JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 26 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 26 July 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let f : R → R be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to : [4 marks]

(A) 4

(B) 10

(C) 11

(D) 16

Correct Answer: (B) 10

Solution:

Concept:

- Continuity permits a limit to pass through a function: if $t_n \to 0$, then $f(t_n) \to f(0)$.
- The equation relates arguments differing by a factor of three. Iterating towards zero produces a convergent geometric sum; the constant value $f(0)$ is then fixed by the given datum.

Step 1: Iterate the relation towards zero For an arbitrary real number t, substitute $x = t/3$ into the equation. This gives $f(t) - f(t/3) = t/3$. Repeating with $t/3, t/3^2, \ldots$, and adding the first n equalities, all intermediate function values cancel. Thus

$$f(t) - f(t/3^n) = \sum_{j=1}^{n} \frac{t}{3^j} = \frac{t}{2}(1 - 3^{-n}).$$

This identity works for positive and negative t; no differentiability assumption is being made.

Step 2: Use continuity and determine the constant As $n \to \infty$, the argument $t/3^n$ tends to zero. Since the function is continuous there, the preceding equality becomes

$$f(t) - f(0) = \frac{t}{2}.$$

Now use the specified value: $7 = f(8) = 4 + f(0)$, so $f(0) = 3$. Hence the entire function is $f(t) = t/2 + 3$. Substitution also verifies $f(3x) - f(x) = x$, so this formula satisfies both requirements.

Step 3: Evaluate the requested value and compare The value at the new argument is

$$f(14) = \frac{14}{2} + 3 = 10.$$

The alternatives 4, 11 and 16 cannot satisfy the derived formula at 14. In particular, the difference $f(14) - f(8)$ must be $(14 - 8)/2 = 3$, which selects 10 directly from the options.

Step 4: Conclusion Therefore $f(14) = 10$. Hence, the correct option is (B).

Quick Tip to Solve: For a dilation equation, telescope at arguments $t, t/3, t/3^2, \ldots$ and use continuity at zero to remove any undetermined oscillation.

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $\mathrm{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true? [4 marks]

(A) $\arg z_2 = \pi - \tan^{-1}3$

(B) $\arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

(C) $|z_2| = \sqrt{10}$

(D) $|2z_1 - z_2| = 5$

Correct Answer: (D) $|2z_1 - z_2| = 5$

Solution:

Concept:

- A right isosceles triangle has two perpendicular legs of equal length. Multiplication of a complex vector by i or $-i$ rotates it through a right angle without changing its length.
- Because OB is the hypotenuse, the right angle is at A. For $z = x + iy$, its modulus is $\sqrt{x^2 + y^2}$, and its argument must be placed in the correct quadrant.

Step 1: Find the admissible location of B The vectors along the two legs from A are $-z_1$ and $z_2 - z_1$. They must differ by a factor i or $-i$. Equivalently, the two possible locations are

$$z_2 = z_1 \pm iz_1.$$

With $z_1 = 1 + 2i$, we have $iz_1 = -2 + i$. The candidates are $-1 + 3i$ and $3 + i$. The given condition $\mathrm{Re}\, z_2 < 0$ therefore selects $z_2 = -1 + 3i$.

Step 2: Check the first three statements The selected point lies in quadrant II, with the ratio of the absolute imaginary to real coordinates equal to 3. Therefore

$$\arg z_2 = \pi - \tan^{-1} 3.$$

Next,

$$z_1 - 2z_2 = (1 + 2i) - 2(-1 + 3i) = 3 - 4i.$$

This number lies in quadrant IV, so its principal argument is $-\tan^{-1}(4/3)$. Also $|z_2| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$. Thus (A), (B) and (C) are each true and cannot answer a question asking for the false statement.

Step 3: Test the remaining modulus Calculate the expression in (D) before taking its modulus:

$$2z_1 - z_2 = 2(1 + 2i) - (-1 + 3i) = 3 + i.$$

Consequently $|2z_1 - z_2| = \sqrt{3^2 + 1^2} = \sqrt{10}$, which is not 5. The condition on the real part was essential: choosing the other triangle would test a different point.

Step 4: Conclusion The statement $|2z_1 - z_2| = 5$ is false; its actual value is $\sqrt{10}$. Hence, the correct option is (D).

Quick Tip to Solve: Locate the right angle from the named hypotenuse first, then generate the two vertices by a quarter-turn and apply the sign condition.

3. If the system of linear equations.

$$8x + y + 4z = -2$$

$$x + y + z = 0$$

$$\lambda x - 3y = \mu$$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x+y+4z+2=0$ is : [4 marks]

(A) $3\sqrt{5}$

(B) 4

(C) $\frac{26}{9}$

(D) $\frac{10}{3}$

Correct Answer: (D) $\frac{10}{3}$

Solution:

Concept:

- Infinitely many solutions require the third equation to impose no additional restriction on the line determined by two independent equations.
- The distance from (x_0, y_0, z_0) to $Ax + By + Cz + D = 0$ is $|Ax_0 + By_0 + Cz_0 + D|/\sqrt{A^2 + B^2 + C^2}$.

Step 1: Eliminate z from the first two equations The first two equations are independent because their coefficient rows are not proportional. Subtract four times the second equation from the first:

$$(8x + y + 4z) - 4(x + y + z) = -2 - 4(0),$$

which reduces to $4x - 3y = -2$. We can also parametrize their common line by taking $x = t$. Then

$$y = \frac{4t + 2}{3}, \qquad z = -\frac{7t + 2}{3}.$$

Here t is free, explaining the infinite set before the third equation is imposed.

Step 2: Impose consistency for every value of the parameter Insert these coordinates into $\lambda x - 3y = \mu$. It becomes

$$(\lambda - 4)t - 2 = \mu.$$

For infinitely many values of t to remain possible, the coefficient of t must vanish and the constants must agree. Therefore $\lambda = 4$ and $\mu = -2$. If $\lambda \neq 4$, this equation would instead determine just one value of t; if $\lambda = 4$ but $\mu \neq -2$, there would be no solution.

Step 3: Apply the distance formula The requested point is $P = (4, -2, -1/2)$. The normal to the

plane is $(8, 1, 4)$, whose length is $\sqrt{64+1+16} = 9$. Thus

$$d = \frac{|8(4) + (-2) + 4(-1/2) + 2|}{9} = \frac{|30|}{9} = \frac{10}{3}.$$

This is different from $3\sqrt{5}$, 4 and $26/9$, ruling out the other listed distances.

Step 4: Conclusion The distance is $10/3$ units. Hence, the correct option is (D).

Quick Tip to Solve: When one equation omits a variable, eliminate that variable from the other two and compare the resulting equation before computing any distance.

4. The odd natural number a, such that the area of the region bounded by y = 1, y = 3, x = 0, x = y^a is $\frac{364}{3}$, is equal to : [4 marks]

(A) 3

(B) 5

(C) 7

(D) 9

Correct Answer: (B) 5

Solution:

Concept:

- For horizontal strips, the area between a right boundary $x_R(y)$ and a left boundary $x_L(y)$ is $\int [x_R(y) - x_L(y)]\, dy$.
- On $1 \le y \le 3$, the quantity y^a is positive for every natural number a. Thus $x = y^a$ is the right boundary and $x = 0$ is the left boundary.

Step 1: Set up the horizontal-strip integral The horizontal boundaries are already given as $y = 1$ and $y = 3$, so integration with respect to y avoids inverting the curve. A strip at height y has width $y^a - 0$. Therefore the bounded area is

$$A = \int_1^3 y^a\, dy = \left[\frac{y^{a+1}}{a+1}\right]_1^3 = \frac{3^{a+1} - 1}{a+1}.$$

There is no subtraction of a second curved region: the vertical axis forms the entire left boundary.

Step 2: Use the prescribed area The condition is

$$\frac{3^{a+1} - 1}{a+1} = \frac{364}{3}.$$

Test the odd candidate $a = 5$. Its area is

$$\frac{3^6 - 1}{6} = \frac{729 - 1}{6} = \frac{728}{6} = \frac{364}{3},$$

so it satisfies the requirement exactly. This step keeps the lower-limit contribution of 1, which cannot

be neglected even though the upper power is large.

Step 3: Establish uniqueness and exclude the other options For $y > 1$, the function y^a strictly increases when a increases. Its integral over $[1, 3]$ therefore also strictly increases with a. Hence at most one natural exponent can have the prescribed area. Explicitly, $a = 3$ gives $(81 - 1)/4 = 20$, which is too small; $a = 7$ and $a = 9$ give areas larger than the area at 5. Thus none of the other odd numbers listed works.

Step 4: Conclusion The odd natural number is $a = 5$. Hence, the correct option is (B).

Quick Tip to Solve: Choose strips parallel to the axis for which both bounding curves are already written explicitly; here horizontal strips give y^a immediately.

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5. Consider two G.Ps. 2, 2^2, 2^3, ... and 4, 4^2, 4^3, ... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to : [4 marks]

(A) 560

(B) 1540

(C) 1330

(D) 2600

Correct Answer: (C) 1330

Solution:

Concept:

- The geometric mean of positive numbers is the appropriate root of their product. Products of powers of the same base are simplified by adding their exponents.
- The standard sums are $\sum_{k=1}^{n} k = n(n+1)/2$ and $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$. The number of terms n must be a positive integer.

Step 1: Express both products as powers of two The first progression contains $2^1, 2^2, \ldots, 2^{60}$. Its product is $2^{60 \cdot 61/2} = 2^{1830}$. The other contains $4^1, 4^2, \ldots, 4^n$, whose product is $4^{n(n+1)/2} = 2^{n(n+1)}$. Therefore the geometric mean of all the terms is

$$2^{\{1830+n(n+1)\}/(60+n)}.$$

The two progressions are counted in full, including any numerical values occurring in both.

Step 2: Determine the admissible number of terms Equate this exponent with $225/8$:

$$8[1830 + n(n+1)] = 225(60 + n).$$

After expansion and collection,

$$8n^2 - 217n + 1140 = (n - 20)(8n - 57) = 0.$$

The roots are $n = 20$ and $n = 57/8$. Only 20 is an integer and can count terms of a progression. As a

check, $[1830 + 20 \cdot 21]/80 = 2250/80 = 225/8$, exactly as required.

Step 3: Evaluate the requested sum Separate the factor $n - k$:

$$\sum_{k=1}^{n} k(n-k) = n\sum_{k=1}^{n} k - \sum_{k=1}^{n} k^2 = \frac{n(n+1)(n-1)}{6}.$$

At $n = 20$, this is $20 \cdot 21 \cdot 19/6 = 1330$. The final term at $k = n$ contributes zero, consistent with the original sum. The alternatives 560, 1540 and 2600 do not equal this exact value.

Step 4: Conclusion The required sum is 1330. Hence, the correct option is (C).

Quick Tip to Solve: Convert every term to the same exponential base before taking a geometric mean; reject nonintegral roots when solving for a number of terms.

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6. If the function $f(x) = \begin{cases} \frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x - \cos x} & , \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\} \\ k & , \quad x = 0 \end{cases}$ is continuous at x = 0, then k is equal to: [4 marks]

(A) 1

(B) -1

(C) e

(D) 0

Correct Answer: (A) 1

Solution:

Concept:

- Continuity at the exceptional point requires the defined value $k = f(0)$ to equal the two-sided limit of the other expression as $x \to 0$.
- Use $\log(uv) = \log u + \log v$, $\lim_{u\to 0} \log(1+u)/u = 1$, and $\lim_{x\to 0} \sin x/x = 1$, with angles measured in radians.

Step 1: Simplify the numerator and denominator exactly Both logarithm arguments are positive near zero; in fact $x^2 \pm x + 1 = (x \pm 1/2)^2 + 3/4 > 0$. Their product is

$$(1 - x + x^2)(1 + x + x^2) = (1 + x^2)^2 - x^2 = 1 + x^2 + x^4.$$

The numerator is therefore $\log(1 + x^2 + x^4)$. Meanwhile,

$$\sec x - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}.$$

These transformations apply for nonzero x sufficiently close to zero.

Step 2: Factor the expression into standard limits Put $u = x^2 + x^4$. The original quotient can now be

written as

$$\frac{\log(1+u)}{u}\,(1+x^2)\left(\frac{x}{\sin x}\right)^2\cos x.$$

As $x \to 0$, we also have $u \to 0$, so the four factors tend respectively to 1, 1, 1 and 1. Thus the two-sided limit exists and equals 1. In particular, both the numerator and denominator vanish to second order; setting them separately to zero would give an indeterminate form rather than the answer.

Step 3: Apply continuity and check the options For continuity, the value placed at zero must agree with the limiting value. Hence $k = 1$. The value -1 has the wrong sign, e confuses a logarithmic limit with an exponential one, and 0 ignores the equal order of vanishing of numerator and denominator. None of these alternatives gives the computed limit.

Step 4: Conclusion The required value is $k = 1$. Hence, the correct option is (A).

Quick Tip to Solve: When a sum of logarithms and $\sec x - \cos x$ both vanish, combine the logarithms and replace the denominator by $\sin^2 x/\cos x$.

7. If $f(x) = \begin{cases} x+a & , \quad x \le 0 \\ |x-4| & , \quad x > 0 \end{cases}$ **and** $g(x) = \begin{cases} x+1 & , \quad x < 0 \\ (x-4)^2 + b & , \quad x \ge 0 \end{cases}$ **are continuous on R, then** $(gof)(2) + (fog)(-2)$ **is equal to :** [4 marks]

(A) -10

(B) 10

(C) 8

(D) -8

Correct Answer: (D) -8

Solution:

Concept:

- A piecewise function is continuous at its joining point when its left-hand limit, right-hand limit and assigned value all agree.
- Composition is evaluated from the inside out: $(g \circ f)(x) = g(f(x))$. The branch of the outer function is chosen using the inner value, not the original input.

Step 1: Determine a and b from continuity For f, the left-hand expression gives $f(0) = a$, whereas the right-hand limit is $|0-4| = 4$. Therefore $a = 4$. For g, the left-hand limit at zero is 1 and the value from its nonnegative branch is $16 + b$. Thus

$$16 + b = 1 \quad \Rightarrow \quad b = -15.$$

Each separate branch is continuous on its interval, including the absolute-value function, so matching at zero establishes the required continuity on the whole real line.

Step 2: Evaluate the first composition Start with the inner function at 2. As $2 > 0$, its absolute-value

branch applies:

$$f(2) = |2 - 4| = 2.$$

The resulting input to g is nonnegative, so use its quadratic branch with the determined value of b:

$$(g \circ f)(2) = g(2) = (2 - 4)^2 - 15 = 4 - 15 = -11.$$

The parameter a does not enter this inner evaluation, because the original input lies in the positive branch of f.

Step 3: Evaluate the second composition and add For the inner input $-2 < 0$, use the linear branch of g: $g(-2) = -2 + 1 = -1$. This is negative, so the linear branch of f now applies:

$$(f \circ g)(-2) = f(-1) = -1 + 4 = 3.$$

The sum asked for is consequently $-11 + 3 = -8$. The other listed values -10, 10 and 8 disagree with these branch-by-branch computations; in particular, reversing the sign of the total would incorrectly select 8.

Step 4: Conclusion The value of $(g \circ f)(2) + (f \circ g)(-2)$ is -8. Hence, the correct option is (D).

Quick Tip to Solve: After finding the joining constants, write each inner function value on its own line and use its sign to select the correct outer branch.

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8. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$.

Then the set of all values of b, for which f(x) has maximum value at x = 1, is : [4 marks]

(A) $(-6, -2)$

(B) $(2, 6)$

(C) $[-6, -2) \cup (2, 6]$

(D) $\left[-\sqrt{6}, -2\right) \cup \left(2, \sqrt{6}\right]$

Correct Answer: (C) $[-6, -2) \cup (2, 6]$

Solution:

Concept:

- A maximum at a joining point must dominate both pieces; continuity at that point is not a necessary condition.
- For a differentiable polynomial, a positive derivative throughout an interval makes the function strictly increasing there. The logarithm also requires a positive argument.

Step 1: Find the largest value on the left piece For $x \le 1$, differentiate the polynomial:

$$f'(x) = 3x^2 - 2x + 10 = 3\left(x - \frac{1}{3}\right)^2 + \frac{29}{3} > 0.$$

Thus this entire piece is strictly increasing, and its largest value occurs at its included right endpoint. Substitution gives $f(1) = 1 - 1 + 10 - 7 = 3$. Every point to the left therefore gives a value smaller than 3.

Step 2: Bound every value on the right piece For $x > 1$, the slope is -2, so the right-hand values decrease as x increases. Their supremum is the right-hand limit

$$-2 + \log_2(b^2 - 4).$$

They are all at most 3 precisely when this supremum is at most 3. Equality is allowed: the point $x = 1$ is excluded from this piece, so its values are then strictly below 3. Hence

$$\log_2(b^2 - 4) \leq 5.$$

Step 3: Combine the logarithmic restriction and the bound The argument must satisfy $b^2 - 4 > 0$. Since the logarithm to base 2 is increasing, the other inequality gives $b^2 - 4 \leq 32$. Combining them,

$$4 < b^2 \leq 36.$$

Both positive and negative values of b are possible because the function depends only on b^2. Options (A) and (B) each omit a whole valid interval and exclude valid outer endpoints; (D) imposes an unnecessarily small upper bound.

Step 4: Conclusion The required set is $[-6, -2) \cup (2, 6]$. Hence, the correct option is (C).

Quick Tip to Solve: At a discontinuous join, compare the included value with the supremum on the open side; do not impose continuity unnecessarily.

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9. Let $\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3$ be three mutually exclusive events such that $\mathrm{P}(\mathrm{E}_1) = \frac{2+3\mathrm{p}}{6}, \mathrm{P}(\mathrm{E}_2) = \frac{2-\mathrm{p}}{8}$ and $\mathrm{P}(\mathrm{E}_3) = \frac{1-\mathrm{p}}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(\mathrm{p}_1 + \mathrm{p}_2)$ is equal to : [4 marks]

(A) $\frac{2}{3}$

(B) $\frac{5}{3}$

(C) $\frac{5}{4}$

(D) 1

Correct Answer: (B) $\frac{5}{3}$

Solution:

Concept:

- Every probability lies between 0 and 1, and probabilities of mutually exclusive events add to the probability of their union.
- The three events need not be exhaustive, so their sum is at most 1 rather than necessarily equal to 1.

Step 1: Apply the bounds to each probability The first probability gives

$$0 \le \frac{2+3p}{6} \le 1 \quad \Longrightarrow \quad -\frac{2}{3} \le p \le \frac{4}{3}.$$

The second similarly gives $-6 \le p \le 2$, and the third gives $-1 \le p \le 1$. Their common interval is therefore $[-2/3, 1]$. These individual bounds alone do not yet use mutual exclusivity.

Step 2: Use the probability of the union Because there are no pairwise overlaps, the sum is

$$P(E_1 \cup E_2 \cup E_3) = \frac{4(2+3p) + 3(2-p) + 12(1-p)}{24} = \frac{26-3p}{24}.$$

A union cannot have probability greater than 1. Thus $26 - 3p \le 24$, giving $p \ge 2/3$. Combining with the previous interval yields $2/3 \le p \le 1$. No strict inequality is needed because zero event probabilities and a union of probability 1 are permitted.

Step 3: Add the two attainable endpoints At $p = 2/3$, the three probabilities are $2/3, 1/6, 1/6$, whose sum is 1. At $p = 1$, they are $5/6, 1/8, 0$, whose sum is $23/24$. Thus both endpoints satisfy all conditions and can represent disjoint events. Consequently $p_1 = 1$ and $p_2 = 2/3$, so their sum is $5/3$. Options (A), (C), and (D) are not this sum; in particular (A) is only the lower endpoint.

Step 4: Conclusion $p_1 + p_2 = 1 + \frac{2}{3} = \frac{5}{3}$. Hence, the correct option is (B).

Quick Tip to Solve: For disjoint events, impose the union bound after checking individual probabilities; forgetting it often gives a false lower endpoint.

10. A point P moves so that the sum of squares of its distances from the points $(1, 2)$ and $(-2, 1)$ is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral ACBD is equal to : [4 marks]

(A) $\frac{9}{2}$

(B) $\frac{3\sqrt{17}}{2}$

(C) $\frac{3\sqrt{17}}{4}$

(D) 9

Correct Answer: (B) $\frac{3\sqrt{17}}{2}$

Solution:

Concept:

- The squared distance from (x, y) to (a, b) is $(x-a)^2 + (y-b)^2$.
- A quadrilateral with perpendicular diagonals of lengths d_1, d_2 has area $\frac{1}{2}d_1 d_2$. Here the diagonals lie on the coordinate axes.

Step 1: Form the locus equation Write $P = (x, y)$. The prescribed sum of squared distances gives

$$(x-1)^2 + (y-2)^2 + (x+2)^2 + (y-1)^2 = 14.$$

Expanding and collecting terms produces

$$2x^2 + 2y^2 + 2x - 6y + 10 = 14, \qquad x^2 + y^2 + x - 3y - 2 = 0.$$

This is a circle. Its intersections with each axis can now be found by putting the other coordinate equal to zero.

Step 2: Calculate the lengths of the two diagonals On the x-axis, $y = 0$, so $x^2+x-2 = (x+2)(x-1) = 0$. The two points have coordinates $(-2, 0)$ and $(1, 0)$, and hence $AB = 3$. On the y-axis, $x = 0$, giving

$$y^2 - 3y - 2 = 0, \qquad y = \frac{3 \pm \sqrt{17}}{2}.$$

The distance between these intercepts is their coordinate difference, so $CD = \sqrt{17}$. Both pairs straddle the origin, which lies inside the quadrilateral.

Step 3: Use the perpendicular diagonals In the order ACBD, the opposite vertex pairs are A,B and C,D. Thus AB and CD are the diagonals, and they meet at a right angle at the origin. Splitting into the four triangles around that intersection, or applying the perpendicular-diagonal formula, gives

$$\text{Area} = \frac{1}{2}(3)(\sqrt{17}) = \frac{3\sqrt{17}}{2}.$$

The factor of one-half is essential. Option (C) uses an extra factor of one-half, while (A) and (D) do not use the actual length $\sqrt{17}$ of the vertical diagonal.

Step 4: Conclusion The area of ACBD is $\frac{3\sqrt{17}}{2}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: When four vertices lie in pairs on the coordinate axes, find the two intercept distances and use half their product.

11. The length of the perpendicular from the point $(1, -2, 5)$ on the line passing through $(1, 2, 4)$ and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is : [4 marks]

(A) $\sqrt{\frac{21}{2}}$

(B) $\sqrt{\frac{9}{2}}$

(C) $\sqrt{\frac{73}{2}}$

(D) 1

Correct Answer: (A) $\sqrt{\frac{21}{2}}$

Solution:

Concept:

- The line of intersection of two planes is perpendicular to both plane normals, so a direction vector is their cross product.
- If a line passes through A with direction $\mathbf{d}$, the squared perpendicular distance of P is $|\overrightarrow{AP}|^2 - (\overrightarrow{AP} \cdot \mathbf{d})^2/|\mathbf{d}|^2$.

Step 1: Find a direction vector for the line The two given planes have normals $\mathbf{n}_1 = (1, 1, -1)$ and $\mathbf{n}_2 = (1, -2, 3)$. Their intersection, and hence every parallel line, has direction

$$\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = (1, -4, -3).$$

Indeed, its dot product with each normal is zero. The constant term -5 locates the intersection but does not affect its direction. The required line uses this direction and passes through $A = (1, 2, 4)$.

Step 2: Resolve the displacement along and across the line For the given point $P = (1, -2, 5)$,

$$\mathbf{w} = \overrightarrow{AP} = (0, -4, 1), \qquad |\mathbf{w}|^2 = 17, \quad |\mathbf{d}|^2 = 26, \quad \mathbf{w} \cdot \mathbf{d} = 16 - 3 = 13.$$

The square of the component of $\mathbf{w}$ along the line is $13^2/26 = 13/2$. The perpendicular component is orthogonal to it, so the Pythagorean theorem subtracts this component from $|\mathbf{w}|^2$.

Step 3: Evaluate the perpendicular distance Consequently the required squared distance is

$$h^2 = 17 - \frac{169}{26} = 17 - \frac{13}{2} = \frac{21}{2}, \qquad h = \sqrt{\frac{21}{2}}.$$

As a check, the foot occurs at parameter $t = (\mathbf{w} \cdot \mathbf{d})/|\mathbf{d}|^2 = 1/2$, and the remaining displacement is orthogonal to $\mathbf{d}$. The other displayed distances do not have squared value $21/2$; in particular the distance is less than $|AP| = \sqrt{17}$, excluding option (C).

Step 4: Conclusion The perpendicular length is $\sqrt{\frac{21}{2}}$ units. Hence, the correct option is (A).

Quick Tip to Solve: For a line specified by two plane equations, cross the normals first; use the stated point to locate the parallel line.

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12. Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}, \alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to : [4 marks]

(A) $\frac{15}{2}$

(B) 8

(C) $\frac{13}{2}$

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- The scalar projection of $\mathbf{u}$ on $\mathbf{v}$ is $(\mathbf{u} \cdot \mathbf{v})/|\mathbf{v}|$; the direction vector must be normalised.
- Compute a cross product component by component in the stated order, because reversing that order changes its sign.

Step 1: Calculate the cross product With $\mathbf{a} = (\alpha, 1, -1)$ and $\mathbf{b} = (2, 1, -\alpha)$, expansion gives

$$\mathbf{a} \times \mathbf{b} = (1 - \alpha)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}.$$

For example, the last component is $\alpha \cdot 1 - 1 \cdot 2 = \alpha - 2$. Keeping this component and the sign of the middle component correct is necessary before taking the projection.

Step 2: Apply the scalar projection condition The target vector is $\mathbf{v} = (-1, 2, -2)$, with length $\sqrt{1 + 4 + 4} = 3$. Its dot product with the cross product is

$$\begin{aligned}(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{v} &= -(1 - \alpha) + 2(\alpha^2 - 2) - 2(\alpha - 2) \\ &= 2\alpha^2 - \alpha - 1.\end{aligned}$$

Thus the stated projection is $(2\alpha^2 - \alpha - 1)/3 = 30$. The denominator 3 distinguishes the projection from an unnormalised dot product.

