JEE Main 2021 Paper 1 (B.E./B.Tech) Jul 25 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 3 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 July 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$, $x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is : [4 marks]

(A) increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

(B) decreasing in $\left(0, \frac{\pi}{2}\right)$

(C) increasing in $\left(-\frac{\pi}{6}, 0\right)$

(D) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Correct Answer: (D) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Solution:

Concept:

- A differentiable function is strictly increasing where its derivative is positive and strictly decreasing where its derivative is negative. The derivative must be examined separately on intervals separated by its zeros.
- Use the chain rule for powers of sine: $\frac{d}{dx}(\sin^n x) = n\sin^{n-1} x\cos x$. Factoring the derivative makes its sign easier to determine than expanding individual terms.

Step 1: Differentiate and factor the result Differentiating the given polynomial in $\sin x$, we obtain

$$\begin{aligned} f'(x) &= 12\sin^3 x\cos x + 30\sin^2 x\cos x + 12\sin x\cos x \\ &= 6\sin x\cos x(2\sin x + 1)(\sin x + 2). \end{aligned}$$

The constant term contributes zero. We need the signs of these four factors on the specified domain, rather than just the value of the derivative at one point.

Step 2: Determine the sign on each interval For $-\pi/6 < x < 0$, we have $-1/2 < \sin x < 0$, while $\cos x > 0$, $2\sin x + 1 > 0$ and $\sin x + 2 > 0$. Thus $f'(x) < 0$ throughout this interval. For $0 < x < \pi/2$, all four factors are positive, so $f'(x) > 0$. The function therefore decreases up to zero and increases after zero. This rejects (A), which requires increase over the entire domain; (B), which reverses the positive-interval behaviour; and (C), which reverses the negative-interval behaviour.

Step 3: Conclusion The function is decreasing in $(-\pi/6, 0)$. Hence, the correct option is (D).

Quick Tip to Solve: Factor the derivative completely; on this domain only the sign of $\sin x$ changes inside the interval.

2. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is : [4 marks]

(A) 6

(B) 4

(C) 2

(D) 8

Correct Answer: (A) 6

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the sum of its first m terms is $S_m = \frac{m}{2}[2a + (m-1)d]$.
- The relation between sums with different numbers of terms determines a relation between a and d. Substitute that relation before forming the requested ratio, retaining the factors arising from the number of terms.

Step 1: Use the given relation between the sums Writing out both sides of $S_{3n} = 3S_{2n}$ gives

$$\frac{3n}{2}[2a + (3n-1)d] = 3n[2a + (2n-1)d].$$

Since n is a positive integer, division by $3n/2$ is legitimate. Rearranging yields

$$2a + (3n-1)d = 4a + (4n-2)d, \qquad 2a + (n-1)d = 0.$$

Consequently $2a = -(n-1)d$; this relation also means that the sum of the first n terms is zero.

Step 2: Calculate the two required sums and divide Substituting the relation into each sum separately,

$$S_{2n} = n[-(n-1)d + (2n-1)d] = n^2 d,$$
$$S_{4n} = 2n[-(n-1)d + (4n-1)d] = 6n^2 d.$$

The ratio in the question presupposes $S_{2n} \neq 0$, so we can cancel $n^2 d$. It equals 6, rather than 4, 2 or 8. In particular, merely dividing the numbers of terms would give 2, but that ignores the change in the average term of an arithmetic progression.

Step 3: Conclusion $\frac{S_{4n}}{S_{2n}} = 6$. Hence, the correct option is (A).

Quick Tip to Solve: After finding $2a = -(n-1)d$, write $S_{kn} = \frac{1}{2}k(k-1)n^2 d$ to compare any two multiples quickly.

3. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is : [4 marks]

(A) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(B) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

(C) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

(D) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

Correct Answer: (A) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

Solution:

Concept:

- The centroid of a triangle is the arithmetic mean of the coordinates of its three vertices. If the two fixed vertices are the foci of a central conic, their midpoint is the centre of the conic.
- Complete squares to find the centre. Express the moving vertex coordinates in terms of the centroid coordinates, then substitute into the original curve to eliminate the moving vertex.

Step 1: Locate the centre and express the centroid Completing squares in the given equation gives

$$16(x+1)^2 - 9(y-2)^2 = 144.$$

Thus the centre is $C = (-1, 2)$. The foci are symmetric about this point, so their coordinate sums are -2 and 4, respectively; their individual positions are unnecessary. Let the moving point be $P = (u, v)$ and the centroid be $G = (x, y)$. The centroid formula gives

$$x = \frac{u-2}{3}, \qquad y = \frac{v+4}{3},$$

whence $u = 3x + 2$ and $v = 3y - 4$.

Step 2: Eliminate the moving point and compare the options Since P belongs to the original hyperbola,

$$16(u+1)^2 - 9(v-2)^2 = 144.$$

Substitution and division by nine yield

$$16(x+1)^2 - 9(y-2)^2 = 16.$$

Expanding carefully, including the constant terms, gives

$$16x^2 - 9y^2 + 32x + 36y - 36 = 0.$$

This transformation is invertible, so every point of this locus corresponds to a point of the original hyperbola. Options (B) and (D) interchange the quadratic coefficients and therefore the transverse-axis orientation. Option (C) has the right quadratic terms but the wrong constant, giving a different size.

Step 3: Conclusion The locus is $16x^2 - 9y^2 + 32x + 36y - 36 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: The centroid lies one third of the way from the centre to P; shrink both semiaxes by three without calculating the foci individually.

4. Let f : R → R be defined as

$$f(x) = \begin{cases} \frac{\lambda\left|x^2-5x+6\right|}{\mu(5x-x^2-6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}}, & x > 2 \\ \mu, & x = 2 \end{cases}$$

where [x] is the greatest integer is than or equal to x. If f is continuous at x = 2, then $\lambda + \mu$ is equal to : [4 marks]

(A) e(−e + 1)

(B) e(e − 2)

(C) 1

(D) 2e − 1

Correct Answer: (A) e(−e + 1)

Solution:

Concept:

- Continuity at $x = 2$ requires both one-sided limits to exist and equal $f(2) = \mu$. The two formulae must be simplified using the sign information appropriate to their respective sides.
- Immediately to the right of two, $[x] = 2$. The standard trigonometric limit $\lim_{h\to 0} \tan h/h = 1$ then evaluates the exponent in the right-hand branch.

Step 1: Evaluate the left-hand limit For $x < 2$, both factors of $x^2-5x+6 = (x-2)(x-3)$ are negative, so their product is positive. Therefore the absolute value equals x^2-5x+6. The denominator contains its negative:

$$5x - x^2 - 6 = -(x^2 - 5x + 6).$$

For points sufficiently near two on its left, cancellation gives

$$f(x) = -\frac{\lambda}{\mu}, \qquad \lim_{x\to 2^-} f(x) = -\frac{\lambda}{\mu}.$$

Continuity must therefore impose $-\lambda/\mu = \mu$, provided the denominator parameter is nonzero.

Step 2: Evaluate the right-hand limit and solve the parameters Write $h = x - 2$. For $0 < h < 1$, $[x] = 2$, so $x - [x] = h$. Hence

$$\lim_{x\to 2^+} f(x) = \exp\left(\lim_{h\to 0^+} \frac{\tan h}{h}\right) = e.$$

Matching this with the assigned value gives $\mu = e$, which is indeed nonzero. The left-hand condition then gives $\lambda = -\mu^2 = -e^2$. Thus $\lambda + \mu = e - e^2 = e(1 - e)$. The other three expressions do not equal this value; in particular, keeping the wrong sign when removing the absolute value would incorrectly change λ to a positive number.

Step 3: Conclusion $\lambda + \mu = e(1 - e)$. Hence, the correct option is (A).

Quick Tip to Solve: Near 2^+, replace $[x]$ by 2 before taking the limit; near 2^-, resolve the absolute value before cancelling.

5. The value of the definite integral $\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}}$ is : [4 marks]

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{12}$

(D) $\frac{\pi}{18}$

Correct Answer: (C) $\frac{\pi}{12}$

Solution:

Concept:

- The reflection identity $\int_a^b g(x)\,dx = \int_a^b g(a+b-x)\,dx$ follows from the substitution $u = a+b-x$, with both the limits and the differential reversed.
- If the reflected integrand and the original add to one, twice the required integral equals the length of the interval. Here the bounds sum to $\pi/4$, so the reflected tangent becomes a cotangent.

Step 1: Reflect the variable about the midpoint Let

$$I = \int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}}.$$

Under $x = \pi/4 - u$, the lower and upper limits interchange and $dx = -du$. Thus, after reversing the limits again,

$$I = \int_{\pi/24}^{5\pi/24} \frac{du}{1+\sqrt[3]{\cot 2u}}.$$

On this interval $2u$ lies strictly between zero and $\pi/2$, so the tangent and its real cube root are positive. In particular no zero denominator or branch ambiguity is introduced by reciprocating.

Step 2: Add the two forms and simplify Putting $t = \sqrt[3]{\tan 2x}$, the reflected integrand is $t/(1+t)$, since $\sqrt[3]{\cot 2x} = 1/t$. Adding the two integral representations therefore gives

$$2I = \int_{\pi/24}^{5\pi/24} \left(\frac{1}{1+t} + \frac{t}{1+t}\right) dx = \frac{5\pi}{24} - \frac{\pi}{24} = \frac{\pi}{6}.$$

Consequently $I = \pi/12$. Option (B) is twice the answer and results from forgetting to divide by two after adding. Options (A) and (D) also fail the reflection identity, which fixes the integral exactly without finding an antiderivative.

Step 3: Conclusion The definite integral equals $\pi/12$. Hence, the correct option is (C).

Quick Tip to Solve: When bounds sum to $\pi/4$ and the integrand involves $\tan 2x$, test reflection first: it swaps tangent with its reciprocal.

6. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity $\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+.....+\frac{1}{a-nb}=\alpha n+\beta n^2+\gamma n^3$, then the value of γ is : [4 marks]

(A) $\frac{a^2+b}{3a^3}$
(B) $\frac{a+b}{3a^2}$
(C) $\frac{b^2}{3a^3}$
(D) $\frac{a+b^2}{3a^3}$

Correct Answer: (C) $\frac{b^2}{3a^3}$

Solution:

Concept:

- Write $1/(a-kb)=a^{-1}(1-kb/a)^{-1}$, then use $(1-z)^{-1}=1+z+z^2+\cdots$. Retain terms through the second power of b/a, as stipulated in the question.
- The sums $\sum_{k=1}^{n}k=n(n+1)/2$ and $\sum_{k=1}^{n}k^2=n(n+1)(2n+1)/6$ convert the expansion to a polynomial in n. Compare its cubic coefficient with γ.

Step 1: Expand each reciprocal to the required order For the kth denominator, the retained terms are

$$\frac{1}{a-kb}\simeq\frac{1}{a}+\frac{kb}{a^2}+\frac{k^2b^2}{a^3}.$$

The neglected terms begin with the cube of kb/a. The question's approximation is understood for the terms in the given sum; for a numerical use with large n, nb/a must also remain sufficiently small. Summing the retained expression over $k=1,\ldots,n$ gives

$$\sum_{k=1}^{n}\frac{1}{a-kb}\simeq\frac{n}{a}+\frac{b}{a^2}\frac{n(n+1)}{2}+\frac{b^2}{a^3}\frac{n(n+1)(2n+1)}{6}.$$

Step 2: Collect powers of n and identify the cubic term Expand the finite-sum polynomials and group equal powers:

$$\sum_{k=1}^{n}\frac{1}{a-kb}\simeq\left(\frac{1}{a}+\frac{b}{2a^2}+\frac{b^2}{6a^3}\right)n+\left(\frac{b}{2a^2}+\frac{b^2}{2a^3}\right)n^2+\frac{b^2}{3a^3}n^3.$$

Only the sum of squares generates a cubic power of n; the constant and linear-in-k terms cannot contribute. Therefore the coefficient γ is $b^2/(3a^3)$. Each of (A), (B) and (D) includes an extra term which cannot arise in the cubic coefficient of this retained expansion.

Step 3: Conclusion $\gamma=\frac{b^2}{3a^3}$. Hence, the correct option is (C).

Quick Tip to Solve: To obtain only the coefficient of n^3, track the k^2b^2/a^3 term: the leading coefficient of $\sum k^2$ is $1/3$.

7. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$ then, the minimum value of $y(x), x \in (-\sqrt{2}, \sqrt{2})$ is equal to : [4 marks]

(A) $(2 - \sqrt{3}) - \log_e 2$

(B) $(2 + \sqrt{3}) + \log_e 2$

(C) $(1 + \sqrt{3}) - \log_e (\sqrt{3} - 1)$

(D) $(1 - \sqrt{3}) - \log_e (\sqrt{3} - 1)$

Correct Answer: (D) $(1 - \sqrt{3}) - \log_e (\sqrt{3} - 1)$

Solution:

Concept:

- For an equation involving e^{y-x}, the substitution $z = y - x$ removes the constant part of dy/dx and produces a separable equation.
- After using the initial condition, differentiate the resulting explicit function. A minimum on an open interval requires checking the derivative sign and the behaviour toward both endpoints, rather than evaluating only a stationary point.

Step 1: Solve the differential equation using the initial value Set $z = y - x$. Then $z' = y' - 1 = xe^z$, so

$$e^{-z}\,dz = x\,dx, \qquad -e^{-z} = \frac{x^2}{2} + C.$$

Since $z(0) = 0$, the constant is $C = -1$, giving

$$e^{-z} = 1 - \frac{x^2}{2}, \qquad y = x - \ln\left(1 - \frac{x^2}{2}\right).$$

The logarithm is defined throughout the prescribed interval $(-\sqrt{2}, \sqrt{2})$, because its argument is positive there. Differentiating this expression also recovers the given differential equation and checks the sign in the integration.

Step 2: Locate and evaluate the minimum Differentiating yields

$$y' = 1 + \frac{2x}{2 - x^2} = \frac{2 + 2x - x^2}{2 - x^2}.$$

The denominator is positive on the domain. The numerator vanishes at $1 \pm \sqrt{3}$, but only $x_0 = 1 - \sqrt{3}$ lies in that domain. It is negative before x_0 and positive after x_0. Also $y \to +\infty$ at both ends, so this is the global minimum. At x_0,

$$1 - \frac{x_0^2}{2} = \sqrt{3} - 1, \qquad y_{\min} = 1 - \sqrt{3} - \ln(\sqrt{3} - 1).$$

Option (C) has the wrong linear term, while (A) and (B) have a different logarithm and do not equal this minimum. The second algebraic root must not be used because it lies outside the interval.

Step 3: Conclusion The minimum value is $(1 - \sqrt{3}) - \ln(\sqrt{3} - 1)$. Hence, the correct option is (D).

Quick Tip to Solve: After setting $z = y - x$, integrate $e^{-z}dz = x\,dx$; check the negative sign in the antiderivative of e^{-z}.

8. The area (in sq. units) of the region, given by the set $\{(x, y) \in R \times R | x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is : [4 marks]

(A) $\frac{8}{3}$
(B) $\frac{17}{3}$
(C) $\frac{13}{3}$
(D) $\frac{7}{3}$

Correct Answer: (D) $\frac{7}{3}$

Solution:

Concept:

- For vertical strips, the area between an upper curve and a lower curve is $A = \int_a^b (y_{\text{upper}} - y_{\text{lower}})\, dx$. The integration limits must also satisfy every inequality defining the region.
- The intersection points determine where the line $y = 4 - 2x$ lies above the parabola $y = 2x^2$. The additional condition $x \geq 0$ discards the part to the left of the vertical axis.

Step 1: Find the allowed horizontal interval The lower bound must not exceed the upper bound. Thus

$$2x^2 \leq 4 - 2x \quad \Longleftrightarrow \quad x^2 + x - 2 \leq 0.$$

Factoring gives $(x + 2)(x - 1) \leq 0$, which holds for $-2 \leq x \leq 1$. Intersect this interval with $x \geq 0$, so the required strips run only from $x = 0$ to $x = 1$. Throughout this interval the line is the upper boundary and the parabola is the lower boundary.

Step 2: Integrate the difference of the boundaries The height of a strip is $4 - 2x - 2x^2$, and hence

$$\begin{aligned} A &= \int_0^1 (4 - 2x - 2x^2)\, dx \\ &= \left[4x - x^2 - \frac{2x^3}{3}\right]_0^1 \\ &= 4 - 1 - \frac{2}{3} = \frac{7}{3}. \end{aligned}$$

Using the entire interval between both intersections would include points with negative x, which the set excludes. None of the values in (A), (B) or (C) equals this definite integral.

Step 3: Conclusion The required area is $\boxed{7/3}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Find the curve intersections first, then intersect their interval with extra restrictions such as $x \geq 0$ before integrating.

9. Let g : N $\to$ N be defined as

g(3n + 1) = 3n + 2,

g(3n + 2) = 3n + 3,

g(3n + 3) = 3n + 1, for all n $\geq$ 0.

Then which of the following statements is true? [4 marks]

(A) There exists an onto function f : N $\to$ N such that fog = f

(B) There exists a one-one function f : N $\to$ N such that fog = f

(C) gogog = g

(D) There exists a function : f : N $\to$ N such that gof = f

Correct Answer: (A) There exists an onto function f : N $\to$ N such that fog = f

Solution:

Concept:

- The function g cycles each three-element block: $3n+1 \mapsto 3n+2 \mapsto 3n+3 \mapsto 3n+1$. Consequently g^3 is the identity, while g itself has no fixed point.
- The equation $f \circ g = f$ requires f to be constant on each cycle. Different cycles may have different values, so this condition does not prevent surjectivity.

Step 1: Construct an onto function satisfying the equation For every $n \geq 0$, define

$$f(3n+1) = f(3n+2) = f(3n+3) = n+1.$$

All three numbers in a block have the same image. Since g only permutes these numbers within the same block, applying g first does not alter the value of f. Thus $f(g(m)) = f(m)$ for every positive integer m. Every positive integer k is attained by taking $m = 3k-2$, so this f is onto $\mathbb{N}$. This proves the existence claimed in option (A).

Step 2: Test the remaining claims If f were one-one and $f(g(m)) = f(m)$, injectivity would force $g(m) = m$, contradicting the three-cycles. Thus (B) is false. Also $g \circ g \circ g$ is the identity, not g: at 1 their values are 1 and 2 respectively, so (C) is false. Finally, $g \circ f = f$ would require every value $f(m)$ to be a fixed point of g. There are no such positive integers, so no function with the property in (D) exists. Distinguishing which side contains the composition is essential here.

Step 3: Conclusion An onto function constant on each three-element cycle satisfies $f \circ g = f$. Hence, the correct option is (A).

Quick Tip to Solve: For $f \circ g = f$, make f constant on the cycles of g; for $g \circ f = f$, look instead for fixed points of g.

10. Let $f : [0, \infty) \to [0, \infty)$ be defined as $f(x) = \int_0^x [y]dy$
where [x] is the greatest integer less than or equal to x. Which of the following is true? [4 marks]

(A) f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points.

(B) f is both continuous and differentiable except at the integer points in $[0, \infty)$.

(C) f is continuous everywhere except at the integer points in $[0, \infty)$.

(D) f is differentiable at every point in $[0, \infty)$.

Correct Answer: (A) f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points.

Solution:

Concept:

- Integrating a locally bounded step function produces a continuous function. On an open interval where $\lfloor x \rfloor$ is constant, the integral is linear with that constant as its derivative.
- At a positive integer, compare the left and right slopes rather than applying the fundamental theorem of calculus at a discontinuity of the integrand.

Step 1: Write the integral on each unit interval Let n be a nonnegative integer and let $n \le x < n + 1$. The completed unit intervals contribute $0 + 1 + \cdots + (n - 1)$; the remaining interval has height n. Therefore

$$f(x) = \frac{n(n-1)}{2} + n(x - n).$$

This is a straight-line expression on each interval. At a positive integer n, the value from the interval on the left is

$$\frac{(n-1)(n-2)}{2} + (n - 1) = \frac{n(n-1)}{2},$$

which equals the value from the interval on the right. Thus the pieces join continuously at every integer.

Step 2: Compare the slopes at joining points For $n < x < n + 1$, differentiation gives $f'(x) = n$. At a positive integer n, the left-hand slope is $n - 1$, whereas the right-hand slope is n, so the ordinary derivative does not exist there. At the domain endpoint 0 the function is continuous and its right derivative is 0; the interior nondifferentiability occurs at the positive integers. Thus integration removes the jumps in the function values but leaves corners. Option (A) gives the intended continuity and differentiability description. Options (B) and (C) incorrectly exclude continuity at integers, and (D) overlooks the unequal one-sided slopes.

Step 3: Conclusion The function is continuous throughout $[0, \infty)$, with corners at positive integers and differentiability between them. Hence, the correct option is (A).

Quick Tip to Solve: For the integral of a step function, check joined function values for continuity and adjacent step heights for differentiability.

11. The values of a and b, for which the system of equations

2x + 3y + 6z = 8

x + 2y + az = 5

3x + 5y + 9z = b

has no solution, are : [4 marks]

(A) a = 3, b $\neq$ 13

(B) a $\neq$ 3, b $\neq$ 13

(C) a $\neq$ 3, b = 3

(D) a = 3, b = 13

Correct Answer: (A) a = 3, b $\neq$ 13

Solution:

Concept:

- A system is inconsistent when elimination produces an equation $0 = c$ with $c \neq 0$. If the coefficient determinant is nonzero, a unique solution exists for every right-hand side.
- Here the first two rows have independent x, y coefficients, since $\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$. Their sum can be compared directly with the third row.

Step 1: Eliminate the first two variables from the third equation Adding the first and second equations gives

$$3x + 5y + (6 + a)z = 13.$$

Subtract this equation from the third printed equation. The resulting condition is

$$(3 - a)z = b - 13.$$

If $a \neq 3$, this determines exactly one value $z = (b - 13)/(3 - a)$. Substitution into the first two equations then determines x, y uniquely because their two-variable determinant is 1. Therefore none of the cases with $a \neq 3$ can give no solution, regardless of the value of b.

Step 2: Examine the singular case Set $a = 3$. The eliminated equation becomes $0 = b - 13$. When $b \neq 13$, this is impossible, proving inconsistency. When $b = 13$, the third equation is exactly the sum of the first two; the free variable z can be chosen arbitrarily and the remaining variables found from the independent pair. Thus this latter case has infinitely many solutions, rather than no solution. This excludes option (D), while the nonsingular argument excludes both (B) and (C). The coefficient relation alone is insufficient: its right-hand sides must also be compared.

Step 3: Conclusion The system has no solution precisely when $\boxed{a = 3,\ b \neq 13}$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever one coefficient row is the sum of others, compare the corresponding constants too: matching constants give dependence, unequal constants give inconsistency.

12. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability than B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set : [4 marks]

(A) $\{x \in R : |x - 3| < 1\}$

(B) $\{x \in R : |x - 2| \leq 1\}$

(C) $\{x \in R : |x - 1| < 1\}$

(D) $\{x \in R : |x - 5| \leq 1\}$

Correct Answer: (A) $\{x \in R : |x - 3| < 1\}$

Solution:

Concept:

- Each distinct ball independently has four equally likely boxes. For membership in the specified box B_3, the success probability is $1/4$, and the failure probability is $3/4$.
- The number of balls in B_3 has the binomial distribution with $n = 9$. Exactly three successes have probability $\binom{9}{3}(1/4)^3(3/4)^6$.

Step 1: Count the favourable assignments There are 4^9 equally likely assignments of the nine distinct balls to four labelled boxes. Choose the three balls entering B_3 in $\binom{9}{3} = 84$ ways. Each of the other six balls must avoid B_3, so each has three choices, independently. Consequently

$$P(B_3 \text{ contains exactly 3 balls}) = \frac{84 \cdot 3^6}{4^9}.$$

There is no restriction on the occupancy of the other boxes; they may be empty. Counting only configurations in which all four boxes are nonempty would describe a different event.

Step 2: Extract the coefficient and check its interval Equating this probability with the form in the question gives

$$k\frac{3^9}{4^9} = \frac{84 \cdot 3^6}{4^9}, \qquad k = \frac{84}{27} = \frac{28}{9}.$$

The four listed sets are, in order, $(2, 4)$, $[1, 3]$, $(0, 2)$, and $[4, 6]$. Since $3 < 28/9 < 4$, the value belongs to $(2, 4)$ alone. In particular, it is just above 3, so the closed upper bound in (B) still excludes it. It is too large for (C) and too small for (D).

Step 3: Conclusion We obtain $k = 28/9 \in \{x \in \mathbb{R} : |x - 3| < 1\}$. Hence, the correct option is (A).

Quick Tip to Solve: When a binomial probability is written as a multiple of $(3/4)^9$, divide symbolically first; the common powers of 4 cancel immediately.

13. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is : [4 marks]

(A) 2

(B) 4

(C) 6

(D) 1

Correct Answer: (A) 2

Solution:

Concept:

- The substitution $t = e^x$ is a bijection from real x to positive t. Thus real roots of the exponential equation correspond exactly to positive roots of its polynomial in t.
- Descartes' rule of signs bounds the number of positive polynomial roots by the number of sign changes, counting multiplicities. Sign changes of the polynomial value on disjoint intervals establish the existence of distinct roots.

Step 1: Convert the equation and bound the root count Set $t = e^x > 0$. The equation becomes

$$P(t) = t^6 - t^4 - 2t^3 - 12t^2 + t + 1 = 0.$$

Ignoring the missing t^5 coefficient, the nonzero coefficient signs in descending order are

$$+, -, -, -, +, +.$$

There are exactly two sign changes. Descartes' rule therefore allows at most two positive roots, with multiplicity included. Negative polynomial roots are irrelevant because no real exponential e^x can equal a negative number. Counting all real roots of P, without its domain restriction, would not answer the question.

