (3k-1)^2 \\ &= (x - (3k-1))^2 - k^2 + 6k - 8 \\ &= (x - (3k-1))^2 + 1 - (k-3)^2. \end{aligned}$$

The square is nonnegative and becomes zero at the real point $x = 3k - 1$. Therefore the remaining term is the exact minimum.

Step 2: Apply strict positivity and the integer restriction To have $Q(x) > 0$ for every real x, require

$$1 - (k-3)^2 > 0 \quad \Longleftrightarrow \quad |k-3| < 1 \quad \Longleftrightarrow \quad 2 < k < 4.$$

The only integer in this open interval is $k = 3$. For this value, $Q(x) = (x-8)^2 + 1$, which is indeed always positive. The alternatives $k = 2$ and $k = 4$ have minimum zero and fail the strict inequality; $k = 0$ has minimum -8.

Step 3: Conclusion The required integer is $\boxed{k = 3}$. Hence, the correct option is (C).

Quick Tip to Solve: Complete the square and impose a positive minimum. For a strict all-real inequality, boundary parameter values giving a zero minimum must be excluded.

5. The total number of positive integral solutions (x, y, z) such that xyz = 24 is : [4 marks]

(A) 36

(B) 24

(C) 45

(D) 30

Correct Answer: (D) 30

Solution:

Concept:

- Unique prime factorisation allows an ordered factorisation $xyz = N$ to be counted by independently assigning the exponent of each prime among x, y, z.
- The number of nonnegative solutions of $a+b+c=r$ is $\binom{r+2}{2}$. Zero exponents are allowed because each variable is positive but may equal 1.

Step 1: Distribute the exponent of each prime Factor the specified product as $24 = 2^3 \cdot 3$. Write

$$x = 2^{a_1}3^{b_1}, \quad y = 2^{a_2}3^{b_2}, \quad z = 2^{a_3}3^{b_3},$$

where all exponents are nonnegative integers. No other prime can divide any variable, since it would then divide 24. The product condition is precisely

$$a_1 + a_2 + a_3 = 3, \qquad b_1 + b_2 + b_3 = 1.$$

The variables form an ordered triple, so assigning a prime power to x is different from assigning it to y.

Step 2: Multiply the independent counts By stars and bars, the first exponent equation has $\binom{5}{2} = 10$ solutions. The second has $\binom{3}{2} = 3$ solutions: the single factor 3 belongs to exactly one of the three variables. Each distribution of the powers of 2 can be combined with any distribution of the factor 3, and unique factorisation ensures no resulting triple is counted twice. Therefore

$$\#\{(x, y, z) \in \mathbb{N}^3 : xyz = 24\} = 10 \times 3 = 30.$$

The alternatives 36, 24 and 45 do not equal this product; treating the factors as an unordered partition or forbidding a variable from being 1 would count a different problem.

Step 3: Conclusion There are $\boxed{30}$ positive integral solutions. Hence, the correct option is (D).

Quick Tip to Solve: For ordered triples with product $\prod p_i^{r_i}$, multiply $\binom{r_i+2}{2}$ over the primes; positivity of the variables permits zero prime exponents.

6. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point : [4 marks]

(A) (4, 4)

(B) (5, 5)

(C) (5, 4)

(D) (4, 5)

Correct Answer: (B) (5, 5)

Solution:

Concept:

- The tangent slope is dy/dx, so the given condition is a first-order linear differential equation. For $y\prime + P(x)y = Q(x)$, an integrating factor is $e^{\int P(x)\,dx}$.
- The initial point determines the integration constant. The slope expression is defined only for $x \neq 2$; the resulting algebraic curve can still be checked at the proposed points.

Step 1: Put the equation in linear form Rearrange the slope condition and simplify the part independent of y:

$$\frac{dy}{dx} - \frac{y}{x-2} = x - 2 + \frac{4}{x-2}.$$

On an interval avoiding 2, an integrating factor is $1/(x-2)$. Multiplying gives an exact derivative:

$$\frac{d}{dx}\left(\frac{y}{x-2}\right) = 1 + \frac{4}{(x-2)^2}.$$

Integrating both sides yields $y/(x-2) = x - 2 - 4/(x-2) + C$, or

$$y = (x-2)^2 - 4 + C(x-2).$$

Step 2: Use the origin and check the options The origin means $x = 0, y = 0$. Substitution gives $0 = 4 - 4 - 2C$, hence $C = 0$. The corresponding algebraic curve is

$$y = (x-2)^2 - 4 = x^2 - 4x.$$

For $x = 4$, it gives $y = 0$, excluding both (4,4) and (4,5). For $x = 5$, it gives $y = 25 - 20 = 5$, selecting (5,5) rather than (5,4). As a direct check, its derivative is $2x - 4$, and the stated slope becomes $(2x^2 - 8x + 8)/(x-2) = 2(x-2)$ wherever it is defined. Thus the equation satisfies the prescribed slope condition away from its stated singular denominator.

Step 3: Conclusion The curve $y = x^2 - 4x$ passes through $\boxed{(5,5)}$. Hence, the correct option is (B).

Quick Tip to Solve: After finding the integrating factor, apply the initial point before testing options. Substitute the final curve back into the slope formula to check signs.

7. Let f, g : N $\to$ N such that f(n + 1) = f(n) + f(1) $\forall$ n$\in$N and g be any arbitrary function. Which of the following statements is NOT true? [4 marks]

(A) If g is onto, then fog is one-one

(B) f is one-one

(C) If f is onto, then f(n) = n $\forall$n$\in$N

(D) If fog is one-one, then g is one-one

Correct Answer: (A) If g is onto, then fog is one-one

Solution:

Concept:

- Iterating $f(n+1) = f(n) + f(1)$ on the positive natural numbers gives $f(n) = nf(1)$. Since $f(1) \in \mathbb{N}$, this multiplier is positive.
- A composition with an injective outer function is injective exactly when its inner function is injective. Surjectivity of the inner function alone does not imply injectivity.

Step 1: Establish the form and properties of f Put $c = f(1) \geq 1$. The recurrence gives $f(2) = 2c$, and induction gives $f(n) = cn$ for every positive integer n. If $f(m) = f(n)$, then $cm = cn$, so $m = n$; statement (B) is true. If f is onto $\mathbb{N}$, the value 1 must occur as cn. Positive integers c, n can satisfy this only when $c = 1$, giving $f(n) = n$. Thus (C) is also true.

Step 2: Test the two composition claims If $f \circ g$ is one-one and $g(m) = g(n)$, applying f gives $f(g(m)) = f(g(n))$, forcing $m = n$. Therefore (D) is true. To disprove (A), take $f(n) = n$ and

$$g(n) = \left\lceil \frac{n}{2} \right\rceil .$$

This g is onto: each positive integer r is the image of both $2r - 1$ and $2r$. It is not one-one, since $g(1) = g(2) = 1$. Here $f \circ g = g$, so the composition is not one-one even though g is onto. This counterexample satisfies the given recurrence and all domain and codomain conditions.

Step 3: Conclusion The statement that is NOT true is: if g is onto, then $f \circ g$ is one-one. Hence, the correct option is (A).

Quick Tip to Solve: Derive $f(n) = nf(1)$ first. To distinguish onto from one-one, use $g(n) = \lceil n/2 \rceil$, which reaches every natural number but repeats each value.

8. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is : [4 marks]

(A) $\frac{3}{2\sqrt{2}}$

(B) $3\sqrt{2}$

(C) $\frac{1}{2\sqrt{2}}$

(D) $\frac{3}{\sqrt{2}}$

Correct Answer: (A) $\frac{3}{2\sqrt{2}}$

Solution:

Concept:

- Write $z = x + iy$ and $\bar{z} = x - iy$ to turn complex equations into Cartesian lines.
- Every normal to a circle passes through its centre. Its radius is the perpendicular distance from the centre to any tangent.

Step 1: Find the two normal lines Substitute the Cartesian forms into the first equation. The real parts cancel and the imaginary parts give

$$2y - x = 0.$$

For the second equation, the sum of the two complex products is $i(2x + 4y)$. Thus

$$2x + 4y - 4 = 0, \qquad x + 2y = 2.$$

These are distinct normal lines, so their intersection gives the circle centre; their slopes alone do not give the radius.

Step 2: Locate the centre From $x = 2y$ and $x + 2y = 2$, we obtain $y = 1/2$, $x = 1$. Therefore the centre is $O = (1, 1/2)$. Substitution in both normal equations verifies the intersection.

Step 3: Measure the distance to the tangent The tangent equation becomes

$$iz + \bar{z} + 1 + i = (x - y + 1) + i(x - y + 1) = 0.$$

Hence its Cartesian form is $x - y + 1 = 0$. Using the point-to-line distance formula,

$$r = \frac{|1 - 1/2 + 1|}{\sqrt{1^2 + (-1)^2}} = \frac{3}{2\sqrt{2}}.$$

The numerator is $3/2$, not $1/2$, because the tangent has constant term $+1$. Options B and D also omit the required normalization or multiply the distance incorrectly.

Step 4: Conclusion The radius is $\boxed{3/(2\sqrt{2})}$. Hence, the correct option is (A).

Quick Tip to Solve: Find the centre from the intersection of the normals first; then use its distance from the tangent, including the line constant.

9. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$ **is equal to :** [4 marks]

(A) $\frac{1}{2}$

(B) 1

(C) 0

(D) $\frac{1}{e}$

Correct Answer: (B) 1

Solution:

Concept:

- For $u \geq 0$, $0 \leq \log(1+u) \leq u$. Taking logarithms controls a positive sequence raised to a growing power.
- The harmonic sum $H_n = \sum_{k=1}^{n} 1/k$ satisfies $H_n \leq 1 + \log n$, so $H_n/n \to 0$.

Step 1: Name the harmonic sum and take logarithms Set

$$a_n = \left(1 + \frac{H_n}{n^2}\right)^n.$$

The base is positive and greater than one for every positive integer n. Therefore logarithms are defined, and

$$\log a_n = n \log\left(1 + \frac{H_n}{n^2}\right).$$

The exponent growing to infinity does not by itself determine the answer: the rate at which the base tends to one is decisive.

Step 2: Bound the logarithm Apply the logarithmic inequality with $u = H_n/n^2$:

$$0 \leq \log a_n \leq \frac{H_n}{n} \leq \frac{1+\log n}{n}.$$

The harmonic bound follows by comparing $1/2 + \cdots + 1/n$ with $\int_1^n dx/x$. Since the last expression tends to zero, the squeeze theorem gives $\log a_n \to 0$.

Step 3: Return to the original sequence Continuity of the exponential gives

$$\lim_{n\to\infty} a_n = \exp\left(\lim_{n\to\infty} \log a_n\right) = e^0 = 1.$$

All three other options are less than one, whereas every a_n is greater than one; this is a useful independent check. In particular, the familiar exponential limit does not yield $1/e$: the effective exponent H_n/n tends to zero, rather than to -1.

Step 4: Conclusion The required limit is $\boxed{1}$. Hence, the correct option is (B).

Quick Tip to Solve: For a power $(1+u_n)^n$, first check nu_n; here it equals H_n/n, which tends to zero.

10. Let α be the angle between the lines whose direction cosines satisfy the equations l + m − n = 0 and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is : [4 marks]

(A) $\frac{3}{8}$

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $\frac{5}{8}$

Correct Answer: (D) $\frac{5}{8}$

Solution:

Concept:

- Direction cosines satisfy $l^2 + m^2 + n^2 = 1$. A nonzero proportional triple can be used as direction ratios.
- The cosine of the angle between directions $\mathbf{a}, \mathbf{b}$ is $|\mathbf{a} \cdot \mathbf{b}|/(|\mathbf{a}||\mathbf{b}|)$.

Step 1: Find the possible directions The linear constraint gives $n = l + m$. Substitute it into the quadratic constraint:

$$l^2 + m^2 - (l + m)^2 = -2lm = 0.$$

Thus either $l = 0$ or $m = 0$. If $l = 0$, then $n = m$, giving direction ratios $(0, 1, 1)$. If $m = 0$, then $n = l$, giving $(1, 0, 1)$. They cannot both vanish because direction cosines have unit squared sum.

Step 2: Compute the angle between the two lines Take $\mathbf{a} = (0, 1, 1)$ and $\mathbf{b} = (1, 0, 1)$. Their dot product is one and each has length $\sqrt{2}$, so

$$|\cos\alpha| = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}.$$

Hence $\cos^2\alpha = 1/4$ and $\sin^2\alpha = 3/4$. Reversing either direction changes the sign of the cosine but leaves both fourth powers unchanged.

Step 3: Evaluate the requested expression Square these two squared trigonometric values and add:

$$\sin^4\alpha + \cos^4\alpha = \left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{9+1}{16} = \frac{5}{8}.$$

Option B is only $\sin^2\alpha$, not the required sum. Option C would require equal squared sine and cosine, which is inconsistent with $\cos^2\alpha = 1/4$; option A likewise does not satisfy the computed identity.

Step 4: Conclusion The value is $\boxed{5/8}$. Hence, the correct option is (D).

Quick Tip to Solve: Eliminate one direction cosine before normalizing; the product $lm = 0$ immediately reveals the two directions.

11. The value of the integral

$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}\,d\theta$ **is :** [4 marks]

(A) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$

(B) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

(C) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$

(D) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$

Correct Answer: (C) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$

Solution:

Concept:

- Use $\sin 2\theta = 2\sin\theta\cos\theta$ and $1-\cos 2\theta = 2\sin^2\theta$, on intervals where the original denominator is nonzero.
- A substitution $t=\sin\theta$ turns the trigonometric integral into a polynomial times a square root. Factor the polynomial before choosing a further substitution.

Step 1: Reduce the integrand Write $s=\sin\theta$. Cancelling $2s^2$ in the original integrand gives

$$I=\int \cos\theta\,(s^6+s^4+s^2)\sqrt{2s^4+3s^2+6}\,d\theta.$$

With $t=\sin\theta$, this becomes

$$I=\int t^2(t^4+t^2+1)\sqrt{2t^4+3t^2+6}\,dt.$$

Step 2: Recognize a differentiated product Let $Q=2t^4+3t^2+6$. Direct differentiation gives

$$\frac{d}{dt}\left(t^3Q^{3/2}\right)=3t^2\sqrt{Q}\left(Q+\frac{t}{2}Q'\right).$$

Since $Q'=8t^3+6t$, the bracket equals $6t^4+6t^2+6$. Therefore

$$I=\frac{t^3}{18}(2t^4+3t^2+6)^{3/2}+C.$$

This form differentiates to the given integrand on every interval of its domain.

Step 3: Identify the printed option and its branch The quantity inside option C expands using $\cos^2\theta=1-s^2$:

$$11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta=2s^6+3s^4+6s^2=s^2Q.$$

Thus option C is the displayed primitive on intervals with $\sin\theta>0$. Strictly, real $(s^2Q)^{3/2}=|s|^3Q^{3/2}$; on intervals with $\sin\theta<0$, its negative is needed. The signed expression derived above resolves this branch restriction. Options A, B and D contain different polynomials or derivative signs and do not reproduce the integrand on the positive-sine interval.

Step 4: Conclusion The primitive is $\boxed{\frac{\sin^3\theta}{18}(2\sin^4\theta+3\sin^2\theta+6)^{3/2}+C}$. The matching printed expression, on $\sin\theta>0$, is option (C); for negative sine its sign must be reversed. Hence, the correct option is (C).

Quick Tip to Solve: When replacing $t\sqrt{Q}$ by $\sqrt{t^2Q}$, check the sign of t; an absolute value can change an antiderivative.

12. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is : [4 marks]

(A) $\frac{1}{72}$
(B) $\frac{5}{216}$
(C) $\frac{1}{36}$
(D) $\frac{1}{54}$

Correct Answer: (B) $\frac{5}{216}$

Solution:

Concept:

- A quadratic equation has equal roots precisely when its discriminant b^2-4ac is zero.
- Three independent fair die throws give $6^3=216$ equally likely ordered triples (a,b,c); exchanging a and c generally gives a new outcome.

Step 1: Restrict the middle coefficient All coefficients lie in $\{1,2,3,4,5,6\}$, so $a\neq 0$ and the equation is always genuinely quadratic. Equal roots require

$$b^2=4ac.$$

In particular b must be even. We therefore need only consider $b=2,4,6$; an odd square cannot be divisible by four.

Step 2: Enumerate every admissible ordered triple For $b=2$, we have $ac=1$, giving only $(a,c)=(1,1)$. For $b=4$, we have $ac=4$, giving $(1,4),(2,2),(4,1)$. For $b=6$, we have $ac=9$, giving only $(3,3)$. The factorizations $(1,9),(9,1)$ are impossible because a die cannot show nine. Thus the favourable triples are

$$(1,2,1),\ (1,4,4),\ (2,4,2),\ (4,4,1),\ (3,6,3).$$

Step 3: Divide by the full sample-space size There are five favourable ordered triples, each with probability $1/216$. Consequently

$$P(\text{equal roots})=\frac{5}{216}.$$

The other proposed fractions correspond to three, six, or four favourable outcomes respectively and conflict with the explicit list. Do not count the three possible even values of b alone: each can admit a different number of coefficient pairs.

Step 4: Conclusion The probability is $\boxed{5/216}$. Hence, the correct option is (B).

Quick Tip to Solve: For coefficients drawn from a small finite set, use the discriminant and enumerate ordered factor pairs rather than estimating probabilities separately.

13. The image of the point (3, 5) in the line x − y + 1 = 0, lies on : [4 marks]

(A) $(x-4)^2 + (y-4)^2 = 8$

(B) $(x-4)^2 + (y+2)^2 = 16$

(C) $(x-2)^2 + (y-2)^2 = 12$

(D) $(x-2)^2 + (y-4)^2 = 4$

Correct Answer: (D) $(x-2)^2 + (y-4)^2 = 4$

Solution:

Concept:

- The reflection of (x_0, y_0) in $ax+by+c=0$ is (x_0-2ad, y_0-2bd), where $d = (ax_0+by_0+c)/(a^2+b^2)$.
- To check whether a point lies on a circle, substitute its coordinates in the circle equation and compare both sides.

Step 1: Reflect the given point Here $a = 1$, $b = -1$, $c = 1$ and $(x_0, y_0) = (3, 5)$. Hence

$$d = \frac{3-5+1}{1+1} = -\frac{1}{2}.$$

The reflected point is therefore

$$x' = 3 - 2(1)(-1/2) = 4, \qquad y' = 5 - 2(-1)(-1/2) = 4.$$

Thus the image is $P' = (4, 4)$, rather than the original point or merely its projection on the line.

Step 2: Check the geometry of the reflection The midpoint of $(3, 5)$ and $(4, 4)$ is $(7/2, 9/2)$, which satisfies $x - y + 1 = 0$. The segment joining the points has slope -1, whereas the reflecting line has slope 1; the segment is perpendicular to the line. These two facts verify that the image coordinates have the correct signs.

Step 3: Test each proposed circle At $(4, 4)$, the left-hand sides of options A, B, C and D are respectively

$$0, \qquad 36, \qquad (4-2)^2 + (4-2)^2 = 8, \qquad (4-2)^2 + (4-4)^2 = 4.$$

Their required right-hand sides are $8, 16, 12, 4$, respectively. Only the last equality holds. The image happens to be the centre of circle A, so it cannot lie on that circle with nonzero radius.

Step 4: Conclusion The image is $(4, 4)$, which lies on $\boxed{(x-2)^2 + (y-4)^2 = 4}$. Hence, the correct option is (D).

Quick Tip to Solve: After reflecting a point, verify its midpoint lies on the mirror line; then test the image in every proposed locus.

14. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in : [4 marks]

(A) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(B) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(C) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(D) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Correct Answer: (D) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Solution:

Concept:

- Factor $\sin x + \tan x = \sin x(1 + \cos x)/\cos x$, with $\cos x \neq 0$.
- Except where $\cos x = -1$, the factor $1 + \cos x$ is positive. Thus the sign is determined by $\tan x$, while zeros and poles must be excluded.

Step 1: Factor the expression and track its domain Taking $x = 2\theta$, the inequality is

$$\frac{\sin 2\theta(1 + \cos 2\theta)}{\cos 2\theta} > 0.$$

The expression is undefined when $\cos 2\theta = 0$. When $\cos 2\theta = -1$, its numerator is zero and the strict inequality fails. Elsewhere $1 + \cos 2\theta > 0$, so we require sine and cosine of 2θ to have the same sign.

Step 2: Solve over the doubled interval Because $0 \leq \theta \leq 2\pi$, the doubled angle lies in $[0, 4\pi]$. Sine and cosine have the same nonzero sign in the first and third quadrants of each complete revolution. Thus

$$2\theta \in (0, \pi/2) \cup (\pi, 3\pi/2) \cup (2\pi, 5\pi/2) \cup (3\pi, 7\pi/2).$$

Every endpoint is excluded: it is either a zero or a point where the tangent is undefined.

Step 3: Convert back and compare the options Divide every endpoint by two:

$$\theta \in (0, \pi/4) \cup (\pi/2, 3\pi/4) \cup (\pi, 5\pi/4) \cup (3\pi/2, 7\pi/4).$$

This is exactly option D. Options B and C include poles such as $\theta = \pi/4$ or intervals where the expression is negative. Option A omits an entire valid interval near π and extends its final interval beyond the correct boundary.

Step 4: Conclusion The solution set is $\boxed{(0, \pi/4) \cup (\pi/2, 3\pi/4) \cup (\pi, 5\pi/4) \cup (3\pi/2, 7\pi/4)}$. Hence, the correct option is (D).

Quick Tip to Solve: Solve the sign chart in the doubled angle first, then halve the interval endpoints; always remove tangent poles and strict-inequality zeros.

15. A man is observing, from the top of a tower, a boat speeding towards the lower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is : [4 marks]

(A) $10(\sqrt{3}+1)$

(B) $10(\sqrt{3}-1)$

(C) 10

(D) $10\sqrt{3}$

Correct Answer: (A) $10(\sqrt{3}+1)$

Solution:

Concept:

- The angle of depression from the tower equals the angle of elevation from the boat, since the horizontal lines at the observer and the water surface are parallel.
- For a right triangle with tower height h, horizontal distance d and elevation α, $\tan\alpha = h/d$. Uniform speed means that distances travelled are proportional to elapsed times.

Step 1: Express the two distances in terms of the height Let O be the foot of the tower and let its height be $h > 0$. The boat moves along the straight line towards O, first reaching A and then B. Using the two angles gives

$$OA = \frac{h}{\tan 30^\circ} = \sqrt{3}h, \qquad OB = \frac{h}{\tan 45^\circ} = h.$$

Thus the first observed stretch is $AB = OA - OB = (\sqrt{3}-1)h$. The smaller angle corresponds to the larger distance, as expected.

Step 2: Use uniform speed for the remaining journey The boat covers AB in 20 seconds, so its speed is

$$v = \frac{(\sqrt{3}-1)h}{20}.$$

The remaining distance is $BO = h$, not AB. Therefore the required time is

$$t = \frac{h}{v} = \frac{20}{\sqrt{3}-1} = \frac{20(\sqrt{3}+1)}{3-1} = 10(\sqrt{3}+1)\ \text{s}.$$

The height cancels because both travelled distances scale with it. The other three options are not this positive ratio; in particular the remaining distance exceeds AB, so the time must exceed 20 seconds.

