JEE Main 2021 Paper 1 (B.E./B.Tech) Mar 16 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 2 (March), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 16 March 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}$ + (tan x) y = sin x, $0 \le x \le \frac{\pi}{3}$, with y(0) = 0, then $y\left(\frac{\pi}{4}\right)$ equal to : [4 marks]

(A) $\frac{1}{2}\log_e 2$

(B) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(C) $\log_e 2$

(D) $\frac{1}{4}\log_e 2$

Correct Answer: (B) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

Solution:

Concept:

- A first-order linear equation $y\prime + P(x)y = Q(x)$ has integrating factor $\exp(\int P(x)\,dx)$. Multiplication by this factor makes the left side a product derivative.
- Here $P(x) = \tan x$. Throughout the given interval $\cos x > 0$, so the integrating factor is $\sec x$; no absolute-value or singularity issue arises.

Step 1: Find the integrating factor Integrate the coefficient of the dependent variable:

$$\int \tan x\,dx = -\ln(\cos x), \qquad \text{IF} = \sec x.$$

Multiplying the complete differential equation by this factor gives

$$\sec x\, y\prime + \sec x \tan x\, y = \tan x.$$

The two terms on the left are exactly the derivative of $y \sec x$, including the derivative of the integrating factor.

Step 2: Integrate and apply the initial value Consequently,

$$y \sec x = \int \tan x\,dx = -\ln(\cos x) + C.$$

At $x = 0$, both $y \sec x$ and $-\ln(\cos x)$ vanish, so $C = 0$. Hence

$$y = -\cos x \ln(\cos x).$$

This expression also directly satisfies the prescribed zero initial value.

Step 3: Evaluate at the specified point Use $\cos(\pi/4) = 1/\sqrt{2}$ and $\ln(1/\sqrt{2}) = -\frac{1}{2}\ln 2$:

$$y(\pi/4) = \frac{\ln 2}{2\sqrt{2}}.$$

The other options omit or incorrectly change the cosine factor that remains after dividing by the integrating factor.

Step 4: Conclusion $y(\pi/4) = \dfrac{\ln 2}{2\sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: After integrating a linear equation, divide by the integrating factor before substituting the requested value of x.

2. Let f be a real valued function, defined on R $-$ {-1, 1} and given by
f(x) = 3 $\log_e \left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}$.
Then in which of the following intervals, function f(x) is increasing? [4 marks]

(A) $(-\infty, -1) \cup \left([\frac{1}{2}, \infty) - \{1\}\right)$
(B) $(-\infty, \infty) - \{-1, 1)$
(C) $(-\infty, \frac{1}{2}] - \{-1\}$
(D) $(-1, \frac{1}{2}]$
Correct Answer: (A) $(-\infty, -1) \cup \left([\frac{1}{2}, \infty) - \{1\}\right)$

Solution:

Concept:

- A differentiable function is increasing on intervals where its derivative is positive. A zero derivative at an isolated endpoint may be included.
- For nonzero $u(x)$, $(\ln|u|)' = u'/u$. The excluded points $x = \pm 1$ must remain outside the domain.

Step 1: Differentiate the function Write the logarithm locally as $\ln|x-1| - \ln|x+1|$. This is valid on each component of the domain and gives

$$f'(x) = 3\left(\frac{1}{x-1} - \frac{1}{x+1}\right) + \frac{2}{(x-1)^2}.$$

The last term has a positive sign because differentiating $-2(x-1)^{-1}$ produces $+2(x-1)^{-2}$.

Step 2: Combine the fractions and analyse the sign Taking the common denominator yields

$$f'(x) = \frac{6(x-1) + 2(x+1)}{(x-1)^2(x+1)} = \frac{4(2x-1)}{(x-1)^2(x+1)}.$$

Away from $x = 1$, the squared factor is positive. Therefore only the signs of $2x - 1$ and $x + 1$ matter. They are both negative for $x < -1$, opposite for $-1 < x < 1/2$, and both positive for $x > 1/2$.

Step 3: Match the increasing intervals The increasing pieces are $(-\infty, -1)$, $[1/2, 1)$, and $(1, \infty)$; at $1/2$ the derivative is zero. The union notation in option (A) lists these pieces, with 1 removed. Options (B) and (C) include points between -1 and $1/2$, where the derivative is negative. Option (D) consists of the decreasing piece rather than the increasing pieces.

Step 4: Conclusion The increasing intervals are $(-\infty, -1) \cup ([1/2, \infty) \setminus \{1\})$, understood component-wise. Hence, the correct option is (A).

Quick Tip to Solve: Factor the derivative completely and retain the poles in the sign chart; the squared denominator factor cannot change its sign.

3. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r} \cdot \left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$ and $\vec{r} \cdot \left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$, $\alpha \in$ R, then the value of $\alpha + |\vec{r}|^2$ is equal to : [4 marks]

(A) 13

(B) 11

(C) 9

(D) 15

Correct Answer: (D) 15

Solution:

Concept:

- The cross product is antisymmetric: $\mathbf{b} \times \mathbf{r} = -\mathbf{r} \times \mathbf{b}$. Thus the given relation makes $\mathbf{r}$ parallel to $\mathbf{a} + \mathbf{b}$.
- Once this direction is known, write $\mathbf{r} = t(\mathbf{a} + \mathbf{b})$ and use the two scalar-product equations to find the scale and parameter.

Step 1: Reduce the vector relation to a direction Moving the terms to one side gives

$$\mathbf{r} \times (\mathbf{a} + \mathbf{b}) = \mathbf{0}.$$

Here

$$\mathbf{a} + \mathbf{b} = 3\hat{i} - \hat{j} + 2\hat{k},$$

which is nonzero. Hence

$$\mathbf{r} = t(3\hat{i} - \hat{j} + 2\hat{k}).$$

The zero vector is ruled out by the first dot product, whose value is 3.

Step 2: Use both dot products The first equation now reads

$$t(3\alpha - 2 + 2) = 3, \qquad t\alpha = 1.$$

The second equation gives

$$t(6 - 5 - 2\alpha) = -1, \qquad t - 2t\alpha = -1.$$

Substitute $t\alpha = 1$ to obtain $t - 2 = -1$, so $t = 1$ and then $\alpha = 1$. These values satisfy both original scalar conditions, without dividing by a possibly zero parameter.

Step 3: Calculate the squared magnitude It follows that $\mathbf{r} = (3, -1, 2)$, and therefore

$$|\mathbf{r}|^2 = 3^2 + (-1)^2 + 2^2 = 14.$$

The requested sum, rather than the magnitude alone, is

$$\alpha + |\mathbf{r}|^2 = 1 + 14 = 15.$$

Values 13, 11 and 9 do not satisfy this magnitude calculation together with the derived scalar constraints.

Step 4: Conclusion $\alpha + |\mathbf{r}|^2 = 15$. Hence, the correct option is (D).

Quick Tip to Solve: Reverse the cross product before combining terms: here it is the sum $\mathbf{a} + \mathbf{b}$, rather than their difference, that fixes the direction.

4. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to : [4 marks]

(A) $\frac{1}{\sqrt{6}}$

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\sqrt{\frac{2}{3}}$

Correct Answer: (A) $\frac{1}{\sqrt{6}}$

Solution:

Concept:

- A perpendicular from a point to a line is orthogonal to the line direction. This condition determines the unknown direction ratio.
- For lines through $\mathbf{p}, \mathbf{q}$ with nonparallel directions $\mathbf{u}, \mathbf{v}$, the shortest distance is $|(\mathbf{q} - \mathbf{p}) \cdot (\mathbf{u} \times \mathbf{v})| / |\mathbf{u} \times \mathbf{v}|$.

Step 1: Determine the first line direction Let $P = (4, 3, 8)$ and its foot be $H = (3, 5, 7)$. The vector $P - H = (1, -2, 1)$ is perpendicular to the direction $(l, 3, 4)$. Thus

$$l - 6 + 4 = 0, \qquad l = 2.$$

Since the foot lies on the first line, we can use H as its known point; the separate values of a and b are unnecessary.

Step 2: Construct a common normal Take $\mathbf{u} = (2, 3, 4)$, $\mathbf{v} = (3, 4, 5)$, and the point $Q = (2, 4, 5)$ on the second line. Their cross product is

$$\mathbf{u} \times \mathbf{v} = (15 - 16, 12 - 10, 8 - 9) = (-1, 2, -1).$$

Its magnitude is $\sqrt{6}$, so the lines are not parallel and the scalar-triple-product formula applies.

Step 3: Project the connecting vector We have

$$Q - H = (-1, -1, -2),$$

and its scalar product with the common normal is

$$(-1, -1, -2) \cdot (-1, 2, -1) = 1 - 2 + 2 = 1.$$

Hence

$$d = \frac{|1|}{\sqrt{6}} = \frac{1}{\sqrt{6}}.$$

The other distances result from an incorrect normalisation or projection; the denominator must be the length of the cross product, not its squared length.

Step 4: Conclusion The shortest distance is $1/\sqrt{6}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the given perpendicular foot as a point on the line; this avoids solving for the two unnecessary intercept parameters.

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5. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to : [4 marks]

(A) 717

(B) 795

(C) 1890

(D) 1173

Correct Answer: (A) 717

Solution:

Concept:

- The multiplication principle counts selections with one independent choice from each specified side. Points are in side interiors, so no point belongs to two sides.
- A triangle uses three different sides, whereas a quadrilateral of the prescribed kind uses one point from each of all four sides. The selected interior points form nondegenerate polygons.

Step 1: Count the triangles by their occupied sides There are four choices of three sides. Using the side populations $5, 7, 6, 9$, the respective products are

$$5 \cdot 7 \cdot 6 = 210, \qquad 5 \cdot 7 \cdot 9 = 315,$$

$$5 \cdot 6 \cdot 9 = 270, \qquad 7 \cdot 6 \cdot 9 = 378.$$

Every admissible triangle appears in exactly one of these categories, determined by the side that has no selected vertex. Therefore

$$\alpha = 210 + 315 + 270 + 378 = 1173.$$

No factorial divisor is needed, because each selected set is already determined by its side choices.

Step 2: Count the quadrilaterals Choose one point on each of the four sides independently:

$$\beta = 5 \cdot 7 \cdot 6 \cdot 9 = 1890.$$

Joining these points in cyclic side order forms a quadrilateral. Interior placement excludes coincident vertices at the corners, and a line cannot meet three different side interiors of a rectangle, so no collinear triangle needs to be subtracted.

Step 3: Form the requested difference The expression asked for is the difference, not either of the two counts alone:

$$\beta - \alpha = 1890 - 1173 = 717.$$

Thus options (C) and (D) reproduce individual counts rather than their difference, and option (B) does not follow from the complete sum over all four triples of sides.

Step 4: Conclusion $\beta - \alpha = 717$. Hence, the correct option is (A).

Quick Tip to Solve: For four side populations, the triangle count is the sum of the four triple products and the quadrilateral count is their full product.

6. Let f : S $\to$ S where S = (0, ∞) be a twice differentiable function such that f(x + 1) = xf(x). If g : S $\to$ R be defined as g(x) = $\log_e$ f(x), then the value of |g"(5) − g"(1)| is equal to : [4 marks]

(A) 1

(B) $\frac{187}{144}$

(C) $\frac{197}{144}$

(D) $\frac{205}{144}$

Correct Answer: (D) $\frac{205}{144}$

Solution:

Concept:

- Since $f(x) > 0$ on its domain, taking its natural logarithm is valid and converts the multiplicative recurrence into an additive one.
- Differentiate the logarithmic recurrence twice, then telescope the resulting second-derivative differences over integer arguments. No explicit formula for f is needed.

Step 1: Take logarithms and differentiate From $f(x+1) = xf(x)$, positivity gives

$$g(x+1) = \ln x + g(x).$$

Differentiate with respect to x. The derivative of $x + 1$ is 1, so

$$g\prime(x+1) = \frac{1}{x} + g\prime(x).$$

Differentiating again yields

$$g\prime\prime(x+1) - g\prime\prime(x) = -\frac{1}{x^2}.$$

The negative sign is essential: it comes from the derivative of $1/x$.

Step 2: Telescope between 1 and 5 Apply this last identity for $x = 1, 2, 3, 4$ and add. The intermediate

second derivatives cancel, leaving

$$g''(5) - g''(1) = -\left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right).$$

Only four differences are involved: the final endpoint 5 arises from the recurrence at $x = 4$, so there is no $1/25$ term.

Step 3: Take the absolute value All the terms in the parentheses are positive. With common denominator 144,

$$|g''(5) - g''(1)| = \frac{144 + 36 + 16 + 9}{144} = \frac{205}{144}.$$

The other fractions do not equal the sum of the four reciprocal squares; the value 1 retains only the first increment and neglects the remaining three.

Step 4: Conclusion $|g''(5) - g''(1)| = 205/144$. Hence, the correct option is (D).

Quick Tip to Solve: Logarithms simplify multiplicative functional equations; differentiate first and telescope afterwards to eliminate unknown function values.

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7. Consider the integral

$I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}} dx$,

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to :

[4 marks]

(A) 45 (e − 1)

(B) 45 (e + 1)

(C) 9 (e + 1)

(D) 9 (e − 1)

Correct Answer: (A) 45 (e − 1)

Solution:

Concept:

- On each interval $[n, n+1)$, the greatest integer function is the constant $[x] = n$. Split the integral at every integer to remove it.
- Values at isolated integer endpoints do not affect a definite integral. Substitute $x = n + t$ on each unit interval so the remaining exponential integral is identical.

Step 1: Split into ten unit intervals For $n = 0, 1, \ldots, 9$, replace $[x]$ by n on $[n, n+1)$. Then

$$I = \sum_{n=0}^{9} \int_n^{n+1} ne^{n+1-x}\, dx.$$

The $n = 0$ term vanishes. The endpoint $x = 10$ is a single point and contributes no extra integral or separate term, so summing through $n = 9$ is correct.

Step 2: Evaluate a general interval contribution Set $t = x - n$, so the bounds become 0 and 1, and

the exponent simplifies to $1-t$. Thus

$$\int_n^{n+1} ne^{n+1-x}\,dx = n\int_0^1 e^{1-t}\,dt = n[-e^{1-t}]_0^1 = n(e-1).$$

The negative antiderivative produces a difference of endpoint values, not $e+1$. The same factor $e-1$ multiplies each integer coefficient.

Step 3: Sum the coefficients Consequently,

$$I = (e-1)\sum_{n=0}^{9} n = (e-1)\frac{9\cdot 10}{2} = 45(e-1).$$

The options with $e+1$ have an incorrect integration sign. The remaining option $9(e-1)$ keeps only the last interval coefficient, while the integral requires all ten intervals and hence the arithmetic-series sum.

Step 4: Conclusion $I = 45(e-1)$. Hence, the correct option is (A).

Quick Tip to Solve: For a floor-function integral, substitute the fractional part $t = x - n$ on each unit interval; the exponential dependence on n may cancel.

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8. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2 - x^2, x > 0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2} = \frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves C_1 and C_2 is equal to : [4 marks]

(A) $\frac{\pi}{4} + 1$

(B) $\pi + 1$

(C) $\pi - 1$

(D) $\frac{\pi}{2} - 1$

Correct Answer: (D) $\frac{\pi}{2} - 1$

Solution:

Concept:

- Set $u = y^2$ in the first equation to obtain a linear differential equation. The condition $(1, 1)$ fixes the integration constant.
- For the second equation, reverse the roles of the variables and use $v = x^2$. The resulting curves are unit circles; their enclosed lens can be evaluated by sectors.

Step 1: Find the first curve Since $u\prime = 2yy\prime$, the equation becomes $xu\prime = u - x^2$. Dividing by x^2, which is valid for $x > 0$, gives $(u/x)\prime = -1$. Hence $y^2 = Cx - x^2$. Substituting $(1, 1)$ yields $C = 2$, so

$$C_1: \quad (x-1)^2 + y^2 = 1.$$

The branch through the given point is the upper semicircle. Its closure reaches the origin.

Step 2: Find the second curve Writing the equation as $2xy\,dx/dy = x^2 - y^2$, put $v = x^2$. Then $yv' = v - y^2$, and integration gives $x^2 = Dy - y^2$. The common point yields $D = 2$. Thus

$$C_2 : \quad x^2 + (y-1)^2 = 1.$$

Subtracting the two circle equations gives $x = y$; substitution gives the intersection points $(0,0)$ and $(1,1)$.

Step 3: Evaluate the common lens The centres are $(1,0)$ and $(0,1)$, and each radius is one. At either centre the radii to the two intersection points are perpendicular. Consequently the lens is the sum of two circular segments, each equal to a quarter-circle sector minus a right triangle with unit legs:

$$A = 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{2} - 1.$$

The restriction $x > 0$ excludes only the endpoint at the origin and does not alter the area. The other expressions do not equal this sector-minus-triangle sum.

Step 4: Conclusion The enclosed area is $\boxed{\pi/2 - 1}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: After solving, identify the two unit circles: two quarter sectors minus two unit right triangles give the lens immediately.

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9. Let P(x) = x^2 + bx + c be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx$ = 1 and P(x) leaves remainder 5 when it is divided by (x − 2). Then the value of 9(b + c) is equal to : [4 marks]

(A) 9

(B) 11

(C) 7

(D) 15

Correct Answer: (C) 7

Solution:

Concept:

- Integrate a polynomial term by term: $\int_0^1 x^2\,dx = 1/3$, $\int_0^1 x\,dx = 1/2$, and $\int_0^1 1\,dx = 1$.
- The remainder on division by $x - a$ equals $P(a)$. The integral and remainder conditions therefore give two independent linear equations for b, c.

Step 1: Use the definite-integral condition Insert $P(x) = x^2 + bx + c$ in the given integral. The coefficients b, c are constants, so

$$1 = \left[\frac{x^3}{3} + \frac{bx^2}{2} + cx\right]_0^1 = \frac{1}{3} + \frac{b}{2} + c.$$

Thus $b/2 + c = 2/3$. Notice that the linear term contributes $b/2$, not b; evaluating the polynomial at

one would not give its integral.

Step 2: Apply the remainder theorem The divisor is $x-2$, so the stated remainder requires $P(2)=5$. Therefore

$$4+2b+c=5, \qquad 2b+c=1.$$

Subtract the integral equation from this equation:

$$\frac{3b}{2}=1-\frac{2}{3}=\frac{1}{3}, \qquad b=\frac{2}{9}.$$

It follows that $c=1-2b=5/9$. Both coefficients are real, as required.

Step 3: Compute and check the requested expression The question asks for nine times the sum of the coefficients, not either coefficient separately:

$$9(b+c)=9\left(\frac{2}{9}+\frac{5}{9}\right)=7.$$

As a check, the integral is $1/3+1/9+5/9=1$, while $P(2)=4+4/9+5/9=5$. Thus both original conditions hold simultaneously. The alternatives 9, 11 and 15 are excluded by this uniquely determined pair of coefficients.

Step 4: Conclusion $\boxed{9(b+c)=7}$. Hence, the correct option is (C).

Quick Tip to Solve: Turn the integral into one linear equation and the remainder into the other before solving for the coefficients.

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10. Let A = {2, 3, 4, 5, ..., 30} and '$\simeq$' be an equivalence relation on A $\times$ A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to : [4 marks]

(A) 5

(B) 6

(C) 8

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- The relation $(a,b)\simeq(c,d)$ means $ad=bc$. Because the entries are positive, this is equality of the ratios $a/b=c/d$.
- An integer pair with ratio $4:3$ must have the form $(4k,3k)$ for an integer k, since 4 and 3 are coprime. Both coordinates must belong to A.

Step 1: Translate equivalence to a divisibility condition Let $(a,b)\in A\times A$ be equivalent to $(4,3)$. In

the defining equation take $(c, d) = (4, 3)$. We obtain

$$3a = 4b.$$

Since 3 and 4 share no factor greater than one, $4 \mid a$ and $3 \mid b$. Hence $a = 4k$ and $b = 3k$ for the same positive integer k. This parameterisation describes every pair and introduces no duplicates.

Step 2: Impose the bounds on both coordinates The set includes every integer from 2 through 30. Thus

$$2 \le 4k \le 30, \qquad 2 \le 3k \le 30.$$

For positive integral k, the lower bounds require $k \ge 1$. The upper bounds give $k \le 7$ and $k \le 10$, respectively. Their intersection is therefore $k \in \{1, 2, 3, 4, 5, 6, 7\}$. The first coordinate supplies the tighter upper bound.

Step 3: List the admissible class members The seven ordered pairs are

$$(4, 3), (8, 6), (12, 9), (16, 12),$$
$$(20, 15), (24, 18), (28, 21).$$

Each satisfies $3a = 4b$, and both entries lie in the specified set. The next candidate $(32, 24)$ fails because 32 exceeds 30. The pair $(4, 3)$ itself must be counted: equivalence relations are reflexive. Excluding it would incorrectly produce 6; reversing a pair generally changes the ratio and supplies no additional member.

Step 4: Conclusion There are $\boxed{7}$ ordered pairs in the equivalence class. Hence, the correct option is (D).

Quick Tip to Solve: Reduce the target ratio first, write all pairs as integer multiples, and apply the tighter coordinate bound.

11. The least value of |z| where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}|5\sqrt{7}+9i|, i=\sqrt{-1}$$

, is equal to : [4 marks]

(A) 8

(B) 3

(C) 2

(D) $\sqrt{5}$

Correct Answer: (B) 3

Solution:

Concept:

- Write $r = |z| \geq 0$. Then $||z|+1| = r+1 > 0$, so no sign ambiguity occurs when multiplying by this denominator.
- Use $|a+bi| = \sqrt{a^2+b^2}$, $e^{u\ln 2} = 2^u$, and the strict increase of 2^u to simplify the inequality.

Step 1: Evaluate the constant on the right The modulus inside the logarithm is

$$|5\sqrt{7}+9i| = \sqrt{175+81} = 16.$$

Since $\sqrt{2} = 2^{1/2}$ and $16 = 2^4$, the change-of-base formula gives

$$\log_{\sqrt{2}} 16 = \frac{4\ln 2}{(1/2)\ln 2} = 8.$$

The right-hand side is therefore the real number 8, rather than a complex logarithm.

Step 2: Reduce the exponential inequality For $r = |z|$, the left-hand side is $2^{(r+3)(r-1)/(r+1)}$. As $8 = 2^3$ and the base 2 is greater than one, the inequality is equivalent to

$$\frac{(r+3)(r-1)}{r+1} \geq 3.$$

The denominator is strictly positive for all permitted r, so multiplying by it preserves the direction of the inequality.

Step 3: Solve on the modulus domain Expanding and collecting terms gives

$$r^2+2r-3 \geq 3r+3,$$
$$r^2-r-6 \geq 0,$$
$$(r-3)(r+2) \geq 0.$$

On the real line this holds for $r \leq -2$ or $r \geq 3$. A modulus cannot be negative, leaving precisely $r \geq 3$. Equality is attained, for example by $z = 3$. The value 8 is allowed but is not least, while 2 and $\sqrt{5}$ lie below the threshold and fail.

Step 4: Conclusion The least modulus is $\boxed{3}$. Hence, the correct option is (B).

Quick Tip to Solve: Replace the modulus by a nonnegative variable before simplifying absolute values or multiplying inequalities.

12. Let $\alpha \in$ R be such that the function $f(x) = \begin{cases} \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$ is continuous at x = 0, where {x} = x − [x] is the greatest integer less than or equal to x. Then : [4 marks]

(A) no such α exists

(B) $\alpha = 0$

(C) $\alpha = \frac{\pi}{4}$

(D) $\alpha = \frac{\pi}{\sqrt{2}}$

Correct Answer: (A) no such α exists

Solution:

Concept:

- For $t = \{x\} = x - [x]$, the right limit $x \to 0^+$ gives $t \to 0^+$, whereas the left limit $x \to 0^-$ gives $t \to 1^-$.
- Use $\arccos(1-u) \sim \sqrt{2u}$ as $u \to 0^+$ and $\arcsin u \sim u$ as $u \to 0$. Continuity requires the two one-sided limits to agree with α.

