JEE Main 2021 Paper 1 (B.E./B.Tech) Feb 25 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 1 (February), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 February 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The shortest distance between the line x − y = 1 and the curve $x^2 = 2y$ is : [4 marks]

(A) 0

(B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{2}$

Correct Answer: (B) $\frac{1}{2\sqrt{2}}$

Solution:

Concept:

- The perpendicular distance of a point (x, y) from $ax + by + c = 0$ is $|ax + by + c|/\sqrt{a^2 + b^2}$. Minimising this point-to-line distance over all points of the curve gives the shortest distance between the two sets.
- Every point of the parabola $x^2 = 2y$ can be written as $(t, t^2/2)$, where t is real. Completing a square identifies the global minimum without a local-extremum assumption.

Step 1: Express the distance using one parameter The given line is $x - y - 1 = 0$. Substitute the general point of the parabola into the distance formula:

$$d(t) = \frac{|t - t^2/2 - 1|}{\sqrt{2}}.$$

The expression inside the modulus is always negative, since $t^2/2 - t + 1 = ((t-1)^2 + 1)/2 > 0$. Thus the modulus cannot vanish and the line does not intersect the parabola.

Step 2: Minimise the completed square Remove the modulus using the established sign and complete the square:

$$d(t) = \frac{t^2 - 2t + 2}{2\sqrt{2}} = \frac{(t-1)^2 + 1}{2\sqrt{2}} \geq \frac{1}{2\sqrt{2}}.$$

Equality holds at $t = 1$, giving the point $(1, 1/2)$ on the curve. The perpendicular foot on the infinite line exists, so this lower bound is actually attained. Option (A) would require an intersection, while (C) and (D) exceed this attained minimum.

Step 3: Conclusion The shortest distance is $\boxed{1/(2\sqrt{2})}$. Hence, the correct option is (B).

Quick Tip to Solve: Parameterise the parabola first, then minimise the point-to-line distance; complete the square before differentiating.

2. A function f(x) is given by $f(x) = \frac{5^x}{5^x+5}$, then the sum of the series $f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+.....+f\left(\frac{39}{20}\right)$ is equal to : [4 marks]

(A) $\frac{39}{2}$
(B) $\frac{19}{2}$
(C) $\frac{49}{2}$
(D) $\frac{29}{2}$

Correct Answer: (A) $\frac{39}{2}$

Solution:

Concept:

- For a sum of function values at equally spaced arguments, pair arguments reflected about the midpoint. Here the first and last arguments sum to 2, so the useful identity involves $f(t) + f(2-t)$.
- The function can be written as $f(t) = 5^{t-1}/(1+5^{t-1})$. Reciprocating the positive quantity 5^{t-1} produces complementary fractions.

Step 1: Establish the complementary identity Let $u = 5^{t-1} > 0$. Then $5^{1-t} = 1/u$, and hence

$$f(t) = \frac{u}{1+u}, \qquad f(2-t) = \frac{1/u}{1+1/u} = \frac{1}{1+u}.$$

Adding these equations gives $f(t) + f(2-t) = 1$ for every real t. The reflecting argument is $2-t$, rather than $1-t$, because the constant in the original denominator is 5.

Step 2: Pair the terms and include the centre There are 39 terms, with indices $k = 1, 2, \ldots, 39$. Pair $k/20$ with $(40-k)/20$ for $k = 1, \ldots, 19$. These are 19 disjoint pairs, each contributing 1. The only remaining term is the midpoint:

$$f(20/20) = f(1) = \frac{5}{5+5} = \frac{1}{2}.$$

Therefore

$$S = 19 + \frac{1}{2} = \frac{39}{2}.$$

The other offered half-integers result from an incorrect pair count; none equals this complete count of all 39 terms.

Step 3: Conclusion The required sum is $\boxed{39/2}$. Hence, the correct option is (A).

Quick Tip to Solve: When the arguments run from $1/20$ to $39/20$, pair indices whose sum is 40 and leave the midpoint $f(1)$ once.

3. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions form the set A to the set A $\times$ B. Then : [4 marks]

(A) 2y = 273x

(B) y = 91x

(C) 2y = 91x

(D) y = 273x

Correct Answer: (C) 2y = 91x

Solution:

Concept:

- A one-one or injective function assigns different images to different domain elements. From a domain of r distinct elements to a codomain of n elements, the count is ${}^nP_r = n(n-1)\cdots(n-r+1)$.
- The Cartesian product contains ordered pairs, so $|A \times B| = |A||B|$. Its elements form the codomain of the second family of functions; the domain remains A.

Step 1: Count injections into each codomain Label the three elements of A in any fixed order. For a function into B, choose the first image in 5 ways, the second in 4 remaining ways, and the third in 3 remaining ways. Thus

$$x = 5 \times 4 \times 3 = 60.$$

Since $|A \times B| = 3 \times 5 = 15$, the corresponding choices for an injection from A into $A \times B$ are $15, 14, 13$. Therefore

$$y = 15 \times 14 \times 13 = 2730.$$

No division by $3!$ is made: swapping the images of two labelled domain elements changes the function.

Step 2: Compare the counts in the requested form Dividing the two counts and cancelling factors gives

$$\frac{y}{x} = \frac{15 \cdot 14 \cdot 13}{5 \cdot 4 \cdot 3} = \frac{91}{2}.$$

Consequently $2y = 91x$. To check directly, $2y = 5460$ and $91x = 91 \cdot 60 = 5460$. Options (A) and (D) use the wrong coefficient 273; option (B) misses the factor of 2 on the left. Only the stated relation fits both exact counts.

Step 3: Conclusion The counts satisfy $\boxed{2y = 91x}$. Hence, the correct option is (C).

Quick Tip to Solve: For injective functions count ordered image selections, and calculate the cardinality of the entire Cartesian-product codomain before applying permutations.

4. The integral $\int \frac{e^{3\log_e 2x}+5e^{2\log_e 2x}}{e^{4\log_e x}+5e^{3\log_e x}-7e^{2\log_e x}}dx$, x > 0, is equal to : (where c is a constant of integration)

[4 marks]

(A) $\log_e\sqrt{x^2+5x-7}+c$

(B) $4\log_e|x^2+5x-7|+c$

(C) $\frac{1}{4}\log_e|x^2+5x-7|+c$

(D) $\log_e|x^2+5x-7|+c$

Correct Answer: (B) $4\log_e|x^2+5x-7|+c$

Solution:

Concept:

- For positive z, the exponential and natural logarithm are inverse functions: $e^{k\log_e z}=z^k$. The condition $x>0$ makes the logarithms of x and $2x$ well defined.
- If $u(x)\neq 0$, then $\int u'(x)/u(x)\,dx=\log_e|u(x)|+c$, on any interval where the integrand is defined. Match the numerator to the derivative after simplifying all powers.

Step 1: Simplify the exponential factors The numerator becomes $(2x)^3+5(2x)^2=8x^3+20x^2=4x^2(2x+5)$. The denominator becomes $x^4+5x^3-7x^2=x^2(x^2+5x-7)$. Since $x>0$, cancellation of x^2 is valid:

$$I=\int\frac{4(2x+5)}{x^2+5x-7}\,dx.$$

The original expression is undefined at the positive zero $(-5+\sqrt{53})/2$ of the quadratic; integration is considered separately on either side of that point.

Step 2: Integrate and verify the coefficient Choose $u=x^2+5x-7$, so that $du=(2x+5)\,dx$. Then

$$I=4\int\frac{du}{u}=4\log_e|x^2+5x-7|+c.$$

Differentiating this result gives $4(2x+5)/(x^2+5x-7)$, exactly the simplified integrand. Option (A) would have coefficient $1/2$ when defined, option (C) coefficient $1/4$, and option (D) coefficient 1; each has the wrong derivative.

Step 3: Conclusion The integral is $\boxed{4\log_e|x^2+5x-7|+c}$. Hence, the correct option is (B).

Quick Tip to Solve: Read $e^{3\log_e(2x)}$ as $(2x)^3$, cancel the common x^2, and compare the remaining numerator with the denominator derivative.

5. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is : [4 marks]

(A) $\frac{2}{9}$

(B) $\frac{1}{5}$

(C) $\frac{122}{297}$

(D) $\frac{97}{297}$

Correct Answer: (D) $\frac{97}{297}$

Solution:

Concept:

- All elements of A are equally likely, so the required probability is the number of favourable four-digit numbers divided by the total count with exactly one digit equal to 7.
- A four-digit number cannot start with zero. A first digit different from 7 has 8 choices, whereas a non-leading digit different from 7 has 9 choices. Remainder 2 modulo 5 requires a units digit of 2 or 7.

Step 1: Count the sample space by the position of 7 If the thousands digit is 7, each remaining position has 9 choices, giving $9^3 = 729$ numbers. If the sole 7 occurs in one of the other three positions, its position has 3 choices, the leading digit has 8 choices, and the other two non-leading digits have 9 choices each. Thus

$$|A| = 729 + 3 \cdot 8 \cdot 9^2 = 2673.$$

These cases are disjoint because each number has exactly one 7.

Step 2: Count the two possible last digits For units digit 7, none of the first three digits may be 7, giving $8 \cdot 9^2 = 648$ favourable numbers. For units digit 2, the sole 7 lies among the first three digits. A leading 7 gives $9^2 = 81$; a 7 in either middle position gives $2 \cdot 8 \cdot 9 = 144$. Hence

$$N = 648 + 81 + 144 = 873, \qquad P = \frac{873}{2673} = \frac{97}{297}.$$

Neither $1/5$ nor $2/9$ accounts for the imposed exactly-one-7 condition, and $122/297$ does not equal the favourable-count ratio.

Step 3: Conclusion The required probability is $\boxed{97/297}$. Hence, the correct option is (D).

Quick Tip to Solve: Conditioning on exactly one special digit makes final digits nonuniform; count the cases ending in 2 and 7 separately.

6. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is : [4 marks]

(A) 3

(B) 2

(C) 4

(D) 1

Correct Answer: (B) 2

Solution:

Concept:

- Each root r of $r^2 - 6r - 2 = 0$ satisfies $r^2 = 6r + 2$. Multiplying this identity by a power of r yields a recurrence for its powers.
- Subtracting the power identities for α and β produces the same linear recurrence for $a_n = \alpha^n - \beta^n$. This avoids computing large individual powers of the roots.

Step 1: Derive the required relation Multiply the equation for α by α^8, and the equation for β by β^8:

$$\alpha^{10} = 6\alpha^9 + 2\alpha^8, \qquad \beta^{10} = 6\beta^9 + 2\beta^8.$$

Subtracting the second equality from the first gives

$$a_{10} = 6a_9 + 2a_8.$$

Thus the particular numerator in the question simplifies immediately to $a_{10} - 2a_8 = 6a_9$, with the minus sign cancelling the positive recurrence term.

Step 2: Check the denominator and simplify The roots are $3 + \sqrt{11}$ and $3 - \sqrt{11}$. They are distinct, and their ninth powers cannot be equal because the real function $t \mapsto t^9$ is strictly increasing. Hence $a_9 \neq 0$, regardless of which root is named α. Division is therefore legitimate:

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{6a_9}{3a_9} = 2.$$

Interchanging the roots changes the sign of both numerator and denominator and leaves this result unchanged. The proposed values $3, 4, 1$ all contradict the exact recurrence identity.

Step 3: Conclusion The requested value is $\boxed{2}$. Hence, the correct option is (B).

Quick Tip to Solve: When differences of high powers involve quadratic roots, derive the two-term recurrence from the root equation before evaluating any powers.

7. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to : [4 marks]

(A) $\frac{1+\sqrt{3}}{2}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1-\sqrt{3}}{2}$

Correct Answer: (A) $\frac{1+\sqrt{3}}{2}$

Solution:

Concept:

- The sum-to-product identity gives $\cos x + \cos y = 2\cos((x+y)/2)\cos((x-y)/2)$, while $\cos(x+y) = 2\cos^2((x+y)/2) - 1$.
- The domain $0 < x, y < \pi$ implies $|(x-y)/2| < \pi/2$, so the cosine of the half-difference is positive and at most 1. Equality in the resulting upper bound determines both angles.

Step 1: Bound the expression and locate equality Put $u = (x+y)/2$, $v = (x-y)/2$, and $c = \cos u$. The given expression becomes

$$2c\cos v - 2c^2 + 1 = \frac{3}{2}.$$

If $c \le 0$, its left side is at most 1, because $\cos v > 0$. Thus $c > 0$, and using $\cos v \le 1$ gives

$$2c\cos v - 2c^2 + 1 \le 2c - 2c^2 + 1 = \frac{3}{2} - 2(c - \tfrac{1}{2})^2 \le \frac{3}{2}.$$

Since the given value is exactly the final upper bound, equality is required at both steps.

Step 2: Recover the angles and evaluate the target Equality in the square bound requires $c = 1/2$, and equality in the first bound requires $\cos v = 1$. With $u \in (0, \pi)$ and $v \in (-\pi/2, \pi/2)$, these conditions give $u = \pi/3$ and $v = 0$. Therefore $x = u + v = \pi/3$ and $y = u - v = \pi/3$, so

$$\sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1+\sqrt{3}}{2}.$$

The alternatives $1/2$, $\sqrt{3}/2$, and $(1-\sqrt{3})/2$ respectively omit a contribution or use its wrong sign.

Step 3: Conclusion The required value is $\boxed{(1+\sqrt{3})/2}$. Hence, the correct option is (A).

Quick Tip to Solve: A trigonometric expression equal to its maximum often forces equality in two bounds; keep the equality conditions to determine both angles uniquely.

8. cosec$\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to : [4 marks]

(A) $\frac{75}{56}$

(B) $\frac{65}{56}$

(C) $\frac{56}{33}$

(D) $\frac{65}{33}$

Correct Answer: (B) $\frac{65}{56}$

Solution:

Concept:

- Use the principal values of the inverse functions. Put $u=\cot^{-1}5$ and $v=\cos^{-1}(4/5)$; both angles lie in the first quadrant.
- The addition formula is $\sin(2u+v)=\sin 2u\cos v+\cos 2u\sin v$. Cosecant is the reciprocal of sine, provided the sine is nonzero.

Step 1: Find the trigonometric ratios Since $\cot u=5$, we have $\tan u=1/5$. Therefore the double-angle identities give

$$\sin 2u=\frac{2\tan u}{1+\tan^2 u}=\frac{5}{13},\qquad \cos 2u=\frac{1-\tan^2 u}{1+\tan^2 u}=\frac{12}{13}.$$

Also $\cos v=4/5$, and the principal angle is acute, so the positive root must be chosen:

$$\sin v=\sqrt{1-\frac{16}{25}}=\frac{3}{5}.$$

Using a negative square root would contradict the range of the inverse cosine here.

Step 2: Apply the addition formula and take the reciprocal Substitute the four ratios without approximating either angle:

$$\begin{aligned}\sin(2u+v)&=\frac{5}{13}\frac{4}{5}+\frac{12}{13}\frac{3}{5}\\&=\frac{20+36}{65}=\frac{56}{65}.\end{aligned}$$

Thus $\csc(2u+v)=65/56$. The value $75/56$ would require an incorrect denominator when combining the two products. The other proposed values, $56/33$ and $65/33$, have reciprocals $33/56$ and $33/65$, respectively, neither of which equals the sine just obtained. In particular, the requested quantity is cosecant, so stopping at $56/65$ would answer a different question.

Step 3: Conclusion The required value is $\boxed{65/56}$. Hence, the correct option is (B).

Quick Tip to Solve: For sums involving inverse trigonometric functions, compute exact sine and cosine ratios first; numerical angle evaluation is unnecessary.

9. Let A be a 3 $\times$ 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to : [4 marks]

(A) 64

(B) 16

(C) 128

(D) 80

Correct Answer: (A) 64

Solution:

Concept:

- For an $n \times n$ matrix, $\det(kA) = k^n \det A$, because all n rows are multiplied by k.
- A determinant is linear in any one row. Multiplying that row by a scalar multiplies the determinant by the same scalar; adding another row to it does not change the determinant.

Step 1: Evaluate the determinant before the row operation Let $C = 2A$, the matrix on which the stated operation is performed. Since the order of A is three, scalar multiplication by 2 multiplies each of its three rows:

$$\det C = \det(2A) = 2^3 \det A = 8 \times 4 = 32.$$

This factor is 2^3, rather than just 2. The notation $2A$ scales the entire matrix, whereas the later operation changes only its second row. Keeping the two operations separate avoids mixing their determinant factors.

Step 2: Use row linearity on the second row Write the rows of C as C_1, C_2, C_3. The matrix B has rows $C_1, 2C_2 + 5C_3, C_3$. Hence

$$\begin{aligned} \det B &= 2\det(C_1, C_2, C_3) + 5\det(C_1, C_3, C_3) \\ &= 2\det C + 5 \times 0 = 2 \times 32 = 64. \end{aligned}$$

The second determinant is zero because two of its rows are identical. Thus adding the multiple of the third row contributes no extra term. The option 16 misses the full scaling of all three rows; 128 introduces one extra factor of two. The value 80 treats the coefficient 5 as an independent determinant multiplier even though its determinant contribution is zero. Only 64 incorporates both stated operations correctly.

Step 3: Conclusion $\det B = \boxed{64}$. Hence, the correct option is (A).

Quick Tip to Solve: First account for scaling the whole matrix, then handle each row operation; adding a multiple of another row never supplies a new determinant factor.

10. If $\alpha, \beta \in$ R are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to : [4 marks]

(A) -7

(B) 7

(C) 3

(D) -3

Correct Answer: (A) -7

Solution:

Concept:

- Nonreal roots of a polynomial with real coefficients occur as a conjugate pair. Thus a root $1 - 2i$ forces the other root to be $1 + 2i$.
- For the monic quadratic $z^2 + \alpha z + \beta = 0$, the sum of roots is $-\alpha$ and their product is β.

Step 1: Find the sum and product of the roots The coefficients α and β are real, so complex conjugation of the given equation shows that $1 + 2i$ is also a root. These are distinct, and a quadratic has exactly two roots counting multiplicity. Their sum is

$$(1 - 2i) + (1 + 2i) = 2.$$

Their product is found using $i^2 = -1$:

$$(1 - 2i)(1 + 2i) = 1 - (2i)^2 = 1 + 4 = 5.$$

The positive product can also be recognised as the square of the modulus of $1 - 2i$, which is $1^2 + 2^2$.

Step 2: Identify the coefficients and subtract in the requested order Comparing with the sum and product formulas gives

$$-\alpha = 2, \qquad \beta = 5, \qquad \alpha = -2.$$

It follows that

$$\alpha - \beta = -2 - 5 = -7.$$

As an independent substitution check,

$$(1 - 2i)^2 - 2(1 - 2i) + 5 = (-3 - 4i) - 2 + 4i + 5 = 0.$$

Thus both coefficients satisfy the original root condition. The value 7 reverses the order of subtraction, while -3 would result from using $+2$ for α instead of -2, since $2 - 5 = -3$. The option 3 also fails because the correctly computed coefficient difference is negative and equals -7. The root-sum sign is therefore essential to selecting the correct option.

Step 3: Conclusion $\alpha = -2$, $\beta = 5$, and $\boxed{\alpha - \beta = -7}$. Hence, the correct option is (A).

Quick Tip to Solve: For a real monic quadratic, pair the complex root with its conjugate and remember that the coefficient of the linear term is the negative root sum.

11. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where a, $x \in R$ and a > 0, is equal to : [4 marks]

(A) $a + \frac{1}{a}$

(B) 2a

(C) a + 1

(D) $2\sqrt{a}$

Correct Answer: (D) $2\sqrt{a}$

Solution:

Concept:

- For positive numbers p, q, the arithmetic-geometric mean inequality gives $p + q \geq 2\sqrt{pq}$, with equality exactly when $p = q$.
- Here the two positive terms are $p = a^{a^x}$ and $q = a^{1-a^x}$. Their product is the constant a, independent of x.

Step 1: Obtain a lower bound independent of the variable Since $a > 0$, real powers of a are positive. Multiplying the two terms adds their exponents:

$$pq = a^{a^x + 1 - a^x} = a.$$

Therefore AM-GM applies directly and yields

$$f(x) = p + q \geq 2\sqrt{a}.$$

A lower bound is a candidate for the minimum, but it is necessary to check whether some allowed real x actually attains it. Otherwise the inequality alone would not establish the requested minimum.

Step 2: Check that equality is attainable For $a \neq 1$, equality requires equal powers of the same positive base different from one:

$$a^x = 1 - a^x, \qquad a^x = \frac{1}{2}.$$

This equation has the real solution

$$x = \frac{\log(1/2)}{\log a}.$$

At that value each summand is $a^{1/2} = \sqrt{a}$, so the bound is attained. If $a = 1$, both summands equal 1 for every real x, and the same answer gives $2\sqrt{1} = 2$. The alternatives are not identities for the minimum: for example, at $a = 4$ they give $17/4$, 8 and 5, whereas equality above gives 4. Although some options coincide for particular bases, only $2\sqrt{a}$ gives the minimum for every permitted positive base.

Step 3: Conclusion The minimum value is $\boxed{2\sqrt{a}}$. Hence, the correct option is (D).

Quick Tip to Solve: When a sum of two positive exponential terms has a constant product, try AM-GM and then solve its equality condition to prove the bound is attained.

12. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x \, dx$, then : [4 marks]

(A) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

(B) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in A.P.

(C) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(D) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in G.P.

Correct Answer: (A) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

Solution:

Concept:

- Combine integrals over identical limits before integrating: $\cot^n x + \cot^{n+2} x = \cot^n x \csc^2 x$.
- The substitution $t = \cot x$ gives $dt = -\csc^2 x \, dx$; on the stated interval its limits change from 1 to 0.

Step 1: Evaluate the sum of integrals in general form For the positive integers needed here, use $1 + \cot^2 x = \csc^2 x$:

$$I_n + I_{n+2} = \int_{\pi/4}^{\pi/2} \cot^n x \csc^2 x \, dx$$

$$= -\int_1^0 t^n \, dt = \int_0^1 t^n \, dt = \frac{1}{n+1}.$$

The negative derivative reverses the limits; the result is positive, as it should be because the original integrand is nonnegative throughout the interval.

Step 2: Test each proposed progression Putting $n = 2, 3, 4$ gives

$$I_2 + I_4 = \frac{1}{3}, \quad I_3 + I_5 = \frac{1}{4}, \quad I_4 + I_6 = \frac{1}{5}.$$

Their reciprocals are $3, 4, 5$, with equal successive differences of 1, proving that they form an arithmetic progression. The sums themselves do not form an arithmetic progression because their successive differences are $-1/12$ and $-1/20$. The reciprocals do not form a geometric progression because $4/3 \neq 5/4$. Finally, the three numbers in option (D) are $1/3, 1/16, 1/5$. For a geometric progression the square of the middle term must equal the product of the outer terms, but $1/256 \neq 1/15$. This eliminates each remaining statement directly.

Step 3: Conclusion The reciprocals are $3, 4, 5$, which are in A.P. Hence, the correct option is (A).

Quick Tip to Solve: For powers of cotangent, add terms whose exponents differ by two; the resulting cosecant-squared factor makes the substitution immediate.

13. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is : [4 marks]

(A) $\frac{14}{45}$
(B) $\frac{8}{45}$
(C) $\frac{7}{45}$
(D) $\frac{28}{45}$

Correct Answer: (D) $\frac{28}{45}$

Solution:

Concept:

- Bayes theorem gives $P(S \mid D) = P(S \cap D)/P(D)$, where S is the smoker and non-vegetarian group and D is the disorder.
- The three given groups are mutually exclusive and exhaust all 400 people, so the total disorder probability is the sum of their population fractions times their respective disorder probabilities.

