JEE Main 2021 Paper 1 (B.E./B.Tech) Jul 20 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 3 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 20 July 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let a be a positive real number such that $\int_0^a e^{x-[x]}dx = 10e - 9$ where [x] is the greatest integer less than or equal to x. Then a is equal to: [4 marks]

(A) $10 - \log_e(1+e)$

(B) $10 + \log_e 2$

(C) $10 + \log_e 3$

(D) $10 + \log_e(1+e)$

Correct Answer: (B) $10 + \log_e 2$

Solution:

Concept:

- The fractional part of a real number is $\{x\} = x - [x]$. On each interval $[k, k+1)$, substitute $t = x - k$, so every complete unit interval contributes the same integral.
- The integrand $e^{\{x\}}$ is positive. Its cumulative integral is therefore strictly increasing, which makes any value of the upper limit obtained from the equation unique.

Step 1: Find the contribution of a complete interval For an integer $k \geq 0$, the floor is k except at the right endpoint, whose value does not affect the integral. Thus

$$\int_k^{k+1} e^{x-[x]}\,dx = \int_0^1 e^t\,dt = e - 1.$$

Ten complete intervals contribute $10e - 10$. The prescribed value is $10e - 9$, which exceeds this by exactly 1.

Step 2: Locate and evaluate the remaining interval The next complete interval contributes $e - 1 > 1$, so the upper limit lies strictly between 10 and 11. Write $a = 10 + r$, where $0 < r < 1$. Then

$$10(e-1) + \int_{10}^{10+r} e^{x-10}\,dx = 10e - 9.$$

Hence

$$10e - 10 + e^r - 1 = 10e - 9, \qquad e^r = 2.$$

Therefore $r = \ln 2$, which indeed belongs to $(0, 1)$.

Step 3: Check the alternatives The value $10 + \ln 2$ gives the additional integral $2 - 1 = 1$, as required. Option A lies below 10 and cannot produce a value larger than the integral up to 10. Options C and D lie above 11, since $\ln 3 > 1$ and $\ln(1+e) > 1$, and would make the integral too large.

Step 4: Conclusion

$$a = 10 + \ln 2.$$

Hence, the correct option is (B).

Quick Tip to Solve: For an integrand involving $x - [x]$, count complete unit intervals first and integrate only the remaining fractional interval.

2. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are : [4 marks]

(A) 10, 11

(B) 3, 18

(C) 8, 13

(D) 1, 20

Correct Answer: (A) 10, 11

Solution:

Concept:

- For n observations with mean $\bar{x}$, the sum is $n\bar{x}$. The variance in this question is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$.
- Two unknown observations can be recovered from their sum and sum of squares by using $(u+v)^2 = u^2 + v^2 + 2uv$, followed by the quadratic equation with roots u, v.

Step 1: Use the mean to find the missing sum Let the two remaining observations be u and v. The total of the six observations is

$$6 \times 6.5 = 39.$$

The four known observations add to $2 + 4 + 5 + 7 = 18$, so

$$u + v = 39 - 18 = 21.$$

This condition alone does not select an option: all four listed pairs add to 21.

Step 2: Use the variance to find the missing squares Apply the variance formula with divisor 6, rather than the sample-variance divisor 5:

$$\sum x_i^2 = 6\left(10.25 + 6.5^2\right) = 6(10.25 + 42.25) = 315.$$

The known squares sum to

$$2^2 + 4^2 + 5^2 + 7^2 = 4 + 16 + 25 + 49 = 94.$$

Consequently $u^2 + v^2 = 315 - 94 = 221$.

Step 3: Recover and verify the observations Now

$$2uv = (u+v)^2 - (u^2 + v^2) = 441 - 221 = 220,$$

giving $uv = 110$. The missing observations are the roots of

$$t^2 - 21t + 110 = (t - 10)(t - 11) = 0.$$

Thus they are 10 and 11, both distinct from the known observations. Their squares add to 221, whereas the other listed pairs give 333, 233 and 401 respectively, so those pairs fail the variance condition.

Step 4: Conclusion The remaining observations are 10 and 11. Hence, the correct option is (A).

Quick Tip to Solve: When two observations are missing, convert mean and variance into their sum and product; checking only the sum cannot distinguish these options.

3. The value of the integral $\int_{-1}^{1} \log_e(\sqrt{1-x}+\sqrt{1+x})dx$ is equal to: [4 marks]

(A) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$

(B) $2\log_e 2 + \frac{\pi}{4} - 1$

(C) $\log_e 2 + \frac{\pi}{2} - 1$

(D) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$

Correct Answer: (C) $\log_e 2 + \frac{\pi}{2} - 1$

Solution:

Concept:

- The integrand is even, because replacing x by $-x$ interchanges the two square roots. Therefore the integral over $[-1, 1]$ is twice the integral over $[0, 1]$.
- Squaring the positive expression gives $(\sqrt{1-x}+\sqrt{1+x})^2 = 2 + 2\sqrt{1-x^2}$. Integration by parts simplifies the resulting logarithm.

Step 1: Reduce the logarithm using symmetry Write the required integral as I. Since both roots are nonnegative and their sum is positive throughout the interval, the logarithm identity is valid:

$$\ln(\sqrt{1-x}+\sqrt{1+x}) = \tfrac{1}{2}\ln\big(2+2\sqrt{1-x^2}\big).$$

Combining this with evenness yields

$$I = \ln 2 + J, \qquad J = \int_0^1 \ln(1+\sqrt{1-x^2})\,dx.$$

The constant contributes $\ln 2$, not twice that value.

Step 2: Integrate the remaining term by parts Take $u = \ln(1+\sqrt{1-x^2})$ and $dv = dx$. The boundary term $[x\ln(1+\sqrt{1-x^2})]_0^1$ is zero at both ends. Differentiating on $(0, 1)$ gives

$$J = \int_0^1 \frac{x^2}{\sqrt{1-x^2}(1+\sqrt{1-x^2})}\,dx.$$

The endpoint derivative is singular but the integral converges; the calculation can first be done up to $b < 1$ and then passed to the limit.

Step 3: Simplify and evaluate Put $s = \sqrt{1-x^2}$. Since $x^2 = 1 - s^2 = (1-s)(1+s)$, the integrand simplifies to $1/s - 1$. Hence

$$J = \int_0^1 \left(\frac{1}{\sqrt{1-x^2}} - 1\right) dx = \big[\sin^{-1} x - x\big]_0^1 = \frac{\pi}{2} - 1.$$

Thus $I = \ln 2 + \pi/2 - 1$. The other choices have different constant or logarithmic coefficients and do not equal this evaluated expression.

Step 4: Conclusion

$$I = \ln 2 + \frac{\pi}{2} - 1.$$

Hence, the correct option is (C).

Quick Tip to Solve: Square a sum of conjugate square roots inside a logarithm, then use evenness before integration by parts; the factor of two cancels neatly.

4. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to : [4 marks]

(A) 56×3^{25}

(B) 56×3^{24}

(C) 52×3^{24}

(D) 28×3^{25}

Correct Answer: (C) 52×3^{24}

Solution:

Concept:

- Rescale a quadratic when its coefficients involve powers of a common positive number. Here $r = 3^{1/4}$ turns the equation into the standard cube-root-of-unity equation.
- The roots ω, ω^2 of $t^2 + t + 1 = 0$ satisfy $\omega^3 = 1$, with $\omega \neq 1$. Powers divisible by 3 therefore simplify immediately.

Step 1: Normalize the quadratic Let $x = rt$, where $r = 3^{1/4}$. Since $r^2 = 3^{1/2}$, dividing the given equation by r^2 gives

$$t^2 + t + 1 = 0.$$

Its two distinct roots are ω and ω^2, the nonreal cube roots of unity. Thus, in either order,

$$\alpha = r\omega, \qquad \beta = r\omega^2.$$

This representation preserves both the sum and product of the original roots.

Step 2: Evaluate the relevant powers Both exponents 12 and 96 are divisible by 3. Therefore the root-of-unity factors become 1:

$$\alpha^{12} = r^{12}\omega^{12} = 3^3 = 27, \qquad \beta^{12} = 27.$$

Similarly,

$$\alpha^{96} = r^{96}\omega^{96} = 3^{24}, \qquad \beta^{96} = 3^{24}.$$

Keeping the fourth-root scale is essential: replacing it by a square root would produce the wrong power of 3.

Step 3: Substitute and compare the choices The two terms in the requested sum are equal. Substituting the values just found gives

$$\begin{aligned}\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1) &= 3^{24}(27 - 1) + 3^{24}(27 - 1) \\ &= 2 \times 26 \times 3^{24} \\ &= 52 \times 3^{24}.\end{aligned}$$

When every option is expressed as a multiple of 3^{24}, the coefficients are 168, 56, 52 and 84 respectively. Only the third coefficient agrees.

Step 4: Conclusion The required value is 52×3^{24}. Hence, the correct option is (C).

Quick Tip to Solve: Before computing large powers of quadratic roots, rescale the equation and look for roots of unity; then reduce the exponents modulo their order.

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5. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, a∈R be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to : [4 marks]

(A) 36

(B) 24

(C) 45

(D) 18

Correct Answer: (A) 36

Solution:

Concept:

- Every real square matrix has the unique decomposition $A = P + Q$, where $P = (A + A^T)/2$ is symmetric and $Q = (A - A^T)/2$ is skew-symmetric.
- For a matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$, the determinant is $ps - qr$. A skew-symmetric matrix of order two with off-diagonal entries $t, -t$ has determinant t^2.

Step 1: Form the two matrices The transpose interchanges the off-diagonal entries 3 and a. Adding and subtracting it gives

$$P = \begin{pmatrix} 2 & (3+a)/2 \\ \frac{3+a}{2} & 0 \end{pmatrix}, \qquad Q = \begin{pmatrix} 0 & (3-a)/2 \\ \frac{a-3}{2} & 0 \end{pmatrix}.$$

The diagonal entries of Q vanish as required, and its off-diagonal entries are negatives of one another.

Step 2: Use the prescribed determinant Computing the determinant of Q, including the minus sign in the two-by-two formula, yields

$$\det Q = \frac{(3-a)^2}{4} = 9.$$

Thus $(3-a)^2 = 36$, giving $3 - a = 6$ or $3 - a = -6$. The two admissible real values are

$$a = -3 \quad \text{or} \quad a = 9.$$

Both must be retained because the question asks for all possible determinant values.

Step 3: Find the sum before taking its modulus For the symmetric matrix,

$$\det P = -\frac{(3+a)^2}{4}.$$

When $a = -3$, this is 0; when $a = 9$, it is -36. Their sum is -36, and its modulus is

$$|0 - 36| = 36.$$

Direct substitution also verifies $\det Q = 9$ in each case. Values 24, 45 and 18 do not equal the modulus of this complete two-case sum; using only one sign of the square-root equation is not justified.

Step 4: Conclusion The modulus of the sum of all possible values of $\det P$ is 36. Hence, the correct option is (A).

Quick Tip to Solve: Use the standard transpose decomposition, retain both signs when solving the squared determinant condition, and take the modulus only after summing.

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6. If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z) [4 marks]

(A) $\frac{\pi}{4}$

(B) $-\frac{3\pi}{4}$

(C) $-\frac{\pi}{4}$

(D) $\frac{3\pi}{4}$

Correct Answer: (B) $-\frac{3\pi}{4}$

Solution:

Concept:

- If $z = re^{i\theta}$ and $\omega = se^{i\phi}$, then $\bar{z}\omega = rse^{i(\phi-\theta)}$. Conjugating the first factor reverses its argument.
- The modulus condition gives $rs = |z\omega| = 1$. The principal argument of a complex number is determined by its quadrant, with the representative in $(-\pi, \pi]$.

Step 1: Evaluate the conjugate product Write $\theta = \arg z$ and $\phi = \arg \omega$. Both numbers are nonzero because their product has modulus 1. The given difference is $\theta - \phi = 3\pi/2$, so

$$\bar{z}\omega = e^{i(\phi-\theta)} = e^{-3\pi i/2} = i.$$

Although $-3\pi/2$ is not a principal argument, its complex exponential equals that of $\pi/2$. The required product therefore lies on the positive imaginary axis.

Step 2: Simplify the quotient Substitute $\bar{z}\omega = i$ and multiply numerator and denominator by the conjugate of the denominator:

$$\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega} = \frac{1 - 2i}{1 + 3i} = \frac{(1 - 2i)(1 - 3i)}{(1 + 3i)(1 - 3i)}.$$

The numerator is $1 - 5i + 6i^2 = -5 - 5i$, and the denominator is $1 + 9 = 10$. Hence the quotient is $-\frac{1}{2} - \frac{1}{2}i$.

Step 3: Choose the principal angle The real and imaginary parts are equal and negative, placing the

quotient in the third quadrant. Its reference angle is $\pi/4$. The angle $5\pi/4$ represents that direction, but is outside the principal interval, so subtract 2π:

$$\arg\left(-\tfrac{1}{2}-\tfrac{1}{2}i\right)=-\frac{3\pi}{4}.$$

The positive first-quadrant angle and the two angles with opposite signs of real and imaginary parts fail the quadrant check.

Step 4: Conclusion The principal argument is $-3\pi/4$. Hence, the correct option is (B).

Quick Tip to Solve: For conjugate products reverse the first argument, then simplify the quotient into Cartesian form before choosing its principal angle.

7. Let [x] denote the greatest integer $\leq$ x, where x $\in$ R. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is (− ∞, a)]∪ [b, c) ∪ [4, ∞), a < b < c, then the value of a + b + c is : [4 marks]

(A) 8

(B) 1

(C) −2

(D) −3

Correct Answer: (C) −2

Solution:

Concept:

- A real square root requires its radicand to be nonnegative, while its denominator must be nonzero. Both conditions apply before interpreting the floor-function intervals.
- If $n=[x]$, then $n\leq x<n+1$. Consequently an upper restriction on the integer floor usually gives an open upper endpoint for the corresponding real interval.

Step 1: Solve the inequality in the integer parameter Let $n=[x]$ and $m=|n|$. The domain conditions are

$$\frac{m-2}{m-3}\geq 0, \qquad m\neq 3.$$

The numerator vanishes at 2 and the denominator at 3. A sign chart gives $m\leq 2$ or $m>3$. Since m is a nonnegative integer, this is exactly

$$|n|\leq 2 \quad \text{or} \quad |n|\geq 4.$$

The value $|n|=2$ is allowed because the square root is then zero.

Step 2: Translate each floor range into real intervals The condition $|n|\leq 2$ means $-2\leq[x]\leq 2$, which is equivalent to $-2\leq x<3$. The condition $|n|\geq 4$ means $[x]\leq -4$ or $[x]\geq 4$. These give $x<-3$ and $x\geq 4$, respectively. In particular, $x=-3$ is excluded because its floor is -3, whereas numbers just below -3 have floor -4 and are allowed.

Step 3: Identify the requested endpoints The complete domain is therefore

$$(-\infty, -3) \cup [-2, 3) \cup [4, \infty).$$

Comparing its ordered endpoints with the prescribed form gives $a = -3$, $b = -2$ and $c = 3$, satisfying $a < b < c$. Thus

$$a + b + c = -3 - 2 + 3 = -2.$$

The other options do not equal this sum. Including either excluded floor value ± 3 would incorrectly introduce points where the original denominator is zero.

Step 4: Conclusion The requested sum is -2. Hence, the correct option is (C).

Quick Tip to Solve: First solve in $|[x]|$, remembering it is an integer; then convert each floor inequality to an interval and check the negative endpoints carefully.

8. Let y = y(x) be the solution of the differential equation $x \tan\left(\frac{y}{x}\right) dy = \left(y \tan\left(\frac{y}{x}\right) - x\right) dx$, $-1 \le x \le 1$, $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves x = 0, $x = \frac{1}{\sqrt{2}}$ and y = y(x) in the upper half plane is : [4 marks]

(A) $\frac{1}{8}(\pi - 1)$
(B) $\frac{1}{12}(\pi - 3)$
(C) $\frac{1}{4}(\pi - 2)$
(D) $\frac{1}{6}(\pi - 1)$

Correct Answer: (A) $\frac{1}{8}(\pi - 1)$

Solution:

Concept:

- For a homogeneous differential equation, substitute $v = y/x$, so $y = xv$ and $dy/dx = v + x\, dv/dx$. This separates the ratio from the independent variable.
- The required upper-half-plane area is $A = \int_0^{1/\sqrt{2}} y(x)\, dx$. Integration by parts evaluates the inverse-cosine integral.

Step 1: Reduce the equation using the ratio Divide the equation by dx, substitute $y = xv$, and cancel the common term on both sides. For positive x, this gives

$$x \tan v \left(v + x\frac{dv}{dx}\right) = xv \tan v - x, \qquad \tan v\, dv = -\frac{dx}{x}.$$

Integration yields $-\log|\cos v| = -\log x + C$, or $\cos v = C_1 x$. The initial condition gives $v = (\pi/6)/(1/2) = \pi/3$, hence $C_1 = 1$.

Step 2: Choose the upper branch and evaluate its area The branch through the given point is $y = x\cos^{-1} x$, positive on the integration interval. Its limiting value at $x = 0$ is zero. With $a = 1/\sqrt{2}$,

integration by parts gives

$$A = \frac{a^2}{2}\cos^{-1} a + \frac{1}{2}\int_0^a \frac{x^2}{\sqrt{1-x^2}}\,dx.$$

Put $x = \sin t$ in the remaining integral; its upper limit is $\pi/4$. Thus

$$A = \frac{\pi}{16} + \frac{1}{2}\left[\frac{t}{2} - \frac{\sin 2t}{4}\right]_0^{\pi/4} = \frac{\pi}{16} + \frac{\pi}{16} - \frac{1}{8}.$$

The factor x in $y = x\cos^{-1} x$ must remain in the area integrand; integrating only the inverse cosine gives a different region.

Step 3: Conclusion The required area is $\boxed{(\pi-1)/8}$ square units. The other three expressions do not equal the evaluated integral. Hence, the correct option is (A).

Quick Tip to Solve: After substituting $v = y/x$, cancel the repeated $v\tan v$ term before integrating. Apply the initial condition to the ratio, not directly to y.

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9. Let $A = [a_{ij}]$ be a 3 × 3 matrix, where $a_{ij} = \begin{cases} 1 & , \quad if\ i = j \\ -x & , \quad if\ |i-j| = 1 \\ 2x+1 & , \quad otherwise. \end{cases}$

Let a function f : R → R be defined as f(x) = det(A). Then the sum of maximum and minimum values of f on R is equal to: [4 marks]

(A) $-\frac{20}{27}$
(B) $\frac{88}{27}$
(C) $\frac{20}{27}$
(D) $-\frac{88}{27}$

Correct Answer: (D) $-\frac{88}{27}$

Solution:

Concept:

- A differentiable function has a local extremum at a stationary point where its derivative changes sign. The second derivative distinguishes a local maximum from a local minimum.
- Write out the matrix from its entry rule before expanding its determinant. A cubic with nonzero leading coefficient is unbounded on the real line, so local and absolute extrema must be distinguished.

Step 1: Expand the determinant The diagonal entries are one, the entries adjacent to the diagonal are $-x$, and the two remaining corner entries are $2x+1$. Therefore

$$A = \begin{pmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{pmatrix}.$$

For a symmetric matrix of this form, expansion gives

$$f(x) = 1 - 2x^2 - (2x+1)^2 + 2x^2(2x+1) = 4x^3 - 4x^2 - 4x.$$

In particular, the mixed product contributes with a positive sign because it contains two factors $-x$.

Step 2: Find and classify the stationary points Differentiate the polynomial:

$$f\prime(x) = 12x^2 - 8x - 4 = 4(3x+1)(x-1).$$

The stationary points are $x = -1/3$ and $x = 1$. Since $f\prime\prime(x) = 24x - 8$, its values there are -16 and 16, respectively. Hence the first point is a local maximum and the second is a local minimum. Their values are

$$f(-1/3) = \frac{20}{27}, \qquad f(1) = -4 = -\frac{108}{27}.$$

Their sum is $-88/27$; it is not the maximum value alone or the sign-reversed sum.

Step 3: Distinguish local extrema from global bounds The polynomial tends to $+\infty$ as $x \to +\infty$ and to $-\infty$ as $x \to -\infty$. Thus it has no absolute maximum or minimum on $\mathbb{R}$. The maximum and minimum referred to by the listed options are its local stationary extrema.

Step 4: Conclusion The sum of the local maximum and local minimum values is $\boxed{-88/27}$. Under this local-extrema interpretation, the correct option is (D). There are no absolute extrema on $\mathbb{R}$.

Quick Tip to Solve: Find the stationary values, rather than adding their x-coordinates. For a cubic, state explicitly that these extrema are local.

10. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is : [4 marks]

(A) 1

(B) 2

(C) 4

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- The real square root $\sqrt{u}$ requires $u \geq 0$. A real inverse sine additionally requires its argument to lie in $[-1, 1]$.
- The inverse tangent and inverse sine use their principal values. Domain restrictions should be imposed before applying angle identities or taking trigonometric functions of both sides.

Step 1: Intersect the real-domain restrictions For the first square root, the necessary condition is

$$x(x+1) = x^2 + x \geq 0.$$

For the second term, its square-root argument is $x^2 + x + 1$, which is always positive because it equals

$(x+1/2)^2+3/4$. The inverse sine requires this square root to be no greater than one. Consequently

$$x^2+x+1\le 1 \quad\Longrightarrow\quad x^2+x\le 0.$$

Both inequalities hold only when $x^2+x=0$. The only real candidates are therefore $x=-1$ and $x=0$. No real number outside this pair can satisfy the original equation because at least one term would fail its real-domain condition.

Step 2: Test the surviving candidates in the original equation For either candidate, the first radicand equals zero and the second radicand equals one. The left-hand side is then

$$\tan^{-1}0+\sin^{-1}1=0+\frac{\pi}{2}=\frac{\pi}{2}.$$

This does not equal the required value $\pi/4$. Both candidates must be rejected. Squaring or transforming the equation without this substitution could incorrectly retain one or both endpoints; satisfying the domain is necessary but does not itself solve the equation.

Step 3: Conclusion There are $\boxed{0}$ real roots. The options giving one, two or four roots are excluded by the domain intersection and the endpoint check. Hence, the correct option is (D).

Quick Tip to Solve: When $\sqrt{u}$ and $\sin^{-1}\sqrt{u+1}$ occur together, their real domains force $u=0$. Test that boundary value before doing any trigonometric algebra.

11. Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}dx+\left(\frac{y}{x}\right)dy=0$, y(1) = −1. Then the value of $(y(3))^2$ is equal to : [4 marks]

(A) $1-4e^3$

(B) $1-4e^6$

(C) $1+4e^3$

(D) $1+4e^6$

Correct Answer: No listed option is possible for a real solution; its squared value lies between zero and one.

Solution:

Concept:

- A real differential equation containing $\sqrt{1-y^2}$ requires $-1\le y\le 1$. Therefore every real solution value satisfies $0\le y^2\le 1$.
- Dividing by $\sqrt{1-y^2}$ excludes the boundary solutions $y=\pm 1$. Check these solutions in the original equation before using separation of variables.

Step 1: Check the supplied initial value without division At the initial value $y(1)=-1$, the square root is zero. The constant function $y(x)=-1$ has $dy=0$, so both terms in the printed equation vanish for every positive x. It satisfies the initial condition and gives $(y(3))^2=1$. In particular, separation that divides by the square root loses a valid solution at the very initial condition being prescribed.

Step 2: Compare every listed value with the real-domain bound For any real solution defined at $x=3$, the square root in the differential equation enforces $0\le(y(3))^2\le 1$. Since $e^3>1$ and $e^6>1$,

options (A) and (B), namely $1-4e^3$ and $1-4e^6$, are negative. Options (C) and (D), namely $1+4e^3$ and $1+4e^6$, exceed one. Thus none can represent the requested real squared value.

Step 3: Locate the invalid formal manipulation On an interval where $|y|<1$, separation gives

$$\frac{y\,dy}{\sqrt{1-y^2}} = -xe^x\,dx, \qquad \sqrt{1-y^2} = (x-1)e^x + C.$$

Formally imposing the initial condition gives $C=0$. At $x=3$, this would require $\sqrt{1-y^2} = 2e^3 > 1$, impossible for real y. Squaring produces $1-4e^6$, but this formal expression does not define a real solution through to three. It cannot justify selecting (B).

Step 4: Conclusion As printed, no listed option is correct for a real solution. In particular, the valid constant solution gives $(y(3))^2 = 1$, while every listed expression lies outside $[0,1]$. The item has no defensible unique listed answer.

Quick Tip to Solve: At a boundary initial value, test constant solutions before dividing by a radical. A squared real answer cannot be negative, and here it cannot exceed one either.

12. Let 'a' be a real number such that the function f(x) = ax^2 + 6x − 15, x ∈ R is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function g(x) = ax^2 − 6x + 15, x∈R has a : [4 marks]

(A) local maximum at x = $-\frac{3}{4}$
(B) local minimum at x = $-\frac{3}{4}$
(C) local maximum at x = $\frac{3}{4}$
(D) local minimum at x = $\frac{3}{4}$

Correct Answer: (A) local maximum at x = $-\frac{3}{4}$

Solution:

Concept:

- For a differentiable quadratic, a transition from increasing to decreasing occurs at its vertex, where the first derivative is zero and the leading coefficient is negative.
- For $p(x) = ax^2 + bx + c$, the vertex has abscissa $-b/(2a)$. A negative second derivative identifies a local maximum, which is also the absolute maximum of this quadratic.

