JEE Main 2021 Paper 1 (B.E./B.Tech) Mar 17 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 2 (March), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 17 March 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to : [4 marks]

(A) 240

(B) 360

(C) 333

(D) 364

Correct Answer: (C) 333

Solution:

Concept:

- Any three distinct non-collinear points determine exactly one triangle. Thus we count all unordered triples first and remove precisely the collinear triples.
- The selected points lie strictly inside the sides, so no vertex of the original triangle is included. A straight line other than a side can meet the boundary of a triangle in at most two distinct points.

Step 1: Count all triples The total number of available points is $3 + 5 + 6 = 14$. Since a triangle does not depend on the order of its vertices, use combinations rather than permutations:

$$N_{\text{all}} = \binom{14}{3} = \frac{14 \cdot 13 \cdot 12}{6} = 364.$$

This count includes triples lying on the same side, which have zero area and therefore do not form triangles.

Step 2: Remove the degenerate triples The only possible collinear triples are those on one side of the triangle. Their numbers on AB, BC, CA, respectively, are

$$\binom{3}{3} = 1, \qquad \binom{5}{3} = 10, \qquad \binom{6}{3} = 20.$$

These three sets of triples are disjoint because the points are interior to their respective sides. Hence

$$N = 364 - (1 + 10 + 20) = 333.$$

Choosing one point from each side alone would omit valid triangles with two vertices on one side. Option (D) counts degenerate triples as well; the other numerical alternatives do not result from the required subtraction.

Step 3: Conclusion The total number of triangles is 333. Hence, the correct option is (C).

Quick Tip to Solve: For points on the sides of a convex triangle, subtract same-side triples from the combination of all points; include triangles with two points on a side.

2. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to : [4 marks]

(A) $\frac{1}{18}$
(B) $\frac{1}{3}$
(C) $\frac{1}{9}$
(D) $\frac{1}{6}$

Correct Answer: (A) $\frac{1}{18}$

Solution:

Concept:

- At an odd place, $P(0) = 1/3$ and $P(1) = 2/3$; at an even place both digits have probability $1/2$.
- For the independent digit generation in the question, the probability of a prescribed consecutive pattern is the product of its position-wise digit probabilities.

Step 1: Translate the requested pattern The event that the two-digit block 10 is followed by the block 01 means the consecutive four-digit pattern 1001. Suppose its first digit occupies an odd place. The four positions then have parities odd, even, odd and even, in that order. The required digits at these places are $1, 0, 0, 1$. We must include the probabilities of all four digits, rather than only the second two-digit block.

Step 2: Multiply the four probabilities For an odd starting position,

$$P(1001) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{18}.$$

For an even starting position the parities reverse, giving

$$P(1001) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{18}.$$

Thus the unknown starting parity does not affect the result. The values $1/3, 1/9, 1/6$ fail to include the correct four-factor product; for example, $1/6$ is the probability of just one of the relevant two-digit blocks, rather than their specified concatenation. In particular, adding the two conditional probabilities for odd and even starting positions without weighting by the probabilities of those parities would double-count the alternatives. For either parity distribution $p, 1 - p$, the total is $p/18 + (1 - p)/18 = 1/18$.

Step 3: Conclusion The four-digit pattern has probability $1/18$ under independent generation. This differs from the supplied key value $1/9$: the latter adds the odd-start and even-start probabilities without their parity weights, whereas each possible starting parity gives $1/18$. Hence, the correct option is (A).

Quick Tip to Solve: Write the complete digit string before multiplying probabilities, and check both starting parities when the digit law alternates with position.

3. Let f : R → R be defined as f(x) = e^{-x}sinx. If F : [0, 1] → R is a differentiable function with that F(x) = $\int_0^x f(t)dt$, then the value of $\int_0^1 (F'(x) + f(x))e^x dx$ lies in the interval [4 marks]

(A) $\left[\frac{331}{360}, \frac{334}{360}\right]$
(B) $\left[\frac{330}{360}, \frac{331}{360}\right]$
(C) $\left[\frac{335}{360}, \frac{336}{360}\right]$
(D) $\left[\frac{327}{360}, \frac{329}{360}\right]$

Correct Answer: (B) $\left[\frac{330}{360}, \frac{331}{360}\right]$

Solution:

Concept:

- For continuous f, the fundamental theorem gives $F\prime(x) = f(x)$ when $F(x) = \int_0^x f(t)\,dt$.
- The alternating expansion of $\cos 1$ gives strict upper and lower bounds by truncating after successive terms. Angles in these calculus formulas are in radians.

Step 1: Simplify the integrand Here $f(x) = e^{-x}\sin x$ is continuous, so the two terms in the parentheses are equal:

$$\begin{aligned} I &= \int_0^1 (F\prime(x) + f(x))e^x\,dx \\ &= 2\int_0^1 e^{-x}\sin x\, e^x\,dx \\ &= 2\int_0^1 \sin x\,dx = 2(1-\cos 1). \end{aligned}$$

The exponential factors cancel exactly; no numerical quadrature or separate evaluation of F is necessary.

Step 2: Locate the value using exact bounds The cosine series has decreasing alternating terms at 1. Hence

$$1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} < \cos 1 < 1 - \frac{1}{2} + \frac{1}{24}.$$

Subtracting these bounds from 1 and multiplying by 2 reverses their order:

$$\frac{330}{360} < I < \frac{331}{360}.$$

For reference, $I \approx 0.919395$. The entire result lies in option (B). Option (A) begins at the strict upper bound $331/360$; option (C) is larger still, and option (D) ends below the lower bound $330/360$.

Step 3: Conclusion The integral lies in $[330/360, 331/360]$. Hence, the correct option is (B).

Quick Tip to Solve: Use $F\prime = f$ first: the integral collapses to a simple trigonometric integral, and two cosine-series bounds identify its narrow interval.

4. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in C : |z - 1| \leq \sqrt{2}\}$

$S_2 = \{z \in C : Re((1 - i)z) \geq 1\}$

$S_3 = \{z \in C : Im(z) \leq 1\}$

Then the set $S_1 \cap S_2 \cap S_3$: [4 marks]

(A) has exactly three elements

(B) is a singleton

(C) has infinitely many elements

(D) has exactly two elements

Correct Answer: (C) has infinitely many elements

Solution:

Concept:

- Write $z = x + iy$, where x, y are real. The inequality $|z - 1| \leq \sqrt{2}$ describes the closed disk $(x - 1)^2 + y^2 \leq 2$.
- To prove that an intersection is infinite, it suffices to exhibit a whole nontrivial line segment inside it; finding all boundary intersections is unnecessary.

Step 1: Express the two half-plane conditions Multiply the complex numbers explicitly:

$$(1 - i)(x + iy) = (x + y) + i(y - x).$$

Therefore $\mathrm{Re}((1-i)z) = x+y$, and the second set is $x+y \geq 1$. Also $\mathrm{Im}\, z = y$, so the third set requires $y \leq 1$. We thus need simultaneous satisfaction of

$$(x - 1)^2 + y^2 \leq 2, \qquad x + y \geq 1, \qquad y \leq 1.$$

The inequalities include their boundaries.

Step 2: Exhibit infinitely many common elements Take $y = 0$ and let x range through the real interval $[1, 2]$. Then

$$(x - 1)^2 \leq 1 < 2, \qquad x + 0 \geq 1, \qquad 0 \leq 1.$$

Thus every real complex number $z = x$, for $1 \leq x \leq 2$, belongs to all three sets. This interval contains infinitely many distinct real numbers, and hence supplies infinitely many elements of the intersection. Exhibiting just a few common boundary points would not determine the size of the set. In particular the intersection cannot have exactly one, two or three elements, ruling out each of the finite alternatives.

Step 3: Conclusion The set $S_1 \cap S_2 \cap S_3$ has infinitely many elements. Hence, the correct option is (C).

Quick Tip to Solve: For a disk intersected with half-planes, test a horizontal or vertical segment; a single surviving interval proves infinitude without a sketch.

5. The number of solutions of the equation

$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$**, for x∈[−1, 1], and [x] denotes the greatest integer less than or equal to x, is :** [4 marks]

(A) 0

(B) Infinite

(C) 2

(D) 4

Correct Answer: (A) 0

Solution:

Concept:

- The square brackets denote the floor function, not merely grouping: $[u]=\lfloor u\rfloor$. With $x\in[-1,1]$, put $t=x^2\in[0,1]$.
- Use principal inverse-trigonometric values: $\sin^{-1}0=0$, $\sin^{-1}1=\pi/2$, $\cos^{-1}(-1)=\pi$, and $\cos^{-1}0=\pi/2$.

Step 1: Evaluate the two floor functions The only change in the floors occurs at $t=2/3$. For $0\le t<2/3$,

$$0\le t+\frac{1}{3}<1,\qquad -1<t-\frac{2}{3}<0,$$

so the floors are 0 and -1. For $2/3\le t\le 1$,

$$1\le t+\frac{1}{3}\le\frac{4}{3},\qquad 0\le t-\frac{2}{3}\le\frac{1}{3},$$

so the floors are 1 and 0. The second case includes the threshold exactly, so no boundary solution is missed.

Step 2: Compare with the required right-hand side In the first case the left side is

$$\sin^{-1}0+\cos^{-1}(-1)=0+\pi=\pi.$$

In the second case it is

$$\sin^{-1}1+\cos^{-1}0=\frac{\pi}{2}+\frac{\pi}{2}=\pi.$$

Thus throughout the prescribed interval the equation reduces to $t=\pi$. But $t=x^2\le 1<\pi$, which is impossible. This excludes the endpoints as well as the interior. The symmetry $x\leftrightarrow -x$ creates no solutions, so neither two nor four solutions exist; a constant left side also does not imply infinitely many solutions when it never equals the right side.

Step 3: Conclusion There are 0 real solutions in $[-1,1]$. Hence, the correct option is (A).

Quick Tip to Solve: When floors involve x^2, set $t=x^2$ and partition its bounded range at the floor thresholds before applying inverse-trigonometric values.

6. If the curve y = y(x) is the solution of the differential equation

$2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}dx$, **x > 0 which passes through the point** $(1, 1 - \frac{4}{3}\log_e 2)$**, then the value of y(16) is equal to :** [4 marks]

(A) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

(B) $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

(C) $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

(D) $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

Correct Answer: (A) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)dx}$. Here $x > 0$, so fractional powers and $\log x$ are well defined.
- After multiplication by the integrating factor, the left-hand side becomes a product derivative; the given point then determines the integration constant.

Step 1: Put the equation in linear form Divide by $2(x^2+x^{5/4})$, noting that $(x+x^{1/4})/(x^2+x^{5/4}) = 1/x$:

$$y' - \frac{y}{2x} = \frac{x}{1 + x^{3/4}}.$$

The integrating factor is $x^{-1/2}$, giving

$$\frac{d}{dx}(yx^{-1/2}) = \frac{x^{1/2}}{1 + x^{3/4}}.$$

Set $u = x^{1/4}$. Then $dx = 4u^3du$ and the integral becomes

$$\int \frac{4u^5}{1 + u^3}\, du = \frac{4}{3}u^3 - \frac{4}{3}\log(1 + u^3) + C.$$

Here we used $u^5/(1 + u^3) = u^2 - u^2/(1 + u^3)$.

Step 2: Use the initial point and evaluate The resulting solution is

$$\frac{y}{\sqrt{x}} = \frac{4}{3}x^{3/4} - \frac{4}{3}\log(1 + x^{3/4}) + C.$$

At $x = 1$, the given ordinate yields $1 - \frac{4}{3}\log 2 = \frac{4}{3} - \frac{4}{3}\log 2 + C$, hence $C = -1/3$. At $x = 16$, $\sqrt{x} = 4$ and $x^{3/4} = 8$, so

$$y(16) = 4\left(\frac{32}{3} - \frac{4}{3}\log 9 - \frac{1}{3}\right) = 4\left(\frac{31}{3} - \frac{8}{3}\log 3\right).$$

Option (B) omits the outer factor $\sqrt{16}$; options (C) and (D) have the wrong logarithmic sign.

Step 3: Conclusion We obtain $y(16) = 4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$. Hence, the correct option is (A).

Quick Tip to Solve: Cancel the common $1 + x^{3/4}$ factor in the coefficient of y first, then substitute $u = x^{1/4}$ only in the integrating-factor integral.

7. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, x, y∈R, x > 0, be such that $\left|\overrightarrow{PQ}\right| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, z∈R, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to : [4 marks]

(A) 2

(B) 9

(C) 7

(D) 1

Correct Answer: (B) 9

Solution:

Concept:

- Perpendicular vectors have zero dot product. Their squared difference norm is then the sum of their squared norms: $|\mathbf{b} - \mathbf{a}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2$.
- Three position vectors are coplanar precisely when their scalar triple product is zero: $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 0$.

Step 1: Determine the first two coordinates Let $\mathbf{a} = (x, y, -1)$ and $\mathbf{b} = (-1, 2, 3x)$. Orthogonality gives

$$\mathbf{a} \cdot \mathbf{b} = -x + 2y - 3x = 0 \quad \Rightarrow \quad y = 2x.$$

Using the stated distance and the zero dot product,

$$\begin{aligned} 20 &= |\mathbf{b} - \mathbf{a}|^2 \\ &= (x^2 + y^2 + 1) + (1 + 4 + 9x^2) \\ &= 10x^2 + y^2 + 6 = 14x^2 + 6. \end{aligned}$$

Thus $x^2 = 1$. Since $x > 0$, select $x = 1$ and hence $y = 2$, rather than retaining the negative root.

Step 2: Apply coplanarity to find the remaining coordinate The now known vectors are $\mathbf{a} = (1, 2, -1)$ and $\mathbf{b} = (-1, 2, 3)$. Their cross product is

$$\mathbf{a} \times \mathbf{b} = (8, -2, 4).$$

For $\mathbf{c} = (3, z, -7)$, coplanarity therefore requires

$$8(3) - 2z + 4(-7) = 0, \qquad -4 - 2z = 0,$$

giving $z = -2$. Finally

$$x^2 + y^2 + z^2 = 1 + 4 + 4 = 9.$$

The alternatives $2, 7, 1$ do not equal this sum. The negative sign of z disappears on squaring, but the coplanarity equation must still be used to establish its magnitude.

Step 3: Conclusion The requested dimensionless sum is 9. Hence, the correct option is (B).

Quick Tip to Solve: Exploit perpendicularity in the distance formula to avoid expanding coordinate differences, then use a cross product as the normal to the coplanar plane.

8. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where α, β, γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of α + β + γ is equal to : [4 marks]

(A) 0

(B) 10

(C) 20

(D) 25

Correct Answer: (A) 0

Solution:

Concept:

- The greatest integer function is constant between its jump points. In particular, $[\sin(2\pi t)] = 0$ for $0 < t < 1/2$, and it equals -1 for $1/2 < t < 1$, apart from isolated exceptional points.
- Changing the value of a bounded integrand at finitely many points does not change its definite integral. The fractional part $x - [x]$ repeats with period one.

Step 1: Reduce the integral to one unit interval Write $x = k + t$, where $k = 0, 1, \ldots, 9$ and $0 < t < 1$. Then $x - [x] = t$ and $\sin(2\pi x) = \sin(2\pi t)$. All ten intervals therefore give the same contribution. Hence

$$I = 10 \int_0^1 [\sin(2\pi t)] e^{-t}\, dt.$$

The isolated points where the sine is zero or is exactly one do not affect the integral.

Step 2: Integrate on the two half intervals On the first half, the sine lies between zero and one, so its floor is zero almost everywhere. On the second half, it lies between minus one and zero, so its floor is minus one. Therefore

$$\begin{aligned} I &= -10 \int_{1/2}^1 e^{-t}\, dt \\ &= 10 \left[e^{-t}\right]_{1/2}^1 \\ &= 10e^{-1} - 10e^{-1/2}. \end{aligned}$$

Comparing the coefficients gives $\alpha = 10$, $\beta = -10$, and $\gamma = 0$. The alternatives 10, 20 and 25 fail because they do not include the cancellation between the two exponential coefficients.

Step 3: Conclusion Thus $\alpha + \beta + \gamma = 10 - 10 + 0 = 0$. Hence, the correct option is (A).

Quick Tip to Solve: First replace the floor of the sine by its piecewise constant values; then use the period of the fractional part to avoid ten separate integrations.

9. The value of the limit

$\lim\limits_{\theta\to 0}\frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ **is equal to :** [4 marks]

(A) 0

(B) $-\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $-\frac{1}{4}$

Correct Answer: (B) $-\frac{1}{2}$

Solution:

Concept:

- Use $\cos^2\theta = 1-\sin^2\theta$ and the period π of the tangent to move the numerator angle near zero.
- The double-angle identity $\sin(2u) = 2\sin u\cos u$ gives an exact simplification before taking the limit. Angles are measured in radians.

Step 1: Introduce an angle tending to zero Put $u = \pi\sin^2\theta$. As $\theta\to 0$, we have $u\to 0$, while $\pi\cos^2\theta = \pi - u$. Consequently,

$$\tan(\pi\cos^2\theta) = \tan(\pi-u) = -\tan u, \qquad \sin(2\pi\sin^2\theta) = \sin(2u).$$

The negative sign follows from the tangent in the second quadrant and must be retained. For sufficiently small nonzero θ, both the original ratio and the following cancellation are defined.

Step 2: Simplify exactly and evaluate Using the sine double-angle formula,

$$\frac{-\tan u}{\sin(2u)} = -\frac{\sin u/\cos u}{2\sin u\cos u} = -\frac{1}{2\cos^2 u}.$$

Since $\cos u\to 1$, the required value is $-1/2$. The limit is not zero because both numerator and denominator vanish to the same order. A factor of two, rather than four, comes from the double angle; this rules out both quarter-valued alternatives. In addition, the positive quarter has the wrong sign: near zero the denominator is positive and the numerator is negative.

Step 3: Conclusion The given limit is $-\frac{1}{2}$, a dimensionless value. Hence, the correct option is (B).

Quick Tip to Solve: For tangent angles approaching a multiple of π, first subtract that multiple. Here the identity $\tan(\pi-u) = -\tan u$ makes the sign immediate.

10. Consider the function f : R → R defined by

$f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x|, x \neq 0 \\ 0, \ x = 0 \end{cases}$ **. Then f is :** [4 marks]

(A) not monotonic on $(-\infty, 0)$ and $(0, \infty)$

(B) monotonic on $(0, \infty)$ only

(C) monotonic on $(-\infty, 0)$ only

(D) monotonic on $(-\infty, 0) \cup (0, \infty)$

Correct Answer: (A) not monotonic on $(-\infty, 0)$ and $(0, \infty)$

Solution:

Concept:

- On each side of zero the absolute value has a fixed expression: $|x| = x$ for $x > 0$, and $|x| = -x$ for $x < 0$. Differentiate the two expressions separately.
- A differentiable monotone increasing function cannot have a negative derivative, and a differentiable monotone decreasing function cannot have a positive derivative. Exhibiting both signs on an interval rules out monotonicity.

Step 1: Check the positive half-line For $x > 0$, differentiation by the product and chain rules gives

$$f(x) = 2x - x\sin(1/x), \qquad f\prime(x) = 2 - \sin(1/x) + \frac{\cos(1/x)}{x}.$$

At $x = 1/(2n\pi)$, with $n \geq 1$, this derivative equals $2 + 2n\pi > 0$. At $x = 1/((2n+1)\pi)$, it equals $2 - (2n+1)\pi < 0$. Both sets of points lie in $(0, \infty)$, so the function is neither increasing nor decreasing throughout that half-line.

Step 2: Check the negative half-line For $x < 0$, use the other expression of the absolute value:

$$f(x) = -2x + x\sin(1/x), \qquad f\prime(x) = -2 + \sin(1/x) - \frac{\cos(1/x)}{x}.$$

At $x = -1/(2n\pi)$, the derivative is $-2 + 2n\pi > 0$. At $x = -1/((2n+1)\pi)$, it is $-2 - (2n+1)\pi < 0$. Thus monotonicity also fails on $(-\infty, 0)$. Options (B) and (C) each incorrectly claim monotonicity on one half-line, while (D) incorrectly claims it on their union.

Step 3: Conclusion The function is not monotonic on either $(-\infty, 0)$ or $(0, \infty)$. Hence, the correct option is (A).

Quick Tip to Solve: For oscillations containing $1/x$, test derivative signs at reciprocal multiples of π; the sine vanishes and the cosine becomes exactly 1 or -1.

11. The value of $\lim_{n\to\infty} \frac{[r]+[2r]+\ldots+[nr]}{n^2}$, where r is a non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to : [4 marks]

(A) r

(B) $\frac{r}{2}$

(C) 0

(D) 2r

Correct Answer: (B) $\frac{r}{2}$

Solution:

Concept:

- For every real t, including negative values, $t-1<[t]\leq t$. Thus the error made by replacing a floor with its argument has magnitude less than one.
- The sum of the first n positive integers is $n(n+1)/2$. A total error bounded by n disappears after division by n^2.

Step 1: Bound every summand Apply the floor inequality with $t=kr$, for each integer k from 1 to n:

$$kr-1<[kr]\leq kr.$$

Adding all these inequalities preserves their direction, regardless of the sign of r, and gives

$$\frac{rn(n+1)}{2}-n<\sum_{k=1}^{n}[kr]\leq\frac{rn(n+1)}{2}.$$

This avoids the incorrect assumption that the floor of a negative number is obtained by simply discarding its decimal part.

Step 2: Divide by the dominant power of n Since $n^2>0$, divide the bounds to obtain

$$\frac{r}{2}\left(1+\frac{1}{n}\right)-\frac{1}{n}<\frac{\sum_{k=1}^{n}[kr]}{n^2}\leq\frac{r}{2}\left(1+\frac{1}{n}\right).$$

Both outer expressions approach $r/2$. By the squeeze theorem, the expression between them has the same limit. No restriction that r be positive or integral is needed. The factor one-half arises from the arithmetic sum, so r and $2r$ have the wrong leading coefficient. The answer cannot be zero because the problem specifies $r\neq 0$.

Step 3: Conclusion The required limit is $\frac{r}{2}$. Hence, the correct option is (B).

Quick Tip to Solve: In a floor sum divided by n^2, bound each floor error by one: the combined error is only $O(n)$, so the ordinary sum determines the limit.

12. If x, y, z are in arithmetic progression with common difference d, x $\neq$ 3d, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is : [4 marks]

(A) 72

(B) 12

(C) 36

(D) 6

Correct Answer: (A) 72

Solution:

Concept:

- For an arithmetic progression with first term x and common difference d, the next two terms are $y = x + d$ and $z = x + 2d$. Hence $z - 2y + x = 0$.
- Adding multiples of other rows to a row leaves a determinant unchanged. A row operation adapted to the arithmetic progression produces two zeros and makes cofactor expansion immediate.

Step 1: Eliminate the first and third entries of the last row Let the determinant be D. Apply $R_3 \leftarrow R_3 - 2R_2 + R_1$. Its first entry becomes $5 - 8 + 3 = 0$; its last entry becomes $z - 2y + x = 0$. Therefore

$$D = \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & x+d \\ 0 & k-6\sqrt{2} & 0 \end{vmatrix}.$$

Only the middle entry in the last row can contribute to the expansion. This operation uses the given progression rather than introducing unnecessary unknowns.

Step 2: Expand and use the nonzero factor The cofactor sign at position $(3, 2)$ is negative, so

$$\begin{aligned} D &= -(k - 6\sqrt{2}) \begin{vmatrix} 3 & x \\ 4 & x+d \end{vmatrix} \\ &= -(k - 6\sqrt{2})\{3(x + d) - 4x\} \\ &= (k - 6\sqrt{2})(x - 3d). \end{aligned}$$

The problem gives $D = 0$ and $x \neq 3d$, so the second factor cannot vanish. Thus $k = 6\sqrt{2}$, and squaring gives $k^2 = 36 \times 2 = 72$. The alternatives 12, 36 and 6 do not square the entire value $6\sqrt{2}$ correctly; the determinant requires its exact unsquared value first.

Step 3: Conclusion The required value is $k^2 = 72$. Hence, the correct option is (A).

Quick Tip to Solve: Whenever three rows contain arithmetic progressions, try $R_3 - 2R_2 + R_1$; the second difference vanishes and can reduce a three-by-three determinant to one factor.

13. Let y = y(x) be the solution of the differential equation

$\cos x(3\sin x+\cos x+3)dy=(1+y\sin x(3\sin x+\cos x+3))dx, 0\le x\le\frac{\pi}{2}, y(0)=0$. **Then, $y\left(\frac{\pi}{3}\right)$ is equal to :** [4 marks]

(A) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$

(B) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

(C) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

(D) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$

Correct Answer: (C) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

Solution:

Concept:

- A linear equation $y\prime+P(x)y=Q(x)$ is solved by multiplying by its integrating factor $e^{\int P(x)\,dx}$. Here the factor is $\cos x$ on the interval needed to reach $\pi/3$.
- The tangent half-angle substitution $t=\tan(x/2)$ gives $\sin x=2t/(1+t^2)$, $\cos x=(1-t^2)/(1+t^2)$ and $dx=2\,dt/(1+t^2)$.

