JEE Main 2021 Paper 1 (B.E./B.Tech) Feb 26 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 1 (February), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 26 February 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where x∈(0, 1) is : [4 marks]

(A) $\frac{10!}{\sqrt{3}(5!)^2}$

(B) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(C) $\frac{10!}{3(5!)^2}$

(D) $\frac{2.10!}{3(5!)^2}$

Correct Answer: (B) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

Solution:

Concept:

- The general binomial term in $(A+B)^{10}$ is $T_{r+1} = \binom{10}{r}A^{10-r}B^r$. The term independent of a variable has exponent zero in that variable.
- For a positive differentiable function, its maximum can be found by setting its derivative to zero and checking the sign on either side. Here $0 < x < 1$, so all fractional powers are real and positive.

Step 1: Identify the term independent of t Substitute $A = tx^{1/5}$ and $B = (1-x)^{1/10}/t$. Then the general term is

$$T_{r+1} = \binom{10}{r} t^{10-2r} x^{(10-r)/5} (1-x)^{r/10}.$$

The exponent of t vanishes only if $10 - 2r = 0$, giving $r = 5$. Hence the required term is $\binom{10}{5} x\sqrt{1-x}$. The coefficient is fixed; only the last two factors vary with x.

Step 2: Maximise the remaining function Let $f(x) = x\sqrt{1-x}$. Differentiation gives

$$f\prime(x) = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}.$$

The denominator is positive in the given interval. Thus $f\prime(x) > 0$ for $x < 2/3$ and $f\prime(x) < 0$ for $x > 2/3$. This proves a maximum at $x = 2/3$; the limiting endpoint values are both zero.

Step 3: Evaluate and compare the options At the maximum,

$$f(2/3) = \frac{2}{3}\sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}}.$$

Multiplying by $\binom{10}{5} = 10!/(5!)^2$ gives $2 \cdot 10!/[3\sqrt{3}(5!)^2]$. Option A misses the factor $2/3$, while C and D omit the square-root factor from evaluating $\sqrt{1-x}$.

Step 4: Conclusion The maximum term independent of t is $\frac{2 \cdot 10!}{3\sqrt{3}(5!)^2}$, attained at $x = 2/3$. Hence, the correct option is (B).

Quick Tip to Solve: First balance the powers of t; only then maximise the coefficient as a function of x.

2. The value of $\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$ **is :** [4 marks]

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{2}{\sqrt{3}}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Concept:

- The addition formulae expand $\sin(a+h)$ and $\cos(a+h)$. At $a=\pi/6$, the constant cosine terms in the numerator cancel.
- The standard radian limit is $\lim_{h\to 0}\sin h/h = 1$, and $\cos h \to 1$. These limits apply after cancelling the numerator algebraically.

Step 1: Expand the shifted angles Using $\sin(\pi/6)=1/2$ and $\cos(\pi/6)=\sqrt{3}/2$, we obtain

$$\sqrt{3}\sin(\pi/6+h) = \frac{\sqrt{3}}{2}\cos h + \frac{3}{2}\sin h,$$
$$\cos(\pi/6+h) = \frac{\sqrt{3}}{2}\cos h - \frac{1}{2}\sin h.$$

Subtracting the second line from the first gives exactly $2\sin h$. Thus there is no nonzero constant term in the numerator; direct substitution in the original ratio would have produced an indeterminate form.

Step 2: Separate the standard limit Including the factor 2 outside the braces, the expression becomes

$$\frac{4\sin h}{\sqrt{3}\,h(\sqrt{3}\cos h - \sin h)} = \frac{4}{\sqrt{3}}\frac{\sin h}{h}\frac{1}{\sqrt{3}\cos h - \sin h}.$$

The last denominator tends to $\sqrt{3}$, which is nonzero. Hence ordinary product and quotient limit rules apply after this rearrangement. The notation $\cos h$ and $\sin h$ denotes trigonometric functions of the small angle.

Step 3: Evaluate and reject the other values Taking limits yields

$$\frac{4}{\sqrt{3}}\cdot 1\cdot\frac{1}{\sqrt{3}} = \frac{4}{3}.$$

The value $2/3$ would discard the outer factor 2. The reciprocal $3/4$ reverses the quotient, and $2/\sqrt{3}$ misses a factor from the denominator. None of these equals the simplified limit.

Step 4: Conclusion The required limit is $4/3$. Hence, the correct option is (A).

Quick Tip to Solve: Expand the two shifted-angle terms together; their cosine terms cancel before any limiting operation.

3. The value of $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ (a+2)(a+3) & a+3 & 1\\ (a+3)(a+4) & a+4 & 1\end{vmatrix}$ **is :** [4 marks]

(A) -2

(B) 0

(C) (a + 2)(a + 3)(a + 4)

(D) (a + 1)(a + 2)(a + 3)

Correct Answer: (A) -2

Solution:

Concept:

- Adding a multiple of one row to another leaves a determinant unchanged. Such row operations can expose a triangular or sparse determinant without expanding polynomial products.
- Expansion along a column with two zero entries reduces the determinant to one 2×2 minor, with the appropriate cofactor sign.

Step 1: Take consecutive row differences Write the original rows as R_1, R_2, R_3. First replace R_3 by $R_3 - R_2$, and then replace R_2 by $R_2 - R_1$, using the unchanged original first and second rows where required. Since

$$(a+2)(a+3) - (a+1)(a+2) = 2(a+2),$$

and $(a+3)(a+4) - (a+2)(a+3) = 2(a+3)$, the determinant becomes

$$D = \begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ 2(a+2) & 1 & 0\\ 2(a+3) & 1 & 0\end{vmatrix}.$$

The last column has now become particularly convenient for expansion.

Step 2: Expand the sparse column The only nonzero entry in column 3 is the top entry, whose cofactor sign is $(-1)^{1+3} = +1$. Therefore

$$D = \begin{vmatrix}2(a+2) & 1\\ 2(a+3) & 1\end{vmatrix} = 2(a+2) - 2(a+3) = -2.$$

The parameter cancels identically, so the result holds for every real value of a; no division by a possibly zero expression has been used.

Step 3: Check the parameter-independent result For example, at $a = 0$ the rows are $(2, 2, 1)$, $(6, 3, 1)$, and $(12, 4, 1)$; direct expansion again gives -2. This is a check, while the row-difference derivation establishes the result for all a. Option B is excluded because the minor is nonzero. Options C and D depend on a and cannot equal the constant determinant identically.

Step 4: Conclusion The determinant equals -2, independently of a. Hence, the correct option is (A).

Quick Tip to Solve: With consecutive polynomial rows and a constant last column, take row differences before expanding the determinant.

4. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\,dx$ is : [4 marks]

(A) 2π

(B) $\frac{\pi}{2}$

(C) 4π

(D) $\frac{\pi}{4}$

Correct Answer: (D) $\frac{\pi}{4}$

Solution:

Concept:

- On symmetric limits, substituting $x = -u$ replaces $f(x)$ by $f(-x)$ without changing the integral. Adding the two forms can remove an exponential denominator.
- The identity $\cos^2 x = (1 + \cos 2x)/2$ evaluates the remaining elementary integral. The integrand is continuous because $1 + 3^x > 0$.

Step 1: Reflect the integral Let

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1 + 3^x}\,dx.$$

Use $x = -u$, with $dx = -du$, and reverse the transformed limits. Since $\cos^2(-u) = \cos^2 u$, this gives

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 u}{1 + 3^{-u}}\,du.$$

The variable of integration is a dummy variable, so it can now be renamed x.

Step 2: Add the two equal representations For every real x,

$$\frac{1}{1 + 3^x} + \frac{1}{1 + 3^{-x}} = \frac{1}{1 + 3^x} + \frac{3^x}{1 + 3^x} = 1.$$

Consequently

$$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x\,dx = \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_{-\pi/2}^{\pi/2} = \frac{\pi}{2}.$$

Both endpoint sine terms vanish. Dividing by 2 is essential because the addition involved two copies of the original integral.

Step 3: Compare with the choices Thus $I = \pi/4$. In particular, $\pi/2$ is the value of the integral after adding the reflected copy, not the original integral. Also, $0 < 1/(1 + 3^x) < 1$, so the original integral is strictly smaller than $\int_{-\pi/2}^{\pi/2} \cos^2 x\,dx = \pi/2$. This immediately rules out 2π, $\pi/2$, and 4π.

Step 4: Conclusion The integral is $\pi/4$. Hence, the correct option is (D).

Quick Tip to Solve: For a denominator $1 + a^x$ on symmetric limits, add the integral obtained by replacing x with $-x$.

5. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is : [4 marks]

(A) 35

(B) 42

(C) 82

(D) 77

Correct Answer: (D) 77

Solution:

Concept:

- A seven-digit number made from 1, 2, and 3 can be counted by specifying how many positions contain each digit. Equal digits are indistinguishable, but their positions matter.
- If n_1, n_2, n_3 count digits 1, 2, 3, then $n_1 + n_2 + n_3 = 7$ and the digit sum is $n_1 + 2n_2 + 3n_3$. Subtracting these constraints isolates the excess above seven ones.

Step 1: Find all possible digit multiplicities The two conditions are

$$n_1 + n_2 + n_3 = 7, \qquad n_1 + 2n_2 + 3n_3 = 10.$$

Subtracting yields $n_2 + 2n_3 = 3$. All counts must be nonnegative integers. If $n_3 = 0$, then $n_2 = 3$ and $n_1 = 4$. If $n_3 = 1$, then $n_2 = 1$ and $n_1 = 5$. Values $n_3 \geq 2$ force a negative n_2, so there are exactly two disjoint cases.

Step 2: Count the arrangements in each case For four ones and three twos, choose the three positions containing 2:

$$N_1 = \binom{7}{3} = 35.$$

For five ones, one two and one three, choose the position of 3 in 7 ways and then the position of 2 in 6 ways:

$$N_2 = 7 \cdot 6 = 42 = \frac{7!}{5!1!1!}.$$

Every arrangement is a valid seven-digit integer because no permitted digit is zero. The two cases have different digit multiplicities and cannot overlap.

Step 3: Add the exhaustive cases Therefore the total is $N_1 + N_2 = 35 + 42 = 77$. Option A counts only the first case, and option B counts only the second case. Option C, 82, is not the sum of the two exhaustive disjoint counts. Treating the three extra units as unrestricted increments would also allow a digit 4, which is forbidden here.

Step 4: Conclusion There are 77 such seven-digit integers. Hence, the correct option is (D).

Quick Tip to Solve: Subtract the sum of seven ones first: the remaining excess 3 must be made from increments 1 and 2 only.

6. Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, −1) is the set : [4 marks]

(A) $S = \{(x, y)|x^2 + y^2 = \sqrt{2}\}$

(B) $S = \{(x, y)|x^2 + y^2 = 2\}$

(C) $S = \{(x, y)|x^2 + y^2 = 1\}$

(D) $S = \{(x, y)|x^2 + y^2 = 4\}$

Correct Answer: (B) $S = \{(x, y)|x^2 + y^2 = 2\}$

Solution:

Concept:

- An equivalence class consists of all elements related to a fixed element. Here two points are related exactly when their distances from the origin are equal.
- The distance of (x, y) from the origin is $\sqrt{x^2 + y^2}$. A locus of constant positive distance from the origin is a circle centred at the origin.

Step 1: Compute the fixed distance For the specified point $P = (1, -1)$, its distance from the origin $O = (0, 0)$ is

$$OP = \sqrt{1^2 + (-1)^2} = \sqrt{2}.$$

The sign of the second coordinate disappears on squaring; it does not decrease the distance. Thus the class must contain all points at radius $\sqrt{2}$, including the original point itself.

Step 2: Translate the relation into a locus Let $Q = (x, y)$ be any member of the class. The condition $P\,R\,Q$ is precisely

$$\sqrt{x^2 + y^2} = \sqrt{2}.$$

Both sides are nonnegative, so squaring is reversible and yields $x^2 + y^2 = 2$. Conversely, any point satisfying that equation has distance $\sqrt{2}$ from the origin and is related to P. This proves that the equation describes the complete equivalence class, rather than just some of its members.

Step 3: Check the alternatives and the relation Equality of distances is reflexive, symmetric, and transitive because equality of real numbers has these properties. Thus using an equivalence class here is valid. Option A confuses radius $\sqrt{2}$ with radius squared. Option C describes radius 1, and option D describes radius 2. None contains $(1, -1)$, whereas every equivalence class must contain its representative. Only option B passes this necessary check.

Step 4: Conclusion The class of $(1, -1)$ is $S = \{(x, y) : x^2 + y^2 = 2\}$. Hence, the correct option is (B).

Quick Tip to Solve: Test the representative point in each proposed class: an equivalence class must always contain that point.

7. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\leq$ x, is : [4 marks]

(A) 100e

(B) 100(e − 1)

(C) 100(1 + e)

(D) 100(1 − e)

Correct Answer: (B) 100(e − 1)

Solution:

Concept:

- For any integer n, the greatest integer $[x]$ equals $n-1$ throughout $[n-1, n)$. Therefore $x-[x]$ is the fractional part of x and runs from 0 to 1 on each such interval.
- Changing an integrand at finitely many endpoints does not change its definite integral. This allows the constant floor value on each half-open unit interval to be used in the integration.

Step 1: Simplify one unit-interval integral Fix an integer $n \in \{1, 2, \ldots, 100\}$. For $n-1 \leq x < n$, we have $[x] = n-1$. Hence

$$\int_{n-1}^{n} e^{x-[x]}\,dx = \int_{n-1}^{n} e^{x-(n-1)}\,dx.$$

At $x = n$, the floor jumps to n, but a value at a single endpoint has no effect on the integral. We therefore integrate the displayed smooth expression over the whole unit interval.

Step 2: Use the same substitution for every interval Put $u = x-(n-1)$. Then $du = dx$; the lower limit becomes 0 and the upper limit becomes 1. Thus

$$\int_{n-1}^{n} e^{x-[x]}\,dx = \int_{0}^{1} e^{u}\,du = [e^{u}]_{0}^{1} = e-1.$$

This result is independent of n. It reflects the fact that $e^{x-[x]}$ repeats the same shape on each consecutive unit interval rather than growing exponentially across all 100 intervals.

Step 3: Sum and compare the options There are exactly 100 terms, so

$$\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}\,dx = 100(e-1).$$

Option A omits the lower-limit contribution 1 in each interval. Option C adds that contribution instead of subtracting it. Option D is negative since $e > 1$, whereas every integrand and every integral in the sum is positive. These checks distinguish the result unambiguously.

Step 4: Conclusion The required sum is $100(e-1)$. Hence, the correct option is (B).

Quick Tip to Solve: For integrals of a function of $x-[x]$, shift every complete unit interval to $[0, 1]$.

8. The intersection of three lines x − y = 0, x + 2y = 3 and 2x + y = 6 is a : [4 marks]

(A) Right angled triangle

(B) Equilateral triangle

(C) None of the above

(D) Isosceles triangle

Correct Answer: (D) Isosceles triangle

Solution:

Concept:

- Two distinct nonparallel lines intersect at a vertex. Find all three pairwise intersections before classifying the triangle.
- The squared distance between points is $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$. Equal squared lengths establish an isosceles triangle without square roots.
- A right triangle has its largest squared side equal to the sum of the other two; an equilateral triangle has all three sides equal.

Step 1: Find the three vertices For $x - y = 0$, put $x = y$. Its intersection with $x + 2y = 3$ is $U = (1, 1)$, while its intersection with $2x + y = 6$ is $V = (2, 2)$. To intersect the remaining two lines, subtract $2x + 4y = 6$ from $2x + y = 6$, obtaining $y = 0$, hence $W = (3, 0)$. The vertices are distinct, so the lines enclose a triangle rather than meeting at one point.

Step 2: Compare the squared side lengths Use the coordinates in the distance formula separately for each side:

$$UV^2 = (2 - 1)^2 + (2 - 1)^2 = 2,$$

$$VW^2 = (3 - 2)^2 + (0 - 2)^2 = 5,$$

$$WU^2 = (1 - 3)^2 + (1 - 0)^2 = 5.$$

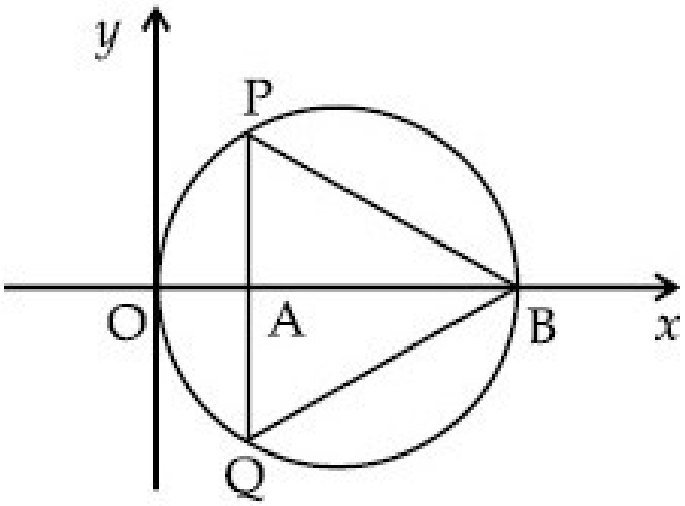
Thus $VW = WU = \sqrt{5}$, whereas $UV = \sqrt{2}$. Exactly two sides are equal. This also agrees with the symmetry of the equations of the two sides that meet at W.

Step 3: Eliminate the other classifications The unequal squared lengths 2 and 5 rule out an equilateral triangle. The largest squared length is 5, but the sum of the other two squared lengths is $2 + 5 = 7$, so the Pythagorean criterion for a right triangle fails. Since the isosceles classification is available, the option saying none of the above is also excluded.

Step 4: Conclusion The three lines form an isosceles triangle with side lengths $\sqrt{5}, \sqrt{5}, \sqrt{2}$. Hence, the correct option is (D).

Quick Tip to Solve: Find pairwise vertices and compare squared distances; two equal values identify the isosceles triangle immediately.

9. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is : [4 marks]



(A) $24\sqrt{2}$

(B) $24\sqrt{3}$

(C) $26\sqrt{2}$

(D) $26\sqrt{3}$

Correct Answer: (B) $24\sqrt{3}$

Solution:

Concept:

- The diagram shows OB as the horizontal diameter, with $O = (0,0)$ and $B = (13,0)$. The perpendicular chord PQ passes through $A = (1,0)$.
- A circle with diameter endpoints $(0,0)$ and $(d,0)$ has equation $x^2 + y^2 = dx$. Its perpendicular chord at abscissa x has half-length $\sqrt{x(d-x)}$.
- The area of a triangle is one-half its base times its perpendicular altitude. For base PQ, the altitude from B is AB.

Step 1: Write the circle equation The centre is the midpoint of the diameter OB, namely $(13/2, 0)$, and the radius is $13/2$. Thus

$$\left(x - \frac{13}{2}\right)^2 + y^2 = \left(\frac{13}{2}\right)^2, \qquad x^2 + y^2 = 13x.$$

This uses the diameter in the figure; OB is not a radius measured from the centre. The origin is at the left endpoint of that diameter.

Step 2: Find the chord length Since $OA = 1$ and PQ is perpendicular to the horizontal line OB, both chord endpoints have $x = 1$. Substitute into the circle equation:

$$1 + y^2 = 13, \qquad y^2 = 12.$$

Therefore $P = (1, 2\sqrt{3})$ and $Q = (1, -2\sqrt{3})$. The chord length is the difference of these ordinates:

$$PQ = 2\sqrt{3} - (-2\sqrt{3}) = 4\sqrt{3}.$$

The chord is bisected by the diameter because it is perpendicular to it.

Step 3: Calculate the triangle area and compare The perpendicular distance from B to the vertical chord is $AB = OB - OA = 13 - 1 = 12$. Consequently

$$[PQB] = \frac{1}{2}(4\sqrt{3})(12) = 24\sqrt{3}.$$

Options containing $\sqrt{2}$ contradict the chord calculation $y^2 = 12$. The value $26\sqrt{3}$ would result from incorrectly using the whole diameter 13 as the altitude instead of the distance 12 from B to the chord.

Step 4: Conclusion The area of triangle PQB is $24\sqrt{3}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For a chord perpendicular to a diameter, use $y^2 = x(d - x)$ and take the triangle altitude from the far endpoint to the chord.

10. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is : [4 marks]

(A) 6

(B) 4

(C) 1

(D) 12

Correct Answer: (B) 4

Solution:

Concept:

- A real symmetric matrix of order two has the form $A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$. Here a, b, d must all be integers.
- The sum of diagonal entries is the trace. For this symmetric matrix, $\mathrm{tr}(A^2) = a^2 + d^2 + 2b^2$, a sum of nonnegative integers.
- When an integer sum of nonnegative squares equals one, exactly one square is one and all the others are zero.

Step 1: Compute the diagonal entries of the square Multiply the general symmetric matrix by itself, keeping the off-diagonal entry equal in both positions:

$$A^2 = \begin{pmatrix} a^2 + b^2 & ab + bd \\ ab + bd & b^2 + d^2 \end{pmatrix}.$$

The hypothesis concerns the sum of the two diagonal entries of this square, not the square of the trace

of A. Therefore its exact algebraic condition is

$$a^2 + d^2 + 2b^2 = 1.$$

In particular, there can be no cancellation among the three terms.

Step 2: Use integrality to restrict the entries If the integer b were nonzero, then $b^2 \geq 1$, and the term $2b^2$ alone would be at least two. This is impossible. Hence $b = 0$ and the remaining equation becomes $a^2 + d^2 = 1$. The only integer possibilities are

$$(a, d) = (1, 0),\ (-1, 0),\ (0, 1),\ (0, -1).$$

Each possibility gives a distinct symmetric matrix and satisfies the original trace condition.

Step 3: Count the matrices and check the alternatives The matrices are

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}.$$

Thus there are four in total. A count of one misses the independent diagonal position and sign choices; counts of six or twelve wrongly admit nonzero off-diagonal entries or extra integer square combinations.

Step 4: Conclusion There are exactly 4 such symmetric integer matrices. Hence, the correct option is (B).

Quick Tip to Solve: For a symmetric real matrix, the trace of its square is the sum of squares of all entries; off-diagonal entries are counted twice.

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11. If $\frac{\sin^1 x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$; $0 < x < 1$,

then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ **is :** [4 marks]

(A) $\frac{1-y^2}{2y}$

(B) $\frac{1-y^2}{y\sqrt{y}}$

(C) $1 - y^2$

(D) $\frac{1-y^2}{1+y^2}$

Correct Answer: The literal exponent 1 does not denote an inverse sine; option D follows under the inverse-sine reading.

Solution:

Concept:

- The usual inverse-trigonometric identity is $\sin^{-1} x + \cos^{-1} x = \pi/2$ for $-1 \leq x \leq 1$. It requires inverse sine, not the ordinary function $\sin x$.
- For $\theta = \tan^{-1} y$, the double-angle identity gives $\cos(2\theta) = (1 - y^2)/(1 + y^2)$.
- The printed first numerator is $\sin^1 x$. Taken literally, this means $\sin x$, so the stated inverse-angle

sum cannot automatically be applied.

Step 1: Distinguish the printed expression from inverse sine Let the common ratio in the printed equality be t, with the denominators defined and nonzero. On the literal reading,

$$at = \sin x, \qquad bt = \cos^{-1} x, \qquad ct = \tan^{-1} y.$$

Adding the first two equations yields $(a+b)t = \sin x + \cos^{-1} x$, rather than $\pi/2$. Thus the argument demanded in the question is literally

$$\frac{\pi c}{a+b} = \frac{\pi \tan^{-1} y}{\sin x + \cos^{-1} x}.$$

It generally still depends on x, whereas every offered answer depends only on y.

Step 2: Check the conventional inverse-sine reading If the first numerator is read as $\sin^{-1} x$, the same common-ratio procedure instead gives

$$(a+b)t = \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}.$$

Combining this with $ct = \tan^{-1} y$ produces

$$\frac{\pi c}{a+b} = 2\tan^{-1} y.$$

This explains the standard double-angle structure of the options; the condition $0 < x < 1$ lies within both inverse functions on their principal real domains.

Step 3: Evaluate the intended expression and compare the options Under that inverse-sine reading, take $\theta = \tan^{-1} y$ and use

$$\cos\left(\frac{\pi c}{a+b}\right) = \cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-y^2}{1+y^2}.$$

This is option (D). Options (A) and (B) have incorrect denominators for the cosine double-angle identity; option (C) omits the necessary normalization by $1+y^2$. These eliminations apply to the inverse-sine interpretation, not to the literal first power printed in the question.