Step 3: Solve and retain the positive root Multiplying by 3 and bringing all terms together gives

$$2\alpha^2 - \alpha - 91 = 0 \quad \Longrightarrow \quad (\alpha - 7)(2\alpha + 13) = 0.$$

The roots are 7 and $-13/2$. Since $\alpha > 0$, only 7 remains. Substitution verifies that the dot product is $98 - 7 - 1 = 90$, and dividing by 3 gives exactly 30. None of $15/2, 8, 13/2$ satisfies this factored equation, so options (A), (B), and (C) are excluded.

Step 4: Conclusion $\alpha = 7$. Hence, the correct option is (D).

Quick Tip to Solve: Take the cross product first, dot with the target direction, and divide by that direction vector's length before solving.

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13. If $\frac{dy}{dx} + 2y \tan x = \sin x$, $0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of $y(x)$ is : [4 marks]

(A) $\frac{1}{8}$

(B) $\frac{3}{4}$

(C) $\frac{1}{4}$

(D) $\frac{3}{8}$

Correct Answer: (A) $\frac{1}{8}$

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, the integrating factor is $e^{\int P(x)\,dx}$, turning the left side into a product derivative.
- On $0 < x < \pi/2$, $0 < \cos x < 1$. A quadratic in $\cos x$ can therefore be maximised by completing the square.

Step 1: Integrate using the appropriate factor Here $P(x) = 2\tan x$, so the integrating factor is

$$e^{\int 2\tan x\,dx} = e^{-2\log(\cos x)} = \sec^2 x.$$

Multiplying the equation by this factor gives

$$\frac{d}{dx}(y\sec^2 x) = \sin x \sec^2 x.$$

The antiderivative on the right is $\sec x$, since $d(\sec x)/dx = \sin x/\cos^2 x$. Hence $y\sec^2 x = \sec x + C$.

Step 2: Use the initial value to determine the solution At $x = \pi/3$, the given value is $y = 0$, while $\sec(\pi/3) = 2$. Therefore $0 = 2 + C$, giving $C = -2$. Multiply the integrated equation by $\cos^2 x$ to obtain

$$y(x) = \cos x - 2\cos^2 x.$$

This satisfies both the differential equation and the initial value. The domain matters when checking whether the maximum found algebraically is actually attained.

Step 3: Complete the square and check attainment Put $u = \cos x$, so $0 < u < 1$. Then

$$y = u - 2u^2 = \frac{1}{8} - 2\left(u - \frac{1}{4}\right)^2 \leq \frac{1}{8}.$$

Equality occurs at $u = 1/4$, which is inside the allowed interval. It corresponds to $x = \cos^{-1}(1/4)$, so this is an attained maximum, not merely an endpoint limit. The other options $3/4, 1/4, 3/8$ are all larger than this upper bound and therefore cannot occur as the maximum.

Step 4: Conclusion The maximum value is $\frac{1}{8}$, attained when $\cos x = 1/4$. Hence, the correct option is (A).

Quick Tip to Solve: After solving this differential equation, maximise the quadratic in $\cos x$; completing the square avoids another derivative.

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14. $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ **is equal to :** [4 marks]

(A) 1

(B) 2

(C) $\frac{1}{4}$

(D) $\frac{5}{4}$

Correct Answer: (B) 2

Solution:

Concept:

- For $\theta = \tan^{-1} t$, $\tan 2\theta = 2t/(1 - t^2)$. Tangent addition gives $\tan(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B)$.
- The principal value of $\sec^{-1}(\sqrt{5}/2)$ is acute, so its cosine is $2/\sqrt{5}$ and its tangent is $1/2$.

Step 1: Find the tangents of the component angles Let $A = 2\tan^{-1}(1/5)$, $B = \sec^{-1}(\sqrt{5}/2)$, and

$C = 2\tan^{-1}(1/8)$. The double-angle formula gives

$$\tan A = \frac{2/5}{1 - 1/25} = \frac{5}{12}, \qquad \tan C = \frac{2/8}{1 - 1/64} = \frac{16}{63}.$$

For the middle angle, $\sec^2 B - 1 = 1/4$. Its principal acute branch selects $\tan B = 1/2$, rather than the negative square root.

Step 2: Combine the two double angles first Using their rational tangents,

$$\begin{aligned}\tan(A + C) &= \frac{5/12 + 16/63}{1 - (5/12)(16/63)} \\ &= \frac{169/252}{169/189} = \frac{3}{4}.\end{aligned}$$

The denominator is nonzero, so this addition is valid. Grouping these angles first keeps the calculation small; no decimal angle approximations or inverse-function evaluations are necessary.

Step 3: Add the remaining angle and identify the option Apply the tangent addition identity once more:

$$\tan(A + B + C) = \frac{3/4 + 1/2}{1 - (3/4)(1/2)} = \frac{5/4}{5/8} = 2.$$

Here again the denominator is nonzero. The fractions $5/4$ and $5/8$ are respectively the numerator and denominator of the final addition formula, rather than its result. Thus option (D) gives only an intermediate numerator; options (A) and (C) also disagree with the exact quotient. All angles were handled on their prescribed inverse-function branches.

Step 4: Conclusion The value of the expression is 2. Hence, the correct option is (B).

Quick Tip to Solve: In sums of inverse-trigonometric angles, compute rational tangents first and combine a convenient pair before adding the last angle.

15. Let A be a 2 × 2 matrix with det (A) = − 1 and det ((A + I) (Adj (A) + I)) = 4. Then the sum of the diagonal elements of A can be : [4 marks]

(A) -1

(B) 2

(C) 1

(D) $-\sqrt{2}$

Correct Answer: (B) 2

Solution:

Concept:

- The trace of a square matrix is the sum of its diagonal entries. Write $t = \operatorname{tr} A$, so the quantity requested is t.
- For a matrix M of order two, $\det(M + I) = \det M + \operatorname{tr} M + 1$, and determinants multiply: $\det(XY) = \det X \det Y$.

Step 1: Express the adjugate using the entries Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then $\operatorname{adj} A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, whose trace is also $a + d = t$. Its determinant is $ad - bc = \det A = -1$. Thus the adjugate has the same trace and determinant as A in this order; no assumption that A is diagonal is needed.

Step 2: Apply the shifted determinant identity For either matrix, the determinant after adding the identity is its determinant plus its trace plus one. Consequently,

$$\det(A + I) = -1 + t + 1 = t, \qquad \det(\operatorname{adj} A + I) = t.$$

Substitution into the given condition yields

$$4 = \det\big((A + I)(\operatorname{adj} A + I)\big) = t \cdot t = t^2.$$

Hence the possible traces are $t = 2$ and $t = -2$.

Step 3: Compare the listed values Among the four options, only 2 is one of these possible traces. The values -1 and 1 would give determinant product 1, while $-\sqrt{2}$ would give product 2; none gives 4. Existence is also consistent: a real diagonal matrix with entries $1 + \sqrt{2}$ and $1 - \sqrt{2}$ has trace 2 and determinant -1.

Step 4: Conclusion The sum of the diagonal elements can be $\boxed{2}$. Hence, the correct option is (B).

Quick Tip to Solve: For a matrix of order two, replace each shifted determinant by determinant plus trace plus one before expanding any matrix product.

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16. If $a = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{2n}{n^2+k^2}$ and $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}$, $x \in (0, 1)$, then : [4 marks]

(A) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(B) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$

(C) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(D) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

Correct Answer: (C) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

Solution:

Concept:

- A sum of the form $\frac{1}{n}\sum_{k=1}^{n} g(k/n)$ tends to $\int_0^1 g(t)\,dt$ when g is continuous on $[0, 1]$.
- The half-angle identity gives $(1-\cos x)/(1+\cos x) = \tan^2(x/2)$. The positive square root is selected using the given domain.

Step 1: Convert the limit into an integral Divide each denominator by n^2 to expose the subinterval width $1/n$:

$$a = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{2}{1 + (k/n)^2} = \int_0^1 \frac{2}{1+t^2}\,dt.$$

The function has no singularity on this interval. Therefore

$$a = 2[\tan^{-1} t]_0^1 = \frac{\pi}{2}, \qquad \frac{a}{2} = \frac{\pi}{4}.$$

Step 2: Evaluate the function and derivative Since $0 < x < 1$, the angle $x/2$ is positive and lies in the first quadrant. Thus $f(x) = \tan(x/2)$, without a negative sign or absolute-value ambiguity. Differentiating with the chain rule,

$$f\prime(x) = \frac{1}{2}\sec^2(x/2).$$

At $x = a/2 = \pi/4$, put $u = \tan(\pi/8) = \sqrt{2} - 1$. Then

$$f(a/2) = \sqrt{2} - 1, \qquad f\prime(a/2) = \frac{1}{2}(1 + u^2) = 2 - \sqrt{2}.$$

Step 3: Test the proposed relations The two values satisfy $\sqrt{2}(\sqrt{2} - 1) = 2 - \sqrt{2}$, so $\sqrt{2}f(a/2) = f\prime(a/2)$. Option (A) has an extra factor of two. The product in (B) is $(\sqrt{2} - 1)(2 - \sqrt{2}) = 3\sqrt{2} - 4$, not $\sqrt{2}$. Option (D) reverses the required ratio and is also false. The evaluation point $\pi/4$ lies in the specified domain.

Step 4: Conclusion $\boxed{\sqrt{2}f(a/2) = f\prime(a/2)}$. Hence, the correct option is (C).

Quick Tip to Solve: First convert the limit to a Riemann integral, then simplify the square root with the half-angle identity before differentiating.

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17. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does NOT pass through the point : [4 marks]

(A) (25, 10)

(B) (20, 12)

(C) (30, 8)

(D) (15, 13)

Correct Answer: (D) (15, 13)

Solution:

Concept:

- Perpendicular nonvertical lines have slopes whose product is -1. Implicit differentiation gives the tangent slope to each conic.
- The normal at a regular point is perpendicular to the tangent there. A candidate point belongs to the normal exactly when it satisfies its line equation.

Step 1: Find the point on the parabola The line $2x + 2y = 5$ has slope -1, so the stated parabola

tangent has slope 1. Differentiate $y^2 = 24x$:

$$2y\frac{dy}{dx} = 24, \qquad \frac{dy}{dx} = \frac{12}{y}.$$

At (α, β), this gives $12/\beta = 1$, hence $\beta = 12$. Substitution in the parabola yields $144 = 24\alpha$, so $\alpha = 6$.

Step 2: Determine the hyperbola normal The hyperbola is consequently $x^2/36 - y^2/144 = 1$, and the specified point is $(10, 16)$. It does lie on the curve because $100/36 - 256/144 = 1$. Differentiating the hyperbola gives

$$\frac{2x}{36} - \frac{2y}{144}\frac{dy}{dx} = 0, \qquad \frac{dy}{dx} = \frac{4x}{y}.$$

The tangent slope at $(10, 16)$ is $5/2$, so the normal slope is $-2/5$. Its equation is

$$y - 16 = -\frac{2}{5}(x - 10), \qquad 2x + 5y = 100.$$

Step 3: Substitute all four candidate points For (A), $2(25) + 5(10) = 100$; for (B), $2(20) + 5(12) = 100$; for (C), $2(30) + 5(8) = 100$. Thus these three points lie on the normal. For (D), however, $2(15) + 5(13) = 95 \neq 100$, so this point fails the necessary line equation. The wording asks for the point the normal does not pass through, so select this last point.

Step 4: Conclusion The normal does not pass through $\boxed{(15, 13)}$. Hence, the correct option is (D).

Quick Tip to Solve: After finding a normal, put it in integer-coefficient form and test every listed point; keep track of the word NOT.

18. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $\mathrm{P}(X = 1) = \frac{4}{243}$, then $\mathrm{P}(X = 4$ or $5)$ is equal to : [4 marks]

(A) $\frac{5}{9}$
(B) $\frac{64}{81}$
(C) $\frac{16}{27}$
(D) $\frac{145}{243}$

Correct Answer: (C) $\frac{16}{27}$

Solution:

Concept:

- For $X \sim \mathrm{Bin}(n, p)$, the mean is np, variance is $np(1-p)$, and $P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$.
- The events $X = 4$ and $X = 5$ are disjoint. Their probabilities must therefore be added, rather than multiplied.

Step 1: Recover the success probability The given probability $P(X = 1) = 4/243 > 0$ ensures $np > 0$, so division of variance by mean is valid. Writing $q = 1 - p$,

$$q = \frac{npq}{np} = \frac{\alpha/3}{\alpha} = \frac{1}{3}, \qquad p = \frac{2}{3}.$$

This uses both the stated mean and variance without needing the value of α itself.

Step 2: Determine the number of trials Use the probability of exactly one success:

$$\frac{4}{243} = npq^{n-1} = n\frac{2}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{2n}{3^n}.$$

The positive integer $n = 6$ satisfies this equation, since $12/729 = 4/243$. It is unique: for positive integers, the ratio of consecutive terms $n/3^n$ is $(n+1)/(3n) < 1$, so the sequence is strictly decreasing. Thus the distribution is $\mathrm{Bin}(6, 2/3)$.

Step 3: Add the two required probabilities Both contributions have denominator $3^6 = 729$:

$$P(X = 4) = \binom{6}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 = \frac{240}{729},$$
$$P(X = 5) = \binom{6}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) = \frac{192}{729}.$$

Their sum is $432/729 = 16/27$. The options $5/9$, $64/81$, and $145/243$ are different from this exact sum; in particular, neither single term alone is the required union probability.

Step 4: Conclusion $\boxed{P(X = 4 \text{ or } 5) = 16/27}$. Hence, the correct option is (C).

Quick Tip to Solve: For binomial questions, variance divided by mean gives the failure probability immediately; use a supplied point probability to identify the trial count.

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19. Let $S = \left\{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\right\}$. **Then** $n(s) + \sum\limits_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right)$ **is equal to:** [4 marks]

(A) 0

(B) -2

(C) -4

(D) 12

Correct Answer: (C) -4

Solution:

Concept:

- For positive numbers with fixed product, AM-GM gives a lower bound for their sum, with equality only when the numbers are equal.
- The identities $\sin^2\theta + \cos^2\theta = 1$ and $\sec u \csc u = 2/\sin(2u)$ reduce the exponential condition and the sum.

Step 1: Use the fixed product to solve the condition Let $u = 8^{2\sin^2\theta}$ and $v = 8^{2\cos^2\theta}$. Both are positive, and

$$uv = 8^{2(\sin^2\theta + \cos^2\theta)} = 8^2 = 64.$$

Consequently $u + v \geq 2\sqrt{64} = 16$. The equation in the question attains exactly this lower bound, forcing $u = v = 8$. Since the base $8 > 1$, equality of the powers gives $2\sin^2\theta = 2\cos^2\theta = 1$.

Step 2: List every permitted angle On the closed interval $[0, 2\pi]$, the solutions of $\sin^2\theta = 1/2$ are

$$S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}.$$

Neither endpoint satisfies this condition, and each listed angle does. Hence $n(S) = 4$. Keeping the complete interval avoids accidentally counting only the first-quadrant solution.

Step 3: Evaluate the contribution of each angle For any listed angle, 4θ is an odd multiple of π, so $\cos(4\theta) = -1$. With $z = \pi/4 + 2\theta$,

$$\sec z \csc z = \frac{2}{\sin(2z)} = \frac{2}{\sin(\pi/2 + 4\theta)} = \frac{2}{\cos 4\theta} = -2.$$

The sine and cosine of z are both nonzero, so every summand is defined. The sum has four contributions and is therefore -8. Adding the cardinality gives $4 - 8 = -4$, which rules out 0, -2, and 12.

Step 4: Conclusion The required value is $\boxed{-4}$. Hence, the correct option is (C).

Quick Tip to Solve: When exponential terms have a fixed product and their sum equals twice its square root, use the equality condition in AM-GM instead of solving exponentials separately.

20. The statement $(\sim(\mathrm{p} \Leftrightarrow \sim \mathrm{q})) \wedge \mathrm{q}$ is : [4 marks]

(A) a tautology

(B) a contradiction

(C) equivalent to $(p \Rightarrow q) \wedge q$

(D) equivalent to $(p \Rightarrow q) \wedge p$

Correct Answer: (D) equivalent to $(p \Rightarrow q) \wedge p$

Solution:

Concept:

- A biconditional $p \Leftrightarrow q$ is true exactly when the two statements have the same truth value. Negating a biconditional reverses this condition.
- The implication $p \Rightarrow q$ is equivalent to $\neg p \vee q$. The conjunction with another statement can be simplified using distributive and complement laws.

Step 1: Simplify the negated biconditional The statement $p \Leftrightarrow \neg q$ says that p and q have opposite truth values. Its negation therefore says that p and q have the same truth value:

$$\neg(p \Leftrightarrow \neg q) \equiv (p \Leftrightarrow q).$$

Conjoining with q forces q to be true, and the biconditional then forces p to be true as well. Thus the

whole statement is equivalent to $p \wedge q$.

Step 2: Compare the two proposed equivalences For option (D), distribute the conjunction over the implication:

$$\begin{aligned}(p \Rightarrow q) \wedge p &\equiv (\neg p \vee q) \wedge p \\ &\equiv (\neg p \wedge p) \vee (q \wedge p) \\ &\equiv p \wedge q.\end{aligned}$$

This equals the original expression. In contrast, option (C) simplifies to $(\neg p \vee q) \wedge q \equiv q$, by absorption. Taking p false and q true makes (C) true but the original expression false, so (C) is not equivalent.

Step 3: Rule out a constant truth value The original expression is true when both p and q are true, so it is not a contradiction. It is false when q is false, so it is not a tautology either. These checks exclude (B) and (A), respectively, and agree with the reduced form $p \wedge q$. The final choice therefore follows from a full logical equivalence, not from one matching truth-table row.

Step 4: Conclusion The statement is equivalent to $\boxed{(p \Rightarrow q) \wedge p}$. Hence, the correct option is (D).

Quick Tip to Solve: When a conjunction already contains q, evaluate the other factor under $q =$ true; then compare with the options using implication as a disjunction.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. If for some $\mathrm{p, q, r} \in \mathbf{R}$, not all have same sign, one of the roots of the equation $\left(\mathrm{p}^2+\mathrm{q}^2\right) x^2-2\mathrm{q}(\mathrm{p}+\mathrm{r})x+\mathrm{q}^2+\mathrm{r}^2=0$ is also a root of the equation $x^2+2x-8=0$, then $\frac{\mathrm{q}^2+\mathrm{r}^2}{\mathrm{p}^2}$ is equal to _____,

[4 marks]

Solution:

Concept:

- For real quantities, a sum of two squares vanishes only when each square vanishes separately. This can give stronger information than the quadratic formula.
- A common root must satisfy both equations. Factoring the simpler equation identifies the only possible values to substitute.

Step 1: Factor the simpler equation The second equation gives

$$x^2 + 2x - 8 = (x-2)(x+4) = 0,$$

so a common root must be either 2 or -4. Before substituting these values, rewrite the first quadratic in a way that uses the reality of p, q, r. Its left-hand side is exactly

$$(px - q)^2 + (qx - r)^2,$$

because expansion produces $(p^2+q^2)x^2 - 2q(p+r)x + q^2 + r^2$, matching every coefficient.

Step 2: Use the real sum-of-squares condition At a real common root, both squares must be zero:

$$px - q = 0, \qquad qx - r = 0.$$

Hence $q = px$ and $r = qx = px^2$. The parameter p cannot be zero here: if it were zero these relations would force $q = r = 0$, incompatible with the stipulated sign condition and with a genuine quadratic equation. Thus division by p^2 is legitimate.

Step 3: Choose the root using the sign condition If $x = 2$, then $q = 2p$ and $r = 4p$, so all three nonzero parameters have the same sign, contrary to the hypothesis. Therefore the common root is $x = -4$, giving $q = -4p$ and $r = 16p$. These parameters do satisfy the mixed-sign requirement: p and r have the same sign while q has the opposite sign. Consequently

$$\frac{q^2 + r^2}{p^2} = \frac{16p^2 + 256p^2}{p^2} = 16 + 256 = 272.$$

The requested result is a dimensionless ratio, so no physical unit is attached.

Step 4: Conclusion The required numerical value is $\boxed{272}$.

Quick Tip to Solve: Look for a sum-of-squares factorization when a real quadratic has coefficients such as sums of squared parameters; use the sign restriction only after identifying its common roots.

22. The number of 5-digit natural numbers, such that the product of their digits is 36 , is _____.

[4 marks]

Solution:

Concept:

- The product is $36 = 2^2 3^2$, so no digit can be zero or contain any prime factor other than 2 and 3. The only possible digits are therefore 1, 2, 3, 4, 6 and 9.
- For a multiset of five digits with repetitions $r_1, r_2, \ldots$, the number of arrangements is $5!/(r_1! r_2! \cdots)$. Every arrangement is a five-digit natural number because none of these digits is zero.

Step 1: List all possible non-unit digit multisets Ignore the digits equal to 1 temporarily. If a 9 occurs, the remaining product is 4, which can be obtained as 4 or as $2 \cdot 2$. Without a 9, the two factors of 3 must come from two digits selected from 3 and 6. This gives the following complete list of non-unit multisets:

$$\{9, 4\}, \quad \{9, 2, 2\}, \quad \{6, 6\}, \quad \{6, 3, 2\}, \quad \{4, 3, 3\}, \quad \{3, 3, 2, 2\}.$$

No additional 2, 3 or composite digit can be included, as that would increase the product beyond the required prime exponents. Fill every unused position with a digit 1.

Step 2: Count the arrangements for each multiset The six five-digit multisets and their counts are

$$\{9,4,1,1,1\}: \frac{5!}{3!} = 20, \quad \{9,2,2,1,1\}: \frac{5!}{2!2!} = 30,$$
$$\{6,6,1,1,1\}: \frac{5!}{2!3!} = 10, \quad \{6,3,2,1,1\}: \frac{5!}{2!} = 60,$$
$$\{4,3,3,1,1\}: \frac{5!}{2!2!} = 30, \quad \{3,3,2,2,1\}: \frac{5!}{2!2!} = 30.$$

These cases cannot overlap because the digit multisets are different. Their total is $20 + 30 + 10 + 60 + 30 + 30 = 180$.

Step 3: Conclusion The required number of five-digit natural numbers is $\boxed{180}$. This is a count, so it has no physical unit.

Quick Tip to Solve: Partition the prime exponents among the non-unit digits first, then add enough 1s to reach the required number of places.

23. The number of distinct real roots of the equation $x^5\left(x^3 - x^2 - x + 1\right) + x\left(3x^3 - 4x^2 - 2x + 4\right) - 1 = 0$ is _____. [4 marks]

Solution:

Concept:

- Factoring a polynomial separates its possible zeros into simpler equations. A repeated factor changes multiplicity but does not add a distinct real root.
- A continuous, strictly increasing function changes sign at most once. If it has opposite signs at two points, the intermediate value theorem guarantees exactly one root between them.

Step 1: Factor the printed polynomial Let the left-hand side be $F(x)$. Expanding its two terms and collecting powers gives

$$F(x) = x^8 - x^7 - x^6 + x^5 + 3x^4 - 4x^3 - 2x^2 + 4x - 1.$$

Its factorisation is

$$F(x) = (x-1)^2(x+1)(x^5 + 3x - 1).$$

One can check the identity without root approximations: $(x-1)^2(x+1) = x^3 - x^2 - x + 1$; multiplying this cubic by $x^5 + 3x - 1$ gives exactly the expanded polynomial above. Thus $x = 1$ and $x = -1$ are real roots of the original equation. The repeated factor $(x-1)^2$ contributes just one distinct value.

Step 2: Count the real zeros of the remaining factor Put $g(x) = x^5 + 3x - 1$. For every real x,

$$g'(x) = 5x^4 + 3 > 0.$$

Consequently g is strictly increasing on the whole real line, so it cannot have two different real zeros. Moreover, $g(0) = -1$ and $g(1) = 3$, so continuity proves that one zero lies strictly between 0 and 1. This zero differs from both 1 and -1; directly, $g(1) = 3$ and $g(-1) = -5$, so neither known root is

repeated in this factor. There are no other factors from which a real root could arise.

Step 3: Conclusion The distinct real roots are -1, 1, and the unique zero of $x^5 + 3x - 1$ in $(0, 1)$. Hence the requested number is $\boxed{3}$, a dimensionless count.

Quick Tip to Solve: After factoring, count distinct root values rather than multiplicities; use a positive derivative to handle an otherwise awkward odd-degree factor.

24. If

$$n(2n+1)\int_0^1 (1-x^n)^{2n}\,\mathrm{d}x = 1177\int_0^1 (1-x^n)^{2n+1}\,\mathrm{d}x,$$

then $n \in \mathbf{N}$ is equal to _____. [4 marks]

Solution:

Concept:

- Integration by parts can relate two integrals whose positive integer powers of $1 - x^n$ differ by one, without evaluating either integral explicitly.
- For $n \in \mathbf{N}$, the integrands are nonnegative on $[0, 1]$ and positive in its interior. Their integrals are positive, which permits cancellation when forming the required ratio.

Step 1: Obtain a recurrence between the two integrals Denote

$$A = \int_0^1 (1-x^n)^{2n}\,\mathrm{d}x, \qquad B = \int_0^1 (1-x^n)^{2n+1}\,\mathrm{d}x.$$

Apply integration by parts to B, taking $u = (1 - x^n)^{2n+1}$ and $\mathrm{d}v = \mathrm{d}x$. Then $v = x$ and

$$\mathrm{d}u = -n(2n+1)x^{n-1}(1-x^n)^{2n}\,\mathrm{d}x.$$

The boundary term $[x(1 - x^n)^{2n+1}]_0^1$ vanishes at both endpoints. Therefore

$$B = n(2n+1)\int_0^1 x^n(1-x^n)^{2n}\,\mathrm{d}x.$$

Since $x^n = 1 - (1 - x^n)$, the integral on the right is exactly $A - B$.

Step 2: Use the given coefficient and solve for the natural number The recurrence becomes

$$n(2n+1)A = \{n(2n+1)+1\}B.$$

But the question gives $n(2n + 1)A = 1177B$. Since $B > 0$, comparing these equalities gives

$$\begin{aligned} n(2n+1)+1 &= 1177, \\ 2n^2 + n - 1176 &= 0, \\ (n-24)(2n+49) &= 0. \end{aligned}$$

The candidates are 24 and $-49/2$. The second is negative and is not a natural number, so only the

first satisfies the stated domain. As a check, $24(49)+1=1177$, exactly the printed coefficient.

