JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 25 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 July 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The total number of functions,

$f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4, 5, 6\}$ **such that** $f(1) + f(2) = f(3)$**, is equal to :** [4 marks]

(A) 60

(B) 90

(C) 108

(D) 126

Correct Answer: (B) 90

Solution:

Concept:

- A function assigns exactly one image to each domain element; different inputs may have the same image unless injectivity is required.
- The relation $f(3) = f(1) + f(2)$ fixes the third image once the first two are chosen. The fourth image remains independent.

Step 1: Count the ordered first two images Write $a = f(1)$ and $b = f(2)$. Both are positive integers and their sum must lie in the codomain, so $2 \le a + b \le 6$. For a fixed sum s, the ordered pairs are $(1, s-1), (2, s-2), \ldots, (s-1, 1)$, giving $s - 1$ choices. Hence the number of pairs is

$$1 + 2 + 3 + 4 + 5 = 15.$$

The pairs are ordered because the inputs 1 and 2 are distinct.

Step 2: Choose the remaining free image For each of these 15 pairs, $f(3)$ has exactly one allowed value, namely their sum. Independently, $f(4)$ can be any of the six codomain elements. Thus the product rule gives

$$N = 15 \times 1 \times 6 = 90.$$

There is no requirement that all images be distinct or that every codomain element occur. The other totals 60, 108 and 126 do not equal this exhaustive count.

Step 3: Conclusion The total number of functions is 90. Hence, the correct option is (B).

Quick Tip to Solve: Count the constrained images first, then multiply by the choices for inputs absent from the relation.

2. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to : [4 marks]

(A) -4

(B) -1

(C) 1

(D) 4

Correct Answer: (B) -1

Solution:

Concept:

- The geometric-series identity gives $(x-1)(x^4 + x^3 + x^2 + x + 1) = x^5 - 1$. Thus every root of the quartic is a fifth root of unity other than 1.
- If $r^5 = 1$, integer powers of r may be reduced modulo 5. The sum of the roots of a monic polynomial equals the negative coefficient of the next-highest power.

Step 1: Reduce the large exponent Let r be any of $\alpha, \beta, \gamma, \delta$. Multiplying its equation by $r - 1$ gives $r^5 - 1 = 0$. Also 1 is not a root of the quartic, since its value there is 5. As

$$2021 = 5 \cdot 404 + 1, \qquad r^{2021} = (r^5)^{404} r = r,$$

the requested expression is simply the sum of the four roots. This reduction applies to each root separately, irrespective of its complex argument.

Step 2: Apply the sum of roots By Vieta's formula for $x^4 + x^3 + x^2 + x + 1$,

$$\alpha + \beta + \gamma + \delta = -1.$$

Therefore their 2021st powers also sum to -1. The value 4 would result from incorrectly reducing the exponent to 0 modulo 5; the actual remainder is 1. The alternatives -4 and 1 also disagree with the exact root sum.

Step 3: Conclusion The required sum is -1. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the exponent modulo the roots-of-unity order before computing any individual complex root.

3. For $n \in \mathbf{N}$, let

$$S_n = \left\{ z \in \mathbf{C} : |z - 3 + 2i| = \frac{n}{4} \right\}$$

and

$$T_n = \left\{ z \in \mathbf{C} : |z - 2 + 3i| = \frac{1}{n} \right\}.$$

Then the number of elements in the set

$$\{ n \in \mathbf{N} : S_n \cap T_n = \phi \}$$

is: [4 marks]

(A) 0

(B) 2

(C) 3

(D) 4

Correct Answer: The displayed empty-intersection condition holds for infinitely many positive integers, so none of the finite options is correct.

Solution:

Concept:

- The locus $|z - z_0| = r$ is the circumference of a circle with centre z_0 and radius r, not its interior.
- Two circles with centre distance d intersect exactly when $|r_1 - r_2| \le d \le r_1 + r_2$, including tangency.

Step 1: Identify the centres and test small integers The centres are $3 - 2i$ and $2 - 3i$, separated by $d = |1 + i| = \sqrt{2}$. Their radii are $r_1 = n/4$ and $r_2 = 1/n$. For $n = 1, 2, 3, 4$, the sums of radii are respectively $5/4, 1, 13/12, 5/4$, all smaller than $\sqrt{2}$. Thus these four values give empty intersection. For $n = 5$, $|r_1 - r_2| = 21/20 < \sqrt{2} < 29/20 = r_1 + r_2$; for $n = 6$, the corresponding bounds are $4/3 < \sqrt{2} < 5/3$. Both 5 and 6 therefore give nonempty intersection.

Step 2: Check all larger integers For $n \ge 7$,

$$r_1 - r_2 = \frac{n}{4} - \frac{1}{n} \ge \frac{7}{4} - \frac{1}{7} = \frac{45}{28} > \sqrt{2}.$$

The inequality follows because $n/4 - 1/n$ increases with positive n; the last comparison can be checked by squaring positive quantities. One circumference then lies strictly inside the other without meeting it. Consequently every integer $n \ge 7$ also satisfies the displayed empty-intersection condition. This produces infinitely many qualifying integers, so none of the four finite choices can describe that set.

Step 3: Conclusion For the printed condition $S_n \cap T_n = \varnothing$, the set of qualifying positive integers is infinite; no listed option is correct. The value 2 counts the nonempty intersections, occurring only at $n = 5, 6$, and therefore does not answer the displayed equality. The recorded key value 4 counts the first four qualifying integers but omits every integer from 7 onwards, so it cannot be correct for the printed condition.

Quick Tip to Solve: Check large radii before counting integer cases: strict containment also makes two circumferences disjoint.

4. The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

$$3(\sin 3\theta)x - y + z = 2$$

$$3(\cos 2\theta)x + 4y + 3z = 3$$

$$6x + 7y + 7z = 9$$

has no solution, is : [4 marks]

(A) 6

(B) 7

(C) 8

(D) 9

Correct Answer: (B) 7

Solution:

Concept:

- An inconsistent system arises when elimination produces $0 = c$ with $c \neq 0$. If elimination determines every variable, the system has a unique solution.
- The identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ converts the resulting trigonometric equation into a polynomial in $\sin\theta$.

Step 1: Eliminate the last two variables Subtract the first equation and twice the second equation from the third. The coefficients of y and z cancel, yielding

$$\big(6 - 3\sin 3\theta - 6\cos 2\theta\big)x = 9 - 2 - 6 = 1.$$

The coefficient matrix in y, z of the first two equations has determinant $(-1)(3) - (1)(4) = -7 \neq 0$. Thus a nonzero coefficient of x gives a unique solution, whereas a zero coefficient gives the contradiction $0 = 1$. No other inconsistency is possible.

Step 2: Solve the inconsistency condition and count The required condition is $\sin 3\theta + 2\cos 2\theta = 2$. Put $s = \sin\theta$ and use $\cos 2\theta = 1 - 2s^2$. Then

$$3s - 4s^3 - 4s^2 = 0 \quad \Longleftrightarrow \quad s(2s - 1)(2s + 3) = 0.$$

Since $-1 \leq s \leq 1$, the possibilities are $s = 0$ and $s = 1/2$. In $(0, 4\pi)$, the first gives $\pi, 2\pi, 3\pi$, and the second gives $\pi/6, 5\pi/6, 13\pi/6, 17\pi/6$. The endpoints are excluded, and these sets do not overlap. Hence the total is $3 + 4 = 7$, ruling out 6, 8 and 9.

Step 3: Conclusion The system has no solution for exactly 7 values of θ. Hence, the correct option is (B).

Quick Tip to Solve: Look for a row combination cancelling two variables before expanding a full determinant.

5. If $\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$**, then** $8(\alpha+\beta)$ **is equal to :** [4 marks]

(A) 4

(B) -8

(C) -4

(D) 8

Correct Answer: (C) -4

Solution:

Concept:

- A finite limit requires the coefficient of the unbounded linear term in n to vanish.
- Rationalising $\sqrt{n^2-n-1}-n$ gives its finite limiting value without an infinite-series approximation.

Step 1: Determine the coefficient of the growing term Let $L_n=\sqrt{n^2-n-1}+n\alpha+\beta$. Since $L_n\to 0$, also $L_n/n\to 0$. Divide by $n>0$:

$$\frac{L_n}{n}=\sqrt{1-\frac{1}{n}-\frac{1}{n^2}}+\alpha+\frac{\beta}{n}\longrightarrow 1+\alpha.$$

Therefore $\alpha=-1$. Any other value leaves a nonzero multiple of n, preventing the stated finite limit.

Step 2: Rationalise and determine the constant With $\alpha=-1$, rationalisation gives

$$\sqrt{n^2-n-1}-n=\frac{-n-1}{\sqrt{n^2-n-1}+n}=\frac{-1-1/n}{\sqrt{1-1/n-1/n^2}+1}\longrightarrow -\frac{1}{2}.$$

Thus the original limit is $-1/2+\beta=0$, so $\beta=1/2$. Finally

$$8(\alpha+\beta)=8\left(-1+\frac{1}{2}\right)=-4.$$

A check is that the leading positive radical cancels $-n$, and the remaining negative half cancels the positive β; changing either sign would violate the zero limit. Hence 4, -8 and 8 are excluded.

Step 3: Conclusion The required value is -4. Hence, the correct option is (C).

Quick Tip to Solve: Cancel the leading growth first; rationalise only the remaining radical difference to find its constant limit.

6. If the absolute maximum value of the function $f(x) = \left(x^2 - 2x + 7\right) \mathrm{e}^{\left(4x^3 - 12x^2 - 180x + 31\right)}$ in the interval $[-3, 0]$ is $f(\alpha)$, then : [4 marks]

(A) $\alpha = 0$

(B) $\alpha = -3$

(C) $\alpha \in (-1, 0)$

(D) $\alpha \in (-3, -1]$

Correct Answer: (B) $\alpha = -3$

Solution:

Concept:

- A continuous function on a closed interval attains an absolute maximum. A strictly decreasing function attains it at the left endpoint.
- For $f(x) = p(x)e^{g(x)}$, the product rule gives $f\prime(x) = e^{g(x)}[p\prime(x) + p(x)g\prime(x)]$; the exponential factor is always positive.

Step 1: Differentiate and factor the exponent derivative Here $p(x) = x^2 - 2x + 7 = (x-1)^2 + 6 > 0$ and $g(x) = 4x^3 - 12x^2 - 180x + 31$. Therefore

$$g\prime(x) = 12x^2 - 24x - 180 = 12(x-5)(x+3),$$

and

$$f\prime(x) = e^{g(x)}\left[2x - 2 + 12p(x)(x-5)(x+3)\right].$$

Factoring the quadratic in the exponent derivative avoids solving the much higher-degree equation obtained by expanding this entire bracket.

Step 2: Determine the sign throughout the interval For every $x \in [-3, 0]$, we have $2x - 2 < 0$, $p(x) > 0$, $x - 5 < 0$, and $x + 3 \geq 0$. Thus the second term in the bracket is nonpositive, while the first is strictly negative, giving $f\prime(x) < 0$ everywhere on the interval. In particular the derivative remains negative even at $x = -3$, where the product term vanishes. Hence f is strictly decreasing and its largest value is $f(-3) = 22e^{355}$. Neither the right endpoint nor any point in either listed interior interval can be the maximiser.

Step 3: Conclusion The absolute maximum occurs at $\alpha = -3$. Hence, the correct option is (B).

Quick Tip to Solve: For a positive polynomial times an exponential, inspect the signs of the two derivative contributions before finding roots.

7. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point $\mathrm{P}(-2, 0)$ and cuts the y-axis at the point Q, where y' is equal to 3 . Then the local maximum value of $y(x)$ is: [4 marks]

(A) $\frac{27}{4}$

(B) $\frac{29}{4}$

(C) $\frac{37}{4}$

(D) $\frac{9}{2}$

Correct Answer: (A) $\frac{27}{4}$

Solution:

Concept:

- Touching the x-axis at $x = -2$ gives both $y(-2) = 0$ and $y\prime(-2) = 0$. At the y-axis the input is $x = 0$.
- Critical points solve $y\prime(x) = 0$. A negative second derivative identifies a strict local maximum, whose value is found by substitution into y, not into $y\prime$.

Step 1: Use the slope and tangency conditions The derivative is $y\prime(x) = 3ax^2 + 2bx + c$, so the slope 3 at the y-axis gives $c = 3$. Now $y(-2) = 0$ gives $-8a + 4b = 1$, and $y\prime(-2) = 0$ gives $12a - 4b = -3$. Adding yields $4a = -2$, so $a = -1/2$, and then $b = -3/4$. Thus

$$y(x) = -\frac{x^3}{2} - \frac{3x^2}{4} + 3x + 5.$$

These coefficients satisfy both the zero value and horizontal tangent at the prescribed point.

Step 2: Locate and evaluate the maximum Differentiating and factoring,

$$y\prime(x) = -\frac{3}{2}(x + 2)(x - 1), \qquad y\prime\prime(x) = -3x - \frac{3}{2}.$$

The critical points are -2 and 1. Since $y\prime\prime(-2) = 9/2 > 0$, the tangency point is a local minimum. Since $y\prime\prime(1) = -9/2 < 0$, the local maximum is attained at $x = 1$, with value

$$y(1) = -\frac{1}{2} - \frac{3}{4} + 3 + 5 = \frac{27}{4}.$$

The alternatives $29/4, 37/4, 9/2$ do not equal this value; $9/2$ is the magnitude of the second derivative here, not the height of the curve.

Step 3: Conclusion The local maximum value of the curve is $27/4$. Hence, the correct option is (A).

Quick Tip to Solve: A tangency to the axis provides two equations: zero height and zero slope; use both before differentiating again.

8. The area of the region given by

$A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}\}$ **is :** [4 marks]

(A) $\frac{31}{8}$

(B) $\frac{17}{6}$

(C) $\frac{19}{6}$

(D) $\frac{27}{8}$

Correct Answer: (B) $\frac{17}{6}$

Solution:

Concept:

- For a vertically described region, area is the integral of upper boundary minus lower boundary. Both inequalities defining the minimum must hold simultaneously.
- The upper boundary changes where the two straight lines meet; the parabola is the lower boundary throughout the allowed interval.

Step 1: Find the permissible interval The condition $x^2 \le \min\{x + 2, 4 - 3x\}$ means both $x^2 \le x + 2$ and $x^2 \le 4 - 3x$. Factoring separately gives

$$(x - 2)(x + 1) \le 0, \qquad (x + 4)(x - 1) \le 0.$$

Thus the common interval is $[-1, 1]$, rather than the whole interval between the intersections with either single line. Outside this intersection at least one proposed upper boundary lies below the parabola, so no point belongs to the region.

Step 2: Split at the crossing of the lines The two lines meet when $x + 2 = 4 - 3x$, giving $x = 1/2$. For $-1 \le x \le 1/2$, the smaller line is $x + 2$; for $1/2 \le x \le 1$, it is $4 - 3x$. Therefore

$$\begin{aligned}\mathcal{A} &= \int_{-1}^{1/2} (x + 2 - x^2)\, dx + \int_{1/2}^{1} (4 - 3x - x^2)\, dx \\ &= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{1/2} + \left[4x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{1/2}^{1} \\ &= \frac{9}{4} + \frac{7}{12} = \frac{17}{6}.\end{aligned}$$

The other listed fractions do not equal this sum; using a single upper line throughout would count points excluded by the minimum.

Step 3: Conclusion The area is $\boxed{17/6}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For a minimum of two upper curves, first intersect their allowed intervals with the lower curve, then split where the upper curves cross.

9. For any real number x, let $[x]$ denote the largest integer less than equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by $f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$. Then the value of $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x \, dx$ is : [4 marks]

(A) 4

(B) 2

(C) 1

(D) 0

Correct Answer: (A) 4

Solution:

Concept:

- On each interval $[n, n+1)$, write $x = n + t$ with $0 \le t < 1$, so $[x] = n$. Then the definition becomes a simple linear function of t.
- The identity $\cos \pi(n+t) = (-1)^n \cos \pi t$ handles the parity of every integer, including negative ones. Values at isolated integer endpoints do not affect the integral.

Step 1: Evaluate one unit interval of each parity If n is even, $f(n+t) = 1 - t$; if n is odd, $f(n+t) = t$. Consequently, the contributions to the integral of $f(x) \cos \pi x$ are

$$I_e = \int_0^1 (1-t) \cos \pi t \, dt, \qquad I_o = -\int_0^1 t \cos \pi t \, dt.$$

Integration by parts gives

$$\int_0^1 t \cos \pi t \, dt = \left[\frac{t \sin \pi t}{\pi} + \frac{\cos \pi t}{\pi^2} \right]_0^1 = -\frac{2}{\pi^2}.$$

Also $\int_0^1 \cos \pi t \, dt = 0$, so both I_e and I_o equal $2/\pi^2$. The sign from cosine on an odd interval must be retained.

Step 2: Sum all twenty contributions There are twenty unit intervals from -10 to 10, ten with even left endpoint and ten with odd left endpoint. Since their contributions agree,

$$\int_{-10}^{10} f(x) \cos \pi x \, dx = 20 \cdot \frac{2}{\pi^2} = \frac{40}{\pi^2}.$$

Multiplying by the factor requested in the question yields

$$\frac{\pi^2}{10} \cdot \frac{40}{\pi^2} = 4.$$

Neither halving the interval nor cancelling the two parities is justified: the parity change in f compensates for the cosine sign. Thus 2, 1 and 0 are excluded.

Step 3: Conclusion The required value is $\boxed{4}$. Hence, the correct option is (A).

Quick Tip to Solve: Substitute $x = n + t$ before integrating a floor-function expression: the parity and the fractional part then separate immediately.

10. The slope of the tangent to a curve $C : y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to : [4 marks]

(A) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$

(B) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

(C) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

(D) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

Correct Answer: (B) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

Solution:

Concept:

- The slope of the tangent is dy/dx. Integrating it determines the curve up to an additive constant, which is fixed by a given point.
- For the remaining rational expression in exponentials, use $t = e^x > 0$. The antiderivative $\int dt/(2t^2+9) = (3\sqrt{2})^{-1}\tan^{-1}(\sqrt{2}t/3)$ uses the principal inverse tangent.

Step 1: Integrate the specified slope The numerator can be separated because

$$2e^{2x} - 6e^{-x} + 9 = e^{2x}(2 + 9e^{-2x}) - 6e^{-x}.$$

Hence

$$\frac{dy}{dx} = e^{2x} - \frac{6e^{-x}}{2 + 9e^{-2x}}.$$

With $t = e^x$, the second integral becomes $-6\int dt/(2t^2+9)$. Therefore the curve has the equation

$$y = \frac{1}{2}e^{2x} - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}e^x}{3}\right) + K.$$

The coefficient $\sqrt{2}$ follows from integrating the quadratic denominator; losing it would change the second point condition.

Step 2: Use both points and solve for the exponential Put $\theta = \tan^{-1}(\sqrt{2}/3)$. At $x = 0$, the given ordinate gives $K = \pi/(2\sqrt{2}) + \sqrt{2}\theta$. At the second point, the $e^{2\alpha}/2$ terms cancel, leaving

$$\tan^{-1}\left(\frac{\sqrt{2}e^{\alpha}}{3}\right) = \frac{\pi}{4} + \theta.$$

Since $0 < \theta < \pi/4$, this angle lies in the range of the principal inverse tangent. Taking tangents,

$$\frac{\sqrt{2}e^{\alpha}}{3} = \frac{1 + \sqrt{2}/3}{1 - \sqrt{2}/3} = \frac{3+\sqrt{2}}{3-\sqrt{2}}.$$

Thus e^{α} includes the additional factor $3/\sqrt{2}$; option A omits it, and options C and D use a different tangent ratio and fail the derived condition.

Step 3: Conclusion We obtain $e^{\alpha} = \boxed{\frac{3}{\sqrt{2}}\frac{3+\sqrt{2}}{3-\sqrt{2}}}$. Hence, the correct option is (B).

Quick Tip to Solve: Before integrating a complicated quotient, subtract its obvious exponential term; the remainder often becomes a standard inverse-tangent integral.

11. The general solution of the differential equation $\left(x-y^2\right)\mathrm{d}x + y\left(5x+y^2\right)\mathrm{d}y = 0$ is : [4 marks]

(A) $\left(y^2+x\right)^4 = \mathrm{C}\left|\left(y^2+2x\right)^3\right|$

(B) $\left(y^2+2x\right)^4 = C\left|\left(y^2+x\right)^3\right|$

(C) $\left|\left(y^2+x\right)^3\right| = \mathrm{C}\left(2y^2+x\right)^4$

(D) $\left|\left(y^2+2x\right)^3\right| = C\left(2y^2+x\right)^4$

Correct Answer: (A) $\left(y^2+x\right)^4 = \mathrm{C}\left|\left(y^2+2x\right)^3\right|$

Solution:

Concept:

- The occurrence of $y\,dy$ together with y^2 suggests $z = y^2$, so $dz = 2y\,dy$. This converts the equation into a homogeneous equation in x, z.
- Use $z = vx$ on a region where $x \neq 0$, separate the variables, and integrate the resulting rational function by partial fractions. Absolute values are needed in logarithms.

Step 1: Reduce to a separable equation After substituting $z = y^2$ and multiplying by two, the equation is

$$2(x-z)\,dx + (5x+z)\,dz = 0.$$

Put $z = vx$, giving $dz = v\,dx + x\,dv$. Collecting terms yields

$$\begin{aligned}[2-2v+5v+v^2]\,dx + x(5+v)\,dv &= 0,\\ (v+1)(v+2)\,dx + x(v+5)\,dv &= 0.\end{aligned}$$

Thus, away from the factors divided out,

$$\frac{dx}{x} + \left(\frac{4}{v+1} - \frac{3}{v+2}\right)dv = 0.$$

Step 2: Integrate and restore the original variables Integration gives

$$\ln|x| + 4\ln|v+1| - 3\ln|v+2| = K.$$

Use $v = z/x$; the powers of $|x|$ cancel. Exponentiating and replacing z by y^2 gives

$$\left(y^2+x\right)^4 = C\left|\left(y^2+2x\right)^3\right|.$$

A direct differentiation of $4\ln|y^2+x| - 3\ln|y^2+2x|$ has numerator proportional to the original differential expression, confirming the family. Reversing the exponents as in B does not give that expression; C and D introduce $2y^2+x$, a different factor. The divided-out curve $y^2+2x=0$ also solves the original equation separately; $y^2+x=0$ is included by $C=0$. These special curves do not change which listed general family is correct.

Step 3: Conclusion The listed general solution is $\boxed{(y^2+x)^4 = C|(y^2+2x)^3|}$. Hence, the correct option is (A).

Quick Tip to Solve: When $y\,dy$ appears beside y^2, try $z=y^2$, and check any factor lost while separating for an additional special solution.

12. A line, with the slope greater than one, passes through the point $A(4,3)$ and intersects the line $x-y-2=0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line : [4 marks]

(A) $2x+y=9$

(B) $3x-2y=7$

(C) $x+2y=6$

(D) $2x-3y=3$

Correct Answer: (C) $x+2y=6$

Solution:

Concept:

- A point on $x-y-2=0$ can be parameterised as $B=(t,t-2)$. Substituting this in the distance formula finds all candidates for the intersection point.
- The distance condition may give two points. The independent slope restriction must be applied before testing which proposed line contains the required point.

Step 1: Find the two points at the specified distance Using $A=(4,3)$ and $AB=\sqrt{29}/3$,

$$(t-4)^2+(t-5)^2=\frac{29}{9}.$$

Multiplying out and simplifying gives

$$9t^2-81t+170=0, \qquad (3t-10)(3t-17)=0.$$

Hence the two candidates are $B_1=(10/3,4/3)$ and $B_2=(17/3,11/3)$. Both satisfy the distance and the original line equation, so distance alone cannot select the point.

Step 2: Apply the slope restriction and test the options The slopes of the corresponding lines through A are

$$m_1=\frac{4/3-3}{10/3-4}=\frac{5}{2}, \qquad m_2=\frac{11/3-3}{17/3-4}=\frac{2}{5}.$$

Only $m_1 > 1$, so $B = (10/3, 4/3)$. At this point the left sides of the four proposed equations are

$$2x + y = 8 \neq 9, \quad 3x - 2y = \frac{22}{3} \neq 7,$$
$$x + 2y = 6, \qquad 2x - 3y = \frac{8}{3} \neq 3.$$

Thus the selected point lies on the third proposed line and on none of the other three. The positive slope condition is essential: using the other root would contradict the stated geometry even though its distance from A is correct.

Step 3: Conclusion $B = (10/3, 4/3)$ lies on $\boxed{x + 2y = 6}$. Hence, the correct option is (C).

Quick Tip to Solve: When a line intersects a circle in two candidate points, use the slope or direction condition before evaluating the answer options.

13. Let the locus of the centre $(\alpha, \beta), \beta > 0$, of the circle which touches the circle $x^2 + (y-1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is:

[4 marks]

(A) $\frac{32\sqrt{2}}{3}$
(B) $\frac{40\sqrt{2}}{3}$
(C) $\frac{64}{3}$
(D) $\frac{32}{3}$

Correct Answer: (C) $\frac{64}{3}$

Solution:

Concept:

- A circle centred at (α, β), with $\beta > 0$, tangent to the x-axis has radius β. External tangency means the distance between centres equals the sum of radii.
- Once the locus is found, its intersections with $y = 4$ give the limits. The enclosed area is the integral of the horizontal line minus the lower parabolic boundary.

Step 1: Derive the locus from external tangency The fixed circle $x^2 + (y-1)^2 = 1$ has centre $(0, 1)$ and radius 1. The moving circle has centre (α, β) and radius β. Therefore external tangency requires

$$\sqrt{\alpha^2 + (\beta - 1)^2} = \beta + 1.$$

Both sides are nonnegative, so squaring introduces no sign ambiguity here:

$$\alpha^2 + \beta^2 - 2\beta + 1 = \beta^2 + 2\beta + 1, \qquad \alpha^2 = 4\beta.$$

Thus the locus is the upward parabola $y = x^2/4$, with its vertex excluded by $\beta > 0$. Removing one boundary point does not alter the area enclosed with the horizontal line.

Step 2: Integrate between the intersections At $y = 4$, the parabola gives $x^2 = 16$, hence the endpoints

have abscissae -4 and 4. Throughout this interval the line is above the parabola. By symmetry,

$$\begin{aligned}\mathcal{A} &= \int_{-4}^{4}\left(4-\frac{x^2}{4}\right)dx \\ &= 2\left[4x-\frac{x^3}{12}\right]_0^4 \\ &= 2\left(16-\frac{16}{3}\right)=\frac{64}{3}.\end{aligned}$$

The width is 8 and the maximum height is 4, consistent with an area smaller than 32. The value $32/3$ counts only half the symmetric region; the two expressions containing $\sqrt{2}$ do not follow from the actual intersections and are unequal to the integral.

