JEE Main 2021 Paper 1 (B.E./B.Tech) Mar 16 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 2 (March), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 16 March 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), (0, β) and (0, 0) is equal to : [4 marks]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) 1

(D) $2\sqrt{2}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept:

- A counterclockwise rotation through θ sends (x, y) to $(x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$. The minus sign in the first coordinate fixes the direction of rotation.
- A right triangle with perpendicular side lengths b and h has area $\frac{1}{2}bh$. Here the vertical side has length β and the horizontal side has length α, since the rotated vector lies in the first quadrant.

Step 1: Rotate the given vector Put $x = \sqrt{3}$, $y = 1$ and $\theta = 45°$. Both sine and cosine equal $1/\sqrt{2}$, so the new coordinates are

$$\alpha = \frac{\sqrt{3}-1}{\sqrt{2}}, \qquad \beta = \frac{\sqrt{3}+1}{\sqrt{2}}.$$

Both are positive, as the question requires. As a check, the initial vector makes an angle of $30°$ with the positive horizontal axis; the new angle is $75°$, still in the first quadrant.

Step 2: Use the perpendicular sides of the triangle The vertices $(0, 0)$ and $(0, \beta)$ lie on the vertical axis, while (α, β) is horizontally displaced from the latter. Thus

$$\text{Area} = \frac{1}{2}\alpha\beta = \frac{1}{2}\left(\frac{(\sqrt{3}-1)(\sqrt{3}+1)}{2}\right) = \frac{1}{2}\left(\frac{3-1}{2}\right) = \frac{1}{2}.$$

The product $\alpha\beta = 1$ is twice the area, so option (C) misses the triangle factor. Neither $1/\sqrt{2}$ nor $2\sqrt{2}$ equals the computed area; those distractors do not use both perpendicular side lengths correctly.

Step 3: Conclusion The area of the triangle is $\boxed{\frac{1}{2}}$ square unit. Hence, the correct option is (B).

Quick Tip to Solve: Rotate the coordinates first, then multiply the new components. In this question the conjugate factors $(\sqrt{3}-1)(\sqrt{3}+1)$ simplify immediately.

2. Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to : [4 marks]

(A) $\cot^{-1}\left(\frac{3}{2}\right)$

(B) $\frac{\pi}{2}$

(C) $\tan^{-1}(3)$

(D) $\tan^{-1}\left(\frac{3}{2}\right)$

Correct Answer: (A) $\cot^{-1}\left(\frac{3}{2}\right)$

Solution:

Concept:

- For positive $u > v$, $\tan^{-1} u - \tan^{-1} v = \tan^{-1}\left(\frac{u-v}{1+uv}\right)$. Both sides are in $(0, \pi/2)$, so there is no additional multiple of π.
- A telescoping sum cancels adjacent terms. Since $3/2 > 1$, its positive powers tend to infinity, and the principal inverse tangent of these powers tends to $\pi/2$.

Step 1: Express one summand as a difference Choose $u = (3/2)^{r+1}$ and $v = (3/2)^r$. Their difference is $\frac{1}{2}(3/2)^r$, and their product is $(3/2)^{2r+1}$. Multiplying numerator and denominator by 2^{2r+1} gives

$$\frac{u-v}{1+uv} = \frac{6^r}{2^{2r+1}+3^{2r+1}}.$$

Consequently each summand equals $\tan^{-1}(3/2)^{r+1} - \tan^{-1}(3/2)^r$. Here the powers are the arguments of the inverse tangents, not powers of the angles.

Step 2: Cancel the intermediate terms and take the limit Writing out the differences from $r = 1$ through $r = k$ leaves only the last positive term and the first negative term:

$$S_k = \tan^{-1}\left((3/2)^{k+1}\right) - \tan^{-1}(3/2).$$

Therefore

$$\lim_{k\to\infty} S_k = \frac{\pi}{2} - \tan^{-1}(3/2) = \cot^{-1}(3/2).$$

The limit is strictly between zero and $\pi/2$, ruling out (B). It equals $\tan^{-1}(2/3)$, which differs from both $\tan^{-1} 3$ and $\tan^{-1}(3/2)$; thus (C) and (D) also fail.

Step 3: Conclusion The required limit is $\boxed{\cot^{-1}(3/2)}$. Hence, the correct option is (A).

Quick Tip to Solve: When the denominator contains consecutive odd powers of two bases and the numerator their product, test an inverse-tangent difference with consecutive powers of their ratio.

3. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, −1, 2) and (−2, 1, −2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is : [4 marks]

(A) $\sqrt{171}$

(B) $\sqrt{227}$

(C) $\sqrt{482}$

(D) $\sqrt{5}$

Correct Answer: (A) $\sqrt{171}$

Solution:

Concept:

- A line through a point P with direction vector $\mathbf{d}$ has position vector $\mathbf{r} = \mathbf{p} + t\mathbf{d}$. At an intersection, the three corresponding coordinates of the two lines agree.
- A vector perpendicular to two nonparallel directions is parallel to their cross product. Its prescribed length fixes its magnitude but allows two opposite directions; both must be checked when finding the distance of the final point from the origin.

Step 1: Find the intersection point The two lines are

$$(x, y, z) = (3, -1, 2) + t(4, -1, 2),$$
$$(x, y, z) = (1, 2, -4) + s(-2, 1, -2).$$

The second coordinates give $s = -3 - t$. Substituting in the first-coordinate equation gives $3 + 4t = 7 + 2t$, hence $t = 2$ and $s = -5$. Both third coordinates then equal 6. Thus the common point is $T = (11, -3, 6)$.

Step 2: Determine both possible perpendicular displacements The cross product of the two direction vectors is

$$(4, -1, 2) \times (-2, 1, -2) = (0, 4, 2) = 2(0, 2, 1).$$

Since $|(0, 2, 1)| = \sqrt{5}$, the required displacement is $\overrightarrow{TA} = \pm(0, 2, 1)$. The possible points are therefore $A = (11, -1, 7)$ and $A = (11, -5, 5)$. Their squared distances from the origin are respectively

$$11^2 + (-1)^2 + 7^2 = 171, \qquad 11^2 + (-5)^2 + 5^2 = 171.$$

Thus the directional choice does not change the requested modulus. The alternatives $\sqrt{227}$, $\sqrt{482}$ and $\sqrt{5}$ have the wrong squared distances; $\sqrt{5}$ is only the displacement length.

Step 3: Conclusion The modulus of the position vector of A is $\boxed{\sqrt{171}}$ units for either permitted displacement. Hence, the correct option is (A).

Quick Tip to Solve: Keep both signs of a prescribed perpendicular displacement. Here $(11, -3, 6) \cdot (0, 2, 1) = 0$, so either sign gives the same distance from the origin.

4. The number of elements in the set $\{x \in R : (|x| - 3)\,|x + 4| = 6\}$ is equal to : [4 marks]

(A) 4

(B) 2

(C) 3

(D) 1

Correct Answer: (B) 2

Solution:

Concept:

- An absolute value is replaced by its signed expression on intervals separated by its zeros. Here the breakpoints are -4 and 0, from $|x+4|$ and $|x|$.
- The right side is positive, while $|x+4| \geq 0$. Therefore a solution must satisfy $|x| - 3 > 0$, so only $x > 3$ or $x < -3$ needs examination; also $x = -4$ cannot work.

Step 1: Solve on the positive interval For $x > 3$, both absolute values have positive arguments, giving

$$(x-3)(x+4) = 6 \quad \Longrightarrow \quad x^2 + x - 18 = 0.$$

The roots are $(-1 \pm \sqrt{73})/2$. Only $(-1+\sqrt{73})/2 > 3$ lies in this interval. The negative root was obtained using the wrong sign for $|x|$, so it cannot be retained from this case.

Step 2: Examine the two negative intervals For $x < -4$, the equation becomes

$$(-x-3)(-x-4) = 6 \quad \Longrightarrow \quad (x+1)(x+6) = 0.$$

Only $x = -6$ belongs to $x < -4$; -1 does not. For $-4 < x < -3$, it becomes

$$(-x-3)(x+4) = 6 \quad \Longrightarrow \quad x^2 + 7x + 18 = 0.$$

Its discriminant is $49 - 72 = -23$, so this interval contributes no real root. At $x = -4$ the product is zero, and all points with $|x| \leq 3$ give a nonpositive product. These checks exhaust the real line. Counting the two accepted roots rules out 4, 3 and 1 in the other options.

Step 3: Conclusion The set is $\left\{-6, \frac{-1+\sqrt{73}}{2}\right\}$, so it contains $\boxed{2}$ elements. Hence, the correct option is (B).

Quick Tip to Solve: Use positivity before splitting absolute values: here it removes the entire interval $[-3, 3]$. Accept a quadratic root only if it lies in the interval used to derive it.

5. Let a complex number z, |z| $\neq$ 1, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of |z| is equal to _____. [4 marks]

(A) 5

(B) 8

(C) 6

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- The modulus $r = |z|$ is a nonnegative real number. The stated exclusion $r \neq 1$ keeps the denominator nonzero, and $(r+11)/(r-1)^2$ is then strictly positive.
- For a base a in $(0,1)$, the logarithm is decreasing: $\log_a u \leq c$ means $u \geq a^c$. Reversing this inequality is essential before clearing the positive squared denominator.

Step 1: Remove the decreasing logarithm Let $r = |z| \geq 0$ with $r \neq 1$. The base is $a = 1/\sqrt{2}$, so $a^2 = 1/2$. The given inequality is equivalent to

$$\frac{r+11}{(r-1)^2} \geq \frac{1}{2}.$$

Because $(r-1)^2 > 0$, multiplying does not reverse the inequality again. We obtain

$$2r + 22 \geq r^2 - 2r + 1 \quad \Longleftrightarrow \quad r^2 - 4r - 21 \leq 0.$$

Step 2: Find the admissible interval and its largest value Factor the quadratic as $(r-7)(r+3)$. Since it opens upwards, the inequality holds for $-3 \leq r \leq 7$. Intersecting this interval with the modulus domain and the exclusion gives

$$r \in [0,1) \cup (1,7].$$

In particular, $r = 7$ is allowed: the logarithm argument is $18/36 = 1/2$, so the original logarithm equals 2 exactly. A complex number such as $z = 7$ attains this modulus. Values 5 and 6 are admissible but are not the largest. Value 8 violates the derived inequality, so it cannot be the answer.

Step 3: Conclusion The largest possible modulus is $\boxed{7}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the modulus as a nonnegative real variable, and mark a logarithm with base below one as decreasing before transforming its inequality.

6. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$**. Then, the system of linear equations** $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ **has :** [4 marks]

(A) Exactly two solutions

(B) Infinitely many solutions

(C) A unique solution

(D) No solution

Correct Answer: (D) No solution

Solution:

Concept:

- If $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, direct multiplication gives $B^2 = 2B$. Induction then gives $B^n = 2^{n-1}B$ for every positive integer n.
- A matrix equation is consistent only if its right-hand side lies in the column space of the coefficient matrix. The two rows of B are negatives of one another, so every vector in its image has components whose sum is zero.

Step 1: Compute the eighth power efficiently The matrix in the question is $A = iB$. Scalars commute with matrices, and $i^8 = 1$, hence

$$A^8 = i^8 B^8 = 2^7 B = 128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

There is no need to multiply eight matrices individually. The dependence of the rows is preserved by this power, so one should check consistency rather than infer a unique solution from the number of equations.

Step 2: Compare the two scalar equations The required system reads

$$128(x - y) = 8, \qquad 128(-x + y) = 64.$$

Adding the equations would give $0 = 72$, a contradiction. Equivalently the first equation requires $x - y = 1/16$, whereas the second requires $x - y = -1/2$. No pair of values, real or complex, can satisfy both. The singular coefficient matrix rules out a unique solution; consistency would have produced infinitely many solutions, but the incompatible right-hand side rules that out too. Exactly two solutions is also impossible here, since a linear system cannot have precisely two distinct solutions without their entire affine line.

Step 3: Conclusion The given linear system is inconsistent and has $\boxed{\text{no solution}}$. Hence, the correct option is (D).

Quick Tip to Solve: Look for a fixed row relation before evaluating a high matrix power. Opposite rows require opposite entries on the right-hand side, which 8 and 64 are not.

7. If n is the number of irrational terms in the expansion of $\left(3^{1/4}+5^{1/8}\right)^{60}$, then (n − 1) is divisible by : [4 marks]

(A) 30

(B) 8

(C) 7

(D) 26

Correct Answer: (D) 26

Solution:

Concept:

- The binomial expansion of $(a+b)^{60}$ has 61 terms indexed by $r=0,1,\ldots,60$, with general term $T_{r+1}=\binom{60}{r}a^{60-r}b^{r}$.
- A product of rational powers of distinct primes is rational only when each prime exponent is an integer. The nonzero integer binomial coefficient cannot remove a fractional exponent of 3 or 5.

Step 1: Identify all the rational terms With $a=3^{1/4}$ and $b=5^{1/8}$, the general term is

$$T_{r+1}=\binom{60}{r}3^{(60-r)/4}5^{r/8}.$$

It is rational precisely when $(60-r)/4$ and $r/8$ are both integers. To justify necessity, raising a proposed rational term to the eighth power makes the exponents of the primes 3 and 5 integral; the valuation of each prime in an eighth power of a rational must be divisible by eight, forcing those original exponents to be integers. Thus no cancellation between different primes can create extra rational terms.

Step 2: Count the remaining terms and check divisibility The condition on 5 requires r to be divisible by 8. Every such r also makes $60-r$ divisible by 4. In the allowed range the indices are

$$r=0,8,16,24,32,40,48,56,$$

which gives eight rational terms. Therefore the number of irrational terms is $n=61-8=53$, so

$$n-1=52=2\times 26.$$

It is not divisible by 30, by 8, or by 7: the respective remainders are 22, 4 and 3. The endpoint $r=60$ is not rational because its exponent of 5 is $60/8$, not an integer.

Step 3: Conclusion Here $n=53$, and $n-1=52$ is divisible by $\boxed{26}$. Hence, the correct option is (D).

Quick Tip to Solve: Count rational terms first using divisibility of both prime exponents, then subtract from 61. Remember that the binomial index starts at zero, so both endpoints must be considered.

8. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is : [4 marks]

(A) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

(B) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(C) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

(D) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

Correct Answer: (A) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Solution:

Concept:

- The chord of a circle with centre at the origin is perpendicular to the radius through its midpoint. If its midpoint is (h, k), its equation is $hx + ky = h^2 + k^2$.
- A line $lx + my = n$ is tangent to $x^2/a^2 - y^2/b^2 = 1$ when $n^2 = a^2l^2 - b^2m^2$. This condition also includes vertical tangents.

Step 1: Express the chord using its midpoint Let $P(h, k)$ be the midpoint. The radius OP is a normal to the chord, so the normal vector is (h, k). Because the chord passes through P, its equation is

$$hx + ky = h^2 + k^2.$$

This follows equally from the circle chord formula $T = S_1$: $hx + ky - 25 = h^2 + k^2 - 25$. The circle radius cancels from the equation, but determines whether the line actually cuts the circle.

Step 2: Apply the hyperbola tangency condition Here $a^2 = 9$, $b^2 = 16$, and the line coefficients are $l = h$, $m = k$, $n = h^2 + k^2$. Hence

$$(h^2 + k^2)^2 = 9h^2 - 16k^2.$$

Replacing the moving coordinates h, k by x, y gives the required algebraic locus. The positive sign of the $16y^2$ term after rearrangement comes from the negative square in the hyperbola, not from the circle.

Step 3: Compare the printed equations The equation is $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$. Option (B) has an incorrect coefficient 144; (C) interchanges the hyperbola parameters; (D) has the wrong sign. Strictly the origin is omitted, since it would give no chord line; all nonzero points of this curve satisfy $x^2 + y^2 \leq 9 < 25$, so the corresponding tangent cuts the circle in a genuine chord.

Step 4: Conclusion The algebraic locus is $\boxed{(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0}$, with the origin excluded from the geometric locus. Hence, the correct option is (A).

Quick Tip to Solve: Write the chord in midpoint form first, then substitute its three line coefficients directly into the conic tangency condition.

9. Let the functions f : R → R and g : R → R be defined as :

$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ **and**

$$g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$$

Then, the number of points in R where (fog) (x) is NOT differentiable is equal to : [4 marks]

(A) 0

(B) 3

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Concept:

- A piecewise smooth function can fail to be differentiable only at its joining points; differentiability first requires continuity.
- In $f(g(x))$, the outer branch changes where $g(x) = 0$, and the inner formula changes at $x = 1$. Both locations must be checked.

Step 1: Find the branches of the composite For $x < 1$, $g(x) = x^3$, whose sign changes at zero. For $x \geq 1$, $g(x) = 3x - 2 \geq 1$, so it is always nonnegative. Consequently, writing $H = f \circ g$,

$$H(x) = \begin{cases} x^3 + 2, & x < 0, \\ x^6, & 0 \leq x < 1, \\ (3x-2)^2, & x \geq 1. \end{cases}$$

Every expression is polynomial on the interior of its interval. Thus there are no possible difficulties away from 0 and 1. In particular, solving $3x - 2 = 0$ gives $2/3$, but that formula for g is not valid there and introduces no additional joining point.

Step 2: Check continuity at zero The left-hand limit of H at zero is 2, whereas $H(0) = 0$, and the right-hand limit is zero. Therefore H is discontinuous at zero and hence cannot be differentiable there. Checking only the limiting polynomial derivatives would miss this jump.

Step 3: Check the derivatives at one At $x = 1$, both branches take value 1, so the composite is continuous. The left derivative is $6x^5$ evaluated at one, namely 6. The right derivative is $6(3x - 2)$ evaluated at one, also 6. Thus it is differentiable at one. The only failure is at zero: zero failures, two failures and three failures are ruled out by these explicit checks.

Step 4: Conclusion There is exactly $\boxed{1}$ point of non-differentiability, namely $x = 0$. Hence, the correct option is (C).

Quick Tip to Solve: For a composite piecewise function, solve the outer switching equation within each valid inner branch before testing continuity and derivatives.

10. Consider three observations a, b, and c such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d, then which of the following is true? [4 marks]

(A) $b^2 = 3(a^2 + c^2) + 9d^2$

(B) $b^2 = 3(a^2 + c^2) - 9d^2$

(C) $b^2 = 3(a^2 + c^2 + d^2)$

(D) $b^2 = a^2 + c^2 + 3d^2$

Correct Answer: (B) $b^2 = 3(a^2 + c^2) - 9d^2$

Solution:

Concept:

- Adding the same constant to every observation shifts the mean by that constant and leaves all deviations from the mean unchanged. Thus the standard deviation remains unchanged.
- For three observations with mean $\bar{x}$, the population variance is $\sigma^2 = (a^2 + b^2 + c^2)/3 - \bar{x}^2$. Here the prescribed standard deviation is d.

Step 1: Remove the common translation The observations are $a + 2, b + 2, c + 2$. Their mean is $(a + b + c)/3 + 2$. Subtracting this mean from any observation cancels the added 2. Therefore a, b, c have exactly the same standard deviation d. We must use the variance of three observations with divisor 3, as in the CBSE definition, rather than the sample-variance divisor 2.

Step 2: Use the given relation in the mean Since $b = a + c$, the total is $a + b + c = 2b$, so $\bar{x} = 2b/3$. The variance formula gives

$$d^2 = \frac{a^2 + b^2 + c^2}{3} - \frac{4b^2}{9}$$
$$= \frac{3a^2 - b^2 + 3c^2}{9}.$$

Multiplying by nine and rearranging,

$$b^2 = 3(a^2 + c^2) - 9d^2.$$

The negative sign before the variance term results from moving it to the other side of the equality.

Step 3: Check the alternatives Option (B) matches both the coefficient and the sign. Option (A) reverses the sign, while (C) also puts the variance on the wrong side and with the wrong coefficient. Option (D) has an incorrect coefficient for the sum of squares. For example, $a = c = 1, b = 2$ gives $d^2 = 2/9$ and verifies the derived equality $4 = 6 - 2$.

Step 4: Conclusion $\boxed{b^2 = 3(a^2 + c^2) - 9d^2}$. Hence, the correct option is (B).

Quick Tip to Solve: Ignore a common additive shift when computing standard deviation; use the given relation to simplify the mean before expanding the variance.

11. The range of a∈R for which the function f(x) = (4a − 3)(x + $\log_e$ 5) + 2(a − 7) $\cot\left(\frac{x}{2}\right)$ $\sin^2\left(\frac{x}{2}\right)$, x ≠ 2nπ, n∈N has critical points, is : [4 marks]

(A) $[1, \infty)$

(B) $(-3, 1)$

(C) $\left[-\frac{4}{3}, 2\right]$

(D) $(-\infty, -1]$

Correct Answer: No option is exact: the printed cotangent excludes every stationary point at a=2; the correct range is [-4/3,2), whereas keyed option (C) includes 2.

Solution:

Concept:

- At a point belonging to the domain where a function is smooth, a critical point requires $f\prime(x) = 0$. A point excluded from the domain is not a critical point.
- The identity $2\cot(x/2)\sin^2(x/2) = \sin x$ is valid wherever the original cotangent is defined. It does not restore the excluded values $x = 2k\pi$.

Step 1: Simplify and differentiate on the original domain For each point where the printed expression is defined,

$$f(x) = (4a - 3)(x + \ln 5) + (a - 7)\sin x.$$

Consequently,

$$f\prime(x) = 4a - 3 + (a - 7)\cos x.$$

The constant $\ln 5$ disappears on differentiation. There are no other interior points where the derivative fails to exist. If $a = 7$, the derivative is 25, so this parameter produces no critical point.

Step 2: Solve the cosine feasibility inequality For $a \neq 7$, stationarity requires

$$\cos x = \frac{3 - 4a}{a - 7}.$$

The preliminary condition $|\cos x| \leq 1$ yields

$$(4a - 3)^2 \leq (a - 7)^2 \quad \Longleftrightarrow \quad (3a + 4)(5a - 10) \leq 0,$$

or $-4/3 \leq a \leq 2$. For interior parameters the cosine lies strictly between -1 and 1, so admissible solutions exist. At $a = -4/3$, it equals -1; odd multiples of π are allowed and give critical points.

Step 3: Check the excluded endpoint At $a = 2$, stationarity requires $\cos x = 1$, whose only real solutions are $x = 2k\pi$. The cotangent in the original function is undefined at every such point. Thus none is a critical point of the printed function. The exact range is $[-4/3, 2)$. Option (C) gives the preliminary closed interval, but incorrectly includes 2; the other options also fail. A smooth extension across the missing points would answer a different domain question.

Step 4: Conclusion For the function on its printed domain, the exact range is $\boxed{[-4/3, 2)}$, so no printed option is fully correct. The keyed choice (C), $[-4/3, 2]$, includes the inadmissible endpoint $a = 2$.

Quick Tip to Solve: After solving a trigonometric range inequality, substitute both endpoint parameters into the derivative and check the original function domain.

12. If for x $\in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$, then the value of n is equal to : [4 marks]

(A) 16

(B) 9

(C) 12

(D) 20

Correct Answer: (C) 12

Solution:

Concept:

- For positive quantities u, v, $\log_{10} u + \log_{10} v = \log_{10}(uv)$. The given first-quadrant interval makes both sine and cosine positive.
- The identity $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$ connects the product determined by the first equation to the sum in the second equation.

Step 1: Find the sine-cosine product Combining the first pair of logarithms gives

$$\log_{10}(\sin x \cos x) = -1.$$

Exponentiating with base ten,

$$\sin x \cos x = 10^{-1} = \frac{1}{10}.$$

This product is positive, as required by $0 < x < \pi/2$. There is no need to determine the two possible first-quadrant angles individually, since the next relation depends only on their sine-cosine sum.

Step 2: Calculate the square of the sum Use the Pythagorean identity to obtain

$$(\sin x + \cos x)^2 = 1 + 2\left(\frac{1}{10}\right) = \frac{6}{5}.$$

The sum is positive, so logarithms and squaring preserve the required relation without any sign ambiguity. Multiplying the second given equation by two gives

$$\log_{10}\left((\sin x + \cos x)^2\right) = \log_{10} n - 1 = \log_{10}(n/10).$$

Equality of logarithms of positive numbers implies $6/5 = n/10$.

Step 3: Determine the parameter and compare Thus $n = 10(6/5) = 12$. This satisfies $n > 0$. Each other candidate produces a different squared sum: $16/10$, $9/10$, or $20/10$, rather than $6/5$. In particular, the value 9 would give a squared sum below one, impossible for two positive sine and cosine values.

Step 4: Conclusion The required value is $\boxed{n = 12}$. Hence, the correct option is (C).

