JEE Main 2021 Paper 1 (B.E./B.Tech) Feb 26 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 1 (February), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 26 February 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let f(x) be a differentiable function at x = a with f'(a) = 2 and f(a) = 4.

Then $\lim_{x\to a} \frac{xf(a)-af(x)}{x-a}$ equals : [4 marks]

(A) 4 − 2a

(B) 2a + 4

(C) a + 4

(D) 2a − 4

Correct Answer: (A) 4 − 2a

Solution:

Concept:

- Differentiability at a means $\lim_{x\to a}(f(x)-f(a))/(x-a) = f\prime(a)$. This also guarantees continuity at that point.
- Rewrite a mixed numerator by adding and subtracting a suitable term so that one part contains $x-a$ and the other contains the difference quotient of f.

Step 1: Rearrange the numerator exactly Add and subtract $af(a)$ inside the numerator:

$$xf(a) - af(x) = (x-a)f(a) - a\big(f(x)-f(a)\big).$$

For every $x \neq a$, division by $x-a$ therefore yields

$$\frac{xf(a)-af(x)}{x-a} = f(a) - a\frac{f(x)-f(a)}{x-a}.$$

The minus sign belongs to the entire function increment; keeping it is essential for the correct answer.

Step 2: Apply differentiability and substitute Taking the limit now uses only the definition of the derivative at the specified point:

$$L = f(a) - af\prime(a).$$

The given values are $f(a) = 4$ and $f\prime(a) = 2$, so

$$L = 4 - 2a.$$

No information about the derivative away from a is needed. In particular, applying a rule requiring differentiability on a whole interval is unnecessary; the stated pointwise hypothesis suffices.

Step 3: Check the sign and eliminate the alternatives An equivalent check is the local expansion $f(x) = 4 + 2(x-a) + o(x-a)$. Inserting this into the numerator leaves $(4-2a)(x-a) + o(x-a)$, which gives the same limit. The expression $2a+4$ reverses the sign of the derivative contribution, $a+4$ uses the wrong coefficient, and $2a-4$ reverses the overall result. They are not identities in the arbitrary real parameter a, although some options may coincide at isolated values.

Step 4: Conclusion The limit equals $4-2a$. Hence, the correct option is (A).

Quick Tip to Solve: For $xf(a) - af(x)$, first write $(x-a)f(a) - a(f(x)-f(a))$ to expose the derivative definition.

2. Let A(1, 4) and B(1, −5) be two points. Let P be a point on the circle $(x-1)^2 + (y-1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points, P, A and B lie on : [4 marks]

(A) a straight line

(B) an ellipse

(C) a parabola

(D) a hyperbola

Correct Answer: (A) a straight line

Solution:

Concept:

- For a point $P = (x, y)$, use squared Euclidean distances, avoiding unnecessary square roots.
- The circle constraint $(x-1)^2 + (y-1)^2 = 1$ reduces a quadratic expression to a linear function of y. Its extreme ordinate determines the maximizing point.

Step 1: Express the quantity to be maximized With $A = (1, 4)$ and $B = (1, -5)$, the sum is

$$S = (x-1)^2 + (y-4)^2 + (x-1)^2 + (y+5)^2.$$

Expanding only the ordinates gives

$$S = 2(x-1)^2 + 2y^2 + 2y + 41.$$

These are squared distances, so maximizing their sum is the task; a constant-sum-of-distances ellipse property does not apply.

Step 2: Use the circle equation and maximize From the circle,

$$(x-1)^2 + y^2 - 2y = 0, \qquad (x-1)^2 + y^2 = 2y.$$

Substitution in S gives

$$S = 6y + 41.$$

The circle has centre $(1, 1)$ and radius 1, hence $0 \le y \le 2$. Since the coefficient of y is positive, the maximum is attained at $y = 2$. The circle equation then forces $(x-1)^2 = 0$, so the maximizing point is $P = (1, 2)$, with $S_{\max} = 53$.

Step 3: Identify the geometric relation The three points are $P = (1, 2)$, $A = (1, 4)$ and $B = (1, -5)$. Each has the same abscissa and therefore belongs to the vertical straight line $x = 1$. Directly, $PA^2 = 4$ and $PB^2 = 49$, confirming the maximum value 53. The appropriate relation among the three specified points is collinearity; none of the nondegenerate conic descriptions in the other options expresses this common straight-line relation.

Step 4: Conclusion The maximizing point is $P = (1, 2)$, and P, A, B lie on the straight line $x = 1$. Hence, the correct option is (A).

Quick Tip to Solve: For a sum of squared distances restricted to a circle, expand and use the circle equation before differentiating.

3. Consider the following system of equations :

x + 2y − 3z = a

2x + 6y − 11z = b

x − 2y + 7z = c,

where a, b and c are real constants. Then the system of equations : [4 marks]

(A) has no solution for all a, b and c

(B) has a unique solution when 5a = 2b + c

(C) has infinite number of solutions when 5a = 2b + c

(D) has a unique solution for all a, b and c

Correct Answer: (C) has infinite number of solutions when 5a = 2b + c

Solution:

Concept:

- A system has infinitely many solutions when its coefficient rank and augmented rank agree and are smaller than the number of unknowns.
- Row elimination reveals both the dependence of the coefficient equations and the corresponding necessary relation among their right-hand sides.

Step 1: Eliminate the first unknown Subtract twice the first equation from the second, and subtract the first equation from the third. The resulting equations are

$$2y - 5z = b - 2a, \qquad -4y + 10z = c - a.$$

The left side of the latter is negative twice the left side of the former. Hence consistency requires

$$c - a = -2(b - 2a), \qquad 5a = 2b + c.$$

If this condition fails, the same linear expression would have two different values, so the system has no solution.

Step 2: Count the independent equations The first coefficient row $(1, 2, -3)$ and the reduced row $(0, 2, -5)$ are independent. There are exactly two independent equations for three unknowns when the consistency condition holds. Thus the coefficient rank is 2, and the augmented rank is also 2. A free variable remains, which rules out a unique solution even in the consistent case.

Step 3: Exhibit the family of solutions Set $z = t$, where t is any real number. From the reduced second equation and then the first equation,

$$y = \frac{b - 2a + 5t}{2}, \qquad x = a - 2y + 3t = 3a - b - 2t.$$

These values satisfy the third equation exactly when $c = 5a - 2b$. Every real t therefore yields a solution under the stated condition. Option (A) fails because consistent choices exist, and options (B) and (D) incorrectly claim uniqueness despite the free parameter.

Step 4: Conclusion The system has infinitely many solutions when $5a = 2b + c$, and no solution otherwise. Hence, the correct option is (C).

Quick Tip to Solve: When an eliminated row is a multiple of another, impose the same multiple on the right-hand side before deciding consistency.

4. A natural number has prime factorization given by $n = 2^x3^y5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisions of n, including 1, is : [4 marks]

(A) 11

(B) 6

(C) 12

(D) 6x

Correct Answer: (C) 12

Solution:

Concept:

- If $n = p_1^{a_1} \cdots p_k^{a_k}$, a positive divisor is obtained by choosing each prime exponent independently between zero and its corresponding exponent.
- An odd divisor contains no factor of 2. Thus only the choices of exponents of 3 and 5 contribute to the requested count.

Step 1: Determine the two odd-prime exponents The reciprocal condition can be combined with the given sum because y and z are nonzero:

$$\frac{1}{y} + \frac{1}{z} = \frac{y+z}{yz} = \frac{5}{6}.$$

Since $y + z = 5$, this gives $yz = 6$. Therefore y, z are roots of

$$u^2 - 5u + 6 = 0 = (u-2)(u-3).$$

The additional condition $y > z$ fixes $y = 3$ and $z = 2$, rather than the reverse assignment.

Step 2: Count the allowed exponent choices An arbitrary positive divisor has the form $2^i3^j5^k$, where $0 \le i \le x$, $0 \le j \le 3$, and $0 \le k \le 2$. It is odd precisely when $i = 0$. Thus there are four possible values $j = 0, 1, 2, 3$ and three possible values $k = 0, 1, 2$. By independent multiplication,

$$N_{\text{odd}} = (3+1)(2+1) = 12.$$

The choice $j = k = 0$ yields the divisor 1, which the question explicitly includes.

Step 3: Check the counting conventions Unique prime factorization ensures that distinct exponent pairs give distinct divisors and that every odd divisor appears once. Subtracting one would give 11, but would wrongly exclude 1. Multiplying yz gives 6, which ignores the zero-exponent choices. A result involving x, such as $6x$, cannot be correct because the exponent of 2 is forced to zero in every odd divisor.

Step 4: Conclusion There are 12 odd divisors of n, including 1. Hence, the correct option is (C).

Quick Tip to Solve: For odd divisors, discard the factor of 2 entirely and multiply one more than each remaining prime exponent.

5. If 0 < a, b < 1, and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of

$(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots\ldots$ **is :** [4 marks]

(A) $\log_e 2$

(B) e

(C) $\log_e\left(\frac{e}{2}\right)$

(D) $e^2 = 1$

Correct Answer: (A) $\log_e 2$

Solution:

Concept:

- For $|u| < 1$, the convergent logarithmic expansion is $\log(1+u) = u - u^2/2 + u^3/3 - u^4/4 + \cdots$.
- The tangent addition formula converts the inverse-tangent condition into an algebraic relation between $a+b$ and ab. Positivity keeps the logarithm arguments valid.

Step 1: Sum the two logarithmic expansions Both a and b lie strictly between zero and one, so the logarithmic expansions converge. Grouping the two convergent series term by term gives

$$S = \log(1+a) + \log(1+b) = \log\big((1+a)(1+b)\big).$$

This recognizes the alternating powers correctly: replacing the series with a geometric sum or with an exponential would give a different expression.

Step 2: Use the angle condition Put $\alpha = \tan^{-1} a$ and $\beta = \tan^{-1} b$. The given sum is $\alpha + \beta = \pi/4$, so

$$1 = \tan(\alpha+\beta) = \frac{a+b}{1-ab}.$$

Here $ab < 1$, making the denominator positive and nonzero. Hence

$$a + b = 1 - ab, \qquad a + b + ab = 1.$$

Consequently

$$(1+a)(1+b) = 1 + a + b + ab = 2.$$

This step uses the sum condition exactly, without needing to find the individual numbers.

Step 3: Evaluate and compare the choices Substituting the product into the logarithm gives $S = \log_e 2$. As a numerical sanity check, the terms sum to a positive value below one, approximately 0.693. The value e is larger than two and cannot fit this logarithmic result. Also $\log(e/2) = 1 - \log 2$ is a different value; the last displayed option does not equal the computed series. The convergence restrictions justify the expansions and prevent any ambiguity from analytic continuation.

Step 4: Conclusion The required series equals $\log_e 2$. Hence, the correct option is (A).

Quick Tip to Solve: Alternating powers divided by their exponents suggest $\log(1+u)$; combine the logarithms before using the trigonometric relation.

6. Let $f(x) = \int_0^x e^t f(t)dt + e^x$ be a differentiable function for all x∈R. Then f(x) equals : [4 marks]

(A) $e^{(e^x-1)}$

(B) $2e^{e^x} - 1$

(C) $2e^{e^x-1} - 1$

(D) $e^{e^x} - 1$

Correct Answer: (C) $2e^{e^x-1} - 1$

Solution:

Concept:

- The fundamental theorem of calculus differentiates an integral with variable upper limit: $\frac{d}{dx}\int_0^x g(t)\,dt = g(x)$ for continuous g.
- A separable equation for $f(x)+1$ follows after differentiating. The original equation also supplies the initial value, which determines the integration constant.

Step 1: Find the differential equation and initial value The integral is zero at its coincident limits, so $f(0) = e^0 = 1$. Differentiating both sides of the given equation gives

$$f\prime(x) = e^x f(x) + e^x = e^x\big(f(x)+1\big).$$

The additional term e^x must be differentiated too; omitting it would incorrectly give a homogeneous equation.

Step 2: Solve and determine the constant Write $u(x) = f(x)+1$. Then $u\prime = e^x u$, with $u(0) = 2$. Separation, or the integrating-factor formula, gives

$$u(x) = C\exp(e^x).$$

At zero, $2 = Ce$, so $C = 2/e$. Consequently

$$f(x) = 2\exp(e^x - 1) - 1.$$

This solution is differentiable for every real x, as required.

Step 3: Check the result in the integral equation For this expression, $f\prime(t) = e^t(f(t)+1)$. Thus

$$\int_0^x e^t f(t)\,dt = \int_0^x \big(f\prime(t) - e^t\big)\,dt = f(x) - f(0) - e^x + 1 = f(x) - e^x.$$

Adding e^x restores $f(x)$, so both the initial condition and the original equation hold. The other displayed choices do not simultaneously have the required initial value and derivative relation.

Step 4: Conclusion Therefore $f(x) = 2e^{e^x-1} - 1$. Hence, the correct option is (C).

Quick Tip to Solve: Obtain the value at the lower integration limit before differentiating; it fixes the otherwise undetermined constant.

7. For x > 0, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to : [4 marks]

(A) $\frac{1}{2}$

(B) -1

(C) 0

(D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- Under the substitution $t = 1/u$, one has $dt = -du/u^2$ and $\log t = -\log u$. Both sign changes must be retained.
- A definite integral with upper limit below its lower limit is an oriented integral. Transforming the bounds explicitly prevents an incorrect extra minus sign.

Step 1: Write both terms with their limits By definition,

$$f(e) = \int_1^e \frac{\log t}{1+t}\, dt, \qquad f(1/e) = \int_1^{1/e} \frac{\log t}{1+t}\, dt.$$

The second integrand is negative between $1/e$ and 1, while its integration direction is reversed. Therefore the second integral is positive, which already warns against concluding that the two terms cancel.

Step 2: Transform the reciprocal endpoint In the second integral, put $t = 1/u$. When $t = 1$, $u = 1$; when $t = 1/e$, $u = e$. Hence

$$f(1/e) = \int_1^e \frac{(-\log u)(-1/u^2)}{1 + 1/u}\, du = \int_1^e \frac{\log u}{u(u+1)}\, du.$$

Rename the variable in the first integral to u and add. The rational factors simplify as

$$\frac{1}{1+u} + \frac{1}{u(1+u)} = \frac{1}{u}.$$

This cancellation is the reason the sum has an elementary evaluation even though the individual integrals are less convenient.

Step 3: Evaluate the combined integral The requested sum is therefore

$$f(e) + f(1/e) = \int_1^e \frac{\log u}{u}\, du = \left[\frac{(\log u)^2}{2}\right]_1^e = \frac{1}{2}.$$

The lower endpoint contributes zero and $\log e = 1$. The negative choice contradicts positivity; zero would incorrectly treat the reciprocal terms as negatives, and one misses the factor $1/2$ from integrating the logarithm.

Step 4: Conclusion The sum is $\frac{1}{2}$. Hence, the correct option is (A).

Quick Tip to Solve: When endpoints are reciprocals and the denominator contains $1+t$, transform one integral using $t = 1/u$ and add the integrands.

8. Let $A = \{1, 2, 3, ..., 10\}$ and $f : A \to A$ be defined as

$$f(k) = \begin{cases} k+1 & if\ k\ is\ odd \\ k & if\ k\ is\ even \end{cases}$$

Then the number of possible functions $g : A \to A$ such that $gof = f$ is : [4 marks]

(A) 5^5

(B) 10^5

(C) 5!

(D) ${}^{10}C_5$

Correct Answer: (B) 10^5

Solution:

Concept:

- For functions on a finite set, $(g \circ f)(k) = g(f(k))$. Thus the condition constrains the values of g only at points attained by f.
- The image of f is the set of even elements of A. Choices for function values at different unconstrained inputs are independent.

Step 1: Find the image of the given function The odd elements are sent to the next even element, while each even element is fixed. Explicitly, $f(1) = f(2) = 2$, $f(3) = f(4) = 4$, and similarly for the other pairs. Hence

$$f(A) = \{2, 4, 6, 8, 10\}.$$

No odd integer is an output of f. This distinction between the domain and the image is essential when imposing a condition on a composition.

Step 2: Apply the composition condition For every $k \in A$, the equality $g(f(k)) = f(k)$ says that the even output $f(k)$ must be fixed by g. Therefore

$$g(2) = 2, \quad g(4) = 4, \quad g(6) = 6, \quad g(8) = 8, \quad g(10) = 10.$$

These five values have exactly one choice each. Conversely, any function satisfying these five equalities automatically satisfies the required composition condition for all ten inputs.

Step 3: Count the remaining choices The values $g(1), g(3), g(5), g(7), g(9)$ do not occur in $g(f(k))$, so each may independently be any of the ten elements of A. Repeated outputs are allowed because g is not required to be one-to-one. The multiplication principle gives

$$N = 1^5 \cdot 10^5 = 100000.$$

The count 5^5 wrongly restricts the available outputs, while $5!$ imposes an unnecessary bijection and the binomial coefficient counts subsets instead of functions.

Step 4: Conclusion There are 10^5 possible functions g. Hence, the correct option is (B).

Quick Tip to Solve: First identify the image of the inner function. A composition identity constrains the outer function on that image; count its other values freely.

9. Let A_1 be the area of the region bounded by the curves y = sinx, y = cosx and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sinx, y = cosx, x-axis and x = $\frac{\pi}{2}$ in the first quadrant. Then, [4 marks]

(A) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

(B) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(C) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

(D) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

Correct Answer: (A) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

Solution:

Concept:

- The curves intersect where $\sin x = \cos x$, namely $x = \pi/4$ in the first quadrant.
- Area between two graphs is the integral of the upper ordinate minus the lower ordinate. The lower curve switches from sine to cosine at their intersection.

Step 1: Compute the area adjacent to the y-axis Between $x = 0$ and $x = \pi/4$, cosine lies above sine. The region enclosed by the two curves and the y-axis therefore has area

$$\begin{aligned} A_1 &= \int_0^{\pi/4} (\cos x - \sin x)\, dx \\ &= [\sin x + \cos x]_0^{\pi/4} = \sqrt{2} - 1. \end{aligned}$$

This is positive, as expected, since the curves meet to the right of the y-axis.

Step 2: Integrate the region along the x-axis The region bounded below by the x-axis follows $y = \sin x$ from 0 to $\pi/4$, then $y = \cos x$ from $\pi/4$ to $\pi/2$. Thus its upper boundary is the smaller of the two ordinates. Splitting at the intersection gives

$$\begin{aligned} A_2 &= \int_0^{\pi/4} \sin x\, dx + \int_{\pi/4}^{\pi/2} \cos x\, dx \\ &= \left(1 - \frac{1}{\sqrt{2}}\right) + \left(1 - \frac{1}{\sqrt{2}}\right) \\ &= 2 - \sqrt{2}. \end{aligned}$$

Using the difference of the curves on the whole interval would describe a different region.

Step 3: Compare the two areas Their sum and ratio simplify to

$$A_1 + A_2 = (\sqrt{2} - 1) + (2 - \sqrt{2}) = 1,$$

$$\frac{A_1}{A_2} = \frac{\sqrt{2} - 1}{2 - \sqrt{2}} = \frac{1}{\sqrt{2}},$$

since $2 - \sqrt{2} = \sqrt{2}(\sqrt{2} - 1)$. Consequently the areas are unequal, excluding (B), and their ratio is neither $1 : 2$ nor $1 : 1$. The sum also excludes the expression $1 + \sqrt{2}$ in (C).

Step 4: Conclusion $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$. Hence, the correct option is (A).

Quick Tip to Solve: Split at $\pi/4$. For the region next to the x-axis integrate the lower curve, whereas the y-axis region requires the difference of the curves.

10. Let $f(x) = \sin^{-1}x$ and $g(x) = \frac{x^2-x-2}{2x^2-x-6}$. If $g(2) = \lim_{x\to 2} g(x)$, then the domain of the function fog is : [4 marks]

(A) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

(B) $(-\infty, -2] \cup [-1, \infty)$

(C) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

(D) $(-\infty, -1] \cup [2, \infty)$

Correct Answer: (A) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

Solution:

Concept:

- The real function $\sin^{-1} t$ is defined exactly for $-1 \le t \le 1$. Both endpoints are included.
- A common factor may be cancelled away from its zero. Here the stated limiting definition restores the removable hole at $x = 2$, while a genuine pole remains excluded.

Step 1: Simplify and extend the inner function Factor both polynomials:

$$g(x) = \frac{(x-2)(x+1)}{(x-2)(2x+3)} = \frac{x+1}{2x+3} \quad (x \neq 2, -3/2).$$

The condition in the question supplies the missing value

$$g(2) = \lim_{x\to 2} \frac{x+1}{2x+3} = \frac{3}{7}.$$

Thus the simplified expression represents the extended function for every $x \neq -3/2$, including $x = 2$. At $-3/2$ the denominator still vanishes but the numerator does not.

Step 2: Impose the inverse sine restriction The composite exists precisely when

$$\left|\frac{x+1}{2x+3}\right| \le 1, \qquad x \neq -\frac{3}{2}.$$

Because the squared denominator is positive at every allowed point, squaring and multiplying by that denominator preserves the inequality:

$$(x+1)^2 \le (2x+3)^2.$$

Move the smaller square to the right and factor the difference:

$$(2x+3)^2 - (x+1)^2 = (x+2)(3x+4) \ge 0.$$

Step 3: Solve the sign condition and check endpoints The zeros are -2 and $-4/3$. A product of these two increasing linear factors is nonnegative outside the interval between its roots. Hence

$$x \le -2 \quad \text{or} \quad x \ge -\frac{4}{3}.$$

The pole $-3/2$ already lies in the excluded middle interval. At -2, $g(x) = 1$; at $-4/3$, $g(x) = -1$, so

both endpoints belong. The restored point 2 also belongs. The other options incorrectly admit points between -2 and $-4/3$, or exclude permitted inputs.

Step 4: Conclusion The domain is $(-\infty, -2] \cup [-4/3, \infty)$. Hence, the correct option is (A).

Quick Tip to Solve: After filling a removable hole, solve $|g(x)| \leq 1$ by comparing squares; keep every remaining genuine denominator zero excluded.

11. Let f : R $\to$ R be defined as

$$f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), if\ x < -1 \\ |ax^2 + x + b|, if\ -1 \leq x \leq 1 \\ \sin(\pi x), if\ x > 1 \end{cases}$$

If f(x) is continuous on R, then a + b equals : [4 marks]

(A) -3

(B) 3

(C) -1

(D) 1

Correct Answer: (C) -1

Solution:

Concept:

- Each formula is continuous on its own interval. Continuity of the piecewise function therefore needs to be checked only at the joining points $x = -1$ and $x = 1$.
- At a joining point, the common function value must equal both one-sided limits. An absolute value equals zero only when its argument equals zero.

Step 1: Use continuity at the right joining point For $x > 1$, the formula is $\sin(\pi x)$. Its right-hand limit at 1 is

$$\lim_{x \to 1^+} f(x) = \sin \pi = 0.$$

The middle branch includes $x = 1$, so $f(1) = |a + 1 + b|$. Matching this value with the right-hand limit gives

$$|a + b + 1| = 0 \quad \Longrightarrow \quad a + b = -1.$$

There is no positive-or-negative alternative when an absolute value equals zero.

Step 2: Verify the left joining point The left-hand branch approaches

$$\lim_{x \to -1^-} 2\sin\left(-\frac{\pi x}{2}\right) = 2\sin\frac{\pi}{2} = 2.$$

The middle formula at -1 gives

$$f(-1) = |a - 1 + b| = |(a + b) - 1|.$$

Substituting $a + b = -1$ makes this $|-2| = 2$, exactly the required limit. Hence the condition obtained at 1 is compatible with continuity at -1, and no second value of the sum is needed.

Step 3: Confirm global continuity and reject the other sums Polynomials are continuous, taking an absolute value preserves continuity, and sine is continuous. Once both joins match, there can be no other discontinuity. The value -1 therefore suffices for continuity on all of $\mathbb{R}$. For the other proposed sums, -3, 3, and 1, the middle value at 1 would respectively be 2, 4, and 2, which do not equal the right-hand limit zero. They are all excluded by the same endpoint check.

Step 4: Conclusion The required sum is $a + b = -1$. Hence, the correct option is (C).

Quick Tip to Solve: Check the join whose neighbouring branch tends to zero first: an absolute value equal to zero fixes its entire argument without a sign case.

12. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to : [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept:

- For endpoints $(3, 2)$ and (u, v), their midpoint is $((u + 3)/2, (v + 2)/2)$.
- A circle written as $(x - h)^2 + (y - k)^2 = r^2$ has centre (h, k) and radius r, the positive square root of the constant.