Step 3: Conclusion The bounded area is $\boxed{64/3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: For a circle touching the x-axis, replace its radius by the positive y-coordinate of its centre before using the distance-between-centres condition.

14. Let ABC be a triangle such that $\overrightarrow{BC}=\vec{a},\overrightarrow{CA}=\vec{b},\overrightarrow{AB}=\vec{c},|\vec{a}|=6\sqrt{2},|\vec{b}|=2\sqrt{3}$ and $\vec{b}\cdot\vec{c}=12$. Consider the statements :

$$(S1):|(\vec{a}\times\vec{b})+(\vec{c}\times\vec{b})|-|\vec{c}|=6(2\sqrt{2}-1)$$

$$(S2):\angle ACB=\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

Then [4 marks]

(A) both (S1) and (S2) are true

(B) only (S1) is true

(C) only (S2) is true

(D) both (S1) and (S2) are false

Correct Answer: (C) only (S2) is true

Solution:

Concept:

- The three directed sides around a triangle add to zero: $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. This relation simplifies both cross products and the squared lengths.
- The angle at C is the angle between $\overrightarrow{CA}=\vec{b}$ and $\overrightarrow{CB}=-\vec{a}$, so its cosine is $-\vec{a}\cdot\vec{b}/(|\vec{a}||\vec{b}|)$.

Step 1: Evaluate the first statement From $\vec{a}=-(\vec{b}+\vec{c})$, taking squared lengths gives

$$72=12+|\vec{c}|^2+2(12),\qquad |\vec{c}|=6.$$

The two cross products in S1 have the same second vector. Combining them before taking the magnitude,

$$(\vec{a}\times\vec{b})+(\vec{c}\times\vec{b})=(\vec{a}+\vec{c})\times\vec{b}=(-\vec{b})\times\vec{b}=\vec{0}.$$

Hence the left side of S1 is $0-6=-6$, whereas its asserted right side $6(2\sqrt{2}-1)$ is positive. Statement S1 is false. Taking the magnitudes of the two cross products separately before adding would be incorrect.

Step 2: Evaluate the angle in the second statement Again using $\vec{a}=-\vec{b}-\vec{c}$,

$$\vec{a}\cdot\vec{b}=-|\vec{b}|^2-\vec{c}\cdot\vec{b}=-12-12=-24.$$

Therefore the actual angle between CA and CB satisfies

$$\cos\angle ACB=\frac{-\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{24}{(6\sqrt{2})(2\sqrt{3})}=\sqrt{\frac{2}{3}}.$$

The triangle angle is in $[0,\pi]$, so it equals the principal inverse cosine appearing in S2. Statement S2 is true. Options A and B wrongly affirm S1, and option D wrongly denies S2. The reversal from BC to CB is necessary to obtain the positive cosine.

Step 3: Conclusion Only S2 is true. Hence, the correct option is (C).

Quick Tip to Solve: Add directed side vectors before evaluating cross products, and reverse any vector whose direction does not start at the vertex of the angle being measured.

15. If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2+\alpha x+\beta>0$, for all $x\in\mathbf{R}$, is : [4 marks]

(A) $\frac{17}{36}$
(B) $\frac{4}{9}$
(C) $\frac{1}{2}$
(D) $\frac{19}{36}$

Correct Answer: (A) $\frac{17}{36}$

Solution:

Concept:

- A monic real quadratic is strictly positive for every real input exactly when its discriminant is negative. Here the leading coefficient is already positive.
- Two independent throws give $6\times6=36$ equally likely ordered pairs (α,β). The first and second throws have different roles in the quadratic.

Step 1: Convert positivity into a counting condition Completing the square makes the strict condition explicit:

$$x^2+\alpha x+\beta=\left(x+\frac{\alpha}{2}\right)^2+\beta-\frac{\alpha^2}{4}.$$

Its minimum over all real x is $\beta - \alpha^2/4$, so we require $4\beta > \alpha^2$. Equality is excluded because then the polynomial has a zero, contrary to the strict inequality in the question.

Step 2: Count the permissible second throws For each possible first throw, list only the die values that satisfy the strict inequality:

α	Allowed values of β	Count
1	$1, 2, 3, 4, 5, 6$	6
2	$2, 3, 4, 5, 6$	5
3	$3, 4, 5, 6$	4
4	$5, 6$	2
5	None	0
6	None	0

In particular, $(2, 1)$ and $(4, 4)$ have zero discriminant and cannot be included.

Step 3: Divide by the size of the sample space The favourable count is $6 + 5 + 4 + 2 = 17$. Thus

$$P = \frac{17}{36}.$$

The alternatives $4/9 = 16/36$, $1/2 = 18/36$, and $19/36$ would require 16, 18, and 19 favourable ordered outcomes respectively, so each conflicts with the explicit count.

Step 4: Conclusion The required probability is $\boxed{17/36}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the minimum of the quadratic or a strict negative discriminant, and count ordered die outcomes; do not include a double real root.

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16. Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point $(2, -5, 11)$, then d^2 is equal to : [4 marks]

(A) $\frac{147}{2}$

(B) 96

(C) $\frac{32}{3}$

(D) 54

Correct Answer: (C) $\frac{32}{3}$

Solution:

Concept:

- A plane containing a line has a normal vector perpendicular to the direction vector of that line. Perpendicular planes have perpendicular normal vectors.
- The distance from (x_0, y_0, z_0) to $ax + by + cz + k = 0$ is $|ax_0 + by_0 + cz_0 + k|/\sqrt{a^2 + b^2 + c^2}$.

Step 1: Find the normal of the second plane The two lines through the origin have direction vectors

$\mathbf{u} = (2, 3, 5)$ and $\mathbf{v} = (3, 7, 8)$. They are not parallel, so they determine a plane whose normal is

$$\mathbf{m} = \mathbf{u} \times \mathbf{v} = (24 - 35, 15 - 16, 14 - 9) = (-11, -1, 5).$$

Only a normal is needed; the equation of this second plane need not be found.

Step 2: Construct the required plane The given contained line has direction $\mathbf{a} = (9, -1, -5)$ and passes through $(3, -4, 7)$. The required normal must be perpendicular to both $\mathbf{a}$ and $\mathbf{m}$, so take

$$\mathbf{a} \times \mathbf{m} = (-10, 10, -20), \qquad \mathbf{n} = (1, -1, 2).$$

Indeed, $\mathbf{n} \cdot \mathbf{a} = 9 + 1 - 10 = 0$ and $\mathbf{n} \cdot \mathbf{m} = -11 + 1 + 10 = 0$. Consequently

$$P: \ (x - 3) - (y + 4) + 2(z - 7) = 0, \quad x - y + 2z - 21 = 0.$$

Step 3: Evaluate the squared distance At $(2, -5, 11)$, the plane expression is $2 + 5 + 22 - 21 = 8$. Hence

$$d^2 = \frac{8^2}{1^2 + (-1)^2 + 2^2} = \frac{64}{6} = \frac{32}{3}.$$

Options (A), (B), and (D), namely $147/2$, 96, and 54, do not equal this distance squared; the denominator must be the squared norm of the plane normal.

Step 4: Conclusion The squared distance is $\boxed{32/3}$. Hence, the correct option is (C).

Quick Tip to Solve: Find the second plane normal first, then cross it with the contained line direction; square the distance formula to avoid unnecessary radicals.

17. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is : [4 marks]

(A) $\frac{33}{2^{32}}$
(B) $\frac{33}{2^{29}}$
(C) $\frac{33}{2^{28}}$
(D) $\frac{33}{2^{27}}$

Correct Answer: (C) $\frac{33}{2^{28}}$

Solution:

Concept:

- For a binomial random variable X with parameters n, p, its mean is $\mu = np$ and variance is $v = np(1 - p) = \mu(1 - p)$.
- The probability of exactly r successes is $\binom{n}{r} p^r (1 - p)^{n-r}$. The events of one and two successes are disjoint, so their probabilities add.

Step 1: Identify the mean and variance The given sum and product imply

$$\mu + v = 24, \qquad \mu v = 128.$$

Thus the two numbers are roots of

$$t^2 - 24t + 128 = (t - 16)(t - 8) = 0.$$

For a nondegenerate binomial distribution, $0 < v < \mu$, since $0 < 1 - p < 1$. Therefore the mean is 16 and the variance is 8; interchanging them would produce an impossible negative success probability.

Step 2: Recover the binomial parameters Use the ratio of variance to mean:

$$1 - p = \frac{v}{\mu} = \frac{8}{16} = \frac{1}{2}, \qquad p = \frac{1}{2}.$$

Since $np = 16$, it follows that $n = 32$. These values check both data: the mean is 16, the variance is 8, their sum is 24, and their product is 128.

Step 3: Add the two required probabilities For a fair success probability, every specified pattern of 32 trials has probability 2^{-32}. There are $\binom{32}{1} = 32$ patterns with one success and $\binom{32}{2} = 496$ with two. Hence

$$P(X = 1 \text{ or } 2) = \frac{32 + 496}{2^{32}} = \frac{528}{2^{32}} = \frac{33}{2^{28}}.$$

The common factor 16 in 528 reduces the exponent by four. Options (A), (B), and (D) retain the wrong power of two after this reduction.

Step 4: Conclusion The required probability is $\boxed{33/2^{28}}$. Hence, the correct option is (C).

Quick Tip to Solve: When mean and variance are supplied symmetrically, identify the larger as the mean before solving for the binomial parameters.

18. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is : [4 marks]

(A) 4

(B) 6

(C) 8

(D) 12

Correct Answer: (C) 8

Solution:

Concept:

- An absolute value is nonnegative, so $|\cos x| = \sin x$ requires $\sin x \ge 0$. Squaring alone loses this necessary sign restriction.
- On each interval of length 2π, the sine is nonnegative in the upper half-cycle, and $\sin^2 x = \cos^2 x$ has two roots there.

Step 1: Solve the equation with the correct sign Squaring gives

$$\cos^2 x = \sin^2 x, \qquad \cos 2x = 0.$$

The squared equation has candidates $x = \pi/4 + k\pi/2$, but the original equation rejects any candidate with negative sine. In the basic period $[0, 2\pi)$, the valid roots are $\pi/4$ and $3\pi/4$. The candidates $5\pi/4$ and $7\pi/4$ have negative sine and fail the original equation.

Step 2: Place both families inside the given interval The full valid families are

$$x = \frac{\pi}{4} + 2k\pi \quad \text{or} \quad x = \frac{3\pi}{4} + 2k\pi, \qquad k \in \mathbb{Z}.$$

For the first family, $-4 \le 1/4 + 2k \le 4$, giving $k = -2, -1, 0, 1$. For the second, $-4 \le 3/4 + 2k \le 4$, giving the same four integer values. The families are disjoint because their offsets differ by $\pi/2$, not a multiple of 2π.

Step 3: Check endpoints and count There are four roots in each family, so the total is $4 + 4 = 8$. Neither endpoint $x = \pm 4\pi$ is a root: there $|\cos x| = 1$ while $\sin x = 0$. The interval spans four complete periods, each with two roots. Thus 4 and 6 undercount, while 12 includes extraneous or miscounted roots; only 8 agrees with both the family count and the periodicity check.

Step 4: Conclusion There are $\boxed{8}$ solutions in the stated closed interval. Hence, the correct option is (C).

Quick Tip to Solve: After squaring an equation involving an absolute value, restore the sign condition from the original equation before counting periodic roots.

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19. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that $QR = 15$ m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is : [4 marks]

(A) $5(2\sqrt{3} + 3)$ m

(B) $5(\sqrt{3} + 3)$ m

(C) $10(\sqrt{3} + 1)$ m

(D) $10(2\sqrt{3} + 1)$ m

Correct Answer: (A) $5(2\sqrt{3} + 3)$ m

Solution:

Concept:

- For an angle of elevation θ, a vertical height and its horizontal distance satisfy $\tan\theta =$ height/distance.
- The angle subtended by a vertical segment above eye level is the difference of the elevations of its upper and lower endpoints. Here all elevations are measured from the same horizontal ground line.

Step 1: Find the horizontal distance to the tower Let $AQ = d$ and let the full tower height be $PQ = h$. Since $QR = 15\,\text{m}$ and $\angle RAQ = 60^\circ$, the right triangle AQR gives

$$\tan 60^\circ = \frac{15}{d}, \qquad d = \frac{15}{\sqrt{3}} = 5\sqrt{3}\,\text{m}.$$

The point R lies between the base and top, so the top has a larger elevation than R.

Step 2: Find the elevation of the top The segment PR subtends 15° at A, meaning $\angle PAR = 15^\circ$. It is not the elevation of the top. Thus

$$\angle PAQ = 60^\circ + 15^\circ = 75^\circ.$$

The tangent addition formula gives

$$\tan 75^\circ = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2 + \sqrt{3}.$$

Step 3: Calculate and compare the full heights Apply the tangent ratio in triangle AQP:

$$h = d\tan 75^\circ = 5\sqrt{3}(2+\sqrt{3}) = 5(2\sqrt{3}+3)\,\text{m}.$$

It is approximately $32.32\,\text{m}$, above the known 15-m point as required. Options (B), (C), and (D) give approximately 23.66 m, 27.32 m, and 44.64 m respectively; with $d = 5\sqrt{3}\,\text{m}$, none has the required tangent $2+\sqrt{3}$ for the top elevation.

Step 4: Conclusion The tower height is $\boxed{5(2\sqrt{3}+3)\,\text{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish the angle subtended by the upper segment from the top elevation; add it to the lower point elevation before applying the tangent ratio.

20. Which of the following statements is a tautology ? [4 marks]

(A) $((\sim \text{p}) \vee \text{q}) \Rightarrow \text{p}$

(B) $p \Rightarrow ((\sim p) \vee q)$

(C) $((\sim p) \vee q) \Rightarrow q$

(D) $q \Rightarrow ((\sim p) \vee q)$

Correct Answer: (D) $q \Rightarrow ((\sim p) \vee q)$

Solution:

Concept:

- A tautology is true for every assignment of truth values to its variables. A single false assignment disproves that a statement is a tautology.
- An implication $r \Rightarrow s$ is equivalent to $(\sim r) \vee s$. It is false only when its antecedent is true and its consequent is false.

Step 1: Prove the fourth statement is always true For option (D), use the implication identity:

$$q \Rightarrow ((\sim p) \vee q) \equiv (\sim q) \vee (\sim p) \vee q.$$

The disjunction contains both q and $\sim q$, whose disjunction is always true. Therefore this expression is true independently of p. Equivalently, when q is true, the consequent contains the true disjunct q; when q is false, the implication is automatically true.

Step 2: Give counterexamples for the other statements Option (A) fails at $p = \mathrm{F}, q = \mathrm{F}$: its antecedent $(\sim p) \vee q$ is true but its consequent p is false. Option (B) fails at $p = \mathrm{T}, q = \mathrm{F}$: its antecedent is true and $(\sim p) \vee q$ is false. Option (C) fails at $p = \mathrm{F}, q = \mathrm{F}$: its antecedent is true and consequent q is false. These explicit assignments rule out all three alternatives.

Step 3: Check the complete logical distinction The truth of a statement for some assignments is insufficient; a tautology must survive every assignment. The counterexamples above establish that (A), (B), and (C) each have a false row in a truth table, whereas the complementary pair $q \vee (\sim q)$ proves that every row of (D) is true. No assumption relating p and q is present or needed.

Step 4: Conclusion The tautology is $q \Rightarrow ((\sim p) \vee q)$. Hence, the correct option is (D).

Quick Tip to Solve: For implications with a disjunction as consequent, look for the antecedent itself inside that disjunction; it often proves the implication immediately.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ **and** $B = A - I$. **If** $\omega = \frac{\sqrt{3}i-1}{2}$, **then the number of elements in the set** $\{n \in \{1, 2, \ldots, 100\} : A^n + (\omega B)^n = A + B\}$ **is equal to _____.** [4 marks]

Solution:

Concept:

- If $A^2 = A$, then $A^n = A$ for all positive integers n. For $B = A - I$, this also implies $B^2 = -B$.
- The number $\omega = (-1 + i\sqrt{3})/2$ is a primitive cube root of unity. Thus $\omega^3 = 1$, while $-\omega = e^{-i\pi/3}$ has order six.

Step 1: Compute the matrix power identities Direct multiplication gives

$$A^2 = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} = A.$$

For example, the first row of the product is $(4 - 1 - 1, -2 + 1, -2 + 1) = (2, -1, -1)$; the other two

rows are $(2-1,-1+1,-1)$ and $(2-1,-1,-1+1)$, respectively. Hence $A^n = A$. Also

$$B^2 = (A-I)^2 = A - 2A + I = I - A = -B,$$

so induction gives $B^n = (-1)^{n-1}B$ for every $n \geq 1$.

Step 2: Reduce the matrix equality to a scalar condition Since scalar multiplication commutes with matrix powers, the required equation becomes

$$A + \omega^n(-1)^{n-1}B = A + B.$$

The matrix B is nonzero; for instance, its first diagonal entry is 1. Therefore the equality holds exactly when

$$\omega^n(-1)^{n-1} = 1, \qquad (-\omega)^n = -1.$$

This reduction is valid even though B is singular: a nonzero matrix multiplied by a scalar is zero only when that scalar is zero.

Step 3: Count the admissible integers With $-\omega = e^{-i\pi/3}$, the scalar condition is $e^{-in\pi/3} = -1$, equivalent to $n \equiv 3 \pmod 6$. The admissible integers from 1 to 100 are

$$3, 9, 15, \ldots, 99.$$

Their count is

$$(99-3)/6 + 1 = 17.$$

The value 99 satisfies both odd parity and divisibility by three, while the next possible integer 105 lies outside the allowed range.

Step 4: Conclusion The set has $\boxed{17}$ elements. This is a dimensionless count.

Quick Tip to Solve: Check whether the given matrix is idempotent before computing powers; combining that identity with the order of a root of unity reduces the task to a congruence.

22. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is _____. [4 marks]

Solution:

Concept:

- Dictionary rank equals one plus the number of distinct arrangements that occur before the given word. Compare the letters from left to right, preserving every prefix already fixed.
- The letters in MANKIND are A, D, I, K, M, N, N. Because N occurs twice, arrangements of a remaining collection with two N letters are counted by dividing the factorial by 2!.

Step 1: Count words whose first letter precedes M The letters smaller than M are A, D, I and K. For any one of these first letters, six letters remain, including two identical N letters. Hence the

contribution is

$$4 \cdot \frac{6!}{2!} = 4 \cdot 360 = 1440.$$

Fix M. Its next letter in the required word is A, which is the smallest remaining letter, so the second position contributes nothing.

Step 2: Continue through the fixed prefixes After MA, the required third letter is N. The available smaller letters are D, I and K; each leaves four letters with two N letters, giving

$$3 \cdot \frac{4!}{2!} = 36.$$

After MAN, the next letter is K. Putting D or I instead leaves three distinct letters, contributing $2 \cdot 3! = 12$. After MANK, the next letter is I; putting D instead contributes $2! = 2$. After MANKI, the next letter is N; putting D instead contributes $1! = 1$. The final D gives no further contribution.

Step 3: Convert the preceding count to rank The number of words strictly before MANKIND is

$$1440 + 36 + 12 + 2 + 1 = 1491.$$

Ranks begin at one, so include the target word itself once. Repeated N letters have already been accounted for in the first two blocks; dividing by two again later would incorrectly reduce the count.

Step 4: Conclusion The serial number of MANKIND is $1491 + 1 = \boxed{1492}$. This is a dimensionless rank.

Quick Tip to Solve: At each position, count only smaller available letters. Recalculate the repeated-letter divisor for the letters that remain, then add one for the word itself.

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23. If the maximum value of the term independent of t in the expansion of $\left(\mathrm{t}^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{\mathrm{t}}\right)^{15}, x \geqslant 0$, is K, then $8\ \mathrm{K}$ is equal to _____. [4 marks]

Solution:

Concept:

- In the binomial expansion, choosing the second summand r times gives $T_{r+1} = \binom{15}{r}(t^2 x^{1/5})^{15-r}((1-x)^{1/10}/t)^r$. A term independent of t has zero net exponent of t.
- The real tenth root $(1-x)^{1/10}$, together with $x \geq 0$, restricts the real-valued expression to $0 \leq x \leq 1$. On this interval $x(1-x) = 1/4 - (x - 1/2)^2$.

Step 1: Identify the constant term in t The power of t in the general term is

$$2(15 - r) - r = 30 - 3r.$$

Setting this equal to zero gives $r = 10$, an allowed integer between zero and fifteen. No other term has zero exponent, so only the eleventh term contributes to the quantity being maximised.

Step 2: Simplify its dependence on x For $r = 10$, the two powers of x and $1 - x$ become

$$(x^{1/5})^5 = x, \qquad ((1-x)^{1/10})^{10} = 1 - x.$$

Thus the term independent of t is

$$\binom{15}{10} x(1-x) = 3003x(1-x).$$

The binomial coefficient can be checked as $\binom{15}{5} = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11/(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 3003$.

Step 3: Maximise the quadratic and compute the requested multiple Since

$$x(1-x) = \frac{1}{4} - \left(x - \frac{1}{2}\right)^2 \le \frac{1}{4},$$

the upper bound is attained at $x = 1/2$, which lies in the permitted interval. The endpoints give zero and cannot be maxima. Consequently

$$K = \frac{3003}{4}, \qquad 8K = 8 \cdot \frac{3003}{4} = 6006.$$

Step 4: Conclusion The requested value is $\boxed{8K = 6006}$, a dimensionless number.

Quick Tip to Solve: First make the exponent of t zero; maximise only that term. The identity x(1-x)=1/4-(x-1/2) squared avoids differentiating the entire expansion.

24. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12\,bx + 6\,b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _____. [4 marks]

Solution:

Concept:

- For roots u, v of $x^2 - Sx + P = 0$, Vieta gives $u + v = S$, $uv = P$, and hence $1/u + 1/v = S/P$. The products here are nonzero because $a, b \neq 0$.
- Four numbers in arithmetic progression may be written $A, A + d, A + 2d, A + 3d$, where d is the common difference. Their order must match the given order of the reciprocals.

Step 1: Find the reciprocal sums from Vieta For the first quadratic, $p + r = 8a$ and $pr = 2a$, so

$$\frac{1}{p} + \frac{1}{r} = \frac{8a}{2a} = 4.$$

For the second quadratic, $q + s = -12b$ and $qs = 6b$, giving

$$\frac{1}{q} + \frac{1}{s} = \frac{-12b}{6b} = -2.$$

These are the sums of the first and third, and of the second and fourth terms of the specified progression.

Step 2: Determine the four reciprocal values Put $1/p = A$, $1/q = A + d$, $1/r = A + 2d$, and $1/s = A + 3d$. The sums above give

$$2A + 2d = 4, \qquad 2A + 4d = -2.$$

Subtracting yields $2d = -6$, hence $d = -3$ and $A = 5$. The reciprocals in the required order are therefore

$$\frac{1}{p} = 5, \quad \frac{1}{q} = 2, \quad \frac{1}{r} = -1, \quad \frac{1}{s} = -4.$$

Step 3: Recover a and b from reciprocal products Their alternating products satisfy

$$\frac{1}{pr} = 5(-1) = -5 = \frac{1}{2a}, \qquad \frac{1}{qs} = 2(-4) = -8 = \frac{1}{6b}.$$

Thus $a^{-1} = -10$ and $b^{-1} = -48$. Both give nonzero real parameters and distinct real roots, so they satisfy the assumptions rather than being extraneous algebraic possibilities.

Step 4: Conclusion Therefore $a^{-1} - b^{-1} = -10 - (-48) = \boxed{38}$, a dimensionless value.

Quick Tip to Solve: When reciprocals of quadratic roots form an AP, compute their alternating sums with Vieta before attempting to solve either quadratic.

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25. Let $f(x) = \begin{cases} \left|4x^2 - 8x + 5\right|, & \text{if } 8x^2 - 6x + 1 \geqslant 0 \\ \left[4x^2 - 8x + 5\right], & \text{if } 8x^2 - 6x + 1 < 0, \end{cases}$ **where** $[\alpha]$ **denotes the greatest integer less than or equal to** α**. Then the number of points in R where** f **is not differentiable is _____.** [4 marks]

Solution:

Concept:

- Write $g(x) = 4x^2 - 8x + 5 = 4(x-1)^2 + 1 > 0$. Therefore the absolute-value branch is simply the smooth polynomial $g(x)$, with no cusp from the modulus.
- The greatest-integer branch applies where $8x^2 - 6x + 1 = (4x-1)(2x-1) < 0$, namely $1/4 < x < 1/2$. Check its integer crossings and both switching endpoints separately.

Step 1: Locate the jump inside the floor-function interval On $(1/4, 1/2)$, $g'(x) = 8x - 8 < 0$, so g decreases strictly from $13/4$ towards 2. The only integer crossed in this open interval is 3. Solving $g(x) = 3$ gives

$$4(x-1)^2 = 2, \qquad x = 1 \pm \frac{1}{\sqrt{2}}.$$

Only $c = 1 - 1/\sqrt{2}$ lies in $(1/4, 1/2)$. The floor is 3 just to the left of c, including at c, and 2 immediately to its right. Hence f is discontinuous, and therefore not differentiable, at c.

Step 2: Check both branch-switching points At $x = 1/4$, the polynomial branch gives $f(1/4) = 13/4$, whereas the right limit from the floor branch is 3. This jump gives another point of non-differentiability.

At $x = 1/2$, the value is $g(1/2) = 2$; the left limit is also 2, so the function is continuous there. However the left-hand branch is locally constant, with derivative zero, while the right derivative is

$$g'(1/2) = 8(1/2) - 8 = -4.$$

The unequal one-sided derivatives make $1/2$ a third such point.

Step 3: Exclude other points Outside the closed switching interval the function is a polynomial. Inside it, away from c, the floor is constant on each of its two open pieces. Each of these functions is differentiable. The other solution $1 + 1/\sqrt{2}$ of $g(x) = 3$ lies on the polynomial branch and creates no jump.

Step 4: Conclusion The points are $1/4$, $1 - 1/\sqrt{2}$, and $1/2$. Their number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: For a piecewise floor function, check integer crossings only on the interval where the floor is active. At its endpoints, compare values first and one-sided derivatives second.

