JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 25 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 25 July 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. For $z \in \mathbb{C}$ if the minimum value of $(|z - 3\sqrt{2}| + |z - p\sqrt{2}i|)$ is $5\sqrt{2}$, then a value Question: of p is _____. [4 marks]

(A) 3

(B) $\frac{7}{2}$

(C) 4

(D) $\frac{9}{2}$

Correct Answer: (C) 4

Solution:

Concept:

- The modulus $|z - a|$ is the distance between the points representing z and a in the Argand plane.
- The triangle inequality gives $|z - a| + |z - b| \geq |a - b|$. Equality is attained when the variable point lies on the line segment joining the fixed points.

Step 1: Identify the fixed points Write $a = 3\sqrt{2}$ and $b = p\sqrt{2}\,i$. These represent $(3\sqrt{2}, 0)$ and $(0, p\sqrt{2})$. Their horizontal and vertical separations are $3\sqrt{2}$ and $|p|\sqrt{2}$, respectively. Thus the distance between them is

$$|a - b| = \sqrt{(3\sqrt{2})^2 + (p\sqrt{2})^2} = \sqrt{18 + 2p^2}.$$

Step 2: Use the attained minimum For every complex number z, the sum in the question is at least this fixed-point distance. Taking $z = a$, for example, makes the first modulus zero and the second exactly $|a - b|$, so the lower bound really is the minimum. Hence

$$\sqrt{18 + 2p^2} = 5\sqrt{2}.$$

$$18 + 2p^2 = 50 \quad \Longrightarrow \quad p^2 = 16.$$

The possible real values are $p = 4$ and $p = -4$; the question asks for a value, so only the listed positive value needs to be selected. The other listed values $3, 7/2, 9/2$ have squares different from 16, and therefore do not give the prescribed minimum.

Step 3: Conclusion A value of p is 4. Hence, the correct option is (C).

Quick Tip to Solve: For the minimum sum of two complex-plane distances, replace the variable path by the straight-line distance between the two fixed points.

2. The number of real values of λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

has no solutions, is [4 marks]

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Concept:

- A nonsingular coefficient matrix gives a unique solution. No solution is possible only when elimination removes a variable coefficient but leaves a contradictory constant.
- Treat $t = \lambda^2 - |\lambda|$ as one coefficient first; solve for λ only after identifying the inconsistent value of t.

Step 1: Eliminate the first two variables Let the first two equations be E_1 and E_2, and put $t = \lambda^2 - |\lambda|$. To reproduce the x, y coefficients of the third equation, solve

$$2a + b = 3, \qquad -3a + 3b = -1.$$

This gives $a = 10/9$, $b = 7/9$. The same linear combination of the first two equations is therefore

$$3x - y + \frac{43}{9}z = -4,$$

because $5(10/9) - 7/9 = 43/9$ and $9(10/9) - 18(7/9) = -4$.

Step 2: Find and count the exceptional parameters Subtract this relation from the third equation:

$$\left(t - \frac{43}{9}\right) z = 20.$$

If $t \neq 43/9$, this determines z, and the first two equations then determine x, y, since their 2×2 determinant is $9 \neq 0$. If $t = 43/9$, however, the equation becomes $0 = 20$, which is impossible. Set $u = |\lambda| \geq 0$. The exceptional condition becomes

$$u^2 - u - \frac{43}{9} = 0, \qquad u = \frac{3 \pm \sqrt{181}}{6}.$$

Only the plus sign is nonnegative. This positive value of u produces the two distinct real values $\lambda = \pm(3 + \sqrt{181})/6$. There are neither zero nor one such values, and the negative root for u cannot produce two additional values.

Step 3: Conclusion There are 2 real values of λ for which the system has no solution. Hence, the correct option is (C).

Quick Tip to Solve: Reduce the parameter to one coefficient, check the augmented equation for contradiction, and enforce nonnegativity when substituting an absolute value.

3. The number of bijective functions $f : \{1, 3, 5, 7, \ldots, 99\} \to \{2, 4, 6, 8, \ldots .100\}$, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq \ldots..f(99)$, is _____. [4 marks]

(A) ${}^{50}P_{17}$

(B) ${}^{50}P_{33}$

(C) $33! \times 17!$

(D) $\frac{50!}{2}$

Correct Answer: (B) ${}^{50}P_{33}$

Solution:

Concept:

- Both sets have 50 elements, so unrestricted bijections number $50!$. A bijection assigns distinct images to distinct inputs.
- Once a subset of images is chosen for an ordered chain, a required strictly decreasing order fixes their assignment uniquely.

Step 1: Count the constrained inputs The inputs whose images are constrained form the arithmetic progression $3, 9, 15, \ldots, 99$, with common difference 6. The number of terms is

$$r = \frac{99 - 3}{6} + 1 = 17.$$

Although the condition uses weak inequalities, equality of any two images is impossible for a bijection. Thus these 17 images must in fact occur in strictly decreasing order. The remaining $50 - 17 = 33$ inputs have no relative-order restriction.

Step 2: Choose images and complete the bijection Choose which 17 of the 50 even numbers are the images of the constrained inputs. There are $\binom{50}{17}$ choices. Arrange each chosen subset in decreasing order and assign it to $3, 9, \ldots, 99$; there is exactly one way, not $17!$ ways. The remaining 33 outputs may be assigned bijectively to the remaining inputs in $33!$ ways. Therefore

$$N = \binom{50}{17} 33! = \frac{50!}{17!} = {}^{50}P_{33}.$$

Option (A) divides by $33!$, which is the wrong denominator. Option (C) does not count the choice of output subset correctly. Option (D) removes only one factor of 2, whereas fixing the relative order of 17 distinct images removes a factor of $17!$.

Step 3: Conclusion The required number is $\frac{50!}{17!} = {}^{50}P_{33}$. Hence, the correct option is (B).

Quick Tip to Solve: If the relative order of the images of r specified inputs is fixed, divide the unrestricted number of bijections by r factorial.

4. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is [4 marks]

(A) 1

(B) 4

(C) 6

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- Congruent integers have congruent positive integral powers. Reduce each base modulo the divisor before handling a large exponent.
- Powers of 2 modulo 9 repeat with period 6, since $2^6 = 64 \equiv 1 \pmod 9$.

Step 1: Reduce the two bases The first base satisfies $11 \equiv 2 \pmod 9$. For the second base, either division or the digit-sum rule gives $1011 \equiv 3 \pmod 9$. Consequently

$$11^{1011} + 1011^{11} \equiv 2^{1011} + 3^{11} \pmod 9.$$

The second term is divisible by 9, because $3^{11} = 9 \cdot 3^9$. It contributes remainder zero; it is not necessary to calculate this enormous power.

Step 2: Use the power cycle and choose the remainder Write $1011 = 6(168) + 3$. Then

$$2^{1011} = (2^6)^{168}\, 2^3 \equiv 1^{168} \cdot 8 \equiv 8 \pmod 9.$$

Equivalently, the remainders of successive powers of 2 are $2, 4, 8, 7, 5, 1$, and the exponent occupies the third position of the cycle. Adding the second term therefore leaves remainder 8. A remainder on division by 9 must lie between 0 and 8, so this congruence already gives the required remainder without further adjustment. The values 1, 4 and 6 in the other options are not congruent to 8 modulo 9; they cannot be the remainder of the same integer. In particular, reducing the exponent by the divisor itself would be an invalid shortcut here.

Step 3: Conclusion The remainder is 8. Hence, the correct option is (D).

Quick Tip to Solve: Reduce bases modulo 9, discard any term containing a factor of 9, and reduce the remaining exponent by the actual power-cycle length.

5. $\lim\limits_{x\to\frac{\pi}{4}} \frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ **is equal to** [4 marks]

(A) 14

(B) 7

(C) $14\sqrt{2}$

(D) $7\sqrt{2}$

Correct Answer: (A) 14

Solution:

Concept:

- The identity $(\sin x + \cos x)^2 = 1 + \sin 2x$ expresses the denominator using the same quantity as the numerator.
- For $a \neq t$, $a^n - t^n = (a-t)(a^{n-1} + a^{n-2}t + \cdots + t^{n-1})$. This cancels the vanishing factor without repeated differentiation.

Step 1: Introduce a common variable Put $t = \cos x + \sin x$ and $a = \sqrt{2}$. As $x \to \pi/4$, we have $t \to a$. Since $a^7 = 8\sqrt{2}$ and $\sin 2x = t^2 - 1$, the quotient becomes

$$\frac{a^7 - t^7}{\sqrt{2}(2 - t^2)} = \frac{a^7 - t^7}{\sqrt{2}(a-t)(a+t)}.$$

Near $\pi/4$, the equality $t = a$ occurs only at the limiting point, so cancelling $a - t$ is legitimate on a punctured neighbourhood.

Step 2: Cancel before evaluating the limit Factor the seventh-power difference to obtain

$$\frac{a^6 + a^5t + a^4t^2 + a^3t^3 + a^2t^4 + at^5 + t^6}{\sqrt{2}(a+t)}.$$

All seven numerator terms tend to $a^6 = (\sqrt{2})^6 = 8$. The denominator tends to $\sqrt{2}(2\sqrt{2}) = 4$, which is nonzero. Hence

$$L = \frac{7 \cdot 8}{4} = 14.$$

The factors of $\sqrt{2}$ cancel fully; retaining one of them would incorrectly give either radical-valued option. The value 7 would miss the factor 2 between the limiting numerator and denominator. This calculation also avoids treating the original zero-over-zero expression as a direct substitution.

Step 3: Conclusion The limit equals 14. Hence, the correct option is (A).

Quick Tip to Solve: When powers of sin x + cos x appear with sin 2x, replace the latter by the square of the former minus 1 before taking the limit.

6. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$, then $P(A \mid B') + P(B \mid A')$ is equal to : [4 marks]

(A) $\frac{3}{4}$

(B) $\frac{5}{8}$

(C) $\frac{5}{4}$

(D) $\frac{7}{8}$

Correct Answer: (B) $\frac{5}{8}$

Solution:

Concept:

- The addition formula is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. It determines the overlap from the given marginal probabilities.
- For a conditioning event C with positive probability, $P(A \mid C) = P(A \cap C)/P(C)$. Complements change both the numerator and the denominator.

Step 1: Find the overlap and exclusive parts Using the addition formula gives

$$P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{2} = \frac{1}{30}.$$

Therefore the part of A outside B has probability

$$P(A \cap B') = \frac{1}{3} - \frac{1}{30} = \frac{3}{10},$$

and the part of B outside A has probability

$$P(B \cap A') = \frac{1}{5} - \frac{1}{30} = \frac{1}{6}.$$

These are joint probabilities, so they must still be divided by the appropriate conditioning probabilities.

Step 2: Calculate and add the conditional probabilities The complement probabilities are $P(B') = 4/5$ and $P(A') = 2/3$, both nonzero. Hence

$$P(A \mid B') = \frac{3/10}{4/5} = \frac{3}{8},$$

$$P(B \mid A') = \frac{1/6}{2/3} = \frac{1}{4}.$$

The requested sum is thus

$$\frac{3}{8} + \frac{1}{4} = \frac{5}{8}.$$

The values $3/4$ and $7/8$ are larger than this explicitly computed sum, and $5/4$ is also incompatible with the two calculated contributions. No independence assumption is used: indeed independence would predict overlap $1/15$, whereas the data give $1/30$.

Step 3: Conclusion The required sum is $\frac{5}{8}$. Hence, the correct option is (B).

Quick Tip to Solve: For conditioning on a complement, first subtract the overlap from the numerator event and then divide by the complement probability.

7. Let $[t]$ denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101} \left([\sin(\pi x)] + e^{[\cos(2\pi x)]}\right) dx$ is equal to [4 marks]

(A) $\frac{52(1-e)}{e}$

(B) $\frac{52}{e}$

(C) $\frac{52(2+e)}{e}$

(D) $\frac{104}{e}$

Correct Answer: (B) $\frac{52}{e}$

Solution:

Concept:

- Except at isolated points, the floor of a sine or cosine is 0 where it is positive and -1 where it is negative.
- Changing a bounded integrand at finitely many points does not change its definite integral. Integrate the two terms separately using their periods.

Step 1: Integrate the sine floor The interval has length $101 - (-3) = 104$, exactly 52 periods of $\sin(\pi x)$, whose period is 2. In each period the sine is positive for length 1 and negative for length 1. Its floor is respectively 0 and -1. At zeros and at extrema the floor may have exceptional values, but there are only finitely many such points in this interval and they do not affect integration. Thus

$$\int_{-3}^{101} [\sin(\pi x)]\, dx = 52(-1) = -52.$$

Step 2: Integrate the exponential term and combine The function $\cos(2\pi x)$ has period 1. In each unit interval its positive and negative regions each have length $1/2$. Consequently $e^{[\cos(2\pi x)]}$ equals 1 over half the interval and e^{-1} over the other half, apart from isolated points. There are 104 full unit periods, so

$$\int_{-3}^{101} e^{[\cos(2\pi x)]}\, dx = 104\left(\frac{1}{2} + \frac{1}{2e}\right) = 52 + \frac{52}{e}.$$

Adding the two integrals leaves $52/e$. Option (A) would omit the positive-half contribution 52; option (C) adds an extra 104 relative to the cancellation, and option (D) doubles the surviving contribution.

Step 3: Conclusion The integral equals $\frac{52}{e}$. Hence, the correct option is (B).

Quick Tip to Solve: For floors of sine and cosine, ignore isolated exceptional points and measure the positive and negative portions of one complete period.

8. Let the point $P(\alpha, \beta)$ be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to : [4 marks]

(A) -14

(B) 42

(C) -22

(D) 14

Correct Answer: (D) 14

Solution:

Concept:

- The perpendicular distance of (x, y) from $Ax + By + C = 0$ is $|Ax + By + C|/\sqrt{A^2 + B^2}$. The absolute value determines distance, while its sign identifies the side of the line.
- To interpret above and below without a sign error, first express each line as $y = mx + c$. Substitute the appropriate signed distance into the two linear equations.

Step 1: Choose the correct signs using the given sides For L_1, the equation is $y = (3x + 12)/4$. Since the point is below this line, $3\alpha - 4\beta + 12 > 0$. Its distance is one, so

$$3\alpha - 4\beta + 12 = \sqrt{3^2 + (-4)^2} = 5.$$

For L_2, we have $y = -(8x + 11)/6$. Above this line means $8\alpha + 6\beta + 11 > 0$. Therefore

$$8\alpha + 6\beta + 11 = \sqrt{8^2 + 6^2} = 10.$$

Both numerators are positive here, despite the different words above and below, because the coefficients of y have opposite signs.

Step 2: Solve the simultaneous equations The equations reduce to

$$3\alpha - 4\beta = -7, \qquad 8\alpha + 6\beta = -1.$$

Multiply the first by three and the second by two, then add:

$$25\alpha = -23, \qquad \alpha = -\frac{23}{25}.$$

Substitution gives

$$\beta = \frac{53}{50}, \qquad \alpha + \beta = \frac{7}{50}.$$

Thus $100(\alpha + \beta) = 14$. The other proposed values $-14, 42, -22$ do not satisfy the two signed-distance equations, so they must be rejected.

Step 3: Conclusion

$$100(\alpha + \beta) = 14.$$

Hence, the correct option is (D).

Quick Tip to Solve: Rewrite both lines in slope-intercept form before choosing distance signs; above does not always mean the same numerator sign.

9. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is : [4 marks]

(A) $\frac{5}{7}$

(B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$

(D) $\frac{2\sqrt{5}}{7}$

Correct Answer: (A) $\frac{5}{7}$

Solution:

Concept:

- For $x^2/a^2+y^2/b^2=1$, its intercepts on the coordinate axes determine the squares of its semiaxes. An intercept may be negative, but its square is positive.
- If the larger squared semiaxis is a^2, the focal distance satisfies $c^2=a^2-b^2$, and the eccentricity is $e=c/a$.

Step 1: Find the two axis intersections The ellipse meets the first line on the x-axis, where $y=0$. Substituting in the line equation gives

$$\frac{x}{7}=1, \qquad x=7.$$

The point $(7,0)$ is on the ellipse, and hence $49/a^2=1$, giving $a^2=49$. For the second line the stated intersection is on the y-axis, so set $x=0$:

$$-\frac{y}{2\sqrt{6}}=1, \qquad y=-2\sqrt{6}.$$

Substitution of $(0,-2\sqrt{6})$ into the ellipse gives $b^2=24$. The minus sign only places the intersection on the negative side of the axis; it does not make a semiaxis negative.

Step 2: Compute the focal distance and eccentricity Since $49>24$, the major axis is along the x-axis and its semiaxis is $a=7$. Thus

$$c^2=49-24=25, \qquad c=5.$$

Consequently

$$e=\frac{c}{a}=\frac{5}{7}.$$

The value $2\sqrt{6}/7$ is the ratio of the minor semiaxis to the major semiaxis, not the eccentricity. The other values $3/7$ and $2\sqrt{5}/7$ would imply focal squares 9 and 20, respectively; neither equals the required difference 25.

Step 3: Conclusion The eccentricity is $\boxed{5/7}$. Hence, the correct option is (A).

Quick Tip to Solve: Axis intersections reveal the semiaxes directly; use the difference of their squares to find the focal distance.

10. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is : [4 marks]

(A) $\frac{32}{9}$

(B) $\frac{18}{5}$

(C) $\frac{27}{4}$

(D) $\frac{27}{10}$

Correct Answer: (D) $\frac{27}{10}$

Solution:

Concept:

- For an ellipse, the squared focal distance is the difference of its squared semiaxes: $c^2=a_e^2-b_e^2$. Confocal conics have the same foci and hence the same value of c.
- For a hyperbola in standard form $x^2/A^2-y^2/B^2=1$, we have $c^2=A^2+B^2$, and its full latus rectum has length $2B^2/A$.

Step 1: Normalize the hyperbola and locate the common foci The ellipse has $a_e^2=16$ and $b_e^2=7$, so

$$c^2=16-7=9.$$

Its foci are $(\pm3,0)$. Multiply the entire hyperbola equation by 25 before reading its semiaxes:

$$\frac{x^2}{144/25}-\frac{y^2}{\alpha/25}=1.$$

Thus $A^2=144/25$, $A=12/5$, and $B^2=\alpha/25$. Reading 144 itself as A^2 would ignore the right-hand side of the printed equation and produce the wrong conic.

Step 2: Use confocality and calculate the latus rectum The hyperbola must have $c^2=9$ as well. Therefore

$$\frac{144}{25}+\frac{\alpha}{25}=9,\qquad \alpha=81.$$

This gives $B^2=81/25$, which is positive as required. Its latus rectum length is

$$\frac{2B^2}{A}=\frac{2(81/25)}{12/5}=\frac{162}{25}\frac{5}{12}=\frac{27}{10}.$$

This is the full chord length through a focus perpendicular to the transverse axis. The other listed lengths $32/9, 18/5, 27/4$ do not equal this expression with the normalized semiaxes and are therefore excluded.

Step 3: Conclusion The length of the latus rectum is $\boxed{27/10}$. Hence, the correct option is (D).

Quick Tip to Solve: First make the right-hand side equal to one; only then identify the hyperbola semiaxes and use the confocal condition.

11. The shortest distance between the lines $\frac{x+7}{-6} = \frac{y-6}{7} = z$ and $\frac{7-x}{2} = y-2 = z-6$ is : [4 marks]

(A) $2\sqrt{29}$

(B) 1

(C) $\sqrt{\frac{37}{29}}$

(D) $\frac{\sqrt{29}}{2}$

Correct Answer: (A) $2\sqrt{29}$

Solution:

Concept:

- For lines $\vec{r} = \vec{p} + t\vec{u}$ and $\vec{r} = \vec{q} + s\vec{v}$, the shortest distance is $d = |(\vec{q} - \vec{p}) \cdot (\vec{u} \times \vec{v})|/|\vec{u} \times \vec{v}|$, provided the directions are not parallel.
- The cross product is perpendicular to both line directions, so projecting the vector between any pair of points onto its unit direction gives the required distance.

Step 1: Extract points and direction vectors carefully Write the first line with parameter t:

$$x = -7 - 6t, \quad y = 6 + 7t, \quad z = t.$$

Thus take $\vec{p} = (-7, 6, 0)$, $\vec{u} = (-6, 7, 1)$. For the second line put each printed ratio equal to s:

$$x = 7 - 2s, \quad y = 2 + s, \quad z = 6 + s.$$

Hence $\vec{q} = (7, 2, 6)$ and $\vec{v} = (-2, 1, 1)$. In particular, the numerator $7 - x$ makes the first direction component negative, not positive. The joining vector is $\vec{q} - \vec{p} = (14, -4, 6)$.

Step 2: Evaluate the scalar triple product The common perpendicular direction is

$$\vec{u} \times \vec{v} = (6, 4, 8), \qquad |\vec{u} \times \vec{v}| = \sqrt{36 + 16 + 64} = 2\sqrt{29}.$$

The dot product is

$$(14, -4, 6) \cdot (6, 4, 8) = 84 - 16 + 48 = 116.$$

Therefore

$$d = \frac{116}{2\sqrt{29}} = 2\sqrt{29}.$$

The nonzero cross product confirms the lines are not parallel, and the nonzero triple product confirms they are skew. Values 1, $\sqrt{37/29}$, and $\sqrt{29}/2$ do not agree with this perpendicular projection and hence cannot be the shortest distance.

Step 3: Conclusion The shortest distance is $\boxed{2\sqrt{29}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a numerator such as 7-x, parameterize first; this prevents reversing just one direction component by mistake.

12. Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is : [4 marks]

(A) $\frac{2}{\sqrt{21}}$

(B) $2\sqrt{\frac{3}{7}}$

(C) $\frac{2}{3}\sqrt{\frac{7}{3}}$

(D) $\frac{2}{3}$

Correct Answer: (A) $\frac{2}{\sqrt{21}}$

Solution:

Concept:

- The identity $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$ relates the magnitudes and dot product without requiring every component of $\vec{b}$.
- The scalar projection of $\vec{b}$ on a nonzero vector $\vec{d}$ is $(\vec{b} \cdot \vec{d})/|\vec{d}|$. Here the direction of projection is $\vec{d} = \vec{a} - \vec{b}$, not $\vec{a}$.

Step 1: Find the squared magnitude of the unknown vector The given vector has

$$|\vec{a}|^2 = 1^2 + (-1)^2 + 2^2 = 6.$$

Since $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$, its squared magnitude is $4 + 1 = 5$. The given dot product is 3. Apply the identity to obtain

$$6|\vec{b}|^2 = 5 + 3^2 = 14, \qquad |\vec{b}|^2 = \frac{7}{3}.$$

These two supplied products determine precisely the scalar information needed for the projection, so solving for all three components of $\vec{b}$ is unnecessary.

Step 2: Compute the projection along the difference vector Expand the required dot product:

$$\vec{b} \cdot (\vec{a} - \vec{b}) = 3 - \frac{7}{3} = \frac{2}{3}.$$

Also

$$|\vec{a} - \vec{b}|^2 = 6 + \frac{7}{3} - 2(3) = \frac{7}{3}.$$

Dividing by the magnitude of the direction gives

$$\frac{\vec{b} \cdot (\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|} = \frac{2/3}{\sqrt{7/3}} = \frac{2}{\sqrt{21}}.$$

Option D is only the numerator before normalization. Options B and C involve incorrect magnitude factors and do not equal the computed scalar projection; the denominator must be the positive magnitude $\sqrt{7/3}$.

Step 3: Conclusion The projection is $\boxed{2/\sqrt{21}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use the dot-cross magnitude identity first; a scalar projection often needs only magnitudes and dot products, not unknown components.

13. If the mean deviation about median for the numbers 3, 5, 7, 2k, 12, 16, 21, 24, arranged in the ascending order, is 6 then the median is : [4 marks]

(A) 11.5

(B) 10.5

(C) 12

(D) 11

Correct Answer: (D) 11

Solution:

Concept:

- For eight observations in ascending order, the median is the average of the fourth and fifth observations. Here those observations are $2k$ and 12.
- Mean deviation about a median M is $\frac{1}{8}\sum_{i=1}^{8}|x_i - M|$. Four observations lie on each side of M, so the four negative and four positive occurrences of the median cancel in the sum.

