JEE Main 2021 Paper 1 (B.E./B.Tech) Jul 27 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2021, Session 3 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 July 2021, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. If the mean and variance of the following data : 6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a - b)^2$ is equal to : [4 marks]

(A) 24

(B) 12

(C) 32

(D) 16

Correct Answer: (D) 16

Solution:

Concept:

- For observations with mean $\bar{x}$, the population variance is $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$. Here $n = 8$, and the variance uses all eight observations.
- The identity $(a-b)^2 = 2(a^2+b^2) - (a+b)^2$ determines the required square without finding the two observations separately.

Step 1: Use the mean to find the sum The six known observations have sum $6+10+7+13+12+12 = 60$. Since the mean of eight observations is 9, their total must be 72. Thus

$$60 + a + b = 8 \cdot 9 = 72, \qquad a + b = 12.$$

Both unknown observations must be retained in the count; dividing the known sum by six would not use the stated data.

Step 2: Use the variance to find the sum of squares The known squares add to $36 + 100 + 49 + 169 + 144 + 144 = 642$. Substitute into the variance formula:

$$\frac{642 + a^2 + b^2}{8} - 81 = \frac{37}{4}.$$

Consequently

$$642 + a^2 + b^2 = 8\left(81 + \frac{37}{4}\right) = 722, \qquad a^2 + b^2 = 80.$$

Step 3: Eliminate the individual observations Combine the sum and sum of squares:

$$(a-b)^2 = 2(80) - 12^2 = 160 - 144 = 16.$$

As a check, $ab = (144 - 80)/2 = 32$, so the two observations are 4 and 8 in either order. Their difference is ± 4, whose square is 16; neither changing the order nor taking the negative difference changes the result. The remaining values 24, 12 and 32 therefore do not satisfy both moments.

Step 4: Conclusion The required value is $\boxed{16}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the sum and sum of squares from the two moments, then use the difference-square identity instead of solving for each observation.

2. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product

$$(\vec{a} + \vec{b}) \times \left(\left(\vec{a} \times \left((\vec{a} - \vec{b}) \times \vec{b}\right)\right) \times \vec{b}\right)$$

is equal to: [4 marks]

(A) $5(34\hat{i} - 5\hat{j} + 3\hat{k})$
(B) $7(34\hat{i} - 5\hat{j} + 3\hat{k})$
(C) $7(30\hat{i} - 5\hat{j} + 7\hat{k})$
(D) $5(30\hat{i} - 5\hat{j} + 7\hat{k})$

Correct Answer: (B) $7(34\hat{i} - 5\hat{j} + 3\hat{k})$

Solution:

Concept:

- Compute a nested cross product from the innermost parentheses outward: cross products are not associative.
- For $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$, the components of $\mathbf{u} \times \mathbf{v}$ are found cyclically:

$$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2,\ u_3v_1 - u_1v_3,\ u_1v_2 - u_2v_1).$$

Step 1: Calculate the innermost product Write the given vectors as $\mathbf{a} = (1, 1, 2)$ and $\mathbf{b} = (-1, 2, 3)$. Then $\mathbf{a} - \mathbf{b} = (2, -1, -1)$, giving

$$\mathbf{u} = (\mathbf{a} - \mathbf{b}) \times \mathbf{b} = (-1, -5, 3).$$

For example the first component is $(-1)3 - (-1)2 = -1$; this fixes the orientation of the cross product.

Step 2: Evaluate the next two products Keeping the printed order of the parentheses, obtain

$$\mathbf{v} = \mathbf{a} \times \mathbf{u} = (13, -5, -4).$$

Now cross this result with $\mathbf{b}$:

$$\mathbf{w} = \mathbf{v} \times \mathbf{b} = (-7, -35, 21) = 7(-1, -5, 3).$$

The middle component is $(-4)(-1) - 13(3) = -35$, so its negative sign must not be lost. Replacing a nested cross product by an unparenthesised product would change the calculation.

Step 3: Take the final cross product Since $\mathbf{a} + \mathbf{b} = (0, 3, 5)$, the required vector is

$$\begin{aligned}(\mathbf{a} + \mathbf{b}) \times \mathbf{w} &= (3 \cdot 21 - 5(-35),\ 5(-7),\ -3(-7)) \\ &= (238, -35, 21) \\ &= 7(34, -5, 3).\end{aligned}$$

Its dot product with $(0, 3, 5)$ is $-105 + 105 = 0$, as a cross product requires. Option A has the right direction but the wrong magnitude, while C and D have incompatible component ratios.

Step 4: Conclusion The vector equals $\boxed{7(34\hat{i}-5\hat{j}+3\hat{k})}$. Hence, the correct option is (B).

Quick Tip to Solve: Name each intermediate vector and proceed outward through the brackets; checking orthogonality catches many cross-product sign errors.

3. The value of the definite integral

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$ **is equal to :** [4 marks]

(A) $-\frac{\pi}{2}$

(B) $\frac{\pi}{2\sqrt{2}}$

(C) $-\frac{\pi}{4}$

(D) $\frac{\pi}{\sqrt{2}}$

Correct Answer: (B) $\frac{\pi}{2\sqrt{2}}$

Solution:

Concept:

- On symmetric limits, substituting $x \mapsto -x$ pairs a function with its reflected value.
- The factor $h(x) = \sin^4 x + \cos^4 x$ is even, whereas $g(x) = x\cos x$ is odd. Hence the two logistic factors satisfy $\frac{1}{1+e^{g(x)}} + \frac{1}{1+e^{-g(x)}} = 1$.

Step 1: Pair the reflected integrals Let the given integral be I. Substitution $x = -u$, with the limits reversed, gives

$$I = \int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{-x\cos x})(\sin^4 x + \cos^4 x)}.$$

Add this to the original expression. The exponential factors cancel in the sum, and therefore

$$2I = \int_{-\pi/4}^{\pi/4} \frac{dx}{\sin^4 x + \cos^4 x}.$$

This is an identity for the integrals; the original exponential factor is not pointwise equal to one half.

Step 2: Use the tangent substitution Put $t = \tan x$. Then $dx = dt/(1+t^2)$ and $\sin^4 x + \cos^4 x = (t^4+1)/(1+t^2)^2$. The new limits are $-1, 1$, so

$$2I = \int_{-1}^{1} \frac{1+t^2}{1+t^4}\, dt.$$

An antiderivative on this interval is

$$\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{2}t}{1-t^2}\right).$$

Indeed differentiating the arctangent gives $\sqrt{2}(1+t^2)/(1+t^4)$, before its prefactor is applied.

Step 3: Apply the endpoint limits Approaching the endpoints from within $(-1, 1)$, the arctangent

tends to $-\pi/2$ and $\pi/2$, respectively. Thus

$$2I = \frac{\pi}{\sqrt{2}}, \qquad I = \frac{\pi}{2\sqrt{2}}.$$

The integrand is positive, eliminating A and C. Option D is the reflected-pair integral $2I$, rather than the requested integral itself.

Step 4: Conclusion The integral is $\boxed{\pi/(2\sqrt{2})}$. Hence, the correct option is (B).

Quick Tip to Solve: First pair the integrand at opposite arguments; an odd exponent and an even denominator often remove the exponential exactly.

4. Let C be the set of all complex numbers. Let

$$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$$

$S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ **and**
$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$**.**

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to : [4 marks]

(A) 1

(B) 0

(C) 2

(D) Infinite

Correct Answer: (A) 1

Solution:

Concept:

- Write $z = x + iy$, so that $|z - 3 - 2i|^2 = (x-3)^2 + (y-2)^2$. This is a circle centred at $(3, 2)$ with squared radius 8.
- The real-part condition is $x \geq 5$. Also $z - \bar{z} = 2iy$, so $|z - \bar{z}| \geq 8$ is equivalent to $|y| \geq 4$.

Step 1: Translate all three conditions A point in the intersection must satisfy

$$(x-3)^2 + (y-2)^2 = 8, \qquad x \geq 5, \qquad y \geq 4 \text{ or } y \leq -4.$$

The last condition is the union of two half-planes, not merely the upper one. We examine both against the same circle; the circle condition is an equality rather than the filled disk.

Step 2: Exclude the lower half-plane If $y \leq -4$, then $y - 2 \leq -6$, so $(y-2)^2 \geq 36$. That already exceeds 8 before the nonnegative term $(x-3)^2$ is added. Thus no point of the circle can belong to this lower half-plane. Equivalently its lowest ordinate is $2 - 2\sqrt{2} > -4$.

Step 3: Find the only point in the upper half-plane For $x \geq 5$ and $y \geq 4$, we have $(x-3)^2 \geq 4$ and $(y-2)^2 \geq 4$. Their sum can equal 8 only when both attain their lower bounds. This forces $x = 5, y = 4$. Conversely $(5-3)^2 + (4-2)^2 = 8$ and $|4| = 4$, so $z = 5 + 4i$ is included. The weak inequalities include

this boundary point. There is neither a second point nor an arc; hence the alternatives zero, two and infinitely many are excluded.

Step 4: Conclusion The intersection is $\{5+4i\}$, and its number of elements is $\boxed{1}$. Hence, the correct option is (A).

Quick Tip to Solve: Use lower bounds on the two squared distances: when their minimum sum equals the squared radius, equality fixes both coordinates.

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5. If the area of the bounded region

$R = \left\{(x, y) : \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2\right\}$ **is ,**

$\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$**, then the value of** $(\alpha + \beta - 2\lambda)^2$ **is equal to :** [4 marks]

(A) 8

(B) 2

(C) 4

(D) 1

Correct Answer: λ is undefined; if $\lambda = \gamma$, the value is 2 (B).

Solution:

Concept:

- Area between an upper curve and a lower curve is the integral of their difference. The lower boundary $\max\{0, \ln x\}$ changes at $x = 1$.
- The antiderivatives are $\int 2^x dx = 2^x/\ln 2$ and $\int \ln x\, dx = x\ln x - x$. An undefined parameter cannot be assigned a value without an additional condition.

Step 1: Split the area at the change in boundary On $[1/2, 1]$, the logarithm is nonpositive, so the lower boundary is zero. On $[1, 2]$, it is $\ln x$. The exponential remains above both on these intervals: it is positive, and on $[1, 2]$, $2^x \ge 2 > \ln 2 \ge \ln x$. Therefore

$$A = \int_{1/2}^{1} 2^x\, dx + \int_{1}^{2} (2^x - \ln x)\, dx.$$

Step 2: Evaluate the area exactly Combine the exponential integrals and evaluate the logarithm separately:

$$A = \frac{4 - \sqrt{2}}{\ln 2} - [x\ln x - x]_1^2$$
$$= \frac{4 - \sqrt{2}}{\ln 2} - 2\ln 2 + 1.$$

Comparing with the displayed form gives the natural coefficients $\alpha = 4 - \sqrt{2},\ \beta = -2,\ \gamma = 1$. These coefficients describe the area and do not introduce a parameter named λ.

Step 3: Check the parameter in the requested expression The question asks for $(\alpha + \beta - 2\lambda)^2$, but defines γ in the area expression and gives no value or relation for λ. Taken literally, the requested

value remains

$$(2-\sqrt{2}-2\lambda)^2,$$

which is not uniquely determined. If the intended last symbol was γ, the conditional calculation is

$$(\alpha+\beta-2\gamma)^2=(4-\sqrt{2}-2-2)^2=2.$$

This explains option B, while retaining the distinction between the defined and undefined symbols.

Step 4: Conclusion As written, λ is undefined and no unique option follows. If λ denotes γ, the value is $\boxed{2}$, corresponding to option (B).

Quick Tip to Solve: For a maximum in a boundary, split where its arguments coincide, and check that every symbol in the requested expression is defined.

6. A ray of light through (2, 1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be : [4 marks]

(A) 11x + 7y + 8 = 0 or 11x + 7y − 15 = 0

(B) 11x − 7y − 8 = 0 or 11x + 7y + 15 = 0

(C) 2x − 7y + 29 = 0 or 2x − 7y − 7 = 0

(D) 2x − 7y − 39 = 0 or 2x − 7y − 7 = 0

Correct Answer: (C) 2x − 7y + 29 = 0 or 2x − 7y − 7 = 0

Solution:

Concept:

- Reflection in the y-axis replaces $(2,1)$ by $(-2,1)$; the reflected ray lies on the straight line joining this image to $(5,3)$.
- For an ellipse with semimajor axis a and eccentricity e, the focus is at distance ae from the centre and its directrix at distance a/e. The nearer focus-to-directrix distance is $a(1/e-e)$.

Step 1: Find the reflecting point and first directrix The line joining $(-2,1)$ and $(5,3)$ has slope $2/7$. Thus its equation is

$$y-1=\frac{2}{7}(x+2), \qquad 2x-7y+11=0.$$

It meets the mirror at $P=(0,11/7)$. The incident and reflected slopes have equal magnitudes and opposite signs, as reflection in a vertical mirror requires. This line through $(5,3)$ is the given directrix.

Step 2: Calculate the distance between the directrices With $e=1/3$, the specified nearest distance gives

$$a\left(3-\frac{1}{3}\right)=\frac{8}{\sqrt{53}}, \qquad a=\frac{3}{\sqrt{53}}.$$

The two directrices lie on opposite sides of the centre, each a distance a/e from it. Their separation is

consequently

$$\frac{2a}{e} = \frac{18}{\sqrt{53}}.$$

The given distance is not itself the separation of the directrices.

Step 3: Write both possible parallel lines Any other directrix is parallel to the first and can be written $2x - 7y + c = 0$. Their distance formula gives

$$\frac{|c - 11|}{\sqrt{2^2 + (-7)^2}} = \frac{18}{\sqrt{53}},$$

so $c = 29$ or $c = -7$. Both are possible because the centre may lie on either side of the given directrix. Options A and B have incorrect normal vectors. D includes a line whose constant differs from 11 by 50 rather than 18.

Step 4: Conclusion The other directrix can be $\boxed{2x - 7y + 29 = 0}$ or $\boxed{2x - 7y - 7 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the reflected line by reflecting the source point, then distinguish the nearest focus distance from the full separation of the directrices.

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7. If the coefficients of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to : [4 marks]

(A) 2

(B) -1

(C) 1

(D) -2

Correct Answer: (C) 1

Solution:

Concept:

- The general term in $(u + v)^{11}$ is $\binom{11}{r} u^{11-r} v^r$, for an integer r between 0 and 11.
- First solve for the exponent of x, and only then compare coefficients. A negative term contributes the sign $(-1)^r$, independently of the power of x.

Step 1: Locate the term with power seven In the first expansion the general term is

$$\binom{11}{r}(x^2)^{11-r}(b^{-1}x^{-1})^r = \binom{11}{r} b^{-r} x^{22-3r}.$$

The equation $22 - 3r = 7$ gives $r = 5$, an allowed integer index. Its coefficient is therefore $\binom{11}{5} b^{-5} = 462b^{-5}$. The index counts the number of second-factor choices; it is not the requested exponent itself.

Step 2: Locate the term with power minus seven For the second expansion, the general term is

$$\binom{11}{r}x^{11-r}(-b^{-1}x^{-2})^r = (-1)^r\binom{11}{r}b^{-r}x^{11-3r}.$$

Setting $11-3r=-7$ yields $r=6$. Its sign is positive because 6 is even, and the coefficient is $\binom{11}{6}b^{-6}=462b^{-6}$. Binomial symmetry explains the equal numerical factors for indices 5 and 6.

Step 3: Equate the coefficients and check the root The stated equality becomes

$$\frac{462}{b^5}=\frac{462}{b^6}.$$

Since $b\neq 0$, multiplication by $b^6/462$ is valid and gives $b=1$. At this value both coefficients equal 462. At $b=-1$ they have opposite signs; $b=2$ or -2 also fails the equation because the two denominators differ by a factor of b. No further root arises from a fractional-power operation.

Step 4: Conclusion The required value is $\boxed{b=1}$. Hence, the correct option is (C).

Quick Tip to Solve: For Laurent binomial expansions, write the power of x as a linear expression in the term index before extracting any coefficient.

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8. If $\sin\theta+\cos\theta=\frac{1}{2}$, then 16(sin(2$\theta$) + cos(4$\theta$) + sin(6$\theta$)) is equal to : [4 marks]

(A) 23

(B) -27

(C) -23

(D) 27

Correct Answer: (C) -23

Solution:

Concept:

- The square of a sum connects the given expression to the double angle: $(\sin\theta+\cos\theta)^2=1+\sin 2\theta$.
- Put $u=2\theta$. Then $\cos 2u=1-2\sin^2 u$ and $\sin 3u=3\sin u-4\sin^3 u$; thus all three requested terms depend on the same number.

Step 1: Find the double-angle sine Square the given equality; no choice of quadrant is necessary because the required combination can be expressed in terms of this square. We obtain

$$1+2\sin\theta\cos\theta=\frac{1}{4},\qquad \sin 2\theta=-\frac{3}{4}.$$

The negative sign is essential: replacing the given sum by its absolute value at this stage must not change the resulting double-angle sine.

Step 2: Evaluate the higher angles Apply the double-angle and triple-angle identities with $u=2\theta$:

$$\cos 4\theta=1-2\left(-\frac{3}{4}\right)^2=-\frac{1}{8},$$

$$\sin 6\theta = 3\left(-\frac{3}{4}\right) - 4\left(-\frac{3}{4}\right)^3 = -\frac{9}{16}.$$

These identities also show why determining $\sin\theta$ and $\cos\theta$ separately would be unnecessary work.

Step 3: Combine the terms and compare the options Use a common denominator before multiplying by 16:

$$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = 16\left(\frac{-12-2-9}{16}\right) = -23.$$

The result is negative, ruling out 23 and 27. The magnitude is 23 rather than 27; the latter would require an incorrect higher-angle evaluation.

Step 4: Conclusion The required value is -23. Hence, the correct option is (C).

Quick Tip to Solve: Square the given sine-plus-cosine sum first, then express every higher angle through $\sin 2\theta$.

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9. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in R$, I is a 2 × 2 identity matrix then $4(\alpha - \beta)$ is equal to : [4 marks]

(A) 5

(B) $\frac{8}{3}$

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- For a nonsingular 2×2 matrix, the Cayley–Hamilton identity is $A^2 - (\operatorname{tr} A)A + (\det A)I = 0$.
- Multiplication by A^{-1} expresses the inverse as a linear combination of I and A, allowing its coefficients to be compared directly.

Step 1: Find the trace and determinant The trace is the sum of the diagonal entries, while the determinant includes the sign of the lower-left entry:

$$\operatorname{tr} A = 1 + 4 = 5, \qquad \det A = (1)(4) - (2)(-1) = 6.$$

Since the determinant is nonzero, the inverse in the question exists and multiplication by it is legitimate.

Step 2: Express the inverse using Cayley–Hamilton The characteristic polynomial is $t^2 - 5t + 6$, so the matrix satisfies

$$A^2 - 5A + 6I = 0.$$

Multiply by A^{-1} and rearrange:

$$A - 5I + 6A^{-1} = 0, \qquad A^{-1} = \frac{5}{6}I - \frac{1}{6}A.$$

Hence $\alpha = 5/6$ and $\beta = -1/6$. The negative coefficient of A is a consequence of moving that term to the other side.

Step 3: Evaluate and verify the requested combination Therefore

$$4(\alpha - \beta) = 4\left(\frac{5}{6} + \frac{1}{6}\right) = 4.$$

For an entrywise check, the expression gives $\frac{1}{6}\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$, exactly the adjugate divided by the determinant. Thus the coefficient comparison is consistent with direct inversion. The other listed values do not follow from these uniquely determined coefficients; in particular $8/3$ would result from adding rather than subtracting β.

Step 4: Conclusion The value of $4(\alpha - \beta)$ is 4. Hence, the correct option is (D).

Quick Tip to Solve: For an invertible two-by-two matrix, use $A^{-1} = ((\operatorname{tr} A)I - A)/\det A$ to avoid solving four entry equations.

10. Let $f : \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \to R$ be defined as $f(x) = \begin{cases} (1 + |\sin x|)^{\frac{3a}{|\sin x|}} & , \quad -\frac{\pi}{4} < x < 0 \\ b & , \quad x = 0 \\ e^{\cot 4x / \cot 2x} & , \quad 0 < x < \frac{\pi}{4} \end{cases}$

If f is continuous at x = 0, then the value of 6a + b^2 is equal to : [4 marks]

(A) 1 − e

(B) e − 1

(C) 1 + e

(D) e

Correct Answer: (C) 1 + e

Solution:

Concept:

- Continuity at zero requires the left-hand limit, the right-hand limit and $f(0) = b$ to be equal.
- The standard exponential limit is $\lim_{t\to 0}(1+t)^{k/t} = e^k$; near zero, $\cot(kx) \sim 1/(kx)$.

Step 1: Evaluate the left-hand limit For $x < 0$, put $t = |\sin x|$. As $x \to 0^-$, $t \to 0^+$, so

$$\lim_{x\to 0^-} f(x) = \lim_{t\to 0^+} (1+t)^{3a/t} = e^{3a}.$$

The absolute value causes no extra negative sign: it makes t positive on this approach as well. Taking logarithms would give the same result from $\log(1+t)/t \to 1$.

Step 2: Evaluate the right-hand limit The exponent is a ratio of cotangents, rather than a product. Rewrite it using tangents:

$$\frac{\cot 4x}{\cot 2x} = \frac{\tan 2x}{\tan 4x} \longrightarrow \frac{2}{4} = \frac{1}{2}.$$

Continuity of the exponential function then gives the right-hand limit $e^{1/2} = \sqrt{e}$. Consequently the middle value must be $b = \sqrt{e}$, not e.

Step 3: Impose continuity and evaluate Equating the two one-sided limits gives $e^{3a} = e^{1/2}$. Since the real exponential function is one-to-one, $3a = 1/2$, hence $a = 1/6$. Therefore

$$6a + b^2 = 6\left(\frac{1}{6}\right) + (\sqrt{e})^2 = 1 + e.$$

The options involving $1 - e$ or $e - 1$ have the wrong sign for one contribution, while e alone omits the nonzero contribution $6a = 1$.

Step 4: Conclusion The required value is $1 + e$. Hence, the correct option is (C).

Quick Tip to Solve: Find both one-sided limits independently before using continuity; remember to square $b = \sqrt{e}$ only at the final substitution.

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11. Let y = y(x) be solution of the differential equation
$\log\left(\frac{dy}{dx}\right) = 3x + 4y$**, with y(0) = 0.**
If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$**, then the value of** α **is equal to :** [4 marks]

(A) $-\frac{1}{4}$
(B) $\frac{1}{4}$
(C) 2
(D) $-\frac{1}{2}$

Correct Answer: (A) $-\frac{1}{4}$

Solution:

Concept:

- Exponentiating the equation gives $dy/dx = e^{3x+4y}$, which is positive and can be separated into an x-part and a y-part.
- Integrate $e^{-4y}\,dy = e^{3x}\,dx$, then determine the integration constant using the specified initial value.

Step 1: Separate and integrate The logarithmic equation yields

$$\frac{dy}{dx} = e^{3x}e^{4y}, \qquad e^{-4y}\,dy = e^{3x}\,dx.$$

Integrating both sides produces

$$-\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C.$$

The negative factor on the left comes from the derivative of $-4y$; retaining it prevents a sign error in

the final logarithm.

Step 2: Apply the initial condition At $x = 0$, $y = 0$, so $-1/4 = 1/3 + C$, giving $C = -7/12$. Rearranging the integrated equation gives

$$e^{-4y} = \frac{7 - 4e^{3x}}{3}.$$

At the specified negative value of x, this right side is positive, so the real solution through the initial point remains defined. There is no need to introduce an arbitrary logarithmic branch.

Step 3: Substitute the specified argument For $x = -\frac{2}{3}\log_e 2$, we have $e^{3x} = e^{-2\log_e 2} = 1/4$. Hence

$$e^{-4y} = \frac{7 - 1}{3} = 2, \qquad y = -\frac{1}{4}\log_e 2.$$

Comparing this with $y = \alpha \log_e 2$ yields $\alpha = -1/4$. The positive option contradicts the fact that the increasing solution has a negative value at a negative argument; $-1/2$ and 2 give the wrong exponential value.

Step 4: Conclusion The coefficient is $\alpha = -\frac{1}{4}$. Hence, the correct option is (A).

Quick Tip to Solve: After separation, solve for e^{-4y} before taking logarithms; this makes the initial condition and the special argument easy to use.

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12. Let f : R $\to$ R be a function such that f(2) = 4 and f'(2) = 1. Then, the value of $\lim\limits_{x\to 2} \frac{x^2 f(2) - 4f(x)}{x-2}$ is equal to : [4 marks]

(A) 4

(B) 8

(C) 16

(D) 12

Correct Answer: (D) 12

Solution:

Concept:

- The definition of the derivative is $f\prime(2) = \lim_{x\to 2}[f(x) - f(2)]/(x - 2)$.
- A numerator vanishing at the limiting point can be split into a polynomial difference and a function difference, avoiding any assumption about higher derivatives.

