# JEE Main 2021 Paper 1 (B.E./B.Tech) Jul 20 Shift 2 Question Paper with Solutions PDF

## Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

**Time Allowed: 3 Hours** **Maximum Marks: 300**

### General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 3 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 20 July 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, $-1$ for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

## Mathematics – Section A: Multiple Choice Questions (80 marks)

**1. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to :** [4 marks]

(A) $\frac{-181}{69}$
(B) $\frac{220}{21}$
(C) $\frac{-291}{76}$
(D) $\frac{151}{63}$

**Correct Answer:** (B) $\frac{220}{21}$

**Solution:**

**Concept:**

- For a principal inverse tangent, set $u=\tan^{-1}(3/5)$, so $\tan u=3/5$. The double-angle identity is $\tan 2u=2\tan u/(1-\tan^2 u)$.
- For $v=\sin^{-1}(5/13)$, the principal value is acute. Thus $\cos v=12/13$, with the positive square root, and $\tan v=5/12$.
- The addition formula is $\tan(A+B)=(\tan A+\tan B)/(1-\tan A\tan B)$, provided the denominator is nonzero.

**Step 1:** Find the two tangent values The inverse functions fix the quadrants, so neither tangent has a sign ambiguity. Apply the double-angle formula before adding the second angle:

$$\tan 2u=\frac{2(3/5)}{1-9/25}=\frac{6/5}{16/25}=\frac{15}{8}.$$

The sine triangle for the second angle has opposite side 5, hypotenuse 13 and adjacent side 12, giving $\tan v=5/12$.

**Step 2:** Apply the addition formula Substitute these exact fractions, keeping the subtraction in the denominator:

$$\tan(2u+v)=\frac{15/8+5/12}{1-(15/8)(5/12)}=\frac{55/24}{21/96}=\frac{220}{21}.$$

The denominator is positive and nonzero, so the result is positive. Options (A) and (C) have the wrong sign. Option (D) is positive but does not equal the exact quotient; cross-multiplication distinguishes it from $220/21$.

**Step 3:** Conclusion The required value is $\boxed{220/21}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Find the tangent of each angle separately; for the inverse sine use the 5–12–13 triangle and its principal quadrant.

**2. The lines x = ay − 1 = z − 2 and x = 3y − 2 = bz − 2, (ab ≠ 0) are coplanar, if :** [4 marks]

(A) b = 1, a∈R − {0}

(B) a = 1, b∈R − {0}

(C) a = 2, b = 2

(D) a = 2, b = 3

**Correct Answer:** (A) b = 1, a∈R − {0}

**Solution:**

**Concept:**

- Two lines with points $P, Q$ and direction vectors $\mathbf{u}, \mathbf{v}$ are coplanar exactly when the scalar triple product $(Q - P) \cdot (\mathbf{u} \times \mathbf{v})$ vanishes.
- A chain of equal expressions is parametrised by setting each expression equal to a parameter. The assumption $ab \neq 0$ permits division by both parameters.

**Step 1:** Parametrise the two lines For the first line set $x = ay - 1 = z - 2 = t$. Its coordinates are $(t, (t+1)/a, t+2)$, so choose $P = (0, 1/a, 2)$ and $\mathbf{u} = (1, 1/a, 1)$. For the second set $x = 3y - 2 = bz - 2 = s$. This gives $(s, (s+2)/3, (s+2)/b)$, hence $Q = (0, 2/3, 2/b)$ and $\mathbf{v} = (1, 1/3, 1/b)$. The constants in the chains belong to the coordinates, not to the direction ratios.

**Step 2:** Set the scalar triple product to zero The coplanarity condition is

$$\begin{vmatrix} 0 & 2/3 - 1/a & 2/b - 2 \\ 1 & 1/a & 1 \\ 1 & 1/3 & 1/b \end{vmatrix} = 0.$$

Expanding along its first row and factoring gives

$$-(2/3 - 1/a)(1/b - 1) + (2/b - 2)(1/3 - 1/a) = \frac{b-1}{ab}.$$

Since $ab \neq 0$, this vanishes precisely when $b = 1$. There is no further restriction on nonzero $a$. Option (B) incorrectly fixes $a$ while permitting $b \neq 1$; options (C) and (D) have $b = 2$ and $b = 3$, respectively, so their triple products are nonzero.

**Step 3:** Conclusion The condition is $\boxed{b = 1,\ a \in \mathbb{R} \setminus \{0\}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Parametrise each entire equality chain first; use a point difference and the two direction vectors in the coplanarity determinant.

**3. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$ is equal to :** [4 marks]

(A) $-\pi$

(B) $\pi$

(C) 0

(D) 1

**Correct Answer:** (A) $-\pi$

**Solution:**

**Concept:**

- For an integer $m$, $\lfloor m+t\rfloor = m+\lfloor t\rfloor$. The outer square brackets therefore act on the whole quantity, not just on $x$.
- For nonintegral $x$, $\lfloor x\rfloor + \lfloor -x\rfloor = -1$. Changing an integrand at finitely many points does not change its definite integral.

**Step 1:** Evaluate the outer floor on each half Write $g(x) = \lfloor \lfloor x\rfloor - \sin x\rfloor$. On $0 < x \le \pi/2$, the sine lies in $(0, 1]$, so $\lfloor -\sin x\rfloor = -1$. Thus $g(x) = \lfloor x\rfloor - 1$. On $-\pi/2 < x < 0$, the quantity $-\sin x$ lies strictly between 0 and 1, giving $g(x) = \lfloor x\rfloor$. The isolated endpoint $-\pi/2$, where the latter floor becomes 1, has no effect on the integral.

**Step 2:** Pair opposite arguments and integrate For almost every $x \in (0, \pi/2)$, the preceding formulas give

$$g(x) + g(-x) = \lfloor x\rfloor - 1 + \lfloor -x\rfloor = -2.$$

The points where $x$ is an integer form only a finite set in this interval and may be omitted in integration. Substituting $x \mapsto -x$ on the negative half therefore yields

$$\int_{-\pi/2}^{\pi/2} g(x)\,dx = \int_{0}^{\pi/2} [g(x) + g(-x)]\,dx = \int_{0}^{\pi/2} -2\,dx = -\pi.$$

In particular the integrand is not odd; treating it as odd would incorrectly produce option (C). The positive results in (B) and (D) also contradict the negative paired value.

**Step 3:** Conclusion The integral equals $\boxed{-\pi}$. Hence, the correct option is (A).

**Quick Tip to Solve:** For nested greatest-integer expressions, first remove any integer shift from the outer floor, then pair the values at opposite arguments.

**4. If the real part of the complex number $(1-\cos\theta+2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta\in(0,\pi)$, then the value of the integral $\int_0^{\theta}\sin x dx$ is equal to:** [4 marks]

(A) 1

(B) 2

(C) $-1$

(D) 0

**Correct Answer:** (A) 1

**Solution:**

**Concept:**

- For real $a, b$ with $a^2+b^2\neq 0$, $1/(a+ib)=(a-ib)/(a^2+b^2)$; its real part is $a/(a^2+b^2)$.
- The identity $\sin^2\theta=1-\cos^2\theta$ reduces the real-part condition to an equation in $c=\cos\theta$. Also $\int_0^{\theta}\sin x\,dx=1-\cos\theta$.

**Step 1:** Extract and simplify the real part Put $c=\cos\theta$. Since $0<\theta<\pi$, we have $-1<c<1$; in particular $1-c\neq 0$. Rationalising the reciprocal gives

$$\mathrm{Re}\,\frac{1}{1-\cos\theta+2i\sin\theta}=\frac{1-c}{(1-c)^2+4(1-c^2)}.$$

Factor the denominator as $(1-c)(5+3c)$. Cancelling the nonzero factor is legitimate throughout the specified interval, and the given condition becomes $1/(5+3c)=1/5$.

**Step 2:** Determine the angle and the integral The denominator $5+3c$ is positive for $-1<c<1$, so cross-multiplication yields $5+3c=5$, or $c=0$. On the stated interval this means $\theta=\pi/2$. Hence

$$\int_0^{\theta}\sin x\,dx=[-\cos x]_0^{\theta}=1-\cos\theta=1.$$

The integral must be positive because the sine is positive between 0 and $\theta$, ruling out options (C) and (D). Option (B) would require $\cos\theta=-1$, an excluded endpoint that also fails the derived condition.

**Step 3:** Conclusion The value of the integral is $\boxed{1}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Factor out $1-\cos\theta$ after rationalising; the requested integral is that same quantity, so no elaborate angle calculation is needed.

**5. Let $f : R - \left\{\frac{\alpha}{6}\right\} \to R$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of $\alpha$ for which (fof)(x) = x, for all $x \in R - \left\{\frac{\alpha}{6}\right\}$, is :** [4 marks]

(A) No such $\alpha$ exists

(B) 5

(C) 8

(D) 6

**Correct Answer:** (B) 5

**Solution:**

**Concept:**

- An involution satisfies $f(f(x)) = x$ at every point of its domain, so the second application must also be defined.
- For a rational function, substitute the entire first fraction into the second occurrence and clear the inner denominator before comparing polynomials.

**Step 1:** Compute the composition For $x \neq \alpha/6$, substitution gives

$$f(f(x)) = \frac{5(5x+3)+3(6x-\alpha)}{6(5x+3)-\alpha(6x-\alpha)} = \frac{43x+15-3\alpha}{(30-6\alpha)x+18+\alpha^2}.$$

Requiring this to equal $x$ produces a polynomial identity:

$$(30-6\alpha)x^2 + (\alpha^2-25)x + (3\alpha-15) = 0.$$

It holds on infinitely many real domain points, so each coefficient must vanish. The quadratic coefficient already gives $\alpha = 5$, and this also makes the other two coefficients zero.

**Step 2:** Verify the excluded point and eliminate alternatives For $\alpha = 5$, the composition simplifies to $43x/43 = x$. It remains to verify that no allowed input is mapped to the forbidden value $5/6$. If $(5x+3)/(6x-5) = 5/6$, cross-multiplication would give $30x + 18 = 30x - 25$, an impossibility. Thus the composition is defined for every $x \neq 5/6$, exactly as required. Option (A) is disproved by this explicit construction. Values 8 and 6 in options (C) and (D) leave a nonzero quadratic coefficient and cannot give the identity for all inputs.

**Step 3:** Conclusion The required parameter is $\boxed{\alpha = 5}$. Hence, the correct option is (B).

**Quick Tip to Solve:** After comparing coefficients in a rational composition, check that the first function never maps an allowed input to its excluded pole.

**6. For the natural numbers m, n, if $(1-y)^m(1+y)^n = 1+a_1y+a_2y^2+....+a_{m+n}y^{m+n}$ and $a_1 = a_2 = 10$, then the value of (m + n) is equal to :** [4 marks]

(A) 88

(B) 64

(C) 100

(D) 80

**Correct Answer:** (D) 80

**Solution:**

**Concept:**

- The coefficient of $y$ in $(1-y)^m(1+y)^n$ is $n-m$, because the two linear contributions are $-my$ and $ny$.
- The coefficient of $y^2$ is $\binom{m}{2}+\binom{n}{2}-mn$, including the cross-product of the two linear terms. This equals $((n-m)^2-(m+n))/2$.

**Step 1:** Use the first coefficient Expanding just far enough,

$$(1-y)^m = 1-my+\frac{m(m-1)}{2}y^2+\cdots,$$

$$(1+y)^n = 1+ny+\frac{n(n-1)}{2}y^2+\cdots.$$

The linear coefficient is therefore $a_1 = n-m$. Since the question gives $a_1 = 10$, we have $n-m = 10$. There is no need to expand all $m+n$ powers; only the first two coefficients contain the required information.

**Step 2:** Use the quadratic coefficient and check the integers Multiplication of the truncated expansions gives

$$a_2 = \frac{m(m-1)+n(n-1)}{2} - mn = \frac{(n-m)^2-(m+n)}{2}.$$

Substitute $a_2 = 10$ and $n-m = 10$:

$$10 = \frac{100-(m+n)}{2}, \qquad m+n = 80.$$

Combining the sum and difference gives $n = 45$ and $m = 35$, both natural numbers as stipulated. As a check, $\binom{35}{2}+\binom{45}{2}-35\cdot 45 = 595+990-1575 = 10$. If the sums were 88, 64 or 100 instead, the same coefficient formula would give 6, 18 or 0, respectively, rather than the required 10.

**Step 3:** Conclusion The sum is $\boxed{m+n=80}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For $(1-y)^m(1+y)^n$, remember $a_1 = n-m$ and $2a_2 = (n-m)^2-(m+n)$; these determine the sum immediately.

**7. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 − k), the probability that exactly one of B and C occurs is (1 − 2k), the probability that exactly one of C and A occurs is (1 − k) and the probability of all A, B and C occur simultaneously is $k^2$, where 0 < k < 1. Then the probability that at least one of A, B and C occur is :** [4 marks]

(A) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

(B) greater than $\frac{1}{2}$

(C) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(D) exactly equal to $\frac{1}{2}$

**Correct Answer:** (B) greater than $\frac{1}{2}$

**Solution:**

**Concept:**

- Exactly one of two events is their symmetric difference. For three event indicators, either all are equal, or exactly two of the three pairwise comparisons are different.
- Consequently, the sum of the three pairwise symmetric-difference probabilities equals twice the probability that exactly one or exactly two of the three events occurs.

**Step 1:** Relate pairwise differences to the union Let $p_1$ and $p_2$ denote the probabilities that exactly one and exactly two of $A, B, C$ occur. In an outcome counted by either of these probabilities, two pairs disagree and the third pair agrees. Outcomes where none occurs or all three occur contribute zero pairwise differences. Thus

$$2(p_1 + p_2) = (1 - k) + (1 - 2k) + (1 - k) = 3 - 4k.$$

This identity counts individual outcomes and does not require independence. Therefore $p_1 + p_2 = (3 - 4k)/2$.

**Step 2:** Add the triple intersection and bound the result The union is the disjoint combination of exactly one, exactly two and exactly three occurrences. The last probability is given as $k^2$, hence

$$P(A \cup B \cup C) = p_1 + p_2 + k^2 = \frac{3}{2} - 2k + k^2 = \frac{1}{2} + (1 - k)^2.$$

Because $0 < k < 1$, the square is strictly positive, so the union probability is strictly greater than $1/2$. The supplied probabilities also impose feasibility restrictions, such as $1 - 2k \geq 0$; these do not weaken the strict lower bound. Options (A) and (C) both put the result below $1/2$, and option (D) would require $k = 1$, which is explicitly excluded.

**Step 3:** Conclusion The probability is $\frac{1}{2} + (1 - k)^2 > \frac{1}{2}$. Hence, the correct option is (B).

**Quick Tip to Solve:** When three pairwise exactly-one probabilities are supplied, add them: outcomes with exactly one or exactly two events are each counted twice.

**8. The sum of all the local minimum values of the twice differentiable function f : R → R defined by $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is :** [4 marks]

(A) $-22$

(B) 5

(C) $-27$

(D) 0

**Correct Answer:** (C) $-27$

**Solution:**

**Concept:**

- The quantities $f''(2)$ and $f''(1)$ are constants with respect to $x$. Thus they disappear when the defining cubic is differentiated twice.
- A stationary point is a local minimum when $f'(x) = 0$ and $f''(x) > 0$. The minimum value is the function value, not its argument.

**Step 1:** Determine the derivative constants Differentiate the defining expression twice, treating the derivative values at fixed arguments as constants:

$$f''(x) = 6x - 6.$$

Consequently $f''(2) = 6$ and $f''(1) = 0$. Substituting these back into the original definition gives the explicit polynomial

$$f(x) = x^3 - 3x^2 - 9x.$$

It is essential to substitute these values before evaluating any extremum; they are not additional unknown functions of $x$.

**Step 2:** Classify the stationary points The first derivative factors as

$$f'(x) = 3x^2 - 6x - 9 = 3(x - 3)(x + 1).$$

The only stationary points are $x = -1$ and $x = 3$. At $x = -1$, the second derivative is $-12 < 0$, so this point is a local maximum. At $x = 3$, it is $12 > 0$, so this point is a local minimum. Since the derivative has only these two roots, there are no other local minima to include.

**Step 3:** Evaluate the minimum value At the unique local minimum,

$$f(3) = 27 - 27 - 27 = -27.$$

For comparison, $f(-1) = -1 - 3 + 9 = 5$, which is the local maximum and must not be included in the requested sum. The option $-22$ incorrectly adds both extrema, while zero is the value at $x = 0$, which is not stationary.

**Step 4:** Conclusion The sum of all local minimum values is $\boxed{-27}$. Hence, the correct option is (C).

**Quick Tip to Solve:** First eliminate fixed derivative constants by differentiating twice; then distinguish the location of an extremum from its function value.

**9. Let y = y(x) satisfies the equation $\frac{dy}{dx} - |A| = 0$, for all x > 0, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is :** [4 marks]

(A) $\frac{\pi}{2} + \frac{4}{\pi}$
(B) $\frac{\pi}{2} - \frac{1}{\pi}$
(C) $\frac{3\pi}{2} - \frac{1}{\pi}$
(D) $\frac{\pi}{2} - \frac{4}{\pi}$

**Correct Answer:** (A) $\frac{\pi}{2} + \frac{4}{\pi}$

**Solution:**

**Concept:**

- The notation $|A|$ denotes the determinant of the given matrix. Expanding it produces a linear first-order differential equation in $y$.
- For $y' + P(x)y = Q(x)$, the integrating factor is $e^{\int P(x)\,dx}$. Since $x > 0$, the integrating factor for $P(x) = 1/x$ is $x$.

**Step 1:** Expand the determinant Expand along the first row, retaining the alternating signs of the cofactors:

$$|A| = y\left(-\frac{1}{x}\right) - \sin x(0 - 2) + (0 + 2)$$
$$= -\frac{y}{x} + 2\sin x + 2.$$

The equation $y' - |A| = 0$ therefore becomes

$$y' + \frac{y}{x} = 2\sin x + 2.$$

The determinant bars here do not mean the absolute value of a real-valued expression.

**Step 2:** Integrate after multiplying by the integrating factor Multiplication by $x$ turns the left side into the derivative of $xy$:

$$\frac{d}{dx}(xy) = 2x\sin x + 2x.$$

Integration by parts gives $\int x\sin x\,dx = -x\cos x + \sin x$, so

$$xy = -2x\cos x + 2\sin x + x^2 + C.$$

No logarithm remains because the integrating factor has already absorbed the coefficient $1/x$.

**Step 3:** Apply the initial value and evaluate At $x = \pi$, the initial condition yields

$$\pi(\pi + 2) = 2\pi + \pi^2 + C,$$

so $C = 0$. Dividing the integrated relation by the positive variable $x$ gives

$$y = x - 2\cos x + \frac{2\sin x}{x}.$$

Hence

$$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{4}{\pi}.$$

This satisfies the original condition at $\pi$. Options with a negative reciprocal term have the wrong sign, and the coefficient $3\pi/2$ cannot arise from the $x$ term at this argument.

**Step 4:** Conclusion $\boxed{y(\pi/2) = \pi/2 + 4/\pi}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Expand the determinant before solving the differential equation; a single cofactor sign error changes the reciprocal term in the answer.

**10. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of | a − b | is equal to :** [4 marks]

(A) 9

(B) 11

(C) 7

(D) 1

**Correct Answer:** (D) 1

**Solution:**

**Concept:**

- For six observations, $\bar{x} = (\sum x_i)/6$. The given variance is the population variance $\sigma^2 = (\sum x_i^2)/6 - \bar{x}^2$.
- The identity $(a-b)^2 = 2(a^2+b^2) - (a+b)^2$ finds the absolute difference without needing to solve for the two observations separately.

**Step 1:** Use the mean to find the sum The mean is 10, so the total of all six observations must be 60. The known observations add to

$$7 + 10 + 11 + 15 = 43.$$

Therefore

$$a + b = 60 - 43 = 17.$$

This condition alone does not fix the difference; the variance supplies the second independent relation. In particular, neither unknown observation has to equal the mean.

**Step 2:** Use the variance to find the sum of squares Insert the given variance and mean into the population-variance formula:

$$\frac{7^2 + 10^2 + 11^2 + 15^2 + a^2 + b^2}{6} - 100 = \frac{20}{3}.$$

The four known squares sum to $49 + 100 + 121 + 225 = 495$. Thus

$$495 + a^2 + b^2 = 6\left(100 + \frac{20}{3}\right) = 640,$$

which gives $a^2 + b^2 = 145$. The divisor is six, not five; the question gives variance of the observations, not an unbiased sample estimate.

**Step 3:** Compute and check the absolute difference Using the two symmetric quantities,

$$(a-b)^2 = 2(145) - 17^2 = 290 - 289 = 1.$$

Taking the nonnegative square root yields $|a-b| = 1$. Indeed the observations are $8$ and $9$, in either order, because their sum is $17$ and their squares sum to $145$. These values reproduce both the stated mean and variance. The alternatives $7, 9, 11$ would require a larger sum of squares for the same sum and are therefore excluded.

**Step 4:** Conclusion The required absolute difference is $\boxed{1}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For two missing observations, find their sum and sum of squares, then use the squared-difference identity instead of solving a quadratic.

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**11. Let $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right)dx$, where $f(x) = \log_e\left(x + \sqrt{x^2+1}\right), x \in R$. Then which one of the following is correct?** [4 marks]

(A) g(1) = g(0)

(B) $\sqrt{2}g(1) = g(0)$

(C) $g(1) = \sqrt{2}g(0)$

(D) g(1) + g(0) = 0

**Correct Answer:** (B) $\sqrt{2}g(1) = g(0)$

**Solution:**

**Concept:**

- An odd integrand integrates to zero over an interval symmetric about the origin. Also $\sin(u+v) = \sin u \cos v + \cos u \sin v$ and $\cos(u+v) = \cos u \cos v - \sin u \sin v$.
- The positive factors $\sqrt{1+x^2} + x$ and $\sqrt{1+x^2} - x$ have product one, which determines the parity of their logarithms.

**Step 1:** Establish that the inner function is odd For all real $x$, $\sqrt{x^2+1} > |x|$, so the logarithm is defined. Furthermore,

$$(\sqrt{x^2+1} - x)(\sqrt{x^2+1} + x) = 1.$$

It follows that

$$f(-x) = \log(\sqrt{x^2+1} - x) = -\log(\sqrt{x^2+1} + x) = -f(x).$$

Thus $f$ is odd, $\sin f(x)$ is odd, and $\cos f(x)$ is even.

**Step 2:** Separate the parameter from the integral Expand the cosine of a sum and use symmetry:

$$\begin{aligned} g(t) &= \cos\left(\frac{\pi t}{4}\right)\int_{-\pi/2}^{\pi/2}\cos f(x)\,dx \\ &\quad - \sin\left(\frac{\pi t}{4}\right)\int_{-\pi/2}^{\pi/2}\sin f(x)\,dx \\ &= C\cos\left(\frac{\pi t}{4}\right), \end{aligned}$$

where $C = \int_{-\pi/2}^{\pi/2}\cos f(x)\,dx = g(0)$. There is no need to find an antiderivative for $\cos f(x)$; only the odd part vanishes.

**Step 3:** Compare the two parameter values At $t = 1$, the cosine factor is $1/\sqrt{2}$. Therefore $g(1) = g(0)/\sqrt{2}$. Also $f(\pi/2) = \log(\pi/2 + \sqrt{1+\pi^2/4}) < \pi/2$, so $\cos f(x) > 0$ throughout this interval and $C > 0$. Hence the equal-value, reciprocal-factor and opposite-sign alternatives cannot hold accidentally because of a zero integral.

**Step 4:** Conclusion $\boxed{\sqrt{2}\,g(1) = g(0)}$. Hence, the correct option is (B).

**Quick Tip to Solve:** When a parameter occurs as an added angle, expand the trigonometric function and test the parity of the remaining integrands before integrating.

**12. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line y = x is :** [4 marks]

(A) $(3x - y)^2 + (x - 3y) + 2 = 0$

(B) $2(3x - y)^2 + (x - 3y) + 2 = 0$

(C) $(3x - y)^2 + 2(x - 3y) + 2 = 0$

(D) $2(x - 3y)^2 + (3x - y) + 2 = 0$

**Correct Answer:** (B) $2(3x - y)^2 + (x - 3y) + 2 = 0$

**Solution:**

**Concept:**

- The perpendicular projection of $(u, v)$ onto $y = x$ is $((u + v)/2, (u + v)/2)$. This follows by intersecting $y = x$ with the perpendicular line of slope $-1$ through the point.
- To obtain a locus, express the original point coordinates in terms of the required midpoint, then substitute into the original curve equation.

**Step 1:** Find the perpendicular foot and midpoint Let $P = (u, v)$, with $v = 4u^2 + 1$. The perpendicular through $P$ is $y - v = -(x - u)$. Its intersection with $y = x$ is

$$H = \left(\frac{u+v}{2}, \frac{u+v}{2}\right).$$

If the midpoint of $PH$ is $(x, y)$, the midpoint formula gives

$$x = \frac{3u + v}{4}, \qquad y = \frac{u + 3v}{4}.$$

These are the coordinates of the desired moving point, rather than the perpendicular foot itself.