Step 1: Find the contributions from the three groups The relevant population fractions are $160/400$, $100/400$ and $140/400$. Multiplying by the conditional disorder probabilities gives

$$P(S \cap D) = \frac{160}{400}\frac{35}{100} = \frac{56}{400},$$
$$P(\text{smoker, vegetarian and } D) = \frac{100}{400}\frac{20}{100} = \frac{20}{400},$$
$$P(\text{non-smoker, vegetarian and } D) = \frac{140}{400}\frac{10}{100} = \frac{14}{400}.$$

The numerators 56, 20 and 14 are probability weights proportional to the contributions of each group, rather than an assertion about a particular realised sample.

Step 2: Condition on the observed disorder The selected person is already known to have the disorder, so the denominator must include only its total probability:

$$P(D) = \frac{56 + 20 + 14}{400} = \frac{90}{400}.$$

Consequently,

$$P(S \mid D) = \frac{56/400}{90/400} = \frac{56}{90} = \frac{28}{45}.$$

Using $160/400$ alone would ignore the observed disorder. The fractions $14/45$, $8/45$ and $7/45$ cannot equal the required $56/90$; in particular, $7/45 = 14/90$ is the contribution of the non-smoker vegetarian group after conditioning. The numerator must refer specifically to smokers who are non-vegetarian.

Step 3: Conclusion The probability is $\boxed{28/45}$. Hence, the correct option is (D).

Quick Tip to Solve: For Bayes problems with group sizes, multiply each size by its event rate, then divide the required group weight by the sum of all event weights.

14. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is : [4 marks]

(A) 3

(B) 2

(C) 1

(D) 4

Correct Answer: (C) 1

Solution:

Concept:

- The condition $AA^T = I_2$ means that the rows of A have unit squared lengths and zero mutual dot product.
- An identity between matrices is checked entry by entry. The transpose switches row and column positions without changing the signs of the entries.

Step 1: Multiply the matrix by its transpose For the stated matrix,

$$A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}, \qquad A^T = \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix}.$$

Ordinary row-by-column multiplication gives

$$AA^T = \begin{bmatrix} 1+\alpha^2 & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \alpha^2 + \beta^2 \end{bmatrix}.$$

For example, the top-left entry is $1 \times 1 + (-\alpha)(-\alpha) = 1 + \alpha^2$; the two negative signs make the square positive.

Step 2: Compare with the identity and evaluate the expression The top-left entry of I_2 is 1, so

$$1 + \alpha^2 = 1 \quad \Longrightarrow \quad \alpha = 0.$$

The bottom-right entry then gives

$$\alpha^2 + \beta^2 = 1 \quad \Longrightarrow \quad \beta^2 = 1.$$

The off-diagonal entry is automatically zero when $\alpha = 0$, so these conditions are consistent and β may be either 1 or -1. Both choices give the same fourth power:

$$\alpha^4 + \beta^4 = 0 + (\beta^2)^2 = 1.$$

There is no need to choose a sign for β. The options 2, 3 and 4 contradict the entrywise conditions, since the first fourth power is zero and the second is exactly one. In particular, treating both parameters as independent unit values would incorrectly produce 2.

Step 3: Conclusion The required value is $\boxed{1}$. Hence, the correct option is (C).

Quick Tip to Solve: In an orthogonality condition, inspect the row with a fixed entry of magnitude one first; its remaining entries are then forced to vanish.

15. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is : [4 marks]

(A) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$
(B) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$
(C) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$
(D) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

Correct Answer: (D) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

Solution:

Concept:

- The endpoints of a chord are obtained by solving the curve and line simultaneously. The angle subtended at the origin is the angle between their position vectors.
- For nonzero vectors, $\cos\theta = (\mathbf{u}\cdot\mathbf{v})/(|\mathbf{u}||\mathbf{v}|)$, where $0 \le \theta \le \pi$. The sign of the dot product determines whether the angle is acute or obtuse.

Step 1: Find the two intersection points Substitute $y = 1 - x$ into the ellipse rather than treating the chord as a tangent. This gives

$$x^2 + 2(1-x)^2 = 2 \quad \Longrightarrow \quad 3x^2 - 4x = 0.$$

Thus $x = 0$ or $x = 4/3$, and the endpoints may be named $P = (0, 1)$ and $Q = (4/3, -1/3)$. Both points satisfy $x + y = 1$, as required.

Step 2: Calculate the angle between the position vectors The dot product and lengths are

$$\overrightarrow{OP}\cdot\overrightarrow{OQ} = -\frac{1}{3}, \qquad |OP| = 1, \qquad |OQ| = \frac{\sqrt{17}}{3}.$$

Consequently $\cos\theta = -1/\sqrt{17}$. In particular the required angle is obtuse, so the two options involving subtraction from $\pi/2$ cannot be correct.

Step 3: Identify the inverse-tangent expression Write $\theta = \pi/2 + \phi$, with $0 < \phi < \pi/2$. Then $\sin\phi = 1/\sqrt{17}$, so a right triangle gives $\cos\phi = 4/\sqrt{17}$ and $\tan\phi = 1/4$. Hence $\phi = \tan^{-1}(1/4)$. The remaining obtuse option with $1/3$ would give a different cosine and is excluded.

Step 4: Conclusion The angle is $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$. Hence, the correct option is (D).

Quick Tip to Solve: Check the sign of the endpoint dot product before using a slope formula: a negative dot product selects the obtuse angle.

16. The following system of linear equations

2x + 3y + 2z = 9

3x + 2y + 2z = 9

x − y + 4z = 8 [4 marks]

(A) does not have any solution

(B) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(C) has a unique solution

(D) has infinitely many solutions

Correct Answer: (C) has a unique solution

Solution:

Concept:

- A linear system has a unique solution if its equations determine each unknown without a free parameter. Elementary elimination preserves its solution set.
- An option claiming a property of the solution must be checked using the actual values; consistency alone does not establish a proposed nonlinear identity.

Step 1: Eliminate the common terms Subtract the second equation from the first. The z-terms cancel and we obtain

$$-x + y = 0 \quad \Longrightarrow \quad y = x.$$

This is a necessary condition for every possible solution. Substituting it into the third equation immediately gives $x - x + 4z = 8$, and hence $z = 2$. No arbitrary parameter remains in the value of z.

Step 2: Determine the remaining variables Put $y = x$ and $z = 2$ into the first equation:

$$2x + 3x + 4 = 9 \quad \Longrightarrow \quad 5x = 5 \quad \Longrightarrow \quad x = y = 1.$$

The candidate solution is therefore $(1, 1, 2)$. Check it directly: the three left sides are $2 + 3 + 4 = 9$, $3 + 2 + 4 = 9$, and $1 - 1 + 8 = 8$, respectively. Thus the system is consistent, and elimination has produced exactly one solution.

Step 3: Compare every proposed description The verified solution rules out the claim of no solution. Since all three variables are fixed, infinitely many solutions are impossible. For the proposed expression in option (B), use $\alpha = 1$, $\beta = 1$, and $\gamma = 2$:

$$\alpha + \beta^2 + \gamma^3 = 1 + 1 + 8 = 10 \neq 12.$$

Only the statement that the solution is unique is true.

Step 4: Conclusion The system has the unique solution $(x, y, z) = (1, 1, 2)$. Hence, the correct option is (C).

Quick Tip to Solve: When two equations have nearly identical coefficients, subtract them first; here that directly gives $x = y$ and simplifies the third equation.

17. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is : [4 marks]

(A) $\frac{x^2}{9}-\frac{y^2}{4}=1$

(B) $\frac{x^2}{9}-\frac{y^2}{16}=1$

(C) $\frac{x^2}{9}-\frac{y^2}{25}=1$

(D) $x^2-y^2=9$

Correct Answer: (B) $\frac{x^2}{9}-\frac{y^2}{16}=1$

Solution:

Concept:

- For an ellipse $x^2/a^2+y^2/b^2=1$, with $a>b$, its focal distance satisfies $c^2=a^2-b^2$ and its eccentricity is $e=c/a$.
- A hyperbola with transverse axis on the x-axis has equation $x^2/A^2-y^2/B^2=1$ and eccentricity $E^2=1+B^2/A^2$. Its intercepts on that axis are its vertices.

Step 1: Locate the ellipse foci The given ellipse has $a=5$ and $b=4$. Therefore

$$c=\sqrt{25-16}=3, \qquad e=\frac{3}{5}.$$

Its foci are $(3,0)$ and $(-3,0)$. The hyperbola has the same coordinate axes as its transverse and conjugate axes, so it is centred at the origin with transverse axis along the x-axis.

Step 2: Use the points through which the hyperbola passes Write the hyperbola as $x^2/A^2-y^2/B^2=1$. Since $(3,0)$ lies on it,

$$\frac{9}{A^2}=1 \quad \Longrightarrow \quad A^2=9.$$

These are vertices of the hyperbola, not its foci. The condition on the eccentricities gives $(3/5)E=1$, and hence $E=5/3$.

Step 3: Calculate the conjugate semi-axis Use the hyperbola eccentricity relation:

$$\frac{25}{9}=1+\frac{B^2}{9} \quad \Longrightarrow \quad B^2=16.$$

Thus the required denominators are 9 and 16. Options (A) and (C) have the wrong conjugate semi-axis. Option (D), when divided by 9, has equal denominators 9 and 9 and eccentricity $\sqrt{2}$; it also fails the prescribed eccentricity product.

Step 4: Conclusion The hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$. Hence, the correct option is (B).

Quick Tip to Solve: Passing through an ellipse focus fixes a hyperbola vertex here: set $y=0$ before applying the product of eccentricities.

18. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\ldots\ldots+\frac{n}{(2n+1)^2}\right]$ **is equal to :** [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) 1

(D) $\frac{1}{4}$

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- A sum of the form $\frac{1}{n}\sum_{k=1}^{n} f(k/n)$ tends to $\int_0^1 f(t)\,dt$ when f is continuous on $[0,1]$.
- Adding or deleting a fixed number of summands tending to zero leaves a finite limit unchanged. This is useful for the endpoint terms in the printed sum.

Step 1: Express the terms using a common scale For $k = 1, 2, \ldots, n+1$, write

$$\frac{n}{(n+k)^2} = \frac{1}{n}\frac{1}{(1+k/n)^2}.$$

The printed expression is the sum of these terms together with $1/n$. Separate the last term so that the remaining index range is the standard 1 to n:

$$S_n = \frac{1}{n} + \frac{1}{n}\sum_{k=1}^{n}\frac{1}{(1+k/n)^2} + \frac{n}{(2n+1)^2}.$$

Step 2: Check the endpoint contributions The first isolated term $1/n$ tends to zero. The final isolated term also tends to zero, since

$$0 < \frac{n}{(2n+1)^2} < \frac{1}{4n}.$$

Therefore neither isolated term changes the limit. The function $f(t) = (1+t)^{-2}$ is continuous throughout $[0,1]$, so the middle term is a valid Riemann sum on that interval.

Step 3: Evaluate the definite integral Taking the limit and integrating gives

$$\lim_{n\to\infty} S_n = \int_0^1 \frac{dt}{(1+t)^2} = \left[-\frac{1}{1+t}\right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2}.$$

This exact evaluation rules out $1/3$, 1, and $1/4$. Keeping the extra final term is harmless, but treating every summand as $1/n$ would wrongly ignore its changing denominator.

Step 4: Conclusion The required limit is $\frac{1}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: Rewrite $n/(n+k)^2$ as $(1/n)(1+k/n)^{-2}$; then remove only the finitely many endpoint terms whose limits are zero.

19. The contrapositive of the statement "If you will work, you will earn money" is : [4 marks]

(A) If you will not earn money, you will not work

(B) If you will earn money, you will work

(C) You will earn money, if you will not work

(D) To earn money, you need to work

Correct Answer: (A) If you will not earn money, you will not work

Solution:

Concept:

- For statements p and q, the contrapositive of $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. It is logically equivalent to the original implication.
- The converse $q \Rightarrow p$ and inverse $\neg p \Rightarrow \neg q$ are different constructions. A contrapositive requires both reversal and negation.

Step 1: Name the hypothesis and conclusion Let p mean "you will work" and let q mean "you will earn money". The given sentence states that working is sufficient for earning money, so its logical form is $p \Rightarrow q$. The words following "if" form the hypothesis, and the remaining clause is the conclusion. Fixing these roles first prevents an accidental interchange of the original statement and its converse.

Step 2: Reverse and negate both clauses The contrapositive starts from failure of the original conclusion. Its hypothesis is $\neg q$, meaning "you will not earn money". Its conclusion is $\neg p$, meaning "you will not work". Thus

$$\neg q \Rightarrow \neg p$$

reads: "If you will not earn money, you will not work." Indeed, if one were to work while not earning money, that would contradict the given implication.

Step 3: Rule out the other wordings Option (B) has form $q \Rightarrow p$, the converse, and lacks the required negations. Option (C) says that not working implies earning, namely $\neg p \Rightarrow q$, which is a different assertion. In option (D), working is described as necessary for earning: earning implies working. This is again the converse rather than the contrapositive. Only option (A) both reverses and negates the clauses.

Step 4: Conclusion The contrapositive is "If you will not earn money, you will not work." Hence, the correct option is (A).

Quick Tip to Solve: Write the two clauses as p and q; the contrapositive is always "not q implies not p".

20. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, −1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length : [4 marks]

(A) $\sqrt{\frac{2}{7}}$

(B) $\sqrt{\frac{2}{5}}$

(C) $\sqrt{\frac{2}{3}}$

(D) $\sqrt{\frac{2}{11}}$

Correct Answer: (D) $\sqrt{\frac{2}{11}}$

Solution:

Concept:

- A normal to a plane through three noncollinear points is the cross product of two direction vectors in the plane.
- For vector $\mathbf{v}$ and plane normal $\mathbf{n}$, the squared length of its orthogonal projection on the plane is $|\mathbf{v}|^2 - (\mathbf{v}\cdot\mathbf{n})^2/|\mathbf{n}|^2$. Only the orientation of the plane matters for projecting a vector.

Step 1: Find two plane directions and their normal Using the given point A as reference,

$$\overrightarrow{AB} = (1, 1, -2), \qquad \overrightarrow{AC} = (1, 2, -1).$$

Their cross product is

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = (3, -1, 1).$$

Check that this is perpendicular to both: $3 - 1 - 2 = 0$ and $3 - 2 - 1 = 0$. Its squared magnitude is $9 + 1 + 1 = 11$, so the plane normal is nonzero.

Step 2: Compute the perpendicular component The vector to be projected is $\mathbf{v} = \overrightarrow{OP} = (2, -1, 1)$, with $|\mathbf{v}|^2 = 4 + 1 + 1 = 6$. Its dot product with the normal is

$$\mathbf{v}\cdot\mathbf{n} = 2(3) + (-1)(-1) + 1(1) = 8.$$

The squared length of the component perpendicular to the plane is therefore $8^2/11 = 64/11$. This is the normal component, which must be removed from the total squared length.

Step 3: Obtain the parallel projection length The parallel and perpendicular components are orthogonal. By the Pythagorean relation,

$$|\mathbf{v}_{\parallel}|^2 = 6 - \frac{64}{11} = \frac{66 - 64}{11} = \frac{2}{11}.$$

Taking the nonnegative square root gives the required length. The squared values in the other options, $2/7$, $2/5$, and $2/3$, do not equal this computed squared projection.

Step 4: Conclusion The projection has length $\sqrt{\frac{2}{11}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a plane projection, subtract the squared normal component from the vector norm squared; do not confuse the normal projection with the required planar one.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _____. [4 marks]

Solution:

Concept:

- If an integer leaves remainder 3 on division by 4, it has the form $x = 4k + 3$ for an integer k.
- Every odd integer has square congruent to 1 modulo 8, because the product of two consecutive integers is even. A positive even power is a power of that square.

Step 1: Express the base using the given remainder Write $x = 4k + 3$. Since $2020 = 4 \cdot 505$,

$$2020 + x = 4(k + 505) + 3.$$

In particular the base is odd. Its residue modulo 8 need not be fixed by the question: it can be 3 or 7 according to the parity of $k + 505$. Both possibilities must give the same required remainder before a definite answer can be asserted.

Step 2: Prove the odd-square congruence Let an arbitrary odd integer be $a = 2r + 1$. Expanding its square gives

$$a^2 = 4r(r + 1) + 1.$$

One of $r, r + 1$ is even, so $r(r + 1) = 2s$ for some integer s. Therefore $a^2 = 8s + 1$, or $a^2 \equiv 1 \pmod 8$. This proof applies to either possible residue of the given base and avoids treating a remainder modulo 4 as if it uniquely fixed a remainder modulo 8.

Step 3: Use the even exponent Apply the result to $a = 2020 + x$. The exponent 2022 is twice 1011, so

$$(2020 + x)^{2022} = \left((2020 + x)^2\right)^{1011} \equiv 1^{1011} \equiv 1 \pmod 8.$$

The residue 1 already lies between 0 and 7, so it is the actual Euclidean remainder. No units apply because the answer is an integer remainder, rather than a measured quantity.

Step 4: Conclusion The remainder when $(2020 + x)^{2022}$ is divided by 8 is $\boxed{1}$.

Quick Tip to Solve: For powers modulo 8, check parity first: every odd base squared is congruent to 1, so any positive even exponent gives remainder 1.

22. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is _____.

[4 marks]

Solution:

Concept:

- To test divisibility by 10, it is sufficient to calculate the remainder modulo 10, namely the units digit of the number.
- The units digits of powers of 3 and 7 each repeat with period 4. Thus every positive integer exponent belongs to one of four residue classes modulo 4.

Step 1: Find the repeating units digits Calculate one complete cycle of positive powers. For powers of 3 the units digits are 3, 9, 7, 1; for powers of 7 they are 7, 9, 3, 1. Therefore, for exponents congruent to 1, 2, 3 and 0 modulo 4, respectively, the sums of the units digits are

$$3 + 7 = 10, \qquad 9 + 9 = 18, \qquad 7 + 3 = 10, \qquad 1 + 1 = 2.$$

Only the first and third residue classes give a units digit of zero. The cycles repeat because the fourth powers of both bases are congruent to 1 modulo 10.

Step 2: Identify every admissible exponent The required condition is therefore equivalent to $n \equiv 1$ or $3 \pmod 4$, which simply says that n is odd. In particular, the even exponents cannot qualify: they produce remainder 8 or 2 rather than 0. This classifies all exponents, rather than testing only a few individual numbers.

Step 3: Count the two-digit odd integers The allowed two-digit exponents form the arithmetic progression

$$11, 13, 15, \ldots, 99.$$

Its first term is 11, last term is 99 and common difference is 2. Consequently its number of terms is

$$N = \frac{99 - 11}{2} + 1 = 44 + 1 = 45.$$

The lower two-digit endpoint 10 is even and is correctly excluded; the upper endpoint 99 is included.

Step 4: Conclusion There are $\boxed{45}$ two-digit numbers satisfying the divisibility condition. This is a dimensionless count.

Quick Tip to Solve: Write one four-term units-digit cycle first; here it reduces the whole counting problem to counting odd two-digit exponents.

23. The value of $\int_{-2}^{2} |3x^2 - 3x - 6|dx$ is _____. [4 marks]

Solution:

Concept:

- For a real function g, the expression $|g(x)|$ equals $g(x)$ where it is nonnegative and $-g(x)$ where it is negative.
- A definite integral containing an absolute value must be split at the zeros where the sign changes. An antiderivative of $3x^2 - 3x - 6$ is $F(x) = x^3 - \frac{3}{2}x^2 - 6x$.

Step 1: Locate the sign changes Factor the expression inside the absolute value:

$$3x^2 - 3x - 6 = 3(x - 2)(x + 1).$$

The zeros are -1 and 2. On the interval $[-2, -1]$, both factors are nonpositive and their product is nonnegative. On $[-1, 2]$, the factor $x + 1$ is nonnegative while $x - 2$ is nonpositive, so the product is nonpositive. The zero at 2 is an endpoint and introduces no additional interval.

Step 2: Remove the absolute value on each interval Using the signs just established, the required integral is

$$I = \int_{-2}^{-1} (3x^2 - 3x - 6)\,dx - \int_{-1}^{2} (3x^2 - 3x - 6)\,dx.$$

The minus sign before the second integral is essential: it reverses the negative signed area into its positive contribution. Both resulting integrals are elementary polynomial integrals.

Step 3: Evaluate and add the contributions With $F(x) = x^3 - \frac{3}{2}x^2 - 6x$, the endpoint values are

$$F(-2) = -2, \qquad F(-1) = \frac{7}{2}, \qquad F(2) = -10.$$

Hence

$$\begin{aligned} I &= \big[F(-1) - F(-2)\big] - \big[F(2) - F(-1)\big] \\ &= \left(\frac{7}{2} + 2\right) - \left(-10 - \frac{7}{2}\right) \\ &= \frac{11}{2} + \frac{27}{2} = 19. \end{aligned}$$

Both separate contributions are positive, as required for an integral of an absolute value.

Step 4: Conclusion The value of the definite integral is $\boxed{19}$. No physical unit is specified.

Quick Tip to Solve: Factor the expression inside the modulus before integrating; split only at its roots lying inside the integration interval.

24. A line 'l' passing through origin is perpendicular to the lines

$$l_1 : \overrightarrow{r} = (3+t)\widehat{i} + (-1+2t)\widehat{j} + (4+2t)\widehat{k}$$

$$l_2 : \overrightarrow{r} = (3+2s)\widehat{i} + (3+2s)\widehat{j} + (2+s)\widehat{k}$$

If the co-ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c) then 18(a + b + c) is equal to _____. [4 marks]

Solution:

Concept:

- A direction vector perpendicular to two nonparallel direction vectors is their cross product. The position vectors of points on an intersecting line must satisfy both line equations.
- The squared distance between points is the sum of the squared coordinate differences. The first octant requires all three coordinates to be positive.

Step 1: Find the direction of the line through the origin The given lines have direction vectors $\mathbf{d}_1 = (1, 2, 2)$ and $\mathbf{d}_2 = (2, 2, 1)$. A direction vector for l is

$$\mathbf{d}_1 \times \mathbf{d}_2 = (-2, 3, -2).$$

Its dot product with each given direction vector is zero. Since l passes through the origin, its points have coordinates $(-2\lambda, 3\lambda, -2\lambda)$.

Step 2: Locate the intersection with the first line A point on l_1 has coordinates $(3+t, -1+2t, 4+2t)$. Equating these with the coordinates on l gives $3+t = -2\lambda$ and $-1+2t = 3\lambda$. Solving gives $t = -1$ and $\lambda = -1$. The third coordinate also agrees: $4 + 2t = 2 = -2\lambda$. Thus the intersection is $P = (2, -3, 2)$.

Step 3: Impose the distance and octant conditions Write the required point on l_2 as $Q = (3 + 2s, 3 + 2s, 2 + s)$. The distance condition $PQ = \sqrt{17}$ yields

$$\begin{aligned}(1+2s)^2 + (6+2s)^2 + s^2 &= 17,\\ 9s^2 + 28s + 20 &= 0,\\ (9s+10)(s+2) &= 0.\end{aligned}$$

The root $s = -2$ gives $(-1, -1, 0)$, which is outside the first octant. The other root $s = -10/9$ gives $Q = (7/9, 7/9, 8/9)$, whose coordinates are all positive.

Step 4: Calculate the required expression Therefore $a = 7/9$, $b = 7/9$ and $c = 8/9$. Substituting these values,

$$18(a+b+c) = 18\left(\frac{7+7+8}{9}\right) = 18 \cdot \frac{22}{9} = 44.$$

The octant condition is what selects the correct distance root.

Step 5: Conclusion The required value is $\boxed{44}$, in the coordinate units implicit in the question.

Quick Tip to Solve: Use the cross product only for the unknown line direction; then enforce the intersection and first-octant conditions separately.