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26. Let $a_1 = b_1 = 1, a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geqslant 2$. Then $\sum_{n=1}^{15} a_n.b_n$ is equal to _____. [4 marks]

Solution:

Concept:

- The recurrence $a_n = a_{n-1} + 2$, with $a_1 = 1$, defines the odd-number arithmetic progression $a_n = 2n - 1$.
- The recurrence for b_n accumulates these odd numbers: $b_n = \sum_{j=1}^{n} a_j$. Use $\sum_{n=1}^{N} n^2 = N(N + 1)(2N + 1)/6$ and $\sum_{n=1}^{N} n^3 = [N(N + 1)/2]^2$.

Step 1: Obtain explicit formulae for both sequences Starting from the first term, the AP formula gives

$$a_n = 1 + 2(n - 1) = 2n - 1.$$

Also $b_n - b_{n-1} = a_n$. Adding these differences from 2 to n, and using $b_1 = a_1 = 1$, yields

$$b_n = a_1 + a_2 + \cdots + a_n = \frac{n}{2}\{1 + (2n - 1)\} = n^2.$$

For example $b_2 = 3 + 1 = 4$, in agreement with this formula. This check prevents treating the second sequence as another AP.

Step 2: Expand the product inside the sum Substitute the explicit terms rather than iterating all fifteen recurrences:

$$a_n b_n = (2n - 1)n^2 = 2n^3 - n^2.$$

Consequently

$$\sum_{n=1}^{15} a_n b_n = 2\sum_{n=1}^{15} n^3 - \sum_{n=1}^{15} n^2.$$

Both sums start at one, so the initial term is already included; it must not be added again separately.

Step 3: Evaluate the standard sums With $N = 15$,

$$\sum_{n=1}^{15} n^3 = \left(\frac{15 \cdot 16}{2}\right)^2 = 120^2 = 14400,$$

$$\sum_{n=1}^{15} n^2 = \frac{15 \cdot 16 \cdot 31}{6} = 1240.$$

Thus the required product sum is $2(14400) - 1240 = 28800 - 1240 = 27560$. The summand grows as a cubic, which is consistent with using a fourth-degree summation formula for the leading part.

Step 4: Conclusion Therefore $\sum_{n=1}^{15} a_n b_n = \boxed{27560}$, a dimensionless sum.

Quick Tip to Solve: If successive differences of b are the odd numbers, recognise b as the square numbers. Then expand the product before applying the square and cube sum formulae.

---

27. **If** $\lim\limits_{n\to\infty} \dfrac{(n+1)^{k-1}}{n^{k+1}}[(nk+1)+(nk+2)+\ldots+(nk+n)]$ $= 33 \cdot \lim\limits_{n\to\infty} \dfrac{1}{n^{k+1}} \cdot \left[1^k + 2^k + 3^k + \ldots + n^k\right]$ **, then the integral value of** k **is equal to**

[4 marks]

Solution:

Concept:

- The sum $(nk+1)+\cdots+(nk+n)$ is an arithmetic sum with n terms. Its exact expression makes the first limit elementary.
- For a nonnegative integer k, the second limit is the Riemann sum $\lim_{n\to\infty} \frac{1}{n}\sum_{j=1}^{n}(j/n)^k = \int_0^1 x^k\,dx = 1/(k+1)$.

Step 1: Simplify the first limit Adding the n copies of nk and the integers from 1 to n gives

$$\sum_{j=1}^{n}(nk+j) = n^2k + \frac{n(n+1)}{2}.$$

Multiplication by the given prefactor therefore reduces the expression to

$$\left(1+\frac{1}{n}\right)^{k-1}\left(k+\frac{1}{2}+\frac{1}{2n}\right).$$

For every fixed real k, its limit is $k + 1/2$.

Step 2: Evaluate the second limit and solve the equation For integral $k \geq 0$, the Riemann-sum formula gives

$$k+\frac{1}{2} = \frac{33}{k+1}.$$

Since $k+1>0$, multiplying through and rearranging is valid:

$$(2k+1)(k+1)=66, \qquad 2k^2+3k-65=0,$$

$$(k-5)(2k+13)=0.$$

The roots are 5 and $-13/2$; only 5 is an integer in this domain.

Step 3: Exclude negative integers and verify the result For $k=-1$, the second limit is the harmonic sum and diverges. For any integer $k\le -2$, the positive sum $\sum_{j=1}^{n} j^k$ is at least 1 and is multiplied by n^{-k-1}, which grows without bound. Such values cannot equal the finite first limit. Finally, at $k=5$, the two sides are $11/2$ and $33/6=11/2$, confirming equality.

Step 4: Conclusion The integral value of k is $\boxed{5}$, a dimensionless exponent.

Quick Tip to Solve: Reduce an arithmetic sum exactly before taking its limit. For a sum of powers divided by n to the power k+1, rewrite it as a Riemann sum and check the exponent domain.

28. Let the equation of two diameters of a circle $x^2+y^2-2x+2fy+1=0$ be $2px-y=1$ and $2x+py=4p$. Then the slope m $\in(0,\infty)$ of the tangent to the hyperbola $3x^2-y^2=3$ passing through the centre of the circle is equal to _____. [4 marks]

Solution:

Concept:

- Completing squares gives the circle $(x-1)^2+(y+f)^2=f^2$, whose centre is $(1,-f)$. Every diameter passes through that centre, and an actual circle requires positive radius.
- For the hyperbola $x^2-y^2/3=1$, a nonvertical line $y=mx+c$ is a tangent when $c^2=m^2-3$. This follows by setting the discriminant of its intersection quadratic to zero.

Step 1: Find the possible circle parameters Substitute the centre into both diameter lines:

$$2p+f=1, \qquad 2-pf=4p.$$

Using $f=1-2p$, the second condition gives

$$p(5-2p)=2, \qquad 2p^2-5p+2=0,$$

$$(2p-1)(p-2)=0.$$

The pairs are $(p,f)=(1/2,0)$ and $(2,-3)$. The first would give radius zero and cannot have two diameters of a circle. Thus $f=-3$ and the centre is $(1,3)$.

Step 2: Apply the tangent condition through the centre A line of slope m through $(1,3)$ has equation

$$y-3=m(x-1), \qquad y=mx+(3-m).$$

Putting $y = mx + c$ in the hyperbola gives

$$(3 - m^2)x^2 - 2mcx - (c^2 + 3) = 0.$$

Its discriminant is $12(c^2 + 3 - m^2)$; tangency requires $c^2 = m^2 - 3$. Hence, with $c = 3 - m$,

$$(3 - m)^2 = m^2 - 3, \qquad 12 = 6m.$$

Step 3: Check the positive slope The equation yields $m = 2$, which is positive as requested. Its line is $y = 2x + 1$. Substitution into the hyperbola reduces the intersection equation to $-(x + 2)^2 = 0$, so there is exactly one contact point, $(-2, -3)$, and the line is indeed a tangent rather than a secant or an asymptote.

Step 4: Conclusion The required slope is $\boxed{m = 2}$, a dimensionless ratio.

Quick Tip to Solve: Use the centre-on-diameter condition before finding tangents. Reject a zero-radius circle, then impose a repeated root for the line-hyperbola intersection.

29. The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5}, \frac{6}{5}\right)$ and (ii) the y-axis, is equal to _____. [4 marks]

Solution:

Concept:

- A circle tangent to the parabola at a specified point has its centre on the normal at that point. Its radius is the distance from the centre to the point of contact.
- A circle with centre (h, k) tangent to the y-axis has radius $r = |h|$. Combining these two conditions determines both possible circles.

Step 1: Find the normal at the point of contact Differentiate the printed parabola implicitly to obtain its tangent slope:

$$150x = 320\frac{dy}{dx}, \qquad \frac{dy}{dx} = \frac{15x}{32}.$$

At $T = (8/5, 6/5)$, the tangent slope is $3/4$, so the normal slope is $-4/3$. Hence a centre (h, k) must satisfy

$$k - \frac{6}{5} = -\frac{4}{3}\left(h - \frac{8}{5}\right).$$

Step 2: Impose tangency to the axis The square of the radius, computed using T, must equal h^2. Since the vertical difference is $-4/3$ times the horizontal difference,

$$\left(h - \frac{8}{5}\right)^2 + \frac{16}{9}\left(h - \frac{8}{5}\right)^2 = h^2.$$

Simplifying gives

$$25\left(h - \frac{8}{5}\right)^2 = 9h^2, \qquad 16h^2 - 80h + 64 = 0.$$

Thus $(h-1)(h-4)=0$, so the radii are 1 and 4. Both centres lie on the required normal and their distances from the axis equal these radii.

Step 3: Conclusion The two diameters are 2 and 8, so their sum is

$$\boxed{10}.$$

This is a numerical value in the coordinate units of the question.

Quick Tip to Solve: Place the centre on the normal first, then equate its squared distance from the contact point to its squared distance from the tangent axis.

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30. The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane $\mathrm{P}: ax-y-z=0$, $(a>0)$. If the image of the point $(1,1,-5)$ in the plane P is (α,β,γ), then $\alpha+\beta-\gamma$ is equal to _____. [4 marks]

Solution:

Concept:

- The direction of the common perpendicular to two skew lines is the cross product of their direction vectors. For a line direction $\mathbf{d}$ and plane normal $\mathbf{n}$, the angle θ with the plane obeys $\sin\theta = |\mathbf{d}\cdot\mathbf{n}|/(|\mathbf{d}||\mathbf{n}|)$.
- The cosine formula instead gives the angle with the normal. These angles are complementary. The printed condition must be checked before reflecting the point.

Step 1: Check the printed angle condition The two line directions are $(0,1,1)$ and $(2,2,1)$. Their cross product is

$$\mathbf{d}=(-1,2,-2), \qquad |\mathbf{d}|=3.$$

For the plane $ax-y-z=0$, take $\mathbf{n}=(a,-1,-1)$. Then

$$|\mathbf{d}\cdot\mathbf{n}|=a, \qquad \sin\theta=\frac{a}{3\sqrt{a^2+2}}<\frac{1}{3}.$$

But the angle printed in the question gives

$$\cos^2\theta=\frac{2}{27}, \qquad \sin^2\theta=\frac{25}{27}.$$

Substitution would require $a^2=25(a^2+2)/3$, or $22a^2=-50$. No real positive a satisfies this.

Step 2: Identify the interpretation producing the keyed value If the stated angle is interpreted as the angle with the plane normal, the cosine relation becomes

$$\frac{a}{3\sqrt{a^2+2}}=\sqrt{\frac{2}{27}}.$$

Squaring gives $3a^2 = 2(a^2+2)$, so $a = 2$. For $X = (1, 1, -5)$, reflection in $2x - y - z = 0$ is

$$X' = X - 2\frac{2(1) - 1 - (-5)}{2^2 + (-1)^2 + (-1)^2}(2, -1, -1) = (-3, 3, -3).$$

This yields $\alpha + \beta - \gamma = -3 + 3 + 3 = 3$, a dimensionless coordinate combination.

Step 3: Conclusion As literally printed, the angle with the plane is inconsistent and there is no real plane or uniquely defined image. The keyed numerical value $\boxed{3}$ is obtained only when the stated angle is taken with the plane normal, as shown above; it does not satisfy the literal line-plane condition.

Quick Tip to Solve: Use sine for an angle between a line and a plane, and cosine for its angle with the normal; a quick upper-bound check can expose a complementary-angle error.

Physics – Section A: Multiple Choice Questions (80 marks)

31. If momentum [P], area $[\mathrm{A}]$ and time $[\mathrm{T}]$ are taken as fundamental quantities, then the dimensional formula for coefficient of viscosity is : [4 marks]

(A) $\left[\mathrm{P\,A^{-1}\,T^{0}}\right]$
(B) $\left[\mathrm{P\,A\,T^{-1}}\right]$
(C) $\left[\mathrm{P\,A^{-1}\,T}\right]$
(D) $\left[\mathrm{P\,A^{-1}\,T^{-1}}\right]$

Correct Answer: (A) $\left[\mathrm{P\,A^{-1}\,T^{0}}\right]$

Solution:

Concept:

- For dynamic viscosity η, Newton's law of viscosity is $F = \eta A_s(dv/dy)$, where A_s is the sheared area. Since the velocity gradient has dimensions T^{-1}, $[\eta] = \mathrm{ML^{-1}T^{-1}}$.
- The new fundamental momentum and area have dimensions $[\mathrm{P}] = \mathrm{MLT^{-1}}$ and $[\mathrm{A}] = \mathrm{L}^2$. Express the required quantity as a product of their powers.

Step 1: Equate the dimensional exponents Write $[\eta] = [\mathrm{P}]^p[\mathrm{A}]^q[\mathrm{T}]^r$. In the usual mass-length-time basis this is

$$\mathrm{ML^{-1}T^{-1}} = \mathrm{M}^p\mathrm{L}^{p+2q}\mathrm{T}^{-p+r}.$$

Equating the mass, length and time powers gives

$$p = 1, \qquad p + 2q = -1, \qquad -p + r = -1.$$

Thus $q = -1$ and $r = 0$.

Step 2: Check the alternatives The resulting combination is momentum divided by area, with no additional time factor:

$$[\eta] = [\mathrm{PA^{-1}T^{0}}].$$

Option (B) multiplies by area rather than dividing by it, giving the wrong length dimension. Option (C) introduces an extra time factor and loses the required inverse-time power. Option (D) introduces one more inverse time and instead has the dimensions of pressure.

Step 3: Conclusion The dimensional formula is $\boxed{[\mathrm{PA}^{-1}\mathrm{T}^{0}]}$. Hence, the correct option is (A).

Quick Tip to Solve: Dynamic viscosity has the dimensions of momentum per area; distinguish it from pressure, which is momentum per area per time.

32. Which of the following physical quantities have the same dimensions? [4 marks]

(A) Electric displacement $(\overrightarrow{\mathrm{D}})$ and surface charge density

(B) Displacement current and electric field

(C) Current density and surface charge density

(D) Electric potential and energy

Correct Answer: (A) Electric displacement $(\overrightarrow{\mathrm{D}})$ and surface charge density

Solution:

Concept:

- Electric displacement $\mathbf{D}$ satisfies Gauss's law $\oint \mathbf{D} \cdot d\mathbf{A} = Q_{\text{free}}$, so its dimensions are charge per area.
- Use charge dimension IT, current density as current per area, and potential as energy per charge. Dimensional equality requires every base exponent to agree.

Step 1: Compare electric displacement with surface charge density From the displacement-flux relation,

$$[D] = \frac{[Q]}{\mathrm{L}^2} = \mathrm{ITL}^{-2}.$$

Surface charge density is $\sigma = Q/A_s$, where A_s is surface area. Therefore

$$[\sigma] = \mathrm{ITL}^{-2} = [D].$$

Thus the two quantities in option (A) have exactly the same dimensions, even though one is a vector field and the other is a scalar surface density.

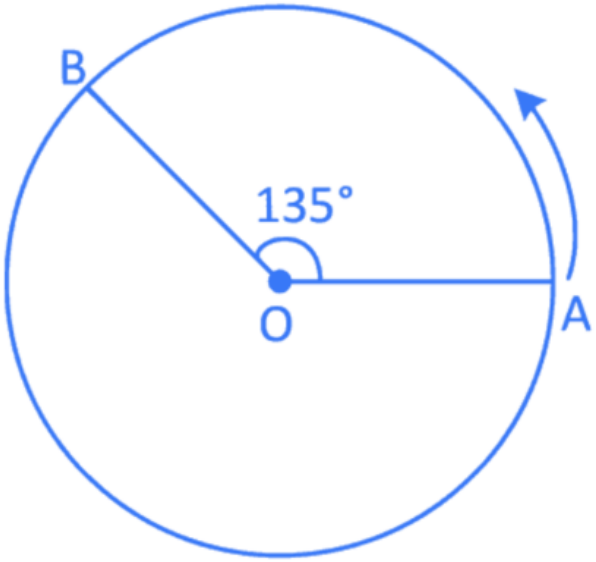
Step 2: Reject the other pairs Displacement current has dimensions I, whereas electric field has dimensions force per charge, $\mathrm{MLT}^{-3}\mathrm{I}^{-1}$; option (B) fails. Current density has dimensions IL^{-2}, while surface charge density has ITL^{-2}; option (C) differs by a time factor. Electric potential has dimensions $\mathrm{ML}^{2}\mathrm{T}^{-3}\mathrm{I}^{-1}$, whereas energy has $\mathrm{ML}^{2}\mathrm{T}^{-2}$. The charge divisor makes option (D) unequal as well.

Step 3: Conclusion Electric displacement and surface charge density both have dimensions $\boxed{\mathrm{ITL}^{-2}}$. Hence, the correct option is (A).

Quick Tip to Solve: Gauss's law for $\mathbf{D}$ immediately gives charge per area; do not confuse electric displacement with displacement current.

33. A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is $60\ \text{m}$, then the magnitude of displacement would be :

(Given $\cos 135^\circ = -0.7$**)** [4 marks]



(A) 42 m

(B) 47 m

(C) 19 m

(D) 40 m

Correct Answer: (B) 47 m

Solution:

Concept:

- Distance along a circular path is its arc length $s = R\theta$, with the central angle in radians. Displacement magnitude is the straight chord joining the start and end points.
- For two radius vectors enclosing angle θ, the cosine rule gives chord length $d = R\sqrt{2(1-\cos\theta)}$. Use the marked 135° arc and the supplied cosine value.

Step 1: Find the radius from the travelled arc The arrow in the figure goes from A to B along the arc subtending 135°. Convert this to radians before using the arc formula:

$$\theta = 135\frac{\pi}{180} = \frac{3\pi}{4}.$$

Since the travelled distance is $60\,\text{m}$,

$$60 = R\frac{3\pi}{4}, \qquad R = \frac{80}{\pi}\,\text{m} \approx 25.46\,\text{m}.$$

The 60 m distance is the curved arc, rather than the straight displacement.

Step 2: Calculate the chord joining A and B Apply the cosine rule to triangle AOB, whose two sides

are radii. With the supplied $\cos 135^\circ = -0.7$,

$$d^2 = R^2 + R^2 - 2R^2(-0.7) = 3.4R^2.$$

Therefore

$$d = \frac{80}{\pi}\sqrt{3.4}\,\text{m} \approx 46.95\,\text{m}.$$

The listed value 47 m matches this result. The other values 42 m, 19 m and 40 m do not follow from both the given arc length and the marked angle; treating degrees as radians or replacing the chord by the radius would be incorrect.

Step 3: Conclusion The magnitude of the displacement is approximately $\boxed{47\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the arc to find the radius first, then use the chord for displacement; arc length requires the central angle in radians.

---

34. A body of mass 0.5 kg travels on straight line path with velocity $v = (3x^2 + 4)$ m/s. The net workdone by the force during its displacement from $x = 0$ to $x = 2$ m is : [4 marks]

(A) 64 J

(B) 60 J

(C) 120 J

(D) 128 J

Correct Answer: (B) 60 J

Solution:

Concept:

- The net work done by all forces equals the change in kinetic energy: $W_{\text{net}} = K_f - K_i = \frac{1}{2}m(v_f^2 - v_i^2)$. This remains valid when speed varies nonlinearly with position.
- The velocity expression is evaluated at the two endpoints. The mass stays constant, so there is no need to determine the time dependence or integrate the force explicitly.

Step 1: Evaluate the endpoint speeds The speed is $v(x) = 3x^2 + 4$, with the numerical position measured in metres and the resulting speed in metres per second. At the initial point and final point, respectively,

$$v_i = v(0) = 4\,\text{m s}^{-1}, \qquad v_f = v(2) = 3(2)^2 + 4 = 16\,\text{m s}^{-1}.$$

Both speeds are positive; the kinetic-energy calculation depends on their squares.

Step 2: Calculate the change in kinetic energy For $m = 0.5\,\text{kg}$, the initial and final kinetic energies are

$$K_i = \frac{1}{2}(0.5)(4)^2 = 4\,\text{J}, \qquad K_f = \frac{1}{2}(0.5)(16)^2 = 64\,\text{J}.$$

Hence

$$W_{\text{net}} = 64 - 4 = 60\,\text{J}.$$

The 64 J alternative is only the final kinetic energy and omits the initial energy. The 120 J result misses the factor $1/2$ in the energy change. The 128 J alternative likewise is not the difference of the two kinetic energies.

Step 3: Conclusion The net work done over the stated displacement is $\boxed{60\,\text{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: When velocity is given as a function of position and only total work is required, evaluate the two kinetic energies instead of integrating acceleration.

35. A solid cylinder and a solid sphere, having same mass M and radius R, roll down the same inclined plane from top without slipping. They start from rest. The ratio of velocity of the solid cylinder to that of the solid sphere, with which they reach the ground, will be : [4 marks]

(A) $\sqrt{\frac{5}{3}}$

(B) $\sqrt{\frac{4}{5}}$

(C) $\sqrt{\frac{3}{5}}$

(D) $\sqrt{\frac{14}{15}}$

Correct Answer: (D) $\sqrt{\frac{14}{15}}$

Solution:

Concept:

- For rolling without slipping, the contact constraint is $v = \omega R$. A body descending vertical height h from rest converts Mgh into translational and rotational kinetic energy.
- The moments of inertia about their symmetry axes are $I_c = \frac{1}{2}MR^2$ for a solid cylinder and $I_s = \frac{2}{5}MR^2$ for a solid sphere. Static friction supplies torque but does no work on a fixed ideal incline.

Step 1: Find each speed using the rolling energy equation Use the same vertical drop h for both bodies. The energy balance is

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v^2}{R^2}, \qquad v^2 = \frac{2gh}{1 + I/(MR^2)}.$$

For the cylinder and sphere this gives

$$v_c^2 = \frac{2gh}{1 + 1/2} = \frac{4gh}{3}, \qquad v_s^2 = \frac{2gh}{1 + 2/5} = \frac{10gh}{7}.$$

The sphere has the smaller dimensionless rotational inertia, so its translational speed is greater.

Step 2: Form and check the requested ratio Divide the squared cylinder speed by the squared sphere

speed before taking the square root:

$$\frac{v_c}{v_s} = \sqrt{\frac{4gh/3}{10gh/7}} = \sqrt{\frac{28}{30}} = \sqrt{\frac{14}{15}}.$$

The mass, radius and drop cancel. Option (A) is greater than one and contradicts the faster sphere. Options (B) and (C) are less than one but have squared ratios $4/5$ and $3/5$, inconsistent with the two inertia factors $3/2$ and $7/5$.

Step 3: Conclusion The dimensionless ratio of cylinder speed to sphere speed is $\boxed{\sqrt{14/15}}$. Hence, the correct option is (D).

Quick Tip to Solve: For rolling from the same height, compare $1 + I/(MR^2)$; the smaller this factor, the larger the final translational speed.

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36. Three identical particles A, B and C of mass $100\ \mathrm{kg}$ each are placed in a straight line with $\mathrm{AB} = \mathrm{BC} = 13\ \mathrm{m}$. The gravitational force on a fourth particle P of the same mass is F, when placed at a distance $13\ \mathrm{m}$ from the particle B on the perpendicular bisector of the line AC. The value of F will be approximately : [4 marks]

(A) 21 G

(B) 100 G

(C) 59 G

(D) 42 G

Correct Answer: (B) 100 G

Solution:

Concept:

- Newton's law gives the attraction between two point masses: $F = Gm_1m_2/r^2$, directed along their joining line.
- For a symmetrical arrangement, equal components perpendicular to the symmetry axis cancel. Components towards the midpoint add. Here all four masses are $m = 100\,\mathrm{kg}$.

Step 1: Identify the distances and directions Put B at the origin, A and C at $(-13, 0)$ and $(13, 0)$, and P at $(0, 13)$, with coordinates in metres. Then $PB = 13\,\mathrm{m}$, whereas $PA = PC = 13\sqrt{2}\,\mathrm{m}$. The forces due to A and C have equal magnitudes, so their horizontal components cancel. Their downward components and the entire force due to B point from P towards B.

Step 2: Resolve and add the attractions The central particle exerts

$$F_B = \frac{G(100)^2}{13^2} = \frac{10000G}{169}.$$

Each outer particle exerts $F_A = F_C = 10000G/338$, at 45° to PB. Hence

$$\begin{aligned} F &= F_B + 2F_A \cos 45^\circ \\ &= \frac{10000G}{169}\left(1 + \frac{1}{\sqrt{2}}\right) \\ &\simeq (59.17 + 41.84)G = 101.01G. \end{aligned}$$

The outer pair alone gives approximately $42G$, while the central particle alone gives approximately $59G$; neither is the resultant.

Step 3: Compare the offered approximations The total is approximately $100G$, so 100 G is the nearest offered value. The 21 G value corresponds to only one of the two equal resolved contributions and omits the other forces. The force is attractive and directed along PB towards B.

Step 4: Conclusion $F \simeq 100G$ in the numerical convention of the options, directed towards B. Hence, the correct option is (B).

Quick Tip to Solve: For masses mirrored about PB, cancel the transverse components before adding forces; do not add the three force magnitudes directly.

37. A certain amount of gas of volume V at 27°C temperature and pressure $2 \times 10^7 \mathrm{Nm}^{-2}$ expands isothermally until its volume gets doubled. Later it expands adiabatically until its volume gets redoubled. The final pressure of the gas will be (Use $\gamma = 1.5$) : [4 marks]

(A) 3.536×10^5 Pa

(B) 3.536×10^6 Pa

(C) 1.25×10^6 Pa

(D) 1.25×10^5 Pa

Correct Answer: (B) 3.536×10^6 Pa

Solution:

Concept:

- For a fixed amount of ideal gas in an isothermal change, PV is constant because T is constant.
- For a reversible adiabatic change, PV^γ is constant. The two stages must be evaluated separately because their pressure-volume laws differ.

Step 1: Find the pressure after the isothermal stage The initial pressure is $P_1 = 2 \times 10^7$ Pa, and the volume changes from V to $2V$ at the initial temperature 27°C. Applying the isothermal law gives

$$P_1 V = P_2(2V), \qquad P_2 = \frac{P_1}{2} = 1.0 \times 10^7 \text{ Pa}.$$

It is not necessary to convert the temperature to kelvin here because it cancels within this constant-temperature relation.

Step 2: Apply the adiabatic law to the second doubling The volume is redoubled from $2V$ to $4V$; this

second stage starts with pressure P_2, not P_1. With $\gamma = 1.5 = 3/2$,

$$P_2(2V)^{3/2} = P_3(4V)^{3/2},$$
$$P_3 = P_2\left(\frac{1}{2}\right)^{3/2}$$
$$= \frac{10^7}{2\sqrt{2}} = 3.5355 \times 10^6\,\text{Pa}.$$

Cooling in the adiabatic expansion makes the pressure fall by more than the factor of two for an isothermal doubling.