**Step 2:** Invert the coordinate relations Form the combinations that eliminate one original coordinate at a time:

$$3x - y = 2u, \qquad 3y - x = 2v.$$

Thus

$$u = \frac{3x - y}{2}, \qquad v = \frac{3y - x}{2}.$$

The transformation is invertible, so eliminating $u, v$ will describe the whole locus and will not introduce a spurious additional branch.

**Step 3:** Insert the parabola equation and compare Substituting into $v = 4u^2 + 1$ gives

$$\frac{3y - x}{2} = 4\left(\frac{3x - y}{2}\right)^2 + 1 = (3x - y)^2 + 1.$$

Multiplying by two and rearranging yields

$$2(3x - y)^2 + x - 3y + 2 = 0.$$

For a check, $P = (0, 1)$ gives midpoint $(1/4, 3/4)$, which satisfies this equation. The coefficients of the squared and linear terms rule out (A) and (C), while (D) interchanges the combinations associated with $u$ and $v$.

**Step 4:** Conclusion The locus is $\boxed{2(3x - y)^2 + (x - 3y) + 2 = 0}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For projection onto $y = x$, average the original coordinates; then form $3x - y$ and $3y - x$ to reverse the midpoint transformation.

**13. The value of k $\in$R, for which the following system of linear equations**

**$3x - y + 4z = 3$,**

**$x + 2y - 3z = -2$**

**$6x + 5y + kz = -3$,**

**has infinitely many solutions, is :** [4 marks]

(A) 3

(B) $-5$

(C) 5

(D) $-3$

**Correct Answer:** (B) $-5$

**Solution:**

**Concept:**

- A system in three unknowns has infinitely many solutions when the coefficient and augmented matrices have equal rank less than three.
- If two equations are independent, the third must be their linear combination, with the same combination holding for both the coefficients and the right-hand constants.

**Step 1:** Find the required linear combination Denote the first two equations by $E_1$ and $E_2$. For the third to be dependent on them, write it as $\alpha E_1 + \beta E_2$. Comparing the coefficients of $x$ and $y$ gives

$$3\alpha + \beta = 6, \qquad -\alpha + 2\beta = 5.$$

Substitution of $\beta = 6 - 3\alpha$ gives $-7\alpha = -7$. Thus $\alpha = 1$ and $\beta = 3$. These values are fixed independently of the unknown coefficient $k$.

**Step 2:** Check the constant and determine the remaining coefficient The corresponding combination of equations is

$$E_1 + 3E_2 : \quad 6x + 5y + (4 - 9)z = 3 + 3(-2).$$

It simplifies to

$$6x + 5y - 5z = -3.$$

The right-hand side is exactly the printed third constant, so dependence is consistent rather than contradictory. Matching the $z$-coefficient therefore requires $k = -5$. A zero coefficient determinant alone would not establish this consistency.

**Step 3:** Confirm that infinitely many solutions remain The first two equations are independent: the determinant of their $x, y$ coefficient block is

$$\begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = 7 \neq 0.$$

Thus their rank is two, and when $k = -5$ the third equation adds no restriction. Taking $z = t$ gives

$$x = \frac{4 - 5t}{7}, \qquad y = \frac{-9 + 13t}{7}, \qquad t \in \mathbb{R}.$$

Every real $t$ gives a solution, proving infinitude. The other listed values make the third equation independent and leave a unique solution instead.

**Step 4:** Conclusion The required value is $\boxed{k = -5}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Compare two convenient coefficient columns to find the dependence weights, and always verify that the same weights reproduce the right-hand side.

**14. In a triangle ABC, if $\left|\overrightarrow{BC}\right| = 3$, $\left|\overrightarrow{CA}\right| = 5$ and $\left|\overrightarrow{BA}\right| = 7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to :** [4 marks]

(A) $\frac{19}{2}$
(B) $\frac{13}{2}$
(C) $\frac{11}{2}$
(D) $\frac{15}{2}$

**Correct Answer:** (C) $\frac{11}{2}$

**Solution:**

**Concept:**

- The scalar projection of $\mathbf{u}$ onto a nonzero vector $\mathbf{v}$ is $(\mathbf{u} \cdot \mathbf{v})/|\mathbf{v}|$. It is a signed length, distinct from the vector projection.
- The identity $|\mathbf{u} - \mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2\mathbf{u} \cdot \mathbf{v}$ extracts a dot product from three side lengths.

**Step 1:** Relate the triangle sides as vectors Choose $\mathbf{u} = \overrightarrow{BA}$ and $\mathbf{v} = \overrightarrow{BC}$, both starting at $B$. Their difference is

$$\mathbf{u} - \mathbf{v} = \overrightarrow{CA}.$$

Therefore $|\mathbf{u}| = 7$, $|\mathbf{v}| = 3$, and $|\mathbf{u} - \mathbf{v}| = 5$. Keeping the starting point common is essential: using $\overrightarrow{AB}$ instead of $\overrightarrow{BA}$ would reverse the sign of the projection.

**Step 2:** Compute the dot product from the side lengths Square the difference identity and substitute the three given lengths:

$$25 = 49 + 9 - 2\mathbf{u} \cdot \mathbf{v}.$$

It follows that

$$\mathbf{u} \cdot \mathbf{v} = \frac{49 + 9 - 25}{2} = \frac{33}{2}.$$

Equivalently, the cosine rule at $B$ gives $\cos B = 33/(2 \cdot 7 \cdot 3) = 11/14$. The angle is acute, so a positive scalar projection is expected. No coordinates or diagram are required to determine it.

**Step 3:** Divide by the magnitude of the projection direction The requested scalar projection on $\overrightarrow{BC}$ is

$$\frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BC}|} = \frac{33/2}{3} = \frac{11}{2}.$$

This equals $7 \cos B$, which is less than 7, as any scalar projection of a vector of length seven must be. Options (A) and (D) exceed this bound immediately. Option (B), though within the bound, does not

satisfy the dot-product identity with the side of length five.

**Step 4:** Conclusion The projection is $\boxed{11/2}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Use vectors from the same vertex, recover their dot product with the cosine rule, and divide by the length of the direction onto which you project.

**15. If $f : R \to R$ is given by $f(x) = x + 1$, then the value of**

$$\lim_{n\to\infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + ..... + f\left(\frac{5(n-1)}{n}\right)\right]$$

**is:** [4 marks]

(A) $\frac{3}{2}$
(B) $\frac{5}{2}$
(C) $\frac{1}{2}$
(D) $\frac{7}{2}$

**Correct Answer:** (D) $\frac{7}{2}$

**Solution:**

**Concept:**

- The expression contains exactly $n$ terms, indexed by $k = 0, 1, \ldots, n-1$. Substitute the specified function before taking the limit.
- For an arithmetic progression, $\sum_{k=0}^{n-1} k = n(n-1)/2$. The factor $1/n$ averages the function values; it is not the interval width $5/n$.

**Step 1:** Substitute the linear function At the argument $5k/n$, the function has value $f(5k/n) = 1 + 5k/n$. Thus the first term is 1 and the last term is $1 + 5(n-1)/n$. Both the constant contribution and the variable contribution must be included. The constant 1 occurs once in every one of the $n$ terms, so its sum is $n$, not 1.

**Step 2:** Evaluate the finite average exactly Using the arithmetic sum gives

$$\frac{1}{n}\sum_{k=0}^{n-1}\left(1 + \frac{5k}{n}\right) = \frac{1}{n}\left[n + \frac{5}{n}\frac{n(n-1)}{2}\right] = 1 + \frac{5(n-1)}{2n}.$$

This equality holds for each positive integer $n$. Keeping the division by $n$ until after summation avoids confusing an average with a Riemann integral.

**Step 3:** Take the limit and check the options Since $(n-1)/n \to 1$, the required limit is $1 + 5/2 = 7/2$. As a check, the average of the first and last terms tends to $(1+6)/2 = 7/2$, because the function values form an arithmetic progression. The value $5/2$ omits the constant contribution, while $3/2$ and $1/2$ do not represent the average over the given range.

**Step 4:** Conclusion The limit equals $\boxed{7/2}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For equally spaced samples of a linear function, average the first and last values before taking the limit.

**16.** **Let $r_1$ and $r_2$ be the radii of the largest and smallest circles, respectively, which pass through the point $(-4, 1)$ and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then a + b is equal to :** [4 marks]

(A) 3

(B) 11

(C) 5

(D) 7

**Correct Answer:** (C) 5

**Solution:**

**Concept:**

- A circle passing through a fixed point has radius equal to the distance from that point to its centre. The allowed centres lie on the given circle.
- If a point is outside a circle of radius $R$, at distance $d$ from its centre, its largest and smallest distances to the circumference are $d + R$ and $d - R$.

**Step 1:** Identify the locus of the centres Complete the squares in the given equation:

$$x^2 + y^2 + 2x + 4y - 4 = 0 \quad \Longrightarrow \quad (x+1)^2 + (y+2)^2 = 9.$$

Consequently the centre-locus has centre $C = (-1, -2)$ and radius $R = 3$. Let the fixed point be $T = (-4, 1)$; it is the point through which each of the required circles must pass.

**Step 2:** Find the extreme radii The distance from $C$ to $T$ is

$$d = \sqrt{(-4+1)^2 + (1+2)^2} = \sqrt{18} = 3\sqrt{2}.$$

Since $d > R$, the point is outside the centre-locus. The nearest and farthest allowed centres lie on the line $CT$, where the triangle inequality attains equality. Therefore

$$r_1 = 3\sqrt{2} + 3, \qquad r_2 = 3\sqrt{2} - 3.$$

The smaller radius is positive, as it must be for a circle.

**Step 3:** Rationalise the ratio Dividing numerator and denominator by 3 and then rationalising gives

$$\frac{r_1}{r_2} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{(\sqrt{2}+1)^2}{2-1} = 3 + 2\sqrt{2}.$$

Hence $a = 3$ and $b = 2$. Their sum is 5, not the individual integer coefficient 3. The other listed sums, 11 and 7, also disagree with the uniquely obtained coefficients.

**Step 4:** Conclusion We obtain $a + b = \boxed{5}$. Hence, the correct option is (C).

**Quick Tip to Solve:** When a centre moves on a circle, use nearest and farthest distances from the fixed point instead of solving a family of circle equations.

**17. Consider the following three statements :**

**(A) If 3 + 3 = 7 then 4 + 3 = 8**

**(B) If 5 + 3 = 8 then earth is flat.**

**(C) If both (A) and (B) are true then 5 + 6 = 17.**

**Then, which of the following statements is correct?** [4 marks]

(A) (A) is false, but (B) and (C) are true

(B) (A) and (C) are true while (B) is false

(C) (A) is true while (B) and (C) are false

(D) (A) and (B) are false while (C) is true

**Correct Answer:** (B) (A) and (C) are true while (B) is false

**Solution:**

**Concept:**

- An implication $p \Rightarrow q$ is false only when its antecedent $p$ is true and its consequent $q$ is false.
- A conjunction is true only when both constituent statements are true. In a nested implication, first evaluate the statements that form its antecedent.

**Step 1:** Evaluate statement (A) The antecedent of statement (A) is $3 + 3 = 7$, which is false because $3 + 3 = 6$. Its consequent $4 + 3 = 8$ is also false because $4 + 3 = 7$. An implication with a false antecedent is true, regardless of its consequent. Thus statement (A) is true. The implication is a logical compound statement; its truth does not assert that either of those arithmetic equalities holds.

**Step 2:** Evaluate statement (B) Here the antecedent $5 + 3 = 8$ is true. The consequent, that the earth is flat, is false. This is precisely the true-antecedent, false-consequent case in the truth table of implication. Statement (B) is therefore false. It is essential to evaluate the conditional itself rather than merely noticing that its first arithmetic equality is correct.

**Step 3:** Evaluate statement (C) and compare The antecedent of (C) says that both (A) and (B) are true. Because (B) is false, this conjunction is false. Its consequent $5 + 6 = 17$ is false as well, but a false antecedent again makes the implication true. Thus the ordered truth values are $(T, F, T)$. The second option states exactly this: (A) and (C) are true while (B) is false. The first, third and fourth options respectively misidentify (A), (C), or both (A) and (B).

**Step 4:** Conclusion The first and third statements are true, while the second statement is false. Hence, the correct option is (B).

**Quick Tip to Solve:** For every conditional, look only for the one falsifying pattern: a true antecedent followed by a false consequent.

**18. Let in a right angled triangle, the smallest angle be $\theta$. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :** [4 marks]

(A) $\frac{\sqrt{5}+1}{4}$

(B) $\frac{\sqrt{5}-1}{2}$

(C) $\frac{\sqrt{2}-1}{2}$

(D) $\frac{\sqrt{5}-1}{4}$

**Correct Answer:** (B) $\frac{\sqrt{5}-1}{2}$

**Solution:**

**Concept:**

- Let the original sides satisfy $0 < a \le b < c$, with $c$ the hypotenuse. Then the smallest angle obeys $\sin\theta = a/c$.
- Reciprocation reverses order. Thus $1/a$, rather than $1/c$, is the hypotenuse of the second right triangle; apply Pythagoras accordingly.

**Step 1:** Write the two Pythagorean equations The original right triangle satisfies $a^2 + b^2 = c^2$. Since the reciprocal sides also form a right triangle, their largest member $1/a$ must satisfy

$$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}.$$

Using $1/c$ as the second hypotenuse would contradict the ordering of positive side lengths. No scale for the sides is required, because the desired sine is a ratio.

**Step 2:** Reduce to an equation for the sine Put $t = a/c = \sin\theta$. From the first equation, $b^2/c^2 = 1 - t^2$. Multiplying the reciprocal equation by $c^2$ gives

$$\frac{1}{t^2} = \frac{1}{1-t^2} + 1 = \frac{2-t^2}{1-t^2}.$$

Since $0 < t < 1$, both denominators are nonzero. Cross-multiplication therefore yields

$$t^4 - 3t^2 + 1 = 0, \qquad t^2 = \frac{3 \pm \sqrt{5}}{2}.$$

**Step 3:** Choose the geometrically admissible root The plus sign gives $t^2 > 1$ and cannot be a sine. The other root is positive and below $1/2$, consistent with the smallest angle of a right triangle. Taking its positive square root gives

$$t = \sqrt{\frac{3-\sqrt{5}}{2}} = \frac{\sqrt{5}-1}{2}.$$

Squaring the last expression verifies the equality. Option (A) exceeds $1/\sqrt{2}$; the smaller numbers in (C) and (D) fail the derived quartic.

**Step 4:** Conclusion $\sin\theta = \boxed{(\sqrt{5}-1)/2}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Before applying Pythagoras to reciprocal sides, identify the new longest side: it is the reciprocal of the original shortest side.

**19. Consider the line L given by the equation**

$\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$.

**Let Q be the mirror image of the point (2, 3, −1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P?** [4 marks]

(A) (−1, 1, 2)

(B) (1, 1, 1)

(C) (1, 1, 2)

(D) (1, 2, 2)

**Correct Answer:** (D) (1, 2, 2)

**Solution:**

**Concept:**

- The line has a point $A = (3, 1, 2)$ and direction vector $\mathbf{d} = (2, 1, 1)$. A plane perpendicular to the line has normal parallel to $\mathbf{d}$.
- If $H$ is the perpendicular projection of a point $T$ onto the mirror line, its reflection is $Q = 2H - T$. The projection parameter is $(T - A) \cdot \mathbf{d}/(\mathbf{d} \cdot \mathbf{d})$.

**Step 1:** Project the given point onto the line Take $T = (2, 3, -1)$. A general point of the line is $H = A + t\mathbf{d}$. Requiring $(T - H) \cdot \mathbf{d} = 0$ gives

$$t = \frac{(-1, 2, -3) \cdot (2, 1, 1)}{2^2 + 1^2 + 1^2} = \frac{-2 + 2 - 3}{6} = -\frac{1}{2}.$$

Hence

$$H = (3, 1, 2) - \tfrac{1}{2}(2, 1, 1) = (2, \tfrac{1}{2}, \tfrac{3}{2}).$$

This point is on the line and the displacement to it is perpendicular to the line.

**Step 2:** Find the reflection and the required plane The midpoint of $T$ and its image is $H$, so

$$Q = 2H - T = (4, 1, 3) - (2, 3, -1) = (2, -2, 4).$$

Use $(2, 1, 1)$ as the plane normal. The plane through $Q$ is

$$2(x - 2) + (y + 2) + (z - 4) = 0, \qquad 2x + y + z = 6.$$

The constant must be evaluated at the reflected point, not at the original point, since only $Q$ is specified as lying on this plane.

**Step 3:** Test every proposed point For the four proposed points, the values of $2x+y+z$ are respectively

$$1, \quad 4, \quad 5, \quad 6.$$

Only $(1, 2, 2)$ produces the required value 6. This substitution check also prevents confusing a point on the mirror line with a point on the required plane. The three other coordinates each violate the plane equation.

**Step 4:** Conclusion The point on $P$ is $\boxed{(1,2,2)}$. Hence, the correct option is (D).

**Quick Tip to Solve:** After finding the reflected point, form the plane using the line direction as its normal and check the options by one dot product each.

**20. If sum of the first 21 terms of the series $\log_{9^{1/2}}x + \log_{9^{1/3}}x + \log_{9^{1/4}}x + .....$, where x $>$ 0 is 504, then x is equal to** [4 marks]

(A) 243

(B) 9

(C) 7

(D) 81

**Correct Answer:** (D) 81

**Solution:**

**Concept:**

- The change-of-base rule gives $\log_{9^{1/k}} x = k \log_9 x$, since $\log(9^{1/k}) = (1/k)\log 9$.
- The denominators in the exponents begin at 2. Therefore the first 21 terms have multipliers $2, 3, \ldots, 22$, whose sum follows the arithmetic progression formula.

**Step 1:** Convert all terms to the same base For each integer $k \geq 2$,

$$\log_{9^{1/k}} x = \frac{\log x}{\log(9^{1/k})} = \frac{\log x}{(1/k)\log 9} = k \log_9 x.$$

All bases are positive and different from 1, and the given $x > 0$ ensures their arguments are valid. In particular, the first three terms become $2\log_9 x$, $3\log_9 x$, and $4\log_9 x$. The reciprocal exponent multiplies the logarithm rather than dividing it.

**Step 2:** Sum the 21 multipliers The last multiplier is $2 + (21 - 1) = 22$. Hence

$$2 + 3 + \cdots + 22 = \frac{21(2+22)}{2} = 252.$$

The stated sum condition is therefore $252 \log_9 x = 504$, giving $\log_9 x = 2$. Starting the multipliers at 1 or ending them at 21 would count a different series and would not reproduce the given first term.

**Step 3:** Recover and verify the argument By the definition of logarithm, $x = 9^2 = 81$. Substitution produces $2(2 + 3 + \cdots + 22) = 2 \times 252 = 504$, exactly as required. The options 243, 9 and 7 have logarithms to base 9 different from 2 and so cannot satisfy this sum. The candidate 81 is positive, so it also satisfies the original domain restriction.

**Step 4:** Conclusion $x = \boxed{81}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For bases with reciprocal exponents, convert to one base first; then count the first and last integer multipliers carefully.

## Mathematics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**21. The number of solutions of the equation**

$\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$**, x > 0, is :** [4 marks]

**Solution:**

**Concept:**

- Factorisation gives $2x^2+7x+5=(x+1)(2x+5)$. For $x>0$, both logarithm bases exceed 1, and all logarithm arguments are positive.
- Use $\log_a(ab)=1+\log_a b$ and the reciprocal-base identity $\log_b a=1/\log_a b$ to reduce the equation to a quadratic in one logarithm.

**Step 1:** Introduce a logarithmic variable with its range Let $t=\log_{x+1}(2x+5)$. Since $2x+5>x+1>1$, the logarithm is strictly larger than 1: $t>1$. This restriction is crucial because not every root of the transformed quadratic need correspond to an allowed positive value of $x$. The factorisation of the first argument gives

$$\log_{x+1}(2x^2+7x+5)=1+t.$$

**Step 2:** Reduce and solve the transformed equation The second term is

$$\log_{2x+5}(x+1)^2=2\log_{2x+5}(x+1)=\frac{2}{t}.$$

Thus the equation becomes $1+t+2/t-4=0$. Multiplication by $t$, which is positive and nonzero, yields

$$t^2-3t+2=0, \qquad (t-1)(t-2)=0.$$

The root $t=1$ is excluded by $t>1$; directly it would require $2x+5=x+1$, giving the inadmissible value $x=-4$.

**Step 3:** Count the positive solutions The remaining root is $t=2$, equivalent to

$$2x+5=(x+1)^2 \quad\Longrightarrow\quad x^2=4.$$

The quadratic gives $x=2$ and $x=-2$, but only $x=2$ satisfies $x>0$. At this value the original logarithms are $\log_3 27=3$ and $\log_9 9=1$, so their sum minus 4 is zero. We have exhausted both possible logarithmic roots and excluded every inadmissible value.

**Step 4:** Conclusion The equation has exactly $\boxed{1}$ solution for $x>0$, namely $x=2$.

**Quick Tip to Solve:** Carry the inequality between the logarithm bases into the substituted variable; here it immediately removes the root $t=1$.

**22. Let a curve y = y(x) be given by the solution of the differential equation $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$. If it intersects y-axis at y = $-1$, and the intersection point of the curve with x-axis is ($\alpha$, 0), then $e^{\alpha}$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- The principal inverse cosine gives $0 \le e^{-x} \le 1$, so the real curve has $x \ge 0$. The half-angle identity is $\cos(\phi/2) = \sqrt{(1+\cos\phi)/2}$ on this range.
- A differential equation of the form $dy = F(x)\,dx$ is integrated directly; the given intercept fixes the constant. The boundary value at $x = 0$ is interpreted by continuity from $x > 0$.

**Step 1:** Simplify the differential equation For $x > 0$, put $u = e^{-x}$. Then $du = -u\,dx$ and $\sqrt{e^{2x}-1} = \sqrt{1-u^2}/u$. Consequently, the given equation becomes

$$\frac{dy}{dx} = \frac{\sqrt{(1+u)/2}\,u}{\sqrt{(1-u)(1+u)}} = \frac{u}{\sqrt{2}\sqrt{1-u}}.$$

Both square roots are positive in the interior of the domain, so cancelling the factor does not introduce a sign choice.

**Step 2:** Integrate and apply the intercept Separating with $du = -u\,dx$, we obtain

$$dy = -\frac{du}{\sqrt{2}\sqrt{1-u}}, \qquad y = \sqrt{2}\sqrt{1-u} + C.$$

At the y-axis, $x = 0$ and $u = 1$. The limiting value of the square-root term is zero. Since the curve passes through $(0, -1)$, the integration constant is $C = -1$. Thus

$$y = \sqrt{2(1-e^{-x})} - 1.$$

**Step 3:** Find the x-axis intersection At $(\alpha, 0)$, substitute $y = 0$ into the curve equation:

$$\sqrt{2(1-e^{-\alpha})} = 1, \qquad 2(1-e^{-\alpha}) = 1.$$

Hence $e^{-\alpha} = 1/2$, which lies in the permitted interval. Squaring is valid here because both sides of the preceding square-root equation are nonnegative.

**Step 4:** Conclusion Therefore $\alpha = \log 2$ and the required numerical value is $\boxed{e^{\alpha} = 2}$.

**Quick Tip to Solve:** Use $u = e^{-x}$ before integrating: the half-angle factor cancels one factor in $\sqrt{1-u^2}$.

**23. For p $>$ 0, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle $\theta$ about origin in counter clockwise direction. If $\tan\theta = \frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of $\alpha$ is equal to _____.**

[4 marks]

**Solution:**

**Concept:**

- A rotation about the origin preserves the length of a vector. Thus the squared magnitudes of the initial and final vectors must be equal.
- For planar vectors $a = (a_x, a_y)$ and $b = (b_x, b_y)$, the directed angle satisfies $\tan\theta = (a_x b_y - a_y b_x)/(a \cdot b)$, provided the dot product is nonzero.

**Step 1:** Determine the positive parameter The squared lengths are

$$|\vec{v}_1|^2 = 3p^2 + 1, \qquad |\vec{v}_2|^2 = 4 + (p+1)^2.$$

Set these equal, as required by a rigid rotation:

$$3p^2 + 1 = p^2 + 2p + 5 \quad \Longrightarrow \quad p^2 - p - 2 = 0.$$

Factoring gives $(p-2)(p+1) = 0$. The condition $p > 0$ excludes $-1$, leaving $p = 2$. Therefore the vectors are $(2\sqrt{3}, 1)$ and $(2, 3)$.

**Step 2:** Calculate the directed tangent The dot product and the signed planar cross product are respectively

$$\vec{v}_1 \cdot \vec{v}_2 = (2\sqrt{3})(2) + (1)(3) = 4\sqrt{3} + 3,$$

$$(\vec{v}_1 \times \vec{v}_2)_z = (2\sqrt{3})(3) - (1)(2) = 6\sqrt{3} - 2.$$

The cross product is positive, which agrees with the stated counterclockwise direction. The dot product is also positive, so the smaller rotation angle is acute. Dividing the sine relation by the cosine relation eliminates the equal magnitudes and yields

$$\tan\theta = \frac{6\sqrt{3} - 2}{4\sqrt{3} + 3}.$$

**Step 3:** Compare the required expression The question writes the same tangent as

$$\tan\theta = \frac{\alpha\sqrt{3} - 2}{4\sqrt{3} + 3}.$$

The denominator is positive and identical in the two expressions. Equating the numerators gives $\alpha\sqrt{3} - 2 = 6\sqrt{3} - 2$, and hence $\alpha = 6$. No inverse trigonometric evaluation is needed.