25. A function f is defined on $[-3, 3]$ as

$f(x) = \begin{cases} \min\{|x|, 2-x^2\}, & -2 \le x \le 2 \\ [|x|], & 2 < |x| \le 3 \end{cases}$ **where [x] denotes the greatest integer $\le$ x. The number of points, where f is not differentiable in $(-3, 3)$ is _____.** [4 marks]

Solution:

Concept:

- A minimum of two differentiable expressions can develop a corner where the active expression changes and the two one-sided derivatives differ.
- Differentiability implies continuity. For a piecewise-defined function, check switching points, junctions and corners; endpoints outside the requested open interval are excluded.

Step 1: Resolve the minimum on the central interval Put $u = |x| \ge 0$. Equality of the competing expressions requires

$$u = 2 - u^2, \qquad (u-1)(u+2) = 0.$$

Only $u = 1$ is admissible. For $|x| < 1$, the smaller expression is $|x|$; for $1 < |x| \le 2$, it is $2 - x^2$. Thus the central definition is the absolute-value function between -1 and 1 and the parabola outside those points up to ± 2.

Step 2: Check the interior corners At $x = 0$, the one-sided derivatives of $|x|$ are -1 and 1, so the derivative fails to exist. At $x = -1$, the parabola on the left has derivative $-2x = 2$, while the absolute-value branch on the right has derivative -1. At $x = 1$, the corresponding derivatives are 1 and -2. The function is continuous at both switching points, but its one-sided derivatives differ. These give three nondifferentiable points: $-1, 0, 1$.

Step 3: Check the outer pieces and junctions For $2 < |x| < 3$, the greatest integer $[|x|]$ equals 2 throughout each open outer interval, so the function is constant and differentiable there. At $x = 2$ and $x = -2$, the central formula gives $2 - x^2 = -2$, whereas the limit from the adjacent outer interval is 2. Both junctions are therefore discontinuous and hence nondifferentiable. The endpoints ± 3 are excluded because the requested interval is open.

Step 4: Count the distinct points All remaining open pieces are polynomials, affine functions or constant functions and are differentiable. There are consequently no further candidates. The complete list is

$$\{-2, -1, 0, 1, 2\},$$

which contains five distinct points.

Step 5: Conclusion The number of points of nondifferentiability in $(-3, 3)$ is $\boxed{5}$. This is a dimensionless count.

Quick Tip to Solve: For a minimum involving an absolute value, first solve the equality of the two branches, then check those switches, zero and every piecewise junction.

26. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}$. $\vec{b}$ is equal to _____. [4 marks]

Solution:

Concept:

- The area of a parallelogram with adjacent vectors $\mathbf{a}$ and $\mathbf{b}$ is $|\mathbf{a} \times \mathbf{b}|$.
- The identity $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$ connects area, magnitudes and scalar product.

Step 1: Express the magnitudes and scalar product From $\mathbf{a} = (1, \alpha, 3)$ and $\mathbf{b} = (3, -\alpha, 1)$, calculate

$$|\mathbf{a}|^2 = 1 + \alpha^2 + 9 = 10 + \alpha^2, \qquad |\mathbf{b}|^2 = 9 + \alpha^2 + 1 = 10 + \alpha^2.$$

Their scalar product is

$$\mathbf{a} \cdot \mathbf{b} = 1 \cdot 3 + \alpha(-\alpha) + 3 \cdot 1 = 6 - \alpha^2.$$

The negative sign in the middle component of $\mathbf{b}$ is important; it makes the middle contribution $-\alpha^2$.

Step 2: Use the given area to determine the parameter square The given area is $8\sqrt{3}$, so its square is 192. Applying the vector identity gives

$$\begin{aligned} 192 &= (10 + \alpha^2)^2 - (6 - \alpha^2)^2 \\ &= \left[(10 + \alpha^2) - (6 - \alpha^2)\right]\left[(10 + \alpha^2) + (6 - \alpha^2)\right] \\ &= (4 + 2\alpha^2)16 = 64 + 32\alpha^2. \end{aligned}$$

Therefore $32\alpha^2 = 128$, giving $\alpha^2 = 4$. Both signs $\alpha = \pm 2$ are possible, but the requested scalar product depends only on the square.

Step 3: Calculate and verify the scalar product Substitution now gives

$$\mathbf{a} \cdot \mathbf{b} = 6 - 4 = 2.$$

As a check, either permitted value of α gives $|\mathbf{a}|^2 = |\mathbf{b}|^2 = 14$. The area identity then produces $14^2 - 2^2 = 192$, exactly the required squared area. Thus the result is consistent with the original geometric constraint.

Step 4: Conclusion The scalar product is $\boxed{2}$ square units when the vector components represent side lengths in units.

Quick Tip to Solve: Use the difference-of-squares form of the area identity; it cancels the fourth powers of the unknown parameter immediately.

27. If $\lim_{x\to 0} \frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of a − 2b is _____. [4 marks]

Solution:

Concept:

- Near zero, the exponential has the expansion $e^{4x} - 1 = 4x + 8x^2 + O(x^3)$. Comparing the first nonzero powers of the numerator and denominator determines whether a finite limit is possible.
- The denominator must define the quotient near zero, so $a = 0$ is not admissible. For $a \neq 0$, its leading term is $4ax^2$.

Step 1: Find the necessary cancellation Insert the expansion in the numerator:

$$ax - (e^{4x} - 1) = (a - 4)x - 8x^2 + O(x^3).$$

The denominator is

$$ax(e^{4x} - 1) = 4ax^2 + 8ax^3 + O(x^4).$$

If $a \neq 4$ and $a \neq 0$, the quotient has a nonzero term $(a - 4)/(4ax)$, which cannot have a finite two-sided limit as $x \to 0$. The value $a = 0$ makes the denominator identically zero, rather than giving a defined limiting function. Consequently the asserted finite limit forces $a = 4$.

Step 2: Evaluate the surviving second-order ratio For $a = 4$, the terms linear in x cancel in the numerator. Hence

$$\begin{aligned} b &= \lim_{x\to 0} \frac{4x - (e^{4x} - 1)}{4x(e^{4x} - 1)} \\ &= \lim_{x\to 0} \frac{-8x^2 + O(x^3)}{16x^2 + O(x^3)} = -\frac{1}{2}. \end{aligned}$$

Division by x^2 makes the finite nonzero leading terms explicit. In particular the limit is negative, because the exponential exceeds its tangent to second order.

Step 3: Form the expression asked for The required combination is now obtained from the same values of a and b:

$$a - 2b = 4 - 2\left(-\frac{1}{2}\right) = 4 + 1 = 5.$$

Keeping the negative sign in b avoids the tempting but incorrect subtraction of 1. No higher-order term affects this result once the leading second-order coefficients have been identified.

Step 4: Conclusion The required dimensionless value is $\boxed{5}$, with $a = 4$ and $b = -\frac{1}{2}$.

Quick Tip to Solve: When a limit has a denominator of order x^2, first cancel any numerator term of order x; only then compare the quadratic coefficients.

28. If the curve, y = y(x) represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to _____. [4 marks]

Solution:

Concept:

- The differential equation can be rewritten as $y' - y/x = -2y^2$ for $x \neq 0$. For a nonzero solution the substitution $v = 1/y$ converts this Bernoulli equation into a linear equation.
- A linear equation $v' + Pv = Q$ is solved with an integrating factor $e^{\int P\,dx}$. The initial point fixes the integration constant on the solution branch containing that point.

Step 1: Determine the point on the curve Solve the two line equations simultaneously:

$$2x - 3y = 1, \qquad 3x + 2y = 8.$$

Multiplying the first by 2 and the second by 3 and adding gives $13x = 26$, hence $x = 2$. Substituting back gives $y = 1$. Thus the required branch passes through $(2, 1)$, which excludes the identically zero solution.

Step 2: Transform and integrate the equation Dividing the given equation by $x\,dx$ yields $y' - y/x = -2y^2$. Let $v = 1/y$, so $v' = -y'/y^2$. Dividing by $-y^2$ gives

$$v' + \frac{v}{x} = 2.$$

For positive x, an integrating factor is x, and therefore

$$(xv)' = 2x, \qquad xv = x^2 + C.$$

Consequently

$$y = \frac{x}{x^2 + C}.$$

The reciprocal substitution is legitimate on a nonzero finite branch of this solution.

Step 3: Fix the constant and evaluate the requested value The point $(2, 1)$ gives $1 = 2/(4 + C)$, hence $C = -2$. The resulting algebraic expression is

$$y(x) = \frac{x}{x^2 - 2}.$$

Its value at 1 is -1, so the requested absolute value is 1. Direct differentiation gives $y' = -(x^2 + 2)/(x^2 - 2)^2$, which satisfies $y' = y/x - 2y^2$ wherever the expression is defined. The expression has poles at $\pm\sqrt{2}$; the initial branch lies in $(\sqrt{2}, \infty)$, while the question evaluates the same formula at 1 on its other regular interval.

Step 4: Conclusion Using the solution formula $y = x/(x^2 - 2)$, the requested value is $\boxed{|y(1)| = 1}$, with no physical unit specified.

Quick Tip to Solve: For a differential equation containing y and y^2, try $1/y$; solve the two lines first so the constant uses the correct initial point.

29. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____. [4 marks]

Solution:

Concept:

- For two differentiable curves meeting at right angles, their finite tangent slopes satisfy $m_1 m_2 = -1$. Evaluate both slopes at the same intersection point.
- Implicit differentiation of $x = y^4$ gives $1 = 4y^3(dy/dx)$, while differentiating $xy = k$ gives $y + x(dy/dx) = 0$. The parameter k is constant along the second curve.

Step 1: Express the two slopes in terms of the ordinate At a nonzero intersection, the first curve gives $x = y^4$. Therefore the two tangent slopes are

$$m_1 = \frac{1}{4y^3}, \qquad m_2 = -\frac{y}{x} = -\frac{1}{y^3}.$$

The origin is not an orthogonal intersection of the two regular curves: when $k = 0$, the equation $xy = 0$ is a pair of axes and is singular there. We therefore apply the tangent condition at a regular intersection, where these expressions are defined.

Step 2: Apply the right-angle condition Multiply the slopes and impose perpendicularity:

$$-\frac{1}{4y^6} = -1 \quad \Longrightarrow \quad y^6 = \frac{1}{4}.$$

This condition is unchanged when the ordinate changes sign, so it will give the same requested even power for either admissible sign of the parameter.

Step 3: Evaluate the requested power directly The intersection also satisfies $k = xy = y^5$. Avoid extracting fractional roots: raise the given expression to the sixth power and use the value of y^6:

$$(4k)^6 = 4^6 y^{30} = 4^6 (y^6)^5 = 4^6 \left(\frac{1}{4}\right)^5 = 4.$$

The sixth power removes the sign of k, so no separate positive and negative cases are needed.

Step 4: Conclusion The required numerical value is $\boxed{4}$.

Quick Tip to Solve: When a question asks for an even power of a parameter, substitute the intersection relation first; here $k = y^5$ converts the target into a power of y^6.

30. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to _____. [4 marks]

Solution:

Concept:

- A nonvertical tangent to $y^2 = 4x$ has equation $y = mx + 1/m$, with point of contact $(1/m^2, 2/m)$, where $m \neq 0$.

- A line is tangent to a circle when its perpendicular distance from the centre equals the radius. Here the circle has centre $(3, 0)$ and radius 3.

Step 1: Impose tangency to the circle Write the parabola tangent as $mx - y + 1/m = 0$. Its distance from $(3, 0)$ is 3, so

$$\frac{|3m + 1/m|}{\sqrt{m^2 + 1}} = 3.$$

Squaring is legitimate because both sides represent nonnegative distances. Expanding and simplifying gives

$$9m^2 + 6 + \frac{1}{m^2} = 9m^2 + 9, \qquad m^2 = \frac{1}{3}.$$

Step 2: Select the first-quadrant tangent and its parabola contact The parabola contact ordinate is $2/m$, so the first-quadrant requirement selects $m = 1/\sqrt{3}$, rather than its negative. The parabola contact is therefore

$$(c, d) = (3, 2\sqrt{3}).$$

The vertical tangent $x = 0$ touches both curves at the origin; its contact is neither in the first quadrant nor distinct, and thus does not meet the question conditions.

Step 3: Find the circle contact and add the abscissae The circle contact is the foot of the perpendicular from its centre to the tangent. For $A = m, B = -1, C = 1/m$, the foot has abscissa

$$a = 3 - \frac{m(3m + 1/m)}{m^2 + 1} = 3 - \frac{2}{4/3} = \frac{3}{2}.$$

Its ordinate is $b = (3m + 1/m)/(m^2 + 1) = 3\sqrt{3}/2 > 0$, confirming that this contact is also in the first quadrant and distinct from $(3, 2\sqrt{3})$. Hence

$$2(a + c) = 2\left(\frac{3}{2} + 3\right) = 9.$$

Step 4: Conclusion The required value is $\boxed{9}$. Interchanging the names of the two contact points leaves their abscissa sum unchanged.

Quick Tip to Solve: Use the parabola tangent in slope form, then apply centre-to-line distance for the circle; the first-quadrant condition determines the slope sign.

Physics – Section A: Multiple Choice Questions (80 marks)

31. An LCR circuit contains resistance of 110Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be : [4 marks]

(A) 1 A

(B) 2.5 A

(C) 2 A

(D) 1.5 A

Correct Answer: (C) 2 A

Solution:

Concept:

- For a series circuit, the impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L$ and $X_C = 1/(\omega C)$. The rms current is V_{rms}/Z.
- In an RL circuit the current lags and $\tan\phi = X_L/R$. In an RC circuit the current leads with $|\tan\phi| = X_C/R$. Each stated phase measurement is at the same supply frequency.

Step 1: Use the phase lag after removing the capacitor With only the capacitor removed, the circuit contains the original resistance and inductor in series. Its phase lag is $45°$, giving

$$\frac{X_L}{R} = \tan 45° = 1, \qquad X_L = R = 110\,\Omega.$$

The inductive reactance refers to the stated angular frequency $300\,\text{rad}\,\text{s}^{-1}$; it is not the reactance at an independently chosen frequency.

Step 2: Use the phase lead after removing the inductor The remaining RC circuit also has a phase angle of magnitude $45°$. Thus

$$\frac{X_C}{R} = 1, \qquad X_C = 110\,\Omega.$$

Restoring both components therefore makes $X_L - X_C = 0$. Their reactive voltage contributions oppose each other, so the full circuit is at series resonance even though neither individual reactance is zero.

Step 3: Compute the rms current and check the options At resonance the impedance is purely resistive:

$$Z = R = 110\,\Omega, \qquad I_{\text{rms}} = \frac{220}{110} = 2\,\text{A}.$$

A current of $1\,\text{A}$ or $1.5\,\text{A}$ would require a larger impedance, contradicting complete reactive cancellation. The proposed $2.5\,\text{A}$ exceeds V/R, the maximum current possible in this passive series circuit.

Step 4: Conclusion The rms current is $\boxed{2\,\text{A}}$. Hence, the correct option is (C).

Quick Tip to Solve: Equal-magnitude RL lag and RC lead angles imply equal reactances at that frequency; restore both components and check for series resonance.

32. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy K_2. Then the ratio of kinetic energies $K_1 : K_2$ will be : [4 marks]

(A) $\frac{\cos^2\beta}{\cos^2\alpha}$
(B) $\frac{\cos\beta}{\cos\alpha}$
(C) $\frac{\sin^2\beta}{\cos^2\alpha}$
(D) $\frac{\cos\beta}{\sin\alpha}$

Correct Answer: (A) $\frac{\cos^2\beta}{\cos^2\alpha}$

Solution:

Concept:

- Between parallel capacitor plates, the electric field and the electric force on an electron are perpendicular to the plates. Hence the component of velocity parallel to the plates remains constant.
- For an electron of mass m_e and speed v, the kinetic energy is $K = \frac{1}{2}m_e v^2$. Because the angles are measured with the plates, the parallel component is $v\cos\theta$.

Step 1: Resolve the entry and exit velocities Let the entry and exit speeds be v_1 and v_2. The force changes the velocity component normal to the plates; there is no force in the direction parallel to their surfaces. Conservation of the parallel component consequently gives

$$v_1\cos\alpha = v_2\cos\beta.$$

The sign of the electron charge determines its deflection direction but does not alter this component equation.

Step 2: Convert the speed ratio into an energy ratio Rearrange the conserved-component relation, then square it because kinetic energy depends on speed squared:

$$\frac{v_1}{v_2} = \frac{\cos\beta}{\cos\alpha}, \qquad \frac{K_1}{K_2} = \frac{\frac{1}{2}m_e v_1^2}{\frac{1}{2}m_e v_2^2} = \frac{\cos^2\beta}{\cos^2\alpha}.$$

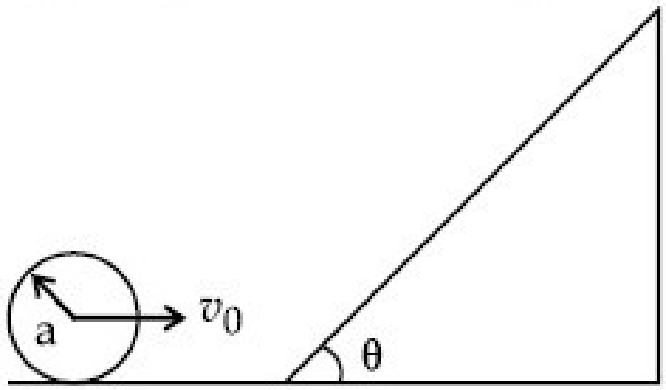
The mass cancels, so neither the plate separation nor the field magnitude is required for this ratio.

Step 3: Compare the listed expressions Option (A) contains the required squared cosine ratio. Option (B) is only the speed ratio and misses the square. Option (C) incorrectly replaces the exit parallel component by a normal component. Option (D) combines a normal component at entry with a parallel component at exit and also misses the squaring needed for energies. As a check, when entry and exit have equal angle magnitudes, the expression gives equal kinetic energies, as expected from equal speed magnitudes.

Step 4: Conclusion The ratio is $\boxed{K_1 : K_2 = \cos^2\beta : \cos^2\alpha}$. Hence, the correct option is (A).

Quick Tip to Solve: First identify the direction with no force. Angles measured from the plates use cosine for the conserved velocity component; square only after finding the speed ratio.

33. A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel? [4 marks]



(A) $\frac{v_0^2}{2g\sin\theta}$

(B) $\frac{7v_0^2}{10g\sin\theta}$

(C) $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$

(D) $\frac{v_0^2}{5g\sin\theta}$

Correct Answer: (B) $\frac{7v_0^2}{10g\sin\theta}$

Solution:

Concept:

- For a uniform solid sphere, the moment of inertia about its centre is $I = \frac{2}{5}ma^2$. Rolling without slipping imposes $\omega = v/a$.
- The total kinetic energy of a rolling body is the sum of translational and rotational energies. Static friction does no work in pure rolling on a fixed surface, so this energy converts into gravitational potential energy during the climb.

Step 1: Calculate both parts of the initial kinetic energy Initially the centre moves with speed v_0 and the sphere rotates with $\omega_0 = v_0/a$. Thus

$$E_{\text{initial}} = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\frac{v_0^2}{a^2} = \frac{7}{10}mv_0^2.$$

The radius cancels because the rolling angular speed is inversely proportional to the radius.

Step 2: Relate the climb distance to the rise in height Let s be the distance travelled upward along the incline. The centre rises through height $h = s\sin\theta$. At the highest point, the translational speed is zero; the no-slip condition also makes its angular speed zero. Conservation of mechanical energy therefore gives

$$mgs\sin\theta = \frac{7}{10}mv_0^2, \qquad s = \frac{7v_0^2}{10g\sin\theta}.$$

This is distance measured along the plane, rather than the vertical height.

Step 3: Check the result against the alternatives Option (B) has the coefficient $7/10$, accounting for both energy contributions. Option (A), with coefficient $1/2$, keeps only translational energy and treats the sphere like a sliding particle. Options (C) and (D) likewise omit part of the initial energy. The result has units of length because v_0^2/g is a length, and it increases for a shallower incline, consistent with the same vertical rise requiring more travel.

Step 4: Conclusion The sphere travels $\boxed{\dfrac{7v_0^2}{10g\sin\theta}}$ up the plane. Hence, the correct option is (B).

Quick Tip to Solve: For rolling bodies, include $\frac{1}{2}I\omega^2$ before equating energy to mgh, and use $h = s\sin\theta$ when the question asks for distance along the incline.

34. An electron of mass m_e and a proton of mass $m_p = 1836\ m_e$ are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{proton}}$ will be : [4 marks]

(A) 1

(B) 1836

(C) $\frac{1}{1836}$

(D) 918

Correct Answer: (B) 1836

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the particle momentum. For nonrelativistic speed v, $p = mv$.
- At equal speeds, momentum is proportional to mass and wavelength is inversely proportional to mass. The question supplies $m_p = 1836m_e$, so no tabulated masses are needed.

Step 1: Write the wavelengths of the two particles If the common speed is v, the two wavelengths are

$$\lambda_e = \frac{h}{m_e v}, \qquad \lambda_p = \frac{h}{m_p v}.$$

The particles have the same speed, not the same momentum or the same kinetic energy. Consequently, it would be incorrect to set their wavelengths equal merely because their speeds agree.

Step 2: Form the ratio in the order requested Divide the electron wavelength by the proton wavelength:

$$\frac{\lambda_e}{\lambda_p} = \frac{h/(m_e v)}{h/(m_p v)} = \frac{m_p}{m_e} = 1836.$$

Both h and the common speed cancel. The lighter electron has smaller momentum at the same speed and thus a much longer wavelength. Even if a common relativistic factor were included for equal speeds, that common factor would cancel in this ratio.

Step 3: Reject the other numerical choices Option (A) would apply to equal momenta, which are not given here. Option (C) is the reciprocal ratio, proton wavelength divided by electron wavelength, so it

reverses the order asked. Option (D), 918, introduces an unjustified factor of two; the factor $1/2$ from kinetic energy is irrelevant because the de Broglie relation uses momentum directly. Hence only the full mass ratio is consistent with the stated equal-speed condition.

Step 4: Conclusion The dimensionless wavelength ratio is $\boxed{\lambda_e/\lambda_p = 1836}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal speeds use the inverse mass dependence of wavelength; for equal kinetic energies a square-root mass dependence would arise instead.

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35. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1 μm. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that: [4 marks]

(A) its size increases, but intensity decreases

(B) its size increases, and intensity increases

(C) its size decreases, but intensity increases

(D) its sizes decreases, and intensity decreases

Correct Answer: (C) its size decreases, but intensity increases

Solution:

Concept:

- Diffraction spreads light more strongly when the aperture is smaller relative to its wavelength. For a circular aperture, the characteristic diffraction angular scale varies inversely with its diameter D.
- Increasing the diameter also increases the aperture area, allowing more incident sunlight to pass through. More transmitted power and a less spread-out central pattern raise its intensity.

Step 1: Determine the change in the pattern size For an aperture of diameter D, the diffraction scale is controlled by the ratio λ/D. In the usual circular-aperture description, the first dark-ring angular radius is approximately $1.22\lambda/D$ when the small-angle approximation applies. The stated tiny pinhole gives very strong spreading, but the qualitative dependence remains: a slightly larger opening produces less spreading. At the same observation distance, the characteristic pattern size therefore decreases.

Step 2: Determine the change in brightness For fixed incident sunlight, increasing the opening area transmits more light. Since the transmitted light also spreads over a smaller angular region, the central diffraction pattern becomes brighter. Thus the intensity increases rather than decreases. This argument compares the same source, observation arrangement and spectral component; no change in the wavelength of the sunlight is implied by changing the aperture.

