JEE Main 2021 Paper 1 (B.E./B.Tech) Mar 18 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 2 (March), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 18 March 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right)$, 0 < x < 2.1, with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to : [4 marks]

(A) $\frac{e^{5/2}}{(1+e^2)^2}$

(B) $\frac{5e^{1/2}}{(e^2+1)^2}$

(C) $-\frac{2e^2}{(1+e^2)^2}$

(D) $\frac{-e^{3/2}}{(e^2+1)^2}$

Correct Answer: (D) $\frac{-e^{3/2}}{(e^2+1)^2}$

Solution:

Concept:

- The substitution $u = 1/(y+1)$ converts this nonlinear differential equation into a first-order linear equation. The initial condition $y(2) = 0$ gives $u(2) = 1$.
- For $u\prime - xu = -e^{x^2/2}$, the integrating factor is $e^{-x^2/2}$. The initial value determines the integration constant before evaluating the requested derivative.

Step 1: Transform the dependent variable Near the initial point, $y + 1 \neq 0$. Write

$$u = \frac{1}{y+1}, \qquad u\prime = -\frac{y\prime}{(y+1)^2}.$$

Dividing the given equation by $(y+1)^2$ and changing sign yields

$$u\prime - xu = -e^{x^2/2}.$$

Thus the apparent quadratic dependence on $y + 1$ has become a linear equation in u.

Step 2: Integrate and apply the initial value Multiply by $e^{-x^2/2}$ to obtain

$$\frac{d}{dx}\left(ue^{-x^2/2}\right) = -1.$$

Integration gives $ue^{-x^2/2} = C - x$. At $x = 2$, $u = 1$, so $C = 2 + e^{-2}$. Hence

$$y + 1 = \frac{e^{-x^2/2}}{2 + e^{-2} - x}.$$

The denominator is positive throughout the specified interval, since $2 + e^{-2} > 2.1$. Therefore this expression is valid at $x = 1$.

Step 3: Substitute into the original differential equation At the required point,

$$y(1) + 1 = \frac{e^{-1/2}}{1 + e^{-2}} = \frac{e^{3/2}}{1 + e^2}.$$

Using the given equation directly avoids a second quotient differentiation:

$$y'(1) = \frac{e^{3/2}}{1+e^2}\left(\frac{e^2}{1+e^2} - 1\right) = -\frac{e^{3/2}}{(1+e^2)^2}.$$

Options (A) and (B) have the wrong sign; option (C) has the wrong numerator even though its sign is negative.

Step 4: Conclusion $y'(1) = -\dfrac{e^{3/2}}{(1+e^2)^2}$. Hence, the correct option is (D).

Quick Tip to Solve: For an equation quadratic in $y+1$, try its reciprocal; after finding $y(1)+1$, reuse the original equation to obtain the derivative.

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2. Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0$, $\lambda, \mu \in R$.

has a non-trivial solution. Then which of the following is true? [4 marks]

(A) $\mu = 6, \lambda \in R$

(B) $\lambda = 3, \mu \in R$

(C) $\mu = -6, \lambda \in R$

(D) $\lambda = 2, \mu \in R$

Correct Answer: (A) $\mu = 6, \lambda \in R$

Solution:

Concept:

- A homogeneous system always has the zero solution. It has a non-trivial solution precisely when its coefficient matrix is singular, that is, when its determinant vanishes.
- The options state parameter families with the other parameter unrestricted. Test whether each proposed family makes the determinant zero for every real value of that unrestricted parameter.

Step 1: Form the coefficient determinant In the variable order x, y, z, the matrix is

$$A = \begin{pmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{pmatrix}.$$

Expanding along the first row and retaining the alternating cofactor signs gives

$$\begin{aligned} \det A &= 4(-3-2) - \lambda(6-\mu) + 2(4+\mu) \\ &= -20 - 6\lambda + \lambda\mu + 8 + 2\mu \\ &= (\lambda+2)(\mu-6). \end{aligned}$$

Thus the full singularity condition is $\lambda = -2$ or $\mu = 6$.

Step 2: Test the listed parameter families If $\mu = 6$, the second factor vanishes for every real λ, exactly as stated in option (A). A direct nonzero solution is $(x, y, z) = (1, 0, -2)$: substitution gives zero in all three equations, independently of λ. For option (B), $\lambda = 3$ gives determinant $5(\mu - 6)$, which is not zero for arbitrary μ. For option (D), $\lambda = 2$ similarly gives $4(\mu - 6)$. Option (C), $\mu = -6$, gives $-12(\lambda + 2)$, which is not zero for arbitrary λ. For example, choosing the remaining parameter as zero disproves each of these unrestricted claims.

Step 3: Interpret the unrestricted condition The other singular family, $\lambda = -2$ with arbitrary μ, is mathematically valid but is absent from the listed options. Consequently, option (A) is a sufficient parameter family, rather than a claim that every singular system must have $\mu = 6$. This distinction prevents discarding the valid choice merely because the determinant has two factors.

Step 4: Conclusion The listed family that guarantees a non-trivial solution is $\mu = 6,\ \lambda \in \mathbb{R}$. Hence, the correct option is (A).

Quick Tip to Solve: Factor the determinant completely, then check the words indicating that the other parameter is arbitrary; one example cannot verify an entire parameter family.

3. The area bounded by the curve $4y^2 = x^2(4 - x)(x - 2)$ is equal to : [4 marks]

(A) $\frac{\pi}{16}$
(B) $\frac{\pi}{8}$
(C) $\frac{3\pi}{2}$
(D) $\frac{3\pi}{8}$

Correct Answer: (C) $\frac{3\pi}{2}$

Solution:

Concept:

- For a curve symmetric about the x-axis, the enclosed area is the integral of the upper ordinate minus the lower ordinate over the interval where the two branches enclose a region.
- The identity $(4 - x)(x - 2) = 1 - (x - 3)^2$ reduces the integral to the standard semicircle form. An odd integrand integrates to zero on symmetric limits.

Step 1: Locate the bounded branches The equation is

$$4y^2 = x^2(4 - x)(x - 2).$$

For $x \neq 0$, real ordinates require $(4 - x)(x - 2) \geq 0$, so $2 \leq x \leq 4$. The point $(0, 0)$ is isolated and contributes no area. On the bounded interval x is positive, giving

$$y_{\pm} = \pm\frac{x}{2}\sqrt{(4 - x)(x - 2)}.$$

The vertical separation of the branches is therefore $x\sqrt{(4 - x)(x - 2)}$, with no remaining factor of one-half.

Step 2: Shift the integration variable The total enclosed area is

$$A = \int_2^4 x\sqrt{(4-x)(x-2)}\,dx.$$

Put $t = x - 3$; then the new limits are -1 and 1, and the radical becomes $\sqrt{1-t^2}$. Thus

$$A = \int_{-1}^{1} (t+3)\sqrt{1-t^2}\,dt.$$

The term $t\sqrt{1-t^2}$ is odd, so its integral is zero. This leaves

$$A = 3\int_{-1}^{1} \sqrt{1-t^2}\,dt.$$

Step 3: Evaluate the standard integral and compare The graph $v = \sqrt{1-t^2}$ is the upper semicircle of radius one. Its integral from -1 to 1 is $\pi/2$. Equivalently, the antiderivative is

$$\frac{1}{2}\left(t\sqrt{1-t^2} + \sin^{-1} t\right).$$

Therefore $A = 3\pi/2$ square units. Options (A), (B) and (D) are smaller and do not equal the full upper-minus-lower integral; retaining a spurious factor of one-half would already undercount the region.

Step 4: Conclusion The area bounded by the curve is $\boxed{3\pi/2}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a squared ordinate, subtract the negative branch from the positive branch before integrating; then shift a quadratic radical to its centre.

4. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to : [4 marks]

(A) 500

(B) 400

(C) 250

(D) 350

Correct Answer: (C) 250

Solution:

Concept:

- Use $\sin^2\alpha + \cos^2\alpha = 1$ to express both fourth powers through one variable. The given equality becomes a perfect square.
- Once the squared sine and cosine are known, $\sec^6\alpha = 1/(\cos^2\alpha)^3$ and $\csc^6\alpha = 1/(\sin^2\alpha)^3$. No choice of quadrant is needed.

Step 1: Reduce the given equation to a quadratic Let $t = \sin^2\alpha$, so $\cos^2\alpha = 1 - t$ and $0 \le t \le 1$. Substitution into the given relation yields

$$15t^2 + 10(1-t)^2 = 6.$$

Expanding and collecting terms gives

$$25t^2 - 20t + 4 = 0 \quad \Longrightarrow \quad (5t-2)^2 = 0.$$

There is only one possible squared sine: $t = 2/5$. Accordingly, $\cos^2\alpha = 3/5$. Both are positive, so neither reciprocal trigonometric function in the target is undefined.

Step 2: Calculate the sixth powers correctly A sixth power is the cube of the corresponding squared quantity. Therefore

$$\sec^6\alpha = \left(\frac{5}{3}\right)^3 = \frac{125}{27}, \qquad \csc^6\alpha = \left(\frac{5}{2}\right)^3 = \frac{125}{8}.$$

These values are independent of the signs of sine and cosine. Taking square roots to determine the quadrant would add work without affecting any even power.

Step 3: Combine the two contributions Multiply each reciprocal power by its own printed coefficient:

$$27\sec^6\alpha + 8\csc^6\alpha = 27\left(\frac{125}{27}\right) + 8\left(\frac{125}{8}\right) = 125 + 125 = 250.$$

The coefficients cancel the denominators exactly. Options (A), (B) and (D), giving $500, 400, 350$, cannot result from these two equal contributions. In particular, doubling their sum once more would produce option (A), but there is no extra factor of two in the expression.

Step 4: Conclusion The requested value is $\boxed{250}$. Hence, the correct option is (C).

Quick Tip to Solve: Replace fourth powers by squares of $\sin^2\alpha$; a perfect-square quadratic often fixes the ratio without finding the angle itself.

5. Let $\overrightarrow{a}$ and $\overrightarrow{b}$ be two non-zero vectors perpendicular to each other and $|\overrightarrow{a}| = |\overrightarrow{b}|$. If $|\overrightarrow{a} \times \overrightarrow{b}| = |\overrightarrow{a}|$, then the angle between the vectors $\left(\overrightarrow{a} + \overrightarrow{b} + \left(\overrightarrow{a} \times \overrightarrow{b}\right)\right)$ and $\overrightarrow{a}$ is equal to : [4 marks]

(A) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

(B) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(C) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(D) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Correct Answer: (D) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Solution:

Concept:

- For perpendicular vectors, $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|$. Their cross product is perpendicular to each original vector.
- The angle between nonzero vectors $\vec{u}, \vec{v}$ obeys $\cos\theta = (\vec{u} \cdot \vec{v})/(|\vec{u}|\,|\vec{v}|)$. Orthogonal components add through the sum of their squared magnitudes.

Step 1: Determine the common magnitude Let $|\vec{a}| = |\vec{b}| = r$, where $r > 0$ because both vectors are nonzero. Since the angle between them is a right angle,

$$|\vec{a} \times \vec{b}| = r^2.$$

The additional condition in the question says $r^2 = r$. Dividing by the positive number r gives $r = 1$. Thus $\vec{a}$, $\vec{b}$, and $\vec{c} = \vec{a} \times \vec{b}$ all have magnitude one and are pairwise perpendicular.

Step 2: Find the magnitude and projection of the sum Write the first vector in the requested angle as $\vec{s} = \vec{a} + \vec{b} + \vec{c}$. Orthogonality makes every mixed dot product zero, so

$$|\vec{s}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 3.$$

Also,

$$\vec{s} \cdot \vec{a} = \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a} = 1.$$

This projection is positive, which confirms that the desired angle is acute.

Step 3: Apply the angle formula and rule out alternatives Consequently,

$$\cos\theta = \frac{1}{\sqrt{3} \cdot 1} = \frac{1}{\sqrt{3}}, \qquad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$$

Option (B) would apply to a sum of only two perpendicular unit vectors, omitting the cross-product component. Option (C) uses the correct number as a sine rather than a cosine: the actual sine is $\sqrt{2/3}$. Option (A) has sine $1/\sqrt{6}$, also inconsistent with this value. All three components must enter the norm.

Step 4: Conclusion The angle is $\boxed{\cos^{-1}(1/\sqrt{3})}$. Hence, the correct option is (D).

Quick Tip to Solve: When two equal perpendicular vectors have cross-product magnitude equal to either length, they are unit vectors; treat their cross product as a third orthogonal unit direction.

6. Let a complex number be w = 1 − $\sqrt{3}$i. Let another complex number z be such that |zw| = 1 and arg(z) − arg(w) = $\frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to : [4 marks]

(A) 4

(B) $\frac{1}{4}$

(C) 2

(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Concept:

- The modulus of a product is the product of the moduli: $|zw| = |z|\,|w|$. The arguments of nonzero complex numbers specify the directions of their position vectors.
- The area of a triangle formed by vectors of lengths r_1, r_2 with included angle θ is $\frac{1}{2}r_1r_2|\sin\theta|$. The factor one-half distinguishes a triangle from a parallelogram.

Step 1: Calculate the two distances from the origin For $w = 1 - \sqrt{3}\,i$, its modulus is

$$|w| = \sqrt{1^2 + (-\sqrt{3})^2} = 2.$$

The condition $|zw| = 1$ therefore gives

$$|z|\,|w| = 1 \quad \Longrightarrow \quad |z| = \frac{1}{2}.$$

Thus the two sides of the triangle drawn from the origin have lengths 2 and $1/2$. The product condition concerns moduli; it does not state that either complex number has modulus one.

Step 2: Use the argument difference as the included angle The prescribed relation is $\arg z - \arg w = \pi/2$. Hence the two position vectors are perpendicular. The area is

$$\Delta = \frac{1}{2}|z|\,|w|\sin\frac{\pi}{2} = \frac{1}{2}\cdot\frac{1}{2}\cdot 2\cdot 1 = \frac{1}{2}.$$

The angle information is essential: the product of the lengths alone would only fix the maximal possible triangle area, and the right angle shows that this maximum is attained here.

Step 3: Check with coordinates and compare the options The principal argument of w is $-\pi/3$, so a compatible argument of z is $\pi/6$. Thus

$$z = \frac{\sqrt{3}}{4} + \frac{1}{4}i.$$

The coordinate area check gives

$$\frac{1}{2}\left|\frac{\sqrt{3}}{4}(-\sqrt{3}) - \frac{1}{4}(1)\right| = \frac{1}{2}\left|-\frac{3}{4} - \frac{1}{4}\right| = \frac{1}{2}.$$

Options (A) and (C) exceed the maximum $\frac{1}{2}|z||w| = 1/2$, while option (B) introduces an additional

unjustified factor of one-half.

Step 4: Conclusion The area of the triangle is $\boxed{1/2}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For a triangle with the origin as a vertex, use the moduli as side lengths and the argument difference as the included angle before calculating coordinates.

7. Let f : R $\to$ R be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x+\sin 2x}{2x}, if\ x < 0 \\ b,\ if\ x = 0 \\ \frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}},\ if\ x > 0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to : [4 marks]

(A) -3

(B) -2

(C) $-\frac{5}{2}$

(D) $-\frac{3}{2}$

Correct Answer: (D) $-\frac{3}{2}$

Solution:

Concept:

- Continuity at zero requires the left-hand limit, the right-hand limit and the assigned value $f(0) = b$ to coincide.
- Use $\sin(kx)/x \to k$ for the left branch and rationalize the difference of square roots in the right branch. Keep the factor b until its cancellation is justified.

Step 1: Evaluate the left-hand limit For $x < 0$, split the quotient into two standard sine limits:

$$\lim_{x\to 0^-} f(x) = \frac{1}{2}\left(\lim_{x\to 0^-} \frac{\sin((a+1)x)}{x} + \lim_{x\to 0^-} \frac{\sin 2x}{x}\right)$$
$$= \frac{1}{2}\big((a+1)+2\big) = \frac{a+3}{2}.$$

This remains valid when $a + 1 = 0$, since the corresponding sine term is then identically zero. Continuity will require this expression to equal b, but the right branch must first determine the value of b.

Step 2: Rationalize the right branch The denominator of the specified right branch contains b, so a

defined branch requires $b \neq 0$. For positive x sufficiently close to zero,

$$\begin{aligned} f(x) &= \frac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}} \\ &= \frac{\sqrt{1 + bx^2} - 1}{bx^2} \\ &= \frac{1}{\sqrt{1 + bx^2} + 1}. \end{aligned}$$

The last equality follows by multiplication by the conjugate. Therefore $\lim_{x \to 0^+} f(x) = 1/2$, independently of the particular nonzero value of b.

Step 3: Impose both continuity conditions Since the value at the origin is b, right continuity gives $b = 1/2$, which satisfies the required nonzero condition. Left continuity then gives

$$\frac{a+3}{2} = \frac{1}{2} \quad \Longrightarrow \quad a = -2.$$

Consequently,

$$a + b = -2 + \frac{1}{2} = -\frac{3}{2}.$$

For these values the radicand on the positive branch is $x + x^3/2 > 0$, so the limiting calculation is valid. The other three offered sums, $-3, -2, -5/2$, disagree with the parameters fixed by the two separate one-sided limits.

Step 4: Conclusion The parameters are $a = -2$, $b = 1/2$, so $\boxed{a + b = -3/2}$. Hence, the correct option is (D).

Quick Tip to Solve: For continuity with a square-root difference, rationalize first to find the right limit, set it equal to the assigned value, then use the sine limit on the left.

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8. Let g(x) = $\int_0^x f(t)dt$, where f is continuous function in [0, 3] such that $\frac{1}{3} \leq$ f(t) ≤ 1 for all t$\in$ [0, 1] and $0 \leq$ f(t) $\leq \frac{1}{2}$ for all t$\in$ (1, 3]. The largest possible interval in which g(3) lies is : [4 marks]

(A) $\left[-1, -\frac{1}{2}\right]$

(B) $\left[-\frac{3}{2}, -1\right]$

(C) [1, 3]

(D) $\left[\frac{1}{3}, 2\right]$

Correct Answer: (D) $\left[\frac{1}{3}, 2\right]$

Solution:

Concept:

- Integration preserves inequalities: if $m \leq f(t) \leq M$ on an interval of length L, its integral lies between mL and ML.
- The integral defining $g(3)$ can be split at $t = 1$, where the bounds on the continuous function change. Values at one endpoint do not change a definite integral.

Step 1: Split the integral at the change in bounds The required value is the accumulated signed area on the entire interval, not the value of the integrand at its right endpoint. Additivity gives

$$g(3) = \int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt.$$

On the first interval the lower and upper bounds are $1/3$ and 1; its length is one. Therefore,

$$\frac{1}{3} \le \int_0^1 f(t)\,dt \le 1.$$

Step 2: Combine the bounds and compare the options On the remaining interval the length is two, and the integrand lies between zero and one half. Hence

$$0 \le \int_1^3 f(t)\,dt \le \frac{1}{2}(3-1) = 1.$$

Adding the two inequalities yields $1/3 \le g(3) \le 2$. Continuity can prevent equality at an extreme, but does not invalidate this closed interval containing every possible value. The negative intervals (A) and (B) are impossible since the integral is positive. Option (C) fails to include permissible values below one and allows values beyond the upper bound. Thus only (D) expresses the bounds obtained from both pieces.

Step 3: Conclusion The stated bounds place $g(3)$ in $\left[\frac{1}{3}, 2\right]$. Hence, the correct option is (D).

Quick Tip to Solve: Split at every point where the stated bound changes, multiply each bound by its interval length, and then add.

9. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to $-$a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to : [4 marks]

(A) 425

(B) 250

(C) 925

(D) 650

Correct Answer: (A) 425

Solution:

Concept:

- The arithmetic mean of equally many observations a and $-a$ is zero. Adding a constant shifts the mean by that constant.
- Standard deviation measures departures from the mean and is unchanged by adding the same constant to every observation. Its square is the average squared departure.

Step 1: Use the new mean to find the shift There are n observations of each type. After the shift, the

two values are $a+b$ and $-a+b$, again each repeated n times. Their mean is

$$\overline{x} = \frac{n(a+b)+n(-a+b)}{2n} = b.$$

The new mean is given as five, so $b = 5$. Notice that the cancelling terms in a do not require a to be positive.

Step 2: Calculate the variance and the required sum The departures of the shifted values from their mean b are exactly a and $-a$. Thus the variance, with all $2n$ observations counted, is

$$\sigma^2 = \frac{na^2 + n(-a)^2}{2n} = a^2.$$

Since the given standard deviation is 20, its square gives $a^2 = 400$. Consequently

$$a^2 + b^2 = 400 + 5^2 = 425.$$

This value is independent of the number of observations because their two multiplicities are equal. The alternatives 250, 925 and 650 cannot satisfy both the variance 400 and the mean 5; none equals the required sum of squares.

Step 3: Conclusion $a^2 + b^2 = 425$. Hence, the correct option is (A).

Quick Tip to Solve: For equally weighted values symmetric about zero, the original standard deviation is $|a|$; a common shift changes only their mean.

10. In a triangle ABC, if $|\overrightarrow{BC}| = 8, |\overrightarrow{CA}| = 7, |\overrightarrow{AB}| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to : [4 marks]

(A) $\frac{25}{4}$
(B) $\frac{127}{20}$
(C) $\frac{85}{14}$
(D) $\frac{115}{16}$

Correct Answer: (C) $\frac{85}{14}$

Solution:

Concept:

- The scalar projection of $\mathbf{u}$ on $\mathbf{v}$ is $\mathbf{u}\cdot\mathbf{v}/|\mathbf{v}|$, the signed component along the direction of $\mathbf{v}$.
- For two sides meeting at a vertex, the cosine rule or the squared length of their difference gives their dot product from the three side lengths.

Step 1: Find the dot product from the third side Put $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{AC}$. Their lengths are ten and seven. The remaining side satisfies $\overrightarrow{BC} = \mathbf{v} - \mathbf{u}$, with length eight. Squaring this identity gives

$$8^2 = 7^2 + 10^2 - 2\mathbf{u}\cdot\mathbf{v}.$$

Therefore,

$$\mathbf{u} \cdot \mathbf{v} = \frac{49 + 100 - 64}{2} = \frac{85}{2}.$$

Step 2: Divide by the length of the projection direction The question asks for the projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$, so the denominator is $|AC| = 7$, rather than $|AB| = 10$. The scalar projection is

$$\frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{85/2}{7} = \frac{85}{14}.$$

The positive sign is consistent with the acute angle at A, since its cosine is $85/140$. The other fractions $25/4$, $127/20$ and $115/16$ do not equal this dot product divided by seven. A vector projection would additionally carry the unit vector along AC, but the scalar options show that its signed length is requested.

Step 3: Conclusion The required projection is $\frac{85}{14}$. Hence, the correct option is (C).

Quick Tip to Solve: In a projection of AB on AC, obtain their dot product using BC, then divide by the length of AC, the direction receiving the projection.

11. Define a relation R over a class of n $\times$ n real matrices A and B as "ARB iff there exists a non-singular matrix P such that PAP^{-1} = B". Then which of the following is true? [4 marks]

(A) R is reflexive, transitive but not symmetric

(B) R is symmetric, transitive but not reflexive.

(C) R is reflexive, symmetric but not transitive

(D) R is an equivalence relation

Correct Answer: (D) R is an equivalence relation

Solution:

Concept:

- A relation is an equivalence relation precisely when it is reflexive, symmetric and transitive on the stated set.
- A nonsingular matrix has an inverse; products and inverses of nonsingular real square matrices remain nonsingular. The order of factors must be preserved in matrix multiplication.

Step 1: Establish reflexivity and symmetry For any real square matrix A, choose the identity matrix I. It is nonsingular and $IAI^{-1} = A$, so ARA. This proves reflexivity without requiring A itself to be invertible. Next suppose ARB. By definition some nonsingular P satisfies $PAP^{-1} = B$. Multiplying on the left by P^{-1} and on the right by P gives

$$A = P^{-1}BP.$$

Set $Q = P^{-1}$. Then Q is nonsingular and $QBQ^{-1} = A$, proving BRA and symmetry.

Step 2: Establish transitivity and select the classification If ARB and BRC, there are nonsingular matrices P, Q such that $B = PAP^{-1}$ and $C = QBQ^{-1}$. Substitute the first equation into the second:

$$C = QPAP^{-1}Q^{-1} = (QP)A(QP)^{-1}.$$

Because QP is nonsingular, this is exactly the condition ARC. All three defining properties therefore hold. Option (A) incorrectly denies symmetry, (B) incorrectly denies reflexivity, and (C) incorrectly denies transitivity. This relation is the familiar similarity relation, and the explicit witnesses above establish every required property.