Step 3: Check the alternatives Rounding gives 3.536×10^6 Pa. Option A has an extra factor-of-ten reduction. Options C and D do not obey the adiabatic volume ratio with exponent $3/2$. Applying the same law to both stages would also give an incorrect result.

Step 4: Conclusion The final pressure is 3.536×10^6 Pa. Hence, the correct option is (B).

Quick Tip to Solve: For successive processes multiply their pressure ratios: here $P_f/P_i = (1/2)(1/2)^{3/2}$.

---

38. Following statements are given :

(A) The average kinetic energy of a gas molecule decreases when the temperature is reduced.

(B) The average kinetic energy of a gas molecule increases with increase in pressure at constant temperature.

(C) The average kinetic energy of a gas molecule decreases with increase in volume.

(D) Pressure of a gas increases with increase in temperature at constant pressure.

(E) The volume of gas decreases with increase in temperature.

Choose the correct answer from the options given below : [4 marks]

(A) (A) and (D) only

(B) (A), (B) and (D) only

(C) (B) and (D) only

(D) (A), (B) and (E) only

Correct Answer: No listed option is literally correct: A alone is true, whereas D contradicts its constant-pressure condition. The supplied key selects option (A).

Solution:

Concept:

- The mean translational kinetic energy per molecule of an ideal gas is $\langle K \rangle = \frac{3}{2}k_{\text{B}}T$. It depends only on absolute temperature.
- The gas law is $PV = Nk_{\text{B}}T$. A statement relating two variables requires a condition on the third variable; a quantity explicitly held constant cannot simultaneously increase.

Step 1: Evaluate the kinetic-energy statements Statement A is correct: reducing absolute temperature reduces $\frac{3}{2}k_{\text{B}}T$. Statement B is false: at constant temperature the average kinetic energy is unchanged,

even if the pressure increases by compression. Statement C gives no temperature or pressure condition. Increasing volume alone does not establish a decrease in temperature: an isothermal expansion leaves the kinetic energy unchanged, whereas an adiabatic expansion can reduce it. Thus C is not a general law.

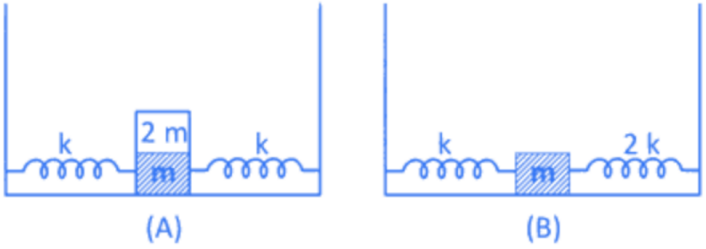
Step 2: Examine the pressure and volume statements literally Statement D says that pressure increases with temperature *at constant pressure*. Its last condition expressly fixes pressure, so the stated increase cannot occur. At constant volume the relation $P \propto T$ would be valid, but that is a different condition from the one printed. Statement E is not a general law either: at constant pressure $V \propto T$, so heating increases volume, and without a constraint its volume trend is undetermined.

Step 3: Distinguish the literal result from the keyed selection Only statement A is unconditionally correct in the wording supplied. Every listed combination includes B, D or E, so none is literally correct. The supplied key selects option A, containing statements A and D. That selection would follow if D used the constant-volume condition; it does not follow from the printed constant-pressure condition. No change of the kinetic-energy formula can resolve this contradiction.

Step 4: Conclusion As printed, only statement A is correct and no listed combination is fully correct. The supplied key selects option (A), but its inclusion of statement D conflicts with the explicit constant-pressure wording; therefore the item has no defensible unique listed answer.

Quick Tip to Solve: Use $\langle K \rangle \propto T$ first, then check exactly which variable is held constant before accepting a pressure or volume statement.

39. In figure (A), mass '$2m$' is fixed on mass 'm' which is attached to two springs of spring constant k. In figure (B), mass 'm' is attached to two springs of spring constant 'k' and '$2k$'. If mass 'm' in (A) and in (B) are displaced by distance 'x' horizontally and then released, then time period T_1 and T_2 corresponding to (A) and (B) respectively follow the relation. [4 marks]



(A) $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$

(B) $\frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$

(C) $\frac{T_1}{T_2} = \sqrt{\frac{2}{3}}$

(D) $\frac{T_1}{T_2} = \frac{\sqrt{2}}{3}$

Correct Answer: (A) $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$

Solution:

Concept:

- For linear simple harmonic motion, $F = -k_{\text{eff}}x$ and $T = 2\pi\sqrt{M/k_{\text{eff}}}$, where M is the total mass moving together.
- When a block is attached between two fixed-wall springs, displacement stretches one spring and compresses the other. Both restoring forces oppose displacement, so their spring constants add.

Step 1: Find the oscillating mass and stiffness in arrangement A The upper block of mass $2m$ is fixed to the lower block of mass m, so the two blocks move together. The oscillating mass is $M_A = 3m$. A horizontal displacement x stretches one spring by x and compresses the other by x. Each gives a restoring force of magnitude kx, giving $F_A = -2kx$. Therefore

$$k_A = 2k, \qquad T_1 = 2\pi\sqrt{\frac{3m}{2k}}.$$

Step 2: Find the corresponding quantities in arrangement B Only the mass m moves in B. Its two spring constants are k and $2k$. Their restoring forces have the same direction for any given displacement, so

$$F_B = -(k + 2k)x = -3kx, \qquad T_2 = 2\pi\sqrt{\frac{m}{3k}}.$$

These springs are not in series: the block displacement changes the length of each spring directly.

Step 3: Take the ratio and check the alternatives Dividing cancels both 2π and the common scale m/k:

$$\frac{T_1}{T_2} = \sqrt{\frac{3m}{2k}\frac{3k}{m}} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}.$$

Option B overlooks part of the mass or stiffness comparison, while C and D invert factors that belong in the numerator. The result is greater than one, consistent with A having a larger moving mass and a smaller effective stiffness.

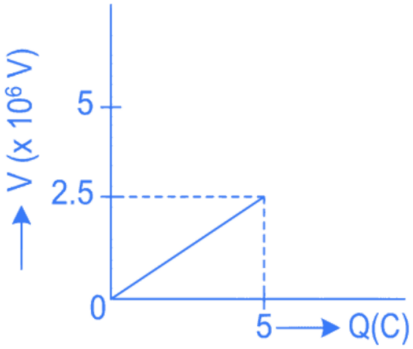
Step 4: Conclusion $T_1/T_2 = 3/\sqrt{2}$. Hence, the correct option is (A).

Quick Tip to Solve: For a block between fixed walls, add the spring constants; separately count every block fixed to and moving with the oscillating mass.

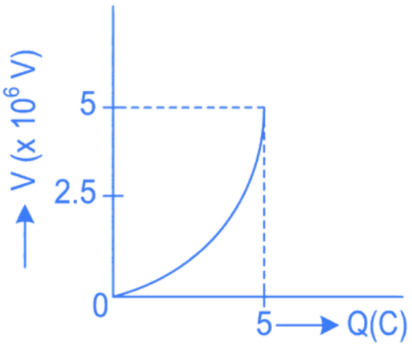
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40. A condenser of $2\,\mu\text{F}$ capacitance is charged steadily from 0 to $5\,\text{C}$. Which of the following graph represents correctly the variation of potential difference (V) across it's plates with respect to the charge (Q) on the condenser? [4 marks]

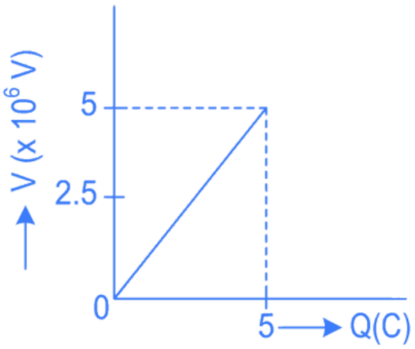
(A) Straight line through the origin reaching $V = 2.5 \times 10^6\,\text{V}$ at $Q = 5\,\text{C}$.



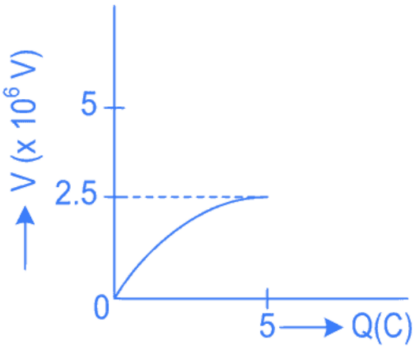
(B) Concave-up curve reaching $V = 5 \times 10^6\,\text{V}$ at $Q = 5\,\text{C}$.



(C) Straight line through the origin reaching $V = 5 \times 10^6\,\text{V}$ at $Q = 5\,\text{C}$.

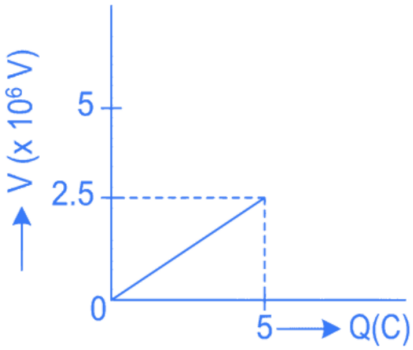


(D) Concave-down curve reaching $V = 2.5 \times 10^6\,\text{V}$ at $Q = 5\,\text{C}$.



Correct Answer: (A)

Straight line through the origin reaching $V = 2.5 \times 10^6\,\text{V}$ at $Q = 5\,\text{C}$.



Solution:

Concept:

- For an ideal capacitor with constant capacitance C, the charge and potential difference obey $Q = CV$. Consequently $V = Q/C$.
- A plot of potential difference against charge has slope $1/C$, measured in volts per coulomb. For a fixed positive capacitance this slope is constant and positive.

Step 1: Write the equation with capacitance in SI units The given capacitance is $C = 2\,\mu\text{F} = 2\times10^{-6}\,\text{F}$. In the absence of any change to geometry or dielectric, charging does not change this capacitance. Hence

$$V = \frac{Q}{2 \times 10^{-6}} = (5 \times 10^{5}\,\text{V C}^{-1})Q.$$

At zero charge the potential difference is zero, so the required graph passes through the origin.

Step 2: Calculate the endpoint of the graph The final charge is $5\,\text{C}$, giving

$$V_f = \frac{5}{2 \times 10^{-6}} = 2.5 \times 10^{6}\,\text{V}.$$

The vertical axis in the options is scaled in $10^6\,\text{V}$, so this endpoint has ordinate 2.5 and abscissa 5. The large voltage follows from the large stated charge and small capacitance; the coulomb unit must not be replaced by microcoulomb.

Step 3: Select the graph using both slope and endpoint Option A is a straight line from the origin to the required endpoint. Options B and D are curved and therefore imply a changing capacitance rather than the given constant value. Option C is linear but reaches $5 \times 10^6\,\text{V}$ at the final charge, twice the required potential difference. A constant charging rate fixes how Q varies with time; the relation between V and Q already follows directly from capacitance.

Step 4: Conclusion The graph is a straight line through the origin ending at $(5\,\text{C}, 2.5 \times 10^6\,\text{V})$. Hence, the correct option is (A).

Quick Tip to Solve: For a V-versus-Q graph, check two facts: its slope is $1/C$, and its endpoint is $Q_{\max}/C$.

41. Two charged particles, having same kinetic energy, are allowed to pass through a uniform magnetic field perpendicular to the direction of motion. If the ratio of radii of their circular paths is $6:5$ and their respective masses ratio is $9:4$. Then, the ratio of their charges will be : [4 marks]

(A) 8 : 5

(B) 5 : 4

(C) 5 : 3

(D) 8 : 7

Correct Answer: (B) 5 : 4

Solution:

Concept:

- For perpendicular motion in a uniform magnetic field, $|q|vB = mv^2/r$, so $r = mv/(|q|B)$.
- If kinetic energy $K = mv^2/2$ is fixed, momentum is $mv = \sqrt{2mK}$. Thus $r = \sqrt{2mK}/(|q|B)$, which is proportional to $\sqrt{m}/|q|$ at fixed K and B.

Step 1: Express the radius ratio with the correct mass dependence The particles have the same kinetic energy, not the same speed. The heavier particle moves more slowly, so using $r \propto m/q$ would be inappropriate. Substituting $v = \sqrt{2K/m}$ into the magnetic-radius formula gives

$$\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}\frac{|q_2|}{|q_1|}.$$

The common magnetic field and common kinetic energy cancel from this ratio.

Step 2: Substitute and solve for the charge ratio With the given ratios $r_1/r_2 = 6/5$ and $m_1/m_2 = 9/4$,

$$\frac{6}{5} = \frac{3}{2}\frac{|q_2|}{|q_1|},$$
$$\frac{|q_1|}{|q_2|} = \frac{3}{2}\frac{5}{6} = \frac{5}{4}.$$

Thus the first particle has charge magnitude one and one-quarter times that of the second.

Step 3: Verify and exclude the other ratios Putting $|q_1|/|q_2| = 5/4$ back gives $r_1/r_2 = (3/2)(4/5) = 6/5$, exactly as required. Ratios 8:5, 5:3 and 8:7 instead predict radius ratios 15:16, 9:10 and 21:16 respectively. Radii determine magnitudes; without curvature directions they do not determine charge signs.

Step 4: Conclusion The ratio of charge magnitudes is $|q_1| : |q_2| = 5 : 4$. Hence, the correct option is (B).

Quick Tip to Solve: At equal kinetic energy use $r \propto \sqrt{m}/|q|$; at equal speed the mass dependence would instead be $r \propto m/|q|$.

42. To increase the resonant frequency in series LCR circuit, [4 marks]

(A) source frequency should be increased.

(B) another resistance should be added in series with the first resistance.

(C) another capacitor should be added in series with the first capacitor.

(D) the source frequency should be decreased.

Correct Answer: (C) another capacitor should be added in series with the first capacitor.

Solution:

Concept:

- At series resonance the inductive and capacitive reactances are equal: $\omega_0 L = 1/(\omega_0 C)$. Hence $f_0 = 1/(2\pi\sqrt{LC})$.
- The circuit's resonance frequency is set by L and C. Changing the driving frequency changes the operating point, while changing series resistance changes the peak current and bandwidth.

Step 1: Identify how the resonant frequency can increase The expression

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

shows that decreasing the inductance or the effective capacitance increases the resonant frequency. The options do not offer a change to L. We must therefore identify a proposed change that decreases the effective capacitance, rather than a change that merely adjusts the source frequency.

Step 2: Calculate the effect of a capacitor added in series If a second positive capacitance C_a is added in series with the existing capacitance C, then

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{1}{C_a}, \qquad C_{\text{eq}} = \frac{CC_a}{C + C_a} < C.$$

Therefore

$$f_0' = \frac{1}{2\pi\sqrt{LC_{\text{eq}}}} > \frac{1}{2\pi\sqrt{LC}} = f_0.$$

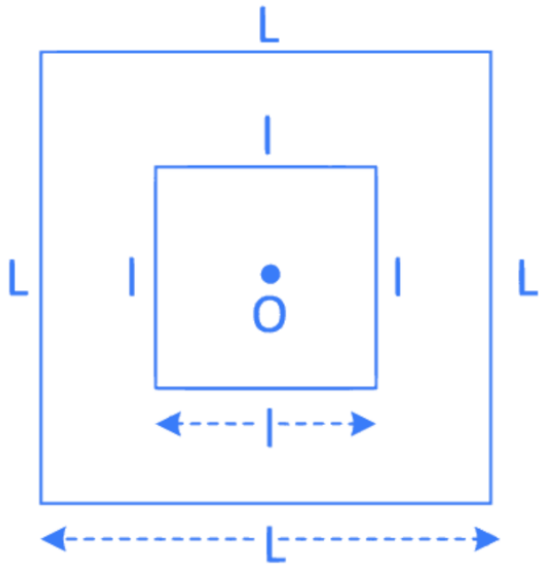
For example, an equal capacitor in series halves C and increases the resonant frequency by a factor $\sqrt{2}$.

Step 3: Rule out changes that leave L and C unchanged Increasing or decreasing the source frequency changes the applied frequency, not the frequency at which this circuit satisfies $X_L = X_C$; thus options A and D do not change its resonance. An extra series resistor changes the impedance at resonance and the sharpness of the current peak. It does not enter the ideal series-current resonance condition, so option B also fails.

Step 4: Conclusion Another capacitor should be added in series with the first capacitor. Hence, the correct option is (C).

Quick Tip to Solve: For series-current resonance, inspect the product LC. A capacitor added in series reduces effective C and therefore raises f_0.

43. A small square loop of wire of side l is placed inside a large square loop of wire $\mathrm{L}(\mathrm{L} >> l)$. Both loops are coplanar and their centres coincide at point O as shown in figure. The mutual inductance of the system is : [4 marks]



(A) $\frac{2\sqrt{2}\mu_0\ \mathrm{L}^2}{\pi l}$

(B) $\frac{\mu_0 l^2}{2\sqrt{2}\pi \mathrm{L}}$

(C) $\frac{2\sqrt{2}\mu_0 l^2}{\pi \mathrm{L}}$

(D) $\frac{\mu_0\ \mathrm{L}^2}{2\sqrt{2}\pi l}$

Correct Answer: (C) $\frac{2\sqrt{2}\mu_0 l^2}{\pi \mathrm{L}}$

Solution:

Concept:

- Mutual inductance is flux linkage per current: $M = \Phi/I$. For a single-turn small loop in a nearly uniform perpendicular field, $\Phi = Bl^2$.
- A finite straight wire produces $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/(4\pi a)$, where a is the perpendicular distance from the observation point.

Step 1: Find the field due to one side Pass a current I through the large square loop of side L. At its centre, the perpendicular distance from any side is $a = L/2$. The line to either endpoint makes 45° with the perpendicular, so the field magnitude from one side is

$$B_1 = \frac{\mu_0 I}{4\pi(L/2)}(\sin 45^\circ + \sin 45^\circ) = \frac{\sqrt{2}\mu_0 I}{2\pi L}.$$

Step 2: Add the fields and calculate the linked flux By the right-hand rule, all four sides produce magnetic fields in the same perpendicular direction at the centre. Since $L \gg l$, this central field is approximately constant over the small loop; use the area of the small loop, not the area of the

current-carrying large loop.

$$B = 4B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi L}, \qquad \Phi = Bl^2 = \frac{2\sqrt{2}\mu_0 I l^2}{\pi L}.$$

Step 3: Divide by the current and compare the options Consequently,

$$M = \frac{\Phi}{I} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}.$$

Options (A) and (D) interchange the roles of the field-producing length and the flux-collecting area, giving an incorrect L^2/l dependence. Option (B) has the right length dependence but is smaller by a factor of eight because the four finite sides have not been combined correctly.

Step 4: Conclusion The mutual inductance is $\boxed{M = 2\sqrt{2}\mu_0 l^2/(\pi L)}$, in henry. Hence, the correct option is (C).

Quick Tip to Solve: For a tiny concentric loop, first calculate the large loop's central field, then multiply by the tiny loop's area and divide by current.

44. The rms value of conduction current in a parallel plate capacitor is $6.9\,\mu$A. The capacity of this capacitor, if it is connected to 230 V ac supply with an angular frequency of $600\,\text{rad/s}$, will be :

[4 marks]

(A) 5 pF

(B) 50 pF

(C) 100 pF

(D) 200 pF

Correct Answer: (B) 50 pF

Solution:

Concept:

- The capacitive reactance at angular frequency ω is $X_C = 1/(\omega C)$. The rms circuit current therefore satisfies $I_{\text{rms}} = V_{\text{rms}}\omega C$.
- The conduction current in the connecting wires equals the displacement current between the capacitor plates. Both have the same rms magnitude in this circuit.

Step 1: Relate the rms current and voltage The supply voltage $230\,\text{V}$ is its rms value, and the given current is also rms. Thus these quantities may be used together without introducing any peak-to-rms conversion. For the capacitor,

$$C = \frac{I_{\text{rms}}}{\omega V_{\text{rms}}}.$$

The given frequency is already angular frequency in radians per second, so there is no additional factor of 2π.

Step 2: Substitute the current in amperes Convert the microampere value before substitution:

$$I_{\text{rms}} = 6.9 \times 10^{-6}\,\text{A}, \qquad \omega V_{\text{rms}} = 600 \times 230 = 138000.$$

$$C = \frac{6.9 \times 10^{-6}}{1.38 \times 10^{5}} = 5.0 \times 10^{-11}\,\text{F} = 50\,\text{pF}.$$

Step 3: Check the alternatives against the current At this voltage and angular frequency, a capacitance of $50\,\text{pF}$ gives $I = 600 \times 50 \times 10^{-12} \times 230 = 6.9\,\mu\text{A}$, as required. Option (A), $5\,\text{pF}$, would give only $0.69\,\mu\text{A}$. Options (C) and (D) would give $13.8\,\mu\text{A}$ and $27.6\,\mu\text{A}$, respectively, and therefore cannot match the specified current.

Step 4: Conclusion The capacitance is $\boxed{50\,\text{pF}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use rms voltage with rms current. When the frequency is given as angular frequency, apply $I = \omega CV$ directly without multiplying by 2π.

45. Which of the following statement is correct? [4 marks]

(A) In primary rainbow, observer sees red colour on the top and violet on the bottom

(B) In primary rainbow, observer sees violet colour on the top and red on the bottom

(C) In primary rainbow, light wave suffers total internal reflection twice before coming out of water drops.

(D) Primary rainbow is less bright than secondary rainbow.

Correct Answer: (A) In primary rainbow, observer sees red colour on the top and violet on the bottom

Solution:

Concept:

- Dispersion in a water drop occurs because its refractive index depends on wavelength. Violet light is refracted more strongly than red light.
- A primary rainbow is formed by refraction on entering the drop, one internal reflection, and refraction on leaving. A secondary rainbow involves two internal reflections.

Step 1: Identify the colour order in the primary bow The rays forming the primary rainbow reach the observer at characteristic angles from the antisolar direction. The red rays form the outer edge at about 42°, while violet forms the inner edge at about 40°. Thus, near the top of the visible arch, the red band lies above the violet band. This is precisely the ordering described by option (A).

Step 2: Distinguish primary and secondary reflection paths The primary path has one internal reflection inside a drop, whereas the secondary path has two. Therefore option (C) is false for the primary bow. Internal reflection is the relevant process; calling it two total internal reflections does not describe the primary ray path. The additional reflection in the secondary path also reverses the observed colour order relative to the primary bow.

Step 3: Compare brightness and eliminate the remaining statements At each encounter with the water surface, some energy leaves the ray path by transmission. A secondary ray undergoes an additional internal reflection, losing more intensity before it reaches the observer; the secondary bow is normally

fainter. Option (D), which says the primary bow is less bright, is therefore false. Option (B) puts violet above red and reverses the primary colour order; that outer-violet ordering belongs to the secondary bow instead.

Step 4: Conclusion In a primary rainbow, red is on the outer or top edge and violet is on the inner or bottom edge. Hence, the correct option is (A).

Quick Tip to Solve: Remember the primary bow as one internal reflection with red outside; the secondary bow has two reflections, reversed colours and lower brightness.

46. Time taken by light to travel in two different materials A and B of refractive indices μ_A and μ_B of same thickness is t_1 and t_2 respectively. If $t_2 - t_1 = 5 \times 10^{-10}$ s and the ratio of μ_A to μ_B is $1:2$. Then, the thickness of material, in meter is: (Given v_{A} and v_{B} are velocities of light in A and B materials respectively.) [4 marks]

(A) $5 \times 10^{-10}\, v_{\mathrm{A}}$ m

(B) 5×10^{-10} m

(C) 1.5×10^{-10} m

(D) $5 \times 10^{-10}\, v_{\mathrm{B}}$ m

Correct Answer: (A) $5 \times 10^{-10}\, v_{\mathrm{A}}$ m

Solution:

Concept:

- For a transparent medium, $v = c/\mu$, where μ is its refractive index and c is the vacuum speed of light.
- For two slabs of equal thickness d, the travel times are $t_1 = d/v_A$ and $t_2 = d/v_B$. Their ratio equals the ratio of refractive indices.

Step 1: Convert the refractive-index ratio to a velocity ratio The specified ratio is $\mu_A : \mu_B = 1:2$. Since refractive index is inversely proportional to the speed in the medium,

$$\frac{v_A}{v_B} = \frac{\mu_B}{\mu_A} = 2, \qquad v_B = \frac{v_A}{2}.$$

The more optically dense material B therefore takes twice as long as A to transmit light through the same thickness.

Step 2: Express the measured delay in terms of thickness Subtract the two transit times and substitute the velocity relation:

$$t_2 - t_1 = \frac{d}{v_B} - \frac{d}{v_A} = \frac{2d}{v_A} - \frac{d}{v_A} = \frac{d}{v_A}.$$

Using the measured time difference gives

$$d = (t_2 - t_1)v_A = (5 \times 10^{-10}\,\mathrm{s})v_A.$$

Step 3: Check units and reject the other expressions When v_A is inserted in metres per second, the

numerical expression for thickness in metres is $5 \times 10^{-10} v_A$, as in option (A). Options (B) and (C) specify fixed lengths even though no absolute refractive index or speed is given; the ratio alone cannot determine a fixed length. Option (D) would give half the required thickness because $v_B = v_A/2$; the equivalent expression using v_B is $10^{-9} v_B$, not $5 \times 10^{-10} v_B$.

Step 4: Conclusion The common thickness is $\boxed{d = (5 \times 10^{-10}\,\text{s}) v_A}$, or $5 \times 10^{-10} v_A$ metres when v_A is in SI units. Hence, the correct option is (A).

Quick Tip to Solve: For equal thicknesses, time is proportional to refractive index. A $1:2$ index ratio makes the delay equal to the entire transit time through the faster slab.

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47. A metal exposed to light of wavelength 800 nm and and emits photoelectrons with a certain kinetic energy. The maximum kinetic energy of photo-electron doubles when light of wavelength 500 nm is used. The workfunction of the metal is : (Take hc $= 1230\,\text{eV} - \text{nm}$). [4 marks]

(A) 1.537 eV

(B) 2.46 eV

(C) 0.615 eV

(D) 1.23 eV

Correct Answer: (C) 0.615 eV

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function and hc/λ is the incident photon energy.
- The work function depends on the metal and remains unchanged when the wavelength changes. Photon energy and work function must be expressed in the same units.

Step 1: Write the photon energies at both wavelengths Use the value supplied in the question, $hc = 1230\,\text{eV nm}$, together with wavelengths in nanometres:

$$E_1 = \frac{1230}{800} = 1.5375\,\text{eV}, \qquad E_2 = \frac{1230}{500} = 2.46\,\text{eV}.$$

The corresponding maximum kinetic energies are $K_1 = 1.5375 - \phi$ and $K_2 = 2.46 - \phi$, in electronvolts.