Step 3: Conclusion The time from B to the tower base is $\boxed{10(\sqrt{3}+1)\ \text{s}}$. Hence, the correct option is (A).

Quick Tip to Solve: For constant speed, use the ratio of the remaining distance to the distance already covered; the unknown tower height cancels.

16. The statement A $\rightarrow$ (B $\rightarrow$ A) is equivalent to : [4 marks]

(A) A $\rightarrow$ (A $\longleftrightarrow$ B)

(B) A $\rightarrow$ (A $\vee$ B)

(C) A $\rightarrow$ (A $\wedge$ B)

(D) A $\rightarrow$ (A $\rightarrow$ B)

Correct Answer: (B) A $\rightarrow$ (A $\vee$ B)

Solution:

Concept:

- A conditional statement satisfies $P \rightarrow Q \equiv \neg P \vee Q$. A tautology is true for every assignment of truth values to its component statements.
- To establish equivalence to a tautology, simplify each candidate or find a truth assignment that makes it false.

Step 1: Simplify the given nested implication Work from the inner implication outwards. First $B \rightarrow A \equiv \neg B \vee A$. Therefore

$$A \rightarrow (B \rightarrow A) \equiv \neg A \vee (\neg B \vee A) \equiv (\neg A \vee A) \vee \neg B \equiv \text{True}.$$

This also has a direct interpretation. If A is false the outer implication is true. If A is true, the consequent $B \rightarrow A$ is true regardless of B. Thus the original statement has no false row in its truth table.

Step 2: Compare all four proposed statements Option (B) has the form

$$A \rightarrow (A \vee B) \equiv \neg A \vee A \vee B \equiv \text{True}.$$

Hence it is also a tautology and is equivalent to the given statement. For completeness, take A true and B false. Then $A \leftrightarrow B$ is false, so option (A) is false. Also $A \wedge B$ is false, so option (C) is false. Finally $A \rightarrow B$ is false, making the outer implication in option (D) false. None of these three can be equivalent to an expression that is always true. This single counterexample disposes of all the distractors without a full four-row table.

Step 3: Conclusion The equivalent statement is $\boxed{A \rightarrow (A \vee B)}$. Hence, the correct option is (B).

Quick Tip to Solve: Replace every implication by a disjunction. If the result contains both a statement and its negation joined by OR, it is a tautology.

17. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it? [4 marks]

(A) (0, 3)

(B) (−6, 0)

(C) (4, 5)

(D) (5, 4)

Correct Answer: (D) (5, 4)

Solution:

Concept:

- For $y^2 = 4ax$, the tangent having nonzero slope m is $y = mx + a/m$. The parameter a is one quarter of the coefficient of x.
- Two finite slopes of perpendicular lines satisfy $m_1 m_2 = -1$. A point lies on a line precisely when its coordinates satisfy the line equation.

Step 1: Find the required tangent equation The given line is $y = -2x + 1$, so its slope is -2. The perpendicular tangent therefore has slope $m = 1/2$. Comparing $y^2 = 6x$ with $y^2 = 4ax$ gives $a = 3/2$. Substitution in the tangent formula yields

$$y = \frac{x}{2} + \frac{3/2}{1/2} = \frac{x}{2} + 3.$$

Equivalently, the tangent is $x - 2y + 6 = 0$. As a check, substituting $x = 2y - 6$ in the parabola gives $y^2 = 12y - 36$, or $(y - 6)^2 = 0$; the repeated intersection confirms tangency.

Step 2: Test each point and identify the exception At $(0, 3)$, the right side $x/2 + 3$ is 3, agreeing with y. At $(-6, 0)$, it is 0, again agreeing with y. At $(4, 5)$, it is 5, so that point also lies on the tangent. But at $(5, 4)$, the right side is $5/2 + 3 = 11/2$, which differs from 4. Equivalently its line residual is $5 - 2(4) + 6 = 3 \neq 0$. The question asks for the point that does NOT lie on the tangent; selecting one of the first three would reverse this condition.

Step 3: Conclusion The point $\boxed{(5, 4)}$ does not lie on the tangent. Hence, the correct option is (D).

Quick Tip to Solve: Find the tangent once, then substitute all candidate coordinates into its simplest linear form; pay attention to the word NOT.

18. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to : [4 marks]

(A) $(-5, -8)$

(B) $(5, -8)$

(C) $(-5, 8)$

(D) $(5, 8)$

Correct Answer: (D) $(5, 8)$

Solution:

Concept:

- A polynomial is continuous on every closed interval and differentiable on its interior. Rolle's theorem additionally requires equal endpoint values, $f(1) = f(2)$.
- The stated stationary point supplies a second equation: $f\prime(4/3) = 0$. Together these two conditions determine the two unknown coefficients.

Step 1: Use the equality of endpoint values For $f(x) = x^3 - ax^2 + bx - 4$, the two endpoint values are

$$f(1) = b - a - 3, \qquad f(2) = 4 - 4a + 2b.$$

Rolle's theorem holds on $[1, 2]$, so these values must be equal. Hence

$$b - a - 3 = 4 - 4a + 2b \quad \Longrightarrow \quad 3a - b = 7.$$

The continuity and differentiability hypotheses add no restrictions here because f is a polynomial. The essential extra information is the specified zero of its derivative.

Step 2: Apply the derivative condition and solve Differentiate to obtain $f\prime(x) = 3x^2 - 2ax + b$. At the supplied point,

$$0 = f\prime\left(\frac{4}{3}\right) = \frac{16}{3} - \frac{8a}{3} + b, \quad \text{so} \quad 16 - 8a + 3b = 0.$$

From the endpoint equation, $b = 3a - 7$. Thus $16 - 8a + 9a - 21 = 0$, giving $a = 5$ and then $b = 8$. Checking, $f(1) = f(2) = 0$ and $f\prime(4/3) = 0$. The point $4/3$ lies inside $(1, 2)$, as required. The other sign combinations in the options fail at least one of the two necessary equations.

Step 3: Conclusion The ordered pair is $\boxed{(a, b) = (5, 8)}$. Hence, the correct option is (D).

Quick Tip to Solve: In a Rolle's theorem coefficient problem, use both equal endpoint values and the specified derivative zero; either condition alone is insufficient.

19. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is TRUE? [4 marks]

(A) a − c = b + d

(B) a + b = c + d

(C) $ab=\frac{c+d}{a+b}$

(D) a − b = c − d

Correct Answer: (D) a − b = c − d

Solution:

Concept:

- At an orthogonal intersection, the tangent directions are perpendicular. Equivalently, the normal vectors, which are the gradients of the two implicit equations, have zero dot product.
- Writing $X=x^2$ and $Y=y^2$ turns the two conic equations into linear relations. The denominators a,b,c,d are nonzero because they occur in the printed equations.

Step 1: Express the orthogonality condition Let (x,y) be a common point. Normal vectors to the curves are proportional to $(x/a,y/b)$ and $(x/c,y/d)$. Their dot product must vanish:

$$\frac{x^2}{ac}+\frac{y^2}{bd}=0.$$

With $X=x^2$, $Y=y^2$, multiply by $abcd$ to obtain $bdX+acY=0$. Meanwhile the first curve gives $bX+aY=ab$, and the second gives $dX+cY=cd$. Using gradients also avoids dividing by y when discussing a tangent slope.

Step 2: Eliminate the intersection coordinates Multiply $bX+aY=ab$ by d and subtract the orthogonality equation. This gives

$$a(d-c)Y=abd \quad\Longrightarrow\quad (d-c)Y=bd.$$

Similarly multiply $dX+cY=cd$ by b and subtract the orthogonality equation:

$$c(b-a)Y=bcd \quad\Longrightarrow\quad (b-a)Y=bd.$$

The right side bd is nonzero, so $Y\neq 0$. Comparing the last two relations yields $d-c=b-a$, or $a-b=c-d$. No assumption about which conic is an ellipse or a hyperbola is needed. The alternatives involve different sums, differences or products and do not express this necessary orthogonality relation.

Step 3: Conclusion The true relation is $\boxed{a-b=c-d}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the dot product of the two gradients for orthogonal implicit curves, then eliminate the squared coordinates with the original equations.

20. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta, y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ **and** $z = \sum_{n=0}^{\infty} \cos^{2n}\theta.\sin^{2n}\phi$ **then :** [4 marks]

(A) xy − z = (x + y)z

(B) xyz = 4

(C) xy + z = (x + y)z

(D) xy + yz + zx = z

Correct Answer: (C) xy + z = (x + y)z

Solution:

Concept:

- An infinite geometric series with first term 1 and ratio r, where $|r| < 1$, has sum $1/(1-r)$. The zero-index term must be included.
- Because $0 < \theta, \phi < \pi/2$, the quantities $\cos^2\theta$, $\sin^2\phi$, and their product all lie strictly between zero and one, so all three series converge.

Step 1: Evaluate the three series Put $u = \cos^2\theta$ and $v = \sin^2\phi$. The three sums become

$$x = \frac{1}{1-u}, \qquad y = \frac{1}{1-v}, \qquad z = \frac{1}{1-uv}.$$

Consequently $u = 1 - 1/x$ and $v = 1 - 1/y$. These substitutions retain the dependence among the sums rather than treating x, y, z as independent quantities. In particular, each sum is greater than one because its first term is one and the remaining terms are positive.

Step 2: Eliminate the common ratios and compare the options Using the expression for z,

$$\frac{1}{z} = 1 - \left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{y}\right) = \frac{1}{x} + \frac{1}{y} - \frac{1}{xy}.$$

Multiply by the positive quantity xyz to obtain $xy = yz + xz - z$, hence $xy + z = (x+y)z$. To reject the other proposed identities directly, choose $u = v = 1/2$, which is allowed by the angle range. Then $x = y = 2$, $z = 4/3$. Option (A) would assert $8/3 = 16/3$; option (B) would assert $16/3 = 4$; and option (D) would equate $28/3$ with $4/3$. Each is false, while the derived identity holds for all allowed angles.

Step 3: Conclusion The required relation is $\boxed{xy + z = (x+y)z}$. Hence, the correct option is (C).

Quick Tip to Solve: Replace each geometric sum by its reciprocal form, then eliminate the two common ratios before expanding any products of sums.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The locus of the point of intersection of the lines $\left(\sqrt{3}\right) kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4\left(\sqrt{3}\right) k = 0$ is a conic, whose eccentricity is _____. [4 marks]

Solution:

Concept:

- For a one-parameter pair of lines, the locus of their intersection is obtained by eliminating the parameter. Multiplication can remove the parameter without introducing a denominator involving the coordinates.
- A hyperbola in standard form $x^2/a^2 - y^2/b^2 = 1$ has eccentricity $e = \sqrt{1 + b^2/a^2}$; the positive term determines the transverse axis.

Step 1: Eliminate the parameter from the line equations The first equation can be written

$$k(\sqrt{3}x + y) = 4\sqrt{3},$$

while the second gives $\sqrt{3}x - y = 4\sqrt{3}k$. Multiply the latter by $\sqrt{3}x + y$ and use the former to find

$$(\sqrt{3}x - y)(\sqrt{3}x + y) = 4\sqrt{3}\,k(\sqrt{3}x + y) = 48.$$

Thus $3x^2 - y^2 = 48$. The value $k = 0$ does not yield an intersection because the first line equation would reduce to $-4\sqrt{3} = 0$, so no valid point was lost by the parameter elimination.

Step 2: Identify the hyperbola and evaluate its eccentricity Dividing the locus equation by 48 yields the standard form

$$\frac{x^2}{16} - \frac{y^2}{48} = 1.$$

Hence $a^2 = 16$ and $b^2 = 48$. For a hyperbola, $c^2 = a^2 + b^2 = 64$, where c is the distance from the centre to either focus. Therefore

$$e = \frac{c}{a} = \frac{8}{4} = 2.$$

As a consistency check, the eccentricity exceeds one, which distinguishes a hyperbola from an ellipse. Conversely any point satisfying this hyperbola has $\sqrt{3}x + y \neq 0$; choosing $k = (\sqrt{3}x - y)/(4\sqrt{3})$ recovers both original line equations, so the complete conic is represented.

Step 3: Conclusion The eccentricity of the locus is $\boxed{2}$.

Quick Tip to Solve: After eliminating a line parameter, divide to make the right side one; use the denominator under the positive squared term as a^2.

22. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _____.

[4 marks]

Solution:

Concept:

- The area enclosed between two curves is the integral of the upper function minus the lower function between their intersections.
- The equation $\sin x = \cos x$ gives $x = \pi/4 + n\pi$, so consecutive intersections are separated by π.

Step 1: Locate consecutive intersections Choose the consecutive intersections $x = \pi/4$ and $x = 5\pi/4$. On this interval, $\sin x - \cos x = \sqrt{2}\sin(x - \pi/4) \geq 0$. Thus sine is the upper curve throughout the interval. The other bounded regions have equal area because shifting by π reverses the sign of both sine and cosine, leaving the absolute difference unchanged.

Step 2: Integrate the vertical difference The required area is therefore

$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx = \big[-\cos x - \sin x \big]_{\pi/4}^{5\pi/4}.$$

At the upper endpoint, both sine and cosine equal $-1/\sqrt{2}$, giving $\sqrt{2}$; at the lower endpoint both equal $1/\sqrt{2}$, giving $-\sqrt{2}$. Consequently, $A = 2\sqrt{2}$. The positive area, rather than a signed integral over an entire period, is what is requested.

Step 3: Calculate the fourth power Square the area first to simplify the radical:

$$A^2 = (2\sqrt{2})^2 = 8, \qquad A^4 = 8^2 = 64.$$

This also shows that choosing any other consecutive pair produces the same requested value.

Step 4: Conclusion The enclosed area is $2\sqrt{2}$ square units, and the required numerical value is $\boxed{A^4 = 64}$.

Quick Tip to Solve: Find the intersections first and integrate only between consecutive ones; integrating over a full period would cancel the signed areas.

23. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _____.

[4 marks]

Solution:

Concept:

- A three-digit number is divisible by 3 exactly when its digit sum is divisible by 3, and by 5 exactly when its last digit is 5.
- For the inclusive condition "either 3 or 5, use inclusion-exclusion: $N(3 \cup 5) = N(3) + N(5) - N(15)$.

Step 1: Count numbers divisible by 3 All digits are nonzero, so every arrangement of three distinct

selected digits is a valid number between 100 and 1000. The three-digit subsets with sum divisible by 3 are

$$\{1,2,3\}, \quad \{1,3,5\}, \quad \{2,3,4\}, \quad \{3,4,5\}.$$

Their sums are respectively 6, 9, 9 and 12. These are the only qualifying subsets among the ten possible choices of three digits. Each has $3! = 6$ arrangements. Hence $N(3) = 4 \times 6 = 24$.

Step 2: Count numbers divisible by 5 The units digit must be 5. Choose the hundreds digit in four ways from $1, 2, 3, 4$, then the tens digit in three ways from the remaining digits. Thus

$$N(5) = 4 \times 3 = 12.$$

The no-repetition condition is imposed separately at both choices.

Step 3: Subtract the common numbers For divisibility by both 3 and 5, use a qualifying digit subset containing 5 and put 5 last. Only $\{1,3,5\}$ and $\{3,4,5\}$ qualify. Each has two orders for its first two digits, so $N(15) = 2 \times 2 = 4$. Therefore

$$N(3 \cup 5) = 24 + 12 - 4 = 32.$$

Subtracting the intersection prevents counting these four numbers twice.

Step 4: Conclusion The total number of three-digit numbers satisfying the condition is $\boxed{32}$.

Quick Tip to Solve: Count digit sets for divisibility by 3, but fix the units digit for divisibility by 5; subtract the overlap once.

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24. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ **and**

$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$**, then** $13(a^2 + b^2)$ **is equal to** [4 marks]

Solution:

Concept:

- For $t = \tan(\theta/2)$, the given matrix is $A = \begin{pmatrix} 0 & -t \\ t & 0 \end{pmatrix}$.
- The inverse of $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ is $\dfrac{1}{ps - qr}\begin{pmatrix} s & -q \\ -r & p \end{pmatrix}$, provided its determinant is nonzero.

Step 1: Find the inverse explicitly The problem defines $t = \tan(\theta/2)$, so work wherever this real quantity exists. We have

$$I_2 - A = \begin{pmatrix} 1 & t \\ -t & 1 \end{pmatrix}, \qquad \det(I_2 - A) = 1 + t^2 > 0.$$

Thus the matrix is invertible on the entire domain of the given expression, and

$$(I_2 - A)^{-1} = \frac{1}{1+t^2}\begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix}.$$

Keeping the signs in the off-diagonal entries is essential.

Step 2: Multiply and identify the entries Since $I_2 + A = \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix}$, multiplication gives

$$(I_2 + A)(I_2 - A)^{-1} = \frac{1}{1+t^2}\begin{pmatrix} 1-t^2 & -2t \\ 2t & 1-t^2 \end{pmatrix}.$$

Comparing corresponding entries with $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, obtain $a = (1-t^2)/(1+t^2)$ and $b = 2t/(1+t^2)$.

Step 3: Evaluate the invariant Rather than evaluating either trigonometric quantity separately, calculate

$$a^2 + b^2 = \frac{(1-t^2)^2 + 4t^2}{(1+t^2)^2} = \frac{1 + 2t^2 + t^4}{(1+t^2)^2} = 1.$$

The numerator is exactly the square of the common denominator base, so the result does not depend on θ.

Step 4: Conclusion Therefore $\boxed{13(a^2+b^2) = 13}$.

Quick Tip to Solve: For a matrix of this rotation form, computing the sum of squares of its first-column entries is simpler than finding the angle.

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25. Let A_1, A_2, A_3, ... be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is _____. [4 marks]

Solution:

Concept:

- For a square of side s, its diagonal is $\sqrt{2}s$ and its area is s^2.
- The side of one square equals the diagonal of the next, so successive side lengths have ratio $1/\sqrt{2}$, while successive areas have ratio $1/2$.

Step 1: Translate the geometric condition Let s_n denote the side length of square A_n. The stated condition gives

$$s_n = \sqrt{2}s_{n+1}, \qquad s_{n+1} = \frac{s_n}{\sqrt{2}}.$$

Starting with $s_1 = 12\,\text{cm}$, induction or the geometric progression formula gives

$$s_n = 12\left(\frac{1}{\sqrt{2}}\right)^{n-1}\ \text{cm}.$$

There are $n-1$ reductions from the first square to the n-th square; using n here would shift the answer by one.

Step 2: Write and solve the area inequality The area of the n-th square is

$$s_n^2 = 144\left(\frac{1}{2}\right)^{n-1}\ \text{cm}^2.$$

We require this area to be strictly less than $1\,\text{cm}^2$, so

$$\frac{144}{2^{n-1}} < 1 \quad\Longleftrightarrow\quad 2^{n-1} > 144.$$

Compare neighboring powers: $2^7 = 128 < 144 < 256 = 2^8$. The smallest possible exponent $n-1$ is therefore 8.

Step 3: Check the first admissible index At $n = 8$, the area is $144/128 = 1.125\,\text{cm}^2$, which fails. At $n = 9$, it is $144/256 = 0.5625\,\text{cm}^2$, which satisfies the strict inequality. Since areas decrease by half each time, every earlier index also fails.

Step 4: Conclusion The smallest required index is $\boxed{n = 9}$.

Quick Tip to Solve: Square the side ratio immediately: areas halve at each stage, and the exponent for the nth square is n minus one.

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26. If the system of equations

kx + y + 2z = 1

3x − y − 2z = 2

−2x −2y −4z = 3

has infinitely many solutions, then k is equal to _____. [4 marks]

Solution:

Concept:

- Infinitely many solutions occur when the equations are consistent but leave at least one variable free.
- Here the coefficients of z are twice those of y in every equation; set $s = y + 2z$ to expose the dependence.

Step 1: Reduce the system to two combinations With $s = y + 2z$, the equations become

$$kx + s = 1, \qquad 3x - s = 2, \qquad -2x - 2s = 3.$$

The last two equations do not involve k, so they first determine the only possible values of x and s for a consistent system. The variable z can remain free once these two quantities are fixed.

Step 2: Solve the two fixed equations From $3x - s = 2$, write $s = 3x - 2$. Substitute this into the

third equation:

$$-2x - 2(3x - 2) = 3 \quad \Longrightarrow \quad -8x + 4 = 3.$$

Therefore

$$x = \frac{1}{8}, \qquad s = \frac{3}{8} - 2 = -\frac{13}{8}.$$

As a check, $3/8 + 13/8 = 2$, and $-2/8 + 26/8 = 3$, so both fixed equations are satisfied.

Step 3: Impose consistency and verify freedom The first equation now requires

$$\frac{k}{8} - \frac{13}{8} = 1 \quad \Longrightarrow \quad k = 21.$$

For this value, choose any real parameter t and set

$$z = t, \qquad y = -\frac{13}{8} - 2t, \qquad x = \frac{1}{8}.$$

All three equations hold for every t, giving infinitely many distinct triples. Any other value of k contradicts the already fixed x and s, so it cannot yield a solution.

Step 4: Conclusion The required parameter value is $\boxed{k = 21}$.

Quick Tip to Solve: When two variable columns are proportional, combine those variables first; then verify consistency rather than using a zero determinant alone.

27. The number of points, at which the function

$f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in R$ is not differentiable, is _____. [4 marks]

Solution:

Concept:

- An absolute value of a smooth expression can fail to be differentiable at a zero of that expression. Away from these zeros, its sign is locally fixed.
- At coincident zeros, derivative jumps from different terms can cancel; compare the total left and right derivatives rather than merely counting roots.

Step 1: List all possible exceptional points The arguments vanish at

$$2x + 1 = 0 \Rightarrow x = -\tfrac{1}{2}, \qquad x + 2 = 0 \Rightarrow x = -2,$$

$$x^2 + x - 2 = (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1.$$

Thus only $-2, -1/2, 1$ need checking. Each term is differentiable at every other point, so their sum is differentiable there.

Step 2: Check cancellation at the shared root Near $x = -2$, $2x + 1 < 0$ and $x - 1 < 0$. Consequently

$$|2x + 1| = -2x - 1, \qquad |(x + 2)(x - 1)| = (1 - x)|x + 2|.$$

Combining the last two absolute-value terms gives

$$-3|x+2|+(1-x)|x+2| = -(x+2)|x+2|.$$

This expression has derivative 0 at -2, because its difference quotient is $-|h| \to 0$. Hence the full function has derivative -2 at -2; it is not an exceptional point.

Step 3: Check the remaining two roots At $x = -1/2$, only $|2x+1|$ has a corner, with derivative changing from -2 to $+2$. The other terms are smooth there, so the nonzero jump of 4 remains. At $x = 1$, only $|x^2+x-2|$ has a corner: the inner derivative is $2x+1 = 3$, so its derivative changes from -3 to $+3$. This nonzero jump of 6 also remains. Thus precisely two points fail differentiability.

