JEE Main 2021 Paper 1 (B.E./B.Tech) Mar 17 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 2 (March), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 17 March 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. In a triangle PQR, the co-ordinates of the points P and Q are (−2, 4) and (4, −2) respectively. If the equation of the perpendicular bisector of PR is 2x − y + 2 = 0, then the centre of the circumcircle of the △PQR is : [4 marks]

(A) (−1, 0)

(B) (1, 4)

(C) (0, 2)

(D) (−2, −2)

Correct Answer: (D) (−2, −2)

Solution:

Concept:

- The circumcentre is equidistant from all three vertices and lies on the perpendicular bisector of each side.
- For a centre (h, k), equality of squared distances avoids square roots: $OP^2 = OQ^2$. The given perpendicular bisector of PR supplies the second linear equation.

Step 1: Find the perpendicular bisector of PQ Let the circumcentre be $O(h, k)$. Since $P = (-2, 4)$ and $Q = (4, -2)$, equate the squared distances:

$$(h+2)^2 + (k-4)^2 = (h-4)^2 + (k+2)^2.$$

Expanding both sides and cancelling the quadratic terms gives

$$4h - 8k = -8h + 4k, \qquad 12h = 12k.$$

Hence the perpendicular bisector of PQ is $k = h$. The same result follows because PQ has midpoint $(1, 1)$, slope -1, and perpendicular-bisector slope 1.

Step 2: Intersect the two bisectors and test the options The centre must also lie on the supplied perpendicular bisector of PR:

$$2h - k + 2 = 0.$$

Substitute $k = h$:

$$h + 2 = 0, \qquad h = -2, \qquad k = -2.$$

At this point, $OP^2 = 0^2 + (-6)^2 = 36$ and $OQ^2 = (-6)^2 + 0^2 = 36$, confirming equal radii. The given PR bisector ensures the radius to R is also equal. Option A fails $h = k$. Option B also fails that condition, and option C fails it as well. Thus only option D lies on both required bisectors; there is no need to determine R explicitly.

Step 3: Conclusion The centre of the circumcircle is $\boxed{(-2, -2)}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the intersection of any two perpendicular bisectors; the third vertex need not be calculated when one bisector is already given.

2. The value of

$\lim_{x\to 0^+} \frac{\cos^{-1}(x-[x]^2).\sin^{-1}(x-[x]^2)}{x-x^3}$, **where [x] denotes the greatest integer $\leq$ x is :** [4 marks]

(A) π

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) 0

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Concept:

- For a right-hand limit at zero, take $0 < x < 1$, so the greatest integer $[x]$ is exactly zero.
- The standard radian limit is $\lim_{u\to 0} \sin^{-1} u/u = 1$, and $\cos^{-1} u$ is continuous at zero with value $\pi/2$.

Step 1: Resolve the greatest integer before taking the limit Because $x \to 0^+$, all sufficiently small admissible x satisfy $0 < x < 1$. Consequently $[x] = 0$, not -1, and the argument $x - [x]^2$ is simply x. The denominator factors as

$$x - x^3 = x(1 - x^2).$$

Thus the expression under the limit becomes

$$\frac{\cos^{-1} x \, \sin^{-1} x}{x(1-x^2)} = \cos^{-1} x \, \frac{\sin^{-1} x}{x} \, \frac{1}{1-x^2}.$$

This factorisation separates the vanishing factor from the two continuous factors. It also shows why direct substitution as a quotient would give an unhelpful zero-over-zero form.

Step 2: Evaluate the three factors Write $t = \sin^{-1} x$. Then $t \to 0^+$, $x = \sin t$, and

$$\frac{\sin^{-1} x}{x} = \frac{t}{\sin t} \longrightarrow 1.$$

The other two limits are

$$\cos^{-1} x \longrightarrow \frac{\pi}{2}, \qquad \frac{1}{1-x^2} \longrightarrow 1.$$

Multiplication gives $\pi/2$. The answer is not zero: although $\sin^{-1} x$ vanishes, its first-order factor x cancels the denominator. Options A and B introduce an incorrect factor of two; the inverse cosine approaches $\pi/2$, while the remaining product approaches one.

Step 3: Conclusion The required limit is $\boxed{\pi/2}$. Hence, the correct option is (C).

Quick Tip to Solve: In a one-sided greatest-integer limit, first freeze the floor function on the appropriate interval, then apply ordinary limits.

3. Which of the following statements is correct for the function g(α) for $\alpha \in$ R such that

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^{\alpha} x}{\cos^{\alpha} x + \sin^{\alpha} x} dx$$

[4 marks]

(A) $g(\alpha)$ is a strictly increasing function

(B) $g(\alpha)$ is an even function

(C) $g(\alpha)$ has an inflection point at $\alpha = -\frac{1}{2}$

(D) $g(\alpha)$ is a strictly decreasing function

Correct Answer: (B) $g(\alpha)$ is an even function

Solution:

Concept:

- The substitution $x = \pi/2 - t$ interchanges sine and cosine and exchanges the bounds $\pi/6$ and $\pi/3$.
- For positive sine and cosine, all real powers are defined. The complementary fractions with numerators $\sin^{\alpha} x$ and $\cos^{\alpha} x$ add to one.

Step 1: Apply the complementary-angle substitution On the entire integration interval, sine and cosine are strictly positive, so the denominator is positive for every real α. Let $x = \pi/2 - t$, giving $dx = -dt$. The lower bound becomes $t = \pi/3$, and the upper bound becomes $t = \pi/6$. Reversing the bounds cancels the minus sign:

$$g(\alpha) = \int_{\pi/6}^{\pi/3} \frac{\cos^{\alpha} t}{\sin^{\alpha} t + \cos^{\alpha} t} \, dt.$$

The integration variable is a dummy variable, so this is a second representation of exactly the original integral.

Step 2: Add the two representations and classify the function Adding the original form and the transformed form produces

$$2g(\alpha) = \int_{\pi/6}^{\pi/3} \frac{\sin^{\alpha} x + \cos^{\alpha} x}{\cos^{\alpha} x + \sin^{\alpha} x} \, dx = \int_{\pi/6}^{\pi/3} 1 \, dx.$$

Hence

$$2g(\alpha) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}, \qquad g(\alpha) = \frac{\pi}{12}.$$

This value is independent of α, so $g(-\alpha) = g(\alpha)$. A constant function is neither strictly increasing nor strictly decreasing, rejecting A and D. Its concavity does not change anywhere, so $-1/2$ is not an inflection point, rejecting C.

Step 3: Conclusion For every real α, $g(\alpha) = \pi/12$, so it is an even function. Hence, the correct option is (B).

Quick Tip to Solve: When the bounds add to $\pi/2$, try replacing x by its complementary angle and adding the resulting integrands.

4. Which of the following is true for y(x) that satisfies the differential equation $\frac{dy}{dx} = xy - 1 + x - y$; y(0) = 0 : [4 marks]

(A) y(1) = 1

(B) y(1) = $e^{-\frac{1}{2}} - 1$

(C) y(1) = $e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

(D) y(1) = $e^{\frac{1}{2}} - 1$

Correct Answer: (B) y(1) = $e^{-\frac{1}{2}} - 1$

Solution:

Concept:

- Factor the right-hand side before separating variables: $xy - 1 + x - y = (x - 1)(y + 1)$.
- Introducing $z = y + 1$ gives the homogeneous equation $z' = (x - 1)z$; the initial condition determines its multiplicative constant.

Step 1: Separate and integrate the differential equation The equation can be written as

$$\frac{dy}{dx} = (x - 1)(y + 1).$$

Near the initial point, $y + 1 = 1$, so division by $y + 1$ is permitted. Separating the variables and integrating gives

$$\frac{dy}{y + 1} = (x - 1)\,dx, \qquad \ln|y + 1| = \frac{x^2}{2} - x + C.$$

Equivalently, solving for $z = y + 1$ gives

$$y + 1 = Ae^{x^2/2 - x}.$$

The constant solution $y = -1$, which separation would exclude, does not satisfy $y(0) = 0$, so it is irrelevant to this initial-value problem.

Step 2: Use the initial condition and evaluate at one At $x = 0$, the given initial value is $y = 0$, giving $1 = A$. Therefore

$$y(x) = e^{x^2/2 - x} - 1.$$

Differentiating confirms

$$y' = (x - 1)e^{x^2/2 - x} = (x - 1)(y + 1),$$

so both the equation and the initial value are satisfied. Now set $x = 1$:

$$y(1) = e^{1/2 - 1} - 1 = e^{-1/2} - 1.$$

This is negative, approximately -0.393. Options A, C and D are positive, so they cannot equal the computed value; option B has exactly the required exponent and subtraction.

Step 3: Conclusion $\boxed{y(1) = e^{-1/2} - 1}$. Hence, the correct option is (B).

Quick Tip to Solve: Factor a bilinear expression in x and y first; shifting y by one can turn the equation into an immediately separable form.

5. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\cdots\cdots\infty}}}}$ is : [4 marks]

(A) $2+\frac{2}{5}\sqrt{30}$

(B) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(C) $5+\frac{2}{5}\sqrt{30}$

(D) $4+\frac{4}{\sqrt{5}}\sqrt{30}$

Correct Answer: (A) $2+\frac{2}{5}\sqrt{30}$

Solution:

Concept:

- An infinite periodic continued fraction repeats its entire tail after one complete period. Name the tail starting with 4, not the tail starting with 5.
- The positive value must satisfy $4 < T < 4 + 1/5$, because the denominator following the initial 4 is strictly greater than 5.

Step 1: Replace the repeated tail by one variable Let the value of the displayed continued fraction be T. After the first 4 and the next 5, the remaining tail begins again with 4 and has exactly the same pattern. Therefore

$$T = 4 + \frac{1}{5 + 1/T}.$$

Since every displayed entry is positive, T is positive; in particular none of the denominators vanishes. Simplify the fractional part:

$$T - 4 = \frac{T}{5T + 1}.$$

Multiply by $5T + 1$ and expand:

$$(T - 4)(5T + 1) = T, \qquad 5T^2 - 20T - 4 = 0.$$

The two-level repetition is essential; treating all entries as equal would give a different quadratic.

Step 2: Choose the root compatible with the continued fraction The quadratic formula gives

$$T = \frac{20 \pm \sqrt{400 + 80}}{10} = \frac{20 \pm 4\sqrt{30}}{10} = 2 \pm \frac{2\sqrt{30}}{5}.$$

The minus root is negative and is excluded by positivity. The plus root is approximately 4.1909, which lies between 4 and 4.2 as required by the original expression. Thus option A satisfies both the quadratic and the magnitude check. Option C exceeds 5, whereas the continued fraction is less than 4.2. Options B and D contain a much larger radical contribution and also exceed this upper bound. The admissible fixed point therefore singles out A.

Step 3: Conclusion The value is $\boxed{2 + \frac{2}{5}\sqrt{30}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use one complete repeating block when replacing a continued-fraction tail, and use a simple bound to reject the extraneous quadratic root.

6. The sum of possible values of x for

$\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ is : [4 marks]

(A) $-\frac{32}{4}$

(B) $-\frac{33}{4}$

(C) $-\frac{31}{4}$

(D) $-\frac{30}{4}$

Correct Answer: (A) $-\frac{32}{4}$

Solution:

Concept:

- The principal ranges are $-\pi/2 < \tan^{-1} u < \pi/2$ and $0 < \cot^{-1} v < \pi$. Taking tangents loses integer multiples of π, so every algebraic candidate needs a branch check.
- For angles A and B, $\tan(A+B) = (\tan A + \tan B)/(1 - \tan A \tan B)$, provided the denominator is nonzero.

Step 1: Find the algebraic candidates Let $A = \tan^{-1}(x+1)$ and $B = \cot^{-1}(1/(x-1))$, where $x \neq 1$. Then $\tan A = x+1$ and $\tan B = x-1$. Taking tangents of the given equation yields

$$\frac{2x}{1-(x+1)(x-1)} = \frac{8}{31}, \qquad \frac{2x}{2-x^2} = \frac{8}{31}.$$

Cross-multiplication gives

$$4x^2 + 31x - 8 = 0 = (4x-1)(x+8).$$

Hence the candidates are $x = 1/4$ and $x = -8$. Their sum alone is not yet the answer: tangent is periodic and can introduce a candidate whose angle differs from the required principal angle by π.

Step 2: Check the principal-angle branches Both candidates are below 1, so

$$B = \pi + \tan^{-1}(x-1).$$

At $x = 1/4$, A is positive and B exceeds $\pi/2$. The sum A+B lies between $\pi/2$ and $3\pi/2$; since its tangent is positive, it lies in $(\pi, 3\pi/2)$. It therefore equals $\pi + \tan^{-1}(8/31)$, and is rejected. At $x = -8$, $A = -\tan^{-1} 7$ and $B = \pi - \tan^{-1} 9$, so

$$A + B = \pi - \tan^{-1} 7 - \tan^{-1} 9.$$

The two positive inverse tangents have sum in $(\pi/2, \pi)$, making A+B lie in $(0, \pi/2)$. Its tangent is $8/31$, so it is exactly the required principal angle. Thus the only possible value is -8.

Step 3: Conclusion The sum of valid values is $\boxed{-8 = -32/4}$. Hence, the correct option is (A). The value $-31/4$ is the sum of both algebraic candidates, but includes $x = 1/4$, which fails the original principal-value equation.

Quick Tip to Solve: Never sum roots obtained by taking tangents until each root has passed the principal-range check in the original inverse-trigonometric equation.

7. The inverse of $y = 5^{\log x}$ is : [4 marks]

(A) $x = 5^{\log y}$

(B) $x = y^{\frac{1}{\log 5}}$

(C) $x = 5^{\frac{1}{\log y}}$

(D) $x = y^{\log 5}$

Correct Answer: (B) $x = y^{\frac{1}{\log 5}}$

Solution:

Concept:

- Taking logarithms uses $\log(a^b) = b \log a$. The same logarithm base must be used consistently throughout.
- For $x > 0$, $5^{\log x} = x^{\log 5}$. Since $\log 5 > 0$ for the usual base, this power function is one-to-one on the positive real axis.

Step 1: Take logarithms and isolate the original variable The given expression contains $\log x$, so its real domain is $x > 0$, and its value y is positive. Take the logarithm of both sides:

$$y = 5^{\log x} \quad \Longrightarrow \quad \log y = (\log 5)(\log x).$$

As $\log 5 \neq 0$, division gives

$$\log x = \frac{\log y}{\log 5}.$$

Taking the antilogarithm, or using the usual power identity, gives

$$x = y^{1/\log 5}.$$

This expresses the original input uniquely in terms of its output, as required by the options.

Step 2: Verify the inverse relation and compare the alternatives Put $x = y^{1/\log 5}$ back into the original expression:

$$5^{\log x} = 5^{(\log y)/\log 5} = y.$$

Thus the relation reverses the original mapping for every positive y. Options A and D are equivalent to $x = y^{\log 5}$; they repeat the original exponent instead of reciprocating it, so their composition with the original power has exponent $(\log 5)^2$, not one. Option C instead puts $\log y$ in a denominator; it is not even defined at $y = 1$, although the original function takes x=1 to y=1. In the conventional notation for an inverse function, one can equivalently write $f^{-1}(t) = t^{1/\log 5}$ for $t > 0$.

Step 3: Conclusion The inverse relation is $\boxed{x = y^{1/\log 5}}$, for $y > 0$. Hence, the correct option is (B).

Quick Tip to Solve: First rewrite an exponential-logarithmic function as a power of x; inversion then replaces that fixed exponent by its reciprocal.

8. Two dies are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is : [4 marks]

(A) $\frac{4}{9}$

(B) $\frac{1}{2}$

(C) $\frac{5}{12}$

(D) $\frac{17}{36}$

Correct Answer: (D) $\frac{17}{36}$

Solution:

Concept:

- The two dice are independent and fair, so each ordered pair of faces is equally likely. There are $6 \times 6 = 36$ possible outcomes.
- For an event E, its probability is $P(E) = n(E)/n(S)$. The permitted labels are $1, 2, 3, 5, 7, 11$, rather than the usual consecutive integers.

Step 1: Count favourable pairs by the first die Fix the first face and list the allowed second faces whose sum is at most 8. If the first face is 1, the second can be 1, 2, 3, 5 or 7: five choices. For a first face of 2, the choices are 1, 2, 3 and 5: four choices. For a first face of 3, the same four choices work, because $3 + 5 = 8$ is allowed. For a first face of 5, the second face can be 1, 2 or 3: three choices. For a first face of 7, only 1 works. A first face of 11 permits no favourable outcome.

Step 2: Divide by the complete sample space The cases are disjoint because they have different first faces. Their total is therefore

$$n(E) = 5 + 4 + 4 + 3 + 1 + 0 = 17.$$

Thus

$$P(E) = \frac{17}{36}.$$

The order of the dice matters: for example, $(1, 7)$ and $(7, 1)$ are distinct outcomes, and both have been counted. The other offered probabilities correspond to 16, 18 and 15 favourable outcomes, respectively; none matches the explicit count of 17.

Step 3: Conclusion The required probability is $\boxed{17/36}$. Hence, the correct option is (D).

Quick Tip to Solve: For unusual dice labels, count valid second faces separately for each first face; include equality at the boundary sum.

9. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where x∈N is equal to : [4 marks]

(A) 3

(B) 1

(C) 4

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- In $(a+b)^n$, the $(r+1)$th term is $\binom{n}{r}a^{n-r}b^r$; the fourth term therefore uses $r = 3$.
- The logarithm $\log_2 x$ requires $x > 0$. Writing $t = \log_2 x$ gives $x = 2^t$, allowing an equation with a variable exponent to become a quadratic.

Step 1: Form the fourth term Here $a = x$, $b = x^{\log_2 x}$ and $n = 7$. Consequently,

$$T_4 = \binom{7}{3} x^4 \left(x^{\log_2 x}\right)^3 = 35x^{4+3\log_2 x}.$$

The stated value 4480 gives

$$x^{4+3\log_2 x} = \frac{4480}{35} = 128 = 2^7.$$

The exponent of the first factor is 4, since choosing three copies of the second term leaves four copies of the first term.

Step 2: Solve and apply the natural-number restriction Put $t = \log_2 x$, so that $x = 2^t$. Equality of powers of 2 gives

$$t(4+3t) = 7, \qquad 3t^2 + 4t - 7 = (t-1)(3t+7) = 0.$$

Hence $t = 1$ or $t = -7/3$, giving $x = 2$ or $x = 2^{-7/3}$. The latter lies strictly between 0 and 1 and is not a natural number. For $x = 2$, the fourth term is $35 \cdot 2^4 \cdot 2^3 = 4480$, as required. The option $x = 1$ gives only 35. The options 3 and 4 exceed 2 and give a larger value of $x^{4+3\log_2 x}$, since both the base and its positive exponent increase for $x \geq 2$.

Step 3: Conclusion The natural-number solution is $\boxed{x = 2}$. Hence, the correct option is (D).

Quick Tip to Solve: Number the first binomial term with r equal to zero; then take a logarithm to handle an exponent containing the same logarithm.

10. The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution if k is equal to : [4 marks]

(A) 0

(B) -1

(C) -2

(D) 1

Correct Answer: (C) -2

Solution:

Concept:

- A square linear system has a unique solution when its coefficient determinant is nonzero. A zero determinant requires a separate consistency test.
- For this symmetric coefficient matrix, the sum of the equations is especially useful: the coefficient of each unknown becomes $k + 2$.

Step 1: Identify the singular parameter values The coefficient matrix can be written as $M = (k-1)I + J$, where J is the three-by-three matrix of ones. The vector $(1, 1, 1)$ has eigenvalue $k + 2$, while vectors with coordinate sum zero have eigenvalue $k - 1$. Equivalently, elementary determinant operations give

$$\det M = (k-1)^2(k+2).$$

Therefore only $k = 1$ and $k = -2$ can fail to have a unique solution. In particular, the options $k = 0$ and $k = -1$ have nonzero determinants and are consistent.

Step 2: Check consistency at each singular value For $k = 1$, all three equations reduce to $x+y+z = 1$, because their right sides are $1, 1, 1$. There are infinitely many solutions, so this value does not satisfy the demand for no solution. For $k = -2$, add the three original equations. The entire left side vanishes, whereas the right side is

$$1 + k + k^2 = 1 - 2 + 4 = 3.$$

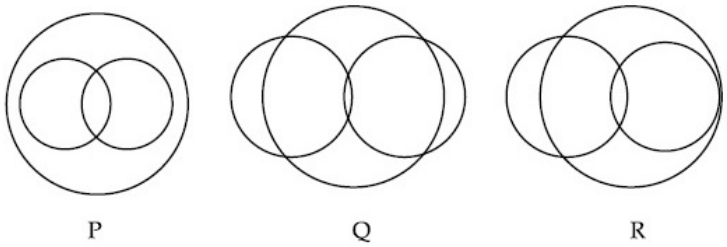
This would require $0 = 3$, which is impossible. More generally the sum reads $(k + 2)(x + y + z) = 1 + k + k^2$, making the contradiction at $k = -2$ immediate. A determinant of zero alone would not have distinguished the consistent value 1 from the inconsistent value -2.

Step 3: Conclusion The system has no solution for $\boxed{k = -2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a symmetric system with equal off-diagonal coefficients, add all equations before doing a full rank calculation.

11. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?

[4 marks]



(A) Q and R

(B) None of these

(C) P and R

(D) P and Q

Correct Answer: (B) None of these

Solution:

Concept:

- Each of the three circles denotes the set of students playing one game. A region inside two circles represents students who play both games.
- The condition that none plays all three requires the triple intersection to be empty. A diagram with a common region inside all three circles fails this condition even if it also has pairwise-only regions.

Step 1: Inspect diagram P In P, the two smaller circles overlap and both lie entirely inside the larger circle. Every point of the lens between the smaller circles therefore also belongs to the large circle. This is a common region of all three game sets. Although P contains regions representing students who play two games, it also contains the prohibited triple-overlap region. Thus P cannot represent the complete statement, which includes the word none.

Step 2: Inspect diagrams Q and R In Q, the left and right circles have a narrow but visible lens near the centre. That lens lies inside the third, larger circle, so there is again a region belonging to all three sets. In R, the overlapping lens of the left and right circles is also inside the larger circle. Thus R likewise has a nonempty triple intersection. The width of the common lens does not affect the logical condition: even a small common region represents a possible all-three membership. Hence Q and R both fail, just as P does. The combinations Q and R, P and R, and P and Q each contain diagrams that violate the requirement; none of the three proposed pairs is valid.

Step 3: Conclusion All three diagrams contain a triple-overlap region, so the answer is $\boxed{\text{None of these}}$. Hence, the correct option is (B).

Quick Tip to Solve: Check the intersection of all three circles first; a tiny central lens is enough to rule out a diagram when no student plays all three games.

12. If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \ldots\ldots$ upto 100 terms, then α is : [4 marks]

(A) 1.02

(B) 1.03

(C) 1.01

(D) 1.00

Correct Answer: (C) 1.01

Solution:

Concept:

- For positive n, the term with argument $2n^2$ can be expressed as a difference of inverse tangents, using $\tan(A-B) = (\tan A - \tan B)/(1+\tan A \tan B)$.
- All inverse cotangents here have positive arguments, so their principal values lie in $(0, \pi/2)$. Matching this range removes any ambiguity from the tangent identity.

Step 1: Rewrite a general term The arguments 2, 8, 18, 32 follow the pattern $2n^2$. Let

$$A_n = \tan^{-1}\frac{1}{2n-1} - \tan^{-1}\frac{1}{2n+1}.$$

Both angles are positive and the first is larger. Therefore $0 < A_n < \pi/2$, and

$$\tan A_n = \frac{\frac{1}{2n-1} - \frac{1}{2n+1}}{1 + \frac{1}{(2n-1)(2n+1)}} = \frac{2}{4n^2} = \frac{1}{2n^2}.$$

It follows that $A_n = \cot^{-1}(2n^2)$, with the correct principal value.