Step 4: Conclusion As printed with $\sin^1 x$, no unique listed expression follows: the result generally retains dependence on x. Under the conventional inverse-sine interpretation $\sin^{-1} x$, the answer is $(1-y^2)/(1+y^2)$, corresponding to option (D).

Quick Tip to Solve: Check the inverse-function exponent before using the complementary inverse-angle identity; its sum is constant only for inverse sine and inverse cosine.

12. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point : [4 marks]

(A) $\left(3, \frac{21}{2}\right)$

(B) (0, 0)

(C) (2, 9)

(D) (2, 2)

Correct Answer: (D) (2, 2)

Solution:

Concept:

- The slope of a differentiable curve is $m(x) = dy/dx$. To locate its local extrema, differentiate the slope, rather than merely setting the original slope equal to zero.
- A stationary slope with $m''(x) < 0$ is a local maximum of the slope. The corresponding ordinate must still be evaluated from the original curve.
- The slope here is a cubic with positive leading coefficient. It is unbounded above on the whole real line, so the available maximum is a local maximum.

Step 1: Form the slope function and locate its stationary points Differentiate the given polynomial term by term:

$$m(x) = y' = 2x^3 - 15x^2 + 36x - 19.$$

To maximize this slope locally, set its derivative equal to zero:

$$m'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3) = 0.$$

The two candidates are $x = 2$ and $x = 3$. These are stationary points of the slope, not necessarily stationary points of the original curve.

Step 2: Classify the slope extrema The second derivative of the slope is $m''(x) = 12x - 30$. At $x = 2$, its value is $-6 < 0$, so the slope has a local maximum there. At $x = 3$, its value is $6 > 0$, so the slope has a local minimum. Equivalently, m' changes from positive to negative at two, and from negative to positive at three. The slope values are $m(2) = 9$ and $m(3) = 8$; neither is an ordinate of the original curve.

Step 3: Find the corresponding point and check the wording Substitute $x = 2$ into the original expression:

$$y(2) = \frac{1}{2}(2^4) - 5(2^3) + 18(2^2) - 19(2) = 8 - 40 + 72 - 38 = 2.$$

Thus the point is $(2, 2)$, not $(2, 9)$, which mistakenly uses the slope as the ordinate. The point at $x = 3$ gives the local minimum slope, while the origin is not a stationary point of the slope. Strictly, $m(x) \to +\infty$ as $x \to +\infty$, so there is no absolute maximum slope over all real x. The listed answer identifies the local maximum.

Step 4: Conclusion The slope has its local maximum at $(2, 2)$. Hence, the correct option is (D), with maximum understood locally; the unrestricted slope has no global maximum.

Quick Tip to Solve: To maximize slope, solve $y'' = 0$, classify with y''', then obtain the ordinate from y, not from y'.

13. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to : [4 marks]

(A) 16

(B) 8

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Growth proportional to the current population is described by $dN/dt = rN$, where r is a constant growth rate per hour.
- The solution with initial count N_0 is $N(t) = N_0 e^{rt}$. Percentage growth over a finite interval determines the exponential factor over that entire interval.
- Natural logarithms convert e^{rt} into rt; the 20 percent growth factor is $1.2 = 6/5$.

Step 1: Determine the growth constant With $N(0) = 1000$, integration of the proportional-growth equation gives $N(t) = 1000e^{rt}$. After two hours the population is $1000(1 + 20/100) = 1200$. Therefore

$$1200 = 1000e^{2r}, \qquad e^{2r} = \frac{6}{5}, \qquad r = \frac{1}{2}\ln\left(\frac{6}{5}\right).$$

The factor of one-half is essential because the stated increase takes place over two hours, not one.

Step 2: Find the time to reach twice the initial population Let T be the time in hours when the count becomes 2000. Substitution into the same exponential law gives

$$2000 = 1000e^{rT}, \qquad rT = \ln 2.$$

Hence

$$T = \frac{\ln 2}{r} = \frac{2\ln 2}{\ln(6/5)}.$$

This result uses continuous proportional growth; assuming a fixed addition of 200 bacteria every two hours would contradict the differential equation because the rate rises as the population rises.

Step 3: Identify the parameter and evaluate the requested square The question writes the time as $T = k/\ln(6/5)$. Comparing numerators gives $k = 2\ln 2$. Consequently

$$\left(\frac{k}{\ln 2}\right)^2 = \left(\frac{2\ln 2}{\ln 2}\right)^2 = 2^2 = 4.$$

This is dimensionless, as required for the offered numerical values. The alternative 2 omits the final squaring; 8 and 16 do not follow from the two-hour exponential calibration.

Step 4: Conclusion The requested value is 4. Hence, the correct option is (D).

Quick Tip to Solve: Calibrate the exponential rate over the stated time interval first; a 20 percent rise in two hours gives $r = \ln(1.2)/2$.

14. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to : [4 marks]

(A) 30

(B) 32

(C) 26

(D) 35

Correct Answer: (D) 35

Solution:

Concept:

- For a geometric progression with first term a and common ratio r, the n-th term is $T_n = ar^{n-1}$.
- Terms equally spaced about a middle term satisfy $T_{n-h}T_{n+h} = T_n^2$. Thus the given product directly determines the magnitude of the fourth term.
- Writing the remaining terms relative to T_4 reduces the unknown ratio to r^2. The increasing condition selects the appropriate root.

Step 1: Find the fourth term with its sign The product in the question is

$$T_3T_5 = (ar^2)(ar^4) = a^2r^6 = T_4^2 = 25.$$

Hence $T_4 = \pm 5$. Since the progression is increasing, its terms cannot alternate in sign, so $r > 0$. Moreover $T_2 + T_6 = T_4(r^{-2} + r^2)$ is positive and the bracket is positive. Therefore $T_4 = 5$, rather than -5. The given sum rules out the negative increasing progression possibility.

Step 2: Determine the increasing ratio Use the given sum relative to the fourth term:

$$5\left(r^2 + \frac{1}{r^2}\right) = \frac{25}{2}, \qquad r^2 + \frac{1}{r^2} = \frac{5}{2}.$$

Put $u = r^2 > 0$. Then

$$2u^2 - 5u + 2 = 0, \qquad (2u - 1)(u - 2) = 0.$$

Thus $u = 1/2$ or $u = 2$. Since T_4 is positive, the progression has positive terms; being increasing therefore requires $r > 1$. Select $r^2 = 2$, not $1/2$.

Step 3: Add the requested even-position terms The sixth and eighth terms are obtained from the fourth by multiplying by r^2 and r^4, respectively. Thus

$$T_4 + T_6 + T_8 = 5(1 + r^2 + r^4) = 5(1 + 2 + 4) = 35.$$

For a direct check, $T_2 = 5/2$ and $T_6 = 10$, whose sum is $25/2$ as given. The other offered sums 30, 32 and 26 are inconsistent with the selected increasing ratio and these terms.

Step 4: Conclusion The sum of the fourth, sixth and eighth terms is 35. Hence, the correct option is (D).

Quick Tip to Solve: Use the middle-term product to obtain T_4 first, then express the required even-position terms using powers of r^2.

15. Let f be any function defined on R and let it satisfy the condition : $|f(x)-f(y)| \leq |(x-y)^2|, \forall (x,y) \in R$

If f(0) = 1, then : [4 marks]

(A) f(x) can take any value in R

(B) $f(x) < 0, \forall x \in R$

(C) $f(x) > 0, \forall x \in R$

(D) $f(x) = 0, \forall x \in R$

Correct Answer: (C) $f(x) > 0, \forall x \in R$

Solution:

Concept:

- The bound $|f(x)-f(y)| \leq |x-y|^2$ controls every difference quotient, without any initial assumption that f is differentiable.
- If a function is differentiable on an interval and its derivative is zero everywhere, the mean value theorem shows that it is constant there.

Step 1: Form the difference quotient at an arbitrary point Fix any real number x, and put $y = x + h$, where $h \neq 0$. The given condition becomes $|f(x+h) - f(x)| \leq h^2$. Dividing by the positive quantity $|h|$ gives

$$\left|\frac{f(x+h) - f(x)}{h}\right| \leq |h|.$$

Both the upper bound and its negative tend to zero as $h \to 0$. The squeeze theorem therefore proves that the derivative exists at every real x, and that $f\prime(x) = 0$.

Step 2: Use the initial value and compare the alternatives For any real $x \neq 0$, apply the mean value theorem between 0 and x. Differentiability gives continuity, so its hypotheses hold. There is a point c between the endpoints such that

$$f(x) - f(0) = f\prime(c)(x - 0) = 0.$$

Thus $f(x) = f(0) = 1$, also true at $x = 0$. Option (A) fails because the value is fixed; (B) has the wrong sign; and (D) contradicts the given initial value. Only (C) is consistent for every real input.

Step 3: Conclusion The function is $f(x) = 1$ for all real x, so $f(x) > 0$ everywhere. Hence, the correct option is (C).

Quick Tip to Solve: When a function difference is bounded by a power of $|x - y|$ greater than one, divide by $|x - y|$ and let the two inputs approach each other.

16. The sum of the infinite series

$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots\ldots$ **is equal to :** [4 marks]

(A) $\frac{9}{4}$

(B) $\frac{13}{4}$

(C) $\frac{15}{4}$

(D) $\frac{11}{4}$

Correct Answer: (B) $\frac{13}{4}$

Solution:

Concept:

- For $|r|<1$, the geometric-series identity is $\sum_{n=1}^{\infty} r^n = r/(1-r)$.
- After its initial term, the displayed series has numerators $2, 7, 12, 17, \ldots$, an arithmetic progression with common difference 5.
- The weighted geometric-series identity is $\sum_{n=1}^{\infty} nr^n = r/(1-r)^2$, again for $|r|<1$.

Step 1: Express the tail using its general term Let S denote the complete sum. The term with denominator 3^n, for $n \geq 1$, has numerator $5n-3$, since this gives 2 at $n=1$ and increases by 5 thereafter. The first standalone term 1 must be retained separately. Hence

$$S = 1 + \sum_{n=1}^{\infty} \frac{5n-3}{3^n} = 1 + 5\sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^n - 3\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n.$$

These series converge absolutely because the geometric factor is $1/3$, so separating and recombining them is valid.

Step 2: Evaluate the geometric sums and simplify Using $r=1/3$, the two sums are

$$\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n = \frac{1}{2}, \qquad \sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^n = \frac{3}{4}.$$

Substitution gives

$$S = 1 + 5\left(\frac{3}{4}\right) - 3\left(\frac{1}{2}\right) = \frac{4+15-6}{4} = \frac{13}{4}.$$

Thus (A), (C) and (D) are excluded by the exact sum. In particular, treating the initial 1 as part of the same numerator progression would give an incorrect constant contribution.

Step 3: Conclusion The infinite series sums to $13/4$. Hence, the correct option is (B).

Quick Tip to Solve: Separate any exceptional first term before writing the numerator as $an+b$; then use the standard geometric and weighted geometric sums.

17. Consider the three planes

$P_1 : 3x + 15y + 21z = 9$,

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true? [4 marks]

(A) P_1 and P_2 are parallel.

(B) P_1, P_2 and P_3 all are parallel.

(C) P_1 and P_3 are parallel.

(D) P_2 and P_3 are parallel.

Correct Answer: (C) P_1 and P_3 are parallel.

Solution:

Concept:

- The coefficients (a, b, c) in a plane $ax + by + cz = d$ form a normal vector. Two planes have parallel directions exactly when their normal vectors are proportional.
- Proportional normals describe distinct parallel planes when their equations, after matching the left sides, have unequal constants; proportional entire equations describe coincident planes.

Step 1: Compare the normal vectors The three normal vectors are

$$\mathbf{n}_1 = (3, 15, 21), \quad \mathbf{n}_2 = (1, -3, -1), \quad \mathbf{n}_3 = (2, 10, 14).$$

The first and third satisfy $\mathbf{n}_1 = \frac{3}{2}\mathbf{n}_3$, because all three component ratios equal $3/2$. For the first and second, the component ratios are $3, -5, -21$, which are unequal. For the third and second, they are $2, -10, -14$, also unequal. Therefore neither pair involving P_2 can be parallel.

Step 2: Confirm that the proportional pair is distinct Divide the equation of P_1 by 3, and that of P_3 by 2. The normalized equations become

$$P_1 : x + 5y + 7z = 3, \qquad P_3 : x + 5y + 7z = \frac{5}{2}.$$

The left sides are identical but the constants differ. Thus these planes cannot share a point and are distinct parallel planes. Option (A) incorrectly pairs P_1 with P_2; option (D) incorrectly pairs P_2 with P_3. Option (B) would require all three normals to be proportional, which they are not.

Step 3: Conclusion Only P_1 and P_3 are parallel. Hence, the correct option is (C).

Quick Tip to Solve: Compare coefficient ratios first, then compare the constant ratio to distinguish parallel planes from coincident planes.

18. If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is : [4 marks]

(A) $-\frac{44}{5}$

(B) $-\frac{39}{5}$

(C) $\frac{44}{5}$

(D) $\frac{39}{5}$

Correct Answer: (C) $\frac{44}{5}$

Solution:

Concept:

- Four points A, B, C, D are coplanar precisely when $\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$, equivalently the determinant of the three displacement vectors is zero.
- For a quadratic $a\lambda^2 + b\lambda + c = 0$, the sum of its two roots is $-b/a$, counting every possible parameter value.

Step 1: Construct displacement vectors from one fixed point Choose $A = (1, 5, 35)$, $B = (7, 5, 5)$, $C = (1, \lambda, 7)$, and $D = (2\lambda, 1, 2)$. Subtracting A from each of the other points gives

$$\overrightarrow{AB} = (6, 0, -30), \quad \overrightarrow{AC} = (0, \lambda - 5, -28), \quad \overrightarrow{AD} = (2\lambda - 1, -4, -33).$$

This subtraction removes the common position vector and leaves the three directions whose scalar triple product must vanish. No division by an expression involving λ has been used, so no special parameter value is lost.

Step 2: Expand the determinant and add its roots The coplanarity condition is

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda - 5 & -28 \\ 2\lambda - 1 & -4 & -33 \end{vmatrix} = 0.$$

Expanding along the first row yields

$$6[-33(\lambda - 5) - 112] + 30(\lambda - 5)(2\lambda - 1) = 0.$$

On simplifying,

$$60\lambda^2 - 528\lambda + 468 = 0, \qquad 5\lambda^2 - 44\lambda + 39 = 0.$$

Indeed this factors as $(\lambda - 1)(5\lambda - 39) = 0$, so the two values are 1 and $39/5$. Their sum is $44/5$; the negative choices (A), (B) and the single-root value in (D) do not give this sum.

Step 3: Conclusion The sum of all possible values of λ is $44/5$. Hence, the correct option is (C).

Quick Tip to Solve: For four-point coplanarity, subtract one point from the other three and use a 3×3 determinant; use Vieta when only a root sum is requested.

19. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is : [4 marks]

(A) $\frac{15}{2^8}$

(B) $\frac{15}{2^{12}}$

(C) $\frac{15}{2^{13}}$

(D) $\frac{15}{2^{14}}$

Correct Answer: (C) $\frac{15}{2^{13}}$

Solution:

Concept:

- With n independent tosses of a fair coin, the number of heads X has probability $P(X = k) = \binom{n}{k}/2^n$ for $0 \le k \le n$.
- Equality of the positive probabilities for seven and nine heads gives equality of the corresponding binomial coefficients; the common denominator cancels.

Step 1: Determine the number of tosses For the stated outcomes to be possible, take $n \ge 9$. Then

$$\binom{n}{7} = \binom{n}{9}.$$

Using factorials and cancelling nonzero common factors gives

$$\frac{\binom{n}{9}}{\binom{n}{7}} = \frac{(n-7)(n-8)}{9 \cdot 8} = 1.$$

Therefore

$$(n-7)(n-8) = 72, \qquad n^2 - 15n - 16 = 0, \qquad (n-16)(n+1) = 0.$$

The number of tosses is a nonnegative integer, so the admissible root is $n = 16$; the algebraic root -1 cannot describe an experiment.

Step 2: Calculate the probability of exactly two heads There are $\binom{16}{2}$ ways to choose the two head positions, and each full sequence has probability 2^{-16}. Thus

$$P(X = 2) = \frac{\binom{16}{2}}{2^{16}} = \frac{16 \cdot 15}{2 \cdot 2^{16}} = \frac{120}{2^{16}} = \frac{15}{2^{13}}.$$

The factor 8 in $120 = 8 \cdot 15$ reduces the exponent from 16 to 13. Hence the denominators in (A), (B), and (D) do not match the exact binomial calculation. This also agrees with the symmetry check $7 + 9 = 16$, as the equal coefficients lie on opposite sides of the centre.

Step 3: Conclusion The probability of getting exactly two heads is $15/2^{13}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal positive binomial probabilities of two distinct head counts with a fair coin, use the symmetry $r + s = n$, then compute the requested coefficient.

20. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)$ is equal to : [4 marks]

(A) $\frac{1}{2}|\vec{a}|^4 \vec{b}$

(B) $\vec{0}$

(C) $\vec{a} \times \vec{b}$

(D) $|\vec{a}|^4 \vec{b}$

Correct Answer: (D) $|\vec{a}|^4 \vec{b}$

Solution:

Concept:

- The vector triple-product identity is $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{w}(\mathbf{u} \cdot \mathbf{v})$. Parentheses matter because the cross product is not associative.
- Here $\mathbf{a} \perp \mathbf{b}$, so $\mathbf{a} \cdot \mathbf{b} = 0$, while $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$.

Step 1: Evaluate the innermost pair of cross products Write $L(\mathbf{v}) = \mathbf{a} \times \mathbf{v}$ to track the repeated operation. Applying the triple-product identity to the innermost two operations gives

$$L^2(\mathbf{b}) = \mathbf{a} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\mathbf{a} \cdot \mathbf{b}) - \mathbf{b}(\mathbf{a} \cdot \mathbf{a}) = -|\mathbf{a}|^2 \mathbf{b}.$$

Thus two successive cross products bring the direction back to that of $\mathbf{b}$, with a negative scalar multiplier. We have not interchanged the order of any two vectors.

Step 2: Apply the same operation twice more The required expression has four applications of L. Since $|\mathbf{a}|^2$ is a scalar independent of the operation, linearity gives

$$\begin{aligned} L^4(\mathbf{b}) &= L^2\left(-|\mathbf{a}|^2 \mathbf{b}\right) \\ &= -|\mathbf{a}|^2 L^2(\mathbf{b}) \\ &= (-|\mathbf{a}|^2)(-|\mathbf{a}|^2)\mathbf{b} \\ &= |\mathbf{a}|^4 \mathbf{b}. \end{aligned}$$

Option (A) introduces an unjustified factor $1/2$. Option (B) is not the general result for nonzero perpendicular vectors. Option (C) points along $\mathbf{a} \times \mathbf{b}$, perpendicular to $\mathbf{b}$, whereas four operations return the original direction.

Step 3: Conclusion The expression equals $|\mathbf{a}|^4 \mathbf{b}$. Hence, the correct option is (D).

Quick Tip to Solve: For repeated cross products with the same perpendicular vector, group the operations in pairs: each pair multiplies the original vector by $-|\mathbf{a}|^2$.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is _____. [4 marks]

Solution:

Concept:

- An absolute value preserves positive portions of a function and reverses its negative portions. Split the integral at zeros where the sign changes.
- On $[0, \pi/2]$, $\sin 2x \geq 0$; on $[\pi/2, \pi]$, $\sin 2x \leq 0$. An antiderivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$.

Step 1: Find the sign change and remove the absolute value Let the required integral be I. The zeros of $\sin 2x$ in the interval are $0, \pi/2, \pi$, and only the middle zero separates a positive lobe from a negative one. Consequently,

$$I = \int_0^{\pi/2} \sin 2x \, dx - \int_{\pi/2}^{\pi} \sin 2x \, dx.$$

The minus sign before the second integral is essential: its original integrand is negative, but the absolute value makes the contribution to I positive. Directly integrating $\sin 2x$ over the whole interval would cancel the two lobes and solve a different problem.

Step 2: Evaluate both positive contributions Use the antiderivative with the chain-rule factor $1/2$:

$$\int_0^{\pi/2} \sin 2x \, dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2} = \frac{1}{2} + \frac{1}{2} = 1,$$
$$-\int_{\pi/2}^{\pi} \sin 2x \, dx = \left[\frac{1}{2}\cos 2x\right]_{\pi/2}^{\pi} = \frac{1}{2} + \frac{1}{2} = 1.$$

Therefore $I = 1 + 1 = 2$. As a check, the substitution $u = 2x$ turns the integral into half the area under $|\sin u|$ over one full period, which is $\frac{1}{2} \cdot 4 = 2$. The integral contains no physical quantities, so its value is dimensionless.

Step 3: Conclusion The required numerical value is $\boxed{2}$ (dimensionless).

Quick Tip to Solve: Before integrating an absolute trigonometric function, mark its zeros and determine its sign on every intervening interval; never allow positive and negative areas to cancel.

22. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _____. [4 marks]

Solution:

Concept:

- The real logarithm $\log_b t$ requires $t > 0$, with a positive base different from one. Both logarithms here must be defined simultaneously.

- Use change of base before equating arguments: $\log_4 t = \frac{1}{2}\log_2 t$. A logarithm to base two is one-to-one on positive arguments.

Step 1: Restrict the domain and change the base The two arguments require $x-1>0$ and $x-3>0$, so the common domain is $x>3$. Convert the first logarithm to base two and multiply the equation by two:

$$\log_2(x-1) = 2\log_2(x-3).$$

Since $x-3$ is positive on the domain, the power rule gives $2\log_2(x-3) = \log_2((x-3)^2)$.

Step 2: Solve and check the candidate roots Injectivity of the logarithm now gives

$$x-1 = (x-3)^2.$$

Expanding and collecting terms,

$$x^2 - 7x + 10 = 0 \quad \Longrightarrow \quad (x-5)(x-2) = 0.$$

The algebraic candidates are $x=5$ and $x=2$. The value two is inadmissible because $x-3=-1$, so its logarithm is not real. For $x=5$, the original left side is $\log_4 4 = 1$, and the right side is $\log_2 2 = 1$; hence it is valid.

Step 3: Conclusion The only real solution is $x=5$. Therefore the number of solutions is $\boxed{1}$, a dimensionless count.

Quick Tip to Solve: Write the common logarithmic domain first; squaring an argument can otherwise hide an inadmissible negative argument.

23. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _____. [4 marks]

Solution:

Concept:

- A cubic root sum involving a high power is simplified by factoring the polynomial and identifying roots of unity, rather than expanding the power.
- Euler's identity gives $\cos\theta + i\sin\theta = e^{i\theta}$, and De Moivre's theorem gives $(e^{i\theta})^n = e^{in\theta}$.

Step 1: Factor the cubic and identify all roots The polynomial vanishes at $x=1$, so divide out $x-1$:

$$x^3 - 2x^2 + 2x - 1 = (x-1)(x^2 - x + 1).$$

The remaining quadratic has discriminant $1-4=-3$. Its roots are

$$\alpha = \frac{1+i\sqrt{3}}{2}, \qquad \beta = \frac{1-i\sqrt{3}}{2}.$$

Both have modulus one. Their arguments are respectively $\pi/3$ and $-\pi/3$, so they are $e^{i\pi/3}$ and $e^{-i\pi/3}$.

Step 2: Reduce the large powers using periodicity The requested sum includes all three roots, counted

once:

$$S = 1^{162} + \alpha^{162} + \beta^{162}.$$

Since $162 = 6 \times 27$, the exponent is a multiple of the period six of each nonreal root. Equivalently,

$$\alpha^{162} = e^{i54\pi} = 1, \qquad \beta^{162} = e^{-i54\pi} = 1.$$

Thus $S = 1 + 1 + 1$. This also verifies that the conjugate imaginary contributions cancel and that the result is real; no numerical approximation to a complex root is needed.

Step 3: Conclusion The sum of the 162nd powers of the three roots is $\boxed{3}$, a dimensionless value.

Quick Tip to Solve: For roots with modulus one, reduce the exponent modulo their angular period before doing any high-power arithmetic.

24. The area bounded by the lines y = || x − 1 | − 2 | is _____. [4 marks]

Solution:

Concept:

- An absolute value folds negative values upward. The graph of $y = \big||x-1|-2\big|$ is piecewise linear and nonnegative.
- Area requires a closed boundary. A single graph of a function does not by itself enclose a region; an additional boundary must be specified. If the intended extra boundary is the x-axis, compute $\int y\,dx$ between the zeros.

