JEE Main 2021 Paper 1 (B.E./B.Tech) Sep 1 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 4 (Aug-Sep), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 1 September 2021, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let f : R → R be a continuous function. Then $\lim_{x\to\frac{\pi}{4}} \frac{\int_{2}^{\sec^2 x} f(x)\,dx}{x^2-\frac{\pi^2}{16}}$ is equal to : [4 marks]

(A) f (2)

(B) 2f (2)

(C) 2f $(\sqrt{2})$

(D) 4f (2)

Correct Answer: (B) 2f (2)

Solution:

Concept:

- For continuous f, the function $F(u) = \int_2^u f(t)\,dt$ is differentiable and $F\prime(u) = f(u)$. The variable inside a definite integral is a dummy variable.
- If the numerator and denominator both vanish, their first-order changes determine the limit when the denominator has nonzero derivative. The chain rule differentiates the variable upper endpoint.

Step 1: Identify the vanishing quantities Put $a = \pi/4$. Since $\sec^2 a = 2$, the integral has coincident endpoints at the limiting point. Thus the numerator tends to zero, as does $x^2 - a^2$. Rewrite the integral as $F(\sec^2 x)$ to avoid confusing its dummy variable with the outer variable x. Continuity is enough to differentiate F; differentiability of f itself is not required.

Step 2: Differentiate the endpoint and evaluate Using the fundamental theorem and the chain rule, the derivative of the numerator is

$$\frac{\pi}{4} f(\sec^2 x)\, 2\sec^2 x \tan x.$$

The denominator has derivative $2x$, whose value at a is $\pi/2 \neq 0$. Therefore the ratio of first-order changes is

$$\frac{(\pi/4) f(2)\, 2(2)(1)}{\pi/2} = 2f(2).$$

The argument is 2, not $\sqrt{2}$, because the upper endpoint is $\sec^2 x$. Options (A) and (D) have incorrect coefficients; option (C) uses the wrong endpoint.

Step 3: Conclusion The limit equals $2f(2)$. Hence, the correct option is (B).

Quick Tip to Solve: First evaluate the upper endpoint at the limiting point, then multiply by its derivative; do not substitute $\sec x$ for $\sec^2 x$.

2. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ **is equal to :**

(The inverse trigonometric functions take the principal values) [4 marks]

(A) $3\pi - 11$

(B) $4\pi - 9$

(C) $4\pi - 11$

(D) $3\pi + 1$

Correct Answer: (C) $4\pi - 11$

Solution:

Concept:

- The principal ranges are $[0, \pi]$ for $\cos^{-1}$, $[-\pi/2, \pi/2]$ for $\sin^{-1}$, and $(-\pi/2, \pi/2)$ for $\tan^{-1}$.
- Use the period and symmetry of each trigonometric function to move its argument into the appropriate principal range. All the given arguments are in radians.

Step 1: Reduce the cosine and sine arguments Cosine is even, so $\cos(-5) = \cos 5$. Since $\pi < 5 < 2\pi$, the angle with that cosine in $[0, \pi]$ is $2\pi - 5$. Also, $6 - 2\pi$ belongs to $(-\pi/2, 0)$, and sine has period 2π. Hence

$$\cos^{-1}(\cos(-5)) = 2\pi - 5, \qquad \sin^{-1}(\sin 6) = 6 - 2\pi.$$

Each reduced angle lies in its required range, so the inverse functions select exactly these values.

Step 2: Reduce the tangent and combine Tangent has period π. The number $12 - 4\pi$ is approximately -0.566, inside $(-\pi/2, \pi/2)$. Therefore

$$\tan^{-1}(\tan 12) = 12 - 4\pi.$$

Keeping the minus sign before this entire term gives

$$(2\pi - 5) + (6 - 2\pi) - (12 - 4\pi) = 4\pi - 11.$$

The cosine and sine contributions alone add to 1. This check excludes options (A) and (D), which have the wrong coefficient of π, and option (B), which has the wrong constant. Inverse functions cannot simply cancel trigonometric functions outside their principal intervals.

Step 3: Conclusion The required principal-value expression is $4\pi - 11$. Hence, the correct option is (C).

Quick Tip to Solve: Reduce each angle separately to the inverse function's own principal interval before adding or subtracting the terms.

3. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then [4 marks]

(A) $n(S_1) = 2, n(S_2) = 2$

(B) $n(S_1) = 1, n(S_2) = 0$

(C) $n(S_1) = 2, n(S_2) = 0$

(D) $n(S_1) = 0, n(S_2) = 2$

Correct Answer: (C) $n(S_1) = 2, n(S_2) = 0$

Solution:

Concept:

- Elimination preserves the solution set and distinguishes unique, inconsistent and dependent systems without dividing by a parameter that could be zero.
- An equation $0 = c$, with $c \neq 0$, makes a system inconsistent. Infinitely many solutions require compatible dependence, rather than merely a singular coefficient matrix.

Step 1: Eliminate the first unknown The first equation gives $x = y + 2z$. Substitute this relation into both remaining equations:

$$3(y + 2z) - ay + 5z = 1 \quad \Longrightarrow \quad (3 - a)y + 11z = 1,$$

$$2(y + 2z) - 2y - az = 7 \quad \Longrightarrow \quad (4 - a)z = 7.$$

This triangular form identifies all exceptional parameter values directly: $a = 3$ and $a = 4$. For every other real a, the last equation determines z, the preceding one determines y, and then the first determines x. Such values yield one solution and belong to neither requested set.

Step 2: Test each exceptional value At $a = 4$, the last reduced equation is $0 = 7$, which is impossible. At $a = 3$, the last equation gives $z = 7$, whereas the other reduced equation requires $11z = 1$. Substitution yields $77 = 1$, again impossible. Therefore

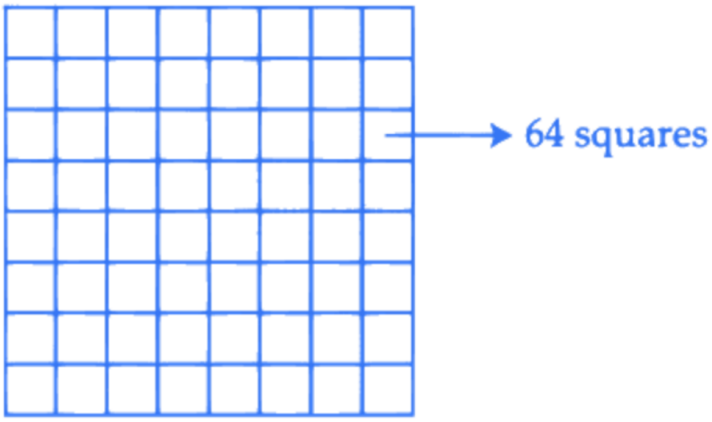
$$S_1 = \{3, 4\}, \qquad S_2 = \varnothing.$$

Neither singular case is compatible, so neither produces a free variable and infinitely many solutions. Options (A) and (D) wrongly include such solutions, and option (B) overlooks one of the two inconsistent cases.

Step 3: Conclusion We obtain $n(S_1) = 2$ and $n(S_2) = 0$. Hence, the correct option is (C).

Quick Tip to Solve: After triangular elimination, test each value making a leading coefficient zero in the original reduced equations before dividing.

4. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is : [4 marks]



(A) $\frac{2}{7}$

(B) $\frac{1}{18}$

(C) $\frac{1}{7}$

(D) $\frac{1}{9}$

Correct Answer: (B) $\frac{1}{18}$

Solution:

Concept:

- Choosing two distinct squares at random from the 64 unit squares gives $\binom{64}{2}$ equally likely unordered pairs.
- Two unit squares share a side precisely when they are horizontal or vertical neighbours. Diagonally touching squares share only a vertex and are not favourable pairs.

Step 1: Count all possible pairs The figure is an 8×8 chessboard containing 64 unit squares. We choose the two squares without replacement; the order in which they are named does not matter. Thus

$$N = \binom{64}{2} = \frac{64 \cdot 63}{2} = 2016.$$

Using unordered pairs throughout prevents an extra factor of two. In particular, the sample space does not include choosing a square twice, because the question asks for two squares.

Step 2: Count the common-side pairs In each row there are seven horizontal neighbour pairs. With eight rows, these contribute $8 \cdot 7 = 56$. The same argument for eight columns gives 56 vertical neighbour pairs. No pair occurs in both groups: one pair cannot lie in both the same row and the same column. Hence

$$N_{\text{fav}} = 56 + 56 = 112, \qquad P = \frac{112}{2016} = \frac{1}{18}.$$

For a check, the sum of the numbers of neighbours of all squares is $4 \cdot 2 + 24 \cdot 3 + 36 \cdot 4 = 224$; dividing by two again gives 112 pairs. Options (A), (C) and (D) exceed this correctly counted ratio, with (D) double-counting favourable unordered pairs.

Step 3: Conclusion The probability of a common side is $1/18$. Hence, the correct option is (B).

Quick Tip to Solve: Count shared internal edges: there are $8 \cdot 7$ in each direction, and each edge identifies exactly one favourable pair.

5. If y = y(x) is the solution curve of the differential equation $x^2dy + \left(y - \frac{1}{x}\right)dx = 0$; x > 0 and y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to : [4 marks]

(A) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

(B) $3 + \frac{1}{\sqrt{e}}$

(C) 3 + e

(D) 3 − e

Correct Answer: (D) 3 − e

Solution:

Concept:

- For $y\prime + P(x)y = Q(x)$, the integrating factor is $I(x) = e^{\int P(x)\,dx}$; multiplication turns the left side into $(Iy)\prime$.
- The initial condition fixes the integration constant. Since $x > 0$, dividing by x^2 and evaluating at $x = 1/2$ are valid.

Step 1: Find and apply the integrating factor Divide the differential equation by x^2dx and rearrange:

$$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}, \qquad I(x) = e^{-1/x}.$$

Consequently

$$\frac{d}{dx}(ye^{-1/x}) = \frac{e^{-1/x}}{x^3}.$$

For the integral, put $u = 1/x$, so $du = -dx/x^2$ and $dx/x^3 = -u\,du$. Integration by parts gives

$$\int \frac{e^{-1/x}}{x^3}\,dx = -\int ue^{-u}\,du = (u+1)e^{-u} + C.$$

Thus $ye^{-1/x} = (1 + 1/x)e^{-1/x} + C$.

Step 2: Use the initial value and substitute Multiplication by $e^{1/x}$ gives

$$y = 1 + \frac{1}{x} + Ce^{1/x}.$$

At $x = 1$, the condition $y(1) = 1$ implies $1 = 2 + Ce$, hence $C = -e^{-1}$. Therefore

$$y(x) = 1 + \frac{1}{x} - e^{1/x-1}, \qquad y(1/2) = 3 - e.$$

As a check, substituting $x = 1$ returns 1. The negative homogeneous contribution excludes the positive-exponential options (B) and (C); option (A) uses an incorrect exponent and rational term. Differentiating this solution also reproduces the given differential equation.

Step 3: Conclusion The required value is $y(1/2) = 3 - e$. Hence, the correct option is (D).

Quick Tip to Solve: For coefficients involving x^{-2} and x^{-3}, the substitution $u = 1/x$ makes the integrating-factor integral simple.

6. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements :

Statement 1 : there exists x_1, $x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

Statement 2 : there exists x_3, $x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$.

Then [4 marks]

(A) both Statement 1 and Statement 2 are true

(B) Statement 1 is false and Statement 2 is true

(C) both Statement 1 and Statement 2 are false

(D) Statement 1 is true and Statement 2 is false

Correct Answer: (A) both Statement 1 and Statement 2 are true

Solution:

Concept:

- A monic cubic with known roots factors into linear terms; its x^2 coefficient determines the sum of all three roots.
- The sign of $f\prime$ determines monotonicity. Existence statements can be proved by explicitly finding points inside the required interval and checking their order.

Step 1: Factor the cubic and prove Statement 1 The roots 2 and 4 and the sum of roots 6 force the third root to be 0. Thus

$$f(x) = x(x-2)(x-4), \qquad f\prime(x) = 3(x-2)^2 - 4.$$

Equivalently, $a = 8$ and $b = 0$. On $(2, 4)$, solving $f\prime(x_1) = -1$ gives $x_1 = 3$; solving $f\prime(x_2) = 0$ gives $x_2 = 2 + 2/\sqrt{3}$. Both points lie in $(2, 4)$, and $3 < 2 + 2/\sqrt{3}$. This explicitly establishes every condition in Statement 1.

Step 2: Check monotonicity and prove Statement 2 Take $x_4 = 2 + 2/\sqrt{3}$. The derivative is negative for $2 < x < x_4$ and positive for $x_4 < x < 4$, so the specified decrease and increase hold. With $t = x - 2$, we have $f(x) = t^3 - 4t$, and hence

$$f(x_4) = -\frac{16}{3\sqrt{3}}.$$

The remaining equality requires $f\prime(x_3) = -8/3$. Substituting the derivative gives

$$3(x_3 - 2)^2 - 4 = -\frac{8}{3} \quad \Longrightarrow \quad x_3 = \frac{8}{3}.$$

This point belongs to $(2, 4)$ and satisfies $x_3 < x_4$; also $2f\prime(x_3) = -16/3 = \sqrt{3}f(x_4)$. Statement 2 is therefore true. Each of options (B), (C) and (D) wrongly declares at least one verified statement false.

Step 3: Conclusion Both Statement 1 and Statement 2 are true. Hence, the correct option is (A).

Quick Tip to Solve: Shift to $t = x - 2$: the cubic becomes $t^3 - 4t$, making both its stationary value and derivative equations easy to evaluate.

7. Let $J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m-1}dx$, $\forall$ n $>$ m and n, m $\in$ N. Consider a matrix $A = [a_{ij}]_{3\times 3}$ where $a_{ij} = \begin{cases} j_{6+i,3} - j_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $\left|adjA^{-1}\right|$ is : [4 marks]

(A) $(15)^2 \times 2^{42}$

(B) $(15)^2 \times 2^{34}$

(C) $(105)^2 \times 2^{38}$

(D) $(105)^2 \times 2^{36}$

Correct Answer: (C) $(105)^2 \times 2^{38}$

Solution:

Concept:

- The determinant of an upper triangular matrix is the product of its diagonal entries; the off-diagonal entries do not affect it.
- For a nonsingular 3×3 matrix B, $\det(\text{adj}\, B) = (\det B)^2$. Taking $B = A^{-1}$ therefore gives $\det(\text{adj}\, A^{-1}) = (\det A)^{-2}$.

Step 1: Simplify the difference of integrals For each row index $i = 1, 2, 3$, combine the two integrals before integrating:

$$J_{6+i,3} - J_{i+3,3} = \int_0^{1/2} \frac{x^{i+6} - x^{i+3}}{x^3 - 1}\, dx.$$

The numerator is $x^{i+3}(x^3 - 1)$. Since $x^3 - 1 \neq 0$ throughout $[0, 1/2]$, cancellation is valid everywhere on the interval. Thus every entry on and above row i's diagonal equals

$$\int_0^{1/2} x^{i+3}\, dx = \frac{1}{(i+4)2^{i+4}}.$$

In particular, the three diagonal entries are $1/(5 \cdot 2^5)$, $1/(6 \cdot 2^6)$ and $1/(7 \cdot 2^7)$.

Step 2: Find the determinant and apply the identity The entries below the diagonal are zero, so

$$\det A = \frac{1}{5 \cdot 6 \cdot 7 \cdot 2^{18}} = \frac{1}{105 \cdot 2^{19}}.$$

This is positive and nonzero, so A^{-1} exists. Apply the adjugate determinant formula:

$$\left|\text{adj}\, A^{-1}\right| = (\det A)^{-2} = 105^2\, 2^{38}.$$

The extra power of two in the determinant comes from $210 = 2 \cdot 105$. Options (A) and (B) omit the factor 7 in the product, while option (D) misses that extra factor of two before squaring.

Step 3: Conclusion The determinant is $105^2 \times 2^{38}$. Hence, the correct option is (C).

Quick Tip to Solve: Combine integrals with the same denominator first; here cancellation turns every needed diagonal entry into an elementary power integral.

8. The area, enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ and the lines $x = 0, x = \frac{\pi}{2}$, is : [4 marks]

(A) $2\sqrt{2}(\sqrt{2} - 1)$

(B) $2(\sqrt{2} + 1)$

(C) $4(\sqrt{2} - 1)$

(D) $2\sqrt{2}(\sqrt{2} + 1)$

Correct Answer: (A) $2\sqrt{2}(\sqrt{2} - 1)$

Solution:

Concept:

- The area between two graphs is the integral of the upper ordinate minus the lower ordinate over the prescribed interval.
- An absolute value changes its formula where its argument changes sign. Here $\cos x - \sin x = 0$ at $x = \pi/4$.

Step 1: Identify the upper curve On $[0, \pi/2]$, both sine and cosine are nonnegative. Therefore $\sin x + \cos x \geq |\cos x - \sin x|$, so the integrand for area is nonnegative throughout. On the first half the absolute value equals $\cos x - \sin x$; on the second half it equals $\sin x - \cos x$. Thus the vertical difference is $2\sin x$ before the midpoint and $2\cos x$ after it.

Step 2: Integrate the two pieces Split at the point where the absolute-value expression changes sign:

$$A = \int_0^{\pi/4} 2\sin x\, dx + \int_{\pi/4}^{\pi/2} 2\cos x\, dx.$$

Each integral is $2(1 - 1/\sqrt{2})$. Consequently,

$$A = 4 - 2\sqrt{2} = 2\sqrt{2}(\sqrt{2} - 1).$$

Adding the two original ordinates instead would count regions outside the required strip. Options (B) and (D) exceed even the integral of the upper curve, which is 2; option (C) is approximately 1.657 and differs from the correctly integrated value, approximately 1.172.

Step 3: Conclusion The required area is $\boxed{2\sqrt{2}(\sqrt{2} - 1)}$ square units. Hence, the correct option is (A).

Quick Tip to Solve: For a sine-cosine absolute value on the first quadrant, split at $\pi/4$; the two area integrals are equal by symmetry.

9. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point (2, −1, 6) is : [4 marks]

(A) $\sqrt{26}$

(B) $2\sqrt{5}$

(C) $2\sqrt{6}$

(D) $4\sqrt{2}$

Correct Answer: (C) $2\sqrt{6}$

Solution:

Concept:

- A line defined as the intersection of two planes can be parametrised by choosing one coordinate and solving for the other two.
- For a point P, a point A on a line with direction $\mathbf{d}$, the perpendicular distance satisfies $h^2 = |\overrightarrow{AP}|^2 - (\overrightarrow{AP} \cdot \mathbf{d})^2/|\mathbf{d}|^2$.

Step 1: Parametrise the given line Put $x = t$. From $3x - z + 4 = 0$, obtain $z = 3t + 4$. Then $3y - 2z - 1 = 0$ gives $y = 2t + 3$. The line is therefore

$$(x, y, z) = (0, 3, 4) + t(1, 2, 3).$$

We may use $A = (0, 3, 4)$ and direction vector $\mathbf{d} = (1, 2, 3)$. Substituting A into both original equations verifies that it lies on their intersection, rather than on only one of the planes.

Step 2: Check the perpendicular and calculate its length For $P = (2, -1, 6)$, the displacement from A is $\overrightarrow{AP} = (2, -4, 2)$. Its scalar product with the line direction is

$$(2, -4, 2) \cdot (1, 2, 3) = 2 - 8 + 6 = 0.$$

Hence AP itself is perpendicular to the line, and A is the foot of the perpendicular. Thus

$$h = \sqrt{2^2 + (-4)^2 + 2^2} = \sqrt{24} = 2\sqrt{6}.$$

The squares of options (A), (B) and (D) are 26, 20 and 32, respectively, none equal to the perpendicular displacement squared, 24.

Step 3: Conclusion The perpendicular distance is $\boxed{2\sqrt{6}}$ coordinate units. Hence, the correct option is (C).

Quick Tip to Solve: After finding a convenient point on a line, first test the displacement dot direction: a zero immediately identifies the perpendicular foot.

10. The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is : [4 marks]

(A) 6

(B) 2

(C) 4

(D) 8

Correct Answer: (A) 6

Solution:

Concept:

- The transformation $T(r) = r^2 - 2$ must send each root of the quadratic into its set of roots.
- A quadratic has either one repeated root or two distinct roots. On a set of two roots, a transformation can fix both, send one to a fixed root, or interchange them.

Step 1: Find the repeated-root and fixed-root cases A fixed root obeys $r^2 - 2 = r$, giving $r = 2$ or $r = -1$. A repeated root must be fixed, giving the two polynomials $(x - 2)^2$ and $(x + 1)^2$. With two distinct fixed roots, the root set is $\{2, -1\}$, giving $(x - 2)(x + 1)$. These account for three coefficient pairs. The same fixed-point equation also shows that complex fixed roots add no further cases.

Step 2: Find roots that eventually reach a fixed root If one distinct root maps to the other and the latter is fixed, use the two possible fixed roots. For fixed root 2, solve $r^2 - 2 = 2$; the distinct predecessor is -2. For fixed root -1, solve $r^2 - 2 = -1$; the distinct predecessor is 1. This adds the root sets $\{-2, 2\}$ and $\{1, -1\}$, hence polynomials $x^2 - 4$ and $x^2 - 1$.

Step 3: Count the genuine two-cycle For distinct interchanged roots r, s, we have $r^2 - 2 = s$ and $s^2 - 2 = r$. Subtraction gives $(r - s)(r + s + 1) = 0$, so $r + s = -1$. Substitution gives $r^2 + r - 1 = 0$; these two roots indeed interchange. They give just one polynomial, $x^2 + x - 1$. All possible maps of a two-element root set have now been covered. The six pairs (a, b) are

$$(-4, 4),\ (2, 1),\ (-1, -2),\ (0, -4),\ (0, -1),\ (1, -1).$$

Thus 2 and 4 omit valid cases, while 8 overcounts root orderings or impossible maps.

Step 4: Conclusion There are $\boxed{6}$ distinct real coefficient pairs. Hence, the correct option is (A).

Quick Tip to Solve: Count invariant root sets under the given transformation, including repeated roots and one-way maps into fixed points; do not count root orderings separately.

11. Let P_1, P_2, ..., P_{15} be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that i +j + k $\neq$ 15, is : [4 marks]

(A) 12

(B) 419

(C) 443

(D) 455

Correct Answer: (C) 443

Solution:

Concept:

- Any three distinct points on a circle form one nondegenerate triangle, so the total number is $\binom{15}{3}$.
- To exclude index sum 15 without counting permutations repeatedly, write the three indices in increasing order $1 \le i < j < k \le 15$.

Step 1: Count all triangles The choice of the vertices is unordered. Therefore

$$N_{\text{all}} = \binom{15}{3} = \frac{15 \cdot 14 \cdot 13}{6} = 455.$$

We must remove exactly those triples whose sum is 15. Counting ordered index triples and subtracting them directly would remove each triangle several times, so fix an increasing ordering before enumerating the exclusions.

Step 2: Enumerate the excluded triples With $k = 15 - i - j$, the requirement $k > j$ gives $2j < 15 - i$. For $i = 1$, the possible j are 2,3,4,5,6, giving five triples. For $i = 2$, the possible j are 3,4,5,6, giving four. For $i = 3$, they are 4,5, giving two. For $i = 4$, only $j = 5$ works, giving one. No larger i is possible, since even $5 + 6 + 7$ exceeds 15. Every resulting k is distinct from the first two and at most 15. Thus the excluded count is $5 + 4 + 2 + 1 = 12$.

Step 3: Subtract and compare the choices The required number is

$$N = 455 - 12 = 443.$$

Option (A), 12, is the excluded count rather than the required count. Option (D), 455, ignores the restriction altogether. Option (B), 419, would subtract 36, which can result from counting the same excluded triangles repeatedly. The increasing-index enumeration counts each forbidden triangle exactly once.

Step 4: Conclusion The number of required distinct triangles is $\boxed{443}$. Hence, the correct option is (C).

Quick Tip to Solve: For a restriction on the sum of vertex indices, count the small forbidden set using $i < j < k$, then subtract from the binomial total.

12. The range of the function,

$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$ **is :** [4 marks]

(A) $\left(0, \sqrt{5}\right)$

(B) $[-2, 2]$

(C) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

(D) [0, 2]

Correct Answer: (D) [0, 2]

Solution:

Concept:

- Use $\cos(A+B) + \cos(A-B) = 2\cos A \cos B$ and $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$.
- For a logarithm with base greater than 1, the logarithmic function is increasing. Apply it to the exact positive range of its argument.

Step 1: Simplify the argument Pair the terms with angle $\pi/4$, whose sum is $\sqrt{2}\cos x$. The difference of the terms with angle $3\pi/4$ is $-\sqrt{2}\sin x$. Thus the complete argument becomes

$$g(x) = 3 + \sqrt{2}\cos x - \sqrt{2}\sin x = 3 + 2\cos(x + \pi/4).$$

The minus sign before the final cosine is essential: pairing it as a sum would change both the amplitude and the range.

Step 2: Find the argument range and take logarithms As x runs over the real numbers, $\cos(x + \pi/4)$ takes every value from -1 to 1, including both endpoints. Hence $1 \le g(x) \le 5$. This also proves the logarithm is defined for all real x. Since $\sqrt{5} > 1$, the order of the endpoints is preserved:

$$\log_{\sqrt{5}} 1 \le f(x) \le \log_{\sqrt{5}} 5, \qquad 0 \le f(x) \le 2.$$

Both endpoints occur at angles where the cosine is respectively -1 and 1. Option (A) wrongly excludes endpoints and has the wrong upper bound; (B) includes negative values although the argument never falls below 1; (C) does not contain the attained value zero.

