JEE Main 2021 Paper 1 (B.E./B.Tech) Jul 22 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 3 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 22 July 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to: [4 marks]

(A) 1862

(B) 1842

(C) 1852

(D) 1872

Correct Answer: (A) 1862

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. The two given sums therefore determine the same pair a, d.
- The requested difference is between two partial sums starting at the first term. Compute both with the recovered progression, or subtract their formulas; it is not the difference of the twentieth and sixth terms.

Step 1: Use the two given partial sums Apply the sum formula separately at $n = 5$ and $n = 10$:

$$\frac{5}{2}(2a + 4d) = 140, \qquad 5(2a + 9d) = 530.$$

Dividing by the numerical factors gives

$$2a + 4d = 56, \qquad 2a + 9d = 106.$$

Subtracting eliminates the first term: $5d = 50$, so $d = 10$. Substitution in the first equation then gives $2a = 16$, hence $a = 8$.

Step 2: Evaluate and subtract the required sums The progression begins $8, 18, 28, \ldots$. Using the same first term and common difference,

$$S_{20} = 10[16 + 19(10)] = 2060, \qquad S_6 = 3[16 + 5(10)] = 198.$$

Therefore

$$S_{20} - S_6 = 2060 - 198 = 1862.$$

As a check, the removed terms are exactly the first six, so this is also the sum of fourteen terms from the seventh term 68 to the twentieth term 198: $14(68 + 198)/2 = 1862$. The other listed values 1842, 1852 and 1872 do not equal this sum.

Step 3: Conclusion $S_{20} - S_6 = \boxed{1862}$. Hence, the correct option is (A).

Quick Tip to Solve: Normalize each given sum by its factor $n/2$ before subtracting; this eliminates a and reveals the common difference immediately.

2. Let f : R → R be defined as $f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x, & x \leq 0 \end{cases}$ **. Then f is increasing function in the interval** [4 marks]

(A) $\left(-\frac{1}{2}, 2\right)$

(B) (0,2)

(C) $\left(-1, \frac{3}{2}\right)$

(D) $(-3, -1)$

Correct Answer: (C) $\left(-1, \frac{3}{2}\right)$

Solution:

Concept:

- A differentiable function is increasing where its derivative is positive. For a piecewise function, calculate the derivative on each branch and check that the branches join without a downward jump.
- The exponential e^x is positive for every real x, so the sign of $3e^x(1+x)$ is controlled solely by $1+x$. A quadratic derivative can be analysed using its two real roots.

Step 1: Find the derivative on each branch For $x > 0$, differentiate the polynomial:

$$f\prime(x) = -4x^2 + 4x + 3 = -(2x-3)(2x+1).$$

This is positive between $-1/2$ and $3/2$; in the actual branch domain it is positive for $0 < x < 3/2$. For $x < 0$, the product rule gives

$$f\prime(x) = 3e^x + 3xe^x = 3e^x(1+x).$$

It is positive for $-1 < x < 0$ and negative for $x < -1$. Thus the two increasing pieces meet at the branch point.

Step 2: Check the join and compare the intervals At the join, $f(0) = 0$ and the right-hand limit of the polynomial is also zero. The left and right derivatives at zero are both 3. Hence the increasing pieces join continuously, giving the increasing interval

$$(-1, 3/2).$$

Options (A) and (B) both extend to 2, and include points greater than $3/2$, where the polynomial derivative is negative. Option (D), $(-3, -1)$, lies where $3e^x(1+x) < 0$, so the function decreases there. Only option (C) stays inside the full increasing region.

Step 3: Conclusion The function is increasing on $\boxed{(-1, 3/2)}$. Hence, the correct option is (C).

Quick Tip to Solve: Intersect each derivative-sign interval with its own branch domain first, then join the pieces only after checking continuity at the boundary.

3. Let y = y(x) be the solution of the differential equation $\cos ec^2 x dy + 2dx = (1 + y\cos 2x)\cos ec^2 x dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0)+1)^2$ is equal to : [4 marks]

(A) $e^{1/2}$

(B) $e^{-1/2}$

(C) e^{-1}

(D) e

Correct Answer: (C) e^{-1}

Solution:

Concept:

- Use $1/\operatorname{cosec}^2 x = \sin^2 x$ and $1 - 2\sin^2 x = \cos 2x$. These identities turn the differential equation into a separable equation for $y + 1$.
- For $y \neq -1$, integrating $dy/(y+1)$ gives $\ln|y+1|$. The initial value fixes the constant and selects the positive branch of $y + 1$.

Step 1: Simplify and separate the equation Divide the given equation by $\operatorname{cosec}^2 x\, dx$ on an interval containing $\pi/4$:

$$y\prime + 2\sin^2 x = 1 + y\cos 2x.$$

Since $1 - 2\sin^2 x = \cos 2x$, this becomes

$$y\prime = (y+1)\cos 2x.$$

The initial value is zero, so this solution is not the constant solution $y = -1$. Separate and integrate:

$$\frac{dy}{y+1} = \cos 2x\, dx, \qquad \ln|y+1| = \frac{1}{2}\sin 2x + C.$$

The factor $1/2$ appears because the angle is $2x$.

Step 2: Apply the initial value and evaluate the square At $x = \pi/4$, we have $y = 0$ and $\sin 2x = 1$. Hence $0 = 1/2 + C$, so $C = -1/2$. As $y + 1$ is positive at the initial point,

$$y + 1 = \exp\left(\frac{\sin 2x - 1}{2}\right).$$

This expression has a continuous extension to zero, even though the original cosecant notation is undefined exactly there. It gives $y(0) + 1 = e^{-1/2}$, and therefore

$$(y(0)+1)^2 = e^{-1}.$$

Option (B) is the unsquared intermediate value; (A) and (D) have positive exponents and fail the initial-value calculation.

Step 3: Conclusion $\boxed{(y(0)+1)^2 = e^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: After simplifying a differential equation, look for a shift such as $u = y + 1$; here it makes separation immediate and prevents losing the final square.

4. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 × 2 matrices. The probability that such formed matrix have all different entries and are non-singular, is : [4 marks]

(A) $\frac{45}{162}$
(B) $\frac{21}{81}$
(C) $\frac{22}{81}$
(D) $\frac{43}{162}$

Correct Answer: (D) $\frac{43}{162}$

Solution:

Concept:

- Four independent dice give 6^4 equally likely ordered outcomes. Assign their values to the four fixed positions of $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Different assignments to positions count separately.
- The matrix is singular precisely when $ad = bc$. Count all outcomes with four distinct entries, then subtract those distinct-entry outcomes whose two diagonal products are equal.

Step 1: Count matrices having four distinct entries There are 6 choices for the first entry, then 5, 4 and 3 choices for the next entries, because repetition is forbidden. Thus the number of distinct-entry matrices is

$$6 \cdot 5 \cdot 4 \cdot 3 = 360.$$

We cannot use $\binom{6}{4}$ alone: it chooses the four face values without placing them in the matrix. Each chosen set has $4!$ possible placements, recovering $\binom{6}{4}4! = 360$. Some of these matrices are singular and must still be excluded.

Step 2: Remove the singular arrangements For four different face values, equal products from disjoint pairs in $\{1, 2, 3, 4, 5, 6\}$ occur only as

$$1 \cdot 6 = 2 \cdot 3, \qquad 2 \cdot 6 = 3 \cdot 4.$$

For either relation, choose which pair occupies the main diagonal in 2 ways and arrange the two entries on each diagonal in $2!$ ways. Thus each relation gives $2 \cdot 2 \cdot 2 = 8$ singular matrices, for a total of 16. The favourable count is $360 - 16 = 344$, so

$$P = \frac{344}{1296} = \frac{43}{162}.$$

Option (A) counts all distinct matrices without removing singular ones; (B) and (C) give different favourable counts and do not match the exhaustive product check.

Step 3: Conclusion The required probability is $\boxed{43/162}$. Hence, the correct option is (D).

Quick Tip to Solve: For a dice matrix, first count ordered distinct placements, then inspect repeated products of disjoint pairs; equal diagonal products are the only obstruction.

5. If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx = \frac{\alpha\pi^3}{1+4\pi^2}, \alpha \in R$ **where [x] is the greatest integer less than or equal to x, then the value of** α **is :** [4 marks]

(A) 200 $(1 - e^{-1})$

(B) 100 $(1 - e)$

(C) 50 $(e - 1)$

(D) 150 $(e^{-1} - 1)$

Correct Answer: (A) 200 $(1 - e^{-1})$

Solution:

Concept:

- The fractional part $x/\pi - \lfloor x/\pi \rfloor$ repeats with period π, and so does $\sin^2 x$. Therefore the complete integrand has period π, apart from irrelevant endpoint values.
- Use $\sin^2 t = (1 - \cos 2t)/2$ to evaluate one period. For the exponential-trigonometric integral, $\int e^{au}\cos(bu)\,du = e^{au}[a\cos(bu) + b\sin(bu)]/(a^2 + b^2)$.

Step 1: Reduce the integral to one period On the interval $k\pi \le x < (k+1)\pi$, put $x = k\pi + t$, where $0 \le t < \pi$. Then the fractional part equals t/π, and $\sin^2(k\pi + t) = \sin^2 t$. Each of the 100 intervals contributes the same value. With $u = t/\pi$,

$$I = 100\int_0^{\pi} e^{-t/\pi}\sin^2 t\,dt = 100\pi\int_0^1 e^{-u}\sin^2(\pi u)\,du.$$

Values at isolated interval endpoints do not change the integral.

Step 2: Evaluate and identify the coefficient Write $J = \int_0^1 e^{-u}\sin^2(\pi u)\,du$. The double-angle identity gives

$$J = \frac{1}{2}\left[\int_0^1 e^{-u}\,du - \int_0^1 e^{-u}\cos(2\pi u)\,du\right].$$

The first integral is $1 - e^{-1}$. Evaluating the displayed exponential-cosine formula at 0 and 1 gives the second as $(1 - e^{-1})/(1 + 4\pi^2)$. Consequently

$$J = \frac{2\pi^2(1 - e^{-1})}{1 + 4\pi^2}, \qquad I = \frac{200\pi^3(1 - e^{-1})}{1 + 4\pi^2}.$$

Comparing with the question yields $\alpha = 200(1 - e^{-1})$. Options (B) and (D) are negative despite the positive integrand, while (C) has the wrong exponential and coefficient.

Step 3: Conclusion $\boxed{\alpha = 200(1 - e^{-1})}$. Hence, the correct option is (A).

Quick Tip to Solve: When a fractional part appears with a matching periodic numerator, split into equal periods before attempting integration; here 100 periods become one elementary integral.

6. The values of λ and μ such that the system of equations $x+y+z=6$, $3x+5y+5z=26$, $x+2y+\lambda z=\mu$ has no solution, are : [4 marks]

(A) $\lambda = 3$, $\mu = 5$

(B) $\lambda = 3$, $\mu \neq 10$

(C) $\lambda \neq 2$, $\mu = 10$

(D) $\lambda = 2$, $\mu \neq 10$

Correct Answer: (D) $\lambda = 2$, $\mu \neq 10$

Solution:

Concept:

- A system has no solution when elimination produces a contradiction, such as $0 =$ a nonzero constant. A zero coefficient determinant alone is insufficient: it may instead give infinitely many solutions.
- Use the first two equations to determine x and the sum $y+z$. The third equation then tests whether the remaining free parameter can be chosen consistently.

Step 1: Eliminate one variable using the first two equations Multiply $x+y+z=6$ by 3 and subtract it from $3x+5y+5z=26$:

$$2y+2z=26-18=8.$$

Hence $y+z=4$. Substituting into the first equation gives $x=2$. Thus all solutions of the first two equations have the form

$$(x,y,z)=(2,4-z,z).$$

There is still one free parameter, so the third equation must be examined rather than assuming inconsistency.

Step 2: Test the third equation and the options Substitute $x=2$ and $y=4-z$ in $x+2y+\lambda z=\mu$:

$$2+2(4-z)+\lambda z=\mu, \qquad (\lambda-2)z=\mu-10.$$

If $\lambda \neq 2$, a unique value $z=(\mu-10)/(\lambda-2)$ exists for every μ; hence the system is consistent. If $\lambda=2$, the equation reduces to $0=\mu-10$. It is contradictory exactly when $\mu \neq 10$; if $\mu=10$, infinitely many solutions remain. Options (A) and (B) have $\lambda=3$ and therefore allow a solution, and (C) also has $\lambda \neq 2$. Only (D) gives the contradictory case.

Step 3: Conclusion There is no solution exactly when $\boxed{\lambda=2,\ \mu\neq 10}$. Hence, the correct option is (D).

Quick Tip to Solve: Reduce a three-equation consistency question to one equation in the free parameter: zero coefficient with a nonzero right side signals no solution.

7. If the shortest distance between the straight lines $3(x-1)=6(y-2)=2(z-1)$ and $4(x-2)=2(y-\lambda)=(z-3), \lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to : [4 marks]

(A) 3

(B) 2

(C) 5

(D) -1

Correct Answer: (A) 3

Solution:

Concept:

- For lines through $\mathbf{a}_1, \mathbf{a}_2$ with nonparallel direction vectors $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|/|\mathbf{b}_1 \times \mathbf{b}_2|$.
- When the symmetric form multiplies coordinate differences, convert it to parametric form before reading the directions. An absolute-value distance equation may give two real parameters, after which the integral-value condition must be applied.

Step 1: Extract points and direction vectors correctly From $3(x-1)=6(y-2)=2(z-1)$, the directions are proportional to $(1/3, 1/6, 1/2)$, so choose $\mathbf{b}_1 = (2,1,3)$, with point $\mathbf{a}_1 = (1,2,1)$. From $4(x-2)=2(y-\lambda)=z-3$, choose $\mathbf{b}_2 = (1,2,4)$, with point $\mathbf{a}_2 = (2,\lambda,3)$. Their cross product is

$$\mathbf{b}_1 \times \mathbf{b}_2 = (-2,-5,3), \qquad |\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{38}.$$

Thus the lines are nonparallel and the stated distance formula applies.

Step 2: Apply the given distance and retain the integer The displacement between the chosen points is $(1, \lambda-2, 2)$. Its scalar product with the normal vector is

$$-2-5(\lambda-2)+6 = 14-5\lambda.$$

Equating the distance to $1/\sqrt{38}$ gives

$$\frac{|14-5\lambda|}{\sqrt{38}} = \frac{1}{\sqrt{38}}, \qquad |14-5\lambda| = 1.$$

The two possibilities are $\lambda = 13/5$ and $\lambda = 3$. Only 3 is an integer. The other listed integers 2, 5 and -1 give absolute scalar products 4, 11 and 19 respectively, so none has the required distance.

Step 3: Conclusion The integral value is $\boxed{\lambda = 3}$. Hence, the correct option is (A).

Quick Tip to Solve: Coefficients multiplying coordinate differences give reciprocal direction ratios; clear their denominators before computing the cross product.

8. Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to : [4 marks]

(A) 2

(B) 1

(C) 3

(D) 9

Correct Answer: (C) 3

Solution:

Concept:

- A matrix row sum is obtained by multiplying that row by the column vector $u = (1, 1, 1)^T$. Thus the given condition is exactly $Au = u$.
- Matrix multiplication is associative: $A^3u = A(A(Au))$. The sum of all entries equals the sum of the components of the row-sum vector.

Step 1: Express the row conditions as a matrix equation The three stated equations, one for each row of A, combine to give

$$Au = \begin{pmatrix} a_{11} + a_{12} + a_{13} \\ a_{21} + a_{22} + a_{23} \\ a_{31} + a_{32} + a_{33} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = u.$$

This statement does not require that the individual entries be positive or that the matrix be symmetric. The row sums alone determine the desired result.

Step 2: Apply the matrix repeatedly Multiply $Au = u$ on the left by A. Associativity gives

$$A^2u = A(Au) = Au = u, \qquad A^3u = A(A^2u) = Au = u.$$

Consequently, every row of A^3 has sum 1, just as every row of A does. This is preservation of the row-sum condition under taking positive integral powers.

Step 3: Add the three row sums There are three rows, so summing all nine entries means adding the three row sums, not counting each entry as 1:

$$\sum_{i=1}^{3}\sum_{j=1}^{3}(A^3)_{ij} = 1 + 1 + 1 = 3.$$

Option (B) gives only one row sum, while option (D) incorrectly treats all nine entries as 1. Option (A) does not equal the sum of the three known row sums.

Step 4: Conclusion The sum of all entries of A^3 is $\boxed{3}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal row sums, multiply by the all-ones column vector; its eigenvalue immediately gives the row sums of every matrix power.

9. Let [x] denote the greatest integer less than or equal to x. Then, the values of x∈R satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval : [4 marks]

(A) $\left[0, \frac{1}{e}\right)$

(B) $[\log_e 2, \log_e 3)$

(C) [1, e)

(D) $[0, \log_e 2)$

Correct Answer: (D) $[0, \log_e 2)$

Solution:

Concept:

- The greatest integer function satisfies $\lfloor t + 1 \rfloor = \lfloor t \rfloor + 1$ for every real t. A positive exponential has a nonnegative integer part.
- The exponential and natural logarithm are strictly increasing. Therefore inequalities for e^x can be converted to inequalities for x without reversing their directions.

Step 1: Reduce the floor equation to an integer equation Set $m = \lfloor e^x \rfloor$. Since $e^x > 0$, we know m is an integer with $m \geq 0$. The shift identity gives $\lfloor e^x + 1 \rfloor = m + 1$, so the given equation becomes

$$m^2 + (m + 1) - 3 = 0 \quad \Longrightarrow \quad m^2 + m - 2 = 0.$$

The square here is the square of the integer part; it is not the integer part of e^{2x}.

Step 2: Select the admissible integer root Factor the polynomial:

$$(m + 2)(m - 1) = 0.$$

Its possible roots are -2 and 1, but a negative integer part is impossible for a positive exponential. Hence $m = 1$. The defining inequality for this floor value is

$$1 \leq e^x < 2.$$

The left endpoint is included because the floor of 1 is 1, whereas 2 is excluded because its floor is 2.

Step 3: Translate the interval and check the choices Taking natural logarithms yields

$$0 = \ln 1 \leq x < \ln 2.$$

At $x = 0$, the original left side is $1 + 2 - 3 = 0$, confirming inclusion. At $x = \ln 2$, it is $4 + 3 - 3 = 4$, confirming exclusion. Option (B) instead gives floor value 2; option (C) starts beyond the solution interval. Option (A) stops at $1/e$, so it omits valid values between $1/e$ and $\ln 2$.

Step 4: Conclusion The solution set is $\boxed{[0, \ln 2)}$. Hence, the correct option is (D).

Quick Tip to Solve: Replace a repeated floor by a single integer first, using the integer-shift identity before solving inequalities.

10. Let the circle S : $36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then : [4 marks]

(A) $\frac{25}{9} < C < \frac{13}{3}$

(B) $100 < C < 165$

(C) $81 < C < 156$

(D) $100 < C < 156$

Correct Answer: (D) $100 < C < 156$

Solution:

Concept:

- For a circle $(x-h)^2 + (y-k)^2 = r^2$, the distances of its centre from the coordinate axes are $|k|$ and $|h|$. Avoiding both axes requires the radius to be strictly smaller than both distances.
- A point is strictly inside a circle when its squared distance from the centre is less than r^2. The boundary inequalities are strict because neither touching nor lying on the circle is allowed.

Step 1: Find the centre and radius Divide the circle equation by 36 and complete the squares:

$$\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{5}{3}\right)^2 = \frac{181 - C}{36}.$$

Thus the centre is $H = (3/2, -5/3)$, and $r^2 = (181 - C)/36$. Its distances from the y-axis and x-axis are respectively $3/2$ and $5/3$.

Step 2: Impose the condition on the axes The smaller centre-to-axis distance is $3/2$. The circle must therefore satisfy

$$r < \frac{3}{2} \quad \Longrightarrow \quad \frac{181 - C}{36} < \frac{9}{4} \quad \Longrightarrow \quad C > 100.$$

Equality would make the circle tangent to the y-axis and is excluded. This condition also ensures separation from the more distant x-axis.

Step 3: Use the given interior point Solving $x - 2y = 4$ and $2x - y = 5$ gives $P = (2, -1)$. Its squared distance from H is

$$HP^2 = \left(2 - \frac{3}{2}\right)^2 + \left(-1 + \frac{5}{3}\right)^2 = \frac{1}{4} + \frac{4}{9} = \frac{25}{36}.$$

Since P is inside, $25/36 < (181 - C)/36$, giving $C < 156$. Combining both restrictions gives $100 < C < 156$, which also guarantees a positive radius. Option (B) permits values failing the interior-point test; option (C) permits values touching or crossing an axis; option (A) fails the axis condition.

Step 4: Conclusion The required range is $\boxed{100 < C < 156}$. Hence, the correct option is (D).

Quick Tip to Solve: Complete the squares once, then translate each geometric statement into a strict inequality involving the squared radius.

11. Let n denote the number of solutions of the equation $z^2 + 3\overline{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to : [4 marks]

(A) 1

(B) $\frac{4}{3}$

(C) $\frac{3}{2}$

(D) 2

Correct Answer: (B) $\frac{4}{3}$

Solution:

Concept:

- Write a complex number as $z = x + iy$, where x, y are real, so $\overline{z} = x - iy$. A complex expression is zero precisely when its real and imaginary parts both vanish.
- For $|r| < 1$, the infinite geometric series beginning at exponent zero sums to $1/(1 - r)$. Here the ratio is $1/n$, after counting all distinct complex roots.

Step 1: Equate the real and imaginary parts Substitution into the equation gives

$$z^2 + 3\overline{z} = (x^2 - y^2 + 3x) + i(2xy - 3y) = 0.$$

Therefore

$$x^2 - y^2 + 3x = 0, \qquad y(2x - 3) = 0.$$

The imaginary equation produces two cases. Keeping both is essential: restricting z to real numbers would discard valid nonreal solutions.

Step 2: Count every distinct solution If $y = 0$, the real equation becomes $x(x + 3) = 0$, yielding $z = 0$ and $z = -3$. If $2x - 3 = 0$, then $x = 3/2$, and the real equation gives

$$y^2 = \frac{9}{4} + \frac{9}{2} = \frac{27}{4}.$$

This produces $z = 3/2 \pm (3\sqrt{3}/2)i$. The two nonreal numbers are distinct from each other and from the two real roots. Thus there are exactly four solutions, so $n = 4$.

Step 3: Evaluate the infinite sum The ratio is $1/4$, whose modulus is less than 1, and the first term, at $k = 0$, is 1. Hence

$$\sum_{k=0}^{\infty} \frac{1}{n^k} = 1 + \frac{1}{4} + \frac{1}{16} + \cdots = \frac{1}{1 - 1/4} = \frac{4}{3}.$$

Option (A) retains only the first term. Options (C) and (D) would correspond to incorrectly counting three or two solutions, respectively, instead of all four complex solutions.

Step 4: Conclusion There are $n = 4$ complex solutions, and the sum is $\boxed{4/3}$. Hence, the correct option is (B).

Quick Tip to Solve: When an equation contains a conjugate, separate real and imaginary parts before counting roots; the usual polynomial degree rule does not apply directly.

12. If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to : [4 marks]

(A) $\frac{3}{2}$

(B) 2

(C) $\frac{1}{2}$

(D) 1

Correct Answer: (A) $\frac{3}{2}$

Solution:

Concept:

- The real function $\cos^{-1} t$ requires $-1 \le t \le 1$. Since its argument here is a square root, the radicand must lie between 0 and 1.
- A square root in the denominator must be strictly positive, so the principal value of the arcsine must be positive as well as defined. On its real principal branch, $\sin^{-1} t > 0$ exactly when $0 < t \le 1$.

Step 1: Find the restriction from the numerator The numerator is defined when

$$0 \le x^2 - x + 1 \le 1.$$

The lower inequality always holds, because $x^2 - x + 1 = (x - 1/2)^2 + 3/4 > 0$. The upper inequality gives

$$x^2 - x \le 0 \quad \Longrightarrow \quad x(x-1) \le 0 \quad \Longrightarrow \quad x \in [0, 1].$$

A zero numerator at an endpoint is allowed; it does not itself exclude that endpoint.

Step 2: Find the strict denominator restriction The denominator $\sqrt{\sin^{-1}((2x-1)/2)}$ must be real and nonzero. Therefore

$$0 < \sin^{-1}\left(\frac{2x-1}{2}\right) \quad \Longleftrightarrow \quad 0 < \frac{2x-1}{2} \le 1.$$

The upper bound includes the real arcsine domain restriction. Solving these inequalities gives $1/2 < x \le 3/2$. At $x = 1/2$, the denominator is zero, so that endpoint must be removed.

