JEE Main 2021 Paper 1 (B.E./B.Tech) Jul 27 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 3 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 July 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The point P (a, b) undergoes the following three transformations successively :

(a) reflection about the line y = x.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to : [4 marks]

(A) 13

(B) 9

(C) 5

(D) 7

Correct Answer: (B) 9

Solution:

Concept:

- Reflection in the line $y = x$ interchanges the two coordinates: $(a, b) \mapsto (b, a)$. A translation along the positive horizontal direction changes only the first coordinate.
- An anticlockwise rotation through θ sends (X, Y) to $(X\cos\theta - Y\sin\theta, X\sin\theta + Y\cos\theta)$. The order of the transformations must be preserved.

Step 1: Apply the reflection and translation The reflected position is (b, a). Translating this point two units to the right gives $(b + 2, a)$. In particular, the translation is applied after the reflection, so the added 2 belongs to the coordinate containing b, not to the coordinate containing a.

Step 2: Use the final rotated coordinates For $\theta = \pi/4$, both sine and cosine equal $1/\sqrt{2}$. Thus the final point is

$$\left(\frac{b + 2 - a}{\sqrt{2}}, \frac{b + 2 + a}{\sqrt{2}}\right).$$

Equating corresponding coordinates with those given gives

$$b + 2 - a = -1, \qquad b + 2 + a = 7.$$

Adding the equations yields $2b + 4 = 6$, hence $b = 1$. Subtracting the first equation from the second yields $2a = 8$, hence $a = 4$.

Step 3: Evaluate and check the requested expression Consequently

$$2a + b = 2(4) + 1 = 9.$$

As a check, the point immediately before rotation is $(3, 4)$, whose rotated coordinates are $(-1/\sqrt{2}, 7/\sqrt{2})$, exactly as required. The values 13, 5 and 7 do not equal the expression computed from these uniquely determined coordinates.

Step 4: Conclusion The required value is 9. Hence, the correct option is (B).

Quick Tip to Solve: Write the point after each transformation before combining equations; reflection and translation generally cannot be interchanged.

2. A possible value of 'x', for which the ninth term in the expansion of

$$\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}\right\}^{10}$$

in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}$ is equal to 180, is : [4 marks]

(A) 0

(B) -1

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

Concept:

- The $(r+1)$th term of $(A+B)^{10}$, in increasing powers of B, is $\binom{10}{r}A^{10-r}B^r$. The ninth term therefore corresponds to $r=8$.
- For a positive argument u, $3^{\log_3 u}=u$. Introducing $t=5^{x-1}>0$ simplifies both logarithmic expressions while retaining their domains.

Step 1: Simplify the two terms inside the bracket Write $t=5^{x-1}$, so that $25^{x-1}=t^2$. The two quantities in the binomial are

$$A=\sqrt{t^2+7}, \qquad B=(t+1)^{-1/8}.$$

Both are real and positive for every real x, so the logarithms and the negative fractional power introduce no additional restriction.

Step 2: Form the ninth term and solve its equation The required term is

$$T_9=\binom{10}{8}A^2B^8=45\frac{t^2+7}{t+1}.$$

Setting it equal to 180 gives

$$t^2+7=4(t+1), \qquad t^2-4t+3=0.$$

Hence $(t-1)(t-3)=0$, so $t=1$ or $t=3$. No root is lost or introduced because the multiplied denominator $t+1$ is strictly positive.

Step 3: Compare the resulting values with the options The corresponding real values are $x=1$ and $x=1+\log_5 3$. Among the listed choices only $x=1$ occurs. Indeed, it gives $t=1$ and $T_9=45(8)/2=180$. The choices $x=0,-1,2$ give $t=1/5,1/25,5$, none of which is a root of the quadratic.

Step 4: Conclusion A possible listed value is $x=1$. Hence, the correct option is (D).

Quick Tip to Solve: For a ninth binomial term use the exponent 8 on the second quantity, and simplify logarithmic powers before expanding.

3. Let f : R → R be defined as $f(x+y)+f(x-y)=2f(x)f(y), f\left(\frac{1}{2}\right)=-1$. Then, the value of $\sum_{k=1}^{20}\frac{1}{\sin(k)\sin(k+f(k))}$ is equal to : [4 marks]

(A) $\text{cosec}^2(21)\cos(20)\cos(2)$

(B) $\sec^2(1)\sec(21)\cos(20)$

(C) $\text{cosec}^2(1)\,\text{cosec}(21)\sin(20)$

(D) $\sec^2(21)\sin(20)\sin(2)$

Correct Answer: (C) $\text{cosec}^2(1)\,\text{cosec}(21)\sin(20)$

Solution:

Concept:

- The given equation can determine the integer values of f without identifying the function at every real argument. Substituting special values first supplies $f(0)$ and $f(1)$.
- The identity $\cot u-\cot v=\sin(v-u)/(\sin u\sin v)$ converts a reciprocal product into a difference that telescopes. All angles here are in radians.

Step 1: Determine the values at zero and one Set $x=y=0$ to obtain $f(0)=f(0)^2$. Also, putting $x=1/2, y=0$ and using $f(1/2)=-1$ gives $-2=-2f(0)$, so $f(0)=1$. Next set $x=y=1/2$. Then

$$f(1)+f(0)=2f(1/2)^2=2,$$

and therefore $f(1)=1$.

Step 2: Establish every positive integer value With $x=n, y=1$, the functional equation becomes

$$f(n+1)+f(n-1)=2f(n).$$

Starting from $f(0)=f(1)=1$, induction gives $f(n)=1$ for every nonnegative integer n. Thus each denominator in the sum is $\sin k\sin(k+1)$. These factors are nonzero: a positive integer radian angle is not an integer multiple of the irrational number π.

Step 3: Telescope and simplify For each summand,

$$\frac{1}{\sin k\sin(k+1)}=\frac{\cot k-\cot(k+1)}{\sin 1}.$$

Consequently the sum is

$$\frac{\cot 1-\cot 21}{\sin 1}=\frac{\sin 20}{\sin^2 1\sin 21}.$$

This is $\text{cosec}^2(1)\,\text{cosec}(21)\sin(20)$. The other options involve cosine denominators or different endpoint factors and do not match the telescoped expression.

Step 4: Conclusion The sum is $\text{cosec}^2(1)\,\text{cosec}(21)\sin(20)$. Hence, the correct option is (C).

Quick Tip to Solve: For a cosine-type functional equation, find integer values first; then use a cotangent difference to telescope reciprocal sine products.

4. Let C be the set of all complex numbers. Let

$S_1 = \{z \in C : |z - 2| \leq 1\}$ and

$S_2 = \{z \in C : z(1 + i) + \bar{z}(1 - i) \geq 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to : [4 marks]

(A) $\frac{3+2\sqrt{2}}{4}$

(B) $\frac{5+2\sqrt{2}}{2}$

(C) $\frac{3+2\sqrt{2}}{2}$

(D) $\frac{5+2\sqrt{2}}{4}$

Correct Answer: (D) $\frac{5+2\sqrt{2}}{4}$

Solution:

Concept:

- Writing $z = x + iy$ converts a modulus inequality into a disk in the coordinate plane. The real expression $z(1 + i) + \bar{z}(1 - i)$ equals $2(x - y)$.
- The squared distance from $5/2$ is $(x - 5/2)^2 + y^2$. Shifting the disk centre to the origin makes it possible to bound this distance and exhibit a point attaining the bound.

Step 1: Describe the feasible region Put $u = x - 2$ and $v = y$. The first condition becomes $u^2 + v^2 \leq 1$. Expanding the second condition gives $2(x - y) \geq 4$, so $u \geq v$. The objective is

$$F = (u - 1/2)^2 + v^2 = u^2 + v^2 - u + \frac{1}{4}.$$

Thus the constraint is the part of the unit disk lying on the side $u \geq v$.

Step 2: Bound the first shifted coordinate If $u \geq 0$, certainly $u \geq -1/\sqrt{2}$. If $u < 0$, the inequality $v \leq u$ gives $v^2 \geq u^2$. Hence $2u^2 \leq u^2 + v^2 \leq 1$, again implying $u \geq -1/\sqrt{2}$. Combining this bound with the disk condition yields

$$F \leq 1 + \frac{1}{\sqrt{2}} + \frac{1}{4} = \frac{5 + 2\sqrt{2}}{4}.$$

Step 3: Verify that the bound is attained Take $u = v = -1/\sqrt{2}$, which satisfies both constraints with equality. For $z = 2 - 1/\sqrt{2} - i/\sqrt{2}$, the objective equals the bound. Therefore it is a maximum, rather than merely an upper estimate. Option (A) is smaller by $1/2$; options (B) and (C) exceed this rigorously established upper bound, so none can be the maximum.

Step 4: Conclusion The maximum is $\frac{5 + 2\sqrt{2}}{4}$. Hence, the correct option is (D).

Quick Tip to Solve: After centring a disk, express the distance squared as the radius squared plus a linear term; bound both and check equality simultaneously.

5. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x-2y|$ is equal to : [4 marks]

(A) 4

(B) 3

(C) 0

(D) 1

Correct Answer: (C) 0

Solution:

Concept:

- In an arithmetic progression of three terms the middle term is the arithmetic mean of the first and third. Therefore $2x = \tan 20^\circ + \tan 70^\circ$ and $2y = \tan 20^\circ + \tan 50^\circ$.
- Complementary-angle identities give $\tan 70^\circ = \cot 20^\circ$ and $\tan 50^\circ = \cot 40^\circ$. The double-angle formula relates the latter to $t = \tan 20^\circ$.

Step 1: Express both middle terms using one tangent The printed angles $\pi/9, 7\pi/18, 5\pi/18$ are respectively $20^\circ, 70^\circ, 50^\circ$. Set $t = \tan 20^\circ$. It is positive and nonzero. The first arithmetic progression gives

$$2x = t + \frac{1}{t}.$$

For the second, use $\tan 50^\circ = \cot 40^\circ$, giving $2y = t + \cot 40^\circ$.

Step 2: Apply the double-angle identity carefully Since $\tan 40^\circ = 2t/(1-t^2)$, its reciprocal is

$$\cot 40^\circ = \frac{1-t^2}{2t}.$$

It follows that

$$4y = 2t + 2\cot 40^\circ = 2t + \frac{1-t^2}{t} = t + \frac{1}{t}.$$

The right side is exactly $2x$, not x; retaining this factor of two is essential when comparing the two progressions.

Step 3: Evaluate the absolute difference The equality $4y = 2x$ implies $x = 2y$. Therefore

$$|x - 2y| = 0.$$

Both original progressions are well-defined because none of their tangent arguments is an odd multiple of $\pi/2$. The exact cancellation rules out 4, 3 and 1; no decimal approximation of the trigonometric values is needed.

Step 4: Conclusion The required absolute difference is 0. Hence, the correct option is (C).

Quick Tip to Solve: Convert the large angles to complementary angles, then use the double-angle formula to compare the AP means exactly.

6. Let the mean and variance of the frequency distribution

$$\begin{array}{lcccc} x: & x_1 = 2 & x_2 = 6 & x_3 = 8 & x_4 = 9 \\ f: & 4 & 4 & \alpha & \beta \end{array}$$

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be :

[4 marks]

(A) 4

(B) 5

(C) $\frac{17}{3}$

(D) $\frac{16}{3}$

Correct Answer: (C) $\frac{17}{3}$

Solution:

Concept:

- For frequencies f_i, the mean is $\overline{x} = \sum f_i x_i / \sum f_i$, and the variance is $\sum f_i (x_i - \overline{x})^2 / \sum f_i$. Each occurrence of a value contributes to these weighted sums.
- Changing a value from 8 to 7 changes all α occurrences of that value. The frequency total remains fixed while the weighted sum decreases by α.

Step 1: Use the given mean to obtain one equation The total frequency is $N = 8 + \alpha + \beta$, and the weighted sum is $32 + 8\alpha + 9\beta$. The mean 6 gives

$$32 + 8\alpha + 9\beta = 6(8 + \alpha + \beta),$$

or

$$2\alpha + 3\beta = 16.$$

The two frequencies must also be consistent with the stated variance.

Step 2: Use deviations from the mean The squared deviations of 2, 6, 8 and 9 from 6 are 16, 0, 4 and 9. Thus

$$\frac{64 + 4\alpha + 9\beta}{8 + \alpha + \beta} = 6.8 = \frac{34}{5}.$$

Multiplying out yields $14\alpha - 11\beta = 48$. Multiply the mean equation by 11 and this last equation by 3; adding gives $64\alpha = 320$. Hence $\alpha = 5$, $\beta = 2$, and $N = 15$.

Step 3: Calculate the revised mean The original weighted sum was $6 \times 15 = 90$. Replacing the value 8 by 7 at frequency 5 reduces this sum by 5, leaving 85. Therefore

$$\overline{x}_{\text{new}} = \frac{85}{15} = \frac{17}{3}.$$

The decrease is only $5/15 = 1/3$, which also checks the result. The options 4, 5 and $16/3$ imply different decreases and do not follow from the unchanged frequencies.

Step 4: Conclusion The new mean is $\frac{17}{3}$. Hence, the correct option is (C).

Quick Tip to Solve: When one class value changes, update the weighted sum by its frequency times the change; do not reduce the total frequency.

7. The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to : [4 marks]

(A) $\frac{16}{3}$

(B) $\frac{2}{3}$

(C) $\frac{9}{2}$

(D) $\frac{4}{3}$

Correct Answer: (C) $\frac{9}{2}$

Solution:

Concept:

- The area between curves written as y-functions is the integral of the upper function minus the lower function between their intersection abscissae. The intersection points determine the complete bounded interval.
- Here the line is $y = x + 2$ and the parabola is $y = x^2$. Their difference is a quadratic, whose sign must be checked before using it as the height of a vertical strip.

Step 1: Find the intersections and determine which curve is above Equating the two ordinates gives

$$x^2 = x + 2, \qquad (x - 2)(x + 1) = 0.$$

Thus the bounded region extends from $x = -1$ to $x = 2$, with endpoints $(-1, 1)$ and $(2, 4)$. On this interval,

$$x + 2 - x^2 = (2 - x)(x + 1) \geq 0.$$

Therefore the line lies above the parabola throughout the interior of the region.

Step 2: Integrate the vertical strip heights The required geometric area is

$$A = \int_{-1}^{2} (x + 2 - x^2)\, dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}.$$

At the upper limit the antiderivative is $2 + 4 - 8/3 = 10/3$. At the lower limit it is $1/2 - 2 + 1/3 = -7/6$. Both endpoints are needed; omitting the lower contribution would not give the enclosed area.

Step 3: Subtract the endpoint values and compare Hence

$$A = \frac{10}{3} - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2}.$$

The value is positive, as expected from the upper-minus-lower integrand. It is distinct from $16/3$, $2/3$ and $4/3$, so those options cannot represent the bounded area. The unit is square units because the calculation integrates a vertical length against a horizontal length.

Step 4: Conclusion The enclosed area is $\frac{9}{2}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Factor the difference of the two curves first: its roots give the limits and its sign identifies the upper curve.

8. Let y = y(x) be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, $x > 2$. If $y(3) = 3$, then $y(4)$ is equal to : [4 marks]

(A) 4

(B) 12

(C) 8

(D) 16

Correct Answer: (B) 12

Solution:

Concept:

- A linear differential equation has the form $y' + P(x)y = Q(x)$; multiplication by the integrating factor $I = e^{\int P(x)\,dx}$ makes its left side $(Iy)'$.
- The initial condition fixes the constant of integration. Here $x > 2$, so x, $x^2 - 1$ and the integrating factor are nonzero throughout the required interval.

Step 1: Write the equation in linear form Divide the given equation by $x - x^3 = x(1 - x^2)$. Moving the term containing y to the left gives

$$y' + \frac{1 + x^2}{x(x^2 - 1)}y = \frac{3x^3}{x^2 - 1}.$$

The coefficient of y can be written as

$$P(x) = -\frac{1}{x} + \frac{2x}{x^2 - 1}.$$

This decomposition makes its integral a difference of logarithms, with no sign ambiguity since $x > 2$.

Step 2: Integrate after multiplying by the integrating factor The integrating factor is

$$I = \exp[-\log x + \log(x^2 - 1)] = \frac{x^2 - 1}{x}.$$

Consequently,

$$\frac{d}{dx}\left(\frac{x^2 - 1}{x}y\right) = 3x^2, \qquad \frac{x^2 - 1}{x}y = x^3 + C.$$

At $x = 3$, $y = 3$, so $8 = 27 + C$, giving $C = -19$.

Step 3: Evaluate the requested value and test the options At $x = 4$, the solution therefore satisfies

$$\frac{15}{4}y(4) = 64 - 19 = 45, \qquad y(4) = 12.$$

The values 4, 8 and 16 would make the left side 15, 30 and 60, respectively, rather than the required 45. Thus none of those alternatives obeys the solution determined by the initial condition.

Step 4: Conclusion $y(4) = 12$. Hence, the correct option is (B).

Quick Tip to Solve: Factor $x - x^3$ before finding the integrating factor; the factor $(x^2 - 1)/x$ turns this equation directly into a total derivative.

9. The value of

$\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ **is equal to :** [4 marks]

(A) 0

(B) 4

(C) -4

(D) -1

Correct Answer: (C) -4

Solution:

Concept:

- For a differentiable function g, $g(u) = g(0) + g'(0)u + o(u)$ near zero; here $g(u) = (1+u)^{1/8}$.
- The standard limit $\sin x/x \to 1$ connects the small argument $\sin x$ with x. The two eighth roots remain real and differentiable near zero.

Step 1: Find the first-order change of each radical At $u = 0$, the derivative of $(1+u)^{1/8}$ is $1/8$. Therefore

$$(1-\sin x)^{1/8} = 1 - \tfrac{1}{8}\sin x + o(\sin x),$$
$$(1+\sin x)^{1/8} = 1 + \tfrac{1}{8}\sin x + o(\sin x).$$

Subtract in exactly the printed order: the radical with the minus sign inside comes first. The constant terms cancel and the two linear terms add negatively.

Step 2: Compute the quotient limit Let $D(x) = (1-\sin x)^{1/8} - (1+\sin x)^{1/8}$. The preceding expansions give

$$\frac{D(x)}{x} = -\frac{1}{4}\frac{\sin x}{x} + o(1) \longrightarrow -\frac{1}{4}.$$

The limit is nonzero, so its reciprocal is legitimate:

$$\lim_{x\to 0}\frac{x}{D(x)} = \frac{1}{-1/4} = -4.$$

Equivalently, differentiating the denominator at zero gives $-1/8 - 1/8 = -1/4$, the same first-order coefficient.

Step 3: Exclude the other choices For small positive x, the first radical is less than the second, so the quotient is negative; this immediately rules out the positive value 4. Since numerator and denominator vanish to the same first order, the quotient is not zero. The slope magnitude is $1/4$, so the quotient magnitude is 4, not 1.

Step 4: Conclusion The required limit is -4. Hence, the correct option is (C).

Quick Tip to Solve: For a difference of equal-index roots near 1, compute the denominator derivative at zero first; retain the printed subtraction order to get the sign right.

10. Two sides of a parallelogram are along the lines 4x + 5y = 0 and 7x + 2y = 0. If the equation of one of the diagonals of the parallelogram is 11x + 7y = 9, then other diagonal passes through the point : [4 marks]

(A) (1, 2)

(B) (2, 2)

(C) (2, 1)

(D) (1, 3)

Correct Answer: (B) (2, 2)

Solution:

Concept:

- The two given side lines intersect at a vertex O. A diagonal not through O joins points A and B on those two sides.
- Diagonals of a parallelogram bisect one another. With $O = (0, 0)$, its opposite vertex is the vector sum $A + B$, so the other diagonal has direction $A + B$.

Step 1: Find the two vertices on the given diagonal The side lines $4x + 5y = 0$ and $7x + 2y = 0$ meet at $O = (0, 0)$. Since $11x + 7y = 9$ does not contain O, it is the diagonal joining the other two adjacent vertices. On the first side, $y = -4x/5$, hence

$$11x - \frac{28}{5}x = 9 \quad \Rightarrow \quad A = (5/3, -4/3).$$

On the second side, $y = -7x/2$, hence

$$11x - \frac{49}{2}x = 9 \quad \Rightarrow \quad B = (-2/3, 7/3).$$

Step 2: Use the common midpoint of the diagonals The midpoint of AB is

$$M = \left(\frac{5/3 - 2/3}{2}, \frac{-4/3 + 7/3}{2}\right) = (1/2, 1/2).$$

The other diagonal passes through O and M. Its equation is therefore $y = x$. This also follows from the opposite vertex $A + B = (1, 1)$; both coordinates are equal, so its line with the origin has slope 1.

Step 3: Test each proposed point The point $(2, 2)$ satisfies $y = x$. The points $(1, 2)$, $(2, 1)$ and $(1, 3)$ have unequal coordinates, so none lies on this diagonal. The diagonal is a line, so the proposed point need not itself be a vertex or lie inside the parallelogram.

Step 4: Conclusion The other diagonal passes through $(2, 2)$. Hence, the correct option is (B).

Quick Tip to Solve: When adjacent side lines pass through the origin, intersect the stated diagonal with both sides and add the resulting position vectors to obtain the second diagonal direction.

11. Let $\alpha = \max_{x \in R}\{8^{2\sin 3x}.4^{4\cos 3x}\}$ and $\beta = \min_{x \in R}\{8^{2\sin 3x}.4^{4\cos 3x}\}$. If $8x^2+bx+c=0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of c − b is equal to : [4 marks]

(A) 42

(B) 47

(C) 43

(D) 50

Correct Answer: (A) 42

Solution:

Concept:

- For real t, the expression $a\sin t + b\cos t$ ranges from $-\sqrt{a^2+b^2}$ to $\sqrt{a^2+b^2}$. Both endpoint values are attained.
- For a quadratic $Ax^2+Bx+C=0$, the sum and product of its roots are $-B/A$ and C/A, respectively.

Step 1: Express the product as a power of 2 Use $8 = 2^3$ and $4 = 2^2$. The expression being maximised and minimised becomes

$$8^{2\sin 3x}\,4^{4\cos 3x} = 2^{6\sin 3x + 8\cos 3x}.$$

As x ranges over the reals, so does $3x$. The exponent has amplitude $\sqrt{6^2+8^2} = 10$, hence it ranges over $[-10, 10]$. Since the base 2 exceeds 1, maximising the exponent also maximises the exponential.

Step 2: Find the required fifth-root pair The extreme values are

$$\alpha = 2^{10}, \qquad \beta = 2^{-10}.$$

Their positive real fifth roots are consequently

$$\alpha^{1/5} = 2^2 = 4, \qquad \beta^{1/5} = 2^{-2} = \frac{1}{4}.$$

Their product is 1 and their sum is $4 + 1/4 = 17/4$. It is these fifth roots, rather than α and β themselves, that are the quadratic roots.

Step 3: Apply the coefficient relations For the leading coefficient 8,

$$-\frac{b}{8} = \frac{17}{4} \;\Rightarrow\; b = -34, \qquad \frac{c}{8} = 1 \;\Rightarrow\; c = 8.$$

Thus $c - b = 8 - (-34) = 42$. As a check the polynomial is $8(x-4)(x-1/4) = 8x^2 - 34x + 8$. The remaining options 47, 43 and 50 do not match these coefficients.

Step 4: Conclusion $c - b = 42$. Hence, the correct option is (A).

Quick Tip to Solve: Convert exponential products to a common base before optimising; then take the requested roots before applying the sum and product formulas.

12. Let $f : [0, \infty) \to [0, 3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$$

Then which of the following is true? [4 marks]

(A) f is continuous everywhere but not differentiable exactly at one point in (0, ∞)

(B) f is differentiable everywhere in (0, ∞)

(C) f is not continuous exactly at two points in (0, ∞)

(D) f is continuous everywhere but not differentiable exactly at two points in (0, ∞)

Correct Answer: (B) f is differentiable everywhere in (0, ∞)

Solution:

Concept:

- A running maximum of $\sin t$ follows sine until its first maximum, then stays at 1 as long as that maximum remains in the permitted interval.
- For a piecewise smooth function, continuity and differentiability need special checking only at its joining points. Matching function values and one-sided derivatives establish differentiability there.

Step 1: Rewrite the running maximum explicitly On $[0, \pi/2]$, sine is increasing, so $\max_{0 \le t \le x} \sin t = \sin x$. For $\pi/2 \le x \le \pi$, the interval already contains $t = \pi/2$, where sine attains its largest possible value 1. Thus

$$f(x) = \begin{cases} \sin x, & 0 \le x \le \pi/2, \\ 1, & \pi/2 < x \le \pi, \\ 2 + \cos x, & x > \pi. \end{cases}$$

Each formula is smooth in the interior of its own interval; only $\pi/2$ and π require a join check.

Step 2: Check the join at the first maximum At $x = \pi/2$, both adjoining function values are 1. The left derivative is $\cos(\pi/2) = 0$; the right derivative of the constant branch is 0. Therefore this join is continuous and differentiable. Although the formula changes here, there is no corner: the sine curve has a horizontal tangent at its maximum.

Step 3: Check the join at pi and choose the statement At $x = \pi$, the left branch and the assigned value give 1, while the right limit is $2 + \cos \pi = 1$. The left derivative is 0, and the right derivative is $-\sin \pi = 0$. This join is also differentiable. Hence there is no discontinuity or nondifferentiable point anywhere in $(0, \infty)$. Options claiming one or two exceptional points are all false. At the domain endpoint 0, the function is right-continuous, which is the relevant endpoint continuity condition.