Step 1: Insert the given function value Using $f(2) = 4$, the numerator becomes $4x^2 - 4f(x)$. It is zero at $x = 2$, but substituting directly gives the indeterminate form $0/0$, not the value of the limit. Add and subtract 16 to expose the two differences:

$$4x^2 - 4f(x) = 4(x^2 - 4) - 4(f(x) - 4).$$

Step 2: Recognise the derivative quotient For $x \neq 2$, divide the identity by $x-2$:

$$\frac{x^2 f(2) - 4f(x)}{x-2} = 4(x+2) - 4\frac{f(x)-f(2)}{x-2}.$$

The first quotient factors exactly as a difference of squares. The second is precisely the difference quotient defining $f\prime(2)$. The existence of the given derivative is enough to take its limit, so no global formula for f is needed.

Step 3: Take the limits and check the options Now use $f\prime(2) = 1$:

$$\lim_{x\to 2} \frac{x^2 f(2) - 4f(x)}{x-2} = 4(2+2) - 4(1) = 16 - 4 = 12.$$

The value 16 retains only the polynomial contribution, while 4 retains only the size of the derivative contribution. The value 8 also fails the exact difference-quotient calculation. As an additional local check, differentiability gives $f(2+h) = 4 + h + o(h)$, making the numerator $12h + o(h)$, with the same limit.

Step 4: Conclusion The limit equals 12. Hence, the correct option is (D).

Quick Tip to Solve: Add and subtract the numerator value at the limiting point to reveal a standard derivative quotient; this avoids an unnecessary use of L'Hopital's rule.

13. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P, Q} is equal to : [4 marks]

(A) {(4, 0), (0, 6)}
(B) $\{(2+2\sqrt{2}, 3-\sqrt{5}), (2-2\sqrt{2}, 3+\sqrt{5})\}$
(C) $\{(2+2\sqrt{2}, 3+\sqrt{5}), (2-2\sqrt{2}, 3-\sqrt{5})\}$
(D) {(−1, 5), (5, 1)}

Correct Answer: (D) {(−1, 5), (5, 1)}

Solution:

Concept:

- A circle passing through the origin with centre $C(2,3)$ has radius $CO = \sqrt{2^2+3^2} = \sqrt{13}$.
- A vector perpendicular to $(2,3)$ is $(-3,2)$. Since these two vectors have the same length, it immediately gives the required radius vectors.

Step 1: Find the radius and the perpendicular direction The radius is fixed by the known point O:

$$r^2 = (2-0)^2 + (3-0)^2 = 13.$$

The direction vector of OC is $(2,3)$. Rotating it through a right angle gives $(-3,2)$, and its negative gives the other direction on the same perpendicular line. Their dot products with $(2,3)$ vanish because $-6+6=0$.

Step 2: Locate the two intersections Both CP and CQ must lie along this perpendicular line through the centre. Each must also have length $r = \sqrt{13}$. Since $|(-3, 2)| = \sqrt{13}$, no scale factor is needed. The two possible points are therefore

$$C + (-3, 2) = (-1, 5), \qquad C - (-3, 2) = (5, 1).$$

These are distinct diametrically opposite points, as required by the question.

Step 3: Verify the geometry and reject the distractors The displacement vectors from the centre are $(-3, 2)$ and $(3, -2)$; each has squared length 13 and dot product zero with OC. Option (A) fails the perpendicularity test: its displacements have squared length 13 but dot product ± 5, so they are not perpendicular. The radical-coordinate options have squared displacement length 13, but their dot products are $\pm(4\sqrt{2} - 3\sqrt{5})$ or $\pm(4\sqrt{2} + 3\sqrt{5})$, both nonzero. Only the stated integer points meet both conditions.

Step 4: Conclusion The set is $\{(-1, 5), (5, 1)\}$. Hence, the correct option is (D).

Quick Tip to Solve: Rotate the centre-to-origin vector through a right angle; its unchanged length supplies the two perpendicular radius vectors directly.

14. Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to [4 marks]

(A) 10

(B) 100

(C) 50

(D) 160

Correct Answer: (C) 50

Solution:

Concept:

- For roots α, β of $x^2 + cx + d = 0$, Vieta's relations give $\alpha + \beta = -c$ and $\alpha\beta = d$.
- The identity $u^2 + v^2 = (u + v)^2 - 2uv$ can be used repeatedly to find sums of even powers without explicitly solving for complex roots.

Step 1: Find the sum of the squares Here $c = 20^{1/4}$ and $d = \sqrt{5}$. Thus

$$\alpha + \beta = -20^{1/4}, \qquad \alpha\beta = \sqrt{5}.$$

Squaring the sum gives $(\alpha + \beta)^2 = \sqrt{20} = 2\sqrt{5}$, so

$$\alpha^2 + \beta^2 = 2\sqrt{5} - 2\sqrt{5} = 0.$$

The cancellation is exact; the fourth root in the coefficient must not be mistaken for a square root.

Step 2: Find the sum of the fourth powers Apply the same identity to α^2 and β^2. Their product is

$(\alpha\beta)^2 = 5$, hence

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = 0 - 10 = -10.$$

Although this sum is negative, it is consistent because the original quadratic has nonreal conjugate roots; no positivity condition applies to each complex fourth power.

Step 3: Find the eighth powers and compare The product $\alpha^4\beta^4 = (\alpha\beta)^4 = 25$. Therefore

$$\alpha^8 + \beta^8 = (\alpha^4 + \beta^4)^2 - 2\alpha^4\beta^4 = 100 - 50 = 50.$$

Option 100 would omit the cross-product subtraction at the last stage. The value 10 belongs to the magnitude of the fourth-power sum, rather than the eighth-power sum. The remaining value 160 is inconsistent with the exact recurrence and Vieta's relations.

Step 4: Conclusion The required sum is 50. Hence, the correct option is (C).

Quick Tip to Solve: For an eighth-power sum of quadratic roots, successively compute the second-, fourth- and eighth-power sums using Vieta's relations.

15. The probability that a randomly selected 2-digit number belongs to the set {n ∈ N : (2^n − 2) is a multiple of 3} is equal to : [4 marks]

(A) $\frac{1}{6}$
(B) $\frac{2}{3}$
(C) $\frac{1}{2}$
(D) $\frac{1}{3}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept:

- For a uniform random choice, probability equals the number of favourable integers divided by the total number of possible integers.
- Work modulo 3: because $2 \equiv -1 \pmod 3$, powers of 2 alternate between the residues 2 and 1.

Step 1: Determine the allowable exponents The divisibility condition is $2^n - 2 \equiv 0 \pmod 3$, or $2^n \equiv 2 \pmod 3$. Since $2^n \equiv (-1)^n \pmod 3$, this is true precisely when n is odd. For an even exponent, the residue of $2^n - 2$ is $1 - 2 \equiv 2 \pmod 3$, so it is not divisible by 3. This condition concerns the exponent and does not require the selected number itself to be divisible by 3.

Step 2: Count the two-digit choices The sample space consists of all integers from 10 through 99, inclusive, and hence contains $99 - 10 + 1 = 90$ equally likely choices. The favourable numbers are $11, 13, 15, \ldots, 99$, an arithmetic progression with common difference 2. Its number of terms is

$$N = \frac{99 - 11}{2} + 1 = 45.$$

Therefore the desired probability is

$$P = \frac{45}{90} = \frac{1}{2}.$$

The values $1/6$ and $1/3$ would impose an additional divisibility restriction that is absent here; $2/3$ does not match the equal split between odd and even exponents.

Step 3: Conclusion The required probability is $\boxed{1/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For powers modulo 3, replace 2 by -1; then the divisibility question reduces immediately to the parity of the exponent.

16. Let $A = \{(x, y) \in R \times R | 2x^2 + 2y^2 - 2x - 2y = 1\}$, $B = \{(x, y) \in R \times R | 4x^2 + 4y^2 - 16y + 7 = 0\}$ and $C = \{(x, y) \in R \times R | x^2 + y^2 - 4x - 2y + 5 \leq r^2\}$.

Then the minimum value of |r| such that $A \cup B \subseteq C$ is equal to [4 marks]

(A) $\frac{3+\sqrt{10}}{2}$

(B) $\frac{2+\sqrt{10}}{2}$

(C) $\frac{3+2\sqrt{5}}{2}$

(D) $1 + \sqrt{5}$

Correct Answer: (C) $\frac{3+2\sqrt{5}}{2}$

Solution:

Concept:

- Completing squares identifies the centre and radius of a circle or circular disk.
- The greatest distance from a fixed point to a circle is the distance between the centres plus the radius of that circle.

Step 1: Identify the three centres Dividing the first equation by 2 and completing squares gives

$$(x - \tfrac{1}{2})^2 + (y - \tfrac{1}{2})^2 = 1.$$

Thus A is the circumference with centre $O_A = (1/2, 1/2)$ and radius 1. Similarly,

$$x^2 + (y - 2)^2 = \frac{9}{4},$$

so B has centre $O_B = (0, 2)$ and radius $3/2$. Finally, the defining inequality of C is

$$(x - 2)^2 + (y - 1)^2 \leq r^2.$$

Its centre is fixed at $O = (2, 1)$, and its radius is $|r|$.

Step 2: Compare the two required radii To contain every point of A, the disk must have radius at least

$$OO_A + 1 = \sqrt{(\tfrac{3}{2})^2 + (\tfrac{1}{2})^2} + 1 = \frac{\sqrt{10}}{2} + 1.$$

To contain every point of B, it needs radius at least

$$OO_B + \frac{3}{2} = \sqrt{2^2 + (-1)^2} + \frac{3}{2} = \sqrt{5} + \frac{3}{2}.$$

These bounds are attained at the farthest point on each circumference along the line of centres. The second bound is larger, since both $\sqrt{5} > \sqrt{10}/2$ and $3/2 > 1$. Taking their maximum therefore gives a necessary and sufficient radius. Option (B) is only the first bound, while (A) and (D) are too small to reach the farthest point of B.

Step 3: Conclusion The minimum value is $\boxed{(3 + 2\sqrt{5})/2}$. Hence, the correct option is (C).

Quick Tip to Solve: For a disk with a prescribed centre, enclosing another circle requires centre-distance plus its radius; take the largest such requirement.

17. The value of $\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to : [4 marks]

(A) $5 + \log_e\left(\frac{3}{2}\right)$
(B) $2 - \log_e\left(\frac{2}{3}\right)$
(C) $3 + 2\log_e\left(\frac{2}{3}\right)$
(D) $1 + 2\log_e\left(\frac{3}{2}\right)$

Correct Answer: (D) $1 + 2\log_e\left(\frac{3}{2}\right)$

Solution:

Concept:

- A midpoint Riemann sum $\frac{1}{n}\sum_{j=1}^{n} f((j-1/2)/n)$ tends to $\int_0^1 f(x)\,dx$ when f is continuous.
- Dividing numerator and denominator by the same factor exposes a sum in the standard integral form.

Step 1: Express the summand using midpoint samples Let $x_j = (2j-1)/(2n) = (j-1/2)/n$. These are the midpoints of the n equal subintervals of $[0,1]$, each of width $1/n$. Dividing numerator and denominator of the given fraction by $2n$ gives

$$\frac{(2j-1)+8n}{(2j-1)+4n} = \frac{x_j + 4}{x_j + 2}.$$

The function $f(x) = (x+4)/(x+2)$ has no pole on $[0,1]$, so it is continuous there and the midpoint limit is valid.

Step 2: Evaluate the resulting definite integral The required limit is therefore

$$L = \int_0^1 \frac{x+4}{x+2}\,dx = \int_0^1 \left(1 + \frac{2}{x+2}\right) dx.$$

Integrating and inserting the endpoints,

$$L = [x + 2\ln(x+2)]_0^1 = 1 + 2\ln 3 - 2\ln 2 = 1 + 2\ln\frac{3}{2}.$$

The coefficient 2 in front of the logarithm comes from the remainder after division, not from the subinterval width. As a check, the summand decreases from nearly 2 to nearly $5/3$, so its average must lie between these values; $L \approx 1.811$ does. The other displayed expressions are distinct from this exact integral.

Step 3: Conclusion The limit is $\boxed{1 + 2\log_e(3/2)}$. Hence, the correct option is (D).

Quick Tip to Solve: When a sum contains $2j-1$, try midpoint samples $(2j-1)/(2n)$ before choosing the integration limits.

18. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to : [4 marks]

(A) $P \vee Q$

(B) $P \wedge \sim Q$

(C) $\sim (P \Rightarrow Q)$

(D) $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

Correct Answer: (D) $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

Solution:

Concept:

- An implication $X \Rightarrow Y$ is false only when X is true and Y is false.
- Logical equivalence means that two compound statements have the same truth value for every assignment of their variables.

Step 1: Simplify the given implication By distributivity, the antecedent becomes

$$(P \vee Q) \wedge \sim P = (P \wedge \sim P) \vee (Q \wedge \sim P) = Q \wedge \sim P.$$

Whenever this antecedent is true, Q is already true. Thus $(Q \wedge \sim P) \Rightarrow Q$ is a tautology. Equivalently, implication elimination gives

$$\sim (Q \wedge \sim P) \vee Q = (\sim Q \vee P) \vee Q,$$

which is true because it includes $\sim Q \vee Q$.

Step 2: Find the option that is also always true For option (D), eliminate the inner implication and apply De Morgan law:

$$\sim (P \Rightarrow Q) = \sim (\sim P \vee Q) = P \wedge \sim Q.$$

Hence that option is the biconditional

$$(P \wedge \sim Q) \Leftrightarrow (P \wedge \sim Q),$$

which is true under every truth assignment. The remaining options are not tautologies: $P \vee Q$ is false when both are false, while $P \wedge \sim Q$ and $\sim (P \Rightarrow Q)$ are false when P is false. Therefore only the fourth option has exactly the same truth table as the given statement.

Step 3: Conclusion The equivalent statement is $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$. Hence, the correct option is (D).

Quick Tip to Solve: First decide whether the entire implication is a tautology; a biconditional between identical Boolean expressions is then an immediate match.

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19. Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of $a + b + c$ is equal to : [4 marks]

(A) 3

(B) 8

(C) 5

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Two perpendicular planes have perpendicular normal vectors, so their normal vectors have zero dot product.
- A plane equation can be multiplied by a common nonzero scalar; a prescribed constant term fixes that scale.

Step 1: Find the direction of the required normal The normals of the two given planes are $\mathbf{n}_1 = (2, 1, -1)$ and $\mathbf{n}_2 = (1, -1, -1)$. The required normal (a, b, c) is perpendicular to both. Thus

$$2a + b - c = 0, \qquad a - b - c = 0.$$

The second relation gives $b = a - c$. Substitution into the first gives $3a - 2c = 0$, and hence $c = 3a/2$ and $b = -a/2$. Consequently

$$(a, b, c) = k(2, -1, 3).$$

This also follows by taking a cross product of the two known normals.

Step 2: Use the point to fix the scale The required plane passes through $(-1, 0, -2)$. Inserting these coordinates in $ax + by + cz + 8 = 0$ yields

$$-a - 2c + 8 = 0, \quad \text{or} \quad a + 2c = 8.$$

With $a = 2k$ and $c = 3k$, this becomes $8k = 8$, giving $k = 1$. Thus $a = 2, b = -1, c = 3$, and

$$a + b + c = 2 - 1 + 3 = 4.$$

Direct checks give $(2,-1,3)\cdot(2,1,-1)=0$ and $(2,-1,3)\cdot(1,-1,-1)=0$. Also the plane value at the given point is $-2-6+8=0$. These checks exclude a sign or normalization error and distinguish 4 from the other proposed sums.

Step 3: Conclusion The required sum is $\boxed{4}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the normal direction first, but do not sum its components until the point condition fixes its scale relative to the constant term.

20. Two tangents are drawn from the point P(−1, 1) to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to : [4 marks]

(A) 2

(B) $(3\sqrt{2}+2)$

(C) 4

(D) $3(\sqrt{2}-1)$

Correct Answer: (C) 4

Solution:

Concept:

- A radius to a point of tangency is perpendicular to the tangent, forming a right triangle with the external point and centre.
- Equal chords subtend equal minor central angles; the area of a right triangle is half the product of its perpendicular legs.

Step 1: Find the angle subtended by the contact chord Completing squares gives

$$(x-1)^2+(y-3)^2=4.$$

Thus the centre is $O=(1,3)$, radius $R=2$, and

$$OP=\sqrt{(-1-1)^2+(1-3)^2}=2\sqrt{2}.$$

In right triangle OAP, $\cos\angle AOP = OA/OP = 1/\sqrt{2}$, so $\angle AOP = 45^\circ$. The two contact points lie symmetrically on opposite sides of OP; therefore $\angle AOB = 90^\circ$, and

$$AB = 2R\sin 45^\circ = 2\sqrt{2}.$$

Step 2: Locate the other equal chord and calculate area The condition $AD = AB$ says that the chord AD also subtends a minor central angle of 90°. From the endpoint A, the two such points are B itself and the point diametrically opposite B. For the nondegenerate triangle in the question, $D \neq B$, so BD is a diameter and $BD = 4$. By the angle in a semicircle, $\angle BAD = 90^\circ$. Its equal perpendicular

legs have lengths $AB = AD = 2\sqrt{2}$; hence

$$\text{Area}(ABD) = \frac{1}{2}(2\sqrt{2})(2\sqrt{2}) = 4.$$

This is also consistent with the right-triangle relation $AB^2 + AD^2 = 8+8 = BD^2$. The other numerical options do not equal this area.

Step 3: Conclusion The area of triangle ABD is $\boxed{4}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: If equal chords from one point each subtend a right central angle, their other distinct endpoints are diametrically opposite.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. For real numbers α and β, consider the following system of linear equations : $x + y - z = 2$, $x + 2y + \alpha z = 1$, $2x - y + z = \beta$. If the system has infinite solutions, then $\alpha + \beta$ is equal to _____. [4 marks]

Solution:

Concept:

- Elementary row operations preserve the solution set of a linear system.
- Infinitely many solutions require a dependent equation with consistent constants, leaving at least one free variable.

Step 1: Eliminate the same variable from two equations Write the equations as

$$x + y - z = 2, \qquad x + 2y + \alpha z = 1, \qquad 2x - y + z = \beta.$$

Subtract the first equation from the second, and twice the first from the third. The reduced equations involving only y, z are

$$y + (\alpha + 1)z = -1, \qquad -3y + 3z = \beta - 4.$$

The first of these always has a nonzero coefficient of y. Therefore, to leave a free variable, the second must be a scalar multiple of it; otherwise the pair determines unique y, z, and the first original equation then determines unique x.

Step 2: Impose dependence and verify consistency The coefficient of y fixes the multiplier as -3. Comparing coefficients of z and the right-hand constants gives

$$3 = -3(\alpha + 1), \qquad \beta - 4 = (-3)(-1) = 3.$$

Thus $\alpha = -2$ and $\beta = 7$. To confirm that this yields infinitely many solutions rather than inconsistency, choose $z = t$ freely. The reduced first equation becomes $y - z = -1$, giving $y = t - 1$. The original first equation then gives $x = 3$. Substitution into the other two produces $3 + 2(t - 1) - 2t = 1$ and $6 - (t - 1) + t = 7$, respectively, for every real t. Hence the parameter choice really does leave one

free variable.

Step 3: Conclusion The parameters are $\alpha = -2$, $\beta = 7$, so $\boxed{\alpha + \beta = 5}$.

Quick Tip to Solve: A zero determinant alone does not guarantee infinitely many solutions; compare the augmented constants after elimination to rule out inconsistency.

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22. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- The squared magnitude of a cross product satisfies $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$. This relates the given cross and dot products to the length of the unknown vector.
- The vector triple-product identity is $\vec{a} \times (\vec{a} \times \vec{b}) = \vec{a}(\vec{a} \cdot \vec{b}) - \vec{b}|\vec{a}|^2$. The length of the projection of $\vec{b}$ on a nonzero vector $\vec{d}$ is $|\vec{b} \cdot \vec{d}|/|\vec{d}|$.

Step 1: Find the magnitude of the unknown vector Here $\vec{a} = (1, 1, 1)$, $\vec{c} = (0, 1, -1)$, so $|\vec{a}|^2 = 3$ and $|\vec{c}|^2 = 2$. Since $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$, the cross-product identity gives

$$2 = 3|\vec{b}|^2 - 1, \qquad |\vec{b}|^2 = 1.$$

These equations use the magnitudes of the actual given vectors; no assumption of perpendicularity between $\vec{a}$ and $\vec{b}$ is needed.

Step 2: Evaluate the projection on the required direction Put $\vec{d} = \vec{a} \times \vec{c}$. By the triple-product identity,

$$\vec{d} = \vec{a} - 3\vec{b}, \qquad \vec{b} \cdot \vec{d} = 1 - 3 = -2.$$

Directly, $\vec{d} = (-2, 1, 1)$, hence $|\vec{d}|^2 = 6$. The projection length is nonnegative, so the negative dot product is taken in absolute value:

$$l = \frac{2}{\sqrt{6}}, \qquad 3l^2 = 3\frac{4}{6} = 2.$$

The sign of the scalar projection indicates the opposite direction, but does not change its length.

Step 3: Conclusion The projection has length $l = \sqrt{2/3}$. Therefore the required numerical value is $\boxed{3l^2 = 2}$.

Quick Tip to Solve: Use the triple-product identity to express the projection direction in terms of the given vectors; this avoids solving three component equations for the unknown vector.

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23. **If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to _____.** [4 marks]

Solution:

Concept:

- Three real numbers A, B, C are in arithmetic progression exactly when $2B = A + C$. A logarithm is defined only when its argument is positive.
- For a fixed valid logarithm base, $2\log u = \log(u^2)$ and $\log v + \log w = \log(vw)$. Equality of logarithms then gives equality of their positive arguments.

Step 1: Impose the logarithmic domain and progression condition Write $t = 2^x$, which is always positive. The two variable logarithms require $t - 5 > 0$ and $t - 7/2 > 0$. Together these give $t > 5$. Applying twice the middle term equals the sum of the outer terms,

$$2\log_3(t - 5) = \log_3 2 + \log_3(t - 7/2).$$

All arguments are positive on the stated domain, so the usual product and power laws are legitimate.

Step 2: Solve the resulting quadratic and check the candidates Combining the logarithms and equating their arguments gives

$$(t - 5)^2 = 2(t - 7/2) = 2t - 7.$$

Thus

$$t^2 - 12t + 32 = 0, \qquad (t - 4)(t - 8) = 0.$$

The candidate $t = 4$ is inadmissible: it makes $t - 5 = -1$, so the middle logarithm is not real. The remaining value $t = 8$ gives $2^x = 8 = 2^3$, hence $x = 3$. As a direct check, the three positive arguments become $2, 3, 9/2$; the product of the first and third is 9, the square of the middle. Therefore their logarithms do indeed form the required progression.

Step 3: Conclusion Only $t = 8$ lies in the logarithmic domain, giving the unique real solution $\boxed{x = 3}$.

Quick Tip to Solve: Check logarithm arguments before accepting roots of the algebraic equation. Squaring the middle argument can hide an inadmissible negative value.

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24. **Let the domain of the function $f(x) = \log_4\left(\log_5\left(\log_3(18x - x^2 - 77)\right)\right)$ be (a, b). Then the value of the integral $\int_a^b \frac{\sin^3 x}{\sin^3 x + \sin^3(a+b-x)}\,dx$ is equal to _____.** [4 marks]

Solution:

Concept:

- For nested logarithms, enforce positivity from the outside inward. Since the bases exceed one, $\log_b u > 0$ is equivalent to $u > 1$.
- The reflection identity $\int_a^b h(x)\,dx = \int_a^b h(a + b - x)\,dx$ allows paired numerators with a common

symmetric denominator to be added.

Step 1: Determine the exact domain endpoints For the outer logarithm to be real, $\log_5(\log_3(18x - x^2 - 77)) > 0$. As base 5 exceeds one, this requires $\log_3(18x - x^2 - 77) > 1$. Consequently

$$18x - x^2 - 77 > 3.$$

Completing the square yields

$$(x-9)^2 < 1, \qquad 8 < x < 10.$$

This strict inequality also ensures all inner logarithms are defined. Therefore $a = 8$, $b = 10$, and $a + b = 18$; neither endpoint belongs to the domain.

Step 2: Add the reflected integral Let I denote the requested integral. Reflection about the midpoint gives

$$I = \int_8^{10} \frac{\sin^3(18-x)}{\sin^3 x + \sin^3(18-x)}\,dx.$$

The denominator is nonzero throughout this interval. Indeed, with $u = \sin x$, $v = \sin(18 - x)$, we have $u + v = 2\sin 9\cos(x - 9) > 0$ for $8 \le x \le 10$; also $u^2 - uv + v^2 > 0$. Hence $u^3 + v^3 \neq 0$, and adding the two ordinary integrals is valid. Their numerators add to their denominator:

$$2I = \int_8^{10} 1\,dx = 2.$$

Step 3: Conclusion The domain is $(a, b) = (8, 10)$, and the required definite integral is $\boxed{I = 1}$.