Step 1: Evaluate the right-hand limit When $0 < x < 1$, $[x] = 0$, so $t = x$. Define

$$g(t) = \frac{\arccos(1-t^2)\arcsin(1-t)}{t(1-t^2)}.$$

As $t \to 0^+$, $\arccos(1-t^2)/t \to \sqrt{2}$, $\arcsin(1-t) \to \pi/2$, and $1 - t^2 \to 1$. Hence

$$\lim_{x \to 0^+} f(x) = \frac{\pi}{\sqrt{2}}.$$

The square-root asymptotic follows directly from $1 - \cos u \sim u^2/2$.

Step 2: Evaluate the left-hand limit When $-1 < x < 0$, $[x] = -1$, so $t = 1 + x \to 1^-$. Put $s = 1 - t \to 0^+$. Now $1 - t^2 = 2s - s^2$, and

$$g(1-s) = \frac{\arccos(2s-s^2)\arcsin s}{(1-s)s(2-s)}.$$

The arccosine tends to $\pi/2$, while $\arcsin s/s \to 1$. Thus

$$\lim_{x \to 0^-} f(x) = \frac{\pi}{4}.$$

The denominator has leading term $2s$, which is essential for the factor one quarter.

Step 3: Compare the limits The two finite one-sided limits differ: $\pi/\sqrt{2} \neq \pi/4$. Assigning a value only at $x = 0$ cannot remove this jump. Choosing either of the displayed limit values matches only one side, and choosing zero matches neither. Therefore no real choice of α makes the function continuous at the origin.

Step 4: Conclusion $\boxed{\text{No such } \alpha \text{ exists}}$. Hence, the correct option is (A).

Quick Tip to Solve: For fractional-part expressions at an integer, calculate both sides: the fractional part approaches zero from the right and one from the left.

13. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy

$\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ **is equal to :** [4 marks]

(A) 2

(B) 0

(C) 3

(D) 1

Correct Answer: (C) 3

Solution:

Concept:

- All principal inverse sines are defined when $-1 \le x \le 1$. On this interval the auxiliary angles $A=\arcsin(3x/5)$, $B=\arcsin(4x/5)$ have nonnegative cosines.
- Use $\sin(A+B)=\sin A\cos B+\cos A\sin B$. Any solutions found after taking sine must be checked in the original principal-value equation.

Step 1: Separate the zero solution At $x=0$, all three inverse sines are zero, so this is a solution. For $x\neq 0$, take sine of the proposed equality and use the nonnegative principal cosines:

$$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x.$$

Division by the nonzero value x is valid for either sign.

Step 2: Restrict the nonzero possibilities Writing $u=x^2\in(0,1]$ and multiplying by 25 gives

$$3\sqrt{25-16u}+4\sqrt{25-9u}=25.$$

Each square root decreases strictly with u throughout this interval. Their positive weighted sum is therefore strictly decreasing. At $u=1$ it equals $3\cdot 3+4\cdot 4=25$, while at every $u<1$ it is larger than 25. Thus the only possible nonzero solutions are $x=1$ and $x=-1$. No repeated squaring or extra roots are needed.

Step 3: Check principal values explicitly Let $\theta=\arcsin(3/5)$, an acute angle. Since $\cos\theta=4/5$, its complement satisfies $\arcsin(4/5)=\pi/2-\theta$. Therefore at $x=1$, the left side equals $\pi/2=\arcsin 1$. Oddness of principal inverse sine verifies $x=-1$ as well. Together with zero, the solution set is $\{-1,0,1\}$. The alternatives 0, 1 and 2 omit verified solutions.

Step 4: Conclusion The number of real solutions is $\boxed{3}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep zero before dividing by x, then use monotonicity in x squared and check the endpoint angles in their principal ranges.

14. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to : [4 marks]

(A) $\sqrt{10}$

(B) $\sqrt{6}$

(C) $\sqrt{11}$

(D) $\sqrt{7}$

Correct Answer: (B) $\sqrt{6}$

Solution:

Concept:

- The intercept length on an axis is the distance between the two real roots obtained by setting the other coordinate equal to zero. For a monic quadratic this distance is the square root of its discriminant.
- A line is tangent to a circle exactly when its perpendicular distance from the centre equals the radius. The requested distance is from the origin to that tangent line.

Step 1: Determine the circle coefficients On the x-axis, $y = 0$, so the endpoints satisfy $x^2 + ax + c = 0$. On the y-axis the equation is $y^2 + 2ay + c = 0$. The given intercept lengths therefore yield

$$a^2 - 4c = 8, \qquad 4a^2 - 4c = 20.$$

Subtracting gives $3a^2 = 12$. Since $a < 0$, we must take $a = -2$, and then $4 - 4c = 8$ gives $c = -1$.

Step 2: Find the centre and allowed tangent direction The circle is

$$x^2 + y^2 - 2x - 4y - 1 = 0, \qquad (x - 1)^2 + (y - 2)^2 = 6.$$

Its centre is $(1, 2)$ and radius is $\sqrt{6}$. The line $x + 2y = 0$ has slope $-1/2$. A perpendicular tangent has slope 2 and can be written as $2x - y + k = 0$. This direction condition fixes the ratio of its normal coefficients.

Step 3: Apply tangency and calculate the distance The centre-to-line distance is

$$\frac{|2(1) - 2 + k|}{\sqrt{2^2 + (-1)^2}} = \frac{|k|}{\sqrt{5}}.$$

Equating it to the radius gives $|k| = \sqrt{30}$. For either of these two parallel tangents, the origin-to-line distance is also $|k|/\sqrt{5}$, hence it equals $\sqrt{6}$. Both tangents have the same distance because the origin lies on their central parallel line $2x - y = 0$. None of $\sqrt{10}, \sqrt{11}, \sqrt{7}$ equals this required value.

Step 4: Conclusion The shortest required distance is $\boxed{\sqrt{6}}$. Hence, the correct option is (B).

Quick Tip to Solve: Obtain a and c from chord discriminants, then write the tangent with the required slope before applying the distance formula.

15. Let A(−1, 1), B(3, 4) and C(2, 0) be given three points.

A line y = mx, m > 0, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of △ABC and △PQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to : [4 marks]

(A) 1

(B) 3

(C) 2

(D) $\frac{4}{15}$

Correct Answer: (A) 1

Solution:

Concept:

- Triangles sharing the angle at C, with their other vertices on the same two lines, have area ratio equal to the product of their directed side ratios in absolute value.
- Write $P = C + t(A - C)$ and $Q = C + s(B - C)$. Then $[CPQ]/[CAB] = |ts|$. Intersect each parametrised line with $y = mx$.

Step 1: Locate both intersections From the given coordinates, $P = (2 - 3t, t)$ and $Q = (2 + s, 4s)$. Substitution in the line equation gives

$$t = m(2 - 3t), \qquad 4s = m(2 + s).$$

Hence

$$t = \frac{2m}{1 + 3m}, \qquad s = \frac{2m}{4 - m}.$$

The value $m = 4$ is excluded because the line would be parallel to BC, so there would be no finite intersection Q.

Step 2: Apply the required area ratio The condition is $|ts| = 1/3$, not a ratio of the lengths alone. Therefore

$$12m^2 = |(1 + 3m)(4 - m)|.$$

For $0 < m < 4$, this becomes

$$12m^2 = 4 + 11m - 3m^2, \qquad (m - 1)(15m + 4) = 0.$$

Only $m = 1$ is positive. For $m > 4$, the equation instead gives $9m^2 + 11m + 4 = 0$, which has no positive root. Thus an intersection outside the sides gives no extra admissible answer.

Step 3: Check the selected slope For $m = 1$, the parameters are $t = 1/2$ and $s = 2/3$, whose product is $1/3$. The other proposed positive slopes $3, 2, 4/15$ do not satisfy the derived area equation. This check also confirms that the desired triangle is nondegenerate and that both intersections lie on the sides for the selected value.

Step 4: Conclusion The required slope is $\boxed{m = 1}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the product of the two side parameters for an area ratio; do not equate either single side ratio to the area ratio.

16. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to : [4 marks]

(A) $\frac{4}{9}$

(B) $\frac{9}{56}$

(C) $\frac{11}{27}$

(D) $\frac{3}{7}$

Correct Answer: (A) $\frac{4}{9}$

Solution:

Concept:

- A number is divisible by 3 exactly when its digit sum is divisible by 3. The available digits have sum $0+1+\cdots+6=21$.
- A six-digit integer cannot begin with zero. Count equally likely valid integers, rather than treating the omitted digit as uniformly distributed.

Step 1: Count the sample space Choose the leading digit in 6 ways from $1,2,\ldots,6$. Fill the remaining five positions by an ordered selection from the remaining six digits. Thus

$$N = 6\,{}^{6}P_{5} = 6\cdot 720 = 4320.$$

Equivalently, there are ${}^{7}P_{6} - {}^{6}P_{5} = 5040 - 720$ arrangements after excluding leading zero. All these integers are the outcomes in the probability experiment.

Step 2: Identify and count favourable omissions If digit d is omitted, the retained sum is $21-d$. Hence the omitted digit must be $0, 3$, or 6. On omitting zero, all six retained digits are nonzero, giving $6! = 720$ integers. On omitting either 3 or 6, zero remains. Each such set gives $6!-5! = 720-120 = 600$ valid integers. The favourable total is therefore

$$F = 720 + 600 + 600 = 1920.$$

The three omitted-digit cases are disjoint, so their counts can be added.

Step 3: Form the probability Therefore

$$P(A) = \frac{1920}{4320} = \frac{4}{9}.$$

Simply using three acceptable omitted digits among seven would give $3/7$, but that ignores the different number of admissible leading-digit arrangements when zero is omitted. The values $9/56$ and $11/27$ also disagree with the direct favourable and total counts.

Step 4: Conclusion The probability is $\boxed{4/9}$. Hence, the correct option is (A).

Quick Tip to Solve: When zero is among the available digits, subtract arrangements beginning with zero separately for each chosen digit set.

17. The maximum value of

$$f(x)=\begin{vmatrix}\sin^2 x & 1+\cos^2 x & \cos 2x\\ 1+\sin^2 x & \cos^2 x & \cos 2x\\ \sin^2 x & \cos^2 x & \sin 2x\end{vmatrix},\ x\in R \textbf{ is :}$$ [4 marks]

(A) $\sqrt{5}$

(B) $\frac{3}{4}$

(C) 5

(D) $\sqrt{7}$

Correct Answer: (A) $\sqrt{5}$

Solution:

Concept:

- Subtracting one row from another does not change a determinant. Here both first rows differ from the third row by particularly simple vectors.
- For real θ, the maximum of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2+b^2}$, with equality attained for a suitable angle.

Step 1: Simplify the determinant by row operations Put $a=\sin^2 x$, $b=\cos^2 x$, $c=\cos 2x$, and $d=\sin 2x$. The identity $a+b=1$ will be used after expanding. Apply $R_1\to R_1-R_3$ and $R_2\to R_2-R_3$:

$$f(x)=\begin{vmatrix}0 & 1 & c-d\\ 1 & 0 & c-d\\ a & b & d\end{vmatrix}.$$

These operations involve no multiplication of rows, so no extra sign or numerical factor is introduced.

Step 2: Expand and use the identity Expansion along the first row gives

$$f(x)=-\{d-a(c-d)\}+b(c-d).$$

Consequently

$$f(x)=(a+b)(c-d)-d=c-2d=\cos 2x-2\sin 2x.$$

The vertical bars in the question denote a determinant, rather than a further absolute value of its result. Even an absolute value would have the same maximum here because this sinusoid attains both opposite extremes.

Step 3: Find and verify the amplitude By the Cauchy–Schwarz inequality,

$$\cos 2x-2\sin 2x\le\sqrt{1^2+(-2)^2}\sqrt{\cos^2 2x+\sin^2 2x}=\sqrt{5}.$$

Equality holds when $\cos 2x=1/\sqrt{5}$ and $\sin 2x=-2/\sqrt{5}$, which is possible for real x. Thus the bound is attained. The value 5 is the square of the amplitude, while $3/4$ and $\sqrt{7}$ do not equal the attained maximum.

Step 4: Conclusion The maximum value is $\boxed{\sqrt{5}}$. Hence, the correct option is (A).

Quick Tip to Solve: For determinants containing repeated trigonometric entries, subtract a common row first and use the amplitude only after simplifying.

18. If (x, y, z) be an arbitrary point lying on a plane P which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$ **is equal to :** [4 marks]

(A) 3

(B) 39

(C) -45

(D) 0

Correct Answer: (A) 3

Solution:

Concept:

- The intercept form of a plane with nonzero intercepts a, b, c is $x/a + y/b + z/c = 1$.
- If $u+v+w=0$, the factorisation $u^3+v^3+w^3-3uvw=(u+v+w)(u^2+v^2+w^2-uv-vw-wu)$ gives $u^3+v^3+w^3=3uvw$.

Step 1: Translate the plane constraint The three intercepts are all 42. Hence the plane is

$$\frac{x}{42}+\frac{y}{42}+\frac{z}{42}=1, \qquad x+y+z=42.$$

Set $u=x-11$, $v=y-19$, and $w=z-12$. Since $11+19+12=42$, these shifted coordinates satisfy $u+v+w=0$. This shift is the reason the apparently unrelated constants in the denominators are useful.

Step 2: Combine the three rational terms At every point where the displayed expression is defined, $uvw \neq 0$. Using the common denominator $u^2v^2w^2$,

$$\frac{u}{v^2w^2}+\frac{v}{u^2w^2}+\frac{w}{u^2v^2}=\frac{u^3+v^3+w^3}{u^2v^2w^2}=\frac{3}{uvw}.$$

The cubic identity follows from the plane constraint, so this simplification is valid for every admissible point, rather than a selected numerical example.

Step 3: Cancel the remaining rational contribution The last fraction in the question is

$$\frac{x+y+z}{14uvw}=\frac{42}{14uvw}=\frac{3}{uvw}.$$

It exactly cancels the sum just obtained. The entire expression is therefore $3+3/(uvw)-3/(uvw)=3$. Points on the plane where a denominator vanishes are excluded from the expression; they are not additional cases with a different value. The alternatives $39, -45, 0$ cannot result after this cancellation.

Step 4: Conclusion The expression equals $\boxed{3}$ wherever defined. Hence, the correct option is (A).

Quick Tip to Solve: When shifted coordinates sum to zero, combine cubic numerators and immediately use the identity giving three times their product.

19. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line y = x. Then the equation of tangent to C at P(2, 1) is : [4 marks]

(A) x − y = 1

(B) 2x + y = 5

(C) x + 3y = 5

(D) x + 2y = 4

Correct Answer: (A) x − y = 1

Solution:

Concept:

- Reflection in the line $y = x$ interchanges the two coordinates: the image of (u, v) is (v, u).
- Implicit differentiation gives the tangent slope of a smooth curve. A tangent at (x_1, y_1) must both pass through that point and have the derivative slope there.

Step 1: Find the reflected locus A point (u, v) on the original parabola satisfies $v^2 = 4u$. Its reflected coordinates are $x = v$ and $y = u$, so substitution gives

$$C: \quad x^2 = 4y.$$

The original parabola opens rightwards, while its reflected image opens upwards. This coordinate exchange is essential: using the tangent formula for $y^2 = 4x$ directly at the given point would refer to the wrong curve.

Step 2: Calculate the slope at the stated point First verify $P = (2, 1)$ belongs to the reflected locus: $2^2 = 4 \cdot 1$. Differentiating $x^2 = 4y$ gives

$$2x = 4\frac{dy}{dx}, \qquad \frac{dy}{dx} = \frac{x}{2}.$$

At $x = 2$, the slope is therefore 1. The point is a smooth point of this parabola, so this derivative determines its unique tangent.

Step 3: Write and compare the tangent equations The point-slope form yields

$$y - 1 = 1(x - 2), \qquad x - y = 1.$$

All four proposed lines happen to pass through $(2, 1)$, so incidence alone does not decide the answer. Their slopes are respectively $1, -2, -1/3, -1/2$. Only the first agrees with the calculated derivative. As an additional tangency check, substituting $y = x - 1$ in $x^2 = 4y$ gives $(x - 2)^2 = 0$, a repeated intersection at the point of contact.

Step 4: Conclusion The tangent is $\boxed{x - y = 1}$. Hence, the correct option is (A).

Quick Tip to Solve: Reflect the curve by swapping x and y before differentiating; checking that a candidate line passes through the point is not enough.

20. If the points of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to : [4 marks]

(A) 12

(B) 10

(C) 6

(D) 5

Correct Answer: (A) 12

Solution:

Concept:

- At a common point, both conic equations and the extra condition $y^2 = 3x^2$ must hold simultaneously.
- The parameter restriction $b > 4$ must be applied after solving the resulting polynomial; an algebraic root at the boundary is inadmissible.

Step 1: Use the circle and the extra condition At every stipulated intersection, substituting $y^2 = 3x^2$ into the circle gives

$$x^2 + 3x^2 = 4b, \qquad x^2 = b.$$

It follows that $y^2 = 3b$. Since $b > 4$, both squares are positive, so these relations describe real nonzero coordinates. There is no need to choose signs of x or y, because both original equations depend only on their squares.

Step 2: Substitute in the ellipse The ellipse condition now becomes

$$\frac{b}{16} + \frac{3b}{b^2} = 1, \qquad \frac{b}{16} + \frac{3}{b} = 1.$$

As $b > 0$, multiplication by $16b$ is legitimate and yields

$$b^2 - 16b + 48 = 0, \qquad (b-4)(b-12) = 0.$$

The two algebraic possibilities are 4 and 12. The strict given inequality discards 4, leaving 12.

Step 3: Verify the admissible value For $b = 12$, the squares are $x^2 = 12$, $y^2 = 36$. They satisfy the ellipse because $12/16 + 36/144 = 3/4 + 1/4 = 1$, and the circle because $12 + 36 = 48 = 4b$. Conversely, subtracting the equations in the variables x^2, y^2 fixes these same squares, so the actual intersection points obey the required relation. None of $10, 6, 5$ solves the parameter polynomial.

Step 4: Conclusion The parameter is $\boxed{b = 12}$. Hence, the correct option is (A).

Quick Tip to Solve: Work with the squared coordinates when intersecting centred conics; impose the extra locus condition before eliminating the parameter.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in A.P., where a, b > 0. Then 72(a + b) is equal to _____. [4 marks]

Solution:

Concept:

- For three consecutive terms in a geometric progression, the square of the middle term equals the product of the outer terms.
- For three consecutive terms in an arithmetic progression, twice the middle term equals the sum of the outer terms. Positivity selects the admissible algebraic root.

Step 1: Express the geometric-progression condition The consecutive terms are $1/16, a, b$, so

$$a^2 = \frac{b}{16}, \qquad b = 16a^2.$$

Because $a, b > 0$, their reciprocals exist and all later multiplication by a and b is valid. The order of the reciprocal terms in the second progression must be preserved: its middle term is $1/b$, not $1/a$.

Step 2: Apply the arithmetic-progression condition The terms $1/a, 1/b, 6$ are in A.P., giving

$$\frac{2}{b} = \frac{1}{a} + 6.$$

Insert $b = 16a^2$, then multiply through by $8a^2$:

$$\frac{1}{8a^2} = \frac{1}{a} + 6, \qquad 48a^2 + 8a - 1 = 0.$$

Factoring gives

$$(12a - 1)(4a + 1) = 0.$$

Thus $a = 1/12$ or $a = -1/4$. The negative root violates the explicit positivity requirement and is rejected.

Step 3: Find the second term and requested value For $a = 1/12$,

$$b = 16\left(\frac{1}{12}\right)^2 = \frac{1}{9}.$$

Check the two progressions: the G.P. ratios are $(1/12)/(1/16) = 4/3$ and $(1/9)/(1/12) = 4/3$. The reciprocal sequence is $12, 9, 6$, whose common difference is -3. Finally,

$$72(a + b) = 72\left(\frac{1}{12} + \frac{1}{9}\right) = 72 \cdot \frac{7}{36} = 14.$$

Both original conditions and the positivity constraint have therefore been checked directly.

Step 4: Conclusion The required numerical value is $\boxed{14}$.

Quick Tip to Solve: Write the middle-term identities first, and check the reciprocals in their printed order before solving the resulting quadratic.

22. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\beta\tan^{-1}\left(\frac{\gamma(x^2+1)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

where C is an arbitrary constant, then the value of $10(\alpha+\beta\gamma+\delta)$ is equal to _____.

[4 marks]

Solution:

Concept:

- An indefinite-integral identity must remain true after differentiation on any interval where its expressions are defined. A constant of integration disappears under this test.
- Every nonconstant term on the proposed right-hand side is a differentiable function of $u = x + 1/x$. At $x = 1$, $u = 2$ and $u\prime = 1 - 1/x^2 = 0$.

Step 1: Differentiate the proposed right-hand side Put $t = \tan^{-1} u$. Near $x = 1$, $t > 0$, so the printed real logarithm $\log_e t$ is well defined. Its derivative is $t\prime/t$, and

$$t\prime = \frac{u\prime}{1+u^2}.$$

Likewise,

$$\frac{d}{dx}\tan^{-1}(\gamma u) = \frac{\gamma u\prime}{1+\gamma^2u^2}.$$

For any finite real $\alpha, \beta, \gamma, \delta$, differentiation of the whole printed right-hand side therefore yields

$$u\prime\left[\frac{\alpha}{(1+u^2)\tan^{-1}u}+\frac{\beta\gamma}{1+\gamma^2u^2}+\frac{\delta}{1+u^2}\right].$$

All denominators are nonzero at $x = 1$, and the common factor $u\prime$ makes this derivative zero there.

Step 2: Compare with the literal integrand The numerator of the printed integrand at $x = 1$ is $\tan^{-1} 2$, since $x^2 - 1 = 0$. Its denominator is $5\tan^{-1} 2$. Thus the derivative of an actual antiderivative at that point must be

$$\frac{(1^2-1)+\tan^{-1}2}{(1^4+3\cdot1^2+1)\tan^{-1}2}=\frac{1}{5}.$$

This contradicts the zero derivative forced by the displayed right-hand side. The issue cannot be removed by choosing the integration constant or by adjusting any of the four real parameters. The given expressions are smooth near 1, so this is an exact local contradiction, not an endpoint or logarithm-domain issue.

Step 3: Conclusion As printed, no real parameters satisfy the stated antiderivative identity, so $10(\alpha+\beta\gamma+\delta)$ is undefined. The supplied answer key gives 6, but it does not follow from this printed identity.

Quick Tip to Solve: Before comparing coefficients in a proposed antiderivative, differentiate it at a point where its inner derivative is zero; a mismatch disproves the identity.

23. Let f : R → R and g : R → R be defined as

$f(x) = \begin{cases} x + a, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases}$ **and**

$g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 1)^2 + b, & x \geq 0 \end{cases}$,

where a, b are non-negative real numbers. If (gof) (x) is continuous for all x ∈ R, then a + b is equal to _____. [4 marks]

Solution:

Concept:

- A composition must be checked at every point where the inner function changes formula and where its value crosses a discontinuity of the outer function.
- The only possible break of g is at input zero: $g(0^-) = 1$, whereas $g(0) = 1 + b$. Since $a \geq 0$, separate $a = 0$ and $a > 0$.

Step 1: Examine the case $a > 0$ For negative x, the inner value is $f(x) = x + a$, which crosses zero at the negative point $x = -a$. Continuity there requires the two limits of g at zero to agree. Thus $1 = 1 + b$, giving $b = 0$. With this value, g is continuous on the whole real line. At the remaining boundary $x = 0$, the left limit of the composition is $g(a) = (a - 1)^2$, while its value and right limit equal $g(1) = 0$. Hence $(a - 1)^2 = 0$, so $a = 1$. This gives $(a, b) = (1, 0)$.