Step 2: Locate two distinct positive roots Evaluate the polynomial at simple nonnegative points:

$$P(0) = 1, \qquad P(1) = -12, \qquad P(3) = 490.$$

As a polynomial it is continuous. The intermediate value theorem gives one root in $(0, 1)$ and another in $(1, 3)$. Both roots are positive and distinct; neither endpoint is a root. The upper bound of two established above means there can be no further positive roots. Applying $x = \ln t$ to these two roots gives exactly two distinct real values of x. Thus the proposed counts 4, 6 and 1 are all excluded without needing explicit radical expressions for the roots.

Step 3: Conclusion There are exactly $\boxed{2}$ real roots of the given equation. Hence, the correct option is (A).

Quick Tip to Solve: After substituting $t = e^x$, count only positive polynomial roots; combine a sign-rule upper bound with two sign-changing intervals to make the count exact.

14. Let an ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus F(α, 0), $\alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to : [4 marks]

(A) $\frac{8}{3}$

(B) $\frac{4}{3}$

(C) $\frac{16}{3}$

(D) 3

Correct Answer: (C) $\frac{16}{3}$

Solution:

Concept:

- For an ellipse with major axis along the x-axis, $e^2 = 1 - b^2/a^2$ and the positive focus is $(ae, 0)$. The supplied point determines its scale after the eccentricity fixes the axis ratio.
- Both the ellipse and a circle centred on its major axis are symmetric about that axis. Intersections with the same abscissa x have ordinates $\pm y$, so their squared separation is $4y^2$.

Step 1: Determine the ellipse and the circle Since $e^2 = 1/3$, we have $b^2 = 2a^2/3$. Substitute the point $(\sqrt{3/2}, 1)$:

$$\frac{3}{2a^2} + \frac{1}{b^2} = \frac{3}{2a^2} + \frac{3}{2a^2} = 1.$$

Hence $a^2 = 3$, $b^2 = 2$, and $ae = 1$. The ellipse and circle therefore have equations

$$\frac{x^2}{3} + \frac{y^2}{2} = 1, \qquad (x-1)^2 + y^2 = \frac{4}{3}.$$

The positive focus, rather than the negative one, is used as explicitly specified.

Step 2: Find the valid common abscissa From the ellipse, $y^2 = 2 - 2x^2/3$. Insert this into the circle:

$$\begin{aligned}(x-1)^2 + 2 - \frac{2x^2}{3} &= \frac{4}{3},\\ x^2 - 6x + 5 &= 0,\\ (x-1)(x-5) &= 0.\end{aligned}$$

The root $x = 5$ is outside the ellipse's range $|x| \le \sqrt{3}$, and would give negative y^2. It must be rejected. At $x = 1$, $y^2 = 4/3$, so the two actual points are $(1, 2/\sqrt{3})$ and $(1, -2/\sqrt{3})$.

Step 3: Compute the squared chord length The chord is vertical, with length equal to the difference of the two ordinates:

$$PQ = \frac{4}{\sqrt{3}}, \qquad PQ^2 = \frac{16}{3}.$$

Squaring the radius alone or using only one ordinate would give a smaller quantity, not the requested squared distance between the two intersection points. The values $8/3$, $4/3$, and 3 therefore do not match the chord calculation.

Step 4: Conclusion The squared distance is $\boxed{PQ^2 = 16/3}$. Hence, the correct option is (C).

Quick Tip to Solve: For intersections symmetric about the major axis, calculate y^2 directly and use $PQ^2 = 4y^2$; reject algebraic roots outside the ellipse.

15. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is : [4 marks]

(A) $8(2+2\sqrt{3}+\sqrt{2})$

(B) $8(\sqrt{6}+\sqrt{2}+2)$

(C) $8(\sqrt{2}+2+\sqrt{3})$

(D) $8(\sqrt{6}-\sqrt{2}+2)$

Correct Answer: (B) $8(\sqrt{6}+\sqrt{2}+2)$

Solution:

Concept:

- The two extreme lines of sight to a spherical balloon are tangents to its circular cross-section through the observer and centre. A radius to a point of tangency is perpendicular to the tangent.
- The line joining the observer to the centre bisects the angle between the tangents. For centre distance d, radius r and half-angle θ, $\sin\theta = r/d$.

Step 1: Find the distance to the centre Let the centre be C and a point of tangency be T. The angle subtended by the whole balloon is $60°$, so $\angle TAC = 30°$. In right triangle ACT, $CT = 16$ m and AC is the hypotenuse. Therefore

$$\sin 30° = \frac{16}{AC}, \qquad AC = \frac{16}{1/2} = 32\,\text{m}.$$

The radius is not the height of the centre, and the full subtended angle must not be used in this triangle.

Step 2: Compute the vertical height of the centre The elevation of C is $75°$, measured from the horizontal through the eye. Thus its vertical height above that horizontal is

$$h_C = 32\sin 75° = 32\frac{\sqrt{6}+\sqrt{2}}{4} = 8(\sqrt{6}+\sqrt{2})\,\text{m}.$$

Here $\sin 75° = \sin(45°+30°)$, so both radical terms have positive signs.

Step 3: Add the radius to reach the top The highest point is vertically 16 m above C, so

$$H = h_C + 16 = 8(\sqrt{6}+\sqrt{2}+2)\,\text{m}.$$

Option (D) uses the difference of the radicals, which corresponds to the wrong trigonometric value. Options (A) and (C) do not give the vertical component of a 32 m line at elevation $75°$.

Step 4: Conclusion The required height is $\boxed{8(\sqrt{6}+\sqrt{2}+2)\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a sphere subtending an angle, first halve that angle to find the centre distance; then resolve vertically and add the radius.

16. Let the vectors

$(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k}, (1+b)\hat{i}+2b\hat{j}-b\hat{k}$ **and** $(2+b)\hat{i}+2b\hat{j}+(1-b)\hat{k}$**,** $a,b,c, \in R$ **be co-planar. Then which of the following is true?** [4 marks]

(A) 2b = a + c

(B) 3c = a + b

(C) a = b + 2c

(D) 2a = b + c

Correct Answer: (A) 2b = a + c

Solution:

Concept:

- Three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are coplanar exactly when their scalar triple product $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ is zero.
- Computing the cross product of the two simpler vectors first reduces the amount of determinant expansion and avoids unnecessary polynomial terms.

Step 1: Find the cross product of the last two vectors Write $\mathbf{v} = (1+b, 2b, -b)$ and $\mathbf{w} = (2+b, 2b, 1-b)$. Their cross product is

$$\begin{aligned}\mathbf{v} \times \mathbf{w} = \big(&2b(1-b) + 2b^2,\\ &-[(1+b)(1-b) + b(2+b)],\\ &2b(1+b) - 2b(2+b)\big)\\ &= (2b, -1-2b, -2b).\end{aligned}$$

In the second component, the minus sign belongs to the cofactor, so it applies to the entire bracket.

Step 2: Apply the coplanarity condition The first vector is $\mathbf{u} = (2+a+b, a+2b+c, -b-c)$. Taking the dot product gives

$$\begin{aligned}0 &= 2b(2+a+b) - (1+2b)(a+2b+c)\\ &\quad + 2b(b+c)\\ &= 4b + 2ab + 2b^2 - a - 2b - c\\ &\quad - 2ab - 4b^2 - 2bc + 2b^2 + 2bc\\ &= 2b - a - c.\end{aligned}$$

Thus all quadratic and mixed terms cancel, leaving a linear relation. No division by b was used, so the conclusion also covers $b = 0$.

Step 3: Identify the relation that must hold The necessary and sufficient condition is $2b = a + c$. For example, $a = 2, b = 1, c = 0$ satisfies it and makes the scalar triple product zero, but violates each of $3c = a + b$, $a = b + 2c$, and $2a = b + c$. Hence none of the other three relations is forced by coplanarity.

Step 4: Conclusion Coplanarity gives $\boxed{2b = a + c}$. Hence, the correct option is (A).

Quick Tip to Solve: In a scalar triple product, take the cross product of the simplest two vectors first and simplify before taking the dot product.

17. The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to : [4 marks]

(A) $\sim$ q

(B) q

(C) p

(D) $\sim$ p

Correct Answer: (D) $\sim$ p

Solution:

Concept:

- An implication $p \Rightarrow q$ is equivalent to $\sim p \vee q$: it is false only when p is true and q is false.
- The distributive identity $(A \vee B) \wedge (A \vee C) = A \vee (B \wedge C)$, together with $q \wedge \sim q = \mathrm{F}$, simplifies the given conjunction.

Step 1: Rewrite both implications For the first implication, $p \Rightarrow q \equiv \sim p \vee q$. For the second implication, the antecedent is q and the consequent is $\sim p$, so

$$q \Rightarrow \sim p \equiv \sim q \vee \sim p.$$

Therefore the complete expression becomes

$$(\sim p \vee q) \wedge (\sim p \vee \sim q).$$

Both brackets contain $\sim p$; the q terms have opposite truth values.

Step 2: Use the distributive law Taking $A = \sim p$, $B = q$, and $C = \sim q$, we obtain

$$\begin{aligned}(\sim p \vee q) \wedge (\sim p \vee \sim q) &\equiv \sim p \vee (q \wedge \sim q) \\ &\equiv \sim p \vee \mathrm{F} \\ &\equiv \sim p.\end{aligned}$$

This is an equivalence for every assignment of p and q, rather than for just one selected truth-value pair.

Step 3: Check the result and eliminate the other options If p is false, the first implication has a false antecedent and is true, while the second has the true consequent $\sim p$ and is also true. If p is true, the two implications require q and $\sim q$ simultaneously, which is impossible. Thus the result depends only on p and is its negation. It cannot equal p; and since q may be either true or false while p remains false, it cannot equal either q or $\sim q$.

Step 4: Conclusion The expression is equivalent to $\boxed{\sim p}$. Hence, the correct option is (D).

Quick Tip to Solve: When two disjunctions share one proposition and their remaining propositions are negations of each other, distribute to eliminate the latter pair.

18. The sum of all values of x in [0, 2π], for which sin x + sin 2x + sin 3x + sin 4x = 0, is equal to : [4 marks]

(A) 8π

(B) 11π

(C) 12π

(D) 9π

Correct Answer: (D) 9π

Solution:

Concept:

- The sum-to-product formula $\sin A + \sin B = 2\sin((A+B)/2)\cos((A-B)/2)$ can factor a sum of several sine terms.
- For a product equal to zero, solve each factor separately on the stated closed interval. Count distinct roots once and include both endpoints when they satisfy the equation.

Step 1: Factor the trigonometric sum Pair the first term with the last and the two middle terms together. Then

$$\sin x + \sin 4x = 2\sin\frac{5x}{2}\cos\frac{3x}{2},$$
$$\sin 2x + \sin 3x = 2\sin\frac{5x}{2}\cos\frac{x}{2}.$$

Using the sum formula for the two cosines, the original expression is

$$2\sin\frac{5x}{2}\left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 4\sin\frac{5x}{2}\cos x\cos\frac{x}{2}.$$

Step 2: List all the distinct roots From $\sin(5x/2) = 0$, the roots in $[0, 2\pi]$ are

$$0,\ \frac{2\pi}{5},\ \frac{4\pi}{5},\ \frac{6\pi}{5},\ \frac{8\pi}{5},\ 2\pi.$$

From $\cos x = 0$, add $\pi/2$ and $3\pi/2$. From $\cos(x/2) = 0$, add π. These nine roots are all distinct. In particular, π is not one of the multiples $2k\pi/5$, because that would require the integer k to be $5/2$.

Step 3: Add the root values The first six roots form an arithmetic progression with average π, so their sum is 6π. The remaining roots contribute

$$\frac{\pi}{2} + \frac{3\pi}{2} + \pi = 3\pi.$$

Hence the total is 9π. The values 8π, 11π, and 12π do not equal the sum of the complete distinct-root list; endpoints and the separate cosine factors must all be retained.

Step 4: Conclusion The sum of all solutions in the given interval is $\boxed{9\pi}$. Hence, the correct option is (D).

Quick Tip to Solve: Factor a sine sum before solving, then form a union of the factor-root sets so that no endpoint is missed and no repeated root is counted twice.

19. Let a parabola b be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of $\triangle$SOR is equal to : [4 marks]

(A) $16\sqrt{2}$

(B) 16

(C) 32

(D) $8\sqrt{2}$

Correct Answer: (B) 16

Solution:

Concept:

- A right-opening parabola with vertex $(h, 0)$ and focus $(h + a, 0)$ has equation $y^2 = 4a(x - h)$. The parameter a is the vertex-to-focus distance.
- A straight line is tangent to a parabola when substituting its equation produces a quadratic with a repeated root. The triangle area is one half of its base times its perpendicular height.

Step 1: Write the parabola and a line through the origin The vertex is $(2, 0)$ and the focus is $(4, 0)$, so $a = 4 - 2 = 2$. Therefore the parabola is

$$y^2 = 8(x - 2).$$

A nonvertical line through O has equation $y = mx$. Substitution gives

$$m^2x^2 - 8x + 16 = 0.$$

The vertical line through O is $x = 0$, which does not meet this right-opening parabola, and hence is not an additional tangent.

Step 2: Find the tangent slopes and contact points For tangency the discriminant must vanish:

$$(-8)^2 - 4(m^2)(16) = 64 - 64m^2 = 0.$$

Thus $m = 1$ or $m = -1$. For either slope, the repeated root is

$$x = \frac{8}{2m^2} = 4.$$

The two contact points are consequently $S = (4, 4)$ and $R = (4, -4)$. Each satisfies $y^2 = 8(x - 2)$, confirming that both points lie on the parabola.

Step 3: Calculate the triangle area The segment SR is vertical, has length $4 - (-4) = 8$, and lies on $x = 4$. The perpendicular distance from O to this base is 4. Hence

$$[\triangle SOR] = \frac{1}{2} \times 8 \times 4 = 16.$$

The radical-valued alternatives do not use the correct vertical base and horizontal altitude. The value 32 omits the factor $1/2$ in the triangle-area formula.

Step 4: Conclusion The area is $\boxed{16 \text{ square units}}$. Hence, the correct option is (B).

Quick Tip to Solve: For tangents drawn from the origin, put y=mx into the conic and set the discriminant to zero; the repeated root gives the contact point.

20. Let the foot of perpendicular from a point P(1, 2, −1) to the straight line $L : \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane x + y + 2z = 0 which meets L at point Q. If α is the acute angle between the lines PN and PQ, then cosα is equal to _____. [4 marks]

(A) $\frac{1}{\sqrt{5}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{2\sqrt{3}}$

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

Concept:

- A point on a line through the origin with direction $\mathbf{d}$ is $t\mathbf{d}$. The perpendicular projection of $\mathbf{p}$ onto this line has parameter $t = (\mathbf{p} \cdot \mathbf{d})/(\mathbf{d} \cdot \mathbf{d})$.
- A line parallel to a plane has direction perpendicular to the plane normal. The acute angle between two lines satisfies $\cos\alpha = |\mathbf{u} \cdot \mathbf{v}|/(|\mathbf{u}||\mathbf{v}|)$.

Step 1: Find the perpendicular foot N The symmetric equation of L means $x = t, y = 0, z = -t$, so its direction vector is $\mathbf{d} = (1, 0, -1)$. The zero denominator specifies the fixed coordinate $y = 0$. For $P = (1, 2, -1)$,

$$t_N = \frac{(1, 2, -1) \cdot (1, 0, -1)}{1^2 + 0^2 + (-1)^2} = \frac{2}{2} = 1.$$

Hence $N = (1, 0, -1)$ and $\overrightarrow{PN} = (0, -2, 0)$, which is indeed perpendicular to $\mathbf{d}$.

Step 2: Use the parallel-plane condition to locate Q Write $Q = (t, 0, -t)$. Then $\overrightarrow{PQ} = (t-1, -2, 1-t)$. The normal to $x + y + 2z = 0$ is $\mathbf{n} = (1, 1, 2)$. Since PQ is parallel to that plane,

$$\begin{aligned} \overrightarrow{PQ} \cdot \mathbf{n} &= (t-1) - 2 + 2(1-t) \\ &= -t - 1 = 0. \end{aligned}$$

Thus $t = -1$, giving $Q = (-1, 0, 1)$ and $\overrightarrow{PQ} = (-2, -2, 2)$.

Step 3: Evaluate the cosine of the acute angle The two direction-vector lengths are 2 and $\sqrt{4+4+4} = 2\sqrt{3}$, while their dot product is 4. Therefore

$$\cos\alpha = \frac{|4|}{2(2\sqrt{3})} = \frac{1}{\sqrt{3}}.$$

The other values $1/\sqrt{5}$, $\sqrt{3}/2$, and $1/(2\sqrt{3})$ fail this direct dot-product calculation; in particular the

length of PQ is $2\sqrt{3}$, not $\sqrt{3}$ or $2\sqrt{5}$.

Step 4: Conclusion $\boxed{\cos\alpha = 1/\sqrt{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: Parameterise the intersection point on the known line, impose a zero dot product with the plane normal, and only then calculate the angle.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let y = y(x) be solution of the following differential equation $e^y\frac{dy}{dx} - 2e^y\sin x + \sin x\cos^2 x = 0, y\left(\frac{\pi}{2}\right) = 0$ **If** $y(0) = \log_e(\alpha + \beta e^{-2})$**, then** $4(\alpha+\beta)$ **is equal to _____.** [4 marks]

Solution:

Concept:

- The substitution $v = e^y$ gives $v\prime = e^y y\prime$, converting the given equation into a first-order linear equation in v.
- For $v\prime + P(x)v = Q(x)$, multiplication by the integrating factor $e^{\int P(x)\,dx}$ makes the left side the derivative of the product of v and the integrating factor.

Step 1: Transform the equation and find its integrating factor With $v = e^y$, the equation becomes

$$v\prime - 2\sin x\, v = -\sin x\cos^2 x.$$

The initial condition is $v(\pi/2) = e^0 = 1$. Since $\int -2\sin x\,dx = 2\cos x$, take the integrating factor $e^{2\cos x}$. We obtain

$$\frac{d}{dx}\left(ve^{2\cos x}\right) = -\sin x\cos^2 x\, e^{2\cos x}.$$

Step 2: Integrate using u=cos x Put $u = \cos x$, so $du = -\sin x\,dx$. The right-hand integral is

$$\begin{aligned}\int u^2e^{2u}\,du &= \frac{1}{2}u^2e^{2u} - \int ue^{2u}\,du\\ &= e^{2u}\left(\frac{u^2}{2} - \frac{u}{2} + \frac{1}{4}\right) + C.\end{aligned}$$

Dividing by $e^{2\cos x}$ yields

$$v = \frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + Ce^{-2\cos x}.$$

Differentiation of the integrated expression gives u^2e^{2u}, which checks both signs and the coefficients.

Step 3: Apply the initial value and evaluate at zero At $x = \pi/2$, the cosine is zero, so $1 = 1/4 + C$, giving $C = 3/4$. At $x = 0$,

$$e^{y(0)} = \frac{1}{2} - \frac{1}{2} + \frac{1}{4} + \frac{3}{4}e^{-2} = \frac{1}{4} + \frac{3}{4}e^{-2}.$$

Therefore

$$y(0) = \log_e \left(\frac{1}{4} + \frac{3}{4} e^{-2} \right).$$

Comparing with the requested form identifies $\alpha = 1/4$ and $\beta = 3/4$. The logarithm has a positive argument, as required for a real solution.

Step 4: Conclusion We obtain $\alpha + \beta = 1$, so the required dimensionless value is $\boxed{4(\alpha + \beta) = 4}$.

Quick Tip to Solve: When e to the power y multiplies the derivative of y, try v=e to the power y before seeking an integrating factor.

22. Consider the following frequency distribution:

Class	10–20	20–30	30–40	40–50	50–60
Frequency	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to _____. [4 marks]

Solution:

Concept:

- The grouped-data median is $M = l + \dfrac{N/2 - C}{f} h$, where l is the lower boundary of the median class, C the preceding cumulative frequency, f its frequency and h its width.
- The total frequency equals the sum of the five class frequencies. The median class contains the observation at cumulative position $N/2$.

Step 1: Identify the median class The given median 45 lies in the class 40–50. Thus its lower boundary is $l = 40$, its width is $h = 10$, and its frequency is $f = 30$. The total is $N = 584$, so the median position is $N/2 = 292$. The cumulative frequency before this class includes the first three classes, giving $C = \alpha + 110 + 54 = \alpha + 164$.

Step 2: Find the unknown first frequency Substitute these quantities into the median formula, retaining the cumulative frequency rather than using only the preceding class frequency:

$$45 = 40 + \frac{292 - (\alpha + 164)}{30} \times 10.$$

Therefore $5 = (128 - \alpha)/3$, so $128 - \alpha = 15$, and hence $\alpha = 113$. As a check, the cumulative frequency before the median class is 277, and after it is 307; the position 292 is indeed between these values.

Step 3: Use the total and evaluate the difference The sum condition gives

$$113 + 110 + 54 + 30 + \beta = 584.$$

The four known frequencies total 307, so $\beta = 277$. Both unknown frequencies are nonnegative, as required. Finally,

$$|\alpha - \beta| = |113 - 277| = 164.$$

Step 4: Conclusion The required absolute difference is $\boxed{164}$, a dimensionless count.

Quick Tip to Solve: Use the given median to identify its class first; the cumulative frequency in the formula includes every earlier class.

23. **Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to _____.** [4 marks]

Solution:

Concept:

- A vector perpendicular to $\mathbf{p} + \mathbf{q}$ and $\mathbf{p} - \mathbf{q}$ satisfies two zero dot-product equations.
- The Euclidean magnitude of $\mathbf{r} = (\alpha, \beta, \gamma)$ is $\sqrt{\alpha^2 + \beta^2 + \gamma^2}$; its specified value fixes the scale after the perpendicular direction is found.

Step 1: Form the two orthogonality equations From the printed vectors, $\mathbf{p} = (2, 3, 1)$ and $\mathbf{q} = (1, 2, 1)$. Consequently,

$$\mathbf{p} + \mathbf{q} = (3, 5, 2), \qquad \mathbf{p} - \mathbf{q} = (1, 1, 0).$$

Taking the dot product with $\mathbf{r} = (\alpha, \beta, \gamma)$ yields

$$3\alpha + 5\beta + 2\gamma = 0, \qquad \alpha + \beta = 0.$$

These equations use the direction vectors themselves; their magnitudes do not enter perpendicularity.

Step 2: Express the perpendicular direction using one parameter The second equation gives $\beta = -\alpha$. Substitution into the first gives $3\alpha - 5\alpha + 2\gamma = 0$, or $\gamma = \alpha$. Thus every common perpendicular has the form

$$\mathbf{r} = \alpha(1, -1, 1).$$

This direction is nonzero when $\alpha \neq 0$. Direct substitution checks both dot products: $3 - 5 + 2 = 0$ and $1 - 1 = 0$.

Step 3: Apply the magnitude condition Its squared magnitude is $\alpha^2 + (-\alpha)^2 + \alpha^2 = 3\alpha^2$. Since the specified magnitude is $\sqrt{3}$, we have $3\alpha^2 = 3$, so $|\alpha| = 1$. Either sign gives a valid vector: $(1, -1, 1)$ or $(-1, 1, -1)$. In both cases,

$$|\alpha| + |\beta| + |\gamma| = 1 + 1 + 1 = 3.$$

Step 4: Conclusion The required dimensionless value is $\boxed{3}$.

Quick Tip to Solve: For perpendicularity to a sum and difference, write the two dot-product equations; the given length only sets the final scale.

24. **The ratio of the coefficient of the middle term in the expansion of $(1 + x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1 + x)^{19}$ is _____.** [4 marks]

Solution:

Concept:

- The general term in $(1 + x)^n$ is $T_{r+1} = \binom{n}{r}x^r$; the term number is one more than the exponent.
- An even exponent gives one middle term, whereas an odd exponent gives two middle terms.

Pascal's identity is $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$.

Step 1: Locate the middle coefficient for exponent twenty The expansion of $(1+x)^{20}$ contains 21 terms. Its middle term is the eleventh term, so the exponent of x in it is 10. Hence the numerator of the requested ratio is

$$[x^{10}](1+x)^{20} = \binom{20}{10}.$$

The problem asks for its coefficient, so the factor x^{10} is not included in the ratio.

Step 2: Identify the two middle coefficients for exponent nineteen The expansion of $(1+x)^{19}$ contains 20 terms, whose middle positions are the tenth and eleventh. Their exponents are 9 and 10, respectively. The denominator is therefore

$$\binom{19}{9} + \binom{19}{10}.$$

The two coefficients are equal by binomial symmetry, although the corresponding terms have different powers of x. The word sum in the question requires both coefficients.

Step 3: Compare the numerator and denominator Pascal's identity gives

$$\binom{19}{9} + \binom{19}{10} = \binom{20}{10}.$$

Equivalently, multiplying $(1+x)^{19}$ by $1+x$ shows that the coefficient of x^{10} in the product is the sum of these adjacent coefficients. Thus the ratio equals

$$\frac{\binom{20}{10}}{\binom{19}{9} + \binom{19}{10}} = 1.$$

No large factorial calculation is necessary.

Step 4: Conclusion The required ratio is $\boxed{1}$, or $1:1$.

Quick Tip to Solve: Convert middle term positions to exponents before applying Pascal's identity; the sum of adjacent coefficients belongs to the next row.

25. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _____. [4 marks]

Solution:

Concept:

- Choosing r students from n distinct students has $\binom{n}{r}$ possibilities; order is irrelevant.
- For fixed numbers selected from disjoint classes, multiply the three binomial counts. Add the counts of mutually exclusive allowed class-size patterns.

Step 1: List all permissible class-size patterns Let a, b, c be the selected numbers from classes 10, 11 and 12. The requirements are $a+b+c=10$, $a, b, c \geq 2$, and $a+b \leq 5$. Thus $a+b$ can only be 4 or 5. If it is 4, the only pair is $(a, b)=(2,2)$ and $c=6$. If it is 5, the pairs are $(2,3)$ and $(3,2)$, with $c=5$. All these choices respect the class populations $5, 6, 8$.