Step 3: Conclusion The relation is reflexive, symmetric and transitive, and hence an equivalence relation. Hence, the correct option is (D).

Quick Tip to Solve: For similarity, use I for reflexivity, the inverse change-of-basis matrix for symmetry, and the product of change-of-basis matrices for transitivity.

12. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to : [4 marks]

(A) $7\sqrt{2}$

(B) $\frac{9}{\sqrt{2}}$

(C) $2\sqrt{2}$

(D) $3\sqrt{2}$

Correct Answer: (B) $\frac{9}{\sqrt{2}}$

Solution:

Concept:

- In an equilateral triangle the centroid, incenter and circumcenter coincide. Thus the distance from the given centroid to any side is the inradius r.
- The perpendicular distance of (x_0, y_0) from $ax + by + c = 0$ is $|ax_0 + by_0 + c|/\sqrt{a^2 + b^2}$. For an equilateral triangle the circumradius is $R = 2r$.

Step 1: Calculate the inradius from the side equation The shared centre is the origin $(0, 0)$, and the specified side lies along $x + y - 3 = 0$. Its distance from the origin is therefore

$$r = \frac{|0 + 0 - 3|}{\sqrt{1^2 + 1^2}} = \frac{3}{\sqrt{2}}.$$

This distance is perpendicular to the side. It is not the distance to a vertex, so it represents the smaller radius r, rather than R. The absolute value ensures the radius is positive.

Step 2: Use the median ratio to combine the radii The altitude of an equilateral triangle is also its median, angle bisector and perpendicular bisector. The centroid divides that median in the ratio $2 : 1$, measured from the vertex. Hence the distance from centre to vertex is twice the distance from centre

to the opposite side:

$$R = 2r = \frac{6}{\sqrt{2}}.$$

It follows that

$$R + r = 3r = \frac{9}{\sqrt{2}}.$$

Option (B) gives this value. Neither $7\sqrt{2}$, $2\sqrt{2}$ nor $3\sqrt{2}$ equals $9/\sqrt{2}$; the last of these is actually the circumradius alone. The orientation of the triangle on either side of the line does not affect these distances.

Step 3: Conclusion The sum of the radii is $R + r = \frac{9}{\sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: When an equilateral triangle has its centroid specified, use its distance to a side as r directly; the required sum R+r is then 3r.

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13. Let f : R − {3} → R − {1} be defined by f(x) = $\frac{x-2}{x-3}$.
Let g : R → R be given as g(x) = 2x − 3. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to : [4 marks]

(A) 3

(B) 5

(C) 2

(D) 7

Correct Answer: (B) 5

Solution:

Concept:

- To find an inverse function, set $y = f(t)$, solve for t, and express that result as a function of y. An inverse is not the reciprocal of the original function.
- The domain of f^{-1} is the range of f, so the variable in the required equation must satisfy $x \neq 1$. Multiplying by a denominator requires checking this restriction.

Step 1: Obtain the two inverse functions For $y = (t-2)/(t-3)$, multiply by $t-3$ and collect the terms involving t:

$$yt - 3y = t - 2, \qquad (y-1)t = 3y - 2.$$

Thus $f^{-1}(x) = (3x-2)/(x-1)$, where $x \neq 1$. Similarly, solving $y = 2t - 3$ gives $g^{-1}(x) = (x+3)/2$. These are inverse functions, rather than reciprocals or negative powers of their outputs.

Step 2: Solve the equation with its domain restriction Substitute the inverse expressions in the stated equation and multiply by $2(x-1)$:

$$\frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2},$$

$$2(3x-2) + (x+3)(x-1) = 13(x-1).$$

Expanding and collecting terms gives

$$x^2 - 5x + 6 = 0 = (x - 2)(x - 3).$$

Both roots 2 and 3 satisfy $x \neq 1$. At these values the inverse-function sums are respectively $4 + 5/2$ and $7/2 + 3$, both $13/2$. The requested sum of roots is five, not either root individually. Thus the alternatives three and two select individual roots, while seven does not equal the root sum.

Step 3: Conclusion The valid values of x are 2 and 3, whose sum is 5. Hence, the correct option is (B).

Quick Tip to Solve: After forming the inverse equation, retain x not equal to 1; the coefficient of x in the resulting quadratic gives the required sum immediately.

14. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to : [4 marks]

(A) 7000

(B) 1000

(C) 3000

(D) 5000

Correct Answer: (C) 3000

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, the sum of its first m terms is $T_m = \frac{m}{2}[2a + (m - 1)d]$.
- The difference $S_2 - S_1$ is the sum of the terms in positions $2n + 1$ through $4n$. Comparing sum formulae can eliminate both unknown progression parameters.

Step 1: Express the given difference using the progression parameters Apply the arithmetic-progression sum formula at $m = 2n$ and $m = 4n$, respectively:

$$S_1 = n[2a + (2n - 1)d],$$

$$S_2 = 2n[2a + (4n - 1)d].$$

Their difference is

$$\begin{aligned} S_2 - S_1 &= n\{4a + (8n - 2)d - 2a - (2n - 1)d\} \\ &= n[2a + (6n - 1)d] = 1000. \end{aligned}$$

The stated thousand is therefore the sum of a middle block of $2n$ terms, not the sum of all the first $4n$ terms.

Step 2: Relate the required sum to that difference The first $6n$ terms have sum

$$T_{6n} = \frac{6n}{2}[2a + (6n-1)d] = 3n[2a + (6n-1)d].$$

The bracket and the factor n are exactly those found in the given difference. Consequently,

$$T_{6n} = 3(S_2 - S_1) = 3(1000) = 3000.$$

No separate values of a, d or n are needed. The other alternatives 7000, 1000 and 5000 do not satisfy this identity; in particular, 1000 is only the given middle-block sum, whereas the required sum covers three times as many equally centered terms.

Step 3: Conclusion The sum of the first $6n$ terms is 3000. Hence, the correct option is (C).

Quick Tip to Solve: Compare S(6n) directly with S(4n)-S(2n): their central term average is identical, and their numbers of terms are in the ratio 3 to 1.

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $0 \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to : [4 marks]

(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{8}$
(D) $\frac{\pi}{4}$

Correct Answer: (A) $\frac{\pi}{6}$

Solution:

Concept:

- The tangent to $x^2/a^2 + y^2/b^2 = 1$ at $(a\cos\theta, b\sin\theta)$ is $x\cos\theta/a + y\sin\theta/b = 1$.
- For positive intercepts, minimise their sum as a differentiable function on $0 < \theta < \pi/2$. A sign change of the derivative from negative to positive establishes the minimum.

Step 1: Find both intercepts Here $a = 3\sqrt{3}$ and $b = 1$. The tangent is therefore

$$\frac{x\cos\theta}{3\sqrt{3}} + y\sin\theta = 1.$$

Setting $y = 0$ gives the horizontal intercept $3\sqrt{3}\sec\theta$; setting $x = 0$ gives the vertical intercept $\csc\theta$. Both are positive in the stated interval, so there is no need for absolute values.

Step 2: Differentiate the sum Write $S = 3\sqrt{3}\sec\theta + \csc\theta$. Then

$$S' = \frac{3\sqrt{3}\sin\theta}{\cos^2\theta} - \frac{\cos\theta}{\sin^2\theta}.$$

Multiplying by the positive number $\sin^2\theta\cos^2\theta$ shows that the sign of S' is the sign of $3\sqrt{3}\sin^3\theta -$

$\cos^3\theta$. Hence the stationary condition gives

$$\tan^3\theta = \frac{1}{3\sqrt{3}}, \qquad \tan\theta = \frac{1}{\sqrt{3}}.$$

Step 3: Confirm the minimum and compare options The tangent function increases strictly throughout this interval. Thus the derivative is negative before $\theta = \pi/6$ and positive afterward. Also, the sum diverges at both endpoints, confirming a global minimum. The other listed angles $\pi/3, \pi/8, \pi/4$ do not satisfy the stationary condition, so none can give the minimum.

Step 4: Conclusion The required angle is $\boxed{\theta = \pi/6}$. Hence, the correct option is (A).

Quick Tip to Solve: For a tangent in parametric form, obtain the intercept sum first; its derivative reduces to a cube of $\tan\theta$.

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16. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points : [4 marks]

(A) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

(B) (1, $\pm$ 2)

(C) $\left(2, \pm\frac{3}{2}\right)$

(D) (0, $\pm\sqrt{3}$)

Correct Answer: (C) $\left(2, \pm\frac{3}{2}\right)$

Solution:

Concept:

- Two circles touching internally have centre distance equal to the difference of their radii; externally touching circles have centre distance equal to their sum.
- The locus of points with a fixed sum of distances from two fixed foci is an ellipse, with distance sum $2a$ and $b^2 = a^2 - c^2$.

Step 1: Translate the two tangency conditions Let the variable circle have centre $C = (x, y)$ and radius $r > 0$. The given circles have centres $O = (0, 0)$, $A = (2, 0)$ and radii 3 and 1. Since the variable circle lies internally tangent to the first and externally tangent to the second,

$$CO = 3 - r, \qquad CA = 1 + r.$$

Adding eliminates the unknown radius and gives $CO + CA = 4$.

Step 2: Write the ellipse equation The foci are separated by 2, so the midpoint is $(1, 0)$, $c = 1$ and $a = 2$. Therefore $b^2 = 4 - 1 = 3$, and the locus is

$$\frac{(x-1)^2}{4} + \frac{y^2}{3} = 1.$$

This relation also allows recovery of $r = 3 - CO$, so the selected points must correspond to a positive radius.

Step 3: Test the listed pairs At $(2, \pm 3/2)$, the left side is $1/4 + (9/4)/3 = 1$, so both points lie on the ellipse. Their distance from O is $5/2$, giving $r = 1/2 > 0$; their distance from A is $3/2 = 1 + r$, verifying both contacts. The corresponding left sides for options (A), (B), and (D) are respectively $23/48$, $4/3$, and $5/4$, none equal to 1.

Step 4: Conclusion The locus passes through $\boxed{(2, \pm 3/2)}$. Hence, the correct option is (C).

Quick Tip to Solve: Add the internal and external centre-distance equations: the variable radius cancels and reveals an ellipse.

17. Consider a hyperbola H : $x^2 - 2y^2 = 4$. Let the tangent at a point P(4, $\sqrt{6}$) meet the x-axis at Q and latus rectum at R(x_1, y_1), $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to : [4 marks]

(A) $\sqrt{6} - 1$

(B) $\frac{7}{\sqrt{6}} - 2$

(C) $4\sqrt{6} - 1$

(D) $4\sqrt{6}$

Correct Answer: (B) $\frac{7}{\sqrt{6}} - 2$

Solution:

Concept:

- For $x^2/a^2 - y^2/b^2 = 1$, the foci are $(\pm c, 0)$ with $c^2 = a^2 + b^2$. Its latus recta lie on $x = \pm c$.
- The tangent at (x_0, y_0) is $xx_0/a^2 - yy_0/b^2 = 1$. A triangle with its base on the horizontal axis has area one-half base times vertical height.

Step 1: Find the focus and tangent Write the equation as $x^2/4 - y^2/2 = 1$, so $a^2 = 4$, $b^2 = 2$ and $c = \sqrt{6}$. Since $P = (4, \sqrt{6})$ has positive horizontal coordinate, the nearer focus is $F = (\sqrt{6}, 0)$. The tangent at P is

$$\frac{4x}{4} - \frac{\sqrt{6}y}{2} = 1, \qquad 2x - \sqrt{6}y = 2.$$

Notice that P indeed satisfies the hyperbola, since $16 - 12 = 4$.

Step 2: Find the two intersections Putting $y = 0$ into the tangent gives $Q = (1, 0)$. The specified positive latus rectum has $x = \sqrt{6}$. Its intersection with the tangent is consequently

$$R = \left(\sqrt{6}, \frac{2\sqrt{6} - 2}{\sqrt{6}}\right) = \left(\sqrt{6}, 2 - \frac{2}{\sqrt{6}}\right).$$

The height is positive; the negative-focus latus rectum is excluded by $x_1 > 0$.

Step 3: Calculate and compare the area The base QF has length $\sqrt{6} - 1$, while FR is vertical and

has length $2 - 2/\sqrt{6}$. Thus

$$[QFR] = \frac{1}{2}(\sqrt{6}-1)\left(2 - \frac{2}{\sqrt{6}}\right)$$
$$= \frac{(\sqrt{6}-1)^2}{\sqrt{6}} = \frac{7}{\sqrt{6}} - 2.$$

Numerically this is about 0.858, whereas the other options are about 1.449, 8.798 and 9.798; they cannot equal this area.

Step 4: Conclusion The area is $\boxed{7/\sqrt{6} - 2}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: The focus and its latus-rectum intersection share an x-coordinate, making the triangle right-angled and avoiding a determinant.

18. If P and Q are two statements, then which of the following compound statement is a tautology?

[4 marks]

(A) ((P $\Rightarrow$ Q) $\wedge \sim$ Q) $\Rightarrow$ (P $\wedge$ Q)

(B) ((P $\Rightarrow$ Q) $\wedge \sim$ Q) $\Rightarrow$ Q

(C) ((P $\Rightarrow$ Q) $\wedge \sim$ Q) $\Rightarrow$ P

(D) ((P $\Rightarrow$ Q) $\wedge \sim$ Q) $\Rightarrow \sim$ P

Correct Answer: (D) ((P $\Rightarrow$ Q) $\wedge \sim$ Q) $\Rightarrow \sim$ P

Solution:

Concept:

- A tautology is true for every assignment of truth values to its elementary statements. An implication is false only when its antecedent is true and its consequent is false.
- The identity $P \Rightarrow Q \equiv \neg P \vee Q$, together with distributivity, can simplify a compound antecedent before evaluating the complete statement.

Step 1: Simplify the common antecedent All four choices have the same antecedent $A = (P \Rightarrow Q) \wedge \neg Q$. Using the implication identity,

$$A = (\neg P \vee Q) \wedge \neg Q$$
$$= (\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)$$
$$= \neg P \wedge \neg Q.$$

Thus this antecedent is true precisely in the case P false and Q false. In every other case, all four implications are automatically true because their antecedent is false.

Step 2: Check the only decisive assignment For $P = \mathrm{F}$ and $Q = \mathrm{F}$, the antecedent is true. The consequents in (A), (B) and (C) are $P \wedge Q$, Q and P; each is false for this assignment. Hence each of these three complete implications is false at least once, which excludes all of them from being tautologies.

Step 3: Establish the remaining implication The consequent of option (D) is $\neg P$. Whenever the

antecedent $\neg P \wedge \neg Q$ is true, this consequent must also be true. Whenever the antecedent is false, the implication remains true by its truth-table definition. Therefore option (D) is true in all four possible assignments. This is the familiar valid inference called modus tollens: from an implication and the denial of its consequent, infer the denial of its antecedent.

Step 4: Conclusion $\boxed{((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P}$ is a tautology. Hence, the correct option is (D).

Quick Tip to Solve: For options with the same antecedent, examine only assignments making that antecedent true; all other rows are automatic.

19. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to : [4 marks]

(A) $\frac{40}{243}$
(B) $\frac{128}{625}$
(C) $\frac{80}{243}$
(D) $\frac{32}{625}$

Correct Answer: (D) $\frac{32}{625}$

Solution:

Concept:

- If $X \sim \text{Bin}(n, p)$, then $\Pr(X = r) = \binom{n}{r} p^r q^{n-r}$, where $q = 1 - p$ is the failure probability.
- The ratio of successive binomial probabilities cancels large common powers: $\Pr(X = r+1)/\Pr(X = r) = (n - r)p/((r + 1)q)$.

Step 1: Use the supplied probability ratio For five independent trials, the two given equations are

$$5pq^4 = 0.4096, \qquad 10p^2q^3 = 0.2048.$$

Both probabilities are nonzero, so division by pq^4 is legitimate. Dividing the second equation by the first gives

$$\frac{2p}{q} = \frac{0.2048}{0.4096} = \frac{1}{2}, \qquad \frac{p}{q} = \frac{1}{4}.$$

This step determines the success odds without taking any high-order roots.

Step 2: Recover the trial probabilities Using $p + q = 1$, put $p = q/4$. Then $5q/4 = 1$, giving $q = 4/5$ and $p = 1/5$. As a consistency check,

$$5\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4 = \frac{256}{625} = 0.4096,$$

exactly the given probability of one success. The probability of two successes is $128/625 = 0.2048$, confirming the same values.

Step 3: Evaluate three successes and exclude distractors There are $\binom{5}{3} = 10$ positions for the three

successes. Hence

$$\Pr(X=3) = 10\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2 = \frac{160}{3125} = \frac{32}{625}.$$

Equivalently, the next-probability ratio is $\Pr(X=3)/\Pr(X=2) = p/q = 1/4$. Option (B) repeats the two-success probability instead, while (A) and (C) do not have the value obtained from the uniquely determined success probability.

Step 4: Conclusion The probability of exactly three successes is $\boxed{32/625 = 0.0512}$. Hence, the correct option is (D).

Quick Tip to Solve: Divide adjacent binomial probabilities first; the success odds emerge directly, with most powers cancelling.

20. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to : [4 marks]

(A) $\frac{1}{\sqrt{3}}$

(B) $2\sqrt{3}$

(C) $\sqrt{3}$

(D) $\frac{2\sqrt{3}}{3}$

Correct Answer: (B) $2\sqrt{3}$

Solution:

Concept:

- For a vertical pole of height h and a point at horizontal distance d from its foot, $\tan\alpha = h/d$, where α is the elevation angle.
- The unique point equidistant from the three non-collinear vertices of a triangle is its circumcentre. That common distance is the circumradius.

Step 1: Relate height to the three ground distances Let O denote the foot of the pole and T its top, with $OT = h$. Because the pole stands vertically, the triangles AOT, BOT, and COT are right-angled at O. At each of A, B, C, the angle of elevation is $\pi/3$, so

$$\frac{h}{OA} = \frac{h}{OB} = \frac{h}{OC} = \tan\frac{\pi}{3} = \sqrt{3}.$$

The same positive height appears in every fraction, and therefore $OA = OB = OC$.

Step 2: Identify the common horizontal distance The vertices form a triangle, hence they are non-collinear. Their equidistant point O is consequently the circumcentre of $\triangle ABC$. The given circumradius is 2, so $OA = OB = OC = 2$. The additional statement that the pole lies inside the park is consistent with this circumcentre lying inside the triangle; it does not change the radius or the trigonometric relation.

Step 3: Substitute and compare the options Using any one of the right triangles,

$$h = OA\tan\frac{\pi}{3} = 2\sqrt{3}.$$

Dividing the radius by $\sqrt{3}$, instead of multiplying it, would give option (D), but would reverse the opposite-to-adjacent ratio. Option (C) omits the radius factor 2; option (A) is neither the required product nor a consistent use of the given distance. The calculated height has the same length unit as the circumradius.

Step 4: Conclusion The pole has height $\boxed{2\sqrt{3}}$ length units. Hence, the correct option is (B).

Quick Tip to Solve: Equal elevation angles to a fixed top mean equal ground distances from the foot; immediately identify the circumcentre.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. **If** $\sum_{r=1}^{10} r!(r^3+6r^2+2r+5) = \alpha(11!)$**, then the value of** α **is equal to _____.** [4 marks]

Solution:

Concept:

- A sum containing $r!$ and a polynomial in r often telescopes after expressing its summand as $(r+1)!g(r) - r!g(r-1)$.
- Choose the degree of g one below that of the polynomial: multiplication by $r+1$ raises its degree by one, allowing coefficient comparison.

Step 1: Find a suitable quadratic We seek a quadratic $g(r) = r^2 + ar + b$ for which

$$(r+1)g(r) - g(r-1) = r^3 + 6r^2 + 2r + 5.$$

Expanding the left side carefully gives

$$r^3 + ar^2 + (b+2)r + (a-1).$$

Comparison of the coefficients yields $a = 6$, $b = 0$; the constant is then $a - 1 = 5$, agreeing with the required expression. Thus $g(r) = r^2 + 6r$.

Step 2: Rewrite each factorial summand Using $(r+1)! = (r+1)r!$, the identity becomes

$$r!(r^3+6r^2+2r+5) = (r+1)!(r^2+6r) - r!\big((r-1)^2+6(r-1)\big).$$

For example, when $r = 1$, the right side is $2!\cdot 7 - 1!\cdot 0 = 14$, matching the original first term. This check also establishes the lower boundary contribution explicitly.

Step 3: Cancel the interior terms and evaluate the boundary Summing from $r = 1$ through 10, the

negative term at each index cancels the positive term at the preceding index. Only the last positive and first negative terms remain:

$$\sum_{r=1}^{10} r!(r^3+6r^2+2r+5) = 11!(10^2+6\cdot 10) - 1!(0^2+6\cdot 0)$$
$$= 160\,(11!).$$

Comparing this with the given definition $\alpha(11!)$, and dividing by the nonzero number $11!$, determines α exactly.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{\alpha = 160}$.

Quick Tip to Solve: For factorial times a cubic, try a difference of consecutive factorials multiplied by a quadratic; only boundary terms survive.

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22. If f(x) and g(x) are two polynomials such that the polynomial P(x) = f(x^3) + x g(x^3) is divisible by x^2 + x + 1, then P(1) is equal to _____. [4 marks]

Solution:

Concept:

- A polynomial divisible by x^2+x+1 has zero remainder when evaluated at either primitive cube root of unity. Such a root ω satisfies $\omega^3 = 1$ and $\omega \neq 1$.
- Polynomial division can also be performed by reducing powers modulo x^2+x+1. Since $x^3-1 = (x-1)(x^2+x+1)$, every polynomial in x^3 reduces to its value at 1.

Step 1: Reduce the expression modulo the divisor Write $f(t) = \sum a_k t^k$ and $g(t) = \sum b_k t^k$. Because $x^3 \equiv 1 \pmod{x^2+x+1}$, every term x^{3k} reduces to 1. Therefore

$$P(x) = f(x^3) + xg(x^3) \equiv f(1) + xg(1) \pmod{x^2+x+1}.$$

The expression on the right is already of degree at most one, so it is exactly the remainder, rather than merely another unreduced expression.

Step 2: Use divisibility to set both coefficients to zero The given divisibility says the remainder is the zero polynomial. A nonzero linear polynomial cannot be divisible by a quadratic. Hence the constant coefficient and the coefficient of x both vanish:

$$f(1) = 0, \qquad g(1) = 0.$$

Equivalently, substituting the distinct roots ω, ω^2 gives two linear equations, whose difference forces $g(1) = 0$.

Step 3: Evaluate the requested polynomial value Only after establishing the two separate zeros do we substitute $x = 1$ into the original formula:

$$P(1) = f(1^3) + 1\cdot g(1^3) = f(1) + g(1) = 0 + 0 = 0.$$

One must not conclude $P(1) = 0$ merely from divisibility: the divisor itself equals 3 at 1. It is the special arrangement of powers in P that makes the result follow.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{0}$.

Quick Tip to Solve: Reduce x^3 to 1 modulo $x^2 + x + 1$; the remaining linear polynomial must vanish identically.

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23. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right]^{10}$, x ≠ 1, is equal to _____. [4 marks]

Solution:

Concept:

- Before expanding, factor differences and sums of powers. Here $a^3 + 1 = (a + 1)(a^2 - a + 1)$, while $x - 1 = (\sqrt{x} - 1)(\sqrt{x} + 1)$.
- In a binomial expansion, the term independent of x is the term whose total exponent is zero. Its coefficient includes the binomial coefficient and the sign of the selected term.

Step 1: Simplify both fractions first With $a = x^{1/3}$, the numerator $x + 1 = a^3 + 1$ factors to give

$$\frac{x+1}{x^{2/3} - x^{1/3} + 1} = x^{1/3} + 1.$$

For the second fraction, factor $x - \sqrt{x} = \sqrt{x}(\sqrt{x} - 1)$. On the original real domain $x > 0$, $x \neq 1$, cancellation is legitimate, and

$$\frac{x-1}{x-\sqrt{x}} = \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)} = 1 + x^{-1/2}.$$

Thus the bracket is $x^{1/3} - x^{-1/2}$; the two constant terms cancel.

Step 2: Write the general term with its exponent Choose the negative term $-x^{-1/2}$ exactly r times from the ten factors. The corresponding term is

$$T_{r+1} = \binom{10}{r}(-1)^r x^{(10-r)/3 - r/2}, \qquad 0 \le r \le 10.$$

The exponent is $(20 - 5r)/6$. This decreases strictly with r, so at most one term has zero exponent; there is no possibility of several different constant terms needing to be added.