Step 4: Conclusion The function is not differentiable at $x = -1/2$ and $x = 1$. The required number is $\boxed{2}$.

Quick Tip to Solve: When several modulus arguments vanish at the same point, add their derivative jumps: the apparent corner may cancel.

28. Let $\overrightarrow{a} = \hat{i}+2\hat{j}-\hat{k}$, $\overrightarrow{b} = \hat{i}-\hat{j}$ and $\overrightarrow{c} = \hat{i}-\hat{j}-\hat{k}$ be three given vectors. If $\overrightarrow{r}$ is a vector such that $\overrightarrow{r}\times\overrightarrow{a} = \overrightarrow{c}\times\overrightarrow{a}$ and $\overrightarrow{r}.\overrightarrow{b} = 0$, then $\overrightarrow{r}.\overrightarrow{a}$ is equal to _____. [4 marks]

Solution:

Concept:

- If a nonzero vector $\vec{a}$ satisfies $\vec{u}\times\vec{a} = \vec{0}$, then $\vec{u}$ is parallel to $\vec{a}$.
- The dot-product condition $\vec{r}\cdot\vec{b} = 0$ fixes the parallel component; use bilinearity to solve for its scalar coefficient.

Step 1: Use the cross-product equation Transpose the given cross products:

$$(\vec{r}-\vec{c})\times\vec{a} = \vec{0}.$$

Since $\vec{a} = (1,2,-1) \neq \vec{0}$, there is a real number λ for which

$$\vec{r} = \vec{c}+\lambda\vec{a}.$$

This expresses every vector satisfying the cross-product equation, including the case $\vec{r} = \vec{c}$ when $\lambda = 0$. It avoids solving three redundant cross-product component equations.

Step 2: Apply the perpendicularity condition For $\vec{b} = (1,-1,0)$ and $\vec{c} = (1,-1,-1)$, calculate

$$\vec{c}\cdot\vec{b} = 1+1+0 = 2, \qquad \vec{a}\cdot\vec{b} = 1-2+0 = -1.$$

Therefore

$$0 = \vec{r}\cdot\vec{b} = 2-\lambda \quad \Longrightarrow \quad \lambda = 2.$$

The scalar is uniquely fixed because $\vec{a}\cdot\vec{b} \neq 0$.

Step 3: Evaluate and check the required product We obtain $\vec{r} = \vec{c}+2\vec{a} = (3,3,-3)$, which indeed

has dot product $3 - 3 = 0$ with $\vec{b}$. Its difference from $\vec{c}$ is exactly $2\vec{a}$, so it also satisfies the original cross-product condition. Finally,

$$\vec{r} \cdot \vec{a} = 3(1) + 3(2) + (-3)(-1) = 3 + 6 + 3 = 12.$$

Equivalently, $\vec{c} \cdot \vec{a} = 0$ and $|\vec{a}|^2 = 6$, giving $0 + 2(6) = 12$.

Step 4: Conclusion The required scalar product is $\boxed{\vec{r} \cdot \vec{a} = 12}$.

Quick Tip to Solve: Equal cross products with the same nonzero vector mean their difference is parallel to that vector; introduce just one scalar parameter.

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29. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = −1 and x = 1. If $\lim_{x \to 0} \frac{f(x)}{x^3} = 1$, then $5.f(2)$ is equal to _____. [4 marks]

Solution:

Concept:

- The finite limit $\lim_{x \to 0} f(x)/x^3 = 1$ fixes the coefficients below degree three and the coefficient of x^3.
- At an interior extremum of a polynomial, its derivative vanishes. Use $f\prime(1) = f\prime(-1) = 0$ to determine the remaining coefficients.

Step 1: Use the limit to write the polynomial Write the monic polynomial initially with all seven coefficients. Dividing by x^3, any nonzero constant, linear or quadratic term would make the two-sided limit nonfinite. Therefore these three coefficients vanish, while the cubic coefficient is one. Thus, with real constants a, b,

$$f(x) = x^6 + ax^5 + bx^4 + x^3.$$

Only two coefficients remain unknown; the leading coefficient is already fixed as unity.

Step 2: Apply the two stationary-point conditions Differentiate and substitute the stated extrema:

$$f\prime(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2.$$

$$9 + 5a + 4b = 0, \qquad -3 + 5a - 4b = 0.$$

Adding gives $10a = -6$, so $a = -3/5$. Substitution then gives $b = -3/2$. The conditions genuinely give extrema: the second derivatives at 1 and -1 are respectively 6 and 18, both positive.

Step 3: Evaluate the required multiple Insert $x = 2$, retaining both negative coefficient terms:

$$f(2) = 64 - \frac{3}{5}(32) - \frac{3}{2}(16) + 8 = \frac{144}{5}.$$

Multiplying this value by the requested factor five removes the denominator. The limit has fixed the cubic coefficient, so no arbitrary constant remains.

Step 4: Conclusion The required numerical value is $\boxed{5f(2) = 144}$.

Quick Tip to Solve: Use the limit before differentiating: it eliminates three low-degree coefficients and fixes the cubic term immediately.

30. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, **where x, y and z are real numbers such that x + y + z $>$ 0 and xyz = 2. If** $A^2 = I_3$, **then the value of** $x^3 + y^3 + z^3$ **is _____.** [4 marks]

Solution:

Concept:

- Matrix multiplication gives the diagonal entries $x^2+y^2+z^2$ and the off-diagonal entries $xy+yz+zx$ of A^2.
- The identity $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$ connects the requested expression to these entries.

Step 1: Extract the equations implied by the matrix Because $A^2 = I_3$, comparison of a diagonal and an off-diagonal entry yields

$$x^2+y^2+z^2=1, \qquad xy+yz+zx=0.$$

Consequently $(x+y+z)^2=1$. The stated positivity selects $x+y+z=1$, rather than the negative root. These implications use the actual matrix pattern, not an assumption that the matrix is diagonal.

Step 2: Compute the formally implied answer Apply the cubic identity and the given product:

$$x^3+y^3+z^3=3xyz+(x+y+z)(1-0)=6+1=7.$$

This is the algebraic value obtained by combining the printed equations, and is the numerical entry accepted by the answer key.

Step 3: Check existence under the printed real-number condition There is nevertheless an inconsistency in the premises. For real x, y, z, the arithmetic-geometric mean inequality on their squares gives

$$x^2y^2z^2 \le \left(\frac{x^2+y^2+z^2}{3}\right)^3 = \frac{1}{27}.$$

But $xyz = 2$ would give $x^2y^2z^2 = 4$, which is impossible. Thus no real triple satisfies all the stated conditions. The formal calculation must not be mistaken for an existing real example.

Step 4: Conclusion The answer-key numerical value is $\boxed{7}$, obtained formally from the equations. Strictly, the printed real-number premises are inconsistent, so there is no admissible real matrix.

Quick Tip to Solve: Compare diagonal and off-diagonal entries first; then check whether a stated product is compatible with the resulting sum of squares.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is _____ and their speed is _____, in the ratio. [4 marks]

(A) 2 : 1 : 1 and 4 : 2 : 1

(B) 4 : 2 : 1 and 2 : 1 : 1

(C) 1 : 2 : 4 and 2 : 1 : 1

(D) 1 : 2 : 4 and 1 : 1 : 2

Correct Answer: (A) 2 : 1 : 1 and 4 : 2 : 1

Solution:

Concept:

- For nonrelativistic particles with equal momentum p, speed is $v = p/m$.
- The magnetic-force magnitude is $F = qvB\sin\theta$. For the same field and angle, equal momentum makes F proportional to q/m.

Step 1: List the masses and charges Take the proton mass as m and its charge as e. To the accuracy intended, a deuteron has mass $2m$ and charge e, while an alpha particle has mass $4m$ and charge $2e$. The alpha particle contains two protons and two neutrons; its charge is twice, not four times, the proton charge.

Step 2: Find the speed ratio Since the same momentum is specified, divide that momentum by each mass:

$$v_p : v_d : v_\alpha = \frac{p}{m} : \frac{p}{2m} : \frac{p}{4m} = 4 : 2 : 1.$$

The heavier alpha particle moves slowest. A ratio based on equal kinetic energy would instead involve square roots of masses, but equal kinetic energy is not given here.

Step 3: Combine charge and speed for the force ratio Using a common angle to the uniform field, the factors $B\sin\theta$ cancel:

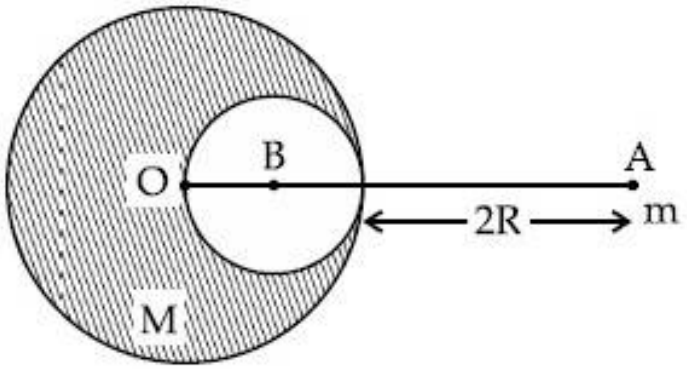
$$F_p : F_d : F_\alpha = e\frac{p}{m} : e\frac{p}{2m} : 2e\frac{p}{4m} = 2 : 1 : 1.$$

The deuteron and alpha particle have the same charge-to-mass ratio, explaining their equal forces. Options with forces increasing as $1 : 2 : 4$ ignore the inverse dependence of speed on mass; option (B) also fails the speed ratio. The listed answer presumes the usual common nonzero inclination to the field.

Step 4: Conclusion The magnetic forces are in the ratio $2 : 1 : 1$, and the speeds are in the ratio $4 : 2 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: For equal momentum, write $v \propto 1/m$ and $F \propto q/m$; this avoids confusing the force ratio with the speed ratio.

32. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is [4 marks]



(A) 36 : 25

(B) 41 : 50

(C) 50 : 41

(D) 25 : 36

Correct Answer: (C) 50 : 41

Solution:

Concept:

- Outside a uniform sphere, its gravitational attraction equals that of a point mass at its centre: $F = GMm/r^2$.
- A cavity is handled by subtracting the attraction of the removed sphere. Its mass scales as the cube of its radius for uniform density.

Step 1: Read the cavity geometry Let O be the original centre and B the centre of the removed sphere. The radius of the cavity is $R/2$, and it touches the outer surface on the side toward the particle. Hence $OB = R/2$. The particle is at $OA = 3R$, giving

$$BA = 3R - \frac{R}{2} = \frac{5R}{2}.$$

Both centres and the particle are collinear, so the two gravitational attractions have the same direction before subtraction.

Step 2: Calculate the removed contribution If the original full sphere has mass M, the cavity would contain mass

$$M_c = M\left(\frac{R/2}{R}\right)^3 = \frac{M}{8}.$$

The original attraction is $F_1 = GMm/(9R^2)$. The removed material would attract the external particle with force

$$F_c = \frac{G(M/8)m}{(5R/2)^2} = \frac{GMm}{50R^2}.$$

The particle lies outside both spheres, so the point-mass rule is valid for both terms.

Step 3: Subtract and form the ratio Superposition gives

$$F_2 = \frac{GMm}{R^2}\left(\frac{1}{9} - \frac{1}{50}\right) = \frac{41GMm}{450R^2}.$$

Thus $F_1 : F_2 = (1/9) : (41/450) = 50 : 41$. The remaining force must be smaller than the original, which immediately excludes the ratios below one. Subtracting only one-eighth of the original force would be wrong because the cavity centre is nearer the particle.

Step 4: Conclusion The required ratio is $\boxed{F_1 : F_2 = 50 : 41}$. Hence, the correct option is (C).

Quick Tip to Solve: Treat a cavity as a negative sphere, and measure the particle distance from the cavity centre before applying the inverse-square law.

33. An α particle and a proton are accelerated from rest by a potential difference of 200V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is : [4 marks]

(A) 8

(B) 2.8

(C) 7.8

(D) 3.8

Correct Answer: (B) 2.8

Solution:

Concept:

- Acceleration from rest through a potential difference V gives a particle kinetic energy $K = qV$.
- For nonrelativistic motion, $p = \sqrt{2mqV}$ and the de Broglie wavelength is $\lambda = h/p$.

Step 1: Express both wavelengths using the same potential The accelerating potential is $200\,\mathrm{V}$ for each particle, but their kinetic energies are not equal because their charges differ. A proton acquires energy eV, whereas an alpha particle acquires energy $2eV$. These energies are small compared with the rest energies, so the nonrelativistic relation between momentum and kinetic energy applies.

Step 2: Insert mass and charge ratios An alpha particle has approximately four times the proton mass and twice its charge. Therefore

$$\lambda_p = \frac{h}{\sqrt{2m_p eV}}, \qquad \lambda_\alpha = \frac{h}{\sqrt{2(4m_p)(2e)V}}.$$

On dividing, the common constants h, e, V and the factor two cancel. It is the product of mass and charge that determines the ratio, not either factor alone.

Step 3: Evaluate and compare the options The requested ratio is

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{8} = 2\sqrt{2} \simeq 2.828.$$

The nearest listed value is 2.8. The number eight is the ratio of the products mq, hence the ratio

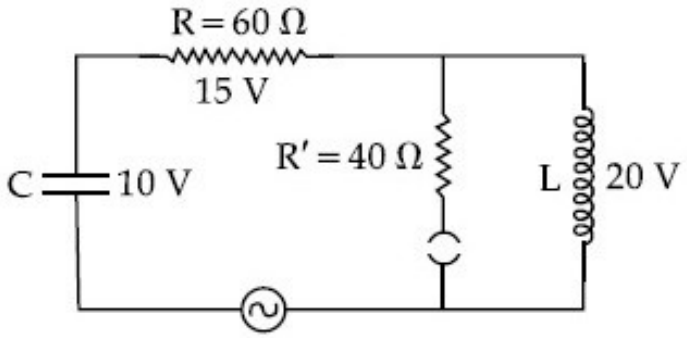
of squared momenta, rather than the wavelength ratio. The values 7.8 and 3.8 satisfy neither the square-root dependence nor the stated mass and charge ratios.

Step 4: Conclusion The wavelength ratio is $2\sqrt{2} \simeq \boxed{2.8}$. Hence, the correct option is (B).

Quick Tip to Solve: At equal accelerating voltage, use $\lambda \propto 1/\sqrt{mq}$; equal voltage does not mean equal kinetic energy for different charges.

34. The angular frequency of alternating current in a L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.

[4 marks]



(A) 0.8 H and 150 μF

(B) 0.8 H and 250 μF

(C) 1.33 H and 150 μF

(D) 1.33 H and 250 μF

Correct Answer: The displayed central source branch leaves the inductor current unspecified; only the capacitance is determined. Option (B) is the intended equal-current interpretation.

Solution:

Concept:

- The resistor voltage fixes the current in its own branch: $I_R = V_R/R$. The capacitor is in series with that resistor, so $I_C = I_R$.
- For ideal reactive components, $X_C = 1/(\omega C)$, $X_L = \omega L$, and voltage magnitude equals current magnitude times reactance. Currents in separate branches cannot be equated without additional information.

Step 1: Determine the capacitor current and capacitance The labelled drop across the $60\,\Omega$ resistor is $15\,\text{V}$, so $I_R = 15/60 = 0.25\,\text{A}$. The left-hand path contains the resistor and capacitor with no junction between them, hence their currents are equal. From the capacitor drop of $10\,\text{V}$,

$$X_C = \frac{10}{0.25} = 40\,\Omega, \qquad C = \frac{1}{100(40)} = 250\,\mu\text{F}.$$

This capacitance follows from the actual diagram independently of the right-hand branch.

Step 2: Check whether the inductance is determined The central branch contains $40\,\Omega$ and a source symbol, and joins the same top and bottom nodes as the inductor. Current can therefore enter or leave the junction through that branch. Kirchhoff's current law relates the branch-current phasors; it does not require the inductor current to equal $0.25\,\text{A}$. The source voltage and phase in the central branch are unspecified. With only the inductor drop $20\,\text{V}$, its inductance satisfies

$$L = \frac{20}{100|I_L|}.$$

Since $|I_L|$ is undetermined, this equation gives no unique inductance.

Step 3: Identify the intended option conditionally If the intended simplified interpretation takes $|I_L| = 0.25\,\text{A}$, then

$$L = \frac{20}{100(0.25)} = 0.8\,\text{H}.$$

Together with the definite capacitance, this is the pair in option (B). The options containing $150\,\mu\text{F}$ contradict the left-branch data. However, selecting between $0.8\,\text{H}$ and $1.33\,\text{H}$ is not justified by the depicted topology alone.

Step 4: Conclusion The diagram determines $\boxed{C = 250\,\mu\text{F}}$, but does not uniquely determine L. The intended answer is option (B), $0.8\,\text{H}$ and $250\,\mu\text{F}$, only under the additional equal-current interpretation.

Quick Tip to Solve: Trace every junction before assigning a common current: the resistor and capacitor share a current, but the central branch prevents the same inference for the inductor.

35. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is _____. [4 marks]

(A) 1.0 m

(B) 0.15 m

(C) 0.2 m

(D) 0.1 m

Correct Answer: (D) 0.1 m

Solution:

Concept:

- The magnetic field at axial distance x from a circular coil of radius a is $B(x) = \mu_0 N I a^2/[2(a^2 + x^2)^{3/2}]$.
- For two points on the same coil axis, all constant numerator factors cancel in their field ratio. Retain the exponent $3/2$ until after taking the ratio.

Step 1: Write the ratio at the specified positions The nearer point is at $x_1 = 0.05\,\text{m}$, the farther point

at $x_2 = 0.20\,\text{m}$. Since the field decreases with increasing axial distance, the stated ratio is

$$\frac{B_1}{B_2} = \left(\frac{a^2 + 0.20^2}{a^2 + 0.05^2}\right)^{3/2} = 8.$$

The current, number of turns and permeability are not needed separately. Reversing numerator and denominator would give a ratio smaller than one, contradicting the stated order.

Step 2: Remove the fractional power and solve Raise both sides to the power $2/3$. Because $8^{2/3} = 4$, this gives

$$a^2 + 0.0400 = 4(a^2 + 0.0025).$$

Collecting terms,

$$0.0400 - 0.0100 = 3a^2, \qquad a^2 = 0.0100\,\text{m}^2.$$

The radius is a length and must be the positive square root; the negative algebraic root is not a physical alternative.

Step 3: Verify against the original field law For $a = 0.10\,\text{m}$, the two denominator bases are $0.0125\,\text{m}^2$ and $0.0500\,\text{m}^2$. Their ratio is four, and $4^{3/2} = 8$, exactly the required field ratio. None of the other listed radii produces this ratio. In particular, a much larger radius would make the two fields nearly equal, so $1.0\,\text{m}$ cannot be appropriate.

Step 4: Conclusion The radius of the circular coil is $\boxed{0.10\,\text{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: For axial-field ratios, take the $2/3$ power first; the resulting equation is linear in the squared radius.

36. Match List - I with List - II :

	List I		List II
(a)	**h (Planck's constant)**	**(i)**	$[ML T^{-1}]$
(b)	**E (kinetic energy)**	**(ii)**	$[ML^2T^{-1}]$
(c)	**V (electric potential)**	**(iii)**	$[ML^2T^{-2}]$
(d)	**P (linear momentum)**	**(iv)**	$[ML^2I^{-1}T^{-3}]$

Choose the correct answer from the options given below : [4 marks]

(A) (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)

(B) (a) → (i), (b) → (ii), (c) → (iv), (d) → (iii)

(C) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

(D) (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)

Correct Answer: (A) (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)

Solution:

Concept:

- Dimensions express a physical quantity in terms of mass M, length L, time T and electric current I. Numerical constants do not affect dimensions.

- Use defining equations: $E = \frac{1}{2}mv^2$, $E = h\nu$, $V = W/q$, and $p = mv$. Electric charge has dimensions $[q] = [IT]$.

Step 1: Determine the mechanical dimensions For kinetic energy, velocity has dimensions $[LT^{-1}]$, hence $[E] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$. Thus (b) matches (iii). Since frequency has dimensions $[T^{-1}]$, the photon relation gives

$$[h] = \frac{[E]}{[\nu]} = [ML^2T^{-1}],$$

so (a) matches (ii). Planck's constant is energy multiplied by time, not energy alone.

Step 2: Find potential and momentum Electric potential is work per unit charge. Therefore

$$[V] = \frac{[ML^2T^{-2}]}{[IT]} = [ML^2I^{-1}T^{-3}].$$

Thus (c) matches (iv). Linear momentum is mass times velocity, so $[p] = [MLT^{-1}]$, giving (d) matched with (i). These four matches independently identify a unique row.

Step 3: Compare the option sets Option (A) contains every derived match. Option (B) gives Planck's constant the dimensions of momentum and gives energy the dimensions of action. Option (C) interchanges energy and action. Option (D) also assigns electrical-current dimensions to kinetic energy. Each violates at least one defining relation, so none can replace (A).

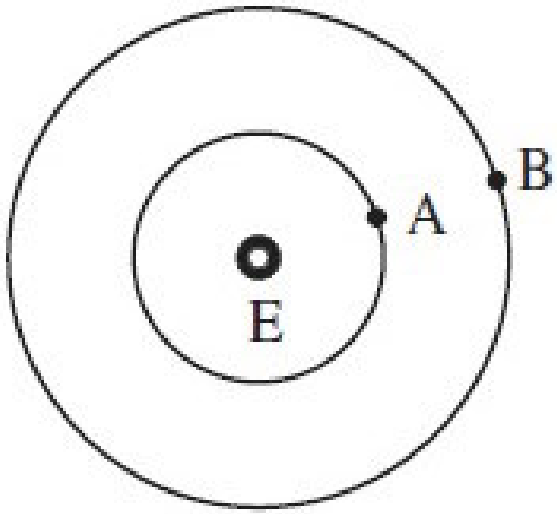
Step 4: Conclusion Planck's constant matches (ii), kinetic energy matches (iii), electric potential matches (iv), and linear momentum matches (i). Hence, the correct option is (A).

Quick Tip to Solve: Remember action as energy times time, and voltage as energy divided by current times time; the remaining two matches then follow immediately.

37. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.

If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:

[Given : radius of earth = 6400 km, mass of earth = 6×10^{24} kg] [4 marks]



(A) 1.33×10^3 s

(B) 4.24×10^2 s

(C) 3.33×10^2 s

(D) 4.24×10^3 s

Correct Answer: (A) 1.33×10^3 s

Solution:

Concept:

- For a circular orbit gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$. The orbital radius r is measured from the centre of the Earth.
- The period is $T = 2\pi\sqrt{r^3/(GM)}$. Satellite mass cancels, so the stated masses do not affect either period.

Step 1: Convert the heights to orbital radii Add the Earth's radius to each altitude before applying the period formula:

$$r_A = (6400 + 600)\,\text{km} = 7.0 \times 10^6\,\text{m},$$

$$r_B = (6400 + 1600)\,\text{km} = 8.0 \times 10^6\,\text{m}.$$

Using $G = 6.67 \times 10^{-11}\,\text{N m}^2\text{kg}^{-2}$ and $M = 6 \times 10^{24}\,\text{kg}$, we have $GM = 4.002 \times 10^{14}\,\text{m}^3\text{s}^{-2}$.

