JEE Main 2021 Paper 1 (B.E./B.Tech) Mar 18 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 2 (March), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 18 March 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The number of integral values of m so that the abscissa of point of intersection of lines 3x + 4y = 9 and y = mx + 1 is also an integer, is : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 0

Correct Answer: (B) 2

Solution:

Concept:

- Substitution eliminates one coordinate from two simultaneous line equations. An intersection exists only when the resulting coefficient of the remaining coordinate is nonzero.
- If an integer quotient has a fixed nonzero integer numerator, its integer denominator must divide that numerator. Positive and negative divisors must both be tested.

Step 1: Eliminate the ordinate Substitute $y = mx + 1$ in $3x + 4y = 9$. This gives $(4m + 3)x = 5$, and hence

$$x = \frac{5}{4m + 3}.$$

For integral m, the denominator cannot vanish, since $4m + 3 = 0$ would require $m = -3/4$.

Step 2: Test all possible divisors For x to be integral, $4m+3$ must belong to $\{1, -1, 5, -5\}$. Solving for m gives respectively $-1/2, -1, 1/2, -2$. Only -1 and -2 are integers. Their abscissae are respectively -5 and -1, so both really satisfy the requirement. No other divisor is possible; counting only positive denominators would incorrectly discard both admissible cases.

Step 3: Conclusion There are exactly 2 integral values of m. Hence, the correct option is (B).

Quick Tip to Solve: List both signs of every divisor before imposing the congruence $4m + 3 \equiv 3 \pmod 4$.

2. The solutions of the equation $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi)$**, are** [4 marks]

(A) $\frac{\pi}{12}, \frac{\pi}{6}$

(B) $\frac{\pi}{6}, \frac{5\pi}{6}$

(C) $\frac{5\pi}{12}, \frac{7\pi}{12}$

(D) $\frac{7\pi}{12}, \frac{11\pi}{12}$

Correct Answer: (D) $\frac{7\pi}{12}, \frac{11\pi}{12}$

Solution:

Concept:

- A matrix whose entries are a fixed column repeated in every column, with 1 added to each diagonal entry, has determinant $1 + u_1 + u_2 + u_3$. This also follows by subtracting the first column from the other columns and expanding.
- The identity $\sin^2 x + \cos^2 x = 1$ simplifies the determinant before solving the resulting sine equation.

Step 1: Reduce the determinant Put $s = \sin^2 x$, $c = \cos^2 x$ and $t = 4\sin 2x$. Direct determinant expansion gives

$$\det \begin{pmatrix} 1+s & s & s \\ c & 1+c & c \\ t & t & 1+t \end{pmatrix} = 1 + s + c + t = 2 + 4\sin 2x.$$

Thus the required zero determinant is equivalent to $\sin 2x = -1/2$.

Step 2: Apply the prescribed interval Since $0 < x < \pi$, the doubled angle lies in $(0, 2\pi)$. Sine is negative in the third and fourth quadrants, so

$$2x = \frac{7\pi}{6}, \frac{11\pi}{6}, \qquad x = \frac{7\pi}{12}, \frac{11\pi}{12}.$$

The positive-sine angles in the first two options cannot work. The pair in the third option includes an angle with positive $\sin 2x$, so it is not the solution set either.

Step 3: Conclusion The solutions are $7\pi/12$ and $11\pi/12$. Hence, the correct option is (D).

Quick Tip to Solve: For a repeated-column perturbation of the identity, simplify the determinant first instead of expanding trigonometric products.

3. Let α, β, γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, (a, b, c $\in$ R and a, b $\neq 0$). If the system of equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is [4 marks]

(A) 5

(B) 3

(C) 1

(D) 0

Correct Answer: (B) 3

Solution:

Concept:

- A homogeneous system of three linear equations has a non-trivial solution precisely when its coefficient determinant is zero.
- For real numbers, $\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha = \frac{1}{2}[(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2]$.

Step 1: Factor the singularity condition The determinant of the given coefficient matrix is

$$-(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha).$$

Vieta's relation gives $\alpha+\beta+\gamma = -a \neq 0$. Therefore the second factor must vanish; it is not permissible to use the first factor to conclude $a = 0$.

Step 2: Use reality and Vieta relations The second factor is a sum of three nonnegative squares divided by two. Its vanishing forces $\alpha = \beta = \gamma = r$. Consequently $a = -3r$ and $b = 3r^2$. Since $b \neq 0$, division by r^2 is valid, and

$$\frac{a^2}{b} = \frac{9r^2}{3r^2} = 3.$$

Thus the remaining numerical options do not satisfy the coefficient relations forced by the real equal roots.

Step 3: Conclusion The required ratio is 3. Hence, the correct option is (B).

Quick Tip to Solve: After factoring a cyclic determinant, use the real-root condition to turn the quadratic factor into a sum of squares.

4. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to (where c is a constant of integration) [4 marks]

(A) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

(B) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(C) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

(D) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

Correct Answer: (A) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

Solution:

Concept:

- When a composite function and the derivative of its inner argument occur together, substitution converts the integral to a standard elementary integral.
- The quadratic under the denominator radical satisfies $4x^2-4x+6=(2x-1)^2+5$, so the two radicals in the integrand are identical.

Step 1: Choose the common radical as variable Let $u=\sqrt{(2x-1)^2+5}$. The radicand is at least 5, so this substitution and its derivative are defined for every real x. Differentiating gives

$$du=\frac{2(2x-1)}{\sqrt{(2x-1)^2+5}}dx.$$

Thus the non-trigonometric part of the given integrand equals $du/2$.

Step 2: Integrate and verify the result The integral becomes

$$\frac{1}{2}\int\cos u\,du=\frac{1}{2}\sin u+c.$$

Returning to x gives $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$. Its derivative is $\frac{1}{2}\cos u\,u\prime$, exactly the given integrand. The cosine alternatives would differentiate to a negative sine factor. The alternative involving $2x+1$ has the wrong inner argument and derivative, so it cannot represent this antiderivative.

Step 3: Conclusion The integral is $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$. Hence, the correct option is (A).

Quick Tip to Solve: Complete the square in the denominator first, then differentiate the common radical to identify the factor of two.

5. The equation of one of the straight lines which passes through the point (1, 3) and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, y + 1 = 3$\sqrt{2}$ x is : [4 marks]

(A) $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

(B) $5\sqrt{2}x + 4y - \left(15 + 4\sqrt{2}\right) = 0$

(C) $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

(D) $4\sqrt{2}x - 5y - \left(5 + 4\sqrt{2}\right) = 0$

Correct Answer: (A) $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

Solution:

Concept:

- For finite slopes m and m_0, the acute angle between the lines obeys $\tan\theta = |(m - m_0)/(1 + mm_0)|$, provided the denominator is nonzero.
- A line of slope m through (x_1, y_1) has equation $y - y_1 = m(x - x_1)$. Both the slope condition and the specified point must hold.

Step 1: Solve the slope condition The given line has slope $3\sqrt{2}$, and the required angle has tangent $\sqrt{2}$. Hence

$$m - 3\sqrt{2} = \pm\sqrt{2}(1 + 3\sqrt{2}m).$$

With the plus sign, $-5m = 4\sqrt{2}$, giving $m = -4\sqrt{2}/5$. With the minus sign, $7m = 2\sqrt{2}$, giving the other possible slope $2\sqrt{2}/7$.

Step 2: Form and check the offered line Using the first slope through $(1, 3)$,

$$y - 3 = -\frac{4\sqrt{2}}{5}(x - 1),$$

or $4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$. Substitution of $(1, 3)$ gives zero. The other three offered equations fail this point test: their left sides give respectively $\sqrt{2} - 3$, 15, and -20. They are therefore excluded even before testing their angles.

Step 3: Conclusion One required line is $4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$. Hence, the correct option is (A).

Quick Tip to Solve: Test the given point in each proposed equation first; then verify the angle for the surviving line.

6. If $\lim_{x\to 0} \frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is [4 marks]

(A) $\frac{1}{6}$

(B) $\frac{1}{2}$

(C) 6

(D) 2

Correct Answer: (D) 2

Solution:

Concept:

- Near zero, the inverse trigonometric expansions in radians are $\sin^{-1} x = x + x^3/6 + O(x^5)$ and $\tan^{-1} x = x - x^3/3 + O(x^5)$.
- When the linear terms cancel, the cubic coefficients determine a quotient whose denominator is a constant times x^3. Terms of order x^5 contribute zero to its limit.

Step 1: Find the leading numerator term Subtracting the expansions, with attention to the negative cubic term of arctangent, gives

$$\sin^{-1} x - \tan^{-1} x = \left(\frac{1}{6} + \frac{1}{3}\right) x^3 + O(x^5) = \frac{x^3}{2} + O(x^5).$$

The coefficient is a sum because the second expression is subtracted. Using a difference of the two positive fractions would give the wrong sign or magnitude.

Step 2: Evaluate the limit and requested expression Divide by the entire denominator $3x^3$:

$$L = \lim_{x\to 0} \left(\frac{1}{6} + O(x^2)\right) = \frac{1}{6}.$$

The question asks for $6L + 1$, rather than for L itself. Thus $6L + 1 = 6(1/6) + 1 = 2$. The first option is the intermediate limit, the second is the numerator's cubic coefficient, and neither is the requested final expression; 6 also does not follow from the evaluation.

Step 3: Conclusion The value of $6L + 1$ is 2. Hence, the correct option is (D).

Quick Tip to Solve: Keep the factor 3 in the denominator and distinguish the auxiliary limit L from the final expression asked.

7. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then the value of p is equal to : [4 marks]

(A) 1

(B) $-\frac{5}{4}$

(C) $\frac{4}{5}$

(D) -1

Correct Answer: (D) -1

Solution:

Concept:

- A rotation of rectangular Cartesian axes preserves the length of a vector, although its individual components change.
- Therefore the sum of the squares of the old components equals the sum of the squares of the new components. This eliminates the unspecified rotation angle.

Step 1: Equate the squared magnitudes The original components give $|\vec{a}|^2 = (3p)^2 + 1^2 = 9p^2 + 1$. The new components give $|\vec{a}|^2 = (p+1)^2 + (\sqrt{10})^2 = (p+1)^2 + 10$. Equating these expressions yields

$$9p^2 + 1 = p^2 + 2p + 11, \qquad 4p^2 - p - 5 = 0.$$

Rotation does not allow either length to be multiplied by a scale factor.

Step 2: Solve and compare with the choices Factor the quadratic:

$$(p+1)(4p-5) = 0.$$

Thus magnitude preservation allows $p = -1$ or $p = 5/4$. Among the printed choices, only -1 occurs; the negative value $-5/4$ is not the positive root. For $p = -1$, the old components are $(-3, 1)$ and the new ones are $(0, \sqrt{10})$, both of length $\sqrt{10}$. A counterclockwise rotation of the axes can produce these components, so the listed value is admissible. The values 1 and $4/5$ fail the squared-length equation.

Step 3: Conclusion The value among the given options is $p = -1$. Hence, the correct option is (D).

Quick Tip to Solve: Use invariance of squared length to remove the rotation angle; then compare every quadratic root with the actual options.

8. If the equation $a|z|^2 + \overline{\overline{\alpha}z + \alpha\overline{z}} + d = 0$ represents a circle where a, d are real constants then which of the following condition is correct? [4 marks]

(A) $|\alpha|^2 - ad \neq 0$

(B) $|\alpha|^2 - ad > 0$ and a$\in$R $-$ {0}

(C) $|\alpha|^2 - ad \geq 0$ and a$\in$R

(D) $\alpha = 0$, a, d$\in R^+$

Correct Answer: (B) $|\alpha|^2 - ad > 0$ and a$\in$R $-$ {0}

Solution:

Concept:

- A circle in the complex plane has the form $|z - z_0|^2 = r^2$, where z_0 is its centre and the squared radius must be strictly positive for a non-degenerate real circle.
- The sum $\overline{\alpha}z + \alpha\overline{z}$ is real because its two terms are complex conjugates. Conjugating that entire sum therefore leaves it unchanged.

Step 1: Exclude the linear case The coefficient of $|z|^2$ must be nonzero: if $a = 0$, only a linear equation in the real and imaginary parts remains, which cannot represent a circle. Since the question gives real a, we require $a \in \mathbb{R} \setminus \{0\}$. Dividing the equation by this nonzero coefficient gives

$$|z|^2 + \frac{\overline{\alpha}}{a}z + \frac{\alpha}{a}\overline{z} + \frac{d}{a} = 0.$$

Step 2: Complete the complex square Use the product of a complex number and its conjugate to combine the quadratic and linear terms:

$$\left|z + \frac{\alpha}{a}\right|^2 = \frac{|\alpha|^2 - ad}{a^2}.$$

Thus the centre is $-\alpha/a$. The denominator on the right is positive, so a positive radius is equivalent to $|\alpha|^2 - ad > 0$. Equality gives only one point; a negative value gives no points. Option (A) permits the negative case and omits the requirement on a; option (C) permits a point and $a = 0$. In option (D), $\alpha = 0$ and positive a, d make the squared radius negative.

Step 3: Conclusion The necessary and sufficient condition is $|\alpha|^2 - ad > 0$ together with $a \in \mathbb{R} \setminus \{0\}$. Hence, the correct option is (B).

Quick Tip to Solve: After completing the square, check both the nonzero quadratic coefficient and a strictly positive squared radius.

9. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further center of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a : [4 marks]

(A) Rhombus

(B) Square

(C) Rectangle

(D) Parallelogram

Correct Answer: (B) Square

Solution:

Concept:

- For $x^2+y^2+2gx+2fy+c=0$, the centre is $(-g,-f)$. The constant term affects the radius, rather than the centre.
- A quadrilateral with four equal sides and four right angles is a square. Its classification follows from the centres, not from the points where the circles intersect.

Step 1: Extract the four centres Compare each equation with the general circle equation, or complete its squares. The centres, in the stated joining order, are

$$M=(0,0), \quad N=(1,0), \quad O=(1,1), \quad P=(0,1).$$

For example, circle O becomes $(x-1)^2+(y-1)^2=1$, confirming its centre is $(1,1)$. Circle N is centred at $(1,0)$, rather than $(2,0)$, because the coefficient of x equals minus twice the centre coordinate.

Step 2: Check lengths and angles The consecutive displacement vectors are

$$\overrightarrow{MN}=(1,0), \quad \overrightarrow{NO}=(0,1), \quad \overrightarrow{OP}=(-1,0), \quad \overrightarrow{PM}=(0,-1).$$

All have length 1. Consecutive vectors have zero dot product, so every interior angle is a right angle; opposite vectors also show opposite sides are parallel. These checks establish a square. A square is also a special rhombus, rectangle and parallelogram, but among the listed classifications the square states all the established properties and is the most specific answer.

Step 3: Conclusion The centres joined in the given order form a unit square. Hence, the correct option is (B).

Quick Tip to Solve: Read each centre by halving and reversing the linear coefficients, then compare consecutive displacement vectors.

10. The real valued function

$f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x-[x]}}$**, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :** [4 marks]

(A) all real except integers

(B) all non-integers except the interval [-1, 1]

(C) all integers except 0, -1, 1

(D) all real except the interval [-1, 1]

Correct Answer: (B) all non-integers except the interval [-1, 1]

Solution:

Concept:

- The real inverse cosecant $\operatorname{cosec}^{-1} x$ is defined for $x \leq -1$ or $x \geq 1$. It denotes an inverse function here, not the reciprocal of cosecant.
- A square root in a denominator requires its radicand to be strictly positive. The fractional part $x - \lfloor x \rfloor$ lies in $[0, 1)$ for every real x.

Step 1: Impose the numerator condition The numerator $\operatorname{cosec}^{-1} x$ is real precisely when $|x| \geq 1$. Hence all values strictly between -1 and 1 are initially excluded. The endpoints themselves give real inverse-cosecant values, so they must be tested separately against the denominator rather than discarded at this stage.

Step 2: Exclude the denominator zeros For a non-integer, the fractional part satisfies $0 < x - \lfloor x \rfloor < 1$, so its square root is real and nonzero. At every integer, including negative integers, the fractional part is zero and division is undefined. Combining the two independent conditions gives

$$\{x \in \mathbb{R} : |x| \geq 1\} \setminus \mathbb{Z} = \big((-\infty, -1) \cup (1, \infty)\big) \setminus \mathbb{Z}.$$

The endpoints $-1, 1$ are excluded because they are integers. Option (A) incorrectly includes non-integers such as $1/2$; option (D) incorrectly includes integers such as 2. Option (C) contains only integers, each of which makes the denominator zero.

Step 3: Conclusion The domain consists of all non-integers outside $[-1, 1]$. Hence, the correct option is (B).

Quick Tip to Solve: Intersect the inverse-trigonometric domain with the strict denominator condition; fractional parts vanish at every integer.

11. If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions :

f + g, f − g, f/g, g/f, g − f where $(f \pm g)(x) = f(x) \pm g(x), (f/g)x = \frac{f(x)}{g(x)}$ [4 marks]

(A) $0 \leq x \leq 1$

(B) $0 \leq x < 1$

(C) $0 < x < 1$

(D) $0 < x \leq 1$

Correct Answer: (C) $0 < x < 1$

Solution:

Concept:

- The real square root $\sqrt{u}$ requires $u \geq 0$. A sum or difference is defined on the intersection of the component domains.
- A quotient additionally requires its denominator to be nonzero. A common domain must satisfy the restrictions of every operation in the list simultaneously.

Step 1: Find where both functions are real From $f(x) = \sqrt{x}$, we require $x \geq 0$. From $g(x) = \sqrt{1-x}$, we require $x \leq 1$. Thus $f + g$, $f - g$ and $g - f$ are all real on $[0, 1]$. Reversing the order of subtraction adds no condition, since it merely changes the sign of an already defined real value.

Step 2: Apply both quotient restrictions For f/g, the denominator $\sqrt{1-x}$ vanishes at $x = 1$, giving the domain $[0, 1)$. For g/f, the denominator $\sqrt{x}$ vanishes at $x = 0$, giving the domain $(0, 1]$. The common domain is the intersection

$$[0, 1] \cap [0, 1) \cap (0, 1] = (0, 1).$$

Every point strictly between the endpoints makes both roots positive, so all five functions exist there. At 0 the quotient g/f is undefined; at 1 the quotient f/g is undefined. Thus option (A) retains two forbidden endpoints, (B) retains 0, and (D) retains 1.

Step 3: Conclusion The common domain is $0 < x < 1$. Hence, the correct option is (C).

Quick Tip to Solve: For simultaneous reciprocal quotients, remove the zeros of both component functions after finding their common real domain.

12. If $f(x) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively : [4 marks]

(A) $\frac{1}{2}, \frac{1}{2}$

(B) $\frac{1}{2}, -\frac{3}{2}$

(C) $\frac{5}{2}, -\frac{3}{2}$

(D) $-\frac{1}{2}, \frac{3}{2}$

Correct Answer: (D) $-\frac{1}{2}, \frac{3}{2}$

Solution:

Concept:

- Differentiability implies continuity. At a junction of two smooth formulas, their limiting values and their one-sided derivatives must both agree.
- The absolute value must be resolved by sign: $1/|x| = 1/x$ for $x > 0$, while $1/|x| = -1/x$ for $x < 0$. The only junctions here are $x = 1$ and $x = -1$.

Step 1: Match the values at the junctions At either endpoint the outer formula gives $f(1) = f(-1) = 1$. The inner polynomial tends to $a + b$ at both points. Continuity is therefore equivalent to

$$a + b = 1.$$

It is necessary to impose this before comparing derivative formulas: otherwise matching formal slopes alone would not make the piecewise function differentiable.

Step 2: Match derivatives and verify both signs At $x = 1$, the derivative of $1/x$ on the right is $-1/x^2$, hence -1, while the inner derivative on the left is $2ax$, hence $2a$. Thus $2a = -1$, so $a = -1/2$. Continuity then gives $b = 3/2$. At $x = -1$, the outer formula is $-1/x$ with derivative $1/x^2 = 1$, and the inner derivative is $-2a = 1$, confirming the second junction too. Away from these two points each applicable formula is smooth. Option (A) satisfies continuity but has the wrong slope at 1; (B) fails continuity; (C) satisfies continuity but gives the wrong slope. Only (D) meets both requirements everywhere.

Step 3: Conclusion The required values are $a = -\frac{1}{2}$ and $b = \frac{3}{2}$, respectively. Hence, the correct option is (D).

Quick Tip to Solve: Use continuity first, then match slopes at one junction and check the absolute-value sign at the other junction.

13. Let $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ **and** $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$**. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) − Tr(B) has value equal to** [4 marks]

(A) 1

(B) 2

(C) 0

(D) 3

Correct Answer: (B) 2

Solution:

Concept:

- The trace of a square matrix is the sum of its diagonal entries. It is linear: $\mathrm{Tr}(rA+sB)=r\,\mathrm{Tr}(A)+s\,\mathrm{Tr}(B)$.
- Only diagonal entries contribute to the trace. Consequently, one can solve for the two unknown traces without determining any of the individual matrix entries.

Step 1: Take the trace of both equations Write $u=\mathrm{Tr}(A)$ and $v=\mathrm{Tr}(B)$. The first displayed matrix has diagonal entries 1, -3, and 1, whose sum is -1. The second has diagonal entries 2, -1, and 2, whose sum is 3. By linearity, the given matrix equations therefore imply

$$u+2v=-1, \qquad 2u-v=3.$$

Off-diagonal entries such as 6 and -5 have no effect on these equations.

Step 2: Solve the two scalar equations From the first equation $u=-1-2v$. Substitute this into the second equation to obtain

$$2(-1-2v)-v=3 \quad \Rightarrow \quad -5v=5 \quad \Rightarrow \quad v=-1.$$

Hence $u=1$, and the requested difference is

$$\mathrm{Tr}(A)-\mathrm{Tr}(B)=u-v=1-(-1)=2.$$

Checking gives $1+2(-1)=-1$ and $2(1)-(-1)=3$, so both trace equations hold. Therefore the value corresponds to option (B), rather than the individual trace $u=1$ in (A). Options (C) and (D) do not satisfy the computed difference.

Step 3: Conclusion The requested difference of traces is 2. Hence, the correct option is (B).

Quick Tip to Solve: Take traces before solving matrix equations when the final expression only involves traces; ignore every off-diagonal entry.

14. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is : [4 marks]

(A) 26664

(B) 122664

(C) 122234

(D) 22264

Correct Answer: (A) 26664

Solution:

Concept:

- The digits 1, 2, 2 and 3 form $4!/2! = 12$ distinct four-digit arrangements. Dividing by $2!$ avoids counting interchanges of the two identical 2s as new numbers.
- For a sum of all arrangements, count each digit in a fixed place and multiply that place contribution by $1000 + 100 + 10 + 1$. There is no leading-zero restriction in this problem.

Step 1: Count each possible digit in one position Fix any one of the four positions. If that position contains 1, the remaining digits 2, 2, 3 can be arranged in $3!/2! = 3$ ways. The same count holds when the fixed digit is 3. If the fixed digit is 2, the remaining digits are 1, 2, 3, all different, so there are $3! = 6$ arrangements. These counts add to $3 + 3 + 6 = 12$, exactly the total number of distinct numbers.