Step 4: Conclusion f is differentiable everywhere in $(0, \infty)$. Hence, the correct option is (B).

Quick Tip to Solve: Replace a running maximum by its actual piecewise formula first; a switch of formulas does not create a corner when both one-sided slopes are zero.

13. Let N be the set of natural numbers and a relation R on N be defined by $R = \{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$. Then the relation R is : [4 marks]

(A) symmetric but neither reflexive nor transitive

(B) reflexive but neither symmetric nor transitive

(C) reflexive and symmetric, but not transitive

(D) an equivalence relation

Correct Answer: (B) reflexive but neither symmetric nor transitive

Solution:

Concept:

- A relation is reflexive when every (x, x) belongs to it, symmetric when (x, y) always implies (y, x), and transitive when $(x, y), (y, z)$ imply (x, z).
- Factoring the defining polynomial reveals which pairs belong to the relation. One explicit counterexample suffices to disprove symmetry or transitivity.

Step 1: Factor the defining equation Group its terms as

$$\begin{aligned} x^3 - 3x^2y - xy^2 + 3y^3 &= x^2(x - 3y) - y^2(x - 3y) \\ &= (x - 3y)(x - y)(x + y). \end{aligned}$$

For positive natural numbers, $x + y > 0$, so membership means either $x = y$ or $x = 3y$. If a convention includes zero in the natural numbers, the only additional possibility from $x + y = 0$ is $(0, 0)$, already included by $x = y$; the conclusion is unchanged.

Step 2: Check reflexivity and disprove symmetry Every natural number satisfies $x = x$, so every pair (x, x) lies in R. Hence R is reflexive. However $(3, 1) \in R$ because $3 = 3 \cdot 1$, whereas $(1, 3) \notin R$: 1 is neither 3 nor $3 \cdot 3$. Thus reversing a related pair need not preserve membership, and the relation is not symmetric.

Step 3: Disprove transitivity and compare the options Take $(9, 3) \in R$ and $(3, 1) \in R$, both by the condition $x = 3y$. For transitivity we would need $(9, 1) \in R$. But 9 is neither 1 nor $3 \cdot 1$, so that pair is absent. The relation is therefore not transitive. The options asserting symmetry fail this counterexample check, and an equivalence relation would need all three properties, which are not present.

Step 4: Conclusion R is reflexive but neither symmetric nor transitive. Hence, the correct option is (B).

Quick Tip to Solve: For polynomial relations, factor first and use small integer pairs to test reversals and chains; avoid trying to prove a property before identifying the actual pair conditions.

14. Consider a circle C which touches the y-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to : [4 marks]

(A) $\sqrt{53}$

(B) 9

(C) 8

(D) $\sqrt{82}$

Correct Answer: (B) 9

Solution:

Concept:

- At a point where a circle touches a line, its radius is perpendicular to that tangent. A circle tangent to the y-axis at $(0, 6)$ therefore has centre $(h, 6)$ and radius $r = |h|$.
- A chord at perpendicular distance d from the centre has half-length $\sqrt{r^2 - d^2}$, by the right triangle formed by the centre, chord midpoint and chord endpoint.

Step 1: Identify the chord and its distance from the centre The intercept cut off on the x-axis is the full chord length, given as $6\sqrt{5}$. The centre has vertical coordinate 6 because the tangent radius at $(0, 6)$ is horizontal. Its perpendicular distance from the x-axis is therefore 6, independent of whether the circle lies to the right or left of the y-axis. Half the intercept is $3\sqrt{5}$.

Step 2: Use the right triangle for half the chord The perpendicular from the centre bisects this horizontal chord. Applying Pythagoras to either half gives

$$r^2 = 6^2 + (3\sqrt{5})^2 = 36 + 45 = 81.$$

The radius is positive, so $r = 9$. Equivalently, the equation is $(x - h)^2 + (y - 6)^2 = h^2$; setting $y = 0$ gives endpoints $x = h \pm \sqrt{h^2 - 36}$, whose separation is $2\sqrt{r^2 - 36}$.

Step 3: Check the remaining radii With $r = 9$, the intercept is $2\sqrt{81 - 36} = 6\sqrt{5}$, exactly as required. The radii $\sqrt{53}$, 8 and $\sqrt{82}$ instead produce chord lengths $2\sqrt{17}$, $4\sqrt{7}$ and $2\sqrt{46}$, respectively. None equals the stated intercept, so those alternatives are excluded.

Step 4: Conclusion The radius of C is 9. Hence, the correct option is (B).

Quick Tip to Solve: Treat an axis intercept as the full chord length. Use half of it in Pythagoras with the centre-to-axis distance to avoid a factor-of-two error.

15. Let f : (a, b) $\to$ R be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function g(x). If f(x) = 0 has exactly five distinct roots in (a, b), then g(x)g'(x) = 0 has at least :

[4 marks]

(A) twelve roots in (a, b)

(B) five roots in (a, b)

(C) seven roots in (a, b)

(D) three roots in (a, b)

Correct Answer: (C) seven roots in (a, b)

Solution:

Concept:

- For a differentiable function, Rolle's theorem gives a zero of its derivative strictly between two distinct zeros of the function.
- The fundamental theorem of calculus gives $f\prime(x) = g(x)$. Apply Rolle's theorem again to g to count zeros of $g\prime$.

Step 1: Order the five roots Let the five distinct roots be $a < x_1 < x_2 < x_3 < x_4 < x_5 < b$. On each closed interval $[x_j, x_{j+1}]$, the function is continuous and is differentiable inside, with equal endpoint values zero. Thus Rolle's theorem produces points $u_j \in (x_j, x_{j+1})$, for $j = 1, 2, 3, 4$, such that $f\prime(u_j) = g(u_j) = 0$. Their intervals are disjoint, so these are four distinct zeros.

Step 2: Count the intervening derivative zeros Apply the same theorem to g on $[u_1, u_2]$, $[u_2, u_3]$ and $[u_3, u_4]$. There are points $v_j \in (u_j, u_{j+1})$ at which $g\prime(v_j) = 0$. These three points are distinct from all four u_j, because they lie strictly between successive ones. Therefore $g(x)g\prime(x) = 0$ holds at each of the seven distinct points $u_1, v_1, u_2, v_2, u_3, v_3, u_4$. We need only a guaranteed lower count; extra roots may exist. The counts three and five miss guaranteed roots, while twelve is not established by the hypotheses.

Step 3: Conclusion There are at least $4 + 3 = 7$ distinct roots in (a, b). Hence, the correct option is (C).

Quick Tip to Solve: Interlace the zeros: five zeros of a function give four derivative zeros and then three second-derivative zeros; strict intervals prevent double counting.

16. For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$, lies on the plane x + 2y − z = 8, then $\alpha - \beta$ is equal to : [4 marks]

(A) 5

(B) 9

(C) 3

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- Write each symmetric line using its own parameter. At the point of intersection, corresponding coordinates must agree.
- A point on $x + 2y - z = 8$ must satisfy that equation after its coordinates are substituted.

Step 1: Match the two line parameters Use parameters t and s for the first and second lines respectively. Their points are $(\alpha + t, 1 + 2t, 1 + 3t)$ and $(4 + \beta s, 6 + 3s, 7 + 3s)$. Matching the last two coordinates gives

$$2t - 3s = 5, \qquad 3t - 3s = 6.$$

Subtract the first equation from the second to obtain $t = 1$. Substitution then gives $2 - 3s = 5$, so $s = -1$. The intersection consequently has $y = 3$ and $z = 4$.

Step 2: Use the plane to recover the constants Let its remaining coordinate be x. The plane equation gives

$$x + 2(3) - 4 = 8 \quad \Longrightarrow \quad x = 6.$$

The first line now yields $\alpha + 1 = 6$, hence $\alpha = 5$. The second gives $4 - \beta = 6$, hence $\beta = -2$, which satisfies the nonzero condition. Both lines therefore pass through $(6, 3, 4)$, and this point also satisfies the plane equation, providing a direct check on every substitution. The requested difference is $5 - (-2) = 7$; the alternatives 5, 9 and 3 do not equal this difference. In particular, 5 is the value of α alone, not the quantity asked for.

Step 3: Conclusion $\alpha - \beta = 7$. Hence, the correct option is (D).

Quick Tip to Solve: First solve the coordinate equations that contain neither unknown constant; use the plane only after those two coordinates are known.

17. A student appeared in an examination consisting of 8 true-false type questions. The student guesses the answers with equal probability. the smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is : [4 marks]

(A) 5

(B) 6

(C) 3

(D) 4

Correct Answer: (A) 5

Solution:

Concept:

- The number X of correct independent guesses follows $\mathrm{Bin}(8, 1/2)$, with $P(X = k) = \binom{8}{k}/256$.
- The distribution is symmetric about four: $P(X = k) = P(X = 8 - k)$. Tail probabilities decrease as their lower cutoff increases.

Step 1: Compare the two central tails The middle probability is

$$P(X = 4) = \frac{\binom{8}{4}}{2^8} = \frac{70}{256}.$$

Symmetry pairs outcomes below four with those above four, so

$$P(X \geq 5) = \frac{1 - P(X = 4)}{2} = \frac{93}{256} < \frac{1}{2}.$$

Thus a cutoff of five correct answers has the required strict inequality. This calculation removes the central mass before dividing the remainder into its equal lower and upper tails.

Step 2: Check that the cutoff is the smallest Including the middle outcome instead gives

$$P(X \geq 4) = \frac{93 + 70}{256} = \frac{163}{256} > \frac{1}{2}.$$

For any integer $n \leq 4$, the event $X \geq 4$ is a subset of $X \geq n$, so its probability cannot decrease; no such cutoff works. A cutoff of six also has a probability below one half, but it is larger than the first acceptable cutoff five. The alternatives three and four retain too much probability, while six fails the requirement of being the smallest. The wording at least n includes the endpoint $X = n$, which is why the central probability must be handled explicitly.

Step 3: Conclusion The smallest value is $n = 5$. Hence, the correct option is (A).

Quick Tip to Solve: For an even number of fair binary trials, remove the central probability: the tail strictly above the midpoint is below one half, while the tail including it is above one half.

18. Which of the following is the negation of the statement "for all M $>$ 0, there exists x$\in$S such that x $\geq$ M" ? [4 marks]

(A) there exists M $>$ 0, such that x $<$ M for all x$\in$S

(B) there exists M $>$ 0, there exists x$\in$S such that x $\geq$ M

(C) there exists M $>$ 0, there exists x$\in$S such that x $<$ M

(D) there exists M $>$ 0, such that x $\geq$ M for all x$\in$S

Correct Answer: (A) there exists M $>$ 0, such that x $<$ M for all x$\in$S

Solution:

Concept:

- Negation interchanges the quantifiers: $\neg(\forall u\, P) = \exists u\, \neg P$ and $\neg(\exists u\, P) = \forall u\, \neg P$.
- For real numbers, the negation of $x \geq M$ is the strict inequality $x < M$. Restrictions on quantified variables remain the same.

Step 1: Write and negate the quantifiers The given statement is

$$\forall M > 0, \quad \exists x \in S : \; x \geq M.$$

To say that this is false, we must be able to find at least one positive threshold for which its inner assertion fails. Thus its negation starts with $\exists M > 0$, followed by there is no $x \in S$ satisfying $x \geq M$. The threshold need not work for every positive M; a single counterexample threshold is sufficient.

Step 2: Negate the inner assertion and compare There being no element with $x \geq M$ means that every element of S is below that threshold:

$$\exists M > 0, \quad \forall x \in S : \; x < M.$$

This is exactly the statement in (A). Option (B) retains an existential element and the original inequality; it does not deny the original assertion. Option (C) merely finds one element below a positive threshold and says nothing about other elements. Option (D) changes the quantifier on x but leaves the inequality unnegated. For example, the original describes elements reaching arbitrarily large positive thresholds, whereas its negation gives one positive upper bound strictly above every element.

Step 3: Conclusion The negation is: there exists $M > 0$ such that $x < M$ for all $x \in S$. Hence, the correct option is (A).

Quick Tip to Solve: Negate a quantified statement from left to right: flip every quantifier and negate the final predicate, preserving the variable domains.

19. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta\left(0 < \theta < \frac{\pi}{2}\right)$, then the value of 1 + tanθ is equal to : [4 marks]

(A) $\sqrt{3}+1$

(B) 2

(C) 1

(D) $\frac{\sqrt{3}+1}{\sqrt{3}}$

Correct Answer: (B) 2

Solution:

Concept:

- The vector triple-product identity is $\mathbf{b} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{b} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{b} \cdot \mathbf{b})$.
- For a unit vector $\mathbf{b}$, this expression is the negative component of $\mathbf{c}$ perpendicular to $\mathbf{b}$; its magnitude is $|\mathbf{c}| \sin\theta$.

Step 1: Resolve the double cross product Since $|\mathbf{b}| = 1$ and $|\mathbf{c}| = 2$, the triple-product identity gives

$$\mathbf{a} = 2\cos\theta\,\mathbf{b} - \mathbf{c}.$$

Taking its squared magnitude and using $\mathbf{b} \cdot \mathbf{c} = 2\cos\theta$, we obtain

$$|\mathbf{a}|^2 = 4\cos^2\theta + 4 - 8\cos^2\theta = 4\sin^2\theta.$$

This also follows by noticing that the parallel component of $\mathbf{c}$ cancels. It is the perpendicular component, rather than the parallel component, that survives the double cross product.

Step 2: Use the angle restriction The given magnitude $|\mathbf{a}| = \sqrt{2}$ yields

$$2 = 4\sin^2\theta, \qquad \sin^2\theta = \frac{1}{2}.$$

Because $0 < \theta < \pi/2$, sine and cosine are both positive. Hence $\sin\theta = \cos\theta = 1/\sqrt{2}$, giving $\theta = \pi/4$ and $\tan\theta = 1$. The desired expression is therefore $1 + 1 = 2$. Option (C) would correspond to a zero tangent, excluded by the angle restriction, while the tangent values implicit in (A) and (D) are $\sqrt{3}$ and $1/\sqrt{3}$; neither satisfies the calculated value $\sin^2\theta = 1/2$.

Step 3: Conclusion $1 + \tan\theta = 2$. Hence, the correct option is (B).

Quick Tip to Solve: When the repeated vector in a double cross product is a unit vector, read off the perpendicular component of the third vector before doing coordinate algebra.

20. Let A and B be two 3 × 3 real matrices such that ($A^2 - B^2$) is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to : [4 marks]

(A) 2

(B) 4

(C) 1

(D) 0

Correct Answer: (D) 0

Solution:

Concept:

- Matrix multiplication need not commute. Factor only products whose factor order is preserved, and use $\det(XY) = \det X \det Y$.
- For a linear map on a three-dimensional space, its kernels stabilize by the third power: $\ker T^3 = \ker T^5$. Once two consecutive kernels are equal, every later kernel is equal too.

Step 1: Show that the common fifth power is singular Put $D = A^2 - B^2$, which is invertible. The two given equalities imply

$$A^3D = A^5 - A^3B^2 = B^5 - A^2B^3 = -DB^3.$$

Taking determinants, with matrix order three, and cancelling the nonzero $\det D$, gives $(\det A)^3 = -(\det B)^3$. Also $A^5 = B^5$ gives $(\det A)^5 = (\det B)^5$. Since the determinants are real, the latter equality implies $\det A = \det B$. Combined, these show that both determinants are zero. Thus the common matrix $C = A^5 = B^5$ is singular and has a nonzero vector v in its kernel.

Step 2: Find a vector annihilated by the required sum We have $A^5v = B^5v = 0$. For any matrix of order three, the increasing chain of kernels $\ker T \subseteq \ker T^2 \subseteq \ker T^3 \subseteq \cdots$ has stabilized by exponent three: every strict inclusion increases the dimension, and there are only three dimensions available. Therefore $A^3v = 0$ and $B^3v = 0$. It follows that

$$(A^3 + B^3)v = 0, \qquad v \neq 0.$$

Consequently the required matrix is singular and its determinant is zero. The alternatives 2, 4 and 1 are all nonzero and would require an invertible matrix, contradicting this nonzero kernel vector. This argument never assumes that A and B commute or uses an invalid scalar factorisation for their sum.

Step 3: Conclusion $\det(A^3 + B^3) = 0$. Hence, the correct option is (D).

Quick Tip to Solve: Preserve factor order in matrix identities. To prove a determinant is zero, a shared nonzero kernel vector is enough; computing the entries is unnecessary.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to _____. [4 marks]

Solution:

Concept:

- To find the real part of a complex quotient, multiply numerator and denominator by the conjugate of the denominator.
- The restriction $0 < \theta < \pi/2$ makes $\cos\theta$ positive and fixes the angle once $\cos^2\theta$ is known.

Step 1: Rationalise the denominator Write $c = \cos\theta$. Multiply by $1 + 3ic$:

$$z = \frac{(3+2ic)(1+3ic)}{1+9c^2} = \frac{3-6c^2+11ic}{1+9c^2}.$$

Here the real contribution of the product of the imaginary terms is $6i^2c^2 = -6c^2$, while the two imaginary cross terms add to $11ic$. The denominator $1 + 9c^2$ is strictly positive, so the real part can vanish only when its real numerator is zero. Thus

$$3 - 6c^2 = 0 \quad \Longrightarrow \quad \cos^2\theta = \frac{1}{2}.$$

Step 2: Evaluate the requested expression The angle lies in the first quadrant, so $\cos\theta = 1/\sqrt{2}$ and $\theta = \pi/4$. Therefore $3\theta = 3\pi/4$, and

$$\sin^2(3\theta) + \cos^2\theta = \sin^2\left(\frac{3\pi}{4}\right) + \cos^2\left(\frac{\pi}{4}\right) = \frac{1}{2} + \frac{1}{2} = 1.$$

The first term must be evaluated at three times the angle; it cannot initially be combined with the second by the identity $\sin^2 u + \cos^2 u = 1$, since their arguments differ. Substitution into the rationalised form also checks that z is purely imaginary at this angle. The requested value is a dimensionless number, because it is a sum of squared trigonometric ratios.

Step 3: Conclusion The value of $\sin^2(3\theta) + \cos^2\theta$ is 1 (dimensionless).

Quick Tip to Solve: After rationalising, set only the real numerator to zero; check the angle domain before substituting into trigonometric multiples.

22. If $\int_0^{\pi} (\sin^3 x)e^{-\sin^2 x}dx = \alpha - \frac{\beta}{e}\int_0^1 \sqrt{t}e^t dt$, then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- The symmetry $\sin(\pi - x) = \sin x$ converts an integral over $[0, \pi]$ into twice the integral over $[0, \pi/2]$. This keeps the substitution $u = \cos x$ one-to-one.
- Integration by parts relates $J = \int_0^1 \sqrt{t}\, e^t\, dt$ to $K = \int_0^1 e^{u^2}\, du$. Neither integral needs an elementary antiderivative to find the requested coefficients.

Step 1: Use symmetry and substitute Let the given integral be I. On the first half of the interval, put $u = \cos x$, so $du = -\sin x\, dx$, $\sin^2 x = 1 - u^2$, and the endpoints change from 1 to 0. Thus

$$I = 2\int_0^{\pi/2} \sin^3 x\, e^{-\sin^2 x}\, dx = \frac{2}{e}\int_0^1 (1 - u^2)e^{u^2}\, du.$$

The factor $2/e$ includes both the symmetry and e^{-1}; it must not be omitted.

Step 2: Reduce the squared term Using $d(ue^{u^2})/du = e^{u^2} + 2u^2e^{u^2}$, write

$$\int_0^1 u^2e^{u^2}\, du = \frac{e - K}{2}, \qquad I = \frac{3K}{e} - 1.$$

Now integrate J by parts with $\sqrt{t}$ as the differentiated factor:

$$J = e - \frac{1}{2}\int_0^1 \frac{e^t}{\sqrt{t}}\, dt = e - K.$$

In the last integral, $t = u^2$ gives $dt/\sqrt{t} = 2\, du$. Its integrable endpoint singularity causes no boundary contribution.

Step 3: Compare the coefficients Substitute $K = e - J$ into the expression for I:

$$I = \frac{3}{e}(e - J) - 1 = 2 - \frac{3}{e}J.$$

This is exactly the form in the question, so $\alpha = 2$ and $\beta = 3$.

Step 4: Conclusion Therefore $\alpha + \beta = 2 + 3 = \boxed{5}$. This is a dimensionless numerical value.

Quick Tip to Solve: Apply symmetry first, then relate the two remaining integrals by parts; evaluating either non-elementary integral is unnecessary.

23. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^{x} + 1 = 0$ is equal to _____.

[4 marks]

Solution:

Concept:

- The substitution $t = e^x$ is a bijection from real x to positive t. Counting real solutions in x is therefore the same as counting positive roots in t.
- A reciprocal quartic becomes a quadratic in $s = t + 1/t$ after division by t^2. For positive t, the arithmetic-geometric mean inequality gives $s \geq 2$.

Step 1: Convert the exponential equation Set $t = e^x > 0$. The equation becomes

$$t^4 - t^3 - 4t^2 - t + 1 = 0.$$

Since t is strictly positive, division by t^2 loses no admissible solution. Pair the reciprocal terms to obtain

$$t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) - 4 = 0.$$

This pairing is preferable to treating the four powers as unrelated terms.

Step 2: Solve the quadratic in the reciprocal sum Put $s = t + 1/t$. The identity $t^2 + t^{-2} = s^2 - 2$ reduces the equation to

$$s^2 - s - 6 = (s - 3)(s + 2) = 0.$$

The possibility $s = -2$ is excluded because $t > 0$ forces $s \geq 2$. Only $s = 3$ is allowed. Counting both quadratic roots in s without this domain check would give an incorrect answer.

Step 3: Count the positive roots For $s = 3$, multiply $t + 1/t = 3$ by t:

$$t^2 - 3t + 1 = 0, \qquad t = \frac{3 + \sqrt{5}}{2} \text{ or } \frac{3 - \sqrt{5}}{2}.$$

Both values are positive, distinct and nonzero, so they correspond to exactly two distinct real numbers $x = \ln t$. The discarded branch $s = -2$ would give $t = -1$, which cannot equal an exponential of a real number.

Step 4: Conclusion The equation has exactly $\boxed{2}$ real roots. A root count is dimensionless.

Quick Tip to Solve: For reciprocal polynomials in an exponential, substitute a positive variable first and enforce its domain after using the reciprocal sum.

24. Let y = y(x) be the solution of the differential equation dy = $e^{\alpha x + y}$ dx; $\alpha \in$ N. If y($\log_e 2$) = $\log_e 2$ and y(0) = $\log_e\left(\frac{1}{2}\right)$, then the value of α is equal to _____. [4 marks]

Solution:

Concept:

- The equation $dy/dx = e^{\alpha x+y}$ is separable: $e^{-y}\,dy = e^{\alpha x}\,dx$. Since $\alpha \in \mathbb{N}$, $\alpha > 0$ and division by it is valid.
- Both prescribed values of the same solution must satisfy its integrated equation with the same integration constant. They determine a scalar equation for α.

Step 1: Integrate and apply the initial value Separating variables and integrating gives

$$-e^{-y} = \frac{e^{\alpha x}}{\alpha} + C.$$

It is useful to absorb the constant in the form

$$e^{-y} = C_1 - \frac{e^{\alpha x}}{\alpha}.$$

At $x = 0$, the condition $y(0) = \ln(1/2)$ means $e^{-y(0)} = 2$. Hence $C_1 = 2 + 1/\alpha$, and

$$e^{-y} = 2 - \frac{e^{\alpha x} - 1}{\alpha}.$$

The minus sign comes from integrating e^{-y}, and is essential.

Step 2: Use the second specified point At $x = \ln 2$, the value $y = \ln 2$ gives $e^{-y} = 1/2$, while $e^{\alpha \ln 2} = 2^{\alpha}$. Therefore

$$\frac{1}{2} = 2 - \frac{2^{\alpha} - 1}{\alpha}, \qquad 2^{\alpha} - 1 = \frac{3\alpha}{2}.$$

The integer $\alpha = 2$ satisfies this equation because $4 - 1 = 3$.

Step 3: Check uniqueness and the solution domain For positive integers define $f(a) = 2^a - 1 - 3a/2$. Then $f(1) = -1/2$, $f(2) = 0$, and

$$f(a+1) - f(a) = 2^a - \frac{3}{2} > 0 \quad (a \geq 1).$$

Thus no other positive integer is a root. With $\alpha = 2$, the solution has $e^{-y} = (5 - e^{2x})/2$, which stays positive between 0 and $\ln 2$, so both data belong to one real solution.

Step 4: Conclusion The required parameter is $\boxed{\alpha = 2}$, a dimensionless positive integer.

Quick Tip to Solve: Apply the initial value to the exponential of minus the dependent variable; this avoids taking logarithms until the parameter is known.

25. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to _____. [4 marks]

Solution:

Concept:

- Prime factorization gives $10^{10}11^{11}13^{13} = 2^{10}5^{10}11^{11}13^{13}$. Every positive divisor has a unique selection of exponents within these ranges.
- A divisor of the form $4n+1$ is odd and congruent to 1 modulo 4. Modulo 4, both 5 and 13 are 1, whereas 11 is -1.

Step 1: Write the general divisor Let a divisor be

$$d = 2^a 5^b 11^c 13^r,$$

where $0 \le a \le 10$, $0 \le b \le 10$, $0 \le c \le 11$, and $0 \le r \le 13$. These exponents are independent because the four bases are distinct primes. Since $4n+1$ is odd, necessarily $a = 0$. All divisors containing a positive power of 2 can therefore be excluded immediately.