Step 3: Set the exponent to zero and compute the coefficient For independence of x,

$$\frac{10-r}{3} - \frac{r}{2} = 0 \quad \Longrightarrow \quad 20 - 5r = 0 \quad \Longrightarrow \quad r = 4.$$

This integer lies in the permitted range. Its sign is positive because $(-1)^4 = 1$, and

$$T_5 = \binom{10}{4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210.$$

Step 4: Conclusion The term independent of x is the dimensionless constant $\boxed{210}$.

Quick Tip to Solve: Cancel the power factorizations before applying the binomial theorem; then solve only the exponent equation.

24. Let y = y(x) be the solution of the differential equation xdy − ydx = $\sqrt{(x^2-y^2)}dx$, x ≥ 1, with y(1) = 0. If the area bounded by the line x = 1, x = e^{π}, y = 0 and y = y(x) is $\alpha e^{2\pi} + \beta$, then the value of 10(α + β) is equal to _____. [4 marks]

Solution:

Concept:

- For a homogeneous equation use $v = y/x$ and $t = \log x$, with $x \geq 1$. The equation then becomes $dv/dt = \sqrt{1-v^2}$, where the square root is nonnegative and a real solution must satisfy $-1 \leq v \leq 1$.
- The nonnegative square root forces v to be nondecreasing. Thus a separated sine solution is valid only while its derivative is nonnegative; extending it beyond that interval can solve the squared equation but not the printed equation.

Step 1: Transform the differential equation and apply the initial value With $y = xv$, we have $xy\prime - y = x^2v\prime$. Since $x > 0$, the printed equation gives

$$x\frac{dv}{dx} = \sqrt{1-v^2}, \qquad \frac{dv}{dt} = \sqrt{1-v^2}, \qquad v(0) = 0.$$

Separation before the first boundary value gives $\arcsin v = t$, so $v = \sin t$ for $0 \leq t \leq \pi/2$. At $t = \pi/2$, v reaches 1. It cannot subsequently decrease because its derivative is nonnegative, and it cannot exceed 1 while remaining real. Hence $v = 1$ for $\pi/2 \leq t \leq \pi$.

Step 2: Integrate the area of the literal real solution The unique continuation is

$$y(x) = \begin{cases} x\sin(\log x), & 1 \leq x \leq e^{\pi/2}, \\ x, & e^{\pi/2} \leq x \leq e^{\pi}. \end{cases}$$

It is continuously differentiable at the joining point: both formulas give $y = x$ and slope 1 there. It is nonnegative, so the geometric area is

$$\begin{aligned} A &= \int_1^{e^{\pi/2}} x\sin(\log x)\,dx + \int_{e^{\pi/2}}^{e^{\pi}} x\,dx \\ &= \left[\frac{x^2}{5}\{2\sin(\log x) - \cos(\log x)\}\right]_1^{e^{\pi/2}} + \frac{e^{2\pi} - e^{\pi}}{2} \\ &= \frac{1}{2}e^{2\pi} - \frac{1}{10}e^{\pi} + \frac{1}{5}. \end{aligned}$$

In particular there is an additional e^{π} contribution compared with the two-term form suggested in the question.

Step 3: Explain the published numerical key and the branch defect The stated key 4 is obtained if

one formally continues $y = x\sin(\log x)$ all the way to e^{π}. Its area would be

$$A_{\text{formal}} = \frac{e^{2\pi}+1}{5}, \qquad 10(\alpha+\beta) = 4.$$

However, for $\pi/2 < \log x < \pi$, the left side of the original equation is $x\cos(\log x) < 0$, while its right side is $x|\cos(\log x)| > 0$. Thus that extension is not a solution of the printed equation. If the literal area is written with $\alpha = 1/2$ and $\beta = 1/5 - e^{\pi}/10$, the expression becomes $7 - e^{\pi}$, not 4. Without restrictions on the constants, a representation of one number as $\alpha e^{2\pi} + \beta$ is itself not unique.

Step 4: Conclusion The published numerical key is $\boxed{4}$, obtained from the formal sine continuation. The equation with the printed nonnegative square root instead gives area $\boxed{\frac{1}{2}e^{2\pi} - \frac{1}{10}e^{\pi} + \frac{1}{5}}$; therefore 4 is not justified by a real solution on the entire stated interval. The literal wording has a square-root branch defect.

Quick Tip to Solve: After separating a square-root differential equation, substitute the candidate back into the original equation and check signs on the entire interval.

---

25. Let I be an identity matrix of order 2 × 2 and P = $\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of n∈N for which P^n = 5I − 8P is equal to _____. [4 marks]

Solution:

Concept:

- The Cayley-Hamilton theorem states that a square matrix satisfies its characteristic polynomial. For a 2×2 matrix, this polynomial is $\lambda^2 - (\operatorname{tr} P)\lambda + \det P$.
- Once a quadratic relation expresses P^2 in terms of I and P, all higher powers can be reduced to the form $aI + bP$. Eigenvalues provide a useful uniqueness check for the exponent.

Step 1: Find the quadratic relation satisfied by the matrix The trace and determinant of the given matrix are

$$\operatorname{tr} P = 2 - 3 = -1, \qquad \det P = 2(-3) - (-1)(5) = -1.$$

Its characteristic polynomial is therefore $\lambda^2 + \lambda - 1$. Applying Cayley-Hamilton gives

$$P^2 + P - I = 0, \qquad P^2 = I - P.$$

For a direct check, multiplication gives $P^2 = \begin{pmatrix} -1 & 1 \\ -5 & 4 \end{pmatrix}$, which is indeed $I - P$.

Step 2: Reduce successive powers without multiplying full matrices Multiply by P and substitute

$P^2 = I - P$ at each stage:

$$P^3 = P - P^2 = 2P - I,$$
$$P^4 = 2P^2 - P = 2I - 3P,$$
$$P^5 = 2P - 3P^2 = 5P - 3I,$$
$$P^6 = 5P^2 - 3P = 5I - 8P.$$

The final expression agrees exactly with the specified right-hand side, including both coefficients and the negative sign before $8P$. Thus $n = 6$ is a valid natural-number exponent.

Step 3: Check that no other natural exponent works One eigenvalue is $\lambda = (-1 + \sqrt{5})/2$, which lies strictly between 0 and 1. Since the target matrix equals P^6, any other solution would imply $\lambda^n = \lambda^6$. Positive powers of a fixed number strictly between 0 and 1 are distinct, so this forces $n = 6$. This also prevents an accidental match arising from a periodic matrix power.

Step 4: Conclusion The required natural number is $\boxed{n = 6}$.

Quick Tip to Solve: Use $P^2 = I - P$ repeatedly; tracking two scalar coefficients is faster and safer than repeated full matrix multiplication.

---

26. Let f : R $\to$ R satisfy the equation f(x + y) = f(x) . f(y) for all x, y $\in$R and f(x) $\neq$ 0 for any x$\in$R. If the function f is differentiable at x = 0 and f'(0) = 3, then $\lim_{h\to 0} \frac{1}{h}(f(h) - 1)$ is equal to _____. [4 marks]

Solution:

Concept:

- For a differentiable function, $f\prime(0) = \lim_{h\to 0}[f(h) - f(0)]/h$. The requested limit becomes this derivative once $f(0)$ is established.
- The multiplicative identity $f(x + y) = f(x)f(y)$, together with nonvanishing of f, determines $f(0) = 1$. Nonvanishing is essential when dividing the identity by a function value.

Step 1: Determine the value of the function at the origin Set $x = y = 0$ in the given equation. It gives

$$f(0) = f(0)^2 \quad \Longrightarrow \quad f(0)\big(f(0) - 1\big) = 0.$$

The premise says $f(x) \neq 0$ for every real x, in particular at zero. Therefore the zero possibility is excluded and

$$f(0) = 1.$$

The same conclusion follows by setting $y = 0$, giving $f(x) = f(x)f(0)$, and cancelling the nonzero value $f(x)$.

Step 2: Identify the exact derivative quotient Now replace 1 in the requested numerator by $f(0)$:

$$\lim_{h\to 0} \frac{f(h) - 1}{h} = \lim_{h\to 0} \frac{f(0 + h) - f(0)}{h}.$$

This is precisely the definition of the derivative at zero. The differentiability hypothesis ensures that

the two-sided limit exists, so no continuity or exponential representation has to be assumed separately.

Step 3: Use the supplied derivative and check consistency The problem directly specifies $f\prime(0) = 3$, hence the quotient tends to 3. For an additional consistency check, differentiability at zero allows

$$f(h) = f(0) + f\prime(0)h + o(h) = 1 + 3h + o(h).$$

Consequently $(f(h) - 1)/h = 3 + o(h)/h \to 3$. Finding $f(x)$ for every real input would be unnecessary work: the requested quantity is local to zero.

Step 4: Conclusion The required dimensionless limit is $\boxed{3}$.

Quick Tip to Solve: In a multiplicative functional equation, find $f(0)$ first; the target quotient may already be the given derivative.

---

27. Let P(x) be a real polynomial of degree 3 which vanishes at x = −3. Let P(x) have local minima at x = 1, local maxima at x = −1 and $\int_{-1}^{1} P(x)dx$ = 18, then the sum of all the coefficients of the polynomial P(x) is equal to _____. [4 marks]

Solution:

Concept:

- A differentiable polynomial has zero first derivative at an interior local maximum or minimum. A cubic has a quadratic derivative, so its two distinct stationary points determine the derivative up to a nonzero factor.
- The sum of all coefficients of a polynomial equals its value at $x = 1$. On a symmetric interval, the integral of an odd polynomial vanishes, leaving only the constant term here.

Step 1: Reconstruct the cubic from the stationary points The stationary points at -1 and 1 imply

$$P\prime(x) = k(x + 1)(x - 1) = k(x^2 - 1), \qquad k \neq 0.$$

Integrating with respect to x,

$$P(x) = k\left(\frac{x^3}{3} - x\right) + c.$$

The minimum at 1 and maximum at -1 require $k > 0$: indeed $P\prime\prime(x) = 2kx$, which is positive at 1 and negative at -1.

Step 2: Use the integral to find the constant term The nonconstant expression $x^3/3 - x$ is odd. Therefore its integral over $[-1, 1]$ is zero, and

$$18 = \int_{-1}^{1} P(x)\,dx = \int_{-1}^{1} c\,dx = 2c.$$

Thus $c = 9$. This is not a claim that all cubic terms vanish on every interval; cancellation holds here because the endpoints are symmetric about zero.

Step 3: Use the root and evaluate the coefficient sum The polynomial vanishes at -3. Substitution

gives

$$0 = k\left(\frac{-27}{3} + 3\right) + 9 = -6k + 9, \qquad k = \frac{3}{2}.$$

Hence $P(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 9$. Its coefficient sum is

$$P(1) = \frac{1}{2} - \frac{3}{2} + 9 = 8.$$

Finally $P''(1) = 3 > 0$ and $P''(-1) = -3 < 0$, confirming that the reconstructed polynomial has the extrema in the stated order.

Step 4: Conclusion The sum of all coefficients is the dimensionless value $\boxed{8}$.

Quick Tip to Solve: Recover the derivative from its roots, use symmetry for the integral, and evaluate $P(1)$ instead of listing coefficients.

---

28. Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-1}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to _____. [4 marks]

Solution:

Concept:

- A plane containing a line contains its direction vector. If the plane is parallel to another line, that second direction vector is also perpendicular to the plane normal.
- The cross product of two nonparallel vectors lying in a plane gives a normal vector. The plane through a known point $\mathbf{r}_0$ is then $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$.

Step 1: Extract the two directions and a point in the plane From the first symmetric line equation, take the point $A = (1, -6, -5)$ and direction $\mathbf{a} = (3, 4, 2)$. The second line has direction $\mathbf{b} = (4, -3, 7)$. Its point $(1, 2, -5)$ need not belong to the plane, because only parallelism to that line is prescribed. Thus use A to locate the plane and both direction vectors to fix its orientation.

Step 2: Calculate the plane normal and its equation The directions are not scalar multiples. Their cross product is

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} = (4 \cdot 7 - 2(-3),\ 2 \cdot 4 - 3 \cdot 7,\ 3(-3) - 4 \cdot 4) = (34, -13, -25).$$

It satisfies $\mathbf{n} \cdot \mathbf{a} = 102 - 52 - 50 = 0$ and $\mathbf{n} \cdot \mathbf{b} = 136 + 39 - 175 = 0$, checking both requirements. The plane through A is therefore

$$34(x - 1) - 13(y + 6) - 25(z + 5) = 0.$$

Step 3: Substitute the given point and compute the absolute value The point $(1, -1, \alpha)$ belongs to this plane, so

$$34(1 - 1) - 13(-1 + 6) - 25(\alpha + 5) = 0.$$

Simplifying, $-65 - 25\alpha - 125 = 0$, hence

$$25\alpha = -190, \qquad \alpha = -\frac{38}{5}.$$

The question asks for the magnitude of 5α, not α itself:

$$|5\alpha| = |-38| = 38.$$

The negative sign in α is therefore necessary even though the requested final numerical value is positive.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{38}$.

Quick Tip to Solve: Use the two line directions for a cross product, but take the plane point only from the line it actually contains.

---

29. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$. If $\sum_{k=0}^{10} (2^2 + 3k)^{10} C_k = \alpha.3^{10} + \beta.2^{10}, \alpha, \beta \in R$, then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- The binomial theorem gives $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. Differentiating $(1 + x)^n$ and then multiplying by x evaluates sums weighted by k.
- The printed first term is $2^2 = 4$, independent of k. The summation therefore contains no 2^k factor.

Step 1: Separate the two sums Write the sum as $S = 4\sum_{k=0}^{10} \binom{10}{k} + 3\sum_{k=0}^{10} k\binom{10}{k}$. The first sum is 2^{10}. For the second, differentiate the binomial expansion to obtain $\sum k\binom{10}{k}x^{k-1} = 10(1 + x)^9$. At $x = 1$, its value is $10 \cdot 2^9$.

Step 2: Evaluate and interpret the coefficients Consequently,

$$S = 4 \cdot 2^{10} + 30 \cdot 2^9 = 19 \cdot 2^{10}.$$

The natural coefficient comparison used for the intended numerical answer is $\alpha = 0, \beta = 19$. Strictly, because α, β are only required to be real and the bases are fixed numbers, the displayed equation alone does not determine them uniquely. For any real t, $\alpha = t, \beta = 19 - t(3/2)^{10}$ also satisfies it. This qualification does not change the evaluated sum.

Step 3: Conclusion The conventional coefficient reading gives $\alpha + \beta = \boxed{19}$, the stated numerical key. Literally, unrestricted real α, β do not give a unique sum without that coefficient convention.

Quick Tip to Solve: First check whether the exponent contains the summation index: a fixed 2^2 must be taken outside the sum.

30. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)-b=0$ be (−3, 5, 2). Then, the value of | a + b | is equal to _____. [4 marks]

Solution:

Concept:

- The segment joining a point to its mirror image is parallel to the plane normal. Its midpoint lies on the reflecting plane.
- For $2x-y+z-b=0$, a normal vector is $(2,-1,1)$. These two conditions determine a and b independently.

Step 1: Use the normal direction Let $P=(1,3,a)$ and $P\prime=(-3,5,2)$. Their displacement is

$$P\prime-P=(-4,2,2-a).$$

Since the first two components are $-2(2,-1)$, the third must also be -2. Thus $2-a=-2$, so $a=4$. This uses direction ratios, rather than setting the original point on the plane.

Step 2: Place the midpoint on the plane The midpoint is $M=(-1,4,3)$. Substitution in the plane gives

$$2(-1)-4+3-b=0 \quad\Longrightarrow\quad b=-3.$$

As a check, the signed plane expressions at the endpoints are 6 and -6, and the displacement is normal to the plane. Thus the midpoint condition and equal opposite distances both hold.

Step 3: Conclusion Therefore $a=4$, $b=-3$, and the required dimensionless value is $\boxed{|a+b|=1}$.

Quick Tip to Solve: For a reflected pair, first compare the joining vector with the plane normal; then use the midpoint to find the plane constant.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field $\left(\vec{E}\right)$ and magnetic field $\left(\vec{B}\right)$ components. [4 marks]

(A) E_x, B_z or E_z, B_x

(B) E_x, B_y or E_y, B_x

(C) E_y, B_y or E_z, B_z

(D) E_y, B_x or E_x, B_y

Correct Answer: (A) E_x, B_z or E_z, B_x

Solution:

Concept:

- In a plane electromagnetic wave in free space, both $\vec{E}$ and $\vec{B}$ are perpendicular to the propagation direction and to one another.
- The direction of energy transport is given by $\vec{E}\times\vec{B}$. Signs of the individual components select the required sense along the propagation axis.

Step 1: Remove longitudinal components Propagation is along the y-axis. Hence $E_y = 0$ and $B_y = 0$: neither field can have a component along the direction of travel. Options B and D each include a y-component, so neither listed pair is admissible. Option C also contains longitudinal components, and its alternative E_z, B_z has parallel fields.

Step 2: Choose perpendicular transverse axes The remaining transverse plane is the xz-plane. Taking E_x with B_z, or E_z with B_x, makes the fields perpendicular and puts their cross product along the y-axis. For travel specifically in positive y, the relative signs are chosen accordingly: for example $\vec{E} = E_0\hat{z}$, $\vec{B} = B_0\hat{x}$ gives positive y. The option specifies component axes, not fixed positive signs.

Step 3: Conclusion The allowed component pairs are E_x, B_z or E_z, B_x. Hence, the correct option is (A).

Quick Tip to Solve: Cross out any option containing a field component along the propagation axis before applying the right-hand rule.

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32. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately : [Take g = 10 ms^{-2}, the radius of earth, R = 6400 $\times$ 10^3 m, Take π = 3.14] [4 marks]

(A) 84 minutes

(B) 1200 minutes

(C) 60 minutes

(D) does not change

Correct Answer: (A) 84 minutes

Solution:

Concept:

- At the equator, circular motion requires inward acceleration $\omega^2 R$. A body floats relative to the surface when its normal reaction becomes zero.
- At this limiting spin, gravity alone supplies the required centripetal acceleration: $g = \omega^2 R$. The spin period is $T = 2\pi/\omega$.

Step 1: Apply the zero-reaction condition Taking inward as positive, the radial force equation is $mg - N = m\omega^2 R$. Setting $N = 0$ gives $\omega = \sqrt{g/R}$. Thus

$$T = 2\pi\sqrt{\frac{R}{g}}.$$

Mass cancels, so the limiting period is the same for all bodies at the equator. The given g is used as the gravitational acceleration in this idealized calculation.

Step 2: Substitute the stated values With $R = 6.4 \times 10^6\,\text{m}$, $g = 10\,\text{m s}^{-2}$, and $\pi = 3.14$,

$$T = 6.28\sqrt{6.4 \times 10^5} = 6.28(800) = 5024\,\text{s}.$$

Converting seconds to minutes gives $5024/60 = 83.73\,\text{min}$, approximately 84 minutes. A 60-minute or 1200-minute period does not satisfy the balance, and increasing the angular speed necessarily changes the day length.

Step 3: Conclusion The day would last approximately $\boxed{84 \text{ minutes}}$. Hence, the correct option is (A).

Quick Tip to Solve: Floating means zero normal reaction, not zero gravity: equate gravitational acceleration to the required centripetal acceleration.

---

33. An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :

[4 marks]

(A) 1 : 1

(B) 3 : 1

(C) 2 : 1

(D) 1 : 2

Correct Answer: (B) 3 : 1

Solution:

Concept:

- Linear momentum is conserved for an isolated collision. Kinetic energy is conserved only for an elastic collision; that property is not stated in the question.
- The coefficient of restitution is the relative separation speed divided by the relative approach speed. It determines which mass ratio can produce specified final speeds.

Step 1: Write the momentum and restitution equations Let the initial speed of m_1 be $u > 0$. After impact let m_1 rebound with velocity $-v$, while m_2 moves with $+v$. Momentum conservation gives

$$m_1 u = (m_2 - m_1)v.$$

Therefore $u = (m_2/m_1 - 1)v$. The relative separation speed is $2v$, so

$$e = \frac{2v}{u} = \frac{2}{m_2/m_1 - 1}.$$

Step 2: Check the conventional elastic answer and the missing assumption For an elastic collision, $e = 1$, giving $m_2/m_1 = 3$. Ratios 1, 2, and $1/2$ cannot give the stated opposite equal speeds in an ordinary passive collision with $0 \le e \le 1$: the ratio 2 would require $e = 2$, while ratios at most 1 give no positive incident speed for the chosen final directions. However, $m_2/m_1 = 5$, $e = 1/2$ is a physically possible inelastic example with the same final-speed condition. Thus the wording itself does not uniquely determine the ratio; it requires the elastic assumption implicit in the conventional answer.

Step 3: Conclusion With the conventional elastic-collision assumption, $m_2 : m_1 = \boxed{3:1}$, corresponding to option (B). As printed, the collision type is unspecified, so a unique mass ratio cannot be

inferred from the stated data alone.

Quick Tip to Solve: Equal final speeds do not imply an elastic collision. Check whether energy conservation or a restitution coefficient is actually given.

34. In a series LCR circuit, the inductive reactance (X_L) is 10Ω and the capacitive reactance (X_C) is 4Ω. The resistance (R) in the circuit is 6Ω. The power factor of the circuit is : [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{2\sqrt{2}}$

Correct Answer: (C) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The net reactance of a series circuit is $X = X_L - X_C$, because inductive and capacitive voltage contributions are opposite in phase.
- The impedance magnitude is $Z = \sqrt{R^2 + X^2}$, and the power factor is $\cos\phi = R/Z$. It is dimensionless and lies between zero and one.

Step 1: Find the net reactance and impedance Using the given reactances,

$$X = 10 - 4 = 6\,\Omega.$$

Hence

$$Z = \sqrt{6^2 + 6^2} = 6\sqrt{2}\,\Omega.$$

The circuit is net inductive, so current lags voltage. Adding the reactances would be incorrect, since the two reactive voltages point in opposite directions in the phasor representation.

Step 2: Calculate and compare the power factor The resistance-to-impedance ratio is

$$\cos\phi = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

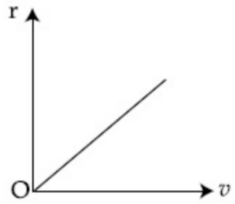
Equivalently, $\tan\phi = X/R = 1$, so the phase angle is 45°. Option A would correspond to a 60° angle and option B to 30°, neither of which matches equal resistance and net reactance. Option D is half the required ratio and does not follow from R/Z.

Step 3: Conclusion The power factor is $\boxed{1/\sqrt{2}}$. Hence, the correct option is (C).

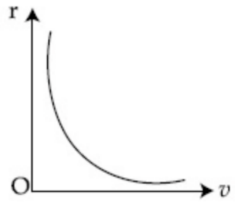
Quick Tip to Solve: When the net reactance equals the resistance, the impedance triangle is isosceles and the power factor is immediately $1/\sqrt{2}$.

35. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = -\frac{C}{r}$, where C is a positive constant. The correct radius − velocity graph of the particle's motion is : [4 marks]

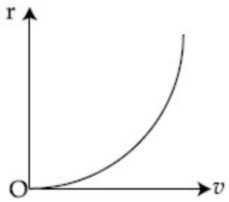
(A)



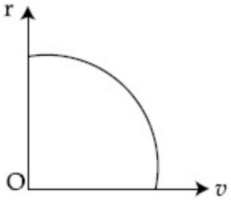
(B)



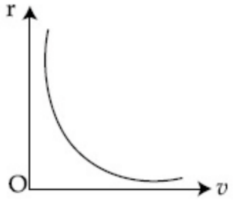
(C)



(D)



Correct Answer: (B)



Solution:

Concept:

- For a central potential energy $U(r)$, the radial force is $F_r = -dU/dr$, with a negative sign denoting attraction.
- A circular orbit requires inward force magnitude mv^2/r. Here $C > 0$ and $m > 0$, so the radius is positive for every nonzero orbital speed.