Quick Tip to Solve: For a fraction whose denominator is a function at x plus the same function at a+b-x, reflect the integral before attempting antiderivatives.

25. Let $f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}, x \in [0, \pi]$**. Then the maximum value of f(x) is equal to _____.** [4 marks]

Solution:

Concept:

- Adding a multiple of one row to another does not change a determinant. Row differences can isolate the constant offsets while leaving just one row of trigonometric entries.
- For $x \in [0, \pi]$, the value of $\cos 2x$ ranges from -1 to 1. A linear expression in $\cos 2x$ reaches an endpoint value.

Step 1: Simplify the determinant using row differences Write $s = \sin^2 x$, $c = \cos^2 x$, and $d = \cos 2x$, with $s + c = 1$. First apply $R_2 \leftarrow R_2 - R_1$ and $R_3 \leftarrow R_3 - R_1$. Since both use the unchanged first row,

the determinant becomes

$$f(x) = \begin{vmatrix} s & c-2 & d \\ 2 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix}.$$

All entries in the bottom two rows are now constants, so expansion is short and avoids multiplying complicated trigonometric terms.

Step 2: Expand and maximize on the specified interval Expanding along the first row, with the alternating cofactor signs, gives

$$\begin{aligned} f(x) &= s(2) - (c-2)(2) + d(4) \\ &= 2s - 2c + 4 + 4d. \end{aligned}$$

Because $c - s = \cos 2x = d$, we obtain

$$f(x) = 4 + 2\cos 2x.$$

Thus $-1 \le \cos 2x \le 1$ implies $2 \le f(x) \le 6$. The upper bound is attained when $\cos 2x = 1$, namely at $x = 0$ and $x = \pi$, both of which are included in the given closed interval. At $x = \pi/2$, the expression is 2 instead, confirming that the other cosine endpoint supplies the minimum.

Step 3: Conclusion The maximum is attained at $x = 0$ and $x = \pi$, and its value is $\boxed{6}$.

Quick Tip to Solve: When determinant rows differ by constant entries, subtract a common row first. Keep the cofactor sign of the middle term when expanding.

26. Let $F : [3, 5] \to R$ be a twice differentiable function on (3, 5) such that $F(x) = e^{-x}\int_3^x (3t^2 + 2t + 4F'(t))dt$. If $F'(4) = \frac{\alpha e^{\beta} - 224}{(e^{\beta}-4)^2}$, then α + β is equal to _____. [4 marks]

Solution:

Concept:

- Multiplying an integral equation by its exponential prefactor can expose an integral whose unknown derivative integrates directly. The fundamental theorem gives $\int_3^x F\prime(t)\,dt = F(x) - F(3)$.
- Once $F(x)$ is isolated as a quotient, apply the quotient rule. Retain the exponential in exact form to compare the result with the requested parametrization.

Step 1: Find the initial value and isolate the function At the lower endpoint $x = 3$, the integral has identical limits and is zero, giving $F(3) = 0$. Multiply the given equation by e^x and evaluate each integral:

$$\begin{aligned} e^x F(x) &= \int_3^x (3t^2 + 2t)\,dt + 4(F(x) - F(3)) \\ &= x^3 + x^2 - 36 + 4F(x). \end{aligned}$$

Hence

$$(e^x - 4)F(x) = x^3 + x^2 - 36, \qquad F(x) = \frac{x^3 + x^2 - 36}{e^x - 4}.$$

On $[3, 5]$, $e^x > 4$, so the denominator does not vanish; isolating the function is valid throughout the interval.

Step 2: Differentiate and identify the constants The quotient rule gives

$$F\prime(x) = \frac{(3x^2 + 2x)(e^x - 4) - (x^3 + x^2 - 36)e^x}{(e^x - 4)^2}.$$

At $x = 4$, the polynomial numerator before differentiation is $64 + 16 - 36 = 44$, while its derivative is $48 + 8 = 56$. Therefore

$$F\prime(4) = \frac{56(e^4 - 4) - 44e^4}{(e^4 - 4)^2} = \frac{12e^4 - 224}{(e^4 - 4)^2}.$$

Comparing with the stated form identifies $\alpha = 12$ and $\beta = 4$, without any numerical approximation to the exponential.

Step 3: Conclusion The constants are $\alpha = 12$, $\beta = 4$. Thus $\boxed{\alpha + \beta = 16}$.

Quick Tip to Solve: Integrate the term containing the unknown derivative before differentiating the integral equation; it can reduce the problem directly to an algebraic formula for the function.

27. Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions f : S → S such that f(m . n) = f(m) . f(n) for every m, n ∈ S and m . n ∈ S is equal to _____. [4 marks]

Solution:

Concept:

- The condition is required only for products that remain in $S = \{1, 2, 3, 4, 5, 6, 7\}$. List these products to distinguish independent function values from forced ones.
- Every output must be an integer between 1 and 7. Multiplicativity forces values at composite arguments from those at their factors, while positivity fixes the value at 1.

Step 1: Identify all constraints and independent choices From $f(1) = f(1)^2$ and $f(1) \in S$, we obtain $f(1) = 1$. The only products of two integers at least 2 that are at most 7 are $2 \cdot 2 = 4$ and $2 \cdot 3 = 3 \cdot 2 = 6$. Thus

$$f(4) = f(2)^2, \qquad f(6) = f(2)f(3).$$

Let $a = f(2)$ and $b = f(3)$. The values $f(5)$ and $f(7)$ are unconstrained except for belonging to S, so each has seven choices. Products involving 1 are automatically satisfied, and every other product is outside S.

Step 2: Count the admissible pairs and combine the free choices The condition $a^2 = f(4) \leq 7$ restricts the positive integer a to 1 or 2. If $a = 1$, then $ab = b \leq 7$, so all seven values $b = 1, \ldots, 7$ work. If $a = 2$, the condition $2b \leq 7$ allows exactly $b = 1, 2, 3$. There are therefore $7 + 3 = 10$ admissible pairs. For each pair, $f(1), f(4), f(6)$ are uniquely fixed, while $f(5), f(7)$ can be chosen independently. Every such assignment satisfies every required product, and different assignments define different functions.

Hence

$$N = 10 \cdot 7 \cdot 7 = 490.$$

Step 3: Conclusion There are exactly $\boxed{490}$ functions satisfying the stated multiplicative condition.

Quick Tip to Solve: List the products that stay inside the finite domain before counting. A prime argument need not have a free image if its products with other arguments also remain in the domain.

---

28. If $y = y(x), y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation $\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0$, with y(0) = 0, then $5y'\left(\frac{\pi}{2}\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- The sum identity $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$ separates the dependence on x and y in the differential equation.
- Since $0 \le y < \pi/2$, $\cos y > 0$, and multiplication by $\sec y$ is legitimate. Use $\frac{d}{dx}\tan y = \sec^2 y\, y'$ to integrate.

Step 1: Separate and integrate the equation The given equation becomes

$$\sec y\, y' = 2\sin x \cos y.$$

Divide by the positive factor $\cos y$, giving

$$\sec^2 y\, y' = 2\sin x.$$

The left-hand side is exactly the derivative of $\tan y$, so integration with respect to x yields

$$\tan y = -2\cos x + C.$$

Apply $y(0) = 0$: $0 = -2 + C$, hence $C = 2$. Therefore the solution in the stipulated range satisfies

$$\tan y = 2(1 - \cos x).$$

The specified range selects the nonnegative principal inverse tangent and avoids any branch ambiguity.

Step 2: Evaluate the derivative at the required argument Differentiate the implicit solution, or use the separated differential equation:

$$y' = 2\sin x \cos^2 y = \frac{2\sin x}{1 + \tan^2 y}.$$

At $x = \pi/2$, $\sin x = 1$ and $\cos x = 0$, so $\tan y = 2$. Consequently

$$y'(\pi/2) = \frac{2}{1+2^2} = \frac{2}{5}.$$

This value is finite and positive, consistent with the original equation because $y = \arctan 2$ lies strictly between 0 and $\pi/2$. Multiplying the derivative by the requested factor of 5 now gives the desired numerical value.

Step 3: Conclusion The derivative is $y\prime(\pi/2) = 2/5$, and hence $\boxed{5y\prime(\pi/2) = 2}$.

Quick Tip to Solve: After applying the sine-sum identity, look for the derivative of tan y. Compute cos squared y from tan y rather than evaluating an inverse trigonometric function.

---

29. Let $f : [0, 3] \to R$ be defined by $f(x) = \min\{x - [x], 1 + [x] - x\}$ where [x] is the greatest integer less than or equal to x. Let P denote the set containing all x $\in$ [0, 3] where f i discontinuous, and Q denote the set containing all x $\in$ (0, 3) where f is not differentiable. Then the sum of number of elements in P and Q is equal to _____. [4 marks]

Solution:

Concept:

- Write the fractional part as $t = x - [x]$. Then $0 \le t < 1$, and $f(x) = \min(t, 1 - t)$ is a triangular piecewise linear function.
- Continuity requires the appropriate one-sided limits to equal the value; differentiability at an interior junction additionally requires equal left and right slopes.

Step 1: Resolve the minimum on each unit interval For $k = 0, 1, 2$ and $k \le x < k + 1$, compare $x - k$ with $k + 1 - x$. They are equal at $x = k + \frac{1}{2}$. Thus

$$f(x) = \begin{cases} x - k, & k \le x \le k + \frac{1}{2}, \\ k + 1 - x, & k + \frac{1}{2} \le x < k + 1. \end{cases}$$

The two expressions agree at the midpoint, where the common value is $1/2$.

Step 2: Check continuity at all possible junctions At an integer k, $f(k) = 0$. The limit from the preceding unit interval is also zero, and the limit from the following interval is zero. At 0 and 3 use only the limit from within the domain: both equal zero. At each half-integer the two linear pieces meet at $1/2$. Everywhere else the function is linear locally. Hence there is no discontinuity anywhere in $[0, 3]$, so $P = \varnothing$.

Step 3: Count changes of slope inside the open interval At $1/2, 3/2, 5/2$, the slope changes from $+1$ to -1. At the interior integers 1 and 2, it changes from -1 to $+1$. These five points are not differentiable. All other interior points lie within one linear segment and are differentiable. Endpoints 0 and 3 are excluded from the definition of Q.

Step 4: Conclusion $Q = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\}$ and $P = \varnothing$. Therefore $\boxed{|P| + |Q| = 0 + 5 = 5}$.

Quick Tip to Solve: For a triangular fractional-part function, check both the peaks at half-integers and the valleys at integers; continuity does not imply a matching slope.

---

30. Let a plane P pass through the point (3, 7, −7) and contain the line, $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to _____. [4 marks]

Solution:

Concept:

- A plane containing a line contains its direction vector and every vector joining a point of the line to another point of the plane.
- The cross product of two independent in-plane vectors gives a normal (a, b, c). For $ax+by+cz+d = 0$, the origin-to-plane distance is $|d|/\sqrt{a^2+b^2+c^2}$.

Step 1: Extract two directions in the plane Put the common ratio of the given line equal to t:

$$x = 2 - 3t, \qquad y = 3 + 2t, \qquad z = -2 + t.$$

Thus $A = (2, 3, -2)$ lies on the line and $\mathbf{u} = (-3, 2, 1)$ is its direction. The given point is $B = (3, 7, -7)$, so the joining vector is $\mathbf{v} = B - A = (1, 4, -5)$. Both vectors lie in P. Since they are not parallel, they determine a unique normal direction.

Step 2: Find and verify the plane equation Take their cross product:

$$\mathbf{u} \times \mathbf{v} = (2(-5) - 4,\ 1 - 15,\ -12 - 2) = (-14, -14, -14).$$

An equivalent simpler normal is $(1, 1, 1)$. The plane through A is therefore

$$(x - 2) + (y - 3) + (z + 2) = 0, \qquad x + y + z - 3 = 0.$$

Check B: $3+7-7 = 3$, so it lies on the plane. Substitution of the line gives $(2-3t)+(3+2t)+(-2+t) = 3$ for every t, confirming that the entire line is contained in it.

Step 3: Calculate the required squared distance Use the coefficients of the plane, rather than the distance to one of its particular points:

$$d = \frac{|-3|}{\sqrt{1^2+1^2+1^2}} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

Squaring removes the radical and gives the quantity asked for. The sign chosen for the normal has no effect because the numerator is an absolute value.

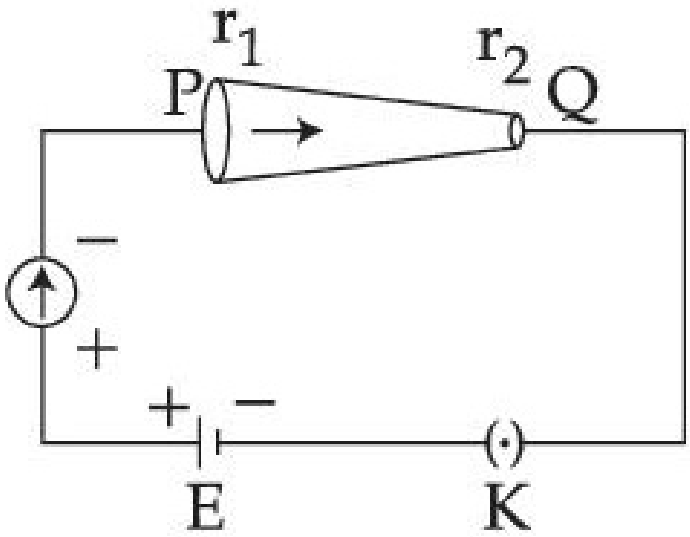
Step 4: Conclusion The required value is $\boxed{d^2 = 3}$.

Quick Tip to Solve: After finding a plane from a line and a point, substitute the line parametrically; every parameter term must cancel.

Physics – Section A: Multiple Choice Questions (80 marks)

31. In the given figure, a battery of emf E is connected across a conductor PQ of length 'l' and different area of cross-sections having radii r_1 and r_2 ($r_2 < r_1$).

Choose the correct option as one moves from P to Q : [4 marks]



(A) Drift velocity of electron increases.

(B) Electric field decreases.

(C) Electron current decreases.

(D) All of these

Correct Answer: (A) Drift velocity of electron increases.

Solution:

Concept:

- In a steady conductor without branches, charge conservation makes the magnitude of the current I the same at every cross-section.
- For a material with carrier density n and electronic charge magnitude e, $I = neAv_d$. Also $J = I/A$ and the local electric field is $\mathcal{E} = \rho J$.

Step 1: Compare the cross-sections along the conductor The conductor tapers from radius r_1 at P to the smaller radius r_2 at Q. Its cross-sectional area $A = \pi r^2$ therefore decreases along that direction. In the steady state charge cannot keep accumulating in a slice of the conductor, so the same charge per unit time must cross every slice. Consequently neither the electronic current magnitude nor the conventional current magnitude decreases on moving towards Q.

Step 2: Use the current relation to obtain the drift speed For the same conductor material, n and e remain constant. Hence

$$v_d = \frac{I}{neA}, \qquad \frac{v_{d,Q}}{v_{d,P}} = \frac{A_P}{A_Q} = \frac{r_1^2}{r_2^2} > 1.$$

Here drift velocity in the option refers to its magnitude. The electron drift direction is opposite to the

conventional current direction, but that does not change the comparison of speeds at P and Q.

Step 3: Check the other statements The current density grows as the cross-section becomes smaller. For fixed resistivity,

$$\mathcal{E} = \rho \frac{I}{A}$$

also increases towards Q. Thus the statement that the field decreases is false. The statement that electronic current decreases is false by continuity. Since those statements are false, the all-of-these option is also excluded. A changing drift speed in a tapered wire is entirely consistent with a constant total current.

Step 4: Conclusion The electron drift speed increases from P to Q. Hence, the correct option is (A).

Quick Tip to Solve: In a tapered wire first keep I constant, then compare v_d, J and the local field through their inverse dependence on cross-sectional area.

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32. The number of molecules in one litre of an ideal gas at 300 K and 2 atmospheric pressure with mean kinetic energy 2×10^{-9} J per molecules is : [4 marks]

(A) 0.75×10^{11}

(B) 3×10^{11}

(C) 1.5×10^{11}

(D) 6×10^{11}

Correct Answer: (C) 1.5×10^{11}

Solution:

Concept:

- The kinetic-theory pressure relation is $PV = \frac{2}{3} N\overline{K}$, where N is the number of molecules and $\overline{K}$ is the mean translational kinetic energy per molecule.
- Convert litres to cubic metres and atmospheres to pascals before using an energy in joules; $\mathrm{Pa\,m^3} = \mathrm{J}$.

Step 1: Convert the pressure-volume product The volume is $V = 1\,\mathrm{L} = 10^{-3}\,\mathrm{m^3}$. Taking $1\,\mathrm{atm} \simeq 10^5\,\mathrm{Pa}$, the pressure is $P = 2 \times 10^5\,\mathrm{Pa}$. Therefore

$$PV = (2 \times 10^5)(10^{-3}) = 200\,\mathrm{J}.$$

Using $1.013 \times 10^5\,\mathrm{Pa}$ instead changes only the rounding to the listed option.

Step 2: Calculate N from the stated molecular energy Use the explicitly supplied $\overline{K} = 2 \times 10^{-9}\,\mathrm{J}$:

$$N = \frac{3PV}{2\overline{K}} = \frac{3(200)}{2(2 \times 10^{-9})} = 1.5 \times 10^{11}.$$

The factors of two are essential: the total translational energy is $N\overline{K} = 3PV/2$, not PV or 3PV. This excludes the alternative molecular counts, which differ by factors of two or four.

Step 3: Check the simultaneous temperature statement For an ideal gas in thermal equilibrium,

the additional relation is $\overline{K} = 3k_BT/2$. At 300 K it gives about 6.21×10^{-21} J, which is inconsistent with the stated 2×10^{-9} J. Thus the printed data cannot all describe one equilibrium ideal gas. The listed answer follows from the pressure, volume and explicitly supplied mean energy; imposing 300 K instead would give about 4.8×10^{22} molecules, absent from the options.

Step 4: Conclusion Using the stated mean molecular kinetic energy gives $\boxed{N = 1.5 \times 10^{11}}$ molecules. Hence, the correct option is (C). The stated temperature and mean energy are mutually inconsistent.

Quick Tip to Solve: Use $N = 3PV/(2\overline{K})$ when molecular energy is supplied, and separately check $\overline{K} = 3k_BT/2$ if a temperature is also given.

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33. The relative permittivity of distilled water is 81. The velocity of light in it will be :
(Given μ_r = 1) [4 marks]

(A) 4.33 $\times$ 10^7 m/s

(B) 2.33 $\times$ 10^7 m/s

(C) 3.33 $\times$ 10^7 m/s

(D) 5.33 $\times$ 10^7 m/s

Correct Answer: (C) 3.33 $\times$ 10^7 m/s

Solution:

Concept:

- The electromagnetic-wave speed in a medium is $v = 1/\sqrt{\mu\varepsilon}$, where μ and ε are its permeability and permittivity.
- With $\mu = \mu_r\mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$ and $c = 1/\sqrt{\mu_0\varepsilon_0}$, this becomes $v = c/\sqrt{\mu_r\varepsilon_r}$.

Step 1: Determine the refractive-index factor The problem gives $\varepsilon_r = 81$ and $\mu_r = 1$. Therefore the refractive-index factor corresponding to these supplied material parameters is

$$\sqrt{\mu_r\varepsilon_r} = \sqrt{1 \times 81} = 9.$$

Both parameters are relative quantities and dimensionless. The permittivity enters under a square root: dividing the vacuum speed directly by 81 would not satisfy the electromagnetic-wave speed formula.

Step 2: Substitute the vacuum speed Using $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$,

$$v = \frac{3.00 \times 10^8}{9} = \frac{1}{3} \times 10^8 = 3.33 \times 10^7\,\mathrm{m\,s^{-1}}.$$

The value is smaller than c, as expected for the supplied positive relative parameters with product greater than one. No extra density or mechanical elastic constant is needed for this electromagnetic calculation.

Step 3: Compare with all listed values The value $3.33 \times 10^7\,\mathrm{m\,s^{-1}}$ is the third option. The first, second and fourth values do not result from dividing c by 9; for example their implied factors c/v differ from the required value 9. The calculation uses the permittivity specified in the question, so substituting a

different frequency-dependent material value would answer a different numerical problem.

Step 4: Conclusion The speed obtained from the supplied parameters is $\boxed{3.33 \times 10^7\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: For a non-magnetic medium, take the square root of relative permittivity before dividing the vacuum speed.

34.

List-I	List-II
(a) MI of the rod (length L, Mass M, about an axis $\perp$ to the rod passing through the midpoint)	**(i)** $8ML^2/3$
(b) MI of the rod (length L, Mass 2M, about an axis $\perp$ to the rod passing through one of its end)	**(ii)** $ML^2/3$
(c) MI of the rod (length 2L, Mass M, about an axis $\perp$ to the rod passing through its midpoint)	**(iii)** $ML^2/12$
(d) MI of the rod (Length 2L, Mass 2M, about an axis $\perp$ to the rod passing through one of its end)	**(iv)** $2ML^2/3$

Choose the correct answer from the options given below: [4 marks]

(A) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

(B) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(C) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(D) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Correct Answer: (C) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Solution:

Concept:

- For a uniform thin rod of mass m and length l, the moment of inertia about a perpendicular axis through its midpoint is $I_c = ml^2/12$.
- The parallel-axis theorem gives $I_{\text{end}} = I_c + m(l/2)^2 = ml^2/3$ for a perpendicular axis through an end. Use the actual mass and length in each row.

Step 1: Evaluate the two rods of length L For row (a), the mass is M and the axis passes through the midpoint. Thus

$$I_a = \frac{ML^2}{12},$$

which is List-II entry (iii). For row (b), the mass is 2M and the axis passes through an end. Consequently

$$I_b = \frac{(2M)L^2}{3} = \frac{2ML^2}{3},$$

which is entry (iv). The different axis location matters as much as the mass: the end-axis formula has coefficient 1/3, rather than 1/12.

Step 2: Evaluate both rods of length 2L For row (c), use mass M, length 2L and the midpoint axis:

$$I_c = \frac{M(2L)^2}{12} = \frac{4ML^2}{12} = \frac{ML^2}{3}.$$

This is entry (ii). For row (d), the mass is 2M, the length is 2L and the axis is through an end:

$$I_d = \frac{(2M)(2L)^2}{3} = \frac{8ML^2}{3},$$

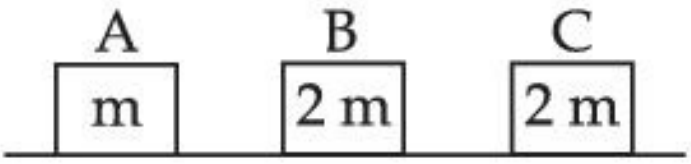
which is entry (i). Doubling length multiplies its squared contribution by four, while doubling mass contributes another factor two.

Step 3: Match the complete sequence The sequence is therefore (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i). The first two listed options already fail row (a), since they assign (ii) instead of (iii). The fourth option gets (a) and (b) right but interchanges the values for (c) and (d). Only the third option satisfies all four independently calculated moments of inertia.

Step 4: Conclusion The matching established above is the third listed sequence. Hence, the correct option is (C).

Quick Tip to Solve: Write the midpoint or end formula first, then replace m and l for each rod; square the full length 2L before simplifying.

35. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, 2m and 2m respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is : [4 marks]



(A) 6 m/s

(B) 9 m/s

(C) 4 m/s

(D) 3 m/s

Correct Answer: (D) 3 m/s

Solution:

Concept:

- In an isolated one-dimensional collision momentum is conserved. An elastic collision also has unit coefficient of restitution, so relative separation speed equals relative approach speed.
- In a completely inelastic collision the two bodies stick and have one common final velocity. Kinetic energy need not be conserved in this second collision.

Step 1: Find B's speed after the elastic collision Take the original motion of A as positive. Initially $u_A = 9\,\mathrm{m\,s^{-1}}$ and $u_B = 0$. If their velocities just after the first collision are v_A, v_B, momentum conservation gives

$$9m = mv_A + 2mv_B, \qquad 9 = v_A + 2v_B.$$

Elastic restitution gives $v_B - v_A = 9$. Solving these equations yields $v_B = 6\,\mathrm{m\,s^{-1}}$ and $v_A = -3\,\mathrm{m\,s^{-1}}$. A rebounds while B moves towards C.