Step 1: Put the equation in linear form On $0\le x<\pi/2$, divide by $\cos x(3\sin x+\cos x+3)$:

$$y\prime-y\tan x=\frac{\sec x}{3\sin x+\cos x+3}.$$

The integrating factor is $\cos x$, giving

$$\frac{d}{dx}(y\cos x)=\frac{1}{3+3\sin x+\cos x}.$$

The initial condition $y(0)=0$ therefore determines the integral from zero to x, with no arbitrary constant remaining.

Step 2: Integrate using the half-angle variable Put $t=\tan(x/2)$. The denominator becomes $2(t+1)(t+2)/(1+t^2)$. Consequently,

$$\begin{aligned}y\cos x&=\int_0^t\frac{du}{(u+1)(u+2)}\\&=\int_0^t\left(\frac{1}{u+1}-\frac{1}{u+2}\right)du\\&=\log\left(\frac{2(t+1)}{t+2}\right).\end{aligned}$$

All logarithm arguments here are positive, since $t\ge0$.

Step 3: Evaluate at the requested point At $x = \pi/3$, we have $t = 1/\sqrt{3}$ and $\cos x = 1/2$. Thus

$$y(\pi/3) = 2\log\left(\frac{2(\sqrt{3}+1)}{2\sqrt{3}+1}\right)$$
$$= 2\log\left(\frac{10+2\sqrt{3}}{11}\right).$$

This is exactly option (C). Option (B) has a negative logarithm argument and is not real. The arguments in (A) and (D) are different from the exact positive ratio obtained above, so neither satisfies this initial-value calculation.

Step 4: Conclusion Thus $y(\pi/3) = 2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: After applying the integrating factor, use $t = \tan(x/2)$ when the denominator is a constant plus both sine and cosine; the resulting rational expression often factors.

14. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to : [4 marks]

(A) 20

(B) 14

(C) 16

(D) 11

Correct Answer: (B) 14

Solution:

Concept:

- For the parabola $y^2 = 4(x-5)$, implicit differentiation gives $2yy' = 4$. The tangent slope at a point is therefore $2/y$.
- A line $y = mx + c$ is tangent to $x^2/a^2 + y^2/b = 1$ precisely when $c^2 = a^2m^2 + b$. Equivalently, substitution of the line into the ellipse gives a quadratic with zero discriminant.

Step 1: Find the tangent to the parabola At the stated point $(6, 2)$, the slope is $m = 2/2 = 1$. The point also lies on the parabola because $2^2 = 4(6) - 20 = 4$. Its tangent is thus

$$y - 2 = x - 6, \qquad y = x - 4.$$

This fixes both slope and intercept of the line L; it is not enough merely to impose that an arbitrary slope-one line touch the ellipse.

Step 2: Impose tangency to the ellipse Substitute $y = x - 4$ into $x^2/2 + y^2/b = 1$, with $b > 0$ for an ellipse. Multiplying by $2b$ gives

$$(b+2)x^2 - 16x + 32 - 2b = 0.$$

Tangency means a repeated intersection, so its discriminant vanishes:

$$256 - 4(b+2)(32-2b) = 0$$
$$64 - (b+2)(32-2b) = 0$$
$$2b(b-14) = 0.$$

The root $b = 0$ is inadmissible because the original ellipse contains division by b. Hence $b = 14$. The tangent formula also confirms $16 = 2 + 14$. Values 20, 16 and 11 fail this exact tangency condition.

Step 3: Conclusion The ellipse parameter is $b = 14$. Hence, the correct option is (B).

Quick Tip to Solve: Once the parabola fixes the line $y = mx + c$, use $c^2 = a^2m^2 + b$ to test ellipse tangency directly; check that the resulting axis parameter is positive.

---

15. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is : [4 marks]

(A) 3 : 1

(B) 9 : 4

(C) 2 : 1

(D) 11 : 4

Correct Answer: (B) 9 : 4

Solution:

Concept:

- The radius to a point of tangency is perpendicular to the tangent. Equal tangents from an external point imply that the line joining that point to the centre bisects the angle between them.
- Triangles with the same base have areas in the ratio of their perpendicular heights. If $M = AB \cap PC$, the required ratio is $PM : CM$.

Step 1: Find the circle and the half-angle Complete the squares to obtain $(x-1)^2 + (y-2)^2 = 1$. Thus the centre is $C = (1, 2)$ and the radius is $r = 1$. Write $\theta = \angle APB$. Since its tangent is positive and $0 < \theta < \pi$, the angle is acute. Therefore $\cos\theta = 5/13$, rather than its negative value. The half-angle formula gives

$$\sin^2(\theta/2) = \frac{1 - 5/13}{2} = \frac{4}{13}.$$

Step 2: Compute the perpendicular heights The right triangle CAP gives $CP = r/\sin(\theta/2) = \sqrt{13}/2$. The chord of contact AB is perpendicular to CP; the right triangles also give $CM \cdot CP = r^2$. Consequently

$$CM = \frac{2}{\sqrt{13}}, \qquad PM = CP - CM = \frac{9}{2\sqrt{13}}.$$

The point M is between C and P, so these are positive perpendicular distances to the common base

AB.

Step 3: Compare the two areas The common factor $AB/2$ cancels when the triangle areas are divided:

$$\frac{[PAB]}{[CAB]} = \frac{PM}{CM} = \frac{9/(2\sqrt{13})}{2/\sqrt{13}} = \frac{9}{4}.$$

Thus neither 3, 2, nor $11/4$ gives the required height ratio; those values cannot represent the area quotient.

Step 4: Conclusion The ratio is $9 : 4$, a dimensionless area ratio. Hence, the correct option is (B).

Quick Tip to Solve: For two tangents, use their common chord as the base; then the area ratio becomes a ratio of distances along the angle bisector.

16. The number of solutions of the equation x + 2tanx = $\frac{\pi}{2}$ in the interval [0, 2π] is : [4 marks]

(A) 4

(B) 3

(C) 2

(D) 5

Correct Answer: (B) 3

Solution:

Concept:

- A continuous strictly increasing function takes a specified value at most once on each interval of continuity. Existence follows when its endpoint values or one-sided limits lie on opposite sides of that value.
- The tangent function is undefined at $\pi/2$ and $3\pi/2$ in the given interval, so the root count must be made separately on the three remaining pieces.

Step 1: Establish strict monotonicity Set $f(x) = x + 2\tan x - \pi/2$. Wherever this function is defined,

$$f\prime(x) = 1 + 2\sec^2 x > 0.$$

Thus it is strictly increasing on each of $[0, \pi/2)$, $(\pi/2, 3\pi/2)$, and $(3\pi/2, 2\pi]$. The two asymptotes are not allowed values of x; clearing a denominator must not accidentally count them as roots.

Step 2: Count the roots on the first two pieces On the first piece, $f(0) = -\pi/2 < 0$, whereas $f(x) \to +\infty$ as $x \to (\pi/2)^-$. Continuity gives a zero and strict increase makes it unique. On the middle piece the limits are

$$-\infty = \lim_{x\to(\pi/2)^+} f(x), \qquad \lim_{x\to(3\pi/2)^-} f(x) = +\infty.$$

There is exactly one more root in this piece.

Step 3: Check the final piece and endpoints Immediately to the right of $3\pi/2$, the function tends to

$-\infty$, while

$$f(2\pi) = 2\pi - \pi/2 = 3\pi/2 > 0.$$

This gives exactly one root in the final piece. Neither endpoint is a root, and the excluded asymptotes add none. There are therefore $1 + 1 + 1 = 3$ roots; counts 2, 4, and 5 contradict the one-root result on the three separate increasing branches.

Step 4: Conclusion The equation has exactly 3 solutions in $[0, 2\pi]$. Hence, the correct option is (B).

Quick Tip to Solve: Split a tangent equation at every asymptote before applying monotonicity; a positive derivative alone does not connect separate branches.

---

17. The value of $\sum_{r=0}^{6} \left({}^6C_r \,.\, {}^6C_{6-r}\right)$ is equal to : [4 marks]

(A) 924

(B) 1024

(C) 1124

(D) 1324

Correct Answer: (A) 924

Solution:

Concept:

- The coefficient of x^k in $(1+x)^n$ is $\binom{n}{k}$. In a product, coefficients of a fixed power are obtained by adding all products whose powers sum to that power.
- Vandermonde's identity is $\sum_r \binom{m}{r}\binom{n}{k-r} = \binom{m+n}{k}$, with terms outside the valid index ranges treated as zero.

Step 1: Recognise the coefficient convolution Expand two identical factors $(1+x)^6$. To obtain x^6 from their product, choose x^r from the first factor and x^{6-r} from the second. The coefficient contributed by that choice is $\binom{6}{r}\binom{6}{6-r}$. Summing over $r = 0, 1, \ldots, 6$ gives precisely the expression in the question, without omitting either endpoint term.

Step 2: Combine the two binomial expansions The product is

$$(1+x)^6(1+x)^6 = (1+x)^{12}.$$

Hence the requested sum equals the coefficient of x^6 in this last expansion, namely

$$\binom{12}{6} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 924.$$

This coefficient method accounts for each possible pair of selected powers exactly once.

Step 3: Verify with symmetry Using $\binom{6}{6-r} = \binom{6}{r}$, a direct check gives

$$1^2 + 6^2 + 15^2 + 20^2 + 15^2 + 6^2 + 1^2 = 924.$$

In particular, this is a sum of squares of binomial coefficients, not the square of their sum. The latter would incorrectly count cross terms. Only 924 matches the exact coefficient; 1024, 1124, and 1324 do not.

Step 4: Conclusion The sum equals 924. Hence, the correct option is (A).

Quick Tip to Solve: Whenever two binomial coefficients have complementary lower indices, look for the coefficient of a fixed power in a product of binomial expansions.

18. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3, 4) meet x-axis and y-axis at points P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to : [4 marks]

(A) $\frac{585}{66}$
(B) $\frac{625}{72}$
(C) $\frac{529}{64}$
(D) $\frac{125}{72}$

Correct Answer: (B) $\frac{625}{72}$

Solution:

Concept:

- The tangent to $x^2 + y^2 = a^2$ at (x_1, y_1) is $xx_1 + yy_1 = a^2$. Its axis intercepts determine the side lengths of the right triangle formed with the origin.
- For a right triangle along the positive coordinate axes, the incentre is (ρ, ρ), where $\rho = (a+b-c)/2$ is its inradius. A different circle centred there and passing through the origin has radius $\sqrt{2}\rho$.

Step 1: Determine the triangle The given contact point $(3, 4)$ lies on the circle because $3^2 + 4^2 = 25$. Its tangent is therefore

$$3x + 4y = 25.$$

Setting one coordinate to zero gives $P = (25/3, 0)$ and $Q = (0, 25/4)$. The triangle OPQ is right angled at O, with positive legs $a = 25/3$ and $b = 25/4$.

Step 2: Find the incentre The hypotenuse length is

$$c = \sqrt{\left(\frac{25}{3}\right)^2 + \left(\frac{25}{4}\right)^2} = \frac{125}{12}.$$

Its inradius is

$$\rho = \frac{a+b-c}{2} = \frac{100/12 + 75/12 - 125/12}{2} = \frac{25}{12}.$$

Equal perpendicular distances to the two axes put the incentre at $I = (25/12, 25/12)$, inside the triangle.

Step 3: Use the distance from the origin The circle asked for is centred at I and passes through O; it

is not the incircle of OPQ. Its radius is $r = IO$, so

$$r^2 = \left(\frac{25}{12}\right)^2 + \left(\frac{25}{12}\right)^2 = \frac{1250}{144} = \frac{625}{72}.$$

This also explains why simply squaring the inradius would be incorrect. The other three fractions differ from the squared distance just computed and are excluded.

Step 4: Conclusion The required squared radius is $r^2 = 625/72$ square coordinate units. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the triangle's inradius from the radius of a new circle centred at its incentre: use the specified point on the new circle.

---

19. If the Boolean expression $(p \wedge q) \odot (p \otimes q)$ is a tautology, then $\odot$ and $\otimes$ are respectively given by : [4 marks]

(A) $\vee, \rightarrow$

(B) $\rightarrow, \rightarrow$

(C) $\wedge, \vee$

(D) $\wedge, \rightarrow$

Correct Answer: (B) $\rightarrow, \rightarrow$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its variables. An implication $u \rightarrow v$ is false only when u is true and v is false.
- To disprove a proposed tautology, one counterexample is enough. To establish one, check all possible assignments or show its only possible false case cannot occur.

Step 1: Test the pair of implications For option (B), substitute $\odot = \rightarrow$ and $\otimes = \rightarrow$. The expression becomes

$$(p \wedge q) \rightarrow (p \rightarrow q).$$

If $p \wedge q$ is false, the outer implication is automatically true. If $p \wedge q$ is true, then both p and q are true, so $p \rightarrow q$ is true as well. Thus the outer implication cannot have a true antecedent and a false consequent.

Step 2: Check all truth assignments The truth values can also be recorded explicitly:

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Every entry of the final column is true, verifying the tautology independently of any single example.

Step 3: Eliminate the remaining operator pairs Option (A) yields $(p \wedge q) \vee (p \to q)$, which is false at $p = \mathrm{T}, q = \mathrm{F}$, since both disjuncts are then false. For options (C) and (D), the outer operator is conjunction. Taking $p = \mathrm{F}, q = \mathrm{F}$ makes the first conjunct $p \wedge q$ false, so the entire expression is false whatever the second conjunct is. Hence none of those three pairs gives a tautology.

Step 4: Conclusion The operators are respectively $\to$ and $\to$. Hence, the correct option is (B).

Quick Tip to Solve: For an outer implication, first examine the assignments that make its antecedent true; all remaining assignments already make the implication true.

---

20. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to : [4 marks]

(A) 21

(B) 19

(C) 18

(D) 20

Correct Answer: (B) 19

Solution:

Concept:

- The foot H of a perpendicular from X to the line $A + t\mathbf{d}$ is obtained from $(X - H) \cdot \mathbf{d} = 0$. The reflection in that line is $X\prime = 2H - X$.
- A plane containing a line has a normal perpendicular to its direction. A second vector from a point on that line to a further point of the plane gives the normal through a cross product.

Step 1: Reflect the given point For the mirror line use $A = (-1, 3, -2)$ and $\mathbf{d} = (2, 1, -1)$. With $X = (2, 3, 1)$,

$$t = \frac{(X - A) \cdot \mathbf{d}}{\mathbf{d} \cdot \mathbf{d}} = \frac{(3, 0, 3) \cdot (2, 1, -1)}{6} = \frac{1}{2}.$$

Therefore $H = (0, 7/2, -5/2)$ and the reflected point is

$$X\prime = 2H - X = (-2, 4, -6).$$

The segment joining X to $X\prime$ is bisected by the mirror line and is perpendicular to it.

Step 2: Obtain a normal to the plane The line that lies in the required plane has point $B = (2, 1, -1)$ and direction $\mathbf{v} = (3, -2, 1)$. The negative second component is essential because its numerator is $1 - y$. Another vector in the plane is $X\prime - B = (-4, 3, -5)$. Hence

$$\mathbf{n} = \mathbf{v} \times (X\prime - B) = (7, 11, 1).$$

Its dot products with both in-plane vectors are zero, confirming its role as a normal.

Step 3: Normalize the equation and compare The plane through B with this normal is

$$7(x-2)+11(y-1)+(z+1)=0,$$

or $7x+11y+z=24$. It also contains $X\prime$, since $-14+44-6=24$. The right-hand side already has the required normalization, so $\alpha=7$, $\beta=11$, and $\gamma=1$. Their sum is 19, excluding the listed values 21, 18, and 20.

Step 4: Conclusion $\alpha+\beta+\gamma=7+11+1=19$. Hence, the correct option is (B).

Quick Tip to Solve: Rewrite every symmetric line in parametric form before reflecting or taking cross products; a reversed numerator changes the direction sign.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ and $B=\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\neq\begin{bmatrix}0\\ 0\end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad − bc is equal to _____. [4 marks]

Solution:

Concept:

- If a square matrix sends a nonzero vector to itself, then 1 is an eigenvalue. Equivalently, the homogeneous system $(A-I)B=0$ has a nonzero solution.
- A square homogeneous system has a nonzero solution only if its coefficient matrix is singular. For a two by two matrix, this gives $\det(A-I)=0$, directly relating its trace and determinant.

Step 1: Write the nonzero homogeneous system The condition $AB=B$ can be rearranged as

$$(A-I)B=0, \qquad A-I=\begin{pmatrix}a-1 & b\\ c & d-1\end{pmatrix}.$$

Since the given column $B=(\alpha,\beta)^{\mathsf{T}}$ is not the zero column, it is a nontrivial solution. If $A-I$ were invertible, multiplying by its inverse would force $B=0$, a contradiction. Therefore $A-I$ must be singular.

Step 2: Expand the determinant condition The determinant of the coefficient matrix is consequently zero:

$$\det(A-I)=(a-1)(d-1)-bc=0.$$

Expanding every term, rather than identifying $\det A$ with $\det(A-I)$, gives

$$ad-a-d+1-bc=0.$$

Rearranging isolates precisely the expression requested in the question:

$$ad - bc = a + d - 1.$$

No separate values of a, b, c, d, or the nonzero entries of B, are needed.

Step 3: Substitute the trace and check Use the given trace $a + d = 2021$:

$$ad - bc = 2021 - 1 = 2020.$$

As a consistency check, the characteristic polynomial is $\lambda^2 - 2021\lambda + (ad - bc)$. Its root $\lambda = 1$ yields the same value. This does not require A to be diagonalizable or to have distinct eigenvalues; the existence of the stated nonzero fixed vector is sufficient.

Step 4: Conclusion The required determinant is $\boxed{2020}$. It is a dimensionless number, in agreement with the specified numerical value.

Quick Tip to Solve: For $AB = B$ with nonzero B, set $\det(A - I) = 0$; for a two by two matrix this immediately gives determinant equals trace minus one.

22. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n, x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _____. [4 marks]

Solution:

Concept:

- The general term of a binomial expansion is $T_{r+1} = \binom{n}{r} x^{n-r}(a/x^2)^r = \binom{n}{r} a^r x^{n-3r}$, where r starts from zero.
- The coefficient of a term is its numerical factor, including the power of a. A term independent of x must have exponent $n - 3r = 0$.

Step 1: Translate the coefficient ratios The third, fourth and fifth terms correspond to $r = 2, 3, 4$, respectively. Their coefficients are $\binom{n}{2}a^2$, $\binom{n}{3}a^3$, and $\binom{n}{4}a^4$. The given nonzero ratio implies $a \neq 0$ and $n \geq 4$. Divide consecutive coefficients to cancel their common factors:

$$\frac{(n-2)a}{3} = \frac{8}{12} = \frac{2}{3}, \qquad \frac{(n-3)a}{4} = \frac{3}{8}.$$

Step 2: Determine the expansion parameters The two equations are $(n - 2)a = 2$ and $(n - 3)a = 3/2$. Subtracting the second from the first gives $a = 1/2$. Substitution into the first gives $n - 2 = 4$, hence $n = 6$. As a check, the three coefficients become $15/4, 5/2, 15/16$; multiplication by $16/5$ gives exactly $12, 8, 3$.

Step 3: Select the zero-power term With $n = 6$, the power of x is $6 - 3r$. Thus only $r = 2$ produces a constant term; it is the third term, not the middle term. Its value is

$$T_3 = \binom{6}{2}\left(\frac{1}{2}\right)^2 x^0 = 15 \cdot \frac{1}{4} = \frac{15}{4} = 3.75.$$

Step 4: Conclusion The exact term independent of x is $\boxed{15/4 = 3.75}$. Rounded to the nearest integer, the numerical entry is $\boxed{4}$.

Quick Tip to Solve: Divide adjacent binomial coefficients before solving for parameters; then impose the zero-exponent condition separately.

---

23. Let tanα, tanβ and tanγ; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, n$\in$N be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha+\cos 3\beta+\cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2$ is equal to _____. [4 marks]

Solution:

Concept:

- When the circumcentre is the origin and the circumradius is R, the position vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ of the vertices have equal length R. The orthocentre has position vector $\mathbf{u}+\mathbf{v}+\mathbf{w}$.
- The triple-angle identity is $\cos 3t = 4\cos^3 t - 3\cos t$. If three numbers sum to zero, their cubes sum to three times their product.

Step 1: Use the orthocentre position Write the vertices as $A = (R\cos\alpha, R\sin\alpha)$, $B = (R\cos\beta, R\sin\beta)$, and $C = (R\cos\gamma, R\sin\gamma)$. To verify the orthocentre formula, the vector from A to $\mathbf{u}+\mathbf{v}+\mathbf{w}$ is $\mathbf{v}+\mathbf{w}$. Its scalar product with $\mathbf{v}-\mathbf{w}$, the direction of BC, is $|\mathbf{v}|^2 - |\mathbf{w}|^2 = 0$. Thus this point lies on each altitude.

Step 2: Apply the y-axis condition The orthocentre has zero first coordinate because it lies on the y-axis. Since $R > 0$, this gives $\cos\alpha + \cos\beta + \cos\gamma = 0$. Set $p = \cos\alpha$, $q = \cos\beta$, $r = \cos\gamma$. Using $p^3+q^3+r^3-3pqr = (p+q+r)(p^2+q^2+r^2-pq-qr-rp)$, obtain $p^3+q^3+r^3 = 3pqr$.

Step 3: Evaluate the trigonometric quotient Apply the triple-angle formula to each numerator term and add. The linear terms vanish and the cubic sum is already known:

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 4(p^3+q^3+r^3) - 3(p+q+r) = 12pqr.$$

The stated angle restrictions make each cosine nonzero, so division by their product is valid. The unsquared quotient is therefore 12.

Step 4: Conclusion The required value is $\boxed{12^2 = 144}$, a dimensionless number.

Quick Tip to Solve: For a triangle with circumcentre at the origin, convert an orthocentre axis condition into a sum of vertex coordinates before using trigonometric identities.

---

24. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- A vector in the plane spanned by the nonparallel vectors $\mathbf{a}$ and $\mathbf{b}$ can be written $\mathbf{x} = \lambda\mathbf{a} + \mu\mathbf{b}$. Perpendicular vectors have zero scalar product.
- The scalar projection of $\mathbf{x}$ on $\mathbf{a}$ is $\mathbf{x} \cdot \mathbf{a}/|\mathbf{a}|$. The squared magnitude is the sum of the squares of Cartesian components.

Step 1: Impose perpendicularity Let $\mathbf{c} = 3\hat{i} + 2\hat{j} - \hat{k}$. Compute the relevant products directly from the given components:

$$\mathbf{a} \cdot \mathbf{c} = 6 - 2 - 1 = 3, \qquad \mathbf{b} \cdot \mathbf{c} = 3 + 4 + 1 = 8.$$

Therefore $\mathbf{x} \cdot \mathbf{c} = 0$ gives the linear equation $3\lambda + 8\mu = 0$. This condition fixes the direction of $\mathbf{x}$ within the specified plane.

Step 2: Use the projection to fix the magnitude We have $|\mathbf{a}| = \sqrt{4 + 1 + 1} = \sqrt{6}$, $\mathbf{a} \cdot \mathbf{a} = 6$, and $\mathbf{b} \cdot \mathbf{a} = 2 - 2 - 1 = -1$. Thus

$$\frac{6\lambda - \mu}{\sqrt{6}} = \frac{17\sqrt{6}}{2} \quad \Longrightarrow \quad 6\lambda - \mu = 51.$$

From perpendicularity, $\mu = -3\lambda/8$. Substitution gives $51\lambda/8 = 51$, hence $\lambda = 8$, $\mu = -3$.

Step 3: Calculate the squared length Now $\mathbf{x} = 8(2, -1, 1) - 3(1, 2, -1) = (13, -14, 11)$. The vector satisfies the perpendicularity check $39 - 28 - 11 = 0$, and its product with $\mathbf{a}$ is $26 + 14 + 11 = 51$, reproducing the specified projection. Finally,

$$|\mathbf{x}|^2 = 13^2 + (-14)^2 + 11^2 = 169 + 196 + 121 = 486.$$

Step 4: Conclusion The required squared magnitude is $\boxed{486}$, in squared coordinate units.

Quick Tip to Solve: Use a linear combination for the plane condition; perpendicularity and scalar projection then give two simple linear equations.

---

25. Let $I_n = \int_1^e x^{19}(\log|x|)^n dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equals to _____. [4 marks]

Solution:

Concept:

- Integration by parts states $\int u\,dv = uv - \int v\,du$. Choose the logarithmic power as u, so its exponent decreases after differentiation.
- On $[1, e]$, $\log|x| = \log x$, and the endpoint values of $\log x$ are 0 and 1. The same boundary term

appears for all positive exponents.