Step 1: Express the moving endpoint through the midpoint Let the moving point on the given unit circle be $P = (u, v)$, so $u^2 + v^2 = 1$. Write its midpoint with the fixed point as $M = (x, y)$. The midpoint equations are

$$x = \frac{u + 3}{2}, \qquad y = \frac{v + 2}{2}.$$

Solving them for the moving endpoint yields

$$u = 2x - 3, \qquad v = 2y - 2.$$

This inversion ensures that the locus is derived from precisely the prescribed endpoints.

Step 2: Substitute in the original circle equation Every permitted moving endpoint satisfies the unit-circle condition. Substitution gives

$$(2x - 3)^2 + (2y - 2)^2 = 1.$$

Factor four from the two squares and divide by it:

$$\left(x - \frac{3}{2}\right)^2 + (y - 1)^2 = \frac{1}{4}.$$

Conversely, for every midpoint satisfying this equation, the reconstructed point $(2x-3, 2y-2)$ lies on the original circle. Thus the equation describes the entire locus, with no extraneous points.

Step 3: Read the radius and check the scale The centre of the midpoint circle is $(3/2, 1)$, and its squared radius is $1/4$. Therefore

$$r = \sqrt{\frac{1}{4}} = \frac{1}{2}.$$

Geometrically the midpoint map is a scaling by $1/2$, together with a translation, so the original unit radius must become $1/2$. The option $1/4$ confuses radius with radius squared; radius 1 ignores the midpoint scaling, while $1/3$ has no relation to the scale factor.

Step 4: Conclusion The locus is a circle of radius $r = 1/2$. Hence, the correct option is (B).

Quick Tip to Solve: A midpoint locus is the original locus scaled by one half about the fixed endpoint. Distinguish the radius from the constant r^2 in its equation.

---

13. If vectors $\overrightarrow{a_1} = x\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{a_2} = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is : [4 marks]

(A) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$

(B) $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$

(C) $\frac{1}{\sqrt{2}}\left(\hat{i} - \hat{j}\right)$

(D) $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} - \hat{k}\right)$

Correct Answer: (A) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$

Solution:

Concept:

- Two nonzero vectors are collinear when one is a scalar multiple of the other. Comparing their corresponding components gives the necessary relations.
- A unit vector parallel to a nonzero vector $\mathbf{v}$ is $\mathbf{v}/|\mathbf{v}|$ or its negative. A possible direction only needs one admissible set of component values.

Step 1: Compare the collinear components Write

$$(1, y, z) = \lambda(x, -1, 1).$$

Component comparison gives

$$\lambda x = 1, \qquad y = -\lambda, \qquad z = \lambda.$$

In particular $\lambda \neq 0$ and $x = 1/\lambda$. Therefore the vector whose direction is sought has the form

$$\mathbf{v} = \left(\frac{1}{\lambda}, -\lambda, \lambda\right).$$

It always satisfies $y = -z$, and the product $xy = -1$ is negative.

Step 2: Construct an admissible unit vector Take the allowed value $\lambda = 1$. Then $x = 1$, $y = -1$, $z = 1$, and the original two vectors are both $(1, -1, 1)$, so they are indeed collinear. The requested vector is also $(1, -1, 1)$, with magnitude

$$|\mathbf{v}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}.$$

Dividing by this magnitude gives the unit vector

$$\hat{\mathbf{v}} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}.$$

Step 3: Check why the other directions cannot occur For option (B), the first component is zero, but $x = 1/\lambda$ is never zero; no scalar multiple can have the required first component. Option (C) has zero third component, whereas $z = \lambda \neq 0$. In option (D), the first two components have the same sign, so every nonzero scalar multiple has positive product of those components. This contradicts $xy = -1$. Thus only the first listed direction is possible, even though the data do not specify a unique value of λ.

Step 4: Conclusion A possible unit vector is $(\hat{i} - \hat{j} + \hat{k})/\sqrt{3}$. Hence, the correct option is (A).

Quick Tip to Solve: Use one common scalar for collinear vectors. For a question asking for a possible direction, test an admissible scalar and then normalize.

14. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is : [4 marks]

(A) $\frac{1}{7}$
(B) $\frac{4}{7}$
(C) $\frac{6}{7}$
(D) $\frac{3}{7}$

Correct Answer: (D) $\frac{3}{7}$

Solution:

Concept:

- A decimal integer is divisible by 2 exactly when its last digit is even. Among the given digits, only 4 is even.
- The number of distinct arrangements of repeated objects is the factorial of their total count divided by the factorials of the multiplicities. All digits here are nonzero, so there is no leading-zero restriction.

Step 1: Count all distinct seven-digit numbers There are two copies of 3, three copies of 4, and two copies of 5. Permuting all seven positions and removing the repetitions among identical digits gives

$$N = \frac{7!}{2!\,3!\,2!} = 210.$$

Every arrangement is a valid seven-digit integer, because none begins with zero. When identical copies

are temporarily labelled, each distinct number is generated by the same number $2!3!2!$ of labelled arrangements, so the distinct outcomes are equally likely.

Step 2: Count the numbers with an even last digit An even number must end in 4. Fix one 4 in the units position. The remaining six positions contain two copies each of 3, 4, and 5, so

$$N_{\text{even}} = \frac{6!}{2!\,2!\,2!} = 90.$$

We do not multiply this by three: the three copies of 4 are indistinguishable, and fixing a different identical copy does not create a new integer.

Step 3: Form the probability and check by symmetry The desired probability is

$$P = \frac{N_{\text{even}}}{N} = \frac{90}{210} = \frac{3}{7}.$$

As a second check, label the seven physical digits temporarily. Each is equally likely to occupy the last position, and three of them are 4, giving $3/7$. Removing the labels preserves this probability since every distinct arrangement had the same multiplicity. The fractions $1/7$, $4/7$, and $6/7$ count neither the three even copies nor the correct complementary event.

Step 4: Conclusion The probability that the number is divisible by 2 is $3/7$. Hence, the correct option is (D).

Quick Tip to Solve: For divisibility by two, inspect only the units digit. Its distribution reflects the multiplicities of the available digits, not just their distinct values.

---

15. Let L be a line obtained from the intersection of two planes x + 2y + z = 6 and y + 2z = 4. If point P(α, β, γ) is the foot of perpendicular from (3, 2, 1) on L, then the value of 21(α + β + γ) equals : [4 marks]

(A) 102

(B) 142

(C) 136

(D) 68

Correct Answer: (A) 102

Solution:

Concept:

- The intersection of two nonparallel planes is a line. Expressing its coordinates in terms of one parameter gives a point and a direction vector.
- At the nearest point on a line, the vector from that point to the given point is perpendicular to the line direction; their scalar product is zero.

Step 1: Parametrize the intersection Put $z = t$. The second plane gives $y = 4 - 2t$, and substitution in

the first plane gives $x = 6 - 2(4 - 2t) - t = -2 + 3t$. Thus every point of L has the form

$$P(t) = (-2, 4, 0) + t(3, -2, 1).$$

The nonzero vector $\mathbf{d} = (3, -2, 1)$ is a direction vector of the intersection. This parametrization satisfies both plane equations for every real t.

Step 2: Apply perpendicularity Let $Q = (3, 2, 1)$. Then $Q - P(t) = (5 - 3t, -2 + 2t, 1 - t)$. The foot satisfies

$$\begin{aligned} (Q - P(t)) \cdot \mathbf{d} &= 0, \\ 3(5 - 3t) - 2(-2 + 2t) + (1 - t) &= 0, \\ 20 - 14t &= 0. \end{aligned}$$

Hence $t = 10/7$. Because the squared distance is a quadratic with positive leading coefficient $\mathbf{d} \cdot \mathbf{d} = 14$, this stationary point is the unique perpendicular foot.

Step 3: Evaluate the requested expression Substituting this parameter gives

$$(\alpha, \beta, \gamma) = \left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right).$$

Therefore

$$21(\alpha + \beta + \gamma) = 21\frac{34}{7} = 102.$$

The coordinates also give $\alpha + 2\beta + \gamma = 6$ and $\beta + 2\gamma = 4$, providing a direct check. The other three proposed values do not equal the evaluated expression.

Step 4: Conclusion The requested value is 102. Hence, the correct option is (A).

Quick Tip to Solve: Parametrize an intersection line using the variable with the simplest plane equation, then impose one dot-product equation for the foot.

16. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is : [4 marks]

(A) An equilateral triangle having each of its side of length $\sqrt{3}$r.

(B) An equilateral triangle of height $\frac{2r}{3}$.

(C) A right angle triangle having two of its sides of length 2r and r.

(D) An isosceles triangle with base equal to 2r.

Correct Answer: (A) An equilateral triangle having each of its side of length $\sqrt{3}$r.

Solution:

Concept:

- For a triangle inscribed in a circle of radius r, its base lies on a chord and its third vertex lies on the circle. Area is half the product of the base and the perpendicular height.
- For a fixed chord, the greatest height is obtained at the endpoint of the perpendicular diameter on the opposite side. A single-variable maximum then gives the optimal chord.

Step 1: Express the greatest area for a chord Choose any chord as the base and let d be its signed

distance from the centre, with the third vertex on the opposite side. The largest possible height above this chord is $h = r + d$, where $-r < d < r$. It occurs at the farthest endpoint of the diameter perpendicular to the chord. The chord half-length is $\sqrt{r^2 - d^2}$, so the largest area available for this base is

$$\Delta = (r + d)\sqrt{r^2 - d^2} = h\sqrt{2rh - h^2}, \qquad 0 < h < 2r.$$

Any other position of the third vertex gives no greater height, so maximizing this expression gives the global maximum over inscribed triangles.

Step 2: Maximize the area using differentiation Since the area is positive, maximizing its square is equivalent to maximizing the area itself. Write

$$f(h) = \Delta^2 = 2rh^3 - h^4.$$

Then

$$f'(h) = 6rh^2 - 4h^3 = 2h^2(3r - 2h).$$

This is positive for $0 < h < 3r/2$ and negative for $3r/2 < h < 2r$. Thus the unique maximum occurs at $h = 3r/2$, or $d = r/2$. The resulting base length is

$$2\sqrt{r^2 - r^2/4} = \sqrt{3}\, r.$$

The third vertex is above the chord midpoint, and each remaining side has length

$$\sqrt{(\sqrt{3}r/2)^2 + (3r/2)^2} = \sqrt{3}r.$$

Step 3: Identify the triangle and eliminate the other choices All three sides are equal, so the maximizing triangle is equilateral with side $\sqrt{3}\, r$. Its height is $3r/2$, whereas (B) gives the incorrect height $2r/3$. A right triangle as in (C) does not have all sides equal. In (D), the base is a diameter and the angle at the opposite vertex is a right angle; it likewise cannot attain the equality case. The maximum area is $3\sqrt{3}\, r^2/4$.

Step 4: Conclusion The maximum-area triangle is equilateral with side $\sqrt{3}\, r$. Hence, the correct option is (A).

Quick Tip to Solve: For an inscribed triangle, first maximize the height above a chosen chord; its optimal area then depends on just the chord distance from the centre.

17. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to : [4 marks]

(A) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(B) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

(C) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

(D) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

Correct Answer: (B) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

Solution:

Concept:

- The odd and even reciprocal factorial sums are $\sinh 1 = (e - e^{-1})/2$ and $\cosh 1 = (e + e^{-1})/2$. These are simply the odd and even parts of the exponential series.
- A polynomial numerator over a factorial can be decomposed into falling-factorial factors, which cancel successive factors in the denominator. All the series here converge absolutely.

Step 1: Decompose the numerator Seek constants so that

$$n^2 + 6n + 10 = \frac{1}{4}(2n)(2n+1) + \frac{11}{4}(2n+1) + \frac{29}{4}.$$

Expanding the right side gives $n^2 + (1/2 + 11/2)n + 11/4 + 29/4 = n^2 + 6n + 10$, confirming the identity. Dividing by $(2n+1)!$ gives

$$\frac{n^2 + 6n + 10}{(2n+1)!} = \frac{1}{4(2n-1)!} + \frac{11}{4(2n)!} + \frac{29}{4(2n+1)!}.$$

Step 2: Sum each factorial sequence For $n \geq 1$, the first denominator sequence begins with $1!$, the second with $2!$, and the third with $3!$. Consequently

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)!} = \sinh 1,$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n)!} = \cosh 1 - 1,$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)!} = \sinh 1 - 1.$$

The subtraction in the third line removes the missing $1/1!$ term, while the second removes $1/0!$.

Step 3: Collect the exponential terms The desired sum is

$$\begin{aligned} S &= \frac{1}{4}\sinh 1 + \frac{11}{4}(\cosh 1 - 1) + \frac{29}{4}(\sinh 1 - 1) \\ &= \frac{15}{2}\sinh 1 + \frac{11}{4}\cosh 1 - 10 \\ &= \frac{41}{8}e - \frac{19}{8}e^{-1} - 10. \end{aligned}$$

This matches (B). The other choices have the wrong sign of the negative-exponential term, the constant, or the positive-exponential term.

Step 4: Conclusion The sum is $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$. Hence, the correct option is (B).

Quick Tip to Solve: After cancelling factorial factors, write out the first denominator in each new series; this prevents missing the low-order terms that must be subtracted.

18. If the mirror image of the point (1, 3, 5) with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals : [4 marks]

(A) 39

(B) 41

(C) 47

(D) 43

Correct Answer: (C) 47

Solution:

Concept:

- For a plane $\mathbf{n} \cdot \mathbf{x} = d$, its normal vector $\mathbf{n}$ is perpendicular to the plane. A point and its mirror image differ by a multiple of this vector.
- If Q is reflected to Q', their midpoint lies in the plane. Hence $Q' = Q - 2(\mathbf{n} \cdot Q - d)\mathbf{n}/\|\mathbf{n}\|^2$.

Step 1: Calculate the normal displacement Here $Q = (1, 3, 5)$, $\mathbf{n} = (4, -5, 2)$, and $d = 8$. Evaluate the signed plane expression at the point:

$$\mathbf{n} \cdot Q - d = 4 - 15 + 10 - 8 = -9.$$

The normal has squared length

$$\|\mathbf{n}\|^2 = 4^2 + (-5)^2 + 2^2 = 45.$$

The negative value means the correction towards the plane is in the positive normal direction. Reflection requires twice that correction, not just the displacement to the foot.

Step 2: Find the reflected coordinates The reflected point is therefore

$$\begin{aligned} Q' &= (1, 3, 5) - \frac{2(-9)}{45}(4, -5, 2) \\ &= (1, 3, 5) + \frac{2}{5}(4, -5, 2) \\ &= \left(\frac{13}{5}, 1, \frac{29}{5}\right). \end{aligned}$$

To verify the construction, the midpoint is $(9/5, 2, 27/5)$. Substitution gives $4(9/5) - 5(2) + 2(27/5) = 8$, so it lies on the plane; the displacement of the reflected point from the original is parallel to the

normal.

Step 3: Evaluate the coordinate sum Thus

$$5(\alpha+\beta+\gamma)=5\left(\frac{13}{5}+1+\frac{29}{5}\right)=5\frac{47}{5}=47.$$

In particular, the factor of two in the reflection formula is essential: using only the perpendicular foot would yield a different value. The proposed values 39, 41, and 43 do not agree with the verified reflected coordinates.

Step 4: Conclusion The requested expression equals 47. Hence, the correct option is (C).

Quick Tip to Solve: Compute the signed plane expression once; subtract twice that expression times the normal divided by its squared length to obtain a reflection.

19. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then : [4 marks]

(A) Both F_1 and F_2 are not tautologies

(B) F_1 and F_2 both are tautologies

(C) F_1 is not a tautology but F_2 is a tautology

(D) F_1 is a tautology but F_2 is not a tautology

Correct Answer: (C) F_1 is not a tautology but F_2 is a tautology

Solution:

Concept:

- A tautology is true for every assignment of truth values to its variables. One counterexample suffices to show an expression is not a tautology.
- The implication $P \rightarrow Q$ is equivalent to $\neg P \vee Q$. The laws $P \vee \neg P = \mathrm{T}$ and associativity of disjunction simplify tautology checks.

Step 1: Test the first expression Take $A = \mathrm{T}$, $B = \mathrm{T}$, and $C = \mathrm{T}$. Then $\neg A = \neg B = \neg C = \mathrm{F}$. In particular,

$$A \wedge \neg B = \mathrm{F}, \qquad \neg C \wedge (A \vee B) = \mathrm{F}.$$

Both conjunctions and the final disjunct $\neg A$ are false. Therefore

$$F_1(\mathrm{T}, \mathrm{T}, \mathrm{T}) = \mathrm{F}.$$

This single assignment disproves the claim that F_1 is a tautology. We do not have to find its values at all eight assignments to establish that conclusion.

Step 2: Simplify the second expression Remove the implication before combining disjunctions:

$$\begin{aligned} F_2 &= (A \vee B) \vee (B \to \neg A) \\ &= (A \vee B) \vee (\neg B \vee \neg A) \\ &= (A \vee \neg A) \vee (B \vee \neg B) \\ &= \mathrm{T}. \end{aligned}$$

The final result holds for all four assignments of A and B, so F_2 is a tautology. Notice that the negation of the antecedent B, not B itself, enters when the implication is rewritten.

Step 3: Match the two results to the options The first expression fails at the all-true assignment, whereas the second is identically true. Option (A) incorrectly denies the tautological nature of F_2. Option (B) incorrectly asserts that F_1 is always true. Option (D) reverses both conclusions. Only option (C) states the two properties in the correct order.

Step 4: Conclusion F_1 is not a tautology, but F_2 is a tautology. Hence, the correct option is (C).

Quick Tip to Solve: Try an all-true assignment to refute complicated expressions quickly; for expressions involving implication, first replace it by a disjunction.

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20. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line x + 2y = 4 at x = −2, then the value of y, for which the point (3, y) lies on the curve, is : [4 marks]

(A) $-\frac{18}{19}$
(B) $-\frac{4}{3}$
(C) $\frac{18}{35}$
(D) $-\frac{18}{11}$

Correct Answer: (A) $-\frac{18}{19}$

Solution:

Concept:

- The given slope gives $dy/dx = y^2 + y/x$ for $x \neq 0$. Setting $v = 1/y$ converts this Bernoulli equation into a first-order linear equation when $y \neq 0$.
- A linear equation $v' + P(x)v = Q(x)$ is solved with an integrating factor $e^{\int P(x)\,dx}$; here one may use x on an interval excluding zero.

Step 1: Obtain the initial point and linear equation At $x = -2$, the line $x + 2y = 4$ gives $y = 3$. Thus the specified curve passes through $(-2, 3)$. Since this value is nonzero, the identically zero solution is excluded. With $v = 1/y$, we have $v' = -y'/y^2$, and hence

$$v' = -1 - \frac{v}{x}, \qquad v' + \frac{v}{x} = -1.$$

Step 2: Integrate and determine the constant Multiplication by x gives

$$(xv)' = -x.$$

On integrating,

$$xv = -\frac{x^2}{2} + C, \qquad \frac{x}{y} = -\frac{x^2}{2} + C.$$

At the specified point,

$$-\frac{2}{3} = -2 + C, \qquad C = \frac{4}{3}.$$

Therefore the algebraic equation of the specified curve is

$$y = \frac{x}{4/3 - x^2/2} = \frac{6x}{8 - 3x^2}.$$

Its differential-equation representation is considered on intervals avoiding zero and the poles; the requested point is evaluated on this same algebraic curve.

Step 3: Evaluate and check the requested value Substitute $x = 3$ in the curve equation:

$$y = \frac{18}{8 - 27} = -\frac{18}{19}.$$

A direct derivative check gives

$$y' = \frac{48 + 18x^2}{(8 - 3x^2)^2} = y^2 + \frac{y}{x} \quad (x \neq 0),$$

confirming that the obtained expression satisfies the printed slope wherever the equation is defined. Each of the other proposed values fails $3/y = 4/3 - 9/2 = -19/6$, the relation determined by the initial point.

Step 4: Conclusion At $x = 3$, the specified curve gives $y = -\frac{18}{19}$. Hence, the correct option is (A).

Quick Tip to Solve: When the slope contains both y^2 and y/x, try $v = 1/y$; multiplying the resulting equation by x produces a direct product derivative.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is _____.

[4 marks]

Solution:

Concept:

- The factorization $18 = 2 \cdot 3^2$ shows that $\gcd(N, 18) = 3$ requires exactly one common factor of 3 and no common factor of 2.

- Equivalently, write $N = 3k$. Then $\gcd(3k, 18) = 3\gcd(k, 6)$, so the condition becomes $\gcd(k, 6) = 1$. Such integers have residues 1 or 5 modulo 6.

Step 1: Translate the four-digit range A four-digit positive integer satisfies $1000 \le N \le 9999$. For $N = 3k$, the corresponding integer range is

$$\left\lceil \frac{1000}{3} \right\rceil \le k \le \left\lfloor \frac{9999}{3} \right\rfloor, \qquad 334 \le k \le 3333.$$

Merely counting all multiples of 3 would also include even multiples and multiples of 9. Both must be excluded to make the gcd exactly 3.

Step 2: Count the allowed residue classes The values $k \equiv 1 \pmod 6$ run from 337 to 3331. Their number is

$$\frac{3331 - 337}{6} + 1 = 500.$$

The values $k \equiv 5 \pmod 6$ run from 335 to 3329 and also number

$$\frac{3329 - 335}{6} + 1 = 500.$$

The two classes are disjoint, so there are $500 + 500 = 1000$ acceptable integers k, and multiplication by 3 gives a one-to-one correspondence with the requested four-digit numbers.

Step 3: Verify with inclusion and exclusion As a separate check, there are $3333 - 333 = 3000$ multiples of 3 in the four-digit range. Among them, $1666 - 166 = 1500$ are multiples of 6 and $1111 - 111 = 1000$ are multiples of 9. Their overlap consists of $555 - 55 = 500$ multiples of 18. Thus inclusion and exclusion gives

$$3000 - 1500 - 1000 + 500 = 1000,$$

confirming the residue-class count and ensuring that neither endpoint has been incorrectly included.

Step 4: Conclusion The total number of four-digit integers satisfying $\gcd(N, 18) = 3$ is $\boxed{1000}$.

Quick Tip to Solve: Divide both the integer and the gcd condition by 3 first; counting numbers coprime to 6 is a simple count of residues 1 and 5 modulo 6.

22. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation

$A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to _____.

[4 marks]

Solution:

Concept:

- The lower triangular matrix has diagonal entries $1, 2, -1$. A polynomial in a triangular matrix has the same polynomial evaluated at each diagonal entry.
- The non-diagonal part on coordinates one and three is the matrix $B = \begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix}$, which satisfies $B^2 = I$. Even and odd powers therefore alternate.

Step 1: Use the first diagonal entry The first diagonal entry of every positive power of A is 1. Comparing the first diagonal entries of the stated matrix equation gives

$$1 + \alpha + \beta = 1, \qquad \beta = -\alpha.$$

The third diagonal entry gives the identical condition, since the powers 20 and 19 are even and odd respectively. Thus it does not introduce a contradictory extra equation.

Step 2: Use the middle diagonal entry The second coordinate is independent of the other two coordinates, and its diagonal entry in A^k is 2^k. Hence

$$2^{20} + \alpha 2^{19} + 2\beta = 4.$$

Substitute $\beta = -\alpha$ and factor:

$$\alpha(2^{19} - 2) = 4 - 2^{20} = -2(2^{19} - 2).$$

Since $2^{19} - 2 \neq 0$, it follows that $\alpha = -2$ and $\beta = 2$.

Step 3: Check the non-diagonal entry On the first and third coordinates, $B^{20} = I$ and $B^{19} = B$. With the obtained coefficients, the combination is $I - 2B + 2B = I$. On the second coordinate it is $2^{20} - 2 \cdot 2^{19} + 4 = 4$. Thus all entries, including the lower-left entry, satisfy the given equation; comparison of diagonal entries has not missed a condition.

Step 4: Conclusion Therefore $\beta - \alpha = 2 - (-2) = \boxed{4}$.

Quick Tip to Solve: For this triangular matrix, compare diagonal entries first, then check the small block whose square is the identity.

23. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of p_n^2 is _____. [4 marks]

Solution:

Concept:

- From $\alpha + \beta = 1$ and $\alpha\beta = -1$, both numbers are roots of $t^2 - t - 1 = 0$. Thus each root obeys $t^2 = t + 1$.
- Adding the corresponding power identities gives a recurrence for $p_k = \alpha^k + \beta^k$. This avoids computing either irrational root explicitly.