Step 2: Apply the doubling condition and solve The shorter wavelength produces twice the original maximum kinetic energy, so

$$2.46 - \phi = 2(1.5375 - \phi).$$

$$2.46 - \phi = 3.075 - 2\phi \quad \Longrightarrow \quad \phi = 0.615\,\text{eV}.$$

This is positive and smaller than both photon energies, so photoemission is possible in both trials.

Step 3: Verify the kinetic energies and eliminate distractors The result gives $K_1 = 0.9225\,\text{eV}$ and $K_2 = 1.845\,\text{eV}$, exactly twice as large. Option (A) is approximately the energy of an $800\,\text{nm}$ photon, rather than the work function. Option (B) is the energy of a $500\,\text{nm}$ photon. Option (D), $1.23\,\text{eV}$, would give kinetic energies $0.3075\,\text{eV}$ and $1.23\,\text{eV}$, whose ratio is four rather than two.

Step 4: Conclusion The metal's work function is $\boxed{0.615\,\text{eV}}$. Hence, the correct option is (C).

Quick Tip to Solve: For $K_2 = 2K_1$, eliminate kinetic energy immediately: $\phi = 2hc/\lambda_1 - hc/\lambda_2$. Keep the given value of hc throughout.

48. The momentum of an electron revolving in n^{th} orbit is given by :
(Symbols have their usual meanings) [4 marks]

(A) $\frac{\text{nh}}{2\pi\text{r}}$
(B) $\frac{nh}{2r}$
(C) $\frac{\text{nh}}{2\pi}$
(D) $\frac{2\pi r}{\text{nh}}$

Correct Answer: (A) $\frac{\text{nh}}{2\pi\text{r}}$

Solution:

Concept:

- Bohr's quantisation condition for a circular orbit is $L = mvr = n\hbar = nh/(2\pi)$, where n is a positive integer and h is Planck's constant.
- Linear momentum has magnitude $p = mv$. For circular motion, the magnitude of orbital angular momentum is $L = rp$, since the radius is perpendicular to the instantaneous velocity.

Step 1: Relate angular momentum to linear momentum Let r be the radius of the specified nth orbit. The angular momentum is the cross product $\vec{L} = \vec{r} \times \vec{p}$; because the momentum is tangential to the orbit, its magnitude is $L = rp$. The quantity $nh/(2\pi)$ is therefore angular momentum and must be divided by r to obtain linear momentum.

Step 2: Substitute the quantisation condition Combine $rp = nh/(2\pi)$ with the radius of the orbit:

$$p = \frac{L}{r} = \frac{nh}{2\pi r}.$$

This gives the magnitude of the instantaneous tangential electron momentum. Its direction changes continuously as the electron revolves, while its magnitude remains constant in a Bohr stationary circular orbit.

Step 3: Check dimensions and the remaining choices Planck's constant has units $\text{kg m}^2\,\text{s}^{-1}$, so dividing it by radius gives kg m s^{-1}, the required momentum units. Option (B) omits the factor π in the Bohr condition. Option (C) is $n\hbar$ itself and represents angular momentum rather than linear momentum. Option (D) is the reciprocal of the correct expression and therefore has inverse-momentum units. Only option (A) has both the correct dimensions and coefficient.

Step 4: Conclusion The electron momentum in the nth orbit is $\boxed{p = nh/(2\pi r)}$. Hence, the correct option is (A).

Quick Tip to Solve: Bohr quantisation fixes angular momentum $rp = n\hbar$; divide by the orbit radius once to obtain linear momentum, keeping the factor 2π.

49. The magnetic moment of an electron (e) revolving in an orbit around nucleus with an orbital angular momentum is given by : [4 marks]

(A) $\vec{\mu}_{\mathrm{L}} = \frac{\vec{\mathrm{eL}}}{2\ \mathrm{m}}$

(B) $\vec{\mu}_{\mathrm{L}} = -\frac{\vec{\mathrm{eL}}}{2\ \mathrm{m}}$

(C) $\vec{\mu}_{l} = -\frac{\vec{eL}}{\mathrm{m}}$

(D) $\vec{\mu}_{l} = \frac{2\vec{\mathrm{eL}}}{\mathrm{m}}$

Correct Answer: (B) $\vec{\mu}_{\mathrm{L}} = -\frac{\vec{\mathrm{eL}}}{2\ \mathrm{m}}$

Solution:

Concept:

- A particle of charge q moving in a circular orbit constitutes a current loop. Its orbital magnetic moment and angular momentum satisfy $\vec{\mu}_L = (q/2m)\vec{L}$.
- For an electron, $q = -e$, with $e > 0$ denoting the magnitude of its charge. The conventional-current direction is opposite to the electron's direction of revolution.

Step 1: Obtain the orbital current and magnetic moment For an electron orbit of radius r and speed v, the period is $T = 2\pi r/v$. The magnitude of the equivalent current is $I = e/T$. Multiplying it by the area of the circular orbit gives

$$\mu_L = I\pi r^2 = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}.$$

Step 2: Relate the magnitude to angular momentum The magnitude of orbital angular momentum is $L = mvr$. Consequently,

$$\mu_L = \frac{e}{2m}L.$$

This equation gives only magnitudes. The orbital angular momentum direction follows the electron's motion, while the magnetic moment follows conventional current around the loop. Since the electron has negative charge, the two vectors are antiparallel.

Step 3: Insert the sign and compare all options Including this direction explicitly gives

$$\vec{\mu}_L = -\frac{e}{2m}\vec{L}.$$

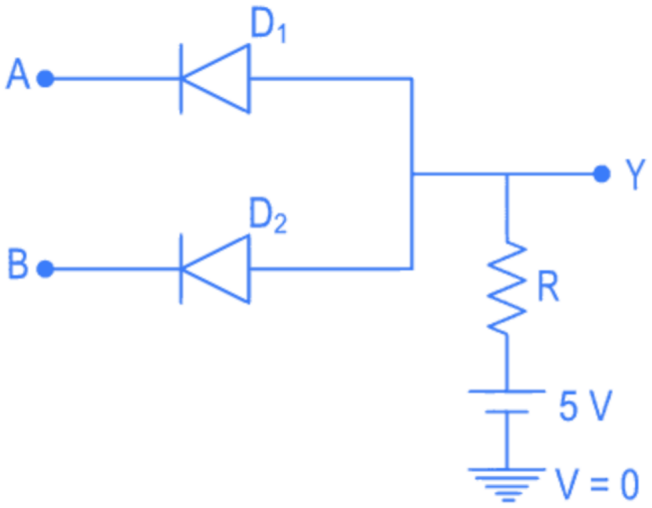
Option (A) has the correct magnitude but makes the vectors parallel, as for a positive charge. Option (C) has the correct negative sign but twice the orbital coefficient; the orbital relation contains $2m$, not m. Option (D) has both the wrong sign and a magnitude four times the correct value. No spin contribution is being requested: the relation here is for orbital angular momentum alone.

Step 4: Conclusion The orbital magnetic moment is $\boxed{\vec{\mu}_L = -e\vec{L}/(2m)}$, antiparallel to the electron's orbital angular momentum. Hence, the correct option is (B).

Quick Tip to Solve: First use $\vec{\mu} = (q/2m)\vec{L}$, then substitute the signed charge. For an electron, using $q = -e$ supplies the required antiparallel direction.

50. In the circuit, the logical value of $A = 1$ or $B = 1$ when potential at A or B is 5 V and the logical value of $A = 0$ or $B = 0$ when potential at A or B is 0 V.

The truth table of the given circuit will be : [4 marks]



(A)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

(B)

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

(C)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

(D)

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

Correct Answer: (A)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

Solution:

Concept:

- A diode conducts when its anode is at a higher potential than its cathode; in this circuit the cathodes face the input terminals.
- The resistor pulls the output towards $5\,\mathrm{V}$, but a conducting diode connected to a low input clamps the output low. The logic levels are 0 V and 5 V.

Step 1: Check all input combinations For $A = B = 0$, both input diodes can conduct and the output is clamped low. For $A = 1, B = 0$, the diode connected to B conducts and holds Y low; the other diode cannot pull Y high against this clamp. Similarly, for $A = 0, B = 1$, the diode connected to A conducts and Y remains low.

Step 2: Identify the high-output state For $A = B = 1$, neither diode provides a path to a low-potential input. The pull-up resistor therefore makes the output high. In the printed row order, the outputs are $0, 0, 0, 1$. Thus the circuit implements $Y = A \cdot B$, an AND gate. The OR table incorrectly makes either single-high input sufficient; the all-zero table misses the both-high state, and the NAND table reverses every output.

Step 3: Conclusion The output is 1 only when both inputs are 1. Hence, the correct option is (A).

Quick Tip to Solve: Check diode orientation before naming the gate: input-side cathodes with an output pull-up implement positive-logic AND.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A car is moving with speed of $150\ \mathrm{km/h}$ and after applying the break it will move $27\ \mathrm{m}$ before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling _____ m distance. [4 marks]

Solution:

Concept:

- For uniform acceleration, $v^2 = u^2 + 2as$, where u and v are initial and final speeds, a is the signed acceleration, and s is the displacement.
- For a fixed braking retardation of magnitude $b > 0$, stopping means $v = 0$, so $s = u^2/(2b)$. Stopping distance is proportional to the square of the initial speed.

Step 1: Compare the two stopping motions Let the original speed be $u_1 = 150\,\mathrm{km\,h^{-1}}$, and let the new speed be $u_2 = u_1/3 = 50\,\mathrm{km\,h^{-1}}$. The same braking conditions give the same value of b. Consequently,

$$\frac{s_2}{s_1} = \frac{u_2^2/(2b)}{u_1^2/(2b)} = \left(\frac{u_2}{u_1}\right)^2 = \frac{1}{9}.$$

No conversion of the speed units is necessary because their ratio is dimensionless.

Step 2: Calculate the reduced distance The first stopping distance is $s_1 = 27\,\mathrm{m}$. Therefore,

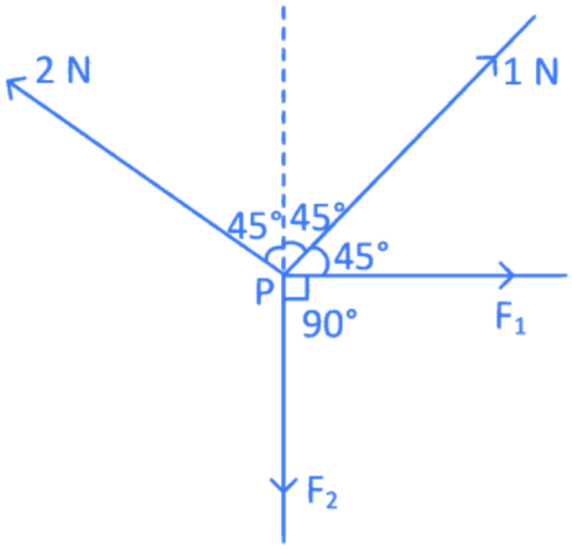
$$s_2 = 27\left(\frac{1}{9}\right) = 3\,\mathrm{m}.$$

Dividing 27 by 3 would be incorrect: that would assume a linear dependence on speed, whereas the kinetic energy to be removed varies as its square.

Step 3: Conclusion The car stops after travelling $\boxed{3\,\mathrm{m}}$. The numerical entry is 3.

Quick Tip to Solve: For unchanged braking force, divide the stopping distance by nine when the initial speed is divided by three.

52. Four forces are acting at a point P in equilibrium as shown in figure. The ratio of force F_1 to F_2 is $1 : x$ where $x =$ _____. [4 marks]



Solution:

Concept:

- A point is in equilibrium when both Cartesian components of the resultant force vanish: $\sum F_x = 0$ and $\sum F_y = 0$.
- The 1 N force points at $45°$ above the positive horizontal; the 2 N force points at $135°$. Their vertical components add, while their horizontal components oppose each other.

Step 1: Balance horizontal components Take rightwards as positive x and upwards as positive y. The force F_1 acts rightwards. The horizontal balance is

$$F_1 + 1\cos 45° - 2\cos 45° = 0.$$

Hence

$$F_1 = (2-1)\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\,\mathrm{N}.$$

The larger inclined force has a leftward component; this is why a rightward force is needed.

Step 2: Balance vertical components and form the ratio The downward force F_2 balances both upward components, so

$$F_2 = 1\sin 45^\circ + 2\sin 45^\circ = \frac{3}{\sqrt{2}}\,\text{N}.$$

Therefore,

$$F_1 : F_2 = \frac{1}{\sqrt{2}} : \frac{3}{\sqrt{2}} = 1 : 3.$$

Matching the required form $1 : x$ gives x directly; it is a dimensionless ratio, not a force.

Step 3: Conclusion $\boxed{x = 3}$, with $F_1 : F_2 = 1 : 3$.

Quick Tip to Solve: For two forces at symmetric 45-degree directions, use their difference horizontally and their sum vertically.

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53. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force F, its length increases by 5 cm. Another wire of the same material of length 4L and radius 4r is pulled by a force 4F under same conditions. The increase in length of this wire is _____ cm. [4 marks]

Solution:

Concept:

- Within the elastic limit, Young modulus is $Y = (F/A)/(\Delta L/L)$, giving the extension $\Delta L = FL/(AY)$.
- A circular wire has cross-sectional area $A = \pi r^2$. Wires of the same material under the same conditions have the same Y, so extension scales as FL/r^2.

Step 1: Compare the geometric and force factors For the original wire,

$$\Delta L_1 = \frac{FL}{\pi r^2 Y} = 5\,\text{cm}.$$

The second wire has $F_2 = 4F$, $L_2 = 4L$, and $r_2 = 4r$. Its cross-sectional area is therefore $A_2 = \pi(4r)^2 = 16\pi r^2$, not four times the original area.

Step 2: Calculate the extension ratio Substitute these factors into the extension formula:

$$\Delta L_2 = \frac{(4F)(4L)}{\pi(4r)^2 Y} = \frac{16FL}{16\pi r^2 Y} = \Delta L_1.$$

Equivalently,

$$\frac{\Delta L_2}{\Delta L_1} = 4 \times 4 \times \frac{1}{4^2} = 1.$$

Although the tensile stress falls to one quarter, the wire is four times as long, so these effects compensate exactly in its total extension.

Step 3: Conclusion The increase in the second wire's length is $\boxed{5\,\text{cm}}$. The numerical entry is 5.

Quick Tip to Solve: In extension ratios, square the radius multiplier before combining it with force and length multipliers.

54. A unit scale is to be prepared whose length does not change with temperature and remains $20\ \text{cm}$, using a bimetallic strip made of brass and iron each of different length. The length of both components would change in such a way that difference between their lengths remains constant. If length of brass is $40\ \text{cm}$ and length of iron will be _____ cm. $(\alpha_{\text{iron}} = 1.2 \times 10^{-5}\ \text{K}^{-1}$ and $\alpha_{\text{brass}} = 1.8 \times 10^{-5}\ \text{K}^{-1})$. [4 marks]

Solution:

Concept:

- Linear thermal expansion is $\Delta L = \alpha L \Delta T$, with alpha the coefficient of linear expansion.
- The difference of two lengths remains unchanged only if their absolute increases are equal: $\alpha_i L_i \Delta T = \alpha_b L_b \Delta T$. It is not necessary for their fractional increases to be equal.

Step 1: Impose the temperature-independence condition Let L_i be the iron length and $L_b = 40\,\text{cm}$ the brass length. After a temperature change,

$$L_i(T) - L_b(T) = (L_i - L_b) + (\alpha_i L_i - \alpha_b L_b)\Delta T.$$

To remove the temperature-dependent term, require $\alpha_i L_i = \alpha_b L_b$. The common temperature change cancels.

Step 2: Find the iron length and check the scale Using the given coefficients,

$$L_i = \frac{\alpha_b}{\alpha_i} L_b = \frac{1.8 \times 10^{-5}}{1.2 \times 10^{-5}} \times 40 = 60\,\text{cm}.$$

The initial difference is $60 - 40 = 20\,\text{cm}$, exactly the required scale length. Since both components expand by the same amount, that 20 cm difference remains unchanged. Iron must be longer because its expansion coefficient is smaller.

Step 3: Conclusion The iron component must have length $\boxed{60\,\text{cm}}$. The numerical entry is 60.

Quick Tip to Solve: For a constant difference of lengths, set the products alpha times length equal, then check the required initial difference.

55. The volume charge density of a sphere of radius $6\ \text{m}$ is $2\,\mu\text{C}\,\text{cm}^{-3}$. The number of lines of force per unit surface area coming out from the surface of the sphere is _____ $\times 10^{10}\,\text{NC}^{-1}$.
[Given : Permittivity of vacuum $\epsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2\ \text{N}^{-1} - \text{m}^{-2}$) [4 marks]

Solution:

Concept:

- For a uniformly charged sphere, Gauss law gives $E(4\pi R^2) = Q/\varepsilon_0$, because the electric field is radial and constant in magnitude on the surface.
- The enclosed charge is $Q = \rho(4\pi R^3/3)$, so the outward electric flux per unit surface area, or electric field, is $E = \rho R/(3\varepsilon_0)$.

Step 1: Convert the charge density to SI units The density is $2\,\mu\text{C}\,\text{cm}^{-3}$. Since $1\,\mu\text{C} = 10^{-6}\,\text{C}$ and

$1\,\mathrm{cm}^3 = 10^{-6}\,\mathrm{m}^3$,

$$\rho = \frac{2 \times 10^{-6}\,\mathrm{C}}{10^{-6}\,\mathrm{m}^3} = 2\,\mathrm{C\,m^{-3}}.$$

The two conversion factors cancel; retaining an extra factor of 10^{-6} would give the wrong field.

Step 2: Calculate the field and extract the coefficient With $R = 6\,\mathrm{m}$ and the given permittivity,

$$E = \frac{2 \times 6}{3 \times 8.85 \times 10^{-12}} = 4.5198 \times 10^{11}\,\mathrm{N\,C^{-1}}.$$

In the requested form this is

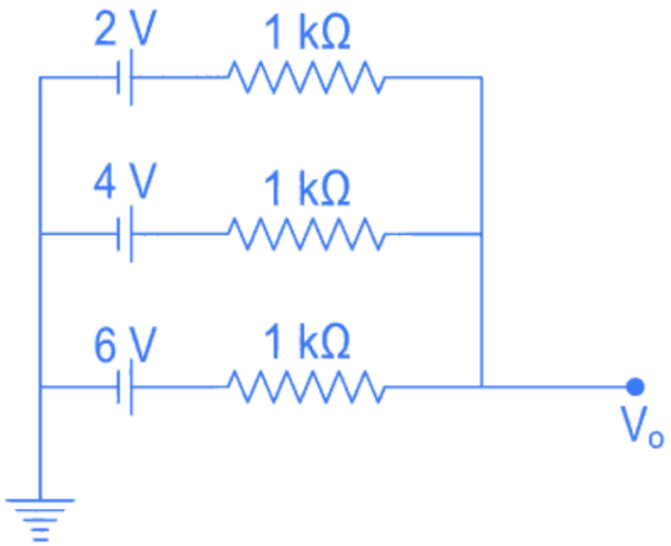
$$E = 45.198 \times 10^{10}\,\mathrm{N\,C^{-1}}.$$

Rounding the numerical coefficient to the integer response gives 45. The field points outwards because the charge density is positive.

Step 3: Conclusion The surface field is approximately $45 \times 10^{10}\,\mathrm{N\,C^{-1}}$, directed outward. The required numerical entry is $\boxed{45}$.

Quick Tip to Solve: Convert a density as charge divided by volume; microcoulomb per cubic centimetre equals coulomb per cubic metre.

56. In the given figure, the value of V_o will be _____ V. [4 marks]



Solution:

Concept:

- At a junction in a steady circuit, the algebraic sum of currents is zero (Kirchhoff current law).
- The output is unloaded. Each branch contains a cell followed by a $1\,\mathrm{k\Omega}$ resistor, and the three cells have their positive terminals towards the resistors.

Step 1: Write the three branch currents The common left terminal is grounded. Thus the potentials

immediately before the resistors are respectively 2 V, 4 V, and 6 V. Taking each branch current towards the output node as positive gives

$$I_1 = \frac{2 - V_o}{1000}, \qquad I_2 = \frac{4 - V_o}{1000}, \qquad I_3 = \frac{6 - V_o}{1000},$$

with currents in amperes when the voltages are in volts. A negative current simply means that branch carries current away from the output.

Step 2: Apply the node equation No current leaves through the open output terminal, so

$$I_1 + I_2 + I_3 = 0.$$

The equal denominators cancel:

$$(2 - V_o) + (4 - V_o) + (6 - V_o) = 0, \qquad 12 - 3V_o = 0.$$

Hence $V_o = 4\,\text{V}$. As a check, the 6 V branch supplies 2 mA, the 2 V branch receives 2 mA, and the 4 V branch carries no current. These branch currents satisfy the junction law.

Step 3: Conclusion The output voltage relative to ground is $\boxed{V_o = 4\,\text{V}}$. The numerical entry is 4.

Quick Tip to Solve: An unloaded node fed by equal resistors sits at the arithmetic mean of the three source potentials.

57. Eight copper wire of length l and diameter d are joined in parallel to form a single composite conductor of resistance R. If a single copper wire of length $2l$ have the same resistance (R) then its diameter will be _____ d. [4 marks]

Solution:

Concept:

- The resistance of a uniform wire is $R_w = \rho L/A$, where ρ is its resistivity, L its length and A its cross-sectional area. For a circular wire, $A = \pi D^2/4$.
- For eight identical resistances connected in parallel, the potential difference across every branch is the same and the total current is eight times the branch current. Their equivalent resistance is therefore $R_w/8$.

Step 1: Find the resistance of the parallel combination Let the resistance of one of the original wires be r. Each has length l, diameter d and area $A = \pi d^2/4$, so

$$r = \frac{\rho l}{A} = \frac{4\rho l}{\pi d^2}.$$

Using the parallel-resistance relation, rather than adding the resistances in series, gives

$$\frac{1}{R} = \frac{8}{r}, \qquad R = \frac{r}{8} = \frac{\rho l}{2\pi d^2}.$$

The copper resistivity is common to all eight wires. The parallel arrangement increases the effective conducting area, which explains why its resistance is smaller than that of one wire.

Step 2: Equate the new wire resistance to this value Write the diameter of the single wire of length $2l$ as D. Since it is also copper, its resistance is

$$R' = \frac{4\rho(2l)}{\pi D^2} = \frac{8\rho l}{\pi D^2}.$$

The required equality $R' = R$ gives

$$\frac{8\rho l}{\pi D^2} = \frac{\rho l}{2\pi d^2} \quad \Longrightarrow \quad D^2 = 16d^2.$$

A diameter is positive, so $D = 4d$. As a check, this diameter makes the new area $16A$; its doubled length then gives $R' = 2\rho l/(16A) = r/8$, exactly the parallel resistance.

Step 3: Conclusion The single wire must have diameter $\boxed{4d}$. Thus the dimensionless number to enter in the blank is $\boxed{4}$.

Quick Tip to Solve: Use $R \propto L/D^2$: eight parallel wires reduce resistance by eight, while doubling length requires an additional factor of two in area. Take the square root only after finding that area factor.

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58. The energy band gap of semiconducting material to produce violet (wavelength = 4000$\overset{o}{A}$) LED is _____ eV. (Round off to the nearest integer). [4 marks]

Solution:

Concept:

- In an LED, radiative recombination of an electron and a hole releases a photon with energy approximately equal to the semiconductor band gap: $E_g = h\nu = hc/\lambda$.
- Use a consistent wavelength unit. The conversions are 1 angstrom $= 10^{-10}\,\mathrm{m}$ and $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$.

Step 1: Convert the wavelength to SI units The specified violet wavelength is 4000 angstrom. Hence

$$\lambda = 4000 \times 10^{-10}\,\mathrm{m} = 4.000 \times 10^{-7}\,\mathrm{m} = 400\,\mathrm{nm}.$$

The gap must supply the energy of a photon at this wavelength. A smaller gap would correspond to a longer wavelength, while a larger gap would correspond to a shorter wavelength. This inverse relation also helps check the numerical result.

Step 2: Calculate the gap and then round it Substitute $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$ and $c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$:

$$E_g = \frac{(6.626 \times 10^{-34})(3.00 \times 10^{8})}{4.000 \times 10^{-7}} = 4.9695 \times 10^{-19}\,\mathrm{J}.$$

Convert the energy per photon into electron volts by dividing by the joule value of one electron volt:

$$E_g = \frac{4.9695 \times 10^{-19}}{1.602 \times 10^{-19}} \simeq 3.102\,\text{eV}.$$

Equivalently, $E_g(\text{eV}) \simeq 1240/\lambda(\text{nm}) = 1240/400 = 3.10$. Both calculations give the same energy. Retain the decimal value until the last step because the problem explicitly asks for the nearest integer.

Step 3: Conclusion The required band gap is approximately $3.10\,\text{eV}$, which rounds to $\boxed{3\,\text{eV}}$. The number to enter is $\boxed{3}$.

Quick Tip to Solve: For LEDs, use $E_g(\text{eV}) \simeq 1240/\lambda(\text{nm})$. Convert angstroms to nanometres before substituting, and round only the final energy.

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59. An observer is riding on a bicycle and moving towards a hill at $18\,\text{kmh}^{-1}$. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 640 Hz and velocity of the sound in air is 320 m/s, the beat frequency between the two sounds heard by observer will be _____ Hz. [4 marks]

Solution:

Concept:

- For a stationary sound source and an observer moving at speed u, the observed frequency is $f(v+u)/v$ when approaching the incoming wave source, and $f(v-u)/v$ when receding. Here v is the sound speed in air.
- A stationary reflecting hill does not change the frequency of the wave in the air. The beat frequency heard by one observer is the absolute difference between the frequencies of the two waves reaching that observer.

Step 1: Find the directly heard frequency The observer rides away from the source behind him. Convert his speed first:

$$u = 18 \times \frac{1000}{3600} = 5\,\text{m}\,\text{s}^{-1}.$$

The direct wave propagates from behind in the same direction as the observer's velocity. The rate at which the observer meets its wavefronts is therefore reduced:

$$f_{\text{direct}} = 640\frac{320-5}{320} = 2(315) = 630\,\text{Hz}.$$

The observer's motion changes what he hears, not the frequency emitted by the stationary source into the surrounding air.