Step 3: Intersect the restrictions and identify the endpoints Both numerator and denominator must be valid simultaneously. Their intervals intersect as

$$[0, 1] \cap (1/2, 3/2] = (1/2, 1].$$

At $x = 1$, the numerator is $\cos^{-1} 1 = 0$ and the denominator is positive, so the function is indeed defined. Thus $\alpha = 1/2$ and $\beta = 1$, with sum $3/2$. The other options do not equal the sum of these verified endpoints; using only the arcsine restriction or allowing a zero denominator would give an incorrect domain.

Step 4: Conclusion The domain is $(1/2, 1]$, and $\boxed{\alpha + \beta = 3/2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a square root in a denominator, use a strict positivity condition before intersecting with the numerator domain.

13. Let f : R → R be defined as $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right), & x \neq 0 \\ \alpha, & x = 0 \end{cases}$

If f is continuous at x = 0, then α is equal to : [4 marks]

(A) 1

(B) 3

(C) 0

(D) 2

Correct Answer: (A) 1

Solution:

Concept:

- Continuity at zero requires the assigned value α to equal the two-sided limit of the expression for nonzero x.
- The standard limits $\sin x/x \to 1$ and $\log(1+t)/t \to 1$ as their variables approach zero reduce the trigonometric and logarithmic factors. Exponentials are continuous at zero.

Step 1: Identify the small logarithmic increment Write the argument of the logarithm as $R(x)$. Subtracting 1 using the common denominator gives

$$R(x) - 1 = \frac{2xe^{-2x} + 2xe^{-x} - x^2e^{-2x}}{(1 - xe^{-x})^2}.$$

Therefore

$$\frac{R(x) - 1}{x} = \frac{2e^{-2x} + 2e^{-x} - xe^{-2x}}{(1 - xe^{-x})^2} \longrightarrow 4.$$

In particular, $R(x) \to 1$; its numerator and denominator are positive in a sufficiently small neighbourhood of zero, so the real logarithm is defined there.

Step 2: Evaluate the logarithmic limit Applying the standard logarithmic limit to $t = R(x) - 1$ gives

$$\frac{\log R(x)}{x} = \frac{\log R(x)}{R(x) - 1}\,\frac{R(x) - 1}{x} \longrightarrow 1 \cdot 4 = 4.$$

This calculation retains both linear contributions in the logarithm argument, one from the numerator and one from the reciprocal denominator. Omitting either contribution would change the resulting constant.

Step 3: Combine the factors and impose continuity Use $1 - \cos 2x = 2\sin^2 x$ to rearrange the nonzero branch:

$$\frac{x^3 \log R(x)}{(1 - \cos 2x)^2} = \frac{1}{4}\left(\frac{x}{\sin x}\right)^4 \frac{\log R(x)}{x}.$$

Both component limits exist from either side, so the limit is $(1/4) \cdot 1 \cdot 4 = 1$. Continuity forces α to be this limit. Values 3, 0 and 2 would leave the assigned value different from the two-sided limit and hence cannot make the function continuous.

Step 4: Conclusion Continuity requires $\boxed{\alpha = 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a logarithm whose argument tends to 1, compute the first-order limit of its argument minus 1 before handling the outer powers.

14. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is : [4 marks]

(A) $\frac{-1+\sqrt{5}}{2}$

(B) $\frac{-1+\sqrt{8}}{2}$

(C) $\frac{-1+\sqrt{3}}{2}$

(D) $\frac{-1+\sqrt{6}}{2}$

Correct Answer: (A) $\frac{-1+\sqrt{5}}{2}$

Solution:

Concept:

- An ellipse with semiaxes $a > b > 0$ has eccentricity $e = \sqrt{1 - b^2/a^2}$. Its centre is the midpoint of its two foci.
- For an ellipse with major semiaxis A, minor semiaxis B, and focal distance c, the relations are $A^2 = B^2 + c^2$ and $e = c/A$.

Step 1: Determine the axes of the second ellipse The first ellipse has endpoints $(\pm a, 0)$ on its major axis and $(0, \pm b)$ on its minor axis. The latter are the foci of E_2, so E_2 is centred at the origin with a vertical major axis and focal distance b. Since it passes through $(\pm a, 0)$, its horizontal minor semiaxis is a. Hence its vertical major semiaxis is

$$A = \sqrt{a^2 + b^2}.$$

Step 2: Express the common eccentricity in two ways Let the common eccentricity be e, with $0 < e < 1$. From E_1,

$$b^2 = a^2(1 - e^2).$$

From E_2,

$$e^2 = \frac{b^2}{a^2 + b^2} = \frac{1 - e^2}{2 - e^2}.$$

Multiplication is valid because $2 - e^2 > 0$. It gives $e^4 - 3e^2 + 1 = 0$.

Step 3: Select the geometrically valid root Factor the equation as

$$(e^2 + e - 1)(e^2 - e - 1) = 0.$$

For $0 < e < 1$, the second factor is negative, so $e^2 + e - 1 = 0$, whose admissible root is

$$e = \frac{\sqrt{5} - 1}{2}.$$

The other root of that quadratic is negative. Options (B), (C), and (D) fail the derived equal-eccentricity equation, so they cannot describe this pair of ellipses.

Step 4: Conclusion The common eccentricity is $\boxed{(\sqrt{5} - 1)/2}$. Hence, the correct option is (A).

Quick Tip to Solve: Locate the second ellipse from its foci first: here its major axis is vertical, so the first ellipse's major semiaxis becomes its minor semiaxis.

15. Let L be the line of intersection of planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})=2$. If $P(\alpha,\beta,\gamma)$ is the foot of perpendicular on L from the point (1, 2, 0), then the value of $35(\alpha+\beta+\gamma)$ is equal to : [4 marks]

(A) 101

(B) 119

(C) 143

(D) 134

Correct Answer: (B) 119

Solution:

Concept:

- The direction of the intersection of two planes is perpendicular to both plane normals. Here the normals are $\mathbf{n}=(1,-1,2)$ and $\mathbf{m}=(2,1,-1)$.
- The perpendicular displacement from a point to the intersection line lies in the span of the two normals. The foot must also satisfy both plane equations.

Step 1: Express the perpendicular foot Let $Q=(1,2,0)$. Write the position vector of the foot as $P=Q+u\mathbf{n}+v\mathbf{m}$, where u,v are real numbers. Every vector in this normal plane is perpendicular to the direction of the intersection line. The remaining requirement is that P actually lie on both given planes. Since $\mathbf{n}^2=\mathbf{m}^2=6$, $\mathbf{n}\cdot\mathbf{m}=-1$, $Q\cdot\mathbf{n}=-1$ and $Q\cdot\mathbf{m}=4$, taking dot products gives

$$-1+6u-v=2, \qquad 4-u+6v=2.$$

Step 2: Solve the two simultaneous equations The equations reduce to $6u-v=3$ and $-u+6v=-2$. From the first, $v=6u-3$. Substituting into the second gives $35u=16$, hence $u=16/35$ and $v=-9/35$. Therefore

$$P=(1,2,0)+\frac{16}{35}(1,-1,2)-\frac{9}{35}(2,1,-1)=\left(\frac{33}{35},\frac{45}{35},\frac{41}{35}\right).$$

Step 3: Check the location and evaluate the sum The first plane check is $(33-45+82)/35=2$, and the second is $(66+45-41)/35=2$. Thus the computed point belongs to the intersection and its displacement from Q is perpendicular to that line. Consequently

$$35(\alpha+\beta+\gamma)=33+45+41=119.$$

The other proposed values 101, 143 and 134 disagree with this verified sum.

Step 4: Conclusion The required value is 119. Hence, the correct option is (B).

Quick Tip to Solve: For the foot on an intersection line, express the displacement as a linear combination of the two plane normals; this avoids finding a separate parametrisation of the line.

16. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then a possible value of $[\vec{a} \quad \vec{b} \quad \vec{c}] + [\vec{a} \quad \vec{b} \quad \vec{d}] + [\vec{a} \quad \vec{c} \quad \vec{d}]$ is equal to : [4 marks]

(A) -42

(B) -40

(C) -29

(D) -38

Correct Answer: (A) -42

Solution:

Concept:

- Coplanarity with two independent vectors means $\mathbf{a} = u\mathbf{b} + v\mathbf{c}$. A scalar triple product $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ then vanishes.
- Perpendicularity is expressed by a zero dot product, while the prescribed magnitude fixes the scale up to its sign. Triple products retain this sign.

Step 1: Use the perpendicularity condition Here $\mathbf{b} = (2, 1, 1)$, $\mathbf{c} = (1, -1, 1)$ and $\mathbf{d} = (3, 2, 6)$. Their dot products with $\mathbf{d}$ are

$$\mathbf{b} \cdot \mathbf{d} = 6 + 2 + 6 = 14, \qquad \mathbf{c} \cdot \mathbf{d} = 3 - 2 + 6 = 7.$$

Writing $\mathbf{a} = u\mathbf{b} + v\mathbf{c}$, the condition $\mathbf{a} \cdot \mathbf{d} = 0$ gives $14u + 7v = 0$, so $v = -2u$. Thus $\mathbf{a} = u(\mathbf{b} - 2\mathbf{c}) = u(0, 3, -1)$.

Step 2: Fix the magnitude and compute the cross products The vector $(0, 3, -1)$ has squared magnitude 10. Hence $|\mathbf{a}| = \sqrt{10}$ requires $u^2 = 1$, allowing $u = 1$ or -1. Direct determinant expansion gives

$$\mathbf{b} \times \mathbf{d} = (4, -9, 1), \qquad \mathbf{c} \times \mathbf{d} = (-8, -3, 5).$$

Their sum is $(-4, -12, 6)$.

Step 3: Evaluate the requested sum with its sign The first triple product is zero by coplanarity. Linearity in the second and third terms therefore gives

$$\begin{aligned} S &= \mathbf{a} \cdot (\mathbf{b} \times \mathbf{d} + \mathbf{c} \times \mathbf{d}) \\ &= u(0, 3, -1) \cdot (-4, -12, 6) = -42u. \end{aligned}$$

Thus the possible values are -42 and 42. The question asks for a possible value, so the negative value is sufficient. The values $-40, -29, -38$ cannot result from either allowed sign.

Step 4: Conclusion A possible value is -42, attained when $\mathbf{a} = (0, 3, -1)$. Hence, the correct option is (A).

Quick Tip to Solve: Use coplanarity first to replace an unknown three-component vector by two coefficients, then use perpendicularity before applying the length condition.

17. Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true? [4 marks]

(A) $\vec{a} \times \left((\vec{b} + \vec{c}) \times (\vec{b} \times \vec{c}) \right) = \vec{0}$

(B) Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2

(C) $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] + \left[\vec{c} \quad \vec{a} \quad \vec{b}\right] = 8$

(D) $\left|3\vec{a} + \vec{b} - 2\vec{c}\right|^2 = 51$

Correct Answer: (D) $\left|3\vec{a} + \vec{b} - 2\vec{c}\right|^2 = 51$

Solution:

Concept:

- A nonzero cross product is perpendicular to both of its factors. Thus $\mathbf{a} \times \mathbf{b} = \mathbf{c}$ and $\mathbf{b} \times \mathbf{c} = \mathbf{a}$, with $|\mathbf{a}| = 2$, imply that these three vectors are pairwise perpendicular and nonzero.
- For perpendicular vectors, cross-product magnitudes multiply. The identity $\mathbf{x} \times (\mathbf{y} \times \mathbf{z}) = \mathbf{y}(\mathbf{x} \cdot \mathbf{z}) - \mathbf{z}(\mathbf{x} \cdot \mathbf{y})$ also evaluates nested products.

Step 1: Determine all three lengths Write $B = |\mathbf{b}|$ and $C = |\mathbf{c}|$. Pairwise perpendicularity gives $C = 2B$ from the first cross product and $2 = BC$ from the second. Therefore $2 = 2B^2$, so $B = 1$ and $C = 2$. Neither $\mathbf{b}$ nor $\mathbf{c}$ is zero. This also implies $\mathbf{b} + \mathbf{c} \neq \mathbf{0}$, since they are perpendicular nonzero vectors.

Step 2: Check the projection and scalar triple products Since $\mathbf{b} \times \mathbf{c} = \mathbf{a}$, the scalar projection in option (B) is $\mathbf{a} \cdot \mathbf{a}/|\mathbf{a}| = 2$, so (B) is true. Also $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \mathbf{a} \cdot \mathbf{a} = 4$. Cyclic permutation preserves a triple product, giving $[\mathbf{c}\,\mathbf{a}\,\mathbf{b}] = 4$. Their sum is 8, making (C) true.

Step 3: Check both remaining statements literally Using the given cross product in option (A), the left side becomes

$$\mathbf{a} \times ((\mathbf{b} + \mathbf{c}) \times \mathbf{a}) = |\mathbf{a}|^2(\mathbf{b} + \mathbf{c}) = 4(\mathbf{b} + \mathbf{c}) \neq \mathbf{0}.$$

Hence (A) is false as printed. For (D), all mixed dot products vanish, so

$$|3\mathbf{a} + \mathbf{b} - 2\mathbf{c}|^2 = 9(4) + 1 + 4(4) = 53,$$

which is not 51. Thus (D) is also false; no single choice is uniquely selected by the literal statements.

Step 4: Conclusion Option (D), the keyed option, is false because the squared magnitude is 53, not 51. However, option (A) is false as well, since its left side is $4(\mathbf{b} + \mathbf{c}) \neq \mathbf{0}$. The printed question therefore has two defensible answers, (A) and (D), rather than a unique answer. Hence, the correct option is (D) according to the key, with (A) also false as printed.

Quick Tip to Solve: When two cross products form a cycle, first establish orthogonality and lengths; then test every option, especially a nested cross product whose order controls its sign.

18. Which of the following Boolean expressions is not a tautology? [4 marks]

(A) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(B) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

(C) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

(D) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Correct Answer: (D) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its variables. To disprove tautology, it is enough to exhibit one assignment making the expression false.
- The implication $u \Rightarrow v$ is equivalent to $\neg u \vee v$. A disjunction containing both p and $\neg p$, or both q and $\neg q$, is always true.

Step 1: Simplify the first two expressions For option (A), replace both implications to obtain

$$(\neg p \vee q) \vee (q \vee p) = \neg p \vee p \vee q = \mathrm{T}.$$

The pair $\neg p \vee p$ makes it true irrespective of q. For option (B), the same procedure gives

$$(\neg q \vee p) \vee (q \vee p) = \neg q \vee q \vee p = \mathrm{T}.$$

Thus both (A) and (B) are tautologies, although each individual implication need not always be true.

Step 2: Check the third expression Option (C) simplifies as

$$(\neg p \vee \neg q) \vee (q \vee p) = \neg p \vee p \vee \neg q \vee q = \mathrm{T}.$$

Both a proposition and its negation occur, so there is no assignment under which the whole disjunction can be false. Consequently (C) is also a tautology and cannot answer the question.

Step 3: Find a counterexample for the last expression In option (D), $\neg p \Rightarrow q$ is $p \vee q$, and $\neg q \Rightarrow p$ is $q \vee p$. The complete expression is therefore simply $p \vee q$. At $p = \mathrm{F}$ and $q = \mathrm{F}$, both implications have true antecedents and false consequents. Each implication, and hence their disjunction, is false. This explicit counterexample proves that the last expression fails the defining condition for a tautology.

Step 4: Conclusion Only option (D) is not a tautology: it reduces to $p \vee q$, which is false when both variables are false. Hence, the correct option is (D).

Quick Tip to Solve: Eliminate every implication before constructing a truth table. Complementary literals often reveal a tautology immediately and leave just one expression to test.

19. The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to [4 marks]

(A) 11

(B) 7

(C) 5

(D) 9

Correct Answer: (C) 5

Solution:

Concept:

- For every real number $t \in [-1, 1]$, the inequality $t^7 \leq t^2$ holds, with equality exactly when $t = 0$ or $t = 1$. For negative nonzero t, its seventh power is negative while its square is positive.
- The identity $\sin^2 x + \cos^2 x = 1$ bounds the sum of the seventh powers. Equality of the sums requires equality for each of the two individual inequalities.

Step 1: Establish the equality condition For $0 < t < 1$, multiplying by additional factors smaller than one gives $t^7 < t^2$; at $t = 0, 1$, equality holds. For $-1 \leq t < 0$, $t^7 < 0 < t^2$. Applying this result separately to sine and cosine yields

$$\sin^7 x + \cos^7 x \leq \sin^2 x + \cos^2 x = 1.$$

Neither individual difference $t^2 - t^7$ can be negative, so the required sum equals one only if both differences are zero.

Step 2: Identify the allowed sine and cosine pairs Both $\sin x$ and $\cos x$ must belong to $\{0, 1\}$. Their squared sum is one, so the only possible ordered pairs are $(0, 1)$ and $(1, 0)$. Values involving -1 do not qualify: their odd seventh powers remain negative. The allowed angles are therefore $x = 2k\pi$ or $x = \pi/2 + 2k\pi$, for integers k.

Step 3: Count carefully on the closed interval In $[0, 4\pi]$, the first family gives $0, 2\pi, 4\pi$, while the second gives $\pi/2, 5\pi/2$. These two lists are disjoint and contain three and two angles respectively. Both endpoints are included and satisfy the original equation. The total is $3 + 2 = 5$. Counts 7, 9 and 11 would require further angles where one of the strict inequalities above holds, so they cannot be correct.

Step 4: Conclusion There are exactly 5 solutions on $[0, 4\pi]$. Hence, the correct option is (C).

Quick Tip to Solve: For high odd powers of sine and cosine, compare each term with its square. Equality conditions can reduce a difficult trigonometric equation to a few axis points.

20. Let a line L : 2x + y = k, k > 0 be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to : [4 marks]

(A) 12

(B) -12

(C) 24

(D) -24

Correct Answer: (D) -24

Solution:

Concept:

- A line is tangent to a nondegenerate conic when substitution produces a quadratic with a repeated root. Thus its discriminant must vanish.
- The same line must meet the tangent conditions for both curves. The condition $k > 0$ resolves the sign after the hyperbola determines k^2.

Step 1: Apply tangency to the hyperbola From the line, $y = k - 2x$. Substitute this into $x^2 - y^2 = 3$:

$$x^2 - (k - 2x)^2 = 3 \quad \Longrightarrow \quad 3x^2 - 4kx + k^2 + 3 = 0.$$

At tangency there is one repeated intersection, so

$$(-4k)^2 - 4(3)(k^2 + 3) = 0.$$

Simplification gives $16k^2 - 12k^2 - 36 = 0$, hence $k^2 = 9$. Since $k > 0$, we must take $k = 3$.

Step 2: Apply the second tangent condition The line is now $2x + y = 3$, or $x = (3 - y)/2$. Substitute into $y^2 = \alpha x$ to obtain

$$2y^2 + \alpha y - 3\alpha = 0.$$

Its discriminant must also vanish, giving

$$\alpha^2 - 4(2)(-3\alpha) = \alpha^2 + 24\alpha = 0.$$

Thus $\alpha(\alpha + 24) = 0$. The value $\alpha = 0$ produces the degenerate equation $y^2 = 0$, not the parabola stated in the question. Therefore $\alpha = -24$.

Step 3: Verify the sign and compare the options With $\alpha = -24$, the intersection equation is $2y^2 - 24y + 72 = 2(y - 6)^2 = 0$, showing a repeated root at $y = 6$. The line then gives $x = -3/2$, and $y^2 = 36 = (-24)(-3/2)$, confirming the contact point lies on both. The alternatives $12, -12, 24$ do not satisfy $\alpha(\alpha + 24) = 0$, so none can make this line tangent to the stated parabola.

Step 4: Conclusion The required parameter is $\alpha = -24$. Hence, the correct option is (D).

Quick Tip to Solve: Substitute the common line into each conic and set each discriminant to zero; retain the sign restrictions and discard values that collapse the conic.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions f : A → A such that f(1) + f(2) = 3 − f(3) is equal to [4 marks]

Solution:

Concept:

- A bijection from an eight-element set to itself assigns eight distinct images, so counting such functions is equivalent to counting permutations of the set.
- The condition on the first three images is $f(1) + f(2) + f(3) = 3$. Injectivity forces these three nonnegative integers to be distinct before the remaining images can be assigned.

Step 1: Determine the first three images Let $a = f(1)$, $b = f(2)$, and $c = f(3)$. The stated equation becomes $a + b + c = 3$. Since the function is bijective, a, b, c are pairwise distinct members of $\{0, 1, 2, 3, 4, 5, 6, 7\}$. The smallest sum of three distinct nonnegative integers is $0 + 1 + 2 = 3$. Equality is possible only for the unordered set $\{0, 1, 2\}$: replacing any of these ordered smallest values by a larger one increases the sum. Thus the images of 1, 2 and 3 are precisely these three values.

Step 2: Count their order and complete each function The values $0, 1, 2$ may be distributed among the three distinct inputs $1, 2, 3$ in $3! = 6$ ways. Every such ordering satisfies the equation because their sum remains three. The remaining inputs are $0, 4, 5, 6, 7$; the unused images are $3, 4, 5, 6, 7$. Assigning these five images bijectively to those five inputs is an arbitrary permutation, giving $5! = 120$ completions for each initial ordering.

Step 3: Use the product rule and check uniqueness The choices of the first three ordered images and of the remaining five images are independent once the allowed sets have been fixed. Therefore

$$N = 3!\,5! = 6 \times 120 = 720.$$

No function is counted twice: its ordered values at 1, 2 and 3 select exactly one of the six initial cases, and its values on the other five inputs select exactly one completion. Conversely every counted assignment is a bijection and satisfies the sum, so the count is exhaustive.

Step 4: Conclusion The required number of bijective functions is $\boxed{720}$. This is a dimensionless count.

Quick Tip to Solve: When distinct nonnegative images have a very small prescribed sum, compare that sum with the minimum possible sum before enumerating ordered triples.

22. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _____. [4 marks]

Solution:

Concept:

- A number greater than $10,000$ must have at least five decimal digits. With only five distinct available digits and no repetition, it must have exactly five digits.
- The first digit of a five-digit number cannot be zero. Once it is chosen, arrange the four remaining distinct digits in the four remaining positions using $4!$ permutations.

Step 1: Determine the possible lengths The available digits are $0, 2, 4, 6, 8$, each usable at most once. Therefore a number can contain no more than five digits. Every number with four or fewer digits is at most 9999, so none can exceed the given bound. Consequently every admissible number uses all five digits, and it remains only to exclude arrangements with a leading zero. We are counting ordinary decimal numbers, so a string beginning with zero does not count as a five-digit number.

Step 2: Choose the leading digit and arrange the rest The first position has four choices: $2, 4, 6$ or 8. For each choice, the remaining digits, including zero, can occupy the last four positions freely. The multiplication principle gives

$$N = 4 \times 4! = 4 \times 24 = 96.$$

Every number counted here is at least $20,468$, the smallest arrangement starting with the smallest nonzero digit, and thus certainly exceeds $10,000$. Conversely, every allowed number above the bound has a nonzero first digit and occurs exactly once in this count. As a check, all $5! = 120$ arrangements minus the $4! = 24$ arrangements beginning with zero also give $120 - 24 = 96$.

Step 3: Conclusion The required number of numbers is $\boxed{96}$. This is a dimensionless count.

Quick Tip to Solve: First fix the number of digits from the threshold; then handle zero separately in the leading position, where it is forbidden.

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23. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3 × 3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is _____. [4 marks]

Solution:

Concept:

- Left multiplication by A exchanges the first two rows, while right multiplication by A exchanges the first two columns.
- The equality $AB = BA$ gives entrywise equalities. Count independent entries after imposing all these equalities; each independent entry has five choices.

Step 1: Compare the two products Write a completely general matrix as

$$B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}.$$

Multiplying on the left and right gives

$$AB = \begin{pmatrix} d & e & f \\ a & b & c \\ g & h & i \end{pmatrix}, \qquad BA = \begin{pmatrix} b & a & c \\ e & d & f \\ h & g & i \end{pmatrix}.$$

These products show precisely why exchanging rows is different from exchanging columns: the entries must agree at the same positions after the two operations. Equating corresponding entries gives $d = b$, $e = a$, $f = c$, and $h = g$. The other equations repeat these conditions, while the bottom-right entry imposes no condition on i.

Step 2: Count the free parameters Thus the full family of commuting matrices is

$$B = \begin{pmatrix} a & b & c \\ b & a & c \\ g & g & i \end{pmatrix}.$$

There are exactly five independent entries, namely a, b, c, g, i. Each may independently be any element of $\{1, 2, 3, 4, 5\}$; entries are allowed to repeat, so no permutation factor is appropriate. Hence

$$N = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^5 = 3125.$$

There are no further restrictions: substituting this displayed family into both products exchanges equal patterns and yields identical matrices. Different choices of these five parameters give different matrices because each parameter is visible in at least one fixed position of B.