Step 2: Calculate both periods and subtract The period increases with orbital radius, so the requested difference must be positive. Substitution gives

$$T_A = 2\pi\sqrt{\frac{(7.0 \times 10^6)^3}{4.002 \times 10^{14}}} \simeq 5817\,\text{s},$$

$$T_B = 2\pi\sqrt{\frac{(8.0\times10^6)^3}{4.002\times10^{14}}} \simeq 7107\,\text{s}.$$

Thus $T_B - T_A \simeq 1290\,\text{s}$, approximately $1.3\times10^3\,\text{s}$.

Step 3: Choose the corresponding listed value The listed value $1.33\times10^3\,\text{s}$ is the closest to this result; modest numerical rounding in the listed answer does not change the selection. The two answers of order a few hundred seconds are too small, while $4.24\times10^3\,\text{s}$ is much too large. In particular, using only the altitudes in place of the centre-to-centre radii would describe different orbits.

Step 4: Conclusion The period difference is approximately $1.3\times10^3\,\text{s}$, corresponding to the listed $1.33\times10^3\,\text{s}$. Hence, the correct option is (A).

Quick Tip to Solve: For satellite periods always use Earth radius plus altitude; the satellite masses cancel before any numerical substitution.

38. Two coherent light sources having intensity in the ratio 2x produce an interference pattern. The ratio $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ will be : [4 marks]

(A) $\frac{2\sqrt{2x}}{2x+1}$

(B) $\frac{2\sqrt{2x}}{x+1}$

(C) $\frac{\sqrt{2x}}{x+1}$

(D) $\frac{\sqrt{2x}}{2x+1}$

Correct Answer: (A) $\frac{2\sqrt{2x}}{2x+1}$

Solution:

Concept:

- For coherent waves, interference extrema are $I_{max} = (\sqrt{I_1}+\sqrt{I_2})^2$ and $I_{min} = (\sqrt{I_1}-\sqrt{I_2})^2$. Amplitudes, rather than intensities, add.
- The dimensionless visibility is $\mathcal{V} = (I_{max}-I_{min})/(I_{max}+I_{min})$. The stated intensity ratio is $I_1/I_2 = 2x$, with $x \geq 0$.

Step 1: Express the two source intensities with one scale Let $I_2 = I_0$ and $I_1 = 2xI_0$, where $I_0 > 0$. The respective amplitudes are proportional to $\sqrt{I_0}$ and $\sqrt{2xI_0}$. Therefore

$$I_{max} = I_0(\sqrt{2x}+1)^2, \qquad I_{min} = I_0(\sqrt{2x}-1)^2.$$

This explicitly retains the square root required when converting an intensity ratio into an amplitude ratio.

Step 2: Subtract and add the extrema Expanding the squares, their difference contains only the interference term:

$$I_{max} - I_{min} = 4I_0\sqrt{2x}.$$

Their sum cancels the interference terms:

$$I_{\max} + I_{\min} = 2I_0(2x + 1).$$

Dividing cancels the arbitrary common intensity scale, giving

$$\mathcal{V} = \frac{2\sqrt{2x}}{2x + 1}.$$

Step 3: Check the limiting case and alternatives When $2x = 1$, the sources have equal intensities and the minima are zero, so visibility must be unity. The derived expression satisfies this test and never exceeds one for nonnegative x, since $2\sqrt{2x} \leq 2x + 1$. Option (D) is smaller by a factor of two; (B) and (C) use the wrong denominator for the stated intensity ratio.

Step 4: Conclusion $\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{2x}}{2x + 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For an intensity ratio $r : 1$, use visibility $2\sqrt{r}/(r + 1)$; do not substitute the intensity ratio directly as an amplitude ratio.

---

39. An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is : [4 marks]

(A) $\frac{u+v}{2}$

(B) $\sqrt{\frac{v^2-u^2}{2}}$

(C) $\frac{v-u}{2}$

(D) $\sqrt{\frac{v^2+u^2}{2}}$

Correct Answer: (D) $\sqrt{\frac{v^2+u^2}{2}}$

Solution:

Concept:

- With constant acceleration a, speed after displacement s obeys $w^2 = u^2 + 2as$. Squared speed is linear in displacement.
- For a rigid train of length L, the middle point passes a fixed post after the train advances $L/2$, while the last compartment passes after an advance L.

Step 1: Relate the endpoint speeds to the train length Take the instant when the engine reaches the signal post as the initial instant. Every part of the translating train then has speed u. By the time the rear reaches that post, the engine and every other point have moved forward by the full train length L, with common speed v. Hence

$$v^2 = u^2 + 2aL, \qquad aL = \frac{v^2 - u^2}{2}.$$

This equation uses the displacement between crossing events, not the separation of different particles at one instant.

Step 2: Evaluate the speed at the halfway displacement Let w be the speed when the train's middle reaches the post. The displacement from the initial configuration is $L/2$. Thus

$$w^2 = u^2 + 2a\frac{L}{2} = u^2 + aL.$$

Substituting the endpoint relation gives

$$w^2 = u^2 + \frac{v^2 - u^2}{2} = \frac{u^2 + v^2}{2}.$$

The forward speed is the positive square root.

Step 3: Distinguish spatial and temporal averages The arithmetic average $(u + v)/2$ gives the speed halfway in time for uniform acceleration, whereas this question specifies halfway along the train's displacement. Option (B) omits the initial contribution u^2, and option (C) gives only half the speed increment. As a useful check, if $u = v$, our expression returns that common speed, while (B) and (C) return zero.

Step 4: Conclusion The speed is $\sqrt{\frac{u^2 + v^2}{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: At a midpoint in displacement, average the squared endpoint speeds; at a midpoint in time, average the endpoint speeds themselves.

---

40. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : When a rod lying freely is heated, no thermal stress is developed in it.

Reason R : On heating, the length of the rod increases.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) A is true but R is false

(B) A is false but R is true

(C) Both A and B are true but R is NOT the correct explanation of A

(D) Both A and R are true and R is the correct explanation of A

Correct Answer: (C) Both A and B are true but R is NOT the correct explanation of A

Solution:

Concept:

- A freely heated rod has natural thermal strain $\epsilon_{\text{th}} = \alpha\Delta T$, where α is its coefficient of linear expansion.
- Thermal stress arises when a constraint prevents the natural thermal expansion. In one dimension, $\sigma = Y(\epsilon_{\text{actual}} - \alpha\Delta T)$, with tensile stress taken positive.

Step 1: Evaluate the assertion The words "lying freely" mean that neither end is constrained against expansion. With a uniform temperature rise, the rod is able to acquire its natural thermal strain. Its actual strain therefore equals $\alpha\Delta T$, and

$$\sigma = Y(\alpha\Delta T - \alpha\Delta T) = 0.$$

There is no opposing reaction force at the ends and hence no thermal stress. Assertion A is true; a change of length by itself does not imply elastic stress.

Step 2: Evaluate the reason For the ordinary rod with positive expansion coefficient, heating increases its length by

$$\Delta L = \alpha L \Delta T.$$

Thus Reason R is true: an ordinary rod expands on heating. This establishes the truth of R but does not yet address what determines stress.

Step 3: Test the explanatory connection The absence of stress depends specifically on the absence of a constraint: the rod attains its natural thermal strain without an opposing end force. The reason states the general fact of thermal expansion, but does not state this mechanical condition. Thermal stress is decided by whether natural expansion is prevented, rather than by heating or a length change alone. Thus the reason is true but does not supply the complete explanation of the assertion. The third option denotes the both-true, no-explanation choice (its A and B refers to A and R in context). The alternatives declaring either statement false are excluded.

Step 4: Conclusion Both statements are true, but the reason does not explain the assertion. Hence, the correct option is (C).

Quick Tip to Solve: Ask whether expansion is prevented: free expansion changes length without stress, whereas a rigid constraint converts thermal strain into stress.

41. The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72nd division on circular scale coincides with the reference line. The radius of the wire is : [4 marks]

(A) 1.80 mm

(B) 0.90 mm

(C) 0.82 mm

(D) 1.64 mm

Correct Answer: (C) 0.82 mm

Solution:

Concept:

- The least count of a screw gauge is pitch divided by the number of circular divisions. The measured wire thickness is its diameter, not its radius.
- A circular-scale zero below the reference line indicates a positive zero error. The correction is the

negative of that error, so it must be subtracted from the observed reading.

Step 1: Calculate least count and zero error The screw advances $1\,\text{mm}$ per full rotation, which contains 100 scale divisions. Therefore

$$\text{LC} = \frac{1}{100} = 0.01\,\text{mm}.$$

With the jaws closed, the zero is 8 divisions below the reference line, corresponding to a positive apparent reading of

$$e = 8(0.01) = 0.08\,\text{mm}.$$

The instrument overreads even when the true thickness is zero; this is why the correction must be negative.

Step 2: Correct the measured diameter The first linear-scale division is visible, giving the main-scale reading $1\,\text{mm}$. The circular-scale reading is $72 \times 0.01 = 0.72\,\text{mm}$. Consequently

$$d_{\text{observed}} = 1 + 0.72 = 1.72\,\text{mm},$$

$$d_{\text{true}} = d_{\text{observed}} - e = 1.72 - 0.08 = 1.64\,\text{mm}.$$

This is the separation of the jaws and therefore the diameter of the wire.

Step 3: Convert diameter into radius The required radius is half the corrected diameter:

$$r = \frac{1.64}{2} = 0.82\,\text{mm}.$$

Option (D) is the diameter rather than the radius. Adding the error would give $1.80\,\text{mm}$ and then $0.90\,\text{mm}$, explaining the other two distractors. Neither is correct because the positive instrumental offset must be removed.

Step 4: Conclusion The radius of the wire is $0.82\,\text{mm}$. Hence, the correct option is (C).

Quick Tip to Solve: Correct the screw-gauge diameter for zero error first, then divide by two; doing only one of these operations selects a distractor.

42. If the time period of a two meter long simple pendulum is 2s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is : [4 marks]

(A) 16 m/s^2

(B) $2\pi^2$ ms^{-2}

(C) π^2 ms^{-2}

(D) 9.8 ms^{-2}

Correct Answer: (B) $2\pi^2$ ms^{-2}

Solution:

Concept:

- For a simple pendulum performing small-amplitude oscillations, $T = 2\pi\sqrt{L/g}$. Here L is the effective length from the suspension point to the bob's centre.
- The acceleration due to gravity at the specified location is inferred from the measured length and period; it need not equal the usual Earth-surface approximation $9.8\,\mathrm{m\,s^{-2}}$.

Step 1: Rearrange the period relation Square the small-angle period formula to isolate the acceleration:

$$T^2 = 4\pi^2\frac{L}{g}, \qquad g = \frac{4\pi^2 L}{T^2}.$$

This also gives a dimensional check: L/T^2 has units of acceleration and the numerical coefficient $4\pi^2$ is dimensionless. The mass of the bob is absent because it cancels from the equation of motion.

Step 2: Substitute the given length and period The question specifies $L = 2\,\mathrm{m}$ and $T = 2\,\mathrm{s}$, so

$$g = \frac{4\pi^2(2)}{2^2} = 2\pi^2\,\mathrm{m\,s^{-2}}.$$

Numerically this is about $19.74\,\mathrm{m\,s^{-2}}$. The relatively large value is a consequence of the supplied data, and is not a reason to replace the period or length by a familiar terrestrial value.

Step 3: Check the alternatives against the period Substituting $g = 2\pi^2$ back into the formula gives $T = 2\pi\sqrt{2/(2\pi^2)} = 2\,\mathrm{s}$, as required. The value π^2 would instead give $2\sqrt{2}\,\mathrm{s}$; 9.8 would give about $2.84\,\mathrm{s}$. The value 16 also fails the exact period relation. Only the listed $2\pi^2$ satisfies both given measurements.

Step 4: Conclusion The acceleration due to gravity is $2\pi^2\,\mathrm{m\,s^{-2}}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the factor $4\pi^2$ when rearranging a pendulum period, and use the measured data rather than assuming the usual value of gravity.

43. A diatomic gas, having $C_p = \frac{7}{2}R$ and $C_v = \frac{5}{2}R$, is heated at constant pressure. The ratio dU : dQ : dW : [4 marks]

(A) 5 : 7 : 3

(B) 3 : 7 : 2

(C) 5 : 7 : 2

(D) 3 : 5 : 2

Correct Answer: (C) 5 : 7 : 2

Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature: $dU = nC_v\,dT$, where n is the amount of gas in moles.
- At constant pressure the supplied heat is $dQ = nC_p\,dT$, and the expansion work done by the gas is $dW = P\,dV = nR\,dT$.

Step 1: Write the energy change and heat supplied The heat capacities given in the question are molar quantities. For the same small temperature rise $dT > 0$, use the same number of moles in both expressions. Thus

$$dU = \frac{5}{2}nR\,dT, \qquad dQ = \frac{7}{2}nR\,dT.$$

The gas is heated, so its internal energy and the heat supplied are both positive. Constant pressure does not mean constant internal energy; the increase in temperature still raises the molecular kinetic energy.

Step 2: Calculate the expansion work and form the ratio From the ideal-gas equation at constant pressure, $P\,dV = nR\,dT$. Therefore

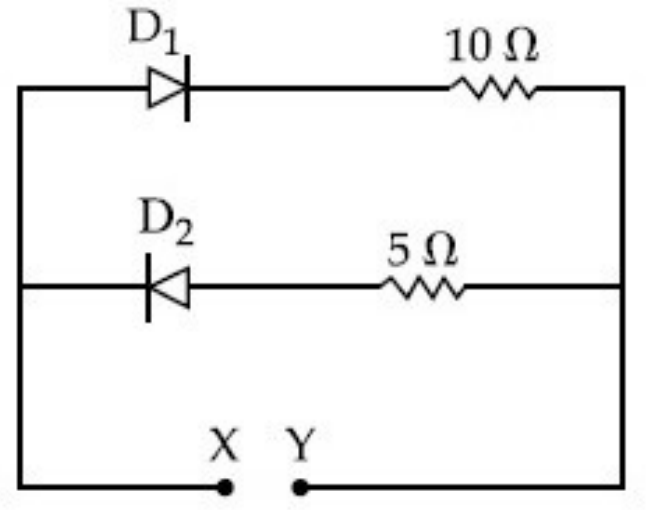
$$dW = nR\,dT, \qquad dU : dQ : dW = \frac{5}{2} : \frac{7}{2} : 1 = 5 : 7 : 2.$$

The first law supplies a useful independent check: $dQ = dU + dW$, and indeed $7 = 5 + 2$ in the reduced ratio. Option (A) fails this energy check because $5 + 3 \neq 7$. Options (B) and (D) use a different heat-capacity ratio and do not reproduce the specified diatomic values. The common factor $nR\,dT/2$ cancels, so the result does not depend on the amount of gas or the size of the temperature increase.

Step 3: Conclusion The required ratio is $\boxed{dU : dQ : dW = 5 : 7 : 2}$. Hence, the correct option is (C).

Quick Tip to Solve: For constant-pressure heating, write $dU : dQ : dW = C_v : C_p : R$ first, and check that the middle entry equals the sum of the other two.

44. A 5V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X. [4 marks]



(A) ~ 0.5 A

(B) ~ 1.5 A

(C) ~ 0.43 A

(D) ~ 0.86 A

Correct Answer: (C) ~ 0.43 A

Solution:

Concept:

- A normal forward-biased silicon diode has an approximately $0.7\,\text{V}$ drop; reverse current is neglected in the elementary circuit model.
- For a conducting series branch, Kirchhoff's voltage law gives $I = (V - V_D)/R$. Battery current is the sum of the currents in its parallel branches.

Step 1: Identify the conducting branch The positive battery terminal is at X, so the left rail is $5\,\text{V}$ above the right rail at Y. In the upper branch, the anode of D_1 is on the left and its cathode bar is on the right. Consequently D_1 is forward biased. The lower diode D_2 points the opposite way and is reverse biased. Its branch is effectively open, so the $5\,\Omega$ resistor does not provide a second conducting path.

Step 2: Apply the voltage law to the upper branch The voltage available across the upper $10\,\Omega$ resistor is the battery voltage less the silicon junction drop:

$$V_R = 5 - 0.7 = 4.3\,\text{V}.$$

The current through this resistor, and hence the current delivered by the battery, is

$$I = \frac{4.3}{10} = 0.43\,\text{A}.$$

Using an ideal zero-drop diode would give $0.5\,\text{A}$, which explains the tempting option (A), but the question specifies normal silicon diodes. The value $0.86\,\text{A}$ would instead use the lower $5\,\Omega$ branch, which is blocked for this polarity. Adding currents through both resistors would incorrectly treat a reverse-biased diode as conducting; it cannot justify option (B).

Step 3: Conclusion The battery supplies approximately $\boxed{0.43\,\text{A}}$. Hence, the correct option is (C).

Quick Tip to Solve: Mark the cathode bar and the battery polarity before reducing a diode circuit; subtract $0.7\,\text{V}$ only in the conducting silicon branch.

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45. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is : [4 marks]

(A) 13 cm

(B) 18.4 cm

(C) 16.6 cm

(D) 14.8 cm

Correct Answer: (D) 14.8 cm

Solution:

Concept:

- An air column above water acts as a pipe closed at one end. At its first resonance, the effective air-column length is $\lambda/4$, with $\lambda = v/f$.
- The open-end correction is approximately $e = 0.6r$, where r is the tube radius; effective length is $L + e$, not the scale reading L.

Step 1: Find the quarter-wavelength The water surface is a displacement node and the open mouth is approximately a displacement antinode. First resonance therefore corresponds to the fundamental mode, rather than to half a wavelength. Using the given frequency and sound speed,

$$\lambda = \frac{336}{504} = \frac{2}{3}\,\text{m}, \qquad \frac{\lambda}{4} = \frac{1}{6}\,\text{m} = 16.67\,\text{cm}.$$

This is the effective acoustic length, which extends slightly beyond the physical mouth of the tube.

Step 2: Subtract the open-end correction The diameter is $6\,\text{cm}$, so the radius is $3\,\text{cm}$. Thus

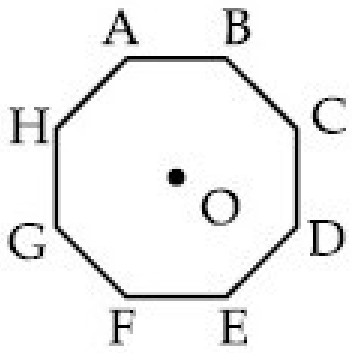
$$e = 0.6(3) = 1.8\,\text{cm}, \qquad L = 16.67 - 1.8 = 14.87\,\text{cm}.$$

The scale zero coincides with the tube top, so the water-level reading is exactly this physical air-column length measured downward. Among the given approximate values it is $14.8\,\text{cm}$. Option (C) neglects the end correction, while option (B) adds it instead of subtracting it. Option (A) is obtained by incorrectly using the diameter as the radius in $0.6r$; that doubles the correction. All distances have been converted to the same unit before subtraction.

Step 3: Conclusion The first-resonance water-level reading is approximately $\boxed{14.8\,\text{cm}}$. Hence, the correct option is (D).

Quick Tip to Solve: Calculate $v/(4f)$ for first resonance and then subtract $0.6r$; use the radius, even when the problem supplies the diameter.

46. In an octagon ABCDEFGH of equal side, what is the sum of
$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$,
if, $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ [4 marks]



(A) $-16\hat{i} - 24\hat{j} + 32\hat{k}$
(B) $16\hat{i} + 24\hat{j} - 32\hat{k}$
(C) $16\hat{i} + 24\hat{j} + 32\hat{k}$
(D) $16\hat{i} - 24\hat{j} + 32\hat{k}$
Correct Answer: (B) $16\hat{i} + 24\hat{j} - 32\hat{k}$

Solution:

Concept:

- For any vertex P, the triangle law gives $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$.
- In the regular octagon shown, opposite vertices have equal and opposite position vectors about its centre O. Hence the sum of the eight centre-to-vertex vectors is zero.

Step 1: Include the zero vector to use all eight vertices Let S denote the requested sum of vectors from A to the other seven vertices. Adding $\overrightarrow{AA} = \vec{0}$ leaves this sum unchanged. For each of the eight vertices use the same decomposition through O:

$$S = \sum_{P=A}^{H} \overrightarrow{AP} = 8\overrightarrow{AO} + \sum_{P=A}^{H} \overrightarrow{OP}.$$

The figure shows the symmetric octagon with O at its centre. The pairs A and E, B and F, C and G, and D and H lie oppositely about O, so each pair of position vectors cancels.

Step 2: Substitute the given centre vector The centre-to-vertex sum vanishes, giving

$$S = 8\overrightarrow{AO} = 8(2\hat{i} + 3\hat{j} - 4\hat{k}).$$

Therefore

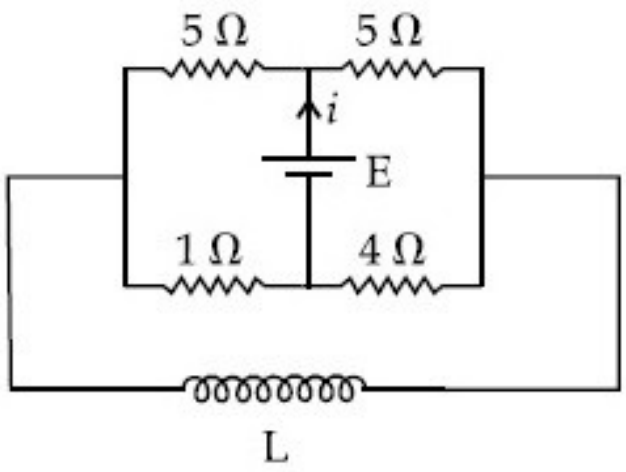
$$S = 16\hat{i} + 24\hat{j} - 32\hat{k}.$$

The coefficient is eight even though only seven nonzero vectors were explicitly listed: including the A term introduces both $\overrightarrow{AO}$ and $\overrightarrow{OA}$, which cancel in that term. Reversing AO to OA would reverse every sign and give option (A). Options (C) and (D) change selected component signs without any valid vector operation; multiplication by positive eight preserves all three original component signs.

Step 3: Conclusion The sum is $\boxed{16\hat{i} + 24\hat{j} - 32\hat{k}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a regular n-gon, the sum from one vertex to all other vertices is n times the vector from that vertex to the centre; include the zero self-vector to see why.

47. The current (i) at time t = 0 and t = ∞ respectively for the given circuit is : [4 marks]



(A) $\frac{18E}{55}, \frac{5E}{18}$
(B) $\frac{5E}{18}, \frac{18E}{55}$
(C) $\frac{5E}{18}, \frac{10E}{33}$
(D) $\frac{10E}{33}, \frac{5E}{18}$

Correct Answer: (C) $\frac{5E}{18}, \frac{10E}{33}$

Solution:

Concept:

- An ideal inductor prevents an instantaneous change of current. Immediately after connecting an initially unenergised DC circuit, its branch carries zero current and acts as an open circuit.
- After a long time under DC conditions, $dI_L/dt = 0$, so the ideal inductor has zero voltage and acts as a short circuit. The marked current i is the battery current.