Step 3: Conclusion The range is $\boxed{[0, 2]}$. Hence, the correct option is (D).

Quick Tip to Solve: Simplify all trigonometric terms into a single sinusoid before finding a logarithmic range, and check whether the base preserves or reverses order.

13. Let $a_1, a_2, \ldots\ldots\ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to : [4 marks]

(A) 57

(B) 72

(C) 48

(D) 36

Correct Answer: (B) 72

Solution:

Concept:

- In a 21-term arithmetic progression, the middle term is the average: $S_{21} = 21a_{11}$. Write terms symmetrically about this middle term.
- If the common difference is nonzero, $1/(a_n a_{n+1}) = (1/d)(1/a_n - 1/a_{n+1})$, so the sum telescopes.

Step 1: Determine the middle term From $S_{21} = 189$, obtain $a_{11} = 189/21 = 9$. If d denotes the common difference, then

$$a_1 = 9 - 10d, \quad a_{21} = 9 + 10d, \quad a_6 = 9 - 5d, \quad a_{16} = 9 + 5d.$$

First rule out $d = 0$: that would make all terms 9 and the given reciprocal sum $20/81$, which is not $4/9$. We may therefore divide by d in the next step.

Step 2: Telescope the reciprocal sum All denominators in the given sum are nonzero. Applying partial fractions gives

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{21}}\right)$$
$$= \frac{a_{21} - a_1}{d\, a_1 a_{21}} = \frac{20}{a_1 a_{21}} = \frac{4}{9}.$$

Therefore $a_1 a_{21} = 45$. On the other hand this endpoint product equals $81 - 100d^2$, so $100d^2 = 36$ and $d^2 = 9/25$.

Step 3: Calculate the requested symmetric product Using the two terms situated five places on either side of the middle,

$$a_6 a_{16} = (9 - 5d)(9 + 5d) = 81 - 25d^2 = 81 - 9 = 72.$$

The sign of the common difference does not affect this product. The other proposed values 57, 48 and 36 all contradict the determined value $25d^2 = 9$. Note that the endpoint product 45 is an intermediate result, not the requested product.

Step 4: Conclusion The required product is $\boxed{a_6 a_{16} = 72}$. Hence, the correct option is (B).

Quick Tip to Solve: For an odd-length AP with a known sum, centre the terms about the middle; symmetric products then depend only on the square of the common difference.

14. The function f(x), that satisfies the condition

$f(x) = x + \int_0^{\pi/2} \sin x . \cos y \, f(y) \, dy$**, is :** [4 marks]

(A) $x + \frac{2}{3}(\pi - 2)\sin x$

(B) $x + (\pi + 2)\sin x$

(C) $x + \frac{\pi}{2}\sin x$

(D) $x + (\pi - 2)\sin x$

Correct Answer: (D) $x + (\pi - 2)\sin x$

Solution:

Concept:

- In an integral over y, a factor depending only on x is constant with respect to integration and can be taken outside.
- The definite integral $C = \int_0^{\pi/2} \cos y \, f(y) \, dy$ is a number, independent of x. This reduces the equation to $f(x) = x + C \sin x$.

Step 1: Reduce to an equation for one constant Substitute $f(y) = y + C \sin y$ into the definition of C. This gives

$$C = \int_0^{\pi/2} y \cos y \, dy + C \int_0^{\pi/2} \sin y \cos y \, dy.$$

The unknown function has disappeared: only the scalar C remains. This substitution is valid for the same argument y because the original identity holds for all inputs.

Step 2: Evaluate the two definite integrals Integration by parts, with $u = y$ and $dv = \cos y \, dy$, gives

$$\int_0^{\pi/2} y \cos y \, dy = [y \sin y + \cos y]_0^{\pi/2} = \frac{\pi}{2} - 1.$$

Also,

$$\int_0^{\pi/2} \sin y \cos y \, dy = \left[\frac{\sin^2 y}{2}\right]_0^{\pi/2} = \frac{1}{2}.$$

Therefore $C = \pi/2 - 1 + C/2$, which uniquely determines $C = \pi - 2$.

Step 3: Substitute and check the coefficient It follows that $f(x) = x + (\pi - 2)\sin x$. Direct substitution into the defining integral returns $C = (\pi - 2)/2 + (\pi - 2)/2 = \pi - 2$, confirming the identity. Every proposed function is of the form $x + k \sin x$, but its coefficient must satisfy $k = \pi/2 - 1 + k/2$. The coefficients in (A), (B) and (C) do not satisfy this equation, so none gives the required function.

Step 4: Conclusion The function is $\boxed{f(x) = x + (\pi - 2)\sin x}$. Hence, the correct option is (D).

Quick Tip to Solve: When the integral kernel splits into an x-factor times a y-factor, name the definite y-integral as a constant and solve its consistency equation.

15. Let the acute angle bisector of the two planes x − 2y − 2z + 1 = 0 and 2x − 3y − 6z + 1 = 0 be the plane P. Then which of the following points lies on P? [4 marks]

(A) $\left(3, 1, -\frac{1}{2}\right)$

(B) $\left(-2, 0, -\frac{1}{2}\right)$

(C) (0, 2, −4)

(D) (4, 0, −2)

Correct Answer: (B) $\left(-2, 0, -\frac{1}{2}\right)$

Solution:

Concept:

- The bisector planes consist of points with equal perpendicular distances from the two given planes. Divide each plane expression by the length of its normal before equating distances.
- For normals making an acute angle, the sum of the normalized plane expressions gives the bisector whose normal lies between their unit normals; this is the acute-angle bisector of the planes.

Step 1: Normalize the two plane equations Write $L_1 = x - 2y - 2z + 1$ and $L_2 = 2x - 3y - 6z + 1$. Their normal vectors and lengths are

$$\mathbf{n}_1 = (1, -2, -2), \quad |\mathbf{n}_1| = 3,$$
$$\mathbf{n}_2 = (2, -3, -6), \quad |\mathbf{n}_2| = 7.$$

Thus the two possible bisectors are $L_1/3 = \pm L_2/7$. The denominators are essential: the original expressions are signed distances only after normalization.

Step 2: Select the acute-angle bisector The dot product is $\mathbf{n}_1 \cdot \mathbf{n}_2 = 2 + 6 + 12 = 20 > 0$, so these normals make an acute angle. The sum of their unit normals bisects that angle. Hence the required plane is

$$\frac{L_1}{3} + \frac{L_2}{7} = 0.$$

Multiplying by 21 gives

$$7(x - 2y - 2z + 1) + 3(2x - 3y - 6z + 1) = 0,$$

so P is $13x - 23y - 32z + 10 = 0$.

Step 3: Test the listed points At $(-2, 0, -1/2)$, the left side is $-26 + 0 + 16 + 10 = 0$, so this point lies on P. For the other three points, in the order (A), (C), (D), the same expression gives $42, 92, 126$, respectively. All are nonzero, so none of those points belongs to the required bisector.

Step 4: Conclusion The point $\boxed{(-2, 0, -1/2)}$ lies on the acute-angle bisector. Hence, the correct option is (B).

Quick Tip to Solve: Normalize both plane equations first. Use the dot product of the displayed normals to choose the appropriate bisector, then test the options in its linear equation.

16. Which of the following is equivalent to the Boolean expression p $\wedge \sim$ q ? [4 marks]

(A) $\sim$ (q $\rightarrow$ p)

(B) $\sim$ p $\rightarrow \sim$ q

(C) $\sim$ (p $\rightarrow \sim$ q)

(D) $\sim$ (p $\rightarrow$ q)

Correct Answer: (D) $\sim$ (p $\rightarrow$ q)

Solution:

Concept:

- An implication $p \rightarrow q$ is false exactly when p is true and q is false. Algebraically, $p \rightarrow q \equiv \sim p \vee q$.
- De Morgan's law changes the negation of a disjunction into the conjunction of the negations: $\sim (A \vee B) \equiv (\sim A) \wedge (\sim B)$.

Step 1: Express the target truth condition The expression $p \wedge \sim q$ is true only for $p = \mathrm{T}$, $q = \mathrm{F}$. An equivalent expression must have exactly this same truth condition, not merely agree for one other choice of truth values. Since this is precisely the failure condition of $p \rightarrow q$, its negation is the natural candidate.

Step 2: Simplify the negated implication Replace the implication by a disjunction, and then apply De Morgan's law:

$$\begin{aligned} \sim (p \rightarrow q) &\equiv \sim (\sim p \vee q) \\ &\equiv \sim (\sim p) \wedge \sim q \\ &\equiv p \wedge \sim q. \end{aligned}$$

Double negation restores p, so option (D) gives the target for every assignment of truth values.

Step 3: Check the remaining expressions Option (A) is $\sim (q \rightarrow p) \equiv q \wedge \sim p$, which reverses the required true and false variables. Option (B) is $(\sim p) \rightarrow (\sim q) \equiv p \vee \sim q$; an OR permits more true cases than the target AND. Option (C) is $\sim (p \rightarrow \sim q) \equiv p \wedge q$, which requires q to be true rather than false. For example, at $p = \mathrm{T}, q = \mathrm{T}$, (B) and (C) are true but the target is false. Thus none of (A)–(C) is equivalent.

Step 4: Conclusion $\boxed{p \wedge \sim q \equiv \sim (p \rightarrow q)}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the unique false case of an implication: true antecedent and false consequent. Negating the implication gives exactly that conjunction.

17. If n is the number of solutions of the equation

$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$ **and S is the sum of all these solutions, then the ordered pair (n, S) is :** [4 marks]

(A) (3, 13π / 9)

(B) (2, 2π / 3)

(C) (2, 8π / 9)

(D) (3, 5π / 3)

Correct Answer: (A) (3, 13π / 9)

Solution:

Concept:

- Use the product-to-sum identity $2\sin A\sin B=\cos(A-B)-\cos(A+B)$ to simplify the two sine factors.
- The triple-angle identity $\cos 3x=4\cos^3 x-3\cos x$ converts the resulting cubic expression into a basic cosine equation. Its solutions must be restricted to the given interval.

Step 1: Reduce the product of the sine terms For $A=\pi/4+x$ and $B=\pi/4-x$, their sum is $\pi/2$ and their difference is $2x$. Therefore

$$\sin(\pi/4+x)\sin(\pi/4-x)=\frac{\cos 2x}{2}.$$

The expression inside the parentheses in the question becomes

$$4\sin(\pi/4+x)\sin(\pi/4-x)-1=2\cos 2x-1=4\cos^2 x-3.$$

These identities hold throughout $[0,\pi]$; no division or squaring has introduced any restrictions or extra roots.

Step 2: Solve the triple-angle equation on the interval The given equation is now

$$2\cos x(4\cos^2 x-3)=1 \quad\Longleftrightarrow\quad \cos 3x=\frac{1}{2}.$$

Since $0\le x\le\pi$, the angle $3x$ lies in $[0,3\pi]$. In that interval, cosine is $1/2$ at precisely

$$3x=\frac{\pi}{3},\quad \frac{5\pi}{3},\quad \frac{7\pi}{3}.$$

The next positive candidate $11\pi/3$ exceeds 3π, and the endpoints are not solutions. Dividing these three values by 3 gives $x=\pi/9, 5\pi/9, 7\pi/9$.

Step 3: Count and add the solutions There are three distinct roots, and their sum is

$$S=\frac{\pi+5\pi+7\pi}{9}=\frac{13\pi}{9}.$$

Options (B) and (C) miss one of the roots. Option (D) has the right count but an incorrect sum; $5\pi/3=15\pi/9$, not $13\pi/9$.

Step 4: Conclusion $\boxed{(n, S) = (3, 13\pi/9)}$. Hence, the correct option is (A).

Quick Tip to Solve: After using a multiple-angle identity, transform the interval along with the angle. Here counting roots for $3x \in [0, 3\pi]$ prevents missing the third solution.

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18. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to : [4 marks]

(A) $\frac{75}{8}$
(B) $\frac{125}{16}$
(C) $\frac{25}{2}$
(D) $\frac{15}{2}$

Correct Answer: (B) $\frac{125}{16}$

Solution:

Concept:

- For a vertical parabola with vertex (h, k), the form is $(x-h)^2 = 4a(y-k)$, with directrix $y = k - a$.
- The normal is perpendicular to the tangent: if the tangent slope is nonzero, its normal slope is the negative reciprocal. The squared distance is $(\Delta x)^2 + (\Delta y)^2$.

Step 1: Find the equation and the point P The vertex has $h = 1/2, k = 3/4$, and the directrix satisfies $k - a = 1/2$. Hence $a = 1/4$, so $4a = 1$, and the parabola opens upwards:

$$(x - 1/2)^2 = y - 3/4.$$

Substitute the given line $x = -1/2$. Then $(x - 1/2)^2 = 1$, giving $y = 7/4$. Thus $P = (-1/2, 7/4)$. This point is determined uniquely because this parabola expresses y as a single-valued function of x.

Step 2: Write the normal and find its second intersection Differentiating gives $dy/dx = 2(x - 1/2)$. At P, the tangent slope is -2, so the normal slope is $1/2$. Its equation is

$$y - \frac{7}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right), \qquad y = \frac{x}{2} + 2.$$

Substitute into the parabola:

$$(x - 1/2)^2 = x/2 + 5/4,$$
$$x^2 - \frac{3}{2}x - 1 = 0,$$
$$(2x + 1)(x - 2) = 0.$$

The root $x = -1/2$ recovers P; the other root is $x = 2$, giving $Q = (2, 3)$. We must use this second root, because the question asks for the point where the normal meets the parabola again.

Step 3: Evaluate the squared chord length The coordinate differences are $2 - (-1/2) = 5/2$ and

$3 - 7/4 = 5/4$. Therefore

$$(PQ)^2 = \left(\frac{5}{2}\right)^2 + \left(\frac{5}{4}\right)^2 = \frac{25}{4} + \frac{25}{16} = \frac{125}{16}.$$

The values $75/8, 25/2, 15/2$ in (A), (C), (D) do not equal this sum of squares.

Step 4: Conclusion $\boxed{(PQ)^2 = 125/16}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: Use the directrix to determine a, then differentiate the translated parabola directly. When finding a normal chord, discard the known intersection only after factoring.

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19. Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3 \cdot (n-3) + \ldots\ldots + (n-1) \cdot 1$, $n \geq 4$.

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to : [4 marks]

(A) $\frac{e-1}{3}$

(B) $\frac{e-2}{6}$

(C) $\frac{e}{3}$

(D) $\frac{e}{6}$

Correct Answer: (A) $\frac{e-1}{3}$

Solution:

Concept:

- Expand $r(n-r) = nr - r^2$ and use $\sum_{r=1}^{m} r = m(m+1)/2$ and $\sum_{r=1}^{m} r^2 = m(m+1)(2m+1)/6$.
- Cancel factorial factors before summing. The exponential series is $e = \sum_{k=0}^{\infty} 1/k!$; the lower index determines which initial terms must be removed.

Step 1: Obtain a closed expression for S_n The displayed finite sum can be written as $S_n = \sum_{r=1}^{n-1} r(n-r)$. Its two standard sums give

$$\begin{aligned} S_n &= n\frac{n(n-1)}{2} - \frac{(n-1)n(2n-1)}{6} \\ &= \frac{n(n-1)\{3n-(2n-1)\}}{6} \\ &= \frac{n(n-1)(n+1)}{6}. \end{aligned}$$

This formula also checks directly at $n = 4$: the terms are $3 + 4 + 3 = 10$, agreeing with $4 \cdot 3 \cdot 5/6$.

Step 2: Simplify each term of the infinite series Since $n! = n(n-1)(n-2)!$, cancel the two consecutive

factors:

$$\frac{2S_n}{n!} - \frac{1}{(n-2)!} = \frac{n+1}{3(n-2)!} - \frac{1}{(n-2)!}$$
$$= \frac{n-2}{3(n-2)!}$$
$$= \frac{1}{3(n-3)!}.$$

Here $n \geq 4$, so all the displayed factorials are defined and the cancellation is valid. It is simpler to combine the terms before summing than to evaluate two separate factorial series.

Step 3: Shift the index and evaluate Set $k = n - 3$. The first term $n = 4$ corresponds to $k = 1$, so the required sum is

$$\frac{1}{3}\sum_{k=1}^{\infty}\frac{1}{k!} = \frac{1}{3}(e-1).$$

The series converges absolutely, as does the exponential series. Option (C) incorrectly retains the $k = 0$ term. Options (B) and (D) use an incorrect factor of $1/6$; the coefficient after simplifying the original summand is $1/3$.

Step 4: Conclusion The sum is $\boxed{(e-1)/3}$. Hence, the correct option is (A).

Quick Tip to Solve: First simplify the factorial summand to one reciprocal factorial. Write the new lower index explicitly; starting at $k = 1$ subtracts 1 from the exponential series.

20. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to : [4 marks]

(A) $\frac{5}{2\sqrt{3}}$
(B) $\frac{2}{\sqrt{3}}$
(C) $\frac{4}{\sqrt{3}}$
(D) 2

Correct Answer: (B) $\frac{2}{\sqrt{3}}$

Solution:

Concept:

- At a common point, differentiate each curve implicitly to find its tangent slope. For finite slopes m_1, m_2, the acute angle satisfies $\tan\theta = |(m_1 - m_2)/(1 + m_1 m_2)|$.
- Use the first-quadrant condition to take positive square roots when solving the intersection equations. The acute angle requires the absolute value of the slope ratio.

Step 1: Locate the intersection in the first quadrant The two equations are $x^2/9 + y^2 = 1$ and $x^2 + y^2 = 3$. Subtracting the first from the second yields

$$\frac{8}{9}x^2 = 2, \qquad x^2 = \frac{9}{4}.$$

Then $y^2 = 3 - 9/4 = 3/4$. The first-quadrant intersection is therefore

$$(x, y) = \left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \qquad \frac{x}{y} = \sqrt{3}.$$

Both coordinates are nonzero, so both tangent slopes below are finite and the standard slope formula applies.

Step 2: Calculate the two slopes Differentiate the ellipse and circle separately:

$$\frac{2x}{9} + 2y\, m_e = 0, \quad m_e = -\frac{x}{9y} = -\frac{\sqrt{3}}{9},$$
$$2x + 2y\, m_c = 0, \quad m_c = -\frac{x}{y} = -\sqrt{3}.$$

The factor 9 belongs only to the ellipse slope. Omitting it would incorrectly make the two tangents parallel. Their product is $m_e m_c = 1/3$, so the denominator in the angle formula is positive and nonzero.

Step 3: Apply the angle formula and compare the options Substitute the slopes:

$$\tan\theta = \left|\frac{-\sqrt{3}/9 + \sqrt{3}}{1 + 1/3}\right|$$
$$= \frac{8\sqrt{3}}{9} \cdot \frac{3}{4} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}.$$

Option (A), $5/(2\sqrt{3})$, has a different numerator; option (C) is twice the result; and option (D), 2, omits the remaining factor $1/\sqrt{3}$. None agrees with the exact slope calculation.

Step 4: Conclusion $\boxed{\tan\theta = 2/\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: Find x/y at the common point before computing slopes. Both derivatives then simplify immediately, and the angle formula avoids finding either tangent equation.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let X be a random variable with distribution.

x	-2	-1	3	4	6
$P(X=x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\sigma^2$ is equal to: [4 marks]

Solution:

Concept:

- The probabilities of every value in a discrete distribution sum to 1. The mean is $E(X) = \sum xP(X = x)$, so these give two linear equations for the unknown probabilities.
- The variance is $\sigma^2 = E(X^2) - [E(X)]^2$. Square the values of the variable when finding $E(X^2)$, but keep their probabilities unchanged.

Step 1: Use the total probability and the given mean From the probability row,

$$a + b = 1 - \left(\frac{1}{5} + \frac{1}{3} + \frac{1}{5}\right) = \frac{4}{15}.$$

The mean is 2.3, which gives

$$-\frac{2}{5} - a + 3\left(\frac{1}{3}\right) + 4\left(\frac{1}{5}\right) + 6b = \frac{23}{10}.$$

The known contributions add to $7/5$, hence $-a + 6b = 9/10$. Adding this to $a + b = 4/15$ gives $7b = 7/6$. Therefore

$$b = \frac{1}{6}, \qquad a = \frac{4}{15} - \frac{1}{6} = \frac{1}{10}.$$

Both probabilities are nonnegative, and together with the three given probabilities their sum is 1.

Step 2: Compute the second moment The squared values corresponding to $-2, -1, 3, 4, 6$ are $4, 1, 9, 16, 36$. Thus

$$\begin{aligned} E(X^2) &= 4\left(\frac{1}{5}\right) + 1\left(\frac{1}{10}\right) + 9\left(\frac{1}{3}\right) + 16\left(\frac{1}{5}\right) + 36\left(\frac{1}{6}\right) \\ &= \frac{4}{5} + \frac{1}{10} + 3 + \frac{16}{5} + 6 = \frac{131}{10} = 13.1. \end{aligned}$$

The negative values of X make positive contributions here because the variable is squared. The mean itself is not squared term by term.

Step 3: Find the variance and the requested multiple Use the square of the given mean, $(2.3)^2 = 5.29$:

$$\sigma^2 = 13.1 - 5.29 = 7.81, \qquad 100\sigma^2 = 781.$$

The variance is positive as required. Since the question supplies no physical unit for X, the requested result is a pure numerical value.

Step 4: Conclusion The variance is $\sigma^2 = 7.81$, so the required numerical answer is $\boxed{100\sigma^2 = 781}$.

Quick Tip to Solve: Solve for unknown probabilities using normalization and the mean before finding the second moment. For variance, subtract the square of the mean from the mean of the squares.

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22. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3$, x $\in$ R. Then the natural number n for which $\lim\limits_{x\to 1} \frac{x^n f(1) - f(x)}{x-1} = 44$ is _____. [4 marks]

Solution:

Concept:

- The derivative at a point is the limit of the difference quotient: $g\prime(1) = \lim_{x\to 1}[g(x) - g(1)]/(x-1)$. This applies to every polynomial.
- When a numerator vanishes at $x = 1$, first separate it into standard difference quotients. Here x^n has derivative nx^{n-1}, so its difference quotient tends to n.

Step 1: Evaluate the fixed polynomial data At the point approached by x, the polynomial gives

$$f(1) = 1 + 2 + 1 + 2 + 3 = 9.$$

Differentiate term by term, remembering to multiply each monomial by its exponent:

$$f\prime(x) = 6x^5 + 8x^3 + 3x^2 + 2, \qquad f\prime(1) = 19.$$

The constant term differentiates to zero. These two values determine the required limit.

Step 2: Separate the numerator into derivative forms Add and subtract $f(1)$ in the numerator without changing it:

$$\frac{x^n f(1) - f(x)}{x-1} = f(1)\frac{x^n - 1}{x-1} - \frac{f(x) - f(1)}{x-1}.$$

Both quotients now have finite derivative limits. Consequently the given limit is $9n - 19$. This separation also preserves the minus sign before the derivative of f.

Step 3: Solve for the natural number The specified limiting value imposes

$$9n - 19 = 44 \implies 9n = 63 \implies n = 7.$$

This is a natural number, as required. Substitution checks the result directly: $9(7) - 19 = 44$, so the computed value satisfies the original condition.

Step 4: Conclusion The required natural number is $\boxed{7}$. This is a dimensionless integer.

Quick Tip to Solve: For a limit involving $x^n f(1) - f(x)$ at 1, split off $f(1)(x^n - 1)$; the limit becomes $nf(1) - f\prime(1)$.

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23. If for the complex numbers z satisfying | z − 2 − 2i | ≤ 1, the maximum value of | 3iz + 6 | is attained at a + ib, then a + b is equal to _____. [4 marks]

Solution:

Concept:

- The inequality $|z - z_0| \leq r$ represents a closed disc in the complex plane, with centre z_0 and radius r. Its boundary is included.
- The modulus $|uv| = |u||v|$, and the triangle inequality gives $|u+v| \leq |u| + |v|$. Equality holds when the nonzero summands point in the same direction.

Step 1: Rewrite the quantity as a distance Factor out $3i$ from the given expression. Since $3i(-2i) = 6$,

$$|3iz + 6| = |3i(z - 2i)| = 3|z - 2i|.$$

Thus we must find the point of the disc centred at $2 + 2i$, with radius 1, which is farthest from $2i$. The factor 3 changes the maximum value, but does not change where that maximum is attained.

Step 2: Find the sharp upper bound Write $z - 2i = (z - 2 - 2i) + 2$. The constraint and the triangle inequality imply

$$|z - 2i| \leq |z - 2 - 2i| + 2 \leq 1 + 2 = 3.$$

Therefore $|3iz + 6| \leq 9$. To attain both inequalities, the displacement $z - 2 - 2i$ must have modulus 1 and point along the positive real direction, the same direction as the real number 2.

Step 3: Identify the point and add its coordinates The equality condition gives

$$z - 2 - 2i = 1 \quad \Longrightarrow \quad z = 3 + 2i.$$

It lies on the permitted boundary because $|(3 + 2i) - 2 - 2i| = 1$. At this point $3iz + 6 = 9i$, whose modulus is indeed 9. Therefore $a = 3$ and $b = 2$, with no choice of a different boundary direction giving a larger value.