Step 2: Examine the case $a = 0$ For $x < 0$, $f(x) = x < 0$, and hence $g(f(x)) = x + 1$. Its left limit at zero is 1. For $x \geq 0$, $f(x) = |x - 1| \geq 0$, so the composition is $(|x - 1| - 1)^2 + b$, a continuous function on that half-line. Its value at zero is b. Therefore continuity requires $b = 1$, giving $(a, b) = (0, 1)$. These two cases exhaust the non-negative possibilities. In each case the composition is continuous elsewhere because its constituent formulas are continuous there.

Step 3: Conclusion Both admissible parameter pairs give $\boxed{a + b = 1}$.

Quick Tip to Solve: Do not check only $x = 0$: the negative branch of f also crosses the outer function's breakpoint at $x = -a$ when a is positive.

24. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	**10**	**2**	**2**
Observation II	**n**	**3**	**1**

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _____. [4 marks]

Solution:

Concept:

- For observations with mean μ and variance σ^2, the mean square is $\sigma^2 + \mu^2$. Combine sums and sums of squares before subtracting the square of the combined mean.
- The total number of observations is $10+n$; the combined variance uses this total as its denominator.

Step 1: Calculate the combined first and second moments The first set has sum $10(2) = 20$ and sum of squares $10(2 + 2^2) = 60$. The second set has sum $3n$ and sum of squares $n(1 + 3^2) = 10n$. Consequently, the combined mean and mean square are respectively

$$\mu = \frac{20 + 3n}{10 + n}, \qquad M_2 = \frac{60 + 10n}{10 + n}.$$

The difference between the set means must be included; simply averaging the two variances would discard this between-set contribution.

Step 2: Impose the specified variance and select a valid size Using $M_2 - \mu^2 = 17/9$, we obtain

$$\frac{60 + 10n}{10 + n} - \frac{(20 + 3n)^2}{(10 + n)^2} = \frac{17}{9}.$$

Multiplication by $9(10 + n)^2$ gives

$$9(n^2 + 40n + 200) = 17(n^2 + 20n + 100).$$

$$8n^2 - 20n - 100 = 0, \qquad 4(2n + 5)(n - 5) = 0.$$

Thus $n = 5$ or $n = -5/2$. A set size must be positive, so only 5 is allowed. As a check, for $n = 5$, $\mu = 7/3$, $M_2 = 22/3$, and their variance difference is $22/3 - 49/9 = 17/9$, as required.

Step 3: Conclusion The second set contains $\boxed{n = 5}$ observations.

Quick Tip to Solve: Combine raw sums and sums of squares; the difference of the two group means contributes to the combined variance.

25. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and k∈R.

If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2 + 1)\, b_2^2 \neq -2b_1b_2$, then the value of k is _____. [4 marks]

Solution:

Concept:

- From $A = XB$, compute both entries of A explicitly before squaring. The resulting scalar identity must be analysed with the exclusion exactly as printed.
- An equality for particular column vectors does not imply a matrix identity for all vectors. A requested unique parameter must be forced by the stated hypotheses.

Step 1: Factor the norm condition Matrix multiplication gives

$$a_1 = \frac{b_1 - b_2}{\sqrt{3}}, \qquad a_2 = \frac{b_1 + kb_2}{\sqrt{3}}.$$

Substitution into $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and multiplication by 3 yield

$$2b_1^2 + 2(k-1)b_1b_2 + (k^2+1)b_2^2 = 2b_1^2 + 2b_2^2.$$

Therefore

$$(k-1)\left[2b_1b_2 + (k+1)b_2^2\right] = 0.$$

The second factor contains $k + 1$, whereas the printed exclusion contains $k^2 + 1$. Consequently that exclusion does not, in general, rule out the second factor being zero.

Step 2: Exhibit two different admissible values For $k = 1$, take $B = (0, 1)^T$. Then $A = (-1, 1)^T/\sqrt{3}$, so the norm equality holds and the exclusion is $2 \neq 0$. For a different value, take $k = 2$ and $B = (-3/2, 1)^T$. This gives

$$A = \frac{1}{\sqrt{3}}(-5/2, 1/2)^T, \qquad a_1^2 + a_2^2 = \frac{13}{6}.$$

Meanwhile $\frac{2}{3}(b_1^2 + b_2^2) = \frac{2}{3}(9/4 + 1) = 13/6$, and the printed exclusion is satisfied because $(k^2 + 1)b_2^2 = 5 \neq 3 = -2b_1b_2$. Thus both 1 and 2 satisfy every stated hypothesis with appropriate real column vectors. A universal norm identity for every B would impose a stronger condition, but that quantifier is absent from the question.

Step 3: Conclusion The value of k is not uniquely determined by the printed assumptions. The supplied key gives $k = 1$, which is admissible, but $k = 2$ is an explicit counterexample to uniqueness.

Quick Tip to Solve: When factoring a parameter equation, compare the excluded expression with the factor exactly; a squared parameter cannot silently replace a linear one.

26. Let n be a positive integer. Let

$A = \sum_{k=0}^{n} (-1)^{kn} {}^{n}C_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$**. If**

$63A = 1 - \frac{1}{2^{30}}$**, then n is equal to _____.** [4 marks]

Solution:

Concept:

- The binomial theorem gives $\sum_{k=0}^{n}(-1)^k\binom{n}{k}t^k = (1-t)^n$. Apply it separately to each of the five terms inside the brackets.
- For $n > 0$, put $r = 2^{-n}$, so $0 < r < 1$. The resulting five terms form a geometric progression.

Step 1: Evaluate the five binomial sums Interchanging the two finite sums is valid. The five bases in the question are $1 - 2^{-j}$ for $j = 1, 2, 3, 4, 5$. Therefore each corresponding binomial sum equals $(2^{-j})^n$, and

$$A = 2^{-n} + 2^{-2n} + 2^{-3n} + 2^{-4n} + 2^{-5n} = r + r^2 + r^3 + r^4 + r^5.$$

This conversion incorporates both the alternating signs and every binomial coefficient; the large-looking expression becomes a sum of just five positive numbers.

Step 2: Find and justify the integer value Try $n = 6$, so $r = 1/64$. The geometric-series formula yields

$$A = \frac{r(1-r^5)}{1-r} = \frac{1-2^{-30}}{63}.$$

Hence $63A = 1 - 2^{-30}$, precisely the given equality. To establish uniqueness, observe that every term 2^{-jn}, for $j = 1, \ldots, 5$, decreases strictly as the positive integer n increases. Their sum therefore also decreases strictly. A prescribed value of A can be attained for at most one positive integer. Since 6 attains it, no other value can satisfy the condition.

Step 3: Conclusion The required positive integer is $\boxed{n = 6}$.

Quick Tip to Solve: Whenever alternating binomial coefficients multiply powers of several bases, sum each base separately using (1-t) raised to n.

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27. Let $\vec{c}$ be a vector perpendicular to the vectors, $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. If $\vec{c} \cdot \left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$ then the value of $\vec{c} \cdot \left(\vec{a} \times \vec{b}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- Two non-parallel vectors determine a normal direction given by their cross product. A vector perpendicular to both is a scalar multiple of that cross product.
- Use the given dot product to determine the scalar; then evaluate the requested dot product using the squared magnitude of the cross product.

Step 1: Find a common perpendicular direction Write $\vec{a} = (1, 1, -1)$ and $\vec{b} = (1, 2, 1)$. Direct expansion gives

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}.$$

This vector is nonzero, so the two given vectors are non-parallel. Hence their common perpendicular vectors form a one-dimensional line, and we can write $\vec{c} = t(3, -2, 1)$ for a real scalar t. The signs in the cross product matter because its direction is fixed by the order $\vec{a} \times \vec{b}$.

Step 2: Use the supplied dot product and evaluate the target The additional vector is $(1, 1, 3)$. Therefore

$$\vec{c} \cdot (1, 1, 3) = t(3 - 2 + 3) = 4t = 8,$$

so $t = 2$ and $\vec{c} = (6, -4, 2)$. This indeed satisfies $\vec{c} \cdot \vec{a} = 6 - 4 - 2 = 0$ and $\vec{c} \cdot \vec{b} = 6 - 8 + 2 = 0$. Finally,

$$\vec{c} \cdot (\vec{a} \times \vec{b}) = t|\vec{a} \times \vec{b}|^2 = 2(9 + 4 + 1) = 28.$$

The final dot product is a scalar; it has no spatial direction attached to it.

Step 3: Conclusion The requested scalar product is $\boxed{28}$.

Quick Tip to Solve: For a vector perpendicular to two known vectors, write it immediately as a scalar multiple of their cross product.

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28. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _____. [4 marks]

Solution:

Concept:

- For two sides enclosing angle A, the area is $\Delta = \frac{1}{2}(AB)(AC)\sin A$. Equality with half their product forces a right angle.
- In a right triangle the circumradius is half the hypotenuse, while the inradius is $r = \Delta/s$, where s is the semiperimeter.

Step 1: Determine the included angle and third side The given sides meet at vertex A. Substituting their lengths and the area into the area formula gives

$$30 = \frac{1}{2}(5)(12)\sin A = 30\sin A.$$

Thus $\sin A = 1$. Because A is an interior angle of a non-degenerate triangle, $0 < A < \pi$, and the only possibility is $A = 90^\circ$. Hence the sides of lengths 5 cm and 12 cm are the perpendicular legs, and the remaining side BC is the hypotenuse. Pythagoras gives

$$BC = \sqrt{5^2 + 12^2} = \sqrt{169} = 13\,\text{cm}.$$

Step 2: Calculate both radii and form the requested combination The circumcentre of a right triangle is the midpoint of its hypotenuse, so

$$R = \frac{13}{2}\,\text{cm}, \qquad 2R = 13\,\text{cm}.$$

The semiperimeter is

$$s = \frac{5+12+13}{2} = 15\,\text{cm}.$$

Using $\Delta = rs$, we obtain $r = 30/15 = 2\,\text{cm}$. This agrees with the right-triangle formula $r = (5+12-13)/2$. Therefore the requested sum is $2R + r = 13 + 2 = 15\,\text{cm}$. Both terms have dimensions of length, consistent with the requested unit.

Step 3: Conclusion The required value is $\boxed{2R + r = 15\,\text{cm}}$.

Quick Tip to Solve: Compare the given area with half the product of the two known sides first; equality identifies a right triangle immediately.

29. If the distance of the point (1, −2, 3) from the plane x + 2y − 3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to _____. [4 marks]

Solution:

Concept:

- The distance measured in a specified direction is the length of the segment from the point to its intersection with the plane, along a line parallel to that direction.
- The direction ratios are read after giving the symmetric denominators a consistent sign: $(2-y)/m = (y-2)/(-m)$. A displacement $t(a,b,c)$ has length $|t|\sqrt{a^2+b^2+c^2}$.

Step 1: Write the line through the given point The given line has direction vector $(3, -m, 1)$; its position does not matter because only parallelism is required. Therefore the line through $P = (1, -2, 3)$ can be written as

$$x = 1 + 3t, \qquad y = -2 - mt, \qquad z = 3 + t.$$

This uses the point specified in the question, rather than the point on the given reference line.

Step 2: Find its intersection with the plane Substitute these coordinates into the plane equation. The constant term is $1 - 4 - 9 + 10 = -2$, while the coefficient of t is $3 - 2m - 3 = -2m$. Hence

$$-2 - 2mt = 0, \qquad t = -\frac{1}{m}.$$

The case $m = 0$ gives a line parallel to the plane with no intersection, so it cannot give the finite stated distance.

Step 3: Use the specified segment length The square of the segment length is

$$|PQ|^2 = t^2(3^2 + m^2 + 1) = \frac{m^2 + 10}{m^2}.$$

Equating this to $7/2$ gives

$$2m^2 + 20 = 7m^2, \qquad m^2 = 4.$$

Both signs of m are admissible and produce the same length. The usual perpendicular-distance formula would answer a different question here.

Step 4: Conclusion Thus $m = \pm 2$, so the required numerical value is $\boxed{|m| = 2}$.

Quick Tip to Solve: For a distance measured parallel to a line, intersect that parallel through the point with the plane; do not use perpendicular distance.

30. $S_n(x) = \log_{a^{1/2}}x + \log_{a^{1/3}}x + \log_{a^{1/6}}x + \log_{a^{1/11}}x + \log_{a^{1/18}}x + \log_{a^{1/27}}x +$ ..... up to n-terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _____. [4 marks]

Solution:

Concept:

- Change of base gives $\log_{a^{1/d}} x = d \log_a x$, for $a > 1$ and $x > 0$. Thus each logarithm contributes its base denominator as a coefficient.
- The denominator sequence $2, 3, 6, 11, 18, 27, \ldots$ has differences $1, 3, 5, 7, 9, \ldots$, so its k-th term is $(k-1)^2 + 2 = k^2 - 2k + 3$.

Step 1: Sum the coefficients Use the standard sums of the first n integers and their squares:

$$\begin{aligned} C_n &= \sum_{k=1}^{n} (k^2 - 2k + 3) \\ &= \frac{n(n+1)(2n+1)}{6} - n(n+1) + 3n \\ &= \frac{n(2n^2 - 3n + 13)}{6}. \end{aligned}$$

It follows that $S_n(x) = C_n \log_a x$. The first term is included: omitting the initial coefficient 2 would change both equations.

Step 2: Determine the two logarithms For $n = 24$, the coefficient is $C_{24} = 4372$, whereas $C_{12} = 530$. The given equations therefore give

$$\log_a x = \frac{1093}{4372} = \frac{1}{4}, \qquad \log_a(2x) = \frac{265}{530} = \frac{1}{2}.$$

The same coefficient formula applies to the argument $2x$; changing the argument does not change the denominator sequence.

Step 3: Eliminate the unknown argument Subtracting the first logarithm from the second removes x:

$$\log_a 2 = \log_a(2x) - \log_a x = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

Consequently $a^{1/4} = 2$, so $a = 2^4$. As a check, this gives $x = a^{1/4} = 2$, and both original sums have positive, valid arguments.

Step 4: Conclusion The required value of the base is $\boxed{a = 16}$.

Quick Tip to Solve: Convert every logarithm to base a first; then subtract the equations for x and 2x to eliminate x.

Physics – Section A: Multiple Choice Questions (80 marks)

31. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively. (Density of water : 10^3 kg/m^3) [4 marks]

(A) Steady flow to unsteady flow

(B) Unsteady to steady flow

(C) Remains turbulent flow

(D) Remains steady flow

Correct Answer: (D) Remains steady flow

Solution:

Concept:

- For flow through a circular tap, the mean speed is $v = Q/(\pi r^2)$, where Q is volume flow rate and r is the radius.
- The Reynolds number based on diameter is $\mathrm{Re} = \rho v(2r)/\eta = 2\rho Q/(\pi r\eta)$. Values well below the transition range, roughly 2000, describe steady laminar flow.

Step 1: Convert the given quantities to SI units The radius is $r = 0.5\,\mathrm{cm} = 5.0 \times 10^{-3}\,\mathrm{m}$, and $\eta = 10^{-3}\,\mathrm{Pa\,s}$. Since one litre is $10^{-3}\,\mathrm{m}^3$ and one minute is 60 seconds,

$$Q_1 = \frac{0.18 \times 10^{-3}}{60} = 3.0 \times 10^{-6}\,\mathrm{m^3 s^{-1}},$$

$$Q_2 = \frac{0.48 \times 10^{-3}}{60} = 8.0 \times 10^{-6}\,\mathrm{m^3 s^{-1}}.$$

The density is $10^3\,\mathrm{kg\,m^{-3}}$.

Step 2: Calculate and compare Reynolds numbers Using the diameter, not the radius, in the definition gives

$$\mathrm{Re}_1 = \frac{2(10^3)(3.0 \times 10^{-6})}{\pi(5.0 \times 10^{-3})(10^{-3})} \approx 382,$$

$$\mathrm{Re}_2 = \frac{2(10^3)(8.0 \times 10^{-6})}{\pi(5.0 \times 10^{-3})(10^{-3})} \approx 1019.$$

Both values are below the onset of turbulent flow. Increasing the rate increases the Reynolds number, but does not necessarily change the nature of flow.

Step 3: Select the description of the flow The initial flow is laminar and the final flow is also laminar. Therefore neither a steady-to-unsteady transition nor its reverse occurs. The description that the flow remains turbulent is also inconsistent with both calculated Reynolds numbers. Among the listed

descriptions only remaining steady fits the two endpoints.

Step 4: Conclusion The water remains in steady flow. The stated values give Reynolds numbers below the transition range at both rates, so they support (D), in disagreement with the supplied key claiming a transition. Hence, the correct option is (D).

Quick Tip to Solve: Use the diameter in Reynolds number, and convert L/min to cubic metres per second before comparing with the laminar limit.

32. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index? Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively. [4 marks]

(A) Zero

(B) Infinite

(C) 1

(D) $\frac{R_1R_2}{R_1-R_2}$

Correct Answer: (B) Infinite

Solution:

Concept:

- The thin-lens maker formula in a medium is $1/f = (n_\ell/n_m - 1)(1/R_1 - 1/R_2)$, where n_ℓ and n_m are the lens and surrounding-medium indices.
- Optical power depends on the refractive-index contrast as well as the surface curvatures. Equal indices remove refraction at each lens surface.

Step 1: Calculate the relative refractive index The material of the lens has index $n_\ell = 1.4$, and the surrounding medium also has index $n_m = 1.4$. Thus

$$\frac{n_\ell}{n_m} = \frac{1.4}{1.4} = 1.$$

The given designation as a converging lens describes its action with a suitable surrounding medium; it does not guarantee a positive power in every medium. The absolute index 1.4 cannot be inserted in place of the relative index in the immersed-lens formula.

Step 2: Find the optical power Substitution in the lens maker formula yields

$$\frac{1}{f} = (1-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0.$$

Consequently the focal length is infinite, whatever the finite surface radii may be. Parallel incident rays remain parallel because no change in optical index occurs at either boundary. There is no finite point at which those rays converge.

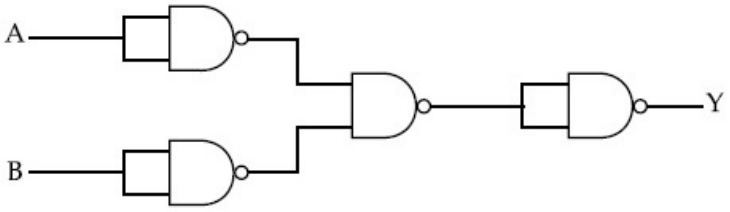
Step 3: Compare with the alternatives Zero focal length would imply unbounded power, the opposite of the result here. The proposed finite value 1 and the expression involving only the radii also fail to

include the vanishing index contrast. It is the power $1/f$, rather than the focal length f, that becomes zero when the two refractive indices are equal.

Step 4: Conclusion The lens has zero power and an infinite focal length. Hence, the correct option is (B).

Quick Tip to Solve: For an immersed lens, replace the material index by its index relative to the surrounding medium before applying lens maker formula.

33. The following logic gate is equivalent to : [4 marks]



(A) NOR Gate

(B) NAND Gate

(C) OR Gate

(D) AND Gate

Correct Answer: (A) NOR Gate

Solution:

Concept:

- A NAND gate gives the complement of the AND of its inputs: $\overline{AB}$. If both inputs are joined, the output is $\overline{AA} = \overline{A}$, so the gate acts as an inverter.
- De Morgan's law is $\overline{\overline{A}\,\overline{B}} = A + B$, where multiplication denotes AND and addition denotes OR.

Step 1: Reduce the two input gates The diagram shows the two inputs of the upper first-stage NAND joined to A, and the two inputs of the lower one joined to B. Their outputs are therefore

$$X_1 = \overline{A \cdot A} = \overline{A}, \qquad X_2 = \overline{B \cdot B} = \overline{B}.$$

The small circles on the gate outputs indicate inversion; ignoring those circles would give a different circuit.

Step 2: Reduce the middle and final gates The middle NAND takes X_1 and X_2, giving

$$X_3 = \overline{X_1 X_2} = \overline{\overline{A}\,\overline{B}} = A + B.$$

The final NAND again has its two inputs tied together. It inverts this middle-stage output, so

$$Y = \overline{X_3 X_3} = \overline{A + B}.$$

The last inverter must be included: the middle stage alone would give OR.

Step 3: Identify the equivalent function The output is 1 only for $A = B = 0$; it is 0 for input pairs $(0,1),(1,0),(1,1)$. This is the NOR truth table. OR has the opposite output, AND is high for $(1,1)$, and NAND is high for three input pairs. Each of those alternatives therefore differs from the complete circuit.

Step 4: Conclusion The complete network implements $Y = \overline{A+B}$, a NOR gate. Hence, the correct option is (A).

Quick Tip to Solve: Treat a NAND with its two inputs tied together as NOT, then simplify the circuit one stage at a time.

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34. Statement I : A cyclist is moving on an unbanked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve. (g = 9.8 m/s^2)

Statement II : If the road is banked at an angle of 45°, cyclist can cross the curve of 2m radius with the speed of 18.5 kmh^{-1} without slipping.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Statement I is incorrect and statement II is correct

(B) Both statement I and statement II are true

(C) Statement I is correct and statement II is incorrect

(D) Both statement I and statement II are false

Correct Answer: (B) Both statement I and statement II are true

Solution:

Concept:

- On an unbanked road, static friction supplies centripetal force. The no-slip limit is $v \le \sqrt{\mu_s r g}$.
- On a banked road, friction can act up or down the slope. The upper limit is $v_{\max}^2 = rg(\tan\theta + \mu_s)/(1 - \mu_s \tan\theta)$; the lower limit is $v_{\min}^2 = rg(\tan\theta - \mu_s)/(1 + \mu_s \tan\theta)$ when positive.

Step 1: Test statement I on the horizontal road For $\mu_s = 0.2$, $r = 2\,\text{m}$, and $g = 9.8\,\text{m s}^{-2}$,

$$v_{\max} = \sqrt{0.2 \times 2 \times 9.8} = 1.98\,\text{m s}^{-1} \approx 7.13\,\text{km h}^{-1}.$$

The stated speed is $7/3.6 = 1.944\,\text{m s}^{-1}$, below this limit. The required friction is less than its limiting value, so the cyclist does not slip. Statement I is correct.

Step 2: Test statement II on the banked road Here $\tan 45^\circ = 1$, and the same coefficient applies. The admissible limits are

$$v_{\max} = \sqrt{19.6\frac{1.2}{0.8}} = \sqrt{29.4} \approx 5.422\,\text{m s}^{-1},$$

$$v_{\min} = \sqrt{19.6\frac{0.8}{1.2}} \approx 3.615\,\text{m s}^{-1}.$$

The proposed $18.5\,\text{km h}^{-1}$ equals $5.139\,\text{m s}^{-1}$, between these limits. Static friction can prevent slipping,

so statement II is correct.

Step 3: Choose the pair of truth values Both statements meet their respective no-slip conditions. The frictionless design speed $\sqrt{rg\tan\theta}$ is not the only allowed speed when friction is present. Thus it would be incorrect to reject statement II merely because 18.5 km/h exceeds the design speed. All choices declaring either statement false are excluded.

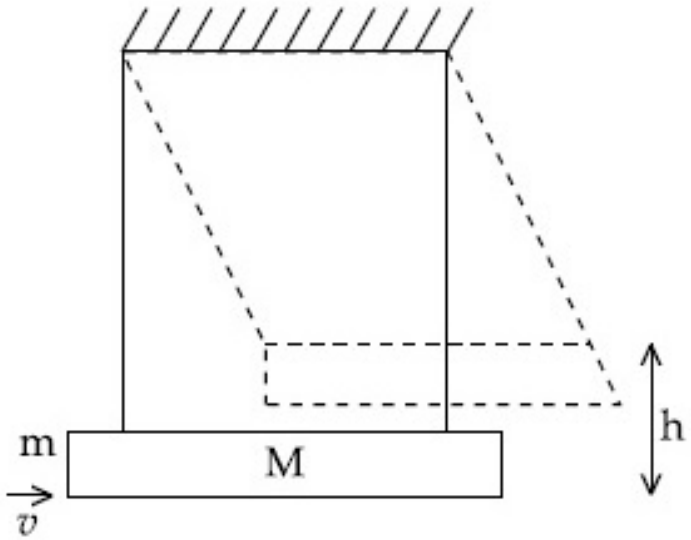
Step 4: Conclusion Both statement I and statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: Convert km/h to m/s first, and use the frictional banked-road speed range rather than only the frictionless design speed.