Step 1: Determine the coefficient from the given turning point The stated increase before $x = 3/4$ and decrease after it imply a stationary maximum at that point. Differentiate the first quadratic:

$$f\prime(x) = 2ax + 6.$$

Its continuity and sign change require $f\prime(3/4) = 0$. Hence

$$\frac{3a}{2} + 6 = 0, \qquad a = -4.$$

This also verifies the sign condition: $f\prime(x) = -8x + 6$ is positive to the left of $3/4$ and negative to the right. Thus the coefficient is consistent with both intervals, not merely with a candidate stationary

point.

Step 2: Find and classify the vertex of the second quadratic Substitute the same coefficient into the second function:

$$g(x) = -4x^2 - 6x + 15, \qquad g\prime(x) = -8x - 6.$$

The stationary point satisfies $-8x - 6 = 0$, giving $x = -3/4$. Moreover, $g\prime\prime(x) = -8 < 0$, so this point is a local maximum. Completing the square confirms the result:

$$g(x) = -4\left(x + \frac{3}{4}\right)^2 + \frac{69}{4}.$$

The square is nonnegative, so the vertex is indeed the highest point. Reversing the linear coefficient moves the vertex across the vertical axis, but does not change the downward-opening shape.

Step 3: Conclusion The function g has a local maximum at $\boxed{x = -3/4}$. The minimum alternatives conflict with its negative second derivative, and $x = 3/4$ is not stationary for g. Hence, the correct option is (A).

Quick Tip to Solve: Changing the sign of the linear term reflects a quadratic in the y-axis: its vertex changes sign while its maximum or minimum character stays the same.

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13. Let a function f : R → R be defined as $f(x) = \begin{cases} \sin x - e^x & if \quad x \le 0 \\ a + [-x] & if \quad 0 < x < 1 \\ 2x - b & if \quad x \ge 1 \end{cases}$

where [x] is the greatest integer less than or equal to x. If f is continuous on R, then (a + b) is equal to: [4 marks]

(A) 4

(B) 3

(C) 2

(D) 5

Correct Answer: (B) 3

Solution:

Concept:

- A piecewise function is continuous at a joining point when its left-hand limit, right-hand limit and assigned value are equal.
- The greatest integer function is a floor: for $0 < x < 1$, one has $-1 < -x < 0$, hence $\lfloor -x \rfloor = -1$, not zero.

Step 1: Apply continuity at zero On the middle interval, the expression is constant because $\lfloor -x \rfloor = -1$. It therefore becomes $a - 1$. The branch including zero gives

$$f(0) = \sin 0 - e^0 = -1.$$

The left-hand limit is also -1, since sine and exponential are continuous. The right-hand limit from the middle branch is $a - 1$. Equating these yields

$$a - 1 = -1, \qquad a = 0.$$

Using $\lfloor 0 \rfloor = 0$ in the middle branch would be incorrect: that branch is defined only for positive x, and the right-hand limit uses negative numbers approaching zero inside its floor.

Step 2: Apply continuity at one and check the remaining intervals From the middle interval, the left-hand limit at one is $a - 1 = -1$. The branch that includes one is $2x - b$, so its value and right-hand limit there are $2 - b$. Continuity therefore requires

$$2 - b = -1, \qquad b = 3.$$

Each open branch is continuous on its own interval: the first is a difference of continuous functions, the middle is constant, and the third is linear. Thus the two matching conditions suffice for continuity everywhere; there is no extra interior jump of the floor function on $(0, 1)$.

Step 3: Conclusion The parameters are $a = 0$ and $b = 3$, giving $\boxed{a + b = 3}$. The other listed sums fail one of the continuity conditions. Hence, the correct option is (B).

Quick Tip to Solve: For a floor involving a negative argument, first locate the argument between consecutive integers. Here every middle-interval value of $-x$ has floor -1.

14. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is : [4 marks]

(A) $\frac{1}{66}$
(B) $\frac{1}{11}$
(C) $\frac{1}{9}$
(D) $\frac{2}{11}$

Correct Answer: (B) $\frac{1}{11}$

Solution:

Concept:

- For equally likely distinct arrangements, probability equals the number of favourable arrangements divided by the total number of arrangements.
- For a word of n letters with repeated multiplicities $r_1, r_2, \ldots$, the number of distinct arrangements is $n!/(r_1!r_2!\cdots)$. A unique letter is equally likely to occupy any of the n positions.

Step 1: Count all distinguishable arrangements The word EXAMINATION contains eleven letters: E, X, A, M, I, N, A, T, I, O, N. The letters A, I and N occur twice each; E, X, M, T and O occur once each. Therefore the sample space of distinct words, including words with no meaning, contains

$$N = \frac{11!}{2!\,2!\,2!}$$

arrangements. Counting only meaningful dictionary words would describe a different experiment. The repetitions must be accounted for, but M itself is not repeated.

Step 2: Fix the fourth position and form the ratio Place the unique M in the fourth position. The remaining ten positions are filled using the other ten letters, with A, I and N still repeated twice each. Hence the favourable count is

$$N_M = \frac{10!}{2!\,2!\,2!}.$$

The required probability is

$$P = \frac{N_M}{N} = \frac{10!}{11!} = \frac{1}{11}.$$

The repetition factors cancel because fixing M does not change any repeated-letter multiplicity. Equivalently, the eleven possible positions for the single M have equal counts of arrangements, and exactly one is the prescribed fourth position. This symmetry also checks that the favourable event has not been overcounted.

Step 3: Conclusion The probability is $\boxed{1/11}$. The choices $1/66$, $1/9$ and $2/11$ do not match the arrangement ratio; in particular, the numerator is not doubled because M appears only once. Hence, the correct option is (B).

Quick Tip to Solve: When a particular letter appears once, its chance of occupying a specified position is $1/n$, even if other letters repeat. Factorial counting provides an immediate check.

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15. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in R$, is : [4 marks]

(A) $\frac{7}{36}$
(B) $\frac{2}{9}$
(C) $\frac{1}{6}$
(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{2}{9}$

Solution:

Concept:

- A quadratic with positive leading coefficient is strictly positive for every real argument exactly when its discriminant is negative. A zero discriminant is excluded because the quadratic then vanishes at its repeated root.
- For a uniformly selected integer, probability is the number of favourable integers divided by the total number of integers. Both endpoints of a closed integer interval are counted.

Step 1: Impose positivity for every real value The coefficient of x^2 is positive, so only the discriminant condition remains. Here the coefficient of x is $2(a+4)$, while the constant is $64 - 5a$. Thus

$$D = [2(a+4)]^2 - 4(64 - 5a) = 4(a^2 + 13a - 48).$$

The required strict inequality is therefore $a^2 + 13a - 48 < 0$.

Step 2: Locate and count the admissible integers Factor the expression without rounding its roots:

$$a^2 + 13a - 48 = (a + 16)(a - 3).$$

It is negative between its roots, so $-16 < a < 3$. Intersect this interval with the specified range $[-5, 30]$. The favourable values are $-5, -4, -3, -2, -1, 0, 1, 2$, giving eight integers. The upper endpoint $a = 3$ must not be included: its discriminant is zero, so strict positivity fails.

Step 3: Form the probability and compare the options The full sample space has $30 - (-5) + 1 = 36$ integers. Hence

$$P = \frac{8}{36} = \frac{2}{9}.$$

The alternatives $7/36$, $1/6 = 6/36$, and $1/4 = 9/36$ would require seven, six, and nine favourable values respectively; none matches the strict-inequality count.

Step 4: Conclusion The required probability is $\boxed{2/9}$. Hence, the correct option is (B).

Quick Tip to Solve: For positivity of a monic quadratic on the whole real line, use a strictly negative discriminant before counting integer parameter values.

16. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to : [4 marks]

(A) $q \Rightarrow p$

(B) $p \Rightarrow q$

(C) $\sim q \Rightarrow p$

(D) $p \Rightarrow \sim q$

Correct Answer: (B) $p \Rightarrow q$

Solution:

Concept:

- The implication $A \Rightarrow B$ is equivalent to $\neg A \vee B$. It is false only when its antecedent is true and its consequent is false.
- De Morgan's law gives $\neg(p \wedge \neg q) = \neg p \vee q$. Repeating a proposition in a disjunction does not change its truth value.

Step 1: Remove the outer implication Take $A = p \wedge \neg q$ and $B = q \vee \neg p$. Applying the implication identity to the whole expression, rather than to just one of its constituent propositions, gives

$$(p \wedge \neg q) \Rightarrow (q \vee \neg p) \equiv \neg(p \wedge \neg q) \vee (q \vee \neg p).$$

This step preserves the truth value for every choice of p and q.

Step 2: Simplify the disjunction By De Morgan's law, the negation of the antecedent is $\neg p \vee q$, which is already the consequent with its terms reversed. Consequently,

$$(\neg p \vee q) \vee (q \vee \neg p) \equiv \neg p \vee q.$$

Repeated copies of q and $\neg p$ may be removed by the idempotent law. Finally $\neg p \vee q \equiv p \Rightarrow q$.

Step 3: Distinguish the other implications The resulting expression is false only for p true and q false. Option (A), $q \Rightarrow p$, is instead false for q true and p false. Option (C), $\neg q \Rightarrow p$, is false when both are false. Option (D), $p \Rightarrow \neg q$, is false when both are true. These different false cases rule out all three alternatives.

Step 4: Conclusion The Boolean expression is equivalent to $\boxed{p \Rightarrow q}$. Hence, the correct option is (B).

Quick Tip to Solve: When the consequent is the negation of the antecedent, the implication simplifies to that consequent; here both reduce to the disjunction of not-p and q.

17. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is : [4 marks]

(A) $10 + 6\sqrt{2}$

(B) $8 + 2\sqrt{2}$

(C) $6 + 8\sqrt{3}$

(D) $4 + 2\sqrt{3}$

Correct Answer: (C) $6 + 8\sqrt{3}$

Solution:

Concept:

- For sides $a = BC$, $b = CA$, $c = AB$ opposite angles A, B, C, the extended sine rule is $a/\sin A = b/\sin B = c/\sin C = 2R$.
- The area is $\Delta = abc/(4R)$. Angles selected from inverse sine must also satisfy the positive-angle sum of a triangle.

Step 1: Find the possible angle opposite the given side Here $c = 5$, $R = 5$, and $\cos B = 3/5$. Since B is an interior acute angle, $\sin B = 4/5$. The extended sine rule gives

$$\sin C = \frac{c}{2R} = \frac{5}{10} = \frac{1}{2}.$$

The two possible interior angles are $C = 30^\circ$ and $C = 150^\circ$.

Step 2: Choose the valid triangle and calculate its remaining sides Since $\cos B = 3/5 < \sqrt{3}/2$, we have $B > 30^\circ$. Thus $C = 150^\circ$ would make $B + C > 180^\circ$, which is impossible. Therefore $C = 30^\circ$, $A = 150^\circ - B$, and

$$\sin A = \sin(B + 30^\circ) = \frac{4}{5}\frac{\sqrt{3}}{2} + \frac{3}{5}\frac{1}{2} = \frac{4\sqrt{3} + 3}{10}.$$

It follows that $a = 2R\sin A = 4\sqrt{3} + 3$, while $b = 2R\sin B = 8$.

Step 3: Compute the area and identify its exact form Use the three sides and the specified circumradius:

$$\Delta = \frac{(4\sqrt{3} + 3)(8)(5)}{4(5)} = 2(4\sqrt{3} + 3) = 6 + 8\sqrt{3}.$$

This result is approximately 19.856. Option (A) is approximately 18.485, option (B) approximately 10.828, and option (D) approximately 7.464; none equals the derived exact value. The radical $\sqrt{3}$ arises from the valid angle $30°$.

Step 4: Conclusion The area is $\boxed{6+8\sqrt{3}}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Use the given circumradius to get the sine of the angle opposite a known side, and reject the supplementary angle using the triangle angle sum.

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18. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is : [4 marks]

(A) ${}^{100}C_{16}$

(B) ${}^{100}C_{15}$

(C) $-{}^{100}C_{16}$

(D) $-{}^{100}C_{15}$

Correct Answer: (B) ${}^{100}C_{15}$

Solution:

Concept:

- The factorisation $(1-x)(1+x+x^2)=1-x^3$ turns the given product into a binomial in x^3, with one factor $1-x$ left over.
- In $(1-x^3)^{100}$, the term indexed by k is $(-1)^k\binom{100}{k}x^{3k}$. Its exponents are all divisible by three.

Step 1: Rewrite the polynomial before expanding Separate one factor from the power of $1-x$, then pair the remaining hundred factors with the hundred quadratic factors:

$$(1-x)^{101}(1+x+x^2)^{100}=(1-x)\{(1-x)(1+x+x^2)\}^{100}.$$

The bracket equals $1-x^3$, so the polynomial is $(1-x)(1-x^3)^{100}$. This exact identity avoids multiplying two long expansions.

Step 2: Identify which part can produce the target exponent Write the expression as $(1-x^3)^{100}-x(1-x^3)^{100}$. Since 256 is not divisible by three, the first summand contributes nothing to x^{256}. In the second summand, the needed power inside the binomial is x^{255}. Therefore $3k=255$, so $k=85$. The associated binomial coefficient carries the sign $(-1)^{85}=-1$.

Step 3: Include the outside sign and use binomial symmetry The factor outside that binomial term is $-x$, which supplies a second negative sign. Hence the desired coefficient is

$$-(-1)^{85}\binom{100}{85}=\binom{100}{85}=\binom{100}{15}.$$

The symmetry identity uses $100-85=15$. The index 16 in options (A) and (C) does not give exponent 255, and the negative sign in option (D) omits the additional minus sign from $-x$.

Step 4: Conclusion The coefficient of x^{256} is $\boxed{\binom{100}{15}}$. Hence, the correct option is (B).

Quick Tip to Solve: Before expanding products involving a geometric-sum factor, combine it with one factor of 1 minus x; then use the target exponent modulo three.

19. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ is : [4 marks]

(A) $\frac{2}{3}$

(B) 4

(C) 3

(D) $\frac{3}{2}$

Correct Answer: (D) $\frac{3}{2}$

Solution:

Concept:

- The identity $|\mathbf{c} - \mathbf{a}|^2 = |\mathbf{c}|^2 + |\mathbf{a}|^2 - 2\mathbf{a} \cdot \mathbf{c}$ converts the given distance condition into an equation for a magnitude.
- For vectors $\mathbf{u}, \mathbf{v}$ at angle ϕ, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin \phi$. The angle supplied here is with $\mathbf{a} \times \mathbf{b}$, not with $\mathbf{a}$ itself.

Step 1: Determine the magnitude of the unknown vector Let $r = |\mathbf{c}|$, so the given dot product is $\mathbf{a} \cdot \mathbf{c} = r$. Also

$$|\mathbf{a}|^2 = 2^2 + 1^2 + (-2)^2 = 9.$$

Squaring the distance condition, whose two sides are nonnegative, gives

$$8 = r^2 + 9 - 2r, \qquad r^2 - 2r + 1 = 0.$$

Thus $(r - 1)^2 = 0$ and $|\mathbf{c}| = 1$. No coordinate solution for $\mathbf{c}$ is needed.

Step 2: Calculate the known cross product Using $\mathbf{a} = (2, 1, -2)$ and $\mathbf{b} = (1, 1, 0)$, evaluate each component:

$$\mathbf{a} \times \mathbf{b} = (2, -2, 1).$$

For instance its last component is $2 \cdot 1 - 1 \cdot 1 = 1$. Its magnitude is therefore

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{2^2 + (-2)^2 + 1^2} = 3.$$

Step 3: Apply the supplied angle to the final cross product The vectors in the final product are $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c}$, whose included angle is $\pi/6$. Hence

$$|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}| = 3(1) \sin \frac{\pi}{6} = \frac{3}{2}.$$

Option (C) would omit the sine factor, while option (A) is the reciprocal of the result. Option (B) does not follow from either of the correctly computed magnitudes. Taking a cosine would calculate a dot-product magnitude instead.

Step 4: Conclusion The required magnitude is $\boxed{3/2}$. Hence, the correct option is (D).

Quick Tip to Solve: When dot product and vector separation are given together, square the separation first to find the unknown magnitude before attempting coordinates.

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20. Let the tangent to the parabola S : $y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to : [4 marks]

(A) $\frac{25}{2}$

(B) $\frac{35}{2}$

(C) $\frac{15}{2}$

(D) 25

Correct Answer: (A) $\frac{25}{2}$

Solution:

Concept:

- Implicit differentiation of $y^2 = 2x$ gives $dy/dx = 1/y$. At a point where this slope is nonzero, the normal slope is its negative reciprocal.
- The area of a triangle with two displacement vectors (u_1, u_2), (v_1, v_2) from one vertex is $\frac{1}{2}|u_1v_2 - u_2v_1|$.

Step 1: Find the tangent and its intercept At $P = (2, 2)$, the tangent slope is $1/2$. Its equation is

$$y - 2 = \frac{1}{2}(x - 2), \qquad y = \frac{x}{2} + 1.$$

Set $y = 0$ to locate its meeting with the x-axis: $x = -2$. Therefore $Q = (-2, 0)$. The negative intercept is essential; replacing it by a positive value would change the triangle.

Step 2: Find the second meeting of the normal with the parabola The normal slope is -2, giving $y - 2 = -2(x - 2)$, or $y = 6 - 2x$. Write $x = (6 - y)/2$ and substitute in $y^2 = 2x$:

$$y^2 = 6 - y, \qquad (y - 2)(y + 3) = 0.$$

The root $y = 2$ reproduces the original point P, so the other intersection has $y = -3$ and $x = (6 + 3)/2 = 9/2$. Thus $R = (9/2, -3)$.

Step 3: Evaluate the area using a determinant Use Q as the common starting vertex. The displacement vectors are $P - Q = (4, 2)$ and $R - Q = (13/2, -3)$, so

$$\Delta = \frac{1}{2}\left|4(-3) - 2\left(\frac{13}{2}\right)\right| = \frac{1}{2}|-25| = \frac{25}{2}.$$

The absolute value removes orientation, ensuring positive area. Option (D), 25, is the determinant magnitude before dividing by two. Options (B) and (C) disagree with the actual normal intersection and determinant.

Step 4: Conclusion The area of triangle PQR is $\boxed{25/2}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: When a normal meets the parabola again, factor out the known point first; the other root gives the second intersection needed for the area.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to _____. [4 marks]

Solution:

Concept:

- Mutually perpendicular nonzero vectors have zero pairwise dot products. The square of the magnitude of their sum is therefore the sum of their squared magnitudes.
- The angle between vectors follows from $\cos\theta = (\mathbf{u}\cdot\mathbf{v})/(|\mathbf{u}||\mathbf{v}|)$. The double-angle identity is $\cos 2\theta = 2\cos^2\theta - 1$.

Step 1: Compute the magnitude of the sum Let the common positive magnitude of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be m, and set $\mathbf{s} = \mathbf{a} + \mathbf{b} + \mathbf{c}$. Expanding its scalar square gives

$$|\mathbf{s}|^2 = 3m^2 + 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) = 3m^2.$$

Every mixed product is zero because the corresponding vectors are perpendicular. Hence $|\mathbf{s}| = \sqrt{3}\,m$.

Step 2: Find the common angle with the sum Taking the dot product with $\mathbf{a}$ gives $\mathbf{a}\cdot\mathbf{s} = \mathbf{a}\cdot\mathbf{a} = m^2$. Therefore

$$\cos\theta = \frac{m^2}{m(\sqrt{3}m)} = \frac{1}{\sqrt{3}}, \qquad \cos^2\theta = \frac{1}{3}.$$

The same calculation with $\mathbf{b}$ or $\mathbf{c}$ gives the identical angle, as stated. The magnitude cancels, so the requested value cannot depend on the scale of the vectors.

Step 3: Evaluate the double-angle expression First calculate the cosine of twice the angle, then square it:

$$\cos 2\theta = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3},$$

$$36\cos^2 2\theta = 36\left(-\frac{1}{3}\right)^2 = 36\left(\frac{1}{9}\right) = 4.$$

The notation asks for the square of $\cos(2\theta)$; it does not ask for $\cos^2\theta$ or for $\cos(2\theta^2)$. Keeping that distinction avoids an incorrect factor in the final evaluation.

Step 4: Conclusion The requested numerical value is $\boxed{4}$.

Quick Tip to Solve: For equal mutually perpendicular vectors, the direction cosine of each with their sum is one over the square root of the number of vectors.

22. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ **and B = 7A^{20} − 20A^{7} + 2I, where I is an identity matrix of order 3 × 3. If B = [b$_{ij}$], then b$_{13}$is equal to _____.** [4 marks]

Solution:

Concept:

- Write the upper triangular matrix as $A = I + N$. Since its strictly upper triangular part satisfies $N^3 = 0$, the binomial expansion terminates after the quadratic term.
- The entry in row 1 and column 3 of I and of N is zero; only the N^2 contribution can affect the requested entry.

Step 1: Identify the nilpotent part Subtract the identity matrix from the given matrix:

$$N = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad N^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The only two-step route from the first row to the third column multiplies the two entries -1, giving $(N^2)_{13} = 1$. A further multiplication gives the zero matrix, so all powers from the third onwards vanish.

Step 2: Expand the required matrix powers Because I commutes with N, the ordinary binomial theorem applies to their sum. For every positive integer m,

$$A^m = I + mN + \binom{m}{2} N^2.$$

In particular, $(A^{20})_{13} = \binom{20}{2} = 190$, whereas $(A^7)_{13} = \binom{7}{2} = 21$. It is unnecessary to calculate the other eight entries of either power.

Step 3: Combine the entries with their coefficients Matrix addition and scalar multiplication act entry by entry. Also $I_{13} = 0$, since this is an off-diagonal position. Hence

$$b_{13} = 7(190) - 20(21) + 2(0) = 1330 - 420 = 910.$$

Both negative superdiagonal entries have already contributed their product in N^2; inserting another minus sign here would be incorrect.

Step 4: Conclusion The required numerical value is $\boxed{910}$.

Quick Tip to Solve: For a triangular matrix with ones on its diagonal, subtract the identity first and check which nilpotent power reaches the requested entry.

23. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ is _____.

[4 marks]

Solution:

Concept:

- The term indexed by r in $(u+v)^{120}$ is $\binom{120}{r}u^{120-r}v^{r}$, where r is an integer from 0 to 120 inclusive.
- A product of rational powers of distinct primes is rational precisely when each prime exponent is an integer. The nonzero integer binomial coefficient cannot remove an irrational prime power.

Step 1: Express a general term using prime bases Take $u=4^{1/4}=2^{1/2}$ and $v=5^{1/6}$. The general term becomes

$$T_{r+1}=\binom{120}{r}2^{(120-r)/2}5^{r/6}.$$

Using the prime base 2 is essential: requiring the original exponent of 4 to be an integer would wrongly discard rational terms such as $4^{1/2}=2$.

Step 2: Impose both rationality conditions The primes 2 and 5 are distinct, so their fractional powers cannot cancel one another. Thus we need

$$\frac{120-r}{2}\in\mathbb{Z},\qquad \frac{r}{6}\in\mathbb{Z}.$$

The second condition says r is a multiple of 6. Such an r is automatically even, and since 120 is even, the first condition then holds as well. There are no further restrictions from the binomial coefficient.

Step 3: Count all admissible indices Write $r=6k$. The permitted interval $0\le r\le 120$ gives $0\le k\le 20$, with k integral. Therefore the allowed indices are

$$r=0,6,12,\ldots,120.$$

There are $(120-0)/6+1=21$ such indices. The added 1 includes both end terms; both are rational, namely 2^{60} and 5^{20}.

Step 4: Conclusion The number of rational terms is $\boxed{21}$.

Quick Tip to Solve: Reduce composite bases to prime powers before imposing integer-exponent conditions, and include both endpoint terms in the count.

24. If the shortest distance between the lines $\vec{r_1}=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$, $\lambda\in$ R, $\alpha>0$ and $\vec{r_2}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$, $\mu\in$ R is 9, then α is equal to _____.

[4 marks]

Solution:

Concept:

- For lines through position vectors $\mathbf{a}_1,\mathbf{a}_2$ with nonparallel direction vectors $\mathbf{b}_1,\mathbf{b}_2$, their shortest distance is $d=|(\mathbf{a}_2-\mathbf{a}_1)\cdot(\mathbf{b}_1\times\mathbf{b}_2)|/|\mathbf{b}_1\times\mathbf{b}_2|$.
- The cross product is perpendicular to both lines. Projecting the joining vector onto this common normal removes its components parallel to the two lines.

Step 1: Read the point and direction vectors From the equations take

$$\mathbf{a}_1 = (\alpha, 2, 2), \qquad \mathbf{b}_1 = (1, -2, 2),$$
$$\mathbf{a}_2 = (-4, 0, -1), \quad \mathbf{b}_2 = (3, -2, -2).$$

In particular, the second line has zero j-coordinate in its fixed point. Thus $\mathbf{a}_2 - \mathbf{a}_1 = (-4 - \alpha, -2, -3)$. The direction parameters may vary independently and do not belong in these fixed vectors.