Step 2: Impose the residue condition For the remaining odd divisors, reduce the prime factors modulo 4:

$$d = 5^b 11^c 13^r \equiv 1^b(-1)^c 1^r = (-1)^c \pmod 4.$$

Consequently $d \equiv 1 \pmod 4$ if and only if c is even. The permitted exponents are $c = 0, 2, 4, 6, 8, 10$, giving six choices. There is no parity restriction on b or r, since every power of 5 or 13 remains 1 modulo 4.

Step 3: Multiply the independent choices The exponent b has 11 possible values, from 0 through 10, and r has 14 possible values, from 0 through 13. Thus

$$\#\{d : d \equiv 1 \pmod 4\} = 1 \times 11 \times 6 \times 14 = 924.$$

Every counted divisor is a positive integer congruent to 1 modulo 4, so $n = (d-1)/4$ is indeed a non-negative integer. The divisor $d = 1$ is included, corresponding to $n = 0$; no endpoint needs to be removed.

Step 4: Conclusion The number of divisors of the required form is $\boxed{924}$. This count has no unit.

Quick Tip to Solve: For divisors in a residue class, first remove forbidden prime factors and then count exponent parities rather than listing the divisors.

26. Let A = {n ∈ N | $n^2 \leq$ n + 10,000}, B = {3k + 1 | k∈ N} an dC = {2k | k∈N}, then the sum of all the elements of the set A ∩(B − C) is equal to _____. [4 marks]

Solution:

Concept:

- The inequality defining A restricts the positive integer n. The set difference $B - C$ retains members of B that are not even, because C is the set of positive even integers.
- For $3k + 1$ to be odd, k must be even. The permitted values then form an arithmetic progression, whose sum is the number of terms times the average of the endpoints.

Step 1: Determine the finite set Consider $g(n) = n^2 - n$. For positive integers, $g(n+1) - g(n) = 2n > 0$, so the defining expression increases strictly. At the two integers around the upper boundary,

$$100^2 - 100 = 9900 \leq 10000, \qquad 101^2 - 101 = 10100 > 10000.$$

It follows that $A = \{1, 2, \ldots, 100\}$. Using just $n^2 \leq 10000$ without checking the linear term would not establish this boundary rigorously.

Step 2: Remove the even members of the progression An element of B is $3k+1$ with $k \in \mathbb{N} = \{1, 2, \ldots\}$. It survives subtraction of C precisely when it is odd. This requires k even, so write $k = 2r$, where $r \geq 1$. Thus the retained numbers have the form $6r + 1$. Intersecting with A gives

$$6r + 1 \leq 100 \quad \Longrightarrow \quad 1 \leq r \leq 16.$$

Hence the intersection is $\{7, 13, 19, \ldots, 97\}$.

Step 3: Sum the surviving elements There are 16 terms, the first is 7 and the last is 97. Their common difference is 6, which agrees with the endpoint check $7 + 15 \cdot 6 = 97$. Therefore

$$\sum_{n \in A \cap (B-C)} n = \frac{16}{2}(7 + 97) = 8 \cdot 104 = 832.$$

The number 1 is not a member of B, since obtaining it would require $k = 0$, outside the natural-number range used here.

Step 4: Conclusion The required sum is $\boxed{832}$, which is dimensionless.

Quick Tip to Solve: Translate a set difference into a parity restriction before intersecting with a finite interval; the remaining sequence is easy to sum.

27. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and M = A + A^2 + A^3 + ... + A^{20}, then the sum of all the elements of the matrix M is equal to _____. [4 marks]

Solution:

Concept:

- Write the upper triangular matrix as $A = I + N$. Here $N^3 = 0$, so the binomial formula for $(I+N)^k$ terminates after the quadratic term.
- The sum of all entries is linear: the sum of entries of $M = \sum_{k=1}^{20} A^k$ equals the sum, over k, of the entry sums of the individual powers.

Step 1: Identify the nilpotent part Subtract the identity from the given matrix:

$$N = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad N^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad N^3 = 0.$$

Thus repeated multiplication need not be carried out twenty times. Because I and N commute, the ordinary binomial coefficients apply to their matrix sum.

Step 2: Find the entries of a general power For each integer $k \geq 1$,

$$A^k = I + kN + \binom{k}{2} N^2 = \begin{pmatrix} 1 & k & k(k+1)/2 \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}.$$

In particular, the top-right entry is $k + \binom{k}{2}$, not just $\binom{k}{2}$, since N itself has a nonzero top-right entry. The sum of all entries in this power is

$$3 + 2k + \frac{k(k+1)}{2}.$$

Step 3: Add the entry sums Use $\sum_{k=1}^{20} k = 210$ and $\sum_{k=1}^{20} k^2 = 20 \cdot 21 \cdot 41/6 = 2870$. Then

$$\begin{aligned} \text{sum of entries of } M &= 60 + 2(210) + \frac{1}{2}(2870 + 210) \\ &= 60 + 420 + 1540 \\ &= 2020. \end{aligned}$$

This includes all three diagonal entries for each of the twenty powers; the entries below the diagonal remain zero throughout. It also agrees with the individual upper entries of M, namely 210, 210 and 1540.

Step 4: Conclusion The sum of all the elements of M is $\boxed{2020}$, with no physical unit.

Quick Tip to Solve: For an upper triangular matrix with diagonal entries 1, subtract the identity and check nilpotency before computing powers.

28. **Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$, then $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}$ is equal to _____.** [4 marks]

Solution:

Concept:

- The dot product is the sum of products of corresponding components. The two prescribed dot products therefore give algebraic conditions on the integers α and β.
- The scalar triple product $(\vec{a} \times \vec{b}) \cdot \vec{c}$ can be evaluated by first taking the ordered cross product. Its sign depends on preserving the specified vector order.

Step 1: Extract the parameter equations In component form, $\vec{a} = (1, -\alpha, \beta)$, $\vec{b} = (3, \beta, -\alpha)$, and $\vec{c} = (-\alpha, -2, 1)$. Thus

$$\vec{a} \cdot \vec{b} = 3 - 2\alpha\beta = -1 \quad \Longrightarrow \quad \alpha\beta = 2.$$

Similarly,

$$\vec{b} \cdot \vec{c} = -3\alpha - 2\beta - \alpha = 10 \quad \Longrightarrow \quad 2\alpha + \beta = -5.$$

Both negative signs from the last components must be retained.

Step 2: Determine the integer pair Substitute $\beta = -5 - 2\alpha$ into $\alpha\beta = 2$:

$$2\alpha^2 + 5\alpha + 2 = 0, \qquad (2\alpha + 1)(\alpha + 2) = 0.$$

The root $\alpha = -1/2$ is excluded by the integer condition. Hence $\alpha = -2$ and $\beta = -1$. The vectors are consequently

$$\vec{a} = (1, 2, -1), \qquad \vec{b} = (3, -1, 2), \qquad \vec{c} = (2, -2, 1).$$

Their dot products are $3 - 2 - 2 = -1$ and $6 + 2 + 2 = 10$, confirming the parameter choice.

Step 3: Evaluate the ordered triple product Compute the cross product in the stated order:

$$\begin{aligned} \vec{a} \times \vec{b} &= \big(2 \cdot 2 - (-1)(-1),\ (-1)3 - 1 \cdot 2,\ 1(-1) - 2 \cdot 3\big) \\ &= (3, -5, -7). \end{aligned}$$

Taking its dot product with $\vec{c}$ now gives

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = 3(2) + (-5)(-2) + (-7)(1) = 6 + 10 - 7 = 9.$$

Reversing the first two vectors would change the sign and would answer a different expression.

Step 4: Conclusion The required scalar triple product is $\boxed{9}$. The vectors carry no specified physical units, so this value is dimensionless.

Quick Tip to Solve: Use the integer restriction while solving the dot-product equations, and preserve the order of the cross product to avoid a sign error.

29. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points. Q(3, −4, −5) and R(2, −3, 1) and the plane 2x + y + z = 7, is equal to _____. [4 marks]

Solution:

Concept:

- A line through two points has direction equal to the difference of their position vectors. Its parametric form gives all three coordinates using the same parameter.
- The intersection with a plane satisfies both its line equations and the plane equation. The distance between two points is the square root of the sum of their squared coordinate differences.

Step 1: Parameterise the line through Q and R The direction from $Q(3, -4, -5)$ to $R(2, -3, 1)$ is $(-1, 1, 6)$. Thus a general point S on the line is written as

$$S = (3 - t, -4 + t, -5 + 6t).$$

The choice $t = 0$ gives Q, while $t = 1$ gives R, which checks the signs of the direction components. The required intersection is on the full line; it need not lie between the two given points.

Step 2: Impose the plane equation Substitute all three coordinates into $2x + y + z = 7$:

$$2(3 - t) + (-4 + t) + (-5 + 6t) = 7.$$

Collecting constants and parameter terms gives $-3 + 5t = 7$, so $t = 2$. Hence the intersection is $S = (1, -2, 7)$. As a check, $2(1) - 2 + 7 = 7$, so this point really lies in the given plane.

Step 3: Calculate the distance from P The displacement from S to $P(3, 4, 4)$ is $(2, 6, -3)$. Therefore

$$PS = \sqrt{(3 - 1)^2 + (4 + 2)^2 + (4 - 7)^2} = \sqrt{4 + 36 + 9} = 7.$$

This is the distance to the particular intersection point, rather than the perpendicular distance to the plane or to the line.

Step 4: Conclusion The required distance is $\boxed{7}$ coordinate units. The numerical entry is 7.

Quick Tip to Solve: Find the line-plane intersection first. A point-to-plane distance formula would answer a different question here.

30. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, −4), one focus at (4, −4) and one vertex at (5, −4). If mx − y = 4, m > 0 is a tangent to the ellipse E, then the value of $5m^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For a horizontal ellipse, $c^2 = a^2 - b^2$, where a is the semimajor axis, b the semiminor axis and c the centre-to-focus distance.
- For $X^2/a^2 + Y^2/b^2 = 1$, the line $Y = mX + k$ is tangent precisely when $k^2 = a^2m^2 + b^2$. This

condition follows by making the intersection quadratic have a repeated root.

Step 1: Find the equation of the ellipse The centre is $(3, -4)$. The focus $(4, -4)$ and vertex $(5, -4)$ are both to its right, so the major axis is horizontal. Their distances from the centre give $c = 1$ and $a = 2$. Consequently $b^2 = 4 - 1 = 3$, and the ellipse is

$$\frac{(x-3)^2}{4} + \frac{(y+4)^2}{3} = 1.$$

The axes being parallel to the coordinate axes means there is no rotation term.

Step 2: Translate the tangent into centred coordinates Put $X = x - 3$ and $Y = y + 4$. The proposed line $mx - y = 4$ is $y = mx - 4$. Thus

$$Y = m(X + 3) = mX + 3m.$$

The intercept relative to the centre is therefore $3m$, not the original intercept -4. Use this translated intercept when applying the standard tangent condition.

Step 3: Apply the tangency condition Tangency requires

$$(3m)^2 = 4m^2 + 3.$$

Hence $9m^2 - 4m^2 = 3$, or $m^2 = 3/5$. The given condition $m > 0$ selects $m = \sqrt{3/5}$. The requested expression depends only on its square, so there is no sign ambiguity in its final value.

Step 4: Conclusion We obtain $\boxed{5m^2 = 3}$. The required numerical entry is 3.

Quick Tip to Solve: When using a tangent formula for a shifted ellipse, translate the line as well as the ellipse; its new intercept is essential.

Physics – Section A: Multiple Choice Questions (80 marks)

31. An electron and proton are separated by a large distance. The electron starts approaching the proton with energy 3 eV. The proton captures the electron and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4000$\overset{o}{A}$. What is the maximum kinetic energy of the emitted photoelectron? [4 marks]

(A) 7.61 eV

(B) 1.41 eV

(C) 3.3 eV

(D) No photoelectron would be emitted

Correct Answer: (B) 1.41 eV

Solution:

Concept:

- The hydrogen energy levels are $E_n = -13.6/n^2\,\text{eV}$, measured from an electron and proton at rest at infinite separation.
- Energy conservation gives the capture photon energy as the initial electron energy minus the final bound-state energy. Einstein's equation is $K_{\max} = E_\gamma - \phi$, with threshold work function $\phi = hc/\lambda_0$.

Step 1: Calculate the energy of the capture photon The second excited state is $n = 3$, since the ground state is $n = 1$ and the first excited state is $n = 2$. Its energy is

$$E_3 = -\frac{13.6}{9} = -1.511\,\text{eV}.$$

The approaching electron initially has positive continuum energy $3\,\text{eV}$. The captured atom has negative energy, so the emitted photon carries

$$E_\gamma = 3 - (-1.511) = 4.511\,\text{eV}.$$

Subtracting the binding energy magnitude from 3 would reverse the energy released by capture.

Step 2: Subtract the metal work function The threshold wavelength is $4000\,\text{Å} = 400\,\text{nm}$. Using $hc \simeq 1240\,\text{eV nm}$,

$$\phi = \frac{1240}{400} = 3.10\,\text{eV}.$$

Since the incident photon exceeds this threshold, emission occurs and

$$K_{\max} = 4.511 - 3.10 = 1.411\,\text{eV}.$$

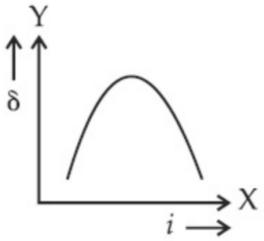
The value $7.61\,\text{eV}$ incorrectly adds the work function. The value $3.3\,\text{eV}$ does not follow the capture energy balance, and the no-emission choice is excluded by $E_\gamma > \phi$.

Step 3: Conclusion The maximum photoelectron kinetic energy is $\boxed{1.41\,\text{eV}}$. Hence, the correct option is (B).

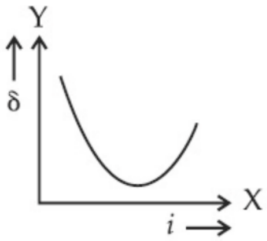
Quick Tip to Solve: For capture from the continuum, add the initial kinetic energy to the magnitude of the final binding energy before applying the photoelectric equation.

32. The expected graphical representation of the variation of angle of deviation 'δ' with angle of incidence 'i' in a prism is : [4 marks]

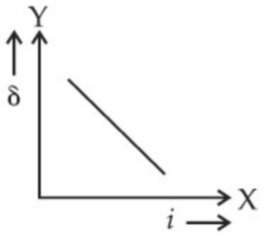
(A)



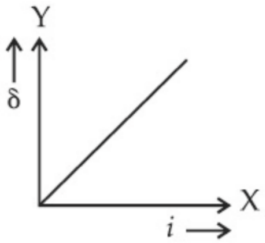
(B)



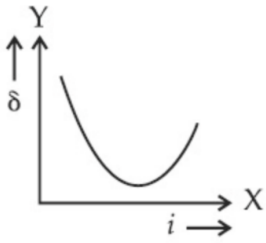
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For a prism of angle A, the deviation is $\delta = i+e-A$, where i and e are the incidence and emergence angles.

- At minimum deviation the optical path is symmetric: $i = e$ and the internal angles are $r_1 = r_2 = A/2$. Optical reversibility pairs the two incidence angles on opposite sides of the minimum.

Step 1: Trace the deviation as incidence increases In the range where the ray emerges from the second face, an increase of incidence from the low-incidence side initially reduces the deviation. The emerging ray moves toward the symmetric path. At that symmetric path the deviation reaches its minimum $\delta_{\min}$. Further increase of incidence takes the ray away from symmetry and makes the deviation rise again. Thus the graph decreases first and increases afterwards; it has a minimum, rather than a maximum.

Step 2: Compare the four plotted alternatives Option (B) has exactly this trough-shaped variation: its falling branch leads to a minimum, followed by a rising branch. The curve need not be an exact parabola; the required feature is the minimum-deviation shape. Option (A) instead shows a maximum and predicts the opposite local behaviour. Option (C) decreases throughout and misses the rise after the symmetric position. Option (D) increases throughout and misses the initial decrease. The nonzero minimum is also consistent with a prism of nonzero refracting angle and refractive index different from that of its surroundings.

Step 3: Conclusion The graph with a minimum deviation is the one shown in option (B). Hence, the correct option is (B).

Quick Tip to Solve: Remember the symmetric prism path at minimum deviation: a deviation-versus-incidence graph must fall and then rise.

33. A raindrop with radius R = 0.2 mm falls from a cloud at a height h = 2000 m above the ground. Assume that the drop is spherical throughout its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is :

[Density of water f_w = 1000 kg m^{-3} and Density of air f_a = 1.2 kg m^{-3}, g = 10 m/s^2, Coefficient of viscosity of air = 1.8 $\times$ 10^{-5} Nsm^{-2}] [4 marks]

(A) 250.6 ms^{-1}

(B) 43.56 ms^{-1}

(C) 4.94 ms^{-1}

(D) 14.4 ms^{-1}

Correct Answer: (C) 4.94 ms^{-1}

Solution:

Concept:

- Stokes' law gives the viscous drag on a sphere as $F_d = 6\pi\eta Rv$, where η is the dynamic viscosity and R its radius.
- At terminal speed the net force is zero. With buoyancy neglected as instructed, the downward weight $(4/3)\pi R^3\rho_w g$ balances the upward viscous drag.

Step 1: Set up the terminal-force balance Once terminal speed is reached, the acceleration vanishes

even though the drop is moving. Equating drag and weight gives

$$6\pi\eta R v_t = \frac{4}{3}\pi R^3 \rho_w g, \qquad v_t = \frac{2R^2\rho_w g}{9\eta}.$$

The air density does not enter this expression because the buoyant force has explicitly been neglected. The cloud height also does not determine the terminal speed; it describes the available fall distance.

Step 2: Convert the radius and substitute Use $R = 0.2\,\text{mm} = 2.0 \times 10^{-4}\,\text{m}$, so $R^2 = 4.0 \times 10^{-8}\,\text{m}^2$. Therefore

$$v_t = \frac{2(4.0 \times 10^{-8})(1000)(10)}{9(1.8 \times 10^{-5})} = 4.938\ldots\,\text{m s}^{-1}.$$

The radius must be converted before squaring. The much larger values in (A), (B) and (D) do not satisfy this viscous force balance. A free-fall calculation from the height would ignore air resistance and would therefore calculate a different speed, not the terminal speed requested.

Step 3: Conclusion The terminal speed in the stated viscous-drag model is $\boxed{4.94\,\text{m s}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: At terminal speed balance forces, rather than using fall height in a free-fall energy formula. Convert millimetres to metres before squaring the radius.

34. One mole of an ideal gas is taken through an adiabatic process where the temperature rises from 27° C to 37° C. If the ideal gas is composed of polyatomic molecule that has 4 vibrational modes, which of the following is true? [R = 8.314 J mol^{-1} k^{-1}] [4 marks]

(A) work done by the gas is close to 332 J

(B) work done on the gas is close to 582 J

(C) work done by the gas is close to 582 J

(D) work done on the gas is close to 332 J

Correct Answer: (B) work done on the gas is close to 582 J

Solution:

Concept:

- For an ideal gas, $\Delta U = nC_V\Delta T$. A nonlinear polyatomic molecule has three translational and three rotational quadratic degrees of freedom.
- Each active vibrational normal mode contributes R to the molar heat capacity, since it stores both kinetic and potential energy. In an adiabatic process $Q = 0$, so $W_{\text{by}} = -\Delta U$ and $W_{\text{on}} = \Delta U$.

Step 1: Count the heat-capacity contributions Translation contributes $3R/2$ and rotation contributes $3R/2$, giving $3R$ before vibration. The four vibrational modes each contribute R. Thus

$$C_V = \frac{3}{2}R + \frac{3}{2}R + 4R = 7R.$$

Equivalently, the total number of quadratic terms is $3 + 3 + 2(4) = 14$, so $C_V = 14R/2$. Counting only one quadratic term per vibrational mode would omit its potential-energy contribution.

Step 2: Apply the first law with the correct sign The temperature change is $37 - 27 = 10\,\mathrm{K}$; a Celsius temperature difference has the same numerical value as a kelvin difference. For one mole,

$$\Delta U = (1)(7)(8.314)(10) = 581.98\,\mathrm{J}.$$

The internal energy increases despite no heat transfer, so work must be done on the gas. Accordingly $W_{\text{on}} \simeq 582\,\mathrm{J}$ and $W_{\text{by}} \simeq -582\,\mathrm{J}$. Options (A) and (C) have the wrong direction of work, while (D) has the wrong magnitude.

Step 3: Conclusion The work done on the gas is close to $\boxed{582\,\mathrm{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: Count a vibrational mode twice in the quadratic degrees of freedom, and use a temperature rise during an adiabatic process to check the work sign.

---

35. An object of mass 0.5 kg is executing simple harmonic motion. It amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}s$ starting from mean position. Assume that the initial phase of the oscillation is zero. [4 marks]

(A) 0.62 J

(B) 6.2×10^{-3} J

(C) 1.2×10^{3} J

(D) 6.2×10^{3} J

Correct Answer: (A) 0.62 J

Solution:

Concept:

- For SHM starting from the mean position with zero phase, $x = A\sin(\omega t)$, where $\omega = 2\pi/T$.
- The potential energy referred to the equilibrium position is $U = \frac{1}{2}m\omega^2 x^2$. At maximum displacement it equals the total oscillation energy.

Step 1: Find the displacement at the stated time The amplitude is $A = 5\,\mathrm{cm} = 0.05\,\mathrm{m}$ and the period is $T = 0.2\,\mathrm{s}$. At a quarter period after crossing the mean position,

$$\omega t = \frac{2\pi}{T}\frac{T}{4} = \frac{\pi}{2}, \qquad x = A\sin\frac{\pi}{2} = A.$$

The object is therefore at an extreme position, where its velocity is zero. All its oscillation energy is potential energy at this instant; starting instead at an extreme position would give a different phase and would not match the stated initial condition.

Step 2: Evaluate the potential energy in SI units The angular frequency is $\omega = 2\pi/0.2 = 10\pi\,\mathrm{s}^{-1}$. Hence

$$U = \frac{1}{2}(0.5)(10\pi)^2(0.05)^2 = 0.0625\pi^2\,\mathrm{J} \simeq 0.617\,\mathrm{J}.$$

This rounds to $0.62\,\mathrm{J}$. Option (B) is smaller by a factor of about 100, as can result from an incorrect amplitude conversion. The large energies in (C) and (D) are inconsistent with the mass, centimetre-scale amplitude and the calculated angular frequency.

Step 3: Conclusion The potential energy is $\boxed{0.62\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: A quarter period after leaving the mean position, an SHM oscillator reaches an extreme: use the full energy with amplitude expressed in metres.

36. Match List I with List II.

	List - I		List - II
(a)	**Capacitance, C**	**(i)**	$M^1L^1T^{-3}A^{-1}$
(b)	**Permittivity of free space,** ε_0	**(ii)**	$M^{-1}L^{-3}T^4A^2$
(c)	**Permeability of free space,** μ_0	**(iii)**	$M^{-1}L^{-2}T^4A^2$
(d)	**Electric field, E**	**(iv)**	$M^1L^1T^{-2}A^{-2}$

Choose the correct answer from the options given below [4 marks]

(A) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

(B) (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)

(C) (a) → (iv), (b) → (ii), (c) → (iii), (d) → (i)

(D) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

Correct Answer: (A) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

Solution:

Concept:

- Charge has dimensions $[Q] = AT$, while force has dimensions $[F] = MLT^{-2}$. Electric potential is work per unit charge.
- The dimensions of a quantity follow from its defining equation. In particular, $C = Q/V$, $C = \varepsilon_0 A_p/d$, and the field of a straight wire obeys $B = \mu_0 I/(2\pi r)$. Here A_p is plate area.

Step 1: Find capacitance and permittivity Since potential is energy per charge,

$$[V] = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}.$$

Consequently,

$$[C] = \frac{AT}{ML^2T^{-3}A^{-1}} = M^{-1}L^{-2}T^4A^2.$$

This is entry (iii). For a parallel-plate capacitor, $\varepsilon_0 = Cd/A_p$, so

$$[\varepsilon_0] = [C]L^{-1} = M^{-1}L^{-3}T^4A^2,$$

which is entry (ii). Distinguish plate area from the base unit symbol A for electric current.

Step 2: Find permeability and electric field The magnetic force equation $F = qvB$ gives $[B] = MT^{-2}A^{-1}$. Therefore the straight-wire relation yields

$$[\mu_0] = \frac{[B]L}{A} = MLT^{-2}A^{-2},$$

which is entry (iv). Finally, electric field is force per charge:

$$[E] = \frac{MLT^{-2}}{AT} = MLT^{-3}A^{-1},$$

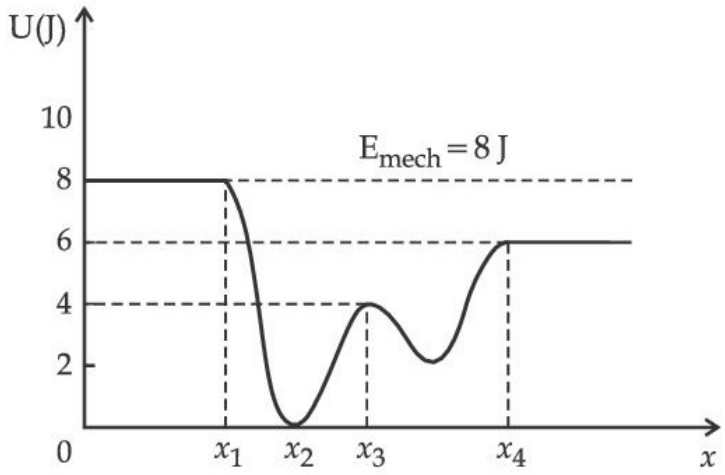
which is entry (i). Thus all four matches are fixed independently. Option B interchanges permittivity and permeability; C gives capacitance the permeability dimensions; D also assigns incompatible dimensions to capacitance and permittivity.