Step 4: Conclusion The maximum is attained at $a + ib = 3 + 2i$, so $\boxed{a + b = 5}$, a dimensionless value.

Quick Tip to Solve: Factor the linear complex expression first, then move from the reference point through the disc centre to the farthest point on its boundary.

24. Let the points of intersections of the lines x − y + 1 = 0, x − 2y + 3 = 0 and 2x − 5y + 11 = 0 are the mid points of the sides of a triangle △ABC. Then, the area of the △ABC is _____. [4 marks]

Solution:

Concept:

- The intersections of the three given lines form the medial triangle, since its vertices are the midpoints of the sides of the required triangle.
- A medial triangle is similar to the original triangle with side ratio $1 : 2$. Its area is therefore one quarter of the original area. For vectors (u_1, u_2) and (v_1, v_2), triangle area is $\frac{1}{2}|u_1v_2 - u_2v_1|$.

Step 1: Find the three pairwise intersections From $x - y + 1 = 0$, write $x = y - 1$. Substitution into $x - 2y + 3 = 0$ gives $y = 2$, so the first intersection is $P = (1, 2)$. Substitution instead into $2x - 5y + 11 = 0$ gives $2(y - 1) - 5y + 11 = 0$, hence $Q = (2, 3)$. Finally, the second line gives $x = 2y - 3$; using the third gives $2(2y - 3) - 5y + 11 = 0$, hence $R = (7, 5)$.

Step 2: Calculate the area of the medial triangle Use P as the common starting point for the two side vectors:

$$Q - P = (1, 1), \qquad R - P = (6, 3).$$

The determinant is $1 \cdot 3 - 1 \cdot 6 = -3$. Its sign reflects orientation only, so

$$[PQR] = \frac{1}{2}|-3| = \frac{3}{2}.$$

This is the area of the triangle whose vertices are the given side midpoints, rather than the required original triangle.

Step 3: Scale the area to the original triangle The midpoint theorem makes each side of PQR half the corresponding side of ABC. Areas scale as the square of the side ratio. Therefore

$$[ABC] = 4[PQR] = 4\left(\frac{3}{2}\right) = 6.$$

The nonzero determinant confirms that the three midpoints form a genuine triangle and hence determine a nondegenerate original triangle.

Step 4: Conclusion The area of $\triangle ABC$ is $\boxed{6 \text{ square units}}$.

Quick Tip to Solve: When line intersections are side midpoints, compute their triangle area by a determinant and multiply by 4; reconstructing all original vertices is unnecessary.

25. Let f(x) be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for k = 2, 3, 4, 5. Then the value of 52 − 10f(10) is equal to : [4 marks]

Solution:

Concept:

- If a polynomial vanishes at distinct numbers $r_1, \ldots, r_m$, it contains the factor $(x - r_1) \cdots (x - r_m)$. A nonzero polynomial of degree m is then a constant multiple of that product.
- Multiplying $f(k) = -2/k$ by k suggests the polynomial $g(x) = xf(x) + 2$, whose prescribed values become zeros. Since f has degree 3, g has degree 4.

Step 1: Convert the four conditions into roots For each $k = 2, 3, 4, 5$, the given equation implies

$$g(k) = kf(k) + 2 = k\left(-\frac{2}{k}\right) + 2 = 0.$$

Hence these four distinct values are roots of the degree-four polynomial g. We may therefore write

$$xf(x) + 2 = C(x-2)(x-3)(x-4)(x-5),$$

for a constant C. This step uses all four conditions simultaneously and avoids solving separately for the four coefficients of f.

Step 2: Determine the proportionality constant The value $g(0)$ is known even though $f(0)$ itself is not: the factor x makes $g(0) = 2$. Substituting zero gives

$$2 = C(-2)(-3)(-4)(-5) = 120C,$$

so $C = 1/60$. The product is positive because it contains four negative factors.

Step 3: Evaluate the requested expression Now put $x = 10$ into the polynomial identity:

$$10f(10) + 2 = \frac{(10-2)(10-3)(10-4)(10-5)}{60} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{60} = 28.$$

It follows that $10f(10) = 26$. The requested quantity is thus

$$52 - 10f(10) = 52 - 26 = 26.$$

Keeping $10f(10)$ together avoids an unnecessary intermediate decimal.

Step 4: Conclusion The required dimensionless value is $\boxed{26}$.

Quick Tip to Solve: When polynomial data have the form $f(k) = c/k$, form $xf(x) - c$: the given arguments become roots immediately.

---

26. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is _____.

[4 marks]

Solution:

Concept:

- Dictionary order is determined at the first position where two words differ. Count valid arrangements that first differ from FARMER by having a smaller available letter.
- The letters are ordered $A < E < F < M < R$, with two identical R letters. For a freely arranged multiset, divide the factorial count by 2!; subtract arrangements in which the two R letters form one adjacent block.

Step 1: Count words beginning before F If the first letter is A, the remaining letters are E, F, M, R, R. There are $5!/2! = 60$ arrangements altogether. Those with adjacent R letters treat RR as one object, giving $4! = 24$ forbidden arrangements. Thus 36 valid words begin with A. The same calculation gives 36 valid words beginning with E. These account for $36 + 36 = 72$ words preceding FARMER.

Step 2: Count smaller continuations of the prefix FA After fixing F, the next letter in FARMER is A, which is already the smallest remaining letter, so no earlier words arise there. After FA, the third target letter is R. A smaller third letter can be E or M. With prefix FAE, the remaining M, R, R can be arranged in $3!/2! - 2! = 1$ valid way, namely RMR. With prefix FAM, the remaining E, R, R similarly have just one valid arrangement, RER. The running count of earlier words is now 74.

Step 3: Finish the prefix count and include FARMER Fix FAR. The fourth target letter is M, and the only available smaller letter is E. With prefix FARE, the remaining M and R can appear in either order, giving FAREMR and FARERM. Both obey the restriction, adding 2. After FARM, the remaining letters are E and R; the target next letter E is already smallest. Therefore precisely

$$72 + 1 + 1 + 2 = 76$$

permitted arrangements precede FARMER. The word itself is permitted because its R letters are separated, so its serial number is $76 + 1 = 77$.

Step 4: Conclusion The serial number of FARMER in the restricted dictionary list is $\boxed{77}$.

Quick Tip to Solve: Count words preceding the target by the first differing letter, enforce the RR restriction within each prefix, and add 1 only at the end.

---

27. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to _____. [4 marks]

Solution:

Concept:

- A vector in the plane spanned by two nonparallel vectors can be written as a linear combination: $\vec{v} = \alpha\vec{a} + \beta\vec{b}$. Perpendicular vectors have zero dot product.
- The scalar projection of $\vec{v}$ on $\vec{a}$ is $(\vec{v} \cdot \vec{a})/|\vec{a}|$. Also $|2\vec{v}|^2 = 4|\vec{v}|^2$; the scale factor is squared.

Step 1: Apply the perpendicularity condition Let $\vec{c} = (3, 2, -1)$, and write $\vec{v} = \alpha\vec{a} + \beta\vec{b}$. The relevant dot products are

$$\vec{a} \cdot \vec{c} = 2(3) - 1(2) + 2(-1) = 2, \qquad \vec{b} \cdot \vec{c} = 1(3) + 2(2) + (-1)(-1) = 8.$$

Therefore $\vec{v} \cdot \vec{c} = 0$ becomes $2\alpha + 8\beta = 0$, so $\alpha = -4\beta$. Thus

$$\vec{v} = \beta(\vec{b} - 4\vec{a}) = \beta(-7, 6, -9).$$

Step 2: Use the given projection to determine the scale The length of $\vec{a} = (2, -1, 2)$ is $\sqrt{4 + 1 + 4} = 3$. Also

$$(-7, 6, -9) \cdot (2, -1, 2) = -14 - 6 - 18 = -38.$$

The projection condition consequently gives

$$\frac{-38\beta}{3} = 19 \quad \Longrightarrow \quad \beta = -\frac{3}{2}.$$

Hence $\vec{v} = (21/2, -9, 27/2)$. Its dot product with $\vec{a}$ is 57, which checks the projection $57/3 = 19$.

Step 3: Calculate the squared magnitude requested It is simplest to use the scaled vector form directly:

$$|\vec{v}|^2 = \left(-\frac{3}{2}\right)^2 \left((-7)^2 + 6^2 + (-9)^2\right) = \frac{9}{4}(49 + 36 + 81) = \frac{9}{4} \cdot 166.$$

Therefore

$$|2\vec{v}|^2 = 4 \cdot \frac{9}{4} \cdot 166 = 9 \cdot 166 = 1494.$$

Even if projection were interpreted as an unsigned length, the opposite vector has the same squared magnitude.

Step 4: Conclusion The required value is $\boxed{1494 \text{ square units}}$.

Quick Tip to Solve: Use perpendicularity first to reduce two vector coefficients to one, then use scalar projection to find its scale; square the factor 2 in the final norm.

28. Let [t] denote the greatest integer $\leq$ t. The number of points where the function $f(x) = [x]\left|x^2 - 1\right| + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1], x \in (-2, 2)$ is not continuous is _____. [4 marks]

Solution:

Concept:

- On an open interval between consecutive integers, $[x]$ is constant and $[x+1] = [x] + 1$. The remaining polynomial, absolute-value and sine expressions are continuous there.
- Only the interior integers $-1, 0, 1$ need one-sided tests. A jump in a greatest-integer factor can be cancelled by another term, so these points must be checked individually.

Step 1: Reduce the function on each unit interval Put $m = [x]$. The function becomes

$$f(x) = m|x^2 - 1| + \sin\left(\frac{\pi}{m+3}\right) - m - 1.$$

In $(-2, 2)$, the possible values of m are $-2, -1, 0, 1$, so $m+3$ is never zero. Within each of the intervals bounded by $-2, -1, 0, 1, 2$, all components of this expression are continuous. The endpoints -2 and 2 do not belong to the domain.

Step 2: Check the cancellation at minus one As $x \to -1$, the absolute-value factor tends to zero. On the left $m = -2$, while on the right and at the point $m = -1$. Consequently

$$f(-1^-) = 0 + \sin\pi - (-2) - 1 = 1,$$

$$f(-1^+) = f(-1) = 0 + \sin(\pi/2) - (-1) - 1 = 1.$$

Thus the apparent jumps cancel, and f is continuous at -1.

Step 3: Test zero and one At zero, $|x^2 - 1| \to 1$. The left side has $m = -1$, and the right side has $m = 0$, giving

$$f(0^-) = 0, \qquad f(0^+) = f(0) = \frac{\sqrt{3}}{2} - 1.$$

These differ. At one, the absolute-value factor tends to zero, and m changes from 0 to 1, so

$$f(1^-) = \frac{\sqrt{3}}{2} - 1, \qquad f(1^+) = f(1) = \frac{\sqrt{2}}{2} - 2.$$

These also differ. There are no further candidate points inside the domain.

Step 4: Conclusion The discontinuities occur exactly at $x = 0$ and $x = 1$. Their number is the dimensionless integer $\boxed{2}$.

Quick Tip to Solve: Use $[x + 1] = [x] + 1$ before checking jumps; at $x = -1$, the sine jump cancels the greatest-integer jump exactly.

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29. A man starts walking from the point P(−3, 4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50\left((PR)^2 + (RQ)^2\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- At a constant speed, minimum travel time is equivalent to minimum total distance $PR + RQ$. The expression to be evaluated is the sum of squared distances at that minimizing point, not the quantity being minimized.
- Reflecting an endpoint in a straight boundary preserves its distance from every point on that boundary. The triangle inequality then makes the straight reflected path the shortest path.

Step 1: Reflect the destination Reflect $Q(0, 2)$ in the x-axis to obtain $Q'(0, -2)$. For every $R = (r, 0)$, we have $RQ = RQ'$. Hence $PR + RQ = PR + RQ' \geq PQ'$, with equality precisely when P, R, Q' lie on one straight segment. Because the endpoints have opposite signs of y-coordinate, this segment actually crosses the x-axis and gives an admissible path.

Step 2: Locate the turning point The line from $P(-3, 4)$ to $Q'(0, -2)$ has slope $(-2-4)/(0+3) = -2$. Its equation is $y - 4 = -2(x + 3)$, or $y = -2x - 2$. Setting $y = 0$ gives $R = (-1, 0)$. This chooses the route minimizing total distance; directly minimizing $PR^2 + RQ^2$ would be a different optimization problem.

Step 3: Evaluate the requested expression Using the distance formula with the original endpoints gives

$$PR^2 = (-1 + 3)^2 + (0 - 4)^2 = 4 + 16 = 20, \qquad RQ^2 = (0 + 1)^2 + (2 - 0)^2 = 1 + 4 = 5.$$

Therefore

$$50(PR^2 + RQ^2) = 50(20 + 5) = 1250.$$

The unsquared lengths also check the reflection condition: $PR + RQ = 2\sqrt{5} + \sqrt{5} = 3\sqrt{5} = PQ'$.

Step 4: Conclusion The required numerical value is $\boxed{1250}$, attained for the turning point $R = (-1, 0)$.

Quick Tip to Solve: For the shortest broken path touching an axis, reflect one endpoint first; evaluate any squared-distance expression only after finding the optimal turning point.

---

30. If the sum of the coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is _____. [4 marks]

Solution:

Concept:

- The sum of all coefficients of a polynomial in two variables is obtained by setting both variables equal to 1. For $(x + y)^n$, this sum is 2^n.
- The coefficient of $x^{n-r}y^r$ is $\binom{n}{r}$. Consecutive binomial coefficients have ratio $\binom{n}{r+1}/\binom{n}{r} = (n - r)/(r + 1)$, which identifies the largest one.

Step 1: Determine the exponent Put $x = y = 1$ in the expansion. This adds every coefficient, since each monomial then has value 1. The given sum therefore yields

$$2^n = 4096 = 2^{12}, \qquad n = 12.$$

The exponent is a non-negative integer in the binomial expansion, so there is no additional possible exponent to consider.

Step 2: Identify the central coefficient For $n = 12$, the ratio of the coefficient at index $r + 1$ to that at index r is $(12 - r)/(r + 1)$. It exceeds 1 for the steps up to $r = 5$, and is less than 1 from $r = 6$ onward. Thus the coefficients rise to $\binom{12}{6}$ and then fall. Equivalently, the symmetry $\binom{12}{r} = \binom{12}{12-r}$ places the unique maximum at the middle index 6.

Step 3: Calculate the maximum Evaluate the central coefficient without including the powers of x or y:

$$\binom{12}{6} = \frac{12!}{6!6!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 924.$$

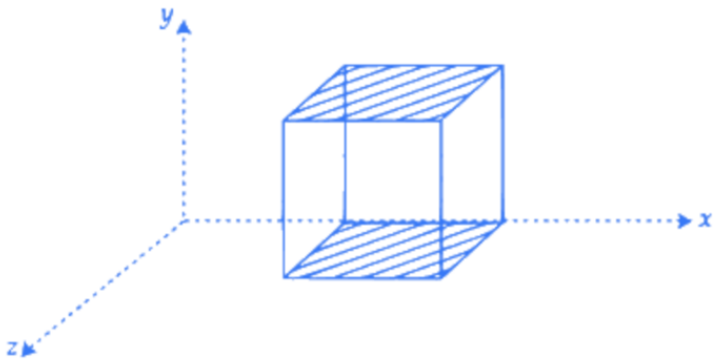
This is the coefficient of x^6y^6. The requested quantity is a coefficient, so it is a number independent of the values of x and y, unlike the greatest term in a numerical binomial expansion.

Step 4: Conclusion The greatest coefficient in the expansion is $\boxed{924}$.

Quick Tip to Solve: Set x and y to 1 to recover n from the coefficient sum; for an even n, the largest coefficient is the single central one.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is : [4 marks]



(A) 3.8×10^{-11} C

(B) 8.3×10^{-11} C

(C) 3.8×10^{-12} C

(D) 8.3×10^{-12} C

Correct Answer: (B) 8.3×10^{-11} C

Solution:

Concept:

- Gauss's law relates the net outward electric flux through a closed surface to its enclosed charge: $Q_{\text{enc}} = \varepsilon_0 \oint \vec{E} \cdot d\vec{A}$.
- Here the field is parallel to the y-axis and varies with y. Only the two faces perpendicular to that axis contribute flux; the four remaining faces have zero normal field component.

Step 1: Identify the contributing faces The lower face in the figure lies at $y = 0$, and the upper face is at $y = a = 0.5\,\text{m}$. Both have area $a^2 = 0.25\,\text{m}^2$. The outward normal is upward on the upper face and downward on the lower face. Thus $\Phi = a^2[E_y(a) - E_y(0)]$, rather than the sum of the magnitudes on both faces. The position of the cube along x or z does not change this result because the field depends only on y.

Step 2: Calculate the net flux and charge Substitute $E_y = 150y^2$:

$$E_y(0) = 0, \qquad E_y(0.5) = 150(0.5)^2 = 37.5\,\text{N C}^{-1}.$$

Hence

$$\Phi = 0.25(37.5) = 9.375\,\text{N m}^2\,\text{C}^{-1},$$

and

$$Q_{\text{enc}} = (8.85 \times 10^{-12})(9.375) = 8.30 \times 10^{-11}\,\text{C}.$$

The positive outward flux confirms a positive net enclosed charge.

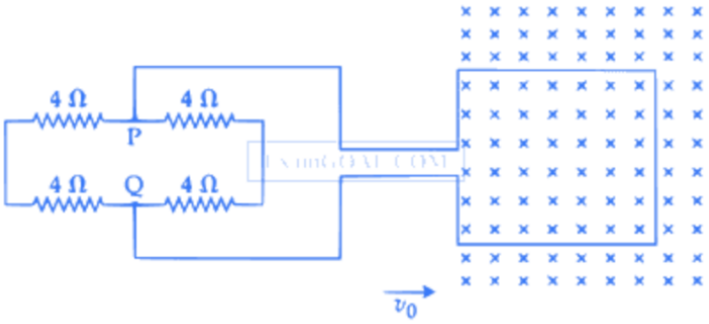
Step 3: Compare the options Option (B) matches the calculated charge to the stated precision. Option

(A) has a different numerical coefficient and cannot follow from the face area and field difference. Options (C) and (D) are one order of magnitude too small. Multiplying the field by the cube volume would also be incorrect: flux requires an area integral, while the volume method would require the divergence $\nabla \cdot \vec{E}$.

Step 4: Conclusion The charge enclosed by the cube is $\boxed{8.3 \times 10^{-11}\,\text{C}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a field along one coordinate axis, compute the field difference on the two perpendicular faces and multiply by their area.

32. A square loop of side 20 cm and resistance 1Ω is moved towards right with a constant speed v_0. The right arm of the loop is in a uniform magnetic field of 5T. The field is perpendicular to the plane of the loop and is going into it. The loop is connected to a network of resistors each of value 4Ω. What should be the value of v_0 so that a steady current of 2 mA flows in the loop? [4 marks]



(A) 1 m/s

(B) 1 cm/s

(C) 10^2 m/s

(D) 10^{-2} cm/s

Correct Answer: (B) 1 cm/s

Solution:

Concept:

- A conductor of length l, moving at speed v perpendicular to a uniform magnetic field B, develops motional emf $\mathcal{E} = Blv$. Here the active conductor is the right vertical arm.
- Resistors in series add, while two parallel resistances have equivalent resistance $R_1R_2/(R_1 + R_2)$. The loop resistance must be added to the external network resistance.

Step 1: Reduce the external network The connections to the loop are the midpoint nodes P and Q. Between P and Q there are two paths: the left path contains the two left $4\,\Omega$ resistors in series, and the right path contains the two right $4\,\Omega$ resistors in series. Each path therefore has resistance $8\,\Omega$,

and the paths are in parallel. Thus

$$R_{PQ} = 8 \parallel 8 = \frac{8 \times 8}{8 + 8} = 4\,\Omega.$$

Including the resistance of the moving loop gives $R_{\text{total}} = 4 + 1 = 5\,\Omega$.

Step 2: Find the required speed The desired current is $I = 2\,\text{mA} = 2 \times 10^{-3}\,\text{A}$. Ohm's law requires

$$\mathcal{E} = IR_{\text{total}} = (2 \times 10^{-3})(5) = 0.010\,\text{V}.$$

Convert the side length to $l = 0.20\,\text{m}$. Since $B = 5\,\text{T}$,

$$v_0 = \frac{\mathcal{E}}{Bl} = \frac{0.010}{5(0.20)} = 0.010\,\text{m s}^{-1} = 1\,\text{cm s}^{-1}.$$

The increasing inward flux determines the current direction by Lenz's law, but the magnitude calculation uses the positive emf magnitude.

Step 3: Check the listed values Option (B) gives the required speed. A speed of $1\,\text{m s}^{-1}$, option (A), produces a current 100 times larger. Option (C) is larger still. Option (D), $10^{-2}\,\text{cm s}^{-1}$, is 100 times too small. Retaining the 1-ohm loop resistance is essential for the stated current.

Step 4: Conclusion The required speed is $\boxed{v_0 = 1\,\text{cm s}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Trace the actual terminal nodes before reducing a resistor drawing; include the moving conductor's own resistance in the total.

---

33. The temperature of an ideal gas in 3-dimensions is 300 K. The corresponding de-Broglie wavelength of the electron approximately at 300 K, is :

[m_e = mass of electron = 9×10^{-31} kg, h = Planck constant = $6.6 \times 6.6 \times 10^{-34}$ Js, k_B = Boltzmann constant = 1.38×10^{-23} JK^{-1}] [4 marks]

(A) 6.26 nm

(B) 8.46 nm

(C) 2.26 nm

(D) 3.25 nm

Correct Answer: The printed extra factor of 6.6 in h gives 41.2 nm; no option matches. The key selects A using the conventional value of h.

Solution:

Concept:

- For three-dimensional translational thermal motion, the mean kinetic energy is $\frac{3}{2}k_BT$. Equating this to $p_{\text{rms}}^2/(2m_e)$ gives $p_{\text{rms}} = \sqrt{3m_ek_BT}$.
- The de Broglie wavelength corresponding to this representative momentum is $\lambda = h/p_{\text{rms}}$. The numerical value of h printed in the question must be distinguished from the usual Planck constant.

Step 1: Calculate the thermal momentum At $T = 300\,\mathrm{K}$, use the given electron mass and Boltzmann constant:

$$3m_e k_B T = 3(9 \times 10^{-31})(1.38 \times 10^{-23})(300) = 1.1178 \times 10^{-50}.$$

Hence

$$p_{\mathrm{rms}} = \sqrt{1.1178 \times 10^{-50}} \simeq 1.0573 \times 10^{-25}\,\mathrm{kg\,m\,s^{-1}}.$$

The factor 3 is necessary because the gas has translational motion in three dimensions; using only one component would give a different momentum.

Step 2: Use the constant exactly as printed The question prints $h = 6.6 \times 6.6 \times 10^{-34}\,\mathrm{J\,s} = 4.356 \times 10^{-33}\,\mathrm{J\,s}$. Taking this value literally gives

$$\lambda = \frac{4.356 \times 10^{-33}}{1.0573 \times 10^{-25}} \simeq 4.12 \times 10^{-8}\,\mathrm{m} = 41.2\,\mathrm{nm}.$$

None of the four listed wavelengths, 6.26, 8.46, 2.26 and 3.25 nm, equals this result. Thus the given numerical constant and the options do not support a unique correct listed answer.

Step 3: Explain the keyed option With the conventional value $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$, the same momentum gives $\lambda \simeq 6.24\,\mathrm{nm}$, approximately the listed $6.26\,\mathrm{nm}$ in option (A). The final key selects (A), which is consistent with that conventional value. The extra factor of 6.6 in the printed data changes the wavelength by that factor and cannot be ignored in a literal calculation. Options (B), (C) and (D) agree with neither calculation.

Step 4: Conclusion Using the printed value of h gives $\boxed{\lambda \simeq 41.2\,\mathrm{nm}}$, so no listed option is correct literally. The key selects (A), $6.26\,\mathrm{nm}$, obtained when h is taken as the conventional $6.6 \times 10^{-34}\,\mathrm{J\,s}$.

Quick Tip to Solve: Use $\lambda = h/\sqrt{3mk_BT}$ for the rms thermal momentum, and check supplied constants when no option matches the literal result.

---

34. A body of mass 'm' dropped from a height 'h' reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is : [4 marks]

(A) -0.68 mgh

(B) mgh

(C) 1.64 mgh

(D) 0.64 mgh

Correct Answer: (A) -0.68 mgh

Solution:

Concept:

- The work-energy theorem states that the total work done by all forces equals the change in kinetic energy: $W_g + W_{\mathrm{air}} = K_f - K_i$.
- For a body falling through height h, gravity does positive work mgh. Air resistance opposes the downward motion and therefore does negative work, reducing the kinetic energy gained.

Step 1: Find the final kinetic energy The body is dropped, so its initial speed and kinetic energy are zero. Its final speed is $v = 0.8\sqrt{gh}$. Squaring the complete expression gives $v^2 = 0.64gh$, not $0.8gh$. Thus

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}m(0.64gh) = 0.32mgh, \qquad K_i = 0.$$

The reference level for potential energy can be the ground; the initial potential energy is then mgh.