Step 2: Count selections with four students from the first two classes For the pattern $(2,2,6)$, choose independently from each class:

$$N_4 = \binom{5}{2}\binom{6}{2}\binom{8}{6} = 10 \times 15 \times 28 = 4200.$$

There is no additional ordering factor, since selecting the same ten students in a different order does not create another selection.

Step 3: Count the remaining cases and recover k For the patterns $(2,3,5)$ and $(3,2,5)$, the count is

$$\begin{aligned} N_5 &= \left(\binom{5}{2}\binom{6}{3} + \binom{5}{3}\binom{6}{2}\right)\binom{8}{5} \\ &= (10 \times 20 + 10 \times 15) \times 56 \\ &= 350 \times 56 = 19600. \end{aligned}$$

These cases cannot overlap because they select different numbers from the classes. Thus the total is $N = 4200 + 19600 = 23800$. The question writes this count as $100k$, giving $k = 23800/100 = 238$.

Step 4: Conclusion The total number of selections is 23800, so the required dimensionless value is $\boxed{k = 238}$.

Quick Tip to Solve: First restrict the combined selection from classes 10 and 11 to four or five; only three class-size patterns then remain.

---

26. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- Each root t of $t^2 + 5\sqrt{2}\,t + 10 = 0$ satisfies $t^{n+2} + 5\sqrt{2}\,t^{n+1} + 10t^n = 0$.
- Subtracting this identity for the two roots gives the same linear recurrence for $P_n = \alpha^n - \beta^n$. Factoring the requested expression exposes adjacent instances of that recurrence.

Step 1: Derive the recurrence without computing large powers Multiply the quadratic identity for α by α^n, do the same for β, and subtract. This gives

$$P_{n+2} + 5\sqrt{2}\,P_{n+1} = -10P_n.$$

In particular,

$$P_{20} + 5\sqrt{2}\,P_{19} = -10P_{18}, \qquad P_{19} + 5\sqrt{2}\,P_{18} = -10P_{17}.$$

The second relation uses $n = 17$, and the first uses $n = 18$.

Step 2: Factor the numerator and denominator The expression to be evaluated can be written as

$$R = \frac{P_{17}(P_{20} + 5\sqrt{2}\,P_{19})}{P_{18}(P_{19} + 5\sqrt{2}\,P_{18})}.$$

Substituting the two recurrence relations gives

$$R = \frac{-10P_{17}P_{18}}{-10P_{18}P_{17}}.$$

The numerator and denominator are now visibly identical; no explicit seventeenth or twentieth power is needed.

Step 3: Check that cancellation is valid The roots are $\alpha = (-5\sqrt{2}+\sqrt{10})/2$ and $\beta = (-5\sqrt{2}-\sqrt{10})/2$. They are distinct negative real numbers with $|\beta| > |\alpha| > 0$. Their powers therefore have unequal magnitudes for every positive integer, so P_{17} and P_{18} are both nonzero. The original denominator equals $-10P_{18}P_{17} \neq 0$, justifying cancellation and giving $R = 1$.

Step 4: Conclusion The value of the given dimensionless expression is $\boxed{1}$.

Quick Tip to Solve: When powers of quadratic roots appear together, derive their recurrence first and factor common terms before calculating any roots.

27. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$, where x ≠ 0, 1 is equal to _____. [4 marks]

Solution:

Concept:

- The identity $u^3 + 1 = (u + 1)(u^2 - u + 1)$ simplifies the first fraction, while a difference of squares simplifies the second.
- In a binomial expansion involving fractional powers of x, a term is independent of x precisely when its total exponent is zero.

Step 1: Simplify the expression inside the power For real values where the printed expression is defined, $x > 0$ and $x \neq 1$. Put $u = x^{1/3}$. Then

$$\frac{x+1}{x^{2/3} - x^{1/3} + 1} = x^{1/3} + 1.$$

Also $x - 1 = (\sqrt{x} - 1)(\sqrt{x} + 1)$ and $x - \sqrt{x} = \sqrt{x}(\sqrt{x} - 1)$. Since $x \neq 0, 1$, cancellation is valid and the second fraction becomes $1 + x^{-1/2}$. Hence the bracket is $x^{1/3} - x^{-1/2}$.

Step 2: Write a general binomial term and solve its exponent condition The term obtained by choosing

the negative second term r times in $(x^{1/3} - x^{-1/2})^{10}$ is

$$T_{r+1} = (-1)^r \binom{10}{r} x^{(10-r)/3 - r/2} = (-1)^r \binom{10}{r} x^{(20-5r)/6}.$$

For this term to be independent of x, require $20 - 5r = 0$, giving $r = 4$. This is an allowed integer index between zero and ten, and the linear exponent condition makes it the unique constant term.

Step 3: Evaluate the coefficient with its sign With $r = 4$, the sign is positive because four negative factors are selected. Thus

$$T_5 = (-1)^4 \binom{10}{4} x^0 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210.$$

The requested object is the term itself; its power of x is zero, so no variable factor remains.

Step 4: Conclusion The term independent of x is $\boxed{210}$.

Quick Tip to Solve: Simplify both fractions before expanding, then set the combined fractional exponent to zero and retain the binomial sign.

---

28. Let $S = \left\{ n \in N \left| \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in R \right. \right\}$**, where i =** $\sqrt{-1}$**. Then the number of 2-digit numbers in the set S is _____.** [4 marks]

Solution:

Concept:

- If $A^n B = B$ for every real 2×2 matrix B, choosing $B = I$ forces $A^n = I$; this condition is also sufficient.
- A short cycle of powers of A determines the possible exponents. Count the resulting multiples within the inclusive two-digit interval $10 \le n \le 99$.

Step 1: Reduce the universal condition to an identity matrix Let

$$A = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}.$$

The condition in the question must hold for all real entries a, b, c, d. In particular, set $a = d = 1$ and $b = c = 0$. This gives $A^n I = I$, so $A^n = I$. Conversely, an exponent satisfying $A^n = I$ works for every permitted matrix, because multiplication by the identity leaves it unchanged.

Step 2: Find exactly which exponents give the identity Direct multiplication yields

$$A^2 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} = iI.$$

For an even exponent $n = 2m$, we have $A^n = i^m I$. This equals I exactly when m is divisible by four,

equivalently when n is divisible by eight. An odd power has the form $A^{2m+1} = i^m A$, whose diagonal entries are zero; it cannot equal I. Therefore no other positive exponents satisfy the condition, and $S = \{8, 16, 24, \ldots\}$.

Step 3: Count the two-digit members The first two-digit multiple of eight is $16 = 8 \times 2$, while the last is $96 = 8 \times 12$. Hence the allowed multipliers are the integers from two through twelve inclusive. Their number is

$$12 - 2 + 1 = 11.$$

Equivalently, $\lfloor 99/8 \rfloor - \lfloor 9/8 \rfloor = 12 - 1 = 11$. This excludes the one-digit member eight and includes ninety-six.

Step 4: Conclusion The number of two-digit integers in S is $\boxed{11}$.

Quick Tip to Solve: Test a condition valid for every matrix on the identity first; then compute a few powers to expose the exact period.

---

29. If the value of

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \ldots . upto\, \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots . upto\, \infty\right)}$$

is l, then l^2 is equal to _____. [4 marks]

Solution:

Concept:

- For $|x| < 1$, the geometric sum is $\sum_{n=1}^{\infty} x^n = x/(1-x)$, and differentiating it gives $\sum_{n=1}^{\infty} nx^n = x/(1-x)^2$.
- In the base, the numerators after the initial 1 form the arithmetic progression $2, 6, 10, \ldots$, whose nth term is $4n - 2$. The logarithm in the exponent must be evaluated separately.

Step 1: Evaluate the series in the exponent The common ratio is $1/3$, whose magnitude is below 1, so the series converges. Its sum is

$$U = \frac{1/3}{1 - 1/3} = \frac{1}{2}.$$

Hence the exponent is $\log_{1/4}(1/2) = 1/2$, since $(1/4)^{1/2} = 1/2$. A logarithm with base below 1 is valid: its base is positive and differs from 1.

Step 2: Sum the arithmetic-geometric base Write the entire base as S, retaining its initial constant term:

$$\begin{aligned} S &= 1 + \sum_{n=1}^{\infty} \frac{4n - 2}{3^n} \\ &= 1 + 4\frac{1/3}{(1 - 1/3)^2} - 2\frac{1/3}{1 - 1/3} \\ &= 1 + 3 - 1 = 3. \end{aligned}$$

This calculation accounts for every term: for $n = 1, 2, 3$, the summand gives $2/3, 6/9, 10/27$, as printed.

Step 3: Apply the exponent and square The given positive base raised to its logarithmic exponent yields $l = S^{1/2} = \sqrt{3}$. Consequently,

$$l^2 = (\sqrt{3})^2 = 3.$$

The requested quantity is the square of l, rather than l itself; no rounding is needed.

Step 4: Conclusion The required numerical value is $\boxed{3}$ (dimensionless).

Quick Tip to Solve: For numerators in an arithmetic progression, express them as $an + b$ and combine the standard sums of x^n and nx^n. Evaluate a nested logarithm separately.

---

30. Let $M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}$. Define f : M $\to$ Z, as f(A) = det(A), for all A$\in$M, where z is set of all integers. Then the number of A$\in$M such that f(A) = 15 is equal to _____.

[4 marks]

Solution:

Concept:

- For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant is $ad - bc$. Each entry is an integer from -3 to 3, so each product lies between -9 and 9.
- The positions in a matrix are distinct. Thus ordered factor pairs, including their signs, must be counted; the choices for (a, d) and (b, c) are independent once their products are fixed.

Step 1: Find the possible product values Put $x = ad$ and $y = bc$. The equation is $x - y = 15$. The available products of two permitted entries are

$$\{-9, -6, -4, -3, -2, -1, 0, 1, 2, 3, 4, 6, 9\}.$$

Because $y \geq -9$, we need $x \geq 6$. The only available positive values at least 6 are 6 and 9. Therefore the only possibilities are

$$(ad, bc) = (6, -9) \quad \text{or} \quad (9, -6).$$

Values 7 and 8 cannot be made from two permitted entries.

Step 2: Count the ordered factor pairs Product 6 has the four ordered pairs

$$(2, 3),\ (3, 2),\ (-2, -3),\ (-3, -2).$$

Product -9 has two: $(3, -3), (-3, 3)$. This case gives $4 \times 2 = 8$ matrices. Product 9 has two ordered pairs, $(3, 3), (-3, -3)$, while product -6 has four: $(2, -3), (3, -2), (-2, 3), (-3, 2)$. This second case gives $2 \times 4 = 8$ matrices.

Step 3: Combine the disjoint cases The two cases cannot overlap because they require different values of ad. Each ordered pair fixes its two matrix positions uniquely, so no additional permutation

factor is needed. Adding the counts gives

$$N = 8 + 8 = 16.$$

Zero entries cannot occur in a counted matrix because neither required product is zero.

Step 4: Conclusion The number of matrices satisfying $f(A) = 15$ is $\boxed{16}$.

Quick Tip to Solve: For a small entry set, count possible values of ad and bc first, then multiply their ordered factor-pair counts; avoid listing every matrix.

Physics – Section A: Multiple Choice Questions (80 marks)

31. For a gas $C_P - C_V = R$ in a state P and $C_P - C_V = 1.10\ R$ in a state Q, T_P and T_Q are the temperatures in two different states P and Q respectively. Then [4 marks]

(A) $T_P = T_Q$

(B) $T_P < T_Q$

(C) $T_P = 0.9\ T_Q$

(D) $T_P > T_Q$

Correct Answer: (D) $T_P > T_Q$

Solution:

Concept:

- For an ideal gas, Mayer's relation for molar heat capacities is $C_P - C_V = R$. It is the high-temperature, weak-interaction limit of a real gas.
- For a real gas the difference can exceed R because intermolecular attractions affect the expansion work. Such departures from ideal behaviour become more pronounced on lowering the temperature.

Step 1: Compare each state with the ideal-gas limit At P the given difference is exactly R, whereas at Q it is $1.10R$, a positive departure of $0.10R$. Thus P corresponds to the more nearly ideal state, and Q shows a stronger influence of attractive interactions. Merely changing the number of translational degrees of freedom does not change the ideal-gas difference: it remains R.

Step 2: Relate the departure to temperature The usual real-gas interpretation can be illustrated with the van der Waals equation. Neglecting excluded volume for this illustration gives $P = RT/V_m - a/V_m^2$. The thermodynamic identity then gives

$$C_P - C_V = -T\frac{(\partial P/\partial T)^2_{V_m}}{(\partial P/\partial V_m)_T} = \frac{R}{1 - 2a/(RTV_m)}.$$

As thermal energy increases, the attraction correction becomes less important and the difference approaches R; a larger departure is associated with the lower-temperature state in this comparison.

Step 3: Choose the temperature ordering Therefore the intended ordering is $T_P > T_Q$. Equality in option (A) does not represent the contrasting ideality stated here. Options (B) and (C) put P at the

lower temperature and reverse the expected trend. The 10 percent change in heat-capacity difference is not itself a 10 percent temperature relation, so it cannot justify the numerical factor 0.9.

Step 4: Conclusion P is the higher-temperature, more nearly ideal state: $\boxed{T_P > T_Q}$. Hence, the correct option is (D).

Quick Tip to Solve: Do not convert a percentage departure of $C_P - C_V$ directly into a temperature ratio. Use the approach toward Mayer's relation at higher temperature.

---

32. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Moment of inertia of a circular disc of mass 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y and I_z respectively. The respectively radii of gyration about all the three axes will be the same.

Reason R : A rigid body making rotational motion has fixed mass and shape. In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are correct but R is NOT the correct explanation of A.

(B) A is not correct but R is correct.

(C) A is correct but R is not correct.

(D) Both A and R are correct and R is the correct explanation of A.

Correct Answer: (B) A is not correct but R is correct.

Solution:

Concept:

- The radius of gyration about a specified axis is defined by $I = Mk^2$, so $k = \sqrt{I/M}$. It depends on the axis as well as on the body.
- For a uniform circular disc, a central diameter in its plane has moment of inertia $MR^2/4$. The central axis normal to the plane has moment of inertia $MR^2/2$.

Step 1: Test the assertion using the three axes By the symmetry of the disc, the two perpendicular central in-plane diameters have equal moments, $I_x = I_y = MR^2/4$. The perpendicular-axis theorem for a plane lamina also gives

$$I_z = I_x + I_y = \frac{MR^2}{2}.$$

Hence

$$k_x = k_y = \frac{R}{2}, \qquad k_z = \frac{R}{\sqrt{2}}.$$

Since $R/2 \neq R/\sqrt{2}$, the three radii of gyration are not all equal. The assertion is false.

Step 2: Test the reason independently A rigid body retains its shape: distances between its particles remain fixed. Its mass also stays fixed during ordinary rotation. Thus the stated reason is true. These fixed properties do not imply equal distances of particles from every possible axis; that axis dependence is precisely why the moments and radii of gyration can differ.

Step 3: Match the truth values to the options The truth values are A false and R true, which is option

(B). Options (A) and (D) both require a true assertion and are therefore excluded. Option (C) reverses both truth values. There is no need to treat the true rigid-body statement as an explanation of an assertion already disproved by the explicit moments of inertia.

Step 4: Conclusion Assertion A is not correct, but Reason R is correct. Hence, the correct option is (B).

Quick Tip to Solve: Check k using $I = Mk^2$ for each axis. A symmetric disc has equal values for its two diameters, but a different value for the normal axis.

---

33. What should be the order of arrangement of de-Broglie wavelength of electron (λ_e), an α-particle (λ_a) and proton (λ_p) given that all have the same kinetic energy? [4 marks]

(A) $\lambda_e = \lambda_p = \lambda_\alpha$

(B) $\lambda_e < \lambda_p < \lambda_\alpha$

(C) $\lambda_e > \lambda_p > \lambda_\alpha$

(D) $\lambda_e = \lambda_p > \lambda_\alpha$

Correct Answer: (C) $\lambda_e > \lambda_p > \lambda_\alpha$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of momentum.
- For nonrelativistic motion, $K = p^2/(2m)$. At the same kinetic energy, $p = \sqrt{2mK}$, and therefore $\lambda \propto 1/\sqrt{m}$.

Step 1: Compare the masses of the three particles An electron is much lighter than a proton, while an alpha particle consists of two protons and two neutrons and has approximately four times the proton mass. Thus

$$m_e < m_p < m_\alpha \simeq 4m_p.$$

The particles are compared at equal kinetic energy, not equal velocity or equal momentum; this distinction determines which form of the wavelength formula is useful.

Step 2: Convert the mass ordering into a wavelength ordering Substitute the momentum expressed through kinetic energy:

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

Since h and K have the same values in all three comparisons, a larger mass makes the denominator larger and the wavelength smaller. In particular,

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} > 1, \qquad \frac{\lambda_p}{\lambda_\alpha} \simeq 2 > 1.$$

It follows that $\lambda_e > \lambda_p > \lambda_\alpha$.

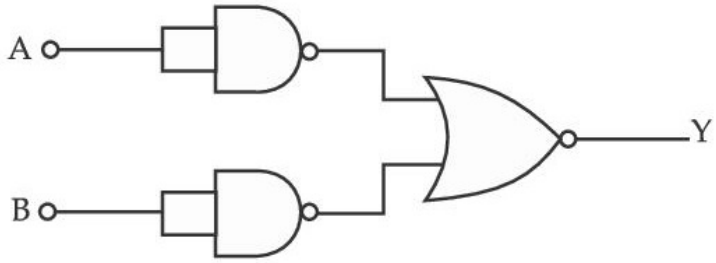
Step 3: Check the alternative orderings Option (A) would require equal momenta despite the different

masses at fixed kinetic energy. Option (B) gives the opposite of the inverse-square-root mass relation. Option (D) incorrectly makes the electron and proton wavelengths equal. Only option (C) agrees with both successive comparisons. The particle charges play no role here because kinetic energy is already specified.

Step 4: Conclusion The required ordering is $\boxed{\lambda_e > \lambda_p > \lambda_\alpha}$. Hence, the correct option is (C).

Quick Tip to Solve: At equal kinetic energy compare $1/\sqrt{m}$; at equal momentum all de Broglie wavelengths would instead be equal.

34. Identify the logic operation carried out. [4 marks]



(A) OR

(B) AND

(C) NOR

(D) NAND

Correct Answer: (B) AND

Solution:

Concept:

- A NAND gate produces the complement of the AND output. If its two inputs are tied together at A, its output is $\overline{A \cdot A} = \overline{A}$.
- The last gate is a NOR gate, so its output is the complement of the OR of its two inputs. De Morgan's law states $\overline{X + Z} = \overline{X}\,\overline{Z}$.

Step 1: Reduce the two input gates The upper gate in the figure has an AND-shaped body and an output bubble, identifying it as NAND. Both its inputs are connected to A, so its output is $\overline{A}$. Likewise the lower NAND gate has both inputs connected to B, so its output is $\overline{B}$. The connection boxes are wires tying each pair of inputs together; they do not introduce an additional independent input.

Step 2: Apply the final NOR operation The final gate has the OR shape with an output inversion bubble. Therefore

$$Y = \overline{\overline{A} + \overline{B}}.$$

Applying De Morgan's theorem and double complementation gives

$$Y = \overline{\overline{A}}\,\overline{\overline{B}} = A \cdot B.$$

This is the AND function of the original inputs, despite the use of NAND and NOR components internally.

Step 3: Verify the truth table and reject the other gates For input pairs $(A, B) = (0,0), (0,1), (1,0), (1,1)$, the resulting output sequence is $0, 0, 0, 1$. An OR gate would also output 1 for either mixed pair, so (A) is excluded. A NOR gate would output 1 at $(0,0)$, while a NAND gate would output 0 at $(1,1)$; therefore (C) and (D) are excluded. The only matching operation is AND.

Step 4: Conclusion The circuit implements $\boxed{Y = A \cdot B}$, the AND operation. Hence, the correct option is (B).

Quick Tip to Solve: A NAND with tied inputs acts as NOT. Replace those gates first, then use De Morgan's law across the output inversion.

---

35. A particle of mass 4M at rest disintegrates into two particles of mass M and 3M respectively having non zero velocities. The ratio of de-Broglie wavelength of particle of mass M to that of mass 3M will be : [4 marks]

(A) 1 : 3

(B) 3 : 1

(C) 1 : $\sqrt{3}$

(D) 1 : 1

Correct Answer: (D) 1 : 1

Solution:

Concept:

- With no external impulse, the total linear momentum is conserved during the disintegration. The parent particle is initially at rest, so its initial momentum is zero.
- The de Broglie wavelength depends on the magnitude of momentum: $\lambda = h/|\mathbf{p}|$. Opposite momentum directions do not make the wavelengths negative.

Step 1: Apply momentum conservation to the two fragments Let the velocities of the fragments with masses M and 3M be $\mathbf{v}_1$ and $\mathbf{v}_2$. Since the parent of mass 4M is at rest,

$$M\mathbf{v}_1 + 3M\mathbf{v}_2 = \mathbf{0}.$$

Thus $\mathbf{p}_1 = -\mathbf{p}_2$, so the two fragment momenta have equal magnitudes and opposite directions. Their velocities have different magnitudes, $v_1 = 3v_2$, but that does not change the momentum equality.

Step 2: Form the wavelength ratio Write the two matter wavelengths in terms of those momentum magnitudes:

$$\lambda_M = \frac{h}{p_1}, \qquad \lambda_{3M} = \frac{h}{p_2}.$$

Consequently,

$$\frac{\lambda_M}{\lambda_{3M}} = \frac{p_2}{p_1} = 1.$$

Both momenta are nonzero, as specified in the question, so this ratio is well defined.

Step 3: Check why the mass-based alternatives fail Options (A) and (B) would follow from comparing masses while incorrectly holding speed fixed. The speeds are not equal here; they are inversely proportional to the fragment masses. Option (C) resembles a fixed-kinetic-energy comparison, but the fragments do not have equal kinetic energies: $K = p^2/(2m)$ gives $K_M = 3K_{3M}$. The equality of momenta is the relevant fact, so only the ratio 1:1 is possible.

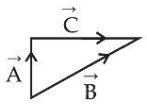
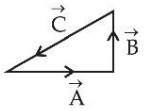
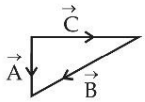
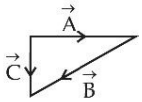
Step 4: Conclusion The de Broglie wavelength ratio is $\boxed{\lambda_M : \lambda_{3M} = 1:1}$. Hence, the correct option is (D).

Quick Tip to Solve: For two fragments from a parent initially at rest, start with equal and opposite momenta. Their matter wavelengths are equal even when their masses differ.

---

36. Match List - I with List - II

Choose the correct answer from the options given below : [4 marks]

List I	List II
(a) $\vec{C} - \vec{A} - \vec{B} = 0$	(i) 
(b) $\vec{A} - \vec{C} - \vec{B} = 0$	(ii) 
(c) $\vec{B} - \vec{A} - \vec{C} = 0$	(iii) 
(d) $\vec{A} + \vec{B} = -\vec{C}$	(iv) 

(A) (a) → (iv), (b) → (i), (c) → (iii), (d) → (ii)

(B) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

(C) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

(D) (a) → (i), (b) → (iv), (c) → (ii), (d) → (iii)

Correct Answer: (B) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

Solution:

Concept:

- The triangle law adds two vectors by placing the tail of the second at the head of the first; the resultant joins the initial tail to the final head.
- A vector may be shifted parallel to itself without changing it. Reversing its arrow changes its sign, so the arrowheads in each sketch determine the equation.

Step 1: Read diagrams (iv) and (iii) In (iv), follow the rightward arrow $\vec{A}$, then the diagonal arrow $\vec{B}$ down to the lower-left vertex. The resultant is the downward arrow $\vec{C}$. Thus $\vec{A}+\vec{B}=\vec{C}$, or $\vec{C}-\vec{A}-\vec{B}=0$, giving (a) to (iv). In (iii), following $\vec{C}$ rightward and then $\vec{B}$ diagonally back gives the downward vector $\vec{A}$. Hence $\vec{A}-\vec{C}-\vec{B}=0$, giving (b) to (iii).

Step 2: Read diagrams (i) and (ii) In (i), the upward vector $\vec{A}$, followed by the rightward vector $\vec{C}$, gives the diagonal vector $\vec{B}$. Therefore $\vec{B}-\vec{A}-\vec{C}=0$, giving (c) to (i). In (ii), $\vec{A}$ runs rightward, $\vec{B}$ upward, and $\vec{C}$ closes the triangle back to its start. Thus $\vec{A}+\vec{B}+\vec{C}=0$, giving (d) to (ii). Options A, C and D each assign at least one diagram an equation inconsistent with these arrow directions.

Step 3: Conclusion The vector equations give the matching listed in the second choice. Hence, the correct option is (B).

Quick Tip to Solve: Follow the arrowheads around each triangle before comparing equations: a closed head-to-tail loop sums to zero.

---

37. A parallel plate capacitor with plate area 'A' and distance of separation 'd' is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as :

$\varepsilon(x)=\varepsilon_0+kx$, **for** $\left(0<x\le\frac{d}{2}\right)$

$\varepsilon(x)=\varepsilon_0+k(d-x)$, **for** $\left(\frac{d}{2}\le x\le d\right)$ [4 marks]

(A) $\left(\varepsilon_0+\frac{kd}{2}\right)^{2/kA}$

(B) $\frac{kA}{2\ln\left(\frac{2\varepsilon_0+kd}{2\varepsilon_0}\right)}$

(C) 0

(D) $\frac{kA}{2}\ln\left(\frac{2\varepsilon_0}{2\varepsilon_0-kd}\right)$

Correct Answer: (B) $\frac{kA}{2\ln\left(\frac{2\varepsilon_0+kd}{2\varepsilon_0}\right)}$

Solution:

Concept:

- A dielectric slice of thickness dx, normal to the electric field, has capacitance $dC=\varepsilon(x)A/dx$. Successive slices are in series, so their reciprocal capacitances add.
- The two halves have mirror-symmetric permittivity profiles. Their voltage drops are equal for a fixed plate charge, allowing the integral over one half to be doubled.