Step 2: Find the reflected frequency and their difference The hill is stationary relative to the air, so it receives and reflects the source frequency $640\,\text{Hz}$. The reflected wave travels back from the hill towards the cyclist. Because the cyclist is moving towards the hill, he meets these wavefronts more rapidly:

$$f_{\text{reflected}} = 640\frac{320+5}{320} = 2(325) = 650\,\text{Hz}.$$

Consequently,

$$f_{\text{beat}} = |650 - 630| = 20\,\text{Hz}.$$

There is no extra moving-reflector shift: the reflecting object is the fixed hill. The two opposite signs arise solely from the cyclist moving away from the direct source and towards the reflected sound.

Step 3: Conclusion The beat frequency is $\boxed{20\,\text{Hz}}$. Thus the required numerical entry is $\boxed{20}$.

Quick Tip to Solve: For a fixed source and fixed reflector with the cyclist between them, subtract $f(v-u)/v$ from $f(v+u)/v$: the beat frequency reduces directly to $2fu/v$.

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60. The required height of a TV tower which can cover the population of 6.03 lakh is h. If the average population density is 100 per square km and the radius of earth is $6400\ \text{km}$, then the value of h will be _____ m. [4 marks]

Solution:

Concept:

- For a transmitting tower of height $h \ll R_E$, the line-of-sight range over Earth is $d \simeq \sqrt{2R_E h}$, where R_E is Earth's radius. The coverage region is approximated by a circle of area $A = \pi d^2$.
- The population covered equals population density multiplied by the coverage area. Use the same length unit for Earth radius, range and tower height until the final conversion.

Step 1: Calculate the area needed to cover the population One lakh is 10^5, so the stated population is

$$P = 6.03 \times 10^5 = 603000.$$

At a uniform average density of $100\,\text{km}^{-2}$, the required coverage area is

$$A = \frac{P}{\text{density}} = \frac{603000}{100} = 6030\,\text{km}^2.$$

This is an area, rather than the radius of the coverage circle. Its range satisfies $d^2 = A/\pi$, giving $d \simeq 43.8\,\text{km}$. Using the population directly as a length would not be dimensionally consistent.

Step 2: Relate that area to the tower height The tangent from the tower top to Earth's surface gives

$$(R_E + h)^2 = R_E^2 + d^2.$$

Since the tower height is very small compared with $6400\,\text{km}$, neglect h^2 to obtain $d^2 \simeq 2R_E h$. Therefore

$$A = 2\pi R_E h, \qquad h = \frac{6030}{2\pi(6400)}\,\text{km} \simeq 0.14995\,\text{km}.$$

Finally convert kilometres into metres:

$$h \simeq 0.14995 \times 1000 = 149.95\,\text{m} \simeq 150\,\text{m}.$$

The result satisfies $h/R_E \ll 1$, so the approximation used for the radio horizon is self-consistent.

Step 3: Conclusion The required height is $\boxed{h \simeq 150\,\text{m}}$. The numerical entry is $\boxed{150}$.

Quick Tip to Solve: Combine the coverage relations before computing: $h = P/(2\pi R_E \,\text{density})$. With radius in kilometres and density per square kilometre, the resulting height is in kilometres.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. SO_2Cl_2 **on reaction with excess of water results into acidic mixture**

$$SO_2Cl_2 + 2H_2O \rightarrow H_2SO_4 + 2HCl$$

16 moles of $NaOH$ **is required for the complete neutralisation of the resultant acidic mixture. The number of moles of** SO_2Cl_2 **used is :** [4 marks]

(A) 16

(B) 8

(C) 4

(D) 2

Correct Answer: (C) 4

Solution:

Concept:

- Each mole of SO_2Cl_2 hydrolyses according to $SO_2Cl_2 + 2\,H_2O \longrightarrow H_2SO_4 + 2\,HCl$. Thus one mole produces one mole of sulphuric acid and two moles of hydrochloric acid.
- Complete neutralisation consumes two moles of $NaOH$ per mole of the diprotic acid H_2SO_4, and one mole of $NaOH$ per mole of the monoprotic acid HCl.

Step 1: Count the alkali required by one mole of reactant Neutralise both acids in the mixture, not just hydrochloric acid:

$$H_2SO_4 + 2\,NaOH \longrightarrow Na_2SO_4 + 2\,H_2O,$$

$$2\,HCl + 2\,NaOH \longrightarrow 2\,NaCl + 2\,H_2O.$$

The one mole of H_2SO_4 uses two moles of alkali. The accompanying two moles of HCl use two more. Hence each mole of SO_2Cl_2 is responsible for consumption of $2 + 2 = 4$ moles of $NaOH$. Excess hydrolysis water supplies the reaction medium and is not another acid to neutralise.

Step 2: Apply the mole ratio and test the choices If x moles of SO_2Cl_2 were used, complete neutralisation requires $4x$ moles of $NaOH$. The given consumption is 16 moles, so

$$4x = 16, \qquad x = \frac{16}{4} = 4\,\text{mol}.$$

As a direct check, 4 moles form 4 moles of H_2SO_4, requiring 8 moles of alkali, and 8 moles of HCl, requiring another 8: the total is 16. Option (A), 16 moles of reactant, would need 64 moles of alkali; option (B), 8 moles, would need 32; option (D), 2 moles, would need only 8. Only option (C) matches the stated neutralisation.

Step 3: Conclusion The amount of SO_2Cl_2 used is $\boxed{4\,\text{mol}}$. Hence, the correct option is (C).

Quick Tip to Solve: First count total acidic protons produced per mole of the hydrolysed compound: H_2SO_4 contributes two and $2HCl$ contributes two, so divide the alkali moles by four.

62. Which of the following sets of quantum numbers is not allowed? [4 marks]

(A) $\text{n} = 3, 1 = 2,\ \text{m}_\text{l} = 0,\ \text{s} = +\frac{1}{2}$

(B) $\text{n} = 3, \text{l} = 2,\ \text{m}_\text{l} = -2,\ \text{s} = +\frac{1}{2}$

(C) $\text{n} = 3, \text{l} = 3,\ \text{m}_\text{l} = -3,\ \text{s} = -\frac{1}{2}$

(D) $\text{n} = 3, \text{l} = 0,\ \text{m}_1 = 0,\ \text{s} = -\frac{1}{2}$

Correct Answer: (C) $\text{n} = 3, \text{l} = 3,\ \text{m}_\text{l} = -3,\ \text{s} = -\frac{1}{2}$

Solution:

Concept:

- The principal quantum number is a positive integer n. For a fixed n, the azimuthal quantum number is restricted to $\ell = 0, 1, \ldots, n-1$; it cannot equal n.
- For an allowed ℓ, the magnetic quantum number is an integer from $-\ell$ to $+\ell$. The electron spin projection can be $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$. The question denotes this projection by s.

Step 1: Check the azimuthal quantum number first All four choices specify $n = 3$. The only allowed azimuthal values for this shell are therefore

$$\ell = 0, 1, 2.$$

These correspond to the $3s$, $3p$ and $3d$ subshells. In option (C), $\ell = 3$, which violates $\ell \leq n - 1 = 2$. An f subshell has $\ell = 3$ and first becomes possible at $n = 4$, so there is no $3f$ subshell. The value $m_\ell = -3$ cannot rescue an inadmissible value of ℓ.

Step 2: Verify that the other three sets satisfy every rule For option (A), interpret the azimuthal entry in its displayed quantum-number list as $\ell = 2$. Its magnetic value $m_\ell = 0$ is within -2 to $+2$, and its positive spin projection is allowed. For option (B), $\ell = 2$, $m_\ell = -2$ and $m_s = +\frac{1}{2}$ are also allowed; the boundary value -2 is included in the magnetic range.

For option (D), $\ell = 0$ forces $m_\ell = 0$, which is exactly the listed value, and $m_s = -\frac{1}{2}$ is permitted. Thus (A), (B) and (D) all represent possible one-electron states in the third shell. In (C), the spin projection itself is allowed and -3 would be a valid magnetic value for an allowed f orbital, but the pairing $n = 3, \ell = 3$ is forbidden.

Step 3: Conclusion The set $n = 3, \ell = 3, m_\ell = -3, m_s = -\frac{1}{2}$ is not allowed because $\ell < n$ is required. Hence, the correct option is (C).

Quick Tip to Solve: Check in the order n, then $\ell < n$, then $|m_\ell| \leq \ell$, then spin. The second check immediately eliminates the forbidden third-shell f state.

63. The depression in freezing point observed for a formic acid solution of concentration $0.5\ \text{mL L}^{-1}$ is 0.0405°C. Density of formic acid is $1.05\ \text{g mL}^{-1}$. The Van't Hoff factor of the formic acid solution is nearly : (Given for water $k_f = 1.86\ \text{k kg mol}^{-1}$) [4 marks]

(A) 0.8

(B) 1.1

(C) 1.9

(D) 2.4

Correct Answer: (C) 1.9

Solution:

Concept:

- Freezing-point depression obeys $\Delta T_f = iK_f m$, where i is the van't Hoff factor and m is the molality of the solute. Thus $i = \Delta T_f/(K_f m)$.
- The concentration is given as a liquid volume per litre of solution. Convert this to solute mass with density, then to moles with the molar mass of formic acid HCOOH, $46\ \text{g mol}^{-1}$. For this very dilute aqueous solution, a litre contains approximately $1\ \text{kg}$ of solvent.

Step 1: Convert the volume concentration into molality Take one litre of the solution. It contains $0.5\,\text{mL}$ of formic acid, whose mass is

$$w = (0.5\,\text{mL})(1.05\,\text{g mL}^{-1}) = 0.525\,\text{g}.$$

Using $M(\text{HCOOH}) = 2(1) + 12 + 2(16) = 46\,\text{g mol}^{-1}$, the amount is

$$n = \frac{0.525}{46} = 0.011413\,\text{mol}.$$

The acid occupies only a tiny fraction of a litre, so the water mass is approximately $1\,\text{kg}$. Consequently

$$m \simeq 0.011413\,\text{mol kg}^{-1}.$$

The supplied $0.5\,\text{mL L}^{-1}$ is not itself a molality and must not be inserted directly into the freezing-point equation.

Step 2: Calculate the particle factor and compare the options If each molecule remained undissociated, the expected depression would be

$$K_f m = 1.86(0.011413) \simeq 0.02123\,\text{K}.$$

The observed depression is larger, so the effective particle factor is

$$i = \frac{0.0405}{0.02123} \simeq 1.908.$$

A Celsius temperature difference has the same numerical value as a kelvin difference. The dimensionless result is therefore nearly 1.9, option (C). The factors 0.8, 1.1 and 2.4 would predict depressions of approximately 0.01698, 0.02335 and 0.05095 K respectively, all inconsistent with the observation.

A factor between one and two is also consistent with partial dissociation of the monoprotic acid into two ionic species; association would instead reduce the particle count.

Step 3: Conclusion The van't Hoff factor is $\boxed{i \simeq 1.9}$, a dimensionless quantity. Hence, the correct option is (C).

Quick Tip to Solve: For a solute concentration stated in $\mathrm{mL\,L^{-1}}$, multiply by density and divide by molar mass before using $\Delta T_f = iK_f m$. Here the dilute-solution approximation makes molarity and molality nearly equal.

---

64. $20\ \mathrm{mL}$ **of** $0.1\ \mathrm{M}\ \mathrm{NH_4OH}$ **is mixed with** $40\ \mathrm{mL}$ **of** $0.05\mathrm{MHCl}$**. The** pH **of the mixture is nearest to :**
(Given : $\mathrm{K_b\,(NH_4OH)} = 1 \times 10^{-5}, \log 2 = 0.30, \log 3 = 0.48, \log 5 = 0.69, \log 7 = 0.84, \log 11 = 1.04$)

[4 marks]

(A) 3.2

(B) 4.2

(C) 5.2

(D) 6.2

Correct Answer: (C) 5.2

Solution:

Concept:

- First compare acid and base amounts using $n = MV$, with volume in litres. Equal amounts of a strong acid and a weak base leave their salt, rather than excess acid or a basic buffer.
- Ammonium ion is a weak acid: $K_a = K_w/K_b$. For salt concentration C, its hydrolysis gives $[\mathrm{H^+}] \simeq \sqrt{K_a C}$ when ionisation is small.

Step 1: Compare the reacting amounts The weak base amount is $0.1 \times 0.020 = 0.002\,\mathrm{mol}$, and the hydrochloric acid amount is $0.05 \times 0.040 = 0.002\,\mathrm{mol}$. They therefore neutralise completely in the one-to-one reaction $\mathrm{NH_4OH + HCl \longrightarrow NH_4Cl + H_2O}$. Neither reagent remains in excess. In particular, treating the mixture as a solution of unreacted strong acid would give an incorrect, excessively low pH.

Step 2: Find the ammonium salt concentration The total solution volume is $0.020 + 0.040 = 0.060\,\mathrm{L}$, so

$$C = \frac{0.002}{0.060} = \frac{1}{30}\,\mathrm{mol\,L^{-1}}.$$

With $K_w = 10^{-14}$ and the given $K_b = 10^{-5}$, the conjugate acid has $K_a = 10^{-9}$. The chloride ion does not cause appreciable hydrolysis.

Step 3: Calculate and compare the pH For $\mathrm{NH_4^+ + H_2O \rightleftharpoons NH_3 + H_3O^+}$,

$$\mathrm{pH} = \tfrac{1}{2}(\mathrm{p}K_a - \log C) = \tfrac{1}{2}(9 + \log 30).$$

Since $\log 30 = 1 + 0.48 = 1.48$, the result is 5.24. The estimated ionisation, about $5.8 \times 10^{-6}\,\mathrm{M}$, is negligible relative to C, validating the approximation. Of 3.2, 4.2, 5.2 and 6.2, only 5.2 is nearest to this result.

Step 4: Conclusion The mixture has $\mathrm{pH} \simeq 5.24$, nearest to 5.2. Hence, the correct option is (C).

Quick Tip to Solve: At exact weak-base/strong-acid equivalence, calculate the conjugate-acid salt pH; the solution is acidic even though no strong acid remains.

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65. The IUPAC nomenclature of an element with electronic configuration [Rn] $5\mathrm{f}^{14}6\mathrm{d}^{1}7\mathrm{s}^{2}$ is : [4 marks]

(A) Unnilbium

(B) Unnilunium

(C) Unnilquadium

(D) Unniltrium

Correct Answer: (D) Unniltrium

Solution:

Concept:

- An electronic configuration gives the atomic number by adding every electron to those in the noble-gas core. Radon contributes 86 electrons.
- The systematic temporary IUPAC name joins the digit roots of the atomic number, followed by ium. The relevant roots are un for 1, nil for 0 and tri for 3.

Step 1: Count the electrons beyond radon The configuration is $[\mathrm{Rn}]\,5f^{14}6d^{1}7s^{2}$. It contains fourteen electrons in the f subshell, one in the d subshell and two in the s subshell beyond the radon core. Hence

$$Z = 86 + 14 + 1 + 2 = 103.$$

The superscripts, rather than the principal quantum numbers 5, 6 and 7, are the electron counts. Adding the principal quantum numbers would not give an atomic number.

Step 2: Construct the systematic name For the digits of 103, write the roots in their printed order:

$$1 \longrightarrow \text{un}, \qquad 0 \longrightarrow \text{nil}, \qquad 3 \longrightarrow \text{tri}.$$

Joining these roots with the ending ium yields un-nil-tri-ium, contracted in the standard spelling to Unniltrium. The corresponding systematic symbol is Unt. The question offers the systematic digit-based names, so this is the nomenclature to compare.

Step 3: Eliminate the other proposed names Unnilbium corresponds to 102, since bi denotes 2. Unnilunium corresponds to 101, with the final un denoting 1. Unnilquadium corresponds to 104, with quad denoting 4. Each conflicts with the electron count 103. Unniltrium alone encodes the required atomic number. The permanent name of element 103 is lawrencium; that does not change which of the four offered systematic names matches the stated configuration.

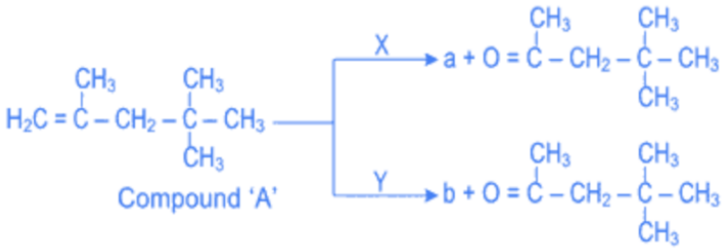
Step 4: Conclusion The stated electronic configuration has $Z = 103$, whose systematic name is Unniltrium. Hence, the correct option is (D).

Quick Tip to Solve: Count electrons first, then translate the atomic-number digits into roots. Do not infer the systematic name directly from the last occupied subshell.

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66. A compound 'A' on reaction with 'X' and 'Y' produces the same major product but different by product 'a' and 'b'. Oxidation of 'a' gives a substance produced by ants.

'X' and 'Y' respectively are [4 marks]



(A) $KMnO_4/H^+$ and dil. $KMnO_4$, 273 K

(B) $KMnO_4$ (dilute), 273 K and $KMnO_4/H^+$

(C) $KMnO_4/H^+$ and $O_3, H_2O/Zn$

(D) $O_3, H_2O/Zn$ and $KMnO_4/H^+$

Correct Answer: (D) $O_3, H_2O/Zn$ and $KMnO_4/H^+$

Solution:

Concept:

- Reductive ozonolysis cleaves an alkene into carbonyl compounds. A terminal $=CH_2$ carbon yields methanal, $HCHO$, while a doubly carbon-substituted alkene carbon gives a ketone.
- Hot acidic permanganate also cleaves the double bond but oxidises the terminal carbon completely to CO_2. Cold dilute permanganate instead produces a vicinal diol.

Step 1: Identify the terminal alkene The displayed compound is $CH_2{=}C(CH_3)CH_2C(CH_3)_3$. Splitting its double bond leaves the substituted carbon attached to a methyl group and the $CH_2C(CH_3)_3$ group. It therefore gives the same ketone, $CH_3COCH_2C(CH_3)_3$, in both cleavage reactions. The terminal CH_2 group is the source of the differing by-products.

Step 2: Use the clue about ants to identify X Ants produce formic acid, $HCOOH$. A substance that oxidises to formic acid is methanal:

$$HCHO + [O] \longrightarrow HCOOH.$$

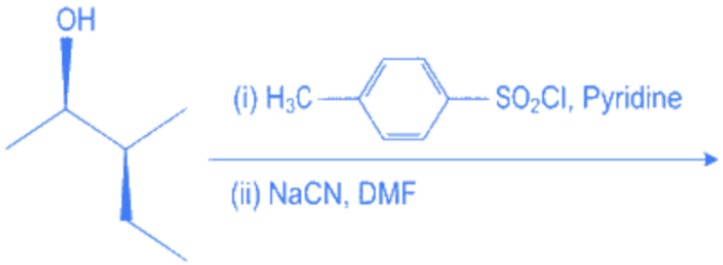
Thus the by-product a is methanal, requiring reductive ozonolysis with O_3 followed by Zn/H_2O. This treatment preserves the aldehyde instead of oxidising it completely. Consequently X must be the ozone/reductive-work-up reagent pair.

Step 3: Assign Y and rule out the alternatives Acidic $KMnO_4$ cleaves the same terminal alkene and gives the ketone together with CO_2, so Y is $KMnO_4/H^+$. Option C reverses the two reagents: its by-product a would be carbon dioxide, which does not oxidise to formic acid. Options A and B each contain cold dilute permanganate at 273 K; that gives a diol rather than the displayed cleavage ketone. Only D has both the required order and the appropriate reaction types.

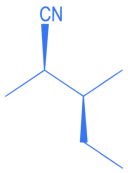
Step 4: Conclusion $X = O_3$, followed by Zn/H_2O, and $Y = KMnO_4/H^+$; a is methanal and b is carbon dioxide. Hence, the correct option is (D).

Quick Tip to Solve: For a terminal alkene, distinguish the one-carbon products: reductive ozonolysis gives methanal, whereas vigorous oxidative cleavage gives carbon dioxide.

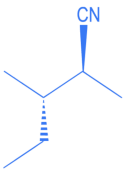
67. Most stable product of the following reaction is: [4 marks]



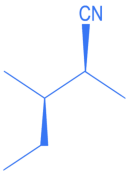
(A) CN on a solid wedge at the left centre; ethyl on a solid wedge at the right centre.



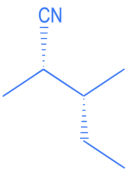
(B) CN on a solid wedge at the right centre; ethyl on a dashed wedge at the left centre.



(C) CN on a solid wedge at the right centre; ethyl on a solid wedge at the left centre.

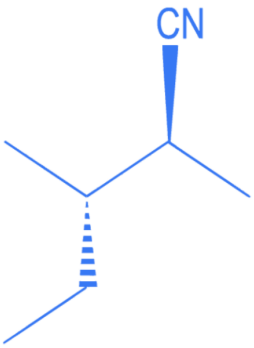


(D) CN on a dashed wedge at the left centre; ethyl on a dashed wedge at the right centre.



Correct Answer: (B)

CN on a solid wedge at the right centre; ethyl on a dashed wedge at the left centre.



Solution:

Concept:

- Tosyl chloride in pyridine converts an alcohol into a tosylate by reaction at oxygen. The carbon–oxygen bond at the stereocentre is retained, so this activation step preserves its spatial configuration.
- Cyanide in polar aprotic DMF is an effective nucleophile for an S_N2 displacement. Backside attack inverts the reacting stereocentre while leaving a neighbouring stereocentre unchanged.

Step 1: Activate the alcohol without changing its carbon geometry The starting alcohol has an OH group drawn on a solid wedge at the left-hand stereocentre. The adjacent stereocentre bears an ethyl group on a solid wedge. With para-toluenesulfonyl chloride and pyridine, the alcohol oxygen

attacks sulfur and loses its proton, producing the corresponding alkyl tosylate. This replaces the oxygen-bound hydrogen by a tosyl group; it does not break the bond connecting the stereogenic carbon to oxygen. Thus neither carbon stereocentre is inverted in this first step.

Step 2: Displace tosylate with cyanide In the second step, CN^- attacks the carbon bearing the tosylate from the opposite side as the leaving group departs. The carbon skeleton is unchanged, and the product is a nitrile, not an isocyanide: $NaCN$ supplies carbon-end attack under these conditions. There is one inversion, specifically at the carbon that initially bore OH. The neighbouring carbon does not participate in bond breaking and retains its configuration.

Step 3: Compare the stereochemical options When the products are oriented consistently, option B alone combines inversion at the substituted carbon with retention at its neighbour. Option A simply replaces the wedged OH by a wedged CN in the same orientation, so it depicts retention at the reacting centre. Option C changes the neighbouring stereocentre in addition to the required substitution geometry. Option D, with both relevant bonds dashed in the original orientation, inverts both centres. Remember that reflecting the zigzag drawing on the page changes left and right; compare three-dimensional configurations rather than wedge labels in isolation.

Step 4: Conclusion Tosylation followed by cyanide displacement gives the nitrile with inversion only at the original alcohol carbon, represented by option (B). Hence, the correct option is (B).

Quick Tip to Solve: Count stereochemical changes step by step: tosylation retains the carbon geometry, then one backside substitution causes one inversion; an untouched neighbouring centre must retain its configuration.

68. Which one of the following reactions does not represent correct combination of substrate and product under the given conditions? [4 marks]

(A) $p\text{-}CH_3C_6H_4COCl \xrightarrow{H_2/Pd\cdot BaSO_4} CH_3C_6H_4CHO$

(B) $p\text{-}CH_3C_6H_4CN \xrightarrow[2.\ H_2O]{1.\ DIBAL-H} CH_3C_6H_4CHO$

(C) $p\text{-}CH_3C_6H_4CO_2C_2H_5 \xrightarrow[2.\ H_2O]{1.\ AlH(i\text{-}Bu)_2} CH_3C_6H_4CHO$

(D) $p\text{-}CH_3C_6H_4CH_2OH \xrightarrow[H_2SO_4,\ H_2O]{Na_2Cr_2O_7} CH_3C_6H_4CHO$

Correct Answer: (D) $p\text{-}CH_3C_6H_4CH_2OH \xrightarrow[H_2SO_4,\ H_2O]{Na_2Cr_2O_7} CH_3C_6H_4CHO$

Solution:

Concept:

- Acid chlorides undergo Rosenmund reduction with hydrogen over poisoned palladium to form aldehydes. DIBAL-H can reduce nitriles and esters selectively to aldehydes after hydrolytic work-up.
- Acidified aqueous dichromate is a strong oxidant. It converts a primary alcohol through the aldehyde stage to the carboxylic acid; it does not normally stop at the aldehyde in aqueous acid.

Step 1: Check the acid-chloride and nitrile reductions In A, the substrate is para-methylbenzoyl chloride. $H_2/Pd\cdot BaSO_4$ is the Rosenmund reagent, which converts the acid chloride group into $-CHO$ while leaving the aromatic methyl group unchanged. Therefore the displayed para-methylbenzaldehyde is correct. In B, DIBAL-H partially reduces the nitrile group; hydrolysis of the

resulting intermediate gives the same aldehyde. This is also a standard correct substrate–product combination.

Step 2: Check the ester reduction Option C shows the para-methylbenzoic acid ethyl ester treated with $\mathrm{AlH(i\text{-}Bu)_2}$, namely DIBAL-H, followed by water. Controlled partial reduction of an ester with this reagent yields an aldehyde after work-up. The ethoxy group is removed during the conversion, while the aromatic ring and its methyl substituent remain. Thus the proposed para-methylbenzaldehyde fits the selective reduction shown; this is not the incorrect combination.

Step 3: Identify the oxidation that proceeds too far In D the substrate is para-methylbenzyl alcohol, with a primary $-\mathrm{CH_2OH}$ group. Aqueous $\mathrm{Na_2Cr_2O_7/H_2SO_4}$ first produces the aldehyde, but the aldehyde hydrates in water and is readily oxidised further. The expected product at this functional group is para-methylbenzoic acid, $\mathrm{p\text{-}CH_3C_6H_4COOH}$, rather than the aldehyde drawn. An aldehyde-selective oxidation would require a milder suitable reagent or removal under controlled conditions. The given aqueous strong oxidant therefore makes D the incorrect representation.

Step 4: Conclusion Option D incorrectly shows a primary alcohol stopping at the aldehyde under aqueous acidified dichromate; the carboxylic acid is expected. Hence, the correct option is (D).

Quick Tip to Solve: For primary alcohol oxidations, inspect both the oxidant and the medium. Strong aqueous chromium(VI) conditions normally continue beyond the aldehyde to the acid.