Step 2: Apply momentum conservation to B and C B of mass 2m now meets stationary C of mass 2m. Because their collision is completely inelastic, they move together with common velocity v. Hence

$$(2m)(6) + (2m)(0) = (2m + 2m)v.$$

It follows that

$$v = \frac{12m}{4m} = 3\,\mathrm{m\,s^{-1}}.$$

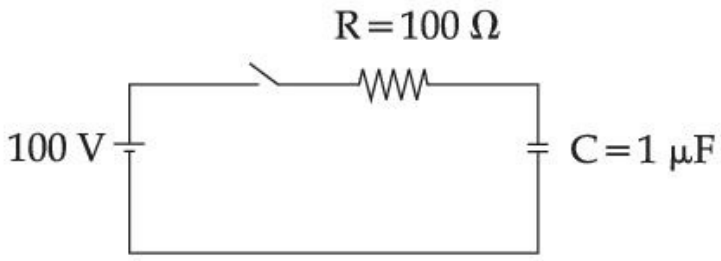
The mass m cancels, as expected when only mass ratios are given. This velocity is positive, so the requested speed of C is also 3 in SI units.

Step 3: Distinguish intermediate and final speeds The option 6 is B's intermediate speed, before it shares momentum with the equal-mass object C. The value 9 is A's initial speed and does not remain unchanged after transferring momentum. The value 4 satisfies neither of the two collision equations. Conserving kinetic energy in the second collision would be incorrect because the bodies stick together; momentum conservation alone gives their common speed.

Step 4: Conclusion The final speed of C is $\boxed{3\,\mathrm{m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Treat consecutive collisions separately: use restitution for the elastic event, then a common velocity for the sticking event.

36. A capacitor of capacitance C = 1 μF is suddenly connected to a battery of 100 volt through a resistance R = 100 Ω. The time taken for the capacitor to be charged to get 50 V is :
[Take ln 2 = 0.69] [4 marks]



(A) 1.44×10^{-4} s
(B) 3.33×10^{-4} s
(C) 0.69×10^{-4} s
(D) 0.30×10^{-4} s
Correct Answer: (C) 0.69×10^{-4} s

Solution:

Concept:

- For an initially uncharged capacitor charged through a resistance, the capacitor voltage is $V_C = V_0(1 - e^{-t/(RC)})$, where V_0 is the battery voltage.
- The time constant $\tau = RC$ has units of seconds. Charging is exponential because the current decreases as the voltage across the capacitor rises.

Step 1: Find the circuit time constant The capacitor is initially uncharged, and the battery supplies a constant voltage of $100\,\text{V}$. Convert the capacitance to farads before multiplying by the resistance:

$$\tau = RC = (100)(1 \times 10^{-6}) = 10^{-4}\,\text{s}.$$

This is a characteristic charging time; it is not itself the time required to reach every specified fraction of the final voltage.

Step 2: Apply the half-voltage condition The required voltage is $50\,\text{V}$, exactly half of the final voltage. Substitute this condition in the charging law:

$$50 = 100(1 - e^{-t/\tau}) \quad \Rightarrow \quad e^{-t/\tau} = \frac{1}{2}.$$

Taking natural logarithms gives

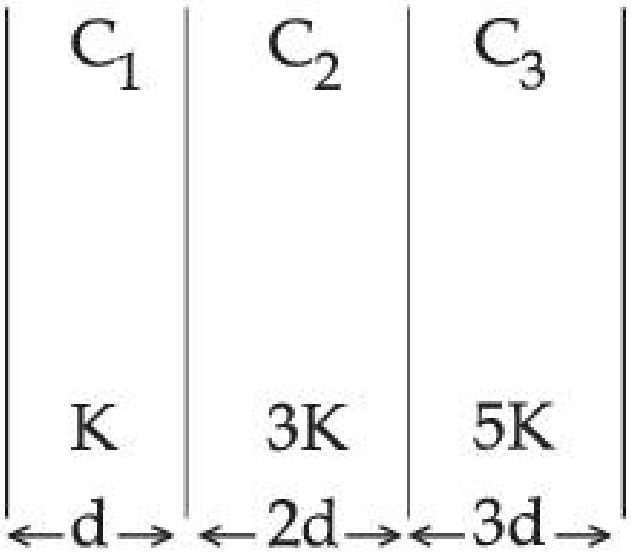
$$t = \tau \ln 2 = (10^{-4})(0.69) = 0.69 \times 10^{-4}\,\text{s}.$$

This time is smaller than one time constant, as expected: at one time constant the capacitor has already reached about 63% of its final voltage. The other listed coefficients do not satisfy $e^{-t/\tau} = 1/2$; in particular, replacing $\ln 2$ by its reciprocal would give the misleading coefficient 1.44.

Step 3: Conclusion The capacitor reaches $50\,\text{V}$ after $0.69 \times 10^{-4}\,\text{s}$. Hence, the correct option is (C).

Quick Tip to Solve: For half of the final charging voltage, use $t = RC \ln 2$; the half-charge time is also the half-voltage time.

37. In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be :
(Given area of plate = A) [4 marks]



(A) $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$
(B) $\frac{15}{6}\frac{K\varepsilon_0 A}{d}$
(C) $\frac{25}{6}\frac{K\varepsilon_0 A}{d}$
(D) $\frac{9}{6}\frac{K\varepsilon_0 A}{d}$

Correct Answer: (A) $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$

Solution:

Concept:

- A dielectric slab of thickness t, relative permittivity K_i, and plate area A contributes $C_i = K_i\varepsilon_0 A/t$.
- Dielectric layers placed successively along the electric field have a common displacement and act as capacitors in series: $1/C = \sum_i 1/C_i$.

Step 1: Identify the series layers The diagram shows three layers occupying the full plate area, with thicknesses $d, 2d, 3d$ and relative permittivities $K, 3K, 5K$, respectively. The layers are traversed successively between the two outer plates, so their potential drops add. They are not three parallel capacitors, since they do not each span the complete plate separation. Their individual capacitances are

$$C_1 = \frac{K\varepsilon_0 A}{d}, \qquad C_2 = \frac{3K\varepsilon_0 A}{2d}, \qquad C_3 = \frac{5K\varepsilon_0 A}{3d}.$$

Step 2: Add reciprocal capacitances Using the series rule and factoring out the common dimensions gives

$$\frac{1}{C} = \frac{d}{K\varepsilon_0 A}\left(1 + \frac{2}{3} + \frac{3}{5}\right) = \frac{d}{K\varepsilon_0 A}\frac{34}{15}.$$

Therefore

$$C = \frac{15}{34}\frac{K\varepsilon_0 A}{d}.$$

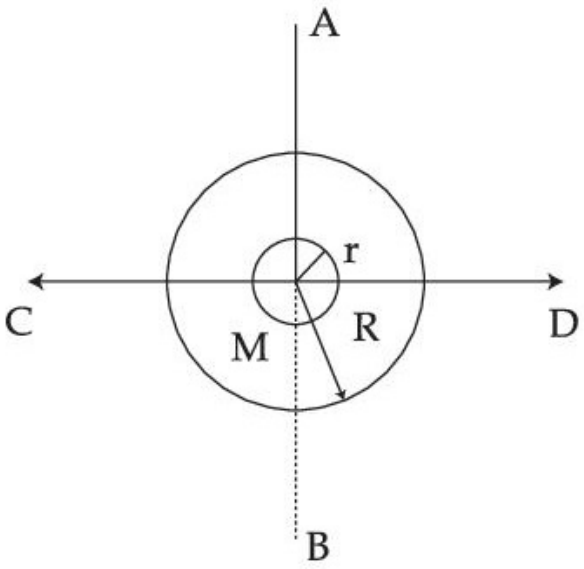
As a check, a series combination must have capacitance smaller than every constituent. This result

is smaller than C_1, the smallest constituent. Each of the other three options exceeds C_1, so none can represent this stack. The total physical thickness is $6d$, but one cannot simply average the three dielectric constants arithmetically; the voltage drops depend on thickness divided by permittivity.

Step 3: Conclusion The capacity is $C = \frac{15}{34}\frac{K\varepsilon_0 A}{d}$. Hence, the correct option is (A).

Quick Tip to Solve: For dielectric layers along the field, add t_i/K_i, then use $C = \varepsilon_0 A/\sum(t_i/K_i)$.

38. The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (Which is $\perp$ to the plane of the disc and passing through its centre) of MI of smaller disc around one of its diameters lying on its plane? Given 'M' is the mass of the larger disc. (MI stands for moment of inertia) [4 marks]



(A) $R^2 : r^2$

(B) $2r^4 : R^4$

(C) $2R^2 : r^2$

(D) $2R^4 : r^4$

Correct Answer: (D) $2R^4 : r^4$

Solution:

Concept:

- For a uniform solid disc of mass m and radius a, the moment of inertia about its central perpendicular axis is $ma^2/2$.
- By symmetry and the perpendicular-axis theorem, the moment of inertia about a diameter is $ma^2/4$. Equal mass per unit area implies mass proportional to radius squared.

Step 1: Express the smaller mass in terms of the larger mass Let the common surface mass density be

σ. The larger disc has mass $M = \sigma\pi R^2$, whereas the smaller disc has mass $m = \sigma\pi r^2$. Thus

$$m = M\frac{r^2}{R^2}.$$

The two discs cannot be treated as having equal masses merely because their mass per unit area is equal. This extra radius dependence is essential in the requested ratio.

Step 2: Use the specified axes and divide The larger disc is considered about the axis perpendicular to its plane through its centre, as specified in the wording, so

$$I_{\text{large}} = \frac{1}{2}MR^2.$$

The smaller disc is considered about one diameter in its own plane. Its two perpendicular diameters have equal inertia and their sum is the perpendicular-axis inertia, giving

$$I_{\text{small}} = \frac{1}{4}mr^2 = \frac{Mr^4}{4R^2}.$$

Consequently

$$\frac{I_{\text{large}}}{I_{\text{small}}} = \frac{MR^2/2}{Mr^4/(4R^2)} = \frac{2R^4}{r^4}.$$

Options containing only squared radii omit the mass scaling. The option with $2r^4/R^4$ reverses that scaling and does not give the required larger-to-smaller ratio.

Step 3: Conclusion The required ratio is $I_{\text{large}} : I_{\text{small}} = 2R^4 : r^4$. Hence, the correct option is (D).

Quick Tip to Solve: For similar laminae with the same surface density, inertia scales as the fourth power of size; also check whether the axis is a diameter or a normal.

39. In Young's double slit experiment, if the source of light changes from orange to blue then :

[4 marks]

(A) the central bright fringe will become a dark fringe.

(B) the distance between consecutive fringes will decrease.

(C) the distance between consecutive fringes will increases.

(D) the intensity of the minima will increase.

Correct Answer: (B) the distance between consecutive fringes will decrease.

Solution:

Concept:

- For two coherent slits separated by d, with a screen at distance D, the fringe width is $\beta = \lambda D/d$ in the usual small-angle approximation.
- Blue light has a shorter wavelength than orange light. At the central point the path difference is zero, independent of wavelength.

Step 1: Compare the two fringe widths Only the colour of the source is changed; the slit separation

and screen distance are fixed. Write the fringe widths before and after the change:

$$\beta_{\text{orange}} = \frac{\lambda_{\text{orange}} D}{d}, \qquad \beta_{\text{blue}} = \frac{\lambda_{\text{blue}} D}{d}.$$

Dividing eliminates the apparatus dimensions:

$$\frac{\beta_{\text{blue}}}{\beta_{\text{orange}}} = \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}} < 1.$$

Therefore consecutive bright fringes move closer together. Consecutive dark fringes also have this same reduced separation.

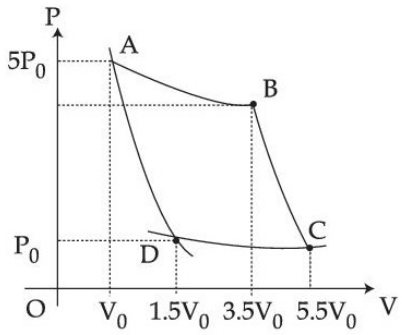
Step 2: Check the remaining statements The central bright fringe corresponds to zero path difference. Changing wavelength leaves that path difference zero, so it does not convert the centre into a dark fringe. Thus option (A) is false. Option (C) gives the opposite wavelength dependence to $\beta \propto \lambda$. In the ideal equal-intensity two-slit experiment, destructive interference gives zero intensity at a minimum for either colour. A colour change by itself therefore does not imply increased minimum intensity, ruling out (D). No numerical wavelength values are needed because the ordering of blue and orange wavelengths already decides the comparison.

Step 3: Conclusion The distance between consecutive fringes decreases on changing from orange to blue light. Hence, the correct option is (B).

Quick Tip to Solve: With the apparatus unchanged, shorter wavelength means narrower fringes; the zero-path-difference central fringe stays bright.

40. In the reported figure, there is a cyclic process ABCDA on a sample of 1 mol of a diatomic gas. The temperature of the gas during the process A $\rightarrow$ B and C $\rightarrow$ D are T_1 and T_2 ($T_1 > T_2$) respectively. Choose the correct option out of the following for work done if processes BC and DA are adiabatic.

[4 marks]



(A) $W_{AB} = W_{DC}$

(B) $W_{AD} = W_{BC}$

(C) $W_{BC} + W_{DA} > 0$

(D) $W_{AB} < W_{CD}$

Correct Answer: (B) $W_{AD} = W_{BC}$

Solution:

Concept:

- Using work done by the gas as positive, the first law is $\Delta U = Q - W$. On an adiabatic path $Q = 0$, so $W = nC_V(T_i - T_f)$.
- For one mole of an ideal diatomic gas in the usual temperature range, $C_V = 5R/2$. Internal energy depends only on temperature, not the particular path.

Step 1: Compare the two adiabatic expansions Points A and B lie on the hot isotherm T_1, while points C and D lie on the cold isotherm T_2. The path BC is an adiabatic expansion from T_1 to T_2. The reversed path AD is likewise an adiabatic expansion from the same initial temperature to the same final temperature. Hence

$$W_{BC} = C_V(T_1 - T_2), \qquad W_{AD} = C_V(T_1 - T_2).$$

Both works are positive because $T_1 > T_2$, and both equal $\frac{5}{2}R(T_1 - T_2)$. Equality follows directly from equal internal-energy decreases and zero heat transfer.

Step 2: Check the signs of the other proposed comparisons In the actual cyclic direction, DA is an adiabatic compression, so

$$W_{DA} = -C_V(T_1 - T_2), \qquad W_{BC} + W_{DA} = 0.$$

This rules out option (C). AB is an isothermal expansion, with positive work; CD is an isothermal compression, with negative work, so $W_{AB} < W_{CD}$ is false. For a reversible cycle with two isotherms joined by adiabats, $V_B/V_A = V_C/V_D$. Thus the expansion works AB and DC are $RT_1 \ln(V_B/V_A)$ and $RT_2 \ln(V_C/V_D)$; the first exceeds the second since $T_1 > T_2$, ruling out (A).

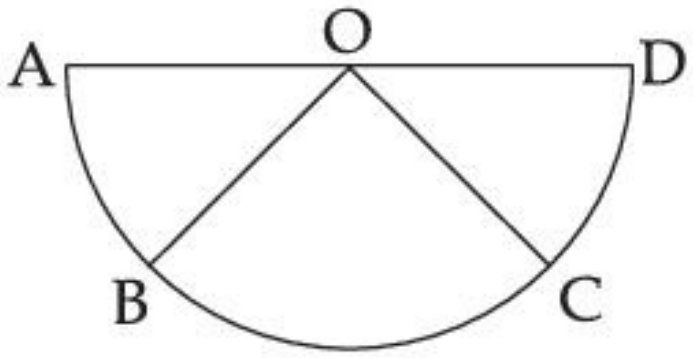
Step 3: Conclusion The equal works are $W_{AD} = W_{BC}$. Hence, the correct option is (B).

Quick Tip to Solve: For an adiabatic ideal-gas process, compare endpoint temperatures first: the work by the gas is its loss of internal energy.

41. Assertion A : If A, B, C, D are four points on a semi-circular are with centre at 'O' such that $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right| = \left|\overrightarrow{CD}\right|$, then $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

Reason R : Polygon law of vector addition yields $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AD} = 2\overrightarrow{AO}$

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]



(A) A is correct but R is not correct.

(B) A is not correct but R is correct.

(C) Both A and R are correct and R is the correct explanation of A.

(D) Both A and R are correct but R is not the correct explanation of A.

Correct Answer: (A) A is correct but R is not correct.

Solution:

Concept:

- The displacement from A to a point P can be written $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$ by the triangle law.
- As A and D are the ends of a diameter with midpoint O, $\overrightarrow{AD} = 2\overrightarrow{AO}$. The polygon law adds successive directed sides, preserving their directions.

Step 1: Evaluate Assertion A Expand the two displacements ending at B and C through O, and use the diameter relation for AD:

$$\begin{aligned}\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} &= (\overrightarrow{AO} + \overrightarrow{OB}) + (\overrightarrow{AO} + \overrightarrow{OC}) + 2\overrightarrow{AO} \\ &= 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}.\end{aligned}$$

This is exactly the asserted identity, so A is correct. The equal-chord condition is consistent with the figure, but is not needed for this identity: the diameter and displacement relations already establish it.

Step 2: Evaluate Reason R independently The successive path A to B to C to D has net displacement AD. Therefore

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}.$$

Adding the extra AD vector in the printed reason gives

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AD} = 2\overrightarrow{AD} = 4\overrightarrow{AO}.$$

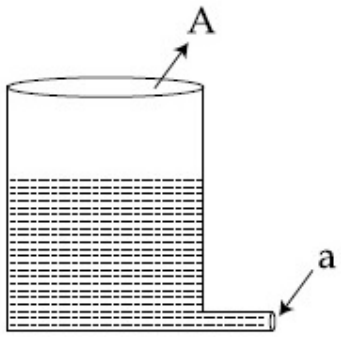
The reason prints $2\overrightarrow{AO}$ instead, which is only $\overrightarrow{AD}$, half the actual sum. Thus R is incorrect. Options

(B), (C), and (D) all assign an incorrect truth value to at least one of the two statements, so the explanation-link question does not arise.

Step 3: Conclusion Assertion A is correct, but Reason R is not correct. Hence, the correct option is (A).

Quick Tip to Solve: Add successive arrows by their endpoints first. An extra copy of the final displacement doubles the sum; it must not be silently dropped.

42. A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a $<<$ A) : [4 marks]



(A) $\frac{A}{2a}$

(B) None of these

(C) $\frac{2a}{A}$

(D) $\frac{a}{A}$

Correct Answer: (C) $\frac{2a}{A}$

Solution:

Concept:

- For an open vessel with liquid depth h, Torricelli's law gives the efflux speed $v = \sqrt{2gh}$ when the outlet area is much smaller than the free-surface area.
- The horizontal momentum carried away per second is $\dot{m}v = \rho a v^2$, producing an equal opposite recoil force on the vessel. The maximum static friction is μN.

Step 1: Calculate the recoil force from the jet Let ρ be the density of the liquid and h its current depth above the outlet. Since $a \ll A$, the downward speed of the free surface can be neglected in Bernoulli's equation. The outlet and the free surface are both at atmospheric pressure, so

$$v^2 = 2gh.$$

The volume flowing out per second is av, and hence the mass flow rate is ρav. The emerging stream

carries horizontal momentum at the rate

$$F = \dot{m}v = \rho a v^2 = 2\rho a g h.$$

This is the magnitude of the reaction force tending to slide the vessel opposite to the jet.

Step 2: Balance the recoil with limiting friction The vessel is light, so its own weight is negligible compared with the liquid weight. The normal reaction of the floor is therefore

$$N = \rho A h g.$$

To keep the vessel stationary the available static friction must obey

$$\mu N \geq F \quad \Rightarrow \quad \mu \rho A h g \geq 2\rho a g h.$$

Cancel the common positive factors to obtain $\mu \geq 2a/A$. The depth and density cancel, so the same threshold applies as the liquid level falls. Option (A) reverses the area ratio; option (D) omits the factor two in $v^2 = 2gh$. Thus the explicitly listed value in (C) rules out the none-of-these choice.

Step 3: Conclusion The minimum coefficient of friction is $\mu_{\min} = \dfrac{2a}{A}$. Hence, the correct option is (C).

Quick Tip to Solve: Jet recoil is mass flow rate times exit speed, $\rho a v^2$; substituting $v^2 = 2gh$ before balancing friction preserves the factor two.

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43. A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by : [4 marks]

(A) y = a

(B) $y = \frac{a}{\sqrt{2}}$

(C) $y = \frac{a\sqrt{3}}{2}$

(D) $y = \frac{a}{2}$

Correct Answer: (D) $y = \frac{a}{2}$

Solution:

Concept:

- For SHM about the mean position, the restoring force is proportional to displacement: $F = -ky$, with $k = m\omega^2$. The displacement is measured from the equilibrium position.
- The total mechanical energy is $E = \frac{1}{2}ka^2$, and the potential energy at displacement y is $U = \frac{1}{2}ky^2$. Hence $K/E = 1 - y^2/a^2$.

Step 1: Find the potential-energy fraction The total energy is the sum of kinetic and potential energies. At the specified instant, kinetic energy occupies three quarters of the total, leaving one quarter as elastic potential energy. Thus

$$U = E - K = E - \frac{3E}{4} = \frac{E}{4}.$$

Step 2: Solve for the displacement Substitute the SHM expressions for total and potential energies. The same stiffness occurs in numerator and denominator and cancels:

$$\frac{U}{E} = \frac{\frac{1}{2}ky^2}{\frac{1}{2}ka^2} = \frac{y^2}{a^2} = \frac{1}{4}.$$

Therefore $y = \pm a/2$. The energy fixes the magnitude of displacement, since potential energy is even in y. Of the positive displacements listed, $a/2$ is the required value.

Step 3: Check the other listed displacements At $y = a$, all the energy is potential and the kinetic energy is zero, excluding (A). At $y = a/\sqrt{2}$, the kinetic fraction is $1/2$, excluding (B). At $y = a\sqrt{3}/2$, the kinetic fraction is $1/4$, excluding (C). Only $y = a/2$ gives the required kinetic fraction $3/4$.

Step 4: Conclusion The displacement magnitude is $a/2$ (the two signed positions are $y = \pm a/2$). Hence, the correct option is (D).

Quick Tip to Solve: Use $K/E = 1 - (y/a)^2$ to convert an SHM energy fraction directly into a displacement magnitude; energy alone does not fix the sign.

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44. Two capacitors of capacities 2C and C are joined in parallel and charged up to potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. The potential difference across the capacitors will now be : [4 marks]

(A) $\frac{V}{K+2}$
(B) $\frac{V}{K}$
(C) $\frac{3V}{K+2}$
(D) $\frac{3V}{K}$

Correct Answer: (C) $\frac{3V}{K+2}$

Solution:

Concept:

- Capacitors in parallel have the same potential difference and their capacitances add. The initial equivalent capacitance is $2C + C = 3C$.
- After the battery is disconnected, the connected capacitor combination is isolated, so its total free charge is conserved. Filling the C capacitor with dielectric changes its capacitance to KC.

Step 1: Find the initial total charge While the battery is connected, each capacitor has potential difference V. Thus their individual charges are $2CV$ and CV. Adding the charges on the corresponding connected plates gives

$$Q_{\text{initial}} = 2CV + CV = 3CV.$$

Removing the battery prevents exchange of charge with an external reservoir, but charge can still redistribute between the capacitors through their connecting wires.

Step 2: Use the new equivalent capacitance The dielectric is inserted only in the capacitor whose original capacitance was C. Its new capacitance is KC, whereas the other stays at $2C$. Since the

parallel connection remains, the final equivalent capacitance and common voltage are

$$C_{\text{final}} = 2C + KC = (K+2)C,$$

$$V_{\text{final}} = \frac{Q_{\text{initial}}}{C_{\text{final}}} = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}.$$

Step 3: Check the alternatives and limiting case With no dielectric effect, $K = 1$ must reproduce the original voltage V. Option (A) would give $V/3$ and (D) would give $3V$, so both fail immediately. Although (B) gives V at $K = 1$, it incorrectly treats the entire capacitance as multiplied by K. The unchanged $2C$ branch must appear in the denominator.

Step 4: Conclusion The final common potential difference is $3V/(K+2)$. Hence, the correct option is (C).

Quick Tip to Solve: For disconnected parallel capacitors, conserve the total charge on the combination; do not conserve the charge of each capacitor separately.

45. A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions. [4 marks]

(A) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(B) $\frac{1}{3}$

(C) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(D) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Correct Answer: (C) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Solution:

Concept:

- For upward projection with initial speed u, take upward as positive and use $s = ut - \frac{1}{2}gt^2$, where g is the positive magnitude of gravitational acceleration.
- The maximum height is $h = u^2/(2g)$. A height below the maximum is crossed twice, first while rising and later while falling; both times are measured from the same launch instant.

Step 1: Relate launch speed to maximum height At the highest point the vertical velocity is zero. Using $v^2 = u^2 - 2gh$, we obtain $u^2 = 2gh$. At the requested height $h/3$, the position equation becomes

$$\frac{h}{3} = ut - \frac{1}{2}gt^2, \qquad gt^2 - 2ut + \frac{2h}{3} = 0.$$

This quadratic has the two crossing times as its roots.