Step 1: Express the median and split the absolute deviations The given ascending order ensures $7 \le 2k \le 12$. Therefore

$$M = \frac{2k+12}{2} = k+6.$$

The lower four observations are $3, 5, 7, 2k$, and the upper four are $12, 16, 21, 24$. Subtract each lower observation from the median and subtract the median from each upper observation. This respects the signs inside every absolute value:

$$\begin{aligned}\sum|x_i - M| &= (4M - 3 - 5 - 7 - 2k) \\ &\quad + (12 + 16 + 21 + 24 - 4M) \\ &= 58 - 2k.\end{aligned}$$

Step 2: Apply the given deviation and check the order Since the mean deviation is 6, the sum of the eight absolute deviations is 48. Thus

$$\frac{58-2k}{8} = 6 \quad \Longrightarrow \quad 58 - 2k = 48 \quad \Longrightarrow \quad k = 5.$$

Hence $2k = 10$, which lies between 7 and 12, as required. The median is 11. A direct check gives deviations $8, 6, 4, 1, 1, 5, 10, 13$, whose sum is 48. The other proposed medians $11.5, 10.5, 12$ correspond respectively to $k = 5.5, 4.5, 6$; their mean deviations are $47/8, 49/8, 46/8$, none equal to 6.

Step 3: Conclusion The median is $\boxed{11}$. Hence, the correct option is (D). The associated key's value, 12, cannot satisfy the printed data: it would give mean deviation $46/8$, not 6.

Quick Tip to Solve: With an even number of sorted observations, subtract the sum of the lower half from that of the upper half to obtain the total absolute deviation about a median.

14. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ **is equal to :** [4 marks]

(A) $\frac{3}{16}$

(B) $\frac{1}{16}$

(C) $\frac{1}{32}$

(D) $\frac{9}{32}$

Correct Answer: (B) $\frac{1}{16}$

Solution:

Concept:

- Complementary angles satisfy $\sin(\pi/2 - x) = \cos x$. This converts the five odd multiples with denominator 22 into cosines with denominator 11.
- The double-angle identity $\sin 2x = 2\sin x \cos x$ gives a product of cosines as a ratio of two sine products. Supplementary angles have equal sine, allowing the factors to be reordered exactly.

Step 1: Rewrite the five sine factors as cosines In the printed expression the numerators are $1, 3, 5, 7, 9$. Taking complements changes these into $10, 8, 6, 4, 2$, all over 22. Reordering the resulting factors, let

$$P = \prod_{j=1}^{5} \cos\frac{j\pi}{11}.$$

The required expression is therefore $2P$. Every factor is positive because each angle $j\pi/11$, for $1 \le j \le 5$, lies strictly between 0 and $\pi/2$.

Step 2: Cancel the doubled-angle sine products Apply the double-angle formula separately for all five values of j:

$$2^5 P = \frac{\prod_{j=1}^{5} \sin(2j\pi/11)}{\prod_{j=1}^{5} \sin(j\pi/11)}.$$

The numerator contains sines with numerators $2, 4, 6, 8, 10$. Using $\sin(\pi - x) = \sin x$, these become numerators $2, 4, 5, 3, 1$, respectively. They are precisely the five denominator factors in a different order. None is zero, so cancellation is valid and yields

$$2^5 P = 1, \qquad P = \frac{1}{32}, \qquad 2P = \frac{1}{16}.$$

Option (C) is the product P before multiplying by the printed leading 2. Options (A) and (D) introduce factors of 3 or 9 unsupported by the exact cancellation.

Step 3: Conclusion The printed expression equals $\boxed{1/16}$. Hence, the correct option is (B). The associated key value $9/32$ contradicts the exact identity $2^5 P = 1$ derived above.

Quick Tip to Solve: For a product of sines at odd multiples of $\pi/22$, take complements first; then use doubling and supplementary angles to cancel a whole product at once.

15. The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to [4 marks]

(A) $\frac{7}{87}$

(B) $\frac{7}{29}$

(C) $\frac{14}{87}$

(D) $\frac{21}{29}$

Correct Answer: (B) $\frac{7}{29}$

Solution:

Concept:

- Partial fractions turn a product of two linear factors into a difference. Here the factors differ by 4, so $\frac{3}{(4n-1)(4n+3)} = \frac{3}{4}\left(\frac{1}{4n-1} - \frac{1}{4n+3}\right)$.
- In a telescoping sum, a negative term cancels the positive term at the next index. Only the first positive and last negative fractions remain.

Step 1: Decompose the general term The numerator of the difference is $(4n+3) - (4n-1) = 4$. Multiplying that difference by $3/4$ therefore reproduces the numerator 3 exactly. Thus, writing the requested sum as S, we obtain

$$S = \frac{3}{4}\sum_{n=1}^{21}\left(\frac{1}{4n-1} - \frac{1}{4n+3}\right).$$

The coefficient is $3/4$, rather than 3; retaining it is essential to the final value.

Step 2: Cancel and simplify the endpoints Expanding the first and last terms displays all cancellations:

$$S = \frac{3}{4}\left[\left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{11}\right) + \cdots + \left(\frac{1}{83} - \frac{1}{87}\right)\right].$$

Consequently,

$$S = \frac{3}{4}\left(\frac{1}{3} - \frac{1}{87}\right) = \frac{3}{4} \cdot \frac{28}{87} = \frac{7}{29}.$$

The terminal denominator is $4(21) + 3 = 87$. Options (A), (C) and (D) have different values and cannot equal this exact endpoint difference.

Step 3: Conclusion The sum is $\boxed{\frac{7}{29}}$. Hence, the correct option is (B).

Quick Tip to Solve: Write the first two and final differences before cancelling: this exposes both the coefficient and the correct final denominator.

16. $\lim\limits_{n\to\infty} \frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+\ \ldots\ +\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$ **is equal to** [4 marks]

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) -2

Correct Answer: (C) 2

Solution:

Concept:

- Put $N = 2^n$, so $N \to \infty$. Reversing the index turns the displayed sum into $N^{-1/2}\sum_{j=1}^{N-1} j^{-1/2}$.
- For the positive decreasing function $f(x) = x^{-1/2}$, integrals above and below its rectangular sums give rigorous bounds. The squeeze theorem determines the limit even though the corresponding Riemann integrand is unbounded at its endpoint.

Step 1: Reverse the summation index Write the finite expression as S_N. Its last denominator has $1-(N-1)/N$, so the index runs from 1 to $N-1$, not to N. For each term, use $1-r/N = (N-r)/N$:

$$S_N = \frac{1}{N}\sum_{r=1}^{N-1}\frac{1}{\sqrt{1-r/N}}$$
$$= \frac{1}{\sqrt{N}}\sum_{r=1}^{N-1}\frac{1}{\sqrt{N-r}} = \frac{1}{\sqrt{N}}\sum_{j=1}^{N-1}\frac{1}{\sqrt{j}}.$$

All terms are defined and positive. Including an additional term with $r = N$ would introduce a zero denominator and would be invalid.

Step 2: Bound the sum by elementary integrals Since $x^{-1/2}$ decreases, rectangles with left endpoints bound the integral from above, while all terms after the first are bounded by the preceding unit-interval integrals. Consequently, for $N \geq 2$,

$$\int_1^N \frac{dx}{\sqrt{x}} \leq \sum_{j=1}^{N-1}\frac{1}{\sqrt{j}} \leq 1 + \int_1^{N-1}\frac{dx}{\sqrt{x}}.$$

Evaluating these integrals and dividing by $\sqrt{N}$ gives

$$2 - \frac{2}{\sqrt{N}} \leq S_N \leq 2\sqrt{1-\frac{1}{N}} - \frac{1}{\sqrt{N}}.$$

Both bounds tend to 2, so the squeeze theorem applies also along $N = 2^n$. Positivity rules out (D), and the two convergent bounds rule out the smaller positive values in (A) and (B).

Step 3: Conclusion The required limit is $\boxed{2}$. Hence, the correct option is (C). The associated key's value, 1, is incompatible with the rigorous lower bound $2 - 2/\sqrt{N}$, which tends to 2.

Quick Tip to Solve: Reverse the index in sums containing $\sqrt{1-r/N}$. A decreasing-function integral bound handles the endpoint singularity without assuming a bounded Riemann integrand.

17. Let a smooth curve $y = f(x)$ be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points $(1, 2)$ and $(8, 1)$, then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to [4 marks]

(A) $2\log_e 2$

(B) 4

(C) 1

(D) $4\log_e 2$

Correct Answer: (B) 4

Solution:

Concept:

- Direct proportionality of the slope to $-y/x$ gives $dy/dx = -ky/x$, where k is a constant fixed by the two given points.
- On the positive x-interval containing the specified points, separation gives $\ln|y| = -k\ln x + C$. A nonzero initial value determines a power-law solution.

Step 1: Integrate the slope condition Separate the variables and integrate:

$$\frac{dy}{y} = -k\frac{dx}{x}, \qquad \ln|y| = -k\ln x + C.$$

The curve passes through $(1, 2)$, so its solution on $x > 0$ is

$$y = 2x^{-k}.$$

This solution stays positive, consistent with the two given ordinates. Substitution into the original equation verifies $y' = -k(2x^{-k})/x$, so the integrated expression obeys precisely the stated proportionality.

Step 2: Determine the constant and evaluate the ordinate Using $(8, 1)$,

$$1 = 2\cdot 8^{-k}, \qquad 8^k = 2, \qquad k = \frac{\ln 2}{\ln 8} = \frac{1}{3}.$$

Thus the curve is $y = 2x^{-1/3}$. At the required argument,

$$y\left(\frac{1}{8}\right) = 2\left(\frac{1}{8}\right)^{-1/3} = 2\cdot 2 = 4.$$

Its absolute value is also 4. Option (C) is the ordinate at $x = 8$, rather than at $x = 1/8$. Options (A) and (D) incorrectly retain logarithms after solving for the exponent; the logarithms simplify to the ratio $1/3$, not a factor in the ordinate.

Step 3: Conclusion $|y(1/8)| = \boxed{4}$. Hence, the correct option is (B).

Quick Tip to Solve: When slope is proportional to y/x, expect a power law; use a point with $x = 1$ first to obtain its coefficient immediately.

18. The tangents at the points $A(1,3)$ and $B(1,-1)$ on the parabola $y^2-2x-2y=1$ meet at the point P. Then the area (in unit ^{2}) of the triangle PAB is : [4 marks]

(A) 4

(B) 6

(C) 7

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- Implicit differentiation of the parabola gives the tangent slope at a specified point. The tangent line through (x_0,y_0) is $y-y_0=m(x-x_0)$.
- The segment joining points with the same abscissa is vertical. Its length is a convenient triangle base, and the perpendicular height is a horizontal distance.

Step 1: Find both tangent equations Differentiate $y^2-2x-2y=1$ with respect to x:

$$2y\frac{dy}{dx}-2-2\frac{dy}{dx}=0, \qquad \frac{dy}{dx}=\frac{1}{y-1}.$$

At $A(1,3)$, the slope is $1/2$; at $B(1,-1)$, it is $-1/2$. Therefore the tangents are

$$y-3=\frac{1}{2}(x-1), \qquad y+1=-\frac{1}{2}(x-1).$$

The signs are opposite, as expected because the points are equally far above and below the symmetry axis $y=1$.

Step 2: Intersect the tangents and calculate the area Rewrite the two equations as $2y=x+5$ and $2y=-x-1$. Equating them gives $2x=-6$, so

$$P=(-3,1).$$

The vertical base AB lies on $x=1$ and has length $|3-(-1)|=4$. The perpendicular distance from P to that line is $|-3-1|=4$. Hence

$$[PAB]=\frac{1}{2}\cdot 4\cdot 4=8 \text{ square units.}$$

Options (A), (B) and (C) do not equal the base-height result. In particular, 4 is only a length here, not the triangle area.

Step 3: Conclusion The area of $\triangle PAB$ is $\boxed{8 \text{ square units}}$. Hence, the correct option is (D).

Quick Tip to Solve: When two tangency points share the same x-coordinate, use their vertical separation as the base and avoid a longer determinant calculation.

19. A plane E is perpendicular to the two planes $2x-2y+z=0$ and $x-y+2z=4$, and passes through the point $P(1,-1,1)$. If the distance of the plane E from the point $Q(a,a,2)$ is $3\sqrt{2}$, then $(PQ)^2$ is equal to : [4 marks]

(A) 9

(B) 12

(C) 21

(D) 33

Correct Answer: (C) 21

Solution:

Concept:

- Two planes are perpendicular exactly when their normal vectors are perpendicular. A normal to a plane perpendicular to two given planes is parallel to the cross product of their normals.
- The distance from (x_0, y_0, z_0) to $Ax+By+Cz+D=0$ is $|Ax_0+By_0+Cz_0+D|/\sqrt{A^2+B^2+C^2}$.

Step 1: Determine the plane through the given point The given plane normals are $\mathbf{n}_1=(2,-2,1)$ and $\mathbf{n}_2=(1,-1,2)$. Their cross product is

$$\mathbf{n}_1\times\mathbf{n}_2=(-3,-3,0).$$

Thus we may take the normal to E as $(1,1,0)$, and its equation has form $x+y+d=0$. Since $P=(1,-1,1)$ lies on it, $d=0$. Therefore $E: x+y=0$. Checking dot products, $(1,1,0)\cdot(2,-2,1)=0$ and $(1,1,0)\cdot(1,-1,2)=0$, verifies both perpendicularity conditions.

Step 2: Apply the distance condition and find the squared length The distance of $Q=(a,a,2)$ from E is

$$\frac{|a+a|}{\sqrt{2}}=\sqrt{2}|a|=3\sqrt{2}.$$

It follows that $|a|=3$ and $a^2=9$; either sign is allowed. Now

$$\begin{aligned} PQ^2&=(a-1)^2+(a+1)^2+(2-1)^2\\ &=2a^2+3=18+3=21. \end{aligned}$$

The linear terms in a cancel, so both possible points Q produce the same result. Option (A) gives only a^2, and options (B) and (D) fail the full three-coordinate distance calculation.

Step 3: Conclusion $(PQ)^2=\boxed{21}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Keep a^2 from the absolute-value distance condition; expanding the squared length may eliminate the need to choose either sign of a.

20. Consider the following statements:

P : Ramu is intelligent.

Q : Ramu is rich.

R : Ramu is not honest.

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as: [4 marks]

(A) $((P \wedge (\sim R)) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee R))$

(B) $((P \wedge R) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(C) $((P \wedge R) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(D) $((P \wedge (\sim R)) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee R))$

Correct Answer: (D) $((P \wedge (\sim R)) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee R))$

Solution:

Concept:

- The proposition R already means that Ramu is not honest. Therefore honesty is represented by $\sim R$, not by R.
- A biconditional $X \leftrightarrow Y$ is false when exactly one of X, Y is true. Its negation is $(X \wedge \sim Y) \vee (Y \wedge \sim X)$.

Step 1: Translate the original sentence correctly Set $X = P \wedge (\sim R)$, expressing that Ramu is intelligent and honest. Set $Y = \sim Q$, expressing that Ramu is not rich. The original statement is thus

$$(P \wedge \sim R) \leftrightarrow (\sim Q).$$

Negating it means that the two sides have different truth values. It does not mean that both sides are false, and it does not mean that the words in each side are individually negated without regard to the connective.

Step 2: Expand the two ways the biconditional can fail Using the exclusive-or form,

$$\sim (X \leftrightarrow Y) = (X \wedge \sim Y) \vee (Y \wedge \sim X).$$

Here $\sim Y = Q$, and De Morgan's law gives $\sim X = (\sim P) \vee R$. Substitution yields

$$\big((P \wedge \sim R) \wedge Q\big) \vee \big((\sim Q) \wedge ((\sim P) \vee R)\big).$$

This is option (D). Options (A) and (C) incorrectly join the two mutually exclusive failure cases by AND; each would require both Q and $\sim Q$. Option (B) replaces honesty by R and therefore negates a different statement.

Step 3: Conclusion The required negation is $\big((P \wedge \sim R) \wedge Q\big) \vee \big((\sim Q) \wedge ((\sim P) \vee R)\big)$. Hence, the correct option is (D).

Quick Tip to Solve: Name each side of an if-and-only-if statement first. Its negation is the OR of the two cases in which exactly one side holds.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A$: either $1 \notin T$ or $2 \in T\}$ and $C = \{T \subseteq A : T$ the sum of all the elements of T is a prime number $\}$. Then the number of elements in the set $B \cup C$ is _____. [4 marks]

Solution:

Concept:

- For the universe $\mathcal{P}(A)$ of all subsets, $|\mathcal{P}(A)| = 2^7 = 128$. Negate the OR defining B to describe the much smaller complement.
- The disjoint union $B \cup C = B \dot{\cup} (B^c \cap C)$ avoids counting any subset twice. In the complement, membership of 1 and 2 is fixed.

Step 1: Count the subsets that belong to B The condition for failing membership in B is $1 \in T$ and $2 \notin T$. Each of $3, 4, 5, 6, 7$ can then be chosen freely, giving

$$|B^c| = 2^5 = 32, \qquad |B| = 128 - 32 = 96.$$

It remains to add only those members of C that are outside B. Every such subset is $T = \{1\} \cup S$, where $S \subseteq \{3, 4, 5, 6, 7\}$. Its element sum is $1 + \sum_{s \in S} s$.

Step 2: Enumerate the prime-sum subsets outside B The possible prime sums are

$$2, 3, 5, 7, 11, 13, 17, 19, 23.$$

Sums 2 and 3 are impossible because the smallest available positive element of S is 3. The remaining choices are exhaustively listed below:

Sum of T	S	Count
5	$\{4\}$	1
7	$\{6\}$	1
11	$\{4, 6\}, \{3, 7\}$	2
13	$\{3, 4, 5\}, \{5, 7\}$	2
17	$\{4, 5, 7\}, \{3, 6, 7\}$	2
19	$\{3, 4, 5, 6\}, \{5, 6, 7\}$	2
23	$\{4, 5, 6, 7\}$	1

These contribute $1 + 1 + 2 + 2 + 2 + 2 + 1 = 11$ new subsets. The largest possible sum is 26, so there is no omitted larger prime. Therefore

$$|B \cup C| = 96 + 11 = 107.$$

Step 3: Conclusion The required number of elements is $\boxed{107}$. This is a dimensionless count of subsets.

Quick Tip to Solve: When a union involves all subsets and an OR condition, count the complement of that OR first; then add only prime-sum subsets lying in the complement.

22. Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0) = p, p \neq 0$, and $f(1) = \frac{1}{3}$. If the equations $f(x) = 0$ and $f \circ f \circ f \circ f(x) = 0$ have a common real root, then $f(-3)$ is equal to _____. [4 marks]

Solution:

Concept:

- For a monic quadratic, write $f(x) = x^2 + bx + p$. The values at zero and one determine its coefficients in terms of the single unknown p.
- If r is a common root, then $f(r) = 0$; successive compositions therefore follow the orbit $r \mapsto 0 \mapsto p \mapsto f(p) \mapsto f(f(p))$.

Step 1: Express the quadratic in terms of p The condition at one gives $1+b+p = 1/3$, so $b = -p-2/3$. Thus

$$f(x) = x^2 - \left(p + \frac{2}{3}\right) x + p.$$

This incorporates both supplied values and keeps the leading coefficient equal to one.

Step 2: Use the common-root condition Let r be the stated common real root. Applying the four copies of f, beginning with the innermost one, gives

$$f(r) = 0, \qquad f(f(r)) = p.$$

The next value is particularly simple:

$$f(p) = p^2 - \left(p + \frac{2}{3}\right) p + p = \frac{p}{3}.$$

Consequently the fourth iterate vanishes precisely when

$$f\left(\frac{p}{3}\right) = \frac{p^2}{9} - \frac{p^2}{3} - \frac{2p}{9} + p = \frac{p(7-2p)}{9} = 0.$$

Since the question excludes $p = 0$, we obtain $p = 7/2$.

Step 3: Evaluate the requested value and check existence Now $b = -25/6$. The discriminant is $b^2 - 4p = 625/36 - 14 = 121/36 > 0$, so the quadratic really has real roots, as required. Substitution at -3 gives

$$f(-3) = 9 + 3\left(p + \frac{2}{3}\right) + p = 11 + 4p = 25.$$

The nonzero solution for p is therefore compatible with the common real root condition.

Step 4: Conclusion The required value is $\boxed{f(-3) = 25}$.

Quick Tip to Solve: For a common root of a polynomial and its iterate, track the orbit of zero instead of expanding the high-degree composition.

23. Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in \mathbb{R}$. If for some $n \in \mathbb{N}, A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then $n + a + b$ is equal to _____. [4 marks]

Solution:

Concept:

- Writing $A = I + N$, with N strictly upper triangular of order three, gives $N^3 = 0$. The binomial expansion therefore stops after the quadratic term.
- The entries of equal matrices are equal. In particular, the first superdiagonal determines na and nb, while the top-right entry determines n.

Step 1: Expand the matrix power For the given matrix,

$$N = \begin{pmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix}, \qquad N^2 = \begin{pmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Since $N^3 = 0$ and I commutes with N,

$$A^n = I + nN + \frac{n(n-1)}{2}N^2.$$

This formula applies to every positive integer n, including one, and avoids repeated matrix multiplication.

Step 2: Compare the three nonzero off-diagonal entries The entries in positions $(1, 2)$ and $(2, 3)$ give

$$na = 48, \qquad nb = 96.$$

As n is positive, $a = 48/n$ and $b = 96/n$. The $(1, 3)$ entry contains both the linear contribution na and the quadratic contribution from N^2:

$$48 + \frac{n(n-1)}{2}\frac{48 \cdot 96}{n^2} = 2160.$$

Hence

$$48 + 2304\frac{n-1}{n} = 2160, \qquad \frac{2304}{n} = 192, \qquad n = 12.$$

Step 3: Recover the parameters It follows that $a = 48/12 = 4$ and $b = 96/12 = 8$. Checking the top-right entry gives $12 \cdot 4 + \binom{12}{2} \cdot 4 \cdot 8 = 48 + 2112 = 2160$, so the three entries agree simultaneously. Therefore

$$n + a + b = 12 + 4 + 8 = 24.$$

Step 4: Conclusion The requested sum is $\boxed{24}$.

Quick Tip to Solve: For a unit upper triangular matrix of order three, use the nilpotent part: only I, N and N squared contribute to any power.

24. The sum of the maximum and minimum values of the function $f(x) = |5x - 7| + \left[x^2 + 2x\right]$ in the interval $\left[\frac{5}{4}, 2\right]$, where $[t]$ is the greatest integer $\leq t$, is _____. [4 marks]

Solution:

Concept:

- The greatest-integer function satisfies $\lfloor t \rfloor \leq t$, and its value changes only when its argument crosses an integer. It must not be treated as a differentiable function.
- An absolute value is nonnegative. Bounds for each term, together with points attaining the bounds, can establish global extrema without listing every interval.

Step 1: Bound the greatest-integer term from below On $[5/4, 2]$, the function $g(x) = x^2 + 2x$ is increasing because $g'(x) = 2x + 2 > 0$. Its smallest value is

$$g\left(\frac{5}{4}\right) = \frac{25}{16} + \frac{5}{2} = \frac{65}{16} > 4.$$

Thus $\lfloor x^2 + 2x \rfloor \geq 4$, while $|5x - 7| \geq 0$. Therefore $f(x) \geq 4$ throughout the interval.

Step 2: Find a point attaining the lower bound The absolute-value term vanishes at $x = 7/5$, which belongs to the specified interval. At this point,

$$x^2 + 2x = \frac{49}{25} + \frac{14}{5} = \frac{119}{25} = 4.76.$$

Its greatest integer is four, so $f(7/5) = 4$. The lower bound is attained, making the global minimum exactly four, rather than merely an infimum.

Step 3: Establish and attain the upper bound Throughout the interval, $x^2 + 2x \leq 8$, so its greatest integer is at most eight. Also $5x - 7$ ranges from $-3/4$ to 3, and hence $|5x - 7| \leq 3$. Combining these inequalities gives $f(x) \leq 11$. At the included endpoint $x = 2$, both upper bounds are attained:

$$f(2) = |10 - 7| + \lfloor 4 + 4 \rfloor = 3 + 8 = 11.$$

Thus the maximum is eleven and the sum of the two attained extrema is $11 + 4 = 15$.