Step 1: Derive a recurrence For a positive integer n, choose $u = (\log x)^n$ and $dv = x^{19}dx$. Then $du = n(\log x)^{n-1}dx/x$ and $v = x^{20}/20$. Integration by parts gives

$$I_n = \left[\frac{x^{20}(\log x)^n}{20}\right]_1^e - \frac{n}{20}\int_1^e x^{19}(\log x)^{n-1}dx.$$

The upper endpoint contributes $e^{20}/20$, while the lower contributes zero for $n > 0$.

Step 2: Write the two consecutive relations Multiplying the recurrence by 20 yields $20I_n = e^{20} - nI_{n-1}$. We need I_{10}, I_9, and I_8, so apply it first with $n = 10$ and then with $n = 9$:

$$20I_{10} = e^{20} - 10I_9, \qquad 20I_9 = e^{20} - 9I_8.$$

There is no need to evaluate any integral explicitly; the boundary constant can be eliminated.

Step 3: Eliminate the common boundary term Rearrange the second equation as $e^{20} = 20I_9 + 9I_8$, and substitute into the first. This gives

$$20I_{10} = 20I_9 + 9I_8 - 10I_9 = 10I_9 + 9I_8.$$

Comparing this positive-integer coefficient relation with the given form identifies $\alpha = 10$ and $\beta = 9$. Keeping the minus sign in the integration-by-parts recurrence is essential; changing it would produce the wrong coefficient of I_9.

Step 4: Conclusion Therefore $\alpha - \beta = 10 - 9 = \boxed{1}$, a dimensionless value.

Quick Tip to Solve: When adjacent logarithmic-power integrals occur, derive one recurrence and eliminate its common endpoint term using the next exponent.

---

26. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to : [4 marks]

Solution:

Concept:

- Three terms A, B, C are in arithmetic progression when $2B = A + C$. Logarithm rules convert a sum into a product only when the logarithm arguments are positive.
- A determinant can be expanded along a row or column containing zero. The sign pattern along the first row is $+, -, +$.

Step 1: Convert the progression into an algebraic equation Put $t = 4^x$. The logarithm $\log_{10}(t - 2)$

requires $t > 2$; the other logarithm is then also defined. The progression condition is

$$2\log_{10}(t-2) = 1 + \log_{10}\left(t + \frac{18}{5}\right).$$

Since $1 = \log_{10} 10$, combine the logarithms and use their injectivity on positive arguments:

$$(t-2)^2 = 10\left(t + \frac{18}{5}\right) = 10t + 36.$$

Step 2: Solve and check the admissible value Expansion gives $t^2 - 4t + 4 = 10t + 36$, hence $t^2 - 14t - 32 = 0$. Factor this as $(t-16)(t+2) = 0$. The root $t = -2$ is impossible because $t = 4^x > 0$, and also violates the logarithm domain. Therefore $4^x = 16$, giving $x = 2$. At this value the progression is $1, \log_{10} 14, \log_{10}(98/5)$; its middle-term condition follows from $14^2 = 10(98/5)$.

Step 3: Evaluate the determinant One may first simplify the determinant for a general x. Expanding along its first row gives

$$\begin{aligned} D &= (2x-1)(-x) - (x-1)(-x^2) + x^2 \\ &= -2x^2 + x + x^3 - x^2 + x^2 \\ &= x^3 - 2x^2 + x = x(x-1)^2. \end{aligned}$$

Now insert the admissible value $x = 2$. This also avoids confusing the determinant bars with absolute-value bars: the determinant is signed and must be expanded algebraically.

Step 4: Conclusion The requested determinant is $D = 2(2-1)^2 = \boxed{2}$.

Quick Tip to Solve: Check the logarithm domain before retaining roots; then use the zero in the second row to make the determinant expansion shorter.

27. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to _____. [4 marks]

Solution:

Concept:

- For a data set, $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$. A group mean gives its sum when multiplied by the number of observations.
- For a shift by a constant d, $(x+d)^2 = x^2 + 2dx + d^2$. Different shifts on different groups change the overall variance, even though a common shift would not.

Step 1: Recover the original totals The first $2n$ observations have sum $12n$, and the last n observations have sum $3n$. The total sum is $15n$, so the original mean is $15n/(3n) = 5$. Since the original variance is 4, the second moment is $4 + 5^2 = 29$. Thus the sum of squares of all $3n$ observations is $87n$. We do not need the individual group variances.

Step 2: Compute the new sum and sum of squares Adding 1 to the first group increases the total

sum by $2n$; subtracting 1 from the last decreases it by n. Hence the new mean is $16n/(3n) = 16/3$. The change in sum of squares in the first group is $2(12n) + 2n = 26n$. In the second group it is $-2(3n) + n = -5n$. These follow by expanding each shifted square before summing; the observations themselves need not be known.

Step 3: Form the new variance The new sum of squares is therefore $87n + 26n - 5n = 108n$, and the new second moment is $108n/(3n) = 36$. Apply the variance formula with the new mean, not the old mean:

$$k = 36 - \left(\frac{16}{3}\right)^2 = \frac{324 - 256}{9} = \frac{68}{9}.$$

The increase is reasonable: the group means move from $6, 3$ to $7, 2$, so their separation grows and increases the between-group contribution to variance.

Step 4: Conclusion Consequently, the requested value is $9k = \boxed{68}$, a dimensionless numerical value.

Quick Tip to Solve: For different constant shifts on groups, update the total sum and sum of squares; a single overall-shift invariance argument is invalid.

---

28. Let $f : [-1, 1] \to \mathbb{R}$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where $a, b, c \in \mathbb{R}$ such that $f(-1) = 2$, $f'(-1) = 1$, and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \leq \alpha$, $x \in [-1, 1]$, then the least value of α is equal to _____. [4 marks]

Solution:

Concept:

- For $f(x) = ax^2 + bx + c$, the derivatives are $f'(x) = 2ax + b$ and $f''(x) = 2a$. A constant second derivative has the same maximum at every point of an interval.
- The least upper bound α satisfying $f(x) \leq \alpha$ on a closed interval is the maximum value of f there. If the derivative is positive throughout, that maximum is at the right endpoint.

Step 1: Determine the quadratic coefficient The stated maximum of the second derivative on $(-1, 1)$ is $1/2$. Since the function is quadratic, its second derivative is the constant $2a$, so

$$2a = \frac{1}{2} \quad \Longrightarrow \quad a = \frac{1}{4}.$$

This condition is on the second derivative, whereas $f'(-1) = 1$ is the separate first-derivative condition. Keeping those two conditions distinct is necessary to determine the coefficients.

Step 2: Use the value and slope at the left endpoint The slope condition gives $-2a + b = 1$. With $a = 1/4$, this becomes $-1/2 + b = 1$, hence $b = 3/2$. Next, $f(-1) = a - b + c = 2$ gives

$$c = 2 - \frac{1}{4} + \frac{3}{2} = \frac{13}{4}.$$

Thus $f(x) = x^2/4 + 3x/2 + 13/4$. Substituting $x = -1$ back gives $(1 - 6 + 13)/4 = 2$, and the derivative there is $-1/2 + 3/2 = 1$, confirming both endpoint conditions.

Step 3: Locate the maximum on the full interval The first derivative is $f'(x) = x/2 + 3/2$. For every

$x \in [-1, 1]$, it lies between 1 and 2, so it is strictly positive. Hence f increases throughout the interval, with no interior stationary maximum. Its largest value is attained at $x = 1$:

$$f(1) = \frac{1}{4} + \frac{3}{2} + \frac{13}{4} = \frac{20}{4} = 5.$$

Any smaller bound fails at that endpoint, while 5 bounds every value of the increasing function.

Step 4: Conclusion The least possible upper bound is $\boxed{\alpha = 5}$.

Quick Tip to Solve: For a quadratic, a prescribed maximum of the second derivative fixes its constant value immediately; test the first derivative to locate the function maximum.

---

29. Let f : [−3, 1] → R be given as

$$f(x) = \begin{cases} \min\{(x+6), x^2\}, -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\}, \ 0 \le x \le 1. \end{cases}$$

If the area bounded by y = f(x) and x-axis is A, then the value of 6A is equal to _____. [4 marks]

Solution:

Concept:

- The required area is $A = \int_{-3}^{1} f(x)\,dx$, since both expressions chosen by the minimum or maximum are nonnegative on the stated intervals.
- For a minimum or maximum, first find intersection points and then identify which expression is selected on each subinterval. Integrating both candidate curves over the whole interval would count the wrong region.

Step 1: Locate the switching point Solve $x^2 = x + 6$:

$$x^2 - x - 6 = (x - 3)(x + 2) = 0.$$

Only $x = -2$ lies in $[-3, 0]$. At $x = -3$, the line has value 3 and the parabola has value 9, so the minimum is $x + 6$ on $[-3, -2]$. At $x = -1$, $x^2 = 1 < x + 6 = 5$, so the minimum is x^2 on $[-2, 0]$.

Step 2: Identify the positive-interval branch For $0 \le x \le 1$, we have $x^2 \le x \le \sqrt{x}$. Consequently the maximum is $\sqrt{x}$ throughout this interval. Every selected branch is nonnegative, so signed integration already gives the geometric area; no subtraction or absolute-value correction is needed.

Step 3: Integrate the three pieces Use each branch only on its own interval:

$$\begin{aligned} A &= \int_{-3}^{-2} (x+6)\,dx + \int_{-2}^{0} x^2\,dx + \int_{0}^{1} \sqrt{x}\,dx \\ &= \left[\frac{x^2}{2} + 6x\right]_{-3}^{-2} + \left[\frac{x^3}{3}\right]_{-2}^{0} + \left[\frac{2x^{3/2}}{3}\right]_{0}^{1} \\ &= \frac{7}{2} + \frac{8}{3} + \frac{2}{3} = \frac{41}{6}. \end{aligned}$$

Step 4: Conclusion The enclosed area is $A = \frac{41}{6}$ square units. Therefore the requested dimensionless numerical value is $\boxed{6A = 41}$.

Quick Tip to Solve: Find intersections before integrating a minimum or maximum; here only the negative interval switches branches, at $x = -2$.

---

30. Let P be an arbitrary point having sum of the squares of the distances from the planes x + y + z = 0, lx − nz = 0 and x − 2y + z = 0, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of l − n is equal to _____. [4 marks]

Solution:

Concept:

- The squared distance of (x, y, z) from $ax+by+cz = 0$ is $(ax+by+cz)^2/(a^2+b^2+c^2)$. Normalising the normal vector is essential.
- If the locus is a sphere centred at the origin, its quadratic form has no mixed terms and equal coefficients of x^2, y^2, z^2. A nonzero plane normal is required, so $l^2 + n^2 > 0$.

Step 1: Form the equation of the locus The given sum of squared distances yields

$$\frac{(x+y+z)^2}{3} + \frac{(lx-nz)^2}{l^2+n^2} + \frac{(x-2y+z)^2}{6} = 9.$$

The first and third planes have perpendicular normals because $(1,1,1)\cdot(1,-2,1) = 0$. Their squared projections therefore account for two mutually perpendicular directions.

Step 2: Combine the fixed-plane terms Expanding just the first and third contributions gives

$$\frac{(x+y+z)^2}{3} + \frac{(x-2y+z)^2}{6} = \frac{x^2}{2} + y^2 + \frac{z^2}{2} + xz.$$

The xy and yz terms cancel exactly. The remaining squared distance must complete this expression to $x^2 + y^2 + z^2$ for the stated spherical locus.

Step 3: Compare the mixed coefficient The coefficient of xz in the complete sum must be zero:

$$1 - \frac{2ln}{l^2+n^2} = 0.$$

Thus

$$l^2 + n^2 = 2ln \quad \Longrightarrow \quad (l-n)^2 = 0.$$

Hence $l = n \neq 0$. This also gives $l^2/(l^2+n^2) = n^2/(l^2+n^2) = 1/2$, so both diagonal coefficients become 1. As a check, the middle squared distance is then $(x-z)^2/2$, precisely the missing perpendicular projection.

Step 4: Conclusion The required condition is $l = n$, which reproduces the sphere exactly. Hence the requested dimensionless value is $\boxed{l - n = 0}$.

Quick Tip to Solve: For sums of squared distances from planes through the origin, compare the normalised projection quadratic form with the desired sphere.

31. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and from through a total distance of 6 cm. What will be the mathematical expression of this travelling wave? [4 marks]

(A) Y(x, t) = 0.03 [sin 5.1x − (0.2×10^3)t]

(B) Y(x, t) = 0.03 [sin 5.1x − (1.5×10^3)t]

(C) Y(x, t) = 0.06 [sin 5.1x − (1.5×10^3)t]

(D) Y(x, t) = 0.06 [sin 0.8x − (0.5×10^3)t]

Correct Answer: (B) Y(x, t) = 0.03 [sin 5.1x − (1.5×10^3)t]

Solution:

Concept:

- A sinusoidal wave travelling along the positive x-direction is $Y(x,t) = a\sin(kx - \omega t + \phi)$. Here a is amplitude, $k = \omega/v$, and $\omega = 2\pi f$.
- The distance between the two extreme positions is $2a$. It must not be confused with the total path $4a$ travelled during one complete oscillation.

Step 1: Find the amplitude The stated 6 cm is the full excursion from one extreme position to the other. Thus

$$2a = 6\,\text{cm}, \qquad a = 3\,\text{cm} = 0.03\,\text{m}.$$

Taking $a = 0.06\,\text{m}$ would double the stated excursion. The options use zero initial phase, which is permissible because no phase or initial displacement is specified.

Step 2: Calculate the angular frequency and wave number For $f = 245\,\text{Hz}$ and $v = 300\,\text{m}\,\text{s}^{-1}$,

$$\omega = 2\pi(245) = 490\pi \simeq 1.54 \times 10^3\,\text{rad}\,\text{s}^{-1},$$
$$k = \frac{490\pi}{300} \simeq 5.13\,\text{rad}\,\text{m}^{-1}.$$

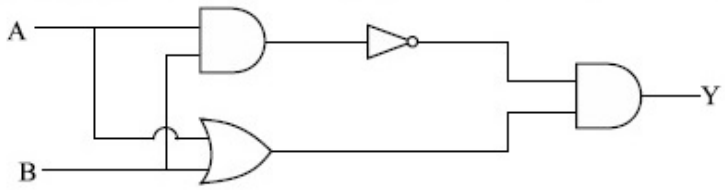
These are represented in the options by 1.5×10^3 and 5.1, respectively. The minus sign between the spatial and temporal phases makes a constant phase move towards increasing x.

Step 3: Compare all options Option (B) has amplitude $0.03\,\text{m}$, wave number $5.1\,\text{rad}\,\text{m}^{-1}$, and angular frequency $1.5 \times 10^3\,\text{rad}\,\text{s}^{-1}$. Option (A) uses 0.2×10^3 for the angular frequency and therefore has the wrong frequency and speed. Option (C) doubles the amplitude. Option (D) has the wrong amplitude as well as the wrong angular frequency and wave number. The phase is understood as the complete argument of the sine.

Step 4: Conclusion The appropriate wave is $Y(x,t) = 0.03\sin[5.1x - (1.5 \times 10^3)t]$, in SI units, to the precision used in the options. Hence, the correct option is (B).

Quick Tip to Solve: Check amplitude, angular frequency and propagation sign separately: a, $2\pi f$, and $kx - \omega t$ determine the answer.

32. Which one of the following will be the output of the given circuit? [4 marks]



(A) XOR Gate

(B) NOR Gate

(C) NAND Gate

(D) AND Gate

Correct Answer: (A) XOR Gate

Solution:

Concept:

- An AND gate produces AB, an OR gate produces $A + B$, and a NOT gate complements its input. Multiplication and addition here denote Boolean AND and OR.
- The exclusive-OR operation is $A \oplus B = \overline{A}B + A\overline{B}$; it is 1 precisely when the two inputs differ.

Step 1: Read the two parallel branches Both inputs A and B enter the upper AND gate, whose output is AB. The triangular inverter following it changes this to $\overline{AB}$. The lower gate receives the same inputs and is an OR gate, so its output is $A + B$. The small bridge where wires cross indicates that the crossing is not an extra connection.

Step 2: Combine the branch outputs The final gate is an AND gate. Its output is therefore

$$Y = (A + B)\overline{AB}.$$

Applying De Morgan's law and distributing gives

$$\begin{aligned} Y &= (A + B)(\overline{A} + \overline{B}) \\ &= A\overline{B} + \overline{A}B, \end{aligned}$$

because $A\overline{A} = B\overline{B} = 0$. This is the standard exclusive-OR expression, not the ordinary OR expression.

Step 3: Verify the truth table and reject the alternatives For input pairs $(A, B) = (0, 0), (0, 1), (1, 0), (1, 1)$, the outputs are respectively $0, 1, 1, 0$. This confirms that exactly one input must be high. NOR would give $1, 0, 0, 0$; NAND would give $1, 1, 1, 0$; AND would give $0, 0, 0, 1$. Thus none of the other listed gates has the observed output pattern. In particular the inverter acts on the upper AND output alone, not on the final output.

Step 4: Conclusion The circuit implements $Y = A\overline{B} + \overline{A}B = A \oplus B$, an XOR gate. Hence, the correct option is (A).

Quick Tip to Solve: Write the expression at each intermediate gate before simplifying; $(A+B)\overline{AB}$ immediately identifies XOR.

33. Match List - I with List - II

	List - I		List - II
(a)	**Phase difference between current and voltage in a purely resistive AC circuit**	**(i)**	**$\frac{\pi}{2}$; current leads voltage**
(b)	**Phase difference between current and voltage in a pure inductive AC circuit**	**(ii)**	**zero**
(c)	**Phase difference between current and voltage in a pure capacitive AC circuit**	**(iii)**	**$\frac{\pi}{2}$; current lags voltage**
(d)	**Phase difference between current and voltage in an LCR series circuit**	**(iv)**	$\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

(B) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(C) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(D) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

Correct Answer: (A) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

Solution:

Concept:

- In a resistor, voltage and current are in phase. In a pure inductor, current lags voltage by $\pi/2$; in a pure capacitor, current leads voltage by $\pi/2$.
- For a series LCR circuit, the signed phase of current relative to applied voltage is $\phi_I = \tan^{-1}[(X_C - X_L)/R]$, where $X_L = \omega L$ and $X_C = 1/(\omega C)$.

Step 1: Match the resistor and inductor For the resistor, $v = iR$, so the two sinusoidal quantities reach their maxima together and the phase difference is zero. Hence (a) matches (ii). For the inductor, $v = L\,di/dt$. Differentiation advances the sinusoidal voltage phase by $\pi/2$ relative to current, so current lags voltage by $\pi/2$. Hence (b) matches (iii).

Step 2: Match the capacitor and series circuit For the capacitor, $i = C\,dv/dt$, which advances current by $\pi/2$ relative to voltage. Hence (c) matches (i). The LCR impedance is $Z = R + j(X_L - X_C)$. Its argument is the phase of voltage relative to current; reversing the reference gives

$$\phi_I = \tan^{-1}\left(\frac{X_C - X_L}{R}\right).$$

Therefore (d) matches (iv). This sign convention also correctly predicts a positive current lead when capacitive reactance dominates.

Step 3: Select the complete matching The resulting order is $(a) \to (ii)$, $(b) \to (iii)$, $(c) \to (i)$, $(d) \to (iv)$. Only option (A) preserves all four assignments. Option (B) incorrectly assigns a phase lead to a resistor. Option (C) assigns the general LCR expression to a pure capacitor. Option (D) assigns that expression to a pure inductor and reverses the capacitor behaviour. These mismatches rule out the remaining options.

Step 4: Conclusion The matches in the printed row order are (ii), (iii), (i), and (iv). Hence, the correct option is (A).

Quick Tip to Solve: Remember that current lags in L and leads in C; then check whether the stated LCR phase refers to current or to voltage.

34. The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra is lying in the visible region? [4 marks]

(A) Brackett series

(B) Balmer series

(C) Paschen series

(D) Lyman series

Correct Answer: (B) Balmer series

Solution:

Concept:

- Hydrogen emits a photon when an electron drops from n_i to a lower level n_f. A spectral series groups transitions with the same final principal quantum number.
- The Rydberg relation is $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where $n_i > n_f$ and $R_H \simeq 1.097 \times 10^7\ \mathrm{m}^{-1}$.

Step 1: Identify the final levels of the series The series are named according to the lower energy level: Lyman has $n_f = 1$, Balmer has $n_f = 2$, Paschen has $n_f = 3$, and Brackett has $n_f = 4$. Larger drops to the ground state release more photon energy and therefore produce shorter wavelengths; drops ending at the higher final levels generally give longer infrared wavelengths.

Step 2: Check a characteristic Balmer wavelength The first Balmer transition is $n_i = 3$ to $n_f = 2$. Substituting gives

$$\frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R_H}{36},$$

$$\lambda = \frac{36}{5R_H} \simeq 6.56 \times 10^{-7}\ \mathrm{m} = 656\ \mathrm{nm}.$$

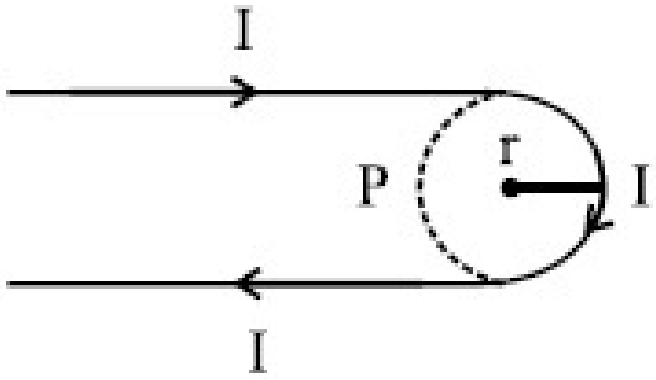
This is the familiar red hydrogen line in the visible region. Other prominent Balmer lines, including about 486 nm and 434 nm, also lie in the visible region. The series approaches a limit near 365 nm at its short-wavelength end.

Step 3: Eliminate the other series Brackett transitions end at $n_f = 4$ and are infrared; Paschen transitions end at $n_f = 3$ and are also infrared. Lyman transitions end at $n_f = 1$ and are ultraviolet. Therefore the characteristic visible hydrogen series among the choices is Balmer. The answer refers to the standard classification of its visible lines; it does not require every member arbitrarily close to the series limit to remain visible.

Step 4: Conclusion The hydrogen series associated with the visible region is the Balmer series, corresponding to transitions ending at $n_f = 2$. Hence, the correct option is (B).

Quick Tip to Solve: Associate the final levels $1, 2, 3, 4$ with Lyman, Balmer, Paschen and Brackett; Balmer is the visible series.

35. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle? [4 marks]



(A) $\frac{\mu_0 I}{4\pi r}(2-\pi)$
(B) $\frac{\mu_0 I}{2\pi r}(2-\pi)$
(C) $\frac{\mu_0 I}{4\pi r}(2+\pi)$
(D) $\frac{\mu_0 I}{2\pi r}(2+\pi)$

Correct Answer: (C) $\frac{\mu_0 I}{4\pi r}(2+\pi)$

Solution:

Concept:

- At the centre of a circular arc of angle θ radians and radius r, the magnetic field is $B_{\text{arc}} = \mu_0 I\theta/(4\pi r)$.
- Each straight semi-infinite segment at perpendicular distance r contributes $\mu_0 I/(4\pi r)$. Magnetic fields must be added with directions determined by the right-hand rule.

Step 1: Evaluate the semicircular contribution The curved portion is a semicircle, so $\theta = \pi$. Its contribution at P, the centre, is

$$B_{\text{arc}} = \frac{\mu_0 I\pi}{4\pi r} = \frac{\mu_0 I}{4r}.$$

The current enters along the upper straight segment, follows the right-hand semicircle clockwise, and exits along the lower segment. The right-hand rule therefore gives the semicircular field into the page.

Step 2: Evaluate the two straight contributions Each long straight arm extends from the tangent point towards the left. The perpendicular from P meets an arm at its endpoint, so the limiting angles in the finite-wire formula are 0 and $90°$:

$$B_{\text{arm}} = \frac{\mu_0 I}{4\pi r}(\sin 0 + \sin 90°) = \frac{\mu_0 I}{4\pi r}.$$

For the upper arm, current is rightward and P lies below it; for the lower arm, current is leftward and P lies above it. Both fields point into the page.

Step 3: Add the fields and test the choices All three contributions reinforce:

$$\begin{aligned} B &= 2B_{\text{arm}} + B_{\text{arc}} \\ &= \frac{2\mu_0 I}{4\pi r} + \frac{\mu_0 I\pi}{4\pi r} \\ &= \frac{\mu_0 I}{4\pi r}(2+\pi). \end{aligned}$$

Options (A) and (B) incorrectly subtract contributions which have the same direction, and even contain the negative factor $2-\pi$ despite asking for magnitude. Option (D) doubles the correct result by treating each semi-infinite arm as a full infinite wire and likewise doubling the arc contribution.