Step 2: Find the two roots Apply the quadratic formula and then replace $2gh$ with u^2:

$$t_{1,2} = \frac{u \mp \sqrt{u^2 - 2gh/3}}{g} = \frac{u}{g}\left(1 \mp \sqrt{\frac{2}{3}}\right).$$

The minus sign gives the earlier upward crossing, because it lies before the top time u/g. The plus sign gives the later downward crossing, after the ball passes its maximum height.

Step 3: Form the ratio and check the choices The common factor u/g cancels:

$$\frac{t_{\text{up}}}{t_{\text{down}}} = \frac{1-\sqrt{2/3}}{1+\sqrt{2/3}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}.$$

Numerically this ratio is about 0.101. The values in (A), (B) and (D) are approximately 0.172, 0.333 and 0.268, respectively, and do not equal the ratio of the two quadratic roots.

Step 4: Conclusion The ratio of the upward-crossing time to the downward-crossing time is $(\sqrt{3}-\sqrt{2})/(\sqrt{3}+\sqrt{2})$. Hence, the correct option is (C).

Quick Tip to Solve: At height αh, the two launch-to-crossing times are $(u/g)(1 \mp \sqrt{1-\alpha})$; take their ratio before inserting numbers.

46. A 0.07 H inductor and a 12Ω resistor are connected in series to a 220V, 50 Hz ac source. The approximate current in the circuit and the phase angle between current and source voltage are respectively. [Take π as $\frac{22}{7}$] [4 marks]

(A) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$
(B) 88 A and $\tan^{-1}\left(\frac{11}{6}\right)$
(C) 0.88 A and $\tan^{-1}\left(\frac{11}{6}\right)$
(D) 8.8 A and $\tan^{-1}\left(\frac{6}{11}\right)$

Correct Answer: (A) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$

Solution:

Concept:

- In a series resistor-inductor circuit, the inductive reactance is $X_L = \omega L = 2\pi f L$, and the impedance magnitude is $Z = \sqrt{R^2 + X_L^2}$.
- The rms current is $I = V/Z$. The current lags the source voltage by angle ϕ, where $\tan\phi = X_L/R$. Resistance and reactance combine at right angles in the impedance plane.

Step 1: Calculate the inductive reactance Use the frequency $f = 50\,\text{Hz}$, inductance $L = 0.07\,\text{H}$, and the given value $\pi = 22/7$:

$$X_L = 2\left(\frac{22}{7}\right)(50)(0.07) = 22\,\Omega.$$

The reactance exceeds the resistance $12\,\Omega$, so the phase lag should be greater than 45°, which already distinguishes the reciprocal-angle choice.

Step 2: Determine impedance and rms current The resistor voltage is in phase with current, whereas the inductor voltage leads it by 90°. Their magnitudes therefore add by Pythagoras rather than ordinary addition:

$$Z = \sqrt{12^2 + 22^2} = \sqrt{628}\,\Omega \approx 25.06\,\Omega,$$

$$I = \frac{220}{\sqrt{628}}\,\text{A} \approx 8.78\,\text{A} \approx 8.8\,\text{A}.$$

The source value is the standard rms voltage of the AC supply, so the result is an rms current.

Step 3: Find the phase angle and exclude alternatives The phase relation is

$$\tan\phi = \frac{22}{12} = \frac{11}{6}, \qquad \phi = \tan^{-1}\left(\frac{11}{6}\right).$$

The circuit is inductive, so current lags voltage. Options (B) and (C) have currents differing by a factor of ten from the impedance result. Option (D) uses the reciprocal ratio R/X_L and would give an angle below 45°, contrary to $X_L > R$.

Step 4: Conclusion The current is approximately $8.8\,\text{A}$, lagging the voltage by $\tan^{-1}(11/6)$. Hence, the correct option is (A).

Quick Tip to Solve: Compute X_L first, then use the impedance triangle: its hypotenuse gives current and its opposite-to-adjacent ratio gives the lag angle.

47. Two identical tennis balls each having mass 'm' and charge 'q' are suspended from a fixed point by threads of length 'l'. What is the equilibrium separation when each thread makes a small angle 'θ' with the vertical? [4 marks]

(A) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{2}}$

(B) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

(C) $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g}\right)^{\frac{1}{3}}$

(D) $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{\frac{1}{3}}$

Correct Answer: (B) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

Solution:

Concept:

- Each ball is acted on by its weight mg, the tension T along its thread, and the horizontal electrostatic repulsion $F_e = q^2/(4\pi\varepsilon_0 x^2)$, where x is the separation.
- By symmetry each ball is displaced horizontally by $x/2$. For a small angle in radians, $\sin\theta \simeq \tan\theta \simeq \theta$, and geometry gives $\sin\theta = x/(2l)$.

Step 1: Resolve the equilibrium forces The vertical component of tension balances gravity, and its horizontal component balances the repulsion:

$$T\cos\theta = mg, \qquad T\sin\theta = \frac{q^2}{4\pi\varepsilon_0 x^2}.$$

Divide the horizontal equation by the vertical equation to eliminate the unknown tension. This gives

$$\tan\theta = \frac{q^2}{4\pi\varepsilon_0 mgx^2}.$$

Both balls obey the same equations because their masses and charges are identical.

Step 2: Apply small-angle geometry and solve From the two equal thread lengths, $x = 2l\sin\theta$. For the stated small-angle equilibrium, substitute $\tan\theta \simeq x/(2l)$ into the force ratio:

$$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 mgx^2}.$$

Multiply by $2lx^2$ and take the positive cube root, since separation is a positive distance:

$$x^3 = \frac{q^2 l}{2\pi\varepsilon_0 mg}, \qquad x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

Step 3: Check scaling and exclude the other options The quantity $q^2/(\varepsilon_0 mg)$ has dimensions of length squared. Multiplication by l gives length cubed, requiring a cube root and excluding the square root in (A). Options (C) and (D) introduce additional powers of thread length, mass or gravity absent from the force-balance equation and are dimensionally inconsistent as lengths. The approximation remains valid when the resulting x is much smaller than $2l$.

Step 4: Conclusion The equilibrium separation is $x = \left(q^2 l/(2\pi\varepsilon_0 mg)\right)^{1/3}$. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate tension by dividing horizontal and vertical balances, then replace the small angle by $x/(2l)$; the inverse-square force produces a cubic equation.

48. Assertion A : If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R :

Least Count = $\frac{Pitch}{Total\ divisions\ on\ circular\ scale}$

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is not correct but R is correct.

(B) Both A and R are correct and R is the correct explanation of A.

(C) A is correct but R is not correct.

(D) Both A and R are correct and R is NOT the correct explanation of A.

Correct Answer: (A) A is not correct but R is correct.

Solution:

Concept:

- The pitch of a screw gauge is the axial distance advanced by the screw during one complete rotation of its circular scale.
- The least count is the displacement for one circular-scale division: $\text{LC} = \text{pitch}/n$, where n is the total number of divisions in one full rotation. Unit conversion must be applied after this division.

Step 1: Evaluate Assertion A from the given data The screw travels $5\,\text{mm}$ in five complete rotations. Therefore the pitch is the travel per rotation, not the full travel:

$$p = \frac{5\,\text{mm}}{5} = 1\,\text{mm}.$$

There are fifty divisions around the circular scale, so one division advances the screw by

$$\text{LC} = \frac{1\,\text{mm}}{50} = 0.02\,\text{mm} = 0.002\,\text{cm}.$$

Here $1\,\text{mm} = 0.1\,\text{cm}$. The asserted least count $0.001\,\text{cm}$ is half the actual value, so Assertion A is false.

Step 2: Evaluate Reason R independently The stated reason divides the pitch by the total divisions on the circular scale. One full rotation advances the screw by one pitch, and each of the equally spaced divisions represents $1/50$ of that rotation. Thus the formula in R is precisely the definition of the instrument least count. Reason R is true; applying it correctly produces $0.002\,\text{cm}$, rather than the numerical value claimed in A.

Step 3: Choose the matching assertion-reason combination The required truth values are false for A and true for R. Option (A) states exactly that combination. Options (B) and (D) both call A true, so neither can hold, irrespective of the explanatory relationship. Option (C) reverses both truth values. Because A is false, there is no valid claim that R explains A as a true assertion.

Step 4: Conclusion The least count is $0.002\,\text{cm}$. Assertion A is not correct, but Reason R is correct. Hence, the correct option is (A).

Quick Tip to Solve: First divide the total screw travel by the number of complete turns to get pitch; only then divide by circular-scale divisions and convert units.

49. If 'f' denotes the ratio of the number of nuclei decayed (N_d) to the number of nuclei at t = 0 (N_0) then for a collection of radioactive nuclei, the rate of change of 'f' with respect to time is given as : [λ is the radioactive decay constant] [4 marks]

(A) $-\lambda\ (1 - e^{-\lambda t})$

(B) $\lambda\ (1 - e^{-\lambda t})$

(C) $\lambda e^{-\lambda t}$

(D) $-\lambda e^{-\lambda t}$

Correct Answer: (C) $\lambda e^{-\lambda t}$

Solution:

Concept:

- The number of undecayed nuclei follows $N(t) = N_0 e^{-\lambda t}$, where N_0 is the initial population and $\lambda > 0$ is the decay constant.
- The number already decayed is $N_d = N_0 - N(t)$, so its fraction relative to the fixed initial population is $f = N_d/N_0$. Distinguish this increasing fraction from the decreasing surviving fraction N/N_0.

Step 1: Express the fraction of nuclei already decayed The total initial population is partitioned into nuclei that remain and nuclei that have decayed. Subtract the surviving number from N_0, then divide by the constant initial population:

$$N_d = N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t}),$$

$$f(t) = \frac{N_d}{N_0} = 1 - e^{-\lambda t}.$$

The fraction starts at zero and tends to one as time increases, as required for cumulative decay.

Step 2: Differentiate the fraction with respect to time The derivative of the constant term is zero. The exponential derivative introduces a factor $-\lambda$, which cancels the minus sign already present:

$$\frac{df}{dt} = -\frac{d}{dt}e^{-\lambda t} = -(-\lambda)e^{-\lambda t} = \lambda e^{-\lambda t}.$$

This derivative has units of inverse time, while f itself is dimensionless. Equivalently, $df/dt = \lambda N/N_0$, so the rate is proportional to the number still available to decay.

Step 3: Check the sign and initial-time behaviour The decayed fraction increases, so its derivative must be positive: the negative expressions in (A) and (D) are excluded. At $t = 0$, the rate is λ, because the full initial population is still radioactive. Option (B) would instead give zero initially and increase towards λ, reversing the actual decrease of decay rate. Option (C) has the correct initial value and tends to zero at long times.

Step 4: Conclusion The rate of change is $df/dt = \lambda e^{-\lambda t}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the decayed fraction as one minus the surviving fraction before differentiating; the two minus signs then ensure a positive rate.

50. A body takes 4 min. to cool from 61° C to 59° C. If the temperature of the surroundings is 30° C, the time taken by the body to cool from 51° C to 49° C is : [4 marks]

(A) 4 min.

(B) 3 min.

(C) 8 min.

(D) 6 min.

Correct Answer: (D) 6 min.

Solution:

Concept:

- Newton's law of cooling states that the rate of fall of temperature is proportional to the excess temperature above the surroundings: $-dT/dt = k(T - T_s)$, where k is constant for the same body and conditions.
- For a small temperature interval, use the mean body temperature to estimate the cooling rate. Here both intervals are only $2°C$, so the same temperature fall takes a time inversely proportional to the mean excess temperature.

Step 1: Find the excess temperatures The surroundings remain at $30°C$. In the first interval the mean body temperature is $(61 + 59)/2 = 60°C$, giving an excess of $30°C$. In the second interval the mean temperature is $50°C$, giving an excess of $20°C$. The second interval therefore has a smaller cooling rate.

Step 2: Compare equal temperature falls The first cooling rate is $2/4 = 0.5\ °C\,min^{-1}$. Newton's law gives the second rate as $0.5(20/30) = 1/3\ °C\,min^{-1}$. Hence

$$t_2 = \frac{2}{1/3} = 6 \text{ min.}$$

Equivalently, $t_2/t_1 = 30/20$. A time of 4 minutes would incorrectly assume equal cooling rates, while 3 minutes would make cooling faster nearer room temperature. The value 8 minutes overestimates the change in rate.

Step 3: Check the small-interval approximation Integrating the cooling law gives the exact ratio

$$\frac{t_2}{t_1} = \frac{\ln(21/19)}{\ln(31/29)} \simeq 1.5007.$$

Thus the exact time is approximately 6.003 minutes, agreeing with the listed answer to the precision of the options.

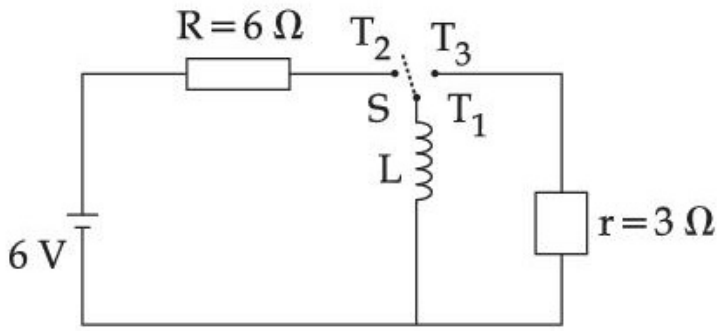
Step 4: Conclusion The required cooling time is 6 min. Hence, the correct option is (D).

Quick Tip to Solve: For equal small falls in temperature, compare the mean excess temperatures above the surroundings, then invert that ratio to compare cooling times.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. Consider an electrical circuit containing a two way switch 'S'. Initially S is open and then T_1 is connected to T_2. As the current in R = 6Ω attains a maximum value of steady state level, T_1 is disconnected from T_2 and immediately connected to T_3. Potential drop across r = 3Ω resistor immediately after T_1 is connected to T_3 is _____ V. (Round off to the Nearest Integer) [4 marks]



Solution:

Concept:

- An ideal inductor acts as a short circuit after a sufficiently long time with a direct-current source, because its induced voltage is $L\,di/dt$ and the steady current has $di/dt = 0$.
- The current through an inductor cannot change instantaneously under a finite voltage. Its value just after switching equals the current just before switching; the resistor voltage then follows from Ohm's law.

Step 1: Find the steady current before switching With T_1 connected to T_2, the 6 V battery, the $6\,\Omega$ resistor and the inductor form the conducting loop. The $3\,\Omega$ resistor is disconnected at its upper end, so it does not affect this steady current. Treating the ideal inductor as a wire at steady state gives

$$I_0 = \frac{6}{6} = 1\ \text{A}.$$

The current flows from the battery through the $6\,\Omega$ resistor into the inductor.

Step 2: Apply current continuity at the switch Immediately after T_1 is moved to T_3, the battery and $6\,\Omega$ resistor are excluded from the inductor loop. The inductor is now connected to the $3\,\Omega$ resistor. The magnetic energy stored in the inductor supplies the resistor current, and

$$I(0^+) = I(0^-) = 1\ \text{A}.$$

Setting this current to zero immediately would require an impulsive, unbounded inductor voltage.

Step 3: Calculate the requested potential drop The same 1 A current passes through the $3\,\Omega$ discharge resistor at that instant. Therefore

$$V_r(0^+) = I(0^+)r = 1 \times 3 = 3\ \text{V}.$$

Subsequently the current decays as $I_0 e^{-rt/L}$, but neither that decay time nor the numerical inductance is needed for the immediate voltage.

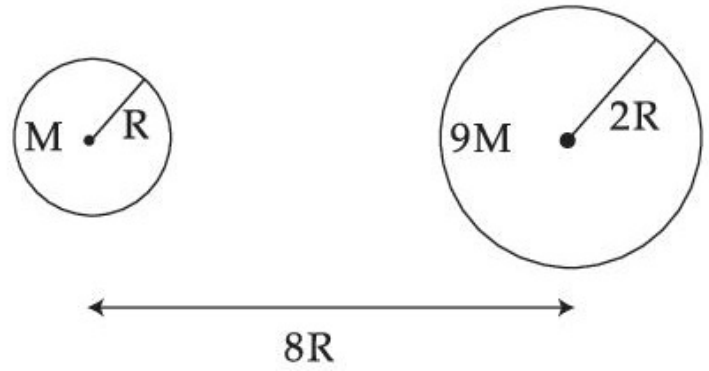
Step 4: Conclusion The immediate potential drop across the $3\,\Omega$ resistor is 3 V. The nearest-integer entry is $\boxed{3}$.

Quick Tip to Solve: First determine the old steady inductor current; carry that current unchanged across the switching instant, then multiply by the new resistance.

---

52. Suppose two planets (spherical in shape) in radii R and 2R, but mass M and 9M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is _____.

[Given : The two planets are fixed in their position] [4 marks]



Solution:

Concept:

- Outside a spherical planet, the gravitational potential per unit mass is $-GM/r$. Potentials add algebraically, so both planets must be included at the launch point and throughout the flight.
- The minimum launch kinetic energy must overcome the maximum of the gravitational potential along the path. Equal potential at the destination alone is insufficient if an intervening potential barrier is higher.

Step 1: Locate the potential maximum Let x be the distance from the centre of the smaller planet, measured toward the larger planet. Between their surfaces, $R \le x \le 6R$, the potential is

$$\phi(x) = -\frac{GM}{x} - \frac{9GM}{8R - x}.$$

At the stationary point the opposite gravitational attractions balance:

$$\frac{GM}{x^2} = \frac{9GM}{(8R - x)^2}.$$

Thus $8R - x = 3x$ and $x = 2R$. The potential rises up to this point and then falls; this is the barrier the satellite must cross.

Step 2: Evaluate the launch and barrier potentials The satellite launches from the facing surface of the smaller planet, where $x = R$ and its distance from the larger centre is $7R$. Hence

$$\phi_i = -\frac{GM}{R} - \frac{9GM}{7R} = -\frac{16GM}{7R}.$$

At $x = 2R$, the distances are $2R$ and $6R$, giving

$$\phi_{\max} = -\frac{GM}{2R} - \frac{9GM}{6R} = -\frac{2GM}{R}.$$

The potential increase is therefore $2GM/(7R)$.

Step 3: Apply conservation of mechanical energy At the threshold, the initial kinetic energy per unit mass equals this potential increase:

$$\frac{v^2}{2} = \phi_{\max} - \phi_i = \frac{2GM}{7R}, \qquad v = \sqrt{\frac{4GM}{7R}}.$$

The satellite mass cancels. Strictly the threshold trajectory approaches the equilibrium point asymptotically; an arbitrarily small excess over this speed crosses it and then accelerates toward the second surface. This is the usual limiting minimum-speed condition.

Step 4: Conclusion Comparison with $v = \sqrt{(a/7)(GM/R)}$ gives $\boxed{a = 4}$.

Quick Tip to Solve: For motion between gravitating bodies, find the zero-force point and test the potential barrier there before comparing the two surface energies.

---

53. In Bohr's atomic model, the electron is assumed to revolve in a circular orbit of radius 0.5 $\overset{o}{A}$. If the speed of electron is 2.2 $\times$ 10^6 m/s, then the current associated with the electron will be _____ $\times$ 10^{-2} mA. [Take π as $\frac{22}{7}$] [4 marks]

Solution:

Concept:

- A charge completing one circular revolution every period T constitutes a current of magnitude $I = e/T$. For an orbit of radius r traversed at speed v, the period is $T = 2\pi r/v$.
- Thus $I = ev/(2\pi r)$. Use $e = 1.6 \times 10^{-19}$ C, convert angstroms to metres, and distinguish amperes from the requested unit 10^{-2} mA.

Step 1: Retain the stated speed and convert the radius The printed speed is $2.2 \times 16^6\,\mathrm{m\,s^{-1}}$; retaining that expression gives $16^6 = 16{,}777{,}216$. The orbital radius is

$$r = 0.5 \times 10^{-10} = 5 \times 10^{-11}\ \mathrm{m}.$$

The circumference is $2\pi r$. One electron passes a fixed point per revolution, so its charge magnitude

divided by the revolution period gives the current magnitude. Conventional current is opposite to the electron's motion.

Step 2: Calculate the current for the printed value With $\pi = 22/7$, substitution yields

$$I = \frac{(1.6 \times 10^{-19})(2.2 \times 16^{6})}{2(22/7)(5 \times 10^{-11})} = 0.01879048192 \text{ A}.$$

Since $10^{-2}\,\text{mA} = 10^{-5}\,\text{A}$, the numerical coefficient requested in those units is

$$\frac{I}{10^{-5}\,\text{A}} = 1879.048192.$$

The unit conversion is essential: expressing the current directly in mA would instead give 18.79048192.

Step 3: Compare with the supplied answer key The supplied key gives 112. That value follows if the speed were $2.2 \times 10^{6}\,\text{m s}^{-1}$, because then

$$I = \frac{(1.6 \times 10^{-19})(2.2 \times 10^{6})}{2(22/7)(5 \times 10^{-11})} = 1.12 \times 10^{-3}\ \text{A} = 112 \times 10^{-2}\ \text{mA}.$$

However, 16^{6} and 10^{6} are different numerical inputs. The stated expression must be retained when answering the literal question.

Step 4: Conclusion For the printed speed $2.2 \times 16^{6}\,\text{m s}^{-1}$, the required coefficient is $\boxed{1879.048192}$. The supplied key value 112 corresponds to $2.2 \times 10^{6}\,\text{m s}^{-1}$, rather than the printed speed.

Quick Tip to Solve: Calculate charge per revolution divided by the orbital period, then convert the result to the exact current unit requested in the blank.

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54. A particle of mass 9.1×10^{-31} kg travels in a medium with a speed of 10^{6} m/s and a photon of a radiation of linear momentum 10^{-27} kg m/s travels in vacuum. The wavelength of photon is _____ times the wavelength of the particle. [4 marks]

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, with h Planck's constant and p the magnitude of linear momentum. It applies to the matter particle as well as to the photon.
- For the slowly moving massive particle, $p = mv$. A ratio of wavelengths cancels Planck's constant and reverses the ratio of momenta, avoiding unnecessary numerical wavelength calculations.

Step 1: Determine the matter-particle momentum The particle has mass $9.1 \times 10^{-31}\,\text{kg}$ and speed $10^{6}\,\text{m s}^{-1}$. Its speed is much smaller than the speed of light, so the usual nonrelativistic momentum is appropriate:

$$p_{\text{particle}} = mv = (9.1 \times 10^{-31})(10^{6}) = 9.1 \times 10^{-25}\ \text{kg m s}^{-1}.$$

The reference to motion in a medium does not change this stated relation between mechanical

momentum, mass and speed.

Step 2: Form the wavelength ratio The photon momentum is already supplied as $p_\gamma = 10^{-27}\,\mathrm{kg\,m\,s^{-1}}$; it need not be calculated from its energy or frequency. Applying the same momentum-wavelength relation to the two objects gives

$$\frac{\lambda_\gamma}{\lambda_{\text{particle}}} = \frac{h/p_\gamma}{h/p_{\text{particle}}} = \frac{p_{\text{particle}}}{p_\gamma}.$$

The inverse proportionality ensures that the object with smaller momentum has the longer wavelength.

Step 3: Substitute and check the powers of ten Therefore

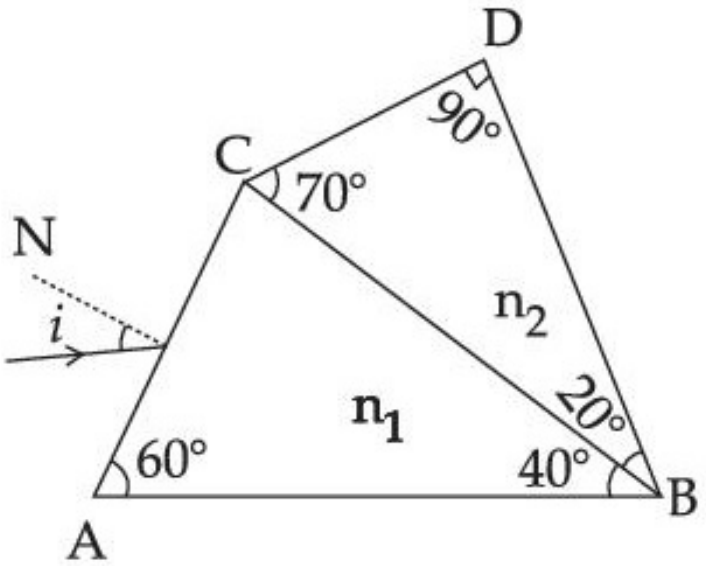
$$\frac{\lambda_\gamma}{\lambda_{\text{particle}}} = \frac{9.1 \times 10^{-25}}{10^{-27}} = 9.1 \times 10^2 = 910.$$

The ratio is dimensionless because the momentum units cancel. Its being greater than one is consistent with the photon having the smaller specified momentum; inverting the ratio would give a wavelength ordering contrary to $\lambda = h/p$.

Step 4: Conclusion The photon wavelength is 910 times the matter-particle wavelength. The required numerical entry is $\boxed{910}$.