Step 2: Find a common normal Calculate the cross product component by component:

$$\mathbf{b}_1 \times \mathbf{b}_2 = (8, 8, 4) = 4(2, 2, 1).$$

Its magnitude is $\sqrt{64 + 64 + 16} = 12$, so the direction vectors are not parallel. The dot product in the numerator is

$$(-4 - \alpha, -2, -3) \cdot (8, 8, 4) = -60 - 8\alpha.$$

Step 3: Use the distance and the sign condition Substitute the prescribed distance 9:

$$\frac{|60 + 8\alpha|}{12} = 9, \qquad |60 + 8\alpha| = 108.$$

The two algebraic possibilities are $60 + 8\alpha = 108$ and $60 + 8\alpha = -108$, giving $\alpha = 6$ and $\alpha = -21$, respectively. The printed condition $\alpha > 0$ selects the first. Substitution gives $108/12 = 9$, as required.

Step 4: Conclusion The permitted value is $\boxed{\alpha = 6}$.

Quick Tip to Solve: Keep the absolute value until the final equation, then apply any sign condition to eliminate the extraneous branch.

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25. Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and x = $\sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to _____. [4 marks]

Solution:

Concept:

- The tangent to $x^2 + 4y^2 = 5$ at $(1, 1)$ is obtained by replacing squared coordinates with products: $x + 4y = 5$.
- On $1 \le x \le \sqrt{5}$, the relevant ellipse branch is $y = \frac{1}{2}\sqrt{5 - x^2}$. The bounded area is the integral of the upper ordinate minus the lower ordinate.

Step 1: Set up the area integral The tangent has ordinate $y_T = (5 - x)/4$, while the ellipse has ordinate $y_E = \sqrt{5 - x^2}/2$. They agree at $x = 1$, and the tangent lies above the upper ellipse branch throughout the interval to its right. Both ordinates are nonnegative there. Hence

$$A = \int_1^{\sqrt{5}} \left(\frac{5 - x}{4} - \frac{\sqrt{5 - x^2}}{2} \right) dx.$$

We use only the upper branch because it is the arc passing through the specified point of tangency.

Step 2: Evaluate the two contributions The straight-line part is

$$\left[\frac{5x}{4}-\frac{x^2}{8}\right]_1^{\sqrt{5}}=\frac{5\sqrt{5}}{4}-\frac{7}{4}.$$

For the curved part use the standard circular-segment antiderivative:

$$\int\frac{\sqrt{5-x^2}}{2}\,dx=\frac{x\sqrt{5-x^2}}{4}+\frac{5}{4}\sin^{-1}\frac{x}{\sqrt{5}}.$$

At the upper endpoint the radical vanishes; at the lower endpoint the algebraic term is $1/2$. Thus this contribution equals

$$\frac{5}{4}\cos^{-1}\frac{1}{\sqrt{5}}-\frac{1}{2},$$

using $\sin^{-1}t+\cos^{-1}t=\pi/2$ on the relevant real interval.

Step 3: Compare the coefficients Subtract the curved part from the straight-line part:

$$A=\frac{5}{4}\sqrt{5}-\frac{5}{4}-\frac{5}{4}\cos^{-1}\frac{1}{\sqrt{5}}.$$

Therefore $\alpha=5/4$, $\beta=-5/4$, and $\gamma=-5/4$. Their signed sum is $-5/4$; the question asks for its absolute value, not the sum of the absolute values.

Step 4: Conclusion The required value is $\boxed{|\alpha+\beta+\gamma|=\frac{5}{4}=1.25}$.

Quick Tip to Solve: Write tangent minus ellipse before integrating, and convert the inverse sine to the printed inverse cosine before reading coefficients.

---

26. Let a, b, c, d in arithmetic progression with common difference λ. If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}=2$, **then value of λ^2 is equal to _____.** [4 marks]

Solution:

Concept:

- For an arithmetic progression with common difference λ, write $b=a+\lambda$, $c=a+2\lambda$, and $d=a+3\lambda$.
- Adding multiples of other rows to one row preserves a determinant. A row with two zeros then permits a quick cofactor expansion.

Step 1: Rewrite the entries using the common difference The first-column entries simplify to $x-2\lambda$, $x-1$, and $x+2\lambda$, respectively. The remaining columns are consecutive progression values added to x.

Consequently the determinant is

$$D = \begin{vmatrix} x-2\lambda & x+a+\lambda & x+a \\ x-1 & x+a+2\lambda & x+a+\lambda \\ x+2\lambda & x+a+3\lambda & x+a+2\lambda \end{vmatrix}.$$

Keep the printed constant -1 in the middle first-column entry; it is not a progression parameter.

Step 2: Create a sparse first row Perform the single row replacement $R_1 \leftarrow R_1 + R_3 - 2R_2$. This adds a linear combination of the other rows to the first, so the determinant is unchanged. Its new first entry is

$$(x-2\lambda) + (x+2\lambda) - 2(x-1) = 2.$$

Its second and third entries are both zero because each corresponding column is an arithmetic progression. Expanding along that row gives

$$D = 2\left[(x+c)^2 - (x+b)(x+d)\right].$$

Step 3: Evaluate the remaining quadratic difference Put $t = x + c$. Since $b = c - \lambda$ and $d = c + \lambda$, the two factors in the product become $t - \lambda$ and $t + \lambda$. Hence

$$D = 2\left[t^2 - (t-\lambda)(t+\lambda)\right] = 2\lambda^2.$$

The given determinant equals 2, so $2\lambda^2 = 2$. Neither a nor x affects the result, and the sign of λ is immaterial because only its square is requested.

Step 4: Conclusion The required value is $\boxed{\lambda^2 = 1}$.

Quick Tip to Solve: When columns contain successive progression terms, combine rows to annihilate those columns before expanding the determinant.

27. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsman and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsman and 1 wicketkeeper, is _____. [4 marks]

Solution:

Concept:

- Players are distinct and the three stated categories are disjoint. Selecting fixed category counts (b, t, w) therefore gives $\binom{6}{b}\binom{7}{t}\binom{2}{w}$ teams.
- An unordered team is counted with combinations. Mutually exclusive category-count cases can then be added without double counting.

Step 1: Enumerate all feasible category counts Let b, t, w denote the numbers of bowlers, batsmen and wicketkeepers. The constraints are

$$b + t + w = 11, \qquad b \geq 4, \quad t \geq 5, \quad w \geq 1.$$

The minimum counts add to $4+5+1=10$, leaving precisely one extra player to allocate. It can be a bowler, a batsman, or a wicketkeeper, giving only the triples

$$(5,5,1), \qquad (4,6,1), \qquad (4,5,2).$$

Each respects the available category totals. No other triple can meet both the minimum requirements and the total of 11.

Step 2: Count teams in each case For five bowlers, five batsmen and one wicketkeeper, the count is

$$\binom{6}{5}\binom{7}{5}\binom{2}{1} = 6 \cdot 21 \cdot 2 = 252.$$

For four bowlers, six batsmen and one wicketkeeper, it is

$$\binom{6}{4}\binom{7}{6}\binom{2}{1} = 15 \cdot 7 \cdot 2 = 210.$$

For four bowlers, five batsmen and both wicketkeepers, it is

$$\binom{6}{4}\binom{7}{5}\binom{2}{2} = 15 \cdot 21 \cdot 1 = 315.$$

The factors multiply because choices from separate categories can be combined independently.

Step 3: Add the disjoint selections Every allowed team belongs to exactly one of the three triples, so there is no overlap to subtract. The total number is

$$252 + 210 + 315 = 777.$$

There is no additional factor of $11!$, since the positions or batting order of the chosen players are not being assigned. Similarly, the two selected wicketkeepers in the last case form an unordered subset.

Step 4: Conclusion The number of possible teams is $\boxed{777}$.

Quick Tip to Solve: Add the minimum category counts first: the small number of remaining places often makes the complete case list immediate.

---

28. If the value of $\lim\limits_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to _____. [4 marks]

Solution:

Concept:

- For a positive base tending to 1, take logarithms: if $L = \lim f(x)^{g(x)}$, then $\log L = \lim g(x)\log f(x)$, provided the latter limit exists.
- Near zero, $\cos x = 1 - x^2/2 + O(x^4)$, $\sqrt{1+u} = 1 + u/2 + O(u^2)$, and $\log(1+u) = u + O(u^2)$.

Step 1: Expand the product inside the base For sufficiently small real x, $\cos 2x > 0$, so its real square

root and the logarithm used below are defined. First,

$$\cos 2x = 1 - 2x^2 + O(x^4), \qquad \sqrt{\cos 2x} = 1 - x^2 + O(x^4).$$

Multiplying by the expansion of $\cos x$ gives

$$\cos x\sqrt{\cos 2x} = \left(1 - \frac{x^2}{2} + O(x^4)\right)\left(1 - x^2 + O(x^4)\right) = 1 - \frac{3}{2}x^2 + O(x^4).$$

Both quadratic contributions are needed; expanding only one factor would give the wrong coefficient.

Step 2: Take the logarithm of the limit expression Let $F(x) = 2 - \cos x\sqrt{\cos 2x}$. The preceding result yields

$$F(x) = 1 + \frac{3}{2}x^2 + O(x^4), \qquad \log F(x) = \frac{3}{2}x^2 + O(x^4).$$

The base is positive near zero and tends to 1, so taking real logarithms is legitimate. For the expression in the question,

$$\log\left(F(x)^{(x+2)/x^2}\right) = \frac{x+2}{x^2}\left(\frac{3}{2}x^2 + O(x^4)\right).$$

Step 3: Evaluate the exponent and identify the parameter Cancel x^2 in the leading term. The error term becomes $(x+2)O(x^2)$, which tends to zero. Therefore

$$\lim_{x\to 0} \log\left(F(x)^{(x+2)/x^2}\right) = \lim_{x\to 0}(x+2)\left(\frac{3}{2} + O(x^2)\right) = 3.$$

Continuity of the exponential function now gives the original limit as e^3. Since the stated value is e^a with real parameter a, equality of these exponentials implies $a = 3$.

Step 4: Conclusion The required value is $\boxed{a = 3}$.

Quick Tip to Solve: For a base approaching 1 and an unbounded exponent, find the leading term of the logarithm; here only the quadratic term survives.

---

29. Let P be a plane passing through the points (1, 0, 1), (1, −2, 1) and (0, 1, −2). Let a vector $\vec{d} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{d}$ is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{d}\,.\,(\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals _____. [4 marks]

Solution:

Concept:

- A vector parallel to a plane is perpendicular to its normal. Two independent displacement vectors joining points on the plane give a normal by their cross product.
- For vectors $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$, perpendicularity means $\mathbf{u} \cdot \mathbf{v} = 0$. The given nonzero dot product fixes the magnitude and sign of the required vector.

Step 1: Find a normal to the plane Call the three given points A, B, C, in their stated order. Their

displacement vectors are

$$\overrightarrow{AB} = (0, -2, 0), \qquad \overrightarrow{AC} = (-1, 1, -3).$$

They are not parallel, and their cross product is

$$\overrightarrow{AB} \times \overrightarrow{AC} = (6, 0, -2) = 2(3, 0, -1).$$

Thus we may use $\mathbf{n} = (3, 0, -1)$ as a plane normal; its scale is immaterial.

Step 2: Apply the two perpendicularity conditions Because $\mathbf{a} = (\alpha, \beta, \gamma)$ is parallel to the plane,

$$3\alpha - \gamma = 0, \qquad \gamma = 3\alpha.$$

Its perpendicularity to $(1, 2, 3)$ gives

$$\alpha + 2\beta + 3\gamma = 0.$$

Substitution now gives $10\alpha + 2\beta = 0$, so $\beta = -5\alpha$. Therefore every vector meeting the first two requirements is $\alpha(1, -5, 3)$.

Step 3: Fix the scale and evaluate the expression The remaining condition is not another zero dot product: it fixes the scale through

$$\alpha + \beta + 2\gamma = 2.$$

Hence $\alpha - 5\alpha + 6\alpha = 2$, so $\alpha = 1$, $\beta = -5$, and $\gamma = 3$. As a check, $(1, -5, 3) \cdot (1, 2, 3) = 1 - 10 + 9 = 0$. Finally

$$(\alpha - \beta + \gamma)^2 = (1 - (-5) + 3)^2 = 9^2 = 81.$$

Step 4: Conclusion The required numerical value is $\boxed{81}$.

Quick Tip to Solve: First find the direction using the two zero dot products; use the nonzero dot product only at the end to determine the scale.

30. Let y = mx + c, m > 0 be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}$ (m + c) is equal to _____. [4 marks]

Solution:

Concept:

- For $y^2 = 4ax$, the focus is $(a, 0)$. A focal chord is a chord whose supporting line passes through this focus.
- A line is tangent to a circle exactly when the perpendicular distance from the centre to the line equals the radius. For $Ax + By + C = 0$, that distance from (h, k) is $|Ah + Bk + C|/\sqrt{A^2 + B^2}$.

Step 1: Impose passage through the focus Compare $y^2 = -64x$ with $y^2 = 4ax$. This gives $a = -16$, so the focus is $(-16, 0)$. Since $y = mx + c$ is a focal chord, substituting the focus gives

$$0 = -16m + c, \qquad c = 16m.$$

The positive-slope condition will choose the positive root for m after the tangency equation is solved.

Step 2: Use tangency to the circle The circle $(x+10)^2+y^2=4$ has centre $(-10,0)$ and radius 2. Write the line as $mx-y+c=0$. The perpendicular distance condition is

$$\frac{|-10m+c|}{\sqrt{m^2+1}}=2.$$

Insert $c=16m$ and use $m>0$ to obtain

$$\frac{6m}{\sqrt{m^2+1}}=2.$$

Both sides are positive, so squaring introduces no sign ambiguity.

Step 3: Solve for the slope and requested value Squaring yields

$$36m^2=4(m^2+1), \qquad 32m^2=4, \qquad m^2=\frac{1}{8}.$$

Thus $m=1/(2\sqrt{2})$ and $c=16m=8/\sqrt{2}$. The sum is $m+c=17/(2\sqrt{2})$, and therefore

$$4\sqrt{2}(m+c)=4\sqrt{2}\frac{17}{2\sqrt{2}}=34.$$

The negative root would also describe a tangent through the focus but violates the explicitly stated positive slope.

Step 4: Conclusion The required numerical value is $\boxed{34}$.

Quick Tip to Solve: A focal-chord condition immediately relates the intercept and slope. Then use centre-to-line distance rather than solving a line-circle quadratic.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A deuteron and an alpha particle having equal kinetic energy enter perpendicularly into a magnetic field. Let r_d and r_α be their respective radii of circular path. The value of $\frac{r_d}{r_\alpha}$ is equal to : [4 marks]

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Correct Answer: (C) $\sqrt{2}$

Solution:

Concept:

- A charged particle entering a uniform magnetic field perpendicularly follows a circle because $|q|vB=mv^2/r$. Thus $r=p/(|q|B)$, where $p=mv$.
- For a nonrelativistic particle of kinetic energy K, momentum is $p=\sqrt{2mK}$. At fixed energy and

field, the radius is therefore proportional to $\sqrt{m}/|q|$, rather than to mass alone.

Step 1: Identify the nuclear masses and charges A deuteron has one proton and one neutron, giving mass approximately $m_d = 2u$ and charge $q_d = e$. An alpha particle has two protons and two neutrons, giving $m_\alpha = 4u$ and charge $q_\alpha = 2e$. Here u is the atomic mass unit and e is the elementary charge. Using the standard nuclear mass ratios is appropriate for this comparison.

Step 2: Form the radius ratio at equal kinetic energy Use the radius formula separately for the two particles:

$$r_d = \frac{\sqrt{2m_dK}}{eB}, \qquad r_\alpha = \frac{\sqrt{2m_\alpha K}}{2eB}.$$

The field and common kinetic energy cancel in the ratio, giving

$$\frac{r_d}{r_\alpha} = 2\sqrt{\frac{m_d}{m_\alpha}} = 2\sqrt{\frac{2}{4}} = \sqrt{2}.$$

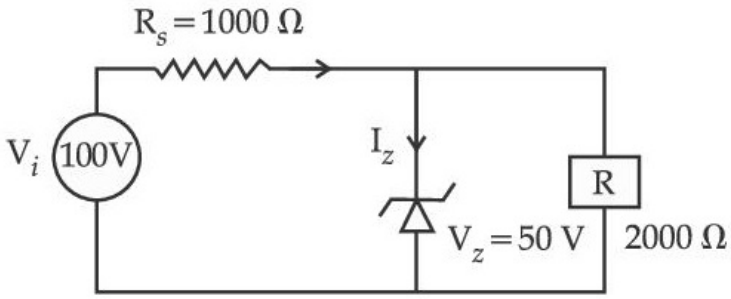
Although the alpha particle has greater momentum at equal energy, its doubled charge reduces its radius more strongly.

Step 3: Check the alternative ratios Equal kinetic energy does not mean equal speeds or equal momenta. The ratio 1 would overlook the different mass-to-charge dependence, while 2 would retain the charge factor but omit the square-root mass factor. The ratio $1/\sqrt{2}$ is the reciprocal of the required result and corresponds to reversing the order of the radii. The deuteron thus follows the larger circle.

Step 4: Conclusion $\boxed{r_d/r_\alpha = \sqrt{2}}$. Hence, the correct option is (C).

Quick Tip to Solve: At equal kinetic energy use $r \propto \sqrt{m}/q$; at equal speed the dependence would instead be $r \propto m/q$.

32. For the circuit shown below, calculate the value of I_z : [4 marks]



(A) 25 mA

(B) 0.1 A

(C) 0.15 A

(D) 0.05 A

Correct Answer: (A) 25 mA

Solution:

Concept:

- A reverse-biased Zener diode in breakdown maintains its specified voltage across the parallel load. The series resistor limits the total supply current.
- Kirchhoff's current law at the output junction gives $I_s = I_R + I_z$. The load current must be subtracted from the series-resistor current to obtain the Zener current.

Step 1: Check that breakdown can occur From the circuit, the supply is $100\,\text{V}$, the series resistance is $1000\,\Omega$, the load is $2000\,\Omega$, and the Zener voltage is $50\,\text{V}$. If the Zener were open, the divider output would be

$$V_{\text{out}} = 100\frac{2000}{1000+2000} = 66.7\,\text{V}.$$

This exceeds the breakdown voltage, so the reverse-biased diode can conduct and clamp the output to $50\,\text{V}$.

Step 2: Calculate the supply and load currents The series resistor has a voltage drop of $100-50 = 50\,\text{V}$, so its current is

$$I_s = \frac{100-50}{1000} = 0.050\,\text{A}.$$

The load is parallel to the Zener and therefore also has $50\,\text{V}$ across it. Its current is

$$I_R = \frac{50}{2000} = 0.025\,\text{A}.$$

These are different currents because some of the supply current leaves the junction through the load branch.

Step 3: Apply current conservation at the junction The downward Zener current is the remainder after the load current is supplied:

$$I_z = I_s - I_R = 0.050 - 0.025 = 0.025\,\text{A} = 25\,\text{mA}.$$

It is positive, consistent with the assumed breakdown state. The value $0.05\,\text{A}$ in option (D) is the total series current, not the Zener current. The larger currents $0.1\,\text{A}$ and $0.15\,\text{A}$ cannot satisfy this junction equation with the stated series resistor and clamped voltage.

Step 4: Conclusion The Zener current is $\boxed{I_z = 25\,\text{mA}}$, downward as shown. Hence, the correct option is (A).

Quick Tip to Solve: In a Zener regulator, calculate the series current and load current separately before subtracting; the series current is rarely the Zener current.

33. A current of 5 A is passing through a non-linear magnesium wire of cross-section 0.04 m^2. At every point the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is :

(Resistivity of magnesium $\rho = 44 \times 10^{-8}$ Ωm) [4 marks]

(A) 11×10^{-5} V/m

(B) 11×10^{-3} V/m

(C) 11×10^{-7} V/m

(D) 11×10^{-2} V/m

Correct Answer: (A) 11×10^{-5} V/m

Solution:

Concept:

- Current is the flux of the current-density vector through a cross-section: $I = \int \mathbf{J} \cdot d\mathbf{A}$. With uniform magnitude and a fixed angle θ to the area normal, $I = JA\cos\theta$.
- For an isotropic conductor, the local form of Ohm's law is $\mathbf{E} = \rho\mathbf{J}$, so the electric-field magnitude is $E = \rho J$. The resistivity must be used in SI units.

Step 1: Use the component normal to the area The direction of the area vector is perpendicular to the cross-section. The stated angle of 60° is between this normal and $\mathbf{J}$, so the normal component is $J\cos 60^\circ$, rather than $J\sin 60^\circ$. Insert $I = 5\,\text{A}$ and $A = 0.04\,\text{m}^2$:

$$5 = J(0.04)\cos 60^\circ.$$

Hence

$$J = \frac{5}{0.04 \times 0.5} = 250\,\text{A m}^{-2}.$$

Step 2: Convert current density to electric field The given resistivity is $\rho = 44 \times 10^{-8}\,\Omega\,\text{m}$. Applying the local relation gives

$$E = \rho J = (44 \times 10^{-8})(250) = 11000 \times 10^{-8}\,\text{V m}^{-1}.$$

Equivalently,

$$E = 1.1 \times 10^{-4}\,\text{V m}^{-1} = 11 \times 10^{-5}\,\text{V m}^{-1}.$$

The units follow because $(\Omega\,\text{m})(\text{A m}^{-2}) = \text{V m}^{-1}$.

Step 3: Check the exponent and geometric factor The area is already in square metres, so no extra centimetre-to-metre conversion is needed. Omitting the cosine would halve the field, and changing the exponent of the resistivity would shift the answer by powers of ten. Options (B) and (D) are too large by factors of 100 and 1000; option (C) is too small by a factor of 100. Only the first listed value agrees with both the vector-current relation and Ohm's law.

Step 4: Conclusion The field magnitude is $\boxed{11 \times 10^{-5}\,\text{V/m}}$. Hence, the correct option is (A).

Quick Tip to Solve: The angle in current-flux problems is measured from the area normal. Divide by $A\cos\theta$, then multiply by resistivity.

34. The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _____ $\times 10^3$ kg m/s^2. [Given cos30° = 0.87, μ_s = 0.2] [4 marks]

(A) 12.4

(B) 7.2

(C) 6.96

(D) 10.2

Correct Answer: (D) 10.2

Solution:

Concept:

- At the maximum speed before outward skidding, static friction is limiting and acts down the bank. Its magnitude is $f = \mu_s N$, where N is the normal reaction.
- The vehicle has no vertical acceleration. The vertical component of the normal is $N\cos\theta$, while down-slope friction has a downward component $f\sin\theta$. The horizontal components provide the centripetal force.

Step 1: Choose the friction direction for maximum speed At a speed above the frictionless design speed, the vehicle tends to slide up the bank, outward from the centre of the turn. Static friction must oppose this tendency, so at the maximum possible speed it acts down the slope. Its limiting value is $f = 0.2N$. Friction contributes inward and downward components.

Step 2: Balance the vertical forces Take vertically upward as positive. The normal makes an angle $\theta = 30°$ with the vertical. There is no vertical motion, so

$$N\cos\theta - f\sin\theta - mg = 0.$$

Replacing f by its limiting value gives

$$N(\cos\theta - \mu_s\sin\theta) = mg.$$

Therefore

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}.$$

The radius is unnecessary for this vertical balance.

Step 3: Substitute the numerical values and compare Use $m = 800\,\text{kg}$, $g = 9.8\,\text{m s}^{-2}$, the supplied $\cos 30° = 0.87$, and $\sin 30° = 0.5$. Thus

$$N = \frac{800 \times 9.8}{0.87 - 0.2 \times 0.5} = \frac{7840}{0.77} \approx 10182\,\text{N}.$$

To the precision of the options, this is $10.2 \times 10^3\,\text{kg m s}^{-2}$. Projecting weight alone is incorrect because friction also has a vertical component.

Step 4: Conclusion The normal reaction is approximately $\boxed{10.2 \times 10^3\,\text{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: For maximum speed on a banked road, friction acts down the slope; the vertical denominator is $\cos\theta - \mu\sin\theta$.

35. Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is : [4 marks]

(A) $v_A = v_B \neq v_C$

(B) $\frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$

(C) $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

(D) $v_A = v_B = v_C = 0$

Correct Answer: (C) $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

Solution:

Concept:

- For molecules of one species in a gas at equilibrium, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_BT$. Here m is the mass of one molecule, k_B is Boltzmann's constant, and T is the absolute temperature.
- All species in an equilibrium mixture have the same temperature and average translational kinetic energy. Their RMS speeds need not be equal: $v_{\text{rms}} = \sqrt{3k_BT/m}$.

Step 1: Compare the speeds at the common temperature The mixture is at normal temperature and pressure, so species A, B and C share the same nonzero absolute temperature. Applying the RMS formula to each species gives

$$v_A = \sqrt{\frac{3k_BT}{m_A}}, \quad v_B = \sqrt{\frac{3k_BT}{m_B}}, \quad v_C = \sqrt{\frac{3k_BT}{m_C}}.$$

Since $m_A < m_B < m_C$, the positive reciprocals of the masses have the reverse order. Taking positive square roots preserves that order, so

$$v_A > v_B > v_C > 0.$$

Step 2: Express the result in the form of the options For positive quantities, taking reciprocals reverses an inequality. Consequently

$$\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}.$$

One may also write the speed ratio as

$$v_A : v_B : v_C = \frac{1}{\sqrt{m_A}} : \frac{1}{\sqrt{m_B}} : \frac{1}{\sqrt{m_C}}.$$

Pressure does not supply a separate species-dependent factor in this thermal-speed comparison; the common temperature is the controlling condition.