Step 4: Conclusion The magnitude at P is $\boxed{B = \frac{\mu_0 I}{4\pi r}(2+\pi)}$, in tesla for SI inputs; its direction is into the page. Hence, the correct option is (C).

Quick Tip to Solve: For a hairpin wire, separate the semicircle and the two semi-infinite arms, then use the right-hand rule before adding magnitudes.

36. Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then [4 marks]

(A) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$

(B) $v_1^2 + v_2^2 = \frac{2h}{m}[f_1 + f_2]$

(C) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(D) $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$

Correct Answer: (D) $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$

Solution:

Concept:

- Einstein's photoelectric equation is $hf = \phi + K_{\max}$, where h is Planck's constant, f the incident frequency and ϕ the work function of the emitting material.
- For a nonrelativistic photoelectron of mass m, the maximum kinetic energy is $K_{\max} = \frac{1}{2}mv^2$. Identical photocathodes have the same work function, so subtraction eliminates it.

Step 1: Write the equations for both photocathodes For the first illumination, the photon energy supplies the work function and the outgoing electron's kinetic energy. For the second illumination the work-function term is unchanged. Thus

$$hf_1 = \phi + \frac{1}{2}mv_1^2, \qquad hf_2 = \phi + \frac{1}{2}mv_2^2.$$

The speeds here are the maximum photoelectron speeds associated with the two frequencies; photons in excess of the threshold energy provide the electron's kinetic energy.

Step 2: Subtract and compare the proposed relations Subtract the second equation from the first:

$$h(f_1 - f_2) = \frac{m}{2}(v_1^2 - v_2^2).$$

Rearrangement gives

$$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2).$$

Option (A) incorrectly replaces a difference of squares by the square of a difference. Adding the original equations instead gives $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2) - \frac{4\phi}{m}$, so (B) omits the work function. Option (C) also incorrectly combines the speeds and omits this energy cost.

Step 3: Conclusion The common work function cancels only on subtracting the energy equations. Therefore $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$. Hence, the correct option is (D).

Quick Tip to Solve: Subtract Einstein's equations for identical surfaces before manipulating the speeds; the kinetic-energy term contains the square of each speed.

37. If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_P}{C_V}\right)$ then the value of β is : [4 marks]

(A) 1.02

(B) 1.35

(C) 1.2

(D) 1.25

Correct Answer: (C) 1.2

Solution:

Concept:

- Each independent quadratic term contributes $\frac{1}{2}RT$ to the molar internal energy. A nonlinear polyatomic molecule has three translational and three rotational degrees of freedom.
- An active vibrational mode has kinetic and potential energy terms, so each mode contributes RT per mole. For an ideal gas, Mayer's relation is $C_P - C_V = R$.

Step 1: Count the energy contributions The translational and rotational contributions together give six quadratic degrees of freedom. Two vibrational modes add four more quadratic terms, since the displacement and velocity of each vibration each store energy. Consequently the effective number is

$$f = 3 + 3 + 2(2) = 10.$$

This is the standard polyatomic-gas model of the question, with the stated vibrational modes active. Counting each vibration only once would underestimate the internal energy.

Step 2: Find the two molar heat capacities For one mole, the internal energy and its temperature derivative are

$$U = \frac{f}{2}RT = 5RT, \qquad C_V = \frac{dU}{dT} = 5R.$$

At constant pressure, some supplied heat also performs expansion work. Therefore

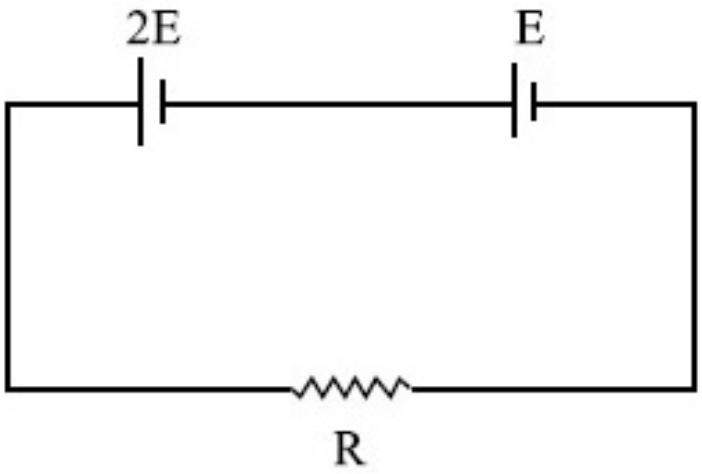
$$C_P = C_V + R = 6R, \qquad \beta = \frac{C_P}{C_V} = \frac{6}{5} = 1.2.$$

The gas quantity does not affect this ratio. The values 1.02, 1.35 and 1.25 would correspond through $\beta = 1 + 2/f$ to effective counts 100, approximately 5.71, and 8, respectively; none matches ten quadratic terms.

Step 3: Conclusion The molar heat capacities are $C_V = 5R$ and $C_P = 6R$, giving the dimensionless ratio $\beta = 1.2$. Hence, the correct option is (C).

Quick Tip to Solve: Count two quadratic contributions for every vibrational mode, then use $\gamma = 1 + 2/f$; a vibration contributes both kinetic and potential energy.

38. Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is [4 marks]



(A) $r_1 - r_2$

(B) $\frac{r_1}{2} - r_2$

(C) $\frac{r_1}{2} + r_2$

(D) $r_1 + r_2$

Correct Answer: (B) $\frac{r_1}{2} - r_2$

Solution:

Concept:

- A discharging cell of emf $\mathcal{E}$ and internal resistance r has terminal voltage $V = \mathcal{E} - Ir$, where I is the current it supplies.
- The diagram connects the two cells in series aiding, so their emfs add. Their internal resistances and the external resistance also add in the loop.

Step 1: Calculate the current in the complete loop The long plate of each cell is on the same side in the circuit, so the two sources drive the same circulating current. The net emf is $2E + E = 3E$, and Kirchhoff's loop rule gives

$$3E = I(R + r_1 + r_2), \qquad I = \frac{3E}{R + r_1 + r_2}.$$

All three resistances carry this same current because there is only one closed loop and no parallel branch.

Step 2: Impose zero terminal voltage on the first cell The first cell has emf $2E$. Its terminal voltage becomes zero when the internal voltage drop equals this emf:

$$2E - Ir_1 = 0, \qquad I = \frac{2E}{r_1}.$$

Equate this with the loop current and cancel the nonzero E:

$$\frac{3}{R + r_1 + r_2} = \frac{2}{r_1}.$$

$$3r_1 = 2R + 2r_1 + 2r_2, \qquad R = \frac{r_1}{2} - r_2.$$

Option (A) has the wrong coefficient of r_1; (C) has the wrong sign of r_2; (D) cannot satisfy the zero-voltage condition. A nonnegative external resistance requires $r_1 \geq 2r_2$.

Step 3: Conclusion The required external resistance is $R = \frac{r_1}{2} - r_2$, provided this value is physically nonnegative. Hence, the correct option is (B).

Quick Tip to Solve: First set the specified cell's terminal voltage to zero to obtain the current; only then combine that condition with the current of the entire series loop.

39. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved? [4 marks]

(A) Both, inducting reactance and current will be doubled.

(B) Inductive reactance will be doubled and current will be halved.

(C) Both, inductive reactance and current will be halved.

(D) Inductive reactance will be halved and current will be doubled.

Correct Answer: (D) Inductive reactance will be halved and current will be doubled.

Solution:

Concept:

- For a pure inductor with inductance L, the inductive reactance is $X_L = \omega L = 2\pi f L$, where f is the source frequency.
- At a fixed rms applied voltage V, the rms current is $I = V/X_L$. Thus reactance is proportional to frequency, whereas current is inversely proportional to frequency.

Step 1: Apply the change in frequency to the reactance Let the original frequency be f. Its new value is $f' = f/2$, while the inductance is unchanged. Hence

$$X_L' = 2\pi f' L = 2\pi \left(\frac{f}{2}\right) L = \frac{X_L}{2}.$$

The inductive opposition decreases as the alternating current changes more slowly. This rules out both (A) and (B), which claim that the reactance increases.

Step 2: Calculate the resulting current The comparison holds the applied voltage amplitude, or equivalently its rms value, fixed. Substitute the new reactance into the ac form of Ohm's law:

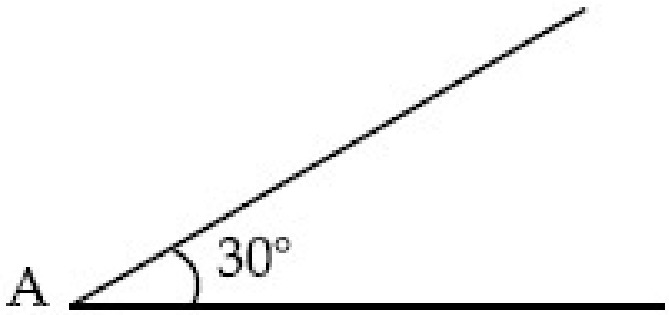
$$I' = \frac{V}{X_L'} = \frac{V}{X_L/2} = 2\frac{V}{X_L} = 2I.$$

Although the current continues to lag the voltage by $\pi/2$, that phase relation does not change the magnitude calculation. Option (C) gets the reactance change right but makes current decrease in the same proportion, contradicting $I = V/X_L$. Only option (D) combines the halved opposition with the doubled current. These ratios apply to a purely inductive circuit, without an additional resistance altering its impedance.

Step 3: Conclusion On halving the frequency at fixed applied voltage, the inductive reactance is halved and the current is doubled. Hence, the correct option is (D).

Quick Tip to Solve: Write $X_L \propto f$ and $I \propto 1/f$ side by side before selecting a change; the two quantities move in opposite directions.

40. A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A? [4 marks]



(A) 0.60 s

(B) 0.52 s

(C) 0.80 s

(D) 0.57 s

Correct Answer: (D) 0.57 s

Solution:

Concept:

- For a uniform solid sphere, $I_{\text{cm}} = \frac{2}{5}mr^2$. Rolling without slipping imposes $v = r\omega$ and couples translational and rotational motion.
- The acceleration of a freely rolling body down an incline is $a = g\sin\theta/(1 + I_{\text{cm}}/(mr^2))$. Ideal static friction enables rolling without dissipating mechanical energy.

Step 1: Find the acceleration along the incline Take the downhill direction as positive. Combining the centre-of-mass equation $ma = mg\sin\theta - f$ with the torque equation $fr = I_{\text{cm}}\alpha$ and $a = r\alpha$ yields

$$a = \frac{g\sin\theta}{1 + I_{\text{cm}}/(mr^2)} = \frac{5}{7}g\sin 30^\circ = \frac{5g}{14}.$$

The downhill acceleration is constant during both ascent and descent. Neither the mass 2 kg nor the radius 0.5 m remains in this expression, because the moment of inertia scales as mr^2.

Step 2: Calculate the complete return time Use the uphill direction for displacement and take the initial speed as $u = 1\,\text{m}\,\text{s}^{-1}$. The displacement from A is

$$s = ut - \frac{1}{2}at^2.$$

At the return to A, $s = 0$. Excluding the initial instant gives

$$t = \frac{2u}{a} = \frac{28}{5g}\,\text{s}.$$

With $g = 9.8\,\text{m}\,\text{s}^{-2}$, $t = 0.5714\,\text{s}$, or 0.57 s. Using $g = 10$ gives 0.56 s and selects the same nearest option. Values 0.60, 0.52 and 0.80 s do not match the solid-sphere rolling acceleration. Counting only the ascent would give half the required time.

Step 3: Conclusion The sphere returns to A after approximately $0.57\,\text{s}$. Hence, the correct option is (D).

Quick Tip to Solve: For a return to the launch position under constant acceleration, use $t = 2u/a$. For rolling, obtain a using rotational inertia before substituting.

41. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is [4 marks]

(A) $\frac{K_1}{K_2}$

(B) $\sqrt{\frac{K_1}{K_2}}$

(C) $\frac{K_2}{K_1}$

(D) $\sqrt{\frac{K_2}{K_1}}$

Correct Answer: (D) $\sqrt{\frac{K_2}{K_1}}$

Solution:

Concept:

- A mass m attached to an ideal spring of constant K oscillates about equilibrium with angular frequency $\omega = \sqrt{K/m}$. Gravity shifts the equilibrium position but does not change this frequency.
- For simple harmonic motion with amplitude A, the maximum speed is $v_{\max} = \omega A$, attained as the mass passes through its equilibrium position.

Step 1: Write the maximum speed of each particle Use A_A and A_B for the amplitudes of particles A and B, respectively, to distinguish the first amplitude from the particle's name. The two equal masses have frequencies

$$\omega_A = \sqrt{\frac{K_1}{m}}, \qquad \omega_B = \sqrt{\frac{K_2}{m}}.$$

Their maximum speeds therefore are $A_A\sqrt{K_1/m}$ and $A_B\sqrt{K_2/m}$. These speeds depend on amplitude as well as spring stiffness, so equal mass alone does not make them equal.

Step 2: Use the equality and isolate the amplitude ratio The given equality requires

$$A_A\sqrt{\frac{K_1}{m}} = A_B\sqrt{\frac{K_2}{m}}.$$

Cancel the common mass factor and divide by $A_B\sqrt{K_1}$:

$$\frac{A_A}{A_B} = \sqrt{\frac{K_2}{K_1}}.$$

This also follows from equal maximum kinetic energies, which imply $\frac{1}{2}K_1A_A^2 = \frac{1}{2}K_2A_B^2$. Options (A) and (C) omit the square root, while (B) reverses the required ratio. A stiffer first spring must have a smaller amplitude for equal maximum speed, confirming the direction of the result.

Step 3: Conclusion The ratio of the amplitude of A to that of B is $\sqrt{K_2/K_1}$. Hence, the correct option is (D).

Quick Tip to Solve: When maximum SHM speeds are equal, set ωA equal; amplitudes vary inversely as the square root of the spring constants for equal masses.

42. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is x = 0 at t = 0; then its displacement after time (t = 1) is : [4 marks]

(A) $v_0 + g + f$

(B) $v_0 + \frac{g}{2} + \frac{F}{3}$

(C) $v_0 + 2g + 3F$

(D) $v_0 + \frac{g}{2} + F$

Correct Answer: (B) $v_0 + \frac{g}{2} + \frac{F}{3}$

Solution:

Concept:

- Instantaneous velocity is the rate of change of position: $v = dx/dt$. Displacement over an interval is therefore the definite integral of velocity over that interval.
- Integrate each power using $\int t^n\,dt = t^{n+1}/(n+1)$. Here v_0, g and F are constants; in particular, Ft^2 is a velocity term rather than a constant acceleration.

Step 1: Integrate the velocity with the initial condition At an intermediate time τ, the velocity is $v_0 + g\tau + F\tau^2$. The initial position is zero, so

$$x(t) - x(0) = \int_0^t (v_0 + g\tau + F\tau^2)\,d\tau.$$

Integration term by term gives

$$x(t) = v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}.$$

Each term vanishes at $t = 0$, so no further integration constant is needed. Differentiating this expression returns the prescribed velocity, confirming all coefficients.

Step 2: Evaluate at the specified time For $t = 1$, the result in the time units used by the question is

$$x(1) - x(0) = v_0 + \frac{g}{2} + \frac{F}{3}.$$

The factors 1/2 and 1/3 arise because the velocity contains powers t and t^2, respectively. Option (A) simply substitutes into velocity, rather than integrating it. Option (C) does not follow the integration rule and increases the coefficients. Option (D) integrates the linear term correctly but omits division by three for the quadratic term. With SI coefficients and a one-second interval, the displacement is expressed in metres.

Step 3: Conclusion The displacement after the stated unit time is $v_0 + \frac{g}{2} + \frac{F}{3}$, in the corresponding length unit. Hence, the correct option is (B).

Quick Tip to Solve: For a velocity polynomial, integrate before inserting the final time. Divide the coefficient of each time power by its new exponent.

43. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take g = 10 ms^{-2}) [4 marks]

(A) 2.50 ms^{-1}

(B) 3.0 ms^{-1}

(C) 2.0 ms^{-1}

(D) 3.50 ms^{-1}

Correct Answer: (A) 2.50 ms^{-1}

Solution:

Concept:

- Average speed over the complete bouncing motion is total distance divided by total elapsed time; it is not the average of the initial and final speeds.
- For a fall through height h, the time is $t_0 = \sqrt{2h/g}$. Successive rebound heights form a geometric series of ratio $r = 0.81$, whereas their flight times have ratio $\sqrt{r} = 0.9$.

Step 1: Sum the distances The initial descent contributes $h = 5\,\text{m}$. Every rebound height is covered twice, once upwards and once downwards. Thus

$$S = h + 2h(r + r^2 + \cdots) = h\frac{1+r}{1-r} = 5\frac{1.81}{0.19} = \frac{905}{19}\,\text{m}.$$

Step 2: Sum the flight times The first fall takes $t_0 = \sqrt{2(5)/10} = 1\,\text{s}$. Each subsequent upward and downward leg has the time associated with its rebound height. Hence

$$T = t_0 + 2t_0(0.9 + 0.9^2 + \cdots) = 1 + \frac{2(0.9)}{1-0.9} = 19\,\text{s}.$$

Step 3: Divide the complete distance by the complete time Both geometric sums converge, even though there are infinitely many bounces. Their ratio is

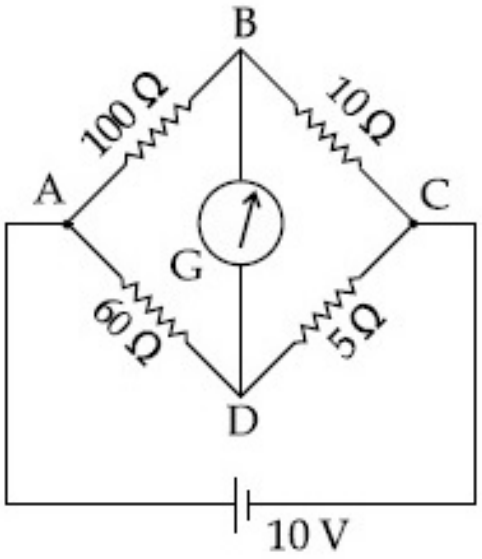
$$v_{\text{av}} = \frac{905/19}{19} = \frac{905}{361} = 2.507\,\text{m s}^{-1}.$$

The listed value $2.50\,\text{m s}^{-1}$ is the intended approximation. The values 3.0, 2.0 and 3.50 do not agree with the sum; using rebound-height ratios directly for time would give a wrong result.

Step 4: Conclusion The average speed is approximately $2.50\,\text{m s}^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the height ratio for the distance series and its square root for the time series; the first drop is counted only once.

44. The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10V is maintained across AC. [4 marks]



(A) 4.87 μA

(B) 4.87 mA

(C) 2.44 mA

(D) 2.44 μA

Correct Answer: (B) 4.87 mA

Solution:

Concept:

- The galvanometer bridge is not balanced because $100/10 \neq 60/5$, so a nonzero current flows through BD.
- The Thevenin voltage across BD is its open-circuit potential difference. The Thevenin resistance is calculated after replacing the ideal voltage source across AC by a short circuit.

Step 1: Find the open-circuit potential difference Take $V_A = 10\,\text{V}$ and $V_C = 0$. With the galvanometer removed, the upper and lower arms are separate potential dividers.

$$V_B = 10\frac{10}{100+10} = \frac{10}{11}\,\text{V}, \qquad V_D = 10\frac{5}{60+5} = \frac{10}{13}\,\text{V}.$$

Thus $V_{\text{th}} = V_B - V_D = 20/143\,\text{V}$; B is at the higher potential.

Step 2: Calculate the resistance seen from BD When A and C are shorted, the two resistances meeting B are in parallel, as are the two meeting D. These parallel equivalents are in series between B and D:

$$R_{\text{th}} = \frac{100(10)}{110} + \frac{60(5)}{65} = \frac{100}{11} + \frac{60}{13} = \frac{1960}{143}\,\Omega.$$

Step 3: Reconnect the galvanometer Its $15\,\Omega$ resistance must be added to this equivalent, rather than

ignored. Therefore

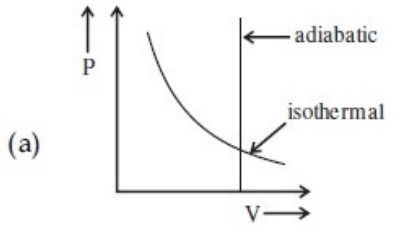
$$I_G = \frac{20/143}{15 + 1960/143} = \frac{20}{4105}\,\text{A} = 4.872\,\text{mA}.$$

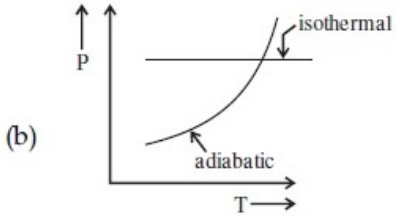
Current flows from B to D. The microampere alternatives are smaller by three orders of magnitude, and 2.44 mA is about half the actual current; neither follows from the complete bridge resistance.

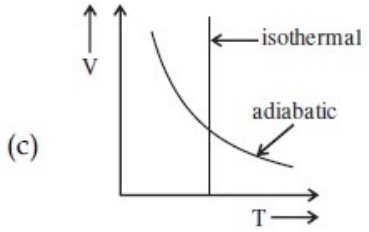
Step 4: Conclusion The galvanometer current is $4.87\,\text{mA}$, directed from B to D. Hence, the correct option is (B).

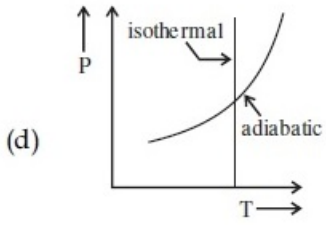
Quick Tip to Solve: For an unbalanced bridge, find the open-circuit voltage across the galvanometer and divide by its resistance plus the Thevenin resistance.

45. Which one is the correct option for the two different thermodynamic processes? [4 marks]









(A) (a) only

(B) (c) and (d)

(C) (b) and (c)

(D) (c) and (a)

Correct Answer: (B) (c) and (d)

Solution:

Concept:

- An isothermal process has constant absolute temperature. It is a vertical line whenever temperature is on the horizontal axis, and obeys $PV =$ constant on a P–V diagram.
- For an ideal gas undergoing a reversible adiabatic process, $PV^{\gamma} =$ constant, with $\gamma > 1$. Together with $PV = nRT$, this gives $TV^{\gamma-1} =$ constant and $P \propto T^{\gamma/(\gamma-1)}$.

Step 1: Check graph (a) The falling hyperbola labelled isothermal is consistent with $P \propto 1/V$. However, the vertical line labelled adiabatic would keep volume constant. A nontrivial adiabatic path on P–V axes is a falling curve, steeper than the isotherm through the same point, not a vertical line. Thus (a) as a complete diagram is incorrect.

Step 2: Check graph (b) On P–T axes an isothermal path must have fixed T and therefore be vertical. The horizontal path drawn in (b) fixes P instead; it is isobaric, not isothermal. Although its rising adiabatic curve has the right qualitative trend, the wrong isothermal label makes (b) incorrect.

Step 3: Check graphs (c) and (d) In (c), the vertical line on V–T axes is correctly isothermal. The adiabatic relation gives

$$V \propto T^{-1/(\gamma-1)},$$

which decreases as T increases, matching the curve. In (d), the vertical isotherm on P–T axes is again correct and

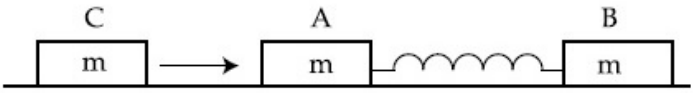
$$P \propto T^{\gamma/(\gamma-1)}$$

has an exponent greater than one, giving the increasing convex adiabatic curve shown. Thus both (c) and (d) are correct. Options involving (a) or (b) fail the constant-temperature test.

Step 4: Conclusion Graphs c and d correctly represent both processes. Hence, the correct option is (B).

Quick Tip to Solve: Identify the axes before judging a process: an isotherm is vertical on a graph with T on the horizontal axis.

46. Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is [4 marks]



(A) $\sqrt{\frac{mv}{K}}$
(B) $\sqrt{\frac{m}{2K}}$
(C) $\sqrt{\frac{mv}{2K}}$
(D) $v\sqrt{\frac{m}{2K}}$

Correct Answer: Collision elasticity is unspecified; option D follows for a perfectly elastic C–A collision.

Solution:

Concept:

- With no horizontal external force, momentum is conserved. The relative kinetic energy of the two equal masses A and B can be stored as spring energy at their maximum compression.
- The speed imparted to A depends on the coefficient of restitution e of the C–A collision. A smooth floor removes friction but does not specify whether that collision is elastic.

Step 1: Retain the unspecified collision parameter Immediately before collision, C has speed v and A is at rest. The spring impulse during the short collision is negligible in the usual impulsive model. Momentum and restitution give

$$v_C + v_A = v, \qquad v_A - v_C = ev.$$

Consequently $v_A = (1+e)v/2$, while B initially remains at rest. The question does not state a value of e.