Step 3: Combine the two changes and test the options The required combination is smaller size and greater intensity. Options (A) and (B) predict a larger pattern, contrary to the inverse aperture dependence of diffraction. Option (D) predicts reduced intensity despite the enlarged light-admitting area and reduced spreading. Option (C) alone gives both changes correctly. Sunlight contains multiple wavelengths, so the pattern can have wavelength-dependent features, but the direction of the aperture-size effect is the same for each component.

Step 4: Conclusion The diffraction pattern becomes smaller and more intense. Hence, the correct option is (C).

Quick Tip to Solve: When an aperture widens, separate two effects: diffraction spreading decreases while transmitted light increases; both favour a brighter central region.

36. In a ferromagnetic material, below the curie temperature, a domain is defined as : [4 marks]

(A) a macroscopic region with zero magnetization.

(B) a macroscopic region with saturation magnetization.

(C) a macroscopic region with randomly oriented magnetic dipoles.

(D) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction.

Correct Answer: (B) a macroscopic region with saturation magnetization.

Solution:

Concept:

- A ferromagnetic domain is a region in which exchange interactions align the atomic magnetic moments in a common direction below the Curie temperature.
- Magnetization is magnetic moment per unit volume: $\vec{M} = \sum \vec{\mu}/V$. Alignment gives spontaneous saturation magnetization within a domain.

Step 1: Identify what is aligned inside one domain Below the Curie temperature, the interaction between neighbouring atomic moments favours parallel alignment. A domain contains a very large collection of these aligned moments. Their magnetic moments add rather than cancel, so an individual domain is strongly and spontaneously magnetized even before an external field is applied. The direction of this magnetization is uniform within the ideal domain.

Step 2: Distinguish a domain from the entire specimen An unmagnetized specimen can contain domains pointing in different directions. Its resultant magnetization may therefore be zero even though each separate domain has saturation magnetization. Option (A) confuses this specimen-wide cancellation with the state inside one domain. Random dipoles in (C) do not describe an ordered ferromagnetic domain, and alternating opposite moments in (D) describe a different kind of magnetic ordering. Only (B) states the required property.

Step 3: Conclusion A domain is a macroscopic region with saturation magnetization. Hence, the correct option is (B).

Quick Tip to Solve: Apply the alignment test within a single domain; cancellation between different domains does not make each domain unmagnetized.

37. Given below are two statements :

Statement I : In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II : In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (C) Statement I is true but Statement II is false.

Solution:

Concept:

- In classical thermal equilibrium, molecular kinetic degrees of freedom have Maxwell-Boltzmann statistics, with thermal weighting proportional to $e^{-E/(k_{\mathrm{B}}T)}$.
- Equipartition gives an average energy $k_{\mathrm{B}}T/2$ for each active quadratic degree of freedom. A linear diatomic molecule has three translational and two rotational degrees of freedom.

Step 1: Evaluate the statistical statement At the temperature where molecular rotation is classically active, the angular-velocity components have Maxwell-type distributions, just as the linear-velocity components do. Rotational energies consequently follow the corresponding equilibrium distribution. Thus Statement I expresses the Maxwell-Boltzmann statistical behaviour of rotational motion and is true in the classical kinetic-theory setting of this question.

Step 2: Compare the two mean energies For a rigid diatomic molecule, rotation about the two axes perpendicular to the bond contributes, whereas rotation about the bond has negligible moment of inertia. Therefore

$$\langle E_{\mathrm{rot}}\rangle = 2\left(\frac{1}{2}k_{\mathrm{B}}T\right) = k_{\mathrm{B}}T, \qquad \langle E_{\mathrm{trans}}\rangle = \frac{3}{2}k_{\mathrm{B}}T.$$

These averages differ. Moreover, equipartition concerns averages, not equality for every individual molecule at every instant. Statement II is false. Options (A) and (D) wrongly accept it, while (B) wrongly rejects Statement I.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Count active degrees of freedom before comparing energies, and distinguish a thermal average from an individual molecule's instantaneous energy.

38. Match List I with List II.

	List I		List II
(a)	Rectifier	(i)	Used either for stepping up or stepping down the a.c. voltage
(b)	Stabilizer	(ii)	Used to convert a.c. voltage into d.c. voltage
(c)	Transformer	(iii)	Used to remove any ripple in the rectified output voltage
(d)	Filter	(iv)	Used for constant output voltage even when the input voltage or load current change

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(B) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(C) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(D) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Correct Answer: (A) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Solution:

Concept:

- A rectifier uses one-way conduction to convert alternating current or voltage into a unidirectional, generally pulsating, output.
- A filter smooths rectified output; a stabilizer maintains the output voltage against changes in supply or load. A transformer changes an alternating voltage by electromagnetic induction.

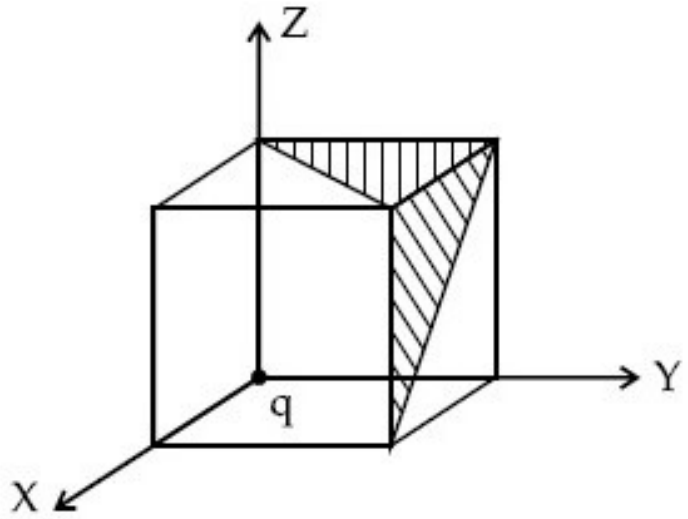
Step 1: Match rectification and voltage stabilization The rectifier is the conversion stage from a.c. to d.c., giving (a)-(ii). Its output can still have a periodic ripple; rectification alone does not guarantee a smooth or constant voltage. A stabilizer acts to keep the delivered output voltage constant when the input voltage fluctuates or the load current changes. This gives (b)-(iv), rather than the transformer function in (i).

Step 2: Match transformation and filtering A transformer uses the turns ratio of its primary and secondary coils to step an a.c. voltage up or down: $V_s/V_p = N_s/N_p$ in the ideal case. Thus (c)-(i). A filter suppresses the alternating ripple superposed on the rectified d.c. output, giving (d)-(iii). The complete matching is therefore (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii). Option (B) interchanges rectifier and filter, (C) assigns transformer action to the stabilizer, and (D) additionally assigns filtering to the transformer. Only (A) preserves all four functions.

Step 3: Conclusion The rectifier converts a.c. to d.c.; the stabilizer maintains voltage; the transformer changes a.c. voltage; and the filter removes ripple. Hence, the correct option is (A).

Quick Tip to Solve: Remember the power-supply sequence: rectify to make the output unidirectional, filter to smooth it, and stabilize to hold its voltage constant.

39. A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ through the shaded area is : [4 marks]



(A) $\frac{q}{24\varepsilon_0}$
(B) $\frac{q}{48\varepsilon_0}$
(C) $\frac{q}{4\varepsilon_0}$
(D) $\frac{q}{8\varepsilon_0}$

Correct Answer: (A) $\frac{q}{24\varepsilon_0}$

Solution:

Concept:

- Gauss's law gives the total outward electric flux through a closed surface enclosing charge q: $\Phi = q/\varepsilon_0$.
- For a charge at a cube corner, place eight identical cubes around that corner. The charge is then at the centre of the resulting larger cube, so symmetry distributes flux equally among equivalent faces.

Step 1: Find the flux through one remote face The larger cube has six equivalent faces, each carrying flux $q/(6\varepsilon_0)$. Each such large face consists of four congruent original-cube faces with equal flux by reflection symmetry. Hence one face of the original cube that does not contain the charge carries

$$\Phi_{\text{face}} = \frac{1}{4}\frac{q}{6\varepsilon_0} = \frac{q}{24\varepsilon_0}.$$

The three faces through the charged corner contribute zero flux because the electric field is tangent to them away from that corner.

Step 2: Add the two shaded triangular contributions The drawing shades one triangular half of the top remote face and one triangular half of the right remote face. On each square, exchanging its two coordinates parallel to the face maps the diagonal halves into each other without changing the normal electric-field component. Each triangle therefore carries half the full-face flux. The two shaded halves

together carry

$$\Phi_{\text{shaded}} = 2\left(\frac{1}{2}\frac{q}{24\varepsilon_0}\right) = \frac{q}{24\varepsilon_0}.$$

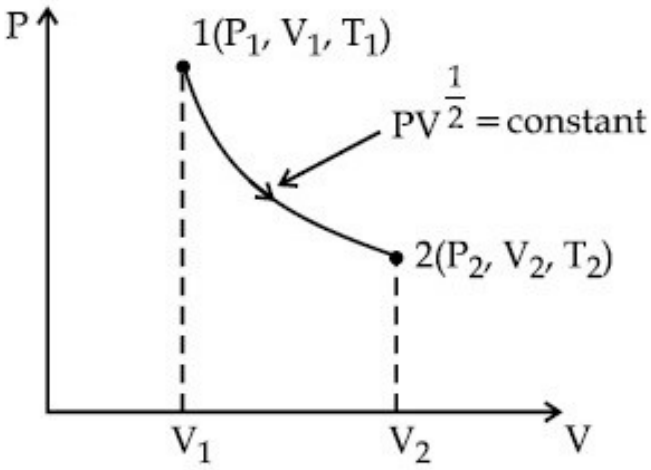
Option (B) counts only one triangle. Options (C) and (D) are too large: (D) is the flux through all three remote faces of the original cube, and (C) exceeds even that total.

Step 3: Conclusion The flux through the complete shaded area is $q/(24\varepsilon_0)$. Hence, the correct option is (A).

Quick Tip to Solve: Count every shaded triangle first: two symmetry-equivalent half faces together carry the flux of one complete remote face.

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40. Thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is : [4 marks]



(A) $\sqrt{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2}$

(D) 2

Correct Answer: (A) $\sqrt{2}$

Solution:

Concept:

- For one mole of an ideal gas, $PV = RT$, so the temperature ratio is $T_2/T_1 = P_2V_2/(P_1V_1)$.
- The curve in the figure specifies $PV^{1/2} =$ constant. This process law, rather than a constant-pressure or constant-temperature assumption, fixes the pressure ratio.

Step 1: Obtain the final pressure from the plotted process Apply the same process constant at the two marked equilibrium states:

$$P_1\sqrt{V_1} = P_2\sqrt{V_2}.$$

Since $V_2 = 2V_1$, division gives

$$\frac{P_2}{P_1} = \sqrt{\frac{V_1}{V_2}} = \frac{1}{\sqrt{2}}.$$

The pressure drops during expansion, as the downward-sloping curve indicates. This ratio alone is not the requested temperature ratio.

Step 2: Combine pressure and volume in the ideal gas equation The amount of gas and the gas constant cancel when the ideal-gas equations at states 1 and 2 are divided:

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{V_2}{V_1} = \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}.$$

Equivalently, the process implies $T \propto \sqrt{V}$. Thus the gas warms even though its pressure falls, because the increase of volume more than compensates for the pressure decrease. Option (B) gives the pressure ratio, not the temperature ratio; (C) reverses the volume ratio, and (D) would require constant pressure.

Step 3: Conclusion The required dimensionless temperature ratio is $T_2/T_1 = \sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a process $PV^n =$ constant, combine it with $PV = RT$ to obtain $T \propto V^{1-n}$.

41. The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from n = 2 to n = 1 state is : [4 marks]

(A) 194.8 nm

(B) 490.7 nm

(C) 913.3 nm

(D) 121.8 nm

Correct Answer: (D) 121.8 nm

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\,\text{eV}$. In emission, the photon energy is the initial atomic energy minus the lower final atomic energy.
- Photon energy and wavelength satisfy $E_\gamma = hc/\lambda$. For the hydrogen spectrum, $1/\lambda = R_\text{H}(1/n_f^2 - 1/n_i^2)$.

Step 1: Calculate the emitted photon energy Here the electron falls from $n_i = 2$ to $n_f = 1$. The energies are $E_2 = -3.4\,\text{eV}$ and $E_1 = -13.6\,\text{eV}$, so

$$E_\gamma = E_2 - E_1 = (-3.4) - (-13.6) = 10.2\,\text{eV}.$$

The positive difference is the energy carried away by the photon. This is the first Lyman-series line, whose wavelength lies in the ultraviolet region.

Step 2: Convert the photon energy into a wavelength Using $hc \simeq 1240\,\text{eV nm}$,

$$\lambda = \frac{hc}{E_\gamma} = \frac{1240}{10.2}\,\text{nm} \simeq 121.6\,\text{nm}.$$

The Rydberg formula gives the same result: $1/\lambda = (3/4)R_\text{H}$. With the rounding of the listed answers, the matching choice is $121.8\,\text{nm}$. The alternatives 194.8, 490.7, and $913.3\,\text{nm}$ would correspond to substantially smaller photon energies, so none can represent the $10.2\,\text{eV}$ transition specified here.

Step 3: Conclusion The wavelength is approximately $121.6\,\text{nm}$, corresponding to the listed value $121.8\,\text{nm}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify the final level first: transitions ending at $n = 1$ are Lyman lines, and the $2 \to 1$ line is near $122\,\text{nm}$.

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42. Y = A sin(ωt + ϕ_0) is the time-displacement equation of a SHM. At t = 0 the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be: [4 marks]

(A) $\frac{5\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{6}$

Correct Answer: (A) $\frac{5\pi}{6}$

Solution:

Concept:

- For $Y = A\sin(\omega t + \phi_0)$, the initial displacement is $Y(0) = A\sin\phi_0$.
- Differentiating displacement gives velocity $v = A\omega\cos(\omega t + \phi_0)$. The sign of the initial velocity determines which phase with the correct sine is appropriate.

Step 1: Use the given initial displacement At $t = 0$, substitute $Y = A/2$ into the displacement equation and divide by the positive amplitude:

$$\sin\phi_0 = \frac{1}{2}.$$

In one full cycle, the two possibilities are $\phi_0 = \pi/6$ and $\phi_0 = 5\pi/6$. Merely taking the principal inverse-sine value would leave the direction of motion unchecked.

Step 2: Select the phase using the initial direction The particle is initially moving in the negative direction along its oscillation axis, so its velocity is negative. Since $A\omega > 0$,

$$v(0) = A\omega\cos\phi_0 < 0 \quad \Longrightarrow \quad \cos\phi_0 < 0.$$

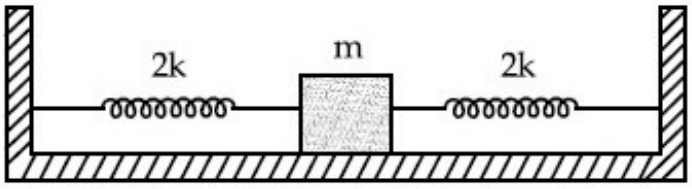
The required phase is therefore in the second quadrant, giving $\phi_0 = 5\pi/6$. Option (D), $\pi/6$, gives the correct positive displacement but positive velocity. Options (B) and (C) both have sine $\sqrt{3}/2$, giving

displacement $\sqrt{3}A/2$, not $A/2$. The displacement and velocity conditions together uniquely select option (A), modulo complete cycles.

Step 3: Conclusion The initial phase is $\phi_0 = 5\pi/6$ radians, modulo 2π. Hence, the correct option is (A).

Quick Tip to Solve: Use displacement to fix the sine and velocity direction to fix the cosine; together their signs locate the correct phase quadrant.

43. Two identical springs of spring constant '2k' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is : [4 marks]



(A) $2\pi\sqrt{\frac{m}{k}}$
(B) $\pi\sqrt{\frac{m}{k}}$
(C) $2\pi\sqrt{\frac{m}{2k}}$
(D) $\pi\sqrt{\frac{m}{2k}}$
Correct Answer: (B) $\pi\sqrt{\frac{m}{k}}$

Solution:

Concept:

- For a displacement x from equilibrium, each spring exerts a restoring force given by Hooke's law, $F = -Kx$. Here the stiffness of each individual spring is $K = 2k$.
- The equation of simple harmonic motion is $m\ddot{x} = -k_{\text{eff}}x$, so $\omega = \sqrt{k_{\text{eff}}/m}$ and $T = 2\pi/\omega$.

Step 1: Add the restoring forces Displace the block a distance x to the right. The left spring stretches by x and pulls the block leftward with magnitude $2kx$. The right spring is compressed by the same distance and also pushes it leftward with magnitude $2kx$. Consequently both forces oppose the displacement and add, rather than cancel:

$$F_{\text{net}} = -2kx - 2kx = -4kx.$$

A leftward displacement reverses both forces and gives the same restoring coefficient.

Step 2: Calculate the oscillation period The effective stiffness is therefore $k_{\text{eff}} = 4k$. Substituting it in the SHM expression gives

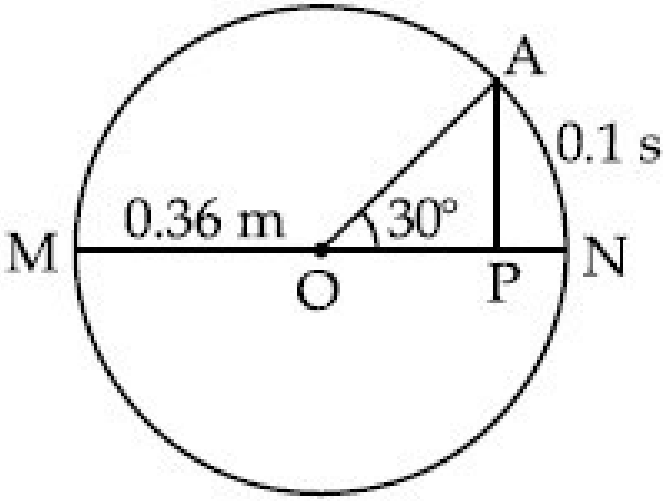
$$\omega = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}, \qquad T = \frac{2\pi}{2\sqrt{k/m}} = \pi\sqrt{\frac{m}{k}}.$$

Option (A) would treat the stiffness as k; option (C) uses only one spring of stiffness $2k$; option (D) would require a stiffness of $8k$. None describes the two spring forces in the figure.

Step 3: Conclusion The time period is $\boxed{T = \pi\sqrt{m/k}}$. Hence, the correct option is (B).

Quick Tip to Solve: When a block lies between two fixed springs, displace it once mentally: extension on one side and compression on the other both restore it, so their stiffnesses add.

44. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be : [4 marks]



(A) 9.87 N

(B) 0.49 N

(C) 50 N

(D) 100 N

Correct Answer: (A) 9.87 N

Solution:

Concept:

- The projection of uniform circular motion of radius R onto a diameter executes SHM with amplitude R and angular frequency equal to the circular angular speed ω.
- The restoring force per unit mass equals acceleration: $F/m = -\omega^2 x$. Its magnitude is maximum at an endpoint, where $|x| = R$.

Step 1: Convert the angular displacement to radians The circular point covers $30^\circ = \pi/6$ radians in $0.1\,\text{s}$, at uniform speed. Thus

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/6}{0.1} = \frac{5\pi}{3}\,\text{rad}\,\text{s}^{-1}.$$

Angular displacement must be in radians before using the SHM acceleration formula; using the

numerical angle 30 would give an incorrect result.

Step 2: Evaluate the acceleration at M At M the projection is at the left endpoint of the diameter, so its distance from O is the amplitude $R = 0.36\,\mathrm{m}$. The restoring acceleration points from M toward O and has magnitude

$$\frac{|F|}{m} = \omega^2 R = \left(\frac{5\pi}{3}\right)^2 (0.36) = \pi^2 \simeq 9.87\,\mathrm{m\,s^{-2}}.$$

The listed numerical value 9.87 agrees with option (A); 0.49, 50 and 100 do not equal this maximum acceleration. The printed options use N, but the physical quantity requested is force per unit mass, whose unit is $\mathrm{N\,kg^{-1}}$.

Step 3: Conclusion The restoring force per unit mass at M is $\boxed{9.87\,\mathrm{N\,kg^{-1}}}$, directed toward O. The matching listed value is 9.87. Hence, the correct option is (A).

Quick Tip to Solve: For a circular-motion projection, read the amplitude directly as the circle radius and use radians per second to compute ω. At either endpoint use $a_{\max} = \omega^2 R$.

45. If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of : [4 marks]

(A) $[MLT^{-1}]$

(B) $[MLT^{0}]$

(C) $[M^0L^0T^0]$

(D) $[LC^{-1}]$

Correct Answer: (C) $[M^0L^0T^0]$

Solution:

Concept:

- Write Coulomb's constant as $k_e = 1/(4\pi\varepsilon_0)$. Coulomb's law is $F = k_e e^2/r^2$, which determines the dimensions of $k_e e^2$ without introducing an electrical base dimension.
- Planck's relation $E = h\nu$ gives $[h] = [ML^2T^{-1}]$; the speed of light has dimensions $[c] = [LT^{-1}]$. Numerical constants such as 4π carry no dimensions.

Step 1: Find the dimensions of the numerator Rearrange Coulomb's law as $k_e e^2 = Fr^2$. Since force has dimensions $[MLT^{-2}]$ and squared separation has dimensions $[L^2]$,

$$\left[\frac{|e|^2}{4\pi\varepsilon_0}\right] = [ML^3T^{-2}].$$

The absolute value changes neither the units nor the power of charge; the numerator is not itself a force, because the factor r^{-2} is absent.

Step 2: Divide by the dimensions of hc The denominator has dimensions

$$[hc] = [ML^2T^{-1}][LT^{-1}] = [ML^3T^{-2}].$$

Consequently the numerator and denominator cancel exactly:

$$\left[\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}\right] = \frac{[ML^3T^{-2}]}{[ML^3T^{-2}]} = [M^0L^0T^0].$$

Options (A) and (B) retain mass and length powers, while option (D) retains length and inverse charge. All are excluded because every base dimension has cancelled.

Step 3: Conclusion The given quantity is dimensionless, with dimensions $\boxed{[M^0L^0T^0]}$. Hence, the correct option is (C).

Quick Tip to Solve: In electrostatic dimension questions, replace $k_e e^2$ by Fr^2; this avoids separately tracking the units of charge and permittivity.

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46. For extrinsic semiconductors; when doping level is increased; [4 marks]

(A) Fermi-level of both p-type and n-type semiconductors will go upward for T > T_FK and downward for T < T_FK, where T_F is Fermi temperature.

(B) Fermi-level of p-type semiconductor will go upward and Fermi-level of n-type semiconductors will go downward

(C) Fermi-level of p and n-type semiconductors will not be affected.

(D) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type semiconductor will go upward.

Correct Answer: (D) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type semiconductor will go upward.

Solution:

Concept:

- The Fermi level E_F determines the equilibrium occupation of electronic energy states. Increasing donor doping increases the electron concentration, while increasing acceptor doping increases the hole concentration.
- In the nondegenerate extrinsic range at fixed temperature, $n = N_C \exp[-(E_C - E_F)/(k_BT)]$ and $p = N_V \exp[-(E_F - E_V)/(k_BT)]$, where E_C and E_V are the conduction- and valence-band edges.

Step 1: Determine the shift for n-type material Donor atoms supply extra electrons to the conduction band. Solving the electron-concentration relation for the Fermi level gives

$$E_F = E_C + k_BT \ln\left(\frac{n}{N_C}\right).$$

At the same temperature, increasing the donor concentration increases n, making the logarithm larger. Thus E_F rises toward the conduction band. An unchanged level would not produce the required increase in occupied electron states.