Step 3: Conclusion Capacitance matches (iii), permittivity matches (ii), permeability matches (iv), and electric field matches (i). Hence, the correct option is (A).

Quick Tip to Solve: Start with $[Q] = AT$ and $[V] = ML^2T^{-3}A^{-1}$; capacitance and permittivity then differ by exactly one inverse length.

37. Given below is the plot of a potential energy function U(x) for a system, in which a particle is in one dimensional motion, while a conservative force F(x) acts on it. Suppose that E_{mech} = 8 J, the incorrect statement for this system is :

[where K.E. = kinetic energy] [4 marks]



(A) at $x > x_4$ K.E. is constant throughout the region.

(B) at $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.

(C) at $x = x_2$, K.E. is greatest and the particle is moving at the fastest speed.

(D) at $x = x_3$, K.E. = 4 J.

Correct Answer: (B) at $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.

Solution:

Concept:

- For motion under a conservative force, $E = K + U$ is constant. Here $K(x) = 8 - U(x)$, with both energies measured in joules.
- For a fixed mass, $v = \sqrt{2K/m}$. Zero kinetic energy means zero speed, rather than motion at a small nonzero speed. The conservative force is $F = -dU/dx$.

Step 1: Read the kinetic energies from the graph For $x > x_4$, the potential is the constant $6\,\text{J}$. Hence

$$K = 8 - 6 = 2\,\text{J},$$

so option A is true. At $x = x_2$, the potential reaches its lowest displayed value, zero. Thus $K = 8\,\text{J}$, the greatest available kinetic energy, and the speed is greatest there. This verifies option C. At $x = x_3$, the graph gives $U = 4\,\text{J}$, so $K = 8 - 4 = 4\,\text{J}$, verifying option D. These readings use the vertical energy scale, not the slope of the potential curve.

Step 2: Test the statement about the left plateau For $x < x_1$, the potential plateau is exactly $U = 8\,\text{J}$, equal to the total mechanical energy. Therefore

$$K = 8 - 8 = 0, \qquad v = 0.$$

The particle is at rest in this region; it is not moving at a slow speed. The horizontal plateau also has zero slope and hence zero force, consistent with a particle initially at rest remaining at rest there. Although zero is the smallest kinetic energy, option B combines that fact with the assertion that the particle is moving. That assertion makes the complete statement incorrect. The other three options correctly describe their respective graph values.

Step 3: Conclusion For $x < x_1$, the kinetic energy and speed are both zero. The statement that the particle is moving there is incorrect. Hence, the correct option is (B).

Quick Tip to Solve: On a potential-energy graph subtract each ordinate from the total energy; when the two coincide, the speed is exactly zero.

38. A 100Ω resistance, a 0.1 μF capacitor and an inductor are connected in series across a 250 V supply at variable frequency. Calculate the value of inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 Hz. [4 marks]

(A) 0.70 H

(B) 70.3 mH

(C) 7.03×10^{-5} H

(D) 70.3 H

Correct Answer: (D) 70.3 H

Solution:

Concept:

- At series resonance, the inductive and capacitive reactances cancel: $\omega_0 L = 1/(\omega_0 C)$. Thus $L = 1/(4\pi^2 f_0^2 C)$.
- The resistance controls the resonant current, while the resonance frequency of an ideal series LCR circuit depends on L and C. Convert capacitance to farads before substitution.

Step 1: Convert the capacitance and write the resonance condition The given capacitor has

$$C = 0.1\,\mu\text{F} = 0.1 \times 10^{-6}\,\text{F} = 10^{-7}\,\text{F}.$$

The angular frequency is $\omega_0 = 2\pi f_0 = 120\pi\,\mathrm{rad\,s^{-1}}$. The impedance magnitude is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

At resonance the expression in parentheses vanishes, leaving $Z = R$. Accordingly the given $100\,\Omega$ resistance and $250\,\mathrm{V}$ supply are not needed to determine the inductance. They would instead determine the resonant current.

Step 2: Substitute and compare the choices Solving the reactance equality for inductance gives

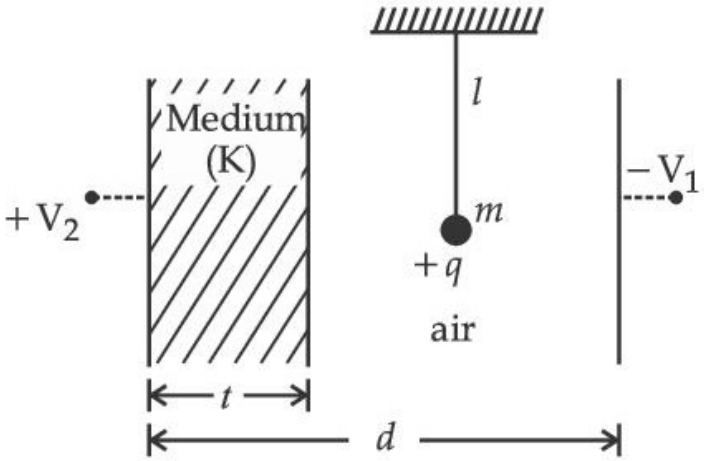
$$L = \frac{1}{(120\pi)^2(10^{-7})} = \frac{1}{1.4212 \times 10^{-2}}\,\mathrm{H} \approx 70.36\,\mathrm{H}.$$

Rounding to the precision used among the options gives $70.3\,\mathrm{H}$. As a check, $\omega_0 L$ is approximately $2.65 \times 10^4\,\Omega$, equal to $1/(\omega_0 C)$. Option A is smaller by about a factor of one hundred. Option B is $0.0703\,\mathrm{H}$, one thousand times too small, and option C is smaller still. None would cancel the capacitive reactance at $60\,\mathrm{Hz}$.

Step 3: Conclusion The required inductance is approximately $70.3\,\mathrm{H}$. Hence, the correct option is (D).

Quick Tip to Solve: Use $L = 1/(4\pi^2 f^2 C)$ with capacitance in farads; confusing microfarads with millifarads produces a large error.

39. A simple pendulum of mass 'm', length 'l' and charge '+ q' suspended in the electric field produced by two conducting parallel plates as shown. The value of deflection of pendulum in equilibrium position will be [4 marks]



(A) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_2-V_1)}{(C_1+C_2)(d-t)}\right]$

(B) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_2-V_1)}{(C_1+C_2)(d-t)}\right]$

(C) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$

(D) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$

Correct Answer: (C) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$

Solution:

Concept:

- At equilibrium the horizontal electric force and vertical weight give $T\sin\theta = qE$ and $T\cos\theta = mg$, hence $\tan\theta = qE/(mg)$.
- The dielectric slab and air gap act as capacitors in series. Write $C_1 = \varepsilon_0 A/(d-t)$ for the air gap and $C_2 = K\varepsilon_0 A/t$ for the dielectric layer, where A is plate area.

Step 1: Find the voltage across the air gap The left plate is marked $+V_2$ and the right plate $-V_1$. Their potential difference is therefore the sum

$$V_{\text{total}} = V_2 - (-V_1) = V_1 + V_2.$$

The two series layers carry equal charge magnitude. Their equivalent capacitance and charge are

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}, \qquad Q = C_{\text{eq}}(V_1 + V_2).$$

The air-gap voltage is the charge divided by the air-gap capacitance:

$$V_{\text{air}} = \frac{Q}{C_1} = \frac{C_2(V_1 + V_2)}{C_1 + C_2}.$$

The bob is in air, so the relevant field is $E = V_{\text{air}}/(d-t)$, not the field inside the dielectric.

Step 2: Balance the forces on the bob The electric field points from the positive left plate towards the negative right plate, and a positive bob deflects in that direction. Dividing the horizontal equilibrium equation by the vertical one eliminates tension:

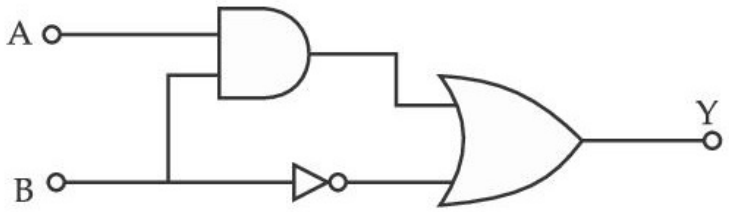
$$\tan\theta = \frac{qE}{mg} = \frac{q}{mg}\frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d-t)}.$$

Equivalently, substituting the layer capacitances gives $E = (V_1 + V_2)/(d - t + t/K)$, which checks the series-layer result. Options A and B incorrectly subtract the magnitudes of oppositely signed plate potentials. Option D uses the wrong voltage-division numerator for the air layer.

Step 3: Conclusion The deflection towards the negative plate is $\theta = \tan^{-1}\left[\frac{q}{mg}\frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d-t)}\right]$. Hence, the correct option is (C).

Quick Tip to Solve: First use the actual signed plate potentials, then divide the total voltage between the dielectric and air layers before balancing forces.

40. Find the truth table for the function Y of A and B represented in the following figure. [4 marks]



(A)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

(B)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

(C)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(D)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Correct Answer: (B)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

Solution:

Concept:

- An AND gate outputs AB, a NOT gate outputs the complement $\overline{B}$, and an OR gate outputs 1 if either of its inputs is 1.
- Read the connections before simplifying: the upper path receives A and B, while the lower path inverts B. Thus $Y = AB + \overline{B}$.

Step 1: Determine the expression for the output The branch from input B feeds one input of the upper AND gate, whose other input is A. Its output is consequently AB. The lower branch of B passes through an inverter, identified by the triangle and output bubble, so that signal becomes $\overline{B}$. The final

gate is an OR gate; there is no inversion bubble at its output. Therefore

$$Y = AB + \overline{B}.$$

Using distributivity, $AB + \overline{B} = (A + \overline{B})(B + \overline{B}) = A + \overline{B}$. This equivalent expression is a useful check, but the unsimplified one follows the actual circuit directly.

Step 2: Evaluate every row of the truth table For $(A, B) = (0, 0)$, the inverter supplies 1 and the output is 1. For $(0, 1)$, both final-gate inputs are zero, so the output is zero. For $(1, 0)$, the inverter again supplies 1; for $(1, 1)$, the AND gate supplies 1. Thus

A	B	AB	$\overline{B}$	Y
0	0	0	1	1
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

The output sequence is $1, 0, 1, 1$. Only B has all four entries. A, C and D already fail the first row, because a zero on B must make the inverter output 1 and hence force Y to 1.

Step 3: Conclusion The truth-table outputs in the order $00, 01, 10, 11$ are $1, 0, 1, 1$. Hence, the correct option is (B).

Quick Tip to Solve: If an OR gate receives $\overline{B}$, every row with $B = 0$ must have output 1, regardless of the other branch.

41. Figure A and B shown to long straight wires of circular cross-section (a and b with a < b), carrying current I which is uniformly distributed across the cross-section. The magnitude of magnetic field B varies with radius r and can be represented as : [4 marks]

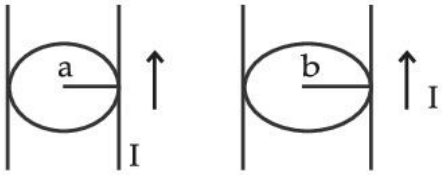
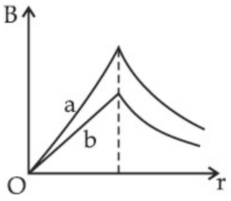
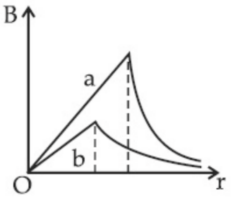


Fig. A Fig. B

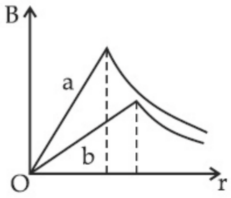
(A)



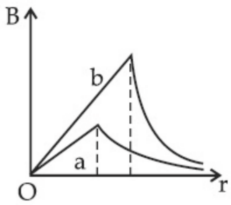
(B)



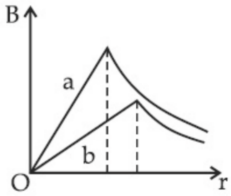
(C)



(D)



Correct Answer: (C)



Solution:

Concept:

- Ampere's circuital law gives $B(2\pi r) = \mu_0 I_{\text{enc}}$ for a long straight wire. Uniform current density makes the enclosed current proportional to the enclosed cross-sectional area.
- For a wire of radius s, $B = \mu_0 I r/(2\pi s^2)$ inside and $B = \mu_0 I/(2\pi r)$ outside. The field reaches its maximum at the surface $r = s$.

Step 1: Compare the two internal fields For the thinner wire of radius a, the current enclosed by an Amperian circle of radius $r < a$ is Ir^2/a^2. Thus

$$B_a(r) = \frac{\mu_0 I}{2\pi a^2} r.$$

For the thicker wire of radius b, similarly,

$$B_b(r) = \frac{\mu_0 I}{2\pi b^2} r \qquad (r < b).$$

Both graphs begin at the origin and initially rise linearly. Since $a < b$, the slope for a is greater than the slope for b. The thinner wire reaches its surface first, at the smaller radial coordinate a, where its linear rise turns into an inverse-radius decrease.

Step 2: Locate the peaks and the external branches The two surface maxima are

$$B_a(a) = \frac{\mu_0 I}{2\pi a}, \qquad B_b(b) = \frac{\mu_0 I}{2\pi b}.$$

The a peak is therefore both to the left of and higher than the b peak. Beyond each surface the corresponding field decreases as $1/r$; at a common point with $r > b$, both fields have the same value because the total currents are equal. Among the sketches, C has the required ordering of the peaks and the initial slopes. A places the peaks at the same radius, B places the a peak farther right, and D makes the thicker wire have the greater initial slope and peak. These contradict the derived relations.

Step 3: Conclusion The thinner wire has a steeper initial rise and a higher peak at a smaller radius. Hence, the correct option is (C).

Quick Tip to Solve: For equal total currents, the surface peak is proportional to $1/s$, while the internal slope is proportional to $1/s^2$.

42. Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is : [4 marks]

(A) $\sqrt{\frac{G}{2R^3}}$

(B) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$

(C) $\frac{1}{2R}\sqrt{\frac{1}{G}}$

(D) $\frac{2G}{R^3}$

Correct Answer: (B) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$

Solution:

Concept:

- Two equal masses revolve about their common centre of mass. If each circular orbit has radius R, their separation is $2R$.
- Newton's gravitational force Gm^2/r^2 supplies the centripetal force $m\omega^2 R$ on each particle. The orbital radius and the separation enter different parts of the equation.

Step 1: Write the attraction using the separation Because the masses are identical, the centre of mass lies midway between them. Each mass remains a distance R from that midpoint, and they occupy opposite ends of a diameter. The mutual attraction on either one is therefore

$$F_g = \frac{Gm^2}{(2R)^2} = \frac{Gm^2}{4R^2}.$$

This force points towards the other mass and hence towards the centre of the circular path. No second force of equal size is added on that same particle: the equal and opposite gravitational force acts on the other particle.

Step 2: Equate the force to the centripetal requirement For either mass the centripetal force is $m\omega^2 R$, using its own orbit radius. Equating magnitudes gives

$$m\omega^2 R = \frac{Gm^2}{4R^2}, \qquad \omega^2 = \frac{Gm}{4R^3}.$$

Putting $m = 1\,\text{kg}$, as specified,

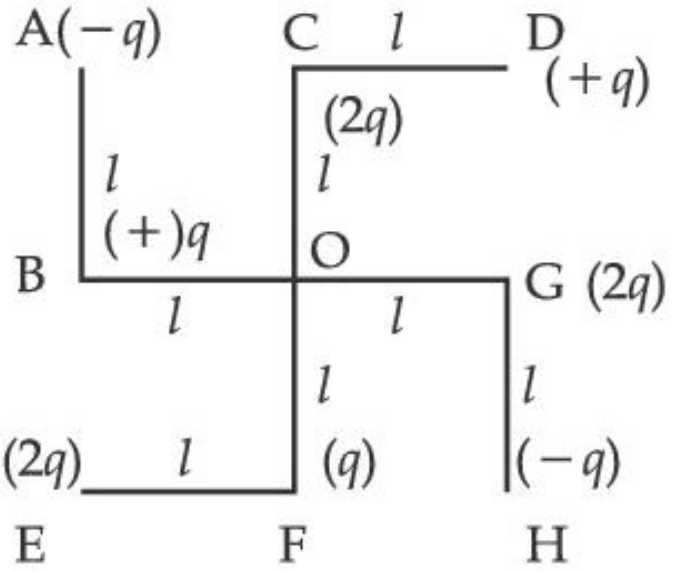
$$\omega = \frac{1}{2}\sqrt{\frac{Gm}{R^3}} = \frac{1}{2}\sqrt{\frac{G}{R^3}},$$

where the final option notation incorporates the given unit mass. Option A has the wrong numerical factor because it does not retain the full squared separation. C has the wrong dependence on G, while D has the form of an angular-speed squared rather than an angular speed. Only B satisfies the individual force balance.

Step 3: Conclusion The angular speed of each particle is $\omega = \frac{1}{2}\sqrt{G/R^3}\,\text{rad}\,\text{s}^{-1}$, with each mass equal to $1\,\text{kg}$ as given. Hence, the correct option is (B).

Quick Tip to Solve: For a binary system of equal masses, distinguish the separation $2R$ in gravity from the orbit radius R in centripetal force.

43. What will be the magnitude of electric field at point O as shown in the figure? Each side of the figure is l and perpendicular to each other? [4 marks]



(A) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l^2)}\left(2\sqrt{2}-1\right)$

(C) $\frac{q}{4\pi\varepsilon_0(2l)^2}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2l^2}\left(\sqrt{2}\right)$

Correct Answer: (B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l^2)}\left(2\sqrt{2}-1\right)$

Solution:

Concept:

- For a point charge, the field at displacement $\mathbf{r}$ from the charge is $\mathbf{E} = kQ\mathbf{r}/r^3$, where $k = 1/(4\pi\varepsilon_0)$. Fields add as vectors, not as magnitudes.
- Positive charges produce fields away from themselves, and negative charges produce fields towards themselves. The axial charges are at distance l, while the four diagonal charges are at distance $\sqrt{2}l$ from O.

Step 1: Add the fields of the axial charges Take O as the origin, right as positive x and up as positive y. The charges at B and G are $+q$ and $+2q$, so their net field is leftward. The charges at C and F are $+2q$ and $+q$, so their net field is downward. Thus

$$\mathbf{E}_{\text{axial}} = -\frac{kq}{l^2}(\hat{\mathbf{i}} + \hat{\mathbf{j}}).$$

Each component follows from subtraction of oppositely directed fields; the four charges do not cancel.

Step 2: Combine the diagonal charges The equal negative charges at A and H are opposite each other about O, so their fields cancel. D carries $+q$ and E carries $+2q$; both lie on the other diagonal, at distance $\sqrt{2}l$. Their net field points from E towards D and has magnitude $kq/(2l^2)$. Its two Cartesian

components are equal:

$$\mathbf{E}_{\text{diagonal}} = \frac{kq}{2\sqrt{2}l^2}(\hat{\mathbf{i}} + \hat{\mathbf{j}}).$$

Step 3: Find the magnitude of the resultant Adding the two contributions leaves equal negative components. Therefore

$$E = \sqrt{2}\frac{kq}{l^2}\left(1 - \frac{1}{2\sqrt{2}}\right) = \frac{kq}{2l^2}(2\sqrt{2} - 1).$$

Options A and C omit the vector combination, while D gives only a different multiple of the axial contribution. Only B contains the required subtraction of the diagonal field.

Step 4: Conclusion

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{2l^2}(2\sqrt{2} - 1)}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Cancel opposite equal charges first, then add the remaining fields component by component; diagonal charges are at distance $\sqrt{2}l$.

---

44. A physical quantity 'y' is represented by the formula $y = m^2r^{-4}g^x l^{-\frac{3}{2}}$

If the percentage errors found in y, m, r, l and g are 18, 1, 0.5, 4 and p respectively, then find the value of x and p. [4 marks]

(A) 5 and ± 2

(B) 4 and ± 3

(C) $\frac{16}{3}$ and $\pm\frac{3}{2}$

(D) 8 and $\pm$ 2

Correct Answer: (C) $\frac{16}{3}$ and $\pm\frac{3}{2}$

Solution:

Concept:

- For a product of powers, the maximum fractional error is the sum of the absolute powers multiplied by the fractional errors of the independent quantities. Negative powers therefore contribute positively to the error bound.
- Writing percentage error as ϵ, the relation $y = m^2r^{-4}g^x l^{-3/2}$ gives $\epsilon_y = 2\epsilon_m + 4\epsilon_r + |x|\epsilon_g + \frac{3}{2}\epsilon_l$. The sign of a measurement deviation is not its error magnitude.

Step 1: Insert the known percentage errors The given percentage uncertainties are $\epsilon_y = 18$, $\epsilon_m = 1$, $\epsilon_r = 0.5$, $\epsilon_l = 4$, and $\epsilon_g = |p|$. Hence

$$18 = 2(1) + 4(0.5) + |x|\,|p| + \frac{3}{2}(4).$$

The contributions from mass, radius and length are respectively 2, 2 and 6 percentage points. Their total is 10, leaving 8 percentage points for the factor involving g.

Step 2: Test the proposed pairs The required condition is

$$|x|\,|p| = 18 - 10 = 8.$$

In option A, $5 \times 2 = 10$; in option B, $4 \times 3 = 12$; and in option D, $8 \times 2 = 16$. None gives the required contribution. Option C gives

$$\frac{16}{3} \times \frac{3}{2} = 8,$$

which exactly reproduces the stated total uncertainty of 18 percent.

Step 3: Interpret the result correctly The uncertainty in g has magnitude $3/2$ percent and may represent a positive or negative deviation, explaining the printed $\pm$ notation. The error data alone constrain the product $|xp|$, rather than determining two independent unknowns uniquely. Among the four offered pairs, however, precisely one satisfies that constraint, so the multiple-choice selection is unambiguous.

Step 4: Conclusion The listed consistent pair is $\boxed{x = \frac{16}{3},\ p = \pm\frac{3}{2}}$, with p expressed as a percentage error. Hence, the correct option is (C).

Quick Tip to Solve: Use absolute exponents in error propagation, subtract the known contributions, and test the remaining product against each proposed pair.

45. An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by : [4 marks]

(A) $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(B) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$

(C) $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(D) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

Correct Answer: (D) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

Solution:

Concept:

- Power is the rate at which kinetic energy increases: $P = dK/dt$, with $K = \frac{1}{2}mv^2$. For constant mass and constant power, kinetic energy grows linearly with time.
- Position follows from integrating velocity: $s(t) - s(0) = \int_0^t v(\tau)\,d\tau$. Constant power does not imply constant force or constant acceleration.

Step 1: Obtain velocity from the energy supplied The vehicle begins at rest, so its initial kinetic energy is zero. In time t, the engine supplies energy Pt. With the stated ideal motion this becomes kinetic energy:

$$\frac{1}{2}mv^2 = Pt, \qquad v(t) = \sqrt{\frac{2P}{m}}\, t^{1/2}.$$

The positive root describes motion in the direction of the engine force. Velocity grows as the square

root of time, rather than linearly as it would under constant acceleration.

Step 2: Integrate the velocity The vehicle starts from the origin, so the integration constant is fixed by $s(0) = 0$. Use a dummy time variable to write

$$s(t) = \sqrt{\frac{2P}{m}} \int_0^t \tau^{1/2}\, d\tau = \frac{2}{3}\sqrt{\frac{2P}{m}}\, t^{3/2}.$$

Combining the numerical factor with the square root gives

$$\frac{2}{3}\sqrt{\frac{2P}{m}} = \sqrt{\frac{8P}{9m}}.$$

Step 3: Check the coefficient and the time dependence Differentiating the position recovers $v = \sqrt{2Pt/m}$, and then $\frac{1}{2}mv^2 = Pt$, so the result obeys the constant-power condition. Option A has the wrong coefficient, B has the wrong exponent of time, and C incorrectly places mass in the numerator. Option D agrees with both the integration and the energy check. The units also work: $\sqrt{P/m}\, t^{3/2}$ has dimensions of length.

Step 4: Conclusion

$$\boxed{s(t) = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: For constant power, integrate energy first to find $v \propto \sqrt{t}$, then integrate once more to obtain $s \propto t^{3/2}$.