Step 2: Add the place-value contributions The sum of the digits occurring in the chosen position across all 12 numbers is

$$1 \cdot 3 + 2 \cdot 6 + 3 \cdot 3 = 24.$$

By symmetry the same digit sum occurs in the units, tens, hundreds and thousands positions. Hence the sum of the numbers is

$$24(1000 + 100 + 10 + 1) = 24 \cdot 1111 = 26664.$$

As a check, the average digit in each position is $24/12 = 2$, making the average number 2222 and the total $12 \cdot 2222 = 26664$. Options (B) and (C) even exceed $12 \cdot 3221$, an upper bound from the largest possible arrangement; option (D) disagrees with the exact place count.

Step 3: Conclusion The sum of all distinct four-digit numbers is 26664. Hence, the correct option is (A).

Quick Tip to Solve: With repeated digits, count how often each value occupies one place; treat identical copies as indistinguishable.

15. The value of $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$ is equal to [4 marks]

(A) 1.5 + $\sqrt{3}$

(B) 2 + $\sqrt{3}$

(C) 3 + 2$\sqrt{3}$

(D) 4 + $\sqrt{3}$

Correct Answer: (A) 1.5 + $\sqrt{3}$

Solution:

Concept:

- A periodic continued fraction repeats after one complete period. If its positive value is x, replacing the repeated tail by x produces an algebraic equation.
- The entries alternate between 3 and 4, so the complete repeating block is $x = 3 + 1/(4 + 1/x)$, rather than a one-entry repetition.

Step 1: Identify the repeated tail Let the given positive expression be x. After the first 3 and the next 4, the remaining tail is again the original expression. Therefore

$$x = 3 + \frac{1}{4 + 1/x} = 3 + \frac{x}{4x + 1}.$$

Every finite truncation has positive entries, so the relevant limiting value must be positive. In fact the outer denominator exceeds 4, giving the useful bound $3 < x < 13/4$.

Step 2: Solve the resulting quadratic Multiply by the nonzero positive denominator and collect the terms:

$$(x - 3)(4x + 1) = x,$$
$$4x^2 - 12x - 3 = 0,$$
$$x = \frac{12 \pm \sqrt{144 + 48}}{8} = \frac{3}{2} \pm \sqrt{3}.$$

The negative-sign root is negative and cannot represent the fraction. The positive-sign root is approximately 3.232 and lies inside the bound found above. The other displayed positive choices all exceed 3.25, so they cannot satisfy the fraction.

Step 3: Conclusion The value is $\boxed{\frac{3}{2} + \sqrt{3}}$. Hence, the correct option is (A).

Quick Tip to Solve: Replace a periodic continued fraction only after its entire repeating block; then use positivity to discard the extraneous quadratic root.

16. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is : [4 marks]

(A) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(B) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(C) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(D) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

Correct Answer: (C) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

Solution:

Concept:

- A differential equation for a one-parameter family is obtained by differentiating once and eliminating the arbitrary constant.
- For the family $y^2 = 4a(x + a)$, the parameter a is constant while y depends on x. Write $p = dy/dx$.

Step 1: Differentiate the family Differentiating both sides with respect to x, keeping a constant, gives

$$2yp = 4a, \qquad a = \frac{yp}{2}.$$

The chain rule supplies the factor p on the left. Only one differentiation is needed because the original family contains exactly one independent arbitrary parameter.

Step 2: Eliminate the constant and simplify Substitute $a = yp/2$ into the original relation:

$$y^2 = 4\left(\frac{yp}{2}\right)\left(x + \frac{yp}{2}\right),$$
$$y^2 = 2xyp + y^2p^2.$$

On an arc where $y \neq 0$, division by y and rearrangement give

$$yp^2 + 2xp - y = 0.$$

This equation extends by continuity to any point of such an arc where the derivative exists. The coefficient of $2xp$ is positive and that of y is negative; changing either sign gives the incorrect alternatives A or B. Alternative D also loses the square on the first derivative.

Step 3: Conclusion The required equation is $\boxed{y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Solve the differentiated relation for the parameter first, and substitute into the original family before cancelling factors.

17. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+....+\frac{1}{(201)^2-1}$ **is equal to** [4 marks]

(A) $\frac{101}{404}$

(B) $\frac{25}{101}$

(C) $\frac{101}{408}$

(D) $\frac{99}{400}$

Correct Answer: (B) $\frac{25}{101}$

Solution:

Concept:

- Difference of squares factors as $m^2-1=(m-1)(m+1)$. Partial fractions can turn a sum into cancelling consecutive differences.
- The odd integers from 3 through 201 are $2k+1$ with $k=1,\ldots,100$. Keeping these endpoints prevents losing the final term.

Step 1: Express the general term as a difference Write the required sum as

$$S=\sum_{k=1}^{100}\frac{1}{(2k+1)^2-1}.$$

Since $(2k+1)^2-1=4k(k+1)$, the summand becomes

$$\frac{1}{4k(k+1)}=\frac{1}{4}\left(\frac{1}{k}-\frac{1}{k+1}\right).$$

This decomposition is exact: combining the two fractions leaves numerator 1 over $k(k+1)$.

Step 2: Cancel the intermediate terms Substitution now gives

$$\begin{aligned}S&=\frac{1}{4}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{100}-\frac{1}{101}\right)\right]\\&=\frac{1}{4}\left(1-\frac{1}{101}\right)=\frac{25}{101}.\end{aligned}$$

Every intermediate reciprocal occurs once with each sign. The upper denominator 101 comes from the final odd number 201. Option A equals $1/4$, the infinite-sum limit rather than this finite sum; options C and D do not equal the surviving endpoint difference.

Step 3: Conclusion The sum is $\boxed{\frac{25}{101}}$. Hence, the correct option is (B).

Quick Tip to Solve: For reciprocals of consecutive odd squares minus one, use $(2k+1)^2-1=4k(k+1)$ and keep only the uncancelled endpoints.

18. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. Then $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to [4 marks]

(A) $2^{20}(2^{20} - 21)$

(B) $2^{19}(2^{20} - 21)$

(C) $2^{19}(2^{20} + 21)$

(D) $2^{20}(2^{20} + 21)$

Correct Answer: (B) $2^{19}(2^{20} - 21)$

Solution:

Concept:

- For a polynomial $P(x) = \sum a_k x^k$, the sum of all odd-indexed coefficients is $[P(1) - P(-1)]/2$.
- The requested list stops at index 37, so the odd coefficient a_{39} must be subtracted from the complete odd-indexed sum.

Step 1: Find the complete odd-indexed sum Let $P(x) = (1 + x + 2x^2)^{20}$. At $x = 1$, all coefficients add; at $x = -1$, the odd-indexed terms change sign. Thus

$$P(1) = 4^{20} = 2^{40},$$
$$P(-1) = 2^{20},$$
$$a_1 + a_3 + \cdots + a_{39} = \frac{2^{40} - 2^{20}}{2} = 2^{39} - 2^{19}.$$

The even-indexed coefficients cancel in this subtraction.

Step 2: Remove the omitted highest odd coefficient To obtain degree 39 from twenty factors of maximum degree 2, select x from one factor and $2x^2$ from each of the other nineteen. There are twenty choices for that factor, so

$$a_{39} = 20 \cdot 2^{19}.$$

No factor can contribute degree 0 here, because that would lower the total degree by at least two. Therefore

$$a_1 + a_3 + \cdots + a_{37} = 2^{39} - 2^{19} - 20 \cdot 2^{19}$$
$$= 2^{19}(2^{20} - 21).$$

A factor 2^{20} or a plus sign before 21 does not follow from the coefficient subtraction, ruling out the remaining choices.

Step 3: Conclusion The requested sum is $\boxed{2^{19}(2^{20} - 21)}$. Hence, the correct option is (B).

Quick Tip to Solve: After using $P(1) - P(-1)$, compare the final index in the question with the polynomial degree; subtract any omitted odd coefficients.

19. If α, β are natural numbers such that
$100^{\alpha} - 199\beta$ = (100)(100) + (99)(101) + (98)(102) + ... + (1)(199), then the slope of the line passing through (α, β) and origin is : [4 marks]

(A) 540

(B) 550

(C) 530

(D) 510

Correct Answer: (B) 550

Solution:

Concept:

- The products can be written as $(100-k)(100+k) = 10000 - k^2$, with $k = 0, 1, \ldots, 99$. Use $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$.
- The line from the origin to (α, β) has slope $m = \beta/\alpha$. The proposed slopes can therefore be tested in the given integer relation.

Step 1: Evaluate the sum of products There are 100 products, including the first product $100 \cdot 100$. Pairing the two factors symmetrically about 100 gives

$$\begin{aligned} S &= \sum_{k=0}^{99} (10000 - k^2) \\ &= 100 \cdot 10000 - \frac{99 \cdot 100 \cdot 199}{6} \\ &= 100^3 - 199 \cdot 1650 = 671650. \end{aligned}$$

Hence $\alpha = 3, \beta = 1650$ satisfies the stated equation and gives slope $1650/3 = 550$.

Step 2: Check the proposed slopes against the integer condition To verify the choice without assuming a unique natural-number representation, put $\beta = m\alpha$. Then

$$100^{\alpha} - 199m\alpha = 671650.$$

Each offered m lies between 510 and 550. For $\alpha = 1, 2$, the left side is less than 10000, hence cannot equal 671650. For $\alpha \geq 4$, even its lower bound $100^{\alpha} - 199 \cdot 550\alpha$ already exceeds 99 million at 4 and increases thereafter. Thus a listed slope requires $\alpha = 3$. Substitution yields $m = (100^3 - 671650)/(199 \cdot 3) = 550$. The other offered slopes fail the equation. Naturalness alone need not make the representation unique; the supplied choices select this value.

Step 3: Conclusion Among the supplied choices, the slope is $\boxed{550}$, attained by $(\alpha, \beta) = (3, 1650)$. Hence, the correct option is (B).

Quick Tip to Solve: Rewrite symmetrically paired products as a constant minus a square, and test slope choices through $\beta = m\alpha$ rather than assuming an exponent is uniquely determined.

20. Choose the correct statement about two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$ [4 marks]

(A) circles have same centre

(B) circles have no meeting point

(C) circles have only one meeting point

(D) circles have two meeting points

Correct Answer: (C) circles have only one meeting point

Solution:

Concept:

- The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.
- Two circles of radii r_1, r_2, with distinct centres distance d apart, touch externally when $d = r_1 + r_2$.

Step 1: Find the centres and radii Complete the squares in the two equations:

$$(x-5)^2 + (y-5)^2 = 9,$$
$$(x-11)^2 + (y-5)^2 = 9.$$

Thus the centres are $C_1 = (5, 5)$ and $C_2 = (11, 5)$, and each radius is 3. The centres are different, which already rules out option A. Both circles are real because their squared radii are positive.

Step 2: Compare the centre distance with the radii The centres have equal vertical coordinates, so

$$d = \sqrt{(11-5)^2 + (5-5)^2} = 6 = r_1 + r_2.$$

Hence they touch externally and have precisely one common point. That point lies halfway between the equal-radius centres, namely $(8, 5)$. Substituting it into either completed-square equation gives $9 = 9$, confirming tangency. No meeting point would require $d > 6$, and two meeting points would require $0 < d < 6$; neither condition holds here.

Step 3: Conclusion The circles have only one meeting point, $\boxed{(8, 5)}$. Hence, the correct option is (C).

Quick Tip to Solve: Complete the squares before judging intersections: compare the centre distance with both the sum and difference of the radii.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is :

[4 marks]

Solution:

Concept:

- Count occurrences separately in the units, tens and hundreds places. A number containing several 3s contributes once for each such position.
- Writing the numbers from 000 to 999 with three positions makes each digit occur equally often in every position. Leading zeros do not add any occurrences of digit 3.

Step 1: Count the occurrences in one position Consider all 1000 three-position strings from 000 through 999. Fix the units digit as 3. Each of the hundreds and tens positions can be any of the ten digits, giving

$$10 \times 10 = 100$$

occurrences in the units place. The identical count holds for the tens place and for the hundreds place, because after fixing a 3 in either position the remaining two positions are still independently free.

Step 2: Add the positions and check the endpoints Adding the three position counts gives

$$100 + 100 + 100 = 300.$$

This is an occurrence count, so a number such as 333 must be counted three times, once for each written 3. The position method does exactly this and needs no subtraction for overlap. The string 000 contributes zero occurrences, while the additional endpoint 1000 also contributes zero. Deleting 000 and including 1000 therefore leaves the total unchanged. Suppressing leading zeros from shorter strings does not affect their 3s either, so the count is exactly the one for the integers as normally written.

Step 3: Conclusion The digit 3 is written $\boxed{300}$ times.

Quick Tip to Solve: For digit occurrences, count by position and allow overlap; inclusion-exclusion is needed for counting numbers containing a digit, not for counting its total appearances.

22. Let f(x) and g(x) be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x)^2 dx$ is

[4 marks]

Solution:

Concept:

- The expression $f(x)^2$ is the square of $f(x)$; it is distinct from $f(x^2)$. Functional equations must be applied to the argument that is actually present.

- The first relation evaluates f only at nonnegative arguments. Therefore its values at negative arguments are unrestricted by either given relation.

Step 1: Construct functions satisfying both relations For every $t \geq 0$, set $f(t) = 24t - 64$, and for every real x, set $g(x) = -4x^3 + 24x^2 - 64$. Direct expansion gives $g(4-x) = 4x^3 - 24x^2 + 64 = -g(x)$. Consequently,

$$f(x^2) + g(4-x) = 24x^2 - 64 + 4x^3 - 24x^2 + 64 = 4x^3.$$

Thus these functions satisfy both conditions for every real x, regardless of the values assigned to $f(t)$ when $t < 0$.

Step 2: Exhibit two different values of the requested integral First define $f(t) = 0$ for $t < 0$. The integral printed in the question is then

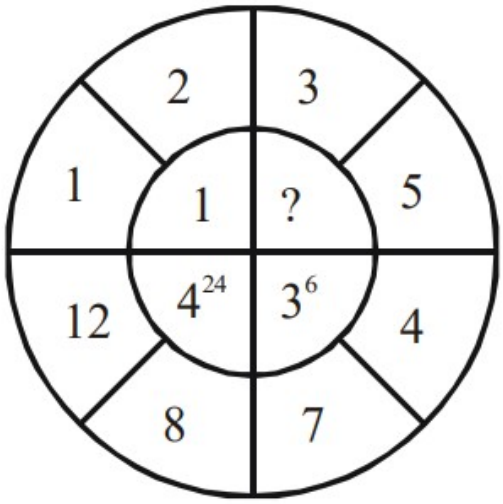
$$\int_{-4}^{4} f(x)^2\,dx = \int_{0}^{4} (24x - 64)^2\,dx = \left[192x^3 - 1536x^2 + 4096x\right]_0^4 = 4096.$$

Instead define $f(t) = 1$ for $t < 0$, keeping exactly the same nonnegative values and the same g. Both given relations still hold, but now the requested integral is $4096 + 4 = 4100$. A value at the single endpoint $x = 0$ does not affect either integral.

Step 3: Conclusion The printed integral $\int_{-4}^{4} f(x)^2\,dx$ has no unique value under the stated assumptions. In particular, 4096 and 4100 are both possible. The supplied answer-key value 512 is therefore not implied by the question as printed.

Quick Tip to Solve: Check whether a square belongs to the argument or to the function value before applying a functional equation; these are different quantities.

23. The missing value in the following figure is [4 marks]



Solution:

Concept:

- Compare the two outer entries in each quadrant and use their positive difference d.
- The three completed quadrants follow the rule $d^{d!}$, where $d!$ denotes the factorial of the positive integer d.

Step 1: Identify the rule from the completed quadrants In the upper-left quadrant the entries are 2 and 1, whose difference is $d = 1$. Its inner entry is $1^{1!} = 1$. In the lower-right quadrant the outer entries are 7 and 4, so $d = 3$; the inner entry is $3^{3!} = 3^6$, exactly as shown. In the lower-left quadrant the outer entries are 12 and 8, giving $d = 4$; the inner entry is $4^{4!} = 4^{24}$, again matching the figure. The factorial explains both of the nontrivial printed exponents simultaneously.

Step 2: Apply the same rule to the missing quadrant The upper-right quadrant contains the outer entries 5 and 3. Their positive difference is

$$d = |5 - 3| = 2.$$

The exponent prescribed by the completed quadrants is $d! = 2! = 2$, and hence the missing inner entry is

$$d^{d!} = 2^{2!} = 2^2 = 4.$$

Using the difference alone would give 2 and would fail to reproduce the exponents in the lower quadrants. Adding or multiplying the two outer entries also fails to match the three supplied inner entries.

Step 3: Conclusion The missing value in the figure is $\boxed{4}$.

Quick Tip to Solve: Use the two nontrivial exponents first: 6 and 24 suggest factorials of the corresponding differences 3 and 4.

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24. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is : [4 marks]

Solution:

Concept:

- For the quadratic $z^2 + az + 12 = 0$, Vieta's relations give $z_1 + z_2 = -a$ and $z_1 z_2 = 12$.
- If $0, z_1, z_2$ form an equilateral triangle, then $|z_1| = |z_2| = |z_1 - z_2| = r$, and the angle between the position vectors is 60°.

Step 1: Find the common side length The distances from the origin to the two roots are sides of the equilateral triangle, so both root moduli equal a positive number r. Taking the modulus of their product gives

$$r^2 = |z_1||z_2| = |z_1 z_2| = 12.$$

Hence $r = \sqrt{12} = 2\sqrt{3}$. This step uses the product rather than assuming that the roots are real. Two distinct real roots and the origin would be collinear and could not form the stated triangle.

Step 2: Calculate the modulus of the sum The vectors representing z_1 and z_2 enclose an angle of 60°. Therefore the squared magnitude of their sum is

$$|z_1 + z_2|^2 = r^2 + r^2 + 2r^2 \cos 60^\circ = 3r^2 = 36.$$

Equivalently, the parallelogram identity gives $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$; substituting the three equal side lengths yields the same result. Finally $-a = z_1 + z_2$, and a minus sign does not change a complex modulus.

Step 3: Conclusion Thus $|a| = |z_1 + z_2| = \sqrt{36} = \boxed{6}$.

Quick Tip to Solve: For an equilateral triangle with one vertex at the origin, the sum of the other two position vectors has magnitude $\sqrt{3}$ times a side.

25. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is _____. [4 marks]

Solution:

Concept:

- The arithmetic mean is the sum of all observations divided by their number: $\overline{x} = S/n$.
- Replacing one teacher keeps the number of teachers fixed at 25. Only the retiring teacher's age is removed and the new teacher's age is added.

Step 1: Find the total ages before and after replacement The original group contains 25 teachers with mean age 40 years. Its total age is therefore

$$S_{\text{old}} = 25 \times 40 = 1000 \text{ years}.$$

After one teacher retires and one teacher is appointed, there are still 25 teachers. The new mean of 39 years gives

$$S_{\text{new}} = 25 \times 39 = 975 \text{ years}.$$

These totals refer to the sum of individual ages, not to the age of any one teacher. The one-year fall in the mean corresponds to a fall of 25 years in this total.

Step 2: Write and solve the replacement equation Let the newly appointed teacher's age be x years. Remove the retiring teacher's age of 60 years from the original total, then add x:

$$1000 - 60 + x = 975.$$

Thus

$$x = 975 - 940 = 35.$$

As a direct check, the remaining 24 teachers have total age 940 years, and including a 35-year-old teacher gives $940 + 35 = 975$ years. Dividing by 25 returns the stated new mean, 39 years. Subtracting just one year from 60 would incorrectly treat the change in the group mean as the change in one

observation.

Step 3: Conclusion The newly appointed teacher is $\boxed{35 \text{ years}}$ old.

Quick Tip to Solve: With a fixed group size, multiply the change in mean by the number of members to obtain the change in the replaced observation.

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26. If $f(x) = \int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \geq 0), f(0) = 0$ and $f(1) = \frac{1}{K}$, then the value of K is [4 marks]

Solution:

Concept:

- The quotient rule gives $(u/v)\prime = (u\prime v - uv\prime)/v^2$. A squared denominator often suggests reversing this rule.
- The condition $f(0) = 0$ determines the additive constant of integration. Since $x \geq 0$, the denominator $x^2 + 1 + 2x^7$ is strictly positive.

Step 1: Recognize the required numerator Let $D(x) = x^2 + 1 + 2x^7$, so that $D\prime(x) = 2x + 14x^6$. Differentiate the quotient with numerator x^7:

$$\frac{d}{dx}\left(\frac{x^7}{D(x)}\right) = \frac{7x^6 D(x) - x^7 D\prime(x)}{D(x)^2}.$$

Expanding the numerator before cancelling is useful here:

$$7x^8 + 7x^6 + 14x^{13} - 2x^8 - 14x^{13} = 5x^8 + 7x^6.$$

This is exactly the numerator of the given integrand, including both coefficients and powers.

Step 2: Use the initial condition and evaluate at one The antiderivative is consequently

$$f(x) = \frac{x^7}{x^2 + 1 + 2x^7} + C.$$

At $x = 0$, the fraction is zero, and the condition $f(0) = 0$ forces $C = 0$. At $x = 1$,

$$f(1) = \frac{1}{1 + 1 + 2} = \frac{1}{4}.$$

The question writes this value as $1/K$, so equating the reciprocals gives $K = 4$. The positive denominator ensures the calculation is valid throughout the prescribed nonnegative domain; there is no pole between the initial point and the evaluation point.

Step 3: Conclusion Therefore $f(1) = 1/4 = 1/K$, and $\boxed{K = 4}$.

Quick Tip to Solve: For a denominator squared, try differentiating a quotient with a numerator whose power is one higher than the lowest numerator power in the integrand.

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27. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is _____. [4 marks]

Solution:

Concept:

- The curve $x^2y^2 = 1$ consists of $xy = 1$ and $xy = -1$. Every vertex and every side midpoint must satisfy $|xy| = 1$.
- For a square centered at the origin, if one vertex is (a, b), its consecutive vertices are $(-b, a), (-a, -b), (b, -a)$, obtained by quarter-turn rotations.

Step 1: Locate the center of the square Two distinct endpoints in the same quadrant on $|xy| = 1$ cannot have their midpoint on that curve: the function $y = 1/x$, or its signed version in that quadrant, is strictly convex or concave. Thus every side must cross at least one coordinate axis. The two axes can intersect the boundary of a convex square at most four times in total; therefore each side crosses exactly one axis, and the vertices occupy the four quadrants in order. Opposite vertices consequently have the same product xy. Write the center as (h, k) and the offsets of an opposite pair as $\pm(u, v)$. Equality of their products yields $hv + ku = 0$. The other opposite pair has an independent offset and gives the same linear relation. These two independent equations force $h = k = 0$.

Step 2: Apply the vertex and midpoint conditions Take consecutive vertices $A = (a, b)$, $B = (-b, a)$. The vertex condition gives $a^2b^2 = 1$. The midpoint of AB is $M = ((a - b)/2, (a + b)/2)$. Since M is also on the curve,

$$\left(\frac{a^2 - b^2}{4}\right)^2 = 1, \qquad (a^2 - b^2)^2 = 16.$$

Combining the two identities,

$$(a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2 = 16 + 4 = 20.$$

Step 3: Compute the requested square of the area The squared side length is

$$AB^2 = (a + b)^2 + (b - a)^2 = 2(a^2 + b^2).$$

The area of a square equals its side length squared, so its area is $2(a^2 + b^2)$. Squaring that area gives

$$\big(\text{Area}(ABCD)\big)^2 = 4(a^2 + b^2)^2 = 4 \times 20 = 80.$$

Both the vertex condition and the midpoint condition have been used; the vertices alone would not determine this quantity.