Step 1: Derive the recurrence Multiply the identity $t^2 = t + 1$ by t^{n-1}, which is valid for $n \geq 1$. Neither root is zero because their product is -1. For each root this gives $t^{n+1} = t^n + t^{n-1}$. Adding the identities for α and β, obtain

$$p_{n+1} = p_n + p_{n-1}.$$

In particular, both terms supplied in the question belong to the same recurrence.

Step 2: Find the intervening power sum Use the values on either side of p_n:

$$29 = p_n + 11 \quad \Longrightarrow \quad p_n = 18.$$

The subtraction is appropriate here: the larger-index term is the sum of the previous two terms, rather than their difference. No choice of sign for the two roots affects this calculation because the power sum is symmetric in them.

Step 3: Square and verify consistency The requested quantity is the square of the sum, so

$$p_n^2 = 18^2 = (20 - 2)^2 = 400 - 80 + 4 = 324.$$

It is not $\alpha^{2n} + \beta^{2n}$, which would omit the cross term. As a check that the supplied values can occur, the recurrence starting with $p_0 = 2$, $p_1 = 1$ gives $2, 1, 3, 4, 7, 11, 18, 29$. Thus the adjacent values 11 and 29 indeed enclose 18.

Step 4: Conclusion The required numerical value is $\boxed{324}$.

Quick Tip to Solve: Convert a quadratic equation for the roots into a recurrence for their power sums; use adjacent sums before evaluating the roots.

---

24. Let z be those complex numbers which satisfy

$|z + 5| \leq 4$ and $z(1 + i) + \bar{z}(1 - i) \geq -10$, $i = \sqrt{-1}$.

If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _____. [4 marks]

Solution:

Concept:

- Write $z = x + iy$. The modulus condition describes a closed disk, and a real linear expression in z and $\bar{z}$ describes a half-plane.

- The required expression $|z+1|^2 = (x+1)^2 + y^2$ is a squared distance. A translation to the centre of the disk makes both constraints easier to compare.

Step 1: Translate the constraints We have $z(1+i) + \overline{z}(1-i) = 2(x-y)$. Thus

$$(x+5)^2 + y^2 \le 16, \qquad x - y \ge -5.$$

Put $u = x+5$, $v = y$. The feasible region is $u^2 + v^2 \le 16$, $u \ge v$, and the objective becomes

$$F = (u-4)^2 + v^2 = u^2 + v^2 - 8u + 16.$$

Step 2: Bound the smallest possible translated coordinate For $u < 0$, the condition $v \le u$ implies $v^2 \ge u^2$. Consequently $2u^2 \le u^2 + v^2 \le 16$, giving $u \ge -2\sqrt{2}$. If $u \ge 0$, the same lower bound is automatic. Therefore every feasible point satisfies

$$F \le 16 - 8u + 16 \le 32 + 16\sqrt{2}.$$

This bound uses both the disk radius and the half-plane restriction; taking the farthest point of the full disk alone would violate the second constraint.

Step 3: Show that the bound is attained Take $u = v = -2\sqrt{2}$. Then $u^2 + v^2 = 16$ and $u = v$, so both constraints hold with equality. This corresponds to $z = -5 - 2\sqrt{2} - 2\sqrt{2}\,i$. At this point

$$|z+1|^2 = (-4 - 2\sqrt{2})^2 + (-2\sqrt{2})^2 = 32 + 16\sqrt{2}.$$

Thus the upper bound is an actual maximum, not only a formal estimate. Comparing the rational and irrational coefficients gives $\alpha = 32$, $\beta = 16$.

Step 4: Conclusion The maximum is $32 + 16\sqrt{2}$, and hence $\alpha + \beta = \boxed{48}$.

Quick Tip to Solve: Translate the disk to the origin and bound the coordinate appearing linearly in the expanded squared distance.

---

25. If the arithmetic mean and geometric mean of the pth and qth terms of the sequence $-16, 8, -4, 2, \ldots\ldots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to _____. [4 marks]

Solution:

Concept:

- The sequence is geometric with first term -16 and common ratio $-\frac{1}{2}$. Its kth term is $T_k = -16(-\frac{1}{2})^{k-1}$.
- For positive numbers the arithmetic mean is at least the nonnegative geometric mean. Once the two means are known, their sum and product determine the original terms.

Step 1: Identify the two means Factor the equation satisfied by the means:

$$4x^2 - 9x + 5 = (4x - 5)(x - 1) = 0.$$

Its roots are $5/4$ and 1. Equal means would require both selected terms to be 1 or both to be $5/4$. Neither value occurs in the sequence: its positive terms are $8, 2, 1/2, 1/8, \ldots$. Thus the two means must be distinct. Since the arithmetic mean is not smaller than the geometric mean,

$$A = \frac{5}{4}, \qquad G = 1.$$

The positive arithmetic mean and positive product also show that the two sequence terms are positive, so the usual mean inequality is applicable.

Step 2: Recover the sequence terms Write the selected terms as $a = T_p$, $b = T_q$. The definitions of the means imply

$$a + b = 2A = \frac{5}{2}, \qquad ab = G^2 = 1.$$

Therefore a and b are the two roots of

$$t^2 - \frac{5}{2}t + 1 = 0 \quad \Longleftrightarrow \quad (2t - 1)(t - 2) = 0.$$

Hence their unordered values are 2 and $1/2$. Negative terms cannot provide these positive sum and product values.

Step 3: Locate the two positions List the terms until both recovered values appear:

$$T_1 = -16,\ T_2 = 8,\ T_3 = -4,\ T_4 = 2,\ T_5 = -1,\ T_6 = \frac{1}{2}.$$

Their absolute values strictly decrease because the ratio has magnitude $1/2$, so neither recovered value occurs at another position. Thus $\{p, q\} = \{4, 6\}$. Interchanging which index is called p does not change their sum.

Step 4: Conclusion Therefore $p + q = 4 + 6 = \boxed{10}$.

Quick Tip to Solve: Find the means from the quadratic, then reconstruct the terms using their sum and product before finding their indices.

26. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, for m, $n \geq 1$, and

$\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+1}}dx = \alpha I_{m,n} \alpha \in R$, then α equals _____. [4 marks]

Solution:

Concept:

- The stated integral $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ is positive for all $m, n \geq 1$. Thus the equality defines α as a ratio for each chosen pair of parameters.
- A fixed numerical answer for arbitrary parameters must hold for every allowed pair. Two exact evaluations giving different ratios disprove the existence of a universal value without changing the stated exponent.

Step 1: Evaluate one admissible parameter pair Take $m = n = 1$. Then

$$I_{1,1} = \int_0^1 1\,dx = 1.$$

The other integral, with the denominator exactly as stated, is

$$\int_0^1 \frac{1+1}{(1+x)^2}\,dx = \left[-\frac{2}{1+x}\right]_0^1 = 1.$$

Therefore this admissible pair requires $\alpha = 1$. This agrees with the supplied key for this particular pair, but does not yet establish a parameter-independent identity.

Step 2: Test a second admissible pair Now take $m = 1$, $n = 2$, which also satisfies every parameter restriction. Here

$$I_{1,2} = \int_0^1 (1-x)\,dx = \frac{1}{2}.$$

Meanwhile the numerator is $1 + x$, and the denominator is still $(1 + x)^2$, since its exponent is $m + 1$. Thus

$$\int_0^1 \frac{1+x}{(1+x)^2}\,dx = \int_0^1 \frac{dx}{1+x} = \ln 2.$$

The equality therefore requires $\alpha = 2\ln 2$, which is different from 1.

Step 3: Interpret the two exact results The same literal formula produces distinct values of α for two allowed choices. Consequently it cannot determine a single numerical value independent of m, n. All the integrals used here are continuous on the closed interval and have elementary antiderivatives, so there is no endpoint or convergence exception. An identity with a different denominator exponent would be a different problem and cannot be substituted for the given one.

Step 4: Conclusion As stated, no unique constant α is determined: it is 1 for $(m, n) = (1, 1)$ and $2\ln 2$ for $(1, 2)$. The supplied key value 1 is therefore not valid for all the permitted parameters.

Quick Tip to Solve: Before applying a claimed integral identity with arbitrary parameters, test two small parameter pairs and retain the denominator exponent exactly.

27. Let $X_1, X_2, \ldots\ldots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i-\alpha)=36$ and $\sum_{i=1}^{18}(X_i-\beta)^2=90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha-\beta|$ is _____. [4 marks]

Solution:

Concept:

- For N observations with mean μ, $\sum(X_i-c)=N(\mu-c)$. The sum of squared deviations about any origin c is $\sum(X_i-\mu)^2+N(\mu-c)^2$.
- The population standard deviation used for a complete set of observations is $\sigma=\sqrt{N^{-1}\sum(X_i-\mu)^2}$. Here $N=18$ and $\sigma=1$.

Step 1: Locate the first origin relative to the mean Denote the mean of the eighteen observations by μ. The first supplied relation becomes

$$18(\mu-\alpha)=36 \quad\Longrightarrow\quad \mu-\alpha=2.$$

Thus $\alpha=\mu-2$. This conclusion uses the sum of deviations, not the squared deviations, so the sign of the displacement is determined.

Step 2: Use the variance to locate the second origin Since $\sigma^2=1$, we know $\sum(X_i-\mu)^2=18$. Expand $X_i-\beta=(X_i-\mu)+(\mu-\beta)$, sum the squares, and use $\sum(X_i-\mu)=0$. Then

$$90=18+18(\mu-\beta)^2.$$

Consequently

$$(\mu-\beta)^2=4, \qquad \mu-\beta=\pm 2.$$

Both signs must initially be retained because squaring removes information about direction.

Step 3: Apply the distinctness condition The positive sign would give $\beta=\mu-2=\alpha$, which is excluded explicitly. We must therefore take $\mu-\beta=-2$, or $\beta=\mu+2$. The two origins lie on opposite sides of the mean, each at distance 2. Their separation is

$$|\alpha-\beta|=|(\mu-2)-(\mu+2)|=4.$$

Substitution checks the squared-deviation sum: $18+18\cdot 4=90$, as required.

Step 4: Conclusion The requested value is $\boxed{4}$.

Quick Tip to Solve: Use the mean as an intermediate origin, and retain both square-root signs until the condition that the two origins differ is applied.

28. Let a be an integer such that all the real roots of the polynomial
$2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval (a, a + 1). Then, |a| is equal to _____.

[4 marks]

Solution:

Concept:

- A polynomial is continuous on the whole real line. Opposite signs at two endpoints guarantee a zero between them by the intermediate value theorem.
- If the derivative is strictly positive everywhere, the polynomial is strictly increasing and can have at most one real zero. Factoring the derivative is more useful here than solving the fifth-degree equation.

Step 1: Factor the derivative Let $f(x) = 2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$. Differentiation gives

$$f\prime(x) = 10x^4 + 20x^3 + 30x^2 + 20x + 10.$$

Complete the square in polynomial form:

$$f\prime(x) = 10(x^2 + x + 1)^2.$$

Indeed, expanding the square gives $x^4+2x^3+3x^2+2x+1$, matching every coefficient of the derivative after multiplication by 10.

Step 2: Prove uniqueness of the real root The quadratic in the square is always positive, since

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} > 0.$$

Hence $f\prime(x) > 0$ for every real x; there is no interval on which the polynomial decreases or remains constant. Therefore f is strictly increasing and has at most one real root. This step is necessary because merely locating one zero would not establish where all real roots lie.

Step 3: Bracket the root between consecutive integers Evaluate the given polynomial at -2 and -1:

$$f(-2) = -64 + 80 - 80 + 40 - 20 + 10 = -34 < 0,$$

$$f(-1) = -2 + 5 - 10 + 10 - 10 + 10 = 3 > 0.$$

Continuity now guarantees a root in $(-2, -1)$. By strict monotonicity this is the only real root. Neither endpoint is a root, so the open interval stipulated in the question is appropriate. Thus the required consecutive-integer interval is $(a, a + 1) = (-2, -1)$.

Step 4: Conclusion Therefore $a = -2$, and $|a| = \boxed{2}$.

Quick Tip to Solve: To bracket every real root of an odd-degree polynomial, first seek a positive-square factorisation of its derivative, then test consecutive integers.

29. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, −3) and (4, $-2\sqrt{2}$), and given that a − $2\sqrt{2}$ b = 3, then (a^2 + b^2 + ab) is equal to _____. [4 marks]

Solution:

Concept:

- The normal at a smooth point is perpendicular to the tangent. If every normal passes through a fixed point, the distance of the curve from that point remains constant.
- A circle with centre (a, b) has equation $(x-a)^2+(y-b)^2=r^2$. Equal distances of two given points from the centre give a linear equation in its coordinates.

Step 1: Identify the curve At a point (x, y), the normal slope is $(y-b)/(x-a)$. Perpendicularity gives $y\prime = -(x-a)/(y-b)$. Therefore

$$(x-a)\,dx+(y-b)\,dy=0.$$

Integrating gives $(x-a)^2+(y-b)^2=r^2$. This derivation also extends across points where the slope formula has a zero denominator by continuity. Thus the required fixed point is the centre of the circle.

Step 2: Use the two points The points $(3,-3)$ and $(4,-2\sqrt{2})$ have the same squared distance from the centre:

$$(3-a)^2+(-3-b)^2=(4-a)^2+(-2\sqrt{2}-b)^2.$$

Expanding and cancelling the common terms gives

$$18-6a+6b=24-8a+4\sqrt{2}b,$$

$$a+(3-2\sqrt{2})b=3.$$

The second relation supplied in the question is $a-2\sqrt{2}b=3$. Subtracting it from the preceding equation gives $3b=0$, so $b=0$ and then $a=3$.

Step 3: Evaluate the expression Substitute these centre coordinates, rather than the coordinates of a point on the curve:

$$a^2+b^2+ab=3^2+0^2+3(0)=9.$$

As a check, both given points lie at distance 3 from $(3,0)$: their squared distances are 9 and $1+8=9$.

Step 4: Conclusion The fixed point is $(a, b)=(3,0)$, and hence the required numerical value is $\boxed{9}$.

Quick Tip to Solve: When all normals pass through one fixed point, first write the circle equation. Subtract the equations for two given points to eliminate the radius.

30. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is _____. [4 marks]

Solution:

Concept:

- A tangent of slope m to the ellipse $x^2/a^2 + y^2/b^2 = 1$ has intercept satisfying $c^2 = a^2m^2 + b^2$ in $y = mx + c$.
- The distance of the origin from $y = mx + c$ is $|c|/\sqrt{1+m^2}$. Tangency to a circle centred at the origin makes this distance equal to its radius.

Step 1: Write the two tangency conditions The ellipse equation becomes

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

Thus $a^2 = 9$ and $b^2 = 4$. A nonvertical tangent $L : y = mx + c$ must satisfy

$$c^2 = 9m^2 + 4.$$

This condition follows as well by substituting the line into the ellipse and setting the discriminant of the resulting quadratic to zero. The other curve is

$$x^2 + y^2 = \frac{31}{4},$$

a circle of radius $\sqrt{31}/2$. Tangency to this circle requires

$$\frac{c^2}{1+m^2} = \frac{31}{4}.$$

Step 2: Eliminate the intercept Equate the two expressions for c^2:

$$9m^2 + 4 = \frac{31}{4}(1+m^2).$$

Multiplication by four and collection of terms gives

$$36m^2 + 16 = 31 + 31m^2,$$

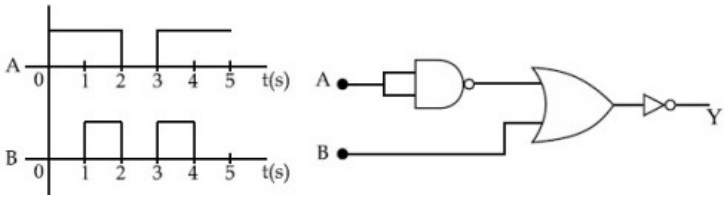
$$5m^2 = 15, \qquad m^2 = 3.$$

The intercept is then $c^2 = 31$, so the lines with $m = \pm\sqrt{3}$ and suitable signs of $c = \pm\sqrt{31}$ satisfy both conditions. Vertical ellipse tangents have $|x| = 3$, which is different from the circle radius, so they do not supply any missing common tangent.

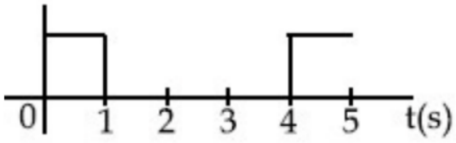
Step 3: Conclusion The square of the slope of the common tangent is $\boxed{3}$.

Quick Tip to Solve: For concentric ellipse and circle tangent problems, use the same line $y = mx + c$ in both tangency conditions and eliminate c^2.

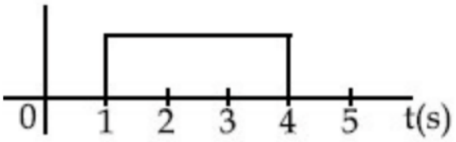
31. Draw the output signal Y in the given combination of gates. [4 marks]



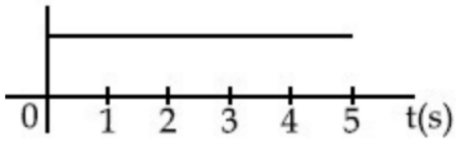
(A) High on $(0, 1)$ and $(4, 5)$; low on $(1, 4)$.



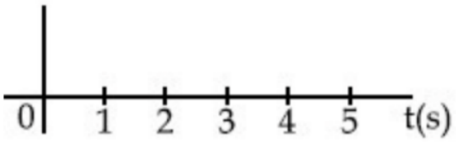
(B) High on $(1, 4)$; low on $(0, 1)$ and $(4, 5)$.



(C) High throughout $(0, 5)$.

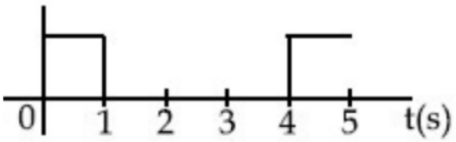


(D) Low throughout $(0, 5)$.



Correct Answer: (A)

High on $(0, 1)$ and $(4, 5)$; low on $(1, 4)$.



Solution:

Concept:

- A NAND gate with both inputs connected to the same signal A produces $\overline{A \cdot A} = \overline{A}$.
- The final NOT gate reverses the OR output. De Morgan's law gives $\overline{\overline{A} + B} = A\overline{B}$. The output is high only when $A = 1$ and $B = 0$.

Step 1: Reduce the combination of gates In the first gate both inputs are tied together, so it functions as an inverter. Its output is $\overline{A}$. This and the second input B enter the OR gate, giving $\overline{A} + B$. The inverter at the right then gives

$$Y = \overline{\overline{A} + B} = A\overline{B}.$$

The truth table is therefore

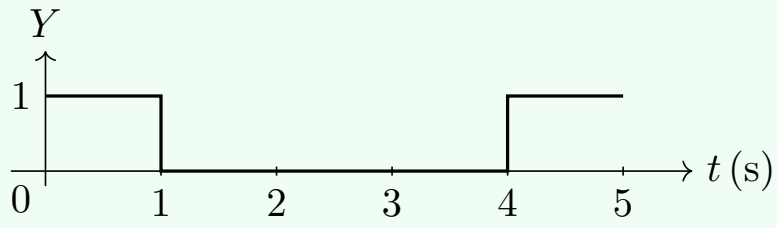
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

In particular, merely taking the OR of the original two signals would ignore both inversions and would give an incorrect waveform.

Step 2: Apply the result in each interval The first signal is high from 0 to $2\,\mathrm{s}$, low from 2 to $3\,\mathrm{s}$, and high from 3 to $5\,\mathrm{s}$. The second is high on $(1, 2)$ and $(3, 4)$. Hence

Interval (s)	$(0,1)$	$(1,2)$	$(2,3)$	$(3,4)$	$(4,5)$
Y	1	0	0	0	1

This is the waveform in option (A). Option (B) instead stays high through the central intervals, while (C) and (D) are constant signals; each disagrees with at least one row of the interval table. Ideal switching instants are represented by vertical transitions.



Output waveform

Step 3: Conclusion The output is high during $0 < t < 1\,\mathrm{s}$ and $4 < t < 5\,\mathrm{s}$, and low between these intervals. Hence, the correct option is (A).

Quick Tip to Solve: Reduce the gates algebraically before reading the pulses. For $Y = A\overline{B}$, select only the portions of the high A pulses where B is low.

32. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5 $\times$ 10^{-4} Wb/m^2 and the angle of dip is 60°. The emf induced between the tips of the plane wings will be _____. [4 marks]

(A) 88.37 mV

(B) 62.50 mV

(C) 54.125 mV

(D) 108.25 mV

Correct Answer: (D) 108.25 mV

Solution:

Concept:

- A conductor of length l moving with speed v across a perpendicular magnetic field develops motional emf $\mathcal{E} = Blv$.
- For horizontal wings moving horizontally, only the vertical component $B_v = B\sin\delta$ contributes to the wingtip emf; δ is the angle of dip.

Step 1: Resolve the field and convert the speed The velocity is along the forward direction and the wings are transverse to it, both horizontal. The force per unit charge is $\mathbf{v}\times\mathbf{B}$. Its component along the wings arises from the vertical field; the horizontal field can only produce a vertical force component. Therefore use

$$B_v = (2.5\times10^{-4})\sin 60^\circ\ \mathrm{T}.$$

Convert the given speed consistently to SI units:

$$v = 180\frac{1000}{3600} = 50\,\mathrm{m\,s^{-1}}.$$

The wing separation is the full span $l = 10\,\mathrm{m}$, not half of it.

Step 2: Calculate and compare the options The magnitude of the potential difference is

$$\mathcal{E} = B_v lv = (2.5\times10^{-4})\frac{\sqrt{3}}{2}(10)(50).$$

Thus

$$\mathcal{E} = 0.108253\,\mathrm{V} = 108.253\,\mathrm{mV}.$$

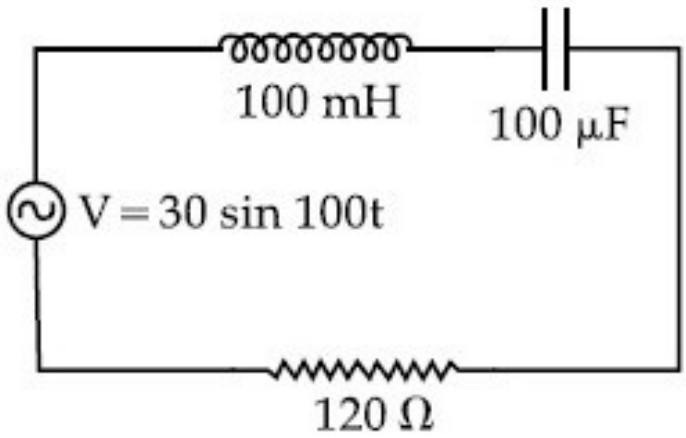
To the precision of the options this is $108.25\,\mathrm{mV}$. Using $B\cos 60^\circ$ would incorrectly give $62.50\,\mathrm{mV}$, option (B). Option (C) is half the correct value, as would result from using a half-span. Option (A) does not agree with the vertical component and SI speed. The direction of the emf depends on flight direction, but its magnitude does not require that information.

Step 3: Conclusion The induced wingtip emf is $\boxed{108.25\,\mathrm{mV}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a horizontally flying aircraft, use the vertical geomagnetic component $B\sin\delta$ and convert kilometres per hour to metres per second before multiplying.

33. Find the peak current and resonant frequency of the following circuit (as shown in figure).

[4 marks]



(A) 2 A and 100 Hz

(B) 2 A and 50 Hz

(C) 0.2 A and 100 Hz

(D) 0.2 A and 50 Hz

Correct Answer: (D) 0.2 A and 50 Hz

Solution:

Concept:

- The peak current in a series LCR circuit is $I_0 = V_0/Z$, where $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$.
- The resonant angular frequency is $\omega_r = 1/\sqrt{LC}$, and the resonant frequency in hertz is $f_r = \omega_r/(2\pi)$.

Step 1: Find the impedance at the driving frequency The source is $V = 30\sin 100t$, so its peak voltage is $30\,\text{V}$ and its angular frequency is $100\,\text{rad}\,\text{s}^{-1}$. The components are $L = 100\,\text{mH} = 0.1\,\text{H}$, $C = 100\,\mu\text{F} = 10^{-4}\,\text{F}$, and $R = 120\,\Omega$. Thus

$$X_L = 100(0.1) = 10\,\Omega, \qquad X_C = \frac{1}{100(10^{-4})} = 100\,\Omega.$$

$$Z = \sqrt{120^2 + (10 - 100)^2} = \sqrt{22500} = 150\,\Omega.$$

The peak current at the stated driving frequency is consequently $I_0 = 30/150 = 0.2\,\text{A}$.