Step 2: Separate the work of air friction The net work is the kinetic-energy gain, but gravity is only one contributor. Therefore

$$mgh + W_{\text{air}} = 0.32mgh,$$

which gives

$$W_{\text{air}} = 0.32mgh - mgh = -0.68mgh.$$

Equivalently, mechanical energy falls from mgh to $0.32mgh$, so 68 percent of the initial mechanical energy is dissipated by air resistance. This sign agrees with a force opposing the displacement.

Step 3: Reject the other choices Option (A) has both the correct coefficient and negative sign. Option (B), mgh, is the work done by gravity, not by air friction. Options (C) and (D) are positive, whereas a resistive force cannot add mechanical energy during this fall. In addition, the coefficient 0.64 in option (D) comes from $v^2/(gh)$ and omits the factor one-half in kinetic energy. All expressions have energy units, so the force accounting decides the answer.

Step 4: Conclusion The work done by air friction is $\boxed{-0.68mgh}$. Hence, the correct option is (A).

Quick Tip to Solve: Compute the final kinetic energy first, then subtract gravity's work; the difference is the negative work of the resisting force.

35. The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option : [4 marks]

(A) $R_1 > R_2$ and $H_1 = H_2$

(B) $R_1 = R_2$ and $H_1 < H_2$

(C) $R_1 < R_2$ and $H_1 < H_2$

(D) $R_1 = R_2$ and $H_1 = H_2$

Correct Answer: (B) $R_1 = R_2$ and $H_1 < H_2$

Solution:

Concept:

- For a projectile launched at speed u and angle θ, returning to its launch level, the horizontal range is $R = u^2 \sin(2\theta)/g$.
- The maximum height above the launch point is $H = u^2 \sin^2\theta/(2g)$. Complementary launch angles give equal ranges but generally different heights.

Step 1: Compare the ranges Both projectiles have the same initial speed, so the factors u^2/g are

identical. Their launch angles satisfy $42^\circ + 48^\circ = 90^\circ$. Accordingly,

$$R_1 = \frac{u^2 \sin 84^\circ}{g}, \qquad R_2 = \frac{u^2 \sin 96^\circ}{g}.$$

Since $\sin 96^\circ = \sin(180^\circ - 96^\circ) = \sin 84^\circ$, the two ranges are equal. This equality follows from complementary angles, without requiring the angles themselves to be equal.

Step 2: Compare the maximum heights The heights depend on the initial vertical velocity components. On the interval from 0° to 90°, sine increases, so $\sin 42^\circ < \sin 48^\circ$. Both values are positive; squaring preserves the inequality. Therefore

$$\frac{H_1}{H_2} = \frac{\sin^2 42^\circ}{\sin^2 48^\circ} < 1, \qquad H_1 < H_2.$$

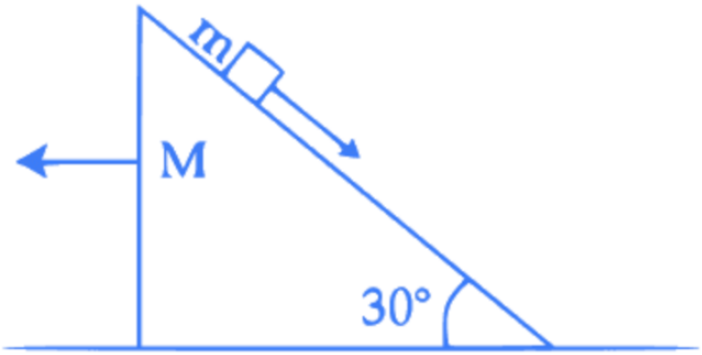
The higher-angle projectile has greater vertical launch speed, remains in the air longer, and reaches a greater peak, while its smaller horizontal speed compensates to give the same range.

Step 3: Select the consistent relation Option (B) combines $R_1 = R_2$ and $H_1 < H_2$, exactly as found. Options (A) and (C) incorrectly make the ranges unequal. Option (D) gets the ranges right but incorrectly makes the heights equal. Equal range must not be confused with equal peak height; the two formulas depend on different trigonometric functions of the launch angle.

Step 4: Conclusion The correct relations are $\boxed{R_1 = R_2 \text{ and } H_1 < H_2}$. Hence, the correct option is (B).

Quick Tip to Solve: For complementary launch angles, use symmetry of $\sin 2\theta$ for the range and monotonicity of $\sin\theta$ for the height.

36. A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is : Given m = 8 kg, M = 16 kg Assume all the surfaces shown in the figure to be frictionless. [4 marks]



(A) $\frac{4}{3}g$
(B) $\frac{6}{5}g$
(C) $\frac{3}{5}g$
(D) $\frac{2}{3}g$

Correct Answer: (D) $\frac{2}{3}g$

Solution:

Concept:

- With no external horizontal force, the total horizontal momentum of the block and wedge is conserved. The block can move down the incline while the wedge recoils. The requested acceleration is measured along the slope in the wedge frame, not in the ground frame.
- Let a_r be the block's acceleration relative to the wedge, down the 30° slope, and let A be the magnitude of the wedge's acceleration to the left.

Step 1: Apply horizontal momentum conservation Take rightwards as positive. The horizontal acceleration of the block relative to the ground is $a_r \cos\theta - A$, whereas that of the wedge is $-A$. Therefore

$$(M+m)A = ma_r \cos\theta.$$

Thus the wedge recoil must be included; setting $A = 0$ would incorrectly treat the wedge as fixed.

Step 2: Project Newton's law down the slope The normal force has no component along the incline. Projecting the block's ground acceleration down the incline gives

$$m(a_r - A\cos\theta) = mg\sin\theta.$$

Eliminate A using the first equation:

$$a_r\left(1 - \frac{m\cos^2\theta}{M+m}\right) = g\sin\theta.$$

Hence

$$a_r = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta}.$$

Step 3: Substitute the masses and inclination For $m = 8\,\text{kg}$, $M = 16\,\text{kg}$, and $\theta = 30^\circ$,

$$a_r = \frac{24g(1/2)}{16 + 8(1/4)} = \frac{12g}{18} = \frac{2g}{3}.$$

The result exceeds the fixed-wedge value $g/2$, as expected because the wedge recoils. The values in (A), (B) and (C) do not satisfy the coupled equations.

Step 4: Conclusion The block accelerates down the incline relative to the wedge with magnitude $\boxed{2g/3}$. Hence, the correct option is (D).

Quick Tip to Solve: For a freely moving wedge, first use horizontal momentum conservation; the acceleration relative to the wedge is not simply $g\sin\theta$.

37. Due to cold weather a 1 m water pipe of cross-sectional area 1 cm^2 is filled with ice at $-10°C$. Resistive heating is used to melt the ice. Current of 0.5A is passed through 4 kΩ resistance. Assuming that all the heat produced is used for melting, what is the minimum time required? (Given latent heat of fusion for water/ice = 3.33 $\times$ 10^5 J kg^{-1}, specific heat of ice = 2 $\times$ 10^3 J kg^{-1} and density of ice = 10^3 kg/m^3 [4 marks]

(A) 0.353 s

(B) 35.3 s

(C) 3.53 s

(D) 70.6 s

Correct Answer: (B) 35.3 s

Solution:

Concept:

- Joule heating supplies power $P = I^2R$. With no heat loss, the minimum heating time is the required heat divided by this power.
- Ice initially below its melting point first needs sensible heat $mc\Delta T$, then latent heat mL_f. Melting alone would omit the warming stage.

Step 1: Calculate the mass of ice The pipe has length $l = 1\,\text{m}$ and cross-sectional area $A = 1\,\text{cm}^2 = 10^{-4}\,\text{m}^2$. Thus

$$V = Al = 10^{-4}\,\text{m}^3, \qquad m = \rho V = 10^3 \times 10^{-4} = 0.1\,\text{kg}.$$

Convert the area before multiplying by the density; an area conversion involves the square of the length conversion.

Step 2: Add the warming and melting energies The temperature rise to the melting point is $10\,\text{K}$. Using the supplied specific heat and latent heat,

$$Q_1 = 0.1(2 \times 10^3)(10) = 2000\,\text{J},$$
$$Q_2 = 0.1(3.33 \times 10^5) = 33300\,\text{J},$$
$$Q = Q_1 + Q_2 = 35300\,\text{J}.$$

No further energy is needed to warm the liquid because the task is just to melt all the ice.

Step 3: Divide the energy by the electrical power The resistance is $4\,\text{k}\Omega = 4000\,\Omega$, so

$$P = (0.5)^2(4000) = 1000\,\text{W}, \qquad t = \frac{35300}{1000} = 35.3\,\text{s}.$$

Options (A) and (C) are too small to supply the required energy, while (D) is twice the calculated minimum time.

Step 4: Conclusion The minimum time required is $\boxed{35.3\,\text{s}}$. Hence, the correct option is (B).

Quick Tip to Solve: When ice starts below zero, include both $mc\Delta T$ and mL_f before dividing by I^2R.

38. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2, without any significant error, his observation are as following.

Physical Quantity	Least count of the Equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then the fractional error in the measurement of Y is : [4 marks]

(A) 0.0083

(B) 0.0155

(C) 0.155

(D) 0.083

Correct Answer: (B) 0.0155

Solution:

Concept:

- For a product of powers, the maximum fractional error is the sum of the fractional errors of the measured factors, multiplied by their absolute powers.
- Here $Y = MgL^3/(4bd^3\delta)$. The constant 4 is exact and the problem says the uncertainty in g is negligible. Use each instrument's least count as its measurement error.

Step 1: Write the error formula Taking the logarithmic differential and adding magnitudes for the limiting error gives

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L} + \frac{\Delta b}{b} + 3\frac{\Delta d}{d} + \frac{\Delta \delta}{\delta}.$$

Although b, d, δ occur in the denominator, their error contributions are positive. They must not be subtracted in a maximum-error calculation.

Step 2: Put each uncertainty and observation in the same units Use grams for the mass pair and millimetres for all length pairs:

$$\frac{\Delta M}{M} = \frac{1}{2000} = 0.0005,$$
$$3\frac{\Delta L}{L} = 3\frac{1}{1000} = 0.003,$$
$$\frac{\Delta b}{b} = \frac{0.1}{40} = 0.0025,$$
$$3\frac{\Delta d}{d} = 3\frac{0.01}{4} = 0.0075,$$
$$\frac{\Delta \delta}{\delta} = \frac{0.01}{5} = 0.002.$$

Step 3: Add the contributions and compare the options Adding the five positive terms,

$$\frac{\Delta Y}{Y} = 0.0005 + 0.003 + 0.0025 + 0.0075 + 0.002 = 0.0155.$$

This is a fractional error of 1.55%, not a fraction of 0.155. Option (A) misses significant weighted contributions, while (C) and (D) have incorrect magnitudes.

Step 4: Conclusion The fractional error in Young's modulus is $\boxed{0.0155}$. Hence, the correct option is (B).

Quick Tip to Solve: For maximum fractional errors, add the absolute-power-weighted terms and convert numerator and denominator of each ratio to identical units.

39. Two resistors $R_1 = (4 \pm 0.8)\ \Omega$ and $R_2 = (4 \pm 0.4)\ \Omega$ are connected in parallel. The equivalent resistance of their parallel combination will be : [4 marks]

(A) $(4 \pm 0.4)\ \Omega$

(B) $(2 \pm 0.4)\ \Omega$

(C) $(2 \pm 0.3)\ \Omega$

(D) $(4 \pm 0.3)\ \Omega$

Correct Answer: (C) $(2 \pm 0.3)\ \Omega$

Solution:

Concept:

- The equivalent resistance is $R = R_1R_2/(R_1 + R_2)$. Equal nominal resistances of $4\,\Omega$ give a nominal parallel resistance of $2\,\Omega$.
- Propagate limiting uncertainties using partial derivatives of the full expression. The numerator and denominator depend on the same measured resistances, so treating their errors as independent overestimates the uncertainty.

Step 1: Find the nominal equivalent resistance Substitute the central values:

$$R = \frac{4 \times 4}{4 + 4} = 2\,\Omega.$$

A parallel resistance is less than either component resistance. This immediately excludes (A) and (D), which both retain the incorrect central value $4\,\Omega$.

Step 2: Differentiate with respect to the two measurements Keeping one resistance fixed while varying the other,

$$\frac{\partial R}{\partial R_1} = \frac{R_2^2}{(R_1 + R_2)^2}, \qquad \frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1 + R_2)^2}.$$

At $R_1 = R_2 = 4\,\Omega$, both sensitivities equal $1/4$. The limiting absolute uncertainty is therefore

$$\Delta R = \frac{1}{4}\Delta R_1 + \frac{1}{4}\Delta R_2.$$

Step 3: Evaluate the uncertainty and report the result Using $\Delta R_1 = 0.8\,\Omega$ and $\Delta R_2 = 0.4\,\Omega$,

$$\Delta R = \frac{0.8 + 0.4}{4} = 0.3\,\Omega.$$

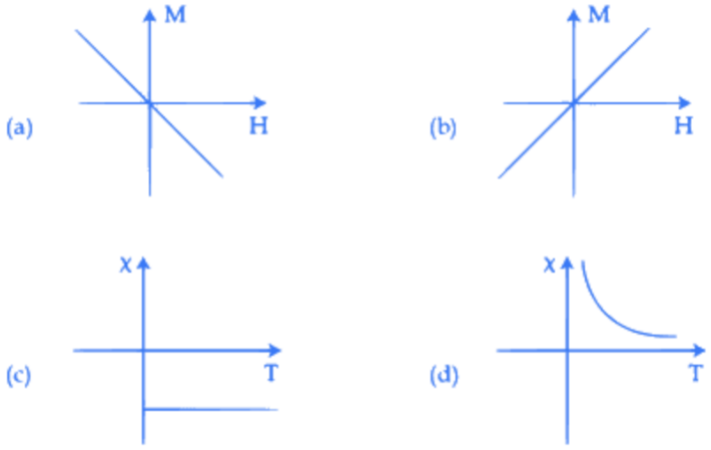
Hence the combined fractional uncertainty is $0.3/2 = 0.15$. Option (B) has the right nominal resistance but an uncertainty of $0.4\,\Omega$, which does not follow from either sensitivity coefficient.

Step 4: Conclusion The parallel combination is $\boxed{R = (2 \pm 0.3)\,\Omega}$. Hence, the correct option is (C).

Quick Tip to Solve: When the same variable appears in both numerator and denominator, differentiate the complete expression before adding limiting errors.

40. Following plots show magnetization (M) vs magnetizing field (H) and magnetic susceptibility (χ) vs temperature (T) graph :

Which of the following combination will be represented by a diamagnetic material? [4 marks]



(A) (a), (c)

(B) (a), (d)

(C) (b), (d)

(D) (b), (c)

Correct Answer: (A) (a), (c)

Solution:

Concept:

- For a linear magnetic material, $M = \chi H$, where M is magnetization, H is magnetizing field and χ is magnetic susceptibility. In the plotted curves, zero susceptibility lies on the horizontal temperature axis.
- A diamagnetic material has a small negative susceptibility: the induced magnetization opposes the applied field. Its susceptibility is approximately independent of temperature.

Step 1: Identify the magnetization graph In an M-versus-H plot, the slope is χ. A negative slope

means that positive H produces negative M, so the magnetization opposes the field. Plot (a) has this negative slope, while plot (b) has a positive slope. Thus (a), rather than (b), is the magnetization graph appropriate to a diamagnetic substance.

Step 2: Identify the susceptibility graph Plot (c) is a horizontal line below the temperature axis: its susceptibility is negative and does not vary appreciably with temperature. Both features agree with diamagnetism. Plot (d), however, has positive susceptibility that falls with increasing temperature; this is the characteristic Curie-law trend of a paramagnetic material, $\chi = C/T$.

Step 3: Compare all proposed combinations The correct pair must contain the negative-slope magnetization plot (a) and the negative, temperature-independent susceptibility plot (c). Option (B) incorrectly pairs (a) with the positive-temperature-dependent plot (d). Option (C) uses two paramagnetic-type plots, and option (D) incorrectly uses the positive-slope plot (b). Only (A) matches both required diamagnetic properties.

Step 4: Conclusion A diamagnetic material is represented by plots a and c. Hence, the correct option is (A).

Quick Tip to Solve: Read the sign of the M-against-H slope first; diamagnetism then requires a negative susceptibility with nearly no temperature dependence.

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41. A glass tumbler having inner depth of 17.5 cm is kept on a table. A student starts pouring water (μ = 4/3) into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled? [4 marks]

(A) 11.7 cm

(B) 10 cm

(C) 7.5 cm

(D) 8.75 cm

Correct Answer: (B) 10 cm

Solution:

Concept:

- For viewing from air approximately normally through a plane water surface, apparent depth equals real depth divided by the refractive index: $h_{\text{app}} = h/\mu$.
- The empty air column above the water suffers no apparent-depth reduction. The tumbler looks half filled when the apparent water depth equals the height of this empty air column.

Step 1: Express the real and apparent depths Let h be the actual height of water above the bottom and $H = 17.5\,\text{cm}$ the full inner depth. The distance from the top rim to the water surface is $H - h$. The bottom appears raised when seen through water, so the visible water-column depth is

$$h_{\text{app}} = \frac{h}{\mu} = \frac{3h}{4}.$$

The air space $H - h$ remains its actual size.

Step 2: Apply the condition for appearing half filled The water surface divides the apparent interior into equal filled and empty portions. Consequently,

$$\frac{3h}{4} = 17.5 - h.$$

Solving without equating h directly to half the real tumbler depth gives

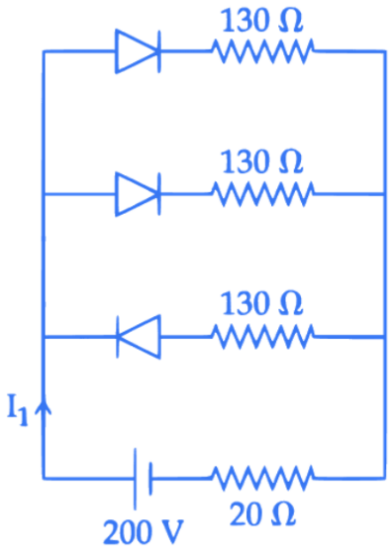
$$\frac{7h}{4} = 17.5, \qquad h = \frac{4(17.5)}{7} = 10\,\text{cm}.$$

Step 3: Check the interpretation and exclude the other values At $h = 10\,\text{cm}$, the water looks $7.5\,\text{cm}$ deep and the air column is also $7.5\,\text{cm}$, verifying the half-filled appearance. The value $8.75\,\text{cm}$ is half the real depth and ignores refraction. Neither $11.7\,\text{cm}$ nor $7.5\,\text{cm}$ makes the two apparent portions equal.

Step 4: Conclusion The tumbler is actually filled to $\boxed{10\,\text{cm}}$ above its bottom. Hence, the correct option is (B).

Quick Tip to Solve: For a vessel that looks half filled from above, equate apparent liquid depth to the actual empty height: $h/\mu = H - h$.

42. In the given figure, each diode has a forward bias resistance of 30Ω and infinite resistance in reverse bias. The current I_1 will be : [4 marks]



(A) 3.75 A

(B) 2.35 A

(C) 2 A

(D) 2.73 A

Correct Answer: (C) 2 A

Solution:

Concept:

- A diode conducts in forward bias and is an open circuit in reverse bias. Here a conducting diode contributes the stated resistance $30\,\Omega$, rather than zero resistance. The positive terminal is the longer battery plate, so this bias choice follows the printed polarity.
- The battery has its positive terminal on the left. Therefore the left rail is at the higher potential and the two upper diodes conduct from left to right; the lowest diode is reverse biased.

Step 1: Reduce the three diode branches In each of the two upper branches, the forward-biased diode is in series with a $130\,\Omega$ resistor. Each conducting branch therefore has resistance

$$R_b = 30 + 130 = 160\,\Omega.$$

The third diode points in the opposite direction. Its infinite reverse resistance opens that branch, so its $130\,\Omega$ resistor carries no current and contributes no conducting parallel path.

Step 2: Calculate the total circuit resistance The two identical conducting branches are in parallel between the rails:

$$R_p = \frac{160 \times 160}{160 + 160} = 80\,\Omega.$$

The $20\,\Omega$ resistor in the battery branch is in series with this equivalent load, so the complete loop resistance is

$$R_{\text{total}} = 80 + 20 = 100\,\Omega.$$

Step 3: Find the current in the left supply lead The arrow I_1 lies below all the diode junctions, and thus denotes the total supply current rather than the current in one conducting branch. Ohm's law gives

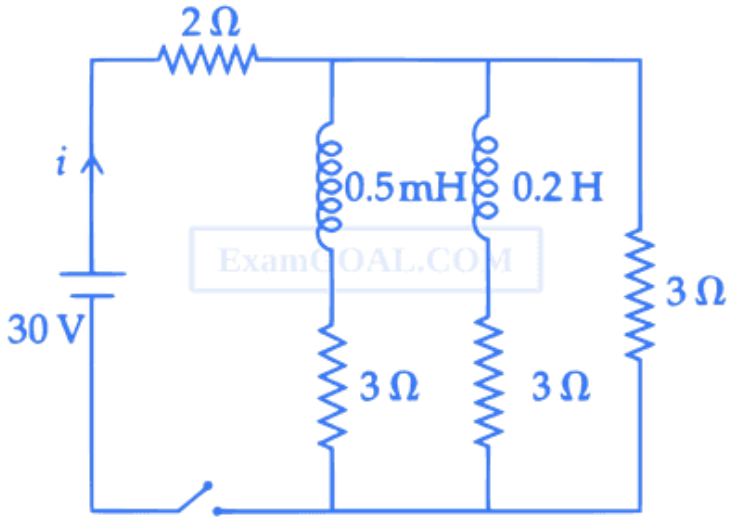
$$I_1 = \frac{200}{100} = 2\,\text{A}.$$

The voltage across the rails is $2 \times 80 = 160\,\text{V}$, giving $1\,\text{A}$ through each conducting branch. Their sum verifies the supply current. The other three options do not satisfy this network reduction.

Step 4: Conclusion The current indicated by the arrow is $\boxed{I_1 = 2\,\text{A}}$, upward. Hence, the correct option is (C).

Quick Tip to Solve: First mark each diode as conducting or open, then include its forward resistance and check whether the indicated current is a branch current or the total.

43. For the given circuit the current i through the battery when the key in closed and the steady state has been reached is _____. [4 marks]



(A) 6 A

(B) 25 A

(C) 10 A

(D) 0 A

Correct Answer: (C) 10 A

Solution:

Concept:

- For an ideal inductor, the induced voltage is $L\,di/dt$. In the steady state of a DC circuit the current is constant, so this voltage is zero and each inductor acts as a connecting wire.
- Resistances in parallel have the same potential difference and add by reciprocals. The equivalent resistance of the parallel network must then be added to the resistor in series with the battery.

Step 1: Reduce the inductor branches The switch has been closed for long enough for the transient currents to settle. Thus neither the $0.5\,\mathrm{mH}$ inductor nor the $0.2\,\mathrm{H}$ inductor contributes a steady voltage drop. Each of their branches retains its $3\,\Omega$ resistor. Together with the right-hand $3\,\Omega$ resistor, there are three equal resistors in parallel. The different inductances affect the approach to equilibrium, rather than the final resistance.

Step 2: Calculate the battery current The parallel combination and total resistance are

$$\frac{1}{R_p} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1, \qquad R_p = 1\,\Omega.$$

$$R_{\mathrm{eq}} = 2 + 1 = 3\,\Omega, \qquad i = \frac{30}{3} = 10\,\mathrm{A}.$$

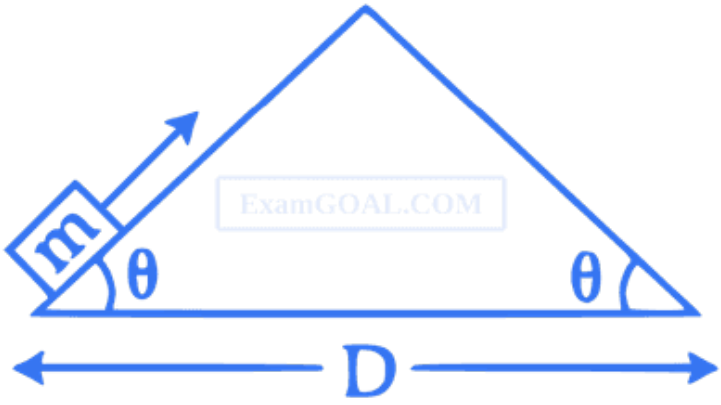
As a check, the parallel network has a voltage of $10\,\mathrm{V}$, so its three branch currents are each $10/3\,\mathrm{A}$, which add to the battery current. The values $6\,\mathrm{A}$ and $25\,\mathrm{A}$ do not satisfy this resistance calculation. Zero current would incorrectly treat the inductors as open circuits.

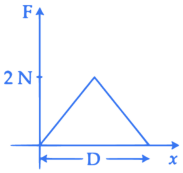
Step 3: Conclusion The steady battery current is $\boxed{10\ \text{A}}$. Hence, the correct option is (C).

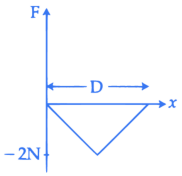
Quick Tip to Solve: For a DC steady state, replace ideal inductors by wires first, but retain the resistors that are in series with them.

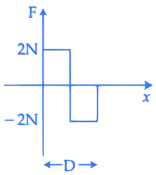
44. An object of mass 'm' is being moved with a constant velocity under the action of an applied force of 2N along a frictionless surface with following surface profile.