**Step 4:** Conclusion The required value is $\boxed{\alpha = 6}$.

**Quick Tip to Solve:** Find the parameter using equal lengths first; then use cross product divided by dot product to obtain the directed tangent.

**24. Consider a triangle having vertices A(−2, 3), B(1, 9) and C(3, 8). If a line L passing through the circum-centre of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number $\alpha$ is _____.** [4 marks]

**Solution:**

**Concept:**

- The circumcentre of a triangle is equidistant from its three vertices. In particular, it lies on the perpendicular bisector of each side.
- A line through the midpoint of a segment and its circumcentre is its perpendicular bisector, unless the circumcentre coincides with that midpoint. Check this exceptional case rather than assuming uniqueness.

**Step 1:** Locate the midpoint and slope For $B = (1, 9)$ and $C = (3, 8)$, their midpoint is

$$M = \left(\frac{1+3}{2}, \frac{9+8}{2}\right) = \left(2, \frac{17}{2}\right).$$

The slope of $BC$ is $(8-9)/(3-1) = -1/2$. Hence the perpendicular slope is 2. Every point equidistant from $B$ and $C$, including the circumcentre, is on the line of slope 2 through $M$.

**Step 2:** Confirm the required line is unique The circumcentre cannot be $M$ itself: its squared distances from $B$ and $A = (-2, 3)$ would be

$$MB^2 = (-1)^2 + \left(\frac{1}{2}\right)^2 = \frac{5}{4},$$

$$MA^2 = (-4)^2 + \left(-\frac{11}{2}\right)^2 = \frac{185}{4}.$$

These are unequal. Therefore the line joining the circumcentre to $M$ is well defined and is precisely the perpendicular bisector of $BC$. We do not need the actual coordinates of the circumcentre.

**Step 3:** Find the y-intercept Use the point-slope equation with point $M$ and slope 2:

$$y - \frac{17}{2} = 2(x - 2), \qquad y = 2x + \frac{9}{2}.$$

At the y-axis $x = 0$, giving $(0, 9/2)$. Comparing this with the stated point $(0, \alpha/2)$, we have $\alpha/2 = 9/2$. Multiplication by 2 gives the desired real number.

**Step 4:** Conclusion The line is $y = 2x + 9/2$, and $\boxed{\alpha = 9}$.

**Quick Tip to Solve:** The circumcentre condition often lets you use a perpendicular bisector directly, avoiding the calculation of the circumcentre itself.

**25. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is $(\alpha, \beta)$, then $2(\alpha + \beta)$ is equal to _____.**

[4 marks]

**Solution:**

**Concept:**

- Minimising distance is equivalent to minimising its square, because the square-root function is strictly increasing on nonnegative numbers.
- The equation $y^2 = 6x$ allows every real point of the parabola to be parameterised by $y = t$, $x = t^2/6$. A global minimum must be checked, rather than just a stationary point.

**Step 1:** Express the squared distance in one variable Let the point on the curve be $P = (t^2/6, t)$ with $t \in \mathbb{R}$. Its squared distance from the fixed point $Q = (3, 3/2)$ is

$$D(t) = \left(\frac{t^2}{6} - 3\right)^2 + \left(t - \frac{3}{2}\right)^2.$$

Using $t$ over the whole real line includes both the upper and lower halves of the parabola. Thus no candidate is lost by choosing this parameter.

**Step 2:** Differentiate and find the stationary point Differentiate the two squares using the chain rule:

$$D'(t) = 2\left(\frac{t^2}{6} - 3\right)\frac{t}{3} + 2\left(t - \frac{3}{2}\right).$$

The linear terms cancel, giving

$$D'(t) = \frac{t^3}{9} - 3 = \frac{t^3 - 27}{9}.$$

Therefore the only real stationary point is $t = 3$. Since $t^3 - 27$ is negative for $t < 3$ and positive for $t > 3$, the squared distance decreases up to this value and increases afterwards.

**Step 3:** Identify the minimum and evaluate the expression The sign change proves that $t = 3$ is the global minimum on $\mathbb{R}$; also $D(t) \to \infty$ as $|t| \to \infty$. The nearest point has

$$\alpha = \frac{3^2}{6} = \frac{3}{2}, \qquad \beta = 3.$$

It indeed satisfies $\beta^2 = 9 = 6\alpha$. Substitute these coordinates into the requested expression:

$$2(\alpha + \beta) = 2\left(\frac{3}{2} + 3\right) = 9.$$

**Step 4:** Conclusion The nearest point is $(\alpha, \beta) = (3/2, 3)$, so $\boxed{2(\alpha + \beta) = 9}$.

**Quick Tip to Solve:** For a sideways parabola, parameterise by y and minimise squared distance; this avoids square roots and covers both branches.

**26. Let a function g : [ 0, 4 ] $\to$ R be defined as**

$g(x) = \begin{cases} \max\limits_{0 \le t \le x} \{t^3 - 6t^2 + 9t - 3), & 0 \le x \le 3 \\ 4 - x, & 3 < x \le 4 \end{cases}$ **, then the number of points in the interval (0, 4) where g(x) is NOT differentiable, is _____.** [4 marks]

**Solution:**

**Concept:**

- Write $f(t) = t^3 - 6t^2 + 9t - 3$. The expression $\max_{0 \le t \le x} f(t)$ records the largest value attained anywhere in the interval, not merely the value at its right endpoint.
- Differentiability at an interior joining point requires continuity and equality of left and right derivatives. A switch in formula alone does not necessarily create a corner.

**Step 1:** Analyse the cubic on its interval Differentiate the polynomial:

$$f'(t) = 3t^2 - 12t + 9 = 3(t - 1)(t - 3).$$

It is positive on $(0, 1)$ and negative on $(1, 3)$. The relevant values are $f(0) = -3$, $f(1) = 1$, and $f(3) = -3$. Thus before $x = 1$, the largest value in $[0, x]$ occurs at $t = x$. Once $x$ reaches $1$, the maximum remains the already attained value $1$, even though the cubic subsequently decreases.

**Step 2:** Write the running maximum explicitly The complete function can therefore be written as

$$g(x) = \begin{cases} x^3 - 6x^2 + 9x - 3, & 0 \le x \le 1, \\ 1, & 1 < x \le 3, \\ 4 - x, & 3 < x \le 4. \end{cases}$$

Each of these expressions is differentiable in the interior of its own interval. Only $x = 1$ and $x = 3$ need separate one-sided checks. The endpoints $0$ and $4$ are excluded by the interval in the question.

**Step 3:** Check both interior junctions At $x = 1$, both values are $1$. The left derivative is $f'(1) = 0$, while the right derivative of the constant branch is also $0$. Hence $g$ is differentiable there. At $x = 3$, both values again tend to $1$, so the function is continuous. However, the left derivative is $0$ and the right derivative of $4 - x$ is $-1$. They are unequal, so $g$ is not differentiable at $3$.

**Step 4:** Conclusion The only point of non-differentiability in $(0, 4)$ is $x = 3$. The required number is $\boxed{1}$.

**Quick Tip to Solve:** For a running maximum, retain earlier peak values. Then test one-sided slopes at every place where the formula changes.

**27.** **If** $\lim\limits_{x\to 0}\frac{\alpha x e^x-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2 x}=10, \alpha,\beta,\gamma\in R$**, then the value of** $\alpha+\beta+\gamma$ **is** _____.

[4 marks]

**Solution:**

**Concept:**

- Near zero, $e^x=1+x+x^2/2+O(x^3)$, $\log(1+x)=x-x^2/2+x^3/3+O(x^4)$, and $e^{-x}=1-x+O(x^2)$.
- The denominator $x\sin^2 x=x^3+O(x^5)$. A finite limit therefore requires the coefficients of $x$ and $x^2$ in the numerator to vanish; its cubic coefficient then gives the limit.

**Step 1:** Expand the numerator to cubic order Multiplying each series by its accompanying power of $x$, we get

$$\alpha x e^x=\alpha x+\alpha x^2+\frac{\alpha}{2}x^3+O(x^4),$$
$$-\beta\log(1+x)=-\beta x+\frac{\beta}{2}x^2-\frac{\beta}{3}x^3+O(x^4),$$
$$\gamma x^2e^{-x}=\gamma x^2-\gamma x^3+O(x^4).$$

Thus the numerator is

$$(\alpha-\beta)x+\left(\alpha+\frac{\beta}{2}+\gamma\right)x^2+\left(\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right)x^3+O(x^4).$$

**Step 2:** Impose the finite-limit conditions Since the denominator has leading term $x^3$, any nonzero linear coefficient would produce an unbounded $1/x^2$ term. After removing it, a nonzero quadratic coefficient would still give a $1/x$ term. Hence

$$\alpha-\beta=0,\qquad \alpha+\frac{\beta}{2}+\gamma=0.$$

These imply $\beta=\alpha$ and $\gamma=-3\alpha/2$.

**Step 3:** Match the cubic coefficient to the limit After the lower powers vanish, the quotient tends to its cubic coefficient:

$$\frac{\alpha}{2}-\frac{\beta}{3}-\gamma=10.$$

Substitution gives $\alpha/2-\alpha/3+3\alpha/2=5\alpha/3=10$. Therefore $\alpha=6$, $\beta=6$, and $\gamma=-9$. These values cancel both lower coefficients and give exactly the required limiting value, so they satisfy the original limit.

**Step 4:** Conclusion Therefore $\boxed{\alpha+\beta+\gamma=6+6-9=3}$.

**Quick Tip to Solve:** Match the order of the denominator first: cancel every lower power in the numerator before equating the leading coefficient.

**28. Let $A = \{a_{ij}\}$ be a 3 × 3 matrix,**

**where** $a_{ij} = \begin{cases} (-1)^{j-i} & if \quad i < j, \\ 2 & if \quad i = j, \\ (-1)^{i+j} & if \quad i > j \end{cases}$

**then** $\det(3Adj(2A^{-1}))$ **is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- For an invertible $n \times n$ matrix $B$, $\det(\text{adj}\, B) = (\det B)^{n-1}$. This follows by taking determinants in $B\,\text{adj}\, B = (\det B)I$.
- For a scalar $k$ and an $n \times n$ matrix $B$, $\det(kB) = k^n \det B$. Also $\det(A^{-1}) = 1/\det A$. Here the order is $n = 3$.

**Step 1:** Construct the matrix from the entry rule The diagonal entries are 2. Above the diagonal, the signs are $(-1)^{j-i}$, giving $-1, 1, -1$ in positions $(1,2), (1,3), (2,3)$. Below it, $(-1)^{i+j}$ gives the matching signs. Therefore

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$

Expanding its determinant along the first row gives

$$\det A = 2(4-1) + (-2+1) + (1-2) = 6 - 1 - 1 = 4.$$

In particular, the determinant is nonzero, as required for the inverse in the question.

**Step 2:** Find the determinant inside the adjugate Let $B = 2A^{-1}$. Scaling a matrix of order three by 2 scales its determinant by $2^3$, rather than by 2. Hence

$$\det B = \det(2A^{-1}) = 2^3 \det(A^{-1}) = \frac{8}{4} = 2.$$

Using the adjugate identity with order three now gives

$$\det(\text{adj}\, B) = (\det B)^2 = 2^2 = 4.$$

**Step 3:** Apply the outer scalar factor The requested matrix is $3\,\text{adj}\, B$, still of order three. Its determinant is therefore

$$\det(3\,\text{adj}\, B) = 3^3 \det(\text{adj}\, B) = 27 \times 4 = 108.$$

This calculation applies the inner scalar 2, the adjugate operation, and the outer scalar 3 in their stated order. Computing the inverse entries or all nine cofactors would add unnecessary work and is not needed to establish the exact value.

**Step 4:** Conclusion The determinant requested in the question is $\boxed{108}$.

**Quick Tip to Solve:** Use determinant identities before computing an inverse: a scalar contributes its cube and an adjugate contributes the square for order three.

**29. For k $\in$ N, let $\frac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)} = \sum_{K=0}^{20} \frac{A_k}{\alpha+k}$, where $\alpha > 0$. Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to _____.** [4 marks]

**Solution:**

**Concept:**

- For distinct linear factors, the coefficient of $1/(\alpha+k)$ is obtained by multiplying the rational identity by $\alpha + k$ and evaluating the resulting polynomial identity at $\alpha = -k$.
- The product of the remaining differences is $\prod_{j\neq k}(j-k) = (-1)^k k!(20-k)!$. Factorial ratios allow nearby coefficients to be compared without evaluating large factorials.

**Step 1:** Extract the general coefficient Multiply the given equality by the common denominator. The resulting polynomial identity holds for every positive $\alpha$, so its two polynomials are identical. Consequently it may be evaluated at negative integers even though the original rational expression has poles there. At $\alpha = -k$, every term except the one containing $A_k$ vanishes. Hence

$$1 = A_k \prod_{j=0,\, j\neq k}^{20} (j-k), \qquad A_k = \frac{(-1)^k}{k!(20-k)!}.$$

There are exactly $k$ negative factors, which accounts for the alternating sign.

**Step 2:** Form the required ratios Use the formula for adjacent indices, cancelling the common factorials before doing arithmetic:

$$\frac{A_{14}}{A_{13}} = -\frac{7}{14} = -\frac{1}{2}, \qquad \frac{A_{15}}{A_{14}} = -\frac{6}{15} = -\frac{2}{5}.$$

The first ratio is negative while their product is positive. Thus

$$\frac{A_{15}}{A_{13}} = \left(-\frac{2}{5}\right)\left(-\frac{1}{2}\right) = \frac{1}{5}.$$

**Step 3:** Evaluate the expression Add the two ratios using their signs, and only then square:

$$\frac{A_{14}+A_{15}}{A_{13}} = -\frac{1}{2} + \frac{1}{5} = -\frac{3}{10}.$$

$$100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2 = 100 \cdot \frac{9}{100} = 9.$$

The square removes the final negative sign; it does not justify ignoring the alternating signs earlier.

**Step 4:** Conclusion The required numerical value is $\boxed{9}$.

**Quick Tip to Solve:** Use $A_{k+1}/A_k = -(20-k)/(k+1)$ to obtain nearby partial-fraction coefficients quickly.

**30.** Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to _____. [4 marks]

**Solution:**

**Concept:**

- For a recurrence with constant coefficients, multiply each term by a power of $x$ and sum. Shifting the index converts the recurrence into an algebraic equation for $S(x) = \sum_{n=1}^{\infty} a_n x^n$.
- The initial terms must be removed explicitly when a shifted sum begins at $n = 3$. Here the required weight is $2^{-3n} = (1/8)^n$, so the final substitution is $x = 1/8$.

**Step 1:** Express the shifted sums Write $S = S(x) = a_1 x + a_2 x^2 + a_3 x^3 + \cdots$. Multiply the recurrence by $x^{n+2}$ and sum for $n \geq 1$:

$$\sum_{n=1}^{\infty} a_{n+2} x^{n+2} = 2x \sum_{n=1}^{\infty} a_{n+1} x^{n+1} + x^2 \sum_{n=1}^{\infty} a_n x^n.$$

Since $a_1 = a_2 = 1$, this becomes

$$S - x - x^2 = 2x(S - x) + x^2 S.$$

The subtraction on the left retains neither of the first two initial terms.

**Step 2:** Solve for the generating function Collect the terms containing $S$:

$$S(1 - 2x - x^2) = x - x^2, \qquad S(x) = \frac{x(1-x)}{1 - 2x - x^2}.$$

The recurrence has characteristic roots $1 + \sqrt{2}$ and $1 - \sqrt{2}$. Therefore its terms grow at most as a constant times $(1 + \sqrt{2})^n$. The series converges absolutely when $|x| < 1/(1 + \sqrt{2})$; in particular $x = 1/8$ lies safely inside this interval.

**Step 3:** Substitute the required weight At $x = 1/8$, numerator and denominator have the same convenient denominator:

$$S(1/8) = \frac{(1/8)(7/8)}{1 - 1/4 - 1/64} = \frac{7/64}{47/64} = \frac{7}{47}.$$

Thus multiplying the given infinite sum by 47 gives exactly 7, without needing to list the sequence or approximate its tail.

**Step 4:** Conclusion The requested value is $47 \sum_{n=1}^{\infty} a_n/2^{3n} = \boxed{7}$.

**Quick Tip to Solve:** For weighted recurrence sums, shift the generating function and subtract the initial terms before collecting coefficients.

**31. If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be :** [4 marks]

(A) 100%

(B) 200%

(C) 300%

(D) 400%

**Correct Answer:** (A) 100%

**Solution:**

**Concept:**

- For a body of fixed mass $m$ in nonrelativistic motion, kinetic energy and momentum magnitude obey $K = p^2/(2m)$. Thus momentum magnitude is proportional to the square root of kinetic energy.
- Percentage change compares the increase with the original value: $100(p_f - p_i)/p_i$. It differs from expressing the final value as a percentage of the original.

**Step 1:** Relate the initial and final momenta Let the initial momentum magnitude be $p_i$ and the final magnitude be $p_f$. Since the same body is involved, its mass is unchanged. Divide the energy equations to eliminate that mass:

$$\frac{K_f}{K_i} = \frac{p_f^2/(2m)}{p_i^2/(2m)} = \left(\frac{p_f}{p_i}\right)^2.$$

The stated energy factor is four, so the positive momentum magnitudes satisfy

$$\frac{p_f}{p_i} = \sqrt{4} = 2.$$

Hence the final momentum magnitude is twice its original value, rather than four times it.

**Step 2:** Calculate the percentage increase The increase is the final magnitude minus the original magnitude:

$$\Delta p = p_f - p_i = 2p_i - p_i = p_i.$$

Consequently

$$\frac{\Delta p}{p_i} \times 100\% = 100\%.$$

The result is independent of the mass and initial speed. The question supplies no change in direction, and its energy comparison determines the momentum magnitude.

**Step 3:** Distinguish the other percentages The final momentum is 200 percent of its initial magnitude, but that is not the percentage change asked for. An increase of 200 percent would mean a final magnitude of $3p_i$, requiring nine times the kinetic energy. Increases of 300 and 400 percent would similarly give final momenta $4p_i$ and $5p_i$, requiring energy factors of 16 and 25. None matches the stated factor of four.

**Step 4:** Conclusion The momentum magnitude increases by $\boxed{100\%}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Take the square root of the energy factor, then subtract one before converting the momentum increase to a percentage.

**32. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time $t_1$. If he remains stationary on a moving escalator then the escalator takes him up in time $t_2$. The time taken by him to walk up on the moving escalator will be :** [4 marks]

(A) $\frac{t_1 t_2}{t_2 - t_1}$

(B) $\frac{t_1 + t_2}{2}$

(C) $\frac{t_1 t_2}{t_2 + t_1}$

(D) $t_2 - t_1$

**Correct Answer:** (C) $\frac{t_1 t_2}{t_2 + t_1}$

**Solution:**

**Concept:**

- For a fixed path length $L$, speed is distance divided by time. A boy walking on an escalator has a speed relative to its steps, while the steps have their own speed relative to the ground.
- When both velocities point up the escalator, the ground speed is their sum: $v = u + w$. The length of the escalator cancels when the separate journey times are used.

**Step 1:** Find the two individual speeds Let the distance along the escalator be $L$. On the stationary escalator, the boy covers this distance in $t_1$, so his walking speed relative to the steps is

$$u = \frac{L}{t_1}.$$

When he stands still on moving steps, the escalator alone takes him through $L$ in $t_2$. Its speed relative to the ground is therefore

$$w = \frac{L}{t_2}.$$

Use the same walking speed relative to the steps when he walks on the moving escalator.

**Step 2:** Add the upward velocities The boy walks in the same direction as the moving escalator. Hence his speed relative to the ground is

$$v = u + w = \frac{L}{t_1} + \frac{L}{t_2} = \frac{L(t_1 + t_2)}{t_1 t_2}.$$

The time for the full journey is distance divided by this combined speed:

$$t = \frac{L}{v} = \frac{t_1 t_2}{t_1 + t_2}.$$

The arbitrary length $L$ has cancelled, as it must because both stated times refer to that same length.

**Step 3:** Check the physical limits This time is smaller than either $t_1$ or $t_2$, since walking and escalator motion help each other. If $t_1 = t_2 = t_0$, the two equal speeds give $t = t_0/2$. A difference in the denominator corresponds to opposing motion and can even diverge for equal times. The arithmetic mean does not give the doubled-speed result, while the difference $t_2 - t_1$ would incorrectly give zero when the two times are equal.

**Step 4:** Conclusion The time is $\boxed{t_1t_2/(t_1+t_2)}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For two simultaneous motions covering the same path in times $t_1, t_2$, add reciprocal times: $1/t = 1/t_1 + 1/t_2$.

**33. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is :** [4 marks]

(A) 1.5

(B) 2.0

(C) 0.7

(D) 3.0

**Correct Answer:** (D) 3.0

**Solution:**

**Concept:**

- For circular motion around the same central mass $M$, equating gravitational force to centripetal force gives $T = 2\pi\sqrt{r^3/(GM)}$. Therefore $T \propto r^{3/2}$.
- For a small fractional radius change, the binomial approximation gives $\Delta T/T \simeq (3/2)\Delta r/r$. Here the fractional increase in orbital radius is 0.02.

**Step 1:** Derive the dependence on radius Let the satellite mass be $m$, orbital speed be $v$, and orbital radius be $r$. In a circular orbit,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}, \qquad v = \sqrt{\frac{GM}{r}}.$$

Since the satellite travels a circumference $2\pi r$ in one period,

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}.$$

Both satellites orbit Earth, so the central mass and the constant of proportionality are identical. Their own masses cancel.

**Step 2:** Compare the two periods With $r_1 = R$ and $r_2 = 1.02R$,

$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = (1.02)^{3/2}.$$

Use $(1+x)^{3/2} \simeq 1 + (3/2)x$ for $x = 0.02$. Then

$$\frac{T_2 - T_1}{T_1} \simeq \frac{3}{2}(0.02) = 0.03.$$

Thus the outer satellite has the longer period, with a percentage increase of approximately three

percent.

**Step 3:** Check the approximation against the options The exact factor is about 1.0301495, giving a percentage difference of about 3.01495 percent relative to the inner satellite. The options are evidently rounded at the stated level, so this selects 3.0. A 2.0 percent answer would incorrectly make period proportional to radius; 1.5 percent confuses the exponent with the percentage itself. The value 0.7 is also incompatible with the $r^{3/2}$ dependence.

**Step 4:** Conclusion The percentage difference is approximately $\boxed{3.0\%}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For small circular-orbit changes, multiply the radius percentage change by $3/2$ to obtain the period percentage change.

**34. With what speed should a galaxy move outward with respect to earth so that the sodium-D line at wavelength 5890 $\overset{o}{A}$ is observed at 5896 $\overset{o}{A}$ ?** [4 marks]

(A) 306 km/sec

(B) 322 km/sec

(C) 296 km/sec

(D) 336 km/sec

**Correct Answer:** (A) 306 km/sec

**Solution:**

**Concept:**

- A source receding at speed $v \ll c$ produces a redshift satisfying $\Delta\lambda/\lambda_0 \simeq v/c$, where $\lambda_0$ is the emitted wavelength.
- The wavelength increases from 5890 to 5896 angstroms, so $\Delta\lambda = 6$ angstroms. The wavelength ratio is dimensionless, and identical wavelength units cancel.

**Step 1:** Identify the shift and its sign The observed wavelength is larger than the emitted wavelength. Thus the spectral line is redshifted, consistently with the stated outward motion of the galaxy. The wavelength difference is

$$\Delta\lambda = 5896 - 5890 = 6 \text{ angstroms.}$$

The fractional shift is

$$z = \frac{\Delta\lambda}{\lambda_0} = \frac{6}{5890} \simeq 1.0187 \times 10^{-3}.$$

This is much smaller than one, so the nonrelativistic Doppler approximation is appropriate for the precision of the choices.

**Step 2:** Calculate the recession speed Use $c = 3.0 \times 10^5\,\mathrm{km\,s^{-1}}$. Substituting into $v \simeq cz$ gives

$$v = 3.0 \times 10^5 \times \frac{6}{5890}\ \mathrm{km\,s^{-1}} \simeq 305.6\ \mathrm{km\,s^{-1}}.$$

Rounding to the nearest speed offered gives $306\,\mathrm{km\,s^{-1}}$. There is no need to convert the two wavelengths to metres because their ratio uses matching units.

**Step 3:** Check the choice and approximation The other proposed speeds, 322, 296 and 336 kilometres per second, do not result from the measured six-angstrom increase. In particular the calculated value is close to 306, differing only through rounding. The relativistic longitudinal relation would give $v/c = ((1+z)^2 - 1)/((1+z)^2 + 1)$; its correction here is far too small to change the selected option. The inferred positive recession speed is also consistent with the redshift sign.

**Step 4:** Conclusion The required outward speed is approximately $\boxed{306\,\mathrm{km\,s^{-1}}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** For a small spectral shift, compute $c\Delta\lambda/\lambda_0$, keeping both wavelengths in the same unit.