Step 2: Telescope all one hundred terms Consecutive inverse-tangent terms cancel, leaving

$$\cot^{-1}\alpha = \sum_{n=1}^{100} A_n = \tan^{-1} 1 - \tan^{-1}\frac{1}{201}.$$

This angle is positive and less than $\pi/4$, so its cotangent is well defined and positive. Taking its tangent gives

$$\frac{1}{\alpha} = \frac{1 - 1/201}{1 + 1/201} = \frac{200}{202} = \frac{100}{101}.$$

Hence $\alpha = 101/100 = 1.01$. The values 1.02 and 1.03 would give different endpoint ratios, while 1.00 would require the final subtracted angle to vanish; it is small but nonzero for a finite sum.

Step 3: Conclusion The required value is $\boxed{\alpha = 1.01}$. Hence, the correct option is (C).

Quick Tip to Solve: The pattern 2n squared suggests inverse tangents with consecutive odd denominators; retain the last endpoint after cancellation.

13. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to : [4 marks]

(A) 5

(B) 2

(C) 4

(D) 6

Correct Answer: (C) 4

Solution:

Concept:

- A single match is specified by one player from Team A and one from Team B. The multiplication principle counts the permitted cross-team pairs.
- Boys play boys and girls play girls. These two classes of matches are disjoint, so their counts are added; mixed-gender pairs are excluded.

Step 1: Count the boys and girls separately Team A has 7 boys and Team B has 4 boys. Every boy in A can meet each boy in B, giving

$$N_{\text{boys}} = 7 \cdot 4 = 28.$$

Team A has n girls and Team B has 6 girls. Every girl in A can meet each girl in B, so

$$N_{\text{girls}} = n \cdot 6 = 6n.$$

There is no division by two: the first member of a pair is identified by Team A and the second by Team B, so a match is counted exactly once. Nor do we use a combination of all players, since that would include forbidden same-team or mixed-gender pairs.

Step 2: Use the specified total and check the options The two disjoint categories account for all permitted single matches. Thus

$$28 + 6n = 52, \qquad 6n = 24, \qquad n = 4.$$

Substitution confirms that the girls contribute $4 \cdot 6 = 24$ matches, which together with the 28 boys matches gives exactly 52. For the other offered numbers, $n = 5$ gives 58 matches, $n = 2$ gives 40 and $n = 6$ gives 64. None meets the stated total. The count concerns possible individual pairings, so no arrangement of an entire tournament schedule is required.

Step 3: Conclusion Team A contains $\boxed{4}$ girls. Hence, the correct option is (C).

Quick Tip to Solve: When matches must cross two teams, multiply the eligible team sizes for each category and then add the disjoint categories.

14. The area of the triangle with vertices A(z), B(iz) and C(z + iz) is : [4 marks]

(A) 1

(B) $\frac{1}{2}|\ z\ |^2$

(C) $\frac{1}{2}|\ z + iz\ |^2$

(D) $\frac{1}{2}$

Correct Answer: (B) $\frac{1}{2}|\ z\ |^2$

Solution:

Concept:

- Multiplication by i rotates a complex vector through 90° without changing its length: $|iz| = |z|$.
- The vector from a point with complex coordinate u to one with coordinate v is $v - u$. A triangle with perpendicular legs of lengths a and b has area $ab/2$.

Step 1: Find the two sides meeting at C The given coordinates are $A = z$, $B = iz$ and $C = z + iz$. Subtract the coordinate of C from those of A and B:

$$\overrightarrow{CA} = z - (z + iz) = -iz, \qquad \overrightarrow{CB} = iz - (z + iz) = -z.$$

Their moduli are both $|z|$. Also, multiplication of $-z$ by i gives $-iz$, so these two sides are perpendicular. Thus the triangle is right isosceles at C. If $z = 0$, all vertices coincide and the area is zero; the same formula below continues to hold.

Step 2: Calculate and compare the areas Using the two perpendicular sides,

$$\text{Area}(ABC) = \frac{1}{2}|\overrightarrow{CA}|\,|\overrightarrow{CB}| = \frac{1}{2}|z|^2.$$

This also follows by writing $z = x + iy$, for which the two perpendicular vectors are $(y, -x)$ and $(-x, -y)$, each of squared length $x^2 + y^2$. The constant options 1 and $1/2$ cannot describe every z because scaling z scales the area by the square of the scale factor. Moreover, $|z + iz|^2 = |1 + i|^2|z|^2 = 2|z|^2$, so option (C) equals $|z|^2$, twice the triangle area for nonzero z.

Step 3: Conclusion The triangle has area $\boxed{\frac{1}{2}|z|^2}$. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the coordinate of the same vertex from the other two; multiplication by i then reveals perpendicular sides immediately.

15. The line 2x − y + 1 = 0 is a tangent to the circle at the point (2, 5) and the centre of the circle lies on x − 2y = 4. Then, the radius of the circle is : [4 marks]

(A) $5\sqrt{3}$

(B) $4\sqrt{5}$

(C) $3\sqrt{5}$

(D) $5\sqrt{4}$

Correct Answer: (C) $3\sqrt{5}$

Solution:

Concept:

- The radius through a point of contact is perpendicular to the tangent. The normal vector to a line $ax + by + c = 0$ is (a, b).
- The centre must satisfy both its given locus and the normal through the contact point. The radius is then the distance from the centre to that point.

Step 1: Locate the centre on the normal Let the contact point be $P = (2, 5)$. It lies on the tangent because $2(2) - 5 + 1 = 0$. The tangent has normal vector $(2, -1)$, so every possible centre has the form $C = (2 + 2t, 5 - t)$. This uses perpendicularity rather than assuming that the centre lies on the tangent itself.

Step 2: Apply the locus of the centre Substitute the coordinates of C in $x - 2y = 4$:

$$2 + 2t - 2(5 - t) = 4.$$

Thus $4t - 8 = 4$, giving $t = 3$. Therefore the centre is $C = (8, 2)$. Checking the locus gives $8 - 2(2) = 4$, as required.

Step 3: Calculate the radius and compare The radius is CP, so

$$r = \sqrt{(8 - 2)^2 + (2 - 5)^2} = \sqrt{36 + 9} = 3\sqrt{5}.$$

As a separate check, the distance of $(8, 2)$ from the tangent is $15/\sqrt{5} = 3\sqrt{5}$. The values $5\sqrt{3}$, $4\sqrt{5}$, and $5\sqrt{4} = 10$ do not equal this distance, so the other options cannot be radii of the required circle.

Step 4: Conclusion The radius is $3\sqrt{5}$ coordinate units. Hence, the correct option is (C).

Quick Tip to Solve: Write the centre as the contact point plus a multiple of the tangent normal; the centre locus then gives one linear equation.

16. If the Boolean expression (p $\Rightarrow$ q) $\Leftrightarrow$ (q * ($\sim$p) is a tautology, then the boolean expression (p * ($\sim$q)) is equivalent to : [4 marks]

(A) q $\Rightarrow$ p

(B) p $\Rightarrow$ q

(C) p $\Rightarrow \sim$ q

(D) $\sim$q $\Rightarrow$ p

Correct Answer: (A) q $\Rightarrow$ p

Solution:

Concept:

- A biconditional is a tautology precisely when the expressions on its two sides have identical truth values for every assignment.
- The implication identity is $u \Rightarrow v \equiv \neg u \vee v$. Disjunction is commutative, so its two inputs may be interchanged.

Step 1: Identify the connective The given tautology says that $q * (\neg p)$ agrees with $p \Rightarrow q$ for every choice of p, q. But

$$p \Rightarrow q \equiv \neg p \vee q \equiv q \vee \neg p.$$

As $(q, \neg p)$ ranges over all four Boolean input pairs, this determines the truth table of $*$: it is disjunction $\vee$. It is not conjunction, since the implication is true when $p = q = \text{false}$, whereas $q \wedge \neg p$ is false there.

Step 2: Rewrite the required expression Using the same connective gives

$$p * (\neg q) \equiv p \vee \neg q \equiv \neg q \vee p \equiv q \Rightarrow p.$$

This expression is false only when q is true and p is false. It is true for each of the other three assignments.

Step 3: Exclude the other implications Option (B), $p \Rightarrow q$, is false at $p = \text{true}, q = \text{false}$, where the required expression is true. Option (C), $p \Rightarrow \neg q$, is false when both propositions are true, while the required expression is true. Option (D), $\neg q \Rightarrow p$, is false when both are false, where the required expression is again true. Hence none has the required truth table.

Step 4: Conclusion The equivalent expression is $q \Rightarrow p$. Hence, the correct option is (A).

Quick Tip to Solve: Convert an implication to disjunction first; the input truth table identifies the unknown Boolean connective without guessing.

17. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is : [4 marks]

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{3}$

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Concept:

- Multiply a matrix by itself to form A^2; the square of a matrix is not obtained by squaring every entry separately.
- The determinant of a diagonal matrix equals the product of its diagonal entries. A real square is zero only when its base is zero.

Step 1: Compute the matrix square Put $s = \sin\alpha$. Direct multiplication gives

$$A^2 = \begin{pmatrix} 0 & s \\ s & 0 \end{pmatrix}\begin{pmatrix} 0 & s \\ s & 0 \end{pmatrix} = \begin{pmatrix} s^2 & 0 \\ 0 & s^2 \end{pmatrix} = s^2 I.$$

The off-diagonal entries vanish because each contributing product contains a zero. The diagonal entries are both s^2.

Step 2: Use the determinant condition Subtracting $I/2$ changes each diagonal entry by $-1/2$. Therefore

$$\det\left(A^2 - \tfrac{1}{2}I\right) = \left(\sin^2\alpha - \tfrac{1}{2}\right)^2 = 0.$$

Consequently $\sin^2\alpha = 1/2$. This is the condition to check, and it does not require $A^2 - I/2$ to be inverted; its determinant is expressly zero.

Step 3: Test the listed angles For $\alpha = \pi/4$, $\sin^2\alpha = 1/2$, so the determinant vanishes. The other three choices give $\sin^2(\pi/6) = 1/4$, $\sin^2(\pi/2) = 1$, and $\sin^2(\pi/3) = 3/4$. Their corresponding diagonal differences from $1/2$ are nonzero, hence their squared determinants are nonzero. Although the trigonometric equation has further solutions, the question asks only for a possible value among these options.

Step 4: Conclusion A possible value is $\alpha = \pi/4$. Hence, the correct option is (A).

Quick Tip to Solve: For a matrix with zeros on the diagonal and equal off-diagonal entries, its square is a scalar multiple of the identity.

18. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.
If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to : [4 marks]

(A) 10

(B) 8

(C) 13

(D) 12

Correct Answer: (D) 12

Solution:

Concept:

- The cross product is distributive: $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = \vec{r} \times (\vec{a} - \vec{b})$.
- If a cross product with a nonzero vector is zero, the vectors are parallel. A dot-product constraint then fixes their scalar proportionality.

Step 1: Reduce the cross-product equation The given equality implies $\vec{r} \times (\vec{a} - \vec{b}) = \vec{0}$. Here

$$\vec{a} - \vec{b} = (2-7)\hat{i} + (-3-1)\hat{j} + (4+6)\hat{k} = -5\hat{i} - 4\hat{j} + 10\hat{k}.$$

This vector is nonzero, so write $\vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$. The supplied nonzero dot product also rules out $\vec{r} = \vec{0}$.

Step 2: Determine the scalar multiple Take the dot product with $\hat{i} + 2\hat{j} + \hat{k}$:

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = \lambda(-5 - 8 + 10) = -3\lambda.$$

Since the given value is -3, it follows that $\lambda = 1$. Thus $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$. Substitution in the first dot product indeed gives -3.

Step 3: Evaluate the requested dot product Use the component products with the requested vector, whose last component is 1, not 4:

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = (-5)(2) + (-4)(-3) + (10)(1) = -10 + 12 + 10 = 12.$$

The positive middle term follows from multiplying two negative components. The values 10, 8 and 13 do not equal this sum, so options (A), (B) and (C) are excluded.

Step 4: Conclusion The requested scalar product equals 12. Hence, the correct option is (D).

Quick Tip to Solve: Equal cross products with the same first vector immediately make that vector parallel to the difference of the other two vectors.

19. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is : [4 marks]

(A) x + 3z = 0

(B) 3x − z = 0

(C) x + 3z = 10

(D) 3x + z = 6

Correct Answer: (B) 3x − z = 0

Solution:

Concept:

- A general plane has equation $ax + by + cz + d = 0$, with its normal (a, b, c) nonzero.
- Containing an entire axis means that every point of that axis satisfies the plane equation, which is stronger than passing through one point on it.

Step 1: Impose the complete axis condition Every point of the y-axis has coordinates $(0, t, 0)$, where t is any real number. Substituting in the general plane gives

$$bt + d = 0 \quad \text{for every real } t.$$

Taking $t = 0$ gives $d = 0$, and then taking any nonzero t gives $b = 0$. Hence the required plane must have the form $ax + cz = 0$. It passes through the origin and its equation is independent of y.

Step 2: Use the additional point The point $(1, 2, 3)$ must also lie in the plane. Therefore $a + 3c = 0$, so $a = -3c$. A nonzero normal requires $c \neq 0$, since otherwise $a = b = c = 0$. Dividing the plane equation by $-c$ now gives

$$3x - z = 0.$$

The second coordinate of the specified point imposes no further condition because the plane already contains the whole y-axis.

Step 3: Check all the options Option (B) gives $3(1) - 3 = 0$ at the point and gives zero at every $(0, t, 0)$. Option (A) fails at the specified point because $1 + 3(3) = 10 \neq 0$. Options (C) and (D) do not contain the origin, since their right sides are 10 and 6; therefore they cannot contain the y-axis. Passing through the point alone is insufficient, which is why (C) and (D) are excluded.

Step 4: Conclusion The plane is $3x - z = 0$. Hence, the correct option is (B).

Quick Tip to Solve: A plane containing the y-axis has no y-term and no constant term; substitute the extra point only after imposing these conditions.

20. Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$ [4 marks]

(A) Distance between two centres is the average of radii of both the circles.

(B) Both circles pass through the centre of each other.

(C) Circles have two intersection points.

(D) Both circle's centers lie inside region of one another.

Correct Answer: (D) Both circle's centers lie inside region of one another.

Solution:

Concept:

- Completing squares in $x^2+y^2+2gx+2fy+c=0$ gives the centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$.
- Two circles intersect in two points when $|r_1-r_2|<d<r_1+r_2$, where d is their centre separation. A point lies strictly inside a circle only when its distance from the centre is less than the radius.

Step 1: Find both centres and radii Complete squares in each equation:

$$(x-5)^2+(y-5)^2=9.$$

$$(x-8)^2+(y-5)^2=9.$$

Thus $C_1=(5,5)$, $C_2=(8,5)$, and $r_1=r_2=3$. Their separation is $d=\sqrt{(8-5)^2+(5-5)^2}=3$.

Step 2: Test the first three statements The average of the radii is $(3+3)/2=3=d$, so (A) is true. Each centre is exactly a radius away from the other centre, so each lies on the other circumference; this proves (B). Moreover $0=|r_1-r_2|<3<6=r_1+r_2$, so there are two intersection points and (C) is true. For an explicit check, subtracting the circle equations gives $x=13/2$, and then $y=5\pm3\sqrt{3}/2$, two distinct values.

Step 3: Distinguish boundary from interior Statement (D) says that both centres lie inside the region of the other circle. Their distances from the opposite centres equal 3, not a number strictly below 3. They therefore lie on the boundary rather than in the interior. It is this strict distinction that makes (D) the incorrect statement; the circles intersecting does not by itself put their centres inside each other.

Step 4: Conclusion The centres lie on each other's circumference, so statement (D) is incorrect. Hence, the correct option is (D).

Quick Tip to Solve: Compare centre separation with each radius carefully: equality means on the circle, whereas a strict inequality is needed for inside.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If $(2021)^{3762}$ is divided by 17, then the remainder is _____. [4 marks]

Solution:

Concept:

- Replacing a base by a congruent integer preserves the remainder of every nonnegative integer power.
- Powers may be reduced using a short modular cycle: if $a^m \equiv 1 \pmod{n}$, complete groups of m factors contribute a remainder of 1.

Step 1: Reduce the base modulo 17 Divide 2021 by 17:

$$2021 = 17(118) + 15.$$

Thus $2021 \equiv 15 \equiv -2 \pmod{17}$. Since the exponent 3762 is even, $(-2)^{3762} = 2^{3762}$. It is useful to retain the negative representative only until this parity check; an odd exponent would retain a minus sign.

Step 2: Establish a short power cycle Calculate $2^4 = 16 \equiv -1 \pmod{17}$, hence $2^8 \equiv 1 \pmod{17}$. Now divide the exponent by 8:

$$3762 = 8(470) + 2.$$

Consequently

$$2^{3762} = (2^8)^{470}2^2 \equiv 1^{470} \cdot 4 \equiv 4 \pmod{17}.$$

No large power has to be evaluated: each group of eight powers has remainder 1 and only the final two powers remain.

Step 3: Confirm the remainder range The division algorithm requires the remainder on division by 17 to be one of $0, 1, \ldots, 16$. The congruence gives 4, which is already in this range, so no adjustment is needed. As an alternative cycle check, $3762 = 4(940) + 2$ and $2^4 \equiv -1$ give $(-1)^{940}2^2 = 4$, the same result. The question asks for a pure integer remainder, so it has no physical unit.

Step 4: Conclusion The remainder when 2021^{3762} is divided by 17 is $\boxed{4}$.

Quick Tip to Solve: Look for a small power congruent to -1; squaring it creates a short cycle and reduces a very large exponent immediately.

22. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations

$(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{Probability\ of\ occurrence\ of\ E_1}{Probability\ of\ occurrence\ of\ E_3}$ is equal to _____. [4 marks]

Solution:

Concept:

- For independent events, a single-event-only probability equals its occurrence probability multiplied by the non-occurrence probabilities of the other events.
- The odds of an event with probability r are $r/(1-r)$. Dividing an only-event probability by the probability of no event isolates these odds.

Step 1: Introduce the three odds Put $r_i = P(E_i)$, and let $u = r_1/(1-r_1)$, $v = r_2/(1-r_2)$, and $w = r_3/(1-r_3)$. Independence gives $p = (1-r_1)(1-r_2)(1-r_3)$, and hence $\alpha = pu$, $\beta = pv$, $\gamma = pw$. All probabilities are strictly between zero and one, so division by p and the odds is permitted.

Step 2: Translate the two equations Substitute these expressions into the given equations and cancel p^2:

$$u - 2v = uv, \qquad v - 3w = 2vw.$$

Thus $u(1-v) = 2v$ and $v(1-2w) = 3w$. Their positive right-hand sides also show that the denominators used below are nonzero.

Step 3: Recover the occurrence probabilities First solve for v, then for u:

$$v = \frac{3w}{1-2w}, \qquad u = \frac{2v}{1-v} = \frac{6w}{1-5w}.$$

An odds value t corresponds to probability $t/(1+t)$. Therefore

$$r_1 = \frac{6w}{1+w}, \qquad r_3 = \frac{w}{1+w}.$$

The common positive factor cancels when taking their ratio; the requested quantity is a ratio of occurrence probabilities, rather than of the only-event probabilities.

Step 4: Conclusion The required dimensionless ratio is $P(E_1)/P(E_3) = 6$.

Quick Tip to Solve: Normalize the only-event probabilities by the none-event probability first; this converts independent-event equations into simple equations in odds.

23. The maximum value of z in the following equation z = 6xy + y^2, where 3x + 4y $\leq$ 100 and 4x + 3y $\leq$ 75 for x $\geq$ 0 and y $\geq$ 0 is _____. [4 marks]

Solution:

Concept:

- For a fixed nonnegative y, the expression $6xy + y^2$ increases with x. Its maximum therefore lies on the largest feasible x for that y.
- A quadratic $ay - by^2$, with $b > 0$, attains its maximum at its vertex. Check that the vertex satisfies every original inequality.

Step 1: Identify the active constraint The condition $4x + 3y \leq 75$, together with nonnegativity, gives $0 \leq y \leq 25$ and $0 \leq x \leq (75 - 3y)/4$. Throughout this triangle,

$$3x + 4y \leq \frac{3(75 - 3y)}{4} + 4y = \frac{225 + 7y}{4} \leq 100.$$

Thus the other inequality is automatically satisfied, including at the endpoints. It does not further restrict the feasible region.

Step 2: Maximize along the boundary For $y > 0$, choose $x = (75 - 3y)/4$, since increasing x increases the objective. At $y = 0$, the objective is zero, which cannot exceed the interior positive values. Substitution gives

$$z \leq 6y\frac{75 - 3y}{4} + y^2 = \frac{225}{2}y - \frac{7}{2}y^2.$$

Complete the square:

$$z \leq \frac{50625}{56} - \frac{7}{2}\left(y - \frac{225}{14}\right)^2.$$

Step 3: Check feasibility and evaluate Equality occurs at

$$y = \frac{225}{14}, \qquad x = \frac{375}{56}.$$

Both coordinates are nonnegative and $y < 25$. Also $4x + 3y = 75$, while $3x + 4y = 4725/56 < 100$, so this point is admissible. The exact maximum is $50625/56 = 904.017857\ldots$. The numerical entry corresponding to this maximum, rounded to the nearest integer, is 904.

Step 4: Conclusion The exact maximum is $\boxed{50625/56}$. Its nearest-integer numerical entry is $\boxed{904}$.

Quick Tip to Solve: Before optimizing, test whether one linear constraint already implies the other; then reduce the objective to a quadratic on the active boundary.

24. If [.] represents the greatest integer function, then the value of $\left|\int_0^{\sqrt{\frac{\pi}{2}}} \left[[x^2] - \cos x\right] dx\right|$ is _____. [4 marks]

Solution:

Concept:

- The greatest integer $\lfloor t \rfloor$ is constant whenever t remains between two consecutive integers. Split an integral at the points where that integer changes.
- Changing finitely many values of a bounded integrand does not change its definite integral. The absolute value is applied after evaluating the signed integral.

Step 1: Split at the change in the inner floor Let $L = \sqrt{\pi/2}$. Since $1 < L < \pi/2$, the interval of integration lies where $\cos x$ is positive, except that $\cos 0 = 1$. For $0 \le x < 1$, one has $\lfloor x^2 \rfloor = 0$. For $1 \le x \le L$, one has $1 \le x^2 \le \pi/2 < 2$, so $\lfloor x^2 \rfloor = 1$. No further jump of the inner floor occurs.

Step 2: Evaluate the outer floor on each interval On $0 < x < 1$, positivity and strict decrease of cosine give $-1 < -\cos x < 0$, so

$$\lfloor \lfloor x^2 \rfloor - \cos x \rfloor = -1.$$

At $x = 0$ the value is also -1. On $1 \le x \le L$, one has $0 < 1 - \cos x < 1$, so the outer floor is zero. In particular, the trigonometric term never makes the outer floor positive on the latter interval.

Step 3: Integrate the constant pieces The entire nested-floor integral is therefore

$$I = \int_0^1 (-1)\, dx + \int_1^{\sqrt{\pi/2}} 0\, dx = -1.$$

The jump at $x = 1$ affects no area and hence needs no extra contribution. Finally take the absolute value requested outside the integral:

$$|I| = |-1| = 1.$$

Step 4: Conclusion The required dimensionless value is $\boxed{1}$.

Quick Tip to Solve: Handle nested floors from the inside out: first find the jumps of the inner floor, then bound the remaining expression between consecutive integers.

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25. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of | $a^2 - b^2$ | is _____ . [4 marks]

Solution:

Concept:

- For $-1 \le u \le 1$, the principal angle $\arccos u$ lies in $[0, \pi]$, so $\sin(\arccos u) = \sqrt{1-u^2}$ with the nonnegative square root.
- The derivative of 2^x is $2^x \ln 2$. Reduce a rational coefficient to lowest terms before minimizing an expression involving its integer numerator and denominator.

Step 1: Simplify the inverse trigonometric expression Put $t = 2^x > 0$. Then the argument of the inverse cosine is $(1-t^2)/(1+t^2)$, which always lies strictly between -1 and 1. Using the principal-angle identity gives

$$f(x) = \sqrt{1 - \left(\frac{1-t^2}{1+t^2}\right)^2} = \sqrt{\frac{4t^2}{(1+t^2)^2}} = \frac{2t}{1+t^2}.$$

The last equality uses $t > 0$; no absolute-value sign may be discarded without that observation.