Step 1: Reduce the initial circuit At $t = 0^+$, remove the open inductor branch. Between the battery terminals there are then two resistive paths: the left path has $5 + 1 = 6\,\Omega$, and the right path has $5 + 4 = 9\,\Omega$. These paths are in parallel, hence

$$R_0 = \frac{6 \times 9}{6 + 9} = \frac{18}{5}\,\Omega, \qquad i(0) = \frac{E}{R_0} = \frac{5E}{18}.$$

Both paths draw current from the battery; the answer is not the current in just one arm.

Step 2: Reduce the steady-state circuit At $t \to \infty$, the inductor connects the left and right outer nodes with an ideal wire. The two upper resistors now share both endpoints and are parallel; the two lower resistors are also parallel. These two equivalent groups are in series across the battery:

$$R_\infty = (5 \parallel 5) + (1 \parallel 4) = \frac{5}{2} + \frac{4}{5} = \frac{33}{10}\,\Omega.$$

Thus

$$i(\infty) = \frac{E}{R_\infty} = \frac{10E}{33}.$$

Option (B) gives the correct initial value but an incorrect final reduction. Option (A) reverses a wrong pair, and (D) reverses the actual chronological order. As a check, shorting an added passive branch reduces the equivalent resistance, so the final battery current is slightly larger than the initial current.

Step 3: Conclusion The currents are $\boxed{i(0) = 5E/18, \quad i(\infty) = 10E/33}$. Hence, the correct option is (C).

Quick Tip to Solve: For a DC inductor, redraw the circuit twice: open initially and short finally. Merge nodes joined by the final short before deciding which resistors are parallel.

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48. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are correct and R is the correct explanation of A

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Correct Answer: (C) A is correct but R is not correct

Solution:

Concept:

- Escape speed from the surface of a spherical planet is $v_e = \sqrt{2GM/R}$, obtained by setting the total mechanical energy at launch equal to zero.

- Equal escape speeds constrain the ratio M/R; they do not separately require equal masses or constrain the product MR.

Step 1: Evaluate Assertion A For the two planets, equal escape speeds imply

$$\frac{2GM_1}{R_1} = \frac{2GM_2}{R_2}, \qquad \frac{M_1}{R_1} = \frac{M_2}{R_2}.$$

This relation is possible with unequal masses. For example, if planet B has twice the mass and twice the radius of planet A, the mass-to-radius ratios, and therefore the escape speeds, are equal. Assertion A is consequently a physically possible and correct statement; equal escape speeds do not force the planets to have the same mass.

Step 2: Evaluate Reason R The reason instead claims $M_1R_1 = M_2R_2$. This is not the condition derived from the escape-speed formula. In the explicit example $M_2 = 2M_1$ and $R_2 = 2R_1$, the product is $M_2R_2 = 4M_1R_1$, even though both escape speeds are identical. Hence the claimed necessity in R is false.

Step 3: Select the assertion-reason combination A is correct but R is incorrect, so R cannot explain A. Options (A) and (B) both require R to be true and are eliminated. Option (D) reverses the truth values and is eliminated as well. The underlying explanation of the assertion is the dependence on M/R, not on MR. An increase in mass can be offset by the same proportional increase in radius in the escape-speed formula.

Step 4: Conclusion Assertion A is correct and Reason R is not correct. Hence, the correct option is (C).

Quick Tip to Solve: Write the escape-speed formula before judging a verbal relation: equal v_e fixes M/R, whereas surface gravity fixes M/R^2.

49. Two radioactive substances X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the half life of Y. After three half lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to : [4 marks]

(A) $\frac{1}{8}$
(B) $\frac{3}{1}$
(C) $\frac{1}{3}$
(D) $\frac{8}{1}$

Correct Answer: (D) $\frac{8}{1}$

Solution:

Concept:

- After a time t, the undecayed population is $N(t) = N(0)\,2^{-t/T}$, where T is the half-life of that substance.
- For equal elapsed time, the substance with the shorter half-life undergoes more halvings. Compare the remaining fractions before equating the populations.

Step 1: Count the half-lives elapsed for each substance Write the half-life of Y as T. The half-life of X is then $T/2$, since it is stated to be half that of Y. The observation is made after $t = 3T$. In this same time Y experiences three half-lives, but X experiences

$$\frac{t}{T_X} = \frac{3T}{T/2} = 6$$

half-lives. It is essential to use the same elapsed physical time for both samples; using three half-lives for both would not describe the experiment.

Step 2: Equate the remaining numbers of nuclei The surviving populations at the observation time are

$$N_X = \frac{N_1}{2^6} = \frac{N_1}{64}, \qquad N_Y = \frac{N_2}{2^3} = \frac{N_2}{8}.$$

The problem states that these numbers are equal, so

$$\frac{N_1}{64} = \frac{N_2}{8} \quad \Longrightarrow \quad \frac{N_1}{N_2} = \frac{64}{8} = 8.$$

X decays faster and therefore must have started with more nuclei to finish with the same number. This immediately rules out the ratios below one in (A) and (C). Option (B) incorrectly relates the initial ratio to the number three of Y half-lives instead of using exponential decay. A substitution check with $N_1 = 8N_2$ gives $8N_2/64 = N_2/8$, confirming the equality.

Step 3: Conclusion The initial population ratio is $\boxed{N_1/N_2 = 8/1}$. Hence, the correct option is (D).

Quick Tip to Solve: When half-lives differ, first convert the common elapsed time into each sample's own number of half-lives; the difference here is three extra halvings for X.

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50. Given below are two statements :

Statement I : A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II : The side band frequencies are 1002 kHz and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- In ordinary amplitude modulation, a sinusoidal message of frequency f_m produces sidebands at $f_c + f_m$ and $f_c - f_m$, where f_c is the carrier frequency.

- The transmission bandwidth is the separation of the two sidebands: $B = (f_c + f_m) - (f_c - f_m) = 2f_m$. The carrier frequency itself does not set this bandwidth.

Step 1: Put the frequencies in common units The carrier frequency is $f_c = 1\,\text{MHz} = 1000\,\text{kHz}$, and the speech-signal frequency is $f_m = 2\,\text{kHz}$. Frequency addition must use the same units; adding 2 directly to 1 MHz would be incorrect.

Step 2: Check Statement I using the bandwidth The modulated spectrum extends by the message frequency on each side of the carrier. Hence

$$B = 2f_m = 2(2\,\text{kHz}) = 4\,\text{kHz}.$$

Statement I is therefore true. A bandwidth of 2 kHz would account for only one sideband, whereas the usual AM signal contains both.

Step 3: Check Statement II and select the option The upper and lower sideband frequencies are

$$f_{\text{USB}} = 1000 + 2 = 1002\,\text{kHz}, \qquad f_{\text{LSB}} = 1000 - 2 = 998\,\text{kHz}.$$

Thus Statement II is also true. Their separation is $1002 - 998 = 4\,\text{kHz}$, independently confirming the bandwidth. Options (A) and (C) incorrectly reject one true statement, while (B) incorrectly rejects both.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: Convert MHz to kHz first, then place the two sidebands equally far from the carrier; their separation gives the AM bandwidth.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A small bob tied at one end of a thin string of length 1 m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is _____ m/s. (Take g = 10 m/s^2) [4 marks]

Solution:

Concept:

- For a bob moving in a vertical circle, the string tension and the radial component of gravity together provide the centripetal force mv^2/r.
- Conservation of mechanical energy between the highest and lowest points gives $v_b^2 - v_t^2 = 4gr$, since their height difference is $2r$.

Step 1: Write the radial equations at the two extremes Let T_b, T_t denote bottom and top tensions, and v_b, v_t the corresponding speeds. Taking the inward radial direction at each position,

$$T_b - mg = \frac{mv_b^2}{r}, \qquad T_t + mg = \frac{mv_t^2}{r}.$$

The tension is maximum at the bottom and minimum at the top: both the speed and the inward contribution required from the string vary in this sense.

Step 2: Use energy to find the tension difference The bob loses potential energy $2mgr$ on descending from top to bottom. Thus

$$\tfrac{1}{2}m(v_b^2 - v_t^2) = 2mgr.$$

Subtracting the radial equations now gives

$$T_b - T_t = \frac{m(v_b^2 - v_t^2)}{r} + 2mg = 6mg.$$

This relation includes both the speed change and the change in gravity's radial direction.

Step 3: Apply the specified ratio Since $T_b : T_t = 5 : 1$, put $T_b = 5T_t$. Then $4T_t = 6mg$, so $T_t = \tfrac{3}{2}mg$. At the top,

$$\frac{mv_t^2}{r} = T_t + mg = \tfrac{5}{2}mg, \qquad v_t^2 = \tfrac{5}{2}gr = \tfrac{5}{2}(10)(1) = 25.$$

The positive square root gives the speed; the mass cancels out.

Step 4: Conclusion The velocity magnitude at the highest point is $v_t = 5\,\mathrm{m\,s^{-1}}$. The required numerical entry is 5.

Quick Tip to Solve: For a complete vertical circle without energy loss, first use $T_b - T_t = 6mg$; combine this with the given tension ratio before finding the top speed.

52. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is _____. [4 marks]

Solution:

Concept:

- For a monatomic ideal gas, internal energy is $U = \tfrac{3}{2}nRT$, where n is the amount of gas and R is the gas constant.
- When the insulated container stops and the gas settles, its initial bulk kinetic energy becomes internal energy: $\tfrac{1}{2}nMv^2 = \tfrac{3}{2}nR\Delta T$, with molar mass M expressed in $\mathrm{kg\,mol^{-1}}$.

Step 1: Interpret the stated atomic mass in SI units The mass $4.0\,\mathrm{u}$ is the mass of one monatomic gas particle. It corresponds to the molar mass

$$M = 4.0\,\mathrm{g\,mol^{-1}} = 4.0 \times 10^{-3}\,\mathrm{kg\,mol^{-1}}.$$

Atomic mass units cannot be inserted as kilograms in an SI energy calculation. This conversion is essential because the given speed is in metres per second.

Step 2: Equate the lost bulk kinetic energy to internal energy gain For n moles initially travelling at

$v = 30\,\mathrm{m\,s^{-1}}$, thermalisation of the bulk motion in the insulated container gives

$$\frac{1}{2}nMv^2 = \frac{3}{2}nR\Delta T, \qquad \Delta T = \frac{Mv^2}{3R}.$$

The amount n cancels, so neither the gas quantity nor the container volume is required.

Step 3: Evaluate the numerator and compare the stated key Substituting the SI molar mass yields

$$\Delta T = \frac{(4.0 \times 10^{-3})(30)^2}{3R} = \frac{3.6}{3R}\,\mathrm{K}.$$

Thus comparison with $x/(3R)$ gives $x = 3.6$. The listed key value 3600 results from using 4 instead of 0.004 as the SI molar mass, introducing a factor of 1000. That value would correspond to $M = 4\,\mathrm{kg\,mol^{-1}}$, inconsistent with the printed $4.0\,\mathrm{u}$.

Step 4: Conclusion For the question as printed, $\Delta T = 3.6/(3R)\,\mathrm{K}$, so $x = 3.6$. The listed key is 3600, but it does not include the required gram-to-kilogram conversion.

Quick Tip to Solve: When a molecular mass is quoted in u, use the same numerical value in grams per mole, then convert to kilograms per mole before calculating kinetic energy.

53. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is _____ cm. [4 marks]

Solution:

Concept:

- For a thin lens in air, $1/f = 1/v - 1/u$ and the signed linear magnification is $m = v/u$. Distances follow the Cartesian sign convention.
- For an object at positive distance s to the left of a convex lens, $u = -s$ and $m = f/(f - s)$. Equal image sizes mean equal magnitudes $|m|$, not necessarily equal signs.

Step 1: Express the two magnifications For the object at 20 cm,

$$m_{20} = \frac{f}{f - 20}.$$

For the object at 10 cm,

$$m_{10} = \frac{f}{f - 10}.$$

The object sizes are the same, so equal image sizes require $|m_{20}| = |m_{10}|$. An inverted real image and an erect virtual image can satisfy this even though their signed magnifications differ.

Step 2: Solve the equal-size condition For a convex lens $f > 0$, cancelling f gives

$$|f - 20| = |f - 10|.$$

Squaring and simplifying,

$$(f-20)^2=(f-10)^2, \qquad -40f+400=-20f+100,$$

so $20f=300$ and $f=15\,\text{cm}$. Geometrically, the focal length lies midway between the two object distances.

Step 3: Check both image positions and sizes With $f=15\,\text{cm}$, the first object is beyond the focus and the second is inside it. The magnifications are

$$m_{20}=15/(15-20)=-3, \qquad m_{10}=15/(15-10)=+3.$$

Thus one image is real and inverted, the other virtual and erect, but both are three times the object size. Their image distances are $+60\,\text{cm}$ and $-30\,\text{cm}$, respectively, consistent with the lens equation.

Step 4: Conclusion The convex lens has focal length $15\,\text{cm}$. The required numerical entry is 15.

Quick Tip to Solve: Equal-sized real and virtual images from the same convex lens occur at object distances equally spaced on opposite sides of the focal point.

54. A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by V = 3t volt. (where t is in second). If the voltage is applied when t = 0, then the energy stored in the coil after 4 s is _____J. [4 marks]

Solution:

Concept:

- For an ideal inductor of constant inductance L, the applied voltage satisfies $V=L\,di/dt$. A time-dependent voltage must be integrated to obtain the current.
- The magnetic energy stored in the coil is $U=\frac{1}{2}Li^2$. With negligible resistance, the electrical work supplied is stored magnetically.

Step 1: Write the current-rate equation The coil is initially unenergised when the source is applied at $t=0$, so $i(0)=0$. Using $L=2\,\text{H}$ and $V(t)=3t\,\text{V}$,

$$2\frac{di}{dt}=3t, \qquad \frac{di}{dt}=\frac{3}{2}t.$$

There is no resistive voltage drop to subtract. The increasing voltage produces an increasing rate of change of current.

Step 2: Integrate the voltage over the stated interval Integrate from the switching instant to time t:

$$i(t)-i(0)=\int_0^t \frac{3}{2}t\,dt=\frac{3}{4}t^2.$$

Here the integration variable represents time; at $t=4\,\text{s}$,

$$i(4)=\frac{3}{4}(4)^2=12\,\text{A}.$$

Using the final voltage as though it acted constantly throughout four seconds would overestimate the current.

Step 3: Calculate and verify the stored energy The final magnetic energy is

$$U = \frac{1}{2}(2)(12)^2 = 144\,\text{J}.$$

As a check, the instantaneous input power is $P = Vi = (3t)(3t^2/4) = 9t^3/4$. Therefore the work supplied from zero to four seconds is

$$\int_0^4 P\,dt = \frac{9}{16}(4)^4 = 144\,\text{J},$$

which equals the stored energy because the resistance is negligible.

Step 4: Conclusion The energy stored in the coil after $4\,\text{s}$ is $144\,\text{J}$. The numerical entry is 144.

Quick Tip to Solve: For a ramp voltage across an ideal inductor, integrate V/L first; substitute the resulting current into $Li^2/2$ only afterwards.

55. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as

$$U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$$

where, α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = _____. [4 marks]

Solution:

Concept:

- The radial force associated with a potential energy $U(r)$ is $F(r) = -dU/dr$. An equilibrium separation satisfies $dU/dr = 0$.
- An equilibrium is stable when the potential has a local minimum, so $d^2U/dr^2 > 0$. An additive constant in potential energy changes neither force nor equilibrium separation.

Step 1: Differentiate the given potential For

$$U(r) = \alpha r^{-10} - \beta r^{-5} - 3,$$

differentiation gives

$$\frac{dU}{dr} = -10\alpha r^{-11} + 5\beta r^{-6}.$$

The constant term -3 has zero derivative and therefore cannot affect the equilibrium. Both α and β are positive, and the physical distance r is positive.

Step 2: Find the finite equilibrium distance Setting the derivative equal to zero and multiplying by

r^{11} gives

$$-10\alpha + 5\beta r^5 = 0, \qquad r^5 = \frac{2\alpha}{\beta}.$$

Hence the positive finite equilibrium separation is

$$r_0 = \left(\frac{2\alpha}{\beta}\right)^{1/5}.$$

The fifth root is required because the exponents of the repulsive and attractive terms differ by five.

Step 3: Check stability and identify the requested exponent The second derivative is

$$U''(r) = 110\alpha r^{-12} - 30\beta r^{-7}.$$

At $r = r_0$, use $\beta r_0^5 = 2\alpha$ to obtain

$$U''(r_0) = \frac{110\alpha - 30\beta r_0^5}{r_0^{12}} = \frac{50\alpha}{r_0^{12}} > 0.$$

Thus the stationary point is a stable equilibrium. In the reduced exponent $a/b = 1/5$, the integers are $a = 1$ and $b = 5$.

Step 4: Conclusion The equilibrium separation is $(2\alpha/\beta)^{1/5}$. Therefore the required numerator is $a = 1$.

Quick Tip to Solve: Ignore additive constants when finding equilibrium; differentiate first, then compare the final power with the reduced fraction requested in the question.

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56. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to y − z plane) to that of the surface of area 0.3 m^2 (parallel to x − z plane) is a : b, where a = _____ [Here $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively.] [4 marks]

Solution:

Concept:

- The electric flux of a uniform field through a plane surface is $\Phi = \mathbf{E} \cdot \mathbf{A} = E_n A$, where E_n is the field component normal to the surface.
- A surface parallel to the yz-plane has its normal along the x-axis, whereas a surface parallel to the xz-plane has its normal along the y-axis. Tangential field components contribute no flux.

Step 1: Identify the field components that cross the surfaces The field is

$$\mathbf{E} = \frac{3}{5}E_0\hat{\mathbf{i}} + \frac{4}{5}E_0\hat{\mathbf{j}}.$$

For the first surface, parallel to the yz-plane, only $E_x = 3E_0/5$ crosses it. For the second surface, parallel to the xz-plane, only $E_y = 4E_0/5$ crosses it. The field magnitude E_0 cannot simply be

multiplied by each area because neither normal is along the whole field.

Step 2: Evaluate the two flux magnitudes Choose the normals along the positive coordinate axes, or equivalently compare the magnitudes of the fluxes. With areas $A_1 = 0.2\,\mathrm{m}^2$ and $A_2 = 0.3\,\mathrm{m}^2$,

$$\Phi_1 = \frac{3}{5}E_0(0.2) = 0.12E_0, \qquad \Phi_2 = \frac{4}{5}E_0(0.3) = 0.24E_0.$$

When E_0 is in $\mathrm{N\,C^{-1}}$, these fluxes are in $\mathrm{N\,m^2\,C^{-1}}$.

Step 3: Reduce the ratio to the required integers The common factor E_0 cancels in the ratio:

$$\frac{\Phi_1}{\Phi_2} = \frac{(3/5)(0.2)}{(4/5)(0.3)} = \frac{0.12}{0.24} = \frac{1}{2}.$$

Thus the reduced ratio is $a : b = 1 : 2$. Comparing areas alone would give $2 : 3$, while comparing field components alone would give $3 : 4$; both the component and its corresponding area are necessary.

Step 4: Conclusion The flux ratio is $1 : 2$, so the required value is $a = 1$.

Quick Tip to Solve: Write the normal direction beside each plane first: $yz \to x$ and $xz \to y$. Then multiply that field component by its own surface area.

57. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be _____ nR, where n denotes number of moles of a gas. [4 marks]

Solution:

Concept:

- For a quasistatic change, the work done by the gas is $W = \int_{V_i}^{V_f} P\,dV$. The pressure must be integrated along the specified process; it is not constant here.
- For an ideal gas, $PV = nRT$. Combining this equation with the process law $P = kV^3$ connects each endpoint volume with its temperature.

Step 1: Express temperature in terms of volume Write the initial and final temperatures on the absolute scale: $T_i = 373\,\mathrm{K}$ and $T_f = 573\,\mathrm{K}$. The temperature difference is therefore $200\,\mathrm{K}$; using Celsius for this difference gives the same number, but the gas law itself uses kelvin. Combining the two relations gives

$$kV^4 = nRT.$$

Thus $kV_i^4 = nRT_i$ and $kV_f^4 = nRT_f$. Since the final temperature is larger and k is positive, the final volume is larger: the gas expands and its work is positive.

Step 2: Integrate the pressure and use the endpoint relation The volume dependence contributes a factor of one fourth upon integration:

$$W = \int_{V_i}^{V_f} kV^3\,dV = \frac{k}{4}(V_f^4 - V_i^4).$$

Now replace each endpoint product by its ideal-gas value:

$$W = \frac{nR}{4}(T_f - T_i) = \frac{nR}{4}(573 - 373) = 50nR.$$

It would be incorrect to use the isobaric formula $nR\Delta T$, because the given process has changing pressure. No heat capacity is needed to calculate this boundary work.

Step 3: Conclusion The work done by the gas is $W = 50nR\,\text{J}$ when R is expressed in SI units. The number to enter in the blank is $\boxed{50}$.

Quick Tip to Solve: For a process $P = kV^m$, combine $PV = nRT$ with the work integral to get $W = nR\Delta T/(m+1)$, provided $m \neq -1$.

58. 512 identical drops of mercury are charged to a potential of 2V each. The drops are joined to form a single drop. The potential of this drop is _____ V. [4 marks]

Solution:

Concept:

- The potential at the surface of an isolated spherical conducting drop is $V = k_e q/r$, with potential zero at infinity and $k_e = 1/(4\pi\varepsilon_0)$.
- When identical drops combine, their total charge and total liquid volume are conserved. Radius therefore grows as the cube root of the number of drops, whereas charge grows in direct proportion to it.

Step 1: Find the radius and charge of the combined drop Let each small drop have radius r and charge q. Its given potential is $V_0 = k_e q/r = 2\,\text{V}$. If R denotes the radius of the large drop, volume conservation gives

$$\frac{4\pi}{3}R^3 = 512\frac{4\pi}{3}r^3, \qquad R = 512^{1/3}r = 8r.$$

There is no loss of charge in the stated coalescence, so the charge of the combined drop is $Q = 512q$. Potentials of separate drops are not simply added; the size of the final conductor must also be included.

Step 2: Calculate the final surface potential Apply the spherical-conductor potential formula to the final drop:

$$V_f = \frac{k_e Q}{R} = \frac{k_e(512q)}{8r} = 64\frac{k_e q}{r} = 64V_0.$$

With the given initial potential,

$$V_f = 64 \times 2 = 128\,\text{V}.$$

The radius increase reduces the enhancement from a factor of 512 to a factor of 64. As a consistency check, the final capacitance is eight times that of one drop, since a sphere has capacitance proportional to its radius; dividing 512 times the charge by eight times the capacitance again gives 64 times the original potential.

Step 3: Conclusion The potential of the combined drop is $\boxed{128\,\text{V}}$. The required numerical entry is 128.