Step 4: Conclusion The square of the area of $ABCD$ is $\boxed{80}$.

Quick Tip to Solve: Use a quarter-turn to write adjacent vertices, then apply the curve equation to their midpoint; the identity relating sum and difference of squares finishes the calculation.

28. The number of solutions of the equation

$|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval [0, 2π] is [4 marks]

Solution:

Concept:

- The functions $\cot x$ and $1/\sin x$ are defined only when $\sin x \neq 0$. Thus $0, \pi, 2\pi$ are excluded from the stated interval.
- Split the absolute value according to the sign of $\cot x$: it equals $\cot x$ for nonnegative cotangent and $-\cot x$ for negative cotangent.

Step 1: Eliminate the nonnegative-cotangent case If $\cot x \geq 0$, the equation becomes

$$\cot x = \cot x + \frac{1}{\sin x}.$$

Subtracting $\cot x$ requires $1/\sin x = 0$, which is impossible for any defined real value of $\sin x$. This also rules out the boundary cases $\cot x = 0$, namely $x = \pi/2$ and $x = 3\pi/2$, because their cosecants are nonzero.

Step 2: Solve and filter the negative-cotangent case If $\cot x < 0$, the equation becomes

$$-\cot x = \cot x + \csc x, \qquad -2\frac{\cos x}{\sin x} = \frac{1}{\sin x}.$$

Multiplication by $\sin x$ is valid on the domain and gives $\cos x = -1/2$. In $[0, 2\pi]$, the candidates are $2\pi/3$ and $4\pi/3$. At $2\pi/3$, cosine is negative and sine positive, so cotangent is negative, as required. At $4\pi/3$, both sine and cosine are negative, making cotangent positive; that candidate violates the case condition and must be rejected. The surviving angle is not a singular endpoint.

Step 3: Conclusion The only solution is $x = 2\pi/3$. Hence the number of solutions is $\boxed{1}$.

Quick Tip to Solve: After solving an absolute-value case, check its sign condition; otherwise $4\pi/3$ would be counted incorrectly here.

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29. The equation of the planes parallel to the plane x − 2y + 2z − 3 = 0 which are at unit distance from the point (1, 2, 3) is ax + by + cz + d = 0. If (b − d) = k(c − a), then the positive value of k is : [4 marks]

Solution:

Concept:

- Parallel planes have proportional normal vectors, so the required family can be written as $x - 2y + 2z + d = 0$. Multiplying all coefficients by the same nonzero constant does not change the plane or the ratio defining k.
- The perpendicular distance of (x_0, y_0, z_0) from $ax + by + cz + d = 0$ is $|ax_0 + by_0 + cz_0 + d|/\sqrt{a^2 + b^2 + c^2}$. The absolute value allows planes on both sides of the given point.

Step 1: Write the parallel family The given plane has normal vector $(1, -2, 2)$, whose magnitude is

$\sqrt{1+4+4} = 3$. Therefore choose $a = 1, b = -2, c = 2$ and leave only the constant term undetermined. At the specified point, the coordinate contribution is $1 - 2(2) + 2(3) = 3$; the constant -3 in the original plane need not remain in a parallel plane.

Step 2: Apply the unit-distance condition The required distance is one, giving

$$\frac{|3+d|}{3} = 1 \quad \Rightarrow \quad |3+d| = 3.$$

Thus $3 + d = 3$ or $3 + d = -3$, so $d = 0$ or $d = -6$. These give the two planes $x - 2y + 2z = 0$ and $x - 2y + 2z - 6 = 0$. Substitution into the distance formula verifies unit distance for each.

Step 3: Evaluate the coefficient relation Here $c - a = 2 - 1 = 1$, so the relation becomes $-2 - d = k$. For $d = 0$, $k = -2$, whereas for $d = -6$, $k = 4$. Only the second value meets the stated positivity condition; both distance branches must be considered before choosing it.

Step 4: Conclusion The positive value is $\boxed{k = 4}$.

Quick Tip to Solve: Normalize parallel-plane coefficients first, and retain both signs in the distance equation until the requested sign of the final parameter selects a branch.

30. Let the plane ax + by + cz + d = 0 bisect the line joining the points (4, −3, 1) and (2, 3, −5) at the right angles. If a, b, c, d are integers, then the minimum value of ($a^2 + b^2 + c^2 + d^2$) is _____. [4 marks]

Solution:

Concept:

- A plane that bisects a line segment at right angles passes through its midpoint and has a normal vector parallel to the segment. This is the locus of points equidistant from the two endpoints.
- The coefficients of a plane can be multiplied by any common nonzero factor. For integer coefficients, the primitive integer normal gives the smallest possible sum of squared coefficients.

Step 1: Find the midpoint and normal Let the endpoints be $P = (4, -3, 1)$ and $Q = (2, 3, -5)$. Their midpoint is

$$H = \left(\frac{4+2}{2}, \frac{-3+3}{2}, \frac{1-5}{2}\right) = (3, 0, -2).$$

A vector along the joining line is $P - Q = (2, -6, 6) = 2(1, -3, 3)$. We can therefore use $(1, -3, 3)$ as the plane normal, removing a common factor of two.

Step 2: Form the plane through the midpoint The point-normal equation gives

$$(x - 3) - 3(y - 0) + 3(z + 2) = 0,$$

which simplifies to $x - 3y + 3z + 3 = 0$. Its normal is along the original segment and its value at the midpoint is zero, so it satisfies both required geometrical conditions. Merely passing through the midpoint without this normal would not ensure perpendicular bisection.

Step 3: Minimize over integer representations Every representation of this same plane is $(a, b, c, d) =$

$t(1,-3,3,3)$, with $t \neq 0$. Since its first coefficient is $a = t$, integer coefficients require nonzero integer t. Hence

$$a^2 + b^2 + c^2 + d^2 = (1+9+9+9)t^2 = 28t^2.$$

The smallest possible t^2 is one, attained for either overall sign of the primitive coefficients.

Step 4: Conclusion The required minimum is $\boxed{28}$.

Quick Tip to Solve: After forming a perpendicular bisector plane, divide out the coefficient gcd before summing squares; the overall sign has no effect.

Physics – Section A: Multiple Choice Questions (80 marks)

31. An oil drop of radius 2 mm with a density 3g cm^{-3} is held stationary under a constant electric field 3.55 $\times$ 10^5 V m^{-1} in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess? (consider g = 9.81 m/s^2) [4 marks]

(A) 48.8 $\times$ 10^{11}

(B) 1.73 $\times$ 10^{10}

(C) 17.3 $\times$ 10^{10}

(D) 1.73 $\times$ 10^{12}

Correct Answer: (B) 1.73 $\times$ 10^{10}

Solution:

Concept:

- For a stationary charged drop, the upward electric force balances its weight: $|q|E = mg$. The excess electron count is the magnitude $n = |q|/e$, where $e = 1.6 \times 10^{-19}\,\text{C}$.
- A spherical drop has mass $m = \rho(4\pi r^3/3)$. All lengths and densities must first be converted to SI units so that the force balance is consistent with the field in volts per metre.

Step 1: Convert the data and calculate the mass The radius is $r = 2 \times 10^{-3}\,\text{m}$, and $3\,\text{g cm}^{-3} = 3 \times 10^3\,\text{kg m}^{-3}$. Therefore

$$m = \frac{4}{3}\pi(2 \times 10^{-3})^3(3 \times 10^3) \simeq 1.0053 \times 10^{-4}\,\text{kg}.$$

The cubed radius is $8 \times 10^{-9}\,\text{m}^3$, an important power-of-ten check. The given radius is in millimetres, so it must not be treated as a micrometre radius.

Step 2: Balance the forces and count electrons At equilibrium, the magnitude of the electric force equals the downward gravitational force. Using the supplied acceleration,

$$|q| = \frac{mg}{E} = \frac{(1.0053 \times 10^{-4})(9.81)}{3.55 \times 10^5} \simeq 2.778 \times 10^{-9}\,\text{C}.$$

Thus

$$n = \frac{2.778 \times 10^{-9}}{1.6 \times 10^{-19}} \simeq 1.736 \times 10^{10}.$$

The negative sign of excess electronic charge determines the required field direction, but the number of electrons is positive.

Step 3: Match the magnitude to the options To the precision of the choices this is 1.73×10^{10}. Option (C) is ten times larger, option (D) is a hundred times larger, and option (A) also has the wrong magnitude. No additional force or buoyancy correction is specified in the question, so the stated weight-electric balance is used.

Step 4: Conclusion The drop possesses approximately $\boxed{1.73 \times 10^{10}}$ excess electrons. Hence, the correct option is (B).

Quick Tip to Solve: Use $n = 4\pi\rho r^3 g/(3eE)$ in SI units; a length conversion error is cubed because the drop mass depends on volume.

32. Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionization potential of this hydrogen atom will be : [4 marks]

(A) 13.6 eV

(B) 2815.2 eV

(C) 331.2 eV

(D) 27.2 eV

Correct Answer: (B) 2815.2 eV

Solution:

Concept:

- In the Bohr approximation, the bound-state energy of a hydrogen-like atom is proportional to the orbiting particle mass when the nucleus is treated as fixed: $E_n \propto -mZ^2/n^2$.
- Ionization from the ground state requires positive energy equal to the magnitude of E_1. Hydrogen with an electron requires $13.6\,\text{eV}$; the replacement particle has the same charge magnitude, so only the stated mass factor changes.

Step 1: Identify what changes in the energy formula The Coulomb attraction is unchanged because the muon and electron have equal charges, and the hydrogen nucleus still has $Z = 1$. The Bohr energy formula has the form

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}.$$

Thus the mass occurs linearly, rather than inversely or quadratically. A larger orbiting mass produces a more tightly bound ground state and correspondingly raises the ionization energy.

Step 2: Scale the hydrogen ground-state binding energy Using $m_\mu = 207 m_e$, the ionization energy in the fixed-nucleus model is

$$I_\mu = \frac{m_\mu}{m_e} I_{\text{H}} = 207(13.6) = 2815.2\,\text{eV}.$$

The energy is positive because ionization supplies energy to lift the particle from a negative bound energy to zero at infinity. This is the mass scaling intended by the supplied data and options.

Step 3: Check the alternatives and terminology Option (A) leaves the mass change out, while (D) merely doubles the usual hydrogen value. Option (C) does not equal the required factor of 207. Strictly, ionization potential is measured in volts and has numerical value $2815.2\,\mathrm{V}$ for a singly charged particle; the choices state the equivalent ionization energy in electronvolts. A finite-nuclear-mass refinement would use reduced mass, outside the fixed-nucleus approximation represented by these choices.

Step 4: Conclusion The ionization energy is $\boxed{2815.2\,\mathrm{eV}}$, corresponding to ionization potential $2815.2\,\mathrm{V}$. Hence, the correct option is (B).

Quick Tip to Solve: For a fixed hydrogen nucleus and unchanged charge, scale the Bohr binding energy directly with the orbiting particle mass.

---

33. A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\vec{B} = 2.0 \times 10^{-8}\hat{k}T$. (where, $\hat{k}$ is unit vector along z-direction) What is $\vec{E}$ at this point? [4 marks]

(A) 0.6 $\hat{j}$ V/m

(B) 6.0 $\hat{k}$ V/m

(C) 6.0 $\hat{j}$ V/m

(D) 0.6 $\hat{k}$ V/m

Correct Answer: (C) 6.0 $\hat{j}$ V/m

Solution:

Concept:

- In a plane electromagnetic wave in vacuum, the electric and magnetic fields are perpendicular to each other and to the propagation direction. Their instantaneous magnitudes satisfy $E = cB$.
- The energy-flow direction is that of $\vec{E} \times \vec{B}$. The right-handed Cartesian basis satisfies $\hat{j} \times \hat{k} = \hat{i}$, fixing the electric-field sign once propagation and magnetic-field directions are given.

Step 1: Calculate the electric-field magnitude The magnetic field magnitude at the stated event is $2.0 \times 10^{-8}\,\mathrm{T}$. With $c = 3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$,

$$E = cB = (3.0 \times 10^{8})(2.0 \times 10^{-8}) = 6.0\,\mathrm{V\,m^{-1}}.$$

The frequency is not needed for this ratio in vacuum: it would determine wavelength through $c = f\lambda$, but not the ratio of the field magnitudes.

Step 2: Determine the vector direction The wave travels along positive x, while $\vec{B}$ is along positive z. Hence the electric field must point along positive y, since

$$(+\hat{j}) \times (+\hat{k}) = +\hat{i}.$$

A field along negative y would reverse the energy flow, and a field along z would be parallel to the magnetic field, violating transversality. The magnitude calculation alone is therefore insufficient to

choose an option.

Step 3: Eliminate the incorrect choices Options (A) and (D) give $0.6\,\mathrm{V\,m^{-1}}$, a factor of ten too small from mishandling the powers of ten. Option (B) has the right magnitude but places the field along $\hat{k}$, the magnetic-field direction. Only option (C) satisfies both the magnitude and cross-product conditions simultaneously.

Step 4: Conclusion At the specified point, $\boxed{\vec{E} = 6.0\hat{j}\,\mathrm{V\,m^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $E = cB$ for magnitude, then check $\vec{E} \times \vec{B}$ against the propagation direction; the right magnitude alone does not identify the vector.

---

34. A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become : [4 marks]

(A) $\omega\frac{M}{M+m}$
(B) $\omega\frac{M+2m}{M}$
(C) $\omega\frac{M}{M+2m}$
(D) $\omega\frac{M-2m}{M+2m}$

Correct Answer: (C) $\omega\frac{M}{M+2m}$

Solution:

Concept:

- When no external torque acts about the rotation axis, the total angular momentum about that axis is conserved, including during attachment of the particles.
- A thin ring rotating about its symmetry axis has moment of inertia Mr^2. A point mass at perpendicular distance r adds mr^2, and moments of inertia add for bodies rotating together.

Step 1: Write the initial angular momentum Initially the ring alone rotates at angular speed ω, so its moment of inertia and angular momentum are

$$I_i = Mr^2, \qquad L_i = Mr^2\omega.$$

The usual attachment condition means the particles bring no initial angular momentum about this axis. The attachment can dissipate rotational kinetic energy, but internal forces do not change the system angular momentum about a torque-free axis.

Step 2: Find the moment of inertia after attachment The two particles are placed at diametrically opposite points of the ring. Each is at the same distance r from the rotation axis, giving

$$I_f = Mr^2 + mr^2 + mr^2 = (M + 2m)r^2.$$

Opposite locations do not make their inertia contributions cancel: both contributions are positive and depend on squared distance. The symmetric placement also keeps the center of mass on the axis.

Step 3: Apply conservation and compare options If the final angular speed is ω_f, conservation requires

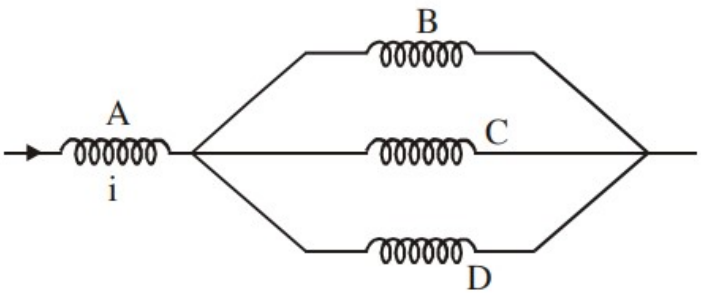
$$Mr^2\omega = (M + 2m)r^2\omega_f, \qquad \omega_f = \frac{M}{M + 2m}\omega.$$

The radius cancels because all three masses lie at that radius. Option (A) counts only one added particle; (B) predicts an increase despite increased inertia; (D) incorrectly subtracts mass in the numerator and can become negative. The derived expression approaches ω as $m \to 0$ and decreases as the attached masses increase, as expected.

Step 4: Conclusion The final angular speed is $\boxed{\omega_f = \omega\frac{M}{M + 2m}}$. Hence, the correct option is (C).

Quick Tip to Solve: For particles attached to a spinning body, conserve angular momentum rather than kinetic energy and add every mr^2 contribution with a positive sign.

35. Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3T, the field at the center of C would be : (Assume that the magnetic field is confined with in the volume of respective solenoid). [4 marks]



(A) 12T

(B) 6T

(C) 9T

(D) 1T

Correct Answer: (D) 1T

Solution:

Concept:

- The field inside a long solenoid is $B = \mu_0 nI$, where n is turns per unit length. Identical solenoids therefore have magnetic fields proportional to their respective currents.
- Equal parallel branches carry equal currents. Kirchhoff's junction rule states that the current entering a junction equals the sum leaving it; the current through a preceding series component is the total branch current.

Step 1: Read the connection in the figure Solenoid A lies before the junction and carries the total current i. After that junction, B, C and D form three separate branches between the same two junctions.

Since the solenoids are identical, those branches have equal electrical characteristics, so symmetry gives equal currents through B, C and D. They are parallel components, not a series chain.

Step 2: Divide the total current between the branches Writing $I_B = I_C = I_D$ and applying the junction rule gives

$$i = I_B + I_C + I_D = 3I_C, \qquad I_C = \frac{i}{3}.$$

The current i flows through A before it splits; it must not be assigned unchanged to all three branches. The common branch voltage is the same across B, C and D, while their identical response ensures the equal division.

Step 3: Relate the magnetic fields The given assumption confines each field to its own solenoid, so fields from B or D do not need to be added at the center of C. Equal turn densities give

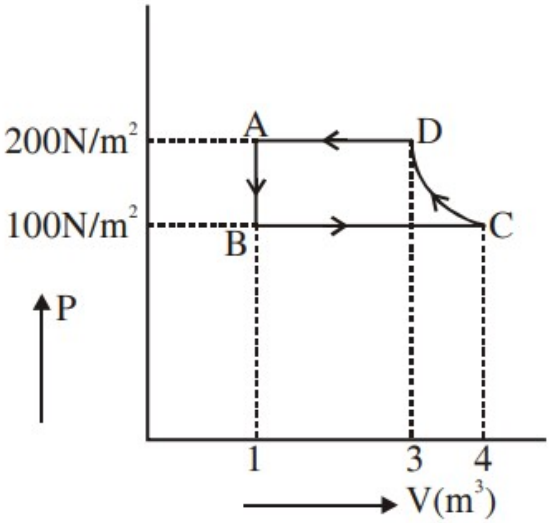
$$\frac{B_C}{B_A} = \frac{I_C}{I_A} = \frac{1}{3}, \qquad B_C = \frac{3\,\text{T}}{3} = 1\,\text{T}.$$

The alternatives 6 T, 9 T and 12 T would require more current in C than in A, inconsistent with dividing the total current among three identical branches. Their larger values cannot follow from the stated field-confinement assumption.

Step 4: Conclusion The magnetic field at the center of C is $\boxed{1\,\text{T}}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify series and parallel connections before using the solenoid field formula; for identical solenoids, the field ratio is simply the current ratio.

36. The P-V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (use γ = 1.4) : [4 marks]



(A) −500 J

(B) −400 J

(C) 400 J

(D) 200 J

Correct Answer: (A) −500 J

Solution:

Concept:

- For an ideal gas undergoing an adiabatic process, heat supplied is zero: $Q = 0$. With work positive when done by the gas, the first law gives $W = -\Delta U$.
- Using $U = PV/(\gamma - 1)$, the endpoint expression is $W_{i\to f} = (P_iV_i - P_fV_f)/(\gamma - 1)$. A product of pressure in pascals and volume in cubic metres is energy in joules.

Step 1: Read the initial and final states The arrow on the curved path points from C towards D. Thus C is the initial state and D is the final state, not the reverse. From the plotted coordinates, $P_C = 100\,\text{Pa}$, $V_C = 4\,\text{m}^3$, $P_D = 200\,\text{Pa}$ and $V_D = 3\,\text{m}^3$. Consequently,

$$P_CV_C = 400\,\text{J}, \qquad P_DV_D = 600\,\text{J}.$$

The decrease in volume also tells us that work by the gas must be negative.

Step 2: Apply the adiabatic energy relation Use the given $\gamma = 1.4$, so that $\gamma - 1 = 0.4$. Substitution of the stated endpoints gives

$$W_{CD} = \frac{400 - 600}{1.4 - 1} = \frac{-200}{0.4} = -500\,\text{J}.$$

The internal energy increases by $500\,\text{J}$; with no heat input, this increase is supplied by work done on the gas. The positive choices have the wrong sign for compression, and $-400\,\text{J}$ omits the correct endpoint difference and heat-capacity factor.

Step 3: Conclusion The work done by the gas in the stated adiabatic process CD is $\boxed{-500\,\text{J}}$. Hence, the correct option is (A).

Quick Tip to Solve: For adiabatic work, read the arrow first and use initial PV minus final PV; compression should give negative work by the gas.

37. In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 $\overset{o}{A}$ is :- [4 marks]

(A) 1178×10^{-9} m

(B) 1178×10^{-6} m

(C) 1178×10^{-12} m

(D) 5890×10^{-7} m

Correct Answer: (B) 1178×10^{-6} m

Solution:

Concept:

- For two coherent slits separated by d, the bright-fringe positions in the small-angle approximation are $y_n = n\lambda D/d$, where D is the screen distance.

- The fringe width is $\beta = \lambda D/d$. The separation of orders n_1 and n_2 on the same side of the centre is $|n_2 - n_1|\beta$.

Step 1: Convert the wavelength and slit separation The wavelength is 5890 angstrom, and one angstrom is $10^{-10}\,\mathrm{m}$. Hence

$$\lambda = 5890 \times 10^{-10}\,\mathrm{m} = 5.890 \times 10^{-7}\,\mathrm{m}.$$

The slit separation is $d = 0.5\,\mathrm{mm} = 5.0 \times 10^{-4}\,\mathrm{m}$, while $D = 0.5\,\mathrm{m}$. Keeping all three lengths in metres avoids mixing millimetres and angstroms.

Step 2: Count the fringe intervals The first and third bright fringes are separated by two fringe widths, since $3 - 1 = 2$. First calculate

$$\beta = \frac{(5.890 \times 10^{-7})(0.5)}{5.0 \times 10^{-4}} = 5.890 \times 10^{-4}\,\mathrm{m}.$$

Therefore

$$y_3 - y_1 = 2\beta = 1.178 \times 10^{-3}\,\mathrm{m} = 1178 \times 10^{-6}\,\mathrm{m}.$$

Options (A) and (C) are much too small because of incorrect powers of ten. Option (D) gives one fringe width rather than two. The screen distance is much larger than the slit separation, consistent with the approximation used.

Step 3: Conclusion The distance is $\boxed{1178 \times 10^{-6}\,\mathrm{m}}$, or $1.178\,\mathrm{mm}$. Hence, the correct option is (B).

Quick Tip to Solve: Between the first and third bright fringes there are two intervals, so calculate $2\lambda D/d$, not $3\lambda D/d$.