Step 2: Find the natural resonant frequency Resonance is a property of the circuit and is distinct from the frequency of the specified source:

$$f_r = \frac{1}{2\pi\sqrt{(0.1)(10^{-4})}} \approx 50.33\,\text{Hz}.$$

The options round this to $50\,\text{Hz}$. The 100 in the source is an angular frequency, not a frequency in hertz, and is not the resonant value. Options (A) and (B) overstate the current by a factor of ten; option (C) has the correct current but the wrong resonant frequency. At resonance the peak

current would instead be $30/120 = 0.25\,\text{A}$, which is not what the first part asks for at the given source frequency.

Step 3: Conclusion The peak current is $\boxed{0.2\,\text{A}}$ and the resonant frequency is approximately $\boxed{50\,\text{Hz}}$. Hence, the correct option is (D).

Quick Tip to Solve: Read ω from the sine argument, use it to calculate the actual current, and independently calculate $f_r = 1/(2\pi\sqrt{LC})$.

---

34. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ wil be : [4 marks]

(A) $\frac{a_1+a_2}{a_2}$

(B) $\frac{a_1+a_2}{a_1}$

(C) $\frac{a_2}{a_1}$

(D) $\frac{a_1}{a_2}$

Correct Answer: (C) $\frac{a_2}{a_1}$

Solution:

Concept:

- For constant acceleration, $v = u + at$. Acceleration and retardation must carry opposite signs in a fixed positive direction.
- The maximum speed reached at the end of the accelerating stage is the initial speed of the retarding stage. The start and final speeds are both zero.

Step 1: Relate the peak speed to acceleration Take the initial direction of travel as positive and denote the speed at the changeover by v_m. During the first stage the scooter starts from rest with positive acceleration a_1, so

$$v_m = 0 + a_1 t_1 = a_1 t_1.$$

Here a_1 and a_2 denote positive magnitudes. The second stage has acceleration $-a_2$, since the scooter is slowing down while moving in the positive direction. There is no discontinuous jump in speed at the transition between the two stages.

Step 2: Use the stopping condition Apply the same velocity equation to the second stage:

$$0 = v_m - a_2 t_2, \qquad v_m = a_2 t_2.$$

Equating the two expressions for the same changeover speed gives

$$a_1 t_1 = a_2 t_2, \qquad \frac{t_1}{t_2} = \frac{a_2}{a_1}.$$

This is also the condition that the total change in velocity over both stages is zero. It does not assert that the distances in the two stages are equal. If $a_1 = a_2$, the two times are equal, so their ratio must be one. Options (A) and (B) would then give two and are excluded. Option (D) reverses the dependence: a larger initial acceleration should require a shorter accelerating time for a fixed peak

speed, which agrees with a_1 in the denominator.

Step 3: Conclusion The correct time ratio is $\boxed{t_1/t_2 = a_2/a_1}$. Hence, the correct option is (C).

Quick Tip to Solve: Introduce the shared maximum speed and express it once as $a_1 t_1$ and once as $a_2 t_2$; equating these avoids a sign error.

35. Given below are two statements:

Statement I : An electric dipole is placed at the center of a hollow sphere. The flux of the electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r ($<$ R) is zero but the electric flux passing through this closed spherical surface of radius r is not zero..

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Statement I is false but Statement II is true

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are false

Correct Answer: (C) Statement I is true but Statement II is false

Solution:

Concept:

- Gauss's law states $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\varepsilon_0$. Zero net flux means zero enclosed net charge, and does not require zero field.
- Inside the material of a conductor in electrostatic equilibrium, $\mathbf{E} = 0$. Any closed surface entirely within that material consequently has zero electric flux.

Step 1: Test Statement I An electric dipole contains charges $+q$ and $-q$. A surrounding sphere encloses net charge $q - q = 0$, so its total flux is zero. Nevertheless, the two fields do not cancel at a general point. For an ideal dipole outside its centre,

$$E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta},$$

which is nonzero for nonzero dipole moment p and finite r. In particular both its axial and equatorial fields are nonzero. In the usual physical meaning of the statement, the dipole's field in the surrounding space is nonzero although its signed flux integrates to zero. Statement I is true; cancellation of the outward and inward flux contributions is different from local field cancellation.

Step 2: Test Statement II A solid metallic sphere in electrostatic equilibrium carries its excess charge on the outer surface. A concentric Gaussian sphere of radius $r < R$ lies inside the metal and encloses

no excess charge. More directly, the field vanishes at each point of that Gaussian surface, so

$$\Phi_E = \oint \mathbf{0} \cdot d\mathbf{A} = 0.$$

Thus the claim that the field is zero but flux is nonzero is self-contradictory. The first clause of Statement II is correct, but its second clause is false, making the complete statement false. Options (A) and (B) incorrectly accept Statement II, while option (D) incorrectly rejects Statement I.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Always distinguish zero net flux from zero local field. A dipole has the former; a Gaussian surface inside an electrostatic conductor has both.

36. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where C/V = λ ? [4 marks]

(A) $[M^{-3}L^{-4}I^3T^7]$

(B) $[M^{-2}L^{-3}I^2T^6]$

(C) $[M^{-2}L^{-4}I^3T^7]$

(D) $[M^{-1}L^{-3}I^{-2}T^{-7}]$

Correct Answer: (C) $[M^{-2}L^{-4}I^3T^7]$

Solution:

Concept:

- Capacitance is charge stored per unit potential difference: $C = Q/V$. Here capacity means electrical capacitance, rather than charge itself.
- Charge has dimension $[Q] = [IT]$, while potential difference is energy per charge. Dimensional exponents add under multiplication and subtract under division.

Step 1: Find the dimension of voltage Electrical work is charge multiplied by voltage, so voltage has the dimensions of energy divided by charge. Since energy has dimensions $[ML^2T^{-2}]$, we obtain

$$[V] = \frac{[ML^2T^{-2}]}{[IT]} = [ML^2T^{-3}I^{-1}].$$

The current exponent is negative because a larger charge for the same work means a smaller voltage.

Step 2: Express the required ratio using charge The required quantity is capacitance divided by voltage, not merely capacitance. Substitute $C = Q/V$ before combining exponents:

$$\lambda = \frac{C}{V} = \frac{Q}{V^2}, \qquad [\lambda] = \frac{[IT]}{[M^2L^4T^{-6}I^{-2}]}.$$

Thus the mass and length powers are both negative, while the current and time powers become positive.

Step 3: Combine powers and compare the choices Collecting the dimensions separately gives

$$[\lambda] = [M^{-2}L^{-4}I^{3}T^{7}].$$

Option (A) has an extra inverse mass factor. Option (B) gives incorrect length, current and time powers, and option (D) does not have the required inverse-square mass dependence. Only (C) agrees with all four exponents simultaneously.

Step 4: Conclusion The dimensions of $\lambda = C/V$ are $\boxed{[M^{-2}L^{-4}I^{3}T^{7}]}$. Hence, the correct option is (C).

Quick Tip to Solve: First replace capacitance by Q/V; then the requested ratio becomes Q/V^2, which avoids confusing capacitance with charge.

37. The recoil speed of a hydrogen atom after it emits a photon in going from n = 5 state to n = 1 state will be : [4 marks]

(A) 4.34 m/s

(B) 2.19 m/s

(C) 3.25 m/s

(D) 4.17 m/s

Correct Answer: (D) 4.17 m/s

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\,\text{eV}$. A downward transition releases the positive difference between the initial and final energies.
- A photon of energy E_γ has momentum $p_\gamma = E_\gamma/c$. Momentum conservation gives the initially stationary atom equal and opposite recoil momentum.

Step 1: Find the transition energy For the transition from $n = 5$ to $n = 1$, the energy released is

$$\Delta E = 13.6\left(1 - \frac{1}{25}\right) = 13.056\,\text{eV}.$$

In SI units this is

$$\Delta E = 13.056(1.602 \times 10^{-19}) \simeq 2.092 \times 10^{-18}\,\text{J}.$$

The factor $24/25$ must be retained: the atom does not start from the ionisation limit.

Step 2: Apply conservation of momentum Initially the total momentum is zero. After emission, the atom recoils opposite to the photon, so their momentum magnitudes satisfy

$$m_{\text{H}}v = \frac{E_\gamma}{c}.$$

The recoiling object is the hydrogen atom, with mass approximately $1.67 \times 10^{-27}\,\text{kg}$, rather than the electron alone. The recoil kinetic energy is negligible compared with the photon energy at the

precision of these options, so $E_\gamma \simeq \Delta E$.

Step 3: Calculate the recoil speed Substitute the energy and atomic mass:

$$v = \frac{2.092 \times 10^{-18}}{(1.67 \times 10^{-27})(3.00 \times 10^{8})} \simeq 4.17\,\mathrm{m\,s^{-1}}.$$

This is a nonrelativistic speed, consistent with using momentum $m_{\mathrm{H}}v$. The values 2.19 and 3.25 do not match the transition energy; 4.34 corresponds approximately to using the full 13.6 eV ionisation energy and omitting the initial-level correction.

Step 4: Conclusion The recoil speed is $\boxed{4.17\,\mathrm{m\,s^{-1}}}$, opposite to the photon direction. Hence, the correct option is (D).

Quick Tip to Solve: For atomic recoil use $v = \Delta E/(mc)$, with the mass of the whole atom and the actual level-energy difference.

38. A wire of 1Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in resistance to the nearest integer is : [4 marks]

(A) 76%

(B) 12.5%

(C) 25%

(D) 56%

Correct Answer: (D) 56%

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.
- In uniform stretching, the wire retains its volume AL. With temperature and material unchanged, resistivity remains constant, so R is proportional to L^2.

Step 1: Relate the new length and cross section The original length is $L = 1\,\mathrm{m}$, and a 25% increase gives

$$L_{\mathrm{new}} = 1.25L.$$

Since stretching redistributes the same material, volume conservation gives

$$A_{\mathrm{new}}L_{\mathrm{new}} = AL, \qquad A_{\mathrm{new}} = \frac{A}{1.25} = 0.80A.$$

The reduction of area is essential: increasing length alone would underestimate the resistance change.

Step 2: Calculate the resistance ratio Use the resistance formula for the initial and final wire,

cancelling the unchanged resistivity:

$$\frac{R_{\text{new}}}{R} = \frac{\rho L_{\text{new}}/A_{\text{new}}}{\rho L/A} = \frac{L_{\text{new}}}{L}\frac{A}{A_{\text{new}}} = (1.25)^2 = 1.5625.$$

Thus the original $1\,\Omega$ resistance becomes $1.5625\,\Omega$. The length factor and the inverse-area factor both increase resistance, so their effects multiply.

Step 3: Convert the increase to a percentage The fractional change is measured relative to the original resistance, not the final one:

$$\frac{R_{\text{new}} - R}{R} \times 100 = (1.5625 - 1) \times 100 = 56.25\%.$$

To the nearest integer this is 56%. The 25% choice counts only the length change, while 12.5% and 76% do not follow the constant-volume relation. For a finite extension, the exact square is needed rather than the small-change approximation $\Delta R/R \simeq 2\Delta L/L$.

Step 4: Conclusion The resistance increases by 56.25%, or $\boxed{56\%}$ to the nearest integer. Hence, the correct option is (D).

Quick Tip to Solve: For unchanged volume and resistivity, multiply resistance by the square of the length factor: here $R_{\text{new}}/R = (1.25)^2$.

39. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors. [4 marks]

(A) $\vec{b} = \vec{a} + 2\vec{c}$
(B) $\vec{b} = \vec{a} - 2\,(\vec{a}\,.\,\vec{c})\vec{c}$
(C) $\vec{b} = 2\vec{a} + \vec{c}$
(D) $\vec{b} = \vec{a} - \vec{c}$

Correct Answer: (B) $\vec{b} = \vec{a} - 2\,(\vec{a}\,.\,\vec{c})\vec{c}$

Solution:

Concept:

- Resolution of a vector along a unit normal $\vec{c}$ gives the normal component $(\vec{a}\cdot\vec{c})\vec{c}$. The remaining component is tangent to the reflecting surface.
- Specular reflection preserves the tangential component of the propagation direction and reverses its normal component. This expresses equality of the angles of incidence and reflection.

Step 1: Resolve the incident direction Because $\vec{c}$ is a unit vector, write

$$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}, \qquad \vec{a}_{\perp} = (\vec{a}\cdot\vec{c})\vec{c}, \qquad \vec{a}_{\parallel} = \vec{a} - (\vec{a}\cdot\vec{c})\vec{c}.$$

The incident direction points towards the surface, so its projection onto the outward normal is negative. The vector resolution already includes that sign and no separate angle-sign convention is required.

Step 2: Reverse the normal projection The reflected ray propagates away from the surface with

unchanged tangential projection. Therefore

$$\vec{b} = \vec{a}_{\parallel} - \vec{a}_{\perp} = \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}.$$

The factor two subtracts the original normal component once to remove it, and once more to install its opposite. Merely subtracting one unit normal would not accomplish this for general incidence.

Step 3: Check limiting incidence and the options At normal incidence $\vec{a} = -\vec{c}$, so the relation gives $\vec{b} = \vec{c}$, as required. At grazing incidence $\vec{a} \cdot \vec{c} = 0$, so it gives $\vec{b} = \vec{a}$. It also preserves unit length because reflection changes only the sign of one orthogonal component. The other choices add or subtract fixed unit vectors without the angle-dependent projection and fail these general reflection conditions.

Step 4: Conclusion The vector reflection relation is $\boxed{\vec{b} = \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}}$. Hence, the correct option is (B).

Quick Tip to Solve: Reflection changes only the normal component: subtract twice the incident vector projection onto the unit normal.

40. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as U = 3PV + 4. The gas is : [4 marks]

(A) either monoatomic or diatomic.

(B) monoatomic only.

(C) polyatomic only.

(D) diatomic only.

Correct Answer: (C) polyatomic only.

Solution:

Concept:

- For an ideal gas, $PV = nRT$. Equipartition gives the temperature-dependent internal energy $U = (f/2)nRT + U_0$, where f is the number of active molecular degrees of freedom.
- The additive constant U_0 fixes the arbitrary energy zero. Heat capacity depends on the derivative of internal energy, so a constant offset does not change the inferred degrees of freedom.

Step 1: Isolate the temperature-dependent energy The stated relation is $U = 3PV + 4$. On using the ideal-gas equation, this becomes

$$U = 3nRT + 4.$$

The constant 4 has the units of energy implied by the relation and is independent of temperature. Consequently

$$C_V = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V = 3R.$$

It would be incorrect to interpret the constant as an additional temperature-dependent contribution.

Step 2: Determine the active degrees of freedom Compare the result with the equipartition expression:

$$\frac{f}{2}R = 3R, \qquad f = 6.$$

In the usual rigid-molecule classification used here, a monatomic gas has three translational degrees of freedom and $C_V = 3R/2$. A diatomic gas has three translational and two rotational degrees of freedom, giving $C_V = 5R/2$. Neither has the coefficient of nRT specified in this question.

Step 3: Identify the molecular classification A nonlinear polyatomic molecule has three translational and three rotational degrees of freedom, so its total is six when vibrational excitation is neglected. Its temperature-dependent internal energy is therefore $3nRT = 3PV$, matching the stated coefficient exactly. The choice saying either monatomic or diatomic is excluded for the same heat-capacity reason as the individual monatomic and diatomic choices. The arbitrary energy offset does not alter that comparison.

Step 4: Conclusion The coefficient implies six active degrees of freedom and identifies the gas as polyatomic in the standard equipartition model. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate U with respect to temperature before classifying a gas; a constant energy offset contributes nothing to C_V.

41. The length of metallic wire is l_1 when tension in it is T_1. It is l_2 when the tension is T_2. The original length of the wire will be : [4 marks]

(A) $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$
(B) $\frac{l_1 + l_2}{2}$
(C) $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$
(D) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

Correct Answer: (D) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

Solution:

Concept:

- Within the elastic limit, a wire obeys Hooke's law: extension is proportional to applied tension. For natural length L_0, Young's modulus Y and area A, $l - L_0 = TL_0/(YA)$.
- The measured length is therefore a linear function of tension, $l = L_0 + kT$, with constant compliance $k = L_0/(YA)$. The natural length is its zero-tension intercept.

Step 1: Write the two linear relations Let L_0 be the original length before applying tension. Under the two stated tensions, Hooke's law gives

$$l_1 = L_0 + kT_1, \qquad l_2 = L_0 + kT_2.$$

The material and initial geometry are the same in both measurements, so the proportionality constant k is common. The measured lengths themselves are not proportional to tension; their extensions above L_0 are.

Step 2: Eliminate the elastic compliance Multiply the first equation by T_2 and the second by T_1:

$$T_2 l_1 = T_2 L_0 + k T_1 T_2, \qquad T_1 l_2 = T_1 L_0 + k T_1 T_2.$$

Subtracting cancels the unknown stretching term and leaves

$$T_2 l_1 - T_1 l_2 = (T_2 - T_1) L_0.$$

This elimination avoids having to know either Young's modulus or the cross-sectional area.

Step 3: Solve and check the expression For distinct measurement tensions,

$$L_0 = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}.$$

As a check, take $T_1 = 0$: then l_1 is already the natural length, and this formula correctly reduces to $L_0 = l_1$. The ordinary average of the lengths cannot give the unloaded length when both tensions are positive. The plus-sign weighted average in (C) and the reversed numerator arrangement in (A) also fail the linear intercept calculation.

Step 4: Conclusion The original length is $\boxed{\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}}$. Hence, the correct option is (D).

Quick Tip to Solve: Treat measured length as $l = L_0 + kT$; eliminate k using the two measurements to obtain the zero-load intercept.

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42. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by : [4 marks]

(A) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$

(B) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$

(C) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$

(D) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

Correct Answer: (A) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$

Solution:

Concept:

- For a projectile launched at speed u and angle θ, eliminating time gives $y = x\tan\theta - gx^2/(2u^2\cos^2\theta)$, with upward vertical distance taken positive.
- The initial trajectory slope is $\tan\theta$. Maximum height occurs at the vertex of the downward-opening trajectory, where $dy/dx = 0$.

Step 1: Compare the initial slope The stated trajectory is

$$y = \alpha x - \beta x^2.$$

At the launch point $x = 0$, the slope is

$$\left.\frac{dy}{dx}\right|_{x=0} = \alpha.$$

The velocity is tangent to the trajectory, so this slope is the ratio of initial vertical to horizontal velocity. Hence $\tan\theta = \alpha$, giving $\theta = \tan^{-1}\alpha$ for the usual forward launch. Equivalently this follows by comparing the linear coefficients in the standard trajectory equation.

Step 2: Locate the vertex of the parabola Differentiate with respect to horizontal distance:

$$\frac{dy}{dx} = \alpha - 2\beta x.$$

For $\beta > 0$, the trajectory is concave downward since $d^2y/dx^2 = -2\beta < 0$. Its maximum is attained when the slope vanishes, at

$$x_{\text{top}} = \frac{\alpha}{2\beta}.$$

This identifies the horizontal location of the top, which must still be substituted into the trajectory to find the vertical height.

Step 3: Substitute to obtain the height The maximum height above the point of projection is

$$H = \alpha\frac{\alpha}{2\beta} - \beta\left(\frac{\alpha}{2\beta}\right)^2 = \frac{\alpha^2}{4\beta}.$$

Both results coincide only in option (A). Option (B) has the correct angle but an incorrect height factor. Options (C) and (D) use the curvature coefficient in the angle, although the angle is fixed by the initial slope; their height expressions are also incorrect. Dimensionally, α is dimensionless and β is inverse length, consistent with the final height.

Step 4: Conclusion The required pair is $\boxed{\theta = \tan^{-1}\alpha, \quad H = \frac{\alpha^2}{4\beta}}$. Hence, the correct option is (A).

Quick Tip to Solve: For $y = \alpha x - \beta x^2$, read the launch slope from α, and find the height by substituting the vertex $x = \alpha/(2\beta)$.

43. Given below are two statements : one is labeled as Assertion A and the other is labeled as Reason R.

Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is true but R is false

(B) A is false but R is true

(C) Both A and R are true and R is the correct explanation of A

(D) Both A and R are true but R is NOT the correct explanation of A

Correct Answer: (D) Both A and R are true but R is NOT the correct explanation of A

Solution:

Concept:

- A simple microscope is a convex lens used with the object inside its focal length; it forms an erect, enlarged virtual image.
- Angular magnification compares the angle with the microscope to the angle made by the unaided object at the least distance of distinct vision, $D = 25\,\text{cm}$.

Step 1: Evaluate the assertion The eye is placed close to the lens. A ray from the top of the object through the optical centre travels essentially undeviated. Its backward extension passes through the top of the virtual image. Thus the object and its virtual image subtend the same angle at the lens and approximately at the eye. This geometrical statement does not say that the microscope has unit angular magnification. Assertion A is true.

Step 2: Evaluate the reason Without the lens, a small object cannot be viewed distinctly much closer than the near point. The lens permits the object to be held within its focal length, closer than $25\,\text{cm}$, while producing a viewable virtual image. The object then subtends a larger angle than it would at the unaided near point. Hence the reason correctly describes why angular magnification is obtained; R is true.

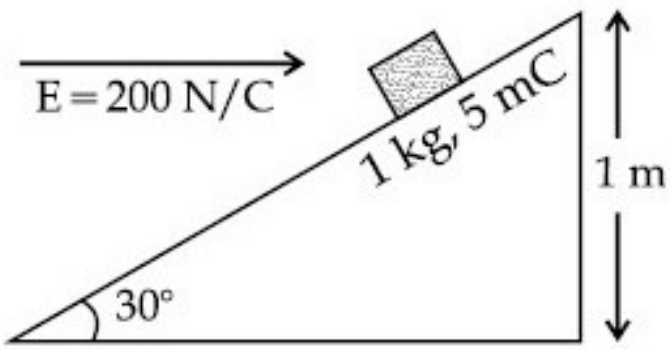
Step 3: Test the explanatory link The equality in A follows from the undeviated optical-centre ray. R instead compares viewing with the lens to unaided viewing at the near point. These are different comparisons, so R does not explain the equality in A. Options declaring either statement false are eliminated, and the option asserting a correct explanatory link is also eliminated.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. This differs from the supplied key: the optical-centre ray explains the equality, while R explains the increase relative to unaided viewing. Hence, the correct option is (D).

Quick Tip to Solve: For microscope assertion questions, distinguish the object-image angle at the lens from the magnification comparison with unaided viewing at 25 cm.

44. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\frac{N}{C}$ as shown in the figure. A body of mass 1 kg and charge 5 mC is allowed to slide down from rest at a height of 1 m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.

[g = 9.8 m/s^2; $\sin 30^\circ = \frac{1}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$] [4 marks]



(A) 0.46 s

(B) 0.92 s

(C) 1.3 s

(D) 2.3 s

Correct Answer: (C) 1.3 s

Solution:

Concept:

- The electric force on a positive charge is $F_e = qE$, in the direction of the electric field.
- Kinetic friction has magnitude μN and acts up the plane when the body slides down. Resolve all forces both parallel and perpendicular to the incline.

Step 1: Resolve the normal forces The incline rises to the right and the field points right. Therefore the electric force has an up-slope component and a component pressing the body into the plane. Here $q = 5 \times 10^{-3}\,\text{C}$, so $F_e = qE = 1\,\text{N}$. The normal reaction is consequently

$$N = mg\cos 30^\circ + F_e \sin 30^\circ = 9.8\frac{\sqrt{3}}{2} + \frac{1}{2} \simeq 8.987\,\text{N}.$$

Using only $mg\cos 30^\circ$ would underestimate friction.

Step 2: Calculate the acceleration down the plane Take the down-slope direction as positive. Weight drives the motion, whereas the electric force and friction oppose it. Thus

$$ma = mg\sin 30^\circ - F_e \cos 30^\circ - \mu N.$$

Substituting the values gives

$$a = 4.9 - 0.8660 - 0.2(8.987) \simeq 2.237\,\text{m s}^{-2}.$$

The positive result is consistent with downward sliding from rest, and all forces are constant.