Quick Tip to Solve: When one logarithmic equation fixes a sine-cosine product and another contains their sum, square the sum using the Pythagorean identity.

13. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is : [4 marks]

(A) $\frac{39}{50}$

(B) $\frac{3}{4}$

(C) $\frac{22}{425}$

(D) $\frac{52}{867}$

Correct Answer: (A) $\frac{39}{50}$

Solution:

Concept:

- Bayes theorem updates a prior probability after an observed event: $P(N \mid E) = P(N)P(E \mid N)/P(E)$.
- A standard pack has 13 spades and 39 non-spades. The missing card is taken to be uniformly random, and two cards are sampled without replacement from the remaining 51.

Step 1: Define the alternatives and their priors Let N mean that the missing card is not a spade and S mean that it is a spade. Their probabilities are

$$P(N) = \frac{39}{52} = \frac{3}{4}, \qquad P(S) = \frac{13}{52} = \frac{1}{4}.$$

Let E denote observing two spades. If N occurs there are 13 spades left; if S occurs there are only 12. Therefore the observation is somewhat more likely when the missing card is a non-spade.

Step 2: Compute the two likelihoods As the draw is without replacement,

$$P(E \mid N) = \frac{\binom{13}{2}}{\binom{51}{2}}, \qquad P(E \mid S) = \frac{\binom{12}{2}}{\binom{51}{2}}.$$

The common denominator cancels when Bayes theorem is applied. Since $\binom{13}{2} = 78$ and $\binom{12}{2} = 66$,

$$P(N \mid E) = \frac{3(78)}{3(78) + 66} = \frac{234}{300} = \frac{39}{50}.$$

This is greater than the original probability $3/4$, in agreement with the likelihood comparison.

Step 3: Check the interpretation Once two spades are observed, the missing card is equally likely to be any of the other 50 cards. Of those, 39 are non-spades, giving the same $39/50$. Option (B) is merely the prior before observing the draw. The small probabilities in (C) and (D) cannot represent this posterior and disagree with the explicit conditional calculation.

Step 4: Conclusion The probability that the missing card is not a spade is $\boxed{39/50}$. Hence, the correct option is (A).

Quick Tip to Solve: After identifying drawn cards, exchangeability makes the missing card uniform among the unobserved cards; use Bayes theorem to confirm this shortcut.

14. If y = y(x) is the solution of the differential equation,

$\frac{dy}{dx} + 2y\tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$**, then the maximum value of the function y(x) over R is equal to:**

[4 marks]

(A) $\frac{1}{8}$

(B) 8

(C) $-\frac{15}{4}$

(D) $\frac{1}{2}$

Correct Answer: (A) $\frac{1}{8}$

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, an integrating factor is $\exp(\int P(x)\,dx)$. Multiplication turns the left side into the derivative of a product.
- Completing the square gives a global bound for a quadratic in $\cos x$, provided the value attaining the bound is in the allowable cosine range.

Step 1: Find and apply the integrating factor On an interval where $\cos x \neq 0$,

$$\text{IF} = \exp\left(\int 2\tan x\,dx\right) = \sec^2 x.$$

Thus

$$\frac{d}{dx}(y\sec^2 x) = \sin x \sec^2 x.$$

Since the derivative of $\sec x$ is $\sin x/\cos^2 x$, integration gives $y\sec^2 x = \sec x + C$, or

$$y = \cos x + C\cos^2 x.$$

The interval containing the initial point $\pi/3$ is $(-\pi/2, \pi/2)$, on which all these operations are valid.

Step 2: Use the initial value At $x = \pi/3$, the cosine is $1/2$, and $y = 0$. Consequently,

$$0 = \frac{1}{2} + \frac{C}{4} \quad \Longrightarrow \quad C = -2.$$

The solution expression is therefore $y(x) = \cos x - 2\cos^2 x$.

Step 3: Complete the square and verify attainment Put $t = \cos x$. Then

$$y = t - 2t^2 = \frac{1}{8} - 2\left(t - \frac{1}{4}\right)^2 \leq \frac{1}{8}.$$

Equality holds when $\cos x = 1/4$, which is achievable even in the initial-value interval. Thus the maximum is attained. The same expression extended over all real x has this identical maximum; the differential equation itself excludes zeros of $\cos x$. Values 8 and $1/2$ exceed the upper bound, while $-15/4$ is not the maximum.

Step 4: Conclusion The maximum value is $\boxed{1/8}$, attained when $\cos x = 1/4$. Hence, the correct option is (A).

Quick Tip to Solve: After solving a trigonometric linear differential equation, optimize in t=cos x and verify that the vertex lies in the permitted range.

15. Which of the following Boolean expression is a tautology? [4 marks]

(A) $(p \wedge q) \vee (p \rightarrow q)$

(B) $(p \wedge q) \vee (p \vee q)$

(C) $(p \wedge q) \rightarrow (p \rightarrow q)$

(D) $(p \wedge q) \wedge (p \rightarrow q)$

Correct Answer: (C) $(p \wedge q) \rightarrow (p \rightarrow q)$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its propositions. An implication $u \rightarrow v$ is false only when u is true and v is false.
- Use $p \rightarrow q \equiv \neg p \vee q$ to simplify an implication. Alternatively, test all four ordered pairs (p, q), since there are only two propositions.

Step 1: Simplify the implication in option (C) The antecedent is $p \wedge q$, so both p and q must be true whenever it holds. In that case the consequent $p \rightarrow q$ is also true. If the antecedent is false, the implication is true by definition. Algebraically,

$$\begin{aligned}(p \wedge q) \rightarrow (p \rightarrow q) &\equiv \neg(p \wedge q) \vee (\neg p \vee q) \\ &\equiv \neg p \vee \neg q \vee \neg p \vee q \\ &\equiv \neg p \vee (\neg q \vee q) \\ &\equiv \mathrm{T}.\end{aligned}$$

Step 2: Eliminate the other three expressions For option (A), choose $p = \mathrm{T}$, $q = \mathrm{F}$. Then both $p \wedge q$ and $p \rightarrow q$ are false, so their disjunction is false. For option (B), take both propositions false; its two disjuncts are false. For option (D), the same choice makes $p \wedge q$ false, so the conjunction is false regardless of its other factor. Thus each rejected option has a definite counterexample, whereas option (C) remains true in every case.

Step 3: Conclusion Only $(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology. Hence, the correct option is (C).

Quick Tip to Solve: For an implication, look first for a true antecedent and false consequent; if that combination is impossible, the expression is a tautology.

16. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than : [4 marks]

(A) $\frac{1}{2}$

(B) 1

(C) -1

(D) $-\frac{1}{2}$

Correct Answer: (B) 1

Solution:

Concept:

- For $y^2 = 4cx$, the point with parameter t is $(ct^2, 2ct)$. Its normal is $y = -tx + 2ct + ct^3$. Here $4c = 2$, so $c = 1/2$.
- Three distinct real normals through a point correspond to three distinct real values of the normal parameter. A repeated root describes a coincident normal and must not be counted twice.

Step 1: Insert the point into the normal equation Substitute $(x, y) = (a, 0)$ and $c = 1/2$ in the normal equation:

$$0 = -at + t + \frac{t^3}{2}.$$

Multiplication by 2 and factorisation give

$$t\left(t^2 + 2 - 2a\right) = 0.$$

Thus one normal always has $t = 0$, namely the axis $y = 0$. The remaining possible normal parameters satisfy $t^2 = 2(a - 1)$. Keeping the parabola parameter distinct from the coordinate a avoids confusing the focal length with the given point.

Step 2: Require three distinct real parameters The other two parameters are $t = \pm\sqrt{2(a-1)}$. They are real and different from zero precisely when $a > 1$. At $a = 1$, all three algebraic roots collapse to zero, so there is just one distinct normal. If $a < 1$, the other two roots are nonreal. For $a > 1$, the two nonzero parameters have opposite signs and give two different sloping normals in addition to the axis. The weaker lower bounds $1/2, -1, -1/2$ do not guarantee the existence of three normals; for example $a = 3/4$ satisfies all those bounds but gives no additional real parameter.

Step 3: Conclusion The required threshold is $\boxed{a > 1}$. Hence, the correct option is (B).

Quick Tip to Solve: For a point on the axis of a parabola, factor the normal cubic by its zero root and require the remaining quadratic to have two nonzero real roots.

17. Let P be a plane lx + my + nz = 0 containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(−3, −6, 1) and B(2, 4, −3) in ratio k : 1 then the value of k is equal to : [4 marks]

(A) 2

(B) 3

(C) 1.5

(D) 4

Correct Answer: (A) 2

Solution:

Concept:

- A plane through the origin has equation $lx + my + nz = 0$. To contain a line, its normal must be perpendicular to the line direction, and a point of the line must satisfy the plane equation.
- If X divides AB internally in the ratio $AX : XB = k : 1$, then $X = (A + kB)/(k + 1)$. A linear plane expression can be applied directly to this formula.

Step 1: Determine the plane containing the given line Write the common ratio as t. Then $x = 1 - t$, $y = -4 + 2t$, $z = -2 + 3t$. Thus a point on the line is $(1, -4, -2)$ and its direction is $(-1, 2, 3)$. The two containment conditions are

$$l - 4m - 2n = 0, \qquad -l + 2m + 3n = 0.$$

Adding gives $-2m + n = 0$, so $n = 2m$, and then $l = 8m$. Taking $m = 1$ gives the nonzero normal $(8, 1, 2)$, and the plane is $8x + y + 2z = 0$. Substituting the parametric line verifies that both the constant and the coefficient of t vanish.

Step 2: Use the internal division ratio Let $F(x, y, z) = 8x + y + 2z$. At the endpoints,

$$F(A) = 8(-3) - 6 + 2 = -28, \qquad F(B) = 16 + 4 - 6 = 14.$$

The opposite signs show that the plane meets the segment internally. Since $F(X) = 0$, linearity of the section formula gives

$$\frac{-28 + 14k}{k + 1} = 0 \quad \Longrightarrow \quad k = 2.$$

As a direct check, the corresponding point is $X = (A + 2B)/3 = (1/3, 2/3, -5/3)$, which satisfies the plane. The alternatives 3, 1.5 and 4 make $-28 + 14k$ nonzero and therefore cannot give the intersection.

Step 3: Conclusion The plane divides the segment in the ratio $2 : 1$, so $\boxed{k = 2}$. Hence, the correct option is (A).

Quick Tip to Solve: Once the plane is known, evaluate its linear expression at the endpoints; the internal ratio equals the ratio of the opposite absolute values.

18. If for a > 0, the feet of perpendiculars from the points A(a, -2a, 3) and B(0, 4, 5) on the plane lx + my + nz = 0 are points C(0, $-$a, -1) and D respectively, then the length of line segment CD is equal to : [4 marks]

(A) $\sqrt{41}$

(B) $\sqrt{55}$

(C) $\sqrt{31}$

(D) $\sqrt{66}$

Correct Answer: (D) $\sqrt{66}$

Solution:

Concept:

- The vector joining a point to its perpendicular foot is parallel to the plane normal. Because the plane passes through the origin, that normal is perpendicular to the position vector of every point in the plane.
- The projection of a point with position vector $\mathbf{b}$ onto $\mathbf{n} \cdot \mathbf{r} = 0$ is $\mathbf{b} - (\mathbf{n} \cdot \mathbf{b})\mathbf{n}/\|\mathbf{n}\|^2$.

Step 1: Find the parameter and a plane normal The perpendicular vector from C to A is $\overrightarrow{CA} = (a, -a, 4)$. Since $C = (0, -a, -1)$ lies on the plane through the origin, its position vector is perpendicular to this normal. Therefore

$$(a, -a, 4) \cdot (0, -a, -1) = a^2 - 4 = 0.$$

The condition $a > 0$ selects $a = 2$. A normal is then $(2, -2, 4)$, which may be scaled to $\mathbf{n} = (1, -1, 2)$. Hence the plane is $x - y + 2z = 0$; indeed $C = (0, -2, -1)$ satisfies it.

Step 2: Project the second point and calculate the distance For $B = (0, 4, 5)$, we have $\mathbf{n} \cdot B = -4 + 10 = 6$ and $\|\mathbf{n}\|^2 = 1 + 1 + 4 = 6$. Consequently,

$$D = (0, 4, 5) - (1, -1, 2) = (-1, 5, 3).$$

This point satisfies $-1 - 5 + 6 = 0$, and $B - D = (1, -1, 2)$ is parallel to the normal, confirming that it is the required foot. Now $D - C = (-1, 7, 4)$, so

$$CD^2 = (-1)^2 + 7^2 + 4^2 = 66.$$

The other proposed lengths have squares 41, 55 and 31; none equals the squared distance of the two verified feet.

Step 3: Conclusion The required length is $\boxed{CD = \sqrt{66}}$. Hence, the correct option is (D).

Quick Tip to Solve: When one perpendicular foot is supplied on a plane through the origin, first dot its position vector with the perpendicular vector to determine any unknown parameter.

19. Let [x] denote greatest integer less than or equal to x. If for n∈N,

$(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$,

then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$ **is equal to :** [4 marks]

(A) 2^{n-1}

(B) n

(C) 2

(D) 1

Correct Answer: The printed second sum has $a_{2j}+1$, giving 6 for odd n and 2 for even n; no listed option works for all n.

Solution:

Concept:

- For $P(x) = \sum_{r=0}^{3n} a_r x^r$, let E be the sum of even-index coefficients and O the sum of odd-index coefficients. Then $P(1) = E + O$ and $P(-1) = E - O$.
- The displayed second summand is $a_{2j}+1$, with the 1 outside the subscript. It is different from a_{2j+1}; coefficient indices must be read literally.

Step 1: Compute the full even-index sum For $P(x) = (1-x+x^3)^n$, substitution gives

$$P(1) = (1-1+1)^n = 1, \qquad P(-1) = (1+1-1)^n = 1.$$

Thus $E + O = 1$ and $E - O = 1$, whence $E = 1$ and $O = 0$. The leading coefficient is $a_{3n} = 1$. These identities hold for every positive integer n, without expanding the polynomial.

Step 2: Evaluate the expression as displayed Put $H = \sum_{j=0}^{\lfloor (3n-1)/2 \rfloor} a_{2j}$. The printed expression, whose final $+1$ is outside the subscript and the summation term, is $E + 4H + 1$. For odd n, the degree $3n$ is odd and H includes all even-index coefficients, so $H = E = 1$. Its value is then $1 + 4 + 1 = 6$. For even n, the last even coefficient $a_{3n} = 1$ is excluded, so $H = E - a_{3n} = 0$; the value is $1 + 0 + 1 = 2$. For example, $n = 1$ already gives 6, which is not among the listed options. Therefore the displayed expression has no single constant answer. Reading the second term instead as the odd-coefficient sum $\sum a_{2j+1}$ would give $E + 4O = 1$, but that changes the printed subscript.

Step 3: Conclusion As displayed, the value is 6 for odd n and 2 for even n. There is no single correct listed option for all $n \in \mathbb{N}$. The value 1 belongs to the different expression with a_{2j+1} in the second sum.

Quick Tip to Solve: Use evaluations at 1 and -1 to separate coefficient sums, but inspect whether a nearby +1 is inside the subscript before identifying an odd-index sum.

20. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval [0, π] is equal to : [4 marks]

(A) 2

(B) 3

(C) 4

(D) 8

Correct Answer: (C) 4

Solution:

Concept:

- The identity $\sin^2 x + \cos^2 x = 1$ converts the two exponential terms into positive numbers with a fixed product.
- After solving for $t = 81^{\sin^2 x}$, recover $\sin^2 x$ using powers of 3, then count each angle only once within the given closed interval.

Step 1: Reduce the equation to a quadratic Let $t = 81^{\sin^2 x} > 0$. Since $\cos^2 x = 1 - \sin^2 x$, the other term is $81/t$. Thus

$$t + \frac{81}{t} = 30 \quad \Longrightarrow \quad t^2 - 30t + 81 = 0.$$

Factoring yields $(t-3)(t-27) = 0$, so $t = 3$ or $t = 27$. Both lie in the permitted range $[1, 81]$; neither is rejected by the exponential substitution. Since $81 = 3^4$, these two cases become $4\sin^2 x = 1$ or $4\sin^2 x = 3$.

Step 2: Enumerate the angles in the interval For $\sin^2 x = 1/4$, sine is nonnegative on $[0, \pi]$, so $\sin x = 1/2$, giving $x = \pi/6, 5\pi/6$. For $\sin^2 x = 3/4$, we similarly have $\sin x = \sqrt{3}/2$, giving $x = \pi/3, 2\pi/3$. These four angles are distinct. The endpoints have $\sin^2 x = 0$ and produce $1 + 81 = 82$, not 30. At each listed root the exponential terms are 3 and 27 in one order or the other, so their sum is indeed 30. Counting only one of the two quadratic roots would incorrectly give 2, while including angles outside the interval would overcount to 8.

Step 3: Conclusion There are $\boxed{4}$ roots in $[0, \pi]$. Hence, the correct option is (C).

Quick Tip to Solve: For complementary squared trigonometric exponents, multiply the two exponential terms first; the constant product often turns their fixed sum into a quadratic.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let the curve y = y(x) be the solution of the differential equation, $\frac{dy}{dx}$ = 2(x + 1). If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to _____. [4 marks]

Solution:

Concept:

- Integrating $dy/dx = 2(x+1)$ gives an upward-opening parabola $y = (x+1)^2 + C$, where the integration constant is determined by the bounded area.
- A nonzero region bounded solely by this parabola and the x-axis requires two distinct intercepts. Write $C = -b^2$ with $b > 0$; the intercepts are $-1-b$ and $-1+b$.

Step 1: Express the bounded region using its half-width Integration gives $y = x^2 + 2x + C_0 = (x+1)^2 + C$. A bounded region of positive area exists only when the vertex lies below the x-axis, hence $C < 0$. Set $C = -b^2$. Between the two roots, the curve is below the axis, so the positive area is obtained by integrating $-y$, rather than y. With $u = x + 1$, the symmetric limits become $-b$ and b. This gives

$$A = \int_{-b}^{b} (b^2 - u^2)\, du = 2\left[b^2 u - \frac{u^3}{3}\right]_0^b = \frac{4b^3}{3}.$$

Step 2: Determine the constant and evaluate the curve Equate this area to the specified numerical value:

$$\frac{4b^3}{3} = \frac{4\sqrt{8}}{3} \quad \Longrightarrow \quad b^3 = \sqrt{8} = 2\sqrt{2}.$$

Since $(\sqrt{2})^3 = 2\sqrt{2}$ and $b > 0$, we obtain $b = \sqrt{2}$, hence $C = -2$. Therefore $y = (x+1)^2 - 2$, and

$$y(1) = (1+1)^2 - 2 = 4 - 2 = 2.$$

As a check, its derivative is $2(x+1)$, its intercepts are $-1 \pm \sqrt{2}$, and the area formula gives $4(\sqrt{2})^3/3 = 4\sqrt{8}/3$, agreeing with both conditions.

Step 3: Conclusion The required numerical value is $\boxed{y(1) = 2}$.

Quick Tip to Solve: Complete the square after integrating; the half-distance between the intercepts gives the area of this parabolic segment immediately as four-thirds of its cube.

22. Let f : R → R be a continuous function such that f(x) + f(x + 1) = 2, for all x∈R.

If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^3 f(x)dx$, then the value of $I_1 + 2I_2$ is equal to _____. [4 marks]

Solution:

Concept:

- The identity $f(x) + f(x+1) = 2$ pairs the integrals over two adjacent unit intervals. Continuity guarantees that all the definite integrals used below exist.
- Replacing x by $x+1$ and subtracting shows that $f(x+2) = f(x)$. Thus the function has period 2, though it need not be constant.

Step 1: Integrate across a pair of unit intervals For any real number a, integrate the given identity from a to $a+1$. In the second integral put $u = x+1$; its limits become $a+1$ and $a+2$. Therefore

$$\int_a^{a+1} f(x)\,dx + \int_{a+1}^{a+2} f(u)\,du = \int_a^{a+1} 2\,dx = 2.$$

The adjacent intervals join, so the integral over every interval of length 2 is exactly 2. This conclusion does not require us to find the individual values of either unit-interval integral.

Step 2: Evaluate the two given integrals The interval $[0, 8]$ is the union of four adjacent intervals of length 2, whereas $[-1, 3]$ is the union of two such intervals. Applying the preceding result separately to each interval gives

$$I_1 = \int_0^2 f + \int_2^4 f + \int_4^6 f + \int_6^8 f = 4(2) = 8,$$

$$I_2 = \int_{-1}^1 f + \int_1^3 f = 2(2) = 4.$$

Here each abbreviated integral has integrand $f(x)\,dx$. Finally combine these values with the coefficient specified in the question: $I_1 + 2I_2 = 8 + 2(4) = 16$.

Step 3: Conclusion The required numerical value is $\boxed{16}$.

Quick Tip to Solve: Integrate a shift identity over one shift length: adjoining the two intervals often determines a full-period integral without determining the function.

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23. If the normal to the curve y(x) = $\int_0^x (2t^2 - 15t + 10)dt$ at a point (a, b) is parallel to the line x + 3y = −5, a > 1, then the value of | a + 6b | is equal to _____. [4 marks]

Solution:

Concept:

- By the fundamental theorem of calculus, the derivative of $y(x) = \int_0^x (2t^2 - 15t + 10)\,dt$ is $y\prime(x) = 2x^2 - 15x + 10$. This derivative is the tangent slope.
- For a nonhorizontal tangent, the normal slope is the negative reciprocal of the tangent slope. Parallel lines have the same slope, and the specified condition $a > 1$ selects the permitted point.

Step 1: Find the abscissa using the normal slope Write the given line as $y = -x/3 - 5/3$, so its slope is $-1/3$. The normal at (a, b) must consequently have slope $-1/3$; the tangent slope there is 3. Hence

$$2a^2 - 15a + 10 = 3,$$

$$2a^2 - 15a + 7 = 0, \qquad (2a - 1)(a - 7) = 0.$$

The candidates are $a = 1/2$ and $a = 7$. Since the question explicitly requires $a > 1$, retain only $a = 7$. Substituting 7 in the derivative gives $98 - 105 + 10 = 3$, confirming the slope condition.

Step 2: Calculate the ordinate and the required expression Integrate the polynomial with the lower limit 0, rather than adding an arbitrary integration constant:

$$y(x) = \left[\frac{2t^3}{3} - \frac{15t^2}{2} + 10t\right]_0^x = \frac{2x^3}{3} - \frac{15x^2}{2} + 10x.$$

At $x = 7$,

$$b = \frac{686}{3} - \frac{735}{2} + 70 = \frac{1372 - 2205 + 420}{6} = -\frac{413}{6}.$$

Therefore $a + 6b = 7 - 413 = -406$. The absolute-value signs in the question convert this negative intermediate result to its positive magnitude.

Step 3: Conclusion Thus $|a + 6b| = \boxed{406}$.

Quick Tip to Solve: For a normal parallel to a given line, convert its slope to the tangent slope first; apply the condition on the abscissa before evaluating the integral.

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24. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _____. [4 marks]

Solution:

Concept:

- Four consecutive initial terms of an arithmetic progression have one common difference; four consecutive initial terms of a geometric progression have one common ratio.
- A common term must satisfy both progressions and lie within the last term of each finite series. Congruences modulo 5 efficiently identify the powers of 2 that belong to the arithmetic progression.

Step 1: Identify and extend the two progressions The supplied set contains the four arithmetic initial terms $11, 16, 21, 26$, with common difference 5, and the four geometric initial terms $4, 8, 16, 32$, with common ratio 2. Thus the terms have forms

$$a_n = 11 + 5(n - 1), \qquad g_k = 4 \cdot 2^k \quad (k = 0, 1, 2, \ldots).$$

The arithmetic terms are congruent to 1 modulo 5. Their largest four-digit term is 9996, since the next such term is 10001. The largest four-digit geometric term is $4 \cdot 2^{11} = 8192$, since the next one, 16384, has five digits. Consequently it suffices to examine $0 \le k \le 11$, with the additional arithmetic

lower bound 11.

Step 2: Count the powers meeting the arithmetic condition A geometric term is also arithmetic exactly when $4 \cdot 2^k \equiv 1 \pmod 5$ and it is at least 11. The successive powers of 2 modulo 5 repeat as $1, 2, 4, 3$, with period 4. Hence

$$2^{k+2} \equiv 1 \pmod 5 \quad \Longleftrightarrow \quad k+2 \equiv 0 \pmod 4.$$

Within $0 \le k \le 11$, the allowed exponents are $k = 2, 6, 10$. They yield the common terms $16, 256, 4096$, respectively. All three exceed 11 and are below both finite upper bounds; the next candidate has $k = 14$ and already exceeds the allowed geometric range.