Step 3: Eliminate the other listed relations Option (A) incorrectly makes the speeds of A and B equal despite their unequal masses. Option (B) reverses the correct ordering of the reciprocal speeds and would incorrectly make the heaviest molecules fastest. Option (D) gives zero molecular speeds, which is incompatible with the positive absolute temperature at normal conditions. Equality holds for the average translational kinetic energies, not for the speeds of unequal-mass molecules.

Step 4: Conclusion The correct relation is $\boxed{1/v_A < 1/v_B < 1/v_C}$. Hence, the correct option is (C).

Quick Tip to Solve: At a common temperature, heavier molecules move more slowly: compare $v \propto 1/\sqrt{m}$, then reverse the order if the options use reciprocal speeds.

36. The amount of heat needed to raise the temperature of 4 moles of rigid diatomic gas from 0° C to 50° C when no work is done is _____. (R is the universal gas constant). [4 marks]

(A) 500 R

(B) 250 R

(C) 750 R

(D) 175 R

Correct Answer: (A) 500 R

Solution:

Concept:

- The first law of thermodynamics is $Q = \Delta U + W$, where W is work done by the gas. When no work is done, the supplied heat equals its increase of internal energy.
- A rigid diatomic molecule has three translational and two rotational degrees of freedom. Vibrations are excluded by rigidity, so $f = 5$ and the molar heat capacity is $C_V = fR/2 = 5R/2$.

Step 1: Identify the relevant heat capacity The condition is explicitly zero work, so there is no energy expenditure on expansion. Therefore use $\Delta U = nC_V\Delta T$, rather than $nC_P\Delta T$. For this ideal rigid diatomic gas the internal energy depends only on temperature, and its five active degrees of freedom determine C_V.

Step 2: Calculate the temperature interval and heat A temperature difference has the same numerical value on the Celsius and Kelvin scales. Hence $\Delta T = 50 - 0 = 50\,\mathrm{K}$, while $n = 4\,\mathrm{mol}$. Substituting gives

$$Q = 4\left(\frac{5R}{2}\right)(50) = 500R.$$

Here R has units of energy per mole per kelvin, so the resulting quantity is an energy.

Step 3: Check the alternatives The value $250R$ omits a factor of two from the amount of gas or temperature interval. Neither $750R$ nor $175R$ follows from the five degrees of freedom and four moles stated in the question. Using a monatomic or a constant-pressure heat capacity would also describe a different physical situation.

Step 4: Conclusion The required heat is $Q = 500R$ (in joules when SI units are used for R). Hence, the correct option is (A).

Quick Tip to Solve: When the question states that no work is done, calculate the internal-energy change first; for a rigid diatomic gas use five active degrees of freedom.

37. A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is : [4 marks]

(A) $12\sqrt{2}$ m

(B) 20 m

(C) 3 m

(D) 15 m

Correct Answer: (D) 15 m

Solution:

Concept:

- Velocities relative to the ground are obtained by vector addition: $\vec{v}_{bg} = \vec{v}_{ba} + \vec{v}_{ag}$, where the subscripts denote butterfly, air and ground.
- North-East makes an angle of 45° with each horizontal compass axis. A constant resultant velocity produces displacement $\vec{s} = \vec{v}_{bg}t$.

Step 1: Resolve the butterfly velocity Take east as positive x and north as positive y. The butterfly velocity relative to the air has magnitude $4\sqrt{2}\,\mathrm{m\,s^{-1}}$. Therefore

$$v_{ba,x} = 4\sqrt{2}\cos 45^\circ = 4, \qquad v_{ba,y} = 4\sqrt{2}\sin 45^\circ = 4.$$

Both components are positive because the direction is North-East.

Step 2: Add the wind and obtain displacement The wind is from north to south, so its components are $(0, -1)\,\mathrm{m\,s^{-1}}$. Thus the ground velocity is $(4, 3)\,\mathrm{m\,s^{-1}}$, whose magnitude is $\sqrt{4^2 + 3^2} = 5\,\mathrm{m\,s^{-1}}$. In three seconds,

$$\vec{s} = (12\hat{\mathbf{i}} + 9\hat{\mathbf{j}})\,\mathrm{m}, \qquad |\vec{s}| = \sqrt{12^2 + 9^2} = 15\,\mathrm{m}.$$

Step 3: Distinguish vector addition from scalar addition The wind reduces only the northward component; it does not reduce the whole butterfly speed by $1\,\mathrm{m\,s^{-1}}$. The option $12\sqrt{2}\,\mathrm{m}$ ignores wind, while $3\,\mathrm{m}$ is just the displacement associated with wind alone. The proposed $20\,\mathrm{m}$ does not equal the magnitude of the resultant displacement.

Step 4: Conclusion The magnitude of the resultant displacement in three seconds is $15\,\mathrm{m}$. Hence, the correct option is (D).

Quick Tip to Solve: Resolve a North-East velocity into equal east and north components, then add a north-to-south wind only to the north component.

38. A certain charge Q is divided into two parts q and (Q − q). How should the charges Q and q be divided so that q and (Q − q) placed at a certain distance apart experience maximum electrostatic repulsion? [4 marks]

(A) Q = 2q

(B) Q = 4q

(C) Q = 3q

(D) Q = $\frac{q}{2}$

Correct Answer: (A) Q = 2q

Solution:

Concept:

- At fixed separation r, the repulsive force between charges q and $Q-q$ is $F = kq(Q-q)/r^2$, where $k = 1/(4\pi\varepsilon_0)$.
- For a fixed total positive charge Q, the product of two positive parts is greatest when the parts are equal. This can be established by completing a square or differentiation.

Step 1: Express the force in one variable The total charge and separation remain fixed while the division changes. Therefore the only varying factor in Coulomb's law is $q(Q-q) = Qq - q^2$, with $0 \le q \le Q$. At either endpoint one part vanishes and the force is zero, so the maximum must lie between the endpoints.

Step 2: Find and verify the maximum Complete the square:

$$q(Q-q) = \frac{Q^2}{4} - \left(q - \frac{Q}{2}\right)^2.$$

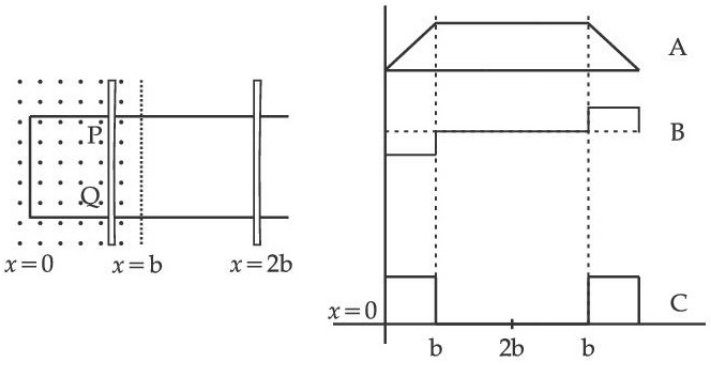
The squared term is nonnegative, so the largest possible product is $Q^2/4$, attained only at $q = Q/2$. Equivalently, $dF/dq = k(Q-2q)/r^2 = 0$ and $d^2F/dq^2 = -2k/r^2 < 0$, confirming a maximum rather than a minimum.

Step 3: Interpret the charge division and options The two charges must each be $Q/2$, so $Q = 2q$. If $Q = 4q$ or $Q = 3q$, the parts are unequal and the squared term above is positive, reducing the force. The relation $Q = q/2$ makes $q > Q$, so the other part becomes negative and the interaction is attractive rather than the required repulsion.

Step 4: Conclusion Maximum repulsion occurs for equal parts: $q = Q - q = Q/2$, or $Q = 2q$. Hence, the correct option is (A).

Quick Tip to Solve: For a fixed sum of two like charges and fixed separation, maximise their product by dividing the total charge equally.

39. The arm PQ of a rectangular conductor is moving from x = 0 to x = 2b outwards and then inwards from x = 2b to x = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from x = 0 to x = b. Identify the graph showing the variation of different quantities with distance. [4 marks]



(A) A-Flux, B-Power dissipated, C-EMF

(B) A-Flux, B-EMF, C-Power dissipated

(C) A-Power dissipated, B-Flux, C-EMF

(D) A-EMF, B-Power dissipated, C-Flux

Correct Answer: (B) A-Flux, B-EMF, C-Power dissipated

Solution:

Concept:

- Magnetic flux through the loop is $\Phi = B\ell \min(x, b)$, where ℓ is the length of the sliding arm and only the strip $0 \le x \le b$ contains magnetic field.
- Faraday's law gives $\mathcal{E} = -d\Phi/dt$. For loop resistance R, dissipated power is $P = \mathcal{E}^2/R$, which cannot be negative.

Step 1: Match the flux curve As the arm travels outwards from $x = 0$ to $x = b$, the area enclosing field increases linearly, so the flux increases linearly from zero to $B\ell b$. Between b and $2b$, no extra field-filled area is enclosed; the flux remains constant. On returning, it remains constant until the arm re-enters the field and then decreases to zero. This is precisely the rise, plateau and fall shown by graph A.

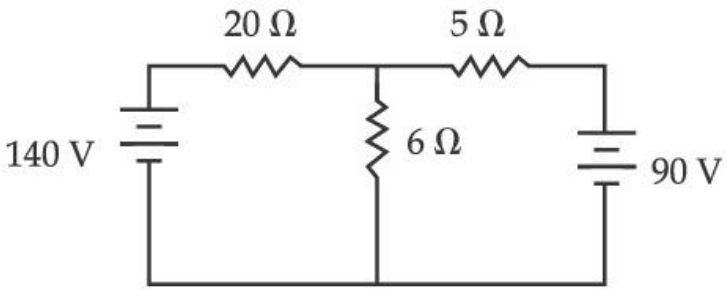
Step 2: Match the signed emf curve While the arm crosses the magnetic strip at constant speed v, the magnitude of induced emf is $B\ell v$. Outside that strip the flux is constant, so emf is zero. During the inward crossing the flux changes in the opposite sense; consequently the emf reverses sign. Graph B has opposite signed pulses at the two crossings, identifying it as emf.

Step 3: Match power and eliminate other assignments Squaring the emf removes its sign. The power is $(B\ell v)^2/R$ during each crossing and zero between them. Graph C has two positive pulses, so it represents power dissipated. Assigning B to power would imply negative dissipated power; assigning C to signed emf would miss the reversal. Graph A cannot be power or emf because both vanish when flux is constant.

Step 4: Conclusion The correspondence is A: flux, B: emf and C: power dissipated. Hence, the correct option is (B).

Quick Tip to Solve: Use three signatures: flux follows enclosed field area, emf changes sign on reversal, and resistive power remains nonnegative.

40. The value of current in the 6 Ω resistance is : [4 marks]



(A) 4A

(B) 8A

(C) 10A

(D) 6A

Correct Answer: (C) 10A

Solution:

Concept:

- Kirchhoff's current law requires the algebraic sum of currents at a junction to be zero. Across a resistor, the current from potential V_1 to V_2 is $(V_1 - V_2)/R$.
- An ideal battery fixes its terminal potential difference. Read its polarity from the longer plate, and select the common bottom wire as zero potential.

Step 1: Assign the three node potentials The longer battery plates in the circuit are at the upper terminals, making the left upper node $+140\,\text{V}$ and the right upper node $+90\,\text{V}$ relative to the bottom wire. Let the junction of the $20\,\Omega$, $5\,\Omega$ and $6\,\Omega$ resistors have potential V. The central branch therefore carries current $V/6$ downwards.

Step 2: Apply junction conservation and solve Current entering the central node from the left and right branches equals the current leaving through the central resistor:

$$\frac{140 - V}{20} + \frac{90 - V}{5} = \frac{V}{6}.$$

Multiplying by 60 gives

$$420 - 3V + 1080 - 12V = 10V, \qquad 25V = 1500.$$

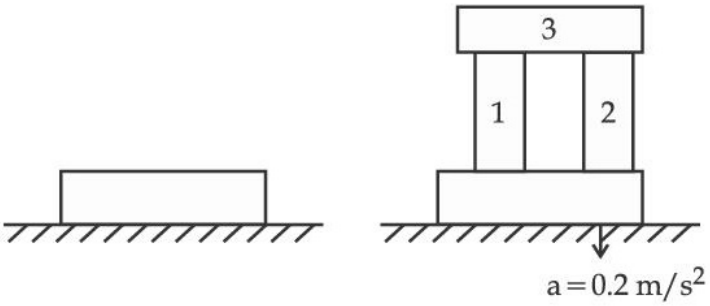
Thus $V = 60\,\text{V}$ and $I_6 = 60/6 = 10\,\text{A}$.

Step 3: Check the branch currents The left branch supplies $(140 - 60)/20 = 4\,\text{A}$, while the right branch supplies $(90 - 60)/5 = 6\,\text{A}$. Their sum is $10\,\text{A}$, exactly the central branch current. The alternatives 4 A and 6 A refer to the individual side branches; 8 A satisfies neither the node equation nor the voltage drop across the central resistance.

Step 4: Conclusion The current through the $6\,\Omega$ resistor is $10\,\text{A}$, directed from the top junction towards the bottom wire. Hence, the correct option is (C).

Quick Tip to Solve: For a circuit with two sources meeting at one junction, choose the common wire as zero and write one node equation instead of two mesh equations.

41. A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s^2. The normal reaction R' by the floor if mass of the iron cylinders are equal and of 20 kg each, is _____ N. [Take g = 10 m/s^2 and μ_s = 0.2] [4 marks]



(A) 686

(B) 684

(C) 714

(D) 716

Correct Answer: (A) 686

Solution:

Concept:

- For an entire accelerating system, internal contact forces cancel. The net external vertical force is the total mass multiplied by the common vertical acceleration.
- For downward acceleration a, weight Mg acts downwards and support reaction R upwards, so $Mg - R = Ma$ and $R = M(g - a)$.

Step 1: Choose the complete assembly as the system The assembly consists of the steel block of mass $10\,\text{kg}$ plus three cylinders of mass $20\,\text{kg}$ each. Its total mass is therefore

$$M = 10 + 3(20) = 70\,\text{kg}.$$

All parts move down together at the stated acceleration. Forces exchanged between cylinders and block occur in equal and opposite pairs and are internal to this system.

Step 2: Apply the vertical force equation Taking downward as positive, the external forces are the total weight and the upward reaction from the supporting floor. Newton's second law gives

$$70(10) - R\prime = 70(0.2).$$

Consequently,

$$R\prime = 700 - 14 = 686\,\text{N}.$$

The support force is smaller than the static total weight because the assembly accelerates downwards.

Step 3: Explain the role of the supplied friction coefficient The coefficient of static friction does not enter this vertical equation: the specified acceleration is already given, and internal contact forces

cannot alter the net external-force balance of the full assembly. The options 714 N and 716 N exceed the total weight and would correspond to an upward, rather than downward, net acceleration. The option 684 N uses an incorrect mass or acceleration reduction.

Step 4: Conclusion The normal reaction exerted by the floor is $R\prime = 686\,\mathrm{N}$. Hence, the correct option is (A).

Quick Tip to Solve: When several bodies move together vertically, combine their masses and use apparent weight $M(g - a)$ for downward acceleration; internal contacts need not be resolved.

42. A nucleus of mass M emits γ -ray photon of frequency 'v'. The loss of internal energy by the nucleus is :

[Take 'c' as the speed of electromagnetic wave] [4 marks]

(A) hv

(B) $hv\left[1+\frac{hv}{2Mc^2}\right]$

(C) $hv\left[1-\frac{hv}{2Mc^2}\right]$

(D) 0

Correct Answer: (B) $hv\left[1+\frac{hv}{2Mc^2}\right]$

Solution:

Concept:

- A photon of frequency ν carries energy $E_\gamma = h\nu$ and momentum $p_\gamma = h\nu/c$. Momentum conservation requires an initially stationary nucleus to recoil oppositely.
- For a heavy nucleus moving nonrelativistically, its recoil kinetic energy is $K = p^2/(2M)$. The released internal energy supplies both photon energy and recoil kinetic energy.

Step 1: Use momentum conservation Before emission the nucleus is at rest, so the total momentum is zero. After emission the photon has momentum of magnitude $h\nu/c$; the nucleus must have equal and opposite momentum. Therefore

$$p_N = \frac{h\nu}{c}, \qquad K_N = \frac{p_N^2}{2M} = \frac{h^2\nu^2}{2Mc^2}.$$

The recoil energy is positive regardless of the chosen direction axis.

Step 2: Add the two energy contributions The loss of internal nuclear energy is not just the energy observed in the photon. Part of that loss becomes kinetic energy of the recoiling nucleus. Hence

$$\Delta E_{\text{internal}} = h\nu + K_N = h\nu + \frac{h^2\nu^2}{2Mc^2} = h\nu\left(1+\frac{h\nu}{2Mc^2}\right).$$

This uses the usual heavy-nucleus approximation implicit in the options, with recoil speed much smaller than c.

Step 3: Check sign and limiting behaviour The expression must exceed $h\nu$, since the same internal transition must supply the positive recoil energy as well. Thus option A neglects recoil, option C

incorrectly subtracts recoil energy and option D violates energy conservation. As M tends to infinity the recoil term tends to zero, recovering photon energy alone as expected.

Step 4: Conclusion The loss of internal energy is $h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$. Hence, the correct option is (B).

Quick Tip to Solve: In photon emission, conserve momentum first to obtain the recoil energy, then add that energy to the photon energy when finding the internal-energy loss.

43. The radiation corresponding to 3 $\rightarrow$ 2 transition of a hydrogen atom falls on a gold surface to generate photoelectrons. The electrons are passed through a magnetic field of 5×10^{-4} T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work function of the metal is : (Mass of electron = 9.1×10^{-31} kg) [4 marks]

(A) 1.36 eV

(B) 1.88 eV

(C) 0.82 eV

(D) 0.16 eV

Correct Answer: (C) 0.82 eV

Solution:

Concept:

- A hydrogen transition emits a photon of energy $E_\gamma = 13.6(1/n_f^2 - 1/n_i^2)\,\text{eV}$. Here the electron falls from level 3 to level 2.
- Einstein's photoelectric equation is $E_\gamma = \phi + K_{\max}$, where ϕ is the work function. In a perpendicular magnetic field, $p = eBr$ and $K = p^2/(2m_e)$.

Step 1: Find the incident photon energy The transition energy is the difference between the two hydrogen energy levels, not the ionisation energy of either level. Thus

$$E_\gamma = 13.6\left(\frac{1}{4}-\frac{1}{9}\right) = 13.6\frac{5}{36} = 1.8889\,\text{eV}.$$

This is the energy available for removing an electron and supplying its kinetic energy.

Step 2: Use the largest orbit to find the maximum kinetic energy The magnetic force supplies centripetal force, so $evB = m_e v^2/r$. The largest radius corresponds to the largest momentum and hence the fastest photoelectron. Using $r = 7 \times 10^{-3}\,\text{m}$,

$$p = eBr = (1.6\times10^{-19})(5\times10^{-4})(7\times10^{-3}) = 5.6\times10^{-25}\,\text{kg m s}^{-1}.$$

Therefore

$$K_{\max} = \frac{(5.6\times10^{-25})^2}{2(9.1\times10^{-31})} = 1.7231\times10^{-19}\,\text{J} = 1.0769\,\text{eV}.$$

Step 3: Subtract the kinetic energy The energy spent in escaping from the metal is

$$\phi = 1.8889 - 1.0769 = 0.8120\,\text{eV} \approx 0.82\,\text{eV}.$$

The value 1.88 eV is approximately the photon energy itself; it would incorrectly leave no kinetic energy. The values 1.36 eV and 0.16 eV do not satisfy the measured orbit radius.

Step 4: Conclusion The work function is approximately $0.82\,\text{eV}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert the orbit energy from joules to electronvolts before subtracting it from the hydrogen transition energy.

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44. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A} \cdot \vec{B} = \left|\vec{A} \times \vec{B}\right|$. Then the value of $\left|\vec{A} - \vec{B}\right|$ will be : [4 marks]

(A) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

(B) $\sqrt{A^2 + B^2}$

(C) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

(D) $\sqrt{A^2 + B^2 + 2AB}$

Correct Answer: (C) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

Solution:

Concept:

- For vectors of magnitudes A, B and angle θ, the scalar product is $AB\cos\theta$ and the cross-product magnitude is $AB\sin\theta$, with $0 \le \theta \le \pi$.
- The squared magnitude of a vector difference follows from taking its scalar product with itself: $|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2AB\cos\theta$.

Step 1: Determine the angle between the vectors For nonzero vectors the supplied equality gives

$$AB\cos\theta = AB\sin\theta \quad \Longrightarrow \quad \cos\theta = \sin\theta.$$

Because the right side is a magnitude, the scalar product cannot be negative. Hence the angle is in the first quadrant and $\theta = 45^\circ$. Squaring the relation without this sign check could wrongly retain an obtuse angle. If either vector is zero, the final expression remains valid directly.

Step 2: Expand the difference and substitute the angle Expanding the dot product keeps the negative cross term:

$$\begin{aligned} |\vec{A} - \vec{B}|^2 &= (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) \\ &= A^2 + B^2 - 2\vec{A} \cdot \vec{B} \\ &= A^2 + B^2 - 2AB\frac{1}{\sqrt{2}} \\ &= A^2 + B^2 - \sqrt{2}AB. \end{aligned}$$

Taking the nonnegative square root gives the required magnitude. The positive mixed term in option A belongs to the vector sum for this angle. Option B would require perpendicular vectors, whereas

option D describes an antiparallel configuration for the difference. Neither satisfies the given relation for nonzero vectors.

Step 3: Conclusion

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - \sqrt{2}AB}.$$

Hence, the correct option is (C).

Quick Tip to Solve: First determine the sign of the dot product; then use the minus cross term for a vector difference.

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45. The entropy of any system is given by $S = \alpha^2\beta \ln\left[\frac{\mu kR}{J\beta^2} + 3\right]$ where α and β are the constants. μ, J, k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively.

[Take $S = \frac{dQ}{T}$]

Choose the incorrect option from the following : [4 marks]

(A) α and J have the same dimensions.

(B) S and α have different dimensions

(C) S, β, k and μR have the same dimensions

(D) α and k have the same dimensions

Correct Answer: (D) α and k have the same dimensions

Solution:

Concept:

- The argument of a logarithm must be dimensionless. Terms added inside that argument must have the same dimensions, so $\mu kR/(J\beta^2)$ is dimensionless.
- Entropy has dimensions of energy divided by temperature: $[S] = ML^2T^{-2}\Theta^{-1}$. Both Boltzmann constant k and μR have these dimensions.

Step 1: Identify the dimensions of the known quantities The mechanical equivalent of heat converts between two units of the same physical quantity, energy. It therefore has no physical dimensions: $[J] = 1$. The mole factor cancels the amount-of-substance denominator in the molar gas constant, giving

$$[\mu R] = [k] = ML^2T^{-2}\Theta^{-1} = [S].$$

This uses the supplied entropy relation $S = dQ/T$; the temperature in this relation is thermodynamic temperature.

Step 2: Find the dimensions of beta and alpha From the logarithm argument,

$$\frac{[\mu R][k]}{[J][\beta]^2} = 1.$$

Since $[J] = 1$ and each numerator factor has entropy dimensions, $[\beta]^2 = [S]^2$, so $[\beta] = [S]$. The

logarithm itself is dimensionless, and the outer equation gives

$$[S] = [\alpha]^2[\beta] = [\alpha]^2[S].$$

Consequently $[\alpha] = 1$: alpha is dimensionless.

Step 3: Check all four statements Alpha and J are both dimensionless, so A is true. Entropy is dimensional whereas alpha is dimensionless, so B is true. Entropy, beta, Boltzmann constant and the product of the mole number with R have the same dimensions, so C is true. Boltzmann constant is not dimensionless; thus the assertion that alpha and k have the same dimensions is false. The question asks for this incorrect statement.

Step 4: Conclusion α is dimensionless but $[k] = ML^2T^{-2}\Theta^{-1}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the dimensionless logarithm argument first, and only then compare the prefactor with entropy.

46. AC voltage V(t) = 20 sinωt of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m^2. The amplitude of the oscillating displacement current for the applied AC voltage is _____. [Take ε_0 = 8.85 $\times$ 10^{-12} F/m] [4 marks]

(A) 55.58 μA

(B) 21.14 μA

(C) 27.79 μA

(D) 83.37 μA

Correct Answer: (C) 27.79 μA

Solution:

Concept:

- For an ideal parallel-plate capacitor in vacuum, $C = \varepsilon_0 A/d$. The plate separation must be expressed in metres.
- Displacement current equals $\varepsilon_0\, d\Phi_E/dt = C\, dV/dt$. If $V = V_0 \sin\omega t$, its current amplitude is $I_0 = \omega C V_0$, where $\omega = 2\pi f$.

Step 1: Calculate the capacitance Using the given area and gap,

$$C = \frac{(8.85 \times 10^{-12})(1)}{2 \times 10^{-3}} = 4.425 \times 10^{-9}\,\text{F}.$$

The electric field is uniform in the ideal parallel-plate model, so its flux through the plate area is AV/d. Differentiating this flux gives the same capacitance factor as above; there is no need for conduction of charge across the gap.