Step 2: Differentiate and substitute the point Apply the quotient rule and the chain rule, with $dt/dx = t\ln 2$:

$$f'(x) = \frac{2(1-t^2)}{(1+t^2)^2}\, t\ln 2.$$

At $x = 1$, we have $t = 2$, so

$$f'(1) = \frac{2(1-4)\cdot 2}{(1+4)^2}\ln 2 = -\frac{12}{25}\ln 2.$$

This is negative, consistent with $2t/(1+t^2)$ decreasing when $t > 1$.

Step 3: Minimize over integer representations Comparison with $-\frac{b}{a}\ln 2$ yields $b/a = 12/25$. Since 12 and 25 are coprime, every integer representation has $a = 25n$, $b = 12n$, for a nonzero integer n. Consequently

$$|a^2 - b^2| = |625 - 144|n^2 = 481n^2.$$

The smallest allowed n^2 is one, attained by either simultaneous sign choice for a, b.

Step 4: Conclusion The minimum dimensionless value is $\boxed{481}$.

Quick Tip to Solve: Simplify a sine of an inverse cosine before differentiating; positivity of the exponential fixes the square-root sign immediately.

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26. If the function $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is _____. [4 marks]

Solution:

Concept:

- Continuity at zero requires the assigned value $f(0)$ to equal the limit of the displayed quotient as $x \to 0$.
- The identity $\cos u - \cos v = -2\sin((u+v)/2)\sin((u-v)/2)$, together with $\sin t/t \to 1$, separates the cancellation in the numerator.

Step 1: Factor the difference of cosines For nonzero x, write

$$\cos(\sin x) - \cos x = 2\sin\left(\frac{x + \sin x}{2}\right)\sin\left(\frac{x - \sin x}{2}\right).$$

The sign is positive because the second factor in the usual difference formula changes sign when $\sin x - x$ is replaced by $x - \sin x$. This factorization exposes the orders of the two small quantities.

Step 2: Compute the two small-quantity limits Let $U = (x + \sin x)/2$ and $V = (x - \sin x)/2$. Then

$$\frac{U}{x} \longrightarrow 1.$$

Also $\lim_{x\to 0}(x - \sin x)/x^3 = 1/6$. For example, applying L'Hopital's rule once reduces this limit to $\lim(1 - \cos x)/(3x^2) = 1/6$, using $1 - \cos x = 2\sin^2(x/2)$. Thus $V/x^3 \to 1/12$. Both U, V tend to zero, so the standard sine ratio limit applies to each.

Step 3: Impose continuity Combine the factors without subtracting nearly equal terms:

$$\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} = 2 \lim_{x\to 0} \frac{\sin U}{U} \frac{\sin V}{V} \frac{U}{x} \frac{V}{x^3}$$
$$= 2 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{1}{12} = \frac{1}{6}.$$

Continuity therefore demands $f(0) = 1/6$. Comparing this with the prescribed $f(0) = 1/k$ gives the value of k.

Step 4: Conclusion The required dimensionless constant is $\boxed{k = 6}$.

Quick Tip to Solve: For a quotient with a fourth-order cancellation, factor a difference of cosines into sine factors to reveal first-order and third-order contributions.

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27. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of det(A^4) + det(A^{10} − (Adj(2A))^{10}) is equal to _____.

[4 marks]

Solution:

Concept:

- For any square matrix, $\det(A^m) = (\det A)^m$. The determinant of a triangular matrix is the product of its diagonal entries.
- Powers of an upper triangular matrix remain upper triangular, with each diagonal entry raised to the corresponding power. Determinants do not distribute over subtraction.

Step 1: Evaluate the first determinant The displayed matrix is upper triangular, so

$$\det A = 2(-1) = -2.$$

Therefore

$$\det(A^4) = (-2)^4 = 16.$$

This part needs no multiplication of four full matrices; the determinant product identity supplies it directly.

Step 2: Find the adjugate and compare diagonal entries First scale the whole matrix, then take its

adjugate:

$$2A = \begin{pmatrix} 4 & 6 \\ 0 & -2 \end{pmatrix}, \qquad B = \text{Adj}(2A) = \begin{pmatrix} -2 & -6 \\ 0 & 4 \end{pmatrix}.$$

For a two-by-two matrix, the adjugate swaps the diagonal entries and negates the off-diagonal entries. Both A^{10} and B^{10} are upper triangular. Their diagonal entries are respectively

$$(2^{10}, (-1)^{10}) = (1024, 1), \qquad ((-2)^{10}, 4^{10}) = (1024, 1048576).$$

The off-diagonal entries need not be evaluated to determine the determinant.

Step 3: Take the determinant of the difference Subtraction preserves upper triangular form. Hence

$$A^{10} - B^{10} = \begin{pmatrix} 0 & * \\ 0 & 1 - 1048576 \end{pmatrix},$$

where the asterisk denotes an irrelevant upper-right entry. Its determinant is zero because one diagonal entry is zero. This computes the determinant of the actual difference, rather than incorrectly subtracting the two determinants. Adding the first contribution gives $16 + 0$.

Step 4: Conclusion The required dimensionless value is $\boxed{16}$.

Quick Tip to Solve: When triangular matrix powers appear inside a determinant, inspect the diagonal first; a matching diagonal entry can make the difference singular without full multiplication.

---

28. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$, $\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and

$$\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$$

such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{b} \cdot \vec{c} = -3$, then $\frac{1}{3}\left(\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- The dot product of vectors written in an orthonormal basis is the sum of the products of matching components. Use the two given dot products to determine the unknown components.
- The scalar triple product $(\vec{a} \times \vec{b}) \cdot \vec{c}$ equals the determinant having $\vec{a}, \vec{b}, \vec{c}$ as its rows, in that order.

Step 1: Use the dot-product conditions From $\vec{a} = (\alpha, \beta, 3)$ and $\vec{b} = (-\beta, -\alpha, -1)$, the first condition gives

$$-2\alpha\beta - 3 = 1, \qquad \alpha\beta = -2.$$

Using $\vec{c} = (1, -2, -1)$, the second condition gives

$$-\beta + 2\alpha + 1 = -3, \qquad \beta = 2\alpha + 4.$$

Both equations follow directly from the printed component signs; the final $+1$ in the second equation is the product $(-1)(-1)$.

Step 2: Determine the vectors Substitute the linear relation into the product equation:

$$\alpha(2\alpha + 4) = -2 \quad \Longrightarrow \quad 2(\alpha + 1)^2 = 0.$$

Thus $\alpha = -1$ and $\beta = 2$. There is a repeated root, so there is no second set of components to test. The vectors are

$$\vec{a} = (-1, 2, 3), \qquad \vec{b} = (-2, 1, -1), \qquad \vec{c} = (1, -2, -1).$$

Checking gives $\vec{a} \cdot \vec{b} = 2 + 2 - 3 = 1$ and $\vec{b} \cdot \vec{c} = -2 - 2 + 1 = -3$, as required.

Step 3: Evaluate the ordered triple product Compute the cross product in the stated order:

$$\vec{a} \times \vec{b} = (-5, -7, 3).$$

Consequently

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -5 + 14 - 3 = 6.$$

The question asks for one third of this scalar, so the required result is $6/3 = 2$. Reversing the order of the cross product would reverse the sign and would not answer the printed expression.

Step 4: Conclusion The requested dimensionless value is $\boxed{2}$.

Quick Tip to Solve: Use dot products to settle unknown components before taking a cross product, and preserve the printed vector order to avoid reversing the scalar triple product sign.

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29. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 24x - 10y + 160 = 0$ is _____. [4 marks]

Solution:

Concept:

- The circle $(x - h)^2 + (y - k)^2 = r^2$ has centre (h, k) and radius r. Completing squares identifies these quantities without estimating them from a sketch.
- For two externally disjoint circles with centre separation $d > r_1 + r_2$, their minimum point-to-point distance is $d - r_1 - r_2$. The closest points lie on the segment joining the centres.

Step 1: Complete squares in both equations For the first circle, add the squares of half the two linear coefficients:

$$x^2 - 10x + y^2 - 10y + 41 = 0 \quad \Longrightarrow \quad (x - 5)^2 + (y - 5)^2 = 9.$$

Thus $C_1 = (5, 5)$ and $r_1 = 3$. Similarly, the second equation becomes

$$(x - 12)^2 + (y - 5)^2 = 144 + 25 - 160 = 9.$$

Hence $C_2 = (12, 5)$ and $r_2 = 3$. Both radii are real and equal; the constant terms themselves are not the radii squared.

Step 2: Compare the separation with the radii The centre distance is

$$d = \sqrt{(12-5)^2 + (5-5)^2} = 7.$$

Since $7 > 3 + 3$, the circles do not intersect and neither encloses the other. For arbitrary points on them, the triangle inequality gives

$$P_1P_2 \geq C_1C_2 - C_1P_1 - C_2P_2 = 7 - 3 - 3 = 1.$$

This lower bound is attained by $P_1 = (8, 5)$ and $P_2 = (9, 5)$, which lie on the respective circles and face each other along the line of centres. Therefore the bound is the actual minimum, rather than merely an estimate.

Step 3: Conclusion The minimum distance between the two circles is $\boxed{1}$ coordinate unit. The numerical entry is 1.

Quick Tip to Solve: First decide whether the circles intersect, lie apart, or are nested; subtract the radii from the centre distance only in the externally disjoint case.

30. If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is _____. [4 marks]

Solution:

Concept:

- If two planes are $P_1 = 0$ and $P_2 = 0$, every finite member $P_1 + \lambda P_2 = 0$ contains their common line: both expressions vanish at every point of that line.
- A further point fixes the parameter. The final coefficients must then be scaled to match the stated constant term, since multiplying a plane equation by a nonzero number does not change its plane.

Step 1: Use the plane pencil and impose the point Write the required member as

$$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0.$$

At $(-2, 1, 3)$, the first bracket equals

$$2(-2) - 7(1) + 4(3) - 3 = -2,$$

and the second bracket equals

$$3(-2) - 5(1) + 4(3) + 11 = 12.$$

Thus the point condition gives $-2 + 12\lambda = 0$, so $\lambda = 1/6$. This uses the given point only after ensuring

the whole line of intersection is already included.

Step 2: Normalize the constant and evaluate the expression Multiply the pencil equation by six to remove its fraction:

$$6(2x - 7y + 4z - 3) + (3x - 5y + 4z + 11) = 0.$$

Collecting coefficients gives

$$15x - 47y + 28z - 7 = 0.$$

Its constant is already the specified -7, so $a = 15$, $b = -47$, and $c = 28$. As a check, substitution of the extra point yields $-30 - 47 + 84 - 7 = 0$. Finally,

$$2a + b + c - 7 = 30 - 47 + 28 - 7 = 4.$$

Using the unscaled coefficients of the pencil would produce the same geometric plane but the wrong requested numerical expression.

Step 3: Conclusion The plane is $15x - 47y + 28z - 7 = 0$, and the required numerical value is $\boxed{4}$.

Quick Tip to Solve: Always match the constant term in the requested plane equation before reading off coefficients for a numerical expression.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now? [4 marks]

(A) 14.41 ms^{-1}

(B) 19.0 ms^{-1}

(C) 4.47 ms^{-1}

(D) 1.00 ms^{-1}

Correct Answer: (C) 4.47 ms^{-1}

Solution:

Concept:

- Translational kinetic energy is $K = \frac{1}{2}mv^2$, where m is mass and v is speed. For the same ball, mass cancels when final and initial energies are compared.
- An energy percentage is a percentage of speed squared, not of speed. Thus a remaining fraction f of kinetic energy corresponds to a speed ratio $\sqrt{f}$.

Step 1: Find the remaining kinetic energy The initial speed is $u = 20\,\mathrm{m\,s^{-1}}$, with $m = 0.5\,\mathrm{kg}$. Its initial kinetic energy is

$$K_i = \frac{1}{2}(0.5)(20)^2 = 100\,\mathrm{J}.$$

The statement says the ball retains 5%, rather than loses 5%, of this energy. Therefore

$$K_f = \frac{5}{100} K_i = 5\,\text{J}.$$

The obstacle changes the kinetic energy; frictionlessness of the floor does not require the collision itself to conserve the ball's kinetic energy.

Step 2: Convert energy to speed and test the options Apply the kinetic-energy formula after the deflection:

$$\frac{1}{2}(0.5)v^2 = 5 \quad \Longrightarrow \quad v^2 = 20 \quad \Longrightarrow \quad v = \sqrt{20} = 4.472\ldots\,\text{m}\,\text{s}^{-1}.$$

Equivalently, $v/u = \sqrt{0.05}$. Option (B), $19.0\,\text{m}\,\text{s}^{-1}$, incorrectly treats the small remaining energy fraction like a small loss of speed. Option (D), $1.00\,\text{m}\,\text{s}^{-1}$, is 5% of the original speed and would retain only 0.25% of the energy. Option (A) also gives a much larger energy ratio, approximately $(14.41/20)^2 = 0.519$, not 0.05.

Step 3: Conclusion The final speed is $\boxed{4.47\,\text{m}\,\text{s}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: When the same object retains a given fraction of kinetic energy, take the square root of that fraction to obtain its speed ratio.

32. A polyatomic ideal gas has 24 vibrational modes. What is the value of γ? [4 marks]

(A) 1.37

(B) 1.30

(C) 1.03

(D) 10.3

Correct Answer: (C) 1.03

Solution:

Concept:

- Each quadratic energy term contributes $\frac{1}{2}k_BT$ per molecule. A vibrational normal mode has both kinetic and potential energy, so an active mode contributes k_BT, or R per mole per kelvin to C_V.
- A nonlinear polyatomic molecule has three translational and three rotational degrees of freedom. With s active vibrational modes, $C_V = (3+s)R$, $C_P = C_V + R$, and $\gamma = C_P/C_V$.

Step 1: Count all energy contributions The translational contribution is $3R/2$ and the rotational contribution is also $3R/2$. They together give $3R$. For the specified 24 vibrational modes, count two quadratic terms per mode rather than one:

$$C_{V,\text{vib}} = 24R, \qquad C_V = 3R + 24R = 27R.$$

The corresponding effective quadratic degree count is $f = 3 + 3 + 2(24) = 54$, which gives the same result through $C_V = fR/2$. The calculation uses the equipartition description of the modes stated in the question.

Step 2: Use the ideal-gas heat-capacity relation For an ideal gas, Mayer's relation gives

$$C_P - C_V = R, \qquad C_P = 28R.$$

Consequently,

$$\gamma = \frac{C_P}{C_V} = \frac{28R}{27R} = \frac{28}{27} = 1.037037\ldots.$$

Among the printed options, 1.03 is the value corresponding to this result. Options 1.37 and 1.30 imply far fewer active energy terms and therefore do not represent the large vibrational contribution. Option 10.3 is impossible here: with $C_V = 27R$, the difference $C_P - C_V = R$ makes their ratio only slightly above unity. Even if a linear rotational count were considered, the large vibrational contribution would still select the same option.

Step 3: Conclusion The equipartition value is $\gamma = 28/27 \simeq 1.037$, corresponding to the listed value 1.03. Hence, the correct option is (C).

Quick Tip to Solve: Count each active vibrational mode twice in the quadratic degree count; many modes make C_V large and γ close to one.

---

33. Two ideal polyatomic gases at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If F_1 and F_2, m_1 and m_2, n_1 and n_2 be the degrees of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is : [4 marks]

(A) $\frac{n_1F_1T_1+n_2F_2T_2}{F_1+F_2}$
(B) $\frac{n_1F_1T_1+n_2F_2T_2}{n_1F_1+n_2F_2}$
(C) $\frac{n_1T_1+n_2T_2}{n_1+n_2}$
(D) $\frac{n_1F_1T_1+n_2F_2T_2}{n_1+n_2}$

Correct Answer: (B) $\frac{n_1F_1T_1+n_2F_2T_2}{n_1F_1+n_2F_2}$

Solution:

Concept:

- For n molecules with F active quadratic degrees of freedom, the internal energy is $U = \frac{1}{2}nFk_BT$. Here n_1, n_2 are molecule counts, not numbers of moles.
- With no energy loss and no external work, total internal energy before mixing equals that after mixing. The common equilibrium temperature is weighted by each gas's heat capacity, proportional to nF.

Step 1: Write the initial and final internal energies Initially the gases are at different temperatures, so their combined energy is

$$U_i = \frac{1}{2}n_1F_1k_BT_1 + \frac{1}{2}n_2F_2k_BT_2.$$

Let T denote their common final temperature. The molecule counts and the degrees of freedom of

each species remain unchanged on ideal mixing, giving

$$U_f = \frac{1}{2}n_1F_1k_BT + \frac{1}{2}n_2F_2k_BT = \frac{1}{2}k_B(n_1F_1 + n_2F_2)T.$$

The given masses are unnecessary because the numbers of molecules already specify the amount of each gas.

Step 2: Conserve energy and compare the choices Set $U_i = U_f$ and cancel the common nonzero factor $k_B/2$:

$$n_1F_1T_1 + n_2F_2T_2 = (n_1F_1 + n_2F_2)T.$$

Thus

$$T = \frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}.$$

Option (A) lacks molecule-count factors in the denominator and fails when both sample sizes are scaled together. Option (C) is valid only in the special case $F_1 = F_2$, not for the general gases stated. Option (D) fails to divide out the degrees-of-freedom weights: if both gases initially have the same temperature, it would generally predict a different temperature. Option (B) passes that equal-temperature check and lies between the two initial temperatures for positive heat capacities.

Step 3: Conclusion The mixture temperature is $\boxed{T = (n_1F_1T_1 + n_2F_2T_2)/(n_1F_1 + n_2F_2)}$. Hence, the correct option is (B).

Quick Tip to Solve: Use heat-capacity weights, not mass weights alone: for a molecule count n_i, the appropriate weight is n_iF_i.

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34. If an electron is moving in the n^{th} orbit of the hydrogen atom, then its velocity (v_n) for the n^{th} orbit is given as : [4 marks]

(A) $v_n \propto \frac{1}{n}$

(B) $v_n \propto n^2$

(C) $v_n \propto n$

(D) $v_n \propto \frac{1}{n^2}$

Correct Answer: (A) $v_n \propto \frac{1}{n}$

Solution:

Concept:

- In the Bohr model of hydrogen, Coulomb attraction provides centripetal force: $m_ev_n^2/r_n = k_ee^2/r_n^2$, where $k_e = 1/(4\pi\varepsilon_0)$.
- The quantization condition is $m_ev_nr_n = n\hbar$. Combining the force law with this condition gives the dependence on principal quantum number without confusing speed with orbital radius or energy.

Step 1: Combine Coulomb force and angular momentum The force equation can be written

$$m_ev_n^2r_n = k_ee^2.$$

The left side is the product of v_n and $m_e v_n r_n$. Substituting Bohr's angular-momentum condition gives

$$v_n(n\hbar) = k_e e^2.$$

Therefore

$$v_n = \frac{k_e e^2}{n\hbar}.$$

All quantities in the numerator and $\hbar$ are constants for hydrogen. The principal quantum number is the only changing factor in this expression.

Step 2: Identify the dependence and eliminate the alternatives The speed consequently varies inversely with n: doubling the orbital quantum number halves the electron's speed in this model. The result also agrees with the familiar radius scaling $r_n \propto n^2$, since the Coulomb-force relation gives $v_n \propto r_n^{-1/2}$.

Option (B), proportional to n^2, confuses the radius scaling with speed. Option (C), proportional to n, predicts an increase in speed in larger orbits and contradicts the combined equations. Option (D), proportional to $1/n^2$, confuses the speed with the magnitude of the energy, which depends on v_n^2. The word velocity in this orbital question refers to the magnitude of the orbital velocity; its direction continuously changes along the orbit.

Step 3: Conclusion The orbital speed obeys $\boxed{v_n \propto 1/n}$. Hence, the correct option is (A).

Quick Tip to Solve: Keep the Bohr scalings distinct: radius varies as n^2, speed as $1/n$, and the magnitude of energy as $1/n^2$.

35. Two identical metal wires of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is : [4 marks]

(A) $\frac{2K_1K_2}{K_1+K_2}$
(B) $\frac{K_1+K_2}{K_1K_2}$
(C) $\frac{K_1+K_2}{2K_1K_2}$
(D) $\frac{K_1K_2}{K_1+K_2}$

Correct Answer: (A) $\frac{2K_1K_2}{K_1+K_2}$

Solution:

Concept:

- A uniform wire of length L, cross-sectional area A, and conductivity K has thermal resistance $R_{\text{th}} = L/(KA)$. Its steady heat current is temperature difference divided by thermal resistance.
- Series wires carry the same steady heat current, and their temperature drops add. Therefore their thermal resistances add. The equivalent uniform wire has the full combined length $2L$, not the length of only one component.

Step 1: Add the thermal resistances Since the wires are identical in dimensions, let each have length

L and area A. Their different materials have conductivities K_1 and K_2, so

$$R_1 = \frac{L}{K_1 A}, \qquad R_2 = \frac{L}{K_2 A}.$$

For a series connection,

$$R_{\text{series}} = R_1 + R_2 = \frac{L}{A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right).$$

The interface reaches a temperature at which the heat entering one wire equals that leaving the other; no independent parallel heat paths are present.

Step 2: Match the equivalent wire and test the result Write the resistance of an equivalent wire with the same area and the total length:

$$R_{\text{eq}} = \frac{2L}{K_{\text{eff}} A}.$$

Equating this to the series resistance and cancelling L/A,

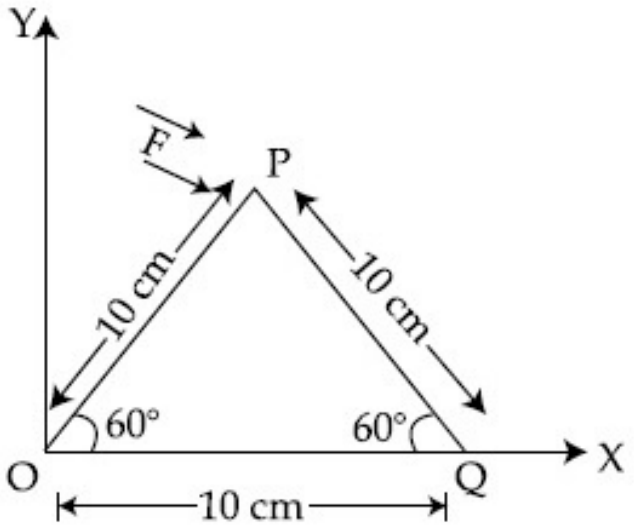
$$\frac{2}{K_{\text{eff}}} = \frac{1}{K_1} + \frac{1}{K_2} \quad \Longrightarrow \quad K_{\text{eff}} = \frac{2K_1 K_2}{K_1 + K_2}.$$

If $K_1 = K_2 = K$, this correctly returns K, as it must for a longer piece of the same material. Option (D) would give $K/2$ because it omits the doubled equivalent length. Options (B) and (C) have dimensions of inverse conductivity rather than conductivity, so neither can be the requested effective material property.

Step 3: Conclusion The effective thermal conductivity is $\boxed{2K_1K_2/(K_1 + K_2)}$. Hence, the correct option is (A).

Quick Tip to Solve: Add thermal resistances first, then use the total length when converting the resistance back to an effective conductivity.

36. A triangular plate is shown. A force $\vec{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are : [4 marks]



(A) $-15 + 20\sqrt{3}, 15 + 20\sqrt{3}$

(B) $15 - 20\sqrt{3}, 15 + 20\sqrt{3}$

(C) $15 + 20\sqrt{3}, 15 - 20\sqrt{3}$

(D) $-15 - 20\sqrt{3}, 15 - 20\sqrt{3}$

Correct Answer: (D) $-15 - 20\sqrt{3}, 15 - 20\sqrt{3}$

Solution:

Concept:

- The torque of a force about a point is $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ runs from that reference point to the point of application.
- For vectors in the xy-plane, $\tau_z = xF_y - yF_x$; positive and negative signs denote anticlockwise and clockwise moments respectively.