Step 4: Conclusion The sum of the maximum and minimum values is $\boxed{15}$.

Quick Tip to Solve: For sums involving an absolute value and a floor, first find simple bounds and then check whether one point attains each bound.

25. Let $y = y(x)$ be the solution of the differential equation
$\frac{dy}{dx} = \frac{4y^3+2yx^2}{3xy^2+x^3}, y(1) = 1$.
If for some $n \in \mathbb{N}, y(2) \in [n-1, n)$, then n is equal to _____. [4 marks]

Solution:

Concept:

- The right-hand side is homogeneous of degree zero in x, y. Put $v = y/x$, so $y = vx$ and $dy/dx = v + x\,dv/dx$.
- On the branch through $(1, 1)$, both x and v are positive. Separating variables and integrating logarithmic derivatives yields the implicit solution.

Step 1: Reduce to a separable equation Substitute $y = vx$ in the differential equation to obtain

$$v + x\frac{dv}{dx} = \frac{4v^3 + 2v}{3v^2 + 1}.$$

Subtracting v and combining the numerator gives

$$x\frac{dv}{dx} = \frac{v(v^2 + 1)}{3v^2 + 1}.$$

The initial condition is $v(1) = 1$; thus the positive solution branch is the relevant one.

Step 2: Integrate and impose the initial condition Separating the variables gives

$$\frac{3v^2 + 1}{v(v^2 + 1)}\,dv = \frac{dx}{x}.$$

Use the decomposition

$$\frac{3v^2 + 1}{v(v^2 + 1)} = \frac{1}{v} + \frac{2v}{v^2 + 1}.$$

Integration yields $\log v + \log(v^2 + 1) = \log x + C$, and hence $v(v^2 + 1) = Kx$. At $x = 1, v = 1$, we find $K = 2$, so

$$v^3 + v = 2x.$$

The function on the left is strictly increasing, with derivative $3v^2 + 1 > 0$; it determines a unique positive continuous branch for positive x.

Step 3: Locate the value at x equal to two At $x = 2$, the equation becomes $v^3 + v = 4$. At $v = 1$, the left side is two; at $v = 3/2$, it is $39/8 > 4$. Strict increase therefore gives $1 < v < 3/2$. Multiplying by $x = 2$,

$$2 < y(2) < 3.$$

The interval $[n - 1, n)$ containing this value has $n - 1 = 2$. No explicit cubic radical is needed.

Step 4: Conclusion The value lies in $[2, 3)$, so $\boxed{n = 3}$.

Quick Tip to Solve: When only an integer interval for a differential-equation solution is needed, bound the implicit root instead of solving the cubic exactly.

26. Let f be a twice differentiable function on $\mathbb{R}$. If $f'(0) = 4$ and

$$f(x) + \int_0^x (x-t)f'(t)\,dt = (e^{2x} + e^{-2x})\cos 2x + \frac{2}{a}x,$$

then $(2a+1)^5a^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For $I(x) = \int_0^x (x-t)f\prime(t)\,dt$, Leibniz differentiation gives $I\prime(x) = \int_0^x f\prime(t)\,dt$; the boundary term vanishes because $x - t = 0$ at $t = x$.
- At zero, an integral with coincident limits vanishes. Thus differentiating the identity once and evaluating at zero isolates the parameter without solving for f.

Step 1: Differentiate the identity once Write the right-hand nonpolynomial term as $h(x) = (e^{2x} + e^{-2x})\cos 2x$. The given identity reads

$$f(x) + I(x) = h(x) + \frac{2}{a}x.$$

The parameter a is nonzero because it appears in a denominator. The assumptions on f justify differentiating under the integral sign. Since the integrand at the upper endpoint has the zero factor $x - x$,

$$f\prime(x) + \int_0^x f\prime(t)\,dt = h\prime(x) + \frac{2}{a}.$$

Step 2: Evaluate the derivative at the origin The product and chain rules give

$$h\prime(x) = \left(2e^{2x} - 2e^{-2x}\right)\cos 2x - 2\left(e^{2x} + e^{-2x}\right)\sin 2x.$$

At zero the first parenthesis is zero and the sine is also zero, so $h\prime(0) = 0$. In addition, $\int_0^0 f\prime(t)\,dt = 0$. Using the supplied value $f\prime(0) = 4$, we obtain

$$4 = 0 + \frac{2}{a}, \qquad a = \frac{1}{2}.$$

No information about $f(0)$ is needed for this calculation; differentiating the linear term retains precisely the unknown constant.

Step 3: Substitute into the requested expression For $a = 1/2$, we have $2a + 1 = 2$ and $a^2 = 1/4$. Therefore

$$(2a+1)^5a^2 = 2^5 \cdot \frac{1}{4} = \frac{32}{4} = 8.$$

This value follows directly from the identity and the initial derivative condition.

Step 4: Conclusion The required value is $\boxed{8}$.

Quick Tip to Solve: A kernel containing x minus t vanishes at the moving endpoint; differentiate once and evaluate at zero before attempting to find the whole function.

27. Let $a_n = \int_{-1}^{n} \left(1 + \frac{x}{2} + \frac{x^2}{3} + \ldots\ldots + \frac{x^{n-1}}{n}\right) dx$ for every n $\in$ N. Then the sum of all the elements of the set {n $\in$ N : $a_n \in$ (2, 30)} is _____. [4 marks]

Solution:

Concept:

- The integrand is the finite polynomial $P_n(x) = \sum_{k=0}^{n-1} x^k/(k+1)$, so it can be integrated term by term.
- On $[-1, 0]$, successive terms alternate with decreasing magnitudes; grouping neighbouring terms shows that the sum is positive. On nonnegative x, every term is nonnegative.

Step 1: Calculate the first three values For $n = 1$,

$$a_1 = \int_{-1}^{1} 1\, dx = 2,$$

which is excluded because the interval $(2, 30)$ is open. For $n = 2$,

$$a_2 = \left[x + \frac{x^2}{4}\right]_{-1}^{2} = \frac{15}{4}.$$

For $n = 3$,

$$a_3 = \left[x + \frac{x^2}{4} + \frac{x^3}{9}\right]_{-1}^{3} = 4 + 2 + \frac{28}{9} = \frac{82}{9}.$$

Both $15/4$ and $82/9$ lie strictly between two and thirty.

Step 2: Exclude every n at least four Let $x = -u$, where $0 \le u \le 1$. The magnitudes $u^k/(k+1)$ decrease. Pairing each positive term with its following negative term shows $P_n(-u) \ge 1 - u/2 \ge 1/2$ for $n \ge 2$. Thus the integral over $[-1, 0]$ is positive. For $n \ge 4$, the interval $[0, 4]$ lies inside $[0, n]$, and the first four terms are present, with all remaining terms nonnegative. Consequently

$$a_n > \int_{0}^{4} \left(1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4}\right) dx.$$

The integral on the right is

$$4 + 4 + \frac{64}{9} + 16 = \frac{280}{9} > 30.$$

This bound rules out every larger positive integer in a single argument.

Step 3: Form the requested set and its sum The first value is an excluded endpoint; the next two are inside the required open interval; and all subsequent values exceed its upper endpoint. Hence the complete set is $\{2, 3\}$. The problem asks for the sum of its indices, not the sum of the integral values:

$$2 + 3 = 5.$$

Step 4: Conclusion The sum of all elements of the set is $\boxed{5}$.

Quick Tip to Solve: After checking small indices, use positive-term bounds on a fixed subinterval to exclude all larger indices at once.

28. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point $(-2, 3)$ be A. Then 8A is equal to _____. [4 marks]

Solution:

Concept:

- For an implicit curve $F(x, y) = 0$, the tangent slope is $dy/dx = -F_x/F_y$ when $F_y \neq 0$. The normal slope is the negative reciprocal of the tangent slope.
- The tangent, normal and x-axis form a triangle. Its base is the distance between the two x-intercepts and its perpendicular height is the absolute y-coordinate of their common point.

Step 1: Differentiate the curve implicitly Let

$$F(x, y) = 4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14.$$

Its partial derivatives are

$$F_x = 12x^2 - 3y^2 + 12x - 5y + 9, \qquad F_y = -6xy - 5x - 16y.$$

At $(-2, 3)$, these give

$$F_x = 48 - 27 - 24 - 15 + 9 = -9, \qquad F_y = 36 + 10 - 48 = -2.$$

Thus the tangent slope is $-(-9)/(-2) = -9/2$, while the normal slope is $2/9$.

Step 2: Find the intersections with the x-axis Both lines pass through $(-2, 3)$. Their equations are

$$y - 3 = -\frac{9}{2}(x + 2), \qquad y - 3 = \frac{2}{9}(x + 2).$$

For the tangent, set $y = 0$ to get $x + 2 = 2/3$, so its intercept is $-4/3$. For the normal, setting $y = 0$ gives $x + 2 = -27/2$, so its intercept is $-31/2$. Both intercepts are negative, and the normal intercept lies farther to the left.

Step 3: Calculate the triangular area The base is the absolute difference of those intercepts:

$$B = \left| -\frac{4}{3} + \frac{31}{2} \right| = \frac{85}{6}.$$

The common vertex is three units above the x-axis, so the altitude is three. Therefore

$$A = \frac{1}{2} \cdot \frac{85}{6} \cdot 3 = \frac{85}{4}.$$

Finally the expression asked for is

$$8A = 8 \cdot \frac{85}{4} = 170.$$

Using the intercept difference avoids any need to draw or integrate the triangular region.

Step 4: Conclusion The required value is $\boxed{8A = 170}$.

Quick Tip to Solve: For tangent-normal triangles with the x-axis, find the two intercepts and use the point height as the triangle altitude.

29. Let $x = \sin(2\tan^{-1}\alpha)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S = \{a \in R : y^2 = 1 - x\}$, then $\sum_{\alpha\in S} 16\alpha^3$ is equal to _____. [4 marks]

Solution:

Concept:

- For a principal inverse tangent, $\theta = \tan^{-1}(4/3)$ lies in the first quadrant. Thus its cosine is positive and can be obtained from a right triangle of sides 3, 4 and 5.
- The half-angle identity is $\sin^2(\theta/2) = (1 - \cos\theta)/2$, while $\sin(2\tan^{-1}\alpha) = 2\alpha/(1 + \alpha^2)$ for every real α.

Step 1: Evaluate the fixed value of y squared Put $\theta = \tan^{-1}(4/3)$. Since $0 < \theta < \pi/2$, we have $\cos\theta = 3/5$. Hence

$$y^2 = \frac{1 - 3/5}{2} = \frac{1}{5}.$$

The condition defining the set therefore becomes $x = 1 - y^2 = 4/5$; there is no sign ambiguity because the condition uses the square of y.

Step 2: Find every admissible value of alpha Use the double-angle identity and cross-multiply by the strictly positive quantity $1 + \alpha^2$:

$$\frac{2\alpha}{1 + \alpha^2} = \frac{4}{5} \quad \Longrightarrow \quad 2\alpha^2 - 5\alpha + 2 = 0.$$

Factoring gives $(2\alpha - 1)(\alpha - 2) = 0$, so $S = \{1/2, 2\}$. Both numbers satisfy the original expression: each gives x equal to 4/5. Consequently neither root is introduced by the algebra.

Step 3: Sum the requested cubes The summation includes both real roots, each once. Therefore

$$\sum_{\alpha\in S} 16\alpha^3 = 16\left[\left(\frac{1}{2}\right)^3 + 2^3\right] = 16\left(\frac{1}{8} + 8\right) = 130.$$

In particular, using just the smaller root would omit a valid member of the specified set.

Step 4: Conclusion The required numerical value is $\boxed{130}$. This is dimensionless, as all the quantities are trigonometric or algebraic numbers.

Quick Tip to Solve: Find y squared directly with the half-angle identity; then the alpha equation is quadratic and both roots must be included.

30. If the circles $x^2+y^2+6x+8y+16=0$ and $x^2+y^2+2\left(3-\sqrt{3}\right)x+2\left(4-\sqrt{6}\right)y=k+6\sqrt{3}+8\sqrt{6}$, $k>0$, touch internally at the point $P(\alpha,\beta)$, then $\left(\alpha+\sqrt{3}\right)^2+\left(\beta+\sqrt{6}\right)^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- Completing squares gives the centre and radius of each circle. For internally tangent circles, the separation of their centres equals the difference of their radii.
- The contact point lies on the line joining the centres, on the side of the smaller centre away from the larger centre. This fixes the sign when computing its coordinates.

Step 1: Find the centres and radii The first equation becomes

$$(x+3)^2+(y+4)^2=9.$$

Thus $C_1=(-3,-4)$ and $r_1=3$. Completing squares in the second equation gives centre $C_2=(-3+\sqrt{3},-4+\sqrt{6})$ and

$$r_2^2=k+6\sqrt{3}+8\sqrt{6}+(3-\sqrt{3})^2+(4-\sqrt{6})^2=k+34.$$

Also $C_1C_2=\sqrt{3+6}=3$. Since k is positive, the second circle is the larger one.

Step 2: Apply internal tangency and locate the contact Internal tangency requires $r_2-r_1=C_1C_2$, so $r_2=6$ and $k=2$, consistent with k being positive. The vector from the larger centre to the smaller centre is $(-\sqrt{3},-\sqrt{6})$, of length 3. Moving a further distance $r_1=3$ in that same direction reaches their common point:

$$P=C_1+(-\sqrt{3},-\sqrt{6})=(-3-\sqrt{3},-4-\sqrt{6}).$$

The distance of this point from C2 is 6, independently checking its membership of the larger circle.

Step 3: Evaluate the specified expression We have $\alpha+\sqrt{3}=-3$ and $\beta+\sqrt{6}=-4$. Substitution now removes the radicals:

$$(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2=(-3)^2+(-4)^2=25.$$

Choosing the opposite point of the smaller circle would fail the larger-circle distance condition.

Step 4: Conclusion The required value is $\boxed{25}$, in squared coordinate units.

Quick Tip to Solve: For internal contact, draw the centre line mentally: the contact is beyond the smaller centre as viewed from the larger one.

31. The electric current in a circular coil of 2 turns produces a magnetic induction B_1 at its centre. The coil is unwound and in rewound into a circular coil of 5 tuns and the same current produces a magnetic induction B_2 at its centre. The ratio of $\frac{B_2}{B_1}$ is [4 marks]

(A) $\frac{5}{2}$
(B) $\frac{25}{4}$
(C) $\frac{5}{4}$
(D) $\frac{25}{2}$

Correct Answer: (B) $\frac{25}{4}$

Solution:

Concept:

- At the centre of a circular coil of N turns and radius r, carrying current I, $B = \mu_0 NI/(2r)$. Each turn contributes in the same direction.
- Rewinding preserves the total wire length L, so $L = 2\pi Nr$. With the same current and wire, the radius decreases as the turn count increases.

Step 1: Relate the two radii using the wire length Let r1 be the radius of the two-turn coil and r2 the radius of the five-turn coil. The same wire is used without any change of length. Thus

$$2\pi(2)r_1 = 2\pi(5)r_2, \qquad \frac{r_1}{r_2} = \frac{5}{2}.$$

The new radius is smaller; simply multiplying the magnetic field by the turn ratio while holding the radius fixed would miss this change.

Step 2: Take the ratio of the central fields Both coils carry the same current, so the permeability and current cancel:

$$\frac{B_2}{B_1} = \frac{\mu_0(5)I/(2r_2)}{\mu_0(2)I/(2r_1)} = \frac{5}{2}\frac{r_1}{r_2} = \frac{5}{2}\frac{5}{2} = \frac{25}{4}.$$

Equivalently, eliminating r in favour of L gives $B = \mu_0 \pi I N^2/L$, making the square dependence explicit. The result exceeds one, consistent with both more turns and a smaller radius increasing the field.

Step 3: Compare with the listed ratios Option (A), 5/2, accounts only for the extra turns and ignores the reduced radius. Option (C), 5/4, and option (D), 25/2, also do not equal the square of the turn ratio; they would require an incorrect radius relation. The derived dimensionless ratio equals only the value in option (B).

Step 4: Conclusion $\boxed{B_2/B_1 = 25/4}$. Hence, the correct option is (B).

Quick Tip to Solve: When the same wire is rewound at unchanged current, use B proportional to the square of the number of turns.

32. A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension 7.5×10^{-4} Ncm^{-1}. The radius of drop in cm will be :

(g = 10 ms^{-2}) [4 marks]

(A) $\frac{15}{\sqrt{(2\rho-\sigma)}}$

(B) $\frac{15}{\sqrt{(\rho-\sigma)}}$

(C) $\frac{3}{2\sqrt{(\rho-\sigma)}}$

(D) $\frac{3}{20\sqrt{(2\rho-\sigma)}}$

Correct Answer: (A) $\frac{15}{\sqrt{(2\rho-\sigma)}}$

Solution:

Concept:

- The weight of a spherical drop is $\rho(4\pi r^3/3)g$. When it is half immersed, the buoyant force is $\sigma(2\pi r^3/3)g$. Densities are used in SI units.
- The upward supporting surface-tension force at the equatorial contact circle is $2\pi rT$, where T is the surface tension. Vertical equilibrium balances this force plus buoyancy against weight.

Step 1: Write the vertical force balance For a drop of radius r, half the volume displaces the surrounding liquid. Hence

$$\frac{4}{3}\pi r^3\rho g = \frac{2}{3}\pi r^3\sigma g + 2\pi rT.$$

The tension acts around the contact circumference, not over the spherical surface area. Dividing through by $2\pi r$ and collecting the density terms yields

$$\frac{r^2 g}{3}(2\rho - \sigma) = T, \qquad r^2 = \frac{3T}{g(2\rho-\sigma)}.$$

The condition $2\rho > \sigma$ supplies the positive residual weight supported by tension.

Step 2: Convert the tension and radius units The given tension is per centimetre. In SI units it is

$$T = 7.5 \times 10^{-4} \times 100 = 0.075\,\mathrm{N\,m^{-1}}.$$

Using $g = 10\,\mathrm{m\,s^{-2}}$ gives

$$r = \sqrt{\frac{3(0.075)}{10(2\rho-\sigma)}}\,\mathrm{m} = \frac{0.15}{\sqrt{2\rho-\sigma}}\,\mathrm{m} = \frac{15}{\sqrt{2\rho-\sigma}}\,\mathrm{cm}.$$

Here the numerical density values are in $\mathrm{kg\,m^{-3}}$, consistent with the SI substitution.

Step 3: Check the alternatives Options (B) and (C) have the wrong density combination: half immersion makes the displaced volume half the drop volume and therefore produces $2\rho - \sigma$. Option (D) has the right density combination but the wrong coefficient for SI density values. The centimetre radius derived above matches option (A).

Step 4: Conclusion The radius is $\boxed{15/\sqrt{2\rho-\sigma}\ \mathrm{cm}}$ when the densities are in SI units. Hence, the

correct option is (A).

Quick Tip to Solve: Write weight minus half-volume buoyancy first, then convert the tension from N per cm to N per m before finding the radius.

33. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is t = 0.005 s, then what is the force exerted on the ball due to each other? [4 marks]

(A) 100 N

(B) 200 N

(C) 300 N

(D) 400 N

Correct Answer: (B) 200 N

Solution:

Concept:

- The average contact force on one ball is its momentum change divided by contact time: $\vec{F}_{\text{av}} = \Delta\vec{p}/\Delta t$. A reversal of velocity changes both its sign and its momentum.
- The two contact forces are equal in magnitude and opposite in direction by Newton's third law. The force on either ball is obtained from that ball's own mass and velocity change.

Step 1: Choose a direction and calculate the momentum change Take the initial direction of one ball as positive. Its initial velocity is $u = +10\,\text{m}\,\text{s}^{-1}$, whereas after rebounding its final velocity is $v = -10\,\text{m}\,\text{s}^{-1}$. Therefore

$$\Delta p = m(v - u) = 0.05(-10 - 10) = -1.0\,\text{kg}\,\text{m}\,\text{s}^{-1}.$$

The impulse magnitude is 1.0 N s. Although the speed is unchanged, the velocity is reversed; hence the momentum change is not zero.

Step 2: Divide the impulse by the contact duration The mean force over the specified contact interval is

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{-1.0}{0.005} = -200\,\text{N}.$$

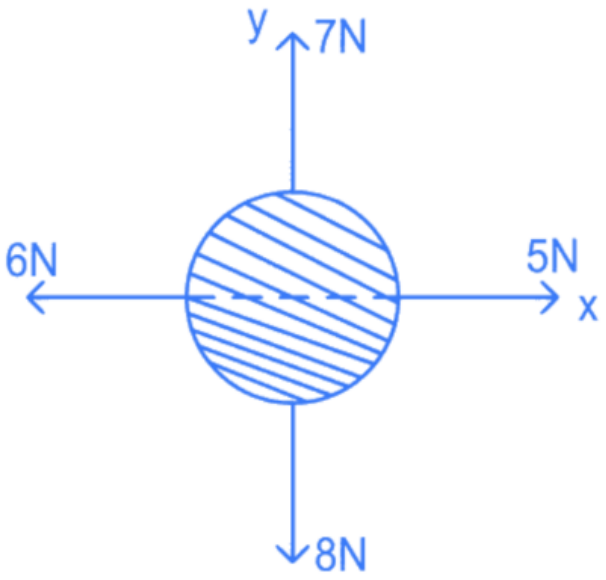
The negative sign says that the force on the chosen ball opposes its initial motion. The other ball has the same mass and opposite velocity change, so its mean force is +200 N in this coordinate convention. These are the third-law pair, not two forces to be added on one ball.

Step 3: Identify the requested magnitude The magnitude of the contact force on each ball is consequently 200 N. Option (A), 100 N, would follow from stopping the ball rather than reversing it. Options (C) and (D), 300 N and 400 N, overestimate the impulse on a single ball. The instantaneous contact force need not be constant; the data determine its average during contact.

Step 4: Conclusion The average force exerted on either ball by the other has magnitude $\boxed{200\,\text{N}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a rebound at unchanged speed, the momentum change magnitude is twice mv; do not use the unchanged speed as zero velocity change.

34. For a free body diagram shown in the figure, the four forces are applied in the 'x' and 'y' directions. What additional force must be applied and at what angle with positive x-axis so that the net acceleration of body is zero? [4 marks]



(A) $\sqrt{2}N, 45°$

(B) $\sqrt{2}N, 135°$

(C) $\frac{2}{\sqrt{3}}N, 30°$

(D) $2N, 45°$

Correct Answer: (A) $\sqrt{2}N, 45°$

Solution:

Concept:

- Zero acceleration requires the resultant external force to vanish, by $\sum \vec{F} = m\vec{a}$. Horizontal and vertical components can be balanced independently.
- The additional force must be the negative of the resultant of the four given forces. Its angle is determined by both component signs, not by the tangent ratio alone.

Step 1: Resolve the forces shown in the figure The diagram gives 5 N to the right, 6 N to the left, 7 N upward and 8 N downward. Taking rightward and upward as positive, their components sum to

$$F_x = 5 - 6 = -1\,\text{N}, \qquad F_y = 7 - 8 = -1\,\text{N}.$$

Thus the given forces together point diagonally down and left. Their nonzero resultant would accelerate the body in that direction unless another force balances it.

Step 2: Find the balancing vector and its direction The required extra force must satisfy $\vec{F}_{\text{extra}} + \vec{F}_{\text{given}} = 0$. Therefore

$$\vec{F}_{\text{extra}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}})\,\text{N}.$$

Its magnitude is

$$|\vec{F}_{\text{extra}}| = \sqrt{1^2 + 1^2} = \sqrt{2}\,\text{N}.$$

Both components are positive and equal, so it is in the first quadrant, at $\theta = \tan^{-1}(1/1) = 45^\circ$ counterclockwise from the positive x-axis.

Step 3: Verify equilibrium and reject the other choices Adding this force makes the horizontal total $-1 + 1 = 0$ N and the vertical total $-1 + 1 = 0$ N. Option (B) points into the second quadrant and supplies the wrong horizontal component. Option (C) has unequal components at 30 degrees and cannot cancel equal residual components. Option (D) points correctly but has magnitude 2 N rather than square root of 2 N, so each component would be too large.

Step 4: Conclusion Apply $\boxed{\sqrt{2}\,\text{N}}$ at $\boxed{45^\circ}$ to the positive x-axis. Hence, the correct option is (A).