Step 3: Conclusion There are exactly $\boxed{3}$ common terms: $16, 256, 4096$.

Quick Tip to Solve: Convert membership in an arithmetic progression to a remainder condition, then exploit the repeating residues of the geometric powers.

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25. If $\lim\limits_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2$, then a + b + c is equal to _____. [4 marks]

Solution:

Concept:

- Near zero, $e^x = 1 + x + x^2/2 + O(x^3)$, $e^{-x} = 1 - x + x^2/2 + O(x^3)$, and $\cos x = 1 - x^2/2 + O(x^4)$.
- The denominator $x \sin x$ has leading term x^2. A finite limit therefore requires the numerator's constant and linear coefficients to vanish before its quadratic coefficient is compared.

Step 1: Impose the conditions for a finite limit Let $N(x) = ae^x - b\cos x + ce^{-x}$. Combining the expansions gives

$$N(x) = (a - b + c) + (a - c)x + \frac{a + b + c}{2}x^2 + O(x^3).$$

Also, $x \sin x = x^2 + O(x^4)$. If the constant term of $N(x)$ were nonzero, dividing by x^2 could not produce a finite limit. Thus $a - b + c = 0$. After imposing this, a nonzero linear term would give a term proportional to $1/x$, so we must also have $a - c = 0$. These cancellation conditions are necessary; simply substituting zero would miss them.

Step 2: Match the remaining quadratic coefficient Once the constant and linear terms have vanished, divide numerator and denominator by x^2:

$$\frac{N(x)}{x \sin x} = \frac{(a + b + c)/2 + O(x)}{1 + O(x^2)}.$$

Its limit is therefore $(a + b + c)/2$. The given limit equals 2, so $a + b + c = 4$. As a consistency check, the earlier equations imply $c = a$ and $b = 2a$; hence $4a = 4$, giving $a = c = 1$ and $b = 2$. With these values the numerator has leading term $2x^2$, as required by the denominator's leading term x^2.

Step 3: Conclusion The requested sum is $a + b + c = \boxed{4}$.

Quick Tip to Solve: When a denominator vanishes quadratically, cancel the numerator's constant and linear terms before reading the coefficient of its quadratic term.

26. Let z and ω be two complex numbers such that $\omega = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and Re(ω) has minimum value. Then, the minimum value of n $\in$ N for which ω^n is real, is equal to _____. [4 marks]

Solution:

Concept:

- Write $z = x + iy$, where x, y are real. The condition $|(z+i)/(z-3i)| = 1$ equates the distances from z to $-i$ and $3i$.
- For a nonzero complex number $\omega = \rho(\cos\theta + i\sin\theta)$, its positive integer power is real exactly when $n\theta$ is an integer multiple of π.

Step 1: Use the modulus condition and minimise the real part Squaring the equality $|z+i| = |z-3i|$ gives

$$x^2 + (y+1)^2 = x^2 + (y-3)^2.$$

The squared terms simplify to $8y = 8$, so $y = 1$. This also guarantees $z \neq 3i$, as needed for the original denominator. Since $z\bar{z} = x^2 + y^2$,

$$\omega = x^2 + y^2 - 2x - 2iy + 2 = (x-1)^2 + 2 - 2i$$

after using $y = 1$. Thus $\mathrm{Re}(\omega) = (x-1)^2 + 2$ has its minimum 2 precisely when $x = 1$. The required complex number is consequently $\omega = 2 - 2i$.

Step 2: Find the first real positive integer power The complex number $2 - 2i$ has modulus $2\sqrt{2}$ and argument $-\pi/4$, so

$$\omega^n = (2\sqrt{2})^n \left(\cos\frac{n\pi}{4} - i\sin\frac{n\pi}{4}\right).$$

Its imaginary part vanishes exactly when $n\pi/4$ is an integer multiple of π, or n is divisible by 4. The least positive such integer is 4. Directly, $\omega^2 = -8i$ and $\omega^4 = -64$; the second power remains imaginary whereas the fourth is real. The third power has argument $-3\pi/4$ and is not real either.

Step 3: Conclusion The minimum positive integer is $n = \boxed{4}$.

Quick Tip to Solve: Turn an equal-modulus condition into a line in the complex plane, minimise the resulting real quadratic, then use the argument to test real powers.

27. The total number of 3 $\times$ 3 matrices A having entries from the set {0, 1, 2, 3} such that the sum of all the diagonal entries of AA^T is 9, is equal to _____. [4 marks]

Solution:

Concept:

- The trace of AA^T, the sum of its diagonal entries, equals the sum of the squares of all nine entries of A. Thus we must count nine ordered slots with entries in $\{0, 1, 2, 3\}$ whose squares total 9.
- If n_j denotes the number of entries equal to j, then $n_1 + 4n_2 + 9n_3 = 9$. Distinct placements of these entries give distinct matrices.

Step 1: Separate the cases involving an entry equal to 3 An entry 3 contributes its entire square, 9, to

the required sum. Therefore, if a 3 occurs, it occurs exactly once and all other entries are 0. There are 9 choices of its slot, giving 9 matrices. If no 3 occurs, write $n_1 + 4n_2 = 9$. Since n_1 and n_2 are nonnegative integers and there are only nine slots, the only possible counts are

$$(n_2, n_1) = (0, 9), (1, 5), (2, 1).$$

The remaining slots contain zeros. A third entry equal to 2 is impossible because it would contribute 12 by itself.

Step 2: Count each remaining arrangement and add For $(n_2, n_1) = (0, 9)$, every entry is 1, giving one matrix. For $(1, 5)$, choose the slot of the 2 and then the five slots of the ones from the remaining eight:

$$\binom{9}{1}\binom{8}{5} = 9 \cdot 56 = 504.$$

For $(2, 1)$, choose the two slots containing 2 and then the one slot containing 1 from the remaining seven:

$$\binom{9}{2}\binom{7}{1} = 36 \cdot 7 = 252.$$

All these cases are disjoint because their entry counts differ, and together they exhaust the square-sum equation. Including the matrices with one 3, the total is $9 + 1 + 504 + 252 = 766$.

Step 3: Conclusion The total number of matrices is $\boxed{766}$.

Quick Tip to Solve: Replace the trace of AA^T by the sum of entry squares; count the possible value multiplicities first, then their placements.

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28. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\,\beta$, where α, β are integers, then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- Place the unit square at $A = (0, 0), B = (1, 0), C = (1, 1), D = (0, 1)$. A circle inside the angle at A, tangent to both axes, has centre (r, r) and radius r.
- Tangency of a line to a circle means that the perpendicular distance from the centre to that line equals the radius. Use this with the tangent through C and its intercept E on AB.

Step 1: Determine the radius of the circle in the square The first circle has centre A and radius 1. The second lies in the corner between the two sides and touches it internally; its centre is distance $r\sqrt{2}$ from A. Thus

$$r\sqrt{2} + r = 1, \qquad r = \sqrt{2} - 1.$$

In particular $r^2 = 3 - 2\sqrt{2} = 1 - 2r$. The other positive-radius tangency arrangement, with $r = \sqrt{2} + 1$, places the circle beyond the square and its tangents from C do not meet the side segment AB; it does

not meet the stated geometry.

Step 2: Use the tangent intercept to find the segment Write $E = (e, 0)$, with $0 \le e \le 1$. The line through E and C is

$$x - (1 - e)y - e = 0.$$

Its distance from (r, r) equals r, so

$$\frac{e(1 - r)}{\sqrt{1 + (1 - e)^2}} = r.$$

Both sides are nonnegative in the side segment. Squaring and collecting terms gives

$$e^2(1 - 2r) + 2r^2e - 2r^2 = 0.$$

Since $1 - 2r = r^2 > 0$, division by r^2 yields $e^2 + 2e - 2 = 0$. Only $e = \sqrt{3} - 1$ lies in $[0, 1]$; the negative root is rejected. Hence

$$EB = 1 - e = 2 - \sqrt{3}.$$

Matching this to $\alpha + \sqrt{3}\,\beta$ gives $\alpha = 2$ and $\beta = -1$.

Step 3: Conclusion Thus $\alpha + \beta = 2 - 1 = \boxed{1}$.

Quick Tip to Solve: For a circle tangent to perpendicular sides, use centre coordinates (r, r); express the other tangent through its axis intercept and apply the distance formula.

29. Let f : (0, 2) $\to$ R be defined as f(x) = $\log_2\left(1 + \tan\left(\frac{\pi x}{4}\right)\right)$. Then, $\lim\limits_{n\to\infty} \frac{2}{n}\left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \ldots + f(1)\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- A right-endpoint Riemann sum satisfies $\lim_{n\to\infty} n^{-1} \sum_{k=1}^{n} f(k/n) = \int_0^1 f(x)\,dx$ when the endpoint extension of f is continuous.
- The reflection identity $\int_0^1 f(x)\,dx = \int_0^1 f(1 - x)\,dx$ often simplifies logarithmic integrals. Here all logarithm arguments on this interval are positive.

Step 1: Convert the sum into an integral Write the summand as $f(k/n)$, with mesh size $1/n$. Although the stated domain is open at zero, the right-hand limit there is $\log_2 1 = 0$, so a continuous extension is available. Hence the required limit is

$$L = 2I, \qquad I = \int_0^1 \log_2\left(1 + \tan\frac{\pi x}{4}\right) dx.$$

The factor 2 multiplies the integral; it does not change the upper endpoint to 2.

Step 2: Reflect the variable about the midpoint For $t = \tan(\pi x/4)$, use the tangent subtraction formula:

$$\tan\frac{\pi(1 - x)}{4} = \frac{1 - t}{1 + t}.$$

Therefore

$$f(1-x) = \log_2 \frac{2}{1+t} = 1 - f(x).$$

Both logarithms are well-defined for $0 \le x \le 1$, since $0 \le t \le 1$. This identity is exact and avoids integrating the logarithm directly.

Step 3: Add the equal integrals Reflection gives $I = \int_0^1 f(1-x)\,dx$. Adding it to the original form yields

$$2I = \int_0^1 \{f(x) + f(1-x)\}\,dx = \int_0^1 1\,dx = 1.$$

Thus $I = 1/2$, and multiplication by the prefactor 2 gives the requested limit.

Step 4: Conclusion The limit is $2I = \boxed{1}$.

Quick Tip to Solve: For a logarithm containing $1 + \tan(\pi x/4)$ on $[0, 1]$, pair x with $1 - x$; their function values add to 1.

30. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ **and** $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$ **where** $\omega = \frac{-1+i\sqrt{3}}{2}$**, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to _____.**

[4 marks]

Solution:

Concept:

- Similar matrices have equal determinants: $\det(P^{-1}BP) = \det B$ for invertible P. Also $\det(B^2) = (\det B)^2$.
- The given ω is a non-real cube root of unity, so $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. These identities simplify polynomial expressions in ω.

Step 1: Remove the similarity transformation Since $P^{-1}I_3P = I_3$, subtracting the identity gives

$$P^{-1}AP - I_3 = P^{-1}(A - I_3)P.$$

Taking determinants cancels $\det P$ with $\det P^{-1}$. Consequently,

$$\det\big((P^{-1}AP - I_3)^2\big) = \{\det(A - I_3)\}^2.$$

There is no need to compute the entries of P^{-1}; their complexity is irrelevant to this invariant.

Step 2: Evaluate the smaller determinant The matrix to use is

$$A - I_3 = \begin{pmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{pmatrix}.$$

Expanding along its first row gives

$$\begin{aligned} \det(A - I_3) &= (\omega^2 + 2\omega) - 7\omega + \omega^3 \\ &= \omega^2 - 5\omega + 1. \end{aligned}$$

The final cofactor contributes $\omega^2\omega = \omega^3$, with a positive sign.

Step 3: Use the cube-root identities and square Using $\omega^2 + 1 = -\omega$ simplifies the determinant to -6ω. Therefore

$$\{\det(A - I_3)\}^2 = (-6\omega)^2 = 36\omega^2.$$

The question asks for the coefficient of ω^2, not the determinant -6ω before squaring. Since $\omega \neq 0$, the coefficients can be equated directly.

Step 4: Conclusion Comparing with $\alpha\omega^2$ gives $\boxed{\alpha = 36}$.

Quick Tip to Solve: Whenever $P^{-1}AP - I$ occurs inside a determinant, rewrite it as $P^{-1}(A - I)P$ before doing any matrix multiplication.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be :
(Assume that atmospheric pressure is 1×10^5 Pa density of water is 10^3 kg m^{-3}, g = 10 ms^{-2})

[4 marks]

(A) $\frac{200}{5}\%$
(B) $\frac{200}{3}\%$
(C) $\frac{3}{200}\%$
(D) $\frac{5}{200}\%$

Correct Answer: (B) $\frac{200}{3}\%$

Solution:

Concept:

- Absolute pressure at depth h in water is $P = P_0 + \rho gh$, where P_0 is atmospheric pressure and ρgh is gauge pressure.
- Percentage increase is $100(P_2 - P_1)/P_1$; the denominator is the original absolute pressure when the question refers to the total pressure acting on the submarine.

Step 1: Separate the water contribution The stated pressure includes the atmospheric part. Thus the

pressure due to the water column at the first depth is

$$\rho g h = P_1 - P_0 = (3-1) \times 10^5 = 2 \times 10^5 \,\text{Pa}.$$

For confirmation, the supplied $\rho = 10^3 \,\text{kg}\,\text{m}^{-3}$ and $g = 10 \,\text{m}\,\text{s}^{-2}$ give $h = 20 \,\text{m}$. Doubling the depth doubles this water contribution alone.

Step 2: Calculate the new total pressure At depth $2h$, the atmosphere still contributes the same pressure:

$$P_2 = P_0 + 2\rho g h = 10^5 + 4 \times 10^5 = 5 \times 10^5 \,\text{Pa}.$$

Hence the increase is $P_2 - P_1 = 2 \times 10^5$ Pa, rather than 3×10^5 Pa. Treating the whole initial pressure as proportional to depth would incorrectly double the atmospheric term too.

Step 3: Express the increase as a percentage Using the original pressure in the denominator,

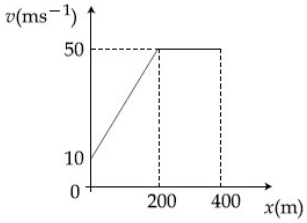
$$\frac{P_2 - P_1}{P_1} \times 100\% = \frac{2 \times 10^5}{3 \times 10^5} \times 100\% = \frac{200}{3}\%.$$

Option (A) uses 5 in the denominator and therefore measures the change relative to the final pressure. The tiny fractions in (C) and (D) are reciprocals unrelated to the defined percentage increase.

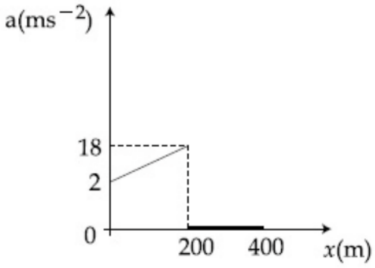
Step 4: Conclusion The percentage increase is $\boxed{200/3\%}$, approximately 66.7%. Hence, the correct option is (B).

Quick Tip to Solve: Subtract atmospheric pressure before scaling with depth, then add it back before comparing absolute pressures.

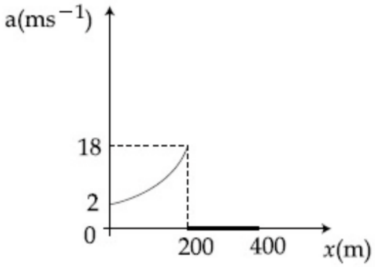
32. The velocity-displacement graph describing the motion of bicycle is shown in the figure. The acceleration-displacement graph of the bicycle's motion is best described by : [4 marks]



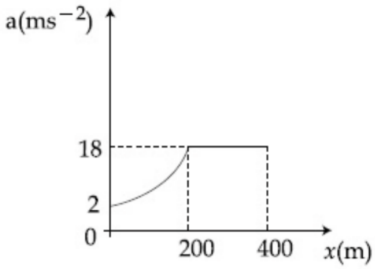
(A)



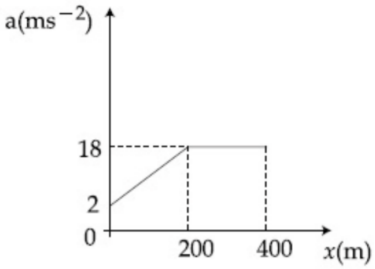
(B)



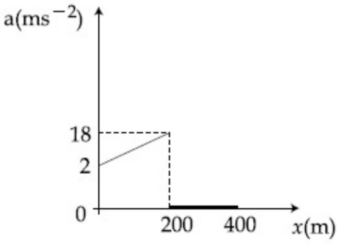
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- For a velocity given as a function of displacement, the chain rule gives $a = dv/dt = (dv/dx)(dx/dt) = v\,dv/dx$. The slope dv/dx alone is not acceleration.
- A straight rising segment in a v-versus-x graph has constant dv/dx, so acceleration is proportional to v on that segment. A horizontal velocity segment has zero acceleration.

Step 1: Read the rising velocity segment The graph rises from $v = 10\,\mathrm{m\,s^{-1}}$ at $x = 0$ to $v = 50\,\mathrm{m\,s^{-1}}$ at $x = 200\,\mathrm{m}$. Thus

$$\frac{dv}{dx} = \frac{50 - 10}{200} = 0.2\,\mathrm{s^{-1}}, \qquad v = 10 + 0.2x.$$

Here x is measured in metres and v in metres per second. The graph is linear in displacement, so its slope must be multiplied by the instantaneous velocity.

Step 2: Find acceleration on both intervals For $0 < x < 200\,\mathrm{m}$,

$$a = v\frac{dv}{dx} = 0.2(10 + 0.2x) = 2 + 0.04x.$$

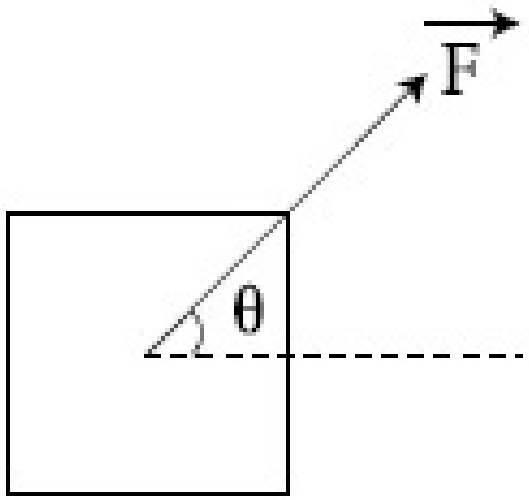
Acceleration therefore rises linearly from 2 to $10\,\mathrm{m\,s^{-2}}$. For $200 < x < 400\,\mathrm{m}$, velocity is constant at $50\,\mathrm{m\,s^{-1}}$, hence $dv/dx = 0$ and $a = 0$. At the sharp joining point the idealized graph has different one-sided derivatives.

Step 3: Compare the graph shapes and numerical scale Only option (A) shows a straight increasing acceleration segment followed by zero acceleration. The curved segments in (B) and (C) do not follow from the constant first slope; (C) and (D) also incorrectly retain nonzero acceleration during constant velocity. However, the options label the upper acceleration as 18, whereas the supplied velocity graph gives 10. Thus (A) is the best qualitative description but its printed numerical endpoint is inconsistent.

Step 4: Conclusion The correct acceleration is $a = 2 + 0.04x$ for $0 < x < 200\,\mathrm{m}$, then $a = 0$. Hence, the correct option is (A) by shape; its printed 18 should be 10 for quantitative agreement.

Quick Tip to Solve: For a velocity-displacement plot, multiply the local slope by velocity. Check both the graph shape and its endpoint values.

33. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_k. then, the block's acceleration 'a' is given by : (g is acceleration due to gravity) [4 marks]



(A) $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$
(B) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$
(C) $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$
(D) $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$
Correct Answer: (B) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

Solution:

Concept:

- Resolve the applied force into horizontal $F\cos\theta$ and upward vertical $F\sin\theta$ components. The block stays in vertical equilibrium while it remains in contact with the floor.
- Kinetic friction has magnitude $f_k = \mu_k N$ and opposes the sliding motion. Apply $\sum F_x = ma$ along the floor and $\sum F_y = 0$ vertically.

Step 1: Find the normal reaction The diagram shows a pulling force directed upward and to the right. With upward taken as positive, vertical equilibrium gives

$$N + F\sin\theta - mg = 0, \qquad N = mg - F\sin\theta.$$

The upward force reduces the floor reaction. The formula applies while $N \geq 0$, as required by the statement that the block slides along the floor. Replacing this by $mg + F\sin\theta$ would describe a downward push.

Step 2: Subtract friction from the horizontal pull For the indicated forward sliding, friction acts backward and has magnitude

$$f_k = \mu_k(mg - F\sin\theta).$$

Therefore Newton's second law horizontally gives

$$ma = F\cos\theta - \mu_k(mg - F\sin\theta).$$

Both the direction of friction and the reduction of normal force must be retained; simply taking friction as $\mu_k mg$ would omit the vertical component of the pull.

Step 3: Divide by the mass and check the signs Dividing through by m yields

$$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right).$$

Option (A) adds friction in the forward direction. Option (C) reverses the horizontal pulling term. Option (D) increases rather than decreases the normal force. As a useful check, for $\theta = 0$ this reduces to the standard result $a = F/m - \mu_k g$.

Step 4: Conclusion The acceleration is $\boxed{a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right)}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the normal force from the vertical components first; an upward pull reduces both the normal force and kinetic friction.

34. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is : [4 marks]

(A) 9.859×10^{-2} N

(B) 0.0314 N

(C) 9.859×10^{-4} N

(D) 6.28×10^{-3} N

Correct Answer: (C) 9.859×10^{-4} N

Solution:

Concept:

- Uniform circular motion requires inward acceleration $a_c = v^2/r = \omega^2 r$. Constant speed does not imply zero acceleration because the velocity direction changes.
- For period T, $\omega = 2\pi/T$. The vertical side wall supplies the horizontal radial force, whereas the bottom support balances the weight vertically.

Step 1: Convert the quantities to SI units The mass and groove radius are

$$m = 200\,\text{g} = 0.200\,\text{kg}, \qquad r = 20\,\text{cm} = 0.200\,\text{m}.$$

One complete revolution takes $T = 40\,\mathrm{s}$. The angular speed is therefore

$$\omega = \frac{2\pi}{40} = \frac{\pi}{20}\,\mathrm{rad\,s^{-1}}.$$

The required reaction is from the side wall, so it must be associated with radial acceleration rather than with the vertical weight.

Step 2: Calculate the radial normal force The inward normal force is

$$N = m\omega^2 r = 0.200\left(\frac{\pi}{20}\right)^2(0.200)$$
$$= \frac{\pi^2}{10000}\,\mathrm{N} \approx 9.87 \times 10^{-4}\,\mathrm{N}.$$

Equivalently, $v = 2\pi r/T = \pi/100\,\mathrm{m\,s^{-1}}$, and substitution into mv^2/r gives the same result. The small force is reasonable because the revolution takes a long time.

Step 3: Match the magnitude with the options The value agrees with $9.859 \times 10^{-4}\,\mathrm{N}$ to the numerical precision used in the listed choices. Option (A) is larger by a factor of 100. Option (B), approximately 0.0314, is the numerical value of the speed in SI units, not the force. Option (D) also does not satisfy $N = m(2\pi/T)^2 r$. Keeping both the mass and radius in SI units prevents these scale errors.

Step 4: Conclusion The normal force is approximately $\boxed{9.86 \times 10^{-4}\,\mathrm{N}}$. Hence, the correct option is (C).

Quick Tip to Solve: When a period is supplied, use $N = 4\pi^2 mr/T^2$ directly and convert grams and centimetres before substituting.

35. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be : [4 marks]

(A) $\sqrt{3}T$
(B) $\sqrt{\frac{2}{3}}T$
(C) $\frac{T}{\sqrt{3}}$
(D) $\sqrt{\frac{3}{2}}T$

Correct Answer: (B) $\sqrt{\frac{2}{3}}T$

Solution:

Concept:

- For small oscillations, a pendulum of length l has period $T = 2\pi\sqrt{l/g_{\text{eff}}}$, where g_{eff} is the effective gravitational acceleration in the support's frame.
- In a lift accelerating upward with acceleration a, the inertial force in the lift frame points downward. Thus it adds to gravity, giving $g_{\text{eff}} = g + a$.

Step 1: Identify the effective gravity The lift accelerates upward at $a = g/2$. In the lift frame the bob

experiences a downward inertial force $mg/2$, in addition to its downward weight mg. The resultant effective downward force is $3mg/2$, so

$$g_{\text{eff}} = g + \frac{g}{2} = \frac{3g}{2}.$$

This is the same effective gravity that sets the equilibrium tension and the restoring torque for a small angular displacement.