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35. A large block of wood of mass M = 5.99 kg is hanging from two long massless cords. A bullet of mass m = 10 g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance h = 9.8 cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is :

(take g = 9.8 ms^{-2}) [4 marks]



(A) 831.4 m/s

(B) 811.4 m/s

(C) 841.4 m/s

(D) 821.4 m/s

Correct Answer: (A) 831.4 m/s

Solution:

Concept:

- During the short embedding collision, horizontal momentum is conserved: $mu = (M + m)V$. Kinetic energy is not conserved because the collision is perfectly inelastic.

- During the subsequent upward swing, mechanical energy is conserved: $\frac{1}{2}(M+m)V^2 = (M+m)gh$. These are separate physical stages.

Step 1: Use the height to determine the post-collision speed Convert the bullet mass and rise before substitution: $m = 10\,\mathrm{g} = 0.010\,\mathrm{kg}$ and $h = 9.8\,\mathrm{cm} = 0.098\,\mathrm{m}$. At the highest point the combined mass is momentarily at rest. During the swing, the cords exert tension perpendicular to the motion and do no work. Therefore

$$V = \sqrt{2gh} = \sqrt{2(9.8)(0.098)} = \sqrt{1.9208} \approx 1.38593\,\mathrm{m\,s^{-1}}.$$

Step 2: Apply momentum conservation to the impact The block is initially at rest, and the bullet embeds in it, so immediately after impact both move together. With the long cords initially vertical, the external horizontal impulse during the brief collision is negligible. Thus

$$u = \frac{M+m}{m}V = \frac{5.99 + 0.010}{0.010}(1.38593).$$

The total moving mass is exactly $6.00\,\mathrm{kg}$, giving the multiplying factor 600, not 599.

Step 3: Evaluate and select the matching speed The result is

$$u \approx 600(1.38593) = 831.56\,\mathrm{m\,s^{-1}}.$$

The listed answer is about $831.4\,\mathrm{m\,s^{-1}}$. This is the closest option; the alternatives differ by much more than the small numerical discrepancy in the listed value. Conserving kinetic energy across the embedding collision would incorrectly ignore the impact energy loss.

Step 4: Conclusion The bullet speed is approximately $8.32 \times 10^2\,\mathrm{m\,s^{-1}}$, matching $831.4\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Work backwards from the rise using energy, then use momentum for the collision; never conserve kinetic energy across embedding.

36. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2 s? [4 marks]

(A) $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y-axis

(B) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(C) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

(D) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

Correct Answer: (A) $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y-axis

Solution:

Concept:

- The instantaneous direction of motion is the direction of velocity, not of acceleration. Substitute the given time into all three components of $\vec{v}$.
- If a vector makes an angle θ_y with the positive y-axis, its parallel component is v_y and the magnitude perpendicular to that axis is $\sqrt{v_x^2 + v_z^2}$. Thus $\tan\theta_y = \sqrt{v_x^2 + v_z^2}/v_y$ when $v_y > 0$.

Step 1: Evaluate the instantaneous velocity At $t = 2\,\text{s}$, the three components are

$$v_x = 0.5(2)^2 = 2, \qquad v_y = 3(2) = 6, \qquad v_z = 9 \quad (\text{m s}^{-1}).$$

Therefore $\vec{v}(2) = 2\hat{i} + 6\hat{j} + 9\hat{k}$. The nonzero z-component means that the direction cannot be found by treating the motion as two-dimensional in the xy-plane. The wording about acceleration does not replace the given velocity expression.

Step 2: Resolve relative to the stated axis The component along the y-axis is 6, while its perpendicular component lies in the xz-plane and has magnitude

$$v_{\perp y} = \sqrt{2^2 + 9^2} = \sqrt{85}.$$

Consequently

$$\tan\theta_y = \frac{\sqrt{85}}{6}, \qquad \theta_y = \tan^{-1}\left(\frac{\sqrt{85}}{6}\right).$$

Option B would require a different ratio. For the positive x-axis the actual tangent is $\sqrt{6^2 + 9^2}/2 = \sqrt{117}/2$, so neither C nor D gives that angle. This also checks the importance of retaining all three velocity components.

Step 3: Conclusion The mosquito moves at $\tan^{-1}(\sqrt{85}/6)$ from the positive y-axis at 2 s. Hence, the correct option is (A).

Quick Tip to Solve: For an angle from a coordinate axis, divide the magnitude of the other two components by the component along that axis.

37. Red light differs from blue light as they have : [4 marks]

(A) Different frequencies and same wavelengths

(B) Different frequencies and different wavelengths

(C) Same frequencies and different wavelengths

(D) Same frequencies and same wavelengths

Correct Answer: (B) Different frequencies and different wavelengths

Solution:

Concept:

- Colour identifies the frequency of visible light: blue light has a higher frequency than red light. Frequency is fixed by the source and does not change merely on crossing an interface.
- For electromagnetic radiation in vacuum, $c = \nu\lambda$, where c is the common speed, ν the frequency and λ the wavelength. A larger frequency therefore corresponds to a smaller wavelength.

Step 1: Compare the frequencies of the colours Red and blue occupy different portions of the visible spectrum. Red is at the lower-frequency end and blue at the higher-frequency end, so

$$\nu_{\text{blue}} > \nu_{\text{red}}.$$

They cannot have the same frequency: waves of the same frequency in the visible range correspond to the same spectral colour. Thus the two options that assert equal frequencies, C and D, already fail this basic colour comparison.

Step 2: Use the wave relation to compare wavelengths Comparing them in vacuum, both waves travel at c. Therefore

$$\lambda_{\text{red}} = \frac{c}{\nu_{\text{red}}}, \qquad \lambda_{\text{blue}} = \frac{c}{\nu_{\text{blue}}}, \qquad \lambda_{\text{red}} > \lambda_{\text{blue}}.$$

Their wavelengths are different as well as their frequencies. Option A incorrectly keeps the wavelength unchanged while changing the frequency, which would violate the common-vacuum-speed relation. Passage into an ordinary transparent medium does not make the two colours have the same source frequency; it changes their speeds and wavelengths according to that medium.

Step 3: Conclusion Red and blue light have different frequencies and different wavelengths; red has the longer wavelength. Hence, the correct option is (B).

Quick Tip to Solve: Remember the order red to violet: frequency increases while vacuum wavelength decreases; the speed in vacuum remains the same.

38. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths? (m_p = 1.00727u m_e = 0.00055u) [4 marks]

(A) 41.4 : 1

(B) $(1860)^2$: 1

(C) 1860 : 1

(D) 43 : 1

Correct Answer: (D) 43 : 1

Solution:

Concept:

- A particle of charge magnitude e, accelerated from rest through a potential difference V, gains kinetic energy eV. Electron and proton have equal charge magnitudes.
- For nonrelativistic motion, $p = \sqrt{2meV}$ and the de Broglie relation is $\lambda = h/p = h/\sqrt{2meV}$. At the same accelerating potential, wavelength varies as $m^{-1/2}$.

Step 1: Form the wavelength ratio before substituting The electron and proton acquire the same kinetic energy, $100\,\text{eV}$, rather than the same speed or momentum. For the two particles,

$$\frac{\lambda_e}{\lambda_p} = \frac{h/\sqrt{2m_e eV}}{h/\sqrt{2m_p eV}} = \sqrt{\frac{m_p}{m_e}}.$$

The constants h, e and the common voltage cancel, so there is no need to calculate either absolute wavelength. At this energy the nonrelativistic expression is entirely adequate for both particles.

Step 2: Evaluate the mass dependence and select the nearest value Using the supplied masses in the same atomic-mass units gives

$$\frac{m_p}{m_e} = \frac{1.00727}{0.00055} \simeq 1831.4, \qquad \frac{\lambda_e}{\lambda_p} \simeq \sqrt{1831.4} \simeq 42.8.$$

Hence the nearest ratio is $43 : 1$. The electron is lighter, so its wavelength must be larger, consistent with this ordering. A ratio proportional to the mass or to its square, as in C or B, uses the wrong power. Option A is numerically farther from the square root of the stated mass ratio.

Step 3: Conclusion The ratio of electron wavelength to proton wavelength is approximately $43 : 1$. Hence, the correct option is (D).

Quick Tip to Solve: Equal accelerating voltages give equal kinetic energies for an electron and proton; take the square root of the inverse mass ratio for their wavelengths.

39. Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. (k = 1.38×10^{-23} JK^{-1}) [4 marks]

(A) 32 nm

(B) 58 nm

(C) 86 nm

(D) 102 nm

Correct Answer: (D) 102 nm

Solution:

Concept:

- For hard-sphere molecules of collision diameter d and number density n, the mean free path is $\lambda = 1/(\sqrt{2}\pi d^2 n)$. The factor $\sqrt{2}$ accounts for relative molecular motion.
- The ideal-gas equation in molecular form is $P = nkT$. Eliminating number density gives $\lambda = kT/(\sqrt{2}\pi d^2 P)$, with absolute temperature in kelvin and SI units throughout.

Step 1: Convert the data and substitute the formula The temperature and diameter must first be expressed as

$$T = 27 + 273 = 300\,\mathrm{K}, \qquad d = 0.3\,\mathrm{nm} = 3.0 \times 10^{-10}\,\mathrm{m}.$$

The pressure is 1.01×10^5 Pa. Thus

$$\lambda = \frac{(1.38 \times 10^{-23})(300)}{\sqrt{2}\,\pi(3.0 \times 10^{-10})^2(1.01 \times 10^5)}.$$

The stated molecular diameter is already the collision diameter, so it should not be doubled as though it were a radius.

Step 2: Calculate and compare the options The numerator is 4.14×10^{-21} in SI units, and the denominator is approximately

$$1.414 \times 3.142 \times 9.0 \times 10^{-20} \times 1.01 \times 10^5 \simeq 4.04 \times 10^{-14}.$$

Therefore

$$\lambda \simeq 1.025 \times 10^{-7}\,\mathrm{m} = 102.5\,\mathrm{nm}.$$

This is closest to $102\,\mathrm{nm}$. The values 32, 58 and 86 nm do not follow from the supplied collision diameter and pressure. The result is much greater than a single molecular diameter, as expected for a dilute gas.

Step 3: Conclusion The mean free path is approximately $102\,\mathrm{nm}$. Hence, the correct option is (D).

Quick Tip to Solve: In the mean-free-path formula, square the diameter in metres and use kelvin; only convert the final length back to nanometres.

40. A charge Q is moving $\overrightarrow{dl}$ distance in the magnetic field $\overrightarrow{B}$. Find the value of work done by $\overrightarrow{B}$. [4 marks]

(A) Zero

(B) -1

(C) Infinite

(D) 1

Correct Answer: (A) Zero

Solution:

Concept:

- The magnetic part of the Lorentz force on charge Q moving with velocity $\vec{v}$ is $\vec{F}_B = Q\vec{v} \times \vec{B}$. It is perpendicular to the instantaneous velocity.
- The work along an infinitesimal displacement is $dW = \vec{F} \cdot d\vec{l}$, and the displacement of the moving charge satisfies $d\vec{l} = \vec{v}\,dt$. A force perpendicular to motion transfers no energy.

Step 1: Write the work as a scalar product Use the magnetic force, rather than the magnetic-field vector itself, in the definition of mechanical work:

$$dW_B = \left(Q\vec{v} \times \vec{B}\right) \cdot d\vec{l}.$$

Because the displacement during the time interval dt is along $\vec{v}$, this becomes

$$dW_B = Q\left(\vec{v} \times \vec{B}\right) \cdot \vec{v}\,dt.$$

The vector $\vec{v} \times \vec{B}$ is perpendicular to $\vec{v}$ whenever it is nonzero. If the two vectors are parallel, the force itself vanishes and the same conclusion follows.

Step 2: Use orthogonality along the whole trajectory The scalar product of perpendicular vectors is zero at every instant, so

$$dW_B = 0, \qquad W_B = \int dW_B = 0.$$

A magnetic force can change the direction of the velocity and hence bend the trajectory, but it cannot change the speed or kinetic energy of a point charge through this magnetic part of the Lorentz force. Neither the value nor the sign of Q alters this result. The nonzero options -1 and 1, as well as infinite work, contradict the vanishing instantaneous magnetic power $\vec{F}_B \cdot \vec{v}$.

Step 3: Conclusion The work done by the magnetic field on the moving charge is zero. Hence, the correct option is (A).

Quick Tip to Solve: For a point charge, check magnetic power directly: $(Q\vec{v} \times \vec{B}) \cdot \vec{v} = 0$, even when the path is curved.

41. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1 kg (measured using a scale of least count = 1 g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment? [4 marks]

(A) 0.14%

(B) 9%

(C) 1.4%

(D) 0.9%

Correct Answer: (C) 1.4%

Solution:

Concept:

- Young's modulus is stress divided by strain: $Y = FL/(A\ell) = MgL/(\pi r^2 \ell)$, where ℓ is the extension and L the original length.
- For the maximum fractional error in a product or quotient, add the absolute fractional errors multiplied by their powers. Here $\Delta Y/Y = \Delta M/M + \Delta L/L + 2\Delta r/r + \Delta \ell/\ell$, treating g and π as constants.

Step 1: Calculate the four fractional uncertainties Use the given least counts as the absolute measurement uncertainties, keeping each numerator and denominator in the same units:

$$\frac{\Delta M}{M} = \frac{1\,\text{g}}{1000\,\text{g}} = 0.001, \qquad \frac{\Delta L}{L} = \frac{1\,\text{mm}}{1000\,\text{mm}} = 0.001.$$

$$\frac{\Delta r}{r} = \frac{0.001\,\text{cm}}{0.2\,\text{cm}} = 0.005, \qquad \frac{\Delta \ell}{\ell} = \frac{0.001\,\text{cm}}{0.5\,\text{cm}} = 0.002.$$

The radius enters as a square because the cross-sectional area is πr^2.

Step 2: Combine the errors and express the percentage The maximum fractional error is therefore

$$\frac{\Delta Y}{Y} = 0.001 + 0.001 + 2(0.005) + 0.002 = 0.014.$$

Multiplying by 100 gives the percentage error:

$$100\frac{\Delta Y}{Y} = 1.4\%.$$

Although radius and elongation appear in the denominator of Y, their error contributions add when finding the maximum uncertainty; they do not cancel the numerator contributions. Omitting the factor two for radius gives an incorrect error estimate. Options A, B and D do not match the sum obtained from all four stated least counts.

Step 3: Conclusion The fractional error is 0.014, equivalent to 1.4%. Hence, the correct option is (C).

Quick Tip to Solve: Write the measured quantity as a product of powers first; the radius contributes twice its fractional error because area contains r^2.

42. A resistor develops 500 J of thermal energy in 20 s when a current of 1.5A is passed through it. If the current is increased from 1.5A to 3A, what will be the energy developed in 20 s. [4 marks]

(A) 1000 J

(B) 2000 J

(C) 1500 J

(D) 500 J

Correct Answer: (B) 2000 J

Solution:

Concept:

- Joule's heating law gives thermal energy $H = I^2Rt$, where I is current, R resistance and t the duration of current flow.
- For the same resistor and the same time interval, R and t are unchanged. Therefore the energy ratio is $H_2/H_1 = (I_2/I_1)^2$, so heating depends on the square of current rather than directly on current.

Step 1: Identify the fixed quantities and take the ratio Initially $H_1 = 500\,\text{J}$, $I_1 = 1.5\,\text{A}$, and $t_1 = 20\,\text{s}$. In the second case the same resistor carries $I_2 = 3\,\text{A}$ for $t_2 = 20\,\text{s}$. Applying the heating law to both cases gives

$$\frac{H_2}{H_1} = \frac{I_2^2 R t_2}{I_1^2 R t_1} = \left(\frac{3}{1.5}\right)^2 = 4.$$

Both the resistance and the duration cancel; no separate value of the resistance is required.

Step 2: Calculate the energy and check the alternatives The current doubles, so the heating rate and the energy over the fixed interval both become four times their original values:

$$H_2 = 4H_1 = 4(500) = 2000\,\text{J}.$$

As an independent check, the initial power is $500/20 = 25\,\text{W}$, giving $R = 25/(1.5)^2 = 100/9\,\Omega$. At $3\,\text{A}$, the power is $9(100/9) = 100\,\text{W}$, which yields $100(20) = 2000\,\text{J}$. Option A incorrectly assumes heating is linear in current. C does not follow the square law, while D would require the heating to stay unchanged despite the larger current.

Step 3: Conclusion The thermal energy developed in $20\,\text{s}$ is $2000\,\text{J}$. Hence, the correct option is (B).

Quick Tip to Solve: For unchanged resistance and time, doubling current quadruples Joule heat; compare currents before calculating resistance.

43. Find out the surface charge density at the intersection of point x = 3 m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space. [4 marks]

(A) 0.424 nC m^{-2}

(B) 4.0 nC m^{-2}

(C) 47.88 C/m

(D) 0.07 nC m^{-2}

Correct Answer: (A) 0.424 nC m^{-2}

Solution:

Concept:

- For an infinitely long uniform line charge, cylindrical symmetry gives the radial electric field $E = \lambda/(2\pi\varepsilon_0 r)$, where r is perpendicular distance from the line.
- The normal electric flux density is $D_n = \varepsilon_0 E_n$, measured in $\mathrm{C\,m^{-2}}$. This is the surface-density quantity used in the options; it is distinct from the field strength measured in newtons per coulomb.

Step 1: Locate the point and its normal The intersection is $(3, 0, 0)$, at perpendicular distance $r = 3\,\mathrm{m}$ from the charged z-axis. Since the line charge is positive, its field there points along the positive x-axis. The normal to the plane $x = 3\,\mathrm{m}$ is also along that axis, so the field is normal to the plane and no cosine factor reduces its normal component.

Step 2: Apply the cylindrical Gauss law A coaxial cylinder of radius r and length L encloses charge λL. The end faces carry no flux because their normals point along the line. Hence $E(2\pi rL) = \lambda L/\varepsilon_0$, giving the normal electric flux density

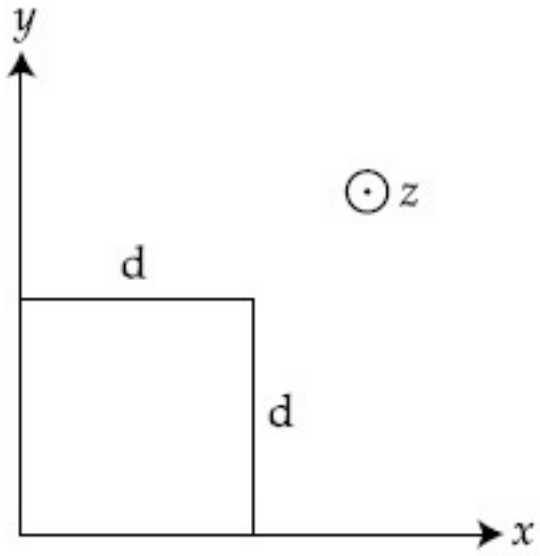
$$D_n = \varepsilon_0 E = \frac{\lambda}{2\pi r} = \frac{8}{2\pi(3)}\,\mathrm{nC\,m^{-2}} = 0.4244\,\mathrm{nC\,m^{-2}}.$$

The factor ε_0 cancels: including it in the final denominator would calculate electric field rather than the requested density. The values 4.0 and 0.07 do not satisfy this cylindrical flux relation, and option (C) also has the wrong dimensions for an areal density.

Step 3: Conclusion The normal electric flux density, termed surface charge density in the question, is $0.424\,\mathrm{nC\,m^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a line charge use cylindrical area $2\pi rL$; divide charge per unit length by $2\pi r$ to obtain electric flux density.

44. The magnetic field in a region is given by $\vec{B} = B_o\left(\frac{x}{a}\right)\hat{k}$. A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v}$ = $v_0\hat{i}$. The emf induced in the loop is : [4 marks]



(A) $\frac{B_o v_o d^2}{2a}$

(B) $\frac{B_o v_o^2 d}{2a}$

(C) $\frac{B_o v_o d}{2a}$

(D) $\frac{B_o v_o d^2}{a}$

Correct Answer: (D) $\frac{B_o v_o d^2}{a}$

Solution:

Concept:

- Faraday's law gives $\mathcal{E} = -d\Phi_B/dt$, with $\Phi_B = \int \vec{B}\cdot d\vec{A}$. A field varying with position must be integrated over the loop.
- For rigid translation along x, let the left edge have coordinate $X(t)$. Then $dX/dt = v_0$, while both the width and height of the square remain d.

Step 1: Integrate the field over the moving square The square is in the xy-plane and its area normal is along z, parallel to the magnetic field. A narrow vertical strip at coordinate x has area $d\,dx$. The field on that strip is $B_0 x/a$; it cannot be replaced everywhere by its value at a single fixed edge. The instantaneous magnetic flux is therefore

$$\Phi_B = \int_X^{X+d} \frac{B_0 x}{a}\, d\, dx = \frac{B_0 d}{2a}\left[(X+d)^2 - X^2\right].$$

Step 2: Differentiate the changing flux Expanding the difference first isolates the only time-dependent quantity:

$$\Phi_B = \frac{B_0 d^2}{a}X + \frac{B_0 d^3}{2a}.$$

The second term is constant as the loop translates. Thus

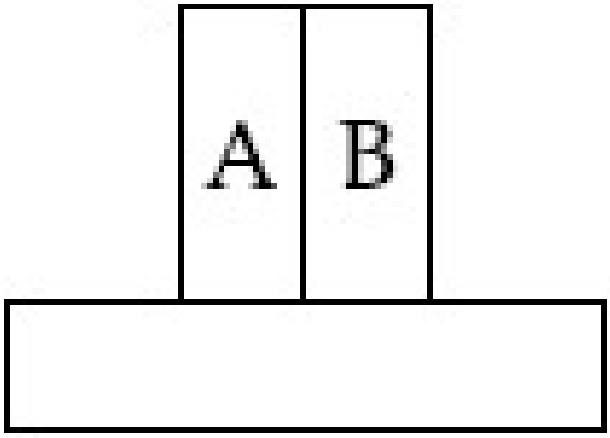
$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \frac{B_0 d^2}{a}\frac{dX}{dt} = \frac{B_0 v_0 d^2}{a}.$$

The integration factor $1/2$ disappears on differentiating the difference of the two moving edges. Option (A) retains an erroneous factor of two. Options (B) and (C) have incorrect dependence on velocity or loop size; the emf must be linear in speed and proportional to the loop area for a constant field gradient.

Step 3: Conclusion The magnitude of the induced emf is $\boxed{B_0 v_0 d^2/a}$. Its direction opposes the increasing outward magnetic flux. Hence, the correct option is (D).

Quick Tip to Solve: For a translating rectangular loop in a constant field gradient, use $|\mathcal{E}| = v \times \text{area} \times |dB/dx|$.

45. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold bath, it will : [4 marks]



(A) Neither bend nor shrink

(B) Bend towards the left

(C) Not bend but shrink

(D) Bend towards the right

Correct Answer: (B) Bend towards the left

Solution:

Concept:

- For a temperature change ΔT, a freely changing metal strip would have length change $\Delta L = \alpha L \Delta T$, where α is its linear expansion coefficient.

- Bonded metals must stay joined. Unequal natural length changes therefore produce bending: the metal that becomes shorter lies on the inner, concave side of the bent strip.