Step 3: Conclusion The required number of matrices is $\boxed{3125}$, a dimensionless count.

Quick Tip to Solve: For a permutation matrix, interpret multiplication as row and column swaps; then count the free entry groups rather than the nine original positions.

24. Consider the following frequency distribution :

Class :	0-6	6-12	12-18	18-24	24-30
Frequency :	a	b	12	9	5

If mean = $\frac{309}{22}$ and median = 14, then the value $(a-b)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For grouped data, $\bar{x} = \sum f_j x_j / \sum f_j$, where x_j is the midpoint of a class and f_j its frequency.
- The median formula is $M = l + [(N/2 - c)/f]h$, where c is the cumulative frequency before the median class, f is its frequency and h its width.

Step 1: Use the median to obtain the total frequency The total frequency is $N = a + b + 12 + 9 + 5 = a + b + 26$. Since the median is 14, the median class is 12–18. Thus $l = 12$, $h = 6$, $f = 12$, and the cumulative frequency before it is $c = a + b$. Substitute these quantities:

$$14 = 12 + \frac{N/2 - a - b}{12} \times 6.$$

It follows that $N/2 - a - b = 4$. Replacing N by $a + b + 26$ gives

$$\frac{a + b + 26}{2} - a - b = 4 \quad \Longrightarrow \quad a + b = 18.$$

Consequently $N = 44$, so the median position is $N/2 = 22$.

Step 2: Use the mean and solve the two equations The class midpoints are $3, 9, 15, 21, 27$. Therefore the given mean yields

$$\frac{3a + 9b + 12(15) + 9(21) + 5(27)}{44} = \frac{309}{22}.$$

The known contributions sum to $180 + 189 + 135 = 504$, while $44(309/22) = 618$. Hence $3a + 9b = 114$, or $a + 3b = 38$. Subtracting $a + b = 18$ gives $2b = 20$, so $b = 10$ and $a = 8$. Both frequencies are positive. The cumulative frequency before the median class is 18, and after it is 30, enclosing the median position 22, which confirms the class selection. Finally,

$$(a - b)^2 = (8 - 10)^2 = (-2)^2 = 4.$$

Step 3: Conclusion The required value is $\boxed{4}$, a dimensionless number.

Quick Tip to Solve: Use the median first: it fixes the sum of the two unknown frequencies, after which the mean gives an easy linear system.

25. The sum of all the elements in the set {n∈ {1, 2, ..., 100} | H.C.F. of n and 2040 is 1} is equal to _____. [4 marks]

Solution:

Concept:

- Factor the fixed integer into primes: an integer is coprime to it exactly when it is divisible by none of those primes.
- To sum integers satisfying several exclusions, use inclusion and exclusion on their sums. For a prime p, its positive multiples up to 100 sum to $p\,k(k+1)/2$, where $k = \lfloor 100/p \rfloor$.

Step 1: Reduce the coprimality condition The prime factorisation is

$$2040 = 2^3 \times 3 \times 5 \times 17.$$

Thus $\gcd(n, 2040) = 1$ means that n is not divisible by $2, 3, 5$, or 17. First retain the odd integers up to 100. There are 50 of these and their sum is $50^2 = 2500$. Among the odd integers, the multiples of 3 are $3, 9, \ldots, 99$, a progression with 17 terms and sum $17(3+99)/2 = 867$. The odd multiples of 5 are $5, 15, \ldots, 95$, with 10 terms and sum $10(5+95)/2 = 500$.

Step 2: Apply inclusion and exclusion and remove the last prime The odd multiples of both 3 and 5 have been subtracted twice. They are $15, 45, 75$, whose sum is 135; add this sum back once. The sum of integers coprime to 30 in the range is therefore

$$2500 - 867 - 500 + 135 = 1268.$$

Now inspect the multiples of 17 up to 100: $17, 34, 51, 68, 85$. Of these, 34 and 68 were already excluded as even, 51 as a multiple of 3, and 85 as a multiple of 5. Only 17 remains in the current sum, so it must be removed exactly once. Hence the desired sum is

$$1268 - 17 = 1251.$$

The integer 1 is included throughout, correctly, since its H.C.F. with every positive integer is 1.

Step 3: Conclusion The sum of all elements of the given set is $\boxed{1251}$, a dimensionless integer.

Quick Tip to Solve: After excluding the smallest prime divisors, inspect the few multiples of a larger prime; subtract only those still present to avoid double subtraction.

26. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is _____. [4 marks]

Solution:

Concept:

- For a region described by upper and lower curves, its area is $\int (y_{\text{upper}} - y_{\text{lower}})\,dx$. Use the nonnegative roots because only the upper half-plane is required.
- The downward parabola is $y = (1 - x^2)/2$. The other two have upper branches $y = 2\sqrt{1-x}$ and

$y = 2\sqrt{1+x}$, and the entire boundary is symmetric about the y-axis.

Step 1: Identify the enclosed boundary The two sideways parabolas meet where $4 - 4x = 4 + 4x$, so their upper intersection is $(0, 2)$. The downward parabola meets the right boundary at $(1, 0)$ and the left boundary at $(-1, 0)$. For example, substituting $y = (1 - x^2)/2$ into $y^2 = 4(1 - x)$ gives

$$(x - 1)\big((x - 1)(x + 1)^2 + 16\big) = 0.$$

On the relevant interval $[-1, 1]$, the second factor is positive, so the only such intersection is $x = 1$; symmetry gives the other endpoint. On $0 \le x \le 1$, the top of the bounded region is $2\sqrt{1 - x}$ and its bottom is $(1 - x^2)/2$. The left half is its mirror image.

Step 2: Integrate one half and double it The required area is

$$\mathcal{A} = 2\int_0^1 \left[2\sqrt{1-x} - \frac{1-x^2}{2}\right] dx.$$

The two integrals are

$$\int_0^1 2\sqrt{1-x}\,dx = \left[-\frac{4}{3}(1-x)^{3/2}\right]_0^1 = \frac{4}{3},$$

$$\int_0^1 \frac{1-x^2}{2}\,dx = \left[\frac{x}{2} - \frac{x^3}{6}\right]_0^1 = \frac{1}{3}.$$

Subtracting the area under the lower parabola from that under the upper branch, and then accounting for the other half, yields

$$\mathcal{A} = 2\left(\frac{4}{3} - \frac{1}{3}\right) = 2.$$

Every point integrated lies on or above $y = 0$, as required by the upper-half-plane restriction.

Step 3: Conclusion The bounded area in the upper half-plane is $\boxed{2 \text{ square units}}$.

Quick Tip to Solve: Exploit the left-right symmetry, but first identify which branch forms the upper boundary on each side of the vertical axis.

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27. Let f : R → R be a function defined as $f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & if \quad |x| \le 2 \\ 0 & if \quad |x| > 2 \end{cases}$

Let g : R → R be given by $g(x) = f(x + 2) - f(x - 2)$**. If n and m denote the number of points in R where g is not continuous and not differentiable, respectively, then n + m is equal to _____.**

[4 marks]

Solution:

Concept:

- The function f is a triangular continuous function: its value matches zero at both endpoints $x = \pm 2$. A difference of shifted continuous functions remains continuous.
- A piecewise linear function fails to be differentiable exactly at a joining point where its left and

right slopes differ. A candidate corner can disappear when shifted terms cancel.

Step 1: Count discontinuities Within $[-2, 0]$, $f(x) = 3 + 3x/2$, and within $[0, 2]$, $f(x) = 3 - 3x/2$. Outside this interval it is zero. At -2 and 2, the inside expression equals the outside value zero, and at 0 both expressions equal 3. Thus f is continuous everywhere. Each translate, $f(x+2)$ and $f(x-2)$, is consequently continuous on $\mathbb{R}$, as is their difference g. Therefore the number of discontinuities is $n = 0$.

Step 2: Compare the slopes at all possible corners The shifted breakpoints can occur only at $-4, -2, 0, 2, 4$. Writing the difference explicitly gives

$$g(x) = \begin{cases} 0, & x \le -4, \\ \frac{3}{2}(x+4), & -4 \le x \le -2, \\ -\frac{3}{2}x, & -2 \le x \le 2, \\ \frac{3}{2}(x-4), & 2 \le x \le 4, \\ 0, & x \ge 4. \end{cases}$$

At $x = 0$, both inner expressions join into the same line $-3x/2$; hence this candidate point is differentiable. The successive slopes across the remaining intervals are $0, 3/2, -3/2, 3/2, 0$. They change at $-4, -2, 2, 4$, so these four and only these four points are not differentiable. At each such point the function itself is still continuous: only the slope jumps. Thus $m = 4$, and

$$n + m = 0 + 4 = 4.$$

Step 3: Conclusion The required value is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: Combine the two shifted formulas before counting corners: at a shared breakpoint, slope changes can cancel, as they do here at zero.

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28. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _____. [4 marks]

Solution:

Concept:

- The general term of $(u+v)^{10}$ is $T_{k+1} = \binom{10}{k} u^{10-k} v^k$, with k an integer from 0 to 10.
- A constant term has exponent zero on x. Its coefficient must also equal the stated value, so both the exponent condition and coefficient condition must be satisfied.

Step 1: Write the general term and its exponent For $u = 2x^r$ and $v = x^{-2}$,

$$T_{k+1} = \binom{10}{k} (2x^r)^{10-k} (x^{-2})^k = \binom{10}{k} 2^{10-k} x^{r(10-k)-2k}.$$

Thus a constant term requires

$$r(10-k) - 2k = 0.$$

Its coefficient is $\binom{10}{k}2^{10-k}$. Notice that $k = 10$ produces x^{-20}, not a constant, regardless of r. The relevant index must therefore lie from 0 to 9, and the given coefficient determines that index before the exponent equation determines r.

Step 2: Determine the coefficient index and solve for the power For completeness, the coefficients $\binom{10}{k}2^{10-k}$, in order for $k = 0, 1, \ldots, 10$, are

$$1024,\ 5120,\ 11520,\ 15360,\ 13440,\ 8064,\ 3360,\ 960,\ 180,\ 20,\ 1.$$

The stated coefficient 180 occurs only at $k = 8$, because $\binom{10}{8}2^2 = 45 \times 4 = 180$. Substituting this index into the zero-exponent condition gives

$$r(10-8) - 2(8) = 0 \quad \Longrightarrow \quad 2r - 16 = 0 \quad \Longrightarrow \quad r = 8.$$

Checking directly, the ninth term is $\binom{10}{8}(2x^8)^2(x^{-2})^8 = 45 \cdot 4 \cdot x^{16-16} = 180$, exactly a constant. Therefore the coefficient and the power of x both meet the question conditions.

Step 3: Conclusion The required exponent is $\boxed{r = 8}$, which is dimensionless.

Quick Tip to Solve: Find the binomial index from the required coefficient first; then set its power of the variable to zero to determine the unknown exponent.

---

29. Let y = y(x) be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)dy$, y(1) = 1. If the domain of y = y(x) is an open interval (α, β), then $|\alpha + \beta|$ is equal to _____.

[4 marks]

Solution:

Concept:

- The shifted variables $X = x + 2$ and $Y = y + 1$ reduce the equation to $X\,dY/dX = Xe^{Y/X} + Y$. Its ratio substitution is $u = Y/X$, with $X \neq 0$.
- A real logarithm requires a positive argument. The maximal open interval containing the initial point is bounded both by the singular point $X = 0$ and by the zero of the expression inside the logarithm defining u.

Step 1: Make the ratio substitution Put $y + 1 = u(x + 2)$. Then $y\prime = u + (x + 2)u\prime$. Substitution in the printed differential equation gives

$$u + (x+2)u\prime = e^u + u, \qquad e^{-u}du = \frac{dx}{x+2}.$$

The initial condition gives $u(1) = 2/3$. The factor $x + 2$ is positive on the interval containing $x = 1$, so no absolute-value ambiguity remains in the logarithm there.

Step 2: Integrate and impose the initial condition Integration yields $-e^{-u} = \ln(x+2) + C$. Evaluating at $x = 1$ and subtracting gives

$$e^{-u} = e^{-2/3} + \ln\frac{3}{x+2}.$$

Consequently the literal solution is

$$y = -1 - (x+2)\ln\left(e^{-2/3} + \ln\frac{3}{x+2}\right).$$

At $x = 1$, the inner logarithm has argument $e^{-2/3}$, giving $y(1) = 1$, as required.

Step 3: Determine the maximal real interval The outer logarithm requires $e^{-2/3} + \ln(3/(x+2)) > 0$, and the inner logarithm requires $x + 2 > 0$. Thus

$$0 < x + 2 < 3\exp(e^{-2/3}).$$

The maximal interval through the initial point has $\alpha = -2$ and $\beta = 3\exp(e^{-2/3}) - 2$. The right endpoint makes the logarithm diverge; the left endpoint is a singular point of the given equation. Neither endpoint can be included in the required open interval.

Step 4: Conclusion For the equation and initial condition printed here,

$$\boxed{|\alpha + \beta| = 3\exp(e^{-2/3}) - 4 \approx 1.013}.$$

This does not equal the listed key value 4: direct integration and substitution into the printed initial condition give the expression above. If an integer entry is required, its nearest integer is 1.

Quick Tip to Solve: After solving a shifted homogeneous equation, impose both logarithm positivity conditions before extracting the interval; the integration constant is fixed by the shifted ratio, not by y alone.

30. The number of elements in the set {n ∈ {1, 2, 3, ..., 100} | $(11)^n > (10)^n + (9)^n$} is _____. [4 marks]

Solution:

Concept:

- Dividing a positive exponential inequality by 11^n preserves its direction. The problem becomes $(10/11)^n + (9/11)^n < 1$.
- For a base strictly between zero and one, its positive integral powers strictly decrease. Therefore the sum of these two powers is strictly decreasing in n; once it drops below one, it remains below one.

Step 1: Locate the transition between consecutive integers Check the small powers around the expected crossing without logarithmic approximations:

$$11^4 = 14641, \qquad 10^4 + 9^4 = 10000 + 6561 = 16561.$$

Hence $n = 4$ fails the required strict inequality. For the next integer,

$$11^5 = 161051, \qquad 10^5 + 9^5 = 100000 + 59049 = 159049.$$

Since $161051 > 159049$, the value $n = 5$ satisfies it. These are exact integer comparisons, so the small

difference at the crossing causes no rounding uncertainty.

Step 2: Prove that no further transitions are possible Define $f(n) = (10/11)^n + (9/11)^n$. Each base is positive and less than one. Thus

$$f(n+1) = \frac{10}{11}\left(\frac{10}{11}\right)^n + \frac{9}{11}\left(\frac{9}{11}\right)^n < f(n).$$

Because $f(4) > 1$, every positive integer $n \le 4$ also has $f(n) > 1$ and fails. Because $f(5) < 1$, every integer $n \ge 5$ has $f(n) < 1$ and satisfies the condition. This monotonicity argument covers the entire stated set, rather than relying on a few numerical trials.

Step 3: Count the admissible values The required subset is exactly $\{5, 6, 7, \ldots, 100\}$. Counting both endpoints gives

$$100 - 5 + 1 = 96.$$

The plus one is essential because both the first valid exponent and the final exponent belong to the set. There is no equality at either tested boundary, so the strict inequality does not remove an endpoint from this count.

Step 4: Conclusion The number of elements is $\boxed{96}$. This is a dimensionless count of integers and agrees with the stated numerical key.

Quick Tip to Solve: Normalize by the largest base, prove monotonicity, and test just the two consecutive exponents at the crossing; then count inclusively.

Physics – Section A: Multiple Choice Questions (80 marks)

31. What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$? [4 marks]

(A) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$

(B) $(\hat{i} + \hat{j})$

(C) $\sqrt{2}(\hat{i} + \hat{j})$

(D) $2(\hat{i} + \hat{j} + \hat{k})$

Correct Answer: (B) $(\hat{i} + \hat{j})$

Solution:

Concept:

- The vector projection of $\vec{A}$ on nonzero $\vec{B}$ is $\text{proj}_{\vec{B}}\,\vec{A} = (\vec{A} \cdot \vec{B})\vec{B}/|\vec{B}|^2$. It is parallel to the vector on which we project.
- The scalar projection $(\vec{A} \cdot \vec{B})/|\vec{B}|$ is a length, whereas the options here are vectors. The vector projection includes multiplication by the unit vector along $\vec{B}$.

Step 1: Calculate the dot product and squared magnitude Using the orthonormal Cartesian basis, the components are $\vec{A} = (1, 1, 1)$ and $\vec{B} = (1, 1, 0)$. Therefore

$$\vec{A} \cdot \vec{B} = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 0 = 2, \qquad |\vec{B}|^2 = 1^2 + 1^2 + 0^2 = 2.$$

The component of $\vec{A}$ in the $\hat{k}$ direction contributes nothing to this dot product because $\vec{B}$ has zero $\hat{k}$ component.

Step 2: Form the projected vector and verify it Substituting in the projection formula gives

$$\text{proj}_{\vec{B}}\,\vec{A} = \frac{2}{2}(\hat{i}+\hat{j}) = \hat{i}+\hat{j}.$$

Its magnitude is $\sqrt{2}$, consistent with the scalar projection $2/\sqrt{2}$. As an independent geometric check, subtract it from $\vec{A}$: the remainder is $\hat{k}$, perpendicular to $\vec{B}$. Thus the decomposition has exactly the required parallel and perpendicular components.

Step 3: Eliminate the other proposed vectors Options (A) and (D) contain a nonzero $\hat{k}$ component, so neither is parallel to $\vec{B}$. Option (C) is parallel to $\vec{B}$, but its magnitude is $\sqrt{2}\sqrt{2} = 2$, rather than the required $\sqrt{2}$. It incorrectly attaches the scalar projection to the unnormalized vector $\vec{B}$. Only option (B) has both the correct direction and magnitude.

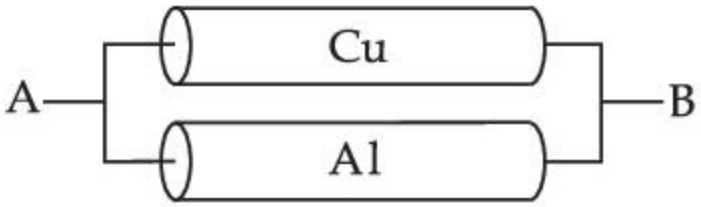
Step 4: Conclusion The projection is $\boxed{\hat{i}+\hat{j}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a vector projection, divide the dot product by the squared magnitude of the target vector. Check that the remaining component is perpendicular to that target.

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32. A Copper (Cu) rod of length 25 cm and cross-sectional area 3 mm^2 is joined with a similar Aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B.

(Take Resistivity of Copper = 1.7 $\times$ 10^{-8} Ωm and Resistivity of Aluminium = 2.6 $\times$ 10^{-8} Ωm) [4 marks]



(A) 0.0858 mΩ

(B) 1.420 mΩ

(C) 0.858 mΩ

(D) 2.170 mΩ

Correct Answer: (C) 0.858 mΩ

Solution:

Concept:

- The resistance of a uniform rod is $R = \rho L/A$, where ρ is its resistivity, L its length and A its cross-sectional area.
- The figure connects both left ends to A and both right ends to B. Hence the copper and aluminium rods have the same potential difference and are in parallel: $R_{\text{eq}}^{-1} = R_{\text{Cu}}^{-1} + R_{\text{Al}}^{-1}$.

Step 1: Convert the dimensions and find each resistance Both similar rods have $L = 25\,\text{cm} = 0.25\,\text{m}$ and $A = 3\,\text{mm}^2 = 3 \times 10^{-6}\,\text{m}^2$. Using the two given resistivities,

$$R_{\text{Cu}} = \frac{1.7 \times 10^{-8} \times 0.25}{3 \times 10^{-6}} = 1.4167 \times 10^{-3}\,\Omega,$$

$$R_{\text{Al}} = \frac{2.6 \times 10^{-8} \times 0.25}{3 \times 10^{-6}} = 2.1667 \times 10^{-3}\,\Omega.$$

In milliohms these are 1.4167 and 2.1667 respectively.

Step 2: Combine the two parallel branches Because current can flow from A to B through either rod, the equivalent resistance is the product divided by the sum:

$$R_{\text{eq}} = \frac{R_{\text{Cu}} R_{\text{Al}}}{R_{\text{Cu}} + R_{\text{Al}}} = \frac{1.4167 \times 2.1667}{1.4167 + 2.1667}\,\text{m}\Omega \approx 0.8566\,\text{m}\Omega.$$

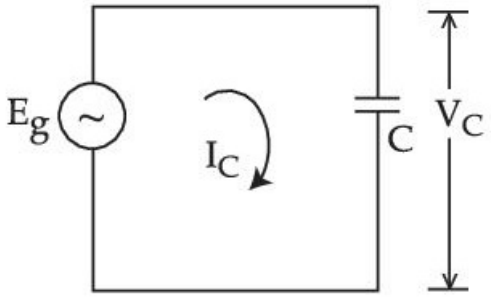
Equivalently, combine the resistivities first as $R_{\text{eq}} = (L/A)\rho_{\text{Cu}}\rho_{\text{Al}}/(\rho_{\text{Cu}} + \rho_{\text{Al}})$. The closest stated option is $0.858\,\text{m}\Omega$, with the small difference due to the numerical values used in the options.

Step 3: Check the scale and eliminate alternatives The result must be smaller than the resistance of the better conducting copper branch, $1.4167\,\text{m}\Omega$. Options (B) and (D) are approximately the individual rod resistances, so they cannot be the parallel combination. Option (A) is smaller by a factor of about ten and does not follow from the stated dimensions. Option (C) has the correct parallel-combination scale and is the uniquely matching choice.

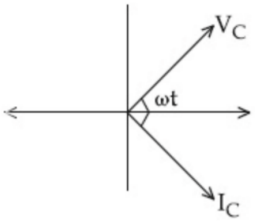
Step 4: Conclusion The combination has $R_{AB} \approx 0.857\,\text{m}\Omega$, corresponding to the listed value $\boxed{0.858\,\text{m}\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: Read the connections before using the material data: rods sharing both end nodes are parallel, so the answer must be smaller than either individual rod resistance.

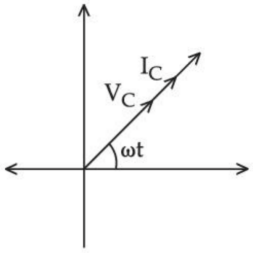
33. **In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0} \sin\omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is** [4 marks]



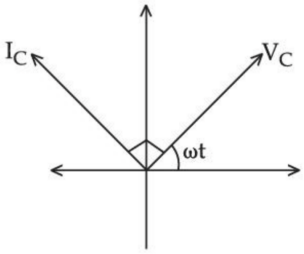
(A)



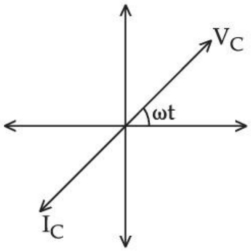
(B)



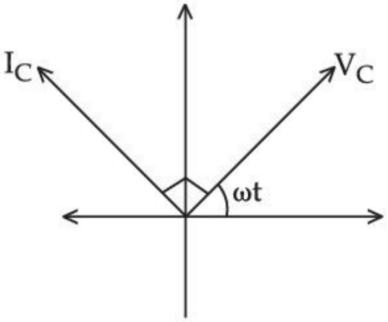
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- For an ideal capacitor, charge and voltage obey $q = CV_C$, and the current is $I_C = dq/dt = C\,dV_C/dt$. Differentiation fixes the phase relation.
- In the usual anticlockwise rotating phasor convention, a phasor at a larger angular position leads. A pure capacitive current leads its voltage by $\pi/2$, rather than lagging it.

Step 1: Differentiate the voltage to determine the current The circuit contains only the AC generator and the capacitor. Thus the capacitor voltage has the generator frequency and can be written

$$V_C = V_0 \sin \omega t.$$

Its current is

$$I_C = C\frac{dV_C}{dt} = \omega C V_0 \cos \omega t = I_0 \sin\left(\omega t + \frac{\pi}{2}\right),$$

where $I_0 = \omega C V_0$. At a zero crossing of the increasing voltage, the current is positive and maximal; this gives a direct time-domain check of the lead.