Step 2: Compare the two periods Before the lift accelerates,

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

During uniform upward acceleration, with the same pendulum length,

$$T\prime = 2\pi\sqrt{\frac{l}{3g/2}}.$$

Taking the ratio eliminates both l and 2π:

$$\frac{T\prime}{T} = \sqrt{\frac{g}{3g/2}} = \sqrt{\frac{2}{3}}.$$

The bob's mass cancels from the pendulum equation, so no mass value is needed.

Step 3: Check the physical trend and choices Greater effective gravity produces a stronger restoring acceleration for the same angular displacement, so the period must decrease. This rules out (A) and (D), which exceed T. Option (C) would require effective gravity $3g$, rather than the correct $3g/2$. The period depends on the lift's acceleration, not merely on the fact that the lift is moving upward; uniform upward velocity alone would leave it unchanged.

Step 4: Conclusion The new period is $\boxed{T\prime = \sqrt{\frac{2}{3}}\,T}$. Hence, the correct option is (B).

Quick Tip to Solve: For an accelerating lift, replace g by $g + a$ for upward acceleration, then compare periods using the square-root ratio.

36. The angle of deviation through a prism is minimum when

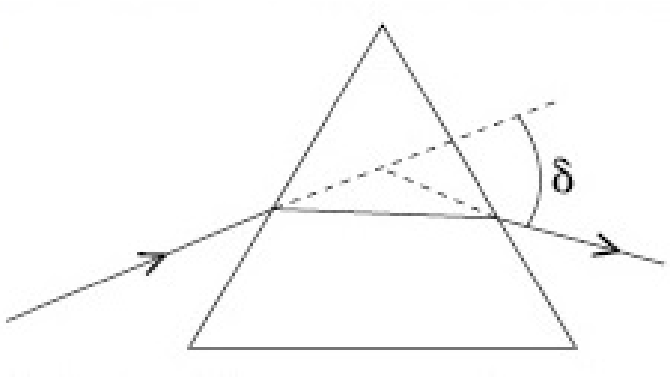
(A) Incident ray and emergent ray are symmetric to the prism.

(B) The refracted ray inside the prism becomes parallel to its base

(C) Angle of incidence is equal to that of the angle of emergence

(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below : [4 marks]



(A) Statements (B) and (C) are true

(B) Only statements (A) and (B) are true

(C) Statements (A), (B) and (C) are true

(D) Only statement (D) is true

Correct Answer: (C) Statements (A), (B) and (C) are true

Solution:

Concept:

- For a prism, the deviation is $\delta = i + e - A$, where i and e are the angles of incidence and emergence and A is the prism angle.
- At minimum deviation the optical path is symmetric: $i = e$ and the internal refraction angles satisfy $r_1 = r_2 = A/2$.

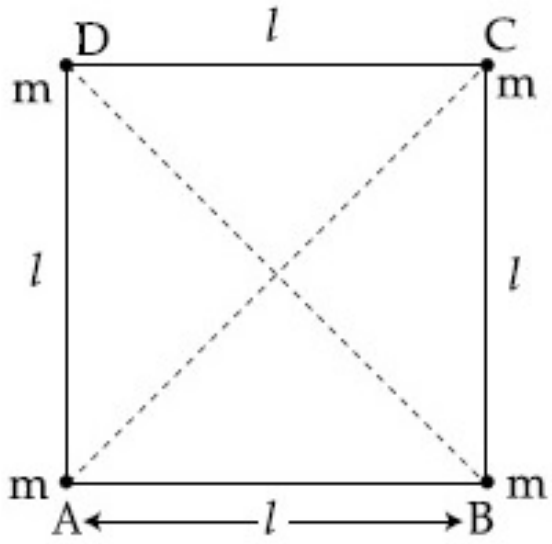
Step 1: Apply the symmetric-path condition As incidence is varied, deviation first decreases and then increases. At its minimum the reversible path coincides with its symmetric counterpart, so the incident and emergent rays make equal angles with their respective face normals. Thus statement (A) describes the symmetric geometry and statement (C) gives its angular condition; both are true.

Step 2: Check the internal ray and the remaining statement For the symmetric prism shown, equal internal angles place the refracted ray parallel to the base, so statement (B) is also true. Statement (D) instead requires $e = 2i$, which is incompatible with the minimum-deviation condition $e = i$ for this refracting prism. The choices that retain only two of the three true statements are incomplete, while the choice containing only (D) is false.

Step 3: Conclusion The first three statements are true, while the fourth is false. Hence, the correct option is (C).

Quick Tip to Solve: At minimum deviation remember the pair $i = e$ and $r_1 = r_2 = A/2$; use the illustrated prism geometry to identify the internal ray.

37. Four equal masses, m each are placed at the comers of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be : [4 marks]



(A) $\sqrt{3}$ ml^2

(B) 2 ml^2

(C) ml^2

(D) 3 ml^2

Correct Answer: (D) 3 ml^2

Solution:

Concept:

- The moment of inertia of point masses is $I = \sum m_j d_j^2$, where d_j is the perpendicular distance of each mass from the rotation axis.
- For a square of side l, a diagonal makes 45° with each side; distances to a parallel axis must be measured perpendicular to that diagonal.

Step 1: Find the perpendicular distances Take the corners as $A = (0,0)$, $B = (l,0)$, $C = (l,l)$, and $D = (0,l)$. The diagonal DB has equation $x+y = l$, so the required parallel axis through A is $x+y = 0$. The distance formula gives distances $0, l/\sqrt{2}, \sqrt{2}l, l/\sqrt{2}$ for A, B, C, D respectively. In particular, the mass at A contributes zero, but the opposite corner C does not.

Step 2: Add all four contributions Substituting the squared distances,

$$I = m(0)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m(\sqrt{2}l)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 = 3ml^2.$$

The answer has the required dimensions of mass times length squared. The value ml^2 belongs to the central diagonal, whereas shifting the axis through A adds a further $2ml^2$; the other listed coefficients do not include the correct distances.

Step 3: Conclusion The moment of inertia is $\boxed{3ml^2}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the axis through A as $x + y = 0$; this makes the four perpendicular distances immediate and prevents confusing it with diagonal DB.

38. In thermodynamics, heat and work are : [4 marks]

(A) Path functions

(B) Point functions

(C) Extensive thermodynamics state variables

(D) Intensive thermodynamic state variables

Correct Answer: (A) Path functions

Solution:

Concept:

- A state or point function is fixed by the thermodynamic state. Its change depends only on the initial and final states; internal energy U is an example.
- Heat Q and work W are energy transfers during a process. For work done by the gas the first law is $\Delta U = Q - W$.

Step 1: Test whether work is fixed by endpoints For a quasistatic expansion, $W = \int_{V_i}^{V_f} P\,dV$, the area under the process curve on a pressure-volume graph. Different curves connecting the same endpoint states generally have different areas. Hence work depends on the route followed, and is a path function rather than a property assigned to one equilibrium state.

Step 2: Use the first law to classify heat The first law gives $Q = \Delta U + W$. Between fixed states ΔU is fixed, but work can vary with the path; therefore heat varies correspondingly. Neither heat nor work is a point function. The extensive/intensive classification in the remaining options describes thermodynamic state variables, so it does not make either transfer a state variable. A cyclic process can have nonzero net heat and work even though the state returns to its starting point.

Step 3: Conclusion Both heat and work are path functions. Hence, the correct option is (A).

Quick Tip to Solve: For a closed cycle, a state function has zero net change, but the enclosed pressure-volume area can give nonzero work and heat.

39. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}$d, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation : [4 marks]

(A) $C' = \frac{3+K}{4K}C_0$

(B) $C' = \frac{4+K}{3}C_0$

(C) $C' = \frac{4K}{K+3}C_0$

(D) $C' = \frac{4}{3+K}C_0$

Correct Answer: (C) $C' = \frac{4K}{K+3}C_0$

Solution:

Concept:

- The original air capacitance is $C_0 = \varepsilon_0 A/d$, where A is plate area and d is plate separation.
- A dielectric slab spanning the full area but only part of the separation produces two layers in series. Their effective separation is $(d - t) + t/K$.

Step 1: Resolve the capacitor into its two layers The slab thickness is $t = 3d/4$, leaving an air gap $d/4$. Both layers carry the same free charge and their potential drops add, so they are equivalent to series capacitors, not parallel ones. Their capacitances are $C_{\text{air}} = 4C_0$ and $C_{\text{slab}} = 4KC_0/3$. The remaining air gap cannot be ignored even when K is large.

Step 2: Combine the layers and check the result For series capacitances,

$$\frac{1}{C\prime} = \frac{1}{4C_0} + \frac{3}{4KC_0} = \frac{K+3}{4KC_0}.$$

Therefore $C\prime = 4KC_0/(K+3)$. With $K = 1$ this reduces to C_0, as it must when the slab behaves like air. As $K \to \infty$, it tends to $4C_0$, the capacitance set by the surviving air gap. The other expressions fail these checks or invert the series result.

Step 3: Conclusion The new capacitance is $\boxed{C\prime = \frac{4K}{K+3}C_0}$. Hence, the correct option is (C).

Quick Tip to Solve: For a slab of thickness t spanning the full plate area, replace d by $d - t + t/K$; use series addition along the field direction.

40. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time,

$\vec{B}$ = 8.0 $\times$ 10^{-8} $\hat{z}$T. The value of electric field at this point is :

(speed of light = 3 $\times$ 10^8 ms^{-1})

$\hat{x}$, $\hat{y}$, $\hat{z}$ are unit vectors along x, y and z directions. [4 marks]

(A) 2.6 $\hat{x}$ V/m

(B) $-24\ \hat{x}$ V/m

(C) 24 $\hat{x}$ V/m

(D) $-2.6\ \hat{y}$ V/m

Correct Answer: (B) $-24\ \hat{x}$ V/m

Solution:

Concept:

- For a plane electromagnetic wave in vacuum, the instantaneous field magnitudes obey $E = cB$, and the electric and magnetic fields are mutually perpendicular.
- The direction of propagation is the direction of $\vec{E} \times \vec{B}$, given by the Poynting vector. This fixes the sign of the electric field.

Step 1: Calculate the electric-field magnitude Using the magnetic field at the specified point,

$$E = cB = (3 \times 10^8)(8.0 \times 10^{-8}) = 24\,\mathrm{V\,m^{-1}}.$$

The factors of ten cancel. The frequency is unnecessary because the local ratio of the two fields in vacuum is always c; no wavelength or time phase needs to be calculated.

Step 2: Determine the direction and exclude the alternatives The wave travels in the positive y direction and $\vec{B}$ points along positive z. Since $\hat{x} \times \hat{z} = -\hat{y}$, a positive-x electric field would send the wave in the negative-y direction. Instead, $(-\hat{x}) \times \hat{z} = +\hat{y}$, so the electric field points along negative x. The value $2.6\,\mathrm{V/m}$ has the wrong magnitude, while a y-directed electric field is not transverse to propagation.

Step 3: Conclusion The electric field is $\boxed{\vec{E} = -24\hat{x}\,\mathrm{V/m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Find the magnitude with E=cB, then check the sign with $\vec{E} \times \vec{B}$; the right magnitude alone does not select the answer.

41. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms^{-1}, the speed at the farthest point is : [4 marks]

(A) 3.0×10^3 m/s

(B) 6.0×10^3 m/s

(C) 1.5×10^3 m/s

(D) 4.5×10^3 m/s

Correct Answer: (A) 3.0×10^3 m/s

Solution:

Concept:

- The gravitational force from the Sun is central, so its torque about the Sun is zero and the comet's angular momentum is conserved.
- At the nearest and farthest points of an elliptical orbit, the radial velocity vanishes and velocity is perpendicular to the radius, so $L = mrv$ at both points.

Step 1: Relate the two extreme-point speeds Let r_n, v_n denote distance and speed at the nearest point and r_f, v_f those at the farthest point. Angular momentum conservation gives $mr_nv_n = mr_fv_f$, hence $v_f = v_nr_n/r_f$. The comet mass cancels. This inverse-distance relation applies at these turning points; it should not be replaced by the circular-orbit relation $v \propto r^{-1/2}$.

Step 2: Substitute the distances and compare options The distance ratio is

$$\frac{r_n}{r_f} = \frac{8.0 \times 10^{10}}{1.6 \times 10^{12}} = 0.05.$$

Thus

$$v_f = (6 \times 10^4)(0.05) = 3.0 \times 10^3 \,\text{m/s}.$$

The farthest speed is twenty times smaller, consistent with equal areas swept in equal times. The other speeds do not preserve the product rv between the given extremes of the same orbit.

Step 3: Conclusion The speed at the farthest point is $\boxed{3.0 \times 10^3 \,\text{m/s}}$. Hence, the correct option is (A).

Quick Tip to Solve: At perihelion and aphelion the velocity is tangential, so compare the products rv directly; no solar mass or gravitational constant is needed.

42. A conducting wire of length 'l', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.

If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be : [4 marks]

(A) $4\frac{VA}{\rho l}$

(B) $\frac{3}{4}\frac{VA}{\rho l}$

(C) $\frac{1}{4}\frac{VA}{\rho l}$

(D) $\frac{1}{4}\frac{\rho l}{VA}$

Correct Answer: (C) $\frac{1}{4}\frac{VA}{\rho l}$

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho l/A$, where ρ is its resistivity, l its length and A its cross-sectional area.
- For the same applied potential difference, Ohm's law gives $I = V/R = VA/(\rho l)$. Resistivity is unchanged for the same material at the same temperature.

Step 1: Calculate the new resistance The new length is $l\prime = 2l$ and the new area is $A\prime = A/2$. Applying both changes together gives

$$R\prime = \rho\frac{2l}{A/2} = 4\frac{\rho l}{A} = 4R.$$

Doubling the length doubles the resistance, and halving the area doubles it again. These factors multiply; they must not be subtracted or averaged.

Step 2: Apply the battery voltage to the new wire The wire remains connected to the same battery, so the applied potential difference is V. Therefore

$$I\prime = \frac{V}{R\prime} = \frac{V}{4R} = \frac{1}{4}\frac{VA}{\rho l}.$$

The current decreases, excluding the fourfold-increase choice. A factor of three quarters does not follow from the geometric changes. Finally, $\rho l/(VA)$ has dimensions of inverse current, so its quarter cannot be the required current.

Step 3: Conclusion The resultant current is $\boxed{I\prime = \frac{1}{4}\frac{VA}{\rho l}}$. Hence, the correct option is (C).

Quick Tip to Solve: At fixed voltage, current scales as A/l; halving A and doubling l each contribute a factor of one half to the current.

43. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is : [4 marks]

(A) $U_e = U_m$

(B) $U_e \neq U_m$

(C) $U_e < U_m$

(D) $U_e > U_m$

Correct Answer: (A) $U_e = U_m$

Solution:

Concept:

- The electric energy density in vacuum is $u_e = \frac{1}{2}\varepsilon_0 E^2$; the magnetic energy density is $u_m = B^2/(2\mu_0)$. These are energies per unit volume, measured in $\mathrm{J\,m^{-3}}$.
- For a travelling electromagnetic wave in free space, $E = cB$ at every point and time, and $c^2 = 1/(\mu_0\varepsilon_0)$. The two fields oscillate in phase.

Step 1: Relate the instantaneous energy densities Substitute the wave relation into the electric contribution, keeping the factor of one-half in both expressions:

$$u_e = \frac{1}{2}\varepsilon_0(cB)^2 = \frac{1}{2}\varepsilon_0\frac{B^2}{\mu_0\varepsilon_0} = \frac{B^2}{2\mu_0} = u_m.$$

Thus the equality follows directly from the propagation speed in vacuum; it does not depend on the amplitude, frequency or polarization of the wave.

Step 2: Average over an oscillation and compare the options Since the instantaneous quantities are equal, averaging either over the same time interval preserves the equality. For sinusoidal peak fields E_0, B_0, this gives

$$U_e = \frac{1}{4}\varepsilon_0 E_0^2, \qquad U_m = \frac{B_0^2}{4\mu_0}.$$

The mean electric and magnetic contributions are therefore equal halves of the total mean energy density. Option (B) denies this equality, while (C) and (D) incorrectly assign a larger contribution to one field. These inequalities do not apply to the specified freely travelling wave.

Step 3: Conclusion $\boxed{U_e = U_m}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $E = cB$ before averaging: equality of instantaneous electric and magnetic energy densities immediately gives equality of their averages.

44. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation : [4 marks]

(A) Phase

(B) Frequency

(C) Amnplitude

(D) Intensity

Correct Answer: (B) Frequency

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ν is the incident frequency and ϕ is the metal's work function.
- The stopping potential V_s is the magnitude of reverse potential just sufficient to stop the fastest photoelectrons: $eV_s = K_{\max}$. The emitting surface is held fixed while comparing properties of the radiation.

Step 1: Express the stopping potential in terms of frequency An electron absorbs a photon of energy $h\nu$. Part of that energy is required to escape the surface, and the maximum remaining kinetic energy is $h\nu - \phi$. A retarding electric field must remove this kinetic energy to reduce the photocurrent to zero. Therefore, above the threshold frequency $\nu_0 = \phi/h$,

$$V_s = \frac{h\nu - \phi}{e} = \frac{h}{e}(\nu - \nu_0).$$

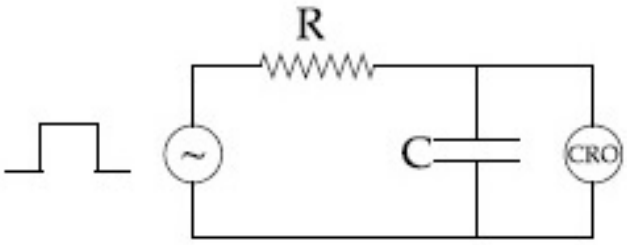
For the same metal, the work function is constant, so increasing frequency increases the stopping potential linearly.

Step 2: Distinguish energy per photon from the number of photons At fixed frequency, increasing intensity increases the number of incident photons per unit time. It increases the number of emitted electrons and hence the saturation photocurrent, but not their maximum kinetic energy. Changing amplitude likewise changes the intensity rather than the energy of an individual photon. The phase of the incident wave does not enter Einstein's energy balance. Thus phase, amplitude and intensity cannot determine the stopping potential in this experiment; frequency does. Below threshold there is no photoemission to stop.

Step 3: Conclusion The stopping potential depends on the frequency of the incident radiation. Hence, the correct option is (B).

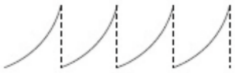
Quick Tip to Solve: Separate the two photoelectric controls: frequency sets maximum electron energy and stopping potential, while intensity mainly sets photocurrent.

45. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to : [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (D) 

Solution:

Concept:

- The CRO is connected across the capacitor, so it measures the capacitor voltage V_C, rather than the resistor voltage. The capacitor voltage cannot change discontinuously for a finite circuit current.
- For a constant input level V_i, the RC circuit obeys $RC\,dV_C/dt + V_C = V_i$. Its solution is $V_C(t) = V_i + [V_C(0) - V_i]e^{-t/(RC)}$.

Step 1: Follow the response during the high level Each flat part of the square input is a constant voltage. When the input switches to its high value V_H, the capacitor starts from the voltage left by the preceding low interval and charges through the resistor:

$$V_C(t) = V_H + [V_C(0) - V_H]e^{-t/(RC)}.$$

The output rises rapidly at first and then more slowly as it approaches V_H. Its upward segment is therefore curved with a decreasing positive slope. The finite resistance prevents an instantaneous jump in capacitor voltage.

Step 2: Follow the low level and select the waveform When the input switches to the low value V_L, the stored charge cannot disappear instantly. Instead the capacitor voltage approaches the low level

exponentially:

$$V_C(t) = V_L + [V_C(0) - V_L]e^{-t/(RC)}.$$

The output falls continuously, initially more steeply and then more slowly. Repeating these two intervals produces alternating rounded charging and discharging segments, as shown in (D). The abrupt downward resets in (A), (B) and (C) contradict continuity of the capacitor voltage. A strictly straight ramp is also only an approximation in a restricted short-time limit, rather than the rounded response displayed here.

Step 3: Conclusion The CRO displays successive exponential charging and discharging curves across the capacitor, matching (D). Hence, the correct option is (D).

Quick Tip to Solve: First locate the CRO terminals. Across a capacitor, voltage stays continuous at a square-wave edge; across a resistor, the response can jump.

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46. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is : [4 marks]

(A) $\frac{10a}{n}$
(B) $\frac{10na}{(n-1)}$
(C) $\left(\frac{n-1}{10n}\right)a$
(D) $\frac{10a}{(n-1)}$

Correct Answer: (A) $\frac{10a}{n}$

Solution:

Concept:

- The least count of a direct vernier is the difference between one main-scale division (MSD) and one vernier-scale division (VSD). It measures the smallest displacement resolved by comparing the two scales.
- Here one MSD is $a\,\text{cm}$, while n VSD equal $(n-1)$ MSD. Convert centimetres to millimetres using $1\,\text{cm} = 10\,\text{mm}$.

Step 1: Find the length of one vernier division The stated coincidence means that the total length of n vernier divisions equals the length of $n-1$ main-scale divisions. Thus

$$n\,\text{VSD} = (n-1)a\,\text{cm}, \qquad \text{VSD} = \frac{n-1}{n}a\,\text{cm}.$$

One vernier division is slightly shorter than one main-scale division. The difference is what enables the vernier to resolve a fraction of the main-scale interval; the full length of a vernier division is not itself the least count.

Step 2: Take the difference and convert the unit Subtract the two division lengths in the same unit before converting:

$$\text{LC} = a - \frac{n-1}{n}a = \frac{a}{n}\,\text{cm} = \frac{10a}{n}\,\text{mm}.$$

The answer decreases when n increases for fixed a, as expected for a finer vernier. Option (B) is larger than an MSD and cannot be this small division difference. Option (C) does not apply the required centimetre-to-millimetre conversion to the difference. Option (D) incorrectly divides by $n-1$ instead of the number n of vernier intervals used in the coincidence.

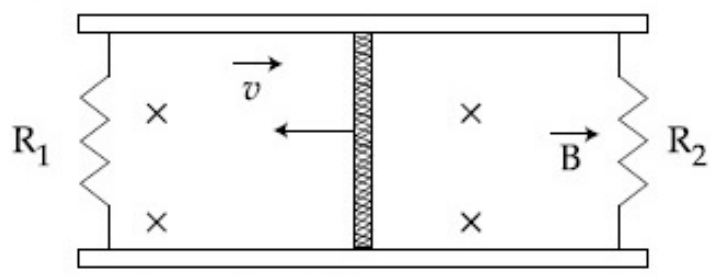
Step 3: Conclusion $\boxed{\text{LC} = \frac{10a}{n}\ \text{mm}}$. Hence, the correct option is (A).

Quick Tip to Solve: If n vernier divisions equal $n-1$ main divisions, the least count is one MSD divided by n; convert its unit only at the end.

---

47. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v.

The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is : [4 marks]



(A) Both I_1 and I_2 are in clockwise direction

(B) I_1 is in clockwise direction and I_2 is in anticlockwise direction

(C) I_1 is in anticlockwise direction and I_2 is in clockwise direction

(D) Both I_1 and I_2 are in anticlockwise direction

Correct Answer: (B) I_1 is in clockwise direction and I_2 is in anticlockwise direction

Solution:

Concept:

- Lenz's law states that an induced current produces a magnetic field opposing the change of flux through its circuit. Clockwise current produces a field into the page; anticlockwise current produces a field out of the page.
- The moving bar forms the right boundary of the left loop containing R_1 and the left boundary of the right loop containing R_2. The uniform external magnetic field points into the page.

Step 1: Apply Lenz law to the left loop As the bar moves left, the area enclosed by the bar and the left resistor decreases. The into-page magnetic flux through this loop therefore decreases. The induced current must generate an into-page field to oppose that decrease. By the right-hand grip rule this requires a clockwise circulation around the left loop. Consequently I_1 is clockwise: in particular, it flows upward through the left resistor and downward through the moving bar.

Step 2: Apply Lenz law independently to the right loop The same leftward motion increases the area between the bar and the right resistor. The into-page external flux through this loop increases. To oppose that increase, the induced magnetic field must point out of the page, requiring an anticlockwise circulation. Thus I_2 is anticlockwise, flowing upward through the right resistor and downward through the bar. The two loop currents consequently reinforce one another in the common moving bar, which is consistent with the downward magnetic force on positive charges from $\mathbf{v} \times \mathbf{B}$. Options (A) and (D) incorrectly give both loops the same circulation; option (C) reverses both Lenz-law results.

Step 3: Conclusion I_1 is clockwise and I_2 is anticlockwise. Hence, the correct option is (B).