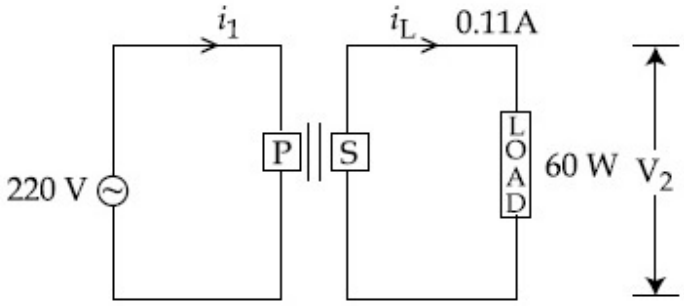
Step 1: Compare the contractions on cooling The bath is cold, so the temperature change is negative: $\Delta T < 0$. The given ordering is $\alpha_A > \alpha_B$. Consequently the free contraction magnitudes satisfy $|\Delta L_A| > |\Delta L_B|$ for the same initial length and temperature change. Metal A therefore tends to become shorter than B. A larger expansion coefficient means a larger contraction as well when the sign of the temperature change is reversed; it does not imply that A expands in a cold bath.

Step 2: Identify the inner side and the bending direction The figure puts A on the left and B on the right, with their lower ends clamped to the support. Since A tends to become the shorter layer, A must occupy the inner side of the curved strip and B the longer outer side. The free upper end therefore turns toward A, which is to the left in the printed diagram. The rigid mounting fixes the lower end but does not prevent bending of the free portion. Neither the no-change choice nor the straight-shrinkage choice can accommodate the unequal natural contractions of two bonded layers. Bending to the right would put the more strongly contracting layer on the outer, longer side, opposite to the required geometry.

Step 3: Conclusion The strip bends toward metal A, that is, toward the left. Hence, the correct option is (B).

Quick Tip to Solve: On cooling, bend toward the metal with the larger expansion coefficient; on heating, bend toward the one with the smaller coefficient.

46. For the given circuit, comment on the type of transformer used. [4 marks]



(A) Auxiliary transformer

(B) Step down transformer

(C) Step-up transformer

(D) Auto transformer

Correct Answer: (C) Step-up transformer

Solution:

Concept:

- A transformer is step-up when its secondary voltage exceeds its primary voltage, and step-down when the secondary voltage is lower.

- For the load data shown, the secondary output power is $P_2 = V_2 I_2$. The voltage comparison, rather than the apparent size of a coil symbol, determines the type.

Step 1: Calculate the secondary voltage The primary is supplied at $V_1 = 220\,\text{V}$. The secondary carries the marked load current $I_2 = 0.11\,\text{A}$, while the load power is $P_2 = 60\,\text{W}$. For the load represented in this elementary transformer circuit, use

$$V_2 = \frac{P_2}{I_2} = \frac{60}{0.11}\,\text{V} \approx 545.5\,\text{V}.$$

The current on the secondary side must be used here, because the indicated power is delivered to that load. Dividing by a primary current would mix quantities belonging to different sides.

Step 2: Compare input and output voltage The ratio is

$$\frac{V_2}{V_1} = \frac{545.5}{220} \approx 2.48 > 1.$$

Thus the device raises the voltage. In the ideal transformer description this also means $N_2/N_1 \approx 2.48$, with a corresponding decrease in current so that power is conserved. A step-down transformer would instead have a ratio below one, so option (B) contradicts the numerical result. An autotransformer uses a common tapped winding, whereas the drawing shows separate primary and secondary circuits. The term auxiliary does not identify the voltage conversion demonstrated by the given ratings, and is not the applicable choice here.

Step 3: Conclusion The secondary voltage is approximately $545\,\text{V}$, greater than $220\,\text{V}$, so this is a step-up transformer. Hence, the correct option is (C).

Quick Tip to Solve: When load power and secondary current are given, compute $V_2 = P_2/I_2$ first, then compare it directly with the supply voltage.

47. The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, ($N_a = 6 \times 10^{23}$/mol). [4 marks]

(A) 240 Ci

(B) 357 Ci

(C) 252 Ci

(D) 535 Ci

Correct Answer: (B) 357 Ci

Solution:

Concept:

- Activity is the decay rate $A = \lambda N$, where $\lambda = \ln 2/T_{1/2}$. The number of radioactive atoms is $N = (m/M)N_A$.
- Use seconds for the half-life to obtain becquerels, and convert with $1\,\text{Ci} = 3.7 \times 10^{10}\,\text{s}^{-1}$. Mass and molar mass must use matching units.

Step 1: Count the radioactive atoms The stated sample mass is $1.50\,\text{mg} = 1.50 \times 10^{-3}\,\text{g}$. With molar mass $198\,\text{g mol}^{-1}$, the amount is

$$n = \frac{1.50 \times 10^{-3}}{198}\,\text{mol} = 7.576 \times 10^{-6}\,\text{mol}.$$

Hence

$$N = nN_A = (7.576 \times 10^{-6})(6 \times 10^{23}) = 4.545 \times 10^{18}.$$

This is the initial number of radioactive nuclei in the sample, not the number decaying in one half-life.

Step 2: Find the activity and compare the printed choices Convert the half-life before substituting:

$$T_{1/2} = 2.7(24)(3600) = 233280\,\text{s}.$$

It follows that

$$\begin{aligned}\lambda &= \frac{0.6931}{233280} = 2.971 \times 10^{-6}\,\text{s}^{-1},\\ A &= (2.971 \times 10^{-6})(4.545 \times 10^{18})\\ &\approx 1.351 \times 10^{13}\,\text{Bq},\\ A &\approx \frac{1.351 \times 10^{13}}{3.7 \times 10^{10}} = 365\,\text{Ci}.\end{aligned}$$

The literal data therefore give about 365 Ci, rather than exactly any printed value. Of the supplied choices, 357 Ci is the nearest, differing by about 2.2 percent. The other values, 240, 252 and 535 Ci, are substantially farther away and cannot be obtained by reasonable rounding of the calculation.

Step 3: Conclusion The calculated activity is approximately $365\,\text{Ci}$. The closest listed value is $357\,\text{Ci}$; hence, the correct option is (B) among the printed choices.

Quick Tip to Solve: Convert milligrams to grams and days to seconds before calculating activity; compare the resulting curies with the options only at the end.

48. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes. [4 marks]

(A) 40 minutes

(B) 60 minutes

(C) 13 minutes

(D) 20 minutes

Correct Answer: (D) 20 minutes

Solution:

Concept:

- The number of undecayed nuclei follows $N(t) = N_0 e^{-\lambda t}$, where $\lambda = \ln 2/T_{1/2}$. A stated percentage decayed must first be converted to the percentage remaining.
- For two instants, dividing the decay equations cancels the original number: $\Delta t = \lambda^{-1} \ln(N_1/N_2)$.

Thus the interval depends on the ratio of remaining populations.

Step 1: Translate the decay percentages into remaining amounts At the first instant, 33 percent has decayed, so $N_1 = 0.67N_0$. At the second instant, 67 percent has decayed, so $N_2 = 0.33N_0$. The second time is later because fewer nuclei remain. The relevant ratio is therefore $N_1/N_2 = 67/33$, close to two. Using 33 and 67 directly as surviving percentages in the wrong order would incorrectly give a negative interval.

Step 2: Calculate the interval using the half-life The decay equations imply

$$\frac{N_2}{N_1} = e^{-\lambda \Delta t}, \qquad \Delta t = \frac{\ln(67/33)}{\lambda}.$$

Since the half-life is 20 minutes,

$$\Delta t = 20\frac{\ln(67/33)}{\ln 2}\,\text{min} \approx 20.43\,\text{min}.$$

The percentages 33 and 67 represent rounded thirds, so this interval is approximately one half-life, namely 20 minutes. Taking the intended fractions as $2/3$ and $1/3$ makes the ratio exactly two and gives precisely 20 minutes. The listed 40 and 60 minutes correspond to two or three half-lives, far too long for a remaining-population ratio near two; 13 minutes is too short.

Step 3: Conclusion The interval is approximately $20\,\text{minutes}$ (20.43 minutes using the percentages literally). Hence, the correct option is (D).

Quick Tip to Solve: Between two radioactive-decay stages, form the ratio of amounts remaining; a ratio near two means a time interval near one half-life.

49. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight? (Assume radius of earth is 6400 km) [4 marks]

(A) 158.2 m

(B) 79.1 m

(C) 19.77 m

(D) 39.55 m

Correct Answer: (D) 39.55 m

Solution:

Concept:

- The horizon distance of an antenna at height $h \ll R$ above a spherical Earth is $d_h \approx \sqrt{2Rh}$, obtained from the right triangle to the tangent point.
- The maximum direct line-of-sight separation of two antennas is the sum of their individual horizon distances. Identical towers carrying identical antennas have equal heights.

Step 1: Combine the two equal horizon ranges Let each antenna height be h and the Earth radius be

R. The line joining the antennas is just tangent to the Earth at the limiting separation. Each antenna contributes $\sqrt{2Rh}$ to the reachable distance, so

$$D = \sqrt{2Rh} + \sqrt{2Rh} = 2\sqrt{2Rh}.$$

Here $D = 45\,\text{km}$. Both antennas have finite and equal heights; treating the transmitter as if it were on the ground would omit its horizon contribution and overestimate the required receiving height.

Step 2: Solve for the common minimum height Squaring the range relation gives $D^2 = 8Rh$. Use consistent kilometre units first:

$$h = \frac{D^2}{8R} = \frac{45^2}{8(6400)}\,\text{km} = 0.0395508\,\text{km} = 39.55\,\text{m}.$$

This height is very small relative to $6400\,\text{km}$, so the horizon approximation is justified. Option (A), about 158.2 m, results from incorrectly allowing only a single antenna horizon for the whole 45 km link. Options (B) and (C) use other incorrect factors in squaring or sharing the total distance. At the calculated equal height, each tower supplies a horizon range of 22.5 km and their ranges add to the required 45 km.

Step 3: Conclusion The minimum receiving antenna height is nearly $\boxed{39.55\,\text{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: For two equal-height antennas remember $D = 2\sqrt{2Rh}$, so the height is $D^2/(8R)$, not $D^2/(2R)$.

50. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to drop to half of its initial value? (ln 2 = 0.693) [4 marks]

(A) 17.32 s

(B) 34.65 s

(C) 0.034 s

(D) 15.01 s

Correct Answer: (B) 34.65 s

Solution:

Concept:

- For a viscous damping force $F_d = -bv$, the equation is $m\ddot{x} + b\dot{x} + kx = 0$. Here b is the damping coefficient, with units of mass per time.
- The amplitude envelope of underdamped motion is $A(t) = A_0 e^{-bt/(2m)}$. The factor of two belongs to amplitude decay; the mechanical energy decays twice as fast.

Step 1: Identify the amplitude decay rate The given decay coefficient is $b = 20\,\text{g}\,\text{s}^{-1}$ and the mass is

$m = 500\,\text{g}$. Keeping both in grams makes their ratio consistent. Therefore the amplitude decay rate is

$$\frac{b}{2m} = \frac{20}{2(500)} = 0.020\,\text{s}^{-1}.$$

Step 2: Set the amplitude equal to half its initial value Using the envelope, rather than the instantaneous displacement, gives

$$\frac{1}{2} = e^{-0.020t}, \qquad 0.020t = \ln 2 = 0.693.$$

Thus $t = 0.693/0.020 = 34.65\,\text{s}$. The value $17.32\,\text{s}$ would result from using the energy decay rate for the amplitude. The very small value $0.034\,\text{s}$ is a unit-conversion error, while $15.01\,\text{s}$ does not satisfy the half-amplitude condition.

Step 3: Conclusion The amplitude becomes half after $\boxed{34.65\,\text{s}}$. Hence, the correct option is (B).

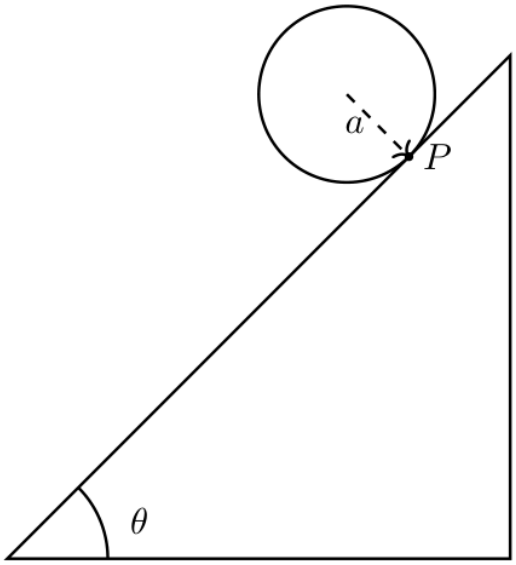
Quick Tip to Solve: For damping quoted in g/s, use mass in grams and remember that the amplitude exponent is $b/(2m)$, not b/m.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}$g sinθ where b is _____. (Round off to the Nearest Integer) (g = acceleration due to gravity, θ = angle as shown in figure)

[4 marks]



Solution:

Concept:

- For rolling without slipping, the translational acceleration a_c and angular acceleration α satisfy $a_c = a\alpha$, where a is the disc radius.
- A solid disc has central moment of inertia $I = \frac{1}{2}ma^2$. Static friction provides the torque needed to increase its angular speed without dissipating mechanical energy.

Step 1: Write the translation and rotation equations Take the downhill direction as positive. The component of weight is $mg\sin\theta$, while static friction f acts uphill. Consequently,

$$ma_c = mg\sin\theta - f.$$

About the disc centre only friction produces a torque, so

$$fa = I\alpha = \frac{1}{2}ma^2\frac{a_c}{a}, \qquad f = \frac{1}{2}ma_c.$$

Step 2: Eliminate friction and identify the coefficient Substitute the rotational result into the translational equation:

$$ma_c = mg\sin\theta - \frac{1}{2}ma_c.$$

Hence

$$\frac{3}{2}ma_c = mg\sin\theta, \qquad a_c = \frac{2}{3}g\sin\theta.$$

The mass and radius cancel, as expected for geometrically similar solid discs. Comparing this expression with the given $(2/b)g\sin\theta$, we obtain $2/b = 2/3$, so $b = 3$.

Step 3: Conclusion The required dimensionless integer is $\boxed{b = 3}$.

Quick Tip to Solve: Use $a_c = g\sin\theta/[1 + I/(ma^2)]$; insert the moment of inertia for a solid disc, not a ring.

52. The energy dissipated by a resistor is 10 mJ in 1 s when an electric current of 2 mA flows through it. The resistance is _____Ω. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- The electrical energy dissipated as heat by a resistor carrying steady current is $E = I^2Rt$. Thus $R = E/(I^2t)$.
- All quantities must be expressed in compatible SI units: energy in joules, current in amperes and time in seconds give resistance in ohms.

Step 1: Convert the given quantities into SI units The prefix milli means 10^{-3}, so the supplied energy and current are

$$E = 10\,\text{mJ} = 10 \times 10^{-3}\,\text{J} = 0.010\,\text{J},$$

$$I = 2\,\text{mA} = 2 \times 10^{-3}\,\text{A}.$$

The time is already in SI units: $t = 1\,\text{s}$. The dissipated power is therefore $E/t = 0.010\,\text{W}$.

Step 2: Apply the heating law and check the result Rearrange the energy relation and substitute:

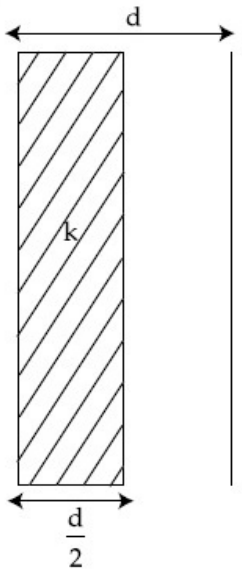
$$R = \frac{0.010}{(2 \times 10^{-3})^2(1)} = \frac{0.010}{4 \times 10^{-6}} = 2500\,\Omega.$$

The current must be squared after conversion to amperes; its square is $4 \times 10^{-6}\,\text{A}^2$, not $4 \times 10^{-3}\,\text{A}^2$. As a check, the voltage across this resistor is $IR = 5\,\text{V}$, and $VI = 0.010\,\text{W}$, reproducing the stated energy in one second.

Step 3: Conclusion The resistance, already an integer, is $\boxed{2500\,\Omega}$.

Quick Tip to Solve: Convert a milliampere current before squaring it; the prefix contributes a factor 10^{-6} to I^2.

53. In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1 m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 m^2 (see fig.) the capacitance of the set-up will be _____ε_0. (Dielectric constant of the material = 3.2) (Round off to the Nearest Integer) [4 marks]



Solution:

Concept:

- A dielectric slab that occupies the full plate area but only part of the separation creates dielectric and air layers in series.
- For plate separation d, slab thickness t, relative permittivity K and area A, the capacitance is $C = \varepsilon_0 A/[d - t + t/K]$, neglecting fringing.

Step 1: Express the series combination of layers The area of both layers is $A = 2\,\mathrm{m}^2$. The air thickness is $d - t = 1 - 0.5 = 0.5\,\mathrm{m}$, and the dielectric thickness is $0.5\,\mathrm{m}$. Their individual capacitances are $C_a = \varepsilon_0 A/(d - t)$ and $C_d = K\varepsilon_0 A/t$. Since the electric field crosses one layer after the other,

$$\frac{1}{C} = \frac{d - t}{\varepsilon_0 A} + \frac{t}{K\varepsilon_0 A}.$$

Step 2: Calculate the coefficient and round it Using $K = 3.2$, the effective separation is

$$d_{\text{eff}} = 0.5 + \frac{0.5}{3.2} = 0.65625\,\mathrm{m}.$$

Therefore

$$C = \frac{2\varepsilon_0}{0.65625} = \frac{64}{21}\varepsilon_0 \approx 3.047619\varepsilon_0.$$

This lies between the all-air capacitance $2\varepsilon_0$ and the fully filled value $6.4\varepsilon_0$, which is a useful physical check. The question asks for the coefficient of ε_0 rounded to the nearest integer, so round only at this final stage.

Step 3: Conclusion The coefficient is $64/21 \approx 3.047619$; the required nearest integer is $\boxed{3}$.

Quick Tip to Solve: A slab across the full area gives layers in series; replace its thickness t by t/K when calculating effective separation.

54. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is _____ (Round off to the Nearest Integer) (M is the mass of earth, R is the radius of earth, G is the gravitational constant) [4 marks]

Solution:

Concept:

- The gravitational potential energy of a uniform solid sphere of mass M and radius R is $U = -3GM^2/(5R)$, with zero energy for completely separated matter at infinity.
- The work needed to disperse the sphere is the increase in potential energy, $W = 0 - U$. The standard Earth model used for this binding-energy calculation is a sphere of uniform density.

Step 1: Build the sphere from concentric thin shells Let the constant density be $\rho = 3M/(4\pi R^3)$. When the assembled sphere has radius r, its mass is $m(r) = 4\pi\rho r^3/3$. Bring in a thin shell of mass $dm = 4\pi\rho r^2 dr$ from infinity. Its potential-energy change is

$$dU = -\frac{Gm(r)\,dm}{r} = -\frac{16\pi^2 G\rho^2}{3} r^4 dr.$$

This construction counts every gravitational pair once.

Step 2: Integrate and reverse the assembly process Integrating from the centre to the surface gives

$$U = -\frac{16\pi^2 G\rho^2}{3}\frac{R^5}{5} = -\frac{3GM^2}{5R}.$$

To break up the Earth completely and leave all its matter infinitely separated, its potential energy must rise from this negative value to zero. Thus the required supplied energy is positive:

$$W = \frac{3}{5}\frac{GM^2}{R}.$$

Comparing with $(x/5)GM^2/R$ fixes $x = 3$. Using GM^2/R directly would incorrectly treat the whole mass as though it were a surface particle.

Step 3: Conclusion The required coefficient is $\boxed{x = 3}$.

Quick Tip to Solve: For a uniform solid sphere, binding energy is $3GM^2/(5R)$; distinguish it from the escape energy of a test particle at the surface.

55. A body of mass 2 kg moves under a force of $\left(2\hat{i}+3\hat{j}+5\hat{k}\right)$N. It starts from rest and was at the origin initially. After 4s, its new coordinates are (8, b, 20). The value of b is _____. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- Newton's second law applies independently along each Cartesian axis: $\vec{a}=\vec{F}/m$. A constant force on a fixed mass produces constant acceleration.
- With initial position and initial velocity both zero, the position after time t is $\vec{r}=\frac{1}{2}\vec{a}t^2$. The same time applies to every component.

Step 1: Calculate the acceleration vector Divide each force component by the mass $m=2\,\mathrm{kg}$:

$$\vec{a}=\frac{2\hat{i}+3\hat{j}+5\hat{k}}{2}=\left(\hat{i}+\frac{3}{2}\hat{j}+\frac{5}{2}\hat{k}\right)\mathrm{m\,s^{-2}}.$$

The acceleration in the y direction is therefore $a_y=1.5\,\mathrm{m\,s^{-2}}$. There is no initial-velocity contribution because the body starts from rest.

Step 2: Evaluate the position at four seconds For $t=4\,\mathrm{s}$, the common kinematic factor is $t^2/2=16/2=8\,\mathrm{s^2}$. Hence

$$\vec{r}=8\left(\hat{i}+\frac{3}{2}\hat{j}+\frac{5}{2}\hat{k}\right)\mathrm{m}=\left(8\hat{i}+12\hat{j}+20\hat{k}\right)\mathrm{m}.$$

The calculated x and z coordinates are 8 and 20, exactly the values supplied in the question. The remaining coordinate is $y=12\,\mathrm{m}$, so the numerical parameter b is 12. No rounding changes this exact result.

Step 3: Conclusion The new coordinates are $(8,12,20)\,\mathrm{m}$, and $\boxed{b=12}$.

Quick Tip to Solve: For a body starting from rest at the origin under constant force, use $\vec{r}=\vec{F}t^2/(2m)$ component by component.

56. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is _____. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- For equal compressibility κ, sound speed satisfies $v=1/\sqrt{\rho\kappa}$. Thus $v_2/v_1=\sqrt{\rho_1/\rho_2}$.
- A closed pipe has only odd harmonics: its first overtone is the third harmonic. An open pipe has all harmonics: its first overtone is the second harmonic. End corrections are neglected.

Step 1: Write the two first-overtone frequencies Let v_1 be sound speed in the closed pipe of length L, and let the open pipe length be ℓ, with sound speed v_2. The fundamental closed-pipe frequency is

$v_1/(4L)$, so its first overtone is

$$f_c = \frac{3v_1}{4L}.$$

The fundamental open-pipe frequency is $v_2/(2\ell)$, so its first overtone is

$$f_o = \frac{2v_2}{2\ell} = \frac{v_2}{\ell}.$$

Step 2: Equate frequencies and insert the density ratio The two overtones have equal frequency, which gives

$$\frac{3v_1}{4L} = \frac{v_2}{\ell}, \qquad \ell = \frac{4}{3}L\frac{v_2}{v_1}.$$

Using equal compressibilities,

$$\ell = \frac{4}{3}L\sqrt{\frac{\rho_1}{\rho_2}}.$$

The density ratio appears in this order because the open-pipe speed is v_2: a smaller ρ_2 raises its speed and requires a longer pipe at the same frequency. Comparing with $(x/3)L\sqrt{\rho_1/\rho_2}$ yields $x = 4$.

Step 3: Conclusion The required dimensionless integer is $\boxed{x = 4}$.

Quick Tip to Solve: First overtone means third harmonic for a closed pipe but second harmonic for an open pipe; write these frequencies before equating them.

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57. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _____°. (Round off to the Nearest Integer) (Find the angle in degrees) [4 marks]

Solution:

Concept:

- The swimmer's velocity relative to the bank is the vector sum of the velocity relative to water and the river velocity. Components along the river must cancel to reach the opposite point.
- If θ is measured from the downstream direction, the upstream component of swimming is represented by $12\cos\theta$, which is negative for an obtuse angle.