Quick Tip to Solve: Subtract opposite forces on each axis first; reverse the resulting vector to obtain the single equilibrant.

35. Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is : [4 marks]

(A) $\frac{n}{n-1}$
(B) $\frac{2n}{2n+1}$
(C) $\frac{n+1}{n}$
(D) $\frac{2n+1}{n}$

Correct Answer: (A) $\frac{n}{n-1}$

Solution:

Concept:

- An isolated conducting sphere of radius R1 has capacitance $C_0 = 4\pi\varepsilon_0 R_1$, measured relative to infinity.
- When a concentric outer conductor of radius R2 is grounded, its potential is zero. The inner and outer spheres form a spherical capacitor with $C = 4\pi\varepsilon_0 R_1 R_2/(R_2 - R_1)$.

Step 1: Find the potential of the inner sphere Let the inner sphere carry charge Q. In the space $R_1 < r < R_2$, Gauss's law gives $E = Q/(4\pi\varepsilon_0 r^2)$. Integrating from the grounded outer surface to the inner surface yields

$$V_1 - 0 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Grounding fixes the outer potential; it does not make the electric field in the gap vanish. Thus the charge-to-potential ratio of the inner sphere increases.

Step 2: Use the specified capacitance enhancement The capacitance is $C = Q/V_1$. Comparing this with the isolated value gives

$$n = \frac{C}{C_0} = \frac{R_2}{R_2 - R_1}.$$

Rearrange without discarding either radius:

$$nR_2 - nR_1 = R_2, \qquad (n-1)R_2 = nR_1.$$

Therefore $R_2/R_1 = n/(n-1)$. Since the outer sphere encloses the inner one, its radius must be greater, and correspondingly n is greater than one.

Step 3: Check the proposed radius ratios Option (B) is less than one for positive n and cannot describe an enclosing sphere. Options (C) and (D) do not satisfy the enhancement equation $n = R_2/(R_2 - R_1)$. Option (A) does satisfy it; it also gives an enhancement tending to one when the outer radius tends to infinity and a large enhancement when the spheres become close.

Step 4: Conclusion $\boxed{R_2/R_1 = n/(n-1)}$. Hence, the correct option is (A).

Quick Tip to Solve: Divide spherical-capacitor capacitance by isolated-sphere capacitance before solving; all electrostatic constants cancel.

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36. The ratio of wavelengths of proton and deuteron accelerated by potential V_p and V_d is 1 : $\sqrt{2}$. Then the ratio of V_p to V_d will be : [4 marks]

(A) 1 : 1

(B) $\sqrt{2}$: 1

(C) 2 : 1

(D) 4 : 1

Correct Answer: (D) 4 : 1

Solution:

Concept:

- A charged particle accelerated from rest through a potential difference V gains kinetic energy qV. In the nonrelativistic regime its momentum is $p = \sqrt{2mqV}$.
- The de Broglie relation is $\lambda = h/p$. A proton and a deuteron both have charge $+e$, while the deuteron has approximately twice the proton mass.

Step 1: Express the wavelength ratio Write the wavelengths using the same charge and Planck constant. These common factors cancel when the ratio is formed:

$$\frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d V_d}{m_p V_p}} = \sqrt{\frac{2V_d}{V_p}}.$$

The heavier particle does not automatically have the shorter wavelength here, since the accelerating potentials can differ.

Step 2: Use the given ratio and compare the choices The stated ratio is $\lambda_p/\lambda_d = 1/\sqrt{2}$. Squaring both sides gives

$$\frac{1}{2} = \frac{2V_d}{V_p}, \qquad V_p = 4V_d.$$

As a check, equal potentials would give a wavelength ratio $\sqrt{2}$, the reverse of the required ratio. A potential ratio of $2:1$ would make the wavelengths equal, while $\sqrt{2}:1$ gives a wavelength ratio $2^{1/4}$. Thus none of the first three choices satisfies the supplied wavelength ratio.

Step 3: Conclusion The required potential ratio is $\boxed{V_p : V_d = 4:1}$. Hence, the correct option is (D).

Quick Tip to Solve: Square a de Broglie wavelength ratio before substituting masses; remember that both proton and deuteron carry the same charge.

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37. For an object placed at a distance 2.4 m from a lens, a sharp focused image is observed on a screen placed at a distance 12 cm from the lens. A glass plate of refractive index 1.5 and thickness 1 cm is introduced between lens and screen such that the glass plate plane faces parallel to the screen. By what distance should the object be shifted so that a sharp focused image is observed again on the screen? [4 marks]

(A) 0.8 m

(B) 3.2 m

(C) 1.2 m

(D) 5.6 m

Correct Answer: (B) 3.2 m

Solution:

Concept:

- For a thin converging lens with a real object at distance U and a real image at distance v, $1/f = 1/U + 1/v$, where both distances denote positive magnitudes.
- A parallel glass plate of thickness t and refractive index μ, inserted into a converging beam, shifts its focus away from the lens by $\delta = t(1 - 1/\mu)$.

Step 1: Find the lens focal length Convert the initial object distance into centimetres: $U = 240\,\text{cm}$, while $v = 12\,\text{cm}$. The focal length is fixed by the original sharp image:

$$\frac{1}{f} = \frac{1}{240} + \frac{1}{12} = \frac{21}{240} = \frac{7}{80}\ \text{cm}^{-1}.$$

Thus $f = 80/7\,\text{cm}$; the lens itself does not change when the plate is inserted.

Step 2: Compensate for the focus shift The glass produces a positive focus shift

$$\delta = 1\left(1 - \frac{1}{1.5}\right) = \frac{1}{3}\,\text{cm}.$$

To leave the final focus at the fixed screen, the lens alone must now converge the rays to $v_0 = 12 - 1/3 = 35/3\,\text{cm}$. Therefore

$$\frac{1}{U_{\text{new}}} = \frac{7}{80} - \frac{3}{35} = \frac{1}{560}, \qquad U_{\text{new}} = 560\,\text{cm}.$$

The object must move away from the lens by $560 - 240 = 320\,\text{cm}$. The value $5.6\,\text{m}$ is its new distance, not its displacement; $0.8\,\text{m}$ and $1.2\,\text{m}$ do not restore focus.

Step 3: Conclusion The object must be shifted $\boxed{3.2\,\text{m}}$ away from the lens. Hence, the correct option is (B).

Quick Tip to Solve: A slab shifts a converging focus outward. Subtract that shift from the screen distance before applying the lens formula again.

38. Light wave travelling in air along x-direction is given by $E_y = 540 \sin \pi \times 10^4 (x - ct)\, Vm^{-1}$. Then, the peak value of magnetic field of wave will be (Given c = 3 $\times$ 10^8 ms^{-1}) [4 marks]

(A) 18×10^{-7} T

(B) 54×10^{-7} T

(C) 54×10^{-8} T

(D) 18×10^{-8} T

Correct Answer: (A) 18×10^{-7} T

Solution:

Concept:

- In a plane electromagnetic wave in air, the electric and magnetic fields oscillate in phase, perpendicular to each other and to the direction of propagation.
- The peak magnitudes obey $E_0 = cB_0$, where c is the propagation speed. The sinusoidal factor varies between -1 and $+1$, so its coefficient is the amplitude.

Step 1: Read the electric field amplitude The given field is $E_y = 540 \sin[\pi \times 10^4 (x - ct)]\,\text{V m}^{-1}$. Its peak electric field is therefore

$$E_0 = 540\,\text{V m}^{-1}.$$

The factor $\pi \times 10^4$ belongs to the phase and controls the wavelength and frequency. It is not an additional multiplier of the amplitude, and no rms conversion is required because the question explicitly asks for a peak value.

Step 2: Calculate the magnetic amplitude and check the exponents Use the supplied wave speed directly:

$$B_0 = \frac{E_0}{c} = \frac{540}{3 \times 10^8} = 180 \times 10^{-8} = 18 \times 10^{-7}\,\text{T}.$$

Equivalently, this is 1.8×10^{-6} T. Multiplying this answer by $3 \times 10^8\,\text{m s}^{-1}$ recovers $540\,\text{V m}^{-1}$, checking the arithmetic and powers of ten. The second option is three times too large, the third is too small by a factor $10/3$, and the fourth is ten times too small.

Step 3: Conclusion The peak magnetic field is $\boxed{18 \times 10^{-7}\,\text{T}}$. Hence, the correct option is (A).

Quick Tip to Solve: Read the amplitude outside the sine first, then divide it by the wave speed; the phase coefficient is irrelevant to this ratio.

39. When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on : [4 marks]

(A) Electromagnetic induction

(B) Resonance in ac circuits

(C) Mutual induction in ac circuits

(D) Interference of electromagnetic waves

Correct Answer: (B) Resonance in ac circuits

Solution:

Concept:

- An LC oscillator has natural angular frequency $\omega_0 = 1/\sqrt{LC}$, or $f_0 = 1/(2\pi\sqrt{LC})$. Its response changes strongly when the circuit moves into or out of resonance.
- A metal object near a detector coil alters its effective inductance and losses through its magnetic response and induced currents. A tuned detector senses the resulting change in its oscillatory response.

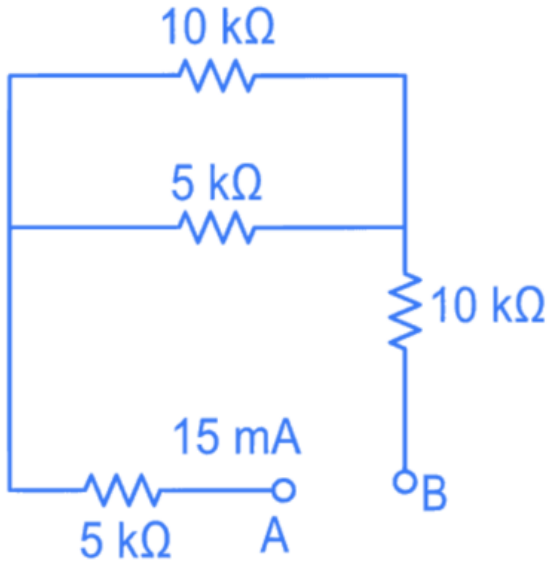
Step 1: Identify the change caused by the metal The detector contains a coil in an alternating-current oscillator or tuned circuit. With no metal nearby, its inductance and capacitance determine the reference oscillation frequency and response. When a conducting or magnetic object enters the coil field, induced currents and magnetic effects modify the coil impedance. In particular, a change in effective L changes the natural frequency, even though the circuit capacitance remains fixed.

Step 2: Relate the alarm to the tuned circuit The apparatus detects the change in frequency or the change in response relative to its tuned condition, and its electronics triggers an audible alarm. Thus the operating principle being tested is resonance in an ac circuit. Electromagnetic induction helps explain why the metal interacts with the coil, but by itself it does not identify the tuned response used for detection in this question. Mutual induction between two prescribed ac coils is not the required description, and interference of freely propagating electromagnetic waves is not the alarm mechanism. The resonance choice directly connects the altered inductance to the detector response.

Step 3: Conclusion The metal detector operates on $\boxed{\text{resonance in ac circuits}}$. Hence, the correct option is (B).

Quick Tip to Solve: When a detector coil is part of a tuned oscillator, connect a metal-induced change in inductance with the resonance frequency.

40. A current of 15 mA flows in the circuit as shown in figure. The value of potential difference between the points A and B will be: [4 marks]



(A) 50 V

(B) 75 V

(C) 150 V

(D) 275 V

Correct Answer: (D) 275 V

Solution:

Concept:

- Resistors are in parallel when both their ends join the same two nodes; they share the same potential difference. Their equivalent resistance obeys $1/R_p = 1/R_1 + 1/R_2$.
- Series sections carry the same total current, so their resistances add. The terminal potential difference in magnitude is $V = IR_{\text{eq}}$, with current in amperes and resistance in ohms.

Step 1: Reduce the parallel pair The upper $10\,\text{k}\Omega$ resistor and middle $5\,\text{k}\Omega$ resistor connect between the left vertical wire and the upper right junction. They are therefore parallel, not series:

$$R_p = \frac{10 \times 5}{10 + 5}\,\text{k}\Omega = \frac{10}{3}\,\text{k}\Omega.$$

The resistor adjacent to A is outside this pair, and the vertical right-hand $10\,\text{k}\Omega$ resistor lies between the right junction and B.

Step 2: Find the complete terminal voltage These three sections are in series between A and B. Their combined resistance is

$$R_{AB} = 5 + \frac{10}{3} + 10 = \frac{55}{3}\,\text{k}\Omega.$$

The total current $15\,\text{mA}$ passes through the two outer resistors and splits only inside the parallel pair.

Hence

$$|V_A - V_B| = 15 \times \frac{55}{3} = 275\,\text{V}.$$

Here $\text{mA} \times \text{k}\Omega = \text{V}$. The values 50, 75, and $150\,\text{V}$ are respectively the drops across the parallel section, the $5\,\text{k}\Omega$ series resistor, and the $10\,\text{k}\Omega$ series resistor, rather than the complete A-to-B drop.

Step 3: Conclusion The potential difference between A and B has magnitude $\boxed{275\,\text{V}}$. Hence, the correct option is (D).

Quick Tip to Solve: Identify nodes before combining resistors. Each individual voltage drop must be included in the full terminal potential difference.

41. The length of a seconds pendulum at a height h = 2R from earth surface will be: (Given R = Radius of earth and acceleration due to gravity at the surface of earth, g = π^2 ms^{-2})

[4 marks]

(A) $\frac{2}{9}$ m

(B) $\frac{4}{9}$ m

(C) $\frac{8}{9}$ m

(D) $\frac{1}{9}$ m

Correct Answer: (D) $\frac{1}{9}$ m

Solution:

Concept:

- Outside a spherical Earth, gravitational acceleration varies inversely as the square of distance from its centre: $g_h = g[R/(R+h)]^2$.
- For a simple pendulum in the small-angle approximation, $T = 2\pi\sqrt{\ell/g_h}$. A seconds pendulum takes one second for a half oscillation, so its complete period is $T = 2\,\text{s}$.

Step 1: Evaluate gravity at the stated height The height $h = 2R$ is measured from the Earth surface, not from its centre. The pendulum is thus at radial distance $R + h = 3R$. Consequently,

$$g_h = g\left(\frac{R}{3R}\right)^2 = \frac{g}{9} = \frac{\pi^2}{9}\,\text{m s}^{-2}.$$

The acceleration is reduced by a factor of nine, not by a factor of four; the latter would correspond to a radial distance of $2R$.

Step 2: Calculate the length for a two-second period Rearranging the period formula,

$$\ell = \frac{g_h T^2}{4\pi^2} = \frac{(\pi^2/9)(2)^2}{4\pi^2} = \frac{1}{9}\,\text{m}.$$

The factor of four from the squared period cancels the factor of four in the denominator. As a useful check, a seconds pendulum at the Earth surface would have length $1\,\text{m}$ with the supplied value of g. Keeping the same period at one ninth of this gravity requires one ninth of that length. The other listed fractions would increase ℓ/g_h and hence produce a period longer than two seconds.

Step 3: Conclusion The required length is $\boxed{\ell = \frac{1}{9}\,\mathrm{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed pendulum period, length is proportional to local gravity. Always convert altitude to distance from the Earth centre.

42. Sound travels in a mixture of two moles of helium and n moles of hydrogen. If rms speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound, then the value of n will be : [4 marks]

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Concept:

- For a gas mixture with mean molar mass M, the effective rms molecular speed satisfies $v_{\mathrm{rms}} = \sqrt{3RT/M}$, while the adiabatic sound speed is $v_s = \sqrt{\gamma RT/M}$.
- Heat capacities add by mole number. Helium is monatomic with molar $C_V = 3R/2$, $C_P = 5R/2$; hydrogen is diatomic with $C_V = 5R/2$, $C_P = 7R/2$ when vibrational modes are inactive.

Step 1: Infer the heat-capacity ratio from the speeds The temperature and mean molar mass cancel in the ratio of the two speeds:

$$\frac{v_{\mathrm{rms}}}{v_s} = \sqrt{\frac{3}{\gamma}} = \sqrt{2}.$$

Squaring this relation gives $3/\gamma = 2$, so the mixture must have $\gamma = 3/2$. It is unnecessary to calculate the mean molar mass separately, because both speed expressions contain the same factor RT/M.

Step 2: Add heat capacities and solve for the mole number For two moles of helium and n moles of hydrogen,

$$C_{V,\mathrm{total}} = \frac{R}{2}(6 + 5n), \qquad C_{P,\mathrm{total}} = \frac{R}{2}(10 + 7n).$$

Thus

$$\frac{10 + 7n}{6 + 5n} = \frac{3}{2}, \qquad 20 + 14n = 18 + 15n, \qquad n = 2.$$

Substitution gives $C_P/C_V = 24/16 = 3/2$, recovering the required speed ratio. For $n = 1, 3, 4$, the heat-capacity ratios are respectively $17/11$, $31/21$, and $19/13$; none equals $3/2$. The resulting mole number is positive and places γ between the pure-gas values $7/5$ and $5/3$, as expected.

Step 3: Conclusion The amount of hydrogen is $\boxed{n = 2}$ moles. Hence, the correct option is (B).

Quick Tip to Solve: Use the speed ratio to find gamma first. For a mixture, add mole-weighted heat capacities before taking their ratio.

43. An object is taken to a height above the surface of earth at a distance $\frac{5}{4}$ R from the centre of the earth. Where radius of earth, R = 6400 km. The percentage decrease in the weight of the object will be : [4 marks]

(A) 36%

(B) 50%

(C) 64%

(D) 25%

Correct Answer: (A) 36%

Solution:

Concept:

- Outside a spherically symmetric Earth, gravitational force is $W(r) = GMm/r^2$, where r is measured from the centre, not from the surface.
- At the surface, $W_0 = GMm/R^2$. The percentage decrease is $100(W_0 - W)/W_0$; it compares the lost weight with the original surface weight.

Step 1: Identify the correct radial distance The stated distance from the centre is $r = 5R/4$. Thus the height above the surface is only $h = r - R = R/4$, although the force calculation can use the central distance directly. Numerically, $r = 8000\,\text{km}$ and $h = 1600\,\text{km}$. Treating $5R/4$ as the height would instead give the wrong distance $9R/4$.

Step 2: Compare the two gravitational forces In the ratio, the mass of Earth, mass of the object and gravitational constant cancel. Therefore the actual value of the radius is unnecessary:

$$\frac{W}{W_0} = \frac{GMm/r^2}{GMm/R^2} = \left(\frac{R}{5R/4}\right)^2 = \frac{16}{25} = 0.64.$$

The object therefore retains 64 percent of its original weight, rather than losing 64 percent.

Step 3: Calculate the fraction lost The fractional loss is the original fraction, unity, minus the remaining fraction:

$$\frac{W_0 - W}{W_0} = 1 - \frac{16}{25} = \frac{9}{25} = 0.36.$$

Hence the percentage decrease is $0.36 \times 100 = 36\%$. Option (C) gives the percentage retained; neither 50 percent nor 25 percent follows the inverse-square law.

Step 4: Conclusion The weight decreases by 36%. Hence, the correct option is (A).

Quick Tip to Solve: First decide whether a quoted distance is a height or a central distance; then square the radius ratio and subtract it from one.

44. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be : [4 marks]

(A) 4.9 J

(B) 9.8 J

(C) 14.7 J

(D) 19.6 J

Correct Answer: (B) 9.8 J

Solution:

Concept:

- When a bullet embeds in a stationary target, the collision is perfectly inelastic: both bodies have the same velocity immediately afterwards.
- Linear momentum in the bullet direction is conserved during the brief collision, but kinetic energy is not. The energy loss is $K_i - K_f$.

Step 1: Convert the mass and conserve momentum The bullet mass is $m = 200\,\mathrm{g} = 0.200\,\mathrm{kg}$, and the sand bag has mass $M = 9.8\,\mathrm{kg}$. The bag is initially at rest, so its initial momentum vanishes. For the short collision interval, the rope does not supply a significant impulse along the bullet motion. With $u = 10\,\mathrm{m\,s^{-1}}$,

$$mu = (M+m)v, \qquad v = \frac{0.2 \times 10}{9.8 + 0.2} = 0.2\,\mathrm{m\,s^{-1}}.$$

Step 2: Calculate kinetic energies on both sides Initially only the bullet has kinetic energy, whereas afterwards the entire combined mass moves with speed v:

$$K_i = \tfrac{1}{2}mu^2 = \tfrac{1}{2}(0.2)(10)^2 = 10\,\mathrm{J},$$

$$K_f = \tfrac{1}{2}(M+m)v^2 = \tfrac{1}{2}(10)(0.2)^2 = 0.2\,\mathrm{J}.$$

One must use the combined mass in the final energy, since the bullet remains embedded.

Step 3: Subtract the final energy from the initial energy The loss of translational kinetic energy is

$$\Delta K = K_i - K_f = 10 - 0.2 = 9.8\,\mathrm{J}.$$

This energy becomes deformation, internal energy and sound during embedding; it is not destroyed. The 4.9 J choice is not the energy difference, and 14.7 J and 19.6 J both exceed the entire initial energy of 10 J and are impossible here.

Step 4: Conclusion The loss of kinetic energy is $9.8\,\mathrm{J}$. Hence, the correct option is (B).

Quick Tip to Solve: For embedding in a stationary target, find the common speed using momentum before calculating the kinetic-energy loss.

45. A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal,

then 'tan θ' will be equal to : [4 marks]

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- For a projectile launched and landing at the same level, the horizontal range is $R = u^2 \sin 2\theta/g$, neglecting air resistance.
- The maximum height above the launch point is $H = u^2 \sin^2 \theta/(2g)$. Both expressions refer to the same launch speed u and gravitational acceleration g.

Step 1: Equate the range and height The given requirement is $R = H$, not equality of the height with half the range. Substituting the two standard expressions gives

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}.$$

The common factor u^2/g cancels. Thus the specified speed $15\,\mathrm{m\,s^{-1}}$ sets the scale of the motion but does not affect the required angle.

Step 2: Expand the double angle and simplify Use $\sin 2\theta = 2 \sin \theta \cos \theta$. The condition becomes

$$4 \sin \theta \cos \theta = \sin^2 \theta.$$

For the nonzero flight with positive range and height, $0 < \theta < 90°$, so division by $\sin \theta \cos \theta$ is valid. It follows that

$$4 \cos \theta = \sin \theta, \qquad \tan \theta = 4.$$

The zero-angle case would give both quantities zero and is not the projected flight described.

Step 3: Check the possible ratios Dividing the general expressions directly gives $H/R = \tan \theta/4$. The values $1/4$, $1/2$ and 2 for the tangent give height-to-range ratios $1/16$, $1/8$ and $1/2$, respectively, rather than unity. Only a tangent of 4 satisfies equality.

Step 4: Conclusion The required value is $\tan \theta = 4$. Hence, the correct option is (D).

Quick Tip to Solve: Remember the ratio $H/R = \tan \theta/4$ for a projectile returning to its launch level; it eliminates both speed and gravity.

46. The maximum error in the measurement of resistance, current and time for which current flows in an electrical circuit are $1\%, 2\%$ and 3% respectively. The maximum percentage error in the detection of the dissipated heat will be : [4 marks]

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (D) 8

Solution:

Concept:

- Joule heating in a resistor is $H = I^2Rt$, where I is current, R resistance and t the duration of current flow.
- For a product of measured quantities, maximum fractional errors add, with each fractional error multiplied by the absolute value of its exponent.

Step 1: Write the dependence on each measured variable The heat depends quadratically on current and linearly on resistance and time. Taking the first-order fractional variation of $H = I^2Rt$ gives

$$\frac{dH}{H} = 2\frac{dI}{I} + \frac{dR}{R} + \frac{dt}{t}.$$

The factor two on the current term is essential: a current error influences the heat twice as strongly as the same fractional resistance or time error.

Step 2: Add the maximum percentage errors For the maximum error, choose the signs so that the magnitudes reinforce rather than cancel. Therefore

$$\left(\frac{\Delta H}{H}\right)_{\max} = 2\frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}.$$

Substituting the given percentage errors,

$$\% \text{ error in } H = 2(2) + 1 + 3 = 8\%.$$

This is the usual first-order maximum-error rule, rather than a statistical root-sum-square uncertainty.