Quick Tip to Solve: For a sliding bar between two closed loops, compare the area change of each loop separately: one shrinks as the other expands, so their circulations are opposite.

48. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is : [4 marks]

(A) $\frac{3RT}{V}$

(B) $\frac{4RT}{V}$

(C) $\frac{88RT}{V}$

(D) $\frac{5}{2}\frac{RT}{V}$

Correct Answer: (D) $\frac{5}{2}\frac{RT}{V}$

Solution:

Concept:

- For an ideal mixture at a common temperature T in volume V, Dalton's law gives $P = \sum_i P_i$, with $P_i V = n_i RT$. Hence $PV = n_{\text{total}} RT$.
- The number of moles is mass divided by molar mass. The molar masses of O_2, N_2 and CO_2 are respectively 32, 28 and $44\,\text{g mol}^{-1}$.

Step 1: Convert each gas mass to its number of moles Use molecular rather than atomic molar masses, since oxygen and nitrogen are present as diatomic gases:

$$n_{O_2} = \frac{16}{32} = \frac{1}{2}\ \text{mol}, \qquad n_{N_2} = \frac{28}{28} = 1\ \text{mol}, \qquad n_{CO_2} = \frac{44}{44} = 1\ \text{mol}.$$

Therefore the total amount is

$$n_{\text{total}} = \frac{1}{2} + 1 + 1 = \frac{5}{2}\ \text{mol}.$$

The different molecular masses affect the conversion from mass to amount, but each mole makes the same contribution RT/V to pressure at this common temperature.

Step 2: Add the partial pressures and compare the options All three gases occupy the same enclosure volume; it is not divided into three separate volumes. Their partial pressures add to give

$$P = \frac{RT}{V}\left(\frac{1}{2} + 1 + 1\right) = \frac{5RT}{2V}.$$

Option (A) would result from treating all three samples as one mole each. Option (B) would require four moles; using atomic masses for both oxygen and nitrogen incorrectly gives $16/16+28/14+44/44 = 4$, since the first two gases are diatomic. Option (C) substitutes the total mass in grams in place of a mole count and therefore does not obey the ideal-gas law with the universal gas constant.

Step 3: Conclusion $\boxed{P = \dfrac{5RT}{2V}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a gas mixture, sum m_i/M_i, not the masses or the number of species; then use the total mole count in $PV = nRT$.

49. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If B_H = 0.4 G, the magnetic moment of the magnet is : (1G = 10^{-4} T) [4 marks]

(A) 2.880 $\times$ 10^2 J T^{-1}

(B) 2.880 J T^{-1}

(C) 2.880 $\times$ 10^3 J T^{-1}

(D) 28.80 J T^{-1}

Correct Answer: (B) 2.880 J T^{-1}

Solution:

Concept:

- With the north pole of the bar magnet toward geographic north, the magnet's equatorial field opposes Earth's horizontal field. A neutral point is where these two fields cancel.
- For a bar magnet of half-length l and magnetic moment M, its equatorial field at centre-distance r is $B_{\text{eq}} = (\mu_0/4\pi)M/(r^2 + l^2)^{3/2}$. This finite-length expression avoids the short-dipole approximation.

Step 1: Locate the neutral point and convert the data The axial field of a north-pointing magnet is in the same direction as Earth's horizontal field, so cancellation occurs on the equatorial line. The pole-to-centre distance is half the stated total length:

$$l = 7\,\text{cm} = 0.07\,\text{m}, \qquad r = 18\,\text{cm} = 0.18\,\text{m}.$$

Also,

$$B_H = 0.4 \times 10^{-4}\,\text{T} = 4.0 \times 10^{-5}\,\text{T}.$$

Since the magnet length is not negligible compared with the neutral-point distance, retain l in the equatorial-field formula.

Step 2: Equate the opposing fields and calculate the moment At the neutral point, the magnitudes are equal. Using $\mu_0/(4\pi) = 10^{-7}$ in SI units gives

$$M = \frac{B_H(r^2 + l^2)^{3/2}}{10^{-7}} = 400(0.18^2 + 0.07^2)^{3/2}.$$

Therefore

$$M = 400(0.0373)^{3/2} \simeq 2.88\,\mathrm{A\,m^2} = 2.88\,\mathrm{J\,T^{-1}}.$$

The factor 400 comes from the ratio of the field in tesla to 10^{-7}; failing to convert gauss introduces powers of ten. The other three options differ from this result by factors of ten, one hundred or one thousand and are inconsistent with the stated SI data.

Step 3: Conclusion The magnetic moment is $\boxed{2.880\,\mathrm{J\,T^{-1}}}$, to the precision of the choices. Hence, the correct option is (B).

Quick Tip to Solve: When a magnet's length is given and is appreciable, retain its half-length in $(r^2 + l^2)^{3/2}$; do not replace this factor by r^3.

50. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be : [4 marks]

(A) 200

(B) 400

(C) 100

(D) 300

Correct Answer: (C) 100

Solution:

Concept:

- For an efficient quarter-wave transmitting antenna, the radiating length is one quarter of the wavelength: $l = \lambda/4$. Here l means the conducting antenna length, not the supporting tower height.
- The support raises the antenna above the ground and can affect line-of-sight coverage. Its height does not add to the electrical length used in the quarter-wave wavelength relation.

Step 1: Identify the radiating length The problem gives a 25 m long antenna mounted on a 75 m tower. The 25 m member is the antenna that carries the oscillating current and radiates the signal. Thus the length to substitute in the quarter-wave relation is $l = 25\,\mathrm{m}$. Adding the tower height would instead describe the elevation of the top above the ground, which is a different quantity.

Step 2: Calculate the wavelength Rearrange the antenna condition before substituting the given length:

$$\lambda = 4l = 4(25) = 100\,\mathrm{m}.$$

The factor of four converts the quarter-wavelength radiating element into one complete wavelength. The tower height of 75 m is unnecessary for this calculation; it would be relevant to a question about transmission range.

Step 3: Compare the options Option (C) gives 100 m, which has a quarter wavelength of exactly 25 m. A wavelength of 200 m would require a 50 m quarter-wave antenna, 400 m would require 100 m, and 300 m would require 75 m. Those lengths do not equal the stated radiating length. In particular, 400 m results from incorrectly using the combined 100 m elevation.

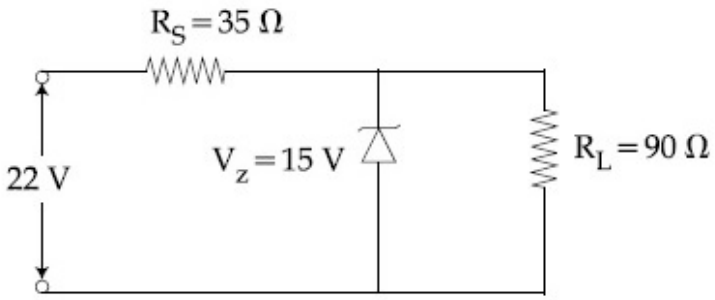
Step 4: Conclusion The transmitted wavelength is $\boxed{100\,\text{m}}$. Hence, the correct option is (C).

Quick Tip to Solve: In an antenna-length question, first separate the radiating element from its support; use $\lambda = 4l$ for the quarter-wave element.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The value of power dissipated across the zener diode (V_z = 15V) connected in the circuit as shown in the figure is x $\times$ 10^{-1} watt.
The value of x, to the nearest integer, is _____. [4 marks]



Solution:

Concept:

- A reverse-biased Zener diode operating in breakdown maintains the load voltage at V_Z. The series resistor therefore carries $I_S = (V_{\text{in}} - V_Z)/R_S$.
- At the output junction, Kirchhoff's current law gives $I_Z = I_S - I_L$, where $I_L = V_Z/R_L$. The Zener power is $P_Z = V_Z I_Z$, not the total power delivered to the parallel combination.

Step 1: Find the current supplied through the series resistor From the circuit, $V_{\text{in}} = 22\,\text{V}$, $R_S = 35\,\Omega$, $R_L = 90\,\Omega$ and $V_Z = 15\,\text{V}$. Taking the Zener in breakdown, the voltage drop across the series resistor is $22 - 15 = 7\,\text{V}$. Hence

$$I_S = \frac{7}{35} = \frac{1}{5}\,\text{A}.$$

This is the total current arriving at the load and Zener junction.

Step 2: Subtract the load current The load resistor is parallel to the Zener, so it also has 15 V across it. Thus

$$I_L = \frac{15}{90} = \frac{1}{6}\,\text{A}, \qquad I_Z = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}\,\text{A}.$$

The positive value confirms that the assumed breakdown state can be maintained. The full series current must not be assigned to the diode, since a substantial fraction flows through the load.

Step 3: Calculate the diode power and the required coefficient The diode dissipates power equal to

its voltage multiplied by its own current:

$$P_Z = 15\left(\frac{1}{30}\right) = 0.5\,\text{W} = 5 \times 10^{-1}\,\text{W}.$$

Comparing this result with $x \times 10^{-1}$ W gives $x = 5$. The value is already an integer, so the requested rounding does not change it.

Step 4: Conclusion The Zener power is 0.5 W, so the requested value is $\boxed{x = 5}$.

Quick Tip to Solve: For a loaded Zener regulator, subtract the load current from the series current before multiplying by the Zener voltage.

---

52. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm. The value of 'x' to the nearest integer is _____.

[4 marks]

Solution:

Concept:

- The fringe width in Young's double-slit experiment is $\beta = \lambda D/d$, where d is the slit separation and D is the distance to the screen.
- Use a consistent set of units before finding λ. The required conversion is $1\,\text{nm} = 10^{-9}\,\text{m}$; fringe width is the separation of adjacent bright fringes, rather than the position of a particular order.

Step 1: Convert the measured distances to metres The given fringe width is $\beta = 6\,\text{mm} = 6 \times 10^{-3}\,\text{m}$, the slit separation is $d = 1\,\text{mm} = 10^{-3}\,\text{m}$, and the screen distance is $D = 10\,\text{m}$. Both millimetre quantities need conversion. The fact that they share a unit does not allow their powers of ten to be discarded when multiplied.

Step 2: Rearrange the fringe-width relation Since the wavelength is the unknown, multiply the fringe-width equation by d/D:

$$\lambda = \frac{\beta d}{D} = \frac{(6 \times 10^{-3})(10^{-3})}{10}\,\text{m} = 6 \times 10^{-7}\,\text{m}.$$

The product in the numerator has units of square metres; division by the screen distance restores metres, as required for a wavelength.

Step 3: Express the answer in nanometres Convert metres to nanometres by multiplying the numerical value by 10^9:

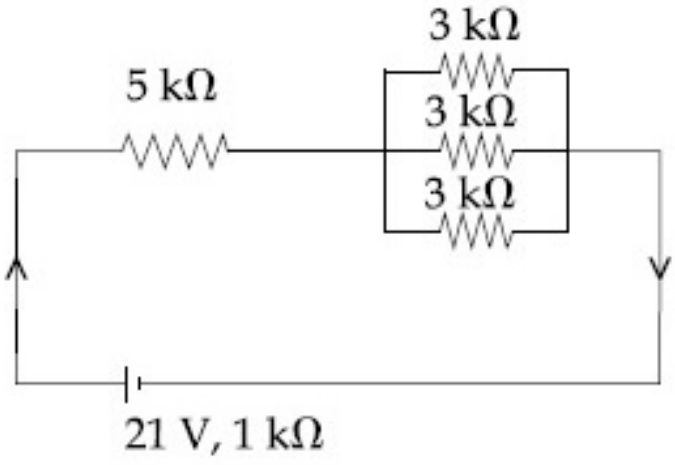
$$\lambda = (6 \times 10^{-7})(10^9)\,\text{nm} = 600\,\text{nm}.$$

This lies in the visible range and is consistent with an optical double-slit experiment. Since the question writes the wavelength as x nm, the coefficient to enter is 600; no extra power of ten accompanies the numerical response.

Step 4: Conclusion The wavelength is $600\,\text{nm}$. Therefore, to the nearest integer, $\boxed{x = 600}$.

Quick Tip to Solve: Write $\lambda = \beta d/D$ first, convert both millimetre distances to metres, then convert the final wavelength to nanometres.

53. In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA. The value of x to the nearest integer is _____. [4 marks]



Solution:

Concept:

- Resistors in parallel have a common potential difference and add through reciprocal resistance: $1/R_p = \sum_i 1/R_i$. Three identical resistors of resistance r give $R_p = r/3$.
- The battery's internal resistance is in series with the external network. The current through the external 5 kilohm resistor is the total battery current, $I = \mathcal{E}/(R_{\text{ext}} + r_{\text{int}})$.

Step 1: Reduce the parallel combination The figure shows three $3\,\text{k}\Omega$ resistors connected between the same two junctions. Thus

$$\frac{1}{R_p} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \quad \text{in } (\text{k}\Omega)^{-1}, \qquad R_p = 1\,\text{k}\Omega.$$

These resistors are parallel, despite their vertical arrangement in the drawing; each begins and ends on the same pair of conductors.

Step 2: Include all series resistances The external 5 kilohm resistor lies before the junction where current divides, so its current is the total circuit current. The battery is labelled 21 V with internal resistance $1\,\text{k}\Omega$. Therefore

$$R_{\text{total}} = 5 + 1 + 1 = 7\,\text{k}\Omega.$$

The last term is the internal resistance. Ignoring it would give a different current and would omit a stated circuit element.

Step 3: Calculate and check the current Apply Ohm's law to the entire circuit:

$$I = \frac{21\,\text{V}}{7\,\text{k}\Omega} = 3\,\text{mA}.$$

The parallel network has a 3 V drop and each 3 kilohm branch carries 1 mA, so their currents sum to 3 mA. The drops across the 5 kilohm resistor, the parallel group and the internal resistance are 15 V, 3 V and 3 V, respectively; they sum to the 21 V emf.

Step 4: Conclusion The current through the $5\,\mathrm{k\Omega}$ resistor is $3\,\mathrm{mA}$, hence $\boxed{x = 3}$.

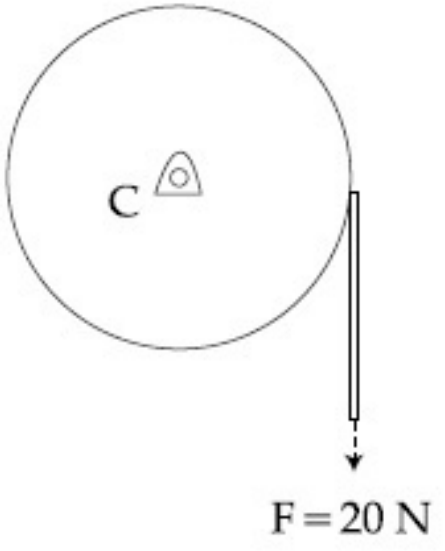
Quick Tip to Solve: When a cell is labelled with both voltage and resistance, include that internal resistance in the series total; volts divided by kilohms gives milliamperes.

54. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F = 20 N through a massless string wrapped around is periphery as shown in the figure.

Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}.

The value of n, to the nearest integer, is _____.

[Given : In one complete revolution, the disk rotates by 6.28 rad] [4 marks]



Solution:

Concept:

- A uniform disk rotating about its central fixed axis has moment of inertia $I = MR^2/2$. A tangential force at its rim exerts torque $\tau = FR$.
- For a constant torque, rotational work is $W = \tau\theta$, with θ in radians. Starting from rest, this equals the final rotational kinetic energy $I\omega^2/2$.

Step 1: Determine the inertia and torque Use $M = 20\,\mathrm{kg}$ and $R = 0.2\,\mathrm{m}$ for the disk:

$$I = \frac{1}{2}(20)(0.2)^2 = 0.4\,\mathrm{kg\,m^2}.$$

The massless string pulls tangentially with constant force 20 N, giving

$$\tau = 20(0.2) = 4\,\mathrm{N\,m}.$$

The central pin prevents translation and exerts no torque about the rotational axis, so only the string

supplies rotational work.

Step 2: Find the angular displacement from energy The disk begins at rest and reaches $\omega = 50\,\mathrm{rad\,s^{-1}}$. Equating work and change in rotational kinetic energy gives

$$4\theta = \frac{1}{2}(0.4)(50)^2 = 500\,\mathrm{J}, \qquad \theta = 125\,\mathrm{rad}.$$

Equivalently, $\alpha = \tau/I = 10\,\mathrm{rad\,s^{-2}}$ and $\omega^2 = 2\alpha\theta$ reproduce the same displacement. Both approaches treat the disk as a solid disk rather than a ring.

Step 3: Convert radians to revolutions The question specifies 6.28 rad in one revolution. Consequently

$$n = \frac{125}{6.28} = 19.904\ldots.$$

This is the total angular displacement divided by the angular displacement per revolution. The requested nearest integer is therefore 20. Rounding the intermediate angular displacement or replacing the disk inertia by MR^2 would change the result incorrectly.

Step 4: Conclusion The disk turns about 19.90 revolutions; to the nearest integer, $\boxed{n = 20}$.

Quick Tip to Solve: For a fixed-axis disk pulled at its rim, set $FR\theta = I\omega^2/2$, then divide the resulting radians by the stated radians per revolution.

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55. The resistance R = $\frac{V}{I}$, where V = (50 ± 2)V and I = (20 ± 0.2)A. The percentage error in R is 'x'%. The value of 'x' to the nearest integer is _____. [4 marks]

Solution:

Concept:

- For a quotient $R = V/I$, the maximum fractional errors add: $\Delta R/R = \Delta V/V + \Delta I/I$. The uncertainty magnitudes are positive even though current occurs in the denominator.
- A percentage error equals the fractional error multiplied by 100. Maximum-error propagation uses a sum of magnitudes, rather than subtracting errors or combining them in quadrature.

Step 1: Identify the measured values and absolute errors The central voltage and its error are $V = 50\,\mathrm{V}$ and $\Delta V = 2\,\mathrm{V}$. For current, $I = 20\,\mathrm{A}$ and $\Delta I = 0.2\,\mathrm{A}$. Thus the relative errors are

$$\frac{\Delta V}{V} = \frac{2}{50} = 0.04, \qquad \frac{\Delta I}{I} = \frac{0.2}{20} = 0.01.$$

They are dimensionless because numerator and denominator have matching units.

Step 2: Add the fractional errors for the quotient Using the maximum fractional-error rule,

$$\frac{\Delta R}{R} = 0.04 + 0.01 = 0.05.$$

Although differentiating V/I gives a negative coefficient for a signed current variation, the largest possible resistance error occurs when the individual effects reinforce one another. This is why the

uncertainty magnitudes add. A subtraction would describe only one particular combination of signed deviations, not the maximum error asked in this context.

Step 3: Convert to percentage error Multiply by 100 to obtain

$$\text{percentage error in } R = 0.05 \times 100 = 5\%.$$

For a check, the nominal resistance is $R = 50/20 = 2.5\,\Omega$, giving the first-order absolute uncertainty $\Delta R = 0.05(2.5) = 0.125\,\Omega$. The numerical blank asks for the percentage coefficient x, rather than this absolute uncertainty.

Step 4: Conclusion The percentage error is 5%, so the nearest-integer response is $\boxed{x = 5}$.

Quick Tip to Solve: For maximum error in a product or quotient, add the relative uncertainty magnitudes; convert the sum to a percentage only at the end.

56. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which R = 8Ω, L = 24 mH and C = 60 μF. The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is _____. [4 marks]

Solution:

Concept:

- At series resonance the inductive and capacitive reactances cancel: $X_L = X_C$. Hence the impedance becomes $Z = R$ and voltage and current have zero phase difference.
- For a sinusoidal voltage with peak value V_0, the rms voltage is $V_{\text{rms}} = V_0/\sqrt{2}$. The average dissipated power at resonance is $P = V_{\text{rms}}^2/R = V_0^2/(2R)$.

Step 1: Use the resonant condition to simplify the impedance The resistor is $R = 8\,\Omega$, while $L = 24\,\text{mH}$ and $C = 60\,\mu\text{F}$. In a series circuit,

$$Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}.$$

At resonance the quantity in parentheses vanishes, leaving $Z = 8\,\Omega$. The supplied inductance and capacitance determine the resonant frequency, but the resonant average power depends only on the resistance and applied rms voltage.

Step 2: Convert the peak voltage to an rms value The stated 250 V is a peak value, not an rms value. Therefore

$$V_{\text{rms}} = \frac{250}{\sqrt{2}}\,\text{V}, \qquad V_{\text{rms}}^2 = \frac{250^2}{2} = 31250\,\text{V}^2.$$

The factor of one half is essential because the time average of the square of a sinusoidal voltage is half its peak square. Using 250 V directly as an rms voltage would double the predicted power.

Step 3: Calculate the mean power and round in kilowatts The power factor is unity at resonance. Thus

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{31250}{8} = 3906.25\,\text{W} = 3.90625\,\text{kW}.$$

The ideal inductor and capacitor exchange energy with the source and do not dissipate average power. Only the resistor contributes to the quoted power loss. Comparing with x kW and rounding the kilowatt value, rather than the watt value, gives 4.

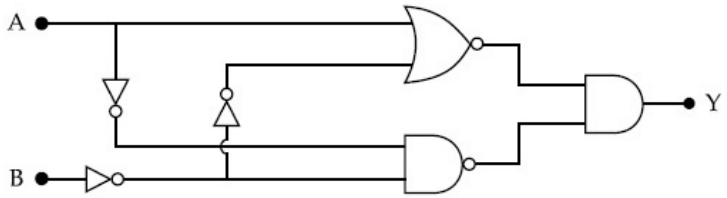
Step 4: Conclusion The average power is $3.90625\,\text{kW}$, giving $\boxed{x = 4}$ to the nearest integer.

Quick Tip to Solve: At series resonance use $P = V_0^2/(2R)$ for a peak voltage; the factor of two distinguishes peak data from rms data.

---

57. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'.

The value of x is _____. [4 marks]



Solution:

Concept:

- A NOT gate reverses its input: $\overline{0} = 1$ and $\overline{1} = 0$. The small circle on a gate output denotes inversion.
- A NOR gate gives $\overline{u + v}$, a NAND gate gives $\overline{uv}$, and the final AND gate gives the product of its two inputs. Here + denotes logical OR.

Step 1: Trace the input inversions The two specified inputs are $A = 0$ and $B = 1$. The lower NOT gate changes B to $\overline{B} = 0$. This signal enters the lower NAND gate and also passes through the upward NOT gate, so the second input of the upper NOR gate is $\overline{\overline{B}} = B = 1$. The NOT gate on the branch from A supplies $\overline{A} = 1$ to the other input of the lower NAND gate. The crossing drawn with a bridge is not a junction; it must not join the A and B branches.

Step 2: Evaluate the two intermediate outputs The upper NOR gate receives 0 and 1. Its OR operation first gives 1, and inversion therefore gives $U = \overline{0 + 1} = 0$. The lower NAND gate receives 1 and 0. Their AND is zero, so the inverted output is $L = \overline{1 \cdot 0} = 1$. These are the inputs of the last gate; the inversions must be applied before that final operation.

Step 3: Apply the final AND operation The rightmost gate has no output bubble and is an AND gate. Consequently

$$Y = U \cdot L = 0 \cdot 1 = 0.$$

An AND gate requires both inputs to be high, whereas the upper input here is low. The output is therefore low even though the lower intermediate output is high.

Step 4: Conclusion The output is $Y = 0$, so the required numerical value is $\boxed{x = 0}$.

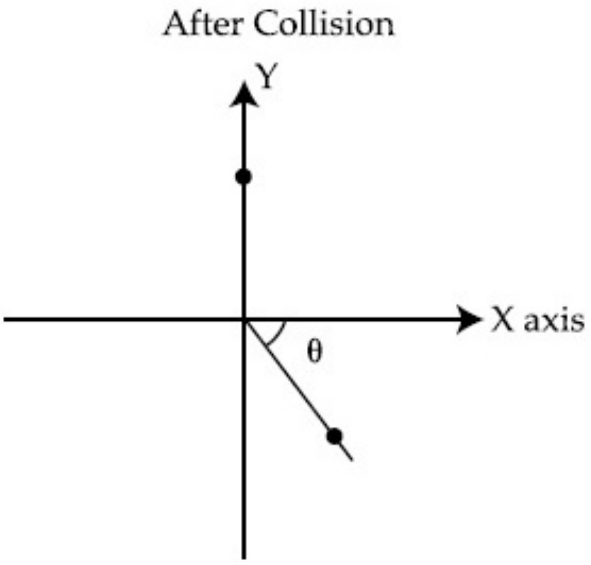
Quick Tip to Solve: Evaluate gates from left to right and write a value on each wire; apply every output bubble before using that output in the next gate.

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58. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m s^{-1} along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure.