Step 1: Obtain the force from the potential Differentiating $U = -C/r$ gives $dU/dr = C/r^2$. Thus

$$F_r = -\frac{C}{r^2}.$$

The force is attractive and its magnitude is C/r^2. Equate this to the centripetal force, rather than equating potential energy directly to kinetic energy:

$$\frac{mv^2}{r} = \frac{C}{r^2}.$$

Step 2: Determine the graph and exclude the alternatives Rearrangement gives

$$r = \frac{C}{mv^2}, \qquad r \propto v^{-2}.$$

The radius decreases continuously as positive speed increases. It diverges as v tends to zero and approaches zero only asymptotically as v becomes large. Graph B has precisely that falling shape. Graphs A and C increase with speed, so both contradict the inverse-square relation. Graph D reaches the axes at finite values, which is also inconsistent with this relation. The graph is for a family of circular orbits, with the constants C and m fixed.

Step 3: Conclusion The required relation is $\boxed{r = C/(mv^2)}$, represented by the decreasing asymptotic curve. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate the potential first, then use centripetal force; for $-C/r$, the orbit radius varies as the inverse square of speed.

36. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is : [4 marks]

(A) $\frac{2L}{M}$
(B) $\frac{L}{2M}$
(C) $\frac{L}{M}$
(D) $\frac{4L}{M}$

Correct Answer: (B) $\frac{L}{2M}$

Solution:

Concept:

- The area swept by the radius vector in a short time is half the area of the parallelogram formed by position and displacement: $dA = \frac{1}{2}|\vec{r} \times d\vec{r}|$.
- Angular momentum about the Sun is $\vec{L} = M(\vec{r} \times \vec{v})$, where M is the planet mass. The gravitational force is central, so its torque about the Sun vanishes.

Step 1: Express the swept area per unit time For an infinitesimal displacement, the sector is locally a

triangle. Dividing its area by the elapsed time gives

$$\frac{dA}{dt} = \frac{1}{2}\left|\vec{r} \times \frac{d\vec{r}}{dt}\right| = \frac{1}{2}|\vec{r} \times \vec{v}|.$$

Only the component of velocity perpendicular to the radius contributes; radial motion sweeps no area.

Step 2: Use the angular momentum magnitude Substitute $|\vec{r} \times \vec{v}| = L/M$:

$$\frac{dA}{dt} = \frac{L}{2M}.$$

This remains constant throughout the elliptical orbit even though the speed and the distance from the Sun vary. The units are $(\mathrm{kg\,m^2\,s^{-1}})/\mathrm{kg} = \mathrm{m^2\,s^{-1}}$, as required for area per time. Options (A), (C) and (D) miss the geometric factor one-half and respectively give four, two and eight times the correct value.

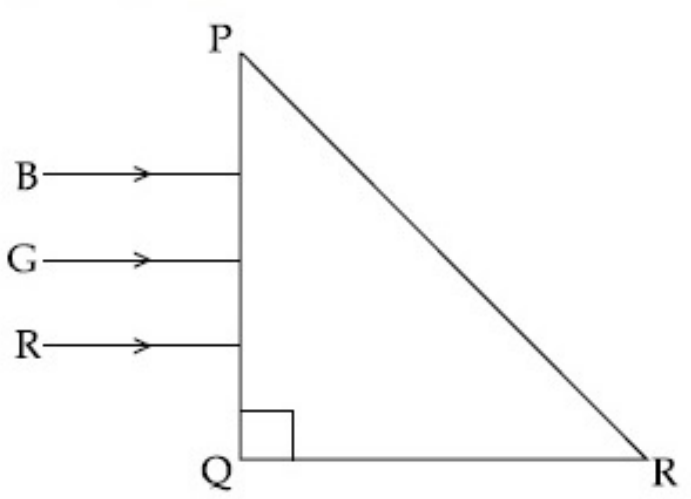
Step 3: Conclusion The magnitude of the areal velocity is $\boxed{L/(2M)}$. Hence, the correct option is (B).

Quick Tip to Solve: Always retain the factor one-half from triangular swept area: areal velocity equals angular momentum divided by twice the mass.

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37. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in the figure.

The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is : [4 marks]



(A) green

(B) blue and green

(C) blue

(D) red

Correct Answer: (D) red

Solution:

Concept:

- A ray incident normally on a prism face enters without changing direction. At the sloping face of the depicted right-isosceles prism, the internal incidence angle is 45°.
- For refraction from a material of index n into air, the critical angle c obeys $\sin c = 1/n$. Total internal reflection occurs when the incidence angle exceeds c.

Step 1: Find the criterion for emergence at PR The horizontal incident rays are perpendicular to PQ, so all three continue horizontally inside the prism. The face PR makes 45° with the horizontal, hence its normal also makes 45° with each ray. Snell's law at PR gives

$$\sin r = n \sin 45^\circ = \frac{n}{\sqrt{2}}.$$

A refracted ray can emerge only if this quantity is at most unity.

Step 2: Compare each refractive index with the threshold The threshold is $\sqrt{2} \simeq 1.414$. For red, $1.27 < \sqrt{2}$, so refraction is possible; its critical angle is about 51.9°, greater than 45°. For green, $1.42 > \sqrt{2}$, giving a critical angle about 44.8°. For blue, $1.49 > \sqrt{2}$, giving about 42.2°. Green and blue therefore undergo total internal reflection at PR. Options (A), (B) and (C) include colours whose incidence angle exceeds their critical angle, so none describes rays emerging through PR.

Step 3: Conclusion Only the red ray emerges through face PR. Hence, the correct option is (D).

Quick Tip to Solve: At a 45° glass-air boundary, compare the index directly with $\sqrt{2}$; an index above this value causes total internal reflection.

38. Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the center is : [4 marks]

(A) $\frac{1}{4}\frac{ML^2}{\pi^2}$
(B) $\frac{1}{2}\frac{ML^2}{\pi^2}$
(C) $\frac{2}{5}\frac{ML^2}{\pi^2}$
(D) $\frac{ML^2}{\pi^2}$

Correct Answer: (D) $\frac{ML^2}{\pi^2}$

Solution:

Concept:

- The length of a semicircular arc of radius r is $L = \pi r$. Bending a wire changes its shape but preserves its total length and mass.
- Moment of inertia about an axis is $I = \int \rho^2 \, dm$, where ρ is perpendicular distance from the axis. For a circular arc about its central perpendicular axis, every element has the same distance r.

Step 1: Relate the radius to the wire length The wire forms only half of a circumference, not a

complete ring. Thus

$$L = \pi r \quad \Rightarrow \quad r = \frac{L}{\pi}.$$

The centre here is the centre of the circle containing the semicircle; it is the point through which the stated perpendicular axis passes. It is not the centre of mass of the semicircular wire.

Step 2: Sum the moments of all wire elements Each element lies in the plane and at radius r from that axis, so

$$I = \int r^2\, dm = r^2 \int dm = Mr^2.$$

Inserting the radius gives

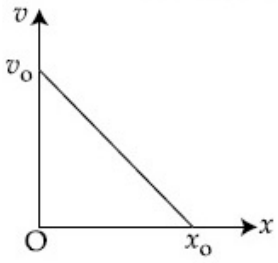
$$I = \frac{ML^2}{\pi^2}.$$

The fact that the shape is a semicircle does not introduce an extra one-half: M is already the mass of this entire wire. Option (A) would incorrectly use the full-ring radius $L/(2\pi)$; (B) resembles the solid-disc factor, and (C) the solid-sphere factor. Neither applies to a thin arc with every element at one radius.

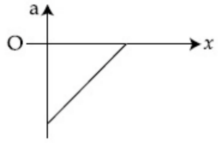
Step 3: Conclusion The required moment of inertia is $\boxed{ML^2/\pi^2}$. Hence, the correct option is (D).

Quick Tip to Solve: For any thin circular arc about the perpendicular axis through its circle centre, use $I = Mr^2$; the arc angle enters only when converting length into radius.

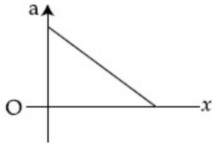
39. The velocity – displacement graph of a particle is shown in the figure.
The acceleration – displacement graph of the same particle is represented by : [4 marks]



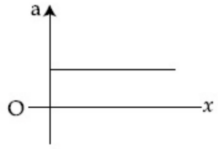
(A) A straight line with positive slope, starting at negative acceleration and reaching zero at positive displacement.



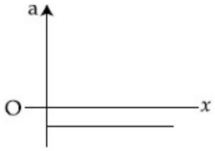
(B) A straight line with negative slope, starting at positive acceleration and reaching zero at positive displacement.



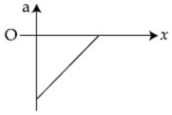
(C) A horizontal line above the displacement axis, representing constant positive acceleration.



(D) A horizontal line below the displacement axis, representing constant negative acceleration.



Correct Answer: (A) A straight line with positive slope, starting at negative acceleration and reaching zero at positive displacement.



Solution:

Concept:

- For velocity specified as a function of position, the chain rule gives $a = dv/dt = (dv/dx)(dx/dt) = v\,dv/dx$. The slope of a velocity-position graph alone is not acceleration.
- The displayed straight line joins $(0, v_0)$ and $(x_0, 0)$. Its slope is constant and negative, $-v_0/x_0$, throughout the shown motion.

Step 1: Write the velocity as a function of position The intercept form of the displayed line gives

$$v(x) = v_0\left(1 - \frac{x}{x_0}\right), \qquad \frac{dv}{dx} = -\frac{v_0}{x_0}.$$

Here v_0 and x_0 are positive. Although velocity decreases uniformly with displacement, it does not decrease uniformly with time, because the particle covers later equal displacement intervals more slowly.

Step 2: Apply the chain rule and identify the graph Multiplying the slope by velocity gives

$$a(x) = -\frac{v_0^2}{x_0}\left(1 - \frac{x}{x_0}\right) = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0}.$$

Thus the graph has a positive slope, a negative vertical intercept, and approaches zero at $x = x_0$. This is exactly the line in (A). Graph (B) has the wrong sign, while (C) and (D) are constant and therefore cannot represent the factor of $v(x)$ in the acceleration. The endpoint represents the limiting value as the speed tends to zero.

Step 3: Conclusion The acceleration increases linearly from $-v_0^2/x_0$ towards zero, remaining negative on the shown interval. Hence, the correct option is (A).

Quick Tip to Solve: On a velocity-position plot, multiply the local slope by the velocity before deciding the sign and shape of the acceleration graph.

40. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
(Molecular weight of oxygen is 32g/mol; R = 8.3 J K^{-1} mol^{-1}) [4 marks]

(A) $\sqrt{\frac{3\pi}{8}}$

(B) $\sqrt{\frac{3}{3}}$

(C) $\sqrt{\frac{8}{3}}$

(D) $\sqrt{\frac{8\pi}{3}}$

Correct Answer: (A) $\sqrt{\frac{3\pi}{8}}$

Solution:

Concept:

- For an ideal gas at absolute temperature T, the rms speed is $v_{\text{rms}} = \sqrt{3RT/M}$, where M is molar mass in kg mol^{-1}.
- The mean molecular speed is $\bar{v} = \sqrt{8RT/(\pi M)}$. It is a mean of speed magnitudes, distinct from the mean vector velocity, which vanishes for gas at rest.

Step 1: Form the ratio before substituting numbers Both speeds contain the same temperature, gas constant and molar mass. Therefore

$$\frac{v_{\text{rms}}}{\bar{v}} = \frac{\sqrt{3RT/M}}{\sqrt{8RT/(\pi M)}} = \sqrt{\frac{3RT}{M}\frac{\pi M}{8RT}}.$$

Cancel R, T and M, which are common positive factors. This avoids unnecessary numerical calculations and possible errors converting the supplied oxygen molar mass.

Step 2: Simplify and check the physical ordering The result is

$$\frac{v_{\text{rms}}}{\bar{v}} = \sqrt{\frac{3\pi}{8}} \simeq 1.085.$$

The ratio is dimensionless and exceeds one, consistent with an rms being greater than the arithmetic mean for a nonconstant distribution of positive speeds. Option (B) equals unity and would require all molecules to have the same speed. Option (C) omits the Maxwell-distribution factor involving π, and option (D) puts that factor in the wrong position; both give values much larger than the actual ratio. The given temperature and oxygen mass determine the individual speeds but not their ratio.

Step 3: Conclusion The ratio is $\boxed{\sqrt{3\pi/8}}$. Hence, the correct option is (A).

Quick Tip to Solve: When comparing two characteristic ideal-gas speeds, take their ratio symbolically first: the temperature and molecular mass cancel.

41. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is : [4 marks]

(A) infinite

(B) $\frac{L}{R}$ ln10

(C) $\frac{L}{R}$ ln2

(D) $\frac{L}{R}$ ln5

Correct Answer: (C) $\frac{L}{R}$ ln2

Solution:

Concept:

- When an initially unenergised RL series circuit is connected to an ideal battery, its current grows as $i(t) = I_\infty(1 - e^{-t/\tau})$, with $I_\infty = \mathcal{E}/R$ and $\tau = L/R$.
- The magnetic energy stored in an inductor is $U = \frac{1}{2}Li^2$. Energy therefore grows as the square of the current fraction, not as the current fraction itself.

Step 1: Convert the energy condition into a current condition The final magnetic energy is $U_{\max} = \frac{1}{2}LI_\infty^2$. Hence

$$\frac{U(t)}{U_{\max}} = \left(\frac{i(t)}{I_\infty}\right)^2 = \left(1 - e^{-t/\tau}\right)^2.$$

At 25 percent of the final energy this ratio equals $1/4$, so the positive, growing current must have reached half of its final value. The negative square root is excluded because the current grows in the battery-driven direction.

Step 2: Solve the exponential equation Using the half-current condition,

$$1 - e^{-t/\tau} = \frac{1}{2} \quad \Rightarrow \quad e^{-t/\tau} = \frac{1}{2}.$$

Taking natural logarithms gives

$$t = \tau \ln 2 = \frac{L}{R} \ln 2.$$

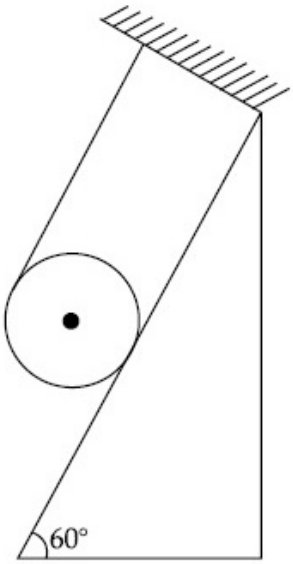
The time is finite: infinite time is needed only to reach the full asymptotic current and energy. At $\tau \ln 10$ the energy fraction is $0.9^2 = 0.81$, and at $\tau \ln 5$ it is $0.8^2 = 0.64$; these rule out (B) and (D). The dimensions of L/R are time, as expected.

Step 3: Conclusion The required time is $\boxed{(L/R)\ln 2}$. Hence, the correct option is (C).

Quick Tip to Solve: Take the square root of an inductor energy fraction before using the exponential current-growth law; one-quarter energy means one-half current.

42. A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :

[The coefficient of static friction, μ_s' is 0.4] [4 marks]



(A) 0

(B) 5 mg

(C) $\frac{7}{2}$ mg

(D) $\frac{mg}{5}$

Correct Answer: (D) $\frac{mg}{5}$

Solution:

Concept:

- The maximum available static friction is $f_{s,\max} = \mu_s N$; the actual static friction is determined by force and torque balance and need not always equal this bound.
- The string in the figure is parallel to the plane. Consequently, the normal reaction is $N = mg\cos 60°$, since tension and friction have no perpendicular component.

Step 1: Calculate the limiting static friction Perpendicular force balance gives

$$N = mg\cos 60° = \frac{mg}{2}.$$

Using the supplied static coefficient,

$$f_{s,\max} = 0.4\frac{mg}{2} = \frac{mg}{5}.$$

This force acts up the plane to oppose the impending downward slipping of the cylinder surface relative to the plane. Options (B) and (C) exceed this static-friction bound, while (A) does not represent the limiting value.

Step 2: Check whether the printed data determine an actual equilibrium force For a cylinder at rest, tension T and friction f produce opposite torques of equal lever arm. Thus torque balance would require $T = f$, while balance along the plane would give $T + f = mg\sin 60°$. Therefore

$$f_{\text{equilibrium}} = \frac{mg\sin 60°}{2} = \frac{\sqrt{3}\,mg}{4} > \frac{mg}{5}.$$

Static equilibrium is impossible with the supplied coefficient. The keyed option therefore identifies the limiting static friction. Once sliding occurs, an actual sliding-friction force would require the kinetic coefficient, which is not supplied; the static coefficient alone cannot specify that subsequent force.

Step 3: Conclusion The limiting static friction is $\boxed{mg/5}$, the value intended by the keyed option. Hence, the correct option is (D) under the limiting-friction interpretation. Literally, the actual force after sliding starts is not uniquely specified without a kinetic-friction coefficient.

Quick Tip to Solve: Check equilibrium before putting $f = \mu_s N$: that product gives the static-friction limit, not every possible actual frictional force.

43. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is : [4 marks]

(A) cos(ωt) + cos(2ωt) + cos(3ωt)

(B) $\sin^2(\omega t)$

(C) sin(ωt) + cos(ωt)

(D) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Correct Answer: The official key selects (D), but (B) also describes SHM about the displaced equilibrium x=1/2 with period pi/omega.

Solution:

Concept:

- A displacement measured from equilibrium executes simple harmonic motion when $\ddot{x} = -\Omega^2 x$, with one fixed angular frequency Ω. Its period is $T = 2\pi/\Omega$.
- An arbitrary constant phase changes the starting position and velocity, but neither the angular frequency nor the period. A sum of different frequencies is generally periodic without being simple harmonic.

Step 1: Identify the required angular frequency The prescribed period gives $\Omega = 2\pi/(\pi/\omega) = 2\omega$. Thus we seek a single sine or cosine with a time-dependent phase of magnitude $2\omega t$. The coefficient multiplying the function is its amplitude, while a constant phase has no effect on the required period.

Step 2: Test the four functions Option (D) is $x = 3\cos(2\omega t - \pi/4)$, so $\ddot{x} = -4\omega^2 x$ and $T = \pi/\omega$. Option (A) contains distinct angular frequencies and is not SHM. Option (C) equals $\sqrt{2}\sin(\omega t + \pi/4)$, with the wrong period $2\pi/\omega$. However, option (B) deserves care:

$$x = \sin^2(\omega t) = \frac{1}{2} - \frac{1}{2}\cos(2\omega t).$$

Writing displacement from its equilibrium as $y = x - 1/2$ gives

$$\ddot{y} = -4\omega^2 y, \qquad T = \frac{\pi}{\omega}.$$

A constant spatial translation does not change the physical character of SHM. Thus (B) is also SHM if the displayed function is a coordinate measured from an arbitrary origin. The official key chooses (D), consistent with the additional convention that the displayed function must itself be displacement from zero equilibrium. That convention is not stated in the question, so the literal wording does not uniquely exclude (B).

Step 3: Conclusion The official keyed answer is option (D). It is SHM about zero with period π/ω, but option (B) is also SHM about $x = 1/2$ with that period. Without specifying an equilibrium-referenced displacement, the wording admits both and is ambiguous.

Quick Tip to Solve: A constant coordinate offset changes the equilibrium position, not the frequency; test $\ddot{x} = -\Omega^2(x - x_0)$ when the origin is unspecified.

44. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where γ is the ratio of specific heats) : [4 marks]

(A) $-\frac{1}{\gamma}\frac{dV}{V}$
(B) $-\gamma\frac{V}{dV}$
(C) $-\gamma\frac{dV}{V}$
(D) $\frac{dV}{V}$

Correct Answer: (C) $-\gamma\frac{dV}{V}$

Solution:

Concept:

- For a reversible adiabatic change of an ideal gas, $PV^\gamma = C$, where P is pressure, V is volume and $\gamma = C_P/C_V > 1$. The usual differential adiabatic relation refers to this quasistatic process.
- A fractional differential change means the infinitesimal change divided by the current value: pressure changes by dP/P, and volume by dV/V.

Step 1: Differentiate the adiabatic equation Keep C and γ constant and differentiate the product:

$$V^\gamma\, dP + \gamma P V^{\gamma-1}\, dV = 0.$$

Divide every term by PV^γ, which is nonzero for the gas state under consideration. This produces

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0.$$

Equivalently, logarithmic differentiation of $\ln P + \gamma \ln V = \ln C$ gives the same result directly.

Step 2: Interpret the sign and eliminate the other expressions Rearranging gives

$$\frac{dP}{P} = -\gamma \frac{dV}{V}.$$

An expansion has $dV > 0$, so the negative sign correctly predicts a pressure decrease. The magnitude of the fractional pressure drop is γ times the fractional volume increase. Option (A) incorrectly replaces γ by its reciprocal. Option (B) uses the reciprocal volume change, which would diverge for an arbitrarily small expansion instead of approaching zero. Option (D) predicts an increase in pressure during expansion and omits the adiabatic factor. These checks agree with the differentiated equation.

Step 3: Conclusion The fractional pressure change is $dP/P = -\gamma\, dV/V$. Hence, the correct option is (C).

Quick Tip to Solve: Logarithmically differentiate a power law: each exponent becomes the coefficient of the corresponding fractional change.

45. A proton and an α-particle, having kinetic energies K_p and K_α respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of α-particle is 2 : 1. The ratio of K_p : K_α is : [4 marks]

(A) 8 : 1

(B) 4 : 1

(C) 1 : 8

(D) 1 : 4

Correct Answer: (B) 4 : 1

Solution:

Concept:

- For nonrelativistic motion perpendicular to a uniform magnetic field, $qvB = mv^2/r$, so $r = p/(|q|B)$, where $p = mv$.
- Using $K = p^2/(2m)$, the kinetic energy is $K = q^2B^2r^2/(2m)$. A proton has charge e and mass m_p; an alpha particle has charge $2e$ and mass approximately $4m_p$.

Step 1: Express the energy ratio in terms of radii Both particles enter the same field at right angles, so they move in circles and the field magnitude cancels in a ratio. Starting with $K = q^2B^2r^2/(2m)$, obtain

$$\frac{K_p}{K_\alpha} = \frac{e^2}{(2e)^2}\frac{m_\alpha}{m_p}\left(\frac{r_p}{r_\alpha}\right)^2.$$

This expression keeps charge, mass and radius factors separate, preventing the alpha particle charge from being confused with its mass number.

Step 2: Substitute the particle properties The charge factor is $1/4$, the mass factor is 4, and the stated

radius ratio is 2. Therefore

$$\frac{K_p}{K_\alpha} = \frac{1}{4} \times 4 \times 2^2 = 4.$$

The charge-squared to mass ratios are equal for these two particles: $e^2/m_p = (2e)^2/(4m_p)$. Consequently their energies are proportional to the squares of their radii in the common field. Option (A) introduces an extra factor of two; options (C) and (D) invert the direction of the energy comparison. Since the proton radius is larger, its energy must also be larger here.

Step 3: Conclusion The kinetic-energy ratio is $K_p : K_\alpha = 4 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: For a proton and an alpha particle, the factors q^2/m are equal, so in a common field compare r^2 directly.

46. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be : [4 marks]

(A) $S_1 - S_2$

(B) $\frac{S_1}{S_2}$

(C) $S_1 \times S_2$

(D) $S_1 + S_2$

Correct Answer: (D) $S_1 + S_2$

Solution:

Concept:

- Entropy is an extensive state property. For two macroscopic parts with negligible interfacial contributions, the entropy of the composite system at the specified state is the sum of their entropies.
- Entropy additivity and entropy production are different statements. Removing a constraint can generate additional entropy if it triggers irreversible equilibration; the inequality $S_1 > S_2$ alone does not establish a temperature or pressure difference.

Step 1: Combine the entropies of the two parts The two given quantities refer to disjoint portions of the same composite system. Their entropy contributions therefore add:

$$S_{\text{composite}} = S_1 + S_2.$$

The piston is a separator, not a rule for subtracting the entropy on one side from the entropy on the other. The fact that $S_1 > S_2$ merely compares two extensive quantities and does not change their addition law; a larger amount of otherwise identical gas can already have a larger entropy.

Step 2: Interpret the piston removal and the options The intended comparison is the additive entropy of the combined gas, represented by option (D). Neither a ratio nor a product is an extensive entropy: S_1/S_2 is dimensionless and $S_1 S_2$ has squared entropy units. The difference $S_1 - S_2$ would incorrectly discard a positive contribution from the second part. Strictly, if removing the piston causes an isolated

gas initially out of equilibrium to relax, its final equilibrium entropy is

$$S_f = S_1 + S_2 + S_{\text{gen}}, \qquad S_{\text{gen}} \geq 0.$$

The question gives no state data to calculate such production. Thus the supplied answer expresses the entropy-additivity result, with equality for no additional entropy production; it must not be interpreted as a universal assertion that all piston removals are reversible.