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38. A particle is travelling 4 time as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, the mass of the particle is : [4 marks]

(A) $\frac{1}{16}$ times the mass of e^-

(B) 8 times the mass of e^-

(C) 16 times the mass of e^-

(D) $\frac{1}{8}$ times the mass of e^-

Correct Answer: (D) $\frac{1}{8}$ times the mass of e^-

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$. In the nonrelativistic form used here, momentum is $p = mv$, so $\lambda = h/(mv)$.
- For two particles, Planck's constant cancels in the wavelength ratio: $\lambda_p/\lambda_e = m_e v_e/(m_p v_p)$. Wavelength is inversely proportional to the product of mass and speed.

Step 1: Insert the two given ratios Let the electron have mass m_e and speed v_e, and let the other particle have mass m_p. The stated speed is $v_p = 4v_e$, while its wavelength is twice the electron

wavelength: $\lambda_p/\lambda_e = 2$. Therefore

$$2 = \frac{m_e v_e}{m_p(4v_e)} = \frac{m_e}{4m_p}.$$

The speed ratio belongs in the denominator; increasing speed alone would shorten, rather than lengthen, the wavelength.

Step 2: Solve and check the mass ratio Rearranging gives

$$8m_p = m_e, \qquad \frac{m_p}{m_e} = \frac{1}{8}.$$

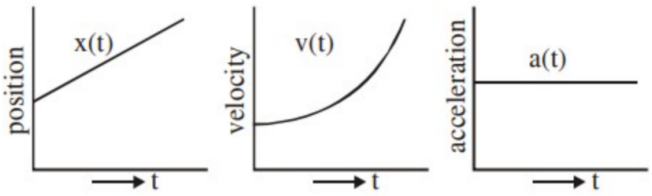
A direct momentum check confirms this: $p_p = (m_e/8)(4v_e) = p_e/2$, so $\lambda_p = 2\lambda_e$, exactly as required. A mass $m_e/16$ would instead produce wavelength ratio four. Masses $8m_e$ and $16m_e$ would yield momentum ratios 32 and 64 and therefore much smaller wavelengths, contradicting the question. The particle must be lighter than the electron to offset its greater speed.

Step 3: Conclusion The particle mass is $\boxed{m_e/8}$, one-eighth of the electron mass. Hence, the correct option is (D).

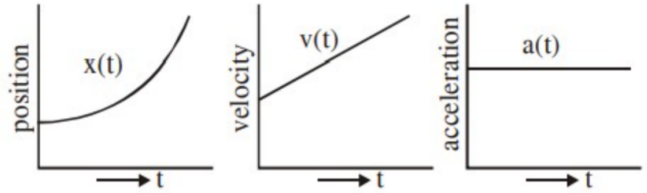
Quick Tip to Solve: Write the wavelength ratio as the inverse momentum ratio before substituting; this prevents accidentally reversing the mass ratio.

39. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by : [4 marks]

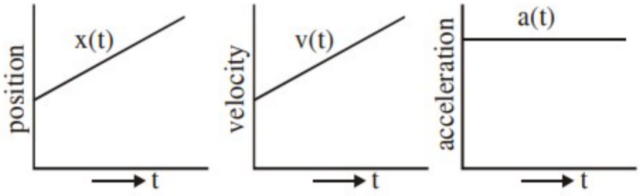
(A)



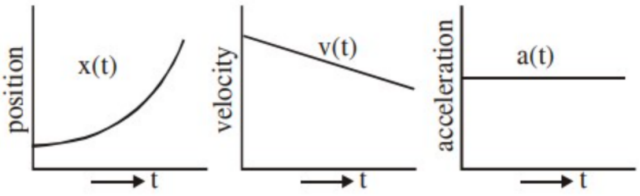
(B)



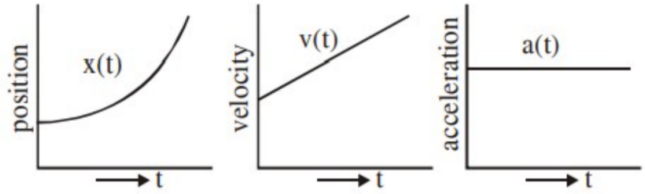
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For constant acceleration a, the velocity and position are $v(t) = u + at$ and $x(t) = x_0 + ut + \frac{1}{2}at^2$.
- Graphs must satisfy both derivative relations: $v = dx/dt$ and $a = dv/dt$. A horizontal acceleration graph requires a straight velocity graph with the corresponding slope.

Step 1: Identify the consistent graph shapes The depicted acceleration is a constant positive value. Therefore velocity must increase linearly with time, and position must curve upward because its slope, the velocity, is increasing. A positive initial velocity is allowed: it is the intercept of the velocity graph. Similarly, the position can begin from any initial coordinate. Option (B) shows precisely these three features: an upward-curving position graph, a rising straight velocity line, and a horizontal positive acceleration line.

Step 2: Eliminate the inconsistent combinations In option (A), position is a straight line, which implies constant velocity, whereas its velocity curve changes slope. These cannot describe the same motion with constant nonzero acceleration. Option (C) again has straight-line position, inconsistent with its increasing velocity. In option (D), the downward velocity slope implies negative acceleration, but the acceleration graph lies above the time axis and the position graph curves upward. The signs and derivatives therefore disagree. Only option (B) satisfies both derivative checks at every displayed time, rather than merely making the acceleration graph horizontal.

Step 3: Conclusion The mutually consistent position, velocity and acceleration plots are those in (B). Hence, the correct option is (B).

Quick Tip to Solve: Check the slopes in sequence: slope of position is velocity, and slope of velocity is acceleration. All three plots must agree.

40. In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is : [4 marks]

(A) 3.9

(B) 8.4

(C) 7.5

(D) 3.0

Correct Answer: (A) 3.9

Solution:

Concept:

- For a uniform conductor, $R = \rho l/A$, while Ohm's law gives $R = V/I$. With diameter d, area is $A = \pi d^2/4$, so $\rho = \pi V d^2/(4Il)$.
- Maximum fractional errors in a product or quotient add in magnitude. A measured quantity raised to a power contributes that power times its fractional error; hence the diameter error is counted twice.

Step 1: Assign errors from the stated precision Take one unit in the last quoted decimal place as the maximum measurement error: $\Delta V = 0.1\,\text{V}$, $\Delta I = 0.01\,\text{A}$, $\Delta l = 0.1\,\text{cm}$, and $\Delta d = 0.01\,\text{mm}$. Their percentage errors are

$$\frac{\Delta V}{V}100 = 2.0\%, \quad \frac{\Delta I}{I}100 = 0.5\%,$$

$$\frac{\Delta l}{l}100 = 1.0\%, \quad \frac{\Delta d}{d}100 = 0.2\%.$$

Each ratio uses the same units in numerator and denominator, so no length-unit conversion is necessary.

Step 2: Add the maximum contributions Apply the error rule to $\rho \propto Vd^2/(Il)$:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta V}{V} + 2\frac{\Delta d}{d} + \frac{\Delta I}{I} + \frac{\Delta l}{l}.$$

Thus the maximum percentage error is

$$2.0 + 2(0.2) + 0.5 + 1.0 = 3.9\%.$$

Denominator errors are added, not subtracted, because the question asks for the largest possible error. The factors π and 4 are exact and introduce no measurement error. The alternatives 8.4, 7.5 and 3.0 do not equal the sum obtained from the stated precision and the squared diameter.

Step 3: Conclusion The maximum permissible percentage error in resistivity is $\boxed{3.9\%}$. Hence, the correct option is (A).

Quick Tip to Solve: Express resistivity in measured quantities first: $\rho \propto Vd^2/(Il)$. Then add errors, remembering to double the diameter contribution.

41. In a series LCR resonance circuit, if we change the resistance only, from a lower to higher value :

[4 marks]

(A) The bandwidth of resonance circuit will increase.

(B) The resonance frequency will increase.

(C) The quality factor will increase.

(D) The quality factor and the resonance frequency will remain constant.

Correct Answer: (A) The bandwidth of resonance circuit will increase.

Solution:

Concept:

- For a series LCR circuit, resonance occurs when $\omega_0 L = 1/(\omega_0 C)$, giving $\omega_0 = 1/\sqrt{LC}$. This resonance frequency is fixed by L and C.
- The angular-frequency bandwidth is $\Delta\omega = R/L$, and the quality factor is $Q = \omega_0 L/R$. Bandwidth measures the separation of the two half-power frequencies.

Step 1: Follow the effect of increasing resistance Only R changes; the inductance and capacitance remain fixed. Since

$$\Delta\omega = \frac{R}{L}, \qquad \Delta f = \frac{R}{2\pi L},$$

increasing the resistance increases the bandwidth in either frequency convention. The resonant current peak becomes less sharp. This is also consistent with the smaller peak current $I_0 = V_0/R$: increasing resistance reduces the peak and broadens the frequency response.

Step 2: Check the resonance frequency and quality factor The resonance condition is cancellation of inductive and capacitive reactances. It gives

$$f_0 = \frac{1}{2\pi\sqrt{LC}},$$

which contains no resistance, so option (B) is false. On the other hand,

$$Q = \frac{\omega_0 L}{R} = \frac{f_0}{\Delta f}$$

decreases as resistance increases, ruling out option (C). Option (D) is also false because only the resonance frequency stays constant, while the quality factor changes. The three relations agree: constant centre frequency with greater bandwidth means a lower quality factor and weaker selectivity.

Step 3: Conclusion The bandwidth increases when the series resistance is increased. Hence, the correct option is (A).

Quick Tip to Solve: For a series circuit remember $\Delta\omega = R/L$: more resistance gives a broader resonance peak and a smaller quality factor.

42. An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is : [4 marks]

(A) 2.5 ms

(B) 25 ms

(C) 2.5 s

(D) 0.25 ms

Correct Answer: (A) 2.5 ms

Solution:

Concept:

- For a resistor connected to a sinusoidal AC source, the current is sinusoidal and in phase with the applied voltage. Its rms magnitude is $I_{\text{rms}} = I_0/\sqrt{2}$.
- The angular frequency is $\omega = 2\pi f$. Measuring time from a positive current maximum gives $i(t) = I_0 \cos(\omega t)$.

Step 1: Find the phase change from peak to rms Set the initial time at the positive maximum, where the current is I_0. At the first subsequent instant when it reaches the rms value,

$$I_0 \cos(\omega t) = \frac{I_0}{\sqrt{2}}.$$

Cancelling the amplitude gives $\cos(\omega t) = 1/\sqrt{2}$. The first decreasing passage therefore has $\omega t = \pi/4$, a phase interval of 45 degrees. Using a sine function without shifting its starting phase would instead measure time from zero current, which is not the requested starting point.

Step 2: Convert the phase interval into time For $f = 50\,\text{Hz}$, the period is

$$T = \frac{1}{f} = 0.020\,\text{s} = 20\,\text{ms}.$$

The phase interval $\pi/4$ is one-eighth of a full cycle, so

$$t = \frac{\pi/4}{2\pi f} = \frac{1}{8f} = \frac{1}{400}\,\text{s} = 2.5\,\text{ms}.$$

The 220 V rating fixes the current amplitude for a given resistance but cancels from this timing ratio. The alternatives 25 ms, 2.5 s and 0.25 ms do not correspond to one-eighth of the 20 ms period.

Step 3: Conclusion The first time interval from maximum current to its rms value is $\boxed{2.5\,\text{ms}}$. Hence, the correct option is (A).

Quick Tip to Solve: On a sinusoid, moving from the peak to $1/\sqrt{2}$ of the peak takes 45 degrees, or one-eighth of a period.

43. Your friend is having eye sight problem. She is not able to see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as :

[4 marks]

(A) Astigmatism

(B) Myopia with Astigmatism

(C) Presbyopia with Astigmatism

(D) Myopia and Hypermetropia

Correct Answer: (B) Myopia with Astigmatism

Solution:

Concept:

- Myopia is defective distant vision: parallel rays from distant objects focus in front of the retina when the eye is relaxed. A concave spherical lens corrects this component.
- Astigmatism results from unequal curvature, and hence unequal refracting power, in different meridians of the cornea or eye lens. Lines in different directions cannot simultaneously form a sharp retinal image.

Step 1: Identify the defect in distant vision The friend cannot see the distant mesh clearly. This identifies the distant-vision difficulty associated with myopia. Myopia alone would make the distant pattern blurred, but would not explain why a uniform grid appears directionally non-uniform and distorted.

Step 2: Use the distortion to identify the additional defect The window mesh contains lines in different directions. Unequal focusing of those directions is characteristic of astigmatism, so the complete diagnosis combines myopia with astigmatism. A cylindrical correction addresses the unequal meridional powers, while the myopic component needs a diverging spherical correction. Option (A) names only the distortion component. Presbyopia in (C) concerns age-related loss of accommodation and chiefly difficulty with near vision; no such symptom is stated. Option (D) does not account for the directional distortion of the mesh.

Step 3: Conclusion The diagnosis is myopia with astigmatism: distant vision is unclear and the regular mesh appears distorted. Hence, the correct option is (B).

Quick Tip to Solve: For a grid, distinguish simple blur from unequal sharpness in different directions: the latter signals astigmatism.

44. A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire. [4 marks]

(A) Loop assumes circular shape with its plane normal to the field.

(B) Loop assumes circular shape with its plane parallel to the field.

(C) Wire gets stretched to become straight.

(D) Shape of the loop remains unchanged.

Correct Answer: (A) Loop assumes circular shape with its plane normal to the field.

Solution:

Concept:

- A current element in a magnetic field experiences the force $d\mathbf{F} = I\, d\boldsymbol{\ell} \times \mathbf{B}$, where I is the current and $d\boldsymbol{\ell}$ points along it.
- A planar loop has magnetic moment $\boldsymbol{\mu} = IA\hat{\mathbf{n}}$, torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$, and potential energy $U = -\boldsymbol{\mu} \cdot \mathbf{B}$.

Step 1: Find the stable orientation The field exerts a torque that aligns the magnetic moment with the external field. Since the magnetic moment is perpendicular to the plane of the loop, alignment makes the plane normal to the magnetic field. A plane parallel to the field would instead place its normal perpendicular to the field and would not be the stable aligned orientation.

Step 2: Determine the shape of the flexible loop With its normal aligned to the field, the magnetic forces on the wire tend to expand the enclosed area. For fixed wire length, the magnetic energy $U = -IAB$ decreases when the area increases. Among closed curves of a given perimeter, the circle encloses the greatest area. The flexible irregular loop therefore tends towards a circle. Becoming a straight wire would eliminate the enclosed area rather than maximize it, and remaining irregular would fail to reach the preferred maximum-area configuration. This reasoning assumes the usual freely deformable loop, with no external constraint fixing its shape.

Step 3: Conclusion The loop assumes a circular shape with its plane normal to the magnetic field. Hence, the correct option is (A).

Quick Tip to Solve: Use torque for the orientation, then maximum enclosed area at fixed perimeter for the shape.

45. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is : [4 marks]

(A) 9 T

(B) 27 T

(C) 12 T

(D) 3 T

Correct Answer: (B) 27 T

Solution:

Concept:

- For a satellite orbiting a central body of mass M, gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$. Here r is measured from the centre of the attracting body.
- The orbital period is $T = 2\pi r/v$, giving Kepler's relation $T^2 = 4\pi^2 r^3/(GM)$ for circular orbits around the same central body.

Step 1: Express the period in terms of the orbital radius Cancel the satellite mass from the force equation and solve for the orbital speed:

$$v = \sqrt{\frac{GM}{r}}.$$

Substitute this into the circumference divided by speed:

$$T(r) = \frac{2\pi r}{\sqrt{GM/r}} = 2\pi\sqrt{\frac{r^3}{GM}}.$$

Thus the period is proportional to $r^{3/2}$, rather than directly proportional to the radius. The satellite's own mass does not enter this result.

Step 2: Take the ratio for the two orbits Both satellites orbit the same central body, so the factors $2\pi/\sqrt{GM}$ cancel. With $r_1 = R$ and $r_2 = 9R$,

$$\frac{T_2}{T_1} = \left(\frac{9R}{R}\right)^{3/2} = 9^{3/2} = (\sqrt{9})^3 = 27.$$

Therefore $T_2 = 27T$. Option (A) would follow from the incorrect scaling $T \propto r$, while option (D) uses only the square root of the radius ratio. Neither includes the extra radius factor from the orbital circumference; option (C) also fails Kepler's law.

Step 3: Conclusion The satellite in the orbit of radius $9R$ has period $27T$. Hence, the correct option is (B).

Quick Tip to Solve: For the same attracting body, cube the radius ratio and then take its square root to obtain the period ratio.

46. The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to be nearest integer is :- [4 marks]

(A) 2%

(B) 3%

(C) 5%

(D) 4%

Correct Answer: (B) 3%

Solution:

Concept:

- Rearranging $T = 2\pi\sqrt{l/g}$ gives $g = 4\pi^2 l/T^2$. The maximum fractional uncertainty is $\Delta g/g = \Delta l/l + 2\Delta T/T$.
- If N oscillations take time t, then $T = t/N$. Counting N as exact gives $\Delta T/T = \Delta t/t$; the clock uncertainty applies to the measured total time.

Step 1: Calculate the relative length uncertainty The length is $l = 10\,\text{cm} = 100\,\text{mm}$, and its accuracy is $\Delta l = 1\,\text{mm}$. Hence

$$\frac{\Delta l}{l} \times 100 = \frac{1}{100} \times 100 = 1\%.$$

The same length unit must be used in numerator and denominator. Dividing 1 mm directly by 10 cm without converting units would overestimate this contribution.

Step 2: Calculate the time contribution and combine the errors The measured time for 200 oscillations is $t = 100\,\text{s}$, with $\Delta t = 1\,\text{s}$. Therefore

$$T = \frac{100}{200} = 0.5\,\text{s}, \qquad \Delta T = \frac{1}{200} = 0.005\,\text{s}.$$

The fractional timing uncertainty is still $0.005/0.5 = 1/100$, or 1%. As time appears squared in the denominator of g, its fractional error enters with magnitude 2:

$$\frac{\Delta g}{g} \times 100 = 1 + 2(1) = 3\%.$$

For maximum error, contributions add regardless of the negative exponent. Option (A) omits the correct length-plus-time combination; (C) and (D) overestimate it.

Step 3: Conclusion The percentage uncertainty in the determination of g is 3%, so the nearest-integer value is $x = 3$. Hence, the correct option is (B).

Quick Tip to Solve: First find the fractional error in the total measured time; averaging over oscillations preserves that fractional error.

47. A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to :- [4 marks]

(A) $t^{2/3}$

(B) $t^{3/2}$

(C) t

(D) $t^{1/2}$

Correct Answer: (B) $t^{3/2}$

Solution:

Concept:

- Constant mechanical power is the rate of work: $P = dW/dt = dK/dt$, when the delivered work changes the kinetic energy of the box.
- For a box of constant mass m, kinetic energy is $K = \frac{1}{2}mv^2$ and velocity is $v = dx/dt$. Initially at rest means the integration constant in the kinetic-energy equation is zero.

Step 1: Find the speed as a function of time Integrate the constant power from the initial instant to time t. With initial speed zero,

$$Pt = K(t) - K(0) = \frac{1}{2}mv^2.$$

Consequently

$$v(t) = \sqrt{\frac{2P}{m}}\, t^{1/2}.$$

The speed is not proportional to time, because constant power does not mean constant force or constant acceleration. Indeed, $F = P/v$ decreases as the box speeds up.

Step 2: Integrate the speed to obtain the displacement Choose the starting position as $x = 0$ and integrate using a separate time variable u:

$$x(t) = \int_0^t \sqrt{\frac{2P}{m}}\, u^{1/2}\, du = \frac{2}{3}\sqrt{\frac{2P}{m}}\, t^{3/2}.$$

Thus the displacement has exponent $3/2$. Option (D), with exponent $1/2$, describes the speed rather than the distance. Option (C) would require constant speed. Option (A) does not result from integrating the kinetic-energy relation. The horizontal-motion model here has the machine's delivered work appearing as kinetic energy; no opposing force is specified.

Step 3: Conclusion The distance moved from rest is proportional to $t^{3/2}$. Hence, the correct option is (B).

Quick Tip to Solve: With constant power, kinetic energy grows as time; take a square root for speed, then integrate once for distance.

48. What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature T? (k_B is Boltzmann constant) [4 marks]

(A) $\frac{1}{2}k_BT$

(B) $\frac{2}{3}k_BT$

(C) $\frac{3}{2}k_BT$

(D) k_BT

Correct Answer: (A) $\frac{1}{2}k_BT$

Solution:

Concept:

- The classical equipartition theorem assigns mean energy $\frac{1}{2}k_BT$ per molecule to every independent quadratic term in the energy at thermal equilibrium.
- A degree of freedom is an independent mode of motion contributing a quadratic energy term. For translation along one axis, that term is $\frac{1}{2}mv_x^2$, where m is the molecular mass.

Step 1: Apply equipartition to a single degree of freedom For motion along one coordinate, the thermal average is

$$\left\langle \frac{1}{2}mv_x^2 \right\rangle = \frac{1}{2}k_BT.$$

The requested quantity refers to one degree of freedom, so it contains only one such contribution. It is a molecular average, using Boltzmann's constant k_B, rather than a molar average using the gas constant R.

Step 2: Distinguish one degree of freedom from the total energy A monatomic ideal gas has three independent translational directions. Summing their contributions gives

$$\langle E_{\text{trans}} \rangle = 3\left(\frac{1}{2}k_BT\right) = \frac{3}{2}k_BT.$$

Therefore option (C) is the total translational energy per molecule, not the energy along one degree of freedom. Option (D) would correspond to two quadratic contributions, such as the kinetic and potential terms of a classical harmonic oscillator. The factor $2/3$ in option (B) is not the equipartition contribution; it can arise in rearranged relations between pressure and total translational energy. None of these replaces the one-term value.

Step 3: Conclusion The average energy along one degree of freedom is $\frac{1}{2}k_BT$ per molecule. Hence, the correct option is (A).

Quick Tip to Solve: Count independent quadratic energy terms: each contributes half of k_BT, not the full three-direction translational energy.

49. Match List - I with List - II.

List - I

(a) 10 km height over earth's surface

(b) 70 km height over earth's surface

(c) 180 km height over earth's surface

(d) 270 km height over earth's surface

List - II

(i) Thermosphere

(ii) Mesosphere

(iii) Stratosphere

(iv) Troposphere [4 marks]

(A) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(B) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(C) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(D) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Correct Answer: No printed option gives the physically correct atmospheric-layer matching.

Solution:

Concept:

- Atmospheric layers classified by temperature are, in increasing altitude, the troposphere, stratosphere, mesosphere and thermosphere. Approximate boundaries are 10–12 km, 50 km and 80–85 km, respectively.
- A matching question must assign every listed height to its actual layer. A proposed option is invalid if even one of its pairings is inconsistent with the layer boundaries.

Step 1: Classify the four printed heights The height 10 km lies in the troposphere in the usual approximate classification, so (a) pairs with (iv). At 70 km the layer is the mesosphere, so (b) pairs with (ii). Both 180 km and 270 km are above the mesopause and lie in the thermosphere; therefore (c) and (d) both pair with (i). The literal matching is

$$(a):(iv), \quad (b):(ii), \quad (c):(i), \quad (d):(i).$$

These heights must not be changed in order to obtain a one-to-one matching.