Quick Tip to Solve: When comparing wavelengths, cancel Planck's constant first: the wavelength ratio is the inverse momentum ratio, with the order carefully preserved.

55. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together (as shown in the figure). n_1 and n_2 depend on λ, the wavelength of light, according to the relation $n_1 = 1.2 + \frac{10.8\times10^{-14}}{\lambda^2}$ and $n_2 = 1.45 + \frac{1.8\times10^{-14}}{\lambda^2}$
The wavelength for which rays incident at any angle on the interface BC pass through without bending at that interface will be _____ nm. [4 marks]



Solution:

Concept:

- Snell's law at the common interface is $n_1 \sin i = n_2 \sin r$. To have no bending for every incident angle, rather than only at normal incidence, the two refractive indices must be equal.
- The given wavelength-dependent refractive indices must be compared at the same wavelength. Solve $n_1(\lambda) = n_2(\lambda)$ with λ in metres, then convert the positive root into nanometres.

Step 1: State the condition at the common interface For rays to pass through BC without changing direction, the angle of refraction must equal the angle of incidence. At a nonzero incidence angle, Snell's law becomes $n_1 \sin i = n_2 \sin i$, which requires $n_1 = n_2$. This equality then guarantees no bending at every incidence angle. Normal incidence alone would not impose this condition, since any two transparent media give an undeviated ray along their common normal.

Step 2: Equate the dispersion relations Using the specified expressions,

$$1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2} = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}.$$

Collecting the wavelength-dependent terms on one side gives

$$\frac{9.0 \times 10^{-14}}{\lambda^2} = 0.25, \qquad \lambda^2 = 3.6 \times 10^{-13}\ \mathrm{m}^2.$$

The prism angles affect refraction at the outside faces, but do not enter the condition for index matching at BC.

Step 3: Take the physical root and check The wavelength is positive, so

$$\lambda = 6.0 \times 10^{-7}\ \mathrm{m} = 600\ \mathrm{nm}.$$

At this wavelength, $\lambda^2 = 36 \times 10^{-14}\ \mathrm{m}^2$, giving $n_1 = 1.2 + 10.8/36 = 1.50$ and $n_2 = 1.45 + 1.8/36 = 1.50$. Thus the two media have exactly matching optical indices at the common interface, confirming the result.

Step 4: Conclusion The required wavelength is $\boxed{600\ \mathrm{nm}}$, so the numerical entry is 600.

Quick Tip to Solve: An interface is optically invisible to rays at all incidence angles when its two media have equal refractive indices at the wavelength considered.

56. A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is _____ m/s. (Young's modulus of rubber = 0.5 $\times$ 10^9 N/m^2) [4 marks]

Solution:

Concept:

- For an elastic element obeying Hooke's law, Young's modulus relates stress to strain: $Y = (F/A)/(x/L)$. Therefore its effective spring constant is $k = YA/L$, where L is the original length.
- The stored strain energy is $U = \frac{1}{2}kx^2$. In the ideal projection model this becomes the stone's kinetic energy $\frac{1}{2}mv^2$, neglecting dissipation and the kinetic energy retained by the rubber.

Step 1: Convert the data and find the effective stiffness The mass is $m = 20\,\text{g} = 0.020\,\text{kg}$. The original rubber length is $L = 0.1\,\text{m}$, its area is $A = 10^{-6}\,\text{m}^2$, and the extension is $x = 0.04\,\text{m}$. Use the original length in the strain, rather than the stretched length. Thus

$$k = \frac{YA}{L} = \frac{(0.5 \times 10^9)(10^{-6})}{0.1} = 5.0 \times 10^3\ \text{N m}^{-1}.$$

Step 2: Calculate the energy released The restoring force rises linearly from zero to kx while stretching, so its average over the extension is $kx/2$. The elastic energy is consequently

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(5.0 \times 10^3)(0.04)^2 = 4.0\ \text{J}.$$

Using final force multiplied by extension without the factor one-half would double the available energy and give an incorrect launch speed.

Step 3: Convert elastic energy into stone motion Equating the released elastic energy to the stone's kinetic energy gives

$$\frac{1}{2}mv^2 = 4.0, \qquad v^2 = \frac{8.0}{0.020} = 400\ \text{m}^2\,\text{s}^{-2}.$$

The positive square root is the speed:

$$v = 20\ \text{m s}^{-1}.$$

Equivalently, $v = x\sqrt{YA/(mL)}$. This expression has speed dimensions, since k/m has dimensions of inverse time squared.

Step 4: Conclusion The projected stone has speed $\boxed{20\ \text{m s}^{-1}}$, giving the numerical entry 20.

Quick Tip to Solve: Replace the stretched rubber by a spring of stiffness YA/L, then equate its one-half stiffness-times-extension-squared energy to kinetic energy.

57. In a uniform magnetic field, the magnetic needle has a magnetic moment 9.85 $\times$ 10^{-2} A/m^2 and moment of inertia 5 $\times$ 10^{-6} kg m^2. If it performs 10 complete oscillations in 5 seconds then the magnitude of the magnetic field is _____ mT. [Take π^2 as 9.85] [4 marks]

Solution:

Concept:

- A magnetic needle displaced through a small angle experiences restoring torque $\tau = -mB\sin\theta \simeq -mB\theta$, where m is its magnetic moment and B is the uniform field.
- The rotational equation $I\ddot{\theta} = -mB\theta$ gives simple harmonic motion with period $T = 2\pi\sqrt{I/(mB)}$. Use the time for one oscillation, not the time for the whole set.

Step 1: Find the oscillation period Ten complete oscillations take $5\,\text{s}$, so the period is

$$T = \frac{5}{10} = 0.5\,\text{s}.$$

The corresponding frequency is $2\,\text{Hz}$. The moment of inertia is $I = 5 \times 10^{-6}\,\text{kg m}^2$, and the stated magnetic-moment magnitude is $m = 9.85 \times 10^{-2}$.

Step 2: Rearrange the period equation Squaring the period expression and isolating the field gives

$$B = \frac{4\pi^2 I}{mT^2}.$$

Substitute the supplied value $\pi^2 = 9.85$:

$$B = \frac{4(9.85)(5 \times 10^{-6})}{(9.85 \times 10^{-2})(0.5)^2} = 8 \times 10^{-3}\,\text{T}.$$

The factor 9.85 cancels, which also provides a quick arithmetic check. A stronger field would increase the restoring torque and shorten the period, consistent with this formula.

Step 3: Conclusion The magnetic field is $8\,\text{mT}$. The numerical entry is $\boxed{8}$.

Quick Tip to Solve: First divide the total time by the number of oscillations; then use $B \propto T^{-2}$, so an error in the period is squared.

58. A radioactive sample has an average life of 30 ms and is decaying. A capacitor of capacitance 200 μF is first charged and later connected with resistor 'R'. If the ratio of charge on capacitor to the activity of radioactive sample is fixed with respect to time then the value of 'R' should be _____ Ω. [4 marks]

Solution:

Concept:

- Activity decays as $A(t) = A_0 e^{-\lambda t}$, and the mean life is $\tau = 1/\lambda$. Mean life is distinct from half-life.
- The charge on a capacitor discharging through a resistance is $Q(t) = Q_0 e^{-t/(RC)}$. The electrical time constant is RC.

Step 1: Impose the constant-ratio condition The ratio at any time is

$$\frac{Q(t)}{A(t)} = \frac{Q_0}{A_0}\exp\left[\left(\lambda - \frac{1}{RC}\right)t\right].$$

For this ratio to remain fixed as the two systems decay, the coefficient of time in the exponential must vanish. Thus $\lambda = 1/(RC)$, or $RC = \tau$. Initial charge and initial activity determine only the constant ratio, not the required resistance.

Step 2: Convert units and calculate resistance The given average life is $30\,\text{ms} = 30 \times 10^{-3}\,\text{s}$, while $C = 200\,\mu\text{F} = 200 \times 10^{-6}\,\text{F}$. Hence

$$R = \frac{\tau}{C} = \frac{30 \times 10^{-3}}{200 \times 10^{-6}} = 150\,\Omega.$$

The units check because $\Omega\,\text{F} = \text{s}$. No factor $\ln 2$ is introduced: that factor would be necessary only if a half-life, rather than a mean life, had been supplied.

Step 3: Conclusion The required resistance is $\boxed{150\,\Omega}$; enter 150.

Quick Tip to Solve: A constant ratio of two exponential decays requires identical time constants. Match RC directly to the radioactive mean life.

59. A transistor is connected in common emitter circuit configuration, the collector supply voltage is 10 V and the voltage drop across a resistor of 1000 Ω in the collector circuit is 0.6 V. If the current gain factor (β) is 24, then the base current is _____ μA. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- In a common-emitter configuration the current gain is $\beta = I_C/I_B$, where I_C and I_B are collector and base currents.
- The collector-resistor voltage drop gives its current by Ohm's law: $I_C = V_R/R_C$. The supply voltage and the resistor voltage drop are different quantities.

Step 1: Find the collector current The stated voltage drop across the $1000\,\Omega$ resistor is $0.6\,\text{V}$. Therefore

$$I_C = \frac{0.6}{1000} = 6 \times 10^{-4}\,\text{A} = 600\,\mu\text{A}.$$

Use 0.6 V here, because that is the voltage actually across this resistor. Using the full 10 V supply would incorrectly ignore the collector-emitter voltage.

Step 2: Use the current gain to find the base current Rearrange the definition of current gain:

$$I_B = \frac{I_C}{\beta} = \frac{600}{24}\,\mu\text{A} = 25\,\mu\text{A}.$$

As a check, multiplying this base current by 24 reproduces the calculated collector current. Kirchhoff's

voltage law also gives $V_{CE} = 10 - 0.6 = 9.4\,\text{V}$, consistent with the supply being shared between the resistor and transistor. The requested base current is already an integer in microamperes, so rounding does not change it.

Step 3: Conclusion The base current is $\boxed{25\,\mu\text{A}}$. The nearest-integer entry is 25.

Quick Tip to Solve: Use the resistor's own voltage drop to calculate I_C, then divide by β; do not divide the supply voltage by the collector resistance.

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60. The amplitude of upper and lower side bands of A.M. wave where a carrier signal with frequency 11.21 MHz, peak voltage 15 V is amplitude modulated by a 7.7 kHz sine wave of 5V amplitude are $\frac{a}{10}V$ and $\frac{b}{10}V$ respectively. Then the value of $\frac{a}{b}$ is _____. [4 marks]

Solution:

Concept:

- For a sinusoidally amplitude-modulated wave, $v = A_c[1 + \mu\cos(\omega_m t)]\cos(\omega_c t)$, where $\mu = A_m/A_c$ is the modulation index.
- Using the product-to-sum identity gives equal-amplitude sidebands at $f_c + f_m$ and $f_c - f_m$. Each sideband amplitude is $\mu A_c/2 = A_m/2$.

Step 1: Determine the modulation index and sideband amplitude The carrier peak amplitude is $A_c = 15\,\text{V}$ and the modulating amplitude is $A_m = 5\,\text{V}$. Thus

$$\mu = \frac{5}{15} = \frac{1}{3}.$$

Expanding the modulated signal produces a carrier term plus two cosine terms. The coefficient of each sideband term is

$$A_{\text{USB}} = A_{\text{LSB}} = \frac{\mu A_c}{2} = \frac{5}{2} = 2.5\,\text{V}.$$

Since $\mu < 1$, this is ordinary modulation without overmodulation.

Step 2: Identify the requested parameters The upper and lower frequencies are $11.21 + 0.0077 = 11.2177\,\text{MHz}$ and $11.21 - 0.0077 = 11.2023\,\text{MHz}$, respectively. Their unequal frequencies do not imply unequal amplitudes. From $a/10 = 2.5$ and $b/10 = 2.5$, we obtain $a = b = 25$. Consequently

$$\frac{a}{b} = \frac{25}{25} = 1.$$

In fact, equal sideband amplitudes alone already establish the ratio, independently of the supplied carrier frequency.

Step 3: Conclusion The requested ratio is $\boxed{a/b = 1}$; enter 1.

Quick Tip to Solve: For single-tone AM, the two sidebands have equal amplitudes. A ratio of their amplitudes is therefore one before any numerical substitution.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Which one of the following compounds will give orange precipitate when treated with 2, 4-dinitrophenyl hydrazine?

[4 marks]

(A) Ethyl 2-hydroxybenzoate, $2\text{-}HOC_6H_4COOC_2H_5$

(B) Ethyl benzoate, $C_6H_5COOC_2H_5$

(C) 2-Hydroxybenzoic acid, $2\text{-}HOC_6H_4COOH$

(D) Acetophenone, $C_6H_5COCH_3$

Correct Answer: (D) Acetophenone, $C_6H_5COCH_3$

Solution:

Concept:

- 2,4-Dinitrophenylhydrazine (2,4-DNP) detects aldehyde and ketone carbonyl groups by forming yellow or orange crystalline hydrazone precipitates.
- A carbonyl group in an ester or carboxylic acid is not an aldehyde or ketone carbonyl. Resonance donation from the attached oxygen changes its chemical reactivity.

Step 1: Classify the four carbonyl compounds Option (A) is ethyl 2-hydroxybenzoate: its $-COOC_2H_5$ group is an ester, despite the additional phenolic hydroxyl. Option (B), ethyl benzoate, is also an ester. Option (C), 2-hydroxybenzoic acid, contains a $-COOH$ carboxyl group. Option (D) is acetophenone, $C_6H_5COCH_3$, whose carbonyl carbon is bonded to a phenyl group and a methyl group; it is a ketone.

Step 2: Apply the carbonyl condensation test Acetophenone undergoes condensation with the terminal $-NH_2$ group of 2,4-DNP, eliminating water and forming a conjugated hydrazone:

$$C_6H_5COCH_3 + H_2NNHC_6H_3(NO_2)_2 \longrightarrow C_6H_5C(CH_3){=}NNHC_6H_3(NO_2)_2 + H_2O.$$

This derivative gives the characteristic orange precipitate. The ester groups in (A) and (B) and the carboxylic acid group in (C) do not respond to the usual 2,4-DNP test. A phenolic hydroxyl group by itself is also insufficient to give this precipitate.

Step 3: Conclusion Acetophenone, $C_6H_5COCH_3$, gives the orange precipitate. Hence, the correct option is (D).

Quick Tip to Solve: Inspect what is attached directly to the carbonyl carbon: a ketone responds to 2,4-DNP, while $-COOR$ and $-COOH$ groups do not.

62. The oxidation states of 'P' in $H_4P_2O_7$, $H_4P_2O_5$ and $H_4P_2O_6$, respectively, are : [4 marks]

(A) 7, 5 and 6

(B) 5, 4 and 3

(C) 5, 3 and 4

(D) 6, 4 and 5

Correct Answer: (C) 5, 3 and 4

Solution:

Concept:

- In a neutral molecule the sum of the oxidation numbers of all atoms is zero. Hydrogen is assigned $+1$, and ordinary oxygen is assigned -2 in these phosphorus oxyacids.
- Let the oxidation state of each phosphorus atom be x. For $H_4P_2O_y$, charge balance gives $4(+1) + 2x + y(-2) = 0$, so $x = y - 2$.

Step 1: Calculate the three phosphorus oxidation states For pyrophosphoric acid, $H_4P_2O_7$,

$$4 + 2x - 14 = 0 \quad \Rightarrow \quad x = +5.$$

For $H_4P_2O_5$,

$$4 + 2x - 10 = 0 \quad \Rightarrow \quad x = +3.$$

For hypophosphoric acid, $H_4P_2O_6$,

$$4 + 2x - 12 = 0 \quad \Rightarrow \quad x = +4.$$

A bond between the two phosphorus atoms in the last compound contributes zero to either atom's oxidation number because the atoms have the same electronegativity.

Step 2: Match the sequence without changing its order The question asks for the values in the order with seven, five and six oxygen atoms, not in increasing order. The resulting sequence is $+5, +3, +4$. Option (A) incorrectly takes oxygen counts as phosphorus oxidation states. Option (B) reverses the last two correct results, while option (D) raises every member by one. Substituting $+5, +3, +4$ separately into the corresponding charge sums gives zero in each case, verifying neutrality.

Step 3: Conclusion The oxidation states are $\boxed{+5, +3, +4}$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: For the family $H_4P_2O_y$, solve once to obtain $x = y - 2$, then insert each oxygen subscript in the stated order.

63. For a reaction of order n, the unit of the rate constant is : [4 marks]

(A) $mol^{1-n}\ L^{1-n}\ s$

(B) $mol^{1-n}\ L^{2n}\ s^{-1}$

(C) $mol^{1-n}\ L^{n-1}\ s^{-1}$

(D) $mol^{1-n}\ L^{1-n}\ s^{-1}$

Correct Answer: (C) $mol^{1-n}\ L^{n-1}\ s^{-1}$

Solution:

Concept:

- The rate law for an overall reaction order n has the dimensional form $r = k[c]^n$, with concentration measured in $\mathrm{mol\,L^{-1}}$.
- Rate has units of concentration per unit time, $\mathrm{mol\,L^{-1}\,s^{-1}}$. The units of k depend on reaction order and follow by dividing the rate units by the concentration factor.

Step 1: Isolate the units of the rate constant From $k = r/[c]^n$,

$$[k] = \frac{\mathrm{mol\,L^{-1}\,s^{-1}}}{(\mathrm{mol\,L^{-1}})^n}.$$

Dividing powers with the same base subtracts the exponents. Therefore

$$[k] = \mathrm{mol^{1-n}\,L^{n-1}\,s^{-1}}.$$

The time exponent stays -1 for every order because no time factor occurs in the concentration term.

Step 2: Check familiar orders and eliminate alternatives For $n = 1$, the concentration factors cancel, leaving $\mathrm{s^{-1}}$. For $n = 2$, the expression becomes $\mathrm{L\,mol^{-1}\,s^{-1}}$; for $n = 0$, it gives $\mathrm{mol\,L^{-1}\,s^{-1}}$. These familiar limits confirm the general expression. Option (A) has the wrong time exponent. Option (B) uses $2n$ as the litre exponent, which fails even the first-order check. Option (D) reverses the required litre exponent; it agrees accidentally at first order but fails at other orders.

Step 3: Conclusion The unit is $\boxed{\mathrm{mol^{1-n}\,L^{n-1}\,s^{-1}}}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the concentration unit as a single factor first: $[k] = [\text{concentration}]^{1-n}\mathrm{s^{-1}}$. Expanding it then fixes both mole and litre powers.

64. Given below are two statements :

Statement I : Aniline is less basic than acetamide.

Statement II : In aniline, the lone pair of electrons on nitrogen atom is delocalized over benzene ring due to resonance and hence less available to a proton.

Choose the most appropriate option; [4 marks]

(A) Statement I is true but statement II is false.

(B) Statement I is false but statement II is true.

(C) Both statement I and statement II are true.

(D) Both statement I and statement II are false.

Correct Answer: (B) Statement I is false but statement II is true.

Solution:

Concept:

- Basicity depends on the availability of an electron pair for accepting a proton. Resonance delocalisation usually reduces the availability of the nitrogen lone pair.
- In an amide, the nitrogen lone pair is conjugated directly with a carbonyl group; this delocalisation is stronger than its conjugation with the benzene ring in aniline.

Step 1: Compare the two nitrogen environments Aniline, $C_6H_5NH_2$, has its nitrogen lone pair conjugated with the aromatic ring. It is consequently less basic than ordinary aliphatic amines. Acetamide, CH_3CONH_2, has its nitrogen immediately attached to a carbonyl carbon. Donation of the nitrogen lone pair towards the oxygen-containing carbonyl system gives the carbon–nitrogen bond partial double-bond character. The pair is therefore still less available for protonation at nitrogen than the pair in aniline.

Step 2: Evaluate each statement Statement I reverses the relevant basicity order: aniline is more basic than acetamide, so Statement I is false. Although amides can accept a proton predominantly at oxygen in sufficiently acidic media, that does not make acetamide more basic than aniline. Statement II correctly describes resonance in aniline and the resulting reduction in lone-pair availability. It is true. The reduction relative to an aliphatic amine must not be confused with a reduction relative to every other nitrogen compound.

Step 3: Match the truth values The required combination is false for Statement I and true for Statement II. Option (A) gives the reverse combination, while (C) and (D) incorrectly assign the same truth value to both statements.

Step 4: Conclusion Statement I is false and Statement II is true. Hence, the correct option is (B).

Quick Tip to Solve: Compare what the nitrogen is conjugated with: an adjacent carbonyl suppresses amide basicity more strongly than the phenyl ring suppresses aniline basicity.

65. The type of hybridisation and magnetic property of the complex $[MnCl_6]^{3-}$, respectively, are :

[4 marks]

(A) sp^3d^2 and diamagnetic

(B) d^2sp^3 and diamagnetic

(C) d^2sp^3 and paramagnetic

(D) sp^3d^2 and paramagnetic

Correct Answer: (D) sp^3d^2 and paramagnetic

Solution:

Concept:

- Find the oxidation state before writing the metal-ion electronic configuration. Chloride contributes a charge of -1 per ligand.
- Chloride is a weak-field ligand. In the valence-bond description, a high-spin octahedral complex uses outer-orbital sp^3d^2 hybridisation; unpaired electrons cause paramagnetism.

Step 1: Find the metal oxidation state Let the oxidation state of manganese be x. For $[\mathrm{MnCl_6}]^{3-}$, charge balance gives

$$x + 6(-1) = -3 \quad \Rightarrow \quad x = +3.$$

Neutral manganese has configuration $[\mathrm{Ar}]3d^5 4s^2$. Removing the two $4s$ electrons and one $3d$ electron gives Mn^{3+} the configuration $[\mathrm{Ar}]3d^4$. The coordination number is six, giving an octahedral arrangement.

Step 2: Determine the spin state and hybridisation The chloride ligands do not cause strong-field pairing of the four metal d electrons. In crystal-field notation the high-spin population is $t_{2g}^3 e_g^1$, with four unpaired electrons. In the elementary valence-bond picture there are not two empty inner $3d$ orbitals available for d^2sp^3 hybridisation without pairing. The six ligand pairs instead occupy hybrids formed from one $4s$, three $4p$ and two $4d$ orbitals: sp^3d^2.

Step 3: Identify the magnetic behaviour Four unpaired electrons make the ion paramagnetic. The spin-only moment would be $\sqrt{4(4+2)} = \sqrt{24}$ Bohr magnetons, consistent with this classification. Both diamagnetic choices fail because the electrons are not all paired, and the inner-orbital paramagnetic choice assigns the wrong hybridisation for the weak-field ligand.

Step 4: Conclusion The complex is outer-orbital, sp^3d^2-hybridised and paramagnetic. Hence, the correct option is (D).

Quick Tip to Solve: Always calculate the metal oxidation state first; then use ligand strength to decide whether an octahedral complex pairs its d electrons.

66. The number of geometrical isomers found in the metal complexes $[PtCl_2(NH_3)_2]$, $[Ni(CO)_4]$, $[Ru(H_2O)_3Cl_3$ and $[CoCl_2(NH_3)_4]^+$ respectively, are : [4 marks]

(A) 1, 1, 1, 1

(B) 2, 1, 2, 2

(C) 2, 0, 2, 2

(D) 2, 1, 2, 1

Correct Answer: (C) 2, 0, 2, 2

Solution:

Concept:

- Geometrical isomers differ in the relative positions of ligands around the same metal. Square-planar MA_2B_2 and octahedral MA_4B_2 complexes have cis and trans forms.
- Octahedral MA_3B_3 complexes have facial and meridional forms. A complex with four identical ligands has no geometrical isomerism.

Step 1: Count the square-planar and tetrahedral cases $[PtCl_2(NH_3)_2]$ is square planar. Its two chloride ligands can occupy adjacent positions (cis) or opposite positions (trans), giving two geometrical isomers. $[Ni(CO)_4]$ is tetrahedral and all four ligands are identical. Every apparent rearrangement is superimposable by rotation, so it exhibits zero geometrical isomers. The count here is the number associated with geometrical isomerism, not the number of distinct formulas or ordinary molecular structures.

Step 2: Count the octahedral cases In octahedral $[Ru(H_2O)_3Cl_3]$, three identical ligands can occupy one triangular face of the octahedron (fac) or lie along a meridian containing the metal (mer). These are two geometrical isomers. In $[CoCl_2(NH_3)_4]^+$, the two chloride ligands can be adjacent, with a 90° angle, or opposite, with a 180° angle. These are the cis and trans forms, again giving two.

Step 3: Compare the ordered sequence Keeping the complexes in the exact order in the question gives $2, 0, 2, 2$. The sequence with a 1 for nickel carbonyl counts its single configuration even though it has no geometrical isomerism, contrary to the convention used in these options. The final cobalt complex certainly has both cis and trans forms, eliminating the choice ending in 1.

Step 4: Conclusion The numbers of geometrical isomers are $2, 0, 2, 2$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Classify the geometry first: square-planar A2B2 gives cis/trans, octahedral A3B3 gives fac/mer, and four identical ligands give no geometrical isomerism.