Quick Tip to Solve: For N identical charged drops, first use $R = N^{1/3}r$; then $V_f = N^{2/3}V_0$, rather than NV_0.

59. A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is _____ $\times 10^{-8}$ H. [4 marks]

Solution:

Concept:

- An electromagnetic wave in air has frequency $f = c/\lambda$, where $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. The receiver must resonate at the transmitted frequency.
- The natural frequency of an ideal LC circuit is $f_0 = 1/(2\pi\sqrt{LC})$. Resonance requires $f_0 = f$, so $L = 1/(4\pi^2 f^2 C)$.

Step 1: Convert the wave data to frequency The wavelength is $960\,\mathrm{m}$, and the capacitance in SI units is $C = 2.56 \times 10^{-6}\,\mathrm{F}$. Therefore

$$f = \frac{3.0 \times 10^8}{960} = 3.125 \times 10^5\,\mathrm{Hz}.$$

The angular frequency is $\omega = 2\pi f$, not just f. The factor 2π must be retained when using $\omega^2 LC = 1$; otherwise the inductance would be too large by $4\pi^2$.

Step 2: Substitute in the resonance condition One convenient form avoids squaring the frequency separately:

$$L = \frac{\lambda^2}{4\pi^2 c^2 C} = \frac{960^2}{4\pi^2 (3.0 \times 10^8)^2 (2.56 \times 10^{-6})}.$$

Here $960^2 = 921600$, and $c^2 C = 2.304 \times 10^{11}$ in SI units. Hence

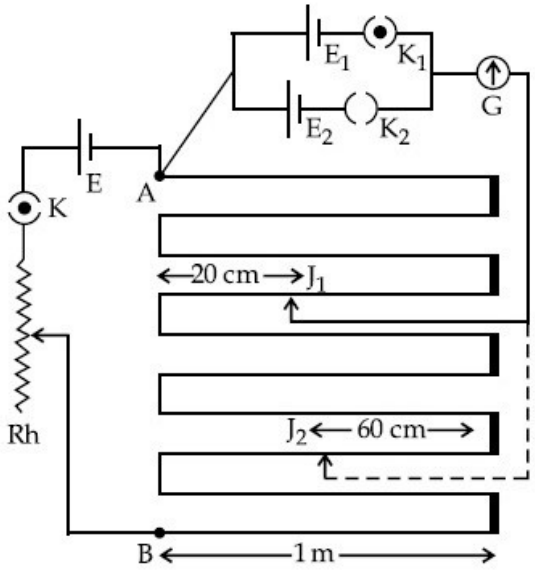
$$L = \frac{10^{-6}}{\pi^2}\,\mathrm{H} \simeq 1.013 \times 10^{-7}\,\mathrm{H} = 10.13 \times 10^{-8}\,\mathrm{H}.$$

Thus the coefficient is about 10.13, which gives 10 to the nearest integer. Using the usual estimate $\pi^2 \simeq 10$ directly gives 10×10^{-8} H.

Step 3: Conclusion The required inductance is approximately 1.01×10^{-7} H. In the specified form, the numerical entry is $\boxed{10}$.

Quick Tip to Solve: Compute $f = c/\lambda$ first, and remember that the LC formula contains angular frequency $2\pi f$; finally convert the answer to the requested power of ten.

60. In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = _____. [4 marks]



Solution:

Concept:

- For a uniform potentiometer wire carrying a steady current, the potential gradient g is constant along its actual conducting length, even when the wire is folded.
- At a null point no current flows through the test cell and galvanometer, so the cell emf equals the potential drop along the balance length: $E_i = g\ell_i$. The common gradient cancels in the ratio.

Step 1: Trace the wire to the first null point Each horizontal stretch in the figure represents $1\,\mathrm{m}$ of the wire. The wire starts at A and alternates rightward and leftward through successive stretches. Point J_1 lies on the fourth stretch, $20\,\mathrm{cm}$ from its left end. The fourth stretch is traversed from right to left, so the distance traversed on it before reaching J_1 is $1.00 - 0.20 = 0.80\,\mathrm{m}$. Including the three preceding complete stretches gives

$$\ell_1 = 3.00 + 0.80 = 3.80\,\mathrm{m}.$$

The straight distance from A to the jockey is not the balance length; we must follow the folded conducting wire.

Step 2: Trace the second balance length and reduce the ratio Point J_2 lies on the eighth stretch, $60\,\mathrm{cm}$ from the right end. This stretch is also traversed right to left. Seven complete stretches precede it, giving

$$\ell_2 = 7.00 + 0.60 = 7.60\,\mathrm{m}.$$

The driver circuit and its current are unchanged when the test cell is selected with the two keys, hence

the same gradient applies:

$$\frac{E_1}{E_2} = \frac{g\ell_1}{g\ell_2} = \frac{3.80}{7.60} = \frac{1}{2}.$$

The fraction must be reduced before reading off its numerator; using 38 as the numerator would not be the intended integer entry.

Step 3: Conclusion In lowest terms, $E_1/E_2 = a/b = 1/2$. Therefore $a = \boxed{1}$ (and $b = 2$).

Quick Tip to Solve: For a folded potentiometer, count complete wire stretches first, then measure the partial stretch in the direction of current from A.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is : [Assume : No cyano complex is formed; K_{sp}(AgCN) = 2.2 × 10^{-16} and K_a(HCN) = 6.2 × 10^{-10}] [4 marks]

(A) 1.9×10^{-5}

(B) 1.6×10^{-6}

(C) 2.2×10^{-16}

(D) 0.625×10^{-6}

Correct Answer: (A) 1.9×10^{-5}

Solution:

Concept:

- For $\mathrm{AgCN(s)} \rightleftharpoons \mathrm{Ag^+ + CN^-}$, the solubility product is $K_{sp} = [\mathrm{Ag^+}][\mathrm{CN^-}]$. The free cyanide concentration is smaller than total dissolved cyanide because it is protonated.
- For $\mathrm{HCN} \rightleftharpoons \mathrm{H^+ + CN^-}$, $K_a = [\mathrm{H^+}][\mathrm{CN^-}]/[\mathrm{HCN}]$. A buffer maintains the specified hydrogen-ion concentration while the salt dissolves.

Step 1: Write the cyanide balance in terms of solubility Let the molar solubility be s. With no cyano complex, $[\mathrm{Ag^+}] = s$, and conservation of dissolved cyanide gives

$$s = [\mathrm{CN^-}] + [\mathrm{HCN}].$$

At pH 3, $[\mathrm{H^+}] = 10^{-3}\,\mathrm{mol\,L^{-1}}$. From the acid equilibrium,

$$[\mathrm{HCN}] = \frac{10^{-3}}{6.2 \times 10^{-10}}[\mathrm{CN^-}], \qquad [\mathrm{CN^-}] = \frac{s}{1 + 10^{-3}/(6.2 \times 10^{-10})}.$$

Thus one must not replace free cyanide by s; most of the dissolved cyanide is HCN in this acidic solution.

Step 2: Calculate the solubility and compare the options Substitute the free cyanide concentration in the solubility product:

$$s^2 = K_{sp}\left(1 + \frac{[\mathrm{H^+}]}{K_a}\right) = (2.2 \times 10^{-16})\left(1 + \frac{10^{-3}}{6.2 \times 10^{-10}}\right).$$

The ratio in parentheses is much larger than one, yielding

$$s \simeq \sqrt{3.548 \times 10^{-10}} = 1.884 \times 10^{-5}\,\text{mol}\,\text{L}^{-1}.$$

This is approximately 1.9×10^{-5}. The smaller alternatives ignore or miscalculate the large enhancement caused by protonation. In particular, K_{sp} itself is a product of concentrations and cannot be identified with the salt's molar solubility.

Step 3: Conclusion The solubility is $x \simeq \boxed{1.9 \times 10^{-5}\,\text{mol}\,\text{L}^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a salt of a weak acid in an acidic buffer, combine the solubility product with the acid equilibrium and use total anion balance before solving for solubility.

62. Which one of the following reactions will not form acetaldehyde? [4 marks]

(A) $CH_3CH_2OH \xrightarrow[573K]{Cu}$

(B) $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$

(C) $CH_2 = CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$

(D) $CH_3CN \xrightarrow[ii)\ H_2O]{i)\ DIBAL-H}$

Correct Answer: (B) $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$

Solution:

Concept:

- Acetaldehyde is ethanal, CH_3CHO. A primary alcohol can give an aldehyde under controlled oxidation or catalytic dehydrogenation, but a strong aqueous acidic oxidant normally takes it further to a carboxylic acid.
- Wacker oxidation converts ethene into acetaldehyde, while controlled reduction of a nitrile with DIBAL-H followed by hydrolysis gives the corresponding aldehyde.

Step 1: Identify the reaction that overoxidises ethanol Option (B) treats ethanol with CrO_3 in H_2SO_4, a strong acidic chromium(VI) oxidising medium. Under these conditions a primary alcohol does not stop at the aldehyde as the final product; the aldehyde is oxidised further to ethanoic acid:

$$\text{CH}_3\text{CH}_2\text{OH} + 2\,[\text{O}] \longrightarrow \text{CH}_3\text{COOH} + \text{H}_2\text{O}.$$

An aldehyde may occur as an intermediate, but the question concerns the product obtained from the stated preparation. This makes (B) the reaction that does not furnish acetaldehyde as its final product.

Step 2: Check why each other reaction produces acetaldehyde In (A), ethanol vapour over copper at $573\,\text{K}$ undergoes dehydrogenation:

$$\text{CH}_3\text{CH}_2\text{OH} \xrightarrow[573\ \text{K}]{\text{Cu}} \text{CH}_3\text{CHO} + \text{H}_2.$$

In (C), the Pd(II)/Cu(II) system in aqueous medium catalyses Wacker oxidation of ethene; its net

reaction is

$$2\,CH_2{=}CH_2 + O_2 \longrightarrow 2\,CH_3CHO.$$

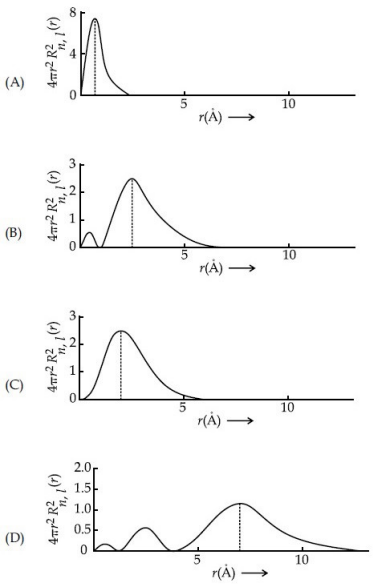
In (D), DIBAL-H carries out controlled partial reduction of CH_3CN; subsequent hydrolysis of the imine-derived intermediate gives CH_3CHO. The carbon atom of the nitrile becomes the aldehyde carbon, so the two-carbon skeleton is retained. Thus (A), (C) and (D) are valid routes to acetaldehyde, unlike the strongly oxidising conditions of (B).

Step 3: Conclusion Ethanol with $CrO_3{-}H_2SO_4$ gives ethanoic acid rather than isolable acetaldehyde as the final product. Hence, the correct option is (B).

Quick Tip to Solve: When comparing aldehyde preparations, distinguish controlled oxidation or dehydrogenation from strong aqueous oxidation, which carries primary alcohols to acids.

63. The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below :

The correct plot for 3s orbital is : [4 marks]



(A) (B)

(B) (A)

(C) (C)

(D) (D)

Correct Answer: (D) (D)

Solution:

Concept:

- The radial probability distribution of a hydrogen orbital is proportional to $r^2|R_{n\ell}(r)|^2$. It is nonnegative, vanishes at the nucleus, and tends to zero far from the nucleus.
- The number of radial nodes is $n - \ell - 1$. A 3s orbital has $n = 3$, $\ell = 0$, and therefore two radial

nodes at finite positive values of r.

Step 1: Determine the shape required for a 3s distribution Substitution in the radial-node formula gives

$$N_{\text{radial}} = 3 - 0 - 1 = 2.$$

Each radial node is a positive radius at which the radial wavefunction, and hence the radial probability distribution, becomes zero. The zero at $r = 0$ caused by the factor r^2, and the limiting zero at infinity, are not counted as radial nodes. Two internal zeros divide the distribution into three nonzero lobes, so the required plot has three peaks separated by two touches of the horizontal axis.

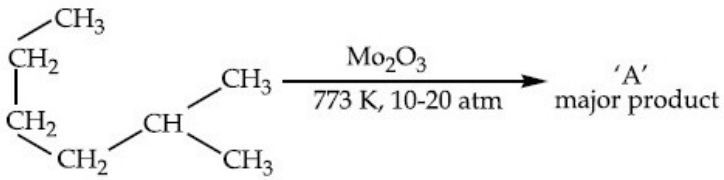
Step 2: Match the labelled plot and then the answer choice Among the four displayed plots, graph (D) shows three radial probability peaks and two internal zeroes. Its outermost peak lies at the largest radius and is the largest peak in the given plot, consistent with the extended 3s orbital. Graphs (A) and (C) have a single lobe and no internal radial node, while graph (B) has two lobes and one internal node. They therefore cannot describe 3s, regardless of their peak heights.

The displayed graph labels and the answer-option letters are separate: option (A) names plot (B), option (B) names plot (A), option (C) names plot (C), and option (D) names plot (D). Here both the required graph label and the correct option happen to be D. Counting internal zeros is more reliable than judging the horizontal position of the maximum alone.

Step 3: Conclusion The 3s orbital requires two radial nodes and three radial probability peaks, as in plot (D). Hence, the correct option is (D).

Quick Tip to Solve: Count only the zeros at finite positive radius: a 3s radial distribution has two such nodes and three lobes; the origin is not an extra radial node.

64. Identify A in the given chemical reaction. [4 marks]



(A) Methylcyclohexane

(B) Benzene

(C) Cyclohexane

(D) Toluene (methylbenzene)

Correct Answer: (D) Toluene (methylbenzene)

Solution:

Concept:

- Catalytic aromatization converts suitable open-chain alkanes containing six or more carbon atoms into aromatic hydrocarbons by ring closure and dehydrogenation.
- The carbon skeleton must be counted before identifying the aromatic product: benzene contains

six carbons whereas toluene contains seven.

Step 1: Identify the starting alkane The displayed chain is $CH_3CH_2CH_2CH_2CH(CH_3)_2$, namely 2-methylhexane. It contains seven carbon atoms in total: six in its longest chain and one methyl substituent. The bent drawing is still an open-chain alkane; a bond is not present between its two ends.

Step 2: Apply catalytic aromatization At 773 K, under the stated pressure and in the presence of Mo_2O_3, the alkane undergoes cyclization followed by loss of hydrogen to form an aromatic ring. The seven-carbon aromatic product is methylbenzene, or toluene. The overall carbon and hydrogen balance is

$$C_7H_{16} \longrightarrow C_7H_8 + 4\,H_2.$$

Step 3: Eliminate the other structures Methylcyclohexane retains a saturated ring and therefore is not the final aromatized product. Benzene and cyclohexane each contain only six carbon atoms and cannot represent the intended seven-carbon product. The conjugated benzene ring with a methyl substituent satisfies both carbon conservation and aromatization.

Step 4: Conclusion The major product A is toluene, $C_6H_5CH_3$. Hence, the correct option is (D).

Quick Tip to Solve: Count the total carbon atoms first, then look for an aromatic ring: a seven-carbon alkane gives a methylbenzene skeleton on aromatization.

---

65. Given below are two statements :

Statement I : CeO_2 can be used for oxidation of aldehydes and ketones.

Statement II : Aqueous solution of $EuSO_4$ is a strong reducing agent.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false.

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (D) Both Statement I and Statement II are true

Solution:

Concept:

- Lanthanoids predominantly exhibit the +3 oxidation state, although some also show +2 or +4 states.
- An oxidizing agent accepts electrons and is reduced; a reducing agent donates electrons and is oxidized. The tendency toward the common +3 state helps identify these roles.

Step 1: Evaluate the cerium statement In CeO_2, cerium is in the +4 oxidation state. Cerium(IV)

compounds act as oxidizing agents by accepting an electron and becoming cerium(III):

$$Ce^{4+} + e^- \longrightarrow Ce^{3+}.$$

Cerium dioxide is used in the oxidation of organic carbonyl compounds, including aldehydes and ketones. Statement I is therefore true; the relevant fact is the oxidizing character of Ce(IV), not a claim that every carbonyl group reacts identically.

Step 2: Evaluate the europium statement In $EuSO_4$, the sulfate ion has charge -2, so europium is in the $+2$ oxidation state. Aqueous europium(II) is readily oxidized to europium(III), donating an electron:

$$Eu^{2+} \longrightarrow Eu^{3+} + e^-.$$

It consequently acts as a strong reducing agent in aqueous solution. Statement II is also true.

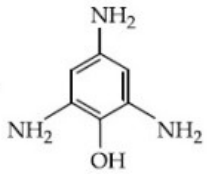
Step 3: Choose the combination The first compound accepts electrons while the second supplies electrons. These are complementary redox roles, and there is no contradiction between the two statements. Options declaring either statement false discard one of these established oxidation-state behaviours; only the option accepting both statements matches the analysis.

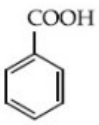
Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

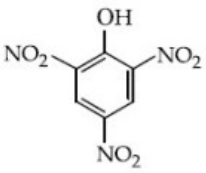
Quick Tip to Solve: Write the metal oxidation state explicitly: Ce(IV) tends to accept an electron, whereas aqueous Eu(II) tends to donate one.

66. Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are :

[4 marks]

A = 

B = 

C = 

(A) C only

(B) B only

(C) B and C only

(D) A and B only

Correct Answer: (C) B and C only

Solution:

Concept:

- An acid stronger than carbonic acid can react with sodium bicarbonate to release carbon dioxide.
- Electron-withdrawing substituents stabilize a phenoxide ion and increase phenol acidity; electron-donating substituents generally decrease it.

Step 1: Examine compound A Compound A is a phenol bearing three amino groups. The $-NH_2$

groups donate electron density through resonance and oppose stabilization of the negatively charged phenoxide ion. This phenol is not sufficiently acidic to displace carbonic acid from bicarbonate. It therefore does not give the characteristic carbon dioxide effervescence.

Step 2: Examine compounds B and C Compound B is benzoic acid, C_6H_5COOH, a carboxylic acid stronger than carbonic acid. It reacts according to

$$C_6H_5COOH + NaHCO_3 \longrightarrow C_6H_5COONa + CO_2 + H_2O.$$

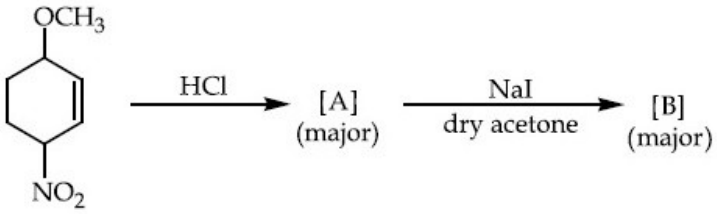
Compound C is 2,4,6-trinitrophenol, or picric acid. Its three nitro groups strongly withdraw electrons and stabilize the picrate ion by inductive and resonance effects. Unlike ordinary phenol, picric acid is sufficiently strong to liberate carbon dioxide from bicarbonate.

Step 3: Compare the offered selections Thus B and C both satisfy the acidity requirement, whereas A does not. Selecting C alone overlooks the carboxyl group of benzoic acid; selecting B alone overlooks the strong acidity of the nitro-substituted phenol. The selection A and B incorrectly treats an electron-rich phenol as a sufficiently strong acid.

Step 4: Conclusion Compounds B and C only liberate carbon dioxide with sodium bicarbonate solution. Hence, the correct option is (C).

Quick Tip to Solve: Do not reject every phenol in the bicarbonate test: several strongly electron-withdrawing nitro groups can make a phenol acidic enough to respond.

67. Identify A and B in the chemical reaction. [4 marks]



A: 2-chloro-1-methoxy-4-nitrocyclohexane;
(A) B: 2-iodo-1-methoxy-4-nitrocyclohexane.
A: 3-chloro-1-methoxy-4-nitrocyclohexane;
(B) B: 3-chloro-1-iodo-4-nitrocyclohexane.
A: 2-chloro-1-methoxy-4-nitrocyclohexane;
(C) B: 3-chloro-1-methoxy-4-nitrocyclohexane.
A: 2-chloro-1-methoxy-4-nitrocyclohexane;
(D) B: 2-chloro-1-iodo-4-nitrocyclohexane.
A: 2-chloro-1-methoxy-4-nitrocyclohexane;

Correct Answer: (A) B: 2-iodo-1-methoxy-4-nitrocyclohexane.

Solution:

Concept:

- Addition of hydrogen chloride to an unsymmetrical alkene proceeds preferentially through the more stable carbocation.

- Sodium iodide in dry acetone effects halogen exchange: iodide replaces an alkyl chloride, with precipitation of sodium chloride driving the reaction.

Step 1: Locate the favoured carbocation Number the ring carbon bearing OCH_3 as C1 and the nitro-bearing carbon as C4. The original double bond is C2=C3. Proton addition to C3 places the positive charge at C2, farther from the strongly electron-withdrawing nitro substituent than a C3 carbocation would be. The alternative orientation would put positive charge immediately next to the nitro-bearing carbon and is less favourable.

Step 2: Identify intermediate A Chloride attacks the preferred C2 carbocation. Therefore A has chlorine at C2, methoxy at C1 and nitro at C4; the former double bond becomes a single bond. The C1 methoxy substituent is retained. This is the chloro product shown in options A, C and D, whereas option B places chlorine at the other carbon of the former double bond.

Step 3: Apply the second reagent With NaI in dry acetone, iodide replaces the chlorine at C2 in the Finkelstein reaction:

$$RCl + NaI \longrightarrow RI + NaCl\downarrow.$$

Here R denotes the substituted cyclohexyl skeleton. This step exchanges the carbon-bound halogen; it neither migrates chlorine to C3 nor replaces the methoxy group at C1. The product B therefore has iodine at C2 and retains both original substituents.

Step 4: Conclusion A is the C2-chloro derivative and B is the corresponding C2-iodo derivative, with methoxy at C1 and nitro at C4 in both. Hence, the correct option is (A).

Quick Tip to Solve: Follow the two reagents separately: HCl fixes the halogen position, then NaI in acetone exchanges Cl for I at that same carbon.

68. Which of the glycosidic linkage between galactose and glucose is present in lactose? [4 marks]

(A) C-1 of galactose and C-6 of glucose

(B) C-1 of glucose and C-6 of galactose

(C) C-1 of galactose and C-4 of glucose

(D) C-1 of glucose and C-4 of galactose

Correct Answer: (C) C-1 of galactose and C-4 of glucose

Solution:

Concept:

- Lactose is a disaccharide composed of galactose and glucose units joined by an oxygen glycosidic bond.
- The glycosidic bond in lactose is a $\beta(1 \rightarrow 4)$ linkage, from the anomeric carbon of galactose to carbon 4 of glucose.