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**35. The length of a metal wire is $l_1$, when the tension in it is $T_1$ and is $l_2$ when the tension is $T_2$. The natural length of the wire is :** [4 marks]

(A) $\sqrt{l_1 l_2}$

(B) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

(C) $\frac{l_1T_2 + l_2T_1}{T_2 + T_1}$

(D) $\frac{l_1 + l_2}{2}$

**Correct Answer:** (B) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

**Solution:**

**Concept:**

- Within the elastic limit, the extension of a uniform wire is proportional to its tension: $l - L = kT$, where $L$ is the natural length and $k = L/(YA)$ is a constant for that wire.
- Two loaded lengths provide two linear equations for the unknown natural length and compliance. Eliminating the compliance is equivalent to finding the zero-tension intercept of a length-versus-tension graph.

**Step 1:** Write the two extension equations Let the unloaded natural length be $L$. For the two stated tensions, Hooke law gives

$$l_1 = L + kT_1, \qquad l_2 = L + kT_2.$$

Here the same positive constant $k$ applies to both measurements, because the wire material and cross-sectional area are unchanged and the elastic approximation is being used. Subtracting the equations yields

$$l_2 - l_1 = k(T_2 - T_1).$$

Thus, for distinct tensions, $k = (l_2 - l_1)/(T_2 - T_1)$.

**Step 2:** Eliminate the extension constant Substitute this value in $L = l_1 - kT_1$:

$$L = l_1 - \frac{T_1(l_2 - l_1)}{T_2 - T_1}.$$

Combining the terms over a common denominator gives

$$L = \frac{l_1T_2 - l_1T_1 - l_2T_1 + l_1T_1}{T_2 - T_1} = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}.$$

The terms proportional to $l_1T_1$ cancel exactly, leaving the zero-load length.

**Step 3:** Test the result and other forms If $T_1 = 0$, the first measured length is already the natural length, and the derived expression reduces to $L = l_1$, as required. The weighted sum in option (C) generally exceeds this zero-load value, while the geometric and arithmetic means of two stretched lengths also fail to extrapolate to zero tension. All options have dimensions of length, so dimensional analysis alone cannot decide this problem; the linear force-extension law is essential.

**Step 4:** Conclusion The natural length is $\boxed{(l_1T_2 - l_2T_1)/(T_2 - T_1)}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For a wire obeying Hooke law, extrapolate the straight line $l = L + kT$ to zero tension to obtain its natural length.

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**36. In an electromagnetic wave the electric field vector and magnetic field vector are given as $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{k}$ respectively. The direction of propagation of electromagnetic wave is along :** [4 marks]

(A) $\left(\hat{k}\right)$

(B) $\hat{j}$

(C) $\left(-\hat{k}\right)$

(D) $\left(-\hat{j}\right)$

**Correct Answer:** (D) $\left(-\hat{j}\right)$

**Solution:**

**Concept:**

- The energy-flow direction of an electromagnetic wave is the direction of the Poynting vector, $\vec{S} = \mu_0^{-1}(\vec{E} \times \vec{B})$. The electric and magnetic fields are transverse to this direction.
- The Cartesian unit vectors obey $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, and $\hat{k} \times \hat{i} = \hat{j}$. Reversing the order reverses the sign.

**Step 1:** Identify the ordered cross product The electric field is along the positive X-axis and the magnetic field is along the positive Z-axis. The propagation direction is obtained from electric field crossed with magnetic field, in that order. Therefore

$$\vec{E} \times \vec{B} = E_0B_0(\hat{i} \times \hat{k}).$$

The amplitudes multiply the vector and do not change its direction. The magnitude of either amplitude is not needed to choose the direction.

**Step 2:** Evaluate the direction and test the options Since $\hat{k} \times \hat{i} = \hat{j}$, anticommutativity gives

$$\hat{i} \times \hat{k} = -\hat{j}, \qquad \vec{S} = -\frac{E_0 B_0}{\mu_0}\hat{j}.$$

Thus energy travels along the negative Y-axis. Both positive and negative Z directions would be parallel to the magnetic field, contradicting transversality, so (A) and (C) fail. Positive Y in (B) results from reversing the order of the two fields. As a check, the wave relation $\vec{B} = c^{-1}\hat{n} \times \vec{E}$ with $\hat{n} = -\hat{j}$ gives a positive Z-directed magnetic field, as required.

**Step 3:** Conclusion The electromagnetic wave propagates along $-\hat{j}$, perpendicular to both given fields. Hence, the correct option is (D).

**Quick Tip to Solve:** Use electric field crossed with magnetic field. Reversing that order gives the opposite propagation direction.

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**37. For a series LCR circuit with R = 100 Ω, L = 0.5 mH and C = 0.1 pF connected across 220V−50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is :** [4 marks]

(A) 0°, resistive circuit

(B) ≈ 90°, predominantly inductive circuit

(C) 0°, resonance circuit

(D) ≈ 90°, predominantly capacitive circuit

**Correct Answer:** (D) ≈ 90°, predominantly capacitive circuit

**Solution:**

**Concept:**

- For a series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$, and $\omega = 2\pi f$. Both reactances and the resistance are measured in ohms.
- If $\phi$ is the voltage phase relative to current, $\tan\phi = (X_L - X_C)/R$. Negative net reactance means a capacitive circuit, in which current leads voltage.

**Step 1:** Convert the components and calculate reactances Use the given frequency, rather than the 220 V supply value, to calculate reactance:

$$\omega = 2\pi(50) = 100\pi \text{ rad s}^{-1}.$$

The component values in SI units are $L = 5 \times 10^{-4}$ H and $C = 10^{-13}$ F. Hence

$$X_L = (100\pi)(5 \times 10^{-4}) \simeq 0.157\ \Omega,$$

$$X_C = \frac{1}{(100\pi)(10^{-13})} \simeq 3.18 \times 10^{10}\ \Omega.$$

The extremely small capacitance makes its reactance overwhelmingly greater than both the inductive reactance and the resistance.

**Step 2:** Determine the phase and circuit character Substitute the reactances and $R = 100\ \Omega$:

$$\tan\phi = \frac{0.157 - 3.18 \times 10^{10}}{100} \simeq -3.18 \times 10^{8}.$$

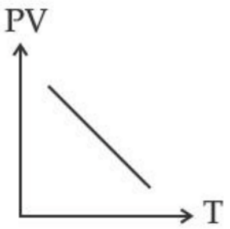
Consequently $\phi$ is very close to $-90°$: the supplied voltage lags the current, or equivalently the current leads the voltage by approximately $90°$. A zero phase would require vanishing net reactance, which is not the case; thus the resistive and resonance descriptions (A) and (C) fail. Predominantly inductive behavior in (B) would require $X_L > X_C$, the opposite inequality. The supply voltage affects current magnitude but does not alter this impedance phase.

**Step 3:** Conclusion The phase difference has magnitude approximately $90°$, with current leading voltage. The circuit is predominantly capacitive. Hence, the correct option is (D).

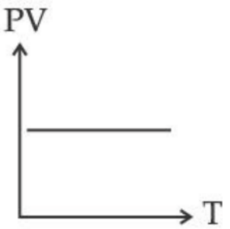
**Quick Tip to Solve:** Convert pF to $10^{-12}$ F carefully; a tiny capacitance at low frequency gives a huge capacitive reactance.

**38. Which of the following graphs represent the behavior of an ideal gas? Symbols have their usual meaning.** [4 marks]

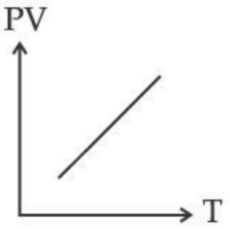
(A)



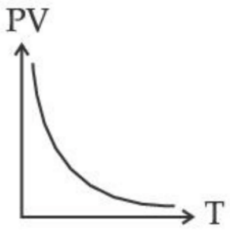

(B)



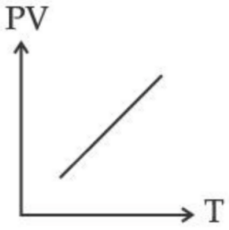

(C)




(D)



**Correct Answer:** (C)




**Solution:**

**Concept:**

- For a fixed amount $n$ of an ideal gas, the equation of state is $PV = nRT$, where $T$ is absolute temperature in kelvin and $R$ is the universal gas constant.
- A plot of the product $PV$ against $T$ therefore has constant positive slope $nR$, and its extrapolation passes through the origin.

**Step 1:** Read the axes before applying a gas law The vertical axis in all four choices represents the product $PV$, not pressure alone, and the horizontal axis represents temperature $T$. For the same sample of gas, $n$ is fixed even if pressure and volume individually change. Rearranging the equation of state gives

$$\frac{PV}{T} = nR = \text{constant}.$$

For example, doubling the absolute temperature doubles the pressure-volume product. This conclusion does not require the process to have constant pressure or constant volume.

**Step 2:** Compare each plotted dependence The graph must consequently be an increasing straight line:

$$PV = (nR)T, \qquad \frac{d(PV)}{dT} = nR > 0.$$

Option (C) is the graph with this positive linear dependence. Option (A) has negative slope and would predict a falling pressure-volume product on heating. Option (B) keeps $PV$ constant while temperature varies, which contradicts the equation of state for a fixed gas sample. Option (D) suggests an inverse dependence on temperature. That resembles neither $PV \propto T$ nor a permissible rearrangement with these axes; the familiar inverse pressure-volume relation applies at fixed temperature with different axes. The drawn segment in (C) is understood as a portion of the straight line that extrapolates to zero absolute temperature.

**Step 3:** Conclusion The product $PV$ increases linearly with absolute temperature, so the increasing straight-line graph represents the ideal gas. Hence, the correct option is (C).

**Quick Tip to Solve:** Read whether the vertical axis says $P$, $V$, or $PV$; Boyle's inverse curve does not describe a $PV$-versus-$T$ plot.

**39. A particle is making simple harmonic motion along the X-axis. If at a distances $x_1$ and $x_2$ from the mean position the velocities of the particle are $v_1$ and $v_2$ respectively. The time period of its oscillation is given as :** [4 marks]

(A) $T = 2\pi\sqrt{\frac{x_2^2+x_1^2}{v_1^2-v_2^2}}$

(B) $T = 2\pi\sqrt{\frac{x_2^2+x_1^2}{v_1^2+v_2^2}}$

(C) $T = 2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2+v_2^2}}$

(D) $T = 2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2-v_2^2}}$

**Correct Answer:** (D) $T = 2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2-v_2^2}}$

**Solution:**

**Concept:**

- In simple harmonic motion, the speed at displacement $x$ satisfies $v^2 = \omega^2(A^2 - x^2)$, where $A$ is amplitude and $\omega$ is angular frequency.
- The period is $T = 2\pi/\omega$. Subtracting the squared-speed equations at two positions eliminates the unknown amplitude.

**Step 1:** Write the speed equations at both positions For the same oscillator, amplitude and angular frequency are unchanged between the two observations. Thus

$$v_1^2 = \omega^2(A^2 - x_1^2), \qquad v_2^2 = \omega^2(A^2 - x_2^2).$$

It is the squared speed that enters the relation, so the formula is valid whether the instantaneous velocities are positive or negative. The distances given in the question enter as squares for the same reason.

**Step 2:** Eliminate amplitude and obtain the period Subtract the second equation from the first:

$$\begin{aligned} v_1^2 - v_2^2 &= \omega^2[(A^2 - x_1^2) - (A^2 - x_2^2)] \\ &= \omega^2(x_2^2 - x_1^2). \end{aligned}$$

Therefore

$$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}, \qquad T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}.$$

For distinct squared distances, numerator and denominator have the same sign, because speed decreases away from the mean position, so the ratio is positive. Its dimensions are time squared. Options (A) and (B) incorrectly contain a sum of squared distances; (C) incorrectly uses a sum of squared speeds. If the two squared distances coincide, the speeds also coincide and these two observations alone do not determine the period; the displayed result applies to the nondegenerate data implicit in the question.

**Step 3:** Conclusion The period is $T = 2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Subtract the two $v^2 = \omega^2(A^2 - x^2)$ equations to remove amplitude before solving for the period.

**40. An electron having de-Broglie wavelength $\lambda$ is incident on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is :** [4 marks]

(A) 0

(B) $\frac{2m^2c^2\lambda^2}{h^2}$

(C) $\frac{2mc\lambda^2}{h}$

(D) $\frac{hc}{mc}$

**Correct Answer:** (C) $\frac{2mc\lambda^2}{h}$

**Solution:**

**Concept:**

- The incident electron has de Broglie momentum $p = h/\lambda$, where $\lambda$ is its matter wavelength and $h$ is Planck's constant.
- In the nonrelativistic treatment, electron kinetic energy is $K = p^2/(2m)$. The maximum-energy X-ray photon receives this entire kinetic energy, so $hc/\lambda_{\min} = K$.

**Step 1:** Express electron kinetic energy using its wavelength The wavelength given belongs to the incident electron, not to the outgoing electromagnetic radiation. First use the de Broglie relation and then the kinetic-energy formula:

$$p = \frac{h}{\lambda}, \qquad K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}.$$

Here $m$ is electron mass. The inverse square dependence is essential: an electron with shorter matter wavelength carries greater kinetic energy and can generate a harder X-ray photon.

**Step 2:** Apply the maximum photon-energy condition The cut-off wavelength is the shortest wavelength in the continuous X-ray spectrum. It occurs when one electron loses all its kinetic energy in producing one photon. Therefore

$$\frac{hc}{\lambda_{\min}} = \frac{h^2}{2m\lambda^2} \quad \Longrightarrow \quad \lambda_{\min} = \frac{2mc\lambda^2}{h}.$$

Finite electron energy cannot produce zero wavelength, excluding (A). The expression in (B) is dimensionless, whereas the result must be a length. Option (D) does not depend on the incident wavelength and therefore cannot reflect the available kinetic energy. Option (C) has the correct dimensions and the required square dependence on electron wavelength. The use of $p^2/(2m)$ is the conventional nonrelativistic X-ray-tube approximation implicit in the supplied alternatives; it is not a formula for photon kinetic energy.

**Step 3:** Conclusion The cut-off wavelength is $\lambda_{\min} = \dfrac{2mc\lambda^2}{h}$. Hence, the correct option is (C).

**Quick Tip to Solve:** Keep the two wavelengths distinct: use the electron wavelength to find kinetic energy, then equate that energy to $hc/\lambda_{\min}$.

**41. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :** [4 marks]

(A) Solid sphere

(B) Solid cylinder

(C) Hollow cylinder

(D) Ring

**Correct Answer:** (B) Solid cylinder

**Solution:**

**Concept:**

- For rolling without slipping, $v = R\omega$, where $R$ is the rolling radius, $v$ is centre-of-mass speed, and $\omega$ is angular speed.
- Translational and rotational kinetic energies are $K_{\text{tr}} = \frac{1}{2}Mv^2$ and $K_{\text{rot}} = \frac{1}{2}I\omega^2$, with $I$ about the central rolling axis.

**Step 1:** Convert the energy ratio to a moment-of-inertia ratio The statement says rotational energy is half the translational energy; it does not say that each contributes half of the total. Their ratio is

$$\frac{K_{\text{rot}}}{K_{\text{tr}}} = \frac{I\omega^2}{Mv^2}.$$

Using the no-slip condition gives

$$\frac{K_{\text{rot}}}{K_{\text{tr}}} = \frac{I}{MR^2} = 0.50, \qquad I = \frac{1}{2}MR^2.$$

The instantaneous speed and incline angle cancel. Hence the identification depends only on how mass is distributed about the rolling axis, not on where the body is on the incline.

**Step 2:** Compare the listed bodies A uniform solid cylinder about its symmetry axis has $I = \frac{1}{2}MR^2$, exactly the required value. A uniform solid sphere has $I = \frac{2}{5}MR^2$, so its ratio is 0.40 rather than 0.50. A thin hollow cylinder and a ring both have $I = MR^2$, giving equal rotational and translational energies rather than the stated ratio. More generally a hollow cylinder of inner radius $r > 0$ has $I = \frac{1}{2}M(R^2 + r^2) > \frac{1}{2}MR^2$, so it cannot give the ratio of a solid cylinder. As a further check, the required body has total energy $K_{\text{tr}} + K_{\text{rot}} = 1.5K_{\text{tr}}$; the rotational fraction of the total is therefore one-third.

**Step 3:** Conclusion The required moment of inertia is $I = \frac{1}{2}MR^2$, which identifies a solid cylinder. Hence, the correct option is (B).

**Quick Tip to Solve:** For pure rolling, the rotational-to-translational energy ratio is simply $I/(MR^2)$; identify the shape from this coefficient.

**42. If time (t), velocity (v), and angular momentum (l) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v and l is :** [4 marks]

(A) $[t^{-1}v^{1}l^{-2}]$

(B) $[t^{1}v^{2}l^{-1}]$

(C) $[t^{-2}v^{-1}l^{1}]$

(D) $[t^{-1}v^{-2}l^{1}]$

**Correct Answer:** (D) $[t^{-1}v^{-2}l^{1}]$

**Solution:**

**Concept:**

- The usual dimensions are $[t] = T$, $[v] = LT^{-1}$, and $[l] = ML^2T^{-1}$, because angular momentum is position crossed with linear momentum.
- To express mass in a new dimensional basis, write $[m] = [t]^a[v]^b[l]^c$ and match the powers of $M$, $L$, and $T$ separately.

**Step 1:** Expand the proposed dimensional expression Let the unknown exponents of time, velocity, and angular momentum be $a, b, c$, respectively. Substitution of their usual dimensions gives

$$\begin{aligned} {}^a[v]^b[l]^c &= T^a(LT^{-1})^b(ML^2T^{-1})^c \\ &= M^cL^{b+2c}T^{a-b-c}. \end{aligned}$$

To represent mass alone, this expression must equal $M^1L^0T^0$. The symbol $l$ denotes angular momentum, not a length, so it supplies both a mass power and two length powers.

**Step 2:** Match powers and verify the result Equating the mass powers gives $c = 1$. The length powers then give $b + 2c = 0$, hence $b = -2$. Finally, the time powers give $a - b - c = 0$, so $a = -1$. Thus

$$[m] = [t^{-1}v^{-2}l].$$

Direct substitution checks the cancellation:

$$T^{-1}(L^{-2}T^2)(ML^2T^{-1}) = M.$$

Option (A) has angular-momentum exponent $-2$, and (B) has exponent $-1$, so neither can provide a single positive power of mass. Option (C) has the right mass power but leaves dimensions $MLT^{-2}$, those of force. Only (D) eliminates both the extra length and time powers. This procedure concerns dimensions; it does not assert a numerical dynamical equation for an arbitrary body.

**Step 3:** Conclusion The dimension of mass is $[t^{-1}v^{-2}l^{1}]$ in the proposed basis. Hence, the correct option is (D).

**Quick Tip to Solve:** Match the mass power first: only angular momentum contains mass here, forcing its exponent to be one.

**43. The correct relation between the degrees of freedom f and the ratio of specific heat $\gamma$ is :** [4 marks]

(A) $f = \frac{2}{\gamma-1}$
(B) $f = \frac{2}{\gamma+1}$
(C) $f = \frac{\gamma+1}{2}$
(D) $f = \frac{1}{\gamma+1}$

**Correct Answer:** (A) $f = \frac{2}{\gamma-1}$

**Solution:**

**Concept:**

- For an ideal gas with $f$ active quadratic degrees of freedom per molecule, equipartition gives molar internal energy $U_m = fRT/2$.
- The molar heat capacities obey Mayer's relation $C_P - C_V = R$, and their dimensionless ratio is $\gamma = C_P/C_V$.

**Step 1:** Express the heat capacities Differentiate the internal energy with respect to temperature at constant volume. With the active degrees of freedom unchanged over the temperature range,

$$C_V = \frac{fR}{2}, \qquad C_P = C_V + R = \frac{(f+2)R}{2}.$$

Both heat capacities are positive; the constant-pressure value is larger because the gas also does expansion work.

**Step 2:** Solve the ratio for the degrees of freedom Divide the expressions to eliminate the gas constant:

$$\gamma = \frac{f+2}{f} = 1 + \frac{2}{f}.$$

Therefore

$$f(\gamma - 1) = 2, \qquad f = \frac{2}{\gamma - 1}.$$

The denominator must involve the difference from one, not the sum. As a check, a monatomic gas has $\gamma = 5/3$, and the result gives $f = 3$; the other three expressions do not reproduce this known case.

**Step 3:** Conclusion The required relation is $\boxed{f = 2/(\gamma - 1)}$. Hence, the correct option is (A).

**Quick Tip to Solve:** Remember $\gamma = 1 + 2/f$; increasing the number of active degrees of freedom moves the ratio towards one.

**44. Consider a binary star system of star A and star B with masses $m_A$ and $m_B$ revolving in a circular orbit of radii $r_A$ an $r_B$, respectively. If $T_A$ and $T_B$ are the time period of star A and star B, respectively, Then :** [4 marks]

(A) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$

(B) $T_A = T_B$

(C) $T_A > T_B$ (if $m_A > m_B$)

(D) $T_A > T_B$ (if $r_A > r_B$)

**Correct Answer:** (B) $T_A = T_B$

**Solution:**

**Concept:**

- Two isolated stars in a circular binary orbit revolve about their common centre of mass while remaining on opposite sides of it.
- The centre-of-mass condition is $m_A r_A = m_B r_B$. The separation between the stars is $d = r_A + r_B$.

**Step 1:** Relate their angular speeds The gravitational force is along the line joining the stars and supplies the centripetal force for each. Writing their angular speeds as $\omega_A$ and $\omega_B$,

$$\frac{G m_A m_B}{d^2} = m_A \omega_A^2 r_A = m_B \omega_B^2 r_B.$$

Since $m_A r_A = m_B r_B$, cancellation shows $\omega_A^2 = \omega_B^2$. Their angular-speed magnitudes are thus equal.

**Step 2:** Compare the orbital periods Each star completes one revolution of the common line of centres in the same time, so

$$T_A = \frac{2\pi}{\omega_A} = \frac{2\pi}{\omega_B} = T_B.$$

A heavier star has a smaller orbital radius, but this changes its linear speed rather than its period. Applying $T \propto r^{3/2}$ separately to these two radii is invalid: that comparison assumes orbits about the same fixed central mass. Neither a larger mass nor a larger radius makes one binary component complete a revolution later than the other.

**Step 3:** Conclusion Both stars have the same orbital period: $\boxed{T_A = T_B}$. Hence, the correct option is (B).

**Quick Tip to Solve:** For a binary system, use the common angular speed first; unequal radii imply unequal linear speeds, not unequal periods.

**45. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to :** [4 marks]

(A) $t^{\frac{3}{2}}$

(B) $t^{\frac{1}{2}}$

(C) $t^{\frac{1}{4}}$

(D) $t^{\frac{3}{4}}$

**Correct Answer:** (A) $t^{\frac{3}{2}}$

**Solution:**

**Concept:**

- Constant mechanical power $P$ means a constant rate of increase of kinetic energy: $P = dK/dt$.
- For mass $m$ moving along the straight horizontal path, $K = mv^2/2$ and $v = dx/dt$. The body starts from rest.

**Step 1:** Find the speed as a function of time The work delivered by the machine during time $t$ is $Pt$. With zero initial kinetic energy, the work-energy relation gives

$$Pt = \frac{1}{2}mv^2, \qquad v(t) = \sqrt{\frac{2P}{m}}\, t^{1/2}.$$

Constant power therefore does not produce constant acceleration. As speed increases, the required driving force $P/v$ decreases.

**Step 2:** Integrate the speed to obtain distance Take the initial position as zero and integrate the positive speed:

$$x(t) = \int_0^t \sqrt{\frac{2P}{m}}\, \tau^{1/2}\, d\tau = \frac{2}{3}\sqrt{\frac{2P}{m}}\, t^{3/2}.$$

The factor involving mass and power is time independent, so the distance scales as $t^{3/2}$. The exponent $1/2$ describes speed rather than distance; the exponents $1/4$ and $3/4$ satisfy neither the energy equation nor its time integral.

**Step 3:** Conclusion The distance moved is proportional to $\boxed{t^{3/2}}$. Hence, the correct option is (A).

**Quick Tip to Solve:** At constant power, first use $K = Pt$ to get $v \propto \sqrt{t}$, then increase the time exponent by one when integrating for distance.

**46. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P}+\vec{Q}$ is n times the magnitude of $\vec{P}-\vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is :** [4 marks]

(A) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$

(B) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

(C) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

(D) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

**Correct Answer:** (D) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

**Solution:**

**Concept:**

- For vectors of equal nonzero magnitude $a$, separated by angle $\theta$, the dot product is $\vec{P}\cdot\vec{Q}=a^2\cos\theta$.
- Squaring a vector sum or difference gives its magnitude squared without introducing an unknown direction. The physical angle lies in $[0,\pi]$.

**Step 1:** Write the squared magnitudes Using the dot product,

$$|\vec{P}+\vec{Q}|^2=2a^2(1+\cos\theta), \qquad |\vec{P}-\vec{Q}|^2=2a^2(1-\cos\theta).$$

The given relation is between magnitudes, so it must be squared before these two expressions are compared. The multiplier on the right consequently becomes $n^2$, not $n$.

**Step 2:** Solve for the angle Substituting and cancelling $2a^2$ gives

$$1+\cos\theta=n^2(1-\cos\theta).$$

Collecting the cosine terms,

$$(n^2+1)\cos\theta=n^2-1, \qquad \theta=\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right).$$

Options involving unsquared $n$ omit the squaring step. The inverse sine expression is also incorrect: for $n=1$, the vectors have equal sum and difference magnitudes and must be perpendicular, whereas that expression gives zero.