Step 2: Identify the phasor with the required lead The voltage phasor shown in the options is in the first quadrant, at the angle marked ωt. To represent a current leading by a quarter cycle, rotate this voltage phasor anticlockwise through $90°$. The resulting current phasor points into the second quadrant. This is exactly the pair of perpendicular arrows in option (C), with I_C ahead of V_C. Their lengths represent different physical quantities, so equality of drawn lengths is not a phase criterion.

Step 3: Reject the other phase arrangements Option (A) places the current clockwise from the voltage by a right angle, representing a lagging current, as for a pure inductor. Option (B) places both phasors on the same ray and therefore describes zero phase difference, as for a resistor. Option (D) places them on opposite rays, representing a phase difference of $180°$. None satisfies the capacitor relation $I_C = C\,dV_C/dt$.

Step 4: Conclusion The current leads the capacitor voltage by $\boxed{90°}$, so the current arrow must be anticlockwise from the voltage arrow. Hence, the correct option is (C).

Quick Tip to Solve: Use $I = C\,dV/dt$: differentiating a sine gives a cosine, which leads by a quarter cycle. Then check the arrow order, not just their perpendicularity.

34. A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. (take g = 9.8 ms^{-2}) [4 marks]

(A) +627.2 J

(B) −62720.0 J

(C) −627.2 J

(D) 784.0 J

Correct Answer: (C) −627.2 J

Solution:

Concept:

- Work by a constant force is $W = \vec{F} \cdot \vec{s} = Fs\cos\theta$, where θ is the angle between that force and the displacement.
- Constant velocity means zero acceleration and hence zero resultant force. The porter therefore applies an upward supporting force equal in magnitude to the downward weight mg. Zero net work does not imply zero work by each individual force.

Step 1: Determine the force exerted by the porter The suitcase has mass $m = 80\,\text{kg}$. Its weight is

$$mg = 80 \times 9.8 = 784\,\text{N}.$$

Since it moves with constant velocity during lowering, the porter must exert $F = 784\,\text{N}$ upwards. A smaller upward force would allow downward acceleration; a larger one would produce upward acceleration, contrary to the given motion. The displacement relevant here is the lowering distance, $80\,\text{cm} = 0.80\,\text{m}$.

Step 2: Calculate work with the correct direction The force by the porter is upward while the suitcase displacement is downward, so $\theta = 180°$. Hence

$$W_{\text{porter}} = 784 \times 0.80 \times \cos 180° = -627.2\,\text{J}.$$

The negative sign means that the porter removes mechanical energy from the suitcase as gravity does positive work. Indeed, gravity does $+mg(0.80) = +627.2\,\text{J}$, so the total work is zero and the kinetic energy stays constant.

Step 3: Compare with all options Option (A) has the magnitude of the gravitational work, but the wrong sign for the work done by the porter. Option (B) uses the centimetre value 80 as though it were metres and is too large by a factor of 100. Option (D), 784, is the numerical value of the supporting force in newtons, rather than the work after multiplying by the displacement. Only option (C) includes both the correct SI distance and the opposite-force sign.

Step 4: Conclusion The work done by the porter in lowering the suitcase is $\boxed{-627.2\,\text{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: Identify whose work is requested. During steady lowering, gravity does positive work and the upward supporting force does equal negative work.

35. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter. The correct statement for this situation is [4 marks]

(A) All of them will have same velocity.

(B) The ring has greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

(C) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

(D) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Correct Answer: (C) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

Solution:

Concept:

- For rolling without slipping, $v = \omega R$. The kinetic energy is the sum of translation and rotation: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.
- For a ring, solid cylinder and solid sphere about their central axes, $I/(mR^2)$ equals 1, $1/2$ and $2/5$, respectively. This dimensionless ratio controls the speed reached from a common height.

Step 1: Relate the speed to the moment of inertia Each body starts at rest and descends through the same vertical height h. For pure rolling on a fixed incline, static friction does no work at the instantaneous contact point. Energy conservation therefore gives

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2}.$$

Writing $k = I/(mR^2)$, solve for the centre-of-mass speed:

$$v^2 = \frac{2gh}{1+k}.$$

The mass cancels, so equal masses are unnecessary. Equal diameter also causes no extra speed factor because R is already incorporated in k.

Step 2: Compare the three squared speeds Substitute the appropriate inertia ratios:

$$v^2_{\text{ring}} = gh, \qquad v^2_{\text{cylinder}} = \frac{4}{3}gh, \qquad v^2_{\text{sphere}} = \frac{10}{7}gh.$$

Since $10/7 > 4/3 > 1$, the speeds, which are positive, satisfy

$$v_{\text{sphere}} > v_{\text{cylinder}} > v_{\text{ring}}.$$

The sphere has the smallest rotational inertia ratio and therefore uses the smallest fraction of the released potential energy in rotation at a given translational speed.

Step 3: Select the matching statement Option (A) is false because the rotational energy fractions

are different. Option (B) incorrectly puts the ring first, although its largest inertia ratio makes it the slowest. Option (D) reverses the sphere and cylinder comparison: $2/5 < 1/2$ means the sphere is faster. Option (C) alone correctly identifies the sphere as fastest and the ring as slowest at the bottom of the same incline.

Step 4: Conclusion The solid sphere has the greatest and the ring the least centre-of-mass velocity at the bottom. Hence, the correct option is (C).

Quick Tip to Solve: For bodies rolling from the same height, compare $I/(mR^2)$: the smallest ratio gives the largest translational speed, regardless of mass.

36. Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at the point?

($\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$) [4 marks]

(A) 2.77×10^{-8} T

(B) 1.96×10^{-8} T

(C) 8.31 T

(D) 5.88 T

Correct Answer: (A) 2.77×10^{-8} T

Solution:

Concept:

- The intensity of a sinusoidal electromagnetic wave is its time-averaged energy flux: $I = \frac{1}{2}\varepsilon_0 c E_0^2$, where E_0 is the peak electric field.
- The peak fields satisfy $E_0 = cB_0$; hence $I = \frac{1}{2}\varepsilon_0 c^3 B_0^2$. The factor one-half distinguishes peak from rms values.

Step 1: Express intensity in terms of the magnetic amplitude Use $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$ and the given ε_0. Solving the intensity relation for the required peak value gives

$$B_0 = \sqrt{\frac{2I}{\varepsilon_0 c^3}}.$$

The measured intensity already represents the cycle average, so it must not be inserted into a formula for instantaneous flux without this factor of two.

Step 2: Substitute and compare the options

$$B_0 = \sqrt{\frac{2(0.092)}{(8.85 \times 10^{-12})(3.0 \times 10^8)^3}} = \sqrt{7.70 \times 10^{-16}} = 2.77 \times 10^{-8}\,\mathrm{T}.$$

The value 1.96×10^{-8} T is the rms magnetic field, not its peak. Values of several tesla are incompatible with this very small intensity and do not satisfy the energy-flux relation.

Step 3: Conclusion The peak magnetic field is $\boxed{2.77 \times 10^{-8}\,\mathrm{T}}$. Hence, the correct option is (A).

Quick Tip to Solve: Check whether an electromagnetic-wave question asks for peak or rms field before using the intensity formula.

37. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is _____ s. [4 marks]

(A) $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

(B) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

(C) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

(D) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

Correct Answer: (D) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

Solution:

Concept:

- For launch at escape speed, the total mechanical energy is zero: $\frac{1}{2}mv^2 - GM_em/r = 0$ at every radius r.
- The acceleration varies with radius, so constant-acceleration kinematics is unsuitable. Use $GM_e = gR_e^2$ and integrate $dt = dr/v(r)$.

Step 1: Find the speed at a general radius The launch just reaches infinity with zero limiting speed. Conservation of energy therefore gives

$$v(r) = \sqrt{\frac{2GM_e}{r}}.$$

The outward velocity remains positive at every finite radius but decreases as the body rises. At the surface this reduces to the escape speed $\sqrt{2gR_e}$.

Step 2: Integrate from the surface to the stated height

$$\begin{aligned} t &= \frac{1}{\sqrt{2GM_e}}\int_{R_e}^{R_e+h} r^{1/2}\,dr \\ &= \frac{2}{3\sqrt{2GM_e}}\left[(R_e+h)^{3/2} - R_e^{3/2}\right] \\ &= \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]. \end{aligned}$$

For small h, this becomes $h/\sqrt{2gR_e}$, as expected from the initial speed. The other coefficients give an incorrect small-height limit: the first is three times too large, the second half the required value and the third three-halves of it.

Step 3: Conclusion

$$\boxed{t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]\ \text{s}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: At escape energy, replace the trajectory problem by the simple relation $v(r) = \sqrt{2GM_e/r}$, then integrate time.

38. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0 $\times$ 10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4N is exerted on the dipole, find out the amount of positive or negative charge of the dipole. [4 marks]

(A) 0.485 mC

(B) 815.1 nC

(C) 8.8 μC

(D) 4.44 μC

Correct Answer: (D) 4.44 μC

Solution:

Concept:

- The electric field of a long straight line charge is $E(r) = \lambda/(2\pi\varepsilon_0 r) = 2k\lambda/r$, directed radially away from a positive line.
- The two equal and opposite dipole charges experience opposite forces. Their unequal distances make their force magnitudes different, leaving a net force equal to their difference.

Step 1: Write the force difference The nearer positive charge is at $r_+ = 0.010\,\mathrm{m}$, and the negative charge is at $r_- = 0.012\,\mathrm{m}$. The field is stronger at the nearer charge. Thus, for charge magnitude q,

$$F = q\,2k\lambda\left(\frac{1}{r_+} - \frac{1}{r_-}\right).$$

Both distances must be converted from millimetres to metres before substitution.

Step 2: Calculate the charge magnitude

$$\begin{aligned}\Delta E &= 2(9.0\times10^{9})(3.0\times10^{-6})\left(100 - \frac{250}{3}\right)\\ &= 9.0\times10^{5}\,\mathrm{N\,C^{-1}},\\ q &= \frac{4}{9.0\times10^{5}} = 4.44\times10^{-6}\,\mathrm{C}.\end{aligned}$$

As a check, the individual forces are about $24\,\mathrm{N}$ and $20\,\mathrm{N}$, leaving $4\,\mathrm{N}$. The other charge magnitudes fail this force-difference check; adding the two forces would incorrectly ignore their opposite directions.

Step 3: Conclusion Each charge has magnitude $\boxed{4.44\,\mu\mathrm{C}}$, with opposite signs. Hence, the correct option is (D).

Quick Tip to Solve: For a radial dipole near a line charge, subtract the fields at the two charges; the uniform-field approximation would miss the net force.

39. A nucleus with mass number 184 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle. [4 marks]

(A) 5.5 MeV

(B) 5.0 MeV

(C) 5.38 MeV

(D) 0.12 MeV

Correct Answer: (C) 5.38 MeV

Solution:

Concept:

- An initially stationary nucleus has zero momentum. After two-body alpha decay, the alpha particle and daughter recoil with equal and opposite momenta.
- For equal momentum magnitude, nonrelativistic kinetic energies are inversely proportional to mass. The released energy Q is shared between both products.

Step 1: Determine the energy-sharing ratio The daughter has mass number $184 - 4 = 180$. With nuclear masses approximated by their mass numbers, let the common momentum magnitude be p. Then

$$\frac{K_\alpha}{K_d} = \frac{p^2/(2m_\alpha)}{p^2/(2m_d)} = \frac{m_d}{m_\alpha} = \frac{180}{4} = 45.$$

The lighter alpha particle receives most, but not all, of the released energy.

Step 2: Use conservation of energy

$$\begin{aligned} Q &= K_\alpha + K_d, \\ K_\alpha &= Q\frac{m_d}{m_d + m_\alpha} = 5.5\frac{180}{184} \\ &= 5.3804\,\text{MeV}. \end{aligned}$$

The recoil energy is $5.5 - 5.3804 = 0.1196\,\text{MeV}$. Thus $0.12\,\text{MeV}$ belongs to the daughter; $5.5\,\text{MeV}$ incorrectly assigns all energy to the alpha particle, and $5.0\,\text{MeV}$ does not obey the momentum-based ratio.

Step 3: Conclusion The alpha-particle kinetic energy is $\boxed{5.38\,\text{MeV}}$. Hence, the correct option is (C).

Quick Tip to Solve: In two-body decay from rest, multiply Q by the other product's mass divided by the total product mass.

40. A bullet of '4 g' mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are : [4 marks]

(A) 0.2 kg ms^{-1}, 0.1 ms^{-1}

(B) 0.4 kg ms^{-1}, 0.05 ms^{-1}

(C) 0.2 kg ms^{-1}, 0.05 ms^{-1}

(D) 0.4 kg ms^{-1}, 0.1 ms^{-1}

Correct Answer: (C) 0.2 kg ms^{-1}, 0.05 ms^{-1}

Solution:

Concept:

- If external impulse during firing is negligible, total momentum of the gun and bullet is conserved. Initially the system is at rest.
- The impulse on the gun equals its change of momentum and is opposite to the momentum acquired by the bullet. Convert the bullet mass to kilograms.

Step 1: Calculate the bullet momentum and gun impulse Take the bullet direction as positive. The bullet mass is $m = 4\,\text{g} = 0.004\,\text{kg}$. Its momentum after firing is

$$p_b = mv = (0.004)(50) = 0.200\,\text{kg m s}^{-1}.$$

The gun therefore receives impulse $J_g = -0.200\,\text{kg m s}^{-1}$, directed backward. Its magnitude is $0.200\,\text{N s}$.

Step 2: Find and check the recoil velocity For gun mass $M = 4\,\text{kg}$,

$$MV + mv = 0 \quad \Rightarrow \quad V = -\frac{0.200}{4} = -0.050\,\text{m s}^{-1}.$$

The gun momentum is then $4(-0.050) = -0.200\,\text{kg m s}^{-1}$, cancelling the bullet momentum exactly. An impulse of 0.4 doubles the available bullet momentum; a recoil speed of 0.1 likewise gives twice the required gun momentum. Only one offered pair satisfies both checks.

Step 3: Conclusion The impulse magnitude is $\boxed{0.2\,\text{kg m s}^{-1}}$, and the recoil speed is $\boxed{0.05\,\text{m s}^{-1}}$, opposite to the bullet. Hence, the correct option is (C).

Quick Tip to Solve: First find bullet momentum; the same magnitude is the gun impulse, and dividing by gun mass gives recoil speed.

41. Statement I : The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement II : At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but Statement II is true

(B) Statement I is true but Statement II is false

(C) Both Statement I and Statement II are false

(D) Both Statement I and Statement II are true

Correct Answer: (B) Statement I is true but Statement II is false

Solution:

Concept:

- Ferromagnetism is cooperative alignment of atomic magnetic moments. Thermal agitation competes with the interaction that maintains this ordered state.
- Above the Curie temperature, spontaneous ferromagnetic order is lost and the material is paramagnetic; domain-wall motion is not the mechanism of this high-temperature transition.

Step 1: Evaluate Statement I As temperature rises, thermal fluctuations weaken spontaneous magnetic order. At the Curie temperature the ferromagnetic state disappears; above it, moments respond to an applied field as in a paramagnet. Thus the stated temperature dependence and the change from ferromagnetism to paramagnetism at sufficiently high temperature are correct.

Step 2: Evaluate Statement II and select the pair A domain wall separates regions having different orientations of an ordered spontaneous magnetization. Above the Curie temperature those ferromagnetic domains and their walls no longer persist as such. The loss of ferromagnetism cannot be described as an increase of domain-wall area at high temperature. Statement II is therefore false. This rejects the choices asserting Statement I is false as well as the choice asserting both statements are true; the valid combination is true for I and false for II.

Step 3: Conclusion Statement I is true, while Statement II is false. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish loss of ferromagnetic order at the Curie temperature from domain-wall rearrangement within the ferromagnetic state.

42. Match List - I with List - II

	List - I		List - II
(a)	$\omega L > \frac{1}{\omega C}$	**(i)**	**Current is in phase with emf**
(b)	$\omega L = \frac{1}{\omega C}$	**(ii)**	**Current lags behind the applied emf**
(c)	$\omega L < \frac{1}{\omega C}$	**(iii)**	**Maximum current occurs**
(d)	**Resonant frequency**	**(iv)**	**Current leads the emf**

Choose the correct answer from the options given below [4 marks]

(A) a(ii), b(i), c(iv), d(iii)

(B) a(ii), b(i), c(iii), d(iv)

(C) a(iii), b(i), c(iv), d(ii)

(D) a(iv), b(iii), c(ii), d(i)

Correct Answer: (A) a(ii), b(i), c(iv), d(iii)

Solution:

Concept:

- For a series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$, and $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- The phase angle of applied voltage relative to current satisfies $\tan\phi = (X_L - X_C)/R$. A positive angle means current lags; a negative angle means current leads.

Step 1: Match the three reactance conditions When $\omega L > 1/(\omega C)$, the circuit is net inductive, so current lags the applied emf: (a) matches (ii). When the two reactances are equal, the net reactance vanishes and $\phi = 0$, so current and emf are in phase: (b) matches (i). When $\omega L < 1/(\omega C)$, the circuit is net capacitive and current leads the emf: (c) matches (iv).

Step 2: Identify the resonance property and eliminate wrong pairs At resonance, $\omega_0 L = 1/(\omega_0 C)$, hence

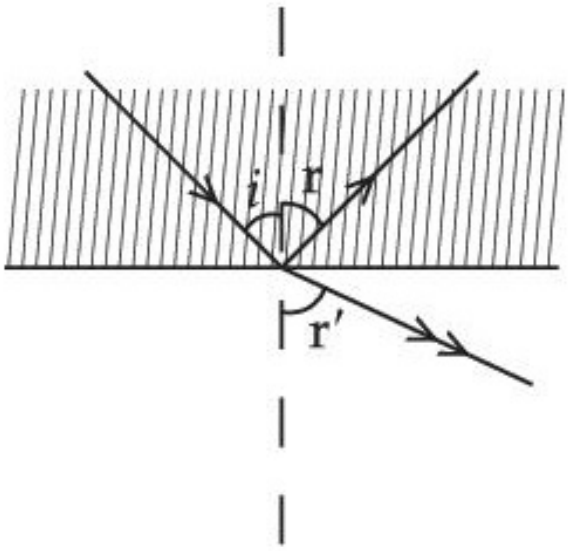
$$\omega_0 = \frac{1}{\sqrt{LC}}, \qquad Z_{\min} = R, \qquad I_{\max} = \frac{V}{R}.$$

Therefore (d) matches (iii). The second offered combination wrongly assigns maximum current to the capacitive inequality; the third assigns it to the inductive inequality. The fourth reverses inductive and capacitive phase behavior. The complete correct matching is unique.

Step 3: Conclusion The matching is $\boxed{\text{a(ii), b(i), c(iv), d(iii)}}$. Hence, the correct option is (A).

Quick Tip to Solve: Remember: inductive means current lags, capacitive means current leads, and resonance means zero phase and maximum current.

43. A ray of light passes from a denser medium to a rarer medium at an angle of incidence i. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by [4 marks]



(A) $\sin^{-1}$ (tan r)

(B) $\sin^{-1}$ (cot r)

(C) $\sin^{-1}$ (tan r')

(D) $\tan^{-1}$ (sin i)

Correct Answer: (A) $\sin^{-1}$ (tan r)

Solution:

Concept:

- Snell's law relates the refractive indices and angles measured from the normal: $n_d \sin i = n_r \sin r\prime$, where $n_d > n_r$.
- The law of reflection gives $r = i$. For incidence from the denser medium, the critical angle c satisfies $\sin c = n_r/n_d$.

Step 1: Use the perpendicular outgoing rays The reflected ray makes angle r with the upward normal, while the refracted ray makes angle $r\prime$ with the downward normal. Their separation is $180^\circ - r - r\prime$. Since this is 90°, their angles obey

$$r + r\prime = 90^\circ, \qquad \sin r\prime = \cos r.$$

Reflection also gives $i = r$; thus the incidence and refraction angles are complementary.

Step 2: Calculate the refractive-index ratio Substitute these relations into Snell's law, retaining the order denser to rarer:

$$\frac{n_r}{n_d} = \frac{\sin i}{\sin r\prime} = \frac{\sin r}{\cos r} = \tan r.$$

Therefore $\sin c = \tan r$. The reciprocal $\cot r$, also equal to $\tan r\prime$, instead gives $n_d/n_r > 1$, so options (B) and (C) cannot be sines of a real critical angle. Option (D) interchanges the roles of sine and

tangent: the condition is on $\sin c$, not on $\tan c$.

Step 3: Conclusion The critical angle is $\boxed{c = \sin^{-1}(\tan r)}$. Hence, the correct option is (A).

Quick Tip to Solve: With reflected and refracted rays perpendicular, first set $i + r\prime = 90°$; then use the index ratio appropriate to the direction of incidence.

44. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is [4 marks]

(A) $\frac{m_e}{m_p}$

(B) 1

(C) $\frac{m_p}{m_e}$

(D) $\sqrt{\frac{m_p}{m_e}}$

Correct Answer: (D) $\sqrt{\frac{m_p}{m_e}}$

Solution:

Concept:

- A particle of charge magnitude e, accelerated from rest through potential difference V, gains kinetic energy eV. Electron and proton have the same charge magnitude.
- For nonrelativistic motion, $p = \sqrt{2mK}$ and the de Broglie wavelength is $\lambda = h/p$, with h Planck's constant.

Step 1: Write the momenta at equal accelerating voltage The work done by the electric field depends on the magnitude of the charge and voltage, not on the mass. Thus

$$K_e = K_p = eV, \qquad p_e = \sqrt{2m_e eV}, \qquad p_p = \sqrt{2m_p eV}.$$

The particles have equal kinetic energies, but their momenta are unequal because momentum at fixed kinetic energy is proportional to the square root of mass.

Step 2: Form the wavelength ratio in the requested order Using wavelength inversely proportional to momentum,

$$\frac{\lambda_e}{\lambda_p} = \frac{h/p_e}{h/p_p} = \frac{p_p}{p_e} = \sqrt{\frac{2m_p eV}{2m_e eV}} = \sqrt{\frac{m_p}{m_e}}.$$

The factors h, $2e$ and V cancel. Since the proton is much heavier, the electron has the longer wavelength. Option (A) is less than one and reverses the physical trend. Option (B) would require equal momenta, whereas the given condition is equal voltage. Option (C) omits the square root that follows from $K = p^2/(2m)$.

Step 3: Conclusion $\boxed{\lambda_e/\lambda_p = \sqrt{m_p/m_e}}$. Hence, the correct option is (D).

Quick Tip to Solve: At equal accelerating voltage and equal charge magnitude, use $\lambda \propto m^{-1/2}$; equal speed would give a different mass dependence.

45. What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T? [4 marks]

(A) $\frac{3}{2}k_BT$

(B) k_BT

(C) $\frac{2}{3}k_BT$

(D) $\frac{1}{2}k_BT$

Correct Answer: (A) $\frac{3}{2}k_BT$

Solution:

Concept:

- The equipartition theorem assigns average energy $\frac{1}{2}k_BT$ to each independent quadratic degree of freedom in thermal equilibrium.
- A monoatomic ideal-gas particle has three translational degrees of freedom, corresponding to velocity components along three perpendicular axes; k_B is Boltzmann's constant.

Step 1: Count the active energy contributions For a monoatomic gas in the usual kinetic-theory model, each particle is treated as a point particle. It has no molecular vibration or rotation contributing to the internal energy. Its kinetic energy is

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2.$$

These are three separate quadratic contributions. Thermal equilibrium makes their mean contributions equal; it does not remove any of the three components.

Step 2: Add the mean energies Apply equipartition to each component and add:

$$\langle E\rangle = \frac{1}{2}k_BT + \frac{1}{2}k_BT + \frac{1}{2}k_BT = \frac{3}{2}k_BT.$$

This is the mean energy per particle, as indicated by the appearance of k_B in the choices. For one mole the corresponding value would be $3RT/2$. Option (D) counts just one translational component and option (B) counts two. Option (C) reverses the factor in the useful relation $k_BT = 2\langle E\rangle/3$; it is not the total mean energy. The result is proportional to absolute temperature and has units of energy.

Step 3: Conclusion The average energy per monoatomic gas particle is $\boxed{\langle E\rangle = \frac{3}{2}k_BT}$. Hence, the correct option is (A).

Quick Tip to Solve: Count quadratic terms in the particle's energy: three translational terms give $3 \times \frac{1}{2}k_BT$.

46. T_0 is the time period of a simple pendulum at a place. if the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is : [4 marks]

(A) 4 T_0

(B) $\frac{1}{4}$ T_0

(C) T_0

(D) 8π T_0

Correct Answer: (B) $\frac{1}{4}$ T_0

Solution:

Concept:

- For small oscillations, a simple pendulum of length L at gravitational acceleration g has period $T = 2\pi\sqrt{L/g}$.
- At the same place, g remains unchanged, so the ratio of periods is the square root of the ratio of lengths: $T_1/T_0 = \sqrt{L_1/L_0}$.