Step 3: Check the alternatives Adding the three quoted errors without their exponents would give 6 percent, option (C), and incorrectly treat heat as linear in current. The 4 percent value accounts only for the squared-current contribution, while 2 percent is merely the original current uncertainty. Both omit the other measured quantities and cannot represent the total maximum error.

Step 4: Conclusion The maximum percentage error in dissipated heat is 8%. Hence, the correct option is (D).

Quick Tip to Solve: Write the measured result as powers of the input quantities first; the exponents become multipliers in its maximum percentage error.

47. Hydrogen atom from excited state comes to the ground state by emitting a photon of wavelength λ. The value of principal quantum number 'n' of the excited state will be : (R : Rydberg constant)

[4 marks]

(A) $\sqrt{\frac{\lambda \mathrm{R}}{\lambda-1}}$

(B) $\sqrt{\frac{\lambda \mathrm{R}}{\lambda \mathrm{R}-1}}$

(C) $\sqrt{\frac{\lambda}{\lambda \mathrm{R}-1}}$

(D) $\sqrt{\frac{\lambda R^2}{\lambda R-1}}$

Correct Answer: (B) $\sqrt{\frac{\lambda \mathrm{R}}{\lambda \mathrm{R}-1}}$

Solution:

Concept:

- The Rydberg formula for a hydrogen emission line is $1/\lambda = R(1/n_f^2 - 1/n_i^2)$, where R has dimensions of inverse length.
- A transition to the ground state has final principal quantum number $n_f = 1$. The initial excited level is the unknown integer $n = n_i > 1$.

Step 1: Apply the ground-state transition formula For emission from n directly to the ground state,

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right).$$

The order of the terms ensures positive photon energy and positive wavelength because the initial state has the higher energy. Dividing by the Rydberg constant gives

$$\frac{1}{\lambda R} = 1 - \frac{1}{n^2}.$$

Step 2: Isolate the initial quantum number Rearrange to collect the reciprocal squared level:

$$\frac{1}{n^2} = 1 - \frac{1}{\lambda R} = \frac{\lambda R - 1}{\lambda R}.$$

Invert and choose the positive square root, since a principal quantum number is positive:

$$n^2 = \frac{\lambda R}{\lambda R - 1}, \qquad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}.$$

A physical finite excited state has $\lambda R > 1$, so the denominator is positive.

Step 3: Check dimensions and the other options The product λR is dimensionless, as a quantum number must be. Option (A) subtracts unity directly from a wavelength and is dimensionally inconsistent. Options (C) and (D) leave respectively dimensions of length and inverse length inside their square roots. Only option (B) has both the required algebra and dimensions. For example, $n = 2$ gives $\lambda R = 4/3$, which the derived expression returns correctly.

Step 4: Conclusion The excited-state quantum number is $n = \sqrt{\lambda R/(\lambda R - 1)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a ground-state hydrogen emission, start with $1/\lambda = R(1 - 1/n^2)$; check that every expression for n is dimensionless.

48. In AM modulation, a signal is modulated on a carrier wave such that maximum and minimum amplitudes are found to be 6 V and 2 V respectively. The modulation index is : [4 marks]

(A) 100%

(B) 80%

(C) 60%

(D) 50%

Correct Answer: (D) 50%

Solution:

Concept:

- For ordinary amplitude modulation without overmodulation, the envelope extremes are $A_{\max} = A_c(1+\mu)$ and $A_{\min} = A_c(1-\mu)$.
- Here A_c is the carrier amplitude and $\mu = A_m/A_c$ is the dimensionless modulation index. Adding and subtracting the extremes eliminates the unknown carrier amplitude.

Step 1: Relate the two observed amplitudes to the envelope The stated 6 V and 2 V are the maximum and minimum amplitudes of the modulated wave, not the separate signal and carrier amplitudes. Their sum is $2A_c$, whereas their difference is $2A_c\mu$. Thus

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}.$$

Since the minimum is positive, this is a normally modulated envelope with $\mu < 1$.

Step 2: Insert the observed values Using the two amplitudes in the same unit makes their ratio dimensionless:

$$\mu = \frac{6-2}{6+2} = \frac{4}{8} = 0.50.$$

Equivalently, the carrier amplitude is $A_c = (6+2)/2 = 4\,\text{V}$, and the signal amplitude is $A_m = (6-2)/2 = 2\,\text{V}$, so $A_m/A_c = 2/4 = 0.50$.

Step 3: Express the index as a percentage The percentage modulation is $100\mu = 50\%$. Full modulation, 100 percent, would require the envelope minimum to reach zero; that does not occur here. An index of 80 percent would give $A_{\min}/A_{\max} = 1/9$, and 60 percent would give $1/4$, whereas the observed ratio is $2/6 = 1/3$. These checks eliminate options (A), (B) and (C).

Step 4: Conclusion The modulation index is 0.50, or 50%. Hence, the correct option is (D).

Quick Tip to Solve: Use difference over sum of the envelope extremes for AM; the extremes themselves are not the signal and carrier amplitudes.

49. An electron with energy 0.1 keV moves at right angle to the earth's magnetic field of 1×10^{-4} Wbm^{-2}. The frequency of revolution of the electron will be :

(Take mass of electron = 9.0×10^{-31} kg) [4 marks]

(A) 1.6×10^5 Hz

(B) 5.6×10^5 Hz

(C) 2.8×10^6 Hz

(D) 1.8×10^6 Hz

Correct Answer: (C) 2.8×10^6 Hz

Solution:

Concept:

- For motion perpendicular to a uniform magnetic field, magnetic force provides the centripetal force: $|q|vB = mv^2/r$. It changes direction but does no work.
- The nonrelativistic cyclotron angular frequency is $\omega = |q|B/m$, and the revolution frequency in cycles per second is $f = \omega/(2\pi)$.

Step 1: Eliminate the speed from the orbital motion From the force balance,

$$r = \frac{mv}{|q|B}.$$

The time for one complete circle is its circumference divided by the speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}.$$

Hence $f = 1/T = |q|B/(2\pi m)$. The given kinetic energy determines the speed and radius, but it cancels from the frequency. At $0.1\,\mathrm{keV}$, the electron is safely in the nonrelativistic regime.

Step 2: Substitute the field and electron constants The field $1 \times 10^{-4}\,\mathrm{Wb\,m^{-2}}$ equals $10^{-4}\,\mathrm{T}$. Use the magnitude of the electron charge, $|q| = 1.6 \times 10^{-19}\,\mathrm{C}$, and the stated mass $9.0 \times 10^{-31}\,\mathrm{kg}$:

$$f = \frac{(1.6 \times 10^{-19})(10^{-4})}{2\pi(9.0 \times 10^{-31})} = 2.83 \times 10^6\,\mathrm{Hz}.$$

Step 3: Choose the matching frequency To the precision of the choices, this is $2.8 \times 10^6\,\mathrm{Hz}$. The negative charge reverses the sense of rotation but does not make the frequency negative. The other offered frequencies do not satisfy the period expression with the stated mass and field; in particular it is essential to retain the 2π that converts angular frequency into cycles per second.

Step 4: Conclusion The frequency of revolution is approximately $2.8 \times 10^6\,\mathrm{Hz}$. Hence, the correct option is (C).

Quick Tip to Solve: In nonrelativistic cyclotron motion the frequency depends on charge, mass and magnetic field; kinetic energy changes only the orbit radius.

50. Let η_1 is the efficiency of an engine at $T_1 = 447^\circ C$ and $T_2 = 147^\circ C$ while η_2 is the efficiency at $T_1 = 947^\circ C$ and $T_2 = 47^\circ C$ The ratio $\frac{\eta_1}{\eta_2}$ will be : [4 marks]

(A) 0.41

(B) 0.56

(C) 0.73

(D) 0.70

Correct Answer: (B) 0.56

Solution:

Concept:

- The efficiency of a reversible heat engine operating between a hot reservoir at T_h and a cold reservoir at T_c is $\eta = 1 - T_c/T_h$. The reservoir temperatures in this relation must be absolute temperatures.
- Convert Celsius temperatures to kelvin using $T(\mathrm{K}) = t(^\circ\mathrm{C}) + 273$. Efficiency is the fraction of heat input converted into useful work and has no unit.

Step 1: Convert both pairs of reservoir temperatures For the first operating condition, the hot and cold temperatures are

$$T_{h1} = 447 + 273 = 720\,\mathrm{K}, \qquad T_{c1} = 147 + 273 = 420\,\mathrm{K}.$$

For the second condition,

$$T_{h2} = 947 + 273 = 1220\,\mathrm{K}, \qquad T_{c2} = 47 + 273 = 320\,\mathrm{K}.$$

Celsius readings cannot be substituted directly because their zero does not represent zero thermal energy.

Step 2: Calculate the two efficiencies and divide Using the heat-engine temperature relation for each pair gives

$$\eta_1 = 1 - \frac{420}{720} = \frac{300}{720} = \frac{5}{12}.$$

$$\eta_2 = 1 - \frac{320}{1220} = \frac{900}{1220} = \frac{45}{61}.$$

Therefore,

$$\frac{\eta_1}{\eta_2} = \frac{5}{12}\frac{61}{45} = \frac{61}{108} = 0.5648\ldots.$$

The second engine has a larger hot temperature and a smaller cold temperature, so its efficiency is larger; the required ratio must be below unity. Option (B) matches the computed ratio to two decimal places. The values 0.41, 0.73 and 0.70 in (A), (C) and (D) do not equal this ratio; in particular, 0.41 and 0.73 are close to the individual efficiencies rather than their quotient.

Step 3: Conclusion The efficiency ratio is $\eta_1/\eta_2 \simeq 0.56$, a dimensionless number. Hence, the correct option is (B).

Quick Tip to Solve: Convert each reservoir temperature to kelvin before forming a ratio; the Celsius temperature difference alone is not enough to determine efficiency.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is _____ ms^{-2} at a point where its velocity is 20 ms^{-1}. [4 marks]

Solution:

Concept:

- Acceleration is the time derivative of velocity, $a = dv/dt$. When velocity is supplied as a function of position, the chain rule gives $a = (dv/dx)(dx/dt) = v\,dv/dx$.
- The statement that velocity increases by $5\,\mathrm{m\,s^{-1}}$ per metre specifies $dv/dx = 5\,\mathrm{s^{-1}}$, rather than a constant time rate of change of velocity.

Step 1: Identify the given spatial rate Let x measure distance along the straight line and let v be the velocity in the direction of motion. The given rate is

$$\frac{dv}{dx} = \frac{5\,\mathrm{m\,s^{-1}}}{1\,\mathrm{m}} = 5\,\mathrm{s^{-1}}.$$

This is a slope on a velocity-position graph. It is not itself the acceleration, whose unit is $\mathrm{m\,s^{-2}}$. As the particle moves faster, it passes through more metres each second and therefore experiences a larger velocity change per second.

Step 2: Apply the chain rule at the specified velocity Since $dx/dt = v$, change the independent variable from time to position:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}.$$

At the point where $v = 20\,\mathrm{m\,s^{-1}}$,

$$a = (20)(5) = 100\,\mathrm{m\,s^{-2}}.$$

The units multiply as $(\mathrm{m\,s^{-1}})(\mathrm{s^{-1}}) = \mathrm{m\,s^{-2}}$, confirming that the expression is an acceleration. Its sign is positive along the motion because the velocity is increasing. No initial position or elapsed time is required: the acceleration at this point follows directly from the local velocity and spatial gradient. A constant dv/dx does not imply constant acceleration because v itself changes.

Step 3: Conclusion The acceleration at the specified point is $\boxed{100\,\mathrm{m\,s^{-2}}}$. The numerical entry is 100.

Quick Tip to Solve: When a rate is given per metre instead of per second, use $a = v\,dv/dx$; check the units before substituting the numerical values.

52. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is _____. [4 marks]

Solution:

Concept:

- The position of the centre of mass is the mass-weighted average $\vec{r}_{\text{cm}} = \sum m_i \vec{r}_i / \sum m_i$. Equal masses therefore contribute equally to each Cartesian coordinate.
- The magnitude of a planar position vector is $|\vec{r}| = \sqrt{x_c^2 + y_c^2}$. The requested parameter x is the coefficient inside the stated square root, not either Cartesian coordinate.

Step 1: Choose axes along the perpendicular sides Place the origin at the right-angle vertex and take the two mutually perpendicular sides along the positive coordinate axes. The centres of the three identical spheres then have position coordinates

$$(0,0), \qquad (3,0), \qquad (0,3) \quad \text{in metres.}$$

This choice uses the supplied side lengths directly and requires no calculation of the hypotenuse. Each sphere has mass M, so the total mass is $3M$. The common mass will cancel in the centre-of-mass calculation.

Step 2: Average the coordinates and find the distance Calculate the horizontal and vertical coordinates separately:

$$x_c = \frac{M(0) + M(3) + M(0)}{3M} = 1\,\text{m},$$

$$y_c = \frac{M(0) + M(0) + M(3)}{3M} = 1\,\text{m}.$$

Thus

$$\vec{r}_{\text{cm}} = (\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}})\,\text{m}.$$

Its magnitude relative to the stated origin is

$$|\vec{r}_{\text{cm}}| = \sqrt{1^2 + 1^2}\,\text{m} = \sqrt{2}\,\text{m}.$$

The symmetry of the equal perpendicular sides also requires the two coordinates to be equal, agreeing with the result. Comparing the distance $\sqrt{2}\,\text{m}$ with the form $\sqrt{x}\,\text{m}$ gives $x = 2$. Do not report 1, which is one coordinate, or $\sqrt{2}$, which is the distance coefficient before squaring.

Step 3: Conclusion The centre of mass is $\sqrt{2}\,\text{m}$ from the origin, so the requested value is $\boxed{x = 2}$. The parameter x is dimensionless.

Quick Tip to Solve: For identical masses, average the coordinates first and then take the magnitude; averaging distances from the origin does not locate the centre of mass.

53. A block of ice of mass 120 g at temperature 0°C is put in 300 g of water at 25°C. The x g of ice melts as the temperature of the water reaches 0°C. The value of x is _____.

[Use specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.5 $\times$ 10^5 Jkg^{-1}] [4 marks]

Solution:

Concept:

- Cooling a mass m_w of water through a temperature fall ΔT releases heat $Q = m_w c_w \Delta T$, where c_w is its specific heat capacity.
- Ice already at $0^\circ\mathrm{C}$ absorbs latent heat $Q = m_i L_f$ while melting at the same temperature. In thermal isolation the heat lost by water equals the heat gained by the ice.

Step 1: Calculate the heat released by the warm water The water mass must be expressed in kilograms because the supplied thermal constants use SI units:

$$m_w = 300\,\mathrm{g} = 0.300\,\mathrm{kg}.$$

It cools from $25^\circ\mathrm{C}$ to $0^\circ\mathrm{C}$, so its temperature drop is $25\,\mathrm{K}$. The available heat is therefore

$$Q_w = (0.300)(4200)(25) = 31500\,\mathrm{J}.$$

The numerical value of a temperature difference is the same in kelvin and degrees Celsius. There is no separate heat term for warming the initial ice because it starts at its melting temperature.

Step 2: Use the available heat to melt ice Let m_i denote the mass that melts in kilograms. With $L_f = 3.5 \times 10^5\,\mathrm{J\,kg^{-1}}$, energy conservation gives

$$m_i L_f = 31500\,\mathrm{J}.$$

Hence

$$m_i = \frac{31500}{3.5 \times 10^5} = 0.090\,\mathrm{kg} = 90\,\mathrm{g}.$$

Check that this is less than the initial $120\,\mathrm{g}$ of ice. Indeed, complete melting would require $(0.120)(3.5 \times 10^5) = 42000\,\mathrm{J}$, more than the warm water supplies. Thus $30\,\mathrm{g}$ of ice remains and an equilibrium mixture of ice and water at $0^\circ\mathrm{C}$ is consistent. No heat remains to raise the temperature of the melt above the melting point.

Step 3: Conclusion The mass of ice melted is $\boxed{90\,\mathrm{g}}$, so the numerical value of x is $\boxed{90}$.

Quick Tip to Solve: Before assuming that all ice melts, compare the warm water's available heat with the latent heat needed for the entire ice mass.

54. $\frac{x}{x+4}$ **is the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its**

(i) third permitted energy level to the second level and

(ii) the highest permitted energy level to the second permitted level.

The value of x will be _____. [4 marks]

Solution:

Concept:

- The permitted hydrogen energy levels are $E_n = -E_0/n^2$, with $E_0 = 13.6\,\text{eV}$. A downward transition emits a photon of positive energy equal to the difference between the initial and final atomic energies.
- The highest limiting permitted level corresponds to $n \to \infty$, where the bound-state energy approaches zero. In a ratio of hydrogen transition energies the common factor E_0 cancels.

Step 1: Calculate the photon energy for each transition For the first transition the initial level is $n_i = 3$ and the final level is $n_f = 2$. The emitted photon energy is

$$\epsilon_1 = E_0\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = E_0\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5E_0}{36}.$$

For the second transition, take the highest level in the limiting sense, $n_i \to \infty$, with the same final level $n_f = 2$:

$$\epsilon_2 = E_0\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{E_0}{4}.$$

Both photon energies are positive, even though the bound atomic energy levels themselves are negative.

Step 2: Form the ratio and solve for the parameter The requested ratio is in the order of the two transitions listed:

$$\frac{\epsilon_1}{\epsilon_2} = \frac{5E_0/36}{E_0/4} = \frac{5}{9}.$$

Use the form specified in the question:

$$\frac{x}{x+4} = \frac{5}{9}.$$

Cross-multiplication gives

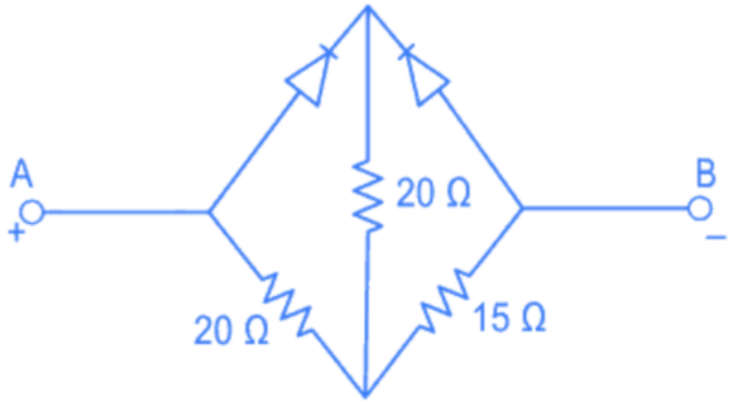
$$9x = 5x + 20, \qquad 4x = 20, \qquad x = 5.$$

The smaller transition energy belongs to the $3 \to 2$ transition, as expected because its initial level lies below the ionisation limit. Consequently its ratio to the limiting transition is less than one. Substitution checks the parameter directly: $5/(5+4) = 5/9$. There is no need to insert the numerical value of E_0.

Step 3: Conclusion The ratio of photon energies is $5/9$, which gives $\boxed{x = 5}$. The requested parameter is dimensionless.

Quick Tip to Solve: When both hydrogen transitions end at the same level, write their reciprocal-square energy differences and cancel the common $13.6\,\text{eV}$ factor.

55. Two ideal diodes are connected in the network as shown in figure. The equivalent resistance between A and B is _____ Ω. [4 marks]



Solution:

Concept:

- An ideal diode is a short circuit under forward bias and an open circuit under reverse bias. Its cathode is indicated by the bar in the circuit symbol.
- Resistors with both ends connected to the same pair of nodes are in parallel; consecutive resistors carrying the same current are in series. Establish diode states before combining resistances.

Step 1: Determine the conducting and blocking diodes Use the polarity shown: terminal A is positive relative to B. Call the upper vertex of the diamond U and the lower vertex L. The left upper diode has its anode towards A and its cathode towards U, so it conducts and makes U equipotential with A. The right upper diode has its cathode towards U and its anode towards B. Since U is at the higher potential, that diode is reverse biased and its branch is open. Thus the two upper branches cannot both be replaced by wires; doing so would incorrectly short A to B.

Step 2: Reduce the remaining resistor network After replacing the left diode by a wire, the vertical $20\,\Omega$ resistor lies between U=A and L. The lower-left $20\,\Omega$ resistor also lies between A and L. They therefore form a parallel pair:

$$R_{AL} = \frac{20 \times 20}{20 + 20} = 10\,\Omega.$$

The lower-right $15\,\Omega$ resistor connects L to B. All current reaching B must pass through it because the right diode blocks the other path. It is in series with the parallel pair:

$$R_{AB} = R_{AL} + 15 = 10 + 15 = 25\,\Omega.$$

For a consistency check, apply a positive voltage V between A and B. The current is $V/25$, and L is at $15V/25$ above B. U remains at A's potential, so the right diode still has a higher cathode potential than anode potential, confirming the assumed reverse bias.

Step 3: Conclusion For the shown polarity, the equivalent resistance between A and B is $\boxed{25\,\Omega}$. The numerical entry is 25.

Quick Tip to Solve: Trace the diode bars and identify which nodes become identical after a conducting diode is shorted; the central resistor may then join a parallel pair.

56. Two parallel plate capacitors of capacity C and 3C are connected in parallel combination and charged to a potential difference 18 V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be _____ V. [4 marks]

Solution:

Concept:

- Parallel capacitors share one potential difference and have total capacitance equal to the sum of their individual capacitances. Their total charge is $Q = C_{\text{eq}}V$.
- Filling a capacitor completely with a dielectric of relative permittivity K multiplies its capacitance by K. With the battery removed, total charge on the connected combination is conserved, although charge can move between its capacitors.

Step 1: Find the charge before disconnecting the battery Initially the capacitors C and $3C$ are connected in parallel across $18\,\text{V}$. Their equivalent capacitance is

$$C_{\text{i}} = C + 3C = 4C.$$

The charge supplied to one set of connected plates is

$$Q_{\text{i}} = C_{\text{i}}V_{\text{i}} = (4C)(18) = 72C.$$

Here C denotes the given capacitance, so $72C$ represents the product of that capacitance and a voltage. The opposite connected plates carry equal and opposite charge. Removal of the battery prevents any further charge supply from an external source.

Step 2: Calculate the new capacitance and common voltage The dielectric is inserted only in the capacitor whose original capacitance is C. Its new capacitance becomes $9C$, while the other capacitor remains $3C$. Since their wires remain connected, the new equivalent capacitance is

$$C_{\text{f}} = 9C + 3C = 12C.$$

Conservation of the total charge gives

$$C_{\text{f}}V_{\text{f}} = Q_{\text{i}}, \qquad V_{\text{f}} = \frac{72C}{12C} = 6\,\text{V}.$$

Individual capacitor charges need not remain at their original values; they redistribute through the connecting wires until the voltage is common. Indeed, their final charges are $(9C)(6) = 54C$ and $(3C)(6) = 18C$, adding to $72C$. The total capacitance has tripled, so the voltage must fall to one third of its original value.

Step 3: Conclusion The final common potential difference across the parallel combination is $\boxed{6\,\text{V}}$. The numerical entry is 6.

Quick Tip to Solve: After the battery is removed, conserve the total charge of the connected combination; do not conserve each capacitor's charge separately while they remain wired together.

57. A convex lens of focal length 20 cm is placed in front of a convex mirror with principal axis coinciding each other. The distance between the lens and mirror is 10 cm. A point object is placed on principal axis at a distance of 60 cm from the convex lens. The image formed by combination coincides the object itself. The focal length of the convex mirror is _____ cm. [4 marks]

Solution:

Concept:

- The thin-lens formula is $1/f = 1/v - 1/u$. With incident light travelling to the right, the real object has negative object distance.
- For the final image to return to the original object, reversibility requires the mirror to return the incident rays along the same paths. A spherical mirror does this for rays directed towards its centre of curvature.

Step 1: Locate the image formed by the lens alone Take the convex lens at $x = 0$, with the object at $x = -60\,\text{cm}$. Its focal length is positive, $f_L = 20\,\text{cm}$. Thus

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-60} = \frac{1}{v} + \frac{1}{60}, \qquad \frac{1}{v} = \frac{1}{30}.$$

The rays leaving the lens converge towards $x = 30\,\text{cm}$, so the lens-alone image is 30 cm to the right of the lens.