Step 3: Conclusion For the entropy-additivity relation tested by the question, $S_{\text{total}} = S_1 + S_2$. Hence, the correct option is (D). Any additional irreversible equilibration would require an entropy-production term not specified here.

Quick Tip to Solve: Add entropies when combining parts; assess irreversible entropy production separately from that bookkeeping step.

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47. Which of the following statements are correct?

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where x is its magnetic susceptibility.

Choose the correct answer from the options given below : [4 marks]

(A) (A) and (B) only

(B) (B) and (C) only

(C) (C) and (E) only

(D) (B) and (D) only

Correct Answer: The key gives (C), but the printed statement (B) is also true; no option contains the full set (B), (C), (E).

Solution:

Concept:

- Ordinary electric charges exist individually, while isolated magnetic monopoles are not part of the standard classical description. Magnetic field lines form closed loops rather than beginning or ending at isolated magnetic poles.
- The magnetic field is confined to an ideal toroid's core. For a linear material,

$$B = \mu_0(H + M) = \mu_0(1 + \chi)H.$$

Perfect diamagnetic exclusion means $B = 0$, hence $\chi = -1$.

Step 1: Evaluate the physical statements Statement (A) reverses the established electric and magnetic monopole facts, so it is false. Statement (B), as written, says that the end and outside fields of a

solenoid cannot be wholly straight and confined. This is true: the external return lines curve around to close the magnetic flux loops, unlike the approximately uniform parallel lines deep inside a long solenoid. Statement (C) is true in the ideal toroid approximation used in this question. Statement (D) is false in the standard bar-magnet diagram, where the internal field is approximately parallel and runs from the south pole toward the north pole. Statement (E) is true, since $B = 0$ for nonzero applied H gives $1 + \chi = 0$.

Step 2: Compare the exhaustive option wording with the key The physically correct set under these standard idealisations is therefore the three statements (B), (C) and (E). None of the printed options lists all three. Option (A) contains the false statement (A), and option (D) contains the false statement (D). Option (B) omits the true statement (E), while option (C) omits the true statement (B); both use the restrictive word only. The official answer key selects option (C), namely (C) and (E) only. That is the keyed response, but the literal wording of (B) prevents this pair from being an exhaustive account of all correct statements. No replacement wording for (B) is needed or justified to establish this discrepancy.

Step 3: Conclusion The official key selects option (C), the pair (C) and (E). Literally, however, statement (B) is also true, so the correct set is (B), (C), (E) and no printed only option is exhaustive. The question as printed has no unambiguous exhaustive option.

Quick Tip to Solve: For a combination question, evaluate every statement before reading the pairs; the word "only makes an omitted true statement significant.

48. The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is : [4 marks]

(A) $\beta = \frac{\alpha}{1+\alpha}$
(B) $\alpha = \frac{\beta}{1-\alpha}$
(C) $\alpha = \frac{\beta}{1+\beta}$
(D) $\beta = \frac{1}{1-\alpha}$

Correct Answer: (C) $\alpha = \frac{\beta}{1+\beta}$

Solution:

Concept:

- The common-base current gain is $\alpha = I_C/I_E$, and the common-emitter current gain is $\beta = I_C/I_B$, where I_C, I_E and I_B are the current magnitudes.
- Kirchhoff's current law at a transistor gives $I_E = I_C + I_B$. The emitter supplies the collector and base current contributions together.

Step 1: Express the emitter current using the base current From the definition of β, write $I_C = \beta I_B$. Substituting into current conservation gives

$$I_E = \beta I_B + I_B = (1 + \beta) I_B.$$

The plus sign is essential: the emitter current is larger than the collector current by the positive

base-current contribution. The current gains are ratios, so their relations are dimensionless.

Step 2: Form the ratio and check limiting behaviour Now divide the collector current by the emitter current:

$$\alpha = \frac{I_C}{I_E} = \frac{\beta I_B}{(1+\beta)I_B} = \frac{\beta}{1+\beta}.$$

Rearrangement also gives $\beta = \alpha/(1-\alpha)$. For a typical $\beta = 99$, these formulae give $\alpha = 0.99$, correctly less than unity. Option (A) wrongly puts a plus sign in the denominator of the inverse relation. Option (B) does not follow from current conservation and gives a right-hand side that is much larger than α in normal operation. Option (D) omits the numerator α: in fact $1/(1-\alpha) = 1+\beta$, not β.

Step 3: Conclusion The required relation is $\alpha = \beta/(1+\beta)$. Hence, the correct option is (C).

Quick Tip to Solve: Begin with $I_E = I_C + I_B$; dividing by either I_E or I_B reconstructs both gain relations without memorising signs.

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49. The decay of a proton to neutron is : [4 marks]

(A) always possible as it is associated only with β^+ decay

(B) possible only inside the nucleus

(C) not possible as proton mass is less than the neutron mass

(D) not possible but neutron to proton conversation is possible

Correct Answer: (B) possible only inside the nucleus

Solution:

Concept:

- An isolated proton cannot spontaneously decay into a neutron, positron and neutrino because the neutron is heavier than the proton, and creation of the positron requires still more energy.
- Inside a nucleus, a proton can convert into a neutron through beta-plus decay or electron capture when the change in total nuclear or atomic masses makes the process energetically allowed.

Step 1: Apply energy conservation to a free proton The beta-plus conversion is

$$p \longrightarrow n + e^+ + \nu_e.$$

The proton rest energy is approximately $938.27\,\mathrm{MeV}$, whereas the neutron rest energy is approximately $939.57\,\mathrm{MeV}$, with another $0.511\,\mathrm{MeV}$ needed for the positron. Thus the products have greater rest energy than an isolated proton can supply. A free proton cannot undergo this spontaneous decay. Charge and baryon-number conservation alone are insufficient; the energy balance must also hold.

Step 2: Include the binding energy of a nucleus For a bound proton, the relevant comparison is between the complete parent and daughter nuclei, including their binding energies. A sufficiently favourable binding-energy change can supply the energy cost of converting the proton. Consequently proton-to-neutron conversion is possible within suitable nuclei, although it is not possible in every nucleus. Option (A) incorrectly claims it is always possible and ignores the energy threshold; electron capture is also another conversion channel. Options (C) and (D) incorrectly extend the free-proton

prohibition to bound protons. Free-neutron beta-minus decay does occur, but that fact does not rule out the reverse conversion in an energetically allowed nuclear transition.

Step 3: Conclusion Proton-to-neutron conversion is possible for suitable bound protons inside a nucleus; it is forbidden as spontaneous decay of a free proton. Hence, the correct option is (B).

Quick Tip to Solve: For nuclear beta conversion, compare whole parent and daughter masses; individual nucleon masses alone settle only the free-particle case.

50. The speed of electrons in a scanning electron microscope is 1×10^7 ms^{-1}. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of : [4 marks]

(A) $\frac{1}{1837}$

(B) 1837

(C) $\sqrt{1837}$

(D) $\frac{1}{\sqrt{1837}}$

Correct Answer: (B) 1837

Solution:

Concept:

- The de Broglie wavelength of a particle is $\lambda = h/p = h/(mv)$ in the nonrelativistic regime, where h is Planck's constant and p is momentum.
- For the same microscope geometry, resolving power varies inversely with the probe wavelength: $\mathcal{R} \propto 1/\lambda$. The proton-to-electron mass ratio is approximately 1837.

Step 1: Compare the momenta at equal speed Both probes have the stated speed $v = 10^7\,\mathrm{m\,s^{-1}}$, so their momenta differ only through their masses. Thus

$$\frac{p_p}{p_e} = \frac{m_p v}{m_e v} = \frac{m_p}{m_e} = 1837.$$

The speed is much smaller than the speed of light; the usual de Broglie relation with nonrelativistic momentum is appropriate here.

Step 2: Convert the wavelength ratio into resolving power The heavier proton has the shorter wavelength:

$$\frac{\lambda_p}{\lambda_e} = \frac{p_e}{p_p} = \frac{1}{1837}.$$

Therefore

$$\frac{\mathcal{R}_p}{\mathcal{R}_e} = \frac{\lambda_e}{\lambda_p} = 1837.$$

Option (A) is the wavelength ratio rather than the resolving-power ratio. The square-root ratios in (C) and (D) belong to a comparison at equal kinetic energy, whereas the question fixes equal speed.

Step 3: Conclusion The resolving power increases by a factor of 1837, a dimensionless ratio. Hence, the correct option is (B).

Quick Tip to Solve: At equal speed use $p \propto m$; at equal kinetic energy use $p \propto \sqrt{m}$. Resolving power follows momentum, not wavelength.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Two wires of same length and thickness having specific resistances 6Ω cm and 3Ω cm respectively are connected in parallel. The effective resistivity is $\rho\Omega$ cm. The value of ρ, to the nearest integer, is _____. [4 marks]

Solution:

Concept:

- A uniform wire has resistance $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.
- Parallel wires have the same potential difference. Their conductances add, and the equivalent composite has length L and total cross-sectional area $2A$.

Step 1: Calculate the parallel resistance Let each original wire have length L and area A. Their resistances are $R_1 = \rho_1 L/A$ and $R_2 = \rho_2 L/A$. Hence

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{L}{A}\frac{\rho_1\rho_2}{\rho_1 + \rho_2}.$$

The given resistivities are $\rho_1 = 6\,\Omega\,\text{cm}$ and $\rho_2 = 3\,\Omega\,\text{cm}$; they can be kept in these common units throughout.

Step 2: Use the full area of the composite The two equal-thickness wires together provide area $2A$, not A. Defining the effective resistivity by $R_{\text{eq}} = \rho_{\text{eff}} L/(2A)$ gives

$$\rho_{\text{eff}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = \frac{2(6)(3)}{6+3} = 4\,\Omega\,\text{cm}.$$

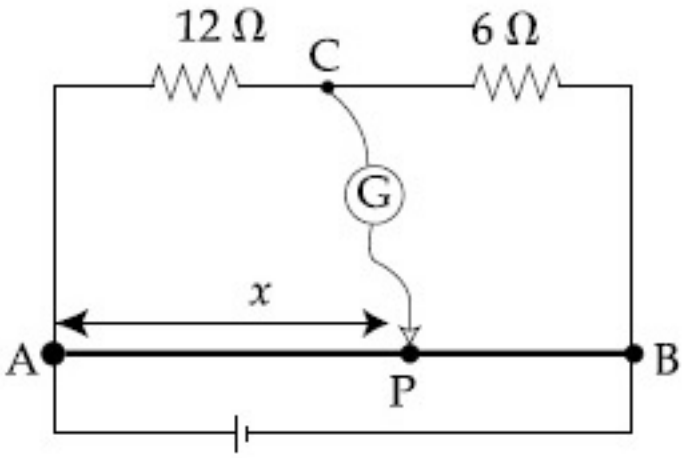
The value 2 would describe the parallel resistance factor referred to the area of just one wire; it is not the effective material resistivity of the combined conductor.

Step 3: Conclusion The effective resistivity is $4\,\Omega\,\text{cm}$. The requested nearest-integer value is $\boxed{\rho = 4}$.

Quick Tip to Solve: When converting a parallel combination back into an effective resistivity, include the sum of the conducting cross-sectional areas.

52. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.

The value of x, to the nearest integer, is _____. [4 marks]



Solution:

Concept:

- A galvanometer shows zero deflection when its two ends have equal potential, so no current passes through its branch.
- For a balanced Wheatstone bridge the resistance ratios of the two paths are equal. In a uniform wire, resistance is proportional to its length.

Step 1: Identify the four bridge arms The upper path from A to B consists of $12\,\Omega$ from A to C and $6\,\Omega$ from C to B. The lower uniform wire AB is divided at P into lengths $AP = x$ and $PB = 72 - x$, in centimetres. Since the galvanometer connects C and P, zero deflection means $V_C = V_P$. The battery is across A and B, so both paths have the same total potential difference.

Step 2: Apply the balance condition and solve With no current in the galvanometer branch, the upper resistors are in series and the current through the lower wire is uniform. Equal fractional voltage drops from A therefore give

$$\frac{R_{AP}}{R_{PB}} = \frac{12}{6} = 2.$$

Because the wire has uniform material and cross-section,

$$\frac{x}{72 - x} = 2 \quad \Rightarrow \quad x = 144 - 2x \quad \Rightarrow \quad 3x = 144.$$

Thus $x = 48\,\text{cm}$; the remaining length is $24\,\text{cm}$, confirming the required $2 : 1$ ratio. Taking the inverse ratio would interchange A and B and contradict the labelled upper arms.

Step 3: Conclusion The jockey lies $\boxed{48\,\text{cm}}$ from A. Hence the requested integer is $x = 48$.

Quick Tip to Solve: Write the upper and lower resistance ratios in the same left-to-right order: A-to-junction over junction-to-B.

53. A galaxy is moving away from the earth at a speed of 286 kms^{-1}. The shift in the wavelength of a redline at 630 nm is x $\times$ 10^{-10} m. The value of x, to the nearest integer, is _____. [Take the value of speed of light c, as 3 $\times$ 10^{8} ms^{-1}] [4 marks]

Solution:

Concept:

- For a source receding with speed $v \ll c$, the fractional wavelength increase is $\Delta\lambda/\lambda \simeq v/c$.
- Recession produces a redshift, so $\Delta\lambda$ is positive. Convert the speed and wavelength to SI units before evaluating the shift.

Step 1: Check the small-speed approximation The galaxy speed and original wavelength are

$$v = 286\,\mathrm{km\,s^{-1}} = 2.86 \times 10^{5}\,\mathrm{m\,s^{-1}}, \qquad \lambda = 630 \times 10^{-9}\,\mathrm{m}.$$

Thus $v/c = (2.86 \times 10^{5})/(3 \times 10^{8}) = 9.533 \times 10^{-4}$. Since this is much less than one, the first-order Doppler relation is sufficient for the requested nearest integer. The emitted wavelength increases because the galaxy is moving away.

Step 2: Calculate the shift in the requested units Use the wavelength shift, rather than the full observed wavelength:

$$\Delta\lambda = \lambda\frac{v}{c} = (630 \times 10^{-9})\frac{2.86 \times 10^{5}}{3 \times 10^{8}} = 6.006 \times 10^{-10}\,\mathrm{m}.$$

Comparing this with $x \times 10^{-10}\,\mathrm{m}$ gives $x = 6.006$, which rounds to 6. Even the relativistic expression changes the result by much less than the amount needed to change this integer.

Step 3: Conclusion The redshift is approximately $6.006 \times 10^{-10}\,\mathrm{m}$. The required nearest-integer coefficient is $\boxed{x = 6}$.

Quick Tip to Solve: First compute v/c, multiply by the original wavelength, and only then extract the coefficient of the power of ten requested.

54. A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is _____. [4 marks]

Solution:

Concept:

- At maximum compression the ball is momentarily at rest, and its initial kinetic energy is stored as elastic potential energy in the spring.
- The spring energy is $U = \frac{1}{2}k\delta^{2}$, where δ is compression from the natural length; the final spring length is $L_0 - \delta$.

Step 1: Determine the maximum compression The ball has mass $m = 4\,\mathrm{kg}$, initial speed $u = 10\,\mathrm{m\,s^{-1}}$,

and initial kinetic energy

$$K_i = \frac{1}{2}mu^2 = \frac{1}{2}(4)(10)^2 = 200\,\text{J}.$$

In the ideal spring collision, mechanical energy is conserved. At the instant of greatest compression, the kinetic energy is zero and the spring stores all $200\,\text{J}$. Therefore

$$\frac{1}{2}k\delta^2 = 200 \quad \Rightarrow \quad \frac{1}{2}(100)\delta^2 = 200.$$

Step 2: Convert compression into the spring length The energy equation gives $\delta^2 = 4\,\text{m}^2$, so the positive compression is $\delta = 2\,\text{m}$. The question asks for the compressed length, not this shortening. With natural length $L_0 = 8\,\text{m}$,

$$x = L_0 - \delta = 8 - 2 = 6\,\text{m}.$$

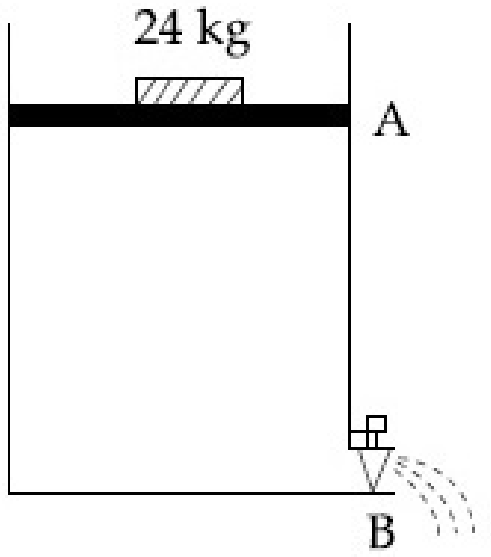
This length is positive and shorter than the natural length, as required. Substitution of $\delta = 2\,\text{m}$ into the spring energy returns $200\,\text{J}$, checking the calculation.

Step 3: Conclusion The maximum compression is $2\,\text{m}$, leaving a spring length of $\boxed{x = 6\,\text{m}}$. The requested integer is 6.

Quick Tip to Solve: Keep separate symbols for compression and final length: spring energy uses the shortening, while the answer here is natural length minus shortening.

55. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross-section area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}.

The value of v, to the nearest integer, is _____. [Take value of g to be 10 ms^{-2}] [4 marks]



Solution:

Concept:

- A load M on a piston of area A adds pressure Mg/A above atmospheric pressure at the water surface.
- Bernoulli's equation combines this pressure head with the height head: $v^2 \simeq 2gh + 2Mg/(\rho_w A)$, for a small outlet and water density $\rho_w = 1000\,\mathrm{kg\,m^{-3}}$.

Step 1: Calculate the additional pressure The tank area is $A = 0.4\,\mathrm{m}^2$, the load is $M = 24\,\mathrm{kg}$, and the surface is $h = 40\,\mathrm{cm} = 0.40\,\mathrm{m}$ above B. The load adds

$$\Delta p = \frac{Mg}{A} = \frac{24(10)}{0.4} = 600\,\mathrm{Pa}.$$

Atmospheric pressure acts above the piston and at the outlet, so it cancels in the pressure difference. The outlet area is $a = 1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$.

Step 2: Combine pressure and gravitational energy Continuity gives a surface speed $u = (a/A)v = 2.5 \times 10^{-4}v$, so its kinetic contribution is negligible. Applying Bernoulli between the surface and B yields

$$\frac{1}{2}\rho_w v^2 = \Delta p + \rho_w g h.$$

Hence

$$v^2 = \frac{2(600)}{1000} + 2(10)(0.40) = 1.2 + 8 = 9.2\,\mathrm{m^2\,s^{-2}},$$

giving $v = \sqrt{9.2} = 3.033\,\mathrm{m\,s^{-1}}$. Retaining the tiny surface-speed correction divides 9.2 by $1 - (a/A)^2$, without affecting the rounded answer.

Step 3: Conclusion The efflux speed is approximately $3.033\,\mathrm{m\,s^{-1}}$. To the nearest integer, $\boxed{v = 3\,\mathrm{m\,s^{-1}}}$.

Quick Tip to Solve: Add the load pressure head $M/(\rho_w A)$ to the water depth before applying the efflux-speed formula.

56. The radius of a sphere is measured to be (7.50 ± 0.85) cm. Suppose the percentage error in its volume is x.

The value of x, to the nearest x, is _____. [4 marks]

Solution:

Concept:

- The volume of a sphere is $V = (4\pi/3)r^3$, with radius r. The numerical factor $4\pi/3$ is exact.
- For a power r^n, the first-order maximum fractional error is $\Delta V/V = |n|\Delta r/r$. Percentage error equals fractional error multiplied by 100.

Step 1: Find the fractional radius error The measured radius is $r = 7.50\,\mathrm{cm}$ with absolute uncertainty

$\Delta r = 0.85\,\text{cm}$. Since the same length unit appears in numerator and denominator, it cancels:

$$\frac{\Delta r}{r} = \frac{0.85}{7.50} = 0.113333\ldots.$$

The corresponding radius percentage error is $11.333\ldots\,\%$. The plus-or-minus sign expresses the uncertainty interval and does not mean that the error should be subtracted from the central measured radius.

Step 2: Propagate the error through the cube Logarithmic differentiation of $V = (4\pi/3)r^3$ gives $dV/V = 3\,dr/r$. The usual measurement-error estimate therefore gives

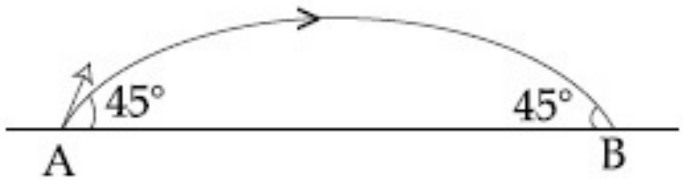
$$\frac{\Delta V}{V} = 3\frac{0.85}{7.50} = 0.34, \qquad x = 0.34(100) = 34\%.$$

The factor three comes from the radius exponent, while the exact geometric prefactor introduces no uncertainty. This is the standard first-order propagated percentage error; directly cubing the upper endpoint would calculate a different finite-interval quantity.

Step 3: Conclusion The percentage error in the volume is $\boxed{34\%}$, so the requested numerical value is $\boxed{x = 34}$.

Quick Tip to Solve: For a product of measured powers, multiply each fractional uncertainty by the magnitude of its exponent before converting to percent.

57. The projectile motion of a particle of mass 5 g is shown in the figure.
The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is x $\times$ 10^{-2} kgms^{-1}. The value of x, to the nearest integer, is _____. [4 marks]



Solution:

Concept:

- Linear momentum is a vector: $\mathbf{p} = m\mathbf{v}$. Its change is $\Delta\mathbf{p} = m(\mathbf{v}_B - \mathbf{v}_A)$, so the velocities must be subtracted component by component.
- In projectile motion without air resistance, horizontal velocity stays constant. At two points at the same height, the vertical velocities have equal magnitudes and opposite signs.

Step 1: Resolve the launch velocity Take the horizontal direction from A to B as positive x, and

vertically upward as positive y. The figure gives a launch angle of 45°, so for $u = 5\sqrt{2}\,\mathrm{m\,s^{-1}}$,

$$u_x = u\cos 45^\circ = 5\,\mathrm{m\,s^{-1}}, \qquad u_y = u\sin 45^\circ = 5\,\mathrm{m\,s^{-1}}.$$

The mass must be converted to SI units: $m = 5\,\mathrm{g} = 5 \times 10^{-3}\,\mathrm{kg}$.

Step 2: Subtract the final and initial velocities A and B are on the same horizontal level. Energy conservation gives equal speeds there, while the downward tangent at B reverses the vertical component. Thus

$$\mathbf{v}_A = 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}}, \qquad \mathbf{v}_B = 5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} \quad (\mathrm{m\,s^{-1}}).$$

Hence $\Delta\mathbf{v} = -10\hat{\mathbf{j}}\,\mathrm{m\,s^{-1}}$. The horizontal contributions cancel; equal speeds do not mean equal momenta because the directions differ.

Step 3: Evaluate the momentum change Using the magnitude of the velocity change,

$$|\Delta\mathbf{p}| = (5 \times 10^{-3})(10) = 5 \times 10^{-2}\,\mathrm{kg\,m\,s^{-1}}.$$

The change points downward, consistent with the net gravitational impulse during the flight. Comparing with the specified form $x \times 10^{-2}\,\mathrm{kg\,m\,s^{-1}}$ identifies the required coefficient.

Step 4: Conclusion The magnitude of the change in momentum is $5 \times 10^{-2}\,\mathrm{kg\,m\,s^{-1}}$. Therefore $\boxed{x = 5}$, already an integer.

Quick Tip to Solve: For launch and landing at the same height, cancel the horizontal momenta first and use $|\Delta p| = 2mu\sin\theta$.

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58. An infinite number of point charges, each carrying 1 μC charge, are placed along the y-axis at y = 1 m, 2 m, 4 m, 8 m .....
The total force on a 1C point charge, placed at the origin, is x $\times$ 10^3 N.
The value of x, to the nearest integer, is _____. [Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2/C^2] [4 marks]

Solution:

Concept:

- By Coulomb's law, the magnitude of the force between charges Q and q separated by distance r is $F = kQq/r^2$, where $k = 1/(4\pi\varepsilon_0)$.
- Electric forces obey vector superposition. Here all source charges lie on the positive y-axis and repel the positive test charge toward negative y, so their magnitudes add.