Step 2: Determine the shift for p-type material Acceptor atoms increase the hole concentration in the

valence band. Rearranging the hole-concentration relation gives

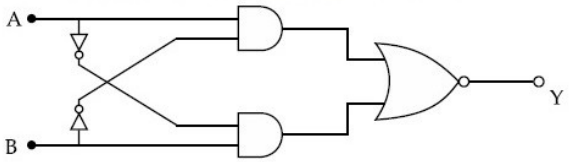
$$E_F = E_V - k_B T \ln\left(\frac{p}{N_V}\right).$$

Increasing p lowers E_F, moving it toward the valence band. Therefore the shifts in the two doping types have opposite directions. Option (B) reverses both shifts; option (C) ignores the change in carrier concentration; option (A) assigns the same direction to both types using a temperature criterion that does not describe this doping change.

Step 3: Conclusion With increased doping, the Fermi level moves downward in a p-type semiconductor and upward in an n-type semiconductor. Hence, the correct option is (D).

Quick Tip to Solve: Draw the conduction band above the valence band: more electrons pull the Fermi level upward toward the conduction band, and more holes move it downward toward the valence band.

47. The truth table for the following logic circuit is : [4 marks]



(A)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(B)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(C)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(D)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Correct Answer: (D)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Solution:

Concept:

- An AND gate gives the product of its inputs; a NOT gate complements its input. The final gate shown has an OR-shaped body with an output bubble, so it is a NOR gate.
- The upper AND gate receives A and $\overline{B}$; the lower AND gate receives $\overline{A}$ and B. The crossing diagonal wires have no junction dot and do not form an electrical connection.

Step 1: Write the Boolean expression The upper product is $A\overline{B}$, and the lower product is $\overline{A}B$. Their sum is the exclusive-OR expression, but the final output is complemented by the NOR bubble:

$$Y = \overline{A\overline{B} + \overline{A}B}.$$

By De Morgan's law, or by testing the four pairs, this equals $AB + \overline{A}\,\overline{B}$, which is the equivalence or XNOR function.

Step 2: Evaluate every input combination For $A = B = 0$, both products entering the NOR gate are zero, so $Y = 1$. For $(A, B) = (0, 1)$ or $(1, 0)$, exactly one AND output is one, so $Y = 0$. For $A = B = 1$, both complemented products are zero, giving $Y = 1$. Hence

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Option (A) is XOR and omits the final complement. Options (B) and (C) respectively depend only on the complement of B and on B, so neither has the required equivalence pattern.

Step 3: Conclusion The output sequence for inputs $00, 01, 10, 11$ is $\boxed{1, 0, 0, 1}$, the XNOR truth table. Hence, the correct option is (D).

Quick Tip to Solve: First identify the final output bubble: the two cross-complemented AND gates generate XOR, and the bubble changes its truth table to XNOR.

48. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is : [4 marks]

(A) 50 m

(B) 25 m

(C) 45 m

(D) 35 m

Correct Answer: (C) 45 m

Solution:

Concept:

- A body released from rest and falling under constant gravitational acceleration travels $s = \frac{1}{2}gt^2$. Both stones are released from rest, but their release times and fall distances differ.
- If the building height is H, the first stone falls through H, while the second stone falls through $H - 25$. The delay between releases is the time taken by the first stone to fall 5 m.

Step 1: Relate the two fall times Let t_0 be the delay and T the first stone's total fall time. The release condition gives

$$t_0 = \sqrt{\frac{2(5)}{g}} = \sqrt{\frac{10}{g}}.$$

Since both stones reach the bottom simultaneously, the second stone falls for $T - t_0$. Accordingly

$$H = \frac{1}{2}gT^2, \qquad H - 25 = \frac{1}{2}g(T - t_0)^2.$$

The 25 m is measured from the building top, not from the first stone's position at that instant.

Step 2: Subtract the equations and find H Subtract the second distance from the first:

$$25 = \frac{1}{2}g\{2Tt_0 - t_0^2\} = gTt_0 - 5.$$

Therefore $gTt_0 = 30$. Using $gt_0^2 = 10$ yields $T/t_0 = 3$, and hence

$$H = \frac{1}{2}g(3t_0)^2 = 9\left(\frac{1}{2}gt_0^2\right) = 9(5) = 45\,\text{m}.$$

The second stone falls 20 m in $2t_0$, consistent with $\frac{1}{2}g(2t_0)^2 = 20$. The alternative heights 50, 25 and 35 m do not satisfy the same arrival-time equations.

Step 3: Conclusion The height of the building is $\boxed{45\,\text{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: For simultaneous arrival after delayed releases, write the two elapsed fall times explicitly. Subtracting the distance equations eliminates the squared total time.

49. The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is : [4 marks]

(A) 400 nm

(B) 329 nm

(C) 309 nm

(D) 382 nm

Correct Answer: (D) 382 nm

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = hc/\lambda - \phi$, where ϕ is the surface work function. The same photosensitive surface has the same work function at both wavelengths.
- Subtracting the two measurements eliminates ϕ: $e(V_2 - V_1) = hc(1/\lambda_2 - 1/\lambda_1)$. A larger stopping potential means a larger photon energy and therefore a shorter wavelength.

Step 1: Find the increase in photon energy The stopping potential increases from $0.710\,\text{V}$ to $1.43\,\text{V}$, giving

$$\Delta V = 1.43 - 0.710 = 0.720\,\text{V}.$$

Thus the photon energy increases by $0.720\,\text{eV}$. The original photon energy, using $hc \simeq 1240\,\text{eV nm}$, is

$$E_1 = \frac{1240}{491} \simeq 2.5255\,\text{eV}.$$

It is the difference of stopping potentials that equals the change in photon energy per electron charge.

Step 2: Calculate the new wavelength The new photon energy is

$$E_2 = E_1 + 0.720 \simeq 3.2455\,\text{eV},$$

so

$$\lambda_2 = \frac{1240}{3.2455}\,\text{nm} \simeq 382.1\,\text{nm}.$$

As a consistency check, 382 nm increases the photon energy by about 0.720 eV. The other choices 400, 329 and 309 nm would give changes of approximately 0.575, 1.244 and 1.488 eV respectively, which do not reproduce the observed stopping-potential increase.

Step 3: Conclusion The new wavelength is approximately $\boxed{382\,\text{nm}}$. Hence, the correct option is (D).

Quick Tip to Solve: For two stopping-potential readings from one surface, subtract Einstein's equations first. This removes the unknown work function and leaves a direct reciprocal-wavelength relation.

50. If a message signal of frequency 'f_m' is amplitude modulated with a carrier signal of frequency 'f_c' and radiated through an antenna, the wavelength of the corresponding signal in air is : [4 marks]

(A) $\frac{c}{f_c+f_m}$
(B) $\frac{c}{f_c}$
(C) $\frac{c}{f_c-f_m}$
(D) $\frac{c}{f_m}$

Correct Answer: (B) $\frac{c}{f_c}$

Solution:

Concept:

- Amplitude modulation varies the amplitude of a high-frequency carrier according to the message. The carrier frequency f_c sets the nominal transmission frequency.
- For a wave travelling in air, its speed is approximately c, so the wavelength belonging to a frequency f is $\lambda = c/f$.

Step 1: Identify the transmission frequency The low-frequency message is placed on the carrier so that an antenna can radiate it efficiently. In ordinary AM terminology, the wavelength of the transmitted signal means its carrier wavelength. The modulation does not replace the carrier frequency by the message frequency.

Step 2: Apply the wave relation and compare the options Using $c = f_c\lambda_c$, we obtain

$$\lambda_c = \frac{c}{f_c}.$$

An AM signal also contains sidebands at $f_c + f_m$ and $f_c - f_m$. Thus options (A) and (C) describe the wavelengths of individual sideband components, rather than the nominal carrier wavelength. Option (D) belongs to the original message and is not the carrier wavelength.

Step 3: Conclusion The corresponding nominal transmitted wavelength is $\boxed{\lambda = c/f_c}$. Hence, the correct option is (B).

Quick Tip to Solve: For the wavelength assigned to an AM transmission, use the carrier frequency; use the sum or difference only for a specified sideband.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$. The value of '$\theta$' will be _____°. [4 marks]

Solution:

Concept:

- The cross product is antisymmetric: $\vec{Q} \times \vec{P} = -\vec{P} \times \vec{Q}$. Its magnitude is $PQ|\sin\theta|$.
- For nonzero vectors, a zero cross product means that the vectors are collinear. The angle must also

satisfy the strict interval printed in the question.

Step 1: Use antisymmetry Substitute the reversed cross product into the given equality:

$$\vec{P} \times \vec{Q} = -\vec{P} \times \vec{Q}.$$

Moving the right side to the left gives $2(\vec{P} \times \vec{Q}) = \vec{0}$, and hence $\vec{P} \times \vec{Q} = \vec{0}$. The equality therefore cannot describe a nonzero cross product.

Step 2: Select the allowed angle Taking magnitudes, $PQ|\sin\theta| = 0$. The angle between vectors presumes that both vectors are nonzero, so $\sin\theta = 0$. In a full revolution the possibilities are $0°$, $180°$, and $360°$. The strict bounds $0° < \theta < 360°$ exclude both endpoints and leave only the antiparallel configuration.

Step 3: Conclusion The required angle is $\boxed{\theta = 180°}$; the numerical entry is 180.

Quick Tip to Solve: When reversing a cross product leaves it unchanged, set it to zero first, then apply the stated angular interval.

52. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be _____%. [4 marks]

Solution:

Concept:

- The transverse wave speed on a stretched string is $v = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length.
- For the same string, μ is unchanged. A small fractional change obeys $\Delta v/v \simeq \frac{1}{2}\Delta T/T$, because the speed depends on the square root of tension.

Step 1: Form the ratio of the speeds An increase of 4% changes the tension from T to $T\prime = 1.04T$. Therefore

$$\frac{v\prime}{v} = \sqrt{\frac{T\prime}{T}} = \sqrt{1.04}.$$

The increase in speed is smaller than the increase in tension; treating the speed as directly proportional to tension would give the wrong result.

Step 2: Evaluate the percentage increase The exact percentage increase is

$$100\left(\sqrt{1.04} - 1\right) = 1.98039\ldots\%.$$

At the precision intended for the numerical entry, this is 2%. Equivalently, the small-change approximation gives $\Delta v/v \simeq (1/2)(0.04) = 0.02$. Both calculations lead to the same rounded numerical entry, while the exact expression identifies the approximation clearly.

Step 3: Conclusion The speed increases by approximately $\boxed{2\%}$ (exactly $1.98039\ldots\%$); enter 2.

Quick Tip to Solve: For a small percentage change in string tension, halve it to estimate the percentage change in transverse wave speed.

53. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be _____. [4 marks]

Solution:

Concept:

- Outside a spherical Earth, gravitational potential energy is $U(r) = -GMm/r$, with zero at infinity. A height above the surface must be added to the Earth's radius to obtain r.
- The escape speed at the surface satisfies $v_e^2 = 2GM/R$. For the minimum projection speed to reach a specified height, the speed at that highest point is zero.

Step 1: Write conservation of mechanical energy The body starts at $r_i = R$ and reaches a height $10R$, so its final distance from the centre is $r_f = 11R$. Neglecting air resistance and Earth's rotation, conservation of energy gives

$$\frac{1}{2}mv_i^2 - \frac{GMm}{R} = -\frac{GMm}{11R}.$$

Using $10R$ as the final centre distance would confuse height with radius.

Step 2: Express the initial speed in terms of escape speed Rearrange the energy equation:

$$\frac{1}{2}mv_i^2 = \frac{GMm}{R}\left(1 - \frac{1}{11}\right) = \frac{10GMm}{11R}.$$

Cancel the body's mass and use $2GM/R = v_e^2$:

$$v_i^2 = \frac{10}{11}v_e^2, \qquad v_i = \sqrt{\frac{10}{11}}\,v_e.$$

The ratio is less than one, as expected because the target altitude is finite rather than infinity.

Step 3: Conclusion In lowest terms, $x/y = 10/11$, so $\boxed{x = 10}$ (dimensionless).

Quick Tip to Solve: For a height h above Earth, the final centre distance is R+h; compare the energy result with the surface escape speed.

54. The peak electric field produced by the radiation coming from the 8W bulb at a distance of 10 m is $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}\frac{V}{m}}$. The efficiency of the bulb is 10% and it is a point source. The value of x is _____. [4 marks]

Solution:

Concept:

- A point source radiating isotropically has mean intensity $I = P_{\text{rad}}/(4\pi r^2)$, where P_{rad} is its radiated power.
- For a sinusoidal electromagnetic wave, the mean intensity and peak electric field are related by $I = E_0^2/(2\mu_0 c)$. The factor of two distinguishes the peak field from the rms field.

Step 1: Find the radiated power and intensity Only 10% of the bulb's $8\,\text{W}$ power is emitted as the

stated radiation. Thus $P_{\text{rad}} = 0.10 \times 8 = 0.8\,\text{W}$. At $r = 10\,\text{m}$, the radiation is spread over a sphere, giving

$$I = \frac{0.8}{4\pi(10)^2} = \frac{0.002}{\pi}\,\text{W m}^{-2}.$$

Using the entire electrical power would ignore the given efficiency.

Step 2: Calculate the peak field and identify its coefficient Solve the electromagnetic intensity relation for E_0:

$$E_0 = \sqrt{2\mu_0 cI} = \sqrt{\frac{0.004\mu_0 c}{\pi}} = \frac{\sqrt{0.4}}{10}\sqrt{\frac{\mu_0 c}{\pi}}\,\text{V m}^{-1}.$$

Comparing this directly with the printed expression $(x/10)\sqrt{\mu_0 c/\pi}\,\text{V m}^{-1}$ gives the required dimensionless coefficient.

Step 3: Conclusion The literal stated power $8\,\text{W}$ and efficiency 10% give $\boxed{x = \sqrt{0.4} = 0.632455\ldots}$, dimensionless. This differs from the supplied key value 2: that value would require $8\,\text{W}$ of radiated power, whereas the printed efficiency gives only $0.8\,\text{W}$. Thus $x = 2$ would follow from a bulb power of $80\,\text{W}$, not the printed $8\,\text{W}$; it does not satisfy the literal data.

Quick Tip to Solve: Apply efficiency before inverse-square spreading, and use the peak-field intensity formula with its factor of two.

---

55. Two identical conducting spheres with negligible volume have 2.1 nC and −0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is _____ $\times$ 10^{-9} N.

[Given : $4\pi\varepsilon_0 = \frac{1}{9\times10^9}$ SI unit] [4 marks]

Solution:

Concept:

- Total electric charge is conserved when conducting spheres touch. Identical spheres have equal capacitances, so equal final potentials imply equal final charges.
- After separation, negligible-sized spheres act as point charges: $F = k|q_1 q_2|/r^2$, with $k = 1/(4\pi\varepsilon_0) = 9 \times 10^9\,\text{N m}^2\,\text{C}^{-2}$.

Step 1: Determine the charge on each sphere after contact The total charge is

$$Q = 2.1\,\text{nC} + (-0.1\,\text{nC}) = 2.0\,\text{nC}.$$

Since the spheres are identical, contact divides this charge equally:

$$q_1' = q_2' = \frac{Q}{2} = 1.0\,\text{nC} = 1.0 \times 10^{-9}\,\text{C}.$$

Both final charges are positive. The initial attraction therefore changes to repulsion after the spheres touch and separate.

Step 2: Substitute the final charges into Coulomb's law At the final centre separation $r = 0.5\,\mathrm{m}$,

$$F = \frac{(9\times10^{9})(1.0\times10^{-9})^{2}}{(0.5)^{2}} = \frac{9\times10^{-9}}{0.25} = 36\times10^{-9}\,\mathrm{N}.$$

The force must be calculated with the redistributed charges, not with the unequal initial charges. The positive charge product confirms a repulsive force of equal magnitude on both spheres.

Step 3: Conclusion The force is $\boxed{36\times10^{-9}\,\mathrm{N}}$, repulsive; the required numerical entry is 36.

Quick Tip to Solve: For identical spheres brought into contact, replace both initial charges by their signed average before applying Coulomb's law.

---

56. The wavelength of an X-ray beam is 10 $\overset{o}{A}$. The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3}h$ kg. The value of x is _____. (h = Planck's constant) [4 marks]

Solution:

Concept:

- A photon of wavelength λ has energy $E = hc/\lambda$. The equivalent mass associated with this energy is obtained from $E = mc^{2}$.
- One angstrom is $10^{-10}\,\mathrm{m}$. Hence the given wavelength 10 angstroms is $10^{-9}\,\mathrm{m}$. Here h denotes Planck's constant as in the question.

Step 1: Relate the photon energy to the stated mass Equate the photon energy to the rest-energy expression for the fictitious particle:

$$mc^{2} = \frac{hc}{\lambda}.$$

Cancel one factor of c and solve for mass:

$$m = \frac{h}{\lambda c}.$$

This is an energy-equivalent mass; the photon itself still has zero rest mass. The use of a fictitious particle allows the question to express the same energy through mc^{2}.

Step 2: Insert the wavelength in SI units Use $\lambda = 10\times10^{-10} = 10^{-9}\,\mathrm{m}$ and $c = 3\times10^{8}\,\mathrm{m\,s^{-1}}$. Then

$$m = \frac{h}{(10^{-9})(3\times10^{8})} = \frac{h}{3\times10^{-1}} = \frac{10}{3}h\,\mathrm{kg},$$

where the last expression uses the SI numerical value of h, as in the printed question. Comparing its coefficient with $(x/3)h\,\mathrm{kg}$ identifies x without substituting the numerical value of Planck's constant.

Step 3: Conclusion The equivalent mass is $\boxed{m = (10/3)h\,\mathrm{kg}}$ in the question's notation, giving $\boxed{x = 10}$.

Quick Tip to Solve: Combine E=hc/lambda with E=mc squared first, then convert angstroms to metres before substituting.

57. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\frac{a}{21} \times 10^{-8}$C. The value of 'a' will be _____. [Given g = 10 ms^{-2}] [4 marks]

Solution:

Concept:

- Each sphere is at rest under three forces: its weight mg downwards, the thread tension T, and the horizontal Coulomb repulsion F. Resolving the tension gives $T\cos\theta = mg$ and $T\sin\theta = F$, where θ is the angle with the vertical.
- For equal charges q separated by r, $F = kq^2/r^2$, with $k = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$. Symmetry gives $\sin\theta = r/(2l)$.

Step 1: Find the thread inclination The separation is shared equally about the vertical through the support. Therefore each sphere is displaced horizontally by $0.10\,\mathrm{m}$. With $l = 0.50\,\mathrm{m}$,

$$\sin\theta = \frac{0.10}{0.50} = \frac{1}{5}, \qquad \cos\theta = \frac{\sqrt{24}}{5}, \qquad \tan\theta = \frac{1}{\sqrt{24}}.$$

The given mass is $10\,\mathrm{mg} = 10^{-5}\,\mathrm{kg}$, so the weight is $10^{-4}\,\mathrm{N}$.

Step 2: Balance the electrical and mechanical forces Dividing the two tension equations removes the unknown tension:

$$F = mg\tan\theta = \frac{10^{-4}}{\sqrt{24}}\,\mathrm{N}.$$

Substitute this in Coulomb's law using the full separation $r = 0.20\,\mathrm{m}$:

$$q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{10^{-4}(0.20)^2}{9 \times 10^9\sqrt{24}}} \approx 9.525 \times 10^{-9}\,\mathrm{C}.$$

Step 3: Read the requested coefficient The question expresses the same charge as $q = (a/21) \times 10^{-8}\,\mathrm{C}$. Hence

$$a = \frac{21q}{10^{-8}} \approx 20.002.$$

Thus the numerical coefficient is 20 to the required integer precision; the small rounding difference comes from the square root in the exact geometry.

Step 4: Conclusion The charge on each sphere is approximately $9.52 \times 10^{-9}\,\mathrm{C}$, so the required dimensionless value is $\boxed{a = 20}$.

Quick Tip to Solve: Use half the separation to obtain the thread angle, but use the complete separation in Coulomb's law. Dividing the two tension equations avoids solving for the tension.

58. Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is n : 2. The value of n will be _____.

[4 marks]

Solution:

Concept:

- For a nonrelativistic particle, $K = p^2/(2m)$, where K is kinetic energy, p is the magnitude of linear momentum, and m is mass.
- When two particles have equal kinetic energies, their momenta are proportional to the square roots of their masses: $p \propto \sqrt{m}$. Equal energy does not imply equal speed or equal momentum when the masses differ.

Step 1: Relate momentum to the common energy Let the common kinetic energy be K, and label the 4 g and 16 g particles by 1 and 2 respectively. Solving the energy equation for the positive momentum magnitude gives

$$p_1 = \sqrt{2m_1K}, \qquad p_2 = \sqrt{2m_2K}.$$

Both momenta contain the same factor $\sqrt{2K}$. The particles need not move in the same direction because the question asks only for magnitudes.

Step 2: Take the ratio in the stated mass order Dividing the two expressions eliminates the unspecified common energy:

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = \frac{1}{2}.$$

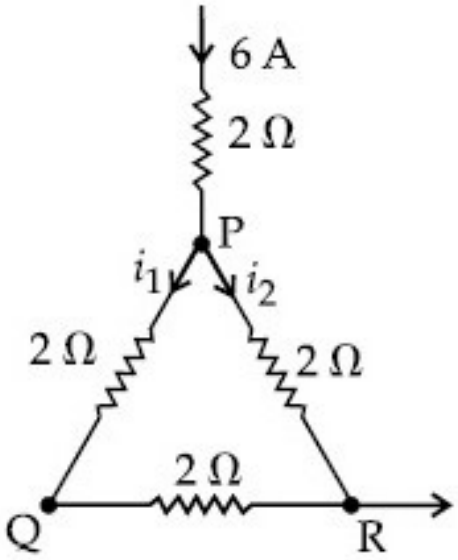
The same mass unit is used in numerator and denominator, so conversion from grams to kilograms is unnecessary for this dimensionless ratio. The heavier particle has greater momentum even though its speed is lower.

Step 3: Match the requested form The result is $p_1 : p_2 = 1 : 2$. Comparing it with the given form $n : 2$, with the 4 g particle written first, immediately gives $n = 1$. As a check, equal kinetic energies also imply $v_1/v_2 = \sqrt{m_2/m_1} = 2$, so $m_1v_1/(m_2v_2) = (4/16) \times 2 = 1/2$, confirming the momentum ratio.

Step 4: Conclusion The ratio of the momentum magnitudes is $1 : 2$, and therefore $\boxed{n = 1}$ (dimensionless).

Quick Tip to Solve: At fixed kinetic energy use the square root of the mass ratio for momentum; the inverse square root gives the speed ratio instead.

59. A current of 6A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R. The currents i_1 in ampere is _____. [4 marks]



Solution:

Concept:

- Kirchhoff's current law states that the total current entering a junction equals the total current leaving it. At P, the supplied current divides into the two paths connecting P and R.
- Parallel paths have the same potential difference. Their currents are inversely proportional to their equivalent resistances. Resistors in series add when no current leaves their intermediate junction.

Step 1: Identify the two parallel paths The direct path P–R contains one $2\,\Omega$ wire. The alternative path P–Q–R contains two $2\,\Omega$ wires in series, because Q has no external connection. Their resistance is

$$R_{PQR} = 2 + 2 = 4\,\Omega.$$

Thus the P–R terminals see a $2\,\Omega$ branch in parallel with a $4\,\Omega$ branch. The marked current i_1 flows from P towards Q, so it is the current in the longer branch.

Step 2: Use the common terminal voltage Let V be the potential difference from P to R. Then

$$i_1 = \frac{V}{4}, \qquad i_2 = \frac{V}{2} = 2i_1.$$

The two branch currents are not equal even though the individual wires have the same resistance: one complete path contains twice as much resistance as the other. The extra resistor on the incoming lead carries the full prescribed current and does not change its division inside the triangle.