Step 2: Convert the relative kinetic energy into spring energy For the isolated A–B pair after the impact, their reduced mass is $\mu = m/2$. At maximum compression their relative speed vanishes; their common centre-of-mass speed does not vanish. Hence

$$\frac{1}{2}Kx_{\max}^2 = \frac{1}{2}\frac{m}{2}v_A^2, \qquad x_{\max} = \frac{(1+e)v}{2}\sqrt{\frac{m}{2K}}.$$

This applies while C remains separated until the first maximum; for example e=1 and e=1/2 both satisfy that condition. Their different compressions explicitly show why m, K and v alone do not fix a unique answer.

Step 3: Explain the intended listed result For a perfectly elastic collision, e=1: equal masses exchange velocities, so A leaves with speed v and C stops. Then

$$x_{\max} = v\sqrt{\frac{m}{2K}},$$

which is option (D). Options (A) and (C) contain $\sqrt{v}$, while (B) omits v altogether; they do not have

the dimensions of length. This dimensional selection identifies the intended expression but does not supply the missing collision condition.

Step 4: Conclusion The literal question is underdetermined because the collision elasticity is unspecified. If the C–A collision is perfectly elastic, the intended result is $x_{\max} = v\sqrt{m/(2K)}$, option (D); for a separating collision with no further impact before compression peaks, it is $(1+e)v\sqrt{m/(2K)}/2$.

Quick Tip to Solve: Subtract the centre-of-mass kinetic energy before equating energy to spring compression, and check whether the collision law has been specified.

47. A geostationary satellite is orbiting around an arbitrary planet 'P' at a height of 11R above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is _____. 'P' has the time period of 24 hours. [4 marks]

(A) 3

(B) 5

(C) $6\sqrt{2}$

(D) $\frac{6}{\sqrt{2}}$

Correct Answer: (A) 3

Solution:

Concept:

- For circular satellites orbiting the same planet, gravitational attraction provides the centripetal force, giving $T^2 = 4\pi^2 r^3/(GM)$. Here r is measured from the planet centre.
- A geostationary satellite has the same orbital period as the rotation period of its planet. Thus the first satellite has $T_1 = 24\,\text{h}$.

Step 1: Convert altitudes into orbital radii The planet radius must be added to each stated height above the surface. For the geostationary satellite,

$$r_1 = R + 11R = 12R.$$

For the second satellite,

$$r_2 = R + 2R = 3R.$$

Thus the orbital-radius ratio is $r_2/r_1 = 1/4$, not the altitude ratio $2/11$. The unknown numerical value of R cancels.

Step 2: Apply Kepler's third law Both satellites experience the same planetary mass M, so dividing their period formulas eliminates G and M:

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3.$$

Taking the positive square root because periods are positive gives

$$T_2 = 24\left(\frac{1}{4}\right)^{3/2}\,\mathrm{h} = 24\frac{1}{8}\,\mathrm{h} = 3\,\mathrm{h}.$$

Step 3: Compare with the choices The smaller orbit must have a shorter period, consistent with this result. The values 5, $6\sqrt{2}$ and $6/\sqrt{2}$ hours do not satisfy the required period ratio of 1/8. No value of the planet mass or its physical radius is required, since only ratios for one attracting body enter the calculation.

Step 4: Conclusion The second satellite has an orbital period of 3 h. Hence, the correct option is (A).

Quick Tip to Solve: In Kepler-law ratios, replace every altitude h by the centre-to-satellite radius R+h before raising it to the power 3/2.

48. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be _____. [Given : density of water is 1000 kgm^{-3} and g = 9.8 ms^{-2}] [4 marks]

(A) 1.44×10^7 Nm^{-2}

(B) 1.44×10^9 Nm^{-2}

(C) 1.96×10^7 Nm^{-2}

(D) 2.26×10^9 Nm^{-2}

Correct Answer: (B) 1.44×10^9 Nm^{-2}

Solution:

Concept:

- Hydraulic stress is the increase in hydrostatic pressure, $\Delta P = \rho g h$. Volumetric strain is the magnitude $|\Delta V|/V$, which is dimensionless.
- The ratio of hydraulic stress to hydraulic strain is the bulk modulus, $B = \Delta P/(|\Delta V|/V)$, expressed in pascals or $\mathrm{N\,m^{-2}}$.

Step 1: Calculate the applied hydraulic stress Convert the depth into metres: $h = 2\,\mathrm{km} = 2000\,\mathrm{m}$. The pressure increase relative to the water surface is

$$\Delta P = (1000)(9.8)(2000) = 1.96 \times 10^7\,\mathrm{N\,m^{-2}}.$$

The atmospheric pressure does not enter this increase; the relevant stress comes from the overlying column of water.

Step 2: Convert percentage compression into strain The stated fractional compression is given as a percentage, so

$$\frac{|\Delta V|}{V} = \frac{1.36}{100} = 0.0136.$$

This number is dimensionless. Using 1.36 directly would make the calculated modulus too small by a factor of 100. The volume change itself is negative, but the conventional modulus uses the positive

magnitude of the compressive strain.

Step 3: Form the stress-to-strain ratio Substitute both quantities in the definition:

$$B = \frac{1.96 \times 10^{7}}{0.0136} = 1.441 \times 10^{9}\,\mathrm{N\,m^{-2}}.$$

Rounding gives $1.44 \times 10^{9}\,\mathrm{N\,m^{-2}}$. Option (A) has the wrong exponent, (C) gives the stress without dividing by strain, and (D) does not follow from the specified compression.

Step 4: Conclusion The hydraulic stress-to-strain ratio is $1.44 \times 10^{9}\,\mathrm{N\,m^{-2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For bulk modulus, divide pressure increase by fractional compression; first convert a percentage strain into a decimal.

---

49. A carrier signal C(t) = 25 sin(2.512 × 10^{10}t) is amplitude modulated by a message signal m(t) = 5 sin(1.57 × 10^{8}t) and transmitted through an antenna. What will be the bandwidth of the modulated signal? [4 marks]

(A) 50 MHz

(B) 8 GHz

(C) 1987.5 MHz

(D) 2.01 GHz

Correct Answer: (A) 50 MHz

Solution:

Concept:

- A sinusoid written as $A\sin(\omega t)$ has ordinary frequency $f = \omega/(2\pi)$. The coefficient of t is angular frequency, not frequency in hertz.
- Conventional amplitude modulation produces sidebands at $f_c - f_m$ and $f_c + f_m$, so its transmission bandwidth is $2f_m$. Signal amplitudes set the modulation index but not the sideband separation.

Step 1: Extract the message frequency The message signal is $m(t) = 5\sin(1.57 \times 10^{8}t)$. Therefore

$$f_m = \frac{1.57 \times 10^{8}}{2\pi}\,\mathrm{Hz} \simeq 2.50 \times 10^{7}\,\mathrm{Hz} = 25\,\mathrm{MHz}.$$

The given coefficient 1.57 is approximately $\pi/2$, which makes this conversion especially simple.

Step 2: Locate the two sidebands The carrier frequency is

$$f_c = \frac{2.512 \times 10^{10}}{2\pi} \simeq 4.00 \times 10^{9}\,\mathrm{Hz} = 4.00\,\mathrm{GHz}.$$

Thus the sidebands lie approximately at 3.975 GHz and 4.025 GHz. The message and carrier amplitudes, 5 and 25, give a modulation index of 0.2; they do not change those frequencies.

Step 3: Calculate the occupied bandwidth Take the difference between the upper and lower sideband

frequencies:

$$\mathrm{BW} = (f_c + f_m) - (f_c - f_m) = 2f_m = 50\,\mathrm{MHz}.$$

The carrier frequency cancels, so a value on the gigahertz scale is not the bandwidth here. Option (B) incorrectly has that scale, and (C) and (D) also fail the required sideband separation. The result is the full width covering both sidebands, rather than the width of only one sideband.

Step 4: Conclusion The bandwidth of the amplitude-modulated signal is $50\,\mathrm{MHz}$. Hence, the correct option is (A).

Quick Tip to Solve: Divide the message angular frequency by 2 pi, then double it for AM bandwidth; do not use the carrier frequency or amplitudes.

50. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion . (take ln 2 = 0.693) [4 marks]

(A) 0.69×10^2 kg s^{-1}

(B) 3.3×10^2 kg s^{-1}

(C) 1.16×10^{-2} kg s^{-1}

(D) 5.7×10^{-3} kg s^{-1}

Correct Answer: (C) 1.16×10^{-2} kg s^{-1}

Solution:

Concept:

- For a viscous damping force $F_d = -bv$, the displacement satisfies $m\ddot{z} + b\dot{z} + kz = 0$. The damping constant b has units $\mathrm{kg\,s^{-1}}$.
- The amplitude envelope is $A(t) = A_0 e^{-bt/(2m)}$. Its decay exponent contains $2m$, whereas the energy envelope decays as $e^{-bt/m}$.

Step 1: Use the amplitude ratio The initial and final amplitudes refer to the same oscillator, so their units cancel in the ratio. Convert the elapsed time before using SI units:

$$t = 2 \times 60 = 120\,\mathrm{s}, \qquad \frac{A(t)}{A_0} = \frac{6}{12} = \frac{1}{2}.$$

The reduction is in amplitude, not in energy; energy would have fallen to one quarter of its initial value.

Step 2: Find the damping constant Substitute the measured ratio into the envelope and take natural logarithms:

$$\frac{1}{2} = e^{-120b/(2\times1)} \quad \Longrightarrow \quad \ln 2 = 60b.$$

Hence

$$b = \frac{2m \ln 2}{t} = \frac{2(1)(0.693)}{120} = 0.01155\,\mathrm{kg\,s^{-1}}.$$

Rounding to the precision of the options gives $1.16 \times 10^{-2}\,\mathrm{kg\,s^{-1}}$.

Step 3: Check the competing options The first two options are far too large: they would suppress the amplitude on a time scale much shorter than two minutes. The last option is approximately $\ln 2/120$; that is the amplitude decay rate $b/(2m)$, and omits the factor $2m$ required to obtain the force damping constant.

Step 4: Conclusion The damping constant is $b = 1.16 \times 10^{-2}\,\mathrm{kg\,s^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the amplitude envelope before taking logarithms; the amplitude decay rate is $b/(2m)$, not b/m.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm. Then the thickness of oleic acid layer will be x $\times$ 10^{-14} m. Where x is _____. [4 marks]

Solution:

Concept:

- The volume of a spherical drop is $V_d = 4\pi r^3/3$. The volume fraction of oleic acid is 0.01, so only this fraction of the delivered solution forms the oleic acid layer.
- For a uniform film of area S and thickness h, volume conservation gives $h = V_{\text{acid}}/S$. Use consistent length units until the final conversion.

Step 1: Calculate the volume of solution delivered Cube the given radius first, preserving the power of ten:

$$r^3 = \frac{3}{40\pi} \times 10^{-9}\,\mathrm{cm^3}.$$

Therefore the volume of one spherical drop is

$$V_d = \frac{4\pi}{3}\frac{3}{40\pi} \times 10^{-9} = 10^{-10}\,\mathrm{cm^3}.$$

For 100 identical drops,

$$V_{\text{solution}} = 100 V_d = 10^{-8}\,\mathrm{cm^3}.$$

The cube root in the radius cancels exactly and no numerical approximation to π is needed.

Step 2: Use the oleic acid concentration The specified concentration is $0.01\,\mathrm{cm^3}$ of acid per $1\,\mathrm{cm^3}$ of solution. The volume of acid spread into the film is consequently

$$V_{\text{acid}} = 0.01 \times 10^{-8} = 10^{-10}\,\mathrm{cm^3}.$$

Using the entire solution volume here would overestimate the oleic acid thickness by a factor of 100.

Step 3: Divide by the film area and convert units The monomolecular film covers $S = 4\,\text{cm}^2$. Thus

$$h = \frac{10^{-10}}{4} = 2.5 \times 10^{-11}\,\text{cm}.$$

Since $1\,\text{cm} = 10^{-2}\,\text{m}$,

$$h = 2.5 \times 10^{-13}\,\text{m} = 25 \times 10^{-14}\,\text{m}.$$

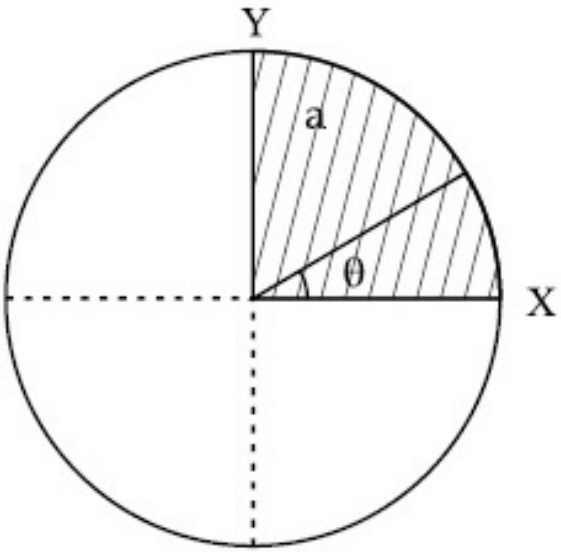
Comparing the coefficient with the form specified in the question gives the required numerical entry.

Step 4: Conclusion The thickness is $25 \times 10^{-14}\,\text{m}$, so $x = 25$ (dimensionless).

Quick Tip to Solve: Cube the drop radius first, multiply by the drop count and acid volume fraction, then divide by the film area.

52. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is _____. (Round off to the Nearest Integer).

[a is an area as shown in the figure] [4 marks]



Solution:

Concept:

- The shaded lamina is a uniform quarter disc of radius a in the first quadrant. Its area is $S = \pi a^2/4$; uniform surface density cancels from the centre-of-mass ratios.
- The coordinates are $\bar{X} = (1/S)\int X\,dS$ and $\bar{Y} = (1/S)\int Y\,dS$. Reflection symmetry about $Y = X$ requires $\bar{X} = \bar{Y}$.

Step 1: Set up the area moment Use polar coordinates in the shaded quadrant: $X = r\cos\theta$, $Y = r\sin\theta$, and $dS = r\,dr\,d\theta$, with $0 \le r \le a$ and $0 \le \theta \le \pi/2$. The letter a labels the radius in the figure, which is the length needed for the stated coordinate expression. For a uniform sheet,

$dm = \sigma\, dS$ and the quadrant mass is $\sigma\pi a^2/4$.

Step 2: Evaluate one coordinate The first area moment about the Y-axis is

$$\int X\, dS = \int_0^{\pi/2} \int_0^a r^2 \cos\theta\, dr\, d\theta.$$

The radial and angular factors separate:

$$\int_0^a r^2\, dr = \frac{a^3}{3}, \qquad \int_0^{\pi/2} \cos\theta\, d\theta = 1.$$

Dividing by the quadrant area therefore gives

$$\bar{X} = \frac{a^3/3}{\pi a^2/4} = \frac{4a}{3\pi}.$$

The result has units of length, as a coordinate must.

Step 3: Use symmetry and identify the coefficient The same integration with $\sin\theta$ gives $\bar{Y} = 4a/(3\pi)$, also guaranteed by the symmetry of the quadrant. Compare

$$(\bar{X}, \bar{Y}) = \left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$$

with the given pair $(xa/(3\pi), xa/(3\pi))$. Thus the coefficient is exactly 4, so rounding to the nearest integer does not change it.

Step 4: Conclusion The centre of mass is $(4a/(3\pi), 4a/(3\pi))$, and the required dimensionless value is $x = 4$.

Quick Tip to Solve: For a uniform quarter disc, integrate one Cartesian coordinate using $dS = r\, dr\, d\theta$; obtain the other by symmetry.

53. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2^{rd}}{3}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. The value of 'x' is _____. [4 marks]

Solution:

Concept:

- For refraction from index n_1 to index n_2 at a spherical surface, $n_2/v - n_1/u = (n_2 - n_1)/R$, with Cartesian signed distances.
- Frequency is unchanged at the interface. Hence $n_2/n_1 = \lambda_1/\lambda_2$, so the stated wavelength ratio determines the refractive index ratio.

Step 1: Determine the object distance and refractive index Take incident light to travel from left to right, with the refracting surface as origin. The real image is behind the surface, so $v = +10\,\text{m}$. Its

distance is two thirds of the object distance:

$$10 = \frac{2}{3}|u| \quad \Longrightarrow \quad |u| = 15\,\text{m}, \qquad u = -15\,\text{m}.$$

The object is in air, so take $n_1 = 1$. Since $\lambda_2 = (2/3)\lambda_1$,

$$n_2 = \frac{\lambda_1}{\lambda_2} = \frac{3}{2}.$$

A shorter wavelength inside corresponds to the larger refractive index.

Step 2: Apply the surface-refraction equation Substitute the signed distances without changing the minus sign in the formula:

$$\frac{3/2}{10} - \frac{1}{-15} = \frac{3/2 - 1}{R}.$$

The left-hand side is positive because both the real image term and the contribution from the negative object distance are positive:

$$\frac{3}{20} + \frac{1}{15} = \frac{9}{60} + \frac{4}{60} = \frac{13}{60}.$$

Thus

$$\frac{1/2}{R} = \frac{13}{60} \quad \Longrightarrow \quad R = \frac{30}{13}\,\text{m}.$$

This positive radius is consistent with the convex surface facing the incident object.

Step 3: Compare with the requested form The question writes the curved-surface radius as $R = (x/13)\,\text{m}$. Comparing this with $30/13\,\text{m}$ fixes the numerator directly. The coefficient x is dimensionless; the radius itself carries metres.

Step 4: Conclusion The radius is $R = 30/13\,\text{m}$, hence $x = 30$.

Quick Tip to Solve: Use the wavelength ratio to find the index, then assign $u < 0$ and $v > 0$ before substituting into the surface formula.

54. The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3 m is E. The electric field intensity produced by the radiation coming from 60W at the same distance is $\sqrt{\frac{x}{5}}$E. Where the value of x = _____. [4 marks]

Solution:

Concept:

- For a source radiating uniformly, the intensity at radius r is $I = P/(4\pi r^2)$, where P is its radiated power.
- The average electromagnetic intensity is proportional to the square of the electric field amplitude: $I = \frac{1}{2}c\varepsilon_0 E_0^2$. The same square-law relation holds when comparing rms fields.

Step 1: Compare intensities at the same distance Both bulbs are observed at $r = 3\,\text{m}$, so the geometrical dilution factor is identical. In the radiation model specified by the question, the powers

therefore give

$$\frac{I_{60}}{I_{100}} = \frac{60/(4\pi r^2)}{100/(4\pi r^2)} = \frac{60}{100} = \frac{3}{5}.$$

There is no need to evaluate the surface area of the sphere numerically because it cancels exactly.

Step 2: Convert the intensity ratio to a field ratio The electric and magnetic fields carry the wave energy, and intensity depends quadratically on either field amplitude. Hence

$$\left(\frac{E_{60}}{E_{100}}\right)^2 = \frac{I_{60}}{I_{100}} = \frac{3}{5}.$$

Taking the positive square root for the field magnitude gives

$$E_{60} = \sqrt{\frac{3}{5}}\, E_{100} = \sqrt{\frac{3}{5}}\, E.$$

Scaling the field directly as $60/100$ would incorrectly treat this quadratic relation as linear.

Step 3: Match the stated coefficient The field in the question is written as $\sqrt{x/5}\, E$. Equating the nonnegative field ratios and squaring gives

$$\frac{x}{5} = \frac{3}{5} \quad \Longrightarrow \quad x = 3.$$

This also has the expected qualitative behaviour: the lower-power bulb produces a smaller field at the same distance, while its field ratio is the square root of its power ratio.

Step 4: Conclusion The field is $\sqrt{3/5}\, E$, so the required value is $x = 3$ (dimensionless).

Quick Tip to Solve: At a fixed distance, power ratios equal intensity ratios; take their square root to compare electric field magnitudes.

55. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3 \frac{N}{C}$. The flux of this field through a rectangular surface area 0.4 m^2 parallel to the Y-Z plane is _____ Nm^2C^{-1}. [4 marks]

Solution:

Concept:

- For a uniform electric field and a plane surface, electric flux is $\Phi = \vec{E} \cdot \vec{S}$, where the area vector is perpendicular to the surface and has magnitude equal to its area.
- A surface parallel to the Y-Z plane has a normal along the X-axis. Only E_x contributes; the Y-component is tangential to the surface.

Step 1: Choose the area vector The rectangular surface has area $S = 0.4\,\mathrm{m}^2$. Taking the normal in the positive X direction gives

$$\vec{S} = 0.4\,\hat{i}\,\mathrm{m}^2.$$

The field is

$$\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}, \qquad E_0 = 4.0 \times 10^3\,\mathrm{N\,C^{-1}}.$$

The absence of an $\hat{k}$ component is irrelevant here; neither a Y nor a Z component can pass normally through this plane.

Step 2: Take the scalar product Use $\hat{i}\cdot\hat{i}=1$ and $\hat{j}\cdot\hat{i}=0$:

$$\Phi=\left(\frac{2}{5}E_0\hat{i}+\frac{3}{5}E_0\hat{j}\right)\cdot(0.4\hat{i})=\frac{2}{5}E_0(0.4).$$

The normal electric field is

$$E_x=\frac{2}{5}(4.0\times10^3)=1.6\times10^3\,\mathrm{N\,C^{-1}}.$$

Therefore

$$\Phi=(1.6\times10^3)(0.4)=640\,\mathrm{N\,m^2\,C^{-1}}.$$

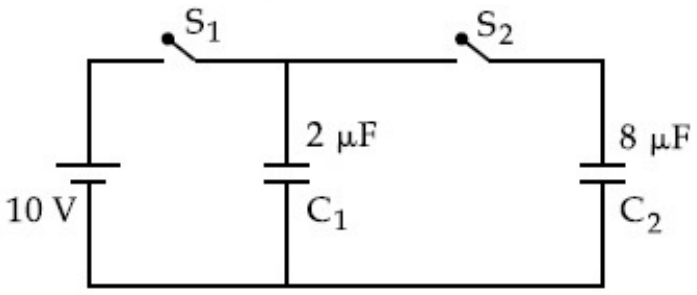
Using the full field magnitude would include a component that runs along the rectangle instead of through it.

Step 3: Interpret the orientation convention For the chosen positive X normal the flux is positive. Reversing the area vector would reverse its sign without changing its magnitude. Since the question requests a numerical flux with no negative normal specified, the conventional positive-normal value, and hence the magnitude, is the required entry.

Step 4: Conclusion The electric flux is $\Phi=640\,\mathrm{N\,m^2\,C^{-1}}$ for the positive X-directed normal.

Quick Tip to Solve: For a plane parallel to a coordinate plane, select the electric field component along the missing coordinate axis.

56. A 2μF capacitor C_1 is first charged to a potential difference of 10V using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8 μF. The charge in C_2 on equilibrium condition is _____ μC. (Round off to the Nearest Integer) [4 marks]



Solution:

Concept:

- An isolated capacitor initially stores charge $Q=CV$. Once the battery is disconnected, the total charge on the connected positive plates is conserved.
- After two capacitors are connected in parallel and equilibrium is reached, both have the same potential difference V_f. Their total capacitance is C_1+C_2.

Step 1: Find the initial charge Initially only $C_1 = 2\,\mu\text{F}$ is charged by the $10\,\text{V}$ battery. Its charge is

$$Q_0 = C_1 V_0 = (2\,\mu\text{F})(10\,\text{V}) = 20\,\mu\text{C}.$$

The other capacitor is uncharged, so its initial contribution to the total positive-plate charge is zero. Opening the battery connection is essential: the battery cannot provide further charge during redistribution.

Step 2: Calculate the final common potential When the connection between the capacitors is made, charge flows until their potential differences become equal. Charge conservation then gives

$$Q_0 = C_1 V_f + C_2 V_f = (C_1 + C_2) V_f.$$

With $C_2 = 8\,\mu\text{F}$,

$$V_f = \frac{20\,\mu\text{C}}{(2+8)\,\mu\text{F}} = 2\,\text{V}.$$

This lies below the original $10\,\text{V}$, as expected when the stored charge is spread across a larger total capacitance.