The value of θ to the nearest integer is _____. [4 marks]



Solution:

Concept:

- In a collision with negligible external impulse, the total vector momentum is conserved. Momentum must be balanced separately along the X and Y directions.
- The initially stationary 20 kg ball splits into equal fragments, so each fragment has mass $10\,\text{kg}$. The first 10 kg ball is at rest after impact and contributes no final momentum.

Step 1: Write the initial momentum Initially only the incoming ball moves, and its velocity is along the positive X-axis. Therefore

$$p_{x,i} = 10(10\sqrt{3}) = 100\sqrt{3}\,\text{kg m s}^{-1}, \qquad p_{y,i} = 0.$$

The fragment moving along positive Y has momentum $10 \times 10 = 100\,\text{kg m s}^{-1}$. The second fragment moves below the positive X-axis as shown; its Y component is negative.

Step 2: Balance the transverse momentum For the second fragment, whose speed is $20\,\text{m s}^{-1}$, the velocity components are $20\cos\theta$ and $-20\sin\theta$. Conservation in the Y direction gives

$$0 = 10(10) - 10(20\sin\theta).$$

Thus

$$\sin\theta = \frac{100}{200} = \frac{1}{2}.$$

The geometry shown places the fragment in the fourth quadrant, so the angle below the positive

X-axis is acute.

Step 3: Check the longitudinal component The X-momentum equation independently requires

$$100\sqrt{3} = 10(20\cos\theta), \qquad \cos\theta = \frac{\sqrt{3}}{2}.$$

Both sine and cosine therefore give $\theta = 30^\circ$; the supplementary sine solution does not satisfy the positive X-momentum. Kinetic energy conservation is not required because fragmentation may change the kinetic energy.

Step 4: Conclusion The angle is $\boxed{\theta = 30^\circ}$. Its nearest-integer numerical value is $\boxed{30}$.

Quick Tip to Solve: For a fragment emitted perpendicular to the initial motion, first cancel the transverse momenta; then use the longitudinal equation to select the correct quadrant.

---

59. The first three spectral lines of H-atom in the Balmer series are given λ_1, λ_2, λ_3 considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x' $\times$ 10^{-1}.

The value of x, to the nearest integer, is _____. [4 marks]

Solution:

Concept:

- The Balmer series consists of transitions ending at $n = 2$. Its first, second and third lines originate from $n = 3, 4, 5$, respectively.
- The Rydberg relation for hydrogen is $1/\lambda = R(1/2^2 - 1/n^2)$, where R is the Rydberg constant. Wavelength is the reciprocal of the corresponding wavenumber.

Step 1: Identify the two transitions The first line is the $3 \to 2$ transition, so

$$\frac{1}{\lambda_1} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}.$$

The third line is the $5 \to 2$ transition, not the $4 \to 2$ transition. Therefore

$$\frac{1}{\lambda_3} = R\left(\frac{1}{4} - \frac{1}{25}\right) = \frac{21R}{100}.$$

Both transitions have the same final energy level, allowing the constant R to cancel.

Step 2: Form the wavelength ratio Because wavelength is inversely proportional to wavenumber, the order of the wavenumbers must be reversed when forming the wavelength ratio. Thus

$$\frac{\lambda_1}{\lambda_3} = \frac{21R/100}{5R/36} = \frac{756}{500} = 1.512.$$

The first Balmer line has the smaller photon energy and hence the longer wavelength, so a ratio greater than one is physically consistent.

Step 3: Extract the requested coefficient The question expresses this ratio as $x \times 10^{-1}$. Hence

$$x \times 10^{-1} = 1.512, \qquad x = 15.12.$$

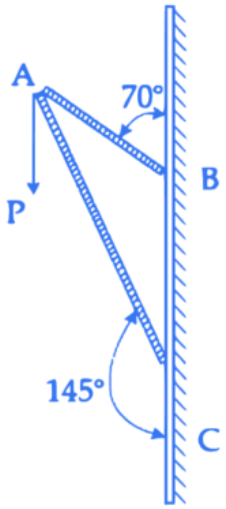
Rounding applies to x, rather than to the wavelength ratio before multiplication by ten. The nearest integer to 15.12 is 15; rounding the ratio itself first would lose the precision required by the question.

Step 4: Conclusion The wavelength ratio is $1.512 = 15.12 \times 10^{-1}$. Therefore the required nearest-integer value is $\boxed{x = 15}$.

Quick Tip to Solve: List the starting levels 3, 4, 5 for the first three Balmer lines, and invert the wavenumber ratio when calculating a wavelength ratio.

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60. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame.
Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame.
The magnitude of the resolved component along the arm AC is xN.
The value of x, to the nearest integer, is _____.
[Given : sin(35°) = 0.573, cos(35°) = 0.819
sin(110°) = 0.939, cos(110°) = − 0.342 J [4 marks]



Solution:

Concept:

- Components along two prescribed oblique directions must add vectorially to the given force; they are not obtained by independent perpendicular projections.
- The diagram makes AB inclined at $20°$ below the positive horizontal, since its angle to the upward vertical at B is $70°$. AC is inclined at $55°$ below the horizontal, since its angle to the downward vertical at C is $145°$.

Step 1: Assign signed components along the rods Let F be the signed component in the direction from A to B, and G the component from A to C. The vertical force has components $(0, -100)\,\mathrm{N}$. Balancing horizontal and vertical components gives

$$F\cos 20^\circ + G\cos 55^\circ = 0,$$

$$F\sin 20^\circ + G\sin 55^\circ = 100.$$

The negative value subsequently obtained for F means its vector points opposite to AB; allowing that sign is essential for oblique resolution.

Step 2: Eliminate the component along AB From the horizontal equation, $F = -G\cos 55^\circ/\cos 20^\circ$. Substitution in the vertical equation gives

$$G\left(\sin 55^\circ - \cos 55^\circ \tan 20^\circ\right) = 100.$$

Using the sine-difference identity, the bracket is $\sin 35^\circ/\cos 20^\circ$. Thus

$$G = \frac{100\cos 20^\circ}{\sin 35^\circ}.$$

This result follows from the printed angles without assuming the two rods are perpendicular.

Step 3: Evaluate and compare with the stated key Using $\cos 20^\circ \simeq 0.9397$ and the supplied $\sin 35^\circ = 0.573$,

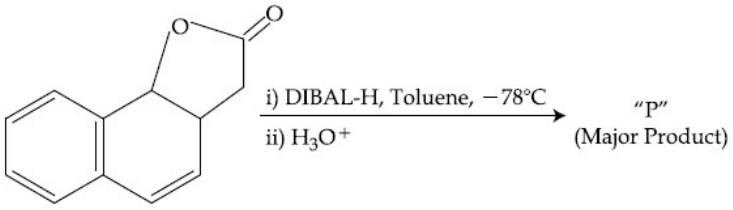
$$G \simeq \frac{93.97}{0.573} = 164.0\,\mathrm{N}.$$

Its horizontal contribution is cancelled by the opposite-directed AB component, and their downward resultant is 100 N. A component of 82 N would give only about 50 N downward after this horizontal cancellation, so it does not resolve the printed 100 N force.

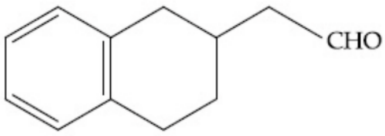
Step 4: Conclusion For the force and angles shown, the component along AC is approximately $\boxed{164\,\mathrm{N}}$, and $\boxed{x = 164}$. The supplied answer key gives 82, which is inconsistent with the printed 100 N force and the displayed rod angles.

Quick Tip to Solve: When resolving along oblique rods, write two Cartesian equations and permit a signed component to be negative; the usual perpendicular-component cosine rule is insufficient.

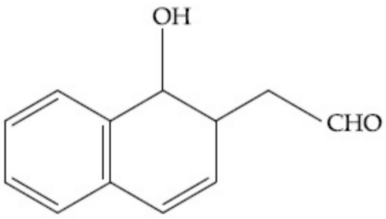
61. The product "P" in the above reaction is : [4 marks]



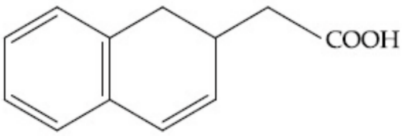
(A) The fused-ring product shown in (A): the non-aromatic ring is saturated, has no hydroxyl group, and bears a $-CH_2CHO$ substituent.



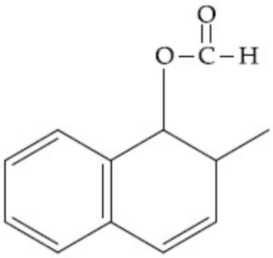
(B) The fused-ring hydroxy aldehyde shown in (B): the original non-aromatic carbon-carbon double bond is retained, the former ring oxygen position gives $-OH$, and the adjacent carbon bears $-CH_2CHO$.



(C) The fused-ring product shown in (C): the original non-aromatic carbon-carbon double bond is retained, no hydroxyl group is present, and the substituent is $-CH_2COOH$.

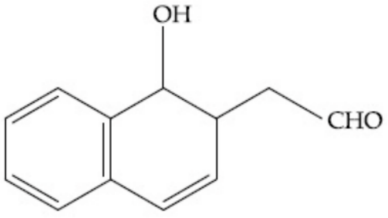


(D) The fused-ring product shown in (D): the original non-aromatic carbon-carbon double bond is retained, and the adjacent positions bear a methyl group and a formate ester $-OCHO$.



Correct Answer: (B)

The fused-ring hydroxy aldehyde shown in (B): the original non-aromatic carbon-carbon double bond is retained, the former ring oxygen position gives $-OH$, and the adjacent carbon bears $-CH_2CHO$.



Solution:

Concept:

- DIBAL-H is diisobutylaluminium hydride. At low temperature it partially reduces an ester carbonyl to the aldehyde oxidation level after hydrolytic work-up.
- A lactone is a cyclic ester. Its partial reduction gives a lactol in equilibrium with the corresponding open-chain hydroxy aldehyde; the carbon framework and an isolated carbon-carbon double bond are retained.

Step 1: Locate the reducible functional group The starting compound contains a five-membered lactone fused to the non-aromatic ring. Its carbonyl carbon is bonded to the ester oxygen on one side and to a methylene carbon on the other. DIBAL-H in toluene at $-78^{\circ}C$ attacks this carbonyl selectively. These are controlled partial-reduction conditions; they do not indicate catalytic hydrogenation of either the aromatic ring or the other carbon-carbon double bond.

Step 2: Follow the two ester fragments during work-up After hydride addition and acidic hydrolysis, opening the lactone converts the former carbonyl carbon into $-CHO$, leaving its bond to the adjacent methylene intact. The carbon attached to the original ester oxygen acquires $-OH$. Consequently the adjacent ring carbon bears $-CH_2CHO$, while the benzylic ring position bears OH. The non-aromatic double bond stays where it was in the starting skeleton.

Step 3: Compare all the products Option (B) has exactly the hydroxy aldehyde framework and retains the alkene. Option (A) incorrectly loses the hydroxyl group and saturates the non-aromatic ring. Option (C) gives a carboxylic acid, corresponding to a different transformation rather than partial hydride reduction, and also lacks the hydroxyl. Option (D) gives a formate ester and methyl group, which would require a different bond-cleavage pattern. None follows the normal ester-carbonyl reduction pathway.

Step 4: Conclusion The major product is the hydroxy aldehyde represented by option (B). Hence, the correct option is (B).

Quick Tip to Solve: For a lactone treated with cold DIBAL-H, open the ester linkage mentally: the carbonyl end becomes CHO and the oxygen-bearing end becomes OH; retain the rest of the skeleton.

62. Given below are two statements : one is labeled as Assertion A and the other is labelled as Reason R :

Assertion A : The H−O−H bond angle in water molecule is 104.5°.

Reason R : The lone pair - lone pair repulsion of electrons is higher than the bond pair - bond pair repulsion.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both A and R are true, and R is the correct explanation of A

(B) A is true but R is false

(C) A is false but R is true

(D) Both A and R are true, but R is not the correct explanation of A

Correct Answer: (A) Both A and R are true, and R is the correct explanation of A

Solution:

Concept:

- The oxygen atom in H_2O has four valence-shell electron pairs: two bonding pairs and two lone pairs. Their electron-pair arrangement is approximately tetrahedral.
- The VSEPR repulsion order is lone pair-lone pair greater than lone pair-bond pair greater than bond pair-bond pair. Lone pairs occupy more angular space than bonding pairs.

Step 1: Check Assertion A A regular tetrahedral arrangement has an angle close to $109.5°$. Water is not a regular tetrahedron of four identical bonds: two of the four electron domains are lone pairs. Its molecular shape is bent, and the measured H-O-H bond angle is approximately $104.5°$. Assertion A therefore states the correct experimental bond angle; the electron-pair geometry must not be confused with the shape defined by the atomic nuclei.

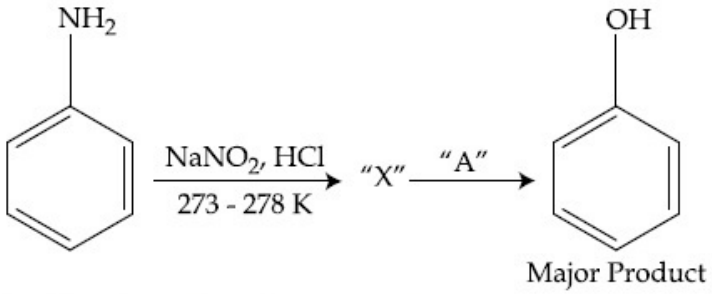
Step 2: Check Reason R A lone pair is localized nearer the central oxygen than the shared electron density of a bond pair. It consequently occupies more space around oxygen and produces stronger electron-pair repulsion. In particular, lone pair-lone pair repulsion exceeds bond pair-bond pair repulsion, as stated in R. Reason R is therefore also true; neither option claiming one of the two statements is false can be selected.

Step 3: Connect the repulsion to the smaller angle The two lone pairs exert greater angular demands than the O-H bonding pairs. The bonding pairs are pushed closer to one another, reducing the angle between the two O-H bonds below the ideal tetrahedral value. The enhanced lone-pair repulsion expressed in R is thus the VSEPR basis for the compressed angle in A. The qualitative model explains the reduction, while the particular numerical value is the observed water bond angle.

Step 4: Conclusion Both Assertion A and Reason R are true, and R supplies the VSEPR explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: Count lone pairs before using the ideal hybridisation angle: two lone pairs compress the H-O-H angle below the tetrahedral value.

63. In the above chemical reaction, intermediate "X" and reagent/condition "A" are : [4 marks]



(A) $X = C_6H_5NO_2$; $A = H_2O$, heat.

(B) $X = C_6H_5N_2{}^+Cl^-$; $A = H_2O/NaOH$.

(C) $X = C_6H_5NO_2$; $A = H_2O/NaOH$.

(D) $X = C_6H_5N_2{}^+Cl^-$; $A = H_2O$, heat.

Correct Answer: (D) $X = C_6H_5N_2{}^+Cl^-$; $A = H_2O$, heat.

Solution:

Concept:

- A primary aromatic amine reacts with nitrous acid at 273–278 K to form an aromatic diazonium salt. Nitrous acid is generated in situ from $NaNO_2$ and HCl.
- Warming aqueous benzenediazonium chloride replaces the diazonium group with hydroxyl, liberating nitrogen and giving phenol. This is hydrolysis of the diazonium salt.

Step 1: Identify intermediate X The amino group of aniline undergoes diazotisation under the stated cold acidic conditions. The overall reaction is

$$C_6H_5NH_2 + NaNO_2 + 2\,HCl \longrightarrow C_6H_5N_2{}^+Cl^- + NaCl + 2\,H_2O.$$

Thus X is benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$. The low temperature stabilizes this reactive intermediate sufficiently for the subsequent substitution. Diazotisation is not nitration, so nitrobenzene is not formed in this step.

Step 2: Choose the hydrolysis condition Heating the diazonium salt with water produces phenol, the displayed final product:

$$C_6H_5N_2{}^+Cl^- + H_2O \xrightarrow{\Delta} C_6H_5OH + N_2 + HCl.$$

Nitrogen is an excellent leaving group, and its escape drives replacement by hydroxyl. Hence the condition A is water and heat. The oxygen in phenol is introduced during this hydrolytic transformation, rather than being supplied by a direct oxidation of the amino group.

Step 3: Eliminate the competing pairs Options (A) and (C) incorrectly identify X as nitrobenzene, so neither can follow from aniline under the stated diazotisation conditions. Option (B) identifies the diazonium salt but pairs it with aqueous sodium hydroxide instead of the standard warm-water hydrolysis; strongly alkaline conditions can form diazotate species. Option (D) correctly supplies both benzenediazonium chloride and water with heating, matching the two-step conversion to phenol.

Step 4: Conclusion X is benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$, and A is H_2O with heat. Hence, the correct option is (D).

Quick Tip to Solve: Cold nitrite plus mineral acid signals diazotisation; warm water then replaces the diazonium group with OH and releases nitrogen.

64. Assertion A : Enol form of acetone [CH_3COCH_3] exists in $<$ 0.1% quantity. However, the enol form of acetyl acetone [$CH_3COCH_2OCCH_3$] exists in approximately 15% quantity.

Reason R : Enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the correct statement : [4 marks]

(A) Both A and R are true but R is not the correct explanation of A

(B) A is true but R is false

(C) A is false but R is true

(D) Both A and R are true and R is the correct explanation of A

Correct Answer: (D) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Keto-enol tautomerism is an equilibrium between a carbonyl compound and its enol, which contains an alkene bonded to an OH group. Greater stabilization of the enol increases its equilibrium proportion.
- An intramolecular hydrogen bond can stabilize an enol when a second suitably placed oxygen atom accepts the hydrogen bond. Conjugation also stabilizes the enol of a beta-dicarbonyl compound.

Step 1: Evaluate the assertion Acetone strongly favours its keto form. Its simple enol has no second carbonyl oxygen to provide chelation, and forming the enol sacrifices the favourable carbonyl bond without comparable extra stabilization. Consequently its enol proportion is below 0.1%. Acetylacetone is a beta-diketone, $CH_3COCH_2COCH_3$; its much larger enol proportion, about 15% in the stated comparison, is consistent with extra stabilization. Assertion A is true.

Step 2: Evaluate the reason In the enol of acetylacetone, the hydroxyl hydrogen and the oxygen of the remaining carbonyl group form an intramolecular $O-H\cdot\cdot O$ hydrogen bond. The favourable six-membered chelate arrangement stabilizes that tautomer. Acetone has only one oxygen, so the corresponding interaction cannot occur in its enol. Reason R is therefore true.

Step 3: Connect the two statements The hydrogen bond selectively lowers the energy of the acetylacetone enol and shifts its keto-enol equilibrium towards that enol relative to acetone. Thus R explains the difference described in A. Options claiming a false statement, or denying the explanatory connection, do not fit this stabilization.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (D).

Quick Tip to Solve: For beta-dicarbonyl enols, look for a six-membered intramolecular hydrogen bond as well as conjugation before comparing enol contents.

65. Given below are two statements :

Statement I : The E^o value for Ce^{4+} / Ce^{3+} is + 1.74 V.

Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is incorrect but statement II is correct

(B) Both statement I and statement II are correct

(C) Both statement I and statement II are incorrect

(D) Statement I is correct but statement II is incorrect

Correct Answer: (D) Statement I is correct but statement II is incorrect

Solution:

Concept:

- A standard reduction potential refers to the reaction as written in the reduction direction. For cerium it is $Ce^{4+} + e^- \longrightarrow Ce^{3+}$.
- A large positive reduction potential indicates a strong tendency of the oxidized species to accept electrons. The common stable oxidation state of lanthanoids in aqueous solution is +3.

Step 1: Read the stated potential The standard reduction potential of the Ce^{4+}/Ce^{3+} couple is $+1.74\,V$, as given in Statement I. This is the accepted value used for this comparison, so Statement I is correct. The positive sign is essential: the stated reaction accepts, rather than releases, an electron.

Step 2: Interpret the stability claim The high positive value means that Ce^{4+} readily undergoes reduction to Ce^{3+}. Thus cerium(IV) acts as an oxidizing agent and cerium(III) is the more stable state in this aqueous redox comparison. An empty 4f subshell in Ce^{4+} can explain the existence of +4 compounds, but does not reverse the experimentally indicated redox tendency. Statement II is incorrect.

Step 3: Choose the matching pair The required combination is true for I and false for II. Option A reverses these truth values; option B incorrectly accepts the stability claim, while option C rejects the correct potential. Comparing both statements separately prevents confusing the existence of the +4 oxidation state with its greater stability in solution.

Step 4: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Write a metal-ion couple as an electron-accepting reaction first: a strongly positive reduction potential identifies the higher oxidation state as an oxidant.

66. In chromotography technique, the purification of compound is independent of : [4 marks]

(A) Solubility of the compound

(B) Mobility or flow of solvent system

(C) Physical state of the pure compound

(D) Length of the column or TLC plate

Correct Answer: (C) Physical state of the pure compound

Solution:

Concept:

- Chromatography separates dissolved components by their different distributions between a stationary phase and a moving mobile phase. Different affinities produce different migration rates.
- The separation depends on adsorption or partition equilibria, solvent properties and the distance and time available for components to move. It does not require the isolated solute to have a particular physical state.

Step 1: Identify the property that controls migration A component that interacts strongly with the stationary phase remains there longer, while a component better accommodated by the mobile phase moves farther. The sample is introduced in a suitable solvent, so a pure compound that was originally a solid or liquid can still be treated by the same separation principle. Its bulk physical state before dissolution is not the basis of chromatographic purification.

Step 2: Check the solvent-related alternatives Solubility matters because the compound must be carried by the mobile phase and partition between phases. The solvent system and its mobility or flow influence how fast material passes through the stationary phase and how well neighbouring bands resolve. Therefore neither option A nor option B describes an independent factor.

Step 3: Check the available separation distance The length of a column or TLC plate supplies the distance over which differences in migration accumulate. It affects the scope for separation and the resolution achieved, so option D cannot be selected either. By contrast, after dissolution, the pure compound's original physical state does not determine the chromatographic mechanism.

Step 4: Conclusion Purification by chromatography is independent of the physical state of the pure compound. Hence, the correct option is (C).

Quick Tip to Solve: Focus on solute interactions with the two phases and the migration conditions; the solid or liquid state of the isolated compound is incidental.

67. A group of 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is : [4 marks]

(A) As

(B) Sb

(C) Bi

(D) P

Correct Answer: (C) Bi

Solution:

Concept:

- Group 15 forms hydrides of the type EH_3. Moving down the group increases atomic size and weakens the E-H bonds, decreasing hydride thermal stability.
- The reducing power of these hydrides generally increases down the group because the less stable hydrides are more readily oxidized or decomposed. Bismuth is the metallic member among the listed elements.

Step 1: Use the hydride trend For the group 15 hydrides the usual reducing-power order is

$$NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3.$$

The larger central atom overlaps less effectively with the hydrogen orbital, so the E-H bond becomes weaker on descending the group. A hydride that more readily loses hydrogen or is oxidized acts as a stronger reducing agent; thus the lowest member in this list gives the strongest reducing hydride.

Step 2: Apply the metallic-character clue Metallic character increases down group 15. Phosphorus is a nonmetal, whereas arsenic and antimony are classified as metalloids in the standard comparison. Bismuth is a metal and forms BiH_3, the least stable hydride in the series. Therefore the metallic clue and the reducing-power clue independently point to the same element.

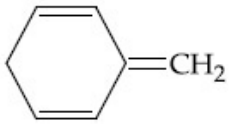
Step 3: Eliminate the other choices PH_3, AsH_3 and SbH_3 are all above BiH_3 in the hydride series and are weaker reducing agents. Antimony may appear tempting because it is low in the group, but bismuth lies still lower and also satisfies the explicit metal requirement.

Step 4: Conclusion The required element is bismuth, Bi, whose hydride is BiH_3. Hence, the correct option is (C).

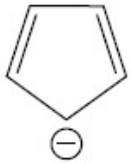
Quick Tip to Solve: For group 15 hydrides, decreasing bond strength down the group means increasing reducing power; then check the element classification given in the question.

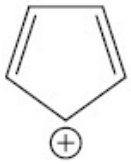
68. Among the following, the aromatic compounds are :

Choose the correct answer from the following options : [4 marks]

(A) 

(B) 

(C) 

(D) 

(A) (B), (C) and (D) only

(B) (B) and (C) only

(C) (A), (B) and (C) only

(D) (A) and (B) only

Correct Answer: (B) (B) and (C) only

Solution:

Concept:

- An aromatic system must be cyclic, planar and continuously conjugated, and must contain $4n + 2$ pi electrons for an integer $n \geq 0$. Counting electrons alone is insufficient without a complete ring of p orbitals.
- Each pi bond contributes two electrons, while a negative charge in a conjugating p orbital contributes a pair. A carbocation supplies an empty p orbital and contributes no electron pair.