Step 3: Use the travel distance The stated metre is the vertical height, not the distance along the plane. The travel distance is

$$s = \frac{h}{\sin 30^\circ} = 2\,\text{m}.$$

Starting from rest, $s = \frac{1}{2}at^2$, and hence

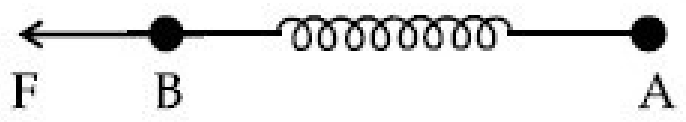
$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{4}{2.237}} \simeq 1.34\,\text{s}.$$

This rounds to the listed value 1.3 s; the other times do not satisfy the same constant-acceleration equation.

Step 4: Conclusion The body takes approximately $1.3\,\text{s}$ to reach the bottom. Hence, the correct option is (C).

Quick Tip to Solve: A horizontal electric force changes the normal reaction as well as the down-slope force; calculate both components before using friction.

45. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in the figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B will be : [4 marks]



(A) $\frac{MF}{F+Ma}$
(B) $\frac{F+Ma}{M}$
(C) $\frac{Ma-F}{M}$
(D) $\frac{F-Ma}{M}$

Correct Answer: (B) $\frac{F+Ma}{M}$

Solution:

Concept:

- A massless spring exerts forces of equal magnitude and opposite direction on the two connected bodies.
- Apply $\sum F = Ma$ to each mass separately, keeping the directions indicated by the motion and the external force.

Step 1: Find the force exerted by the spring The diagram places A on the right and B on the left, with the applied force F directed left on B. The statement that A starts moving away from B means that A accelerates right, with magnitude a. Its only horizontal force is the spring force. Therefore the spring

is pushing A to the right and its magnitude is

$$F_s = Ma.$$

This is the instantaneous force, so no spring constant or extension is needed.

Step 2: Apply the reaction force to B By Newton's third law, the spring pushes B to the left with exactly the same magnitude Ma. The external force on B also acts left. The two forces therefore add rather than subtract. Taking leftward as the positive direction for B gives

$$Ma_B = F + F_s = F + Ma.$$

Thus

$$a_B = \frac{F + Ma}{M}.$$

The acceleration is directed left, away from A at this instant.

Step 3: Check the alternatives The quantity $MF/(F + Ma)$ has dimensions of mass rather than acceleration, so it cannot be correct. The expressions involving a difference between F and Ma incorrectly oppose two forces that act in the same direction on B. As a system check, choose rightward positive: $Ma - Ma_B = -F$, which is satisfied by the result. The internal spring forces cancel from this combined equation, as they must.

Step 4: Conclusion The acceleration of B is $(F + Ma)/M$, directed along the applied force. This differs from the supplied key, which would apply if A accelerated toward B, rather than away as stated. Hence, the correct option is (B).

Quick Tip to Solve: Infer the spring-force direction from the specified acceleration of the other block before deciding whether the forces on the driven block add or subtract.

46. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be : [4 marks]

(A) $\frac{2mgh}{I+2mr^2}$

(B) $\frac{2mgh}{I+mr^2}$

(C) 2gh

(D) $\frac{2gh}{I+mr^2}$

Correct Answer: (B) $\frac{2mgh}{I+mr^2}$

Solution:

Concept:

- With a taut cord unwinding without slipping, the falling mass and wheel satisfy $v = r\omega$.
- Loss of gravitational potential energy becomes the translational kinetic energy of the mass plus the rotational kinetic energy of the wheel.

Step 1: Write the mechanical-energy balance The mass and wheel start from rest. After the mass descends through height h, its gravitational potential energy has decreased by mgh. The wheel rotates about a fixed axis and therefore has rotational, but no centre-of-mass translational, kinetic energy. For an ideal cord and axle,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Tension transfers energy between the falling body and the wheel; it is not an extra energy loss for the combined system.

Step 2: Eliminate the linear speed The cord unwinds at the rim, so its speed equals the tangential speed of the rim. Substituting $v = r\omega$ gives

$$mgh = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(I + mr^2)\omega^2.$$

Solving for the quantity asked for,

$$\omega^2 = \frac{2mgh}{I + mr^2}.$$

Both the wheel inertia and the effective rotational inertia mr^2 of the translating mass oppose the angular acceleration.

Step 3: Check the proposed expressions The coefficient of mr^2 is one because the mass contributes $\frac{1}{2}m(r\omega)^2$, not twice that energy. Thus option A is incorrect. The expression $2gh$ describes a speed squared in free fall and has the wrong dimensions for angular speed squared. Option D omits the mass in the numerator and is dimensionally inconsistent. In the limit $I \to 0$, the result gives $r^2\omega^2 = 2gh$, as expected for free fall.

Step 4: Conclusion The squared angular velocity is $\omega^2 = 2mgh/(I + mr^2)$. Hence, the correct option is (B).

Quick Tip to Solve: For a mass unwinding a wheel, replace the translational term by $\frac{1}{2}mr^2\omega^2$; then add mr^2 to the wheel inertia.

47. A particle executes S.H.M., the graph of velocity as a function of displacement is : [4 marks]

(A) a parabola

(B) a helix

(C) an ellipse

(D) a circle

Correct Answer: (C) an ellipse

Solution:

Concept:

- For simple harmonic motion of amplitude A and angular frequency ω, $x = A\cos(\omega t + \phi)$.
- Eliminating time gives $v^2 = \omega^2(A^2 - x^2)$, where the velocity can have either sign at an interior displacement.

Step 1: Eliminate the phase Differentiate the displacement to obtain

$$v = -A\omega \sin(\omega t + \phi).$$

Squaring the displacement and the velocity expressions, then using $\sin^2\theta + \cos^2\theta = 1$, gives

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1.$$

This directly describes the full velocity-displacement curve during a complete oscillation, including both directions of motion.

Step 2: Identify the curve and intercepts The equation has the form $x^2/a^2 + y^2/b^2 = 1$, the standard equation of an ellipse centred at the origin. Its intercepts are $x = \pm A$ at zero velocity and $v = \pm A\omega$ at zero displacement. The top and bottom halves correspond to opposite velocity signs. Both halves must be retained because the particle crosses each interior position twice in every cycle.

Step 3: Reject the other shapes A parabola would describe v^2 against x, because $v^2 = \omega^2 A^2 - \omega^2 x^2$; that is a different graph from velocity against displacement. A helix is a three-dimensional curve and cannot represent this two-variable relation. A circle is not the general physical form because the two intercept scales are A and $A\omega$, with different units; only after a suitable normalization do the dimensionless coordinates trace a unit circle. The general phase-plane curve is therefore an ellipse.

Step 4: Conclusion The velocity-displacement graph for S.H.M. is an ellipse. Hence, the correct option is (C).

Quick Tip to Solve: Eliminate time and retain both signs of velocity: $x^2/A^2 + v^2/(A\omega)^2 = 1$ immediately identifies the phase-plane ellipse.

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48. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A? [4 marks]

(A) 335 Hz

(B) 345 Hz

(C) 338 Hz

(D) 342 Hz

Correct Answer: (A) 335 Hz

Solution:

Concept:

- The beat frequency of two close frequencies is their absolute difference: $f_b = |f_1 - f_2|$.
- Filing a tuning fork near the prong ends reduces their effective loading and raises the natural frequency.

Step 1: List the initial possibilities The known tuning fork has frequency $340\,\text{Hz}$, and the beat

frequency is initially $5\,\text{Hz}$. Therefore

$$|f_A - 340| = 5,$$

which has two possibilities:

$$f_A = 335\,\text{Hz} \quad \text{or} \quad f_A = 345\,\text{Hz}.$$

The initial beat count alone cannot distinguish whether A has a higher or lower frequency than the known fork.

Step 2: Use the change caused by filing Filing raises the frequency of A. If A initially had $345\,\text{Hz}$, raising it would move it farther above $340\,\text{Hz}$, so the beat rate would increase beyond five. This contradicts the stated decrease to two. If A initially had $335\,\text{Hz}$, raising it toward $340\,\text{Hz}$ reduces the difference, in agreement with the observation. Consequently the initial frequency must be $335\,\text{Hz}$.

Step 3: Distinguish initial and final frequencies A beat frequency of two after filing means that the final frequency can be $338\,\text{Hz}$, or $342\,\text{Hz}$ if the frequency has passed through unison. Both are greater than the initial $335\,\text{Hz}$, so this detail does not change the initial-frequency inference. Options C and D can describe a final frequency but cannot give the initial five beats per second. Option B has already been excluded by the direction of change.

Step 4: Conclusion The original frequency of tuning fork A is $335\,\text{Hz}$. Hence, the correct option is (A).

Quick Tip to Solve: Write both frequencies allowed by the initial beats, then use whether filing moves the unknown frequency toward or away from the reference.

49. Given below are two statements :

Statement I : A second's pendulum has a time period of 1 second.

Statement II : It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are false

(B) Statement I is false but Statement II is true

(C) Both Statement I and Statement II are true

(D) Statement I is true but Statement II is false

Correct Answer: (B) Statement I is false but Statement II is true

Solution:

Concept:

- A seconds pendulum is defined as a pendulum whose complete oscillation has period $T = 2\,\text{s}$.
- Motion from one extreme position to the opposite extreme covers half a complete oscillation and takes $T/2$.

Step 1: Evaluate Statement I The period is the time for a complete repetition of position and direction of motion. A seconds pendulum takes one second for a single swing from one side to the other, but

two seconds to return to the starting extreme. Thus

$$T = 2\,\text{s}.$$

The statement that its time period is one second confuses a half-oscillation with a full oscillation. Statement I is false.

Step 2: Evaluate Statement II Take an extreme position as the start of the cycle. The pendulum reaches the mean position after $T/4$, reaches the opposite extreme after $T/2$, and returns to the initial extreme only after T. For the seconds pendulum,

$$t_{\text{extreme to extreme}} = \frac{T}{2} = \frac{2}{2} = 1\,\text{s}.$$

Statement II therefore correctly describes the time between the two opposite extremes and is true.

Step 3: Match the truth values The pair of truth values is false for I and true for II. Thus neither the choice declaring both statements false nor the choice declaring both true can be correct. The choice with I true and II false reverses both conclusions. This is a question about the definition of the seconds pendulum and the fraction of a cycle represented by a swing; no length, mass or gravitational acceleration needs to be calculated.

Step 4: Conclusion Statement I is false, but Statement II is true. Hence, the correct option is (B).

Quick Tip to Solve: Remember that one swing of a seconds pendulum takes one second; its out-and-back complete oscillation takes two seconds.

50. A radioactive sample is undergoing α decay. At any time t_1, its activity is A and another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample? [4 marks]

(A) $\frac{\ln 5}{t_2 - t_1}$

(B) $\frac{\ln(t_2+t_1)}{2}$

(C) $\frac{t_1 - t_2}{\ln 5}$

(D) $\frac{t_2 - t_1}{\ln 5}$

Correct Answer: (D) $\frac{t_2 - t_1}{\ln 5}$

Solution:

Concept:

- Activity is proportional to the number of undecayed nuclei: $A = \lambda N$, where λ is the positive decay constant.
- Exponential decay gives $A(t) = A_0 e^{-\lambda t}$. The mean or average lifetime is $\tau = 1/\lambda$; it differs from the half-life $T_{1/2} = \ln 2/\lambda$.

Step 1: Compare the activities at the two instants Use the ratio to eliminate the unknown initial activity. Since the later activity is smaller, $t_2 > t_1$. Writing the decay law separately at the two times

gives

$$\frac{A(t_2)}{A(t_1)} = e^{-\lambda(t_2-t_1)} = \frac{A/5}{A} = \frac{1}{5}.$$

The elapsed time, rather than either absolute clock reading, controls the change in activity.

Step 2: Find the decay constant and invert it Taking natural logarithms of the dimensionless ratio gives

$$-\lambda(t_2 - t_1) = -\ln 5, \qquad \lambda = \frac{\ln 5}{t_2 - t_1}.$$

Therefore

$$\tau = \frac{1}{\lambda} = \frac{t_2 - t_1}{\ln 5}.$$

This expression is positive and has units of time. Option (A) is the decay constant itself, with inverse-time units. Option (C) is negative because its numerator reverses the time difference. Option (B) depends on the choice of time origin and does not represent a lifetime.

Step 3: Conclusion The average lifetime is $\boxed{\tau = (t_2 - t_1)/\ln 5}$. Hence, the correct option is (D).

Quick Tip to Solve: For any activity reduction by a factor k, take the elapsed time divided by ln k to obtain the mean lifetime.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$. The value of x is _____. [R = universal gas constant] [4 marks]

Solution:

Concept:

- For one mole of a rigid diatomic ideal gas, three translational and two rotational degrees of freedom contribute, so $C_V = 5R/2$. Vibrational modes are not included in the rigid-molecule model.
- With work done by the gas positive, the first law is $Q = \Delta U + W$, while the molar heat capacity along the specified transformation is $C = Q/\Delta T$ for one mole.

Step 1: Calculate the increase in internal energy The gas receives heat Q and gives back $Q/5$ as mechanical work. The part remaining as internal energy is therefore

$$\Delta U = Q - \frac{Q}{5} = \frac{4Q}{5}.$$

For an ideal gas, internal energy depends only on temperature even when the path is neither isochoric nor isobaric. Thus it is C_V, rather than an assumed constant-pressure heat capacity, that relates this change to temperature.

Step 2: Relate the supplied heat to the temperature rise For one mole,

$$\frac{5R}{2}\Delta T = \frac{4Q}{5}, \qquad \Delta T = \frac{8Q}{25R}.$$

The heat capacity of this particular transformation follows directly:

$$C = \frac{Q}{\Delta T} = \frac{25R}{8}.$$

Comparing with the stated form $xR/8$ gives $x = 25$. Its value lies between $C_V = 20R/8$ and $C_P = 28R/8$, consistent with some supplied heat being converted into positive work.

Step 3: Conclusion The transformation has molar heat capacity $25R/8$, so the required dimensionless value is $\boxed{x = 25}$.

Quick Tip to Solve: Apply the first law before choosing a heat capacity: a specified work fraction determines how much heat raises the internal energy.

52. A particle executes S.H.M. with amplitude 'a', and time period 'T'. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of x is _____. [4 marks]

Solution:

Concept:

- In simple harmonic motion, $v^2 = \omega^2(a^2 - y^2)$, where y is displacement from the mean position, a is amplitude and $\omega = 2\pi/T$.
- The maximum speed is $v_{\max} = \omega a$, attained at the mean position. Squaring the speed ratio gives the fraction of maximum kinetic energy.

Step 1: Insert the stated speed ratio The specified speed is half the maximum speed, so its square is one quarter of the maximum squared speed. Substitution into the SHM relation gives

$$\left(\frac{\omega a}{2}\right)^2 = \omega^2(a^2 - y^2).$$

Since the angular frequency is nonzero, divide through by ω^2. The time period cancels; this question depends on the speed ratio and amplitude rather than on how rapidly the oscillations occur.

Step 2: Solve for the displacement magnitude Rearrange the resulting equation:

$$\frac{a^2}{4} = a^2 - y^2, \qquad y^2 = \frac{3a^2}{4}.$$

Hence the two possible positions are

$$y = \pm\frac{\sqrt{3}\,a}{2}.$$

The question writes the positive magnitude as $\sqrt{x}\,a/2$, so comparing the coefficients gives $x = 3$. A useful energy check is that kinetic energy is one quarter of its maximum, leaving three quarters as elastic potential energy, which is proportional to y^2.

Step 3: Conclusion The displacement magnitude is $\sqrt{3}\,a/2$ (either sign is possible), and the requested value is $\boxed{x = 3}$.

Quick Tip to Solve: Use v squared divided by maximum v squared plus displacement squared divided by amplitude squared equals one; speed and displacement are not linearly related.

53. Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is _____. [4 marks]

Solution:

Concept:

- For a small-amplitude simple pendulum, displacement from the mean can be written $y = a\sin(2\pi t/T)$ when motion starts at the mean towards the positive extreme.
- One complete oscillation traverses a total path $4a$. The path fraction and the elapsed-time fraction differ because the speed varies throughout the motion.

Step 1: Interpret the specified fraction as a path fraction To obtain the keyed result, the stated $5/8$ oscillation is read as travelling $5/8$ of the complete oscillatory path. The corresponding distance is

$$s = \frac{5}{8}(4a) = \frac{5a}{2}.$$

Starting from the mean, the pendulum first travels to the positive extreme and back to the mean, a distance $2a$ in time $T/2$. It must then move a further distance $a/2$ towards the negative extreme.

Step 2: Calculate the remaining phase and time Let u be the time after that return through the mean. Its displacement is $-a\sin(2\pi u/T)$, so the first arrival at $-a/2$ satisfies

$$\sin(2\pi u/T) = \frac{1}{2}, \qquad u = \frac{T}{12}.$$

The required path is therefore completed at

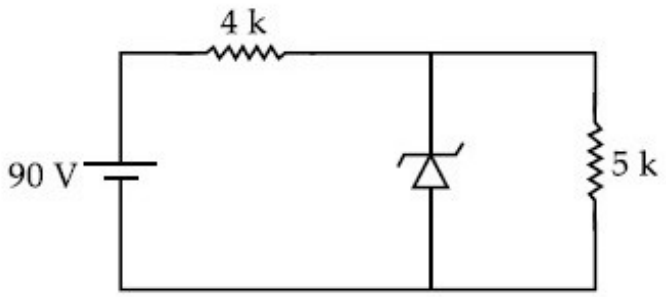
$$t = \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}.$$

In lowest terms this gives $\alpha = 7$, $\beta = 12$. If instead the phrase means a fraction of the phase cycle itself, the time is $5T/8$ and its numerator is 5; that literal convention differs from the distance interpretation underlying the supplied key.

Step 3: Conclusion Using the travelled-path interpretation of the fraction, $t = 7T/12$, so $\boxed{\alpha = 7}$. The phase-fraction interpretation would give $5T/8$; the distinction is necessary because pendulum speed is nonuniform.

Quick Tip to Solve: For a fractional oscillatory path, locate the final position and use its sine phase; multiplying the path fraction by T assumes uniform speed.

54. The zener diode has a V_z = 30V. The current passing through the diode for the following circuit is _____ mA. [4 marks]



Solution:

Concept:

- A reverse-biased ideal zener diode in breakdown maintains its terminal voltage at $V_Z = 30\,\text{V}$. The parallel load therefore has the same voltage.
- Kirchhoff's current law at the junction gives $I_S = I_L + I_Z$: current through the series resistor divides between the load and the zener.

Step 1: Check breakdown and find the series current The source is $90\,\text{V}$, the series resistance is $4\,\text{k}\Omega$, and the parallel load is $5\,\text{k}\Omega$. With the zener absent, the voltage-divider output would be

$$V_{\text{open}} = 90\frac{5}{4+5} = 50\,\text{V}.$$

This exceeds the breakdown voltage, so assuming regulation at $30\,\text{V}$ is consistent. The series resistor then drops $90 - 30 = 60\,\text{V}$, giving

$$I_S = \frac{60}{4000} = 0.015\,\text{A} = 15\,\text{mA}.$$

Step 2: Subtract the load current The load current is determined by its own resistance and the regulated voltage:

$$I_L = \frac{30}{5000} = 0.006\,\text{A} = 6\,\text{mA}.$$

The remaining current flows through the zener:

$$I_Z = I_S - I_L = 15 - 6 = 9\,\text{mA}.$$

This positive result confirms the breakdown assumption. Taking the entire series current as the diode current would neglect the load branch; dividing the supply voltage directly by the load resistance would neglect the series voltage drop.

Step 3: Conclusion The current passing through the zener diode is $\boxed{9\,\text{mA}}$, so the required entry is $\boxed{9}$.

Quick Tip to Solve: First set the parallel-branch voltage equal to the zener voltage; calculate the source and load currents separately before subtracting.

55. The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The workdone when temperature changes by 90K will be xR. The value of x is _____. [R = universal gas constant] [4 marks]

Solution:

Concept:

- For a quasistatic ideal-gas process, work done by the gas is $W = \int P\,dV$. The gas law for n moles is $PV = nRT$.
- The specified relation $V = KT^{2/3}$ fixes the path in the pressure-volume plane. Differentiating its logarithm gives $dV/V = (2/3)\,dT/T$, with constant K.

Step 1: Express the work increment using temperature Differentiate the given relation before inserting the pressure. This avoids treating pressure as constant when it changes along the process:

$$dV = \frac{2}{3}KT^{-1/3}\,dT = \frac{2V}{3T}\,dT.$$

Using $P = nRT/V$, the differential work is

$$dW = \frac{nRT}{V}\frac{2V}{3T}\,dT = \frac{2}{3}nR\,dT.$$

Both the volume and temperature factors cancel. The monatomic character would determine the internal-energy change but is not needed to compute this path-dependent work.

Step 2: Integrate and identify the molar result For a temperature increase of $90\,\mathrm{K}$,

$$W = \frac{2}{3}nR(T_f - T_i) = 60nR.$$

Thus the work per mole is $60R$, corresponding to the keyed coefficient $x = 60$ for one mole. Strictly, the statement specifies only a given mass and does not state its mole number; for an arbitrary amount the coefficient is $60n$. A temperature decrease would reverse the sign of the work. The positive coefficient in the supplied answer therefore uses a temperature rise and the one-mole interpretation.

Step 3: Conclusion The molar work for a $90\,\mathrm{K}$ rise is $60R$; on the one-mole interpretation the requested value is $\boxed{x = 60}$. For an unspecified n-mole sample, the general result is $W = 60nR$.

Quick Tip to Solve: Use logarithmic differentiation of V proportional to a power of T: the work per mole becomes that power times R times the temperature change.

56. 27 similar drops of mercury are maintained at 10V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is _____ times that of a smaller drop. [4 marks]

Solution:

Concept:

- An isolated conducting spherical drop of radius r has capacitance $C = 4\pi\varepsilon_0 r$, potential $V = q/(4\pi\varepsilon_0 r)$, and self-energy $U = q^2/(8\pi\varepsilon_0 r)$.
- On coalescence, total mercury volume and total charge are conserved. Electrostatic energy need not be conserved: the change can be balanced by surface energy and other energy exchanges.

Step 1: Find the final radius and charge Let one small drop have radius r and charge q. The equal initial potentials imply equal charges because all drops have equal radii. On merging 27 identical spheres, volume conservation gives

$$\frac{4}{3}\pi R^3 = 27\frac{4}{3}\pi r^3, \qquad R = 3r.$$

Charge conservation independently gives $Q = 27q$. The final potential is consequently

$$V_{\text{big}} = \frac{27q}{4\pi\varepsilon_0(3r)} = 9V = 90\,\text{V}.$$

It does not stay at the initial value of $10\,\text{V}$.

Step 2: Compare the self-energies Use the charge-squared-over-radius expression for each sphere:

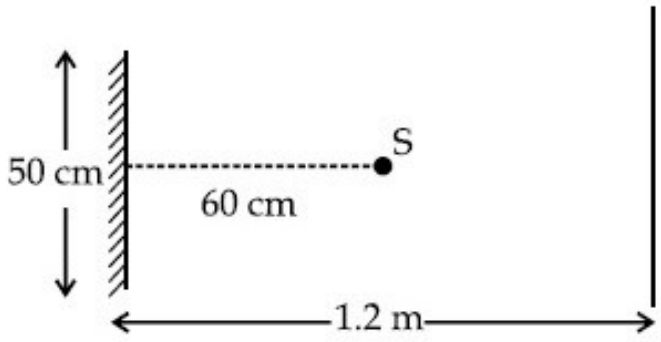
$$\frac{U_{\text{big}}}{U_{\text{small}}} = \frac{Q^2/R}{q^2/r} = \frac{27^2}{3} = 243.$$

Equivalently, $U = \frac{1}{2}qV$ gives the ratio $27 \times 9 = 243$. The comparison requested is with one smaller drop, not the sum of all 27 initial energies; relative to that sum, the ratio would instead be $243/27 = 9$. The numerical starting voltage cancels in the requested dimensionless ratio.

Step 3: Conclusion The self-potential energy of the larger drop is $\boxed{243}$ times that of one smaller drop.

Quick Tip to Solve: For N equal drops, the radius scales as N to the one-third power and the charge as N; hence the self-energy ratio to one drop is N to the five-thirds power.

57. A point source of light S, placed at a distance 60cm in front of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is _____ cm. [4 marks]



Solution:

Concept:

- A plane mirror forms a virtual image as far behind its plane as the object lies in front. An observer sees this image only if the line from the eye to the virtual image intersects the reflecting part of the mirror.
- The limiting eye positions correspond to rays through the two edges of the mirror. Similar triangles give the visible interval on a line parallel to the mirror; all distances must be expressed in the same unit.