Step 2: Apply reversibility at the convex mirror The mirror is at $x = 10\,\text{cm}$, before the rays reach this convergence point. They are therefore directed towards a point $30 - 10 = 20\,\text{cm}$ behind the mirror. If this point is the mirror centre of curvature, each incident ray travels along a normal to the spherical surface. Reflection reverses each ray, and the rays retrace their paths through the lens, forming the image at the original object. Conversely, the lens must receive exactly these returning rays to form the specified image.

Step 3: Convert the radius into focal length The required centre of curvature is therefore 20 cm behind the convex mirror. Its radius of curvature has magnitude

$$R = 30 - 10 = 20\,\text{cm}.$$

For a spherical mirror, $f_M = R/2$, giving

$$f_M = \frac{20}{2} = 10\,\text{cm}.$$

This is the positive focal-length magnitude of a convex mirror; the lens focal length must not be used a second time as the mirror radius.

Step 4: Conclusion The focal length of the convex mirror is $\boxed{10\,\text{cm}}$. The numerical entry is 10.

Quick Tip to Solve: First find the image due to the lens alone. For exact return to the object, place that image at the mirror centre of curvature, then use $f = R/2$.

58. Magnetic flux (in weber) in a closed circuit of resistance 20 Ω varies with time t(s) at $\phi = 8t^2 - 9t + 5$. The magnitude of the induced current at t = 0.25 s will be _____ mA. [4 marks]

Solution:

Concept:

- Faraday's law gives the instantaneous induced emf as $\mathcal{E} = -d\phi/dt$, where ϕ is the magnetic flux linked with the circuit.
- For a closed resistive circuit, the current magnitude is $|I| = |\mathcal{E}|/R$. The derivative must be evaluated at the specified time before converting amperes to milliamperes.

Step 1: Differentiate the given flux The flux is $\phi(t) = 8t^2 - 9t + 5$ in webers, with time in seconds. Differentiate every term:

$$\frac{d\phi}{dt} = 16t - 9.$$

The constant flux contribution 5 Wb has zero derivative and produces no emf. Using ϕ/t would instead give an average-like quantity and would not give the instantaneous current requested.

Step 2: Evaluate the induced emf At $t = 0.25\,\text{s}$,

$$\left.\frac{d\phi}{dt}\right|_{0.25} = 16(0.25) - 9 = 4 - 9 = -5\,\text{Wb}\,\text{s}^{-1}.$$

The flux is decreasing at this instant. Faraday's negative sign gives $\mathcal{E} = +5\,\text{V}$ for the chosen orientation. Since the question asks only for magnitude, the orientation of the current around the loop is unnecessary; the magnitude is 5 V.

Step 3: Use the circuit resistance and convert units The total resistance is $R = 20\,\Omega$. Hence

$$|I| = \frac{5}{20} = 0.25\,\text{A}.$$

Since one ampere equals 10^3 milliamperes,

$$0.25\,\text{A} = 0.25 \times 1000 = 250\,\text{mA}.$$

The unit check is $\text{V}/\Omega = \text{A}$. The negative flux derivative does not make the requested magnitude negative.

Step 4: Conclusion The magnitude of the induced current is $\boxed{250\,\text{mA}}$. The numerical entry is 250.

Quick Tip to Solve: Differentiate the flux first, take the magnitude after substituting the time, and multiply the current in amperes by 1000 to obtain milliamperes.

59. In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 36 cm length of wire. This cell is now replaced by another cell of emf 1.80 V. The difference in balancing length of potentiometer wire in above conditions will be _____ cm. [4 marks]

Solution:

Concept:

- At the balance point of an ideal potentiometer, no current is drawn from the test cell, so its emf is $E = k\ell$, where k is the potential gradient along the wire.
- With the same potentiometer arrangement and unchanged primary current, k is constant. Therefore balancing lengths are directly proportional to the corresponding emfs.

Step 1: Find the potential gradient For the first cell, $E_1 = 1.20\,\text{V}$ balances at $\ell_1 = 36\,\text{cm}$. Thus

$$k = \frac{E_1}{\ell_1} = \frac{1.20}{36} = \frac{1}{30}\,\text{V cm}^{-1}.$$

The balance condition uses the emf rather than the loaded terminal voltage, because the test-cell branch draws no current at null deflection. Replacing only the test cell leaves the wire potential gradient unchanged.

Step 2: Calculate the new balancing length The second cell has $E_2 = 1.80\,\text{V}$. Dividing the two balance conditions eliminates the common potential gradient:

$$\frac{E_2}{E_1} = \frac{\ell_2}{\ell_1}.$$

Therefore

$$\ell_2 = 36\left(\frac{1.80}{1.20}\right) = 36\left(\frac{3}{2}\right) = 54\,\text{cm}.$$

The larger emf correctly requires a longer length of the same uniform wire.

Step 3: Find the change requested The question asks for the difference between the balancing lengths, not the second balancing length itself. Hence

$$\Delta\ell = \ell_2 - \ell_1 = 54 - 36 = 18\,\text{cm}.$$

As a separate check, the emf increases by $1.80 - 1.20 = 0.60\,\text{V}$, and $\Delta\ell = \Delta E/k = 0.60/(1/30) = 18\,\text{cm}$. Both calculations give the same positive increase.

Step 4: Conclusion The difference in balancing lengths is $\boxed{18\,\text{cm}}$. The numerical entry is 18.

Quick Tip to Solve: Use $\ell_2/\ell_1 = E_2/E_1$, then subtract the original length. A question asking for the difference does not want ℓ_2 itself.

60. Two waves executing simple harmonic motions travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is _____ (degree). [4 marks]

Solution:

Concept:

- Two coherent oscillations of amplitudes a_1, a_2 and phase difference δ have resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta}$.
- For equal amplitudes a, this becomes $A^2 = 2a^2(1 + \cos\delta)$. The usual phase difference is reported as its smallest nonnegative magnitude, between 0° and 180°.

Step 1: Write the resultant-amplitude equation The waves have the same frequency, travel in the same direction, and each has amplitude a. Their relative phase therefore determines the resultant amplitude at a given position. The given amplitude is $A = \sqrt{3}\,a$, so squaring the superposition formula gives

$$(\sqrt{3}\,a)^2 = a^2 + a^2 + 2a^2\cos\delta.$$

Amplitudes are nonzero, allowing cancellation of a^2 without changing the equation.

Step 2: Solve for the cosine of the phase difference Simplifying the preceding equation,

$$3a^2 = 2a^2 + 2a^2\cos\delta,$$

$$3 = 2 + 2\cos\delta, \qquad \cos\delta = \frac{1}{2}.$$

This positive cosine indicates partial constructive interference. The resultant is greater than either individual amplitude but smaller than the maximum $2a$, consistent with the specified $\sqrt{3}\,a$.

Step 3: Express the principal phase difference in degrees The smallest phase separation with cosine $1/2$ is

$$\delta = 60^\circ.$$

The equivalent equal-amplitude identity $A = 2a|\cos(\delta/2)|$ checks this result: $2a\cos 30^\circ = 2a(\sqrt{3}/2) = \sqrt{3}\,a$. A phase difference of 120° would instead give resultant amplitude a, while 0° would give $2a$; neither meets the condition. Phases differing by full cycles describe equivalent oscillations.

Step 4: Conclusion The required phase difference is $\boxed{60^\circ}$. The numerical entry in degrees is 60.

Quick Tip to Solve: For equal amplitudes use $A = 2a|\cos(\delta/2)|$; the angle inside the cosine is half the phase difference.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Match List I with List II:

	List I (molecule)		List II (hybridization; shape)
(A)	XeO_3	**(I)**	sp^3d; **linear**
(B)	XeF_2	**(II)**	sp^3; **pyramidal**
(C)	$XeOF_4$	**(III)**	sp^3d^3; **distorted octahedral**
(D)	XeF_6	**(IV)**	sp^3d^2; **square pyramidal**

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-I, C-IV, D-III

(B) A-II, B-IV, C-III, D-I

(C) A-IV, B-II, C-III, D-I

(D) A-IV, B-II, C-I, D-III

Correct Answer: (A) A-II, B-I, C-IV, D-III

Solution:

Concept:

- VSEPR counts each bond direction as one electron domain, including a multiple bond. Lone pairs occupy domains but are omitted when naming the molecular shape.
- The conventional hybridization assignments for 4, 5, 6 and 7 electron domains are sp^3, sp^3d, sp^3d^2 and sp^3d^3, respectively.

Step 1: Classify xenon trioxide and xenon difluoride In XeO_3, xenon has three bonding directions and one lone pair. Its four domains give the sp^3 assignment and a trigonal pyramidal molecular shape. Therefore A matches II. In XeF_2, two bond pairs and three lone pairs give five domains, assigned sp^3d. The three lone pairs occupy the equatorial positions of a trigonal bipyramid, leaving the two fluorine atoms axial. The molecule is linear, so B matches I.

Step 2: Classify xenon oxytetrafluoride and xenon hexafluoride In $XeOF_4$, the Xe–O bond counts as one direction, alongside four Xe–F bonds. With one lone pair there are six electron domains, assigned sp^3d^2. The electron-domain arrangement is octahedral and the five atomic positions form a square pyramid. Thus C matches IV. In XeF_6, six bond pairs and one lone pair give seven domains, assigned sp^3d^3. The lone pair distorts the six-ligand arrangement; its molecular shape is described as distorted octahedral. Thus D matches III.

Step 3: Compare the complete matching with the options The required sequence is A–II, B–I, C–IV, D–III. Only option (A) has all four assignments. Option (B) wrongly calls XeF_2 square pyramidal and XeF_6 linear. Options (C) and (D) incorrectly assign square pyramidal geometry to XeO_3 and pyramidal geometry to XeF_2. Counting only bonded atoms, rather than including lone pairs in the domain count, would produce these incorrect identifications.

Step 4: Conclusion The matching is $\boxed{\text{A–II, B–I, C–IV, D–III}}$. Hence, the correct option is (A).

Quick Tip to Solve: Count bond directions plus lone pairs for hybridization, then omit lone pairs only when naming the molecular shape.

62. Two solutions A and B are prepared by dissolving 1 g of non-volatile solutes X and Y, respectively in 1 kg of water. The ratio of depression in freezing points for A and B is found to be 1 : 4. The ratio of molar masses of X and Y is [4 marks]

(A) 1 : 4

(B) 1 : 0.25

(C) 1 : 0.20

(D) 1 : 5

Correct Answer: (B) 1 : 0.25

Solution:

Concept:

- For a dilute solution of a non-volatile molecular solute, freezing-point depression is $\Delta T_f = K_f m$, where m is the molality and K_f belongs to the solvent.
- At equal solute masses and equal solvent masses, molality is inversely proportional to solute molar mass. The common water solvent makes the cryoscopic constant cancel in a ratio.

Step 1: Express each molality in terms of molar mass Let the molar masses of X and Y be M_X and M_Y, in $\mathrm{g\,mol^{-1}}$. Each solution contains 1 g of solute in 1 kg of water, so

$$m_A = \frac{1/M_X}{1} = \frac{1}{M_X}, \qquad m_B = \frac{1/M_Y}{1} = \frac{1}{M_Y}.$$

The equal mass of solute does not imply equal numbers of solute particles: the lower-molar-mass substance supplies more moles per gram.

Step 2: Use the given depression ratio For the same solvent,

$$\frac{\Delta T_{f,A}}{\Delta T_{f,B}} = \frac{K_f m_A}{K_f m_B} = \frac{1/M_X}{1/M_Y} = \frac{M_Y}{M_X}.$$

The measured ratio is $1 : 4$, hence $M_Y/M_X = 1/4$, or $M_X/M_Y = 4$. Thus X has the larger molar mass, consistent with its smaller freezing-point depression at equal solute mass.

Step 3: Convert the ratio into the printed option form The required ratio is

$$M_X : M_Y = 4 : 1 = 1 : 0.25.$$

Option (A), $1 : 4$, confuses the depression ratio with the molar-mass ratio and reverses the result. Option (C), $1 : 0.20 = 5 : 1$, predicts a depression ratio of $1 : 5$. Option (D), $1 : 5$, predicts $5 : 1$ for the depression ratio. Therefore neither meets the observed $1 : 4$ depression ratio. The intended dilute-solution treatment uses the solutes as molecular particles without an additional dissociation factor.

Step 4: Conclusion The molar-mass ratio is $\boxed{M_X : M_Y = 1 : 0.25}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal grams of solute in equal kilograms of one solvent, invert the freezing-point-depression ratio to obtain the molar-mass ratio.

63. K_{a_1}, K_{a_2} **and** K_{a_3} **are the respective ionization constants for the following reactions (a), (b) and (c).**

(a) $H_2C_2O_4 \rightleftarrows H^+ + HC_2O_4^-$

(b) $HC_2O_4^- \rightleftarrows H^+ + C_2O_4^{2-}$

(c) $H_2C_2O_4 \rightleftarrows 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2} **and** K_{a_3} **is given as :** [4 marks]

(A) $K_{a_3} = K_{a_1} + K_{a_2}$

(B) $K_{a_3} = K_{a_1} - K_{a_2}$

(C) $K_{a_3} = K_{a_1} \,/\, K_{a_2}$

(D) $K_{a_3} = K_{a_1} \times K_{a_2}$

Correct Answer: (D) $K_{a_3} = K_{a_1} \times K_{a_2}$

Solution:

Concept:

- The equilibrium constant of a reaction is the product of equilibrium constants of reactions that add to give it. This follows by multiplying the mass-action expressions and cancelling intermediates.
- Oxalic acid loses its two protons in successive steps. The hydrogen oxalate ion $HC_2O_4^-$ is an intermediate and cancels when the two ionization reactions are added.

Step 1: Write the two stepwise ionization constants Using the concentration expressions for the reactions as printed,

$$K_{a_1} = \frac{[H^+][HC_2O_4^-]}{[H_2C_2O_4]},$$

$$K_{a_2} = \frac{[H^+][C_2O_4^{2-}]}{[HC_2O_4^-]}.$$

Each expression corresponds to the forward dissociation direction. The two constants are not the same because removing a proton from a negatively charged ion differs from removing one from a neutral molecule.

Step 2: Multiply and cancel the intermediate Multiplying the expressions gives

$$K_{a_1}K_{a_2} = \frac{[H^+][HC_2O_4^-]}{[H_2C_2O_4]} \frac{[H^+][C_2O_4^{2-}]}{[HC_2O_4^-]} = \frac{[H^+]^2[C_2O_4^{2-}]}{[H_2C_2O_4]}.$$

The intermediate hydrogen oxalate concentration cancels. The hydrogen-ion concentration is squared because the overall reaction releases two hydrogen ions per oxalic acid molecule.

Step 3: Identify the overall constant and eliminate alternatives The resulting expression is exactly K_{a_3} for $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$. Thus $K_{a_3} = K_{a_1}K_{a_2}$. Adding reaction equations does not mean adding their equilibrium constants, so (A) is incorrect; subtracting constants has no mass-action basis, so (B) is incorrect. Dividing the constants, as in (C), would correspond to adding one reaction to the reverse of the other, rather than adding the two forward ionizations.

Step 4: Conclusion The relation is $\boxed{K_{a_3} = K_{a_1}K_{a_2}}$. Hence, the correct option is (D).

Quick Tip to Solve: When equilibrium reactions are added, multiply their constants. Reverse a reaction only when you also replace its constant by its reciprocal.

64. The molar conductivity of a conductivity cell filled with 10 moles of 20 mL NaCl solution is Λ_{m1} and that of 20 moles another identical cell heaving 80 mL NaCl solution is Λ_{m2}. The conductivities exhibited by these two cells are same. The relationship between Λ_{m2} and Λ_{m1} is [4 marks]

(A) $\Lambda_{m2} = 2\Lambda_{m1}$

(B) $\Lambda_{m2} = \Lambda_{m1} / 2$

(C) $\Lambda_{m2} = \Lambda_{m1}$

(D) $\Lambda_{m2} = 4\Lambda_{m1}$

Correct Answer: (A) $\Lambda_{m2} = 2\Lambda_{m1}$

Solution:

Concept:

- Molar conductivity is conductivity divided by molar concentration: $\Lambda_m = \kappa/c$ in consistent SI units. Here κ describes the solution conductivity and c is moles per unit volume.
- For two solutions with equal conductivity, the molar conductivities vary inversely with their concentrations; the cell geometry does not introduce an additional factor into this comparison.

Step 1: Calculate the concentration ratio Use the specified mole amounts and solution volumes, keeping the same volume unit in both ratios:

$$c_1 = \frac{10}{20} = 0.5 \text{ mol mL}^{-1}, \qquad c_2 = \frac{20}{80} = 0.25 \text{ mol mL}^{-1}.$$

Thus the second solution has twice as many moles but four times the volume, so it has only half the first concentration. The numerical concentrations are unusually large, but their ratio follows directly from the quantities given; no correction of the stated data is needed.

Step 2: Compare the molar conductivities The stated equality is $\kappa_1 = \kappa_2$, rather than an equality of molar conductivities. Therefore

$$\frac{\Lambda_{m2}}{\Lambda_{m1}} = \frac{\kappa_2/c_2}{\kappa_1/c_1} = \frac{c_1}{c_2} = \frac{0.5}{0.25} = 2.$$

The volume and mole ratios must both enter the calculation. Option (B) reverses the inverse dependence on concentration, option (C) confuses conductivity with molar conductivity, and option (D) uses the fourfold volume increase without accounting for the doubled number of moles.

Step 3: Conclusion $\Lambda_{m2} = 2\Lambda_{m1}$. Hence, the correct option is (A).

Quick Tip to Solve: When conductivities are equal, form the ratio $(n_1/V_1)/(n_2/V_2)$; this immediately gives $\Lambda_{m2}/\Lambda_{m1}$.

65. The first ionization enthalpies of Be, B, N and O follow the order [4 marks]

(A) O $<$ N $<$ B $<$ Be

(B) Be $<$ B $<$ N $<$ O

(C) B $<$ Be $<$ N $<$ O

(D) B $<$ Be $<$ O $<$ N

Correct Answer: (D) B $<$ Be $<$ O $<$ N

Solution:

Concept:

- First ionization enthalpy is the energy needed to remove the most loosely bound electron from an isolated gaseous atom. Across a period it generally increases as effective nuclear attraction increases.
- Subshell energies and electron pairing explain two important exceptions: an electron in a $2p$ orbital is easier to remove than a $2s$ electron, and removing a paired $2p$ electron reduces electron repulsion.

Step 1: Compare beryllium with boron The relevant ground-state electronic configurations are

$$\mathrm{Be} : 1s^2 2s^2, \qquad \mathrm{B} : 1s^2 2s^2 2p^1.$$

Boron loses its lone $2p$ electron, which occupies a higher-energy, less penetrating orbital than the filled $2s$ subshell of beryllium. Consequently the extra proton in boron does not make its first electron removal harder than that in beryllium:

$$I_1(\mathrm{B}) < I_1(\mathrm{Be}).$$

This rules out option (B), which applies the general increase without allowing for the subshell exception.

Step 2: Compare nitrogen with oxygen and combine Nitrogen has $2p^3$, with one electron in each p orbital, whereas oxygen has $2p^4$, with one electron pair. Removing the paired electron from oxygen relieves repulsion and gives a half-filled $2p^3$ configuration. Thus

$$I_1(\mathrm{O}) < I_1(\mathrm{N}).$$

Both nitrogen and oxygen lie well above boron and beryllium because of the substantially greater effective nuclear charge across the same period. Combining the comparisons gives

$$I_1(\mathrm{B}) < I_1(\mathrm{Be}) < I_1(\mathrm{O}) < I_1(\mathrm{N}).$$

Option (A) incorrectly puts oxygen and nitrogen below boron and beryllium; option (C) misses the oxygen-nitrogen pairing exception.

Step 3: Conclusion The order is B $<$ Be $<$ O $<$ N. Hence, the correct option is (D).

Quick Tip to Solve: For second-period ionization enthalpies, check the filled $2s^2$ and half-filled $2p^3$ configurations before applying the general trend.

66. The correct order of energy of absorption for the following metal complexes is :

A : $[Ni(en)_3]^{2+}$, B : $[Ni(NH_3)_6]^{2+}$, C : $[Ni(H_2O)_6]^{2+}$ [4 marks]

(A) C < B < A

(B) B < C < A

(C) C < A < B

(D) A < C < B

Correct Answer: (A) C < B < A

Solution:

Concept:

- For an octahedral transition-metal complex, absorption associated with a d-orbital excitation requires energy related to the crystal field splitting Δ_o. A larger splitting corresponds to a higher absorption energy.
- For the same metal ion and oxidation state, ligand field strength is compared with the spectrochemical series. The relevant sequence is $H_2O < NH_3 < en$, where en is ethane-1,2-diamine.

Step 1: Identify the controlled factors Each complex contains nickel in the $+2$ oxidation state. Water, ammonia and en are neutral ligands, so the overall $2+$ charge is the nickel charge. Each complex also has six donor atoms: three bidentate en ligands in A, six monodentate ammonia ligands in B and six water ligands in C. Thus all three are octahedral Ni^{2+} complexes with the same d^8 electron count. The comparison is therefore governed by ligand field strength, rather than a change in oxidation state, coordination number or metal ion.

Step 2: Order the splitting and excitation energy Water is the weakest field ligand among the three, ammonia is intermediate, and en produces the strongest field. Hence

$$\Delta_o(C) < \Delta_o(B) < \Delta_o(A).$$

Since an absorbed photon supplies excitation energy, $E = h\nu$, the absorption energy increases in the same order as the relevant splitting. It does not increase with absorption wavelength, since $E = hc/\lambda$. Therefore the energy order is $C < B < A$. Option (B) incorrectly places the water complex above the ammonia complex; option (C) places en below ammonia; option (D) places the strongest-field en complex lowest. None of these agrees with the ligand sequence.

Step 3: Conclusion The correct absorption-energy order is $C < B < A$. Hence, the correct option is (A).

Quick Tip to Solve: Hold metal, oxidation state and geometry fixed, then use ligand strength for energy; reverse the order only if the question asks for wavelength.

67. Match List I with List II:

	List I		List II
(A)	Sulphate	(I)	Pesticide
(B)	Fluoride	(II)	Bending of bones
(C)	Nicotine	(III)	Laxative effect
(D)	Sodium arsinite	(IV)	Herbicide

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-II, C-I, D-IV

(D) A-III, B-II, C-IV, D-I

Correct Answer: (C) A-III, B-II, C-I, D-IV

Solution:

Concept:

- Dissolved inorganic ions can cause characteristic health effects when present in excess. Sulphate-rich drinking water has a laxative effect, while excess fluoride can cause skeletal fluorosis.
- Nicotine is a plant-derived insecticidal substance and is classified here as a pesticide. Sodium arsenite is an arsenic-containing chemical used as a herbicide.

Step 1: Match the water contaminants to their effects Read each entry in List I and assign its characteristic description in List II. A high sulphate concentration in drinking water can produce a laxative effect, especially when combined with certain cations. This identifies

$$A \longrightarrow \text{III}.$$

Excess fluoride intake over a prolonged period affects bones and teeth. Skeletal fluorosis can cause deformity, including bending of bones, so

$$B \longrightarrow \text{II}.$$

The two effects must not be interchanged: sulphate does not give the characteristic skeletal fluorosis association, and fluoride is not being matched to the laxative description.

Step 2: Match the agricultural chemicals and select the code Nicotine has insecticidal action and belongs with pesticide, giving $C \rightarrow \text{I}$. Sodium arsenite belongs with herbicide, giving $D \rightarrow \text{IV}$. Although herbicides are broadly a category of pesticides, the separate entries in this matching question call for their specific conventional associations. The full mapping is

$$(A, B, C, D) \longrightarrow (\text{III}, \text{II}, \text{I}, \text{IV}).$$

Option (A) interchanges the first two health effects and the two agricultural uses. Option (B) associates sulphate with herbicide and nicotine with bending of bones. Option (D) gets sulphate and fluoride right but swaps the specific nicotine and sodium arsenite uses.

Step 3: Conclusion The matches are A-III, B-II, C-I and D-IV. Hence, the correct option is (C).

Quick Tip to Solve: Fix sulphate-laxative and fluoride-bones first; then distinguish nicotine insecticide from sodium arsenite herbicide to choose the complete code.