**Step 3:** Conclusion The angle is $\boxed{\cos^{-1}((n^2-1)/(n^2+1))}$. Hence, the correct option is (D).

**Quick Tip to Solve:** For equal vectors, square the sum-to-difference ratio; testing $n=1$ quickly distinguishes inverse cosine from inverse sine.

**47. Two small drops of mercury each of radius R coalesce to form a single large drop. The ratio of total surface energy before and after the change is :** [4 marks]

(A) $2^{\frac{1}{3}} : 1$

(B) $1 : 2^{\frac{1}{3}}$

(C) 2 : 1

(D) 1 : 2

**Correct Answer:** (A) $2^{\frac{1}{3}} : 1$

**Solution:**

**Concept:**

- The surface energy of a liquid drop is $E = \sigma A$, where $\sigma$ is surface tension and $A$ is its free surface area.
- Mercury volume is conserved when two equal spherical drops combine. A liquid drop has one free surface, unlike a thin soap bubble.

**Step 1:** Determine the radius of the combined drop Let the radius after coalescence be $r$. Equating final volume to the sum of the two initial volumes,

$$\frac{4}{3}\pi r^3 = 2\left(\frac{4}{3}\pi R^3\right), \qquad r = 2^{1/3}R.$$

The radius does not double because volume varies as the cube of radius. The larger drop has twice the volume of either original drop.

**Step 2:** Compare the total surface energies The initial energy counts both separate drops, while the final energy counts only the large drop:

$$E_i = 2\sigma(4\pi R^2), \qquad E_f = \sigma(4\pi r^2) = 4\pi\sigma\, 2^{2/3}R^2.$$

Hence

$$\frac{E_i}{E_f} = \frac{2}{2^{2/3}} = 2^{1/3}.$$

The ratio is greater than one because coalescence reduces area and releases surface energy. Its reciprocal would correspond to asking after-to-before; a ratio of two ignores the increased final radius, and one-half has both the wrong magnitude and direction.

**Step 3:** Conclusion The ratio of total surface energy before and after is $\boxed{2^{1/3} : 1}$. Hence, the correct option is (A).

**Quick Tip to Solve:** For $N$ equal drops merging, volume gives radius $N^{1/3}R$, and initial-to-final surface energy is $N^{1/3} : 1$.

**48. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi \times 10^{-7}$ H/m. Absolute permeability of the material of the rod is :** [4 marks]

(A) $4\pi \times 10^{-4}$ H/m

(B) $2\pi \times 10^{-4}$ H/m

(C) $3\pi \times 10^{-4}$ H/m

(D) $\pi \times 10^{-4}$ H/m

**Correct Answer:** (B) $2\pi \times 10^{-4}$ H/m

**Solution:**

**Concept:**

- For a linear magnetic material in SI units, magnetisation is $M = \chi_m H$, where susceptibility $\chi_m$ is dimensionless.
- The magnetic induction satisfies $B = \mu_0(H + M) = \mu H$; therefore relative permeability is $\mu_r = 1 + \chi_m$.

**Step 1:** Convert susceptibility to relative permeability The given susceptibility is $\chi_m = 499$. The relative permeability includes the vacuum contribution as well as the magnetisation response:

$$\mu_r = 1 + 499 = 500.$$

Using 499 directly would neglect the vacuum term. Relative permeability is a dimensionless ratio and is not itself the absolute permeability requested.

**Step 2:** Calculate the absolute permeability Multiply the relative value by the permeability of vacuum supplied in the question:

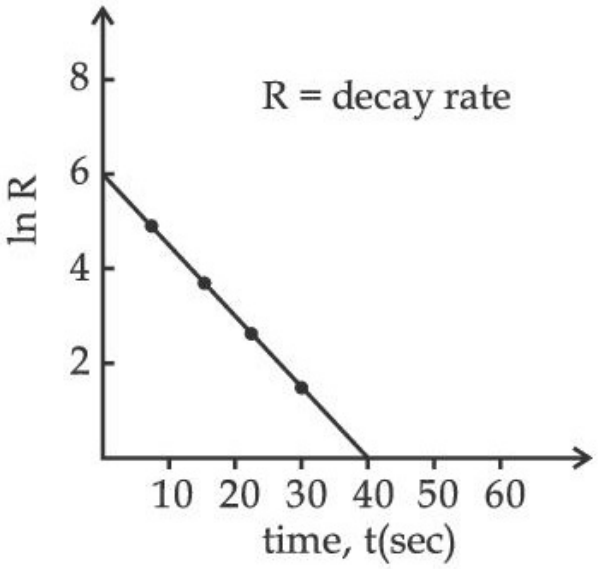
$$\begin{aligned} \mu &= \mu_0 \mu_r \\ &= (4\pi \times 10^{-7})(500)\ \mathrm{H\,m^{-1}} \\ &= 2000\pi \times 10^{-7}\ \mathrm{H\,m^{-1}} \\ &= 2\pi \times 10^{-4}\ \mathrm{H\,m^{-1}}. \end{aligned}$$

The numerical coefficient is two after the power of ten is simplified. The alternatives with coefficients four, three and one do not follow from multiplication by 500. The unit remains henry per metre, since only a dimensionless factor multiplies $\mu_0$.

**Step 3:** Conclusion The absolute permeability is $\boxed{2\pi \times 10^{-4}\ \mathrm{H\,m^{-1}}}$. Hence, the correct option is (B).

**Quick Tip to Solve:** Use $\mu = \mu_0(1 + \chi_m)$; keep the added one until the relative permeability is calculated, then simplify powers of ten.

**49. For a certain radioactive process the graph between In R and t(sec) is obtained as shown in the figure. Then the value of half life for the unknown radioactive material is approximately :** [4 marks]




(A) 9.15 sec

(B) 6.93 sec

(C) 2.62 sec

(D) 4.62 sec

**Correct Answer:** (D) 4.62 sec

**Solution:**

**Concept:**

- Radioactive decay rate follows $R(t) = R_0 e^{-\lambda t}$, with positive decay constant $\lambda$.
- Taking natural logarithms gives $\ln R = \ln R_0 - \lambda t$. Thus a straight-line plot of $\ln R$ against time has slope $-\lambda$, and $T_{1/2} = \ln 2/\lambda$.

**Step 1:** Read the slope of the plotted line The line passes through $(t, \ln R) = (0, 6)$ and $(40, 0)$. Use these well-separated intercepts to reduce reading error:

$$\text{slope} = \frac{0 - 6}{40 - 0} = -0.15 \text{ s}^{-1}.$$

The decay constant is the negative of this slope, so $\lambda = 0.15 \text{ s}^{-1}$. The time-axis intercept is not the half-life: $\ln R = 0$ means $R = 1$ in the graph's rate units, rather than half its initial value.

**Step 2:** Convert the decay constant to half-life At half-life the decay rate is half its original value, hence

$$e^{-\lambda T_{1/2}} = \frac{1}{2}, \qquad T_{1/2} = \frac{0.693}{0.15} = 4.62 \text{ s}.$$

The result depends on the slope, not on the starting rate. The alternatives 9.15, 6.93 and 2.62 seconds would require different decay constants and are inconsistent with the observed decrease of six logarithmic units in forty seconds.

**Step 3:** Conclusion The half-life is approximately $\boxed{4.62\ \text{s}}$. Hence, the correct option is (D).

**Quick Tip to Solve:** On a natural-log activity graph, take the magnitude of the slope as $\lambda$, then divide 0.693 by it; do not halve the plotted logarithm.

**50. At an angle of 30° to the magnetic meridian, the apparent dip is 45°. Find the true dip :** [4 marks]

(A) $\tan^{-1}\sqrt{3}$

(B) $\tan^{-1}\frac{1}{\sqrt{3}}$

(C) $\tan^{-1}\frac{2}{\sqrt{3}}$

(D) $\tan^{-1}\frac{\sqrt{3}}{2}$

**Correct Answer:** (D) $\tan^{-1}\frac{\sqrt{3}}{2}$

**Solution:**

**Concept:**

- The true dip $\delta$ is measured in the magnetic meridian, where the horizontal component of Earth's field is $H$. Its vertical component is $V$, so $\tan\delta = V/H$.
- In a vertical plane making angle $\theta$ with the magnetic meridian, the available horizontal component is $H\cos\theta$, while the vertical component remains $V$.

**Step 1:** Relate apparent and true dip Let the apparent dip in the rotated vertical plane be $\delta_a$. A dip needle aligns with the field projected into that plane. Hence its vertical-to-horizontal ratio is

$$\tan\delta_a = \frac{V}{H\cos\theta} = \frac{\tan\delta}{\cos\theta}.$$

Therefore $\tan\delta = \tan\delta_a\cos\theta$. The cosine multiplies the apparent tangent; dividing by it would reverse the projection relation.

**Step 2:** Insert the given angles Using $\delta_a = 45^\circ$ and $\theta = 30^\circ$,

$$\tan\delta = 1\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}.$$

Thus the true dip is smaller than the apparent dip, as expected because the full horizontal component exceeds its projection. Option (C) is the reciprocal produced by an incorrect projection; (A) and (B) would require different angle data. Only option (D) has the required tangent.

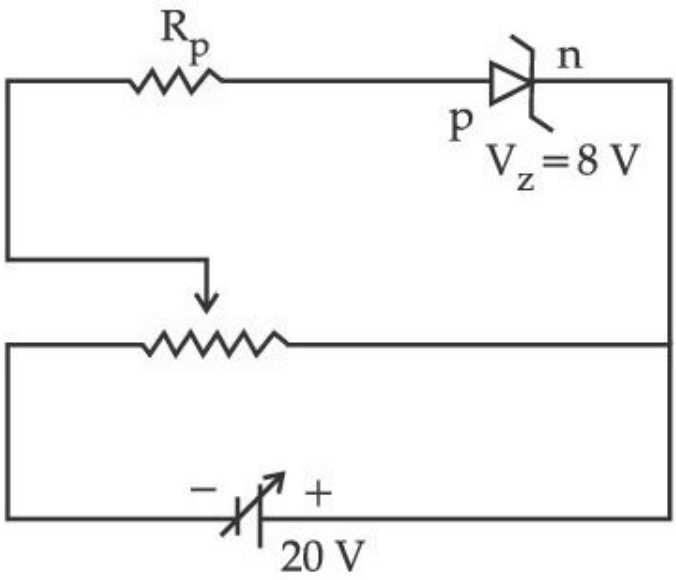
**Step 3:** Conclusion The true dip is $\boxed{\delta = \tan^{-1}(\sqrt{3}/2)}$. Hence, the correct option is (D).

**Quick Tip to Solve:** Remember that rotating away from the magnetic meridian reduces the horizontal component, so the apparent dip must increase.

## Physics – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**51. A zener diode having zener voltage 8 V and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance $R_p$ is ..... $\Omega$.** [4 marks]




**Solution:**

**Concept:**

- In reverse breakdown an ideal zener diode maintains its rated voltage $V_Z$. Its permissible current follows from the power limit $P_{\max} = V_Z I_{Z,\max}$.
- The protective resistor limits the current by dropping the difference between the largest applied voltage and the zener voltage: $R_p = (V_{\max} - V_Z)/I_{Z,\max}$.

**Step 1:** Calculate the maximum safe zener current The diode has $V_Z = 8\,\text{V}$ and maximum dissipation $0.5\,\text{W}$. Therefore

$$I_{Z,\max} = \frac{0.5}{8} = 0.0625\,\text{A} = 62.5\,\text{mA}.$$

This is a current limit, not the current obtained by dividing the supply voltage by the diode power. Exceeding it would exceed the specified heating rating even though the diode voltage remains near its breakdown value.

**Step 2:** Apply the largest divider output The supply across the potential divider is $20\,\text{V}$. At the slider position giving the maximum potential difference across the protective branch, the entire supply is available to that branch. The zener takes $8\,\text{V}$, leaving

$$V_{R_p} = 20 - 8 = 12\,\text{V}.$$

At the limiting safe current,

$$R_p = \frac{12}{0.0625} = 192\,\Omega.$$

A smaller protective resistance would permit more current at this worst-case setting. As a check, $8(12/192) = 0.5\,\text{W}$, exactly the rating.

**Step 3:** Conclusion The required protective resistance is $\boxed{R_p = 192\,\Omega}$. The numerical entry is 192.

**Quick Tip to Solve:** First convert the zener power rating into its maximum current; then use only the excess supply voltage across the series resistor.

**52. A body of mass 'm' is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is _____.** [4 marks]

**Solution:**

**Concept:**

- On an incline of angle $\theta$, the normal reaction is $mg\cos\theta$, and kinetic friction has magnitude $\mu mg\cos\theta$, opposing the direction of motion.
- For motion between rest and a fixed distance $s$ with constant acceleration magnitude $a$, $s = at^2/2$. Both journeys cover the same distance, but their accelerations differ.

**Step 1:** Write the accelerations in both directions During ascent, both gravity along the slope and friction act downhill, giving deceleration magnitude

$$a_u = g(\sin\theta + \mu\cos\theta).$$

During descent gravity acts downhill and friction uphill, so

$$a_d = g(\sin\theta - \mu\cos\theta).$$

The body reaches rest at the upper end and starts its return there. The same path length is therefore $s = a_u t_u^2/2 = a_d t_d^2/2$.

**Step 2:** Use the time ratio to find friction Given $t_u = t_d/2$, the distance relation gives

$$\frac{a_u}{a_d} = \frac{t_d^2}{t_u^2} = 4.$$

Consequently

$$\sin\theta + \mu\cos\theta = 4(\sin\theta - \mu\cos\theta),$$

and $5\mu\cos\theta = 3\sin\theta$. For $\theta = 30°$,

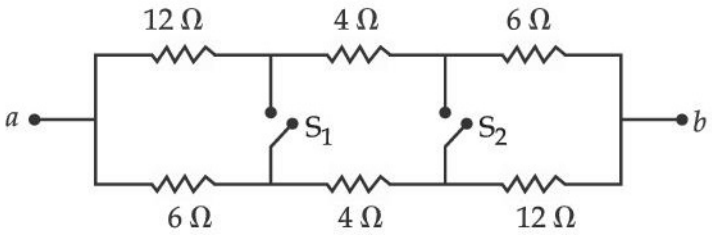
$$\mu = \frac{3}{5}\tan 30° = \frac{\sqrt{3}}{5}.$$

This is less than $\tan 30°$, so downhill motion is possible, consistent with the assumed descent. Comparing with the specified $\sqrt{x}/5$ gives $x = 3$.

**Step 3:** Conclusion The coefficient is $\mu = \sqrt{3}/5$, hence $\boxed{x = 3}$.

**Quick Tip to Solve:** For equal distances ending or starting at rest, acceleration ratios are the inverse squares of the travel-time ratios.

**53. In the given figure switches $S_1$ and $S_2$ are in open condition. The resistance across ab when the switches $S_1$ and $S_2$ are closed is _____ $\Omega$.** [4 marks]




**Solution:**

**Concept:**

- Closing an ideal switch joins its two contacts with negligible resistance, making them a single electrical node.
- Resistors sharing both end nodes are parallel: $R_{\parallel} = R_1 R_2/(R_1 + R_2)$. Equivalent groups traversed successively are in series and their resistances add.

**Step 1:** Identify the nodes after both switches close When $S_1$ closes, the junction after the upper $12\,\Omega$ resistor is joined to the junction after the lower $6\,\Omega$ resistor. Call this node $c$. Similarly, closing $S_2$ joins the two junctions before the final upper $6\,\Omega$ and lower $12\,\Omega$ resistors; call that node $d$. The circuit becomes three consecutive sections: from $a$ to $c$, from $c$ to $d$, and from $d$ to $b$. The switches do not bypass any of these entire sections.

**Step 2:** Reduce each parallel pair and add The first section contains $12\,\Omega$ in parallel with $6\,\Omega$:

$$R_{ac} = \frac{12 \times 6}{12 + 6} = 4\,\Omega.$$

The middle contains the two $4\,\Omega$ resistors in parallel, so $R_{cd} = 2\,\Omega$. The last is $6\,\Omega$ in parallel with $12\,\Omega$, giving $R_{db} = 4\,\Omega$. These three equivalent sections are in series because every terminal current passes through them successively. Hence

$$R_{ab} = 4 + 2 + 4 = 10\,\Omega.$$

**Step 3:** Conclusion The resistance with both switches closed is $\boxed{R_{ab} = 10\,\Omega}$. The numerical entry is 10.

**Quick Tip to Solve:** Redraw closed-switch contact points as single nodes before deciding which resistors are parallel.

**54. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is _____.**

[4 marks]

**Solution:**

**Concept:**

- For rolling without slipping, $v = \omega R$, and gravitational potential energy becomes both translational and rotational kinetic energy.
- A thin ring has moment of inertia $I = mR^2$; a solid cylinder about its symmetry axis has $I = mR^2/2$. Static friction does no work for ideal rolling on a fixed incline.

**Step 1:** Express the speed using the moment of inertia Let both bodies descend through the same vertical height $h$ from rest. Conservation of mechanical energy gives

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2}.$$

On writing $I = kmR^2$, this becomes

$$v^2 = \frac{2gh}{1+k}.$$

Mass cancels separately for each body, so equal material or equal masses are not needed for the speed comparison. The differing distribution of mass determines how much energy is tied up in rotation.

**Step 2:** Compare the ring and cylinder For the ring, $k = 1$, so $v_r^2 = gh$. For the solid cylinder, $k = 1/2$, giving $v_c^2 = 4gh/3$. Therefore

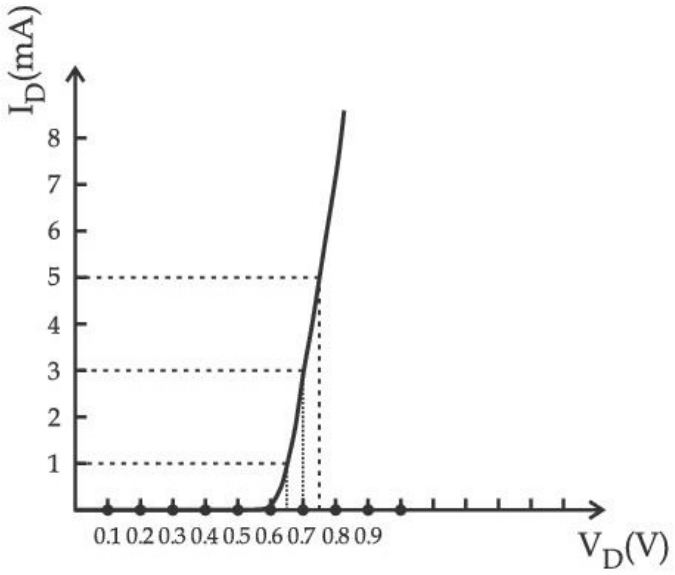
$$\frac{v_r}{v_c} = \sqrt{\frac{gh}{4gh/3}} = \frac{\sqrt{3}}{2}.$$

This is below unity: the ring has the larger rotational inertia per unit $mR^2$, so it acquires a smaller translational speed. Comparing this ratio with the printed $\sqrt{x}/2$ yields $x = 3$. The shared radius also cancels through the rolling constraint.

**Step 3:** Conclusion The speed ratio is $v_r/v_c = \sqrt{3}/2$, and therefore $\boxed{x = 3}$.

**Quick Tip to Solve:** Use $v^2 = 2gh/(1 + I/(mR^2))$; a larger inertia factor means a smaller final centre-of-mass speed.

**55. For the forward biased diode characteristics shown in the figure, the dynamic resistance at $I_D$ = 3 mA will be _____ Ω.** [4 marks]




**Solution:**

**Concept:**

- The dynamic or small-signal resistance is the local differential ratio $r_d = dV_D/dI_D$, evaluated near the stated operating current.
- When current is plotted vertically against voltage horizontally, the plotted slope is conductance $dI_D/dV_D$. The dynamic resistance is its reciprocal, not $V_D/I_D$.

**Step 1:** Read two nearby points on the characteristic The forward characteristic near $I_D = 3\,\text{mA}$ is represented by the straight steep segment in the graph. The indicated nearby points are approximately $(0.65\,\text{V}, 1\,\text{mA})$ and $(0.75\,\text{V}, 5\,\text{mA})$. The operating point at $3\,\text{mA}$ lies between them near $0.70\,\text{V}$. Use the difference between the endpoint readings to obtain the local slope; using the origin would instead describe a static resistance.

**Step 2:** Convert the current interval and evaluate the ratio The voltage increment is $0.75 - 0.65 = 0.10\,\text{V}$, and the current increment is

$$\Delta I_D = (5 - 1)\,\text{mA} = 4 \times 10^{-3}\,\text{A}.$$

Thus

$$r_d \simeq \frac{\Delta V_D}{\Delta I_D} = \frac{0.10}{4 \times 10^{-3}} = 25\,\Omega.$$

Equivalently, the incremental conductance is $0.04\,\text{A/V}$, whose reciprocal is $25\,\Omega$. The static ratio $0.70/0.003$ is much larger and is not the dynamic resistance asked for here.

**Step 3:** Conclusion The dynamic resistance at $I_D = 3\,\text{mA}$ is $\boxed{25\,\Omega}$. The numerical entry is 25.

**Quick Tip to Solve:** On an I-versus-V diode graph, take a small voltage difference divided by the corresponding current difference and convert mA to A.

**56. A series LCR circuit of R = 5Ω, L = 20 mH and C = 0.5 $\mu$F is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is _____ × $10^2$ W.** [4 marks]

**Solution:**

**Concept:**

- For a series LCR circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$ and $X_C = 1/(\omega C)$.
- At resonance $X_L = X_C$, so the impedance is purely resistive, $Z = R$, and the power factor is one. Average power is then $P = V_{\text{rms}}^2/R$.

**Step 1:** Use the resonance condition The frequency of the AC supply is variable, so it can be tuned until the inductive and capacitive reactances cancel. The stated inductance $L = 20\,\text{mH}$ and capacitance $C = 0.5\,\mu\text{F}$ determine that resonant frequency through $\omega_0 = 1/\sqrt{LC}$; they do not contribute a net reactance at resonance. Thus the circuit presents only $R = 5\,\Omega$ to the source. The quoted AC voltage is its rms value, so no extra factor of two is needed.

**Step 2:** Calculate power and express it in the requested form The resonant rms current is

$$I_{\text{rms}} = \frac{250}{5} = 50\,\text{A}.$$

Only the resistor dissipates average power; the ideal inductor and capacitor exchange stored energy with the source. Therefore

$$P = I_{\text{rms}}^2 R = 50^2 \times 5 = 12500\,\text{W}.$$

The same result follows from $250^2/5$. Since the blank multiplies $10^2$ W, write

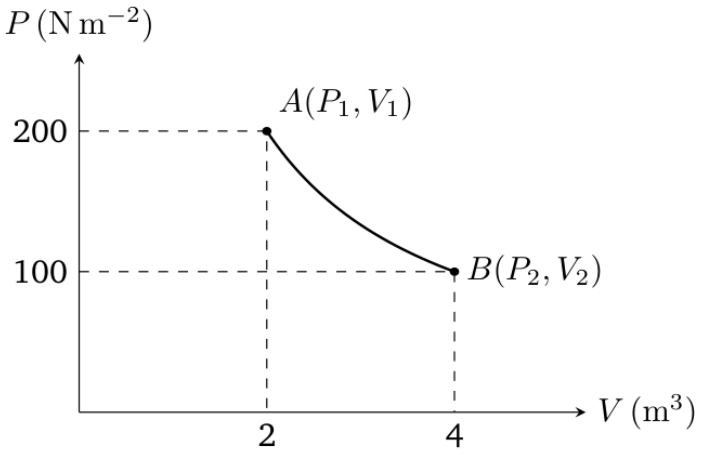
$$12500\,\text{W} = 125 \times 10^2\,\text{W}.$$

It is the coefficient $125$, rather than the full watt value $12500$, that must be entered.

**Step 3:** Conclusion The resonant power is $12500\,\text{W} = \boxed{125 \times 10^2\,\text{W}}$. The numerical entry is $125$.

**Quick Tip to Solve:** At series resonance ignore the cancelling reactances when finding power, and check the power-of-ten factor printed after the blank.

**57. One mole of an ideal gas at 27° is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be _____ $\times 10^{-1}$ J. [Given : R = 8.3 J/mole K, ln2 = 0.6931] (Round off to the nearest integer)** [4 marks]




**Solution:**

**Concept:**

- For a reversible isothermal expansion of an ideal gas, the work done by the gas is $W = nRT\ln(V_B/V_A)$. Here $n$ is the amount of gas, $R$ the gas constant and $T$ the absolute temperature.
- The falling hyperbolic path represents an isothermal expansion. The volume doubles from A to B, so its ratio, rather than the difference of the two volumes, enters the logarithm.

**Step 1:** Identify the temperature and volume ratio The gas initially has temperature $27^\circ\mathrm{C}$. Convert it to the absolute scale before using the ideal-gas work formula:

$$T = 27 + 273 = 300\,\mathrm{K}.$$

The endpoints marked on the horizontal axis give $V_A = 2$ and $V_B = 4$, hence $V_B/V_A = 2$. As the gas expands, $dV > 0$, so the work done by the system is positive. The path is curved, so one must not use a rectangular area or a straight-line trapezium.