Step 3: Conclusion The required natural number is $\boxed{n=24}$. This parameter is dimensionless.

Quick Tip to Solve: For integrals involving adjacent powers, integrate the derivative of $x(1-x^n)^{m+1}$; the endpoint term often disappears.

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25. The equations of the sides AB, BC and CA of a triangle ABC are $2x+y=0, x+\mathrm{p}y=15\mathrm{a}$ and $x-y=3$ respectively. If its orthocentre is $(2,a), -\frac{1}{2}<\mathrm{a}<2$, then p is equal to _____.

[4 marks]

Solution:

Concept:

- The orthocentre lies on each altitude. Thus the line joining a vertex to the orthocentre is perpendicular to the opposite side.
- Perpendicular nonvertical lines have slopes whose product is -1. The coordinates of a vertex must satisfy both side equations through that vertex.

Step 1: Express vertices B and C using the orthocentre The side CA has equation $y=x-3$ and slope 1. Hence the altitude through B and $H=(2,a)$ has slope -1:

$$y-a=-(x-2), \qquad x+y=a+2.$$

Intersecting it with $AB: y=-2x$ gives

$$B=(-a-2,\ 2a+4).$$

Similarly, AB has slope -2, so the altitude through C has slope $1/2$:

$$y-a=\frac{x-2}{2}.$$

Combining this with $CA: y=x-3$ gives $x=2a+4$, and therefore

$$C=(2a+4,\ 2a+1).$$

Both coordinates were obtained using actual altitude conditions, so they incorporate the orthocentre information.

Step 2: Impose the common equation of side BC Both vertices belong to $BC: x+py=15a$. Substitution gives

$$p(2a+4)=16a+2,$$
$$p(2a+1)=13a-4.$$

Eliminate p by cross-multiplication, which does not require division by $2a+1$:

$$(16a+2)(2a+1) = (13a-4)(2a+4),$$
$$6a^2 - 24a + 18 = 0,$$
$$(a-1)(a-3) = 0.$$

The condition $-1/2 < a < 2$ excludes $a = 3$ and retains $a = 1$. Then the first equation gives $6p = 18$, so $p = 3$. For verification, the vertices are $B = (-3, 6)$, $C = (6, 3)$, both on $x + 3y = 15$.

Step 3: Conclusion The required value is $\boxed{p = 3}$, a dimensionless coefficient. The corresponding orthocentre is $(2, 1)$, which satisfies the given bound.

Quick Tip to Solve: Find a vertex by intersecting its own side with the altitude from the known orthocentre; this avoids solving all three vertices first.

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26. Let the function $f(x) = 2x^2 - \log_e x, x > 0$, be decreasing in $(0, a)$ and increasing in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, 8a-1)$ but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is : $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to _____.

[4 marks]

Solution:

Concept:

- A differentiable function decreases where its derivative is negative and increases where its derivative is positive. The transition point determines the parameter a.
- For $y^2 = 4ax$, a point with parameter t is $(at^2, 2at)$. Its tangent is $ty = x + at^2$, and its normal is $y = -tx + 2at + at^3$.

Step 1: Find the parameter from the monotonicity change Differentiate on the given domain $x > 0$:

$$f\prime(x) = 4x - \frac{1}{x} = \frac{4x^2 - 1}{x}.$$

The denominator is positive, so the derivative is negative for $0 < x < 1/2$ and positive for $x > 1/2$. Thus the specified transition is $a = 1/2$. The point through which the tangent passes is consequently $(8a, 8a-1) = (4, 3)$, and the point it must avoid is $(-1/a, 0) = (-2, 0)$.

Step 2: Select the permissible tangent parameter Substitute $(4, 3)$ in $ty = x + at^2$:

$$3t = 4 + \frac{t^2}{2}, \qquad t^2 - 6t + 8 = 0.$$

Hence $t = 2$ or $t = 4$. At $(-2, 0)$, the tangent equation would require $0 = -2 + t^2/2$. This is true for $t = 2$, so that tangent is excluded. For $t = 4$, its right-hand side is 6, so it does not pass through the forbidden point. Therefore the required point is $P = (at^2, 2at) = (8, 4)$.

Step 3: Write the normal in intercept form Using $a = 1/2$ and $t = 4$ in the normal equation gives

$$y = -4x + 2\left(\frac{1}{2}\right)4 + \frac{1}{2}(4^3) = -4x + 36.$$

Thus $4x + y = 36$, or

$$\frac{x}{9} + \frac{y}{36} = 1.$$

The line passes through $(8, 4)$, and its slope -4 is perpendicular to the tangent slope $1/4$, confirming both the point and the normal direction.

Step 4: Conclusion The intercepts are $\alpha = 9$ and $\beta = 36$, so $\boxed{\alpha + \beta = 45}$ in the stated coordinate units.

Quick Tip to Solve: Determine the parabola parameter from the derivative first, then apply the forbidden-point condition before calculating the normal.

27. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point $P(4, 2, 7)$. Then the square of the area of the triangle PQR is _____. [4 marks]

Solution:

Concept:

- A point on a symmetric line is written with a common parameter. Substituting it in a distance equation locates the two specified points.
- If two vertices Q, R are equally distant from P, the segment from P to the midpoint of QR is perpendicular to QR. The area is then one half of the base times this perpendicular height.

Step 1: Find the two line parameters Let the common ratio in the line equation be t. An arbitrary point is

$$X(t) = (2t - 1,\ 3t - 2,\ 2t + 1).$$

Its squared distance from $P = (4, 2, 7)$ is

$$\begin{aligned} PX^2 &= (2t - 5)^2 + (3t - 4)^2 + (2t - 6)^2 \\ &= 17t^2 - 68t + 77. \end{aligned}$$

The required distance is $\sqrt{26}$, so set this expression equal to 26:

$$17t^2 - 68t + 51 = 0, \qquad 17(t - 1)(t - 3) = 0.$$

Thus we may take $Q = X(1) = (1, 1, 3)$ and $R = X(3) = (5, 7, 7)$. Interchanging their names does not change the area.

Step 2: Calculate the base, perpendicular height and squared area The base vector is $\overrightarrow{QR} = (4, 6, 4)$, giving $QR^2 = 16 + 36 + 16 = 68$. Its midpoint is $M = (3, 4, 5)$, so

$$\overrightarrow{MP} = (1, -2, 2), \qquad PM^2 = 1 + 4 + 4 = 9.$$

The dot product $(1,-2,2)\cdot(4,6,4)=4-12+8=0$ explicitly confirms that $PM\perp QR$; thus this is the triangle height, not merely a median. Therefore

$$[PQR]^2=\left(\frac{1}{2}QR\cdot PM\right)^2=\frac{1}{4}(68)(9)=153.$$

As a distance check, $PQ^2=9+1+16=26$ and $PR^2=1+25+0=26$, agreeing with the given equal lengths.

Step 3: Conclusion The square of the area of triangle PQR is $\boxed{153}$ in fourth-power coordinate-length units (the area itself is $\sqrt{153}$ square units).

Quick Tip to Solve: When two points on a line have the same distance from a third point, use the chord midpoint as the foot of the altitude.

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28. The series of positive multiples of 3 is divided into sets : $\{3\},\{6,9,12\},\{15,18,21,24,27\},\ldots$ Then the sum of the elements in the 11^{th} set is equal to _____. [4 marks]

Solution:

Concept:

- The displayed set sizes are the successive odd numbers $1,3,5,\ldots$. The total of the first k odd numbers is k^2.
- The j-th positive multiple of 3 is $3j$. The sum of a finite arithmetic progression is its number of terms multiplied by the average of its first and last terms.

Step 1: Locate the indices belonging to the eleventh set The first set has 1 element, the second has 3, and the third has 5. Thus set k contains $2k-1$ consecutive elements of the sequence $3,6,9,\ldots$. Before the eleventh set, the first ten sets have used

$$1+3+5+\cdots+19=10^2=100$$

elements. The eleventh set contains $2(11)-1=21$ elements, and the cumulative number through it is $11^2=121$. Therefore it consists precisely of the multiples of 3 with indices $101,102,\ldots,121$. This indexing follows the displayed consecutive partition; no multiple is omitted or repeated.

Step 2: Sum the resulting arithmetic progression The first element of the set is $3\times101=303$, and the last is $3\times121=363$. The common difference remains 3, so the number of terms is also confirmed by $(363-303)/3+1=21$. Hence the sum is

$$S_{11}=\frac{21}{2}(303+363)=21\times333=6993.$$

Alternatively, summing indices first gives $101+\cdots+121=21(111)=2331$, and multiplying by 3 again gives 6993. In general the k-th set starts at $3((k-1)^2+1)$ and ends at $3k^2$, making it essential to use square cumulative counts rather than the set number as a sequence index.

Step 3: Conclusion The required sum of the elements in the eleventh set is $\boxed{6993}$. This sum of natural numbers is dimensionless.

Quick Tip to Solve: Odd-sized consecutive groups end at square-number indices: the eleventh group ends at index 121 and begins immediately after index 100.

29. If the coefficients of x and x^2 in the expansion of $(1+x)^p(1-x)^q, p, q \leq 15$, are -3 and -5 respectively, then the coefficient of x^3 is equal to _____. [4 marks]

Solution:

Concept:

- For $f(x) = (1+x)^p(1-x)^q$, logarithmic differentiation gives $(1-x^2)f\prime(x) = [(p-q)-(p+q)x]f(x)$.
- Equating coefficients in this polynomial identity determines successive coefficients without expanding all powers. Write $f(x) = 1 + a_1x + a_2x^2 + a_3x^3 + \cdots$.

Step 1: Use the two supplied coefficients The constant term of the derivative identity gives $a_1 = p - q = -3$. Its coefficient of x gives $2a_2 = (p-q)a_1 - (p+q)$. Substituting $a_1 = -3$ and $a_2 = -5$, we obtain

$$-10 = 9 - (p+q), \qquad p+q = 19.$$

Solving the sum and difference equations gives $p = 8$ and $q = 11$, both satisfying the stated upper bound of 15.

Step 2: Extract the cubic coefficient The coefficient of x^2 on the left is $3a_3 - a_1$, since the factor $-x^2$ multiplies the constant derivative coefficient a_1. On the right it is $(p-q)a_2 - (p+q)a_1$. Hence

$$3a_3 + 3 = (-3)(-5) - 19(-3) = 72, \qquad a_3 = 23.$$

As a direct check, the cubic coefficient in $(1+x)^8(1-x)^{11}$ is $\binom{8}{3} - \binom{8}{2}\binom{11}{1} + \binom{8}{1}\binom{11}{2} - \binom{11}{3} = 56 - 308 + 440 - 165 = 23$. This independent expansion confirms both the signs and the index of the requested term.

Step 3: Conclusion The coefficient of x^3 is $\boxed{23}$, a dimensionless number.

Quick Tip to Solve: Use the logarithmic derivative to obtain a short coefficient recurrence; this avoids a full binomial expansion.

30. Let a curve $y = y(x)$ pass through the point $(3,3)$ and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x(>3)$ be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to _____. [4 marks]

Solution:

Concept:

- The area function $A(x) = \int_3^x y(t)\,dt$ satisfies $A\prime(x) = y(x)$ and $A(3) = 0$ for a continuous curve.
- Differentiating an area equation loses an additive constant. The resulting differential equation must therefore be checked against the original integral condition.

Step 1: Differentiate the given relation Formally set $u = y/x$. Differentiating $A = u^3$ for $x > 3$ gives $y = 3u^2u\prime$. Since $y = xu$ and the positive branch has $u > 0$, this becomes $uu\prime = x/3$. Integration

yields

$$u^2 = \frac{x^2}{3} + C.$$

Using $y(3) = 3$, hence $u(3) = 1$, gives $C = -2$, so the formal positive solution obeys $y^2 = x^4/3 - 2x^2$.

Step 2: Find the formal numerical value and check the area At $y = 6\sqrt{10}$, put $z = x^2$. Then

$$360 = \frac{z^2}{3} - 2z, \qquad z^2 - 6z - 1080 = (z - 36)(z + 30) = 0.$$

The first-quadrant root is $x = 6$. However, integrating $y = 3u^2u'$ from 3 to x gives $A(x) = u(x)^3 - u(3)^3 = (y/x)^3 - 1$, not the printed $(y/x)^3$. Indeed as $x \to 3^+$, the actual area tends to zero whereas the printed right side tends to one. Thus the original area condition and the stated starting point are incompatible for a continuous curve.

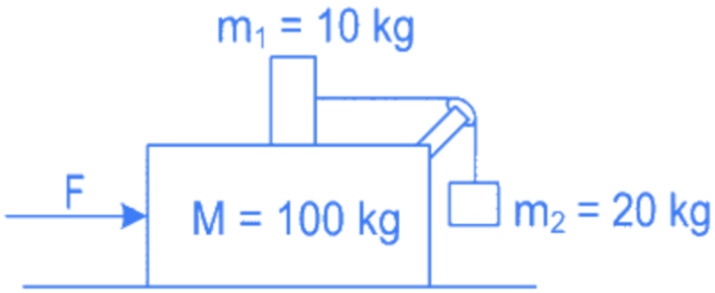
Step 3: Conclusion The formal differentiated calculation gives the answer-key value $\alpha = \boxed{6}$ (dimensionless). Literally, no continuous curve satisfies both printed conditions: the area relation needs an additive constant of -1.

Quick Tip to Solve: After differentiating an area relation, substitute the lower limit back into the integral condition to check its constant.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Three masses $M = 100\ \text{kg},\ m_1 = 10\ \text{kg}$ and $m_2 = 20\ \text{kg}$ are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of $2\ \text{ms}^{-2}$. The value of F is :

(Take $g = 10\ \text{ms}^{-2}$) [4 marks]



(A) 3360 N

(B) 3380 N

(C) 3120 N

(D) 3240 N

Correct Answer: (A) 3360 N

Solution:

Concept:

- A light string over a frictionless pulley has the same tension T throughout. Apply Newton's second law to each moving mass in the ground frame.
- The string length constrains relative motion: when the hanging block rises, the horizontal separation of the top block from the pulley increases by the same amount.

Step 1: Find the string tension and top-block acceleration For the hanging mass, the vertical forces are upward tension and downward weight. Its specified upward acceleration is $2\,\text{m}\,\text{s}^{-2}$, so

$$T - m_2 g = m_2(2), \qquad T = 20(10+2) = 240\,\text{N}.$$

The only horizontal force on m_1 is tension. Therefore its ground-frame horizontal acceleration is $a_1 = T/m_1 = 240/10 = 24\,\text{m}\,\text{s}^{-2}$ to the right. Its acceleration is not equal to that of the large block.

Step 2: Use the length constraint and horizontal momentum balance Let A be the large block's acceleration to the right. The pulley is attached to it. Constant string length gives $A - a_1 = 2$, because the downward string segment shortens as m_2 rises. Thus $A = 26\,\text{m}\,\text{s}^{-2}$. The hanging mass remains against the smooth vertical side and shares this horizontal acceleration. For the entire system, tension and contact forces are internal. Its horizontal force balance is

$$F = (M + m_2)A + m_1 a_1 = 120(26) + 10(24) = 3360\,\text{N}.$$

The other options, 3380 N, 3120 N and 3240 N, do not equal this total. In particular 3120 N omits the horizontal momentum change of the upper block.

Step 3: Conclusion The required applied force is $\boxed{3360\,\mathrm{N}}$ to the right. Hence, the correct option is (A).

Quick Tip to Solve: Apply the string constraint to motion relative to the moving pulley, then sum horizontal momentum changes in the ground frame.

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32. A parallel beam of light of wavelength $900\ \mathrm{nm}$ and intensity $100\,\mathrm{W m^{-2}}$ is incident on a surface perpendicular to the beam. The number of photons crossing $1\ \mathrm{cm^2}$ area perpendicular to the beam in one second is : [4 marks]

(A) 3×10^{16}

(B) 4.5×10^{16}

(C) 4.5×10^{17}

(D) 4.5×10^{20}

Correct Answer: (B) 4.5×10^{16}

Solution:

Concept:

- Intensity I is incident energy per unit area per unit time. Through perpendicular area A in time t, the beam supplies energy IAt.
- A photon of wavelength λ has energy $E_\gamma = hc/\lambda$. Dividing the total energy by this energy gives the number of photons.

Step 1: Convert the area and determine the incident energy The wavelength must be expressed in metres: $\lambda = 900 \times 10^{-9} = 9 \times 10^{-7}\,\mathrm{m}$. The illuminated area is $1\,\mathrm{cm^2} = 10^{-4}\,\mathrm{m^2}$; the square is essential when converting centimetres to metres. During one second, the energy crossing it is

$$E = IAt = 100 \times 10^{-4} \times 1 = 10^{-2}\,\mathrm{J}.$$

Since the area is perpendicular to the beam, its projected area equals its actual area and there is no additional cosine factor.

Step 2: Divide by the energy of one photon With $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$ and $c = 3 \times 10^{8}\,\mathrm{m\,s^{-1}}$,

$$E_\gamma = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{9 \times 10^{-7}} \simeq 2.21 \times 10^{-19}\,\mathrm{J}.$$

Thus

$$N = \frac{10^{-2}}{2.21 \times 10^{-19}} \simeq 4.53 \times 10^{16}.$$

At the precision of the options this is 4.5×10^{16}. Option A has the wrong numerical factor; options C and D are too large by factors of approximately ten and ten thousand. The count is dimensionless, while its rate for this area is the same numerical value in photons per second.

Step 3: Conclusion About $\boxed{4.5 \times 10^{16}}$ photons cross the specified area in one second. Hence, the correct option is (B).

Quick Tip to Solve: Write $N = IAt\lambda/(hc)$ and convert the area before substituting; a square-centimetre conversion affects the answer by four powers of ten.

33. In Young's double slit experiment, the fringe width is 12 mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm): [4 marks]

(A) 16

(B) 9

(C) 48

(D) 12

Correct Answer: (B) 9

Solution:

Concept:

- Fringe width is $\beta = \lambda D/d$, where D is the slit-to-screen distance and d is the slit separation.
- In a uniform medium of refractive index μ, the frequency is unchanged, but the wavelength becomes λ/μ. The geometry remains unchanged when the entire arrangement is immersed.

Step 1: Relate the new fringe width to the old width Let λ_0 denote the wavelength in air. In water the wave speed is c/μ, while its frequency stays fixed by the source. Therefore $\lambda_w = \lambda_0/\mu$. Applying the fringe-width formula to the two situations gives

$$\beta_0 = \frac{\lambda_0 D}{d}, \qquad \beta_w = \frac{\lambda_0 D}{\mu d} = \frac{\beta_0}{\mu}.$$

There is no change in the physical slit spacing or the screen distance, so only the wavelength contributes to the ratio.

Step 2: Substitute and compare all options For $\beta_0 = 12\,\text{mm}$ and $\mu = 4/3$,

$$\beta_w = \frac{12}{4/3} = 12 \times \frac{3}{4} = 9\,\text{mm}.$$

A denser medium reduces the wavelength and hence narrows the fringes. This rules out 16 mm in option A and 48 mm in option C, both of which increase the spacing. The unchanged value 12 mm in option D would require the wavelength in the medium to equal the air wavelength, contrary to the given refractive index. The ratio calculation also shows that numerical values of D, d, and the original wavelength are unnecessary.

Step 3: Conclusion The fringe width in water is $\boxed{9\,\text{mm}}$. Hence, the correct option is (B).

Quick Tip to Solve: For complete immersion in a single medium, divide the air fringe width by the refractive index; the fringe pattern becomes narrower.

34. The magnetic field of a plane electromagnetic wave is given by :

$\vec{B} = 2 \times 10^{-8} \sin\left(0.5 \times 10^3 x + 1.5 \times 10^{11} t\right) \hat{j}$ T.

The amplitude of the electric field would be : [4 marks]

(A) $6\,\mathrm{Vm}^{-1}$ along x-axis

(B) $3\,\mathrm{Vm}^{-1}$ along z-axis

(C) $6\,\mathrm{Vm}^{-1}$ along z-axis

(D) $2 \times 10^{-8}\,\mathrm{Vm}^{-1}$ along z-axis

Correct Answer: (C) $6\,\mathrm{Vm}^{-1}$ along z-axis

Solution:

Concept:

- For a plane wave, the propagation speed is $v = \omega/k$, where ω is the angular frequency and k the wave number.
- The field amplitudes satisfy $E_0 = vB_0$. The electric field, magnetic field and propagation direction are mutually perpendicular, with propagation along $\vec{E} \times \vec{B}$.

Step 1: Read the wave speed and calculate the electric amplitude Compare the phase $0.5 \times 10^3 x + 1.5 \times 10^{11} t$ with $kx + \omega t$. It gives $k = 0.5 \times 10^3\,\mathrm{m}^{-1}$ and $\omega = 1.5 \times 10^{11}\,\mathrm{s}^{-1}$. Consequently,

$$v = \frac{\omega}{k} = \frac{1.5 \times 10^{11}}{0.5 \times 10^3} = 3 \times 10^8\,\mathrm{m\,s}^{-1}.$$

The magnetic amplitude is $B_0 = 2 \times 10^{-8}\,\mathrm{T}$, so $E_0 = vB_0 = (3 \times 10^8)(2 \times 10^{-8}) = 6\,\mathrm{V\,m}^{-1}$.

Step 2: Determine the electric-field axis A constant phase $kx + \omega t$ moves with $dx/dt = -\omega/k$, so the wave propagates along the negative x-direction. Its magnetic field lies along the y-axis. The electric field therefore lies along the z-axis, since $\hat{k} \times \hat{j} = -\hat{i}$. In particular, the electric field may be written $6\sin(kx + \omega t)\hat{k}\,\mathrm{V\,m}^{-1}$. Option A wrongly places it along the propagation axis. Option B has half the required amplitude, while option D repeats the magnetic amplitude numerically and neglects the speed factor.

Step 3: Conclusion The electric-field amplitude is $\boxed{6\,\mathrm{V\,m}^{-1}}$ along the z-axis. Hence, the correct option is (C).

Quick Tip to Solve: Use the phase to find both the speed and propagation direction; then enforce $\vec{E} \times \vec{B}$ along that direction.

35. In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. The ratio $\frac{P_1}{P_2}$ is: [4 marks]

(A) $\frac{1}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{\sqrt{2}}$

(D) 2 : 1

Correct Answer: (B) $\frac{1}{\sqrt{2}}$

Solution:

Concept:

- The power factor of a series circuit is $\cos\phi = R/Z$, where Z is the impedance magnitude and R the resistance.
- For a series RLC circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Equal inductive and capacitive reactances cancel, giving series resonance and unit power factor.

Step 1: Evaluate the power factor of the original LR circuit Before the capacitor is connected, the series impedance is $Z_1 = \sqrt{R^2 + X_L^2}$. Using the given equality $X_L = R$,

$$Z_1 = \sqrt{R^2 + R^2} = \sqrt{2}R, \qquad P_1 = \frac{R}{Z_1} = \frac{1}{\sqrt{2}}.$$

Equivalently, $\tan\phi_1 = X_L/R = 1$, so the current lags the source voltage by 45°. Although a coil stores and returns energy each cycle, only the resistive part contributes to average power.

Step 2: Evaluate the resonant power factor and take the ratio After the specified capacitor is inserted, its reactance equals the inductive reactance: $X_C = X_L$. Thus the net series reactance vanishes even though each reactive element still has a nonzero voltage across it. The new impedance is

$$Z_2 = \sqrt{R^2 + (X_L - X_C)^2} = R, \qquad P_2 = \frac{R}{Z_2} = 1.$$

Therefore $P_1/P_2 = 1/\sqrt{2}$. Option A gives the square of the correct original power factor, not the factor itself. Options C and D exceed unity; they cannot be this ratio because its denominator is one and the original power factor cannot exceed one.

Step 3: Conclusion The requested ratio is $\boxed{P_1/P_2 = 1/\sqrt{2}}$. Hence, the correct option is (B).

Quick Tip to Solve: At series resonance substitute $X_L - X_C = 0$ into impedance before calculating power factor; the reactive voltages cancel as phasors.

36. A charge particle is moving in a uniform magnetic field $(2\hat{i} + 3\hat{j})\,\mathrm{T}$. If it has an acceleration of $(\alpha\hat{i} - 4\hat{j})\,\mathrm{m/s^2}$, then the value of α will be : [4 marks]

(A) 3

(B) 6

(C) 12

(D) 2

Correct Answer: (B) 6

Solution:

Concept:

- The magnetic force on a charge is $\vec{F} = q\vec{v} \times \vec{B}$. The cross product is perpendicular to the magnetic field, whatever the sign of the charge or its velocity.
- Using Newton's second law, $m\vec{a} = \vec{F}$, the magnetic acceleration therefore obeys $\vec{a} \cdot \vec{B} = 0$. This gives a scalar condition without needing the speed or charge-to-mass ratio.

Step 1: Apply perpendicularity The field and acceleration are given in the same Cartesian basis. Their components are $B_x = 2$, $B_y = 3$, $a_x = \alpha$ and $a_y = -4$. There are no stated z-components. Because the force is magnetic, its direction cannot have a component parallel to the field. Thus the dot product must vanish:

$$(\alpha\hat{i} - 4\hat{j}) \cdot (2\hat{i} + 3\hat{j}) = 0.$$

The cross terms vanish because $\hat{i} \cdot \hat{j} = 0$, while each unit vector has unit squared magnitude.

Step 2: Solve the component relation Expanding the dot product gives

$$2\alpha + 3(-4) = 0, \qquad 2\alpha = 12, \qquad \alpha = 6.$$

As a check, the acceleration $6\hat{i} - 4\hat{j}$ gives $12 - 12 = 0$, as required. The alternatives 3, 12 and 2 give dot products -6, 12 and -8, respectively, so none is perpendicular to the specified field. No magnitude equation such as $qvB = ma$ is needed; that equation would introduce unknown quantities unnecessarily.

Step 3: Conclusion The required coefficient is $\boxed{\alpha = 6}$, with the acceleration components expressed in $\mathrm{m\,s^{-2}}$. Hence, the correct option is (B).