Step 1: Choose axes along and across the river Take the positive x-axis along the flow and the positive y-axis towards the opposite bank. The water has velocity $6\,\mathrm{km\,h^{-1}}$ along x. The swimmer has speed $12\,\mathrm{km\,h^{-1}}$ relative to water. Thus the bank-frame components are

$$v_x = 6 + 12\cos\theta, \qquad v_y = 12\sin\theta.$$

The direction asked for is the swimmer's heading, rather than the direction of the resultant velocity.

Step 2: Cancel the downstream displacement The swimmer must arrive at the point directly opposite the starting point. For constant velocities, this requires zero net x-velocity throughout the crossing.

Therefore

$$6 + 12\cos\theta = 0 \quad \Rightarrow \quad \cos\theta = -\frac{1}{2}.$$

Taking the heading towards the opposite bank, $0 < \theta < 180^\circ$, gives $\theta = 120^\circ$. The transverse component remains positive: $12\sin 120^\circ = 6\sqrt{3}\,\text{km h}^{-1}$, so the crossing is physically possible. An angle of 60° measured from downstream would instead add a downstream component and miss the opposite point.

Step 3: Conclusion The required heading makes an angle $\boxed{120^\circ}$ with the direction of river flow. The nearest-integer entry is 120.

Quick Tip to Solve: An opposite-point crossing needs an upstream swimming component equal to the river speed; measuring from downstream makes the heading obtuse.

58. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass as 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be _____° (in degree). (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- For a thin prism, the yellow-light deviation is $\delta = (\mu_y - 1)A$, where A is its refracting angle.
- In an achromatic pair, the prisms face oppositely and their angular dispersions cancel: $\omega_c\delta_c = \omega_f\delta_f$. The net yellow deviation is the difference of the individual deviations.

Step 1: Write the achromatism condition Let δ_c and δ_f denote the positive magnitudes of the yellow deviations of the crown and flint prisms. Their deviations are in opposite directions. Using the given dispersive powers,

$$0.02\delta_c = 0.03\delta_f \quad \Rightarrow \quad \delta_f = \frac{2}{3}\delta_c.$$

The flint prism therefore needs a smaller individual deviation because it disperses light more strongly. Equal prism deviations would not make the combination achromatic.

Step 2: Use the net deviation and find the crown angle The resultant yellow deviation is 2°, so

$$\delta_c - \delta_f = 2^\circ, \qquad \frac{1}{3}\delta_c = 2^\circ.$$

Hence $\delta_c = 6^\circ$ and $\delta_f = 4^\circ$. For crown glass, $\mu_c = 1.5$, giving

$$A_c = \frac{\delta_c}{\mu_c - 1} = \frac{6^\circ}{0.5} = 12^\circ.$$

As a check, the flint angle is $4^\circ/0.6$, and the dispersions are $0.02 \times 6^\circ = 0.03 \times 4^\circ$. Both are 0.12°, while their yellow deviations differ by exactly 2°. All angular relations use degrees consistently.

Step 3: Conclusion The refracting angle of the crown-glass prism is $\boxed{12^\circ}$. The nearest-integer entry is 12.

Quick Tip to Solve: First use dispersive powers to relate the two yellow deviations, subtract them to obtain the net deviation, and only then use the refractive index.

59. A force $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point of x = 2 plane and x-axis. The magnitude of torque of this force about a point (2, 3, 4) is _____. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- Torque about a specified point is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, with $\mathbf{r}$ measured from that point to the point of force application.
- The magnitude of a vector is the square root of the sum of squares of its Cartesian components. Reversing $\mathbf{r}$ reverses torque direction but does not change its magnitude.

Step 1: Locate the force and construct the position vector The x-axis has $y = z = 0$. Its intersection with the plane $x = 2$ is therefore $P = (2, 0, 0)$, not $(0, 0, 0)$. Torque is required about $O = (2, 3, 4)$. The relative position vector is

$$\mathbf{r} = \overrightarrow{OP} = (2-2)\hat{i} + (0-3)\hat{j} + (0-4)\hat{k} = -3\hat{j} - 4\hat{k}.$$

Use this relative vector, since torque depends on the chosen reference point.

Step 2: Evaluate the cross product and magnitude For $\mathbf{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$, the component formula gives

$$\begin{aligned}\boldsymbol{\tau} &= [(-3)4 - (-4)3]\hat{i} \\ &\quad + [(-4)4 - 0(4)]\hat{j} + [0(3) - (-3)4]\hat{k} \\ &= 0\hat{i} - 16\hat{j} + 12\hat{k}.\end{aligned}$$

The x-component vanishes because the y-z components of $\mathbf{r}$ and $\mathbf{F}$ are proportional. Thus

$$|\boldsymbol{\tau}| = \sqrt{0^2 + (-16)^2 + 12^2} = \sqrt{400} = 20.$$

This is the magnitude of the full torque vector, rather than just one component. With position in metres and force in newtons, its unit is $\mathrm{N\,m}$.

Step 3: Conclusion The torque magnitude is $\boxed{20}$, or $20\,\mathrm{N\,m}$ in SI units. The nearest-integer entry is 20.

Quick Tip to Solve: Find the application point geometrically first, then subtract the reference-point coordinates before taking the cross product.

60. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be _____°C. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- An ideal reversible heat engine has Carnot efficiency $\eta = 1 - T_C/T_H$, where both reservoir temperatures must be absolute temperatures in kelvin.

- Temperature conversion is $T(\mathrm{K}) = t(^\circ\mathrm{C}) + 273$ to the precision needed here; a temperature ratio cannot be formed directly from Celsius readings.

Step 1: Convert the source temperature to kelvin The source is at $127^\circ\mathrm{C}$, so

$$T_H = 127 + 273 = 400\,\mathrm{K}.$$

The efficiency is a dimensionless fraction: $60\% = 0.60$. Carnot efficiency describes the fraction of absorbed heat converted into work, while the remaining fraction is rejected to the sink. Thus $1 - \eta = 0.40$ is the required ratio of the sink temperature to the source temperature.

Step 2: Find the sink temperature and convert back Rearrange the efficiency formula before substituting:

$$T_C = (1 - \eta)T_H = 0.40 \times 400 = 160\,\mathrm{K}.$$

The answer is requested in degrees Celsius, not kelvin. Therefore

$$t_C = 160 - 273 = -113^\circ\mathrm{C}.$$

Using the more precise offset 273.15 gives $T_H = 400.15\,\mathrm{K}$, $T_C = 160.06\,\mathrm{K}$ and $t_C = -113.09^\circ\mathrm{C}$, which rounds to the same integer. A negative Celsius sink temperature is admissible: its absolute temperature is positive and below that of the source, as a working heat engine requires.

Step 3: Conclusion The sink temperature should be $\boxed{-113^\circ\mathrm{C}}$, to the nearest integer. The numerical entry is -113.

Quick Tip to Solve: Use kelvin inside the efficiency ratio, and convert the resulting sink temperature back to Celsius only at the end.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Match List - I with List - II :

	List - I Test/Reagents/Observation(s)		List - II Species detected
(a)	Lassaigne's Test	(i)	Carbon
(b)	Cu(II) oxide	(ii)	Sulphur
(c)	Silver nitrate	(iii)	N, S, P, and halogen
(d)	The sodium fusion extract gives black precipitate with acetic acid and lead acetate	(iv)	Halogen Specifically

The correct match is : [4 marks]

(A) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(B) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(C) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(D) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Correct Answer: (C) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Solution:

Concept:

- Sodium fusion converts covalently bound heteroatoms in an organic compound into ionic salts, allowing their qualitative detection in Lassaigne's extract.
- Carbon is detected by oxidation to CO_2, halides by precipitation with $AgNO_3$, and sulphide by formation of black PbS.

Step 1: Match the general test and the carbon reagent Lassaigne's test is the general sodium-fusion method used for detecting nitrogen, sulphur, phosphorus and halogens after appropriate treatment of the extract. Thus (a) matches (iii). Heating an organic compound with copper(II) oxide oxidises its carbon to carbon dioxide. The evolved CO_2 is detected with limewater through formation of $CaCO_3$. Hence (b), CuO, matches (i), carbon.

Step 2: Match halogen and sulphur observations Acidified sodium fusion extract containing halide ions gives a precipitate with silver nitrate:

$$Ag^+ + X^- \longrightarrow AgX\,(s).$$

Thus (c) matches (iv), halogen specifically. In the sulphur test, sodium fusion produces sulphide ions. Lead acetate in acetic acid gives black lead sulphide:

$$Pb^{2+} + S^{2-} \longrightarrow PbS\,(s).$$

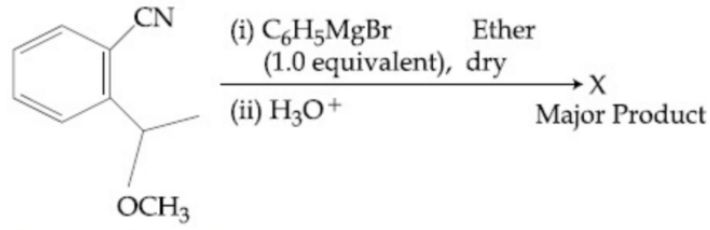
Therefore (d) matches (ii). The complete mapping is (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii). Option A interchanges the halogen and sulphur results; B assigns carbon to Lassaigne's test and halogens to copper oxide; D also mismatches the general test and copper oxide. Only C contains all four correct

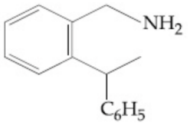
pairs.

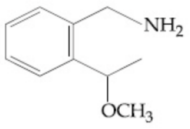
Step 3: Conclusion The complete matching agrees with the third listed combination. Hence, the correct option is (C).

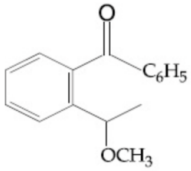
Quick Tip to Solve: Remember black lead sulphide for sulphur and silver-halide precipitates for halogens; these two pairs immediately distinguish the close alternatives.

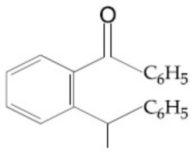
62. The structure of X is : [4 marks]

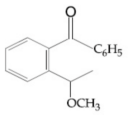


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- A nitrile reacts with one equivalent of a Grignard reagent by nucleophilic addition at its electrophilic carbon. Acidic hydrolysis of the resulting magnesium imine salt produces a ketone.
- An ether group is compatible with a Grignard reagent because it has no acidic O–H proton. The methoxyethyl substituent therefore remains unchanged during the reaction shown.

Step 1: Identify the reactive functional group The starting material has a nitrile attached directly to the benzene ring and an adjacent $-CH(OCH_3)CH_3$ substituent. The phenyl group of C_6H_5MgBr behaves as a carbon nucleophile and attacks the nitrile carbon. It does not reduce the nitrile to a primary amine, and the ether oxygen does not supply an acidic proton that would consume the reagent. Dry ether prevents premature destruction of the Grignard reagent by water.

Step 2: Add phenyl and hydrolyse the imine salt Write Ar for the ortho-substituted phenyl group.

The reaction sequence is

$$\mathrm{Ar{-}CN} \xrightarrow{\mathrm{C_6H_5MgBr}} \mathrm{Ar{-}C(C_6H_5){=}N{-}MgBr} \xrightarrow{\mathrm{H_3O^+}} \mathrm{Ar{-}CO{-}C_6H_5}.$$

The carbon originally in the nitrile becomes the carbonyl carbon of the ketone, and the added phenyl group is bonded to it. Hydrolysis replaces the imine nitrogen by oxygen. The ketone is generated during the aqueous acidic work-up, when the Grignard reagent is destroyed, so further addition to the ketone does not give a tertiary alcohol under this sequence.

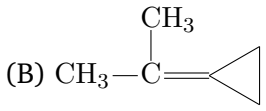
Step 3: Compare the four structures Option C has the required $-\mathrm{COC_6H_5}$ group and retains the adjacent $-\mathrm{CH(OCH_3)CH_3}$ group. Options A and B show $-\mathrm{CH_2NH_2}$, a nitrile-reduction product requiring a reducing agent rather than phenylmagnesium bromide. Option D has replaced the methoxy group at the neighbouring carbon by phenyl; ordinary ether cleavage and substitution of that kind do not occur in this nitrile addition. Thus only C preserves the correct changes and unchanged substituent.

Step 4: Conclusion X is the ortho-substituted aryl phenyl ketone with its methoxyethyl group unchanged, as drawn in C. Hence, the correct option is (C).

Quick Tip to Solve: For a nitrile followed by one equivalent of Grignard reagent and acidic work-up, replace the nitrile nitrogen by oxygen and attach the Grignard carbon group to the nitrile carbon.

63. An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is : [4 marks]

(A) $\mathrm{HC{\equiv}C{-}CH_2{-}CH_3}$

(B) 

(C) $\mathrm{CH_3{-}C{\equiv}C{-}CH_3}$

(D) $\mathrm{(CH_3)_2C{=}C(CH_3)_2}$

Correct Answer: (A) $\mathrm{HC{\equiv}C{-}CH_2{-}CH_3}$

Solution:

Concept:

- Ozonolysis cleaves an unsaturated carbon-carbon bond to give oxygen-containing fragments. Under carbonyl-producing conditions an alkyne gives an alpha-dicarbonyl compound.
- Tollens' reagent, ammoniacal silver nitrate, is reduced by an aldehyde group to metallic silver, producing the silver mirror. A simple ketone does not give this test.

Step 1: Identify which substrate yields an aldehyde-containing product Option A is but-1-yne, $\mathrm{HC{\equiv}C{-}CH_2{-}CH_3}$. Its terminal triple-bond carbon bears hydrogen. Ozonolysis produces the alpha-keto aldehyde $\mathrm{OHC{-}CO{-}CH_2{-}CH_3}$ as the carbonyl product. This molecule has an aldehyde group as well as a ketone group, and its aldehyde group reduces Tollens' reagent. Metallic silver is deposited on the test-tube walls while the aldehyde group is oxidised to carboxylate in the alkaline medium.

Step 2: Eliminate the other hydrocarbons Option C is but-2-yne, $CH_3-C\equiv C-CH_3$. It gives the diketone $CH_3COCOCH_3$, which has no aldehydic hydrogen. Option D is $(CH_3)_2C=C(CH_3)_2$; cleavage gives acetone, a ketone. In option B, the double bond connects a carbon bearing two methyl groups to a cyclopropane-ring carbon. Neither double-bond carbon carries hydrogen, so the carbonyl products are ketonic rather than aldehydic. These products do not account for the stated silver mirror. The relevant observation is the Tollens test on the ozonolysis product A; the silver salt formed directly by a terminal alkyne is a different test and is not the explanation here.

Step 3: Conclusion The hydrocarbon is but-1-yne, $HC\equiv C-CH_2-CH_3$, whose ozonolysis product contains an aldehyde group. Hence, the correct option is (A).

Quick Tip to Solve: Check for a hydrogen on an unsaturated carbon: it identifies an aldehyde-forming carbonyl product and separates a Tollens mirror from a direct alkyne silver-salt test.

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64. The exact volumes of 1 M NaOH solution required to neutralise 50 mL of 1 M H_3PO_3 solution and 100 mL of 2 M H_3PO_2 solution, respectively, are : [4 marks]

(A) 100 mL and 50 mL

(B) 100 mL and 200 mL

(C) 100 mL and 100 mL

(D) 50 mL and 50 mL

Correct Answer: (B) 100 mL and 200 mL

Solution:

Concept:

- Acid basicity is the number of replaceable acidic hydrogen atoms per molecule. In phosphorus oxoacids, hydrogen bonded to oxygen is ionisable, whereas a hydrogen bonded directly to phosphorus is not.
- At complete neutralisation, the moles of hydroxide required equal the moles of acid multiplied by its basicity. Use $n = MV$, with volume in litres.

Step 1: Count the ionisable hydrogens Phosphorous acid has the structure $HP(O)(OH)_2$, containing two O–H groups and one P–H bond. Thus H_3PO_3 is dibasic. Hypophosphorous acid, $H_2P(O)(OH)$, has only one O–H group and is monobasic. The formula containing three hydrogen atoms does not mean that either acid supplies three protons.

Step 2: Calculate each alkali volume For the first acid,

$$n(H_3PO_3) = 1 \times 0.050 = 0.050 \text{ mol}.$$

Hence the hydroxide requirement is $2(0.050) = 0.100$ mol. With 1 M alkali,

$$V_1 = \frac{0.100}{1} = 0.100 \text{ L} = 100 \text{ mL}.$$

For the second acid,

$$n(\mathrm{H_3PO_2}) = 2 \times 0.100 = 0.200 \text{ mol}.$$

Its basicity is one, so

$$V_2 = \frac{0.200}{1} = 0.200 \text{ L} = 200 \text{ mL}.$$

Step 3: Check the alternatives Only the pair 100 mL and 200 mL satisfies both proton balances. Options with 50 mL underestimate the first or second hydroxide requirement, while 100 mL for the second acid neutralises only half its 0.200 mol of acidic hydrogen.

Step 4: Conclusion The required volumes are 100 mL and 200 mL, respectively. Hence, the correct option is (B).

Quick Tip to Solve: For phosphorus oxoacids, count O–H bonds before using $M_1V_1b = M_2V_2$; do not count P–H bonds as acidic.

65. The characteristics of elements X, Y and Z with atomic numbers, respectively, 33, 53 and 83 are : [4 marks]

(A) X and Y are metalloids and Z is a metal.

(B) X and Z are non-metals and Y is a metalloid.

(C) X is a metalloid, Y is a non-metal and Z is a metal.

(D) X, Y and Z are metals.

Correct Answer: (C) X is a metalloid, Y is a non-metal and Z is a metal.

Solution:

Concept:

- An atomic number identifies an element uniquely. Its position in the periodic table then determines whether its usual properties are metallic, non-metallic or intermediate.
- Metallic character generally increases down a group. The heavier group 15 elements therefore need not have the same classification as nitrogen or phosphorus.

Step 1: Identify the three elements The element with atomic number 33 is arsenic, As. It is in period 4 and group 15, with outer configuration $4s^2 4p^3$. Atomic number 53 is iodine, I, a period 5, group 17 halogen with outer configuration $5s^2 5p^5$. Atomic number 83 is bismuth, Bi, in period 6 and group 15, with outer configuration $6s^2 6p^3$.

Step 2: Assign the correct classifications Arsenic lies near the boundary between metals and non-metals and is classified as a metalloid. Iodine is a non-metal; its lustrous solid appearance does not make it a metal. Bismuth is a metal, consistent with the increase in metallic character down group 15 from arsenic to bismuth. Thus the sequence of classifications is metalloid, non-metal, metal.

Step 3: Eliminate the other statements Option (A) incorrectly calls iodine a metalloid. Option (B) incorrectly labels arsenic and bismuth as non-metals and iodine as a metalloid. Option (D) incorrectly treats both arsenic and iodine as metals. The fact that all three elements belong to the p-block does not imply that all three have the same metallic character.

Step 4: Conclusion $X = \text{As}$ is a metalloid, $Y = \text{I}$ is a non-metal and $Z = \text{Bi}$ is a metal. Hence, the correct option is (C).

Quick Tip to Solve: First map atomic numbers to element names; then classify each element. Remember the group 15 change from arsenic metalloid to bismuth metal.

66. Which of the following is least basic? [4 marks]

(A) $(CH_3CO)_2\ddot{N}H$

(B) $(CH_3CO)\ddot{N}HC_2H_5$

(C) $(C_2H_5)_3\ddot{N}$

(D) $(C_2H_5)_2\ddot{N}H$

Correct Answer: (A) $(CH_3CO)_2\ddot{N}H$

Solution:

Concept:

- A nitrogen compound behaves as a base by donating its lone pair to a proton. Basicity decreases when that lone pair is delocalised into a neighbouring carbonyl group.
- An acyl group, CH_3CO^-, withdraws electron density and permits resonance involving nitrogen. An ethyl group instead supplies electron density through its inductive effect.

Step 1: Compare the two carbonyl-containing choices In option (A), nitrogen is bonded to two acetyl groups: $\text{CH}_3\text{CO}-\text{NH}-\text{COCH}_3$. Its lone pair can be delocalised towards either carbonyl group. This imide-type arrangement makes the lone pair particularly unavailable for accepting a proton at nitrogen. In option (B), $\text{CH}_3\text{CO}-\text{NH}-\text{C}_2\text{H}_5$, nitrogen is adjacent to only one carbonyl group. Its lone pair is also delocalised, as in an amide, but it is not flanked by two electron-withdrawing acyl groups.

Step 2: Compare the alkylamines Options (C) and (D) are triethylamine and diethylamine, respectively. They contain no neighbouring carbonyl group into which the nitrogen lone pair can be delocalised. Their ethyl groups donate electron density, so these compounds are much more basic than the acylated nitrogen compounds. Solvation can affect the relative basicities of secondary and tertiary amines, but that does not make either the least basic in this list.

Step 3: Select the strongest lone-pair withdrawal The decisive comparison is therefore between (A) and (B). Two acyl substituents withdraw and delocalise the lone pair more strongly than one acyl substituent plus an ethyl group. Consequently, the diacetyl derivative in (A) has the least tendency to act as a nitrogen base.

Step 4: Conclusion $(\text{CH}_3\text{CO})_2\text{NH}$, with nitrogen attached to two acyl groups, is the least basic. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish an acyl group from an alkyl group: count adjacent carbonyl groups before comparing nitrogen basicity.

67. Ammonolysis of Alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is : [4 marks]

(A) to remove acidic impurities

(B) to remove basic impurities

(C) to activate NH_3 used in the reaction

(D) to increase the reactivity of alkyl halide

Correct Answer: (A) to remove acidic impurities

Solution:

Concept:

- Ammonia attacks an alkyl halide by nucleophilic substitution. The immediate product contains a protonated amine paired with a halide ion.
- A strong base such as NaOH removes the extra proton from an alkylammonium salt and liberates the neutral, free amine.

Step 1: Write the initially formed salt For a primary alkyl halide, the substitution can be represented as

$$\mathrm{RX} + \mathrm{NH_3} \longrightarrow \mathrm{RNH_3}^+ \mathrm{X}^-.$$

Nitrogen has acquired the alkyl group while retaining its hydrogen atoms, so the product is an alkylammonium salt rather than the isolated neutral primary amine. Further alkylation can similarly produce protonated secondary and tertiary amines during the overall process.

Step 2: Show the role of hydroxide Treatment with alkali removes the acidic proton from the ammonium ion:

$$\mathrm{RNH_3}^+ \mathrm{X}^- + \mathrm{NaOH} \longrightarrow \mathrm{RNH_2} + \mathrm{H_2O} + \mathrm{NaX}.$$

The hydroxide ion accepts a proton and forms water, leaving the nitrogen lone pair available in the free amine. The corresponding acid-base step applies to protonated secondary and tertiary amines as well. In the wording of the options, this is removal or neutralisation of acidic impurities.

Step 3: Reject unrelated roles An alkali does not remove basic impurities by neutralisation, so (B) is inappropriate. The added sodium hydroxide is used after ammonolysis, rather than to activate the ammonia initially, excluding (C). Nor is its purpose to make the carbon–halogen bond more reactive in the completed substitution, excluding (D). Its relevant action is acid-base liberation of the amine from its salt.

Step 4: Conclusion NaOH neutralises the acidic ammonium species and releases the free amine. Hence, the correct option is (A).

Quick Tip to Solve: Whenever an amine is obtained as its ammonium salt, add a base to remove the extra proton and obtain the free amine.

68. The secondary structure of protein is stabilised by: [4 marks]

(A) Hydrogen bonding

(B) glycosidic bond

(C) Peptide bond

(D) van der Waals forces

Correct Answer: (A) Hydrogen bonding

Solution:

Concept:

- The secondary structure of a protein describes regular local folding of its polypeptide backbone, especially the alpha helix and beta pleated sheet.
- Backbone carbonyl oxygen atoms act as hydrogen-bond acceptors, and peptide N–H groups act as hydrogen-bond donors. Repeated hydrogen bonds stabilise the folded arrangement.