Step 1: Locate the intercepts and split the expression The printed statement specifies only $y = \big||x-1|-2\big|$. These straight-line pieces form an open W-shaped graph, so no closed region is defined by the stated boundary alone. To examine the usual interpretation with the x-axis as an extra boundary, find the zeros: $|x-1| = 2$, hence $x = -1, 3$. Between them, $y = 2 - |x-1|$. Splitting at $x = 1$,

$$y = \begin{cases} x+1, & -1 \le x \le 1, \\ 3-x, & 1 \le x \le 3. \end{cases}$$

The central vertex is $(1, 2)$.

Step 2: Evaluate the conditional area and identify the ambiguity With the extra x-axis boundary, the enclosed triangle has area

$$\begin{aligned} A &= \int_{-1}^{1} (x+1)\,dx + \int_{1}^{3} (3-x)\,dx \\ &= \left[\frac{x^2}{2} + x\right]_{-1}^{1} + \left[3x - \frac{x^2}{2}\right]_{1}^{3} \\ &= 2 + 2 = 4. \end{aligned}$$

Its base is four and its height two, giving the independent check $A = \frac{1}{2} \times 4 \times 2 = 4$. The exterior rays are unbounded and create no additional finite region. Thus even this conventional added boundary does not yield eight. Without specifying an additional boundary, the original wording has no uniquely

defined enclosed area; a different value cannot be justified by silently adding different lines.

Step 3: Conclusion As printed, the boundary is incomplete and there is no uniquely defined bounded area. If the intended missing boundary is the x-axis, the area is $\boxed{4 \text{ square units}}$. The key value 8 does not follow from the displayed graph.

Quick Tip to Solve: Check that a boundary is closed before finding an area; if an axis must be supplied, state the conditional result explicitly.

25. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, a > 0 is _____. [4 marks]

Solution:

Concept:

- The order is the highest derivative present after eliminating the arbitrary constant. One parameter can be eliminated by differentiating once.
- The degree is the highest power of the highest-order derivative after the equation is made polynomial in derivatives by removing radicals and fractions.

Step 1: Differentiate and eliminate the parameter Write the family as $y^2 = ax + a^{3/2}/2$, where $a > 0$ is constant. Put $p = dy/dx$. Differentiating with respect to x gives

$$2yp = a.$$

Substitution into the original family eliminates a:

$$y^2 = 2xyp + \frac{(2yp)^{3/2}}{2}.$$

Only the first derivative has appeared. There is no need to differentiate again, because the same first derivative already expresses the sole arbitrary parameter.

Step 2: Remove the radical and read the degree Isolate the radical term and square:

$$2(y^2 - 2xyp) = (2yp)^{3/2},$$

$$4(y^2 - 2xyp)^2 = (2yp)^3.$$

Dividing by four gives the polynomial differential equation

$$(y^2 - 2xyp)^2 = 2y^3p^3.$$

Its expanded form is

$$2y^3p^3 - 4x^2y^2p^2 + 4xy^3p - y^4 = 0.$$

Thus the highest derivative is $p = y'$, of order one, and its highest power is three. The original condition $a = 2yp > 0$ restricts the relevant solution branch; squaring does not change how degree is read from this polynomial form.

Step 3: Conclusion The degree is 3 and the order is 1. Their requested difference is $\boxed{3-1=2}$, dimensionless.

Quick Tip to Solve: Eliminate the parameter before identifying the degree; a fractional power of a first derivative must first be removed.

26. If y = y(x) is the solution of the equation
$e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x$, **y(0) = 0; then**
$1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The chain rule gives $\frac{d}{dx}e^{\sin y}=e^{\sin y}\cos y\,dy/dx$, suggesting a substitution that makes the equation linear.
- A linear equation $z\prime+P(x)z=Q(x)$ has integrating factor $e^{\int P(x)\,dx}$. The initial value selects its integration constant.

Step 1: Transform and integrate the equation Let $z=e^{\sin y}$. The given equation becomes

$$z\prime+z\cos x=\cos x.$$

Its integrating factor is $e^{\sin x}$, because the derivative of $\sin x$ is $\cos x$. Multiplication yields

$$\frac{d}{dx}\left(ze^{\sin x}\right)=e^{\sin x}\cos x.$$

Integration with respect to x gives

$$ze^{\sin x}=e^{\sin x}+C,\qquad z=1+Ce^{-\sin x}.$$

This retains the initial condition for determining C, rather than introducing an arbitrary branch for y.

Step 2: Apply the initial value and evaluate the expression At $x=0$, the condition $y(0)=0$ gives $z(0)=e^0=1$. Therefore $1=1+C$, so $C=0$ and $e^{\sin y}=1$. For real y, this means $\sin y=0$. A differentiable solution is continuous; since the values with zero sine are the isolated values $n\pi$, the branch starting at zero remains $y(x)=0$. This constant function also satisfies the original differential equation directly. In particular, all three values at $\pi/6,\pi/3,\pi/4$ vanish, so the requested expression is

$$1+0+\frac{\sqrt{3}}{2}(0)+\frac{1}{\sqrt{2}}(0)=1.$$

Step 3: Conclusion The value of the stated expression is $\boxed{1}$, dimensionless.

Quick Tip to Solve: Recognize the derivative of the composite exponential first; the initial condition then makes the transformed solution constant.

27. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, k∈R is _____. [4 marks]

Solution:

Concept:

- For real x, the range of $a\sin x + b\cos x$ is the closed interval $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$.
- An integer count over a closed interval $[m, n]$ with integer endpoints is $n - m + 1$. Both endpoints must be retained when equality is achievable.

Step 1: Determine the full attainable range Here the amplitude is

$$R = \sqrt{3^2 + 4^2} = 5.$$

Choose an angle ϕ with $\cos\phi = 3/5$ and $\sin\phi = 4/5$. Then

$$3\sin x + 4\cos x = 5\sin(x + \phi).$$

As x runs through the real numbers, $\sin(x + \phi)$ takes every value from minus one to one, including its extrema. Thus the left side takes every real value in $[-5, 5]$; this establishes sufficiency as well as necessity of the range condition.

Step 2: Translate the range and count integers The equation has a real solution exactly when its right side belongs to that interval:

$$-5 \le k + 1 \le 5.$$

Subtract one throughout to obtain

$$-6 \le k \le 4.$$

The integral possibilities are $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$. There are

$$4 - (-6) + 1 = 11$$

such values. The endpoint values are allowed because the sine representation attains both -1 and 1. Using strict inequalities would incorrectly discard two valid integers, while counting only the length of the interval would miss one integer.

Step 3: Conclusion The number of integral values of k is $\boxed{11}$, a dimensionless count.

Quick Tip to Solve: Reduce a sine-cosine sum to its amplitude, shift the entire range by the constant, and count both integer endpoints.

28. If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is _____. [4 marks]

Solution:

Concept:

- Setting $t = \cos x$ converts an equation involving $\cos^2 x$ and $\cos x$ into a quadratic in t.

- On $[0, \pi/2]$, cosine decreases strictly from one to zero. Its range is $[0, 1]$, and each value in that range corresponds to exactly one angle.

Step 1: Factor the quadratic in cosine Put $t = \cos x$ and bring every term to one side:

$$\sqrt{3}\,t^2 - (\sqrt{3} - 1)t - 1 = 0.$$

This factors without approximation as

$$(t - 1)(\sqrt{3}\,t + 1) = 0,$$

since the product expands to $\sqrt{3}t^2 + (1 - \sqrt{3})t - 1$. The two algebraic possibilities are therefore

$$t = 1 \quad \text{or} \quad t = -\frac{1}{\sqrt{3}}.$$

These are candidate cosine values, not yet the number of admissible angles.

Step 2: Use the prescribed interval to count angles The interval restriction is $0 \le x \le \pi/2$, where $0 \le \cos x \le 1$. Hence the negative candidate $-1/\sqrt{3}$ cannot occur in the specified interval, even though it is a legitimate real root of the quadratic. The surviving equation $\cos x = 1$ gives $x = 0$, and strict decrease of cosine on the interval guarantees that there is no second angle. The endpoint zero is included by the square bracket. Direct substitution at $x = 0$ gives left side $\sqrt{3}$ and right side $(\sqrt{3} - 1) + 1 = \sqrt{3}$, confirming it satisfies the original equation.

Step 3: Conclusion The unique solution in the given interval is $x = 0$. The requested number of solutions is $\boxed{1}$, dimensionless.

Quick Tip to Solve: After solving a quadratic in cosine, filter its roots by the cosine range on the stated angle interval before counting angles.

29. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to _____. [4 marks]

Solution:

Concept:

- A plane perpendicular to a line has a normal vector parallel to the direction vector of that line. The direction vector is obtained by subtracting the coordinates of its endpoints in the same order.
- For normal vector $\vec{n} = (a, b, c)$, the plane through (x_0, y_0, z_0) is $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$. Every point on that plane satisfies this equation.

Step 1: Find a normal to the plane Take the displacement from $(-2, -21, 29)$ to $(-1, -16, 23)$:

$$\vec{n} = \big(-1 - (-2), -16 - (-21), 23 - 29\big) = (1, 5, -6).$$

Since the given line is perpendicular to the plane, this vector is a normal to the plane. Using the

reverse displacement would merely multiply the entire plane equation by -1 and would describe the same plane.

Step 2: Write the plane equation and find the parameter The plane passes through $(4, -2, 2)$. Therefore its equation is

$$(x-4)+5(y+2)-6(z-2)=0.$$

Collecting the constant terms gives

$$x+5y-6z+18=0.$$

Now substitute the other point $(\lambda, 2, 1)$, which also lies on it:

$$\lambda+5(2)-6(1)+18=0, \qquad \lambda=-22.$$

As a check, the first point gives $4-10-12+18=0$, and the second gives $-22+10-6+18=0$.

Step 3: Evaluate the requested expression Use the ratio $\lambda/11=-2$ before squaring. The negative sign matters in the linear term:

$$\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4=(-2)^2-4(-2)-4=4+8-4=8.$$

The requested output is the value of this expression, rather than the parameter λ itself.

Step 4: Conclusion The required numerical value is $\boxed{8}$, which is dimensionless.

Quick Tip to Solve: Use the line direction as the plane normal, then compute $\lambda/11$ first to simplify the final substitution.

30. Let m, n∈N and gcd (2, n) = 1. If $30\binom{30}{0}+29\binom{30}{1}+.....+2\binom{30}{28}+1\binom{30}{29}=n.2^m$**, then n + m is equal to _____.**

(Here $\binom{n}{k}={}^nC_k$**)** [4 marks]

Solution:

Concept:

- The binomial theorem gives $\sum_{k=0}^{N}\binom{N}{k}=2^N$. A weighted binomial sum often reduces to this identity after simplifying each summand.
- The factorial formula implies $(N-k)\binom{N}{k}=N\binom{N-1}{k}$ for $0\le k\le N-1$. The condition $\gcd(2,n)=1$ means n is odd, so all factors of two must be included in 2^m.

Step 1: Express the sum with one index The coefficients descend from 30 to 1 while the lower

binomial index ascends from 0 to 29. Thus the given sum is

$$S = \sum_{k=0}^{29} (30-k)\binom{30}{k}.$$

There is no omitted contribution at $k = 30$, because its coefficient would be zero. We seek the odd factor times a power of two, rather than a decimal expansion of this large sum.

Step 2: Reduce to an unweighted binomial sum For each index in the sum, cancel the factorial factor:

$$(30-k)\binom{30}{k} = \frac{(30-k)30!}{k!(30-k)!} = 30\frac{29!}{k!(29-k)!} = 30\binom{29}{k}.$$

Consequently,

$$S = 30\sum_{k=0}^{29}\binom{29}{k} = 30(1+1)^{29} = 30\cdot 2^{29}.$$

This transformation accounts for the endpoints too: $k = 0$ contributes 30 and $k = 29$ contributes 30 in either form.

Step 3: Separate the odd factor The representation $30 \cdot 2^{29}$ has an even prefactor, so it does not yet satisfy the stated gcd condition. Factor out the remaining two:

$$S = (2\cdot 15)2^{29} = 15\cdot 2^{30}.$$

The factor 15 is odd and $\gcd(2, 15) = 1$. Hence $n = 15$ and $m = 30$; moving another factor of two out of 2^{30} would make n even, so this is the required representation.

Step 4: Conclusion Therefore $n + m = 15 + 30 = \boxed{45}$, a dimensionless integer.

Quick Tip to Solve: For a sum weighted by $N - k$, replace each term by $N\binom{N-1}{k}$, then extract every factor of two from its prefactor.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Consider the combination of 2 capacitors C_1 and C_2 with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$. [4 marks]

(A) $\frac{15}{11}$

(B) No Solutions

(C) $\frac{29}{15}$

(D) $\frac{15}{4}$

Correct Answer: (B) No Solutions

Solution:

Concept:

- For two positive capacitances, the parallel equivalent is $C_p = C_1 + C_2$, whereas the series equivalent is $C_s = C_1C_2/(C_1 + C_2)$. Both formulae apply to ideal capacitors.
- The ratio $C_p/C_s = (C_1 + C_2)^2/(C_1C_2)$ is at least 4 for positive capacitances. Equality occurs only when the two capacitances are equal.

Step 1: Translate the stated condition Let $x = C_2/C_1$. The hypothesis requires $x > 1$. Using the series and parallel formulae,

$$\frac{C_p}{C_s} = \frac{(C_1 + C_2)^2}{C_1C_2} = \frac{(1 + x)^2}{x}.$$

The question specifies $C_p = (15/4)C_s$, so the necessary equation is

$$\frac{(1 + x)^2}{x} = \frac{15}{4}.$$

The quotient is dimensionless; no unit choice can change this constraint.

Step 2: Check whether positive capacitances can satisfy it For every positive x,

$$\frac{(1 + x)^2}{x} = x + 2 + \frac{1}{x} = 4 + \frac{(x - 1)^2}{x} \geq 4.$$

Since $x > 1$, the square is nonzero and the ratio is actually greater than 4. The required value $15/4 = 3.75$ is below even the minimum 4, so the stated combination cannot occur. Equivalently, clearing denominators gives $4x^2 - 7x + 4 = 0$, whose discriminant is $49 - 64 = -15 < 0$.

Step 3: Eliminate the proposed numerical ratios Options (A), (C) and (D) propose the positive ratios $15/11$, $29/15$ and $15/4$, respectively. Each is greater than one and therefore makes $x + 2 + 1/x > 4$. None can give $15/4$ for the parallel-to-series equivalent-capacitance quotient. The value $15/4$ printed in the question is a ratio of equivalent capacitances, not automatically the ratio of the component capacitances.

Step 4: Conclusion No positive pair of capacitances satisfies the condition. Hence, the correct option is (B).

Quick Tip to Solve: Before solving for two capacitor values, check $C_p/C_s \geq 4$; any smaller specified ratio is impossible.

32. An alternating current is given by the equation i = i_1 sin ωt + i_2 cos ωt. The rms current will be : [4 marks]

(A) $\frac{1}{\sqrt{2}}\left(i_1^2+i_2^2\right)^{\frac{1}{2}}$

(B) $\frac{1}{\sqrt{2}}(i_1+i_2)$

(C) $\frac{1}{\sqrt{2}}(i_1+i_2)^2$

(D) $\frac{1}{2}\left(i_1^2+i_2^2\right)^{\frac{1}{2}}$

Correct Answer: (A) $\frac{1}{\sqrt{2}}\left(i_1^2+i_2^2\right)^{\frac{1}{2}}$

Solution:

Concept:

- The root mean square current is $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$, where the average is taken over one complete period $T = 2\pi/\omega$. It is the steady current producing the same average Joule heating.
- Over a complete period, $\langle \sin^2 \omega t \rangle = \langle \cos^2 \omega t \rangle = 1/2$ and $\langle \sin \omega t \cos \omega t \rangle = 0$. The sine and cosine components are in quadrature.

Step 1: Square the instantaneous current The stated waveform contains two terms with the same angular frequency but a phase difference of $\pi/2$. Square their sum before averaging:

$$\begin{aligned} i^2 &= (i_1 \sin \omega t + i_2 \cos \omega t)^2 \\ &= i_1^2 \sin^2 \omega t + i_2^2 \cos^2 \omega t + 2 i_1 i_2 \sin \omega t \cos \omega t. \end{aligned}$$

The constants i_1 and i_2 are component amplitudes; adding them directly would incorrectly treat the two currents as in phase.

Step 2: Average over one complete cycle Use the complete-cycle trigonometric averages. The cross term vanishes because $2 \sin \omega t \cos \omega t = \sin 2\omega t$, whose positive and negative contributions cancel:

$$\langle i^2 \rangle = \frac{i_1^2}{2} + \frac{i_2^2}{2} + 0.$$

Taking the nonnegative square root gives

$$I_{\text{rms}} = \sqrt{\frac{i_1^2 + i_2^2}{2}} = \frac{\sqrt{i_1^2 + i_2^2}}{\sqrt{2}}.$$

The result has units of current, as required.

Step 3: Check the alternative expressions Option (B) adds quadrature amplitudes arithmetically and would give the wrong result when both are nonzero. Option (C) squares their sum, so its dimensions are current squared rather than current. Option (D) uses a factor $1/2$ instead of $1/\sqrt{2}$; with $i_2 = 0$, it predicts $i_1/2$, contradicting the rms value $i_1/\sqrt{2}$ of a single sinusoid.

Step 4: Conclusion

$$\boxed{I_{\text{rms}} = \frac{1}{\sqrt{2}} \sqrt{i_1^2 + i_2^2}}.$$

Hence, the correct option is (A).

Quick Tip to Solve: For sine and cosine components at the same frequency, add their amplitude squares and divide by two before taking the square root.

33. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is : [4 marks]

(A) $\frac{4}{5}ma^2$

(B) $\frac{8}{5}ma^2 + mb^2$

(C) $\frac{4}{5}ma^2 + 2mb^2$

(D) $\frac{8}{5}ma^2 + 2mb^2$

Correct Answer: (D) $\frac{8}{5}ma^2 + 2mb^2$

Solution:

Concept:

- A uniform solid sphere of mass m and radius a has moment of inertia $I_{\text{cm}} = 2ma^2/5$ about any diameter through its centre.
- The parallel-axis theorem is $I = I_{\text{cm}} + md^2$, where d is the perpendicular distance between the required axis and the parallel diameter through that sphere. Moments of inertia of distinct bodies about the same axis add.

Step 1: Identify the distances of the four centres The axis lies along one side of the square, in the plane of the four centres. The two centres at the endpoints of that side lie on the axis and therefore have $d = 0$. Each of the remaining two centres is on the opposite side, at perpendicular distance $d = b$. The distance is the side length, not the diagonal $b\sqrt{2}$, because distance to a line is measured along its perpendicular.

Step 2: Add the contributions of the two pairs For the two spheres centred on the chosen side,

$$I_{\text{near}} = 2\left(\frac{2}{5}ma^2\right) = \frac{4}{5}ma^2.$$

For each sphere on the opposite side, the axis parallel to the diameter is displaced by b, giving

$$I_{\text{far}} = 2\left(\frac{2}{5}ma^2 + mb^2\right) = \frac{4}{5}ma^2 + 2mb^2.$$

All four spheres have intrinsic rotational inertia even when their centres lie directly on the rotation axis.

Step 3: Combine and check the listed choices Adding both pairs yields

$$I = I_{\text{near}} + I_{\text{far}} = \frac{8}{5}ma^2 + 2mb^2.$$

Option (A) omits the two distant spheres entirely. Option (B) includes only one displacement contribution mb^2, although two sphere centres are displaced. Option (C) includes both displacement terms but only half the total intrinsic sphere contribution. In the small-radius limit $a \to 0$, the answer correctly becomes the point-mass result $2mb^2$.

Step 4: Conclusion

$$\boxed{I = \frac{8}{5}ma^2 + 2mb^2}$$

in units of mass times length squared. Hence, the correct option is (D).

Quick Tip to Solve: Count the centre-to-axis distances first: here two are zero and two equal b; then add the intrinsic inertia of all four spheres.

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34. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be : [4 marks]

(A) $\frac{2T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$
(B) $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$
(C) $\frac{3T}{rJ}$
(D) $\frac{2T}{rJ}$

Correct Answer: (B) $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

Solution:

Concept:

- A liquid surface has energy $U = TA$, where T is surface tension and A is surface area. A water drop has one liquid-air surface, unlike a soap bubble with two surfaces.
- When drops coalesce, liquid volume is conserved and surface area decreases. The lost surface energy appears as heat. If J is the mechanical equivalent of heat, mechanical energy E corresponds to heat $Q = E/J$ in the chosen heat unit.

Step 1: Relate the number of drops to their radii Let N small drops form the single larger drop. Conservation of water volume gives

$$N\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3, \qquad N = \frac{R^3}{r^3}.$$

The total water volume, before or after combination, is $V = 4\pi R^3/3$. The number of drops is eliminated using this relation; it need not be specified separately.

Step 2: Calculate the decrease in surface energy The initial surface area is $A_i = N4\pi r^2$, whereas the final surface area is $A_f = 4\pi R^2$. Thus the released energy is

$$\begin{aligned} E &= T(A_i - A_f) \\ &= 4\pi T\left(\frac{R^3}{r} - R^2\right). \end{aligned}$$

For coalescence $R > r$, so this expression is positive. Convert to heat by $Q = E/J$, then divide by the

conserved liquid volume:

$$\frac{Q}{V} = \frac{4\pi T(R^3/r - R^2)}{J(4\pi R^3/3)} = \frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right).$$

Step 3: Check the coefficient and limiting cases The factor 3 comes from dividing the spherical surface area by its volume; the factor 2 in options (A) and (D) is inappropriate here. Option (C) drops the final surface-energy term and is only an approximation when R is extremely large compared with r. The full answer must vanish when $R = r$, since then no surface area is lost. Only option (B) has both the correct coefficient and the required subtraction.

Step 4: Conclusion

$$\boxed{\frac{Q}{V} = \frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

in heat units per unit liquid volume. Hence, the correct option is (B).

Quick Tip to Solve: Use volume conservation to remove the drop count, then divide the lost surface energy by the final volume and by J.

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35. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.

[h = 6.63 × 10^{-34} Js and c = 3 × 10^{8} ms^{-1}] [4 marks]

(A) 654 nm and orange colour

(B) 654 nm and red colour

(C) 1046 nm and red colour

(D) 1046 nm and blue colour

Correct Answer: (B) 654 nm and red colour

Solution:

Concept:

- In an LED, electron-hole recombination can emit a photon with energy approximately equal to the semiconductor band gap: $E_\gamma = E_g$. The photon wavelength follows $E_\gamma = hc/\lambda$.
- Convert electronvolts to joules before using SI values of h and c: $1\,\text{eV} = 1.6 \times 10^{-19}$ J. Visible red light has a longer wavelength than orange light; a wavelength near $654\,\text{nm}$ is red.

Step 1: Convert the band gap to SI energy The stated Ga-As-P LED has a band gap of $1.9\,\text{eV}$. The recombination energy available to one emitted photon is therefore

$$E_\gamma = 1.9(1.6 \times 10^{-19}) = 3.04 \times 10^{-19}\,\text{J}.$$

This is the energy of a photon, not an electric potential. The material name identifies the emitting semiconductor; the numerical wavelength is determined here by its specified band gap.

Step 2: Apply the photon energy relation Use the constants supplied in the question:

$$\begin{aligned}\lambda &= \frac{hc}{E_\gamma} \\ &= \frac{(6.63\times10^{-34})(3\times10^{8})}{3.04\times10^{-19}}\,\mathrm{m} \\ &= 6.54\times10^{-7}\,\mathrm{m} \simeq 654\,\mathrm{nm}.\end{aligned}$$

In the exponent calculation, $10^{-34}10^{8}/10^{-19} = 10^{-7}$, so the result is in the visible wavelength range. Since $1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$, the conversion to nanometres multiplies the value in metres by 10^{9}.

Step 3: Identify the colour and reject the alternatives A wavelength of about $654\,\mathrm{nm}$ lies in the red region of the visible spectrum. Thus option (A) has the computed wavelength but the wrong colour. The $1046\,\mathrm{nm}$ wavelength in options (C) and (D) is inconsistent with the $1.9\,\mathrm{eV}$ gap and lies in the infrared region, so it is neither the stated red nor blue visible emission. The selected option must satisfy both the energy calculation and the colour classification.

Step 4: Conclusion The emitted wavelength is approximately $\boxed{654\,\mathrm{nm}}$, and its colour is red. Hence, the correct option is (B).

Quick Tip to Solve: Convert eV to joules once, use hc/E_g, and check the colour from the resulting wavelength rather than the material name.

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36. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value | f | = 40 cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information : [4 marks]

(A) Image is real, same side of convex mirror.

(B) Image is virtual, opposite side of convex mirror.

(C) Image is virtual, opposite side of concave mirror.

(D) Image is real, same side of concave mirror.