Quick Tip to Solve: For acceleration caused only by a magnetic field, set $\vec{a} \cdot \vec{B} = 0$ before using a force-magnitude formula.

37. B_X **and** B_Y **are the magnetic fields at the centre of two coils** X **and** Y **respectively each carrying equal current. If coil** X **has 200 turns and** 20 cm **radius and coil** Y **has 400 turns and** 20 cm **radius, the ratio of** B_X **and** B_Y **is :** [4 marks]

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 4 : 1

Correct Answer: (B) 1 : 2

Solution:

Concept:

- A circular coil of N turns and radius R, carrying current I, produces a central field of magnitude $B = \mu_0 NI/(2R)$. The fields of its turns add.
- For two coils in the same medium, $B_X/B_Y = (N_X/N_Y)(I_X/I_Y)(R_Y/R_X)$. Equal currents and radii leave only the turn ratio.

Step 1: Write the fields for both coils Use the radius, not the diameter, in the circular-coil formula. Both radii are $20\,\text{cm} = 0.20\,\text{m}$. Let the common current be I. The central fields are therefore

$$B_X = \frac{\mu_0(200)I}{2(0.20)}, \qquad B_Y = \frac{\mu_0(400)I}{2(0.20)}.$$

Each additional turn contributes the same field at the centre, so doubling the number of turns doubles the total field when the current and radius remain fixed.

Step 2: Take the ratio and check the options Dividing the two expressions cancels μ_0, the common current, the factor 2 and the common radius:

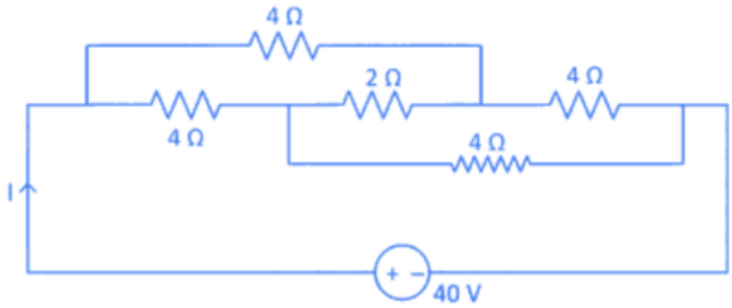
$$\frac{B_X}{B_Y} = \frac{200}{400} = \frac{1}{2}.$$

Thus coil Y produces twice the field of coil X, not the reverse. A ratio of 1:1 would ignore the different numbers of turns. A ratio of 2:1 reverses the requested order X:Y, and 4:1 would incorrectly square the turn ratio as well as reverse it. Magnetic field is linear in the number of turns, so such a squared dependence has no basis here.

Step 3: Conclusion The magnetic-field ratio is $\boxed{B_X : B_Y = 1 : 2}$. Hence, the correct option is (B).

Quick Tip to Solve: In central-field comparisons, cancel equal currents and radii first; the coil with more turns has the larger field in direct proportion.

38. The current I in the given circuit will be : [4 marks]



(A) 10 A

(B) 20 A

(C) 4 A

(D) 40 A

Correct Answer: (A) 10 A

Solution:

Concept:

- In a balanced Wheatstone bridge, the two internal junctions have equal potential. Consequently the resistor joining them carries zero current, even though its resistance is finite.
- Resistances in series add, and the reciprocal of the equivalent resistance of parallel branches equals the sum of their reciprocals. The source current is $I = V/R_{\text{eq}}$.

Step 1: Identify the balanced bridge Call the left and right supply junctions L and R. Call the junction between the left $4\,\Omega$ and central $2\,\Omega$ resistors M, and the junction between the central $2\,\Omega$ and right $4\,\Omega$ resistors N. The upper $4\,\Omega$ arm connects L to N, while the lower $4\,\Omega$ arm connects M to R. Hence each of the two paths L-M-R and L-N-R has two equal $4\,\Omega$ arms. Their division ratios are equal:

$$\frac{4}{4} = \frac{4}{4}.$$

Therefore M and N are at the same potential and no current flows through the $2\,\Omega$ bridge resistor.

Step 2: Reduce the network and find the source current For calculating current drawn from the source, omit the zero-current bridge arm. The remaining branches each have resistance $4 + 4 = 8\,\Omega$, and these two branches are in parallel:

$$R_{\text{eq}} = \frac{8 \times 8}{8 + 8} = 4\,\Omega.$$

The battery is $40\,\text{V}$, so

$$I = \frac{40}{4} = 10\,\text{A}.$$

Each branch carries $5\,\text{A}$, consistent with a $20\,\text{V}$ drop across each $4\,\Omega$ resistor. The other proposed currents 20, 4 and 40 A would require equivalent resistances 2, 10 and $1\,\Omega$, respectively, which this balanced network does not have.

Step 3: Conclusion The current supplied by the battery is $\boxed{I = 10\,\text{A}}$. Hence, the correct option is (A).

Quick Tip to Solve: Before combining resistors in a bridge circuit, compare the arm ratios; a balanced bridge removes its central resistor from the current calculation.

39. The total charge on the system of capacitors $C_1 = 1\mu\text{F}, C_2 = 2\mu\text{F}, \text{C}_3 = 4\mu\text{F}$ and $\text{C}_4 = 3\mu\text{F}$ connected in parallel is :

(Assume a battery of 20 V is connected to the combination) [4 marks]

(A) $200\,\mu\text{C}$

(B) 200 C

(C) $10\,\mu\text{C}$

(D) 10 C

Correct Answer: (A) $200\,\mu\text{C}$

Solution:

Concept:

- Capacitors connected in parallel have the same potential difference across them. Their equivalent capacitance is the sum $C_{\text{eq}} = C_1 + C_2 + C_3 + C_4$.
- The charge delivered by the battery to one side of the capacitor combination is $Q_{\text{total}} = C_{\text{eq}}V = \sum_i C_i V$. The algebraic charge of both plates together is zero; the requested stored charge uses the usual magnitude convention.

Step 1: Add the parallel capacitances All four capacitors are connected across the same $20\,\text{V}$ battery. There is no series voltage division. Therefore

$$C_{\text{eq}} = (1 + 2 + 4 + 3)\,\mu\text{F} = 10\,\mu\text{F}.$$

Keeping the capacitances in microfarads is convenient because multiplying microfarads by volts gives microcoulombs directly. In SI units the same equivalent capacitance is $10 \times 10^{-6}\,\text{F}$.

Step 2: Calculate and verify the stored charge Using the full battery voltage across the equivalent capacitor gives

$$Q_{\text{total}} = C_{\text{eq}}V = (10\,\mu\text{F})(20\,\text{V}) = 200\,\mu\text{C}.$$

Individually the four capacitor charges are 20, 40, 80 and $60\,\mu\text{C}$, which add to the same result. Thus the combination stores $2.0 \times 10^{-4}\,\text{C}$ in the conventional sense. The value 200 C misses the micro prefix by a factor of a million. The values $10\,\mu\text{C}$ and 10 C do not include the required multiplication by the battery voltage and therefore cannot represent the total charge.

Step 3: Conclusion The total stored charge is $\boxed{200\,\mu\text{C}}$. Hence, the correct option is (A).

Quick Tip to Solve: For a parallel capacitor network, add capacitances first and then multiply by the common voltage; $\mu\text{F} \times \text{V} = \mu\text{C}$.

40. When a particle executes Simple Hormonic Motion, the nature of graph of velocity as a function of displacement will be : [4 marks]

(A) Circular

(B) Elliptical

(C) Sinusoidal

(D) Straight line

Correct Answer: (B) Elliptical

Solution:

Concept:

- For simple harmonic motion of amplitude A and angular frequency ω, the displacement and velocity can be written as $x = A\cos\theta$ and $v = -A\omega\sin\theta$, where $\theta = \omega t + \phi$.
- Eliminating the phase uses $\sin^2\theta + \cos^2\theta = 1$. This relates the two instantaneous variables directly, rather than describing either of them as a function of time.

Step 1: Eliminate time from the SHM equations Divide the displacement expression by its amplitude and the velocity expression by the maximum speed:

$$\frac{x}{A} = \cos\theta, \qquad \frac{v}{A\omega} = -\sin\theta.$$

Squaring and adding removes the phase:

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1.$$

Equivalently, $v^2 = \omega^2(A^2 - x^2)$. Both positive and negative velocities occur at an intermediate displacement because the particle passes that position in opposite directions during an oscillation.

Step 2: Identify the curve and reject the other shapes The equation has the standard form of an ellipse centred at the origin in the x-v plane. Its intercepts are $x = \pm A$ at zero velocity and $v = \pm A\omega$ at zero displacement. A sinusoid describes displacement or velocity against time, whereas time has been eliminated here. A straight line would imply a linear velocity-displacement relation, which contradicts the squared relation above. A circle appears only after normalising the axes to x/A and $v/(A\omega)$; it is not the general velocity-versus-displacement curve in the stated variables.

Step 3: Conclusion The velocity-displacement graph for SHM is $\boxed{\text{elliptical}}$. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish time graphs from phase-space graphs: SHM is sinusoidal against time, but eliminating time between displacement and velocity gives an ellipse.

41. 7 mol of a certain monoatomic ideal gas undergoes a temperature increase of $40\ \mathrm{K}$ at constant pressure. The increase in the internal energy of the gas in this process is :

(Given $\mathrm{R} = 8.3\,\mathrm{JK}^{-1}\,\mathrm{mol}^{-1}$) [4 marks]

(A) 5810 J

(B) 3486 J

(C) 11620 J

(D) 6972 J

Correct Answer: (B) 3486 J

Solution:

Concept:

- A monoatomic ideal gas has three translational degrees of freedom. Its internal energy is $U = \frac{3}{2}nRT$, so $\Delta U = \frac{3}{2}nR\Delta T$.
- Internal energy of an ideal gas depends only on temperature. The use of $C_V = \frac{3}{2}R$ in $\Delta U = nC_V\Delta T$ remains valid even when the actual process occurs at constant pressure.

Step 1: Select the internal-energy expression The requested quantity is the increase in internal energy, rather than the heat absorbed. With $n = 7\,\mathrm{mol}$, $\Delta T = 40\,\mathrm{K}$ and $R = 8.3\,\mathrm{J\,K^{-1}\,mol^{-1}}$, use

$$\Delta U = n\left(\frac{3}{2}R\right)\Delta T.$$

A constant-pressure process allows the gas to expand, but this changes the work and heat, not the relation between internal energy and temperature for an ideal gas.

Step 2: Substitute and distinguish heat from internal energy First calculate

$$nR\Delta T = 7 \times 8.3 \times 40 = 2324\,\mathrm{J}.$$

Hence

$$\Delta U = \frac{3}{2}(2324) = 3486\,\mathrm{J}.$$

The units reduce correctly because moles and kelvins cancel those in R. As a first-law check, the expansion work is $W = nR\Delta T = 2324\,\mathrm{J}$, and the heat absorbed at constant pressure is $Q = \Delta U + W = 5810\,\mathrm{J}$. Thus 5810 J is the heat, not the internal-energy increase. The values 11620 J and 6972 J are respectively twice that heat and twice the correct energy increase; neither follows from the three translational degrees of freedom.

Step 3: Conclusion The increase in internal energy is $\boxed{\Delta U = 3486\,\mathrm{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: Even in a constant-pressure process use C_V to find internal-energy change; reserve C_P for the heat supplied at constant pressure.

42. A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be : [4 marks]

(A) P

(B) 8P

(C) 32P

(D) 64P

Correct Answer: (C) 32P

Solution:

Concept:

- For an ideal gas undergoing an isentropic change, PV^{γ} remains constant, where $\gamma = C_P/C_V$. The condition of constant entropy is what selects this pressure-volume relation.
- A monoatomic ideal gas has $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$, hence $\gamma = 5/3$. An isothermal relation $PV =$ constant would require constant temperature instead.

Step 1: Apply the constant-entropy condition Let the initial state be (P, V) and the final state be $(P_f, V/8)$. Although the statement describes the compression as sudden, it explicitly asks for the pressure at constant entropy. Use that stated thermodynamic condition:

$$PV^{5/3} = P_f \left(\frac{V}{8}\right)^{5/3}.$$

Dividing by $(V/8)^{5/3}$ gives

$$\frac{P_f}{P} = \left(\frac{V}{V/8}\right)^{5/3} = 8^{5/3}.$$

The volume decreases, so the pressure must increase; this is also a useful direction check.

Step 2: Evaluate the pressure factor Write 8 as 2^3 before applying the fractional exponent:

$$8^{5/3} = (2^3)^{5/3} = 2^5 = 32.$$

Therefore $P_f = 32P$. The value P would describe no pressure change despite compression. The value 8P follows from an isothermal assumption and omits the temperature rise. The value 64P would require an exponent of 2 rather than the monoatomic value $5/3$. As an independent consistency check, $TV^{\gamma-1} =$ constant gives $T_f/T = 8^{2/3} = 4$, and the ideal-gas equation then gives $P_f/P = (T_f/T)(V/V_f) = 4 \times 8 = 32$.

Step 3: Conclusion Under the specified constant-entropy condition, the final pressure is $\boxed{P_f = 32P}$. Hence, the correct option is (C).

Quick Tip to Solve: For a monoatomic gas remember $\gamma = 5/3$; express volume factors as cubes to evaluate fractional powers such as $8^{5/3}$ quickly.

43. A water drop of radius $1\ \text{cm}$ is broken into 729 equal droplets. If surface tension of water is 75 dyne/ cm, then the gain in surface energy upto first decimal place will be :

(Given $\pi = 3.14$) [4 marks]

(A) 8.5×10^{-4} J

(B) 8.2×10^{-4} J

(C) 7.5×10^{-4} J

(D) 5.3×10^{-4} J

Correct Answer: (C) 7.5×10^{-4} J

Solution:

Concept:

- Surface energy of a liquid drop is $U = TA$, where T is surface tension and A is its single liquid-air surface area. A drop does not have the two surfaces of a soap bubble.
- Breaking a drop conserves its volume, not its area. For n equal daughter drops, $r = R/n^{1/3}$, so their combined area is $n^{1/3}$ times the original area.

Step 1: Find the radius of each droplet The initial radius is $R = 1\,\text{cm}$, and the number of droplets is $n = 729 = 9^3$. Equating the initial volume to the sum of the final volumes gives

$$\frac{4}{3}\pi R^3 = 729\frac{4}{3}\pi r^3, \qquad r = \frac{R}{9}.$$

Each radius is one-ninth of the original, while the total number of drops is 729.

Step 2: Calculate the increase in surface area The original area is $A_i = 4\pi R^2$. The final combined area is

$$A_f = 729 \cdot 4\pi \left(\frac{R}{9}\right)^2 = 9 \cdot 4\pi R^2.$$

Hence

$$\Delta A = (9-1)4\pi R^2 = 32\pi\,\text{cm}^2.$$

The factor is eight times the initial area for the increase; nine times is the final total, which must not be used directly as the increase.

Step 3: Convert the surface work to joules Using the supplied $\pi = 3.14$, the required energy is

$$\Delta U = T\Delta A = 75 \times 32 \times 3.14 = 7536\,\text{erg}.$$

Since $1\,\text{dyne cm} = 1\,\text{erg} = 10^{-7}$ J, this becomes 7.536×10^{-4} J. Rounded to one decimal place in the option coefficient it is 7.5×10^{-4} J; the other three coefficients do not follow from the conserved volume and area increase.

Step 4: Conclusion The gain in surface energy is 7.5×10^{-4} J. Hence, the correct option is (C).

Quick Tip to Solve: For fragmentation into n equal drops, use $\Delta U = 4\pi T R^2(n^{1/3} - 1)$; subtract the original surface energy.

44. The percentage decrease in the weight of a rocket, when taken to a height of $32\ \text{km}$ above the surface of earth will, be :

(**Radius of earth** $= 6400\ \text{km}$) [4 marks]

(A) 1%

(B) 3%

(C) 4%

(D) 0.5%

Correct Answer: (A) 1%

Solution:

Concept:

- For an unchanged rocket mass, gravitational weight is $W = GMm/r^2$, where r is distance from the centre of Earth. The altitude must be added to Earth's radius.
- For a small height $h \ll R$, $g_h = g_0(1 + h/R)^{-2} \simeq g_0(1 - 2h/R)$. Therefore the fractional decrease of weight is approximately $2h/R$.

Step 1: Form the weight ratio Let $W_0 = GMm/R^2$ be the surface weight. At height h,

$$W_h = \frac{GMm}{(R+h)^2}, \qquad \frac{W_h}{W_0} = \left(\frac{R}{R+h}\right)^2.$$

The rocket mass cancels. This question concerns variation of gravitational weight with height, so no fuel consumption or change of mass enters the calculation.

Step 2: Use the small-height approximation The two lengths are already in kilometres, so their ratio needs no conversion:

$$\frac{h}{R} = \frac{32}{6400} = 0.005.$$

Since this is small,

$$\frac{W_0 - W_h}{W_0} \times 100 \simeq \frac{2h}{R} \times 100 = 2(0.005) \times 100 = 1\%.$$

The decrease is twice the percentage change in the distance from Earth's centre to first order, because weight varies inversely as its square.

Step 3: Check the exact result against the options The exact inverse-square calculation gives

$$100\left[1 - \left(\frac{6400}{6432}\right)^2\right] = 0.99255\ldots\%.$$

This agrees with 1% at the precision of the given options. The 0.5% value uses h/R alone and misses the factor two. Values 3% and 4% are too large for this small altitude. The height is measured above the surface, whereas the force law uses the centre-to-rocket separation.

Step 4: Conclusion The percentage decrease in the rocket's weight is approximately 1%. Hence, the correct option is (A).

Quick Tip to Solve: For $h \ll R$, the percentage decrease in gravitational weight is $200h/R$, with h and R in the same units.

45. As per the given figure, two blocks each of mass $250\ \mathrm{g}$ are connected to a spring of spring constant $2\ \mathrm{Nm^{-1}}$. If both are given velocity v in opposite directions, then maximum elongation of the spring is : [4 marks]



(A) $\frac{v}{2\sqrt{2}}$
(B) $\frac{v}{2}$
(C) $\frac{v}{4}$
(D) $\frac{v}{\sqrt{2}}$

Correct Answer: (B) $\frac{v}{2}$

Solution:

Concept:

- For an ideal spring, the elastic energy at elongation x from natural length is $\frac{1}{2}kx^2$. Both moving blocks contribute to the initial kinetic energy.
- Equal masses with equal opposite velocities have zero total momentum. Their centre of mass remains at rest, and at maximum separation the relative speed vanishes.

Step 1: Calculate the initial kinetic energy Each block has mass $m = 250\,\mathrm{g} = 0.250\,\mathrm{kg}$ and speed v. For the initially unstretched spring, the initial elastic energy is zero. Therefore

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2 = 0.250v^2.$$

Counting only one block would underestimate the available energy. The opposite directions cancel momentum, but kinetic energies are scalar positive quantities and add.

Step 2: Identify the instant of maximum elongation At maximum elongation the blocks have zero relative speed. Their common velocity is the centre-of-mass velocity, which is zero because their initial momenta cancel. Both are therefore instantaneously at rest, and the spring stores all the initial kinetic energy.

Step 3: Equate the initial energy to spring energy For total spring elongation $x_{\max}$, conservation of energy gives

$$\frac{1}{2}kx_{\max}^2 = mv^2.$$

Substitute $k = 2\,\mathrm{N\,m^{-1}}$ and $m = 0.250\,\mathrm{kg}$:

$$x_{\max} = v\sqrt{\frac{2m}{k}} = v\sqrt{\frac{2(0.250)}{2}} = \frac{v}{2}.$$

The result is in metres for v in metres per second. The other options fail this energy balance. In particular, $v/4$ is either block's displacement, half the full spring elongation.

Step 4: Conclusion The maximum elongation of the spring is $x_{\max} = v/2$ in SI units. Hence, the correct option is (B).

Quick Tip to Solve: For two equal masses launched oppositely, use the sum of their kinetic energies and calculate the full spring extension, which is twice either block's displacement.

46. A monkey of mass $50\ \text{kg}$ climbs on a rope which can withstand the tension (T) of $350\ \text{N}$. If monkey initially climbs down with an acceleration of $4\ \text{m/s}^2$ and then climbs up with an acceleration of $5\ \text{m/s}^2$. Choose the correct option $(g = 10\ \text{m/s}^2)$. [4 marks]

(A) $T = 700\ \text{N}$ while climbing upward

(B) $T = 350\ \text{N}$ while going downward

(C) Rope will break while climbing upward

(D) Rope will break while going downward

Correct Answer: (C) Rope will break while climbing upward

Solution:

Concept:

- The monkey is acted upon by its weight mg downward and rope tension T upward. Newton's second law must use the direction of acceleration, not merely the direction of motion.
- The rope remains safe only if the tension required for the proposed motion does not exceed $350\,\text{N}$. With $m = 50\,\text{kg}$ and $g = 10\,\text{m s}^{-2}$, the weight is $500\,\text{N}$.

Step 1: Find the tension for downward acceleration For the first motion take downward as positive. The net downward force is $mg - T$, so

$$mg - T = ma_d, \qquad T = m(g - a_d).$$

With $a_d = 4\,\text{m s}^{-2}$,

$$T = 50(10 - 4) = 300\,\text{N}.$$

This is below the rope's limiting tension. Therefore it does not break in this downward-accelerating motion. Its tension is $300\,\text{N}$, rather than the limiting value $350\,\text{N}$.

Step 2: Find the tension needed for upward acceleration Now take upward as positive. For the intended upward acceleration of $5\,\text{m s}^{-2}$,

$$T - mg = ma_u, \qquad T = m(g + a_u).$$

Hence the required tension would be

$$T = 50(10 + 5) = 750\,\text{N}.$$

An upward acceleration requires tension greater than the weight, since the excess upward force supplies the acceleration.

Step 3: Compare with the breaking strength The required upward-motion tension $750\,\text{N}$ exceeds $350\,\text{N}$, so the rope cannot sustain the specified upward acceleration and breaks. The $700\,\text{N}$ claim in (A) is numerically wrong. Option (B) mistakes the breaking limit for the actual downward-motion tension, and (D) contradicts the safe value $300\,\text{N}$. The breaking-strength comparison selects only the upward-motion statement.

Step 4: Conclusion The rope will break while the monkey attempts to climb upward with the stated acceleration. Hence, the correct option is (C).

Quick Tip to Solve: Write $T = m(g+a)$ for upward acceleration and $T = m(g-a)$ for downward acceleration, then compare the required tension with the rope's limit.

47. Two projectiles thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is : [4 marks]

(A) $1 : \sqrt{2}$

(B) $2 : 1$

(C) $\sqrt{2} : 1$

(D) $1 : 2$

Correct Answer: (C) $\sqrt{2} : 1$

Solution:

Concept:

- The vertical component of the launch velocity for speed u and angle θ is $u \sin\theta$. At the highest point, the vertical velocity is zero.
- Under the same downward acceleration g, the time to reach maximum height is $t_{\max} = u \sin\theta / g$, obtained from the vertical velocity equation.

Step 1: Express the common ascent time Let u_1 be the initial speed at 30°, and u_2 that at 45°. From $v_y = u_y - gt$, setting $v_y = 0$ at the highest point gives

$$t_1 = \frac{u_1 \sin 30^\circ}{g}, \qquad t_2 = \frac{u_2 \sin 45^\circ}{g}.$$

The horizontal velocity does not affect the time at which vertical motion reverses. Thus it is the initial vertical component that must match.

Step 2: Equate the vertical components The problem states that the two maximum heights are reached in the same time. Therefore

$$u_1 \sin 30^\circ = u_2 \sin 45^\circ.$$

Using $\sin 30^\circ = 1/2$ and $\sin 45^\circ = 1/\sqrt{2}$,

$$\frac{u_1}{u_2} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

The smaller launch angle requires the larger initial speed in order to produce the same vertical launch component.

Step 3: Check the proposed ratios With $u_1 = \sqrt{2}u_2$, the first vertical component is $\sqrt{2}u_2/2 = u_2/\sqrt{2}$, exactly equal to the second. The reciprocal ratio in (A) would make the first ascent time smaller. Ratios $2 : 1$ and $1 : 2$ also give unequal vertical components when multiplied by their respective sines. Equal ascent time does not require equal total speeds; the launch angles are different.

Step 4: Conclusion The ratio of the initial velocities, in the stated order, is $\sqrt{2} : 1$. Hence, the correct option is (C).

Quick Tip to Solve: Equal times to the highest point mean equal initial vertical velocity components: compare $u \sin\theta$, rather than the complete speeds.

48. A screw gauge of pitch 0.5 mm is used to measure the diameter of uniform wire of length 6.8 cm, the main scale reading is 1.5 mm and circular scale reading is 7 . The calculated curved surface area of wire to appropriate significant figures is :

[Screw gauge has 50 divisions on its circular scale] [4 marks]

(A) 6.8 cm^2

(B) 3.4 cm^2

(C) 3.9 cm^2

(D) 2.4 cm^2

Correct Answer: (B) 3.4 cm^2

Solution:

Concept:

- The least count of a screw gauge is its pitch divided by the number of circular-scale divisions. The diameter equals the main-scale reading plus the circular reading times the least count, when no zero correction is specified.
- A cylindrical wire of diameter d and length l has curved surface area $A = \pi dl$. Curved area excludes the two circular ends.

Step 1: Determine the least count and diameter The screw advances $0.5\,\text{mm}$ in one rotation, divided into 50 equal circular-scale divisions. Consequently

$$\text{LC} = \frac{0.5}{50} = 0.01\,\text{mm}.$$

The circular reading 7 contributes $7(0.01) = 0.07\,\text{mm}$. Thus

$$d = 1.5 + 0.07 = 1.57\,\text{mm} = 0.157\,\text{cm}.$$

No zero error is given, so no additional correction is applied.

Step 2: Calculate the curved surface area The length is $l = 6.8\,\text{cm}$, and the circumference is πd. Multiplying circumference by length,

$$A = \pi dl = \pi(0.157)(6.8)\,\text{cm}^2 = 3.3539\ldots\ \text{cm}^2.$$

Using diameter directly in this formula avoids a needless conversion to radius. If radius were used instead, the formula would be $2\pi rl$ and would give the same result.