67. Presence of which reagent will affect the reversibility of the following reaction, and change it to a irreversible reaction :

$$\mathbf{CH_4 + I_2} \underset{\text{Reversible}}{\overset{h\nu}{\rightleftharpoons}} \mathbf{CH_3 - I + HI}$$ [4 marks]

(A) HOCl

(B) dilute HNO_2

(C) Liquid NH_3

(D) Concentrated HIO_3

Correct Answer: (D) Concentrated HIO_3

Solution:

Concept:

- Photochemical iodination of methane is reversible because the hydrogen iodide formed promotes the reverse reaction.
- An oxidising agent that removes HI shifts the reaction towards the organic iodide by Le Chatelier's principle.

Step 1: Identify the product responsible for reversibility The given equilibrium is

$$CH_4 + I_2 \rightleftharpoons CH_3I + HI.$$

Iodination differs from the more favourable chlorination and bromination of alkanes. The hydrogen iodide formed can take part in the reverse process, so merely irradiating methane and iodine does not drive efficient irreversible conversion. A suitable added reagent must remove the HI without simply supplying more of it.

Step 2: Use iodic acid to consume hydrogen iodide Concentrated iodic acid is an oxidising agent and oxidises iodide in HI to iodine. The balanced reaction is

$$HIO_3 + 5\,HI \longrightarrow 3\,I_2 + 3\,H_2O.$$

In oxidation-state terms, iodine in iodic acid falls from $+5$ to 0, while iodine in five iodide ions rises from -1 to 0. Thus the reagent consumes the hydrogen iodide product and regenerates iodine. Decreasing the concentration of a reaction product drives the original equilibrium towards methyl iodide.

Step 3: Select the specified reagent The standard reagent for rendering this alkane iodination effectively irreversible is concentrated HIO_3, option (D). Liquid ammonia is not this oxidising agent, and dilute nitrous acid is not the prescribed reagent for this transformation. Although hypochlorous acid has oxidising chemistry, it is not the standard selective iodic-acid treatment of this equilibrium and can introduce competing halogen chemistry.

Step 4: Conclusion Concentrated HIO_3 consumes HI, driving iodination forward. Hence, the correct option is (D).

Quick Tip to Solve: For reversible alkane iodination, look for the reagent that removes HI: iodic acid converts it into iodine and water.

68. Which one among the following chemical tests is used to distinguish monosaccharide from disaccharide? [4 marks]

(A) Seliwanoff's test

(B) Iodine test

(C) Barfoed test

(D) Tollen's test

Correct Answer: (C) Barfoed test

Solution:

Concept:

- Barfoed's reagent is copper(II) acetate in dilute acetic acid. Under controlled heating, monosaccharides reduce it more rapidly than disaccharides.
- A brick-red precipitate of Cu_2O appearing promptly in acidic medium is the characteristic positive Barfoed test for a monosaccharide.

Step 1: Choose the distinguishing property Both monosaccharides and reducing disaccharides can possess a reducing end, so a test for reducing character alone cannot distinguish the two classes. The useful difference is their rate of reduction of copper(II) ions in an acidic medium. Monosaccharides reduce Barfoed's reagent rapidly during the prescribed short heating period, producing red copper(I) oxide. Disaccharides react much more slowly; prolonged heating can cause hydrolysis and invalidate the simple distinction.

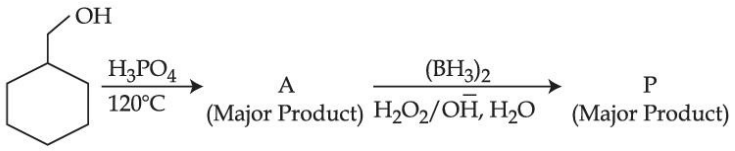
Step 2: Apply Barfoed's test correctly Use equal sample concentrations, add the acidic copper(II) acetate reagent, and heat only for the specified short interval. A rapid red Cu_2O precipitate indicates a monosaccharide. The absence of such a prompt precipitate, while the control responds, distinguishes a disaccharide under the same conditions. The controlled reaction time is essential because this is a kinetic distinction rather than a claim that no disaccharide can ever reduce copper ions.

Step 3: Eliminate the other tests Seliwanoff's test is mainly used to distinguish ketose sugars from aldoses by their different rates of dehydration and colour formation. The iodine test identifies starch through its coloured complex, rather than classifying simple sugars by the number of units. Tollens' reagent detects reducing sugars: both a monosaccharide such as glucose and a reducing disaccharide such as maltose can give a positive result. Therefore these alternatives do not supply the requested general distinction.

Step 4: Conclusion The suitable test is Barfoed's test. Hence, the correct option is (C).

Quick Tip to Solve: Link Barfoed with rapid red Cu2O in acidic solution; do not prolong heating, because disaccharide hydrolysis can spoil the distinction.

69. Consider the above reaction and identify the Product P : [4 marks]



(A) Cyclohexylmethanol

(B) 1-Methylcyclohexan-1-ol

(C) 2-(Hydroxymethyl)cyclohexan-1-ol

(D) 2-Methylcyclohexan-1-ol

Correct Answer: (D) 2-Methylcyclohexan-1-ol

Solution:

Concept:

- Acid-catalysed dehydration of an alcohol favours the more substituted alkene when rearrangement and elimination provide it.
- Hydroboration–oxidation adds water across an alkene in anti-Markovnikov orientation: the hydroxyl group ends on the less substituted alkene carbon.

Step 1: Identify the major dehydration product The starting alcohol is cyclohexylmethanol, with a $-CH_2OH$ group attached to a cyclohexane carbon. In hot phosphoric acid the hydroxyl group is protonated, enabling water loss and rearrangement. A hydride shift from the adjacent ring carbon to the side-chain carbon produces a more stable tertiary carbocation at the ring carbon; the side chain becomes $-CH_3$. Loss of a proton from an adjacent ring carbon gives the more substituted endocyclic alkene, 1-methylcyclohexene, as the major product A. Stopping at the less substituted exocyclic alkene would miss this acid-catalysed preference.

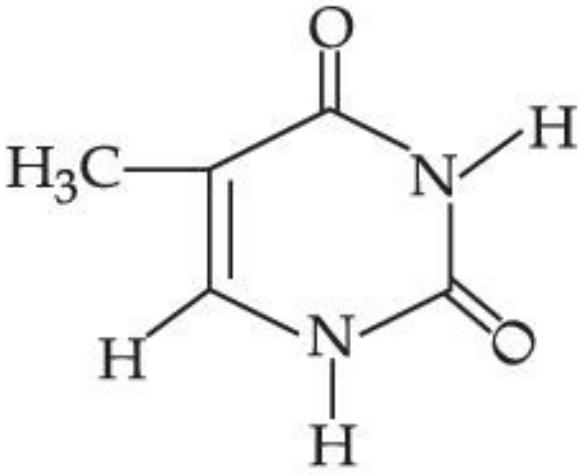
Step 2: Apply the hydroboration orientation In 1-methylcyclohexene, the alkene carbon bearing the methyl group is the more substituted carbon. Borane attaches boron predominantly to the other, less substituted alkene carbon, while hydrogen is delivered to the methyl-bearing carbon. Alkaline hydrogen peroxide replaces the carbon–boron bond by a carbon–oxygen bond with retention, giving an alcohol on the ring carbon adjacent to the methyl-bearing carbon. The constitutional product is 2-methylcyclohexan-1-ol.

Step 3: Compare the four structures Option (D) has adjacent methyl and hydroxyl groups on the ring and matches that product. Option (A) is the original primary alcohol, while (B) places the hydroxyl group on the methyl-bearing carbon, the Markovnikov position. Option (C) contains two hydroxyl groups and would require a different oxidation pattern: hydroboration–oxidation of one alkene does not create this diol.

Step 4: Conclusion Product P is 2-methylcyclohexan-1-ol, the structure with adjacent methyl and hydroxyl substituents. Hence, the correct option is (D).

Quick Tip to Solve: Determine the major alkene after acidic rearrangement first; only then place OH on its less substituted carbon during hydroboration–oxidation.

70. The compound 'A' is a complementary base of _____ in DNA stands. [4 marks]



(A)

(A) Uracil

(B) Guanine

(C) Adenine

(D) Cytosine

Correct Answer: (C) Adenine

Solution:

Concept:

- DNA has the complementary base pairs adenine–thymine and guanine–cytosine. A purine pairs with a pyrimidine.
- Thymine is 5-methyluracil: its six-membered ring has two carbonyl groups and a methyl substituent. Adenine and thymine form two hydrogen bonds.

Step 1: Identify the displayed base The structure labelled A is a single six-membered heterocyclic ring containing two nitrogen atoms. It has two carbonyl groups and a methyl group at the carbon adjacent to the upper carbonyl. These structural features identify thymine, chemically 5-methyluracil. The methyl group distinguishes it from uracil, which has hydrogen at that position. Thus the letter A printed beneath the figure is simply the label assigned to the compound; it must not be interpreted as the abbreviation for adenine.

Step 2: Apply complementary base pairing In DNA, thymine pairs specifically with adenine through two hydrogen bonds. This pairing matches hydrogen-bond donor and acceptor positions while maintaining the nearly uniform width of the double helix, since a single-ring pyrimidine pairs with a two-ring purine. Therefore the blank asks for adenine, even though the pictured compound itself is thymine.

Step 3: Check the alternative names Uracil normally replaces thymine in RNA, so it is not the complementary partner of thymine in DNA. Guanine pairs with cytosine through three hydrogen bonds; choosing either guanine or cytosine would use the other canonical base-pair system. Cytosine

also differs structurally from the displayed base because it has an amino substituent rather than the same pair of carbonyl groups. Only adenine gives the proper thymine complement.

Step 4: Conclusion The displayed compound is thymine, whose complementary DNA base is adenine. Hence, the correct option is (C).

Quick Tip to Solve: Recognise the methyl-bearing pyrimidine as thymine, then recall A–T and G–C; a figure label A does not identify the base as adenine.

71. Staggered and eclipsed conformers of ethane are : [4 marks]

(A) Polymers

(B) Rotamers

(C) Enantiomers

(D) Mirror images

Correct Answer: (B) Rotamers

Solution:

Concept:

- Conformations are spatial arrangements produced by rotation about a single sigma bond, without breaking that bond.
- Ethane has a carbon–carbon sigma bond. Rotation changes the dihedral angle between a C–H bond on one carbon and a C–H bond on the other carbon.

Step 1: Identify the change between the structures In an eclipsed ethane conformation the C–H bonds on the rear carbon lie behind those on the front carbon when viewed along the C–C axis. In a staggered conformation these sets of bonds are displaced by a dihedral angle of 60 degrees. Both arrangements retain the same two carbon atoms, six hydrogen atoms and the same bond connectivity.

Step 2: Classify the isomerism and eliminate other descriptions Rotation about the C–C single bond interconverts the eclipsed and staggered arrangements; therefore they are rotational conformers, commonly called rotamers. They are not polymers, since polymerisation would join repeating monomer units and change molecular size. They are not an enantiomeric pair: enantiomers must be non-superimposable mirror images. Merely changing the torsion angle in ethane does not create such a pair. The staggered conformation is lower in energy because its torsional strain is smaller; this energy difference does not change the classification.

Step 3: Conclusion The staggered and eclipsed conformers of ethane are rotamers. Hence, the correct option is (B).

Quick Tip to Solve: If two structures differ only by rotation about a sigma bond, classify them as conformers or rotamers before considering optical isomerism.

72. Match List - I with List - II :

	List - I		List - II
(a)	**NaOH**	**(i)**	**Acidic**
(b)	$Be(OH)_2$	**(ii)**	**Basic**
(c)	$Ca(OH)_2$	**(iii)**	**Amphoteric**
(d)	$B(OH)_3$		
(e)	$Al(OH)_3$		

Choose the most appropriate answer from the options given below [4 marks]

(A) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(ii), (e)-(iii)

(B) (a)-(ii), (b)-(iii), (c)-(ii), (d)-(i), (e)-(iii)

(C) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(i), (e)-(iii)

(D) (a)-(ii), (b)-(i), (c)-(ii), (d)-(iii), (e)-(iii)

Correct Answer: (B) (a)-(ii), (b)-(iii), (c)-(ii), (d)-(i), (e)-(iii)

Solution:

Concept:

- Metal hydroxides are generally basic, whereas small highly polarising metal ions can form amphoteric hydroxides.
- Amphoteric hydroxides react with both acids and strong bases. Boric acid, $B(OH)_3$, behaves as a Lewis acid by accepting hydroxide from water.

Step 1: Assign the basic and acidic hydroxides Sodium hydroxide and calcium hydroxide release hydroxide ions in water and neutralise acids, so both are basic: (a)-(ii) and (c)-(ii). Boron hydroxide is boric acid, not an alkali. Its aqueous action is represented by

$$B(OH)_3 + 2\,H_2O \rightleftharpoons B(OH)_4^- + H_3O^+.$$

Thus (d)-(i). The presence of OH in a formula alone does not guarantee basic character.

Step 2: Identify the amphoteric members and match the code Beryllium hydroxide and aluminium hydroxide dissolve in acids and also in excess strong alkali through complex formation. For aluminium hydroxide the two reactions are

$$Al(OH)_3 + 3\,H^+ \longrightarrow Al^{3+} + 3\,H_2O,$$

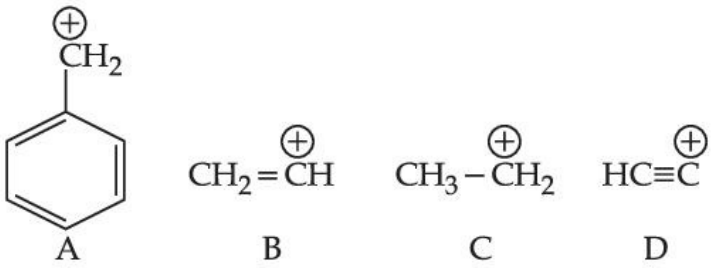
$$Al(OH)_3 + OH^- \longrightarrow [Al(OH)_4]^-.$$

Beryllium hydroxide analogously forms a hydroxo complex in alkali. Therefore (b)-(iii) and (e)-(iii). Option A wrongly calls beryllium hydroxide basic and calcium hydroxide amphoteric; option C makes the same two errors. Option D wrongly calls beryllium hydroxide acidic and boric acid amphoteric.

Step 3: Conclusion Sodium and calcium hydroxides are basic, beryllium and aluminium hydroxides are amphoteric, and boric acid is acidic. Hence, the correct option is (B).

Quick Tip to Solve: In this list, remember the pair Be and Al as amphoteric; Na and Ca hydroxides are bases, while boric acid is acidic.

73. The correct order of stability of given carbocation is : [4 marks]



(A) A > C > B > D

(B) D > B > C > A

(C) D > B > A > C

(D) C > A > D > B

Correct Answer: (A) A > C > B > D

Solution:

Concept:

- Electron donation stabilises a carbocation, while localisation of positive charge on a more electronegative carbon destabilises it.
- Resonance delocalisation is especially effective. An adjacent alkyl group can stabilise an ordinary alkyl carbocation through hyperconjugation and its positive inductive effect.

Step 1: Compare the benzylic and ethyl cations A is the benzyl cation, $C_6H_5CH_2^+$. Its empty p orbital overlaps with the aromatic pi system, allowing positive charge to be delocalised onto the ring. This resonance stabilisation makes A the most stable of the four. C is the ethyl cation, $CH_3CH_2^+$; it has no aromatic resonance, but donation from the adjacent methyl group and hyperconjugation stabilise it relative to the unsaturated cations shown.

Step 2: Compare the unsaturated cations and order all four B is a vinyl cation, with the positive charge directly on a carbon of a carbon–carbon double bond. D has the positive charge directly on an acetylenic carbon of a triple bond. Neither should be confused with an allyl or propargyl cation, where a saturated cationic centre is adjacent to the multiple bond and can gain resonance stabilisation. Increasing s character makes carbon more electronegative and less able to sustain positive charge. Thus the ethynyl cation D is less stable than vinyl cation B. Combining the comparisons gives A greater than C greater than B greater than D. Options B and C put the very unstable D first, and option D incorrectly places ethyl above benzyl.

Step 3: Conclusion The stability order is $A > C > B > D$. Hence, the correct option is (A).

Quick Tip to Solve: Check whether the positive charge is adjacent to a multiple bond or directly on it: benzylic resonance stabilisation must not be assigned to a vinylic cation.

74. The product obtained from the electrolytic oxidation of acidified sulphate solutions, is : [4 marks]

(A) HSO_4^-

(B) HO_3SOOSO_3H

(C) HO_2SOSO_2H

(D) HO_3SOSO_3H

Correct Answer: (B) HO_3SOOSO_3H

Solution:

Concept:

- Electrolytic oxidation removes electrons at the anode. Acidified sulphate or hydrogen sulphate can undergo oxidative coupling.
- Peroxodisulphuric acid has formula $H_2S_2O_8$ and contains a peroxide O–O linkage between two sulphur-containing groups.

Step 1: Write the anodic oxidation Two hydrogen sulphate ions couple at the anode with loss of two electrons:

$$2\,HSO_4^- \longrightarrow H_2S_2O_8 + 2\,e^-.$$

The equation conserves both atoms and charge: the left side has charge minus two, and the two electrons on the right supply the same charge. Equivalently, sulphate ions can be oxidised to peroxodisulphate ions. This is formation of a peroxide linkage, rather than simple acid-base conversion of sulphate.

Step 2: Identify the product among the formulas The acid product is peroxodisulphuric acid, also called Marshall's acid. Its connectivity can be written as $HO{-}S(=O)_2{-}O{-}O{-}S(=O)_2{-}OH$, corresponding to the condensed formula HO_3SOOSO_3H printed in option B. The hydrogen sulphate ion in option A is the reactant species rather than the coupled product. Option D has a single bridging oxygen rather than the required peroxide pair and corresponds to a different sulphur oxoacid framework. Option C also lacks the correct oxygen count and peroxo connectivity. Counting atoms in option B gives two hydrogen atoms, two sulphur atoms and eight oxygen atoms.

Step 3: Conclusion Electrolytic oxidation gives peroxodisulphuric acid, $H_2S_2O_8$, written as HO_3SOOSO_3H. Hence, the correct option is (B).

Quick Tip to Solve: For the electrolytic oxidation of acidified sulphate, look for a product with two sulphur atoms and an O–O peroxide linkage.

75. The parameters of the unit cell of a substance are a = 2.5, b = 3.0, c = 4.0, α = 90°, β = 120°, γ = 90°. The crystal system of the substance is : [4 marks]

(A) Hexagonal

(B) Orthorhombic

(C) Monoclinic

(D) Triclinic

Correct Answer: (C) Monoclinic

Solution:

Concept:

- A crystal system is identified by both edge lengths a, b, c and interaxial angles α, β, γ.
- The monoclinic system has three unequal edge lengths and two right angles; the remaining angle is not a right angle.

Step 1: Compare the supplied edge lengths and angles The lengths are $a = 2.5$, $b = 3.0$, and $c = 4.0$, so all three edges are unequal. The angle data give $\alpha = \gamma = 90°$, while $\beta = 120° \neq 90°$. The numerical value 120 degrees must be associated with the specified angle beta; it cannot be treated as gamma simply to resemble a different crystal system. Two angles are right angles and exactly one is oblique.

Step 2: Apply the defining conditions and eliminate alternatives These conditions identify the monoclinic system:

$$a \neq b \neq c, \qquad \alpha = \gamma = 90°, \quad \beta \neq 90°.$$

Orthorhombic crystals also have three unequal edges, but require all three angles to be 90 degrees, which fails here. Hexagonal crystals require two equal basal edges and the characteristic 120-degree basal angle, so the unequal values of a and b already exclude them. Triclinic crystals have no right-angle constraints, with all three angles oblique in the standard classification; the two right angles supplied here instead meet the monoclinic conditions. Thus edge data and angle data give the same unique classification.

Step 3: Conclusion The substance belongs to the monoclinic crystal system. Hence, the correct option is (C).

Quick Tip to Solve: Use both lengths and angles: three unequal edges with exactly two right angles is the characteristic monoclinic pattern.

76. Which one of the following statements is NOT correct? [4 marks]

(A) Eutrophication indicates that water body is polluted

(B) The dissolved oxygen concentration below 6 ppm inhibits fish growth

(C) Eutrophication leads to increase in the oxygen level in water

(D) Eutrophication leads to anaerobic conditions

Correct Answer: (C) Eutrophication leads to increase in the oxygen level in water

Solution:

Concept:

- Eutrophication is nutrient enrichment of a water body, particularly by nitrates and phosphates, causing excessive algal growth.
- Microbial decomposition of dead organic matter consumes dissolved oxygen; low dissolved oxygen harms aquatic organisms and can produce anaerobic conditions.

Step 1: Trace the oxygen demand after nutrient enrichment Nutrients entering a lake promote an algal bloom. As algae die, aerobic microorganisms decompose the accumulated organic material and use dissolved oxygen. The biochemical oxygen demand rises, while the oxygen available to fish and other aerobic aquatic life falls. Although living algae can photosynthesise in daylight, this does not reverse the overall oxygen depletion associated with severe eutrophication and the decay of the bloom.

Step 2: Select the incorrect statement and check the others Statement C claims eutrophication increases the oxygen level in water. This contradicts the characteristic oxygen depletion caused by decomposition, so it is the statement requested by NOT correct. Statement A is consistent with nutrient pollution of the water body. Statement B expresses the standard environmental-chemistry observation that dissolved oxygen below about 6 ppm inhibits fish growth. Statement D is also consistent: when aerobic oxygen consumption becomes sufficiently extensive, anaerobic conditions develop and anaerobic decomposition can follow. The question asks for an incorrect statement, so these three valid descriptions must be excluded even though they describe harmful effects.

Step 3: Conclusion Eutrophication generally depletes dissolved oxygen rather than increasing it. Hence, the correct option is (C).

Quick Tip to Solve: For eutrophication questions, follow the sequence nutrient input, algal bloom, decomposition, oxygen depletion; then check whether the question asks for the incorrect statement.

77. Given below are two statements :

Statement I : Rutherford's gold foil experiment cannot explain the line spectrum of hydrogen atom.

Statement II : Bohr's model of hydrogen atom contradicts Heisenberg' uncertainty principle.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Statement I is false but statement II is true.

(B) Statement I is true but statement II is false.

(C) Both statement I and statement II are false.

(D) Both statement I and statement II are true.

Correct Answer: (D) Both statement I and statement II are true.

Solution:

Concept:

- A line spectrum requires discrete permitted atomic energies; emitted photon energies obey $h\nu = E_i - E_f$.
- Heisenberg's uncertainty principle requires $\Delta x\,\Delta p_x \geq \hbar/2$. An electron cannot simultaneously have an exact position and an exact momentum component.

Step 1: Evaluate Statement I Rutherford's nuclear model establishes a small positively charged nucleus and electrons outside it. It does not introduce quantised stationary electronic energy levels. Classical orbital motion therefore gives neither a stable set of atomic states nor a mechanism for the discrete frequencies observed in hydrogen. It cannot explain hydrogen's line spectrum. Statement I is consequently true; the gold-foil scattering result itself concerns nuclear structure, not quantisation of electronic energies.

Step 2: Evaluate Statement II and choose the paired verdict Bohr assigns the electron a definite circular orbit with a specified radius and orbital speed. Such a trajectory presupposes simultaneously well-defined position along the path and momentum, which is incompatible with the uncertainty principle. A modern orbital is a probability distribution, not a definite planetary path. Hence the classical-orbit aspect of Bohr's model contradicts Heisenberg's principle, even though Bohr's quantised energy formula successfully accounts for the hydrogen spectrum. Statement II is true. Option A wrongly rejects Statement I; option B wrongly rejects Statement II; option C rejects both true statements. The question asks only for their truth values, not whether one explains the other.

Step 3: Conclusion Both Statement I and Statement II are true. Hence, the correct option is (D).

Quick Tip to Solve: Separate a model's successful energy predictions from its physical picture: Bohr explains hydrogen lines but its definite electron orbit conflicts with uncertainty.

78. Match List - I with List - II :

	List-I(Drug)		List-II(Class of Drug)
(a)	Furacin	(i)	Antibiotic
(b)	Arsphenamine	(ii)	Tranquilizers
(c)	Dimetone	(iii)	Antiseptic
(d)	Valium	(iv)	Synthetic antihistamines

Choose the most appropriate match : [4 marks]

(A) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(B) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(C) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(D) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Correct Answer: (D) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Solution:

Concept:

- Drugs are classified by their therapeutic action. An antiseptic suppresses microbial growth on living tissues, while an antihistamine counteracts the effects of histamine.
- Tranquillizers act on the central nervous system to relieve anxiety. The drug names must be matched individually rather than by the order of their appearance in the lists.