Step 1: Find the force from each successive charge The test charge is $Q = 1\,\mathrm{C}$ and every source charge is $q = 1\,\mu\mathrm{C} = 10^{-6}\,\mathrm{C}$. With the supplied $k = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$, the nearest charge at $r = 1\,\mathrm{m}$ gives

$$F_1 = \frac{(9 \times 10^9)(1)(10^{-6})}{1^2} = 9 \times 10^3\,\mathrm{N}.$$

Doubling the distance reduces the force to one quarter, rather than one half, because Coulomb force follows an inverse-square law.

Step 2: Sum the forces along their common direction The charges at $1, 2, 4, 8, \ldots$ m therefore contribute magnitudes in the sequence

$$F = 9 \times 10^3 \left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots\right) \text{N}.$$

All terms point downward along the negative y-axis. There is no cancellation from any charge on the negative side, since no such charges are present in the question.

Step 3: Evaluate the convergent geometric series The bracket is a geometric series with first term 1 and common ratio $1/4$. Since the ratio has magnitude less than one, its infinite sum exists:

$$S = \frac{1}{1 - 1/4} = \frac{4}{3}, \qquad F = 9 \times 10^3 \frac{4}{3} = 12 \times 10^3 \,\text{N}.$$

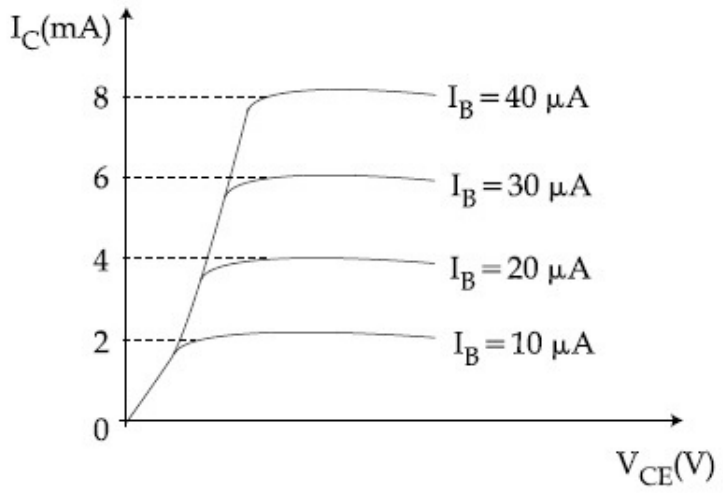
Although there are infinitely many charges, their increasing separations make the total force finite. The required numerical coefficient is obtained by comparing this magnitude with $x \times 10^3$ N.

Step 4: Conclusion The net force is $-12 \times 10^3 \hat{\mathbf{j}}$ N, of magnitude 12×10^3 N. Hence $\boxed{x = 12}$.

Quick Tip to Solve: When successive charge distances double, the successive force terms have ratio $1/4$; sum that force series, not the distance series.

59. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.

The estimated current gain from the figure is _____. [4 marks]



Solution:

Concept:

- The common-emitter current gain measures how a change in base current changes collector current: $\beta = \Delta I_C / \Delta I_B$, evaluated at the same collector-emitter voltage V_{CE}.
- Use the nearly horizontal active-region portions of the output curves. Current gain is a ratio of two

currents, so it is dimensionless after both currents are expressed in the same unit.

Step 1: Read two curves at one collector-emitter voltage Choose a vertical line through the nearly flat parts of the curves, to the right of their knees. From the graph, the curve labelled $I_B = 10\,\mu\text{A}$ has $I_C \approx 2\,\text{mA}$, while the curve labelled $I_B = 20\,\mu\text{A}$ has $I_C \approx 4\,\text{mA}$. Reading them at the same V_{CE} avoids confusing base-current amplification with output-voltage variation.

Step 2: Convert the current increments to compatible units The change between these curves is

$$\Delta I_C = 4 - 2 = 2\,\text{mA} = 2000\,\mu\text{A}, \qquad \Delta I_B = 20 - 10 = 10\,\mu\text{A}.$$

The collector-current scale is in milliamperes, whereas the base-current labels are in microamperes. Dividing the displayed numbers without converting the units would incorrectly give 0.2.

Step 3: Calculate and cross-check the gain The current gain is therefore

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2000\,\mu\text{A}}{10\,\mu\text{A}} = 200.$$

A second check comes from the $I_B = 30\,\mu\text{A}$ and $40\,\mu\text{A}$ curves: their collector currents are about 6 and $8\,\text{mA}$, respectively, giving the same increment ratio. The consistent separation of all the active-region curves supports this estimate.

Step 4: Conclusion The estimated common-emitter current gain is $\boxed{\beta = 200}$, a dimensionless quantity.

Quick Tip to Solve: Read two flat portions at the same V_{CE}, then convert milliamperes to microamperes before taking the current ratio.

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60. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.
(i) ground level
(ii) a height of 5 m.
The increase in antenna range in case (ii) relative to case (i) is n%.
The value of n, to the nearest integer, is _____. [4 marks]

Solution:

Concept:

- For an antenna of height $h \ll R$, where R is Earth's radius, the distance to the horizon is approximately $d = \sqrt{2Rh}$.
- The maximum line-of-sight separation of transmitting and receiving antennas is the sum of their horizon distances: $d_{\max} = \sqrt{2Rh_T} + \sqrt{2Rh_R}$.

Step 1: Write the range for a ground-level receiver The transmitting antenna has height $h_T = 20\,\text{m}$. In case (i), the receiving antenna is at ground level, so $h_R = 0$. The receiving horizon contribution vanishes, giving

$$d_1 = \sqrt{2R(20)} + \sqrt{2R(0)} = \sqrt{40R}.$$

The approximation treats the Earth as spherical and the radio waves as travelling in straight lines, as

appropriate to the stated antenna-range calculation.

Step 2: Write the range for the elevated receiver In case (ii), the transmitter is unchanged but the receiver height is $5\,\mathrm{m}$. The total range becomes

$$d_2 = \sqrt{2R(20)} + \sqrt{2R(5)} = \sqrt{40R} + \sqrt{10R}.$$

Thus the increase is $d_2 - d_1 = \sqrt{10R}$. The additional receiving height contributes its own horizon distance; it should not be added directly to the transmitter height inside a single square root.

Step 3: Find the percentage increase relative to case (i) The reference range is d_1, since the question asks for the increase relative to the ground-level case. Therefore

$$n = 100\frac{d_2 - d_1}{d_1} = 100\frac{\sqrt{10R}}{\sqrt{40R}} = 100\sqrt{\frac{5}{20}} = 50.$$

Earth's radius cancels, so its numerical value is unnecessary. This is a percentage increase in linear range, rather than a percentage increase in coverage area, which would involve squaring distances.

Step 4: Conclusion The antenna range increases by 50%. Hence the requested nearest integer is $\boxed{n = 50}$.

Quick Tip to Solve: For a fixed transmitter, the fractional extra range is $\sqrt{h_R/h_T}$; multiply by 100 for the percentage increase.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Deficiency of vitamin K causes : [4 marks]

(A) Cheilosis

(B) Decrease in blood clotting time

(C) Increase in blood clotting time

(D) Increase in fragility of RBC's

Correct Answer: (C) Increase in blood clotting time

Solution:

Concept:

- Vitamin K is a fat-soluble vitamin needed for the normal synthesis and activation of several blood-clotting proteins, including prothrombin.
- Clotting time is the time needed for blood to form a clot. Reduced availability of functional clotting factors delays clot formation and therefore increases this time.

Step 1: Connect the vitamin with its physiological role Blood clotting depends on a sequence of reactions that ultimately converts soluble fibrinogen into insoluble fibrin. Prothrombin is involved in generating thrombin, which helps bring about this final conversion. Vitamin K supports the normal clotting system, so a deficiency impairs its effectiveness. The result is a greater tendency to bleed because a stable clot forms more slowly.

Step 2: Determine the direction of the time change An impaired clotting process requires longer to produce a clot. Thus vitamin K deficiency causes an *increase* in blood clotting time, exactly as stated in option (C). Option (B) reverses the effect: a decrease in clotting time would mean faster clot formation, whereas the deficiency slows the process. The word "time" is essential when deciding between these two alternatives.

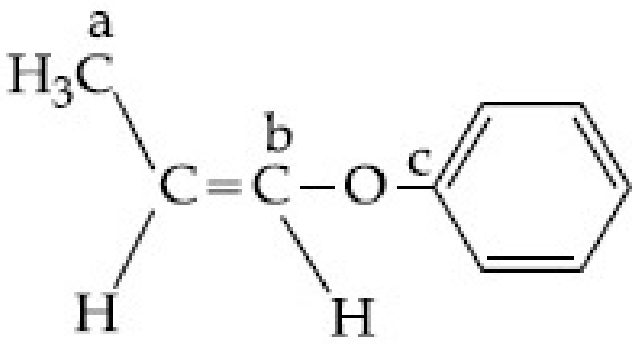
Step 3: Exclude the other vitamin-deficiency symptoms Cheilosis, the inflammation and cracking at the corners of the mouth in option (A), is classically associated with riboflavin (vitamin B_2) deficiency. Increased fragility of red blood cells in option (D) is associated with vitamin E deficiency and impaired antioxidant protection of cell membranes. Neither is the characteristic vitamin K deficiency effect being tested. The clotting-time change uniquely matches the role of vitamin K.

Step 4: Conclusion Deficiency of vitamin K causes an increase in blood clotting time. Hence, the correct option is (C).

Quick Tip to Solve: Link vitamin K to clotting: less effective clotting means more bleeding and a longer clotting time.

62. In the following molecule,

Hybridisation of Carbon a, b, and c respectively are : [4 marks]



(A) sp^3, sp^2, sp^2

(B) sp^3, sp, sp^2

(C) sp^3, sp^2, sp

(D) sp^3, sp, sp

Correct Answer: (A) sp^3, sp^2, sp^2

Solution:

Concept:

- A carbon with four sigma bonds and no pi bond is normally sp^3-hybridised. A carbon with three sigma-bond directions and one unhybridised p orbital is normally sp^2-hybridised.
- Hybridisation is determined locally at the labelled atom. A double bond counts as one sigma-bond direction plus a pi bond, not as two independent sigma-bond directions.

Step 1: Identify the environment of carbon a The atom labelled a is the methyl carbon in $CH_3-CH=CH-O-C_6H_5$. It is attached by three C–H sigma bonds and one C–C sigma bond. These four sigma bonds require four hybrid orbitals directed approximately tetrahedrally, so a is sp^3. Its neighbouring carbon participates in a double bond, but that does not make the methyl carbon itself sp^2.

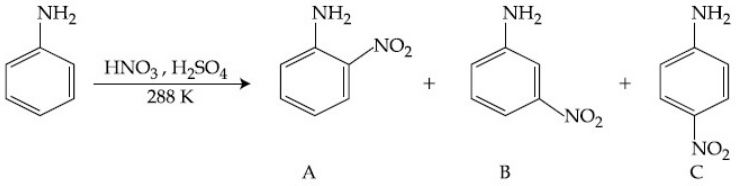
Step 2: Identify the environment of carbon b The atom labelled b is the right-hand carbon of the C=C double bond. It has sigma bonds to the other alkene carbon, to hydrogen, and to oxygen: three sigma-bond directions. Its remaining unhybridised p orbital provides the pi overlap of the alkene. Therefore b is trigonal planar and sp^2, rather than sp. An sp carbon would have two sigma-bond directions and a linear arrangement.

Step 3: Identify carbon c and compare the options The atom labelled c belongs to the benzene ring and is bonded to oxygen and to its two neighbouring ring carbons. It has three sigma-bond directions and an unhybridised p orbital participating in the aromatic pi system, so it is sp^2. The ordered sequence is therefore sp^3, sp^2, sp^2. Option (B) incorrectly assigns sp to b, option (C) incorrectly assigns sp to c, and option (D) makes both of those incorrect assignments.

Step 4: Conclusion The hybridisations of a, b and c are $\boxed{sp^3,\ sp^2,\ sp^2}$, respectively. Hence, the correct option is (A).

Quick Tip to Solve: Count sigma-bond directions at each labelled carbon; being next to a double bond does not by itself change a methyl carbon from sp^3.

63. Consider the given reaction, percentage yield of : [4 marks]



(A) B > C > A

(B) A > C > B

(C) C > A > B

(D) C > B > A

Correct Answer: (D) C > B > A

Solution:

Concept:

- Aniline undergoes electrophilic aromatic nitration with the HNO_3/H_2SO_4 mixture. In the strongly acidic medium, much of the amino group becomes protonated to $-NH_3^+$.
- The neutral $-NH_2$ group directs electrophilic substitution mainly to ortho and para positions, whereas protonated anilinium is meta-directing. Both effects matter in direct nitration.

Step 1: Identify the three labelled products In the reaction figure, A is ortho-nitroaniline, with the

nitro group adjacent to the amino group. B is meta-nitroaniline, and C is para-nitroaniline. The comparison concerns percentage yields under the indicated strongly acidic nitrating conditions at $288\,\mathrm{K}$, so the directing behaviour of isolated neutral aniline is not sufficient to decide the order.

Step 2: Account for protonation in the nitrating mixture The acid-base equilibrium

$$C_6H_5NH_2 + H^+ \rightleftharpoons C_6H_5NH_3{}^+$$

places much of the substrate in its anilinium form. The positively charged $-NH_3{}^+$ group withdraws electron density and directs substitution toward the meta position. Consequently the meta product B is substantial, even though the unprotonated amino group is normally a strong ortho/para director. Ignoring protonation would incorrectly predict that B is least abundant.

Step 3: Compare the known direct-nitration product distribution Direct nitration of aniline gives approximately 51% para product, 47% meta product and 2% ortho product under these conditions. Para substitution is the largest contribution; the meta contribution is close behind because of anilinium formation, while the ortho product is minor. Thus the order is C greater than B greater than A. Option (A) incorrectly puts meta ahead of para, option (B) makes ortho most abundant, and option (C) incorrectly places ortho above meta.

Step 4: Conclusion The percentage yields follow $\boxed{C > B > A}$: para-nitroaniline exceeds meta-nitroaniline, and ortho-nitroaniline is least abundant. Hence, the correct option is (D).

Quick Tip to Solve: In direct nitration of aniline, remember protonation in mixed acid: substantial meta product accompanies para product, while ortho is minor.

64. Given below are two statements :

Statement I : C_2H_5OH and AgCN both can generate nucleophile.

Statement II : KCN and AgCN both will generate nitrile nucleophile with all reaction conditions.

Choose the most appropriate option : [4 marks]

(A) Statement I is false but statement II is true.

(B) Both statement I and statement II are true.

(C) Both statement I and statement II are false.

(D) Statement I is true but statement II is false.

Correct Answer: (D) Statement I is true but statement II is false.

Solution:

Concept:

- A nucleophile donates an available electron pair to an electron-deficient centre. A neutral molecule can be nucleophilic; a negative charge is not essential.
- Cyanide is ambident: reaction through carbon gives a nitrile, $R-CN$, whereas reaction through nitrogen gives an isocyanide, $R-NC$.

Step 1: Evaluate Statement I Ethanol, C_2H_5OH, has lone pairs on oxygen and can act as a neutral

nucleophile. Silver cyanide can also provide a nucleophilic nitrogen end in substitution reactions. Thus the statement that both can generate a nucleophile is true; it does not assert that they have identical nucleophilic strength.

Step 2: Evaluate Statement II KCN is predominantly ionic and supplies free cyanide ions; carbon-end attack normally forms an alkyl nitrile. In largely covalent AgCN, carbon is bonded to silver and the available nitrogen end normally attacks to form an alkyl isocyanide. Thus both reagents do not invariably give nitriles under all reaction conditions. Statement II is false.

Step 3: Select the truth combination The required combination is true for I and false for II. The other three options each assign at least one incorrect truth value: ethanol need not be an anion to donate an electron pair, and silver cyanide must not be treated as an ionic equivalent of potassium cyanide.

Step 4: Conclusion Statement I is true and Statement II is false. Hence, the correct option is (D).

Quick Tip to Solve: For cyanide substitutions, distinguish ionic KCN from largely covalent AgCN before deciding which end attacks.

65. The oxide that shows magnetic property is : [4 marks]

(A) MgO

(B) SiO_2

(C) Mn_3O_4

(D) Na_2O

Correct Answer: (C) Mn_3O_4

Solution:

Concept:

- Unpaired electrons produce a magnetic moment. Species with only paired electrons are diamagnetic and do not show the characteristic magnetic behaviour sought here.
- In a mixed-valence oxide, determine the oxidation states of the metal ions and inspect their electronic configurations.

Step 1: Identify the mixed-valence oxide $\mathrm{Mn_3O_4}$ can be represented as $\mathrm{MnO \cdot Mn_2O_3}$. Charge neutrality therefore corresponds to one $\mathrm{Mn^{2+}}$ and two $\mathrm{Mn^{3+}}$ ions per formula unit. Manganese has atomic number 25; the corresponding configurations are $3d^5$ and $3d^4$, respectively. These ions possess unpaired d electrons and support magnetic behaviour in the solid.

Step 2: Eliminate the other oxides In MgO, $\mathrm{Mg^{2+}}$ has the closed-shell configuration of neon. In $\mathrm{Na_2O}$, $\mathrm{Na^+}$ is also neon-like. The oxide ion has filled shells. $\mathrm{SiO_2}$ is a covalent network with paired bonding electrons and no partially filled transition-metal d shell. These alternatives are diamagnetic rather than the magnetic transition-metal oxide required.

Step 3: Match the listed formula Only manganese oxide contains the open-shell transition-metal ions among the four choices. The inference relies on electronic structure, rather than on oxygen being present, since oxygen occurs in every option.

Step 4: Conclusion The oxide showing the indicated magnetic property is $\mathrm{Mn_3O_4}$. Hence, the correct

option is (C).

Quick Tip to Solve: Check for a partially filled d shell: closed-shell sodium and magnesium ions cannot supply unpaired d electrons.

66. In the reaction of hypobromite with amide, the carbonyl carbon is lost as : [4 marks]

(A) CO_2

(B) CO

(C) HCO_3^-

(D) CO_3^{2-}

Correct Answer: (D) CO_3^{2-}

Solution:

Concept:

- Hypobromite converts a primary amide, $RCONH_2$, to the primary amine RNH_2 with one fewer carbon atom through Hofmann rearrangement.
- The reaction medium is strongly alkaline. Carbon dioxide formed during hydrolysis is trapped by hydroxide as carbonate.

Step 1: Track the carbonyl carbon After N-bromination and rearrangement, the carbonyl carbon appears in an isocyanate, $R{-}N{=}C{=}O$. Hydrolysis gives an unstable carbamic acid intermediate, which releases carbon dioxide and leaves the amine. Therefore this carbon is removed from the organic product; it does not become carbon monoxide.

Step 2: Account for the alkaline medium Any CO_2 produced is consumed in the alkaline solution:

$$CO_2 + 2\,OH^- \longrightarrow {CO_3}^{2-} + H_2O.$$

Thus the final inorganic form of the carbonyl carbon is carbonate. Bicarbonate is not the final form under the excess strong-base conditions of the hypobromite reaction.

Step 3: Verify with the overall equation The usual sodium hydroxide equation is

$$RCONH_2 + Br_2 + 4\,NaOH \longrightarrow RNH_2 + Na_2CO_3 + 2\,NaBr + 2\,H_2O.$$

It conserves the carbonyl carbon in sodium carbonate. Carbon dioxide is an intermediate description of its loss, whereas the question asks for its form in this alkaline reaction mixture.

Step 4: Conclusion The carbonyl carbon is ultimately lost as ${CO_3}^{2-}$. Hence, the correct option is (D).

Quick Tip to Solve: In Hofmann degradation, distinguish CO2 release at the intermediate stage from carbonate formation in excess alkali.

67. The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4 \cdot 5H_2O$, respectively, are : [4 marks]

(A) 5 and 1

(B) 4 and 1

(C) 6 and 5

(D) 6 and 4

Correct Answer: (B) 4 and 1

Solution:

Concept:

- Werner's secondary valency is the coordination number: it counts the ligand donor atoms directly coordinated to the central metal in the usual coordination formulation.
- Water within coordination brackets is coordinated water; water written outside them is water of crystallisation and can be retained by hydrogen bonding.

Step 1: Write the standard hydrate formulation Copper(II) sulphate pentahydrate is conventionally written for this purpose as $[Cu(H_2O)_4]SO_4 \cdot H_2O$. This separates four water molecules associated with the copper coordination entity from one additional water molecule in the crystal. The suffix pentahydrate counts all five waters together; it must not itself be read as a coordination number.

Step 2: Count the secondary valency Each coordinated water molecule is monodentate and donates one oxygen lone pair. Four coordinated water molecules therefore give secondary valency 4 in this standard formulation. The remaining one water molecule is hydrogen bonded in the lattice, so the requested second number is 1.

Step 3: Compare the offered pairs The pair $5, 1$ incorrectly equates the total hydration number with the secondary valency. The pairs $6, 5$ and $6, 4$ do not describe the conventional division into four coordinated waters and one hydrogen-bonded water. The matching pair is consequently $4, 1$.

Step 4: Conclusion The secondary valency is 4 and the number of hydrogen-bonded water molecules is 1. Hence, the correct option is (B).

Quick Tip to Solve: For this hydrate, first write [Cu(H2O)4]SO4.H2O; count inside and outside the brackets separately.

68. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively, are : [4 marks]

(A) neon, sodium and chlorine

(B) argon, chlorine and sodium

(C) chlorine, lithium and sodium

(D) argon, lithium and sodium

Correct Answer: (B) argon, chlorine and sodium

Solution:

Concept:

- First ionization energy is the energy required to remove an electron from an isolated gaseous atom: $\mathrm{M(g)} \longrightarrow \mathrm{M^+(g) + e^-}$.
- Across a period it generally increases with effective nuclear charge. Noble gases have particularly high values, while alkali metals have relatively low values.

Step 1: Translate the condition into inequalities The elements must satisfy $I_1(X) > I_1(\mathrm{Mg})$, $I_1(Y) > I_1(\mathrm{Mg})$, and $I_1(Z) < I_1(\mathrm{Mg})$. In period 3, sodium loses its single $3s$ electron more readily than magnesium loses an electron from its filled $3s^2$ subshell. Chlorine and argon have substantially higher first ionization energies than magnesium.

Step 2: Test the candidate triples The triple argon, chlorine, sodium satisfies all three requirements. Option A fails because sodium cannot be Y and chlorine cannot be Z. Options C and D both put lithium in the Y position; lithium is an alkali metal whose first ionization energy is below that of magnesium, so it fails the required inequality.

Step 3: Confirm the ordering The decisive order among the selected elements is $I_1(\mathrm{Ar}) > I_1(\mathrm{Cl}) > I_1(\mathrm{Mg}) > I_1(\mathrm{Na})$. No precise numerical values are required, and the closed-shell noble gas is correctly placed above magnesium rather than below it.

Step 4: Conclusion $X = \mathrm{Ar}$, $Y = \mathrm{Cl}$, and $Z = \mathrm{Na}$. Hence, the correct option is (B).

Quick Tip to Solve: Turn the wording into three inequalities first; a single low-ionization alkali metal in the Y position rejects a triple.

69. Given below are two statements :

Statement I : Bohr's theory accounts for the stability and line spectrum of Li^+ ion.

Statement II : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but statement II is true.

(B) Both statement I and statement II are true.

(C) Statement I is true but statement II is false.

(D) Both statement I and statement II are false.

Correct Answer: (A) Statement I is false but statement II is true.

Solution:

Concept:

- The elementary Bohr model applies to hydrogen-like species containing exactly one electron, such as H, He^+, and Li^{2+}.
- Splitting of spectral lines in an external magnetic field is the Zeeman effect and is not explained by Bohr's original model.

Step 1: Check the electron count in Statement I Lithium has atomic number 3. The ion Li^+ therefore contains $3 - 1 = 2$ electrons, not one. Electron-electron interaction prevents the elementary one-electron Bohr model from accounting for its stability and line spectrum. The relevant one-electron lithium ion would be Li^{2+}. Statement I, which explicitly names Li^+, is false.

Step 2: Evaluate Statement II Bohr assigned discrete orbital energies but did not include the additional quantum features needed to account for magnetic splitting of spectral lines. Its inability to explain the Zeeman effect is a standard limitation. Hence Statement II is true.

Step 3: Reject the other truth combinations Any option calling Statement I true overlooks the difference between singly and doubly ionized lithium. The option calling both statements false also denies the established magnetic-field limitation. The only consistent combination is false for I and true for II.