Step 1: Locate the point of application The plate is an equilateral triangle of side $10\,\text{cm}$. Taking O as origin, the coordinates of P follow from the angle of $60°$:

$$x_P = 10\cos 60° = 5\,\text{cm}, \qquad y_P = 10\sin 60° = 5\sqrt{3}\,\text{cm}.$$

Point Q has coordinates $(10, 0)\,\text{cm}$. Thus the position vector from Q to P has horizontal component $-5\,\text{cm}$, with the same vertical component. The force components remain $F_x = 4$ and $F_y = -3$; changing the origin does not reverse the force.

Step 2: Evaluate both cross products Using centimetres for the lever arms gives the torque components in newton-centimetres:

$$\tau_{O,z} = 5(-3) - (5\sqrt{3})(4) = -15 - 20\sqrt{3},$$

$$\tau_{Q,z} = (-5)(-3) - (5\sqrt{3})(4) = 15 - 20\sqrt{3}.$$

The upward lever arm combined with the rightward force produces a clockwise contribution about both points. Only the contribution of the downward force changes sign when the origin moves from O to Q. Options A and C reverse at least one required sign, while B gives an incorrect sign for the vertical-force contribution about O. For SI torques multiply both displayed values by 10^{-2}.

Step 3: Conclusion The ordered pair is $(-15-20\sqrt{3},\ 15-20\sqrt{3})\,\mathrm{N\,cm}$, or $(-0.15-0.20\sqrt{3},\ 0.15-0.20\sqrt{3})\,\mathrm{N\,m}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the lever arm from each chosen origin to P before using $xF_y - yF_x$. Keep centimetres until comparing these options, then convert to SI if required.

37. When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is : [4 marks]

(A) $\frac{b-a}{ab}$
(B) $\frac{a+b}{ab}$
(C) $\frac{ab}{a+b}$
(D) $\frac{ab}{b-a}$

Correct Answer: (D) $\frac{ab}{b-a}$

Solution:

Concept:

- A soap bubble has two liquid-air interfaces. Its excess pressure over the outside is $4T/r$, with surface tension T and bubble radius r.
- The common soap film separates the two interior pressures, so its curvature is determined by their difference, not their sum.

Step 1: Compare the pressures inside the bubbles Let the surrounding pressure be p_0. The pressures inside the two spherical portions are

$$p_a = p_0 + \frac{4T}{a}, \qquad p_b = p_0 + \frac{4T}{b}.$$

Because $b > a$, the smaller bubble has the higher pressure. Subtracting the two equations cancels the atmospheric pressure:

$$p_a - p_b = 4T\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{4T(b-a)}{ab}.$$

This is the pressure difference across the common film. The film bulges towards the lower-pressure, larger bubble; its radius is taken as a positive magnitude.

Step 2: Apply the pressure jump to the common film Write R for the radius of curvature of the common soap surface. It also has two interfaces and obeys

$$\frac{4T}{R} = p_a - p_b = \frac{4T(b-a)}{ab}.$$

Cancelling $4T$ and inverting gives

$$\frac{1}{R} = \frac{b-a}{ab}, \qquad R = \frac{ab}{b-a}.$$

Options A and B have dimensions of inverse length, so neither can be a radius. Option C incorrectly adds the reciprocal radii; it would predict a curved common film even when both bubble pressures are equal. Our result instead tends to infinity as b approaches a, correctly describing a flat common film between equal bubbles.

Step 3: Conclusion The radius of curvature is $R = \frac{ab}{b-a}$. Hence, the correct option is (D).

Quick Tip to Solve: For a common film, subtract the two bubble pressures. Equal bubble radii must give a plane interface, so its curvature radius must tend to infinity.

38. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is : [4 marks]

(A) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$

(B) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$

(C) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$

(D) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$

Correct Answer: (C) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$

Solution:

Concept:

- Under constant acceleration, the speed changes linearly with time; the distance is the area under the speed-time graph.
- The peak speed is common to the accelerating and braking stages because the car changes its acceleration without an instantaneous change of velocity.

Step 1: Determine the peak speed Let the car accelerate for t_1 and brake for t_2, reaching peak speed v. The magnitudes α and β are positive. Since each stage begins or ends at rest,

$$v = \alpha t_1 = \beta t_2, \qquad t_1 + t_2 = t.$$

Substitute $t_1 = v/\alpha$ and $t_2 = v/\beta$ into the total-time relation:

$$t = v\left(\frac{1}{\alpha} + \frac{1}{\beta}\right), \qquad v = \frac{\alpha\beta t}{\alpha+\beta}.$$

The intervals are generally unequal; assuming that each occupies half the total time is valid only if the two acceleration magnitudes are equal.

Step 2: Find the distance from the two stages During the first interval the average speed is $v/2$, and

during braking it is also $v/2$, since each velocity-time segment is a straight line. Therefore

$$s = \frac{1}{2}vt_1 + \frac{1}{2}vt_2 = \frac{1}{2}v(t_1 + t_2) = \frac{1}{2}vt.$$

Substituting the peak speed yields

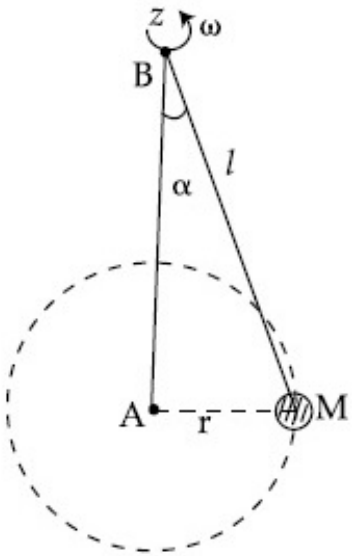
$$s = \frac{\alpha\beta}{2(\alpha + \beta)}t^2.$$

The factor one-half is the area factor of the full triangular speed-time graph. A and B overestimate that area, while D inserts an extra factor of one-half. As a check, setting $\alpha = \beta = a$ gives a peak speed $at/2$ and distance $at^2/4$, precisely the sum of two equal constant-acceleration distances.

Step 3: Conclusion The total distance is $s = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$. Hence, the correct option is (C).

Quick Tip to Solve: Eliminate the two unknown stage times using the common peak speed, then use half the total time multiplied by that peak speed.

39. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about point B is L_B. The correct statement for this system is : [4 marks]



(A) L_A is constant, both in magnitude and direction

(B) L_B is constant in direction with varying magnitude

(C) L_B is constant, both in magnitude and direction

(D) L_A and L_B are both constant in magnitude and direction

Correct Answer: (A) L_A is constant, both in magnitude and direction

Solution:

Concept:

- Angular momentum about a fixed point is $\mathbf{L} = \mathbf{r} \times M\mathbf{v}$. Constancy of its magnitude does not imply constancy of its direction.
- For steady circular motion, write the velocity as $\mathbf{v} = \omega r\,\hat{\boldsymbol{\phi}}$, where r is the fixed circle radius.

Step 1: Evaluate the angular momentum about A The figure places A at the centre of the horizontal circular path. The position of the mass relative to A is $r\hat{\mathbf{r}}$, perpendicular to its tangential velocity. Therefore

$$\mathbf{L}_A = (r\hat{\mathbf{r}}) \times (M\omega r\hat{\boldsymbol{\phi}}) = M\omega r^2\hat{\mathbf{z}}.$$

Both r and ω are constant, and $\hat{\mathbf{z}}$ is a fixed upward direction. Consequently $\mathbf{L}_A$ is constant as a vector, even though the position and velocity vectors themselves continually rotate.

Step 2: Compare the angular momentum about B Let $h = AB = l\cos\alpha$, with $r = l\sin\alpha$. Relative to B the mass has position $r\hat{\mathbf{r}} - h\hat{\mathbf{z}}$. The cross product becomes

$$\mathbf{L}_B = M\omega r^2\hat{\mathbf{z}} + M\omega rh\hat{\mathbf{r}}.$$

Its vertical component is fixed, but its horizontal radial component turns with the mass. Its magnitude is

$$|\mathbf{L}_B| = M\omega r\sqrt{r^2 + h^2} = M\omega rl,$$

which is constant. Its direction, however, varies because $\hat{\mathbf{r}}$ varies. Thus B has the constancies reversed, and C and D incorrectly require the entire vector about B to remain fixed. Only the statement about A correctly describes both magnitude and direction for the conical motion shown.

Step 3: Conclusion $\mathbf{L}_A$ is constant in both magnitude and direction, whereas $\mathbf{L}_B$ has constant magnitude and changing direction. Hence, the correct option is (A).

Quick Tip to Solve: Choose cylindrical unit vectors for circular motion. A fixed coefficient of $\hat{\mathbf{r}}$ still represents a changing vector because that unit vector rotates.

40. A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5A. The magnetic flux density produced by the solenoid is : (permeability of free space = $4\pi \times 10^{-7}$ H/m) [4 marks]

(A) πT

(B) $2 \times 10^{-3}\pi$ T

(C) $10^{-4}\pi$ T

(D) $\frac{\pi}{5}$ T

Correct Answer: (A) πT

Solution:

Concept:

- For a long solenoid with turn density n carrying current I, the magnetic field strength is $H = nI$.

- A linear magnetic core of relative permeability μ_r gives flux density $B = \mu_0\mu_r H = \mu_0\mu_r nI$.

Step 1: Identify the turn density and core factor The stated 1000 turns per metre is already the turn density, not the total number of turns. Use

$$n = 10^3\,\mathrm{m}^{-1}, \qquad I = 5\,\mathrm{A}, \qquad \mu_r = 500.$$

The field strength due to the free winding current is

$$H = nI = 1000 \times 5 = 5000\,\mathrm{A\,m}^{-1}.$$

The permeability of the core is $\mu = 500\mu_0$. The insulation separates adjacent windings electrically; it does not remove the magnetic enhancement by the core. No extra multiplication by a solenoid length is necessary because the turns are specified per unit length.

Step 2: Substitute and simplify the flux density Insert the given permeability of free space into the solenoid expression:

$$B = (4\pi \times 10^{-7})(500)(1000)(5).$$

Combine the purely numerical factors before simplifying the power of ten:

$$500 \times 1000 \times 5 = 2.5 \times 10^6,$$

$$B = (4\pi \times 10^{-7})(2.5 \times 10^6) = 10\pi \times 10^{-1} = \pi\,\mathrm{T}.$$

Option B is the air-core value $\mu_0 nI$, which neglects the relative permeability 500. Options C and D likewise fail the stated product of turn density, current and core permeability. The units check is $(\mathrm{H\,m}^{-1})(\mathrm{A\,m}^{-1}) = \mathrm{T}$, as required for magnetic flux density.

Step 3: Conclusion The magnetic flux density is $B = \pi\,\mathrm{T}$. Hence, the correct option is (A).

Quick Tip to Solve: First calculate the air-core field $\mu_0 nI$, then multiply it by the relative permeability. Treat turns per metre as n, not as a total turn count.

41. The thickness at the centre of a plane convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is 2×10^8 ms^{-1}. The focal length of the lens is _____. [4 marks]

(A) 0.30 cm

(B) 30 cm

(C) 15 cm

(D) 1.5 cm

Correct Answer: (B) 30 cm

Solution:

Concept:

- The refractive index relative to air is $\mu = c/v$, where c and v are the light speeds in vacuum and in the lens.

- For a thin plano-convex lens in air, the lens-maker relation reduces to $1/f = (\mu - 1)/R$, with R the curved surface radius.

Step 1: Find the index and the surface radius Using the given speed,

$$\mu = \frac{3 \times 10^8}{2 \times 10^8} = \frac{3}{2}.$$

The aperture radius is half the diameter, so $a = 3\,\text{cm}$. The central thickness is the sagitta $h = 3\,\text{mm} = 0.3\,\text{cm}$ for the thin plano-convex geometry. The sphere generating the curved face obeys

$$(R - h)^2 + a^2 = R^2, \qquad R = \frac{a^2 + h^2}{2h}.$$

Since $h \ll a$, use the thin-lens approximation $R \simeq a^2/(2h) = 9/0.6 = 15\,\text{cm}$. Retaining the small squared-thickness correction gives $15.15\,\text{cm}$, showing that it has only a small effect at the precision of the options.

Step 2: Apply the lens-maker formula The plane face has infinite radius and hence contributes zero curvature. Taking the positive focal length of the converging lens,

$$f = \frac{R}{\mu - 1} \simeq \frac{15}{0.5} = 30\,\text{cm}.$$

The exact sagitta value would give $30.3\,\text{cm}$, consistent with the offered approximate value. Option C mistakes the radius of curvature for the focal length and misses division by $\mu - 1$. Options A and D contain large unit or aperture errors; a millimetre thickness must first be changed to centimetres when the aperture is in centimetres. The diameter itself must not be used for a, since the right triangle extends only from the axis to the rim.

Step 3: Conclusion The focal length is approximately $30\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the central thickness to the aperture units, halve the diameter, and use $R \simeq a^2/(2h)$ before applying $f = R/(\mu - 1)$.

42. For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal ? [4 marks]

(A) x = $\frac{A}{2}$

(B) x = $\pm$ A

(C) x = $\pm \frac{A}{\sqrt{2}}$

(D) x = 0

Correct Answer: (C) x = $\pm \frac{A}{\sqrt{2}}$

Solution:

Concept:

- For SHM of amplitude A, the total mechanical energy is $E = \frac{1}{2}kA^2$, with effective restoring-force constant $k = m\omega^2$.
- At displacement x from equilibrium, $U = \frac{1}{2}kx^2$ and $K = E - U = \frac{1}{2}k(A^2 - x^2)$.

Step 1: Set the two energies equal The potential energy is measured from equilibrium, where the restoring force vanishes. The total energy is fixed by the turning-point displacement A, where the velocity is zero. At any intermediate point conservation of energy gives

$$K + U = \frac{1}{2}kA^2.$$

For equal kinetic and potential energies, each must be half of the total. Equivalently, equate their displacement-dependent expressions:

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2.$$

The common positive factor $k/2$ cancels. This makes the required displacement independent of the mass and oscillation frequency.

Step 2: Solve for both allowed displacements Rearranging the equality gives

$$A^2 - x^2 = x^2, \qquad 2x^2 = A^2, \qquad x = \pm\frac{A}{\sqrt{2}}.$$

Both signs are required because potential energy depends on x^2, so the energy sharing is identical on the two sides of equilibrium. Option A, $x = A/2$, gives $U = E/4$ and $K = 3E/4$, which are unequal. At the extremes $x = \pm A$ of option B, all the energy is potential and the kinetic energy vanishes. At $x = 0$ of option D, all the energy is kinetic. These limiting checks also show that equal sharing must occur between the equilibrium point and either turning point, rather than at one of them.

Step 3: Conclusion The required displacements are $x = \pm A/\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $U/E = x^2/A^2$. Equal energies mean $U/E = 1/2$, and remember to retain both signs when taking the square root.

43. An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading : [4 marks]

(A) $\frac{I_1+I_2}{\sqrt{2}}$

(B) $\sqrt{\frac{I_1^2-I_2^2}{2}}$

(C) $\sqrt{\frac{I_1^2+I_2^2}{2}}$

(D) $\frac{I_1+I_2}{2\sqrt{2}}$

Correct Answer: (C) $\sqrt{\frac{I_1^2+I_2^2}{2}}$

Solution:

Concept:

- A hot-wire ammeter responds to the heating effect of current. Its reading is the root-mean-square value, $I_{\text{rms}} = \sqrt{\langle I^2 \rangle}$, rather than the algebraic mean current.
- Over a complete period, $\langle \sin^2 \omega t \rangle = \langle \cos^2 \omega t \rangle = 1/2$ and $\langle \sin \omega t \cos \omega t \rangle = 0$. The sine and cosine components are in quadrature.

Step 1: Square the instantaneous current Let the averaging interval be a complete period of the alternating current. Squaring the given expression before averaging preserves its heating effect:

$$I^2 = I_1^2 \sin^2 \omega t + I_2^2 \cos^2 \omega t + 2I_1 I_2 \sin \omega t \cos \omega t.$$

The cross term must be included initially; it vanishes only after the time average is taken.

Step 2: Average and take the square root Using the full-period averages gives

$$\langle I^2 \rangle = \frac{I_1^2}{2} + \frac{I_2^2}{2} + 0.$$

Thus the effective current is the square root of a sum of squares. Options (A) and (D) incorrectly add the amplitudes as though the components were in phase. Option (B) subtracts positive heating contributions and could even become imaginary when $I_2 > I_1$; it cannot represent an ammeter reading.

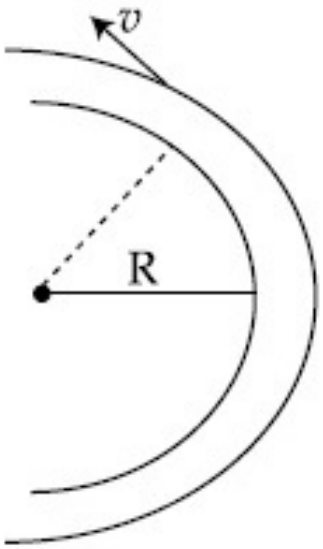
Step 3: Conclusion The hot-wire ammeter reads

$$I_{\text{rms}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For sine and cosine currents of the same frequency, add their squared amplitudes and divide by two before taking the square root.

44. A modern grand - prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_L acting downwards on the car is : (Assume forces on the four tyres are identical and g = acceleration due to gravity) [4 marks]



(A) $m\left(g-\frac{v^2}{\mu_s R}\right)$

(B) $-m\left(g+\frac{v^2}{\mu_s R}\right)$

(C) $m\left(\frac{v^2}{\mu_s R}-g\right)$

(D) $m\left(\frac{v^2}{\mu_s R}+g\right)$

Correct Answer: (C) $m\left(\frac{v^2}{\mu_s R}-g\right)$

Solution:

Concept:

- On a flat track the vertical acceleration is zero. The upward total normal reaction N balances the weight and downward aerodynamic force: $N = mg + F_L$.
- Static friction supplies the horizontal centripetal force. Its limiting value is $\mu_s N$; at the limiting speed, $mv^2/R = \mu_s N$.

Step 1: Balance forces vertically Take the entire car as the system, so N is the sum of the reactions at all four tyres. Equal tyre loads allow us to add them directly. Because the negative lift acts downwards, it increases the normal force rather than reducing it:

$$N - mg - F_L = 0, \qquad N = mg + F_L.$$

The term negative lift describes its direction; F_L here denotes its nonnegative magnitude.

Step 2: Use limiting friction for the turn At the threshold of sideways slipping, the required centripetal

force equals the maximum available friction:

$$\frac{mv^2}{R} = \mu_s(mg + F_L).$$

Rearranging,

$$F_L = \frac{mv^2}{\mu_s R} - mg.$$

This is the minimum downforce required if $v^2 > \mu_s gR$. Below that speed no downforce is necessary, since friction need not always equal its limiting value. Option (A) reverses the required sign; (B) gives a negative magnitude; (D) adds the weight term instead of subtracting it.

Step 3: Conclusion At the limiting speed, the required downward lift is

$$F_L = m\left(\frac{v^2}{\mu_s R} - g\right).$$

Hence, the correct option is (C).

Quick Tip to Solve: Downforce increases the normal reaction. Write $N = mg + F_L$ before using $\mu_s N = mv^2/R$.

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45. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is _____ cm. (least count = 0.01 cm) [4 marks]

(A) 8.58 cm

(B) 8.54 cm

(C) 8.56 cm

(D) 8.36 cm

Correct Answer: (B) 8.54 cm

Solution:

Concept:

- A vernier measurement is the main-scale reading plus the coinciding vernier division multiplied by the least count: $L_{\text{obs}} = \text{MSR} + n\,\text{LC}$.
- The true length equals the observed length minus the zero error. A positive zero error makes the instrument over-read even when the actual separation is zero.

Step 1: Find the observed reading The vernier zero lies between $8.5\,\text{cm}$ and $8.6\,\text{cm}$, so the main-scale mark immediately before it is $8.5\,\text{cm}$. We do not round this up to the next centimetre mark. The sixth vernier division coincides, giving

$$n\,\text{LC} = 6(0.01) = 0.06\,\text{cm}.$$

Therefore

$$L_{\text{obs}} = 8.50 + 0.06 = 8.56\,\text{cm}.$$

Step 2: Subtract the positive zero error Convert the stated error to the same unit as the reading:

$$0.2\,\text{mm} = 0.02\,\text{cm}.$$

The zero correction is therefore $-0.02\,\text{cm}$, not $+0.02\,\text{cm}$. Hence

$$L_{\text{true}} = 8.56 - 0.02 = 8.54\,\text{cm}.$$

Option (A) adds the error, option (C) leaves it uncorrected, and option (D) subtracts $0.20\,\text{cm}$, confusing millimetres with centimetres. The subtraction is smaller than the vernier contribution, so the answer remains above $8.50\,\text{cm}$.

Step 3: Conclusion The correct measurement is $8.54\,\text{cm}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert zero error into the reading unit first; positive zero error is always subtracted from the observed length.

46. A current of 10A exists in a wire of cross-sectional area of 5 mm^2 with a drift velocity of 2×10^{-3} ms^{-1}. The number of free electrons in each cubic meter of the wire is _____. [4 marks]

(A) 625×10^{25}

(B) 1×10^{23}

(C) 2×10^{25}

(D) 2×10^{6}

Correct Answer: (A) 625×10^{25}

Solution:

Concept:

- The magnitude of current in a metal wire is $I = neAv_d$, where n is the free-electron number density, e the elementary charge, A the cross-sectional area and v_d the drift speed.
- Use SI units throughout. Since area contains a squared length, $1\,\text{mm}^2 = 10^{-6}\,\text{m}^2$, while $e = 1.6 \times 10^{-19}\,\text{C}$.

Step 1: Convert the data and isolate the density The given values are

$$I = 10\,\text{A}, \quad A = 5 \times 10^{-6}\,\text{m}^2, \quad v_d = 2 \times 10^{-3}\,\text{m s}^{-1}.$$

Rearranging the drift-current equation gives

$$n = \frac{I}{eAv_d}.$$

Electron drift is opposite to conventional current, but the question asks for a number density, so only

the positive magnitudes enter this equation.

Step 2: Substitute and simplify the powers of ten Calculate the denominator carefully:

$$eAv_d = (1.6 \times 10^{-19})(5 \times 10^{-6})(2 \times 10^{-3}) = 1.6 \times 10^{-27}.$$

Consequently

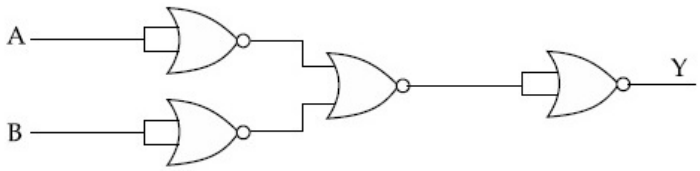
$$n = \frac{10}{1.6 \times 10^{-27}} = 6.25 \times 10^{27}\,\mathrm{m}^{-3} = 625 \times 10^{25}\,\mathrm{m}^{-3}.$$

The coefficient in the first option is not in standard scientific notation, but it is exactly this result. Options (B), (C) and (D) have different magnitudes and do not reproduce $10\,\mathrm{A}$ when substituted in $neAv_d$.

Step 3: Conclusion The wire contains 6.25×10^{27} free electrons per cubic metre, equivalent to $625 \times 10^{25}\,\mathrm{m}^{-3}$. Hence, the correct option is (A).

Quick Tip to Solve: For drift-current calculations, convert square millimetres with 10^{-6}, then compare options after putting them into scientific notation.

47. The output of the given combination gates represents : [4 marks]



(A) AND Gate

(B) NOR Gate

(C) NAND Gate

(D) XOR Gate

Correct Answer: (C) NAND Gate

Solution:

Concept:

- A NOR gate gives the complement of an OR operation: $X\ \mathrm{NOR}\ Z = \overline{X + Z}$. In Boolean notation, $+$ denotes OR and juxtaposition denotes AND.
- Joining both inputs of a NOR gate makes an inverter: $\overline{X + X} = \overline{X}$. De Morgan's law gives $\overline{\overline{A} + \overline{B}} = AB$.