68. Major product of the following reaction is [4 marks]



(A) $CH_3CHBrCOOCH_2CH_2CH_2Br$

(B) $CH_3CHBrCOOCH_2CHBrCH_3$

(C) $BrCH_2CH_2COOCH_2CH_2CH_2Br$

(D) $BrCH_2CH_2COOCH_2CHBrCH_3$

Correct Answer: (D) $BrCH_2CH_2COOCH_2CHBrCH_3$

Solution:

Concept:

- The reactant is allyl acrylate, $CH_2{=}CHCOOCH_2CH{=}CH_2$. It contains two chemically different double bonds: one conjugated with a carbonyl group and one isolated alkene.
- The electron-deficient double bond of an α,β-unsaturated ester undergoes conjugate addition, placing bromine at its β-carbon. The isolated terminal alkene undergoes normal Markovnikov addition of HBr in the absence of peroxide.

Step 1: Add hydrogen bromide to the acrylate end Label the carbon adjacent to the carbonyl group as α, and the terminal carbon of the conjugated double bond as β. Carbonyl conjugation polarizes this system so that nucleophilic bromide addition occurs at the β-carbon; proton transfer restores the carbonyl-containing saturated ester. The net change is

$$CH_2{=}CHCOOR \xrightarrow{HBr} BrCH_2CH_2COOR.$$

Thus the left-hand end becomes a 3-bromopropanoate unit, not a 2-bromopropanoate unit. Treating both double bonds as ordinary isolated alkenes would miss the effect of the electron-withdrawing ester carbonyl. The ester linkage itself stays intact; only its conjugated carbon-carbon double bond is saturated in this addition.

Step 2: Add hydrogen bromide to the allyl end At the other end, $-OCH_2CH{=}CH_2$ is an isolated terminal alkene. Proton addition at its terminal carbon gives the more stable secondary carbocation on the middle carbon; bromide capture gives $-OCH_2CHBrCH_3$. Using both equivalents of HBr, the full product is

$$BrCH_2CH_2COOCH_2CHBrCH_3.$$

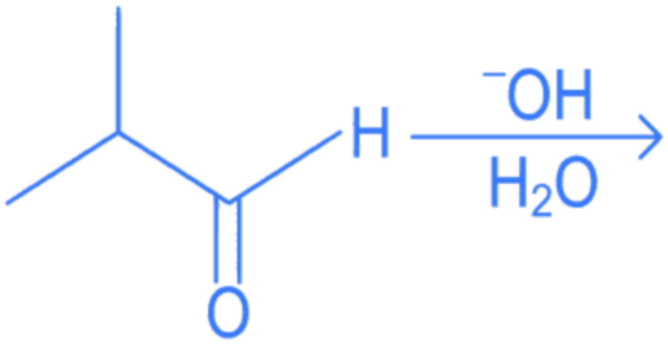
Option (A) has the wrong bromine position on both ends. Option (B) gets the allyl-end Markovnikov

addition right but puts bromine at the acrylate α-carbon. Option (C) has the correct conjugate-addition end but puts bromine at the terminal carbon of the isolated alkene, as if a peroxide-induced radical addition had been specified.

Step 3: Conclusion The major product is $BrCH_2CH_2COOCH_2CHBrCH_3$. Hence, the correct option is (D).

Quick Tip to Solve: In a molecule with two alkenes, classify each separately: carbonyl-conjugated addition at one end and ordinary Markovnikov addition at the other.

69. What is the major product of the following reaction? [4 marks]



(A) $OHC-C(CH_3)_2-CH=C(CH_3)_2$

(B) $OHC-C(CH_3)_2-CH(OH)-CH(CH_3)_2$

(C) $OHC-C(CH_3)_2-CO-CH(CH_3)_2$

(D) $OHC-C(CH_3)=C(CH_3)_2$

Correct Answer: (B) $OHC-C(CH_3)_2-CH(OH)-CH(CH_3)_2$

Solution:

Concept:

- An aldehyde with at least one α-hydrogen undergoes aldol addition in aqueous dilute base. Its enolate attacks the carbonyl carbon of a second molecule and forms a β-hydroxy aldehyde.
- The reactant is 2-methylpropanal, $(CH_3)_2CHCHO$. Its single α-hydrogen permits enolate formation, but the resulting aldol product has no remaining α-hydrogen on that donor carbon.

Step 1: Form the enolate and the new carbon-carbon bond Hydroxide removes the hydrogen from the carbon next to the aldehyde group. The resulting enolate has nucleophilic carbon character at that α-carbon. It attacks the carbonyl carbon of another molecule of 2-methylpropanal. The donor molecule retains its aldehyde group, while the acceptor carbonyl oxygen becomes an alkoxide. The donor α-carbon is now attached to two methyl groups, the retained CHO group and the newly attached acceptor carbon; it therefore has no hydrogen left.

Step 2: Protonate the alkoxide and compare the products Water protonates the alkoxide to give the

aldol addition product:

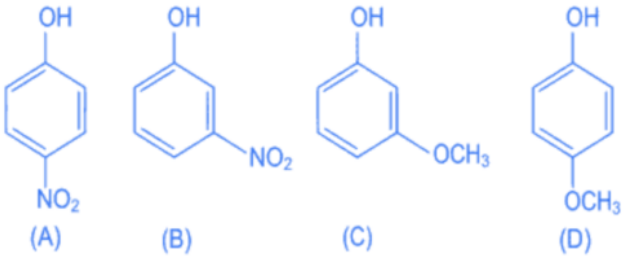
$$2\,(CH_3)_2CHCHO \xrightarrow{OH^-/H_2O} OHC{-}C(CH_3)_2{-}CH(OH){-}CH(CH_3)_2.$$

The product is 3-hydroxy-2,2,4-trimethylpentanal. Its original donor aldehyde remains intact, and the acceptor carbonyl has become the $CH(OH)$ centre. Option (A) replaces this alcohol region by an alkene, despite the lack of the donor α-hydrogen needed for usual aldol dehydration. Option (C) has a ketone at that position, which would require an oxidation step absent from the reagents. Option (D) is an unsaturated aldehyde with a different carbon skeleton and does not preserve the eight-carbon total of the two four-carbon reactant molecules. The aqueous base shown supports aldol addition rather than those transformations.

Step 3: Conclusion The major product is $OHC{-}C(CH_3)_2{-}CH(OH){-}CH(CH_3)_2$, the beta-hydroxy aldehyde. Hence, the correct option is (B).

Quick Tip to Solve: Before dehydrating an aldol product, check whether the donor alpha carbon still has a hydrogen; here its two methyl groups and new bond leave none.

70. Arrange the following in decreasing acidic strength. [4 marks]



(A) A > B > C > D

(B) B > A > C > D

(C) D > C > A > B

(D) D > C > B > A

Correct Answer: (A) A > B > C > D

Solution:

Concept:

- Phenol acidity is governed by the stability of the phenoxide conjugate base. Electron-withdrawing groups stabilize its negative charge and increase acidity; electron-donating groups generally have the opposite effect.
- A nitro group withdraws by both induction and resonance. A methoxy group withdraws inductively but donates by resonance at the ortho and para positions; the positional relation to the phenolic group must therefore be checked.

Step 1: Compare the two nitrophenols From the structures, A is para-nitrophenol and B is meta-nitrophenol. The negative charge of phenoxide is delocalized over oxygen and the ortho and para

positions of the ring. A para nitro group can accept electron density through resonance as well as withdrawing inductively, so it strongly stabilizes the conjugate base. At the meta position the direct resonance interaction with these charge-bearing positions is absent, and the nitro group acts mainly through its inductive effect. Consequently

$$A > B$$

in acidic strength. Both nitro-substituted phenols are more acidic than the methoxy-substituted phenols because nitro is strongly electron withdrawing.

Step 2: Compare the methoxyphenols and combine C is meta-methoxyphenol and D is para-methoxyphenol. In C, the methoxy group's electron-withdrawing inductive contribution is not countered by direct para resonance donation to the phenoxide charge-bearing position. In D, strong resonance donation increases electron density in the conjugate base and makes removal of the phenolic proton less favourable. Therefore $C > D$, and the overall order is

$$A > B > C > D.$$

Option (B) wrongly places meta-nitrophenol above para-nitrophenol, ignoring the extra resonance stabilization in A. Options (C) and (D) put methoxyphenols ahead of nitrophenols and therefore reverse the dominant stabilizing effect of the nitro substituent. Option (D) additionally reverses the para/meta nitro comparison.

Step 3: Conclusion The decreasing acidic-strength order is $A > B > C > D$. Hence, the correct option is (A).

Quick Tip to Solve: Draw the phenoxide resonance positions mentally: a para nitro group accepts charge, whereas a para methoxy group donates electron density and opposes acidity.

71.

$$CH_3CH_2CN \xrightarrow[\text{Ether}]{CH_3MgBr} A \xrightarrow{H_3O^+} B \xrightarrow[HCl]{Zn-Hg} C$$

The correct structure of C is [4 marks]

(A) $CH_3CH_2CH_2CH_3$

(B) $CH_3CH_2COCH_3$

(C) $CH_3CH_2CH(OH)CH_3$

(D) $CH_3CH_2CH{=}CH_2$

Correct Answer: (A) $CH_3CH_2CH_2CH_3$

Solution:

Concept:

- A Grignard reagent adds its alkyl group to the electrophilic carbon of a nitrile; acid hydrolysis of the resulting imine magnesium salt gives a ketone.
- Clemmensen reduction, $Zn-Hg/HCl$, replaces a ketone carbonyl group by CH_2, retaining the carbon skeleton.

Step 1: Form the ketone through the nitrile The starting compound is propanenitrile, CH_3CH_2CN. Its nitrile carbon becomes the carbonyl carbon, while CH_3MgBr supplies a methyl group. Addition in dry ether gives the imine magnesium salt A. Subsequent acid hydrolysis converts its carbon–nitrogen double bond into a carbonyl group:

$$CH_3CH_2CN + CH_3MgBr \longrightarrow CH_3CH_2C(=NMgBr)CH_3$$

$$CH_3CH_2C(=NMgBr)CH_3 \xrightarrow{H_3O^+} CH_3CH_2COCH_3$$

Thus B is butan-2-one, containing four carbon atoms.

Step 2: Apply the final reduction and compare options The last reagent is zinc amalgam with hydrochloric acid, the Clemmensen reducing system. It converts the carbonyl group of B into a methylene group:

$$CH_3CH_2COCH_3 \xrightarrow{Zn-Hg/HCl} CH_3CH_2CH_2CH_3$$

Option B is the intermediate ketone, so it stops one stage too early. Option C is butan-2-ol, which would result from carbonyl-to-alcohol reduction rather than Clemmensen reduction. Option D is but-1-ene and incorrectly retains unsaturation. Only option A is the fully reduced four-carbon alkane.

Step 3: Conclusion C is $CH_3CH_2CH_2CH_3$, butane. Hence, the correct option is (A).

Quick Tip to Solve: Track the nitrile carbon: it becomes the ketone carbonyl carbon and then a methylene carbon under Clemmensen reduction.

72. Glycosidic linkage between C_1 of α-glucose and C_2 of β-fructose is found in [4 marks]

(A) maltose

(B) sucrose

(C) lactose

(D) amylose

Correct Answer: (B) sucrose

Solution:

Concept:

- A glycosidic linkage joins monosaccharide units through an oxygen atom formed by condensation of hydroxyl groups.
- Sucrose contains α-D-glucose and β-D-fructose joined through their respective anomeric carbon atoms, glucose C_1 and fructose C_2.

Step 1: Identify the units and anomeric positions Glucose is an aldose, so its anomeric carbon in the cyclic form is C_1. Fructose is a ketose, so its anomeric carbon is C_2. The question specifies the alpha configuration of glucose and the beta configuration of fructose, as well as the exact pair of participating carbon atoms. This combination identifies the linkage in sucrose, conventionally described as an $\alpha(1 \rightarrow 2)\beta$ glycosidic bond. Both anomeric hydroxyl groups participate, leaving no free hemiacetal or hemiketal group.

Step 2: Eliminate carbohydrates with different units Maltose consists of two glucose units joined mainly by an $\alpha(1 \rightarrow 4)$ linkage; it has no fructose unit. Lactose contains galactose and glucose linked through a $\beta(1 \rightarrow 4)$ bond; its sugar pair and bond positions do not match. Amylose is a linear polysaccharide made from alpha-glucose units connected by $\alpha(1 \rightarrow 4)$ linkages. Sucrose alone contains the stated glucose–fructose pair and joins glucose C_1 to fructose C_2. Participation of both anomeric centres also explains why sucrose is non-reducing.

Step 3: Conclusion The specified glycosidic linkage is present in sucrose. Hence, the correct option is (B).

Quick Tip to Solve: Glucose C1 joined to fructose C2 identifies sucrose; glucose–glucose and galactose–glucose identify maltose and lactose instead.

73. In base vs. acid titration, at the end point methyl orange is present as [4 marks]

(A) quinonoid form

(B) heterocyclic form

(C) phenolic form

(D) benzenoid form

Correct Answer: (A) quinonoid form

Solution:

Concept:

- Methyl orange is an acid-base indicator whose protonated and unprotonated forms have different conjugated structures and colours.
- The alkaline form is yellow and benzenoid; the acidic form is red and quinonoid. Acid addition shifts the indicator equilibrium toward its protonated form.

Step 1: Follow the indicator as acid is added In the stated base-versus-acid titration, the alkaline solution is brought to the methyl-orange end point by addition of acid. Initially, in the basic medium, the indicator is predominantly in its yellow benzenoid form. As acid is added, the hydrogen-ion concentration rises and the indicator becomes protonated. The associated redistribution of electrons in the conjugated system gives the quinonoid form responsible for the acidic colour. The observed endpoint colour lies in the methyl-orange transition region.

Step 2: Select the relevant structural form The form associated with reaching the acid side of the methyl-orange endpoint is the quinonoid form. The benzenoid description applies to the initial alkaline yellow form, so option D does not represent the expected endpoint form in this titration. Methyl orange does not become a heterocyclic indicator at the endpoint; protonation does not create a new heteroatom-containing ring, ruling out B. The phenolic form is also inappropriate: methyl orange is an azo indicator, and its colour change is described by the benzenoid–quinonoid transformation rather than a phenolic structural change. Therefore A is the matching description.

Step 3: Conclusion Methyl orange is present in the quinonoid form at the stated endpoint. Hence, the correct option is (A).

Quick Tip to Solve: For methyl orange, remember the paired descriptions: yellow alkaline benzenoid form and red acidic quinonoid form.

74. For micelle formation, which of the following statements are correct?

A. Micelle formation is an exothermic process.

B. Micelle formation is an endothermic process.

C. The entropy change is positive.

D. The entropy change is negative. [4 marks]

(A) A and D only

(B) A and C only

(C) B and C only

(D) B and D only

Correct Answer: (C) B and C only

Solution:

Concept:

- Micelles form when surfactant molecules aggregate above the critical micelle concentration, placing hydrophobic tails inside and hydrophilic heads in contact with water.
- The textbook treatment of micellisation in water has positive enthalpy and positive entropy changes; spontaneity follows from $\Delta G = \Delta H - T\Delta S$.

Step 1: Determine the heat effect Micelle formation is treated here as an endothermic process, so the enthalpy change is positive:

$$\Delta H > 0.$$

Formation of the aggregate involves rearranging interactions between surfactant molecules and their surrounding water. The heat effect therefore selects statement B and rejects the contradictory statement A. The sign of enthalpy alone does not determine spontaneity; the entropy contribution must also be considered.

Step 2: Account for the entropy increase Before aggregation, water around the separate nonpolar tails has a relatively ordered arrangement. When the tails are buried in a micelle, many of these surrounding water molecules return to the bulk liquid and gain freedom of arrangement. Although the surfactant molecules themselves aggregate, this solvent contribution makes the net entropy change positive in the standard treatment:

$$\Delta S > 0.$$

Thus statement C is correct and D is incorrect. A sufficiently large positive $T\Delta S$ can outweigh positive ΔH, making ΔG negative and allowing spontaneous aggregation under the required conditions. The pair B and C is offered only in option C; every other pair contains at least one incorrect sign.

Step 3: Conclusion Statements B and C are correct: micelle formation is endothermic and the entropy change is positive. Hence, the correct option is (C).

Quick Tip to Solve: Count the released water molecules when reasoning about micellisation entropy; aggregation of solute alone misses the solvent contribution.

75. Given below are two statements.

Statement I : Pig iron is obtained by heating cast iron with scrap iron.

Statement II : Pig iron has a relatively lower carbon content than that of cast iron.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are not correct.

(C) Statement I is correct but Statement II is not correct.

(D) Statement I is not correct but Statement II is correct.

Correct Answer: (B) Both Statement I and Statement II are not correct.

Solution:

Concept:

- Pig iron is the carbon-rich iron obtained from the blast furnace by reduction of iron oxides with carbon monoxide and coke.
- Cast iron is prepared by melting pig iron with scrap iron and coke under controlled conditions; it has a lower carbon content than pig iron.

Step 1: Test the preparation statement Statement I reverses the metallurgical sequence. The blast furnace first yields molten iron containing a high proportion of carbon and other impurities, called pig iron. Cast iron is then made by remelting pig iron with scrap iron and coke using a suitable furnace. Heating cast iron with scrap iron is not the stated preparation of pig iron. The names must therefore be associated with the direction of refining rather than treated as interchangeable grades. Statement I is incorrect.

Step 2: Compare their carbon contents Pig iron typically contains about 4% carbon, whereas cast iron contains a lower proportion, around 3% in the standard textbook comparison. Thus the claim that pig iron has relatively less carbon than cast iron has the inequality reversed:

$$\text{carbon content of pig iron} > \text{carbon content of cast iron.}$$

Statement II is also incorrect. Option A requires both statements to be correct, while C and D require one of them to be correct. Since each statement independently fails its factual check, none of those three options can apply. Option B correctly records that both statements are not correct.

Step 3: Conclusion Both Statement I and Statement II are incorrect. Hence, the correct option is (B).

Quick Tip to Solve: Remember the order blast furnace to pig iron to cast iron; pig iron starts with the higher carbon content.

76. High purity ($>$ 99.95%) dihydrogen is obtained by : [4 marks]

(A) reaction of zinc with aqueous alkali.

(B) electrolysis of acidified water using platinum electrodes.

(C) electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

(D) reaction of zinc with dilute acid.

Correct Answer: (C) electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

Solution:

Concept:

- Electrolysis separates hydrogen evolution at the cathode from oxygen evolution at the anode, avoiding many impurities associated with reactive-metal preparations.
- High-purity dihydrogen, greater than 99.95%, is obtained by electrolysing warm aqueous barium hydroxide solution using nickel electrodes.

Step 1: Identify the specified high-purity method The purity threshold in the question points to the standard electrolytic preparation using warm aqueous $\mathrm{Ba(OH)_2}$ and nickel electrodes. Barium hydroxide provides ions that allow the solution to conduct, while water supplies the hydrogen and oxygen. The half-reactions in alkaline solution are

$$2\,\mathrm{H_2O} + 2\,\mathrm{e^-} \longrightarrow \mathrm{H_2} + 2\,\mathrm{OH^-}$$

at the cathode and

$$4\,\mathrm{OH^-} \longrightarrow \mathrm{O_2} + 2\,\mathrm{H_2O} + 4\,\mathrm{e^-}$$

at the anode. Adding them gives $2\,\mathrm{H_2O} \longrightarrow 2\,\mathrm{H_2} + \mathrm{O_2}$. Hydrogen is collected separately from the oxygen.

Step 2: Compare the other preparations Zinc reacts with aqueous alkali or dilute acid to release hydrogen, so options A and D are valid gas-generating reactions, but ordinary zinc and reagents can introduce impurities; these are not the specified high-purity preparation. Electrolysis of acidified water with platinum electrodes, option B, also generates hydrogen and oxygen, but it is not the textbook method identified with the greater-than-99.95% purity specification in this question. Option C uniquely names the warm barium hydroxide electrolyte and nickel electrodes required by that preparation. The question asks for this particular method, rather than merely any reaction that evolves hydrogen.

Step 3: Conclusion Use electrolysis of warm aqueous barium hydroxide between nickel electrodes. Hence, the correct option is (C).

Quick Tip to Solve: For the stated purity above 99.95%, recall the full combination: warm aqueous barium hydroxide electrolyte and nickel electrodes.

77. The correct order of density is : [4 marks]

(A) Be $>$ Mg $>$ Ca $>$ Sr

(B) Sr $>$ Ca $>$ Mg $>$ Be

(C) Sr $>$ Be $>$ Mg $>$ Ca

(D) Be $>$ Sr $>$ Mg $>$ Ca

Correct Answer: (C) Sr $>$ Be $>$ Mg $>$ Ca

Solution:

Concept:

- Density is mass per unit volume, $\rho = m/V$. Down a group, both atomic mass and atomic volume change, so density need not follow atomic number monotonically.
- Among the listed alkaline earth metals, calcium has the lowest density; strontium is denser than beryllium, magnesium and calcium.

Step 1: Compare the actual relative densities The approximate room-temperature densities, in $\mathrm{g\,cm^{-3}}$, are

$$\rho(\mathrm{Be}) \approx 1.85, \quad \rho(\mathrm{Mg}) \approx 1.74,$$
$$\rho(\mathrm{Ca}) \approx 1.55, \quad \rho(\mathrm{Sr}) \approx 2.64.$$

Hence density decreases from beryllium through magnesium to calcium and then increases at strontium. These values directly test the order without relying on atomic mass alone. The initial decrease reflects the increase in atomic volume relative to the mass increase; farther down the group, the growth in atomic mass produces the higher density.

Step 2: Arrange the metals and eliminate alternatives Arrange the displayed values from largest to smallest:

$$\mathrm{Sr} > \mathrm{Be} > \mathrm{Mg} > \mathrm{Ca}.$$

Option A puts strontium last despite its having the largest density here. Option B assumes a monotonic decrease up the group and incorrectly places calcium above magnesium and beryllium. Option D puts beryllium above strontium, contradicting their approximate densities 1.85 and 2.64. Option C preserves all three required comparisons: strontium exceeds beryllium, beryllium exceeds magnesium, and magnesium exceeds calcium. Thus it is the only listed sequence consistent with the density data.

Step 3: Conclusion The correct density order is $\mathrm{Sr} > \mathrm{Be} > \mathrm{Mg} > \mathrm{Ca}$. Hence, the correct option is (C).

Quick Tip to Solve: Do not assume a monotonic group trend: memorise the density dip from Be to Mg to Ca, followed by the rise at Sr.

78. The total number of acidic oxides from the following list is

NO, N_2O, B_2O_3, N_2O_5, CO, SO_3, P_4O_{10} [4 marks]

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (B) 4

Solution:

Concept:

- An acidic oxide reacts with a base to give a salt, and many acidic oxides form an oxyacid on hydration. Classification depends on chemical behaviour, rather than merely on the presence of oxygen.
- The familiar neutral oxides in this list are NO, N_2O and CO. They must not be counted along with the acid anhydrides.

Step 1: Identify the acidic members The acidic members are B_2O_3, N_2O_5, SO_3 and P_4O_{10}. Their corresponding acids are boric acid, nitric acid, sulfuric acid and phosphoric acid. For example,

$$N_2O_5 + H_2O \longrightarrow 2\,HNO_3$$

$$SO_3 + H_2O \longrightarrow H_2SO_4$$

$$P_4O_{10} + 6\,H_2O \longrightarrow 4\,H_3PO_4$$

Boron oxide is also acidic; its reaction with alkali forms borate. The fact that boron is a metalloid does not make its oxide basic.

Step 2: Count and check the alternatives There are four acidic oxides and three neutral oxides, accounting for all seven entries. The count 3 in (A) omits an acidic member. Counts 5 and 6 in (C) and (D) incorrectly include one or more of the neutral oxides. In particular, NO and N_2O do not become acidic merely because other nitrogen oxides are acidic.

Step 3: Conclusion The total number of acidic oxides is $\boxed{4}$. Hence, the correct option is (B).

Quick Tip to Solve: First cross out the standard neutral trio CO, NO, and N_2O; then classify the remaining oxides.