Step 1: Express the initial and reduced lengths Let the original length be L_0. The initial period is

$$T_0 = 2\pi\sqrt{\frac{L_0}{g}}.$$

The statement gives the new length directly as one-sixteenth of the initial length, not a reduction by one-sixteenth:

$$L_1 = \frac{L_0}{16}.$$

The new pendulum is shorter and should therefore complete each small oscillation faster.

Step 2: Take the ratio of periods Substitute the changed length into the same period formula:

$$T_1 = 2\pi\sqrt{\frac{L_0}{16g}} = \frac{1}{4}\left(2\pi\sqrt{\frac{L_0}{g}}\right) = \frac{T_0}{4}.$$

Equivalently, $T_1/T_0 = \sqrt{1/16} = 1/4$. The square root is essential: the period does not scale directly with length. Option (A) inverts the ratio and predicts a slower oscillation for a shorter pendulum. Option (C) ignores the change in length. Option (D) both increases the period and introduces an extra factor of π, although the common factor 2π must cancel in the period ratio.

Step 3: Conclusion The modified time period is $\boxed{T_1 = T_0/4}$. Hence, the correct option is (B).

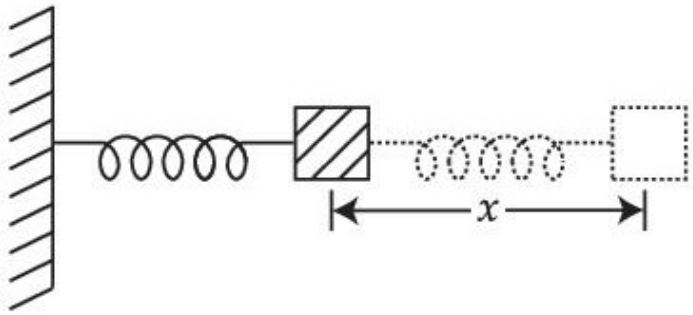
Quick Tip to Solve: For a length multiplied by a factor f, multiply the pendulum period by $\sqrt{f}$; take the ratio before inserting any constants.

47. The motion of a mass on a spring, with spring constant K is as shown in figure.

The equation of motion is given by

x(t) = A sinωt + B cosωt with $\omega = \sqrt{\frac{K}{m}}$

Suppose that at time t = 0, the position of mass is x(0) and velocity v(0), then its displacement can also be represented as x(t) = C cos(ωt − ϕ), where C and ϕ are : [4 marks]



(A) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

(B) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

(C) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

(D) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

Correct Answer: (C) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

Solution:

Concept:

- For the spring oscillator, $\omega = \sqrt{K/m}$ and $x(t) = A\sin\omega t + B\cos\omega t$. Displacement and velocity at $t = 0$ determine the two constants.
- The identity $\cos(\omega t - \phi) = \cos\omega t\cos\phi + \sin\omega t\sin\phi$ connects the amplitude-phase form to the sine-cosine form.

Step 1: Determine the sine and cosine coefficients Differentiate displacement to get velocity:

$$v(t) = A\omega\cos\omega t - B\omega\sin\omega t.$$

At $t = 0$, $\sin 0 = 0$ and $\cos 0 = 1$, so

$$B = x(0), \qquad A = \frac{v(0)}{\omega}.$$

The factor $1/\omega$ is needed to convert the initial velocity into a displacement coefficient.

Step 2: Match the amplitude-phase expression Expand the alternative form:

$$x(t) = C\cos\phi\cos\omega t + C\sin\phi\sin\omega t.$$

Comparing independent sine and cosine coefficients gives

$$C\cos\phi = x(0), \qquad C\sin\phi = \frac{v(0)}{\omega}.$$

Squaring and adding eliminates phase, while dividing gives its tangent:

$$C = \sqrt{x(0)^2 + \frac{v(0)^2}{\omega^2}}, \qquad \tan\phi = \frac{v(0)}{\omega x(0)}.$$

Choose the phase quadrant from the two coefficient equations; the inverse tangent in the option is understood with that convention. Options (A) and (D) incorrectly insert a factor two in the velocity term. Option (B) gives the reciprocal tangent, exchanging the initial sine and cosine conditions.

Step 3: Conclusion $\boxed{C = \sqrt{x(0)^2 + v(0)^2/\omega^2}, \quad \phi = \tan^{-1}[v(0)/(\omega x(0))]}$, with the phase quadrant fixed by the initial conditions. Hence, the correct option is (C).

Quick Tip to Solve: Expand the stated cosine form before choosing a phase formula; the sign and whether sine or cosine is used determine which ratio occurs.

48. Consider a situation in which reserve biased current of a particular P-N junction increases when it is exposed to a light of wavelength $\leq$ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly.

[4 marks]

(A) 1 eV

(B) 4 eV

(C) 0.5 eV

(D) 2 eV

Correct Answer: (D) 2 eV

Solution:

Concept:

- Light creates an electron-hole pair when a photon supplies at least the semiconductor band-gap energy E_g. Reverse bias helps collect the generated charge carriers.
- At the longest effective wavelength λ_0, the threshold relation is $E_g = hc/\lambda_0$. In electronvolts, $hc \simeq 1240\,\text{eV nm}$.

Step 1: Identify the threshold wavelength The reverse-biased current increases for wavelengths up to $621\,\text{nm}$. Photon energy is inversely proportional to wavelength, so the longest effective wavelength has the smallest energy capable of producing the stated electron-hole pairs. Hence

$$\lambda_0 = 621\,\text{nm}, \qquad E_g = \frac{hc}{\lambda_0}.$$

Shorter wavelengths have larger photon energies and can also generate carriers; the threshold is set by the largest wavelength.

Step 2: Evaluate the photon energy and compare the choices Using consistent nanometre units in the convenient photon-energy relation,

$$E_g \simeq \frac{1240}{621}\,\text{eV} = 1.997\,\text{eV} \simeq 2\,\text{eV}.$$

A gap of $1\,\text{eV}$ or $0.5\,\text{eV}$ would instead permit threshold wavelengths near $1240\,\text{nm}$ or $2480\,\text{nm}$, much longer than the stated cutoff. A gap of $4\,\text{eV}$ requires wavelengths near $310\,\text{nm}$ or shorter, and $621\,\text{nm}$ photons could not bridge it. Reverse bias changes the collection current, but it does not replace the photon-energy requirement for pair production.

Step 3: Conclusion The band gap is nearly $\boxed{2\,\text{eV}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a semiconductor optical cutoff, use the longest active wavelength in $E_g(\text{eV}) = 1240/\lambda_0(\text{nm})$.

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49. Choose the correct option [4 marks]

(A) True dip is always equal to apparent dip.

(B) True dip is not mathematically related to apparent dip.

(C) True dip is less than the apparent dip.

(D) True dip is always greater than the apparent dip.

Correct Answer: (C) True dip is less than the apparent dip.

Solution:

Concept:

- True dip δ is the angle the Earth's magnetic field makes with the horizontal in the magnetic meridian. If B_V and B_H are its vertical and horizontal magnitudes, $\tan\delta = B_V/B_H$.
- In a vertical plane at angle θ to the magnetic meridian, the horizontal field component is $B_H\cos\theta$; the vertical component is unchanged.

Step 1: Relate apparent dip to true dip Let δ_a denote the dip observed with the needle constrained to the inclined vertical plane. Its inclination follows the field components within that plane:

$$\tan\delta_a = \frac{B_V}{B_H\cos\theta} = \frac{\tan\delta}{\cos\theta}.$$

Use the acute angle between the two vertical planes, so $0 \le \cos\theta \le 1$. Except when the planes coincide, the horizontal component is smaller than its full meridian value.

Step 2: Compare the inclinations and test the statements For nonzero dip and a plane different from the magnetic meridian, $\cos\theta < 1$, so $\tan\delta_a > \tan\delta$. Tangent increases from 0° to 90°, giving $\delta_a > \delta$. Thus true dip is smaller than the apparent dip in the usual comparison. Option (A) is false because equality holds only in the meridian or degenerate zero-dip case, not always. Option (B) is false because the displayed tangent relation connects the angles. Option (D) reverses the inequality. At $\theta = 0$, both dips coincide; in a perpendicular vertical plane the apparent dip tends to 90° for nonzero vertical field.

Step 3: Conclusion True dip is less than apparent dip for a vertical plane inclined to the magnetic meridian; equality occurs in the magnetic meridian. Hence, the correct option is (C).

Quick Tip to Solve: True dip is the minimum inclination: rotating away from the magnetic meridian reduces the horizontal component but leaves the vertical component unchanged.

50. What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km? The average population density around the tower is 2000/km^2 and the value of $R_e = 6.5 \times 10^6$ m. [4 marks]

(A) Height = 1241 m

Population covered = 7×10^5

(B) Height = 1731 m

Population covered = 1413×10^5

(C) Height = 1800 m

Population covered = 1413×10^8

(D) Height = 1600 m

Population covered = 2×10^5

Correct Answer: (B) Height = 1731 m

Population covered = 1413×10^5

Solution:

Concept:

- Television waves propagate approximately along the line of sight. For an antenna of height h above a spherical Earth of radius R_e, its horizon distance is $d \simeq \sqrt{2R_eh}$, valid when $h \ll R_e$.
- The covered region is a circle of radius d. With uniform population density σ, the covered population is $N = \sigma\pi d^2$; the distance unit must match that in the density.

Step 1: Find the required antenna height Convert the coverage radius to metres: $d = 150 \times 10^3$ m. The line from the antenna to the horizon is tangent to Earth, so the right-triangle relation gives $d^2 \simeq 2R_eh$. Hence

$$h = \frac{(150 \times 10^3)^2}{2(6.5 \times 10^6)} = 1730.77\,\text{m} \simeq 1731\,\text{m}.$$

This height is much smaller than Earth's radius, justifying the horizon approximation.

Step 2: Calculate the covered population Use kilometres for the area because the density is given per square kilometre:

$$N = 2000 \times \pi(150)^2 = 45\pi \times 10^6 \simeq 1.413 \times 10^8.$$

Thus $N \simeq 1413 \times 10^5$ people. Option (A) has both the wrong height and population; (C) has a population larger by a factor of 10^3, and (D) has neither required value. Only (B) gives the calculated pair.

Step 3: Conclusion The antenna height is approximately 1731 m, covering 1413×10^5 people. Hence, the correct option is (B).

Quick Tip to Solve: Calculate height in metres, but calculate the coverage area in square kilometres when the density is per square kilometre.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A ray of light passing through a prism ($\mu = \sqrt{3}$) suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is _____ (in degrees). [4 marks]

Solution:

Concept:

- At minimum deviation, a prism has a symmetric ray path: the two internal refraction angles are equal, $r_1 = r_2 = r$, and the prism angle is $A = r_1 + r_2 = 2r$.
- Snell's law at the air-to-prism surface is $\mu = \sin i / \sin r$. The stated relation between the angles is $i = 2r$; incidence and refraction are measured from the face normal.

Step 1: Use the given angle relation Insert $i = 2r$ and $\mu = \sqrt{3}$ in Snell's law. Since the ray is refracted through the prism, $r \neq 0$, so division by $\sin r$ is valid:

$$\sqrt{3} = \frac{\sin 2r}{\sin r} = \frac{2 \sin r \cos r}{\sin r} = 2 \cos r.$$

Therefore $\cos r = \sqrt{3}/2$. The physical refraction angle lies between zero and 90°, which selects $r = 30^\circ$.

Step 2: Find and check the prism angle The symmetric minimum-deviation path gives

$$A = 2r = 2(30^\circ) = 60^\circ.$$

As a check, $i = 2r = 60^\circ$, and $\sin 60^\circ / \sin 30^\circ = \sqrt{3}$, exactly the given refractive index. The emergence angle equals the incidence angle, so this ray satisfies the required minimum-deviation condition.

Step 3: Conclusion The prism angle is $A = 60^\circ$. The required numerical entry in degrees is $\boxed{60}$.

Quick Tip to Solve: At minimum deviation first set both internal angles equal to half the prism angle; then combine any extra angle condition with Snell's law.

52. Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of Pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1	64.0	8	128.0	16.0
2	64.0	4	64.0	16.0
3	20.0	4	36.0	9.0

(Least count of length = 0.1 cm and Least count for time = 0.1 s)

If E_1, E_2 and E_3 are the percentage errors in 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no. _____. [4 marks]

Solution:

Concept:

- For a simple pendulum, $g = 4\pi^2 L/T^2$. The maximum fractional error is therefore $\Delta g/g = \Delta L/L + 2\Delta T/T$, with the integer oscillation count treated as exact.
- If t is the measured total time for n oscillations, $T = t/n$ and $\Delta T = \Delta t/n$. Thus $\Delta T/T = \Delta t/t$: the least-count error belongs to the total measured time, not separately to every period.

Step 1: Evaluate the errors for students 1 and 2 Using $\Delta L = 0.1\,\text{cm}$ and $\Delta t = 0.1\,\text{s}$, the percentage errors calculated from the printed table are

$$E_1 = 100\left(\frac{0.1}{64.0} + \frac{2(0.1)}{128.0}\right) = 0.3125\%,$$

$$E_2 = 100\left(\frac{0.1}{64.0} + \frac{2(0.1)}{64.0}\right) = 0.46875\%.$$

The lengths are equal, but student 1 measures a longer total interval and therefore has a smaller fractional timing error.

Step 2: Compare the third error For student 3, both the shorter length and shorter measured time increase the error:

$$E_3 = 100\left(\frac{0.1}{20.0} + \frac{2(0.1)}{36.0}\right) \simeq 1.0556\%.$$

Consequently $E_1 < E_2 < E_3$. The comparison uses the given observations consistently; no separate error is attached to the exact count of oscillations.

Step 3: Conclusion Student S_1 obtains the smallest percentage error, 0.3125%. The required student number is $\boxed{1}$, a dimensionless index.

Quick Tip to Solve: For pendulum error comparisons, use the total timing interval in $2\Delta t/t$; timing more oscillations reduces this contribution.

53. Three particles P, Q and R are moving along the vectors $\overrightarrow{A} = \hat{i} + \hat{j}$, $\overrightarrow{B} = \hat{j} + \hat{k}$ and $\overrightarrow{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\overrightarrow{A}$ and $\overrightarrow{B}$. Similarly particle Q is moving normal to the plane which contains vector $\overrightarrow{A}$ and $\overrightarrow{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is _____. [4 marks]

Solution:

Concept:

- The cross product of two nonparallel vectors is perpendicular to the plane containing them. Thus a direction normal to the $\vec{A}, \vec{B}$ plane is $\vec{n}_P = \vec{A} \times \vec{B}$.
- For two direction vectors, $\cos\theta = (\vec{n}_P \cdot \vec{n}_Q)/(|\vec{n}_P||\vec{n}_Q|)$. The specified positive cosine selects the acute angle between the normal directions.

Step 1: Calculate both normal directions Write $\vec{A} = (1, 1, 0)$, $\vec{B} = (0, 1, 1)$, and $\vec{C} = (-1, 1, 0)$. Taking the cross products gives

$$\vec{n}_P = \vec{A} \times \vec{B} = (1, -1, 1), \qquad \vec{n}_Q = \vec{A} \times \vec{C} = (0, 0, 2).$$

Both are nonzero, so the two planes have well-defined normals. Their magnitudes are $\sqrt{3}$ and 2, respectively. Reversing a normal changes its orientation, but not the acute angle fixed by the printed expression.

Step 2: Apply the scalar product formula The scalar product is $(1)(0)+(-1)(0)+(1)(2) = 2$. Therefore

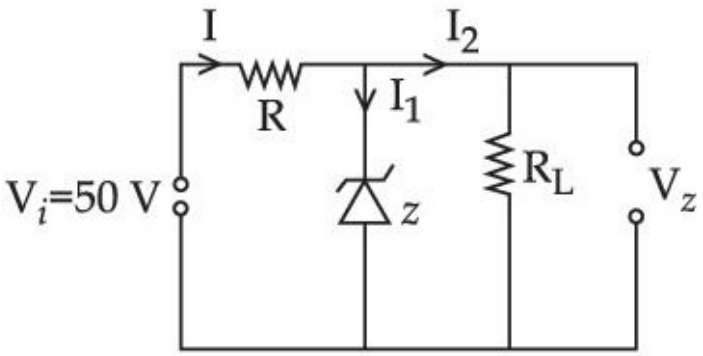
$$\cos\theta = \frac{2}{\sqrt{3}\,(2)} = \frac{1}{\sqrt{3}}, \qquad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$$

Comparison with $\cos^{-1}(1/\sqrt{x})$ gives $\sqrt{x} = \sqrt{3}$, hence $x = 3$. This calculation depends on the two final normal directions, so no collision-force or momentum calculation is needed.

Step 3: Conclusion The required dimensionless value is $\boxed{x = 3}$.

Quick Tip to Solve: For an angle between normals to two planes, form their cross products first and then use their dot product; do not use the angle between the original vectors.

54. In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be _____ Ω. [4 marks]



Solution:

Concept:

- A reverse-biased zener diode in breakdown maintains approximately its zener voltage, here $V_Z = 5\,\text{V}$, across the parallel load. The series resistor absorbs the remaining supply voltage.
- Kirchhoff's current law gives $I = I_Z + I_L$. The greatest zener current occurs at no load, when $I_L = 0$, so the resistor must limit this worst-case current to $I_{Z,\max}$.

Step 1: Find the voltage across the series resistor The diagram places the resistor between the $50\,\text{V}$ supply and the regulated $5\,\text{V}$ output node. Kirchhoff's voltage law gives

$$V_R = V_i - V_Z = 50 - 5 = 45\,\text{V}.$$

The full supply voltage is not across the resistor: the zener branch retains $5\,\text{V}$. This subtraction is essential when calculating the limiting resistance.

Step 2: Impose the maximum-current condition With no load, all resistor current passes through the zener. Convert $90\,\text{mA} = 0.090\,\text{A}$, and require

$$I_Z = \frac{45}{R} \le 0.090\,\text{A} \quad \Longrightarrow \quad R \ge \frac{45}{0.090} = 500\,\Omega.$$

At this boundary the zener current is exactly its stated maximum. A smaller resistance would exceed the rating in the no-load condition; a load, when present, diverts some of the series current away from the zener.

Step 3: Conclusion The minimum required series resistance is $\boxed{500\,\Omega}$.

Quick Tip to Solve: Find the zener-protection resistor using the supply-minus-zener voltage and the no-load current; the load cannot be relied on to protect the diode.

55. The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is _____ nC, if electric flux density of its field is found as

D = e^{-x} sin y $\hat{i}$ − e^{-x} cos y $\hat{j}$ + 2z $\hat{k}$ C/m^2 [4 marks]

Solution:

Concept:

- The electric flux density $\vec{D}$ obeys Gauss's law $\nabla \cdot \vec{D} = \rho_v$, where ρ_v is the free volume charge density in $\mathrm{C\,m^{-3}}$.
- For an incremental volume ΔV around a point, its enclosed charge is $\Delta Q = \rho_v \Delta V$, evaluated at that point. No additional factor of ε_0 is needed because the given vector is $\vec{D}$, rather than $\vec{E}$.

Step 1: Differentiate the flux-density components The stated components are $D_x = e^{-x}\sin y$, $D_y = -e^{-x}\cos y$, and $D_z = 2z$. Hence

$$\begin{aligned}\rho_v &= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \\ &= -e^{-x}\sin y + e^{-x}\sin y + 2 = 2\,\mathrm{C\,m^{-3}}.\end{aligned}$$

The first two terms cancel, so the result is constant and in particular has this value at the origin.

Step 2: Multiply by the enclosed volume The incremental volume is $\Delta V = 2 \times 10^{-9}\,\mathrm{m^3}$. Therefore

$$\Delta Q = (2)(2 \times 10^{-9}) = 4 \times 10^{-9}\,\mathrm{C} = 4\,\mathrm{nC}.$$

The positive divergence means a positive enclosed charge. The conversion uses $1\,\mathrm{nC} = 10^{-9}\,\mathrm{C}$; the requested numerical entry is the coefficient in nanocoulombs.

Step 3: Conclusion The enclosed charge is $\boxed{4\,\mathrm{nC}}$, so the numerical entry is 4.

Quick Tip to Solve: For a given $\vec{D}$, take its divergence directly; differentiate each component only with respect to its matching coordinate.

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56. The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$. Then, the value of | x | is _____. [4 marks]

Solution:

Concept:

- For a uniform wire, each small element has mass $dm = \lambda\, ds$, where λ is its constant linear mass density. Centre-of-mass coordinates are mass-weighted averages of position.
- A semicircular arc is symmetric about the perpendicular bisector of its diameter, so its centre of mass has $x_{\mathrm{CM}} = 0$. The wire is an arc, so the mass element is proportional to arc length rather than area.

Step 1: Integrate the vertical coordinate Take the upper semicircle and parameterize it by $0 \le \theta \le \pi$.

Then $y = R\sin\theta$, $ds = R\,d\theta$, and its total mass is $M = \lambda\pi R$. Thus

$$y_{\text{CM}} = \frac{1}{M}\int y\,dm = \frac{\lambda R^2}{\lambda\pi R}\int_0^{\pi}\sin\theta\,d\theta = \frac{R}{\pi}[-\cos\theta]_0^{\pi} = \frac{2R}{\pi}.$$

The constant density cancels, as expected for a purely geometrical centre of mass.

Step 2: Compare with the given coordinate The upper-arc position is therefore $(0, 2R/\pi)$. Comparison with $(0, xR/\pi)$ gives $x = 2$. If the wire instead occupies the lower half-plane, the same integral gives $y_{\text{CM}} = -2R/\pi$, so $x = -2$. The question asks for $|x|$, making the answer independent of which semicircle is chosen. The result lies between zero and R, as a physical average must.

Step 3: Conclusion The requested dimensionless value is $\boxed{|x| = 2}$.

Quick Tip to Solve: Distinguish a semicircular wire from a lamina: use $dm = \lambda R\,d\theta$ for the arc, giving a distance $2R/\pi$ from the diameter.

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57. In an electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5Ω. However, it provides a potential difference of 1 V across a load resistance of 2Ω. The emf of the cell is given by $\frac{x}{10}V$. Then the value of x is _____. [4 marks]

Solution:

Concept:

- The terminal voltage of a cell delivering current is $V = E - Ir$, where E is its emf and r its internal resistance.
- The external current is $I = V/R$. The emf and internal resistance remain the same for both loads.

Step 1: Calculate the current for each load For the first load, the measured voltage is across the external resistance, so Ohm's law uses that voltage rather than the unknown emf. Thus

$$I_1 = \frac{1.25}{5} = 0.25\,\text{A}.$$

For the second load,

$$I_2 = \frac{1.00}{2} = 0.50\,\text{A}.$$

The smaller load draws the larger current. Consequently its internal voltage drop is larger, consistent with its lower terminal voltage.

Step 2: Eliminate the emf and find the internal resistance The two operating conditions give

$$E = 1.25 + 0.25r, \qquad E = 1.00 + 0.50r.$$

Subtracting eliminates the same cell emf:

$$0.25 = 0.25r \quad \Rightarrow \quad r = 1.00\,\Omega.$$

This is positive, as expected for an ordinary cell delivering electrical power.

Step 3: Determine the requested coefficient Substitute the resistance into either operating equation:

$$E = 1.25 + (0.25)(1.00) = 1.50\,\text{V}.$$

The other condition gives $1.00 + (0.50)(1.00) = 1.50\,\text{V}$, independently checking the result. Comparing with the requested form,

$$\frac{x}{10} = 1.50 \quad \Rightarrow \quad x = 15.$$

Here x is a numerical coefficient, not the emf expressed directly in volts.

Step 4: Conclusion The cell emf is $1.50\,\text{V}$; hence the requested dimensionless value is $\boxed{x = 15}$.

Quick Tip to Solve: Compute both currents first, then use $r = (V_1 - V_2)/(I_2 - I_1)$; do not put the unknown emf in Ohm's law for the external load.

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58. The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of liner expansion of material of track is 10^{-5}/°C. The energy stored per meter in the track is _____ J/m.
(Young's modulus of material of track is 10^{11} Nm^{-2}) [4 marks]

Solution:

Concept:

- A temperature change would produce free longitudinal strain $\alpha\Delta T$, where α is the coefficient of linear expansion.
- When the track cannot expand freely, the magnitude of the elastic strain is $\epsilon = \alpha\Delta T$, and the elastic energy density is $u = \frac{1}{2}Y\epsilon^2$.