Step 1: Reduce the two input stages The upper left NOR gate has both inputs connected to A, so its output is $\overline{A}$. The lower left gate similarly supplies $\overline{B}$. These are inversions of the individual inputs; they are not yet the final output of the circuit. The middle NOR gate then produces

$$X = \overline{\overline{A} + \overline{B}} = AB.$$

Thus the output at the middle connection is AND, but there is still one more gate to include.

Step 2: Include the final inversion and check the truth table The final NOR gate has its inputs joined to X, and hence

$$Y = \overline{X + X} = \overline{X} = \overline{AB}.$$

For input pairs $(A, B) = (0,0), (0,1), (1,0), (1,1)$, the corresponding outputs are $1, 1, 1, 0$. This is the NAND truth table. AND would have outputs $0, 0, 0, 1$, NOR would give $1, 0, 0, 0$, and XOR would give $0, 1, 1, 0$; all differ from the circuit. In particular, ignoring the final gate would incorrectly select AND.

Step 3: Conclusion The equivalent operation is $Y = \overline{AB}$, a NAND gate. Hence, the correct option is (C).

Quick Tip to Solve: Mark each tied-input NOR gate as NOT, simplify the central operation, and remember the inverter at the final output.

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48. Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom? [4 marks]

(A) 8

(B) 6

(C) 1

(D) 4

Correct Answer: (B) 6

Solution:

Concept:

- For a hydrogen-like ion with exactly one bound electron and nuclear charge Z, the Bohr energy is $E_n = -13.6Z^2/n^2\,\text{eV}$, with n a positive integer.
- Carbon has atomic number $Z = 6$. A one-electron carbon ion is C^{5+}; a singly ionized carbon atom is C^+ and has five bound electrons. The hydrogenic formula does not apply to the latter.

Step 1: Identify the condition needed for the energy formula The question says single ionized carbon. Literally that means C^+, whose electrons experience screening and electron-electron interactions. Its level energies cannot be obtained merely by inserting $Z = 6$ into the one-electron formula. The numerical options and the energy comparison correspond to the usual one-electron carbon calculation. We therefore state that interpretation explicitly instead of treating a five-electron ion as hydrogen-like.

Step 2: Perform the hydrogen-like comparison For hydrogen in its ground state, $Z = 1$ and $n = 1$, so $E_H = -13.6\,\text{eV}$. For one-electron carbon, equality requires

$$-13.6\frac{6^2}{n^2} = -13.6, \qquad n^2 = 36, \qquad n = 6.$$

Only the positive root is a permissible principal quantum number. The alternatives $n = 8, 1, 4$ give $Z^2/n^2 = 9/16, 36, 9/4$, respectively, none equal to unity. This calculation establishes option (B) for the hydrogen-like interpretation, while identifying the wording limitation.

Step 3: Conclusion For hydrogen-like carbon C^{5+}, the required level is $n = 6$. Hence, the correct

option is (B) under the one-electron interpretation. Literally, singly ionized carbon is C^+, so the wording does not justify this hydrogenic calculation.

Quick Tip to Solve: Before applying $E_n = -13.6Z^2/n^2$, check that the ion has exactly one electron; its charge is then $Z - 1$.

49. An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is : (c being the velocity of light) [4 marks]

(A) $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$

(B) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

(C) $\left(\frac{E}{2m}\right)^{1/2}$

(D) $c(2mE)^{1/2}$

Correct Answer: (B) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$. For a nonrelativistic electron of kinetic energy E, $p_e = \sqrt{2mE}$.
- A photon obeys $E = p_\gamma c$, so its wavelength is $\lambda_\gamma = hc/E$. The electron and photon have different energy-momentum relations even when their energies are equal.

Step 1: Write each wavelength using its own momentum relation For the electron, the nonrelativistic kinetic-energy relation gives

$$E = \frac{p_e^2}{2m}, \qquad \lambda_e = \frac{h}{\sqrt{2mE}}.$$

For the photon, there is no rest mass and $p_\gamma = E/c$, giving

$$\lambda_\gamma = \frac{hc}{E}.$$

Here the electron energy is understood as kinetic energy in the standard nonrelativistic matter-wave treatment used by these options; one must not apply $E = pc$ to the electron.

Step 2: Form the ratio in the requested order Divide the electron wavelength by the photon wavelength:

$$\frac{\lambda_e}{\lambda_\gamma} = \frac{h}{\sqrt{2mE}}\frac{E}{hc} = \frac{1}{c}\sqrt{\frac{E}{2m}}.$$

Planck's constant cancels because both wavelengths follow the same de Broglie relation. The ratio is dimensionless: $\sqrt{E/m}$ has dimensions of speed. Option (A) reverses the energy-to-mass dependence; option (C) omits the factor $1/c$, and option (D) has the wrong product of energy and mass and cannot be a dimensionless ratio.

Step 3: Conclusion The required ratio is

$$\frac{\lambda_e}{\lambda_\gamma} = \frac{1}{c}\sqrt{\frac{E}{2m}}.$$

Hence, the correct option is (B).

Quick Tip to Solve: Write $p_e = \sqrt{2mE}$ and $p_\gamma = E/c$ separately, then use $\lambda_e/\lambda_\gamma = p_\gamma/p_e$.

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50. A Carnot's engine working between 400 K and 800 K has a work output of 1200 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is : [4 marks]

(A) 2400 J

(B) 1600 J

(C) 1800 J

(D) 3200 J

Correct Answer: (A) 2400 J

Solution:

Concept:

- A Carnot engine is a reversible heat engine. Its efficiency depends only on the absolute temperatures of its hot and cold reservoirs: $\eta = 1 - T_c/T_h$. Temperatures must be expressed in kelvin.
- The thermal efficiency is the ratio of work output to heat absorbed from the source, $\eta = W/Q_h$. The heat rejected to the sink is $Q_c = Q_h - W$, by conservation of energy.

Step 1: Identify the reservoirs The hotter reservoir supplies heat, so $T_h = 800\,\text{K}$; the colder reservoir receives rejected heat, so $T_c = 400\,\text{K}$. Thus the efficiency is

$$\eta = 1 - \frac{400}{800} = \frac{1}{2}.$$

Only half the heat absorbed in a cycle can become work. Interchanging the source and sink temperatures would incorrectly give a negative efficiency.

Step 2: Calculate the absorbed heat and test the options Using the work output $W = 1200\,\text{J}$,

$$Q_h = \frac{W}{\eta} = \frac{1200}{1/2} = 2400\,\text{J}.$$

The rejected heat is $Q_c = 2400 - 1200 = 1200\,\text{J}$, consistent with $Q_c/Q_h = T_c/T_h = 1/2$. Options (B), (C) and (D) would give efficiencies $3/4$, $2/3$ and $3/8$, respectively; none equals the Carnot efficiency for these two reservoirs.

Step 3: Conclusion The heat supplied from the source in each cycle is $\boxed{2400\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: First calculate the Carnot efficiency from the temperature ratio, then divide the work by that efficiency; do not multiply the work by it.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The radius in kilometer to which the present radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 times is _____. [4 marks]

Solution:

Concept:

- The escape speed from the surface of a spherical body of mass M and radius R is $v_e = \sqrt{2GM/R}$, obtained by equating the initial kinetic energy to the increase in gravitational potential energy to infinity.
- Compression changes the radius but leaves the total mass unchanged. Consequently, at fixed mass, escape speed varies as the inverse square root of the surface radius, rather than inversely with radius itself.

Step 1: Write the escape speeds before and after compression Let the compressed radius be r, with initial radius $R = 6400\,\text{km}$. For the same Earth mass and gravitational constant,

$$v_e^2 = \frac{2GM}{R}, \qquad (v_e\prime)^2 = \frac{2GM}{r}.$$

The increase specified is a final escape speed ten times the initial one, so $v_e\prime = 10v_e$. The speed here is the minimum speed needed to escape, not the circular orbital speed.

Step 2: Take the ratio and solve for the new radius Dividing the squared-speed equations eliminates both G and the unknown mass:

$$\left(\frac{v_e\prime}{v_e}\right)^2 = \frac{R}{r} = 10^2 = 100.$$

Hence,

$$r = \frac{R}{100} = \frac{6400}{100}\,\text{km} = 64\,\text{km}.$$

As a check, substitution gives $\sqrt{R/r} = \sqrt{100} = 10$, exactly the required factor. A tenfold radius reduction would produce only a $\sqrt{10}$-fold increase in escape speed.

Step 3: Conclusion The compressed radius is $\boxed{64\,\text{km}}$, so the numerical entry is $\boxed{64}$.

Quick Tip to Solve: For fixed mass, multiply the escape speed by a factor n by dividing the radius by n^2.

52. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is _____. (Assuming the acceleration to be uniform). [4 marks]

Solution:

Concept:

- For uniform angular acceleration, angular velocity changes linearly with time. The mean angular velocity over the interval is therefore the arithmetic mean of its initial and final values.
- Angular displacement is average angular velocity multiplied by elapsed time. When speeds are in revolutions per minute, using time in minutes directly gives the displacement in revolutions, without a factor of 2π.

Step 1: Find the mean rotational speed The given initial and final rates are $900\,\mathrm{rpm}$ and $2460\,\mathrm{rpm}$. Uniform acceleration gives

$$\bar{n} = \frac{900 + 2460}{2} = 1680\,\mathrm{rev\,min^{-1}}.$$

Equivalently, the initial and final angular speeds are 30π and $82\pi\,\mathrm{rad\,s^{-1}}$; their mean is $56\pi\,\mathrm{rad\,s^{-1}}$. This independent conversion helps keep revolutions distinct from radians.

Step 2: Calculate the total revolutions The elapsed time is $26\,\mathrm{s} = 26/60\,\mathrm{min}$. Thus

$$N = \bar{n}\,t = 1680 \times \frac{26}{60} = 28 \times 26 = 728.$$

In angular units the same result follows from $\theta = 56\pi \times 26 = 1456\pi\,\mathrm{rad}$, followed by $N = \theta/(2\pi) = 728$. The average-speed method applies to the rotating member whose speeds are supplied; no separate engine-to-wheel transmission ratio is provided. Using only the initial or final speed throughout would ignore the uniform increase.

Step 3: Conclusion The number of revolutions during the stated interval is $\boxed{728}$ revolutions.

Quick Tip to Solve: Average the two rpm readings first, then multiply by seconds divided by 60; reserve 2π only for conversion to radians.

---

53. A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference V = 12 V between its plates. The charging battery is now disconnected and a porcelin plate with k = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of _____ pJ. (Assume no friction) [4 marks]

Solution:

Concept:

- After the battery is disconnected, the charge Q on an isolated capacitor remains constant. The appropriate stored-energy expression is $U = Q^2/(2C)$, rather than treating its voltage as fixed.
- Insertion of a dielectric of relative permittivity k changes the capacitance from C to kC. At fixed charge, the final electrostatic energy is U_i/k. In the absence of friction, the decrease becomes conserved mechanical energy of the moving dielectric.

Step 1: Calculate the initial electric energy Initially the battery establishes $V = 12\,\text{V}$ across $C = 14\,\text{pF}$. The stored energy is

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(14 \times 10^{-12})(12)^2\,\text{J} = 1008 \times 10^{-12}\,\text{J} = 1008\,\text{pJ}.$$

The fixed charge thereafter is $Q = CV = 168\,\text{pC}$. This charge does not acquire a factor of seven when the dielectric enters, because there is no longer a conducting connection to the battery.

Step 2: Find the energy transferred to the dielectric When the dielectric is fully inserted, $C_f = 7C$, so

$$U_f = \frac{Q^2}{2(7C)} = \frac{U_i}{7} = 144\,\text{pJ}.$$

The electric force pulls the plate inward, and the lost field energy supplies its motion. Therefore its mechanical energy relative to the fully inserted equilibrium is

$$E_{\text{mech}} = U_i - U_f = 1008 - 144 = 864\,\text{pJ}.$$

At full insertion this is kinetic energy; at turning positions it is stored as the corresponding change of interaction energy. With no friction the total mechanical energy remains constant over the oscillation.

Step 3: Conclusion The constant mechanical energy is $\boxed{864\,\text{pJ}}$, so the numerical entry is $\boxed{864}$.

Quick Tip to Solve: For an isolated capacitor, dielectric insertion releases $U_i(1 - 1/k)$; fixed charge makes the final electric energy smaller.

54. The following bodies,

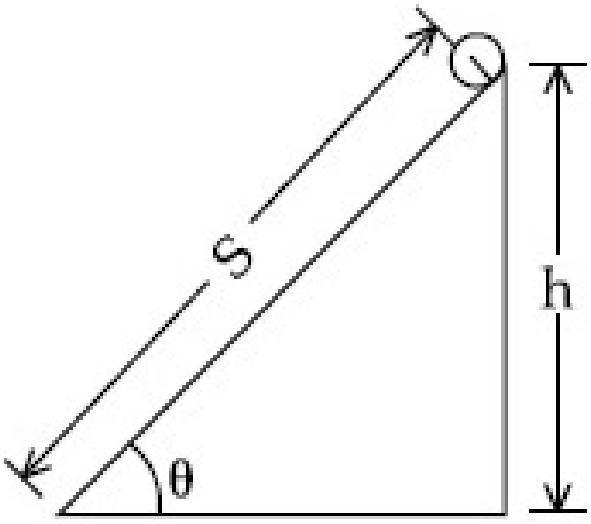
(1) a ring

(2) a disc

(3) a solid cylinder

(4) a solid sphere,

of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is _____. [Mark the body as per their respective numbering given in the question] [4 marks]



Solution:

Concept:

- For a rigid body rolling down an incline without slipping, translational acceleration is $a = g\sin\theta/[1+I/(mR^2)]$, where I is its moment of inertia about its central rotation axis.
- The static friction supplies torque but does no dissipative work for ideal rolling. For bodies released together from rest on the same incline, the one with greatest acceleration reaches the bottom first.

Step 1: Express the acceleration using rotational inertia Along the incline, $mg\sin\theta - f = ma$. About the centre, $fR = I\alpha$, and the no-slip condition gives $a = \alpha R$. Eliminating f yields

$$mg\sin\theta = a\left(m + \frac{I}{R^2}\right), \qquad a = \frac{g\sin\theta}{1 + I/(mR^2)}.$$

Thus the relevant comparison is the dimensionless inertia factor, rather than the common mass or radius separately.

Step 2: Compare all four bodies The inertia factors for a ring, disc, solid cylinder and solid sphere are 1, $1/2$, $1/2$ and $2/5$, respectively. Their accelerations are therefore

$$a_{\text{ring}} = \frac{1}{2}g\sin\theta, \quad a_{\text{disc}} = a_{\text{cylinder}} = \frac{2}{3}g\sin\theta, \quad a_{\text{sphere}} = \frac{5}{7}g\sin\theta.$$

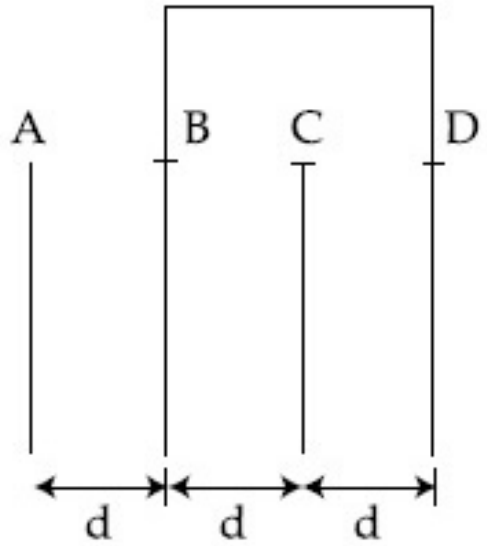
Since $5/7 > 2/3 > 1/2$, the sphere accelerates fastest. For the common incline length s, the travel time $t = \sqrt{2s/a}$ is smallest for that body. The disc and cylinder tie with each other but do not beat the sphere.

Step 3: Conclusion The solid sphere reaches the bottom first. It is body number $\boxed{4}$.

Quick Tip to Solve: For equal-radius rolling bodies, compare $I/(mR^2)$: the smallest value gives the greatest downhill acceleration.

---

55. Four identical rectangular plates with length, l = 2 cm and breadth, b = $\frac{3}{2}$ cm are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{x\varepsilon_0}{d}$. The value of x is _____. (Round off to the Nearest Integer) [4 marks]



Solution:

Concept:

- Each adjacent pair of parallel rectangular plates has capacitance $C_0 = \varepsilon_0 A/d$, neglecting edge effects, where $A = lb$ is the common facing area and d is the separation.
- The wire joining plates B and D makes them one equipotential node. Capacitors between the same two nodes are in parallel; that parallel combination is then in series with the capacitor between A and the joined node.

Step 1: Reduce the arrangement to two capacitors in series The three adjacent gaps are AB, BC and CD, each contributing C_0. Since B and D are connected, the BC and CD capacitors both connect C to the same joined node. They are in parallel, giving $2C_0$. Between A and C, this combination is in series with the AB capacitor:

$$\frac{1}{C_{AC}} = \frac{1}{C_0} + \frac{1}{2C_0} = \frac{3}{2C_0}, \qquad C_{AC} = \frac{2}{3}C_0.$$

It would be incorrect to put all three gaps in series, because the B–D wire changes the circuit connections.

Step 2: Insert the plate area and identify the coefficient The area is $A = (2\,\text{cm})(3/2\,\text{cm}) = 3\,\text{cm}^2$. Consequently,

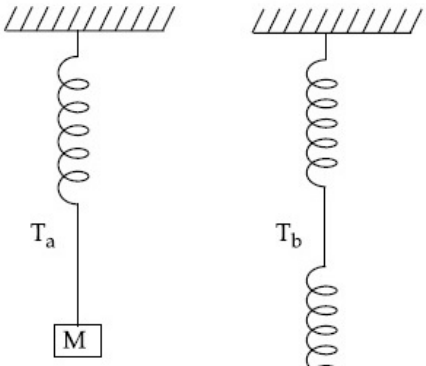
$$C_{AC} = \frac{2}{3}\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0}{d}(2\,\text{cm}^2) = \frac{\varepsilon_0}{d}(2 \times 10^{-4}\,\text{m}^2).$$

The requested numerical coefficient is therefore 2 when the plate area is measured in the given square-centimetre units. The area unit must accompany this coefficient: in a fully SI expression its value is $2 \times 10^{-4}\,\text{m}^2$, so it is not a dimensionless factor multiplying ε_0/d.

Step 3: Conclusion Using the square-centimetre area convention of the question, $\boxed{x = 2}$. The dimensionally explicit result is $\boxed{C_{AC} = \varepsilon_0(2\,\text{cm}^2)/d}$; the numerical entry is 2.

Quick Tip to Solve: First join all plates connected by a wire into one node. Only then decide which adjacent-gap capacitors are parallel or in series.

56. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig. 1 shows one of them and Fig. 2 shows their series combination. The ratios of time period of oscillation of the two SHM is $T_b/T_a = \sqrt{x}$, where value of x is _____. (Round off to the Nearest Integer) [4 marks]



Solution:

Concept:

- A mass M attached to a spring of effective stiffness k_{eff} has period $T = 2\pi\sqrt{M/k_{\text{eff}}}$. The oscillation is about the equilibrium position, so gravity changes the static extension but not this period.
- For two massless springs in series, the same force acts in both while their extensions add. Their effective spring constant satisfies $1/k_{\text{series}} = 1/k_1 + 1/k_2$.

Step 1: Find the stiffness in each arrangement In Fig. 1, only one spring supports the mass, so the effective stiffness is $k_a = k$. In Fig. 2 the two identical springs are in series. If a force F extends each

by F/k, their total extension is $2F/k$. Hence

$$k_b = \frac{F}{2F/k} = \frac{k}{2}.$$

The two springs are not in parallel: their extensions add rather than their restoring forces at the same extension.

Step 2: Take the ratio of the periods The mass is M in both diagrams, and the springs have negligible mass. Thus

$$T_a = 2\pi\sqrt{\frac{M}{k}}, \qquad T_b = 2\pi\sqrt{\frac{M}{k/2}} = 2\pi\sqrt{\frac{2M}{k}}.$$

Taking the ratio cancels 2π, M and k:

$$\frac{T_b}{T_a} = \sqrt{2}.$$

Comparison with the specified form $T_b/T_a = \sqrt{x}$ gives $x = 2$. The longer period in Fig. 2 is physically reasonable because its series arrangement is less stiff and therefore has a smaller restoring force for the same displacement.

Step 3: Conclusion The period ratio is $\boxed{T_b/T_a = \sqrt{2}}$, so the dimensionless numerical value is $\boxed{x = 2}$.

Quick Tip to Solve: Two identical springs in series halve the stiffness, so the period increases by $\sqrt{2}$ for the same attached mass.

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57. The equivalent resistance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is 'p'. If s = np, then the minimum value for n is _____. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- For two positive resistances R_1, R_2, the series resistance is $s = R_1 + R_2$, while the parallel resistance is $p = R_1R_2/(R_1 + R_2)$.
- The required multiplier is the dimensionless ratio $n = s/p$. Its minimum follows from $(R_1 - R_2)^2 \geq 0$, with equality for equal resistors.

Step 1: Form the ratio The same two physical resistors are used in both combinations, so substitute their series and parallel equivalents into the given relation. This gives

$$n = \frac{s}{p} = \frac{(R_1 + R_2)^2}{R_1R_2} = \frac{R_1^2 + 2R_1R_2 + R_2^2}{R_1R_2}.$$

The denominator is positive for ordinary resistors. Therefore the ratio can be bounded without reversing an inequality.

Step 2: Find and attain the lower bound Rewrite the numerator to isolate a nonnegative term:

$$n = 4 + \frac{(R_1 - R_2)^2}{R_1 R_2} \geq 4.$$

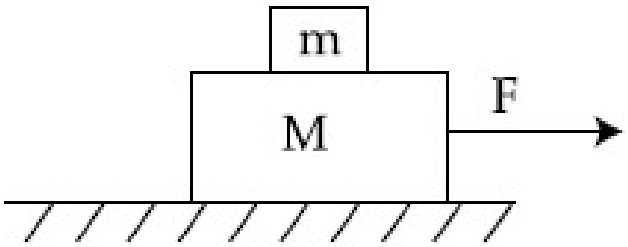
Equality requires $R_1 = R_2 = R$. In that case $s = 2R$ and $p = R/2$, so $s/p = 4$ exactly. Thus the bound is physically attainable, rather than merely a limiting value. Unequal resistors give a strictly positive additional term and cannot produce a smaller multiplier.

Step 3: Conclusion The minimum value is $n_{\min} = 4$. Rounded to the nearest integer, the answer is $\boxed{4}$ (dimensionless).

Quick Tip to Solve: For two positive resistors, write the series-to-parallel ratio as $4 + (R_1 - R_2)^2/(R_1 R_2)$; the minimum occurs when the resistors are equal.

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58. Two blocks (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is _____ N. (Round off to the Nearest Integer) [Take g as 9.8 ms^{-2}] [4 marks]



Solution:

Concept:

- Static friction accelerates the upper block along with the lower block. Its limiting value is $f_{\max} = \mu_s N$, with $N = mg$ because the upper block has no vertical acceleration.
- Treating both blocks as one system eliminates their internal friction force. Since the table is frictionless, the horizontal external force obeys $F = (M + m)a$.

Step 1: Find the limiting common acceleration Only static friction acts horizontally on the upper block; the applied force acts directly on the larger block. For no relative slipping, Newton's second law for the upper block gives $f = ma$. Its normal reaction is $N = mg$. Hence

$$ma \leq \mu_s mg, \qquad a \leq \mu_s g.$$

Using the given coefficient and gravitational acceleration,

$$a_{\max} = \frac{3}{7} \times 9.8 = 4.2\,\mathrm{m\,s^{-2}}.$$

The mass cancels in the acceleration condition, but both masses must still be included when calculating the applied force.

Step 2: Apply Newton's law to both blocks The internal friction forces on the blocks are equal and opposite, so they cancel for the combined system. The total mass is

$$M + m = 4.5 + 0.5 = 5.0\,\text{kg}.$$

At the threshold of slipping,

$$F_{\max} = (M + m)a_{\max} = 5.0 \times 4.2 = 21\,\text{N}.$$

As a check, the friction needed on the upper block is $0.5 \times 4.2 = 2.1\,\text{N}$, equal to $(3/7)(0.5)(9.8)$. A larger applied force would demand friction above its limiting value.