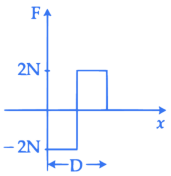
The correct applied force vs distance graph will be : [4 marks]



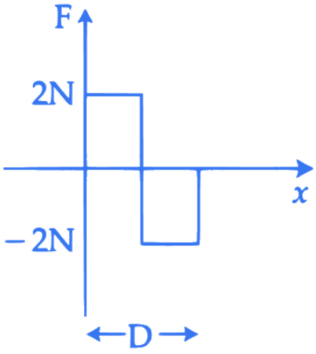
(A) 

(B) 

(C) 

(D) 

Correct Answer: (C)



Solution:

Concept:

- Constant velocity requires zero net force along the direction of motion. On a smooth incline the only tangential forces are the applied force and the component of gravity $mg\sin\theta$.
- The normal reaction acts perpendicular to the surface and therefore does not balance the tangential component of the weight. A straight incline has a constant angle, so the required tangential force is constant on that segment.

Step 1: Find the applied force while climbing Take the positive direction along the motion from left to right across the profile. On the rising straight slope, gravity opposes the motion with component $mg\sin\theta$. To prevent any acceleration, the applied force must balance it:

$$F_{\text{up}} - mg\sin\theta = 0.$$

The stated climbing force is $2\,\text{N}$, hence $mg\sin\theta = 2\,\text{N}$. The force does not increase with height: it depends on the constant slope angle, rather than the gravitational potential energy already gained.

Step 2: Reverse the force on the descending slope On the descending straight slope, gravity now has a component $+mg\sin\theta$ in the direction of motion. The object would accelerate unless the applied force opposes its motion. Therefore

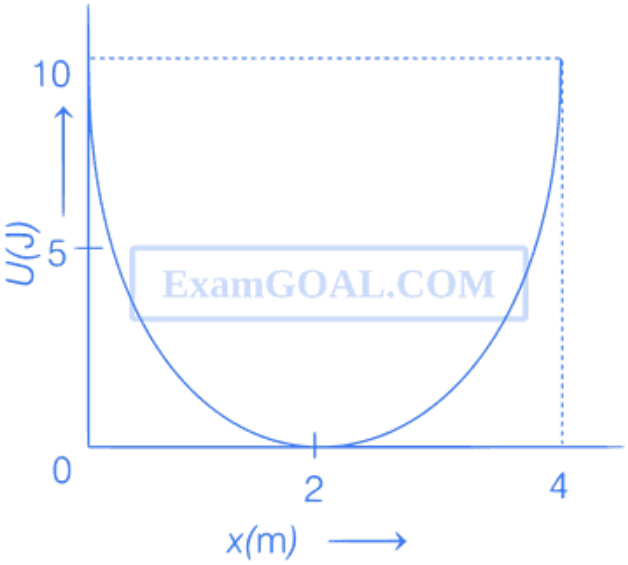
$$F_{\text{down}} + mg\sin\theta = 0, \qquad F_{\text{down}} = -2\,\text{N}.$$

The two equal angles place the peak halfway across the horizontal span D. The force graph is therefore a positive constant followed by an equal negative constant. The triangular graphs in (A) and (B) incorrectly make the force vary continuously on a straight slope; (D) has the signs reversed. The idealized abrupt corner only marks the change between the two straight segments.

Step 3: Conclusion The force is $+2\,\text{N}$ on the ascent and $-2\,\text{N}$ on the descent, as in the third graph. Hence, the correct option is (C).

Quick Tip to Solve: Use the slope angle to find force; use height to find potential energy. A straight ramp gives a constant gravitational component.

45. A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed? [4 marks]



(A) 10 m/s^2

(B) 5 m/s^2

(C) 4 m/s^2

(D) 9.8 m/s^2

Correct Answer: (C) 4 m/s^2

Solution:

Concept:

- Near the equilibrium position x_0, the spring potential energy is $U = \frac{1}{2}k(x - x_0)^2$, where k is the spring constant. Read displacement from the minimum of the graph.
- The periods of an ideal mass-spring oscillator and a small-amplitude simple pendulum are $T_s = 2\pi\sqrt{m/k}$ and $T_p = 2\pi\sqrt{l/g}$, respectively.

Step 1: Obtain the spring constant from the curve The minimum of the potential energy curve occurs at $x_0 = 2\,\text{m}$, where the energy is zero. At $x = 4\,\text{m}$, the energy is $10\,\text{J}$. Thus the displacement from equilibrium is $4 - 2 = 2\,\text{m}$, not $4\,\text{m}$. Substitution gives

$$10 = \frac{1}{2}k(2)^2 = 2k, \qquad k = 5\,\text{N m}^{-1}.$$

The location of the minimum changes the coordinate origin for displacement, but does not change the spring-period formula.

Step 2: Equate the two oscillation periods Both experiments have the same period, so their common

factor 2π cancels:

$$\sqrt{\frac{m}{k}} = \sqrt{\frac{l}{g}}, \qquad \frac{m}{k} = \frac{l}{g}.$$

Consequently, with $m = 5\,\mathrm{kg}$ and $l = 4\,\mathrm{m}$,

$$g = \frac{kl}{m} = \frac{5 \times 4}{5} = 4\,\mathrm{m\,s^{-2}}.$$

As a check, both m/k and l/g are $1\,\mathrm{s^2}$, giving $T = 2\pi\,\mathrm{s}$. The other proposed accelerations, 10, 5, and $9.8\,\mathrm{m\,s^{-2}}$, give different pendulum periods and cannot satisfy the stated equality. No terrestrial value of gravity should be imposed on this planet.

Step 3: Conclusion The acceleration due to gravity is $\boxed{4\,\mathrm{m\,s^{-2}}}$. Hence, the correct option is (C).

Quick Tip to Solve: On a spring-energy graph, measure displacement from the parabola minimum before using $U = \frac{1}{2}k\xi^2$.

46. A capacitor is connected to a 20 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 2 V in 1 μs. The capacitance of the capacitor is _____ μF. Given : $\ln\left(\frac{10}{9}\right) = 0.105$ [4 marks]

(A) 9.52

(B) 0.95

(C) 0.105

(D) 1.85

Correct Answer: (B) 0.95

Solution:

Concept:

- For an initially uncharged capacitor connected to a constant-voltage battery through a resistor, $V_C = V_0(1 - e^{-t/(RC)})$. The product RC is the charging time constant.
- The capacitor voltage approaches the battery voltage exponentially, rather than increasing at a constant rate. The logarithm of the remaining voltage fraction gives $t/(RC)$.

Step 1: Find the exponential fraction The battery voltage is $V_0 = 20\,\mathrm{V}$ and the capacitor voltage after the stated interval is $V_C = 2\,\mathrm{V}$. The capacitor is therefore charged to one tenth of its final voltage:

$$\frac{V_C}{V_0} = \frac{2}{20} = 0.1 = 1 - e^{-t/(RC)}.$$

It follows that

$$e^{-t/(RC)} = 0.9 = \frac{9}{10}, \qquad \frac{t}{RC} = \ln\left(\frac{10}{9}\right) = 0.105.$$

The fraction 0.9 is the remaining fraction, not the fraction already acquired by the capacitor.

Step 2: Calculate and convert the capacitance Rearrange before substituting to keep the time and

capacitance units consistent:

$$C = \frac{t}{0.105R} = \frac{1 \times 10^{-6}}{0.105 \times 10}\,\text{F}.$$

$$C = 0.95238 \times 10^{-6}\,\text{F} = 0.95238\,\mu\text{F}.$$

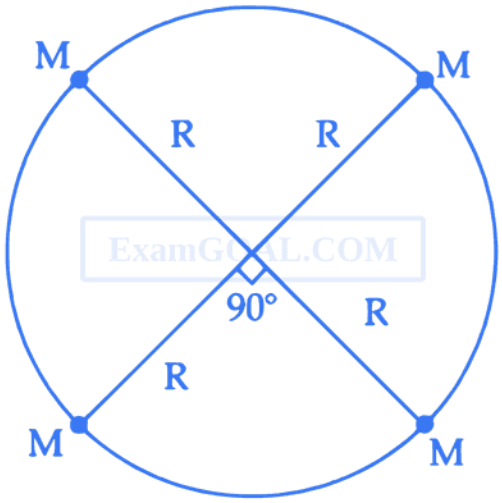
To the precision of the options this is $0.95\,\mu$F. The option 9.52 is too large by a factor of ten. The number 0.105 is a dimensionless logarithm and is not itself the capacitance; 1.85 also fails the exponential relation. As a check, $RC \simeq 9.52\,\mu$s, so the elapsed $1\,\mu$s is about 0.105 of a time constant, consistent with a small initial rise.

Step 3: Conclusion The capacitance is approximately $\boxed{0.95\,\mu\text{F}}$. Hence, the correct option is (B).

Quick Tip to Solve: For capacitor charging, take the logarithm of $1 - V_C/V_0$, rather than of the charged fraction V_C/V_0.

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47. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is : [4 marks]



(A) $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$

(B) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$

(C) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$

(D) $\sqrt{\frac{GM}{R}}$

Correct Answer: (B) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$

Solution:

Concept:

- Each particle needs an inward centripetal force Mv^2/R. Its net gravitational force is the vector sum of the attractions from the other three particles.
- The four equally spaced particles are vertices of a square inscribed in the circle. Adjacent separations

are $\sqrt{2}R$, and the opposite separation is $2R$. Symmetry cancels transverse force components.

Step 1: Add the inward components of gravitational attraction Consider one particle. Each adjacent particle attracts it with force

$$F_a = \frac{GM^2}{(\sqrt{2}R)^2} = \frac{GM^2}{2R^2}.$$

Each of these forces makes 45° with the inward radius. Their transverse components cancel, while their inward components add to

$$2F_a \cos 45^\circ = \frac{GM^2}{\sqrt{2}R^2}.$$

The diametrically opposite particle attracts directly inward with

$$F_o = \frac{GM^2}{(2R)^2} = \frac{GM^2}{4R^2}.$$

This contribution adds to the adjacent pair; it does not subtract.

Step 2: Equate the net force to the centripetal requirement The total inward force is

$$F_{\text{in}} = \frac{GM^2}{R^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = \frac{GM^2}{4R^2}(2\sqrt{2}+1).$$

For uniform circular motion,

$$\frac{Mv^2}{R} = \frac{GM^2}{4R^2}(2\sqrt{2}+1), \qquad v^2 = \frac{GM}{4R}(2\sqrt{2}+1).$$

Taking the positive speed gives the expression in (B). Option (A) incorrectly puts the geometric factor in the denominator. Option (C) subtracts the opposite-particle attraction, even though it is inward. Option (D) neglects the actual separations and vector components of the three forces.

Step 3: Conclusion The speed of each particle is $\boxed{\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a square of attracting masses, add radial force components first; adding the three force magnitudes directly overestimates the result.

48. Electric field of plane electromagnetic wave propagating through a non-magnetic medium is given by

E = 20cos(2 × 10^{10} t − 200x) V/m. The dielectric constant of the medium is equal to : (Take μ_r = 1) [4 marks]

(A) 9

(B) 2

(C) $\frac{1}{3}$

(D) 3

Correct Answer: (A) 9

Solution:

Concept:

- Compare a travelling wave with $E = E_0 \cos(\omega t - kx)$: ω is angular frequency and k is angular wave number. Its phase speed is $v = \omega/k$.
- For an electromagnetic wave in a medium, $v = c/\sqrt{\varepsilon_r \mu_r}$. In a non-magnetic medium $\mu_r = 1$, so the dielectric constant is $\varepsilon_r = (c/v)^2$.

Step 1: Read the phase coefficients and calculate speed The given phase is $2 \times 10^{10} t - 200x$. Thus

$$\omega = 2 \times 10^{10}\,\mathrm{rad\,s^{-1}}, \qquad k = 200\,\mathrm{rad\,m^{-1}}.$$

Their ratio gives

$$v = \frac{\omega}{k} = \frac{2 \times 10^{10}}{2 \times 10^{2}} = 10^{8}\,\mathrm{m\,s^{-1}}.$$

The minus sign between the time and position terms determines the propagation direction. It does not introduce a negative wave speed. The field amplitude $20\,\mathrm{V\,m^{-1}}$ is not needed for the speed or dielectric constant.

Step 2: Use the electromagnetic speed relation With $c = 3 \times 10^{8}\,\mathrm{m\,s^{-1}}$ and $\mu_r = 1$,

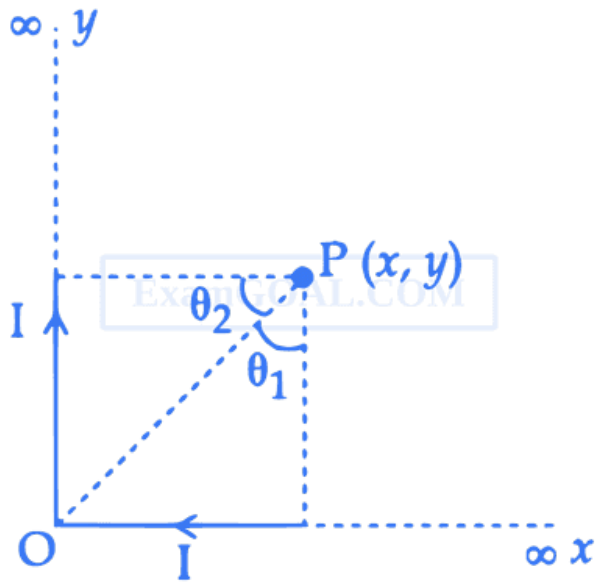
$$\varepsilon_r = \frac{c^2}{v^2 \mu_r} = \left(\frac{3 \times 10^{8}}{10^{8}}\right)^2 = 9.$$

The refractive index is $n = c/v = 3$, whereas the dielectric constant is its square for this non-magnetic medium. Therefore (D), which gives 3, confuses the two quantities. The reciprocal $1/3$ is v/c, not a dielectric constant, and the value 2 cannot reproduce the measured phase speed. Substitution of $\varepsilon_r = 9$ back into $v = c/\sqrt{\varepsilon_r}$ indeed gives $10^{8}\,\mathrm{m\,s^{-1}}$.

Step 3: Conclusion The dielectric constant is $\boxed{\varepsilon_r = 9}$, a dimensionless quantity. Hence, the correct option is (A).

Quick Tip to Solve: Read ω and k directly from the cosine phase, then square c/v to get the dielectric constant when $\mu_r = 1$.

49. There are two infinitely long straight current carrying conductors and they are held at right angles to each other so that their common ends meet at the origin as shown in the figure given below. The ratio of current in both conductor is 1 : 1. The magnetic field at point P is _____. [4 marks]



(A) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$

(B) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(C) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(D) $\frac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}+(x+y)\right]$

Correct Answer: (A) $\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$

Solution:

Concept:

- For a straight wire segment, $B = \mu_0 I(\sin\alpha + \sin\beta)/(4\pi d)$, where d is the perpendicular distance and the endpoint angles are measured from that perpendicular.
- The field direction follows the right-hand rule. In the diagram current flows toward the origin along the positive x-axis and away from the origin along the positive y-axis; both fields at $P(x, y)$ point into the page.

Step 1: Calculate the two wire contributions Let $r = \sqrt{x^2+y^2}$, with $x, y > 0$ as shown. For the horizontal semi-infinite wire, the perpendicular distance is y. The endpoint at infinity contributes sine 1, and the origin contributes x/r. Thus

$$B_h = \frac{\mu_0 I}{4\pi y}\left(1+\frac{x}{r}\right).$$

For the vertical semi-infinite wire, the perpendicular distance is x, while the finite-end sine is y/r. Therefore

$$B_v = \frac{\mu_0 I}{4\pi x}\left(1+\frac{y}{r}\right).$$

The current ratio $1:1$ allows the same symbol I in both expressions.

Step 2: Add fields with their common direction Both contributions are into the page, so their magnitudes add:

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{1}{y} + \frac{1}{x} + \frac{x}{yr} + \frac{y}{xr}\right].$$

Use a common denominator and $x^2 + y^2 = r^2$:

$$B = \frac{\mu_0 I}{4\pi xy}\left[x + y + \frac{x^2 + y^2}{r}\right] = \frac{\mu_0 I}{4\pi xy}(x + y + r).$$

Option (B) has the wrong subtraction and would even give a negative magnitude because $r < x + y$. Options (C) and (D) place xy in the numerator; their dimensions are incompatible with a magnetic field. The plus signs in (A) also agree with the fact that the fields reinforce rather than cancel.

Step 3: Conclusion The field magnitude is $\boxed{\frac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2 + y^2} + x + y\right]}$, directed into the page. Hence, the correct option is (A).

Quick Tip to Solve: Before adding fields of perpendicular wires, use the right-hand rule on each wire; here both contributions reinforce at a first-quadrant point.

50. The half life period of radioactive element x is same as the mean life time of another radioactive element y. Initially they have the same number of atoms. Then : [4 marks]

(A) x-will decay faster than y.

(B) y-will decay faster than x.

(C) x and y have same decay rate initially and later on different decay rate.

(D) x and y decay at the same rate always.

Correct Answer: (B) y-will decay faster than x.

Solution:

Concept:

- Radioactive decay obeys $N(t) = N_0 e^{-\lambda t}$, where λ is the decay constant. The fraction decaying per unit time is governed by λ.
- The half-life is $T_{1/2} = \ln 2/\lambda$, while the mean lifetime is $\tau = 1/\lambda$. A larger decay constant means a shorter lifetime and faster depletion of the sample.

Step 1: Relate the decay constants The given equality is between the half-life of x and the mean lifetime of y, rather than between their half-lives. Therefore,

$$\frac{\ln 2}{\lambda_x} = \frac{1}{\lambda_y} \quad \Longrightarrow \quad \lambda_x = (\ln 2)\lambda_y.$$

Since $\ln 2 \simeq 0.693 < 1$, we have $\lambda_y > \lambda_x$. Thus each nucleus of y has the greater probability of decaying in a fixed short interval.

Step 2: Compare the samples and the options Initially both samples contain N_0 atoms, so their initial activities are $A_x(0) = \lambda_x N_0$ and $A_y(0) = \lambda_y N_0$. Hence $A_y(0) > A_x(0)$. More generally,

$$\frac{N_y(t)}{N_x(t)} = e^{-(\lambda_y - \lambda_x)t} < 1 \quad (t > 0).$$

The sample y therefore loses the same fraction of its nuclei sooner. Option (A) reverses this comparison. Options (C) and (D) are false because even the initial activities are unequal. The statement about faster decay refers to the larger decay constant; it does not assert that the absolute activity of y remains larger at every later instant, since the number of surviving nuclei also changes.

Step 3: Conclusion Element y has the larger decay constant and decays faster than x. Hence, the correct option is (B).

Quick Tip to Solve: Convert half-life and mean lifetime to decay constants before comparing them; the factor $\ln 2$ must not be omitted.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0°C without changing the pressure of the gas. The molecules in the gas rotate but do not oscillate. If the ratio of change in internal energy of the gas to the amount of workdone by the gas is $\frac{x}{10}$. Then the value of x (round off to the nearest integer) is _____. (Given R = 8.31 J mol^{-1} K^{-1}) [4 marks]

Solution:

Concept:

- A diatomic ideal gas with active rotation but no vibration has five quadratic degrees of freedom: three translational and two rotational. Thus $C_V = 5R/2$.
- For an ideal gas, $\Delta U = nC_V\Delta T$. At constant pressure the work done by the gas is $W = P\Delta V = nR\Delta T$, using $PV = nRT$.

Step 1: Calculate the work in the isobaric process The temperature increase of 40.0°C equals an increase of $40.0\,\text{K}$; no 273 is added to a temperature difference. The gas expands because its pressure is fixed. Therefore its work is positive:

$$W = nR\Delta T = 3.00 \times 8.31 \times 40.0 = 997.2\,\text{J}.$$

The amount of gas and the temperature change enter both the work and the internal-energy change.

Step 2: Use the five degrees of freedom Vibrational energy is excluded by the question, so adding vibrational degrees of freedom would overestimate the internal-energy change. Using the translational and rotational contributions gives

$$\Delta U = \frac{5}{2}nR\Delta T = \frac{5}{2}(997.2) = 2493\,\text{J}.$$

Taking the requested ratio,

$$\frac{\Delta U}{W} = \frac{(5/2)nR\Delta T}{nR\Delta T} = \frac{5}{2} = \frac{x}{10}.$$

Thus $x = 10(5/2) = 25$. As a consistency check, the supplied heat is $Q = \Delta U + W = (7/2)nR\Delta T$, the usual constant-pressure result for this diatomic gas.

Step 3: Conclusion The required dimensionless value, rounded to the nearest integer, is $\boxed{x = 25}$.

Quick Tip to Solve: At constant pressure, $\Delta U/W = C_V/R = f/2$; the amount of gas and the temperature rise cancel.

52. The width of one of the two slits in a Young's double slit experiment is three times the other slit. If the amplitude of the light coming from a slit is proportional to the slit-width, the ratio of minimum to maximum intensity in the interference pattern is x : 4 where x is _____. [4 marks]

Solution:

Concept:

- When two coherent waves of amplitudes a_1 and a_2 interfere, intensity is proportional to the square of their resultant amplitude.
- At constructive interference the amplitude is $a_1 + a_2$; at destructive interference its magnitude is $|a_1 - a_2|$. Hence $I_{\min}/I_{\max} = (a_1 - a_2)^2/(a_1 + a_2)^2$.

Step 1: Convert slit widths to amplitudes The problem explicitly states that amplitude is proportional to slit width. If the narrower slit contributes amplitude a, the slit three times as wide contributes amplitude $3a$. Thus

$$a_1 : a_2 = 3 : 1.$$

This is an amplitude ratio, not an intensity ratio. The corresponding individual intensities would be in the ratio $9 : 1$, but these individual intensities must not be directly added to find the interference extrema.

Step 2: Find both interference extrema At a maximum the phase difference is an integral multiple of 2π, so the two amplitudes add. At a minimum their phase difference is an odd multiple of π, so they subtract. With the same proportionality constant K for both intensities,

$$I_{\max} = K(3a + a)^2 = 16Ka^2, \qquad I_{\min} = K(3a - a)^2 = 4Ka^2.$$

Therefore,

$$I_{\min} : I_{\max} = 4 : 16 = 1 : 4.$$

The minima are not completely dark because the two slit amplitudes are unequal: cancellation leaves amplitude $2a$. Comparing the reduced ratio with the prescribed form $x : 4$ directly gives $x = 1$.

Step 3: Conclusion The intensity ratio is $\boxed{I_{\min} : I_{\max} = 1 : 4}$, so the required dimensionless value is $\boxed{x = 1}$.

Quick Tip to Solve: First form the sum and difference of amplitudes, then square them. Unequal amplitudes produce a nonzero minimum.

53. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2×10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x} rad\, h^{-1}$ where x is _____. [4 marks]

Solution:

Concept:

- For circular orbits around the same planet, Kepler's third law gives $T^2 \propto r^3$. The orbital speed is $v = 2\pi r/T$.
- The apparent angular speed of one satellite about the other is the transverse relative velocity divided by their separation: $\omega_{\text{app}} = |v_{\perp,\text{rel}}|/d$. It differs from the difference of their angular speeds about the planet.

Step 1: Find the farther orbital radius Let the nearer orbital radius be $r = 2 \times 10^3\,\text{km}$ and its period be $T_1 = 1\,\text{h}$. The farther satellite has $T_2 = 8\,\text{h}$. Applying Kepler's law,

$$\left(\frac{r_2}{r}\right)^3 = \left(\frac{8}{1}\right)^2 = 64, \qquad r_2 = 4r.$$

Their closest approach occurs when the planet and both satellites lie on the same radial line on the same side of the planet. The separation is then $d = r_2 - r = 3r$.

Step 2: Calculate the transverse relative velocity At closest approach both anticlockwise velocity vectors point along the same tangent. Their magnitudes are

$$v_1 = \frac{2\pi r}{1\,\text{h}}, \qquad v_2 = \frac{2\pi(4r)}{8\,\text{h}} = \frac{\pi r}{\text{h}}.$$

The farther satellite moves more slowly, so its velocity relative to the nearer satellite has transverse magnitude $v_1 - v_2 = \pi r/\text{h}$. Therefore the magnitude of the angular speed of the line of sight is

$$\omega_{\text{app}} = \frac{v_1 - v_2}{3r} = \frac{\pi}{3}\,\text{rad}\,\text{h}^{-1}.$$

The relative line of sight turns clockwise at this instant in axes parallel to fixed inertial axes. The question asks for angular speed, so its positive magnitude is used.

Step 3: Conclusion Comparing $\omega_{\text{app}} = \pi/3\,\text{rad}\,\text{h}^{-1}$ with $\pi/x\,\text{rad}\,\text{h}^{-1}$ gives the dimensionless value $\boxed{x = 3}$.

Quick Tip to Solve: For motion seen from another satellite, use relative transverse velocity over their separation, rather than subtracting planet-centred angular speeds.

54. When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The co-efficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2-1}{\alpha^2}\right)$ where x = _____. [4 marks]

Solution:

Concept:

- A body starting from rest and moving with constant acceleration through distance s satisfies $s = at^2/2$. For the same distance, acceleration is inversely proportional to the square of travel time.
- On a smooth slope at angle θ, $a_s = g\sin\theta$. On a rough slope sliding downward, friction $\mu mg\cos\theta$ acts uphill, so $a_r = g(\sin\theta - \mu\cos\theta)$.