Step 1: Identify the stabilising interaction A peptide chain contains repeated $-CO-NH-$ linkages. A hydrogen bond forms between a carbonyl oxygen of one peptide unit and the hydrogen attached to nitrogen of another peptide unit. These are non-covalent interactions between suitable backbone groups; they do not require formation of new peptide bonds during folding.

Step 2: Relate it to the common secondary structures In an alpha helix, the backbone coils and intrachain hydrogen bonds help maintain the regular helical form. In a beta pleated sheet, hydrogen bonds between neighbouring extended chain segments stabilise their alignment. Thus the same kind of directional interaction explains both principal secondary structures, even though their shapes and hydrogen-bond arrangements differ.

Step 3: Separate the other options from this role A glycosidic bond links sugar units in carbohydrates and is not the defining stabiliser of protein secondary structure. Peptide bonds covalently join amino-acid residues and establish the primary sequence; their existence alone does not specify the helix or sheet. Van der Waals forces can contribute to molecular packing, but hydrogen bonding is the characteristic interaction asked for here.

Step 4: Conclusion The secondary structure of proteins is stabilised principally by hydrogen bonding between backbone peptide groups. Hence, the correct option is (A).

Quick Tip to Solve: Associate peptide bonds with amino-acid sequence, and backbone hydrogen bonds with alpha helices and beta sheets.

69. Fex_2 and Fey_3 are known when x and y are : [4 marks]

(A) x = F, Cl, Br, I and y = F, Cl, Br, I

(B) x = F, Cl, Br and y = F, Cl, Br, I

(C) x = F, Cl, Br, I and y = F, Cl, Br

(D) x = Cl, Br, I and y = F, Cl, Br, I

Correct Answer: (C) x = F, Cl, Br, I and y = F, Cl, Br

Solution:

Concept:

- Iron commonly forms compounds in the +2 and +3 oxidation states. Whether a halide is stable depends on both the iron oxidation state and the reducing strength of the halide ion.
- Among the halide ions, iodide is readily oxidised to iodine. It can therefore reduce an oxidising metal ion instead of remaining paired with that ion in a stable simple salt.

Step 1: List the iron(II) halides In FeX_2, each halogen has oxidation number -1, so iron is in the +2 state. The common iron(II) halides include FeF_2, $FeCl_2$, $FeBr_2$ and FeI_2. Thus the allowed list for x contains all four halogens: F, Cl, Br and I. This already eliminates options that omit fluoride or iodide from the iron(II) list.

Step 2: Check the iron(III) iodide exception In FeY_3, iron would be in the +3 state. Fluoride, chloride and bromide form the corresponding iron(III) halides. However, iodide reduces iron(III) to iron(II), while itself being oxidised:

$$2\,Fe^{3+} + 2\,I^- \longrightarrow 2\,Fe^{2+} + I_2.$$

Consequently simple FeI_3 is not a stable member of the usual iron(III) halide series.

Step 3: Compare the lists with the choices The required combination is therefore $x = F, Cl, Br, I$ and $y = F, Cl, Br$. Options (A) and (D) wrongly include iodide in the +3 series, while (B) both excludes iodide from the +2 series and includes it in the +3 series. Only option (C) has both lists correct.

Step 4: Conclusion $x = F, Cl, Br, I$, whereas $y = F, Cl, Br$. Hence, the correct option is (C).

Quick Tip to Solve: For a proposed metal iodide in a higher oxidation state, check whether iodide reduces the metal ion and is oxidised to iodine.

70. Identify the elements X and Y using the ionisation energy values given below :

	Ionization energy (kJ/mol)	
	$1st$	$2nd$
X	495	4563
Y	731	1450

[4 marks]

(A) X = F; Y = Mg

(B) X = Mg; Y = F

(C) X = Na; Y = Mg

(D) X = Mg; Y = Na

Correct Answer: (C) X = Na; Y = Mg

Solution:

Concept:

- The first ionisation energy removes an electron from a gaseous neutral atom; the second removes an electron from its gaseous singly charged cation.
- A very large jump after the first ionisation signals that the neutral atom has one valence electron. Removing the next electron then breaks into a stable inner-shell configuration.

Step 1: Use the sharp jump for X For X, the first and second ionisation energies are 495 and $4563\ \text{kJ mol}^{-1}$. The second is over nine times the first. This is typical of sodium:

$$\text{Na} : [\text{Ne}]3s^1.$$

The first electron is easily removed from the outer 3s orbital, but Na^+ has the stable neon configuration. Its next electron must be removed from an inner shell, requiring much more energy.

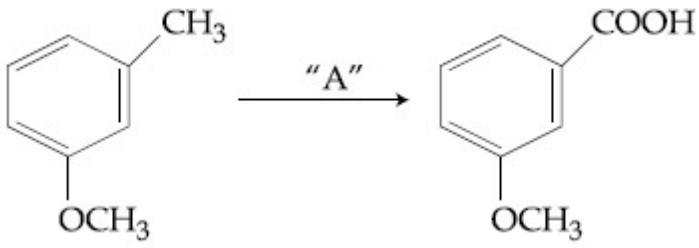
Step 2: Identify Y from its two accessible electrons For Y, the values are 731 and $1450\ \text{kJ mol}^{-1}$. Both correspond to removing valence-shell electrons, with the expected increase for a positively charged ion. Magnesium has configuration $[\text{Ne}]3s^2$, so its first two ionisations remove the two 3s electrons. Its very large jump would occur at the third ionisation, after the neon-like Mg^{2+} ion is formed.

Step 3: Eliminate the wrong element assignments Fluorine has a strongly held valence electron and cannot account for the low first value of 495. Magnesium also does not show the enormous first-to-second jump seen for X. Sodium, in turn, cannot fit Y because its second ionisation is the much larger inner-shell value. Thus both numerical patterns independently support the same pair.

Step 4: Conclusion $X = \text{Na}$ and $Y = \text{Mg}$. Hence, the correct option is (C).

Quick Tip to Solve: Locate the large jump in successive ionisation energies: a jump after removal of one electron indicates one valence electron.

71. In the above reaction, the reagent "A" is : [4 marks]



(A) $LiAlH_4$

(B) $NaBH_4$, H_3O^+

(C) HCl, Zn − Hg

(D) Alkaline $KMnO_4$, H^+

Correct Answer: (D) Alkaline $KMnO_4$, H^+

Solution:

Concept:

- An alkyl side chain attached to a benzene ring is oxidised to a carboxyl group by hot alkaline $KMnO_4$, provided the benzylic carbon carries a hydrogen.
- The oxidation first gives a carboxylate salt in alkaline solution. Subsequent acidification with H^+ gives the free carboxylic acid.

Step 1: Identify the group that changes The starting compound has a methyl group and a methoxy group on the benzene ring. In the product the methyl group has become $-COOH$, while the methoxy substituent retains its position and identity. The required transformation is therefore benzylic side-chain oxidation, rather than reduction of a carbonyl group or cleavage of an ether. The methyl carbon has benzylic hydrogen atoms, so it satisfies the condition for permanganate oxidation.

Step 2: Choose the oxidant and work-up Alkaline $KMnO_4$ oxidises the side chain to the corresponding aromatic carboxylate. Acidification then supplies the proton of the carboxylic acid. Writing Ar for the methoxy-substituted aromatic group, the change can be summarised as

$$\mathrm{Ar{-}CH_3} \xrightarrow{\text{alkaline } \mathrm{KMnO_4}} \mathrm{Ar{-}COO^-} \xrightarrow{\mathrm{H^+}} \mathrm{Ar{-}COOH}.$$

The carbon skeleton of the aromatic ring is retained.

Step 3: Reject the reducing reagents $LiAlH_4$ and $NaBH_4$ are reducing agents, so neither provides the oxidation needed here. Acidic zinc amalgam, Zn(Hg)/HCl, is used for Clemmensen reduction of a carbonyl group to a methylene group; it also cannot oxidise the methyl substituent to a carboxyl group. Thus options A, B and C have the wrong redox role.

Step 4: Conclusion The reagent A is alkaline $KMnO_4$ followed by acidification with H^+. Hence, the correct option is (D).

Quick Tip to Solve: Compare the side chain before and after reaction: $-CH_3$ becoming $-COOH$ signals strong benzylic oxidation followed by acid work-up.

72. Arrange the following metal complex/compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.

(Atomic numbers Ce = 58, Gd = 64 and Eu = 63)

(a) $(NH_4)_2[Ce(NO_3)_6]$

(b) $Gd(NO_3)_3$ and

(c) $Eu(NO_3)_3$ [4 marks]

(A) (a) < (b) < (c)

(B) (b) < (a) < (c)

(C) (a) < (c) < (b)

(D) (c) < (a) < (b)

Correct Answer: (C) (a) < (c) < (b)

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_B$, where n is the number of unpaired electrons and μ_B is the Bohr magneton.
- Find the oxidation state before counting electrons. For high-spin $4f$ configurations, occupy the seven f orbitals singly before pairing.

Step 1: Determine the metal oxidation states In $(NH_4)_2[Ce(NO_3)_6]$, two ammonium ions require the complex anion to have charge -2. If the cerium oxidation state is x, then

$$x + 6(-1) = -2 \quad \Rightarrow \quad x = +4.$$

Both neutral nitrates $Gd(NO_3)_3$ and $Eu(NO_3)_3$ contain three nitrate anions, so their metal ions are Gd^{3+} and Eu^{3+}, respectively.

Step 2: Count the unpaired electrons Ce^{4+} has the xenon-core configuration $[Xe]4f^0$, so compound (a) has no unpaired f electron. Gd^{3+} has $[Xe]4f^7$, with all seven f orbitals singly occupied, so (b) has seven unpaired electrons. Eu^{3+} has $[Xe]4f^6$, giving six unpaired electrons in the specified high-spin treatment.

Step 3: Compare the requested moments The spin-only values are

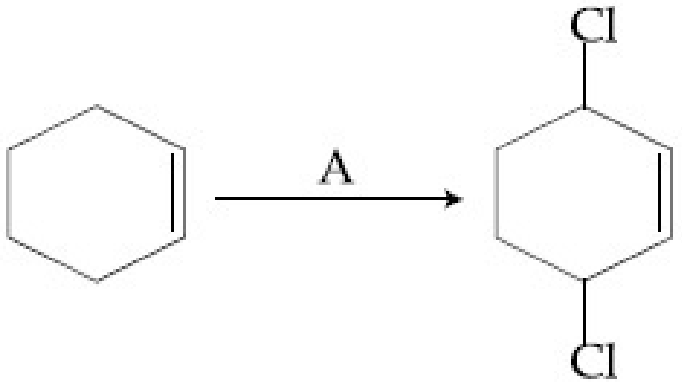
$$\mu_a = 0, \qquad \mu_b = \sqrt{63}\,\mu_B, \qquad \mu_c = \sqrt{48}\,\mu_B.$$

Hence (a) is least and (b) is greatest, giving (a) < (c) < (b). The alternative orders place either the closed-shell cerium ion above an open-shell ion or europium above gadolinium. The question explicitly asks for spin-only values, so orbital contributions are not included in this comparison.

Step 4: Conclusion The cerium complex has the smallest spin-only moment, followed by europium nitrate and then gadolinium nitrate. Hence, the correct option is (C).

Quick Tip to Solve: Balance the nitrate charges first: cerium is +4 here, whereas gadolinium and europium are +3. Count unpaired electrons only after that step.

73. Identify the reagent(s) 'A' and condition(s) for the reaction [4 marks]



(A) A = HCl, $ZnCl_2$

(B) A = HCl; Anhydrous $AlCl_3$

(C) A = Cl_2; UV light

(D) A = Cl_2; dark, Anhydrous $AlCl_3$

Correct Answer: (C) A = Cl_2; UV light

Solution:

Concept:

- Chlorine under ultraviolet light undergoes homolytic cleavage, generating chlorine radicals that can replace allylic hydrogen atoms.
- An allylic carbon is a saturated carbon adjacent to a carbon-carbon double bond. Its radical is stabilised by resonance, favouring allylic substitution under radical conditions.

Step 1: Locate the substitutions in the product The reactant is cyclohexene. The product retains the double bond at the same side of the ring and has chlorine substituents on the two adjacent allylic ring carbons. The change shown is replacement of allylic carbon-hydrogen bonds by carbon-chlorine bonds. Addition across the double bond would remove that double bond and put chlorine on its two carbons, which is not the product drawn.

Step 2: Select radical-generating conditions Ultraviolet light initiates the homolytic cleavage

$$Cl_2 \xrightarrow{h\nu} 2\,Cl\cdot .$$

A chlorine radical abstracts an allylic hydrogen to produce HCl and an allylic radical. That radical reacts with another chlorine molecule to form an allylic carbon-chlorine bond and regenerate a chlorine radical. Further substitution at the other allylic position gives the dichloro product shown, while preserving the alkene bond.

Step 3: Distinguish the other reagent sets HCl/$ZnCl_2$ is associated with replacement of an alcohol hydroxyl group, but cyclohexene has no hydroxyl group. HCl with anhydrous $AlCl_3$ does not provide the radical chlorination conditions needed for the depicted pair of allylic substitutions. Chlorine in the dark with a Lewis acid is also not the stated photochemical radical pathway; electrophilic addition would consume the alkene. Thus A, B and D do not explain retention of the double bond together

with allylic chlorination.

Step 4: Conclusion Reagent A is Cl_2 under UV light. Hence, the correct option is (C).

Quick Tip to Solve: If chlorine replaces hydrogens adjacent to an alkene while the double bond survives, look for radical conditions such as UV light.

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74. Statement I : Sodium hydride can be used as an oxidising agent.
Statement II : The lone pair of electrons on nitrogen in pyridine makes it basic.
Choose the CORRECT answer from the options given below : [4 marks]

(A) Statement I is false but statement II is true
(B) Both statement I and statement II are true
(C) Statement I is true but statement II is false
(D) Both statement I and statement II are false

Correct Answer: (A) Statement I is false but statement II is true

Solution:

Concept:

- An oxidising agent accepts electrons and is itself reduced. In sodium hydride, NaH, hydrogen is present as hydride, H^-, with oxidation state -1.
- A Lewis base donates a lone pair. In pyridine, the nitrogen lone pair occupies an orbital outside the aromatic pi system and is available for protonation.

Step 1: Evaluate Statement I Sodium hydride is a hydride reagent and a strong base, not an oxidising reagent in the stated chemical context. Its hydrogen is already in the -1 oxidation state, the lowest usual oxidation state of hydrogen. Hydride can lose electron density or transfer to an electrophile; that is reducing behaviour, rather than acceptance of electrons as required for an oxidising agent. Sodium hydride is especially used to remove acidic protons with liberation of hydrogen gas. Thus Statement I is false.

Step 2: Evaluate Statement II Pyridine contains an sp^2-hybridised nitrogen atom. Its perpendicular p orbital supplies an electron to the aromatic pi system, whereas the lone pair resides in an in-plane hybrid orbital. That pair is not needed to maintain the aromatic sextet. It can be donated to H^+, forming the pyridinium ion:

$$C_5H_5N + H^+ \longrightarrow C_5H_5NH^+.$$

This availability of the lone pair makes pyridine basic. Thus Statement II is true.

Step 3: Match the two truth values The correct combination is false for Statement I and true for Statement II. Option B incorrectly treats hydride as an oxidising agent; C also reverses the correct judgement of pyridine; D incorrectly denies the basic role of the nitrogen lone pair. Only option A records both evaluations correctly.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: For pyridine, distinguish the nitrogen p electron in the aromatic system from the separate lone pair that accepts a proton.

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75. The INCORRECT statement regarding the structure of C_{60} is : [4 marks]

(A) The five-membered rings are fused only to six-membered rings.

(B) It contains 12 six-membered rings and 24 five-membered rings.

(C) Each carbon atom forms three sigma bonds.

(D) The six-membered rings are fused to both six and five-membered rings.

Correct Answer: (B) It contains 12 six-membered rings and 24 five-membered rings.

Solution:

Concept:

- Buckminsterfullerene, C_{60}, has a closed cage shaped like a truncated icosahedron, with carbon atoms at its 60 vertices.
- Each carbon is bonded to three neighbouring carbons through sigma bonds. The cage contains 12 pentagons and 20 hexagons; pentagons do not share edges with one another.

Step 1: Check the ring count independently Let P and H denote the numbers of pentagonal and hexagonal faces. Three edges meet at every carbon vertex, giving $E = 60 \times 3/2 = 90$. Euler's relation for the cage yields $F = 2 - 60 + 90 = 32$, so $P + H = 32$. Counting face edges, each edge belongs to two faces, so

$$5P + 6H = 2E = 180.$$

Subtracting five times the first relation gives $H = 20$, followed by $P = 12$. Thus the statement claiming 12 six-membered rings and 24 five-membered rings is incorrect.

Step 2: Verify the other structural statements In the C_{60} cage each pentagon is surrounded by hexagons, so the five-membered rings are fused only to six-membered rings: A is correct. Each carbon has three neighbours and forms three sigma bonds, making C correct. Each hexagon shares edges with both pentagons and other hexagons, so D is also correct. The remaining p orbitals take part in the cage's pi bonding; this does not add extra sigma bonds at a carbon. The word INCORRECT in the question requires selecting the false ring-count statement rather than one of these true bonding or fusion statements.

Step 3: Conclusion C_{60} contains 20 six-membered and 12 five-membered rings, not the numbers stated in B. Hence, the correct option is (B).

Quick Tip to Solve: For fullerene, remember 12 pentagons and 20 hexagons; Euler's relation plus edge counting verifies both numbers without a picture.

76. Which of the following reduction reaction CANNOT be carried out with coke? [4 marks]

(A) $ZnO \rightarrow Zn$

(B) $Fe_2O_3 \rightarrow Fe$

(C) $Cu_2O \rightarrow Cu$

(D) $Al_2O_3 \rightarrow Al$

Correct Answer: (D) $Al_2O_3 \rightarrow Al$

Solution:

Concept:

- A metal oxide can be reduced by carbon only when coupling oxide decomposition with carbon oxidation gives a negative overall Gibbs energy at the operating temperature.
- In an Ellingham diagram, the very low oxide-formation line of a strongly electropositive metal indicates an exceptionally stable oxide that common carbon reduction cannot practically decompose.

Step 1: Identify the exceptionally stable oxide Aluminium has a high affinity for oxygen, and Al_2O_3 is a very stable oxide. Its formation has a much more negative Gibbs energy than that of the oxides commonly reduced with coke. Consequently, carbon oxidation cannot compensate for the energy required to remove oxygen from alumina under the ordinary metallurgical conditions considered here. Aluminium is therefore obtained by electrolytic reduction of alumina dissolved in molten cryolite.

Step 2: Check the oxides that coke can reduce Zinc oxide is reduced at sufficiently high temperature by carbon, for example

$$ZnO + C \longrightarrow Zn + CO.$$

Iron oxides are reduced in a blast furnace using coke and the carbon monoxide generated from it. Copper(I) oxide can also be reduced by carbon because copper has a much smaller affinity for oxygen than aluminium. These reactions are thermodynamically accessible at suitable temperatures; they cannot be placed in the same category as alumina.

Step 3: Select the impossible coke reduction The requested exception is therefore $Al_2O_3 \longrightarrow Al$, option D. Options A, B and C describe oxides for which carbon or its generated reducing gas can remove oxygen. Simply observing that carbon is a reducing agent is insufficient: the relative stability of the specific oxide determines whether the combined process is favourable. Trying carbon reduction of alumina also risks carbide formation rather than obtaining aluminium metal cleanly.

Step 4: Conclusion The reduction $Al_2O_3 \longrightarrow Al$ cannot be carried out with coke. Hence, the correct option is (D).

Quick Tip to Solve: Remember aluminium as an electrolytic-extraction metal: the exceptional stability of alumina rules out ordinary coke reduction.

77. Which of the following polymer is used in the manufacture of wood laminates? [4 marks]

(A) Melamine formaldehyde resin

(B) Urea formaldehyde resin

(C) cis-poly isoprene

(D) Phenol and formaldehyde resin

Correct Answer: (B) Urea formaldehyde resin

Solution:

Concept:

- Urea-formaldehyde resin is a condensation polymer formed from urea and formaldehyde. Its cross-linked structure makes it a thermosetting resin.
- Thermosetting resins can act as binders and adhesives in wood products because curing creates a rigid network that does not soften readily on reheating.

Step 1: Recall the use associated with the resin Urea-formaldehyde resin is the polymer conventionally specified for the manufacture of wood laminates. Urea reacts with formaldehyde to produce intermediate methylol derivatives, followed by condensation and network formation. On curing, the resin binds wooden layers together and produces a rigid bonded material. The question asks for this standard application rather than merely whether a polymer can have some use in a composite material.

Step 2: Compare the other named polymers Melamine-formaldehyde resin is commonly associated with durable, hard, scratch-resistant moulded articles and surface materials such as unbreakable crockery. Although related resins occur in broader laminate technology, the standard textbook association for wood laminates in this list is urea-formaldehyde. Cis-polyisoprene is natural rubber: it is an elastomer with flexible chains, suited to elastic behaviour rather than serving as the specified rigid wood-laminate adhesive.

Step 3: Separate phenolic resin from the requested answer Phenol-formaldehyde resin, including Bakelite, is another thermosetting condensation polymer. Its familiar applications include electrical switches, insulating components and handles because it is hard and resistant to heat. It is therefore chemically distinct from the urea-formaldehyde resin singled out by the stated application. Comparing both the monomer pair and the conventional end use identifies option B, rather than selecting any thermosetting resin indiscriminately.

Step 4: Conclusion Urea-formaldehyde resin is used in the manufacture of wood laminates. Hence, the correct option is (B).

Quick Tip to Solve: Pair the standard applications: urea-formaldehyde with wood laminates, melamine-formaldehyde with crockery, and cis-polyisoprene with rubber.

78. The green house gas/es is (are) :

(A) Carbon dioxide

(B) Oxygen

(C) Water vapour

(D) Methane

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A) and (C) only

(B) (A) and (B) only

(C) (A), (C) and (D) only

(D) (A) only

Correct Answer: (C) (A), (C) and (D) only

Solution:

Concept:

- Greenhouse gases absorb outgoing terrestrial infrared radiation through molecular vibrations that change their dipole moment. Absorption and re-emission retain heat in the lower atmosphere.
- The greenhouse effect depends on infrared absorption, not simply on whether a gas is abundant or participates in respiration. Carbon dioxide, methane and water vapour are important greenhouse gases.

Step 1: Evaluate the carbon-containing gases Carbon dioxide, statement (A), has infrared-active bending and asymmetric stretching vibrations. Although its equilibrium molecule has no permanent dipole, those vibrations can absorb radiation. Methane, statement (D), also has infrared-active vibrational modes and is a potent greenhouse gas. Both must be included in the selected combination.

Step 2: Distinguish water vapour from oxygen Water vapour, statement (C), is a greenhouse gas because its polar molecules absorb infrared radiation in several bands. Molecular oxygen, statement (B), is a homonuclear diatomic molecule and is not counted among the greenhouse gases in this classification. Its abundance in air does not imply the infrared absorption characteristic of greenhouse gases.

Step 3: Match the complete set The true entries are therefore (A), (C) and (D). The first option omits methane; the second includes oxygen and omits two greenhouse gases; the last retains carbon dioxide alone. Only the third option contains all three true entries without oxygen.

Step 4: Conclusion The greenhouse gases are carbon dioxide, water vapour and methane. Hence, the correct option is (C).

Quick Tip to Solve: When selecting greenhouse gases, check infrared-active molecular vibrations; do not confuse a gas needed for life with a greenhouse gas.

79. The correct statements about H_2O_2 are :

(A) used in the treatment of effluents.

(B) used as both oxidising and reducing agents.