Step 2: Check every printed option against the layers Option (A) correctly associates 10 km with the troposphere and 270 km with the thermosphere, but incorrectly places 70 km in the stratosphere and 180 km in the mesosphere. Option (B) incorrectly puts 10 km in the thermosphere. Option (C) incorrectly puts 10 km in the stratosphere and 270 km in the troposphere. Option (D) incorrectly puts 10 km in the mesosphere. Consequently none of the four options is fully correct for the printed heights. The supplied answer key selects (A), but its middle two matches contradict the standard temperature-layer classification; the literal question does not support that selection.

Step 3: Conclusion No printed option is scientifically correct for the stated heights. The supplied key selects (A), but the literal correct matching is (a)-(iv), (b)-(ii), (c)-(i), (d)-(i).

Quick Tip to Solve: Check atmospheric matches against the approximate boundaries 12, 50 and 85 km; above the mesopause, both listed high altitudes are thermospheric.

50. A radioactive sample disintegrates via two independent decay processes having half lives $T_{1/2}^{(1)}$ and $T_{1/2}^{(2)}$ respectively. The effective half-life $T_{1/2}$ of the nuclei is : [4 marks]

(A) None of the above

(B) $T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$

(C) $T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$

(D) $T_{1/2} = \frac{T_{1/2}^{(1)} + T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}}$

Correct Answer: (C) $T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$

Solution:

Concept:

- For independent competing decay modes of the same nucleus, the decay constants add: $\lambda = \lambda_1 + \lambda_2$. Each constant measures the probability of decay per unit time through that channel.
- The half-life and decay constant are related by $T_{1/2} = \ln 2/\lambda$. Half-lives themselves are not additive, because either channel can remove a surviving nucleus.

Step 1: Add the rates of loss Let N be the number of undecayed nuclei at time t. The first channel removes nuclei at rate $\lambda_1 N$, while the second removes them at rate $\lambda_2 N$. Therefore the total loss rate is

$$-\frac{dN}{dt} = (\lambda_1 + \lambda_2)N.$$

Integrating gives $N = N_0 e^{-(\lambda_1+\lambda_2)t}$; this is one exponential with the effective decay constant equal to the sum.

Step 2: Express the result using half-lives Write $T_1 = T_{1/2}^{(1)}$ and $T_2 = T_{1/2}^{(2)}$. Substitution gives

$$\frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2}, \qquad \frac{1}{T_{1/2}} = \frac{1}{T_1} + \frac{1}{T_2}.$$

Thus the effective half-life is smaller than either individual half-life. Option (B) instead predicts an increased lifetime. Option (D) is dimensionless and diverges for equal half-lives, whereas two equal channels must halve the lifetime. Option (A) is excluded because option (C) gives the derived expression.

Step 3: Conclusion The effective half-life is

$$\boxed{T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}}.$$

Hence, the correct option is (C).

Quick Tip to Solve: For simultaneous decay channels, add reciprocal half-lives. As a quick check, two channels with the same half-life T give the effective half-life $T/2$.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}$s. The value of 'a' to the nearest integer is _____. [4 marks]

Solution:

Concept:

- Measure time from a mean-position crossing towards the positive extreme. The displacement is then $y = A\sin(\omega t)$, where A is amplitude and ω is angular frequency.
- The angular frequency is $\omega = 2\pi/T$. The first arrival at a specified displacement is found from the smallest positive phase satisfying the displacement equation.

Step 1: Choose the initial phase The particle begins the interval at the mean position, so its displacement is zero at $t = 0$. Choosing its initial velocity in the positive direction permits the sine form without any extra phase constant. With the given period,

$$T = 2\,\text{s}, \qquad \omega = \frac{2\pi}{2} = \pi\,\text{rad}\,\text{s}^{-1}.$$

The amplitude is not given, but it will cancel because the required displacement is a fraction of it.

Step 2: Find the first time to reach half amplitude At the requested first arrival, $y = A/2$, so

$$\frac{A}{2} = A\sin(\pi t), \qquad \sin(\pi t) = \frac{1}{2}.$$

During travel from the mean to the adjacent extreme, the phase lies between zero and $\pi/2$. The appropriate phase is therefore $\pi/6$, rather than $5\pi/6$, which describes a later return towards the mean. Hence

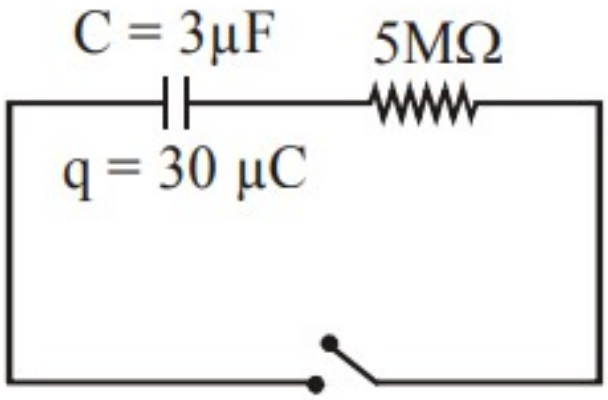
$$\pi t = \frac{\pi}{6}, \qquad t = \frac{1}{6}\,\text{s}.$$

Comparing this with $t = 1/a\,\text{s}$ directly gives the requested dimensionless integer. Uniform-speed reasoning cannot be used because the speed decreases as the particle moves away from the mean.

Step 3: Conclusion The time is $1/6\,\text{s}$, so $\boxed{a = 6}$. It is already an integer and needs no rounding.

Quick Tip to Solve: From the mean position, use a sine function: half amplitude corresponds to a phase of $\pi/6$, so the first travel time is $T/12$.

52. The circuit shown in the figure consists of a charged capacitor of capacity 3 μF and a charge of μC. At time t = 0, when the key is closed, the value of current flowing through the 5 MΩ resistor is 'x' μ-A. The value of 'x to the nearest integer is _____. [4 marks]



Solution:

Concept:

- The potential difference across a capacitor is $V_C = q/C$, where q is its stored charge and C its capacitance.
- A capacitor voltage cannot change instantaneously through a finite resistance. Immediately after the switch closes, the resistor therefore has the initial capacitor voltage across it, and $I(0^+) = V_C(0)/R$.

Step 1: Read the initial capacitor voltage The circuit labels give $q_0 = 30\,\mu\text{C}$ and $C = 3\,\mu\text{F}$. Use these values before considering any later discharge. Since both charge and capacitance contain the same micro prefix,

$$V_C(0) = \frac{q_0}{C} = \frac{30 \times 10^{-6}}{3 \times 10^{-6}} = 10\,\text{V}.$$

This is the voltage available to drive current around the closed capacitor–resistor loop. The capacitor is the source of the initial electrical energy; there is no separate battery in this circuit.

Step 2: Apply Ohm's law just after closing The resistance is $R = 5\,\text{M}\Omega = 5 \times 10^6\,\Omega$. Thus the initial current magnitude is

$$I(0^+) = \frac{10}{5 \times 10^6} = 2 \times 10^{-6}\,\text{A} = 2\,\mu\text{A}.$$

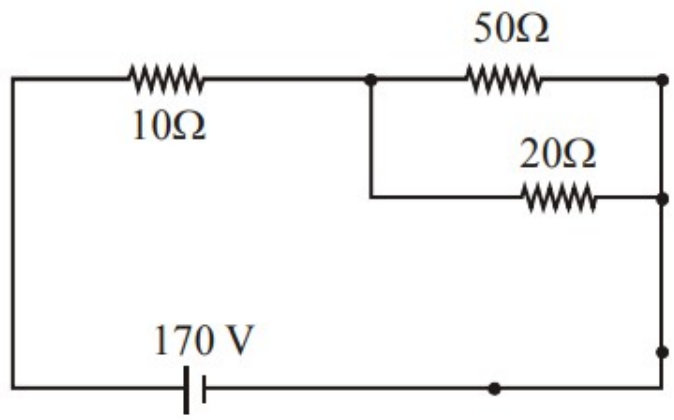
At later times it decreases as $I(t) = I(0^+)e^{-t/(RC)}$, with $RC = 15\,\text{s}$. This confirms that the asked value is the maximum initial discharge current, rather than the zero current eventually obtained after the stored charge has dissipated. Current direction depends on the initial capacitor polarity, but its magnitude is fixed by the stated charge and resistance.

Step 3: Conclusion The initial current through the $5\,\text{M}\Omega$ resistor is $2\,\mu\text{A}$. Therefore $\boxed{x = 2}$.

Quick Tip to Solve: For the first instant of an RC discharge, calculate q_0/C first and divide by R. A megaohm resistance converts a voltage in volts directly into a current in microamperes.

53. The voltage across the 10Ω resistor in the given circuit is x volt.

The value of 'x' to the nearest integer is _____. [4 marks]



Solution:

Concept:

- Resistors connected between the same pair of nodes are parallel. Their equivalent resistance satisfies $1/R_p = 1/R_1 + 1/R_2$.
- A series resistor carries the total circuit current. Ohm's law gives $I = V/R_{\text{eq}}$, and the voltage across that resistor is IR.

Step 1: Reduce the parallel branch The diagram shows the $50\,\Omega$ and $20\,\Omega$ resistors connected to the same two nodes: the junction after the $10\,\Omega$ resistor and the right-hand return wire. They are therefore parallel, not series. Their equivalent resistance is

$$R_p = \frac{50 \times 20}{50 + 20} = \frac{100}{7}\,\Omega.$$

The $10\,\Omega$ resistor is in series with this combination because every current leaving the source must pass through it before dividing between the two parallel branches.

Step 2: Find the total current and required voltage The total resistance across the $170\,\text{V}$ source is

$$R_{\text{eq}} = 10 + \frac{100}{7} = \frac{170}{7}\,\Omega.$$

Hence the current through the source and the $10\,\Omega$ resistor is

$$I = \frac{170}{170/7} = 7\,\text{A}, \qquad V_{10} = I(10) = 70\,\text{V}.$$

As a check, the remaining voltage is $170 - 70 = 100\,\text{V}$ across each parallel resistor. Their currents are $100/50 = 2\,\text{A}$ and $100/20 = 5\,\text{A}$, which add to the same $7\,\text{A}$. Both the voltage sum and the junction current balance are satisfied.

Step 3: Conclusion The voltage across the $10\,\Omega$ resistor is $70\,\text{V}$. Thus the requested integer is $\boxed{x = 70}$.

Quick Tip to Solve: Identify parallel branches by their endpoints, then reduce them before using the voltage divider. Here $V_{10} = 170 \times 10/(10 + 100/7)$ gives the answer directly.

54. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of a : b. Then a/b can be expressed as 1/x where x is _____. [4 marks]

Solution:

Concept:

- Young's modulus is $Y = (F/A)/(\Delta L/L)$. Rearranging gives the extension $\Delta L = FL/(AY)$ for a wire within its elastic limit.
- For wires of the same material under the same force, Y and F cancel in ratios. The cross-sectional area is $A = \pi r^2$, so area varies as the square of radius.

Step 1: Form the extension ratio Denote the original lengths by L_A, L_B and cross-sectional areas by A_A, A_B. The same $2\,\mathrm{N}$ force acts on each wire and the material is identical, so

$$\frac{\Delta L_A}{\Delta L_B} = \frac{FL_A/(A_A Y)}{FL_B/(A_B Y)} = \frac{L_A}{L_B}\frac{A_B}{A_A}.$$

The force value is not needed numerically because it is common to both experiments. It is the extensions, not the original lengths, that are stated to be $2\,\mathrm{mm}$ and $4\,\mathrm{mm}$.

Step 2: Use the squared radius ratio Since $r_B = 4r_A$,

$$\frac{A_B}{A_A} = \frac{\pi(4r_A)^2}{\pi r_A^2} = 16.$$

Substitute the extension ratio using the same units in numerator and denominator:

$$\frac{2}{4} = 16\frac{L_A}{L_B}, \qquad \frac{L_A}{L_B} = \frac{1}{32}.$$

The larger-radius wire has sixteen times the area and would stretch less at equal length. To stretch twice as much under the same force it must instead be thirty-two times as long. This physical check confirms the ratio orientation and avoids mistaking the radius ratio for the area ratio.

Step 3: Conclusion The length ratio is $a : b = L_A : L_B = 1 : 32$. Therefore $a/b = 1/32 = 1/x$, giving $\boxed{x = 32}$.

Quick Tip to Solve: Write $L \propto \Delta L\, r^2$ when material and force are common. Then $L_A/L_B = (2/4)(1/4)^2$, which keeps the area factor in the correct direction.

55. A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is _____. [4 marks]

Solution:

Concept:

- The swimmer's velocity relative to the ground is the vector sum of the swimmer's velocity relative to water and the water's velocity relative to the ground.
- To reach the point directly opposite the start, the resultant velocity component parallel to the river flow must be zero. The perpendicular component carries the swimmer across the river.

Step 1: Resolve the swimming velocity Take the positive horizontal direction along the river flow. The given swimming speed relative to water is $10\,\mathrm{m\,s^{-1}}$, directed at 120° to that positive direction. Its component along the flow is

$$v_{s,x} = 10\cos 120^\circ = 10\left(-\frac{1}{2}\right) = -5\,\mathrm{m\,s^{-1}}.$$

The negative sign means that this component points upstream. The transverse component is $10\sin 120^\circ = 5\sqrt{3}\,\mathrm{m\,s^{-1}}$, which is nonzero and enables the crossing.

Step 2: Cancel the downstream drift Let the flow speed be $x\,\mathrm{m\,s^{-1}}$. The river contributes only a downstream component, so the swimmer's ground-frame velocity along the river is

$$v_{g,x} = x - 5.$$

Reaching the point directly opposite requires no net displacement in this direction. Since these velocities are constant during the crossing, the horizontal resultant itself must vanish:

$$x - 5 = 0, \qquad x = 5.$$

It would be incorrect to equate the flow speed to the full swimming speed; only the upstream projection cancels the current. The transverse speed affects the crossing time, but the river width and crossing time are unnecessary for determining the flow speed.

Step 3: Conclusion The speed of the river flow is $5\,\mathrm{m\,s^{-1}}$. Hence the nearest-integer value is $\boxed{x = 5}$.

Quick Tip to Solve: For a crossing without drift, cancel the along-river components: $v_{\text{flow}} = -v_{\text{swim}}\cos\theta$. Measure θ from the flow direction, keeping the cosine sign.

56. A parallel plate capacitor has plate area 100 m^2 and plate separation of 10 m. The space between the plates is filled up to a thickness 5 m with a material of dielectric constant of 10. The resultant capacitance of the system is 'x' pF.

The value of $\varepsilon_0 = 8.85 \times 10^{-12}$ $F.m^{-1}$.

The value of 'x' to the nearest integer is _____. [4 marks]

Solution:

Concept:

- A dielectric slab filling the plate area but only part of the plate separation forms dielectric and air layers in series along the electric field.
- For plate area A, total gap d, slab thickness t, and dielectric constant K, the equivalent capacitance is $C = \varepsilon_0 A/[(d-t) + t/K]$, neglecting edge effects.

Step 1: Identify the two series layers The plate area is $100\,\mathrm{m}^2$ and their separation is $10\,\mathrm{m}$. A slab of thickness $5\,\mathrm{m}$ and dielectric constant 10 leaves an air thickness of $10 - 5 = 5\,\mathrm{m}$. The same free charge faces both layers, while their voltage drops add, so

$$\frac{1}{C} = \frac{5}{\varepsilon_0 A} + \frac{5}{10\varepsilon_0 A}.$$

Adding the two capacitances directly would correspond to side-by-side regions spanning the full gap, which is not the stated thickness arrangement.

Step 2: Calculate and round the capacitance The effective electrical separation is

$$d_{\text{eff}} = 5 + \frac{5}{10} = 5.5\,\mathrm{m}.$$

Use the supplied permittivity without altering its exponent:

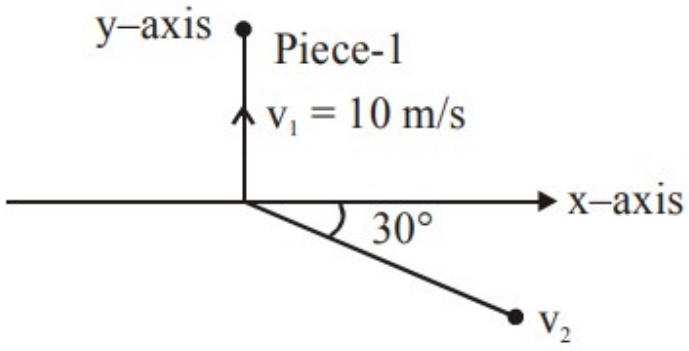
$$C = \frac{8.85 \times 10^{-12} \times 100}{5.5} = 160.909\ldots \times 10^{-12}\,\mathrm{F}.$$

Since $1\,\mathrm{pF} = 10^{-12}\,\mathrm{F}$, the capacitance is $160.909\ldots\,\mathrm{pF}$. For comparison, the empty capacitor would have $88.5\,\mathrm{pF}$ and a completely filled gap would give $885\,\mathrm{pF}$. The calculated intermediate value lies between these limits, as expected for partial filling. Rounding is applied only after conversion to the requested unit.

Step 3: Conclusion The resultant capacitance is approximately $160.909\,\mathrm{pF}$. Therefore its nearest-integer coefficient is $\boxed{x = 161}$.

Quick Tip to Solve: For a slab inserted along the field, replace its physical thickness t by the reduced electrical thickness t/K, then add the remaining air thickness before calculating capacitance.

57. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m/s along the x-axis, hits another ball of mass 20 kg which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along y-axis with a speed of 10 m/s. The second piece starts moving at an angle of 30° with respect to the x-axis. The velocity of the ball moving at 30° with x-axis is x m/s. The configuration of pieces after collision is shown in the figure below. The value of x to the nearest integer is _____. [4 marks]



Solution:

Concept:

- In the absence of an external impulse, total linear momentum is conserved separately along the two perpendicular axes.
- The initially stationary ball splits into two equal masses of $10\,\mathrm{kg}$. Momentum depends on velocity components, not just speed.

Step 1: Write the initial momentum The incoming ball has mass $10\,\mathrm{kg}$ and velocity $10\sqrt{3}\,\mathrm{m\,s^{-1}}$ along the positive x-axis. Therefore

$$p_{x,i} = 100\sqrt{3}\,\mathrm{kg\,m\,s^{-1}}, \qquad p_{y,i} = 0.$$

The first ball is at rest after collision, so it contributes no final momentum.

Step 2: Resolve the fragment velocities The first fragment moves vertically upwards at $10\,\mathrm{m\,s^{-1}}$. The second moves below the x-axis, as shown, with components $x\cos 30°$ and $-x\sin 30°$. Conservation of horizontal momentum gives

$$100\sqrt{3} = 10x\cos 30° = 5\sqrt{3}x,$$

so $x = 20$.

Step 3: Check the vertical component The total final vertical momentum must vanish:

$$10(10) - 10x\sin 30° = 0.$$

With $x = 20$, the two terms are $+100$ and -100, which cancel. Thus both component equations give the same speed. Kinetic energy need not be conserved during collision and disintegration; momentum conservation is the relevant principle.

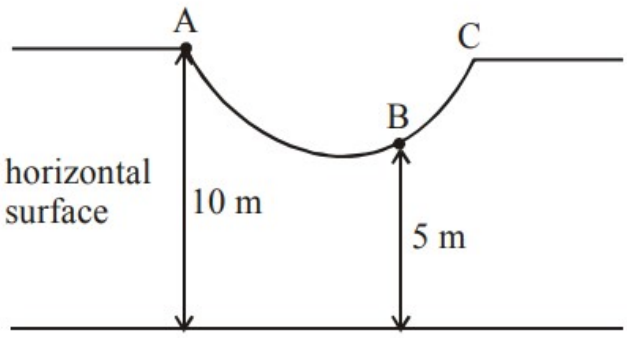
Step 4: Conclusion The speed of the fragment directed at $30°$ below the x-axis is $20\,\mathrm{m\,s^{-1}}$. The

required nearest-integer value is $\boxed{20}$.

Quick Tip to Solve: Use the initially zero transverse momentum first: equal fragment masses imply equal and opposite vertical velocity components.

58. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take g = 10 m/s^2)

The value of 'x' to the nearest integer is _____. [4 marks]



Solution:

Concept:

- For motion under gravity on a smooth fixed track, the normal reaction does no work, so kinetic plus gravitational potential energy is conserved.
- Gravitational potential energy is $U = mgh$. Only the difference in vertical height matters, irrespective of the track shape.

Step 1: Read the heights from the diagram The particle starts almost from rest at A, at height $h_A = 10\,\text{m}$. Point B is at height $h_B = 5\,\text{m}$ above the same horizontal datum. Its vertical drop from A to B is therefore

$$\Delta h = 10 - 5 = 5\,\text{m}.$$

The small initial displacement merely starts the motion and supplies negligible kinetic energy.

Step 2: Apply energy conservation The loss of potential energy becomes the kinetic energy at B:

$$mgh_A = mgh_B + \frac{1}{2}mv_B^2.$$

Cancelling the common mass gives

$$v_B^2 = 2g(h_A - h_B) = 2(10)(5) = 100\,\text{m}^2\,\text{s}^{-2}.$$

Therefore $v_B = 10\,\text{m}\,\text{s}^{-1}$, taking the positive square root because speed is nonnegative.

Step 3: Check the energy balance For the given mass of $10\,\text{kg}$, the potential-energy decrease is

$10 \times 10 \times 5 = 500\,\text{J}$. The calculated kinetic energy is $\frac{1}{2} \times 10 \times 10^2 = 500\,\text{J}$, confirming the result. Using the full height of A instead of the height drop would incorrectly treat B as lying at the datum.

Step 4: Conclusion The speed at B is $10\,\text{m s}^{-1}$, so the required nearest-integer value is $\boxed{10}$.

Quick Tip to Solve: For a curved smooth track, ignore the path length and use only the vertical drop between the initial and final points.

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59. A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is 'x' N. The value of 'x' to the nearest integer is _____. [4 marks]

Solution:

Concept:

- Uniform deceleration allows the constant-acceleration equation $v^2 = u^2 + 2as$ to be used along the direction of motion.
- The effective retarding force is the net force opposing motion. Its magnitude is $F = m|a|$, and its work equals the change in kinetic energy.

Step 1: Convert the penetration distance The bullet travels $50\,\text{cm} = 0.50\,\text{m}$ before stopping. Choose its initial direction as positive. The data are

$$u = 10\,\text{m s}^{-1}, \quad v = 0, \quad m = 0.1\,\text{kg}, \quad s = 0.50\,\text{m}.$$

The final speed is zero because the bullet stops within the block.

Step 2: Find the acceleration and force Using the kinematic relation,

$$0 = 10^2 + 2a(0.50), \qquad a = -100\,\text{m s}^{-2}.$$

The negative sign indicates acceleration opposite to the initial velocity. Newton's second law then gives

$$F_x = ma = 0.1(-100) = -10\,\text{N}.$$

The question asks for magnitude, so the answer is positive.

Step 3: Verify using the work done The initial kinetic energy is

$$K_i = \frac{1}{2}(0.1)(10)^2 = 5\,\text{J}.$$

A constant retarding force of magnitude $10\,\text{N}$ acting over $0.50\,\text{m}$ does work $-10 \times 0.50 = -5\,\text{J}$. This removes exactly the initial kinetic energy, independently checking the calculation. Keeping the distance in centimetres would give an incorrect numerical force.