**Step 2:** Evaluate the logarithmic work Along an isotherm, $P = nRT/V$; therefore integrating the pressure over the volume change gives

$$W = \int_{V_A}^{V_B} P\,dV = nRT\ln\frac{V_B}{V_A}.$$

Using the stated amount, temperature and constants,

$$W = 1 \times 8.3 \times 300 \times 0.6931 = 1725.819\,\mathrm{J}.$$

The question expresses this energy as a coefficient of $10^{-1}$ J, so the coefficient is $1725.819/0.1 = 17258.19$. Round this coefficient, not the energy in joules, to the nearest integer.

**Step 3:** Conclusion The work done by the gas is $1725.819\,\mathrm{J}$, or $17258.19 \times 10^{-1}\,\mathrm{J}$. The required integer is $\boxed{17258}$.

**Quick Tip to Solve:** For isothermal expansion, first extract the volume ratio from the graph; convert the final energy into the requested power-of-ten unit before rounding.

**58. A certain metallic surface is illuminated by monochromatic radiation of wavelength $\lambda$. The stopping potential for photoelectric current for this radiation is $3V_0$. If the same surface is illuminated with a radiation of wavelength $2\lambda$, the stopping potential is $V_0$. The threshold wavelength of this surface for photoelectric effect is _____ $\lambda$.** [4 marks]

**Solution:**

**Concept:**

- Einstein's photoelectric equation is $hc/\lambda = \phi + eV_s$, where $\phi$ is the work function and $V_s$ the stopping potential.
- The work function belongs to the metallic surface and is unchanged when the wavelength changes. At threshold, emitted electrons have zero maximum kinetic energy and $\phi = hc/\lambda_0$.

**Step 1:** Write the equations for both illuminations Let $E = hc/\lambda$ denote the energy of a photon in the first beam. The measured stopping potential is $3V_0$, so

$$E = \phi + 3eV_0.$$

Doubling the wavelength halves the photon energy. The second stopping potential is $V_0$, giving

$$\frac{E}{2} = \phi + eV_0.$$

The stopping potential is a measure of maximum electron kinetic energy; it is not proportional to wavelength. The same work function must be retained in both equations.

**Step 2:** Eliminate the stopping potential and obtain the threshold Subtract the second equation from the first:

$$\frac{E}{2} = 2eV_0, \qquad eV_0 = \frac{E}{4}.$$

Substitute this result in the second equation:

$$\phi = \frac{E}{2} - \frac{E}{4} = \frac{E}{4} = \frac{hc}{4\lambda}.$$

At threshold, all the photon energy is used to overcome the work function, with no surplus kinetic energy. Hence

$$\lambda_0 = \frac{hc}{\phi} = 4\lambda.$$

This is longer than either incident wavelength, consistent with both beams producing photoelectrons. No numerical value of $h$, $c$, $e$, or $V_0$ is needed because the requested result is a ratio.

**Step 3:** Conclusion The threshold wavelength is $\boxed{4\lambda}$. Thus the number to fill in the blank is $\boxed{4}$.

**Quick Tip to Solve:** When the same metal is illuminated twice, subtract its two Einstein equations to eliminate the work function before finding the threshold wavelength.

**59. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is _____.** [4 marks]

**Solution:**

**Concept:**

- With constant angular acceleration, angular displacement equals the average angular speed multiplied by the elapsed time: $\theta = (\omega_i + \omega_f)t/2$.
- The same average-speed rule can be used directly in revolutions per second. One revolution corresponds to $2\pi$ radians; dividing both angular speeds by this factor gives the rotation frequency.

**Step 1:** Convert both speeds to revolutions per second The initial and final speeds are quoted per minute whereas the duration is quoted in seconds. Put them in compatible units:

$$f_i = \frac{600}{60} = 10\,\text{rev}\,\text{s}^{-1}, \qquad f_f = \frac{1800}{60} = 30\,\text{rev}\,\text{s}^{-1}.$$

The acceleration is uniform, so the frequency increases linearly with time, rather than jumping immediately to its final value. Its mean over this interval is therefore the arithmetic mean of its two endpoint values.

**Step 2:** Use the average rotation frequency The average frequency is

$$f_{\text{av}} = \frac{10 + 30}{2} = 20\,\text{rev}\,\text{s}^{-1}.$$

Consequently the number of rotations during $10\,\text{s}$ is

$$N = f_{\text{av}} t = 20 \times 10 = 200.$$

As an independent check, the angular speeds are $20\pi$ and $60\pi\,\text{rad}\,\text{s}^{-1}$, so $\alpha = 4\pi\,\text{rad}\,\text{s}^{-2}$. Then $\theta = 20\pi(10) + \frac{1}{2}(4\pi)(10)^2 = 400\pi\,\text{rad}$, which is $400\pi/(2\pi) = 200$ complete rotations. Using the final speed for the whole time would overcount; using the initial speed would undercount.

**Step 3:** Conclusion The body makes $\boxed{200}$ rotations during the acceleration interval.

**Quick Tip to Solve:** For a uniformly accelerated rotor, average the initial and final rpm, then multiply by the time expressed in minutes to get revolutions directly.

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**60. A radioactive substance decays to $\left(\frac{1}{16}\right)^{th}$ of its initial activity in 80 days. The half life of the radioactive substance expressed in days is _____.** [4 marks]

**Solution:**

**Concept:**

- For a radioactive isotope, activity is proportional to the number of undecayed nuclei: $A = \lambda N$. Its decay law is $A/A_0 = e^{-\lambda t}$.
- The half-life $T_{1/2}$ is the interval in which activity halves. After $n$ half-lives, the remaining activity fraction is $(1/2)^n$, where $n = t/T_{1/2}$.

**Step 1:** Express the surviving activity as a power of one half The stated fraction is the activity that remains, not the activity that has been lost. Thus

$$\frac{A}{A_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^4.$$

The activity therefore passes through four successive halvings: $A_0$ to $A_0/2$, then to $A_0/4$, then to $A_0/8$, and finally to $A_0/16$. Each interval has the same duration, because radioactive decay has a constant decay probability per nucleus.

**Step 2:** Divide the total elapsed time by the number of half-lives Four half-lives occupy the given duration of $80$ days, giving

$$4T_{1/2} = 80\,\text{days}, \qquad T_{1/2} = 20\,\text{days}.$$

One can also obtain this result from the exponential law: $e^{-80\lambda} = 1/16$, so $80\lambda = \ln 16 = 4\ln 2$. Therefore $T_{1/2} = \ln 2/\lambda = 80/4$ days. It would be incorrect to divide the time by 16: the surviving fraction decreases geometrically, not linearly. The initial activity cancels because only its ratio to the final activity is needed.

**Step 3:** Conclusion The radioactive substance has half-life $\boxed{20\,\text{days}}$. The required numerical entry is $\boxed{20}$.

**Quick Tip to Solve:** Write the remaining activity as a power of $1/2$; the exponent counts the half-lives that have elapsed.

## Chemistry – Section A: Multiple Choice Questions (80 marks)

**61. Which one of the following pairs of isomers is an example of metamerism?** [4 marks]

(A) $CH_3CH_2CH_2CH_2CH_3$ and $C(CH_3)_4$

(B) $CH_3COC_6H_5$ and $C_6H_5CH_2CH_2CHO$

(C) $C_6H_5CH_2CH_2OH$ and $C_6H_5CH(OH)CH_3$

(D) $CH_3CH_2CH_2CH_2COCH_2CH_3$ and $CH_3CH_2CH_2COCH_2CH_2CH_3$

**Correct Answer:** (D) $CH_3CH_2CH_2CH_2COCH_2CH_3$ and $CH_3CH_2CH_2COCH_2CH_2CH_3$

**Solution:**

**Concept:**

- Metamers have the same molecular formula and the same functional group, but different distributions of carbon atoms on its two sides. Ketones, ethers and secondary amines commonly show this structural isomerism.
- For a ketone written as $RCOR'$, changing the two alkyl groups while retaining their total carbon count can produce metamerism. Merely branching a hydrocarbon or moving an OH group is a different classification.

**Step 1:** Compare the two ketones in option D The first displayed ketone is $CH_3CH_2CH_2CH_2COCH_2CH_3$, heptan-3-one: the carbonyl group has a butyl group on one side and an ethyl group on the other.

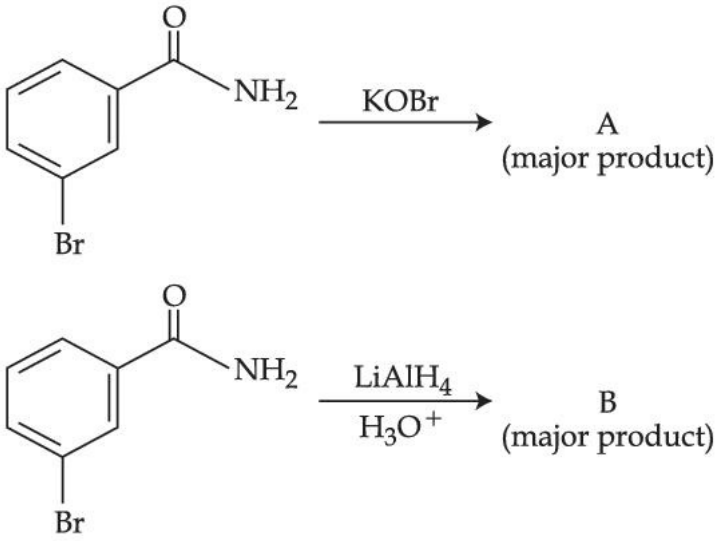
The second is $CH_3CH_2CH_2COCH_2CH_2CH_3$, heptan-4-one, with a propyl group on each side. Both have molecular formula $C_7H_{14}O$ and the same ketone functional group. Their side-group carbon distributions are $4+2$ and $3+3$, so this pair satisfies the definition of metamers.

**Step 2:** Exclude the remaining displayed pairs Option A compares straight-chain pentane with a branched pentane structure; the change concerns the carbon skeleton, so it is chain isomerism, with no divalent functional group separating two alkyl groups. Option C compares the two phenylethanol structures, in which the location of the hydroxyl group relative to the phenyl group changes; it does not redistribute carbon groups across a ketone or ether linkage. Option B contains an aromatic ketone and an aldehyde, so their functional groups differ and they cannot be a metameric pair. Only option D retains both the molecular formula and the carbonyl functional group while changing the groups on its sides.

**Step 3:** Conclusion Heptan-3-one and heptan-4-one are metamers. Hence, the correct option is (D).

**Quick Tip to Solve:** For a proposed metameric ketone pair, count the carbons on each side of the carbonyl carbon: the total stays fixed while the partition changes.

**62. In the above reactions, product A and product B respectively are :** [4 marks]




(A) Aniline and 3-bromobenzylamine

(B) 3-Bromoaniline and benzylamine

(C) Aniline and benzylamine

(D) 3-Bromoaniline and 3-bromobenzylamine

**Correct Answer:** (D) 3-Bromoaniline and 3-bromobenzylamine

**Solution:**

**Concept:**

- Hofmann bromamide degradation converts a primary amide $RCONH_2$ into $RNH_2$; the carbonyl carbon is lost. Hypobromite, here $KOBr$, is the rearrangement reagent.

- Lithium aluminium hydride reduces a primary amide to $RCH_2NH_2$. Its carbonyl carbon remains in the product as a methylene carbon. The aryl bromine substituent is retained in these transformations.

**Step 1:** Identify product A from the hypobromite reaction The starting compound is 3-bromobenzamide. Take R to be the 3-bromophenyl group. The Hofmann rearrangement follows

$$RCONH_2 \xrightarrow{KOBr} RNH_2.$$

Loss of the amide carbonyl carbon places the amino group directly on the aromatic ring at the position formerly attached to the carbonyl group. The bromine remains meta to it. Product A is therefore 3-bromoaniline, not aniline: the reaction removes the carbonyl carbon, not the bromine on the ring.

**Step 2:** Identify product B and compare the pairs With $LiAlH_4$ followed by acidic work-up, the amide carbonyl is reduced instead of being lost:

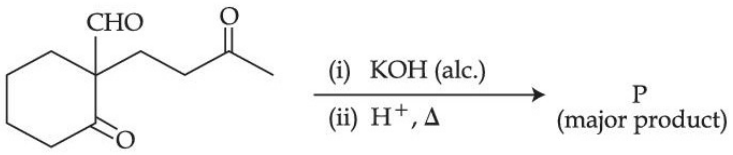
$$RCONH_2 \xrightarrow[H_3O^+]{LiAlH_4} RCH_2NH_2.$$

For the same 3-bromophenyl group, product B is 3-bromobenzylamine, $3\text{-}BrC_6H_4CH_2NH_2$. Its amino nitrogen is attached to a methylene group, so this is a benzylamine rather than an aniline. Option A omits bromine from A; option B omits bromine from B; option C omits it from both. Option D alone retains the aryl bromine in both products while distinguishing carbon loss in the first reaction from carbon retention in the second.

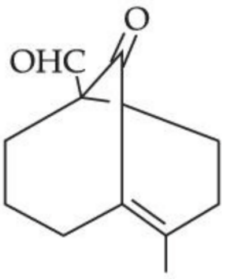
**Step 3:** Conclusion Products A and B are 3-bromoaniline and 3-bromobenzylamine, respectively. Hence, the correct option is (D).

**Quick Tip to Solve:** In an amide conversion, check the carbon count: Hofmann degradation loses the carbonyl carbon, whereas $LiAlH_4$ preserves it as $CH_2$.

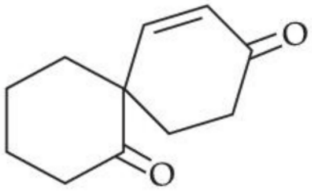
**63.** **The major product (P) in the following reaction is :** [4 marks]



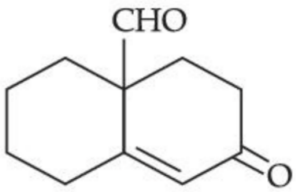

(A)



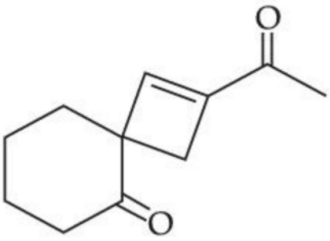

(B)



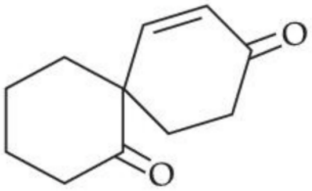

(C)




(D)




**Correct Answer:** (B)



**Solution:**

**Concept:**

- Alcoholic KOH produces an enolate at an alpha carbon of a carbonyl compound. An intramolecular aldol addition is favoured when the enolate can attack an aldehyde to form a five- or six-membered ring.
- Acidic heating dehydrates the beta-hydroxy carbonyl addition product to an alpha,beta-unsaturated carbonyl compound. An aldehyde is generally more susceptible to nucleophilic addition than a ketone.

**Step 1:** Choose the enolate that closes a six-membered ring The substrate contains an aldehyde attached to the ring junction and a side chain terminating in a methyl ketone. Deprotonation at the terminal methyl group of this ketone gives an enolate. Its carbon attacks the aldehyde carbon intramolecularly. Counting along the chain from that alpha carbon through the side-chain carbonyl and the two methylene groups to the junction carbon, then to the aldehyde carbon, gives a six-membered ring. This new ring shares only the original junction carbon with the cyclohexanone ring: the skeleton is spiro, not fused.

**Step 2:** Dehydrate the aldol product and select its structure The aldehyde oxygen becomes the hydroxyl group in the aldol addition intermediate. Acid and heat remove water from that group and the adjacent alpha carbon, introducing a double bond conjugated with the side-chain ketone. The original cyclohexanone carbonyl remains unchanged. Thus the final structure consists of the original cyclohexanone ring joined at a spiro carbon to a six-membered unsaturated ketone ring, exactly as shown in option B. Option A has a bridged skeleton and retains the aldehyde; option C is fused and also retains the aldehyde, corresponding to attack on the less reactive ring ketone instead. Option D would require an unfavourable four-membered ring. These do not match the preferred six-membered closure onto the aldehyde.

**Step 3:** Conclusion The major product is the spirocyclic alpha,beta-unsaturated ketone shown in option (B). Hence, the correct option is (B).

**Quick Tip to Solve:** For intramolecular aldol problems, count the proposed new ring atoms before drawing the product; favour a six-membered closure onto an aldehyde over a strained small ring.

**64. A solution is 0.1 M in $Cl^-$ and 0.001 M in $CrO_4^{2-}$. Solid $AgNO_3$ is gradually added to it. Assuming that the addition does not change in volume and $K_{sp}(AgCl) = 1.7 \times 10^{-10}$ $M^2$ and $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12}$ $M^3$.**

**Select correct statement from the following :** [4 marks]

(A) AgCl precipitates first because its $K_{sp}$ is high.

(B) $Ag_2CrO_4$ precipitates first as its $K_{sp}$ is low.

(C) $Ag_2CrO_4$ precipitates first because the amount of $Ag^+$ needed is low.

(D) AgCl will precipitate first as the amount of $Ag^+$ needed to precipitate is low.

**Correct Answer:** (D) AgCl will precipitate first as the amount of $Ag^+$ needed to precipitate is low.

**Solution:**

**Concept:**

- A salt starts to precipitate when its ionic product reaches its solubility product, $K_{sp}$. The relevant comparison is the threshold concentration of the common ion, not the numerical values of the two solubility products alone.
- For $AgCl$, $K_{sp} = [Ag^+][Cl^-]$; for $Ag_2CrO_4$, $K_{sp} = [Ag^+]^2[{CrO_4}^{2-}]$.

**Step 1:** Calculate the chloride threshold Before any solid forms, the given chloride concentration is $0.1\,\text{mol}\,\text{L}^{-1}$. The silver concentration at the onset of silver chloride precipitation is therefore

$$[Ag^+]_{AgCl} = \frac{1.7 \times 10^{-10}}{0.1} = 1.7 \times 10^{-9}\,\text{mol}\,\text{L}^{-1}.$$

Only a very small amount of dissolved silver is required to make this ionic product reach saturation.

**Step 2:** Calculate the chromate threshold For silver chromate, account for the square of the silver-ion concentration in the equilibrium expression:

$$[Ag^+]_{Ag_2CrO_4} = \sqrt{\frac{1.9 \times 10^{-12}}{0.001}} = \sqrt{1.9 \times 10^{-9}} = 4.36 \times 10^{-5}\,\text{mol}\,\text{L}^{-1}.$$

This concentration is much larger than the chloride threshold, so silver chloride reaches saturation first as silver nitrate is added.

**Step 3:** Assess the proposed reasons Option (A) names the first precipitate but gives an invalid reason: a larger solubility product alone does not determine the order for salts with different stoichiometries. Options (B) and (C) incorrectly place silver chromate first. Option (D) correctly compares the required silver-ion concentrations.

**Step 4:** Conclusion $AgCl$ precipitates first because it needs the lower concentration of $Ag^+$. Hence, the correct option is (D).

**Quick Tip to Solve:** Calculate the critical common-ion concentration for each salt; do not directly compare $K_{sp}$ values for different salt stoichiometries.

**65. Outermost electronic configuration of a group 13 element, E, is $4s^2$, $4p^1$. The electronic configuration of an element of p-block period-five placed diagonally to element, E is :**

[4 marks]

(A) [Kr] $3d^{10}$ $4s^2$ $4p^2$

(B) [Ar] $3d^{10}$ $4s^2$ $4p^2$

(C) [Xe] $5d^{10}$ $6s^2$ $6p^2$

(D) [Kr] $4d^{10}$ $5s^2$ $5p^2$

**Correct Answer:** (D) [Kr] $4d^{10}$ $5s^2$ $5p^2$

**Solution:**

**Concept:**

- For a main-group element, the largest principal quantum number in its outer electronic configuration identifies its period. A valence configuration $ns^2np^1$ places an element in group 13.
- A diagonal move to the next period and next p-block group increases both the period number and the group number by one. The group-14 valence pattern is $ns^2np^2$.

**Step 1:** Locate the given element The configuration $4s^24p^1$ has highest principal quantum number $n = 4$, so E is in period four. It has three valence electrons and belongs to group 13, as stated. This is gallium, whose full abbreviated configuration is $[\mathrm{Ar}]\,3d^{10}4s^24p^1$. The filled inner d shell does not change its p-block group.

**Step 2:** Move diagonally into period five The p-block element in period five diagonally below and to the right of gallium is tin in group 14. Its valence shell must therefore be $5s^25p^2$. Starting with the krypton core, the filled $4d$ subshell is also included:

$$\mathrm{Sn}: \quad [\mathrm{Kr}]\,4d^{10}5s^25p^2.$$

The electron count is $36 + 10 + 2 + 2 = 50$, which independently identifies tin.

**Step 3:** Reject the other configurations Option (B) describes germanium in period four, horizontally beside gallium rather than diagonally in period five. Option (C) uses a sixth-period valence shell, so it cannot meet the period-five condition. Option (A) incorrectly appends already filled lower subshells to the krypton core and retains a fourth-period valence pattern. Only option (D) has the required core and fifth-period valence shell.

**Step 4:** Conclusion The required configuration is $[\mathrm{Kr}]\,4d^{10}5s^25p^2$, corresponding to tin. Hence, the correct option is (D).

**Quick Tip to Solve:** Locate the original element by its highest $n$, then move one period down and one p-block group right before writing the new configuration.

**66. Metallic sodium does not react normally with :** [4 marks]

(A) gaseous ammonia

(B) But-2-yne

(C) Ethyne

(D) tert-butyl alcohol

**Correct Answer:** (B) But-2-yne

**Solution:**

**Concept:**

- Metallic sodium can replace sufficiently acidic hydrogen and liberate hydrogen gas. In alcohols the replaceable hydrogen is bonded to oxygen; in a terminal alkyne it is bonded directly to an sp-hybridised carbon.
- An internal alkyne has no terminal $C{\equiv}C{-}H$ group. Having a carbon-carbon triple bond is therefore insufficient, by itself, for the usual sodium acetylide reaction.

**Step 1:** Inspect the two alkynes Ethyne has the structure $H{-}C{\equiv}C{-}H$, so it contains terminal acetylenic hydrogens. Sodium can replace these hydrogens to give an acetylide with liberation of hydrogen. But-2-yne instead has the structure $CH_3{-}C{\equiv}C{-}CH_3$. Both carbons of its triple bond are attached to carbon groups, and neither bears an acetylenic hydrogen. The methyl hydrogens are not equivalent to terminal sp-carbon hydrogens and are not displaced in this normal test.

**Step 2:** Check ammonia and the alcohol With ammonia, sodium can form sodium amide, with hydrogen evolution under the usual conditions for this reaction:

$$2\,Na + 2\,NH_3 \longrightarrow 2\,NaNH_2 + H_2.$$

Tert-butyl alcohol retains an O–H bond despite the tertiary carbon centre. It reacts to give sodium tert-butoxide:

$$2\,Na + 2\,(CH_3)_3COH \longrightarrow 2\,(CH_3)_3CONa + H_2.$$

Thus steric crowding around the carbon does not remove the alcohol hydrogen that sodium replaces.

**Step 3:** Identify the nonreacting choice Options (A), (C) and (D) possess replaceable hydrogen in the relevant functional group and can undergo the stated sodium reaction. Option (B), the internal alkyne, lacks the required terminal hydrogen. The distinction is the position of the triple bond and the hydrogen attached to it, rather than simply whether the molecule is unsaturated.

**Step 4:** Conclusion Metallic sodium does not normally react with but-2-yne in this acidic-hydrogen substitution reaction. Hence, the correct option is (B).

**Quick Tip to Solve:** For the sodium test on an alkyne, look specifically for $-C{\equiv}C{-}H$; hydrogens on neighbouring alkyl groups do not count.

**67. Spin only magnetic moment of an octahedral complex of $Fe^{2+}$ in the presence of a strong field ligand in BM is :** [4 marks]

(A) 4.89

(B) 2.82

(C) 0

(D) 3.46

**Correct Answer:** (C) 0

**Solution:**

**Concept:**

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where $n$ is the number of unpaired electrons. Completely paired electrons contribute zero spin-only moment.
- An octahedral ligand field splits the five d orbitals into the lower $t_{2g}$ set and upper $e_g$ set. With a strong-field ligand, the splitting exceeds the pairing cost and a low-spin arrangement is favoured.

**Step 1:** Count the d electrons of iron(II) Neutral iron has configuration $[\text{Ar}]\,3d^6 4s^2$. Formation of $Fe^{2+}$ removes the two $4s$ electrons first, leaving $[\text{Ar}]\,3d^6$. The d-electron count is therefore six; it is not four, and the oxidation state must be applied before deciding how electrons occupy the split levels.

**Step 2:** Fill the strong-field levels In the octahedral strong field, electrons pair within the three lower-energy $t_{2g}$ orbitals instead of entering the higher-energy $e_g$ orbitals. The resulting low-spin configuration is

$$(t_{2g})^6(e_g)^0.$$

Each of the three lower orbitals contains a pair of electrons, and the upper two orbitals are empty. Consequently there are no unpaired electrons: $n = 0$.

**Step 3:** Evaluate the magnetic moment Substitute the unpaired-electron count into the spin-only expression:

$$\mu = \sqrt{0(0+2)} = 0\,\text{BM}.$$

The complex is diamagnetic in this description. The value near $4.89\,\text{BM}$ would belong to four unpaired electrons in a high-spin d-six arrangement, which contradicts the strong-field condition. The other nonzero choices similarly require unpaired electrons absent from the low-spin configuration.