Step 1: Relate the two accelerations using the times For the smooth slope, the given travel time is T, while on the rough slope it is αT. Both motions begin from rest and cover the same distance. Consequently,

$$\frac{1}{2}a_s T^2 = \frac{1}{2}a_r(\alpha T)^2 \quad \Longrightarrow \quad a_r = \frac{a_s}{\alpha^2} = \frac{g\sin\theta}{\alpha^2}.$$

Because $\alpha > 1$, this acceleration is smaller than the smooth-slope acceleration, as expected for opposing friction.

Step 2: Determine the coefficient of friction Equate the acceleration from Newton's second law to that from the timing comparison:

$$g(\sin\theta - \mu\cos\theta) = \frac{g\sin\theta}{\alpha^2}.$$

Cancel g and isolate μ:

$$\mu\cos\theta = \sin\theta\left(1 - \frac{1}{\alpha^2}\right), \qquad \mu = \tan\theta\frac{\alpha^2-1}{\alpha^2}.$$

Putting $\theta = 30°$, for which $\tan 30° = 1/\sqrt{3}$, gives

$$\mu = \frac{1}{\sqrt{3}}\left(\frac{\alpha^2-1}{\alpha^2}\right).$$

The expression is positive for $\alpha > 1$ and tends to zero as $\alpha \to 1$, providing checks on its sign and time dependence.

Step 3: Conclusion Comparison with the given form $\mu = \frac{1}{\sqrt{x}}\frac{\alpha^2-1}{\alpha^2}$ gives the dimensionless value $\boxed{x = 3}$.

Quick Tip to Solve: For equal distances from rest, square the time ratio before comparing accelerations; friction then follows from the downhill force balance.

55. The average translational kinetic energy of N_2 gas molecules at .....°C becomes equal to the K.E. of an electron accelerated from rest through a potential difference of 0.1 volt. (Given k_B = 1.38 $\times$ 10^{-23} J/K) (Fill the nearest integer). [4 marks]

Solution:

Concept:

- The average translational kinetic energy per molecule of an ideal gas is $\langle K_{\text{tr}} \rangle = 3k_BT/2$, with absolute temperature T in kelvin.
- An electron accelerated from rest through potential difference V gains kinetic energy $K_e = eV$, where the magnitude of its charge is $e = 1.60 \times 10^{-19}\,\text{C}$.

Step 1: Express the electron energy in joules The applied potential difference is $0.1\,\text{V}$, so the electron gains

$$K_e = eV = (1.60 \times 10^{-19})(0.1) = 1.60 \times 10^{-20}\,\text{J}.$$

This is $0.1\,\text{eV}$, not $0.1\,\text{J}$. The gas is nitrogen, but only its translational kinetic energy is requested. Rotational degrees of freedom therefore do not enter the expression; using the total diatomic molecular energy would answer a different question.

Step 2: Equate the two energies and convert temperature Set the mean translational molecular energy equal to the electron kinetic energy and solve for the absolute temperature:

$$\frac{3}{2}k_BT = 1.60 \times 10^{-20}\,\text{J}, \qquad T = \frac{2(1.60 \times 10^{-20})}{3(1.38 \times 10^{-23})}\,\text{K}.$$

Simplifying the powers of ten and the numerical factor gives

$$T = \frac{3.20}{4.14} \times 10^3\,\text{K} \simeq 772.95\,\text{K}.$$

The blank is in degrees Celsius, so subtract 273 (or 273.15):

$$t \simeq 772.95 - 273 \simeq 499.95^\circ\text{C}.$$

Using 273.15 also gives the same nearest integer. Rounding is performed after the kelvin-to-Celsius conversion.

Step 3: Conclusion The required temperature, to the nearest integer, is $\boxed{500^\circ\text{C}}$.

Quick Tip to Solve: Use $3k_BT/2$ for translational energy even for a diatomic gas, and convert the final absolute temperature to Celsius.

56. A uniform heating wire of resistance 36Ω is connected across a potential difference of 240 V. The wire is then cut into half and potential difference of 240V is applied across each half separately. The ratio of power dissipation in first case to the total power dissipation in the second case would be 1 : x, where x is _____ [4 marks]

Solution:

Concept:

- For a resistance R maintained at potential difference V, the dissipated power is $P = V^2/R$. The potential difference is specified across each piece here.
- For a uniform wire of unchanged material and cross-sectional area, $R = \rho L/A$; cutting its length in half therefore halves its resistance.

Step 1: Find the power in the uncut wire The original wire has resistance $36\,\Omega$ and potential difference $240\,\text{V}$. Its power dissipation is

$$P_1 = \frac{240^2}{36} = \frac{57600}{36} = 1600\,\text{W}.$$

The relevant comparison is with the combined power of both cut pieces, rather than with one piece alone. The material and cross-sectional area are unchanged when the wire is cut, so resistivity and area cancel in the resistance ratio.

Step 2: Add the power dissipated in the two halves Each half has resistance $R_h = 36/2 = 18\,\Omega$. A potential difference of $240\,\text{V}$ is applied across each half separately, so each dissipates

$$P_h = \frac{240^2}{18} = 3200\,\text{W} = 2P_1.$$

There are two such halves. Their combined power is

$$P_2 = 2P_h = 6400\,\text{W} = 4P_1.$$

Thus

$$P_1 : P_2 = 1600 : 6400 = 1 : 4.$$

Using V^2/R shows why the power rises when resistance falls at fixed voltage. Treating the halves as series-connected across a single 240 V supply would keep the original resistance, but that is not the connection stated in the question.

Step 3: Conclusion The original power to the combined power of the halves is $\boxed{1:4}$. Hence the required dimensionless value is $\boxed{x = 4}$.

Quick Tip to Solve: Each half has half the resistance and twice the original power at the same voltage; add the contributions of both halves.

57. A steel rod with y = 2.0 $\times$ 10^{11} Nm^{-2} and α = 10^{-5} $°C^{-1}$ of length 4 m and area of cross-section 10 cm^2 is heated from 0°C to 400°C without being allowed to extend. The tension produced in the rod is x $\times$ 10^5 N where the value of x is _____. [4 marks]

Solution:

Concept:

- A rod prevented from expanding develops a thermal stress of magnitude $\sigma = Y\alpha\Delta T$. Here Y is Young's modulus and α is the coefficient of linear expansion.
- The restraining force has magnitude $F = \sigma A$, where the cross-sectional area must be expressed in square metres. Heating under restraint produces compression.

Step 1: Find the suppressed thermal strain The temperature rise is $\Delta T = 400 - 0 = 400\,°\text{C}$. If free, the rod would expand by $\Delta L = \alpha L \Delta T$. Hence the strain that the constraint suppresses is

$$\frac{\Delta L}{L} = \alpha\Delta T = 10^{-5}(400) = 4.0 \times 10^{-3}.$$

The length cancels, so the given 4 m does not enter the force calculation.

Step 2: Calculate the force Convert the area before applying Young's law:

$$A = 10\,\text{cm}^2 = 10 \times 10^{-4} = 10^{-3}\,\text{m}^2.$$

Therefore

$$F = Y\alpha\Delta T A = (2.0 \times 10^{11})(4.0 \times 10^{-3})(10^{-3}) = 8.0 \times 10^5\,\text{N}.$$

This is the magnitude of the internal force and the restraining force.

Step 3: Conclusion The required numerical coefficient is $\boxed{x = 8}$. The force magnitude is 8.0×10^5 N; physically, it is compressive for a rod heated without expansion.

Quick Tip to Solve: For a fully constrained rod use $F = YA\alpha\Delta T$; convert cm^2 using a factor of 10^{-4}.

58. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity, ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is _____ ms^{-1}. (Take g = 10 ms^{-2}) [4 marks]

Solution:

Concept:

- With a fixed frictionless pivot, the rod rotates about its lower end. Its moment of inertia about that end is $I = mL^2/3$.
- Loss of gravitational potential energy becomes rotational kinetic energy: $mg\Delta h = I\omega^2/2$. The free end has speed $v = \omega L$.

Step 1: Find the fall of the centre of mass Initially the uniform rod points vertically upward, so its centre of mass is $L/2$ above the pivot. At the lowest position it points vertically downward, so the

centre is $L/2$ below the pivot. Thus the total vertical fall is

$$\Delta h = \frac{L}{2} + \frac{L}{2} = L = 0.6\,\text{m}.$$

The tiny push starts the unstable rod falling; no finite additional kinetic energy is specified.

Step 2: Apply rotational energy conservation The fixed pivot does no work. Using the full downward position rather than the horizontal position gives

$$mgL = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2.$$

Cancel m and use $v = \omega L$:

$$v^2 = 6gL = 6(10)(0.6) = 36\,\text{m}^2\text{s}^{-2}.$$

Taking the positive square root gives the speed of the free end.

Step 3: Conclusion The required speed is $\boxed{6\,\text{m}\,\text{s}^{-1}}$, so the numerical entry is $\boxed{6}$.

Quick Tip to Solve: At the lowest position below the pivot, the centre of mass has fallen by the full rod length, not half its length.

59. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of 72 kmh^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is _____ $\times$ 10^5 N/m. [4 marks]

Solution:

Concept:

- The initial kinetic energy is $K = mv^2/2$. If 90 percent is dissipated by friction, only 10 percent remains available as elastic potential energy.
- A spring compressed by s stores $U = ks^2/2$, where k is the spring constant and s is the change of length, not its original length.

Step 1: Calculate the initial and surviving energies Convert the initial speed into SI units:

$$v = 72 \times \frac{1000}{3600} = 20\,\text{m}\,\text{s}^{-1}.$$

Therefore the initial kinetic energy of the stated moving mass is

$$K = \frac{1}{2}(40000)(20)^2 = 8.0 \times 10^6\,\text{J}.$$

Friction dissipates $0.90K$, leaving

$$U = 0.10K = 8.0 \times 10^5\,\text{J}$$

to be stored in the compressed spring at rest.

Step 2: Solve for the spring constant The compression is explicitly given as $s = 1.0\,\mathrm{m}$. The original 1.5 m length does not replace this compression. Equating the remaining kinetic energy to spring energy gives

$$\frac{1}{2}k(1.0)^2 = 8.0 \times 10^5, \qquad k = 1.6 \times 10^6\,\mathrm{N\,m^{-1}}.$$

Re-express the result using the requested power of ten:

$$k = 16 \times 10^5\,\mathrm{N\,m^{-1}}.$$

Step 3: Conclusion The spring constant is $\boxed{16 \times 10^5\,\mathrm{N\,m^{-1}}}$, and the required numerical entry is $\boxed{16}$.

Quick Tip to Solve: Use only the energy left after friction, and use the actual compression in $ks^2/2$, not the original spring length.

60. A carrier wave with amplitude of 250 V is amplitude modulated by a sinusoidal base band signal of amplitude 150V. The ratio of minimum amplitude to maximum amplitude for the amplitude modulated wave is 50 : x, then value of x is _____. [4 marks]

Solution:

Concept:

- For a sinusoidally amplitude-modulated carrier, the envelope extremes are $A_{\max} = A_c + A_m$ and $A_{\min} = A_c - A_m$, provided $A_m < A_c$.
- The modulation index is $\mu = A_m/A_c$. Equivalently the envelope ratio is $(1-\mu)/(1+\mu)$; both approaches give the same result.

Step 1: Find the envelope extremes The carrier amplitude is $A_c = 250\,\mathrm{V}$ and the modulating-signal amplitude is $A_m = 150\,\mathrm{V}$. Thus $\mu = 150/250 = 0.6 < 1$, so there is no overmodulation and the minimum envelope remains positive. The extremes are

$$A_{\min} = 250 - 150 = 100\,\mathrm{V}, \qquad A_{\max} = 250 + 150 = 400\,\mathrm{V}.$$

These are envelope amplitudes rather than the instantaneous positive and negative carrier peaks.

Step 2: Match the stated ratio The required minimum-to-maximum ratio is

$$\frac{A_{\min}}{A_{\max}} = \frac{100}{400} = \frac{1}{4}.$$

The question expresses this same ratio as $50 : x$, hence

$$\frac{50}{x} = \frac{1}{4} \quad \Rightarrow \quad x = 4(50) = 200.$$

The ratio and its coefficient are dimensionless; the voltage units cancel between the two amplitudes.

Step 3: Conclusion The ratio is $50 : 200$, so the required value is $\boxed{x = 200}$.

Quick Tip to Solve: For AM envelope ratios, subtract the signal amplitude from the carrier for the minimum and add it for the maximum.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Experimentally reducing a functional group cannot be done by which one of the following reagents? [4 marks]

(A) Pt-C/H_2

(B) Na/H_2

(C) Pd-C/H_2

(D) Zn/H_2O

Correct Answer: (B) Na/H_2

Solution:

Concept:

- Catalytic hydrogenation uses molecular hydrogen with a suitable metal surface, such as platinum or palladium, to reduce an appropriate unsaturated functional group.
- A reducing reagent must provide an effective chemical route for electron or hydrogen transfer. The identity of the metal, the hydrogen source and the medium all matter.

Step 1: Recognise the catalytic hydrogenation systems The combinations Pt-C/H_2 and Pd-C/H_2 represent supported platinum and palladium catalysts with hydrogen. Their surfaces activate adsorbed hydrogen and enable its addition to suitable reducible functional groups. Thus the systems in the first and third choices are familiar reduction reagents. Their presence rules them out as answers to the negative wording, "cannot be done.

Step 2: Distinguish sodium from the listed hydrogenation catalysts Na/H_2, as written, is not a standard functional-group reduction system: sodium is not a hydrogenation catalyst comparable to platinum or palladium. Do not confuse it with sodium in alcohol or sodium in liquid ammonia, which are different reducing systems with essential solvent conditions. Zinc in the aqueous reducing system of the fourth choice is the metal-mediated reduction alternative; it does not justify treating sodium with molecular hydrogen as catalytic hydrogenation.

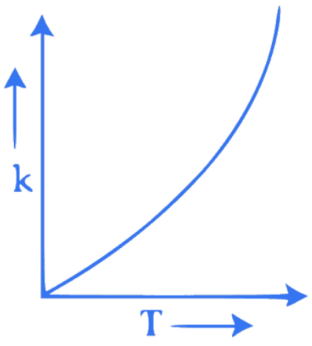
Step 3: Conclusion The unsuitable reagent as listed is Na/H_2. Hence, the correct option is (B).

Quick Tip to Solve: When a reagent contains sodium, check its stated medium; sodium/alcohol or sodium/liquid ammonia cannot be substituted for Na/H_2.

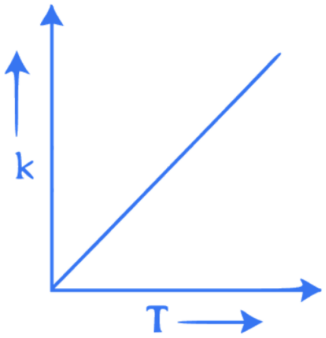
62. Which one of the following given graphs represents the variation of rate constant k with temperature T for an endothermic reaction?

[4 marks]

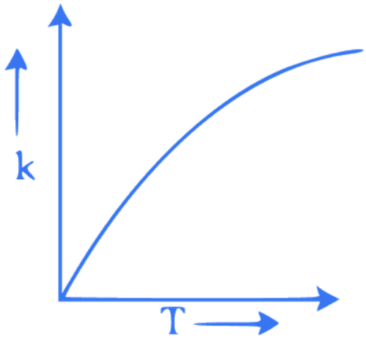
(A)



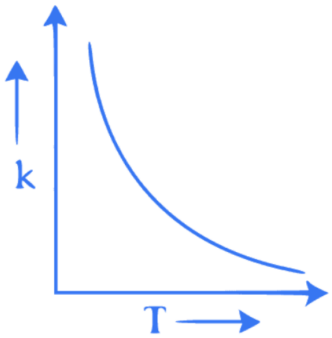
(B)



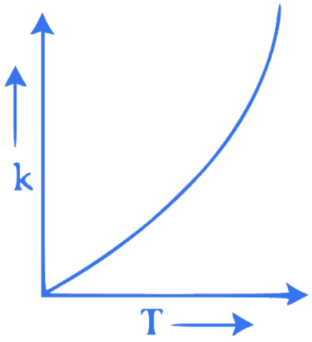
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where E_a is the activation energy and T is absolute temperature.

- For an ordinary activated endothermic reaction, $E_a > 0$. Increasing temperature increases the fraction of collisions able to overcome the activation barrier and increases the rate constant.

Step 1: Determine how the rate constant changes With the Arrhenius prefactor treated as constant over the plotted temperature range,

$$\frac{d\ln k}{dT} = \frac{E_a}{RT^2} > 0.$$

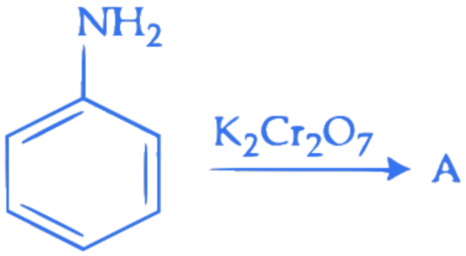
Hence k increases with temperature. The dependence is exponential in $-1/T$, rather than proportional to T. Over the usual temperature range for the schematic graph, the rate constant rises increasingly rapidly as the reaction becomes thermally accessible.

Step 2: Compare all four curves The first graph shows the increasing, upward-curving variation conventionally used for the Arrhenius dependence in the ordinary reaction-temperature range. The second graph instead asserts a linear dependence $k \propto T$. The third depicts a saturating, downward-curving increase over its displayed range, while the fourth depicts a decrease. These do not represent the intended activated-reaction trend in this set of schematic choices. Endothermicity itself should not be confused with a linear relation between k and temperature.

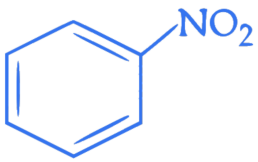
Step 3: Conclusion The first, upward-curving increasing graph represents the intended variation. Hence, the correct option is (A).

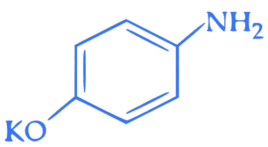
Quick Tip to Solve: Distinguish a plot of k against T from the straight-line Arrhenius plot of $\ln k$ against $1/T$.

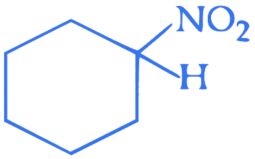
63. Identify A in the following reaction. [4 marks]



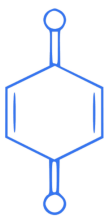
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- Aniline, $C_6H_5NH_2$, is readily oxidised by strong oxidants such as potassium dichromate.
- Under the standard strong oxidation conditions tested here, the product is para-benzoquinone, also called cyclohexa-2,5-diene-1,4-dione.

Step 1: Identify the reactant and oxidant The figure shows an amino group directly attached to a benzene ring, so the starting compound is aniline rather than an aliphatic amine. The reagent over the arrow is $K_2Cr_2O_7$, potassium dichromate. This indicates oxidation, not diazotisation, electrophilic nitration, or reduction of the aromatic ring. Strong oxidation of aniline gives the quinonoid product with two opposite carbonyl groups.

Step 2: Match the product with the structures The first structure has two $C{=}O$ groups at opposite positions on the six-membered ring and two remaining carbon-carbon double bonds: it is para-benzoquinone. The second structure is nitrobenzene, which is not the standard dichromate oxidation product asked here. The third retains the amino group and introduces a potassium phenoxide group, so it does not show the quinone transformation. The fourth is a saturated nitro-substituted ring and would require a different set of transformations, including loss of aromatic unsaturation.

Step 3: Conclusion The product A is para-benzoquinone, $C_6H_4O_2$, represented by the first structure. Hence, the correct option is (A).

Quick Tip to Solve: For aniline with a strong oxidant, look for the para-quinone ring with two opposite carbonyl groups rather than a nitro substituent.

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64. In the following sequence of reactions a compound A, (molecular formula $C_6H_{12}O_2$) with a straight chain structure gives a C_4 carboxylic acid. A is : [4 marks]

$$A \xrightarrow[H_3O^+]{LiAlH_4} B \xrightarrow{Oxidation} C_4 \text{ - carboxylic acid}$$

(A) $CH_3CH_2COOCH_2CH_2CH_3$

(B) $CH_3CH_2CH(OH)CH_2OCH{=}CH_2$

(C) $CH_3CH_2CH_2COOCH_2CH_3$

(D) $CH_3CH_2CH_2OCH{=}CHCH_2OH$

Correct Answer: (C) $CH_3CH_2CH_2COOCH_2CH_3$

Solution:

Concept:

- Lithium aluminium hydride reduces an ester to two primary alcohols after acidic work-up. The acyl carbon remains in the alcohol derived from the acid portion.
- Oxidation of a primary alcohol gives the carboxylic acid with the same number of carbon atoms: $RCH_2OH \xrightarrow{O} RCOOH$.

Step 1: Trace the carbon atoms through the sequence The final product is a straight-chain four-

carbon carboxylic acid, butanoic acid. Its immediate alcohol precursor must therefore be butan-1-ol, $CH_3CH_2CH_2CH_2OH$. Ester reduction gives this alcohol when the ester contains the butanoyl group $CH_3CH_2CH_2CO^-$. The carbonyl carbon is reduced to a terminal CH_2OH group; no carbon atom is removed in this conversion.

Step 2: Identify the ester and compare the alternatives Option (C) is ethyl butanoate, with four carbons in its acid portion and two in its ethoxy portion. Its reduction is

$$CH_3CH_2CH_2COOCH_2CH_3 \xrightarrow[H_3O^+]{LiAlH_4} CH_3CH_2CH_2CH_2OH + CH_3CH_2OH.$$

Oxidation of the first alcohol gives butanoic acid. Option (A), propyl propanoate, instead produces propan-1-ol from both portions and hence a three-carbon acid on oxidation. Options (B) and (D) are unsaturated hydroxy ethers rather than the required ester; they do not supply a butanoyl group for this two-step conversion. The six-carbon molecular formula is satisfied by ethyl butanoate, $C_6H_{12}O_2$.

Step 3: Conclusion Compound A is ethyl butanoate, $CH_3CH_2CH_2COOCH_2CH_3$. Its four-carbon alcohol product gives the required four-carbon acid on oxidation. Hence, the correct option is (C).

Quick Tip to Solve: For ester reduction followed by oxidation, count the carbonyl carbon as part of the acid-side chain; it is retained throughout.

65. The Crystal Field Stabilization Energy (CFSE) and magnetic moment (spin-only) of an octahedral aqua complex of a metal ion (M^{z+}) are $-0.8\Delta_o$ and 3.87 BM, respectively.

Identify (M^{z+}) : [4 marks]

(A) V^{3+}

(B) Cr^{3+}

(C) Mn^{4+}

(D) Co^{2+}

Correct Answer: (D) Co^{2+}

Solution:

Concept:

- For an octahedral complex, each t_{2g} electron contributes $-0.4\Delta_o$, while each e_g electron contributes $+0.6\Delta_o$ to CFSE.
- The spin-only moment is $\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons. Water normally gives the high-spin arrangement for these ions.

Step 1: Infer the number of unpaired electrons The measured spin-only value is 3.87 BM. Taking three unpaired electrons gives

$$\mu = \sqrt{3(3+2)} = \sqrt{15} = 3.873 \text{ BM}.$$

Thus an acceptable ion must have three unpaired electrons in its aqua complex. This condition alone does not distinguish all choices: both a d^3 ion and a high-spin d^7 ion can have three unpaired

electrons.

Step 2: Use CFSE to distinguish the configurations Co^{2+} has configuration $3d^7$. Its high-spin octahedral distribution is $t_{2g}^5 e_g^2$, with one unpaired electron in the lower set and two in the upper set. Therefore

$$\text{CFSE} = 5(-0.4\Delta_o) + 2(0.6\Delta_o) = -0.8\Delta_o.$$

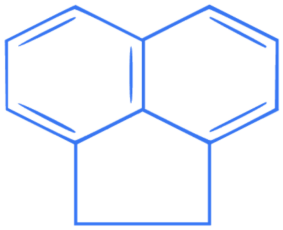
By comparison, V^{3+} is d^2: it has this same CFSE but only two unpaired electrons, giving $\sqrt{8}$ BM. Both Cr^{3+} and Mn^{4+} are d^3: their three electrons occupy t_{2g}, giving $-1.2\Delta_o$, not the required value. Only cobalt(II) satisfies both observations simultaneously.

Step 3: Conclusion The ion is Co^{2+}, whose high-spin $t_{2g}^5 e_g^2$ configuration gives CFSE $-0.8\Delta_o$ and moment $\sqrt{15} \approx 3.87$ BM. Hence, the correct option is (D).

Quick Tip to Solve: Use magnetic moment and CFSE together: either quantity alone can leave more than one possible d-electron count.

66. Which one of the following compounds is aromatic in nature? [4 marks]

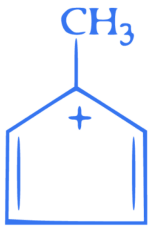
(A)



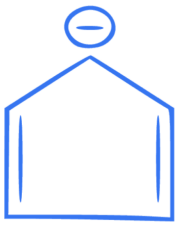
(B)



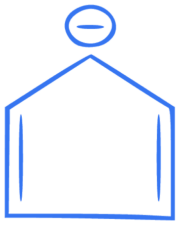
(C)



(D)



Correct Answer: (D)



Solution:

Concept:

- An aromatic ring has a continuous cyclic array of overlapping p orbitals and obeys the $4n + 2$ pi-electron rule when planar.
- A negative charge on an appropriate conjugated ring atom can contribute a lone pair of two pi electrons; an empty p orbital on a carbocation contributes zero electrons.