Correct Answer: (B) Image is virtual, opposite side of convex mirror.

Solution:

Concept:

- For a spherical mirror, transverse magnification is $m = h_i/h_o = -v/u$, where u and v are Cartesian object and image distances. An erect image has positive magnification.
- The mirror equation is $1/f = 1/u + 1/v$. For a real object in front of a mirror, $u < 0$; a convex mirror has $f > 0$ and its virtual image lies behind the mirror.

Step 1: Determine the magnification The object and image have heights 100 cm and 25 cm, respectively. Their same orientation fixes the positive sign, so

$$m = \frac{25}{100} = \frac{1}{4} = -\frac{v}{u}, \qquad v = -\frac{u}{4}.$$

Since the object is in front of the mirror, its distance is negative. Hence the image distance is positive:

the image lies behind the mirror and is virtual, not real.

Step 2: Identify the mirror and check its distances Substitute the relation between image and object distances into the mirror formula:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{-u/4} = -\frac{3}{u}.$$

Thus $f = -u/3 > 0$, which identifies a convex mirror. With $f = 40\,\text{cm}$, this gives $u = -120\,\text{cm}$ and $v = 30\,\text{cm}$. These distances reproduce $m = 1/4$. A concave mirror can produce an erect virtual image, but that image is enlarged, whereas this image is diminished. Options (A) and (D) incorrectly say real; (C) incorrectly identifies the mirror as concave.

Step 3: Conclusion The mirror is convex and its image is virtual, erect and behind the mirror, on the side opposite to the object. Hence, the correct option is (B).

Quick Tip to Solve: An erect diminished image of a real object immediately identifies a convex mirror; use the sign of magnification before substituting distances.

37. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be : [4 marks]

(A) 0.50 mm

(B) 0.25 mm

(C) 1 mm

(D) 0.75 mm

Correct Answer: (B) 0.25 mm

Solution:

Concept:

- For two coherent slits separated by d, the small-angle fringe width on a screen at distance D is $\beta = \lambda D/d$. It is the distance between consecutive bright fringes or consecutive dark fringes.
- All lengths must be expressed in a common unit before substitution. The formula applies here because the slit separation is much smaller than the screen distance.

Step 1: Convert the stated distances The wavelength is $500\,\text{nm}$, the slit separation is $2\,\text{mm}$, and the screen distance is one metre. Converting into SI units gives

$$\lambda = 500 \times 10^{-9}\,\text{m} = 5.0 \times 10^{-7}\,\text{m}, \quad d = 2.0 \times 10^{-3}\,\text{m}, \quad D = 1.0\,\text{m}.$$

Here $D/d = 500$, consistent with the usual small-angle arrangement. We need the separation of adjacent fringes, so no additional factor of two is required.

Step 2: Calculate and compare the fringe width Apply the fringe-width expression directly:

$$\beta = \frac{(5.0 \times 10^{-7})(1.0)}{2.0 \times 10^{-3}} = 2.5 \times 10^{-4}\,\text{m}.$$

Since one metre contains 1000 millimetres,

$$\beta = 2.5 \times 10^{-4} \times 10^{3} = 0.25\,\text{mm}.$$

Option (A), 0.50 mm, is twice this separation; (C), 1 mm, is four times it; and (D), 0.75 mm, is three times it. None of these matches the adjacent-fringe separation for the stated slit distance.

Step 3: Conclusion The fringe separation is $\boxed{0.25\,\text{mm}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use metres for wavelength and slit separation, calculate the width once, then convert the final result into millimetres to avoid powers-of-ten errors.

38. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period : [4 marks]

(A) $2\pi\sqrt{\frac{R}{g}}$

(B) $\frac{g}{2\pi R}$

(C) $\frac{2\pi R}{g}$

(D) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

Correct Answer: (A) $2\pi\sqrt{\frac{R}{g}}$

Solution:

Concept:

- In the standard uniform-density model of Earth, the gravitational acceleration at an interior position $\boldsymbol{r}$ is $\boldsymbol{a} = -(g/R)\boldsymbol{r}$, where g is surface gravity and R is Earth's radius.
- Only the component of acceleration along a frictionless straight tunnel drives motion along it. An equation $\ddot{x} = -\omega^2 x$ describes simple harmonic motion with period $T = 2\pi/\omega$.

Step 1: Resolve gravity along the tunnel Let O be the Earth's centre and C the midpoint of the chord, so $OC = R/2$. Measure a particle's displacement x along the tunnel from C. Its position vector has perpendicular component OC and parallel component x. Because interior gravity is proportional to the position vector, its component along the tunnel is

$$a_x = -\frac{g}{R}x.$$

The normal reaction of the wall balances the perpendicular component; the frictionless wall contributes no tangential force.

Step 2: Read the frequency and test the choices The equation of motion becomes

$$\ddot{x} + \frac{g}{R}x = 0, \qquad \omega^2 = \frac{g}{R}.$$

Therefore

$$T = 2\pi\sqrt{\frac{R}{g}}.$$

The offset $R/2$ affects the available amplitude, whose maximum is $\sqrt{R^2 - (R/2)^2} = \sqrt{3}R/2$, but does not affect the coefficient of x. Options (B) and (C) have dimensions of inverse time squared and time squared respectively. Option (D) is the ordinary frequency $1/T$, rather than the time period.

Step 3: Conclusion The motion along the chord has time period $\boxed{2\pi\sqrt{R/g}}$, independent of the chord's offset in this model. Hence, the correct option is (A).

Quick Tip to Solve: Project the interior acceleration onto the tunnel: the component is always $-gx/R$, so the midpoint is the equilibrium position and every chord has the same period.

---

39. In a typical combustion engine the workdone by a gas molecule is given by $W = \alpha^2\beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be : [4 marks]

(A) $[M^0LT^0]$

(B) $[MLT^{-1}]$

(C) $[MLT^{-2}]$

(D) $[M^2LT^{-2}]$

Correct Answer: (A) $[M^0LT^0]$

Solution:

Concept:

- The argument of an exponential is dimensionless. Thus the dimensions in $\beta x^2/(kT)$ must cancel; kT has dimensions of energy.
- The exponential factor itself has no dimensions, so the prefactor $\alpha^2\beta$ must have the dimensions of work, $[ML^2T^{-2}]$. Here the dimensional symbol T denotes time, distinct from the temperature in the question.

Step 1: Find the dimensions of beta The displacement x has dimensions $[L]$, and the product of Boltzmann constant with absolute temperature is an energy. For the exponent to be dimensionless,

$$[\beta][x]^2 = [kT] = [ML^2T^{-2}].$$

Consequently

$$[\beta] = \frac{[ML^2T^{-2}]}{[L^2]} = [MT^{-2}].$$

In SI units beta therefore has units of joules per square metre, equivalently kilograms per square second. Its dimensions must be found before using the overall work equation.

Step 2: Use the dimensions of the prefactor From the stated expression for work,

$$[W] = [\alpha]^2[\beta].$$

Substituting the dimensions just obtained gives

$$[\alpha]^2 = \frac{[ML^2T^{-2}]}{[MT^{-2}]} = [L^2], \qquad [\alpha] = [L].$$

Hence there is no mass or time factor. Option (B) has the dimensions of momentum and option (C) those of force; neither can produce work when its square is multiplied by beta. Option (D) also introduces an unwanted mass dependence and has the wrong time power.

Step 3: Conclusion The required dimensions are $\boxed{[M^0LT^0]}$, so alpha has the dimensions of length. Hence, the correct option is (A).

Quick Tip to Solve: In an exponential formula, first make the exponent dimensionless to find one unknown constant; then use the prefactor to find the other.

40. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor : [4 marks]

(A) $\frac{1}{2}, 2\sqrt{2}$
(B) $\frac{1}{4}, 2\sqrt{2}$
(C) $\frac{1}{2}, \sqrt{2}$
(D) $\frac{1}{4}, \sqrt{2}$

Correct Answer: (C) $\frac{1}{2}, \sqrt{2}$

Solution:

Concept:

- For two springs joined in series, the same force acts through both springs and their extensions add. The effective spring constant satisfies $1/K_s = 1/K_a + 1/K_b$.
- For an unchanged oscillating mass m, the period is $T = 2\pi\sqrt{m/K}$. Therefore the period changes as the inverse square root of the effective spring constant.

Step 1: Find the combined spring constant Each spring has constant K_1. Under a force F, each extends by F/K_1. Because they are in series, the total extension is

$$x = \frac{F}{K_1} + \frac{F}{K_1} = \frac{2F}{K_1}.$$

Defining $F = K_s x$ therefore gives

$$K_s = \frac{K_1}{2}, \qquad \frac{K_s}{K_1} = \frac{1}{2}.$$

The pair is less stiff than either individual spring, because both extensions contribute to the displace-

ment of the attached mass.

Step 2: Compare the oscillation periods For the same attached mass, the original and new periods are

$$T_1 = 2\pi\sqrt{\frac{m}{K_1}}, \qquad T_s = 2\pi\sqrt{\frac{m}{K_s}}.$$

Dividing eliminates the common mass and numerical factor:

$$\frac{T_s}{T_1} = \sqrt{\frac{K_1}{K_s}} = \sqrt{2}.$$

Thus the constant halves and the period grows by the factor $\sqrt{2}$. Option (A) introduces an extra factor of two into the period. Options (B) and (D) incorrectly reduce the stiffness to one quarter; two identical springs in series reduce it to one half, not one quarter.

Step 3: Conclusion The factors are $\boxed{1/2,\ \sqrt{2}}$ for spring constant and time period, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Keep the mass fixed when comparing periods: halving stiffness increases the period by the square root of two, not by a factor of two.

41. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by : [4 marks]

(A) $T \propto R^{\frac{4}{3}}$

(B) $T \propto R^{\frac{5}{2}}$

(C) $T \propto R^{\frac{3}{2}}$

(D) $T \propto R^2$

Correct Answer: (D) $T \propto R^2$

Solution:

Concept:

- For a particle of fixed mass m in uniform circular motion, the inward radial force has magnitude $F = mv^2/R$, where v is speed and R is the orbital radius.
- The revolution period is circumference divided by speed: $T = 2\pi R/v$. The stated force law is $F = C/R^3$, with the proportionality constant C fixed when comparing circular orbits.

Step 1: Obtain the radius dependence of speed The force providing the centripetal acceleration is proportional to the inverse cube of radius. Introduce a positive constant C and equate the magnitudes:

$$\frac{mv^2}{R} = \frac{C}{R^3}.$$

Multiplying by R/m,

$$v^2 = \frac{C}{mR^2}, \qquad v = \sqrt{\frac{C}{m}}\frac{1}{R}.$$

The speed is uniform along any one circular orbit, but its value changes between different orbital radii according to $v \propto R^{-1}$.

Step 2: Determine the period scaling Substitute the speed into the circumference-over-speed formula:

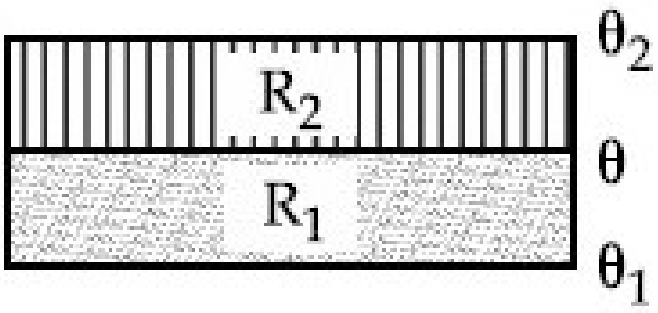
$$T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{m}{C}}\,R^2.$$

Hence the required radius power is two. This also follows by substituting $v = 2\pi R/T$ into the force equation, which gives $4\pi^2 mR/T^2 = C/R^3$, and therefore $T^2 \propto R^4$. Options (A) and (B) have different powers and cannot satisfy this equation. Option (C), $R^{3/2}$, would correspond to an inverse-square force, rather than the inverse-cube force specified here.

Step 3: Conclusion The time period varies as $\boxed{T \propto R^2}$. Hence, the correct option is (D).

Quick Tip to Solve: For a central force $F \propto R^{-n}$, substitute $v = 2\pi R/T$ into the centripetal equation to obtain $T \propto R^{(n+1)/2}$.

42. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by : [4 marks]



(A) $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$
(B) $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$
(C) $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$
(D) $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$

Correct Answer: (A) $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$

Solution:

Concept:

- In steady conduction through two layers in series, the same heat current passes through both. Thermal resistance is defined by $H = \Delta\theta/R$, where H is the heat transferred per unit time.
- The junction cannot accumulate heat in the steady state. Its temperature is therefore found by

equating the heat currents through R_1 and R_2, using the temperature at each layer's own outer surface.

Step 1: Write the two heat currents The figure places resistance R_1 between the lower temperature label θ_1 and the junction θ, while R_2 lies between the junction and θ_2. Taking heat flow from the first boundary to the second as positive,

$$H = \frac{\theta_1 - \theta}{R_1} = \frac{\theta - \theta_2}{R_2}.$$

This signed relation is valid whichever boundary is hotter; the current simply changes sign if the temperatures are reversed. Resistances remain positive.

Step 2: Solve for the junction temperature Cross-multiply and collect the terms containing the unknown temperature:

$$R_2\theta_1 - R_2\theta = R_1\theta - R_1\theta_2,$$

$$\theta(R_1 + R_2) = R_2\theta_1 + R_1\theta_2.$$

Therefore

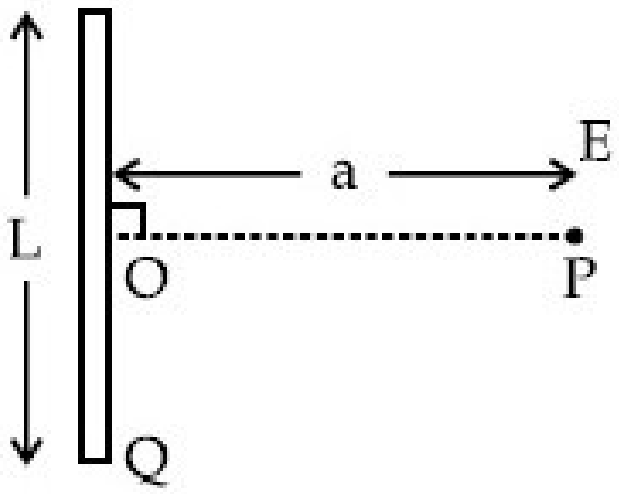
$$\theta = \frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}.$$

Option (B) pairs each boundary temperature with the wrong resistance; for example, as R_1 tends to zero the junction must approach θ_1, which (B) fails. Options (C) and (D) have a difference of resistances in the denominator and become singular when the resistances are equal, although then the temperature is simply the average.

Step 3: Conclusion The junction temperature is $\boxed{\theta = (\theta_1 R_2 + \theta_2 R_1)/(R_1 + R_2)}$. Hence, the correct option is (A).

Quick Tip to Solve: The boundary connected through the smaller resistance pulls the junction temperature closer to itself; check your result by setting that resistance to zero.

43. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is a = $\frac{\sqrt{3}}{2}L$. [4 marks]



(A) $\frac{Q}{4\pi\varepsilon_0 L^2}$

(B) $\frac{Q}{3\pi\varepsilon_0 L^2}$

(C) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$

(D) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$

Correct Answer: (C) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$

Solution:

Concept:

- For a uniform wire the linear charge density is $\lambda = Q/L$. Each element produces a field according to Coulomb's law, with $k = 1/(4\pi\varepsilon_0)$.
- On the perpendicular bisector, components parallel to the rod cancel in symmetric pairs. Only components along the line from its centre to P add.

Step 1: Resolve the field of an element Place the rod along the y-axis, centred at the origin, and P at $(a, 0)$. An element of length dy carries $dq = \lambda\, dy$, at distance $r = \sqrt{a^2 + y^2}$ from P. Its field magnitude is $k\lambda\, dy/r^2$, and its component toward P along the bisector is

$$dE_x = \frac{k\lambda a\, dy}{(a^2 + y^2)^{3/2}}.$$

The y-components cancel between elements at y and -y, so no component along the rod remains.

Step 2: Integrate over the finite length Adding the bisector components from both halves gives

$$E = 2k\lambda a \int_0^{L/2} \frac{dy}{(a^2 + y^2)^{3/2}} = \frac{k\lambda L}{a\sqrt{a^2 + L^2/4}}.$$

The endpoint factor must be retained: the finite rod cannot be replaced by an infinite line or a point charge at this distance.

Step 3: Substitute the specified distance Here $a = \sqrt{3}L/2$, so $\sqrt{a^2 + L^2/4} = L$. Therefore

$$E = \frac{kQ}{(\sqrt{3}L/2)L} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}.$$

Option A uses an incorrect coefficient, while B is the field obtained by incorrectly treating the whole rod as a point charge at its centre. Option D also fails to retain the finite-length angular factor.

Step 4: Conclusion The electric field has magnitude $E = \dfrac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$, directed along OP away from a positively charged wire. Hence, the correct option is (C).

Quick Tip to Solve: Use $E = kQ/[a\sqrt{a^2 + (L/2)^2}]$ on a finite rod's perpendicular bisector; check the two endpoint distances before simplifying.

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44. A planet revolving in elliptical orbit has :

A. a constant velocity of revolution.

B. has the least velocity when it is nearest to the sun.

C. its areal velocity is directly proportional to its velocity.

D. areal velocity is inversely proportional to its velocity.

E. to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below : [4 marks]

(A) D only

(B) E only

(C) C only

(D) A only

Correct Answer: (B) E only

Solution:

Concept:

- Gravity is a central force: its line of action passes through the Sun. The torque about the Sun therefore vanishes and angular momentum is conserved.
- Areal velocity is the area swept by the radius vector per unit time: $dA/dt = |\mathbf{r} \times \mathbf{v}|/2 = \ell/(2m)$, where ℓ is the conserved angular momentum.

Step 1: Apply angular-momentum conservation The gravitational force is parallel to the radius vector joining the Sun to the planet, so

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = 0, \qquad \frac{d\boldsymbol{\ell}}{dt} = 0.$$

Consequently $dA/dt = \ell/(2m)$ is constant throughout the elliptical orbit. This is Kepler's second law:

equal areas are swept in equal times, even though the orbital speed changes.

Step 2: Test the statements about speed Statement A is false because both the direction of velocity and, in an ellipse, its magnitude change. Statement B is false because the planet is fastest at perihelion, its closest point to the Sun. At perihelion and aphelion the velocity is perpendicular to the radius, giving

$$r_{\mathrm{p}} v_{\mathrm{p}} = r_{\mathrm{a}} v_{\mathrm{a}}.$$

Since $r_{\mathrm{p}} < r_{\mathrm{a}}$, the speed at perihelion is greater.

Step 3: Distinguish areal velocity from orbital velocity Statements C and D assert direct or inverse variation of the areal velocity with the changing orbital velocity. Both are false: the changing radius and transverse velocity combine to leave their cross product constant. Statement E states this constant-area property correctly. Thus option A (D only), option C (C only), and option D (A only) each select a false statement.

Step 4: Conclusion Only statement E is true: the planet follows its elliptical trajectory with constant areal velocity about the Sun. Hence, the correct option is (B).

Quick Tip to Solve: For a central force, start with zero torque. Constant angular momentum immediately gives constant areal velocity, rather than constant speed.

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45. Given below are two statements : one is labeled as Assertion A and the other is labelled as Reason R.

Assertion A : An electron microscope can achieve better resolving power than an optical microscope.

Reason R : The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) A is false but R is true.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) Both A and R are true and R is the correct explanation of A.

(D) A is true but R is false.

Correct Answer: (C) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- A moving electron has de Broglie wavelength $\lambda = h/p$. For an electron accelerated nonrelativistically through potential V, $\lambda = h/\sqrt{2m_e eV}$.
- The diffraction-limited minimum resolvable separation scales with wavelength, approximately $d_{\min} \propto \lambda/\mathrm{NA}$. A smaller minimum separation means greater resolving power.

Step 1: Evaluate Assertion A An optical microscope uses visible wavelengths of roughly 400–700 nm. Diffraction prevents it from distinguishing arbitrarily close points: the images of two points spread into diffraction patterns. An electron microscope can distinguish much smaller separations because

the probing electrons have a much shorter wavelength. Thus the assertion that it can achieve better resolving power is true.

Step 2: Evaluate Reason R Electrons accelerated by an electron gun have substantial momentum. For example, at an accelerating potential of 100 V the nonrelativistic estimate gives

$$\lambda = \frac{h}{\sqrt{2m_e e(100\,\mathrm{V})}} \simeq 0.123\,\mathrm{nm}.$$

This is already thousands of times shorter than visible-light wavelengths. Higher practical accelerating potentials shorten the wavelength still further, with relativistic corrections when necessary. Hence Reason R is true.

Step 3: Check whether the reason explains the assertion The short electron wavelength directly reduces the diffraction limit of resolution. It is therefore the physical explanation for the greater resolving power in the assertion, rather than an unrelated true fact. Option A incorrectly calls the assertion false; option D incorrectly calls the reason false. Option B accepts both statements but denies their explanatory connection, contrary to the wavelength dependence of diffraction. Real instruments have lens aberrations, but these do not remove the stated capability.

Step 4: Conclusion Both Assertion A and Reason R are true, and the much shorter electron wavelength correctly explains the superior resolving power. Hence, the correct option is (C).

Quick Tip to Solve: In a resolution question, translate greater resolving power into smaller $d_{\min}$; shortening wavelength lowers the diffraction limit.

46. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m $<<$ M, body 'Q' will have a maximum speed equal to '2u' after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is correct but R is not correct.

(B) A is not correct but R is correct.

(C) Both A and R are correct and R is the correct explanation of A.

(D) Both A and R are correct but R is NOT the correct explanation of A.

Correct Answer: (C) Both A and R are correct and R is the correct explanation of A.

Solution:

Concept:

- In an isolated elastic collision, total linear momentum and total kinetic energy are conserved. These two conservation laws determine both final velocities.

- In a head-on elastic collision, the relative separation speed equals the relative approach speed. For a target initially at rest, $v_Q - v_P = u$.

Step 1: Evaluate the conservation-law statement Let v_P and v_Q denote the velocities after collision along the incident direction. Momentum and kinetic-energy conservation give

$$Mu = Mv_P + mv_Q,$$

$$\frac{1}{2}Mu^2 = \frac{1}{2}Mv_P^2 + \frac{1}{2}mv_Q^2.$$

These are exactly the conditions stated in Reason R, so the reason is correct. They apply to the two-body system during the short isolated collision.

Step 2: Derive the target velocity Rearranging the two equations gives $M(u - v_P) = mv_Q$ and $M(u - v_P)(u + v_P) = mv_Q^2$. Dividing for a nontrivial collision yields $u + v_P = v_Q$. Substituting into momentum conservation,

$$Mu = M(v_Q - u) + mv_Q,$$

$$v_Q = \frac{2M}{M + m}u = \frac{2u}{1 + m/M}.$$

Thus the target can move faster than the incident body's initial speed without violating energy conservation, because its mass is much smaller.

Step 3: Interpret the light-target limit For $m \ll M$, the ratio m/M is negligible and $v_Q \simeq 2u$. The maximum limiting speed is $2u$, approached as $m/M \to 0$; the exact speed for a finite positive target mass is slightly smaller. This is the light-target limit intended in Assertion A. The assertion is therefore correct in its stated approximation, and it follows directly from the two conservation laws in R. Options A and B incorrectly reject a true statement, while D incorrectly denies the explanation.

Step 4: Conclusion In the light-target limit, $v_Q = 2Mu/(M + m) \simeq 2u$. Both A and R are correct, and R correctly explains A. Hence, the correct option is (C).

Quick Tip to Solve: For an elastic collision with a stationary target, use $v_Q = 2Mu/(M + m)$; test $m = M$ and $m/M \to 0$ as quick checks.

47. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be : [4 marks]

(A) $\frac{\rho K}{P}$

(B) $\frac{PK}{\rho}$

(C) $\frac{\rho P}{K}$

(D) $\frac{K}{\rho P}$

Correct Answer: (C) $\frac{\rho P}{K}$

Solution:

Concept:

- The bulk modulus is $K = -V\,dP/dV$. For a small uniform pressure increase P, the fractional decrease in volume is $-\Delta V/V \simeq P/K$.
- The sample's mass remains constant during compression. Since $\rho = m/V$, the fractional density change is opposite to the fractional volume change: $d\rho/\rho = -dV/V$.

Step 1: Relate the density change to the volume change Initially the sample has mass m, volume V, and density $\rho = m/V$. Applying pressure on all sides compresses the sample; no mass is added or removed. Differentiating the fixed-mass relation gives

$$d\rho = -\frac{m}{V^2}\,dV = -\frac{\rho}{V}\,dV.$$

Thus a negative volume change produces a positive density change, as expected physically.