---

46. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0 $\times$ 10^3 km. Find the mass of Mars.

$$\left\{Given\ \frac{4\pi^2}{G} = 6 \times 10^{11} N^{-1} m^{-2} kg^2\right\}$$

[4 marks]

(A) 5.96 $\times$ 10^{19} kg

(B) 3.25 $\times$ 10^{21} kg

(C) 7.02 $\times$ 10^{25} kg

(D) 6.00 $\times$ 10^{23} kg

Correct Answer: (D) 6.00 $\times$ 10^{23} kg

Solution:

Concept:

- For a moon in a circular orbit of radius r and period T, gravity provides centripetal acceleration: $GM/r^2 = 4\pi^2 r/T^2$. Thus the planet mass is $M = (4\pi^2/G)r^3/T^2$.
- The moon mass cancels between the gravitational force and the required centripetal force. Only its

orbital radius and period are needed; all inputs must be converted to mutually consistent SI units.

Step 1: Convert the orbital data to SI units The quoted orbital radius is measured from the centre of Mars, so it is used directly without adding a planetary radius. Convert kilometres to metres and hours to seconds:

$$r = 9.0 \times 10^3\,\text{km} = 9.0 \times 10^6\,\text{m},$$

$$T = 7\,\text{h} + 30\,\text{min} = 7.5 \times 3600 = 2.70 \times 10^4\,\text{s}.$$

This conversion is essential because the supplied value of $4\pi^2/G$ uses SI units.

Step 2: Substitute into the orbital relation Using the value supplied in the question gives

$$M = (6 \times 10^{11})\frac{(9.0 \times 10^6)^3}{(2.70 \times 10^4)^2}.$$

Evaluate the cube and square separately:

$$r^3 = 7.29 \times 10^{20}\,\text{m}^3, \qquad T^2 = 7.29 \times 10^8\,\text{s}^2.$$

Their ratio is $1.00 \times 10^{12}\,\text{m}^3\text{s}^{-2}$, so the mass follows without further approximation.

Step 3: Compare the orders of magnitude The mass is therefore $6.00 \times 10^{23}\,\text{kg}$. The units reduce to kilograms because $1/G$ has dimensions $\text{kg}\,\text{s}^2\text{m}^{-3}$. Options A and B are far too small and C is too large relative to the value obtained directly from the given orbit. The presence of a second moon does not alter the calculation for this moon in the stated planet-dominated orbit.

Step 4: Conclusion

$$\boxed{M_{\text{Mars}} = 6.00 \times 10^{23}\,\text{kg}}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Convert radius and period before cubing or squaring them; a kilometre-to-metre error becomes a factor of 10^9 in the inferred mass.

47. A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0\left[1 - \left(\frac{t-T}{T}\right)^2\right]$$

Where F_0 and T are constants. The force acts only for the time interval 2T. The velocity v of the particle after time 2T is : [4 marks]

(A) $2F_0T/M$

(B) $F_0T/2M$

(C) $4F_0T/3M$

(D) $F_0T/3M$

Correct Answer: (C) $4F_0T/3M$

Solution:

Concept:

- Impulse equals change in momentum: $\int F\,dt = M(v-u)$. Since the force direction is fixed, this is a one-dimensional signed relation along that direction.
- The particle starts at rest, so $u = 0$. A time dependent force must be integrated; multiplying the maximum force by the total time would incorrectly treat the force as constant.

Step 1: Write the impulse over the full duration The force acts from $t = 0$ to $t = 2T$, and vanishes at both endpoints. Its maximum is F_0 at $t = T$. Therefore

$$Mv = F_0\int_0^{2T}\left[1 - \frac{(t-T)^2}{T^2}\right]dt.$$

The integrand is nonnegative throughout the interval, so the final velocity is in the direction of the applied force.

Step 2: Perform the definite integration An antiderivative of the bracket is $t - (t-T)^3/(3T^2)$. Hence

$$Mv = F_0\left[t - \frac{(t-T)^3}{3T^2}\right]_0^{2T}.$$

The upper endpoint contributes $2T - T/3 = 5T/3$, while the lower endpoint contributes $T/3$. Subtracting gives

$$Mv = F_0\left(\frac{5T}{3} - \frac{T}{3}\right) = \frac{4F_0T}{3}.$$

Step 3: Check the result against the force profile Dividing the impulse by mass gives the velocity. Equivalently, the mean force is $(4F_0T/3)/(2T) = 2F_0/3$, smaller than the peak F_0, as required. Option A would use the peak force over the entire duration, while B and D give too small an impulse. Only C equals the exact area under the force-time parabola. The dimensions F_0T/M are those of velocity.

Step 4: Conclusion

$$\boxed{v = \frac{4F_0T}{3M}}$$

in the direction of the force. Hence, the correct option is (C).

Quick Tip to Solve: For a varying force, find the area under its force-time graph. A symmetric parabolic pulse has a mean force smaller than its peak.

48. The resistance of a conductor at 15°C is 16Ω and at 100°C is 20Ω. What will be the temperature coefficient of resistance of the conductor? [4 marks]

(A) $0.010^{\circ}C^{-1}$

(B) $0.033^{\circ}C^{-1}$

(C) $0.003^{\circ}C^{-1}$

(D) $0.042^{\circ}C^{-1}$

Correct Answer: (C) $0.003^{\circ}C^{-1}$

Solution:

Concept:

- In the linear temperature model, $R_t = R_0(1 + \alpha_0 t)$, where R_0 is resistance at $0^{\circ}C$ and α_0 is the coefficient referred to that temperature.
- Two measured resistance-temperature pairs determine the slope $R_0\alpha_0$ and the intercept R_0. A positive slope means the conductor resistance increases as its temperature rises.

Step 1: Form equations for the two measurements Using the temperatures on the Celsius scale gives

$$16 = R_0(1 + 15\alpha_0), \qquad 20 = R_0(1 + 100\alpha_0).$$

Subtracting eliminates the constant term:

$$4 = 85R_0\alpha_0.$$

The resistance increase per degree is therefore $R_0\alpha_0 = 4/85\,\Omega\,^{\circ}C^{-1}$. This slope is not itself the fractional temperature coefficient.

Step 2: Eliminate the unknown zero degree resistance Take the ratio of the two equations, or cross-multiply directly:

$$20(1 + 15\alpha_0) = 16(1 + 100\alpha_0).$$

Thus

$$20 + 300\alpha_0 = 16 + 1600\alpha_0, \qquad \alpha_0 = \frac{4}{1300} = 0.0030769\,^{\circ}C^{-1}.$$

The implied value $R_0 = 16/(1 + 15\alpha_0)$ is about $15.29\,\Omega$, consistent with the lower resistance at zero degrees.

Step 3: Select the appropriately rounded option The coefficient rounds to $0.003\,^{\circ}C^{-1}$, matching C.

The coefficients in A, B and D are much larger and do not reproduce both measured resistances in the same linear law. If one instead refers the coefficient to 15°C, its value is $4/(16 \times 85) = 0.00294\,^\circ\text{C}^{-1}$, which also rounds to the same listed option. The conventional zero-degree reference gives the exact calculation above.

Step 4: Conclusion The temperature coefficient is approximately $\boxed{0.003\,^\circ\text{C}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\alpha_0 = (R_2 - R_1)/(R_1 t_2 - R_2 t_1)$ when the two measured resistances are given at nonzero Celsius temperatures.

49. Two Carnot engines A and B operate in series such that engine A absorbs heat at T_1 and rejects heat to a sink at temperature T. Engine B absorbs half of the heat rejected by Engine A and rejects heat to the sink at T_3. When workdone in both the cases is equal, to value of T is : [4 marks]

(A) $\frac{2}{3}T_1 + \frac{3}{2}T_3$
(B) $\frac{1}{3}T_1 + \frac{2}{3}T_3$
(C) $\frac{3}{2}T_1 + \frac{1}{3}T_3$
(D) $\frac{2}{3}T_1 + \frac{1}{3}T_3$

Correct Answer: (D) $\frac{2}{3}T_1 + \frac{1}{3}T_3$

Solution:

Concept:

- For a reversible Carnot engine between absolute temperatures T_h and T_c, efficiency is $1 - T_c/T_h$ and rejected-to-absorbed heat ratio is T_c/T_h.
- The second engine receives only half of the heat rejected by the first. This factor must multiply its heat input before its work is evaluated. All temperatures in the Carnot relations are in kelvin.

Step 1: Find the work and rejected heat of engine A Let engine A absorb heat Q from the reservoir at T_1. Its sink temperature is T, so its rejected heat and work output are

$$Q_{A,\text{out}} = Q\frac{T}{T_1}, \qquad W_A = Q\left(1 - \frac{T}{T_1}\right).$$

The heat passed on to engine B is half of this rejected heat, not half of Q. Therefore

$$Q_{B,\text{in}} = \frac{QT}{2T_1}.$$

Step 2: Compute engine B work and equate the outputs Engine B operates between T and T_3, so

$$W_B = \frac{QT}{2T_1}\left(1 - \frac{T_3}{T}\right) = \frac{Q(T - T_3)}{2T_1}.$$

The condition $W_A = W_B$ now becomes

$$Q\frac{T_1 - T}{T_1} = Q\frac{T - T_3}{2T_1}.$$

Cancel the common nonzero factor Q/T_1 and solve the resulting linear equation.

Step 3: Solve and check the intermediate temperature The equation is $2(T_1 - T) = T - T_3$, giving

$$3T = 2T_1 + T_3, \qquad T = \frac{2T_1 + T_3}{3}.$$

For $T_1 > T_3$, this lies strictly between the hot and cold reservoir temperatures, as an intermediate sink must. Substitution gives both works equal to $Q(T_1 - T_3)/(3T_1)$. Options A and C do not even give coefficients summing to one; B reverses the weights and fails the equal-work equation. D satisfies both checks.

Step 4: Conclusion

$$\boxed{T = \frac{2}{3}T_1 + \frac{1}{3}T_3}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Track the actual heat entering each engine before applying efficiency; the half-transfer factor changes the weights in the intermediate temperature.

50. Consider the following statements :

A. Atoms of each element emit characteristics spectrum.

B. According to Bohr's Postulate, an electron in a hydrogen atom, revolves in a certain stationary orbit.

C. The density of nuclear matter depends on the size of the nucleus.

D. A free neutron is stable but a free proton decay is possible.

E. Radioactivity is an indication of the instability of nuclei.

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C, D and E

(B) A, B and E only

(C) B and D only

(D) A, C and E only

Correct Answer: (B) A, B and E only

Solution:

Concept:

- Discrete atomic energy levels produce characteristic spectral lines: a photon has energy $h\nu = E_i - E_f$. Bohr postulated non-radiating stationary orbits for the hydrogen atom.
- Nuclear radii obey $R = R_0 A^{1/3}$, so nuclear mass and volume both scale with mass number A.

Radioactive emission indicates an unstable nucleus.

Step 1: Test the atomic statements A is true because each element has its own energy-level structure and hence a characteristic spectrum. B is true: in the Bohr model an electron occupies an allowed stationary orbit without continuously losing energy as radiation; emission or absorption accompanies a transition.

Step 2: Test the nuclear statements The density is $\rho \simeq Am_n/[(4\pi/3)R_0^3A] = 3m_n/(4\pi R_0^3)$, approximately independent of nuclear size, so C is false. A free neutron is unstable and undergoes beta decay, whereas a free proton is stable in the ordinary nuclear-physics description, so D is false. E is true because radioactivity is a spontaneous transformation of an unstable nucleus.

Step 3: Compare the combinations Only A, B and E survive. The combinations in options (A) and (D) incorrectly include C; option (C) incorrectly includes D. Thus all three alternative combinations contain a false statement.

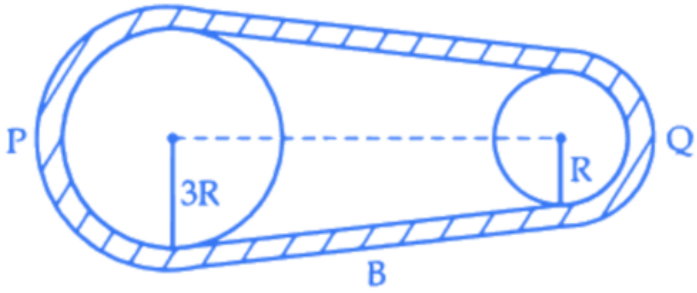
Step 4: Conclusion The correct statements are A, B and E only. Hence, the correct option is (B).

Quick Tip to Solve: For nuclear density, substitute the radius law before deciding whether size matters; the factor A cancels.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_1}{I_2}\right)$ will be x : 1. The value of x will be _____. [4 marks]



Solution:

Concept:

- A taut belt moving without slipping has the same linear speed at the rims of both wheels: $v = R_P\omega_P = R_Q\omega_Q$.
- The rotational kinetic energy of a rigid wheel is $K = \frac{1}{2}I\omega^2$, where I is its moment of inertia about its axle.

Step 1: Relate the angular speeds The figure labels the large wheel P with radius $3R$ and the smaller wheel Q with radius R. Equality of the belt speeds gives $3R\omega_P = R\omega_Q$, hence $\omega_Q = 3\omega_P$. The smaller wheel therefore makes three rotations while the larger makes one; equal rim speeds do not mean

equal angular speeds.

Step 2: Apply equality of rotational energy With I_1 for P and I_2 for Q, the given equal energies imply

$$\frac{1}{2}I_1\omega_P^2 = \frac{1}{2}I_2\omega_Q^2.$$

Substitution yields $I_1\omega_P^2 = 9I_2\omega_P^2$, or $I_1/I_2 = 9$. This direction of the ratio is consistent: the wheel rotating more slowly requires the larger inertia to store the same kinetic energy.

Step 3: Conclusion The ratio is $I_1 : I_2 = 9 : 1$, so $\boxed{x = 9}$. The requested ratio coefficient is dimensionless.

Quick Tip to Solve: For equal rotational energies, the inertia ratio is the inverse square of the angular-speed ratio.

52. The difference in the number of waves when yellow light propagates through air and vacuum columns of the same thickness is one. The thickness of the air column is _____ mm. [Refractive index of air = 1.0003, wavelength of yellow light in vacuum = 6000 $\overset{o}{A}$] [4 marks]

Solution:

Concept:

- The frequency of light does not change when it enters a medium. For vacuum wavelength λ_0, its wavelength in air of index μ is λ_0/μ.
- The number of wavelengths in a geometrical thickness d is $N = d/\lambda$. The extra number in air is therefore $(\mu - 1)d/\lambda_0$.

Step 1: Express the wave-number difference For equal column thicknesses, $N_{\text{air}} = \mu d/\lambda_0$ and $N_{\text{vac}} = d/\lambda_0$. Air has the shorter wavelength and therefore contains more waves. The stated difference of one gives

$$\frac{(\mu - 1)d}{\lambda_0} = 1, \qquad d = \frac{\lambda_0}{\mu - 1}.$$

This uses the excess optical path, rather than the entire optical path through air.

Step 2: Convert and substitute The given vacuum wavelength is $6000\,\text{Å} = 6000 \times 10^{-10}\,\text{m} = 6.0 \times 10^{-7}\,\text{m}$. Also $\mu - 1 = 1.0003 - 1 = 3.0 \times 10^{-4}$. Thus

$$d = \frac{6.0 \times 10^{-7}}{3.0 \times 10^{-4}} = 2.0 \times 10^{-3}\,\text{m} = 2.0\,\text{mm}.$$

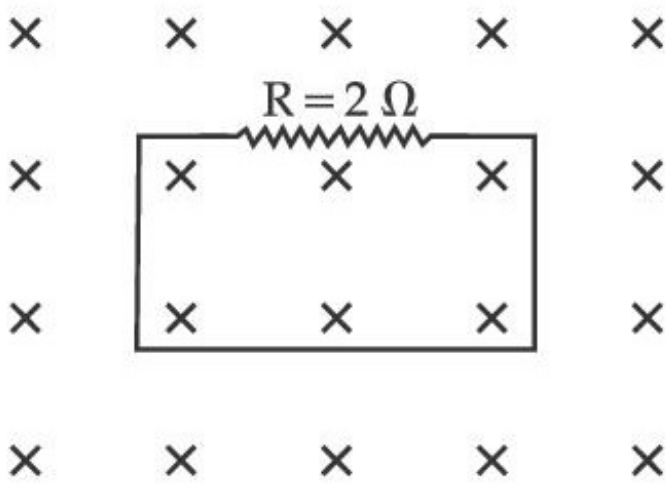
The units are converted to millimetres only after evaluating the thickness.

Step 3: Conclusion The thickness of the air column is $\boxed{2\,\text{mm}}$; the required numerical entry is 2.

Quick Tip to Solve: One extra wave means an excess optical path of one vacuum wavelength: use (index minus one) times thickness.

53. In the given figure the magnetic flux through the loop increases according to the relation $\phi_B(t) = 10t^2 + 20t$, where ϕ_B is in milliwebers and t is in seconds.

The magnitude of current through R = 2Ω resistor at t = 5 s is _____ mA. [4 marks]



Solution:

Concept:

- Faraday's law gives the magnitude of the emf in one loop as $|\mathcal{E}| = |d\Phi_B/dt|$. The minus sign in its signed form specifies the Lenz-law direction.
- For the resistive loop, Ohm's law gives $I = |\mathcal{E}|/R$. A flux derivative in milliwebers per second is an emf in millivolts.

Step 1: Differentiate the magnetic flux The printed time dependence is $\Phi_B(t) = 10t^2 + 20t$, with flux in milliwebers and time in seconds. The instantaneous rate, rather than Φ_B/t, determines the current:

$$\frac{d\Phi_B}{dt} = 20t + 20 \quad \text{mWb s}^{-1}.$$

At $t = 5\,\text{s}$, this becomes $20(5) + 20 = 120\,\text{mWb s}^{-1}$, giving an emf magnitude of $120\,\text{mV} = 0.120\,\text{V}$.

Step 2: Use the loop resistance The figure contains a single resistor of resistance $2\,\Omega$, with the remaining wire treated as ideal. Hence

$$I = \frac{0.120}{2} = 0.060\,\text{A} = 60\,\text{mA}.$$

The increasing flux into the page would induce a counterclockwise current to oppose that increase. Its sense does not alter the requested magnitude.

Step 3: Conclusion The current through the resistor at $5\,\text{s}$ has magnitude $\boxed{60\,\text{mA}}$.

Quick Tip to Solve: Differentiate flux first; dividing flux by time gives an average rate and generally misses the instantaneous emf.

54. A particle executes simple harmonic motion represented by displacement function as

x(t) = A sin(ωt + ϕ)

If the position and velocity of the particle at t = 0 s are 2 cm and 2ω cm s^{-1} respectively, then its amplitude is $x\sqrt{2}$ cm where the value of x is _____. [4 marks]

Solution:

Concept:

- For $x(t) = A\sin(\omega t + \phi)$, differentiation gives $v(t) = A\omega\cos(\omega t + \phi)$.
- The identity $\sin^2\phi + \cos^2\phi = 1$ eliminates the unknown phase: $A^2 = x_0^2 + (v_0/\omega)^2$.

Step 1: Use the initial conditions At zero time, the stated displacement gives $A\sin\phi = 2\,\text{cm}$. The positive initial velocity gives $A\omega\cos\phi = 2\omega\,\text{cm}\,\text{s}^{-1}$, so $A\cos\phi = 2\,\text{cm}$. Dividing the velocity by angular frequency converts it to a length. It is not appropriate to identify either initial displacement or initial speed alone with the amplitude.

Step 2: Eliminate the phase and find the coefficient Square the two equations and add them:

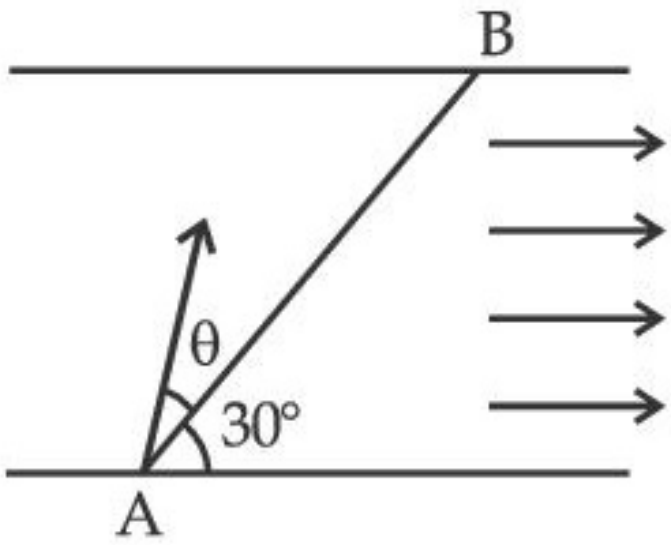
$$A^2(\sin^2\phi + \cos^2\phi) = 2^2 + 2^2 = 8\,\text{cm}^2.$$

The amplitude is the positive root, $A = \sqrt{8}\,\text{cm} = 2\sqrt{2}\,\text{cm}$. Both sine and cosine of the initial phase are positive and equal, so $\phi = \pi/4$ modulo 2π, providing a consistency check on the initial direction of motion.

Step 3: Conclusion The amplitude is $2\sqrt{2}\,\text{cm}$. Comparing with the stated form, $\boxed{x = 2}$; this coefficient is dimensionless.

Quick Tip to Solve: Eliminate phase with displacement squared plus (velocity divided by angular frequency) squared.

55. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be _____°, so that the swimmer reaches point B. [4 marks]



Solution:

Concept:

- The swimmer's velocity relative to the bank is the vector sum of the swimming velocity relative to water and the river velocity.
- To reach B, the resultant velocity must lie along AB. Its component perpendicular to AB must therefore vanish.

Step 1: Resolve perpendicular to the required path Let the common magnitude of the river speed and the swimmer's speed relative to water be u. The river flows at 30° to AB. The figure shows the swimmer aiming on the opposite side of AB, at the required angle θ. Their perpendicular components oppose each other, so the condition for no departure from AB is

$$u \sin\theta = u \sin 30^\circ.$$

Consequently $\sin\theta = 1/2$.

Step 2: Select the forward-moving direction The angle drawn in the figure is acute, giving $\theta = 30^\circ$. The components along AB then add to $u\cos 30^\circ + u\cos 30^\circ = \sqrt{3}u > 0$, so the resultant actually points towards B. The supplementary mathematical choice 150° would cancel the along-AB components and give zero resultant for equal speeds; it cannot carry the swimmer across. Equivalently, the resultant of two equal non-opposite vectors bisects their included angle.

Step 3: Conclusion The swimmer should aim at $\boxed{\theta = 30^\circ}$ to line AB, on the side indicated in the figure.

Quick Tip to Solve: Cancel velocities perpendicular to the desired track, then check that the remaining component points towards the destination.

56. For the circuit shown, the value of current at time t = 3.2 s will be _____ A.
[Voltage distribution V(t) is shown by Fig. (1) and the circuit is shown in Fig. (2)] [4 marks]

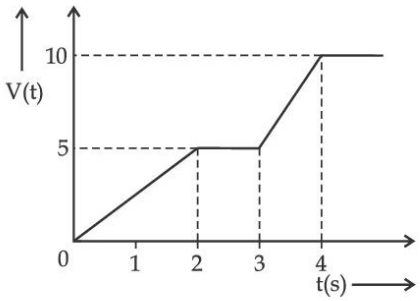


Figure 1

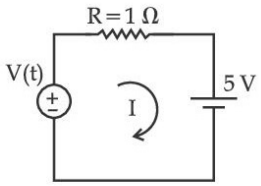


Figure 2

Solution:

Concept:

- On a straight segment of a voltage-time graph, the source voltage is obtained by linear interpolation between the endpoints.

- Kirchhoff's loop law equates the net emf to IR. The 5 V cell and the variable source have positive terminals at the top, so their emfs oppose around this loop.

Step 1: Read the source voltage at the given time The graph is flat at $5\,\mathrm{V}$ between 2 s and 3 s, then rises linearly to $10\,\mathrm{V}$ at 4 s. At $3.2\,\mathrm{s}$, use the rising segment, whose slope is $(10-5)/(4-3) = 5\,\mathrm{V\,s^{-1}}$. Thus

$$V(3.2) = 5 + 5(3.2-3) = 6\,\mathrm{V}.$$

Using the preceding constant segment would miss the voltage increase after 3 s.

Step 2: Apply the loop equation Take clockwise current as shown. Passing upwards through the variable source gives a rise of $6\,\mathrm{V}$; crossing the fixed cell downwards gives a drop of $5\,\mathrm{V}$. The resistor has $R = 1\,\Omega$. Hence

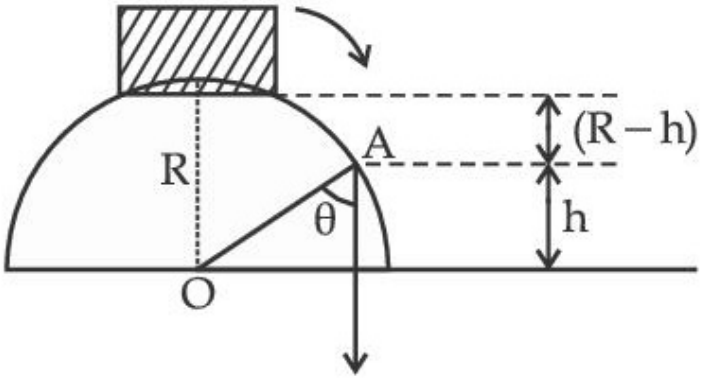
$$6 - I(1) - 5 = 0, \qquad I = 1\,\mathrm{A}.$$

The positive result confirms the indicated clockwise direction. With no inductor or capacitor present, there is no transient storage term to include.

Step 3: Conclusion The current at $t = 3.2\,\mathrm{s}$ is $\boxed{1\,\mathrm{A}}$, clockwise as indicated.

Quick Tip to Solve: First identify the correct time interval on the graph, then subtract opposing source voltages before dividing by resistance.

