JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 26 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 26 July 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The minimum value of the sum of the squares of the roots of $x^2 + (3 - a)x + 1 = 2a$ is: [4 marks]

(A) 4

(B) 5

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Concept:

- For a monic quadratic $x^2 + bx + c = 0$ with roots α, β, Vieta's relations give $\alpha + \beta = -b$ and $\alpha\beta = c$.
- The required symmetric expression is $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$. A real square is nonnegative, so completing the square identifies its minimum and the equality case.

Step 1: Put the equation in standard form Move the constant on the right to the left, without changing the coefficient of x:

$$x^2 + (3 - a)x + (1 - 2a) = 0.$$

Thus the sum of its roots is $a - 3$, while their product is $1 - 2a$. In particular, the product is not the constant 1 visible on the original left-hand side; the right-hand side contributes to it.

Step 2: Complete the square in the parameter Substitute the two Vieta relations into the requested expression:

$$\begin{aligned} \alpha^2 + \beta^2 &= (a - 3)^2 - 2(1 - 2a) \\ &= a^2 - 6a + 9 - 2 + 4a \\ &= a^2 - 2a + 7 = (a - 1)^2 + 6. \end{aligned}$$

The term $(a - 1)^2$ is nonnegative for every real a, and it vanishes at $a = 1$. This establishes both the lower bound and the parameter attaining it.

Step 3: Check attainability and compare the options At $a = 1$, the quadratic becomes $x^2 + 2x - 1 = 0$. Its discriminant is $2^2 - 4(1)(-1) = 8 > 0$, so the equality case is valid even if real roots are required. The roots are $-1 \pm \sqrt{2}$, whose squared sum is 6. Values 4 and 5 lie below the established bound. Although 8 can occur for other parameters, it is not the minimum.

Step 4: Conclusion The minimum sum of the squares of the roots is $\boxed{6}$, attained when $a = 1$. Hence, the correct option is (C).

Quick Tip to Solve: First move every term to one side before applying Vieta: here the root product is $1 - 2a$, not 1. Then complete the square in a.

2. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then : [4 marks]

(A) $x + 2y - 4 = 0$

(B) $x^2 + y - 4 = 0$

(C) $x + 2y + 4 = 0$

(D) $x^2 - y + 3 = 0$

Correct Answer: (C) $x + 2y + 4 = 0$

Solution:

Concept:

- The modulus condition $|z| = 2$, for $z = x + iy$, gives the circle $x^2 + y^2 = 4$.
- Equality of distances from i and $-5i$ gives their perpendicular bisector. Equivalently, square the two nonnegative moduli and simplify their Cartesian expressions.

Step 1: Convert the modulus conditions to equations The first condition gives

$$\sqrt{x^2 + y^2} = 2 \quad \Longrightarrow \quad x^2 + y^2 = 4.$$

For the second condition, write

$$|z - i| = \sqrt{x^2 + (y - 1)^2}, \qquad |z + 5i| = \sqrt{x^2 + (y + 5)^2}.$$

Both square roots are nonnegative, so squaring their equality introduces no extraneous sign possibility.

Step 2: Find the intersection of the loci Subtract the equal squared distances:

$$\begin{aligned} x^2 + (y - 1)^2 &= x^2 + (y + 5)^2, \\ y^2 - 2y + 1 &= y^2 + 10y + 25, \\ -12y = 24 \quad &\Longrightarrow \quad y = -2. \end{aligned}$$

Substituting into $x^2 + y^2 = 4$ gives $x^2 + 4 = 4$, hence $x = 0$. Therefore the only possible complex number is $z = -2i$, rather than an entire line or circle.

Step 3: Test the proposed equations For $(x, y) = (0, -2)$, the expression in option (C) is $x + 2y + 4 = 0 - 4 + 4 = 0$. In contrast, the left-hand sides in (A), (B), and (D) become respectively

$$-8, \qquad -6, \qquad 5,$$

so none of those equations is satisfied. As a direct check, $|-2i| = 2$ and the distances from $-2i$ to i and $-5i$ are both 3, confirming the original conditions.

Step 4: Conclusion The point is $(0, -2)$, which satisfies $\boxed{x + 2y + 4 = 0}$. Hence, the correct option is (C).