Step 2: Apply the bulk-modulus relation Using the definition of bulk modulus for the applied small pressure,

$$P \simeq -K\frac{\Delta V}{V}, \qquad -\frac{\Delta V}{V} \simeq \frac{P}{K}.$$

Substitution into the density relation gives

$$\Delta\rho \simeq -\rho\frac{\Delta V}{V} = \frac{\rho P}{K}.$$

This is the linear elastic, small-strain result requested by the options; products of small changes are neglected.

Step 3: Check the dependence and eliminate alternatives The increase must vanish when the applied pressure vanishes and become smaller when the material is harder to compress, meaning K is larger. Option A varies inversely with P and increases with K, so both trends are wrong. Option B also increases with K and has incorrect density dimensions. Option D has reciprocal-density dimensions. Only option C has density units, grows with pressure, and decreases with increasing bulk modulus. The sign is positive because the volume decreases.

Step 4: Conclusion The magnitude of the increase in density is $\Delta\rho = \frac{\rho P}{K}$ in the small-compression approximation. Hence, the correct option is (C).

Quick Tip to Solve: At constant mass, use $\Delta\rho/\rho = -\Delta V/V$. Pair it with P/K to avoid reversing the density change.

48. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is : [4 marks]

(A) 7 : 135

(B) 7 : 108

(C) 1 : 9

(D) 1 : 3

Correct Answer: (A) 7 : 135

Solution:

Concept:

- For hydrogen, the Rydberg formula is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$. A spectral series fixes the lower level n_f.
- The Lyman series ends at $n_f = 1$; the Paschen series ends at $n_f = 3$. The first member starts one level above the lower level, and subsequent members increase the upper level by one.

Step 1: Identify the two transitions The first Lyman member is $2 \to 1$, the second is $3 \to 1$, and the third is $4 \to 1$. Thus λ_1 belongs to $4 \to 1$, not $3 \to 1$. The first Paschen member is $4 \to 3$, so λ_2 belongs to that transition. Although both upper levels equal 4, the lower energy levels differ.

Step 2: Compute the reciprocal wavelengths Applying the Rydberg formula separately gives

$$\frac{1}{\lambda_1} = R_H\left(1 - \frac{1}{16}\right) = \frac{15R_H}{16},$$

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R_H}{144}.$$

The Lyman transition has the larger energy difference and therefore the shorter wavelength, which provides a useful check on the direction of the ratio.

Step 3: Invert before taking the wavelength ratio The wavelengths are $\lambda_1 = 16/(15R_H)$ and $\lambda_2 = 144/(7R_H)$. Consequently,

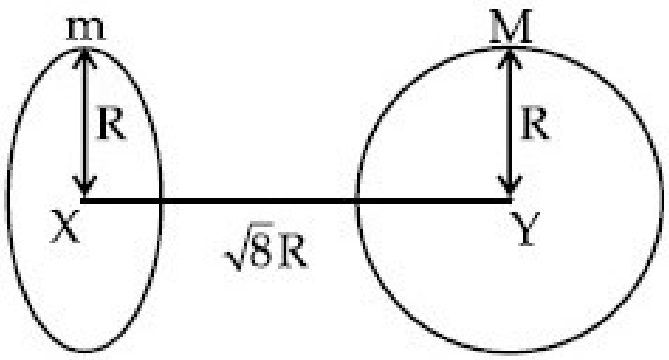
$$\frac{\lambda_1}{\lambda_2} = \frac{16}{15R_H}\frac{7R_H}{144} = \frac{7}{135}.$$

The common Rydberg constant cancels, so no numerical value is needed. Options B, C, and D do not equal this exact ratio: B misses the Lyman factor, while the simple ratios in C and D ignore the differences of inverse squared levels. Taking the ratio of the wavenumbers in the same order would instead give the reciprocal and be incorrect.

Step 4: Conclusion The wavelength ratio is $\lambda_1 : \lambda_2 = 7 : 135$. Hence, the correct option is (A).

Quick Tip to Solve: Count series members by starting at $n_i = n_f + 1$, then invert the Rydberg wavenumbers when forming a wavelength ratio.

49. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}$R is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'. [4 marks]



(A) $\frac{2\sqrt{2}}{3}.\frac{GMm}{R^2}$

(B) $\frac{\sqrt{8}}{9}.\frac{GmM}{R}$

(C) $\frac{\sqrt{8}}{27}.\frac{GmM}{R^2}$

(D) $\frac{1}{3\sqrt{8}}.\frac{GMm}{R^2}$

Correct Answer: (C) $\frac{\sqrt{8}}{27}.\frac{GmM}{R^2}$

Solution:

Concept:

- A spherically symmetric mass produces an external gravitational field as if its entire mass were at its centre. Every point of the ring lies outside the sphere here.
- For a uniform ring, force components perpendicular to its axis cancel in opposite pairs. Axial components add, giving $F = GMmx/(x^2 + R^2)^{3/2}$ at axial separation x.

Step 1: Use the shell theorem at each ring element Let x be the centre-to-centre distance. A mass element dm on the ring lies a distance $s = \sqrt{x^2 + R^2}$ from the sphere's centre. Here $x = \sqrt{8}R$, so $s = 3R > R$: each ring element is external to the spherical mass. Its attraction has magnitude $dF = GM\,dm/s^2$, directed toward the sphere's centre. The shell theorem therefore applies to the force on each element, without replacing the ring itself by a point mass.

Step 2: Add the axial components The angle of each force to the line joining the centres satisfies $\cos\theta = x/s$. Transverse components cancel by the symmetry of the ring, leaving

$$dF_x = \frac{GMx\,dm}{s^3}.$$

Both x and s are constant around the ring. Integrating dm over its full mass m gives

$$F = \frac{GMmx}{(x^2 + R^2)^{3/2}}.$$

By Newton's third law, the sphere experiences an equal and opposite attraction.

Step 3: Substitute the separation and compare options With $x = \sqrt{8}R$, the denominator is $(8R^2 + R^2)^{3/2} = 27R^3$. Hence

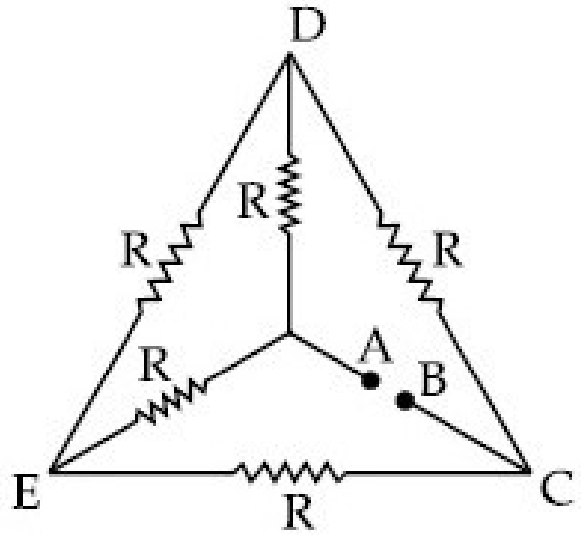
$$F = \frac{GMm\sqrt{8}R}{27R^3} = \frac{\sqrt{8}}{27}\frac{GMm}{R^2}.$$

Option B has $1/R$ instead of $1/R^2$, so it is dimensionally wrong. Options A and D retain incorrect geometric coefficients; neither includes the correct combination of the element distance $3R$ and its axial projection $\sqrt{8}/3$.

Step 4: Conclusion The attractive force along the line joining the centres is $F = \frac{\sqrt{8}}{27}\frac{GMm}{R^2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a ring, use the element distance $\sqrt{x^2 + R^2}$ and multiply by the projection $x/\sqrt{x^2 + R^2}$; do not use GMm/x^2.

50. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is : [4 marks]



(A) $\frac{3R}{2}$

(B) $\frac{R}{2}$

(C) 2R

(D) R

Correct Answer: (D) R

Solution:

Concept:

- A Wheatstone bridge is balanced when the resistance ratios in its two arms are equal. The junctions at the same potential then carry no current between them.
- For resistors in series, resistances add; two equal resistances in parallel have half the resistance of

either branch. An open gap between terminals is not a conducting wire.

Step 1: Identify the two symmetric paths Terminal A is connected to the central junction, whereas terminal B is connected to C. From A to C, the upper path is A–D–C and the lower path is A–E–C. Each contains two resistors of resistance R, so each path has resistance $2R$. The fifth resistor joins D and E; it is not directly across the measuring terminals.

Step 2: Use symmetry to remove the bridge current Apply a test potential difference between A and C. Since both paths have identical resistances on either side of their intermediate junctions, D and E are each at the midpoint potential. Hence $V_D = V_E$, and the current through the D–E resistor is $(V_D - V_E)/R = 0$. Removing this inactive bridge resistor does not change the terminal current.

Step 3: Combine the conducting branches and test the options The remaining two branches are in parallel, giving

$$R_{AB} = \frac{(2R)(2R)}{2R + 2R} = R.$$

Option (C), $2R$, counts only one branch. Option (B), $R/2$, wrongly treats the branches as single resistors. Option (A), $3R/2$, results from a series-parallel reduction that ignores the equal-potential junctions.

Step 4: Conclusion The equivalent resistance between A and B is $\boxed{R}$. Hence, the correct option is (D).

Quick Tip to Solve: With equal arms, locate the two equal-potential junctions first: the resistor joining them carries no current.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be _____ N. [g = 10 m/s^2] [4 marks]

Solution:

Concept:

- The reading of a spring balance is the normal reaction N, called apparent weight; the gravitational force remains mg.
- Newton's second law along the upward vertical direction is $N - mg = ma_y$. A downward lift acceleration makes a_y negative.

Step 1: Determine the mass from the stationary reading When the lift is stationary its acceleration is zero. The balance then supports the full gravitational force, and its kilogram-calibrated reading gives the person's mass as $m = 60\,\text{kg}$. With the stated $g = 10\,\text{m}\,\text{s}^{-2}$, the gravitational force is

$$mg = 60 \times 10 = 600\,\text{N}.$$

The mass stays the same when the lift begins accelerating; only the force exerted by the balance

changes.

Step 2: Apply the vertical force equation Take upward as positive. The upward normal reaction is N, the downward gravitational force is mg, and the acceleration is $-1.8\,\mathrm{m\,s^{-2}}$. Thus

$$N - mg = -ma, \qquad N = m(g - a).$$

A downward acceleration requires a downward net force, so the normal reaction must be smaller than the true weight. Using a plus sign would instead describe upward acceleration.

Step 3: Calculate the force measured by the balance Substitute the given mass and acceleration:

$$N = 60(10 - 1.8) = 60 \times 8.2 = 492\,\mathrm{N}.$$

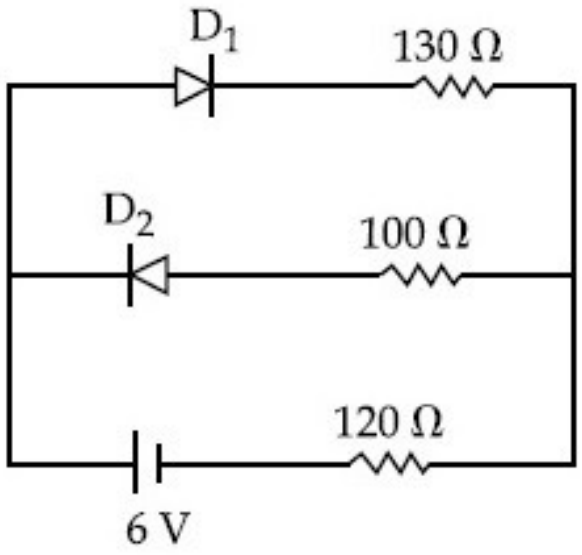
The result is positive because the downward acceleration is less than g, so contact with the balance is maintained. The question asks for the force in newtons rather than the reduced kilogram reading.

Step 4: Conclusion The apparent weight measured by the spring balance is $\boxed{492\,\mathrm{N}}$.

Quick Tip to Solve: For a downward-accelerating lift use $m(g - a)$; the gravitational weight itself remains mg.

52. The circuit contains two diodes each with a forward resistance of 50Ω and with infinite reverse resistance. If the battery voltage is 6V, the current through the 120Ω resistance is _____ mA.

[4 marks]



Solution:

Concept:

- A forward-biased diode conducts with the stated forward resistance, whereas a reverse-biased diode with infinite resistance acts as an open circuit.
- Once the conducting path is identified, Kirchhoff's loop law gives $I = V/R_{\text{total}}$. The series current

also passes through the battery branch resistor.

Step 1: Identify the conducting diode The longer battery plate is on the left, so the left-hand rail is at higher potential than the right-hand rail. Diode D_1 is oriented to conduct from left to right and is therefore forward biased. Diode D_2 points in the opposite direction, so it is reverse biased. Its infinite reverse resistance prevents current in the middle branch, including the $100\,\Omega$ resistor.

Step 2: Replace the diodes by their circuit models Replace D_1 by its forward resistance of $50\,\Omega$ and leave the D_2 branch open. The closed loop contains the $6\,\text{V}$ battery, the $120\,\Omega$ resistor in its own branch, and the upper branch with $130\,\Omega$ in series with $50\,\Omega$. Consequently,

$$R_{\text{loop}} = 120 + 130 + 50 = 300\,\Omega.$$

The $100\,\Omega$ resistor must not be included because its branch carries no current.

Step 3: Find the current through the battery branch There is only one conducting loop, so its current is the current through the $120\,\Omega$ resistor. Applying the loop equation gives

$$6 - I(300) = 0, \qquad I = \frac{6}{300} = 0.020\,\text{A}.$$

Convert amperes to milliamperes by multiplying by 10^3: $0.020\,\text{A} = 20\,\text{mA}$. No additional diode threshold voltage is specified, so the stated resistance model is used.

Step 4: Conclusion The current through the $120\,\Omega$ resistor is $\boxed{20\,\text{mA}}$.

Quick Tip to Solve: Determine diode bias before combining resistors; an open diode removes its entire series branch.

53. In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be _____. [4 marks]

Solution:

Concept:

- At resonance a series LCR circuit has angular frequency $\omega_0 = 1/\sqrt{LC}$, where L is inductance and C is capacitance.
- The quality factor is $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$. Therefore changing inductance changes both L and the resonant frequency.

Step 1: Write the initial and final quality factors Initially,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 100.$$

The stated changes mean $L\prime = 2L$ and $R\prime = R/2$, while capacitance is unchanged. The final circuit is evaluated at its own resonance, rather than at the old resonant frequency. Its quality factor is therefore $Q\prime = (1/R\prime)\sqrt{L\prime/C}$.

Step 2: Form a ratio to cancel the unchanged quantities Dividing the final expression by the initial one eliminates the unknown numerical values of L, C, and R:

$$\frac{Q\prime}{Q} = \frac{R}{R\prime}\sqrt{\frac{L\prime}{L}} = 2\sqrt{2}.$$

Halving resistance doubles the quality factor, and doubling inductance supplies an additional factor $\sqrt{2}$. Multiplying by four would incorrectly keep the original resonant frequency fixed.

Step 3: Evaluate the new dimensionless factor Substitution of the initial value gives

$$Q\prime = 100(2\sqrt{2}) = 200\sqrt{2} \approx 282.8427.$$

Quality factor is a ratio of energy measures, so it is dimensionless. Expressing the numerical response as the nearest integer gives 283. The exact value is retained before rounding to avoid accumulating intermediate rounding errors.

Step 4: Conclusion The new quality factor is $\boxed{200\sqrt{2} \approx 283}$, dimensionless.

Quick Tip to Solve: At resonance use $Q \propto \sqrt{L}/R$ when capacitance is fixed; do not treat resonant frequency as unchanged.

54. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form y = − 0.21 sin (x + 30t) is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is x $\times$ 10^{-2} N. Value of x is _____. (Round off to the nearest integer) [4 marks]

Solution:

Concept:

- A sinusoidal wave $y = A\sin(kx \pm \omega t)$ has speed $v = \omega/k$, independent of its amplitude and overall sign.
- For a stretched uniform wire, $v = \sqrt{T/\mu}$, so the tension is $T = \mu v^2$, with μ expressed in $\mathrm{kg\,m^{-1}}$.

Step 1: Convert the mass per unit length The given linear mass density is $0.135\,\mathrm{g\,cm^{-1}}$. Converting the numerator and denominator separately gives

$$\mu = \frac{0.135 \times 10^{-3}\,\mathrm{kg}}{10^{-2}\,\mathrm{m}} = 0.0135\,\mathrm{kg\,m^{-1}}.$$

Since one metre contains one hundred centimetres, the density is also $13.5\,\mathrm{g\,m^{-1}}$. Keeping the original numerical density with SI wave speed would give the wrong tension.

Step 2: Extract the wave speed from the phase Comparing $y = -0.21\sin(x + 30t)$ with the standard wave form gives $k = 1\,\mathrm{rad\,m^{-1}}$ and $\omega = 30\,\mathrm{rad\,s^{-1}}$. Therefore

$$v = \frac{\omega}{k} = 30\,\mathrm{m\,s^{-1}}.$$

The plus sign in the phase indicates travel toward decreasing position, but tension depends on the

square of the speed. The minus sign before the amplitude changes the phase, not the speed.

Step 3: Calculate the tension and the requested coefficient Use the string-wave relation:

$$T = \mu v^2 = 0.0135(30)^2 = 0.0135 \times 900 = 12.15\,\text{N}.$$

The question defines a numerical coefficient by $T = x \times 10^{-2}\,\text{N}$. Hence

$$x = \frac{12.15}{10^{-2}} = 1215.$$

This coefficient is already an integer, so the stated rounding instruction makes no further change.

Step 4: Conclusion The tension is $12.15\,\text{N} = 1215 \times 10^{-2}\,\text{N}$; the requested value is $\boxed{x = 1215}$.

Quick Tip to Solve: Convert linear density to kilograms per metre first, then use the phase coefficients to obtain $v = \omega/k$.

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55. In an electrical circuit, a battery is connected to pass 20C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15V. The workdone by the battery is _____J. [4 marks]

Solution:

Concept:

- Potential difference is work transferred per unit charge: $V = W/q$. Thus moving charge q through a maintained potential difference V transfers energy $W = qV$.
- One volt equals one joule per coulomb, so multiplying charge in coulombs by potential difference in volts gives work directly in joules.

Step 1: Identify the transferred charge and voltage The total charge passing through the battery is $q = 20\,\text{C}$, and the maintained potential difference between its terminals is $V = 15\,\text{V}$. The word maintained means that this voltage remains constant during the charge transfer. Every coulomb therefore receives $15\,\text{J}$ of electrical energy from the battery. We need the total energy supplied, rather than the rate at which it is supplied.

Step 2: Apply the definition of potential difference For a small charge transfer dq, the work supplied is $dW = V\,dq$. With constant voltage this integrates to

$$W = \int_0^{20} 15\,dq = Vq.$$

Equivalently, the result follows immediately from $V = W/q$. A factor of one-half is not appropriate: that factor occurs when charging an initially uncharged capacitor whose voltage rises from zero, whereas this battery maintains a fixed voltage.

Step 3: Substitute and check the dimensions The work is

$$W = 20 \times 15 = 300\,\text{J}.$$

The dimensional check is $\mathrm{C} \times \mathrm{V} = \mathrm{C} \times \mathrm{J\,C^{-1}} = \mathrm{J}$. The time interval is unnecessary because the question asks for work; time would only be required to calculate average power $P = W/t$. The positive result represents energy supplied by the battery to the circuit.

Step 4: Conclusion The work done by the battery is $\boxed{300\,\mathrm{J}}$.

Quick Tip to Solve: For a battery holding its voltage constant, multiply total charge by voltage; elapsed time is needed only for power.

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56. A boy pushes a box of mass 2 kg with a force $\vec{F} = \left(20\hat{i} + 10\hat{j}\right)\,N$ on a frictionless surface. If the box was initially at rest, then _____ m is displacement along the x-axis after 10s. [4 marks]

Solution:

Concept:

- Newton's second law applies independently to Cartesian components: $a_x = F_x/m$. With no friction, the applied horizontal force determines the horizontal acceleration.
- For constant acceleration, displacement along an axis is $\Delta x = u_x t + \frac{1}{2}a_x t^2$. A perpendicular force component does not enter this component equation.

Step 1: Resolve the applied force along the required axis The force is $\vec{F} = (20\hat{i} + 10\hat{j})\,\mathrm{N}$, so its x component is $F_x = 20\,\mathrm{N}$. The mass is $m = 2\,\mathrm{kg}$. The requested displacement is specifically along the x-axis, so the magnitude $\sqrt{20^2 + 10^2}$ must not be used in place of F_x. Any force balance or acceleration in the perpendicular direction does not alter the independent x-component equation.

Step 2: Calculate the constant horizontal acceleration The surface is frictionless, so there is no frictional force opposing motion in x. Newton's second law gives

$$a_x = \frac{F_x}{m} = \frac{20}{2} = 10\,\mathrm{m\,s^{-2}}.$$

The applied force and mass are constant, so this acceleration stays constant throughout the stated $10\,\mathrm{s}$ interval. The box starts from rest, which means its initial x-component of velocity is $u_x = 0$.

Step 3: Use the displacement equation For elapsed time $t = 10\,\mathrm{s}$,

$$\Delta x = u_x t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(10)(10)^2 = 500\,\mathrm{m}.$$

As a check, the final x velocity is $a_x t = 100\,\mathrm{m\,s^{-1}}$. Its average over the interval is $50\,\mathrm{m\,s^{-1}}$, giving the same displacement $50 \times 10 = 500\,\mathrm{m}$. The displacement is positive because the force points along positive x.

Step 4: Conclusion The displacement along the x-axis after $10\,\mathrm{s}$ is $\boxed{500\,\mathrm{m}}$.

Quick Tip to Solve: For a displacement along one axis, use that force component in $\Delta x = F_x t^2/(2m)$ when starting from rest.

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57. A radiation is emitted by 1000W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is x $\times$ 10^{-1} V/m. Value of x is _____. (Rounded off to the nearest integer) [Take $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2N^{-1} m^{-2}, c = 3×10^8 ms^{-1}] [4 marks]

Solution:

Concept:

- The efficiency gives the fraction of electrical power converted to radiation. At distance r, an isotropic point source spreads this power over area $4\pi r^2$.
- For a sinusoidal electromagnetic wave, the average intensity is $I = \frac{1}{2}\varepsilon_0 c E_0^2$, where E_0 is the peak electric field. The factor one-half distinguishes peak and rms fields.

Step 1: Find the useful radiated power The input power is not the power to use directly in the wave formula. Only 1.25 per cent becomes the radiation specified in the question. Thus

$$P_{\text{rad}} = \frac{1.25}{100}(1000) = 12.5\,\text{W}.$$

The rest of the electrical power is not included in the stated radiative efficiency.

Step 2: Calculate the intensity at the observation point At $r = 2\,\text{m}$, the area of the sphere centred on the bulb is $4\pi(2)^2 = 16\pi\,\text{m}^2$. Consequently

$$I = \frac{12.5}{16\pi} = 0.24868\,\text{W m}^{-2}.$$

Using the spherical area, rather than a circular cross-section, accounts for emission in all directions.

Step 3: Recover the peak field and the requested coefficient Solve the average-intensity relation for the peak amplitude and insert the given constants:

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(0.24868)}{(8.85 \times 10^{-12})(3 \times 10^8)}} = 13.687\,\text{V m}^{-1}.$$

Since $E_0 = x \times 10^{-1}\,\text{V m}^{-1}$, the coefficient is $x = 136.87$, which must be rounded only at the end.

Step 4: Conclusion The peak electric field is approximately $13.69\,\text{V m}^{-1}$, so the requested nearest-integer coefficient is $\boxed{x = 137}$ (dimensionless).

Quick Tip to Solve: For a bulb-radiation calculation, apply efficiency before dividing by spherical area; use the factor 2 when recovering a peak field from average intensity.

58. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is x $\times$ 10^{-1} atm. Value of x is _____. [4 marks]

Solution:

Concept:

- For an ideal gas, $PV = nRT$. In a rigid isolated container, removing a partition does no external volume work and conserves total internal energy.
- For the same ideal gas in both chambers, $U = nC_VT$, so energy conservation gives $(n_1 + n_2)T_f = n_1T_1 + n_2T_2$. Combining this with the ideal-gas equation eliminates the unknown initial temperatures.

Step 1: Express the initial thermal energies The two temperatures need not initially be equal: the supplied pressures, volumes and mole numbers determine them. Write

$$n_1RT_1 = P_1V_1, \qquad n_2RT_2 = P_2V_2.$$

For the same gas, its common molar heat capacity cancels from the energy balance. There is no need to calculate the temperatures individually or to take an unweighted average of the two pressures.