57. A small block slides down from the top of hemisphere of radius R = 3 m as shown in the figure. The height 'h' at which the block will lose contact with the surface of the sphere is _____ m. (Assume there is no friction between the block and the hemisphere) [4 marks]



Solution:

Concept:

- On a smooth fixed hemisphere, the normal reaction does no work; the loss of gravitational potential energy equals the gain of kinetic energy.
- Contact is lost when the normal reaction first becomes zero. The remaining inward component of weight must then supply the centripetal force.

Step 1: Express the speed in terms of height Measure height h above the horizontal plane through the centre O, as in the figure. The block starts from rest at the top, at height R. At a later point its

vertical fall is $R - h$. With no friction, mechanical energy is conserved:

$$mgR = mgh + \frac{1}{2}mv^2, \qquad v^2 = 2g(R - h).$$

The reference level of potential energy is arbitrary, but the height difference here must be the vertical drop, not the distance travelled along the curved surface.

Step 2: Apply the radial contact condition Let ϕ be the angle the outward radius makes with the upward vertical. Geometry gives $h = R\cos\phi$. The inward radial equation while the block is still touching the surface is

$$mg\cos\phi - N = \frac{mv^2}{R}.$$

At separation $N = 0$, so $v^2 = gR\cos\phi = gh$. Equating this to the energy result eliminates both the mass and gravitational acceleration:

$$gh = 2g(R - h) \quad \Longrightarrow \quad 3h = 2R.$$

For $R = 3\,\text{m}$, this gives $h = 2\,\text{m}$. The drop from the top is only $1\,\text{m}$; it is not the height requested.

Step 3: Conclusion The block loses contact at a height $\boxed{h = 2\,\text{m}}$ above the plane through the centre. The numerical entry is $\boxed{2}$.

Quick Tip to Solve: Use energy for the speed and set the normal reaction to zero for separation; distinguish height above the centre from the fall below the top.

58. The K_α X-ray of molybdenum has wavelength 0.071 nm. If the energy of a molybdenum atoms with a K electron knocked out is 27.5 keV, the energy of this atom when an L electron is knocked out will be _____ keV. (Round off to the nearest integer)
[h = 4.14 $\times$ 10^{-15} eVs, c = 3 $\times$ 10^8 ms^{-1}] [4 marks]

Solution:

Concept:

- A K_α photon is emitted when an electron from the L shell fills a K-shell vacancy.
- The photon energy is the difference between the energies of the atom with a K vacancy and with an L vacancy: $E_\gamma = E_K - E_L = hc/\lambda$.

Step 1: Calculate the emitted photon energy The supplied Planck constant is in electron-volt seconds, so the result of hc/λ is directly in electron-volts when wavelength is expressed in metres. Convert the wavelength first:

$$\lambda = 0.071\,\text{nm} = 7.1 \times 10^{-11}\,\text{m}.$$

Consequently,

$$E_\gamma = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{7.1 \times 10^{-11}}\,\text{eV} = 1.7493 \times 10^4\,\text{eV} = 17.493\,\text{keV}.$$

The factor of one thousand in converting eV to keV is essential; both atomic energies must be compared in the same unit.

Step 2: Find the energy of the L-vacancy state The K-vacancy state is the higher-energy initial state. When an L electron drops into the K vacancy, the atom releases the photon and is left with an L vacancy. Conservation of energy therefore gives

$$E_K = E_L + E_\gamma.$$

Using the given K-vacancy energy,

$$E_L = 27.5 - 17.493 = 10.007\,\text{keV}.$$

Adding the photon energy would reverse the direction of the transition and make the final state higher than the initial state, contradicting emission. The instruction asks for the nearest integer, so round only after performing this subtraction.

Step 3: Conclusion The energy of the atom with an L electron knocked out is approximately $10.007\,\text{keV}$, which rounds to $\boxed{10\,\text{keV}}$. The numerical entry is $\boxed{10}$.

Quick Tip to Solve: For a characteristic X-ray emission, subtract the photon energy from the initial vacancy-state energy; keep eV and keV consistent.

59. The water is filled upto height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth 'h' below the water level. The value of 'h' for which the emerging steam of water strikes the ground at the maximum range is _____ m. [4 marks]

Solution:

Concept:

- Torricelli's theorem gives the speed $v = \sqrt{2gh}$ of water leaving a small hole at depth h below a free surface.
- The horizontal range is the efflux speed multiplied by the fall time from the hole to the ground. The hole height is $12 - h$, not h.

Step 1: Combine efflux speed and fall time Write the water-surface height as $H = 12\,\text{m}$. The hole is h below that surface and hence $H - h$ above ground. The free surface and emerging jet are both at atmospheric pressure. Bernoulli's equation, neglecting the free-surface speed in a large tank, gives

$$v = \sqrt{2gh}.$$

The jet initially moves horizontally, so its initial vertical velocity is zero. Its time to reach the ground follows from vertical free fall:

$$H - h = \frac{1}{2}gt^2, \qquad t = \sqrt{\frac{2(H-h)}{g}}.$$

Horizontal and vertical motion must be combined: a deep hole gives a faster jet but also a shorter flight time.

Step 2: Maximise the horizontal range The range X is

$$X = vt = 2\sqrt{h(H-h)}, \qquad X^2 = 4(Hh - h^2).$$

Maximising X is equivalent to maximising its nonnegative square. Complete the square:

$$h(H-h) = \frac{H^2}{4} - \left(h - \frac{H}{2}\right)^2.$$

The largest value occurs at $h = H/2$, which lies inside the physical interval $0 < h < H$. At either end the range vanishes, confirming this is a maximum. Thus $h = 12/2 = 6\,\text{m}$; the hole is also $6\,\text{m}$ above the ground.

Step 3: Conclusion The required depth below the water surface is $\boxed{h = 6\,\text{m}}$. The numerical entry is $\boxed{6}$.

Quick Tip to Solve: For a horizontal jet, maximise the product of depth below the surface and height above ground; these two distances must be equal.

60. The maximum amplitude for an amplitude modulated wave is found to be 12V while the minimum amplitude is found to be 3V. The modulation index is 0.6x where x is _____. [4 marks]

Solution:

Concept:

- For ordinary amplitude modulation without overmodulation, the envelope extremes are $A_{\max} = A_c(1+\mu)$ and $A_{\min} = A_c(1-\mu)$.
- The modulation index $\mu = A_m/A_c$ is dimensionless. Adding and subtracting the envelope extremes isolates the carrier and modulating amplitudes.

Step 1: Recover the carrier and modulating amplitudes Here $A_{\max} = 12\,\text{V}$ and $A_{\min} = 3\,\text{V}$. Adding the two envelope equations removes the modulating part:

$$2A_c = A_{\max} + A_{\min} = 12 + 3 = 15\,\text{V}, \qquad A_c = 7.5\,\text{V}.$$

Subtracting the equations gives twice the modulating amplitude:

$$2A_m = A_{\max} - A_{\min} = 12 - 3 = 9\,\text{V}, \qquad A_m = 4.5\,\text{V}.$$

Neither envelope extreme by itself is the carrier amplitude; taking their average is what identifies the carrier correctly.

Step 2: Evaluate the index and the requested multiplier Divide the modulating amplitude by the

carrier amplitude, or use the equivalent envelope formula:

$$\mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{12-3}{12+3} = \frac{9}{15} = 0.6.$$

This is below unity, consistent with the assumed nonnegative AM envelope. The question expresses this same index as $0.6x$, so

$$0.6x = 0.6 \quad \Longrightarrow \quad x = 1.$$

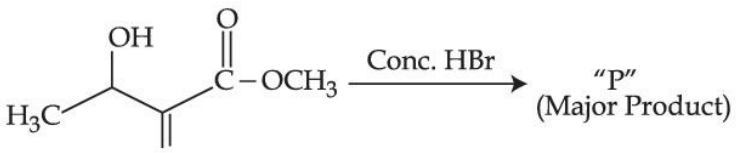
The voltage units cancel in the ratio. As a check, $7.5(1+0.6) = 12$ and $7.5(1-0.6) = 3$, reproducing both measured amplitudes.

Step 3: Conclusion The modulation index is 0.6, so the required dimensionless value is $\boxed{x = 1}$.

Quick Tip to Solve: Find the modulation index as envelope difference divided by envelope sum, then solve for the multiplier explicitly requested.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Consider the above reaction, the major product "P" formed is : [4 marks]



(A) $CH_3CH(Br)C(=CH_2)COOCH_3$

(B) $CH_3CH{=}C(CH_2Br)COOCH_3$

(C) $CH_3CH(OBr)C(=CH_2)COOCH_3$

(D) $CH_3CH(Br)C(=CH_2)COBr$

Correct Answer: (B) $CH_3CH{=}C(CH_2Br)COOCH_3$

Solution:

Concept:

- Protonation converts an alcohol's hydroxyl group into water, a good leaving group. An allylic carbocation can delocalise its positive charge over two positions.
- Resonance-assisted substitution may shift the double bond. Compare the stabilisation of the product alkenes, including conjugation with a neighbouring carbonyl group.

Step 1: Form the allylic intermediate The substrate is $CH_3CH(OH)C(=CH_2)COOCH_3$. Its hydroxyl-bearing carbon is next to a carbon-carbon double bond. Concentrated HBr protonates the hydroxyl oxygen, after which water leaves. The resulting allylic cation has the resonance forms

$$CH_3CH^+\,C(=CH_2)COOCH_3 \longleftrightarrow CH_3CH{=}C(CH_2{}^+)COOCH_3.$$

These are two representations of one delocalised intermediate, not separate molecules in equilibrium. Bromide can attack at the original alcohol carbon or at the terminal carbon carrying positive charge

in the second resonance form.

Step 2: Select the rearranged major product Attack at the terminal carbon produces the rearranged bromide:

$$CH_3CH{=}C(CH_2Br)COOCH_3.$$

Its internal double bond is trisubstituted, whereas the original terminal methylene is disubstituted. Both arrangements are conjugated with the ester carbonyl, but the additional alkyl substitution stabilises the rearranged alkene, favouring option (B). Option (A) represents unrearranged replacement at the original hydroxyl carbon and retains the less substituted terminal methylene arrangement. Option (C) places bromine on oxygen as an alkyl hypobromite; that is not the substitution product of an alcohol with HBr. Option (D) converts the ester to an acyl bromide as well as replacing the hydroxyl group; ordinary concentrated aqueous HBr does not accomplish this acyl-halide preparation. The methyl ester group is retained in the selected product.

Step 3: Conclusion The major product is the rearranged allylic bromide $CH_3CH{=}C(CH_2Br)COOCH_3$, with a conjugated alkene-ester system. Hence, the correct option is (B).

Quick Tip to Solve: For an allylic alcohol in strong acid, draw both carbocation resonance forms before choosing where bromide attacks; check product conjugation.

62. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : $SO_2(g)$ is absorbed to a large extent than $H_2(g)$ on activated charcoal.

Reason R : $SO_2(g)$ has a higher critical temperature than $H_2(g)$.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are correct but R is not the correct explanation of A.

(B) Both A and R are correct and R is the correct explanation of A.

(C) A is not correct but R is correct.

(D) A is correct but R is not correct.

Correct Answer: (B) Both A and R are correct and R is the correct explanation of A.

Solution:

Concept:

- Physical adsorption of gases on activated charcoal results from intermolecular attractions at its large surface area.
- At comparable conditions, gases that liquefy more readily are generally adsorbed more strongly. A higher critical temperature indicates stronger intermolecular attractions.

Step 1: Evaluate Assertion A The statement compares uptake of SO_2 and H_2 by the same adsorbent. Activated charcoal offers a large surface on which gas molecules can accumulate. Sulfur dioxide is much more readily liquefied and more strongly attracted than hydrogen, so it is retained to a greater extent. Thus Assertion A is correct. In this surface-chemistry context the printed word “absorbed”

refers to the gas uptake by adsorption on charcoal; the comparison being tested is the relative extent of adsorption.

Step 2: Evaluate Reason R Sulfur dioxide has a much higher critical temperature than hydrogen. Critical temperature is the highest temperature at which a gas can be liquefied by pressure. A high value reflects stronger attractive forces between molecules, whereas hydrogen has very weak intermolecular forces and an exceptionally low critical temperature. Hence Reason R is also correct. This comparison is about ease of liquefaction and physical attraction, not a compulsory chemical reaction with the carbon surface.

Step 3: Check the explanatory connection The same stronger attractions that make SO_2 easier to liquefy also make its physical adsorption on charcoal more favourable. Therefore R supplies the reason for the greater uptake stated in A. Option (A) wrongly denies this explanatory link. Option (C) rejects the correct adsorption comparison, and option (D) rejects the correct critical-temperature comparison. Both statements and the causal link are retained only in option (B).

Step 4: Conclusion Both Assertion A and Reason R are correct, and R is the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: For physical adsorption of gases, compare ease of liquefaction: a higher critical temperature generally means stronger adsorption on the same surface.

63. The correct order of first ionisation enthalpy is : [4 marks]

(A) Mg $<$ S $<$ Al $<$ P

(B) Mg $<$ Al $<$ S $<$ P

(C) Al $<$ Mg $<$ S $<$ P

(D) Mg $<$ Al $<$ P $<$ S

Correct Answer: (C) Al $<$ Mg $<$ S $<$ P

Solution:

Concept:

- First ionisation enthalpy is the energy required to remove an electron from an isolated gaseous neutral atom. It generally rises across a period as effective nuclear charge increases.
- Filled subshells and half-filled subshells introduce exceptions. Removing a 3p electron from aluminium is easier than removing a 3s electron from magnesium; paired 3p electrons lower sulfur's value relative to phosphorus.

Step 1: Compare aluminium with magnesium Ignoring the common neon core, magnesium is $3s^2$ and aluminium is $3s^2 3p^1$. The electron removed from aluminium occupies the higher-energy, more shielded 3p subshell. Magnesium must lose an electron from its filled 3s subshell. This subshell change outweighs the general increase in effective nuclear charge between these neighbours, so

$$I_1(\text{Al}) < I_1(\text{Mg}).$$

Any proposed order placing magnesium below aluminium therefore misses the first important excep-

tion to the simple left-to-right trend.

Step 2: Compare sulfur with phosphorus and assemble the order Phosphorus has configuration $3s^2 3p^3$, with one electron in each p orbital and a stable half-filled p subshell. Sulfur has $3s^2 3p^4$, requiring one pair in a p orbital. Repulsion within that pair makes an electron easier to remove from sulfur than from phosphorus:

$$I_1(\mathrm{S}) < I_1(\mathrm{P}).$$

Both have substantially higher ionisation enthalpy than magnesium, because their valence electrons experience the greater effective nuclear charge later in the period. Combining the comparisons gives

$$\mathrm{Al} < \mathrm{Mg} < \mathrm{S} < \mathrm{P}.$$

Options (A), (B) and (D) all put magnesium below aluminium. Option (D) additionally reverses the phosphorus-sulfur exception. Only option (C) satisfies both subshell effects and the overall periodic trend.

Step 3: Conclusion The correct order of first ionisation enthalpy is $\boxed{\mathrm{Al} < \mathrm{Mg} < \mathrm{S} < \mathrm{P}}$. Hence, the correct option is (C).

Quick Tip to Solve: Across Period 3, flag the two exceptions before ordering values: aluminium is below magnesium, and sulfur is below phosphorus.

---

64. Given below are two statements :

Statement I : Hyperconjugation is a permanent effect.

Statement II : Hyperconjugation in ethyl cation $\left(CH_3 - \overset{+}{C}H_2\right)$ **involves the overlapping of** $C_{sp^2} - H_{1s}$ **bond with empty 2p orbital of other carbon.**

Choose the correct option : [4 marks]

(A) Both statement I and statement II are false

(B) Statement I is incorrect but statement II is true

(C) Statement I is correct but statement II is false

(D) Both statement I and statement II are true.

Correct Answer: (C) Statement I is correct but statement II is false

Solution:

Concept:

- Hyperconjugation is delocalisation of electrons of an adjacent sigma bond into a vacant or partially filled p orbital or an appropriate pi system. It stabilises the molecule or ion without requiring the approach of an external reagent.
- In an alkyl carbocation, the positively charged carbon is approximately sp^2 hybridised and has an empty p orbital. The neighbouring saturated carbon supplying a C–H sigma bond is sp^3 hybridised.

Step 1: Evaluate Statement I Hyperconjugation is part of the electronic structure of the species itself. For example, ethyl cation is stabilised by delocalisation from the C–H bonds of its methyl

group into the vacant p orbital of the neighbouring cationic carbon. This electron delocalisation is present without an attacking reagent, unlike an electromeric effect induced during attack. Thus hyperconjugation is a permanent electronic effect, and Statement I is true.

Step 2: Identify the donor bond in Statement II Write ethyl cation as $CH_3-CH_2^+$. The methyl carbon has four sigma bonds and is sp^3 hybridised. Hence the donating bond is formed from a carbon sp^3 orbital and a hydrogen 1s orbital, not a carbon sp^2 orbital. The empty 2p orbital belongs to the adjacent positively charged carbon. The statement correctly mentions an empty p acceptor but assigns the wrong hybridisation to the donor carbon, so Statement II is false. Options A and B incorrectly reject Statement I; option D incorrectly accepts Statement II.

Step 3: Conclusion Statement I is correct but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: Label the donor and acceptor carbons separately: the donor methyl carbon is sp^3, while the carbocation centre is sp^2.

---

65. Given below are two statements :

Statement I : $[Mn(CN)_6]^{3-}$, $[Fe(CN)_6]^{3-}$ **and** $[Co(C_2O_4)_3]^{3-}$ **are d^2sp^3 hybridised.**

Statement II : $[MnCl)_6]^{3-}$ **and** $[FeF_6]^{3-}$ **are paramagnetic and have 4 and 5 unpaired electrons, respectively.**

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Statement I is correct but statement II is false

(B) Both statement I and Statement II are false

(C) Statement I is incorrect but statement II is true

(D) Both statement I and statement II are true

Correct Answer: (D) Both statement I and statement II are true

Solution:

Concept:

- Find the metal oxidation state before assigning its d-electron count. In all five complexes here, the metal is in oxidation state +3: Mn^{3+} is d^4, Fe^{3+} is d^5, and Co^{3+} is d^6.
- Strong-field octahedral complexes can pair electrons in the lower t_{2g} set and use inner 3d orbitals, giving d^2sp^3 hybridisation in the valence-bond description. Weak-field ligands favour high-spin configurations.

Step 1: Check the three complexes in Statement I Cyanide is a strong-field ligand. The low-spin configurations of the Mn(III) and Fe(III) cyanide complexes are respectively t_{2g}^4 and t_{2g}^5, with both upper e_g orbitals vacant. Thus two inner d orbitals can participate in d^2sp^3 hybridisation. In the Co(III) oxalate complex, the large splitting associated with Co(III) gives the low-spin d^6 arrangement t_{2g}^6; it also has two vacant inner d orbitals. Therefore all three are inner-orbital octahedral complexes, and Statement I is true.

Step 2: Count unpaired electrons in Statement II Chloride and fluoride give high-spin configurations

for these Mn(III) and Fe(III) complexes. For d^4, the octahedral arrangement is $t_{2g}^3 e_g^1$, containing four unpaired electrons. For d^5, it is $t_{2g}^3 e_g^2$, containing five unpaired electrons. Both complexes are therefore paramagnetic, so Statement II is true. Option A wrongly rejects the second statement, option C wrongly rejects the first, and option B rejects both correct statements.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: Use oxidation state, d count, then ligand strength in that order; never count unpaired electrons from the neutral atom.

---

66. To an aqueous solution containing ions such as Al^{3+}, Zn^{2+}, Ca^{2+}, Fe^{3+}, Ni^{2+}, Ba^{2+} and Cu^{2+} was added conc. HCl, followed by H_2S.

The total number of cations precipitated during this reaction is/are : [4 marks]

(A) 1

(B) 3

(C) 4

(D) 2

Correct Answer: (A) 1

Solution:

Concept:

- Passing H_2S through an acidic solution supplies only a small concentration of S^{2-}, because hydrogen ions suppress the dissociation of the weak acid H_2S.
- A metal sulphide precipitates only when its ionic product exceeds its solubility product. Group-II sulphides such as CuS are sufficiently insoluble to precipitate in acidic medium; group-IV sulphides need a higher sulphide concentration.

Step 1: Check the effect of adding hydrochloric acid The listed ions do not include the usual group-I cations such as silver, lead(II), or mercury(I), whose chlorides precipitate on adding hydrochloric acid. Thus none of the seven listed cations is removed at this initial chloride-precipitation stage. In particular, the chlorides of calcium, barium, zinc, nickel, aluminium, copper(II), and iron(III) remain in aqueous solution under the qualitative-analysis conditions.

Step 2: Identify the sulphide precipitate in acid On passing H_2S, copper(II) is precipitated as black copper sulphide:

$$Cu^{2+} + H_2S \longrightarrow CuS\,(s) + 2\,H^{+}.$$

Zinc and nickel require the greater sulphide concentration of alkaline medium for their group-IV precipitation. Aluminium and iron are ordinarily separated as hydroxides at a later stage, not as sulphides here. Calcium and barium also remain dissolved under these conditions. Iron(III) may oxidise hydrogen sulphide to sulphur while becoming iron(II), but deposition of elemental sulphur is not precipitation of a cation. Therefore only copper counts; totals of two, three, or four incorrectly include ions belonging to later analytical groups.

Step 3: Conclusion Only Cu^{2+} precipitates, so the total number of cations precipitated is 1. Hence,

the correct option is (A).

Quick Tip to Solve: For acidic H_2S, first look for group-II ions; do not count elemental sulphur as a precipitated metal ion.

67. Compound A gives D-Galactose and D-Glucose on hydrolysis. The compound A is : [4 marks]

(A) Amylose

(B) Sucrose

(C) Maltose

(D) Lactose

Correct Answer: (D) Lactose

Solution:

Concept:

- Hydrolysis of a disaccharide cleaves its glycosidic linkage and identifies the two monosaccharide units from which it is built. The identities of the products distinguish sugars that share the same molecular formula.
- Lactose contains one D-galactose unit and one D-glucose unit joined by a $\beta(1 \rightarrow 4)$ glycosidic linkage. It is a reducing disaccharide because the glucose anomeric carbon remains free.

Step 1: Match the stated hydrolysis products The question specifies D-galactose together with D-glucose. These are the two constituent monosaccharides of lactose, the sugar found in milk. Cleavage of its glycosidic bond consumes one water molecule and releases one molecule of each monosaccharide:

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow C_6H_{12}O_6 + C_6H_{12}O_6.$$

Although both products have formula $C_6H_{12}O_6$, they are different sugars; their names, rather than molecular formula alone, provide the decisive identification.

Step 2: Eliminate the other carbohydrates Amylose is the essentially unbranched component of starch. Its glucose units are linked mainly through alpha-1,4 bonds, and complete hydrolysis produces glucose only, not galactose. Sucrose hydrolyses into D-glucose and D-fructose, so its second product disagrees with the question. Maltose contains two glucose units and therefore gives D-glucose alone on complete hydrolysis. Lactose uniquely supplies the stated galactose–glucose pair among the four options. The question asks for the parent carbohydrate, not for either individual product, so choosing glucose-related polymers from just one product would be insufficient.

Step 3: Conclusion Compound A is lactose, which hydrolyses into D-galactose and D-glucose. Hence, the correct option is (D).

Quick Tip to Solve: Remember the product pairs: lactose gives galactose + glucose; sucrose gives glucose + fructose; maltose gives glucose + glucose.

68.

$$R - CN \xrightarrow[(ii)H_2O]{(i)DIBAL-H} R - Y$$

Consider the above reaction and identify "Y" [4 marks]

(A) $-CH_2NH_2$

(B) $-CONH_2$

(C) $-CHO$

(D) $-COOH$

Correct Answer: (C) $-CHO$

Solution:

Concept:

- Diisobutylaluminium hydride (DIBAL-H) is a selective reducing reagent. Controlled reduction of a nitrile followed by aqueous work-up gives an aldehyde, retaining the nitrile carbon in the carbon skeleton.
- The sequence matters: hydride first produces an aluminium-bound imine intermediate, and water then hydrolyses that intermediate to the carbonyl compound. This differs from complete nitrile reduction to a primary amine.

Step 1: Follow the controlled hydride reduction The starting functional group is $\mathrm{R{-}C{\equiv}N}$. DIBAL-H transfers hydride to the electrophilic nitrile carbon, and the resulting nitrogen-containing intermediate remains coordinated to aluminium until work-up. This controlled reduction stops before the full conversion of the nitrile carbon to a methylene group. The carbon initially present in the nitrile becomes the carbonyl carbon of the aldehyde; the alkyl or aryl group R remains attached throughout.

Step 2: Apply aqueous work-up and compare options Water hydrolyses the reduced imine intermediate, replacing its carbon–nitrogen linkage by the aldehyde carbonyl. The net functional-group change is

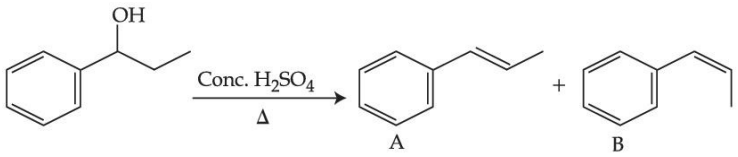
$$\mathrm{R{-}CN} \xrightarrow[\mathrm{H_2O}]{\mathrm{DIBAL{-}H}} \mathrm{R{-}CHO}.$$

Consequently the group Y attached to R is $-\mathrm{CHO}$. The $-\mathrm{CH_2NH_2}$ option would correspond to exhaustive reduction, for example with lithium aluminium hydride, rather than this selective work-up. An amide, $-\mathrm{CONH_2}$, is associated with partial hydrolysis of a nitrile rather than its DIBAL-H reduction. A carboxylic acid, $-\mathrm{COOH}$, is obtained by extensive hydrolysis and also does not represent the reducing sequence shown. Only the aldehyde option follows both steps in their stated order.