Step 3: Conclusion The maximum horizontal force is $\boxed{21\,\text{N}}$, which is already an integer.

Quick Tip to Solve: First impose $ma \le \mu_s mg$ on the upper block; then multiply the resulting acceleration limit by the combined mass, not just the lower mass.

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59. If 2.5×10^{-6} N average force is exerted by a light wave on a non-reflecting surface of 30 cm^2 area during 40 minutes of time span, the energy flux of light just before it falls on the surface is _____ W/cm^2. (Round off to the Nearest Integer)

(Assume complete absorption and normal incidence conditions are there) [4 marks]

Solution:

Concept:

- For complete absorption at normal incidence, radiation pressure is $P_{\text{rad}} = I/c$, where I is energy flux (power per unit area) and c is the speed of light.
- The average force is $F = P_{\text{rad}}A = IA/c$. Thus $I = Fc/A$; the duration cancels because both momentum transfer and incident energy grow with exposure time.

Step 1: Relate force to incident power A light energy E carries momentum E/c. Complete absorption transfers all of that momentum to the surface, rather than reversing it as reflection would. During a time interval t,

$$F = \frac{\Delta p}{t} = \frac{E}{ct}, \qquad \frac{E}{t} = Fc.$$

The incident average power is therefore

$$\mathcal{P} = (2.5 \times 10^{-6})(3.0 \times 10^{8}) = 750\,\text{W}.$$

The supplied exposure of 40 minutes would be needed to find total energy, but the question asks for energy flux, which is already a rate.

Step 2: Divide by the illuminated area The illuminated area is $30\,\text{cm}^2$. Keeping this area unit directly

gives the requested unit without a final conversion:

$$I = \frac{\mathcal{P}}{A} = \frac{750}{30} = 25\,\mathrm{W\,cm^{-2}}.$$

Equivalently, $A = 30 \times 10^{-4} = 3.0 \times 10^{-3}\,\mathrm{m^2}$, yielding $2.5 \times 10^{5}\,\mathrm{W\,m^{-2}}$. Dividing this latter value by 10^4 again gives $25\,\mathrm{W\,cm^{-2}}$. No factor of two is appropriate, since the surface is non-reflecting.

Step 3: Conclusion The incident energy flux, to the nearest integer, is $\boxed{25\,\mathrm{W\,cm^{-2}}}$.

Quick Tip to Solve: Absorption gives $F = IA/c$, while perfect reflection gives $F = 2IA/c$. Check the surface condition before using either expression.

60. For VHF signal broadcasting, _____ km^2 of maximum service area will be covered by an antenna tower of height 30 m, if the receiving antenna is placed at ground. Let radius of the earth be 6400 km. (Round off to the Nearest Integer) (Take π as 3.14) [4 marks]

Solution:

Concept:

- VHF broadcasting is limited by the line of sight to the horizon. For a tower of height $h \ll R$, its horizon range is $d = \sqrt{2Rh}$, where R is the Earth's radius.
- A receiving antenna at ground level contributes zero additional horizon range in this model. The service region has area $A = \pi d^2$, using consistent units for R and h.

Step 1: Calculate the squared horizon range The line joining the tower top to the horizon is tangent to the Earth. The right triangle formed by the Earth's centre, the horizon point and the tower top gives

$$d^2 = (R + h)^2 - R^2 = 2Rh + h^2 \simeq 2Rh.$$

The h^2 term is negligible since the tower is only 30 m high compared with a radius of 6400 km. Convert height before substituting:

$$h = 30\,\mathrm{m} = 0.030\,\mathrm{km}, \qquad d^2 = 2(6400)(0.030) = 384\,\mathrm{km^2}.$$

Thus $d \simeq 19.60\,\mathrm{km}$. The receiving aerial is at ground, so its height does not extend this range.

Step 2: Find the circular service area Broadcasting reaches the same horizon distance in every horizontal direction in the ideal line-of-sight model. Therefore the maximum service region is approximated by a circle of radius d, and

$$A = \pi d^2 = 3.14 \times 384 = 1205.76\,\mathrm{km^2}.$$

Rounding this area, rather than prematurely rounding the range, gives $1206\,\mathrm{km^2}$. Retaining the negligible h^2 term would add only about $0.003\,\mathrm{km^2}$ and would not alter the required integer.

Step 3: Conclusion The maximum service area, to the nearest integer, is $\boxed{1206\,\mathrm{km^2}}$.

Quick Tip to Solve: For a ground-level receiver, use $A = 2\pi Rh$ directly; convert tower height to kilometres when the Earth radius is given in kilometres.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The absolute value of the electron gain enthalpy of halogens satisfies : [4 marks]

(A) Cl > Br > F > I

(B) Cl > F > Br > I

(C) I > Br > Cl > F

(D) F > Cl > Br > I

Correct Answer: (B) Cl > F > Br > I

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for adding an electron to an isolated gaseous atom. Halogen electron gain enthalpies are negative, so their absolute values measure the magnitude of energy released.
- Although electron attraction generally weakens down a group, fluorine is exceptional: its compact $2p$ shell gives strong electron–electron repulsion for the incoming electron.

Step 1: Compare fluorine with chlorine Adding an electron to chlorine releases more energy than adding one to fluorine. The new electron in fluorine must enter a very compact $2p$ orbital, where repulsion from the existing electrons is strong. Chlorine accepts the electron into the more spacious $3p$ shell. Thus chlorine has the greater absolute electron gain enthalpy despite fluorine being the most electronegative element. Typical magnitudes are about $349\,\mathrm{kJ\,mol^{-1}}$ for chlorine and $328\,\mathrm{kJ\,mol^{-1}}$ for fluorine; the required comparison is therefore $\mathrm{Cl} > \mathrm{F}$.

Step 2: Complete the order and check the choices Below chlorine, increasing atomic size and shielding reduce attraction for the added electron. Bromine and iodine have smaller energy-release magnitudes, about 325 and $295\,\mathrm{kJ\,mol^{-1}}$, respectively. The resulting order is

$$|\Delta_{\mathrm{eg}}H(\mathrm{Cl})| > |\Delta_{\mathrm{eg}}H(\mathrm{F})| > |\Delta_{\mathrm{eg}}H(\mathrm{Br})| > |\Delta_{\mathrm{eg}}H(\mathrm{I})|.$$

Option (A) wrongly places bromine above fluorine. Option (C) makes iodine the greatest, contrary to the down-group trend. Option (D) overlooks the fluorine anomaly and incorrectly puts fluorine above chlorine.

Step 3: Conclusion The correct absolute-value order is $\boxed{\mathrm{Cl} > \mathrm{F} > \mathrm{Br} > \mathrm{I}}$. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish electron gain enthalpy from electronegativity: fluorine is most electronegative, but chlorine releases the most energy when a halogen atom gains an electron.

62. What is the spin-only magnetic moment value (BM) of a divalent metal ion with atomic number 25, in it's aqueous solution? [4 marks]

(A) zero

(B) 5.26

(C) 5.0

(D) 5.92

Correct Answer: (D) 5.92

Solution:

Concept:

- Atomic number 25 identifies manganese, whose neutral ground-state configuration is $[\mathrm{Ar}]\,3d^5 4s^2$. Forming Mn^{2+} removes the two $4s$ electrons first.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{BM}$, where n is the number of unpaired electrons and BM denotes the Bohr magneton.

Step 1: Count the unpaired electrons The divalent ion has the electronic configuration

$$\mathrm{Mn} : [\mathrm{Ar}]\,3d^5 4s^2, \qquad \mathrm{Mn}^{2+} : [\mathrm{Ar}]\,3d^5.$$

In aqueous solution, water is a weak-field ligand and the d^5 ion is high spin. By Hund's rule, the five d electrons occupy different orbitals before pairing. In an octahedral description this is $t_{2g}^3 e_g^2$, with one electron in each of the five orbitals. Consequently there are five unpaired electrons, $n = 5$, rather than five paired electrons or a closed shell.

Step 2: Evaluate the spin-only expression Substitute the unpaired-electron count, not the atomic number or the ionic charge:

$$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.916\ldots\ \mathrm{BM} \simeq 5.92\,\mathrm{BM}.$$

Option (A), zero, would require all electrons to be paired and is incompatible with high-spin d^5. Option (B), $5.26\,\mathrm{BM}$, does not follow from $n = 5$ in the spin-only formula. Option (C), $5.0\,\mathrm{BM}$, confuses the number of unpaired electrons with the magnetic moment; the factor $n + 2$ must be included.

Step 3: Conclusion The spin-only magnetic moment is $\boxed{5.92\,\mathrm{BM}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a transition-metal cation, remove the outer s electrons before the d electrons, count unpaired electrons, and then use $\sqrt{n(n+2)}$.

63. Hoffmann bromomide degradation of benzamide gives product A, which upon heating with $CHCl_3$ and NaOH gives product B.

The structures of A and B are : [4 marks]

(A) A: aniline ($C_6H_5NH_2$); B: phenyl isocyanide (C_6H_5NC).

(B) A: 4-bromoaniline; B: 2-amino-5-bromobenzaldehyde.

(C) A: 4-bromobenzamide; B: 2-formyl-4-bromobenzamide.

(D) A: aniline; B: 2-aminobenzaldehyde.

Correct Answer: (A) A: aniline ($C_6H_5NH_2$); B: phenyl isocyanide (C_6H_5NC).

Solution:

Concept:

- Hofmann bromamide degradation converts a primary amide $RCONH_2$ into a primary amine RNH_2 containing one fewer carbon atom; the carbonyl carbon is lost.
- A primary amine heated with chloroform and alkali undergoes the carbylamine reaction to give an isocyanide $R-NC$, with the organic group bonded to nitrogen, not carbon.

Step 1: Identify product A from benzamide The starting material is benzamide, $C_6H_5CONH_2$. In the Hofmann rearrangement, the phenyl group migrates and the carbonyl carbon is eliminated, giving aniline:

$$C_6H_5CONH_2 + Br_2 + 4\,NaOH \longrightarrow C_6H_5NH_2 + 2\,NaBr + Na_2CO_3 + 2\,H_2O.$$

Therefore product A is $C_6H_5NH_2$. The reaction is degradation of the amide group, rather than substitution of bromine onto the ring. Thus the proposed brominated products in options (B) and (C) do not represent the required product A.

Step 2: Apply the carbylamine reaction to aniline Aniline is a primary aromatic amine, so it responds to the carbylamine test on heating with chloroform and sodium hydroxide:

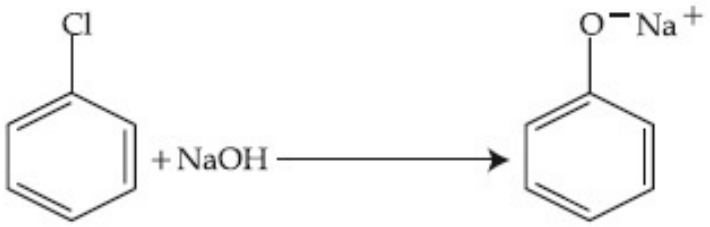
$$C_6H_5NH_2 + CHCl_3 + 3\,NaOH \longrightarrow C_6H_5NC + 3\,NaCl + 3\,H_2O.$$

Product B is phenyl isocyanide, C_6H_5-NC. Its nitrogen atom is attached to the phenyl group. This is different from benzonitrile, C_6H_5-CN, in which the carbon atom is attached to the ring. Option (A) shows the correct aniline/isocyanide pair. Option (D) incorrectly proposes formylation of the ring; the stated reagents acting on a primary amine give an isocyanide instead.

Step 3: Conclusion Product A is aniline, $C_6H_5NH_2$, and product B is phenyl isocyanide, C_6H_5NC. Hence, the correct option is (A).

Quick Tip to Solve: Follow the functional group through both reactions: amide loses its carbonyl carbon to form a primary amine, then $CHCl_3/NaOH$ changes $-NH_2$ into $-NC$.

64. The above reaction requires which of the following reaction conditions? [4 marks]



(A) 623 K, Cu, 300 atm

(B) 623 K, 300 atm

(C) 573 K, 300 atm

(D) 573 K, Cu, 300 atm

Correct Answer: (B) 623 K, 300 atm

Solution:

Concept:

- The carbon–chlorine bond of chlorobenzene is resistant to ordinary hydrolysis because resonance gives it partial double-bond character; the bonded carbon is also sp^2-hybridised.
- The Dow process uses aqueous NaOH at $623\,\mathrm{K}$ under $300\,\mathrm{atm}$. Sodium phenoxide is obtained before acidification.

Step 1: Identify the transformation The figure shows chlorobenzene reacting with sodium hydroxide to form sodium phenoxide. This is hydrolytic replacement of chlorine by an oxygen-containing group, not the substitution of an ordinary alkyl chloride. The strong aryl carbon–chlorine bond explains why dilute alkali at room temperature is ineffective. The oxygen in the product carries a negative charge paired with sodium, so the product is the salt rather than free phenol.

Step 2: Apply the required conditions The prescribed conditions for this conversion are $623\,\mathrm{K}$ and $300\,\mathrm{atm}$, with no copper catalyst specified. The net substitution is

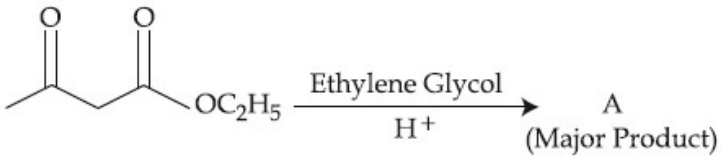
$$\mathrm{C_6H_5Cl} + 2\,\mathrm{NaOH} \longrightarrow \mathrm{C_6H_5ONa} + \mathrm{NaCl} + \mathrm{H_2O}.$$

Copper-assisted conversion of aryl halides in other named reactions must not be confused with this alkaline hydrolysis. Option (A) adds copper unnecessarily; (C) and (D) use $573\,\mathrm{K}$, rather than the standard temperature of this process. Option (D) also adds copper.

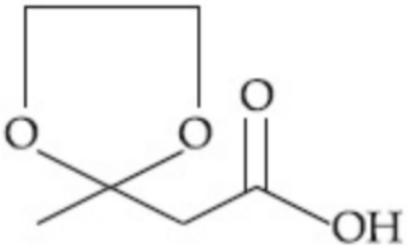
Step 3: Conclusion The required conditions are $623\,\mathrm{K}$, $300\,\mathrm{atm}$. Hence, the correct option is (B).

Quick Tip to Solve: For chlorobenzene to sodium phenoxide, recall Dow conditions: 623 K and 300 atm; acidification would give phenol.

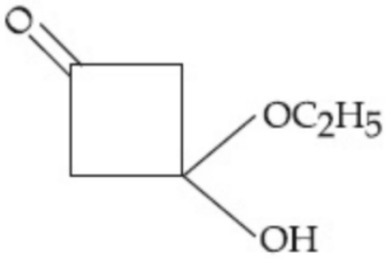
65. The product "A" in the above reaction is : [4 marks]



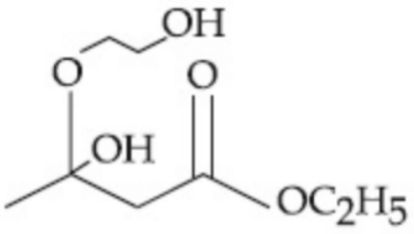
(A) Cyclic ethylene ketal of acetoacetic acid (carboxylic acid group).



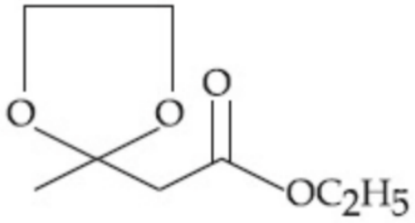
(B) 3-Ethoxy-3-hydroxycyclobutan-1-one.



(C) Open-chain ethylene glycol hemiketal of ethyl acetoacetate.

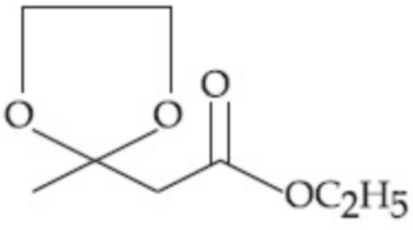


(D) Cyclic ethylene ketal of ethyl acetoacetate (ethyl ester group retained).



Correct Answer: (D)

Cyclic ethylene ketal of ethyl acetoacetate (ethyl ester group retained).



Solution:

Concept:

- A ketone reacts with ethylene glycol, $HOCH_2CH_2OH$, in acid to give a five-membered cyclic ketal and water.
- In ethyl acetoacetate, $CH_3COCH_2COOC_2H_5$, the ketone carbonyl is protected preferentially; the ester group remains an ester.

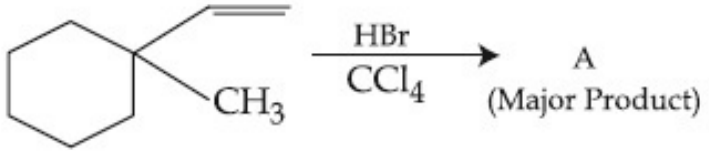
Step 1: Locate the reacting carbonyl The starting molecule is a beta-keto ester. Its left-hand carbonyl is a ketone, bonded to a methyl group and to $CH_2COOC_2H_5$; the right-hand carbonyl belongs to an ester. Acid protonates the ketone oxygen, making that carbonyl carbon more susceptible to nucleophilic attack by an alcohol oxygen of ethylene glycol. Addition first gives a hemiketal, which still contains an OH group at the former carbonyl carbon.

Step 2: Complete ring formation and compare the products After proton transfers and loss of water, the second hydroxyl group attacks intramolecularly. The resulting five-membered ring contains the former ketone carbon, two oxygen atoms and the two methylene carbons of glycol. The former carbonyl carbon remains attached to methyl and $CH_2COOC_2H_5$. Both glycol oxygens are bonded to it, so the final product is a ketal rather than a hemiketal. Option (D) shows precisely this structure with the ethyl ester unchanged. Option (A) incorrectly replaces the ethyl ester by a carboxylic acid; (B) creates a carbon ring absent from this mechanism; (C) stops at the open-chain hemiketal rather than the stable cyclic protection product.

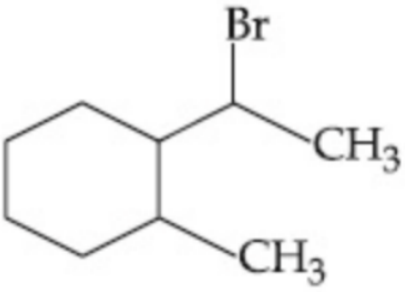
Step 3: Conclusion The major product is the cyclic ethylene ketal of the ketone group, retaining $COOC_2H_5$. Hence, the correct option is (D).

Quick Tip to Solve: With glycol and acid, circle the ketone carbonyl first: it becomes a five-membered ketal, while the ester survives.

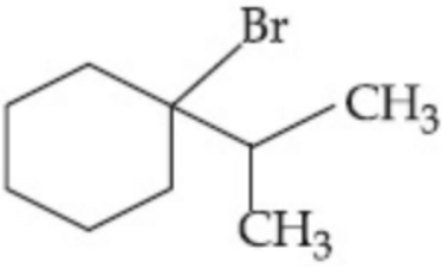
66. Product "A" in the above chemical reaction is : [4 marks]



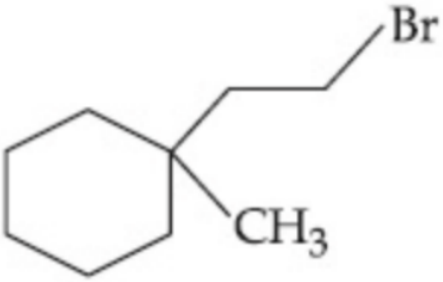
(A) 1-(1-Bromoethyl)-2-methylcyclohexane.



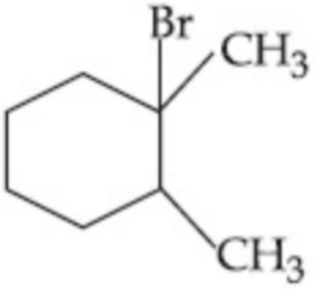
(B) 1-Bromo-1-(propan-2-yl)cyclohexane.



(C) 1-(2-Bromoethyl)-1-methylcyclohexane.

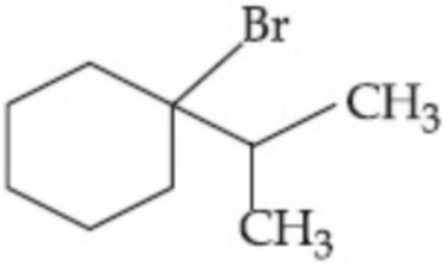


(D) 1-Bromo-1,2-dimethylcyclohexane.



Correct Answer: (B)

1-Bromo-1-(propan-2-yl)cyclohexane.



Solution:

Concept:

- In the absence of peroxide, HBr adds to an alkene by electrophilic protonation followed by bromide capture of a carbocation.
- A neighbouring alkyl group can undergo a 1,2-shift if that produces a more stable carbocation; the migrating group carries its bonding electron pair.

Step 1: Generate the initial carbocation Label the substituted ring carbon as C1. It bears methyl and a vinyl group, $-CH=CH_2$, and has no hydrogen. Protonation at the terminal alkene carbon gives the more substituted, secondary carbocation $C_1-CH^+-CH_3$. Protonation in the reverse direction would create a less stable primary carbocation. Since no peroxide is present, the anti-Markovnikov radical route is not applicable; carbon tetrachloride acts as solvent.

Step 2: Rearrange before bromide capture The methyl group attached to C1 migrates with its bond pair to the adjacent positively charged carbon. That side-chain carbon becomes $CH(CH_3)_2$, while the positive charge moves to C1. C1 now has three carbon attachments, giving a tertiary carbocation instead of the initial secondary one. Bromide attacks this ring carbocation to form 1-bromo-1-(propan-2-yl)cyclohexane, the structure in (B). Option (C) places bromine at the terminal carbon and would require radical regioselectivity. Option (A) places the original methyl on a neighbouring ring carbon without the required bond movement. Option (D) has a different carbon skeleton and does not represent the methyl-shift product. The rearrangement conserves every carbon of the starting material.

Step 3: Conclusion The major product is 1-bromo-1-(propan-2-yl)cyclohexane after a 1,2-methyl shift. Hence, the correct option is (B).

Quick Tip to Solve: After Markovnikov protonation, inspect the neighbouring carbon for a methyl shift before trapping the carbocation with bromide.

67. Which of the following compound CANNOT act as a Lewis base? [4 marks]

(A) SF_4

(B) NF_3

(C) ClF_3

(D) PCl_5

Correct Answer: (D) PCl_5

Solution:

Concept:

- A Lewis base donates an available electron pair to form a coordinate bond; central-atom lone pairs provide the usual test for these molecules.
- Count electrons left on the central atom after forming its single bonds: a lone pair can remain even when the atom is bonded to electronegative halogens.

Step 1: Count the available central-atom lone pairs Sulfur has six valence electrons. In SF_4, four

are used in the four S–F bonds, leaving two electrons, or one lone pair, on sulfur. Nitrogen has five valence electrons; three N–F bonds in NF_3 leave one lone pair on nitrogen. Chlorine has seven valence electrons; three Cl–F bonds in ClF_3 leave four nonbonding electrons, or two lone pairs. Thus all three have a central-atom pair that can in principle be donated, although their base strengths need not be equal.

Step 2: Identify the molecule without a donor pair Phosphorus has five valence electrons, and all five are involved in P–Cl bonding in molecular PCl_5. There is no lone pair on the central phosphorus. Consequently it cannot serve as the electron-pair donor in the sense tested here. Instead, it can accept a chloride electron pair, forming PCl_6^-, which illustrates Lewis acidity. Options (A), (B) and (C) fail the word CANNOT because their central atoms retain lone pairs. The electronegativity of fluorine can weaken donation in NF_3, but weak basicity is distinct from having no central donor pair at all.

Step 3: Conclusion PCl_5 cannot act as the Lewis base among the listed compounds. Hence, the correct option is (D).

Quick Tip to Solve: For this comparison, count central-atom lone pairs; do not confuse a weak donor such as NF3 with the absence of a donor pair.