Step 2: Differentiate the applied voltage The specified 20 V is the peak voltage, not an rms value. The angular frequency is

$$\omega = 2\pi(50) = 100\pi\,\text{rad}\,\text{s}^{-1}.$$

Therefore

$$I_d(t) = C(20\omega \cos \omega t), \qquad I_0 = (4.425 \times 10^{-9})(20)(100\pi).$$

The cosine describes the quarter-cycle lead of the capacitor current over the sine voltage; the amplitude is its positive coefficient.

Step 3: Convert the amplitude to microamperes Numerically,

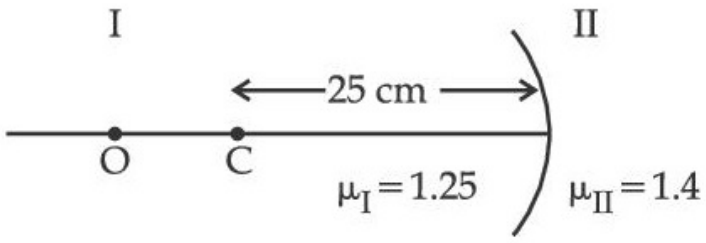
$$I_0 = 2.7803 \times 10^{-5}\,\text{A} = 27.803\,\mu\text{A}.$$

Rounding with the conventional approximation for pi used in the options gives 27.79 microamperes. Option A is twice this value and D is three times it; option B also fails the product $2\pi f C V_0$. No rms conversion is appropriate because the requested quantity is an amplitude.

Step 4: Conclusion The displacement-current amplitude is approximately $27.79\,\mu\text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: For a sinusoidal capacitor voltage, multiply its peak by $2\pi f C$; do not divide the peak by $\sqrt{2}$.

47. Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is : [4 marks]



(A) 55.44 cm

(B) 18.23 cm

(C) 9.52 cm

(D) 37.58 cm

Correct Answer: (D) 37.58 cm

Solution:

Concept:

- Refraction from index n_1 to index n_2 at a spherical surface obeys $n_2/v - n_1/u = (n_2 - n_1)/R$ with Cartesian signed distances.
- Take the direction of incident light from region I to region II as positive. An object or centre of curvature on the incident side then has a negative coordinate.

Step 1: Read the refractive indices and signs The figure gives $n_1 = 1.25$ in region I and $n_2 = 1.4$ in region II. The object is 40 cm to the left of the surface, hence $u = -40\,\text{cm}$. The centre C is also to the left, 25 cm from the surface, hence $R = -25\,\text{cm}$. The negative radius is essential: the surface is

concave as seen by the incident rays. All distances may be kept in centimetres because they enter the same equation.

Step 2: Substitute in the spherical-refraction equation Using the signed quantities gives

$$\frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4 - 1.25}{-25}.$$

Thus

$$\frac{1.4}{v} + 0.03125 = -0.006, \qquad \frac{1.4}{v} = -0.03725\,\text{cm}^{-1}.$$

Solving for the image coordinate,

$$v = -\frac{1.4}{0.03725} = -37.58\,\text{cm}.$$

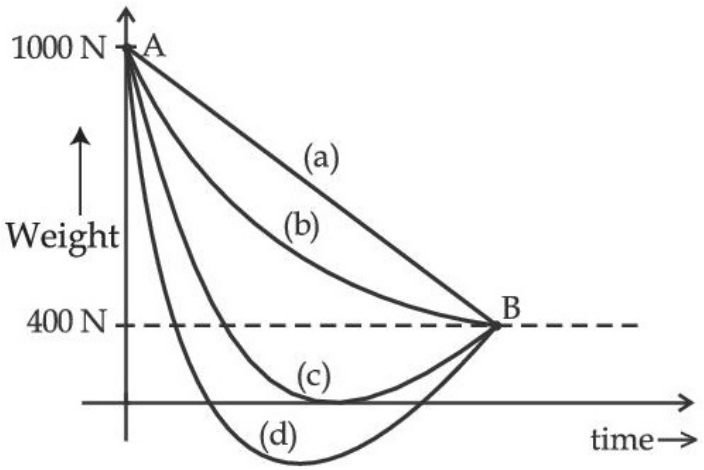
The negative coordinate describes an image on the object side; it is a virtual image formed by backward extensions of the refracted rays.

Step 3: Compare the distance with the options The question asks for distance from the surface, so report the magnitude, 37.58 cm, together with its side. The values 55.44 cm, 18.23 cm and 9.52 cm do not solve the signed refraction equation. In particular, changing the sign of R because the light enters a denser medium is incorrect: the sign is fixed by the actual position of C in the figure.

Step 4: Conclusion The image is $37.58\,\text{cm}$ from the surface in region I (virtual). Hence, the correct option is (D).

Quick Tip to Solve: Mark the incident-light direction first; both the object and the centre lie on its negative side in this figure.

48. A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s^2 and 4 m/s^2 respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time. [4 marks]



(A) (b)

(B) (c)

(C) (d)

(D) (a)

Correct Answer: (B) (c)

Solution:

Concept:

- Gravitational weight is the magnitude of the net gravitational force. Along the route between Earth and Mars, their attractions act in opposite directions.
- Each planetary attraction decreases with the square of distance from that planet. A point between the planets has equal and opposite attractions, giving zero net weight.

Step 1: Calculate the endpoint weights The passenger mass stays $100\,\text{kg}$ throughout the journey; mass itself does not decrease with distance. At the Earth end the specified surface gravity gives

$$W_E = mg_E = 100(10) = 1000\,\text{N}.$$

At the Mars end it gives

$$W_M = mg_M = 100(4) = 400\,\text{N}.$$

The graph must start at 1000 N and end at 400 N, as the supplied curves do.

Step 2: Identify the minimum weight between the planets While travelling away from Earth, its attraction weakens. Mars attracts the passenger towards Mars, opposite to Earth's pull along the connecting line. Before reaching Mars there is a point where

$$\frac{GM_Em}{r_E^2} = \frac{GM_Mm}{r_M^2}.$$

There the resultant gravitational force is zero. Beyond it, the direction of the resultant reverses and its magnitude grows towards the final 400 N. Because weight here is a magnitude, it cannot become negative.

Step 3: Select the curve with the correct behaviour Curve (c) falls from 1000 N to zero and subsequently rises to 400 N. Curves (a) and (b) never reach the zero-force point and therefore miss the cancellation of the two attractions. Curve (d) shows negative weight, which is unsuitable for a graph of force magnitude. The graph is qualitative: no constant spaceship speed or detailed time law is required to distinguish the zero minimum from the other displayed possibilities.

Step 4: Conclusion The curve with a zero minimum best represents the passenger's gravitational weight. Hence, the correct option is (B).

Quick Tip to Solve: Check both endpoints and the cancellation point; the magnitude of net gravitational weight reaches zero but never becomes negative.

---

49. The value of tension in a long thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are l_1 and l_2 respectively. The actual length of the metal wire is : [4 marks]

(A) $\frac{l_1+l_2}{2}$

(B) $\sqrt{T_1T_2l_1l_2}$

(C) $\frac{T_1l_2-T_2l_1}{T_1-T_2}$

(D) $\frac{T_1l_1-T_2l_2}{T_1-T_2}$

Correct Answer: (C) $\frac{T_1l_2-T_2l_1}{T_1-T_2}$

Solution:

Concept:

- Within the elastic limit, extension is proportional to tensile force: $\Delta l = TL/(AY)$, where L is the natural length, A the cross-sectional area and Y Young's modulus.
- The measured length is natural length plus extension. Thus length is a linear function of tension, and its zero-tension intercept is the required actual length.

Step 1: Write the two extension equations Let L denote the unstretched length of the same wire. In the usual small-strain approximation its area and Young modulus are constant, so

$$l_1 = L + \frac{T_1L}{AY}, \qquad l_2 = L + \frac{T_2L}{AY}.$$

Both equations contain the same elastic coefficient $L/(AY)$. Neither observed stretched length alone equals L unless its associated tension vanishes. We therefore eliminate this common coefficient instead of averaging the two lengths.

Step 2: Eliminate the elastic coefficient Multiply the first equation by T_2 and the second by T_1:

$$T_2l_1 = T_2L + \frac{T_1T_2L}{AY}, \qquad T_1l_2 = T_1L + \frac{T_1T_2L}{AY}.$$

Subtracting cancels the extension terms exactly, giving

$$T_1 l_2 - T_2 l_1 = (T_1 - T_2)L.$$

For the two distinct tensions specified, divide by their difference to obtain L.

Step 3: Check the result and reject the distractors The ratio has units of length as required. If $T_2 = 0$, the result reduces to $L = l_2$, which correctly recovers the unloaded length. The arithmetic average in A is generally still a stretched length. Option B has units of force times length, not length. Option D does not give the zero-tension intercept; with $T_2 = 0$ it gives l_1, the length under the other tension. These checks confirm both the order and the signs in the numerator.

Step 4: Conclusion

$$L = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Treat measured length versus tension as a straight line and eliminate its slope to obtain the zero-load intercept.

50. A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after which one third of the material remains ? (Take ln 3 = 1.1) [4 marks]

(A) 740 years

(B) 1110 years

(C) 700 years

(D) 340 years

Correct Answer: (A) 740 years

Solution:

Concept:

- For independent simultaneous decay channels, the probability per unit time of removal of a nucleus is the sum of their decay constants: $\lambda = \lambda_1 + \lambda_2$.
- A partial half-life T_i is related to its decay constant by $\lambda_i = \ln 2/T_i$. The surviving number follows $N = N_0 e^{-\lambda t}$.

Step 1: Combine the two decay channels Both emissions remove nuclei from the same original radioactive material. They are competing decay routes, not two successive stages, so their rates must be added. With time measured in years,

$$\lambda = \frac{\ln 2}{1400} + \frac{\ln 2}{700} = \frac{3 \ln 2}{1400} \text{ year}^{-1}.$$

The corresponding effective half-life is $1400/3$ years, shorter than either partial half-life. This provides a useful physical check.

Step 2: Apply the required surviving fraction One third remains when $N/N_0 = 1/3$, rather than when one third has decayed. Therefore

$$\frac{1}{3} = e^{-\lambda t}, \qquad t = \frac{\ln 3}{\lambda} = \frac{1400 \ln 3}{3 \ln 2}.$$

Using the supplied $\ln 3 = 1.1$ and $\ln 2 \simeq 0.693$,

$$t = \frac{1400(1.1)}{3(0.693)} \simeq 740.7 \text{ years}.$$

The closest listed value is 740 years. The values 1110 and 700 years do not satisfy the exponential with the combined decay constant; 340 years is even shorter than the effective half-life and cannot leave only one third.

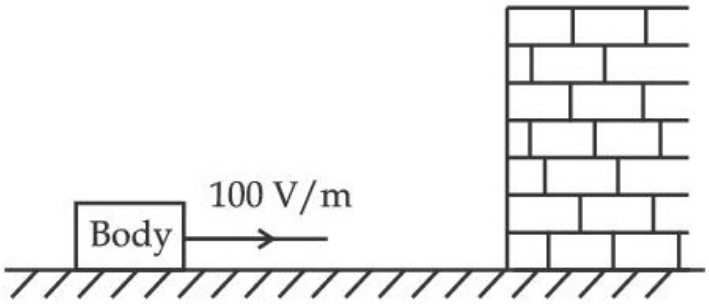
Step 3: Conclusion The required time is approximately 740 years. Hence, the correct option is (A).

Quick Tip to Solve: For simultaneous independent radioactive decay modes, add decay constants, not half-lives; then use the fraction remaining in the exponential.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A body having specific charge 8 μC/g is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be _____ s. [4 marks]



Solution:

Concept:

- The electric acceleration on a charged body is $a = qE/m$. Specific charge means the ratio q/m, so the mass need not be known separately.
- A perfectly elastic impact with a fixed wall reverses velocity without changing its magnitude. The electric force continues to point towards the wall after impact.

Step 1: Find the acceleration and first arrival time Convert the specific charge to SI units before

applying the electric-force equation:

$$\frac{q}{m} = 8\ \mu\text{C/g} = 8 \times 10^{-3}\ \text{C/kg}.$$

The acceleration towards the wall is constant:

$$a = \frac{q}{m}E = (8 \times 10^{-3})(100) = 0.8\ \text{m/s}^2.$$

The initial separation is $s = 0.10$ m, and the body starts from rest. Hence

$$s = \frac{1}{2}at_0^2, \qquad t_0 = \sqrt{\frac{2(0.10)}{0.8}} = 0.50\ \text{s}.$$

Its speed immediately before the first collision is $at_0 = 0.40$ m/s.

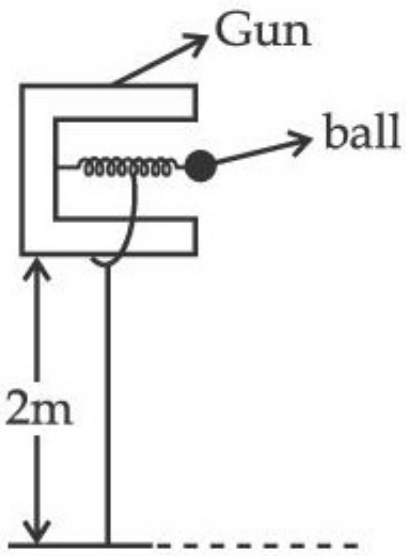
Step 2: Follow the return motion to the initial state The elastic collision reverses the speed to 0.40 m/s away from the wall. The acceleration is still directed towards the wall, so the body decelerates until it stops. The time to stop is $v/a = 0.40/0.8 = 0.50$ s. Its distance travelled away from the wall is $v^2/(2a) = 0.10$ m, exactly the original separation. It is now at the same position and has the same zero velocity as at the start; the subsequent motion repeats. Thus one period contains the approach to the wall and the return to the outer turning point:

$$T = 2t_0 = 1.00\ \text{s}.$$

Step 3: Conclusion The time period is $\boxed{1\ \text{s}}$, so the numerical entry is 1.

Quick Tip to Solve: Count a complete cycle by returning to the same position and velocity. For a constant-force wall bounce, the first wall-arrival time is only half the period.

52. In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is _____ m. (g = 10 m/s^2). [4 marks]



Solution:

Concept:

- An ideal compressed spring stores elastic energy $U = \frac{1}{2}kx^2$. On release into a horizontal barrel, this becomes the ball's launch kinetic energy $\frac{1}{2}mv^2$.
- For a horizontal launch, initial vertical velocity is zero. The vertical fall determines flight time through $h = \frac{1}{2}gt^2$, while horizontal range is $d = vt$.

Step 1: Calculate the horizontal launch speed The spring constant is $k = 100\ \mathrm{N/m}$, compression is $x = 0.05\ \mathrm{m}$, and ball mass is $m = 100\ \mathrm{g} = 0.10\ \mathrm{kg}$. Equating spring energy to launch kinetic energy gives

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2, \qquad v = x\sqrt{\frac{k}{m}}.$$

Consequently,

$$v = 0.05\sqrt{\frac{100}{0.10}} = \sqrt{2.5}\ \mathrm{m/s}.$$

The initial spring energy is 0.125 J. No gravitational energy term is needed during the horizontal release; the height affects the subsequent fall.

Step 2: Find the flight time and locate the box The ball leaves horizontally from height $h = 2\ \mathrm{m}$, so the vertical component of its initial velocity is zero. With the stated $g = 10\ \mathrm{m/s^2}$,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{4}{10}} = \sqrt{0.4}\ \mathrm{s}.$$

There is no horizontal acceleration after launch, so the required ground distance from the vertical line through the muzzle is

$$d = vt = \sqrt{2.5}\sqrt{0.4} = \sqrt{1} = 1\ \mathrm{m}.$$

The two square roots multiply exactly, so rounding the speed or the flight time separately is unnecessary. The box must intercept the ball at this horizontal displacement.

Step 3: Conclusion The required distance is $\boxed{d = 1\ \mathrm{m}}$, so the numerical entry is 1.

Quick Tip to Solve: For a horizontal spring launch, combine energy and projectile motion directly: $d = x\sqrt{2kh/(mg)}$. Keep exact factors until the final multiplication.

---

53. An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to _____ cm. [4 marks]

Solution:

Concept:

- For a simple magnifying lens forming the image at the least distance of distinct vision D, the angular magnification is $M_s = 1 + D/f_o$.

- For a compound microscope in normal adjustment, with the final image at infinity, $M = (L/f_o)(D/f_e)$ in the standard tube-length approximation.

Step 1: Determine the focal length of the original lens The near-point distance is $D = 25\ \text{cm}$. The given lens used as a simple microscope has magnification 6 with the image at the near point. Therefore

$$6 = 1 + \frac{25}{f_o}, \qquad f_o = \frac{25}{5} = 5\ \text{cm}.$$

The additive 1 belongs to the near-point simple-microscope formula. Using D/f_o at this stage would instead describe the relaxed-eye setting and would give a different focal length. The same lens is now used as the objective of the two-lens arrangement.

Step 2: Apply the new total magnification The desired total magnification is twice the earlier value, so $M = 2(6) = 12$. Convert the tube length to the same units as the focal lengths: $L = 0.6\ \text{m} = 60\ \text{cm}$. Since the final image is at infinity, the eyepiece magnification is D/f_e, without an additive 1. In the usual microscope tube-length formula,

$$12 = \frac{60}{5}\frac{25}{f_e} = 12\frac{25}{f_e}.$$

Thus $25/f_e = 1$, giving $f_e = 25\ \text{cm}$. Here L/f_o is the approximate objective magnification used in the standard compound-microscope expression; all the lengths in this substitution are in centimetres.

Step 3: Conclusion The focal length of the eyepiece is $\boxed{25\ \text{cm}}$, so the numerical entry is 25.

Quick Tip to Solve: Use $1 + D/f$ for a simple microscope at the near point, but D/f for an eyepiece whose final image is at infinity. Convert the tube length before substituting.

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54. A circular disc reaches from top to bottom of an inclined plane of length 'L'. When it slips down the plane, it makes time 't_1'. When it rolls down the plane, it takes time t_2. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of x will be _____. [4 marks]

Solution:

Concept:

- For frictionless sliding on an incline of angle θ, the acceleration is $a_s = g\sin\theta$. Starting from rest, the travel time over length L is $\sqrt{2L/a_s}$.
- For rolling without slipping, $a_r = g\sin\theta/[1 + I/(mR^2)]$. A uniform circular disc has $I = \frac{1}{2}mR^2$.

Step 1: Compare the sliding and rolling accelerations In the sliding comparison, gravitational potential energy becomes translational kinetic energy, so the acceleration down the plane is

$$a_s = g\sin\theta.$$

For pure rolling, some of the released gravitational energy becomes rotational kinetic energy as well.

With the disc's moment of inertia $I = mR^2/2$,

$$a_r = \frac{g\sin\theta}{1+1/2} = \frac{2}{3}g\sin\theta.$$

Static friction supplies the rolling torque and, for an ideal fixed incline with no slipping, does no work at the contact point. The smaller centre-of-mass acceleration means that rolling takes longer.

Step 2: Take the ratio of travel times Both motions start from rest at the top and cover the same incline length L. Using $L = at^2/2$ for each gives

$$t_1 = \sqrt{\frac{2L}{g\sin\theta}}, \qquad t_2 = \sqrt{\frac{2L}{(2/3)g\sin\theta}}.$$

Therefore

$$\frac{t_2}{t_1} = \sqrt{\frac{a_s}{a_r}} = \sqrt{\frac{3}{2}}.$$

The specified form is $\sqrt{3/x}$. Comparing the positive quantities under the square roots gives $3/x = 3/2$, hence $x = 2$. The incline length, disc mass, disc radius and angle cancel, which is why none of those numerical values is needed.

Step 3: Conclusion The time ratio is $\sqrt{3/2}$, so $\boxed{x = 2}$.

Quick Tip to Solve: For equal distances from rest, times vary as $1/\sqrt{a}$. Insert the moment-of-inertia factor before taking the square root, rather than comparing accelerations directly.

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55. The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time t = 0 and $y = \frac{1}{1+(x-2)^2}$ at t = 1 s, where x and y are in metres. The shape of wave does not change during the propagation. The velocity of the wave will be _____ m/s. [4 marks]

Solution:

Concept:

- A wave of unchanging shape moving at speed v in the positive x-direction has the form $y(x,t) = F(x - vt)$, where F is its complete initial profile.
- Two snapshots must be exact horizontal translations of each other. A translation can shift a singularity or a maximum, but cannot turn a singular profile into a bounded one.

Step 1: Test the profiles exactly as written The stated initial profile is $F(x) = 1/(1+x)^2$. If this profile translates without changing shape, the later profile must be

$$y(x,t) = \frac{1}{(1+x-vt)^2}.$$

At $t = 1$ s, this would be $1/(1+x-v)^2$. It has a singularity at $x = v - 1$. However, the supplied later profile $1/[1+(x-2)^2]$ has a strictly positive denominator and is finite for every real x. Thus no real constant velocity makes the two complete profiles translations of one another.

Step 2: Check the key value and state the limitation The listed answer key gives $2\ \mathrm{m/s}$, the speed associated with a shift of 2 m in 1 s. That value would follow if the initial profile were $1/(1+x^2)$, since replacing x by $x-2t$ would then produce the stated second snapshot at 1 s. It does not follow from the printed $1/(1+x)^2$. For example, substituting $v=2$ into the actual initial profile gives $y(x,1)=1/(x-1)^2$, not $1/[1+(x-2)^2]$. Even a coefficient comparison of the two quadratic denominators rules out equality, independently of the singularity argument. The initial expression must therefore be distinguished from the conditional value suggested by the key.

Step 3: Conclusion As printed, the profiles are inconsistent with propagation of unchanged shape, so no constant wave velocity satisfies both. The key's value $2\ \mathrm{m/s}$ is obtained only for the different initial profile $1/(1+x^2)$.

Quick Tip to Solve: Before reading speed from a shifted bracket, verify that the entire denominator has the same functional form in both snapshots; parentheses and squares are essential.

56. In an LCR series circuit, an inductor 30 mH and a resistor 1 Ω are connected to an AC source of angular frequency 300 rad/s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x}\times 10^{-3}$ F. Then the value of x is _____. [4 marks]

Solution:

Concept:

- The inductive and capacitive reactances are $X_L=\omega L$ and $X_C=1/(\omega C)$. Both have units of resistance.
- For a series LCR circuit the voltage phase relative to current satisfies $\tan\phi=(X_L-X_C)/R$. A leading current means a net capacitive circuit, so $\phi<0$.

Step 1: Use the direction of the phase difference The current leads the applied voltage by 45°. Equivalently, the voltage lags the current, so $\phi=-45^\circ$ in the impedance convention. Therefore

$$\frac{X_L-X_C}{R}=\tan(-45^\circ)=-1, \qquad X_C=X_L+R.$$

This sign is essential: choosing $X_L-X_C=+R$ would instead describe a lagging current. Convert the inductance to henry before calculating reactance:

$$L=30\ \mathrm{mH}=0.030\ \mathrm{H}, \qquad X_L=(300)(0.030)=9\ \Omega.$$

Step 2: Determine the capacitance and compare the given form With $R=1\ \Omega$, the necessary capacitive reactance is

$$X_C=9+1=10\ \Omega.$$

Hence

$$C=\frac{1}{\omega X_C}=\frac{1}{300(10)}\ \mathrm{F}=\frac{1}{3}\times 10^{-3}\ \mathrm{F}.$$

Comparing this result with the question's specified $(1/x)\times 10^{-3}$ F gives $x=3$. As a check, this capacitance gives $X_L-X_C=9-10=-1\ \Omega$, so $\tan\phi=-1$. The resulting voltage lag is exactly 45°,

confirming the required current lead.

Step 3: Conclusion The capacitance is $\frac{1}{3} \times 10^{-3}$ F, so $\boxed{x = 3}$.

Quick Tip to Solve: Attach the sign to the impedance phase: a leading current requires $X_C > X_L$. At a 45-degree lead the excess capacitive reactance equals R.

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57. A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of 'x' will be _____. [4 marks]

Solution:

Concept:

- With no external horizontal impulse, total linear momentum is conserved. The rod can both translate and rotate after an off-centre impact.
- Angular momentum about the rod centre is conserved during impact. For a uniform thin rod, $I = ML^2/12$. A completely elastic collision also conserves total kinetic energy.

Step 1: Find the translational speed Let the speed of the rod centre after collision be V, and its angular speed be ω. The incoming particle stops, so all its initial linear momentum is transferred to the rod. Thus

$$mu = MV, \qquad V = \frac{mu}{M}.$$

The frictionless support supplies no horizontal impulse; it does not prevent translation of the rod. Treating the rod as rotating about a fixed end would therefore use the wrong moment of inertia.

Step 2: Find the angular speed The particle strikes an end, a perpendicular distance $L/2$ from the rod centre. Its initial angular momentum about that centre is $muL/2$. After impact the particle is stationary, while the rod has spin angular momentum $I\omega$. Therefore

$$\frac{muL}{2} = \frac{ML^2}{12}\omega, \qquad \omega = \frac{6mu}{ML}.$$

The rod centre has no orbital angular momentum about its own instantaneous position.