Step 1: Count electrons in the simple monocyclic options Option (D) is the cyclopentadienyl anion. Its two double bonds provide four pi electrons and the negatively charged carbon provides a p-orbital lone pair, adding two. The ring therefore has six pi electrons,

$$4 + 2 = 6 = 4(1) + 2,$$

delocalised over five ring atoms. It is aromatic. Option (B), cyclobutadiene, has only four pi electrons and is antiaromatic in the planar conjugated description. Option (C), a methyl-substituted cyclopentadienyl cation, also has four ring pi electrons: the positive ring carbon supplies an empty p orbital, not an electron pair. The methyl substituent does not convert this into a six-pi-electron aromatic ring.

Step 2: Examine the fused system and the accepted choices Option (A) contains a naphthalene-type conjugated framework with five double bonds and ten pi electrons. The saturated bridge below the fused rings does not eliminate that aromatic naphthalene framework. Consequently, (A) also represents an aromatic compound, whereas the four-pi-electron rings in (B) and (C) do not. The singular wording of the question therefore does not yield a unique aromatic choice. The final answer key accepts both (A) and (D); selecting the cyclopentadienyl anion is a directly demonstrable correct choice.

Step 3: Conclusion Option (D), the cyclopentadienyl anion, is aromatic with six pi electrons. Option (A) also contains an aromatic ten-pi-electron framework, and the official final key accepts both (A) and (D). Hence, the correct option is (D).

Quick Tip to Solve: Count a conjugated anionic lone pair as two pi electrons, but count an empty carbocation p orbital as zero; also inspect fused aromatic substructures.

67. In the given chemical reaction, colors of the Fe^{2+} and Fe^{3+} ions, are respectively :

$5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5Fe^{3+}$ [4 marks]

(A) Yellow, Orange

(B) Yellow, Green

(C) Green, Orange

(D) Green, Yellow

Correct Answer: (D) Green, Yellow

Solution:

Concept:

- The common aqueous colours are pale green for iron(II), Fe^{2+}, and yellow for iron(III), Fe^{3+}, in the usual qualitative description.
- In acidic permanganate oxidation, iron(II) loses an electron to form iron(III), while permanganate is reduced to manganese(II).

Step 1: Identify the oxidation states in the reaction The iron half-reaction is

$$Fe^{2+} \longrightarrow Fe^{3+} + e^-.$$

Thus the first species named in the question is the ferrous ion before oxidation, and the second is the ferric ion after oxidation. Five iron(II) ions supply the five electrons required for the reduction

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O.$$

This accounts for the coefficients and confirms that the question asks for the colours of two iron ions, rather than those of the oxidising agent.

Step 2: Assign the two iron colours in the requested order The characteristic colour associated with aqueous ferrous salts is pale green, so the first entry must be green. Ferric solutions are commonly described as yellow in this examination context, so the second entry must be yellow. Option (D) has exactly this order. Options (A) and (B) incorrectly start with yellow for iron(II), while option (C) incorrectly gives orange for iron(III). The strong purple colour of permanganate does not change this identification; it belongs to MnO_4^-, a different species. Likewise, the oxidation-state increase from +2 to +3 must not be confused with a change from green to the orange colour of dichromate.

Step 3: Conclusion The colours of Fe^{2+} and Fe^{3+} are green and yellow, respectively. Hence, the correct option is (D).

Quick Tip to Solve: For a redox colour question, first list the exact ions requested; do not substitute the colour of another reagent in the equation.

68. The stereoisomers that are formed by electrophilic addition of bromine to trans-but-2-ene is/are :

[4 marks]

(A) 2 enantiomers and 2 mesomers

(B) 2 identical mesomers

(C) 2 enantiomers

(D) 1 racemic and 2 enantiomers

Correct Answer: (B) 2 identical mesomers

Solution:

Concept:

- Bromine adds across an alkene through a bridged bromonium ion, followed by backside attack of bromide. The net addition is anti.
- A meso compound has stereogenic centres but is achiral because its internal symmetry makes its mirror image superposable.

Step 1: Apply anti addition to the trans alkene Trans-but-2-ene has its two methyl groups on opposite sides of the carbon-carbon double bond. Bromonium-ion formation can begin from either face, but bromide opens the three-membered intermediate from the opposite face. The two bromine atoms therefore add anti across the original double bond. The product skeleton is

$$\mathrm{CH_3CHBrCHBrCH_3},$$

namely 2,3-dibromobutane. The two middle carbon atoms become stereogenic centres, so the configuration at both centres must be tracked rather than treating the reaction as unrestricted random addition.

Step 2: Recognise the identity of the two face products For this trans alkene, anti addition produces the configurations $(2R, 3S)$ and $(2S, 3R)$ when the two possible approaches are labelled separately. Because the ends of the molecule are identical, these descriptions are superposable representations of the same meso compound. They are not a pair of distinct enantiomers and do not constitute a racemic mixture. Thus the printed phrase "2 identical mesomers" in option (B) refers to the identical meso products from the two approaches. Options (A), (C) and (D) wrongly include distinct enantiomeric products for this stereospecific addition. In contrast, anti addition to the cis alkene would produce an enantiomeric pair.

Step 3: Conclusion Both approaches give the same meso-2,3-dibromobutane, represented as $(2R, 3S)$ or $(2S, 3R)$. This corresponds to the printed choice "2 identical mesomers". Hence, the correct option is (B).

Quick Tip to Solve: Remember the pairing for bromination: trans alkene plus anti addition gives the meso product when the two alkene ends are identical.

69. In the following sequence of reactions,

The compounds B and C respectively are :- [4 marks]

$$C_3H_6 \xrightarrow{H^+/H_2O} A \xrightarrow[\text{dil KOH}]{KIO} B + C$$

(A) CI_3COOK, HCOOH

(B) CI_3COOK, CH_3I

(C) CH_3I, HCOOK

(D) CHI_3, CH_3COOK

Correct Answer: (D) CHI_3, CH_3COOK

Solution:

Concept:

- Acid-catalysed hydration of propene follows Markovnikov orientation and gives propan-2-ol, $CH_3CH(OH)CH_3$.
- Alkaline hypoiodite gives the iodoform reaction with methyl ketones and with secondary alcohols containing the $CH_3CH(OH)^-$ group.

Step 1: Identify the alcohol formed by hydration The starting alkene C_3H_6 is propene in the given sequence. Addition of water in acid places the hydroxyl group on the middle carbon, giving

$$CH_3CH{=}CH_2 + H_2O \xrightarrow{H^+} CH_3CH(OH)CH_3.$$

Hence A is propan-2-ol. This alcohol has a methyl group directly attached to its hydroxyl-bearing carbon, which is the structural requirement for a positive iodoform reaction after oxidation to a methyl ketone.

Step 2: Follow oxidation and haloform cleavage Under the alkaline hypoiodite conditions, propan-2-ol is first oxidised to propanone, CH_3COCH_3. Successive iodination of a methyl group adjacent to the carbonyl, followed by hydroxide attack and cleavage, gives yellow iodoform and acetate. The standard net ketone reaction is

$$CH_3COCH_3 + 3\,I_2 + 4\,KOH \longrightarrow CHI_3 + CH_3COOK + 3\,KI + 3\,H_2O.$$

Therefore B and C in the listed product order are CHI_3 and CH_3COOK. Options (A) and (B) propose triiodoacetate rather than haloform cleavage products; option (C) proposes methyl iodide and formate, which do not arise from this methyl-ketone reaction. The carbonyl-containing two-carbon fragment remains as acetate.

Step 3: Conclusion The products are B = iodoform, CHI_3, and C = potassium acetate, CH_3COOK. Hence, the correct option is (D).

Quick Tip to Solve: Convert a secondary alcohol of the form methyl-CH(OH)-R mentally into methyl-CO-R, then split it into iodoform and the R-carboxylate.

70. Given below are two statements :

Statement I : The nucleophilic addition of sodium hydrogen sulphite to an aldehyde or a ketone involves proton transfer to form a stable ion.

Statement II : The nucleophilic addition of hydrogen cyanide to an aldehyde or a ketone yields amine as final product.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement are true.

(B) Statement I is true but Statement II is false.

(C) Statement I is false but Statement II is true.

(D) Both Statement I and Statement II are false.

Correct Answer: (B) Statement I is true but Statement II is false.

Solution:

Concept:

- A carbonyl carbon is electrophilic. Nucleophilic attack breaks the carbon-oxygen pi bond and gives a tetrahedral oxygen-containing intermediate.
- Proton transfer can stabilise the addition product without changing the nitrogen in a cyanide group into an amine.

Step 1: Evaluate the hydrogen sulphite statement Hydrogen sulphite adds to the electrophilic carbonyl carbon through its sulphur-containing nucleophilic centre. The carbonyl oxygen initially acquires negative charge, and proton transfer gives the stable alpha-hydroxysulphonate ion. Schematically,

$$R_2CO + HSO_3^- \longrightarrow R_2C(OH)SO_3^-.$$

The product retains a negative charge on the sulphonate group and is isolated as its sodium salt. Thus the statement about proton transfer forming a stable ion is correct: proton transfer does not require the whole product to be electrically neutral.

Step 2: Evaluate the hydrogen cyanide statement Cyanide attacks the carbonyl carbon, producing an alkoxide bearing a newly attached nitrile group. Protonation of oxygen then gives a cyanohydrin:

$$R_2CO + HCN \longrightarrow R_2C(OH)CN.$$

The $-CN$ group remains a nitrile in this addition product; no reduction of its carbon-nitrogen triple bond has occurred. An amine would require a further chemical transformation, not merely nucleophilic addition of hydrogen cyanide. Statement II is therefore false. Option (A) fails because it makes II true, option (C) reverses both truth values, and option (D) wrongly rejects the true sulphite statement.

Step 3: Conclusion Statement I is true, while Statement II is false because hydrogen cyanide gives a cyanohydrin rather than an amine. Hence, the correct option is (B).

Quick Tip to Solve: After carbonyl addition, identify which group is actually present: cyanohydrins retain a nitrile group; amines require additional reduction chemistry.

71. Which one of the following gives the most stable Diazonium salt? [4 marks]

(A) $CH_3CH_2CH_2NH_2$ (propan-1-amine)

(B) p-Toluidine (4-methylaniline), p-$CH_3C_6H_4NH_2$

(C) N-Methylaniline, $C_6H_5NHCH_3$

(D) $(CH_3)_2CHNH_2$ (propan-2-amine)

Correct Answer: (B) p-Toluidine (4-methylaniline), p-$CH_3C_6H_4NH_2$

Solution:

Concept:

- Diazotisation converts a primary amine, RNH_2, into a diazonium salt with the group $-N_2^+$. The stability depends strongly on whether this group is bonded directly to an aromatic ring.
- Arenediazonium ions are resonance-stabilised by conjugation with the benzene ring. Ordinary aliphatic diazonium ions readily lose the very stable nitrogen molecule and are much less stable.

Step 1: Classify the four amines Option (A) is propan-1-amine and option (D) is propan-2-amine. Both are primary aliphatic amines. Option (B), p-toluidine, is a primary aromatic amine because its NH_2 group is attached directly to the benzene ring. Option (C), N-methylaniline, is a secondary aromatic amine: nitrogen bears a phenyl group and a methyl group.

Step 2: Compare their reactions with nitrous acid The primary aromatic amine in (B) undergoes diazotisation at low temperature to give p-methylbenzenediazonium salt. For example, with hydrochloric acid the transformation is

$$\text{p-}CH_3C_6H_4NH_2 \xrightarrow[273\text{-}278\,K]{NaNO_2/HCl} \text{p-}CH_3C_6H_4N_2^+\,Cl^-.$$

The diazonium group is conjugated with the aromatic ring. In contrast, the aliphatic diazonium intermediates from (A) and (D) decompose easily with loss of N_2. A secondary amine such as (C) normally forms an N-nitroso compound rather than the corresponding primary-amine diazonium salt.

Step 3: Conclusion p-Toluidine gives the most stable diazonium salt among the listed amines, owing to the aromatic diazonium structure. Hence, the correct option is (B).

Quick Tip to Solve: First check that the amine is primary; then check whether its amino nitrogen is directly bonded to an aromatic ring.

72. The potassium ferrocyanide solution gives a Prussian blue colour, when added to : [4 marks]

(A) $CoCl_3$

(B) $FeCl_2$

(C) $CoCl_2$

(D) $FeCl_3$

Correct Answer: (D) $FeCl_3$

Solution:

Concept:

- Potassium ferrocyanide is $K_4[Fe(CN)_6]$; its complex anion is hexacyanidoferrate(II), $[Fe(CN)_6]^{4-}$. The iron inside this complex has oxidation state $+2$.
- Ferric ions, Fe^{3+}, react with ferrocyanide ions to produce the characteristic deep Prussian blue ferric ferrocyanide precipitate.

Step 1: Identify the ion needed for the blue precipitate The analytical test requires free ferric ions in the solution being tested. Ferric chloride supplies these ions by dissociation:

$$FeCl_3 \longrightarrow Fe^{3+} + 3\,Cl^-.$$

The chloride ions are spectators; the distinctive colour is produced by the iron cyanide coordination solid formed from ferric ions and ferrocyanide.

Step 2: Write the precipitation reaction and eliminate alternatives The idealised ionic and molecular equations are

$$4\,Fe^{3+} + 3\,[Fe(CN)_6]^{4-} \longrightarrow Fe_4[Fe(CN)_6]_3(s)$$

$$4\,FeCl_3 + 3\,K_4[Fe(CN)_6] \longrightarrow Fe_4[Fe(CN)_6]_3(s) + 12\,KCl.$$

The blue product is ferric ferrocyanide, commonly called Prussian blue. Option (B), $FeCl_2$, supplies ferrous rather than ferric ions, so it is not this direct ferric-ion test. Options (A) and (C) contain cobalt and cannot supply the required ferric ions. Do not confuse potassium ferrocyanide used here with potassium ferricyanide, which is a different reagent.

Step 3: Conclusion Potassium ferrocyanide gives the Prussian blue reaction with $FeCl_3$, which supplies Fe^{3+}. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish ferrocyanide, with complexed Fe(II), from the free Fe(III) ions required for its Prussian blue test.

73. Number of paramagnetic oxides among the following given oxides is _____. Li_2O, CaO, Na_2O_2, KO_2, MgO and K_2O [4 marks]

(A) 1

(B) 2

(C) 3

(D) 0

Correct Answer: (A) 1

Solution:

Concept:

- A substance is paramagnetic when it has one or more unpaired electrons. Closed-shell ions with all electrons paired are diamagnetic.
- Normal oxide, peroxide and superoxide contain different oxygen species: O^{2-}, O_2^{2-} and O_2^{-}, respectively. Their electron configurations must be checked separately.

Step 1: Group the compounds by their oxygen ions The normal oxides Li_2O, CaO, MgO and K_2O contain O^{2-}, which has the closed-shell configuration of neon. Their metal cations, Li^+, Ca^{2+}, Mg^{2+} and K^+, are also closed shell. All four compounds therefore have paired electrons and are diamagnetic.

Step 2: Compare peroxide with superoxide and count Na_2O_2 is sodium peroxide and contains O_2^{2-}. Its two antibonding π^* orbitals are fully occupied, so no unpaired electron remains. It is diamagnetic. KO_2, however, contains the superoxide ion O_2^{-}. Addition of one electron to molecular oxygen leaves three electrons in the two antibonding π^* orbitals: one orbital is doubly occupied and the other singly occupied. Hence superoxide has one unpaired electron and is paramagnetic. Only KO_2 qualifies; counting the peroxide as paramagnetic would incorrectly give two, while zero would miss the superoxide.

Step 3: Conclusion There is exactly $\boxed{1}$ paramagnetic oxide in the list, namely KO_2. Hence, the correct option is (A).

Quick Tip to Solve: For alkali-metal oxygen compounds, identify the anion first: oxide and peroxide are paired, whereas superoxide has one unpaired electron.

74. Identify the element for which electronic configuration in +3 oxidation state is $[\mathrm{Ar}]3d^5$: [4 marks]

(A) Ru

(B) Mn

(C) Co

(D) Fe

Correct Answer: (D) Fe

Solution:

Concept:

- The symbol $[\mathrm{Ar}]$ represents 18 electrons. Thus the configuration $[\mathrm{Ar}]3d^5$ contains $18 + 5 = 23$ electrons.
- A tripositive ion has lost three electrons from its neutral atom. For first-row transition elements the $4s$ electrons are removed before the $3d$ electrons during ion formation.

Step 1: Recover the atomic number from the ion The given configuration belongs to the +3 ion, not to the neutral atom. Consequently the neutral atom must contain three more electrons:

$$Z = (18 + 5) + 3 = 26.$$

The element of atomic number 26 is iron, Fe. Its neutral configuration is $[\mathrm{Ar}]3d^6 4s^2$.

Step 2: Check ionisation and the other choices Iron loses its two $4s$ electrons first and then one $3d$ electron:

$$\mathrm{Fe} \longrightarrow \mathrm{Fe}^{3+} + 3\,\mathrm{e}^-, \qquad [\mathrm{Ar}]3d^6 4s^2 \longrightarrow [\mathrm{Ar}]3d^5.$$

This gives the required half-filled $3d$ subshell. Manganese starts from $[\mathrm{Ar}]3d^5 4s^2$, so its +3 ion is $[\mathrm{Ar}]3d^4$; its +2 ion, not +3 ion, has $3d^5$. Cobalt has atomic number 27 and gives $[\mathrm{Ar}]3d^6$ as a +3 ion. Ruthenium is a second-row transition metal and involves the $4d$ shell and a krypton core, so it cannot have the specified argon-core configuration.

Step 3: Conclusion The required element is $\boxed{\mathrm{Fe}}$, because Fe^{3+} has configuration $[\mathrm{Ar}]3d^5$. Hence, the correct option is (D).

Quick Tip to Solve: Count electrons in the given ion and add its positive charge to recover the atomic number; do not confuse Mn(II) with Fe(III).

75. Water sample is called cleanest on the basis of which one of the BOD values given below : [4 marks]

(A) 11 ppm

(B) 15 ppm

(C) 3 ppm

(D) 21 ppm

Correct Answer: (C) 3 ppm

Solution:

Concept:

- Biochemical oxygen demand (BOD) measures the dissolved oxygen consumed by microorganisms while decomposing biodegradable organic matter in a water sample under specified conditions.
- A larger BOD indicates more oxygen-demanding organic contamination. For comparable measurements, a smaller BOD indicates cleaner water with less biodegradable pollution.

Step 1: Relate cleanliness to oxygen demand Microorganisms use oxygen to oxidise biodegradable organic substances. A sample containing more such matter supports greater microbial oxygen consumption, so its BOD is higher. Thus cleanliness and BOD vary in opposite directions for the comparison asked here. BOD is oxygen demanded by decomposition; it must not be confused with the amount of dissolved oxygen already present in the sample.

Step 2: Compare all four reported values Arrange the given values in increasing order:

$$3\,\text{ppm} < 11\,\text{ppm} < 15\,\text{ppm} < 21\,\text{ppm}.$$

The sample with BOD $3\,\text{ppm}$ has the smallest demand for oxygen and hence the least biodegradable organic pollution among these choices. The samples at 11, 15 and $21\,\text{ppm}$ all demand more oxygen and are correspondingly less clean by this criterion. Choosing $21\,\text{ppm}$ would reverse the required relationship by treating oxygen demand as oxygen availability. The question explicitly bases the comparison on BOD, so the minimum of these values determines the answer.

Step 3: Conclusion The cleanest sample on the stated BOD basis has BOD $\boxed{3\,\text{ppm}}$. Hence, the correct option is (C).

Quick Tip to Solve: When comparing BOD values measured under the same conditions, choose the smallest value for the cleanest water.

76. Calamine and Malachite, respectively, are the ores of : [4 marks]

(A) Nickel and Aluminium

(B) Zinc and Copper

(C) Copper and Iron

(D) Aluminium and Zinc

Correct Answer: (B) Zinc and Copper

Solution:

Concept:

- An ore is a naturally occurring mineral from which a metal can be extracted economically. Identifying the metal in the mineral formula determines the metal to which an ore belongs.
- Calamine is the carbonate ore of zinc, $ZnCO_3$. Malachite is basic copper carbonate, commonly written $CuCO_3 \cdot Cu(OH)_2$ or $Cu_2CO_3(OH)_2$.

Step 1: Identify the metal in calamine The formula $ZnCO_3$ contains zinc as its metal constituent, so calamine is a zinc ore. Carbonate ores can be calcined to obtain an oxide; for calamine this conversion is

$$ZnCO_3 \xrightarrow{\text{heat}} ZnO + CO_2.$$

This chemical composition rules out nickel, copper and aluminium as the first entry of the required pair.

Step 2: Identify malachite and retain the stated order Malachite contains copper in its basic carbonate structure, $Cu_2CO_3(OH)_2$. On heating it yields copper(II) oxide:

$$Cu_2CO_3(OH)_2 \xrightarrow{\text{heat}} 2\,CuO + CO_2 + H_2O.$$

The copper in this mineral can subsequently be recovered through appropriate metallurgical treatment. Therefore the order demanded by the word respectively is calamine followed by malachite, hence zinc followed by copper. Option (A) assigns nickel and aluminium, neither of which fits these formulas. Option (C) incorrectly calls calamine a copper ore and malachite an iron ore. Option (D) assigns aluminium to calamine and zinc to malachite; both assignments are wrong.

Step 3: Conclusion Calamine and malachite are ores of $\boxed{\text{zinc and copper}}$, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Anchor the names to formulas: calamine is zinc carbonate, whereas the green basic carbonate malachite contains copper.

77. **Match List - I with List - II :**

List - I (Colloid Preparation Method)	List - II (Chemical Reaction)
(a) Hydrolysis	**(i)** $2\,AuCl_3 + 3\,HCHO + 3\,H_2O \longrightarrow 2\,Au\,(sol) + 3\,HCOOH + 6\,HCl$
(b) Reduction	**(ii)** $As_2O_3 + 3\,H_2S \longrightarrow As_2S_3(sol) + 3\,H_2O$
(c) Oxidation	**(iii)** $SO_2 + 2\,H_2S \longrightarrow 3\,S\,(sol) + 2\,H_2O$
(d) Double Decomposition	**(iv)** $FeCl_3 + 3\,H_2O \longrightarrow Fe(OH)_3(sol) + 3\,HCl$

Choose the most appropriate answer from the options given below. [4 marks]

(A) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(B) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)

(C) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(D) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Correct Answer: (B) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)

Solution:

Concept:

- Chemical condensation methods prepare colloidal particles by hydrolysis, reduction, oxidation or double decomposition. Classify the actual reaction rather than the name of the sol alone.
- Hydrolysis uses water to form a sparingly soluble hydroxide; reduction lowers an oxidation state, oxidation raises one, and double decomposition exchanges ions to form an insoluble dispersed product.

Step 1: Match hydrolysis and reduction In reaction (iv), ferric chloride reacts with water to form ferric hydroxide sol:

$$FeCl_3 + 3\,H_2O \longrightarrow Fe(OH)_3(sol) + 3\,HCl.$$

This is hydrolysis, so (a)-(iv). In reaction (i), formaldehyde reduces gold(III) chloride to colloidal gold:

$$2\,AuCl_3 + 3\,HCHO + 3\,H_2O \longrightarrow 2\,Au\,(sol) + 3\,HCOOH + 6\,HCl.$$

Gold changes from oxidation state +3 to 0; hence this is reduction and (b)-(i).

Step 2: Match oxidation and double decomposition Reaction (iii) is

$$SO_2 + 2\,H_2S \longrightarrow 3\,S\,(sol) + 2\,H_2O.$$

Sulfur in H_2S changes from -2 to 0, so oxidation of hydrogen sulfide produces the sulfur sol and (c)-(iii). Reaction (ii),

$$As_2O_3 + 3\,H_2S \longrightarrow As_2S_3(sol) + 3\,H_2O,$$

forms arsenic sulfide by double decomposition, giving (d)-(ii). Only option (B) contains all four matches. Option (A) incorrectly calls gold reduction hydrolysis; option (C) misassigns reduction to the arsenic sulfide reaction; option (D) also assigns hydrolysis and oxidation incorrectly.

Step 3: Conclusion The matching is hydrolysis–iv, reduction–i, oxidation–iii and double decomposition–ii. Hence, the correct option is (B).

Quick Tip to Solve: Look for water producing a hydroxide for hydrolysis, a metal ion becoming metal for reduction, and sulfide becoming sulfur for oxidation.

78. Monomer units of Dacron polymer are : [4 marks]

(A) ethylene glycol and phthalic acid

(B) ethylene glycol and terephthalic acid

(C) glycerol and terephthalic acid

(D) glycerol and phthalic acid

Correct Answer: (B) ethylene glycol and terephthalic acid

Solution:

Concept:

- Dacron, also called Terylene or polyethylene terephthalate (PET), is a condensation polyester. Its chain contains ester linkages, $-CO-O-$, between alcohol-derived and acid-derived units.
- A linear polyester forms from a diol containing two hydroxyl groups and a dicarboxylic acid containing two carboxyl groups. Esterification eliminates water while joining successive monomers.