Step 1: Locate the virtual image The point source is centred on the mirror and is 60 cm in front of it. Its virtual image therefore lies 60 cm behind the mirror, on the normal through its centre. The observer walks along a line 1.2 m, or 120 cm, in front of the mirror. Thus the perpendicular separation between the image and the observer line is $60 + 120 = 180\,\text{cm}$.

Step 2: Use the two edge rays Join the virtual image to the upper and lower edges of the 50 cm wide mirror and extend these straight lines to the observer line. These lines bound every position from which a reflected ray can reach the eye. The triangle with its base on the mirror has height 60 cm, whereas the larger similar triangle ending at the observer line has height 180 cm. If its base is L, then

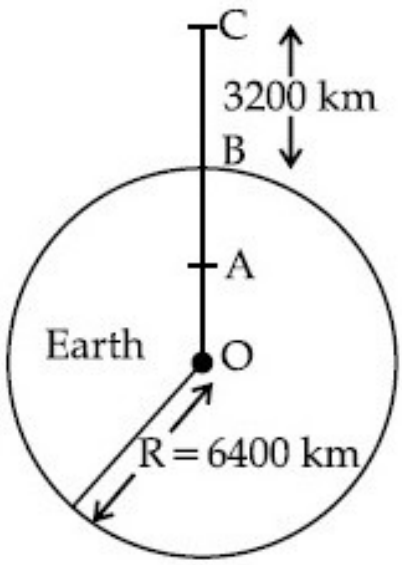
$$\frac{L}{50} = \frac{180}{60} = 3, \qquad L = 150\,\text{cm}.$$

The source is centred, so the two limits lie 75 cm on either side of the central normal. Beyond either limit, the required ray would strike the wall outside the mirror.

Step 3: Conclusion The distance between the extreme positions is $\boxed{150\,\text{cm}}$. The required numerical entry is 150.

Quick Tip to Solve: Construct the virtual image first, then project both mirror edges from it onto the observer line; the enlargement factor is total image-to-eye distance divided by image-to-mirror distance.

58. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is _____. [4 marks]



Solution:

Concept:

- For the standard uniform-density Earth model, gravitational acceleration inside the Earth varies linearly with distance from the centre: $g(r) = g_B r/R$, where R is the Earth radius and $g_B = GM/R^2$ is its surface value.
- Outside a spherical Earth, its mass acts at the centre: $g(r) = GM/r^2 = g_B(R/r)^2$. An altitude must be added to the radius before applying this inverse-square relation.

Step 1: Calculate gravity at C The figure gives $R = 6400\,\text{km}$ and $BC = 3200\,\text{km}$. Hence the distance of C from the centre is

$$OC = OB + BC = 6400 + 3200 = 9600\,\text{km} = \frac{3}{2}R.$$

Therefore

$$\frac{g_C}{g_B} = \left(\frac{R}{OC}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

This is smaller than surface gravity, as required by the statement.

Step 2: Find the interior radius and its complementary depth Write $OA = r$. The specified equality $g_A = g_C$, together with the interior linear dependence, gives

$$g_B\frac{r}{R} = \frac{4}{9}g_B \quad \Rightarrow \quad OA = \frac{4}{9}R.$$

Point A lies on radius OB, so its depth below B is

$$AB = OB - OA = R - \frac{4}{9}R = \frac{5}{9}R.$$

Consequently $OA : AB = (4R/9) : (5R/9) = 4 : 5$. The ratio concerns the interior radius and the remaining part of the radius, rather than the altitude of C. Both interior and exterior values are indeed $4g_B/9$, which independently checks the equality.

Step 3: Conclusion In lowest integer terms, $OA : AB = 4 : 5 = x : y$. Hence $\boxed{x = 4}$.

Quick Tip to Solve: At a given exterior altitude, find g/g_B first; the same fraction is the interior radius OA/R, and its complement gives the depth fraction.

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59. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is x : y. The value of x is _____. [4 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_\gamma - \phi$, where ϕ is the metal's work function and E_γ is the incident photon energy.
- The maximum nonrelativistic electron speed obeys $K_{\max} = \frac{1}{2}m_e v_{\max}^2$. Thus a speed ratio is the square root of the corresponding kinetic-energy ratio, not the photon-energy ratio.

Step 1: Subtract the work function in each case Let the work function of the same metal be $\phi > 0$. Both photon energies exceed threshold, so photoelectrons are emitted in both exposures. For the first stream $E_1 = 2\phi$, whereas for the second $E_2 = 10\phi$. The largest available kinetic energies are therefore

$$K_1 = 2\phi - \phi = \phi, \qquad K_2 = 10\phi - \phi = 9\phi.$$

The work function must be subtracted separately from each photon energy, because part of each photon's energy is spent liberating an electron from the surface.

Step 2: Convert the energy ratio into a velocity ratio The emitted particles in both cases are electrons, so their mass is the same. Using their maximum speeds gives

$$\frac{v_1}{v_2} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{\phi}{9\phi}} = \frac{1}{3}.$$

Equivalently, $v_1 = \sqrt{2\phi/m_e}$ and $v_2 = 3\sqrt{2\phi/m_e}$. The ratio is dimensionless, and neither the metal's unknown work function nor the electron mass needs a numerical value. A ratio $1 : 5$ would incorrectly use photon energies directly; a ratio $1 : 9$ would incorrectly equate speed with kinetic energy.

Step 3: Conclusion The maximum velocities in the stated order are $v_1 : v_2 = 1 : 3 = x : y$. Thus the required entry is $\boxed{x = 1}$.

Quick Tip to Solve: For photon energies $a\phi$ and $b\phi$ on the same metal, use $v_1 : v_2 = \sqrt{a-1} : \sqrt{b-1}$.

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60. If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are _____. [4 marks]

Solution:

Concept:

- An amplitude-modulated carrier at frequency f_c has an upper sideband extending to $f_c + f_{m,\max}$ and a lower sideband extending to $f_c - f_{m,\max}$, where $f_{m,\max}$ is the highest modulating frequency.
- The required bandwidth of an ordinary double-sideband AM channel is $B = 2f_{m,\max}$. Separate stations need nonoverlapping channels, so their number is the available bandwidth divided by the bandwidth per station.

Step 1: Find the bandwidth occupied by one station The highest modulating frequency is $5\,\text{kHz}$. It produces a frequency extent of 5 kHz on each side of the carrier. Hence the complete AM spectrum spans

$$B = (f_c + 5\,\text{kHz}) - (f_c - 5\,\text{kHz}) = 10\,\text{kHz}.$$

The carrier lies inside this interval and does not require an additional separate 5 kHz channel. Taking only one sideband would give too small a bandwidth for conventional AM transmission.

Step 2: Fit independent channels into the available band The total available band is $90\,\text{kHz}$. Assuming the ideal channel allocation implicit in the question, with no extra guard intervals specified, the maximum number of channels is

$$N = \left\lfloor \frac{90\,\text{kHz}}{10\,\text{kHz}} \right\rfloor = 9.$$

The units cancel before counting stations. Nine channels together occupy $9 \times 10 = 90\,\text{kHz}$, exactly the available bandwidth, whereas a tenth would require 100 kHz and cannot fit. Using $90/5 = 18$ would count only half of each station's two-sided spectrum.

Step 3: Conclusion The number of AM broadcast stations accommodated is $\boxed{9}$.

Quick Tip to Solve: For conventional AM, double the highest audio frequency before dividing the available broadcast bandwidth; take the integer part if the division is not exact.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Match List - I with List - II.

	List - I (Molecule)		List - II (Bond order)
(a)	Ne_2	**(i)**	**1**
(b)	N_2	**(ii)**	**2**
(c)	F_2	**(iii)**	**0**
(d)	O_2	**(iv)**	**3**

Choose the correct answer from the options given below : [4 marks]

(A) (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)

(B) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

(C) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

(D) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

Correct Answer: (C) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

Solution:

Concept:

- Molecular orbital bond order is $\mathrm{BO} = (N_b - N_a)/2$, where N_b and N_a count electrons in bonding and antibonding orbitals. Equal populations give zero net bond order.
- For second-period homonuclear diatomic species, core bonding and antibonding electrons cancel. The net bond order can therefore be calculated from valence molecular orbitals; filling antibonding orbitals reduces it.

Step 1: Determine the four bond orders For $\mathrm{Ne_2}$, all valence bonding and antibonding orbitals are filled: $N_b = N_a = 8$, so its bond order is zero. For $\mathrm{N_2}$, the valence counts are $N_b = 8$, $N_a = 2$, giving $(8-2)/2 = 3$. For $\mathrm{F_2}$, the additional electrons fill antibonding orbitals: $N_b = 8$, $N_a = 6$, giving bond order 1. For $\mathrm{O_2}$, $N_b = 8$, $N_a = 4$, giving bond order 2. The zero for the hypothetical neon dimer refers to its elementary MO bond order, not to a normal covalent neon molecule.

Step 2: Match each result to the printed list List II assigns (i) to 1, (ii) to 2, (iii) to 0 and (iv) to 3. Hence the complete mapping is

$$(a)\ \mathrm{Ne_2} \longrightarrow (iii),$$
$$(b)\ \mathrm{N_2} \longrightarrow (iv),$$
$$(c)\ \mathrm{F_2} \longrightarrow (i),$$
$$(d)\ \mathrm{O_2} \longrightarrow (ii).$$

Only option (C) gives all four pairs. Option (A) incorrectly assigns neon a single bond, (B) assigns nitrogen zero bond order, and (D) assigns nitrogen a single bond and fluorine a triple bond. These fail the electron population calculation.

Step 3: Conclusion The bond orders of neon, nitrogen, fluorine and oxygen dimers are respectively 0, 3, 1 and 2. Hence, the correct option is (C).

Quick Tip to Solve: Remember the sequence $\mathrm{N_2}, \mathrm{O_2}, \mathrm{F_2}, \mathrm{Ne_2}$: their elementary MO bond orders decrease as $3, 2, 1, 0$.

62. Ceric ammonium nitrate and $CHCl_3$/alc. KOH are used for the identification of functional groups present in _____ and _____ respectively. [4 marks]

(A) amine, alcohol

(B) alcohol, phenol

(C) alcohol, amine

(D) amine, phenol

Correct Answer: (C) alcohol, amine

Solution:

Concept:

- Ceric ammonium nitrate is a qualitative reagent for the alcoholic hydroxyl group. Alcohols give the characteristic red coloration with the yellow ceric ammonium nitrate reagent.
- Chloroform and alcoholic potassium hydroxide constitute the carbylamine test for primary amines. Heating produces an isocyanide with a characteristic unpleasant odour; secondary and tertiary amines do not give this test.

Step 1: Identify the group tested by the first reagent The first blank must correspond to an alcohol, since ceric ammonium nitrate is used as an alcohol test. The characteristic colour change involves interaction of the alcoholic oxygen with the cerium reagent. Therefore options (A) and (D), which put amine first, cannot give the correct order. The question asks for functional groups in the same sequence as the reagents, so the order of the two entries must be retained.

Step 2: Identify the group tested by the second reagent The combination $CHCl_3$ and alcoholic KOH, on heating with a primary amine, produces an isocyanide:

$$RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O.$$

Thus the second blank is amine, more precisely a primary amine. This is the carbylamine reaction, not a general alcohol identification test. Option (B) puts phenol second, but the familiar chloroform/alkali reaction of phenol is the Reimer–Tiemann formylation under its appropriate conditions, not the primary-amine identification reaction specified here. Among the listed pairs, only (C) has alcohol followed by amine. Both aliphatic and aromatic primary amines can respond to the carbylamine test.

Step 3: Conclusion The functional groups are alcohol and amine, respectively; the second test specifically identifies primary amines. Hence, the correct option is (C).

Quick Tip to Solve: Associate ceric ammonium nitrate with alcohol coloration and $CHCl_3$/alcoholic KOH with the primary-amine carbylamine test.

63. In $\overset{1}{C}H_2 = \overset{2}{C} = \overset{3}{C}H - \overset{4}{C}H_3$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively, are : [4 marks]

(A) sp^2, sp, sp^2, sp^3

(B) sp^2, sp^2, sp^2, sp^3

(C) sp^3, sp, sp^3, sp^3

(D) sp^2, sp^3, sp^2, sp^3

Correct Answer: (A) sp^2, sp, sp^2, sp^3

Solution:

Concept:

- The hybridisation of a carbon is found from its number of sigma-bond directions: two give sp, three give sp^2, and four give sp^3. Each double bond contributes one sigma bond and one pi bond.
- The middle carbon of a cumulated pair of double bonds uses two perpendicular unhybridised p orbitals for the two pi bonds. Its two sigma bonds are formed with sp hybrid orbitals and have a linear arrangement.

Step 1: Count the bonding directions at carbons 1 and 2 The numbered chain is $CH_2{=}C{=}CH{-}CH_3$, with carbon numbers 1 through 4 from left to right. Carbon 1 has two C–H sigma bonds and one C–C sigma bond within its double bond. Its three sigma directions require sp^2 hybridisation; its remaining p orbital makes the pi bond to carbon 2. Carbon 2 is double-bonded to both carbon 1 and carbon 3. It has only two sigma directions and uses two unhybridised p orbitals for the two separate pi bonds, so it is sp, not sp^2 or sp^3.

Step 2: Complete the chain and compare the options Carbon 3 has a double bond to carbon 2, a single bond to carbon 4 and a single bond to hydrogen. These are three sigma directions, so carbon 3 is sp^2. Carbon 4 is the methyl carbon with three C–H bonds and one C–C bond, all sigma bonds, giving sp^3. Therefore the ordered sequence is

$$sp^2, \quad sp, \quad sp^2, \quad sp^3.$$

Only option (A) has this sequence. Option (B) incorrectly makes the central carbon trigonal, (C) incorrectly makes the terminal double-bonded carbons tetrahedral, and (D) gives the central carbon four sigma directions despite there being only two.

Step 3: Conclusion Carbons 1, 2, 3 and 4 have hybridisations sp^2, sp, sp^2, sp^3, respectively. Hence, the correct option is (A).

Quick Tip to Solve: For a cumulated diene, count sigma-bond directions rather than total bond lines: the carbon shared by both double bonds is sp.

64. **A. Phenyl methanamine**

B. N,N-Dimethylaniline

C. N-Methyl aniline

D. Benzenamine

Choose the correct order of basic nature of the above amines. [4 marks]

(A) D > C > B > A

(B) A > C > B > D

(C) D > B > C > A

(D) A > B > C > D

Correct Answer: (D) A > B > C > D

Solution:

Concept:

- An amine is basic because the lone pair on nitrogen can accept a proton. Greater availability of that pair generally means greater basic strength.
- In anilines the nitrogen lone pair is conjugated with the benzene ring; an intervening saturated carbon prevents this delocalisation in benzylamine.

Step 1: Separate the aliphatic amine Phenylmethanamine is benzylamine, $C_6H_5CH_2NH_2$. Its nitrogen is attached to a saturated CH_2 carbon, not directly to the aromatic ring. Its lone pair is therefore not tied up in ring resonance. It is more basic than each of the three anilines, so A must come first; this immediately excludes the orders beginning with D.

Step 2: Compare the substituted anilines Benzenamine D is aniline, $C_6H_5NH_2$. An N-methyl group supplies electron density through its positive inductive effect, increasing the availability of the nitrogen pair relative to aniline. N,N-dimethylaniline B has two such methyl groups, while N-methylaniline C has one. For these specified amines the order is $B > C > D$. The benzyl group in A does not make it an aniline: the location of the nitrogen is crucial. Combining the comparisons gives $A > B > C > D$, rather than the sequence that places C above B.

Step 3: Conclusion The basicity order is A > B > C > D. Hence, the correct option is (D).

Quick Tip to Solve: Check whether nitrogen is directly bonded to the aromatic ring before counting its alkyl substituents.

65. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : In TlI_3, isomorphous to CsI_3, the metal is present in +1 oxidation state.

Reason R : Tl metal has fourteen f electrons in its electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is correct but R is not correct

(B) A is not correct but R is correct

(C) Both A and R are correct but R is NOT the correct explanation of A

(D) Both A and R are correct and R is the correct explanation of A

Correct Answer: (C) Both A and R are correct but R is NOT the correct explanation of A

Solution:

Concept:

- Oxidation state follows charge balance after identifying the actual ions. A polyiodide ion must not be replaced by three separate iodide ions.
- Thallium has electronic configuration $[\mathrm{Xe}]\,4f^{14}5d^{10}6s^{2}6p^{1}$, so the statement about fourteen f electrons is true.

Step 1: Evaluate the assertion The given isomorphism with CsI_3 identifies a triiodide structure. Just as caesium triiodide contains Cs^+ and $I_3{}^-$, thallium triiodide is represented as $Tl^+\,I_3{}^-$. The total charge of the three-iodine unit is minus one, not minus three. Charge neutrality therefore requires thallium to be in oxidation state +1. Assertion A is correct.

Step 2: Evaluate the reason The filled $4f$ subshell in the ground-state configuration contains fourteen electrons. Reason R is consequently correct as a statement about the atom. It is not a direct identification of the anion in this compound.

Step 3: Test the explanatory link The +1 assignment in this particular formula is established by the polyiodide constitution, supported by the stated isomorphism. Merely counting f electrons does not show that iodine occurs as $I_3{}^-$, and cannot by itself establish the oxidation state from the empirical formula. Although inert-pair effects help explain the stability of Tl(I) more generally, R as worded is not the correct explanation of the assertion. The alternatives declaring either statement false must be rejected.

Step 4: Conclusion Both A and R are correct, but R is not the correct explanation of A. Hence, the correct option is (C).

Quick Tip to Solve: For a triiodide salt, first assign the charge of the whole $I_3{}^-$ ion; do not assign three independent iodide ions.

66. Which of the following forms of hydrogen emits low energy β^- particles? [4 marks]

(A) Tritium ${}^{3}_{1}\mathrm{H}$

(B) Proton H^{+}

(C) Protium ${}^{1}_{1}\mathrm{H}$

(D) Deuterium ${}^{2}_{1}\mathrm{H}$

Correct Answer: (A) Tritium ${}^{3}_{1}\mathrm{H}$

Solution:

Concept:

- Isotopes have the same atomic number but different neutron numbers. Protium, deuterium and tritium all have one proton, with zero, one and two neutrons respectively.
- In beta-minus decay a neutron changes into a proton with emission of an electron and an electron antineutrino; the mass number remains unchanged.

Step 1: Identify the radioactive isotope Tritium, ${}^{3}_{1}\mathrm{H}$, is the radioactive isotope among the listed forms of hydrogen. Its nucleus contains one proton and two neutrons. It decays with emission of low-energy β^- particles, producing helium-3. The emitted beta particle is an electron created in the nuclear transition; it is not an orbital electron stripped from a hydrogen atom.

Step 2: Write the decay and exclude the other forms The nuclear equation is

$$ {}^{3}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}_e. $$

The charge numbers balance as $1 = 2 - 1$, and the mass numbers balance as $3 = 3 + 0$. Protium and deuterium are stable hydrogen isotopes and do not undergo this low-energy beta-minus decay. A proton, H^{+}, has no neutron and is not the radioactive hydrogen isotope requested. Thus neither ionisation of hydrogen nor merely having an extra neutron is sufficient: the relevant isotope is specifically tritium.

Step 3: Conclusion Tritium, ${}^{3}_{1}\mathrm{H}$, emits low-energy beta-minus particles. Hence, the correct option is (A).

Quick Tip to Solve: Use beta-minus bookkeeping: the mass number stays fixed while the atomic number increases by one.

67. Which pair of oxides is acidic in nature? [4 marks]

(A) CaO, SiO_2

(B) B_2O_3, CaO

(C) B_2O_3, SiO_2

(D) N_2O, BaO

Correct Answer: (C) B_2O_3, SiO_2

Solution:

Concept:

- Acidic oxides react with bases to form salts. Oxides of nonmetals and some metalloids commonly show this behaviour.
- Metal oxides such as CaO and BaO are basic, whereas N_2O is classified as a neutral oxide.

Step 1: Test boron and silicon oxides Boron oxide B_2O_3 and silicon dioxide SiO_2 are both acidic oxides. Their reactions with sodium hydroxide illustrate the classification:

$$B_2O_3 + 2\,NaOH \longrightarrow 2\,NaBO_2 + H_2O$$

$$SiO_2 + 2\,NaOH \longrightarrow Na_2SiO_3 + H_2O.$$

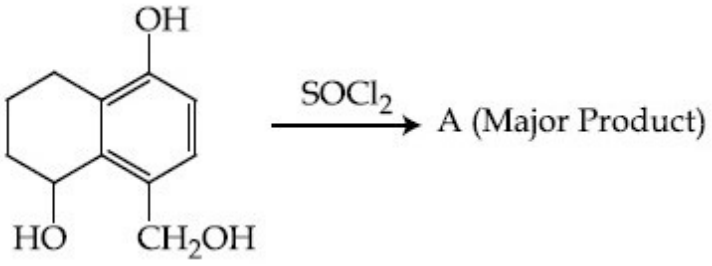
Each consumes a base to form the corresponding salt. An oxide need not dissolve readily in water to count as acidic; silica is an important example.

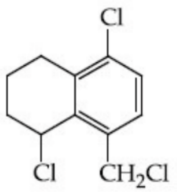
Step 2: Check every proposed pair The first pair contains CaO, a basic oxide, so the presence of acidic silica does not make both members acidic. The second pair again contains basic CaO, despite its acidic boron oxide. The third pair contains the two acidic oxides already verified. The final pair consists of neutral N_2O and basic BaO, so neither member qualifies as an acidic oxide here. In particular, the fact that an oxide contains nitrogen does not automatically make it acidic: its oxidation state and chemistry matter.

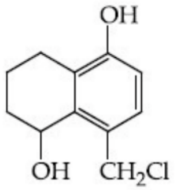
Step 3: Conclusion The acidic pair is B_2O_3 and SiO_2. Hence, the correct option is (C).

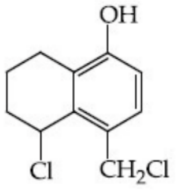
Quick Tip to Solve: Test both members of each pair; one acidic oxide alongside a basic oxide does not satisfy the question.

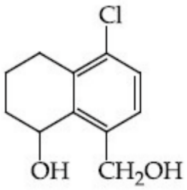
68. Identify A in the given reaction. [4 marks]



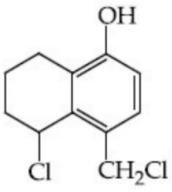
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Thionyl chloride converts an aliphatic alcohol into the corresponding alkyl chloride: $\mathrm{ROH} + \mathrm{SOCl_2} \longrightarrow \mathrm{RCl} + \mathrm{SO_2} + \mathrm{HCl}$.
- An aromatic phenolic C–O bond does not undergo the same ordinary alcohol-to-chloride substitu-

tion; distinguish phenolic OH from an OH on a saturated carbon.

Step 1: Classify the three hydroxyl groups The upper OH group is bonded directly to a carbon of the benzene ring and is phenolic. The lower-left OH is on a saturated carbon of the fused ring: it is an aliphatic, benzylic secondary alcohol. The CH_2OH substituent is a primary benzylic alcohol. Although all three contain an O–H bond, their carbon–oxygen bonds do not have the same reactivity under the specified conditions.

Step 2: Apply substitution only at the alcohol sites The saturated-ring $C-OH$ group is replaced by $C-Cl$, and the primary CH_2OH substituent becomes CH_2Cl. The phenolic OH remains unchanged because its aryl C–O bond has partial double-bond character and ordinary nucleophilic replacement at the aromatic carbon is not the alcohol reaction of thionyl chloride. The carbon skeleton and aromatic ring remain intact.

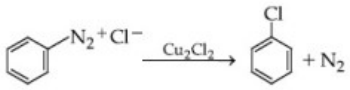
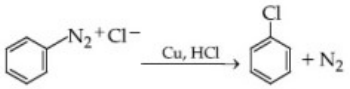
Step 3: Compare the four structures The required structure must therefore retain the upper phenolic OH and show chlorine at both aliphatic alcohol positions. The first and fourth structures incorrectly replace the phenolic OH by Cl. The second structure retains the saturated-ring OH, so it fails to show reaction at both ordinary alcohol sites. Only the third structure has the required selective pattern.

Step 4: Conclusion The phenolic OH remains, while the secondary alcohol and CH_2OH group become chlorides. Hence, the correct option is (C).