Step 1: Set up the reciprocal-capacitance integral Let the free plate charge be Q. Since every slice has the same area and carries the same displacement, $E(x)=Q/[A\varepsilon(x)]$. Integrating the field gives $V=(Q/A)\int_0^d dx/\varepsilon(x)$, and therefore

$$\frac{1}{C}=\frac{V}{Q}=\frac{2}{A}\int_0^{d/2}\frac{dx}{\varepsilon_0+kx}.$$

The second half becomes the same integral on substituting $u=d-x$.

Step 2: Evaluate and invert The antiderivative is $k^{-1}\ln(\varepsilon_0 + kx)$. Thus

$$\frac{1}{C} = \frac{2}{kA}\ln\left(\frac{\varepsilon_0 + kd/2}{\varepsilon_0}\right).$$

Inverting gives

$$C = \frac{kA}{2\ln\left(\frac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}.$$

The logarithm must appear in the denominator. Option A has an inadmissible dimensional exponent; C wrongly predicts zero capacitance; D has the wrong logarithm and multiplication instead of inversion. As $k \to 0$, the result tends to $\varepsilon_0 A/d$, as required.

Step 3: Conclusion

$$C = \frac{kA}{2\ln\left(\frac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}.$$

Hence, the correct option is (B).

Quick Tip to Solve: When permittivity varies along plate separation, integrate $dx/\varepsilon(x)$, not $\varepsilon(x)\,dx$; those layers are in series.

38. A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If l_1 and l_2 are the lengths of the gas column, before and after the expansion respectively, then the value of $\frac{T_1}{T_2}$ will be : [4 marks]

(A) $\left(\frac{l_1}{l_2}\right)^{\frac{2}{3}}$

(B) $\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$

(C) $\frac{l_2}{l_1}$

(D) $\frac{l_1}{l_2}$

Correct Answer: (B) $\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$

Solution:

Concept:

- For a monoatomic ideal gas, $C_V = 3R/2$, $C_P = 5R/2$, and the adiabatic index is $\gamma = C_P/C_V = 5/3$.
- The ideal-gas adiabatic relation used for the gas-column expansion is $TV^{\gamma-1} =$ constant. With a fixed cylinder cross-section S, the volume is $V = Sl$.

Step 1: Relate temperature to gas-column length For the initial and final adiabatic states, write

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}.$$

The same cylinder cross-section applies at both positions of the piston, so $V_1 = Sl_1$ and $V_2 = Sl_2$. On dividing by $T_2 V_1^{\gamma-1}$,

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{l_2}{l_1}\right)^{\gamma-1}.$$

The cross-sectional area cancels; no numerical value for it is needed.

Step 2: Insert the monoatomic exponent and check the alternatives Since $\gamma - 1 = 5/3 - 1 = 2/3$, the temperature ratio is

$$\frac{T_1}{T_2} = \left(\frac{l_2}{l_1}\right)^{2/3}.$$

Expansion gives $l_2 > l_1$, and the gas cools, so $T_1/T_2 > 1$. Options A and D give a ratio below one and reverse this cooling trend. Option C has the correct trend but uses exponent one instead of the monoatomic exponent $2/3$; it therefore does not satisfy the stated adiabatic relation.

Step 3: Conclusion

$$\frac{T_1}{T_2} = \left(\frac{l_2}{l_1}\right)^{2/3}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For an ideal-gas adiabatic length problem, replace volume by length only after checking that the cylinder cross-section is fixed.

39. A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option. [4 marks]

(A) angle of refraction is 24.41°

(B) angle of refraction is 30°

(C) refraction is not possible

(D) angle of refraction is 53.4°

Correct Answer: (C) refraction is not possible

Solution:

Concept:

- Snell's law is $n_1 \sin i = n_2 \sin r$, where angles are measured from the normal. A propagating refracted ray requires $\sin r \leq 1$.
- For light travelling from a denser medium to a rarer medium, the critical angle obeys $\sin i_c = n_2/n_1$. Incidence above this angle produces total internal reflection.

Step 1: Test whether a refracted angle exists The incident medium is diamond, so $n_1 = 2.42$; the emergent medium is air, so $n_2 = 1$. At $i = 30°$, Snell's law would require

$$\sin r = \frac{2.42}{1} \sin 30° = 2.42 \times \frac{1}{2} = 1.21.$$

No real angle has sine greater than one. Consequently, there is no propagating refracted ray into the air at this incidence angle.

Step 2: Confirm the critical-angle condition The critical angle is

$$i_c = \sin^{-1}\left(\frac{1}{2.42}\right) \approx 24.41^\circ.$$

Because $30^\circ > 24.41^\circ$, the incident ray undergoes total internal reflection. The value 24.41° in option A is the critical angle, not the refraction angle for the given ray. Options B and D also imply refracted rays and fail Snell's law. The given wavelength identifies the laser but is unnecessary once the refractive indices at that wavelength are supplied.

Step 3: Conclusion Refraction into air is not possible; the ray undergoes total internal reflection. Hence, the correct option is (C).

Quick Tip to Solve: Before calculating an inverse sine in a denser-to-rarer problem, check whether $(n_1/n_2)\sin i$ exceeds one.

40. Two wires of same length and radius are joined end to end and loaded. The Young's modulii of the materials of the two wires are Y_1 and Y_2. The combination behaves as a single wire then its Young's modulus is : [4 marks]

(A) $Y = \frac{2Y_1Y_2}{3(Y_1+Y_2)}$
(B) $Y = \frac{2Y_1Y_2}{Y_1+Y_2}$
(C) $Y = \frac{Y_1Y_2}{2(Y_1+Y_2)}$
(D) $Y = \frac{Y_1Y_2}{Y_1+Y_2}$
Correct Answer: (B) $Y = \frac{2Y_1Y_2}{Y_1+Y_2}$

Solution:

Concept:

- For a wire of length L, area A, Young's modulus Y and tensile force F, the extension is $\Delta L = FL/(AY)$, within the elastic limit.
- Wires joined end to end carry the same tensile force, and their extensions add. The effective wire has total original length $2L$, with the same cross-sectional area as either wire.

Step 1: Add the individual extensions Let each original wire have length L and cross-sectional area $A = \pi r^2$. Under the common applied load F, the extensions are

$$\Delta L_1 = \frac{FL}{AY_1}, \qquad \Delta L_2 = \frac{FL}{AY_2}.$$

Their combined extension is therefore

$$\Delta L = \frac{FL}{A}\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right).$$

The force is common because the wires are in series, rather than sharing the load as parallel wires would.

Step 2: Equate this to the extension of the effective wire The equivalent wire has length $2L$, so $\Delta L = F(2L)/(AY)$. Equating the two expressions and cancelling FL/A gives

$$\frac{2}{Y} = \frac{1}{Y_1} + \frac{1}{Y_2}, \qquad Y = \frac{2Y_1Y_2}{Y_1 + Y_2}.$$

This is the harmonic mean of the two moduli. If $Y_1 = Y_2 = Y_0$, it correctly reduces to Y_0. Options A, C and D give $Y_0/3$, $Y_0/4$ and $Y_0/2$, respectively, and fail this identical-material check.

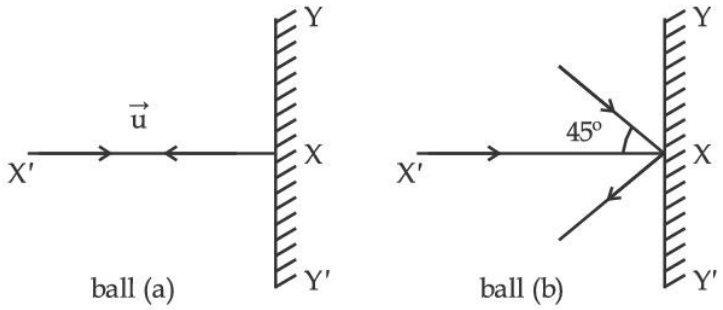
Step 3: Conclusion

$$Y = \frac{2Y_1Y_2}{Y_1 + Y_2}.$$

Hence, the correct option is (B).

Quick Tip to Solve: For series wires, add extensions and use the combined original length when computing strain; forgetting $2L$ loses a factor of two.

41. Two billiard balls of equal mass 30g strike a rigid wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball 'a' and ball 'b' by the wall along 'X' direction is : [4 marks]



(A) 1 : 1

(B) $\sqrt{2}$: 1

(C) 2 : 1

(D) 1 : $\sqrt{2}$

Correct Answer: (B) $\sqrt{2}$: 1

Solution:

Concept:

- Impulse equals change of momentum: $\vec{J} = m(\vec{v} - \vec{u})$. Elastic reflection reverses the component normal to a smooth rigid wall while preserving its magnitude.
- For incidence at angle θ to the wall normal, the magnitude of the normal impulse is $J_x = 2mu\cos\theta$. The component parallel to the wall does not contribute to this impulse.

Step 1: Compute the impulse on ball a The figure shows ball a moving perpendicular to the wall, along the X direction, so $\theta_a = 0$. Convert the data: $m = 30\,\text{g} = 0.030\,\text{kg}$ and $u = 108\,\text{km h}^{-1} = 30\,\text{m s}^{-1}$. Its normal velocity changes from $+u$ to $-u$. Thus the magnitude of the normal momentum change is

$$J_{a,x} = 2mu = 2(0.030)(30) = 1.8\,\text{N s}.$$

Step 2: Compute ball b's normal impulse and form the ratio Ball b is incident at 45° to the X direction, which is the normal to the wall. Its normal speed is $u\cos 45^\circ = u/\sqrt{2}$. Hence

$$J_{b,x} = 2mu\cos 45^\circ = \frac{1.8}{\sqrt{2}}\,\text{N s}.$$

Therefore

$$J_{a,x} : J_{b,x} = 1.8 : \frac{1.8}{\sqrt{2}} = \sqrt{2} : 1.$$

Equal masses and speeds do not give equal impulses when the normal components differ, excluding A. Option C misses the cosine factor, while D reverses the required order of the two balls.

Step 3: Conclusion The magnitude ratio of the X-directed impulses is $\sqrt{2} : 1$. Hence, the correct option is (B).

Quick Tip to Solve: On reflection, double the momentum component normal to the wall, rather than doubling the full momentum for an oblique collision.

---

42. In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin\omega t$; where d_0, ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as : [4 marks]

(A) $\frac{2\lambda D(d_0)}{(d_0^2 - a_0^2)}$

(B) $\frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$

(C) $\frac{\lambda D}{d_0^2} a_0$

(D) $\frac{\lambda D}{d_0 + a_0}$

Correct Answer: (B) $\frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$

Solution:

Concept:

- In Young's double-slit experiment, fringe width is $\beta = \lambda D/d$, with wavelength λ, screen distance D, and slit separation d.
- Since $-1 \le \sin\omega t \le 1$, the separation ranges from $d_0 - a_0$ to $d_0 + a_0$, taking a_0 as the positive amplitude and $d_0 > a_0$. Fringe width decreases as separation increases.

Step 1: Identify the largest and smallest fringe widths The smallest slit separation occurs at $\sin\omega t = -1$, giving the widest fringes:

$$\beta_{\max} = \frac{\lambda D}{d_0 - a_0}.$$

Conversely, $\sin\omega t = 1$ gives the largest slit separation and the narrowest fringes:

$$\beta_{\min} = \frac{\lambda D}{d_0 + a_0}.$$

The frequency ω sets when these extrema occur, but not their magnitudes.

Step 2: Subtract the extrema exactly Taking their difference over a common denominator gives

$$\begin{aligned}\Delta\beta &= \lambda D\left(\frac{1}{d_0 - a_0} - \frac{1}{d_0 + a_0}\right)\\ &= \lambda D\frac{(d_0 + a_0) - (d_0 - a_0)}{d_0^2 - a_0^2}\\ &= \frac{2\lambda D a_0}{d_0^2 - a_0^2}.\end{aligned}$$

Option A incorrectly remains nonzero when $a_0 = 0$, even though then the width cannot vary. Option C lacks a factor of two even in the small-amplitude limit; D is merely the minimum width, not the difference. No small-amplitude approximation is needed.

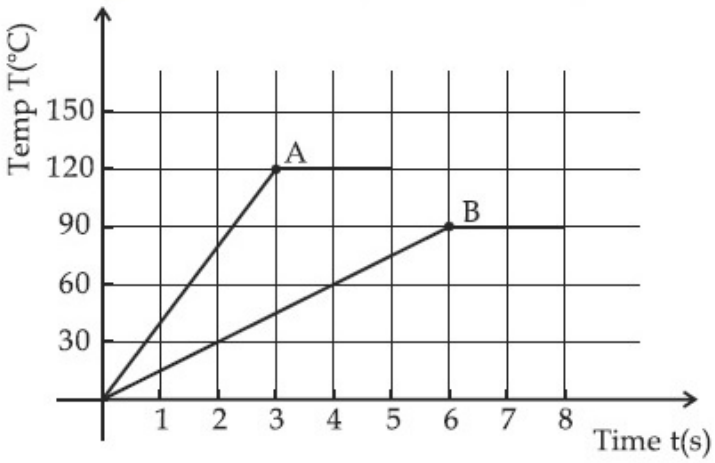
Step 3: Conclusion

$$\beta_{\max} - \beta_{\min} = \frac{2\lambda D a_0}{d_0^2 - a_0^2}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Find both extreme slit separations first, then invert them for fringe widths; the smaller separation produces the larger width.

43. Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is : [4 marks]



(A) $\frac{8}{3}$
(B) $\frac{3}{8}$
(C) $\frac{3}{4}$
(D) $\frac{4}{3}$

Correct Answer: (B) $\frac{3}{8}$

Solution:

Concept:

- For a body of mass m, heat supplied without a change of state is $Q = mc\Delta T$, where c is its specific heat capacity.
- At constant heating power P, $P = mc\,dT/dt$. For equal masses supplied with the same power, specific heat is inversely proportional to the slope of the temperature-time graph.

Step 1: Read the two slopes from the graph The straight line for A rises from the origin to $120^\circ\mathrm{C}$ in $3\,\mathrm{s}$; its slope is therefore

$$s_A = \frac{120}{3} = 40\,\mathrm{K\,s^{-1}}.$$

The line for B reaches $90^\circ\mathrm{C}$ in $6\,\mathrm{s}$, so

$$s_B = \frac{90}{6} = 15\,\mathrm{K\,s^{-1}}.$$

Temperature differences in degrees Celsius have the same numerical value as differences in kelvin.

Step 2: Invert the slope ratio Both bodies have the same mass and are heated at the same rate under similar conditions. Hence

$$mc_A s_A = mc_B s_B = P, \qquad \frac{c_A}{c_B} = \frac{s_B}{s_A} = \frac{15}{40} = \frac{3}{8}.$$

A heats faster, so its specific heat must be smaller. Option (A), $8/3$, is the slope ratio rather than its inverse. Options (C) and (D) do not use both the temperature rise and the corresponding elapsed time; comparing endpoint temperatures alone would be incorrect.

Step 3: Conclusion The specific heat capacities satisfy $c_A : c_B = 3 : 8$. Hence, the correct option is (B).

Quick Tip to Solve: On an equal-power heating graph, first calculate each rise divided by its own time; then invert their slope ratio to obtain the specific heat ratio.

44. A linearly polarized electromagnetic wave in vacuum is

$$E = 3.1\cos\left[(1.8)z - (5.4\times10^6)t\right]\hat{i}N/C$$

is incident normally on a perfectly reflecting wall at z = a. Choose the correct option [4 marks]

(A) The wavelength is 5.4 m

(B) The frequency of electromagnetic wave is 54×10^4 Hz.

(C) The transmitted wave will be $3.1\cos\left[(1.8)z - (5.4\times10^6)t\right]\hat{i}N/C$

(D) The reflected wave will be $3.1\cos\left[(1.8)z + (5.4\times10^6)t\right]\hat{i}N/C$

Correct Answer: No option is literally generally correct: the printed coefficients contradict vacuum propagation and (D) omits the wall-dependent reflection phase. Option (D) is the intended direction-of-propagation choice.

Solution:

Concept:

- A phase $kz - \omega t$ travels along positive z, whereas $kz + \omega t + \phi$ travels along negative z; ϕ is a constant phase.
- Wavelength and frequency are obtained from $\lambda = 2\pi/k$ and $f = \omega/(2\pi)$, not directly from k and ω. A perfectly reflecting wall transmits no wave.

Step 1: Test the wavelength, frequency and transmission statements The printed coefficients are $k = 1.8\,\mathrm{rad\,m^{-1}}$ and $\omega = 5.4\times10^6\,\mathrm{rad\,s^{-1}}$. They give

$$\lambda = \frac{2\pi}{1.8} \simeq 3.49\,\mathrm{m}, \qquad f = \frac{5.4\times10^6}{2\pi} \simeq 8.59\times10^5\,\mathrm{Hz}.$$

Thus (A) and (B) are false. Perfect reflection also rules out the nonzero transmitted field in (C).

Step 2: Identify the reversal of propagation and retain the phase qualification The reflected field must travel back along negative z, so its phase has the form $kz + \omega t + \phi$. This is the direction change represented by (D). For a conducting reflecting surface, requiring zero total tangential electric field at $z = a$ gives the more complete expression

$$\mathbf{E}_r = -3.1\cos[kz + \omega t - 2ka]\,\hat{\mathbf{i}}\ \mathrm{N/C}.$$

The constant phase depends on the reflecting plane; a reversed propagation sign alone does not specify this boundary phase. Also, the printed coefficients give $\omega/k = 3.0\times10^6\,\mathrm{m/s}$, which is inconsistent with the stated vacuum medium.

Step 3: Conclusion No option is literally generally correct as printed: the coefficients contradict vacuum propagation, and the reflected field needs a wall-dependent constant phase. Option (D) is the intended direction-of-propagation answer, understood only up to that phase.

Quick Tip to Solve: Use the signs multiplying position and time to determine direction, but use the wall boundary condition to fix any constant reflection phase.

45. Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4^{th} second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap ? (Take g = 9.8 m/s^2) [4 marks]

(A) 3 drops / 2 sconds

(B) 2 drops / second

(C) 1 drop / second

(D) 1 drop / 7 seconds

Correct Answer: (C) 1 drop / second

Solution:

Concept:

- A drop released from rest falls through $y = \frac{1}{2}gt^2$ after time t, neglecting air resistance.
- If successive releases are separated by τ, the younger drop has been falling for $t - \tau$ when the older one has been falling for t. The release rate is $1/\tau$.

Step 1: Write the spacing at the observation time At the stated instant, the older droplet has fallen for $4\,\mathrm{s}$. Its immediately following droplet has fallen for $(4 - \tau)\,\mathrm{s}$. Both distances are measured downward from the same tap, so

$$34.3 = \frac{9.8}{2}\left[4^2 - (4 - \tau)^2\right].$$

This compares positions at the same instant, rather than distances travelled by one drop in consecutive seconds.

Step 2: Solve for the physical release interval Dividing by 4.9 and expanding gives

$$7 = 16 - (16 - 8\tau + \tau^2) = 8\tau - \tau^2,$$

$$\tau^2 - 8\tau + 7 = 0, \qquad (\tau - 1)(\tau - 7) = 0.$$

The next drop is already in flight, so $0 < \tau \leq 4\,\mathrm{s}$. Thus $\tau = 1\,\mathrm{s}$; the root $7\,\mathrm{s}$ would make its age negative and is inadmissible.

Step 3: Check the rate against the alternatives The rate is $1/\tau = 1$ drop per second. With a one-second interval, the two distances are $4.9(16) = 78.4\,\mathrm{m}$ and $4.9(9) = 44.1\,\mathrm{m}$; their difference is exactly $34.3\,\mathrm{m}$. Two drops per second and three drops per two seconds give different spacings; one drop per seven seconds is the rejected root.

Step 4: Conclusion The droplets leave at a rate of 1 drop per second. Hence, the correct option is (C).

Quick Tip to Solve: For successive falling drops, use different falling times at one common observation instant, and reject any interval that gives a negative age for the younger drop.

46. The minimum and maximum distances of a planet revolving around the sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be : [4 marks]

(A) $\frac{v_0 x_1^2}{x_2^2}$

(B) $\frac{v_0 x_2^2}{x_1^2}$

(C) $\frac{v_0 x_1}{x_2}$

(D) $\frac{v_0 x_2}{x_1}$

Correct Answer: (D) $\frac{v_0 x_2}{x_1}$

Solution:

Concept:

- The gravitational force on a planet is directed toward the Sun. Its torque about the Sun vanishes, so the angular momentum $L = mrv_\perp$ is conserved.
- At the closest and farthest points of an elliptical orbit, velocity is perpendicular to the radius. The speed is maximum at the closest point and minimum at the farthest point.

Step 1: Associate each extreme speed with its distance The minimum Sun-planet distance is x_1, and the maximum distance is x_2, with $x_2 \geq x_1$. The given minimum speed v_0 occurs at the farthest point, $r = x_2$. Write the unknown maximum speed at $r = x_1$ as $v_{\max}$. At these turning points the radial component of velocity is zero, so the complete speed contributes to angular momentum.

Step 2: Apply conservation about the Sun Equating angular momenta at the two apsides gives

$$mx_1 v_{\max} = mx_2 v_0.$$

The planet mass cancels, leaving

$$v_{\max} = v_0 \frac{x_2}{x_1}.$$

This is equivalent to Kepler's equal-area law: the areal velocity is $\frac{1}{2} r v_\perp$, which remains constant throughout the orbit.

Step 3: Rule out the alternative ratios The factor x_2/x_1 is at least one, so the result properly exceeds or equals v_0. Options (A) and (C) would instead reduce the speed when $x_2 > x_1$. Option (B) contains an unnecessary square: angular momentum is proportional to rv, not $r^2 v$. A square appears only when expressing the areal velocity in terms of angular speed, $r^2\dot{\theta}/2$, because $v = r\dot{\theta}$.

Step 4: Conclusion The maximum speed is $v_0 x_2/x_1$. Hence, the correct option is (D).

Quick Tip to Solve: At perihelion and aphelion, conserve rv. Use $r^2\dot{\theta}$ only if the unknown is angular speed rather than linear speed.

47. Some nuclei of a radioactive material are undergoing radioactive decay. The time gap between the instances when a quarter of the nuclei have decayed and when half of the nuclei have decayed is given as :

(where λ is the decay constant) [4 marks]

(A) $\frac{1}{2}\frac{\ln 2}{\lambda}$

(B) $\frac{\ln 2}{\lambda}$

(C) $\frac{2\ln 2}{\lambda}$

(D) $\frac{\ln \frac{3}{2}}{\lambda}$

Correct Answer: (D) $\frac{\ln \frac{3}{2}}{\lambda}$

Solution:

Concept:

- For a radioactive sample, the surviving population follows $N(t) = N_0 e^{-\lambda t}$, where $\lambda > 0$ is its decay constant.
- A fraction described as decayed must be subtracted from one to obtain the fraction remaining. Time intervals can then be found by taking the logarithm of the ratio of surviving populations.

Step 1: Convert the two decayed fractions to populations Let t_1 be the instant when one quarter has decayed. Three quarters remain, so

$$N(t_1) = \frac{3}{4}N_0, \qquad e^{-\lambda t_1} = \frac{3}{4}.$$

Let t_2 be the instant when one half has decayed. Then

$$N(t_2) = \frac{1}{2}N_0, \qquad e^{-\lambda t_2} = \frac{1}{2}.$$

The requested quantity is $t_2 - t_1$, rather than the elapsed time from the start until half decay.

Step 2: Eliminate the initial population and calculate the interval Dividing the two equations gives

$$e^{-\lambda(t_2 - t_1)} = \frac{1/2}{3/4} = \frac{2}{3}.$$

Taking the natural logarithm yields

$$-\lambda(t_2 - t_1) = \ln\frac{2}{3}, \qquad t_2 - t_1 = \frac{1}{\lambda}\ln\frac{3}{2}.$$

The result is positive, since $\ln(3/2) > 0$. Its units are time because λ has units of inverse time.

Step 3: Check the half-life alternatives The half-life $\ln 2/\lambda$ would describe a decrease from N_0 to $N_0/2$, or from any population to half of that population. Here the decrease is from $3N_0/4$ to $N_0/2$, a factor $2/3$, not $1/2$. Thus neither one half, one, nor two half-lives in options (A), (B) and (C) gives the required logarithmic factor.

Step 4: Conclusion The time gap is $\frac{\ln(3/2)}{\lambda}$. Hence, the correct option is (D).

Quick Tip to Solve: For an interval between two decay percentages, form remaining-before divided by remaining-after inside the logarithm; do not use the ratio of percentages decayed.

48. The half-life of ^{198}Au is 3 days. If atomic weight of ^{198}Au is 198 g/mol then the activity of 2 mg of ^{198}Au is [in disintegration/second] : [4 marks]

(A) 2.67×10^{12}

(B) 6.06×10^{18}

(C) 32.36×10^{12}

(D) 16.18×10^{12}

Correct Answer: (D) 16.18×10^{12}

Solution:

Concept:

- Activity is the number of nuclear disintegrations per unit time: $A = \lambda N$. For half-life $T_{1/2}$, the decay constant is $\lambda = \ln 2/T_{1/2}$.
- The number of radioactive atoms is $N = (m/M)N_A$, where sample mass m and molar mass M must use the same mass unit, and $N_A \simeq 6.02 \times 10^{23}\,\text{mol}^{-1}$.

Step 1: Find the number of gold nuclei The sample mass is $2\,\text{mg} = 2 \times 10^{-3}\,\text{g}$, while the molar mass is $198\,\text{g mol}^{-1}$. Assuming the stated sample consists of the radioactive isotope,

$$N = \frac{2 \times 10^{-3}}{198}(6.02 \times 10^{23}) \simeq 6.08 \times 10^{18}.$$

This is a count of nuclei, not a rate; it cannot itself be the activity.