Step 1: Find the thermal strain The change of temperature is $10\,^\circ\text{C}$, so it may be used directly with the given coefficient per degree Celsius. The magnitude of the free thermal strain would be

$$\epsilon = \alpha\Delta T = (10^{-5})(10) = 10^{-4}.$$

For the constrained railway track, this is also the magnitude of the mechanical strain that opposes thermal expansion. Heating produces compression; cooling produces tension. The stored energy depends on the square and is positive in either case.

Step 2: Calculate the energy density Within the elastic limit, stress is $\sigma = Y\epsilon$. Hence

$$\sigma = (10^{11})(10^{-4}) = 10^{7}\,\text{N}\,\text{m}^{-2}.$$

The stress rises linearly from zero as the strain develops, so the work per unit volume is the triangular area under the stress-strain line:

$$u = \frac{1}{2}\sigma\epsilon = \frac{1}{2}(10^{7})(10^{-4}) = 500\,\text{J}\,\text{m}^{-3}.$$

Using only $\sigma\epsilon$ would incorrectly double this energy.

Step 3: Convert volume density into energy per length A track segment of length L has volume AL. Therefore

$$U = uAL, \qquad \frac{U}{L} = uA.$$

Substitute the stated cross-sectional area:

$$\frac{U}{L} = (500)(0.01) = 5\,\mathrm{J\,m^{-1}}.$$

The cross section multiplies the volume energy density; it does not divide it. This conversion also verifies the required units.

Step 4: Conclusion The stored elastic energy per metre of the track is $\boxed{5\,\mathrm{J\,m^{-1}}}$.

Quick Tip to Solve: For a constrained uniform rod, use $U/L = \frac{1}{2}YA(\alpha\Delta T)^2$; square the whole thermal strain, including the temperature change.

59. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}v_0$. Then the value of x is _____.

[4 marks]

Solution:

Concept:

- For rolling without slipping, the centre speed and angular speed satisfy $v_0 = \omega R$, with wheel radius R.
- The ground-frame velocity of a rim point is the vector sum of translation of the centre and rotation about the centre.

Step 1: Locate the rim point and its rotational velocity The point is at the same height as the wheel centre. Its radius from the centre is therefore horizontal: it is at either the frontmost or rearmost position on the circular rim. The rotational velocity is tangent to the circle, hence vertical at either of these two positions. Its magnitude is

$$v_{\mathrm{rot}} = \omega R = v_0.$$

The two possible points have opposite vertical velocities, but this changes only direction, not the speed sought.

Step 2: Add the perpendicular velocity components The centre moves horizontally with velocity of magnitude v_0. Since the rotational velocity here is vertical, the two contributions are perpendicular. Thus the magnitude of their vector sum is

$$v = \sqrt{v_0^2 + (\omega R)^2} = \sqrt{v_0^2 + v_0^2} = \sqrt{2}\,v_0.$$

Adding the two magnitudes directly would give $2v_0$, which applies at the top of the wheel, where both velocities point horizontally in the same direction. It does not apply at the centre-height points.

Step 3: Compare with the given form The question expresses the ground-frame speed as $\sqrt{x}\,v_0$.

Therefore

$$\sqrt{x}\,v_0 = \sqrt{2}\,v_0 \quad \Rightarrow \quad x = 2.$$

As a consistency check, this intermediate speed lies between zero at the instantaneous contact point and $2v_0$ at the top. Both side points have the same speed because reversing the vertical component preserves its square.

Step 4: Conclusion The rim point moves at $\sqrt{2}\,v_0$, so the dimensionless coefficient is $\boxed{x = 2}$.

Quick Tip to Solve: At centre height the rotational velocity is vertical and the centre velocity is horizontal; use Pythagoras instead of adding their magnitudes.

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60. In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is _____° C. [4 marks]

Solution:

Concept:

- Newton's cooling law is $dT/dt = -k(T - T_a)$, where T_a is the constant ambient temperature and k the cooling constant.
- The excess temperature decays exponentially: $T - T_a = (T_0 - T_a)e^{-kt}$. Equal time intervals multiply the excess by the same factor.

Step 1: Measure temperatures relative to the room The surroundings remain at $25\,^\circ\text{C}$. Initially the excess temperature is

$$\theta_0 = 75 - 25 = 50\,^\circ\text{C}.$$

After the first five minutes it is

$$\theta_1 = 65 - 25 = 40\,^\circ\text{C}.$$

It is this excess, rather than the Celsius temperature itself, that determines the cooling rate. Thus the factor for one five-minute interval is

$$f = \frac{\theta_1}{\theta_0} = \frac{40}{50} = \frac{4}{5}.$$

Step 2: Apply the same factor to the next interval The next interval has the same duration and cooling constant, so the exponential factor is again $4/5$. Hence

$$\theta_2 = \theta_1 f = 40\left(\frac{4}{5}\right) = 32\,^\circ\text{C}.$$

Equivalently, $\theta_2 = \theta_0 f^2 = 50(4/5)^2 = 32\,^\circ\text{C}$. This method avoids calculating k or taking logarithms. The drop in the second interval is $8\,^\circ\text{C}$, smaller than the first drop of $10\,^\circ\text{C}$, because the body is closer to room temperature.

Step 3: Recover the actual body temperature Add the room temperature back to the final excess:

$$T_2 = T_a + \theta_2 = 25 + 32 = 57\,^\circ\text{C}.$$

Subtracting another ten degrees from 65 would incorrectly treat cooling as linear. Multiplying 65 itself by $4/5$ would also be wrong: Newton's law scales $T - T_a$, not the absolute Celsius reading.

Step 4: Conclusion At the end of the next five minutes the body temperature is $\boxed{57\,^\circ\mathrm{C}}$.

Quick Tip to Solve: For successive equal cooling intervals, form a geometric progression of excess temperatures above the surroundings, then add the room temperature back.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which one of the following statements for D.I. Mendeleeff, is incorrect? [4 marks]

(A) He authored the textbook - Principles of Chemistry

(B) At the time, he proposed Periodic Table of elements structure of atom was known.

(C) Element with atomic number 101 is named after him.

(D) He invented accurate barometer.

Correct Answer: (B) At the time, he proposed Periodic Table of elements structure of atom was known.

Solution:

Concept:

- Mendeleev's original periodic classification related the properties of elements to their atomic masses and chemical similarities.
- The detailed internal structure of the atom was established through later discoveries; it was not known when his periodic table was proposed.

Step 1: Identify what was known when the table was proposed Mendeleev proposed his periodic table in 1869. He arranged elements using atomic masses and recurring chemical properties, leaving gaps and predicting properties of undiscovered elements. This achievement did not depend on a known arrangement of electrons, protons and neutrons. The electron was identified only later, and the nuclear model and electronic explanations of periodicity came after that. Therefore the statement that the structure of the atom was already known at the time of his proposal reverses the historical order.

Step 2: Check the remaining biographical statements Option (A) is correct: Mendeleev authored the textbook *Principles of Chemistry*; his systematic treatment of the elements was closely associated with his work on that book. Option (C) is also correct: element 101 is mendelevium, symbol Md, named in his honour. Option (D), referring to his work on an accurate barometer, is an accepted biographical contribution and is not the incorrect statement sought. None of these claims asserts that atomic structure had been determined before the periodic table.

Step 3: Select the incorrect claim The question asks for the incorrect statement, so a true contribution must not be selected merely because it is less familiar. Option (B) alone says that atomic structure was known when Mendeleev proposed the table. His successful classification preceded that knowledge; modern explanations using atomic number and electronic configuration belong to later developments.

Step 4: Conclusion The claim that atomic structure was known when Mendeleev proposed his table is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: For historical periodic-table questions, distinguish the basis of Mendeleev's table, atomic mass, from the later atomic-number and electronic-configuration explanation.

62. Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point)? [4 marks]

(A) Simple distillation

(B) Steam ditillation

(C) Fractional distillation

(D) Reduced pressure distillation

Correct Answer: (D) Reduced pressure distillation

Solution:

Concept:

- A liquid boils when its equilibrium vapour pressure becomes equal to the external pressure above it.
- Reducing external pressure lowers the boiling temperature, allowing a heat-sensitive high-boiling liquid to distil without reaching its decomposition temperature.

Step 1: Connect the purification problem with boiling pressure The organic liquid has a high normal boiling point and decomposes close to that temperature. Heating it to its atmospheric-pressure boiling point would therefore cause chemical change rather than clean physical separation. A purification method must bring about vaporisation at a lower temperature while preserving the compound. Since boiling occurs when vapour pressure equals external pressure, lowering the pressure allows this condition to be reached earlier during heating.

Step 2: Choose reduced-pressure distillation In reduced-pressure, or vacuum, distillation a pump lowers the pressure in the apparatus. The liquid then boils below its normal boiling point; its vapour is condensed and collected. The important control is the pressure, not a reaction that changes the substance. This directly addresses both clues in the question: high boiling point and decomposition on strong heating. It is the standard suitable technique for such thermally unstable liquids.

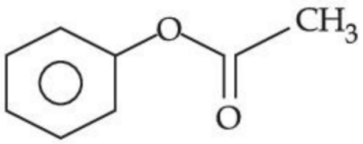
Step 3: Eliminate the other techniques Option (A), simple distillation at ordinary pressure, still requires heating near the normal boiling point and does not avoid the stated decomposition. Option (C), fractional distillation, primarily separates miscible liquids with nearby boiling points; adding a fractionating column does not itself lower the boiling temperature. Option (B), steam distillation, requires the appropriate steam volatility and immiscibility with water. Those special conditions are not given, so it is not the general answer to this problem. Option (D) directly supplies the required reduction of boiling temperature.

Step 4: Conclusion The compound should be purified by reduced pressure distillation. Hence, the correct option is (D).

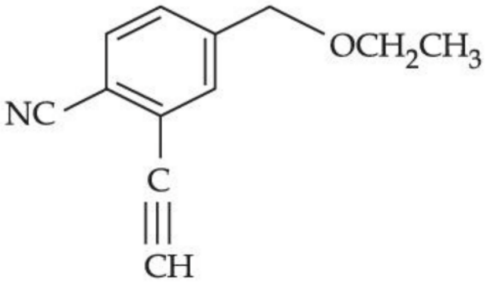
Quick Tip to Solve: When a high-boiling liquid decomposes near its normal boiling point, lower the pressure to lower the boiling temperature; fractionation alone does not solve that problem.

63. Which of the following compounds will provide a tertiary alcohol or reaction with excess of CH_3MgBr followed by hydrolysis? [4 marks]

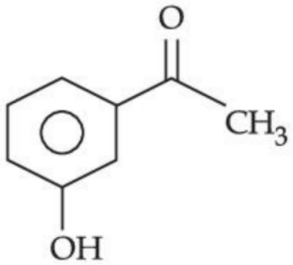
(A)



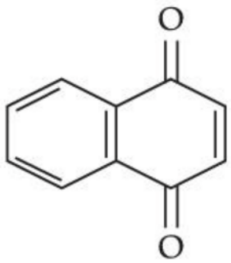
(B)



(C)



(D)



Correct Answer: At least options (A) and (C) both give tertiary alcohols, so the printed single-choice question has no unique answer.

Solution:

Concept:

- An ester reacts with two equivalents of a Grignard reagent and, after hydrolysis, normally yields a tertiary alcohol: $\mathrm{RCOOR_1} \xrightarrow[\mathrm{H_3O^+}]{2\,\mathrm{CH_3MgBr}} \mathrm{RC(OH)(CH_3)_2}$.
- A ketone gains a carbon group on Grignard addition, yielding a tertiary alcohol after hydrolysis. Acidic hydrogen atoms consume reagent first, but excess reagent can still react with a carbonyl group.

Step 1: Check the ester in option A Option (A) is phenyl acetate, $\mathrm{CH_3COOC_6H_5}$. The first methyl addition gives a tetrahedral intermediate which eliminates phenoxide to form acetone. A second

methyl addition then gives a tertiary alkoxide:

$$CH_3COOC_6H_5 \xrightarrow{CH_3MgBr} (CH_3)_2CO \xrightarrow{CH_3MgBr} (CH_3)_3COMgBr.$$

Hydrolysis produces tert-butyl alcohol, $(CH_3)_3COH$. Its hydroxyl-bearing carbon is bonded to three carbon groups, so (A) satisfies the printed demand.

Step 2: Check the hydroxy ketone in option C Option (C) contains a phenolic hydroxyl group and an acetyl group. Some methylmagnesium bromide first deprotonates the phenol, releasing methane. Because the question explicitly gives excess reagent, this acid-base reaction does not prevent addition to the ketone. Addition to the acetyl carbonyl, followed by hydrolysis, produces the side chain $ArC(OH)(CH_3)_2$, where Ar is the hydroxyphenyl group. The alcohol carbon has the aryl group and two methyl groups attached; this is also a tertiary alcohol. Thus (C) independently satisfies the demand.

Step 3: Compare the other structures and assess uniqueness Option (B) has a nitrile, an ether and a terminal alkyne. The terminal alkyne consumes Grignard reagent by deprotonation; addition to the nitrile followed by hydrolysis gives a ketone, rather than directly giving a tertiary alcohol. Option (D) has ketone carbonyl groups in a quinone framework; carbonyl addition can give tertiary alcohol centres after hydrolysis. In any case, the definite ester reaction of (A) and ketone reaction of (C) already establish that the literal wording does not single out one option. Selecting only one of them would discard another chemically valid answer.

Step 4: Conclusion As worded, the question has multiple defensible choices: at least (A) and (C) yield tertiary alcohols with excess CH_3MgBr followed by hydrolysis. There is no unique correct option.

Quick Tip to Solve: Count the carbon groups attached to the final hydroxyl-bearing carbon. With excess Grignard reagent, remember that acidic OH consumes one equivalent without stopping later carbonyl addition.

64. Which of the following compounds does not exhibit resonance? [4 marks]

(A) $CH_3CH_2OCH{=}CH_2$

(B) $C_6H_5CH_2OH$ (benzyl alcohol)

(C) $CH_3CH_2CH_2CONH_2$

(D) $CH_3CH_2CH{=}CHCH_2NH_2$

Correct Answer: (D) $CH_3CH_2CH{=}CHCH_2NH_2$

Solution:

Concept:

- Resonance requires adjacent orbitals that can overlap continuously: a lone pair beside a double bond, a conjugated ring, or a conjugated carbonyl group.
- A saturated tetrahedral carbon between a lone pair and a double bond interrupts their conjugation. Ordinary single-bond hyperconjugation is distinguished from the resonance being tested here.

Step 1: Check the oxygen-containing options In option (A), the oxygen atom in $CH_3CH_2OCH{=}CH_2$ is directly attached to a vinylic carbon. An oxygen lone pair can overlap with the adjacent carbon-carbon

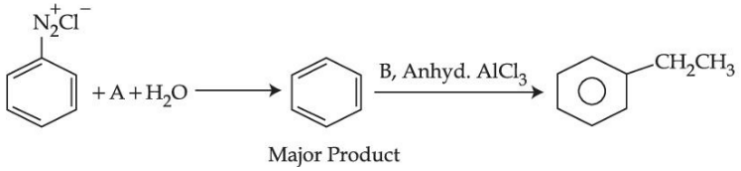
π system, so resonance is possible. Option (B) is benzyl alcohol, $C_6H_5CH_2OH$. The alcohol oxygen is separated from the ring by a methylene group, but the benzene ring itself still has delocalised π electrons. Thus the molecule certainly exhibits aromatic resonance.

Step 2: Compare the two nitrogen-containing options Option (C), $CH_3CH_2CH_2CONH_2$, contains an amide group. Its nitrogen lone pair is directly adjacent to the carbonyl group and can be donated into it, giving a contributor with a carbon-nitrogen double bond and opposite charges on oxygen and nitrogen. In option (D), $CH_3CH_2CH{=}CHCH_2NH_2$, a saturated CH_2 group lies between the amino nitrogen and the alkene. Therefore the nitrogen lone pair is not in conjugation with the double bond, and this isolated alkene has no corresponding delocalised resonance system. This eliminates (A), (B) and (C).

Step 3: Conclusion $CH_3CH_2CH{=}CHCH_2NH_2$ does not exhibit the resonance described above. Hence, the correct option is (D).

Quick Tip to Solve: Trace an uninterrupted chain of overlapping p orbitals; an intervening saturated CH_2 breaks lone-pair conjugation, whereas an aromatic ring still resonates internally.

65. In the chemical reactions given above A and B respectively are : [4 marks]



(A) H_3PO_2 and CH_3CH_2Cl

(B) CH_3CH_2OH and H_3PO_2

(C) H_3PO_2 and CH_3CH_2OH

(D) CH_3CH_2Cl and H_3PO_2

Correct Answer: (A) H_3PO_2 and CH_3CH_2Cl

Solution:

Concept:

- Aryl diazonium salts are reduced by hypophosphorous acid, H_3PO_2, replacing the diazonium group with hydrogen.
- Friedel-Crafts alkylation introduces an alkyl group into benzene using an alkyl halide and anhydrous $AlCl_3$.

Step 1: Identify reagent A from the first conversion The first starting material is benzenediazonium chloride and the first product is benzene. Thus the $-N_2{}^+$ group has been replaced by hydrogen, not hydroxyl or another substituent. Hypophosphorous acid performs this reduction in water. The balanced transformation can be written as

$$C_6H_5N_2Cl + H_3PO_2 + H_2O \longrightarrow C_6H_6 + N_2 + H_3PO_3 + HCl.$$

Consequently, $A = H_3PO_2$. This observation immediately rejects options (B) and (D), which assign a

different first reagent.

Step 2: Identify reagent B from the ethyl group The second conversion changes benzene into ethylbenzene in the presence of anhydrous $AlCl_3$. The required alkyl halide is chloroethane, CH_3CH_2Cl. Its interaction with the Lewis acid generates the electrophile used in aromatic substitution, and the ring retains aromaticity after loss of a proton. The net equation is

$$C_6H_6 + CH_3CH_2Cl \xrightarrow{AlCl_3} C_6H_5CH_2CH_3 + HCl.$$

Ethanol is not the alkyl halide used in this stated Friedel-Crafts reagent combination. Therefore (C), although it identifies A correctly, has the wrong B. Only (A) satisfies both conversions in their given order.

Step 3: Conclusion $A = H_3PO_2$ and $B = CH_3CH_2Cl$. Hence, the correct option is (A).

Quick Tip to Solve: For a reaction sequence, identify each reagent from the group replaced or introduced: diazonium to H suggests H_3PO_2, and benzene to ethylbenzene with $AlCl_3$ suggests ethyl chloride.

66. Isotope(s) of hydrogen which emits low energy β^- particles with $t_{1/2}$ value $>$ 12 years is/are

[4 marks]

(A) Protium

(B) Tritium

(C) Deuterium

(D) Deuterium and Tritium

Correct Answer: (B) Tritium

Solution:

Concept:

- Isotopes of hydrogen have one proton but different neutron numbers: protium ^1H, deuterium ^2H, and tritium ^3H.
- Tritium is radioactive and undergoes low-energy beta-minus decay with a half-life of approximately 12.3 years; protium and deuterium are stable.

Step 1: Apply the radioactive-decay requirement The question requires an isotope that emits β^- particles. A beta-minus decay converts a neutron into a proton, emitting an electron and an electron antineutrino. Tritium has one proton and two neutrons, and it decays into helium-3:

$$^3_1H \longrightarrow ^3_2He + ^0_{-1}e + \bar{\nu}_e.$$

The mass number remains three and charge is conserved. Thus tritium has the specified type of decay, unlike the two stable isotopes.

Step 2: Check the half-life and eliminate alternatives Tritium has a half-life of about 12.3 years, which is greater than the stated twelve-year threshold. Its emitted beta particles also have low energies, so it meets both conditions together. Protium does not undergo the specified radioactive decay, excluding

(A). Deuterium is a stable isotope and does not emit these beta particles, excluding (C). Option (D) incorrectly includes deuterium along with tritium; listing one correct isotope does not make the combined choice correct. Therefore the only acceptable isotope among the options is tritium alone. The isotope name, decay mode, and half-life must all be checked rather than selecting every isotope that contains neutrons.

Step 3: Conclusion Tritium is the isotope with low-energy β^- emission and $t_{1/2} > 12$ years. Hence, the correct option is (B).

Quick Tip to Solve: Remember the stability pattern first: protium and deuterium are stable; tritium is the radioactive hydrogen isotope with a half-life close to twelve years.

67. Match List-I with List-II :

	List-I(Species)		List-II(Hybrid Orbitals)
(a)	SF_4	**(i)**	sp^3d^2
(b)	IF_5	**(ii)**	d^2sp^3
(c)	NO_2^+	**(iii)**	sp^3d
(d)	NH_4^+	**(iv)**	sp^3
		(v)	sp

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(i), (b)-(ii), (c)-(v) and (d)-(iii)

(B) (a)-(ii), (b)-(i), (c)-(iv) and (d)-(v)

(C) (a)-(iii), (b)-(i), (c)-(v) and (d)-(iv)

(D) (a)-(iv), (b)-(iii), (c)-(ii) and (d)-(v)

Correct Answer: (C) (a)-(iii), (b)-(i), (c)-(v) and (d)-(iv)

Solution:

Concept:

- The steric number is the number of sigma-bonding domains plus lone pairs on the central atom; a multiple bond counts as one domain.
- In the conventional hybrid-orbital description, steric numbers 2, 4, 5 and 6 correspond respectively to sp, sp^3, sp^3d and sp^3d^2.

Step 1: Assign the two expanded-octet species In SF_4, sulfur forms four sigma bonds and retains one lone pair. Its steric number is $4 + 1 = 5$, so the conventional hybridisation is sp^3d, corresponding to (iii). The electron-domain geometry is trigonal bipyramidal, while the molecular shape is seesaw. In IF_5, iodine forms five sigma bonds and has one lone pair. Its steric number is six, giving sp^3d^2, corresponding to (i), with square-pyramidal molecular shape. Thus (a)-(iii) and (b)-(i) are fixed.

Step 2: Assign the two cations and compare complete matches The nitronium ion, NO_2^+, has two nitrogen-oxygen bonding domains and no lone pair on nitrogen. Its two double bonds count as two domains, not four; the ion is linear and nitrogen is sp hybridised, giving (c)-(v). In NH_4^+, nitrogen has four nitrogen-hydrogen sigma bonds and no lone pair, hence tetrahedral geometry and

sp^3 hybridisation, giving (d)-(iv). The full match is therefore (a)-(iii), (b)-(i), (c)-(v), (d)-(iv). Option (A) assigns the wrong sulfur and ammonium hybridisations, (B) assigns wrong sulfur, nitronium and ammonium entries, and (D) also conflicts with the domain counts. Only (C) matches all four.

Step 3: Conclusion All four species match the hybridisations established above. Hence, the correct option is (C).

Quick Tip to Solve: Count sigma-bonding domains and lone pairs before reading the hybridisation list; each double bond is only one domain in the steric-number count.

68. Which of the following molecules does not show stereo isomerism? [4 marks]

(A) 3, 4-Dimethylhex-3-ene

(B) 3-Methylhex-1-ene

(C) 3-Ethylhex-3-ene

(D) 4-Methylhex-1-ene

Correct Answer: (C) 3-Ethylhex-3-ene

Solution:

Concept:

- An alkene exhibits geometrical isomerism only if each doubly bonded carbon carries two different substituents.
- A tetrahedral carbon bonded to four different groups is a stereogenic centre and can give optical isomerism even when the alkene itself is terminal.

Step 1: Inspect the alkene substituents in options A and C For 3,4-dimethylhex-3-ene, the structure is $CH_3CH_2C(CH_3){=}C(CH_3)CH_2CH_3$. Each double-bonded carbon has a methyl and an ethyl group, so distinct E/Z arrangements exist. Option (A) therefore shows stereoisomerism. In 3-ethylhex-3-ene, the structure is $CH_3CH_2C(CH_2CH_3){=}CHCH_2CH_3$. Carbon 3 has two identical ethyl groups. Interchanging them cannot generate a distinct geometrical isomer, so this double bond does not meet the E/Z condition.

Step 2: Check the terminal alkenes for optical isomerism For 3-methylhex-1-ene, carbon 3 is attached to H, methyl, vinyl $CH{=}CH_2$, and propyl $CH_2CH_2CH_3$. These four groups differ, making carbon 3 a stereogenic centre; option (B) thus shows optical isomerism. For 4-methylhex-1-ene, carbon 4 carries H, methyl, ethyl, and $CH_2CH{=}CH_2$, again four different groups, so option (D) is also optically active in its individual enantiomeric forms. By contrast, option (C) has no tetrahedral carbon carrying four different groups and its double bond has identical substituents on one carbon. It therefore lacks both of the relevant sources of stereoisomerism.