Step 3: Find the charge on the second capacitor The requested charge is on C_2, rather than the total of both capacitors:

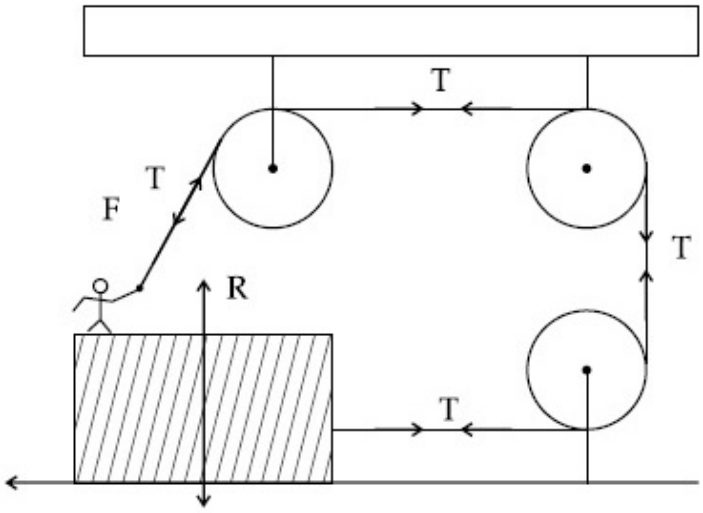
$$Q_2 = C_2 V_f = (8\,\mu\text{F})(2\,\text{V}) = 16\,\mu\text{C}.$$

The remaining charge on C_1 is $2 \times 2 = 4\,\mu\text{C}$, and $16 + 4 = 20\,\mu\text{C}$, confirming conservation. Use charge conservation rather than capacitor energy conservation during redistribution.

Step 4: Conclusion The equilibrium charge on C_2, rounded to the nearest integer, is $16\,\mu\text{C}$.

Quick Tip to Solve: After disconnecting the battery, conserve total charge and divide it by the combined capacitance to obtain the common voltage.

57. A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is _____ N. (Round off to the Nearest Integer) [Take g = 10 ms^{-2}] [4 marks]



Solution:

Concept:

- The ideal rope has uniform tension T. Its end pulls the wood horizontally, and the free segment pulls the boy towards the upper pulley.
- The limiting friction is $\mu_s N$. Both the horizontal and vertical components of the pull on the boy must be included when finding the greatest possible tension.

Step 1: Write equilibrium for a general pulling direction Let the free rope segment make angle θ above the horizontal, with $0 \le \theta \le 90°$, as represented by the upward-right pull. Treat the boy and wood together. Their total weight is $(4+5)g = 90\,\text{N}$. The upward rope component is $T\sin\theta$, so

$$N = 90 - T\sin\theta.$$

The rope pulls the boy horizontally by $T\cos\theta$ and the wood by T; both act to the right. Hence the required opposing friction is $T(1+\cos\theta)$, not just the horizontal component on the boy.

Step 2: Find the largest pull allowed by friction At the limiting condition,

$$T(1+\cos\theta) = 0.5(90 - T\sin\theta),$$

$$T = \frac{45}{1+\cos\theta+0.5\sin\theta}.$$

To find the maximum possible force, choose the pulling direction giving the smallest denominator. On $0 \le \theta \le 90°$, the expression $\cos\theta + 0.5\sin\theta$ has its minimum value 0.5 at the vertical direction: its interior stationary point is a maximum, and the endpoint values are 1 and 0.5. Thus

$$T_{\max} = \frac{45}{1.5} = 30\,\text{N}.$$

In this limiting vertical-pull configuration, $N = 90 - 30 = 60\,\text{N}$, so $\mu_s N = 30\,\text{N}$, exactly balancing the rope pull on the wood. The boy also retains contact since the support from the wood is $40 - 30 = 10\,\text{N} > 0$.

Step 3: Conclusion The maximum possible force exerted by the boy is $\boxed{30\,\text{N}}$, so the requested nearest-integer entry is 30.

Quick Tip to Solve: For an inclined free rope segment, include its horizontal pull on the boy as well as its reduction of the floor load; maximise only after writing both components.

---

58. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be _____. (Round off to the Nearest Integer) [Take g = 10 ms^{-2}] [4 marks]

Solution:

Concept:

- A pulling force at angle θ above the horizontal has components $F\cos\theta$ and $F\sin\theta$. Its upward component reduces the normal reaction.
- At impending sliding, $F\cos\theta = \mu_s N$, with $N = mg - F\sin\theta$. Minimise the required force by choosing its direction.

Step 1: Express the force in terms of its direction Let the body be on the point of moving to the right, with the force directed upwards and rightwards. The vertical acceleration is zero, so

$$N = mg - F\sin\theta.$$

The horizontal component must just equal the limiting static friction. Therefore

$$F\cos\theta = \mu_s(mg - F\sin\theta),$$

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}.$$

A downward inclined force would increase the normal reaction, making motion harder rather than providing the minimum possible pull.

Step 2: Maximise the denominator For constants a, b, the maximum of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2+b^2}$. Here

$$\cos\theta + \mu_s\sin\theta \le \sqrt{1+\mu_s^2}.$$

Equality occurs for $\tan\theta = \mu_s = 1/\sqrt{3}$, namely $\theta = 30^\circ$. Hence

$$F_{\min} = \frac{\mu_s mg}{\sqrt{1+\mu_s^2}} = \frac{10/\sqrt{3}}{\sqrt{1+1/3}} = \frac{10/\sqrt{3}}{2/\sqrt{3}} = 5\,\text{N}.$$

The reaction at this force is $10 - 5\sin 30^\circ = 7.5\,\text{N} > 0$, confirming that the body remains in contact and that the friction model applies.

Step 3: Conclusion The minimum applied force is $\boxed{5\,\mathrm{N}}$, so the requested nearest-integer entry is 5.

Quick Tip to Solve: For a freely chosen pulling direction on a rough horizontal plane, use $F_{\min} = \mu mg/\sqrt{1+\mu^2}$, attained at $\tan\theta = \mu$.

59. A particle of mass m moves in a circular orbit in a central potential field U(r) = $U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals r_n vary with $n^{\frac{1}{\alpha}}$, where α is _____. [4 marks]

Solution:

Concept:

- The central force is $F_r = -dU/dr$. For $U = U_0 r^4$ the inward force magnitude is $4U_0 r^3$, taking $U_0 > 0$ for circular bound motion.
- A circular orbit obeys $mv^2/r = |F_r|$, while Bohr angular-momentum quantisation gives $mvr = n\hbar$.

Step 1: Obtain the speed-radius relation Differentiate the specified potential rather than using the inverse-square force law appropriate to a Coulomb field:

$$F_r = -\frac{d}{dr}(U_0 r^4) = -4U_0 r^3.$$

The negative sign denotes the inward direction. Equating its magnitude to the centripetal-force requirement gives

$$\frac{mv^2}{r} = 4U_0 r^3, \qquad mv^2 = 4U_0 r^4.$$

Thus v is proportional to r^2 for fixed mass and potential coefficient. Neither the hydrogenic $r \propto n^2$ rule nor a fixed-speed orbit applies to this different potential.

Step 2: Impose angular-momentum quantisation From $mvr = n\hbar$, the orbital speed can also be written as

$$v = \frac{n\hbar}{mr}.$$

Substitute this in the circular-orbit equation:

$$m\left(\frac{n\hbar}{mr}\right)^2 = 4U_0 r^4, \qquad \frac{n^2\hbar^2}{m} = 4U_0 r^6.$$

Therefore

$$r_n = \left(\frac{\hbar^2}{4mU_0}\right)^{1/6} n^{1/3}.$$

The coefficient is independent of the positive quantum number n; only its power determines the requested scaling. Comparing this with $r_n \propto n^{1/\alpha}$ gives $1/\alpha = 1/3$.

Step 3: Conclusion The radius varies as $n^{1/3}$, and hence $\boxed{\alpha = 3}$. The exponent parameter is dimensionless.

Quick Tip to Solve: Combine $mv^2/r \propto r^3$ with $mvr = n\hbar$: the resulting $r^6 \propto n^2$ immediately fixes the radius exponent.

60. Seawater at a frequency f = 9×10^2 Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\ \Omega$m. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source V(t) = $V_0 \sin(2\pi ft)$. Then the conduction current density becomes 10^x times the displacement current density after time t = $\frac{1}{800}$s. The value of x is _____. (Given : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2C^{-2})

[4 marks]

Solution:

Concept:

- Ohmic conduction gives $J_c = \sigma E = E/\rho$, where ρ is resistivity. Displacement current density is $J_d = \varepsilon\, dE/dt$.
- For sinusoidal voltage across fixed parallel plates, $E(t) = E_0 \sin\omega t$, where $\omega = 2\pi f$. Their instantaneous ratio includes a factor $\tan\omega t$.

Step 1: Form the ratio at the stated instant The plate separation is constant, so the electric field follows the applied voltage. Consequently

$$J_c = \frac{E_0}{\rho}\sin\omega t, \qquad J_d = \varepsilon\omega E_0 \cos\omega t.$$

Dividing eliminates the unknown voltage amplitude and plate separation:

$$\frac{J_c}{J_d} = \frac{\tan\omega t}{\rho\varepsilon\omega}.$$

With $f = 900\,\text{Hz}$ and $t = 1/800\,\text{s}$,

$$\omega t = 2\pi\frac{900}{800} = \frac{9\pi}{4}, \qquad \tan\omega t = 1.$$

Both sine and cosine are positive at this phase, so the ratio is positive.

Step 2: Evaluate the material and frequency factor The supplied electrostatic constant yields

$$\varepsilon_0 = \frac{1}{36\pi \times 10^9}\,\text{F m}^{-1}.$$

Substitute $\rho = 0.25\,\Omega\,\text{m}$, $\varepsilon = 80\varepsilon_0$, and $\omega = 1800\pi\,\text{s}^{-1}$:

$$\rho\varepsilon\omega = \frac{1}{4} \times \frac{80}{36\pi \times 10^9} \times 1800\pi = 10^{-6}.$$

Hence

$$\frac{J_c}{J_d} = \frac{1}{10^{-6}} = 10^6.$$

This ratio is dimensionless: resistivity times permittivity has dimensions of time, cancelled by angular frequency. Comparing the calculated ratio with the specified 10^x determines the required exponent.

Step 3: Conclusion The conduction current density is 10^6 times the displacement current density at the specified instant, so $\boxed{x = 6}$ (dimensionless).

Quick Tip to Solve: Use the instantaneous sine/cosine ratio, not only the current amplitudes. Here the specified phase makes that extra tangent factor exactly one.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61.

$$C_{12}H_{22}O_{11} + H_2O \xrightarrow{Enzyme\ A} \underset{Glucose}{C_6H_{12}O_6} + \underset{Fructose}{C_6H_{12}O_6}$$

$$\underset{Glucose}{C_6H_{12}O_6} \xrightarrow{Enzyme\ B} 2C_2H_5OH + 2CO_2$$

In the above reactions, the enzyme A and enzyme B respectively are : [4 marks]

(A) Zymase and Invertase

(B) Invertase and Amylase

(C) Amylase and Invertase

(D) Invertase and Zymase

Correct Answer: (D) Invertase and Zymase

Solution:

Concept:

- Enzymes are substrate-specific biological catalysts. Sucrose hydrolysis breaks its glycosidic linkage and produces one molecule each of glucose and fructose.
- Invertase catalyses sucrose hydrolysis; zymase catalyses alcoholic fermentation of glucose to ethanol and carbon dioxide.

Step 1: Identify the hydrolysis enzyme The first reaction consumes water and converts the disaccharide $C_{12}H_{22}O_{11}$ into two monosaccharides, glucose and fructose. It is the hydrolysis of sucrose, catalysed by invertase (also called sucrase):

$$C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{invertase}} C_6H_{12}O_6 + C_6H_{12}O_6.$$

The two products have the same molecular formula but are different sugars. The enzyme A must therefore be invertase. Zymase performs fermentation rather than this hydrolysis. Amylase acts on starch and cannot replace invertase in the displayed conversion of sucrose.

Step 2: Identify the fermentation enzyme and compare the pairs The second conversion transforms glucose into two molecules of ethanol and two of carbon dioxide:

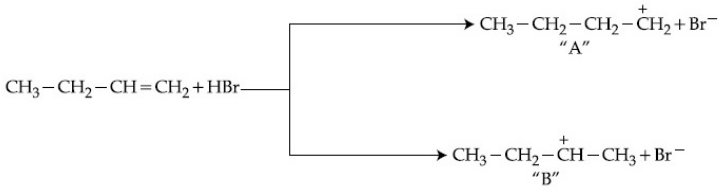
$$C_6H_{12}O_6 \xrightarrow{\text{zymase}} 2\,C_2H_5OH + 2\,CO_2.$$

This is alcoholic fermentation, catalysed by the enzyme complex zymase in yeast, so enzyme B is zymase. The ordered pair is consequently invertase followed by zymase. Option (A) reverses these functions. Option (B) has the correct first enzyme but substitutes starch-digesting amylase for the fermentation enzyme. Option (C) assigns amylase to sucrose hydrolysis and invertase to fermentation, so neither assignment gives the displayed pair of reactions.

Step 3: Conclusion Enzyme A is invertase and enzyme B is zymase. Hence, the correct option is (D).

Quick Tip to Solve: Associate invertase with sucrose becoming glucose plus fructose, and zymase with glucose becoming ethanol plus carbon dioxide.

62. Choose the correct statement regarding the formation of carbocations A and B given. [4 marks]



(A) Carbocation A is more stable and formed relatively at faster rate

(B) Carbocation B is more stable and formed relatively at slow rate

(C) Carbocation B is more stable and formed relatively at faster rate

(D) Carbocation A is more stable and formed relatively at slow rate

Correct Answer: (C) Carbocation B is more stable and formed relatively at faster rate

Solution:

Concept:

- Electrophilic addition of HBr to an unsymmetrical alkene can form different carbocation intermediates depending on which alkene carbon accepts the proton.
- Alkyl groups stabilise carbocations through their electron-releasing inductive effect and hyperconjugation. A secondary carbocation is more stable than a comparable primary carbocation.

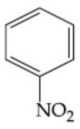
Step 1: Classify the two displayed intermediates The initial alkene is but-1-ene, $CH_3CH_2CH{=}CH_2$. In A, the positive charge lies on its terminal carbon: $CH_3CH_2CH_2CH_2^+$. That carbon is bonded to only one other carbon, so A is a primary carbocation. In B, the positive charge is on the internal carbon: $CH_3CH_2CH^+CH_3$. It is bonded to two carbon groups, making B secondary. Both arise from proton addition to the double bond; the bromide ion is the counter-ion, not the source of the relative stabilisation.

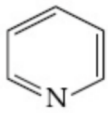
Step 2: Connect stability to the preferred formation pathway The two alkyl groups attached to the charged carbon in B provide greater electron donation and hyperconjugative stabilisation than the single alkyl group in A. Therefore B is lower in energy. Carbocation formation in electrophilic addition is favoured along the pathway having the more stabilised developing positive charge; this pathway has a lower activation barrier and is faster. Thus B is both more stable and formed relatively faster. Options (A) and (D) incorrectly assign the greater stability to the primary carbocation A. Option (B) identifies the more stable ion correctly but incorrectly calls its formation relatively slow. The stability and rate statements agree only in option (C).

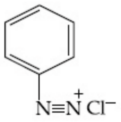
Step 3: Conclusion Carbocation B is the secondary, more stable intermediate and is formed relatively faster. Hence, the correct option is (C).

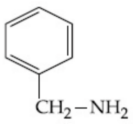
Quick Tip to Solve: Count carbon groups directly bonded to the positively charged carbon; the secondary intermediate explains the normal Markovnikov addition pathway.

63. Nitrogen can be estimated by Kjeldahl's method for which of the following compound? [4 marks]

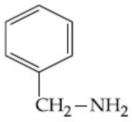
(A) Nitrobenzene, $C_6H_5NO_2$. 

(B) Pyridine. 

(C) Benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$. 

(D) Benzylamine, $C_6H_5CH_2NH_2$. 

Correct Answer: (D)

Benzylamine, $C_6H_5CH_2NH_2$. 

Solution:

Concept:

- Kjeldahl's method estimates nitrogen after digestion converts suitable organic nitrogen into ammonium sulfate; subsequent alkali treatment liberates ammonia for measurement.
- The usual method does not estimate nitrogen in nitro groups, diazonium groups, or heterocyclic rings because these nitrogen atoms are not quantitatively converted into ammonium sulfate.

Step 1: Inspect the chemical environment of each nitrogen Option (A) is nitrobenzene, $C_6H_5NO_2$, whose nitrogen belongs to a nitro group. Option (B) is pyridine; its nitrogen is a constituent of the aromatic six-membered ring. Option (C) is benzenediazonium chloride, $C_6H_5N_2{}^+Cl^-$, containing a diazonium group. These are the standard exclusions for the ordinary Kjeldahl digestion procedure. Merely containing nitrogen is therefore insufficient for a compound to be suitable for this estimation method.

Step 2: Select the nitrogen that can be recovered as ammonia Option (D) is benzylamine, $C_6H_5CH_2NH_2$. Its nitrogen is present as an ordinary primary amino group attached to the side chain, rather than within the aromatic ring. On sulfuric-acid digestion this nitrogen is converted into ammonium sulfate. Treating the digest with excess alkali releases ammonia:

$$(NH_4)_2SO_4 + 2\,NaOH \longrightarrow 2\,NH_3 + Na_2SO_4 + 2\,H_2O.$$

The ammonia can then be trapped in a measured amount of acid to determine nitrogen. Thus (D) satisfies the essential quantitative-conversion requirement. The nitro nitrogen in (A), ring nitrogen in (B), and diazonium nitrogen in (C) do not satisfy it in the ordinary method, which rules out all three alternatives.

Step 3: Conclusion Benzylamine, shown in option (D), is suitable for nitrogen estimation by Kjeldahl's method. Hence, the correct option is (D).

Quick Tip to Solve: For Kjeldahl estimation, first exclude nitro, diazonium and heterocyclic nitrogen; a benzene ring elsewhere in an amine does not itself prevent the method.

64. Amongst the following, the linear species is : [4 marks]

(A) O_3

(B) Cl_2O

(C) N_3^-

(D) NO_2

Correct Answer: (C) N_3^-

Solution:

Concept:

- VSEPR theory predicts molecular shape from the number of bonding regions and lone pairs around the central atom. A multiple bond counts as one region for this purpose.
- Two bonding regions without a central lone pair arrange themselves at $180°$; this gives a linear species. Lone pairs or an odd electron can produce a bent triatomic species.

Step 1: Identify the arrangement in azide The azide ion, $N_3{}^-$, has three nitrogen atoms connected in a chain. A useful resonance contributor is $^-N{=}N^+{=}N^-$, with two double bonds to the central nitrogen. The central atom has two bonding regions and no lone pair. Its sigma-bond framework is therefore sp-hybridised, giving a linear $N{-}N{-}N$ arrangement. Resonance redistributes the charge and bond order; it does not turn the ion into a bent molecule.

Step 2: Compare the other three species Ozone, O_3, has two bonding regions and one lone pair on its central oxygen. Its electron-domain arrangement is trigonal planar but its molecular shape is bent. In Cl_2O, oxygen is the central atom and has two bonds and two lone pairs, so the molecular shape is also bent. Neutral NO_2 is an odd-electron molecule with a bent structure rather than the linear structure of $NO_2{}^+$. Thus none of these three alternatives has the required two-region linear framework. In particular, confusing neutral nitrogen dioxide with nitronium would select the wrong species.

Step 3: Conclusion The linear species is the azide ion, $N_3{}^-$, with a central bond angle of $180°$. Hence, the correct option is (C).

Quick Tip to Solve: Count electron regions on the central atom; distinguish neutral NO_2 from linear $NO_2{}^+$ before assigning a shape.

65. The correct pair(s) of the ambident nucleophiles is(are)

(A) AgCN/KCN

(B) RCOOAg/RCOOK

(C) $AgNO_2/KNO_2$

(D) AgI/KI [4 marks]

(A) (B) and (C) only

(B) (B) only

(C) (A) and (C) only

(D) (A) only

Correct Answer: (C) (A) and (C) only

Solution:

Concept:

- An ambident nucleophile possesses two different atoms through which it can form a new bond to an electrophilic carbon. Cyanide and nitrite are standard examples.
- The ionic or covalent character of a salt can favour a different attacking end. Equivalent oxygen atoms in a carboxylate do not give the usual distinct ambident products tested here.

Step 1: Examine cyanide and nitrite The cyanide ion can attack through carbon to form a nitrile, R−CN, or through nitrogen to form an isocyanide, R−NC. With an alkyl halide, ionic KCN generally gives the nitrile, whereas predominantly covalent AgCN favours the isocyanide. Thus pair (A) displays ambident behaviour. Similarly, nitrite has nitrogen and oxygen donor ends. KNO_2 generally gives an alkyl nitrite, R−ONO, whereas $AgNO_2$ favours a nitroalkane, $R-NO_2$. Hence pair (C) also belongs to the required category.

Step 2: Reject the remaining pairs and select the combination In pair (B), the carboxylate group $RCOO^-$ has resonance-equivalent oxygen atoms. Changing between its silver and potassium salts does not supply the carbon-versus-nitrogen or nitrogen-versus-oxygen ambident alternatives described above. Pair (D) contains iodide, a single-atom nucleophile; it has no second donor atom at all. The qualifying listed pairs are therefore exactly (A) and (C). The option containing (B) and (C) includes a nonqualifying pair, the option containing (B) alone misses both valid pairs, and the option containing (A) alone omits nitrite.

Step 3: Conclusion The cyanide and nitrite pairs are the required ambident nucleophile pairs. Hence, the correct option is (C).

Quick Tip to Solve: For ambident behaviour, name the two different donor atoms and the two different linkage products; merely changing the metal salt is not enough.

66. Given below are two statements :

Statement I : 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol.

Statement II : n-alkanes can be easily oxidised to corresponding alcohols with $KMnO_4$.

Choose the correct option : [4 marks]

(A) Both statement I and statement II are incorrect

(B) Both statement I and statement II are correct

(C) Statement I is correct but statement II is incorrect

(D) Statement I is incorrect but statement II is correct

Correct Answer: (C) Statement I is correct but statement II is incorrect

Solution:

Concept:

- Alkanes are generally resistant to ordinary oxidising agents because their carbon-carbon and carbon-hydrogen bonds are relatively unreactive.
- A tertiary carbon bearing hydrogen is a special site: controlled oxidation can replace its hydrogen by a hydroxyl group and produce a tertiary alcohol.

Step 1: Test Statement I at the tertiary carbon Write 2-methylbutane as $CH_3-CH(CH_3)-CH_2-CH_3$. Carbon 2 is bonded to three carbon groups and one hydrogen, so it is a tertiary carbon. Under the stated oxidation, that tertiary hydrogen is replaced by a hydroxyl group without changing the carbon skeleton. The product is $CH_3-C(OH)(CH_3)-CH_2-CH_3$, named 2-methylbutan-2-ol. This is the special controlled oxidation of an alkane containing a tertiary hydrogen. Therefore Statement I is correct; the alcohol group is attached to carbon 2, not to a terminal carbon.

Step 2: Test the general claim in Statement II A normal alkane contains primary and secondary carbon-hydrogen sites but no tertiary carbon. It is not generally and easily converted into the corresponding alcohol simply by treatment with $KMnO_4$. Such a broad claim ignores the resistance of alkanes to ordinary oxidation and the special role of the tertiary hydrogen in Statement I. Strong or uncontrolled oxidation also need not stop cleanly at a single alcohol. Thus Statement II is incorrect. The options saying both statements are true or both are false each misclassify one statement, while the option saying only Statement II is correct reverses both judgements.

Step 3: Conclusion Statement I is correct, but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Locate a tertiary hydrogen before applying this controlled alkane oxidation; do not extend that special reaction to every straight-chain alkane.

67. During which of the following processes, does entropy decrease?

(A) Freezing of water to ice at $0^\circ C$

(B) Freezing of water to ice at $-10^\circ C$

(C) $N_2(g) + 3\,H_2(g) \longrightarrow 2\,NH_3(g)$

(D) Adsorption of CO (g) on lead surface.

(E) Dissolution of NaCl in water

Choose the correct answer from the options given below: [4 marks]

(A) (A), (C) and (E) only

(B) (B) and (C) only

(C) (A), (B), (C) and (D) only

(D) (A) and (E) only

Correct Answer: (C) (A), (B), (C) and (D) only

Solution:

Concept:

- Entropy measures the dispersal of energy and the number of accessible microscopic arrangements. Freezing and restricting gaseous motion generally lower the entropy of the system.
- For qualitative comparison, a decrease in the number of gas molecules or adsorption of a gas onto a surface usually gives a negative system entropy change.

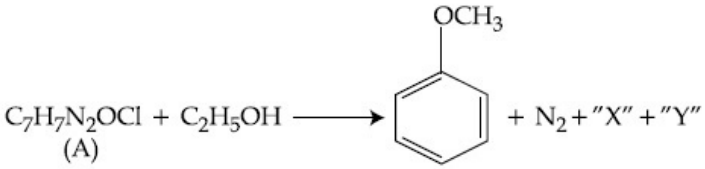
Step 1: Classify freezing and ammonia formation In both (A) and (B), liquid water becomes crystalline ice. The more ordered solid has fewer accessible molecular arrangements, so the entropy change of the water is negative both at $0^\circ C$ and at $-10^\circ C$. At the equilibrium freezing temperature, the total entropy change of system plus surroundings can be zero, but the entropy of the freezing water itself still decreases. In process (C), $N_2(g) + 3\,H_2(g) \longrightarrow 2\,NH_3(g)$, four moles of gas form two moles of gas. This reduces the freedom of molecular distribution and gives a decrease in entropy.