Step 3: Use the elastic energy condition The initial kinetic energy becomes both translational and rotational energy of the rod:

$$\frac{1}{2}mu^2 = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2.$$

Substituting the two speeds gives

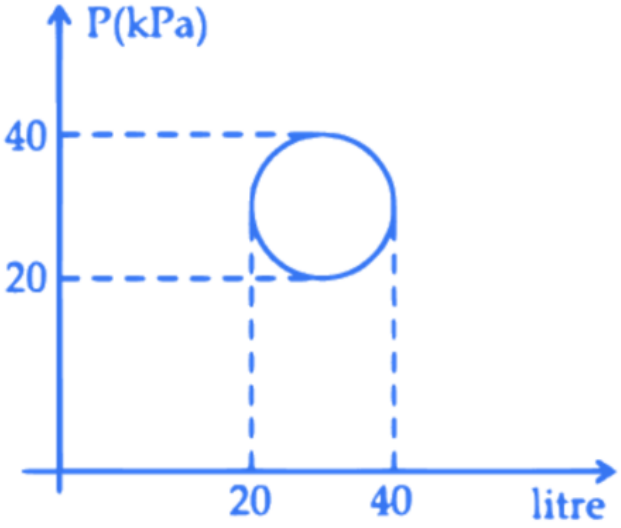
$$\frac{1}{2}mu^2 = \frac{m^2u^2}{2M} + \frac{3m^2u^2}{2M} = \frac{2m^2u^2}{M}.$$

Cancel the nonzero common factors to obtain $M = 4m$.

Step 4: Conclusion The required mass ratio is $m/M = 1/4 = 1/x$. Therefore $\boxed{x = 4}$.

Quick Tip to Solve: For an elastic end impact on a free rod, include both centre-of-mass kinetic energy and rotational kinetic energy; neither contribution can be omitted.

58. In the reported figure, heat energy absorbed by a system in going through a cyclic process is _____ πJ. [4 marks]



Solution:

Concept:

- Internal energy is a state function, so its net change around a closed thermodynamic cycle is zero: $\Delta U_{\text{cycle}} = 0$.
- The first law, $Q = \Delta U + W$, consequently gives net heat equal to net work. The magnitude of work in a pressure-volume cycle is the enclosed area.

Step 1: Read the two semiaxes from the graph The loop extends from 20 to $40\,\text{L}$ horizontally and from 20 to $40\,\text{kPa}$ vertically. Its centre is therefore at $(30\,\text{L}, 30\,\text{kPa})$. The volume semiaxis is $a = (40 - 20)/2 = 10\,\text{L}$, and the pressure semiaxis is $b = (40 - 20)/2 = 10\,\text{kPa}$. Although the curve looks circular on the drawing, its enclosed physical area is computed using the units on each axis, not by measuring the picture.

Step 2: Calculate the enclosed work The area of an ellipse in the pressure-volume plane is the product of its two semiaxes and π. Thus

$$|W| = \pi ab = \pi(10\,\text{L})(10\,\text{kPa}) = 100\pi\,\text{kPa}\,\text{L}.$$

The unit conversion is particularly simple:

$$1\,\text{kPa}\,\text{L} = (10^3\,\text{Pa})(10^{-3}\,\text{m}^3) = 1\,\text{J}.$$

Hence the area represents $100\pi\,\text{J}$. Using the full diameters instead of the semiaxes would overestimate this by a factor of four.

Step 3: Apply the first law over the whole cycle At the end of the cycle the system returns to its original state, so the positive and negative changes of internal energy cancel. Consequently the net heat absorbed, as asked in the question, has the enclosed-area value $Q = 100\pi\,\text{J}$. This is the net heat

for the complete loop, rather than the sum of just the positive heat transfers along individual portions. The requested blank is the coefficient multiplying π.

Step 4: Conclusion The heat absorbed is $\boxed{100\pi\,\text{J}}$, so the number to enter is $\boxed{100}$.

Quick Tip to Solve: On a closed P-V loop use its enclosed area; remember that kPa multiplied by litre already gives joules.

59. A carrier wave $V_c(t) = 160\sin(2\pi \times 10^6 t)$ volts is made to vary between $V_{max} = 200$ V and $V_{min} = 120$ V by a message signal $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ volts. The peak voltage A_m of the modulating signal is _____.

[4 marks]

Solution:

Concept:

- In amplitude modulation, a sinusoidal message changes the carrier envelope about its unmodulated peak amplitude A_c. The maximum and minimum envelope amplitudes are $A_c + A_m$ and $A_c - A_m$.
- The modulation index is $\mu = A_m/A_c = (V_{\max} - V_{\min})/(V_{\max} + V_{\min})$. It describes the fractional change in carrier amplitude, not a frequency ratio.

Step 1: Identify the envelope amplitudes The carrier is $V_c(t) = 160\sin(2\pi \times 10^6 t)\,\text{V}$, so its unmodulated peak is $A_c = 160\,\text{V}$. The envelope varies between $200\,\text{V}$ and $120\,\text{V}$. Their average is $(200 + 120)/2 = 160\,\text{V}$, confirming that the envelope is centred on the given carrier amplitude. The carrier frequency and message frequency determine how rapidly the oscillations and envelope change, respectively; neither alters this amplitude calculation.

Step 2: Determine the message peak At the positive message peak the envelope is $A_c + A_m = 200\,\text{V}$; at the negative message peak it is $A_c - A_m = 120\,\text{V}$. Subtracting these equations isolates twice the message amplitude:

$$2A_m = 200 - 120 = 80\,\text{V}, \qquad A_m = 40\,\text{V}.$$

The full envelope range is a peak-to-peak change, so it must be divided by two to obtain the message peak.

Step 3: Check the modulation index Independently,

$$\mu = \frac{200 - 120}{200 + 120} = \frac{80}{320} = 0.25.$$

Therefore $A_m = \mu A_c = 0.25 \times 160 = 40\,\text{V}$. Since $\mu < 1$, the minimum envelope remains positive and the signal is not overmodulated. Substitution reproduces both limits: $160 + 40 = 200$ and $160 - 40 = 120$.

Step 4: Conclusion The peak voltage of the modulating signal is $\boxed{A_m = 40\,\text{V}}$. The numerical entry is $\boxed{40}$.

Quick Tip to Solve: Take half the difference between the maximum and minimum envelope voltages; their average should reproduce the carrier peak.

60. The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is _____ km/h. [4 marks]

Solution:

Concept:

- A sound reflected by a stationary wall has the same frequency at the wall as the incident sound. The moving car first acts as an approaching source and then as an approaching observer of the reflected wave.
- For sound speed v, source frequency f, and car speed u, the returned frequency heard in the car is $f_{\text{echo}} = f(v+u)/(v-u)$. There are two Doppler shifts.

Step 1: Find the frequency received by the wall The horn emits at $f = 400\,\text{Hz}$, and the car approaches the stationary wall with speed u. Successive wavefronts in front of the moving horn are compressed, so the wall receives

$$f_{\text{wall}} = 400\frac{330}{330-u}.$$

The wall does not move through air, so reflection itself does not change this frequency. It behaves as a stationary source sending the same-frequency wave back towards the car.

Step 2: Include motion of the listener The driver moves towards the returning reflected sound at speed u. For this second part the source is stationary and the observer is moving, giving

$$500 = f_{\text{wall}}\frac{330+u}{330} = 400\frac{330+u}{330-u}.$$

The observed increase from 400 Hz to 500 Hz is thus the double shift of the echo. Using only the approaching-source factor would miss the additional increase caused by the listener moving into the returning wavefronts.

Step 3: Solve and convert the speed Cross-multiply the frequency equation:

$$500(330-u) = 400(330+u).$$

Hence

$$165000 - 500u = 132000 + 400u, \quad 900u = 33000, \quad u = \frac{110}{3}\,\text{m s}^{-1}.$$

Convert metres per second to kilometres per hour using the factor $18/5$:

$$u = \frac{110}{3}\times\frac{18}{5} = 132\,\text{km h}^{-1}.$$

This is less than the sound speed, as required by the expressions used.

Step 4: Conclusion The speed of the car is $\boxed{132\,\text{km h}^{-1}}$. The numerical entry is $\boxed{132}$.

Quick Tip to Solve: For an echo heard by a moving source, apply Doppler twice: source-to-wall first, then wall-to-moving-observer.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. According to the valence bond theory the hybridization of central metal atom is dsp^2 for which one of the following compounds? [4 marks]

(A) $NiCl_2.6H_2O$

(B) $K_2[Ni(CN)_4]$

(C) $[Ni(CO)_4]$

(D) $Na_2[NiCl_4]$

Correct Answer: (B) $K_2[Ni(CN)_4]$

Solution:

Concept:

- In valence bond theory, four-coordinate square-planar complexes use dsp^2 hybrid orbitals, while four-coordinate tetrahedral complexes use sp^3 hybrid orbitals.
- Nickel(II) is $3d^8$. A sufficiently strong ligand such as $\mathrm{CN^-}$ pairs its d electrons, making one inner 3d orbital available for square-planar hybridization.

Step 1: Determine the metal configuration in the cyanide complex In $\mathrm{K_2[Ni(CN)_4]}$, two potassium ions require the complex ion to have charge -2. Four cyanide ligands contribute -4, so nickel is in oxidation state $+2$. Its configuration is therefore $[\mathrm{Ar}]3d^8$. Cyanide is a strong-field ligand and induces pairing of the d electrons. The eight electrons occupy four 3d orbitals in pairs, leaving the fifth inner d orbital vacant.

Step 2: Identify the hybrid orbitals The empty inner 3d orbital combines with the 4s and two 4p orbitals to produce four coplanar dsp^2 hybrid orbitals. Each accepts a ligand lone pair. Thus $\mathrm{[Ni(CN)_4]^{2-}}$ is a square-planar, diamagnetic complex and directly satisfies the stated hybridization. The potassium ions are counterions outside the coordination sphere; they do not contribute orbitals to the metal-ligand bonds.

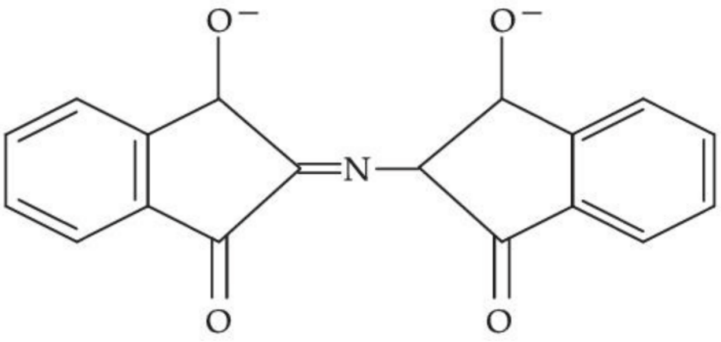
Step 3: Exclude the remaining compounds In hydrated nickel chloride, nickel has a six-coordinate octahedral environment, rather than a four-coordinate square plane. The solid hydrate contains coordinated chloride and water as well as lattice water; its formula does not mean six water ligands surround nickel. In $\mathrm{Ni(CO)_4}$, nickel is in oxidation state zero and is $3d^{10}$; this complex is tetrahedral with sp^3 hybridization. In $\mathrm{Na_2[NiCl_4]}$, chloride is a weak-field ligand, so the nickel(II) complex is also tetrahedral and uses sp^3, rather than an inner-d square-planar arrangement. These eliminate A, C and D.

Step 4: Conclusion The compound with dsp^2 hybridization is $\boxed{\mathrm{K_2[Ni(CN)_4]}}$. Hence, the correct option is (B).

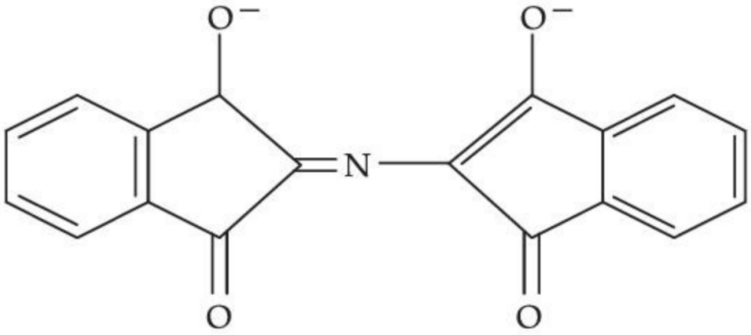
Quick Tip to Solve: For four-coordinate nickel, find the oxidation state first: strong-field cyanide with d8 nickel favours square-planar dsp2, while chloride favours tetrahedral sp3.

62. The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is : [4 marks]

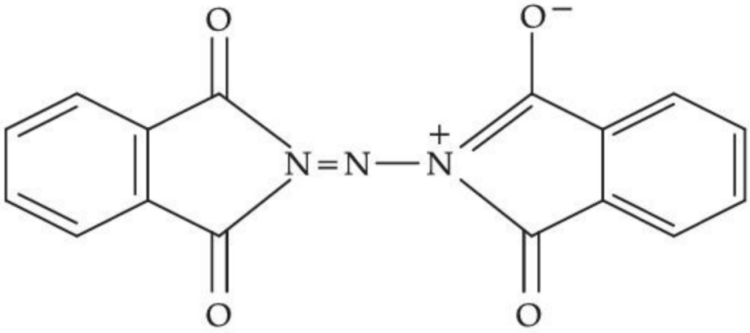
(A)



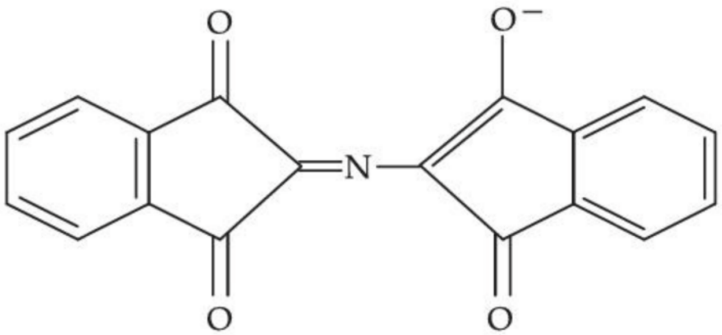
(B)



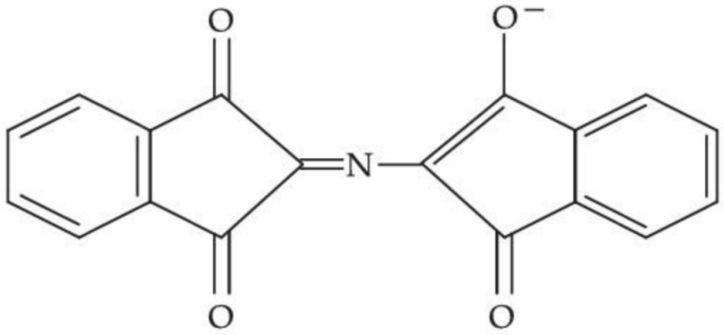
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- Ninhydrin reacts with a primary amino group through oxidative deamination and decarboxylation. The nitrogen ultimately becomes part of the intensely coloured conjugated product called Ruhemann purple.
- The purple product contains two indane-derived ring systems connected through one nitrogen atom. Its conjugated carbonyl-imine-enolate system is responsible for the visible colour.

Step 1: Identify the nitrogen-containing chromophore An amino acid reacts with ninhydrin to release carbon dioxide and the corresponding aldehyde, while its amino nitrogen participates in formation of the coloured product. Reduced ninhydrin and a second ninhydrin-derived unit combine with this nitrogen. The final chromophore therefore has a single nitrogen bridge between two carbon frameworks; it is not a chain of several nitrogen atoms. The violet colour arises from delocalization across this linked system, so the positions of the carbonyl groups and double bonds matter.

Step 2: Compare the oxygen and double-bond arrangement In option D, one indane unit retains two carbonyl groups and is connected at its central carbon to nitrogen by a double bond. The nitrogen is bonded to the central carbon of the other indane unit, which carries the enolate portion: one oxygen is negatively charged and its adjacent carbon-carbon bond is drawn double. Together these features provide the imine/enolate resonance system of Ruhemann purple. The structure has the correct single nitrogen atom and a single negative charge in the representation shown.

Step 3: Reject the altered structures Option A shows two negatively charged oxygen atoms but interrupts the required unsaturation in the second five-membered ring. Option B also changes a carbonyl into an additional singly bonded negatively charged oxygen, producing the wrong oxygen-charge arrangement. Option C introduces a three-nitrogen chain, which is not the nitrogen bridge formed in the ninhydrin test. Option D alone preserves the characteristic carbonyl groups, one enolate oxygen, and the conjugated one-nitrogen linkage. This identification follows the actual bond pattern, rather than just the presence of two fused-ring units.

Step 4: Conclusion The correct structure of Ruhemann purple is the one shown in option D. Hence, the correct option is (D).

Quick Tip to Solve: In the ninhydrin product, look for two indane units joined by one nitrogen and the conjugated carbonyl-imine-enolate pattern; count both nitrogen atoms and oxygen charges.

63. The correct order of intensity of colors of the compounds is : [4 marks]

(A) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$

(B) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$

(C) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

(D) $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

Correct Answer: (C) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

Solution:

Concept:

- Colour intensity depends on the probability of electronic transitions, not simply on the magnitude of crystal-field splitting. Symmetry selection rules strongly influence d-d absorption.
- Octahedral complexes have an inversion centre, making their d-d transitions Laporte-forbidden. Tetrahedral complexes have no inversion centre, so mixing makes these transitions more intense.

Step 1: Compare tetrahedral and octahedral nickel All three complex ions contain nickel(II), which has a $3d^8$ configuration. The weak-field chloride complex $[\mathrm{NiCl_4}]^{2-}$ is tetrahedral. Because a tetrahedron has no centre of inversion, its d-d transitions gain intensity through orbital mixing. The aqua ion $[\mathrm{Ni(H_2O)_6}]^{2+}$ is octahedral; its d-d transitions are restricted by the Laporte selection rule and gain only weaker intensity through effects such as vibrations. The chloride complex consequently has the more intense colour.

Step 2: Place the square-planar cyanide complex Cyanide produces the strong-field, square-planar ion $[\mathrm{Ni(CN)_4}]^{2-}$. The square-planar structure has a centre of inversion, and its relevant d-d bands are weak. In this comparison its colour intensity is less than that of the octahedral aqua complex. The stronger ligand field of cyanide changes transition energies and electron pairing, but it does not imply a larger absorption probability. Colour shade, transition energy, and colour intensity should not be treated as interchangeable quantities.

Step 3: Match the complete sequence The resulting order is

$$[\mathrm{NiCl_4}]^{2-} > [\mathrm{Ni(H_2O)_6}]^{2+} > [\mathrm{Ni(CN)_4}]^{2-}.$$

Option C is precisely this sequence. Option A incorrectly puts the cyanide complex first, and option B incorrectly places the octahedral aqua complex above the tetrahedral chloride complex. Option D starts correctly with chloride but reverses the order of the remaining two ions. Thus comparison of the complete order, not only its first member, is required.

Step 4: Conclusion The correct intensity order is $\boxed{[\mathrm{NiCl_4}]^{2-} > [\mathrm{Ni(H_2O)_6}]^{2+} > [\mathrm{Ni(CN)_4}]^{2-}}$. Hence, the correct option is (C).

Quick Tip to Solve: For colour intensity compare geometry and transition selection rules; a strong-field ligand does not automatically produce the most intense colour.

64. The species given below that does NOT show disproportionation reaction is : [4 marks]

(A) BrO_4^-

(B) BrO^-

(C) BrO_2^-

(D) BrO_3^-

Correct Answer: (A) BrO_4^-

Solution:

Concept:

- Disproportionation is a redox process in which the same element in one oxidation state is both oxidised and reduced.
- An element must have accessible oxidation states on both sides of its initial state. In an oxyanion, oxygen normally has oxidation number -2, and the oxidation numbers sum to the ionic charge.

Step 1: Calculate the oxidation states Write the oxidation number of bromine as x. For $\mathrm{BrO_4}^-$, $x + 4(-2) = -1$, so $x = +7$. Similarly, bromine is $+1$ in BrO^-, $+3$ in $\mathrm{BrO_2}^-$, and $+5$ in $\mathrm{BrO_3}^-$. The number of oxygen atoms therefore provides a direct comparison of the four initial oxidation states.

Step 2: Apply the requirement for simultaneous oxidation and reduction Bromine is a Group 17 element, and $+7$ is its highest normal oxidation state. Perbromate, $\mathrm{BrO_4}^-$, can be reduced but cannot supply a bromine centre that is oxidised beyond $+7$. It therefore cannot undergo disproportionation. The other three oxyanions contain bromine at intermediate oxidation states; they can form a product with a higher bromine oxidation state and another with a lower state. For example, hypobromite can form bromate and bromide:

$$3\,\mathrm{BrO}^- \longrightarrow \mathrm{BrO_3}^- + 2\,\mathrm{Br}^-.$$

Here one bromine goes from $+1$ to $+5$, while two go from $+1$ to -1. Thus options B, C and D do not satisfy the requested negative condition.

Step 3: Conclusion The species that does not show disproportionation is $\mathrm{BrO_4}^-$. Hence, the correct option is (A).

Quick Tip to Solve: For an oxyanion disproportionation question, calculate the central atom oxidation state first; a highest-state species cannot be oxidised further.

65. An inorganic Compound 'X' on treatment with concentrated H_2SO_4 produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated H_2SO_4. Also Compound 'X' gives precipitate 'Y', when its solution in dilute HCl is treated with H_2S gas. The precipitate 'Y' on treatment with concentrated HNO_3 followed by excess of NH_4OH further gives deep blue coloured solution, Compound 'X' is : [4 marks]

(A) $Co(NO_3)_2$

(B) $Pb(NO_2)_2$

(C) $Cu(NO_3)_2$

(D) $Pb(NO_3)_2$

Correct Answer: (C) $Cu(NO_3)_2$

Solution:

Concept:

- The brown-ring test identifies nitrate through reduction to nitric oxide and formation of a brown nitrosyl iron complex.
- Copper(II) gives a black sulfide in acidic medium and a deep blue ammine complex with excess aqueous ammonia. The anion and cation tests must identify the same salt.

Step 1: Identify the anion from the nitrate tests Concentrated H_2SO_4 liberates nitric acid from a nitrate; formation of brown NO_2 fumes is consistent with nitrate chemistry. In the brown-ring test, ferrous ions reduce nitrate to nitric oxide:

$$NO_3^- + 3\,Fe^{2+} + 4\,H^+ \longrightarrow NO + 3\,Fe^{3+} + 2\,H_2O.$$

The nitric oxide then coordinates to iron to form the characteristic brown nitrosyl complex. These observations indicate the nitrate anion in X.

Step 2: Identify the cation using sulfide and ammonia Passing H_2S through the acidic solution precipitates copper as black CuS:

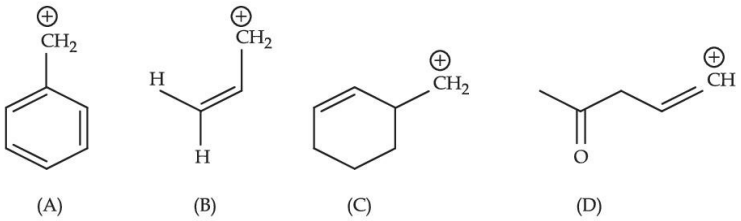
$$Cu^{2+} + H_2S \longrightarrow CuS\downarrow + 2\,H^+.$$

Concentrated nitric acid oxidatively dissolves this sulfide and restores copper(II) in solution. Addition of excess NH_4OH, a source of aqueous ammonia, produces the deep blue tetraamminecopper(II) complex, commonly written $[Cu(NH_3)_4(H_2O)_2]^{2+}$. This colour is the decisive copper test. Lead nitrate may give a sulfide, but it does not yield this deep blue copper complex; cobalt nitrate likewise fails the complete sequence. Lead nitrite has the wrong anion as well as the wrong cation. Combining both independent identifications gives copper(II) nitrate.

Step 3: Conclusion Compound X is $Cu(NO_3)_2$. Hence, the correct option is (C).

Quick Tip to Solve: Use the final confirmatory colour to fix the cation, then combine it with the anion identified by the brown-ring test.

66. Among the given species the Resonance stabilised carbocations are : [4 marks]



(A) (B) (C) (D)

(A) (C) and (D) only

(B) (A), (B) and (D) only

(C) (A) and (B) only

(D) (A), (B) and (C) only

Correct Answer: (C) (A) and (B) only

Solution:

Concept:

- Resonance stabilisation requires an empty p orbital at the positively charged carbon adjacent to a conjugated pi system.
- A saturated tetrahedral carbon interrupts conjugation. A vinylic carbocation is not the same as an allylic carbocation; merely containing a double bond does not establish resonance stabilisation.

Step 1: Identify the conjugated carbocations Species (A) is a benzyl carbocation: the positively charged CH_2^+ group is directly attached to the benzene ring. Its empty p orbital overlaps with the aromatic pi orbitals, and the positive charge can be delocalised to the ortho and para positions. Species (B) is an allyl carbocation, $CH_2{=}CH{-}CH_2^+$. Moving the pi electron pair toward the adjacent empty orbital gives the equivalent contributor $+\,CH_2{-}CH{=}CH_2$. These are valid resonance contributors with unchanged atomic connectivity.

Step 2: Exclude the nonconjugated and vinylic species In species (C), the ring carbon carrying the CH_2^+ substituent is saturated and separates that cationic centre from the ring double bond. There is no uninterrupted chain of p orbitals connecting the positive centre to that double bond. It is therefore not an allylic cation. In species (D), the positive charge is on a carbon of the terminal double bond itself: this is a vinylic carbocation. The carbonyl group is separated from it by a saturated CH_2 unit, so the carbonyl cannot delocalise the positive charge either. Neither (C) nor (D) has the benzyl or allyl resonance stabilisation present in (A) and (B). The alternatives including (C) or (D) must consequently be rejected.