Step 3: Apply the appropriate precision The length $6.8\,\text{cm}$ has two significant figures, and the diameter $0.157\,\text{cm}$ has three. Their product must therefore be reported to two significant figures:

$$A = 3.4\,\text{cm}^2.$$

The other listed areas disagree with πdl. An extra factor two after using the diameter would incorrectly give about $6.7\,\text{cm}^2$.

Step 4: Conclusion The curved surface area to the appropriate significant figures is $3.4\,\text{cm}^2$. Hence, the correct option is (B).

Quick Tip to Solve: First calculate the screw-gauge least count, then convert the diameter to the same length unit as the wire before using $A = \pi dl$.

49. A radio can tune to any station in $6\,\text{MHz}$ to $10\,\text{MHz}$ band. The value of corresponding wavelength bandwidth will be : [4 marks]

(A) 4 m

(B) 20 m

(C) 30 m

(D) 50 m

Correct Answer: (B) 20 m

Solution:

Concept:

- For electromagnetic waves in air, $c = f\lambda$, with $c \simeq 3.0 \times 10^8\,\text{m}\,\text{s}^{-1}$. Frequency and wavelength are inversely related.
- The wavelength bandwidth is the difference between the wavelengths at the two frequency endpoints. It is not the wavelength calculated from the frequency bandwidth.

Step 1: Convert the lower frequency to a wavelength The lower tuning frequency is $f_{\text{min}} = 6\,\text{MHz} = 6 \times 10^6\,\text{Hz}$. It corresponds to the largest wavelength:

$$\lambda_{\text{max}} = \frac{c}{f_{\text{min}}} = \frac{3 \times 10^8}{6 \times 10^6} = 50\,\text{m}.$$

The inverse relation means the order of the wavelength limits is opposite to the order of the frequency limits.

Step 2: Convert the upper frequency to a wavelength The upper frequency is $f_{\text{max}} = 10\,\text{MHz} = 10 \times 10^6\,\text{Hz}$. The corresponding smallest wavelength is

$$\lambda_{\text{min}} = \frac{c}{f_{\text{max}}} = \frac{3 \times 10^8}{10 \times 10^6} = 30\,\text{m}.$$

Thus the radio covers wavelengths from $30\,\text{m}$ to $50\,\text{m}$, even though its tuning range is specified by increasing frequencies.

Step 3: Take the difference of the wavelength limits The required wavelength bandwidth is

$$\Delta\lambda = \lambda_{\text{max}} - \lambda_{\text{min}} = 50 - 30 = 20\,\text{m}.$$

The frequency bandwidth is $10 - 6 = 4\,\text{MHz}$, but its numerical value 4 cannot be treated as a wavelength in metres; this rules out (A). Options (C) and (D) are individual endpoint wavelengths rather than the span between them. Computing $c/(f_{\text{max}} - f_{\text{min}})$ would also be incorrect because taking a reciprocal does not commute with subtraction.

Step 4: Conclusion The corresponding wavelength bandwidth is $20\,\text{m}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert each edge frequency separately with $\lambda = c/f$, then subtract the two wavelengths; never put the frequency difference in the denominator.

50. The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minute. 10 minutes later, the rate becomes 2250 disintegrations per minute. The approximate decay constant is :

(**Take** $\log_{10} 1.88 = 0.274$) [4 marks]

(A) $0.02\,\text{min}^{-1}$

(B) $2.7\,\text{min}^{-1}$

(C) $0.063\,\text{min}^{-1}$

(D) $6.3\,\text{min}^{-1}$

Correct Answer: (C) $0.063\,\text{min}^{-1}$

Solution:

Concept:

- Activity is the disintegration rate, $A = \lambda N$. For one radioactive species the activity follows $A(t) = A(0)e^{-\lambda t}$, with constant decay constant λ.
- Taking logarithms gives $\lambda = \ln[A(0)/A(t)]/t$. Since the elapsed time is in minutes, the resulting unit is min^{-1}.

Step 1: Form the activity ratio The two measured activities refer to the same sample and are separated by ten minutes. Their ratio removes the unknown initial number of nuclei and any common conversion of the activity units. Thus use the earlier rate in the numerator so that the logarithm and decay constant are positive:

$$\frac{A(0)}{A(10)} = \frac{4250}{2250} = \frac{17}{9} \simeq 1.88.$$

The fact that the rate decreases agrees with a positive exponential decay constant.

Step 2: Convert the given logarithm and divide by time The supplied logarithm is to base ten, whereas the exponential decay law requires a natural logarithm. Use $\ln z = 2.303 \log_{10} z$:

$$\lambda = \frac{2.303 \log_{10}(1.88)}{10} = \frac{2.303(0.274)}{10} = 0.0631\,\text{min}^{-1}.$$

Using the unrounded ratio $17/9$ changes the result only slightly and selects the same option. The value 0.02 is too small to produce the observed reduction in ten minutes; 2.7 and 6.3 would make the activity almost vanish over that interval. A check is $e^{-0.0631(10)} \simeq 0.532$, close to $2250/4250$.

Step 3: Conclusion The approximate decay constant is $\boxed{0.063\,\text{min}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: For two activity readings, take the logarithm of their ratio; multiply a common logarithm by 2.303 before dividing by the elapsed time.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1-x)$. The y component vector of the initial velocity is _____ $\hat{j}$. (Take $g = 10\ m/s^2$) [4 marks]

Solution:

Concept:

- For projectile motion with constant downward acceleration and no air resistance, $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$, where u_x, u_y are the initial velocity components.
- Eliminating time gives $y = (u_y/u_x)x - [g/(2u_x^2)]x^2$. The coefficient of x is the initial trajectory slope, which equals the ratio of the vertical and horizontal velocities.

Step 1: Express the given trajectory as a polynomial The horizontal initial velocity is $\hat{i}$, so its magnitude in SI units is $u_x = 1\,\mathrm{m\,s^{-1}}$. Expand the given equation without changing its signs:

$$y = 5x(1-x) = 5x - 5x^2.$$

The positive linear coefficient means the initial vertical velocity is upward. The negative quadratic coefficient describes the subsequent downward curvature under gravity; it does not make the initial vertical component negative.

Step 2: Compare coefficients with the projectile equation Equating the coefficients of x gives

$$\frac{u_y}{u_x} = 5, \qquad u_y = 5(1) = 5\,\mathrm{m\,s^{-1}}.$$

The coefficient of x^2 independently checks the supplied horizontal velocity:

$$\frac{g}{2u_x^2} = \frac{10}{2(1)^2} = 5.$$

Both terms therefore match the same physical motion. Equivalently, differentiating the trajectory gives $dy/dx = 5 - 10x$; at launch $x = 0$, the slope is 5. Since $dy/dx = v_y/v_x$, this again gives $u_y = 5$. The requested quantity is the vertical component vector, so attach $\hat{j}$ rather than reporting the total launch speed $\sqrt{26}\,\mathrm{m\,s^{-1}}$.

Step 3: Conclusion The initial vertical velocity vector is $\boxed{5\hat{j}\,\mathrm{m\,s^{-1}}}$; the numerical entry is $\boxed{5}$.

Quick Tip to Solve: In a trajectory written as $y = ax - bx^2$, use $a = u_y/u_x$; the quadratic term provides a separate check using gravity.

52. A disc of mass $1\ \mathrm{kg}$ and radius R is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}\ \mathrm{rad}\ s^{-1}$ where $x =$ _____. $(g = 10\ \mathrm{ms}^{-2})$ [4 marks]

Solution:

Concept:

- The loss of gravitational potential energy becomes rotational kinetic energy, $K = \frac{1}{2}I\omega^2$, because the fixed axle does no work.
- A uniform disc has moment of inertia $I_d = \frac{1}{2}MR^2$ about its central normal. A small body of mass m fixed at its rim contributes $I_b = mR^2$.

Step 1: Find the potential-energy decrease and total inertia The disc centre stays at the fixed axle, so the gravitational potential energy of the disc does not change. The attached body moves from a height R above the axle to R below it, a vertical fall of $2R$. With $M = m = 1\,\mathrm{kg}$,

$$\Delta U = mg(2R) = 20R.$$

The body and disc rotate together and share one angular speed. Their moments of inertia must be added before applying energy conservation:

$$I = \frac{1}{2}MR^2 + mR^2 = \frac{3}{2}R^2.$$

Using the disc inertia alone would omit the kinetic energy of the attached body.

Step 2: Apply energy conservation and compare the stated form The system starts from rest. At the lowest position, its gain in kinetic energy is the potential-energy decrease:

$$\frac{1}{2}\left(\frac{3}{2}R^2\right)\omega^2 = 20R.$$

Solving for the positive magnitude of angular velocity gives

$$\omega^2 = \frac{80}{3R}, \qquad \omega = \sqrt{\frac{80}{3R}} = 4\sqrt{\frac{5}{3R}}\,\mathrm{rad\,s^{-1}}.$$

The problem writes the same angular speed as $4\sqrt{x/(3R)}$. Comparing equal squared expressions gives $16x/(3R) = 80/(3R)$, hence $x = 5$. The inverse-square-root dependence on radius also agrees with energy scaling: the fall energy is proportional to R, while inertia is proportional to R^2.

Step 3: Conclusion The angular speed is $4\sqrt{5/(3R)}\,\mathrm{rad\,s^{-1}}$, so the required coefficient is $\boxed{x = 5}$.

Quick Tip to Solve: For a mass attached to a rotating disc, include both inertias and use the attached mass's full top-to-bottom fall of $2R$.

53. In an experiment to determine the Young's modulus of wire of a length exactly $1\ \mathrm{m}$, the extension in the length of the wire is measured as $0.4\ \mathrm{mm}$ with an uncertainty of $\pm 0.02\ \mathrm{mm}$ when a load of $1\ \mathrm{kg}$ is applied. The diameter of the wire is measured as $0.4\ \mathrm{mm}$ with an uncertainty of $\pm 0.01\ \mathrm{mm}$. The error in the measurement of Young's modulus $(\Delta \mathrm{Y})$ is found to be $x \times 10^{10}\ \mathrm{Nm}^{-2}$. The value of x is _____. (take $\mathrm{g} = 10\ \mathrm{ms}^{-2}$) [4 marks]

Solution:

Concept:

- For a wire under tensile force F, Young's modulus is $Y = FL/(A\ell) = 4FL/(\pi d^2\ell)$, where d is diameter and ℓ is extension.
- For the maximum first-order uncertainty of a product or quotient, fractional absolute uncertainties add with their powers: $\Delta Y/Y = \Delta L/L + \Delta F/F + 2\Delta d/d + \Delta\ell/\ell$.

Step 1: Calculate the measured Young's modulus The applied force is $F = mg = 1(10) = 10\,\mathrm{N}$. The length is exactly $L = 1\,\mathrm{m}$. Convert both the measured diameter and extension to SI units: $d = \ell = 0.4 \times 10^{-3}\,\mathrm{m}$. Thus

$$Y = \frac{4(10)(1)}{\pi(0.4 \times 10^{-3})^2(0.4 \times 10^{-3})} \simeq 1.989 \times 10^{11}\,\mathrm{N\,m^{-2}}.$$

The diameter is squared because the cross-sectional area is $\pi d^2/4$; using the radius formula without converting diameter would change the result by a factor of four.

Step 2: Propagate the stated uncertainties No uncertainty in load or gravity is specified, and the length is explicitly exact, so only extension and diameter contribute. Their fractional uncertainties are

$$\frac{\Delta\ell}{\ell} = \frac{0.02}{0.4} = 0.05, \qquad \frac{\Delta d}{d} = \frac{0.01}{0.4} = 0.025.$$

The diameter contribution is doubled:

$$\frac{\Delta Y}{Y} = 0.05 + 2(0.025) = 0.10.$$

Therefore the absolute uncertainty is

$$\Delta Y = 0.10(1.989 \times 10^{11}) = 1.989 \times 10^{10}\,\mathrm{N\,m^{-2}} \simeq 2 \times 10^{10}\,\mathrm{N\,m^{-2}}.$$

This is an uncertainty magnitude; the measured modulus may be reported with plus-or-minus this amount. The requested coefficient is dimensionless.

Step 3: Conclusion The error is approximately $\boxed{\Delta Y = 2 \times 10^{10}\,\mathrm{N\,m^{-2}}}$, giving the numerical entry $\boxed{x = 2}$.

Quick Tip to Solve: When a modulus contains d^{-2}, double the fractional diameter uncertainty before adding the extension uncertainty.

54. A composite parallel plate capacitor is made up of two different dielectric materials with different thickness (t_1 and t_2) as shown in figure. The two different dielectric materials are separated by a conducting foil F. The voltage of the conducting foil is V. [4 marks]



Solution:

Concept:

- Dielectric layers stacked along the electric field act as capacitors in series, with $C_i = \varepsilon_0 \varepsilon_{ri} A/t_i$. Their common series charge gives $V_i \propto t_i/\varepsilon_{ri}$.
- The conducting foil is an equipotential surface. Relative to the lower plate connected to the negative battery terminal, its voltage equals the voltage drop across the lower dielectric layer.

Step 1: Identify the layer parameters and voltage ratio The upper layer has relative permittivity $\varepsilon_{r1} = 3$ and thickness $t_1 = 0.5\,\text{mm}$; the lower layer has $\varepsilon_{r2} = 4$ and thickness $t_2 = 1\,\text{mm}$. The battery applies $100\,\text{V}$ between the outer plates. The isolated intermediate foil separates two series capacitances, so their voltage drops add to 100 V. Their ratio is

$$\frac{V_1}{V_2} = \frac{t_1/\varepsilon_{r1}}{t_2/\varepsilon_{r2}} = \frac{0.5/3}{1/4} = \frac{2}{3}.$$

The common area and ε_0 cancel, and the common millimetre unit cancels as well.

Step 2: Find the potential of the foil Let the lower outer plate be the zero-potential reference. Since the upper battery terminal is positive in the diagram, the upper plate is at 100 V and the foil lies at an intermediate positive potential. Write $V_1 = 2k$, $V_2 = 3k$. Then

$$2k + 3k = 100, \qquad k = 20\,\text{V},$$

so

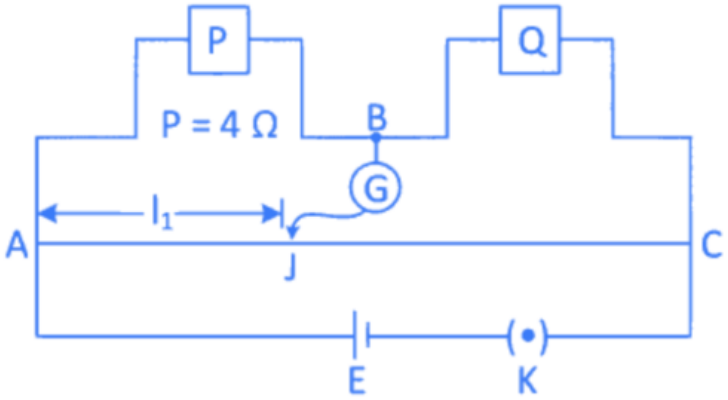
$$V_1 = 40\,\text{V}, \qquad V_2 = 60\,\text{V}.$$

The foil is 60 V above the lower plate. The upper plate is 40 V above the foil. The two drops total the applied 100 V. Equal thicknesses or equal permittivities are not present, so splitting the supply equally would be incorrect. The layer with larger t/ε_r takes the larger voltage drop.

Step 3: Conclusion The conducting foil is at $\boxed{60\,\text{V}}$ relative to the lower negative plate. The required numerical entry is $\boxed{60}$.

Quick Tip to Solve: For dielectric layers in series, divide the total voltage in the ratio $t_1/\varepsilon_{r1} : t_2/\varepsilon_{r2}$, then identify which drop gives the foil potential.

55. Resistances are connected in a meter bridge circuit as shown in the figure. The balancing length l_1 is 40 cm. Now an unknown resistance x is connected in series with P and new balancing length is found to be 80 cm measured from the same end. Then the value of x will be _____ Ω. [4 marks]



Solution:

Concept:

- At the balance point of a metre bridge the galvanometer carries no current, and the gap resistances obey $P/Q = l/(100-l)$ for a uniform 100 cm wire measured from the P end.
- Adding a resistance in series with P replaces that gap resistance by $P + x$; the other gap resistance Q remains unchanged.

Step 1: Use the first balance to determine the other resistance The figure labels the left gap resistance as $P = 4\,\Omega$. The first balance length from the same left end is $l_1 = 40\,\text{cm}$, leaving $100 - 40 = 60\,\text{cm}$ on the right. Therefore

$$\frac{4}{Q} = \frac{40}{60} = \frac{2}{3}, \qquad Q = 6\,\Omega.$$

The total wire length is 100 cm, and the lengths are used as a ratio, so no conversion to metres is necessary. The balance condition concerns the two gap resistances and the wire segments; the source emf cancels out.

Step 2: Apply the new balance after adding the series resistance The new left gap resistance is $4 + x$, and the new balance length is $80\,\text{cm}$ measured from the same end, so the right wire segment is now 20 cm. Hence

$$\frac{4+x}{6} = \frac{80}{20} = 4.$$

Multiplying through and subtracting the original resistance gives

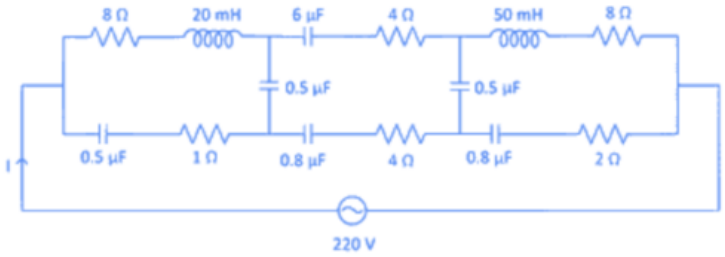
$$4 + x = 24\,\Omega, \qquad x = 20\,\Omega.$$

As a consistency check, the left gap resistance increases from 4 to 24 ohms, so the ratio P/Q rises from $2/3$ to 4. The balance point should consequently move towards the right, as the stated change from 40 cm to 80 cm shows. Taking the new length from the opposite end would invert the ratio and conflict with the explicit wording.

Step 3: Conclusion The unknown series resistance is $\boxed{x = 20\,\Omega}$, so the numerical entry is $\boxed{20}$.

Quick Tip to Solve: Find the unchanged gap resistance from the first balance before using the second balance, and keep the same wire end as the length reference.

56. The effective current I in the given circuit at very high frequencies will be _____ A. [4 marks]



Solution:

Concept:

- For sinusoidal steady state, inductive reactance is $X_L = \omega L$ and capacitive reactance is $X_C = 1/(\omega C)$. At very high frequency an ideal inductor behaves as an open circuit and an ideal capacitor as a short circuit.
- After these limiting replacements, reduce the remaining resistor network. Effective current means rms current, so use $I_{\text{rms}} = V_{\text{rms}}/R_{\text{eq}}$ for the resulting resistive circuit.

Step 1: Replace the reactive elements in their high-frequency limits Both upper outer branches contain inductors (20 mH on the left and 50 mH on the right). Their reactances grow without bound, so those branches carry negligible current. The 8 ohm resistors in those branches are consequently disconnected from the conducting path. All the capacitors, including the two vertical 0.5 microfarad links and the capacitors along the horizontal branches, have reactances approaching zero and join their adjacent nodes. The conducting path therefore enters through the lower-left $1\,\Omega$ resistor and leaves through the lower-right $2\,\Omega$ resistor.

Step 2: Reduce the middle resistors and calculate the current The two vertical capacitor shorts connect the ends of the upper central $4\,\Omega$ resistor to the corresponding ends of the lower central $4\,\Omega$ resistor. Thus these resistors are in parallel, rather than in series:

$$R_{\text{middle}} = \frac{4 \times 4}{4 + 4} = 2\,\Omega.$$

The total resistance seen by the supply is

$$R_{\text{eq}} = 1 + 2 + 2 = 5\,\Omega.$$

The AC source is labelled 220 V, its effective value, so

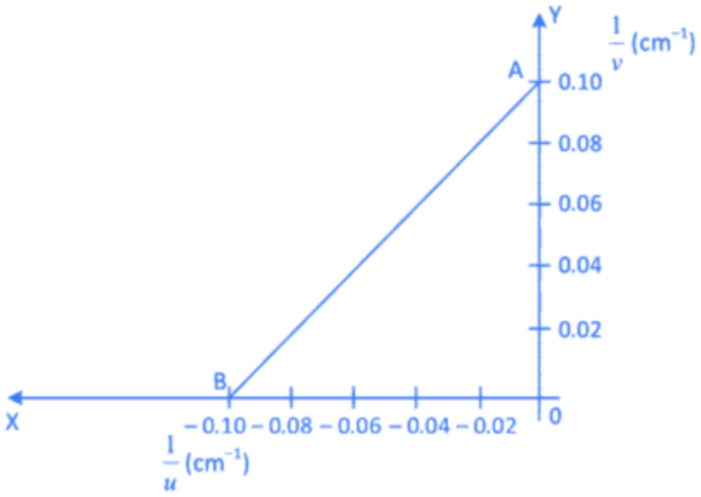
$$I_{\text{rms}} = \frac{220}{5} = 44\,\text{A}.$$

No extra factor of $\sqrt{2}$ is needed because both the requested current and the supply rating are effective values. The exact component capacitances and inductances do not affect this limiting result, provided they are finite and nonzero.

Step 3: Conclusion The effective current at very high frequencies is $\boxed{44\,\text{A}}$. The required numerical entry is $\boxed{44}$.

Quick Tip to Solve: First open every inductor and short every capacitor, then redraw the node connections mentally before deciding which resistors are in parallel.

57. The graph between $\frac{1}{u}$ and $\frac{1}{v}$ for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of lens is 1.5 and its both the surfaces have same radius of curvature R. The value of R will be _____ cm. (where u = object distance, v = image distance) [4 marks]



Solution:

Concept:

- The thin-lens formula in the Cartesian sign convention is $1/v - 1/u = 1/f$. Thus a plot of $1/v$ against $1/u$ has a vertical intercept equal to $1/f$.
- For a thin lens in air, the lens maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$. The two signed radii of a symmetric biconvex lens are opposite, even though their magnitudes are equal.

Step 1: Read the intercept from the graph The vertical axis represents $1/v$ in cm^{-1}. Its intercept is $0.10\,\text{cm}^{-1}$, at the point where $1/u = 0$. This is the case of an object at infinity, for which the image is at the focus. Therefore

$$\frac{1}{f} = 0.10\,\text{cm}^{-1}, \qquad f = 10\,\text{cm}.$$

The horizontal intercept $-0.10\,\text{cm}^{-1}$ is consistent with $u = -f$; its negative sign does not make the focal length negative.

Step 2: Substitute the two surface radii For light travelling from left to right, take $R_1 = +R$ and $R_2 = -R$. Substituting $\mu = 1.5$ gives

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(0.5)}{R} = \frac{1}{R}.$$

Hence the magnitude of either surface radius equals the focal length. With $f = 10\,\text{cm}$, we obtain $R = 10\,\text{cm}$. Using two positive radii would incorrectly cancel the curvatures and describe a different lens.

Step 3: Conclusion The required radius of curvature is $\boxed{R = 10\,\text{cm}}$. The numerical value to enter is $\boxed{10}$.

Quick Tip to Solve: Read the reciprocal-distance intercept first, invert it to obtain the focal length, and only then use the signed radii in the lens maker formula.

58. In the hydrogen spectrum, λ be the wavelength of first transition line of Lyman series. The wavelength difference will be "aλ" between the wavelength of 3^{rd} transition line of Paschen series and that of 2^{nd} transition line of Balmer series where $a =$ _____. [4 marks]

Solution:

Concept:

- The Rydberg formula for hydrogen is $1/\lambda_{n_i \to n_f} = R_H(1/n_f^2 - 1/n_i^2)$, where R_H is the Rydberg constant and $n_i > n_f$.
- Lyman, Balmer and Paschen series terminate at $n_f = 1, 2, 3$, respectively. Within a series the first line starts at $n_f + 1$, so the ordinal line number determines the initial level.

Step 1: Identify the three transitions The first Lyman line is $2 \to 1$, so its wavelength is λ. The third Paschen line is $6 \to 3$, since the first two are $4 \to 3$ and $5 \to 3$. The second Balmer line is $4 \to 2$, following the first line $3 \to 2$. Applying the formula to the given reference wavelength gives

$$\frac{1}{\lambda} = R_H\left(1 - \frac{1}{4}\right) = \frac{3R_H}{4}, \qquad \lambda = \frac{4}{3R_H}.$$

The transition labels must be fixed before comparing wavelengths.

Step 2: Calculate the wavelength difference For the third Paschen line and second Balmer line, respectively,

$$\frac{1}{\lambda_P} = R_H\left(\frac{1}{9} - \frac{1}{36}\right) = \frac{R_H}{12}, \qquad \lambda_P = \frac{12}{R_H},$$

$$\frac{1}{\lambda_B} = R_H\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R_H}{16}, \qquad \lambda_B = \frac{16}{3R_H}.$$

Thus the positive wavelength difference is

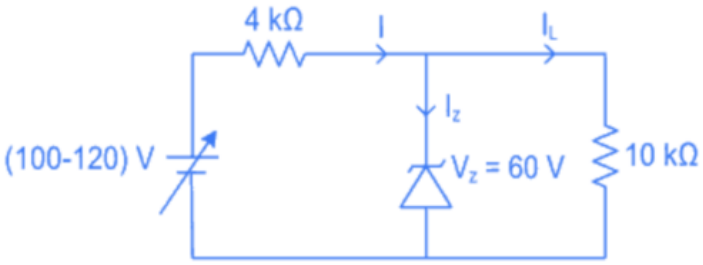
$$\lambda_P - \lambda_B = \frac{36 - 16}{3R_H} = \frac{20}{3R_H} = 5\left(\frac{4}{3R_H}\right) = 5\lambda.$$

The Rydberg constant cancels from the ratio, so no numerical value or wavelength unit is needed for a.

Step 3: Conclusion The difference is 5λ, giving the dimensionless value $\boxed{a = 5}$.

Quick Tip to Solve: For the kth line of a series ending at n, use the transition from n+k to n; compare wavelengths only after inverting the Rydberg expressions.