Step 1: Identify the alcohol component The alcohol used for Dacron is ethylene glycol, $HO-CH_2-CH_2-OH$. Its two hydroxyl groups allow chain growth at both ends. Glycerol has three hydroxyl groups and can introduce branching or cross-linking, so it is not the alcohol in the standard linear Dacron chain. This rules out options (C) and (D).

Step 2: Identify the acid component The acid is terephthalic acid, $HOOC-C_6H_4-COOH$, with the carboxyl groups at the para (1,4) positions of the benzene ring. Condensation with ethylene glycol gives the repeat unit $[-O-CH_2-CH_2-O-CO-C_6H_4-CO-]_n$. Phthalic acid is the ortho (1,2) isomer; confusing these positional isomers would incorrectly select option (A). The name polyethylene terephthalate itself identifies the required acid.

Step 3: Conclusion The monomer units are ethylene glycol and terephthalic acid. Hence, the correct option is (B).

Quick Tip to Solve: Recall Dacron as PET: the ethylene part points to ethylene glycol and terephthalate points to terephthalic acid.

79. Hydrogen peroxide reacts with iodine in basic medium to give : [4 marks]

(A) IO_4^-

(B) IO^-

(C) I^-

(D) IO_3^-

Correct Answer: (C) I^-

Solution:

Concept:

- Hydrogen peroxide can act as either an oxidising agent or a reducing agent. With iodine in alkaline medium, it acts as a reducing agent and is oxidised to molecular oxygen.
- Iodine has oxidation state zero in I_2, while iodide has oxidation state -1. Reduction of iodine therefore requires two electrons per iodine molecule.

Step 1: Write the alkaline half-reactions The reduction half-reaction is $I_2 + 2\,e^- \longrightarrow 2\,I^-$. The oxidation half-reaction of peroxide in basic solution is $H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-$. Oxygen changes from oxidation state -1 in peroxide to zero in O_2, supplying exactly the electrons required to reduce iodine.

Step 2: Combine and identify the iodine product Adding the half-reactions and cancelling the electrons gives

$$I_2 + H_2O_2 + 2\,OH^- \longrightarrow 2\,I^- + 2\,H_2O + O_2.$$

Both atoms and charge are balanced: each side has two iodine atoms and net charge -2. Thus the iodine-containing product is iodide. Hypoiodite, iodate and periodate contain iodine in oxidation states $+1$, $+5$ and $+7$, respectively. Those would represent oxidation of iodine, whereas peroxide reduces iodine under these conditions, so options (A), (B) and (D) do not describe this reaction.

Step 3: Conclusion Hydrogen peroxide reduces iodine to I^- in basic medium. Hence, the correct option is (C).

Quick Tip to Solve: For iodine with alkaline peroxide, remember oxygen evolution: peroxide becomes oxygen and iodine becomes iodide.

80. The oxide without nitrogen-nitrogen bond is : [4 marks]

(A) N_2O

(B) N_2O_4

(C) N_2O_3

(D) N_2O_5

Correct Answer: (D) N_2O_5

Solution:

Concept:

- The presence of a nitrogen–nitrogen bond is determined from molecular connectivity, rather than just the number of nitrogen atoms in a formula.
- In molecular dinitrogen pentoxide, the two nitrogen centres are joined through a bridging oxygen: $O_2N-O-NO_2$. A nitrogen–oxygen–nitrogen bridge is not a direct nitrogen–nitrogen bond.

Step 1: Check the first three oxide structures In nitrous oxide, N_2O, the atoms have the sequence N–N–O, so the two nitrogen atoms are directly bonded. In dinitrogen tetroxide, N_2O_4, dimerisation of two nitrogen dioxide units gives O_2N-NO_2, which has a direct N–N bond. Molecular N_2O_3 also has a direct N–N linkage, conventionally represented by joining the NO and NO_2 fragments. Thus options (A), (B) and (C) fail the requirement.

Step 2: Examine dinitrogen pentoxide Dinitrogen pentoxide is the anhydride of nitric acid. Its molecular framework is $O_2N-O-NO_2$: each nitrogen is attached to oxygen atoms and the two nitrogen centres communicate through the central oxygen. There is no direct N–N linkage. The solid can also be represented as nitronium nitrate, $[NO_2]^+[NO_3]^-$; its separate oxygen-containing ions likewise contain no N–N bond. The absence of that bond therefore singles out the same oxide, rather than relying on an oxidation-state comparison.

Step 3: Conclusion The oxide without a nitrogen–nitrogen bond is N_2O_5. Hence, the correct option is (D).

Quick Tip to Solve: Write the connectivity of N_2O_5 as $O_2N-O-NO_2$; the central oxygen rules out a direct N–N bond.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The molar solubility of $Zn(OH)_2$ in 0.1 M NaOH solution is x $\times$ 10^{-18} M. The value of x is _____ (Nearest integer)

(Given : The solubility product of $Zn(OH)_2$ is 2 $\times$ 10^{-20}) [4 marks]

Solution:

Concept:

- For the dissolution $Zn(OH)_2(s) \rightleftharpoons Zn^{2+}(aq) + 2\,OH^-(aq)$, the solubility product is $K_{sp} = [Zn^{2+}][OH^-]^2$. The pure solid is omitted from this expression.

- If the molar solubility is s, dissolution contributes s of zinc ions and $2s$ of hydroxide ions. The pre-existing hydroxide from sodium hydroxide suppresses dissolution by the common-ion effect.

Step 1: Express the equilibrium concentrations The given sodium hydroxide solution is a strong base of concentration $0.1\,\text{mol}\,\text{L}^{-1}$, so it supplies approximately $0.1\,\text{mol}\,\text{L}^{-1}$ hydroxide ions. Consequently, $[\text{Zn}^{2+}] = s$ and $[\text{OH}^-] = 0.1 + 2s$. Since the solubility product is extremely small, the dissolved hydroxide contribution can be neglected provisionally.

Step 2: Calculate and check the solubility Substituting the common-ion concentration gives

$$2 \times 10^{-20} = s(0.1)^2.$$

Hence

$$s = \frac{2 \times 10^{-20}}{10^{-2}} = 2 \times 10^{-18}\,\text{mol}\,\text{L}^{-1}.$$

The neglected contribution is $2s = 4 \times 10^{-18}\,\text{mol}\,\text{L}^{-1}$, negligible beside $0.1\,\text{mol}\,\text{L}^{-1}$, confirming the approximation. Using $K_{sp} = 4s^3$ would apply to dissolution in pure water, not to this solution containing added hydroxide. Comparing with $s = x \times 10^{-18}\,\text{M}$ gives the required coefficient.

Step 3: Conclusion The molar solubility is $2 \times 10^{-18}\,\text{M}$, so $\boxed{x = 2}$ (nearest integer; dimensionless coefficient).

Quick Tip to Solve: In a strong common-ion solution, use its supplied ion concentration in the solubility product before substituting stoichiometric multiples of solubility.

82. For the reaction, $2NO_2(g) \rightleftharpoons N_2O_4(g)$, when $\Delta S = -176.0\ JK^{-1}$ and $\Delta H = -57.8\ kJ\ mol^{-1}$, the magnitude of ΔG at 298 K for the reaction is _____ kJ mol^{-1}. (Nearest integer) [4 marks]

Solution:

Concept:

- At temperature T, the Gibbs energy change is $\Delta G = \Delta H - T\Delta S$. Enthalpy and the entropy term must be expressed in matching energy units.
- A negative entropy change makes $-T\Delta S$ positive. The magnitude of Gibbs energy is $|\Delta G|$, so it is reported as a nonnegative number even when the actual change is negative.

Step 1: Convert the entropy and find its temperature term The enthalpy change is $\Delta H = -57.8\,\text{kJ}\,\text{mol}^{-1}$. Convert the given entropy change to kilojoules for the reaction as written:

$$\Delta S = -176.0\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1} = -0.1760\,\text{kJ}\,\text{K}^{-1}\,\text{mol}^{-1}.$$

At 298 K,

$$T\Delta S = 298(-0.1760) = -52.448\,\text{kJ}\,\text{mol}^{-1}.$$

The negative sign agrees with the decrease from two gas molecules to one gas molecule.

Step 2: Calculate the signed change and then its magnitude Substitute without dropping the negative

entropy sign:

$$\Delta G = -57.8 - (-52.448) = -5.352\,\text{kJ mol}^{-1}.$$

The favourable negative enthalpy contribution is larger in magnitude than the opposing entropy contribution at this temperature. The question asks for magnitude, giving $|\Delta G| = 5.352\,\text{kJ mol}^{-1}$. Rounding this final value, rather than rounding the intermediate product, gives the nearest integer. No extra factor of two is applied: the thermodynamic data belong to the entire reaction as written.

Step 3: Conclusion The magnitude is $5.352\,\text{kJ mol}^{-1}$, or $\boxed{5\,\text{kJ mol}^{-1}}$ to the nearest integer.

Quick Tip to Solve: Convert entropy to kJ first, retain its sign in $\Delta H - T\Delta S$, and take the absolute value only when the question asks for magnitude.

83. The sum of oxidation states of two silver ions in $[Ag(NH_3)_2]$ $[Ag(CN)_2]$ complex is _____.

[4 marks]

Solution:

Concept:

- An oxidation state is found by adding the formal charges of the metal and its ligands to obtain the charge on the coordination entity. Neutral ammonia contributes zero, whereas each cyanide ligand contributes -1.
- The compound $[Ag(NH_3)_2][Ag(CN)_2]$ is neutral as a whole. Its two silver atoms belong to different coordination entities, so their oxidation states must be included separately in the requested sum.

Step 1: Set up the charge balance for both silver centres Let the silver oxidation states in $[Ag(NH_3)_2]$ and $[Ag(CN)_2]$ be a and b, respectively. The first entity has charge $a + 2(0) = a$, while the second has charge $b + 2(-1) = b - 2$. Neutrality of the complete salt therefore requires

$$a + (b - 2) = 0.$$

This relation directly determines the sum even without assuming the separate charges in advance.

Step 2: Find the sum and identify the usual ionic description Rearranging the neutral-salt equation gives

$$a + b = 2.$$

In the usual description, the ions are $[Ag(NH_3)_2]^+$ and $[Ag(CN)_2]^-$. Their balances are $a + 0 = +1$ and $b - 2 = -1$, hence $a = b = +1$. Their sum is therefore $+1 + (+1) = +2$. A complex ion with a negative overall charge can still have a positive metal oxidation state because negatively charged ligands contribute to its total charge. Counting two ammonia ligands as charged would give an incorrect result.

Step 3: Conclusion The two silver oxidation states are each $+1$, and their sum is $\boxed{2}$ (dimensionless).

Quick Tip to Solve: For the sum of metal oxidation states in a neutral coordination salt, balance all ligand charges across the whole formula first.

84. The number of atoms in 8g of sodium is x $\times$ 10^{23}. The value of x is _____. (Nearest integer)
[Given : N_A = 6.02 $\times$ 10^{23} mol^{-1}
Atomic mass of Na = 23.0 u] [4 marks]

Solution:

Concept:

- The number of moles in a mass m is $n = m/M$, where M is the molar mass. An atomic mass of $23.0\,\mathrm{u}$ for sodium corresponds to molar mass $23.0\,\mathrm{g\,mol^{-1}}$.
- The number of atoms is $N = nN_A$, where the given Avogadro constant is $N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$. Sodium is counted as individual atoms, so no molecular atom-count multiplier is required.

Step 1: Convert the sample mass into moles For $8\,\mathrm{g}$ sodium, substitute the given molar mass:

$$n = \frac{8\,\mathrm{g}}{23.0\,\mathrm{g\,mol^{-1}}} = \frac{8}{23.0}\,\mathrm{mol}.$$

This is about $0.348\,\mathrm{mol}$, less than one mole, so the atom count should be less than one Avogadro number. Retain the fraction to avoid premature rounding of the final coefficient.

Step 2: Calculate the atom count and round the coefficient Multiply by the supplied Avogadro constant:

$$N = \frac{8}{23.0}(6.02 \times 10^{23}) = \frac{48.16}{23.0} \times 10^{23} = 2.0939\ldots \times 10^{23}\ \text{atoms}.$$

The units of moles cancel those in N_A. Comparing this result with $x \times 10^{23}$ atoms identifies $x = 2.0939\ldots$. The requested nearest integer is therefore obtained by rounding the coefficient, not the entire enormous atom count. Neither multiplying the mass by the atomic mass nor treating sodium as diatomic is appropriate.

Step 3: Conclusion The sample contains approximately 2.0939×10^{23} atoms; the requested coefficient is $\boxed{x = 2}$ (nearest integer, dimensionless).

Quick Tip to Solve: When the atom count is written as $x \times 10^{23}$, compute $x = (m/M) \times 6.02$ directly and round only that coefficient.

85. If 80 g of copper sulphate $CuSO_4$. $5H_2O$ is dissolved in deionised water to make 5L of solution. The concentration of the copper sulphate solution is x $\times$ 10^{-3} mol L^{-1}. The value of x is _____.
[Atomic masses Cu : 63.54u, S : 32u, O : 16u, H : 1u] [4 marks]

Solution:

Concept:

- Molarity is the amount of solute in moles divided by the final solution volume in litres: $c = n/V$. The stated 5 L is the solution volume, so no density or separate water-volume calculation is needed.
- The weighed substance is $\mathrm{CuSO_4 \cdot 5\,H_2O}$. Its molar mass includes the five waters of crystallisation; one mole of this hydrate supplies one mole of copper sulphate formula units.

Step 1: Calculate the molar mass of the hydrate Use the supplied atomic masses, including every

oxygen and hydrogen in the water of crystallisation:

$$M(\mathrm{CuSO_4 \cdot 5\,H_2O}) = 63.54 + 32 + 4(16) + 5[2(1) + 16]$$
$$= 159.54 + 90 = 249.54\,\mathrm{g\,mol^{-1}}.$$

The five water molecules contribute 90 grams per mole of hydrate. Omitting them would incorrectly treat the weighed sample as anhydrous copper sulphate and overestimate its amount.

Step 2: Find the amount and concentration The amount dissolved and its concentration are

$$n = \frac{80}{249.54} = 0.32059\,\mathrm{mol},$$
$$c = \frac{0.32059}{5} = 0.064118\,\mathrm{mol\,L^{-1}}.$$

Dissociation in water does not multiply this analytical concentration: the question asks for copper sulphate concentration, not the total concentration of all ions. Express the same concentration using the power of ten specified in the blank:

$$c = 64.118 \times 10^{-3}\,\mathrm{mol\,L^{-1}}.$$

Step 3: Conclusion The coefficient is $x \approx 64.118$, giving the numerical entry $\boxed{64}$. The copper sulphate concentration is approximately $64 \times 10^{-3}\,\mathrm{mol\,L^{-1}}$.

Quick Tip to Solve: For a hydrated salt, include its waters in the weighed molar mass but use the one-to-one mole ratio of hydrate to salt before dividing by final solution volume.

86. A 50 watt bulb emits monochromatic red light of wavelength of 795 nm. The number of photons emitted per second by the bulb is x $\times$ 10^{20}. The value of x is _____. [Given : h = 6.63 $\times$ 10^{-34} Js and c = 3.0 $\times$ 10^{8} ms^{-1}] [4 marks]

Solution:

Concept:

- A photon of wavelength λ has energy $E = h\nu = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- Power is energy emitted per second. If $\dot{N}$ photons are emitted per second, $P = \dot{N}E$, so $\dot{N} = P\lambda/(hc)$. The stated monochromatic emission gives the same energy to every photon.

Step 1: Convert the wavelength and find one photon energy Use SI units with the given constants. A nanometre equals 10^{-9} metre, hence

$$\lambda = 795 \times 10^{-9}\,\mathrm{m} = 7.95 \times 10^{-7}\,\mathrm{m}.$$

Substitute into the photon-energy formula:

$$E = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{7.95 \times 10^{-7}} = 2.5019 \times 10^{-19}\,\mathrm{J}.$$

This is the energy per photon, whereas the bulb emits 50 joules in each second. The two quantities must not be confused when finding a photon count.

Step 2: Divide the emitted energy rate by photon energy The photon emission rate is therefore

$$\begin{aligned}\dot{N} &= \frac{50\,\mathrm{J\,s^{-1}}}{2.5019 \times 10^{-19}\,\mathrm{J}} \\ &= 1.9985 \times 10^{20}\,\mathrm{s^{-1}}.\end{aligned}$$

The joule units cancel and leave inverse seconds, as required for a count per second. Equivalently, direct substitution in $P\lambda/(hc)$ gives the same result. Comparing this with the form $x \times 10^{20}$ photons per second shows that the coefficient is essentially 2 at the precision of the supplied power and speed of light.

Step 3: Conclusion The bulb emits approximately 2×10^{20} photons per second, so $\boxed{x = 2}$.

Quick Tip to Solve: For photons per second, divide power by energy per photon; multiplying power by photon energy cannot give a photon rate. Convert nanometres to metres before substitution.

87. The spin-only magnetic moment value of B_2^+ species is _____ $\times$ 10^{-2} BM. (Nearest integer) [Given : $\sqrt{3}$ = 1.73] [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mathrm{BM}$, where n counts unpaired electrons and BM denotes the Bohr magneton.
- For the lighter second-period diatomic species, including boron, the bonding $\pi(2p_x)$ and $\pi(2p_y)$ orbitals lie below $\sigma(2p_z)$. Degenerate orbitals fill singly before electron pairing, according to Hund's rule.

Step 1: Count and place the electrons Each boron atom has atomic number 5. Neutral $\mathrm{B_2}$ therefore contains ten electrons, while removing one electron to make the singly positive ion gives

$$N_e(\mathrm{B_2}^+) = 2(5) - 1 = 9.$$

The first eight electrons occupy the paired core and 2s molecular orbitals. The configuration of the ion can be written

$$(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\pi 2p)^1.$$

The final electron occupies one member of the degenerate bonding pi pair. Its particular choice of $2p_x$ or $2p_y$ does not change the number of unpaired electrons.

Step 2: Evaluate and rescale the magnetic moment All electrons in the completely filled orbitals are paired and make no net spin-only contribution. The remaining pi electron is unpaired, so $n = 1$. Thus

$$\mu = \sqrt{1(1+2)}\,\mathrm{BM} = \sqrt{3}\,\mathrm{BM}.$$

Using the value explicitly supplied in the question,

$$\mu = 1.73\,\text{BM} = 173 \times 10^{-2}\,\text{BM}.$$

The blank asks for the coefficient of 10^{-2} BM, rather than the moment expressed directly in BM. Therefore 1.73 must be multiplied by 100 before entering the number.

Step 3: Conclusion There is one unpaired electron in B_2^+. Its spin-only magnetic moment is $1.73\,\text{BM}$, and the required nearest-integer coefficient is $\boxed{173}$.

Quick Tip to Solve: Count electrons after applying the ionic charge, fill the molecular orbitals, and count only unpaired electrons. Then convert the moment into the power-of-ten form requested in the blank.

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88. If the conductivity of mercury at 0°C is 1.07×10^6 S m^{-1} and the resistance of a cell containing mercury is 0.243Ω, then the cell constant of the cell is x $\times$ 10^4 m^{-1}. The value of x is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- For a uniform cell, resistance is $R = \rho l/A$, where l is electrode separation, A is the effective area and ρ is resistivity.
- Conductivity is $\kappa = 1/\rho$. Therefore the cell constant $K = l/A$ satisfies $K = \kappa R$. The geometry factor has units of inverse length and is distinct from both resistance and conductance.

Step 1: Relate the measured resistance to cell geometry Conductance is the reciprocal of resistance, $G = 1/R$. Combining this with the conductivity relation gives

$$\kappa = G\frac{l}{A} = \frac{1}{R}\frac{l}{A}.$$

Rearranging gives the required geometry factor directly:

$$K = \frac{l}{A} = \kappa R.$$

Both the conductivity and resistance are specified for mercury in the same cell at the stated temperature, so they can be used together without a temperature correction. No separate electrode dimensions are required.

Step 2: Substitute and express the requested coefficient Keep the given SI conductivity and resistance:

$$\begin{aligned} K &= (1.07 \times 10^6\,\text{S m}^{-1})(0.243\,\Omega) \\ &= 0.26001 \times 10^6\,\text{m}^{-1} \\ &= 2.6001 \times 10^5\,\text{m}^{-1} \\ &= 26.001 \times 10^4\,\text{m}^{-1}. \end{aligned}$$

Here $\text{S}\,\Omega = 1$, so the remaining unit is m^{-1}, which checks the formula. Dividing conductivity by

resistance would have the wrong units for a cell constant. Comparing the last expression with $x \times 10^4\,\mathrm{m}^{-1}$ gives $x = 26.001$ before rounding.

Step 3: Conclusion The cell constant is $2.6001 \times 10^5\,\mathrm{m}^{-1}$. The required coefficient, rounded to the nearest integer, is $\boxed{x = 26}$.

Quick Tip to Solve: Remember the measurement relation as conductivity equals conductance times cell constant. Since conductance is reciprocal resistance, cell constant equals conductivity times resistance.

89. A peptide synthesized by the reactions of one molecule each of Glycine, Leucine, Aspartic acid and Histidine will have _____ peptide linkages. [4 marks]

Solution:

Concept:

- A peptide linkage is the amide group $-\mathrm{CO}-\mathrm{NH}-$ formed when the carboxyl group of one amino acid condenses with the amino group of another, eliminating a water molecule.
- An ordinary linear peptide containing r amino-acid residues has $r - 1$ peptide linkages. Its two ends retain an amino terminus and a carboxyl terminus; a terminal group is not an additional peptide linkage.

Step 1: Identify the number of amino-acid residues One molecule each of glycine, leucine, aspartic acid and histidine supplies four amino-acid residues to the peptide:

$$r = 1 + 1 + 1 + 1 = 4.$$

Their side chains distinguish the residues but do not change this count. Aspartic acid has an additional side-chain carboxyl group and histidine has nitrogen atoms in its side chain. In the ordinary peptide backbone, however, residues join through their alpha-amino and alpha-carboxyl groups. The question does not specify a cyclic peptide or additional side-chain cross-links.

Step 2: Count the successive backbone connections Connecting four initially separate residues into one linear chain requires three joins. After the first two residues join, adding the third residue forms one more linkage, and adding the fourth forms another:

$$N_{\text{peptide}} = (4 - 1) = 3.$$

Equivalently, four residue positions give the three neighbouring connections 1–2, 2–3 and 3–4. Each connection contains one $-\mathrm{CO}-\mathrm{NH}-$ group and produces one water molecule during condensation. Changing the sequence changes which amino acids are neighbours, but every linear sequence of these four residues still has the same three backbone linkages. Counting four would incorrectly include a free terminal group as a bond.

Step 3: Conclusion The product is a tetrapeptide with $\boxed{3}$ peptide linkages. This is a dimensionless count of backbone amide bonds.

Quick Tip to Solve: For a linear peptide, count the amino-acid residues and subtract one. Side-chain functional groups and the free terminal groups do not add backbone peptide linkages.

90. An empty LPG cylinder weighs 14.8 kg. When full, it weighs 29.0 kg and shows a pressure of 3.47 atm. In the course of use at ambient temperature, the mass of the cylinder is reduced to 23.0 kg. The final pressure inside of the cylinder is _____ atm. (Nearest integer)

(Assume LPG of be an ideal gas) [4 marks]

Solution:

Concept:

- The ideal gas equation is $PV = nRT$. In the same rigid cylinder at unchanged ambient temperature, V and T remain constant, so pressure is proportional to the amount of gas.
- For the same gas composition, $n = m/M$, with fixed molar mass M. Thus $P_2/P_1 = m_2/m_1$, where both masses refer to LPG alone, excluding the metal cylinder.

Step 1: Subtract the empty-cylinder mass from both weighings The full and partly used weights both include the container. Its empty mass is 14.8 kg, so the gas masses are

$$m_1 = 29.0 - 14.8 = 14.2\,\text{kg},$$
$$m_2 = 23.0 - 14.8 = 8.2\,\text{kg}.$$

The cylinder has therefore lost 6.0 kg of LPG. Using the ratio $23.0/29.0$ would incorrectly include the unchanging container mass and would not represent the ratio of gas amounts. The mass units can remain kilograms because the same units occur in numerator and denominator.

Step 2: Apply the constant-volume, constant-temperature pressure ratio Under the stated ideal-gas assumption,

$$\frac{P_2}{P_1} = \frac{n_2}{n_1} = \frac{m_2}{m_1}.$$

Use the supplied initial pressure directly:

$$P_2 = 3.47\left(\frac{8.2}{14.2}\right)\text{ atm}$$
$$= 2.0038\,\text{atm}.$$

This is smaller than the initial pressure, as expected after gas has been withdrawn. The question explicitly asks for an ideal-gas treatment, so the required calculation uses the gas-amount ratio rather than the vapour-pressure behaviour of real liquefied LPG. Round only this final pressure to the nearest integer.

Step 3: Conclusion The final pressure is approximately $2.0038\,\text{atm}$, so the nearest-integer answer is $\boxed{2\,\text{atm}}$.

Quick Tip to Solve: In a cylinder-weighing problem, remove the tare mass first. At fixed temperature and volume, pressure follows the mass of gas remaining, not the total cylinder weight.