Step 3: Apply current conservation at P The given current entering P is $6\,\mathrm{A}$, so

$$i_1 + i_2 = 6, \qquad i_1 + 2i_1 = 6.$$

Therefore $i_1 = 2\,\mathrm{A}$ and $i_2 = 4\,\mathrm{A}$. Both branches have a voltage drop of $8\,\mathrm{V}$: $2 \times 4 = 4 \times 2 = 8$. This verifies the parallel-branch condition and the stated arrow directions.

Step 4: Conclusion The current marked along P to Q is $\boxed{i_1 = 2\,\text{A}}$. The numerical entry in ampere is $\boxed{2}$.

Quick Tip to Solve: Compare complete paths between the entry and exit corners. The two-edge path has twice the resistance, so it carries one-third of the supplied current.

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60. A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52K, its efficiency is doubled. The temperature in Kelvin of the source will be _____. [4 marks]

Solution:

Concept:

- A reversible heat engine operating between a source temperature T_H and sink temperature T_C has efficiency $\eta = 1 - T_C/T_H$. Temperatures must be absolute temperatures in kelvin.
- Efficiency is the work output divided by the heat absorbed: $\eta = W/Q_H$. Reducing the sink temperature while leaving the source fixed increases efficiency by the temperature reduction divided by T_H.

Step 1: Translate the initial work fraction The engine converts one-fourth of the supplied heat into work. Therefore its initial efficiency is $\eta_1 = 1/4$. The Carnot relation gives

$$1 - \frac{T_C}{T_H} = \frac{1}{4}, \qquad \frac{T_C}{T_H} = \frac{3}{4}.$$

We do not need the actual amount of heat absorbed, because it cancels in the efficiency ratio.

Step 2: Write the efficiency after cooling the sink The source remains at T_H, while the sink changes from T_C to $T_C - 52$. The doubled efficiency is $\eta_2 = 2\eta_1 = 1/2$. Hence

$$1 - \frac{T_C - 52}{T_H} = \frac{1}{2}.$$

Subtracting the initial efficiency equation from this new equation eliminates T_C:

$$\frac{52}{T_H} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

Step 3: Solve and verify both operating states It follows that $T_H = 52 \times 4 = 208\,\text{K}$. Initially, $T_C = (3/4)208 = 156\,\text{K}$; after cooling, the sink is at $104\,\text{K}$. These yield

$$\eta_1 = 1 - \frac{156}{208} = \frac{1}{4}, \qquad \eta_2 = 1 - \frac{104}{208} = \frac{1}{2}.$$

Both sinks are colder than the source, and the final efficiency is exactly twice the original.

Step 4: Conclusion The source temperature is $\boxed{208\,\text{K}}$. The required numerical entry is $\boxed{208}$.

Quick Tip to Solve: At a fixed source temperature, use $\Delta\eta = \Delta T_C/T_H$ for a decrease in sink temperature; this eliminates the unknown initial sink temperature.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is : [4 marks]

(A) $CH_3COCl/AlCl_3, Br_2/AlBr_3, HNO_3/H_2SO_4, Zn/HCl$

(B) $CH_3COCl/AlCl_3, Zn-Hg/HCl, Br_2/AlBr_3, HNO_3/H_2SO_4$

(C) $Br_2/AlBr_3, CH_3COCl/AlCl_3, HNO_3/H_2SO_4, Zn/HCl$

(D) $HNO_3/H_2SO_4, Br_2/AlCl_3, CH_3COCl/AlCl_3, Zn-Hg/HCl$

Correct Answer: (B) $CH_3COCl/AlCl_3, Zn-Hg/HCl, Br_2/AlBr_3, HNO_3/H_2SO_4$

Solution:

Concept:

- An ethyl substituent activates a benzene ring and directs electrophilic substitution to the ortho and para positions. A carbonyl group directly attached to the ring, such as $-COCH_3$, instead deactivates the ring and is meta-directing.
- Friedel–Crafts acylation with $CH_3COCl/AlCl_3$ gives acetophenone; Clemmensen reduction with $Zn(Hg)/HCl$ converts its carbonyl group into a methylene group, giving ethylbenzene.

Step 1: Install the ethyl group before further substitution Acylate benzene first, then reduce the side-chain carbonyl:

$$C_6H_6 \xrightarrow{CH_3COCl/AlCl_3} C_6H_5COCH_3 \xrightarrow{Zn(Hg)/HCl} C_6H_5CH_2CH_3.$$

This two-step route installs the required two-carbon ethyl group. Reducing before bromination is essential, because the directing influence of an ethyl group differs from that of an acetyl group.

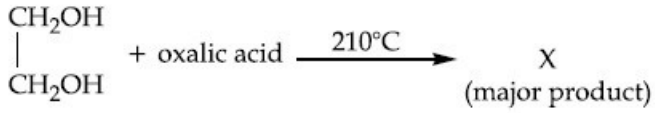
Step 2: Brominate and then nitrate the ring Bromination of ethylbenzene using $Br_2/AlBr_3$ favours the para product over the more crowded ortho product. In para-bromoethylbenzene, nitration with HNO_3/H_2SO_4 is directed mainly by the strongly activating ethyl group. The equivalent positions 2 and 6 are ortho to ethyl, giving 4-bromo-2-nitroethylbenzene when ethyl is numbered as position 1.

Step 3: Reject the other reaction orders Option A brominates while the side chain is still acetyl, so its meta-directing effect places bromine incorrectly; subsequent reduction cannot move bromine. Option C installs acetyl on bromobenzene before nitration, so the acetyl group directs nitration meta to itself rather than to the required ortho position. Option D starts with nitrobenzene, which is strongly deactivated and does not undergo the required Friedel–Crafts acylation. Only option B establishes the ethyl directing group at the correct stage.

Step 4: Conclusion The required order is $CH_3COCl/AlCl_3$, then $Zn(Hg)/HCl$, then $Br_2/AlBr_3$, and finally HNO_3/H_2SO_4. Hence, the correct option is (B).

Quick Tip to Solve: Track the directing group present at each substitution step. Reduce acetyl to ethyl before bromination when the target requires bromine para to the ethyl group.

62. What is 'X' in the given reaction? [4 marks]



(A) $HOCH_2-CHO$

(B) $CH_2=CH-OH$

(C) $CH_2=CH_2$

(D) $OHC-CHO$

Correct Answer: (C) $CH_2=CH_2$

Solution:

Concept:

- Ethylene glycol is a vicinal diol, $HOCH_2CH_2OH$, and oxalic acid is a dicarboxylic acid, $HOOC-COOH$. Esterification between these difunctional molecules can form the cyclic diester ethylene oxalate.
- At the stated high temperature, the cyclic oxalate decomposes to an alkene with loss of two molecules of carbon dioxide. The two carbon atoms originally belonging to ethylene glycol remain together in ethene.

Step 1: Form the cyclic oxalate ester Each hydroxyl group of ethylene glycol esterifies one carboxyl group of oxalic acid. Closing the ring eliminates two molecules of water and gives ethylene oxalate, a cyclic diester containing the sequence $-O-CH_2-CH_2-O-CO-CO-$. The atom balance for this intermediate is

$$C_2H_6O_2 + C_2H_2O_4 \longrightarrow C_4H_4O_4 + 2\,H_2O.$$

The formula $C_4H_4O_4$ belongs to the cyclic ester; it is not the final product X.

Step 2: Decompose the ester at the given temperature On strong heating at $210\,^{\circ}C$, ethylene oxalate fragments with loss of carbon dioxide. The carbon–carbon bond of the glycol fragment becomes a double bond:

$$C_4H_4O_4 \longrightarrow C_2H_4 + 2\,CO_2.$$

This equation conserves four carbon atoms, four hydrogen atoms and four oxygen atoms. Both oxygens attached to the glycol carbon atoms leave with the oxalate-derived carbon atoms, so the organic product has no oxygen and is ethene.

Step 3: Select the structure with the correct carbon skeleton Option C is $CH_2=CH_2$, the ethene produced by this fragmentation. Option A is glycolaldehyde, $HOCH_2CHO$, which would require partial oxidation and retains two oxygens. Option B is vinyl alcohol, $CH_2=CHOH$, which retains an oxygen and corresponds to a different transformation. Option D is glyoxal, $OHC-CHO$, requiring oxidation of both alcohol groups. None of A, B or D represents the deoxygenation through the cyclic oxalate under the stated conditions.

Step 4: Conclusion The major product is $\boxed{CH_2=CH_2}$, ethene, formed by thermolysis of ethylene oxalate. Hence, the correct option is (C).

Quick Tip to Solve: For a vicinal diol heated with oxalic acid, identify the cyclic oxalate first; loss of two CO_2 molecules leaves the corresponding alkene.

63. The solubility of $Ca(OH)_2$ in water is :

[Given : The solubility product of $Ca(OH)_2$ in water = 5.5×10^{-6}] [4 marks]

(A) 1.11×10^{-6}

(B) 1.11×10^{-2}

(C) 1.77×10^{-6}

(D) 1.77×10^{-2}

Correct Answer: (B) 1.11×10^{-2}

Solution:

Concept:

- For the dissolution $\mathrm{Ca(OH)_2(s)} \rightleftharpoons \mathrm{Ca^{2+}(aq)} + 2\,\mathrm{OH^-(aq)}$, the solubility product is $K_{sp} = [\mathrm{Ca^{2+}}][\mathrm{OH^-}]^2$. The solid has unit activity and is omitted.
- If the molar solubility is $s\,\mathrm{mol\,L^{-1}}$, the dissolved salt supplies $[\mathrm{Ca^{2+}}] = s$ and $[\mathrm{OH^-}] = 2s$. Consequently, $K_{sp} = 4s^3$, not s^2 or $2s^3$.

Step 1: Write the equilibrium concentrations One mole of calcium hydroxide releases one mole of calcium ions and two moles of hydroxide ions. Therefore a saturated solution with molar solubility s has the concentrations s and $2s$, respectively. The hydroxide from water autoionisation is negligible compared with the concentration supplied by this base. Substitute the stoichiometric concentrations into the expression for the equilibrium constant:

$$5.5 \times 10^{-6} = s(2s)^2 = 4s^3.$$

Step 2: Calculate the cube root Rearrange before taking the root:

$$s^3 = \frac{5.5 \times 10^{-6}}{4} = 1.375 \times 10^{-6}.$$

Therefore

$$s = (1.375 \times 10^{-6})^{1/3} = 1.112 \times 10^{-2}\,\mathrm{mol\,L^{-1}}.$$

The exponent is divided by three, since the solubility enters the equilibrium expression cubed.

Step 3: Check the value and the alternatives Using $s \approx 0.0111$ gives $[\mathrm{OH^-}] \approx 0.0222\,\mathrm{mol\,L^{-1}}$; the product $0.0111(0.0222)^2$ is approximately 5.47×10^{-6}, consistent with the given value after rounding. Options A and C incorrectly keep a solubility of order 10^{-6}. Option D does not satisfy $4s^3 = 5.5 \times 10^{-6}$. The value in option B has both the correct coefficient and exponent.

Step 4: Conclusion The molar solubility is $\boxed{1.11 \times 10^{-2}\,\mathrm{mol\,L^{-1}}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a salt of the form $M(OH)_2$, write the hydroxide concentration as $2s$ before squaring it; then take a cube root to obtain solubility.

64. Which among the following species has unequal bond lengths? [4 marks]

(A) XeF_4

(B) BF_4^-

(C) SiF_4

(D) SF_4

Correct Answer: (D) SF_4

Solution:

Concept:

- VSEPR theory distinguishes bonding pairs and lone pairs around a central atom. A lone pair repels bonding pairs more strongly than another bonding pair does.
- In a trigonal-bipyramidal electron-domain arrangement, axial and equatorial bonds occupy geometrically different environments. Equivalent ligands do not imply equal bond lengths unless symmetry makes their positions equivalent.

Step 1: Count the electron domains Sulfur has six valence electrons. Four are used in bonding to four fluorine atoms and two remain as one lone pair. Thus SF_4 has five electron domains: four bond pairs and one lone pair. Its electron-domain geometry is trigonal bipyramidal, and the lone pair preferentially occupies an equatorial position to reduce the number of close, $90°$, interactions.

Step 2: Distinguish the sulfur-fluorine bonds The molecular shape of SF_4 is seesaw. Two fluorine atoms are axial and two are equatorial. An axial bond experiences more close interactions with the equatorial electron domains than an equatorial bond does; the axial sulfur-fluorine bonds are therefore longer than the equatorial bonds. There are two distinct sets of bond lengths rather than four equivalent bonds.

Step 3: Eliminate the symmetric alternatives XeF_4 is square planar, with its two lone pairs in opposite positions, so the four xenon-fluorine bonds are symmetry-equivalent. BF_4^- and SiF_4 are regular tetrahedral species with four equivalent central-atom-fluorine bonds. Hence none of (A), (B) and (C) has the inequivalent axial and equatorial bond positions required here.

Step 4: Conclusion SF_4 has unequal axial and equatorial sulfur-fluorine bond lengths. Hence, the correct option is (D).

Quick Tip to Solve: For identical surrounding atoms, check positional equivalence: a seesaw geometry retains two axial and two equatorial bonds.

65. In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment?

(i) $[FeF_6]^{3-}$

(ii) $[Co(NH_3)_6]^{3+}$

(iii) $[NiCl_4]^{2-}$

(iv) $[Cu(NH_3)_4]^{2+}$ [4 marks]

(A) (iii) > (iv) > (ii) > (i)

(B) (ii) > (iii) > (i) > (iv)

(C) (i) > (iii) > (iv) > (ii)

(D) (ii) > (i) > (iii) > (iv)

Correct Answer: (C) (i) > (iii) > (iv) > (ii)

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons. It increases with n.
- Find the metal oxidation state and its d-electron count first; ligand strength and coordination geometry then determine electron pairing.

Step 1: Assign the metal configurations In (i), six fluoride ions and the overall charge -3 give iron in oxidation state $+3$, so Fe^{3+} is d^5. In (ii), ammonia is neutral and cobalt is $+3$, giving d^6. In (iii), four chlorides and charge -2 give Ni^{2+}, which is d^8. In (iv), neutral ammonia and charge $+2$ give Cu^{2+}, which is d^9.

Step 2: Count the unpaired electrons Fluoride is a weak-field ligand, so octahedral $[FeF_6]^{3-}$ is high spin with five unpaired electrons. The cobalt(III)-ammine complex is low spin: its six electrons fill t_{2g}^6, leaving none unpaired. Tetrahedral $[NiCl_4]^{2-}$ has two unpaired electrons. A d^9 copper(II) complex has one unpaired electron.

Step 3: Compare the moments and the options The respective values of n for (i), (ii), (iii), (iv) are $5, 0, 2, 1$. Thus their moments are $\sqrt{35}, 0, \sqrt{8}, \sqrt{3}$ Bohr magnetons. Decreasing order is therefore

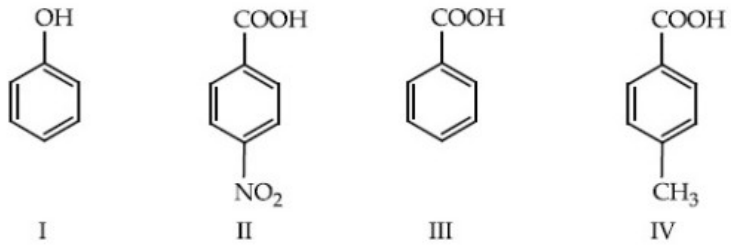
$$(i) > (iii) > (iv) > (ii).$$

Options (B) and (D) incorrectly put diamagnetic (ii) first; (A) puts the highest-moment iron complex last.

Step 4: Conclusion The decreasing spin-only magnetic moment is $(i) > (iii) > (iv) > (ii)$. Hence, the correct option is (C).

Quick Tip to Solve: Rank unpaired-electron counts directly; calculating decimal magnetic moments is unnecessary.

66. The correct order of acid character of the following compounds is : [4 marks]



(A) IV > III > II > I

(B) III > II > I > IV

(C) II > III > IV > I

(D) I > II > III > IV

Correct Answer: (C) II > III > IV > I

Solution:

Concept:

- An acid is stronger when its conjugate base is more stable. Resonance delocalisation and electron-withdrawing substituents stabilise a negative charge.
- Aromatic carboxylic acids are substantially stronger acids than phenol. For para-substituted benzoic acids, a nitro group increases acidity, whereas a methyl group decreases it.

Step 1: Identify the four structures Structure I is phenol, II is para-nitrobenzoic acid, III is benzoic acid and IV is para-methylbenzoic acid. Compare loss of the hydroxyl proton in each case. The three carboxylic acids form carboxylate ions in which negative charge is delocalised between two oxygen atoms; the two principal resonance contributors are equivalent.

Step 2: Rank the three carboxylic acids The para nitro group in II withdraws electron density by both inductive and resonance effects, stabilising the carboxylate conjugate base. II is therefore more acidic than unsubstituted III. The methyl group in IV donates electron density through its inductive effect and hyperconjugation, destabilising the anion relative to benzoate. Hence the carboxylic-acid order is

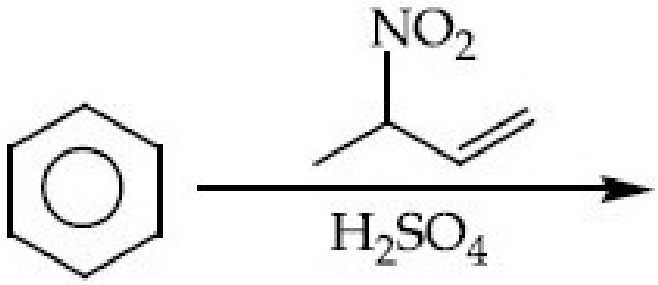
$$\text{II} > \text{III} > \text{IV}.$$

Step 3: Place phenol and reject the other orders Although phenoxide is resonance-stabilised, some contributing structures move the negative charge onto carbon, and its stabilisation is less effective than the equivalent oxygen-based resonance in a carboxylate. Thus phenol I is the weakest of the four. Option (A) incorrectly makes the methyl-substituted acid strongest; (B) puts phenol above a carboxylic acid; (D) puts phenol first. Only (C) respects both comparisons.

Step 4: Conclusion The correct acid-strength order is II > III > IV > I. Hence, the correct option is (C).

Quick Tip to Solve: First separate phenols from carboxylic acids, then compare electron-withdrawing and electron-donating substituents.

67. The major product of the following reaction is : [4 marks]



(A) $C_6H_5CH(CH_3)CH(NO_2)CH_3$

(B) $C_6H_5CH(NO_2)CH(CH_3)_2$

(C) $C_6H_5C(NO_2)(CH_3)CH_2CH_3$

(D) $C_6H_5CH_2CH(NO_2)CH_3$

Correct Answer: (A) $C_6H_5CH(CH_3)CH(NO_2)CH_3$

Solution:

Concept:

- An alkene can be protonated in acid to produce a carbocation, which acts as an electrophile toward benzene. Addition is favoured when protonation avoids a primary carbocation.
- In electrophilic aromatic substitution, the ring forms a carbon-carbon bond to the electrophile; loss of a ring proton restores aromaticity. The carbon skeleton and nitro-group position must be tracked.

Step 1: Locate the protonation site The alkene shown is $CH_3CH(NO_2)CH{=}CH_2$. Protonation at the terminal alkene carbon produces the secondary carbocation

$$CH_3CH(NO_2)CH^+CH_3.$$

Protonation in the opposite direction would produce a primary terminal carbocation and is less favourable. The nitro substituent remains attached to the original second carbon.

Step 2: Attach the benzene ring The benzene pi system attacks the positively charged carbon, giving a sigma complex. Removal of the proton from the attacked ring carbon regenerates aromaticity and the acid catalyst. The resulting side chain is

$$C_6H_5CH(CH_3)CH(NO_2)CH_3.$$

Reading it from either end gives the same four-carbon skeleton: phenyl and nitro are attached to adjacent internal carbons.

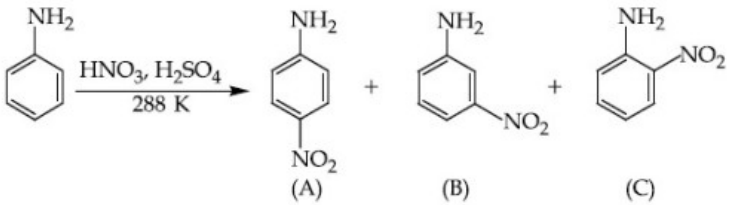
Step 3: Compare all the proposed structures Option (A) has precisely this connectivity. Option (B) puts phenyl on the nitro-bearing carbon with a branched methyl arrangement. Option (C) also attaches phenyl directly to the nitro-bearing carbon and requires a different carbocation position. Such a shift would place positive charge on the strongly electron-withdrawing nitro-bearing centre rather than the directly generated secondary centre. Option (D) has only a three-carbon side chain

and does not conserve the four-carbon alkene skeleton.

Step 4: Conclusion The major product is $C_6H_5CH(CH_3)CH(NO_2)CH_3$. Hence, the correct option is (A).

Quick Tip to Solve: Number the alkene chain before protonation, and retain those carbon labels while attaching the aromatic ring.

68. Correct statement about the given chemical reaction is : [4 marks]



(A) The $-NH_2$ group is ortho and para directive, so product (B) is not possible.

(B) Reaction is possible and compound (B) will be the major product.

(C) Reaction is possible and compound (A) will be the major product.

(D) The reaction will form sulphonated product instead of nitration.

Correct Answer: (C) Reaction is possible and compound (A) will be the major product.

Solution:

Concept:

- The free amino group activates benzene and directs electrophilic substitution to the ortho and para positions by resonance donation of its nitrogen lone pair.
- In strongly acidic nitrating mixture, aniline is partly converted into the anilinium ion, $C_6H_5NH_3^+$, which is deactivating and meta directing. Both free and protonated forms must be considered.

Step 1: Identify the products in the scheme The labelled structures are (A), para-nitroaniline; (B), meta-nitroaniline; and (C), ortho-nitroaniline. The reagent mixture HNO_3/H_2SO_4 generates the nitronium electrophile. At the stated temperature of $288\,K$, direct nitration gives a mixture rather than exclusive ortho/para substitution.

Step 2: Account for the substantial meta product Sulfuric acid protonates the amino nitrogen:

$$C_6H_5NH_2 + H^+ \rightleftharpoons C_6H_5NH_3^+.$$

In the anilinium ion the nitrogen lone pair is no longer available for resonance donation. The positively charged substituent directs incoming electrophiles meta. Therefore the presence of product (B) is chemically possible, and option (A), which declares it impossible, is false.

Step 3: Select the major product and exclude the alternatives The standard direct nitration distribution is approximately para 51%, meta 47%, and ortho 2%. The reactive free-aniline fraction gives ortho/para substitution, with para favoured over the sterically crowded ortho position. Consequently the para product (A) is slightly more abundant than meta (B): statement (C) is correct and statement

(B) is not. The given mixed-acid conditions are nitrating conditions, so statement (D), replacing nitration by sulphonation, is also false.

Step 4: Conclusion The reaction occurs and para-nitroaniline is the major product. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish product labels from option letters: product (A) is para-nitroaniline, but the correct statement is option (C).

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69. Given below are two statements :

Statement I : The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH.

Statement II : The dimethyl glyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is false but Statement II is true.

(B) Statement I is true but Statement II is false.

(C) Both Statement I and Statement II are false.

(D) Both Statement I and Statement II are true.

Correct Answer: (D) Both Statement I and Statement II are true.

Solution:

Concept:

- Dimethylglyoxime is the organic reagent used to identify nickel(II) by formation of a characteristic red chelate precipitate in ammoniacal solution.
- Denticity counts donor atoms available for coordination, while charge describes a particular protonation state. The dimethylglyoxime molecule is neutral and bidentate; its deprotonated dimethylglyoximato form in the nickel precipitate is monoanionic.