Step 1: Identify the donor carbon Galactose is an aldose, so its anomeric carbon is C1. In lactose this carbon forms the glycosidic linkage in the beta configuration. Thus the first end of the bridge must be C1 of galactose, rather than C1 of glucose. This immediately removes the alternatives that reverse the identities of the two monosaccharides.

Step 2: Identify the glucose carbon The hydroxyl group at C4 of glucose provides the other end of the oxygen bridge. The connection is therefore

galactose-C1-O-C4-glucose.

It is a 1-to-4 linkage, not a 1-to-6 linkage. The C6 hydroxymethyl group of glucose is not the group used for this bond.

Step 3: Check the reducing character and options The anomeric C1 of glucose remains free as a hemiacetal, which explains why lactose is a reducing sugar. This is consistent with attachment at glucose C4. Option A incorrectly uses glucose C6; option B combines the wrong donor sugar with a 1-to-6 linkage; option D reverses which monosaccharide contributes its anomeric carbon. Only the stated galactose-C1 and glucose-C4 pair is correct.

Step 4: Conclusion Lactose contains a glycosidic linkage between C1 of galactose and C4 of glucose. Hence, the correct option is (C).

Quick Tip to Solve: Remember lactose as Gal-beta(1 to 4)-Glc: the galactose anomeric carbon is linked, while the glucose anomeric carbon stays free.

69. According to molecular orbital theory, the species among the following that does not exist is :

[4 marks]

(A) O_2^{2-}

(B) Be_2

(C) He_2^-

(D) He_2^+

Correct Answer: (B) Be_2

Solution:

Concept:

- Molecular orbital bond order is $\text{BO} = (N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons.
- In the elementary molecular orbital model, zero bond order indicates no stable chemical bond; positive bond order indicates net bonding.

Step 1: Calculate the bond order of beryllium dimer Each Be atom has two valence electrons. The four valence electrons of Be_2 occupy $(\sigma_{2s})^2(\sigma_{2s}^*)^2$. The core bonding and antibonding orbitals also cancel. Therefore

$$\text{BO}(\text{Be}_2) = \frac{2-2}{2} = 0.$$

The elementary MO model consequently predicts that this species does not exist as a stable bonded molecule.

Step 2: Check the helium molecular ions For He_2^+, three electrons give $(\sigma_{1s})^2(\sigma_{1s}^*)^1$, hence bond order $1/2$. For He_2^-, the fifth electron enters the next bonding orbital after the filled 1s bonding and

antibonding pair. Its bond order is also $(3-2)/2 = 1/2$. Both ions therefore have positive bond order in the model used in this question.

Step 3: Check the peroxide ion The peroxide ion O_2^{2-} has two additional electrons compared with oxygen, and these occupy antibonding pi orbitals. Its bond order decreases from 2 for O_2 to 1. It still has a net single bond. Thus the peroxide ion and both helium ions are ruled out as the zero-bond-order choice.

Step 4: Conclusion According to the molecular orbital bond-order criterion, Be_2 has zero bond order and is the species sought. Hence, the correct option is (B).

Quick Tip to Solve: For a non-existence question, calculate bond order rather than counting electrons alone; equal bonding and antibonding populations give zero.

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70. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is : [4 marks]

(A) 51.63

(B) 50.33

(C) 63.53

(D) 53.33

Correct Answer: (D) 53.33

Solution:

Concept:

- All carbon in an organic compound becomes carbon dioxide on complete combustion; all hydrogen becomes water.
- For a compound containing only C, H and O, oxygen mass is obtained by subtracting the carbon and hydrogen masses from the original sample mass.

Step 1: Find the carbon mass The carbon fraction by mass in CO_2 is $12/44$. Therefore the carbon present in the original sample is

$$m_C = 2.64 \times \frac{12}{44} = 0.72\,\text{g}.$$

This uses only the carbon portion of the product: oxygen added from the combustion atmosphere must not be counted as original sample carbon.

Step 2: Find the hydrogen and oxygen masses The hydrogen mass fraction in H_2O is $2/18$, since each mole contains two grams of hydrogen. Consequently

$$m_H = 1.08 \times \frac{2}{18} = 0.12\,\text{g}.$$

The original sample contains only C, H and O, so its remaining mass is oxygen:

$$m_O = 1.80 - 0.72 - 0.12 = 0.96\,\text{g}.$$

Oxygen in the gaseous combustion products cannot be used directly because some came from the external oxygen supply.

Step 3: Convert the oxygen mass to percentage Divide the oxygen mass by the initial compound mass, not by the total mass of the combustion products:

$$\%O = \frac{0.96}{1.80} \times 100 = 53.333\ldots\%.$$

Rounding to two decimal places gives 53.33. The other offered numbers do not follow from the carbon and hydrogen balances; no empirical-formula calculation is required.

Step 4: Conclusion The percentage of oxygen in the compound is 53.33%. Hence, the correct option is (D).

Quick Tip to Solve: In C-H-O combustion analysis, obtain C from carbon dioxide and H from water, then find O by subtraction from the original sample mass.

71. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are :

[4 marks]

(A) sp^3d^2 and diamagnetic

(B) d^2sp^3 and paramagnetic

(C) sp^3d^2 and paramagnetic

(D) d^2sp^3 and diamagnetic

Correct Answer: (B) d^2sp^3 and paramagnetic

Solution:

Concept:

- For a complex ion, the metal oxidation state follows from the overall charge and the ligand charges. Cyanide, CN^-, is a strong-field ligand that promotes pairing in an octahedral complex.
- An octahedral low-spin d^5 ion has the configuration $t_{2g}^5 e_g^0$. It contains one unpaired electron and uses two inner d orbitals in the valence-bond description.

Step 1: Determine the two metal configurations Let the oxidation state of Mn be a. Since six cyanide ions contribute charge -6, $a - 6 = -4$, giving Mn(II). Similarly, $b - 6 = -3$ gives Fe(III). Manganese has atomic number 25 and iron has atomic number 26. Removing the $4s$ electrons before the $3d$ electrons gives $Mn^{2+} : [Ar]3d^5$ and $Fe^{3+} : [Ar]3d^5$. Thus both complexes have the same metal-ion d-electron count.

Step 2: Apply strong-field pairing and identify the option In the cyanide ligand field the five d electrons occupy the three lower-energy orbitals as two pairs and one single electron. Two inner $3d$ orbitals remain available for octahedral d^2sp^3 hybridisation. The single unpaired electron makes the complexes paramagnetic; pairing does not make a five-electron set completely paired. The outer-orbital sp^3d^2 choices therefore do not describe these strong-field complexes, and either diamagnetic choice contradicts the odd d-electron count.

Step 3: Conclusion The required hybridisation is d^2sp^3 and the magnetic nature is paramagnetic. Hence, the correct option is (B).

Quick Tip to Solve: Calculate the oxidation state before counting d electrons: both Mn(II) and Fe(III) are d5, and even low-spin d5 retains one unpaired electron.

72. The major product of the following chemical reaction is : [4 marks]

$$CH_3CH_2CN \xrightarrow[3)\ Pd/BaSO_4,\ H_2]{1)\ H_3O^+,\ \Delta \quad 2)\ SOCl_2} ?$$

(A) CH_3CH_2CHO

(B) $CH_3CH_2CH_3$

(C) $CH_3CH_2CH_2OH$

(D) $(CH_3CH_2CO)_2O$

Correct Answer: (A) CH_3CH_2CHO

Solution:

Concept:

- Acidic hydrolysis of a nitrile converts its $-CN$ carbon into the carboxyl carbon of a carboxylic acid, without changing the total carbon count.
- Thionyl chloride converts a carboxylic acid into an acyl chloride. Rosenmund reduction, using hydrogen over poisoned palladium on barium sulphate, converts that acyl chloride into an aldehyde.

Step 1: Hydrolyse the nitrile and form the acyl chloride The starting compound is propanenitrile, CH_3CH_2CN. Heating with aqueous acid hydrolyses the nitrile group to give propanoic acid, CH_3CH_2COOH. The original nitrile carbon remains in the carbon chain. Treatment with $SOCl_2$ then replaces the acid hydroxyl group by chlorine:

$$CH_3CH_2COOH + SOCl_2 \longrightarrow CH_3CH_2COCl + SO_2 + HCl.$$

The intermediate entering the last stage is therefore propanoyl chloride, not an alcohol or an anhydride.

Step 2: Apply the selective reduction The last reagents are H_2 and $Pd/BaSO_4$, which identify Rosenmund reduction. The poisoned catalyst limits the reaction to the aldehyde stage:

$$CH_3CH_2COCl + H_2 \longrightarrow CH_3CH_2CHO + HCl.$$

Propane would require much deeper deoxygenation, while propan-1-ol would require reduction beyond the aldehyde. An anhydride is a carboxylic-acid derivative and is not the product of this hydrogenation. Thus the sequence preserves all three carbon atoms and ends in propanal.

Step 3: Conclusion The major product is propanal, CH_3CH_2CHO. Hence, the correct option is (A).

Quick Tip to Solve: When SOCl2 is followed by H2/Pd-BaSO4, track the acid chloride intermediate and stop its reduction at the aldehyde.

73. In which of the following pairs, the outer most electronic configuration will be the same? [4 marks]

(A) Cr^+ and Mn^{2+}

(B) Ni^{2+} and Cu^+

(C) V^{2+} and Cr^+

(D) Fe^{2+} and Co^+

Correct Answer: (A) Cr^+ and Mn^{2+}

Solution:

Concept:

- Transition-metal cations lose their outer $4s$ electrons before their $3d$ electrons. Equal d-electron counts must be checked after ion formation, rather than from neutral-atom atomic numbers alone.
- The neutral configurations include the stable exceptions $Cr : [Ar]3d^5 4s^1$ and $Cu : [Ar]3d^{10}4s^1$. These exceptions are essential when comparing the listed ions.

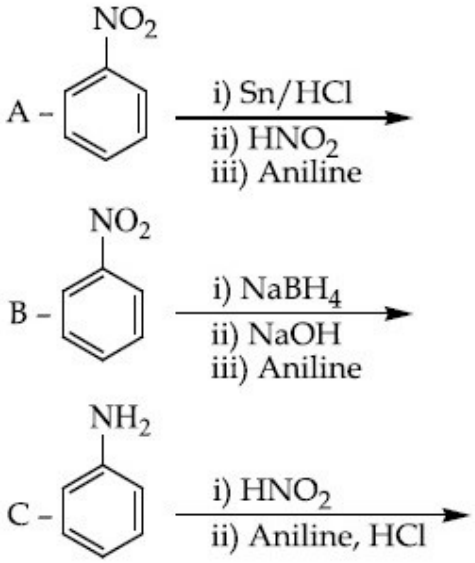
Step 1: Compare chromium and manganese Chromium loses its single $4s$ electron to form Cr^+, leaving $[Ar]3d^5$. Manganese starts as $[Ar]3d^5 4s^2$, and removal of both $4s$ electrons gives $Mn^{2+} : [Ar]3d^5$. Therefore the two ions in the first pair have identical outer electronic configurations, including the same half-filled d subshell.

Step 2: Eliminate the remaining pairs explicitly Nickel is $[Ar]3d^8 4s^2$, so Ni^{2+} is $3d^8$, whereas Cu^+ is $3d^{10}$. Vanadium is $[Ar]3d^3 4s^2$, so V^{2+} is $3d^3$, unlike Cr^+, which is $3d^5$. Finally, Fe^{2+} has $3d^6$, whereas the cobalt ion does not have that configuration: the simple outer-electron removal description gives $3d^7 4s^1$ for Co^+, and its relaxed ground configuration is $3d^8$. Neither description matches $3d^6$. Only the first pair matches.

Step 3: Conclusion Both Cr^+ and Mn^{2+} have the configuration $[Ar]3d^5$. Hence, the correct option is (A).

Quick Tip to Solve: Write the neutral configuration first, remembering Cr and Cu exceptions, and then remove 4s electrons before comparing the ions.

74. Which of the following reaction/s will not give p-aminoazobenzene? [4 marks]



(A) C only

(B) A only

(C) A and B

(D) B only

Correct Answer: (D) B only

Solution:

Concept:

- p-Aminoazobenzene forms by electrophilic azo coupling of a benzenediazonium ion with aniline, predominantly at the para position of the activated aniline ring.
- A successful route must first provide an aromatic primary amine and then generate a diazonium salt with nitrous acid. The coupling medium must retain enough unprotonated aniline for ring activation.

Step 1: Check reaction A In route A, $\mathrm{Sn/HCl}$ reduces the nitro group of nitrobenzene to an amino group. Nitrous acid then diazotises this aromatic primary amine. Coupling the resulting benzenediazonium species with aniline produces the para-substituted azo compound. This supplies the diazonium electrophile and activated coupling partner, so A gives p-aminoazobenzene.

Step 2: Check reactions B and C Route B treats nitrobenzene with $\mathrm{NaBH_4}$, followed by NaOH and aniline. Sodium borohydride under these ordinary conditions does not provide the required nitrobenzene-to-diazonium sequence, and there is no nitrous-acid diazotisation step at all. Merely adding aniline cannot create the azo linkage. Route C starts from aniline itself; nitrous acid forms its diazonium salt, which can then couple with aniline under the stated acid conditions used for this reaction. Thus C, like A, can give the named dye.

Step 3: Conclusion Reaction B alone will not give p-aminoazobenzene. Hence, the correct option is (D).

Quick Tip to Solve: For an azo-dye synthesis, locate the diazonium-forming step first; reduction or addition of aniline alone cannot replace diazotisation.

75. Given below are two statements :

Statement I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Classical reducing smog forms in cool, humid conditions from smoke, fog and sulphur dioxide. Photochemical smog forms in sunlight through reactions involving nitrogen oxides and hydrocarbons.
- Ozone is the triatomic allotrope of oxygen, O_3. It is an oxidising secondary pollutant associated with photochemical smog, rather than the characteristic intermediate of reducing smog.

Step 1: Evaluate Statement I The allotrope mentioned is ozone. Sunlight causes photolysis of nitrogen dioxide, and the oxygen atom produced can combine with molecular oxygen to form ozone. This chemistry is a central part of oxidising photochemical smog. Reducing smog is instead associated with sulphurous smoke and fog, especially in cold conditions. Therefore describing an oxygen allotrope as an important intermediate in the formation of reducing smog assigns ozone to the wrong type of smog. Statement I is false.

Step 2: Evaluate Statement II as a whole Nitrogen oxides do participate in photochemical smog, but the statement groups them with sulphur oxides as contributors to its characteristic formation mechanism. In the standard classification, sulphur dioxide belongs to classical reducing smog, while nitrogen oxides together with hydrocarbons and sunlight produce photochemical smog and its secondary pollutants. Because the combined claim incorrectly includes sulphur oxides in this role, Statement II is false. One correct component of a compound statement does not make the entire statement true; the options must reflect the truth value of each complete sentence.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: Associate ozone, NOx and sunlight with oxidising smog; associate SO2, smoke and fog with classical reducing smog.

76. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is : [4 marks]

(A) ∞

(B) zero

(C) 1

(D) $\frac{1}{n}$

Correct Answer: (D) $\frac{1}{n}$

Solution:

Concept:

- At a fixed temperature, the Freundlich adsorption isotherm relates the mass of gas adsorbed per unit mass of adsorbent to equilibrium pressure: $x_{\text{ads}}/m = kP^{1/n}$.
- The constants k and n depend on the adsorbent, gas and temperature. In the usual moderate-pressure range the exponent satisfies $0 < 1/n < 1$, so adsorption increases sublinearly with pressure.

Step 1: Distinguish the exponent from adsorbed mass The question uses the same letter x in the adsorption ratio and as the exponent of pressure. To avoid confusing their roles, denote adsorbed mass by x_{ads} and the requested exponent by a. The given relation is then $x_{\text{ads}}/m \propto P^a$. The empirical Freundlich expression is

$$\frac{x_{\text{ads}}}{m} = kP^{1/n}.$$

Comparing the powers of the same pressure variable gives $a = 1/n$ directly.

Step 2: Check against the limiting alternatives An exponent of one describes the low-pressure linear dependence, whereas an exponent of zero would make the adsorption independent of pressure, as in a saturation limit. Neither is the general moderate-pressure Freundlich law asked for here. An infinite exponent has no physical role in this isotherm. Taking logarithms confirms the identification:

$$\log\left(\frac{x_{\text{ads}}}{m}\right) = \log k + \frac{1}{n}\log P.$$

The slope of the straight line on a log-log plot is precisely $1/n$, which is the requested power of pressure.

Step 3: Conclusion The pressure exponent is $x = 1/n$. Hence, the correct option is (D).

Quick Tip to Solve: On a plot of log(adsorption per unit mass) against log P, the Freundlich slope is 1/n; use it to identify the exponent.

77. Which statement is correct? [4 marks]

(A) Buna-N is a natural polymer.

(B) Synthesis of Buna-S needs nascent oxygen.

(C) Buna-S is a synthetic and linear thermosetting polymer.

(D) Neoprene is an addition copolymer used in plastic bucket manufacturing.

Correct Answer: (B) Synthesis of Buna-S needs nascent oxygen.

Solution:

Concept:

- Buna-S is a synthetic elastomer obtained by free-radical copolymerisation of 1,3-butadiene and styrene. An oxygen-derived or peroxide radical initiator starts the chain reaction.
- Buna-N is a synthetic butadiene-acrylonitrile copolymer. Neoprene is the addition polymer of chloroprene, so it is a homopolymer rather than a copolymer of two different monomers.

Step 1: Identify the correct polymerisation statement In the synthesis of Buna-S, reactive radicals initiate addition across the carbon-carbon double bonds of the monomers. The statement that its synthesis needs nascent oxygen refers to the oxygen-based radical initiation conventionally used to describe this polymerisation. The resulting material is styrene-butadiene rubber, an important synthetic rubber. Thus the second statement identifies the intended initiating condition for formation of Buna-S.

Step 2: Test each other description Buna-N cannot be classified as natural: it is manufactured from butadiene and acrylonitrile, whereas natural rubber is cis-1,4-polyisoprene. Buna-S is synthetic, but describing it as a linear thermosetting polymer is inconsistent; thermosetting behaviour is associated with an extensively cross-linked network, while Buna-S is classified as an elastomer. Neoprene comes from one monomer, chloroprene, so calling it an addition copolymer is false. It is used for oil-resistant rubber products such as hoses, belts and gaskets, rather than being the standard polymer for plastic buckets. Hence each competing option contains a substantive classification error.

Step 3: Conclusion The correct statement is that synthesis of Buna-S needs nascent oxygen. Hence, the correct option is (B).

Quick Tip to Solve: Identify a rubber by its monomers: Buna-S uses butadiene plus styrene, Buna-N uses butadiene plus acrylonitrile, and neoprene uses chloroprene alone.

78. The correct statement about B_2H_6 is : [4 marks]

(A) Its fragment, BH_3, behaves as a Lewis base.

(B) All $B-H-B$ angles are of 120°.

(C) The two $B-H-B$ bonds are not of same length.

(D) Terminal $B-H$ bonds have less p-character when compared to bridging bonds.

Correct Answer: (D) Terminal $B-H$ bonds have less p-character when compared to bridging bonds.

Solution:

Concept:

- Diborane, B_2H_6, contains four terminal two-centre two-electron bonds and two bridging three-centre two-electron bonds.
- The bridging bonds are electron deficient. Their orbital composition and geometry differ from those of the terminal $B-H$ bonds.

Step 1: Compare terminal and bridging bonding Each hydrogen bridge joins both boron atoms using a three-centre two-electron bond, often called a banana bond. The terminal bonds are conventional two-centre bonds. The terminal boron bonding orbitals have relatively greater s-character, whereas the orbitals used toward the hydrogen bridges have relatively greater p-character. Thus terminal bonds have less p-character than bridging bonds; this is the comparison in option (D).

Step 2: Eliminate the other statements An isolated BH_3 fragment has only six electrons around boron and an empty orbital. It accepts an electron pair, so it behaves as a Lewis acid, not a Lewis base; (A) is false. The bridging $B-H-B$ angle is about 97°, not 120°, so (B) is false. The two equivalent bridges are related by the symmetry of diborane and have equal corresponding bond lengths, so (C) is also false. Terminal bonds are shorter than bridging bonds, but this does not make the two bridges unequal.

Step 3: Conclusion Terminal $B-H$ bonds have less p-character than the bridging bonds. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish terminal bonds from equivalent hydrogen bridges; do not confuse unequal terminal/bridge lengths with unequal bridges.

79. Ellingham diagram is a graphical representation of : [4 marks]

(A) ΔG vs P

(B) ΔG vs T

(C) (ΔG $-$ TΔS) vs T

(D) ΔH vs T

Correct Answer: (B) ΔG vs T

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change of oxide formation, ΔG°, against absolute temperature T.
- The relation $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ explains the approximate straight-line portions when enthalpy and entropy changes vary little with temperature.

Step 1: Identify the plotted quantities In metallurgy the thermodynamic tendency of a metal to form an oxide is compared over a range of temperatures. The vertical coordinate is the Gibbs energy change for the specified oxidation reaction, conventionally normalized to the same amount of oxygen. The horizontal coordinate is temperature. A lower line means a more negative oxide-formation Gibbs energy under the stated standard conditions.

Step 2: Connect the slope to thermodynamics For an interval with no phase change, the line has approximate intercept ΔH° and slope $-\Delta S^\circ$. This is why changes in phase may change the slope. The diagram compares oxide stability and helps identify thermodynamically feasible reducing agents; it does not by itself determine the reaction rate. Option (A) incorrectly puts pressure on the horizontal axis. Option (C) subtracts the entropy term again from Gibbs energy, which is not the plotted function. Option (D) uses enthalpy alone and omits the entropy contribution that changes feasibility with temperature.

Step 3: Conclusion The graph is ΔG versus T. Hence, the correct option is (B).

Quick Tip to Solve: Remember the line equation: Gibbs energy is the ordinate, temperature the abscissa, and minus entropy change the approximate slope.

80. Which of the following equation depicts the oxidizing nature of H_2O_2? [4 marks]

(A) $Cl_2 + H_2O_2 \longrightarrow 2\,HCl + O_2$

(B) $KIO_4 + H_2O_2 \longrightarrow KIO_3 + H_2O + O_2$

(C) $2\,I^- + H_2O_2 + 2\,H^+ \longrightarrow I_2 + 2\,H_2O$

(D) $I_2 + H_2O_2 + 2\,OH^- \longrightarrow 2\,I^- + 2\,H_2O + O_2$

Correct Answer: (C) $2\,I^- + H_2O_2 + 2\,H^+ \longrightarrow I_2 + 2\,H_2O$

Solution:

Concept:

- An oxidizing agent accepts electrons and is itself reduced. Oxygen has oxidation number -1 in H_2O_2.
- Peroxide shows oxidizing behaviour when its oxygen becomes -2, as in water; conversion to O_2, with oxidation number zero, shows reducing behaviour.

Step 1: Inspect the iodide reaction In option (C), iodide changes from oxidation number -1 to zero in iodine. It loses electrons according to $2\,I^- \longrightarrow I_2 + 2\,e^-$. Hydrogen peroxide accepts these electrons in acidic solution:

$$H_2O_2 + 2\,H^+ + 2\,e^- \longrightarrow 2\,H_2O.$$

Combining the half-reactions gives exactly $2\,I^- + H_2O_2 + 2\,H^+ \longrightarrow I_2 + 2\,H_2O$. Thus peroxide oxidizes iodide and is reduced to water.