Quick Tip to Solve: Equal distances from two points on the imaginary axis fix their midpoint ordinate immediately; intersect that horizontal line with the modulus circle.

3.

Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$**, then the value of** $A'BA$ **is:**

[4 marks]

(A) 1224

(B) 1042

(C) 540

(D) 539

Correct Answer: (D) 539

Solution:

Concept:

- The prime denotes transpose: for $A = (1, 1, 1)'$, $A' = (1, 1, 1)$. The product $A'BA$ has size 1×1, so its value is a scalar.
- Right multiplication by a column of ones adds each row of B; left multiplication by the row of ones then adds those row sums. Thus $A'BA$ is the sum of all nine entries.

Step 1: Evaluate the signed entries carefully A minus sign before a square is outside the power: $-10^2 = -(10^2) = -100$, not $(-10)^2$. The same convention applies to -14^2 and -15^2. Therefore

$$B = \begin{pmatrix} 81 & -100 & 121 \\ 144 & 169 & -196 \\ -225 & 256 & 289 \end{pmatrix}.$$

Preserving these signs is essential because the desired scalar uses every entry of the matrix.

Step 2: Multiply by the column of ones Compute the three row sums to obtain

$$BA = \begin{pmatrix} 81 - 100 + 121 \\ 144 + 169 - 196 \\ -225 + 256 + 289 \end{pmatrix} = \begin{pmatrix} 102 \\ 117 \\ 320 \end{pmatrix}.$$

This multiplication is valid because B is 3×3 and A is 3×1. The resulting column contains the contributions of the three rows separately, allowing the arithmetic to be checked before the final addition.

Step 3: Take the final scalar product Now premultiply the column by A':

$$A'BA = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 102 \\ 117 \\ 320 \end{pmatrix} = 102 + 117 + 320 = 539.$$

As an independent check, the positive squares sum to $81 + 121 + 144 + 169 + 256 + 289 = 1060$; the magnitudes of the negative terms sum to $100 + 196 + 225 = 521$, giving $1060 - 521 = 539$. Hence 1224, 1042, and 540 do not equal the matrix product; the last differs by an arithmetic error of one.

Step 4: Conclusion The matrix product is $\boxed{A'BA = 539}$. Hence, the correct option is (D).

Quick Tip to Solve: When both outer vectors consist entirely of ones, add the matrix entries directly, keeping minus signs outside squares such as -10^2.

4. Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y=x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval : [4 marks]

(A) $\left(0, \frac{1}{4}\right)$
(B) $\left(\frac{1}{2}, \frac{3}{4}\right)$
(C) $\left(\frac{1}{4}, \frac{1}{2}\right)$
(D) $\left(\frac{3}{4}, 1\right)$

Correct Answer: (C) $\left(\frac{1}{4}, \frac{1}{2}\right)$

Solution:

Concept:

- The circle has centre $C=(1,-1)$ and radius 1. For a point Q outside it, the closest circle point lies on the ray CQ, at distance 1 from C. Its distance from Q is $CQ-1$.
- Parametrise the parabola by $Q=(t,t^2)$. Minimising $CQ-1$ is equivalent to minimising the squared distance CQ^2. A derivative test and exact bounds locate the required abscissa without solving a cubic explicitly.

Step 1: Reduce the problem to one parameter For $Q=(t,t^2)$, put $d=CQ$. Then

$$d^2=(t-1)^2+(t^2+1)^2=1+(t-1)^2+2t^2+t^4>1.$$

The strict inequality holds because all three nonconstant terms cannot vanish together. Thus every point of the parabola lies outside the circle. For each such Q, the minimum distance to a circle point is $d-1$, so minimising d^2 indeed minimises the original distance.

Step 2: Locate the unique minimising parameter Differentiate the squared distance:

$$\frac{d}{dt}(d^2)=2(t-1)+4t(t^2+1)=2(2t^3+3t-1).$$

Let $g(t)=2t^3+3t-1$. Since $g'(t)=6t^2+3>0$, it has a unique zero t_0; the squared distance decreases before that zero and increases after it. Moreover,

$$g(3/10)=-23/500<0, \qquad g(1/3)=2/27>0,$$

so $3/10<t_0<1/3$. This gives the global minimum, since the squared distance also tends to infinity at either end.