68. Given below are two statements :

Statement I : Potassium permanganate on heating at 573K forms potassium manganate.

Statement II : Both potassium permanganate and potassium manganate are tetrahedral and paramagnetic in nature.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is true but statement II is false

(B) Statement I is false but statement II is true

(C) Both statement I and statement II are false

(D) Both statement I and statement II are true

Correct Answer: (A) Statement I is true but statement II is false

Solution:

Concept:

- Heating potassium permanganate produces potassium manganate, manganese dioxide and oxygen.
- Tetrahedral geometry alone does not determine magnetism. The oxidation state fixes the metal d-electron count; unpaired electrons cause paramagnetism.

Step 1: Test the thermal-decomposition statement At about $573\,K$, potassium permanganate undergoes thermal decomposition according to

$$2\,KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2.$$

The equation conserves potassium, manganese and oxygen: the eight oxygen atoms on the left appear as four, two and two on the right. Potassium manganate is therefore one of the products, even though

it is not the only product. Statement I says that manganate forms, so it is true.

Step 2: Separate the geometry and magnetism claims In MnO_4^-, the four oxide ions contribute -8 and the ion charge is -1, so manganese is +7. Its configuration is $3d^0$, with no unpaired d electrons, and permanganate is diamagnetic. In MnO_4^{2-}, manganese is +6 and has configuration $3d^1$, so manganate is paramagnetic. Both ions are tetrahedral, but the second statement joins that true geometry claim to the false claim that both are paramagnetic. The statement as a whole is therefore false. Options (B) and (C) wrongly reject the observed decomposition; (D) wrongly assigns paramagnetism to the d-zero permanganate ion.

Step 3: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (A).

Quick Tip to Solve: For manganese oxoanions, calculate the oxidation state before magnetism: Mn(VII) is d-zero, whereas Mn(VI) is d-one.

69. Mesityl oxide is a common name of : [4 marks]

(A) 3-Methyl cyclohexane carbaldehyde

(B) 4-Methyl pent-3-en-2-one

(C) 2,4-Dimethyl pentan-3-one

(D) 2-Methyl cyclohexanone

Correct Answer: (B) 4-Methyl pent-3-en-2-one

Solution:

Concept:

- Mesityl oxide is the dehydrated aldol-condensation product of two molecules of acetone.
- Its condensed structure is $CH_3COCH{=}C(CH_3)_2$; in IUPAC naming, the ketone receives the lowest available locant before the double bond.

Step 1: Recover the structure from acetone condensation Acetone first undergoes self-aldol addition to form diacetone alcohol, $CH_3COCH_2C(OH)(CH_3)_2$. On dehydration, an alpha hydrogen and the beta hydroxyl group are eliminated as water. The resulting conjugated enone is

$$CH_3COCH{=}C(CH_3)_2.$$

This has an acyclic carbon skeleton, a ketone carbonyl and an adjacent carbon–carbon double bond. It is the compound traditionally called mesityl oxide.

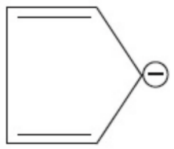
Step 2: Assign the systematic name and reject alternatives Select the longest chain containing the carbonyl and the double bond. It contains five carbon atoms; number from the end nearer the carbonyl so that the ketone carbon is C2. The double bond then lies between C3 and C4, and the additional methyl substituent is attached to C4. The name is therefore 4-methylpent-3-en-2-one, matching (B). Option (A) is a cyclic aldehyde, whereas mesityl oxide is an acyclic ketone. Option (C) describes a saturated ketone with two methyl substituents and lacks the required double bond. Option (D) is a cyclic saturated ketone, again with neither the correct chain nor the conjugated enone unit. Counting the six total carbon atoms also agrees with formation from two acetone molecules.

Step 3: Conclusion Mesityl oxide is 4-methylpent-3-en-2-one. Hence, the correct option is (B).

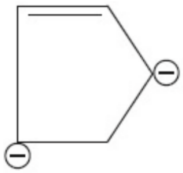
Quick Tip to Solve: Remember mesityl oxide through acetone aldol dehydration, then number the enone chain from the carbonyl end.

70. Which of the following is an aromatic compound? [4 marks]

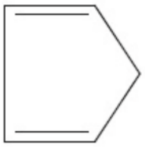
(A) Cyclopentadienyl anion: two double bonds and one negative charge.



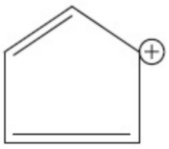
(B) Five-membered ring with one double bond and two negative charges, as drawn.



(C) Cyclopentadiene: two double bonds and a saturated ring carbon.

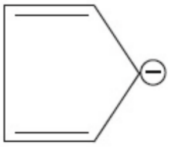


(D) Cyclopentadienyl cation: two double bonds and one positive charge.



Correct Answer: (A)

Cyclopentadienyl anion: two double bonds and one negative charge.



Solution:

Concept:

- An aromatic ring must be cyclic, planar and continuously conjugated, with $4n + 2$ pi electrons for a nonnegative integer n.
- A carbanion lone pair contributes two pi electrons when it occupies the ring p orbital; a carbocation contributes an empty p orbital but no electrons.

Step 1: Count the electrons in the fully conjugated anion Option (A) is the cyclopentadienyl anion.

Its two double bonds contribute four pi electrons. The negatively charged ring carbon supplies a lone pair in a p orbital, adding two more. Every carbon can therefore have a p orbital perpendicular to the planar ring, allowing continuous delocalisation around all five atoms. The count is

$$N_\pi = 4 + 2 = 6 = 4(1) + 2.$$

Both the orbital-continuity condition and the Huckel electron-count condition are fulfilled.

Step 2: Check why the other rings do not qualify In option (B), the ring contains only one double bond and two negatively charged carbons. The remaining saturated carbon interrupts the continuous sequence of p orbitals. A numerical count of six electrons alone cannot make an interrupted ring aromatic. Option (C), neutral cyclopentadiene, likewise contains a saturated methylene carbon and is not continuously conjugated. Option (D) is the cyclopentadienyl cation: the positive carbon provides an empty p orbital, so the two double bonds supply only four pi electrons. A planar fully conjugated four-pi-electron ring would be antiaromatic rather than aromatic. Thus the negative charge in (A) is essential: its electron pair completes the six-electron cyclic system.

Step 3: Conclusion The cyclopentadienyl anion in (A) is aromatic, with six delocalised pi electrons. Hence, the correct option is (A).

Quick Tip to Solve: Check uninterrupted p-orbital overlap first, then count pi electrons; a negative charge contributes two only when its lone pair joins the ring.

71. A central atom in a molecule has two lone pairs of electrons and forms three single bonds. The shape of this molecule is : [4 marks]

(A) trigonal pyramidal

(B) T-shaped

(C) see-saw

(D) planar triangular

Correct Answer: (B) T-shaped

Solution:

Concept:

- VSEPR theory counts electron domains around the central atom: each single bond and each lone pair occupies one domain.
- Five electron domains give a trigonal-bipyramidal electron-pair arrangement. Molecular shape names describe the positions of the bonded atoms, excluding lone pairs.
- Lone pairs repel more strongly than bonding pairs; in a trigonal bipyramid they favour equatorial positions, which have fewer interactions at 90°.

Step 1: Count the electron domains The central atom makes three single bonds and has two lone pairs. Thus its steric number is $3 + 2 = 5$, and its VSEPR formula is AX_3E_2. The five electron domains arrange themselves as a trigonal bipyramid, with three equatorial and two axial sites. We must now remove the lone-pair positions when naming the molecular shape.

Step 2: Place the lone pairs and identify the shape Both lone pairs occupy equatorial sites, leaving one equatorial bond and the two axial bonds. The axial bonds lie opposite each other, while the remaining equatorial bond is approximately perpendicular to them. The three bonded atoms therefore form a T. Trigonal pyramidal would correspond to AX_3E, with four domains; see-saw corresponds to AX_4E, with four bonds. Planar triangular requires three bonding domains and no lone pairs in the simple VSEPR description, so none matches the given count.

Step 3: Conclusion The molecule has a T-shaped geometry, as for ClF_3. Hence, the correct option is (B).

Quick Tip to Solve: Count all electron domains before naming the molecular shape: AX_3E_2 has five domains but only three visible bonds.

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72. Given below are two statements :

Statement I : Retardation factor (R_f) can be measured in meter/centimeter.

Statement II : R_f value of a compound remains constant in all solvents.

Choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is true but statement II is false

(B) Statement I is false but statement II is true

(C) Both statement I and statement II are true

(D) Both statement I and statement II are false

Correct Answer: (D) Both statement I and statement II are false

Solution:

Concept:

- The retardation factor is the ratio of the distance travelled by a solute spot to the distance travelled by the solvent front, each measured from the same origin.
- $$R_f = \frac{\text{distance travelled by solute}}{\text{distance travelled by solvent front}}.$$
  Both distances have the same physical dimension, so their units cancel.
- Chromatographic migration depends on the relative attraction of a solute for the stationary phase and mobile phase; changing the solvent can change that balance.

Step 1: Test the units in Statement I Suppose a spot travels $3\,\text{cm}$ while the solvent front travels $5\,\text{cm}$. Its retardation factor is $3/5 = 0.6$, with no unit. Measuring both distances in metres gives exactly the same ratio. Thus centimetres or metres are units of the distances, not of R_f. Statement I is false.

Step 2: Test the solvent dependence in Statement II If the solvent changes, the solubility of the compound in the mobile phase and its competition for adsorption sites can change. The compound can consequently travel a different fraction of the solvent-front distance. An R_f value is useful for identification only under specified experimental conditions, including the solvent and stationary phase; it is not a constant across all solvents. Statement II is therefore false. Options (A), (B) and (C) each declare at least one false statement true and must be rejected.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (D).

Quick Tip to Solve: For an R_f question, check two things separately: the distance units cancel, and the solvent conditions must be fixed.

73. Which of the following is correct structure of tyrosine? [4 marks]

(A) m-$HOC_6H_4CH_2CH(NH_2)COOH$

(B) p-$HOC_6H_4(CH_2)_3CH(NH_2)COOH$

(C) m-$HOC_6H_4(CH_2)_3CH(NH_2)COOH$

(D) p-$HOC_6H_4CH_2CH(NH_2)COOH$

Correct Answer: (D) p-$HOC_6H_4CH_2CH(NH_2)COOH$

Solution:

Concept:

- Tyrosine is an aromatic α-amino acid. Its α-carbon bears NH_2, H, $COOH$, and a side chain.
- The side chain of tyrosine is $-CH_2-C_6H_4-OH$, with the phenolic hydroxyl group para to the point where the side chain attaches to the ring.
- The identity of the amino acid requires both the correct side-chain length and the correct position of the hydroxyl group on the aromatic ring.

Step 1: Write the connectivity of tyrosine Start with the common amino-acid unit $HOOC-CH(NH_2)^-$. For tyrosine, attach one methylene group and then a phenyl ring bearing a para hydroxyl group. The resulting condensed structure is

$$HOOC-CH(NH_2)-CH_2-C_6H_4-OH \quad \text{(para substitution)}.$$

Thus there must be exactly one CH_2 between the α-carbon and the benzene ring.

Step 2: Compare the four structures Option (D) has one methylene linker and the hydroxyl group opposite the side chain on the ring, so it gives the required para-hydroxyphenylalanine structure. In (A), the side chain is correctly short but the hydroxyl group is meta rather than para. Structures (B) and (C) have a longer carbon chain between the ring and the amino-acid centre; (B) has the para position but the wrong chain length, while (C) also has the wrong hydroxyl position. Neither extra methylene groups nor moving the phenolic group preserves the identity of tyrosine.

Step 3: Conclusion Tyrosine is p-hydroxyphenylalanine. Its one-methylene side chain and para phenolic hydroxyl group match the fourth structure. Hence, the correct option is (D).

Quick Tip to Solve: Identify tyrosine by the pair of features: one CH_2 linker and a phenolic OH para to that linker.

74. Which of the following reaction is an example of ammonolysis? [4 marks]

(A) $C_6H_5COCl + C_6H_5NH_2 \rightarrow C_6H_5CONHC_6H_5$

(B) $C_6H_5CH_2CN \xrightarrow{[H]} C_6H_5CH_2CH_2NH_2$

(C) $C_6H_5CH_2CN \xrightarrow{HCl} C_6H_5\overset{+}{N}H_3Cl^-$

(D) $C_6H_5CH_2Cl + NH_3 \rightarrow C_6H_5CH_2NH_2$

Correct Answer: (D) $C_6H_5CH_2Cl + NH_3 \rightarrow C_6H_5CH_2NH_2$

Solution:

Concept:

- Ammonolysis means cleavage of a bond by ammonia. In preparing amines from alkyl or benzyl halides, ammonia replaces the halogen through nucleophilic substitution.
- Ammonia acts as a nucleophile using its lone pair. The initially formed alkylammonium salt loses a proton to give the amine.
- The characteristic transformation is $\mathrm{R{-}X} \longrightarrow \mathrm{R{-}NH_2}$ using $\mathrm{NH_3}$; reduction of a nitrile and acylation of an amine are different reactions.

Step 1: Identify the reaction with ammonia In option (D), benzyl chloride reacts with ammonia to form benzylamine. The carbon-halogen bond is broken and a carbon-nitrogen bond is formed. Writing the overall stoichiometry with ammonia also accepting the liberated acid gives

$$\mathrm{C_6H_5CH_2Cl + 2\,NH_3 \longrightarrow C_6H_5CH_2NH_2 + NH_4Cl.}$$

The organic transformation displayed in the question is therefore an example of ammonolysis.

Step 2: Distinguish the other transformations Option (A) combines benzoyl chloride with aniline to form an amide; this is acylation of an amine, and ammonia is not the reacting nucleophile. Option (B) uses reducing conditions to convert a nitrile group into an aminoethyl group, so it is nitrile reduction. Option (C) shows treatment with hydrochloric acid and an ammonium chloride product; this is not replacement of halogen by ammonia. The essential reagent-and-bond-change combination for ammonolysis occurs only in (D).

Step 3: Conclusion The conversion of $\mathrm{C_6H_5CH_2Cl}$ into $\mathrm{C_6H_5CH_2NH_2}$ using $\mathrm{NH_3}$ is ammonolysis. Hence, the correct option is (D).

Quick Tip to Solve: Look for ammonia replacing a halogen on carbon; an amine merely appearing as a product does not by itself imply ammonolysis.

75. The INCORRECT statement(s) about heavy water is (are)

(A) used as a moderator in nuclear reactor

(B) obtained as a by-product in fertilizer industry

(C) used for the study of reaction mechanism

(D) has a higher dielectric constant than water

Choose the correct answer from the options given below : [4 marks]

(A) (D) only

(B) (B) and (D) only

(C) (B) only

(D) (C) only

Correct Answer: (A) (D) only

Solution:

Concept:

- Heavy water is D_2O, containing deuterium in place of ordinary hydrogen. Isotopic substitution changes several physical properties while retaining closely related chemistry.
- Heavy water acts as a neutron moderator and is also useful as an isotopic tracer for investigating reaction mechanisms.
- The dielectric constant must be compared at the same temperature: heavy water has a slightly lower dielectric constant than ordinary water.

Step 1: Check the listed applications Statement (A) is correct: heavy water slows fast neutrons in nuclear reactors while absorbing comparatively few of them. Statement (C) is also correct: introducing deuterium and following its position or the associated isotope effect can provide evidence about a reaction pathway. Thus neither use should be selected as an incorrect statement.

Step 2: Check the production statement and physical property Statement (B) is treated as correct in the industrial context of heavy-water production associated with fertilizer manufacture: isotopic enrichment can be linked to hydrogen or ammonia processing in such plants, so heavy water can be obtained as a by-product. Statement (D) is incorrect because its direction of comparison is reversed: D_2O has a lower, not higher, dielectric constant than H_2O at the same temperature. Therefore only statement (D) is false. The alternatives selecting (B), (B) together with (D), or (C) wrongly reject statements that are accepted applications or production facts.

Step 3: Conclusion The dielectric-constant claim is the only incorrect statement. This corresponds to the first answer choice. Hence, the correct option is (A).

Quick Tip to Solve: Separate the statement labels from the option labels: here false statement (D) is selected by answer option (A).

76. With respect to drug-enzyme interaction, identify the wrong statement. [4 marks]

(A) Allosteric inhibitor changes the enzyme's active site

(B) Allosteric inhibitor competes with the enzyme's active site

(C) Competitive inhibitor binds to the enzyme's active site

(D) Non-Competitive inhibitor binds to the allosteric site

Correct Answer: (B) Allosteric inhibitor competes with the enzyme's active site

Solution:

Concept:

- A competitive inhibitor occupies the enzyme active site and competes with the normal substrate for that binding site.
- An allosteric inhibitor binds at a different regulatory site. This binding can change enzyme conformation and hence alter the active site and catalytic activity.
- In the usual textbook description, non-competitive inhibition is associated with binding away from the active site rather than direct competition for its occupancy.

Step 1: Locate the binding sites in the statements Option (C) correctly states that a competitive inhibitor binds to the active site. Its structural similarity to the substrate can allow it to occupy that site, preventing substrate binding. By contrast, an allosteric inhibitor binds to a site distinct from the active site. It need not sit in the substrate pocket to reduce the enzyme reaction rate.

Step 2: Identify the statement with the wrong mechanism Option (A) is correct because allosteric binding can cause a conformational change that changes the active site and weakens substrate binding or catalysis. Option (D) matches the stated non-competitive, allosteric-site model. Option (B), however, assigns active-site competition to an allosteric inhibitor. It confuses two distinct mechanisms: binding elsewhere and changing the active site is not the same as competing for active-site occupancy. Therefore (B) is the wrong statement, while the other three describe the intended enzyme-inhibition mechanisms.

Step 3: Conclusion The claim that an allosteric inhibitor competes with the enzyme's active site is wrong; it binds elsewhere and alters enzyme activity. Hence, the correct option is (B).

Quick Tip to Solve: Ask where the inhibitor binds: the active site indicates competition, while a separate regulatory site indicates an allosteric effect.

77. Reducing smog is a mixture of : [4 marks]

(A) Smoke, fog and O_3

(B) Smoke, fog and N_2O_3

(C) Smoke, fog and $CH_2 = CH - CHO$

(D) Smoke, fog and SO_2

Correct Answer: (D) Smoke, fog and SO_2

Solution:

Concept:

- Classical smog, also called London smog or reducing smog, occurs particularly in cool, humid conditions and contains smoke, fog and sulfur dioxide.
- Photochemical smog forms under sunlight from nitrogen oxides and hydrocarbons; it contains oxidants such as ozone and irritating organic oxidation products.
- The adjective reducing distinguishes the sulfurous classical mixture from the oxidising photochemical mixture.

Step 1: Recognise the reducing-smog components Burning sulfur-containing fuels releases SO_2, while incomplete combustion adds smoke and particulate matter. In moist, foggy air these pollutants produce the classical smog mixture. The required combination is therefore smoke, fog and SO_2, which is exactly the combination in option (D). Its characteristic association is with humid industrial air rather than strong sunlight.

Step 2: Reject the photochemical and unrelated alternatives Option (A) includes ozone, O_3, a strong oxidising constituent of photochemical smog rather than the defining gas of reducing smog. Option (C) includes acrolein, $CH_2{=}CH{-}CHO$, an irritating organic product associated with photochemical pollution. Option (B), involving N_2O_3, does not give the standard sulfur-dioxide-containing classical mixture either. Thus the named type of smog, rather than simply the presence of fog, selects the sulfurous option uniquely.

Step 3: Conclusion Reducing smog is a mixture of smoke, fog and SO_2. Hence, the correct option is (D).

Quick Tip to Solve: Use the contrast: cool, humid London smog contains SO_2; sunny photochemical smog is associated with ozone and organic oxidants.

78. The correct order of conductivity of ions in water is : [4 marks]

(A) $K^+ > Na^+ > Cs^+ > Rb^+$

(B) $Cs^+ > Rb^+ > K^+ > Na^+$

(C) $Na^+ > K^+ > Rb^+ > Cs^+$

(D) $Rb^+ > Na^+ > K^+ > Li^+$

Correct Answer: (B) $Cs^+ > Rb^+ > K^+ > Na^+$

Solution:

Concept:

- Ionic conductivity in water depends on the mobility of the hydrated ion, rather than simply on the radius of the bare ion.
- All the alkali-metal ions listed are singly charged. Their different hydration strengths therefore provide the useful comparison: a strongly hydrated ion drags a larger solvent shell through water.

Step 1: Compare hydration down the group The bare ionic radius increases down Group 1. A smaller ion has greater charge density and attracts water molecules more strongly. Consequently, sodium is more strongly hydrated than potassium, while the larger rubidium and caesium ions have weaker hydration. The effective size relevant to motion in water is the hydrated size; using only the bare radius would incorrectly predict that sodium moves fastest.

Step 2: Apply the aqueous mobility trend The standard alkali-metal trend used here is increasing ionic mobility as hydration decreases down the group. Thus the conductivity order among the ions in the question is

$$Cs^+ > Rb^+ > K^+ > Na^+.$$

Option (A) incorrectly puts potassium above both rubidium and caesium. Option (C) reverses the hydration-based trend and makes sodium the fastest. Option (D) places sodium above potassium, whereas sodium has the larger hydration shell and lower mobility. Only option (B) follows the required aqueous trend throughout. The comparison concerns ions already in solution, not the electrical conductivity of the elemental metals.

Step 3: Conclusion The order is $Cs^+ > Rb^+ > K^+ > Na^+$. Hence, the correct option is (B).

Quick Tip to Solve: For alkali-metal ions in water, compare hydrated size: stronger hydration lowers mobility, so do not rank them using bare ionic size alone.

79. A colloidal system consisting of a gas dispersed in a solid is called a/an : [4 marks]

(A) foam

(B) aerosol

(C) solid sol

(D) gel

Correct Answer: (A) foam

Solution:

Concept:

- A colloid is classified using both the dispersed phase and the dispersion medium. The dispersed phase consists of the distributed particles or bubbles; the medium is the continuous phase.
- Gas dispersed in a liquid or solid belongs to the foam class. When the continuous medium is solid, the more specific name is a solid foam.

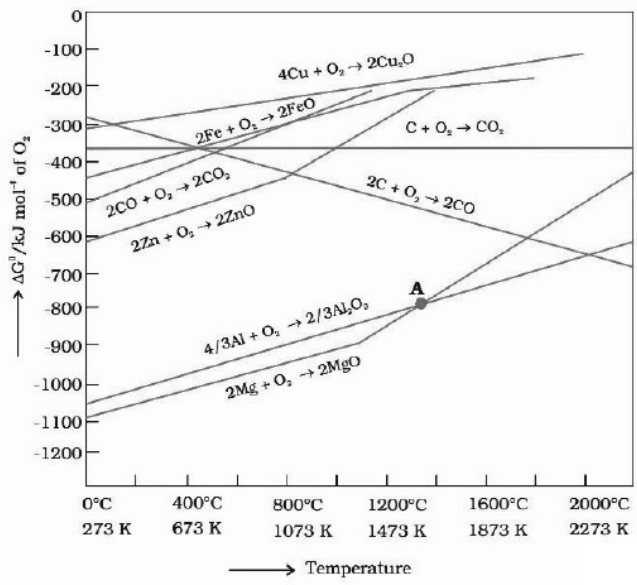
Step 1: Identify the two phases The question explicitly gives gas as the dispersed phase and solid as the dispersion medium. Think of gas bubbles trapped throughout a solid framework, as in pumice stone or foam rubber. The gas is distributed in isolated pockets; it is not the continuous medium. This distinction matters because exchanging the two phases gives a different colloidal class even though the same states of matter are involved.

Step 2: Match the classification and eliminate alternatives Gas dispersed in a solid is a solid foam, which is included under foam in option (A). An aerosol requires a gaseous dispersion medium: examples are liquid droplets in fog or solid particles in smoke. Hence option (B) reverses the role of gas. A solid sol consists of a solid dispersed in another solid, such as a coloured glass, so option (C) does not describe gas bubbles. A gel has a liquid dispersed in a solid medium, such as jelly, and therefore option (D) has the wrong dispersed phase. Classifying by both phases selects foam unambiguously from the alternatives; the solid nature of the medium merely makes its full name solid foam.