**Step 4:** Conclusion The spin-only magnetic moment is $0\,\text{BM}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For octahedral $d^6$, decide strong field versus weak field before counting unpaired electrons: strong field gives $t_{2g}^6$.

**68. Which one of the following species doesn't have a magnetic moment of 1.73 BM, (spin only value)?** [4 marks]

(A) $O_2^+$

(B) CuI

(C) $[Cu(NH_3)_4]Cl_2$

(D) $O_2^-$

**Correct Answer:** (B) CuI

**Solution:**

**Concept:**

- A spin-only moment of $1.73\,\mathrm{BM}$ corresponds to one unpaired electron because $\sqrt{1(1+2)} = \sqrt{3} \approx 1.73$. A closed-shell species has no unpaired electrons and is diamagnetic.
- For copper compounds, determine the oxidation state before the d-electron count. For oxygen molecular ions, removal or addition of an electron changes occupation of the antibonding $\pi^*$ orbitals.

**Step 1:** Count electrons in the copper species In $\mathrm{CuI}$, iodide has charge $-1$, so copper is $\mathrm{Cu}^+$, with configuration $3d^{10}$. All five d orbitals are doubly occupied: $n = 0$ and the spin-only moment is zero. In $[\mathrm{Cu(NH_3)_4}]\mathrm{Cl_2}$, the neutral ammonia ligands and two chloride counterions imply copper(II). The $3d^9$ configuration necessarily leaves one unpaired electron, giving approximately $1.73\,\mathrm{BM}$.

**Step 2:** Check the oxygen molecular ions Neutral $O_2$ has two unpaired electrons in its degenerate antibonding $\pi^*$ orbitals. Removing one electron forms $O_2{}^+$ and leaves one unpaired electron. Adding one electron forms $O_2{}^-$, pairing one of the two electrons and again leaving one unpaired electron. Both ions therefore have the spin-only value

$$\mu = \sqrt{1(1+2)} = \sqrt{3}\,\mathrm{BM} \approx 1.73\,\mathrm{BM}.$$

**Step 3:** Choose the exception Options (A), (C) and (D) each have one unpaired electron. Copper(I) iodide in option (B) instead has the closed d-ten configuration and does not share their magnetic moment. The Roman numeral implicit in the compound is essential: confusing copper(I) with copper(II) would give the wrong magnetic behaviour.

**Step 4:** Conclusion $\mathrm{CuI}$ does not have a spin-only magnetic moment of $1.73\,\mathrm{BM}$; its spin-only moment is zero. Hence, the correct option is (B).

**Quick Tip to Solve:** In copper salts, infer the oxidation state from ligand and counterion charges: $\mathrm{Cu}^+$ is $d^{10}$, while $\mathrm{Cu}^{2+}$ is $d^9$.

**69. Which one of the following statements is not true about enzymes?** [4 marks]

(A) Enzymes are non-specific for a reaction and substrate.

(B) Almost all enzymes are proteins.

(C) Enzymes work as catalysts by lowering the activation energy of a biochemical reaction.

(D) The action of enzymes is temperature and pH specific.

**Correct Answer:** (A) Enzymes are non-specific for a reaction and substrate.

**Solution:**

**Concept:**

- Enzymes are biological catalysts that accelerate biochemical reactions by providing a pathway with lower activation energy. They do not change the equilibrium constant or get consumed in the overall reaction.
- The active site has a particular shape and chemical environment, so enzyme activity is generally specific to a substrate or closely related substrates and to the reaction catalysed. Temperature and pH affect the active-site structure and catalytic groups.

**Step 1:** Examine the specificity claim Option (A) states that enzymes are non-specific for both reaction and substrate. This reverses the characteristic behaviour of enzymes. A suitable substrate interacts with the active site through complementary shape and chemical interactions, and the enzyme promotes a particular transformation. For example, urease catalyses the hydrolysis of urea; it is not a general catalyst for unrelated substrates and reactions.

**Step 2:** Verify the remaining statements Option (B), that almost all enzymes are proteins, is true. The qualifier almost allows for catalytic RNA molecules, or ribozymes, rather than claiming that every catalyst of biological origin must be a protein. Option (C) correctly describes catalysis through lowering the activation-energy barrier. This increases reaction rate without changing the free-energy difference between reactants and products.

**Step 3:** Check environmental dependence Option (D) is also true in the intended sense: an enzyme has a suitable temperature range and an optimum pH for its activity. Changes in pH alter ionisation of catalytic groups, while excessive heating can disrupt the protein structure and denature it. Since the question asks for the statement that is not true, these valid properties must be excluded rather than selected.

**Step 4:** Conclusion The false statement is that enzymes are non-specific for a reaction and substrate. Hence, the correct option is (A).

**Quick Tip to Solve:** For a negative enzyme-property question, remember the combination: substrate specificity, lower activation energy, and an optimum temperature and pH.

**70. The hybridisations of the atomic orbitals of nitrogen in $NO_2^-$, $NO_2^+$ and $NH_4^+$ respectively are.**

[4 marks]

(A) $sp^3$, $sp^2$ and sp

(B) sp, $sp^2$ and $sp^3$

(C) $sp^3$, sp and $sp^2$

(D) $sp^2$, sp and $sp^3$

**Correct Answer:** (D) $sp^2$, sp and $sp^3$

**Solution:**

**Concept:**

- Count sigma-bond directions and lone pairs around the central atom to find its steric number. A multiple bond counts as one direction because its additional pi bond does not introduce a new axis.
- The usual hybridisation assignments are steric number two: $sp$; three: $sp^2$; and four: $sp^3$. Molecular shape must distinguish lone pairs from bonded atoms.

**Step 1:** Analyse the nitrite ion In $NO_2^-$, nitrogen is bonded to two oxygen atoms and has one lone pair. Its resonance structures contain one single and one double N–O bond, but the double bond still contributes only one sigma-bond direction. Nitrogen therefore has two sigma bonds plus one lone pair:

$$\text{steric number} = 2 + 1 = 3 \quad \Rightarrow \quad sp^2.$$

The electron-domain arrangement is trigonal planar and the molecular shape is bent.

**Step 2:** Analyse the nitronium ion For $NO_2^+$, the Lewis structure is O=N=O. Nitrogen forms two sigma bonds and has no lone pair; the two pi bonds do not add to the steric number. Thus

$$\text{steric number} = 2 + 0 = 2 \quad \Rightarrow \quad sp.$$

The nitrogen-centred sigma framework is linear, consistent with the two equivalent bond directions being opposite.

**Step 3:** Analyse ammonium and compare the sequence In $NH_4^+$, nitrogen forms four N–H sigma bonds and has no lone pair. All four electron domains are bonding domains, so its steric number is four and the hybridisation is $sp^3$, with tetrahedral geometry. The requested order is nitrite, nitronium, then ammonium; it gives $sp^2, sp, sp^3$. The other choices assign the wrong number of electron domains to at least one of the first two ions.

**Step 4:** Conclusion The hybridisations are $sp^2$, $sp$ and $sp^3$, respectively. Hence, the correct option is (D).

**Quick Tip to Solve:** For hybridisation, count each sigma bond once and add lone pairs; never count a double bond as two electron-domain directions.

**71. Benzene on nitration gives nitrobenzene in presence of $HNO_3$ and $H_2SO_4$ mixture, where :**

[4 marks]

(A) both $H_2SO_4$ and $HNO_3$ act as a bases

(B) $HNO_3$ acts as an acid and $H_2SO_4$ acts as a base

(C) both $H_2SO_4$ and $HNO_3$ act as an acids

(D) $HNO_3$ acts as a base and $H_2SO_4$ acts as an acid

**Correct Answer:** (D) $HNO_3$ acts as a base and $H_2SO_4$ acts as an acid

**Solution:**

**Concept:**

- In a mixed-acid nitrating mixture, the stronger acid protonates the weaker acid. A Bronsted acid donates a proton, whereas a Bronsted base accepts one; these roles depend on the reaction partner rather than the name of the reagent.
- The electrophile for aromatic nitration is the nitronium ion, $NO_2^+$, generated by protonation of nitric acid and subsequent elimination of water.

**Step 1:** Identify the proton donor Sulfuric acid is stronger than nitric acid in the nitrating mixture, so it transfers a proton to an oxygen atom of nitric acid. The initial acid-base step is

$$HNO_3 + H_2SO_4 \rightleftharpoons H_2NO_3^+ + HSO_4^-.$$

Here $H_2SO_4$ donates $H^+$ and therefore acts as the acid. Nitric acid accepts that proton and therefore acts as the base. Calling nitric acid a base here does not deny that it behaves as an acid in water; it specifies its role relative to sulfuric acid.

**Step 2:** Connect protonation with the attacking species Protonated nitric acid loses water to form the electrophile:

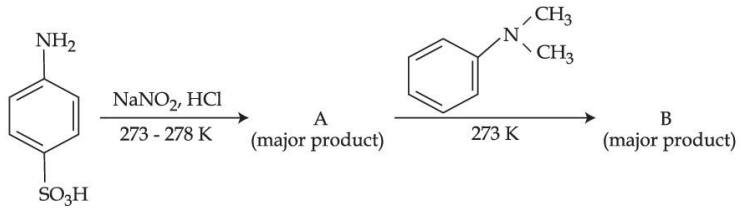
$$H_2NO_3^+ \longrightarrow NO_2^+ + H_2O.$$

Benzene then attacks $NO_2^+$, forms the sigma complex, and loses a proton to restore aromaticity, giving nitrobenzene. Thus the mixture is not described by both reagents acting as acids or both as bases in their mutual proton-transfer step. Option (B) reverses the proton-donor and proton-acceptor roles. Options (A) and (C) miss the relative acid-base behaviour that produces the nitronium ion.

**Step 3:** Conclusion $HNO_3$ acts as a base and $H_2SO_4$ acts as an acid. Hence, the correct option is (D).

**Quick Tip to Solve:** Trace the proton-transfer arrow in the mixed acid: the reagent receiving the proton is the base, even when it is normally called an acid.

**72. Consider the above reaction, compound B is :** [4 marks]




(A) An azo compound with $SO_3H$ para to the azo group on one ring and $N(CH_3)_2$ meta to it on the other.

(B) $C_6H_5-N=N-C_6H_4-N(CH_3)_2$, with the dimethylamino group para to the azo group.

(C) $HO_3S-C_6H_4-N=N-C_6H_4-N(CH_3)_2$, with both terminal groups para to the azo group.

(D) $HO_3S-C_6H_4-C_6H_4-N(CH_3)_2$, with both terminal groups para to the bond joining the rings.

**Correct Answer:** (C) $HO_3S-C_6H_4-N=N-C_6H_4-N(CH_3)_2$, with both terminal groups para to the azo group.

**Solution:**

**Concept:**

- A primary aromatic amine reacts with nitrous acid, generated from $NaNO_2/HCl$, at 273–278 K to form an arenediazonium salt.
- Arenediazonium ions undergo electrophilic azo coupling with activated aromatic rings. The $N(CH_3)_2$ group activates its ring and directs substitution mainly to the available para position.

**Step 1:** Form intermediate A The starting material is sulfanilic acid, which has $NH_2$ and $SO_3H$ para to one another. Diazotisation replaces the amino group by the diazonium group, without removing the sulfonic acid substituent:

$$\text{p-}HO_3S-C_6H_4-NH_2 \xrightarrow{NaNO_2/HCl} \text{p-}HO_3S-C_6H_4-N_2^+.$$

The low temperature stabilises the diazonium intermediate A and suppresses its decomposition. Consequently, any proposed final product lacking the original $SO_3H$ group cannot follow this sequence.

**Step 2:** Couple at the para position Intermediate A acts as the electrophile toward $N,N$-dimethylaniline. Its dimethylamino group donates electron density to the ring, with the free para carbon giving the major coupling product. The nitrogen-nitrogen bond is retained as an azo linkage:

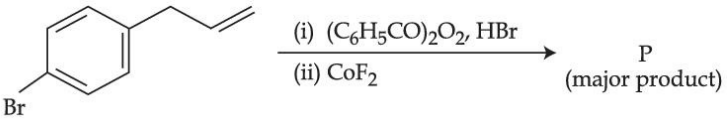
$$HO_3S-C_6H_4-N=N-C_6H_4-N(CH_3)_2.$$

Both the sulfonic acid substituent on the first ring and the dimethylamino substituent on the second ring are para to the azo group. Option (A) incorrectly places the dimethylamino group meta to the azo linkage. Option (B) omits the sulfonic acid group, while (D) replaces the essential azo bridge by a carbon-carbon bond.

**Step 3:** Conclusion B is the para-coupled azo dye $HO_3S-C_6H_4-N=N-C_6H_4-N(CH_3)_2$, with both end groups para to the azo bridge (methyl orange in its acid form). Hence, the correct option is (C).

**Quick Tip to Solve:** Preserve all substituents during diazotisation, then attach the azo group para to the strongly activating dimethylamino group.

**73. Major product P of above reaction, is :** [4 marks]




(A) p-Br$-C_6H_4-$CH(F)$-$CH$=CH_2$

(B) p-F$-C_6H_4-CH_2-CH_2-CH_2$F

(C) p-Br$-C_6H_4-CH_2-$CH(F)$-CH_3$

(D) p-Br$-C_6H_4-CH_2-CH_2-CH_2$F

**Correct Answer:** (D) p-Br$-C_6H_4-CH_2-CH_2-CH_2$F

**Solution:**

**Concept:**

- In the presence of a peroxide, addition of HBr to an unsymmetrical alkene follows the radical, anti-Markovnikov pathway. Bromine ends up on the less substituted alkene carbon.
- Swarts halogen exchange replaces an alkyl chloride or bromide by fluorine using a metallic fluoride such as $CoF_2$. An aryl carbon-halogen bond does not undergo this ordinary alkyl-halide exchange.

**Step 1:** Add HBr across the terminal double bond Write the aromatic group as Ar $= p$-Br$-C_6H_4$. The side chain is Ar$-CH_2-$CH$=CH_2$. Benzoyl peroxide initiates radical addition of HBr. A bromine radical adds to the terminal carbon so that the intermediate radical is on the more substituted internal carbon; abstraction of hydrogen from HBr then gives

$$\text{Ar}-CH_2-\text{CH}=CH_2 \xrightarrow{\text{HBr/peroxide}} \text{Ar}-CH_2-CH_2-CH_2\text{Br}.$$

The side-chain double bond is consumed. The bromine already attached directly to the benzene ring remains in place.

**Step 2:** Exchange the alkyl bromide for fluoride Treatment with $CoF_2$ replaces the terminal side-chain bromine by fluorine:

$$2\,\text{Ar}-CH_2-CH_2-CH_2\text{Br} + CoF_2 \longrightarrow 2\,\text{Ar}-CH_2-CH_2-CH_2\text{F} + CoBr_2.$$

The product is therefore p-Br$-C_6H_4-CH_2-CH_2-CH_2$F. Option (A) retains the alkene and puts fluorine at the benzylic carbon, contrary to addition across the double bond. Option (B) also exchanges aryl bromine, which is not the reaction here. Option (C) places fluorine on the internal carbon, corresponding to the wrong addition orientation.

**Step 3:** Conclusion P is p-Br$-C_6H_4-CH_2-CH_2-CH_2$F, retaining aryl bromine and bearing terminal side-chain fluorine. Hence, the correct option is (D).

**Quick Tip to Solve:** Apply the reagents in order: peroxide puts bromine at the terminal alkene carbon, then cobalt fluoride exchanges only that alkyl bromide.

**74. $Cu^{2+}$ salt reacts with potassium iodide to give** [4 marks]

(A) $Cu_2I_2$

(B) $Cu_2I_3$

(C) CuI

(D) $Cu(I_3)_2$

**Correct Answer:** (C) CuI

**Solution:**

**Concept:**

- Copper(II) oxidises iodide ions to iodine while being reduced to copper(I). The reduced copper ion forms insoluble copper(I) iodide.
- The stable precipitate is written with its simplest stoichiometric formula, $\mathrm{CuI}$. Precipitation of $\mathrm{CuI}$ helps drive the coupled redox reaction.

**Step 1:** Balance the electron transfer Iodide acts as the reducing agent and copper(II) as the oxidising agent. Their half-reactions are

$$2\,\mathrm{Cu^{2+}} + 2\,\mathrm{e^-} \longrightarrow 2\,\mathrm{Cu^{+}}, \qquad 2\,\mathrm{I^-} \longrightarrow \mathrm{I_2} + 2\,\mathrm{e^-}.$$

Thus two copper(II) ions are reduced for each iodine molecule formed. The process cannot be described simply as precipitation of a stable copper(II) iodide, because iodide itself reduces $\mathrm{Cu^{2+}}$. The brown colour that may appear is due to liberated iodine, not a copper(II) iodide precipitate.

**Step 2:** Identify the insoluble copper product The copper(I) ions combine with further iodide:

$$2\,\mathrm{Cu^{+}} + 2\,\mathrm{I^-} \longrightarrow 2\,\mathrm{CuI\,(s)}.$$

Adding the two stages gives

$$2\,\mathrm{Cu^{2+}} + 4\,\mathrm{I^-} \longrightarrow 2\,\mathrm{CuI\,(s)} + \mathrm{I_2}.$$

The copper-containing salt is therefore copper(I) iodide, $\mathrm{CuI}$, which is the intended formula in option (C). $\mathrm{Cu_2I_3}$ is not the precipitate, and the triiodide expression in (D) does not describe this product. Option (A), $\mathrm{Cu_2I_2}$, merely doubles the empirical ratio and is not the customary formula used for the salt; the accepted answer is the simplest formula $\mathrm{CuI}$.

**Step 3:** Conclusion The precipitate is copper(I) iodide, $\mathrm{CuI}$, accompanied by liberation of $\mathrm{I_2}$. Hence, the correct option is (C).

**Quick Tip to Solve:** For copper(II) plus iodide, remember both changes: copper falls to $+1$ and iodide gives iodine; the precipitate is CuI.

**75. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of :** [4 marks]

(A) $HNO_3$

(B) $AgNO_3$

(C) $CuSO_4$

(D) $BaSO_4$

**Correct Answer:** (B) $AgNO_3$

**Solution:**

**Concept:**

- The Carius method estimates chlorine, bromine or iodine in an organic compound by converting the halogen into an insoluble silver halide of known composition.
- Fuming nitric acid oxidises the organic material in a sealed Carius tube, while silver nitrate supplies the $Ag^+$ ions that capture the halide.

**Step 1:** Identify the precipitating reagent The weighed organic compound is heated with fuming $HNO_3$ in the presence of $AgNO_3$. Oxidation destroys the organic framework and converts carbon and hydrogen into oxidised products, while the halogen is collected as $AgX$, where X denotes chlorine, bromine or iodine. The capture step is

$$Ag^+ + X^- \longrightarrow AgX(s).$$

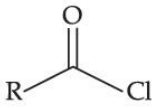
The silver halide precipitate can then be filtered, washed, dried and weighed, giving a directly measurable quantity related to the original halogen content.

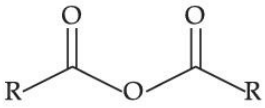
**Step 2:** Distinguish the reagent from the reaction medium Fuming nitric acid is already specified in the question as the oxidising medium. The additional reagent sought is therefore silver nitrate, not another portion of nitric acid. $CuSO_4$ does not provide silver ions and cannot produce the required silver halide. $BaSO_4$ is itself an insoluble sulfate; it is not a halogen-trapping reagent. For a monohalide precipitate, one mole of $AgX$ corresponds to one mole of halogen atoms, so its measured mass permits quantitative estimation rather than merely a qualitative colour test.

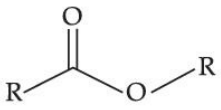
**Step 3:** Conclusion The compound is heated with fuming nitric acid in the presence of $AgNO_3$, which precipitates the halogen as $AgX$. Hence, the correct option is (B).

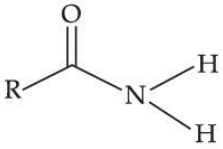
**Quick Tip to Solve:** Link Carius halogen estimation with a weighed silver halide precipitate: the necessary added reagent is silver nitrate.

**76. The correct order of their reactivity towards hydrolysis at room temperature is :** [4 marks]

(A) 

(B) 

(C) 

(D) 

(A) $(A) > (B) > (C) > (D)$

(B) $(D) > (A) > (B) > (C)$

(C) $(D) > (B) > (A) > (C)$

(D) $(A) > (C) > (B) > (D)$

**Correct Answer:** (A) $(A) > (B) > (C) > (D)$

**Solution:**

**Concept:**

- Carboxylic acid derivatives hydrolyse by nucleophilic acyl substitution. Their relative reactivity depends on carbonyl electrophilicity and how readily the attached group can depart.
- For comparable acyl groups, the usual order is acid chloride greater than acid anhydride greater than ester greater than amide. Strong resonance donation and a poor leaving group make amides least reactive.

**Step 1:** Identify the four functional groups The illustrated compounds are (A) an acid chloride, $\mathrm{RCOCl}$; (B) an acid anhydride, $(\mathrm{RCO})_2\mathrm{O}$; (C) an ester, $\mathrm{RCOOR}$; and (D) a primary amide, $\mathrm{RCONH_2}$. Water attacks the carbonyl carbon, after which the tetrahedral intermediate must eliminate a leaving group. Chloride is a good leaving group, and the acid chloride has a strongly electrophilic carbonyl. Accordingly, acid chlorides hydrolyse readily even under mild conditions.

**Step 2:** Compare resonance and leaving-group ability An anhydride can expel a resonance-stabilised carboxylate group and is therefore less reactive than an acid chloride but more reactive than an ester. An ester must expel an alkoxide-derived group, making its hydrolysis less favourable without acid or base catalysis. In an amide, nitrogen donates its lone pair strongly into the carbonyl, reducing electrophilicity, and the corresponding nitrogen leaving group is particularly poor. Consequently amides require the most vigorous conditions among these derivatives. The resulting order is

$$\mathrm{RCOCl} > (\mathrm{RCO})_2\mathrm{O} > \mathrm{RCOOR} > \mathrm{RCONH_2}.$$

The other orders incorrectly place the amide first or put the ester ahead of the anhydride.

**Step 3:** Conclusion The reactivity decreases from acid chloride to acid anhydride to ester to amide. Hence, the correct option is (A).

**Quick Tip to Solve:** Rank acyl derivatives by leaving groups: chloride leaves most easily, then carboxylate, while amide nitrogen is the poorest leaving group.

**77. The single largest industrial application of dihydrogen is :** [4 marks]

(A) Manufacture of metal hybrides

(B) Rocket fuel in space research

(C) In the synthesis of ammonia

(D) In the synthesis of nitric acid

**Correct Answer:** (C) In the synthesis of ammonia

**Solution:**

**Concept:**

- Dihydrogen is used in bulk chemical manufacture, hydrogenation and reduction, as well as in specialised applications such as rocket propulsion.
- The Haber process consumes large quantities of hydrogen to manufacture ammonia: $N_2 + 3\,H_2 \rightleftharpoons 2\,NH_3$. Ammonia is a major feedstock for fertilisers and other nitrogen compounds.

**Step 1:** Identify the bulk chemical process Industrial ammonia synthesis combines nitrogen with hydrogen in the presence of an iron-based catalyst at elevated temperature and pressure. Its stoichiometry is

$$N_2(g) + 3\,H_2(g) \rightleftharpoons 2\,NH_3(g).$$

Three moles of hydrogen are needed per mole of nitrogen, so the very large worldwide scale of ammonia production entails a very large demand for dihydrogen. The question asks for the single largest industrial application, rather than the use with the most striking energy release or technological visibility.

**Step 2:** Compare the other stated uses Metal hydride manufacture is a real chemical application, but it is much smaller in scale than ammonia production. Liquid hydrogen is also an important rocket propellant, yet space research consumes far less hydrogen overall. Nitric acid is normally manufactured by oxidation of ammonia in the Ostwald process; hydrogen is not directly combined with nitrogen and oxygen to make nitric acid in that process. Thus nitric acid production may depend indirectly on ammonia feedstock, but the direct large-scale use of dihydrogen among the listed choices is ammonia synthesis. The distinction between direct hydrogen consumption and a downstream product removes option (D).

**Step 3:** Conclusion The single largest industrial application of dihydrogen is the synthesis of ammonia by the Haber process. Hence, the correct option is (C).

**Quick Tip to Solve:** For the largest chemical use of hydrogen, recall the three hydrogen molecules consumed in each Haber-process reaction with nitrogen.

**78. Consider two chemical reactions (A) and (B) that take place during metallurgical process:**

**(A)** $ZnCO_{3(s)} \xrightarrow{\Delta} ZnO_{(s)} + CO_{2(g)}$

**(B)** $2ZnS_{(s)} + 3O_{2(g)} \xrightarrow{\Delta} 2ZnO_{(s)} + 2SO_{2(g)}$

**The correct option of names given to them respectively is :** [4 marks]

(A) (A) is calcination and (B) is roasting

(B) Both (A) and (B) are producing same product so both are roasting

(C) Both (A) and (B) are producing same product so both are calcination

(D) (A) is roasting and (B) is calcination

**Correct Answer:** (A) (A) is calcination and (B) is roasting

**Solution:**

**Concept:**

- Calcination converts a carbonate or hydrated ore into its oxide by strong heating in the absence or a limited supply of air. The carbonate releases carbon dioxide.
- Roasting heats a sulphide ore in excess air or oxygen. Oxygen is a reactant, and sulphur is usually removed as sulphur dioxide.