Step 3: Conclusion 3-Ethylhex-3-ene does not show stereoisomerism. Hence, the correct option is (C).

Quick Tip to Solve: Do not stop after finding a terminal double bond: check the saturated carbons for four different groups, since optical isomerism can still occur.

69. Which one of the following group-15 hydride is the strongest reducing agent? [4 marks]

(A) AsH_3

(B) BiH_3

(C) PH_3

(D) SbH_3

Correct Answer: (B) BiH_3

Solution:

Concept:

- The stability of group-15 hydrides decreases down the group because the central atom becomes larger and the element-hydrogen bond becomes weaker.
- Reducing character increases as a hydride more readily loses hydrogen and is itself oxidised; this trend is opposite to the hydride stability trend.

Step 1: Establish the bond-strength trend The relevant group order is phosphorus, arsenic, antimony and bismuth. As the central atom becomes larger down the group, overlap of its valence orbital with the hydrogen 1s orbital becomes less effective. The element-hydrogen bond weakens and the corresponding hydride becomes less thermally stable. Bismuthine, BiH_3, is therefore the least stable hydride among the four choices. This bond-strength argument concerns the ease of reducing action and should not be confused with the basicity of the lone pair.

Step 2: Reverse the stability order to compare reducing power The reducing-character order for the listed hydrides is

$$PH_3 < AsH_3 < SbH_3 < BiH_3.$$

A stronger reducing agent is more readily oxidised while it reduces another species. The weak bonds and low stability of bismuthine make it the strongest reducing agent in this series. Thus AsH_3 in (A), PH_3 in (C), and SbH_3 in (D) are all weaker reducers than BiH_3. In particular, choosing phosphine because phosphorus is higher in the group would reverse the required trend. The comparison is made among hydrides of the same group, so the systematic decrease in bond strength supplies the decisive criterion.

Step 3: Conclusion BiH_3 is the strongest reducing agent among the listed group-15 hydrides. Hence, the correct option is (B).

Quick Tip to Solve: For group-15 hydrides, write stability decreasing down the group first, then reverse that order to obtain increasing reducing character.

70. The set having ions which are coloured and paramagnetic both is : [4 marks]

(A) Cu^{2+}, Cr^{3+}, Sc^{+}

(B) Cu^{2+}, Zn^{2+}, Mn^{4+}

(C) Sc^{3+}, V^{5+}, Ti^{4+}

(D) Ni^{2+}, Mn^{7+}, Hg^{2+}

Correct Answer: (A) Cu^{2+}, Cr^{3+}, Sc^{+}

Solution:

Concept:

- Paramagnetism requires at least one unpaired electron. Colour commonly arises from transitions involving partially filled d orbitals.
- For transition-metal ions, electrons are removed from the outer ns orbital before the (n-1)d orbital. A set qualifies only if every ion satisfies both requirements.

Step 1: Test every ion in the first set Cu^{2+} has configuration $[Ar]3d^9$, with one unpaired d electron; its partially filled d shell supports colour. Cr^{3+} is $[Ar]3d^3$, with three unpaired electrons and a partially filled d shell. The printed third ion is Sc^{+}, not Sc^{3+}. Removing one outer electron from scandium gives $[Ar]3d^14s^1$ in the elementary configuration description, retaining unpaired electrons and a partially occupied d subshell. Thus all three ions in (A) meet the stated criteria.

Step 2: Reject the other sets using closed or empty d shells Option (B) includes Zn^{2+}, whose configuration is $3d^{10}$: its d electrons are paired, so it is diamagnetic and does not show ordinary d-d colour. Option (C) contains Sc^{3+}, V^{5+} and Ti^{4+}, each with a d^0 configuration, so there are no unpaired d electrons. Option (D) contains Mn^{7+}, which is d^0, and Hg^{2+}, which is $5d^{10}$; these fail the requirement even though Ni^{2+} itself has a partially filled d shell. Colour due to charge transfer in a compound would not supply the missing paramagnetism. Hence only the first complete set survives both tests.

Step 3: Conclusion The qualifying set is Cu^{2+}, Cr^{3+}, Sc^{+}. Hence, the correct option is (A).

Quick Tip to Solve: Eliminate an entire set as soon as it contains a d^0 or d^{10} ion that fails paramagnetism, and distinguish the printed Sc^{+} from Sc^{3+}.

71. Thiamine and pyridoxine are also known respectively as : [4 marks]

(A) Vitamin B_2 and Vitamin E

(B) Vitamin E and Vitamin B_2

(C) Vitamin B_6 and Vitamin B_2

(D) Vitamin B_1 and Vitamin B_6

Correct Answer: (D) Vitamin B_1 and Vitamin B_6

Solution:

Concept:

- Vitamins are micronutrients required in small quantities. The B-complex contains distinct water-soluble vitamins, each with its own conventional number and chemical name.
- Thiamine is vitamin B_1; pyridoxine is vitamin B_6. Riboflavin is vitamin B_2, whereas vitamin E denotes tocopherols and is fat soluble.

Step 1: Identify thiamine Thiamine belongs to the B-complex and is conventionally designated vitamin B_1. Its active coenzyme form is thiamine pyrophosphate, which participates in carbohydrate metabolism, including oxidative decarboxylation. The name thiamine therefore cannot be interchanged with riboflavin or vitamin E. In the first position of the required ordered pair we must put B_1.

Step 2: Identify pyridoxine and preserve the order Pyridoxine belongs to the vitamin B_6 group. Its coenzyme derivative, pyridoxal phosphate, participates in amino-acid reactions such as transamination. The question says respectively, so the two names must be matched in the given sequence: thiamine first and pyridoxine second. Thus the ordered pair is (B_1, B_6), rather than a reversed or partially correct pair.

Step 3: Eliminate the competing pairs Option (A) incorrectly assigns thiamine to B_2 and pyridoxine to E. Option (B) reverses those same incorrect names. Option (C) assigns thiamine to B_6, which belongs to pyridoxine, and assigns pyridoxine to B_2, which is riboflavin. Only option (D) has both vitamin identities in the required order.

Step 4: Conclusion Thiamine and pyridoxine are vitamin B_1 and vitamin B_6, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Remember the distinct pairings: thiamine–B_1, riboflavin–B_2, pyridoxine–B_6. Check the word respectively before selecting the pair.

72. An organic compound A (C_6H_6O) gives dark green colouration with ferric chloride. On treatment with $CHCl_3$ and KOH, followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC). Identify A, B and C. [4 marks]

(A) A = phenol, C_6H_5OH; B = 2-hydroxybenzaldehyde, o-HOC_6H_4CHO; C = 2-(hydroxymethyl)phenol, o-$HOC_6H_4CH_2OH$.

(B) A = phenol, C_6H_5OH; B = 2-(hydroxymethyl)phenol, o-$HOC_6H_4CH_2OH$; C = 2-hydroxybenzaldehyde, o-HOC_6H_4CHO.

(C) A = 2-(hydroxymethyl)phenol, o-$HOC_6H_4CH_2OH$; B = 2-hydroxybenzaldehyde, o-HOC_6H_4CHO; C = phenol, C_6H_5OH.

(D) A = 2-hydroxybenzaldehyde, o-HOC_6H_4CHO; B = 2-(hydroxymethyl)phenol, o-$HOC_6H_4CH_2OH$; C = phenol, C_6H_5OH.

Correct Answer: (A) A = phenol, C_6H_5OH; B = 2-hydroxybenzaldehyde, o-HOC_6H_4CHO; C = 2-(hydroxymethyl)phenol, o-$HOC_6H_4CH_2OH$.

Solution:

Concept:

- Phenols give a characteristic coloured complex with ferric chloride. The formula C_6H_6O and this test together identify the phenolic compound here.
- The Reimer–Tiemann reaction of phenol with $CHCl_3$ and aqueous alkali, followed by acidification, introduces an aldehyde group mainly ortho to $-OH$. PCC oxidises a primary alcohol group to an aldehyde.

Step 1: Identify compound A from the formula and test A benzene ring bearing one hydroxyl group has the formula C_6H_5OH, or C_6H_6O. Its phenolic hydroxyl group accounts for the ferric-chloride colour test. Therefore A is phenol. The other proposed A structures contain an additional CHO or CH_2OH group and do not have the stated molecular formula; this immediately excludes options (C) and (D).

Step 2: Apply the Reimer–Tiemann transformation Treatment of phenol with chloroform and potassium hydroxide is the Reimer–Tiemann formylation. Acid work-up gives the ortho-hydroxy aldehyde shown in the options:

$$C_6H_5OH \xrightarrow[H^+]{CHCl_3,\,KOH} \text{o-}HOC_6H_4CHO.$$

Thus B is 2-hydroxybenzaldehyde, commonly called salicylaldehyde. It is an aldehyde, not a hydroxymethyl-substituted phenol.

Step 3: Work backwards through PCC oxidation PCC converts the primary alcohol side chain $-CH_2OH$ into $-CHO$, retaining the carbon skeleton and the ortho relationship to the phenolic group. Hence C must be 2-(hydroxymethyl)phenol:

$$\text{o-}HOC_6H_4CH_2OH \xrightarrow{PCC} \text{o-}HOC_6H_4CHO.$$

Option (A) has all three structures in this sequence. Option (B) interchanges the alcohol and aldehyde; PCC does not reduce an aldehyde to that alcohol.

Step 4: Conclusion A is phenol, B is salicylaldehyde and C is 2-(hydroxymethyl)phenol. Hence, the correct option is (A).

Quick Tip to Solve: Use the two independent clues: Reimer–Tiemann makes an ortho aldehyde from phenol, and PCC makes that aldehyde from an ortho hydroxymethyl group.

73. Which one of the following reactions does not occur? [4 marks]

(A) $C_6H_5NH_2 \xrightarrow{(CH_3CO)_2O\ /\ \text{Pyridine}} C_6H_5NHCOCH_3$

(B) $C_6H_5NH_2 \xrightarrow{H_2SO_4} \text{p-}H_2NC_6H_4SO_3H$

(C) $C_6H_5NH_2 \xrightarrow{AlCl_3+CH_3Cl} \text{p-}H_2NC_6H_4CH_3$

(D) $C_6H_5NH_2 \xrightarrow{HNO_3\ /\ H_2SO_4} \text{p-}H_2NC_6H_4NO_2$

Correct Answer: (C) $C_6H_5NH_2 \xrightarrow{AlCl_3+CH_3Cl} \text{p-}H_2NC_6H_4CH_3$

Solution:

Concept:

- Aniline has a basic nitrogen lone pair that undergoes acylation and also forms a strong Lewis acid–base complex with $AlCl_3$.
- Friedel–Crafts alkylation requires an aromatic ring capable of attacking an electrophile under Lewis-acid conditions. Complexation of aniline with the catalyst deactivates the ring, so direct Friedel–Crafts alkylation fails.

Step 1: Check acylation and sulphonation In option (A), the nitrogen of aniline attacks acetic anhydride and is acetylated to give acetanilide, $C_6H_5NHCOCH_3$. Pyridine acts as a base, so this is a normal amine reaction. Option (B) represents sulphonation: aniline forms its hydrogen sulphate first, and the usual heating conditions give sulphanilic acid, p-$H_2NC_6H_4SO_3H$. These are recognised transformations of aniline.

Step 2: Test the proposed Friedel–Crafts alkylation Option (C) proposes direct methylation of the aromatic ring with $CH_3Cl/AlCl_3$. However, the nitrogen lone pair coordinates strongly to the Lewis acid:

$$C_6H_5NH_2 + AlCl_3 \longrightarrow C_6H_5NH_2 \cdot AlCl_3.$$

This interaction ties up the catalyst and gives the nitrogen strong positive character, reducing the ring electron density. Consequently aniline does not undergo the illustrated direct Friedel–Crafts reaction to form para-toluidine.

Step 3: Distinguish nitration from a failed reaction Aniline can undergo nitration with the mixed-acid reagent in option (D). Strong acid protonates much of the amine, so direct nitration gives a mixture and includes appreciable meta product; the para-nitroaniline shown can nevertheless be formed. The presence of complications or other isomers does not mean that nitration fails altogether. The uniquely inapplicable reaction class in the choices is direct Friedel–Crafts alkylation of unprotected aniline.

Step 4: Conclusion The direct formation of para-toluidine from aniline with $CH_3Cl/AlCl_3$ does not occur as shown. Hence, the correct option is (C).

Quick Tip to Solve: For an aromatic amine with an available lone pair, check whether a Lewis-acid catalyst is trapped at nitrogen before applying the Friedel–Crafts rule.

74. Which one of the following 0.06 M aqueous solutions has lowest freezing point? [4 marks]

(A) $Al_2(SO_4)_3$

(B) $C_6H_{12}O_6$

(C) KI

(D) K_2SO_4

Correct Answer: (A) $Al_2(SO_4)_3$

Solution:

Concept:

- Freezing-point depression obeys $\Delta T_f = iK_f m$, where i is the van't Hoff factor, K_f is the solvent constant and m is molality.
- For dilute aqueous solutions of equal stated concentration, compare the number of solute particles produced per formula unit. More particles give a larger depression and therefore a lower freezing point.

Step 1: Count particles for the electrolytes Using the usual complete-dissociation comparison for dilute solutions, aluminium sulphate gives two aluminium ions and three sulphate ions:

$$Al_2(SO_4)_3 \longrightarrow 2\,Al^{3+} + 3\,{SO_4}^{2-}.$$

Its ideal van't Hoff factor is therefore 5. Potassium iodide produces two ions, K^+ and I^-, so its factor is 2. Potassium sulphate produces two potassium ions and one sulphate ion, so its factor is 3.

Step 2: Include the molecular solute Glucose, $C_6H_{12}O_6$, dissolves as neutral molecules and does not dissociate into ions. Its van't Hoff factor is 1. Thus the relative particle concentrations for the common 0.06 M concentration are 0.30, 0.06, 0.12 and 0.18 in the order aluminium sulphate, glucose, potassium iodide and potassium sulphate. The comparison uses the dilute-solution approximation intended in the question.

Step 3: Compare the resulting freezing temperatures The solvent is water in every case, so its freezing-point constant is common. The largest value of i gives the largest positive depression ΔT_f, which is subtracted from the freezing point of pure water. Aluminium sulphate therefore has the lowest freezing point. Glucose, KI and potassium sulphate give smaller depressions because their respective particle counts are only 1, 2 and 3 rather than 5.

Step 4: Conclusion The 0.06 M aqueous solution of $Al_2(SO_4)_3$ has the lowest freezing point. Hence, the correct option is (A).

Quick Tip to Solve: Count all ions per formula unit, including stoichiometric coefficients: $Al_2(SO_4)_3$ gives five particles, not two or three.

75. The water having more dissolved O_2 is : [4 marks]

(A) boiling water

(B) water at 80°C

(C) polluted water

(D) water at 4°C

Correct Answer: (D) water at 4°C

Solution:

Concept:

- At a given gas partial pressure, the equilibrium amount dissolved depends on temperature. Dissolution of oxygen in water is favoured at lower temperature.
- Henry's law may be written $p = K_H x$, where p is oxygen partial pressure and x its dissolved mole fraction. Increasing temperature generally increases K_H for a gas, reducing its solubility at fixed pressure.

Step 1: Compare the temperatures Water at $4^\circ\mathrm{C}$ is much colder than water at $80^\circ\mathrm{C}$ or at its boiling point. Under the same surrounding oxygen pressure, colder water can retain a larger amount of dissolved oxygen. Heating favours escape of dissolved gas into the atmosphere; this is why bubbles of dissolved air can appear on warming water even before vigorous boiling begins.

Step 2: Reject the heated-water choices Boiling water has a very low dissolved oxygen concentration because continuous heating and vapour formation drive gas out of the liquid. Water at $80^\circ\mathrm{C}$ also has substantially lower oxygen solubility than cold water. Thus neither option (A) nor option (B) can exceed the oxygen content of the cold-water choice under the usual comparison conditions.

Step 3: Assess the polluted-water choice Polluted water commonly contains organic matter whose microbial decomposition consumes dissolved oxygen, increasing the oxygen demand and decreasing the oxygen available in the water. It is therefore not the choice associated with high dissolved oxygen. Combining the temperature trend with this oxygen-consumption effect selects the cold water in option (D). The observation concerns dissolved oxygen, not the oxygen atoms chemically bound within water molecules.

Step 4: Conclusion Water at $4^\circ\mathrm{C}$ has the greater dissolved O_2 content among the stated choices. Hence, the correct option is (D).

Quick Tip to Solve: For dissolved gases, reverse the familiar solid-solubility intuition: colder water generally retains more oxygen, while heating and organic pollution reduce it.

76. Match List-I with List-II

	List-I(Elements)		List-II(Properties)
(a)	Ba	(i)	Organic solvent soluble compounds
(b)	Ca	(ii)	Outer electronic configuration $6s^2$
(c)	Li	(iii)	Oxalate insoluble in water
(d)	Na	(iv)	Formation of very strong monoacidic base

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)

(B) (a)-(iv), (b)-(i), (c)-(ii) and (d)-(iii)

(C) (a)-(iii), (b)-(ii), (c)-(iv) and (d)-(i)

(D) (a)-(i), (b)-(iv), (c)-(ii) and (d)-(iii)

Correct Answer: (A) (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)

Solution:

Concept:

- Electronic configuration locates an s-block element by its period and group. Barium has the configuration $[\mathrm{Xe}]6s^2$.
- Lithium shows enhanced covalent character because of its small cation. Calcium oxalate is sparingly soluble in water, while sodium hydroxide is a strong base with one hydroxide ion per formula unit.

Step 1: Match barium and calcium Barium lies in group 2 and period 6. Its two outer electrons occupy the 6s orbital, so Ba matches property (ii), the outer configuration $6s^2$. Calcium is identified by the formation of the water-insoluble oxalate $\mathrm{CaC_2O_4}$, giving the familiar precipitation reaction

$$\mathrm{Ca^{2+} + {C_2O_4}^{2-} \longrightarrow CaC_2O_4(s).}$$

Thus Ca matches (iii).

Step 2: Match lithium and sodium The small $\mathrm{Li^+}$ ion has strong polarising power. Lithium compounds consequently show relatively greater covalent character than the corresponding compounds of other alkali metals; compounds such as lithium chloride have appreciable solubility in suitable organic solvents. This identifies Li with property (i). Sodium forms NaOH, which dissociates strongly in water and supplies one hydroxide ion per formula unit, so Na matches the very strong monoacidic base in (iv).

Step 3: Read the complete code Collecting these independent assignments gives $(a) \to (ii)$, $(b) \to (iii)$, $(c) \to (i)$ and $(d) \to (iv)$. This is option (A). Option (B) incorrectly gives the 6s configuration to lithium; option (C) gives it to calcium; option (D) again gives it to lithium. Since this configuration alone is uniquely barium's among the listed elements, these alternatives are inconsistent even before checking their remaining matches.

Step 4: Conclusion Barium matches property (ii), calcium (iii), lithium (i), and sodium (iv). Hence, the correct option is (A).

Quick Tip to Solve: Begin matching with the uniquely identifying configuration $6s^2$: it fixes Ba immediately, after which calcium oxalate and covalent lithium compounds complete the code.

77. When silver nitrate solution is added to potassium iodide solution then the sol produced is :

[4 marks]

(A) AgI / I^-

(B) AgI / Ag^+

(C) KI / NO_3^-

(D) $AgNO_3$ / NO_3^-

Correct Answer: (A) AgI / I^-

Solution:

Concept:

- A precipitated colloid preferentially adsorbs an ion common to its own lattice. The reagent present in excess determines which common ion is available for adsorption.
- Adding silver nitrate to potassium iodide forms AgI. In the potassium iodide medium, excess iodide ions adsorb on the silver iodide particles, giving a negatively charged sol.

Step 1: Identify the dispersed solid The two salts react to form sparingly soluble silver iodide:

$$\mathrm{AgNO_3(aq) + KI(aq) \longrightarrow AgI(s) + KNO_3(aq).}$$

The dispersed particles are therefore AgI, not potassium iodide or silver nitrate. This immediately rules out options (C) and (D), both of which incorrectly identify a soluble reagent as the colloidal dispersed substance.

Step 2: Determine the adsorbed ion from the addition order The question adds silver nitrate solution to potassium iodide solution. The surrounding medium is consequently rich in iodide during formation of the sol. Iodide is a constituent ion of the AgI lattice and is preferentially adsorbed on its surface. The particles acquire a negative surface charge, represented schematically by $\mathrm{AgI/I^-}$. Potassium ions form the accompanying counter-ion atmosphere rather than the charge-determining layer.

Step 3: Distinguish the positive-sol alternative Option (B), $\mathrm{AgI/Ag^+}$, would correspond to silver iodide particles formed in an excess of silver nitrate, where silver ions are preferentially adsorbed. That is the reverse reagent-excess situation. For the stated addition into potassium iodide, iodide is the adsorbed ion and the negative sol in option (A) is the appropriate representation. The slash indicates the surface-associated ion, not a new bulk compound with that formula.

Step 4: Conclusion The sol is negatively charged silver iodide, represented by $\mathrm{AgI/I^-}$. Hence, the correct option is (A).

Quick Tip to Solve: For a precipitation sol, identify the insoluble solid first, then attach the common ion supplied by the reagent in excess; the sign follows that adsorbed ion.

78. Given below are the statements about diborane

(a) Diborane is prepared by the oxidation of $NaBH_4$ with I_2

(b) Each boron atom is in sp^2 hybridized state

(c) Diborane has one bridged 3 centre-2-electron bond

(d) Diborane is a planar molecule

The option with correct statements is : [4 marks]

(A) (c) and (d) only

(B) (a) only

(C) (c) only

(D) (a) and (b) only

Correct Answer: (B) (a) only

Solution:

Concept:

- Diborane, B_2H_6, is electron deficient: it contains four terminal two-centre two-electron bonds and two bridging three-centre two-electron bonds.
- Each boron has an approximately tetrahedral arrangement of four bonding directions, described using sp^3 hybrid orbitals; the complete molecule is not planar.

Step 1: Check the preparation statement Iodine oxidises borohydride to diborane. A balanced preparation equation is

$$2\,NaBH_4 + I_2 \longrightarrow B_2H_6 + 2\,NaI + H_2.$$

Atoms balance on both sides: two boron atoms and eight hydrogen atoms become six hydrogens in diborane and two in hydrogen gas. Thus statement (a) is correct.

Step 2: Check the hybridisation and bridge count In diborane each boron is attached to two terminal hydrogens and participates in two bridges. The boron bonding framework is approximately sp^3, not the planar sp^2 arrangement of an isolated trigonal boron compound. Hence (b) is false. Each of the two bridging hydrogen atoms forms a separate B–H–B three-centre two-electron bond, so there are two such bonds, not one. Statement (c) is therefore false.

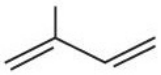
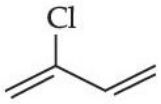
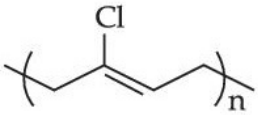
Step 3: Test planarity and eliminate options The two boron atoms and four terminal hydrogen atoms lie in one plane. The two bridging hydrogens lie on opposite sides of that plane, so all eight atoms do not lie in a single plane and (d) is false. Options (A) and (C) include the false statement (c), while option (D) includes the false statement (b). Only option (B) retains the one true statement.

Step 4: Conclusion Only the preparation statement is correct. Hence, the correct option is (B).

Quick Tip to Solve: For diborane, remember four terminal bonds and two hydrogen bridges; the bridge count immediately rules out the claim of only one three-centre bond.

79. Match List-I with List-II :

Choose the correct answer from the options given below : [4 marks]

List - I		List - II	
(a)	Chloroprene	(i)	
(b)	Neoprene	(ii)	
(c)	Acrylonitrile	(iii)	
(d)	Isoprene	(iv)	$CH_2=CH-CN$

(A) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(B) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(C) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(D) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Correct Answer: (B) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Solution:

Concept:

- Chloroprene is 2-chlorobuta-1,3-diene, $CH_2=C(Cl)-CH=CH_2$. Its addition polymer is neoprene.
- Isoprene is 2-methylbuta-1,3-diene, $CH_2=C(CH_3)-CH=CH_2$; acrylonitrile is $CH_2=CH-CN$.

Step 1: Identify the chlorinated monomer and polymer Structure (ii) is a conjugated four-carbon diene with chlorine on its second carbon. It is chloroprene, so (a) matches (ii). Structure (iii) carries a repeating-unit bracket and the subscript n; its chlorine-bearing chain is formed by 1,4-addition of chloroprene. Thus it is neoprene, and (b) matches (iii). A polymer bracket is decisive here: it cannot represent the small-molecule monomer chloroprene.