Step 3: Conclusion The colloid is a solid foam, belonging to the foam class. The question-sheet key selects solid sol, but a solid sol has a solid dispersed phase, while the printed question specifies gas. The phase classification therefore proves (A). Hence, the correct option is (A).

Quick Tip to Solve: Write dispersed phase first and medium second: gas in solid is solid foam, whereas a gaseous medium signals an aerosol.

80. The point of intersection and sudden increase in the slope, in the diagram given below, respectively, indicates : [4 marks]



(A) $\Delta G > 0$ and decomposition of the metal oxide

(B) $\Delta G < 0$ and decomposition of the metal oxide

(C) $\Delta G = 0$ and reduction of the metal oxide

(D) $\Delta G = 0$ and melting or boiling point of the metal oxide

Correct Answer: (D) $\Delta G = 0$ and melting or boiling point of the metal oxide

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy of oxide formation against temperature. For each reaction, $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$, so the line slope is approximately $-\Delta S^\circ$.
- Two oxide-formation lines at the same ordinate have equal formation Gibbs energies. Their difference, which is the Gibbs energy for the corresponding oxygen-transfer reaction, is then zero. A kink in a line marks a change in physical state and hence in reaction entropy.

Step 1: Interpret the intersection correctly At the intersection the two formation reactions have equal Gibbs energies per mole of oxygen. Subtracting one formation reaction from the other gives the metal–oxide displacement reaction. Its Gibbs energy is the difference between the two ordinates, so at the intersection

$$\Delta G^\circ_{\text{transfer}} = 0.$$

This does not say that the plotted formation ordinate itself is zero: point A is visibly below zero on the vertical scale. Options (A) and (B) therefore give the wrong interpretation of the intersection.

Step 2: Explain the abrupt slope change A sudden slope change reflects a discontinuous change in entropy when a participant melts or boils. Because the slope is minus the reaction entropy, a phase transition changes the slope even while the line remains continuous. It is not by itself evidence of chemical decomposition or reduction. Of the given statements, option (D) identifies the required phase-transition interpretation, described there as the melting or boiling point of the metal oxide.

Option (C) incorrectly treats the kink itself as reduction. The intersection and the kink answer different parts of the question and must be interpreted separately.

Step 3: Conclusion The intersection indicates zero Gibbs energy for the corresponding oxygen-transfer reaction, and the kink indicates a melting or boiling transition. Hence, the correct option is (D).

Quick Tip to Solve: At an intersection subtract the two ordinates; at a kink use slope $= -\Delta S^\circ$ and look for a phase transition.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. A certain orbital has n = 4 and $m_L = -3$. The number of radial nodes in this orbital is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- For an orbital with principal quantum number n, the allowed azimuthal quantum numbers are $l = 0, 1, \ldots, n-1$. For a fixed l, the magnetic quantum number satisfies $-l \le m_l \le l$.
- The number of radial nodes is $n - l - 1$; the number of angular nodes is l. Their sum is $n - 1$, which provides a check on the assignment.

Step 1: Determine the only allowed value of l The given principal quantum number is $n = 4$, so l may be 0, 1, 2 or 3. The given magnetic quantum number is $m_l = -3$. Its magnitude cannot exceed l, so $l \ge 3$. Combining this lower bound with $l \le n - 1 = 3$ leaves just

$$l = 3.$$

This is a $4f$ orbital. The negative sign on m_l distinguishes its orientation; it does not make the azimuthal quantum number negative.

Step 2: Calculate and check the radial-node count Substitute $n = 4$ and $l = 3$ into the radial-node formula:

$$N_{\text{radial}} = n - l - 1 = 4 - 3 - 1 = 0.$$

There are three angular nodes because $l = 3$. Thus the total is $0 + 3 = 3 = n - 1$, confirming that no node has been omitted. Zero radial nodes means that there is no spherical radial surface, away from the origin and infinity, on which this orbital wavefunction vanishes. It does not mean that the orbital has no angular nodes. The count is exact and dimensionless, so rounding to the nearest integer leaves it unchanged.

Step 3: Conclusion The orbital has $\boxed{0}$ radial nodes. The required integer is 0.

Quick Tip to Solve: First use $|m_l| \le l \le n - 1$; only then substitute in $n - l - 1$. Do not substitute the signed magnetic quantum number for l.

82. 15 mL of aqueous solution of Fe^{2+} in acidic medium completely reacted with 20 mL of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the Fe^{2+} solution is _____ $\times 10^{-2}$ M. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- In acidic medium dichromate is reduced to Cr^{3+}, accepting six electrons per $Cr_2O_7^{2-}$ ion. Each Fe^{2+} is oxidised to Fe^{3+}, donating one electron.
- Moles of a dissolved species equal molarity times volume in litres, $n = MV$. The balanced reaction therefore fixes a six-to-one mole ratio of iron(II) to dichromate.

Step 1: Balance the electron transfer The half-reactions in acidic solution are

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O$$

$$Fe^{2+} \longrightarrow Fe^{3+} + e^-.$$

Multiply the iron half-reaction by six before adding. The resulting net equation is

$$Cr_2O_7^{2-} + 14\,H^+ + 6\,Fe^{2+} \longrightarrow 2\,Cr^{3+} + 7\,H_2O + 6\,Fe^{3+}.$$

The acidic medium is essential: it supplies the protons used in the dichromate half-reaction.

Step 2: Find the iron concentration and requested coefficient The dichromate amount in 20 mL is

$$n_{\text{dichromate}} = 0.03 \times 0.020 = 6.0 \times 10^{-4}\ \text{mol}.$$

Consequently the 15 mL iron(II) sample contains

$$n_{\text{iron(II)}} = 6(6.0 \times 10^{-4}) = 3.6 \times 10^{-3}\ \text{mol}.$$

Divide by the iron-solution volume, not the total mixed volume:

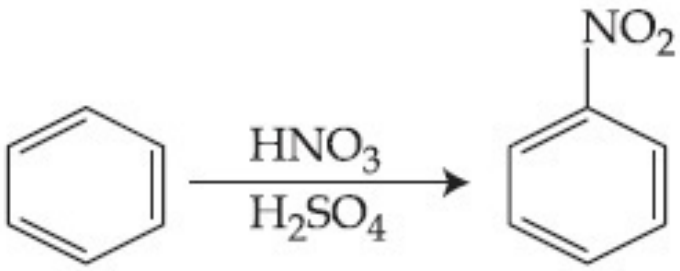
$$M_{\text{iron(II)}} = \frac{3.6 \times 10^{-3}}{0.015} = 0.24\ \text{mol}\,\text{L}^{-1}.$$

Finally $0.24 = 24 \times 10^{-2}$, so the blank asks for 24 rather than 0.24. The coefficient is already an integer.

Step 3: Conclusion The molarity is $\boxed{0.24\ \text{M} = 24 \times 10^{-2}\ \text{M}}$. The required integer is 24.

Quick Tip to Solve: One dichromate ion oxidises six iron(II) ions in acid. After finding molarity, convert it to the power of ten printed beside the blank.

83. In the above reaction, 3.9 g of benzene on nitration gives 4.92 g of nitrobenzene. The percentage yield of nitrobenzene in the above reaction is _____%. (Round off to the Nearest Integer).
(Given atomic mass : C : 12.0 u, H : 1.0 u, O : 16.0 u, N : 14.0 u) [4 marks]



Solution:

Concept:

- Mononitration replaces one hydrogen atom of benzene by a nitro group: $C_6H_6 + HNO_3 \longrightarrow C_6H_5NO_2 + H_2O$. Benzene and nitrobenzene have a one-to-one mole ratio.
- Percentage yield compares actual product mass with the theoretical mass from the limiting reactant: yield $= 100\, m_{\text{actual}}/m_{\text{theoretical}}$. The theoretical mass must be obtained using molar masses, not by directly comparing reactant and product masses.

Step 1: Calculate the molar masses and theoretical product Using the atomic masses provided, the molar masses are

$$M(C_6H_6) = 6(12) + 6(1) = 78\ \text{g mol}^{-1},$$

$$M(C_6H_5NO_2) = 6(12) + 5(1) + 14 + 2(16) = 123\ \text{g mol}^{-1}.$$

The 3.9 g benzene sample therefore contains

$$n(C_6H_6) = \frac{3.9}{78} = 0.050\ \text{mol}.$$

The one-to-one stoichiometry permits 0.050 mol of nitrobenzene at complete conversion. Thus its theoretical mass is

$$m_{\text{theoretical}} = 0.050(123) = 6.15\ \text{g}.$$

Step 2: Compare actual and theoretical yields The actual isolated product is 4.92 g, which is below the theoretical 6.15 g, as a yield below 100 percent requires. Hence

$$\text{percentage yield} = \frac{4.92}{6.15} \times 100 = 80\%.$$

The increase in product molar mass comes from the incoming nitro group, so dividing 4.92 directly by 3.9 would not measure yield. As a separate check, the actual product contains $4.92/123 = 0.040$ mol, and $0.040/0.050 = 0.80$, giving the same result. Rounding does not change this exact integer percentage.

Step 3: Conclusion The percentage yield of nitrobenzene is $\boxed{80\%}$. The required integer is 80.

Quick Tip to Solve: Convert the benzene mass to moles, use the one-to-one reaction ratio, and only then compare actual product with theoretical product.

84. For a certain first order reaction 32% of the reactant is left after 570s. The rate constant of this reaction is _____ $\times 10^{-3}$ s^{-1}. (Round off to the Nearest Integer). [Given : $\log_{10}2 = 0.301$, $\ln 10 = 2.303$] [4 marks]

Solution:

Concept:

- For a first-order reaction, the integrated law is $k = t^{-1}\ln([A]_0/[A]_t)$, where $[A]_0$ is initial concentration and $[A]_t$ is the concentration remaining at time t.
- When the amount remaining is stated as a percentage, the concentration ratio can be formed directly from that percentage. With time in seconds, the rate constant has unit s^{-1}.

Step 1: Use the fraction remaining The statement says that 32 percent is left after 570 s, so

$$\frac{[A]_t}{[A]_0} = \frac{32}{100} = 0.32, \qquad \frac{[A]_0}{[A]_t} = \frac{100}{32} = \frac{25}{8}.$$

The reacted fraction is 68 percent, but this must not replace 32 percent in the integrated law. The law uses reactant remaining, not reactant consumed. Because the remaining fraction is less than one, its reciprocal exceeds one and the logarithm gives a positive rate constant.

Step 2: Evaluate the logarithm and scale the answer Using only the logarithmic constants supplied,

$$\begin{aligned}\log_{10}(100/32) &= 2 - \log_{10}(2^5)\\ &= 2 - 5(0.301) = 0.495.\end{aligned}$$

Convert this common logarithm into a natural logarithm and divide by time:

$$k = \frac{2.303(0.495)}{570} = \frac{1.139985}{570}\ \mathrm{s}^{-1} \approx 1.99997 \times 10^{-3}\ \mathrm{s}^{-1}.$$

The requested blank is the coefficient of $10^{-3}\ \mathrm{s}^{-1}$. Its value is approximately 1.99997, which rounds to 2. A check is $e^{-kt} \approx e^{-1.14} \approx 0.32$, recovering the stated fraction remaining.

Step 3: Conclusion The rate constant is approximately $\boxed{2.00 \times 10^{-3}\ \mathrm{s}^{-1}}$. The required integer is 2.

Quick Tip to Solve: For a first-order decay, put the percentage left in the denominator of the initial-to-final ratio; round the requested coefficient only at the end.

85. 0.01 moles of a weak acid HA ($K_a = 2.0 \times 10^{-6}$) is dissolved in 1.0 L of 0.1 M HCl solution. The degree of dissociation of HA is _____ $\times 10^{-5}$ (Round off to the Nearest Integer).
[Neglect volume change on adding HA. Assume degree of dissociation $<<1$] [4 marks]

Solution:

Concept:

- The acid dissociation constant is $K_a = [\mathrm{H}^+][\mathrm{A}^-]/[\mathrm{HA}]$. Strong hydrochloric acid supplies the common ion H^+, suppressing dissociation of the weak acid.

- If the analytical acid concentration is C and its degree of dissociation is α, then $[\mathrm{A}^-] = C\alpha$ and $[\mathrm{HA}] = C(1-\alpha)$. The degree of dissociation is dimensionless.

Step 1: Write the equilibrium concentrations The volume is unchanged, so $C = 0.01/1.0 = 0.01\,\mathrm{mol\,L^{-1}}$. The fully dissociated $0.1\,\mathrm{M}$ hydrochloric acid already gives $[\mathrm{H}^+] = 0.1\,\mathrm{M}$. Weak-acid dissociation adds 0.01α, giving the exact expression

$$K_a = \frac{(0.1 + 0.01\alpha)(0.01\alpha)}{0.01(1-\alpha)}.$$

Step 2: Apply the small-dissociation approximation The stated condition $\alpha \ll 1$ lets us replace $1-\alpha$ by 1. The extra hydrogen-ion concentration is negligible relative to 0.1 M. Hence

$$2.0 \times 10^{-6} = 0.1\alpha, \qquad \alpha = 2.0 \times 10^{-5}.$$

The added hydrogen-ion concentration is only $2.0 \times 10^{-7}\,\mathrm{M}$, so the approximation is self-consistent. Using $\sqrt{K_a/C}$ would be wrong because that formula omits the strong-acid common ion.

Step 3: Conclusion The degree of dissociation is 2.0×10^{-5} (dimensionless), so the required nearest integer is $\boxed{2}$.

Quick Tip to Solve: With a large common-ion concentration, use $\alpha \simeq K_a/[\mathrm{H}^+]$; the weak-acid concentration cancels.

86. The mole fraction of a solute in a 100 molal aqueous solution is _____ $\times$ 10^{-2}. (Round off to the Nearest Integer).

[Given : Atomic masses : H : 1.0 u, O : 16.0 u] [4 marks]

Solution:

Concept:

- Molality is the number of moles of solute per kilogram of solvent: $m = n_s/(\text{mass of solvent in kg})$. It does not refer to a litre of solution.
- The solute mole fraction is $x_s = n_s/(n_s + n_w)$, where n_w is the amount of water. Mole fraction is dimensionless and lies between zero and one.

Step 1: Choose one kilogram of water For a 100 molal solution, take exactly $1.000\,\mathrm{kg}$ of water. The amount of solute on this basis is $n_s = 100\,\mathrm{mol}$. From the atomic masses, the molar mass of water is

$$M_w = 2(1.0) + 16.0 = 18.0\,\mathrm{g\,mol^{-1}}.$$

Thus the water contains $n_w = 1000/18 = 55.5556\,\mathrm{mol}$. No density or volume assumption is needed.

Step 2: Calculate the fraction and rescale it Include both solute and solvent in the total amount:

$$x_s = \frac{100}{100 + 1000/18} = \frac{1800}{2800} = \frac{9}{14} = 0.642857\ldots.$$

The printed blank multiplies 10^{-2}, so its unrounded value is $0.642857/10^{-2} = 64.2857\ldots$. Round this coefficient only after calculating the mole fraction. Dividing by solvent moles alone would give an amount ratio, not a mole fraction.

Step 3: Conclusion The solute mole fraction is $0.642857\ldots$, so the required nearest-integer coefficient is $\boxed{64}$ in 64×10^{-2}.

Quick Tip to Solve: Use a 1 kg solvent basis for molality; convert that solvent mass to moles before forming the mole fraction.

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87. The standard enthalpies of formation of Al_2O_3 and CaO are -1675 kJ mol^{-1} and -635 kJ mol^{-1} respectively.

For the reaction

3CaO + 2Al $\rightarrow$ 3Ca + Al_2O_3 the standard reaction enthalpy Δ_rH^0 = _____ kJ.

(Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- Hess's law gives $\Delta_r H^\circ = \sum \nu \Delta_f H^\circ(\text{products}) - \sum \nu \Delta_f H^\circ(\text{reactants})$, including the stoichiometric coefficients ν.
- The standard enthalpy of formation of an element in its standard state is zero. Therefore standard calcium metal and aluminium metal contribute zero to the formation-enthalpy sums.

Step 1: Form the product and reactant sums For the reaction $3\,\text{CaO} + 2\,\text{Al} \longrightarrow 3\,\text{Ca} + \text{Al}_2\text{O}_3$, one mole of aluminium oxide occurs among the products and three moles of calcium oxide occur among the reactants. Therefore

$$\sum H_f^\circ(\text{products}) = 3(0) + (-1675) = -1675\,\text{kJ},$$

$$\sum H_f^\circ(\text{reactants}) = 3(-635) + 2(0) = -1905\,\text{kJ}.$$

The coefficients multiply the given molar formation enthalpies.

Step 2: Subtract the reactants from the products The standard enthalpy change for the equation as written is

$$\Delta_r H^\circ = -1675 - (-1905) = +230\,\text{kJ}.$$

Equivalently, decomposing three moles of calcium oxide costs $1905\,\text{kJ}$, while forming one mole of aluminium oxide releases $1675\,\text{kJ}$. The net requirement is $230\,\text{kJ}$, which confirms the positive sign and the endothermic nature of this direction. Reversing the reaction would reverse this sign.

Step 3: Conclusion The standard reaction enthalpy is $\boxed{+230\,\text{kJ}}$; the nearest-integer entry is $\boxed{230}$.

Quick Tip to Solve: Attach stoichiometric coefficients before subtracting formation enthalpies, and retain the minus sign on the reactant sum.

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88. The oxygen dissolved in water exerts a partial pressure of 20 kPa in the vapour above water. The molar solubility of oxygen in water is _____ $\times 10^{-5}$ mol dm^{-3}. (Round off to the Nearest Integer).

[Given : Henry's law constant = $K_H = 8.0 \times 10^4$ kPa for O_2. Density of water with dissolved oxygen = 1.0 kg dm^{-3}] [4 marks]

Solution:

Concept:

- For the given pressure-form Henry constant, Henry's law is $p = K_H x_{O_2}$, where x_{O_2} is a dimensionless mole fraction. This constant does not directly convert pressure into molarity.
- Molar solubility is the amount of dissolved oxygen per unit solution volume. Converting a mole fraction to $\mathrm{mol\,dm^{-3}}$ requires the amount of water, using its molar mass and the supplied density.

Step 1: Find the dissolved oxygen mole fraction The pressure and Henry constant both use kPa, so

$$x_{O_2} = \frac{20}{8.0 \times 10^4} = 2.5 \times 10^{-4}.$$

This is a mole fraction, not yet the requested molar solubility. Its small value confirms that the solution is dilute. In particular, directly writing this value as $\mathrm{mol\,dm^{-3}}$ would omit the solvent-to-volume conversion.

Step 2: Convert to amount per cubic decimetre Take $1\,\mathrm{dm^3}$ of solution. Its mass is 1000 g. If it contains n mol oxygen, its water mass is $1000 - 32n$ g and the water amount is $(1000 - 32n)/18$ mol. Therefore

$$2.5 \times 10^{-4} = \frac{n}{n + (1000 - 32n)/18},$$

$$n = \frac{1000(2.5 \times 10^{-4})}{18 + 14(2.5 \times 10^{-4})} = 0.013886\ldots \text{ mol}.$$

The dilute approximation $n \simeq (2.5 \times 10^{-4})(1000/18)$ gives the same rounded coefficient.

Step 3: Conclusion The molar solubility from the printed data is $0.013886\ldots\ \mathrm{mol\,dm^{-3}} = 1388.6\ldots \times 10^{-5}\,\mathrm{mol\,dm^{-3}}$, so the requested integer is $\boxed{1389}$. The answer-key value 25 corresponds to the mole fraction 2.5×10^{-4}, rather than the molar solubility asked for; it does not include the conversion using water density.

Quick Tip to Solve: Check the Henry constant units: a constant in pressure units gives mole fraction. Multiply by the solvent molarity to obtain dilute-solution molarity.

89. The reaction of white phosphorus on boiling with alkali in inert atmosphere resulted in the formation of product 'A'. The reaction of 1 mol of 'A' with excess of $AgNO_3$ in aqueous medium gives _____ mol(s) of Ag. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- White phosphorus disproportionates in hot alkali: some phosphorus is reduced to phosphine, while the remainder forms hypophosphite. In hypophosphite, $H_2PO_2^-$, phosphorus has oxidation state $+1$.
- Hypophosphite reduces silver ions to silver metal and is oxidised to phosphate in excess oxidant. Phosphorus changes from $+1$ to $+5$, releasing four electrons per phosphorus atom; each silver ion requires one electron.

Step 1: Identify the silver-reducing product The reaction with sodium hydroxide is

$$P_4 + 3\,NaOH + 3\,H_2O \longrightarrow PH_3 + 3\,NaH_2PO_2.$$

The product responsible for depositing silver metal in the stated aqueous reaction is sodium hypophosphite, NaH_2PO_2, denoted A. The alkali cation is a spectator in the redox reaction. Phosphine is the gaseous coproduct; its formation must not cause us to mistake A for a phosphorus oxide.

Step 2: Balance the electron transfer to silver The oxidation half-equation for hypophosphite in acidic bookkeeping is

$$H_2PO_2^- + 2\,H_2O \longrightarrow H_3PO_4 + 3\,H^+ + 4\,e^-.$$

Silver reduction is $4\,Ag^+ + 4\,e^- \longrightarrow 4\,Ag$. Adding them shows that one mole of hypophosphite transfers four moles of electrons and therefore produces four moles of silver. For example, the molecular equation is

$$NaH_2PO_2 + 4\,AgNO_3 + 2\,H_2O \longrightarrow NaH_2PO_4 + 4\,Ag + 4\,HNO_3.$$

Both atoms and net charge balance; excess silver nitrate permits the complete oxidation.

Step 3: Conclusion One mole of A, the hypophosphite product, gives $\boxed{4\,mol}$ of silver. The required integer is $\boxed{4}$.

Quick Tip to Solve: For a reducing phosphorus oxyanion, count the oxidation-state increase of phosphorus; it equals the moles of silver deposited per mole.

90. The pressure exerted by a non-reactive gaseous mixture of 6.4g of methane and 8.8 g of carbon dioxide in a 10L vessel at 27°C is _____ kPa. (Round off to the Nearest Integer).

[Assume gases are ideal, R = 8.314 J mol^{-1} K^{-1}

Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u] [4 marks]

Solution:

Concept:

- For a non-reacting ideal gas mixture, $PV = n_{\text{total}}RT$. The total amount is the sum of the individual gas amounts, as also follows from Dalton's law of partial pressures.
- With $R = 8.314\,\text{J mol}^{-1}\,\text{K}^{-1}$, use volume in cubic metres to obtain pressure in pascals. Absolute temperature is required: $T = t/^\circ\text{C} + 273$ in kelvin for the given data.

Step 1: Convert the masses to mole amounts The given atomic masses give

$$M_{\text{CH}_4} = 12 + 4(1) = 16\,\text{g mol}^{-1}, \qquad M_{\text{CO}_2} = 12 + 2(16) = 44\,\text{g mol}^{-1}.$$

Hence $n_{\text{CH}_4} = 6.4/16 = 0.4\,\text{mol}$ and $n_{\text{CO}_2} = 8.8/44 = 0.2\,\text{mol}$. Since the gases do not react, neither amount changes on mixing, and $n_{\text{total}} = 0.6\,\text{mol}$. Adding the masses and using either one of the molar masses would be incorrect.

Step 2: Apply the gas equation in consistent units The temperature is $27 + 273 = 300\,\text{K}$, and $10\,\text{L} = 0.010\,\text{m}^3$. Substitution gives

$$P = \frac{0.6(8.314)(300)}{0.010} = 149652\,\text{Pa} = 149.652\,\text{kPa}.$$

The individual partial pressures would be $99.768\,\text{kPa}$ for methane and $49.884\,\text{kPa}$ for carbon dioxide, and their sum verifies the total. Round only after converting the total pressure to the requested kPa.

Step 3: Conclusion The mixture pressure is $149.652\,\text{kPa}$, which rounds to $\boxed{150\,\text{kPa}}$. The numerical entry is $\boxed{150}$.

Quick Tip to Solve: Add moles for an ideal gas mixture; using R in joules requires volume in cubic metres before converting Pa to kPa.