Step 2: Apply the final equilibrium equation After the partition is removed, all $3+4 = 7$ moles occupy $V_1 + V_2 = 10.0\,\mathrm{L}$. Multiplying the energy-balance equation by R and using the gas laws gives

$$P_f(V_1 + V_2) = P_1V_1 + P_2V_2.$$

This relation uses the initial pressure-volume products; it is compatible with different initial temperatures and does not assume an isothermal mixing process.

Step 3: Substitute the data and convert the coefficient Keeping pressure in atmospheres and volume in litres is consistent because identical units occur in numerator and denominator:

$$P_f = \frac{(2.0)(4.5) + (3.0)(5.5)}{4.5 + 5.5} = \frac{9.0 + 16.5}{10.0} = 2.55\,\mathrm{atm}.$$

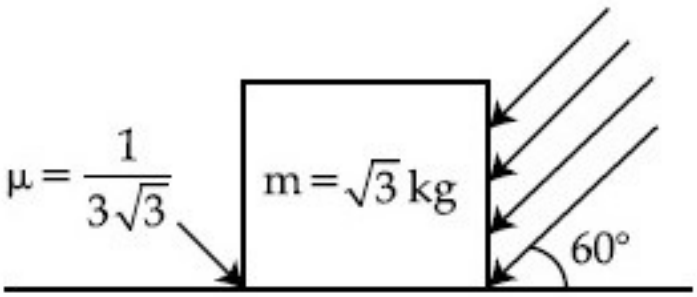
Thus $P_f = x \times 10^{-1}\,\mathrm{atm}$ implies $x = 25.5$. No rounding instruction is printed, so the exact coefficient from the supplied values is retained.

Step 4: Conclusion The common pressure is $\boxed{2.55\,\mathrm{atm}}$, hence $\boxed{x = 25.5}$ (dimensionless). Although the supplied key gives 25, the printed values give $(2 \times 4.5 + 3 \times 5.5)/10 = 2.55$, not 2.50; no rounding instruction makes the coefficient 25.

Quick Tip to Solve: For mixing the same ideal gas in a rigid isolated vessel, add the initial PV products, then divide by total volume; the mole numbers cancel after energy conservation.

59. As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be 3x. The value of x will be _____.

[g = 10 m/s^2; sin60° = $\frac{\sqrt{3}}{2}$; cos60° = $\frac{1}{2}$] [4 marks]



Solution:

Concept:

- Static friction adjusts to oppose impending sliding and reaches $f_{\max} = \mu N$ at the threshold. The normal reaction must include any downward component of the applied force.
- The force drawn at 60° pushes into the block and downward: its horizontal component is $F\cos 60^\circ$, and its downward component is $F\sin 60^\circ$.

Step 1: Resolve the force and obtain the normal reaction The block remains in contact with the horizontal surface, so vertical acceleration is zero. Gravity and the downward component of the push are balanced by the upward normal reaction:

$$N = mg + F\sin 60^\circ = 10\sqrt{3} + \frac{\sqrt{3}}{2}F.$$

The horizontal push tends to move the block left, so friction acts to the right. A formula using $N = mg$ would omit the extra compression of the surface.

Step 2: Use limiting equilibrium in the horizontal direction At the largest push for which there is no sliding, the required horizontal friction just equals its maximum value:

$$F\cos 60^\circ = \mu N.$$

Substituting $\mu = 1/(3\sqrt{3})$ and the vertical result gives

$$\frac{F}{2} = \frac{1}{3\sqrt{3}}\left(10\sqrt{3} + \frac{\sqrt{3}}{2}F\right) = \frac{10}{3} + \frac{F}{6}.$$

Both sides have units of force; the downward component increases the friction capacity as F grows.

Step 3: Solve for the critical force and interpret x Collect the force terms:

$$\frac{F}{2} - \frac{F}{6} = \frac{10}{3} \quad\Longrightarrow\quad \frac{F}{3} = \frac{10}{3} \quad\Longrightarrow\quad F = 10\,\text{N}.$$

For smaller pushes, static friction can supply the needed value; beyond this threshold it cannot. The question writes the critical force as $3x$, hence $3x = 10$ and $x = 10/3$.

Step 4: Conclusion The critical applied force is $\boxed{10\,\text{N}}$. Therefore the requested numerical coefficient is $\boxed{x = 10/3 \simeq 3.33}$.

Quick Tip to Solve: Before using limiting friction, decide whether the angled force presses downward or pulls upward; this fixes the sign of its contribution to the normal reaction.

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60. The maximum and minimum amplitude of an amplitude modulated wave is 16V and 8V respectively. The modulation index for this amplitude modulated wave is x $\times$ 10^{-2}. The value of x is _____. [4 marks]

Solution:

Concept:

- For an amplitude-modulated carrier, the envelope extremes are $A_{\max} = A_c(1 + m)$ and $A_{\min} = A_c(1 - m)$, where m is the dimensionless modulation index.
- Adding the extremes isolates the carrier amplitude; subtracting them isolates twice the modulating amplitude. Hence $m = (A_{\max} - A_{\min})/(A_{\max} + A_{\min})$.

Step 1: Recover the carrier and modulating amplitudes The maximum envelope voltage is 16 V, while the minimum is 8 V. These are envelope extremes, not positive and negative excursions of the carrier. Adding and subtracting their equations gives

$$A_c = \frac{16 + 8}{2} = 12\,\text{V}, \qquad A_m = \frac{16 - 8}{2} = 4\,\text{V}.$$

The modulating amplitude is smaller than the carrier amplitude, consistent with a nonzero minimum envelope and absence of overmodulation.

Step 2: Calculate the modulation index The index is the ratio of modulating amplitude to carrier amplitude:

$$m = \frac{A_m}{A_c} = \frac{4}{12} = \frac{1}{3}.$$

Equivalently, the envelope formula gives

$$m = \frac{16 - 8}{16 + 8} = \frac{8}{24} = 0.3333\ldots.$$

The voltage unit cancels. Using only the difference divided by the maximum would produce a different quantity, not the modulation index.

Step 3: Match the specified power-of-ten form The question specifies $m = x \times 10^{-2}$. Therefore

$$x = 10^2 m = \frac{100}{3} = 33.3333\ldots.$$

To the integer precision used in the supplied answer key this is 33; equivalently the modulation is approximately 33 per cent. The exact index remains one-third, and the factor of 100 converts that

fraction to the coefficient requested in the question.

Step 4: Conclusion The modulation index is $\boxed{m = 1/3 \simeq 0.33}$, dimensionless. Thus $x = 100/3 \simeq 33.33$; the integer value used by the key is $\boxed{33}$.

Quick Tip to Solve: Use envelope difference over envelope sum, then multiply by 100 only when the requested form is x times ten to the power minus two.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. An amine on reaction with benzensulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is : [4 marks]

(A) $CH_3CH_2NH_2$

(B) $CH_3CH_2CH_2NHCH_3$

(C) 

(D) $C_6H_5NHCH_2CH_2CH_3$

Correct Answer: The required amine is diethylamine, $(C_2H_5)_2NH$; no unambiguous printed option gives this structure.

Solution:

Concept:

- Benzenesulphonyl chloride distinguishes primary, secondary and tertiary amines by sulphonamide formation and alkaline solubility.
- A primary sulphonamide retains an acidic N–H bond; a secondary-amine sulphonamide has no N–H bond and is insoluble in aqueous alkali.
- Ammonolysis of ethyl chloride introduces ethyl groups successively onto nitrogen; the secondary amine obtainable this way is diethylamine.

Step 1: Identify the class of amine from the Hinsberg test A primary amine gives $C_6H_5SO_2NHR$, whose N–H hydrogen can be removed by alkali, producing a soluble salt. A secondary amine gives $C_6H_5SO_2NR_2$, which has no N–H hydrogen and therefore does not form that salt. A tertiary amine has no N–H bond with which to form a sulphonamide. Since the question says that a compound forms and is insoluble in alkaline solution, the required starting amine must be secondary.

$$R_2NH + C_6H_5SO_2Cl \longrightarrow C_6H_5SO_2NR_2 + HCl$$

Step 2: Apply the preparation condition Ethyl chloride first gives ethylamine with ammonia. Further alkylation introduces another ethyl group, giving diethylamine. The secondary product must therefore

contain two ethyl groups, not methyl, propyl or phenyl groups:

$$C_2H_5Cl + 2\,NH_3 \longrightarrow C_2H_5NH_2 + NH_4Cl$$

$$C_2H_5NH_2 + C_2H_5Cl + NH_3 \longrightarrow (C_2H_5)_2NH + NH_4Cl$$

Thus the two independent conditions specify $CH_3CH_2NHCH_2CH_3$.

Step 3: Check every printed option literally Option (A), ethylamine, is primary, so its sulphonamide is soluble in alkali. Option (B), methylpropylamine, is secondary and passes the solubility condition, but its methyl and propyl groups cannot arise from ammonolysis of ethyl chloride. Option (D), N-propylaniline, likewise has substituents incompatible with that preparation. The disconnected structure in option (C) displays propyl and ethyl fragments with an NH label; it does not unambiguously depict diethylamine. Even joining those displayed fragments to nitrogen would give ethylpropylamine, which fails the preparation condition. Consequently a unique printed option cannot be defended.

Step 4: Conclusion The chemically required amine is diethylamine, $(C_2H_5)_2NH$. None of the options as printed unambiguously represents it, so this is an ambiguous option set; selecting a letter would require altering a displayed structure.

Quick Tip to Solve: Use both filters: an alkali-insoluble Hinsberg product identifies a secondary amine, and ethyl chloride requires ethyl groups on nitrogen.

62. Which of the following is 'a' FALSE statement ? [4 marks]

(A) Carius method is used for the estimation of nitrogen in an organic compound.

(B) Kjeldahl's method is used for the estimation of nitrogen in an organic compound.

(C) Carius tube is used in the estimation of sulphur in an organic compound.

(D) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture.

Correct Answer: Both (A) and (D) are false as literally printed; (A) is the intended keyed answer.

Solution:

Concept:

- Carius analysis estimates halogens and sulphur after oxidation in a sealed hard-glass tube. Nitrogen is estimated by methods such as Kjeldahl or Dumas, not by Carius analysis.
- In phosphorus analysis, magnesia mixture first precipitates magnesium ammonium phosphate, $MgNH_4PO_4$; ignition then gives magnesium pyrophosphate, $Mg_2P_2O_7$, the weighed final product.

Step 1: Check the nitrogen and sulphur statements Option (A) assigns nitrogen estimation to the Carius method. This is false: Carius oxidation is used for sulphur and halogens, whereas Kjeldahl digestion converts suitable organic nitrogen into ammonium sulphate for subsequent ammonia estimation. Thus (B) is a correct statement of the method, subject to its standard applicability restrictions. In sulphur estimation the Carius tube is used to oxidise sulphur to sulphuric acid, which is determined gravimetrically as barium sulphate, so (C) is also true.

Step 2: Distinguish precipitation from ignition in phosphorus estimation Option (D), as literally printed, says that magnesia mixture precipitates $Mg_2P_2O_7$. The immediate precipitate is instead $MgNH_4PO_4$, often obtained as its hydrated form. Heating the precipitate produces the pyrophosphate:

$$2\,MgNH_4PO_4 \longrightarrow Mg_2P_2O_7 + 2\,NH_3 + H_2O.$$

Consequently magnesium pyrophosphate is the final weighed product, but it is not directly precipitated by adding magnesia mixture.

Step 3: Resolve the wording and the supplied key The supplied single-answer key selects (A), and that choice unequivocally identifies the wrong nitrogen-estimation method. However, reading (D) as an exact statement of the precipitation step makes it false too. If (D) was intended as a compressed description of the final gravimetric determination including ignition, (A) would be the intended unique answer. The missing ignition qualification cannot be silently supplied when assessing the literal wording.

Step 4: Conclusion The intended key answer is (A), because Carius analysis does not estimate nitrogen. Literally, (D) is also false: magnesia mixture precipitates $MgNH_4PO_4$, and $Mg_2P_2O_7$ forms only on ignition. The printed question therefore has no unique false statement.

Quick Tip to Solve: In gravimetric analysis, separate the species first precipitated from the species weighed after ignition; they need not have the same formula.

63. Find A, B and C in the following reactions :

$$NH_3 + A + CO_2 \rightarrow (NH_4)_2CO_3$$

$$(NH_4)_2CO_3 + H_2O + B \rightarrow NH_4HCO_3$$

$$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$$

[4 marks]

(A) $A - O_2; B - CO_2; C - Na_2CO_3$
(B) $A - H_2O; B - O_2; C - NaHCO_3$
(C) $A - H_2O; B - O_2; C - Na_2CO_3$
(D) $A - H_2O; B - CO_2; C - NaHCO_3$
Correct Answer: (D) $A - H_2O; B - CO_2; C - NaHCO_3$

Solution:

Concept:

- The Solvay process uses ammonia, water and carbon dioxide to form ammonium hydrogen carbonate. Its reaction with brine precipitates sparingly soluble sodium hydrogen carbonate.
- Identify each missing substance by atom balance and the carbonate-to-hydrogen-carbonate conver-

sion; the reaction schemes need stoichiometric coefficients when written as balanced equations.

Step 1: Identify the water required in the first reaction Ammonia and carbon dioxide react in water to form ammonium carbonate. The balanced equation is

$$2\,NH_3 + H_2O + CO_2 \longrightarrow (NH_4)_2CO_3.$$

Therefore $A = H_2O$. Oxygen alone cannot provide the missing hydrogen and cannot complete this acid-base formation reaction, which rules out option (A). The omission of coefficients in the supplied scheme does not alter the identity of A.

Step 2: Convert carbonate to hydrogen carbonate Further carbon dioxide, in the presence of water, converts ammonium carbonate into ammonium hydrogen carbonate:

$$(NH_4)_2CO_3 + H_2O + CO_2 \longrightarrow 2\,NH_4HCO_3.$$

Thus $B = CO_2$. This is the familiar carbonate plus carbon dioxide plus water conversion; oxygen does not supply the additional carbon needed to produce two hydrogen carbonate ions. Both options (B) and (C), which assign oxygen to B, fail this check.

Step 3: Identify the sodium precipitate and select the combination Ammonium hydrogen carbonate exchanges ions with sodium chloride in brine:

$$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + NaHCO_3\downarrow.$$

The poorly soluble product at this stage is sodium hydrogen carbonate, so $C = NaHCO_3$, not sodium carbonate. Sodium carbonate is obtained later by heating:

$$2\,NaHCO_3 \longrightarrow Na_2CO_3 + CO_2 + H_2O.$$

Only the last listed combination has water for A, carbon dioxide for B and sodium hydrogen carbonate for C.

Step 4: Conclusion $\boxed{A = H_2O, \quad B = CO_2, \quad C = NaHCO_3}$. Hence, the correct option is (D).

Quick Tip to Solve: In the Solvay sequence, bicarbonate is precipitated first; carbonate is obtained only after heating that bicarbonate.

64. Match List - I with List - II.

	List IElectronic configuration		List II$\Delta_i H$ in kJ mol^{-1}
(a)	$1s^2 2s^2$	**(i)**	**801**
(b)	$1s^2 2s^2 2p^4$	**(ii)**	**899**
(c)	$1s^2 2s^2 2p^3$	**(iii)**	**1314**
(d)	$1s^2 2s^2 2p^1$	**(iv)**	**1402**

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a) $\rightarrow$ (i), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (iii)

(B) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (i), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (iii)

(C) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)

(D) (a) $\rightarrow$ (i), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (ii)

Correct Answer: (C) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)

Solution:

Concept:

- First ionization enthalpy is the energy required for $\mathrm{M(g)} \longrightarrow \mathrm{M^+(g)} + \mathrm{e^-}$. Across a period, increasing effective nuclear charge generally raises this energy.
- A filled $2s$ subshell and a half-filled $2p$ subshell have extra stability. Removing a paired $2p$ electron from oxygen is easier than removing an unpaired electron from nitrogen.

Step 1: Identify the elements The configurations give (a) beryllium, $1s^2 2s^2$; (b) oxygen, $1s^2 2s^2 2p^4$; (c) nitrogen, $1s^2 2s^2 2p^3$; and (d) boron, $1s^2 2s^2 2p^1$. The relevant quantity is the first ionization enthalpy of the gaseous atom, so neither electron affinity nor the energy of removing a second electron is being compared.

Step 2: Match the values and explain the exceptions Boron loses its higher-energy $2p$ electron more easily than beryllium loses a $2s$ electron. Thus boron has 801 and beryllium $899\,\mathrm{kJ\,mol^{-1}}$. Nitrogen has three singly occupied $2p$ orbitals. Oxygen has one pair, whose repulsion facilitates removal; nitrogen therefore has 1402, while oxygen has $1314\,\mathrm{kJ\,mol^{-1}}$. Hence the matching is (a)–(ii), (b)–(iii), (c)–(iv), (d)–(i).

Step 3: Reject the other matchings Option (A) incorrectly assigns beryllium the smallest value and assigns boron the oxygen value. Option (B) puts oxygen at the smallest value and nitrogen below oxygen. Option (D) reverses the nitrogen–oxygen order and also interchanges the appropriate beryllium and boron values. Only (C) respects both subshell exceptions.

Step 4: Conclusion The four species match the values 899, 1314, 1402 and 801 $\mathrm{kJ\,mol^{-1}}$, respectively, in the order given in List I. Hence, the correct option is (C).

Quick Tip to Solve: Identify Be, B, N and O first; remember the two exceptions $I_1(\mathrm{Be}) > I_1(\mathrm{B})$ and $I_1(\mathrm{N}) > I_1(\mathrm{O})$.

65. Given below are two statements :

Statement I : A mixture of chloroform and aniline can be separated by simple distillation.

Statement II : When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (A) Both Statement I and Statement II are true

Solution:

Concept:

- Simple distillation separates volatile liquids when their boiling points differ sufficiently, provided they do not decompose or form a troublesome constant-boiling mixture.
- For two substantially immiscible liquids in steam distillation, the total vapour pressure is approximately the sum of their separate vapour pressures: $p_{\text{total}} = p_{\text{water}} + p_{\text{organic}}$.

Step 1: Assess simple distillation Chloroform boils at about $61\,^{\circ}\text{C}$, whereas aniline boils at about $184\,^{\circ}\text{C}$ at atmospheric pressure. The difference is approximately $123\,^{\circ}\text{C}$, which is large. On heating their mixture, the vapour is strongly enriched in the much more volatile chloroform. It is condensed and collected first, leaving aniline behind. Thus Statement I is true; fractional distillation is mainly needed when boiling points are close.

Step 2: Assess steam distillation Aniline is sufficiently steam-volatile and only sparingly soluble in water. During steam distillation both water and aniline contribute to the vapour pressure. Boiling begins when their combined pressure reaches atmospheric pressure; aniline alone need not supply the whole atmospheric pressure. Consequently it co-distils at a temperature below its normal boiling point. This reduces the temperature at which the organic liquid is isolated and confirms Statement II.

Step 3: Compare all choices Both statements have independently been established as true. Option (B) denies both valid separation principles. Option (C) incorrectly denies the lowering of the co-distillation temperature, and option (D) incorrectly denies the feasibility of separating liquids with such widely different boiling points. Only option (A) gives the required pair of truth values.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (A).

Quick Tip to Solve: Use the large boiling-point difference for simple distillation; use the sum of vapour pressures to explain why steam distillation lowers the operating temperature.

66. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation.

Reason R : Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) A is true but R is false

(C) A is false but R is true

(D) Both A and R are true but R is NOT the correct explanation of A

Correct Answer: No listed option: both literal statements are false; neutral HF must not be confused with symmetric bifluoride.

Solution:

Concept:

- Hydrogen bonding is directional and has electrostatic, induction and donor–acceptor contributions; a pure dipole–dipole description does not exhaust its physical origin.
- A symmetric hydrogen bond has the hydrogen equidistant from its donor and acceptor atoms. Identical terminal elements alone do not establish equality of these two distances.

Step 1: Test the exclusive claim in Assertion A The word "only makes A too restrictive. Hydrogen bonding is often introduced as a particularly strong dipole–dipole interaction, but its directional donor–acceptor contribution also involves electron-density donation from an acceptor lone pair into the antibonding orbital of the donor bond. Treating the elementary electrostatic description as the sole non-covalent contribution is therefore not justified. On a literal reading, Assertion A is false.

Step 2: Test both clauses of Reason R Fluorine is indeed the most electronegative element. However, ordinary molecular HF has a short covalent $\mathrm{F-H}$ bond and a longer intermolecular $\mathrm{H\cdots F}$ contact. The hydrogen is attached more closely to its donor fluorine than to the acceptor fluorine, so that hydrogen bond is asymmetric. A familiar symmetric $\mathrm{F-H-F}$ arrangement occurs in the bifluoride ion, $\mathrm{HF_2}^-$, which must not be identified with neutral HF. Thus the conjunction used as Reason R includes a false claim and is false overall.

Step 3: Compare the available combinations The literal truth values are therefore A false and R false. None of the four listed choices offers this pair: three choices require R to be true, and the remaining choice requires A to be true. Even if the elementary pure-dipole model were used to accept A, the physical asymmetry issue would still require rejecting the printed R. The supplied key selecting A false and R true cannot be established from the ordinary HF statement as written.

Step 4: Conclusion The printed question has no fully defensible option on a literal scientific reading: both A and R are false, whereas this pair is absent. The supplied key (C) would require the HF symmetry claim to be true; symmetric bifluoride, $\mathrm{HF_2}^-$, is a different species.

Quick Tip to Solve: Check the species as well as the elements: a symmetric $\mathrm{F-H-F}$ ion is not interchangeable with asymmetric intermolecular hydrogen bonding in neutral HF.

67. Which one of the following lanthanoids does not form MO_2? [M is lanthanoid metal] [4 marks]

(A) Nd

(B) Dy

(C) Yb

(D) Pr

Correct Answer: (C) Yb

Solution:

Concept:

- In a dioxide MO_2, each oxide ion has oxidation number -2, so the lanthanoid must be in the $+4$ oxidation state.
- Although $+3$ is the characteristic lanthanoid oxidation state, some lanthanoids also form $+2$ or $+4$ compounds. Stability of an oxide must therefore be judged from the accessible oxidation states.

Step 1: Find the required metal oxidation state Charge neutrality for the formula gives $x + 2(-2) = 0$, hence $x = +4$. The question is asking which listed metal does not form the dioxide; it is not asking which metal cannot form an oxide at all. A normal sesquioxide M_2O_3 contains M^{3+} and would not satisfy the stated formula.

Step 2: Examine ytterbium Ytterbium has the outer configuration $[Xe]4f^{14}6s^2$. Loss of its two $6s$ electrons gives the especially stable filled $4f^{14}$ configuration of Yb^{2+}. Ytterbium also forms $+3$ compounds, including its common sesquioxide, but does not provide the $+4$ state required for the listed dioxide. Its tendency toward a stable divalent state must not be confused with the tetravalency needed in MO_2.

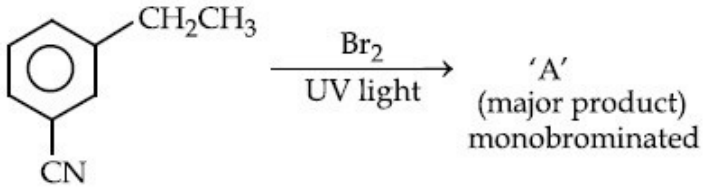
Step 3: Check the remaining options Praseodymium forms a dioxide involving $+4$. Neodymium and dysprosium are also included among lanthanoids known to form dioxides under appropriate conditions; the question does not limit the preparation to ordinary ambient oxidation. Consequently they do not meet the negative condition in the question. The distinctive exception among Nd, Dy, Yb and Pr is Yb.

Step 4: Conclusion Ytterbium does not form MO_2. Hence, the correct option is (C).

Quick Tip to Solve: Convert the oxide formula to an oxidation state first: MO_2 demands $+4$, while ytterbium is notable for its stable $+2$ state.

68. For the given reaction

What is 'A'? [4 marks]



(A) 3-(1-Bromoethyl)benzonitrile, $\mathrm{m\text{-}NC{-}C_6H_4{-}CHBrCH_3}$

(B) 2-Bromo-3-ethylbenzonitrile

(C) 3-Bromo-5-ethylbenzonitrile

(D) 2-Bromo-5-ethylbenzonitrile

Correct Answer: (A) 3-(1-Bromoethyl)benzonitrile, $\mathrm{m\text{-}NC{-}C_6H_4{-}CHBrCH_3}$

Solution:

Concept:

- Ultraviolet light initiates homolytic cleavage of $\mathrm{Br_2}$, giving bromine radicals and a free-radical substitution chain.
- Benzylic radicals are resonance-stabilized. Radical bromination is selective for a benzylic carbon–hydrogen bond rather than ordinary aromatic substitution under these conditions.