59. In the circuit shown below, maximum zener diode current will be _____ mA. [4 marks]



Solution:

Concept:

- In reverse breakdown an ideal Zener diode maintains its rated voltage across the parallel load. Here the regulated output voltage is $V_Z = 60\,\text{V}$.
- Kirchhoff's current law at the output node gives $I = I_L + I_Z$, where I is the series-resistor current, I_L is the load current and I_Z is the Zener current.

Step 1: Find the fixed load current The $10\,\text{k}\Omega$ load is in parallel with the Zener diode, so its voltage is also $60\,\text{V}$ while the diode regulates. Ohm's law gives

$$I_L = \frac{V_Z}{R_L} = \frac{60}{10 \times 10^3}\,\text{A} = 6\,\text{mA}.$$

This current is set by the regulated voltage and load resistance. It must be subtracted from the series current; the entire current through the $4\,\text{k}\Omega$ resistor does not pass through the diode.

Step 2: Maximise the series current and subtract the load The series current is

$$I = \frac{V_{\text{in}} - V_Z}{R_s}.$$

Since the load current and Zener voltage are fixed, I_Z increases with the input voltage. The maximum therefore occurs at $V_{\text{in}} = 120\,\text{V}$, not at $100\,\text{V}$. At that input,

$$I_{\text{max}} = \frac{120 - 60}{4 \times 10^3}\,\text{A} = 15\,\text{mA},$$

$$I_{Z,\text{max}} = I_{\text{max}} - I_L = 15 - 6 = 9\,\text{mA}.$$

At the minimum input the series current is $10\,\text{mA}$, leaving a positive Zener current of $4\,\text{mA}$; hence the breakdown assumption is consistent throughout the stated range.

Step 3: Conclusion The maximum Zener diode current is $\boxed{9\,\text{mA}}$. Enter $\boxed{9}$.

Quick Tip to Solve: For a fixed regulated load, maximum Zener current occurs at maximum input voltage: subtract the load current from the series-resistor current.

60. When a car is approaching the observer, the frequency of horn is 100 Hz. After passing the observer, it is 50 Hz. If the observer moves with the car, the frequency will be $\frac{x}{3}$ Hz where $x =$ _____. [4 marks]

Solution:

Concept:

- For a stationary observer and a source moving at speed v_s in still air, the approaching and receding frequencies are $f_a = f_0 c/(c - v_s)$ and $f_r = f_0 c/(c + v_s)$.
- Here c is the speed of sound and f_0 is the actual horn frequency. An observer moving along with the car has the same velocity as the horn, so there is no Doppler shift between them.

Step 1: Determine the source speed ratio The ratio of the two stationary-observer readings eliminates the unknown horn frequency:

$$\frac{f_a}{f_r} = \frac{c + v_s}{c - v_s} = \frac{100}{50} = 2.$$

Therefore $c + v_s = 2c - 2v_s$, so $3v_s = c$ and $v_s/c = 1/3$. This speed is less than the speed of sound, consistent with the approaching and receding formulas. The larger observed frequency corresponds to approach because successive wavefronts are compressed.

Step 2: Recover the horn frequency and the co-moving reading Substitute the speed ratio into the approaching frequency:

$$100 = \frac{f_0}{1 - 1/3} = \frac{3}{2} f_0, \qquad f_0 = \frac{200}{3}\ \text{Hz}.$$

The receding formula independently gives $f_0/(1 + 1/3) = 50\,\text{Hz}$, confirming the result. If the observer moves with the car, the observer-motion factor cancels the source-motion factor. For an observer ahead of the horn, for example,

$$f_{\text{obs}} = f_0 \frac{c - v_s}{c - v_s} = f_0.$$

Thus the co-moving observer hears $200/3\,\text{Hz}$. This is not the arithmetic mean of the two stationary-observer readings.

Step 3: Conclusion Comparing $f_{\text{obs}} = 200/3\,\text{Hz}$ with $x/3\,\text{Hz}$ gives the dimensionless value $\boxed{x = 200}$.

Quick Tip to Solve: The true frequency is the harmonic mean of the approach and recession readings, and a passenger moving with the source hears that true frequency.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Match List-I with List-II:

	List I (Compound)		List II (Shape)
(A)	**BrF_5**	**(I)**	**bent**
(B)	**$[CrF_6]^{3-}$**	**(II)**	**square pyramidal**
(C)	**O_3**	**(III)**	**trigonal bipyramidal**
(D)	**PCl_5**	**(IV)**	**octahedral**

Choose the correct answer from the options given below: [4 marks]

(A) $(A)-(I),(B)-(II),(C)-(III),(D)-(IV)$

(B) $(A)-(IV),(B)-(III),(C)-(II),(D)-(I)$

(C) $(A)-(II),(B)-(IV),(C)-(I),(D)-(III)$

(D) $(A)-(III),(B)-(IV),(C)-(II),(D)-(I)$

Correct Answer: (C) $(A)-(II),(B)-(IV),(C)-(I),(D)-(III)$

Solution:

Concept:

- VSEPR theory determines molecular shape from bonding domains and lone pairs around the central atom. Lone pairs affect geometry but are not counted as visible vertices of a molecular shape.
- A six-coordinate complex with six equivalent ligand positions has octahedral coordination geometry. The coordination number, rather than the total number of atoms in the formula, is used for a complex ion.

Step 1: Assign the halide and complex geometries In BrF_5, bromine has seven valence electrons. Five bonds leave one lone pair, giving six electron domains: an octahedral electron-domain arrangement and a square-pyramidal molecular shape. Therefore (A) matches (II). In $[CrF_6]^{3-}$, six fluoride ligands surround the chromium centre. Its coordination number is six, and the complex has octahedral geometry, so (B) matches (IV). The lone pair in BrF_5 is the reason its molecular shape differs from the six-coordinate complex.

Step 2: Assign ozone and phosphorus pentachloride and compare options The central oxygen of O_3 has two bonding domains and a lone pair. The electron-domain arrangement is trigonal planar, while the molecular shape defined by the three nuclei is bent. Hence (C) matches (I). In molecular PCl_5, phosphorus forms five bonds and has no lone pair; the five domains give trigonal-bipyramidal geometry, so (D) matches (III). Combining these independently obtained results gives

$$(A)-(II),\quad (B)-(IV),\quad (C)-(I),\quad (D)-(III).$$

Option (A) incorrectly makes BrF_5 bent, option (B) makes it octahedral, and option (D) makes it trigonal bipyramidal. Those three are excluded by the first pairing alone; only option (C) contains all four correct pairings.

Step 3: Conclusion All four independently established pairings agree with the same listed choice. Hence, the correct option is (C).

Quick Tip to Solve: Distinguish electron-domain geometry from molecular shape: six domains with one lone pair give a square pyramid, while six ligand bonds give an octahedron.

62. Match List-I with List-II.

	List I (Processes/Reactions)		List II (Catalyst)
(A)	$2\,SO_2(g) + O_2(g) \longrightarrow 2\,SO_3(g)$	(I)	$Fe\,(s)$
(B)	$4\,NH_3(g) + 5\,O_2(g) \longrightarrow 4\,NO\,(g) + 6\,H_2O\,(g)$	(II)	$Pt(s)–Rh(s)$
(C)	$N_2(g) + 3\,H_2(g) \longrightarrow 2\,NH_3(g)$	(III)	V_2O_5
(D)	Vegetable oil(l) + $H_2 \rightarrow$ Vegetable ghee(s)	(IV)	$Ni\,(s)$

Choose the correct answer from the options given below: [4 marks]

(A) $(A) - (III), (B) - (I), (C) - (II), (D) - (IV)$

(B) $(A) - (III), (B) - (II), (C) - (I), (D) - (IV)$

(C) $(A) - (IV), (B) - (III), (C) - (I), (D) - (II)$

(D) $(A) - (IV), (B) - (II), (C) - (III), (D) - (I)$

Correct Answer: (B) $(A) - (III), (B) - (II), (C) - (I), (D) - (IV)$

Solution:

Concept:

- A catalyst provides an alternative reaction pathway with lower activation energy, increasing the reaction rate without changing its equilibrium constant at a fixed temperature.
- Industrial catalyst matching depends on the particular transformation: oxidation of sulfur dioxide, catalytic oxidation of ammonia, ammonia synthesis, and hydrogenation of an oil use different established catalysts.

Step 1: Match the two oxidation reactions The conversion of SO_2 to SO_3 is the catalytic oxidation step in the contact process for sulfuric acid manufacture. Vanadium(V) oxide, V_2O_5, is the listed catalyst for this conversion. Thus (A) matches (III). The oxidation of ammonia to nitric oxide is the initial catalytic step of the Ostwald process for nitric acid manufacture. It uses platinum-rhodium gauze, represented in the list by Pt(s)–Rh(s). Hence (B) matches (II). These two reactions both involve oxygen but do not therefore share the same catalyst.

Step 2: Match synthesis and hydrogenation and reject the alternatives The reaction $N_2 + 3\,H_2 \longrightarrow 2\,NH_3$ is ammonia synthesis in the Haber process, whose catalyst is finely divided iron. Thus (C) matches (I). Addition of hydrogen to the unsaturated bonds of vegetable oil converts it into the more saturated vegetable ghee; nickel is the catalyst, so (D) matches (IV). The full sequence is therefore

$$(A)–(III), \quad (B)–(II), \quad (C)–(I), \quad (D)–(IV).$$

Option (A) interchanges the catalysts for ammonia oxidation and synthesis, so it fails. Options (C) and (D) both assign nickel to sulfur dioxide oxidation, which is incorrect, and also give other wrong pairings. Option (B) alone preserves all four process-specific assignments. No stoichiometric coefficient changes these catalyst identities.

Step 3: Conclusion All four independently established pairings agree with the same listed choice. Hence, the correct option is (B).

Quick Tip to Solve: Link the industrial process names to catalysts: contact–vanadium oxide, Ostwald–platinum-rhodium, Haber–iron, oil hydrogenation–nickel.

63. Given two statements below :

Statement I : In Cl_2 molecule the covalent radius is double of the atomic radius of chlorine.

Statement II : Radius of anionic species is always greater than their parent atomic radius.

Choose the most appropriate answer from options given below : [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (D) Statement I is incorrect but Statement II is correct.

Solution:

Concept:

- The covalent radius of an atom is defined as half the internuclear distance between two identical atoms joined by a single covalent bond. The full internuclear distance is the bond length.
- An anion is formed by addition of electron(s) to an atom without an increase in nuclear charge. Increased electron repulsion and reduced attraction per electron generally expand the electron cloud.

Step 1: Test the covalent-radius statement For the homonuclear molecule Cl_2, let d_{Cl-Cl} denote the internuclear separation. Each chlorine contributes one covalent radius, and the two contributions are equal:

$$d_{Cl-Cl} = 2r_{cov}(Cl), \qquad r_{cov}(Cl) = \frac{d_{Cl-Cl}}{2}.$$

Thus the distance between the nuclei is twice the covalent radius; the covalent radius is not twice the atomic radius of chlorine. Statement I reverses the meaning of the factor of two and is incorrect. The bond length must not be confused with a single-atom radius.

Step 2: Test the anion statement and select the truth pattern When a neutral atom accepts an electron to form an anion, its number of protons stays the same while the number of electrons increases. The extra electron increases repulsion within the occupied shell, and the unchanged positive nuclear charge is distributed over more electrons. The electron cloud expands. For example, Cl^- is larger than neutral Cl; the comparison is between an ion and its own parent atom, not between unrelated species across a period. In the standard atomic-radius trend tested here, the anionic radius is therefore greater, making Statement II correct. The required pattern is I incorrect, II correct. Option (A) fails because I is false; (B) fails because II is true; (C) reverses both truth values. Only option (D) has the correct pattern.

Step 3: Conclusion Statement I is incorrect, while Statement II is correct. Hence, the correct option is (D).

Quick Tip to Solve: For a homonuclear bond, halve the bond length to get one covalent radius; adding electrons to a fixed nucleus enlarges the parent atom into an anion.

64. Which of the given reactions is not an example of disproportionation reaction? [4 marks]

(A) $2H_2O_2 \rightarrow 2H_2O + O_2$

(B) $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

(C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

(D) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

Correct Answer: (C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

Solution:

Concept:

- A disproportionation reaction oxidises and reduces the same element initially present in one oxidation state. Two products must contain that element in higher and lower oxidation states.
- Oxidation number increases on oxidation and decreases on reduction. In ordinary oxides oxygen has oxidation number -2, but in peroxide it has oxidation number -1.

Step 1: Check oxygen and nitrogen In option (A), oxygen in H_2O_2 starts at -1. It becomes -2 in water and 0 in molecular oxygen. Thus one portion is reduced and another oxidised. In option (B), nitrogen starts at $+4$ in NO_2; it becomes $+5$ in HNO_3 and $+3$ in HNO_2. Therefore both (A) and (B) satisfy the two-direction criterion.

Step 2: Track manganese in the electron half-reaction In option (C), charge balance gives $x - 8 = -1$, so manganese in MnO_4^- is $+7$. In MnO_2, manganese is $+4$. The only manganese transformation is therefore

$$Mn(+7) + 3e^- \longrightarrow Mn(+4).$$

Every manganese atom is reduced; none is oxidised. The explicit electrons on the left also identify this as a reduction half-reaction, rather than disproportionation.

Step 3: Check the remaining manganate reaction In option (D), manganese in $MnO_4{}^{2-}$ is $+6$. Two manganese atoms form permanganate, in which the oxidation number is $+7$, while one forms manganese dioxide, in which it is $+4$. The two atoms oxidised each lose one electron, balancing the two electrons gained by the reduced atom. This is a true disproportionation and is excluded from the requested answer.

Step 4: Conclusion Only the permanganate-to-manganese-dioxide half-reaction involves reduction alone. Hence, the correct option is (C).

Quick Tip to Solve: Write the oxidation state of the same element on both sides: disproportionation requires both an upward and a downward change from the starting state.

65. The dark purple colour of $KMnO_4$ disappears in the titration with oxalic acid in acidic medium. The overall change in the oxidation number of manganese in the reaction is : [4 marks]

(A) 5

(B) 1

(C) 7

(D) 2

Correct Answer: (A) 5

Solution:

Concept:

- In acidic medium, permanganate MnO_4^- oxidises oxalic acid and is reduced to Mn^{2+}. The purple permanganate colour disappears as this reduction proceeds.
- The change in oxidation number is found per manganese atom. A stoichiometric coefficient in the balanced equation does not multiply that per-atom change.

Step 1: Find the initial oxidation number Potassium has oxidation number $+1$ and the four oxygen atoms contribute -8. Since $KMnO_4$ is electrically neutral,

$$1 + x + 4(-2) = 0 \quad \Rightarrow \quad x = +7.$$

Equivalently, the manganese oxidation number follows from the charge -1 of the permanganate ion. This is the strongly oxidising manganese species present before titration.

Step 2: Use the product appropriate to acidic medium In the stated acidic medium the reduction product is Mn^{2+}, not manganese dioxide or manganate. Its oxidation number is $+2$. The balanced half-reaction is

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O.$$

Thus one manganese atom gains five electrons. Oxalic acid supplies those electrons as it is oxidised to carbon dioxide; the acid is essential for the specified manganese product.

Step 3: Calculate the change and eliminate the other values The signed change is final minus initial:

$$\Delta\text{O.N.} = 2 - 7 = -5.$$

Hence the oxidation number decreases by five units. The choices list positive magnitudes, so select 5. The value 7 is merely the initial oxidation number and 2 merely the final one. A change of 1 would describe conversion from permanganate to manganate in a different medium, which is not the acidic oxalic-acid titration specified here.

Step 4: Conclusion The oxidation number of manganese changes from $+7$ to $+2$, a decrease of 5. Hence, the correct option is (A).

Quick Tip to Solve: Before calculating the electron change for permanganate, choose its reduction product from the medium: acidic solution gives Mn^{2+}.

66.

$$\dot{\mathrm{Cl}} + \mathrm{CH_4} \rightarrow \mathrm{A} + \mathrm{B}$$

A and B in the above atmospheric reaction step are : [4 marks]

(A) C_2H_6 and Cl_2

(B) $\dot{C}HCl_2$ and H_2

(C) $\dot{C}H_3$ and HCl

(D) C_2H_6 and HCl

Correct Answer: (C) $\dot{C}H_3$ and HCl

Solution:

Concept:

- A chlorine radical abstracts a hydrogen atom from methane in a propagation step of radical chlorination. A radical has an unpaired electron and can generate another radical while making a stable molecule.
- The carbon skeleton and the numbers of atoms must be conserved in an individual elementary step. Hydrogen abstraction is distinct from the later step that forms chloromethane.

Step 1: Identify the bond broken and the bond formed The reactant Cl carries one unpaired electron. It removes one hydrogen atom from a carbon–hydrogen bond of methane. One electron from that bond accompanies hydrogen and pairs with the chlorine electron to make a hydrogen–chlorine bond; the other remains on carbon. Consequently the carbon fragment retains three hydrogen atoms and one unpaired electron.

Step 2: Write the propagation reaction The resulting products are the methyl radical and hydrogen chloride:

$$\mathrm{Cl}^{\bullet} + \mathrm{CH_4} \longrightarrow \mathrm{CH_3}^{\bullet} + \mathrm{HCl}.$$

There is one carbon atom, four hydrogen atoms and one chlorine atom on each side. One radical is consumed and one is formed, so the radical chain can continue. In the subsequent chlorination step the methyl radical can react with Cl_2 to give chloromethane and regenerate a chlorine radical, but that is a different step.

Step 3: Test the alternative product pairs Options (A) and (D) contain ethane, which has two carbon atoms although the reactants in the stated step contain only one; neither can be the product pair. Ethane can instead arise by combination of two methyl radicals in a termination step. Option (B) contains a radical with two chlorine atoms, but only one chlorine atom is available here; it also proposes hydrogen rather than hydrogen chloride. These atom-balance checks leave only the methyl-radical/hydrogen-chloride pair.

Step 4: Conclusion A and B are $\mathrm{CH_3}^{\bullet}$ and HCl, respectively. Hence, the correct option is (C).

Quick Tip to Solve: For attack of a halogen radical on an alkane, remove one H atom: the products are the alkyl radical and hydrogen halide.

67. Which technique among the following, is most appropriate in separation of a mixture of $100\,\mathrm{mg}$ of p-nitrophenol and picric acid ? [4 marks]

(A) Steam distillation

(B) 2-5 ft long column of silica gel

(C) Sublimation

(D) Preparative TLC (Thin Layer Chromatography)

Correct Answer: (D) Preparative TLC (Thin Layer Chromatography)

Solution:

Concept:

- Chromatography separates components by their different affinities for a stationary phase and a mobile phase. Different interactions produce different migration distances.
- Preparative thin-layer chromatography uses a thicker adsorbent layer than analytical TLC, enabling small isolated quantities to be recovered from separated bands. The scale of the sample helps decide the practical technique.

Step 1: Use the stated amount of material The mixture contains only $100\,\mathrm{mg}$ of material. This is a small preparative-scale separation, so a technique should resolve the components while limiting solvent use, adsorbent consumption and losses during transfer. Preparative TLC is suited to isolating components from such small samples rather than merely identifying their presence.

Step 2: Explain how preparative TLC separates the phenols The two compounds have different numbers of nitro groups: p-nitrophenol has one, whereas picric acid is 2,4,6-trinitrophenol. Their interactions with a polar adsorbent and with the chosen eluent therefore differ. Apply the mixture as a band on a preparative plate and develop it in a suitable solvent system. The compounds travel as separate bands; scrape off each separated adsorbent band and extract the compound with solvent. This provides an actual preparative separation without requiring a long column.

Step 3: Compare the other proposed methods A 2–5 ft silica-gel column is unnecessarily large for $100\,\mathrm{mg}$, increasing solvent use and the risk of sample loss. Steam distillation relies on suitable steam volatility and is not the most appropriate way to resolve this small mixture of strongly interacting phenols; the familiar steam separation of ortho- and para-nitrophenol uses a different volatility contrast. Sublimation likewise requires a useful difference in sublimation behaviour and offers no comparable general resolution here. Preparative TLC directly exploits differential adsorption on the appropriate small scale.

Step 4: Conclusion The most appropriate method is preparative TLC (Thin Layer Chromatography). Hence, the correct option is (D).

Quick Tip to Solve: For an isolation problem, distinguish analytical TLC from preparative TLC and match the separation method to the sample quantity before choosing a large column.

68. The difference in the reaction of phenol with bromine in chloroform and bromine in water medium is due to : [4 marks]

(A) Hyperconjugation in substrate

(B) Polarity of solvent

(C) Free radical formation

(D) Electromeric effect the substrate

Correct Answer: (B) Polarity of solvent

Solution:

Concept:

- The hydroxyl group donates electron density to the aromatic ring by resonance and activates the ortho and para positions towards electrophilic substitution.
- The solvent changes ionisation and the effective activation of phenol and bromine. Polar water promotes rapid bromination much more strongly than chloroform does.

Step 1: Compare the reactions in the two media With bromine in water, phenol reacts readily to form the characteristic white precipitate of 2,4,6-tribromophenol:

$$C_6H_5OH + 3\,Br_2 \longrightarrow C_6H_2Br_3OH + 3\,HBr.$$

Substitution occurs at both ortho positions and the para position because the ring is strongly activated under these conditions. In chloroform, bromination is more controlled and gives mainly para-bromophenol along with some ortho-bromophenol rather than immediate extensive tribromination.

Step 2: Relate the difference to the solvent Water is a much more polar and ionising solvent than chloroform. It favours some phenol ionisation and stabilises ionic species; the phenoxide ion, even when present only in a small amount, is especially strongly activating because its negative charge can be delocalised into the ring. The polar medium also assists polarisation of bromine. In less polar chloroform, these effects are weaker, so the extent of substitution differs. The changed variable in the question is the reaction medium.

Step 3: Exclude the other explanations Hyperconjugation is not the source of the solvent-dependent change: the phenol substrate has not acquired a new alkyl group or a different hyperconjugative framework. These are electrophilic aromatic substitution reactions, so a free-radical mechanism is not the relevant explanation. The electromeric effect describes complete temporary displacement of a pi-electron pair under attack; naming it does not account for the particular difference between water and chloroform. The appropriate distinguishing factor among the options is solvent polarity.

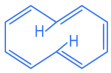
Step 4: Conclusion The different bromination behaviour results from the polarity of the solvent. Hence, the correct option is (B).

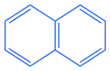
Quick Tip to Solve: When the substrate and reagent are unchanged but substitution changes with the medium, first compare the solvents' polarity and ability to stabilise ions.

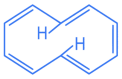
69. Which of the following compounds is not aromatic? [4 marks]

(A) Cyclopentadienyl anion 

(B) Cyclopropenyl cation 

(C) [10]Annulene 

(D) Naphthalene 

Correct Answer: (C) [10]Annulene 

Solution:

Concept:

- An aromatic ring must have a cyclic, planar, continuously conjugated system containing $4n + 2$ pi electrons. Electron count alone does not establish aromaticity if planarity fails.
- A negative charge in a conjugated ring can contribute a lone pair of two pi electrons; a positively charged carbon has an empty p orbital and contributes no pi electron of its own.

Step 1: Check the two charged monocyclic rings Option (A) is the cyclopentadienyl anion. Its two double bonds supply four pi electrons, and the negative charge contributes a further pair in a p orbital. Thus it has six pi electrons in a planar conjugated five-membered ring, satisfying $4n + 2$ with $n = 1$. Option (B) is the cyclopropenyl cation. Its double bond supplies two pi electrons while the cationic carbon provides an empty p orbital, completing cyclic conjugation. The planar three-membered ring satisfies the rule with $n = 0$. Both are aromatic.

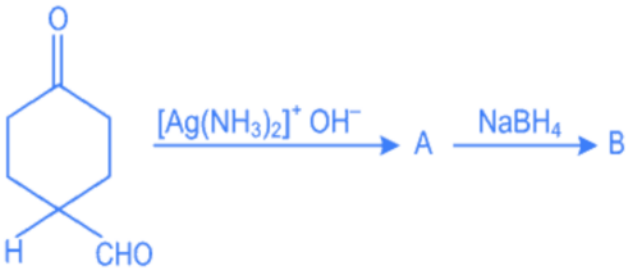
Step 2: Inspect the geometry of the ten-membered ring Option (C) is [10]annulene, with five double bonds and hence ten pi electrons. Although $10 = 4(2) + 2$, the inward-pointing hydrogen atoms shown in its ring crowd one another. A planar arrangement would cause severe steric repulsion. The ring adopts a nonplanar conformation, preventing the continuous parallel p-orbital overlap required for aromaticity. Therefore this compound is nonaromatic; a favourable electron count cannot compensate for its geometry.

Step 3: Compare the fused aromatic system Option (D) is naphthalene. Its fused carbon framework is planar and continuously conjugated, with ten pi electrons delocalised over the system. It therefore meets the aromaticity requirements. Unlike the inward hydrogen atoms of [10]annulene, the fused structure does not force the same obstructing crowding inside a ten-membered monocycle. This eliminates (D), leaving the nonplanar annulene as the sole nonaromatic option.

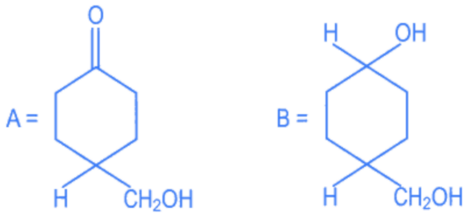
Step 4: Conclusion The nonaromatic compound is [10]annulene, because it fails the planarity requirement despite having ten pi electrons. Hence, the correct option is (C).

Quick Tip to Solve: Apply the aromaticity tests in order: cyclic conjugation, planarity, then $4n + 2$ counting. An annulene may pass the count and still fail the geometry test.

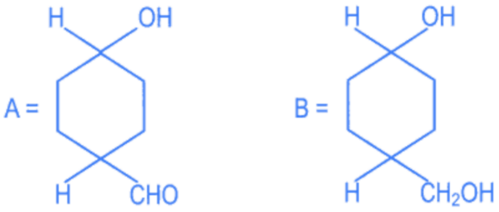
70. The products formed in the following reaction, A and B are [4 marks]



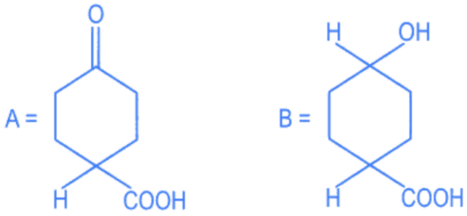
(A) A: 4-(hydroxymethyl)cyclohexan-1-one; B: 4-(hydroxymethyl)cyclohexan-1-ol.