Step 2: Convert the half-life and multiply by the population Because the question asks for disintegrations per second, use

$$T_{1/2} = 3 \times 24 \times 3600 = 259200\,\text{s},$$

$$\lambda = \frac{0.693}{259200} \simeq 2.674 \times 10^{-6}\,\text{s}^{-1}.$$

Therefore

$$A = \lambda N \simeq (2.674 \times 10^{-6})(6.08 \times 10^{18}) \simeq 1.62 \times 10^{13}\,\text{s}^{-1}.$$

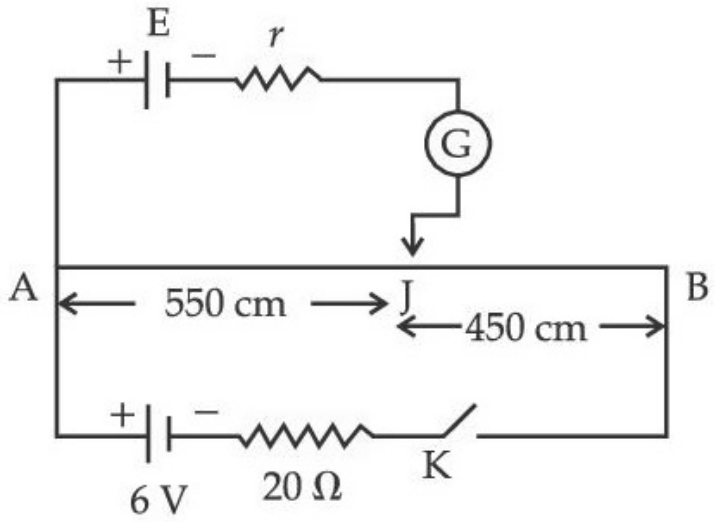
Rounding the constants used in the options gives 16.18×10^{12} disintegrations per second.

Step 3: Compare the choices with the calculation Option (D) agrees with the calculated order and coefficient. Option (B) is approximately the population of nuclei, without the decay probability per second. Option (C) is about twice the activity and option (A) is too small; neither satisfies $A = (\ln 2)N/T_{1/2}$ with the given mass and three-day half-life.

Step 4: Conclusion The activity is approximately 1.62×10^{13} Bq, matching 16.18×10^{12} disintegrations per second. Hence, the correct option is (D).

Quick Tip to Solve: Convert milligrams to grams and days to seconds before evaluating $(m/M)N_A \ln 2/T_{1/2}$; the result then comes directly in becquerels.

49. In the given figure, there is a circuit of potentiometer of length AB = 10 m. The resistance per unit length is 0.1 Ω per cm. Across AB, a battery of emf E and internal resistance 'r' is connected. The maximum value of emf measured by this potentiometer is : [4 marks]



(A) 5 V

(B) 2.25 V

(C) 6 V

(D) 2.75 V

Correct Answer: (A) 5 V

Solution:

Concept:

- The potential gradient along a uniform potentiometer wire is $k = IR/L$. An emf balances at length l when $E = kl$, because the galvanometer branch then carries no current.
- The largest balancing length is the whole wire L, so the maximum measurable emf is its end-to-end potential drop $V_{AB} = IR$, rather than necessarily the driving cell's emf.

Step 1: Calculate the resistance of the potentiometer wire The diagram shows the wire length $AB = 10\,\text{m} = 1000\,\text{cm}$, its resistance per unit length $0.1\,\Omega/\text{cm}$, a $6\,\text{V}$ driving cell and a $20\,\Omega$ series resistor. Thus

$$R_{AB} = 1000(0.1) = 100\,\Omega.$$

The branch containing the cell being tested and galvanometer carries zero current at balance, so its internal resistance r does not enter the driving current.

Step 2: Find the full-wire potential drop With the key closed, the driving circuit resistance is

$$R_{\text{total}} = 100 + 20 = 120\,\Omega, \qquad I = \frac{6}{120} = 0.05\,\text{A}.$$

Hence the voltage across the potentiometer wire is

$$V_{AB} = IR_{AB} = 0.05(100) = 5\,\text{V}.$$

Equivalently, the potential gradient is $k = 5/1000 = 0.005\,\text{V/cm}$.

Step 3: Use the full balancing range rather than the illustrated balance The shown position $AJ = 550\,\text{cm}$ would balance $0.005(550) = 2.75\,\text{V}$, which is option (D), but it is not the maximum range. The remaining $450\,\text{cm}$ corresponds to $2.25\,\text{V}$, option (B). The driving emf $6\,\text{V}$ in option (C) includes the $1\,\text{V}$ drop in the external series resistor. Moving the balancing contact all the way to B gives the maximum of $5\,\text{V}$.

Step 4: Conclusion The maximum emf measurable with this potentiometer is $5\,\text{V}$. Hence, the correct option is (A).

Quick Tip to Solve: For maximum potentiometer range, calculate the voltage across the entire wire. A marked jockey position gives one particular measured emf, not the instrument's full range.

---

50. In amplitude modulation, the message signal

$V_m(t) = 10 \sin(2\pi \times 10^5 t)$ volts and

Carrier signal

$V_C(t) = 20 \sin(2\pi \times 10^7\, t)$ volts

The modulated signal now contains the message signal with lower side band and upper side band frequency, therefore the bandwidth of modulated signal is α kHz. The value of α is : [4 marks]

(A) 200 kHz

(B) 50 kHz

(C) 100 kHz

(D) 0

Correct Answer: (A) 200 kHz

Solution:

Concept:

- For a sinusoid written as $V(t) = V_0 \sin(2\pi f t)$, the coefficient of $2\pi t$ is its frequency f, measured in hertz.
- An amplitude-modulated carrier has sidebands at $f_c - f_m$ and $f_c + f_m$. Its bandwidth is the difference between these two extreme frequencies, namely $2f_m$.

Step 1: Identify the message and carrier frequencies The message signal is $10\sin(2\pi \times 10^5 t)$, so $f_m = 10^5\,\text{Hz} = 100\,\text{kHz}$. Similarly, the carrier frequency is $f_c = 10^7\,\text{Hz} = 10{,}000\,\text{kHz}$. The amplitudes 10 V and 20 V determine the modulation index, not the frequency separation. In fact their ratio is 0.5, so the signal is not overmodulated.

Step 2: Calculate the two sideband frequencies The lower sideband is below the carrier by the message frequency, while the upper sideband is above it by the same amount:

$$f_{\text{LSB}} = 10{,}000 - 100 = 9900\,\text{kHz}, \qquad f_{\text{USB}} = 10{,}000 + 100 = 10{,}100\,\text{kHz}.$$

Thus the complete modulated spectrum extends across both sides of the carrier, not just one side.

Step 3: Find the occupied bandwidth and compare the choices The required bandwidth is

$$\Delta f = f_{\text{USB}} - f_{\text{LSB}} = 10{,}100 - 9900 = 200\,\text{kHz}.$$

The 100 kHz choice is only the separation of either sideband from the carrier. The 50 kHz choice incorrectly brings the amplitude ratio into the bandwidth calculation. Zero would require a zero-frequency message and cannot describe this signal.

Step 4: Conclusion The bandwidth is $200\,\text{kHz}$, hence $\alpha = 200$. Hence, the correct option is (A).

Quick Tip to Solve: For ordinary AM with both sidebands present, read the highest message frequency directly from the sinusoid and double it; the carrier frequency cancels.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A body of mass 2 kg moving with a speed of 4 m/s. makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial peed. The speed of the two body centre of mass is $\frac{x}{10}$ m/s. Then the value of x is _____. [4 marks]

Solution:

Concept:

- In an isolated collision the total linear momentum is conserved. The centre-of-mass velocity equals total momentum divided by total mass and is unchanged by the collision.
- For a one-dimensional elastic collision, the relative speed of separation equals the relative speed of approach: $v_2 - v_1 = u_1 - u_2$.

Step 1: Use the specified final speed Let the initially stationary mass be m. Choose the original direction of the 2 kg body as positive. Its initial velocity is $u_1 = 4\,\text{m s}^{-1}$, the other initial velocity is $u_2 = 0$, and its final velocity is

$$v_1 = \frac{4}{4} = 1\,\text{m s}^{-1}.$$

The positive sign is essential: the problem explicitly says that it continues in the original direction.

Step 2: Apply elasticity and momentum conservation The elastic-collision relation gives

$$v_2 - 1 = 4 - 0, \qquad v_2 = 5\,\text{m s}^{-1}.$$

Conservation of momentum then gives

$$2(4) + m(0) = 2(1) + m(5).$$

Therefore $5m = 6$, or $m = 1.2\,\text{kg}$. As a check, the final kinetic energy is $\frac{1}{2}(2)(1)^2 + \frac{1}{2}(1.2)(5)^2 = 16\,\text{J}$, equal to its initial value.

Step 3: Calculate the centre-of-mass speed The total mass is $2 + 1.2 = 3.2\,\text{kg}$, and the conserved total

momentum is $8\,\mathrm{kg\,m\,s^{-1}}$. Hence

$$V_{\mathrm{CM}} = \frac{8}{3.2} = 2.5\,\mathrm{m\,s^{-1}}.$$

Comparing this with $(x/10)\,\mathrm{m\,s^{-1}}$ gives $x/10 = 2.5$.

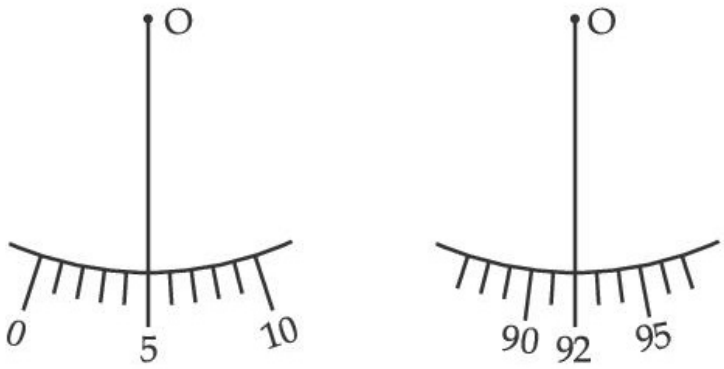
Step 4: Conclusion The centre of mass moves at $2.5\,\mathrm{m\,s^{-1}}$ in the initial direction, so the required dimensionless number is $\boxed{x = 25}$.

Quick Tip to Solve: Use the relative-speed rule to find the second final velocity first; then one momentum equation determines its mass and the centre-of-mass speed.

52. Student A and student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is _____.

[Figure shows position of reference 'O' when jaws of screw gauge are closed]

Given pitch = 0.1 cm. [4 marks]



Screw gauge (A) Screw gauge (B)

Solution:

Concept:

- The least count of a screw gauge equals its pitch divided by the number of circular divisions. Correct reading equals observed reading minus zero error.
- When closed jaws show a small positive circular reading, the zero error is positive. A reading just below the zero mark corresponds to a negative error found using the remaining divisions to 100.

Step 1: Calculate the least count Both instruments have pitch $0.1\,\mathrm{cm}$ and 100 circular divisions. Their common least count is

$$\mathrm{LC} = \frac{0.1}{100} = 0.001\,\mathrm{cm}.$$

Thus each one-division displacement of the reference line represents $0.001\,\mathrm{cm}$. The true radius $0.322\,\mathrm{cm}$ would correspond to a main-scale contribution of $0.3\,\mathrm{cm}$ plus 22 circular divisions for an

error-free gauge.

Step 2: Read the zero errors from the closed-jaw positions For gauge A the reference line meets circular division 5 when the jaws are closed. Its positive zero error is $+5\text{LC} = +0.005\,\text{cm}$. For gauge B it meets division 92; the zero is short by $100 - 92 = 8$ divisions. Its zero error is therefore $-8\text{LC} = -0.008\,\text{cm}$. The errors have opposite signs and must not simply be subtracted as unsigned readings.

Step 3: Recover the observed readings for the same wire Since observed reading equals true reading plus zero error,

$$r_A = 0.322 + 0.005 = 0.327\,\text{cm}, \qquad r_B = 0.322 - 0.008 = 0.314\,\text{cm}.$$

Their main-scale readings are both $0.3\,\text{cm}$, so the final circular readings are 27 divisions and 14 divisions, respectively. The required absolute difference is $|27 - 14| = 13$.

Step 4: Conclusion The absolute difference between the final circular scale readings is $\boxed{13}$ divisions.

Quick Tip to Solve: Convert the closed-jaw reading near 100 to a negative zero error before calculating observed readings; here the opposite errors differ by 5+8 divisions.

53. An inductor of 10 mH is connected to a 20V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after 1 μs is $\frac{x}{100}$ mA. Then x is equal to _____. (Take e^{-1} = 0.37) [4 marks]

Solution:

Concept:

- After a long time on a DC supply an ideal inductor has no induced voltage, so the steady current is $I_0 = V/R$.
- During current decay through resistance R, the RL equation is $L\,dI/dt + RI = 0$, giving $I(t) = I_0 e^{-t/\tau}$, where $\tau = L/R$.

Step 1: Find the current before switching Use the resistance in ohms: $R = 10\,\text{k}\Omega = 10^4\,\Omega$. Once the maximum current has been established,

$$I_0 = \frac{20}{10^4} = 2 \times 10^{-3}\,\text{A} = 2\,\text{mA}.$$

This is the starting current for the decay. It cannot fall instantaneously to zero, because a sudden current change would require an unbounded induced voltage across the inductor.

Step 2: Evaluate the circuit time constant The inductance is $L = 10\,\text{mH} = 10^{-2}\,\text{H}$. Therefore

$$\tau = \frac{L}{R} = \frac{10^{-2}}{10^4} = 10^{-6}\,\text{s} = 1\,\mu\text{s}.$$

The stated observation time is exactly one time constant. The exponent $-t/\tau$ is dimensionless and equals -1; using milliseconds instead of microseconds would give the wrong decay factor.

Step 3: Apply the exponential decay law Using the supplied value $e^{-1} = 0.37$,

$$I(1\,\mu\text{s}) = 2e^{-1}\,\text{mA} = 2(0.37)\,\text{mA} = 0.74\,\text{mA}.$$

The current has decreased to 37 percent of its initial value, as expected after one time constant. Comparing with the given form $(x/100)\,\text{mA}$ gives $x/100 = 0.74$.

Step 4: Conclusion The remaining current is $0.74\,\text{mA}$, so $\boxed{x = 74}$.

Quick Tip to Solve: First check whether the specified time equals L/R; at one time constant the decaying current is simply 0.37 times its initial value.

---

54. A circular conducting coil of radius 1 m is being heated by the change of magnetic field $\vec{B}$ passing perpendicular to the plane in which the coil is laid. The resistance of the coil is 2 $\mu\Omega$. The magnetic field is slowly switched off such that its magnitude changes in time as

$$B = \frac{4}{\pi} \times 10^{-3} T \left(1 - \frac{t}{100}\right)$$

The energy dissipated by the coil before the magnetic field is switched off completely is E = _____ mJ.

[4 marks]

Solution:

Concept:

- For a single circular loop in a perpendicular uniform field, magnetic flux is $\Phi = AB$, where $A = \pi r^2$. Faraday's law gives the induced emf magnitude $\mathcal{E} = A|dB/dt|$.
- The heat dissipated in resistance R is $E = \int I^2 R\,dt = \int \mathcal{E}^2/R\,dt$. A field changing linearly in time produces a constant induced emf.

Step 1: Find the switch-off interval and field slope The factor $1 - t/100$ becomes zero at $t = 100\,\text{s}$. Over the interval from 0 to 100 s,

$$\frac{dB}{dt} = -\frac{4 \times 10^{-3}}{100\pi} = -\frac{4 \times 10^{-5}}{\pi}\,\text{T s}^{-1}.$$

The negative sign describes the decreasing field. For the dissipated energy we use the emf magnitude, since the heat is independent of the direction of the induced current.

Step 2: Calculate the induced emf and power The radius is 1 m, so $A = \pi\,\text{m}^2$. Thus

$$\mathcal{E} = \pi \frac{4 \times 10^{-5}}{\pi} = 4 \times 10^{-5}\,\text{V}.$$

Using $R = 2\,\mu\Omega = 2 \times 10^{-6}\,\Omega$,

$$P = \frac{\mathcal{E}^2}{R} = \frac{16 \times 10^{-10}}{2 \times 10^{-6}} = 8 \times 10^{-4}\,\text{W}.$$

Equivalently the induced current is 20 A, and I^2R gives the same power.

Step 3: Integrate the constant heating power Because both the resistance and field slope are constant, the heating power remains constant until the field reaches zero:

$$E = P\Delta t = (8 \times 10^{-4})(100) = 8 \times 10^{-2}\,\text{J}.$$

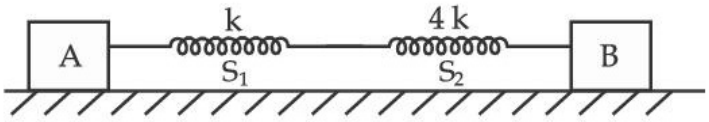
Finally, converting joules to millijoules requires multiplication by 1000, so $8 \times 10^{-2}\,\text{J} = 80\,\text{mJ}$. No further heat is generated by changing flux once the field has become constant at zero.

Step 4: Conclusion The energy dissipated during switch-off is $\boxed{E = 80\,\text{mJ}}$.

Quick Tip to Solve: For a linear field decrease, use E=(A times field slope) squared times duration divided by resistance; keep micro-ohms and millijoules separate.

---

55. In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be _____ rad/s when k = 20 N/m. [4 marks]



Solution:

Concept:

- Two massless springs in series carry equal force. Their equivalent spring constant obeys $1/k_{\text{eq}} = 1/k_1 + 1/k_2$.
- For two free masses connected by a spring, the relative coordinate oscillates with $\omega^2 = k_{\text{eq}}(1/m_A + 1/m_B) = k_{\text{eq}}/\mu$, where $\mu = m_A m_B/(m_A + m_B)$ is the reduced mass.

Step 1: Replace the series combination by one spring The diagram places the springs of constants k and $4k$ in series between the bodies. Their extensions add under a common force, so

$$k_{\text{eq}} = \frac{k(4k)}{k + 4k} = \frac{4k}{5}.$$

With $k = 20\,\text{N m}^{-1}$, this gives $k_{\text{eq}} = 16\,\text{N m}^{-1}$. Treating these springs as parallel would incorrectly add their constants even though they do not share the same extension.

Step 2: Account for the motion of both bodies Convert the masses to kilograms: $m_A = 0.2\,\text{kg}$ and $m_B = 0.8\,\text{kg}$. If q is the total extension from equilibrium, the spring forces on the masses are equal and opposite. Their relative acceleration therefore satisfies

$$\ddot{q} = -k_{\text{eq}}\left(\frac{1}{m_A} + \frac{1}{m_B}\right) q.$$

Both bodies move because neither is fixed to a wall; using just one of their masses would miss this

relative motion.

Step 3: Evaluate the angular frequency The reduced mass is

$$\mu = \frac{0.2 \times 0.8}{0.2 + 0.8} = 0.16\,\text{kg}.$$

Comparing the relative-motion equation with $\ddot{q} = -\omega^2 q$,

$$\omega = \sqrt{\frac{16}{0.16}} = \sqrt{100} = 10\,\text{rad}\,\text{s}^{-1}.$$

The initial extension sets the amplitude, not this angular frequency. The centre of mass has no acceleration because the frictionless surface supplies no horizontal external force.

Step 4: Conclusion The angular frequency of the internal oscillation is $\boxed{10\,\text{rad}\,\text{s}^{-1}}$.

Quick Tip to Solve: For two freely moving blocks, first combine the springs and then use the reduced mass; the sum of the masses is not the oscillating mass of the relative coordinate.

56. The value of aluminium susceptibility is 2.2×10^{-5}. The percentage increase in the magnetic field if space within a current carrying toroid is filled with aluminium is $\frac{x}{10^4}$. Then the value of x is _____.

[4 marks]

Solution:

Concept:

- For a linear magnetic material, magnetisation is $M = \chi H$, where χ is the dimensionless magnetic susceptibility.
- The flux density is $B = \mu_0(H + M) = \mu_0(1 + \chi)H$. At fixed toroid current, $H = nI$ is unchanged, so the fractional increase over the empty toroid is χ.

Step 1: Compare the empty and filled toroid Let B_0 be the magnetic field before inserting aluminium. With air or vacuum inside the toroid, $B_0 = \mu_0 H$. After filling it with aluminium,

$$B = \mu_0(1 + \chi)H = (1 + \chi)B_0.$$

The same current and toroid geometry are used in the comparison, so no current ratio or geometrical factor remains. Aluminium has positive susceptibility, hence its field is slightly greater than the original field.

Step 2: Compute the fractional and percentage increases The change in field is $\Delta B = B - B_0 = \chi B_0$. Therefore

$$\frac{\Delta B}{B_0} = \chi = 2.2 \times 10^{-5}.$$

A percentage increase is 100 times the fractional increase:

$$\text{percentage increase} = 100(2.2 \times 10^{-5}) = 2.2 \times 10^{-3}.$$

Thus the actual increase is 0.0022%, not $2.2 \times 10^{-5}\%$. The latter would confuse the fractional change with its percentage.

Step 3: Match the requested numerical form The question writes the numerical percentage as $x/10^4$. Hence

$$\frac{x}{10^4} = 2.2 \times 10^{-3}, \qquad x = (2.2 \times 10^{-3})(10^4) = 22.$$

This is a small paramagnetic enhancement: the relative permeability is 1.000022, consistent with the very small positive susceptibility. The requested x is a pure numerical coefficient; it has no magnetic-field unit.

Step 4: Conclusion The magnetic field increases by 0.0022%, giving $\boxed{x = 22}$.

Quick Tip to Solve: At fixed toroid current, susceptibility is the fractional increase in B; multiply by 100 before matching a formula stated as a percentage.

---

57. A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at a distance '2 m' when each is carrying same charge 'q'. If the free charged particle is displaced from its equilibrium position through distance 'x' (x $<<$ 1 m). The particle executes SHM. Its angular frequency of oscillation will be _____ $\times$ 10^5 rad/s if q^2 = 10 C^2. [4 marks]

Solution:

Concept:

- For motion along the line joining two identical fixed charges, the nearer charge repels the displaced charge more strongly, producing a restoring force toward the midpoint.
- For small displacement, simple harmonic motion has $F = -m\omega^2 x$. Use Coulomb's constant $k = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ and expand the force to first order in displacement.

Step 1: Write the force after a small displacement Let the fixed charges be at coordinates $-a$ and $+a$, where $a = 1\,\mathrm{m}$, and displace the moving charge a distance $x > 0$ toward the right. Both fixed charges have the same sign as the moving charge. The left charge pushes right and the nearer right charge pushes left, so

$$F = kq^2\left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2}\right].$$

The negative sign of the resultant for positive displacement verifies that this is the stable, longitudinal oscillation.

Step 2: Linearise and identify the angular frequency Because $x \ll a$, use $(1 \pm x/a)^{-2} \simeq 1 \mp 2x/a$. Hence

$$F \simeq -\frac{4kq^2}{a^3}x, \qquad \omega^2 = \frac{4kq^2}{ma^3}.$$

The mass is $1\,\mathrm{mg} = 10^{-6}\,\mathrm{kg}$, not $10^{-3}\,\mathrm{kg}$. Substituting the given squared charge directly gives

$$\omega^2 = \frac{4(9 \times 10^9)(10)}{(10^{-6})(1)^3} = 3.6 \times 10^{17}\,\mathrm{s^{-2}}.$$

Thus $\omega = 6 \times 10^8\,\mathrm{rad\,s^{-1}}$. This is angular frequency; no factor of 2π is needed unless ordinary frequency is requested.

Step 3: Conclusion The angular frequency is $6000 \times 10^5\,\mathrm{rad\,s^{-1}}$. Therefore the number to enter is $\boxed{6000}$.

Quick Tip to Solve: Keep the half-separation a in the force formula until the end; the restoring coefficient is 4kq squared divided by a cubed.

58. An electric bulb rated as 200 W at 100 V is used in a circuit having 200 V supply. The resistance 'R' that must be put in series with the bulb so that the bulb delivers the same power is _____ Ω. [4 marks]

Solution:

Concept:

- A bulb operates at its rated power when its terminal voltage and current equal their rated values. Use $P = VI$ and $P = V^2/R_b$, where R_b is the bulb resistance at its operating temperature.
- In series, the same current passes through the bulb and the added resistor. Kirchhoff's voltage law assigns the remaining supply voltage to the resistor.

Step 1: Find the required bulb current and resistance The label specifies $P_b = 200\,\mathrm{W}$ at $V_b = 100\,\mathrm{V}$. To preserve that operating condition, the current must be

$$I_b = \frac{P_b}{V_b} = \frac{200}{100} = 2\,\mathrm{A}.$$

The bulb resistance under the rated condition is

$$R_b = \frac{V_b^2}{P_b} = \frac{100^2}{200} = 50\,\Omega.$$

This is the operating resistance; using the cold filament resistance would not describe the stated rated power.

Step 2: Allocate the excess voltage to the series resistor The source supplies $200\,\mathrm{V}$, but the bulb must receive only $100\,\mathrm{V}$. Therefore the external resistor must drop

$$V_R = 200 - 100 = 100\,\mathrm{V}.$$

Since its current is the same $2\,\mathrm{A}$, Ohm's law gives

$$R = \frac{V_R}{I_b} = \frac{100}{2} = 50\,\Omega.$$

As a check, the total resistance is $50 + 50 = 100\,\Omega$, so the source current is $200/100 = 2\,\mathrm{A}$. Each equal resistance then receives half the supply voltage, and the bulb power is $I_b^2 R_b = 2^2 \times 50 = 200\,\mathrm{W}$. The resistor also dissipates $200\,\mathrm{W}$; it is not a zero-power voltage-dropping element.

Step 3: Conclusion The required series resistance is $\boxed{50\,\Omega}$. The numerical entry is 50.

Quick Tip to Solve: First calculate rated current from power divided by rated voltage; divide the excess supply voltage by that same current.