Step 3: Conclusion $Y = -\mathrm{CHO}$, and the product is the aldehyde $\mathrm{R{-}CHO}$. Hence, the correct option is (C).

Quick Tip to Solve: For nitriles, distinguish DIBAL-H followed by water (aldehyde) from complete hydride reduction (primary amine).

69. Consider the above reaction, and choose the correct statement : [4 marks]



(A) The reaction is not possible in acidic medium

(B) Both compounds A and B are formed equally

(C) Compound A will be the major product

(D) Compound B will be the major product

Correct Answer: (C) Compound A will be the major product

Solution:

Concept:

- Concentrated sulphuric acid and heat dehydrate an alcohol by converting the hydroxyl group into a good leaving group, followed by elimination. A benzylic carbocation is stabilised by resonance with the benzene ring.
- When geometrical alkene isomers can form, the trans or E isomer is generally more stable than the cis or Z isomer because bulky groups are farther apart. Both products here retain conjugation with the phenyl group.

Step 1: Identify the elimination products The alcohol shown is $C_6H_5-CH(OH)-CH_2-CH_3$. Protonation of its hydroxyl group is followed by loss of water, giving a resonance-stabilised benzylic carbocation. Removal of a beta hydrogen from the adjacent methylene carbon then produces $C_6H_5-CH=CH-CH_3$. Rotation in the cationic intermediate permits elimination leading to either geometrical isomer. Thus the reaction is possible in acidic medium, ruling out option A.

Step 2: Compare A and B using their geometry In the drawing of compound A, the phenyl and methyl substituents lie on opposite sides of the carbon–carbon double bond. A is therefore the E (trans) isomer. In B, these two groups lie on the same side, making B the Z (cis) isomer. The E isomer has less steric repulsion between the phenyl and methyl groups and is thermodynamically more stable. Under heated acid-catalysed dehydration conditions it is favoured, so A is the major product. Both isomers are conjugated alkenes; conjugation alone cannot distinguish them. Their different steric energies rule out equal production as proposed in option B, and the less stable cis product cannot be assigned as the major product in option D.

Step 3: Conclusion Compound A, the E (trans) alkene, is the major product. Hence, the correct option is (C).

Quick Tip to Solve: For geometrical isomers, locate the phenyl and methyl groups across the double bond before comparing steric stability.

70. Match List - I with List - II :

	List - I(compound)		List - II(effect/affected species)
(a)	Carbon monoxide	(i)	Carcinogenic
(b)	Sulphur dioxide	(ii)	Metabolized by pyrus plants
(c)	Polychlorinated biphenyls	(iii)	haemoglobin
(d)	Oxides of Nitrogen	(iv)	Stiffness of flower buds

Choose the correct answer from the options given below : [4 marks]

(A) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

(B) (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)

(C) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)

(D) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Correct Answer: (A) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

Solution:

Concept:

- Environmental pollutants can be matched to their characteristic biological effects. A correct matching option must satisfy all four associations, so a familiar pair is useful for initial elimination but not sufficient for the final choice.
- Carbon monoxide binds haemoglobin strongly; sulphur dioxide damages plant tissues; polychlorinated biphenyls are persistent carcinogenic pollutants; certain plants, including Pyrus, can metabolise nitrogen oxides.

Step 1: Assign the characteristic effects Carbon monoxide forms carboxyhaemoglobin and reduces the oxygen-carrying capacity of blood, so (a) matches haemoglobin, (iii). Sulphur dioxide adversely affects plants, including stiffening of flower buds and premature falling of flowers, so (b) matches (iv). Polychlorinated biphenyls are persistent chlorinated organic pollutants associated with carcinogenic effects, so (c) matches (i). Oxides of nitrogen can be absorbed and metabolised by Pyrus plants, giving (d) matched with (ii).

Step 2: Check the complete matching against the options The required sequence is

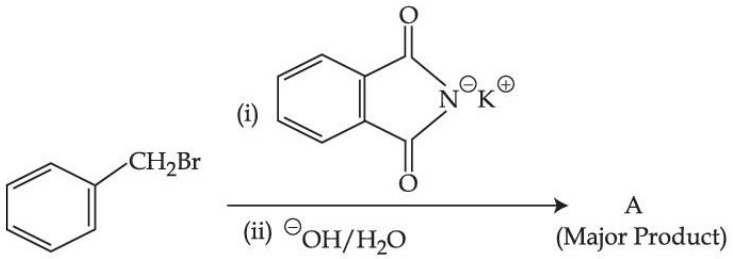
$$(a) \to (iii), \quad (b) \to (iv), \quad (c) \to (i), \quad (d) \to (ii).$$

Option A reproduces every pair. Option B assigns flower-bud stiffness to carbon monoxide and haemoglobin to PCBs, so it fails immediately. Option C assigns carcinogenicity to carbon monoxide and haemoglobin to PCBs while also misplacing the plant effects. Option D gets the first two pairs right but exchanges the last two: it attributes Pyrus metabolism to PCBs and the listed carcinogenic effect to nitrogen oxides. Therefore checking only carbon monoxide or sulphur dioxide would leave a tempting wrong alternative; the PCB and Pyrus pairs settle the choice.

Step 3: Conclusion Carbon monoxide matches haemoglobin; sulphur dioxide matches stiffness of flower buds; PCBs match carcinogenicity; nitrogen oxides match Pyrus metabolism. Hence, the correct option is (A).

Quick Tip to Solve: Fix CO–haemoglobin first, then distinguish the remaining similar options using PCBs–carcinogenic and nitrogen oxides–Pyrus.

71. What is A in the following reaction? [4 marks]



(A) N-benzylbenzamide, $C_6H_5CONHCH_2C_6H_5$

(B) Benzyl alcohol, $C_6H_5CH_2OH$

(C) Phthalimide

(D) Benzylamine, $C_6H_5CH_2NH_2$

Correct Answer: (D) Benzylamine, $C_6H_5CH_2NH_2$

Solution:

Concept:

- Gabriel synthesis converts a suitable alkyl halide into a primary amine through alkylation of potassium phthalimide followed by hydrolysis.
- The nitrogen anion is the nucleophile. Benzyl bromide, $C_6H_5CH_2Br$, has an accessible benzylic carbon and undergoes substitution without breaking the carbon skeleton.

Step 1: Identify the first substitution The starting compound is benzyl bromide, not bromobenzene: bromine is attached to the side-chain CH_2 carbon. The nitrogen of potassium phthalimide attacks that carbon and replaces bromide, giving N-benzylphthalimide. This places the benzyl group on nitrogen and forms the carbon–nitrogen bond required in the product.

Step 2: Hydrolyse the protecting group Treatment with aqueous hydroxide hydrolyses the imide carbonyl groups and releases the primary amine from the phthalic-acid-derived protecting group. The benzyl–nitrogen bond survives. Thus the net change in the organic side chain is

$$C_6H_5CH_2Br \longrightarrow C_6H_5CH_2NH_2.$$

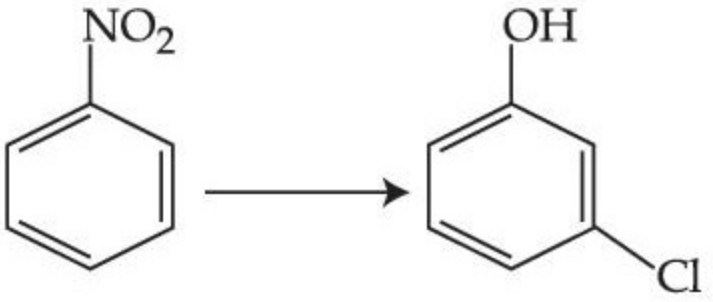
The product nitrogen carries two hydrogen atoms after hydrolysis and proton transfer.

Step 3: Compare the four structures Option (D) is benzylamine and has exactly this connectivity. Option (A) is an amide, which would require retention of a benzoyl group rather than cleavage of the phthalimide protecting group. Option (B) is benzyl alcohol, a substitution product that contains oxygen instead of the introduced nitrogen. Option (C) is unsubstituted phthalimide and does not retain the benzyl group as the required amine product.

Step 4: Conclusion The major product A is benzylamine, $C_6H_5CH_2NH_2$. Hence, the correct option is (D).

Quick Tip to Solve: In Gabriel synthesis, keep the alkyl skeleton unchanged and replace its halogen by NH_2 after hydrolysis.

72. **The correct sequence of correct reagents for the following transformation is :** [4 marks]



(A) (i) Fe, HCl
(ii) Cl_2, HCl
(iii) $NaNO_2$, HCl, 0°C
(iv) H_2O/H^+
(B) (i) Fe, HCl
(ii) $NaNO_2$, HCl, 0°C
(iii) H_2O/H^+
(iv) Cl_2, $FeCl_3$
(C) (i) Cl_2, $FeCl_3$
(ii) Fe, HCl
(iii) $NaNO_2$, HCl, 0°C
(iv) H_2O/H^+
(D) (i) Cl_2, $FeCl_3$
(ii) $NaNO_2$, HCl, 0°C
(iii) Fe, HCl
(iv) H_2O/H^+

Correct Answer: (C) (i) Cl_2, $FeCl_3$
(ii) Fe, HCl
(iii) $NaNO_2$, HCl, 0°C
(iv) H_2O/H^+

Solution:

Concept:

- The nitro group is deactivating and meta-directing in electrophilic aromatic substitution, whereas amino and hydroxyl groups favour ortho and para substitution.
- Reduction changes NO_2 to NH_2; diazotisation followed by aqueous hydrolysis changes aromatic NH_2 into OH at the same ring carbon.

Step 1: Establish the required relative positions The starting ring is nitrobenzene and the target is meta-chlorophenol. Chlorine must therefore be introduced while the nitro group is still present, because NO_2 directs the entering electrophile to the meta position. Use $Cl_2/FeCl_3$ first to obtain meta-chloronitrobenzene. The chlorine position is then fixed for the subsequent conversions.

Step 2: Convert the nitro group into hydroxyl Reduce the nitro group with iron and hydrochloric acid to obtain meta-chloroaniline. Treat this primary aromatic amine with $NaNO_2/HCl$ at $0\,^\circ C$ to form its diazonium salt. Hydrolysis with water in acidic conditions replaces the diazonium group by hydroxyl with loss of nitrogen, giving meta-chlorophenol.

Step 3: Check the order against the options The required order is chlorination, reduction, diazotisation, then hydrolysis. Option (C) gives precisely this sequence. Option (A) reduces first, losing the nitro group's meta-directing effect. Option (B) makes phenol before chlorination and would favour ortho and para products. Option (D) attempts diazotisation before converting the nitro group into an amine; nitrobenzene cannot undergo the required primary-amine diazotisation.

Step 4: Conclusion The sequence is $Cl_2/FeCl_3$, Fe/HCl, $NaNO_2/HCl$ at $0\,^\circ C$, then H_2O/H^+. Hence, the correct option is (C).

Quick Tip to Solve: For a meta-substituted phenol obtained from nitrobenzene, introduce the other substituent before reducing the meta-directing nitro group.

73. Which one of the following set of elements can be detected using sodium fusion extract? [4 marks]

(A) Sulfur, Nitrogen, Phosphorous, Halogens

(B) Phosphorous, Oxygen, Nitrogen, Halogens

(C) Nitrogen, Phosphorous, Carbon, Sulfur

(D) Halogens, Nitrogen, Oxygen, Sulfur

Correct Answer: (A) Sulfur, Nitrogen, Phosphorous, Halogens

Solution:

Concept:

- In sodium fusion, covalently bound heteroatoms are converted into water-soluble ionic compounds so that qualitative tests can be performed on the extract.
- The method detects nitrogen, sulfur and halogens; phosphorus is also detected after conversion to phosphate in the elemental-analysis procedure.

Step 1: List the detectable heteroatoms Fusion converts nitrogen into cyanide, sulfur into sulfide and halogens into sodium halides. These ionic species enter the aqueous extract and can be tested chemically. Nitrogen is identified through the Prussian-blue test, sulfur through sulfide reactions such as the sodium nitroprusside test, and halogens through their silver-halide precipitates after suitable treatment of the extract.

Step 2: Include the phosphorus test Phosphorus is converted into phosphate in the fusion and oxidative work-up used for its qualitative detection. Acidification and treatment with ammonium molybdate give the characteristic yellow ammonium phosphomolybdate precipitate. Thus phosphorus belongs with nitrogen, sulfur and halogens in the set sought here; the presence of phosphorus in the list does not make that option incorrect.

Step 3: Eliminate carbon and oxygen choices The question concerns detection by the sodium-fusion extract, not every possible elemental-analysis method. Carbon is normally detected by oxidation

to carbon dioxide, and oxygen is not identified by a characteristic Lassaigne-extract test. Option (B) includes oxygen, option (C) includes carbon, and option (D) includes oxygen. Only option (A) contains the complete permissible set without inserting carbon or oxygen.

Step 4: Conclusion The detectable set is sulfur, nitrogen, phosphorus and halogens. Hence, the correct option is (A).

Quick Tip to Solve: For a sodium-fusion question, look for heteroatoms that form testable ionic products; reject lists that introduce carbon or oxygen.

74. The number of neutrons and electrons, respectively, present in the radioactive isotope of hydrogen is : [4 marks]

(A) 1 and 1

(B) 3 and 1

(C) 2 and 1

(D) 2 and 2

Correct Answer: (C) 2 and 1

Solution:

Concept:

- For a nuclide ${}^{A}_{Z}X$, the neutron number is $A - Z$, while a neutral atom contains Z electrons.
- Hydrogen has atomic number $Z = 1$. Its common isotopes are protium, deuterium and tritium, and tritium is radioactive.

Step 1: Identify the radioactive isotope The three isotopes have the same nuclear charge but different mass numbers: protium is ${}^{1}_{1}\mathrm{H}$, deuterium is ${}^{2}_{1}\mathrm{H}$, and tritium is ${}^{3}_{1}\mathrm{H}$. Protium and deuterium are stable. The radioactive isotope specified in the question is therefore tritium, whose mass number is three and atomic number is one.

Step 2: Count neutrons and electrons separately The mass number counts protons plus neutrons. Subtract the single proton from the mass number to obtain

$$N = A - Z = 3 - 1 = 2.$$

Since the question specifies an isotope atom and gives no ionic charge, it is electrically neutral. Its one proton is balanced by one electron, so

$$n_e = Z = 1.$$

Isotopic substitution changes the neutron count, not the atomic number or the neutral electron count.

Step 3: Check the ordered pair The requested order is neutrons first and electrons second, giving $(2, 1)$. Option (A), $(1, 1)$, describes deuterium rather than tritium. Option (B) incorrectly uses the mass number three as the neutron count. Option (D) assigns two electrons to a neutral hydrogen atom, which would instead require an extra negative charge.

Step 4: Conclusion Tritium has 2 neutrons and 1 electron. Hence, the correct option is (C).

Quick Tip to Solve: First identify tritium as hydrogen-3, then use neutrons = mass number minus atomic number; neutral hydrogen always has one electron.

75. Match List - I with List - II :

	List - I		List - II
(a)	Li	(i)	photoelectric cell
(b)	Na	(ii)	absorbent of CO_2
(c)	K	(iii)	coolant in fast breeder nuclear reactor
(d)	Cs	(iv)	treatment of cancer
		(v)	bearings for motor engines

Choose the correct answer from the options given below : [4 marks]

(A) (a) - (v), (b) - (i), (c) - (ii), (d) - (iv)

(B) (a) - (v), (b) - (ii), (c) - (iv), (d) - (i)

(C) (a) - (iv), (b) - (iii), (c) - (i), (d) - (ii)

(D) (a) - (v), (b) - (iii), (c) - (ii), (d) - (i)

Correct Answer: (D) (a) - (v), (b) - (iii), (c) - (ii), (d) - (i)

Solution:

Concept:

- Applications of alkali metals and their compounds depend on their characteristic physical properties, chemical reactivity and electron-loss tendencies.
- In this matching question, potassium is associated with its compounds used for carbon-dioxide absorption, while lithium, sodium and caesium have the listed material or electronic uses.

Step 1: Match lithium and sodium Lithium is used in alloys, including bearing materials for motor engines, so (a) matches (v). Sodium is used as a heat-transfer medium in fast breeder nuclear reactors because liquid sodium transfers heat effectively. Thus (b) matches (iii). These two matches already distinguish the correct assignment from alternatives that associate sodium with photoelectric cells or carbon-dioxide absorption.

Step 2: Match potassium and caesium Potassium compounds are used for carbon-dioxide absorption. For example, potassium hydroxide reacts with carbon dioxide to form carbonate and water:

$$2\,\mathrm{KOH} + \mathrm{CO_2} \longrightarrow \mathrm{K_2CO_3} + \mathrm{H_2O}.$$

Consequently (c) matches (ii). Caesium readily loses its outer electron because of its low ionisation enthalpy, making it useful in photoelectric cells; therefore (d) matches (i).

Step 3: Check the entire option The combined assignment is (a)–(v), (b)–(iii), (c)–(ii), (d)–(i), which is option (D). Option (A) misassigns sodium and caesium. Option (B) misassigns sodium and potassium. Option (C) incorrectly assigns lithium to cancer treatment and swaps the potassium and

caesium uses. The cancer-treatment entry is the unused distractor in List II, rather than a required match for these four entries.

Step 4: Conclusion Lithium matches bearings, sodium matches reactor coolant, potassium matches carbon-dioxide absorption, and caesium matches photoelectric cells. Hence, the correct option is (D).

Quick Tip to Solve: Use liquid sodium as reactor coolant and caesium in photoelectric cells as two anchor matches before checking the remaining entries.

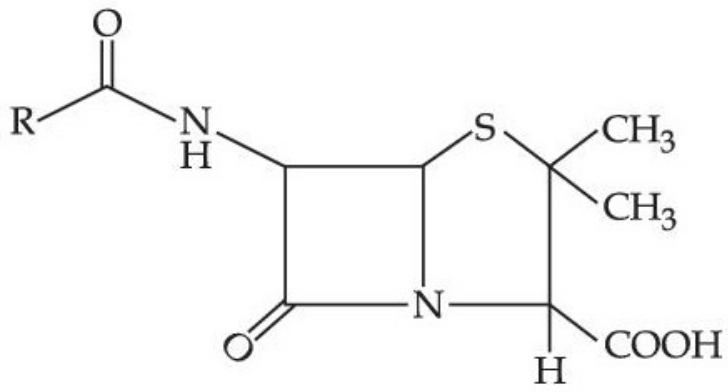
---

76. Given below are two statements :

Statement I : Penicillin is a bacteriostatic type antibiotic.

Statement II : The general structure of Penicillin is :

Choose the correct option : [4 marks]



(A) Both statement I and statement II are false

(B) Statement I is incorrect but statement II is true

(C) Both statement I and statement II are true

(D) Statement I is correct but statement II is false

Correct Answer: (B) Statement I is incorrect but statement II is true

Solution:

Concept:

- A bactericidal antibiotic kills bacteria, whereas a bacteriostatic antibiotic mainly inhibits their growth and multiplication.
- Penicillin is a beta-lactam antibiotic: its characteristic structure contains a four-membered cyclic amide fused to a sulfur-containing five-membered ring.

Step 1: Evaluate the action claimed in Statement I Penicillin inhibits bacterial cell-wall synthesis by interfering with the cross-linking of peptidoglycan. Growing bacteria lose the integrity of their protective wall and can undergo lysis. Its standard classification is therefore bactericidal, not bacteriostatic. Statement I is incorrect. Merely knowing that an antibiotic inhibits a biochemical process does not imply that its final effect is bacteriostatic.

Step 2: Inspect the structure in Statement II The displayed molecule has the characteristic fused ring

system of penicillin: a four-membered beta-lactam ring containing the amide nitrogen and carbonyl, fused to a five-membered sulfur-containing ring. It also shows the carboxylic-acid group, two methyl groups and the variable acylamino side chain $\mathrm{RCONH^-}$. The variable R group accounts for different members of the penicillin family without changing the common nucleus.

Step 3: Combine the truth values The first statement is false because the antibiotic is bactericidal. The second statement is true because the drawing represents the general penicillin skeleton. Option (A) incorrectly rejects the structure, option (C) incorrectly accepts the bacteriostatic classification, and option (D) reverses both truth values. Only option (B) assigns the two statements their correct values.

Step 4: Conclusion Statement I is incorrect but Statement II is true. Hence, the correct option is (B).

Quick Tip to Solve: Separate antibiotic action from structural recognition: penicillin kills bacteria and contains the fused beta-lactam–thiazolidine ring system.

---

77. If the Thompson model of the atom was correct, then the result of Rutherford's gold foil experiment would have been : [4 marks]

(A) All of the α-particles pass through the gold foil without decrease in speed.

(B) α-particles are deflected over a wide range of angles.

(C) All α-particles get bounced back by 180°

(D) α-particles pass through the gold foil deflected by small angles and with reduced speed.

Correct Answer: (D) α-particles pass through the gold foil deflected by small angles and with reduced speed.

Solution:

Concept:

- Thomson's model distributes positive charge throughout the atom, with electrons embedded in it; it has no small, concentrated, massive positive nucleus.
- A heavy positive alpha particle traversing such diffuse charge would experience only weak deflections, unlike the occasional large-angle scattering caused by Rutherford's concentrated nucleus.

Step 1: Apply the diffuse-charge model In Thomson's picture the positive charge is spread through the atomic volume. An alpha particle would not encounter a compact centre with a strong repulsive electric field capable of reversing its direction. Its relatively large momentum also prevents the light electrons from producing large deflections in a single encounter. It should therefore continue mainly forward, suffering only small angular deviations.

Step 2: Account for passage through matter Passage through the foil involves interactions with the distributed charges and electrons. The alpha particles can transfer energy to the material and emerge with a reduced speed. Thus Thomson's model predicts predominantly transmitted particles with small-angle deflections and an energy loss. The prediction is not that the foil exerts absolutely no force and leaves every incident particle's speed unchanged.

Step 3: Compare with each offered observation Option (D) combines small deflections with reduced speed and is the appropriate prediction. Option (A) neglects the energy transferred during passage.

Option (B) permits a wide range of angles, including the large-angle scattering that requires a concentrated positive centre. Option (C) predicts universal backward reflection, which is incompatible with diffuse positive charge and also contradicts the fact that most alpha particles are transmitted even in the actual experiment.

Step 4: Conclusion Alpha particles would pass through with small deflections and reduced speed. Hence, the correct option is (D).

Quick Tip to Solve: To distinguish atomic models in scattering questions, ask whether the positive charge is diffuse or concentrated enough to cause a large-angle deflection.

78. Number of Cl = O bonds in chlorous acid, chloric acid and perchloric acid respectively are :

[4 marks]

(A) 3, 1 and 1

(B) 4, 1 and 0

(C) 1, 1 and 3

(D) 1, 2 and 3

Correct Answer: (D) 1, 2 and 3

Solution:

Concept:

- An oxoacid of chlorine contains a single hydroxyl group, $Cl-O-H$, while the remaining oxygen atoms are represented as terminal $Cl=O$ bonds in its conventional Lewis structure.
- The names chlorous, chloric and perchloric acid correspond to $HClO_2$, $HClO_3$ and $HClO_4$, respectively. Count double bonds, not the total oxygen atoms or the oxidation state of chlorine.

Step 1: Write the chlorous acid structure Chlorous acid is $HClO_2$. One oxygen makes the hydroxyl linkage and the other is terminal. Its conventional structure is $HO-Cl=O$. Thus there is exactly one chlorine–oxygen double bond. The hydroxyl oxygen has two single bonds and is not counted as a double-bonded oxygen.

Step 2: Count the remaining terminal oxygen atoms Chloric acid, $HClO_3$, has one hydroxyl oxygen and two terminal oxygen atoms: $HO-Cl(=O)_2$. Its double-bond count is therefore two. Perchloric acid, $HClO_4$, has one hydroxyl oxygen and three terminal oxygen atoms: $HO-Cl(=O)_3$, giving three double bonds. The increasing oxygen content adds terminal double-bonded oxygen atoms without adding hydroxyl groups.

Step 3: Compare all offered sequences The required ordered triple is $(1, 2, 3)$. Option (A) incorrectly assigns three double bonds to chlorous acid and only one to perchloric acid. Option (B) gives four to chlorous acid despite its having only two oxygen atoms. Option (C) counts only one double bond for chloric acid, omitting one terminal oxygen.

Step 4: Conclusion The numbers of $Cl=O$ bonds are $\boxed{1,\ 2,\ 3}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For these monobasic chlorine oxoacids, reserve one oxygen for the OH group; the remaining oxygen atoms give the conventional Cl=O count.

79. Select the correct statements

(A) Crystalline solids have long range order.

(B) Crystalline solids are isotropic

(C) Amorphous solid are sometimes called pseudo solids.

(D) Amorphous solids soften over a range of temperatures.

(E) Amorphous solids have a definite heat of fusion.