Step 1: Identify the reactive side chain The substrate is 3-ethylbenzonitrile, with an ethyl group $-\mathrm{CH_2CH_3}$ meta to $-\mathrm{CN}$. The carbon of the ethyl group directly attached to the aromatic ring is benzylic. Abstraction of one of its hydrogens gives a secondary benzylic radical, whose unpaired electron is delocalized into the aromatic ring. Abstraction from the terminal methyl group would give a much less stable primary radical lacking this benzylic stabilization.

Step 2: Follow the radical chain Light first causes $\mathrm{Br_2} \xrightarrow{h\nu} 2\,\mathrm{Br^{\bullet}}$. A bromine radical abstracts benzylic hydrogen to produce HBr and the resonance-stabilized radical. That radical then takes bromine from another $\mathrm{Br_2}$ molecule, giving the product and regenerating a bromine radical. The overall side-chain change is

$$\mathrm{Ar{-}CH_2CH_3 + Br_2 \xrightarrow{h\nu} Ar{-}CHBrCH_3 + HBr,}$$

where Ar is the 3-cyanophenyl group. The cyano group and its position on the ring remain unchanged.

Step 3: Compare the four products Option (A) places bromine on the benzylic carbon and therefore matches the selective radical substitution. Options (B), (C) and (D) all put bromine directly on the aromatic ring while leaving the ethyl side chain intact. Such ring substitution is not the major reaction of bromine under the stated UV radical conditions. No Lewis-acid catalyst for electrophilic ring bromination is specified. Thus all three ring-brominated structures are rejected.

Step 4: Conclusion A is 3-(1-bromoethyl)benzonitrile: bromine replaces one benzylic hydrogen in the ethyl side chain. Hence, the correct option is (A).

Quick Tip to Solve: With $\mathrm{Br_2}$ and UV light, inspect benzylic hydrogens first; distinguish side-chain radical substitution from ring bromination with a Lewis acid.

69. Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is : [4 marks]

(A) PbO

(B) PbO_2

(C) $PbSO_4$

(D) Pb_3O_4

Correct Answer: (B) PbO_2

Solution:

Concept:

- The positive active material of a charged lead storage battery is lead dioxide, PbO_2; it is reduced to lead(II) sulfate during discharge.
- Lead(IV) compounds can act as oxidizing agents by accepting electrons to reach the more stable $+2$ state. Lead dioxide also shows amphoteric behaviour.

Step 1: Use the battery clue A charged lead accumulator contains a lead negative electrode, a positive electrode whose active material is PbO_2, and sulfuric acid electrolyte. The compound sought is therefore not merely any lead compound occurring somewhere during the battery cycle. On discharge the positive-electrode reduction is

$$PbO_2 + {SO_4}^{2-} + 4\,H^+ + 2\,e^- \longrightarrow PbSO_4 + 2\,H_2O.$$

This identifies the oxide that accepts electrons and acts as the oxidizing agent.

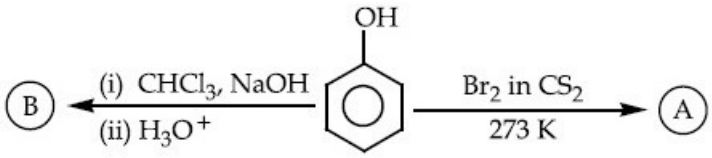
Step 2: Check oxidation state and amphoterism In PbO_2, lead has oxidation number $+4$. Its reduction to $+2$ explains the strong oxidizing property. Its amphoteric character is consistent with reactions in strongly acidic and alkaline media; under strongly alkaline conditions it can form plumbate(IV) species. Thus both the chemical-property clues and the specific electrode-material clue lead to the same dioxide.

Step 3: Reject the alternatives PbO is an amphoteric lead(II) oxide, but it is not the strong oxidizing positive active material. $PbSO_4$ is the discharge product and contains lead in the $+2$ state; it is not the charged positive-electrode oxidant. Pb_3O_4, red lead, is a mixed oxide and is not the compound specified by the simultaneous battery and strong-oxidant clues. It cannot replace PbO_2 in this identification.

Step 4: Conclusion Compound A is lead dioxide, PbO_2. Hence, the correct option is (B).

Quick Tip to Solve: In a charged lead accumulator remember the pair Pb and PbO_2; during discharge both electrodes produce $PbSO_4$.

70. Identify the major products A and B respectively in the following reactions of phenol : [4 marks]



(A) 2-Bromophenol and 4-hydroxybenzaldehyde

(B) 2-Bromophenol and 2-hydroxybenzaldehyde

(C) 4-Bromophenol and 4-hydroxybenzaldehyde

(D) 4-Bromophenol and 2-hydroxybenzaldehyde

Correct Answer: (D) 4-Bromophenol and 2-hydroxybenzaldehyde

Solution:

Concept:

- The hydroxyl group activates the benzene ring toward electrophilic substitution and directs substitution to ortho and para positions.
- Phenol with bromine in a nonpolar solvent at low temperature mainly gives monobromination, whereas chloroform and aqueous alkali followed by acidification carry out Reimer–Tiemann formylation.

Step 1: Identify product A from the bromination conditions The right-hand reaction uses Br_2 in CS_2 at $273\,K$. These controlled, nonpolar conditions give ortho- and para-bromophenol rather than the 2,4,6-tribromo product characteristic of bromine water. Of the monobromo products, para-bromophenol is the major product because substitution at the para position encounters less steric hindrance than substitution adjacent to the hydroxyl group. Therefore A is 4-bromophenol.

Step 2: Identify product B by the named reaction The left-hand sequence is (i) $CHCl_3, NaOH$, then (ii) H_3O^+. Base generates the reactive dichlorocarbene species from chloroform, and phenoxide undergoes predominantly ortho formylation. Hydrolysis and acid work-up yield 2-hydroxybenzaldehyde, or salicylaldehyde. The hydroxyl group remains on the ring and the newly introduced $-CHO$ group is adjacent to it. Thus B is the ortho-hydroxy aldehyde.

Step 3: Match the ordered pair The question asks for A and B respectively, so the order matters. Option (A) has ortho bromination and para formylation; (B) has the correct aldehyde but the wrong major bromophenol; (C) has the correct para-bromophenol but the wrong aldehyde isomer. Option (D) alone combines para-bromophenol with ortho-hydroxybenzaldehyde.

Step 4: Conclusion A is 4-bromophenol and B is 2-hydroxybenzaldehyde (salicylaldehyde). Hence, the correct option is (D).

Quick Tip to Solve: Read the solvent before predicting phenol bromination: cold CS_2 gives mainly para monobromination; Reimer–Tiemann puts $-CHO$ mainly ortho.

71. For the given reaction:

$$CH_3-CH=CHBr \xrightarrow[2.\ \text{Red hot iron tube, } 873\ K]{1.\ NaNH_2} A$$

What is A (major product)? [4 marks]

(A) 1,3,5-Trimethylbenzene (mesitylene)

(B) Benzene

(C) $CH_3-CH=CH-NH_2$

(D) $CH_3CH_2CH_2NH_2$

Correct Answer: (A) 1,3,5-Trimethylbenzene (mesitylene)

Solution:

Concept:

- Sodamide, $NaNH_2$, is a strong base that removes hydrogen halide from a vinyl halide to form an alkyne.
- Passing propyne through a red-hot iron tube at $873\,K$ produces a cyclic trimer: 1,3,5-trimethylbenzene (mesitylene).
- Conservation of carbon atoms provides a quick distinction between benzene from ethyne and mesitylene from propyne.

Step 1: Form the alkyne by dehydrohalogenation The starting compound is $CH_3-CH=CHBr$, a three-carbon vinyl bromide. The strong base removes the hydrogen on the adjacent double-bond carbon while bromide leaves. A second pi bond forms between the same two carbon atoms, giving propyne. This is elimination, not replacement of bromine by an amino group:

$$CH_3-CH=CHBr + NaNH_2 \longrightarrow CH_3-C\equiv CH + NaBr + NH_3$$

The carbon skeleton remains three atoms long. The reagent's nitrogen is not incorporated into the organic product in this step.

Step 2: Cyclotrimerise propyne in the hot tube Three propyne molecules undergo cyclic polymerisation on passing through the red-hot iron tube at the stated temperature. Their six alkyne carbon atoms form the aromatic ring, while the three methyl groups become substituents at alternate ring positions:

$$3\,CH_3-C\equiv CH \xrightarrow[873\ K]{\text{red hot iron}} C_6H_3(CH_3)_3$$

The product is 1,3,5-trimethylbenzene, commonly called mesitylene. The atom balance confirms the result: three C_3H_4 molecules give one C_9H_{12} molecule without losing any carbon or hydrogen atoms.

Step 3: Compare the four choices Option (A) is mesitylene and agrees with the elimination followed by trimerisation sequence. Option (B), benzene, is obtained by trimerising ethyne, not propyne; it would omit the three methyl groups. Option (C) is a vinyl amine and incorrectly treats sodamide as a substitution reagent here, as well as ignoring the high-temperature second step. Option (D), propylamine, would additionally require hydrogenation and nitrogen incorporation that the specified sequence does not provide.

Step 4: Conclusion The major product A is 1,3,5-trimethylbenzene (mesitylene), $C_6H_3(CH_3)_3$. Hence, the correct option is (A).

Quick Tip to Solve: Identify the alkyne first: ethyne trimerises to benzene, whereas propyne retains its three methyl groups and gives mesitylene.

72. Given below are two statements :

Statement I : o-Nitrophenol is steam volatile due to intramolecular hydrogen bonding.

Statement II : o-Nitrophenol has high melting due to hydrogen bonding.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is true but Statement II is false

(C) Statement I is false but Statement II is true

(D) Both Statement I and Statement II are false

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- Intramolecular hydrogen bonding joins a hydrogen-bond donor and acceptor within the same molecule. In o-nitrophenol the adjacent hydroxyl and nitro groups can form this interaction.
- Intermolecular hydrogen bonding associates different molecules and usually increases the energy needed to separate them. Steam volatility and melting behaviour must be distinguished.

Step 1: Evaluate Statement I The ortho arrangement places the phenolic hydrogen close to an oxygen atom of the nitro group. A hydrogen bond therefore forms within one molecule rather than producing an extensive association between different molecules. This reduces intermolecular association relative to p-nitrophenol. Consequently o-nitrophenol is sufficiently volatile to distil with steam, so Statement I is true.

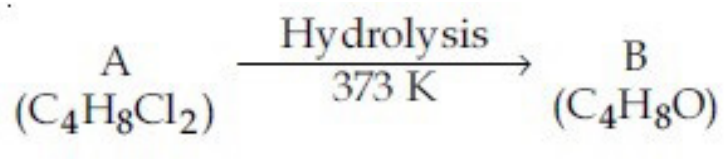
Step 2: Evaluate Statement II The intramolecular bond does not create a strongly linked network throughout the solid. The claim that o-nitrophenol has a high melting point because of hydrogen bonding confuses intramolecular bonding with the intermolecular bonding of p-nitrophenol. The para isomer has the substantially higher melting point, aided by its intermolecular association and more symmetrical crystal packing. Thus Statement II is false.

Step 3: Compare the four combinations Only the combination true for I and false for II survives. Option A incorrectly accepts Statement II, option C reverses both judgements, and option D incorrectly rejects the correct steam-volatility explanation. Hydrogen bonding is present, but its location determines the consequence; its mere presence does not establish the second statement.

Step 4: Conclusion Statement I is true, while Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: For nitrophenol isomers, ask whether the hydrogen bond is within one molecule or between molecules before comparing volatility and melting point.

73. B reacts with Hydroxyl amine but does not give Tollen's test. Identify A and B. [4 marks]



(A) 1, 1-Dichlorobutane and Butanal

(B) 2, 2-Dichlorobutane and Butan-2-one

(C) 1, 1-Dichlorobutane and 2-Butanone

(D) 2, 2-Dichlorobutane and Butanal

Correct Answer: (B) 2, 2-Dichlorobutane and Butan-2-one

Solution:

Concept:

- A geminal dihalide has both halogen atoms on the same carbon. Hydrolysis replaces them by hydroxyl groups; the resulting geminal diol loses water to form a carbonyl compound.
- Hydroxylamine reacts with aldehydes and ketones to form oximes. A negative Tollens test rules out an ordinary aliphatic aldehyde, leaving a ketone here.

Step 1: Identify the carbonyl compound B The formula of B is C_4H_8O. Its reaction with hydroxylamine indicates a carbonyl group, since the carbonyl oxygen is replaced during oxime formation. Butanal and butan-2-one both have this formula and both form oximes. Butanal, however, reduces Tollens reagent to metallic silver, whereas butan-2-one does not. The stated negative Tollens test therefore identifies B as butan-2-one.

Step 2: Recover the starting geminal dihalide To give a ketone with its carbonyl at carbon 2, the two chlorine atoms must initially occupy carbon 2. The conversion is

$$CH_3CCl_2CH_2CH_3 + 2\,H_2O \longrightarrow CH_3COCH_2CH_3 + 2\,HCl + H_2O.$$

Equivalently, replacing the two chlorines first gives $CH_3C(OH)_2CH_2CH_3$, which loses water. Thus A is 2,2-dichlorobutane. Carbon skeleton and carbonyl position are both preserved.

Step 3: Eliminate the other pairs Option A gives butanal, contradicting the negative Tollens test. Option C pairs a terminal geminal dichloride with an internal ketone, which is not the product of its hydrolysis. Option D likewise assigns butanal to an internal geminal dichloride and also fails the Tollens observation. Only option B satisfies the reaction pathway and both chemical tests together.

Step 4: Conclusion A is 2,2-dichlorobutane and B is butan-2-one. Hence, the correct option is (B).

Quick Tip to Solve: Locate the carbon bearing both halogens: after hydrolysis that same carbon becomes the carbonyl carbon.

74. Which of the following vitamin is helpful in delaying the blood clotting? [4 marks]

(A) Vitamin K

(B) Vitamin C

(C) Vitamin E

(D) Vitamin B

Correct Answer: (C) Vitamin E

Solution:

Concept:

- Vitamin K supports the formation of functional blood-clotting factors. Its role promotes normal coagulation rather than delaying it.
- Vitamin E, a fat-soluble antioxidant, is associated with inhibition of platelet aggregation and an increased bleeding tendency at sufficiently high intake. Among the listed vitamins it is the one associated with delayed clotting.

Step 1: Read the direction of the effect The question asks for a vitamin helpful in *delaying* blood clotting. This is the opposite of asking which vitamin is essential for normal clot formation. Vitamin K is essential for the activation of several coagulation proteins, including prothrombin-related pathways; deficiency of it produces a bleeding tendency. Thus its usual physiological action cannot be used to justify selecting it for a delay in clotting.

Step 2: Select the vitamin matching delayed clotting Vitamin E can reduce platelet aggregation and interfere with vitamin-K-dependent coagulation, particularly in excess. These effects explain its association with a longer bleeding tendency rather than accelerated clot formation. Therefore the choice matching the literal word "delaying" is vitamin E. This identifies a biochemical association in this question; it is not a recommendation to use a vitamin to alter coagulation.

Step 3: Check the remaining choices Vitamin C is primarily associated with collagen formation and maintenance of connective tissue; its deficiency causes scurvy and fragile blood vessels, which is distinct from the intended anticoagulant association. The B vitamins chiefly participate in metabolic coenzyme functions and do not supply the stated specific effect. Vitamin K instead supports coagulation. Consequently neither A, B nor D answers the wording as printed.

Step 4: Conclusion For the printed request for *delaying* clotting, vitamin E is the defensible answer. The supplied key selects vitamin K, but vitamin K promotes normal clotting and therefore answers the opposite question. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish "helps blood clotting" (vitamin K) from "delays blood clotting" (vitamin E); the direction of the effect decides this question.

75. The orbital having two radial as well as two angular nodes is : [4 marks]

(A) 3p

(B) 5d

(C) 4d

(D) 4f

Correct Answer: (B) 5d

Solution:

Concept:

- For an atomic orbital with principal quantum number n and angular momentum quantum number l, the number of angular nodes is l. The letters s, p, d and f correspond to $l = 0, 1, 2, 3$.
- The number of radial nodes is $n - l - 1$, and the total number of nodes is $n - 1$. Both given node counts must hold for the same orbital.

Step 1: Use the angular node count Two angular nodes require $l = 2$. Hence the orbital belongs to the d subshell. This immediately excludes 3p, which has one angular node, and 4f, which has three angular nodes. The two remaining possibilities, 5d and 4d, have the required angular symmetry, so their radial node counts must now be compared.

Step 2: Determine the principal quantum number Substitute the radial node count and $l = 2$ into the radial-node formula:

$$n - l - 1 = 2.$$

$$n - 2 - 1 = 2 \quad \Rightarrow \quad n = 5.$$

Thus the required orbital is 5d. Notice that the angular count determines the letter while the radial count then fixes the shell number; adding the two counts also gives four total nodes, so $n - 1 = 4$ independently confirms $n = 5$.

Step 3: Verify every proposed orbital For 3p, the radial count is $3 - 1 - 1 = 1$, and the angular count is 1, so it fails both conditions. For 5d the counts are $5 - 2 - 1 = 2$ radial and 2 angular, exactly as required. For 4d the radial count is only $4 - 2 - 1 = 1$, despite its two angular nodes. For 4f the radial count is zero and its angular count is three. Therefore the correct match is unique.

Step 4: Conclusion The orbital has $n = 5$ and $l = 2$, so it is $5d$. Hence, the correct option is (B).

Quick Tip to Solve: First set l equal to the angular node count; then use $n =$ radial nodes $+ l + 1$.

76. Statements about heavy water are given below :

A. Heavy water is used in exchange reactions for the study of reaction mechanisms.

B. Heavy water is prepared by exhaustive electrolysis of water.

(C) Heavy water has higher boiling point than ordinary water.

(D) Viscosity of H_2O is greater than D_2O.

Choose the most appropriate answer from the options given below : [4 marks]

(A) A, B, and C only

(B) A and C only

(C) A and D only

(D) A and B only

Correct Answer: (A) A, B, and C only

Solution:

Concept:

- Heavy water is D_2O, where D denotes deuterium. Isotope exchange allows deuterium to act as a tracer when studying reaction mechanisms.
- Ordinary water is preferentially decomposed during electrolysis, enriching the residue in heavy water. Heavy water has higher boiling point and viscosity than ordinary water under comparable conditions.

Step 1: Check the use and preparation statements Statement A is correct: hydrogen atoms in suitable exchangeable positions can be replaced by deuterium from heavy water. Following the label helps establish which bonds or hydrogen atoms are involved in a reaction mechanism. Statement B is also correct: repeated, exhaustive electrolysis of ordinary water concentrates the less readily electrolysed heavy-water component in the remaining liquid.

Step 2: Compare the physical properties Statement C is correct. At normal pressure ordinary water boils near $100\,^{\circ}C$, while heavy water boils near $101.4\,^{\circ}C$. Statement D reverses the actual viscosity comparison. At the same ordinary laboratory temperature, D_2O is more viscous than H_2O, so claiming that the viscosity of ordinary water is greater is false. The physical-property comparison must be made at the same temperature.

Step 3: Match the true set to the options The resulting truth pattern is A true, B true, C true and D false. Option A lists exactly A, B and C. Option B leaves out the valid preparation statement B. Option C includes the false viscosity statement D and omits other correct statements. Option D includes two correct statements but omits the also-correct boiling-point statement C. The question asks for the complete correct set, so those incomplete sets do not qualify.

Step 4: Conclusion Statements A, B and C are correct; statement D is false. Hence, the correct option is (A).

Quick Tip to Solve: For heavy water remember the paired comparison: its boiling point and viscosity are both higher than those of ordinary water at comparable conditions.

77. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are : [4 marks]

(A) X = SO_2, Y = Cr_2O_3

(B) X = SO_3, Y = $Cr_2(SO_4)_3$

(C) X = SO_3, Y = Cr_2O_3

(D) X = SO_2, Y = $Cr_2(SO_4)_3$

Correct Answer: (D) X = SO_2, Y = $Cr_2(SO_4)_3$

Solution:

Concept:

- Sulphites release sulphur dioxide on acidification. Sulphur dioxide is a reducing agent because sulphur in oxidation state +4 can be oxidised to +6.
- Acidified dichromate contains orange $Cr_2O_7^{2-}$, which is reduced from chromium(VI) to green chromium(III). In sulphuric acid the resulting salt is chromium(III) sulphate.

Step 1: Identify the gas from the colour change The orange-to-green change of acidified dichromate indicates reduction of chromium(VI) to chromium(III). Sulphur dioxide supplies the reducing action and is itself oxidised to sulphate. For example an acidified sulphite gives

$$SO_3^{2-} + 2\,H^+ \longrightarrow SO_2 + H_2O.$$

Hence X is SO_2, not SO_3. Sulphur in SO_3 is already at +6 and cannot provide the same oxidation to sulphate.

Step 2: Write the balanced redox change The net ionic reaction is

$$Cr_2O_7^{2-} + 3\,SO_2 + 2\,H^+ \longrightarrow 2\,Cr^{3+} + 3\,SO_4^{2-} + H_2O.$$

Each sulphur atom loses two electrons, while the two chromium atoms together gain six electrons, so three molecules of sulphur dioxide are required. In the sulphuric-acid medium the corresponding molecular equation is

$$K_2Cr_2O_7 + 3\,SO_2 + H_2SO_4 \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O.$$

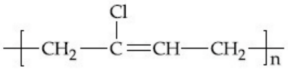
This identifies the green product Y as chromium(III) sulphate.

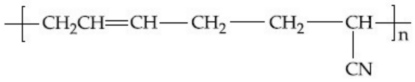
Step 3: Reject the other pairings Options B and C give sulphur trioxide as X and fail the reducing-gas test. Option A correctly names sulphur dioxide but incorrectly gives chromium(III) oxide as the product of this acidic aqueous reaction. Although chromium(III) compounds may be green, the sulphate is the relevant product in the stated sulphuric-acid medium. Only option D supplies both the correct gas and the correct compound.

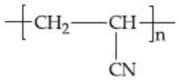
Step 4: Conclusion $X = SO_2$ and $Y = Cr_2(SO_4)_3$. Hence, the correct option is (D).

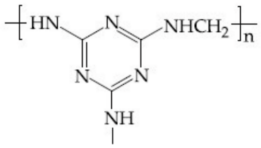
Quick Tip to Solve: Acidified dichromate turning orange to green signals a reducing gas; with sulphur dioxide, name the chromium(III) sulphate formed in the acidic medium.

78. The structure of Neoprene is : [4 marks]

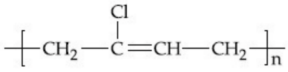
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- Neoprene is polychloroprene, an addition polymer of chloroprene, $CH_2=C(Cl)-CH=CH_2$ (2-chlorobuta-1,3-diene).
- In 1,4-addition of a conjugated diene, the terminal carbon atoms form the polymer backbone links and one double bond remains between the two inner carbon atoms.

Step 1: Identify the monomer and the repeat unit Chloroprene has four carbon atoms in its conjugated chain and chlorine bonded to carbon 2. Polymerisation joins carbon 1 of one molecule to carbon 4 of the next, leaving an internal double bond. Thus its repeat unit is $[-CH_2-C(Cl)=CH-CH_2-]_n$. The chlorine substituent is retained; it is neither removed nor replaced by a nitrile group during the addition process.

Step 2: Compare every displayed structure Option (A) shows precisely this four-carbon repeat unit with a chlorine substituent on an alkene carbon. Option (B) contains a nitrile group along with a butadiene-derived segment and represents a butadiene-acrylonitrile type copolymer rather than polychloroprene. Option (C), $[-CH_2-CH(CN)-]_n$, is polyacrylonitrile: its backbone has two carbons per repeat and its substituent is nitrile. Option (D) contains a nitrogen-rich triazine ring joined through amino/methylene functionality, characteristic of melamine-formaldehyde resin. It is a condensation resin and has neither the chloroprene chain nor a chlorine substituent.

Step 3: Conclusion Neoprene has the repeat unit $[-CH_2-C(Cl)=CH-CH_2-]_n$, shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: For neoprene, locate chlorine directly on the remaining backbone double bond; a nitrile group identifies a different polymer.