59. A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be (g = 10 m/s^2) _____ m/s. [4 marks]

Solution:

Concept:

- With no dissipative force, a pendulum conserves its mechanical energy. The string tension is perpendicular to instantaneous motion and does no work.
- For a pendulum of length l, an angular displacement θ from the downward vertical raises the bob by $h = l(1 - \cos\theta)$.

Step 1: Calculate the rise above the lowest point Take gravitational potential energy as zero at the lowest position, where the speed is $v_0 = 3\,\mathrm{m\,s^{-1}}$. Convert the length before substitution: $50\,\mathrm{cm} = 0.50\,\mathrm{m}$. At $\theta = 60°$, the vertical distance of the bob below the support is $l\cos 60°$, compared with l initially. The rise is therefore

$$h = l - l\cos 60° = 0.50\left(1 - \frac{1}{2}\right) = 0.25\,\mathrm{m}.$$

Use the vertical rise rather than the arc length, since gravitational potential energy depends only on height.

Step 2: Convert lost kinetic energy into potential energy Conservation of energy between the bottom and the specified angle gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh.$$

The mass cancels, and rearrangement yields

$$v^2 = v_0^2 - 2gh = 3^2 - 2(10)(0.25) = 9 - 5 = 4\,\mathrm{m^2\,s^{-2}}.$$

The speed is the nonnegative square root, so $v = 2\,\mathrm{m\,s^{-1}}$. The positive value of v^2 confirms that the bob can reach this angle. Its speed is lower than the bottom speed, as expected while it gains height; taking the opposite sign for potential energy would incorrectly predict acceleration on ascent.

Step 3: Conclusion The speed at 60° to the vertical is $\boxed{2\,\mathrm{m\,s^{-1}}}$. The numerical entry is 2.

Quick Tip to Solve: For pendulum energy questions, use v squared equals bottom speed squared minus 2gl times one minus cosine theta.

60. A particle of mass 'm' is moving in time 't' on a trajectory given by

$$\vec{r} = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$$

Where α and β are dimensional constants.

The angular momentum of the particle becomes the same as it was for t = 0 at time t = _____ seconds. [4 marks]

Solution:

Concept:

- For a particle, angular momentum about the coordinate origin is $\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$, with $\boldsymbol{v} = d\boldsymbol{r}/dt$.
- For planar motion, only the perpendicular component survives: $L_z = m(xv_y - yv_x)$. Equality of angular momentum refers to the vector, including its sign.

Step 1: Differentiate the position components From the prescribed trajectory,

$$x = 10\alpha t^2, \qquad y = 5\beta(t-5).$$

Differentiating each component independently gives

$$v_x = 20\alpha t, \qquad v_y = 5\beta.$$

At $t = 0$, the position need not vanish: $y = -25\beta$. Nevertheless $v_x = 0$, and both the position and velocity lie along the y-axis, so their cross product initially vanishes.

Step 2: Form the cross product and compare with its initial value Substitute the components without replacing the vector product by a product of magnitudes:

$$\begin{aligned} L_z &= m\{(10\alpha t^2)(5\beta) - [5\beta(t-5)](20\alpha t)\} \\ &= m\alpha\beta[50t^2 - 100t(t-5)] \\ &= 50m\alpha\beta\, t(10-t). \end{aligned}$$

Thus $\boldsymbol{L}(0) = \boldsymbol{0}$, and equality with the initial angular momentum requires

$$50m\alpha\beta\, t(10-t) = 0.$$

For the stated nontrivial trajectory, this gives $t = 0$ or $t = 10\,\mathrm{s}$. The first root is the initial instant itself; the requested subsequent time is the other root. At that time, position and velocity again have parallel directions, providing a direct geometric check of the zero cross product.

Step 3: Conclusion The angular momentum returns to its initial value at $\boxed{t = 10\,\mathrm{s}}$. The numerical entry is 10.

Quick Tip to Solve: For a planar trajectory, calculate $xv_y - yv_x$ directly; keeping the sign makes the return-time equation factor easily.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which one of the following species responds to an external magnetic field? [4 marks]

(A) $[Fe(H_2O)_6]^{3+}$

(B) $[Ni(CN)_4]^{2-}$

(C) $[Co(CN)_6]^{3-}$

(D) $[Ni(CO)_4]$

Correct Answer: (A) $[Fe(H_2O)_6]^{3+}$

Solution:

Concept:

- The characteristic response tested here is paramagnetism: a species with unpaired electrons is attracted by an external magnetic field. A complex with all electrons paired is diamagnetic.
- Find the metal oxidation state and d-electron count before using ligand strength and geometry to decide whether electrons pair.

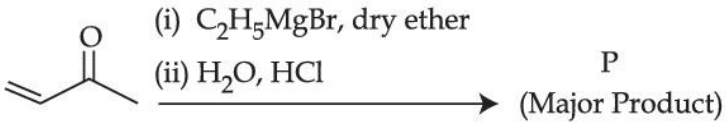
Step 1: Examine the iron aqua complex In $[Fe(H_2O)_6]^{3+}$, water is neutral, so iron is Fe^{3+}, with configuration $3d^5$. Water is a weak-field ligand and the octahedral complex is high spin. Its d-level occupancy is $t_{2g}^3 e_g^2$, with five unpaired electrons. It is therefore strongly paramagnetic and gives the expected magnetic-field response. The spin-only moment is $\sqrt{n(n+2)}\,\mu_B = \sqrt{35}\,\mu_B$ for $n = 5$; the nonzero moment confirms the selection.

Step 2: Rule out the remaining complexes In $[Ni(CN)_4]^{2-}$, nickel is $+2$, hence d^8. Strong-field cyanide produces the square-planar complex with paired electrons, so option (B) is diamagnetic. In $[Co(CN)_6]^{3-}$, cobalt is $+3$, hence d^6; strong-field cyanide gives low-spin t_{2g}^6, again all paired, excluding (C). In neutral $Ni(CO)_4$, CO is neutral and nickel is in oxidation state zero with a d^{10} configuration. All d electrons are paired, excluding (D). Thus only the aqua iron(III) species has unpaired electrons among the listed choices. As usual in this classification, the wording refers to paramagnetic attraction; diamagnetic substances do have a weak opposing induced response, rather than literally no response at all.

Step 3: Conclusion $[Fe(H_2O)_6]^{3+}$ is the paramagnetic species. Hence, the correct option is (A).

Quick Tip to Solve: Apply oxidation state first, d count second, and ligand strength third; a d count alone does not decide whether a complex is paramagnetic.

62. Consider the above reaction, the major product 'P' is : [4 marks]



(A) $CH_2{=}CH{-}C(OH)(CH_3){-}CH_2CH_3$

(B) $CH_3(CH_2)_4{-}C(OH)(CH_3){-}CH_2CH_3$

(C) $CH_3{-}CH(Cl){-}C(OH)(CH_3){-}CH_2CH_3$

(D) $ClCH_2CH_2{-}C(OH)(CH_3){-}CH_2CH_3$

Correct Answer: (A) $CH_2{=}CH{-}C(OH)(CH_3){-}CH_2CH_3$

Solution:

Concept:

- A Grignard reagent $R{-}MgBr$ supplies a strongly nucleophilic carbon group R. Its normal reaction with a ketone is addition to the carbonyl carbon, followed by protonation of the alkoxide.
- With an alpha,beta-unsaturated ketone, the hard Grignard nucleophile favours 1,2-addition. The carbonyl becomes an alcohol while the carbon-carbon double bond remains.

Step 1: Locate the carbonyl and add the ethyl group The starting compound is methyl vinyl ketone, $CH_2{=}CHCOCH_3$. Its carbonyl carbon is bonded to a vinyl group and a methyl group. The reagent C_2H_5MgBr contributes an ethyl group to this carbon. In dry ether the carbonyl pi bond opens to oxygen, giving the magnesium alkoxide

$$CH_2{=}CH{-}C(OMgBr)(CH_3){-}CH_2CH_3.$$

The newly formed carbon-carbon bond joins the ethyl group directly to the former carbonyl carbon. No additional carbon chain is supplied by the ether solvent.

Step 2: Protonate the alkoxide and identify the product The aqueous acid in the second step protonates the alkoxide oxygen to form

$$CH_2{=}CH{-}C(OH)(CH_3){-}CH_2CH_3.$$

This is 3-methylpent-1-en-3-ol, a tertiary allylic alcohol, matching the connectivity in option (A). The product has six carbons, the four of the ketone plus the two supplied by the Grignard reagent. Option (B) contains a longer saturated carbon chain and cannot arise from these reactants. Options (C) and (D) contain chlorine on the carbon skeleton and lack the original double bond. They treat the acid work-up as a separate hydrochlorination reaction, whereas its specified role in this Grignard sequence is protonation of the oxygen anion. Neither chlorinated structure is the expected major product.

Step 3: Conclusion The major product is $CH_2{=}CH{-}C(OH)(CH_3){-}CH_2CH_3$, 3-methylpent-1-en-3-ol. Hence, the correct option is (A).

Quick Tip to Solve: For a Grignard reaction, attach its carbon group at the carbonyl carbon and then replace the alkoxide group by OH during work-up.

63. The water soluble protein is : [4 marks]

(A) Fibrin

(B) Albumin

(C) Myosin

(D) Collagen

Correct Answer: (B) Albumin

Solution:

Concept:

- Globular proteins generally expose polar and charged side chains to water, making many of them water soluble. Albumin is a standard example.
- Fibrous structural proteins have extended, strongly associated chains and are generally water insoluble. Protein shape and biological role help distinguish the listed examples.

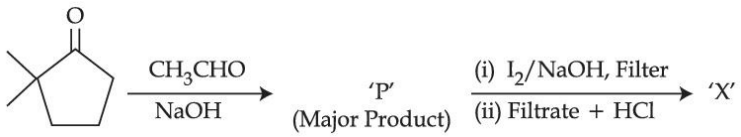
Step 1: Identify the soluble globular protein Albumin is a globular protein found in egg white and blood serum. Its folded structure presents hydrophilic groups to the aqueous surroundings, so water can hydrate the surface and keep the protein dispersed. This makes albumin the water-soluble member of the list. Solubility here refers to the native protein under ordinary aqueous conditions; denaturation or precipitation by a change in conditions is a different process and does not change the standard classification.

Step 2: Compare the other three proteins Fibrin forms the insoluble fibrous network of a blood clot; its role is to form a solid mesh rather than remain dissolved, excluding (A). Myosin is a muscle contractile protein whose filaments and salt-dependent extraction behaviour distinguish it from a readily water-soluble protein, excluding (C). Collagen is a fibrous structural protein of connective tissue, with strongly associated triple-helical units and poor water solubility, excluding (D). Thus the three alternatives are associated with structural or filament-forming arrangements, whereas albumin is the familiar soluble globular protein. Being a protein does not by itself guarantee water solubility: the distribution of polar surface groups and interactions between chains determine the behaviour.

Step 3: Conclusion The water-soluble protein is albumin. Hence, the correct option is (B).

Quick Tip to Solve: Remember albumin as the soluble globular example; collagen, fibrin and muscle filaments are associated with insoluble structural assemblies.

64. Consider the given reaction, the product 'X' is : [4 marks]



(A) 2-(1-Hydroxyethyl)-5,5-dimethylcyclopentan-1-one (the ring ketone bearing a $-CH(OH)CH_3$ group at its other alpha carbon).

(B) 2-Acetyl-5,5-dimethylcyclopentan-1-one (the ring ketone bearing a $-COCH_3$ group at its other alpha carbon).

(C) 2-(1-Hydroxy-2,2-dimethylcyclopentyl)ethanal (the former ring carbonyl carbon bears $-OH$ and $-CH_2CHO$).

(D) 3,3-Dimethyl-2-oxocyclopentane-1-carboxylic acid (the ring ketone bearing a $-COOH$ group at its other alpha carbon).

Correct Answer: (D) 3,3-Dimethyl-2-oxocyclopentane-1-carboxylic acid (the ring ketone bearing a $-COOH$ group at its other alpha carbon).

Solution:

Concept:

- An enolate attacks the electrophilic carbonyl carbon of another molecule in aldol addition. The carbonyl oxygen becomes an alcohol after protonation.
- The iodoform reaction converts a methyl ketone, $RCOCH_3$, into a carboxylate, $RCOO^-$. A secondary alcohol of the form $RCH(OH)CH_3$ first oxidises to that methyl ketone.

Step 1: Identify the carbon skeleton of the aldol product The cyclic ketone has a gem-dimethyl carbon on one side of its carbonyl; that carbon has no alpha hydrogen. Enolate formation therefore takes place on the other side. This enolate adds to the less hindered and more electrophilic carbonyl group of acetaldehyde. The original ring carbonyl remains intact, while the acetaldehyde carbonyl becomes $-CH(OH)CH_3$. Thus the aldol product P has the structure shown in option (A). The newly formed bond joins the ring alpha carbon to the former aldehyde carbonyl carbon.

Step 2: Apply the iodoform reaction and acidify The side chain in P is a methyl secondary alcohol, so iodine in alkali oxidises it to $-COCH_3$, the side chain in option (B). Exhaustive iodination of that methyl group and cleavage then gives the ring-substituted carboxylate and yellow CHI_3. Filtering removes iodoform; adding hydrochloric acid to the filtrate protonates the carboxylate to $-COOH$. The ring ketone is retained because it is not a methyl ketone. Option (A) is only the intermediate P, (B) stops before haloform cleavage, and (C) has the wrong aldol connectivity.

Step 3: Conclusion X is the ring ketone bearing a carboxylic acid group at the other alpha carbon: 3,3-dimethyl-2-oxocyclopentane-1-carboxylic acid. Hence, the correct option is (D).

Quick Tip to Solve: Trace the acetaldehyde carbonyl carbon: after aldol addition it becomes the alcohol carbon, and after the iodoform sequence it becomes the carboxylic acid carbon.

65. The given reaction can occur in the presence of :

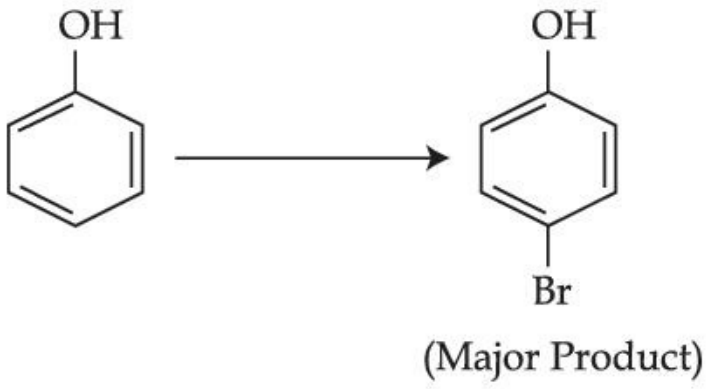
(a) Bromine water

(b) Br_2 in CS_2, 273 K

(c) $Br_2/FeBr_3$

(d) Br_2 in $CHCl_3$, 273 K

Choose the correct answer from the options gien below : [4 marks]



(Major Product)

(A) (b) and (d) only

(B) (a) and (c) only

(C) (b), (c) and (d) only

(D) (a), (b) and (d) only

Correct Answer: (C) (b), (c) and (d) only

Solution:

Concept:

- The hydroxyl group activates the benzene ring by resonance donation and directs electrophilic substitution to the ortho and para positions.
- Bromination conditions determine the extent of substitution: a low-temperature nonpolar medium favours monobromination, whereas bromine water rapidly gives the tribromo derivative.

Step 1: Check bromine water and the nonpolar solvents The displayed product is para-bromophenol, not 2,4,6-tribromophenol. Bromine water, condition (a), causes rapid substitution at both ortho positions and the para position, giving the white precipitate of $2,4,6-C_6H_2Br_3OH$. It therefore does not represent the displayed selective monobromination. In CS_2 or $CHCl_3$ at 273 K, bromination is controlled. The principal monosubstitution product is para-bromophenol, accompanied by a smaller amount of the ortho isomer because para attack is less sterically hindered. Thus (b) and (d) are suitable.

Step 2: Include the Lewis-acid bromination condition Condition (c), $Br_{2/}FeBr_3$, supplies an electrophilic brominating species. The hydroxyl group directs its attack to the ortho and para sites, with the para isomer the major product in the monobromination represented. A Lewis acid is not necessary for the activated phenol ring, but its presence does not exclude the displayed reaction. Therefore the accepted set is (b), (c) and (d). Option (A) unnecessarily excludes (c); options (B) and (D) include bromine water and hence the wrong degree of substitution.

Step 3: Conclusion The suitable conditions are bromine in carbon disulphide at 273 K, bromine with ferric bromide, and bromine in chloroform at 273 K. Hence, the correct option is (C).

Quick Tip to Solve: Before selecting phenol bromination reagents, count the bromines in the product: bromine water points to tribromination, while cold nonpolar conditions point to monobromination.

66. Given below are two statements, one is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) : Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines.

Reason (R) : Aryl halides do not undergo nucleophilic substitution reaction.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) true but (R) is not the correct explanation of (A).

(B) (A) is false but (R) is true.

(C) Both (A) and (R) true and (R) is correct explanation of (A).

(D) (A) is true but (R) is false.

Correct Answer: (C) Both (A) and (R) true and (R) is correct explanation of (A).

Solution:

Concept:

- Gabriel synthesis uses the phthalimide anion as a nitrogen nucleophile; alkylation followed by cleavage gives a primary amine.
- Ordinary aryl halides do not undergo the nucleophilic substitution required for this alkylation under Gabriel synthesis conditions. A direct aryl carbon–nitrogen bond is therefore not installed.

Step 1: Evaluate the assertion Potassium phthalimide reacts with a suitable alkyl halide to form an N-alkylphthalimide. Hydrolysis or hydrazinolysis releases RNH_2, so the method prepares primary aliphatic amines without the overalkylation associated with direct ammonolysis. To prepare an aromatic primary amine such as aniline by the same route, the nucleophile would have to replace halide directly on the aromatic ring. That substitution is not available in the ordinary Gabriel reaction. The assertion is therefore true.

Step 2: Evaluate the reason in the reaction context The carbon–halogen bond in an ordinary aryl halide has partial double-bond character through conjugation and is attached to an sp^2 carbon. It does not undergo the usual displacement by phthalimide anion. Thus the reason is true in the normal reaction conditions meant here. Special activated aromatic substrates or very harsh conditions can permit aromatic nucleophilic substitution, but those are not the Gabriel synthesis under discussion.

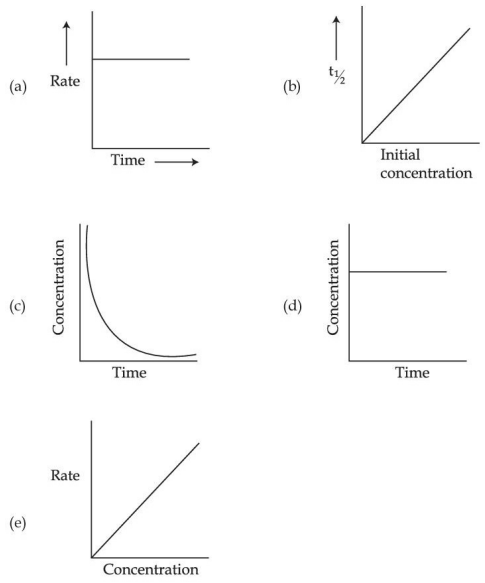
Step 3: Connect the reason to the assertion The failure of aryl halides to undergo the required substitution is exactly why N-arylphthalimide and then an aromatic primary amine are not obtained. Hence the reason explains the assertion. Option (A) incorrectly denies this explanatory connection, option (B) incorrectly calls the assertion false, and option (D) incorrectly calls the contextual reason false.

Step 4: Conclusion The assertion and the reason are both true in Gabriel synthesis conditions, and the reason correctly explains the assertion. Hence, the correct option is (C).

Quick Tip to Solve: For Gabriel questions, test the carbon bearing the leaving group: a suitable alkyl carbon can be displaced, but an ordinary aryl carbon cannot under the synthesis conditions.

67. For the following graphs,

Choose from the options given below, the correct one regarding order of reaction is : [4 marks]



(A) (b) Zero order (c) and (e) First order

(B) (a) and (b) Zero order (e) First order

(C) (b) and (d) Zero order (e) First order

(D) (a) and (b) Zero order (c) and (e) First order

Correct Answer: (B) (a) and (b) Zero order (e) First order

Solution:

Concept:

- For zero-order kinetics, $r = k$, $[A] = [A]_0 - kt$, and $t_{1/2} = [A]_0/(2k)$. The rate is constant while reactant remains.
- For first-order kinetics, $r = k[A]$, $[A] = [A]_0 e^{-kt}$, and $t_{1/2} = \ln 2/k$. The rate is proportional to concentration.

Step 1: Identify the zero-order graphs Graph (a) plots rate against time as a horizontal line. A zero-order reaction has rate k independent of the decreasing reactant concentration, so it has this shape until the reactant is exhausted. Graph (b) shows half-life increasing linearly with initial concentration and passing through the origin. This follows directly from $t_{1/2} = [A]_0/(2k)$, so (b) also represents zero order. Graph (d), by contrast, shows concentration remaining constant as time passes; a reacting zero-order system must show a straight-line decrease in concentration, not a horizontal concentration

line.

Step 2: Identify the first-order rate graph Graph (e) is a straight line through the origin for rate versus concentration. Its slope is k and it expresses $r = k[A]$, the diagnostic first-order rate law. Graph (c) is drawn as a reciprocal-type concentration decay with a very steep initial branch; it is not the finite-intercept exponential first-order decay intended by the options. In any event, a falling curved concentration plot alone is not sufficient to establish first order without its functional relation. The definite first-order classification supplied here is (e).

Step 3: Select the matching combination Combining the unambiguous relations gives (a) and (b) as zero order and (e) as first order. Option (A) omits the valid graph (a); option (C) wrongly assigns the constant-concentration graph (d) to zero order; option (D) additionally assigns graph (c) to first order. Thus the appropriate listed combination is option (B).

Step 4: Conclusion The horizontal rate plot and the linear half-life versus initial-concentration plot represent zero order; the linear rate versus concentration plot represents first order. Hence, the correct option is (B).

Quick Tip to Solve: Use axis labels before curve shape: constant rate is zero order, half-life proportional to initial concentration is zero order, and rate proportional to concentration is first order.

68. Which one of the products of the following reactions does not react with Hinsberg reagent to form sulphonamide? [4 marks]

(A) 4-Nitrobenzonitrile treated with Na/Hg in C_2H_5OH.

(B) Benzonitrile treated with $SnCl_2$ and HCl.

(C) 4-Formylbenzonitrile treated with $LiAlH_4$, followed by H_3O^+.

(D) 3-Methylbenzonitrile treated with H_2/Ni.

Correct Answer: (B) Benzonitrile treated with $SnCl_2$ and HCl.

Solution:

Concept:

- Hinsberg reagent is benzenesulphonyl chloride, $C_6H_5SO_2Cl$. Primary and secondary amines have an N–H bond and form sulphonamides; a product without an amino group does not.
- Complete reduction of a nitrile gives a primary amine, $RCN \longrightarrow RCH_2NH_2$. Stephen reduction with $SnCl_2/HCl$ instead stops at the iminium salt, which gives an aldehyde on hydrolysis.

Step 1: Examine the product of option (B) Benzonitrile with $SnCl_2/HCl$ undergoes Stephen reduction. The immediate product is an iminium hydrochloride, conventionally written $C_6H_5CH{=}NH \cdot HCl$, which on hydrolysis gives benzaldehyde, C_6H_5CHO. This is not a primary or secondary amine and does not give the normal Hinsberg sulphonamide. In particular, the reagent pair must not be confused with a complete nitrile-to-benzylamine reduction.

Step 2: Rule out the three complete reductions In option (A), sodium amalgam in ethanol reduces the nitrile group to an aminomethyl group, $-CH_2NH_2$. Whatever further reduction of the nitro substituent occurs, the product contains a primary amino group and can form a sulphonamide. In option (C),

$LiAlH_4$ reduces the nitrile to $-CH_2NH_2$ and the formyl group to $-CH_2OH$; acidic work-up releases the amino alcohol. Its primary amino group reacts with Hinsberg reagent. In option (D), catalytic hydrogenation with H_2/Ni converts the nitrile into the corresponding methyl-substituted benzylamine, again a primary amine. These three products therefore cannot be the requested exception.

Step 3: Conclusion The product from benzonitrile with $SnCl_2/HCl$ does not form the Hinsberg sulphonamide. Hence, the correct option is (B).

Quick Tip to Solve: For nitrile questions, distinguish Stephen reduction, which leads to an aldehyde, from complete reduction by hydride or hydrogen, which leads to a primary amine.

69. The ionic radii of K^+, Na^+, Al^{3+} and Mg^{2+} are in the order: [4 marks]

(A) $Na^+ < K^+ < Mg^{2+} < Al^{3+}$

(B) $Al^{3+} < Mg^{2+} < K^+ < Na^+$

(C) $Al^{3+} < Mg^{2+} < Na^+ < K^+$

(D) $K^+ < Al^{3+} < Mg^{2+} < Na^+$

Correct Answer: (C) $Al^{3+} < Mg^{2+} < Na^+ < K^+$

Solution:

Concept:

- Isoelectronic ions have the same number of electrons. Within such a series, a larger nuclear charge pulls the same electron cloud inward more strongly and produces a smaller radius.
- An additional occupied principal shell generally increases ionic size. Compare shell counts first, then compare nuclear charge for ions having the same electron configuration.

Step 1: Order the three ten-electron ions Sodium has atomic number 11, magnesium 12 and aluminium 13. Their cations Na^+, Mg^{2+} and Al^{3+} each have 10 electrons with the neon configuration, $1s^2 2s^2 2p^6$. Their nuclear charges are respectively +11, +12 and +13. The strongest attraction is therefore in Al^{3+}, and the weakest is in Na^+. Consequently,

$$r(Al^{3+}) < r(Mg^{2+}) < r(Na^+).$$

The larger positive ionic charge here is accompanied by a larger nuclear charge for the same electron count.