Step 2: Classify adsorption and dissolution In (D), carbon monoxide molecules leave free gaseous motion and become attached to a lead surface. This restriction of translational freedom lowers their entropy. In (E), dissolving NaCl disperses its ions through water; for this process the system entropy increases rather than decreases. Hence the decreasing-entropy processes are (A), (B), (C) and (D). The alternatives containing (E) incorrectly include dissolution, and the alternative containing only (B) and (C) omits freezing at $0^\circ C$ and gas adsorption. The result concerns the named system processes, not a claim that the entropy of the universe decreases.

Step 3: Conclusion The two freezing processes, ammonia formation and adsorption of carbon monoxide decrease entropy. Hence, the correct option is (C).

Quick Tip to Solve: Separate the entropy of the system from that of the universe: water loses entropy on freezing even when freezing occurs reversibly at equilibrium.

68. In the above reaction, the structural formula of (A), "X" and "Y" respectively are : [4 marks]



(A) p-Chlorobenzenediazonium methoxide, CH_3CHO, H_2O

(B) p-Methoxybenzenediazonium chloride, CH_3CHO, HCl

(C) p-Chlorobenzenediazonium methoxide, ethylene oxide, HCl

(D) p-Methoxybenzenediazonium chloride, ethylene oxide, H_2O

Correct Answer: (B) p-Methoxybenzenediazonium chloride, CH_3CHO, HCl

Solution:

Concept:

- Ethanol reduces an arenediazonium salt by replacing the diazonium group with hydrogen; nitrogen gas is evolved while ethanol is oxidised to ethanal.
- For a diazonium chloride, the products also include HCl: $ArN_2^+Cl^- + C_2H_5OH \longrightarrow ArH + N_2 + CH_3CHO + HCl$.

Step 1: Identify the aromatic precursor The aromatic product shown is anisole, $C_6H_5OCH_3$. Its methoxy group must already be present on the starting aromatic ring because ethanol replaces the diazonium group with hydrogen rather than introducing a methoxy group. The formula $C_7H_7N_2OCl$ is consistent with methoxybenzenediazonium chloride: the ring bears OCH_3 and N_2^+, with Cl^- as counterion. Among the drawn choices, the relevant structure is para-methoxybenzenediazonium chloride. The choices showing a ring chlorine and a methoxide counterion do not have the required retained methoxy substituent.

Step 2: Identify the two non-nitrogen products In the reduction, ethanol loses two hydrogen atoms and becomes ethanal, CH_3CHO. One hydrogen replaces the diazonium group on the ring, and the other combines with chloride to form HCl. Thus $X = CH_3CHO$ and $Y = HCl$. The balanced overall reaction is

$$p\text{-}CH_3OC_6H_4N_2Cl + C_2H_5OH \longrightarrow C_6H_5OCH_3 + N_2 + CH_3CHO + HCl.$$

Ethylene oxide is not the oxidation product in this reaction, so the two choices containing that cyclic ether are rejected. Water also does not supply the required chloride-containing product in the correct reduction equation.

Step 3: Conclusion The precursor is para-methoxybenzenediazonium chloride; the other products are ethanal and hydrochloric acid. Hence, the correct option is (B).

Quick Tip to Solve: When ethanol replaces an aryl diazonium group by hydrogen, pair the organic reduction with ethanol oxidation to CH_3CHO; chloride gives HCl.

69. Primary, secondary and tertiary amines can be separated using: [4 marks]

(A) Chloroform and KOH

(B) Acetyl amide

(C) Benzene sulphonic acid

(D) para-Toluene sulphonyl chloride

Correct Answer: (D) para-Toluene sulphonyl chloride

Solution:

Concept:

- The Hinsberg method distinguishes amines by their reaction with an aromatic sulphonyl chloride and by the alkaline solubility of the resulting sulphonamides.
- A primary amine gives a sulphonamide with an acidic nitrogen-hydrogen bond; a secondary amine gives a sulphonamide without that bond; a tertiary amine has no nitrogen-hydrogen bond for this reaction.

Step 1: Apply the sulphonyl chloride test Para-toluene sulphonyl chloride, $p\text{-}CH_3C_6H_4SO_2Cl$, is an aromatic sulphonyl chloride suitable for the Hinsberg separation. A primary amine RNH_2 forms $p\text{-}CH_3C_6H_4SO_2NHR$. This product retains an N–H bond and forms a soluble salt in aqueous alkali. Acidification precipitates the neutral sulphonamide. A secondary amine R_2NH forms $p\text{-}CH_3C_6H_4SO_2NR_2$; it has no N–H hydrogen and remains insoluble in alkali. These different solubilities allow the two derivatives to be separated.

Step 2: Account for the tertiary amine and reject other reagents A tertiary amine R_3N has no nitrogen-hydrogen bond and does not form the corresponding sulphonamide under these conditions. It can be separated by its ability to dissolve in dilute acid as an ammonium salt. Chloroform with KOH is the carbylamine test, useful for detecting a primary amine but not for this complete separation. Acetyl amide is not the required sulphonylating reagent. Benzene sulphonic acid is also not benzene sulphonyl chloride: the acid mainly forms salts and lacks the reactive sulphonyl chloride group needed for the distinguishing sulphonamide derivatives.

Step 3: Conclusion Primary, secondary and tertiary amines can be separated using para-toluene sulphonyl chloride through the Hinsberg method. Hence, the correct option is (D).

Quick Tip to Solve: Check the reagent ending carefully: a sulphonyl chloride forms the diagnostic sulphonamides, whereas a sulphonic acid mainly gives amine salts.

70. Match List - I with List - II :

	List - I		List - II
(a)	$[Co(NH_3)_6][Cr(CN)_6]$	**(i)**	**Linkage isomerism**
(b)	$[Co(NH_3)_3(NO_2)_3]$	**(ii)**	**Solvate isomerism**
(c)	$[Cr(H_2O)_6Cl_3$	**(iii)**	**Co-ordination isomerism**
(d)	$cis-[CrCl_2(ox)_2]^{3-}$	**(iv)**	**Optical isomerism**

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(B) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(C) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(D) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

Correct Answer: (B) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

Solution:

Concept:

- Coordination isomerism exchanges ligands between complex cation and complex anion. Linkage isomerism changes which donor atom of an ambidentate ligand binds to the metal.
- Solvate isomerism changes whether solvent molecules lie inside or outside the coordination sphere. Optical isomers are nonsuperimposable mirror images.

Step 1: Match the first two coordination compounds In (a), $[Co(NH_3)_6][Cr(CN)_6]$, both the cation and anion are coordination entities. Exchanging the ligand sets between the two metals gives a coordination isomer, so (a) matches (iii). In (b), $[Co(NH_3)_3(NO_2)_3]$, the nitrite ligand can bind through nitrogen as nitro or through oxygen as nitrito. These have the same overall composition but a different atom directly attached to cobalt. Therefore (b) exhibits linkage isomerism and matches (i). This criterion is the donor atom, not a change in the total number of ligands.

Step 2: Match the solvated and chiral complexes The chromium aqua chloride in (c) belongs to the familiar hydrated chromium(III) chloride system, in which water and chloride can occupy different positions inside and outside the coordination sphere. It therefore matches solvate isomerism, (ii). The cis complex in (d) contains two bidentate oxalate ligands. Its octahedral arrangement has nonsuperimposable mirror-image forms, giving optical isomerism, (iv). Combining all four matches gives (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv). The first alternative interchanges the first three categories; the third assigns solvate behaviour to (a); and the fourth incorrectly assigns linkage behaviour to the oxalate complex.

Step 3: Conclusion The matching is coordination, linkage, solvate and optical isomerism, respectively, in the order of List I. Hence, the correct option is (B).

Quick Tip to Solve: First look for two complex ions, an ambidentate ligand, water in the formula and a cis chelate arrangement; these identify the four isomerism classes quickly.

71. Fructose is an example of : [4 marks]

(A) Heptose

(B) Pyranose

(C) Ketohexose

(D) Aldohexose

Correct Answer: (C) Ketohexose

Solution:

Concept:

- A monosaccharide is classified by the number of carbon atoms and the nature of its carbonyl group in its open-chain structure.
- A hexose has six carbon atoms. An aldose has an aldehyde group, whereas a ketose has a ketone group. Ring-size names describe a separate structural feature.

Step 1: Identify the carbon skeleton Fructose has molecular formula $C_6H_{12}O_6$. Its open-chain structure can be written as $CH_2OH-CO-(CHOH)_3-CH_2OH$. Counting the two terminal carbon atoms, the carbonyl carbon and the three intermediate CHOH carbons gives six carbon atoms. Therefore the carbon-number suffix is hexose; a heptose would require seven carbons.

Step 2: Locate the carbonyl group and compare the options The carbonyl carbon is the second carbon and is bonded to carbon groups on both sides. It is a ketone group, so fructose is a ketose. Combining this observation with the six-carbon skeleton gives ketohexose. An aldohexose, such as glucose, instead has a terminal aldehyde in its open-chain form. Pyranose refers to a six-membered cyclic ring and does not give the carbonyl-based classification requested here; fructose is characteristically represented in its furanose form in sucrose.

Step 3: Conclusion Fructose is a six-carbon ketose, namely a ketohexose. Hence, the correct option is (C).

Quick Tip to Solve: Classify a sugar from its open chain first: count the carbons, then distinguish a terminal aldehyde from an internal ketone.

72. The common positive oxidation states for an element with atomic number 24, are : [4 marks]

(A) +1 and +3

(B) +2 to +6

(C) +1 to +6

(D) +1 and +3 to +6

Correct Answer: (B) +2 to +6

Solution:

Concept:

- Atomic number identifies the element and its electronic configuration; transition metals can use both outer ns and $(n-1)d$ electrons in bonding.

- Chromium has the ground-state configuration $[\mathrm{Ar}]\,3d^5 4s^1$. Its positive oxidation states span the range $+2$ to $+6$ listed in the options.

Step 1: Identify the element and available valence electrons The element with atomic number 24 is chromium, Cr. Its configuration is $[\mathrm{Ar}]\,3d^5 4s^1$, with six electrons outside the argon core. The partially filled d subshell permits variable oxidation states, so the answer cannot be inferred merely by counting the single $4s$ electron.

Step 2: Check the range against chromium compounds Chromium occurs in the $+2$ state in $\mathrm{CrCl_2}$, in the $+3$ state in $\mathrm{Cr_2O_3}$, and in the $+6$ state in chromate, $\mathrm{CrO_4^{2-}}$, and dichromate, $\mathrm{Cr_2O_7^{2-}}$. Intermediate $+4$ and $+5$ states are also known, although $+3$ and $+6$ are especially important. Thus the offered range is $+2$ to $+6$. Option (A) omits the characteristic higher states. Options (C) and (D) introduce $+1$ into the requested common range; option (D) also omits $+2$.

Step 3: Conclusion The positive oxidation-state range represented for chromium is $+2$ to $+6$. Hence, the correct option is (B).

Quick Tip to Solve: For atomic number 24, recall chromium and its chromate ion: $x + 4(-2) = -2$ gives the characteristic upper state $x = +6$.

---

73. The set that represents the pair of neutral oxides of nitrogen is : [4 marks]

(A) NO and N_2O

(B) N_2O and NO_2

(C) NO and NO_2

(D) N_2O and N_2O_3

Correct Answer: (A) NO and N_2O

Solution:

Concept:

- A neutral oxide is neither an acidic oxide nor a basic oxide under the usual classification. Neutrality here does not mean that the nitrogen oxidation number is zero.
- Among the common nitrogen oxides, N_2O and NO are neutral; N_2O_3 and NO_2 are acidic oxides.

Step 1: Identify the two neutral oxides Nitrous oxide, N_2O, and nitric oxide, NO, do not form acids or bases simply by reacting with water in the usual acid-base classification. They are therefore classified as neutral oxides. Nitrogen has average oxidation states $+1$ and $+2$, respectively, in them; the word neutral concerns acid-base behaviour rather than oxidation state or electrical charge.

Step 2: Eliminate the pairs containing acidic oxides Dinitrogen trioxide is the acid anhydride of nitrous acid:

$$N_2O_3 + H_2O \longrightarrow 2\,HNO_2.$$

Nitrogen dioxide reacts with water to give acids:

$$2\,NO_2 + H_2O \longrightarrow HNO_2 + HNO_3.$$

Hence a pair containing either NO_2 or N_2O_3 is not a pair of neutral oxides. Options (B) and (C) each contain NO_2, while (D) contains N_2O_3. Only option (A) contains both neutral oxides and no acidic oxide.

Step 3: Conclusion The pair of neutral nitrogen oxides is NO and N_2O. Hence, the correct option is (A).

Quick Tip to Solve: Remember N_2O and NO as the neutral exceptions; the higher common nitrogen oxides are acidic.

74. The set of elements that differ in mutual relationship from those of the other sets is : [4 marks]

(A) Li - Na

(B) Be - Al

(C) B - Si

(D) Li - Mg

Correct Answer: (A) Li - Na

Solution:

Concept:

- Elements in the same vertical group show a group relationship, whereas certain diagonally adjacent elements in periods 2 and 3 show a diagonal relationship.
- The principal diagonal pairs are Li–Mg, Be–Al, and B–Si. Similar size and polarising behaviour explain their similarities.

Step 1: Locate the pairs in the periodic table Lithium and sodium both belong to group 1. Lithium is in period 2 and sodium immediately below it is in period 3. Their mutual relationship is therefore vertical, within the alkali-metal group. They share a valence configuration of the form ns^1, which is the ordinary reason for their group resemblance.

Step 2: Compare with the remaining three relationships Beryllium in group 2, period 2 is diagonally adjacent to aluminium in group 13, period 3. Boron in group 13, period 2 is diagonally adjacent to silicon in group 14, period 3. Lithium similarly forms the diagonal pair with magnesium in group 2, period 3. In these cases, changes in size and electronegativity along a period and down a group partly compensate, producing the diagonal relationship. Thus (B), (C) and (D) all belong to one category, while Li–Na belongs to the vertical-group category.

Step 3: Conclusion The Li–Na pair differs because it shows a vertical group relationship, whereas the other pairs show diagonal relationships. Hence, the correct option is (A).

Quick Tip to Solve: Memorise the three diagonal pairs as Li–Mg, Be–Al and B–Si; then identify a same-group pair immediately.

75. For the coagulation of a negative sol, the species below, that has the highest flocculating power is : [4 marks]

(A) Na^+

(B) Ba^{2+}

(C) PO_4^{3-}

(D) SO_4^{2-}

Correct Answer: (B) Ba^{2+}

Solution:

Concept:

- Coagulation of a sol is caused chiefly by ions whose charge is opposite to that of its colloidal particles; these ions neutralise the stabilising surface charge.
- The Hardy–Schulze rule states that greater counter-ion valency gives greater coagulating or flocculating power. A negative sol must therefore be compared using its cations.

Step 1: Select ions of the effective sign The colloidal particles in a negative sol carry negative surface charge. Positive ions approach the particles and reduce the electrostatic repulsion that keeps them dispersed. Therefore the relevant ions among the choices are Na^+ and Ba^{2+}. Phosphate, $PO_4{}^{3-}$, and sulfate, $SO_4{}^{2-}$, have the same sign as the particles; their large negative charges do not make them effective counter-ions for this sol.

Step 2: Compare the counter-ion valencies Sodium has valency one and barium has valency two. By the Hardy–Schulze rule, the divalent barium ion has a much greater tendency to discharge and coagulate a negative sol than the monovalent sodium ion. Consequently Ba^{2+} has the greatest flocculating power of the species listed. Option (A) is a suitable counter-ion but weaker; options (C) and (D) are rejected by charge sign before any magnitude comparison is made.

Step 3: Conclusion For a negative sol, Ba^{2+} is the strongest listed flocculating ion. Hence, the correct option is (B).

Quick Tip to Solve: Apply Hardy–Schulze in two stages: choose the opposite charge first, then choose the largest valency among those counter-ions.

76. Match List - I with List - II.

	List - IChemical Compound		List - IIUsed as
(a)	Sucralose	(i)	Synthetic detergent
(b)	Glyceryl ester of stearic acid	(ii)	Artificial sweetener
(c)	Sodium benzoate	(iii)	Antiseptic
(d)	Bithionol	(iv)	Food preservative

Choose the correct match : [4 marks]

(A) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

(B) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(C) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(D) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Correct Answer: (D) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Solution:

Concept:

- Matching a compound to a use requires recognising its characteristic function: sweetening, preservation, antiseptic action or surfactant action.
- Sucralose is an artificial sweetener, sodium benzoate is a food preservative, and bithionol is an antiseptic. These three unambiguous matches determine the option.

Step 1: Make the direct functional matches Sucralose is a chlorine-containing derivative of sucrose used as an artificial sweetener, giving (a)–(ii). Sodium benzoate suppresses the growth of microorganisms in suitable foods and is used as a food preservative, giving (c)–(iv). Bithionol is an antiseptic that has been incorporated into soaps to impart antiseptic properties, giving (d)–(iii). Antiseptic action concerns microorganisms on living tissue and must not be confused with sweetness or preservation of food.

Step 2: Complete the given matching and eliminate other codes In the classification used by the given lists, the glyceryl ester of stearic acid occupies the remaining surfactant/detergent entry (i), giving (b)–(i). There is an imprecision in this printed pairing: a glyceryl ester of stearic acid is a glyceride (some partial glycerides are emulsifiers), whereas the standard nonionic synthetic detergent example is the polyethylene glycol ester of stearic acid. It would be chemically inaccurate to identify every glyceryl stearate as a synthetic detergent. The other three chemically secure matches nevertheless uniquely fix the intended code among the offered options. The complete code is therefore (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii). Option (A) exchanges the first two uses, (B) incorrectly calls sucralose an antiseptic, and (C) incorrectly calls sucralose a preservative and sodium benzoate a sweetener.

Step 3: Conclusion Sucralose is the sweetener, sodium benzoate the preservative and bithionol the antiseptic. The intended code pairs the glyceryl ester with the remaining detergent entry, subject to the chemical imprecision explained above. Hence, the correct option is (D).

Quick Tip to Solve: Fix the unmistakable matches first: sucralose–sweetener, benzoate–preservative and bithionol–antiseptic uniquely identify this code.

77. **Which of the following statement(s) is (are) incorrect reason for eutrophication?**

(A) excess usage of fertilisers

(B) excess usage of detergents

(C) dense plant population in water bodies

(D) lack of nutrients in water bodies that prevent plant growth

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A) Only

(B) (B) and (D) only

(C) (C) only

(D) (D) only

Correct Answer: (D) (D) only

Solution:

Concept:

- Eutrophication is enrichment of a water body with nutrients, especially nitrogen and phosphorus compounds, causing excessive plant or algal growth.
- Decomposition of the resulting biomass consumes dissolved oxygen. Nutrient deficiency prevents luxuriant growth and is the opposite of nutrient enrichment.

Step 1: Check statements A, B and C Excess fertiliser use supplies nitrates and phosphates that can enter lakes through runoff, so statement (A) gives a valid cause. Excess use and discharge of phosphate-containing detergents also enrich water with nutrients, making statement (B) a valid reason. Dense aquatic plant or algal populations are associated with the eutrophic condition in (C). Their subsequent death and microbial decomposition increase oxygen demand and worsen water quality, so (C) describes the eutrophication process rather than the opposite nutrient condition.

Step 2: Identify the incorrect reason and select the code Statement (D) specifies a lack of nutrients that prevents plant growth. Eutrophication requires excessive nutrient availability, not nutrient scarcity, so (D) is the incorrect reason. The option asking for (A) alone incorrectly rejects fertiliser runoff; the option combining (B) and (D) incorrectly rejects detergents; and the option containing (C) alone does not identify the direct contradiction in (D). Thus the appropriate answer is the option stating that statement (D) alone is incorrect.

Step 3: Conclusion Statement (D) only is the incorrect reason for eutrophication. Hence, the correct option is (D).

Quick Tip to Solve: Underline the word incorrect: eutrophication means nutrient enrichment, so a statement describing nutrient deficiency reverses the process.

78. Match List - I with List - II :

	List - I		List - II
(a)	**Haematite**	**(i)**	$Al_2O_3 \, . \, xH_2O$
(b)	**Bauxite**	**(ii)**	Fe_2O_3
(c)	**Magnetite**	**(iii)**	$CuCO_3 \, . \, Cu(OH)_2$
(d)	**Malachite**	**(iv)**	Fe_3O_4

Choose the correct answer from the options given below : [4 marks]

(A) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(B) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(C) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

(D) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

Correct Answer: (B) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Solution:

Concept:

- An ore is identified by its chemical composition. Haematite and magnetite are iron oxides, bauxite is a hydrated aluminium oxide, and malachite is a basic copper carbonate.
- The familiar formulae are $\mathrm{Fe_2O_3}$, $\mathrm{Fe_3O_4}$, $\mathrm{Al_2O_3 \cdot xH_2O}$, and $\mathrm{CuCO_3 \cdot Cu(OH)_2}$. The hydration number x in bauxite is not needed to establish the matching.

Step 1: Match the iron ores Haematite is ferric oxide, $\mathrm{Fe_2O_3}$, so (a) matches (ii). Magnetite is the mixed-valence iron oxide $\mathrm{Fe_3O_4}$, which may be regarded compositionally as $\mathrm{FeO \cdot Fe_2O_3}$; therefore (c) matches (iv). These are distinct minerals: exchanging their names exchanges the formulae and gives an incorrect match.

Step 2: Match the aluminium and copper ores Bauxite is represented by hydrated alumina, $\mathrm{Al_2O_3 \cdot xH_2O}$, so (b) matches (i). Malachite contains both carbonate and hydroxide associated with copper(II); its composition is $\mathrm{CuCO_3 \cdot Cu(OH)_2}$. Consequently (d) matches (iii). No iron or aluminium oxide can represent this basic carbonate.

Step 3: Check the complete option The combined sequence is (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii). Option (A) incorrectly identifies haematite as hydrated alumina; option (C) incorrectly assigns bauxite the copper carbonate formula; option (D) interchanges the two iron oxide formulae. Only option (B) satisfies all four matches simultaneously.

Step 4: Conclusion All four ore identities agree with the matching sequence in the second listed choice. Hence, the correct option is (B).

Quick Tip to Solve: First fix the two iron ores: haematite is $\mathrm{Fe_2O_3}$ and magnetite is $\mathrm{Fe_3O_4}$. Then identify hydrated alumina and the basic copper carbonate.

79. The functional groups that are responsible for the ion-exchange property of cation and anion exchange resins, respectively, are : [4 marks]

(A) $-SO_3H$ and $-NH_2$

(B) $-NH_2$ and COOH

(C) $-NH_2$ and $-SO_3H$

(D) $-SO_3H$ and $-COOH$

Correct Answer: (A) $-SO_3H$ and $-NH_2$

Solution:

Concept:

- An ion-exchange resin contains fixed functional groups on an insoluble polymer. Mobile counter-ions are exchanged with ions of the same charge from the water passing through it.
- A cation exchanger needs an acidic group that releases H^+, whereas an anion exchanger needs a basic group that can bind a proton and provide exchangeable anions.

Step 1: Identify the cation-exchange group The sulfonic acid group $-SO_3H$ ionises to leave a fixed sulfonate site, $-SO_3{}^-$, on the polymer. Its hydrogen ion can be replaced by a dissolved metal cation. Writing the polymer as R, a representative exchange is

$$2\,R{-}SO_3H + Ca^{2+} \longrightarrow (R{-}SO_3)_2Ca + 2\,H^+.$$

Thus the negatively charged sites retain cations while the polymer itself remains insoluble.

Step 2: Identify the anion-exchange group The amino group $-NH_2$ is basic and becomes protonated in the appropriate resin form. The resulting positively charged site retains anions; its associated counter-anion can be exchanged with anions such as Cl^- or $SO_4{}^{2-}$. Hence an amino-functional resin has the required anion-exchange behaviour. The distinction concerns the charge of the ion exchanged, not the charge of the original neutral functional group.

Step 3: Compare the ordered pairs The question asks for cation and anion exchangers in that order, so the pair is sulfonic acid followed by amino. Option (B) puts the amino group first and a carboxylic acid second; option (C) reverses the correct order. Option (D) puts two acidic groups together, and the carboxylic acid group does not supply the required basic anion-exchange site.

Step 4: Conclusion The required groups are $\boxed{-SO_3H \text{ and } -NH_2}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Remember the fixed-site charge: acidic resins leave negative sites and exchange cations; protonated basic resins leave positive sites and exchange anions.