(C) the two hydroxyl groups lie in the same plane.

(D) miscible with water.

Choose the correct answer from the options given below : [4 marks]

(A) (A), (C) and (D) only

(B) (A), (B), (C) and (D)

(C) (A), (B) and (D) only

(D) (B), (C) and (D) only

Correct Answer: (C) (A), (B) and (D) only

Solution:

Concept:

- In hydrogen peroxide, H_2O_2, oxygen has oxidation number -1, intermediate between 0 in O_2 and -2 in water. It can consequently act as either a reducing or an oxidising agent.
- The peroxide molecule is nonplanar, with a skew arrangement of its O–H bonds about the O–O bond. Hydrogen bonding accounts for its strong affinity for water.

Step 1: Check use and solubility Statement (A) is true: hydrogen peroxide is used in treatment of domestic and industrial effluents, where its oxidising action helps remove pollutants. Statement (D) is also true: it is miscible with water. Extensive hydrogen bonding between peroxide and water molecules favours mixing; peroxide is not an immiscible organic liquid.

Step 2: Check the two redox roles Statement (B) is true. As an oxidising agent, peroxide is reduced to water; as a reducing agent, it is oxidised to oxygen. The corresponding acidic half-reactions are

$$H_2O_2 + 2\,H^+ + 2\,e^- \longrightarrow 2\,H_2O$$

and

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

Thus neither role alone describes all its redox behaviour.

Step 3: Reject the planar structure Statement (C) is false: the two hydroxyl groups are not in the same plane in the usual skew molecular structure. The true set is (A), (B) and (D). Every other listed combination includes the false planar-structure statement, so those combinations must be rejected.

Step 4: Conclusion The treatment use, both redox roles and miscibility statements are correct; the planar-structure statement is false. Hence, the correct option is (C).

Quick Tip to Solve: For peroxide questions, remember oxygen at oxidation state -1 permits both redox roles, while the molecule itself is skew rather than planar.

80. The INCORRECT statements below regarding colloidal solutions is : [4 marks]

(A) The flocculating power of Al^{3+} is more than that of Na^{+}.

(B) A colloidal solutions shows Brownian motion of colloidal particles.

(C) An ordinary filter paper can stop the flow of colloidal particles.

(D) A colloidal solutions shows colligative properties.

Correct Answer: (C) An ordinary filter paper can stop the flow of colloidal particles.

Solution:

Concept:

- Colloidal particles are larger than molecular solutes but smaller than the pores of ordinary filter paper. They pass through ordinary paper and require methods such as ultrafiltration for separation.
- Brownian motion is the irregular motion caused by unequal bombardment by molecules of the dispersion medium. Coagulation is promoted by counter-ions, with greater charge generally giving greater coagulating power.

Step 1: Test the filtration claim An ordinary filter paper does not retain typical colloidal particles: its pores are too large. Therefore the statement that ordinary paper can stop their flow is incorrect. This behaviour distinguishes colloids from coarse suspensions, whose larger suspended particles can be separated by ordinary filtration.

Step 2: Check the remaining physical properties Colloidal particles show Brownian motion, so the second option states a real property. Colloidal dispersions also show colligative properties such as osmotic pressure, although these effects are small at a given mass concentration because colloidal particles have large masses and their number is comparatively small. Small effects are not absent effects.

Step 3: Check coagulation and select the exception For the usual negatively charged sol under comparison, aluminium ions have much greater flocculating power than sodium ions by the Hardy–Schulze rule. Thus the first statement expresses the standard coagulation comparison. The filtration claim remains the intended incorrect statement among the listed properties.

Step 4: Conclusion Ordinary filter paper cannot stop the flow of colloidal particles. Hence, the correct option is (C).

Quick Tip to Solve: Keep ordinary filtration and ultrafiltration separate: colloids pass ordinary paper but can be retained by an appropriate ultrafilter.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d − d transition. The octahedral splitting energy for the above complex is _____ $\times 10^{-19}$ J. (Round off to the Nearest Integer).
h = 6.626 $\times 10^{-34}$ Js; c = 3 $\times 10^{8}$ ms^{-1} [4 marks]

Solution:

Concept:

- In an octahedral complex the five d orbitals split into lower t_{2g} and upper e_g levels. A d–d absorption can promote an electron across this separation, denoted Δ_o.
- The energy absorbed from a photon is $E = h\nu = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is the vacuum wavelength.

Step 1: Identify the energy gap In $[Ti(H_2O)_6]^{3+}$, titanium is in oxidation state +3 and has one d electron. The specified absorption corresponds to excitation from the lower octahedral level to the upper level. Therefore the photon energy gives the splitting per complex, rather than a molar energy. No Avogadro factor is needed because the answer is requested in joules.

Step 2: Convert the wavelength and substitute The wavelength must be in metres to use the supplied SI constants:

$$\lambda = 498\,\text{nm} = 498 \times 10^{-9}\,\text{m}.$$

Hence

$$\Delta_o = \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{498 \times 10^{-9}}\,\text{J}.$$

The metre units cancel, and the remaining unit is joule.

Step 3: Round the requested coefficient The numerical value is

$$\Delta_o = 3.9916 \times 10^{-19}\,\text{J}.$$

The blank multiplies 10^{-19} J, so the quantity to round is 3.9916. Rounding that coefficient to the nearest integer yields 4; it would be incorrect to round the very small energy itself to zero.

Step 4: Conclusion The splitting energy is approximately 4×10^{-19} J, so the required integer is $\boxed{4}$.

Quick Tip to Solve: First express the photon energy using the power of ten printed after the blank; round its coefficient only.

82. A 5.0 m mol dm^{-3} aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell of cell constant 1.3 cm^{-1}. The molar conductivity of this solution is _____ mSm^2 mol^{-1}. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Conductivity is conductance times cell constant: $\kappa = G(l/A)$. Conductance measures the particular cell, whereas conductivity is the material property after accounting for cell geometry.
- Molar conductivity is $\Lambda_m = \kappa/c$, provided conductivity and concentration use compatible length units. In SI, conductivity is in $\mathrm{S\,m^{-1}}$ and concentration in $\mathrm{mol\,m^{-3}}$.

Step 1: Convert the measured quantities to SI The conductance is $G = 0.55\,\mathrm{mS} = 0.55 \times 10^{-3}\,\mathrm{S}$. The cell constant is $1.3\,\mathrm{cm^{-1}} = 130\,\mathrm{m^{-1}}$, since one inverse centimetre equals one hundred inverse metres. The concentration $5.0\,\mathrm{mmol\,dm^{-3}}$ is $5.0\,\mathrm{mol\,m^{-3}}$: both the millimole and cubic decimetre contribute a factor of 10^{-3}.

Step 2: Calculate conductivity Multiply the conductance by the cell constant, rather than dividing:

$$\kappa = (0.55 \times 10^{-3})(130) = 0.0715\,\mathrm{S\,m^{-1}}.$$

This step removes the influence of the dimensions of the measuring cell and gives the conductivity of the solution.

Step 3: Calculate and express molar conductivity Dividing by the molar concentration gives

$$\Lambda_m = \frac{0.0715}{5.0} = 0.0143\,\mathrm{S\,m^2\,mol^{-1}}.$$

Since one siemens equals 10^3 millisiemens, this is $14.3\,\mathrm{mS\,m^2\,mol^{-1}}$. Rounding to the nearest integer in the requested unit yields 14.

Step 4: Conclusion The molar conductivity is $14.3\,\mathrm{mS\,m^2\,mol^{-1}}$, giving the required integer $\boxed{14}$.

Quick Tip to Solve: Convert the cell constant and concentration to SI together; mmol per cubic decimetre has the same numerical value as mol per cubic metre.

83. At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is _____ J. (Round off to the Nearest Integer).

[Given : R = 8.314 J mol^{-1} K^{-1}. Assume, hydrogen is an ideal gas] [Atomic mass of Fe is 55.85 u] [4 marks]

Solution:

Concept:

- The reaction $\mathrm{Fe + 2\,HCl \longrightarrow FeCl_2 + H_2}$ produces one mole of hydrogen per mole of iron consumed. Thus the mass of iron fixes the number of moles of evolved gas.
- For ideal gas expanding against constant external pressure, the positive work done by the gas is

$W_{\text{by}} = P_{\text{ext}}\Delta V = nRT$, when the evolved gas occupies its volume at that pressure. Work on the system has the opposite sign.

Step 1: Calculate the hydrogen amount Using the given atomic mass,

$$n(\text{Fe}) = \frac{50}{55.85}\,\text{mol} = 0.895255\,\text{mol}.$$

The balanced reaction has equal coefficients for iron and hydrogen, so $n(\text{H}_2) = 0.895255\,\text{mol}$. The coefficient of hydrochloric acid does not double the hydrogen amount.

Step 2: Use the ideal-gas expansion relation The absolute temperature is $T = 25 + 273 = 298\,\text{K}$. At the stated constant pressure the gas volume obeys $P_{\text{ext}}V = nRT$. As the gas is generated, its volume increases from no hydrogen volume to this value. Therefore

$$W_{\text{by}} = P_{\text{ext}}\Delta V = nRT.$$

Pressure need not be separately converted because it cancels when the ideal-gas volume is substituted.

Step 3: Substitute and apply the sign convention The positive work performed on the surroundings is

$$W_{\text{by}} = \frac{50}{55.85} \times 8.314 \times 298 = 2218.06\,\text{J}.$$

Using the usual thermochemical convention would give work on the gas as $-2218.06\,\text{J}$. Here the question explicitly asks work done by the gas, so its requested value is positive. Round only after completing the calculation.

Step 4: Conclusion The work done by the hydrogen gas is $\boxed{2218\,\text{J}}$ to the nearest integer.

Quick Tip to Solve: When gas is produced at constant pressure, use its stoichiometric mole amount in nRT and check whether work is asked by or on the gas.

84. The number of orbitals with n = 5, m_1 = +2 is _____. (Round off to the Nearest Integer).

[4 marks]

Solution:

Concept:

- For a given principal quantum number n, the orbital angular momentum quantum number takes values $l = 0, 1, \ldots, n-1$. For each l, the magnetic quantum number ranges from $-l$ to $+l$ in integral steps.
- An orbital is specified by the three quantum numbers (n, l, m_l). Electron spin specifies an electron state within an orbital and does not double the number of orbitals.

Step 1: List the allowed subshells For $n = 5$, the allowed values of l are $0, 1, 2, 3, 4$, corresponding to the 5s, 5p, 5d, 5f and 5g subshells. The condition in the question fixes $m_l = +2$. It does not ask for every orbital in the fifth shell, so the total $n^2 = 25$ is not the quantity to report.

Step 2: Apply the magnetic quantum number restriction An allowed magnetic quantum number must

satisfy $|m_l| \le l$. Since $|m_l| = 2$, subshells with $l = 0$ and $l = 1$ cannot contribute. The remaining values $l = 2, 3, 4$ each permit $m_l = +2$. Each permitted value of l, with fixed n and m_l, identifies exactly one orbital.

Step 3: Count the distinct orbitals The three possible triples are

$$(n, l, m_l) = (5, 2, 2),\ (5, 3, 2),\ (5, 4, 2).$$

Thus there is one qualifying orbital in each of the 5d, 5f and 5g subshells, giving three orbitals in total. Although each orbital may accommodate two electrons of opposite spins, that capacity is irrelevant when counting orbitals. The result is already an integer.

Step 4: Conclusion The number of orbitals with $n = 5$ and $m_l = +2$ is $\boxed{3}$.

Quick Tip to Solve: For fixed n and magnetic quantum number m, count the allowed l values from absolute m through n-1; do not multiply by spin degeneracy.

85. A and B decompose via first order kinetics with half-lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is _____ min. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- For first-order decomposition, the concentration remaining is $[X]_t = [X]_0 e^{-k_X t}$. The rate constant is related to half-life by $k_X = \ln 2 / t_{1/2,X}$.
- For an initially equimolar mixture, the initial concentrations cancel when taking the ratio. A larger half-life means slower decomposition, so A becomes more abundant relative to B even though both concentrations decrease.

Step 1: Write the two independent decay laws The mixture is non-reactive: A and B decompose independently, so neither species changes the rate law of the other. Let their common initial concentration be C_0. Then $[A]_t = C_0 e^{-k_A t}$ and $[B]_t = C_0 e^{-k_B t}$, with time expressed in minutes. The given half-lives give

$$k_A = \frac{\ln 2}{54}, \qquad k_B = \frac{\ln 2}{18} \quad (\text{min}^{-1}).$$

Step 2: Form the required concentration ratio Divide the two expressions before substituting concentrations. This avoids needing the actual initial amount of either substance:

$$\frac{[A]_t}{[B]_t} = e^{(k_B - k_A)t}, \qquad k_B - k_A = \ln 2 \left(\frac{1}{18} - \frac{1}{54} \right) = \frac{\ln 2}{27}.$$

The positive difference is consistent with B disappearing faster. Requiring the ratio to be 16 gives

$$\ln 16 = \frac{t \ln 2}{27}, \qquad t = 27 \frac{4 \ln 2}{\ln 2} = 108 \text{ min}.$$

As a direct check, this is two half-lives of A and six half-lives of B: the fractions remaining are $1/4$ and

$1/64$, whose ratio is 16.

Step 3: Conclusion The required time is 108 min. The nearest-integer numerical answer is $\boxed{108}$.

Quick Tip to Solve: For two independent first-order decays starting equimolar, use the difference of rate constants in the logarithm of their concentration ratio.

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86. In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is _____. (Round off to the Nearest Integer). [Given : Aqueous tension at 287 K = 14 mm of Hg] [4 marks]

Solution:

Concept:

- In Dumas nitrogen estimation the measured gas is molecular nitrogen, N_2. Its mass is obtained from the dry gas volume or from $n = PV/(RT)$.
- Gas collected over water contains water vapour. Subtract the aqueous tension from the observed pressure before calculating the amount of nitrogen; use absolute temperature and mutually consistent pressure and volume units.

Step 1: Correct the nitrogen pressure The collected 30 mL gas is moist. Dalton's law assigns part of its 758 mm Hg pressure to water vapour. With aqueous tension 14 mm Hg, the dry nitrogen pressure is

$$P_{N_2} = 758 - 14 = 744 \text{ mm Hg} = \frac{744}{760} \text{ atm}.$$

The nitrogen volume is $V = 30 \times 10^{-3} = 0.030$ L, and the gas temperature is already absolute, $T = 287$ K. No Celsius conversion is necessary.

Step 2: Calculate nitrogen mass and percentage Use $R = 0.082057 \text{ L atm mol}^{-1} \text{ K}^{-1}$ with these units:

$$n(N_2) = \frac{(744/760)(0.030)}{0.082057(287)} \simeq 1.247 \times 10^{-3} \text{ mol}.$$

Each mole of molecular nitrogen has mass 28 g. This factor already includes both nitrogen atoms; multiplying by two again would double-count them. Thus

$$m_N = (1.247 \times 10^{-3})(28) \simeq 0.03492 \text{ g},$$

$$\%N = \frac{0.03492}{0.1840} \times 100 \simeq 18.98\%.$$

The mass of nitrogen is smaller than the original sample mass, and rounding the percentage only at the end gives the required integer.

Step 3: Conclusion The nitrogen composition is approximately 18.98%, so the nearest-integer answer is $\boxed{19}$.

Quick Tip to Solve: For moist nitrogen subtract aqueous tension first, and use 28 g per mole of nitrogen gas when converting its amount into elemental nitrogen mass.

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87. When 35 mL of 0.15 M lead nitrate solution is mixed with 20 mL of 0.12 M chromic sulphate solution, _____ $\times 10^{-5}$ moles of lead sulphate precipitate out. (Round off to the Nearest Integer).

[4 marks]

Solution:

Concept:

- Lead nitrate supplies one Pb^{2+} ion per formula unit. Chromic sulphate is $Cr_2(SO_4)_3$, so each mole supplies three moles of sulphate ions.
- The precipitation reaction is $Pb^{2+} + {SO_4}^{2-} \longrightarrow PbSO_4(s)$. The smaller available amount of these two reacting ions determines the amount of precipitate.

Step 1: Find the available amounts of reacting ions Convert the solution volumes to litres before using $n = MV$. For lead nitrate,

$$n(Pb(NO_3)_2) = 0.15(0.035) = 0.00525 \text{ mol.}$$

This also equals the available moles of lead ions. The chromic sulphate amount is

$$n(Cr_2(SO_4)_3) = 0.12(0.020) = 0.00240 \text{ mol,}$$

and therefore $n({SO_4}^{2-}) = 3(0.00240) = 0.00720$ mol. Comparing the formula-unit amounts without the factor of three would select the wrong limiting reagent.

Step 2: Apply the precipitation stoichiometry The balanced molecular equation is

$$3\,Pb(NO_3)_2 + Cr_2(SO_4)_3 \longrightarrow 3\,PbSO_4(s) + 2\,Cr(NO_3)_3.$$

Only 0.00525 mol of sulphate is needed to precipitate all the lead, whereas 0.00720 mol is available. Lead ions are therefore limiting, and 0.00195 mol sulphate remains after the stoichiometric precipitation. The 1:1 ionic ratio gives

$$n(PbSO_4) = 0.00525 \text{ mol} = 525 \times 10^{-5} \text{ mol.}$$

Mixing changes the final solution volume but does not change these initial mole amounts; no dilution factor enters the limiting-reagent comparison.

Step 3: Conclusion The amount of lead sulphate precipitated is 525×10^{-5} mol. The required nearest-integer coefficient is $\boxed{525}$.

Quick Tip to Solve: In precipitation calculations compare reacting ion amounts, including every ion supplied by one formula unit, rather than comparing solution molarities alone.

88. Sulphurous acid (H_2SO_3) has $Ka_1 = 1.7 \times 10^{-2}$ and $Ka_2 = 6.4 \times 10^{-8}$. The pH of 0.588 M H_2SO_3 is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Sulphurous acid is diprotic: $H_2SO_3 \rightleftharpoons H^+ + HSO_3^-$, followed by $HSO_3^- \rightleftharpoons H^+ + SO_3^{2-}$. Its two dissociations have very different equilibrium constants.
- The first dissociation supplies almost all the hydrogen ions because $K_{a2} \ll K_{a1}$. Calculate $\text{pH} = -\log_{10}[H^+]$, and round the pH only after obtaining the hydrogen-ion concentration.

Step 1: Solve the first-dissociation equilibrium Let x be the concentration dissociated from the initial $C = 0.588\ \text{mol L}^{-1}$. Neglecting the extremely small second dissociation initially gives $[H^+] = [HSO_3^-] = x$ and $[H_2SO_3] = C - x$. Hence

$$K_{a1} = \frac{x^2}{0.588 - x} = 0.017,$$

$$x^2 + 0.017x - 0.009996 = 0.$$

The positive quadratic root is used because concentration cannot be negative:

$$x = \frac{-0.017 + \sqrt{(0.017)^2 + 4(0.009996)}}{2} \simeq 0.09184\ \text{mol L}^{-1}.$$

Step 2: Check the second dissociation and calculate pH In the second equilibrium, the already present hydrogen ions suppress further dissociation. In particular,

$$\frac{[SO_3^{2-}]}{[HSO_3^-]} = \frac{K_{a2}}{[H^+]} \simeq \frac{6.4 \times 10^{-8}}{0.09184} \simeq 7.0 \times 10^{-7}.$$

This confirms that the extra hydrogen-ion contribution is negligible here. Therefore

$$\text{pH} = -\log_{10}(0.09184) \simeq 1.037.$$

The approximation $x \approx \sqrt{K_{a1}C}$ also suggests a pH near 1, but the quadratic avoids assuming that the roughly 16 percent first dissociation is negligible in the denominator.

Step 3: Conclusion The pH is approximately 1.04. Rounded to the nearest integer, the numerical answer is $\boxed{1}$.

Quick Tip to Solve: For a diprotic acid with widely separated dissociation constants, solve the first equilibrium and then test whether the second contributes appreciably.

89. At 363 K, the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is _____ kPa. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- For an ideal liquid solution, Raoult's law gives the partial pressure of each volatile component: $p_A = x_A p_A^\circ$ and $p_B = x_B p_B^\circ$. The starred pressures refer to the pure liquids at the same temperature.
- Dalton's law gives total vapour pressure $P = p_A + p_B$. The mole fractions in Raoult's law are those of the liquid mixture, not the equilibrium vapour.

Step 1: Calculate the liquid mole fractions The mixture contains one mole of A and two moles of B. Its total liquid amount is three moles, so

$$x_A = \frac{n_A}{n_A + n_B} = \frac{1}{3}, \qquad x_B = \frac{n_B}{n_A + n_B} = \frac{2}{3}.$$

The fractions sum to unity, as required for a binary solution. Since the quantities are already given in moles, no molar masses or densities are needed. Both pure-component vapour pressures are given at 363 K, the temperature of interest.

Step 2: Add the two equilibrium partial pressures Use the given pure-liquid pressures $p_A^\circ = 21$ kPa and $p_B^\circ = 18$ kPa:

$$p_A = \frac{1}{3}(21) = 7 \text{ kPa}, \qquad p_B = \frac{2}{3}(18) = 12 \text{ kPa}.$$

Both liquids are volatile, so both terms must be retained. The total pressure is

$$P = 7 + 12 = 19 \text{ kPa}.$$

This lies between 18 and 21 kPa and is closer to 18 kPa because the liquid contains more B. The same result follows from $P = p_B^\circ + x_A(p_A^\circ - p_B^\circ) = 18 + (1/3)(3)$. A simple unweighted average would incorrectly assume equal mole amounts.

Step 3: Conclusion The ideal mixture has vapour pressure 19 kPa. Its nearest-integer numerical answer is $\boxed{19}$.

Quick Tip to Solve: For two volatile ideal components use the liquid mole-fraction-weighted average of their pure vapour pressures, keeping both contributions.

90. Ga (atomic mass 70 u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is _____ $\times 10^{21}$. (Round off to the Nearest Integer). [Given : N_A = 6.023 $\times$ 10_{23}] [4 marks]

Solution:

Concept:

- A close-packed structure with N atoms has N octahedral voids and $2N$ tetrahedral voids. Thus the total number of these interstitial voids is $3N$, for both hcp and ccp packing.
- The number of atoms in a sample is $N = (m/M)N_A$, where m is the sample mass, M its molar mass and N_A Avogadro's constant. Void counts are dimensionless counts, not volumes.

Step 1: Find the number of gallium atoms The given atomic mass 70 u corresponds to a molar mass of $70\ \mathrm{g\,mol^{-1}}$. Therefore the 0.581 g sample contains

$$n(\mathrm{Ga}) = \frac{0.581}{70} = 0.00830\ \mathrm{mol}.$$

Using the value of Avogadro's constant supplied in the question gives

$$N = 0.00830(6.023 \times 10^{23}) = 4.99909 \times 10^{21}\ \text{atoms}.$$

This counts all the atoms in the macroscopic sample; choosing a conventional hcp unit cell is unnecessary and must not introduce an extra factor of six.

Step 2: Count both kinds of interstitial void For every close-packed atom there is one octahedral void and two tetrahedral voids. Hence

$$N_{\mathrm{oct}} = 4.99909 \times 10^{21}, \qquad N_{\mathrm{tet}} = 9.99818 \times 10^{21}.$$

The total is their sum, not their difference and not the number of only one type:

$$N_{\mathrm{voids}} = 3N = 14.99727 \times 10^{21}.$$

The requested blank is the coefficient multiplying 10^{21}. Round that coefficient, 14.99727, to the nearest integer. No packing-efficiency factor is applied: packing efficiency describes occupied volume, whereas these relationships count the discrete tetrahedral and octahedral sites.

Step 3: Conclusion The total number of voids is 14.99727×10^{21}, approximately 15×10^{21}. The requested coefficient is $\boxed{15}$.

Quick Tip to Solve: First count atoms using mass divided by molar mass; then multiply by three for the total tetrahedral plus octahedral void count in hcp or ccp packing.