Step 2: Place the potassium ion and compare all options Potassium has atomic number 19. After losing one electron, K^+ has the argon configuration, with electrons occupying the third shell. The other three ions have only the first two shells occupied. The extra principal shell makes K^+ larger than Na^+. Combining both comparisons gives aluminium, magnesium, sodium, potassium in increasing radius. Option (A) reverses the neon-like cation trend; option (B) incorrectly puts sodium above potassium; option (D) incorrectly makes potassium the smallest. Only option (C) respects both nuclear attraction and occupied-shell count.

Step 3: Conclusion The increasing order of ionic radii is

$$Al^{3+} < Mg^{2+} < Na^{+} < K^{+}.$$

Hence, the correct option is (C).

Quick Tip to Solve: Count electrons first: the three neon-like ions shrink as nuclear charge increases, while argon-like potassium has an extra occupied shell and is the largest.

70. Which one of the following compounds of Group-14 elements is not known? [4 marks]

(A) $[GeCl_6]^{2-}$

(B) $[Sn(OH)_6]^{2-}$

(C) $[SiCl_6]^{2-}$

(D) $[SiF_6]^{2-}$

Correct Answer: (C) $[SiCl_6]^{2-}$

Solution:

Concept:

- The stability of a six-coordinate complex depends on the size of the central atom as well as ligand size and bonding; membership of a group alone does not guarantee every analogous complex.
- Silicon can form the hexafluorosilicate ion, $[SiF_6]^{2-}$, because fluoride is small and forms strong silicon–fluorine bonds. Six bulkier chlorides are not accommodated in the corresponding familiar silicon complex.

Step 1: Compare the two silicon complexes Both proposed silicon anions would have silicon in oxidation state +4: six monovalent halide ligands and an overall charge of -2 require $x - 6 = -2$, hence $x = +4$. Their distinction therefore cannot be decided by oxidation state. Fluoride is much smaller than chloride and its strong bonding to silicon stabilises the known octahedral $[SiF_6]^{2-}$ ion. The analogous $[SiCl_6]^{2-}$ ion is not a known standard stable species in this Group-14 comparison; packing six larger chlorides around the smaller silicon centre is unfavourable.

Step 2: Check the heavier Group-14 centres Germanium and tin are larger than silicon and can support the relevant higher coordination numbers. Hexachlorogermanate, $[GeCl_6]^{2-}$, is known, so option (A) is excluded. Hexahydroxostannate(IV), $[Sn(OH)_6]^{2-}$, is also known, notably in alkaline chemistry of amphoteric tin compounds, so option (B) is excluded. Option (D), hexafluorosilicate, is a familiar known ion and is excluded as well. Thus only the proposed hexachlorosilicate ion satisfies the question. No inference that silicon can never be six-coordinate is needed or correct.

Step 3: Conclusion The compound not known in the listed comparison is $[SiCl_6]^{2-}$. Hence, the correct option is (C).

Quick Tip to Solve: Contrast ligand sizes within the same central element: known six-coordinate silicon fluoride does not imply a stable six-coordinate silicon chloride; heavier centres accommodate larger ligands more readily.

71. Which one among the following resonating structures is not correct? [4 marks]

(A) $CH_2{=}CH{-}CH^{-}{-}CH^{+}{-}N^{+}(=O){-}O^{-}$

(B) $CH_2{=}CH{-}CH^{+}{-}CH^{-}{-}N^{+}(=O){-}O^{-}$

(C) $^{+}CH_2{-}CH{=}CH{-}CH{=}N^{+}(-O^{-}){-}O^{-}$

(D) $^{+}CH_2{-}CH^{-}{-}CH{=}CH{-}N^{+}(=O){-}O^{-}$

Correct Answer: (A) $CH_2{=}CH{-}CH^{-}{-}CH^{+}{-}N^{+}(=O){-}O^{-}$

Solution:

Concept:

- Resonance contributors retain the atomic skeleton and move only electrons in conjugated bonds or lone pairs. The total charge remains the same.
- The nitro group withdraws electrons by resonance. Its nitrogen already bears a positive formal charge; favourable charge-separated contributors place negative charge on oxygen or on the adjacent carbon rather than positive charge beside this nitrogen.

Step 1: Inspect the nitro nitrogen and adjacent carbon Number the chain from its terminal CH_2 carbon towards nitrogen. In option (A), carbon 3 bears a negative charge and carbon 4 a positive charge, while the directly attached nitro nitrogen is also positive. Thus the structure places two positive centres directly next to one another, at carbon 4 and nitrogen. This is the incorrect charge disposition among the proposed conventional resonance forms: it opposes the electron-withdrawing resonance effect of the nitro group and gives the highly unfavourable adjacent positive centres.

Step 2: Compare the other charge distributions In (B), the carbon next to nitrogen is negative and the preceding carbon is positive, so the charge separation has the opposite orientation. The adjacent carbon can donate its electron pair towards nitrogen while the N=O electrons move to oxygen. In (C), this delocalisation is represented by a carbon–nitrogen double bond, two singly bonded negatively charged oxygen atoms, and positive charge at the remote terminal carbon. In (D), the charge separation is at the other end of the conjugated chain, with a positive terminal carbon and a negative neighbouring carbon. None of (B), (C) or (D) places a positively charged carbon directly beside the positively charged nitro nitrogen as (A) does.

Step 3: Conclusion The unacceptable contributor is the one with adjacent positively charged carbon and nitro nitrogen. Hence, the correct option is (A).

Quick Tip to Solve: For a nitro-conjugated chain, mark the positive nitro nitrogen first, then check the orientation of every proposed charge-separated contributor.

72. The correct order of following 3d metal oxides, according to their oxidation numbers is :

(a) CrO_3, (b) Fe_2O_3, (c) MnO_2, (d) V_2O_5, (e) Cu_2O [4 marks]

(A) (d) > (a) > (b) > (c) > (e)

(B) (a) > (c) > (d) > (b) > (e)

(C) (a) > (d) > (c) > (b) > (e)

(D) (c) > (a) > (d) > (e) > (b)

Correct Answer: (C) (a) > (d) > (c) > (b) > (e)

Solution:

Concept:

- The algebraic sum of oxidation numbers in a neutral compound is zero. Oxygen has oxidation number -2 in each oxide listed here.
- Calculate the oxidation number per metal atom, not the sum for all metal atoms in the formula. Arrange the resulting values in decreasing order.

Step 1: Calculate the five metal oxidation numbers For (a), CrO_3, let chromium have oxidation number x. Then $x + 3(-2) = 0$, giving $x = +6$. For (b), Fe_2O_3, $2x + 3(-2) = 0$, so each iron atom is $+3$. For (c), MnO_2, $x + 2(-2) = 0$, so manganese is $+4$. For (d), V_2O_5, $2x + 5(-2) = 0$, giving vanadium $+5$. Finally, for (e), Cu_2O, $2x - 2 = 0$, so each copper atom is $+1$.

Step 2: Sort the values and test the options The values are therefore $(a) : +6$, $(b) : +3$, $(c) : +4$, $(d) : +5$ and $(e) : +1$. In decreasing order,

$$+6 > +5 > +4 > +3 > +1,$$

hence

$$(a) > (d) > (c) > (b) > (e).$$

Option (A) incorrectly puts vanadium(V) above chromium(VI) and iron(III) above manganese(IV). Option (B) reverses vanadium(V) and manganese(IV). Option (D) puts manganese(IV) above chromium(VI), and also copper(I) above iron(III). Only option (C) respects all five calculated oxidation numbers.

Step 3: Conclusion The decreasing metal oxidation states are chromium(VI), vanadium(V), manganese(IV), iron(III) and copper(I). Hence, the correct option is (C).

Quick Tip to Solve: In a metal oxide, use total oxygen charge first and divide the required positive charge by the number of metal atoms before ordering them.

73. Which one of the following compounds will liberate CO_2, when treated with $NaHCO_3$? [4 marks]

(A) $(CH_3)_3NH^+Cl^-$

(B) $(CH_3)_4N^+OH^-$

(C) $CH_3C(=O)NH_2$

(D) CH_3NH_2

Correct Answer: (A) $(CH_3)_3NH^+Cl^-$

Solution:

Concept:

- An acidic proton can react with hydrogen carbonate to form carbonic acid, which decomposes into water and carbon dioxide; evolution of the gas favours its removal.
- An alkylammonium salt contains an acidic N–H proton. A quaternary ammonium hydroxide and an alkylamine are basic, while the amide shown is not the acidic salt tested by bicarbonate.

Step 1: Identify the acidic option Option (A) is trimethylammonium chloride, $(CH_3)_3NH^+Cl^-$, the hydrochloride salt of trimethylamine. Its positively charged nitrogen carries a hydrogen that can be transferred to hydrogen carbonate, releasing the neutral amine. The acid-base step can be written

$$(CH_3)_3NH^+ + HCO_3^- \rightleftharpoons (CH_3)_3N + H_2CO_3.$$

Carbonic acid is connected to dissolved carbon dioxide and water, and removal of carbon dioxide from the reaction mixture drives the gas-evolving reaction. This acidic ammonium salt is therefore the appropriate choice among the four compounds.

Step 2: Write the net reaction and reject the alternatives The overall gas-evolution reaction is

$$(CH_3)_3NHCl + NaHCO_3 \longrightarrow (CH_3)_3N + NaCl + H_2O + CO_2\uparrow.$$

In option (B), tetramethylammonium hydroxide contains hydroxide and no nitrogen-bound hydrogen; it is basic rather than an acidic ammonium salt. Option (C), acetamide CH_3CONH_2, is a neutral amide and does not give the bicarbonate effervescence test. Option (D), methylamine CH_3NH_2, is a base; it accepts a proton rather than acting as the acid needed for this gas-evolution reaction. Thus the presence of nitrogen alone does not decide the test: the ionic ammonium salt and its proton are decisive.

Step 3: Conclusion Trimethylammonium chloride is the compound that liberates CO_2 on treatment with $NaHCO_3$ among the choices. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish an amine from its hydrochloride salt: the latter has a positively charged nitrogen with an acidic hydrogen, unlike the free amine.

74. An organic compound 'A' C_4H_8 on treatment with $KMnO_4/H^+$ yields compound 'B' C_3H_6O. Compound 'A' also yields compound 'B' an ozonolysis. Compound 'A' is : [4 marks]

(A) 2-Methylpropene

(B) 1-Methylcyclopropane

(C) But-2-ene

(D) Cyclobutane

Correct Answer: (A) 2-Methylpropene

Solution:

Concept:

- Ozonolysis cleaves a carbon–carbon double bond and converts its two alkene carbons into carbonyl groups.
- An alkene carbon bonded to two carbon substituents gives a ketone on cleavage. A terminal CH_2 carbon gives methanal in reductive ozonolysis and is oxidised further by hot acidic permanganate.

Step 1: Infer the carbonyl fragment from its formula The product B has formula C_3H_6O. A three-carbon ketone is propanone, CH_3COCH_3. To form this ketone by cleavage, one carbon of the original double bond must already be bonded to two methyl groups. Because the starting alkene has only four carbons in all, its other alkene carbon must be terminal CH_2. This reconstructs the required alkene as $CH_2=C(CH_3)_2$, which is 2-methylpropene.

Step 2: Verify both reactions and exclude the other structures Ozonolysis of $CH_2=C(CH_3)_2$, followed by the usual reductive work-up, gives propanone and methanal:

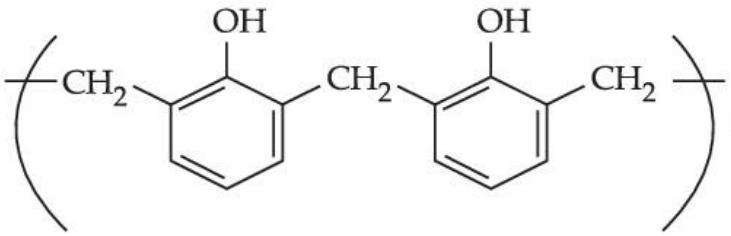
$$CH_2=C(CH_3)_2 \xrightarrow[Zn/H_2O]{O_3} (CH_3)_2CO + HCHO.$$

Acidic permanganate cleavage also gives propanone from the substituted alkene carbon, while the terminal carbon is oxidised to CO_2. Thus both reactions contain the same C_3H_6O product. But-2-ene instead gives two ethanal molecules on reductive ozonolysis and ethanoic acid on strong oxidation, so it does not produce the required three-carbon ketone. 1-Methylcyclopropane and cyclobutane have no carbon–carbon double bond and do not undergo the stated ordinary alkene ozonolysis sequence. Only the branched terminal alkene has the required carbon arrangement.

Step 3: Conclusion Compound A is 2-methylpropene and compound B is propanone. Hence, the correct option is (A).

Quick Tip to Solve: To reconstruct an alkene from a ketone cleavage product, replace the carbonyl oxygen by the missing double-bonded carbon and preserve its two carbon substituents.

75. is a repeating unit for : [4 marks]



(A) Novolac

(B) Buna-N

(C) Acrilan

(D) Neoprene

Correct Answer: (A) Novolac

Solution:

Concept:

- Novolac is a phenol-formaldehyde condensation polymer with phenolic aromatic rings connected by methylene bridges.
- A repeat unit identifies its monomers through the groups retained in the backbone: phenolic OH and aromatic rings indicate phenol, while $-CH_2-$ links arise from formaldehyde.

Step 1: Read the functional groups in the repeating segment The displayed segment contains benzene rings, each carrying a phenolic hydroxyl group, with $-CH_2-$ bridges joining successive rings. The hydroxyl groups remain attached directly to aromatic carbons; they are not pendant alcohol groups on an aliphatic addition-polymer chain. This is the characteristic phenol-formaldehyde framework formed when phenol condenses with formaldehyde at reactive ortho and para positions. The linear resin obtained with phenol in excess under acidic conditions is novolac.

Step 2: Match the framework to the named polymers Novolac therefore matches both diagnostic features: phenolic rings and methylene bridges. Buna-N is a copolymer of 1,3-butadiene and acrylonitrile; it has an aliphatic carbon chain with nitrile-containing units and unsaturation, rather than hydroxyl-substituted benzene rings. Acrilan is polyacrylonitrile, with repeating $-CH_2-CH(CN)-$ units, again lacking the aromatic phenolic framework. Neoprene comes from chloroprene and contains chlorine-substituted diene-derived units, not phenol-formaldehyde links. Although novolac can be further converted to a cross-linked phenolic resin, the linear methylene-linked unit printed here is identified as novolac among these choices.

Step 3: Conclusion The displayed repeating unit belongs to novolac. Hence, the correct option is (A).

Quick Tip to Solve: Look for phenolic rings joined by methylene bridges to recognise phenol-formaldehyde resins; nitrile or chlorine groups point to the alternative addition polymers.

76. Sodium stearate $CH_3(CH_2)_{16}COO^-Na^+$ is an anionic surfactant which forms micelles in oil. Choose the correct statement for it from the following : [4 marks]

(A) It forms spherical micelles with $CH_3(CH_2)_{16}$ - group pointing towards the centre of sphere.

(B) It forms non-spherical micelles with -COO^- group pointing outwards on the surface.

(C) It forms spherical micelles with $CH_3(CH_2)_{16}$ - group pointing outwards on the surface of sphere

(D) It forms non-spherical micelles with $CH_3(CH_2)_{16}$ - group pointing towards the centre.

Correct Answer: (C) It forms spherical micelles with $CH_3(CH_2)_{16}$ - group pointing outwards on the surface of sphere

Solution:

Concept:

- A surfactant has a polar ionic head and a nonpolar hydrocarbon tail. Its orientation depends on which part is compatible with the surrounding solvent.
- In oil, a nonpolar medium, the hydrocarbon tails face the solvent and ionic heads cluster inside. Such aggregates are called reverse micelles.

Step 1: Use the specified solvent to orient the molecules Sodium stearate has a long nonpolar $CH_3(CH_2)_{16}{}^-$ tail and a charged $-COO-$ head, with Na^+ as counterion. The question explicitly places the micelles in oil, not in water. Hydrocarbon tails mix favourably with the nonpolar oil, whereas exposed ionic heads are energetically unfavourable in that medium. The molecules therefore arrange their carboxylate heads towards the interior and their hydrocarbon chains towards the exterior, in contact with oil.

Step 2: Choose the matching statement and address the key For the spherical micelle described in the options, the $CH_3(CH_2)_{16}{}^-$ groups consequently point outwards on its surface, which is option (C). Option (A) puts the tails inward; that is the familiar orientation for ordinary micelles in water, and contradicts the oil medium printed in the question. Options (B) and (D) propose non-spherical aggregates and do not state the required outward hydrocarbon-tail arrangement; (B) also exposes the charged head group. The question-sheet key gives (A), but its orientation would apply to water. With the literal printed word “oil, the reverse-micelle orientation is (C), because nonpolar tails must face the nonpolar continuous phase.

Step 3: Conclusion In oil, the hydrocarbon tails point outwards and the ionic heads point inwards. This proves option (C) for the printed question, despite the question-sheet key selecting the inward-tail arrangement. Hence, the correct option is (C).

Quick Tip to Solve: Before selecting a micelle orientation, underline the solvent: water exposes ionic heads, whereas oil exposes hydrocarbon tails.

77. At 298.2 K the relationship between enthalpy of bond dissociation (in kJ mol^{-1}) for hydrogen (E_H) and its isotope, deuterium (E_D), is best described by : [4 marks]

(A) $E_H = \frac{1}{2}E_D$

(B) $E_H = E_D$

(C) $E_H \simeq E_D - 7.5$

(D) $E_H = 2E_D$

Correct Answer: (C) $E_H \simeq E_D - 7.5$

Solution:

Concept:

- Isotopic substitution barely changes the electronic potential-energy curve, but it changes vibrational energy through the reduced mass.
- The vibrational frequency follows $\nu \propto 1/\sqrt{\mu}$, and the zero-point energy is $h\nu/2$. Deuterium has a lower zero-point energy than hydrogen and therefore a larger dissociation energy measured from its occupied vibrational level.

Step 1: Compare the vibrational ground states For a homonuclear diatomic molecule, the reduced mass is half the mass of one atom. Since deuterium is about twice as massive as hydrogen, $\mu_{D_2} \simeq 2\mu_{H_2}$, giving $\nu_{D_2} \simeq \nu_{H_2}/\sqrt{2}$. Thus the vibrational ground state of D_2 lies lower above the minimum of the shared electronic potential well. More energy is needed to separate D_2 from that ground state into atoms, so $E_D > E_H$.

Step 2: Choose the small difference rather than a large ratio The known bond dissociation enthalpies near room temperature are approximately $436\,\text{kJ}\,\text{mol}^{-1}$ for H_2 and $443.5\,\text{kJ}\,\text{mol}^{-1}$ for D_2. Their difference is therefore about $7.5\,\text{kJ}\,\text{mol}^{-1}$, with deuterium higher. Accordingly,

$$E_H \simeq E_D - 7.5 \quad (\text{kJ}\,\text{mol}^{-1}).$$

Option (B) ignores the isotope-dependent vibrational contribution. Options (A) and (D) incorrectly multiply the entire bond dissociation enthalpy by the isotopic mass ratio; mass affects the comparatively small vibrational correction, not the electronic binding energy by a factor of two.

Step 3: Conclusion The relationship is $E_H \simeq E_D - 7.5$, with the enthalpies in $\text{kJ}\,\text{mol}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: For isotope bond energies, first compare zero-point energies: the heavier isotope vibrates more slowly and usually has a slightly larger dissociation enthalpy.

78. Given below are two statements :

Statement I : None of the alkaline earth metal hydroxides dissolve in alkali.

Statement II : Solubility of alkaline earth metal hydroxides in water increases down the group.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is correct but Statement II is incorrect.

(B) Statement I is incorrect but Statement II is correct.

(C) Statement I and Statement II both are incorrect.

(D) Statement I and Statement II both are correct.

Correct Answer: (B) Statement I is incorrect but Statement II is correct.

Solution:

Concept:

- The hydroxides of Group 2 metals become more basic down the group. Beryllium hydroxide is exceptional because it is amphoteric: it reacts with both acids and strong alkalis.
- Solubility depends on the balance between lattice dissociation and hydration. Down Group 2, the fall in lattice enthalpy favours dissolution of the hydroxides sufficiently to give increasing water solubility.

Step 1: Test the universal claim in Statement I The word "none" asserts that every alkaline earth hydroxide is insoluble in alkali. A single counterexample is sufficient to reject it. Amphoteric beryllium hydroxide dissolves in excess strong alkali by forming a soluble hydroxo complex:

$$\mathrm{Be(OH)_2 + 2\,OH^- \longrightarrow [Be(OH)_4]^{2-}}.$$

The four hydroxide ligands surround beryllium in tetrahydroxoberyllate(II). Thus, although the heavier members are predominantly basic hydroxides, the Group 2 family does contain a hydroxide that reacts with alkali. Statement I is incorrect.

Step 2: Check the solubility trend in Statement II The hydroxide solubility generally increases from the very sparingly soluble lighter members to the more soluble heavier members. In particular, $Mg(OH)_2$ is sparingly soluble, whereas calcium, strontium and barium hydroxides become successively more soluble. The increasing cation size weakens the lattice attraction. The overall energetic balance for dissolving the hydroxides therefore becomes more favourable down the group. Statement II is correct.

Step 3: Match the two judgements to the choices We need the combination "I incorrect, II correct". Option (B) states exactly that. Option (A) reverses both judgements; option (C) incorrectly rejects the increasing-solubility trend; option (D) incorrectly accepts the universal claim despite amphoteric $Be(OH)_2$.

Step 4: Conclusion Statement I is incorrect, but Statement II is correct. Hence, the correct option is (B).

Quick Tip to Solve: For a universal claim about Group 2 hydroxides, test amphoteric beryllium hydroxide before applying the trend for the heavier members.

79. Which one of the following chemical agent is not being used for dry-cleaning of clothes? [4 marks]

(A) H_2O_2

(B) CCl_4

(C) Liquid CO_2

(D) $Cl_2C = CCl_2$

Correct Answer: (A) H_2O_2

Solution:

Concept:

- Dry-cleaning removes grease and other nonpolar stains using a suitable cleaning medium rather than an ordinary aqueous wash. Conventional chlorinated organic solvents dissolve oily material.
- Hydrogen peroxide, H_2O_2, is an oxidising and bleaching agent. Its role in textile treatment must be distinguished from that of a solvent used for dry-cleaning.

Step 1: Identify the conventional dry-cleaning solvents Carbon tetrachloride, CCl_4, was used historically as a solvent for removing grease from fabrics. Tetrachloroethene, also called perchloroethylene, has the structure $Cl_2C{=}CCl_2$; it is a familiar dry-cleaning solvent. Both are chlorinated organic liquids capable of dissolving oily dirt. Their historical use does not make them environmentally desirable, but it does make options (B) and (D) unsuitable answers to a question asking which agent is not used.

Step 2: Recognise the alternative cleaning medium Liquid carbon dioxide can replace conventional chlorinated solvents in dry-cleaning applications. Under suitable pressure it acts as the cleaning medium and can be recovered after use. This is the green-chemistry alternative listed in option (C). Therefore liquid CO_2 is also not the requested exception.

Step 3: Separate bleaching from solvent cleaning Hydrogen peroxide removes coloured stains through oxidation and is used as a bleaching agent. Such chemical bleaching is different from dissolving grease in a dry-cleaning solvent. Among the four listed substances, H_2O_2 is the agent that does not belong to the dry-cleaning solvent group. Do not choose liquid carbon dioxide merely because it is a gas at ordinary atmospheric pressure: the option explicitly specifies its liquid form.

Step 4: Conclusion Hydrogen peroxide, H_2O_2, is not the dry-cleaning agent among these choices. Hence, the correct option is (A).

Quick Tip to Solve: Classify each choice by its role: chlorinated solvents and liquid carbon dioxide dissolve stains, whereas hydrogen peroxide acts primarily by oxidative bleaching.

80. In the leaching of alumina from bauxite, the ore expected to leach out in the process by reacting with NaOH is : [4 marks]

(A) TiO_2

(B) Fe_2O_3

(C) ZnO

(D) SiO_2

Correct Answer: (D) SiO_2

Solution:

Concept:

- In the Bayer process, hot concentrated NaOH dissolves alumina from bauxite as soluble aluminate. Selective leaching separates soluble products from insoluble residue.
- Bauxite also contains impurities such as silica, ferric oxide and titanium dioxide. Silica is an acidic oxide and can react with alkali to produce soluble sodium silicate.

Step 1: Write the desired dissolution of alumina The aluminium-containing part of bauxite reacts with hot sodium hydroxide. In the hydrated representation, the leaching reaction can be written as

$$Al_2O_3 + 2\,NaOH + 3\,H_2O \longrightarrow 2\,Na[Al(OH)_4].$$

The soluble aluminate enters the leach liquor. The question asks which listed material also reacts with sodium hydroxide in this bauxite treatment, so we must identify the impurity that accompanies alumina into solution.

Step 2: Identify the reacting impurity Silica reacts with the alkali to form sodium silicate:

$$SiO_2 + 2\,NaOH \longrightarrow Na_2SiO_3 + H_2O.$$

Consequently, silica can be leached along with alumina. This follows its acidic-oxide behaviour; it is not necessary to suppose that every oxide in the ore dissolves merely because sodium hydroxide is concentrated.

Step 3: Eliminate the remaining choices in the bauxite context Ferric oxide, Fe_2O_3, and titanium dioxide, TiO_2, largely remain in the insoluble residue during the standard bauxite-leaching treatment; they are not the co-leached impurity sought here. Zinc oxide is amphoteric and can react with strong alkali in a separate chemical setting, but it is not the characteristic impurity being removed from bauxite in the stated Bayer-process question. The relevant listed bauxite impurity is therefore SiO_2, not ZnO.

Step 4: Conclusion Silica, SiO_2, reacts with NaOH and enters the leach liquor as sodium silicate. Hence, the correct option is (D).