Quick Tip to Solve: Circle OH groups attached to saturated carbons first; thionyl chloride replaces those without replacing the phenolic OH.

69. Match List - I with List - II.

Choose the correct answer from the options given below : [4 marks]

List-I		List-II	
(a)		(i)	Wurtz reaction
(b)		(ii)	Sandmeyer reaction
(c)	$2CH_3CH_2Cl + 2Na \xrightarrow{\text{Ether}} C_2H_5 - C_2H_5 + 2NaCl$	(iii)	Fittig reaction
(d)	$2C_6H_5Cl + 2Na \xrightarrow{\text{Ether}} C_6H_5 - C_6H_5 + 2NaCl$	(iv)	Gatterman reaction

(A) (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)

(B) (a) → (iii), (b) → (iv), (c) → (i), (d) → (ii)

(C) (a) → (iii), (b) → (i), (c) → (iv), (d) → (ii)

(D) (a) → (ii), (b) → (i), (c) → (iv), (d) → (iii)

Correct Answer: (A) (a) → (ii), (b) → (iv), (c) → (i), (d) → (iii)

Solution:

Concept:

- Diazonium salts undergo replacement of the diazonium group with loss of nitrogen. The reagent distinguishes Sandmeyer and Gattermann reactions.
- Coupling of alkyl halides with sodium in dry ether is the Wurtz reaction; coupling of aryl halides under these conditions is the Fittig reaction.

Step 1: Identify the diazonium substitutions In reaction (a), benzenediazonium chloride reacts with cuprous chloride, written as Cu_2Cl_2, giving chlorobenzene and nitrogen. This is the Sandmeyer reaction, so (a) matches (ii). In reaction (b), copper powder and hydrochloric acid replace the diazonium group by chlorine. These reagents identify the Gattermann reaction, so (b) matches (iv).

Step 2: Identify the sodium couplings Reaction (c) couples two ethyl groups:

$$2\,C_2H_5Cl + 2\,Na \longrightarrow C_2H_5{-}C_2H_5 + 2\,NaCl.$$

An alkyl halide is used, so it is the Wurtz reaction, giving (c) with (i). Reaction (d) couples two phenyl groups from chlorobenzene to form biphenyl:

$$2\,C_6H_5Cl + 2\,Na \longrightarrow C_6H_5{-}C_6H_5 + 2\,NaCl.$$

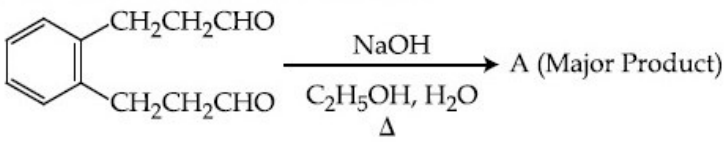
This aryl-halide coupling is the Fittig reaction, giving (d) with (iii).

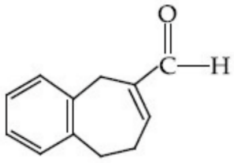
Step 3: Compare the complete matching The required sequence is therefore (ii), (iv), (i), (iii). The other options interchange named substitutions or misidentify alkyl and aryl coupling. Only option (A) has all four matches.

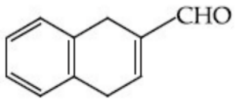
Step 4: Conclusion The sequence of matches is (ii), (iv), (i), (iii), in the order of List I. Hence, the correct option is (A).

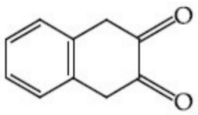
Quick Tip to Solve: First separate diazonium substitution from sodium coupling; then use cuprous chloride versus Cu/HCl, and alkyl versus aryl halide.

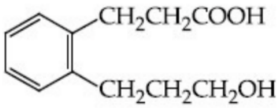
70. Identify A in the following chemical reaction. [4 marks]



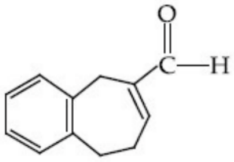
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- An aldehyde with alpha hydrogen forms an enolate in dilute base. The enolate can attack another aldehyde group in the same molecule to form a new carbon–carbon bond.
- Heating an aldol product promotes loss of water to give a conjugated alpha,beta-unsaturated aldehyde. Count the newly formed ring before choosing a product.

Step 1: Locate the reacting carbon atoms The benzene ring carries adjacent $-CH_2CH_2CHO$ side chains. Label one chain ring–a–b–c, where c is its aldehyde carbon, and the other ring–d–e–f, where f is its aldehyde carbon. Hydroxide removes an alpha hydrogen at b, producing an enolate. This nucleophilic carbon attacks aldehyde carbon f of the other side chain.

Step 2: Count the ring produced by cyclisation The new bond is $b - f$. The closed path passes through b, a, the two adjacent aromatic carbons, d, e, f, and back to b: it therefore makes a seven-membered ring fused to benzene. The original aldehyde carbon c is not in this ring; it remains as a formyl substituent on b. This atom count distinguishes the first structure from the tempting six-membered fused-ring alternative.

Step 3: Dehydrate the aldol product After protonation of the alkoxide, heating removes water from the beta-hydroxy aldehyde. The resulting double bond lies between b and f, adjacent to the surviving formyl group. No oxidising reagent is present to produce the diketone option, and these enolisable aldehydes favour aldol condensation rather than the acid/alcohol disproportionation shown in the last structure. The structure with a seven-membered unsaturated ring and a formyl substituent is the first option.

Step 4: Conclusion The product is the seven-membered benzene-fused alpha,beta-unsaturated aldehyde shown in the first structure. Hence, the correct option is (A).

Quick Tip to Solve: Trace the path closed by the new enolate-to-carbonyl bond; count every atom before selecting a fused-ring aldol product.

71. 2, 4-DNP test can be used to identify : [4 marks]

(A) ether

(B) amine

(C) aldehyde

(D) halogens

Correct Answer: (C) aldehyde

Solution:

Concept:

- 2,4-Dinitrophenylhydrazine (2,4-DNP) detects the carbonyl group of aldehydes and ketones by condensation to a coloured hydrazone.
- The terminal amino nitrogen attacks the electrophilic carbonyl carbon, and elimination of water produces a carbon–nitrogen double bond.

Step 1: Identify the reactive group An aldehyde contains the group $\mathrm{R{-}CHO}$, whose carbonyl carbon is electrophilic because oxygen attracts the bonding electrons. The reagent $\mathrm{H_2N{-}NH{-}C_6H_3(NO_2)_2}$ is a substituted hydrazine. Its terminal nitrogen supplies the nucleophile required for addition to this carbonyl group. Thus an aldehyde has precisely the functional group needed for a positive test.

Step 2: Write the condensation and exclude the alternatives The net transformation is

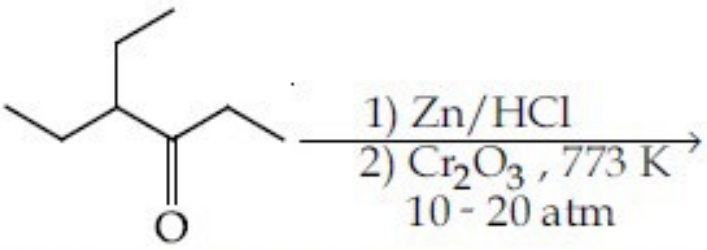
$$\mathrm{R{-}CHO + H_2N{-}NH{-}C_6H_3(NO_2)_2 \longrightarrow R{-}CH{=}N{-}NH{-}C_6H_3(NO_2)_2 + H_2O.}$$

The yellow, orange or red precipitate is the corresponding 2,4-dinitrophenylhydrazone. An ether has a carbon–oxygen–carbon linkage but no carbonyl group; an amine has an amino group; and halogens do not provide an aldehydic or ketonic carbonyl. Those three alternatives therefore do not give this diagnostic condensation. Ketones also respond, but no ketone is among the listed choices.

Step 3: Conclusion Among the given substances, an aldehyde gives the 2,4-DNP test. Hence, the correct option is (C).

Quick Tip to Solve: Look for an aldehydic or ketonic carbonyl before applying the DNP test; it does not distinguish an aldehyde from a ketone.

72. Considering the above reaction, the major product among the following is : [4 marks]



(A) Ethylbenzene

(B) 1,2-Dimethylbenzene

(C) Propiophenone (1-phenylpropan-1-one)

(D) Propylbenzene

Correct Answer: (A) Ethylbenzene

Solution:

Concept:

- Zinc and hydrochloric acid reduce the ketonic carbonyl group to a methylene group without changing the carbon skeleton.
- Heating a suitable six-carbon chain over Cr_2O_3 causes dehydrocyclisation: ring closure followed by loss of hydrogen gives an aromatic ring.

Step 1: Reduce the carbonyl group Read the carbon framework as $CH_3CH_2CH(CH_2CH_3)COCH_2CH_3$. The oxygen belongs to a ketone, rather than to a substituent on an already aromatic ring. In the first treatment its CO unit is converted into CH_2, giving $CH_3CH_2CH(CH_2CH_3)CH_2CH_2CH_3$, namely 3-ethylhexane. All eight carbon atoms are retained; removal of oxygen does not remove the carbonyl carbon.

Step 2: Close the six-membered ring and compare the products The six-carbon continuous chain undergoes aromatisation at 773 K over Cr_2O_3. The ethyl branch attached to its third carbon remains an ethyl substituent on the new benzene ring:

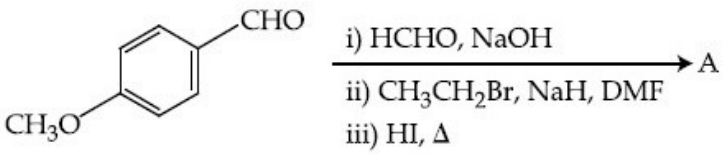
$$C_8H_{18} \longrightarrow C_6H_5CH_2CH_3 + 4\,H_2.$$

Either equivalent ethyl arm at the branching carbon can form the same six-carbon path, so this leads to ethylbenzene. Option (B), the dimethylbenzene, would require a different distribution of the branches. Option (C) retains oxygen despite the initial reduction, while option (D) has nine carbons and violates carbon conservation.

Step 3: Conclusion The major aromatic product is ethylbenzene, $C_6H_5CH_2CH_3$. Hence, the correct option is (A).

Quick Tip to Solve: First replace the ketonic CO by CH2; only then count the six-carbon chain that becomes the benzene ring.

73. Identify A in the given chemical reaction. [4 marks]



(A) Ethyl 4-hydroxybenzoate

(B) 4-Methoxybenzyl alcohol

(C) 4-Hydroxybenzyl alcohol

(D) 4-Hydroxybenzyl iodide

Correct Answer: (D) 4-Hydroxybenzyl iodide

Solution:

Concept:

- An aromatic aldehyde without an alpha hydrogen undergoes crossed Cannizzaro reduction with formaldehyde in concentrated base; formaldehyde is preferentially oxidised to formate.
- Sodium hydride followed by ethyl bromide converts an alcohol into its ethyl ether. Hot HI cleaves both alkyl aryl ethers and benzyl ethers.

Step 1: Follow reduction and ether formation The initial compound is para-methoxybenzaldehyde, $p\text{-}CH_3OC_6H_4CHO$. With $HCHO$ and $NaOH$, its aldehyde group is reduced to CH_2OH, while formaldehyde supplies the oxidation partner. This gives para-methoxybenzyl alcohol, the structure shown in option (B), but that is only the first intermediate. NaH removes the alcohol proton and the resulting alkoxide displaces bromide from CH_3CH_2Br in DMF, yielding $p\text{-}CH_3OC_6H_4CH_2OCH_2CH_3$.

Step 2: Apply hot HI to both ether linkages At the aryl methyl ether, iodide attacks the methyl group after protonation; the aryl–oxygen bond remains, producing a phenolic OH group. At the benzyl ethyl ether, cleavage produces the especially reactive benzyl derivative, and under hot HI the benzylic alcohol or protonated ether is converted into the benzyl iodide. The final substituted benzene therefore carries OH and CH_2I in para positions. Option (A) is an ester and would demand oxidation rather than Cannizzaro reduction. Option (B) stops before etherification and cleavage; option (C) stops at the benzyl alcohol despite the hot HI treatment.

Step 3: Conclusion Product A is para-hydroxybenzyl iodide, $p\text{-}HOC_6H_4CH_2I$. Hence, the correct option is (D).

Quick Tip to Solve: Track both oxygen sites separately: hot HI unmasks the phenol and converts the benzylic ether site into CH2I.

74. The correct order of electron gain enthalpy is : [4 marks]

(A) Te > Se > S > O

(B) S > Se > Te > O

(C) S > O > Se > Te

(D) O > S > Se > Te

Correct Answer: (B) S > Se > Te > O

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for adding an electron to an isolated gaseous atom. For an exothermic addition it has a negative signed value.
- In the order convention used by these options, a greater electron gain means a greater magnitude of energy released; oxygen is anomalous because its compact 2p shell gives strong electron repulsion.

Step 1: Separate the oxygen anomaly Oxygen receives the extra electron in a compact 2p orbital. Repulsion between the incoming electron and the electrons already occupying this small shell reduces the energy released. Sulphur receives the electron in the larger 3p shell, where this crowding is less severe. Consequently sulphur releases more energy on electron addition than oxygen, despite oxygen lying higher in Group 16. This rules out the simple top-to-bottom order in option (D).

Step 2: Order the heavier members and interpret the signs Below sulphur, increasing atomic size and shielding reduce the attraction for the added electron. The magnitude therefore falls from sulphur to selenium to tellurium, while oxygen remains the smallest of these four. Representative signed values in $\mathrm{kJ\,mol^{-1}}$ are approximately $-200, -195, -190, -141$ for S, Se, Te and O respectively. Thus the energy-release magnitude follows S > Se > Te > O. Option (A) reverses the heavier-element trend; option (C) incorrectly places oxygen ahead of selenium and tellurium. Strictly ordering the negative signed enthalpies would reverse the inequalities; the listed choices use the usual magnitude convention.

Step 3: Conclusion The intended decreasing order of electron-gain enthalpy magnitude is S > Se > Te > O. Hence, the correct option is (B).

Quick Tip to Solve: Remember the oxygen exception first, then decrease the energy-release magnitude from S to Se to Te; retain the negative sign when quoting actual enthalpies.

75. Match List - I with List - II.

	List - I		List - II
(a)	**Sucrose**	**(i)**	**β-D-Galactose and β-D-Glucose**
(b)	**Lactose**	**(ii)**	**α-D-Glucose and β-D-Fructose**
(c)	**Maltose**	**(iii)**	**α-D-Glucose and α-D-Glucose**

Choose the correct answer from the options given below : [4 marks]

(A) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (i)

(B) (a) $\rightarrow$ (i), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (ii)

(C) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iii)

(D) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (ii)

Correct Answer: (C) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iii)

Solution:

Concept:

- A disaccharide consists of two monosaccharide units joined through a glycosidic linkage; hydrolysis identifies its constituent sugars.
- Sucrose contains glucose and fructose, lactose contains galactose and glucose, and maltose contains two glucose units.

Step 1: Match sucrose and lactose Sucrose is formed from the anomeric carbon of α-D-glucose and the anomeric carbon of β-D-fructose through an $\alpha(1 \rightarrow 2)\beta$ linkage. Thus (a) matches (ii). Lactose has a $\beta(1 \rightarrow 4)$ linkage joining a galactose unit to a glucose unit, so the galactose–glucose pair in (i) belongs to (b). The free glucose anomeric centre can mutarotate in solution; the list's beta form identifies the stated pair and does not change its monosaccharide identities.

Step 2: Match maltose and eliminate the other codes Maltose contains two D-glucose units connected by an $\alpha(1 \rightarrow 4)$ linkage. Hence the glucose–glucose entry (iii) belongs to (c). Its free reducing end can likewise mutarotate; the given alpha representation is the one used in List II. The full correspondence is

$$(a) \rightarrow (ii), \qquad (b) \rightarrow (i), \qquad (c) \rightarrow (iii).$$

Option (A) assigns two glucose units to sucrose and glucose–fructose to lactose. Option (B) assigns galactose to sucrose. Option (D) gets lactose right but interchanges the sucrose and maltose pairs. Only option (C) keeps all three constitutions correct.

Step 3: Conclusion Sucrose matches (ii), lactose matches (i), and maltose matches (iii). Hence, the correct option is (C).

Quick Tip to Solve: Use the distinguishing unit: fructose identifies sucrose, galactose identifies lactose, and two glucose units identify maltose.

76. Seliwanoff test and Xanthoproteic test are used for the identification of _____ and _____ respectively. [4 marks]

(A) ketoses, proteins

(B) proteins, ketoses

(C) aldoses, ketoses

(D) ketoses, aldoses

Correct Answer: (A) ketoses, proteins

Solution:

Concept:

- Seliwanoff's test distinguishes ketoses from aldoses by their faster acid-catalysed dehydration and reaction with resorcinol.
- The xanthoproteic test detects proteins containing nitration-responsive aromatic amino acid residues, especially tyrosine and tryptophan.

Step 1: Identify the first test Seliwanoff's reagent contains resorcinol and concentrated hydrochloric acid. On heating, a ketose such as fructose rapidly dehydrates to a hydroxymethylfurfural derivative, which condenses with resorcinol to give a characteristic cherry-red colour. Aldoses can react more slowly under prolonged heating, so the test is read promptly. The rapid positive response identifies ketoses, placing ketoses in the first blank rather than proteins or aldoses.

Step 2: Identify the second test and compare the pairs In the xanthoproteic test, concentrated nitric acid nitrates suitable aromatic rings in protein amino acid residues, giving a yellow colour that becomes orange on adding alkali. The test is therefore used for identification of proteins possessing those residues, rather than for identification of a carbohydrate class. The question asks for the two tests in the stated order: Seliwanoff followed by xanthoproteic. Option (B) reverses their functions. Option (C) starts with aldoses and then assigns ketoses to the protein test. Option (D) has the correct first blank but incorrectly identifies the second test with aldoses. Thus only the ketoses–proteins pair is consistent with both chemical reactions.

Step 3: Conclusion The blanks are ketoses and proteins, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Associate Seliwanoff with rapid cherry-red fructose response and xanthoproteic with yellow nitrated aromatic residues in proteins.

77. **Match List - I with List - II.**

	List - I		List - II
(a)	Siderite	(i)	Cu
(b)	Calamine	(ii)	Ca
(c)	Malachite	(iii)	Fe
(d)	Cryolite	(iv)	Al
		(v)	Zn

Choose the correct answer from the options given below : [4 marks]

(A) (a) → (i), (b) → (ii), (c) → (iii), (d) → (iv)

(B) (a) → (iii), (b) → (i), (c) → (v), (d) → (ii)

(C) (a) → (i), (b) → (ii), (c) → (v), (d) → (iii)

(D) (a) → (iii), (b) → (v), (c) → (i), (d) → (iv)

Correct Answer: (D) (a) → (iii), (b) → (v), (c) → (i), (d) → (iv)

Solution:

Concept:

- An ore or mineral is matched to its characteristic metal by writing its chemical formula rather than guessing from its name.
- Siderite is iron carbonate, calamine is zinc carbonate, malachite is basic copper carbonate, and cryolite is sodium hexafluoroaluminate.

Step 1: Match the carbonate minerals Siderite has formula $FeCO_3$, so its characteristic metal is iron: (a) matches (iii). Calamine has formula $ZnCO_3$, hence (b) matches (v), zinc. Malachite is basic copper carbonate, $CuCO_3 \cdot Cu(OH)_2$, also written $Cu_2CO_3(OH)_2$, so (c) matches (i), copper. The carbonate and hydroxide portions do not change which metal is represented by these mineral formulae.

Step 2: Match cryolite and check the complete code Cryolite is Na_3AlF_6, a complex fluoride containing aluminium, giving (d) matches (iv). It is used with alumina in the electrolytic extraction of aluminium to lower the operating melting temperature and improve the bath, although it is not the principal ore supplying aluminium in modern extraction. Calcium, entry (ii), is the unused distractor. The combined code is

$$(a) \to (iii), \quad (b) \to (v), \quad (c) \to (i), \quad (d) \to (iv).$$

Options (A) and (C) already fail by assigning siderite to copper. Option (B) recognises iron in siderite but assigns calamine to copper, malachite to zinc, and cryolite to calcium. Only option (D) matches all four formulae.

Step 3: Conclusion Siderite corresponds to iron, calamine to zinc, malachite to copper, and cryolite to aluminium. Hence, the correct option is (D).

Quick Tip to Solve: Memorise the carbonate formulae FeCO3, ZnCO3 and basic copper carbonate; cryolite's AlF6 unit then completes the matching.

78. Match List - I with List - II.

	List - I		List - II
(a)	Sodium Carbonate	(i)	Deacon
(b)	Titanium	(ii)	Castner-Kellner
(c)	Chlorine	(iii)	van-Arkel
(d)	Sodium hydroxide	(iv)	Solvay

Choose the correct answer from the options given below : [4 marks]

(A) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

(B) (a) → (iv), (b) → (i), (c) → (ii), (d) → (iii)

(C) (a) → (iii), (b) → (ii), (c) → (i), (d) → (iv)

(D) (a) → (i), (b) → (iii), (c) → (iv), (d) → (ii)

Correct Answer: (A) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

Solution:

Concept:

- Industrial processes can be matched by their starting materials and characteristic products, rather than by the similarity of their names.
- The Solvay process produces sodium carbonate, the van Arkel method purifies titanium, the Deacon process produces chlorine, and Castner–Kellner electrolysis produces sodium hydroxide.

Step 1: Match sodium carbonate and titanium In the Solvay process, ammoniated brine is treated with carbon dioxide to precipitate sodium hydrogen carbonate; heating that solid gives sodium carbonate:

$$2\,NaHCO_3 \longrightarrow Na_2CO_3 + CO_2 + H_2O.$$

Hence (a) matches (iv). The van Arkel method forms volatile titanium tetraiodide and then decomposes it on a hot filament, depositing pure titanium:

$$TiI_4 \longrightarrow Ti + 2\,I_2.$$

Thus (b) matches (iii). It is a purification method, rather than an electrolyte-cell process for making an alkali.

Step 2: Match chlorine and sodium hydroxide The Deacon process oxidises hydrogen chloride with oxygen in the presence of a catalyst to form chlorine:

$$4\,HCl + O_2 \longrightarrow 2\,Cl_2 + 2\,H_2O.$$

This gives (c) with (i). The Castner–Kellner process is the mercury-cell process involving electrolysis of brine and subsequent treatment of sodium amalgam to give sodium hydroxide, so (d) matches (ii).

Step 3: Check the complete code The complete mapping is (a) to (iv), (b) to (iii), (c) to (i), and (d) to (ii). The other proposed codes incorrectly exchange at least one process: in particular, neither the Deacon process nor the van Arkel method is the manufacture of sodium carbonate.

Step 4: Conclusion Sodium carbonate matches Solvay, titanium matches van Arkel, chlorine matches

Deacon, and sodium hydroxide matches Castner–Kellner. Hence, the correct option is (A).

Quick Tip to Solve: Anchor Solvay to carbonate and van Arkel to titanium purification; then verify the remaining two matches.

79. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is : [4 marks]

(A) neutral

(B) sometimes positive and sometimes negative

(C) positive

(D) negative

Correct Answer: (C) positive

Solution:

Concept:

- A colloidal particle commonly acquires charge through preferential adsorption of ions from the medium. The adsorbed ion determines the sign of the particle.
- Ferric chloride undergoes hydrolysis in hot water to produce a colloidal ferric hydroxide sol. This familiar sol is positively charged.

Step 1: Identify the colloid formed Addition of ferric chloride to a large quantity of hot water promotes hydrolysis. The overall chemical change is

$$\mathrm{FeCl_3} + 3\,\mathrm{H_2O} \longrightarrow \mathrm{Fe(OH)_3} + 3\,\mathrm{HCl}.$$

The freshly produced ferric hydroxide is distributed as very small colloidal particles. This is a hydrolysis method for preparing a lyophobic sol; it does not mean that every dissolved ion becomes an electrically neutral colloidal particle.

Step 2: Determine the preferentially adsorbed species The surface of ferric hydroxide particles preferentially adsorbs positively charged iron-containing species from the medium, conventionally represented as Fe^{3+}. The particles therefore have a positive surface charge. Chloride ions supply oppositely charged counterions in the surrounding solution, maintaining overall electrical neutrality of the complete sol. Overall neutrality of the dispersion must not be confused with zero charge on each colloidal particle.