Step 4: Conclusion The effective retarding force has magnitude $10\,\text{N}$, so $\boxed{x = 10}$.

Quick Tip to Solve: For stopping over a known distance, use $Fs = \frac{1}{2}mu^2$; this avoids calculating the stopping time.

60. An npn transistor operates as a common emitter amplifier with a power gain of 10^6. The input circuit resistance is 100Ω and the output load resistance is 10 KΩ. The common emitter current gain 'β' will be _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- The common-emitter current gain is $\beta = I_{\text{out}}/I_{\text{in}}$, using the magnitudes of the small-signal currents.
- For resistive input and output circuits, power is I^2R. Hence power gain is $A_P = \beta^2 R_L/R_{\text{in}}$.

Step 1: Relate the current and voltage gains The input voltage is $V_{\text{in}} = I_{\text{in}}R_{\text{in}}$, while the output voltage magnitude is $V_{\text{out}} = I_{\text{out}}R_L$. Thus

$$|A_V| = \beta\frac{R_L}{R_{\text{in}}}.$$

The phase reversal of a common-emitter amplifier affects the voltage sign but does not make the power gain negative.

Step 2: Use the given power gain Power gain is the product of current gain and voltage-gain magnitude:

$$A_P = \beta|A_V| = \beta^2\frac{R_L}{R_{\text{in}}}.$$

Convert the load resistance before substitution:

$$R_L = 10\,\text{k}\Omega = 10^4\,\Omega, \quad R_{\text{in}} = 100\,\Omega.$$

Consequently

$$10^6 = \beta^2\frac{10^4}{100} = 100\beta^2.$$

Step 3: Solve and verify the dimensionless gain It follows that

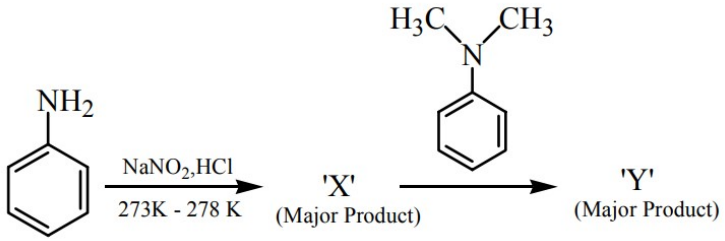
$$\beta^2 = 10^4, \qquad \beta = 100.$$

The positive root is appropriate to the magnitude of current gain. As a check, this gives voltage-gain magnitude $100 \times 100 = 10^4$, and its product with current gain is $10^4 \times 100 = 10^6$, exactly the stated power gain. Treating power gain as voltage gain alone would miss one factor of current gain.

Step 4: Conclusion The common-emitter current gain, rounded to the nearest integer, is $\boxed{\beta = 100}$.

Quick Tip to Solve: Write power gain directly as $\beta^2(R_L/R_{\text{in}})$, then check that both resistances use the same unit.

61. Considering the above reaction, X and Y respectively are : [4 marks]



(A) $C_6H_5N_2^+Cl^-$ and ortho-$C_6H_5-N=N-C_6H_4-N(CH_3)_2$

(B) $C_6H_5N_2^+Cl^-$ and para-$C_6H_5-N=N-C_6H_4-N(CH_3)_2$

(C) C_6H_5Cl and para-$C_6H_5-N=N-C_6H_4-N(CH_3)_2$

(D) C_6H_5Cl and ortho-$C_6H_5-N=N-C_6H_4-N(CH_3)_2$

Correct Answer: (B) $C_6H_5N_2^+Cl^-$ and para-$C_6H_5-N=N-C_6H_4-N(CH_3)_2$

Solution:

Concept:

- A primary aromatic amine reacts with nitrous acid generated from $NaNO_2/HCl$ at 273–278 K to form a stable cold diazonium salt.
- Diazonium ions undergo electrophilic aromatic substitution with strongly activated aromatic rings. The $-N(CH_3)_2$ group directs coupling mainly to the para position when it is free.

Step 1: Identify the intermediate X The starting compound is aniline. Under the cold diazotisation conditions it gives benzenediazonium chloride:

$$C_6H_5NH_2 + NaNO_2 + 2\,HCl \longrightarrow C_6H_5N_2^+Cl^- + NaCl + 2\,H_2O.$$

Thus X retains the diazonium group; it is not chlorobenzene. Replacement of that group by chlorine would require a separate substitution reaction, such as suitable Sandmeyer conditions.

Step 2: Locate the coupling position The second reactant is N,N-dimethylaniline. Its nitrogen lone pair donates electron density into the benzene ring, activating the ortho and para carbons. The electrophilic diazonium species couples predominantly at the less hindered para carbon. The product has an azo linkage joining the phenyl group to the para-substituted dimethylaniline ring:

$$C_6H_5-N=N-C_6H_4-N(CH_3)_2 \quad \text{(para)}.$$

Step 3: Compare the four pairs Option A has the correct diazonium intermediate but shows the ortho azo product rather than the major para product. Options C and D give chlorobenzene as X and therefore fail at the first stage; D also shows ortho coupling. Option B alone combines benzenediazonium chloride with the para azo compound, satisfying both reaction steps.

Step 4: Conclusion X is benzenediazonium chloride and Y is 4-(phenyldiazenyl)-N,N-dimethylaniline. Hence, the correct option is (B).

Quick Tip to Solve: In an azo-coupling sequence, identify the cold diazonium salt first, then choose the free para position of the activated coupling ring.

62. The ionic radius of Na^+ ions is 1.02 $\overset{o}{A}$. The ionic radii (in $\overset{o}{A}$) of Mg^{2+} and Al^{3+}, respectively, are [4 marks]

(A) 1.05 and 0.99

(B) 0.72 and 0.54

(C) 0.85 and 0.99

(D) 0.68 and 0.72

Correct Answer: (B) 0.72 and 0.54

Solution:

Concept:

- Na^+, Mg^{2+} and Al^{3+} are isoelectronic: each has ten electrons and the closed-shell configuration $1s^2 2s^2 2p^6$.
- For an isoelectronic series, increasing nuclear charge pulls the same number of electrons closer, so ionic radius decreases as atomic number increases.

Step 1: Compare electron counts and nuclear charges Sodium has atomic number 11 and loses one electron; magnesium has atomic number 12 and loses two; aluminium has atomic number 13 and loses three. Each ion therefore has ten electrons. The number of occupied shells is the same, while the nuclear charge increases from $+11e$ through $+12e$ to $+13e$. The stronger attraction leads to

$$r(Na^+) > r(Mg^{2+}) > r(Al^{3+}).$$

Step 2: Apply the trend to the options The given sodium radius is 1.02 Å. Both requested radii must be smaller, and the magnesium ion must remain larger than the aluminium ion. Option A gives magnesium as 1.05 Å, exceeding sodium, and is inconsistent. Options C and D reverse the required magnesium–aluminium ordering. Only option B satisfies both inequalities.

Step 3: State the matching ionic radii The listed pair 0.72 Å and 0.54 Å is the standard pair compatible with this series:

$$1.02 > 0.72 > 0.54.$$

The trend identifies the correct offered pair; the exact numerical radii are tabulated ionic-radius values, rather than values obtained by a simple proportional scaling of the sodium radius.

Step 4: Conclusion The ionic radii of Mg^{2+} and Al^{3+} are 0.72 Å and 0.54 Å, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Count electrons before comparing ionic sizes: for equal electron counts, the ion with more protons is smaller.

63. Reaction of Grignard reagent, C_2H_5MgBr with C_8H_8O followed by hydrolysis gives compound "A" which reacts instantly with Lucas reagent to give compound B, $C_{10}H_{13}Cl$. The Compound B is : [4 marks]

(A) $C_6H_5CH(CH_3)CH_2CH_2Cl$

(B) $C_6H_5CH(CH_2CH_3)CH_2Cl$

(C) $C_6H_5C(Cl)(CH_3)CH_2CH_3$

(D) $C_6H_5CH(CH_3)CH(Cl)CH_3$

Correct Answer: (C) $C_6H_5C(Cl)(CH_3)CH_2CH_3$

Solution:

Concept:

- An ethyl Grignard reagent adds an ethyl group to a carbonyl carbon; hydrolysis converts the magnesium alkoxide into an alcohol.
- Lucas reagent, concentrated HCl with anhydrous $ZnCl_2$, converts a readily reacting alcohol into a chloride. A tertiary benzylic alcohol reacts immediately because its carbocation is strongly stabilised.

Step 1: Identify the carbonyl precursor The molecular formula C_8H_8O is consistent with acetophenone, $C_6H_5COCH_3$. Addition of C_2H_5MgBr to this ketone creates a carbon bearing phenyl, methyl and ethyl groups. Hydrolysis gives

$$C_6H_5COCH_3 \xrightarrow[\text{hydrolysis}]{C_2H_5MgBr} C_6H_5C(OH)(CH_3)CH_2CH_3.$$

This is 2-phenylbutan-2-ol, a tertiary benzylic alcohol.

Step 2: Replace the hydroxyl group by chloride In Lucas reagent the hydroxyl group is activated and leaves as water. The resulting carbocation is tertiary and benzylic, so it is stabilised by both alkyl groups and resonance with the phenyl ring. Chloride captures this carbocation at the same central carbon, yielding

$$C_6H_5C(Cl)(CH_3)CH_2CH_3.$$

Its formula is $C_{10}H_{13}Cl$, agreeing with the question.

Step 3: Distinguish the structural options Option C places chlorine, methyl, ethyl and phenyl on the central carbon, exactly as required. Options A and B instead have chlorine on a terminal primary carbon. Option D has chlorine on a neighbouring secondary carbon; it does not preserve the carbonyl carbon as the site receiving the ethyl group and subsequently bearing chlorine. Thus these alternatives do not follow the stated direct Grignard-addition and substitution sequence.

Step 4: Conclusion Compound B is 2-chloro-2-phenylbutane, $C_6H_5C(Cl)(CH_3)CH_2CH_3$. Hence, the correct option is (C).

Quick Tip to Solve: Track the original carbonyl carbon through both steps: Grignard addition attaches ethyl there, and Lucas reagent replaces its OH by Cl.

64. Reagent, 1-napthylamine and sulphanilic acid in acetic acid is used for the detection of [4 marks]

(A) N_2O

(B) NO_3^-

(C) NO

(D) NO_2^-

Correct Answer: (D) NO_2^-

Solution:

Concept:

- The Griess test detects nitrite through diazotisation followed by azo coupling. Sulphanilic acid and 1-naphthylamine are the two organic components of the reagent.
- In an acidic solution, nitrite gives nitrous acid, which diazotises the aromatic amino group of sulphanilic acid. The resulting diazonium species couples with 1-naphthylamine to form a coloured azo compound.

Step 1: Identify the role of the acidic medium Acetic acid provides the acidic conditions in which NO_2^- produces nitrous acid. This is the reactive nitrogen species required for diazotisation; the test is therefore linked directly to nitrite rather than simply to any compound containing nitrogen.

Step 2: Follow diazotisation and coupling Sulphanilic acid is converted into its diazonium derivative. Coupling with 1-naphthylamine then produces a pink or red azo dye, giving the characteristic positive observation. Both organic reagents are needed: one supplies the diazonium component and the other supplies the coupling component.

Step 3: Eliminate the other nitrogen species NO_3^- does not directly give this response; nitrate would first need reduction to nitrite. N_2O and NO are neutral gaseous oxides and are not the anions detected by this direct colour test. Thus none of options (A), (B) or (C) identifies the species to which the stated reagent responds directly.

Step 4: Conclusion The reagent detects nitrite, NO_2^-, by formation of an azo dye. Hence, the correct option is (D).

Quick Tip to Solve: Remember the pair: sulphanilic acid plus 1-naphthylamine is the Griess test for nitrite; nitrate needs prior reduction.

65. A non-reducing sugar "A" hydrolyses to give two reducing mono saccharides. Sugar A is [4 marks]

(A) Fructose

(B) Galactose

(C) Glucose

(D) Sucrose

Correct Answer: (D) Sucrose

Solution:

Concept:

- A reducing sugar has a free anomeric centre capable of generating an open-chain carbonyl form. A glycosidic linkage that blocks both anomeric centres makes a disaccharide non-reducing.
- Hydrolysis cleaves the glycosidic bond of a disaccharide and regenerates the monosaccharide units. Sucrose gives glucose and fructose, both of which behave as reducing sugars under the usual alkaline tests.

Step 1: Use the hydrolysis requirement The sugar must contain two monosaccharide units joined by a hydrolysable glycosidic bond. Fructose, galactose and glucose are already monosaccharides, so they cannot give two simpler monosaccharide units by hydrolysis. This immediately distinguishes sucrose from the other three choices.

Step 2: Explain why the original sugar is non-reducing In sucrose, the anomeric carbon C-1 of glucose is linked through oxygen to the anomeric carbon C-2 of fructose. Both anomeric groups participate in the bond, so neither is free to open into the carbonyl form needed for the reducing-sugar tests.

Step 3: Identify both hydrolysis products Acid or the enzyme invertase cleaves this linkage:

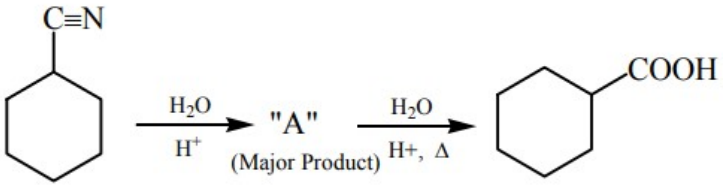
$$C_{12}H_{22}O_{11} + H_2O \longrightarrow C_6H_{12}O_6 + C_6H_{12}O_6.$$

The products are D-glucose and D-fructose. Glucose is an aldose; fructose, although a ketose, is reducing because it can isomerise through an enediol in alkaline medium. Thus the description of both products as reducing is consistent.

Step 4: Conclusion Sugar A is sucrose, which hydrolyses to glucose and fructose. Hence, the correct option is (D).

Quick Tip to Solve: Check whether both anomeric carbons are tied up: sucrose is non-reducing, but its hydrolysis products are reducing.

66. Consider the above chemical reaction and identify product "A" [4 marks]



(A) $C_6H_{11}-CH_2NH_2$

(B) $C_6H_{11}-CH_2NO_2$

(C) $C_6H_{11}-CONH_2$

(D) $C_6H_{11}-CH=NOH$

Correct Answer: (C) $C_6H_{11}-CONH_2$

Solution:

Concept:

- Acidic hydrolysis of a nitrile occurs in stages: the nitrile first becomes an amide and further hydrolysis gives a carboxylic acid. The nitrile carbon remains part of the carbon skeleton.
- For a group R, the sequence is $R-CN \longrightarrow R-CONH_2 \longrightarrow R-COOH$. Water and acid promote hydrolysis, whereas production of a primary amine would require reduction.

Step 1: Identify the starting functional group The ring is attached directly to a nitrile group, so write the substrate as $C_6H_{11}-CN$. The six-membered ring is retained during the reaction; the transformation takes place at the exocyclic nitrile carbon.

Step 2: Stop at the first hydrolysis stage Acid protonates the nitrile nitrogen and makes its carbon more susceptible to attack by water. Hydration and proton transfers give the amide $C_6H_{11}-CONH_2$, cyclohexanecarboxamide. This is the intermediate A. The carbon of the original nitrile becomes the carbonyl carbon; no extra carbon atom is introduced.

Step 3: Check the final product and reject alternatives Further heating with water and acid converts the amide into $C_6H_{11}-COOH$, exactly the final acid shown. Option (A), $C_6H_{11}-CH_2NH_2$, is a nitrile reduction product, not a hydrolysis intermediate. The nitro compound in (B) would require different chemistry, and the oxime in (D) is not produced by simple nitrile hydration. Only (C) continues directly to the displayed acid.

Step 4: Conclusion Product A is cyclohexanecarboxamide, $C_6H_{11}-CONH_2$. Hence, the correct option is (C).

Quick Tip to Solve: For nitrile hydrolysis, retain the nitrile carbon and use the sequence nitrile, amide, carboxylic acid.

67. Match List - I with List - II

	List - I		List - II
(a)	Chlorophyll	(i)	Ruthenium
(b)	Vitamin-B_{12}	(ii)	Platinum
(c)	Anticancer drug	(iii)	Cobalt
(d)	Grubbs catalyst	(iv)	Magnesium

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a) - (iii), (b) - (ii), (c) - (iv), (d) - (i)

(B) (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)

(C) (a) - (iv), (b) - (iii), (c) - (i), (d) - (ii)

(D) (a) - (iv), (b) - (ii), (c) - (iii), (d) - (i)

Correct Answer: (B) (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)

Solution:

Concept:

- The central metal is a characteristic part of many biologically active coordination compounds and industrial catalysts. The ligand environment controls their properties, but the identity of the metal supplies the matching criterion here.
- Chlorophyll contains magnesium, vitamin B12 contains cobalt, cisplatin is a platinum anticancer drug, and Grubbs catalysts contain ruthenium.

Step 1: Match the two biological compounds Chlorophyll contains a central magnesium ion coordinated within its large ring ligand, giving (a)-(iv). Vitamin B12 is a cobalt-containing corrinoid; its cobalt is the source of the name cobalamin. Hence (b)-(iii). These two assignments already eliminate options (A) and (D).

Step 2: Match the medicinal and catalytic examples Cisplatin, the familiar anticancer coordination compound, has platinum as its central metal, giving (c)-(ii). Grubbs catalysts used for olefin metathesis are ruthenium carbene complexes, giving (d)-(i). Ruthenium therefore belongs with the catalyst, not with the anticancer drug in this standard matching question.

Step 3: Compare the complete sequence The ordered matches are (iv), (iii), (ii), (i). Option (B) contains this complete sequence. Option (C) starts correctly with magnesium and cobalt but exchanges platinum and ruthenium in the last two positions; it therefore fails even though its first two entries agree.

Step 4: Conclusion The matched metals are magnesium, cobalt, platinum and ruthenium, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Use the four anchors Mg-chlorophyll, Co-B12, Pt-cisplatin and Ru-Grubbs before looking at the option codes.

68. Match List - I with List - II :

List - I (Chemicals)

(a) Alcoholic potassium hydroxide

(b) Pd/$BaSO_4$

(c) BHC (Benzene hexachloride)

(d) Polyacetylene

List - II (Use / Preparation / Constituent)

(i) Electrodes in batteries

(ii) Obtained by addition reaction

(iii) Used for β - elimination reaction

(iv) Lindlar's catalyst

Choose the most appropriate match : [4 marks]

(A) (a) - (ii), (b) - (i), (c) - (iv), (d) - (iii)

(B) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

(C) (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)

(D) (a) - (ii), (b) - (iv), (c) - (i), (d) - (iii)

Correct Answer: (B) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Solution:

Concept:

- Alcoholic potassium hydroxide promotes beta-elimination of hydrogen halide from suitable haloalkanes. Supported palladium is associated with selective partial hydrogenation of alkynes.
- Benzene hexachloride is produced by photochemical addition of chlorine to benzene. Polyacetylene is a conjugated polymer whose conducting forms have applications as electrode materials.

Step 1: Match the elimination and hydrogenation reagents Alcoholic KOH acts as a base and removes a beta-hydrogen while halide leaves from the neighbouring carbon, so (a)-(iii). Palladium on $BaSO_4$, with suitable poisoning, is the supported catalyst formulation referred to here as Lindlar's catalyst; it enables partial hydrogenation of an alkyne to a cis-alkene. Thus (b)-(iv).

Step 2: Match addition product and conducting polymer Under light, benzene adds three molecules of chlorine to form benzene hexachloride:

$$C_6H_6 + 3\,Cl_2 \xrightarrow{h\nu} C_6H_6Cl_6.$$

Therefore (c)-(ii). Polyacetylene has a conjugated backbone, and doping allows substantial electrical conductivity, making it useful in battery electrode applications. Hence (d)-(i).

Step 3: Verify the combined code The full ordered sequence is (iii), (iv), (ii), (i), appearing in option (B). Option (A) wrongly assigns alcoholic potash to the addition-product entry; option (C) wrongly puts supported palladium into the electrode entry; option (D) again mistakes the role of alcoholic potash. Each fails a direct reagent-function check.

Step 4: Conclusion The matches are beta-elimination, Lindlar catalyst, addition product and battery electrodes, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Anchor alcoholic KOH to elimination and BHC to chlorine addition; then match supported palladium and polyacetylene.

69. Compound with molecular formula C_3H_6O can show : [4 marks]

(A) Positional isomerism

(B) Both positional isomerism and metamerism

(C) Metamerism

(D) Functional group isomerism

Correct Answer: (D) Functional group isomerism

Solution:

Concept:

- Functional group isomers have the same molecular formula but different functional groups. Saturated open-chain aldehydes and ketones share the general formula $C_nH_{2n}O$.
- For C_3H_6O, propanal and propanone give the standard aldehyde-ketone pair. A carbonyl group needs one degree of unsaturation, consistent with the formula.

Step 1: Construct the aldehyde and ketone The aldehyde propanal has the condensed formula CH_3CH_2CHO, while the ketone propanone is CH_3COCH_3. Counting atoms in each gives three carbons, six hydrogens and one oxygen. Their identical molecular formula therefore meets the first requirement for isomerism.

Step 2: Classify the structural difference Propanal contains a terminal aldehyde group with a hydrogen bonded to the carbonyl carbon; propanone contains a ketone group bonded to two carbon groups. The functional group itself changes, so this pair demonstrates functional group isomerism rather than simply moving an unchanged group along a chain.

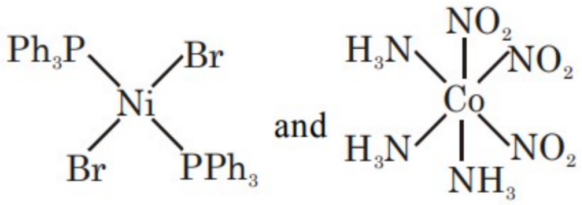
Step 3: Assess the other categories Within the usual aldehyde-ketone comparison, there is only one aldehyde and one ketone with three carbons. Propanone has only the methyl-methyl distribution around its carbonyl group, so there is no alternative ketone metamer at this carbon count. Moving the carbonyl to an end produces an aldehyde, making the standard classification functional rather than positional. Thus (D) is the intended classification, while (A), (B) and (C) do not describe this carbonyl-isomer pair.

Step 4: Conclusion CH_3CH_2CHO and CH_3COCH_3 demonstrate functional group isomerism for C_3H_6O. Hence, the correct option is (D).

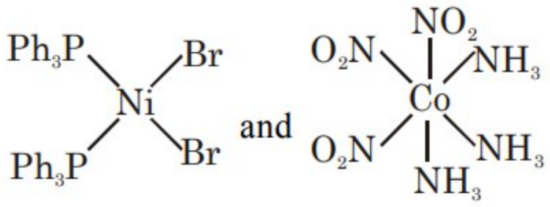
Quick Tip to Solve: Write propanal and propanone explicitly: an aldehyde-ketone change is functional group isomerism, even when drawn by moving C=O.