Step 3: Locate the abscissa of the circle point At the stationary point, $t_0-1=-2t_0(t_0^2+1)$.

Consequently

$$d = (t_0^2 + 1)\sqrt{1 + 4t_0^2}, \qquad x_P = 1 + \frac{t_0 - 1}{d} = 1 - \frac{2t_0}{\sqrt{1 + 4t_0^2}}.$$

The last expression decreases with t_0. Our bounds imply

$$1 - \frac{2}{\sqrt{13}} < x_P < 1 - \frac{3}{\sqrt{34}}.$$

The lower bound exceeds $1/4$ and the upper bound is below $1/2$. Thus P, not merely Q, has abscissa in the third interval. The other three intervals are disjoint from this range.

Step 4: Conclusion The minimising abscissa of P lies in $\boxed{(1/4, 1/2)}$. Hence, the correct option is (C).

Quick Tip to Solve: Minimise the distance from the parabola to the circle centre first, then move one radius toward the parabola to obtain the circle point. Do not confuse their abscissae.

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5. If the maximum value of a, for which the function $f_a(x) = \tan^{-1} 2x - 3ax + 7$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to : [4 marks]

(A) $8 - \frac{9\pi}{4(9+\pi^2)}$

(B) $8 - \frac{4\pi}{9(4+\pi^2)}$

(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$

(D) $8 - \frac{\pi}{4}$

Correct Answer: No listed option equals the value of the inverse-tangent function as printed.

Solution:

Concept:

- For a differentiable function, being non-decreasing on an interval is equivalent here to $f_a\prime(x) \geq 0$ throughout that interval. The derivative of $\tan^{-1}(2x)$ is $2/(1 + 4x^2)$.
- The notation $\tan^{-1}(2x)$ denotes the inverse tangent. Its value at $x = \pi/8$ is $\tan^{-1}(\pi/4)$, which must not be replaced by $\tan(\pi/4) = 1$.

Step 1: Find the upper bound for the parameter Differentiate the function as printed:

$$f_a\prime(x) = \frac{2}{1 + 4x^2} - 3a.$$

On $(-\pi/6, \pi/6)$, the first term decreases as $|x|$ increases. Its infimum, approached at the excluded endpoints, is

$$\frac{2}{1 + \pi^2/9} = \frac{18}{9 + \pi^2}.$$

Therefore $3a \leq 18/(9 + \pi^2)$, or $a \leq 6/(9 + \pi^2)$. Equality is allowed: the derivative remains positive at every interior point. Any larger value makes the derivative negative sufficiently near an endpoint.

Step 2: Evaluate the function without changing the inverse tangent The maximum parameter is

consequently $\bar{a} = 6/(9+\pi^2)$. Direct substitution gives

$$f_{\bar{a}}(\pi/8) = \tan^{-1}(\pi/4) - 3\frac{6}{9+\pi^2}\frac{\pi}{8} + 7$$
$$= 7 + \tan^{-1}(\pi/4) - \frac{9\pi}{4(9+\pi^2)}.$$

This is approximately 7.291. Since $\tan^{-1}(\pi/4) \neq 1$, it is not option (A), which replaces the inverse-tangent contribution by 1.

Step 3: Compare the remaining choices The listed values are approximately 7.625 for (A), 7.899 for (B), 4.608 for (C), and 7.215 for (D). None equals the value obtained from the printed inverse-tangent function. In particular, (D) would follow from the different function $\tan(2x) - 3ax + 7$, but substituting that function would change the question. Thus a unique listed option cannot be justified. The final-key choice (A) disagrees with direct differentiation and substitution for the literal expression.

Step 4: Conclusion For the function as printed, $\boxed{f_{\bar{a}}(\pi/8) = 7 + \tan^{-1}(\pi/4) - \frac{9\pi}{4(9+\pi^2)}}$. None of (A)–(D) is correct; the key choice (A) omits the distinction between $\tan^{-1}(\pi/4)$ and 1.