79. Match List - I with List - II.

	List - I (Ore)		List - II (Element Present)
(a)	Kernite	(i)	Tin
(b)	Cassiterite	(ii)	Boron
(c)	Calamine	(iii)	Fluorine
(d)	Cryolite	(iv)	Zinc

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)

(B) (a) → (iii), (b) → (i), (c) → (ii), (d) → (iv)

(C) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

(D) (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)

Correct Answer: (C) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

Solution:

Concept:

- Mineral formulae identify their constituent elements. Kernite is a hydrated sodium borate, cassiterite is tin dioxide, calamine is zinc carbonate and cryolite is sodium hexafluoroaluminate.
- In a matching question, each ore must be paired with the listed element it actually contains; the order in List II is independent of the order in List I.

Step 1: Write the compositions of the four minerals Kernite has composition $Na_2B_4O_7 \cdot 4\,H_2O$, so it contains boron and gives (a) → (ii). Cassiterite is SnO_2, the principal ore of tin, and gives (b) → (i). Calamine is $ZnCO_3$, so its listed element is zinc and (c) → (iv). Cryolite is Na_3AlF_6; although it also contains sodium and aluminium, the element offered in List II is fluorine, giving (d) → (iii).

Step 2: Check the complete code against the options The required sequence for (a), (b), (c), (d) is therefore (ii), (i), (iv), (iii). Only option (C) contains all four matches in this order. Option (A) incorrectly assigns tin to kernite and fluorine to cassiterite. Option (B) assigns fluorine to kernite and boron to calamine. Option (D) begins and ends correctly but interchanges the tin and zinc matches for cassiterite and calamine. A single correct pair cannot establish a whole matching code; each of its four entries must agree with the corresponding mineral formula.

Step 3: Conclusion Kernite matches boron, cassiterite matches tin, calamine matches zinc and cryolite matches fluorine. Hence, the correct option is (C).

Quick Tip to Solve: Remember the formula anchors: cassiterite is tin dioxide and calamine is zinc carbonate; these immediately separate codes (C) and (D).

80. The presence of ozone in troposphere : [4 marks]

(A) protects us from greenhouse effect

(B) protects us from the UV radiation

(C) protects us from the X-ray radiation

(D) generates photochemical smog

Correct Answer: (D) generates photochemical smog

Solution:

Concept:

- Ozone has different environmental roles in different atmospheric layers. Stratospheric ozone absorbs harmful ultraviolet radiation, whereas ground-level, tropospheric ozone is an oxidising pollutant.
- Photochemical smog forms through sunlight-driven reactions involving nitrogen oxides and volatile organic compounds; ozone is an important secondary pollutant in this mixture.

Step 1: Identify how ozone forms near the ground In polluted air, sunlight photodissociates nitrogen dioxide. The oxygen atom produced then combines with molecular oxygen to form ozone:

$$NO_2 \xrightarrow{\text{sunlight}} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

In clean air, reaction of ozone with nitric oxide can remove it. Hydrocarbon-derived radicals in polluted air convert nitric oxide to nitrogen dioxide without consuming ozone, allowing ozone to accumulate. This oxidising mixture, together with other secondary pollutants, constitutes photochemical smog.

Step 2: Distinguish the proposed effects Option (D) identifies the harmful tropospheric role correctly. Option (B) confuses tropospheric ozone with the protective ozone layer in the stratosphere; the location named in the question is decisive. Option (A) is false because ozone is itself a greenhouse gas and does not protect against the greenhouse effect. Option (C) is also false: protection from X-rays is not the role of tropospheric ozone and does not describe the atmospheric process under consideration. Thus the beneficial reputation of the ozone layer cannot be transferred to ozone present in the air we breathe.

Step 3: Conclusion Tropospheric ozone is a major component of photochemical smog and contributes to its generation. Hence, the correct option is (D).

Quick Tip to Solve: Always identify the layer: stratospheric ozone shields against UV, while tropospheric ozone is a pollutant associated with photochemical smog.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. For a chemical reaction A + B $\rightleftharpoons$ C + D
($\Delta_r H^\Theta$ = 80 kJ mol^{-1}) the entropy change $\Delta_r S^\Theta$ depends on the temperature T (in K) as $\Delta_r S^\Theta$ = 2T (J K^{-1}mol^{-1}).
Minimum temperature at which it will become spontaneous is _____ K. (Integer) [4 marks]

Solution:

Concept:

- At constant temperature and pressure, a reaction is thermodynamically spontaneous in the forward direction when $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ < 0$.
- The threshold temperature is found from $\Delta_r G^\circ = 0$. Energy units must be consistent before substituting an entropy quoted in joules.

Step 1: Express Gibbs energy in consistent units The given enthalpy change is positive:

$$\Delta_r H^\circ = 80\,\text{kJ mol}^{-1} = 80000\,\text{J mol}^{-1}.$$

The prescribed entropy change has numerical value $2T$ in $\text{J K}^{-1}\,\text{mol}^{-1}$, with T entered in kelvin. Hence

$$\Delta_r G^\circ = (80000 - 2T^2)\,\text{J mol}^{-1}.$$

The temperature dependence of the entropy must be substituted before solving: treating the entropy as a fixed value of 2 would miss one factor of temperature.

Step 2: Find the onset and determine the spontaneous side At the limiting temperature, the enthalpy and entropy terms balance:

$$80000 - 2T^2 = 0 \quad \Rightarrow \quad T^2 = 40000.$$

Absolute temperature is positive, so

$$T = 200\,\text{K}.$$

For temperatures below this value, $2T^2 < 80000$ and the Gibbs energy is positive. For temperatures above it, $2T^2 > 80000$ and the Gibbs energy becomes negative. Exactly at the boundary the reaction is at the thermodynamic threshold, rather than having a strictly negative Gibbs energy. The integer blank asks for this onset temperature, not a finite temperature increment beyond it.

Step 3: Conclusion The onset temperature is $\boxed{200\,\text{K}}$, so the required integer is $\boxed{200}$. Strict forward spontaneity occurs for $T > 200\,\text{K}$; at $200\,\text{K}$, $\Delta_r G^\circ = 0$.

Quick Tip to Solve: When entropy is proportional to temperature, the term $T\Delta S$ is quadratic in temperature; solve the zero-Gibbs-energy boundary first.

82. An exothermic reaction X $\rightarrow$ Y has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is -20 kJ, then the activation energy for the reverse reaction in kJ is _____. (Integer answer) [4 marks]

Solution:

Concept:

- An activation energy is the energy difference between the transition state and the reactants of that direction. Forward and reverse reactions pass through the same transition state.
- For a common energy profile, $E_{a,\text{forward}} - E_{a,\text{reverse}} = \Delta E$, where $\Delta E = E_{\text{products}} - E_{\text{reactants}}$.

Step 1: Place the reactants, products and transition state Take the energy of X as zero on the stated reaction basis. Since the forward reaction is exothermic with energy change $-20\,\text{kJ}$, Y lies $20\,\text{kJ}$ below X. The transition state lies $30\,\text{kJ}$ above X because the forward activation energy is $30\,\text{kJ}\,\text{mol}^{-1}$. On a molar energy profile these levels are $E_X = 0$, $E_Y = -20$ and $E_{\text{TS}} = 30$, all in $\text{kJ}\,\text{mol}^{-1}$. The negative energy change lowers the starting level for the reverse reaction.

Step 2: Compute the reverse activation energy The reverse reaction starts at Y and must climb to the same transition state. Therefore

$$E_{a,\text{reverse}} = E_{\text{TS}} - E_Y = 30 - (-20) = 50\,\text{kJ}\,\text{mol}^{-1}.$$

Equivalently, rearranging the barrier relation gives

$$E_{a,\text{reverse}} = E_{a,\text{forward}} - \Delta E = 30 - (-20).$$

Adding the magnitude of the heat released is essential here. Subtracting 20 from 30 would incorrectly place the products above the reactants and describe the opposite sign of reaction energy. The reverse barrier exceeds the forward barrier because the forward process releases energy.

Step 3: Conclusion The reverse activation energy is $\boxed{50\,\text{kJ}\,\text{mol}^{-1}}$ ($50\,\text{kJ}$ on the stated reaction basis), so the required integer is $\boxed{50}$.

Quick Tip to Solve: For an exothermic forward reaction, add the magnitude of the energy released to the forward barrier to obtain the reverse barrier.

83. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is _____. [4 marks]

Solution:

Concept:

- Dichromate and chromate ions are related by an acid-base equilibrium. Adding hydroxide favours yellow chromate, CrO_4^{2-}, rather than orange dichromate, $Cr_2O_7^{2-}$.
- The sum of the oxidation numbers of all atoms equals the ionic charge. Oxygen has oxidation number -2 in both of these oxoanions.

Step 1: Identify the product in alkaline medium Base converts dichromate into chromate according to

$$Cr_2O_7^{2-} + 2\,OH^- \longrightarrow 2\,CrO_4^{2-} + H_2O.$$

This equation conserves both atoms and charge: each side contains two chromium atoms, nine oxygen atoms and two hydrogen atoms, while the total charge is -4. Hydroxide removes the acidic conditions that favour dichromate, shifting the equilibrium towards separated chromate units. No reducing agent is supplied, so the change should not be interpreted as reduction of chromium to a lower oxidation state.

Step 2: Calculate chromium oxidation number in chromate Let the oxidation number of chromium in CrO_4^{2-} be x. The four oxygen atoms contribute $4(-2) = -8$. Equating the sum to the charge of the ion gives

$$x + 4(-2) = -2 \quad \Rightarrow \quad x = +6.$$

As a check, in the original dichromate ion, $2x + 7(-2) = -2$, which also gives $x = +6$. Thus the alkaline conversion changes the oxoanion and its colour but not the chromium oxidation number. The charge -2 belongs to the whole ion and is not the oxidation number of the metal.

Step 3: Conclusion The product is chromate, CrO_4^{2-}, and chromium has oxidation number $\boxed{+6}$. The numerical entry is $\boxed{6}$, a dimensionless oxidation number.

Quick Tip to Solve: First identify chromate as the alkaline product, then use total ionic charge; dichromate-to-chromate conversion is not a redox reaction.

84. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is _____. [4 marks]

Solution:

Concept:

- A terminal carbonyl ligand is bonded to one metal atom; a bridging carbonyl ligand connects two metal atoms. A metal-metal bond does not itself count as a bridging ligand.
- In $Mn_2(CO)_{10}$, the two manganese centres are joined by an Mn–Mn bond, and each manganese bears five terminal carbonyl ligands.

Step 1: Describe the arrangement of the ten ligands Dimanganese decacarbonyl consists of two $Mn(CO)_5$ fragments connected by a single manganese-manganese bond. The ten carbonyl groups are distributed as five terminal ligands on each manganese. Each CO binds through its carbon atom to only one manganese centre, rather than linking both centres. Therefore none of the CO groups needs to be placed across the Mn–Mn bond. The presence of two metal atoms in a molecular formula alone is insufficient to infer bridging ligands; the actual bonding arrangement is required.

Step 2: Check the electron count at each metal CO is a neutral two-electron donor, so five terminal CO ligands supply $5 \times 2 = 10$ electrons to each manganese. Manganese is a group 7 metal, contributing seven valence electrons in the neutral counting method. The Mn–Mn bond supplies one electron to the count at each metal. Thus

$$7 + 5(2) + 1 = 18$$

electrons are counted at each centre. This is consistent with two $Mn(CO)_5$ fragments linked directly by a metal-metal bond. The bonding description has ten terminal CO ligands and zero bridging CO ligands; the direct metal-metal linkage is not an eleventh ligand.

Step 3: Conclusion All ten CO ligands in $Mn_2(CO)_{10}$ are terminal. The number of bridging carbonyl ligands is $\boxed{0}$, a dimensionless count.

Quick Tip to Solve: Do not infer CO bridges from the metal count: $Mn_2(CO)_{10}$ has a direct Mn–Mn bond and five terminal CO ligands on each metal.

85. Consider the following reaction

$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O, E^o = 1.51V$**.**

The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is _____. (Integer answer) [4 marks]

Solution:

Concept:

- One faraday is the charge carried by one mole of electrons: $F = N_A e \simeq 96485\,\mathrm{C\,mol^{-1}}$. The electron coefficient in a balanced half-reaction determines the charge required per mole of reactant.
- In acidic medium manganese changes from oxidation state $+7$ in $\mathrm{MnO_4^-}$ to $+2$ in $\mathrm{Mn^{2+}}$, so each manganese atom gains five electrons.

Step 1: Read the electron requirement The given balanced half-reaction is $\mathrm{MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O}$. Thus one mole of permanganate consumes five moles of electrons. The coefficient of hydrogen ions balances hydrogen and charge; it does not represent the number of electrons transferred. The oxidation-state decrease, $7 - 2 = 5$, independently confirms the electron coefficient.

Step 2: Scale the reaction to five moles Apply the stoichiometric ratio to the specified amount of permanganate, rather than treating five moles of the ion as five moles of electrons.

$$n(e^-) = 5\,\mathrm{mol\ MnO4^-} \times \frac{5\,\mathrm{mol\ e^-}}{1\,\mathrm{mol\ MnO4^-}} = 25\,\mathrm{mol\ e^-}.$$

Therefore $Q = n(e^-)F = 25F$, or approximately $2.41 \times 10^6\,\mathrm{C}$. The standard reduction potential describes the thermodynamic tendency of the half-reaction, not the charge stoichiometry, so the stated $1.51\,\mathrm{V}$ is not needed in this calculation.

Step 3: Conclusion The required quantity of electricity is $\boxed{25\ \text{faradays}}$. The integer answer is $\boxed{25}$.

Quick Tip to Solve: For charge in faradays, multiply reactant moles by electrons accepted per formula unit; do not multiply by the electrode potential.

86. 224 mL of $SO_{2(g)}$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36g of water. The lowering of vapour pressure of solution (assuming the solution in dilute) ($P^o_{(H_2O)}$ − 24 mm of Hg) is x × 10^{-2} mm of Hg, the value of x is _____. (Integer answer) [4 marks]

Solution:

Concept:

- For a dilute solution of non-volatile solutes, Raoult's law gives $\Delta p \simeq p^\circ n_{\text{particles}}/n_{\text{water}}$. Count dissolved ions, not merely formula units.
- Absorption of SO_2 by alkali first gives $NaHSO_3$; additional alkali converts some bisulfite into Na_2SO_3. The specified gas temperature must be used in $n = PV/(RT)$.

Step 1: Calculate the reactant amounts The alkali supplies $n(\text{NaOH}) = 0.100 \times 0.1 = 0.0100\,\text{mol}$. At $298\,\text{K}$ and $1\,\text{atm}$, take $R = 0.08206\,\text{L atm mol}^{-1}\,\text{K}^{-1}$:

$$s = n(SO_2) = \frac{1 \times 0.224}{0.08206 \times 298} = 0.00916\,\text{mol}.$$

There is more than one but less than two moles of hydroxide per mole of sulfur dioxide, so the resulting salt contains both bisulfite and sulfite.

Step 2: Count ions in the resulting salts Let a and b be moles of $NaHSO_3$ and Na_2SO_3, respectively. Conservation of sulfur and sodium gives $a + b = s$ and $a + 2b = 0.0100$. Hence $a = 0.00832$ and $b = 0.00084\,\text{mol}$. With complete ionic dissociation, the total solute-particle amount is

$$n_{\text{particles}} = 2a + 3b = (a + b) + (a + 2b) = s + 0.0100 = 0.01916\,\text{mol}.$$

Step 3: Apply the dilute-solution approximation The solvent amount is $n_{\text{water}} = 36/18 = 2.00\,\text{mol}$. Using the stated pure-water vapour pressure of $24\,\text{mm Hg}$,

$$\Delta p \simeq 24\frac{0.01916}{2.00} = 0.2299\,\text{mm Hg} = 22.99 \times 10^{-2}\,\text{mm Hg}.$$

The integer value is therefore obtained by rounding 22.99.

Step 4: Conclusion $\boxed{x = 23}$, corresponding to $\Delta p \simeq 0.23\,\text{mm Hg}$. This lies within the accepted integer range 18 to 24.

Quick Tip to Solve: At 298 K, use the ideal-gas equation instead of the STP molar volume; count sodium ions plus sulfur-containing anions to obtain total solute particles.

87. A homogeneous ideal gaseous reaction $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$ is carried out in a 25 litre flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm. The value of K_p is x $\times$ 10^{-2}. The value of x is _____. (Integer answer)

[R = 0.08206 dm^3atm $K^{-1}mol^{-1}$] [4 marks]

Solution:

Concept:

- For $\mathrm{AB_2(g)} \rightleftharpoons \mathrm{A\,(g)} + 2\,\mathrm{B\,(g)}$, dissociation of a fraction α of one initial mole gives equilibrium amounts $1-\alpha, \alpha, 2\alpha$, so total moles are $1+2\alpha$.
- An ideal-gas mixture obeys $PV = nRT$. Each partial pressure is $p_i = n_i RT/V$, and $K_p = p_A p_B^2 / p_{AB_2}$ in the pressure convention used by the question.

Step 1: Find the equilibrium total moles Convert the temperature before using the gas law: $27^\circ\mathrm{C} = 300\,\mathrm{K}$. The volume is $25\,\mathrm{dm}^3$, consistent with the stated gas constant.

$$n_{\text{total}} = \frac{PV}{RT} = \frac{1.9 \times 25}{0.08206 \times 300} = 1.92948\,\text{mol}.$$

Since each mole that dissociates increases the total amount by two moles,

$$1 + 2\alpha = 1.92948, \qquad \alpha = 0.464741.$$

This is between zero and one, as required for partial dissociation.

Step 2: Calculate the partial pressures The pressure contribution per mole is $RT/V = 0.98472\,\mathrm{atm\,mol^{-1}}$. Therefore

$$\begin{aligned} p_{AB_2} &= (1-\alpha)RT/V = 0.52708\,\text{atm}, \\ p_A &= \alpha RT/V = 0.45764\,\text{atm}, \\ p_B &= 2\alpha RT/V = 0.91528\,\text{atm}. \end{aligned}$$

Their sum is $1.90000\,\mathrm{atm}$, which checks the composition against the given total pressure.

Step 3: Form the equilibrium constant Substitute each equilibrium partial pressure with its proper stoichiometric power:

$$K_p = \frac{0.45764(0.91528)^2}{0.52708} = 0.72737\,\text{atm}^2.$$

Thus $K_p = 72.737 \times 10^{-2}\,\mathrm{atm}^2$. Retain intermediate precision before rounding the requested numerical coefficient.

Step 4: Conclusion The requested integer coefficient is $\boxed{x = 73}$, within the accepted range 72 to 75. In the usual pressure-based convention, $K_p \simeq 0.727\,\mathrm{atm}^2$.

Quick Tip to Solve: For dissociation into three product molecules, write total moles as $1+2\alpha$; the increase is two, not three, per molecule dissociated.

88. A certain gas obeys P(V_m − b) = RT. The value of $\left(\frac{\partial Z}{\partial P}\right)_T$ is $\frac{xb}{RT}$. The value of x is _____. (Integer answer) (Z : compressibility factor) [4 marks]

Solution:

Concept:

- The compressibility factor for a molar volume V_m is $Z = PV_m/(RT)$. An ideal gas has $Z = 1$, while pressure dependence of Z measures departures from ideal behaviour.
- The subscript T on a partial derivative means that temperature is held fixed. The gas parameter b and the universal gas constant R are also constant in the stated equation of state.

Step 1: Express the molar volume explicitly Begin with the given equation, rather than substituting a different real-gas model:

$$P(V_m - b) = RT.$$

Divide by P and rearrange to obtain

$$V_m = b + \frac{RT}{P}.$$

The first term is a fixed excluded-volume contribution, whereas the second varies inversely with pressure. This form also shows that V_m approaches the ideal-gas expression when the contribution of b is negligible.

Step 2: Write and differentiate the compressibility factor Substitute the entire expression for molar volume into the definition:

$$Z = \frac{P}{RT}\left(b + \frac{RT}{P}\right) = 1 + \frac{Pb}{RT}.$$

At fixed temperature the denominator RT is constant. Consequently,

$$\left(\frac{\partial Z}{\partial P}\right)_T = 0 + \frac{b}{RT}.$$

The derivative is not zero just because the ideal-gas part of Z equals one; the extra pressure-dependent term must also be differentiated.

Step 3: Compare the requested coefficient The question expresses this derivative as $xb/(RT)$. Direct comparison with $b/(RT)$ gives $x = 1$. A dimensional check supports the calculation: $b/(RT)$ has units of inverse pressure, exactly the units of the derivative of dimensionless Z with respect to pressure.

Step 4: Conclusion $\boxed{x = 1}$ (dimensionless), with $(\partial Z/\partial P)_T = b/(RT)$.

Quick Tip to Solve: First express Z as an explicit function of pressure; differentiating the original equation without accounting for changing molar volume can hide the simple linear term.

89. The number of significant figures in 50000.020×10^{-3} is _____. [4 marks]

Solution:

Concept:

- Every nonzero digit is significant. Zeros between significant digits are also significant, and trailing zeros to the right of a decimal point record measurement precision.
- An exact power-of-ten multiplier changes the position of the decimal point, but does not change the number of significant figures in a measured coefficient.

Step 1: Inspect the measured coefficient The coefficient is 50000.020. The decimal point is explicitly present, and the last zero lies to its right. Therefore the precision extends to the thousandths place in this coefficient. The zeros after the initial 5 are not leading zeros; they lie between the nonzero 5 and the nonzero 2. They must be retained when counting significant digits.

Step 2: Count the significant digits Write the digits in their printed order:

$$5,\ 0,\ 0,\ 0,\ 0,\ 0,\ 2,\ 0.$$

There are eight digits altogether. The five zeros between 5 and 2 are significant internal zeros, the digits 5 and 2 are significant nonzero digits, and the final zero is a significant trailing decimal zero. Thus the count is $1 + 5 + 1 + 1 = 8$.

Step 3: Check that the scale factor preserves precision Multiplying by 10^{-3} gives 50.000020, with exactly the same eight significant figures. In normalized scientific notation the value is

$$5.0000020 \times 10^{1}.$$

The coefficient still has eight significant digits. The exponent is an exact scaling factor, so its minus sign and digit 3 in the original expression are not included in the count of significant figures. Nor should the zero after the last 2 be dropped: doing so would imply a different reported precision.

Step 4: Conclusion The stated quantity has $\boxed{8}$ significant figures. The requested integer is $\boxed{8}$.

Quick Tip to Solve: Ignore the exact power of ten, then count from the first nonzero digit through the last explicitly reported decimal digit, including all intervening zeros.

90. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is _____.
[R = 0.0821 L atm $K^{-1}mol^{-1}$] [4 marks]

Solution:

Concept:

- The oxygen molecule is O_2, with molar mass $32\,g\,mol^{-1}$. Convert the adsorbed mass to moles before applying the ideal-gas law.
- The volume represented by n moles at a specified pressure and temperature is $V = nRT/P$. Volume per gram of adsorbent is the total gas volume divided by the mass of platinum.

Step 1: Find the amount of adsorbed oxygen The given $3.12\,\mathrm{g}$ refers to oxygen gas, so use the molecular mass of O_2, not the atomic mass of oxygen:

$$n(O_2) = \frac{3.12}{32} = 0.0975\,\mathrm{mol}.$$

This is the amount adsorbed on the complete $1.2\,\mathrm{g}$ platinum sample. It is not yet an amount per gram of the metal.

Step 2: Calculate the equivalent gas volume Use the given $R = 0.0821\,\mathrm{L\,atm\,K^{-1}\,mol^{-1}}$, $T = 300\,\mathrm{K}$ and $P = 1\,\mathrm{atm}$:

$$V = \frac{nRT}{P} = \frac{0.0975 \times 0.0821 \times 300}{1} = 2.401425\,\mathrm{L}.$$

The pressure units cancel and the resulting volume is in litres. The problem asks for a gas volume at these conditions, so using the STP molar volume would use the wrong temperature.

Step 3: Normalize by the adsorbent mass The adsorbent is platinum; oxygen is the adsorbate. Divide the total equivalent oxygen volume by the specified mass of platinum:

$$\frac{V}{m_{\mathrm{Pt}}} = \frac{2.401425}{1.2} = 2.0011875\,\mathrm{L\,g^{-1}}.$$

This is approximately $2.0\,\mathrm{L}$ for each gram of adsorbent. Dividing instead by the oxygen mass would calculate volume per gram of gas, which is a different quantity.

Step 4: Conclusion The adsorbed oxygen volume is $\boxed{2.0\,\mathrm{L\,g^{-1}}}$ of platinum at the stated conditions. The numerical answer is $\boxed{2}$.

Quick Tip to Solve: Keep adsorbate and adsorbent distinct: convert the oxygen mass to gas volume, then divide by the platinum mass to obtain litres per gram of adsorbent.