70. The correct structures of trans-$[NiBr_2(PPh_3)_2]$ and meridional-$[Co(NH_3)_3(NO_2)_3]$, respectively, are [4 marks]

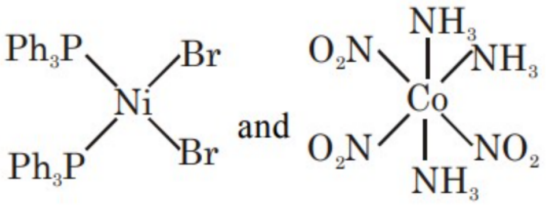
(A)



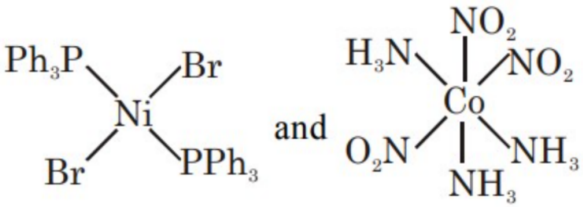
(B)



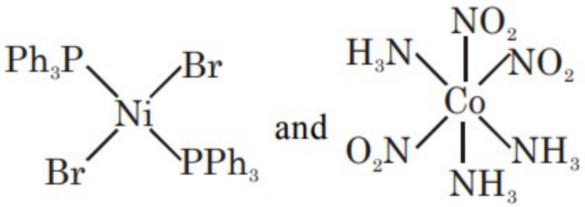
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- In a square-planar complex of type MA_2B_2, trans means identical ligands occupy opposite positions, making a 180° angle at the metal.
- For an octahedral complex of type MA_3B_3, a meridional arrangement places the three identical ligands in a plane through the metal. It contains a trans pair of each ligand type; a facial arrangement instead has all three identical ligands mutually cis.

Step 1: Select the trans nickel structures In the nickel drawing, the two bromide ligands must be diagonally opposite across Ni, and the two triphenylphosphine ligands must also be opposite. Options (A) and (D) satisfy this condition. In (B) and (C), the two bromides are adjacent on the right of the drawing, so those are cis arrangements and are eliminated.

Step 2: Distinguish meridional from facial cobalt Treat the top and bottom bonds in the octahedral projection as an opposite pair; the upper-left and lower-right bonds are opposite, as are the upper-right and lower-left bonds. In (D), two nitro ligands occupy upper-right and lower-left positions and are trans. Two ammine ligands likewise form the other diagonal trans pair. The third ligand of each kind lies on the vertical axis, giving the required meridional arrangement.

Step 3: Reject the remaining alternative In (A), the three nitro ligands lie at the top, upper-right and lower-right positions. None is opposite another nitro ligand, so all three are mutually cis and form a face of the octahedron. This is the facial arrangement, not meridional. Consequently only (D) satisfies both the nickel and cobalt conditions simultaneously.

Step 4: Conclusion Option (D) shows trans nickel and meridional cobalt structures in the stated order. Hence, the correct option is (D).

Quick Tip to Solve: For an octahedral mer isomer, look for an opposite identical-ligand pair; if all three identical ligands are cis, it is fac.

71. A certain orbital has no angular nodes and two radial nodes. The orbital is : [4 marks]

(A) 2s

(B) 3s

(C) 3p

(D) 2p

Correct Answer: (B) 3s

Solution:

Concept:

- An orbital is specified by the principal quantum number n and azimuthal quantum number l. The number of angular nodes is l.
- The number of radial nodes is $n-l-1$, while the total number of nodes is $n-1$. For an s orbital, $l=0$.

Step 1: Identify the subshell The question specifies zero angular nodes. Therefore $l=0$, which identifies an s orbital. A p orbital has $l=1$ and one angular nodal plane, so both 3p and 2p are ruled out before using the radial-node condition. Angular nodes describe angular variation of the wavefunction; they do not depend on the radial-node count.

Step 2: Determine the principal quantum number Insert $l=0$ and the given two radial nodes into the radial-node formula:

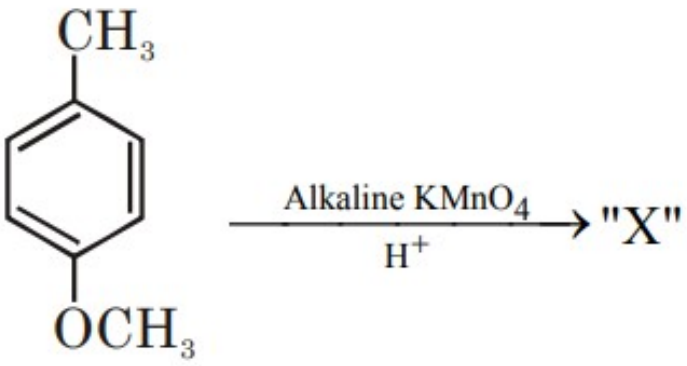
$$n-l-1=2 \quad \Rightarrow \quad n-0-1=2 \quad \Rightarrow \quad n=3.$$

Thus the orbital must belong to the third shell and the s subshell. The proposed 2s orbital has only $2-0-1=1$ radial node and cannot satisfy the question. For 3s the total of two radial and zero angular nodes also equals $n-1=2$, providing a consistency check.

Step 3: Conclusion The orbital is $3s$, with zero angular nodes and two radial nodes. Hence, the correct option is (B).

Quick Tip to Solve: Use angular nodes first to identify the subshell, then substitute into $n-l-1$ to find the shell.

72. Considering the above chemical reaction, identify the product "X" : [4 marks]



(A) 4-Methoxybenzaldehyde, p-$CH_3OC_6H_4CHO$

(B) 4-Methoxybenzyl alcohol, p-$CH_3OC_6H_4CH_2OH$

(C) 4-Methoxybenzoic acid, p-$CH_3OC_6H_4COOH$

(D) 4-Methylphenol, p-$CH_3C_6H_4OH$

Correct Answer: (C) 4-Methoxybenzoic acid, p-$CH_3OC_6H_4COOH$

Solution:

Concept:

- Hot alkaline $KMnO_4$ oxidises an aromatic side chain containing a benzylic hydrogen to a carboxylate group.
- Acidification converts the carboxylate into $COOH$. Under this standard side-chain oxidation the methoxy substituent on the ring remains unchanged.

Step 1: Identify the oxidisable side chain The reactant is para-methylanisole: its benzene ring carries CH_3 and OCH_3 groups in para positions. The methyl carbon bonded directly to the aromatic ring has benzylic hydrogens and is susceptible to vigorous permanganate oxidation. The methoxy group is an ether substituent, not an alkyl side chain attached to the ring through carbon.

Step 2: Apply oxidation followed by acidification The benzylic methyl group is oxidised completely rather than stopping at an alcohol or an aldehyde. In the alkaline medium the immediate product is the carboxylate salt, and the indicated acid work-up gives the free acid:

$$p\text{-}CH_3OC_6H_4CH_3 \xrightarrow{KMnO_4/OH^-} CH_3OC_6H_4COO^- \xrightarrow{H^+} CH_3OC_6H_4COOH.$$

The relative positions of the ring substituents are retained. The aldehyde shown in A and benzyl alcohol shown in B represent incomplete oxidation; D would require cleavage of the methyl ether and does not represent this reaction.

Step 3: Conclusion Product X is 4-methoxybenzoic acid (p-anisic acid), p-$CH_3OC_6H_4COOH$. Hence, the correct option is (C).

Quick Tip to Solve: For strong permanganate oxidation of an alkylbenzene, replace the benzylic side chain by $COOH$ after acid work-up.

73. Match List - I with List - II

	List - I(process)		List - II(catalyst)
(a)	Deacron's process	(i)	ZSM-5
(b)	Contact process	(ii)	$CuCl_2$
(c)	Cracking of hydrocarbons	(iii)	Particles 'Ni'
(d)	Hydrogenation of vegetable oils	(iv)	V_2O_5

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(B) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(C) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(D) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

Correct Answer: (A) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Solution:

Concept:

- A catalyst supplies a lower-activation-energy pathway without changing the equilibrium constant of a reaction.
- Industrial processes use characteristic catalysts: copper(II) chloride in Deacon's process, vanadium(V) oxide in the contact process, zeolites in cracking, and nickel in oil hydrogenation.

Step 1: Match the oxidation processes Deacon's process oxidises hydrogen chloride to chlorine using $CuCl_2$:

$$4\,HCl + O_2 \longrightarrow 2\,Cl_2 + 2\,H_2O.$$

Therefore (a) matches (ii). The contact process oxidises sulphur dioxide to sulphur trioxide using V_2O_5:

$$2\,SO_2 + O_2 \rightleftharpoons 2\,SO_3.$$

Thus (b) matches (iv). These two assignments already select A rather than the options that interchange unrelated catalysts.

Step 2: Match cracking and hydrogenation ZSM-5 is a shape-selective zeolite catalyst associated with hydrocarbon conversion and cracking, so (c) matches (i). Finely divided nickel catalyses addition of hydrogen across carbon-carbon double bonds in vegetable oils, converting them towards saturated fats, so (d) matches (iii). Nickel is effective because reactants adsorb on its surface. Combining all four matches gives (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii). B and C assign the wrong catalysts to the first processes; D reverses copper chloride and vanadium oxide.

Step 3: Conclusion The catalysts are copper(II) chloride, vanadium(V) oxide, ZSM-5 and nickel in the process order given. Hence, the correct option is (A).

Quick Tip to Solve: Anchor the list with contact process–V_2O_5 and oil hydrogenation–Ni, then check the remaining pairs.

74. Given below are two statements : One is labelled as Assertion A and the other labelled as reason R

Assertion A : During the boiling of water having temporary hardness, $Mg(HCO_3)_2$ is converted to $MgCO_3$.

Reason R : The solubility product of $Mg(OH)_2$ is greater than that of $MgCO_3$.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are true but R is not the correct explanation of A

(B) A is true but R is false

(C) Both A and R are true and R is the correct explanation of A

(D) A is false but R is true

Correct Answer: As worded, both A and R are false, and none of the listed options is correct. The supplied key gives (D), but its claim that R is true is inconsistent with the stated inequality.

Solution:

Concept:

- Temporary magnesium hardness is removed on boiling with precipitation of $Mg(OH)_2$, unlike calcium bicarbonate, which yields $CaCO_3$.
- The numerical solubility product of magnesium hydroxide is smaller than that of magnesium carbonate under standard aqueous conditions; comparisons of salts of different stoichiometry must not be treated as direct molar-solubility comparisons.

Step 1: Evaluate Assertion A The overall removal reaction for magnesium bicarbonate during boiling is

$$Mg(HCO_3)_2 \longrightarrow Mg(OH)_2\downarrow + 2\,CO_2\uparrow.$$

Magnesium carbonate may be considered an intermediate in a stepwise description, but it is not the final precipitate in the standard removal of temporary magnesium hardness. Therefore the assertion that boiling converts the bicarbonate to magnesium carbonate as the resulting salt is false. Confusing the magnesium reaction with the calcium bicarbonate reaction leads to an incorrect conclusion.

Step 2: Evaluate Reason R The reason states that the solubility product of $Mg(OH)_2$ is greater than that of $MgCO_3$. This inequality is reversed: typical values are of order 10^{-11} for the hydroxide and 10^{-6} to 10^{-5} for the carbonate. Hence R, exactly as worded, is false. This is a comparison of the stated numerical solubility products, not a claim that molar solubility can always be compared just by their magnitudes.

Step 3: Check the offered combinations A and C require both statements to be true, B requires A true and R false, and D requires A false and R true. None offers the combination A false and R false. The answer key selects D, but that selection requires changing the word greater in R to smaller, which is not what the supplied question says.

Step 4: Conclusion As worded, both A and R are false, and none of the listed options is correct. The supplied key gives (D), but its claim that R is true is inconsistent with the stated inequality.

Quick Tip to Solve: Write the magnesium bicarbonate boiling reaction first, then test the exact direction of the solubility-product inequality.

75. The chemical that is added to reduce the melting point of the reaction mixture during the extraction of aluminium is : [4 marks]

(A) Cryolite

(B) Bauxite

(C) Calamine

(D) Kaolite

Correct Answer: (A) Cryolite

Solution:

Concept:

- Aluminium is extracted electrolytically from alumina dissolved in a molten electrolyte in the Hall-Heroult process.
- Cryolite, Na_3AlF_6, lowers the effective melting temperature of the electrolyte mixture and improves its electrical conductivity.

Step 1: Identify why an additive is required Pure alumina, Al_2O_3, has a very high melting point, about $2050\,^{\circ}C$. Electrolysis of pure molten alumina would therefore require excessive heating and energy. The industrial electrolyte dissolves alumina in molten cryolite, allowing operation near $950\,^{\circ}C$. The additive helps create a conducting melt at a much more manageable temperature rather than supplying a chemical reducing agent.

Step 2: Compare the offered substances Cryolite is sodium hexafluoroaluminate, Na_3AlF_6, and is the required bath additive. Bauxite is the principal ore from which purified alumina is obtained; it is not the additive used to lower the bath melting point. Calamine is associated with zinc ore, usually represented as $ZnCO_3$, so it is unrelated to the aluminium electrolyte. Kaolite is not the cryolite electrolyte additive. The distinction is between the aluminium-bearing feed and the material used to modify the physical properties of the molten bath.

Step 3: Conclusion Cryolite is added to reduce the melting point of the reaction mixture during aluminium extraction. Hence, the correct option is (A).

Quick Tip to Solve: Remember the roles separately: bauxite supplies aluminium oxide; cryolite makes the electrolysis bath workable.

76. The statements that are TRUE :

(A) Methane leads to both global warming and photochemical smog

(B) Methane is generated from paddy fields

(C) Methane is a stronger global warming gas than CO_2

(D) Methane is a part of reducing smog

Choose the most appropriate answer from the options given below : [4 marks]

(A) (A), (B), (C) only

(B) (A) and (B) only

(C) (B), (C), (D) only

(D) (A), (B), (D) only

Correct Answer: (A) (A), (B), (C) only

Solution:

Concept:

- Methane is a greenhouse gas and participates in atmospheric photochemistry that can promote ground-level ozone formation.
- Flooded paddy fields support anaerobic microbial production of methane. Classical reducing smog is associated chiefly with smoke, sulphur dioxide and fog.

Step 1: Test statements A and B Statement A is true. Methane absorbs outgoing infrared radiation and contributes to global warming. Its atmospheric oxidation also participates in photochemical reactions that, in the presence of nitrogen oxides and sunlight, can promote ozone formation and photochemical smog. Methane is not as reactive as many other hydrocarbons, but this does not eliminate its contribution. Statement B is also true: waterlogged paddy soils develop anaerobic conditions in which methanogenic microorganisms generate methane from organic matter.

Step 2: Test statements C and D Statement C is true on the usual equal-mass global-warming-potential comparison: methane is more potent than carbon dioxide over standard comparison periods. This does not mean its total atmospheric contribution exceeds that of carbon dioxide, since concentrations and lifetimes differ. Statement D is false in the standard classification. Reducing, or London-type, smog contains smoke, sulphur dioxide and fog; methane is not its defining component. Thus the true set is A, B and C, while D must be excluded.

Step 3: Select the complete combination The first option contains A, B and C only. The second omits the true statement C, while the third and fourth include the false statement D. Therefore only the first combination accounts for all the statements correctly.

Step 4: Conclusion The first three statements are true, while the reducing-smog statement is false. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish methane's greenhouse and atmospheric-photochemistry effects from the smoke–SO_2–fog mixture of reducing smog.

77. In a binary compound, atoms of element A form a hcp structure and those of element M occupy 2/3 of the tetrahedral voids of the hcp structure. The formula of the binary compound is : [4 marks]

(A) M_2A_3

(B) M_4A_3

(C) M_4A

(D) MA_3

Correct Answer: (B) M_4A_3

Solution:

Concept:

- A close-packed arrangement containing N atoms has $2N$ tetrahedral voids and N octahedral voids, for both hcp and ccp packing.
- The compound formula follows from the occupied sites: reduce the ratio of the numbers of M and A atoms to the smallest whole numbers.

Step 1: Count tetrahedral sites Let the hcp framework contain N atoms of A. The total number of tetrahedral voids is then $2N$. The question specifies that M occupies two-thirds of these sites, not two-thirds as many sites as there are A atoms. Therefore

$$N_{\mathrm{M}} = \frac{2}{3}(2N) = \frac{4N}{3}.$$

The remaining one-third of the tetrahedral sites is empty and contributes no atoms to the compound formula.

Step 2: Reduce the atomic ratio The numbers of the two elements obey

$$\mathrm{M} : \mathrm{A} = \frac{4N}{3} : N = 4 : 3.$$

Equivalently, choose three A atoms as the counting basis: they create six tetrahedral voids, of which four are occupied by M. The formula is written with M first because that is the ordering used in the answer choices. A would reverse the ratio and ignore the fact that there are two tetrahedral sites per framework atom. C and D also have incompatible occupancy ratios; neither corresponds to four M atoms for every three A atoms.

Step 3: Conclusion The empirical formula is $\mathrm{M_4A_3}$. Hence, the correct option is (B).

Quick Tip to Solve: For fractional tetrahedral occupancy f, start with $\mathrm{M} : \mathrm{A} = 2f : 1$, then clear fractions.

78. Match List - I with List - II

	List - I		List - II
(a)	$Ca(OCl)_2$	**(i)**	**Antacid**
(b)	$CaSO_4.\frac{1}{2}H_2O$	**(ii)**	**Cement**
(c)	CaO	**(iii)**	**Bleach**
(d)	$CaCO_3$	**(iv)**	**Plaster of paris**

Choose the most appropriate answer from the options given below : [4 marks]

(A) (a) - (i), (b) - (iv), (c) - (iii), (d) - (ii)

(B) (a) - (iii), (b) - (ii), (c) - (iv), (d) - (i)

(C) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

(D) (a) - (iii), (b) - (ii), (c) - (i), (d) - (iv)

Correct Answer: (C) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Solution:

Concept:

- Calcium compounds have different applications because of their oxidising, setting or acid-neutralising properties. Match each actual compound rather than calcium alone.
- Calcium hypochlorite is an oxidising bleaching agent; calcium sulphate hemihydrate is plaster of Paris. Calcium oxide is a raw material in cement manufacture.

Step 1: Identify the bleach and the setting material $\mathrm{Ca(OCl)_2}$ contains hypochlorite ions and is used for bleaching and disinfection. Thus (a) matches (iii). Plaster of Paris has composition $\mathrm{CaSO_4 \cdot \frac{1}{2}\,H_2O}$; on adding water it sets by forming gypsum, $\mathrm{CaSO_4 \cdot 2\,H_2O}$. Therefore (b) matches (iv). Gypsum is added in cement manufacture to regulate setting, but this does not make plaster of Paris the cement entry in this matching exercise.

Step 2: Match the remaining two uses Quicklime, CaO, is a major constituent used in producing cement, so (c) matches (ii). Calcium carbonate, $\mathrm{CaCO_3}$, neutralises excess stomach acid and is used as an antacid, so (d) matches (i). Its neutralisation can be represented by $\mathrm{CaCO_3 + 2\,HCl \longrightarrow CaCl_2 + CO_2 + H_2O}$. These assignments use all four applications once.

Step 3: Check the alternatives The required sequence is (iii), (iv), (ii), (i). The first distractor incorrectly calls the hypochlorite an antacid. The second and fourth distractors assign the sulphate hemihydrate to cement rather than plaster of Paris, and the fourth also interchanges the uses of calcium oxide and carbonate. Only option C preserves every match.

Step 4: Conclusion The entries match bleach, plaster of Paris, cement and antacid, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Lock in hypochlorite as bleach and the half-hydrate as plaster of Paris first; these two distinctive formulas eliminate all the distractors.

79. Match the list - I with list - II

	List - I(Class of Drug)		List - II(Example)
(a)	Antacid	(i)	Novestrol
(b)	Artificial sweetener	(ii)	Cimetidine
(c)	Antifertility	(iii)	Valium
(d)	Tranquilizers	(iv)	Alitame

[4 marks]

(A) (a) - (ii), (b) - (iv), (c) - (i), (d) - (iii)

(B) (a) - (iv), (b) - (i), (c) - (ii), (d) - (iii)

(C) (a) - (iv), (b) - (iii), (c) - (i), (d) - (ii)

(D) (a) - (ii), (b) - (iv), (c) - (iii), (d) - (i)

Correct Answer: (A) (a) - (ii), (b) - (iv), (c) - (i), (d) - (iii)

Solution:

Concept:

- Drug classes are assigned by their physiological action: antacids reduce gastric acidity, antifertility drugs prevent conception, and tranquilizers act on the central nervous system.
- Artificial sweeteners provide sweetness in place of sugar. Alitame belongs to this class, whereas cimetidine, Novestrol and Valium have different therapeutic roles.

Step 1: Match the antacid and sweetener Cimetidine blocks histamine action at gastric H_2 receptors and reduces acid secretion in the stomach. It is therefore the antacid example, giving (a)-(ii). It need not neutralise acid directly like a carbonate to be classified here as an antacid drug. Alitame is an artificial sweetener, giving (b)-(iv); its role is not sedation or contraception.

Step 2: Match the remaining drug classes Novestrol is used as an antifertility drug, so (c) matches (i). Valium is the trade name of diazepam, a benzodiazepine used as a tranquilizer, so (d) matches (iii). The distinction is between an effect on fertility and an effect on anxiety or nervous-system activity; these examples must not be interchanged.

Step 3: Compare the complete mapping with the options The four matches in list order are (ii), (iv), (i), (iii). The first listed choice has precisely this sequence. The second choice incorrectly makes Alitame an antacid and Novestrol a sweetener. The third choice assigns both the antacid and sweetener incorrectly and makes cimetidine a tranquilizer. The fourth choice gets the first two matches right but swaps Novestrol and Valium in the final pair.

Step 4: Conclusion The examples in class order are cimetidine, Alitame, Novestrol and Valium. Hence, the correct option is (A).

Quick Tip to Solve: Use the distinctive pair Alitame–sweetener and Valium–tranquilizer to check a drug-matching sequence quickly.

80. The number of ionisable hydrogens present in the product obtained from a reaction of phosphorus trichloride and phosphonic acid is : [4 marks]

(A) 3

(B) 0

(C) 2

(D) 1

Correct Answer: (C) 2

Solution:

Concept:

- Phosphonic acid, H_3PO_3, has structure $HP(O)(OH)_2$. Its two hydroxyl hydrogens are ionisable; hydrogen bonded directly to phosphorus is not.
- The reaction with phosphorus trichloride produces pyrophosphorous acid, $H_4P_2O_5$. This is a condensed oxyacid containing a phosphorus–oxygen–phosphorus bridge.

Step 1: Identify the condensed acid Phosphorus trichloride reacts with phosphonic acid to produce pyrophosphorous acid. The product has molecular formula $H_4P_2O_5$, with structural connectivity

$$HO-P(=O)(H)-O-P(=O)(H)-OH.$$

Each phosphorus retains one hydrogen directly attached to phosphorus, while the bridge oxygen links the two phosphorus centres. The oxygen inventory is two terminal doubly bonded oxygens, one bridging oxygen and two hydroxyl oxygens, totalling five.

Step 2: Distinguish the two kinds of hydrogen There are four hydrogen atoms in $H_4P_2O_5$, but they do not have equivalent bonding. Two are the terminal $O-H$ hydrogens and can be released as protons during acid ionisation. The other two occupy $P-H$ bonds and do not ionise as acidic hydrogens in water. Consequently the acid is dibasic, and its number of ionisable hydrogens is two.