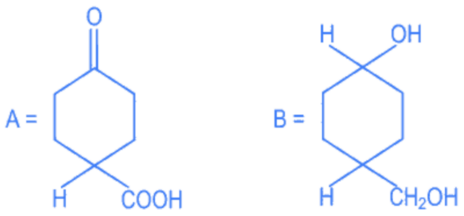
(B) A: 4-hydroxycyclohexane-1-carbaldehyde; B: 4-(hydroxymethyl)cyclohexan-1-ol.



(C) A: 4-oxocyclohexane-1-carboxylic acid; B: 4-hydroxycyclohexane-1-carboxylic acid.

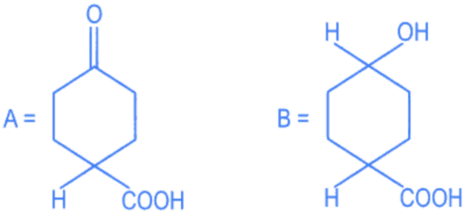


(D) A: 4-oxocyclohexane-1-carboxylic acid; B: 4-(hydroxymethyl)cyclohexan-1-ol.



Correct Answer: (C)

A: 4-oxocyclohexane-1-carboxylic acid; B: 4-hydroxycyclohexane-1-carboxylic acid.



Solution:

Concept:

- Tollens' reagent, $[Ag(NH_3)_2]^+$ in alkaline solution, oxidises an aldehyde to a carboxylate, conventionally represented as the corresponding carboxylic acid in product choices. An ordinary ketone is not oxidised.
- Sodium borohydride, $NaBH_4$, reduces an ordinary ketone to a secondary alcohol but does not ordinarily reduce a carboxylic acid or carboxylate group. Apply successive reagents in the printed order.

Step 1: Identify the two carbonyl functions in the starting material The starting structure contains a ketone carbonyl as part of the cyclohexanone ring and a formyl group, $-CHO$, attached at the opposite ring carbon. These functions respond differently to Tollens' reagent. The hydrogen shown at the formyl-bearing ring carbon is simply part of that carbon's valence; it is not another carbonyl group and does not alter the selective oxidation.

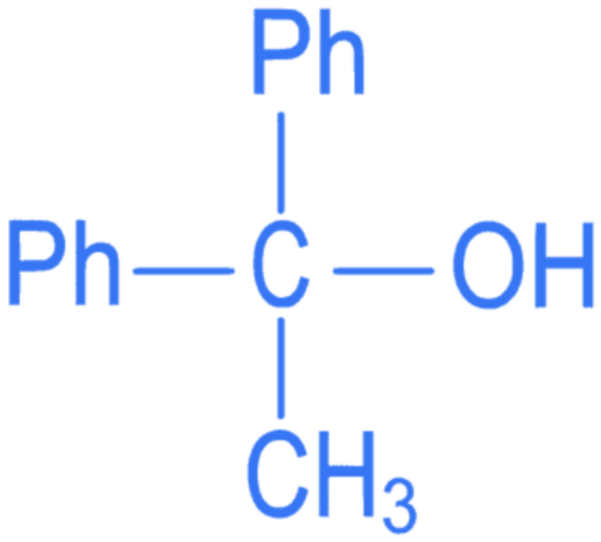
Step 2: Determine product A after Tollens oxidation Tollens' reagent oxidises the aldehyde substituent from $-CHO$ to $-COO^-$ in its alkaline reaction mixture. The acid-form product displayed in the alternatives is $-COOH$. The aldehyde carbon is oxidised, whereas the cyclic ketone carbonyl remains intact; these selectivities identify the first product before the second reagent is considered. Thus A is the compound shown as 4-oxocyclohexane-1-carboxylic acid. Options (A) and (B) are eliminated at this stage: (A) incorrectly reduces the aldehyde to an alcohol, whereas (B) incorrectly reduces the ring ketone while leaving the aldehyde.

Step 3: Determine product B after borohydride reduction Now treat A with $NaBH_4$. Hydride addition to the remaining ketone carbonyl followed by protonation replaces the ring $C{=}O$ by $CH{-}OH$. The carboxyl group is retained because borohydride is not the reagent used for its reduction under these conditions. Consequently B is 4-hydroxycyclohexane-1-carboxylic acid. Option (D) incorrectly converts the carboxyl substituent into $-CH_2OH$; that would require a stronger reducing reagent. Only the third printed pair preserves the oxidised substituent and reduces the ketone.

Step 4: Conclusion A is 4-oxocyclohexane-1-carboxylic acid and B is 4-hydroxycyclohexane-1-carboxylic acid, as represented in the product choices. Hence, the correct option is (C).

Quick Tip to Solve: Mark each functional group before following a reagent sequence: Tollens oxidises $-CHO$, then $NaBH_4$ reduces the ketone while retaining $-COOH$.

71. Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide $(PhMgBr)$ followed by acidic hydrolysis? [4 marks]



(A) $CH_3-C\equiv N$

(B) $C_6H_5-C\equiv N$

(C) $CH_3-C(=O)-O-C_6H_5$

(D) $C_6H_5-C(=O)-CH_3$

Correct Answer: (D) $C_6H_5-C(=O)-CH_3$

Solution:

Concept:

- A Grignard reagent supplies a carbon nucleophile: the phenyl group of C_6H_5MgBr attacks an electrophilic carbonyl carbon. Acidic work-up protonates the resulting magnesium alkoxide.
- A ketone $R-CO-R'$ gives a tertiary alcohol after one addition. The two groups already bonded to the carbonyl carbon remain attached to the alcohol carbon.

Step 1: Work backwards from the alcohol The alcohol carbon bears two phenyl groups and one methyl group. One phenyl group must come from the one mole of phenylmagnesium bromide. Remove that newly introduced phenyl group and replace the carbon–oxygen single bond by a carbonyl bond. The required starting ketone therefore has one phenyl group and one methyl group attached to its carbonyl carbon: acetophenone, $C_6H_5COCH_3$.

Step 2: Apply addition and acidic hydrolysis The carbonyl oxygen coordinates magnesium, and nucleophilic phenyl adds to carbon. The product before work-up is the magnesium alkoxide.

$$C_6H_5COCH_3 \xrightarrow{C_6H_5MgBr} (C_6H_5)_2C(OMgBr)CH_3 \xrightarrow{H_3O^+} (C_6H_5)_2C(OH)CH_3$$

This consumes one equivalent of reagent and gives exactly the displayed alcohol.

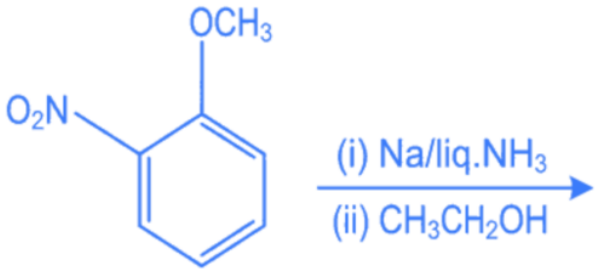
Step 3: Exclude the other functional groups Acetonitrile and benzonitrile, options (A) and (B), form ketones after Grignard addition followed by hydrolysis, rather than the required tertiary alcohol in this single addition sequence. Phenyl acetate, option (C), is an ester: formation of the tertiary alcohol

requires two equivalents of Grignard reagent, first producing a ketone and then adding to it. The stipulated one mole therefore identifies the ketone in (D).

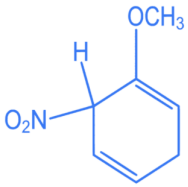
Step 4: Conclusion The reactant is acetophenone, $C_6H_5COCH_3$. Hence, the correct option is (D).

Quick Tip to Solve: For one-equivalent Grignard addition, remove the incoming carbon group from the alcohol carbon and restore C=O to identify the ketone.

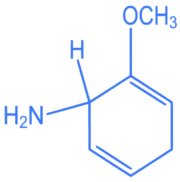
72. The major product of the following reaction is [4 marks]



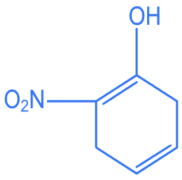
(A) OCH_3 at C1, NO_2 at C2; double bonds C1=C6 and C3=C4.



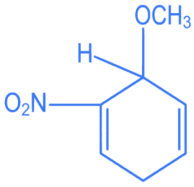
(B) OCH_3 at C1, NH_2 at C2; double bonds C1=C6 and C3=C4.



(C) OH at C1, NO_2 at C2; double bonds C1=C2 and C4=C5.

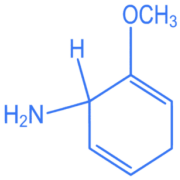


(D) OCH_3 at C1, NO_2 at C2; double bonds C2=C3 and C5=C6.



Correct Answer: (B)

OCH_3 at C1, NH_2 at C2; double bonds C1=C6 and C3=C4.



Solution:

Concept:

- Sodium in liquid ammonia supplies solvated electrons, and ethanol supplies protons. These conditions reduce an aromatic ring to a non-conjugated cyclohexa-1,4-diene.
- The nitro group is reducible under the strongly reducing conditions. The methoxy substituent remains an ether; its substituted ring carbon remains unsaturated in the appropriate Birch product.

Step 1: Identify the changes caused by the reagents The starting material is ortho-nitroanisole: adjacent ring carbons carry OCH_3 and NO_2. The metal–ammonia system is a reducing medium, not an acidic ether-cleavage reagent. Thus the methoxy group is retained. In addition to partial reduction of the aromatic ring, the nitro substituent is reduced to an amino substituent, NH_2.

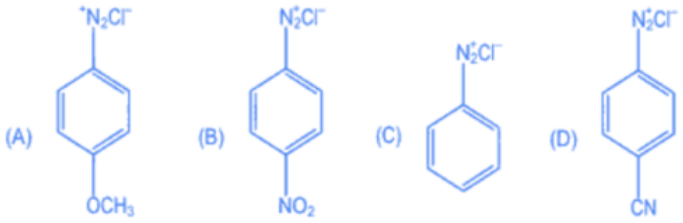
Step 2: Match the ring double bonds Number the methoxy-bearing carbon C1 and its adjacent nitrogen-bearing carbon C2. In the required product the ring has double bonds C1=C6 and C3=C4; C2 and C5 are saturated. The methoxy-bearing carbon is still part of a double bond, whereas C2 now bears a hydrogen and the amino group. The two remaining double bonds are separated, so the product is a non-conjugated diene rather than an aromatic ring.

Step 3: Compare all four structures Option (B) combines both required changes: it retains OCH_3, replaces NO_2 by NH_2, and has the displayed 1,4-diene arrangement. Option (A) has the same ring arrangement but leaves the nitro group unreduced. Option (C) replaces methoxy by hydroxyl and also leaves nitro unchanged. Option (D) leaves nitro unchanged and places the methoxy group on a saturated carbon. Those structures do not represent the major product under the stated conditions.

Step 4: Conclusion The major product is the methoxy-substituted amino cyclohexadiene shown in (B). Hence, the correct option is (B).

Quick Tip to Solve: Check both the ring and other reducible groups in a dissolving-metal question; recognizing a 1,4-diene alone does not distinguish these options.

73. The correct stability order of the following diazonium salt is [4 marks]



(A) (A) > (B) > (C) > (D)
(B) (A) > (C) > (D) > (B)
(C) (C) > (A) > (D) > (B)
(D) (C) > (D) > (B) > (A)

Correct Answer: (B) (A) > (C) > (D) > (B)

Solution:

Concept:

- An arenediazonium ion is conjugated with its benzene ring. Electron donation through the ring stabilizes its electron-deficient diazonium group.
- At the para position, methoxy donates by resonance, whereas cyano and nitro withdraw electron density. Nitro withdraws more strongly than cyano in this comparison.

Step 1: Identify the four substituents The common group is the arenediazonium chloride group. Structure (A) has para-methoxy, (B) has para-nitro, (C) is unsubstituted benzenediazonium chloride, and (D) has para-cyano. Chloride is the same counter-ion in every structure, so it does not decide the relative order. The comparison concerns the effect transmitted by the para substituent through the aromatic ring.

Step 2: Compare donation and withdrawal The oxygen lone pair of the methoxy group can donate into the conjugated system. This resonance contribution stabilizes the diazonium cation despite the oxygen inductive withdrawing effect, placing (A) above the unsubstituted reference (C). Both cyano and nitro are electron-withdrawing groups and reduce this stabilization relative to (C). Nitro has the stronger withdrawing effect here, so the nitro derivative (B) is below the cyano derivative (D).

Step 3: Assemble the order and test the options The resulting sequence is

$$(A) > (C) > (D) > (B).$$

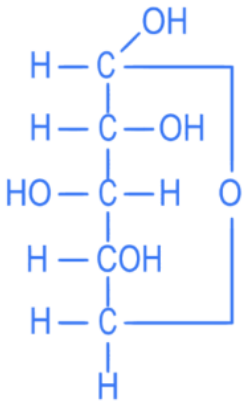
Option (B) reproduces this order. Option (A) incorrectly places the nitro derivative above the unsubstituted and cyano derivatives. Option (C) reverses the methoxy and unsubstituted pair. Option (D) puts the methoxy derivative last, opposite to its stabilizing resonance donation. Thus only one printed sequence has all comparisons in the right direction.

Step 4: Conclusion The methoxy derivative is most stable, followed by unsubstituted, cyano and nitro derivatives, in that order. Hence, the correct option is (B).

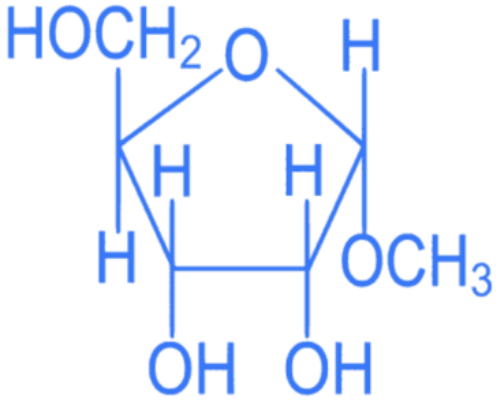
Quick Tip to Solve: For this para-substituted series, start with the unsubstituted diazonium ion, place the resonance donor above it and electron-withdrawing groups below it.

74. Which one of the following is a reducing sugar? [4 marks]

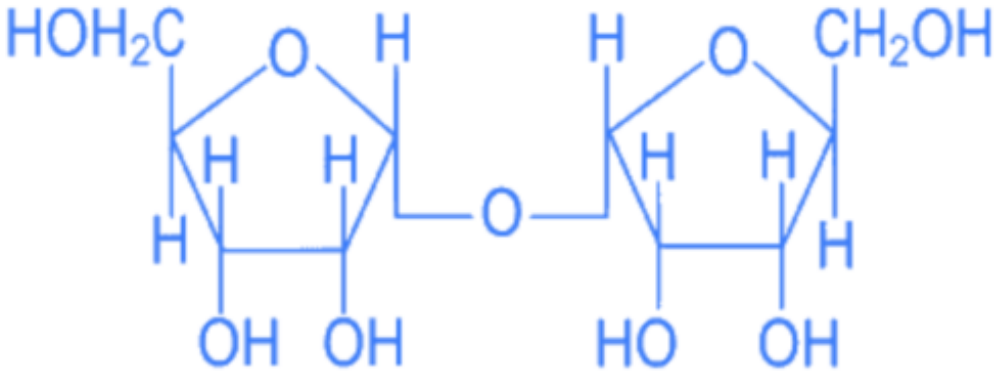
(A) Cyclic sugar with a free anomeric OH group.



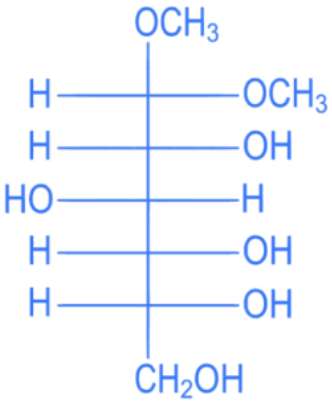
(B) Cyclic methyl glycoside with an anomeric OCH_3 group.



(C) Disaccharide with both anomeric carbons joined through the glycosidic oxygen.

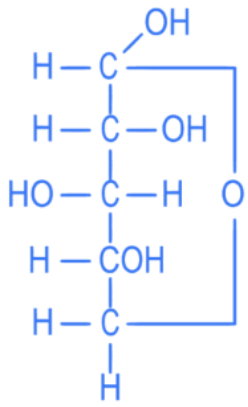


(D) Open-chain acetal with two OCH_3 groups on C1.



Correct Answer: (A)

Cyclic sugar with a free anomeric OH group.



Solution:

Concept:

- A reducing sugar can establish an equilibrium with an open-chain carbonyl form. In a cyclic aldose, a free anomeric hydroxyl group identifies a hemiacetal capable of ring opening.
- Conversion of the anomeric hydroxyl into a glycosidic acetal prevents that end from opening in the usual reducing-sugar test. Ordinary hydroxyl groups elsewhere do not replace a free reducing end.

Step 1: Locate the anomeric carbon in each structure The anomeric carbon is attached to two oxygen atoms in the cyclic sugar: the ring oxygen and an exocyclic oxygen substituent. In option (A), the upper carbon is attached to the ring oxygen and a free OH group. It also bears hydrogen. Thus this carbon is a hemiacetal carbon, and its carbonyl-derived hydroxyl has not been locked as a glycoside.

Step 2: Use ring opening to identify the reducing sugar The hemiacetal in (A) can open by cleavage of the reversible carbon–ring-oxygen bond, regenerating the open-chain aldehyde. That aldehyde can be oxidized while reducing a suitable reagent, such as Tollens reagent or alkaline copper(II) solution. Consequently the fact that (A) is drawn as a ring does not make it non-reducing: equilibrium supplies the reactive carbonyl form.

Step 3: Exclude structures with protected reducing ends Option (B) has an anomeric OCH_3 group and is a methyl glycoside, so the carbonyl-derived carbon is an acetal. In option (C), the glycosidic oxygen joins both anomeric carbons; neither unit has a free hemiacetal end. In option (D), the upper carbon bears two methoxy groups, again an acetal rather than a free aldehyde or hemiacetal. The many other hydroxyl groups in these structures do not make their blocked anomeric centres reducing.

Step 4: Conclusion The structure with the free anomeric hydroxyl group, (A), is a reducing sugar. Hence, the correct option is (A).

Quick Tip to Solve: Inspect the anomeric carbon, not the total number of OH groups: a free hemiacetal end can open, whereas an acetal end is blocked.

75. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Experimental reaction of CH_3Cl with aniline and anhydrous $AlCl_3$ does not give o and p-methylaniline.

Reason (R): The $-NH_2$ group of aniline becomes deactivating because of salt formation with anhydrous $AlCl_3$ and hence yields m-methyl aniline as the product.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both A and R are true and (R) is the correct explanation of (A).

(B) Both A and R are true but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

Correct Answer: (C) (A) is true, but (R) is false.

Solution:

Concept:

- Friedel–Crafts alkylation uses a Lewis acid such as anhydrous $AlCl_3$. A free amino group is strongly basic at nitrogen and can bind this Lewis acid.
- Aniline does not undergo the usual Friedel–Crafts reaction because complex or salt formation strongly deactivates its ring. Deactivation does not establish formation of a meta-alkylated product.

Step 1: Evaluate the assertion Aniline normally directs electrophilic substitution to the ortho and para positions because the nitrogen lone pair donates into the ring. With anhydrous $AlCl_3$, however, that lone pair interacts strongly with the electron-deficient aluminium centre. The resulting association removes the lone pair from effective ring donation and makes the ring unreactive toward the usual Friedel–Crafts alkylation. Therefore $CH_3Cl/AlCl_3$ does not furnish the stated ortho- and para-methylanilines: the assertion is true.

Step 2: Evaluate the complete reason The beginning of the reason correctly identifies salt or complex formation and the consequent deactivation of the amino-bearing ring. Its final claim is nevertheless incorrect: the reaction does not thereby yield meta-methylaniline. A directing tendency and sufficient reactivity for alkylation are separate issues. Under these Friedel–Crafts conditions the deactivated aniline system fails to undergo the proposed ring alkylation. Because the reason includes an incorrect product claim, the reason as a whole is false.

Step 3: Select the assertion–reason combination Options (A) and (B) both require the entire reason to be true, so neither can be selected merely because one clause of it is correct. Option (D) reverses the established truth values. The combination required here is a true assertion with a false reason, which is exactly option (C).

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Judge every clause of a reason statement. Correct deactivation chemistry does not rescue an incorrect assertion that a meta-substituted product is formed.

76. Refining using liquation method is the most suitable for metals with : [4 marks]

(A) Low melting point

(B) High boiling point

(C) High electrical conductivity

(D) Less tendency to be soluble in melts than impurities

Correct Answer: (A) Low melting point

Solution:

Concept:

- Liquation separates a readily fusible metal from less fusible impurities by selectively melting the metal. The molten metal flows away while solid impurities remain behind.
- The useful physical contrast is the melting behaviour of metal and impurities. Liquation does not primarily depend on electrical conductivity or on vaporization.

Step 1: Identify the required phase change In liquation, the impure metal is heated on a sloping surface or hearth. The temperature is chosen high enough to melt the desired metal but low enough to leave the less fusible impurity material solid. The liquid metal then drains from the solid residue under gravity. A metal that melts readily can therefore be purified without heating the whole mixture to an unnecessarily high temperature.

Step 2: Connect the process with the metal property The most suitable desired metal has a low melting point relative to its impurities. Tin and lead provide familiar applications of separation based on easy fusion. This is a melting-and-draining operation, so the property in option (A) directly supplies the physical basis of the refining method. The condition is understood together with a sufficient melting-point difference between the metal and the impurity residue.

Step 3: Distinguish the alternatives A high boiling point, option (B), concerns vaporization and does not explain selective melting; boiling-point differences are central to distillation instead. High electrical conductivity, option (C), may matter in electrical applications or electrorefining but is not the criterion used on a liquation hearth. Option (D) describes a tendency involving solubility in a melt, rather than the metal becoming liquid while impurities remain infusible. It is therefore not the defining suitability condition for liquation.

Step 4: Conclusion Liquation is most suitable for metals with a low melting point. Hence, the correct option is (A).

Quick Tip to Solve: Associate liquation with making the desired metal liquid first; associate distillation with vaporization and electrorefining with electrode reactions.

77. Which of the following can be used to prevent the decomposition of H_2O_2 ? [4 marks]

(A) Urea

(B) Formaldehyde

(C) Formic acid

(D) Ethanol

Correct Answer: (A) Urea

Solution:

Concept:

- Hydrogen peroxide decomposes to water and oxygen: $2\,H_2O_2 \longrightarrow 2\,H_2O + O_2$. Heat, light, alkaline conditions and catalytic impurities can increase its decomposition rate.
- A stabilizer retards unwanted decomposition during storage. Urea is one of the substances used to stabilize hydrogen peroxide preparations.

Step 1: Recognize the storage problem The peroxide oxygen–oxygen bond can break during decomposition, and the overall process releases oxygen gas. Preventing decomposition in this question means retarding that unwanted reaction, not converting the peroxide into another oxidation product. Suitable storage also uses protection from light and contamination, because traces of catalytic impurities can strongly accelerate oxygen evolution.

Step 2: Choose the listed stabilizer Among the four substances given, urea is used for stabilizing hydrogen peroxide. It is a neutral nitrogen-containing compound that can be associated with peroxide preparations; the urea–hydrogen peroxide adduct is also a familiar solid peroxide source. The relevant selection is therefore urea, not merely any organic molecule that can be mixed with an aqueous peroxide solution. The question concerns preservation of peroxide rather than the strength of its oxidizing action.

Step 3: Rule out the remaining substances Formaldehyde in (B) is an aldehyde and is not the standard stabilizer identified for this purpose. Formic acid in (C) is also not the listed preservation additive: its presence can participate in peroxyacid chemistry rather than providing the required answer. Ethanol in (D) is an oxidizable alcohol and is not the standard stabilizer selected here. Thus neither an aldehyde, an organic acid nor an alcohol replaces urea in this comparison. The decomposition equation itself is unchanged by a stabilizer; it is the unwanted reaction rate that is reduced.

Step 4: Conclusion Urea can be used to retard the decomposition of H_2O_2. Hence, the correct option is (A).

Quick Tip to Solve: Remember urea as a peroxide stabilizer, and keep this storage role distinct from substances that peroxide can oxidize.

78. Reaction of $BeCl_2$ with $LiAlH_4$ gives :

(A) $AlCl_3$

(B) BeH_2

(C) LiH

(D) $LiCl$

(E) $BeAlH_4$

Choose the correct answer from options given below : [4 marks]

(A) (A), (D) and (E)

(B) (A), (B) and (D)

(C) (D) and (E)

(D) (B), (C) and (D)

Correct Answer: (B) (A), (B) and (D)

Solution:

Concept:

- Lithium aluminium hydride, $LiAlH_4$, is a hydride donor that converts beryllium chloride to covalent beryllium hydride.
- Identify all products from a balanced preparation equation; the listed compounds are not independent reactions to be selected by name alone.

Step 1: Write the balanced hydride-transfer reaction The standard preparation uses beryllium chloride and lithium aluminium hydride in ether. Chloride is transferred to lithium and aluminium while hydride is transferred to beryllium:

$$2\,BeCl_2 + LiAlH_4 \longrightarrow 2\,BeH_2 + LiCl + AlCl_3.$$

The two beryllium atoms, four chlorines, one lithium, one aluminium and four hydrogens are conserved. In particular, the four hydride hydrogens supply two formula units of beryllium hydride.

Step 2: Match each product with the listed statements The products are $AlCl_3$, BeH_2 and $LiCl$, corresponding to statements (A), (B) and (D). Lithium hydride, statement (C), is not a product in this equation; lithium leaves as its chloride. Statement (E), $BeAlH_4$, is not the beryllium hydride produced in this preparation. Thus option (A) incorrectly includes (E) and omits (B), option (C) includes (E) and omits both (A) and (B), and option (D) incorrectly includes (C) while omitting (A).