69. An organic compound 'A' on reaction with NH_3 followed by heating gives compound B. Which on further strong heating gives compound C ($C_8H_5NO_2$). Compound C on sequential reaction with ethanolic KOH, alkyl chloride and hydrolysis with alkali gives a primary amine. The compound A is : [4 marks]

(A) $\mathrm{1,2-C_6H_4(CHO)_2}$

(B) $\mathrm{1,2-C_6H_4(CHO)(COOH)}$

(C) $\mathrm{1,2-C_6H_4(COOH)_2}$

(D) $\mathrm{1,2-C_6H_4(CHO)(OH)}$

Correct Answer: (C) $\mathrm{1,2-C_6H_4(COOH)_2}$

Solution:

Concept:

- Gabriel synthesis uses phthalimide: its acidic nitrogen hydrogen is removed by ethanolic alkali, the anion is alkylated, and hydrolysis releases a primary amine.
- Two adjacent carboxyl groups on benzene can form a five-membered cyclic imide with ammonia on heating. Phthalimide has the molecular formula $\mathrm{C_8H_5NO_2}$.

Step 1: Identify the compound used in the amine-forming sequence The sequence ethanolic KOH, alkyl chloride, then alkaline hydrolysis is diagnostic of Gabriel phthalimide synthesis. Therefore C must be phthalimide, the cyclic imide with two carbonyl groups on adjacent ring positions and one NH linking them. Its benzene portion contributes $\mathrm{C_6H_4}$, and the imide adds $\mathrm{C_2HNO_2}$, giving the stated total $\mathrm{C_8H_5NO_2}$. This formula supports the identification independently of the reaction name.

Step 2: Trace C back to the starting acid Phthalic acid, $1,2-C_6H_4(COOH)_2$, reacts with ammonia to form ammonium salt species. Heating causes dehydration and amide formation, and stronger heating promotes cyclisation to the imide. The overall change can be represented as

$$C_6H_4(COOH)_2 + NH_3 \longrightarrow C_6H_4(CO)_2NH + 2\,H_2O.$$

The ortho arrangement is essential because it places the two carbonyl-bearing groups close enough to form the favourable five-membered imide ring.

Step 3: Verify the amine synthesis and eliminate other options Potassium phthalimide attacks an appropriate alkyl chloride to give N-alkylphthalimide; hydrolysis then cleaves its acyl–nitrogen bonds and liberates RNH_2. Option C supplies the required two adjacent carboxylic acid groups. Option A has two aldehydes, B has only one acid group and one aldehyde, and D has an aldehyde and a phenolic OH. None of A, B or D provides the dicarboxylic-acid precursor for the described phthalimide formation without additional reactions that are not given.

Step 4: Conclusion A is phthalic acid, $1,2-C_6H_4(COOH)_2$, which yields phthalimide on treatment with ammonia and heating. Hence, the correct option is (C).

Quick Tip to Solve: Recognise ethanolic KOH, alkyl halide and hydrolysis as the Gabriel sequence; then work backward from phthalimide to an ortho-dicarboxylic acid.

70. During the denaturation of proteins, which of these structures will remain intact? [4 marks]

(A) Primary

(B) Secondary

(C) Tertiary

(D) Quaternary

Correct Answer: (A) Primary

Solution:

Concept:

- The primary structure of a protein is the ordered amino-acid sequence linked by covalent peptide bonds. Secondary, tertiary and quaternary structures describe increasingly higher levels of folding and association.
- Denaturation disrupts the interactions maintaining the native conformation, such as hydrogen bonds and many other noncovalent interactions. Ordinary denaturation does not hydrolyse the peptide-bond backbone.

Step 1: Separate sequence from three-dimensional conformation A protein chain contains amino acids joined through $-CO-NH-$ peptide linkages. The order of those residues defines its primary structure. Folding of the chain into helices or sheets, further folding into a compact shape and association of multiple chains are separate structural levels. A change in the overall shape therefore need not change which amino acids are joined to each other or their sequence.

Step 2: Apply the effect of denaturation Heating or suitable changes in chemical conditions can

disrupt the stabilising interactions of the native protein. The chain unfolds, losing its characteristic secondary and tertiary arrangement; where several chains associate, their quaternary organisation may also be disturbed. This loss of the native shape commonly destroys biological activity. It is distinct from hydrolysis, in which peptide bonds are actually broken and the sequence is split into smaller fragments.

Step 3: Compare the four levels offered Because peptide bonds survive ordinary denaturation, primary structure remains intact. Secondary structure depends strongly on organised backbone hydrogen bonding and is affected by unfolding, so B is not the protected level. Tertiary structure is the overall folded form of a chain and is disrupted, eliminating C. Quaternary structure concerns the arrangement of subunits and is also sensitive to denaturing conditions, eliminating D. Thus the structural information that remains through denaturation is the amino-acid sequence represented by A.

Step 4: Conclusion The primary structure remains intact because denaturation does not normally cleave the covalent peptide-bond sequence. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish denaturation from hydrolysis: denaturation unfolds the protein, whereas hydrolysis cuts the peptide backbone and changes its primary structure.

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71. Given below are two statements :

Statement I : On heating with $KHSO_4$, glycerol is dehydrated and acrolein is formed.

Statement II : Acrolein has fruity odour and can be used to test glycerol's presence.

Choose the correct option. [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (C) Statement I is correct but Statement II is incorrect.

Solution:

Concept:

- Glycerol is a trihydric alcohol, $HOCH_2-CHOH-CH_2OH$. Heating it with potassium hydrogen sulphate removes water and forms the unsaturated aldehyde acrolein.
- An observation identifies the product only when its characteristic property is stated correctly. Acrolein has a sharp, pungent, irritating odour; the usual fruity smell is associated with many esters.

Step 1: Check the dehydration statement Potassium hydrogen sulphate acts as a dehydrating agent on heating. Glycerol loses two molecules of water, giving prop-2-enal, commonly called acrolein:

$$HOCH_2-CHOH-CH_2OH \xrightarrow[\text{heat}]{KHSO_4} CH_2=CH-CHO + 2\,H_2O.$$

The three carbon atoms remain in the organic product; removal of water creates its double bond and

aldehyde group. Thus Statement I gives the correct reagent, transformation and product.

Step 2: Check the stated odour and compare the choices The acrolein produced has a characteristic pungent, disagreeable and irritating smell. Its formation is indeed the basis of the acrolein test for glycerol, but describing its odour as fruity is false. Since Statement II joins the odour claim to the test claim, the whole statement is incorrect. Option (A) wrongly accepts the fruity-odour claim. Option (B) rejects the valid dehydration reaction, and option (D) reverses the truth values of both statements. Only the combination in option (C) is consistent with the reaction and the observation.

Step 3: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: For the acrolein test, connect glycerol plus a dehydrating agent with a pungent aldehyde; do not confuse that odour with the fruity smell of esters.

72. Match List I with List II

	List I		List II
(A)	$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$	(I)	Cu
(B)	$CO(g) + 3H_2(g) \rightarrow CH_4(g) + H_2O(g)$	(II)	$Cu/ZnO - Cr_2O_3$
(C)	$CO(g) + H_2(g) \rightarrow HCHO(g)$	(III)	$Fe_xO_y + K_2O + Al_2O_3$
(D)	$CO(g) + 2H_2(g) \rightarrow CH_3OH(g)$	(IV)	Ni

Choose the correct answer from the options given below : [4 marks]

(A) (A) – (II), (B) – (IV), (C) – (I), (D) – (III)

(B) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

(C) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

(D) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

Correct Answer: (C) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Solution:

Concept:

- A catalyst is matched to a particular reaction and desired product; the reactants alone do not determine the correct catalyst. Carbon monoxide and hydrogen can give different products under different catalytic conditions.
- The Haber process uses an iron catalyst with promoters, methanation uses nickel, and methanol manufacture uses a copper-based mixed catalyst. These known associations determine the matching code.

Step 1: Match ammonia and methane formation Reaction (A), $N_2 + 3\,H_2 \longrightarrow 2\,NH_3$, is ammonia synthesis by the Haber process. The iron-containing catalyst system with K_2O and Al_2O_3 promoters is entry (III), so (A)–(III). Reaction (B), $CO + 3\,H_2 \longrightarrow CH_4 + H_2O$, is methanation of carbon monoxide. Nickel promotes this hydrogenation, giving (B)–(IV). The amount of hydrogen and the final product distinguish methanation from the two oxygen-containing products in the other rows.

Step 2: Match formaldehyde and methanol, then test each code For reaction (C), $CO + H_2 \longrightarrow HCHO$,

the copper catalyst is entry (I), so (C)–(I). For reaction (D), $CO + 2\,H_2 \longrightarrow CH_3OH$, the listed copper/zinc oxide/chromium oxide catalyst system is entry (II), so (D)–(II). The complete sequence is therefore (III), (IV), (I), (II). Options (A) and (B) put the methanol catalyst against ammonia and the ammonia catalyst against methanol, contradicting those standard processes. Option (D) exchanges the catalysts for methanation and formaldehyde formation. Option (C) preserves all four associations, including the nickel assignment that distinguishes it from option (D).

Step 3: Conclusion The catalyst-code sequence for the four reactions in order is (III), (IV), (I), (II). Hence, the correct option is (C).

Quick Tip to Solve: Anchor a catalyst-matching problem with Haber process–iron and methanation–nickel, then check the remaining product-specific pairings.

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73. The compound(s) that is (are) removed as slag during the extraction of copper is :

(A) CaO

(B) FeO

(C) Al_2O_3

(D) ZnO

(E) NiO

Choose the correct answer from the options given below: [4 marks]

(A) (C), (D) only

(B) (A), (B), (E) only

(C) (A), (B) only

(D) (B) only

Correct Answer: (D) (B) only

Solution:

Concept:

- A flux reacts with gangue or an impurity oxide to form a fusible slag that can be separated from the molten metal-bearing phase. Acidic silica combines with the basic iron(II) oxide impurity in copper extraction.
- In the standard extraction of copper from copper pyrites, iron sulphide is oxidised preferentially to iron oxide. Silica added during smelting removes this oxide as iron silicate.

Step 1: Identify the oxide formed from the iron impurity Copper pyrites contains iron as well as copper. During the oxidation and smelting stages, iron sulphide gives iron(II) oxide. One way to express this conversion is

$$2\,FeS + 3\,O_2 \longrightarrow 2\,FeO + 2\,SO_2.$$

The resulting FeO must be separated from the copper-bearing material. It is this iron oxide impurity, rather than a copper oxide required for subsequent metal recovery, that is targeted by the silica flux in the usual copper-smelting reaction.

Step 2: Form the slag and select the listed substance Silica reacts with iron(II) oxide to form iron

silicate:

$$\mathrm{FeO} + \mathrm{SiO_2} \longrightarrow \mathrm{FeSiO_3}.$$

The fusible $\mathrm{FeSiO_3}$ is the slag. Thus the substance from the given list that is removed by conversion into this slag is (B), FeO. The answer does not mean free FeO itself is the final silicate phase; it is the impurity consumed in making that phase. Option (A) selects alumina and zinc oxide instead. Options (B) and (C) include calcium oxide in addition to iron oxide, and option (B) also adds nickel oxide; those additions are not the standard silica-flux removal step for copper pyrites. The combination containing only the listed iron oxide is option (D).

Step 3: Conclusion Iron(II) oxide is removed as iron silicate slag. Hence, the correct option is (D).

Quick Tip to Solve: Write the slag-forming equation first: acidic silica plus basic FeO gives iron silicate. Match the impurity consumed, rather than calling the impurity itself the final slag.

74. The reaction of $\mathrm{H_2O_2}$ with potassium permanganate in acidic medium leads to the formation of mainly : [4 marks]

(A) $\mathrm{Mn^{2+}}$

(B) $\mathrm{Mn^{4+}}$

(C) $\mathrm{Mn^{3+}}$

(D) $\mathrm{Mn^{6+}}$

Correct Answer: (A) $\mathrm{Mn^{2+}}$

Solution:

Concept:

- Hydrogen peroxide can act either as an oxidising agent or as a reducing agent. Against strongly oxidising acidified permanganate, it acts as the reducing agent and is itself oxidised to oxygen.
- The reduction product of permanganate depends on the medium. In acidic solution, manganese is reduced from oxidation state $+7$ in $\mathrm{MnO_4^-}$ to $+2$ in $\mathrm{Mn^{2+}}$.

Step 1: Write the two half-reactions in acidic medium The reduction half-reaction is

$$\mathrm{MnO_4^-} + 8\,\mathrm{H^+} + 5\,\mathrm{e^-} \longrightarrow \mathrm{Mn^{2+}} + 4\,\mathrm{H_2O}.$$

The peroxide oxidation half-reaction is

$$\mathrm{H_2O_2} \longrightarrow \mathrm{O_2} + 2\,\mathrm{H^+} + 2\,\mathrm{e^-}.$$

Oxygen changes from oxidation state -1 in peroxide to zero in molecular oxygen, releasing electrons. Manganese accepts five electrons per permanganate ion. Both mass and charge are balanced in each half-reaction, which fixes the identity of the manganese product without relying on the colour change alone.

Step 2: Combine the reactions and distinguish the other products Multiply the first equation by two

and the second by five, then cancel ten electrons and the common hydrogen ions:

$$2\,MnO_4^- + 5\,H_2O_2 + 6\,H^+ \longrightarrow 2\,Mn^{2+} + 5\,O_2 + 8\,H_2O.$$

Hence the main manganese species is Mn^{2+}. Manganese(IV), often present as MnO_2, is characteristic of permanganate reduction in neutral or weakly alkaline medium, not the specified acidic medium. Manganese(VI) as manganate is associated with strongly alkaline conditions. Manganese(III) is not the normal final product of this acidified permanganate-peroxide reaction. These observations eliminate options (B), (D) and (C), respectively.

Step 3: Conclusion The manganese product is mainly Mn^{2+}. Hence, the correct option is (A).

Quick Tip to Solve: For a permanganate redox question, mark the medium first: acidic conditions normally lead to Mn(II), whereas Mn(IV) and Mn(VI) belong to different media.

75. Choose the correct order of density of the alkali metals : [4 marks]

(A) Li < K < Na < Rb < Cs

(B) Li < Na < K < Rb < Cs

(C) Cs < Rb < K < Na < Li

(D) Li < Na < K < Cs < Rb

Correct Answer: (A) Li < K < Na < Rb < Cs

Solution:

Concept:

- The density of a metal is mass per unit volume, $\rho = m/V$. A trend in atomic mass alone cannot establish the density order because atomic volume also changes down a group.
- Alkali-metal densities generally increase down Group 1, but potassium is less dense than sodium. This sodium-potassium exception must be retained when arranging the elements.

Step 1: Compare the lighter three alkali metals Representative densities near room temperature are approximately

$$\rho(Li) = 0.53, \quad \rho(K) = 0.86, \quad \rho(Na) = 0.97\ g\,cm^{-3}.$$

Thus lithium is the least dense, and potassium comes before sodium in an increasing-density list. Although potassium has a greater atomic mass, its atomic volume increases enough that its mass per unit volume is lower. It is therefore incorrect to place these three simply in atomic-number order. All three densities are below that of water under ordinary conditions, but that fact alone does not distinguish sodium from potassium.

Step 2: Add rubidium and caesium and eliminate the alternatives Rubidium and caesium are denser, with representative values about 1.53 and $1.9\ g\,cm^{-3}$, respectively. Combining the comparisons gives

$$Li < K < Na < Rb < Cs.$$

Option (B) overlooks the sodium-potassium exception. Option (C) begins with caesium as the

least dense and reverses the broad trend, contradicting the values. Option (D) also places sodium before potassium and additionally reverses rubidium and caesium. Only option (A) keeps both the exceptional lighter pair and the normal ordering of the heavier metals. The numerical values are supporting checks; the essential conclusion is the relative order.

Step 3: Conclusion The increasing-density order is $\mathrm{Li} < \mathrm{K} < \mathrm{Na} < \mathrm{Rb} < \mathrm{Cs}$. Hence, the correct option is (A).

Quick Tip to Solve: In an increasing-density list for Group 1, write Li first, swap the usual Na–K order to K–Na, and then append Rb and Cs.

76. The geometry around boron in the product 'B' formed from the following reaction is

$$\mathrm{BF_3} + \mathrm{NaH} \xrightarrow{450\ \mathrm{K}} \mathrm{A} + \mathrm{NaF}$$

$$\mathrm{A} + \mathrm{NMe_3} \rightarrow \mathrm{B}$$

[4 marks]

(A) trigonal planar

(B) tetrahedral

(C) pyramidal

(D) square planar

Correct Answer: (B) tetrahedral

Solution:

Concept:

- Boron trifluoride is electron deficient. Its reduction with sodium hydride yields borane units, which associate as diborane; a Lewis base can cleave diborane by donating a lone pair to boron.
- Trimethylamine, $\mathrm{NMe_3}$, has a nitrogen lone pair. In its borane adduct, boron makes three B–H bonds and one coordinate B–N bond, giving four bonding domains and no lone pair on boron.

Step 1: Identify the boron hydride intermediate Heating boron trifluoride with sodium hydride replaces fluorine with hydrogen and forms diborane. A balanced overall representation is

$$2\,\mathrm{BF_3} + 6\,\mathrm{NaH} \longrightarrow \mathrm{B_2H_6} + 6\,\mathrm{NaF}.$$

Thus intermediate A supplies electron-deficient $\mathrm{BH_3}$ units. Free $\mathrm{BH_3}$ has only six electrons around boron and readily accepts an electron pair. It must not be confused with $\mathrm{BH_4^-}$, which would require an additional hydride and a negative charge; the subsequent reaction here is with a neutral electron-pair donor.

Step 2: Form the amine-borane and count the bonds at boron Trimethylamine donates its nitrogen lone pair to a borane unit, giving the amine-borane adduct $\mathrm{Me_3N \cdot BH_3}$. The corresponding reaction is

$$\mathrm{B_2H_6} + 2\,\mathrm{NMe_3} \longrightarrow 2\,\mathrm{Me_3N \cdot BH_3}.$$

In the adduct, boron is bonded to three hydrogen atoms and one nitrogen atom. Four bond-pair directions with no boron lone pair give tetrahedral geometry, commonly described by sp^3 hybridisation. Trigonal planar geometry would describe an isolated three-coordinate boron centre before the donor attaches, so option (A) misses the new bond. Pyramidal geometry requires a three-bond-plus-lone-pair arrangement and is not the environment of boron here. Square planar geometry is also incompatible with this simple four-bond boron centre. Hence options (C) and (D) are excluded.

Step 3: Conclusion Boron in product B, the trimethylamine-borane adduct, has tetrahedral geometry. Hence, the correct option is (B).

Quick Tip to Solve: Count bonds around the atom named in the question after adduct formation: the nitrogen donor adds a fourth bond to boron, changing its geometry to tetrahedral.

77. The interhalogen compound formed from the reaction of bromine with excess of fluorine is a : [4 marks]

(A) hypohalite

(B) halate

(C) perhalate

(D) halite

Correct Answer: The product is neutral BrF_5, an interhalogen; none of the listed oxyanion classes describes it.

Solution:

Concept:

- Excess fluorine converts bromine into bromine pentafluoride, BrF_5. Fluorine is assigned oxidation state -1, so the bromine atom is in oxidation state $+5$.
- Hypohalite, halite, halate and perhalate are classes of oxygen-containing anions, with general forms XO^-, $XO_2{}^-$, $XO_3{}^-$ and $XO_4{}^-$, respectively. A neutral fluoride cannot be placed in any of these classes.

Step 1: Identify the fluoride formed and calculate its oxidation state With excess fluorine, bromine forms the higher fluoride BrF_5, rather than the lower fluoride formed with a smaller fluorine supply:

$$Br_2 + 5\,F_2 \longrightarrow 2\,BrF_5.$$

Let the oxidation state of bromine be x. Because the compound is neutral and all five fluorine atoms have oxidation state -1,

$$x + 5(-1) = 0 \quad \Longrightarrow \quad x = +5.$$

Fluorine fixes the sign unambiguously; it is not appropriate to divide the oxidation state equally between the two different halogens.

Step 2: Check whether any listed species class describes the product The four choices refer to oxyanions. For bromine, hypobromite is BrO^-, bromite is $BrO_2{}^-$, bromate is $BrO_3{}^-$, and perbromate is $BrO_4{}^-$. None is BrF_5: every listed class contains oxygen and carries a negative charge, whereas

the product here contains only bromine and fluorine and is neutral. The bromine oxidation state in bromate and in bromine pentafluoride is the same, $+5$, but equal oxidation states do not make two different chemical species members of the same oxyanion class. Thus choosing halate merely from the value $+5$ would not answer the literal question correctly. There is no oxygen source in the reaction given that could produce an oxyanion. All four options fail as descriptions of its interhalogen product.

Step 3: Conclusion The reaction gives BrF_5, bromine pentafluoride, a neutral interhalogen. No listed option is chemically correct for the question as written. Although the supplied key selects the halate category, BrF_5 contains no oxygen and is not a halate ion; equality of oxidation state with bromate does not establish that classification.

Quick Tip to Solve: Identify the actual product formula before matching a class name: equal oxidation states do not make a neutral fluoride an oxygen-containing anion.

78. The photochemical smog does not generally contain : [4 marks]

(A) NO

(B) NO_2

(C) SO_2

(D) HCHO

Correct Answer: (C) SO_2

Solution:

Concept:

- Photochemical smog forms when nitrogen oxides and volatile organic compounds react in sunlight. It is an oxidising smog, associated with warm, sunny conditions.
- Its characteristic mixture includes nitrogen oxides, ozone, aldehydes such as formaldehyde (HCHO), and peroxyacetyl nitrate (PAN). Sulphur dioxide (SO_2) is associated principally with classical, sulphurous smog.

Step 1: Identify the nitrogen-oxide photochemical cycle Sunlight splits nitrogen dioxide, producing oxygen atoms that form ozone. A simplified sequence is

$$NO_2 \xrightarrow{\text{sunlight}} NO + O, \qquad O + O_2 \longrightarrow O_3.$$

Nitric oxide can react with ozone to regenerate nitrogen dioxide:

$$NO + O_3 \longrightarrow NO_2 + O_2.$$

Thus both NO and NO_2 occur in the atmospheric reaction system. Options (A) and (B) cannot be the substance generally excluded from photochemical smog.

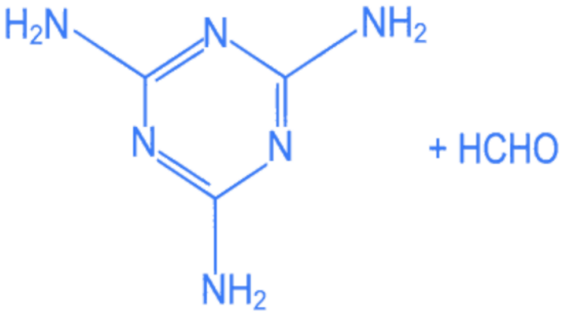
Step 2: Compare the organic and sulphurous components Oxidation of unburnt hydrocarbons produces aldehydes, including HCHO; this rules out option (D). In contrast, SO_2 comes mainly from burning sulphur-containing fuels and characterises classical reducing smog, commonly associated with cool, humid conditions. It is not a characteristic constituent of the photochemical-smog mixture. The

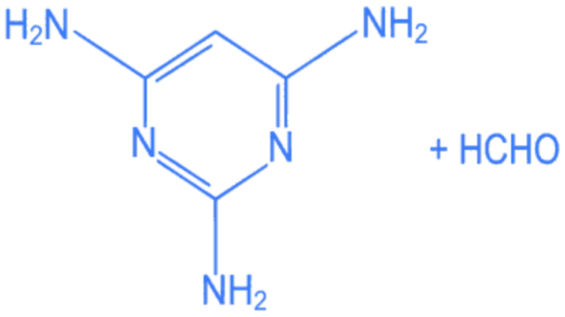
word “generally” distinguishes the characteristic smog components; it does not claim that different pollutants can never coexist in real urban air.

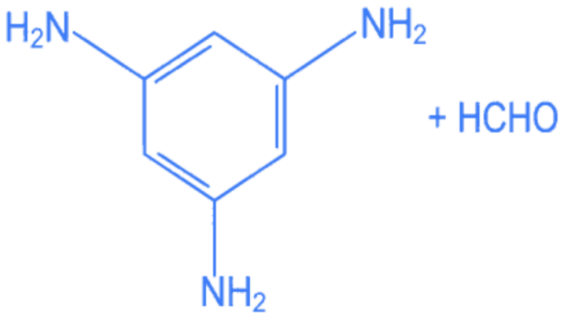
Step 3: Conclusion The substance not generally contained in photochemical smog is SO_2. Hence, the correct option is (C).

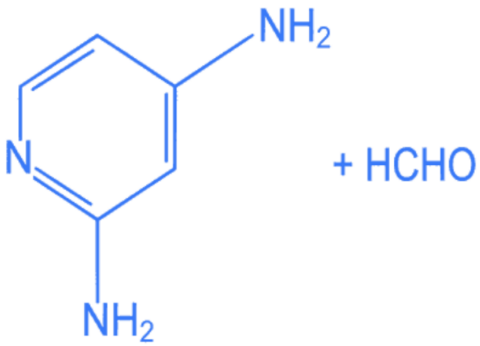
Quick Tip to Solve: Associate sunny photochemical smog with nitrogen oxides, ozone, PAN and aldehydes; associate sulphur dioxide with classical sulphurous smog.

79. Melamine polymer is formed by the condensation of: [4 marks]

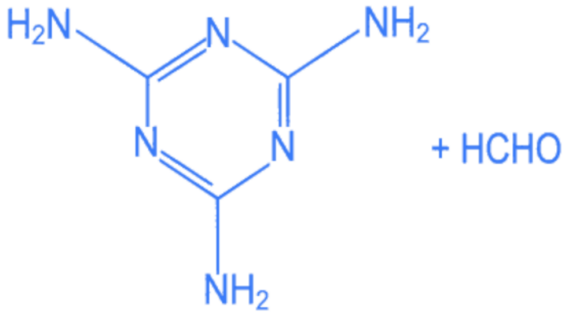
(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- Melamine-formaldehyde resin is a condensation polymer made from melamine and formaldehyde (HCHO). Melamine is 2,4,6-triamino-1,3,5-triazine.
- The melamine ring contains three alternating nitrogen atoms and three carbon atoms bearing amino groups. Its multiple reactive amino sites allow formation of a cross-linked thermosetting network.

Step 1: Identify the melamine monomer in the options Inspect the six-membered ring rather than counting only the external amino groups. Option (A) has three nitrogen atoms within the ring, alternating with the three carbon atoms carrying NH_2 groups. This is melamine. It is accompanied by HCHO, the required second reactant. The other structures change the identity of the ring even where several amino substituents are present.