Step 4: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (A).

Quick Tip to Solve: Count electrons as atomic number minus positive charge before applying any hydrogen-like formula.

70. The oxidation states of nitrogen in NO, NO_2, N_2O and NO_3^- are in the order of : [4 marks]

(A) $N_2O > NO_2 > NO > NO_3^-$

(B) $NO > NO_2 > N_2O > NO_3^-$

(C) $NO_2 > NO_3^- > NO > N_2O$

(D) $NO_3^- > NO_2 > NO > N_2O$

Correct Answer: (D) $NO_3^- > NO_2 > NO > N_2O$

Solution:

Concept:

- The sum of oxidation numbers equals the net charge of a species. Oxygen has oxidation number -2 in each of the compounds and ion listed here.
- For a formula containing two nitrogen atoms, the value obtained by dividing the total nitrogen oxidation number by two is its average oxidation state.

Step 1: Calculate the four oxidation numbers For NO, let nitrogen be x: $x - 2 = 0$, so $x = +2$. For NO_2, $x - 4 = 0$, giving $x = +4$. For N_2O, $2x - 2 = 0$, giving average $x = +1$. For nitrate, $NO_3{}^-$, the total is -1, not zero:

$$x + 3(-2) = -1 \quad \Rightarrow \quad x = +5.$$

Step 2: Arrange them in descending order The numerical comparison is $+5 > +4 > +2 > +1$. Therefore the species must occur as nitrate, nitrogen dioxide, nitric oxide, and nitrous oxide. In nitrous oxide the two nitrogen atoms need not be individually equivalent; the conventional oxidation state asked for from the molecular formula is the average.

Step 3: Check the alternatives Options A and B put nitrate last even though its nitrogen has the highest oxidation number. Option C reverses the first two entries by putting nitrogen dioxide above nitrate. Only the final listed order agrees with all four charge-balance calculations.

Step 4: Conclusion The order is $NO_3{}^- > NO_2 > NO > N_2O$. Hence, the correct option is (D).

Quick Tip to Solve: Use the actual ionic charge for nitrate and divide by two when calculating the average nitrogen oxidation state in N2O.

71. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is : [4 marks]

(A) $C_6H_5CH(CH_3)NH_2$

(B) $C_6H_5NHCH_2CH_3$

(C) $C_6H_5N(CH_3)_2$

(D) $C_6H_5CH_2NHCH_3$

Correct Answer: (A) $C_6H_5CH(CH_3)NH_2$

Solution:

Concept:

- The Hinsberg reagent is benzene sulphonyl chloride, $C_6H_5SO_2Cl$. It distinguishes primary, secondary and tertiary amines by sulphonamide formation and solubility in alkali.
- A primary amine has two hydrogens initially bonded to nitrogen. Its monosulphonamide retains one acidic N–H hydrogen, which can be removed by dilute $NaOH$.

Step 1: Use the alkali-solubility observation The compound B must contain an acidic hydrogen that can form an ionic sodium salt. For a primary amine the reactions can be represented as

$$RNH_2 + C_6H_5SO_2Cl \longrightarrow C_6H_5SO_2NHR + HCl.$$

The sulphonyl group strongly withdraws electrons and stabilises the anion formed when the remaining N–H proton is removed. Consequently, this sulphonamide dissolves in dilute alkali.

Step 2: Inspect the nitrogen in every option In (A), $C_6H_5CH(CH_3)NH_2$, nitrogen is bonded to only one carbon group, so this is a primary amine. It does not matter that the carbon carrying the amino group is a secondary carbon. In (B) and (D), nitrogen is bonded to two carbon groups, so both are secondary amines: their sulphonamides have no N–H hydrogen and are insoluble in dilute alkali. In (C), nitrogen has three carbon substituents and no N–H bond; this tertiary amine does not give the required sulphonamide.

Step 3: Identify the soluble product For (A), take $R = C_6H_5CH(CH_3)$. The product is $C_6H_5SO_2NHR$; treatment with hydroxide gives its ionic salt and water. The simultaneous requirements of reaction with the Hinsberg reagent and solubility of B in dilute sodium hydroxide therefore identify (A) uniquely.

Step 4: Conclusion Compound A is $C_6H_5CH(CH_3)NH_2$, a primary amine. Hence, the correct option is (A).

Quick Tip to Solve: Classify an amine by the number of carbon groups directly attached to nitrogen, not by the degree of the neighbouring carbon.

72. Main products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are :

[4 marks]

(A) 1-Iodonaphthalene and CH_3I

(B) 1-Naphthol and CH_3OH

(C) 1-Iodonaphthalene and CH_3OH

(D) 1-Naphthol and CH_3I

Correct Answer: (D) 1-Naphthol and CH_3I

Solution:

Concept:

- An aryl alkyl ether undergoes acid cleavage at the alkyl–oxygen bond. The aryl–oxygen bond has partial double-bond character from resonance.
- After oxygen is protonated, iodide attacks an unhindered methyl carbon by the S_N2 mechanism; a phenolic group remains on the aromatic ring.

Step 1: Protonate the ether oxygen Write the substrate as $Ar-O-CH_3$, where Ar is the 1-naphthyl group. Hydrogen iodide protonates the oxygen, converting the methoxy group into an oxonium ion. This makes the oxygen-containing fragment a much better leaving group and supplies I^- as the attacking nucleophile. Protonation alone does not replace a substituent on the naphthalene ring.

Step 2: Choose the bond that can undergo substitution The methyl group is accessible to backside attack, so iodide cleaves its bond to oxygen:

$$Ar-O-CH_3 + HI \longrightarrow Ar-OH + CH_3I.$$

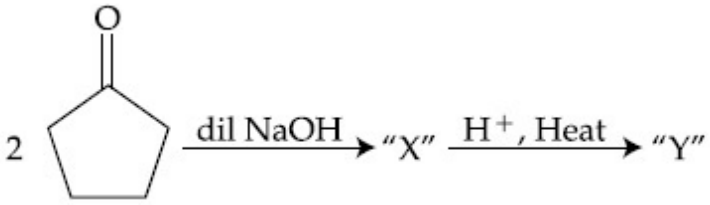
An aromatic sp^2 carbon does not undergo ordinary backside S_N2 displacement. Cleavage at that carbon would also sacrifice the stabilised aryl–oxygen bond; an aryl carbocation route is unfavourable. Thus iodine enters the methyl product, rather than replacing oxygen on the naphthalene ring.

Step 3: Match both products and reject the alternatives The aromatic product is 1-naphthol and the small-molecule product is methyl iodide. Options (A) and (C) incorrectly put iodine on the naphthalene ring. Option (B) has the correct phenol but gives methanol instead of methyl iodide; it overlooks nucleophilic attack by iodide. Only (D) gives both products together while preserving the aryl–oxygen connection.

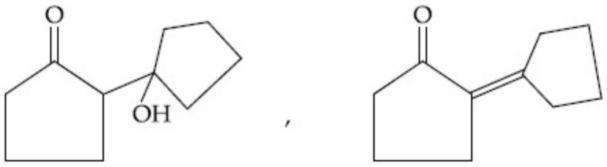
Step 4: Conclusion The main products are 1-naphthol and CH_3I. Hence, the correct option is (D).

Quick Tip to Solve: For an aryl methyl ether with HI, keep oxygen on the aromatic ring and attach iodine to the methyl group.

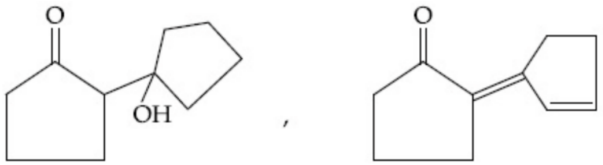
73. Consider the above reaction, the product 'X' and 'Y' respectively are : [4 marks]



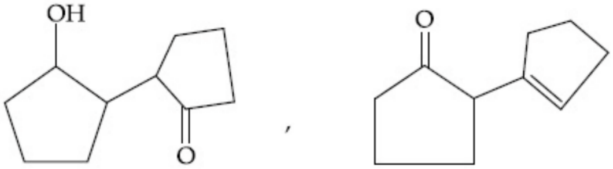
(A) X: the beta-hydroxy ketone with the hydroxyl group on the attacked carbon of the second ring; Y: the conjugated ketone with a double bond joining the two rings, as drawn.



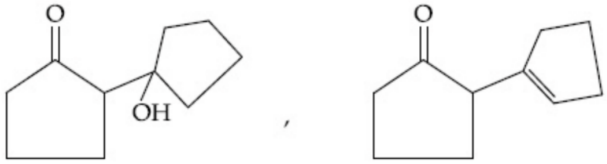
(B) X: the same beta-hydroxy ketone as in (A); Y: the inter-ring double bond together with an additional double bond inside the second ring, as drawn.



(C) X: the two rings joined at their alpha carbons, with hydroxyl on the adjacent carbon of one ring and carbonyl on the other; Y: the ketone with a double bond inside the second ring, as drawn.

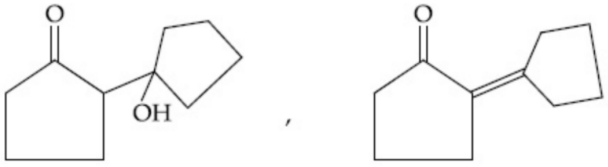


(D) X: the same beta-hydroxy ketone as in (A); Y: the ketone with a double bond inside the second ring, as drawn.



Correct Answer: (A)

X: the beta-hydroxy ketone with the hydroxyl group on the attacked carbon of the second ring; Y: the conjugated ketone with a double bond joining the two rings, as drawn.



Solution:

Concept:

- Cyclopentanone contains alpha hydrogens. Dilute $NaOH$ produces an enolate that adds to the carbonyl carbon of another cyclopentanone molecule.
- Aldol addition forms a beta-hydroxy ketone. On acid treatment and heating, dehydration gives the conjugated alpha,beta-unsaturated ketone.

Step 1: Locate the nucleophile and electrophile One molecule loses an alpha hydrogen to form a resonance-stabilised enolate. Its alpha carbon attacks the carbonyl carbon of a second cyclopentanone molecule. The donor molecule retains its carbonyl group; the acceptor carbonyl oxygen becomes an alkoxide and is then protonated by water. No ring expansion or reduction is involved in this carbon–carbon bond-forming step.

Step 2: Identify the aldol addition product X The new bond joins the alpha carbon of the first ring to the former carbonyl carbon of the second ring. That second carbon bears the hydroxyl group. X is therefore 2-(1-hydroxycyclopentyl)cyclopentan-1-one, the beta-hydroxy ketone shown as X in option (A). The hydroxyl-bearing carbon is part of the second ring, not the carbonyl carbon of the first ring. Both rings retain five members.

Step 3: Dehydrate and compare the product pairs Acid and heat remove water using the hydroxyl group and a hydrogen from the alpha carbon adjacent to the retained carbonyl. The double bond therefore lies between the two rings, conjugated with the surviving carbonyl. Y is 2-cyclopentylidenecyclopentan-1-one. Option (B) introduces an additional double bond without a corresponding additional elimination. The Y structures in (C) and (D) place the double bond inside the second ring and miss the favoured conjugation. The X structure in (C) also has the wrong connectivity for enolate addition.

Step 4: Conclusion X is the beta-hydroxy ketone formed by self-aldol addition. Y is its dehydrated, conjugated ketone, with the double bond joining the two rings. Hence, the correct option is (A).

Quick Tip to Solve: In a self-aldol reaction, mark the enolate alpha carbon and the attacked carbonyl carbon; the dehydration double bond joins these same two carbons.

74. Match List - I with List - II :

	List - I(Class of Chemicals)		List - II(Example)
(a)	Antifertility drug	(i)	Meprobamate
(b)	Antibiotic	(ii)	Alitame
(c)	Tranquilizer	(iii)	Norethindrone
(d)	Artificial Sweetener	(iv)	Salvarsan

Choose the most appropriate match : [4 marks]

(A) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(B) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(C) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(D) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Correct Answer: (B) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Solution:

Concept:

- A drug is classified by its biological action. An antifertility drug, an antibiotic and a tranquilizer have different therapeutic functions.
- Artificial sweeteners supply sweetness at low dosage and are food additives rather than drugs used to prevent pregnancy or treat infection.

Step 1: Match the medicinal compounds Norethindrone is a synthetic progestational agent used in oral contraceptive formulations. Hence antifertility drug (a) matches (iii). Salvarsan is an arsenic-containing antimicrobial historically used for treatment of syphilis, so the listed antibiotic category (b) matches (iv). These two matches already rule out any option assigning norethindrone to antibiotic or assigning Salvarsan to antifertility drug.

Step 2: Match the tranquilizer and sweetener Meprobamate is a tranquilizer used to reduce anxiety and tension, so (c) matches (i). Alitame is a highly potent artificial sweetener, so (d) matches (ii). It is not a tranquilizer: sweetness and central-nervous-system calming action belong to entirely different classes. Similarly, meprobamate cannot be assigned the artificial-sweetener function simply because both are organic compounds.

Step 3: Assemble and check the complete code The complete mapping is

$$(a) \to (iii), \qquad (b) \to (iv), \qquad (c) \to (i), \qquad (d) \to (ii).$$

This is exactly option (B). Option (A) puts alitame under antifertility drugs and meprobamate under sweeteners. Option (C) assigns Salvarsan to antifertility drugs and alitame to tranquilizers. Option (D) has the antibiotic and tranquilizer matches correct but interchanges the antifertility and sweetener examples; a partially correct matching code is insufficient.

Step 4: Conclusion Norethindrone is the antifertility drug, Salvarsan the antibiotic, meprobamate the tranquilizer and alitame the artificial sweetener. Hence, the correct option is (B).

Quick Tip to Solve: Anchor this matching question with alitame as the sweetener and norethindrone as the contraceptive; then check the remaining pair.

75. The charges on the colloidal CdS sol and TiO_2 sol are, respectively : [4 marks]

(A) positive and positive

(B) positive and negative

(C) negative and negative

(D) negative and positive

Correct Answer: (D) negative and positive

Solution:

Concept:

- A colloidal particle can carry a net electrical charge at its surface, with oppositely charged counterions forming the surrounding diffuse layer.
- In the standard classification of sols, metal sulphides such as CdS are negatively charged, whereas metal oxides such as TiO_2 are positively charged.

Step 1: Identify the sulphide sol Cadmium sulphide belongs to the metal-sulphide class of colloids. The CdS particles in the usual sol have a negative surface charge. This sign refers to the dispersed colloidal particles, not to the cadmium ion in the chemical formula and not to the net charge of the complete dispersion. The whole sol, including its counterions, remains electrically neutral.

Step 2: Identify the oxide sol Titanium dioxide is a metal oxide and its usual sol is classified as positively charged. Surface ionisation and adsorption establish the particle charge; assigning the oxidation number of titanium does not by itself calculate the charge of a colloidal particle. The oxide sol is therefore placed in the positive-sol category, opposite to the sulphide sol in this question.

Step 3: Preserve the order and test the choices The question asks for CdS first and titanium dioxide second. Their ordered pair is therefore negative, positive. Option (A) gives the wrong sign for CdS, option (B) reverses both signs, and option (C) assigns the wrong sign to titanium dioxide. Only option (D) retains the standard sulphide/oxide distinction. These are the conventional sol charges used for this classification question; changing preparation conditions is outside the stated comparison.

Step 4: Conclusion The charges are negative on the CdS sol and positive on the TiO_2 sol, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For this standard classification, remember metal sulphide sols as negative and metal oxide/hydroxide sols as positive; keep the question order.

76. In basic medium, H_2O_2 exhibits which of the following reactions?

(A) $Mn^{2+} \rightarrow Mn^{4+}$

(B) $I^2 \rightarrow I^-$

(C) $PbS \rightarrow PbSO_4$

Choose the most appropriate answer from the options given below : [4 marks]

(A) (B) only

(B) (A), (B) only

(C) (A) only

(D) (A), (C) only

Correct Answer: (B) (A), (B) only

Solution:

Concept:

- In H_2O_2, oxygen has oxidation number -1. Peroxide can therefore act as an oxidant when oxygen is reduced to -2, or as a reductant when it is oxidised to 0.
- The reaction medium matters when selecting the standard examples: alkaline peroxide oxidises manganese(II) to manganese(IV) and reduces iodine to iodide.

Step 1: Check reaction (A) Manganese changes from $+2$ to $+4$, so it is oxidised. Hydrogen peroxide accepts the corresponding electrons and produces manganese dioxide in alkaline solution:

$$Mn^{2+} + H_2O_2 + 2\,OH^- \longrightarrow MnO_2 + 2\,H_2O.$$

The manganese oxidation-number rise is two, while the two peroxide oxygens each undergo a one-unit reduction. Thus reaction (A) is an appropriate basic-medium reaction.

Step 2: Check reaction (B) Iodine is reduced from oxidation number zero to -1. Peroxide supplies the electrons by forming oxygen gas:

$$I_2 + H_2O_2 + 2\,OH^- \longrightarrow 2\,I^- + 2\,H_2O + O_2.$$

Both atoms and charge balance: each side has net charge -2. This demonstrates the reducing action of peroxide in alkaline medium, so reaction (B) must also be included.

Step 3: Distinguish the listed lead-sulphide example The conversion $PbS \longrightarrow PbSO_4$ is the familiar oxidation of black lead sulphide to white lead sulphate by peroxide, conventionally listed with its oxidising action outside the alkaline examples being tested. It is not the basic-medium pair requested here. Consequently, selecting only (A) omits the valid iodine reaction, selecting only (B) omits manganese oxidation, and selecting (A),(C) substitutes the lead-sulphide example for the required iodine example.

Step 4: Conclusion Alkaline peroxide oxidises manganese(II) and reduces iodine to iodide; the first two listed reactions are selected. Hence, the correct option is (B).

Quick Tip to Solve: Check both roles of peroxide in alkaline medium: manganese(II) is oxidised, while iodine is reduced to iodide.

77. A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a/an : [4 marks]

(A) Covalent solid

(B) Molecular solid

(C) Ionic solid

(D) Metallic solid

Correct Answer: (A) Covalent solid

Solution:

Concept:

- The melting point and hardness of a solid reflect the strength and extent of the forces holding its structural units together.
- Electrical conduction requires mobile charge carriers. A network covalent solid has localised bonding electrons and generally lacks mobile ions or a sea of electrons.

Step 1: Use the hardness and melting point A very hard solid with a high melting temperature requires strong bonding throughout an extended structure. A network covalent solid meets this description: atoms are linked by directional covalent bonds in a continuous lattice. Melting requires disruption of many strong bonds. This differs from an ordinary molecular solid, whose molecules are held together by comparatively weaker intermolecular attractions.

Step 2: Use the conductivity in both states The solid is an insulator not only before melting but also in its molten state. This eliminates an ionic solid as the best answer: ionic solids do not conduct when ions are fixed in the lattice, but their molten state contains mobile ions and conducts. A metallic solid also fails because its delocalised electrons give electrical conduction in the solid and liquid states. In a typical network covalent substance, the electrons remain involved in bonds rather than serving as freely moving charge carriers.

Step 3: Compare the remaining classes Molecular solids can be electrical insulators, but they do not normally combine the specified hardness with a very high melting point. Network covalent solids provide the best overall fit, as illustrated by diamond-type behaviour. Graphite is a special conducting covalent solid because of its delocalised electrons, but the existence of that exception does not make the general network-covalent classification unsuitable. All three given observations must be used together.

Step 4: Conclusion The substance is most likely a network covalent solid. Hence, the correct option is (A).

Quick Tip to Solve: Use molten-state conductivity to separate ionic solids from covalent networks: melting frees ions in the former, whereas hardness alone cannot distinguish them.

78. Match List - I with List - II :

	List - I		List - II
(a)	Mercury	(i)	Vapour phase refining
(b)	Copper	(ii)	Distillation refining
(c)	Silicon	(iii)	Electrolytic refining
(d)	Nickel	(iv)	Zone refining

Choose the most appropriate answer from the option given below : [4 marks]

(A) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(B) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(C) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

(D) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Correct Answer: (A) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Solution:

Concept:

- Choose a refining method from the physical or chemical property that distinguishes the metal from its impurities. Different metals therefore need different purification processes.
- Distillation uses volatility, electrolytic refining transfers metal through an electrolyte, zone refining uses unequal impurity solubility in solid and liquid phases, and vapour-phase refining uses a volatile compound.

Step 1: Match mercury and copper Mercury is a volatile metal. On heating impure mercury, mercury vapour is collected and condensed, leaving nonvolatile impurities behind. Thus mercury matches distillation refining, (a)-(ii). Copper is purified by electrolytic refining: impure copper is the anode and pure copper is deposited at the cathode. The electrode processes are $\mathrm{Cu} \longrightarrow \mathrm{Cu}^{2+} + 2\,\mathrm{e}^-$ and $\mathrm{Cu}^{2+} + 2\,\mathrm{e}^- \longrightarrow \mathrm{Cu}$. Hence (b)-(iii).

Step 2: Match silicon and nickel Silicon required for semiconductor use must be extremely pure. A molten zone is moved along a solid rod; impurities preferentially remain in the liquid and are swept towards one end. This is zone refining, giving (c)-(iv). Nickel is purified by the Mond vapour-phase process:

$$\mathrm{Ni} + 4\,\mathrm{CO} \longrightarrow \mathrm{Ni(CO)_4}.$$

The volatile carbonyl is separated and decomposed on heating to deposit pure nickel, so (d)-(i).

Step 3: Compare the four combinations The complete sequence is (ii), (iii), (iv), (i). Option B incorrectly interchanges the methods for copper and silicon. Option C assigns vapour-phase refining to silicon and zone refining to nickel. Option D already assigns the wrong method to mercury and also mismatches the remaining metals. Only A contains all four correct associations.

Step 4: Conclusion Mercury matches distillation, copper electrolytic refining, silicon zone refining and nickel vapour-phase refining. Hence, the correct option is (A).

Quick Tip to Solve: Recall one distinctive property for each: volatile mercury, electrolytic copper, ultrapure silicon and nickel carbonyl.

79. Match List - I with List - II :

	List - I		List - II
(a)	Be	(i)	treatment of cancer
(b)	Mg	(ii)	extraction of metals
(c)	Ca	(iii)	incendiary bombs and signals
(d)	Ra	(iv)	windows of X-ray tubes
		(v)	bearings for motor engines

Choose the most appropriate answer from the option given below : [4 marks]

(A) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(B) (a)-(iii), (b)-(iv), (c)-(v), (d)-(ii)

(C) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(v)

(D) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Correct Answer: (D) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Solution:

Concept:

- The uses of Group 2 elements follow from properties such as low X-ray absorption, bright combustion, strong reducing power and radioactivity.
- This is a matching question: each element must be paired with its characteristic use, while the unused entry is a distractor.

Step 1: Identify beryllium and magnesium Beryllium has a low atomic number and allows X-rays to pass with comparatively little absorption. Thin beryllium windows therefore seal X-ray tubes while transmitting the emitted radiation: (a)-(iv). Magnesium burns with an intense white light, which makes it useful in incendiary bombs and signal flares: (b)-(iii). The magnesium use concerns its combustion, not transparency to X-rays.

Step 2: Identify calcium and radium Calcium is a highly electropositive metal and acts as a reducing agent in the extraction of metals from their compounds. It takes up oxygen or halogen while liberating the less electropositive metal, so (c)-(ii). Radium is radioactive; its radiation has been used in the treatment of cancer. Thus (d)-(i). The question asks for the conventional applications associated with these elements; it does not assign the motor-engine-bearing entry to any of the four matches.

Step 3: Eliminate incorrect matchings The required sequence is (iv), (iii), (ii), (i). Option A gets the first two entries right but exchanges the applications of calcium and radium. Options B and C incorrectly associate beryllium with incendiary bombs and magnesium with X-ray windows. In addition, their remaining entries do not recover the calcium-reducing-agent and radium-radiation matches. Option D is the only complete correct matching.

Step 4: Conclusion Beryllium matches X-ray tube windows, magnesium incendiary bombs and signals, calcium metal extraction and radium cancer treatment. Hence, the correct option is (D).

Quick Tip to Solve: Use the distinctive pairs Be–X-ray windows and Ra–radioactivity first; they eliminate the distractors quickly.

80. Given below are two statements :

Statement I : Non-biodegradable wastes are generated by the thermal power plants.

Statement II : Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but statement II is true.

(B) Both statement I and statement II are true.

(C) Both statement I and statement II are false.

(D) Statement I is true but statement is false.

Correct Answer: (B) Both statement I and statement II are true.