Step 2: Check why the other equations do not qualify In (A), chlorine is reduced from zero to -1, while peroxide forms oxygen gas. In (B), iodine in periodate is reduced from $+7$ to $+5$, while peroxide again produces oxygen gas. In (D), iodine is reduced from zero to iodide in alkaline solution, and peroxide is oxidized to oxygen gas. Therefore peroxide is the reducing agent in all three of these reactions. Merely seeing peroxide as a reactant is insufficient; its own oxidation-number change establishes its role.

Step 3: Conclusion Option (C) depicts the oxidizing nature of H_2O_2, since it oxidizes iodide to iodine and is reduced to water. Hence, the correct option is (C).

Quick Tip to Solve: Track peroxide oxygen: a change from minus one to minus two means peroxide acts as an oxidizing agent.

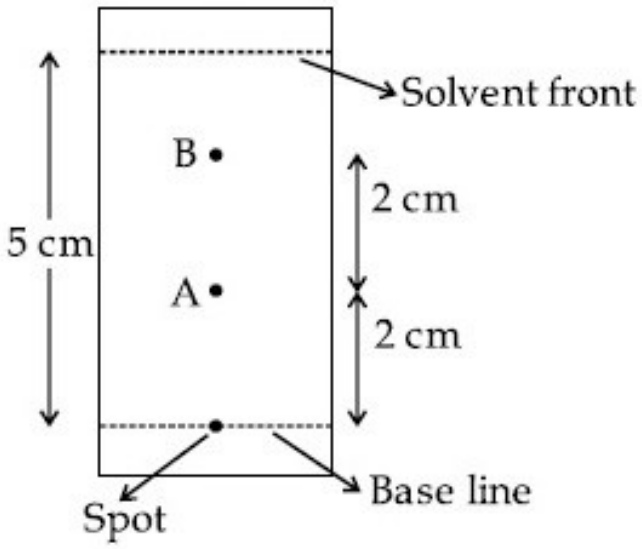
Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Using the provided information in the following paper chromatogram :

Fig : Paper chromatography for compounds A and B.

the calculated R_f value of A _____$\times 10^{-1}$. [4 marks]



Solution:

Concept:

- The retention factor is $R_f = \dfrac{\text{distance travelled by solute from baseline}}{\text{distance travelled by solvent front from baseline}}$.
- Both distances must be measured in the direction of solvent travel from the same starting baseline; R_f is dimensionless.

Step 1: Read the distances for compound A The lower dotted line is the baseline containing the original spot. Compound A is located $2\,\text{cm}$ above this line. The solvent front is the upper dotted line and lies $5\,\text{cm}$ from the baseline. The other labelled $2\,\text{cm}$ interval is the separation of spots A and B; it is not an additional distance travelled by A. Thus the numerator for A is 2 cm, not 4 cm.

Step 2: Calculate and express the requested coefficient Substitute the distances measured from the common baseline:

$$R_f(A) = \frac{2\,\text{cm}}{5\,\text{cm}} = 0.4.$$

The centimetres cancel because the quantity is a ratio. The question expresses this result as a coefficient multiplied by 10^{-1}:

$$0.4 = 4 \times 10^{-1}.$$

The numerical entry is consequently 4 rather than 0.4. As a consistency check, the solute has travelled less far than the solvent front, so its retention factor lies between zero and one.

Step 3: Conclusion $R_f(A) = 0.4 = 4 \times 10^{-1}$. The required numerical entry is $\boxed{4}$.

Quick Tip to Solve: Always measure from the baseline, then convert the ratio to the power-of-ten format requested in the blank.

82. The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be -742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$$NH_2CH_{(S)} + \frac{3}{2}O_{2(g)} \rightarrow N_{2(g)} + O_{2(g)} + H_2O_{(I)}$$

is _____ kJ. (Rounded off to the nearest integer) [Assume ideal gases and R = 8.314 J mol^{-1} K^{-1}] [4 marks]

Solution:

Concept:

- A bomb calorimeter operates at constant volume, so the measured reaction energy is ΔU.
- For ideal gases, $\Delta H = \Delta U + \Delta n_g RT$, where Δn_g counts only gaseous products minus gaseous reactants.

Step 1: Count the gaseous coefficients For the reaction displayed, the gaseous products have total coefficient $1 + 1 = 2$, whereas the gaseous reactant coefficient is $3/2$. The solid reactant and liquid water do not enter the gas count. Hence

$$\Delta n_g = 2 - \frac{3}{2} = \frac{1}{2}.$$

Use this net coefficient change, rather than the total number of species or all stoichiometric coefficients. Since the change is positive, the enthalpy is slightly greater than the internal energy.

Step 2: Calculate enthalpy with consistent units Convert the gas constant to kilojoules before adding the correction to the supplied energy:

$$R = 0.008314\,\text{kJ mol}^{-1}\,\text{K}^{-1}.$$

At $298\,\text{K}$,

$$\begin{aligned}\Delta H &= -742.24 + \tfrac{1}{2}(0.008314)(298)\\ &= -742.24 + 1.238786\\ &= -741.001214\,\text{kJ mol}^{-1}.\end{aligned}$$

The reaction remains exothermic. The problem asks for the magnitude, so take the positive absolute value only after calculating the signed enthalpy. Rounding 741.001214 to the nearest integer gives 741.

Step 3: Conclusion The magnitude is approximately $741\,\text{kJ mol}^{-1}$ for the reaction as written; the required numerical entry is $\boxed{741}$.

Quick Tip to Solve: Count gas coefficients first, convert R to kilojoules, and take the magnitude only after preserving the sign in the enthalpy calculation.

83. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is -325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is -728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is $(-)$ _____ $\times 10^{-1}$ kJ mol^{-1}. [4 marks]

Solution:

Concept:

- Hess law allows consecutive enthalpy changes to be added when their intermediate species cancel.
- Ionization absorbs energy, electron gain has the given negative enthalpy, and the supplied negative lattice enthalpy refers to formation of the solid from gaseous ions.

Step 1: Write the three supplied stages The stated data describe the following sequence:

$$\begin{aligned} \mathrm{Na(g)} &\longrightarrow \mathrm{Na^+(g) + e^-}, & \Delta H_1 &= 495.8 \\ \mathrm{Br(g) + e^-} &\longrightarrow \mathrm{Br^-(g)}, & \Delta H_2 &= -325.0 \\ \mathrm{Na^+(g) + Br^-(g)} &\longrightarrow \mathrm{NaBr(s)}, & \Delta H_3 &= -728.4. \end{aligned}$$

Each energy is in $\mathrm{kJ\,mol^{-1}}$. Adding eliminates the electron and gaseous ions, leaving formation of the ionic solid from the gaseous atoms. The supplied lattice value is already a formation value, so its sign must not be reversed.

Step 2: Add the energies and convert the coefficient The net energy change from the specified stages is

$$\Delta H = 495.8 - 325.0 - 728.4 = -557.6\,\mathrm{kJ\,mol^{-1}}.$$

The release in lattice formation exceeds the net energy needed to produce the ions, so the negative sign is physically consistent. The blank is preceded by a minus sign and followed by 10^{-1}. Therefore

$$-557.6 = -5576 \times 10^{-1}.$$

The blank contains the positive coefficient 5576. No extra atomization or sublimation terms are provided or needed for this stated gaseous-atom sequence.

Step 3: Conclusion The net energy is $-557.6\,\mathrm{kJ\,mol^{-1}}$; the requested coefficient is $\boxed{5576}$.

Quick Tip to Solve: Keep the given lattice-formation sign, add the three stages, then account separately for the minus sign already printed before the blank.

84. 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is _____. (Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- At the phenolphthalein endpoint, all hydroxide is neutralized and carbonate has accepted one proton to become bicarbonate.

- The additional acid from phenolphthalein to methyl orange neutralizes that bicarbonate; its moles equal the original carbonate moles.

Step 1: Use the second endpoint to isolate carbonate The first acid volume includes the contributions of both NaOH and the first protonation of $\mathrm{CO_3}^{2-}$, so it does not directly give the carbonate amount. The subsequent reaction is

$$\mathrm{HCO_3}^- + \mathrm{H}^+ \longrightarrow \mathrm{CO_2} + \mathrm{H_2O}.$$

Each original carbonate ion produces one bicarbonate ion at the first endpoint, and each bicarbonate consumes one proton afterward. Thus the additional $1.5\,\mathrm{mL}$ of $0.1\,\mathrm{mol\,L^{-1}}$ hydrochloric acid gives

$$n(\mathrm{Na_2CO_3}) = 0.1(1.5 \times 10^{-3}) = 1.5 \times 10^{-4}\,\mathrm{mol}.$$

Step 2: Calculate carbonate mass and percentage The molar mass of anhydrous sodium carbonate is $2(23) + 12 + 3(16) = 106\,\mathrm{g\,mol^{-1}}$. Hence

$$m(\mathrm{Na_2CO_3}) = (1.5 \times 10^{-4})(106) = 0.0159\,\mathrm{g}.$$

Relative to the complete $0.4\,\mathrm{g}$ sample, including inert impurities,

$$\text{mass percentage} = \frac{0.0159}{0.4} \times 100 = 3.975\%.$$

Rounding only the final percentage to the nearest integer gives 4. The $17.5\,\mathrm{mL}$ first-endpoint volume would be required to determine the hydroxide amount, but is not needed for this carbonate percentage.

Step 3: Conclusion The weight percentage of $\mathrm{Na_2CO_3}$ is 3.975%, or $\boxed{4}$ on rounding to the nearest integer.

Quick Tip to Solve: For an NaOH/carbonate mixture, the extra acid between the two indicator endpoints measures carbonate alone.

85. In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is _____ mL. (Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- An oxidising agent gains electrons while the reducing agent loses them. In basic medium, balance oxygen with $\mathrm{H_2O}$, hydrogen with $\mathrm{OH^-}$, and charge with electrons.
- In $\mathrm{CrO_4}^{2-}$, chromium is in oxidation state +6; in $\mathrm{Cr(OH)_4}^-$, it is +3. Thus each chromate ion accepts three electrons.

Step 1: Balance the thiosulfate oxidation The two sulfur atoms in one thiosulfate ion have a combined oxidation number of +4. In two sulfate ions their combined oxidation number is +12. Hence one

thiosulfate ion releases eight electrons. The balanced oxidation half-reaction is

$$S_2O_3^{2-} + 10\,OH^- \longrightarrow 2\,SO_4^{2-} + 5\,H_2O + 8\,e^-.$$

This uses both sulfur atoms; treating only one sulfur atom as the reactant would give the wrong mole ratio.

Step 2: Match the electron transfer The reduction half-reaction is

$$CrO_4^{2-} + 4\,H_2O + 3\,e^- \longrightarrow Cr(OH)_4^- + 4\,OH^-.$$

Multiplying oxidation by 3 and reduction by 8 cancels 24 electrons. Therefore three moles of thiosulfate require eight moles of chromate.

Step 3: Calculate the required volume Convert the given thiosulfate volume to litres before finding its amount:

$$n(S_2O_3^{2-}) = 0.25 \times 0.040 = 0.0100 \text{ mol}.$$

$$n(CrO_4^{2-}) = \frac{8}{3}(0.0100) = 0.0266667 \text{ mol}.$$

$$V = \frac{n}{M} = \frac{0.0266667}{0.154} = 0.17316 \text{ L} = 173.16 \text{ mL}.$$

Step 4: Conclusion The required volume, rounded to the nearest integer, is $\boxed{173 \text{ mL}}$.

Quick Tip to Solve: For thiosulfate oxidation to sulfate, count the total oxidation-number change of both sulfur atoms: it is eight electrons per ion.

86. Consider the following chemical reaction.

The number of sp^2 hybridized carbon atom(s) present in the product is _____. [4 marks]

$$CH \equiv CH \xrightarrow[2)\ CO,\ HCl,\ AlCl_3]{1)\ \text{Red hot Fe tube, 873 K}} \text{Product}$$

Solution:

Concept:

- Ethyne undergoes cyclic trimerisation when passed through a red-hot iron tube at about 873 K. Three ethyne molecules form one benzene ring.
- Treatment of benzene with CO and HCl in the presence of $AlCl_3$ introduces a formyl group. This is the Gattermann–Koch formylation of an aromatic ring.

Step 1: Identify the first product The first reagent and temperature describe the conversion of an alkyne to an aromatic ring, rather than simple hydrogenation or hydration. The carbon balance is

$$3\,HC{\equiv}CH \xrightarrow[873\,K]{\text{red-hot Fe}} C_6H_6.$$

Thus the intermediate is benzene. Its six carbon atoms form a planar conjugated ring and each has

three sigma bonds with an unhybridised p orbital.

Step 2: Identify the final product The second treatment is aromatic formylation. One hydrogen atom on benzene is replaced by $-\mathrm{CHO}$, giving benzaldehyde:

$$\mathrm{C_6H_6 + CO \xrightarrow{HCl,\, AlCl_3} C_6H_5CHO.}$$

The aromatic ring remains intact during this electrophilic substitution. The added carbon is the carbonyl carbon of an aldehyde, not a saturated methyl carbon.

Step 3: Count the hybridised carbon atoms All six aromatic ring carbons are sp^2-hybridised. The carbonyl carbon also has three regions of electron density: one bond to the ring, one bond to hydrogen and the carbon–oxygen double bond. It is trigonal planar and sp^2-hybridised as well. Therefore the total is

$$N(sp^2 \text{ carbon}) = 6 + 1 = 7.$$

There are no other carbons in benzaldehyde, so none must be excluded as an sp or sp^3 centre.

Step 4: Conclusion The product is benzaldehyde and contains $\boxed{7}$ sp^2-hybridised carbon atoms.

Quick Tip to Solve: After aromatic formylation, add one carbonyl carbon to the six aromatic carbons when counting the sp^2 centres.

87. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is _____ K. (Rounded off to the nearest integer)

[Given K_b for (H_2O) = 0.52 K kg mol^{-1}] [4 marks]

Solution:

Concept:

- The boiling-point elevation of a dilute solution is $\Delta T_b = iK_bm$, where i is the van't Hoff factor, K_b is the solvent constant and m is the analytical molality.
- If one formula unit produces ν ions and the degree of ionisation is α, then $i = 1 + (\nu - 1)\alpha$. Undissociated formula units must be included in the particle count.

Step 1: Calculate the effective particle count For the electrolyte $\mathrm{A_2B_3}$, complete dissociation would produce two cations and three anions, hence five ions per formula unit. For one initial mole, 60

$$i = (1 - 0.60) + 5(0.60) = 1 + 4(0.60) = 3.40.$$

Using $i = 5$ would incorrectly assume complete ionisation.

Step 2: Calculate the elevation The stated concentration is already molality, so there is no conversion to molarity or solution volume. Substitute $m = 1\ \mathrm{mol\,kg^{-1}}$ and the given solvent constant:

$$\Delta T_b = 3.40 \times 0.52 \times 1 = 1.768\ \mathrm{K}.$$

This positive change means that the solution boils above the boiling point of pure water at the same

external pressure.

Step 3: Find the absolute boiling temperature At 1 atm, pure water boils at approximately 373.15 K. Add the elevation to the solvent boiling point:

$$T_b = 373.15 + 1.768 = 374.918 \text{ K}.$$

Using the school approximation 373 K instead gives 374.768 K; both values round to the same integer. The question asks for the solution boiling point, rather than merely its elevation.

Step 4: Conclusion The boiling point rounded to the nearest integer is $\boxed{375 \text{ K}}$.

Quick Tip to Solve: For partial ionisation, first calculate i; then add iK_bm to the pure solvent boiling point before rounding.

88. Among the following, the number of halide(s) which is/are inert to hydrolysis is _____.

(A) $\mathbf{BF_3}$

(B) $\mathbf{SiCl_4}$

(C) $\mathbf{PCl_5}$

(D) $\mathbf{SF_6}$ [4 marks]

Solution:

Concept:

- Hydrolysis of a covalent halide involves reaction with water and replacement of halogen-containing bonds by oxygen-containing bonds. Its occurrence depends on accessibility, bonding and kinetic stability.
- Among the listed halides, SF_6 is exceptionally inert: its sulfur atom is enclosed by six fluorine atoms in an octahedral arrangement and the S–F bonds are very strong.

Step 1: Check the boron and silicon halides BF_3 is not inert to hydrolysis. Despite the strength of its B–F bonds, boron trifluoride reacts with water, ultimately producing boron–oxygen species. It must not be confused with the much more protected sulfur hexafluoride. Silicon tetrachloride also hydrolyses readily; a usual net representation is

$$SiCl_4 + 2\,H_2O \longrightarrow SiO_2 + 4\,HCl.$$

Thus neither entry (A) nor entry (B) belongs in the inert count.

Step 2: Check phosphorus pentachloride PCl_5 reacts with water. Limited water first gives phosphoryl chloride, while excess water gives phosphoric acid:

$$PCl_5 + H_2O \longrightarrow POCl_3 + 2\,HCl,$$

$$POCl_3 + 3\,H_2O \longrightarrow H_3PO_4 + 3\,HCl.$$

Consequently entry (C) is also excluded. The existence of a molecular halide alone is not evidence

that it resists water.

Step 3: Identify and count the inert halide SF_6 has six strong S–F bonds and a compact octahedral fluorine shell around sulfur. Water cannot readily attack the central atom or initiate substitution under ordinary conditions. It is therefore the hydrolytically inert entry (D). The three other given halides react with water, so counting only the inert members gives

$$N_{\text{inert}} = 1.$$

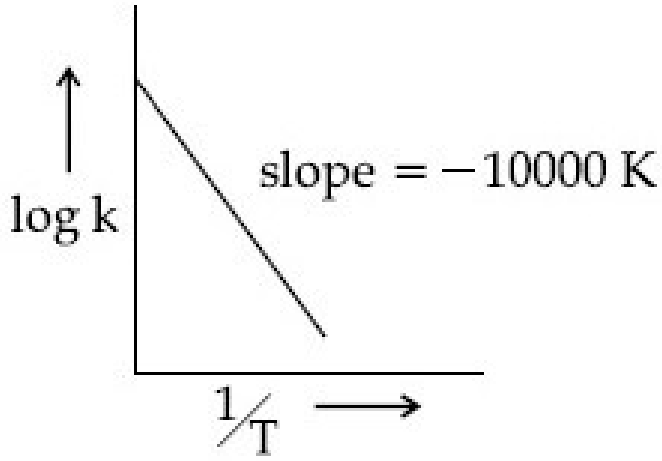
Step 4: Conclusion Only SF_6 is inert to hydrolysis among the given compounds. The required number is $\boxed{1}$.

Quick Tip to Solve: Compare the individual halides rather than using bond polarity alone: the protected octahedral structure makes SF_6 the inert exception.

89. For the reaction, aA + bB $\rightarrow$ cC + dD, the plot of log k vs $\frac{1}{T}$ is given below :

The temperature at which the rate constant of the reaction is 10^{-4} s^{-1} is _____ K. (Rounded off to the nearest integer)

[Given : The rate constant of the reaction is 10^{-5} s^{-1} at 500 K.] [4 marks]



Solution:

Concept:

- The Arrhenius equation in common logarithms is $\log k = \log A - E_a/(2.303RT)$. Hence a plot of $\log k$ against $1/T$ is a straight line.
- The supplied graph has slope -10000 K. For two points on that same line, $\log k_2 - \log k_1 = -10000(1/T_2 - 1/T_1)$, with both temperatures in kelvin.

Step 1: Find the change in logarithm The given first state has $k_1 = 10^{-5}\ \text{s}^{-1}$ and $T_1 = 500$ K. The

target is $k_2 = 10^{-4}\ \mathrm{s}^{-1}$. These rate constants have the same units, so their ratio is dimensionless:

$$\log \frac{k_2}{k_1} = \log(10) = 1.$$

An increase by one decade corresponds to an upward displacement of one unit on the plotted $\log k$ axis.

Step 2: Use the slope to calculate reciprocal temperature Apply the slope directly, without introducing an unnecessary activation-energy calculation:

$$1 = -10000\left(\frac{1}{T_2} - \frac{1}{500}\right).$$

$$\frac{1}{T_2} = \frac{1}{500} - \frac{1}{10000} = 0.00200 - 0.00010 = 0.00190\ \mathrm{K}^{-1}.$$

The reciprocal temperature decreases, which means the temperature increases. This agrees with the larger rate constant for the negatively sloped Arrhenius plot.

Step 3: Invert and round the result Take the reciprocal only after calculating the new horizontal coordinate:

$$T_2 = \frac{1}{0.00190} = 526.3158\ \mathrm{K}.$$

Substitution checks the graph displacement: $-10000(0.00190 - 0.00200) = 1$, so the logarithm indeed changes from -5 to -4. Rounding the temperature, rather than an intermediate reciprocal, gives the requested integer.

Step 4: Conclusion The required temperature is $\boxed{526\ \mathrm{K}}$ to the nearest integer.

Quick Tip to Solve: For an Arrhenius graph, a tenfold increase in k changes $\log k$ by one; divide that change by the slope to find the shift in $1/T$.

90. A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is _____. (Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- For a fixed amount of ideal gas at constant volume, $PV = nRT$ gives $P/T =$ constant. Temperature must be expressed on the absolute kelvin scale.
- The tyre is treated as a fixed-volume container until the limiting pressure is reached. Use the stated pressures consistently in the ratio; the unit psi cancels, so conversion to pascals is unnecessary.

Step 1: Express the initial temperature in kelvin The nitrogen starts at $27^\circ\mathrm{C}$, which corresponds to approximately

$$T_1 = 27 + 273 = 300\ \mathrm{K}.$$

Celsius temperature cannot be used directly in the proportionality: zero Celsius is not zero thermal

energy, and multiplying 27 by a pressure ratio would have no physical basis. The amount of nitrogen is unchanged during the heating.

Step 2: Apply the limiting pressure ratio Take the given initial pressure as $P_1 = 35$ psi and the stated limiting pressure as $P_2 = 40$ psi. At the burst threshold,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}, \qquad T_2 = T_1 \frac{P_2}{P_1}.$$

$$T_2 = 300 \times \frac{40}{35} = 342.857 \text{ K}.$$

The increased pressure requires a higher temperature, as expected when the gas is heated in a container of fixed volume.

Step 3: Convert back to Celsius and round Subtract 273 from the calculated kelvin temperature:

$$t_2 = 342.857 - 273 = 69.857^\circ\text{C}.$$

Using 273.15 consistently in both temperature conversions also gives a value that rounds to 70. The pressure reaches 40 psi at this temperature and exceeds the limit on further heating, so this is the limiting temperature requested.

Step 4: Conclusion The tyre reaches its burst threshold at approximately $\boxed{70^\circ\text{C}}$, rounded to the nearest integer.

Quick Tip to Solve: In a constant-volume gas problem, form the pressure ratio with kelvin temperatures first and convert the final temperature to Celsius only at the end.