Step 3: Conclusion Only the benzyl and allyl species are resonance stabilised. Hence, the correct option is (C).

Quick Tip to Solve: Trace the route from the positive carbon to a pi bond: one intervening saturated carbon breaks conjugation and rules out ordinary allylic resonance.

67. In the given reaction

3-Bromo-2, 2-dimethyl butane $\xrightarrow{C_2H_5OH}$ $\underset{(Major\ Product)}{'A'}$

Product A is : [4 marks]

(A) 2-Ethoxy-3, 3-dimethyl butane

(B) 1-Ethoxy-3, 3-dimethyl butane

(C) 2-Ethoxy-2, 3-dimethyl butane

(D) 2-Hydroxy-3, 3-dimethyl butane

Correct Answer: (C) 2-Ethoxy-2, 3-dimethyl butane

Solution:

Concept:

- Ethanol is a polar protic solvent and a weak neutral nucleophile; ionisation of a suitable alkyl bromide permits a carbocation substitution pathway.
- A neighbouring alkyl group can migrate with its bonding electron pair to convert a less stable carbocation into a more stable one. Nucleophilic attack occurs on the rearranged cation.

Step 1: Generate and rearrange the carbocation The substrate can be written $(CH_3)_3C-CH(Br)-CH_3$. Loss of bromide gives the secondary carbocation $(CH_3)_3C-CH^+-CH_3$, adjacent to a quaternary carbon. That adjacent carbon has no hydrogen, so a hydride shift from it is impossible. A methyl group instead undergoes a 1,2-shift from the quaternary carbon to the cationic carbon, carrying the carbon-carbon bond electron pair. The positive charge moves to the carbon that lost the methyl group, giving the more stable tertiary cation $(CH_3)_2C^+-CH(CH_3)_2$.

Step 2: Capture the rearranged cation with ethanol The oxygen lone pair of ethanol attacks the tertiary cationic centre. The initially formed oxonium ion loses a proton, yielding the neutral ether $(CH_3)_2C(OC_2H_5)-CH(CH_3)_2$. Its longest carbon chain is butane, with ethoxy and methyl at carbon 2 and another methyl at carbon 3. Its name is therefore 2-ethoxy-2,3-dimethylbutane. Option A corresponds to substitution without the favourable skeletal rearrangement, while option B puts the ethoxy group at an unsupported terminal position. Option D is an alcohol rather than the ether formed by trapping with ethanol. The rearranged ether is the listed major product.

Step 3: Conclusion Product A is 2-ethoxy-2,3-dimethylbutane. Hence, the correct option is (C).

Quick Tip to Solve: Before attaching the solvent nucleophile, check whether the initially formed carbocation can gain stability through a neighbouring methyl shift.

68. Compound A is converted to B on reaction with $CHCl_3$ and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are : [4 marks]

(A) primary amine, nitrile compound, conc. HCl

(B) secondary amine, isonitrile compound, conc. NaOH

(C) primary amine, isonitrile compound, conc. HCl

(D) secondary amine, nitrile compound, conc. NaOH

Correct Answer: (C) primary amine, isonitrile compound, conc. HCl

Solution:

Concept:

- The carbylamine test is given by primary amines with chloroform and alcoholic potassium hydroxide, producing an isocyanide.
- An isocyanide has connectivity $R-N\equiv C$, unlike a nitrile, $R-C\equiv N$. Isocyanides are offensive-smelling and toxic, and acid hydrolysis destroys them.

Step 1: Identify A and the reaction product B Only a primary amine gives the characteristic carbylamine reaction under these conditions. Its two nitrogen-bound hydrogens permit formation of the isocyanide group. The overall equation is

$$RNH_2 + CHCl_3 + 3\,KOH \longrightarrow RNC + 3\,KCl + 3\,H_2O.$$

Thus A is a primary amine and B is an isonitrile, also called an isocyanide. Secondary amines do not give this test, so both alternatives naming a secondary amine are eliminated. Calling B a nitrile would also give the wrong carbon-nitrogen connectivity.

Step 2: Choose the reagent that decomposes B Isocyanides undergo hydrolysis in acidic medium. Concentrated hydrochloric acid provides the required acid conditions, converting the isocyanide into the corresponding primary amine salt and formic acid. A convenient overall representation is

$$RNC + 2\,H_2O + HCl \longrightarrow RNH_3Cl + HCOOH.$$

This destroys the toxic isocyanide functional group. Concentrated sodium hydroxide is not the reagent selected for this decomposition in the carbylamine test. Combining the primary-amine starting material, isonitrile product and acid treatment selects one complete matching sequence; the nitrile and secondary-amine alternatives each contradict the characteristic reaction.

Step 3: Conclusion A, B and C are primary amine, isonitrile compound and concentrated HCl, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Remember the carbylamine test as primary amine to RNC; distinguish it from nitrile RCN before choosing the hydrolysis reagent.

69. Identify the incorrect statement from the following [4 marks]

(A) Amylose is a branched chain polymer of glucose

(B) Starch is a polymer of α-D glucose

(C) β-Glycosidic linkage makes cellulose polymer

(D) Glycogen is called as animal starch

Correct Answer: (A) Amylose is a branched chain polymer of glucose

Solution:

Concept:

- Starch contains amylose and amylopectin, both built from alpha-D-glucose. Their distinction is the presence or absence of branching.
- Cellulose contains beta-linked glucose units, while glycogen is the branched glucose-storage polysaccharide characteristic of animals.

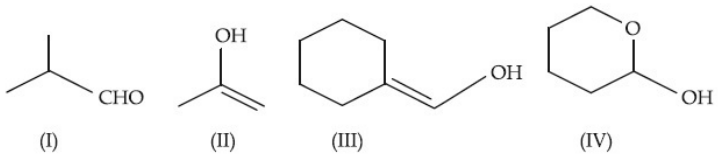
Step 1: Check the statement about amylose Amylose is the essentially unbranched component of starch. Its glucose residues are joined mainly through $\alpha(1 \rightarrow 4)$ glycosidic linkages, producing a chain that can adopt a helical conformation. A helical shape is not a branched molecular chain. Amylopectin, rather than amylose, contains branch points involving $\alpha(1 \rightarrow 6)$ linkages in addition to its $\alpha(1 \rightarrow 4)$ backbone. Therefore the statement that amylose is a branched chain polymer is incorrect.

Step 2: Verify that the remaining statements are correct Starch is indeed a polymer of alpha-D-glucose: both amylose and amylopectin belong to this class, so option B is true. Cellulose is composed of glucose units connected by $\beta(1 \rightarrow 4)$ glycosidic bonds. The beta configuration gives extended chains and permits strong hydrogen bonding between chains, consistent with option C. Glycogen is the storage carbohydrate of animals, especially in liver and muscle, and is commonly called animal starch; hence option D is also true. Although glycogen and amylopectin are both branched, that fact must not be transferred to amylose. Since the question specifically asks for the incorrect statement, the false branching description is the required selection.

Step 3: Conclusion The incorrect statement is that amylose is a branched chain polymer of glucose. Hence, the correct option is (A).

Quick Tip to Solve: Keep the names paired: amylose is unbranched, amylopectin is branched, and glycogen is the more highly branched animal storage polymer.

70. Which among the above compound/s does/do not form Silver mirror when treated with Tollen's reagent? [4 marks]



(I) (II) (III) (IV)

(A) (I), (III) and (IV) only

(B) Only (IV)

(C) Only (II)

(D) (III) and (IV) only

Correct Answer: (C) Only (II)

Solution:

Concept:

- Tollens reagent oxidises aldehydes while its silver(I) complex is reduced to metallic silver, giving the silver mirror.
- A structure may reveal an aldehyde through keto-enol tautomerism or through opening of a cyclic hemiacetal. Evaluate the form present in equilibrium, not just the initially drawn group.

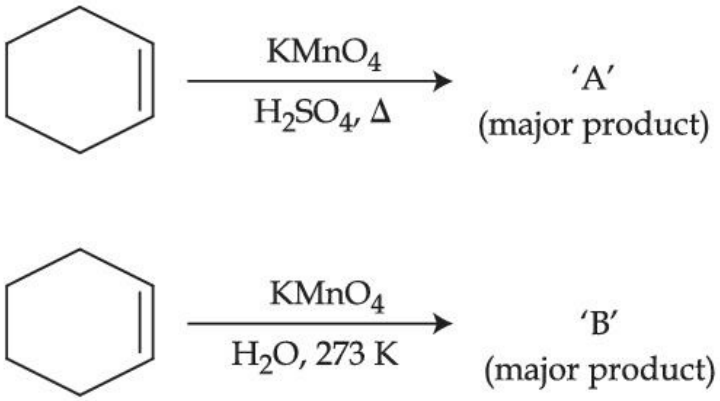
Step 1: Test structures I and II Structure (I) has an explicit aldehyde group, $-CHO$, attached to an isopropyl group. It is oxidised by Tollens reagent and deposits metallic silver. Structure (II) is the enol $CH_3-C(OH)=CH_2$. On keto-enol tautomerism it gives acetone, CH_3COCH_3, an ordinary ketone. It is not an alpha-hydroxy ketone and has no reducing aldehyde group. Therefore (II) does not produce the silver mirror under the test conditions.

Step 2: Check the hidden aldehydes in III and IV In (III), the hydroxyl-bearing alkene carbon carries a hydrogen. Its enol tautomer therefore corresponds to an aldehyde, not a ketone: moving the hydroxyl proton to the adjacent alkene carbon and forming the carbonyl leaves a terminal $-CHO$ group. The aldehyde tautomer is oxidised by Tollens reagent, so (III) gives a positive test. Structure (IV) has a carbon bearing both the ring oxygen bond and an OH group, making it a cyclic hemiacetal. Ring opening regenerates a hydroxy aldehyde, which also reduces Tollens reagent. Thus (I), (III) and (IV) all give silver mirrors. The combinations in options A, B and D incorrectly include one or more of these reducing compounds as negative results; only (II) is nonreducing in this comparison.

Step 3: Conclusion Only compound (II) does not form a silver mirror. Hence, the correct option is (C).

Quick Tip to Solve: For a Tollens test, convert an enol mentally to its carbonyl tautomer and open a hemiacetal before deciding whether an aldehyde is available.

71. For above chemical reactions, identify the correct statement from the following : [4 marks]



(A) Both compound 'A' and compound 'B' are dicarboxylic acids

(B) Both compound 'A' and compound 'B' are diols

(C) Compound 'A' is diol and compound 'B' is dicarboxylic acid

(D) Compound 'A' is dicarboxylic acid and compound 'B' is diol

Correct Answer: (D) Compound 'A' is dicarboxylic acid and compound 'B' is diol

Solution:

Concept:

- Permanganate oxidation of an alkene depends strongly on temperature and medium: cold dilute reagent adds two hydroxyl groups, whereas hot acidic reagent cleaves the double bond.
- On oxidative cleavage, a double-bond carbon bearing hydrogen becomes a carboxylic-acid carbon. A cyclic alkene therefore opens into a chain when both alkene carbons bear hydrogen.

Step 1: Identify the hot acidic oxidation product The starting compound is cyclohexene. In the upper reaction, $KMnO_4$, H_2SO_4 and heat cause oxidative cleavage rather than stopping at hydroxylation. Each carbon of the double bond carries one hydrogen. Both ends therefore become $-COOH$ groups, while the four intervening methylene groups remain connected. Thus A is adipic acid, $HOOC-(CH_2)_4-COOH$, a dicarboxylic acid.

Step 2: Identify the cold oxidation product The lower reaction uses aqueous permanganate at $273\,K$. Under these mild conditions, hydroxyl groups add across the double bond without cleavage of the carbon skeleton. The ring remains intact and B is cis-cyclohexane-1,2-diol. It contains two alcohol groups and no carboxylic-acid group.

Step 3: Compare the alternatives Options (A) and (B) incorrectly give both products the same functional-group class. Option (C) reverses the effect of mild and vigorous oxidation. Only option (D) assigns the dicarboxylic acid to the hot acidic reaction and the diol to the cold aqueous reaction.

Step 4: Conclusion A is a dicarboxylic acid and B is a diol. Hence, the correct option is (D).

Quick Tip to Solve: First read the temperature and medium above the arrow: cold permanganate gives a vicinal diol; hot acidic permanganate cleaves the alkene.

72. Green chemistry in day-to-day life is in the use of : [4 marks]

(A) Chlorine for bleaching of paper

(B) Large amount of water alone for washing clothes

(C) Tetrachloroethene for laundry

(D) Liquified CO_2 for dry cleaning of clothes

Correct Answer: (D) Liquified CO_2 for dry cleaning of clothes

Solution:

Concept:

- Green chemistry seeks to reduce hazardous substances and waste at the source, including replacing harmful solvents with safer alternatives.
- Liquefied CO_2 can be used as a dry-cleaning medium in place of chlorinated organic solvents; it can be recovered when pressure is released.

Step 1: Identify the safer cleaning solvent Traditional dry cleaning may use tetrachloroethene, a chlorinated solvent associated with health and environmental concerns. Using liquefied CO_2, with a suitable detergent, reduces dependence on such hazardous solvents. The carbon dioxide is maintained as a liquid under pressure during cleaning and can be separated from the clothes by changing pressure. This is the green-chemistry example among the choices.

Step 2: Examine the other choices Chlorine bleaching can generate undesirable chlorinated by-products; greener bleaching processes use alternatives such as hydrogen peroxide. Tetrachloroethene itself is the conventional solvent that the greener process replaces, so (C) is unsuitable. Merely consuming a large amount of water is not a green process: it wastes a resource and water alone does not provide an effective general replacement for the cleaning formulation.

Step 3: Apply the criterion to the options The distinguishing feature is substitution of a hazardous chemical by a safer medium, rather than simply using more material or retaining a problematic traditional reagent. Among (A)–(D), the liquefied-carbon-dioxide process best meets that criterion.

Step 4: Conclusion Liquefied CO_2 for dry cleaning is the green-chemistry application. Hence, the correct option is (D).

Quick Tip to Solve: For green-chemistry examples, identify the hazardous reagent being replaced: liquefied carbon dioxide replaces chlorinated dry-cleaning solvents.

73. The set in which compounds have different nature is : [4 marks]

(A) $B(OH)_3$ and H_3PO_3

(B) $B(OH)_3$ and $Al(OH)_3$

(C) NaOH and $Ca(OH)_2$

(D) $Be(OH)_2$ and $Al(OH)_3$

Correct Answer: (B) $B(OH)_3$ and $Al(OH)_3$

Solution:

Concept:

- Acidic compounds react with bases, basic compounds react with acids, and amphoteric hydroxides can react with both acids and strong bases.
- $B(OH)_3$ is acidic, whereas $Al(OH)_3$ and $Be(OH)_2$ are amphoteric. Alkali-metal and ordinary alkaline-earth-metal hydroxides are basic.

Step 1: Classify the boron and phosphorus compounds Boric acid, $B(OH)_3$, behaves as a weak Lewis acid in water by accepting hydroxide:

$$B(OH)_3 + 2\,H_2O \rightleftharpoons B(OH)_4{}^- + H_3O^+.$$

Phosphorous acid, H_3PO_3, is also acidic. Although their structures and acid strengths differ, their broad chemical nature is the same; therefore pair (A) is not the required pair.

Step 2: Classify the metal hydroxides Aluminium hydroxide is amphoteric: it dissolves in acid and in strong alkali, forming aluminium salts and aluminate species respectively. Beryllium hydroxide is also amphoteric, so pair (D) has the same nature. Sodium hydroxide and calcium hydroxide are both bases; their different solubilities do not make their acid-base nature different. Thus pair (C) is also excluded.

Step 3: Select the unlike pair Pair (B) combines acidic boric acid with amphoteric aluminium hydroxide. For example, aluminium hydroxide reacts with hydroxide according to

$$Al(OH)_3 + OH^- \longrightarrow [Al(OH)_4]^-,$$

while it also neutralises acids. This two-sided behaviour distinguishes it from boric acid.

Step 4: Conclusion $B(OH)_3$ is acidic and $Al(OH)_3$ is amphoteric. Hence, the correct option is (B).

Quick Tip to Solve: Keep the exceptions together: beryllium and aluminium hydroxides are amphoteric, but boric acid is acidic despite its hydroxide-like formula.

74. Given below are two statements. One is labelled as Assertion A nd the other is labelled as Reason R.

Assertion A : Sharp glass edge becomes smooth on heating it upto its melting point.

Reason R : The viscosity of glass decreases on melting.

Choose the most appropriate answer from the options given below. [4 marks]

(A) A is true but R is false

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is false but R is true.

(D) Both A and R are true and R is the correct explanation of A.

Correct Answer: (B) Both A and R are true but R is NOT the correct explanation of A.

Solution:

Concept:

- Viscosity measures resistance to flow. Heating glass greatly lowers its viscosity and allows the softened material to move.
- Surface tension tends to reduce surface area and rounds sharp edges once the heated glass is sufficiently mobile.

Step 1: Evaluate Assertion A A sharp edge of glass becomes smooth when strongly heated to its softening or melting range. At an ordinary temperature, glass has such high viscosity that its shape is effectively fixed on the observation timescale. On heating, the edge can deform and round off. The familiar fire-polishing of glass illustrates this effect, so the assertion describes a real phenomenon and is true.

Step 2: Evaluate Reason R The viscosity of glass decreases markedly as its temperature is raised into the softened state. There is then less resistance to movement within the material. Although glass softens over a range rather than possessing the sharp melting point of a pure crystalline solid, the intended heating process in the assertion does lower its viscosity. Reason R is therefore true.

Step 3: Connect the reason to the assertion The rounding is driven by surface tension, which tends to reduce the surface area and hence the surface energy of the softened glass. Decreased viscosity permits flow, but does not itself supply the force or the tendency that makes a sharp edge become rounded. Thus R is a true statement about heated glass, but it is not the correct explanation of the smoothing in A. Options (A) and (C) wrongly reject a true statement. Option (D) wrongly attributes the rounding itself to the viscosity decrease, overlooking the role of surface tension.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A: surface tension causes the smoothing of the softened edge. Hence, the correct option is (B).

Quick Tip to Solve: In glass questions, separate the driving force from mobility: surface tension rounds an edge, and reduced viscosity lets the material flow.

75. Orlon fibres are made up of : [4 marks]

(A) Polyacrylonitrile

(B) Polyesters

(C) Polyamide

(D) Cellulose

Correct Answer: (A) Polyacrylonitrile

Solution:

Concept:

- Orlon is the commercial name associated with polyacrylonitrile, a synthetic addition polymer used to make acrylic fibres.
- An addition polymer forms by opening the carbon-carbon double bond of its monomer without eliminating a small molecule; the side group remains attached to the carbon chain.

Step 1: Identify the monomer and repeat unit The monomer acrylonitrile has the structure $CH_2{=}CH{-}CN$. During addition polymerisation its double bond opens and the monomers link into a saturated carbon backbone. The nitrile group remains as a pendant group on alternate backbone carbons:

$$n\,CH_2{=}CH{-}CN \longrightarrow \left[-CH_2{-}CH(CN){-}\right]_n.$$

The product is polyacrylonitrile, abbreviated PAN.

Step 2: Connect the polymer with the fibre name Polyacrylonitrile can be spun into fibres whose properties make them useful as wool substitutes. These acrylic fibres are known by commercial names including Orlon. The question asks for the chemical identity of that fibre, so it is the polymer made from acrylonitrile, rather than a generic class of condensation polymers.

Step 3: Eliminate the other polymer classes Polyesters, such as terylene, contain ester linkages in the main chain. Polyamides, such as nylon, contain amide linkages. Cellulose is a natural polysaccharide built from glucose units. None has the nitrile-bearing carbon backbone characteristic of Orlon, so options (B), (C) and (D) do not match the named fibre.

Step 4: Conclusion Orlon fibres are made of polyacrylonitrile. Hence, the correct option is (A).

Quick Tip to Solve: Associate Orlon with acrylonitrile and its pendant nitrile group; nylon and terylene belong to different polymer families.

76. Given below are two statements : One is labelled as Assertion A and other is labelled as Reason R.

Assertion A : The dihedral angles in H_2O_2 in gaseous phase is 90.2° and in solid phase is 111.5°.

Reason R : The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.

Choose the most appropriate answer from the options given below for A and R. [4 marks]

(A) A is correct but R is not correct.

(B) Both A and R are correct but R is not the correct explanation of A.

(C) Both A and R are correct and R is the correct explanation of A.

(D) A is not correct but R is correct.

Correct Answer: (D) A is not correct but R is correct.

Solution:

Concept:

- Hydrogen peroxide, H_2O_2, has a non-planar open-book structure. Its dihedral angle describes the relative orientation of the two O–H bonds about the O–O bond.
- The dihedral angle is about 111.5° in the gas phase and 90.2° in the solid phase. Intermolecular interactions alter the preferred conformation in the condensed phase.

Step 1: Evaluate Assertion A The assertion assigns 90.2° to gaseous hydrogen peroxide and 111.5° to solid hydrogen peroxide. These phase assignments are interchanged. The approximately isolated gaseous molecule has the larger dihedral angle, about 111.5°, whereas the solid has an angle about 90.2°. The molecule is non-planar in both cases, but the numerical interchange makes Assertion A false.

Step 2: Evaluate Reason R In the solid, neighbouring hydrogen peroxide molecules interact through hydrogen bonding and other intermolecular forces. The geometry reflects these surrounding interactions in addition to the intramolecular forces within one molecule. In the gaseous phase, intermolecular influence is much weaker. Thus a difference in the intermolecular environment is responsible for the phase-dependent change in dihedral angle, and Reason R is true.

Step 3: Match the truth values The correct combination is a false assertion and a true reason. Options (B) and (C) both wrongly accept the reversed angles, and option (A) also wrongly calls the assertion correct while rejecting the valid reason. Only option (D) matches the independently assessed statements.

Step 4: Conclusion A is not correct because its gaseous and solid angles are reversed, but R is correct. Hence, the correct option is (D).

Quick Tip to Solve: Memorise the phase order rather than the numbers alone: hydrogen peroxide has the larger dihedral angle in the gas phase.

77. Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric acid and P_4O_{10} (in 4 : 1 ratio) is : [4 marks]

(A) acidic

(B) basic

(C) amphoteric

(D) neutral

Correct Answer: (A) acidic

Solution:

Concept:

- P_4O_{10} is a powerful dehydrating agent. Removing water from concentrated nitric acid produces dinitrogen pentoxide, N_2O_5.
- An acid anhydride regenerates its acid on reaction with water. N_2O_5 is the anhydride of nitric acid and is therefore an acidic oxide.

Step 1: Identify the product of dehydration Two molecules of nitric acid lose one molecule of water to give dinitrogen pentoxide. The specified $4 : 1$ molar ratio is consistent with the balanced overall reaction

$$4\,HNO_3 + P_4O_{10} \longrightarrow 2\,N_2O_5 + 4\,HPO_3.$$

Phosphorus pentoxide takes up the elements of water and is converted into metaphosphoric acid in this equation. The nitrogen-containing oxide product to classify is N_2O_5, not a phosphorus oxide.

Step 2: Determine the chemical nature of the oxide Dinitrogen pentoxide reacts with water to form nitric acid:

$$N_2O_5 + H_2O \longrightarrow 2\,HNO_3.$$

This is the defining behaviour of an acidic oxide or acid anhydride. It also reacts with bases to form nitrates, for example

$$N_2O_5 + 2\,NaOH \longrightarrow 2\,NaNO_3 + H_2O.$$

These reactions establish its acidic character directly.

Step 3: Exclude the remaining classifications A basic oxide would neutralise an acid rather than function as the anhydride of a strong acid. An amphoteric oxide would display both acidic and basic behaviour, which is not the classification of this oxide. Neutral nitrogen oxides include nitric oxide and nitrous oxide, but neither is the dehydration product obtained here. Hence the labels in (B), (C) and (D) are unsuitable.

Step 4: Conclusion The oxide formed is N_2O_5, an acidic oxide. Hence, the correct option is (A).

Quick Tip to Solve: When a dehydrating agent acts on an oxyacid, identify its anhydride first; adding water back reveals the oxide's acid-base nature.

78. A s-block element (M) reacts with oxygen to form an oxide of the formula MO_2. The oxide is pale yellow in colour and paramagnetic. The element (M) is : [4 marks]

(A) Mg

(B) Na

(C) Ca

(D) K

Correct Answer: (D) K

Solution:

Concept:

- The nature of an alkali-metal oxide depends on the oxygen species present: oxide, peroxide and superoxide contain O^{2-}, O_2^{2-} and O_2^{-}, respectively.
- Paramagnetism requires at least one unpaired electron. The superoxide ion has one unpaired electron, whereas the peroxide ion has all its electrons paired.

Step 1: Identify the oxygen ion from the formula For an alkali metal with oxidation state $+1$, the formula MO_2 requires an oxygen unit of charge -1. Thus it represents a superoxide, $M^{+}O_2^{-}$, rather than a normal oxide. In contrast, a divalent metal can form a peroxide of the same empirical formula, so the formula alone is insufficient; the magnetic property must also be used.