Choose the most appropriate answer from the options given below. [4 marks]

(A) (A), (B), (E) only

(B) (B), (D) only

(C) (C), (D) only

(D) (A), (C), (D) only

Correct Answer: (D) (A), (C), (D) only

Solution:

Concept:

- Crystalline solids have a regular repeating arrangement extending over long distances. Their physical properties may depend on measurement direction, which is called anisotropy.
- Amorphous solids possess short-range order but no periodic long-range lattice. They soften over a temperature interval and are conventionally described as pseudo solids or supercooled liquids.

Step 1: Test the statements about crystalline solids Statement (A) is correct: repeating structural units produce long-range order throughout an ideal crystal. Statement (B) is incorrect as a general classification: crystalline solids are anisotropic, so properties such as refractive index or electrical conductivity can differ along different crystal directions. Isotropy is the usual characteristic contrasted with this in amorphous solids. One must assess the statements as textbook general properties rather than infer isotropy from a particular highly symmetric example.

Step 2: Test the statements about amorphous solids Statement (C) is correct in the conventional school-level classification: amorphous solids are sometimes termed pseudo solids. Statement (D) is also correct because their irregular arrangement leads to gradual softening over a range rather than a sharp melting point. Statement (E) is incorrect: they do not exhibit a definite heat of fusion associated with melting sharply at one temperature.

Step 3: Match the complete set to the options The true statements are (A), (C) and (D). The option containing (A), (B), (E) includes two false statements, and the option containing (B), (D) includes false (B). The option containing only (C), (D) leaves out the true long-range-order statement (A). Only the last option includes all three true statements and excludes both false statements.

Step 4: Conclusion The long-range order of crystals and the pseudo-solid character and gradual softening of amorphous solids are the correct statements. Hence, the correct option is (D).

Quick Tip to Solve: Pair long-range order with crystals and a softening range with amorphous solids; a sharp melting point and definite fusion heat go together.

80. The addition of silica during the extraction of copper from its sulphide ore :- [4 marks]

(A) converts copper sulphide into copper silicate

(B) converts iron oxide into iron silicate

(C) reduces copper sulphide into metallic copper

(D) reduces the melting point of the reaction mixture

Correct Answer: (B) converts iron oxide into iron silicate

Solution:

Concept:

- A flux reacts with gangue to form an easily separable slag. Silica, SiO_2, is an acidic flux and reacts with the basic iron(II) oxide present during copper extraction.
- In extraction from copper sulphide ores, iron sulphide is preferentially oxidised to FeO. Removal of this iron impurity is distinct from the subsequent conversion of copper compounds into metallic copper.

Step 1: Identify the impurity removed by silica During roasting and smelting, the iron-containing component produces iron(II) oxide. A representative oxidation is

$$2\,FeS + 3\,O_2 \longrightarrow 2\,FeO + 2\,SO_2.$$

The oxide FeO is basic. Added silica supplies the acidic partner required to convert it into a silicate slag. The purpose is selective removal of the iron compound while copper remains in the matte.

Step 2: Write the slag-forming reaction The relevant reaction is

$$FeO + SiO_2 \longrightarrow FeSiO_3.$$

Iron(II) silicate, $FeSiO_3$, is the slag, which separates from the copper-containing molten material. This is an acid–base combination of a basic oxide and an acidic oxide; it is not a reduction of copper sulphide by silica.

Step 3: Exclude the alternative descriptions Option (A) names copper silicate, whereas the impurity removed here is iron oxide and the slag is iron silicate. Option (C) assigns reducing action to silica, which this reaction does not have. Although formation of a fusible slag facilitates separation, option (D) does not identify the specific chemical function asked here. Option (B) states the actual conversion represented by the balanced reaction.

Step 4: Conclusion Silica converts iron oxide into iron silicate slag, $FeSiO_3$. Hence, the correct option is (B).

Quick Tip to Solve: Identify the gangue first: basic FeO needs acidic SiO2, and their combination gives iron silicate slag.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The equilibrium constant for the reaction

$A(s) \rightleftharpoons M(s) + \frac{1}{2}O_2(g)$

is $K_p = 4$. At equilibrium, the partial pressure of O_2 is _____ atm. (Round off to the nearest integer) [4 marks]

Solution:

Concept:

- In a heterogeneous equilibrium, pure solids have unit activity and do not appear as variable factors in the equilibrium constant expression.
- The power of a gaseous species in K_p equals its stoichiometric coefficient in the balanced equation. Thus the coefficient $1/2$ on oxygen produces a square-root dependence on oxygen pressure.

Step 1: Write the equilibrium expression The printed reaction is

$$\mathrm{A\,(s)} \rightleftharpoons \mathrm{M\,(s)} + \frac{1}{2}\,\mathrm{O_2(g)}.$$

Both A and M are pure solids, so their activities are unity while both solid phases are present. Consequently the only pressure-dependent contribution comes from oxygen. Using pressures in atmospheres as in the question, the school-level expression is

$$K_p = (p_{\mathrm{O_2}})^{1/2}.$$

There is no total-pressure sum because oxygen is the only gaseous species in this reaction.

Step 2: Substitute and remove the square root Insert the given value $K_p = 4$:

$$4 = (p_{\mathrm{O_2}})^{1/2}.$$

Squaring both sides gives

$$p_{\mathrm{O_2}} = 4^2 = 16 \text{ atm}.$$

The exponent belongs to the pressure, not to the equilibrium constant before substitution. Thus taking the square root of four would reverse the required algebra. Changing amounts of pure solids does not insert their masses into this calculation.

Step 3: Check the value and the requested reporting The result is positive, as a physical partial pressure must be, and substituting it back gives $\sqrt{16} = 4$, recovering the specified equilibrium constant. The numerical value is already an integer, so rounding causes no change. A value of four atmospheres would instead give $K_p = 2$ and would not satisfy the given data.

Step 4: Conclusion The equilibrium oxygen partial pressure is $\boxed{16 \text{ atm}}$; the number to enter is $\boxed{16}$.

Quick Tip to Solve: Leave pure solids out of Kp, retain the one-half coefficient as an exponent, and square Kp to recover the oxygen pressure.

82. When 400 mL of 0.2 M H_2SO_4 solution is mixed with 600 mL of 0.1 M NaOH solution, the increase in temperature of the final solution is _____ $\times 10^{-2}$ K. (Round off to the nearest integer).

[Use : H^+ (aq) + OH^- (aq) $\rightarrow$ H_2O : $\Delta_\gamma H = -57.1$ kJ mol^{-1}]

Specific heat of H_2O = 4.18 J K^{-1} g^{-1}

density of H_2O = 1.0 g cm^{-3}

Assume no change in volume of solution on mixing. [4 marks]

Solution:

Concept:

- The heat released by neutralisation is the number of moles of water formed multiplied by $57.1\ \text{kJ mol}^{-1}$. Determine the limiting acid/base equivalents before using this value.
- The solution absorbs the released heat according to $q = mc\Delta T$, where m is the mass of the entire final solution and $c = 4.18\ \text{J g}^{-1}\ \text{K}^{-1}$.

Step 1: Find the limiting reagent The acid amount is

$$n(\text{H}_2\text{SO}_4) = 0.400 \times 0.2 = 0.080\ \text{mol}.$$

It can provide $0.160\ \text{mol}$ of acid equivalents. The base provides

$$n(\text{NaOH}) = 0.600 \times 0.1 = 0.060\ \text{mol}.$$

Each mole of hydroxide neutralises one mole of hydrogen ions and forms one mole of water. Hydroxide is limiting, so only $0.060\ \text{mol}$ of water is formed; one must not use all the acid in the heat calculation.

Step 2: Calculate heat and the final solution mass The heat transferred to the solution is

$$q = 0.060 \times 57.1 \times 10^3 = 3426\ \text{J}.$$

The total volume is $400 + 600 = 1000\ \text{mL}$. Using the given density and no volume change, its mass is

$$m = 1000 \times 1.0 = 1000\ \text{g}.$$

The negative reaction enthalpy means heat is evolved, so the solution temperature increases.

Step 3: Convert the rise to the requested coefficient The energy balance gives

$$\Delta T = \frac{3426}{1000 \times 4.18} = 0.8196\ \text{K}.$$

Write this in the printed form:

$$0.8196\ \text{K} = 81.96 \times 10^{-2}\ \text{K}.$$

Round the coefficient 81.96 to the nearest integer only at the end, giving 82. The temperature rise itself must not be rounded first to one kelvin.

Step 4: Conclusion The rise is approximately $0.820\ \text{K} = \boxed{82} \times 10^{-2}\ \text{K}$. The required entry is $\boxed{82}$.

Quick Tip to Solve: Use limiting OH equivalents for neutralisation heat, but the combined mass of both solutions for the heat capacity.

83. $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$

The above reaction is carried out in a vessel starting with partial pressure P_{SO_2} = 250 m bar, P_{O_2} = 750 m bar and P_{SO_3} = 0 bar. When the reaction is complete, the total pressure in the reaction vessel is _____ m bar. (Round off of the nearest integer). [4 marks]

Solution:

Concept:

- At fixed temperature and vessel volume, the ideal-gas relation gives $p_i \propto n_i$. Changes in partial pressures can therefore be written in the same ratios as changes in mole amounts.
- For $2\,SO_2 + O_2 \longrightarrow 2\,SO_3$, consumption of two pressure units of sulphur dioxide consumes one pressure unit of oxygen and produces two pressure units of sulphur trioxide.

Step 1: Determine which gas is limiting The initial partial pressures are 250 mbar for SO_2, 750 mbar for O_2, and zero for SO_3. Complete consumption of 250 mbar worth of SO_2 requires only

$$\frac{250}{2} = 125 \text{ mbar}$$

worth of oxygen. Since 750 mbar of oxygen is available, oxygen is in excess and sulphur dioxide is the limiting reactant. Consuming all oxygen would require more sulphur dioxide than is present.

Step 2: Calculate the final individual pressures All sulphur dioxide is consumed, so its final partial pressure is zero. The remaining oxygen pressure is

$$p_{O_2,f} = 750 - 125 = 625 \text{ mbar}.$$

Because the coefficients of sulphur dioxide and sulphur trioxide are equal, the product pressure is

$$p_{SO_3,f} = 250 \text{ mbar}.$$

This pressure bookkeeping represents mole changes in the same vessel; the product pressure is not doubled again.

Step 3: Add and check the final total Dalton's law gives

$$P_f = 0 + 625 + 250 = 875 \text{ mbar}.$$

Initially the total was 1000 mbar. Each stoichiometric reaction event converts three gas moles into two, so total pressure must decrease. The decrease of 125 mbar agrees with the oxygen-pressure extent of the reaction, independently checking the calculation. The answer already is an integer.

Step 4: Conclusion The final total pressure is $\boxed{875 \text{ mbar}}$; the required integer is $\boxed{875}$.

Quick Tip to Solve: Use partial pressures as mole units in a fixed vessel: 250 mbar of SO2 consumes 125 mbar O2 and produces 250 mbar SO3.

84. 10.0 mL of 0.05 M $KMnO_4$ solution was consumed in a titration with 10.0 mL of given oxalic acid dihydrate solution. The strength of given oxalic acid solution is _____ $\times 10^{-2}$ g/L.
(Round off to the nearest integer) [4 marks]

Solution:

Concept:

- In acidic permanganate titration, MnO_4^- gains five electrons per ion and oxalate loses two electrons per ion. Thus two permanganate ions react with five oxalate units.
- Strength in $g\,L^{-1}$ equals molarity times molar mass. Oxalic acid dihydrate is $H_2C_2O_4 \cdot 2\,H_2O$, with molar mass $126\ g\,mol^{-1}$; the water of crystallisation must be included.

Step 1: Find the amount of permanganate used The titrant volume is $10.0\ mL = 0.0100\ L$, so

$$n(KMnO_4) = 0.05 \times 0.0100 = 5.00 \times 10^{-4}\ mol.$$

The acidic redox stoichiometry is

$$2\,MnO_4^- + 5\,C_2O_4^{2-} + 16\,H^+ \longrightarrow 2\,Mn^{2+} + 10\,CO_2 + 8\,H_2O.$$

This balanced equation supplies the required 2:5 mole ratio; equal titration volumes do not imply equal molarities.

Step 2: Calculate the oxalic acid concentration The amount of oxalic acid in the 10.0 mL aliquot is

$$n = \frac{5}{2}(5.00 \times 10^{-4}) = 1.25 \times 10^{-3}\ mol.$$

Dividing by its aliquot volume gives

$$M = \frac{1.25 \times 10^{-3}}{0.0100} = 0.125\ mol\,L^{-1}.$$

Each formula unit of the dihydrate supplies one oxalic acid unit, so hydration changes its mass per mole but not this titration mole ratio.

Step 3: Convert molarity to the printed strength The molar mass is $2 + 24 + 64 + 36 = 126\ g\,mol^{-1}$. Therefore

$$S = 0.125 \times 126 = 15.75\ g\,L^{-1}.$$

Expressing this as the requested multiple gives

$$15.75 = 1575 \times 10^{-2}.$$

No further numerical rounding changes the coefficient. Using the anhydrous molar mass 90 would incorrectly omit the two waters of crystallisation.

Step 4: Conclusion The solution strength is $15.75\ g\,L^{-1} = \boxed{1575} \times 10^{-2}\ g\,L^{-1}$. The required entry is $\boxed{1575}$.

Quick Tip to Solve: Use the 2:5 permanganate-to-oxalate ratio, then multiply the oxalic acid molarity by 126 for the dihydrate strength.

85. The total number of electrons in all bonding molecular orbitals of O_2^{2-} is _____.
(Round off to the nearest integer) [4 marks]

Solution:

Concept:

- Bonding molecular orbitals have lower energy than the atomic orbitals from which they form. Their occupied electrons must be counted separately from electrons in antibonding orbitals, indicated by a star.
- Each oxygen atom supplies eight electrons. The peroxide ion, $O_2{}^{2-}$, has two additional electrons, so its total electron count is $2(8) + 2 = 18$.

Step 1: Write the occupied molecular orbitals For oxygen and its negative ions, fill the core and valence molecular orbitals in the usual oxygen ordering. The complete configuration of the peroxide ion is

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2(\pi_{2p_x})^2(\pi_{2p_y})^2(\pi_{2p_x}^*)^2(\pi_{2p_y}^*)^2.$$

The two electrons added to neutral oxygen complete the two antibonding pi orbitals. They do not add to the bonding population.

Step 2: Count only the bonding electrons The bonding orbitals are the unstarred sigma and pi orbitals. Include the core bonding orbital because the question asks for all bonding molecular orbitals, rather than only valence orbitals. Thus

$$N_b = 2 + 2 + 2 + 2 + 2 = 10.$$

As a consistency check, the antibonding orbitals contain $2 + 2 + 2 + 2 = 8$ electrons; the total is $10 + 8 = 18$. This also gives bond order $(10 - 8)/2 = 1$, as expected for peroxide.

Step 3: Conclusion The total number of electrons in all bonding molecular orbitals is $\boxed{10}$. It is a dimensionless electron count, and the nearest integer is 10.

Quick Tip to Solve: When the question says all bonding orbitals, include the core σ_{1s} pair as well as the valence bonding electrons; ignore every starred orbital.

86. 3 moles of metal complex with formula $Co(en)_2Cl_3$ gives 3 moles of silver chloride on treatment with excess of silver nitrate. The secondary valency of Co in the complex is _____.
(Round off to the nearest integer) [4 marks]

Solution:

Concept:

- In Werner theory, secondary valency is the coordination number: the number of donor atoms directly attached to the metal ion inside the coordination sphere.
- Ethylenediamine, en, is a neutral bidentate ligand with two nitrogen donor atoms. Chloride outside the coordination sphere is ionisable and precipitates as $AgCl$ with excess $AgNO_3$.

Step 1: Determine the number of ionisable chlorides Three moles of the complex produce three

moles of silver chloride. The amount of precipitate per mole of complex is therefore

$$\frac{n(\text{AgCl})}{n(\text{complex})} = \frac{3}{3} = 1.$$

Since $\text{Ag}^+ + \text{Cl}^- \longrightarrow \text{AgCl(s)}$ is a one-to-one precipitation, exactly one chloride ion per formula unit lies outside the coordination sphere. The remaining two chlorides are directly coordinated to cobalt.

Step 2: Count the donor atoms around cobalt The coordination formula is consequently $[\text{Co(en)}_2\text{Cl}_2]\text{Cl}$. Each of the two en ligands occupies two donor positions, and each of the two coordinated chloride ligands occupies one. Hence

$$\text{Coordination number} = 2 \times 2 + 2 \times 1 = 6.$$

The outer chloride balances the charge of the complex cation but does not contribute to secondary valency. Charge balance gives cobalt oxidation state $+3$; this is its primary valency here, not the secondary valency being asked for.

Step 3: Conclusion The secondary valency of Co is $\boxed{6}$, a dimensionless coordination number. The required nearest integer is 6.

Quick Tip to Solve: Use the moles of precipitated AgCl per mole of complex to locate outer-sphere chloride first; then count donor atoms, with each en contributing two.

87. In a solvent 50% of an acid HA dimerizes and the rest dissociates. The van't Hoff factor of the acid is _____ $\times 10^{-2}$.

(Round off to the nearest integer) [4 marks]

Solution:

Concept:

- The van't Hoff factor i is the number of solute particles actually present divided by the number of formula units initially dissolved. Association reduces particle number, while dissociation increases it.
- Dimerisation follows $2\,\text{HA} \rightleftharpoons (\text{HA})_2$, and dissociation follows $\text{HA} \rightleftharpoons \text{H}^+ + \text{A}^-$. Count the particles contributed by each fraction separately.

Step 1: Choose a convenient starting amount Take one mole of the acid initially. Half of it, or 0.50 mol of HA formula units, dimerises. Two formula units make one dimer, so this portion gives

$$n\big((\text{HA})_2\big) = \frac{0.50}{2} = 0.25 \text{ mol}.$$

The phrase "the rest dissociates" means the other 0.50 mol does not remain as undissociated monomer; it is the portion that forms ions.

Step 2: Add the particles from the dissociated portion Each dissociated HA unit gives one hydrogen ion and one conjugate-base ion. Therefore the remaining half mole produces 0.50 mol of H^+ and 0.50

mol of A^-, a total of 1.00 mol of ionic particles. The complete particle balance is

$$n_{\text{particles}} = 0.25 + 0.50 + 0.50 = 1.25 \text{ mol}.$$

Thus

$$i = \frac{1.25}{1.00} = 1.25 = 125 \times 10^{-2}.$$

One must not add an extra half mole of HA, since the entire initial amount has already been assigned to the two processes.

Step 3: Conclusion The van't Hoff factor is $i = 1.25$, so the number to fill in the blank is $\boxed{125}$. Both the factor and the requested coefficient are dimensionless.

Quick Tip to Solve: For simultaneous association and dissociation, count particles from each fraction: a dimerising fraction contributes half as many particles, and a fully dissociating HA fraction contributes twice as many.

88. The dihedral angle in staggered form of Newman projection of 1, 1, 1-Trichloro ethane is _____ degree. (Round off to the nearest integer) [4 marks]

Solution:

Concept:

- A Newman projection is viewed along a carbon-carbon bond. The three bonds from the front carbon are separated by $120°$, and the three bonds from the rear carbon have the same separation.
- The dihedral or torsional angle is the angle between bonds on adjacent carbons when viewed along their connecting bond. A staggered arrangement places a rear bond midway between two successive front bonds.

Step 1: Identify the groups viewed along the carbon-carbon bond The molecule 1,1,1-trichloroethane has structure CCl_3-CH_3. Looking along its carbon-carbon bond, one carbon presents three chlorine bonds and the other presents three hydrogen bonds. All three chlorine atoms on their carbon are equivalent, as are the three hydrogen atoms on the other carbon. There is therefore no special large-group anti-versus-gauche choice to make for this question.

Step 2: Use the geometry of a staggered projection Keep one front bond at angular position $0°$. The other front bonds appear at $120°$ and $240°$. In the staggered form, the rear bonds lie midway between them, at $60°$, $180°$, and $300°$. The smallest torsional separation of a front bond from an adjacent rear bond is consequently

$$\theta = \frac{120°}{2} = 60°.$$

An eclipsed conformation instead places the corresponding bonds in the same viewing direction, giving $0°$. A selected opposite pair can subtend $180°$, but the conventional staggered dihedral requested here is the minimum adjacent-bond torsion.

Step 3: Conclusion The dihedral angle in the staggered Newman projection is $\boxed{60°}$. The numerical entry, rounded to the nearest integer in degrees, is $\boxed{60}$.

Quick Tip to Solve: In a Newman projection, remember the $120°$ separation on either carbon; staggering rotates one set by half that angle, giving $60°$.

89. For the first order reaction A $\rightarrow$ 2B, 1 mole of reactant A gives 0.2 moles of B after 100 minutes. The half life of the reaction is _____ min. (Round off to the nearest integer).

[Use : ln 2 = 0.69, ln 10 = 2.3]

Properties of logarithms : ln x^y = y ln x;

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

(Round off to the nearest integer) [4 marks]

Solution:

Concept:

- For a first-order reaction, $\ln(n_0/n_t) = kt$, where n_0 and n_t are the initial and remaining reactant amounts and k is the rate constant.
- The half-life is $t_{1/2} = \ln 2/k$. Reaction stoichiometry must first convert product formed into reactant consumed; for $\mathrm{A} \longrightarrow 2\,\mathrm{B}$, one mole of A gives two moles of B.

Step 1: Find the remaining amount of reactant The product amount after 100 minutes is 0.20 mol of B. Hence the amount of A consumed is

$$n_{A,\text{consumed}} = \frac{0.20}{2} = 0.10 \text{ mol}.$$

Starting from one mole, the amount of A remaining is $1.00 - 0.10 = 0.90$ mol. The unreacted fraction is therefore 0.90, not 0.80: using the latter would ignore the coefficient 2 in the balanced reaction.

Step 2: Evaluate the rate constant and half-life Substitute the amount ratio and the elapsed time into the integrated first-order law:

$$k = \frac{1}{100}\ln\left(\frac{1}{0.9}\right) = \frac{0.1053605}{100} = 1.053605 \times 10^{-3}\ \text{min}^{-1}.$$

The logarithm can also be obtained without a table from $-\ln(1-x) = x + x^2/2 + x^3/3 + \cdots$, with $x = 0.1$. Its first five terms give 0.1053603, already sufficient for rounding. Using the question's supplied approximation $\ln 2 = 0.69$,

$$t_{1/2} = \frac{0.69}{1.053605 \times 10^{-3}} = 654.89 \text{ min}.$$

The rate constant has inverse-time units, so the calculated half-life is in minutes.

Step 3: Conclusion Using the supplied value $\ln 2 = 0.69$, the half-life rounded to the nearest integer is $\boxed{655 \text{ min}}$. This lies in the accepted 600–700 min range.

Quick Tip to Solve: Convert product moles to reactant consumption before applying the first-order equation. Here 0.2 mol of B means only 0.1 mol of A has reacted.

90. For the cell

Cu(s) | Cu^{2+} (aq) (0.1 M) || Ag^{+}(aq) (0.01 M) | Ag(s)

the cell potential E_1 = 0.3095 V

For the cell

Cu(s) | Cu^{2+} (aq) (0.01 M) || Ag^{+}(aq) (0.001 M) | Ag(s)

the cell potential = _____ $\times 10^{-2}$ V. (Round off the nearest integer).

[Use : $\frac{2.303RT}{F}$ = 0.059] [4 marks]

Solution:

Concept:

- The cell reaction is $\mathrm{Cu(s) + 2\,Ag^{+}(aq) \longrightarrow Cu^{2+}(aq) + 2\,Ag(s)}$. It transfers two electrons, so the Nernst equation contains $n = 2$.
- For this reaction, $Q = [\mathrm{Cu^{2+}}]/[\mathrm{Ag^{+}}]^2$. The solid metals have unit activity. With the supplied constant, $E = E^\circ - (0.059/2)\log_{10} Q$, using the usual concentration approximation.

Step 1: Compute the two reaction quotients For the first cell, the copper-ion concentration is 0.1 M and the silver-ion concentration is 0.01 M. Thus

$$Q_1 = \frac{0.1}{(0.01)^2} = 10^3.$$

For the second cell, the respective concentrations are 0.01 M and 0.001 M, giving

$$Q_2 = \frac{0.01}{(0.001)^2} = 10^4.$$

Although both concentrations fall tenfold, the square of the silver-ion concentration falls a hundredfold. Consequently the quotient increases by a factor of ten.

Step 2: Subtract the Nernst equations The electrode materials and temperature are unchanged, so E° is identical and cancels on subtraction:

$$E_2 - E_1 = -\frac{0.059}{2}\log_{10}\left(\frac{Q_2}{Q_1}\right) = -0.0295\log_{10}(10) = -0.0295\ \mathrm{V}.$$

Hence

$$E_2 = 0.3095 - 0.0295 = 0.2800\ \mathrm{V} = 28.00 \times 10^{-2}\ \mathrm{V}.$$

The sign is consistent: increasing the reaction quotient lowers the cell potential. Using a one-electron factor would double the potential change incorrectly.

Step 3: Conclusion The second cell potential is $\boxed{0.2800\ \mathrm{V}}$, or 28×10^{-2} V. The nearest-integer number to enter is $\boxed{28}$.

Quick Tip to Solve: For two cells with the same electrodes, subtract the Nernst equations so that the unknown standard potential cancels. Keep the squared silver-ion concentration in the reaction quotient.