79. Match List I with List II :

	List I		List II
(A)	Nylon 6, 6	(I)	Buckets
(B)	Low density polythene	(II)	Non-stick utensils
(C)	High density polythene	(III)	Bristles of brushes
(D)	Teflon	(IV)	Toys

Choose the correct answer from the options given below : [4 marks]

(A) A-III, B-I, C-IV, D-II

(B) A-III, B-IV, C-I, D-II

(C) A-II, B-I, C-IV, D-III

(D) A-II, B-IV, C-I, D-III

Correct Answer: (B) A-III, B-IV, C-I, D-II

Solution:

Concept:

- The use of a polymer follows from its physical properties: tensile strength, flexibility, rigidity, chemical resistance and heat resistance.
- Nylon 6,6 is a polyamide, polythene is a polyolefin, and Teflon is polytetrafluoroethene. Low-density and high-density polythene have different degrees of branching and packing.

Step 1: Match each material to its characteristic use Nylon 6,6 is tough and abrasion-resistant, so it is used for bristles of brushes: A–III. Low-density polythene has extensive branching, lower packing efficiency and greater flexibility, making it suitable for toys: B–IV. High-density polythene has relatively linear chains, closer packing and greater rigidity, so it is used for buckets: C–I. Teflon has strong carbon–fluorine bonds, low surface adhesion and good chemical and thermal resistance, making it useful as a coating for non-stick utensils: D–II.

Step 2: Compare the complete matching with the options The complete correspondence is

$$\mathrm{A-III},\quad \mathrm{B-IV},\quad \mathrm{C-I},\quad \mathrm{D-II}.$$

Only option (B) contains all four links. Option (A) interchanges the polythene applications by putting low-density polythene with buckets. Options (C) and (D) wrongly put nylon with non-stick utensils and Teflon with brush bristles. Matching the distinctive Teflon use first narrows the possibilities, but the LDPE/HDPE distinction is needed to choose uniquely.

Step 3: Conclusion The matching is A–III, B–IV, C–I, D–II. Hence, the correct option is (B).

Quick Tip to Solve: Use Teflon for non-stick coatings and nylon for bristles first; then distinguish flexible LDPE toys from rigid HDPE buckets.

80. Some drugs bind to a site other than the active site of an enzyme. This site is known as [4 marks]

(A) non-active site

(B) allosteric site

(C) competitive site

(D) therapeutic site

Correct Answer: (B) allosteric site

Solution:

Concept:

- An enzyme has an active site where its substrate binds and undergoes the catalysed reaction. Drug molecules can alter enzyme activity by binding either there or at a distinct regulatory site.
- A site spatially different from the active site, at which binding changes the enzyme conformation and influences its activity, is called an allosteric site.

Step 1: Identify the binding mechanism described The decisive phrase is "a site other than the active site. A drug binding at such a regulatory site can change the shape of the enzyme, including the shape or effectiveness of its active site. The substrate may then bind less effectively or the catalytic process may be inhibited. Because the drug occupies a different location, the mechanism is distinguished from direct competition with substrate molecules for the active site. The recognised name of this location is the allosteric site.

Step 2: Eliminate the other names Option (A), non-active site, is only a general descriptive phrase and is not the specific biochemical term requested. Option (C), competitive site, confuses this mechanism with competitive inhibition, in which inhibitor and substrate compete for the active site. Option (D), therapeutic site, is not the name of an enzyme regulatory binding site; therapeutic refers to treatment rather than this structural location. Thus the terminology and the binding mechanism both select option (B).

Step 3: Conclusion The site is the **allosteric site**. Hence, the correct option is (B).

Quick Tip to Solve: Binding away from the active site suggests allosteric regulation; competition for the active site suggests competitive inhibition.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 56.0 L of nitrogen gas is mixed with excess hydrogen gas and it is found that 20 L of ammonia gas is produced. The volume of unused nitrogen gas is found to be _____ L. [4 marks]

Solution:

Concept:

- At the same temperature and pressure, gas volumes are proportional to their amounts in moles. The stoichiometric coefficients of a balanced gaseous equation therefore give reacting volume ratios.

- For ammonia formation, $N_2 + 3\,H_2 \longrightarrow 2\,NH_3$. One volume of nitrogen produces two volumes of ammonia when sufficient hydrogen is available.

Step 1: Find the nitrogen volume consumed The balanced equation gives

$$\frac{V_{N_2,\,\text{used}}}{V_{NH_3,\,\text{formed}}} = \frac{1}{2}.$$

The stated ammonia volume is the volume actually produced, so nitrogen consumption must be inferred from that value, rather than from the initial nitrogen charge. Substitution gives

$$V_{N_2,\,\text{used}} = \frac{20}{2} = 10\,\text{L}.$$

Excess hydrogen ensures that hydrogen availability does not limit this conversion.

Step 2: Subtract consumption from the initial nitrogen The initial nitrogen volume is $56.0\,\text{L}$. The part left unreacted is therefore

$$V_{N_2,\,\text{unused}} = 56.0 - 10 = 46.0\,\text{L}.$$

This is a material balance on nitrogen, not on the total gas mixture. Subtracting $20\,\text{L}$ directly would be incorrect because the ammonia and nitrogen coefficients differ. As a check, $10\,\text{L}$ nitrogen consumes $30\,\text{L}$ hydrogen and yields the specified $20\,\text{L}$ ammonia.

Step 3: Conclusion The volume of unused nitrogen is $\boxed{46\,\text{L}}$. The required numerical entry is 46.

Quick Tip to Solve: Convert product volume to reactant volume using the balanced coefficients before subtracting from the initial volume.

82. When the excited electron of a H atom from n = 5 drops to the ground state, the maximum number of emission lines observed are _____. [4 marks]

Solution:

Concept:

- Each downward transition between two distinct energy levels emits a photon of a definite frequency. The maximum spectral-line count includes all distinct pairs of levels accessible during de-excitation.
- For levels 1 through n, the number of possible transitions is $\binom{n}{2} = n(n-1)/2$. This counts possible lines across an ensemble, rather than the photon count in one particular cascade.

Step 1: List the levels and possible lower destinations The highest populated level is $n = 5$, and the ground state is $n = 1$. Thus five levels are involved: $1, 2, 3, 4, 5$. From level 5 there are four possible lower destinations; from level 4 there are three; from level 3 there are two; and from level 2 there is one. The ground state has no lower level in this set. Each upper–lower pair corresponds to one allowed spectral energy difference in the usual hydrogen line-counting treatment.

Step 2: Add all distinct transitions The maximum number of emission lines is

$$N = 4 + 3 + 2 + 1 = 10.$$

Equivalently,

$$N = \frac{5(5-1)}{2} = \frac{5 \times 4}{2} = 10.$$

Counting only the direct transitions from level 5 would give four and would omit lines arising after intermediate levels become populated. Conversely, the count is not five simply because five energy levels are involved. The requested maximum includes every possible downward pair, with each pair counted once.

Step 3: Conclusion The maximum number of emission lines is $\boxed{10}$, a dimensionless count.

Quick Tip to Solve: For a maximum line count from level n, count pairs of levels with $n(n-1)/2$; do not count only direct jumps from the top level.

83. While performing a thermodynamics experiment, a student made the following observations.

$HCl + NaOH \rightarrow NaCl + H_2O$ $\Delta H = -57.3$ kJ mol^{-1}

$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$ $\Delta H = -55.3$ kJ mol^{-1}

The enthalpy of ionization of CH_3COOH as calculated by the student is _____ kJ mol^{-1}. (nearest integer) [4 marks]

Solution:

Concept:

- For a strong acid and a strong base in dilute aqueous solution, the net neutralization process is $H^+ + OH^- \longrightarrow H_2O$, with the stated enthalpy $-57.3\,\text{kJ mol}^{-1}$.
- A weak acid must ionize as its hydrogen ions are consumed. By Hess's law, its measured neutralization enthalpy includes both ionization and the subsequent formation of water.

Step 1: Write the two contributions for ethanoic acid Ethanoic acid undergoes the ionization

$$CH_3COOH \longrightarrow CH_3COO^- + H^+, \qquad \Delta H_{\text{ion}}.$$

The liberated hydrogen ions then react with hydroxide:

$$H^+ + OH^- \longrightarrow H_2O, \qquad \Delta H = -57.3\,\text{kJ mol}^{-1}.$$

Adding these processes gives the net reaction of ethanoic acid with sodium hydroxide; sodium ions are spectators. Hence the weak-acid neutralization enthalpy is the sum of these two enthalpy changes.

Step 2: Use Hess's law with the correct signs The experimental weak-acid value is $-55.3\,\text{kJ mol}^{-1}$, so

$$-55.3 = \Delta H_{\text{ion}} + (-57.3).$$

Rearranging,

$$\Delta H_{\text{ion}} = -55.3 + 57.3 = +2.0\,\text{kJ mol}^{-1}.$$

The positive sign is physically consistent: energy is absorbed in ionizing the weak acid, making the overall neutralization less exothermic than the strong-acid reaction. Taking the difference of magnitudes without tracking the signs could incorrectly suggest an exothermic ionization.

Step 3: Conclusion The enthalpy of ionization is $\boxed{+2\,\text{kJ mol}^{-1}}$, to the nearest integer. The numerical entry is 2.

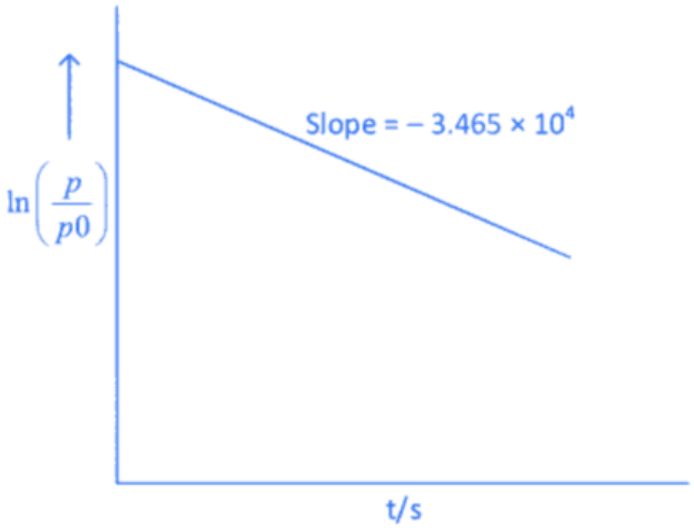
Quick Tip to Solve: Use $\Delta H_{\text{weak acid}} = \Delta H_{\text{ion}} + \Delta H_{\text{strong acid}}$ and retain both negative neutralization signs.

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84. For the decomposition of azomethane.

$CH_3N_2CH_3(g) \rightarrow CH_3CH_3(g) + N_2(g)$, a first order reaction, the variation in partial pressure with time at 600 K is given as

The half life of the reaction is _____ $\times 10^{-5}$ s. [Nearest integer] [4 marks]



Solution:

Concept:

- For first-order decomposition, the reactant partial pressure obeys $\ln(p/p_0) = -kt$, where p_0 is its initial partial pressure and k is the rate constant.
- The slope of a natural-log pressure-ratio plot against time is $-k$. The first-order half-life is $t_{1/2} = \ln 2/k$, independent of the initial pressure.

Step 1: Read the rate constant from the graph The vertical axis is $\ln(p/p_0)$, and the horizontal axis is t/s. The plotted straight-line slope is -3.465×10^4. Comparing with $\ln(p/p_0) = -kt$ gives

$$k = 3.465 \times 10^4\,\text{s}^{-1}.$$

The rate constant itself is positive; the negative slope represents the decreasing reactant pressure. No factor of 2.303 is needed because the graph uses a natural logarithm rather than a common logarithm.

Step 2: Calculate and express the half-life as requested At half-life, $p/p_0 = 1/2$, so

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{3.465 \times 10^4}\,\text{s}.$$

Since $0.693/3.465 = 0.2$,

$$t_{1/2} = 0.2 \times 10^{-4}\,\text{s} = 2.0 \times 10^{-5}\,\text{s}.$$

The blank asks for the multiplier of $10^{-5}\,\text{s}$, so the nearest integer is 2. As a check, $kt_{1/2} = 0.693$, giving $p/p_0 = e^{-0.693} \simeq 1/2$, as required.

Step 3: Conclusion The half-life is $\boxed{2 \times 10^{-5}\,\text{s}}$. The required numerical entry is $\boxed{2}$.

Quick Tip to Solve: Identify whether the graph uses $\ln$ or $\log_{10}$ first; a natural-log slope directly gives $-k$.

85. The sum of number of lone pairs of electrons present on the central atoms of XeO_3, $XeOF_4$ and XeF_6, is _____ [4 marks]

Solution:

Concept:

- Xenon has eight valence electrons. In the usual Lewis structures, each terminal fluorine forms one single bond and each terminal oxygen forms one double bond to xenon.
- A lone pair contains two nonbonding electrons. Count lone pairs on the central xenon atom only; the terminal oxygen and fluorine lone pairs are not included.

Step 1: Count the lone pair in each xenon compound For XeO_3, the three Xe=O bonds account for six of xenon's valence electrons, leaving two as one lone pair. Its central atom therefore has three bonding domains and one lone pair. For $XeOF_4$, one double bond and four single bonds again account for six of xenon's valence electrons. The remaining two electrons form one lone pair. A double bond is one electron domain for shape, although it uses two electron pairs in the Lewis structure.

Step 2: Check the hexafluoride and add the counts In XeF_6, six single bonds use six xenon valence electrons, leaving one lone pair on xenon. Thus this molecule has six bonding domains and one lone pair, consistent with its distorted octahedral molecular geometry. The three central-atom lone-pair counts are consequently

$$N(XeO_3) = 1, \qquad N(XeOF_4) = 1, \qquad N(XeF_6) = 1.$$

Adding these counts, rather than adding every lone pair in the molecules, gives $1 + 1 + 1 = 3$.

Step 3: Conclusion The required sum of lone pairs on the central atoms is $\boxed{3}$.

Quick Tip to Solve: For these neutral xenon oxyfluorides, subtract two per terminal O and one per F from eight, then divide the remainder by two.

86. The spin-only magnetic moment value of M^{3+} ion (in gaseous state) from the pairs Cr^{3+} / Cr^{2+}, Mn^{3+} / Mn^{2+}, Fe^{3+} / Fe^{2+} and Co^{3+} / Co^{2+} that has negative standard electrode potential, is _____ B.M. [Nearest integer] [4 marks]

Solution:

Concept:

- The relevant standard electrode potentials refer to reduction, $M^{3+} + e^- \longrightarrow M^{2+}$. The negative member must be identified before calculating the magnetic moment.
- The spin-only moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons. A gaseous free ion has no ligand-field pairing imposed by a complex.

Step 1: Identify the metal with negative reduction potential For the listed couples, the standard reduction potentials are approximately $-0.41\,\text{V}$ for Cr^{3+}/Cr^{2+}, $+1.51\,\text{V}$ for Mn^{3+}/Mn^{2+}, $+0.77\,\text{V}$ for Fe^{3+}/Fe^{2+}, and $+1.82\,\text{V}$ for Co^{3+}/Co^{2+}. Thus the required trivalent ion is Cr^{3+}. The sign refers to reduction as written; reversing that half-reaction would reverse its potential.

Step 2: Count unpaired electrons and evaluate the moment Chromium has the neutral configuration $[\text{Ar}]\,3d^5 4s^1$. On forming Cr^{3+}, the $4s$ electron is removed first and then two $3d$ electrons, giving $[\text{Ar}]\,3d^3$. By Hund's rule the three d electrons occupy separate orbitals, so $n = 3$.

$$\mu = \sqrt{3(3+2)} = \sqrt{15} = 3.873\,\text{B.M.}$$

The question asks for the nearest integer, so round this value only at the final step.

Step 3: Conclusion The spin-only moment is $3.873\,\text{B.M.}$, or $\boxed{4\,\text{B.M.}}$ to the nearest integer.

Quick Tip to Solve: Choose the negative reduction-potential couple first. For chromium ions, remove the 4s electron before counting the remaining 3d electrons.

87. A sample of 4.5 mg of an unknown monohydric alcohol, R-OH was added to methylmagnesium iodide. A gas is evolved and is collected and its volume measured to be 3.1 mL. The molecular weight of the unknown alcohol is _____ g/mol. [Nearest integer] [4 marks]

Solution:

Concept:

- A Grignard reagent reacts quantitatively with the acidic hydrogen of an alcohol. Methylmagnesium iodide liberates methane: $\text{ROH} + \text{CH}_3\text{MgI} \longrightarrow \text{ROMgI} + \text{CH}_4$.
- A monohydric alcohol has one reactive O–H hydrogen, so one mole of alcohol gives one mole of methane. Using the standard gas molar volume, $V_m = 22.4\,\text{L mol}^{-1}$, converts gas volume to amount.

Step 1: Use the one-to-one methane stoichiometry The methyl group of the Grignard reagent takes a proton from the alcohol and becomes CH_4; it does not become ethane or combine with the carbon skeleton of the alcohol in this acid–base reaction. The stated monohydric nature fixes the ratio at one

methane molecule per alcohol molecule. Convert the measured quantities before calculating:

$$m = 4.5\,\text{mg} = 4.5 \times 10^{-3}\,\text{g}, \qquad V = 3.1\,\text{mL} = 3.1 \times 10^{-3}\,\text{L}.$$

The amount of gas, and therefore of alcohol, is $n = V/V_m$.

Step 2: Calculate the molar mass and round once Using the standard molar gas volume in the gas-volume calculation gives

$$n(\text{ROH}) = n(\text{CH}_4) = \frac{3.1 \times 10^{-3}}{22.4} = 1.3839 \times 10^{-4}\,\text{mol}.$$

Hence

$$M = \frac{m}{n} = \frac{4.5 \times 10^{-3} \times 22.4}{3.1 \times 10^{-3}} = 32.516\,\text{g mol}^{-1}.$$

The numerical result is rounded as requested, rather than rounded during the mole calculation. The ratio would change for an alcohol with more than one reactive hydroxyl group.

Step 3: Conclusion The required molecular weight, to the nearest integer, is $\boxed{33\,\text{g mol}^{-1}}$.

Quick Tip to Solve: For a monohydric alcohol reacting with methylmagnesium iodide, use one mole of methane per mole of alcohol and keep the gas-volume units consistent.

88. The separation of two coloured substances was done by paper chromatography. The distances travelled by solvent front, substance A and substance B from the base line are 3.25 cm, 2.08 cm and 1.05 cm, respectively. The ratio of R_f values of A to B is _____. [4 marks]

Solution:

Concept:

- The retention factor in paper chromatography is $R_f = d_{\text{solute}}/d_{\text{front}}$, with both distances measured from the same baseline along the direction of solvent movement.
- An R_f value and a ratio of two R_f values are dimensionless. When two substances use the same solvent front, that common distance cancels from their ratio.

Step 1: Calculate the retention factors from the baseline The solvent has travelled $3.25\,\text{cm}$, while A and B have travelled $2.08\,\text{cm}$ and $1.05\,\text{cm}$. Therefore

$$R_f(A) = \frac{2.08}{3.25} = 0.6400, \qquad R_f(B) = \frac{1.05}{3.25} = 0.32308.$$

Both factors are below one, as expected for spots lying behind the solvent front. These are distances from the baseline, so no distance needs to be subtracted from the solvent-front position.

Step 2: Take the ratio in the requested order The ratio of A to B means divide A's retention factor by B's, not the reverse:

$$\frac{R_f(A)}{R_f(B)} = \frac{2.08/3.25}{1.05/3.25} = \frac{2.08}{1.05} = 1.98095.$$

Thus A has approximately twice the retention factor of B. The solvent-front distance is essential to

define each factor but cancels because it is identical for the two spots on the same chromatogram. The numerical response is the integer value corresponding to this approximately two-to-one ratio.

Step 3: Conclusion The ratio is $R_f(A) : R_f(B) = 1.98095 : 1 \approx \boxed{2:1}$; the required numerical entry is $\boxed{2}$.

Quick Tip to Solve: For two spots on the same chromatogram, divide their travel distances directly to obtain the ratio of their retention factors.

89. The total number of monobromo derivatives formed by the alkanes with molecular formula C_5H_{12} is (excluding stereo isomers) _____. [4 marks]

Solution:

Concept:

- A monobromo derivative results when exactly one hydrogen in an alkane is replaced by bromine. Hydrogens related by molecular symmetry lead to the same constitutional product.
- The formula C_5H_{12} has three carbon skeletons: pentane, 2-methylbutane and 2,2-dimethylpropane. Count distinct substitution sites for each, excluding stereoisomers as instructed.

Step 1: Count the products from pentane and 2-methylbutane In pentane, positions 1 and 5 are equivalent, as are 2 and 4; position 3 is distinct. Substitution therefore gives 1-bromopentane, 2-bromopentane and 3-bromopentane: three constitutional products. In $CH_3CH(CH_3)CH_2CH_3$, the two methyl groups bonded directly to the branched carbon are equivalent. The terminal methyl at the other end is different, and the methylene and tertiary methine are each distinct. These four classes give four constitutional monobromo products. Rotating a molecule or reversing a written chain does not create a new product.

Step 2: Include the highly symmetric third skeleton In 2,2-dimethylpropane, $C(CH_3)_4$, all four methyl groups are related by symmetry, and all twelve hydrogens are equivalent for constitutional substitution. Its central carbon has no hydrogen to replace. It therefore gives only one monobromo derivative. The total over all alkane isomers is

$$N = 3 \text{ (pentane)} + 4 \text{ (2-methylbutane)} + 1 \text{ (2,2-dimethylpropane)} = 8.$$

Although some products can exist as enantiomers, those pairs are counted once here because stereoisomers are explicitly excluded.

Step 3: Conclusion The total number of monobromo derivatives, excluding stereoisomers, is $\boxed{8}$.

Quick Tip to Solve: List every carbon skeleton first, then group equivalent C–H sites by symmetry. Do not multiply a constitutional product by its number of enantiomers.

90. A sealed flask with a capacity of 2 dm^3 contains 11 g of propane gas. The flask is so weak that it will burst if the pressure becomes 2 MPa. The minimum temperature at which the flask will burst is _____ °C. [Nearest integer]

(Given : R = 8.3 J K^{-1} mol^{-1} , Atomic masses of C and H are 12u and 1u, respectively.) (Assume that propane behaves as an ideal gas.) [4 marks]

Solution:

Concept:

- For a sealed rigid flask containing an ideal gas, $PV = nRT$, where T must be the absolute temperature in kelvin. The amount and volume remain constant until the flask bursts.
- The minimum bursting temperature occurs when the gas pressure first reaches the limiting pressure, $2\,\mathrm{MPa}$. Use SI units with $R = 8.3\,\mathrm{J\,K^{-1}\,mol^{-1}}$.

Step 1: Calculate the gas amount and convert the data The molar mass of propane, $\mathrm{C_3H_8}$, from the supplied atomic masses is

$$M = 3(12) + 8(1) = 44\,\mathrm{g\,mol^{-1}}.$$

The sealed flask therefore contains

$$n = \frac{11}{44} = 0.25\,\mathrm{mol}.$$

Convert the capacity and limiting pressure to the units compatible with the given gas constant:

$$V = 2\,\mathrm{dm^3} = 2 \times 10^{-3}\,\mathrm{m^3}, \qquad P = 2\,\mathrm{MPa} = 2 \times 10^{6}\,\mathrm{Pa}.$$

Here $\mathrm{Pa\,m^3} = \mathrm{J}$, so the units are consistent.

Step 2: Find the threshold temperature and convert to Celsius At the bursting threshold,

$$T = \frac{PV}{nR} = \frac{(2 \times 10^{6})(2 \times 10^{-3})}{(0.25)(8.3)} = 1927.71\,\mathrm{K}.$$

Converting this absolute temperature to degrees Celsius gives

$$t = T - 273.15 = 1654.56\,^{\circ}\mathrm{C}.$$

Using the usual exam conversion $T - 273$ gives $1654.71\,^{\circ}\mathrm{C}$, with the same nearest integer. Because pressure increases monotonically with absolute temperature at fixed volume and amount, this is the first temperature at which the pressure limit is reached.

Step 3: Conclusion The minimum bursting temperature is $\boxed{1655\,^{\circ}\mathrm{C}}$, to the nearest integer.

Quick Tip to Solve: Solve PV=nRT in kelvin first, with MPa converted to Pa and cubic decimetres to cubic metres; convert to Celsius only at the end.