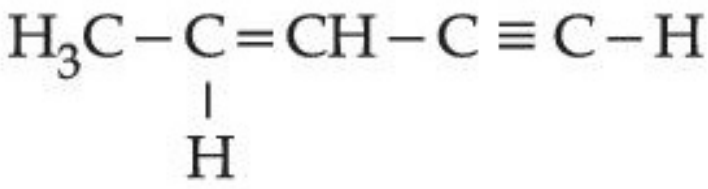
Quick Tip to Solve: In a Bayer-process question, recall the actual bauxite impurities: silica can enter the alkali solution, while iron and titanium oxides form the residue.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The number of sigma bonds in

is _____. [4 marks]



Solution:

Concept:

- Every bonded pair of atoms has one sigma bond. A double bond consists of one sigma and one pi bond, whereas a triple bond consists of one sigma and two pi bonds.
- The displayed open-chain structure is $CH_3-CH=CH-C\equiv CH$. Count the carbon skeleton and the carbon–hydrogen bonds separately to avoid counting multiple bonds more than once.

Step 1: Count the carbon–carbon sigma bonds Label the five carbons from the methyl end as C_1, C_2, C_3, C_4, C_5. The connected pairs are $C_1 - C_2$, $C_2 = C_3$, $C_3 - C_4$ and $C_4 \equiv C_5$. There are four carbon–carbon connections, so there are four carbon–carbon sigma bonds. The double bond contributes only one to this count, and the triple bond also contributes only one; their extra bonds are pi bonds.

Step 2: Count the carbon–hydrogen sigma bonds The left terminal carbon is a methyl group and contributes three C–H sigma bonds. Each alkene carbon has one hydrogen, giving two more. The carbon next to the terminal alkyne carbon already has one single and one triple carbon–carbon bond, so it has no hydrogen. The right terminal alkyne carbon has one hydrogen. Thus

$$N_{\mathrm{C-H},\sigma} = 3 + 1 + 1 + 0 + 1 = 6.$$

Every carbon–hydrogen bond is a sigma bond.

Step 3: Add the two contributions and check the valencies Combining the carbon framework with the hydrogen bonds gives

$$N_\sigma = N_{\mathrm{C-C},\sigma} + N_{\mathrm{C-H},\sigma} = 4 + 6 = 10.$$

As a check, the molecular formula is C_5H_6; the one double and one triple bond supply three pi bonds in total. Counting those pi bonds as additional sigma bonds would incorrectly overcount the carbon skeleton. All five carbons satisfy valency four in the displayed structure.

Step 4: Conclusion The given molecule contains $\boxed{10}$ sigma bonds. The requested number is 10 (a dimensionless bond count).

Quick Tip to Solve: For an acyclic hydrocarbon, first count one sigma bond per carbon connection, then add one for every hydrogen; multiple bonds never add extra sigma bonds.

82. Three moles of AgCl get precipitated when one mole of an octahedral co-ordination compound with empirical formula $CrCl_3.3NH_3.3H_2O$ reacts with excess of silver nitrate. The number of chloride ions satisfying the secondary valency of the metal ion is _____. [4 marks]

Solution:

Concept:

- Primary valency corresponds to oxidation state and can be satisfied by counterions. Secondary valency corresponds to coordination number and is satisfied by ligands inside the coordination sphere.
- On adding excess $AgNO_3$, free chloride counterions precipitate as AgCl. Chloride coordinated directly to the metal is not counted as freely ionisable chloride in this test.

Step 1: Use the precipitation stoichiometry Each mole of free chloride gives one mole of silver chloride:

$$Ag^+ + Cl^- \longrightarrow AgCl(s).$$

One mole of the compound gives three moles of AgCl. Therefore each formula unit releases three chloride ions as counterions. The empirical formula contains only three chlorides in total, so all three have already been accounted for outside the coordination sphere.

Step 2: Build the octahedral coordination sphere An octahedral chromium complex has coordination number six. The formula supplies three ammonia molecules and three water molecules, all of which can act as neutral monodentate ligands. Together they provide the required six donor atoms without using any chloride ligand. The formula is consequently

$$[Cr(NH_3)_3(H_2O)_3]Cl_3.$$

The complex cation has charge $+3$, because its ligands are neutral and chromium is in the $+3$ oxidation state.

Step 3: Distinguish the chloride roles The three external Cl^- counterions balance the complex charge and satisfy the primary valency. The secondary valency is satisfied by three NH_3 ligands and three H_2O ligands. Hence the number of chlorides bound directly to chromium is

$$N_{\text{coordinated chloride}} = 3 - 3 = 0.$$

This result also checks the silver nitrate observation: placing even one chloride inside the coordination sphere would leave fewer than three freely precipitable chlorides per formula unit.

Step 4: Conclusion No chloride ion satisfies the secondary valency. The required number is $\boxed{0}$ (a dimensionless ion count).

Quick Tip to Solve: Subtract the number of precipitable chlorides from the total chloride count before filling the metal's coordination sphere.

83. A source of monochromatic radiation of wavelength 400 nm provides 1000 J of energy in 10 seconds. When this radiation falls on the surface of sodium, x $\times$ 10^{20} electrons are ejected per second. Assume that wavelength 400 nm is sufficient for ejection of electron from the surface of sodium metal. The value of x is _____. (Nearest integer)

(h = 6.626 $\times$ 10^{-34} Js) [4 marks]

Solution:

Concept:

- The energy of one photon of wavelength λ is $E_\gamma = hc/\lambda$, where h is Planck's constant and $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$.
- In the ideal photoelectric counting model, one absorbed photon above threshold ejects one electron. Thus the emitted-electron rate equals the incident-photon rate used in the question.

Step 1: Convert the energy supplied into power The source supplies $1000\,\mathrm{J}$ in $10\,\mathrm{s}$, so its energy delivered per second is

$$P = \frac{1000}{10} = 100\,\mathrm{J\,s^{-1}}.$$

We use this rate rather than the total energy because the question asks for electrons ejected per second. The wavelength in SI units is

$$\lambda = 400\,\mathrm{nm} = 4.00 \times 10^{-7}\,\mathrm{m}.$$

Step 2: Calculate the photon energy Substitute the printed value of Planck's constant:

$$E_\gamma = \frac{(6.626 \times 10^{-34})(3.0 \times 10^8)}{4.00 \times 10^{-7}} = 4.9695 \times 10^{-19}\,\mathrm{J}.$$

The wavelength is stated to be sufficient for emission, so no threshold-frequency calculation is required. The work function would affect the kinetic energy of an emitted electron, whereas here we need the number of incident photons.

Step 3: Find the electron rate and round its coefficient Dividing the energy delivered each second by the energy per photon gives

$$\dot{N} = \frac{100}{4.9695 \times 10^{-19}} = 2.012 \times 10^{20}\,\mathrm{s^{-1}}.$$

Under the stated photoelectric counting model, this is also the rate of electron ejection. Comparing with $x \times 10^{20}$ electrons per second gives $x = 2.012$. The instruction asks for the nearest integer, so only this coefficient is rounded.

Step 4: Conclusion The electron emission rate is approximately $2.012 \times 10^{20}\,\mathrm{s^{-1}}$. Therefore the required nearest-integer coefficient is $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: Convert joules supplied over an interval into joules per second first, then divide by the energy of a single photon; round the coefficient only at the end.

84. CO_2 gas is bubbled through water during a soft drink manufacturing process at 298 K. If CO_2 exerts a partial pressure of 0.835 bar then x m mol of CO_2 would dissolve in 0.9 L of water. The value of x is _____. (Nearest integer)

(Henry's law constant for CO_2 at 298 K is 1.67×10^3 bar) [4 marks]

Solution:

Concept:

- Henry's law in the supplied pressure convention is $p = K_H y$, where y is the mole fraction of dissolved CO_2; the pressure and K_H must use the same units.
- For water near room temperature, use the usual density approximation $1.0\,\mathrm{g\,mL^{-1}}$. Convert the water volume to moles and use $y = n_g/(n_w + n_g)$ before converting the gas amount to millimoles.

Step 1: Calculate the dissolved-gas mole fraction Both pressure quantities are given in bar, so their ratio is directly usable:

$$y = \frac{p}{K_H} = \frac{0.835}{1.67 \times 10^3} = 5.00 \times 10^{-4}.$$

This is a mole fraction, not the number of moles dissolved. Its small value also shows that the dissolved-gas amount is much smaller than the water amount.

Step 2: Convert the water volume into moles The water volume is $0.9\,\mathrm{L} = 900\,\mathrm{mL}$. With the standard density approximation, its mass is $900\,\mathrm{g}$. Using the molar mass $18\,\mathrm{g\,mol^{-1}}$,

$$n_w = \frac{900}{18} = 50\,\mathrm{mol}.$$

This step supplies the solvent amount needed to turn the mole fraction into an actual dissolved quantity. Multiplying the fraction directly by litres would not produce moles.

Step 3: Solve for the dissolved amount and express it in millimoles Let n_g be the dissolved CO_2 amount in moles. Then

$$\frac{n_g}{50 + n_g} = 5.00 \times 10^{-4}, \qquad n_g = \frac{50(5.00 \times 10^{-4})}{1 - 5.00 \times 10^{-4}} = 0.0250125\,\mathrm{mol}.$$

Hence the dissolved amount is $25.0125\,\mathrm{mmol}$. The dilute approximation $n_g \approx y n_w = 0.0250\,\mathrm{mol}$ leads to the same nearest-integer answer. The symbol x in the question is the numerical coefficient of millimoles, not the mole fraction y.

Step 4: Conclusion Approximately $25.0125\,\mathrm{mmol}$ of CO_2 dissolves. Thus the required nearest-integer coefficient is $\boxed{x = 25}$.

Quick Tip to Solve: Keep the Henry-law mole fraction separate from the requested millimole coefficient: calculate the solvent moles, solve for gas moles, then multiply by 1000.

85. For the reaction

A + B $\rightleftharpoons$ 2C

the value of equilibrium constant is 100 at 298 K. If the initial concentration of all the three species is 1 M each, then the equilibrium concentration of C is x $\times$ 10^{-1} M. The value of x is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- For $\mathrm{A}+\mathrm{B} \rightleftharpoons 2\,\mathrm{C}$, the equilibrium constant is $K_c = [\mathrm{C}]^2/([\mathrm{A}][\mathrm{B}])$. Only equilibrium concentrations belong in this expression.
- The reaction quotient has the same concentration form as the equilibrium constant. If $Q_c < K_c$, the reaction proceeds forward, consuming equal amounts of A and B and forming twice that amount of C.

Step 1: Determine the direction of change Initially all three concentrations are $1\,\mathrm{mol\,L^{-1}}$. Hence

$$Q_c = \frac{1^2}{1 \times 1} = 1.$$

Since $1 < 100$, more C must form. Let $y\,\mathrm{mol\,L^{-1}}$ of each reactant be consumed. The stoichiometric coefficients give

$$[\mathrm{A}]_{eq} = [\mathrm{B}]_{eq} = 1 - y, \qquad [\mathrm{C}]_{eq} = 1 + 2y.$$

A physical solution requires $0 \le y < 1$, so both reactant concentrations remain positive.

Step 2: Solve the equilibrium equation Substituting these concentrations into the given equilibrium constant,

$$100 = \frac{(1+2y)^2}{(1-y)^2}.$$

Both concentrations are positive, so the physically meaningful square root is

$$10 = \frac{1+2y}{1-y}.$$

Thus $10 - 10y = 1 + 2y$, giving $12y = 9$ and $y = 0.75$. Therefore

$$[\mathrm{C}]_{eq} = 1 + 2(0.75) = 2.50\,\mathrm{mol\,L^{-1}}.$$

As a check, each reactant is $0.25\,\mathrm{M}$, and $2.50^2/0.25^2 = 100$.

Step 3: Convert to the requested coefficient The concentration is written as $x \times 10^{-1}\,\mathrm{M}$. Therefore

$$x = \frac{2.50}{10^{-1}} = 25.$$

The exponent is part of the requested answer format; entering 2.5 would give the concentration rather than its coefficient.

Step 4: Conclusion The equilibrium concentration of C is $2.50\,\mathrm{M} = 25 \times 10^{-1}\,\mathrm{M}$. The required nearest integer is $\boxed{25}$ (dimensionless coefficient).

Quick Tip to Solve: When A and B start at equal concentrations, they remain equal here. Take the positive square root of the equilibrium expression before solving for the reaction extent.

86. Consider the cell at 25°C

Zn | Zn^{2+} (aq), (1M) || Fe^{3+} (aq), Fe^{2+} (aq) | Pt(s)

The fraction of total iron present as Fe^{3+} ion at the cell potential of 1.500 V is x $\times$ 10^{-2}. The value of x is _____. (Nearest integer)

(Given : $E^0_{Fe^{3+}/Fe^{2+}} = 0.77V$, $E^0_{Zn^{2+}/Zn} = -0.76V$) [4 marks]

Solution:

Concept:

- The cell potential is the cathode reduction potential minus the anode reduction potential. At $25^\circ C$, $E = E^\circ - (0.0591/n)\log_{10} Q$, where n is the number of electrons transferred in the balanced cell reaction.
- The fraction of total dissolved iron present as Fe^{3+} is $[Fe^{3+}]/([Fe^{3+}] + [Fe^{2+}])$. A concentration ratio must first be converted to this fraction.

Step 1: Write the cell reaction and standard potential Zinc is oxidised, while iron(III) is reduced at platinum:

$$Zn \longrightarrow Zn^{2+} + 2\,e^-, \qquad 2\,Fe^{3+} + 2\,e^- \longrightarrow 2\,Fe^{2+}.$$

The overall reaction is $Zn + 2\,Fe^{3+} \longrightarrow Zn^{2+} + 2\,Fe^{2+}$, so $n = 2$. Using the supplied reduction potentials,

$$E^\circ_{cell} = 0.77 - (-0.76) = 1.53\,V.$$

The solid zinc and platinum do not enter the reaction quotient.

Step 2: Obtain the iron concentration ratio Since the zinc-ion concentration is $1\,M$, the reaction quotient is

$$Q = \frac{[Zn^{2+}][Fe^{2+}]^2}{[Fe^{3+}]^2} = \left(\frac{[Fe^{2+}]}{[Fe^{3+}]}\right)^2.$$

The square cancels the factor 2 in the Nernst denominator. With $r = [Fe^{2+}]/[Fe^{3+}]$,

$$1.500 = 1.530 - 0.0591\log_{10} r.$$

Consequently $\log_{10} r = 0.030/0.0591 = 0.5076$, and $r \simeq 3.22$.

Step 3: Convert the ratio to the requested fraction The total iron concentration includes both oxidation states, hence

$$f_{Fe^{3+}} = \frac{1}{1+r} = \frac{1}{1+3.22} \simeq 0.237.$$

Writing $f = x \times 10^{-2}$ gives $x \simeq 23.7$, which rounds to 24. Since the observed potential is slightly below the standard value, the reduced form predominates, consistent with this fraction being below one-half.

Step 4: Conclusion The iron(III) fraction is approximately $0.237 = 23.7 \times 10^{-2}$. The required nearest integer is $\boxed{24}$; the fraction and its coefficient are dimensionless.

Quick Tip to Solve: Use the balanced cell reaction to obtain the squared iron ratio. After finding Fe(II)/Fe(III), use 1/(1+ratio), rather than the inverse ratio alone, for the Fe(III) fraction.

87. At 298 K, the enthalpy of fusion of a solid (X) is 2.8 kJ mol^{-1} and the enthalpy of vaporisation of the liquid (X) is 98.2 kJ mol^{-1}. The enthalpy of sublimation of the substance (X) in kJ mol^{-1} is _____. (in nearest integer) [4 marks]

Solution:

Concept:

- Enthalpy is a state function. By Hess's law, the enthalpy change for a net transformation equals the sum of enthalpy changes along an alternative path between the same initial and final states.
- Sublimation is conversion of solid directly to gas. Fusion converts solid to liquid, and vaporisation converts liquid to gas; adding the last two changes produces the sublimation change when the temperature and substance are the same.

Step 1: Construct a path from solid to gas The required initial state is solid X and the final state is gaseous X, both for the stated temperature of $298\,\mathrm{K}$. The supplied quantities describe the two consecutive changes

$$\mathrm{X(s)} \longrightarrow \mathrm{X(l)}, \qquad \Delta H_{fus} = +2.8\,\mathrm{kJ\,mol^{-1}},$$

$$\mathrm{X(l)} \longrightarrow \mathrm{X(g)}, \qquad \Delta H_{vap} = +98.2\,\mathrm{kJ\,mol^{-1}}.$$

Both quantities are positive because the specified forward phase changes absorb energy. There is no reversal of either step in the solid-to-gas path.

Step 2: Add the phase-change enthalpies Adding the two transformations cancels the intermediate liquid state and gives $\mathrm{X(s)} \longrightarrow \mathrm{X(g)}$. Therefore Hess's law gives

$$\Delta H_{sub} = \Delta H_{fus} + \Delta H_{vap}.$$

Substituting the supplied molar quantities,

$$\Delta H_{sub} = 2.8 + 98.2 = 101.0\,\mathrm{kJ\,mol^{-1}}.$$

No mass or number-of-moles factor is needed: both values are already per mole of the same substance. Subtracting the fusion value would instead correspond to a different net phase change.

Step 3: Apply the requested rounding The question requests the numerical value in $\mathrm{kJ\,mol^{-1}}$, rounded to the nearest integer. The calculated value is exactly 101.0 at the precision supplied, so its nearest-integer value is 101. Its positive sign is consistent with the energy needed to transform the solid to gas.

Step 4: Conclusion The enthalpy of sublimation is $\boxed{101\,\mathrm{kJ\,mol^{-1}}}$, so the required integer is $\boxed{101}$.

Quick Tip to Solve: Write solid to liquid to gas and cancel the liquid state. This makes it clear that fusion and vaporisation enthalpies are added for sublimation, without reversing either sign.

88. When 10 mL of an aqueous solution of Fe^{2+} ions was titrated in the presence of dil H_2SO_4 using diphenylamine indicator, 15 mL of 0.02 M solution of $K_2Cr_2O_7$ was required to get the end point. The molarity of the solution containing Fe^{2+} ions is x $\times$ 10^{-2} M. The value of x is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- In acidic medium, one dichromate ion accepts six electrons and is reduced to two chromium(III) ions. Each iron(II) ion loses one electron on oxidation to iron(III), so one mole of dichromate oxidises six moles of iron(II).
- Moles in a solution are $n = MV$, with volume expressed in litres. At the titration endpoint the reacting amounts satisfy the balanced redox stoichiometry; the indicator locates the endpoint but does not change this mole ratio.

Step 1: Balance the redox stoichiometry The two half-reactions in the stated dilute sulfuric-acid medium are

$$\mathrm{Cr_2O_7^{2-}} + 14\,\mathrm{H^+} + 6\,\mathrm{e^-} \longrightarrow 2\,\mathrm{Cr^{3+}} + 7\,\mathrm{H_2O},$$

$$\mathrm{Fe^{2+}} \longrightarrow \mathrm{Fe^{3+}} + \mathrm{e^-}.$$

Multiplying the iron half-reaction by six and adding gives

$$\mathrm{Cr_2O_7^{2-}} + 14\,\mathrm{H^+} + 6\,\mathrm{Fe^{2+}} \longrightarrow 2\,\mathrm{Cr^{3+}} + 7\,\mathrm{H_2O} + 6\,\mathrm{Fe^{3+}}.$$

Thus the required mole ratio is $n(\mathrm{Fe^{2+}}) : n(\mathrm{Cr_2O_7^{2-}}) = 6 : 1$, not 1:1.

Step 2: Calculate the amount of iron(II) The dichromate volume is $15\,\mathrm{mL} = 0.015\,\mathrm{L}$. Its amount is

$$n(\mathrm{K_2Cr_2O_7}) = 0.02 \times 0.015 = 3.0 \times 10^{-4}\,\mathrm{mol}.$$

Each formula unit supplies one dichromate ion. Therefore the iron(II) in the titrated aliquot is

$$n(\mathrm{Fe^{2+}}) = 6(3.0 \times 10^{-4}) = 1.8 \times 10^{-3}\,\mathrm{mol}.$$

This is the amount in the original 10 mL aliquot; it must not be divided by the volume of titrant.

Step 3: Find the molarity and coefficient Using the iron-solution volume $10\,\mathrm{mL} = 0.010\,\mathrm{L}$,

$$M_{\mathrm{Fe^{2+}}} = \frac{1.8 \times 10^{-3}}{0.010} = 0.18\,\mathrm{mol\,L^{-1}}.$$

Since $0.18 = 18 \times 10^{-2}$, the coefficient requested in the question is already an integer.

Step 4: Conclusion The iron(II) solution has molarity $0.18\,\mathrm{M} = 18 \times 10^{-2}\,\mathrm{M}$. Hence $\boxed{x = 18}$, a dimensionless coefficient.

Quick Tip to Solve: For acidic dichromate titration, use six Fe(II) per dichromate. Divide the resulting iron moles by the iron aliquot volume, not by the volume of dichromate delivered.

89. Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is _____ $\times 10^{-1}$ g. (in nearest integer) [4 marks]

Solution:

Concept:

- Complete combustion of butane produces carbon dioxide and water. A balanced chemical equation relates amounts in moles; its coefficients do not directly relate masses in grams.
- The mass-to-mole relation is $n = m/M$, with molar mass M. Using C = 12, H = 1 and O = 16, the molar masses are $M(C_4H_{10}) = 58\,\text{g mol}^{-1}$ and $M(H_2O) = 18\,\text{g mol}^{-1}$.

Step 1: Balance the complete combustion Balance carbon and hydrogen first, then oxygen:

$$2\,C_4H_{10} + 13\,O_2 \longrightarrow 8\,CO_2 + 10\,H_2O.$$

For every 2 mol of butane, 10 mol of water are produced. Equivalently, each mole of butane gives 5 mol of water. This factor can also be checked directly: the ten hydrogen atoms in each butane molecule make five water molecules. Oxygen is available for complete combustion, so no incomplete-combustion product is included.

Step 2: Convert the water mass to butane amount The water produced has mass $72.0\,\text{g}$. Consequently,

$$n(H_2O) = \frac{72.0}{18} = 4.00\,\text{mol}.$$

Applying the balanced mole ratio gives

$$n(C_4H_{10}) = \frac{2}{10}\,n(H_2O) = \frac{4.00}{5} = 0.800\,\text{mol}.$$

The direction of this ratio matters: fewer butane molecules are needed than water molecules produced.

Step 3: Calculate the mass and requested coefficient The mass of butane consumed is

$$m(C_4H_{10}) = 0.800 \times 58 = 46.4\,\text{g}.$$

The answer blank multiplies $10^{-1}\,\text{g}$. Thus its coefficient is

$$x = \frac{46.4}{10^{-1}} = 464.$$

For a consistency check, 46.4 g corresponds to 0.8 mol butane and therefore to 4 mol, or 72 g, water as required.

Step 4: Conclusion The butane consumed is $46.4\,\text{g} = 464 \times 10^{-1}\,\text{g}$. The required nearest integer is $\boxed{464}$, a dimensionless coefficient.

Quick Tip to Solve: Count hydrogen first: one mole of butane gives five moles of water. After finding the mass, divide by the power of ten printed beside the answer blank.

90. A home owner uses 4.00 $\times$ 10^3 m^3 of methane (CH_4) gas, (assume CH_4 is an ideal gas) in a year to heat his home. Under the pressure of 1.0 atm and 300 K, mass of gas used is x $\times$ 10^5 g. The value of x is _____. (Nearest integer)

(Given R = 0.083 L atm K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- An ideal gas obeys $PV = nRT$, where P is pressure, V volume, n amount, T absolute temperature and R the gas constant. Units must agree with those of the supplied value of R.
- The mass of a gas is $m = nM$. Methane, CH_4, has molar mass $16\,\mathrm{g\,mol^{-1}}$, using atomic masses 12 for carbon and 1 for hydrogen.

Step 1: Express all quantities in compatible units The given gas constant is $R = 0.083\,\mathrm{L\,atm\,K^{-1}\,mol^{-1}}$, so use litres for volume and atmospheres for pressure. Since $1\,\mathrm{m^3} = 10^3\,\mathrm{L}$, the annual methane volume is

$$V = 4.00 \times 10^3 \times 10^3 = 4.00 \times 10^6\,\mathrm{L}.$$

The pressure is already $1.0\,\mathrm{atm}$, and the absolute temperature is $300\,\mathrm{K}$. There is no Celsius-to-kelvin conversion to apply.

Step 2: Calculate the amount and mass of methane Rearrange the ideal gas equation to find the amount used:

$$n = \frac{PV}{RT} = \frac{1.0(4.00 \times 10^6)}{0.083(300)}.$$

The denominator is $24.9\,\mathrm{L\,atm\,mol^{-1}}$, giving

$$n \simeq 1.6064 \times 10^5\,\mathrm{mol}.$$

Multiplying by the methane molar mass,

$$m = nM = (1.6064 \times 10^5)(16) \simeq 2.5703 \times 10^6\,\mathrm{g}.$$

Use the supplied gas constant rather than substituting a different value; premature rounding of the amount is unnecessary.

Step 3: Extract the coefficient and round once The question writes the mass as $x \times 10^5\,\mathrm{g}$. Therefore

$$x = \frac{2.5703 \times 10^6}{10^5} = 25.703.$$

Rounding this coefficient to the nearest integer gives 26. The mass itself is about $2.57 \times 10^6\,\mathrm{g}$; the integer to enter is its coefficient in the specific scale requested.

Step 4: Conclusion The gas used has mass approximately $2.57 \times 10^6\,\mathrm{g} = 25.7 \times 10^5\,\mathrm{g}$. The required nearest integer is $\boxed{26}$, a dimensionless coefficient.

Quick Tip to Solve: Match volume units to R before substitution: cubic metres must become litres here. Keep extra digits until converting the mass to the requested power-of-ten coefficient.