80. One of the by-products formed during the recovery of NH_3 from Solvay process is : [4 marks]

(A) $Ca(OH)_2$

(B) NH_4Cl

(C) $NaHCO_3$

(D) $CaCl_2$

Correct Answer: (D) $CaCl_2$

Solution:

Concept:

- The Solvay process recycles ammonia after sodium hydrogen carbonate has precipitated. The remaining solution contains ammonium chloride, which is treated with slaked lime.
- A strong base liberates ammonia from an ammonium salt. The chloride ions remain with calcium, producing soluble calcium chloride as the by-product.

Step 1: Locate the ammonia-containing stream In the carbonation stage, ammoniated brine reacts with carbon dioxide and water:

$$NaCl + NH_3 + CO_2 + H_2O \longrightarrow NaHCO_3\downarrow + NH_4Cl.$$

Sodium hydrogen carbonate is removed as a precipitate and ultimately heated to give sodium carbonate. The ammonium chloride in the mother liquor must be processed to recover the ammonia for reuse; it is not the by-product of the recovery reaction.

Step 2: Write the ammonia recovery reaction Slaked lime supplies hydroxide ions that convert ammonium ions into ammonia and water. The balanced overall equation is

$$2\,NH_4Cl + Ca(OH)_2 \longrightarrow 2\,NH_3\uparrow + CaCl_2 + 2\,H_2O.$$

There are two ammonium ions for each calcium ion, and the two chloride ions produce one formula unit of calcium chloride. Atom balance therefore directly identifies the calcium salt remaining after ammonia is evolved.

Step 3: Eliminate the other stages and reagents Calcium hydroxide in option (A) is consumed as a reagent in this step. Ammonium chloride in option (B) is the ammonia-containing reactant. Sodium hydrogen carbonate in option (C) is the precipitated intermediate from the earlier carbonation stage, not a product of lime treatment. Calcium chloride in option (D) is the salt formed during the actual ammonia-recovery step.

Step 4: Conclusion The by-product formed during ammonia recovery is $\boxed{CaCl_2}$. Hence, the correct option is (D).

Quick Tip to Solve: When ammonia is recovered from NH_4Cl using lime, pair the two chloride ions with Ca^{2+} to identify the by-product immediately.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. A KCl solution of conductivity 0.14 S m^{-1} shows a resistance of 4.19Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to 1.03Ω. The conductivity of the HCl solution is _____ $\times$ 10^{-2} S m^{-1}. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Conductivity κ and resistance R are related by $\kappa = K_{\text{cell}}/R$, where $K_{\text{cell}} = l/A$ depends only on the geometry of the conductivity cell.
- Using the same cell keeps K_{cell} unchanged. Thus $\kappa_1 R_1 = \kappa_2 R_2$, even though the two solutions have different ionic compositions and resistances.

Step 1: Determine the cell constant using potassium chloride The standard solution has conductivity $\kappa_{\text{KCl}} = 0.14\,\text{S m}^{-1}$ and resistance $R_{\text{KCl}} = 4.19\,\Omega$. Therefore

$$K_{\text{cell}} = \kappa_{\text{KCl}} R_{\text{KCl}} = 0.14 \times 4.19 = 0.5866\,\text{m}^{-1}.$$

The product of siemens and ohms is dimensionless, leaving the required reciprocal-length unit for the cell constant.

Step 2: Calculate the hydrochloric acid conductivity The cell geometry does not change on replacing the solution, so use the same constant with $R_{\text{HCl}} = 1.03\,\Omega$:

$$\kappa_{\text{HCl}} = \frac{0.5866}{1.03} = 0.56951456\ldots\,\text{S m}^{-1}.$$

The resistance is lower, so a larger conductivity is expected. Multiplying by the resistance ratio gives the same result: $0.14(4.19/1.03)$.

Step 3: Express the result in the requested scale The blank is the coefficient of $10^{-2}\,\text{S m}^{-1}$, rather than the conductivity in unscaled SI units. Hence

$$0.56951456\ldots\,\text{S m}^{-1} = 56.951456\ldots \times 10^{-2}\,\text{S m}^{-1}.$$

Rounding this coefficient to the nearest integer gives 57. Rounding the unscaled conductivity first would lose the information requested by the question.

Step 4: Conclusion The required integer is $\boxed{57}$, corresponding to $57 \times 10^{-2}\,\text{S m}^{-1}$ after the specified rounding.

Quick Tip to Solve: For one conductivity cell, keep κR constant. Convert the calculated conductivity to the stated power of ten before rounding the coefficient.

82. On complete reaction of $FeCl_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- In Werner's terminology, secondary valency is the coordination number: the number of donor atoms directly bonded to the central metal ion. It is distinct from primary valency, which corresponds to oxidation state.
- Oxalic acid gives oxalate, $C_2O_4{}^{2-}$, in alkaline solution. Oxalate is a bidentate ligand, coordinating through two oxygen donor atoms to form a chelate ring.

Step 1: Identify the ferric oxalato product Iron(III) chloride supplies Fe^{3+}, while potassium hydroxide neutralises oxalic acid and makes oxalate ions available. Complete complex formation yields potassium tris(oxalato)ferrate(III), $K_3[Fe(C_2O_4)_3]$. The complex ion is $[Fe(C_2O_4)_3]^{3-}$. Its charge follows from $+3 + 3(-2) = -3$, balanced by three potassium ions outside the coordination sphere.

Step 2: Count donor atoms rather than ligand molecules There are three oxalate ligands within the coordination sphere. Each ligand uses two oxygen atoms to bond to the iron centre, so

$$\text{coordination number} = 3 \times 2 = 6.$$

These six donor atoms form the usual octahedral coordination arrangement around iron(III). Counting only the three ligand molecules would ignore their bidentate nature and would give the wrong secondary valency.

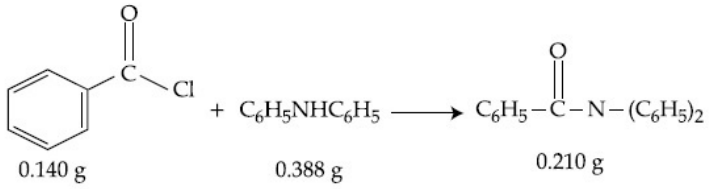
Step 3: Separate secondary valency from oxidation state The iron remains in oxidation state $+3$, because this is complex formation rather than a change in oxidation state. Its primary valency is therefore three, but the question asks for secondary valency. Neither the three potassium counter-ions nor the charges on the ligands are counted as additional donor atoms. The six directly bound oxygen atoms alone determine the requested number.

Step 4: Conclusion The secondary valency of iron in product A is $\boxed{6}$ (a dimensionless coordination number).

Quick Tip to Solve: For an oxalato complex, multiply the number of oxalate ligands by two to obtain the coordination number; do not substitute the oxidation state.

83. Consider the above reaction. The percentage yield of amide product is _____. (Round off to the Nearest Integer).

(Given : Atomic mass : C : 12.0 u, H : 1.0 u, N : 14.0 u, O : 16.0 u, Cl : 35.5 u) [4 marks]



Solution:

Concept:

- Benzoyl chloride reacts with diphenylamine in a 1:1 molar ratio to form the amide and hydrogen chloride. The limiting reagent fixes the maximum possible amount of amide.
- Percentage yield is $100 \times m_{\text{actual}}/m_{\text{theoretical}}$. The theoretical mass must be calculated from the limiting reagent, not from the sum of the reactant masses.

Step 1: Find the reactant molar masses and amounts The acyl chloride is C_6H_5COCl, or C_7H_5OCl, with molar mass

$$M = 7(12) + 5(1) + 16 + 35.5 = 140.5\,\text{g mol}^{-1}.$$

Diphenylamine, $(C_6H_5)_2NH$, is $C_{12}H_{11}N$, with molar mass $12(12) + 11 + 14 = 169\,\text{g mol}^{-1}$. Their amounts are

$$n_{\text{acyl}} = \frac{0.140}{140.5} = 9.9644 \times 10^{-4}\,\text{mol}, \qquad n_{\text{amine}} = \frac{0.388}{169} = 2.2959 \times 10^{-3}\,\text{mol}.$$

Step 2: Identify the limiting reagent and theoretical product mass One mole of either reactant produces one mole of amide, so benzoyl chloride is limiting and diphenylamine is in excess. The amide $C_6H_5CON(C_6H_5)_2$ has formula $C_{19}H_{15}NO$ and molar mass

$$M_{\text{amide}} = 19(12) + 15 + 14 + 16 = 273\,\text{g mol}^{-1}.$$

Thus

$$m_{\text{theoretical}} = \frac{0.140}{140.5} \times 273 = 0.272028\ldots\,\text{g}.$$

Step 3: Compare the isolated mass with the theoretical mass The actual amide mass shown is $0.210\,\text{g}$. Therefore

$$\%\ \text{yield} = \frac{0.210}{0.272028\ldots} \times 100 = 77.198\ldots\,\%.$$

This is below 100 percent, as required for a product mass smaller than the theoretical maximum. The requested nearest-integer percentage is 77; the excess amount of amine must not be used to calculate a larger theoretical yield.

Step 4: Conclusion The percentage yield of the amide is $\boxed{77\%}$ to the nearest integer, so the numerical entry is $\boxed{77}$.

Quick Tip to Solve: Convert both reactant masses to moles first. For a 1:1 acylation, the smaller molar amount sets the theoretical amide mass, after allowing for loss of HCl.

84. The reaction 2A + $B_2 \rightarrow$ 2AB is an elementary reaction.

For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- For an elementary reaction, the reactant stoichiometric coefficients give the powers in the rate law. For $2\,\mathrm{A} + \mathrm{B_2} \longrightarrow 2\,\mathrm{AB}$, the rate is $r = k[\mathrm{A}]^2[\mathrm{B_2}]$.
- Concentration equals amount divided by volume. At unchanged amounts and temperature, reducing volume to one third triples every reactant concentration while leaving the rate constant unchanged.

Step 1: Write the original and compressed concentrations Let the original volume be V, with amounts n_A and n_{B_2}. The original concentrations are $[\mathrm{A}] = n_A/V$ and $[\mathrm{B_2}] = n_{B_2}/V$. After compression the volume is $V/3$, so

$$[\mathrm{A}]' = 3[\mathrm{A}], \qquad [\mathrm{B_2}]' = 3[\mathrm{B_2}].$$

The same specified quantities are being compared; no reactant amount is added by changing the vessel volume.

Step 2: Apply the elementary rate law The original rate is $r = k[\mathrm{A}]^2[\mathrm{B_2}]$. Substitution of the new concentrations gives

$$r' = k\big(3[\mathrm{A}]\big)^2\big(3[\mathrm{B_2}]\big) = 3^2 \times 3\,k[\mathrm{A}]^2[\mathrm{B_2}].$$

Hence

$$\frac{r'}{r} = 3^3 = 27.$$

This uses the explicitly stated elementary character; for a general overall reaction the rate powers could not be inferred from stoichiometry alone.

Step 3: Interpret the factor The total reaction order is $2 + 1 = 3$, so a common concentration multiplication by three increases the rate by the cube of three. The coefficient of product AB does not introduce an extra factor: using a species disappearance or appearance rate consistently in numerator and denominator would cancel its stoichiometric conversion factor. The requested rate factor is therefore 27, not 3 or 9.

Step 4: Conclusion The rate increases by a factor of $\boxed{27}$. This is a dimensionless ratio and is already an integer.

Quick Tip to Solve: For fixed reactant amounts, replacing V by V/f multiplies each concentration by f; the elementary rate then multiplies by f raised to the total order.

85. Consider the reaction

$$N_2O_4\,(g) \rightleftharpoons 2NO_2\,(g)$$

The temperature at which K_C = 20.4 and K_P = 600.1, is _____ K. (Round off to the Nearest Integer). [Assume all gases are ideal and R = 0.0831 L bar K^{-1} mol^{-1}] [4 marks]

Solution:

Concept:

- For an ideal gaseous equilibrium, $K_p = K_c(RT)^{\Delta n_g}$, where Δn_g is the sum of gaseous product coefficients minus the sum of gaseous reactant coefficients.
- The temperature in this relation is absolute temperature. The numerical value of R must use the pressure and volume units associated with the stated equilibrium constants.

Step 1: Count the gaseous stoichiometric coefficients For $\mathrm{N_2O_4(g)} \rightleftharpoons 2\,\mathrm{NO_2(g)}$, there are two moles of gaseous product per one mole of gaseous reactant. Thus

$$\Delta n_g = 2 - 1 = 1.$$

The reaction as written, rather than its reverse or a halved version, fixes this exponent. No liquid or solid contributes to this gaseous mole difference.

Step 2: Isolate the absolute temperature Substitute $\Delta n_g = 1$ into the relation between the constants:

$$K_p = K_cRT, \qquad T = \frac{K_p}{K_cR}.$$

Use $K_p = 600.1$, $K_c = 20.4$ and the given $R = 0.0831\,\mathrm{L\,bar\,K^{-1}\,mol^{-1}}$:

$$T = \frac{600.1}{20.4 \times 0.0831} = \frac{600.1}{1.69524} \simeq 353.99\,\mathrm{K}.$$

The factor RT exceeds one, so $K_p > K_c$, consistent with the positive gaseous mole change. Converting this temperature to Celsius would answer a different demand, since the blank explicitly requires kelvin.

Step 3: Round only the final temperature The unrounded result lies just below $354\,\mathrm{K}$. Retain the given constants during division and round the final result to the nearest integer; premature rounding of R would unnecessarily reduce accuracy.

Step 4: Conclusion The temperature is $\boxed{354\,\mathrm{K}}$; the required numerical entry is $\boxed{354}$.

Quick Tip to Solve: Compute Δn_g before substituting: here it is one, so the ratio K_p/K_c directly equals RT.

86. In the ground state of atomic Fe(Z = 26), the spin-only magnetic moment is _____ $\times 10^{-1}$ BM. (Round off to the Nearest Integer). [Given : $\sqrt{3}$ = 1.73, $\sqrt{2}$ = 1.41] [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu_{\text{so}} = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons and BM denotes the Bohr magneton.
- For a free ground-state atom, obtain n from its neutral-atom electronic configuration and Hund's rule; do not use the configuration of an iron ion.

Step 1: Write the configuration of the neutral atom Iron has atomic number 26, so its ground-state configuration is $[\text{Ar}]\,3d^6 4s^2$. The two electrons in the $4s$ orbital are paired. In the five $3d$ orbitals, the first five electrons occupy separate orbitals with parallel spins. The sixth electron pairs with one of them, leaving four singly occupied $3d$ orbitals. Hence $n = 4$, not six; the total number of d electrons is different from the number of unpaired electrons.

Step 2: Calculate the spin-only moment Insert the unpaired-electron count into the spin-only expression:

$$\mu_{\text{so}} = \sqrt{4(4+2)}\,\text{BM} = \sqrt{24}\,\text{BM} = 2\sqrt{6}\,\text{BM}.$$

Using the supplied values, $\sqrt{6} = \sqrt{2}\sqrt{3} \simeq 1.41 \times 1.73 = 2.4393$, so

$$\mu_{\text{so}} \simeq 4.8786\,\text{BM}.$$

This is the requested spin contribution; no additional orbital contribution is included in a spin-only calculation.

Step 3: Convert to the coefficient requested in the blank The blank multiplies $10^{-1}\,\text{BM}$. If the blank is x, then

$$x \times 10^{-1}\,\text{BM} = 4.8786\,\text{BM}, \qquad x = 48.786.$$

Round this coefficient, rather than the moment in BM, to the nearest integer. Using a more precise square root would also give the same integer.

Step 4: Conclusion The required coefficient is $\boxed{49}$; the moment is approximately $4.9\,\text{BM} = 49 \times 10^{-1}\,\text{BM}$.

Quick Tip to Solve: For $3d^6$, pair one electron after singly filling the five orbitals: four remain unpaired. Convert the moment to the blank's power of ten before rounding.

87. A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation of 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is _____ u. (Round off to the Nearest Integer). [Density of water = 1.0 g cm^{-3}] [4 marks]

Solution:

Concept:

- Boiling-point elevation obeys $\Delta T_b = iK_b m$, where i is the van't Hoff factor and m is molality, measured per kilogram of solvent.
- If one formula unit dissociates into ν particles to degree α, then $i = 1 + (\nu - 1)\alpha$. Equal boiling points for aqueous solutions mean equal elevations above the boiling point of water.

Step 1: Count the particles from the electrolyte Potassium ferrocyanide dissociates as

$$\mathrm{K_4[Fe(CN)_6]} \longrightarrow 4\,\mathrm{K^+} + \mathrm{[Fe(CN)_6]^{4-}}.$$

The complex ion remains a single particle, so one formula unit produces five ions, not separate iron and cyanide ions. At $\alpha = 0.4$,

$$i = 1 + (5 - 1)(0.4) = 2.6.$$

Its analytical molality is $1\,\mathrm{mol\,kg^{-1}}$, hence its effective particle molality is $2.6\,\mathrm{mol\,kg^{-1}}$.

Step 2: Use equality of the boiling-point elevations For the non-electrolytic solute A, $i_A = 1$. Both solutions have water as solvent, so their common K_b cancels:

$$K_b(2.6)(1) = K_b(1)m_A, \qquad m_A = 2.6\,\mathrm{mol\,kg^{-1}}.$$

The water density is not needed because molality is defined using solvent mass, not solution volume.

Step 3: Convert the mass percentage to molar mass Take $100\,\mathrm{g}$ of the solution of A. It contains $18.1\,\mathrm{g}$ solute and $81.9\,\mathrm{g} = 0.0819\,\mathrm{kg}$ water. If M_A is the molar mass in $\mathrm{g\,mol^{-1}}$,

$$2.6 = \frac{18.1/M_A}{0.0819}, \qquad M_A = \frac{18.1}{2.6 \times 0.0819} \simeq 85.0\,\mathrm{g\,mol^{-1}}.$$

The numerical molecular mass in u is the same as this molar-mass value in grams per mole.

Step 4: Conclusion The molecular mass of A is $\boxed{85\,\mathrm{u}}$, corresponding to molar mass $85\,\mathrm{g\,mol^{-1}}$. The required entry is $\boxed{85}$.

Quick Tip to Solve: For weight percent, take 100 g of solution and subtract the solute mass to obtain the solvent mass; using 100 g as the solvent would give the wrong molality.

88. The total number of C-C sigma bond/s in mesityl oxide ($C_6H_{10}O$) is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Every bond between a given pair of carbon atoms contains one sigma bond. A carbon–carbon double bond has one sigma bond and one pi bond, so it still contributes only one C–C sigma bond.
- Mesityl oxide is 4-methylpent-3-en-2-one, with condensed structure $CH_3COCH{=}C(CH_3)_2$. Count carbon–carbon connections in this structure, excluding all C–H and C–O bonds.

Step 1: Identify the carbon skeleton Write the skeleton as $CH_3{-}C(=O){-}CH{=}C(CH_3){-}CH_3$. The five-carbon parent chain contains the carbonyl carbon at position 2 and the double bond between positions 3 and 4. A methyl group is attached to carbon 4. This accounts for all six carbon atoms in $C_6H_{10}O$. The unsaturation consists of one carbonyl double bond and one alkene double bond; the skeleton contains no ring.

Step 2: Count the sigma bonds along the chain The five-carbon main chain has four adjacent carbon pairs: C1–C2, C2–C3, C3–C4 and C4–C5. Each pair contributes one C–C sigma bond, including the C3=C4 pair. Thus the parent chain contributes four such sigma bonds. The additional pi bond of C3=C4 does not increase the sigma count, and the C=O sigma bond is excluded because one of its atoms is oxygen.

Step 3: Include the methyl branch and check the total The sixth carbon, in the methyl branch at C4, is connected to C4 by one further C–C single bond. Consequently

$$N(\text{C}{-}\text{C sigma}) = 4 + 1 = 5.$$

As a check, a connected acyclic skeleton of six carbon atoms has five carbon–carbon connections regardless of whether some connections are double bonds. Only a ring would add an extra connection without introducing another carbon atom.

Step 4: Conclusion The total number of C–C sigma bonds in mesityl oxide is $\boxed{5}$, a dimensionless bond count.

Quick Tip to Solve: Count connected carbon pairs rather than drawn bond strokes: the two strokes in C=C represent one sigma bond and one pi bond.

89. KBr is doped with 10^{-5} mole percent of $SrBr_2$. The number of cationic vacancies in 1g of KBr crystal is _____ 10^{14}. (Round off to the Nearest Integer).

[Atomic Mass : K : 39.1 u, Br : 79.9 u N_A = 6.023 $\times$ 10^{23}] [4 marks]

Solution:

Concept:

- When Sr^{2+} replaces K^+ in KBr, it introduces one excess positive charge. One vacant potassium site compensates this charge, so each incorporated strontium ion produces one cation vacancy.
- A mole percent must be divided by 100 to obtain a mole fraction. The number of vacancies is the

number of host formula units multiplied by the dopant mole fraction for this dilute substitution.

Step 1: Determine the charge-compensation ratio In the KBr lattice the normal cation is K^+. A dopant Sr^{2+} occupies one potassium site but carries twice its positive charge. Removing one additional K^+ from another site restores electroneutrality. Thus the incorporated strontium ion together with one empty cation site replaces two monovalent potassium ions. The vacancy-to-strontium ratio is exactly $1:1$, rather than $2:1$.

Step 2: Convert the amount of dopant and host to particles The dopant concentration is 10^{-5} mole percent, so its mole fraction is

$$x = \frac{10^{-5}}{100} = 10^{-7}.$$

The molar mass of KBr is

$$M(\mathrm{KBr}) = 39.1 + 79.9 = 119.0\,\mathrm{g\,mol^{-1}}.$$

Therefore $1\,\mathrm{g}$ contains $1/119$ mole of KBr formula units. The small dopant concentration permits the stated 1 g host-crystal amount to be used directly at the requested rounding precision.

Step 3: Calculate and scale the vacancy count Using one vacancy for every dopant ion gives

$$N_v = \frac{1}{119} \times 10^{-7} \times 6.023 \times 10^{23} = 5.0613 \times 10^{14}.$$

The blank asks for the coefficient of 10^{14}, so round 5.0613, rather than the full particle number, to the nearest integer. Vacancies are counted lattice sites and do not carry a mass unit.

Step 4: Conclusion The vacancy count is approximately 5.0613×10^{14}; the required coefficient is $\boxed{5}$, meaning 5×10^{14} cationic vacancies after the requested rounding.

Quick Tip to Solve: Two common traps are forgetting that mole percent includes a factor of $1/100$, and assigning two vacancies to each divalent dopant instead of one.

90. The number of chlorine atoms in 20 mL of chlorine gas at STP is _____ 10^{21}. (Round off to the Nearest Integer).

[Assume chlorine is an ideal gas at STP

R = 0.083 L bar mol^{-1} K^{-1}, N_A = 6.023 $\times$ 10^{23}] [4 marks]

Solution:

Concept:

- For an ideal gas, $n = PV/(RT)$. At STP use $T = 273\,\mathrm{K}$ and $P = 1\,\mathrm{bar}$ with the given gas constant in litre–bar units.
- Chlorine gas consists of diatomic Cl_2 molecules. Each molecule contains two chlorine atoms, so the atomic count is $N_{Cl} = 2nN_A$, rather than just nN_A.

Step 1: Put the gas volume into litres The volume given is $20\,\mathrm{mL}$, or

$$V = 20 \times 10^{-3}\,\mathrm{L} = 0.020\,\mathrm{L}.$$

Use litres because $R = 0.083\,\mathrm{L\,bar\,mol^{-1}\,K^{-1}}$. The pressure and temperature are those of STP, so they must be inserted as an absolute pressure and an absolute temperature. No density or molar mass of chlorine is required for this particle-count calculation.

Step 2: Calculate the number of gas molecules Apply the ideal-gas relation:

$$n(\mathrm{Cl_2}) = \frac{1 \times 0.020}{0.083 \times 273} = \frac{0.020}{22.659} \simeq 8.8265 \times 10^{-4}\,\mathrm{mol}.$$

The corresponding number of molecules is

$$N(\mathrm{Cl_2}) = 8.8265 \times 10^{-4} \times 6.023 \times 10^{23} \simeq 5.316 \times 10^{20}.$$

The denominator is the molar gas volume in litres at the stated STP conditions, providing a useful independent check on the scale of the result.

Step 3: Count atoms and round the scaled answer Each $\mathrm{Cl_2}$ molecule supplies two chlorine atoms, so

$$N_{\mathrm{Cl}} = 2(5.316 \times 10^{20}) \simeq 1.0632 \times 10^{21}.$$

The blank is the coefficient multiplying 10^{21}. Its unrounded value is about 1.0632, which rounds to one. Rounding the number of moles at an earlier stage could obscure this conversion from molecules to atoms.

Step 4: Conclusion The number of chlorine atoms is approximately 1.0632×10^{21}. The required rounded coefficient of 10^{21} is $\boxed{1}$, a dimensionless atom count.

Quick Tip to Solve: Write $N_{\mathrm{atoms}} = 2PVN_A/(RT)$ for chlorine: the factor of two converts diatomic molecules to atoms, and the millilitre-to-litre conversion must precede substitution.