**Step 1:** Classify reaction A The first ore is zinc carbonate. Its decomposition is

$$\mathrm{ZnCO_3(s)} \xrightarrow{\Delta} \mathrm{ZnO\,(s) + CO_2(g)}.$$

No oxygen is consumed. Thus this is thermal decomposition of a carbonate, which is calcination.

**Step 2:** Classify reaction B and compare the choices The second ore is zinc sulphide, and it reacts with oxygen:

$$\mathrm{2\,ZnS\,(s) + 3\,O_2(g)} \xrightarrow{\Delta} \mathrm{2\,ZnO\,(s) + 2\,SO_2(g)}.$$

This oxidation in air is roasting. Producing the same solid oxide does not make the two processes identical: the starting ore and the role of oxygen distinguish them. Hence the choices calling both reactions roasting or both calcination fail, and the choice reversing the two names also fails.

**Step 3:** Conclusion Reaction A is calcination and reaction B is roasting. Hence, the correct option is (A).

**Quick Tip to Solve:** Check the ore and oxygen: carbonate losing $\mathrm{CO_2}$ means calcination; sulphide consuming oxygen means roasting.

**79.** **Bakelite is a cross-linked polymer of formaldehyde and :** [4 marks]

(A) PHBV

(B) Buna-S

(C) Novolac

(D) Dacron

**Correct Answer:** (C) Novolac

**Solution:**

**Concept:**

- Phenol reacts with formaldehyde through condensation, initially forming substituted phenols that link through methylene bridges. The reaction conditions control the extent of linking.
- Novolac is the linear phenol-formaldehyde resin. Further treatment with formaldehyde produces the highly cross-linked thermosetting polymer Bakelite.

**Step 1:** Identify the precursor Bakelite contains a three-dimensional network of phenolic units connected by methylene bridges. The listed material that already supplies these phenolic chains is novolac. On further reaction with formaldehyde, links form between different chains, converting the linear resin into a hard, infusible network.

**Step 2:** Eliminate the other polymers PHBV is a biodegradable copolyester of hydroxybutyrate and hydroxyvalerate units. Buna-S is a rubber formed from butadiene and styrene. Dacron is a polyester of ethylene glycol and terephthalic acid. None is the linear phenol-formaldehyde precursor required for the cross-linking reaction in this question. The formaldehyde-plus-novolac combination therefore uniquely gives Bakelite among the choices.

**Step 3:** Conclusion Bakelite is obtained by cross-linking novolac with formaldehyde. Hence, the correct option is (C).

**Quick Tip to Solve:** Remember the sequence: phenol plus formaldehyde gives linear novolac; further formaldehyde cross-links it into Bakelite.

**80.** **Which one of the following gases is reported to retard photosynthesis?** [4 marks]

(A) CO

(B) CFCs

(C) $CO_2$

(D) $NO_2$

**Correct Answer:** (D) $NO_2$

**Solution:**

**Concept:**

- Nitrogen dioxide, $NO_2$, is an air pollutant that damages leaves and is reported to retard the rate of photosynthesis in plants.
- The characteristic biological or environmental effect of each pollutant distinguishes the options;

the question asks specifically about retardation of photosynthesis.

**Step 1:** Identify the gas affecting photosynthesis Photosynthesis depends on healthy leaf tissues and their photosynthetic machinery. Nitrogen dioxide is harmful to vegetation and retards photosynthesis. Therefore the gas sought here is $NO_2$, rather than a gas selected only because it is generally associated with air pollution.

**Step 2:** Distinguish the other effects Carbon monoxide is best known for binding strongly to haemoglobin and reducing oxygen transport in humans and other animals. Chlorofluorocarbons are associated with depletion of the stratospheric ozone layer. Carbon dioxide is a reactant in photosynthesis; its familiar pollution effect is its contribution to global warming when its atmospheric concentration increases. These characteristic effects do not match the specific statement about retarding photosynthesis as directly as nitrogen dioxide does.

**Step 3:** Conclusion The gas reported to retard photosynthesis is $NO_2$. Hence, the correct option is (D).

**Quick Tip to Solve:** Associate $NO_2$ with leaf injury and retarded photosynthesis; keep this separate from CO toxicity and CFC ozone depletion.

## Chemistry – Section B: Numerical Value Questions (20 marks)

*Attempt any 5 of the 10 questions in this section.*

**81. For a given chemical reaction A $\rightarrow$ B at 300 K the free energy change is $-49.4$ kJ mol$^{-1}$ and the enthalpy of reaction is 51.4 kJ mol$^{-1}$. The entropy change of the reaction is _____ JK$^{-1}$ mol$^{-1}$.** [4 marks]

**Solution:**

**Concept:**

- At a specified temperature, Gibbs energy, enthalpy and entropy changes obey $\Delta G = \Delta H - T\Delta S$. Here $T$ is the absolute temperature in kelvin.
- The energy terms must use the same units before subtraction. To obtain entropy in $\mathrm{J\,K^{-1}\,mol^{-1}}$, convert the supplied kilojoule values into joules.

**Step 1:** Rearrange with the correct signs The given changes are $\Delta G = -49.4\,\mathrm{kJ\,mol^{-1}}$ and $\Delta H = +51.4\,\mathrm{kJ\,mol^{-1}}$. Moving the entropy term to the other side gives

$$T\Delta S = \Delta H - \Delta G = 51.4 - (-49.4) = 100.8\,\mathrm{kJ\,mol^{-1}}.$$

The negative free-energy change must be subtracted algebraically, so the two magnitudes add.

**Step 2:** Calculate the entropy change At $T = 300\,\mathrm{K}$,

$$\Delta S = \frac{100.8 \times 10^3}{300} = 336\,\mathrm{J\,K^{-1}\,mol^{-1}}.$$

The positive entropy contribution exceeds the positive enthalpy change: substituting back gives

$51.4 - 300(0.336) = -49.4\,\text{kJ mol}^{-1}$, confirming both the sign and the value.

**Step 3:** Conclusion The entropy change is $\boxed{+336\,\text{J K}^{-1}\,\text{mol}^{-1}}$. The required numerical entry is 336.

**Quick Tip to Solve:** Write $\Delta S = (\Delta H - \Delta G)/T$ first, and put a negative $\Delta G$ in parentheses before subtracting it.

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**82. The wavelength of electrons accelerated from rest through a potential difference of 40 kV is x $\times$ $10^{-12}$ m. The value of x is _____. (Nearest integer)**

**Give : Mass of electron = 9.1 $\times$ $10^{-31}$ kg**

**Charge on an electron = 1.6 $\times$ $10^{-19}$ C**

**Planck's constant = 6.63 $\times$ $10^{-34}$ Js** [4 marks]

**Solution:**

**Concept:**

- An electron accelerated from rest through a potential difference $V$ gains kinetic energy $eV$, where $e$ is the magnitude of its charge.
- Using the usual nonrelativistic de Broglie relation, $\lambda = h/p = h/\sqrt{2meV}$. Use volts, kilograms, coulombs and joule-seconds to obtain metres.

**Step 1:** Convert the potential and obtain momentum The potential is $40\,\text{kV} = 4.0 \times 10^4\,\text{V}$. Therefore

$$K = eV = (1.6 \times 10^{-19})(4.0 \times 10^4) = 6.4 \times 10^{-15}\,\text{J}.$$

Since $K = p^2/(2m)$, the momentum is

$$p = \sqrt{2(9.1 \times 10^{-31})(6.4 \times 10^{-15})} \approx 1.079 \times 10^{-22}\,\text{kg m s}^{-1}.$$

The electron begins at rest, so there is no initial kinetic energy to add.

**Step 2:** Calculate and round the coefficient With the supplied Planck constant,

$$\lambda = \frac{6.63 \times 10^{-34}}{1.079 \times 10^{-22}} \approx 6.14 \times 10^{-12}\,\text{m}.$$

The question defines $\lambda = x \times 10^{-12}\,\text{m}$, so $x \approx 6.14$. Rounding the coefficient, rather than the wavelength in metres, gives the requested nearest integer.

**Step 3:** Conclusion The wavelength is approximately $6.14 \times 10^{-12}\,\text{m}$, giving $\boxed{x = 6}$ to the nearest integer.

**Quick Tip to Solve:** Convert kV to V before using $h/\sqrt{2meV}$; then extract the coefficient of the specified power of ten before rounding.

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**83. The vapour pressures of A and B at 25°C are 90 mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is x $\times$ $10^{-1}$. The value of x is _____. (Nearest integer)** [4 marks]

**Solution:**

**Concept:**

- For an ideal liquid mixture, Raoult law gives each partial pressure as $p_i = x_i^{(l)} p_i^{\circ}$, where $x_i^{(l)}$ is its liquid-phase mole fraction.
- Dalton law gives the vapour-phase mole fraction as $y_i = p_i/(p_A + p_B)$. Liquid and vapour compositions need not be equal because the pure-component vapour pressures differ.

**Step 1:** Calculate the partial pressures The liquid mole fraction of B is $x_B^{(l)} = 1 - 0.6 = 0.4$. At the given temperature,

$$p_A = 0.6(90) = 54\,\text{mm Hg}, \qquad p_B = 0.4(15) = 6\,\text{mm Hg}.$$

Therefore the total equilibrium vapour pressure is $P = 54 + 6 = 60\,\text{mm Hg}$. Both terms use the same pressure unit, so no conversion is needed for their ratio.

**Step 2:** Find the vapour fraction and coefficient The fraction of B in the vapour is

$$y_B = \frac{p_B}{P} = \frac{6}{60} = 0.10.$$

Thus $y_B = 1 \times 10^{-1}$, and the required coefficient is exactly 1. This smaller vapour fraction than the liquid value 0.4 is consistent with B being less volatile: its pure vapour pressure is much lower than that of A.

**Step 3:** Conclusion The mole fraction of B in the vapour phase is $0.10 = 1 \times 10^{-1}$, hence $\boxed{x = 1}$.

**Quick Tip to Solve:** Use Raoult law to get partial pressures first; divide by total pressure only when converting to vapour-phase mole fractions.

**84. 4g equimolar mixture of NaOH and $Na_2CO_3$ contains x g of NaOH and y g of $Na_2CO_3$. The value of x is _____ g. (Nearest integer)** [4 marks]

**Solution:**

**Concept:**

- Equimolar means equal numbers of moles, not equal masses. For a substance, mass equals the number of moles multiplied by its molar mass.
- The molar masses are $M(\text{NaOH}) = 40\,\text{g mol}^{-1}$ and $M(\text{Na}_2\text{CO}_3) = 106\,\text{g mol}^{-1}$, using Na = 23, O = 16, H = 1 and C = 12.

**Step 1:** Express the two masses using equal moles Let the mixture contain $n$ moles of each component. The masses are then $x = 40n$ grams of NaOH and $y = 106n$ grams of $\text{Na}_2\text{CO}_3$. The total mass

condition gives

$$40n + 106n = 4, \qquad n = \frac{4}{146}\,\text{mol}.$$

The common amount is the number of moles of each individual component; the total amount of both together would be $2n$.

**Step 2:** Find the requested mass and round it The sodium hydroxide mass is

$$x = 40\left(\frac{4}{146}\right) = \frac{80}{73} \approx 1.096\,\text{g}.$$

Correspondingly, $y \approx 2.904\,\text{g}$, and these add to 4 g as required. Rounding the requested mass to the nearest integer gives 1 g. Taking half the total mass would incorrectly treat an equimolar mixture as an equal-mass mixture.

**Step 3:** Conclusion The mass of $\text{NaOH}$ is approximately $1.096\,\text{g}$, so the required nearest-integer value is $\boxed{x = 1}$.

**Quick Tip to Solve:** For an equimolar binary mixture, divide its total mass in the ratio of the two molar masses, not in the ratio 1:1.

**85. When 0.15 g of an organic compound was analyzed using Carius method for estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is _____. (Nearest integer)**

**[Atomic mass : Silver = 108, Bromine = 80]** [4 marks]

**Solution:**

**Concept:**

- In the Carius method, halogen in a weighed organic sample is converted quantitatively into silver halide. Each mole of $\text{AgBr}$ contains one mole of bromine atoms.
- The mass fraction of bromine in the precipitate is $M_{\text{Br}}/M_{\text{AgBr}}$. Percentage composition refers to the original organic sample, not to the silver bromide precipitate.

**Step 1:** Find the molar mass of the precipitate Use the atomic masses supplied in the question rather than more precise tabulated values. Silver and bromine occur in a one-to-one ratio in the precipitate, so

$$M_{\text{AgBr}} = 108 + 80 = 188\,\text{g mol}^{-1}.$$

Thus every 188 g of silver bromide represents 80 g of bromine from the sample. The silver contributes to precipitate mass but was not part of the organic compound.

**Step 2:** Calculate the mass of bromine The experimentally collected precipitate has mass 0.2397 g. Apply its bromine mass fraction:

$$m_{\text{Br}} = 0.2397 \times \frac{80}{188} = 0.1020\,\text{g}.$$

This is smaller than the precipitate mass, as required, and is the amount of bromine present in the

original 0.15 g sample.

**Step 3:** Express bromine as a percentage of the sample Divide by the mass of the organic compound and multiply by 100:

$$\%\mathrm{Br} = \frac{0.1020}{0.15} \times 100 = 68.0\%.$$

The denominator must be 0.15 g; dividing by 0.2397 g would instead give the percentage of bromine in silver bromide. No further stoichiometric factor is needed because one bromide ion gives one formula unit of the precipitate.

**Step 4:** Conclusion The percentage of bromine is $68.0\%$, so the requested nearest-integer answer is $\boxed{68}$.

**Quick Tip to Solve:** For Carius analysis, first multiply the silver-halide mass by the halogen-to-silver-halide molar-mass ratio; only then divide by the original sample mass.

**86.**

$$PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$$

**In the above first order reaction the concentration of $PCl_5$ reduces from initial concentration 50 mol $L^{-1}$ to 10 mol $L^{-1}$ in 120 minutes at 300 K. The rate constant for the reaction at 300 K is x $\times$ $10^{-2}$ $min^{-1}$. The value of x is _____. [Given log5 = 0.6989]** [4 marks]

**Solution:**

**Concept:**

- For a first-order decomposition, the integrated rate law is $k = (2.303/t)\log_{10}([A]_0/[A]_t)$, where $t$ is elapsed time and both concentrations use the same units.
- The units of a first-order rate constant are inverse time. Keeping time in minutes gives $k$ in $\mathrm{min}^{-1}$; the coefficient $x$ must then be separated from the stated power of ten.

**Step 1:** Identify the concentration ratio The reacting species is $\mathrm{PCl_5}$, whose concentration decreases from 50 to 10 in 120 minutes at the specified temperature. The ratio in the integrated law is therefore

$$\frac{[\mathrm{PCl_5}]_0}{[\mathrm{PCl_5}]_t} = \frac{50}{10} = 5.$$

The concentration units cancel. Use remaining reactant concentration, not the concentration decrease of 40, in the denominator.

**Step 2:** Evaluate the first-order constant Substitute the given common logarithm directly:

$$k = \frac{2.303}{120}\log_{10} 5 = \frac{2.303 \times 0.6989}{120}.$$

$$k = \frac{1.6095667}{120} = 0.0134131\,\mathrm{min}^{-1}.$$

The factor 2.303 converts a common logarithm into a natural logarithm; it would be omitted only if the logarithm were already natural.

**Step 3:** Match the coefficient to the requested form The statement defines $k = x \times 10^{-2}\,\text{min}^{-1}$. Consequently,

$$x = \frac{0.0134131}{10^{-2}} = 1.34131.$$

The numerical-entry value used by the stated answer key is the integer 1. Keep the calculated coefficient visible so that it is clear how the entry relates to the rate constant, rather than mistaking the full rate constant for $x$.

**Step 4:** Conclusion $k = 1.34131 \times 10^{-2}\,\text{min}^{-1}$, giving $x \approx 1.34$. The integer numerical entry is $\boxed{1}$.

**Quick Tip to Solve:** When a rate constant is written as $x \times 10^{-2}$, calculate $k$ first and divide it by $10^{-2}$. Do not round the logarithm or the rate constant before extracting $x$.

**87. An aqueous solution of $NiCl_2$ was heated with excess sodium cyanide in presence of strong oxidizing agent to form $[Ni(CN)_6]^{2-}$. The total change in number of unpaired electrons on metal centre is _____.** [4 marks]

**Solution:**

**Concept:**

- Nickel has atomic number 28 and neutral configuration $[\text{Ar}]\,3d^8 4s^2$. Remove the outer $4s$ electrons before $3d$ electrons when assigning the configurations of its ions.
- For an octahedral complex, the five metal $d$ orbitals split into lower $t_{2g}$ and higher $e_g$ levels. Cyanide is a strong-field ligand and favours electron pairing when the configuration permits it.

**Step 1:** Count unpaired electrons in the aqueous starting ion In $NiCl_2$, each chloride has charge $-1$, so nickel is in oxidation state $+2$. Aqueous nickel(II) is a $3d^8$ ion, commonly represented by the octahedral aqua ion. Its occupation is

$$t_{2g}^6 e_g^2.$$

The two electrons in the degenerate $e_g$ orbitals occupy separate orbitals, giving two unpaired electrons. The filled $t_{2g}$ set contributes none.

**Step 2:** Find the oxidation state in the product In $[Ni(CN)_6]^{2-}$, all six cyanide ligands each carry charge $-1$. If the oxidation state of nickel is $z$, charge balance gives

$$z + 6(-1) = -2 \quad \Rightarrow \quad z = +4.$$

The strong oxidizing agent therefore changes nickel(II) to nickel(IV). Nickel(IV) has lost two further $d$ electrons and is $3d^6$, not $3d^8$.

**Step 3:** Apply the strong-field electron arrangement Six cyanide ligands give an octahedral environment. With strong-field cyanide, the six $d$ electrons pair in the lower three orbitals:

$$t_{2g}^6 e_g^0.$$

Every occupied orbital is doubly occupied, so the product has zero unpaired electrons. Comparing the

counts gives

$$|n_{\text{final}} - n_{\text{initial}}| = |0 - 2| = 2.$$

This is a decrease in unpaired electrons; the numerical question asks its total magnitude.

**Step 4:** Conclusion The unpaired-electron count decreases from 2 to 0. The total change is therefore $\boxed{2}$.

**Quick Tip to Solve:** Always determine the product metal oxidation state before using ligand strength. Here oxidation changes $d^8$ into $d^6$, and strong-field cyanide then produces a fully paired configuration.

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**88. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.**

**$6OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$**

**A current of xA has to be passed for 10h to produce 10.0g of potassium chlorate. The value of x is _____. (Nearest integer)**

**(Molar mass of $KClO_3$ = 122.6 g $mol^{-1}$, F = 96500 C)** [4 marks]

**Solution:**

**Concept:**

- Faraday's law relates charge to electron amount: $Q = n_e F$. A constant current supplies charge $Q = It$, with $I$ in amperes and $t$ in seconds.
- The given oxidation half-reaction produces one chlorate ion while releasing six electrons. Hence forming one mole of $KClO_3$ requires passage of six moles of electronic charge.

**Step 1:** Read the electron requirement from the equation The stated half-reaction is

$$6\,OH^- + Cl^- \longrightarrow ClO_3^- + 3\,H_2O + 6\,e^-.$$

One mole of chlorate is produced per six moles of electrons. Potassium is the spectator cation, so one mole of $KClO_3$ contains one mole of $ClO_3^-$. The factor is six, not one: chlorine changes oxidation state from $-1$ to $+5$.

**Step 2:** Convert the target product mass to charge Using the supplied molar mass,

$$n_{KClO_3} = \frac{10.0}{122.6} = 0.0815661\,\text{mol}.$$

The required charge is therefore

$$Q = 6 \times \frac{10.0}{122.6} \times 96500 \approx 47226.8\,\text{C}.$$

This is the stoichiometric charge for the stated production process; no separate current-efficiency factor is specified in the question.

**Step 3:** Convert the duration and calculate current The current unit ampere is coulomb per second,

so first express ten hours in seconds:

$$t = 10 \times 60 \times 60 = 36000\,\text{s}.$$

Now apply $I = Q/t$:

$$I = \frac{47226.8}{36000} = 1.31186\,\text{A}.$$

Because the question asks for the nearest integer, round only this final current. The result lies closer to 1 A than to 2 A.

**Step 4:** Conclusion The required current is approximately $1.312\,\text{A}$, so $x = \boxed{1}$ to the nearest integer.

**Quick Tip to Solve:** For electrolytic production, obtain electrons per formula unit from the balanced half-reaction, then use $I = (m/M)zF/t$ with time in seconds.

**89. 100 ml of 0.0018% (w/v) solution of $Cl^-$ ion was the minimum concentration of $Cl^-$ required to precipitate a negative sol in one h. The coagulating value of $Cl^-$ ion is _____ (Nearest integer)**

[4 marks]

**Solution:**

**Concept:**

- The coagulation value is the minimum concentration of the effective electrolyte or coagulating ion, conventionally expressed in millimoles per litre, needed for coagulation in the specified time.
- A concentration of $p\%\,(\text{w/v})$ means $p$ grams of solute in 100 mL of solution. Convert the given chloride-ion mass concentration into molarity before expressing it in millimoles per litre.

**Step 1:** Interpret the percentage concentration The stated 0.0018% $(\text{w/v})$ means that 100 mL of solution contains 0.0018 g of chloride ion. The 100 mL sample therefore has that ion mass, not 0.0018 mol. Scaling to one litre gives

$$c_{\text{mass}} = 0.0018 \times \frac{1000}{100} = 0.018\,\text{g}\,\text{L}^{-1}.$$

The one-hour condition defines when coagulation is observed; it is not a factor by which this concentration is divided.

**Step 2:** Convert to millimolar concentration Using the usual molar mass of chloride ion, approximately $35.5\,\text{g}\,\text{mol}^{-1}$,

$$c = \frac{0.018}{35.5} = 5.0704 \times 10^{-4}\,\text{mol}\,\text{L}^{-1}.$$

$$c = 0.50704\,\text{mmol}\,\text{L}^{-1}.$$

The electron mass is negligible here, so the atomic molar mass of chlorine is appropriate. Rounding this numerical concentration to the nearest integer gives 1.

**Step 3:** Distinguish the numerical entry from the charge statement The concentration calculation above follows the quantity explicitly supplied. There is, however, a charge inconsistency in the wording: a negative sol is ordinarily coagulated by oppositely charged cations, whereas chloride is an

anion. Thus chloride cannot be identified as its usual counter-ion merely from this statement. This does not change the conversion of the stated chloride concentration, which supplies the requested numerical entry and agrees with the given key.

**Step 4:** Conclusion The stated concentration is $0.507\,\text{mmol}\,\text{L}^{-1}$; its nearest-integer numerical entry is $\boxed{1}$. The reference to chloride coagulating a negative sol is charge-inconsistent.

**Quick Tip to Solve:** Convert a w/v percentage to grams per litre by multiplying by ten, divide by molar mass, and then multiply by 1000 to obtain the coagulation concentration in millimoles per litre.

**90. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is _____.** [4 marks]

**Solution:**

**Concept:**

- An FCC conventional unit cell contains eight corner sites and six face-centre sites. A corner atom contributes one eighth to the cell, while a face-centred atom contributes one half.
- A close-packed FCC cell has two tetrahedral voids per lattice atom. Diamond occupies half of those tetrahedral sites with additional carbon atoms, which must be added to the FCC contribution.

**Step 1:** Count the carbon atoms on FCC lattice sites The eight corners are each shared among eight neighbouring unit cells, and the six faces are each shared between two cells. Their effective contributions are

$$N_{\text{corners}} = 8 \times \frac{1}{8} = 1, \qquad N_{\text{faces}} = 6 \times \frac{1}{2} = 3.$$

Hence the FCC framework contributes

$$N_{\text{FCC}} = 1 + 3 = 4.$$

Counting all fourteen drawn corner and face positions as whole atoms would overcount atoms shared across cell boundaries.

**Step 2:** Count the occupied tetrahedral voids For an FCC cell with four effective lattice atoms, the number of tetrahedral voids is twice that number:

$$N_{\text{tetrahedral}} = 2 \times 4 = 8.$$

The question specifies 50% occupancy, so the number of additional carbon atoms is

$$N_{\text{additional}} = \frac{50}{100} \times 8 = 4.$$

These tetrahedral positions lie within the conventional cell and each occupied site contributes one atom to this count.

**Step 3:** Add the two distinct contributions The atoms on the FCC sites and those occupying its

tetrahedral voids are separate populations. Therefore,

$$N_{\text{total}} = N_{\text{FCC}} + N_{\text{additional}} = 4 + 4 = 8.$$

The void count alone is not the total atom count: it only identifies available interstitial positions. Conversely, using all eight voids would contradict the explicitly stated half occupancy. Diamond's tetrahedral bonding is consistent with this FCC-plus-basis description.

**Step 4:** Conclusion The conventional unit cell of diamond contains $\boxed{8}$ carbon atoms.

**Quick Tip to Solve:** For diamond, remember the count as four atoms from the FCC framework plus four atoms from half of its eight tetrahedral holes; distinguish framework atoms from occupied holes.