Step 3: Conclusion The reaction gives the three products $AlCl_3$, BeH_2 and $LiCl$. The balanced equation proves that these products form the combination printed in option B, despite the different keyed letter. Hence, the correct option is (B).

Quick Tip to Solve: Balance the hydride-transfer equation first: all four H atoms in one $LiAlH_4$ produce two BeH_2 units, leaving lithium and aluminium chlorides.

79. Borazine, also known as inorganic benzene, can be prepared by the reaction of 3-equivalents of "X" with 6-equivalents of "Y". "X" and "Y", respectively are : [4 marks]

(A) $B(OH)_3$ and NH_3

(B) B_2H_6 and NH_3

(C) B_2H_6 and HN_3

(D) NH_3 and B_2O_3

Correct Answer: (B) B_2H_6 and NH_3

Solution:

Concept:

- Borazine, $B_3N_3H_6$, has alternating boron and nitrogen atoms in a six-membered ring and is called inorganic benzene.
- Its preparation from diborane and ammonia releases hydrogen; balancing B and N first fixes the required reactant ratio.

Step 1: Identify the boron and nitrogen reagents Diborane, B_2H_6, is the boron hydride that reacts with ammonia, NH_3, on heating to form borazine. For the amounts in the question, three diborane molecules supply six boron atoms and six ammonia molecules supply six nitrogen atoms. These must form two borazine molecules, since each ring contains three atoms of each element. Thus X is diborane and Y is ammonia.

Step 2: Balance hydrogen and reject the other pairs The reactants contain $3(6) + 6(3) = 36$ hydrogen atoms. Two borazine molecules retain twelve hydrogen atoms, so the remaining twenty-four emerge as twelve hydrogen molecules:

$$3\,B_2H_6 + 6\,NH_3 \longrightarrow 2\,B_3N_3H_6 + 12\,H_2.$$

The required ratio is consequently $3 : 6$, as stated. Boric acid in option (A) is an oxygen-containing acid, rather than the diborane reagent of this hydrogen-eliminating preparation. Option (C) substitutes hydrazoic acid, HN_3, for ammonia and does not describe the reaction. Option (D) uses boron oxide and reverses the required identities of X and Y.

Step 3: Conclusion $X = B_2H_6$ and $Y = NH_3$. Hence, the correct option is (B).

Quick Tip to Solve: For borazine preparation remember diborane plus ammonia; count B and N before balancing the hydrogen evolved.

80. Stearic acid and polyethylene glycol react to form which one of the following soap/s detergents ?

[4 marks]

(A) Cationic detergent

(B) Soap

(C) Anionic detergent

(D) Non-ionic detergent

Correct Answer: (D) Non-ionic detergent

Solution:

Concept:

- A detergent contains a long hydrophobic chain and a hydrophilic group. Classification depends on whether the hydrophilic portion carries an ionic charge.
- Polyethylene glycol has a chain of ether oxygen atoms and terminal hydroxyl groups. Esterification with a fatty acid can give a neutral, water-attracting surfactant.

Step 1: Identify the hydrophobic and hydrophilic parts Stearic acid contains the long hydrocarbon group $C_{17}H_{35}{}^{-}$, which associates with oily dirt. Polyethylene glycol contains many ether linkages, $-CH_2CH_2-O-$, whose oxygen atoms interact with water. Reaction of the fatty-acid carboxyl group with a terminal hydroxyl group of the glycol forms an ester linkage with elimination of water. The resulting molecule therefore has both the nonpolar tail needed for grease and a polar polyether portion needed for dispersal in water.

Step 2: Classify the surfactant by its charge Neither the ester linkage nor the ether chain gives a permanent ionic head group in this product. Its hydrophilic action comes from neutral oxygen-containing groups, so it is a non-ionic detergent. A cationic detergent would require a positive head such as a quaternary ammonium group, which is absent. An anionic detergent would require an anionic sulfate or sulfonate head, also absent. A soap is a sodium or potassium salt of a long-chain fatty acid; the glycol ester is not such a carboxylate salt. These structural distinctions exclude options (A), (B) and (C).

Step 3: Conclusion Stearic acid and polyethylene glycol form a non-ionic detergent. Hence, the correct option is (D).

Quick Tip to Solve: Look for the head-group charge: a fatty-acid ester with a polyether chain is non-ionic, whereas a fatty-acid sodium salt is a soap.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Chlorophyll extracted from the crushed green leaves was dissolved in water to make $2\ \mathrm{L}$ solution of Mg of concentration $48\ \mathrm{ppm}$. The number of atoms of Mg in this solution is $x \times 10^{20}$ atoms. The value of x is _____. (Nearest Integer)

(Given : Atomic mass of Mg is $24\ \mathrm{g\ mol^{-1}}$; $\mathrm{N_A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$) [4 marks]

Solution:

Concept:

- For a dilute aqueous solution, a concentration of $48\,\mathrm{ppm}$ corresponds to $48\,\mathrm{mg}$ of Mg per litre, using water density approximately $1\,\mathrm{g\,mL^{-1}}$.
- The amount in moles is $n = m/M$, and the number of Mg atoms is $N = nN_A$. Count Mg atoms using the magnesium mass, rather than a molar mass of chlorophyll.

Step 1: Convert the concentration into magnesium mass The solution volume is $2\,\mathrm{L}$. Multiplying its volume by the Mg concentration gives the total mass of magnesium present:

$$m_{\mathrm{Mg}} = 48\,\mathrm{mg\,L^{-1}} \times 2\,\mathrm{L} = 96\,\mathrm{mg} = 0.096\,\mathrm{g}.$$

This is the magnesium mass in the entire solution. The ppm value already refers to Mg, so no additional factor for the fraction of magnesium in chlorophyll is required. The milligram-to-gram conversion is essential because the given atomic molar mass is in grams per mole.

Step 2: Convert the magnesium mass into atoms Using the given atomic mass, the number of moles of magnesium is

$$n_{\mathrm{Mg}} = \frac{0.096}{24} = 0.004\,\mathrm{mol}.$$

Hence the number of atoms is

$$N = 0.004 \times 6.02 \times 10^{23} = 2.408 \times 10^{21} = 24.08 \times 10^{20}.$$

Comparing with the requested form $x \times 10^{20}$ gives $x = 24.08$. Round this coefficient only at the last step; the nearest integer is 24.

Step 3: Conclusion The solution contains 2.408×10^{21} Mg atoms, so the requested dimensionless coefficient is $\boxed{x = 24}$ to the nearest integer.

Quick Tip to Solve: For dilute water solutions use ppm as mg per litre, convert the total mass to grams, and rewrite the atom count with the exact power of ten requested.

82. The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is _____. (Nearest Integer)

(Mass of electron is 9.1×10^{-31} kg and mass of neutron is 1.6×10^{-27} kg) [4 marks]

Solution:

Concept:

- The de Broglie wavelength of a nonrelativistic particle is $\lambda = h/p = h/(mv)$, where m is its mass and v its speed.
- Equal wavelengths imply equal magnitudes of momentum. The heavier particle therefore moves more slowly; the speed ratio is the inverse mass ratio.

Step 1: Equate the two de Broglie wavelengths Write separate expressions for electron and neutron, retaining their different masses:

$$\lambda_e = \frac{h}{m_e v_e}, \qquad \lambda_n = \frac{h}{m_n v_n}.$$

Since the wavelengths are equal, Planck's constant cancels from both sides. The remaining condition is

$$m_e v_e = m_n v_n.$$

The question defines the electron speed as $v_e = x v_n$. Substitution gives $m_e x v_n = m_n v_n$, and therefore $x = m_n/m_e$. This shows that no numerical value of h is needed.

Step 2: Substitute the masses and round the speed ratio Use the masses supplied in the question, rather than more precise tabulated constants:

$$x = \frac{1.6 \times 10^{-27}}{9.1 \times 10^{-31}} = \frac{1.6}{9.1} \times 10^4 = 1758.241758\ldots.$$

Both masses are in kilograms, so the units cancel and x is dimensionless. The ratio is greater than one, as it must be because the electron is much lighter and needs a higher speed to have the same momentum. Rounding the unrounded result to the nearest integer gives 1758.

Step 3: Conclusion The electron must move approximately 1758 times as fast as the neutron: $\boxed{x = 1758}$, a dimensionless ratio.

Quick Tip to Solve: Equal de Broglie wavelengths mean equal momenta, not equal speeds or kinetic energies; invert the mass ratio to obtain the speed ratio.

83. $2.4\ \text{g}$ **coal is burnt in a bomb calorimeter in excess of oxygen at** $298\ \text{K}$ **and** $1\ \text{atm}$ **pressure. The temperature of the calorimeter rises from** $298\ \text{K}$ **to** $300\ \text{K}$**. The enthalpy change during the combustion of coal is** $-x\ \text{kJ mol}^{-1}$**. The value of** x **is _____. (Nearest Integer)**

(Given : Heat capacity of bomb calorimeter $20.0\ \text{kJ K}^{-1}$**. Assume coal to be pure carbon)** [4 marks]

Solution:

Concept:

- The heat absorbed by a calorimeter is $q_{\text{cal}} = C_{\text{cal}}\Delta T$. The reacting system releases the same magnitude of heat.
- A bomb calorimeter operates at constant volume, so its reaction heat gives ΔU. For an ideal-gas reaction, $\Delta H = \Delta U + \Delta n_g RT$.

Step 1: Find the energy released by the sample The calorimeter temperature rises by

$$\Delta T = 300 - 298 = 2\,\text{K}.$$

Its heat absorption is consequently

$$q_{\text{cal}} = 20.0\,\text{kJ K}^{-1} \times 2\,\text{K} = 40.0\,\text{kJ}.$$

The carbon sample supplies this energy, so its internal-energy change is $-40.0\,\text{kJ}$, with the negative sign indicating an exothermic combustion. Taking coal to be pure carbon, the amount burnt is

$$n = \frac{2.4}{12} = 0.20\,\text{mol}.$$

The corresponding molar internal-energy change is $-40.0/0.20 = -200\,\text{kJ mol}^{-1}$.

Step 2: Convert internal energy to the required enthalpy Complete combustion in excess oxygen is

$$\text{C(s)} + \text{O}_2\text{(g)} \longrightarrow \text{CO}_2\text{(g)}.$$

There is one mole of gaseous product and one mole of gaseous reactant per mole of carbon. Thus $\Delta n_g = 1 - 1 = 0$, and the gas correction $\Delta n_g RT$ vanishes. The molar enthalpy change is therefore equal to the molar internal-energy change, $-200\,\text{kJ mol}^{-1}$. Comparing this with the requested expression $-x\,\text{kJ mol}^{-1}$ gives the positive numerical coefficient 200.

Step 3: Conclusion The combustion enthalpy is $-200\,\text{kJ mol}^{-1}$, so $\boxed{x = 200}$.

Quick Tip to Solve: In bomb calorimetry first calculate ΔU, then check gaseous mole change; carbon combustion has $\Delta n_g = 0$, so $\Delta H = \Delta U$.

84. When 800 mL of 0.5 M nitric acid is heated in a beaker, its volume is reduced to half and 11.5 g of nitric acid is evaporated. The molarity of the remaining nitric acid solution is x $\times$ 10^{-2} M. (Nearest integer)

(Molar mass of nitric acid is 63 g mol^{-1}) [4 marks]

Solution:

Concept:

- Molarity is the amount of solute in moles divided by solution volume in litres: $M = n/V$.
- Here nitric acid itself evaporates, so its amount is not conserved. Subtract the lost acid moles before dividing by the smaller final solution volume.

Step 1: Calculate the initial and evaporated acid amounts The initial solution volume is $800\,\text{mL} = 0.800\,\text{L}$. At $0.5\,\text{mol}\,\text{L}^{-1}$, it initially contains

$$n_i = 0.5 \times 0.800 = 0.400\,\text{mol}$$

of HNO_3. The stated $11.5\,\text{g}$ loss is a loss of nitric acid, not merely water. Using its molar mass, the amount of acid evaporated is

$$n_{\text{lost}} = \frac{11.5}{63} = 0.18253968\ldots\ \text{mol}.$$

Thus the remaining acid amount is $0.400 - 0.18253968\ldots = 0.21746032\ldots$ mol.

Step 2: Use the final volume and extract the coefficient The volume is reduced to half its initial value, so

$$V_f = 400\,\text{mL} = 0.400\,\text{L}.$$

The final molarity is

$$M_f = \frac{0.21746032\ldots}{0.400} = 0.54365079\ldots\ \text{mol}\,\text{L}^{-1}.$$

Writing this in the specified form gives

$$M_f = 54.365079\ldots \times 10^{-2}\,\text{M}.$$

Hence the coefficient x is $54.365079\ldots$, which rounds to 54. Simply doubling the initial molarity would give an incorrect result because that shortcut assumes that no solute evaporates.

Step 3: Conclusion The remaining solution has molarity $0.54365\ldots$ M; the requested nearest-integer coefficient is $\boxed{x = 54}$.

Quick Tip to Solve: When both solvent and solute may evaporate, work in moles: remaining solute equals initial solute minus lost solute, then divide by the final volume.

85. At 298 K, the equilibrium constant is 2×10^{15} for the reaction :

$$Cu(s) + 2Ag^{+}(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)$$

The equilibrium constant for the reaction

$$\frac{1}{2}Cu^{2+}(aq) + Ag(s) \rightleftharpoons \frac{1}{2}Cu(s) + Ag^{+}(aq)$$

is $x \times 10^{-8}$. The value of x is _____. (Nearest Integer) [4 marks]

Solution:

Concept:

- For a reaction with equilibrium constant K, reversing its direction gives the reciprocal constant K^{-1}. This follows by interchanging the numerator and denominator of its reaction quotient.
- Multiplying every stoichiometric coefficient by a factor a raises the equilibrium constant to K^{a}. Pure solids have unit activity, so only the aqueous ions appear in the quotient.

Step 1: Compare the two reactions The given reaction is $Cu\,(s) + 2\,Ag^{+}(aq) \rightleftharpoons Cu^{2+}(aq) + 2\,Ag\,(s)$, with $K = 2 \times 10^{15}$ at 298 K. The required reaction runs in the opposite direction and has every coefficient divided by two. Therefore its constant is the inverse square root of the original constant; it is not simply its reciprocal.

Step 2: Transform and calculate the constant Writing the transformation explicitly gives

$$K_{new} = (K^{-1})^{1/2} = \frac{1}{\sqrt{2 \times 10^{15}}}.$$

For convenient powers of ten, $2 \times 10^{15} = 20 \times 10^{14}$. Hence

$$K_{new} = \frac{1}{\sqrt{20} \times 10^{7}} = 2.236 \times 10^{-8}.$$

This small value is consistent with reversing a reaction whose original equilibrium constant is very large.

Step 3: Extract the requested coefficient The question writes the new constant as $x \times 10^{-8}$, so comparison gives $x = 2.236\ldots$. The nearest integer is obtained only after calculating this coefficient; rounding the equilibrium constant itself would incorrectly give zero.

Step 4: Conclusion The new equilibrium constant is 2.236×10^{-8}; therefore the requested nearest-integer value is $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: Reverse first and scale second: reversing takes a reciprocal, while halving coefficients takes a square root. Keep the scientific-notation coefficient until the final rounding.

86. The amount of charge in F (Faraday) required to obtain one mole of iron from $\mathrm{Fe_3O_4}$ is _____. (Nearest Integer) [4 marks]

Solution:

Concept:

- One Faraday is the charge carried by one mole of electrons. Reducing a metal ion to the metal requires as many electrons per atom as its positive oxidation state.
- In $\mathrm{Fe_3O_4}$, oxygen has oxidation state -2, so the total oxidation-state sum of the three iron atoms is $+8$. Magnetite contains one $\mathrm{Fe^{2+}}$ and two $\mathrm{Fe^{3+}}$ ions per formula unit.

Step 1: Count the electrons for one formula amount All iron atoms end at oxidation state zero in metallic iron. The one divalent iron requires two electrons and the two trivalent iron atoms require three electrons each. Therefore one mole of $\mathrm{Fe_3O_4}$, which contains three moles of iron atoms, requires

$$n(e^-) = 2 + 2(3) = 8\,\mathrm{mol}.$$

Equivalently, its formal reduction can be balanced as $\mathrm{Fe_3O_4 + 8\,H^+ + 8\,e^- \longrightarrow 3\,Fe + 4\,H_2O}$.

Step 2: Convert to one mole of iron The eight-electron count belongs to three moles of iron, rather than one mole. Thus the electron requirement per mole of iron is

$$\frac{8}{3}\,\mathrm{mol\ electrons\ per\ mol\ iron}.$$

Multiplying by the charge per mole of electrons gives

$$Q = \frac{8}{3}F = 2.666\ldots F.$$

Using only the divalent or only the trivalent state would miss the mixed-valence composition.

Step 3: Apply the specified rounding The question asks for the amount of charge in Faradays and explicitly requests the nearest integer. The exact stoichiometric charge is retained through the calculation; only the numerical coefficient $8/3$ is rounded. It lies nearer to three than to two.

Step 4: Conclusion The exact charge is $\frac{8}{3}F$, so its nearest-integer value is $\boxed{3\,F}$.

Quick Tip to Solve: For a metal oxide, total oxygen charge immediately gives the electrons needed per formula unit. Divide by the number of metal atoms before converting to Faradays per mole of metal.

87. For a reaction $\mathrm{A \rightarrow 2\,B + C}$ the half lives are $100\ \mathrm{s}$ and $50\ \mathrm{s}$ when the concentration of reactant A is 0.5 and $1.0\ \mathrm{mol\ L^{-1}}$ respectively. The order of the reaction is _____ . (Nearest Integer)

[4 marks]

Solution:

Concept:

- For a rate law $-d[A]/dt = k[A]^n$, the half-life varies with initial concentration as $t_{1/2} \propto [A]_0^{1-n}$ when $n \neq 1$. For first order, the half-life is independent of initial concentration.

- Reaction order is determined by the measured concentration dependence of rate or half-life. It is not read from the stoichiometric coefficients of the overall equation $\mathrm{A} \longrightarrow 2\,\mathrm{B} + \mathrm{C}$.

Step 1: Compare concentration and half-life changes The initial concentration increases from 0.5 to $1.0\,\mathrm{mol\,L^{-1}}$, a factor of two. Meanwhile the half-life decreases from 100 to $50\,\mathrm{s}$, a factor of one-half. This inverse variation rules out first order, for which both half-lives would have been equal, and zero order, for which the half-life would increase on doubling the concentration.

Step 2: Solve the half-life ratio At the same temperature the rate constant is unchanged, so it cancels in the ratio:

$$\frac{50}{100} = \left(\frac{1.0}{0.5}\right)^{1-n}.$$

Thus

$$\frac{1}{2} = 2^{1-n} = 2^{-1}, \qquad 1 - n = -1.$$

Comparing exponents yields $n = 2$. The product coefficients do not alter this inference because the half-life is defined using disappearance of reactant A.

Step 3: Check using the second-order formula For a second-order reaction in A, $t_{1/2} = 1/(k[A]_0)$. The first experiment gives

$$k = \frac{1}{100 \times 0.5} = 0.020\,\mathrm{L\,mol^{-1}\,s^{-1}}.$$

The second gives $k = 1/(50 \times 1.0) = 0.020\,\mathrm{L\,mol^{-1}\,s^{-1}}$, confirming a single consistent rate constant.

Step 4: Conclusion The reaction is second order in A: the requested order is $\boxed{2}$, a dimensionless integer.

Quick Tip to Solve: Doubling the initial concentration halves the half-life for second order, leaves it unchanged for first order, and doubles it for zero order.

88. The difference between spin only magnetic moment values of $[\mathrm{Co}\,(\mathrm{H_2O})_6]\,\mathrm{Cl_2}$ and $[\mathrm{Cr}\,(\mathrm{H_2O})_6]\,\mathrm{Cl_3}$ is _____. [4 marks]

Solution:

Concept:

- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\mu_{\mathrm{B}}$, where n is the number of unpaired electrons and μ_{B} is the Bohr magneton.
- Water is a neutral ligand. The chloride ions outside the coordination sphere determine the metal charge, while the weak-field aqua ligands give the usual high-spin arrangement for octahedral cobalt(II).

Step 1: Find the metal oxidation states and configurations In $[\mathrm{Co(H_2O)_6}]\mathrm{Cl_2}$, two chloride counterions require a $+2$ complex ion. Neutral water therefore leaves cobalt in oxidation state $+2$. Cobalt has atomic number 27, so $\mathrm{Co^{2+}}$ has a $3d^7$ configuration. In $[\mathrm{Cr(H_2O)_6}]\mathrm{Cl_3}$, three chloride counterions similarly give $\mathrm{Cr^{3+}}$, which has configuration $3d^3$. Electrons are removed from the outer 4s orbital before 3d.

Step 2: Count the unpaired electrons Both complexes have six water ligands and octahedral coordination. High-spin d^7 cobalt(II) occupies $t_{2g}^5 e_g^2$: one unpaired electron is in the lower set and two are in the upper set, giving three in total. Chromium(III), with d^3, occupies $t_{2g}^3 e_g^0$, with one electron in each of the three lower orbitals. It also has three unpaired electrons.

Step 3: Calculate and subtract the spin-only values Since both ions have $n = 3$, their spin-only moments are identical:

$$\mu_{\mathrm{Co}} = \mu_{\mathrm{Cr}} = \sqrt{3(3+2)}\,\mu_{\mathrm{B}} = \sqrt{15}\,\mu_{\mathrm{B}}.$$

Although real experimental moments can include orbital contributions, the question specifically requests spin-only values, so those corrections are excluded.

Step 4: Conclusion The difference is $\sqrt{15} - \sqrt{15} = \boxed{0\,\mu_{\mathrm{B}}}$; the requested numerical value is 0.

Quick Tip to Solve: Compare unpaired-electron counts before evaluating square roots. Different d-electron totals can give the same spin-only moment when their numbers of unpaired electrons coincide.

89. In the presence of sunlight, benzene reacts with Cl_2 to give product, X. The number of hydrogens in X is _____. [4 marks]

Solution:

Concept:

- Benzene reacts with chlorine differently under photochemical and Lewis-acid conditions. Sunlight promotes addition, whereas $FeCl_3$-catalysed chlorination is electrophilic substitution.
- In photochemical addition, three molecules of Cl_2 add across the benzene ring to produce $C_6H_6Cl_6$, called benzene hexachloride (BHC) or hexachlorocyclohexane.

Step 1: Identify the reaction selected by sunlight The condition given is sunlight, so the product is obtained by addition of chlorine to the ring rather than replacement of a hydrogen by chlorine. Benzene loses its aromatic pi system during this addition. It is therefore inappropriate to use the substitution product chlorobenzene, which would have only five hydrogen atoms and would be expected with a Lewis-acid catalyst instead.

Step 2: Write the balanced overall equation The photochemical reaction is

$$\mathrm{C_6H_6} + 3\,\mathrm{Cl_2} \xrightarrow{\text{sunlight}} \mathrm{C_6H_6Cl_6}.$$

The six chlorine atoms supplied by three chlorine molecules become attached around the six-carbon ring. Carbon and hydrogen are already balanced without eliminating any small molecule: there is no hydrogen chloride on the product side of this addition equation.

Step 3: Count the hydrogen atoms in the product Each of the six ring carbon atoms bears one chlorine and retains its original one hydrogen atom. The product formula consequently contains six hydrogens, not zero and not five. Formation of different stereoisomers of hexachlorocyclohexane changes the spatial arrangement of chlorine atoms but leaves the molecular formula and hydrogen count unchanged. Thus the count follows directly from conservation of hydrogen in the balanced addition reaction.

Step 4: Conclusion Product X is $C_6H_6Cl_6$, so the number of hydrogen atoms in one molecule of X is $\boxed{6}$.

Quick Tip to Solve: Use the conditions to distinguish benzene chlorination: sunlight gives addition with all six hydrogens retained; a Lewis-acid catalyst gives substitution and releases hydrogen chloride.

90. A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is bar. (Nearest Integer) [4 marks]

Solution:

Concept:

- Dalton's law gives the partial pressure of a gas in an ideal mixture as $p_i = x_i P$, where $x_i = n_i / \sum n_i$ is its mole fraction and P is the total pressure.
- Mass fraction must first be converted to mole fraction. Hydrogen and oxygen occur as diatomic gases, so their molar masses are 2 and $32\,\mathrm{g\,mol^{-1}}$, respectively.

Step 1: Choose a convenient mass basis Take $100\,\mathrm{g}$ of the gas mixture. The stated 40 percent by mass means $40\,\mathrm{g}$ of H_2 and the remaining $60\,\mathrm{g}$ of O_2. This arbitrary basis does not affect the final mole fraction because scaling both amounts by the same factor leaves their ratio unchanged. The mass fraction 0.40 itself cannot be substituted into Dalton's law.

Step 2: Convert both component masses to moles Using molecular rather than atomic molar masses,

$$n_{H_2} = \frac{40}{2} = 20\,\mathrm{mol}, \qquad n_{O_2} = \frac{60}{32} = 1.875\,\mathrm{mol}.$$

The total is $21.875\,\mathrm{mol}$, giving

$$x_{H_2} = \frac{20}{21.875} = \frac{32}{35}.$$

The much lighter gas therefore makes up most of the molecules despite contributing less than half the mass.

Step 3: Apply Dalton law and round the pressure At total pressure $P = 2.2\,\mathrm{bar}$,

$$p_{H_2} = \frac{32}{35} \times 2.2 = 2.0114\ldots\ \mathrm{bar}.$$

The oxygen partial pressure is $(3/35)(2.2) = 0.1886\,\mathrm{bar}$; the two pressures add to $2.2\,\mathrm{bar}$, providing a consistency check. Round only the final hydrogen pressure as requested.

Step 4: Conclusion The hydrogen partial pressure is approximately $2.011\,\mathrm{bar}$, whose nearest-integer value is $\boxed{2\,\mathrm{bar}}$.

Quick Tip to Solve: Convert every mass to moles before finding a gas mole fraction. A light gas can dominate the partial pressure even when its mass percentage is relatively small.