Step 1: Evaluate the nickel identification test Adding dimethylglyoxime to a solution containing Ni^{2+} in the presence of ammonium hydroxide produces the characteristic red nickel chelate precipitate. The ammoniacal medium provides the appropriate alkaline conditions for the test. This is a standard confirmatory reaction for nickel(II), so Statement I is true. Options (A) and (C), both of which say that Statement I is false, can therefore be eliminated.

Step 2: Identify the reagent and its donor atoms Dimethylglyoxime has the molecular structure $CH_3C(=NOH)C(=NOH)CH_3$. It has no overall ionic charge in this written molecular form, so it is a neutral molecule. It contains two oxime nitrogen atoms with donor lone pairs; these provide two coordination sites and allow chelate formation. This explains the description of dimethylglyoxime as a bidentate neutral ligand in Statement II. On that stated reagent classification, Statement II is also true.

Step 3: Distinguish the coordinated derivative and choose the option One must distinguish the neutral dimethylglyoxime molecule from the deprotonated ligand actually present in the red nickel precipitate. If the neutral reagent is denoted H_2dmg, the bound dimethylglyoximato form is $Hdmg^-$,

and the precipitate is $\mathrm{Ni(Hdmg)_2}$. Its neutrality follows from $+2+2(-1)=0$. This deprotonation does not make the original reagent molecule charged. The question names dimethylglyoxime rather than the dimethylglyoximato ion, so its neutral-reagent classification is the intended statement; option (B) confuses these two protonation forms.

Step 4: Conclusion Both Statement I and Statement II are true: dimethylglyoxime identifies nickel(II) in ammoniacal solution and is a neutral, bidentate reagent. Its coordinated deprotonated derivative is monoanionic. Hence, the correct option is (D).

Quick Tip to Solve: Keep the names distinct: dimethylglyoxime is the neutral molecule; dimethylglyoximato denotes its deprotonated ligand form.

70. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline? [4 marks]

(A) $\mathrm{C_6H_5CN}$

(B) $\mathrm{C_6H_5CONH_2}$

(C) $\mathrm{C_6H_5NHCH_3}$

(D) $\mathrm{C_6H_5NC}$

Correct Answer: (D) $\mathrm{C_6H_5NC}$

Solution:

Concept:

- A primary amine reacts with chloroform and alcoholic potassium hydroxide on heating to form an isocyanide, also called a carbylamine.
- In an isocyanide, the organic group is attached to nitrogen: $\mathrm{R-NC}$. A nitrile has the different connectivity $\mathrm{R-CN}$, with the organic group attached to carbon.

Step 1: Apply the test to the primary aromatic amine Aniline is $\mathrm{C_6H_5NH_2}$, a primary amine because the amino nitrogen is bonded to one phenyl group and two hydrogen atoms. It therefore gives the carbylamine reaction just as a primary aliphatic amine does. The phenyl group remains attached to the original amino nitrogen throughout the transformation.

Step 2: Write the balanced product-forming reaction The transformation with chloroform and alcoholic alkali is

$$\mathrm{C_6H_5NH_2 + CHCl_3 + 3\,KOH \longrightarrow C_6H_5NC + 3\,KCl + 3\,H_2O.}$$

The product is phenyl isocyanide. It has the characteristic unpleasant odour associated with a positive carbylamine test. The carbon introduced from chloroform becomes the terminal carbon of the isocyanide group; the product is not a nitrile.

Step 3: Check the connectivity in every option Option (A) shows $\mathrm{C_6H_5CN}$, benzonitrile, which has phenyl bonded to the carbon of the cyano group and is a constitutional isomer of the required product. Option (B) is benzamide, $\mathrm{C_6H_5CONH_2}$, and would require a carbonyl-forming transformation. Option (C) is N-methylaniline, $\mathrm{C_6H_5NHCH_3}$, a secondary amine rather than an isocyanide. Only option (D),

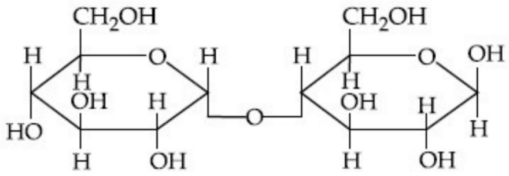
C_6H_5NC, has the required phenyl-nitrogen-carbon connection.

Step 4: Conclusion Aniline forms phenyl isocyanide, C_6H_5NC, in the carbylamine test. Hence, the correct option is (D).

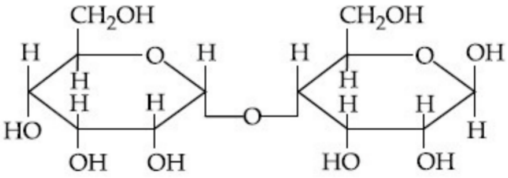
Quick Tip to Solve: For the carbylamine test, preserve the original carbon-nitrogen bond of the primary amine: the product is R-NC rather than R-CN.

71. Which of the following is correct structure of α-anomer of maltose? [4 marks]

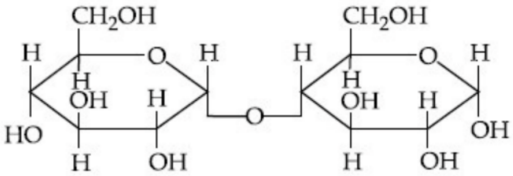
(A)



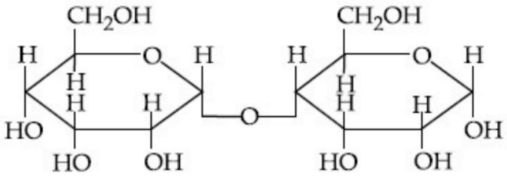
(B)



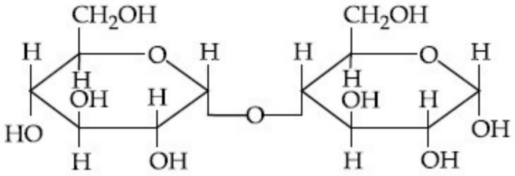
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- Maltose consists of two D-glucopyranose units joined by an $\alpha(1 \rightarrow 4)$ glycosidic bond: C1 of the left glucose is connected through oxygen to C4 of the right glucose.
- The free anomeric carbon is C1 of the right-hand, reducing glucose unit. Its hydroxyl orientation

determines whether the whole maltose molecule is the alpha or beta anomer.

- For a D-glucose Haworth structure, the CH_2OH group is above the ring; an alpha anomeric hydroxyl lies below the ring, trans to that group.

Step 1: Check the glucose stereochemistry In the standard D-glucopyranose Haworth projection shown, the hydroxyl groups at C2, C3 and C4 point down, up and down, respectively, while the C5 hydroxymethyl group points up. Both residues must retain the glucose configuration. Options (B) and (D) put the C3 hydroxyl below the ring, so they do not represent two D-glucose residues and cannot be the requested maltose structure.

Step 2: Check the glycosidic linkage The left anomeric carbon forms the glycosidic bond below its ring, giving an alpha linkage. The oxygen bridge terminates at C4 of the second ring. This is the required $\alpha(1 \rightarrow 4)$ connection; it leaves the right-hand C1 as a hemiacetal carbon capable of anomerism.

Step 3: Identify the free anomeric hydroxyl Option (A) has the right-hand anomeric hydroxyl above the ring and is therefore the beta anomer at the reducing end. In option (C), this hydroxyl points below the ring, whereas the hydroxymethyl group is above. Option (C) thus combines the correct glucose stereochemistry and glycosidic linkage with the requested alpha reducing end.

Step 4: Conclusion The structure in (C) is α-maltose, with an $\alpha(1 \rightarrow 4)$ linkage and a downward free anomeric hydroxyl. Hence, the correct option is (C).

Quick Tip to Solve: For maltose, check two separate features: the fixed alpha glycosidic linkage and the free anomeric hydroxyl on the reducing glucose. The latter decides the alpha or beta anomer.

72. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens? [4 marks]

(A) Nitric acid

(B) Sodium hydroxide

(C) Ammonia

(D) Hydrochloric acid

Correct Answer: (A) Nitric acid

Solution:

Concept:

- Sodium fusion converts covalently bound halogen into ionic sodium halide, NaX, where X denotes Cl, Br or I.
- Addition of $AgNO_3$ detects halide through the insoluble silver salt: $Ag^+ + X^- \longrightarrow AgX\downarrow$. Interfering ions must first be removed.
- If nitrogen or sulphur is present, the sodium extract may also contain cyanide or sulphide, which give interfering precipitates with silver ions.

Step 1: Acidify and boil the sodium extract Treat the sodium extract with nitric acid and boil it before the silver nitrate test. This removes cyanide and sulphide interference by converting them into

volatile products. Thus the subsequent silver precipitate can be attributed to halide rather than to an unrelated silver salt. The treatment precedes addition of the detecting reagent; simply adding silver nitrate to an untreated extract can produce a misleading result.

Step 2: Use a non-interfering acid Nitric acid supplies acidity without introducing a halide ion. Nitrate remains soluble with silver, so it does not itself create the silver-halide precipitate being tested for. Hydrochloric acid would supply chloride even when the original organic compound contained none: $Ag^+ + Cl^- \longrightarrow AgCl\downarrow$. It could therefore generate a false positive.

Step 3: Eliminate the other choices Sodium hydroxide does not acidify the extract or carry out the required removal of the interfering anions; an alkaline medium can also interfere with silver-ion chemistry. Ammonia is unsuitable because it complexes silver ions and can dissolve silver chloride, weakening the test. Only nitric acid performs the pretreatment without adding the species being detected.

Step 4: Conclusion Nitric acid is added to the sodium extract before silver nitrate in the halogen test. Hence, the correct option is (A).

Quick Tip to Solve: Before a precipitation test, choose a pretreatment reagent that removes interference without supplying the ion being tested. For silver halides, use nitric acid rather than hydrochloric acid.

73. The major product of the following reaction is : [4 marks]

$$CH_3CH_2CH{=}CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

(A) $CH_3CH_2CH_2CHO$

(B) $CH_3CH_2C(CHO){=}CH_2$

(C) $CH_3CH_2CH{=}CHCHO$

(D) $CH_3CH_2CH_2CH_2CHO$

Correct Answer: (D) $CH_3CH_2CH_2CH_2CHO$

Solution:

Concept:

- An alkene reacts with CO and H_2 in the presence of a rhodium catalyst by hydroformylation, also called the oxo process.
- Hydroformylation adds H and a formyl group, $-CHO$, across the carbon–carbon double bond. The product is an aldehyde with one more carbon atom than the starting alkene.
- For a terminal alkene, the normal straight-chain aldehyde is the major product in the hydroformylation considered here; the alkene double bond is consumed.

Step 1: Identify and count the starting carbons The displayed substrate is $CH_3CH_2CH{=}CH_2$, but-1-ene. It contains four carbon atoms and one terminal double bond. Carbon monoxide contributes the additional carbon of the aldehyde group, so the expected aldehyde contains five carbons. Hydrogen supplies the hydrogen atoms needed to saturate the double bond while forming the aldehyde.

Step 2: Write the normal hydroformylation product Place the formyl group on the terminal carbon and hydrogen on the adjacent alkene carbon. The resulting normal aldehyde is pentanal:

$$CH_3CH_2CH{=}CH_2 + CO + H_2 \xrightarrow{Rh} CH_3CH_2CH_2CH_2CHO.$$

Carbon, hydrogen and oxygen balance on both sides: the organic product has formula $C_5H_{10}O$. The carbonyl remains as an aldehyde, but the original alkene bond is gone.

Step 3: Compare all four printed products Option (A), butanal, has only four carbons, so it misses the carbon supplied by CO. Options (B) and (C) retain a carbon–carbon double bond and therefore do not describe addition of H and formyl across the original double bond. Option (D), $CH_3CH_2CH_2CH_2CHO$, is the saturated five-carbon normal aldehyde required by the oxo reaction.

Step 4: Conclusion The major product is pentanal, $CH_3CH_2CH_2CH_2CHO$. Hence, the correct option is (D).

Quick Tip to Solve: Recognise CO/H_2 with a Rh catalyst as hydroformylation: add one carbon, convert the alkene to an aldehyde and check that the original double bond is consumed.

74. The correct order of bond dissociation enthalpy of halogens is : [4 marks]

(A) $F_2 > Cl_2 > Br_2 > I_2$

(B) $Cl_2 > Br_2 > F_2 > I_2$

(C) $I_2 > Br_2 > Cl_2 > F_2$

(D) $Cl_2 > F_2 > Br_2 > I_2$

Correct Answer: (B) $Cl_2 > Br_2 > F_2 > I_2$

Solution:

Concept:

- Bond dissociation enthalpy is the energy required to break one mole of gaseous diatomic molecules into gaseous atoms: $X_2(g) \longrightarrow 2\,X\,(g)$.
- Longer halogen bonds generally have poorer orbital overlap, so bond strength decreases from chlorine through bromine to iodine.
- Fluorine is anomalous: strong repulsions between the closely spaced lone pairs of the small fluorine atoms weaken the $F{-}F$ bond.

Step 1: Order chlorine, bromine and iodine As atomic size increases from Cl to Br to I, the bonding orbitals become larger and the internuclear distance increases. The effective overlap decreases, giving

$$D(Cl{-}Cl) > D(Br{-}Br) > D(I{-}I).$$

Thus the iodine molecule has the weakest bond among these three. An order that makes iodine stronger than bromine or chlorine reverses this trend.

Step 2: Place fluorine using its lone-pair exception Although fluorine is smaller and its bond is shorter, its compact valence shell brings the nonbonding electron pairs on the two bonded atoms very close

together. Their repulsion makes the fluorine bond much weaker than a simple size argument predicts. Its dissociation enthalpy is below that of bromine but still above that of iodine. Approximate values in $\mathrm{kJ\,mol^{-1}}$ are 243 for chlorine, 193 for bromine, 159 for fluorine and 151 for iodine.

Step 3: Select the matching option Combining the ordinary down-group trend with the fluorine exception gives

$$\mathrm{Cl_2 > Br_2 > F_2 > I_2}.$$

Option (A) incorrectly places fluorine first. Option (C) reverses the trend and makes iodine strongest. Option (D) correctly places chlorine first but still overestimates the strength of fluorine relative to bromine. Only option (B) has every comparison correct.

Step 4: Conclusion The order is $\mathrm{Cl_2 > Br_2 > F_2 > I_2}$. Hence, the correct option is (B).

Quick Tip to Solve: Do not extrapolate the halogen size trend through fluorine: first order Cl, Br and I normally, then insert weak-bonded fluorine between bromine and iodine.

75. The major components of German Silver are : [4 marks]

(A) Zn, Ni and Ag

(B) Cu, Zn and Ni

(C) Cu, Zn and Ag

(D) Ge, Cu and Ag

Correct Answer: (B) Cu, Zn and Ni

Solution:

Concept:

- An alloy is a metallic mixture whose useful properties differ from those of its individual constituents. Common alloy names do not necessarily name their chemical constituents.
- German silver, also called nickel silver, is a copper-based alloy containing copper, zinc and nickel. The name refers to its silvery appearance rather than to elemental silver.
- Nickel contributes the silvery colour and resistance to corrosion; zinc and copper contribute the practical mechanical and working properties of this alloy.

Step 1: Recall the composition of the named alloy The identifying constituent set of German silver is Cu, Zn and Ni. Different commercial grades may use different proportions, but the question asks for the major elements rather than a particular percentage formula. It is therefore the presence of these three metals, not a fixed numerical ratio, that determines the answer.

Step 2: Distinguish a name from an elemental ingredient The word silver describes the appearance of the alloy. It does not mean that Ag is present as a major component. Likewise, the adjective German does not indicate the presence of germanium, Ge. Using either word as a literal chemical clue would produce the distractors rather than the actual alloy composition.

Step 3: Check each printed set Option (A) contains zinc and nickel but substitutes silver for the essential copper base. Option (C) contains copper and zinc but substitutes silver for nickel. Option (D) introduces germanium and silver while omitting both zinc and nickel. Option (B) alone lists all

three correct major components: copper, zinc and nickel. No addition of silver is required to account for the silvery finish.

Step 4: Conclusion The major components of German silver are Cu, Zn and Ni. Hence, the correct option is (B).

Quick Tip to Solve: Memorise German silver as Cu–Zn–Ni. Its silvery colour is a property of the alloy; the name does not imply silver or germanium in its composition.

76. Water does not produce CO on reacting with : [4 marks]

(A) C

(B) CH_4

(C) C_3H_8

(D) CO_2

Correct Answer: (D) CO_2

Solution:

Concept:

- Steam can act as an oxidising reagent toward hot carbon and hydrocarbons, producing carbon monoxide together with hydrogen.
- The water-gas reaction is $C + H_2O\,(g) \longrightarrow CO + H_2$; steam reforming extends the same idea to hydrocarbons under suitable high-temperature conditions.
- Carbon dioxide already contains carbon in its highest common oxidation state, +4. Water alone does not reduce it to carbon monoxide in the reactions considered here.

Step 1: Check carbon and methane with steam Hot carbon reacts with steam to give water gas:

$$C + H_2O\,(g) \longrightarrow CO + H_2.$$

Methane undergoes catalytic steam reforming:

$$CH_4 + H_2O\,(g) \longrightarrow CO + 3\,H_2.$$

Both reactions explicitly produce CO, so neither carbon in (A) nor methane in (B) satisfies the phrase does not produce CO.

Step 2: Check the higher hydrocarbon Propane similarly undergoes steam reforming. Balancing carbon first gives three CO molecules; balancing oxygen then requires three water molecules, and hydrogen balance gives seven hydrogen molecules:

$$C_3H_8 + 3\,H_2O\,(g) \longrightarrow 3\,CO + 7\,H_2.$$

Carbon atoms, oxygen atoms and hydrogen atoms are conserved, confirming that propane can also yield CO on reacting with steam. Option (C) is therefore excluded.

Step 3: Identify the non-producing reactant Carbon dioxide cannot play the role of a reducing carbon

fuel in these steam reactions. In contact with water it may establish carbonic-acid equilibria, but this does not form CO. Producing CO from CO_2 requires a separate reducing agent or energy input beyond simply reacting it with water under the chemical processes being tested. Thus option (D) is the exception.

Step 4: Conclusion Water does not produce CO on reacting with CO_2 in the stated context. Hence, the correct option is (D).

Quick Tip to Solve: For steam reforming, balance each carbon atom into CO and the remaining hydrogen into H_2. Carbon dioxide is already fully oxidised and is the exception among these choices.

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77. Given below are two statements :

Statement I : α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

Statement II : At room temperature the stable crystalline form of sulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is true but Statement II is false.

(B) Statement I is false but Statement II is true.

(C) Both Statement I and Statement II are true.

(D) Both Statement I and Statement II are false.

Correct Answer: (A) Statement I is true but Statement II is false.

Solution:

Concept:

- Rhombic sulphur, α-sulphur, and monoclinic sulphur, β-sulphur, are crystalline allotropes composed of puckered S_8 rings with different crystal arrangements.
- The transition temperature is approximately $369\,K$ ($95.5\,^\circ C$). Rhombic sulphur is stable below it and monoclinic sulphur above it, within the relevant solid range.
- The transition is enantiotropic: slow heating or cooling allows the two solid forms to interconvert reversibly rather than preserving a metastable form.

Step 1: Evaluate Statement I On slowly heating rhombic sulphur through its transition temperature, monoclinic sulphur becomes the stable form. On slowly cooling monoclinic sulphur below the same temperature, the rhombic form is recovered. Slow changes allow the crystal rearrangement to follow the stable form, so the two allotropes can change reversibly between themselves. Therefore Statement I is true.

Step 2: Evaluate Statement II Room temperature is about $298\,K$, well below the transition at approximately $369\,K$. Consequently the thermodynamically stable crystalline form at room temperature is rhombic sulphur, not monoclinic sulphur. Monoclinic crystals may persist temporarily after preparation, but this does not make them the equilibrium stable room-temperature form. Statement II is therefore false.

Step 3: Match the truth values to the options The required combination is true for Statement I and false for Statement II. Option (A) states exactly this combination. Option (B) reverses both truth values; option (C) incorrectly accepts the room-temperature monoclinic claim; option (D) incorrectly rejects reversible interconversion. Distinguishing crystal stability from temporary persistence is enough to remove these distractors.

Step 4: Conclusion Statement I is true, whereas Statement II is false: rhombic sulphur is stable at room temperature. Hence, the correct option is (A).

Quick Tip to Solve: Use the transition temperature as the anchor: below about $369\,\mathrm{K}$, sulphur is rhombic; above it, the stable solid form is monoclinic. Room temperature belongs to the first range.

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78. The method used for the purification of Indium is : [4 marks]

(A) zone refining

(B) vapour phase refining

(C) liquation

(D) van Arkel method

Correct Answer: (A) zone refining

Solution:

Concept:

- Zone refining separates impurities because their solubility in the molten metal differs from that in the solid metal. The impurities of interest are preferentially retained in the liquid zone.
- The method is used to obtain metals and semiconductors of exceptionally high purity; indium is among the elements purified by this technique.

Step 1: Identify the suitable purity-based method A narrow molten zone is produced in an otherwise solid rod of the impure material. As the heater moves slowly along the rod, fresh solid melts ahead of the zone and purer solid crystallises behind it. Since the impurities prefer the liquid phase, they travel with the molten region instead of entering the newly solidified part to the same extent. Repeating the movement concentrates the impurities near one end, which is removed. This is the principle applied to the purification of indium.

Step 2: Distinguish the other refining processes Vapour phase refining requires formation of a suitable volatile compound followed by its decomposition to regenerate the metal. The van Arkel method is a specific iodide-based vapour phase process, commonly associated with titanium and zirconium. Liquation instead exploits a low melting point so that a fusible metal can flow away from relatively infusible impurities. Those descriptions do not identify the standard high-purity refining method for indium asked here. Thus options (B), (C) and (D) are excluded.

Step 3: Conclusion Indium is purified by zone refining. Hence, the correct option is (A).

Quick Tip to Solve: For high-purity semiconductor materials, recall the moving molten zone: impurities travel with the melt and collect at an end.

79. Which one of the following statements is FALSE for hydrophilic sols? [4 marks]

(A) They do not require electrolytes for stability.

(B) The sols cannot be easily coagulated.

(C) These sols are reversible in nature.

(D) Their viscosity is of the order of that of H_2O.

Correct Answer: (D) Their viscosity is of the order of that of H_2O.

Solution:

Concept:

- A hydrophilic sol is a lyophilic colloid whose dispersed particles have a strong affinity for water. Hydration or solvation provides an important source of stability.
- Lyophilic sols are generally reversible, resist coagulation and have appreciably higher viscosity than their dispersion medium at comparable conditions.

Step 1: Check the stability and reversibility statements Water molecules form a solvating layer around the dispersed particles. This affinity for the medium stabilises the sol without requiring an added electrolyte, so statement (A) is true. It also makes the particles harder to precipitate: ordinary small additions of electrolyte do not readily coagulate the sol, so (B) is true. The dispersed substance can generally be recovered and redispersed in the medium to form the sol again; this is the reversible character described by (C). Consequently none of these three true properties answers a question asking for the false statement.

Step 2: Compare viscosity with that of water In a hydrophilic sol, the solvated particles interact strongly with the water and impede its flow. Many hydrophilic colloids are macromolecular substances, which increase this resistance further. Their viscosity is therefore appreciably higher than the viscosity of pure water, rather than approximately the same as that of water. The property in (D) is characteristic of the contrast with lyophobic sols and is false for the hydrophilic sols considered here. The question concerns viscosity, not density or the ability to dissolve an electrolyte.