Step 2: Identify the nitrile and methyl-substituted diene Structure (iv), $CH_2=CH-CN$, contains the nitrile group attached to a vinyl group, so it is acrylonitrile: (c) matches (iv). Structure (i) has a methyl substituent on the second carbon of the conjugated diene. This is isoprene, so (d) matches (i). The methyl substituent distinguishes isoprene from chloroprene, whose substituent is chlorine.

Step 3: Compare the complete mapping with each option The required sequence is

(a)–(ii), (b)–(iii), (c)–(iv), (d)–(i).

Option (A) incorrectly sends chloroprene to the polymer and acrylonitrile to the chloro diene. Option (C) sends neoprene to the unpolymerised methyl diene and isoprene to the polymer. Option (D) also confuses the polymer and the monomers. Option (B) alone gives all four matches correctly.

Step 4: Conclusion Chloroprene matches ii, neoprene matches iii, acrylonitrile matches iv, and isoprene matches i. Hence, the correct option is (B).

Quick Tip to Solve: First locate the polymer brackets and the nitrile group; these fix neoprene and acrylonitrile before comparing the two substituted diene monomers.

80. Sulphide ion is soft base and its ores are common for metals.

(a) Pb (b) Al (c) Ag (d) Mg [4 marks]

(A) (a) and (c) only

(B) (a) and (d) only

(C) (a) and (b) only

(D) (c) and (d) only

Correct Answer: (A) (a) and (c) only

Solution:

Concept:

- The sulphide ion, S^{2-}, is a large, polarizable soft base. Soft or relatively polarizable metal cations favour bonding to sulphide.
- Hard metal cations such as Al^{3+} and Mg^{2+} preferentially occur with hard oxygen-donor anions, as oxides, silicates and carbonates.

Step 1: Identify metals with common sulphide ores Lead commonly occurs as galena, PbS, and silver occurs as argentite, Ag_2S. Their relatively polarizable cations are compatible with the soft sulphide anion. Thus the listed metals (a) Pb and (c) Ag satisfy the statement. In galena the lead oxidation state is $+2$; in argentite the two silver atoms each have oxidation state $+1$, balancing one sulphide ion.

Step 2: Exclude the hard metal cations Aluminium forms the small, highly charged hard cation Al^{3+}; its important ores include hydrated alumina in bauxite rather than a common sulphide ore. Magnesium similarly forms the hard cation Mg^{2+}, with common occurrences such as magnesite, $MgCO_3$, and dolomite, $CaMg(CO_3)_2$. The question concerns common ores, so merely considering whether an unusual sulphide compound can exist would not answer it.

Step 3: Select the option containing only the accepted pair The valid set is (a) and (c). Option (B) includes magnesium, option (C) includes aluminium, and option (D) again includes magnesium. Each of those choices combines a valid sulphide-forming metal with a hard cation whose common ores are not sulphides. Option (A) is the only pair containing both lead and silver.

Step 4: Conclusion Pb and Ag have common sulphide ores, PbS and Ag_2S. Hence, the correct option is (A).

Quick Tip to Solve: Use galena and argentite as the sulphide-ore checks; small hard cations such as aluminium and magnesium are associated mainly with oxygen-containing ores.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is : [4 marks]

Solution:

Concept:

- Ammonia is a neutral ligand and each outer-sphere chloride has charge -1; the chloride count therefore fixes the charge and oxidation state of cobalt.
- Cobalt has atomic number 27. Co^{2+} is $3d^7$, whereas Co^{3+} is $3d^6$. Octahedral splitting places three t_{2g} orbitals below two e_g orbitals.

Step 1: Find the cobalt oxidation states For $[Co(NH_3)_6]Cl_2$, two chloride ions balance a complex cation of charge $+2$. Since all six ammonia ligands are neutral, cobalt is $+2$ and has seven d electrons. For $[Co(NH_3)_6]Cl_3$, the three chloride ions require a $+3$ complex cation, so cobalt is $+3$ with six d electrons. The chloride ions are counterions, not ligands occupying the six coordination sites.

Step 2: Count unpaired electrons in the cobalt(II) complex In the octahedral cobalt(II) ammine complex, the splitting is insufficient to force the low-spin d^7 arrangement. The high-spin occupancy is

$$t_{2g}^5 e_g^2.$$

The five electrons in the three lower orbitals leave one unpaired electron there, while the two higher orbitals each contain one electron. The cobalt(II) complex therefore has $1 + 2 = 3$ unpaired electrons.

Step 3: Count the cobalt(III) contribution and add The higher $+3$ oxidation state produces a substantially larger octahedral splitting. In $[Co(NH_3)_6]^{3+}$, the six d electrons pair in the three lower orbitals, giving the low-spin configuration

$$t_{2g}^6 e_g^0.$$

It has no unpaired electrons. Therefore the total for the two complexes is $3 + 0 = 3$. Applying the same spin arrangement to both ions would overlook the oxidation-state dependence of crystal-field splitting.

Step 4: Conclusion The total number of unpaired electrons is $\boxed{3}$ (a dimensionless count).

Quick Tip to Solve: Determine the cobalt oxidation state from the outer chloride ions before assigning spin: the Co(III) hexaammine is low spin, whereas the Co(II) complex contributes three unpaired electrons.

82. Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene _____. (Nearest integer) [4 marks]

Solution:

Concept:

- Methylation replaces one hydrogen of benzene by a methyl group: one mole of benzene yields one mole of toluene. The mole ratio is $1 : 1$.

- Percentage yield equals actual product mass divided by theoretical product mass, multiplied by 100. Molar masses are $M(\mathrm{C_6H_6}) = 78$ and $M(\mathrm{C_7H_8}) = 92\,\mathrm{g\,mol^{-1}}$.

Step 1: Convert the benzene mass to a theoretical product amount The benzene supplied is 10 g. Its amount is

$$n(\mathrm{C_6H_6}) = \frac{10\,\mathrm{g}}{78\,\mathrm{g\,mol^{-1}}} = \frac{5}{39}\,\mathrm{mol}.$$

The methylation reaction forms one toluene molecule for each benzene molecule consumed. Hence complete conversion of this benzene would give the same number of moles of toluene, taking the methylating reagent as sufficient as implied by the yield question.

Step 2: Calculate the theoretical mass of toluene Toluene has formula $\mathrm{C_7H_8}$, so its molar mass is $7(12) + 8(1) = 92\,\mathrm{g\,mol^{-1}}$. The theoretical product mass is consequently

$$m_{\text{theoretical}} = \frac{10}{78} \times 92 = 11.7949\,\mathrm{g}.$$

It is greater than the initial benzene mass because methylation incorporates additional carbon and hydrogen from the methylating reagent. The 10 g starting mass is therefore not itself the theoretical toluene mass.

Step 3: Compare the actual and theoretical yields The actual isolated toluene mass is 9.2 g. Thus

$$\text{Percentage yield} = \frac{9.2}{10(92/78)} \times 100 = 78\%.$$

Equivalently, 9.2 g of toluene is 0.100 mol and the theoretical amount is $10/78$ mol; their ratio is 0.78. The value is already an integer, so rounding to the nearest integer leaves it unchanged.

Step 4: Conclusion The percentage yield of toluene is $\boxed{78\%}$. The required numerical entry is 78.

Quick Tip to Solve: Use the molar mass ratio 92/78 to obtain theoretical toluene; dividing 9.2 g by the 10 g benzene mass would incorrectly ignore the methyl group added.

83. The number of acyclic structural isomers (including geometrical isomers) for pentene are _____

[4 marks]

Solution:

Concept:

- An acyclic pentene has molecular formula $\mathrm{C_5H_{10}}$ and one carbon-carbon double bond. Count distinct carbon skeletons and distinct double-bond positions without renaming the same molecule twice.
- An alkene has geometrical isomers only if each carbon of the double bond bears two different groups. A terminal $\mathrm{CH_2}{=}$ group cannot show cis-trans isomerism.

Step 1: List the straight-chain constitutional isomers For the unbranched five-carbon chain there are two distinct positions of the double bond: pent-1-ene and pent-2-ene. Placing the double bond at position 3 or 4 gives pent-2-ene or pent-1-ene after numbering from the nearer end, so these are

duplicates. Pent-1-ene has a terminal $CH_2=$ group and gives only one form. Pent-2-ene has different substituents on both alkene carbons, so it has two geometrical forms, cis and trans.

Step 2: List the branched constitutional isomers The distinct branched alkenes are 2-methylbut-1-ene, 3-methylbut-1-ene and 2-methylbut-2-ene. The first two contain a terminal $CH_2=$ unit, excluding geometrical isomerism. In 2-methylbut-2-ene one double-bond carbon bears two identical methyl groups, so that molecule also has no geometrical pair. Each branched constitution therefore contributes exactly one form.

Step 3: Combine the constitutional and geometrical counts There are five constitutional isomers altogether: two straight-chain and three branched. Only pent-2-ene splits into two geometrical isomers. Since the question explicitly includes geometrical isomers, replace the single pent-2-ene entry by its two forms:

$$N = 1 + 2 + 1 + 1 + 1 = 6.$$

Counting only constitutional isomers would give five, while counting an extra cis-trans pair for a terminal alkene would overcount. Cyclic structures are excluded by the word acyclic.

Step 4: Conclusion Including the geometrical pair of pent-2-ene, the number is $\boxed{6}$ (a dimensionless count).

Quick Tip to Solve: List the five constitutional pentenes first, then add one extra entry because only pent-2-ene has a cis-trans pair.

84. Assume a cell with the following reaction

$$Cu_{(s)} + 2Ag^{+}(1 \times 10^{-3}M) \rightarrow Cu^{2+}(0.250M) + 2Ag_{(s)}$$

$E^{\Theta}_{cell} = 2.97$ **V**

E_{cell} for the above reaction is _____ V. (Nearest integer)

[Given : log 2.5 = 0.3979, T = 298 K] [4 marks]

Solution:

Concept:

- At 298 K the Nernst equation is $E = E^{\circ} - (0.0591/n)\log_{10} Q$, with n the number of transferred electrons and Q the reaction quotient.
- Pure solid copper and silver have unit activity. Thus only the aqueous ion concentrations enter Q; stoichiometric coefficients become powers.

Step 1: Identify electron transfer and form the quotient The oxidation is $\mathrm{Cu} \longrightarrow \mathrm{Cu}^{2+} + 2\,\mathrm{e}^{-}$ and the reduction is $2\,\mathrm{Ag}^{+} + 2\,\mathrm{e}^{-} \longrightarrow 2\,\mathrm{Ag}$. Consequently $n = 2$. For the stated forward reaction,

$$Q = \frac{[\mathrm{Cu}^{2+}]}{[\mathrm{Ag}^{+}]^2} = \frac{0.250}{(1.0 \times 10^{-3})^2} = 2.5 \times 10^{5}.$$

The silver-ion concentration is squared because the balanced reaction consumes two silver ions. The solids are omitted rather than inserted as zero concentrations.

Step 2: Evaluate the logarithm and voltage correction Using the supplied logarithm,

$$\log Q = \log(2.5) + 5 = 0.3979 + 5 = 5.3979.$$

Substitute the supplied standard cell potential, $E^\circ = 2.97\,\text{V}$, without replacing it by a tabulated value:

$$E = 2.97 - \frac{0.0591}{2}(5.3979)\,\text{V}.$$

The correction is $0.1595\,\text{V}$, giving $E \simeq 2.8105\,\text{V}$. Since $Q > 1$, the logarithmic term is positive and the nonstandard cell potential is lower than its stated standard value.

Step 3: Round only the final potential The requested nearest integer is obtained from the final unrounded value, not from the concentration ratio or its logarithm. Since 2.8105 lies between 2.5 and 3.5, it rounds to 3. The small difference between using 0.0591 and 0.0592 in the 298 K coefficient has no effect on this nearest-integer result.

Step 4: Conclusion The cell potential is approximately $2.81\,\text{V}$, so the required nearest-integer answer is $\boxed{3\,\text{V}}$.

Quick Tip to Solve: Write the balanced reaction quotient before substituting: the square on silver-ion concentration and the two-electron denominator are both essential.

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85. Value of K_P for the equilibrium reaction

$N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47.9. The K_C for this reaction at same temperature is _____. (Nearest integer)

(R = 0.083 L bar K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- For a gaseous equilibrium, pressure and concentration equilibrium constants satisfy $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is gaseous product stoichiometry minus gaseous reactant stoichiometry.
- Only gaseous species enter Δn_g. Use the supplied gas constant so that the pressure and volume units used to relate partial pressure to concentration remain consistent.

Step 1: Find the change in gaseous mole number The balanced equilibrium is $\text{N}_2\text{O}_4(\text{g}) \rightleftharpoons 2\,\text{NO}_2(\text{g})$. There are two moles of gaseous product for each mole of gaseous reactant. Therefore

$$\Delta n_g = 2 - 1 = 1.$$

This is a difference of coefficients, not the number of different gas species. Both sides contain a gas, but their total gaseous coefficients are unequal.

Step 2: Relate the two equilibrium constants For each species, $p = cRT$. Thus $p_{\text{NO}_2}^2/p_{\text{N}_2\text{O}_4}$ contains two factors of RT in the numerator and one in the denominator. Accordingly,

$$K_p = K_c RT, \qquad K_c = \frac{K_p}{RT}.$$

This also shows why multiplying by RT would reverse the required conversion.

Step 3: Substitute the temperature and round The temperature is already in kelvin, so no temperature conversion is needed. With the given $R = 0.083\,\mathrm{L\,bar\,K^{-1}\,mol^{-1}}$,

$$RT = 0.083 \times 288 = 23.904,$$

$$K_c = \frac{47.9}{23.904} = 2.00385\ldots.$$

The requested nearest-integer value is obtained only after the division; rounding RT prematurely is unnecessary.

Step 4: Conclusion The concentration equilibrium constant is $K_c \approx 2.004$, so the required nearest-integer entry is $\boxed{2}$.

Quick Tip to Solve: Compute gaseous products minus gaseous reactants before using the relation; here a positive difference means divide K_p by RT to find K_c.

86. If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is _____ kJ. (Nearest integer) [4 marks]

Solution:

Concept:

- A molar enthalpy of combustion gives the heat change when one mole of a substance burns completely under the stated conditions. For n moles, $\Delta H = n\Delta H_c^\circ$.
- An exothermic combustion has negative enthalpy change for the reacting system. The amount of heat generated is its positive magnitude, $Q_{\text{released}} = -\Delta H$.

Step 1: Convert graphite mass to amount Graphite is elemental carbon, so use its molar mass $12\,\mathrm{g\,mol^{-1}}$. The sample is only one gram, not one mole. Its amount is

$$n = \frac{m}{M} = \frac{1\,\mathrm{g}}{12\,\mathrm{g\,mol^{-1}}} = \frac{1}{12}\,\mathrm{mol}.$$

The powder form does not change this mass-to-mole conversion.

Step 2: Scale the supplied molar combustion enthalpy The question supplies $\Delta H_c^\circ = -2.48 \times 10^2 = -248\,\mathrm{kJ\,mol^{-1}}$. Use this value directly for the stated graphite powder. The enthalpy change for the sample is

$$\Delta H = \frac{1}{12} \times (-248) = -20.6667\,\mathrm{kJ}.$$

The mole unit cancels against the inverse-mole unit of the molar enthalpy. A one-gram sample therefore produces much less heat than a one-mole sample.

Step 3: Report the heat released as a positive amount The negative sign describes energy leaving the system during combustion. Since the blank asks for the amount of heat generated, report the

magnitude:

$$Q_{\text{released}} = |\Delta H| = 20.6667\,\text{kJ}.$$

Rounding this value to the nearest integer gives 21, rather than truncating it to 20. No additional factor for oxygen is required because the combustion enthalpy is already quoted per mole of graphite.

Step 4: Conclusion The heat generated is approximately $20.67\,\text{kJ}$, or $\boxed{21\,\text{kJ}}$ to the nearest integer. The numerical entry is $\boxed{21}$.

Quick Tip to Solve: For heat released by a specified mass, divide the molar heat by the molar mass first, and distinguish a negative system enthalpy from the positive heat generated.

87. Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to _____ [4 marks]

Solution:

Concept:

- A neutral atom has as many electrons as its atomic number. Its ground-state configuration distributes those electrons among occupied subshells according to their filling order.
- A p subshell contains three orbitals and can hold six electrons. Counting all p-orbital electrons means adding the occupancies of every occupied p subshell, including the inner shells.

Step 1: Write the complete electronic configuration Vanadium has atomic number $Z = 23$, so the neutral atom contains 23 electrons. Its configuration is

$$\text{V} : 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^3\,4s^2.$$

Equivalently, it is $[\text{Ar}]3d^3 4s^2$. The full form is useful because the argon core itself contains the p electrons we must count.

Step 2: Identify the occupied p subshells The first shell has no p subshell. The occupied p subshells in vanadium are $2p$ and $3p$, and each is completely filled. Thus

$$N(2p) = 6, \qquad N(3p) = 6.$$

The $4p$ subshell is empty in the ground-state atom; the electrons after the argon core occupy $4s$ and $3d$, not $4p$.

Step 3: Add only the p-electron populations The requested total therefore is

$$N_p = N(2p) + N(3p) = 6 + 6 = 12.$$

As a check, the remaining electrons are eight s electrons and three d electrons, giving $12 + 8 + 3 = 23$ overall. Do not count the six occupied p orbitals as six electrons: each of those orbitals contains a pair. Also, the three outer d electrons are excluded because the question specifically asks for p orbitals.

Step 4: Conclusion The vanadium atom has $\boxed{12}$ electrons in p orbitals. This is a dimensionless electron count.

Quick Tip to Solve: Expand the noble-gas core when counting a particular orbital type; the notation [Ar] hides both filled p subshells needed here.

88.

$$N_2O_{5(g)} \rightarrow 2NO_{2(g)} + \frac{1}{2}O_{2(g)}$$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K. The concentration of N_2O_5 after 1 hour was 1.60×10^{-2} mol L^{-1}. The rate constant of the reaction at 318 K is _____ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Given : log 3 = 0.477, log 5 = 0.699] [4 marks]

Solution:

Concept:

- For a first-order reaction, $k = \frac{2.303}{t}\log_{10}\left(\frac{[A]_0}{[A]_t}\right)$. The concentration ratio is dimensionless and k has inverse-time units.
- The integrated rate law uses the concentration of the same reactant at the two times. Product stoichiometric coefficients do not multiply the logarithm when the first-order decay law is written for that reactant.

Step 1: Form the reactant concentration ratio For N_2O_5, the initial and final concentrations give

$$\frac{[A]_0}{[A]_t} = \frac{2.40 \times 10^{-2}}{1.60 \times 10^{-2}} = \frac{3}{2}.$$

The common concentration units and power of ten cancel. One hour is $60\,\text{min}$, which is the time unit required by the requested rate constant.

Step 2: Evaluate the logarithm from the supplied data To use the given common logarithms, first obtain

$$\log 2 = 1 - \log 5 = 1 - 0.699 = 0.301.$$

Hence

$$\log \frac{3}{2} = \log 3 - \log 2 = 0.477 - 0.301 = 0.176.$$

The ratio exceeds one, so the logarithm is positive, as needed for a positive decay constant.

Step 3: Calculate the constant in the requested units Substitute the elapsed time in minutes:

$$k = \frac{2.303}{60}(0.176) = 0.0067555\,\text{min}^{-1}.$$

Equivalently,

$$k = 6.7555 \times 10^{-3}\,\text{min}^{-1}.$$

The blank is the coefficient multiplying 10^{-3}, so round 6.7555, not 0.0067555, to the nearest integer. Using seconds would instead produce an inverse-second constant and would not answer the specified blank.

Step 4: Conclusion The rate constant is approximately $6.76 \times 10^{-3}\,\text{min}^{-1}$. The nearest-integer coefficient in the blank is $\boxed{7}$.

Quick Tip to Solve: Convert the elapsed hour to minutes before substitution, and round the coefficient after expressing k in the exact power-of-ten form printed in the question.

89. If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1}, the molarity of glucose in blood is _____ $\times 10^{-3}$ M. (Nearest integer)

[Given : Atomic mass of C = 12, H = 1, O = 16 u] [4 marks]

Solution:

Concept:

- Molarity is the amount of solute in moles per litre of solution: $C = n/V$. A concentration in grams per litre becomes molarity after division by the solute molar mass.
- The molar mass of a compound is the sum of the atomic masses multiplied by the corresponding subscripts in its molecular formula. Glucose is $C_6H_{12}O_6$.

Step 1: Calculate the molar mass of glucose Use the supplied atomic masses $C = 12$, $H = 1$, and $O = 16$. The six carbon, twelve hydrogen and six oxygen atoms contribute

$$M = 6(12) + 12(1) + 6(16)$$

$$M = 72 + 12 + 96 = 180\,\text{g mol}^{-1}.$$

This is the mass of one mole of glucose molecules, rather than the mass of an individual molecule.

Step 2: Convert grams per litre into moles per litre Consider one litre of blood for convenience. It contains $0.72\,\text{g}$ glucose, so the amount of glucose in this litre is

$$n = \frac{0.72}{180} = 0.004\,\text{mol}.$$

Therefore

$$C = \frac{0.004\,\text{mol}}{1\,\text{L}} = 0.004\,\text{mol L}^{-1}.$$

Equivalently, dividing the given g L^{-1} directly by g mol^{-1} gives the same units.

Step 3: Match the requested numerical form Since $1\,\text{M} = 1\,\text{mol L}^{-1}$, the calculated molarity can be written as

$$C = 4.00 \times 10^{-3}\,\text{M}.$$

The blank asks for the coefficient of $10^{-3}\,\text{M}$, which is exactly 4. No density conversion is required because the concentration already refers to a litre of solution; multiplying or dividing by 1000 again would introduce an erroneous factor.

Step 4: Conclusion The glucose molarity is $\boxed{4 \times 10^{-3}\,\text{M}}$. The nearest-integer entry in the blank is $\boxed{4}$.

Quick Tip to Solve: When mass concentration is already in grams per litre, divide only by molar mass; then read off the coefficient of the power of ten used in the blank.

90. A copper complex crystallising in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3}. The molar mass of copper complex is _____ g mol^{-1}. (Nearest integer)

[Given : N_A = 6.022 $\times$ 10^{23} mol^{-1}] [4 marks]

Solution:

Concept:

- For a cubic unit cell, density is $\rho = \frac{ZM}{N_A a^3}$, where Z is the number of formula units per cell, M the molar mass, and a the cell edge.
- Cubic close packing (CCP) has the face-centred cubic arrangement. Its eight corner contributions and six face contributions give $Z = 8(1/8) + 6(1/2) = 4$.

Step 1: Identify the cell population and convert the edge The CCP lattice therefore contains four formula units of the copper complex per unit cell. The density is in $\mathrm{g\,cm^{-3}}$, so convert the edge to centimetres:

$$1\,\mathrm{nm} = 10^{-7}\,\mathrm{cm},$$

$$a = 0.4518 \times 10^{-7} = 4.518 \times 10^{-8}\,\mathrm{cm}.$$

The factor is cubed when finding the volume; a nanometre-to-metre conversion alone would not be compatible with the given density.

Step 2: Calculate the mass of one unit cell The cubic cell volume is

$$V = a^3 = (4.518 \times 10^{-8})^3$$

$$V = 9.22229 \times 10^{-23}\,\mathrm{cm}^3.$$

Its mass is consequently

$$m_{\mathrm{cell}} = \rho V = 7.62(9.22229 \times 10^{-23})$$

$$m_{\mathrm{cell}} = 7.02738 \times 10^{-22}\,\mathrm{g}.$$

This cell mass belongs to four formula units, not to one formula unit.

Step 3: Convert the cell mass to molar mass Multiply the cell mass by Avogadro's constant and divide by the cell population:

$$M = \frac{\rho a^3 N_A}{Z}$$

$$M = \frac{(7.02738 \times 10^{-22})(6.022 \times 10^{23})}{4}$$

$$M \approx 105.80\,\mathrm{g\,mol^{-1}}.$$

The result concerns the copper complex specified in the question, so it must not be replaced by the atomic molar mass of elemental copper. Round the computed value at the end.

Step 4: Conclusion The molar mass of the copper complex is approximately $105.80\,\mathrm{g\,mol^{-1}}$, or $\boxed{106\,\mathrm{g\,mol^{-1}}}$ to the nearest integer. The entry is $\boxed{106}$.

Quick Tip to Solve: For CCP use $Z = 4$, and convert the cell edge to the length unit in the density before cubing it; a length conversion error becomes cubed in the volume.