Step 1: Test structures A and B In A, the ring contains two internal double bonds and an exocyclic $=CH_2$ group. The remaining saturated ring carbon interrupts conjugation, so there is no continuous cyclic pi-electron path. A is nonaromatic. Structure B is benzene: all six ring atoms have p orbitals and its three double bonds contribute six pi electrons, satisfying $4n + 2$ for $n = 1$. B is aromatic.

Step 2: Count the electrons in C and D C is the cyclopentadienyl anion. Its two double bonds provide four pi electrons, and the negatively charged carbon contributes its lone pair in a p orbital. The conjugated five-membered ring therefore has six pi electrons and is aromatic. D is the cyclopentadienyl cation: its positively charged carbon has an empty p orbital, leaving only four pi electrons. It fails the aromatic electron count and is antiaromatic in its planar conjugated form.

Step 3: Match the aromatic pair Only structures B and C satisfy both continuous conjugation and the six-electron aromatic count. The options including A fail because of interrupted ring conjugation; the option including D incorrectly treats a four-electron cation as aromatic.

Step 4: Conclusion Benzene and the cyclopentadienyl anion are the aromatic compounds. Hence, the correct option is (B).

Quick Tip to Solve: For a five-membered ring with two double bonds, distinguish the anion's extra lone pair from the cation's empty orbital before applying the pi-electron count.

69. Which of the following reactions DOES NOT involve Hoffmann bromamide degradation? [4 marks]

(A) $C_6H_5CH_2CONH_2 \xrightarrow{Br_2, NaOH} C_6H_5CH_2NH_2$

(B) $C_6H_5CH_2COCH_3$ treated successively with (i) Br_2, $NaOH/H^+$; (ii) NH_3, heat; (iii) $LiAlH_4/H_2O$, giving $C_6H_5CH_2NH_2$.

(C) C_6H_5COCl treated with (i) NH_3, $NaOH$; (ii) Br_2, $NaOH$, giving $C_6H_5NH_2$.

(D) C_6H_5CN treated with (i) KOH, H_2O; (ii) Br_2, $NaOH$, giving $C_6H_5NH_2$.

Correct Answer: (B) $C_6H_5CH_2COCH_3$ treated successively with (i) Br_2, $NaOH/H^+$; (ii) NH_3, heat; (iii) $LiAlH_4/H_2O$, giving $C_6H_5CH_2NH_2$.

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide $RCONH_2$ into RNH_2 with Br_2 and aqueous alkali. The carbonyl carbon is removed, so the amine has one fewer carbon atom than the amide.
- $LiAlH_4$ reduction is different: reduction of an amide gives an amine while retaining its carbonyl carbon as a methylene carbon. The identity of the step that makes the amine distinguishes the routes.

Step 1: Recognize the direct bromamide reaction Option A begins with phenylacetamide, $C_6H_5CH_2CONH_2$, and treats it with $Br_2/NaOH$. Its amide group is converted to an amino group with loss of the carbonyl carbon, giving benzylamine. This is the direct characteristic Hofmann bromamide degradation and therefore cannot be the requested exception.

Step 2: Inspect the reduction sequence in B Option B first uses $Br_2/NaOH$ on a methyl ketone, not on a primary amide. That first transformation belongs to the haloform reaction. The subsequent ammonia/heating treatment produces an amide, and the listed final reagent is $LiAlH_4$, which reduces the amide. Thus no step in this sequence treats a primary amide with bromine and alkali to cause Hofmann degradation.

Step 3: Compare the remaining precursor routes In C, benzoyl chloride is first converted to benzamide by ammonia, after which $Br_2/NaOH$ gives aniline by Hofmann degradation. D also places hydrolysis before the bromine/alkali step, corresponding to the amide-intermediate route followed by bromamide degradation. The decisive exception is B: its amine-forming reagent is a hydride reducing agent rather than bromine and alkali.

Step 4: Conclusion The reaction sequence in (B) does not involve Hofmann bromamide degradation. Hence, the correct option is (B).

Quick Tip to Solve: Do not identify Hofmann degradation merely from seeing bromine and alkali: those reagents must act on a primary amide, not on a methyl ketone.

70. Which of the following is Lindlar catalyst? [4 marks]

(A) Partially deactivated palladised charcoal

(B) Zinc chloride and HCl

(C) Sodium and Liquid NH_3

(D) Cold dilute solution of $KMnO_4$

Correct Answer: (A) Partially deactivated palladised charcoal

Solution:

Concept:

- Lindlar catalyst is a partially deactivated or poisoned palladium hydrogenation catalyst. Its reduced activity allows an alkyne to undergo controlled addition of hydrogen to give an alkene.
- The poisoned catalyst promotes syn addition at the surface, normally producing a cis alkene from an internal alkyne and preventing facile further reduction to the alkane.

Step 1: Identify the selective catalyst Palladium ordinarily catalyses hydrogenation efficiently, but partial deactivation reduces its activity sufficiently to stop after one equivalent of hydrogen has added to an alkyne. The usual Lindlar preparation uses supported palladium poisoned with lead compounds and often quinoline. Among the stated choices, the partially deactivated palladised catalyst in A is the one describing this selective hydrogenation catalyst.

Step 2: Distinguish the dissolving-metal reagent Sodium in liquid NH_3, option C, is also used to reduce an alkyne to an alkene, but it operates by a dissolving-metal mechanism and gives the trans alkene. It is neither a supported palladium catalyst nor Lindlar catalyst. The stereochemical outcome is therefore a useful independent check when distinguishing these commonly confused reducing systems.

Step 3: Exclude reagents with other functions $ZnCl_2$ with concentrated HCl is Lucas reagent, used to classify alcohols by their substitution behaviour, so B is unrelated. Cold dilute $KMnO_4$ is an oxidizing reagent used in reactions of unsaturation, not a hydrogenation catalyst, excluding D. Only A contains the characteristic deactivated palladium component.

Step 4: Conclusion The listed Lindlar catalyst is partially deactivated palladised charcoal. Hence, the correct option is (A).

Quick Tip to Solve: Associate poisoned palladium with cis selective hydrogenation; sodium in liquid ammonia gives the contrasting trans product.

71. Which among the following pairs of Vitamins is stored in our body relatively for longer duration?

[4 marks]

(A) Thiamine and Vitamin A

(B) Ascorbic acid and Vitamin D

(C) Thiamine and Ascorbic acid

(D) Vitamin A and Vitamin D

Correct Answer: (D) Vitamin A and Vitamin D

Solution:

Concept:

- Vitamins are classified as fat-soluble or water-soluble according to the medium in which they dissolve and are transported.
- The fat-soluble vitamins A, D, E and K can be retained in liver and fatty tissues. Most water-soluble vitamins need more regular dietary replenishment.

Step 1: Classify the four named vitamins Thiamine is vitamin B1 and ascorbic acid is vitamin C. Both belong to the water-soluble group. Vitamin A and vitamin D belong to the fat-soluble group. The question concerns relative duration of storage, so the relevant distinction is solubility and retention rather than the disease caused by deficiency.

Step 2: Compare the storage behaviour Fat-soluble vitamins are absorbed with fats and can accumulate in tissue stores. These reserves allow vitamins A and D to remain available for a relatively long duration. By contrast, excess thiamine and ascorbic acid are generally excreted in urine, so their continued availability depends more directly on regular intake. This comparison is not a claim that all vitamins have identical retention times within either class.

Step 3: Eliminate pairs containing water-soluble vitamins Option (A) contains thiamine, so it does not contain two vitamins with the required long-term storage behaviour. Option (B) contains ascorbic acid and fails for the same reason. Option (C) contains two water-soluble vitamins. Only option (D) contains two fat-soluble vitamins: A and D.

Step 4: Conclusion Vitamin A and vitamin D are stored in our body for a relatively longer duration. Hence, the correct option is (D).

Quick Tip to Solve: Remember ADEK as the fat-soluble group; then check that both members of a proposed pair belong to it.

72. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Size of Bk^{3+} ion is less than Np^{3+} ion.

Reason R : The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both A and B are true and R is the correct explanation of A

(B) A is false but R is true

(C) A is true but R is false

(D) Both A and B are true but R is not the correct explanation of A

Correct Answer: (C) A is true but R is false

Solution:

Concept:

- Actinoid contraction is the general decrease in atomic and ionic radii across the actinoid series as atomic number increases.
- Poor shielding by added 5f electrons increases effective nuclear attraction. Lanthanoid contraction refers instead to the analogous effect involving the 4f series.

Step 1: Evaluate the assertion Neptunium, Np, has atomic number 93, whereas berkelium, Bk, has atomic number 97. Both ions in the assertion carry the same charge, +3. Moving from Np to Bk across the actinoid series increases nuclear charge, while the added 5f electrons do not shield it efficiently. The general contraction therefore makes the later Bk ion smaller. The assertion that Bk3+ is smaller than Np3+ is true.

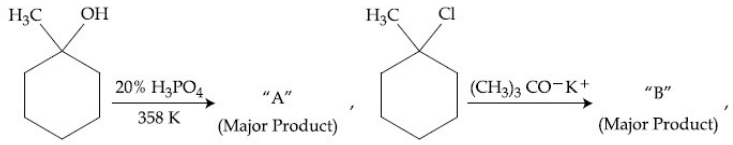
Step 2: Identify the contraction named in the reason The ions being compared are actinoid ions, so their decreasing size is attributed to actinoid contraction. Lanthanoid contraction describes the progressive reduction in size across the lanthanoid series with filling of the 4f subshell. Although both effects arise from imperfect shielding by f electrons, their names identify different series. Substituting the name of one effect for the other makes the stated reason false.

Step 3: Select the correct truth combination The assertion is true and the reason is false. Options claiming that both statements are true cannot be selected, even if one says the reason is not the correct explanation. The option declaring the assertion false is also excluded. The remaining combination is exactly the one in option (C).

Step 4: Conclusion Assertion A is true, but Reason R is false: the size decrease is due to actinoid contraction. Hence, the correct option is (C).

Quick Tip to Solve: Before naming a contraction, identify the series: 4f corresponds to lanthanoids and 5f to actinoids.

73. The products "A" and "B" formed in above reactions are : [4 marks]



(A) A: 1-methylcyclohexene; B: 1-methylcyclohexene.

(B) A: 1-methylcyclohexene; B: methylenecyclohexane.

(C) A: methylenecyclohexane; B: 1-methylcyclohexene.

(D) A: methylenecyclohexane; B: methylenecyclohexane.

Correct Answer: (B) A: 1-methylcyclohexene; B: methylenecyclohexane.

Solution:

Concept:

- Acid-catalysed dehydration of a tertiary alcohol proceeds mainly by E1 elimination and normally favours the more substituted, more stable alkene.
- Potassium tert-butoxide is a bulky strong base. In E2 elimination it preferentially removes an accessible beta hydrogen, often favouring the less substituted Hofmann alkene.

Step 1: Identify the possible elimination sites The alcohol is 1-methylcyclohexan-1-ol and the chloride is 1-chloro-1-methylcyclohexane. The carbon bearing the leaving group is bonded to two neighbouring ring carbons and one methyl group. Loss of a beta hydrogen from an adjacent ring carbon gives 1-methylcyclohexene. Loss from the methyl group gives an exocyclic double bond, methylenecyclohexane.

Step 2: Determine product A under acidic conditions With 20 percent phosphoric acid at 358 K, the hydroxyl group is protonated and water leaves to generate a tertiary carbocation. Beta deprotonation then forms an alkene. The internal double bond of 1-methylcyclohexene is trisubstituted, whereas the exocyclic double bond is less substituted. The more stable internal alkene predominates under these dehydration conditions, so A is 1-methylcyclohexene.

Step 3: Determine product B with the bulky base Potassium tert-butoxide abstracts a beta hydrogen as chloride leaves. Its bulky tert-butyl group makes approach to the less hindered methyl hydrogens favourable. Elimination there gives the exocyclic methylene group. Therefore B is methylenecyclohexane, the Hofmann product. Options with the same product for A and B miss this change of reaction conditions; the option reversing the products also reverses the selectivity.

Step 4: Conclusion A is 1-methylcyclohexene and B is methylenecyclohexane. Hence, the correct option is (B).

Quick Tip to Solve: Compare the reagent before comparing alkene stability: acid dehydration favours Zaitsev substitution, while bulky tert-butoxide favours accessible beta hydrogens.

74. Given below are two statements :

Statement I : H_2O_2 can act as both oxidising and reducing agent in basic medium.

Statement II : In the hydrogen economy, the energy is transmitted in the form of dihydrogen.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both statement I and statement II are false

(B) Statement I is false but statement II is true

(C) Statement I is true but statement II is false

(D) Both statement I and statement II are true

Correct Answer: (D) Both statement I and statement II are true

Solution:

Concept:

- In hydrogen peroxide, oxygen has oxidation state -1. It can be reduced to -2 or oxidised to 0, allowing peroxide to act as either oxidising or reducing agent.
- The hydrogen economy uses dihydrogen as an energy carrier: energy is stored, transported and delivered through hydrogen for use in combustion or fuel cells.

Step 1: Test peroxide as an oxidising agent in basic medium An oxidising agent accepts electrons and is itself reduced. The alkaline reduction half-reaction of peroxide is

$$\mathrm{H_2O_2 + 2\,e^- \longrightarrow 2\,OH^-}.$$

Oxygen changes from -1 in peroxide to -2 in hydroxide. Thus peroxide can oxidise another substance while it is reduced, even in basic solution. The presence of a basic medium does not remove this oxidising ability.

Step 2: Test peroxide as a reducing agent A reducing agent donates electrons and is itself oxidised. In alkaline medium the corresponding peroxide half-reaction is

$$\mathrm{H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-}.$$

Here oxygen changes from -1 to 0. Which role peroxide plays depends on its reaction partner. These two possible oxidation-state changes establish that Statement I is true.

Step 3: Evaluate the hydrogen-economy statement In a hydrogen economy, energy obtained from primary sources is used to produce hydrogen, which can then be transported and used where needed. Dihydrogen carries the energy chemically; in a fuel cell it can combine with oxygen to generate electrical energy and water. Therefore Statement II correctly describes transmission of energy in the form of dihydrogen. It does not imply that hydrogen is a primary energy source.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: For peroxide, track oxygen from -1: movement to -2 shows oxidising action, while movement to 0 shows reducing action.

75. Given below are two statements :

Statement I : Both $CaCl_2 \cdot 6H_2O$ and $MgCl_2 \cdot 8H_2O$ undergo dehydration on heating.

Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both statement I and statement II are false

(B) Statement I is false but statement II is true

(C) Statement I is true but statement II is false

(D) Both statement I and statement II are true

Correct Answer: (A) Both statement I and statement II are false

Solution:

Concept:

- Heating a hydrated salt can cause simple dehydration or hydrolysis. The outcome depends on the polarising power of its metal ion.
- Beryllium oxide is amphoteric; the other Group 2 metal oxides are basic, with basic character generally increasing down the group.

Step 1: Distinguish dehydration from hydrolysis Dehydration removes water of crystallisation without changing the identity of the salt. Calcium chloride hydrate can lose its bound water on heating to give anhydrous calcium chloride. Magnesium ions are smaller and more strongly polarising than calcium ions. Hydrated magnesium chloride therefore tends to undergo hydrolysis on heating, with formation of magnesium oxide and release of hydrogen chloride, rather than cleanly yielding anhydrous magnesium chloride.

Step 2: Evaluate Statement I The statement places both hydrated chlorides under the same simple dehydration behaviour. That is not correct because of the hydrolytic decomposition of the magnesium salt. A representative overall transformation is

$$\mathrm{MgCl_2 \cdot xH_2O \longrightarrow MgO + 2\,HCl + (x\text{-}1)\,H_2O.}$$

The distinction, rather than merely counting the waters of hydration written in the question, makes Statement I false.

Step 3: Evaluate Statement II and compare the options The first clause, that BeO is amphoteric, is correct: it can react with both acids and strong bases. The second clause calls the remaining Group 2 oxides acidic, which is wrong. MgO, CaO, SrO and BaO are basic oxides, and their basic character generally increases as metallic character increases down the group. Because both clauses must hold for Statement II to be true, it is false. Thus neither of the two statements is true.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (A).

Quick Tip to Solve: Do not equate heating every hydrate with dehydration: strongly polarising cations can cause hydrolysis and acid-gas evolution.

76. Match List - I with List - II :

	List - IIndustrial process		List - IIApplication
(a)	Haber's process	(i)	HNO_3 synthesis
(b)	Ostwald's process	(ii)	Aluminium extraction
(c)	Contact process	(iii)	NH_3 synthesis
(d)	Hal-Heroult process	(iv)	H_2SO_4 synthesis

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(B) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(C) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(D) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Correct Answer: (B) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Solution:

Concept:

- Industrial process names identify a characteristic raw material, transformation or product. Match the chemical operation rather than similar-sounding process names.
- Haber, Ostwald, Contact and Hall–Heroult processes correspond respectively to ammonia, nitric acid, sulfuric acid and aluminium production.

Step 1: Match the nitrogen-compound processes Haber's process directly combines nitrogen and hydrogen over an iron catalyst:

$$\mathrm{N_2 + 3\,H_2 \rightleftharpoons 2\,NH_3}.$$

Thus (a) matches (iii), ammonia synthesis. Ostwald's process starts by catalytic oxidation of ammonia to nitric oxide; subsequent oxidation and absorption lead to nitric acid. Its overall industrial product is HNO_3, so (b) matches (i), not ammonia synthesis.

Step 2: Match the sulfuric-acid process The Contact process uses catalytic oxidation of sulfur dioxide to sulfur trioxide:

$$\mathrm{2\,SO_2 + O_2 \rightleftharpoons 2\,SO_3}.$$

Sulfur trioxide is absorbed into concentrated sulfuric acid to form oleum, which is diluted to obtain sulfuric acid. Thus the application in List II is H_2SO_4 synthesis, and (c) matches (iv). The process is not used for aluminium extraction or nitric acid synthesis.

Step 3: Match aluminium extraction and check the full code The Hall–Heroult process electrolyses alumina dissolved in molten cryolite to obtain aluminium. The aluminium ions are reduced at the cathode, so (d) matches (ii). Putting all four independent matches together gives (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii). Option (B) has every match correct. The other options interchange one or more unrelated industrial products and therefore cannot represent the full list.

Step 4: Conclusion Haber produces ammonia, Ostwald produces nitric acid, the Contact process produces sulfuric acid, and Hall–Heroult extracts aluminium. Hence, the correct option is (B).

Quick Tip to Solve: Use the sequence Haber–ammonia, Ostwald–nitric acid, Contact–sulfuric acid, Hall–Heroult–aluminium to check the whole matching code.

77. The type of pollution that gets increased during the day time and in the presence of O_3 is :

[4 marks]

(A) Oxidising smog

(B) Global warming

(C) Acid rain

(D) Reducing smog

Correct Answer: (A) Oxidising smog

Solution:

Concept:

- Photochemical smog develops when nitrogen oxides and hydrocarbons undergo sunlight-driven reactions, generating secondary oxidants such as ozone.
- It is called oxidising smog because ozone and related photochemical products have strong oxidising character. Reducing smog is associated with cool, humid, sulfurous conditions.

Step 1: Connect daytime with the photochemical mechanism Daylight provides the radiation needed to initiate photochemical reactions involving nitrogen dioxide. A simplified reaction sequence is

$$NO_2 \xrightarrow{\text{sunlight}} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

In polluted air containing hydrocarbons, further radical reactions promote accumulation of ozone and other oxidants. Therefore the combination of daytime enhancement and ozone points directly to photochemical smog.

Step 2: Identify the oxidising character Ozone is a powerful oxidising agent, and it is an important component of photochemical smog along with oxidised organic compounds. The term oxidising smog describes this chemical character. It commonly becomes more pronounced in warm, sunny conditions because the relevant reactions require light. This explains both clues supplied in the question instead of treating the mere presence of ozone as a general atmospheric effect.

Step 3: Rule out the other pollution categories Global warming is a longer-term heat-balance effect caused principally by greenhouse gases, rather than the name of this daytime smog. Acid rain is associated with acids formed from sulfur and nitrogen oxides and is not identified by ozone accumulation. Reducing smog is the sulfurous, smoky form associated with cool humid weather; it has the opposite chemical character to the ozone-rich pollution described here.

Step 4: Conclusion The pollution described is oxidising, or photochemical, smog. Hence, the correct option is (A).

Quick Tip to Solve: The paired clues sunlight and ozone identify photochemical oxidising smog; sulfur dioxide and fog instead suggest reducing smog.

78. The process that involves the removal of sulphur from the ores is : [4 marks]

(A) Roasting

(B) Smelting

(C) Leaching

(D) Refining

Correct Answer: (A) Roasting

Solution:

Concept:

- Roasting heats a concentrated sulphide ore strongly in excess air, below its melting point. Oxidation converts the sulphide into an oxide and removes sulphur as gaseous SO_2.
- The purpose of a metallurgical operation distinguishes it from other operations: smelting produces molten metal, leaching dissolves selected constituents, and refining purifies the crude metal.

Step 1: Identify the constituent to remove The question asks specifically for removal of sulphur from an ore. In a sulphide mineral, sulphur is chemically bound to the metal. Heating the mineral in oxygen oxidises the sulphur to a volatile oxide, so that it escapes from the solid ore. This identifies an oxidation treatment in air, rather than simple melting or dissolution.

Step 2: Write a representative roasting reaction For zinc blende, the relevant conversion is

$$2\,\mathrm{ZnS} + 3\,\mathrm{O_2} \longrightarrow 2\,\mathrm{ZnO} + 2\,\mathrm{SO_2}\uparrow .$$

The metal remains in the solid oxide while sulphur leaves as sulphur dioxide. The equation conserves two zinc atoms, two sulphur atoms and six oxygen atoms. It illustrates why removal of sulphur is associated with roasting.

Step 3: Exclude the other processes Smelting uses high-temperature reduction and fusion to obtain a molten metal, commonly with a flux producing slag. Leaching uses a suitable reagent to dissolve the required component selectively. Refining is carried out after extraction to improve metal purity. None of these is the named preliminary oxidation of a sulphide ore in excess air.

Step 4: Conclusion The process is **roasting**. Hence, the correct option is (A).

Quick Tip to Solve: For sulphide ores, look for heating in excess air and evolution of SO_2: these identify roasting. Do not confuse it with calcination in limited or no air.

79. Match List - I with List - II :

	List - IName of oxo acid		List - IIOxidation state of 'P'
(a)	Hypophosphorous acid	(i)	+5
(b)	Orthophosphoric acid	(ii)	+4
(c)	Hypophosphoric acid	(iii)	+3
(d)	Orthophosphorous acid	(iv)	+2
		(v)	+1

Choose the correct answer from the options given below : [4 marks]

(A) (a) - (iv), (b) - (v), (c) - (ii), (d) - (iii)

(B) (a) - (iv), (b) - (i), (c) - (ii), (d) - (iii)

(C) (a) - (v), (b) - (i), (c) - (ii), (d) - (iii)

(D) (a) - (v), (b) - (iv), (c) - (ii), (d) - (iii)

Correct Answer: (C) (a) - (v), (b) - (i), (c) - (ii), (d) - (iii)

Solution:

Concept:

- For a neutral oxoacid, the algebraic sum of oxidation numbers is zero. Hydrogen has oxidation number $+1$ and oxygen -2 in these acids.
- The required formulae are hypophosphorous acid H_3PO_2, orthophosphoric acid H_3PO_4, hypophosphoric acid $H_4P_2O_6$, and orthophosphorous acid H_3PO_3.

Step 1: Calculate the first two oxidation states Let x denote the oxidation state of phosphorus. For hypophosphorous acid,

$$3(+1) + x + 2(-2) = 0 \quad \Rightarrow \quad x = +1.$$

Thus (a) matches (v). For orthophosphoric acid,

$$3(+1) + x + 4(-2) = 0 \quad \Rightarrow \quad x = +5.$$

Thus (b) matches (i). Oxidation number should be obtained from the complete formula, not from the number of ionisable hydrogen atoms.

Step 2: Calculate the remaining oxidation states Hypophosphoric acid contains two equivalent phosphorus atoms. Its balance is

$$4(+1) + 2x + 6(-2) = 0 \quad \Rightarrow \quad 2x = 8, \quad x = +4.$$

Therefore (c) matches (ii). For orthophosphorous acid,

$$3(+1) + x + 3(-2) = 0 \quad \Rightarrow \quad x = +3.$$

Therefore (d) matches (iii). The phosphorus–phosphorus bond in hypophosphoric acid contributes zero to the oxidation-number difference between identical atoms.