Step 3: Compare with the numerical alternatives A count of four would confuse total hydrogen atoms with ionisable hydrogen atoms. Zero would overlook the two hydroxyl groups, and one would count only one end of the symmetric condensed acid. Three is also inconsistent with the connectivity: only two oxygen atoms bear hydrogen. The correct count must therefore be based on the two terminal hydroxyl groups rather than on the formula alone.

Step 4: Conclusion The product $H_4P_2O_5$ contains $\boxed{2}$ ionisable hydrogens. Hence, the correct option is (C).

Quick Tip to Solve: For phosphorus oxyacids, draw the P–H and O–H bonds separately: only the O–H hydrogens determine basicity.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. AX is a covalent diatomic molecule where A and X are second row elements of periodic table. Based on Molecular orbital theory, the bond order of AX is 2.5. The total number of electrons in AX is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Molecular-orbital bond order is $B = (N_b - N_a)/2$, where N_b and N_a count bonding and antibonding electrons. Filled core orbitals cancel in this difference.
- For the oxygen-like filling used for nitric oxide, the valence sequence fills σ_{2s}, σ^*_{2s}, σ_{2p}, the two π_{2p} orbitals, and then π^*_{2p}.

Step 1: Construct the intended heteronuclear molecule For NO, the total electron count is $7 + 8 = 15$. Four are core electrons and eleven are valence electrons. The valence occupancy is

$$(\sigma_{2s})^2 (\sigma^*_{2s})^2 (\sigma_{2p})^2 (\pi_{2p})^4 (\pi^*_{2p})^1 .$$

The bonding valence orbitals contain eight electrons and the antibonding valence orbitals contain three.

Step 2: Calculate its bond order and electron total Substitution gives

$$B = \frac{8-3}{2} = 2.5.$$

Thus the intended oxygen-like example satisfies the stated bond order. Adding the four core electrons to the eleven valence electrons gives $11 + 4 = 15$. This is already an integer, so the requested rounding does not change it.

Step 3: Check the limitation of the wording Bond order alone does not uniquely determine an electron count for all second-period heteronuclear diatomics. The neutral radical CN has thirteen total electrons, nine valence electrons and bond order 2.5 with the nitrogen-like ordering: seven bonding and two antibonding valence electrons. Thus the wording admits this additional example. The accepted intended numerical response is 15, corresponding to NO; uniqueness requires an additional assumption about the species or orbital filling.

Step 4: Conclusion The intended answer is $\boxed{15}$ total electrons, as in NO. Strictly, the stated conditions alone also permit 13 for CN, so the species is not uniquely specified.

Quick Tip to Solve: Count core electrons in a total-electron question, and check the orbital ordering before inferring an electron count from bond order.

82. In order to prepare a buffer solution of pH 5.74, sodium acetate is added to acetic acid. If the concentration of acetic acid in the buffer is 1.0 M, the concentration of sodium acetate in the buffer is _____ M. (Round off to the Nearest Integer). [Given : pKa (acetic acid) = 4.74] [4 marks]

Solution:

Concept:

- A weak acid and its conjugate-base salt form a buffer. For acetic acid and sodium acetate, use $\mathrm{pH} = \mathrm{p}K_a + \log_{10}([\mathrm{CH_3COO^-}]/[\mathrm{CH_3COOH}])$.
- Sodium acetate dissociates to supply one acetate ion per formula unit. In the usual concentration-based buffer approximation, the salt concentration supplies the conjugate-base concentration.

Step 1: Find the required base-to-acid ratio The requested pH is 5.74 and the given acid $\mathrm{p}K_a$ is 4.74. Therefore

$$\log_{10}\frac{[\mathrm{CH_3COO^-}]}{[\mathrm{CH_3COOH}]} = 5.74 - 4.74 = 1.00.$$

Taking the base-ten antilogarithm gives

$$\frac{[\mathrm{CH_3COO^-}]}{[\mathrm{CH_3COOH}]} = 10^{1.00} = 10.$$

A pH above $\mathrm{p}K_a$ requires more conjugate base than undissociated acid, consistent with this ratio.

Step 2: Use the concentration in the final buffer The acid concentration in the buffer is explicitly given as $1.0\,\mathrm{mol\,L^{-1}}$, rather than the concentration of an initial stock solution. Consequently no dilution volume is needed. The sodium acetate concentration required in this same buffer is

$$c_{\mathrm{salt}} = 10(1.0) = 10\,\mathrm{mol\,L^{-1}}.$$

The concentration ratio in the logarithm is dimensionless because both concentrations use the same units.

Step 3: Check and round the answer Substituting the calculated ratio back gives $4.74 + \log_{10}(10/1.0) = 5.74$, which reproduces the target pH. The requested nearest-integer numerical value is therefore 10. This is the ideal concentration-based buffer calculation specified by the data; no activity correction or separate preparation volume is supplied.

Step 4: Conclusion The required sodium acetate concentration is $\boxed{10\,\mathrm{M}}$, so enter $\boxed{10}$.

Quick Tip to Solve: Every one-unit increase of pH above the acid pKa multiplies the required conjugate-base/acid ratio by ten.

83. **2NO(g) + Cl_2(g) $\rightleftharpoons$ 2NOCl(s)**

This reaction was studied at -10° and the following data was obtained

Run	$[NO]_0$	$[Cl_2]_0$	r_0
1	0.10	0.10	0.18
2	0.10	0.20	0.35
3	0.20	0.20	1.40

$[NO]_0$ and $[Cl_2]_0$ are the initial concentrations and r_0 is the initial reaction rate.

The overall order of the reaction is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Write the experimental rate law as $r = k[\mathrm{NO}]^m[\mathrm{Cl_2}]^n$. The overall order is $m + n$; it is determined from data, not assumed from the balanced equation.
- Compare experiments in which one reactant concentration stays fixed. The rate ratio then isolates the power of the concentration that changes.

Step 1: Find the order with respect to chlorine Between runs 1 and 2, $[\mathrm{NO}]$ stays at 0.10 while $[\mathrm{Cl_2}]$ doubles from 0.10 to 0.20. Thus

$$\frac{r_2}{r_1} = \frac{0.35}{0.18} = 1.944\ldots = 2^n.$$

The measured rate approximately doubles, indicating first order in chlorine. If the printed values are used literally, $n = \log(0.35/0.18)/\log 2 \simeq 0.959$, consistent with first order within the precision of the tabulated rates.

Step 2: Find the order with respect to nitric oxide Between runs 2 and 3, $[\mathrm{Cl_2}]$ stays at 0.20 while $[\mathrm{NO}]$ doubles from 0.10 to 0.20. The rate rises from 0.35 to 1.40, exactly fourfold:

$$\frac{1.40}{0.35} = 4 = 2^m \quad \Rightarrow \quad m = 2.$$

Thus the reaction is second order in nitric oxide.

Step 3: Add the partial orders and apply the requested rounding The experimental rate law is approximately $r = k[\mathrm{NO}]^2[\mathrm{Cl_2}]$, giving overall order $2 + 1 = 3$. Even retaining the literal nonintegral estimate from runs 1 and 2 gives $2 + 0.959 = 2.959$, whose nearest integer is 3. The small difference between 0.18 and half of 0.35 therefore does not change the reported result.

Step 4: Conclusion The overall reaction order, rounded to the nearest integer, is $\boxed{3}$.

Quick Tip to Solve: Use runs 2 and 3 first: their exact fourfold rate increase immediately fixes the exponent of NO at two.

84. For the reaction $C_2H_6 \rightarrow C_2H_4 + H_2$
the reaction enthalpy Δ_rH = _____ kJ mol^{-1}. (Round off to the Nearest Integer).
[Given : Bond enthalpies in kJ mol^{-1} : C-C : 347, C = C : 611; C-H : 414, H-H : 436] [4 marks]

Solution:

Concept:

- The bond-enthalpy estimate is $\Delta_r H = \sum D(\text{bonds broken}) - \sum D(\text{bonds formed})$. Bond breaking costs energy, whereas bond formation releases energy.
- Ethane has one carbon–carbon single bond and six carbon–hydrogen bonds. Ethene has one carbon–carbon double bond and four carbon–hydrogen bonds; hydrogen gas has one hydrogen–hydrogen bond.

Step 1: Count reactant and product bonds For $C_2H_6 \longrightarrow C_2H_4 + H_2$, the reactant bond-energy sum is

$$E_{\text{reactant}} = D(\text{C}{-}\text{C}) + 6D(\text{C}{-}\text{H}) = 347 + 6(414) = 2831\,\text{kJ mol}^{-1}.$$

The product bond-energy sum includes the hydrogen molecule:

$$E_{\text{product}} = D(\text{C}{=}\text{C}) + 4D(\text{C}{-}\text{H}) + D(\text{H}{-}\text{H}) = 611 + 4(414) + 436 = 2703\,\text{kJ mol}^{-1}.$$

All bond enthalpies are those supplied in the question.

Step 2: Subtract with the correct sign The net enthalpy is

$$\Delta_r H = 2831 - 2703 = +128\,\text{kJ mol}^{-1}.$$

The positive sign means the conversion of ethane to ethene and hydrogen is endothermic under this bond-enthalpy estimate. Reversing the subtraction would instead calculate the enthalpy of hydrogenating ethene.

Step 3: Check by cancelling unchanged bonds Four carbon–hydrogen bonds occur on both sides and cancel in the difference. The shorter calculation is

$$\Delta_r H = 347 + 2(414) - 611 - 436 = 128\,\text{kJ mol}^{-1}.$$

This agrees with the full inventory and confirms that the newly formed hydrogen–hydrogen bond has not been omitted. The result is already an integer, so nearest-integer rounding leaves it unchanged.

Step 4: Conclusion The reaction enthalpy is $\boxed{+128\,\text{kJ mol}^{-1}}$; enter $\boxed{128}$.

Quick Tip to Solve: Cancel the four unchanged C–H bonds first, but retain the H–H bond formed in the hydrogen molecule.

85. _____ grams of 3-Hydroxy propanal (MW = 74) must be dehydrated to produce 7.8 g of acrolein (MW = 56) (C_3H_4O) if the percentage yield is 64. (Round off to the Nearest Integer).

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u] [4 marks]

Solution:

Concept:

- Dehydration removes one molecule of water from each molecule of 3-hydroxypropanal. The reactant and acrolein therefore have a mole ratio of $1:1$, although their masses are different.
- Percentage yield compares actual product with theoretical product: yield $= 100 m_{\text{actual}}/m_{\text{theoretical}}$. Apply the yield to the product before converting its amount to reactant mass.

Step 1: Write the dehydration stoichiometry The hydroxy aldehyde loses water to form the carbon–carbon double bond of acrolein:

$$\mathrm{HOCH_2CH_2CHO} \longrightarrow \mathrm{CH_2{=}CHCHO} + \mathrm{H_2O}.$$

The supplied molar masses obey $74 = 56 + 18$, confirming that one mole of the starting compound produces one mole of acrolein in the theoretical reaction.

Step 2: Calculate the theoretical amount of acrolein The isolated $7.8\,\mathrm{g}$ is only 64% of the theoretical product. Hence

$$m_{\text{theoretical}} = \frac{7.8}{0.64} = 12.1875\,\mathrm{g}, \qquad n_{\text{acrolein}} = \frac{12.1875}{56} = 0.217634\,\mathrm{mol}.$$

Dividing by the fractional yield increases the required amount; multiplying by it would underestimate the starting material.

Step 3: Convert product moles to reactant mass Because the mole ratio is unity, the required starting amount is also $0.217634\,\mathrm{mol}$. Thus

$$m = \frac{7.8 \times 74}{0.64 \times 56} = 16.1049\,\mathrm{g}.$$

Retain the unrounded value until this final step, then round to the nearest integer as requested.

Step 4: Conclusion The required mass is approximately $16.10\,\mathrm{g}$; the numerical entry is $\boxed{16}$.

Quick Tip to Solve: For a one-to-one conversion, use actual product mass times the reactant/product molar-mass ratio, divided by fractional yield.

86. A reaction of 0.1 mole of Benzylamine with bromomethane gave 23 g of Benzyl trimethyl ammonium bromide. The number of moles of bromomethane consumed in this reaction are n $\times$ 10^{-1}, when n = _____. (Round off to the Nearest Integer).

(Given : Atomic masses : C : 12.0 u, H : 1.0 u, N : 14.0 u, Br : 80.0 u] [4 marks]

Solution:

Concept:

- A primary amine is converted into a quaternary ammonium salt by three successive N-methylations. Benzylamine retains its benzyl group, while three methyl groups are supplied by three molecules of bromomethane.
- For benzyltrimethylammonium bromide, calculate the complete formula mass, including the bromide counter-ion. The product amount then fixes the amount of methylating reagent by reaction stoichiometry.

Step 1: Identify the quaternary ammonium product Benzylamine is $C_6H_5CH_2NH_2$. Its quaternary salt is $[C_6H_5CH_2N(CH_3)_3]Br$: nitrogen is attached to one benzyl group and three methyl groups. Its overall formula is $C_{10}H_{16}NBr$, so

$$M = 10(12) + 16(1) + 14 + 80 = 230\,\text{g mol}^{-1}.$$

The counter-ion is essential when converting the isolated salt mass into moles.

Step 2: Find the amount of salt formed The given product mass corresponds to

$$n_{\text{salt}} = \frac{23}{230} = 0.100\,\text{mol}.$$

This equals the starting amount of benzylamine, so the supplied data correspond to complete conversion of its nitrogen atoms into the quaternary salt; there is no additional yield correction to apply.

Step 3: Count methyl groups and obtain the requested coefficient Each product ion contains three newly introduced methyl groups. Therefore three moles of CH_3Br are consumed per mole of salt:

$$n_{CH_3Br} = 3(0.100) = 0.300\,\text{mol} = 3 \times 10^{-1}\,\text{mol}.$$

Two methylations would stop at a neutral tertiary amine; the third methylation gives the positively charged quaternary nitrogen.

Step 4: Conclusion Bromomethane consumed is $0.300\,\text{mol}$, so the requested coefficient is $\boxed{n = 3}$.

Quick Tip to Solve: Count how many new methyl groups appear on nitrogen: a primary amine needs three methylations to become a quaternary ammonium salt.

87. The total number of unpaired electrons present in the complex $K_3[Cr(oxalate)_3]$ is _____.

[4 marks]

Solution:

Concept:

- Find the oxidation state of the metal before counting its d electrons. Each potassium ion carries charge $+1$, and each oxalate ligand, $C_2O_4{}^{2-}$, carries charge -2.
- Three bidentate oxalate ligands give six donor atoms and an octahedral coordination environment. The lower-energy t_{2g} set has three orbitals, occupied singly before pairing according to Hund's rule.

Step 1: Determine the chromium oxidation state The three potassium ions require the complex ion to have charge -3. If chromium has oxidation state x, the charge balance inside the brackets is

$$x + 3(-2) = -3, \qquad x = +3.$$

Potassium is outside the coordination sphere and contributes no unpaired electrons: K^+ has the closed-shell argon configuration.

Step 2: Obtain the d-electron count Neutral chromium has configuration $[Ar]\,3d^5 4s^1$. Electrons are removed from the $4s$ orbital first during ion formation, followed by two from $3d$. Consequently Cr^{3+} is $[Ar]\,3d^3$. The six negative ligand charges determine oxidation state; they do not mean that six electrons are added to the metal d shell.

Step 3: Fill the octahedral orbitals The three d electrons occupy the three lower-energy orbitals separately:

$$t_{2g}^3 e_g^0.$$

All three electrons are unpaired. For an octahedral d^3 ion, there is no competing arrangement obtained by choosing high spin or low spin: the three lower orbitals can each accommodate one electron without pairing. The closed-shell oxalate ligands do not add another unpaired electron.

Step 4: Conclusion The complex contains $\boxed{3}$ unpaired electrons per formula unit.

Quick Tip to Solve: For octahedral chromium(III), identify the d^3 count first; its three t_{2g} electrons remain unpaired regardless of the ligand-field strength.

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88. 2 molal solution of a weak acid HA has a freezing point of 3.885°C. The degree of dissociation of this acid is _____ $\times 10^{-3}$. (Round off to the Nearest Integer).

[Given : Molal depression constant of water = 1.85 K kg mol^{-1} Freezing point of pure water = 0° C]

[4 marks]

Solution:

Concept:

- For an ideal dilute solution, freezing-point depression is $\Delta T_f = T_f^0 - T_f = iK_f m$. A dissolved nonvolatile solute lowers the freezing point, so a physically valid depression is positive.
- For $HA \rightleftharpoons H^+ + A^-$, dissociation of a fraction α changes one initial solute particle into $1 + \alpha$

particles on average. Thus $i = 1 + \alpha$, with $0 \le \alpha \le 1$.

Step 1: Check the sign of the stated freezing point The question prints the freezing point as $+3.885\,^\circ\mathrm{C}$, while pure water freezes at $0\,^\circ\mathrm{C}$. Taken literally, this gives $\Delta T_f = -3.885\,\mathrm{K}$, a negative van't Hoff factor and an impossible degree of dissociation. Therefore the printed positive sign is inconsistent with freezing-point depression. The accepted numerical result uses $3.885\,\mathrm{K}$ as the depression, corresponding to a freezing point of $-3.885\,^\circ\mathrm{C}$.

Step 2: Calculate the particle multiplication factor Using the positive depression magnitude in the colligative-property relation gives

$$i = \frac{\Delta T_f}{K_f m} = \frac{3.885}{1.85 \times 2} = \frac{3.885}{3.700} = 1.050.$$

The non-dissociating acid would give $3.700\,\mathrm{K}$; the extra $0.185\,\mathrm{K}$ reflects the additional particles produced by ionisation.

Step 3: Express the degree of dissociation in the requested form Since one molecule dissociates into two ions,

$$\alpha = i - 1 = 0.050 = 50 \times 10^{-3}.$$

This is a 5% degree of dissociation, consistent with a weak acid. It is dimensionless; neither molality nor the temperature unit belongs in the final degree of dissociation.

Step 4: Conclusion Using the depression magnitude, the accepted coefficient is $\boxed{50}$, with $\alpha = 0.050$. The literal positive freezing point printed in the question has no physically admissible dissociation value.

Quick Tip to Solve: Always check the sign first: freezing-point depression is positive and requires the solution freezing point to lie below that of pure water.

89. For the reaction

$2Fe^{3+}(aq) + 2I^-(aq) \rightarrow 2Fe^{2+}(aq) + I_2(s)$

the magnitude of the standard molar Gibbs free energy change, $\Delta_r G^o_m$ = − _____ kJ (Round off to the Nearest Integer).

$$\begin{bmatrix} E^o_{Fe^{2+}/Fe(s)} = -0.440V; & E^o_{Fe^{3+}/Fe(s)} = -0.036V \\ E^o_{I_2/2I^-} = 0.539V; & F = 96500C \end{bmatrix}$$

[4 marks]

Solution:

Concept:

- Standard reaction Gibbs energies add, whereas electrode potentials do not add directly. Convert each reduction potential with $\Delta G^\circ = -nFE^\circ$, where n is its transferred-electron count.
- For the overall reaction, two electrons are transferred. After obtaining the Fe^{3+}/Fe^{2+} reduction potential, use $E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$ and express Gibbs energy in kilojoules.

Step 1: Derive the iron(III) to iron(II) potential Combine the reduction $Fe_3{}^{3++}e^- \longrightarrow Fe$, with $E^\circ = -0.036\,V$, and the reverse of $Fe_2{}^{2++}e^- \longrightarrow Fe$, whose reduction potential is $-0.440\,V$. The resulting reaction is $Fe^{3++}e^- \longrightarrow Fe^{2+}$. Its Gibbs energy is

$$\Delta G^\circ = -3F(-0.036) + 2F(-0.440) = -0.772F.$$

Thus the required one-electron reduction potential is $+0.772\,V$, rather than the simple difference of the two supplied potentials.

Step 2: Calculate the standard cell potential Iron(III) is reduced and iodide is oxidised. The iodine reduction potential remains $+0.539\,V$ when used in the cathode-minus-anode expression:

$$E^\circ_{cell} = 0.772 - 0.539 = 0.233\,V.$$

Its positive value indicates that the reaction as written has negative standard Gibbs energy.

Step 3: Use the electron count of the overall reaction Two Fe^{3+} ions each accept one electron, so $n = 2$. Therefore

$$\Delta_r G^\circ_m = -2(96500)(0.233) = -44969\,J\,mol^{-1} = -44.969\,kJ\,mol^{-1}.$$

Rounding only this final result gives a magnitude of 45. The supplied numerical potentials give this value directly; they do not give 46.

Step 4: Conclusion For the potentials printed in the question, $\Delta_r G^\circ_m \approx -45\,kJ\,mol^{-1}$, so the blank is $\boxed{45}$. This differs from the answer-key value 46 because the stated potentials yield $-44.969\,kJ\,mol^{-1}$.

Quick Tip to Solve: Weight each electrode potential by its electron count before combining half-reactions; add Gibbs energies, never raw potentials.

90. Complete combustion of 3g of ethane gives x $\times$ 10^{22} molecules of water. The value of x is _____. (Round off to the Nearest Integer). [Use : N_A = 6.023 $\times$ 10^{23}; Atomic masses in u : C : 12.0; O : 16.0; H : 1.0] [4 marks]

Solution:

Concept:

- In complete combustion, all carbon in ethane forms carbon dioxide and all hydrogen forms water. A balanced equation fixes the mole ratio independently of whether the water is liquid or vapour.
- Convert the given ethane mass to moles with its molar mass, use the stoichiometric ratio to find moles of water, and multiply by the given Avogadro constant to obtain the number of molecules.

Step 1: Balance the combustion equation Ethane is C_2H_6. Each mole supplies two moles of carbon atoms and six moles of hydrogen atoms, so it produces two moles of carbon dioxide and three moles of water:

$$2\,C_2H_6 + 7\,O_2 \longrightarrow 4\,CO_2 + 6\,H_2O.$$

Hence the required ratio is $n_{H_2O} : n_{C_2H_6} = 3 : 1$. Oxygen is supplied in sufficient amount because

combustion is stated to be complete.

Step 2: Find the amount of water formed The molar mass of ethane is

$$M_{C_2H_6} = 2(12) + 6(1) = 30\,\mathrm{g\,mol^{-1}}.$$

Thus $3\,\mathrm{g}$ contains $3/30 = 0.100\,\mathrm{mol}$ of ethane, producing

$$n_{H_2O} = 3(0.100) = 0.300\,\mathrm{mol}.$$

The factor of three comes from molecular stoichiometry, not from the mass of oxygen consumed.

Step 3: Convert moles into molecules Using the supplied value, rather than prematurely rounding Avogadro's constant,

$$N = 0.300(6.023 \times 10^{23}) = 1.8069 \times 10^{23} = 18.069 \times 10^{22}.$$

The coefficient accompanying 10^{22} is therefore 18.069, which rounds to 18. Keep the specified power of ten when extracting the coefficient.

Step 4: Conclusion The number of water molecules is 18.069×10^{22}; therefore $\boxed{x = 18}$ to the nearest integer.

Quick Tip to Solve: Balance hydrogen first: six H atoms in one ethane molecule give three water molecules, then convert the final count to the requested power of ten.