Step 2: Explain condensation and exclude the other rings Formaldehyde first reacts at amino sites to form methylol groups. On condensation these groups link melamine units, with elimination of small molecules such as water, yielding a three-dimensional resin. A representative methylene-bridge formation is

$$\mathrm{R{-}NH{-}CH_2OH + H_2N{-}R \longrightarrow R{-}NH{-}CH_2{-}NH{-}R + H_2O},$$

where R denotes the melamine-ring residue. Option (B) contains only two ring nitrogen atoms; option (C) has an all-carbon ring; option (D) contains only one ring nitrogen atom and two amino substituents. None is melamine, so pairing those compounds with formaldehyde cannot identify the named melamine polymer.

Step 3: Conclusion Melamine condenses with formaldehyde to form melamine-formaldehyde resin. The melamine plus HCHO pair is shown in (A). Hence, the correct option is (A).

Quick Tip to Solve: For melamine, count three alternating nitrogen atoms inside the six-membered ring as well as three external amino groups; the amino groups alone do not identify it.

80. Drugs used to bind to receptors, inhibiting is natural function and blocking a message are called:

[4 marks]

(A) Agonists

(B) Antagonists

(C) Allosterists

(D) Anti histaminists

Correct Answer: (B) Antagonists

Solution:

Concept:

- Receptors are biological macromolecules that bind chemical messengers and initiate a response. Binding and activation are distinct: a molecule may occupy a receptor without producing its normal response.
- An agonist mimics the natural messenger and activates the receptor. An antagonist binds and blocks the normal messenger-mediated action, inhibiting the receptor response.

Step 1: Match the described action to the drug class The decisive words are "inhibiting its natural function" and "blocking a message". These describe a drug that occupies or interferes with the receptor so that the natural chemical messenger cannot produce its usual effect. Such receptor-blocking drugs are called antagonists. The question asks for this general functional classification, rather than for a drug treating one particular condition.

Step 2: Distinguish the remaining options An agonist, option (A), stimulates or reproduces the normal receptor response, so it has the opposite action to the one described. Allosteric action, suggested by option (C), refers to binding at a site different from the messenger's usual binding site; it describes a binding mechanism and does not by itself define every receptor-blocking drug. Antihistamines, option (D), act against histamine-mediated effects at particular histamine receptors. They can be examples of receptor-blocking action, but are a specific therapeutic group, not the general name for drugs that block messages at receptors. No mention of histamine is made here, so that narrower label is not the requested class.

Step 3: Conclusion The general class of receptor-binding drugs that inhibit the natural function and block the message is antagonists. Hence, the correct option is (B).

Quick Tip to Solve: Use the effect on signalling: agonists activate, antagonists block. A named therapeutic group such as antihistamines is more specific than this general classification.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Among the following species

$$N_2,\ N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}$$

the number of species showing diamagnesim is _____. [4 marks]

Solution:

Concept:

- A species is diamagnetic only when every occupied molecular orbital contains paired electrons. An even electron count alone is insufficient, because degenerate orbitals are filled singly before pairing (Hund's rule).
- For the species listed, electron removal or addition changes the highest occupied or lowest vacant valence molecular orbitals. Core electrons are paired and do not affect the classification.

Step 1: Classify the nitrogen species In N_2, the valence configuration is

$$(\sigma_{2s})^2(\sigma_{2s}^*)^2(\pi_{2p_x})^2(\pi_{2p_y})^2(\sigma_{2p_z})^2.$$

All electrons are paired, so N_2 is diamagnetic. Removing one electron produces $N_2{}^+$ with one unpaired electron. Adding one electron gives $N_2{}^-$ with one electron in a π^* orbital. Adding two electrons gives $N_2{}^{2-}$ with one electron in each of the two degenerate π^* orbitals, hence two unpaired electrons. The last three nitrogen species are paramagnetic.

Step 2: Classify the oxygen species and count The two highest electrons of O_2 occupy the degenerate antibonding π^* orbitals singly, so O_2 has two unpaired electrons. Removing one gives $O_2{}^+$ with one unpaired electron. Adding one gives $O_2{}^-$: one π^* orbital is paired but the other remains singly occupied. Adding two gives $O_2{}^{2-}$, with both π^* orbitals fully paired. Only this last oxygen species is diamagnetic. Therefore the complete diamagnetic list is N_2 and $O_2{}^{2-}$, not all the species with even electron counts.

Step 3: Conclusion Two species, N_2 and $O_2{}^{2-}$, are diamagnetic. The required number is $\boxed{2}$ (dimensionless).

Quick Tip to Solve: Check the occupation of both degenerate antibonding orbitals: two added electrons remain unpaired until each orbital is singly occupied.

82. The enthalpy of combustion of propane, graphite and dihydrogen at $298\,\mathrm{K}$ are $-2220.0\,\mathrm{kJ\,mol^{-1}}$, $-393.5\,\mathrm{kJ\,mol^{-1}}$ and $-285.8\,\mathrm{kJ\,mol^{-1}}$ respectively. The magnitude of enthalpy of formation of propane (C_3H_8) is _____ $\mathrm{kJ\,mol^{-1}}$. (Nearest integer) [4 marks]

Solution:

Concept:

- Hess's law states that the enthalpy change depends only on the initial and final states. Two paths from the elements to identical combustion products therefore have equal overall enthalpy changes.
- The standard formation reaction for one mole of propane is $3\,\mathrm{C(graphite)} + 4\,\mathrm{H_2(g)} \longrightarrow \mathrm{C_3H_8(g)}$. The combustion products corresponding to the supplied values are $\mathrm{CO_2(g)}$ and $\mathrm{H_2O\,(l)}$.

Step 1: Construct the direct combustion path from elements Three moles of graphite form three moles of carbon dioxide, and four moles of dihydrogen form four moles of liquid water. Multiply each molar combustion enthalpy by its stoichiometric amount:

$$\begin{aligned}\Delta H_{\text{direct}} &= 3(-393.5) + 4(-285.8)\\ &= -1180.5 - 1143.2\\ &= -2323.7\,\mathrm{kJ\,mol^{-1}}.\end{aligned}$$

This is the enthalpy per mole of propane that could be formed from those amounts of the elements; no extra factor of three or four is applied to propane.

Step 2: Compare with formation followed by propane combustion Combustion of propane is

$$\mathrm{C_3H_8(g)} + 5\,\mathrm{O_2(g)} \longrightarrow 3\,\mathrm{CO_2(g)} + 4\,\mathrm{H_2O\,(l)}.$$

The two-stage path first forms propane, then releases $2220.0\,\mathrm{kJ\,mol^{-1}}$ on its combustion. Thus

$$\begin{aligned}\Delta_f H^\circ(\mathrm{C_3H_8}) - 2220.0 &= -2323.7,\\ \Delta_f H^\circ(\mathrm{C_3H_8}) &= -103.7\,\mathrm{kJ\,mol^{-1}}.\end{aligned}$$

The question asks for the magnitude, so take the absolute value before rounding. The sign of the formation enthalpy is negative, but its magnitude is positive.

Step 3: Conclusion The formation enthalpy is $-103.7\,\mathrm{kJ\,mol^{-1}}$. Its magnitude to the nearest integer is $\boxed{104\,\mathrm{kJ\,mol^{-1}}}$.

Quick Tip to Solve: Write the formation-plus-combustion Hess cycle before inserting signs. When a magnitude is requested, round the absolute value only after completing the calculation.

83. The cell potential for $\mathrm{Zn}\left|\mathrm{Zn}^{2+}(\mathrm{aq})\right|\left|\mathrm{Sn}^{x+}\right|\mathrm{Sn}$ is $0.801\ \mathrm{V}$ at $298\ \mathrm{K}$. The reaction quotient for the above reaction is 10^{-2}. The number of electrons involved in the given electrochemical cell reaction is _____.

(**Given** : $\mathrm{E}^{\circ}_{\mathrm{Zn}^{2+}|\mathrm{Zn}} = -0.763\ \mathrm{V}, \mathrm{E}^{\circ}_{\mathrm{Sn}^{x+}|\mathrm{Sn}} = +0.008\ \mathrm{V}$ **and** $\frac{2.303\mathrm{RT}}{\mathrm{F}} = 0.06\ \mathrm{V}$) [4 marks]

Solution:

Concept:

- The standard cell potential is the cathode reduction potential minus the anode reduction potential. Electrode potentials are intensive quantities and are not multiplied by balancing coefficients.
- At the stated temperature the Nernst equation is $E = E^{\circ} - (0.06/n)\log_{10} Q$, where n is the number of electrons transferred in the cell reaction and Q is its reaction quotient.

Step 1: Calculate the standard cell potential Zinc is the anode in the written cell and tin is the cathode. Using the given reduction potentials,

$$\begin{aligned} E^{\circ}_{\text{cell}} &= E^{\circ}_{\mathrm{Sn}^{x+}/\mathrm{Sn}} - E^{\circ}_{\mathrm{Zn}^{2+}/\mathrm{Zn}} \\ &= 0.008 - (-0.763) \\ &= 0.771\ \mathrm{V}. \end{aligned}$$

The observed potential is $0.801\ \mathrm{V}$, higher than the standard value by $0.030\ \mathrm{V}$. This sign is consistent with $Q < 1$: the negative logarithm makes the Nernst correction positive.

Step 2: Substitute the quotient and isolate the electron count Since $Q = 10^{-2}$, its common logarithm is -2, not 10^{-2}. Substitute directly:

$$\begin{aligned} 0.801 &= 0.771 - \frac{0.06}{n}(-2), \\ 0.030 &= \frac{0.12}{n}, \\ n &= \frac{0.12}{0.030} = 4. \end{aligned}$$

As a consistency check, setting $n = 4$ gives $E = 0.771 + 0.030 = 0.801\ \mathrm{V}$, exactly the supplied potential. The electron count comes from the complete balanced cell reaction; it is not automatically two merely because zinc loses two electrons per atom. Multiple zinc atoms may contribute to one balanced reaction.

Step 3: Conclusion The electrochemical cell reaction involves $\boxed{4}$ electrons. The requested electron-transfer number is dimensionless.

Quick Tip to Solve: First compute the potential difference $E - E^{\circ}$; for $Q = 10^{-2}$, the Nernst correction is $+0.12/n$, which makes the electron count immediate.

84. The half life for the decomposition of gaseous compound A is 240 s when the gaseous pressure was 500 Torr initially. When the pressure was 250 Torr, the half life was found to be 4.0 min. The order of the reaction is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- At fixed temperature, the concentration of a gaseous reactant is proportional to its partial pressure: $[A]_0 = P_0/(RT)$. Changing initial pressure therefore tests the concentration dependence of the half-life.
- For a rate law $-d[A]/dt = k[A]^n$, the half-life varies as $[A]_0^{1-n}$ when $n \neq 1$. A first-order reaction has $t_{1/2} = \ln 2/k$, independent of the initial concentration.

Step 1: Put both half-lives in the same units The first initial pressure is $500\,\text{Torr}$, with half-life $240\,\text{s}$. At the second initial pressure, $250\,\text{Torr}$, the reported half-life is $4.0\,\text{min}$. Convert it before comparing:

$$4.0 \times 60 = 240\,\text{s}.$$

Thus halving the initial pressure and hence the initial concentration leaves the half-life unchanged. The numerical values 240 and 4 cannot be compared directly because they originally use different time units.

Step 2: Determine and check the order Using the pressure form of the half-life relation gives

$$\frac{t_{1/2,2}}{t_{1/2,1}} = \left(\frac{P_{0,2}}{P_{0,1}}\right)^{1-n} = \left(\frac{1}{2}\right)^{1-n} = 1.$$

A positive base different from one can equal one here only if its exponent is zero. Hence $1 - n = 0$, so $n = 1$. This is precisely the first-order result. For comparison, zero order would halve the half-life, while second order would double it. Both contradict the equal observed half-lives. The implied first-order rate constant is

$$k = \frac{0.693}{240} = 2.89 \times 10^{-3}\,\text{s}^{-1}.$$

Step 3: Conclusion The reaction is first order. The requested nearest-integer order is $\boxed{1}$, a dimensionless quantity.

Quick Tip to Solve: Always convert minutes to seconds before comparing half-lives. Equal half-lives at different starting pressures are the signature of first-order kinetics.

85. Consider the following metal complexes :

$$[\mathrm{Co}\,(\mathrm{NH_3})_6]^{3+}$$

$$[\mathrm{CoCl}\,(\mathrm{NH_3})_5]^{2+}$$

$$[\mathrm{Co(CN)_6}]^{3-}$$

$$[\mathrm{Co}\,(\mathrm{NH_3})_5\,(\mathrm{H_2O})]^{3+}$$

The spin-only magnetic moment value of the complex that absorbes light with shortest wavelength is _____ B. M. (Nearest integer) [4 marks]

Solution:

Concept:

- For an electronic transition across the octahedral crystal-field gap, $\Delta_o = hc/\lambda$. Thus the shortest absorbed wavelength corresponds to the largest splitting, not to the smallest splitting.
- The spin-only moment is $\mu = \sqrt{n(n+2)}\,\mathrm{B.M.}$, where n is the number of unpaired electrons. Strong-field ligands can pair the electrons of an octahedral d^6 ion.

Step 1: Compare the ligand fields All four complexes contain cobalt in oxidation state +3: ammonia and water are neutral, chloride contributes -1, and six cyanide ligands contribute -6. Therefore each metal centre is $\mathrm{Co^{3+}}$, with configuration $3d^6$. For the ligands present, the spectrochemical ordering is $\mathrm{Cl^-} < \mathrm{H_2O} < \mathrm{NH_3} < \mathrm{CN^-}$. The six cyanide ligands produce the largest octahedral splitting. Replacing ammonia with water or chloride cannot produce the largest gap among these choices.

Step 2: Fill the orbitals and calculate the moment The complex absorbing the shortest wavelength is consequently $[\mathrm{Co(CN)_6}]^{3-}$. Cyanide is a strong-field ligand, and its Δ_o exceeds the pairing energy for this cobalt(III) ion. The six electrons occupy the lower orbitals as $t_{2g}^6 e_g^0$, with each occupied orbital doubly filled. Thus $n = 0$, and

$$\mu = \sqrt{0(0+2)} = 0\,\mathrm{B.M.}$$

The complex is diamagnetic; a high-spin d^6 count would not describe this cyanide complex.

Step 3: Conclusion The shortest-wavelength absorber is $[\mathrm{Co(CN)_6}]^{3-}$, and its spin-only magnetic moment is $\boxed{0\,\mathrm{B.M.}}$. The required nearest integer is $\boxed{0}$.

Quick Tip to Solve: First rank Δ_o using the ligand field; only then count unpaired electrons in the selected complex. Wavelength varies inversely with the splitting.

86. Among Co^{3+}, Ti^{2+}, V^{2+} and Cr^{2+} ions, one if used as a reagent cannot liberate H_2 from dilute mineral acid solution, its spin-only magnetic moment in gaseous state is _____ B.M. (Nearest integer) [4 marks]

Solution:

Concept:

- A metal ion liberates hydrogen from acid only if it can act as a reductant: the ion is oxidised while $2\,H^+ + 2\,e^- \longrightarrow H_2$. An oxidising ion cannot supply those electrons.
- For a gaseous free ion, use its electron configuration and Hund's rule without imposing ligand-field pairing. The spin-only moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$.

Step 1: Identify the ion that cannot reduce the acid The ions Ti^{2+}, V^{2+} and Cr^{2+} readily undergo oxidation to their +3 states and act as reducing agents in acidic solution. In particular, oxidation of Cr^{2+} yields the relatively stable d^3 chromium(III) ion. In contrast, Co^{3+} is an oxidising species: it tends to accept an electron and form Co^{2+}. It is therefore the ion in the list that cannot serve as the reducing reagent to liberate hydrogen from dilute mineral acid.

Step 2: Count the gaseous ion's unpaired electrons Cobalt has atomic number 27 and neutral configuration $[Ar]3d^7 4s^2$. Remove the two $4s$ electrons and then one $3d$ electron to obtain Co^{3+} : $[Ar]3d^6$. In the gaseous ion the five degenerate d orbitals are filled singly before pairing; the sixth electron pairs in one orbital, leaving four unpaired electrons. Thus

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.90\,\text{B.M.}$$

This must not be replaced by the low-spin count of a cobalt(III) complex, because no ligand field is specified here.

Step 3: Conclusion The ion is gaseous Co^{3+}, with a spin-only magnetic moment of about $4.90\,\text{B.M.}$. The required nearest integer is $\boxed{5}$.

Quick Tip to Solve: The words gaseous state distinguish a free ion from a coordination complex: apply Hund's rule directly rather than assuming the common low-spin cobalt(III) arrangement.

87. While estimating the nitrogen present in an organic compound by Kjeldahl's method, the ammonia evolved from 0.25 g of the compound neutralized 2.5 mL of $2\,M\,H_2SO_4$. The percentage of nitrogen present in organic compound is _____. [4 marks]

Solution:

Concept:

- Kjeldahl estimation converts the nitrogen in the sample into ammonia. Each mole of NH_3 contains one mole of nitrogen atoms, so the trapped ammonia determines the nitrogen mass.
- Sulfuric acid is dibasic. Complete neutralisation follows $2\,NH_3 + H_2SO_4 \longrightarrow (NH_4)_2SO_4$; consequently one mole of H_2SO_4 neutralises two moles of ammonia.

Step 1: Calculate the acid and ammonia amounts Convert the acid volume into litres before

multiplying by molarity:

$$V = 2.5 \times 10^{-3}\,\text{L}, \qquad n(\text{H}_2\text{SO}_4) = 2 \times 2.5 \times 10^{-3} = 5.0 \times 10^{-3}\,\text{mol}.$$

The stated acid amount is neutralised by the ammonia evolved from the entire $0.25\,\text{g}$ sample. Using the balanced reaction gives

$$n(\text{NH}_3) = 2n(\text{H}_2\text{SO}_4) = 1.0 \times 10^{-2}\,\text{mol}.$$

Using a 1:1 acid-to-ammonia ratio would miss one acidic proton and halve the answer.

Step 2: Convert ammonia into the nitrogen percentage Every ammonia molecule contributes one nitrogen atom, not three. Hence

$$n(\text{N}) = 1.0 \times 10^{-2}\,\text{mol}, \qquad m(\text{N}) = (1.0 \times 10^{-2})(14) = 0.14\,\text{g}.$$

Divide by the original organic sample mass, rather than the mass of ammonia or ammonium sulfate:

$$\%\text{N} = \frac{m(\text{N})}{m(\text{sample})} \times 100 = \frac{0.14}{0.25} \times 100 = 56.$$

The nitrogen mass is smaller than the sample mass, as a mass percentage requires.

Step 3: Conclusion The percentage of nitrogen in the organic compound is $\boxed{56\%}$; the numerical entry is $\boxed{56}$.

Quick Tip to Solve: For sulfuric acid, multiply its moles by two before converting ammonia to nitrogen mass; using 14, rather than 17, gives the mass of nitrogen alone.

88. The number of sp^3 hybridised carbons in an acyclic neutral compound with molecular formula C_4H_5N is _____. [4 marks]

Solution:

Concept:

- For a neutral closed-shell formula containing carbon, hydrogen and trivalent nitrogen, the degree of unsaturation is $\text{DBE} = (2C + 2 + N - H)/2$. A double bond contributes one and a triple bond contributes two.
- A tetrahedral carbon with four sigma bonds is sp^3-hybridised. Molecular formula and acyclic character constrain the total unsaturation, but do not generally determine how those multiple bonds are distributed.

Step 1: Calculate the unsaturation and the usual nitrile arrangement For $\text{C}_4\text{H}_5\text{N}$,

$$\text{DBE} = \frac{2(4) + 2 + 1 - 5}{2} = 3.$$

If nitrogen is present as a nitrile, its $\text{C}{\equiv}\text{N}$ bond accounts for two units, leaving one carbon-carbon

double bond in an acyclic structure. For example, $CH_3{-}CH{=}CH{-}C{\equiv}N$ has the required formula. Its methyl carbon is sp^3, its two alkene carbons are sp^2, and its nitrile carbon is sp; it therefore has one sp^3 carbon.

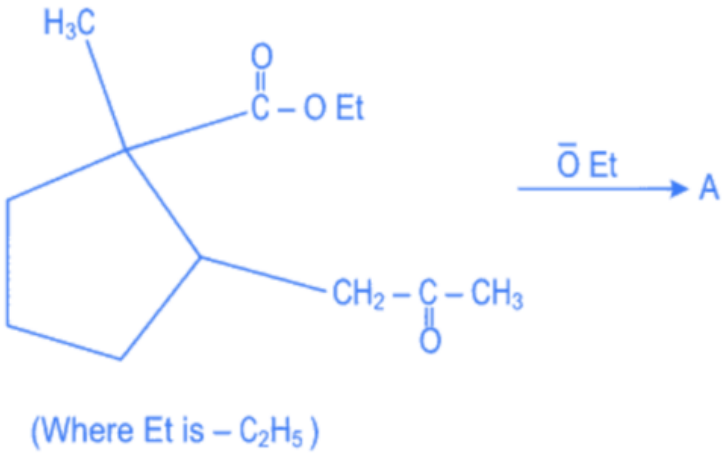
Step 2: Check whether the wording forces that functional group The formula does not specify a nitrile. The neutral acyclic imine $CH_2{=}C{=}CH{-}CH{=}NH$ is also C_4H_5N: its hydrogens total $2{+}0{+}1{+}1{+}1 = 5$. It has three double bonds and satisfies the same DBE. The central allene carbon is sp, while both terminal allene carbons and the imine carbon are sp^2. It therefore has zero sp^3 carbons. Both examples satisfy ordinary neutral valences, so acyclic and neutral do not exclude this counterexample. An added functional-group restriction would be needed to make the count unique.

Step 3: Conclusion The expected alkene-nitrile interpretation gives $\boxed{1}$ sp^3 carbon, agreeing with the numerical key. Literally, however, the printed conditions do not determine a unique count: $CH_2{=}C{=}CH{-}CH{=}NH$ provides a valid zero-sp^3 counterexample.

Quick Tip to Solve: Use DBE to count total unsaturation, then inspect possible functional groups. Do not infer a nitrile group solely from the presence of one nitrogen atom.

89. In the given reaction

The number of chiral carbon/s in product A is _____. [4 marks]



(Where Et is – C_2H_5)

Solution:

Concept:

- An ester and an enolisable ketone in the same molecule can undergo intramolecular Claisen condensation in alkoxide base. Enolate attack on the ester carbonyl forms a carbon-carbon bond and eliminates ethoxide.
- A chiral tetrahedral carbon must have four different substituent paths. Carbonyl carbons are trigonal planar, and a methylene carbon with two identical hydrogens is not a chiral centre.

Step 1: Identify the favourable ring closure Call the ring carbon bearing the methyl and ester groups C_1, and the adjacent ring carbon bearing $-CH_2{-}CO{-}CH_3$ C_2. The terminal ketone methyl group can form an enolate in ethoxide base. Its attack on the ester carbonyl closes a six-membered ring

along the sequence: ester carbonyl carbon, C_1, C_2, side-chain methylene, ketone carbonyl carbon and the original terminal methyl carbon. Loss of EtO^- gives the fused cyclic beta-dicarbonyl framework. Attack from the methylene between C_2 and the ketone would instead require an unfavourable four-membered ring.

Step 2: Count the tetrahedral stereogenic centres in A The two carbons shared by the original five-membered ring and the newly formed six-membered ring are C_1 and C_2. At C_1, the methyl group, the carbonyl-bearing path, and the two inequivalent ring paths are all different. At C_2, hydrogen and the three different paths through the fused skeleton are likewise distinct. Both are therefore chiral carbons. All other saturated carbons in this skeleton are methylene or methyl carbons and have repeated hydrogens; the two carbonyl carbons are sp^2. The carbon derived from the terminal methyl becomes a methylene, not an additional chiral centre. Thus condensation does not add a third tetrahedral stereogenic carbon.

Step 3: Conclusion Product A contains the two stereogenic ring-junction carbons, so the number of chiral carbons is $\boxed{2}$.

Quick Tip to Solve: After a ring-forming condensation, count four distinct substituent paths at each tetrahedral atom; a new carbon-carbon bond does not automatically create a new chiral centre.

90. The pressure of a moist gas at $27^\circ C$ is 4 atm. The volume of the container is doubled at the same temperature. The new pressure of the moist gas is _____ $\times 10^{-1}$ atm. (Nearest integer)
(Given : The vapour pressure of water at $27^\circ C$ is 0.4 atm.) [4 marks]

Solution:

Concept:

- Dalton's law gives $P_{\text{total}} = P_{\text{dry}} + P_{\text{water}}$. At fixed temperature, saturated water-vapour pressure is fixed by the temperature, provided moisture maintains vapour-liquid equilibrium.
- For the fixed amount of dry gas at constant temperature, Boyle's law gives $P_{\text{dry},1}V_1 = P_{\text{dry},2}V_2$. Apply the volume change to the dry-gas component before adding the water-vapour pressure back.

Step 1: Separate the initial gas and vapour pressures The initial total pressure is $4\,\text{atm}$, of which $0.4\,\text{atm}$ is the saturated vapour pressure of water at $27^\circ C$. Hence

$$P_{\text{dry},1} = 4 - 0.4 = 3.6\,\text{atm}.$$

The temperature remains unchanged, so the equilibrium water-vapour pressure after expansion is still $0.4\,\text{atm}$. It would be incorrect to halve the entire original moist-gas pressure: the saturated vapour component is replenished by evaporation as the available volume increases.

Step 2: Double the volume and recombine the components Because $V_2 = 2V_1$, the partial pressure of the dry gas becomes

$$P_{\text{dry},2} = 3.6\frac{V_1}{2V_1} = 1.8\,\text{atm}.$$

Now add the water-vapour component at the same temperature:

$$P_{\text{total},2} = 1.8 + 0.4 = 2.2\,\text{atm}.$$

The requested blank is the coefficient of $10^{-1}\,\text{atm}$, rather than the pressure directly in atmospheres. Since $2.2 = 22 \times 10^{-1}$, that coefficient is already an integer. The result lies above $2\,\text{atm}$, as expected from maintaining the vapour pressure instead of halving it.

Step 3: Conclusion The new pressure is $\boxed{2.2\,\text{atm}} = \boxed{22 \times 10^{-1}\,\text{atm}}$. The required nearest-integer entry is $\boxed{22}$.

Quick Tip to Solve: For a moist gas at fixed temperature, subtract the saturated vapour pressure, apply Boyle's law to the dry gas, then restore the same vapour pressure.