Step 3: Match the complete code The complete sequence is (a)–(v), (b)–(i), (c)–(ii), (d)–(iii). Option A assigns the wrong states to both first acids, option B assigns hypophosphorous acid the incorrect

state +2, and option D assigns orthophosphoric acid the incorrect state +2. Only option C satisfies all four calculated balances.

Step 4: Conclusion The oxidation states in the order listed are +1, +5, +4 and +3. Hence, the correct option is (C).

Quick Tip to Solve: Remember the formula of hypophosphoric acid as $H_4P_2O_6$: its two phosphorus atoms are each +4. Oxidation state and basicity are different quantities.

80. The functions of antihistamine are : [4 marks]

(A) Antiallergic and Analgesic

(B) Antacid and antialergic

(C) Analgesic and antacid

(D) Antiallergic and antidepressant

Correct Answer: (B) Antacid and antialergic

Solution:

Concept:

- Histamine acts through different receptor types. Blocking H_1 receptors reduces allergic responses, while blocking H_2 receptors reduces histamine-stimulated secretion of gastric acid.
- A drug class is identified by its therapeutic mechanism. Analgesics relieve pain, whereas antidepressants treat depressive illness; these are distinct therapeutic categories.

Step 1: Identify the antiallergic function Histamine released during an allergic response produces effects such as itching, sneezing and increased permeability of small blood vessels. Antihistamine drugs such as cetirizine block the relevant histamine receptors and reduce these symptoms. Thus antiallergic activity is one of the functions sought. This does not require the drug to act as a general painkiller.

Step 2: Identify the acid-suppressing function Histamine also stimulates acid secretion in the stomach through a different receptor subtype. Drugs such as cimetidine and ranitidine block these gastric histamine receptors and thereby reduce acid production. In the therapeutic classification used in this question, this is the antacid function. They suppress acid secretion rather than neutralising existing acid in the way that a simple inorganic antacid does.

Step 3: Compare the offered pairs The required pair is therefore antacid and antiallergic. Option A includes analgesic activity instead of acid suppression. Option C also contains analgesic activity and omits the antiallergic action. Option D adds antidepressant activity, which does not define the antihistamine class. Different receptor-selective antihistamines need not all be used for both indications; the question asks for the functions of the class.

Step 4: Conclusion Antihistamines have antacid and antiallergic functions. Hence, the correct option is (B).

Quick Tip to Solve: Link H_1 blockade with allergy and H_2 blockade with gastric acid secretion. This separates the two antihistamine uses from analgesic and antidepressant actions.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of $CoCl_3 \cdot 4NH_3$ is _____. (Round off to the Nearest Integer).

[4 marks]

Solution:

Concept:

- In $CoCl_3 \cdot 4\,NH_3$, the octahedral coordination sphere is $[Co(NH_3)_4Cl_2]^+$, with one chloride outside the sphere. Cobalt has coordination number six.
- Ethylenediamine, abbreviated en, is a neutral bidentate ligand: its two nitrogen donor atoms occupy two coordination sites, replacing two monodentate ammonia ligands.

Step 1: Identify the neutral ligands in the complex The coordination compound is written as $[Co(NH_3)_4Cl_2]Cl$. Four ammonia molecules and two chloride ions are directly bonded to cobalt; the remaining chloride balances the charge outside the coordination sphere. The neutral ligands to be replaced are the four coordinated NH_3 molecules. The coordinated chlorides are anionic ligands and are not included in this replacement count.

Step 2: Count the donor sites supplied by en Each ammonia molecule supplies one nitrogen donor atom and occupies one coordination site. Each ethylenediamine molecule, $H_2NCH_2CH_2NH_2$, supplies two nitrogen donors and occupies two adjacent sites as a chelating ligand. Replacing all four ammonia donors therefore requires

$$n(\text{en}) = \frac{4 \text{ donor sites}}{2 \text{ donor sites per en}} = 2.$$

The four ammonia sites of the trans-dichloro arrangement can be paired to accommodate the two chelate rings.

Step 3: Verify coordination and charge after replacement The substitution can be represented by

$$[Co(NH_3)_4Cl_2]^+ + 2\,\text{en} \longrightarrow [Co(en)_2Cl_2]^+ + 4\,NH_3.$$

Both sides retain six donor sites around cobalt: four nitrogen donors and two chloride donors. The incoming and outgoing nitrogen ligands are neutral, so the complex charge remains $+1$. The count is a number of ligand molecules per complex, not the number of nitrogen atoms.

Step 4: Conclusion Two equivalents of ethylenediamine are needed per equivalent of complex. The required nearest integer is $\boxed{2}$.

Quick Tip to Solve: Count coordination sites before counting ligand molecules: four monodentate ammonia ligands are replaced by two bidentate en ligands.

82. Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities i.e. $\frac{S(A_2X)}{S(MX)}$ = _____. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- Molar solubility s is the amount of formula units dissolving per litre. The individual ion concentrations follow from the stoichiometric coefficients in the dissolution equation.
- For ideal dilute saturated solutions without a common ion, K_{sp} is the product of ion concentrations raised to their stoichiometric powers. Equal solubility products need not mean equal solubilities.

Step 1: Find the solubility of the salt A2X Write the dissolution as

$$\mathrm{A_2X\,(s)} \rightleftharpoons 2\,\mathrm{A^+(aq)} + \mathrm{X^{2-}(aq)}.$$

If its molar solubility is s_1, the concentrations are $[\mathrm{A^+}] = 2s_1$ and $[\mathrm{X^{2-}}] = s_1$. Therefore

$$K_{sp} = (2s_1)^2 s_1 = 4s_1^3 = 4.0 \times 10^{-12}.$$

It follows that

$$s_1^3 = 10^{-12}, \qquad s_1 = 1.0 \times 10^{-4}\,\mathrm{mol\,L^{-1}}.$$

The factor four is essential; it arises because the concentration of the first ion is squared.

Step 2: Find the solubility of the salt MX For the one-to-one salt,

$$\mathrm{MX\,(s)} \rightleftharpoons \mathrm{M^{2+}(aq)} + \mathrm{X^{2-}(aq)}.$$

Writing its molar solubility as s_2, both ion concentrations equal s_2. Hence

$$K_{sp} = s_2^2 = 4.0 \times 10^{-12}, \qquad s_2 = 2.0 \times 10^{-6}\,\mathrm{mol\,L^{-1}}.$$

The formal ionic charges do not change the concentration exponents; those exponents come from the numbers of ions in the balanced dissolution equation.

Step 3: Take the ratio in the requested order The numerator is the solubility of the two-to-one salt, so

$$\frac{S(\mathrm{A_2X})}{S(\mathrm{MX})} = \frac{1.0 \times 10^{-4}}{2.0 \times 10^{-6}} = 50.$$

The concentration units cancel. Reversing the ratio would give 0.02, which is not the quantity requested. No further approximation is needed before rounding.

Step 4: Conclusion The ratio of molar solubilities is $\boxed{50}$, already an integer.

Quick Tip to Solve: Write $K_{sp} = 4s^3$ for a two-to-one salt and $K_{sp} = s^2$ for a one-to-one salt before substituting the shared value.

83. Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and _____ respectively. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Complete combustion converts all carbon of the sample into CO_2 and all hydrogen into H_2O. Elemental mass is found from the corresponding mass fraction in each combustion product.
- The hydrogen mass fraction in water is $2/18$, and the carbon mass fraction in carbon dioxide is $12/44$. Percentage composition uses the original sample mass, here 750 g, as the denominator.

Step 1: Extract the hydrogen mass from water One mole of water has a mass of 18 g and contains 2 g of hydrogen atoms. Thus the 210 g of water produced contains

$$m_{\mathrm{H}} = 210\left(\frac{2}{18}\right) = \frac{70}{3}\,\mathrm{g} = 23.333\ldots\,\mathrm{g}.$$

All of this hydrogen originated in the organic sample. Using the whole mass of water as hydrogen would wrongly include the oxygen supplied during combustion.

Step 2: Calculate the hydrogen percentage Divide the hydrogen mass by the mass of the original compound and multiply by 100:

$$\%\mathrm{H} = \frac{(70/3)}{750} \times 100 = \frac{28}{9} = 3.111\ldots\%.$$

The problem requests rounding to the nearest integer, so the entry is 3. Keep the unrounded mass until this final percentage calculation to avoid unnecessary rounding error.

Step 3: Check the supplied carbon percentage The 420 g of carbon dioxide contains

$$m_{\mathrm{C}} = 420\left(\frac{12}{44}\right) = 114.545\ldots\,\mathrm{g}.$$

Consequently,

$$\%\mathrm{C} = \frac{114.545\ldots}{750} \times 100 = 15.2727\ldots\% \simeq 15.3\%.$$

This agrees with the printed carbon composition and confirms that the combustion-product masses have been interpreted consistently. The sample need not contain only carbon and hydrogen; its remaining mass is not assigned without further information.

Step 4: Conclusion The hydrogen composition is approximately 3.111%. Its nearest-integer value is $\boxed{3}$.

Quick Tip to Solve: Use $\%\mathrm{H} = (m_{\mathrm{H_2O}}/m_{\mathrm{sample}})(2/18)100$; the denominator is the original sample mass, not the mass of the combustion products.

84. For the reaction $A(g) \rightleftharpoons B(g)$ at 495 K, $\Delta_r G^\circ = -9.478$ kJ mol^{-1}.

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B in the equilibrium mixture is _____ millimoles.

(Round off to the Nearest Integer). [R = 8.314 J mol^{-1} K^{-1}; ln 10 = 2.303] [4 marks]

Solution:

Concept:

- The standard reaction Gibbs energy and equilibrium constant satisfy $\Delta_r G^\circ = -RT \ln K$. Energy units must agree with those of the gas constant.
- For $\mathrm{A(g)} \rightleftharpoons \mathrm{B(g)}$, $K = p_B/p_A = n_B/n_A$ in the same ideal-gas mixture. Both species have coefficient one, so the common pressure or volume factor cancels.

Step 1: Determine the equilibrium constant Convert the standard Gibbs energy to joules:

$$\Delta_r G^\circ = -9.478 \times 10^3 \,\mathrm{J\,mol^{-1}}.$$

At 495 K,

$$\ln K = \frac{9478}{8.314 \times 495} = 2.303\ldots.$$

Using the supplied $\ln 10 = 2.303$, we obtain $K \simeq 10$. The negative standard Gibbs energy gives $K > 1$, so products should predominate, providing a useful sign check.

Step 2: Write the equilibrium amounts Initially there are 22 mmol of A and no B. If x mmol of A converts to B, the one-to-one stoichiometry gives

$$n_A = 22 - x, \qquad n_B = x.$$

The total amount remains 22 mmol because one gaseous molecule is converted into one gaseous molecule. For the same container, the ratio of partial pressures equals the ratio of these amounts, and millimoles can be used directly because their units cancel.

Step 3: Solve for the amount of B The equilibrium condition becomes

$$10 = \frac{x}{22 - x}.$$

Multiplication and collection of terms give

$$220 - 10x = x, \qquad 11x = 220, \qquad x = 20\,\mathrm{mmol}.$$

The remaining A is 2 mmol and the ratio $20/2 = 10$ verifies the equilibrium condition. Retaining the small extra digits in the exponential for K also rounds the amount of B to 20 mmol, so the requested integer is unchanged.

Step 4: Conclusion The equilibrium mixture contains $\boxed{20\,\mathrm{mmol}}$ of B. The required numerical entry is $\boxed{20}$.

Quick Tip to Solve: After finding K, a one-to-one conversion starting from pure A gives $n_B = n_0 K/(1+K)$. Check that the answer remains below the initial total amount.

85.

$$2MnO_4^- + bC_2O_4^{2-} + cH^+ \rightarrow xMn^{2+} + yCO_2 + zH_2O$$

If the above equation is balanced with integer coefficients, the value of c is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- In acidic solution, balance a redox reaction by conserving atoms and electric charge separately. Oxygen is balanced with H_2O, hydrogen with H^+, and charge with electrons.
- In MnO_4^-, manganese is in oxidation state $+7$; it becomes Mn^{2+} by accepting five electrons. Each oxalate ion loses two electrons when its two carbon atoms form CO_2.

Step 1: Balance the reduction half-reaction The medium is acidic because hydrogen ions occur among the reactants. First retain one manganese atom on each side, then balance the four oxygen atoms with water and the eight hydrogen atoms with hydrogen ions. Charge balance gives

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O.$$

The left charge is $-1 + 8 - 5 = +2$, equal to the right charge.

Step 2: Balance and combine the oxidation half-reaction The oxalate half-reaction is

$$C_2O_4^{2-} \longrightarrow 2\,CO_2 + 2\,e^-.$$

Multiply the manganese half-reaction by 2 and the oxalate half-reaction by 5, so both involve ten electrons. Add them and cancel the electrons:

$$2\,MnO_4^- + 5\,C_2O_4^{2-} + 16\,H^+ \longrightarrow 2\,Mn^{2+} + 10\,CO_2 + 8\,H_2O.$$

The fixed coefficient 2 of permanganate is now preserved. The total charge is $-2 - 10 + 16 = +4$ on the left and $+4$ on the right; manganese, carbon, oxygen and hydrogen also balance.

Step 3: Conclusion The integer coefficients are $b = 5$, $c = 16$, $x = 2$, $y = 10$, and $z = 8$. Hence the required numerical answer is $\boxed{16}$.

Quick Tip to Solve: For acidic permanganate, remember that each MnO_4^- consumes eight H^+. The given coefficient 2 therefore fixes the hydrogen-ion coefficient immediately at 16.

86. A 6.50 molal solution of KOH (aq.) has a density of 1.89 g cm^{-3}. The molarity of the solution is _____ mol dm^{-3}. (Round off to the Nearest Integer).

[Atomic masses : K : 39.0 u; O : 16.0 u; H : 1.0 u] [4 marks]

Solution:

Concept:

- Molality counts moles of solute per kilogram of solvent, whereas molarity counts moles per litre of solution. The mass of dissolved solute must be included before using the solution density.
- The definitions are $m = n/w_{\text{solvent}}$, $\rho = w_{\text{solution}}/V_{\text{solution}}$, and $M = n/V_{\text{solution}}$. Use w_{solvent} in kilograms for molality and V_{solution} in litres for molarity.

Step 1: Choose one kilogram of water A convenient basis is $1000\,\text{g}$ of water. Since the solution is 6.50 molal, this basis contains $6.50\,\text{mol}$ of KOH. Its molar mass, using the stated atomic masses, is

$$M_{\text{KOH}} = 39.0 + 16.0 + 1.0 = 56.0\,\text{g mol}^{-1}.$$

Thus the dissolved solute has mass

$$w_{\text{solute}} = 6.50 \times 56.0 = 364\,\text{g}.$$

The density applies to the complete solution, whose mass is $1000 + 364 = 1364\,\text{g}$.

Step 2: Convert solution mass to volume and calculate molarity At the given density, the solution volume is

$$V = \frac{1364}{1.89} = 721.69\,\text{cm}^3 = 0.72169\,\text{dm}^3.$$

The mole amount is unchanged by this conversion of the concentration unit. Therefore

$$M = \frac{6.50}{0.72169} = 9.0066\,\text{mol dm}^{-3}.$$

Using only the solvent mass in the density calculation would omit the 364 g of solute and give an incorrect volume. The requested rounding is performed only after the molarity is calculated.

Step 3: Conclusion The molarity is approximately $9.0066\,\text{mol dm}^{-3}$, so the required nearest-integer answer is $\boxed{9}$.

Quick Tip to Solve: Take 1 kg of solvent for any molal solution: the number of solute moles is then the stated molality. Add the solute mass before dividing by density.

87. AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is _____°C. (Round off to the Nearest Integer).

[Given : Molal elevation constant of water K_b = 0.5 K kg mol^{-1} boiling point of pure water = 100°C] [4 marks]

Solution:

Concept:

- Boiling-point elevation is a colligative property: $\Delta T_b = iK_b m$. Here m is the analytical molality, K_b is the solvent constant, and i accounts for changes in particle number on dissociation.
- For a solute producing ν ions per formula unit with fractional dissociation α, the van't Hoff factor is $i = 1 + (\nu - 1)\alpha$. A temperature increment has the same numerical value in kelvin and degrees Celsius.

Step 1: Count particles after partial dissociation The dissociation is

$$\mathrm{AB_2} \longrightarrow \mathrm{A^{2+}} + 2\,\mathrm{B^-}.$$

Thus one dissociated formula unit produces three ions, not two. On a basis of one mole initially, 0.90 mole remains undissociated and 0.10 mole dissociates. The products are 0.10 mole of $\mathrm{A^{2+}}$ and 0.20 mole of $\mathrm{B^-}$. The total amount of solute particles is therefore

$$0.90 + 0.10 + 0.20 = 1.20\,\mathrm{mol},$$

so $i = 1.20$. Equivalently, $i = 1 + 2(0.10) = 1.20$.

Step 2: Calculate the elevation and add it to the normal boiling point Using the supplied molality and boiling-point constant gives

$$\Delta T_b = 1.20 \times 0.5 \times 10.0 = 6.0\,\mathrm{K}.$$

This is an elevation, so it must be added to the pure-water boiling point:

$$T_b = 100 + 6.0 = 106.0\,^\circ\mathrm{C}.$$

Using $i = 3$ would incorrectly assume complete dissociation. Using $i = 1.10$ would omit the second anion; the explicit particle count prevents either error.

Step 3: Conclusion The solution boils at $106\,^\circ\mathrm{C}$. The requested nearest-integer numerical answer is $\boxed{106}$.

Quick Tip to Solve: For $\mathrm{AB_2}$, 10 percent dissociation raises the particle count by 20 percent: use $i = 1 + 2\alpha$, then add the elevation to 100 degrees Celsius.

88. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is _____$\overset{o}{A}$. (Round off to the Nearest Integer).

[Use : $\sqrt{3}$ = 1.73, h = 6.63 $\times$ 10^{-34} Js

m_e = 9.1 $\times$ 10^{-31} kg; c = 3.0 $\times$ 10^8 ms^{-1}; 1eV = 1.6 $\times$ 10^{-19} J] [4 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda_\gamma - \phi$, where ϕ is the threshold energy. Only the photon energy left after overcoming this threshold becomes electron kinetic energy.
- The de Broglie relation for a nonrelativistic electron is $\lambda_e = h/p = h/\sqrt{2m_eK}$. Here K must be in joules when SI values of h and m_e are used; $1\,\text{Å} = 10^{-10}\,\text{m}$.

Step 1: Find the kinetic energy of the emitted electrons Convert the incident wavelength to SI units: $248\,\text{nm} = 248 \times 10^{-9}\,\text{m}$. The photon energy is

$$E_\gamma = \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{248 \times 10^{-9}} = 8.020 \times 10^{-19}\,\text{J}.$$

The threshold energy is

$$\phi = 3.0(1.6 \times 10^{-19}) = 4.80 \times 10^{-19}\,\text{J}.$$

Therefore $K_{\max} = 3.220 \times 10^{-19}\,\text{J}$, approximately $2.01\,\text{eV}$. This is positive, so emission is possible.

Step 2: Apply the matter-wave relation The electron speed corresponding to this energy is much smaller than the speed of light, so the nonrelativistic expression is appropriate. Substituting the stated constants,

$$\lambda_e = \frac{6.63 \times 10^{-34}}{\sqrt{2(9.1 \times 10^{-31})(3.220 \times 10^{-19})}}.$$

The denominator is approximately $7.656 \times 10^{-25}\,\text{kg m s}^{-1}$, giving

$$\lambda_e = 8.66 \times 10^{-10}\,\text{m} = 8.66\,\text{Å}.$$

The incident photon wavelength cannot be used as the electron wavelength: they describe different particles and different momenta.

Step 3: Conclusion The de Broglie wavelength is approximately $8.66\,\text{Å}$, which rounds to $\boxed{9}$ Å. The required numerical entry is 9.

Quick Tip to Solve: Subtract the threshold energy from the photon energy before using $h/\sqrt{2m_eK}$; then convert metres to angstroms by multiplying by 10^{10}.

89. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is 1.0 $\times$ 10^{-3} s^{-1} and the activation energy E_a = 11.488 kJ mol^{-1}, the rate constant at 200 K is _____ $\times$ 10^{-5} s^{-1}. (Round off to the Nearest Integer). (Given : R = 8.314 J mol^{-1} K^{-1})

[4 marks]

Solution:

Concept:

- The two-temperature Arrhenius equation is $\ln(k_2/k_1) = -(E_a/R)(1/T_2 - 1/T_1)$. It compares rate constants without requiring the pre-exponential factor.
- Use absolute temperature in kelvin and matching energy units. Since $R = 8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$, convert $E_a = 11.488\,\mathrm{kJ\,mol^{-1}}$ to $11488\,\mathrm{J\,mol^{-1}}$.

Step 1: Substitute the two temperatures with the correct sign Let $T_1 = 300\,\mathrm{K}$, $k_1 = 1.0 \times 10^{-3}\,\mathrm{s^{-1}}$, and $T_2 = 200\,\mathrm{K}$. The reciprocal-temperature difference is

$$\frac{1}{200} - \frac{1}{300} = \frac{1}{600}\,\mathrm{K^{-1}}.$$

Consequently,

$$\ln\frac{k_2}{k_1} = -\frac{11488}{8.314}\left(\frac{1}{600}\right) \simeq -2.30294.$$

The sign is negative, as required: a lower temperature decreases the rate constant for positive activation energy.

Step 2: Exponentiate and express the requested coefficient Taking the exponential gives

$$\frac{k_2}{k_1} = e^{-2.30294} \simeq 0.09996.$$

Hence

$$k_2 = (1.0 \times 10^{-3})(0.09996) \simeq 9.996 \times 10^{-5}\,\mathrm{s^{-1}}.$$

The question asks for the coefficient multiplying $10^{-5}\,\mathrm{s^{-1}}$, not the whole rate constant as a decimal number. That coefficient is 9.996, which is rounded at the final stage. The near factor-of-ten decrease is consistent with the logarithmic change being close to $-\ln 10$.

Step 3: Conclusion At $200\,\mathrm{K}$, $k \simeq 10 \times 10^{-5}\,\mathrm{s^{-1}}$. Thus the required nearest-integer coefficient is $\boxed{10}$.

Quick Tip to Solve: Check that cooling gives a negative $\ln(k_2/k_1)$. After calculating k_2, divide by the power of ten printed beside the answer blank before rounding.

90. A certain element crystallises in a bcc lattice of unit cell edge length 27 $\overset{o}{A}$. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cel in $\overset{o}{A}$ will be _____. (Round off to the Nearest Integer).

[Assume each lattice point has a single atom]

[Assume $\sqrt{3}$ = 1.73, $\sqrt{2}$ = 1.41] [4 marks]

Solution:

Concept:

- In a body-centred cubic lattice, atoms touch along the body diagonal: $4r = \sqrt{3}\,a_{\text{bcc}}$. In a face-centred cubic lattice they touch along a face diagonal: $4r = \sqrt{2}\,a_{\text{fcc}}$.
- The same element under the same stated conditions is treated as having the same atomic radius r in the two geometrical arrangements. Equating the two expressions for $4r$ gives the relation between their unit-cell edge lengths.

Step 1: Use the bcc contact geometry to obtain the atomic diameter The body diagonal passes through a corner atom, the central atom and the opposite corner atom. Its length contains four atomic radii. Therefore

$$4r = \sqrt{3}\,a_{\text{bcc}} = 1.73 \times 27 = 46.71\,\text{Å}.$$

Thus $r = 11.6775\,$Å using the supplied approximate square root. Only this radius is carried into the fcc contact relation; the numerical edge length itself is not unchanged on changing the arrangement.

Step 2: Apply the fcc contact relation The face diagonal joins two corner atoms through a face-centred atom, and its length also contains four radii. Hence

$$a_{\text{fcc}} = \frac{4r}{\sqrt{2}} = \frac{46.71}{1.41} = 33.13\,\text{Å}.$$

Equivalently, $a_{\text{fcc}}/a_{\text{bcc}} = \sqrt{3}/\sqrt{2}$. The bcc and fcc cells contain different numbers of atoms, but this question relates edge lengths through the unchanged atomic radius, not by imposing equal cell masses or volumes. The ratio exceeds one, as the calculated edge lengths do.

Step 3: Conclusion The fcc unit-cell edge length is approximately $33.13\,$Å. Rounded to the nearest integer, the required answer is $\boxed{33}$ Å, so the numerical entry is 33.

Quick Tip to Solve: Write $4r$ for both structures and eliminate it: $a_{\text{fcc}} = a_{\text{bcc}}\sqrt{3/2}$. Do not equate bcc and fcc unit-cell volumes.