Solution:

Concept:

- Biodegradability describes whether microorganisms can decompose a waste; mineral residues such as ash are not broken down biologically like organic matter.
- Eutrophication is nutrient enrichment of water, especially by phosphates and nitrates, causing excessive algal growth and subsequent oxygen depletion.

Step 1: Evaluate Statement I Thermal power plants burn fuels, notably coal, and generate fly ash and bottom ash containing inorganic mineral material. Such residues are non-biodegradable: microorganisms do not convert these mineral solids into harmless organic decomposition products. Therefore thermal power plants do generate non-biodegradable wastes, and Statement I is true. The presence of gaseous emissions as well does not make the statement about solid wastes false.

Step 2: Evaluate Statement II Detergent formulations may contain phosphate builders. When detergent-containing wastewater enters a water body, these phosphates supply nutrients to algae and aquatic plants. Their rapid growth produces an algal bloom; decomposition of the resulting biomass consumes dissolved oxygen and harms aquatic life. This is eutrophication. A detergent being biodegradable does not imply that its nutrient contribution is absent: breakdown of the surfactant and phosphate enrichment are different issues. In this environmental-chemistry context, Statement II is true.

Step 3: Select the truth combination Both statements are true, so the required combination is B. Option A wrongly rejects the ash-producing nature of thermal power plants. Option C rejects both valid statements. Option D accepts the first statement but overlooks the nutrient enrichment caused by detergent formulations. The question is about the two truth values, rather than whether one statement explains the other.

Step 4: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (B).

Quick Tip to Solve: Do not equate biodegradable with pollution-free: phosphate nutrients can cause eutrophication even when a surfactant decomposes.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[Ag(NH_3)_2]^+$ per aldehyde group to form silver mirror is _____. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- Tollens reagent contains the diamminesilver(I) ion, $[\mathrm{Ag(NH_3)_2}]^+$. Each silver(I) centre accepts one electron when it is reduced to metallic silver.
- An aldehyde group is oxidised to a carboxylate group in the alkaline medium. The oxidation is a two-electron process for each aldehyde group.

Step 1: Balance oxidation of one aldehyde group Write the aldehyde as RCHO, where R does not participate in this oxidation. The balanced alkaline oxidation half-equation is

$$\mathrm{RCHO} + 3\,\mathrm{OH^-} \longrightarrow \mathrm{RCOO^-} + 2\,\mathrm{H_2O} + 2\,\mathrm{e^-}.$$

Both atoms and charge balance: the charge is -3 on either side when the two electrons are included. Hence a single aldehyde group supplies two electrons, rather than one. For an ordinary carbon substituent R, the carbonyl carbon changes from oxidation state $+1$ in the aldehyde to $+3$ in the carboxylate, giving the same two-electron increase.

Step 2: Balance reduction of the silver reagent Each silver ion is reduced from oxidation state $+1$ to zero:

$$[\mathrm{Ag(NH_3)_2}]^+ + \mathrm{e^-} \longrightarrow \mathrm{Ag} + 2\,\mathrm{NH_3}.$$

Multiply this half-equation by two to consume the two electrons released by the aldehyde. Consequently two silver atoms are deposited per aldehyde group, forming the visible silver mirror. The ammonia ligands coordinate silver but do not alter the one-electron requirement per silver centre.

Step 3: Read the requested electron count The question asks for the overall number of electrons transferred per aldehyde group, not the number required by just one silver complex. One aldehyde reduces two silver(I) complexes, transferring two electrons in total. This count is already an integer, so rounding does not change it.

Step 4: Conclusion The overall transfer is $\boxed{2}$ electrons per aldehyde group. The numerical entry is 2 (a dimensionless count).

Quick Tip to Solve: Balance the aldehyde oxidation half-equation first; two silver atoms deposited means two electrons transferred per aldehyde group.

82. 10.0 mL of Na_2CO_3 solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings :

4.8 mL, 4.9 mL, 5.0 mL, 5.0 mL and 5.0 mL

Based on these readings, and convention of titrimetric estimation the concentration of Na_2CO_3 solution is _____ mM.

(Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- In titrimetric estimation, use the concordant readings that reproduce the endpoint, rather than averaging preliminary or nonconcordant trials indiscriminately.
- Complete neutralisation follows $\mathrm{Na_2CO_3 + 2\,HCl \longrightarrow 2\,NaCl + H_2O + CO_2}$. Thus one mole of carbonate consumes two moles of hydrochloric acid.

Step 1: Choose the concordant titre The reported acid volumes are 4.8, 4.9, 5.0, 5.0 and 5.0 mL. Three consecutive measurements agree at 5.0 mL, so the conventional titre used for the calculation is 5.0 mL. Including the initial lower readings in a simple average would ignore the instruction to follow the convention of titrimetric estimation. The volume of the carbonate aliquot is 10.0 mL throughout.

Step 2: Calculate acid and carbonate amounts The moles of hydrochloric acid in the concordant titre are

$$n(\mathrm{HCl}) = 0.2 \times 5.0 \times 10^{-3} = 1.0 \times 10^{-3}\ \mathrm{mol}.$$

Two moles of acid neutralise one mole of carbonate, so

$$n(\mathrm{Na_2CO_3}) = \frac{1.0 \times 10^{-3}}{2} = 5.0 \times 10^{-4}\ \mathrm{mol}.$$

Using a one-to-one ratio would give twice the actual carbonate concentration; the balanced reaction fixes this factor of two.

Step 3: Find the concentration in millimolar units This carbonate amount is present in 10.0×10^{-3} L, giving

$$c = \frac{5.0 \times 10^{-4}}{10.0 \times 10^{-3}} = 0.050\ \mathrm{mol\,L^{-1}}.$$

Since $1\ \mathrm{mol\,L^{-1}} = 1000\ \mathrm{mM}$, the concentration is $50\ \mathrm{mM}$. It is already integral in the requested units, so nearest-integer rounding leaves 50.

Step 4: Conclusion The sodium carbonate concentration is $\boxed{50\ \mathrm{mM}}$. The numerical entry is 50.

Quick Tip to Solve: Use the repeated 5.0 mL titre, then divide the HCl amount by two before dividing by the carbonate aliquot volume.

83. The number of species below that have two lone pairs of electrons in their central atom is _____. (Round off to the Nearest Integer).

SF_4, BF_4^-, ClF_3, AsF_3, PCl_5, BrF_5, XeF_4, SF_6 [4 marks]

Solution:

Concept:

- Count electrons on the central atom after forming the stated single bonds. Terminal fluorine and chlorine atoms form one bond each in these species.
- For a central atom with V valence electrons and b single bonds, the remaining nonbonding electrons are $V - b$, with an extra electron for each negative charge. Divide the remainder by two to obtain the number of lone pairs.

Step 1: Count the first four species In SF_4, sulfur has six valence electrons and four bonds, leaving two electrons or one lone pair. In BF_4^-, boron contributes three electrons and the negative charge supplies one more; all four are used in the four bonds, leaving zero lone pairs. For ClF_3, chlorine has seven valence electrons and three bonds, leaving four electrons, hence two lone pairs. For AsF_3, arsenic has five valence electrons and three bonds, leaving two electrons or one lone pair.

Step 2: Count the remaining four species Phosphorus in PCl_5 uses its five valence electrons in five bonds and has no lone pair. Bromine in BrF_5 has seven valence electrons and five bonds, leaving two electrons or one lone pair. Xenon in XeF_4 has eight valence electrons and four bonds, leaving four electrons or two lone pairs. Finally, sulfur in SF_6 uses six valence electrons for six bonds and has no lone pair.

Step 3: Select exactly the two-lone-pair cases The lone-pair counts in the printed order are $1, 0, 2, 1, 0, 1, 2, 0$. Only ClF_3 and XeF_4 meet the condition. Their shapes are T-shaped and square planar respectively; the different shapes arise from five and six total electron domains, despite the same number of lone pairs. Count the species, rather than adding their lone pairs together.

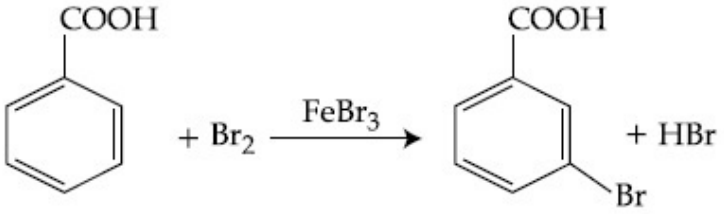
Step 4: Conclusion Exactly ClF_3 and XeF_4 qualify, so the requested number is $\boxed{2}$ species (a dimensionless count).

Quick Tip to Solve: Include ionic charge before counting leftover electrons; in particular, the extra electron makes BF4- have zero central lone pairs.

84. Consider the above reaction where 6.1 g of Benzoic acid is used to get 7.8 g of m-bromo benzoic acid. The percentage yield of the product is _____

(Round off to the Nearest Integer).

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u, Br : 80.0 u] [4 marks]



Solution:

Concept:

- Electrophilic monobromination replaces one ring hydrogen by bromine: one mole of benzoic acid gives one mole of m-bromobenzoic acid.
- Percentage yield compares the actual isolated product mass with the theoretical mass from the reactant: $\%$ yield $= 100\, m_{\text{actual}}/m_{\text{theoretical}}$.

Step 1: Calculate the two molar masses Benzoic acid has formula $C_7H_6O_2$, so its molar mass from the supplied atomic masses is

$$M_{\text{acid}} = 7(12.0) + 6(1.0) + 2(16.0) = 122.0\ \text{g mol}^{-1}.$$

Replacing one hydrogen by bromine gives $C_7H_5BrO_2$, whose molar mass is

$$M_{\text{product}} = 7(12.0) + 5(1.0) + 80.0 + 2(16.0) = 201.0\ \text{g mol}^{-1}.$$

The net mass increase is 79 rather than 80 because a hydrogen atom is replaced.

Step 2: Find the theoretical product mass The amount of benzoic acid used is

$$n = \frac{6.1}{122.0} = 0.0500\ \text{mol}.$$

The reaction has a one-to-one mole ratio between benzoic acid and the monobrominated product. Thus complete conversion would yield 0.0500 mol product, corresponding to

$$m_{\text{theoretical}} = 0.0500 \times 201.0 = 10.05\ \text{g}.$$

The catalyst does not change this mole ratio.

Step 3: Compare actual and theoretical yields The isolated product mass is only 7.8 g. Therefore

$$\%\ \text{yield} = \frac{7.8}{10.05} \times 100 = 77.6119\ldots\%.$$

This value is below 100 percent, consistent with an actual mass smaller than the theoretical maximum. Round the percentage only at the end, as the question asks for the nearest integer; rounding intermediate molar masses or mole amounts prematurely is unnecessary.

Step 4: Conclusion The percentage yield is $77.61\ldots\%$, which rounds to $\boxed{78\%}$. The numerical entry is 78.

Quick Tip to Solve: For a substitution product, subtract the replaced H when finding its formula; then compare product mass with the one-to-one theoretical yield.

85. The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and 426 S cm^2 mol^{-1} respectively. The molar conductivity at infinite dilution of barium sulphate is _____ S cm^2 mol^{-1}. (Round off to the Nearest Integer). [4 marks]

Solution:

Concept:

- At infinite dilution, each ion contributes independently to the molar conductivity of an electrolyte. Its contribution is multiplied by its stoichiometric coefficient in the dissociation equation.
- Kohlrausch's law permits addition and subtraction of limiting molar conductivities when the same ionic contributions cancel. All given conductivities use the same units, $\mathrm{S\,cm^2\,mol^{-1}}$.

Step 1: Write the ionic contributions Let b, s, h, c denote the limiting ionic molar conductivities of $\mathrm{Ba^{2+}}, \mathrm{SO_4{}^{2-}}, \mathrm{H^+}, \mathrm{Cl^-}$, respectively. The given electrolytes therefore give

$$b + 2c = 280, \qquad 2h + s = 860, \qquad h + c = 426.$$

The factor two is required for chloride in barium chloride and for hydrogen ions in sulphuric acid. It is not a factor two on the divalent ionic conductivity itself.

Step 2: Cancel the unwanted ions The desired salt dissociates as $\mathrm{BaSO_4 \longrightarrow Ba^{2+} + SO_4{}^{2-}}$, so its limiting molar conductivity is $b + s$. Add the first two equations and subtract twice the third:

$$\begin{aligned} b + s &= (b + 2c) + (2h + s) - 2(h + c) \\ &= 280 + 860 - 2(426) \\ &= 1140 - 852 = 288. \end{aligned}$$

Both hydrogen-ion and chloride-ion contributions cancel exactly; the remaining expression contains one formula unit of barium sulphate. Low solubility does not prevent applying the limiting-conductivity relation, which concerns complete separation of ions at infinite dilution.

Step 3: Conclusion The molar conductivity at infinite dilution of $\mathrm{BaSO_4}$ is $\boxed{288\ \mathrm{S\,cm^2\,mol^{-1}}}$. The nearest-integer entry is $\boxed{288}$.

Quick Tip to Solve: Construct $\mathrm{BaSO_4}$ as $\mathrm{BaCl_2 + H_2SO_4 - 2HCl}$; the coefficient 2 removes both unwanted ionic contributions.

86. The gas phase reaction $2A(g) \rightleftharpoons A_2(g)$ at 400 K has ΔG^o = + 25.2 kJ mol^{-1}. The equilibrium constant K_C for this reaction is _____ $\times 10^{-2}$. (Round off to the Nearest Integer). [Use : R = 8.3 J mol^{-1} K^{-1}, ln 10 = 2.3 $\log_{10}$ 2 = 0.30, 1 atm = 1 bar] [antilog (−0.3) = 0.501] [4 marks]

Solution:

Concept:

- For a gaseous reaction, $\Delta G^\circ = -RT \ln K_p$, with consistent energy units. A positive standard Gibbs energy makes K_p smaller than unity.
- The concentration and pressure equilibrium constants obey $K_p = K_c(RT)^{\Delta n_g}$. Here Δn_g is gaseous product moles minus gaseous reactant moles.

Step 1: Obtain the pressure equilibrium constant Convert the energy before using $R = 8.3\ \mathrm{J\,mol^{-1}\,K^{-1}}$: $\Delta G^\circ = 25200\ \mathrm{J\,mol^{-1}}$. Thus

$$\ln K_p = -\frac{25200}{8.3(400)} = -7.590.$$

Using the supplied $\ln 10 = 2.3$,

$$\log_{10} K_p = -\frac{7.590}{2.3} \simeq -3.30.$$

Therefore $K_p \simeq 10^{-3}10^{-0.30} = 0.501 \times 10^{-3}$. The small positive value is consistent with the positive standard Gibbs energy.

Step 2: Convert to the concentration constant For $2\,\mathrm{A(g)} \rightleftharpoons \mathrm{A_2(g)}$, the mole change is $\Delta n_g = 1 - 2 = -1$. Consequently $K_p = K_c/(RT)$, or $K_c = K_pRT$. For this conversion use the gas constant in litre-pressure units, $R = 0.083\ \mathrm{L\,bar\,mol^{-1}\,K^{-1}}$, obtained from the supplied value in joules. Hence

$$K_c = (0.501 \times 10^{-3})(0.083)(400) = 0.0166332.$$

Thus $K_c = 1.66332 \times 10^{-2}$ in the usual concentration convention ($\mathrm{L\,mol^{-1}}$). The question requests the integer coefficient of 10^{-2}, so round 1.66332 rather than rounding the small equilibrium constant to zero.

Step 3: Conclusion $K_c \simeq 1.66 \times 10^{-2}\ \mathrm{L\,mol^{-1}}$, so the requested nearest-integer coefficient is $\boxed{2}$.

Quick Tip to Solve: Use $R = 8.3$ in the Gibbs-energy equation but $R = 0.083$ in the pressure-to-concentration conversion; for dimerization, $K_c = K_pRT$.

87. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is _____ min. (Round off to the Nearest Integer). [Use : ln 2 = 0.69; ln 10 = 2.3] [4 marks]

Solution:

Concept:

- For first-order kinetics, $k = \ln 2/t_{1/2}$ and $t = k^{-1} \ln([A]_0/[A]_t)$. A fixed half-life leads to repeated halving of the amount remaining.

- Percentage completion must be converted into the fraction remaining. At 99.9% completion, the remaining fraction is 0.1%, or 10^{-3}, rather than 0.999.

Step 1: State the kinetic interpretation The intended numerical calculation uses first-order kinetics, for which the half-life is independent of the initial concentration. With $t_{1/2} = 1$ min,

$$k = \frac{\ln 2}{1 \text{ min}} = 0.69 \text{ min}^{-1}.$$

The wording does not explicitly specify the reaction order. A single half-life alone does not determine a time to arbitrary completion for every possible rate law, so the following numerical result uses the first-order interpretation implicit in this half-life calculation.

Step 2: Calculate the time from the remaining fraction At 99.9% completion,

$$\frac{[A]_t}{[A]_0} = 1 - \frac{99.9}{100} = 0.001.$$

Substitute this fraction into the integrated first-order equation:

$$\begin{aligned} t &= \frac{1}{0.69} \ln\left(\frac{1}{0.001}\right) \\ &= \frac{\ln(10^3)}{0.69} = \frac{3(2.3)}{0.69} \\ &= 10 \text{ min}. \end{aligned}$$

A separate check is that ten successive half-lives leave $2^{-10} = 1/1024$ of the original amount, approximately 0.098%, corresponding to about 99.902% completion. This agrees with the rounding requested and prevents confusing the percentage reacted with the percentage still present.

Step 3: Conclusion Under the first-order interpretation, the required time is $\boxed{10 \text{ min}}$, giving the nearest-integer entry $\boxed{10}$. Without a kinetic-order assumption, the printed half-life alone does not uniquely fix this time.

Quick Tip to Solve: For first-order reactions, 99.9 percent completion leaves one-thousandth; calculate ln(1000)/ln(2) half-lives, approximately ten.

88. The solubility of $CdSO_4$ in water is 8.0×10^{-4} mol L^{-1}. Its solubility in 0.01 M H_2SO_4 solution is _____ $\times 10^{-6}$ mol L^{-1}. (Round off to the Nearest Integer). (Assume that solubility is much less than 0.01 M) [4 marks]

Solution:

Concept:

- For the sparingly soluble 1:1 electrolyte $CdSO_4$, the dissolution equilibrium is $CdSO_4(s) \rightleftharpoons Cd^{2+}(aq) + {SO_4}^{2-}(aq)$, with $K_{sp} = [Cd^{2+}][{SO_4}^{2-}]$.
- An externally supplied common sulphate ion suppresses dissolution. Under the stated small-solubility approximation, the common-ion concentration is taken from the acid solution and its

change on dissolution is neglected.

Step 1: Find the solubility product in water Let the molar solubility in pure water be $s_0 = 8.0 \times 10^{-4}\ \text{mol L}^{-1}$. Each dissolved formula unit produces one cadmium ion and one sulphate ion, so both concentrations equal s_0. Consequently

$$K_{sp} = s_0^2 = (8.0 \times 10^{-4})^2 = 6.4 \times 10^{-7}.$$

The one-to-one stoichiometry is essential: there is no factor of two in either ionic concentration for this salt.

Step 2: Use the supplied sulphate concentration In the common-ion approximation intended here, the 0.01 M sulphuric acid provides $[SO_4{}^{2-}] \simeq 0.01\ \text{mol L}^{-1}$. Let the salt solubility there be s, so $[Cd^{2+}] = s$. The question explicitly permits neglecting s compared with 0.01 M. Hence

$$s(0.01) = 6.4 \times 10^{-7}, \qquad s = 6.4 \times 10^{-5}\ \text{mol L}^{-1}.$$

Expressing this in the requested power of ten gives $s = 64 \times 10^{-6}\ \text{mol L}^{-1}$. Its ratio to the common-ion concentration is 0.0064, consistent with the stated approximation. Detailed bisulphate speciation is outside this supplied-data model; no acid-dissociation constant is provided.

Step 3: Conclusion The solubility is $\boxed{64 \times 10^{-6}\ \text{mol L}^{-1}}$, and the nearest-integer coefficient to enter is $\boxed{64}$.

Quick Tip to Solve: First square the water solubility for a 1:1 salt, then divide Ksp by the common-ion concentration; convert to the requested exponent only at the end.

89. A solute A dimerizes in water. The boiling point of a 2 molal solution of A is 100.52°C. The percentage association of A is _____. (Round off to the Nearest Integer).

[Use : K_b for water = 0.52 K kg mol^{-1} Boiling point of water = 100°C] [4 marks]

Solution:

Concept:

- Boiling-point elevation depends on the number of dissolved particles: $\Delta T_b = iK_bm$, where i is the van't Hoff factor and m is the molality based on solute formula units added.
- When two monomers associate into one dimer, the particle count falls. If a fraction α of the monomer units associates, the van't Hoff factor is $i = 1 - \alpha/2$.

Step 1: Calculate the observed particle factor The elevation above the pure solvent boiling point is

$$\Delta T_b = 100.52 - 100 = 0.52\ \text{K}.$$

Using the given molality and ebullioscopic constant,

$$i = \frac{\Delta T_b}{K_bm} = \frac{0.52}{0.52(2)} = 0.50.$$

Without association, $i = 1$ would give an elevation of 1.04 K, twice the observed elevation. The measurement therefore indicates a reduced number of solute particles, as expected for association rather than dissociation.

Step 2: Relate the factor to the degree of association Consider initially one mole of monomer units in $2\,\mathrm{A} \rightleftharpoons \mathrm{A_2}$. If the degree of association is α, unassociated monomer amounts to $1-\alpha$ mole, whereas dimers amount to $\alpha/2$ mole, because each dimer uses two monomers. The final number of particle moles is therefore

$$i = (1-\alpha) + \frac{\alpha}{2} = 1 - \frac{\alpha}{2}.$$

Substituting the observed factor gives

$$0.50 = 1 - \frac{\alpha}{2}, \qquad \alpha = 1.00.$$

Thus all monomer units participate in dimers in this model. This reaches the lowest possible particle factor, one-half, for dimerization and is physically consistent with $0 \le \alpha \le 1$.

Step 3: Conclusion The percentage association is $100\alpha = \boxed{100\%}$. The requested nearest-integer entry is $\boxed{100}$.

Quick Tip to Solve: For dimerization, $i = 1 - \alpha/2$; an observed boiling-point elevation exactly half the unassociated value means complete association.

90. A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2. The number of lone pair of electrons present in compound A is _____. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- Partial hydrolysis replaces pairs of Xe–F bonds by Xe=O bonds. The fluorine-rich xenon(VI) precursor $\mathrm{XeF_6}$ gives $\mathrm{XeO_2F_2}$ when two oxygen atoms replace four fluorine atoms.
- The question asks for lone pairs in the entire compound, so count terminal-fluorine lone pairs as well as the lone pair on xenon. Each fluorine in a single bond has three lone pairs.

Step 1: Identify the xenon precursor The partial-hydrolysis reaction is

$$\mathrm{XeF_6} + 2\,\mathrm{H_2O} \longrightarrow \mathrm{XeO_2F_2} + 4\,\mathrm{HF}.$$

Both sides contain one xenon atom, six fluorine atoms, four hydrogen atoms and two oxygen atoms. Thus compound A is $\mathrm{XeF_6}$. Xenon retains oxidation state +6: the two oxygen atoms contribute -4 and the two fluorine atoms contribute -2 in the product. $\mathrm{XeF_4}$, which contains xenon(IV), is not the precursor for this xenon(VI) oxyfluoride.

Step 2: Count all lone pairs in $\mathrm{XeF_6}$ Neutral $\mathrm{XeF_6}$ has $8+6(7) = 50$ valence electrons. Six Xe–F single bonds use 12 electrons, leaving 38 nonbonding electrons. Completing the octet on each terminal fluorine requires six nonbonding electrons, or three lone pairs, per atom. The six fluorines therefore contribute $6(3) = 18$ lone pairs, using 36 electrons. The remaining two electrons form one lone pair

on xenon. Hence

$$N_{\text{lone pairs}} = 18 + 1 = 19.$$

The same total follows directly from dividing the 38 nonbonding electrons by two. Counting only the central atom would give one, but that is not the total in compound A requested here.

Step 3: Conclusion Compound A is XeF_6, which contains $\boxed{19}$ lone pairs in total: eighteen on fluorine atoms and one on xenon. The integer entry is $\boxed{19}$.

Quick Tip to Solve: For total lone pairs in XeF_6, count 3 on each of its 6 fluorines and 1 on Xe; do not confuse the molecular total with the central-atom count.