Step 3: Eliminate the other descriptions Option (A) incorrectly treats the colloidal particles as neutral. Option (D) would require preferential adsorption of negative ions, which is not the result in this preparation. Option (B) suggests an indeterminate sign, whereas ferric hydroxide prepared by hydrolysis of ferric chloride in hot water is the standard positively charged sol. Hence the specified preparation selects a definite positive charge.

Step 4: Conclusion The resulting colloidal particles are positively charged. Hence, the correct option is (C).

Quick Tip to Solve: For hydroxide sols prepared by hydrolysis of a metal chloride, recall the positively charged ferric hydroxide example; distinguish particle charge from neutrality of the whole solution.

80. Calgon is used for water treatment. Which of the following statement is NOT true about Calgon?

[4 marks]

(A) It is also known as Graham's salt.

(B) It is polymeric compound and is water soluble.

(C) It does not remove Ca^{2+} ion by precipitation.

(D) Calgon contains the 2nd most abundant element by weight in the Earth's crust.

Correct Answer: (D) Calgon contains the 2nd most abundant element by weight in the Earth's crust.

Solution:

Concept:

- Calgon is sodium hexametaphosphate, commonly represented as $(NaPO_3)_6$, and is also called Graham's salt.
- It softens water by forming soluble complexes with hardness-producing calcium ions. Complex formation keeps calcium in solution rather than precipitating it.

Step 1: Check the identity and solubility Calgon is a water-soluble condensed or polymeric phosphate of sodium. Sodium hexametaphosphate has the composition $Na_6P_6O_{18}$, usually written $(NaPO_3)_6$. Graham's salt is its familiar name. Thus the descriptions in options (A) and (B) agree with the chemical identity of Calgon and are true statements. Neither its name nor its solubility supplies the requested false statement.

Step 2: Check the mechanism of softening Calcium ions responsible for hardness are sequestered by the polyphosphate species into a soluble complex. The calcium is consequently prevented from reacting as free Ca^{2+} and causing the usual hardness effects. The method does not rely on forming an insoluble calcium salt and filtering it off. Option (C), which says that calcium ions are not removed by precipitation, correctly describes this distinction.

Step 3: Compare the elemental composition The second most abundant element by mass in the Earth's crust is silicon, following oxygen. Calgon contains sodium, phosphorus and oxygen; its formula contains no silicon. The presence of oxygen is irrelevant to the claim about the second most abundant element, because oxygen occupies the first position. Therefore the statement in option (D) fails the direct formula check, while the other three statements remain true.

Step 4: Conclusion Calgon does not contain silicon, the second most abundant element by weight in the Earth's crust. Hence, the correct option is (D).

Quick Tip to Solve: For a NOT-true question about a named compound, write its formula first and check each composition claim literally; Calgon is a phosphate, not a silicate.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. When 12.2 g of benzoic acid is dissolved in 100 g of water, the freezing point of solution was found to be -0.93°C ($K_f(H_2O)$ = 1.86 K kg mol^{-1}). The number (n) of benzoic acid molecules associated (assuming 100% association) is _____. [4 marks]

Solution:

Concept:

- Freezing-point depression is $\Delta T_f = iK_f m$, where m is the molality calculated from the amount of solute before association and i is the van't Hoff factor.
- If every group of n solute molecules associates to form one aggregate, complete association reduces the number of particles by a factor of n, so $i = 1/n$.

Step 1: Calculate the analytical molality Benzoic acid has formula $C_7H_6O_2$, giving molar mass

$$M = 7(12) + 6(1) + 2(16) = 122\,\text{g mol}^{-1}.$$

The given mass therefore corresponds to

$$n_0 = \frac{12.2}{122} = 0.100\,\text{mol}.$$

The mass of solvent, rather than the mass of solution, is used in molality. Since $100\,\text{g} = 0.100\,\text{kg}$, the molality before accounting for association is

$$m = \frac{0.100}{0.100} = 1.00\,\text{mol kg}^{-1}.$$

Step 2: Find the van't Hoff factor Pure water freezes at $0\,^\circ$C, so the observed lowering is positive:

$$\Delta T_f = 0 - (-0.93) = 0.93\,\text{K}.$$

Temperature differences have the same numerical value in kelvin and degree Celsius. Substitution into the colligative-property equation gives

$$i = \frac{0.93}{1.86 \times 1.00} = 0.500.$$

A value below unity agrees with association, since fewer independent solute particles remain.

Step 3: Convert the particle ratio into aggregation number For complete association of n monomer molecules into each aggregate, the initial 0.100 mol becomes $0.100/n$ mol of particles. Thus $i = (0.100/n)/0.100 = 1/n$. With $i = 0.500$,

$$n = \frac{1}{0.500} = 2.$$

The effective species are dimers, and their particle molality is $0.500\,\text{mol kg}^{-1}$, reproducing the mea-

sured lowering.

Step 4: Conclusion Two benzoic acid molecules associate in each aggregate: $\boxed{n = 2}$.

Quick Tip to Solve: Compute molality from monomer molar mass first. Under complete association only, the aggregation number is the reciprocal of the van't Hoff factor.

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82. If the activation energy of a reaction is 80.9 kJ mol^{-1}, the fraction of molecules at 700 K, having enough energy to react to form products is e^{-x}. The value of x is _____. (Rounded off to the nearest integer) [Use R = 8.31 J K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- The fraction of molecules possessing at least the activation energy is represented by the Boltzmann factor $f = e^{-E_a/(RT)}$ in the Arrhenius treatment.
- The exponent is dimensionless. Activation energy must be in joules per mole when R is in joules per kelvin per mole.

Step 1: Identify the exponent The question expresses the fraction of sufficiently energetic molecules as e^{-x}. Comparing this with the energy factor

$$f = \exp\left(-\frac{E_a}{RT}\right)$$

directly gives

$$x = \frac{E_a}{RT}.$$

The negative sign is already displayed before x; it must not be included again in the numerical value of x. Since activation energy and temperature are positive, x is positive and the fraction lies between zero and one.

Step 2: Use consistent energy units The activation energy is $80.9\,\text{kJ mol}^{-1}$, whereas the supplied gas constant is $8.31\,\text{J K}^{-1}\,\text{mol}^{-1}$. Convert the energy before substituting:

$$E_a = 80.9 \times 10^3 = 80900\,\text{J mol}^{-1}.$$

At $700\,\text{K}$,

$$RT = 8.31 \times 700 = 5817\,\text{J mol}^{-1}.$$

The molar energy units cancel in the ratio, as required for an exponential argument.

Step 3: Calculate and round only at the end The resulting exponent is

$$x = \frac{80900}{5817} = 13.9075\ldots.$$

This is closer to 14 than to 13, so the specified nearest-integer value is 14. The unrounded fraction

is approximately $e^{-13.9075}$, a small positive number, which is consistent with an activation barrier substantially larger than the thermal molar energy. The question asks for the exponent rather than this fraction.

Step 4: Conclusion The value of x, rounded to the nearest integer, is $\boxed{14}$.

Quick Tip to Solve: Before using an Arrhenius exponential, match the energy units of E_a and R; then round the exponent only after dividing by RT.

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83. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is _____. [4 marks]

Solution:

Concept:

- Permanganate is a strong oxidising agent. In neutral or mildly alkaline medium it is reduced to manganese dioxide while thiosulphate is oxidised to sulphate.
- The sum of oxidation states in an ion equals its net charge. Oxygen normally has oxidation state -2 in oxoanions such as sulphate.

Step 1: Identify the oxidation product In the stated mildly alkaline medium, $\mathrm{MnO_4}^-$ oxidises $\mathrm{S_2O_3}^{2-}$ to sulphate, so product A is $\mathrm{SO_4}^{2-}$. This product must be identified before assigning a sulphur oxidation state: thiosulphate and tetrathionate are different ions, and using either of their formulae would answer a different chemical question. Both sulphur atoms of thiosulphate ultimately occur in sulphate ions.

Step 2: Confirm the redox change The balanced reaction in this medium can be written as

$$8\,\mathrm{MnO_4}^- + 3\,\mathrm{S_2O_3}^{2-} + \mathrm{H_2O} \longrightarrow 8\,\mathrm{MnO_2} + 6\,\mathrm{SO_4}^{2-} + 2\,\mathrm{OH}^-.$$

There are eight manganese atoms, six sulphur atoms and forty-two oxygen atoms on each side, and the total charge is -14 on both sides. Manganese is reduced from $+7$ to $+4$, while oxidation of thiosulphate supplies the electrons. This confirms sulphate as the sulphur-containing oxidation product.

Step 3: Calculate sulphur oxidation state in sulphate Let the oxidation state of sulphur in $\mathrm{SO_4}^{2-}$ be s. Four oxygen atoms contribute $4(-2) = -8$. Equating the sum to the ion charge gives

$$s + 4(-2) = -2, \qquad s = +6.$$

The requested numerical entry is the magnitude 6; when stating the oxidation state chemically, retain its positive sign. It is not the average sulphur oxidation state in the starting thiosulphate.

Step 4: Conclusion Product A is sulphate, $\mathrm{SO_4}^{2-}$, whose sulphur has oxidation state $\boxed{+6}$. The numerical entry is $\boxed{6}$.

Quick Tip to Solve: Identify the oxidation product first, then apply the charge-sum rule to that ion; do not calculate sulphur oxidation state from the starting thiosulphate formula.

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84. The average S-F bond energy in kJ mol^{-1} of SF_6 is _____. (Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of SF_6(g), S(g) and F(g) are - 1100, 275 and 80 kJ mol^{-1} respectively.] [4 marks]

Solution:

Concept:

- Average bond enthalpy is the enthalpy needed per mole of bonds to separate a gaseous molecule into its gaseous constituent atoms.
- Hess's law gives the atomisation enthalpy as the sum of formation enthalpies of the gaseous atoms minus the formation enthalpy of the gaseous molecule. Six S–F bonds occur in $\mathrm{SF_6}$.

Step 1: Write the gaseous atomisation process The required dissociation is

$$\mathrm{SF_6(g)} \longrightarrow \mathrm{S\,(g)} + 6\,\mathrm{F\,(g)}.$$

All six fluorine atoms must appear in the product total; the given $80\,\mathrm{kJ\,mol^{-1}}$ is the formation enthalpy of one mole of gaseous F atoms. The value for gaseous sulphur is likewise used directly. There is no need to insert an additional fluorine dissociation factor because the supplied formation enthalpy already refers to atomic fluorine.

Step 2: Apply Hess's law with the correct signs Using products minus reactant for the atomisation reaction,

$$\begin{aligned}\Delta H_{\text{atom}} &= \Delta_f H^\circ[\mathrm{S\,(g)}] + 6\Delta_f H^\circ[\mathrm{F\,(g)}] - \Delta_f H^\circ[\mathrm{SF_6(g)}]\\ &= 275 + 6(80) - (-1100)\\ &= 1855\,\mathrm{kJ\,mol^{-1}}.\end{aligned}$$

The negative formation enthalpy of $\mathrm{SF_6}$ becomes a positive contribution when its formation is reversed. A positive atomisation value is physically consistent with breaking bonds.

Step 3: Calculate the average per bond Atomising one mole of $\mathrm{SF_6}$ breaks six moles of S–F bonds. Therefore the average S–F bond enthalpy is

$$\overline{D}(\mathrm{S-F}) = \frac{1855}{6} = 309.1666\ldots\,\mathrm{kJ\,mol^{-1}}.$$

Division by six is essential: 1855 is the total for the molecule, rather than the average for one bond. Rounding at the final step gives 309 in the requested units.

Step 4: Conclusion The average S–F bond energy, rounded to the nearest integer, is $\boxed{309\,\mathrm{kJ\,mol^{-1}}}$.

Quick Tip to Solve: For average bond energy, atomise the gaseous molecule using Hess's law and divide by the number of bonds; reverse the sign of its formation enthalpy.

85. Emf of the following cell at 298K in V is x $\times$ 10^{-2}.

Zn|Zn^{2+}(0.1 M)||Ag^{+} (0.01 M)|Ag

The value of x is _____. (Rounded off to the nearest integer)

[Given : $E^{\theta}_{Zn^{2+}/Zn} = -0.76V; E^{\theta}_{Ag^{2+}/Ag} = +0.80V; \frac{2.303RT}{F} = 0.059$**]** [4 marks]

Solution:

Concept:

- The cell notation identifies zinc as the oxidation electrode and silver as the reduction electrode. The standard cell potential is the cathode reduction potential minus the anode reduction potential.
- For a cell reaction transferring n electrons, the Nernst equation at 298 K is $E = E^{\circ} - (0.059/n)\log_{10} Q$. Pure solid metals have unit activity, so only dissolved-ion concentrations enter Q.

Step 1: Write the balanced cell reaction The half-reactions are $\mathrm{Zn} \longrightarrow \mathrm{Zn^{2+}} + 2\,\mathrm{e^-}$ and $2\,\mathrm{Ag^+} + 2\,\mathrm{e^-} \longrightarrow 2\,\mathrm{Ag}$. Therefore the net reaction is $\mathrm{Zn} + 2\,\mathrm{Ag^+} \longrightarrow \mathrm{Zn^{2+}} + 2\,\mathrm{Ag}$, with $n = 2$. The concentration of silver ions must be squared because two silver ions occur in this balanced reaction.

Step 2: Calculate the standard potential and reaction quotient Use the supplied reduction potentials for the zinc anode and silver cathode:

$$E^{\circ} = 0.80 - (-0.76) = 1.56\,\mathrm{V}.$$

The specified cell contains $\mathrm{Ag^+}$, so its reaction quotient is

$$Q = \frac{[\mathrm{Zn^{2+}}]}{[\mathrm{Ag^+}]^2} = \frac{0.1}{(0.01)^2} = 10^3.$$

Concentrations here are used relative to the standard concentration.

Step 3: Apply the concentration correction Since $\log_{10}(10^3) = 3$,

$$E = 1.56 - \frac{0.059}{2}(3) = 1.4715\,\mathrm{V}.$$

The positive quotient correction lowers the emf below its standard value. Comparing with $E = x \times 10^{-2}\,\mathrm{V}$, obtain $x = 147.15$ before rounding.

Step 4: Conclusion The cell emf is $1.4715\,\mathrm{V}$, giving $x = 147.15$. Rounded to the nearest integer, the required answer is $\boxed{147}$.

Quick Tip to Solve: Balance the electron transfer before forming the Nernst quotient: here the square on silver-ion concentration and the factor 2 in the denominator are both essential.

86. The pH of ammonium phosphate solution, if pk_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is _____. [4 marks]

Solution:

Concept:

- A salt of a weak acid and a weak base undergoes hydrolysis through both its cation and its anion. Its pH depends on the relative acid and base strengths, rather than being fixed at 7 merely because it is a salt.
- In the weak-acid/weak-base salt treatment, $\mathrm{pH} = \frac{1}{2}(\mathrm{p}K_w + \mathrm{p}K_a - \mathrm{p}K_b)$. At ordinary temperature, $\mathrm{p}K_w = 14$; the supplied acid and base constants are used in this expression.

Step 1: Compare the acid and base strengths The given values are $\mathrm{p}K_a = 5.23$ and $\mathrm{p}K_b = 4.75$. A smaller pK corresponds to a larger equilibrium constant, so the weak base is slightly stronger than the weak acid on this comparison. Consequently, their salt should have a pH slightly greater than 7, which provides a useful check on the sign in the formula.

Step 2: Substitute the supplied pK values Using the salt-hydrolysis relation,

$$\mathrm{pH} = 7 + \frac{1}{2}(\mathrm{p}K_a - \mathrm{p}K_b).$$

Thus

$$\mathrm{pH} = 7 + \frac{1}{2}(5.23 - 4.75) = 7 + \frac{0.48}{2} = 7.24.$$

The positive correction of 0.24 is consistent with the relative strengths found above. Interchanging the two pK values would incorrectly predict an acidic solution.

Step 3: Express the numerical result The logarithmic pH is dimensionless. The calculation gives 7.24 from the supplied constants; its nearest whole-number value is 7. This is a nearly neutral, slightly basic solution under the weak-acid/weak-base hydrolysis model used in the question.

Step 4: Conclusion The calculated pH is $\boxed{7.24}$, or approximately $\boxed{7}$ when expressed as a whole number.

Quick Tip to Solve: For a weak-acid/weak-base salt, compare pKa with pKb first: pKa greater than pKb gives a positive correction to pH 7.

87. The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is _____. [ox = oxalate] [4 marks]

Solution:

Concept:

- Oxalate, $C_2O_4{}^{2-}$, is a bidentate ligand, binding through two donor atoms. Two oxalate ligands occupy four coordination sites; bromide and ammonia occupy the remaining two sites of an octahedral complex.
- For octahedral $[M(AA)_2ab]$ complexes, the two unlike monodentate ligands can be cis or trans. Optical isomers are counted separately when the two mirror-image arrangements cannot be

superimposed.

Step 1: Identify the geometrical arrangements The coordination number is $2(2)+1+1=6$, so use an octahedral arrangement. The Br^- and NH_3 ligands may be adjacent, at $90°$, giving the cis form; or opposite, at $180°$, giving the trans form. Rotation of the entire complex does not produce another geometrical isomer.

Step 2: Count the optical forms of each geometry In the cis geometry, the two oxalate chelate rings wind around the metal in two opposite senses. These are the Δ and Λ configurations. They are nonsuperimposable mirror images, so the cis geometry contributes two stereoisomers. In the trans geometry, a mirror plane containing the two unlike monodentate ligands relates the two identical chelate arrangements. It is achiral and contributes only one stereoisomer.

Step 3: Add distinct stereoisomers without double counting Count configurations, rather than ligand labels or positions in a drawing. The total is

$$N = 2\ (\text{cis enantiomers}) + 1\ (\text{trans form}) = 3.$$

The charge of the complex does not introduce an additional geometrical arrangement, and the two equivalent oxalate ligands must not be treated as different ligand types.

Step 4: Conclusion The cis form is an optical pair and the trans form is optically inactive. The number of possible stereoisomers is $\boxed{3}$.

Quick Tip to Solve: For an octahedral bis-chelate complex, first list cis and trans geometries, then test each separately for an optical pair; do not count every geometry twice.

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88. A ball weighing 10 g is moving with a velocity of 90 ms^{-1}. If the uncertainty in its velocity is 5%, then the uncertainty in its position is _____ $\times 10^{-33}$ m. (Rounded off to the nearest integer)
[Given : h = 6.63 $\times 10^{-34}$ Js] [4 marks]

Solution:

Concept:

- The position and momentum uncertainties obey $\Delta x\,\Delta p \geq h/(4\pi)$. For a body with a fixed mass, $\Delta p = m\Delta v$, so the least position uncertainty is $h/(4\pi m\Delta v)$.
- A percentage uncertainty must be applied to the given velocity. SI units require the mass in kilograms, velocity uncertainty in metres per second and Planck constant in joule seconds.

Step 1: Find the absolute velocity uncertainty The speed is $v = 90\,\mathrm{m\,s^{-1}}$, and the uncertainty is 5 percent of this speed. Therefore

$$\Delta v = \frac{5}{100}(90) = 4.5\,\mathrm{m\,s^{-1}}.$$

Use this uncertainty, rather than the full velocity of 90, in the uncertainty relation. The mass is $10\,\mathrm{g} = 0.010\,\mathrm{kg}$.

Step 2: Calculate momentum and position uncertainties The momentum uncertainty is

$$\Delta p = m\Delta v = 0.010(4.5) = 0.045\,\mathrm{kg\,m\,s^{-1}}.$$

At the limiting equality,

$$\Delta x_{\min} = \frac{6.63 \times 10^{-34}}{4\pi(0.045)}\,\mathrm{m} \simeq 1.172 \times 10^{-33}\,\mathrm{m}.$$

The dimensions reduce to metres because $\mathrm{J\,s/(kg\,m\,s^{-1})} = \mathrm{m}$.

Step 3: Round the requested coefficient The blank multiplies $10^{-33}\,\mathrm{m}$, so compare the calculated result with $x \times 10^{-33}\,\mathrm{m}$. This gives $x \simeq 1.172$, whose nearest integer is 1. The tiny positional uncertainty is consistent with the macroscopic mass of the ball.

Step 4: Conclusion The limiting position uncertainty is approximately $1.172 \times 10^{-33}\,\mathrm{m}$. The required rounded coefficient is $\boxed{1}$.

Quick Tip to Solve: Convert grams to kilograms and turn the percentage into an absolute velocity uncertainty before substituting into h divided by 4 pi m delta v.

89. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is _____ g. (Rounded off to the nearest integer)

[Given : Atomic weight in g mol^{-1} - Na : 23; N : 14; O : 16] [4 marks]

Solution:

Concept:

- One mole of $NaNO_3$ contains one mole of sodium ions, so the moles of salt needed equal the moles of Na^+ required. Dissociation changes the number of dissolved particles but not the mass of sodium present.
- The mass of sodium in the full solution equals its mass concentration multiplied by the total volume. Convert milligrams to grams before dividing by the molar mass of sodium.

Step 1: Find the total sodium mass The solution must contain $70.0\,\mathrm{mg}$ of Na^+ in each millilitre, and its final volume is $50\,\mathrm{mL}$. Thus

$$m(Na^+) = 70.0 \times 50 = 3500\,\mathrm{mg} = 3.50\,\mathrm{g}.$$

This is the mass of sodium alone, not the mass of the complete sodium nitrate required.

Step 2: Convert sodium mass to salt amount Using the supplied atomic mass of sodium,

$$n(Na^+) = \frac{3.50}{23}\,\mathrm{mol}.$$

Since $NaNO_3 \longrightarrow Na^+ + NO_3^-$, the mole ratio of sodium nitrate to sodium ions is 1:1. Therefore $n(NaNO_3) = 3.50/23\,\mathrm{mol}$. No factor of 2 is introduced by the two ions formed on dissociation.

Step 3: Calculate the weighed mass The molar mass of sodium nitrate is

$$M(\mathrm{NaNO_3}) = 23 + 14 + 3(16) = 85\,\mathrm{g\,mol^{-1}}.$$

Hence

$$m(\mathrm{NaNO_3}) = \frac{3.50}{23}(85) = 12.9348\,\mathrm{g}.$$

Equivalently, the sodium mass fraction is $23/85$; dividing 3.50 g by this fraction gives the same required salt mass.

Step 4: Conclusion The mass to be weighed is approximately $12.9348\,\mathrm{g}$, which rounds to $\boxed{13\,\mathrm{g}}$.

Quick Tip to Solve: The stated concentration refers to sodium ions, so multiply the total sodium mass by the molar-mass ratio 85/23 to obtain the sodium nitrate mass.

90. The number of octahedral voids per lattice site in a lattice is _____. (Rounded off to the nearest integer) [4 marks]

Solution:

Concept:

- In a close-packed lattice of N constituent particles, the number of octahedral voids is N, whereas the number of tetrahedral voids is $2N$. The requested quantity is the void count divided by the number of lattice sites.
- An octahedral void is surrounded by six neighbouring particles. This coordination number describes the geometry of one void; it is different from the number of voids contributed per lattice particle.

Step 1: Count sites in a conventional close-packed cell Use the face-centred cubic close-packed cell to obtain the ratio explicitly. Eight corner particles contribute $8 \times \frac{1}{8} = 1$ particle, while six face-centred particles contribute $6 \times \frac{1}{2} = 3$ particles. Thus the effective number of occupied lattice sites per unit cell is

$$N_{\text{sites}} = 1 + 3 = 4.$$

Sharing with neighbouring cells must be included when counting both particles and voids.

Step 2: Count octahedral voids with their sharing factors There is one octahedral void at the body centre, entirely inside the cell. There is also one at the centre of each of its twelve edges; an edge is shared by four unit cells. Consequently

$$N_{\text{oct}} = 1 + 12\left(\frac{1}{4}\right) = 4.$$

Counting all edge positions as whole voids would overestimate the number because the same void belongs to four adjacent cells.

Step 3: Form the ratio asked for Divide the effective void count by the effective lattice-site count:

$$\frac{N_{\text{oct}}}{N_{\text{sites}}} = \frac{4}{4} = 1.$$

The same one-to-one result holds for hexagonal close packing. The value 2 would instead describe tetrahedral voids per constituent particle, while 6 is the coordination number around one octahedral void.

Step 4: Conclusion There is one octahedral void per lattice site in the close-packed lattice. The required number is $\boxed{1}$.

Quick Tip to Solve: Remember the close-packing count as N octahedral and 2N tetrahedral voids for N particles; the six neighbours around a void are not its count per particle.