Step 2: Use the colour and magnetic behaviour Potassium burns in excess oxygen to give potassium superoxide, a pale-yellow solid:

$$K + O_2 \longrightarrow KO_2.$$

The O_2^{-} ion has an odd electron count and one unpaired electron in its antibonding molecular orbitals. It therefore explains the observed paramagnetism as well as the stated colour.

Step 3: Eliminate the other metals Sodium characteristically forms Na_2O_2, a peroxide, rather than the stated superoxide. Magnesium and calcium form ordinary oxides MgO and CaO; their possible peroxides contain O_2^{2-}, which is diamagnetic. They do not satisfy the combined formula and paramagnetic clue.

Step 4: Conclusion The element is potassium, K, and its product is KO_2. Hence, the correct option is (D).

Quick Tip to Solve: Use the magnetic clue to distinguish a superoxide from a peroxide: O_2^{-} has one unpaired electron; O_2^{2-} has none.

79. The metal that can be purified economically by fractional distillation method is : [4 marks]

(A) Fe

(B) Zn

(C) Cu

(D) Ni

Correct Answer: (B) Zn

Solution:

Concept:

- Distillation purifies a volatile metal by vaporising it and condensing the vapour, while less volatile impurities remain behind.
- Fractional distillation separates components with different boiling points. For an economical process, the desired metal must vaporise at a practicable temperature without excessive energy expenditure.

Step 1: Compare the volatility of the listed metals Zinc is substantially more volatile than iron, copper and nickel. Its boiling point is about $907\,^{\circ}\mathrm{C}$, whereas those of the other listed metals are much higher. Zinc can therefore be converted into vapour at an industrially convenient temperature and subsequently condensed as the purified metal.

Step 2: Apply the refining principle to zinc On heating impure zinc, volatile zinc enters the vapour phase while nonvolatile impurities remain in the residue. If volatile impurities are present, their different boiling behaviour allows their separation through fractional condensation or distillation. The separation exploits physical volatility; it does not require changing zinc permanently into another compound.

Step 3: Rule out the other choices Iron, copper and nickel are relatively high-boiling metals. Vaporising them for routine purification would demand much higher temperatures and energy, making this method uneconomical compared with their appropriate refining processes. Copper is commonly refined electrolytically, and nickel can be purified by the Mond process involving a volatile carbonyl. The question asks specifically for economical fractional distillation, which points to zinc.

Step 4: Conclusion The metal is zinc, Zn. Hence, the correct option is (B).

Quick Tip to Solve: For a distillation-based metal-refining question, first identify the volatile, relatively low-boiling metal; zinc is the standard choice among Fe, Zn, Cu and Ni.

80. The conditions given below are in the context of observing Tyndall effect in colloidal solutions :

(A) The diameter of the colloidal particles is comparable to the wavelength of light used.

(B) The diameter of the colloidal particles is much smaller than the wavelength of light used.

(C) The diameter of the colloidal particles is much larger than the wavelength of light used.

(D) The refractive indices of the dispersed phase and the dispersion medium are comparable.

(E) The dispersed phase has a very different refractive index from the dispersion medium.

Choose the most appropriate conditions from the options given below : [4 marks]

(A) (A) and (E) only

(B) (C) and (D) only

(C) (A) and (D) only

(D) (B) and (E) only

Correct Answer: (A) (A) and (E) only

Solution:

Concept:

- The Tyndall effect is the scattering of light by colloidal particles, making the path of a beam visible when viewed from the side.
- Two key conditions are a suitable particle size compared with the wavelength of light and a sufficiently large difference between the refractive indices of the dispersed phase and dispersion medium.

Step 1: Assess the particle-size conditions Statement (A) specifies a colloidal-particle diameter comparable to the wavelength used. This provides appreciable scattering and is the appropriate condition in the question. Statement (B), with particles much smaller than the wavelength, describes a regime where scattering becomes very weak, as in a true solution. Statement (C), with particles much larger than the wavelength, is not the stated colloidal size condition.

Step 2: Assess the optical contrast Scattering depends on the optical difference between the particles and the surrounding medium. If their refractive indices are nearly equal, light encounters little optical contrast and the beam is not made strongly visible. Therefore statement (D) is unsuitable, whereas statement (E), requiring very different refractive indices, is appropriate.

Step 3: Combine the two necessary clues The suitable combination is thus (A) together with (E). The remaining options each contain the incorrect refractive-index condition (D), or an unsuitable size condition. Although particle size is essential, size alone is not enough: the dispersed phase must also interact with light differently from the continuous medium. This is why both parts of the selected combination are needed.

Step 4: Conclusion The most appropriate conditions are comparable particle diameter and wavelength together with very different refractive indices. Hence, the correct option is (A).

Quick Tip to Solve: Check both size and optical contrast: a visible Tyndall cone requires suitable colloidal particles and unequal refractive indices, not merely a heterogeneous mixture.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of lone pairs of electrons on the central I atom in I_3^- is _____. [4 marks]

Solution:

Concept:

- The Lewis structure is obtained by counting all valence electrons, including an additional electron for each unit of negative charge.
- VSEPR theory counts bonding domains and lone-pair domains around the central atom. In I_3^-, five electron-pair domains arrange in a trigonal-bipyramidal electron-domain geometry.

Step 1: Count the total valence electrons Each iodine atom contributes seven valence electrons, and the negative charge contributes one additional electron. Therefore the total is

$$N = 3(7) + 1 = 22.$$

Place one iodine at the centre and connect it to the two terminal iodine atoms using two single bonds. These two bonds use four electrons.

Step 2: Complete the terminal octets Each terminal iodine needs three lone pairs in addition to its bond, so the two terminal atoms require $2 \times 6 = 12$ nonbonding electrons. The number left on the central iodine is consequently

$$22 - 4 - 12 = 6.$$

Six nonbonding electrons form $6/2 = 3$ lone pairs on the central atom. The terminal lone pairs must not be included in the requested count.

Step 3: Check the geometry and charge The central iodine has two bonding pairs and three lone pairs, giving the VSEPR description AX_2E_3. Its three lone pairs occupy equatorial positions of a trigonal bipyramid, leaving the two bonded atoms at the axial positions; the ion is linear. The central formal charge is $7 - (6 + 2) = -1$, consistent with the overall charge. This independently confirms the electron allocation.

Step 4: Conclusion The central iodine atom in I_3^- has $\boxed{3}$ lone pairs.

Quick Tip to Solve: Count total electrons first and finish the terminal octets before assigning central lone pairs; the negative charge adds one electron, not one pair.

82. 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is _____ $\times 10^{21}$. (Nearest integer)
($N_A = 6.022 \times 10^{23}$) [4 marks]

Solution:

Concept:

- Molarity is moles of solute per litre of solution, so $n = MV$ when V is expressed in litres.
- The reaction $\mathrm{NaOH} + \mathrm{HCl} \longrightarrow \mathrm{NaCl} + \mathrm{H_2O}$ consumes the base and acid in a one-to-one molar ratio. The excess acid is found by subtracting the reacting moles, then multiplying by Avogadro's constant.

Step 1: Calculate the starting amounts Convert both volumes to litres before multiplying by molarity:

$$n(\mathrm{NaOH}) = 0.5 \times 0.250 = 0.125\,\mathrm{mol},$$

$$n(\mathrm{HCl}) = 1.0 \times 0.500 = 0.500\,\mathrm{mol}.$$

These are the initial chemical amounts; adding the volumes is unnecessary when calculating the total number of residual acid entities.

Step 2: Identify the limiting reagent and remaining acid The balanced equation consumes one mole of hydrochloric acid per mole of sodium hydroxide. Since $0.125 < 0.500$, sodium hydroxide is limiting and consumes $0.125\,\mathrm{mol}$ of acid. Hence

$$n_{\mathrm{remaining}}(\mathrm{HCl}) = 0.500 - 0.125 = 0.375\,\mathrm{mol}.$$

The reaction is complete, but the acid is not exhausted.

Step 3: Convert the excess amount to the requested number Multiply the unreacted acid amount by the specified Avogadro constant:

$$N = 0.375(6.022 \times 10^{23}) = 2.25825 \times 10^{23}.$$

Expressing this on the requested scale gives

$$N = 225.825 \times 10^{21} \approx 226 \times 10^{21}.$$

Hydrochloric acid is dissociated in water; this count represents the unneutralised acid amount expressed in the molecular units requested by the question.

Step 4: Conclusion The unreacted acid corresponds to 226×10^{21} molecules on the stated scale. The nearest-integer entry is $\boxed{226}$.

Quick Tip to Solve: Subtract reacting moles before using Avogadro's constant; then match the power of ten in the answer blank before rounding.

83. The Azimuthal quantum number for the valence electrons of Ga^+ ion is _____.
(Atomic number of Ga = 31) [4 marks]

Solution:

Concept:

- The azimuthal quantum number l specifies the subshell: s, p, d, f correspond to $l = 0, 1, 2, 3$, respectively.
- When forming a positive ion, electrons are removed from the occupied outer orbital of highest energy. The ionic configuration must be written before identifying its valence subshell.

Step 1: Write the neutral gallium configuration Gallium has atomic number 31, so its neutral atom contains 31 electrons. Filling the orbitals gives

$$\mathrm{Ga} : [\mathrm{Ar}]\, 3d^{10}4s^2 4p^1.$$

The outermost shell has principal quantum number $n = 4$; it contains two $4s$ electrons and one $4p$ electron in the neutral atom.

Step 2: Remove the electron to form the cation The singly charged ion is formed by removing one electron. That electron is the lone $4p$ electron, leaving

$$\mathrm{Ga}^+ : [\mathrm{Ar}]\, 3d^{10}4s^2.$$

Thus the ion has 30 electrons and its outermost occupied shell still has $n = 4$, but its valence electrons now occupy only the $4s$ subshell. Using the neutral atom's $4p$ subshell would answer a different question.

Step 3: Assign the azimuthal quantum number For any s subshell, the orbital angular-momentum quantum number is $l = 0$, independently of its principal quantum number. Therefore both remaining valence electrons of Ga^+, which are in $4s^2$, have $l = 0$. The filled $3d^{10}$ subshell belongs to the lower principal shell and does not change the requested outer-shell assignment.

Step 4: Conclusion The valence electrons of Ga^+ are in $4s^2$, so the required azimuthal quantum number is $\boxed{0}$.

Quick Tip to Solve: For ions, remove the appropriate electron first. Here losing $4p^1$ changes the valence subshell from p to s, so l becomes zero.

84. The spin-only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is _____ BM.
[At. no. of Co = 27] [4 marks]

Solution:

Concept:

- The oxidation state of the central metal follows from the total complex charge and the charges of its ligands. Each cyanide ligand has charge -1.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{BM}$, where n is the number of unpaired

electrons. Cyanide is a strong-field ligand and favours low-spin pairing in an octahedral complex.

Step 1: Determine the oxidation state and d-electron count Let the oxidation state of cobalt be x. With six cyanide ligands and overall charge -4,

$$x + 6(-1) = -4 \quad \Rightarrow \quad x = +2.$$

Neutral cobalt has configuration $[\mathrm{Ar}]3d^7 4s^2$; removal of two $4s$ electrons gives Co^{2+} with a $3d^7$ configuration.

Step 2: Apply strong-field octahedral splitting The six ligands produce an octahedral ligand field. Because CN^- is a strong-field ligand, the lower t_{2g} orbitals are filled with paired electrons before the higher e_g orbitals are occupied. For seven d electrons the low-spin distribution is

$$t_{2g}^6 e_g^1.$$

The six lower-level electrons are paired, and the seventh electron remains unpaired, so $n = 1$.

Step 3: Calculate and report the magnetic moment Substitute the unpaired-electron count into the spin-only expression:

$$\mu = \sqrt{1(1+2)} = \sqrt{3}\,\mathrm{BM} \approx 1.73\,\mathrm{BM}.$$

For an integer numerical entry this is reported as 2. A high-spin d^7 assignment would have three unpaired electrons and a different magnetic moment, but it ignores the strong-field cyanide ligand specified here.

Step 4: Conclusion The spin-only magnetic moment is $\sqrt{3}\,\mathrm{BM} \approx 1.73\,\mathrm{BM}$, giving the integer entry $\boxed{2}$.

Quick Tip to Solve: Find oxidation state before counting d electrons, then use ligand strength to decide pairing; the magnetic formula counts unpaired electrons rather than total d electrons.

85. **$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$**

In an equilibrium mixture, the partial pressures are

P_{SO_3} = 43 kPa; P_{O_2} = 530 Pa and P_{SO_2} = 45 kPa. The equilibrium constant K_P = _____ $\times 10^{-2}$. (Nearest integer) [4 marks]

Solution:

Concept:

- For a gaseous equilibrium, the partial-pressure expression has each pressure raised to the corresponding stoichiometric coefficient. Products appear in the numerator and reactants in the denominator.
- Use one common pressure unit before substituting. Here the numerical convention is pressure in kPa, with $530\,\mathrm{Pa} = 0.530\,\mathrm{kPa}$; the requested coefficient multiplies 10^{-2}.

Step 1: Write the equilibrium expression The reaction is $2\,\mathrm{SO_2(g)} + \mathrm{O_2(g)} \rightleftharpoons 2\,\mathrm{SO_3(g)}$. Its

coefficients give

$$K_P = \frac{(p_{SO_3})^2}{(p_{SO_2})^2 p_{O_2}}.$$

The oxygen pressure is not squared because its coefficient is one. The two sulphur-oxide pressures must each be squared; using their ratio without squaring would describe a different reaction.

Step 2: Convert and substitute the pressures Convert oxygen pressure into the same units as the other two pressures before doing any arithmetic:

$$p_{O_2} = \frac{530}{1000} = 0.530\,\text{kPa}.$$

Thus, in the pressure-unit convention of the question,

$$K_P = \frac{43^2}{45^2(0.530)} = \frac{1849}{1073.25} = 1.7228\,\text{kPa}^{-1}.$$

The inverse-pressure unit also follows from the net gas exponent $2 - 2 - 1 = -1$.

Step 3: Extract the requested coefficient Writing the result in the specified form gives

$$1.7228 = 172.28 \times 10^{-2}.$$

The blank is the coefficient 172.28 rounded to its nearest integer, not the constant rounded directly. Keeping several digits until this final conversion avoids a premature-rounding error.

Step 4: Conclusion With pressures expressed in kPa, $K_P \simeq 1.7228\,\text{kPa}^{-1} = 172.28 \times 10^{-2}\,\text{kPa}^{-1}$. The required nearest-integer entry is $\boxed{172}$.

Quick Tip to Solve: Convert the lone Pa pressure to kPa first, then evaluate the squared sulphur-oxide ratio and divide by the oxygen pressure.

86. The number of nitrogen atoms in a semicarbazone molecule of acetone is _____.

[4 marks]

Solution:

Concept:

- Semicarbazide has the connectivity $H_2N{-}NH{-}C(=O){-}NH_2$. It contains three nitrogen atoms: the terminal hydrazino nitrogen, the central nitrogen and the amide nitrogen.
- A ketone forms a semicarbazone by condensation with the terminal $-NH_2$ group of semicarbazide. Water is eliminated, while the complete nitrogen skeleton remains in the product.

Step 1: Identify the reacting groups Acetone is $CH_3{-}C(=O){-}CH_3$. Its carbonyl carbon is attacked by the terminal hydrazino amino group of semicarbazide. The other amino group is part of the amide fragment $-C(=O){-}NH_2$; its lone pair is delocalised towards that carbonyl, so it is not the group used to write the normal semicarbazone linkage.

Step 2: Write the condensation product The oxygen of the acetone carbonyl and two hydrogens from

the reacting amino group leave as one water molecule. The product connectivity is

$$(CH_3)_2C{=}N{-}NH{-}C(=O){-}NH_2.$$

The new carbon–nitrogen double bond replaces the ketone carbon–oxygen double bond. Neither the central $-NH-$ group nor the terminal amide $-NH_2$ group is removed during this condensation.

Step 3: Count all nitrogen atoms Read the product from the acetone-derived carbon towards the amide end. There is one imine nitrogen in $C{=}N$, a second nitrogen in $-NH-$, and a third nitrogen in $-NH_2$. Hence

$$N_{\text{nitrogen atoms per molecule}} = 1 + 1 + 1 = 3.$$

Only hydrogen and oxygen are lost as water; counting only nitrogens bearing hydrogen would incorrectly miss the imine nitrogen.

Step 4: Conclusion The semicarbazone of acetone contains $\boxed{3}$ nitrogen atoms per molecule.

Quick Tip to Solve: Write the semicarbazone tail as $=N-NH-CO-NH_2$; the three separate N symbols immediately give the count.

87. To synthesize 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate _____ equivalents of CH_3MgBr Reagent will be required. (Integer value) [4 marks]

Solution:

Concept:

- An ester normally consumes two equivalents of a Grignard reagent to form a tertiary alcohol after acidic hydrolysis. The first addition produces a ketone; the second addition produces a tertiary alkoxide.
- Ethyl ethanoate contains an acetyl fragment CH_3CO^-. Each equivalent of CH_3MgBr contributes one methyl group to its carbonyl carbon. The ethoxy group leaves during the first stage.

Step 1: Account for the first equivalent The starting ester is $CH_3COOC_2H_5$. Nucleophilic addition of a methyl group from CH_3MgBr gives a tetrahedral intermediate, which eliminates the ethoxy group. The resulting ketone is acetone, CH_3COCH_3. Thus the first equivalent changes the ester into a ketone rather than directly producing the final alcohol. The original acetyl methyl group remains attached to the carbonyl carbon.

Step 2: Account for the second equivalent Acetone reacts further with another equivalent of methylmagnesium bromide. A second methyl group adds to the ketone carbonyl carbon, giving a magnesium tert-butoxide intermediate:

$$(CH_3)_2CO + CH_3MgBr \longrightarrow (CH_3)_3C{-}OMgBr.$$

The resulting central carbon now bears three methyl groups, exactly the carbon skeleton required for 2-methylpropan-2-ol.

Step 3: Complete the work-up and count equivalents Acidic hydrolysis protonates the alkoxide

oxygen:

$$(CH_3)_3C{-}OMgBr \xrightarrow{H_3O^+} (CH_3)_3COH.$$

Hydrolysis supplies the hydroxyl hydrogen, not another methyl group, and is performed after the Grignard additions. For one mole of the tertiary alcohol, one mole of ester requires two moles of CH_3MgBr. Therefore the stoichiometric reagent requirement is two equivalents, not one or three.

Step 4: Conclusion To prepare 1.0 mol of 2-methylpropan-2-ol from ethyl ethanoate, $\boxed{2}$ equivalents of CH_3MgBr are required.

Quick Tip to Solve: For an ester-to-tertiary-alcohol conversion, count one Grignard addition to make the ketone and a second to add to that ketone.

88. The inactivation rate of a viral preparation is proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is _____ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Use : ln 10 = 2.303; $\log_{10}$ 3 = 0.477; property of logarithm : log x^y = y log x] [4 marks]

Solution:

Concept:

- If the inactivation rate is proportional to the amount of active virus N, then $-dN/dt = kN$. This is first-order decay, with integrated equation $k = t^{-1}\ln(N_0/N_t)$.
- A loss of 10 percent means that 90 percent remains active. With time measured in minutes, the first-order rate constant has units min^{-1}.

Step 1: Find the fraction remaining after one minute Let the initial active amount be N_0. In the first minute, $0.10N_0$ becomes inactive, so

$$N_1 = N_0 - 0.10N_0 = 0.90N_0.$$

The integrated law uses the remaining active quantity, not the inactivated quantity. The initial amount cancels in the ratio, so no absolute virus concentration or count is needed.

Step 2: Apply the integrated first-order equation At $t = 1\,\text{min}$,

$$k = \frac{1}{1\,\text{min}}\ln\left(\frac{1}{0.9}\right) = \ln\left(\frac{10}{9}\right)\ \text{min}^{-1}.$$

Using only the supplied logarithmic values,

$$\begin{aligned}\log_{10}(10/9) &= 1 - \log_{10}(3^2)\\ &= 1 - 2(0.477) = 0.046,\\ k &= 2.303(0.046) = 0.105938\,\text{min}^{-1}.\end{aligned}$$

The logarithm is positive, as required for a positive decay constant.

Step 3: Convert to the requested power of ten The result is

$$k = 105.938 \times 10^{-3}\,\text{min}^{-1}.$$

Its coefficient rounds to 106. Approximating the rate constant merely as the fraction lost per minute would give 0.10 and would neglect the exponential nature of first-order decay. The logarithmic equation accounts for the changing active amount throughout the minute.

Step 4: Conclusion $k \simeq 106 \times 10^{-3}\,\text{min}^{-1}$, so the required nearest-integer entry is $\boxed{106}$.

Quick Tip to Solve: For a stated percentage inactivated, first subtract from 100 percent; insert the surviving fraction into $-\ln(f)/t$.

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89. An average person needs about 10000 kJ energy per day. The amount of glucose (molar mass = 180.0 g mol^{-1}) needed to meet this energy requirement is _____ g.
(Use : Δ_CH(glucose) = -2700 kJ mol^{-1}) [4 marks]

Solution:

Concept:

- The molar enthalpy of combustion gives the enthalpy change when one mole of a substance burns completely. A negative value denotes heat released by the reacting system.
- To meet a positive energy requirement, use the magnitude of the combustion enthalpy: $n = E/|\Delta_c H|$. Convert the resulting moles into mass using $m = nM$.

Step 1: Interpret the energy supplied per mole The given combustion enthalpy is $-2700\,\text{kJ}\,\text{mol}^{-1}$. Thus complete oxidation of one mole of glucose releases $2700\,\text{kJ}$ for energy use. The negative sign describes the system losing energy; it does not imply a negative amount of glucose. The stated calculation treats this released energy as the amount available to meet the daily requirement.

Step 2: Determine the number of moles required The daily requirement is $E = 10000\,\text{kJ}$. Equating the total released energy to this requirement gives

$$n\left(2700\,\text{kJ}\,\text{mol}^{-1}\right) = 10000\,\text{kJ}.$$

Hence

$$n = \frac{10000}{2700}\,\text{mol} = 3.7037\,\text{mol}.$$

The energy units cancel, leaving moles. This amount exceeds one mole because the required energy is several times the energy released by a single mole.

Step 3: Convert the amount into grams Using the given molar mass $M = 180.0\,\text{g}\,\text{mol}^{-1}$,

$$m = nM = \frac{10000}{2700}(180.0) = 666.67\,\text{g}.$$

The mole units cancel in this multiplication. As a check, each gram supplies $2700/180 = 15\,\text{kJ}$; multiplying $666.67\,\text{g}$ by this value recovers approximately $10000\,\text{kJ}$. Expressing the mass as a whole

number gives 667 g.

Step 4: Conclusion The glucose requirement is approximately $666.67\,\mathrm{g}$ per day, corresponding to the integer entry $\boxed{667}$.

Quick Tip to Solve: Find the energy released per gram, $2700/180 = 15\,\mathrm{kJ\,g^{-1}}$, then divide the required energy by 15.

90. At 20° C, the vapour pressure of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapour phase at 20° above an equimolar mixture of benzene and methyl benzene is _____ $\times 10^{-2}$. (Nearest integer) [4 marks]

Solution:

Concept:

- For an ideal liquid solution, Raoult's law gives $p_i = x_i P_i^\circ$, where x_i is the liquid-phase mole fraction and P_i° is the pure-component vapour pressure at the same temperature.
- Dalton's law gives the vapour-phase mole fraction $y_i = p_i/p_{\text{total}}$. Equal liquid mole fractions do not imply equal vapour mole fractions when the pure-component vapour pressures differ.

Step 1: Use the equimolar liquid composition An equimolar mixture has equal numbers of moles of benzene and methyl benzene in the liquid. Therefore

$$x_{\mathrm{B}} = x_{\mathrm{T}} = \frac{1}{2}.$$

At the stated temperature, benzene has the greater pure vapour pressure, 70 torr compared with 20 torr. It is the more volatile component, so the vapour should contain a larger benzene fraction than the liquid.

Step 2: Calculate both partial pressures Apply Raoult's law separately to the two components:

$$p_{\mathrm{B}} = \frac{1}{2}(70) = 35\,\mathrm{torr}, \qquad p_{\mathrm{T}} = \frac{1}{2}(20) = 10\,\mathrm{torr}.$$

The total vapour pressure is their sum,

$$p_{\text{total}} = 35 + 10 = 45\,\mathrm{torr}.$$

The values 70 and 20 torr are pure-liquid vapour pressures; they must first be weighted by liquid mole fractions to obtain the partial pressures above the mixture.

Step 3: Find the vapour mole fraction and scale it Dalton's law now gives

$$y_{\mathrm{B}} = \frac{35}{45} = \frac{7}{9} = 0.77778.$$

This is greater than the liquid value 0.5, consistent with preferential evaporation of benzene. In the required form,

$$y_{\mathrm{B}} = 77.778 \times 10^{-2}.$$

The coefficient rounds to 78. The complementary methyl-benzene fraction is $10/45 = 2/9$, and the two fractions correctly add to one.

Step 4: Conclusion The benzene mole fraction in the vapour is $7/9 \simeq 0.77778 = 77.778 \times 10^{-2}$. The required nearest-integer entry is $\boxed{78}$.

Quick Tip to Solve: For an equimolar binary liquid, the vapour fraction simplifies to $P_{\mathrm{B}}^{\circ}/(P_{\mathrm{B}}^{\circ} + P_{\mathrm{T}}^{\circ})$.