Step 3: Conclusion The false statement is that their viscosity is of the order of that of water. Hence, the correct option is (D).

Quick Tip to Solve: Associate lyophilic sols with solvation, reversibility and increased viscosity; read FALSE carefully before choosing.

80. Given below are two statements :

Statement I : The pH of rain water is normally 5.6.

Statement II : If the pH of rain water drops below 5.6, it is called acid rain.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (A) Both Statement I and Statement II are true.

Solution:

Concept:

- Ordinary rainwater dissolves atmospheric carbon dioxide and is mildly acidic. Its typical pH is about 5.6 rather than the pH 7 of pure neutral water.
- Acid rain is conventionally identified by rainwater pH below 5.6, reflecting additional acidity from pollutants such as sulfur and nitrogen oxides.

Step 1: Evaluate Statement I As rain falls through the atmosphere, carbon dioxide dissolves in it and establishes the carbonic-acid equilibria.

$$CO_2 + H_2O \rightleftharpoons H_2CO_3$$

$$H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$$

The hydrogen ions produced make unpolluted rainwater weakly acidic. Under the normal conditions used for this environmental-chemistry definition, the pH is approximately 5.6. It would be incorrect to reject the statement on the ground that all water must have pH 7, because rainwater is exposed to atmospheric gases. Statement I is therefore true.

Step 2: Evaluate Statement II and choose the combination Acidity exceeding this normal carbon-dioxide background is the basis of the acid-rain classification. Oxides of sulfur and nitrogen can be converted into stronger acids in atmospheric moisture, lowering the rainwater pH below 5.6. Thus Statement II gives the accepted threshold and is also true. Since both individual statements are true, the option claiming both false is excluded, as are the two mixed options, each of which incorrectly labels one statement false. No claim that the pH remains fixed during every weather event is needed; the question gives the conventional normal value.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Use 5.6 as the normal-rain reference, not 7: dissolved atmospheric carbon dioxide already makes ordinary rain mildly acidic.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed $[Cu^{2+}]$ and $P_{NO} = P_{NO_2}$), the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of 2x is _____. (Rounded off to the nearest integer)

[Given, $E^o_{Cu^{2+}/Cu} = 0.34$ V, $E^o_{NO_3^-/NO} = 0.96$ V, $E^o_{NO_3^-/NO_2} = 0.79$ V and at 298 K, $\frac{RT}{F}(2.303) = 0.059$]

[4 marks]

Solution:

Concept:

- For a reduction involving n electrons, the Nernst equation at 298 K is $E = E^\circ - (0.059/n)\log Q$. Logarithms are base ten, and activities and gas pressures are relative to their standard states.
- With the copper-ion concentration fixed, the copper electrode potential is identical for both cells. Equal driving potentials therefore require equality of the two nitrate reduction potentials.

Step 1: Write the two balanced reduction equations The competing half-reactions in acid are

$$NO_3^- + 4\,H^+ + 3\,e^- \longrightarrow NO + 2\,H_2O,$$

$$NO_3^- + 2\,H^+ + e^- \longrightarrow NO_2 + H_2O.$$

Write $c = 10^x$ for the numerical molar concentration of nitric acid and use the usual strong-acid approximation $[H^+] = [NO_3^-] = c$. If the common gas pressure relative to the standard pressure is p, the Nernst equations become

$$E_1 = 0.96 - \frac{0.059}{3}\log\frac{p}{c^5}, \qquad E_2 = 0.79 - 0.059\log\frac{p}{c^3}.$$

The same copper potential, including its stated standard value 0.34 V, cancels on equating the cell potentials.

Step 2: Equate the potentials and inspect the specified data Putting $E_1 = E_2$ gives

$$0.17 = \frac{0.118}{3}(2x - \log p),$$

$$2x = \frac{3(0.17)}{0.118} + \log p = 4.322\ldots + \log p.$$

Equal gas pressures alone do not specify p, so the literal data do not determine a unique numerical value. If both gases are taken at standard pressure, $p = 1$ and $2x = 4.322\ldots$, which rounds to 4. A value of 1 would require $p \approx 10^{-3.322}$, which the question does not supply.

Step 3: Conclusion The printed data give $2x = 4.322\ldots + \log(p/p^\circ)$, so equal but unspecified pressures leave the answer undetermined. With the conventional additional standard-pressure assumption, the rounded value is $\boxed{4}$ (dimensionless). The supplied key value 1 does not follow from the stated potentials and conditions.

Quick Tip to Solve: Keep gas-pressure factors in each Nernst quotient: equal pressures do not cancel when the electron counts differ.

82. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is _____ BM. [4 marks]

Solution:

Concept:

- For a transition-metal ion with n unpaired electrons, the spin-only magnetic moment is $\mu_{\text{so}} = \sqrt{n(n+2)}\,\text{BM}$. A Bohr magneton (BM) is the unit used here.
- When transition-metal atoms form cations, electrons are removed from the outer $4s$ orbital before the $3d$ orbitals. The atomic number 29 identifies copper.

Step 1: Obtain the electron configuration of the divalent ion Neutral copper has the ground-state configuration $[\text{Ar}]\,3d^{10}4s^1$. Formation of Cu^{2+} removes two electrons: the $4s$ electron is removed first, followed by one electron from the filled $3d$ subshell. Thus the divalent aqueous ion has configuration $[\text{Ar}]\,3d^9$. Using the configuration of the neutral atom, or removing both electrons from an assumed $4s^2$ configuration, would give an incorrect unpaired-electron count.

Step 2: Count unpaired electrons and calculate the moment A d^9 arrangement contains nine electrons in five d orbitals. Four orbitals are doubly occupied and the remaining orbital is singly occupied; hence $n = 1$. In an aqueous copper(II) ion, crystal-field splitting does not change this single-unpaired-electron count. Substitution in the spin-only formula gives

$$\mu_{\text{so}} = \sqrt{1(1+2)}\,\text{BM} = \sqrt{3}\,\text{BM} \approx 1.73\,\text{BM}.$$

This is the requested spin-only value; orbital contributions are excluded by the wording. If the numerical response is entered as an integer, 1.73 rounds to 2.

Step 3: Conclusion The spin-only magnetic moment is $\boxed{\sqrt{3}\,\text{BM} \approx 1.73\,\text{BM}}$, corresponding to the nearest-integer response $\boxed{2}\,\text{BM}$.

Quick Tip to Solve: For copper(II), remember d^9: one unpaired electron gives $\sqrt{3}$ BM, even though neutral copper is $d^{10}s^1$.

83. If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is _____ molal. (Rounded off to the nearest integer)

[K_b = 0.52 K kg mol^{-1}] [4 marks]

Solution:

Concept:

- The boiling-point elevation is $\Delta T_b = iK_b m$, where m is the analytical molality of the solute and i is the van't Hoff factor.
- For dissociation of one formula unit into two particles, $\text{AB} \longrightarrow \text{A}^+ + \text{B}^-$, the van't Hoff factor is $i = 1 + \alpha$, with α the fraction dissociated.

Step 1: Convert the dissociation percentage into a particle factor The stated 75% dissociation means

$\alpha = 0.75$, not 75. Starting with one mole of AB, the undissociated amount is $1 - \alpha = 0.25$ mol. Dissociation supplies 0.75 mol of each ion, so the total number of moles of solute particles is

$$0.25 + 0.75 + 0.75 = 1.75.$$

Consequently $i = 1.75$. This is less than the complete-dissociation value 2 and greater than the non-dissociating value 1, as expected. The analytical molality describes the original formula units; the factor i accounts separately for the extra particles.

Step 2: Calculate the analytical molality and round at the end Rearrange the elevation equation before substituting the data:

$$m = \frac{\Delta T_b}{iK_b} = \frac{2.5\,\text{K}}{1.75 \times 0.52\,\text{K kg mol}^{-1}}.$$

Since $1.75 \times 0.52 = 0.91$,

$$m = \frac{2.5}{0.91} = 2.74725\ldots\ \text{mol kg}^{-1}.$$

The kelvin units cancel, leaving molality in moles per kilogram of solvent. Rounding this result to the nearest integer, as requested, gives 3. Ignoring dissociation would overestimate the required molality because it would miss the extra particles that elevate the boiling point.

Step 3: Conclusion The molality is approximately $2.75\,\text{mol kg}^{-1}$; the required nearest-integer value is $\boxed{3\,\text{mol kg}^{-1}}$, or 3 molal.

Quick Tip to Solve: For AB dissociating into two ions, use $i = 1 + \alpha$; insert 0.75, then round only the final molality.

84. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal A. The ionization enegy of metal A in kJ mol^{-1} is _____. (Rounded off to the nearest integer)
[h = 6.63 $\times$ 10^{-34} Js, c = 3.00 $\times$ 10^{8} ms^{-1}, N_A = 6.02 $\times$ 10^{23} mol^{-1}] [4 marks]

Solution:

Concept:

- The energy of one photon is $E = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is the wavelength.
- At the ionisation threshold, the photon supplies just the energy needed to remove one electron. Multiplying the energy per atom by Avogadro's constant gives the molar ionisation energy.

Step 1: Convert the wavelength and find the energy per atom The radiation is just sufficient for ionisation, so there is no excess photon energy to assign to the kinetic energy of the outgoing electron. The photon energy therefore equals the ionisation energy of one atom. Convert nanometres to metres:

$$\lambda = 663\,\text{nm} = 663 \times 10^{-9}\,\text{m} = 6.63 \times 10^{-7}\,\text{m}.$$

Using precisely the constants in the question,

$$E = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{6.63 \times 10^{-7}}\,\text{J} = 3.00 \times 10^{-19}\,\text{J}.$$

This is an energy for one ionisation event, not yet for one mole.

Step 2: Convert to the requested molar unit One mole contains 6.02×10^{23} atoms. Multiplying the energy per atom by this number gives

$$E_{\text{mol}} = (3.00 \times 10^{-19})(6.02 \times 10^{23}) = 1.806 \times 10^{5}\,\text{J mol}^{-1}.$$

Since $1\,\text{kJ} = 10^{3}\,\text{J}$,

$$E_{\text{mol}} = 180.6\,\text{kJ mol}^{-1}.$$

The energy is positive because ionisation requires energy input to separate the electron from the atom. Rounding 180.6 to the nearest integer gives 181. Omitting the nanometre conversion or the joule-to-kilojoule conversion would change the answer by large powers of ten.

Step 3: Conclusion The ionisation energy is $180.6\,\text{kJ mol}^{-1}$; the requested rounded value is $\boxed{181\,\text{kJ mol}^{-1}}$.

Quick Tip to Solve: For a threshold wavelength, use hcN_A/λ; convert nm to m first and divide joules by 1000 only at the end.

85. Consider titration of NaOH solution versus 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

(i) 4.5 mL

(ii) 4.5 mL

(iii) 4.4 mL

(iv) 4.4 mL

(v) 4.4 mL

If the volume of oxalic acid taken was 10.0 mL then the molarity of the NaOH solution is _____ M. (Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- Oxalic acid, $H_2C_2O_4$, is dibasic: one mole supplies two acid equivalents in complete neutralisation with NaOH.
- At the end point, acid and base equivalents are equal. Hence $2M_{\text{acid}}V_{\text{acid}} = M_{\text{base}}V_{\text{base}}$, with both volumes in the same unit.

Step 1: Choose the concordant titre The readings settle at $4.4\,\text{mL}$ in three successive trials. Use this repeated, concordant value as the volume of sodium hydroxide delivered from the burette to neutralise the $10.0\,\text{mL}$ acid aliquot. The initial two readings of $4.5\,\text{mL}$ are not the final concordant set; in any event, using them would not change the nearest-integer result.

Step 2: Apply the reaction stoichiometry The balanced neutralisation is

$$H_2C_2O_4 + 2\,NaOH \longrightarrow Na_2C_2O_4 + 2\,H_2O.$$

The amount of oxalic acid is $1.25 \times 0.0100 = 0.0125\,\text{mol}$, so the required amount of sodium hydroxide is $2 \times 0.0125 = 0.0250\,\text{mol}$. Both acidic protons must be counted; a one-to-one mole ratio would underestimate the base concentration by a factor of two.

Step 3: Calculate and round the molarity Divide the base amount by the delivered base volume in litres:

$$M_{NaOH} = \frac{0.0250}{4.4 \times 10^{-3}} = 5.6818\ldots\ \text{mol}\,\text{L}^{-1}.$$

Equivalently, $M_{NaOH} = 2(1.25)(10.0)/4.4$. Round only this final value, rather than rounding the acid concentration or intermediate mole amount.

Step 4: Conclusion The sodium hydroxide concentration is $5.68\,\text{M}$, which rounds to $\boxed{6\,\text{M}}$. The integer to enter is 6.

Quick Tip to Solve: Count the replaceable acidic protons before using a titration formula: oxalic acid needs twice its mole amount of NaOH.

86. Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is _____ kJ mol^{-1}. (Rounded off to the nearest integer) [Use R = 8.314 J mol^{-1}K^{-1}] [4 marks]

Solution:

Concept:

- For an ideal gas, internal energy depends only on temperature, so an isothermal process has $\Delta U = 0$. With the chemistry sign convention, $\Delta U = q + w$.
- At constant external pressure, $w = -P_{ext}\Delta V$, while the specified ideal-gas end states give $V = nRT/P$. A molar heat is the total heat divided by n.

Step 1: Calculate the volume change from the stated end states Convert all pressures consistently to pascals: $P_1 = 2.1 \times 10^6\,\text{Pa}$, $P_2 = 1.3 \times 10^6\,\text{Pa}$, and $P_{ext} = 4.3 \times 10^6\,\text{Pa}$. Then

$$\begin{aligned}\Delta V &= nRT\left(\frac{1}{P_2} - \frac{1}{P_1}\right)\\ &= 5(8.314)(293)\left(\frac{1}{1.3 \times 10^6} - \frac{1}{2.1 \times 10^6}\right)\\ &= 3.56923 \times 10^{-3}\,\text{m}^3.\end{aligned}$$

Step 2: Use the first law and distinguish total from molar heat Because $\Delta U = 0$, the formal heat absorbed is

$$q = -w = P_{ext}\Delta V = 15.3477\,\text{kJ}.$$

This is the heat for all five moles, not heat per mole. The printed unit therefore requires

$$q_{\mathrm{m}} = \frac{q}{5} = 3.06954\,\mathrm{kJ\,mol^{-1}}.$$

It rounds to 3, whereas the total heat rounds to 15.

Step 3: Check the physical and unit consistency The imposed external pressure exceeds even the initial gas pressure, so a free expansion driven only by the stated pressure difference cannot occur; additional mechanical action would be required. The calculation above is the formal first-law result for the specified end states and pressure-volume work prescription. Independently of that mechanical inconsistency, assigning 15 to a quantity labelled per mole would confuse total and molar heat.

Step 4: Conclusion For the unit actually printed, the formal result is $\boxed{3\,\mathrm{kJ\,mol^{-1}}}$ after rounding. The answer key gives 15, which instead equals the rounded total heat in kJ for five moles; it does not have the printed molar unit.

Quick Tip to Solve: Before rounding, inspect whether heat is requested for the entire sample or per mole; dividing by the stated mole number is essential here.

87. The number of compound/s given below which contain/s -COOH group is _____. (Integer answer)

(A) Sulphanilic acid

(B) Picric acid

(C) Aspirin

(D) Ascorbic acid [4 marks]

Solution:

Concept:

- A carboxyl group has the connectivity $-\mathrm{C(=O)OH}$, abbreviated $-\mathrm{COOH}$. An acidic name alone does not prove that this group is present.
- Phenolic, sulfonic-acid and enediol groups can produce acidity without a free carboxyl group. An ester or lactone carbonyl must not be counted as $-\mathrm{COOH}$.

Step 1: Identify the first two acid groups Sulphanilic acid has an amino substituent and a $-\mathrm{SO_3H}$ group on the aromatic ring. Its acid group is sulfonic, not carboxylic, and it contributes zero to the count. Picric acid is 2,4,6-trinitrophenol. Its acidic hydrogen belongs to the phenolic $-\mathrm{OH}$, while three nitro groups stabilise the conjugate base. It also has no $-\mathrm{COOH}$ group.

Step 2: Distinguish aspirin from ascorbic acid Aspirin is acetylsalicylic acid, or 2-acetoxybenzoic acid. Acetylation converts the phenolic hydroxyl of salicylic acid into an ester, but leaves its aromatic $-\mathrm{COOH}$ intact. Therefore aspirin must be counted. Ascorbic acid contains an enediol arrangement in a lactone structure; its acidity is associated with this enediol system. The cyclic ester has no free $-\mathrm{COOH}$, so ascorbic acid must not be counted.

Step 3: Count compounds rather than acidic hydrogens The requested number concerns listed compounds containing at least one carboxyl group, not the number of acidic protons or total oxygen

atoms. The classification is therefore A: no, B: no, C: yes, D: no. Exactly one of the four names satisfies the functional-group test.

Step 4: Conclusion Only aspirin, compound (C), contains a free $-\mathrm{COOH}$ group. The required integer is $\boxed{1}$.

Quick Tip to Solve: Write the acid functional group for each name; phenol, sulfonic acid and lactone should never be counted as carboxylic acid.

88. The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in kJ mol^{-1} is _____. (Rounded off to the nearest integer) [R = 8.314 J K^{-1}mol^{-1}] [4 marks]

Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where E_a is activation energy, R is the gas constant and T is absolute temperature.
- For the same reaction with an effectively constant pre-exponential factor over this interval, $\ln(k_2/k_1) = E_a(1/T_1 - 1/T_2)/R$. Celsius temperatures must first be converted to kelvin.

Step 1: Write the temperature and rate-constant ratios The rate constant becomes five times its initial value, so $k_2/k_1 = 5$. The temperatures are $T_1 = 27 + 273 = 300\,\mathrm{K}$ and $T_2 = 52 + 273 = 325\,\mathrm{K}$. The relevant temperature increase is $25\,\mathrm{K}$, but this difference alone cannot replace either absolute temperature in the reciprocal-temperature expression.

Step 2: Rearrange the two-temperature Arrhenius equation Taking natural logarithms and solving for the activation energy gives

$$\ln 5 = \frac{E_a}{R}\frac{T_2 - T_1}{T_1 T_2},$$
$$E_a = \frac{R \ln 5\, T_1 T_2}{T_2 - T_1}.$$

The right side is positive, consistent with a rate constant that increases when the temperature rises. If common logarithms were used, an additional factor 2.303 would be necessary.

Step 3: Substitute and convert the energy unit Using the supplied gas constant,

$$\begin{aligned} E_a &= \frac{8.314(1.60944)(300)(325)}{25} \\ &= 5.21854 \times 10^4\,\mathrm{J\,mol^{-1}} \\ &= 52.1854\,\mathrm{kJ\,mol^{-1}}. \end{aligned}$$

Divide joules by 1000 before rounding to the nearest integer in the requested unit.

Step 4: Conclusion The activation energy is $52.1854\,\mathrm{kJ\,mol^{-1}}$, hence the required rounded answer is $\boxed{52\,\mathrm{kJ\,mol^{-1}}}$.

Quick Tip to Solve: For a two-temperature Arrhenius question, use kelvin and natural logarithms; keep the factor of 1000 for J to kJ until the end.

89. Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is _____. (Integer answer)

(A) Li

(B) Na

(C) Rb

(D) Cs

[4 marks]

Solution:

Concept:

- Photoelectric emission requires incident photon energy $h\nu$ at least equal to the surface work function ϕ; the maximum kinetic energy is $K_{\max} = h\nu - \phi$.
- Alkali metals have comparatively small work functions. Caesium is the familiar textbook choice for a photoelectric-cell electrode because it readily emits electrons, but this is a practical preference rather than a unique consequence of the words "can be used".

Step 1: Identify the intended textbook selection Among the listed elements, caesium is the standard textbook example associated with photoelectric-cell electrodes. Its outermost electron is weakly held, which favours electron emission under relatively low-energy illumination. On that intended convention, item (D) alone is selected and the expected integer is one. This explains the answer key without treating the four lettered entries as mutually exclusive MCQ options.

Step 2: Check the scope of the printed wording The statement asks which metals can be used, rather than which is the conventional or most suitable choice. Sodium and rubidium also have photoemissive surfaces and can serve in photocathodes. Lithium likewise has a finite photoelectric threshold and can emit when sufficiently energetic radiation is supplied. Actual suitability depends on wavelength, surface preparation, chemical stability and the required sensitivity; those conditions are absent from the question.

Step 3: Distinguish the convention from a universal exclusion The photoelectric threshold law does not single out one alkali metal without a specified incident spectrum or work-function comparison. Thus it would be incorrect to argue that Li, Na and Rb are fundamentally incapable of photoelectric emission. The examination convention gives one for Cs, while a broad literal interpretation allows more than one and, with sufficiently energetic illumination, all four. The statement therefore requires its intended textbook-use convention for a unique integer.

Step 4: Conclusion The intended textbook answer is $\boxed{1}$, corresponding to Cs, and this agrees with the answer key. The unrestricted wording "can be used" is ambiguous: it does not physically exclude the other photoemissive alkali metals without further operating conditions.

Quick Tip to Solve: Use Cs for the conventional textbook application, but distinguish a preferred photocathode material from the general threshold condition for photoemission.

90. The unit cell of copper corresponds to a face centered cube of edge length 3.596 $\overset{o}{A}$ with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is _____. [Molar mass of Cu : 63.54 g; Avogadro Number = 6.022 $\times$ 10^{23}] [4 marks]

Solution:

Concept:

- An FCC cell contains eight corner contributions of $1/8$ atom and six face-centre contributions of $1/2$ atom, so $Z = 4$ atoms per unit cell.
- The density is $\rho = ZM/(N_A a^3)$. For $\mathrm{kg\,m^{-3}}$, use molar mass M in $\mathrm{kg\,mol^{-1}}$ and edge length a in metres.

Step 1: Count the atoms belonging to one cell Each corner atom is shared among eight neighbouring cubes and each face-centred atom among two. Therefore

$$Z = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) = 1 + 3 = 4.$$

Counting the fourteen visible lattice positions as fourteen whole atoms would overestimate the mass; the sharing factors are essential.

Step 2: Convert the data and calculate the cell mass and volume The supplied molar mass and edge length become

$$M = 63.54 \times 10^{-3}\,\mathrm{kg\,mol^{-1}}, \qquad a = 3.596 \times 10^{-10}\,\mathrm{m}.$$

Consequently,

$$m_{\text{cell}} = \frac{4(0.06354)}{6.022 \times 10^{23}} = 4.2205 \times 10^{-25}\,\mathrm{kg},$$

$$V_{\text{cell}} = (3.596 \times 10^{-10})^3 = 4.6499852736 \times 10^{-29}\,\mathrm{m^3}.$$

The angstrom-to-metre conversion must be cubed with the numerical edge length.

Step 3: Divide and check the numerical precision Using the given constants without premature rounding,

$$\rho = \frac{4(0.06354)}{(6.022 \times 10^{23})(3.596 \times 10^{-10})^3} = 9076.2699\,\mathrm{kg\,m^{-3}}.$$

This is equivalently $9.07627\,\mathrm{g\,cm^{-3}}$. To four significant figures it is $9.076 \times 10^3\,\mathrm{kg\,m^{-3}}$; rounding to an integer gives 9076.

Step 4: Conclusion The stated data give $\boxed{\rho \approx 9.07627 \times 10^3\,\mathrm{kg\,m^{-3}}}$, or $9076\,\mathrm{kg\,m^{-3}}$ to the nearest integer. The answer key gives 9077; direct substitution of its printed constants gives 9076.2699, so that one-unit difference is numerical rather than an FCC counting difference.

Quick Tip to Solve: For FCC use four atoms per cell, convert angstroms to metres, and cube the full edge length before dividing; defer rounding until the density.