Step 1: Match the antimicrobial drugs Furacin, also called nitrofurazone, is used as an antiseptic, so (a) matches (iii). Arsphenamine is the historically important antimicrobial drug used to treat syphilis; in the classification offered here it belongs to antibiotics, so (b) matches (i). These two matches already distinguish the required row from the alternatives. In particular, option (A) interchanges the assignments of Furacin and arsphenamine.

Step 2: Match the remaining therapeutic classes Dimetone is a synthetic antihistamine, so (c) matches (iv). Valium is the trade name of diazepam, a tranquillizer, so (d) matches (ii). It is not an antibiotic. Thus option (B), which assigns Valium to antibiotics and Dimetone to tranquillizers, is rejected. Option (C) also assigns Furacin to tranquillizers and Dimetone to antiseptics, so it cannot be correct. All four independently identified pairings occur together only in option (D).

Step 3: Conclusion Furacin is an antiseptic, arsphenamine an antibiotic, Dimetone an antihistamine, and Valium a tranquillizer. Hence, the correct option is (D).

Quick Tip to Solve: Start with the familiar Valium–tranquillizer and Furacin–antiseptic pairs; then check both remaining entries before choosing a matching code.

79. The statement is incorrect about Ellingham diagram is [4 marks]

(A) provides idea about the reaction rate.

(B) provides idea about free energy change.

(C) provides idea about changes in the phases during the reaction.

(D) provides idea about reduction of metal oxide.

Correct Answer: (A) provides idea about the reaction rate.

Solution:

Concept:

- An Ellingham diagram plots the standard Gibbs energy change ΔG° for oxidation against temperature. It is a thermodynamic comparison of competing oxidation and reduction processes.
- The relation $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ explains the approximate straight-line segments and their changes at phase transitions. Reaction rates instead depend on kinetic factors such as activation energy.

Step 1: Identify the information actually shown The vertical coordinate gives the free-energy change for oxide formation at each temperature, so statement (B) is a valid use of the diagram. Changes of phase affect entropy and may change a line's slope; therefore statement (C) is also valid. Comparing oxide-formation lines shows whether oxidation of a proposed reducing agent can make the combined reduction thermodynamically favourable. This is the basis of statement (D).

Step 2: Separate feasibility from speed A negative Gibbs energy change indicates thermodynamic favourability under the relevant conditions; it does not specify how quickly the reaction proceeds. An otherwise favourable reduction may be slow because of a large activation barrier, poor contact between solids, or transport limitations. An Ellingham diagram does not supply activation energies or rate constants. Thus the assertion in (A) that it gives the reaction rate is the incorrect statement requested, whereas the other three describe thermodynamic or phase information.

Step 3: Conclusion An Ellingham diagram does not provide the reaction rate. Hence, the correct option is (A).

Quick Tip to Solve: For an Ellingham question, distinguish whether reduction is feasible from how fast it happens: the diagram answers the thermodynamic question.

80. Given below are two statements : One is labelled as Assertion A and the other labelled as Reason R.

Assertion A : Lithium halides are some what covalent in nature.

Reason R : Lithium posses high polarisation capability.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) A is true but R is false

(B) A is false but R is true

(C) Both A and R are true but R is NOT the correct explanation of A

(D) Both A and R are true and R is the correct explanation of A

Correct Answer: (D) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- According to Fajans' rules, a small cation with high charge density has strong polarising power: it distorts the electron cloud of a neighbouring anion.
- Polarisation shifts electron density towards the region between the ions, increasing covalent character. Lithium is the smallest alkali-metal cation, so its charge density is especially high.

Step 1: Evaluate the assertion Lithium halides have appreciable covalent character compared with the corresponding halides of the larger alkali metals. The statement says somewhat covalent, not wholly covalent, so it is consistent with the coexistence of ionic and covalent contributions to their bonding. The magnitude also varies with the halide: a larger, more polarisable anion generally develops greater covalent character. Hence Assertion A is true.

Step 2: Evaluate the reason The polarising species in the salt is Li^+. Its small ionic radius concentrates its positive charge, producing a strong electric field that distorts the halide electron cloud. Thus the statement that lithium possesses high polarisation capability, understood as the polarising power of its cation in its salts, is true.

Step 3: Check the explanatory connection The distortion caused by Li^+ is precisely the mechanism responsible for increased covalent character in lithium halides. Therefore R explains A directly. Options (A) and (B) incorrectly call one of the true statements false, while (C) incorrectly denies this causal connection.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (D).

Quick Tip to Solve: Apply Fajans' rules to the ions: small cation means high polarising power, whereas large anion means high polarisability.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The density of NaOH solution is 1.2 g cm^{-3}. The molality of this solution is _____ m. (Round off to the Nearest Integer)

[Use : Atomic mass : Na : 23.0 u, O : 16.0 u, H : 1.0 u, Density of H_2O : 1.0 g cm^{-3}] [4 marks]

Solution:

Concept:

- Molality is the amount of dissolved solute per kilogram of solvent: $m = n_{\text{solute}}/w_{\text{solvent,kg}}$. It requires composition, not just total solution mass per volume.
- Density is $\rho = w_{\text{solution}}/V_{\text{solution}}$. The molar mass of NaOH is $23 + 16 + 1 = 40\,\text{g mol}^{-1}$.

Step 1: Choose a basis and identify the missing quantity Take $1.000\,\text{L}$ of solution. Its mass is $1.2 \times 1000 = 1200\,\text{g}$. Let the mass of dissolved NaOH be $x\,\text{g}$; the water mass is then $(1200 - x)\,\text{g}$. The required molality would be

$$m = \frac{x/40}{(1200 - x)/1000} = \frac{25x}{1200 - x}\,\text{mol kg}^{-1}.$$

No value of x, mass percentage, molarity or equivalent composition datum is supplied.

Step 2: Check why the water density does not close the calculation The water density converts a specified water mass to its volume, but that water mass is itself unknown. The volume of a solution need not equal the volume of the water used: dissolution can change the volume. Thus subtracting the mass of one litre of pure water from the mass of one litre of solution does not establish the solute mass in either sample. Setting $x = 200\,\text{g}$ in the expression would yield $5\,\text{mol kg}^{-1}$, but the statement provides no basis for this particular composition.

Step 3: Conclusion The molality cannot be uniquely determined from the stated data. The supplied answer key gives 5, but that value requires an additional composition or volume assumption that is absent from the question.

Quick Tip to Solve: Before converting density into molality, locate a composition datum such as mass percentage or molarity; density alone gives only total mass per solution volume.

82. The conductivity of a weak acid HA of concentration 0.001 mol L^{-1} is 2.0 $\times$ 10^{-5} S cm^{-1}. If Λ_m^o(HA) = 190 S cm^2 mol^{-1}, the ionization constant (K_a) of HA is equal to _____ $\times$ 10^{-6}. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- The molar conductivity is $\Lambda_m = 1000\kappa/c$ when conductivity κ is in S cm^{-1} and concentration c in mol L^{-1}.
- For a weak monoprotic acid, $\alpha = \Lambda_m/\Lambda_m^\circ$. With initial acid concentration c, Ostwald's dilution law

gives $K_a = c\alpha^2/(1-\alpha)$.

Step 1: Calculate the degree of ionisation Use the factor 1000 to convert litres to cubic centimetres consistently with the conductivity units. The measured molar conductivity is

$$\Lambda_m = \frac{1000(2.0 \times 10^{-5})}{0.001} = 20\,\mathrm{S\,cm^2\,mol^{-1}}.$$

Dividing by the limiting value gives

$$\alpha = \frac{20}{190} = \frac{2}{19} = 0.105263.$$

This means about 10.5 percent of the acid molecules ionise at the stated concentration.

Step 2: Apply the equilibrium expression without premature approximation For $\mathrm{HA} \rightleftharpoons \mathrm{H^+ + A^-}$, the equilibrium concentrations are $c(1-\alpha)$, $c\alpha$, and $c\alpha$. Hence

$$K_a = \frac{10^{-3}(2/19)^2}{1-2/19} = \frac{4 \times 10^{-3}}{323} = 1.23839 \times 10^{-5}.$$

Expressing this in the requested form gives 12.3839×10^{-6}. Retaining $1-\alpha$ is necessary here: approximating it as unity would alter the nearest-integer coefficient.

Step 3: Conclusion $K_a = 12.3839 \times 10^{-6}$, so the required nearest-integer value is $\boxed{12}$.

Quick Tip to Solve: When ionisation is around ten percent, keep the denominator $1-\alpha$; the weak-acid approximation may change the rounded numerical answer.

83. 1.46 g of a biopolymer dissolved in a 100 mL water at 300 K exerted an osmotic pressure of 2.42 $\times$ 10^{-3} bar.

The molar mass of the biopolymer is _____ $\times$ 10^4 g mol^{-1}. (Round off to the Nearest Integer)

[Use : R = 0.083 L bar mol^{-1} K^{-1}] [4 marks]

Solution:

Concept:

- For a dilute solution of a non-electrolyte, osmotic pressure obeys $\pi V = nRT$. A biopolymer molecule counts as one solute particle in this relation.
- Using $n = w/M$, the molar mass is $M = wRT/(\pi V)$. The volume unit must match the litre in the given gas constant.

Step 1: Convert the volume and select the equation The solute mass is $w = 1.46\,\mathrm{g}$, the temperature is $T = 300\,\mathrm{K}$, and $100\,\mathrm{mL} = 0.100\,\mathrm{L}$. As usual for this dilute polymer solution, use the stated liquid volume as the solution volume in the osmotic-pressure calculation. The pressure is already in bar, consistent with $R = 0.083\,\mathrm{L\,bar\,mol^{-1}\,K^{-1}}$. No pressure-unit conversion or additional particle factor is required.

Step 2: Evaluate the molar mass and the requested coefficient Substitute into the rearranged

osmotic-pressure equation:

$$\begin{aligned} M &= \frac{1.46 \times 0.083 \times 300}{(2.42 \times 10^{-3})(0.100)} \\ &= \frac{36.354}{2.42 \times 10^{-4}} \\ &= 1.50223 \times 10^{5}\,\mathrm{g\,mol^{-1}}. \end{aligned}$$

In units of $10^4\,\mathrm{g\,mol^{-1}}$, the coefficient is 15.0223. Round this coefficient, rather than the molar mass in grams, because the blank already includes the power of ten. The large value is consistent with a macromolecular solute exerting a very small osmotic pressure.

Step 3: Conclusion The molar mass is approximately $15 \times 10^4\,\mathrm{g\,mol^{-1}}$. The required nearest-integer value is $\boxed{15}$.

Quick Tip to Solve: Write the requested power of ten beside the final molar mass before rounding; here the blank asks for the coefficient of 10^4, not the mass itself.

84. An organic compound is subjected to chlorination to get compound A using 5.0 g of chlorine. When 0.5 g of compound A is reacted with $AgNO_3$ [Carius Method], the percentage of chlorine in compound A is _____ when it forms 0.3849 g of AgCl. (Round off to the Nearest Integer) (Atomic masses of Ag and Cl are 107.87 and 35.5 respectively) [4 marks]

Solution:

Concept:

- In the Carius method, chlorine in the analysed organic sample is recovered as AgCl. Each mole of AgCl contains one mole of chlorine atoms.
- The chlorine mass is the precipitate mass multiplied by $M_{\mathrm{Cl}}/M_{\mathrm{AgCl}}$. Percentage composition refers to the mass of the compound actually analysed.

Step 1: Find the chlorine represented by the precipitate The molar mass of silver chloride is

$$M_{\mathrm{AgCl}} = 107.87 + 35.5 = 143.37\,\mathrm{g\,mol^{-1}}.$$

Therefore the mass of chlorine in $0.3849\,\mathrm{g}$ of silver chloride is

$$w_{\mathrm{Cl}} = 0.3849 \times \frac{35.5}{143.37} = 0.0953055\,\mathrm{g}.$$

The silver contributes most of the precipitate mass, so the whole precipitate mass must not be identified as chlorine.

Step 2: Divide by the analysed compound mass The sample of compound A used for estimation weighs $0.500\,\mathrm{g}$. Thus

$$\%\mathrm{Cl} = \frac{0.0953055}{0.500} \times 100 = 19.0611\%.$$

The $5.0\,\mathrm{g}$ of chlorine mentioned in the preparation is not the denominator: it concerns the chlorination stage, not the mass of A subjected to analysis. The precipitate directly measures the chlorine present in

the analysed portion, without requiring the original compound's structure, conversion, or preparation yield. Rounding is done only after this mass fraction has been calculated.

Step 3: Conclusion Compound A contains approximately 19% chlorine. The required nearest-integer value is $\boxed{19}$.

Quick Tip to Solve: In a Carius calculation, first remove the silver contribution using the molar-mass ratio, then divide by the mass of the sample analysed.

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85. The number of geometrical isomers possible in triamminetrinitrocobalt (III) is X and in trioxalatochromate (III) is Y. Then the value of X + Y is _____.

[4 marks]

Solution:

Concept:

- An octahedral complex of the type $[MA_3B_3]$ has facial and meridional arrangements of its two sets of three monodentate ligands.
- Geometrical isomerism concerns different relative positions of ligands. Optical enantiomers are a different kind of stereoisomerism and must not be counted as geometrical isomers.

Step 1: Identify the first coordination sphere Triamminetrinitrocobalt(III) has coordination sphere $[Co(NH_3)_3(NO_2)_3]$. The three ammine ligands are equivalent to one another, as are the three nitro ligands. Its six donor atoms occupy an octahedral arrangement; hence it has the pattern $[MA_3B_3]$. The ligand name nitro is fixed here, so linkage alternatives are not part of the requested geometrical count.

Step 2: Count the two arrangements In the facial (fac) form, the three identical ligands occupy the corners of one triangular face; each pair in that set is cis. In the meridional (mer) form they lie along a meridian, and one pair is trans. These exhaust the distinct relative placements, giving $X = 2$.

Step 3: Examine the tris-chelate complex Trioxalatochromate(III), $[Cr(C_2O_4)_3]^{3-}$, contains three identical bidentate oxalato ligands. Each ligand occupies two adjacent sites. The resulting tris-chelate has optical forms, conventionally called Δ and Λ, but no distinct geometrical isomers. Thus the number contributing to the geometrical-isomer count is $Y = 0$.

Step 4: Conclusion Therefore $X + Y = 2 + 0 = \boxed{2}$. The optical pair of the oxalato complex does not add to this geometrical count.

Quick Tip to Solve: For $[MA_3B_3]$, recall fac/mer; for three identical bidentate ligands, separate optical isomerism from geometrical isomerism.

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86. In gaseous triethyl amine the "−C−N−C−" bond angle is _____ degree. [4 marks]

Solution:

Concept:

- The nitrogen in a trialkylamine forms three sigma bonds and retains one lone pair. Its local geometry is trigonal pyramidal, with an approximately tetrahedral electron-pair arrangement.
- A lone pair occupies more angular space than a bonding pair, so the bond angle near nitrogen differs from the ideal tetrahedral value $109.5°$. Actual angles depend on the attached groups and physical state.

Step 1: Identify the atoms defining the angle Triethylamine is $N(C_2H_5)_3$. The requested C–N–C angle is measured at nitrogen between the nitrogen-to-carbon bonds of two ethyl groups. It is not the C–C–N angle within an ethyl group, nor is it an angle at a carbon atom. Nitrogen is bonded to three carbon atoms and has one nonbonding electron pair.

Step 2: Apply the molecular geometry The three bonds and lone pair give four electron domains about nitrogen. The molecular shape is pyramidal rather than trigonal planar: nitrogen is not an electron-deficient centre with only three electron domains. Consequently a planar $120°$ answer is inappropriate. The lone pair perturbs the tetrahedral bond directions, while the ethyl groups also influence the precise angle.

Step 3: Use the gaseous trialkylamine angle For gaseous triethylamine, the C–N–C bond angle used for this question is approximately $108°$. This is an empirical molecular-geometry value, not an exact number derivable by subtracting a universal fixed correction from $109.5°$. The specified gaseous state matters because intermolecular interactions in other environments can alter structural parameters.

Step 4: Conclusion The requested gaseous C–N–C bond angle is $\boxed{108°}$; therefore the numerical entry is $\boxed{108}$.

Quick Tip to Solve: First locate the central atom of the named bond angle; in a tertiary amine, count the nitrogen lone pair as well as its three sigma bonds.

87. For water at 100° C and 1 bar,

Δ_{vap} H − Δ_{vap} U = _____ × 10^2 J mol^{-1}. (Round off to the Nearest Integer)

[Use : R = 8.31 J mol^{-1} K^{-1}]

[Assume volume of $H_2O(l)$ is much smaller than volume of $H_2O(g)$. Assume $H_2O(g)$ treated as an ideal gas] [4 marks]

Solution:

Concept:

- Enthalpy is defined by $H = U + pV$; at the same pressure, the difference between enthalpy and internal-energy changes is $\Delta H - \Delta U = p\Delta V$.
- For one mole of ideal vapour, $pV_g = RT$. Neglecting the liquid volume makes the vaporisation difference approximately RT.

Step 1: Relate the two vaporisation quantities Compare one mole of liquid water and one mole of water vapour at the stated temperature and pressure. The defining relation gives

$$\Delta_{\text{vap}}H - \Delta_{\text{vap}}U = p(V_{\text{g}} - V_{\text{l}}).$$

The liquid volume is explicitly assumed much smaller than the gas volume, so its contribution may be neglected. Vaporisation increases the volume, making this difference positive.

Step 2: Substitute the absolute temperature The gas law requires absolute temperature. Thus $T = 100 + 273 = 373\,\text{K}$, and the ideal-gas assumption gives

$$p(V_{\text{g}} - V_{\text{l}}) \simeq pV_{\text{g}} = RT = (8.31)(373) = 3099.63\,\text{J mol}^{-1}.$$

There is no need to insert the pressure separately because the ideal-gas relation has already replaced the product pV_{g}.

Step 3: Convert to the requested coefficient The blank multiplies $10^2\,\text{J mol}^{-1}$, so divide the calculated energy by 100:

$$\frac{3099.63}{10^2} = 30.9963.$$

Rounding this coefficient, rather than the energy in joules, to the nearest integer gives 31. Using $373.15\,\text{K}$ also gives the same rounded entry.

Step 4: Conclusion $\Delta_{\text{vap}}H - \Delta_{\text{vap}}U \simeq 31 \times 10^2\,\text{J mol}^{-1}$. The required entry is $\boxed{31}$.

Quick Tip to Solve: For liquid-to-ideal-gas vaporisation with negligible liquid volume, use $\Delta H - \Delta U = RT$ per mole, then convert to the blank's power of ten.

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88. **$PCl_5 \rightleftharpoons PCl_3 + Cl_2$**

$K_c = 1.844$

3.0 moles of PCl_5 is introduced in a 1 L closed reaction vessel at 380 K. The number of moles of PCl_5 at equilibrium is _____ $\times 10^{-3}$. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- For $PCl_5 \rightleftharpoons PCl_3 + Cl_2$, the concentration equilibrium constant is $K_c = [PCl_3][Cl_2]/[PCl_5]$.
- If x moles of initially pure PCl_5 dissociate, the equilibrium mole amounts are $3-x, x, x$, respectively. Concentrations equal these numerical values in the stated 1 L vessel.

Step 1: Set up the equilibrium equation Initially the vessel contains 3.0 mol of PCl_5 and neither product. Dissociating x mol produces x mol each of PCl_3 and Cl_2. Therefore

$$1.844 = \frac{x^2}{3-x}, \qquad 0 < x < 3.$$

Both product factors are required; replacing the numerator by x would incorrectly omit one species.

Step 2: Solve without premature rounding Rearranging gives

$$x^2 + 1.844x - 5.532 = 0.$$

The physical root is

$$x = \frac{-1.844 + \sqrt{1.844^2 + 4(5.532)}}{2} = 1.604278686.$$

The other root is negative and cannot represent the amount dissociated. Hence the remaining amount is

$$n(\mathrm{PCl_5}) = 3 - x = 1.395721314\,\mathrm{mol}.$$

Substitution in the equilibrium quotient reproduces 1.844.

Step 3: Round the coefficient actually requested Writing the remaining amount as $a \times 10^{-3}\,\mathrm{mol}$ gives

$$a = \frac{1.395721314}{10^{-3}} = 1395.721314.$$

The nearest integer is 1396. Rounding the mole amount first to 1.4 would instead produce 1400, but that loses precision before the explicitly requested final rounding.

Step 4: Conclusion For the printed $K_c = 1.844$, the required coefficient is $\boxed{1396}$, corresponding to $1396 \times 10^{-3}\,\mathrm{mol}$. The supplied key lists 1400; the literal equilibrium calculation gives 1396, so the key value is not the nearest-integer result for the printed data.

Quick Tip to Solve: Keep extra digits in the quadratic root and convert to the blank's scale before rounding; rounding 1.3957 mol to 1.4 mol is too early.

89. The difference between bond orders of CO and $NO^{\oplus}$ is $\frac{x}{2}$ where x = _____. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- Molecular-orbital bond order is $\mathrm{BO} = (N_\mathrm{b} - N_\mathrm{a})/2$, where N_b and N_a are the numbers of bonding and antibonding electrons.
- Isoelectronic diatomic species with the same relevant orbital occupancy have the same bond order. A positive charge removes an electron from the neutral species; it does not add one.

Step 1: Count the electrons in both species Carbon contributes 6 electrons and oxygen contributes 8, so neutral CO has $6 + 8 = 14$ electrons. For $\mathrm{NO^+}$, nitrogen contributes 7 and oxygen contributes 8, followed by subtraction of one electron for the positive charge:

$$N(\mathrm{NO^+}) = 7 + 8 - 1 = 14.$$

Thus CO and $\mathrm{NO^+}$ are isoelectronic. Counting neutral NO instead would give 15 electrons and answer a different question.

Step 2: Determine the bond orders Each species has ten valence electrons. Their closed-shell valence

occupancy contains eight bonding electrons and two antibonding electrons; the lower core bonding and antibonding orbitals cancel in the bond-order difference. Therefore

$$\mathrm{BO(CO)} = \frac{8-2}{2} = 3, \qquad \mathrm{BO(NO^+)} = \frac{8-2}{2} = 3.$$

This is also consistent with the familiar triple-bond descriptions of these isoelectronic species.

Step 3: Match the stated difference to x The difference is zero whichever order of subtraction is used, because both bond orders are equal:

$$\mathrm{BO(CO)} - \mathrm{BO(NO^+)} = 3 - 3 = 0 = \frac{x}{2}.$$

Multiplying by 2 gives $x = 0$. Since the value is already an integer, the requested rounding has no effect.

Step 4: Conclusion The bond-order difference is zero, so $\boxed{x = 0}$.

Quick Tip to Solve: Account for charge before comparing diatomic electron counts: CO and NO^+ both have 14 electrons and bond order 3.

90. CO_2 gas adsorbs on charcoal following Freundlich adsorption isotherm. For a given amount of charcoal, the mass of CO_2 adsorbed becomes 64 times when the pressure of CO_2 is doubled. The value of n in the Freundlich isotherm equation is _____ $\times 10^{-2}$. (Round off to the Nearest Integer) [4 marks]

Solution:

Concept:

- For a fixed mass m of adsorbent, the Freundlich relation is $x/m = kP^{1/n}$, where x is the adsorbed mass and k, n are constants at the given temperature.
- Taking ratios at two pressures cancels the constant k and fixed adsorbent mass: $x_2/x_1 = (P_2/P_1)^{1/n}$. The pressure exponent is $1/n$, not n.

Step 1: Form the ratio for the two states The quantity of charcoal remains fixed, so the reported 64-fold increase in adsorbed carbon dioxide is also a 64-fold increase in x/m. The pressure is doubled. Substituting these ratios in the isotherm gives

$$64 = \frac{x_2}{x_1} = \left(\frac{P_2}{P_1}\right)^{1/n} = 2^{1/n}.$$

No absolute pressure or charcoal mass is necessary because both cancel in the ratio.

Step 2: Solve for the Freundlich parameter Express the mass ratio as a power with the same base: $64 = 2^6$. Comparing exponents in $2^6 = 2^{1/n}$ gives

$$\frac{1}{n} = 6, \qquad n = \frac{1}{6} = 0.166666\ldots.$$

The result follows from the numerical relation supplied in the question; inserting an additional assumed range for the exponent would contradict those given ratios. It is essential to invert 6 to obtain n.

Step 3: Convert to the requested numerical scale The question writes the answer as a coefficient times 10^{-2}. If that coefficient is a, then

$$a \times 10^{-2} = \frac{1}{6} \quad \Longrightarrow \quad a = \frac{100}{6} = 16.666666\ldots.$$

Round only at this final stage. The nearest integer is 17; rounding n itself to an integer before scaling would discard the required information.

Step 4: Conclusion The parameter is $n = 1/6$, so its requested coefficient in units of 10^{-2} is $\boxed{17}$.

Quick Tip to Solve: Use adsorption ratios to cancel the constants, solve for the exponent $1/n$, then invert and apply the power of ten specified beside the blank.