Quick Tip to Solve: Keep inverse tangent and tangent distinct during substitution: $\tan^{-1}(\pi/4)$ is an angle, whereas $\tan(\pi/4) = 1$.

6. Let $\beta = \lim_{x\to 0} \frac{\alpha x - (e^{3x}-1)}{\alpha x(e^{3x}-1)}$ for some $\alpha \in \mathbb{R}$. Then the value of $\alpha + \beta$ is : [4 marks]

(A) $\frac{14}{5}$
(B) $\frac{3}{2}$
(C) $\frac{5}{2}$
(D) $\frac{7}{2}$

Correct Answer: (C) $\frac{5}{2}$

Solution:

Concept:

- A finite limit at zero requires the leading powers of numerator and denominator to balance. The constant parameter α must first be determined from this requirement.
- The exponential expansion is $e^{3x} - 1 = 3x + \frac{9}{2}x^2 + O(x^3)$. Terms through the second power are needed because the denominator contains a product of two quantities that vanish linearly.

Step 1: Identify the possible parameter For $\alpha = 0$, the displayed quotient is undefined throughout its punctured domain, so this choice cannot define the stated real limit. For nonzero α, substitution of the expansion gives

$$\frac{(\alpha-3)x - \frac{9}{2}x^2 + O(x^3)}{3\alpha x^2 + O(x^3)}.$$

If $\alpha \neq 3$, the leading quotient contains $(\alpha-3)/(3\alpha x)$. This has no finite two-sided limit. Therefore the first-order numerator term must cancel, forcing $\alpha = 3$.

Step 2: Evaluate the remaining second-order ratio With the necessary value substituted, the numerator starts at order x^2, exactly as the denominator does:

$$\beta = \lim_{x\to 0} \frac{-\frac{9}{2}x^2 + O(x^3)}{9x^2 + O(x^3)} = -\frac{1}{2}.$$

The minus sign is essential: the positive quadratic term of the exponential is subtracted in the numerator. Dividing numerator and denominator by x^2 justifies taking the ratio of these leading coefficients.

Step 3: Combine the two constants and compare The requested quantity is the sum, not either constant separately:

$$\alpha + \beta = 3 - \frac{1}{2} = \frac{5}{2}.$$

Option (B) would require the wrong negative correction, while (D) corresponds to adding a positive $1/2$. Option (A) is also incompatible with the exact second-order coefficient ratio.

Step 4: Conclusion The finite limit forces $\alpha = 3$ and gives $\beta = -\frac{1}{2}$. Thus $\alpha + \beta = \boxed{\frac{5}{2}}$. Hence, the correct option is (C). A supplied key value of $14/5$, is inconsistent with the exact expansion of this printed limit.

Quick Tip to Solve: Cancel the linear numerator term before evaluating a parameter-dependent limit; retain the next exponential term to determine its finite value.

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7. The value of $\log_e 2\frac{d}{dx}\left(\log_{\cos x} \operatorname{cosec} x\right)$ at $x = \frac{\pi}{4}$ is [4 marks]

(A) $-2\sqrt{2}$

(B) $2\sqrt{2}$

(C) -4

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- The change-of-base formula is $\log_a b = \ln b / \ln a$, valid for $a > 0$, $a \neq 1$, and $b > 0$. These conditions hold near $x = \pi/4$ for $a = \cos x$, $b = \operatorname{cosec} x$.
- Use $\ln(\operatorname{cosec} x) = -\ln(\sin x)$, then differentiate by the quotient rule. The factor $\ln 2$ multiplies the derivative after it is evaluated.

Step 1: Rewrite the variable-base logarithm Let $u(x) = \log_{\cos x}(\operatorname{cosec} x)$. Changing to natural logarithms gives

$$u(x) = \frac{-\ln(\sin x)}{\ln(\cos x)}.$$

The base depends on x, so treating it as a constant when differentiating would miss a denominator-derivative term. Also, at $\pi/4$, both sine and cosine are positive and the base is neither 0 nor 1, so this

representation and differentiation are legitimate.

Step 2: Apply the quotient rule Set $N = -\ln(\sin x)$ and $D = \ln(\cos x)$. Then $N' = -\cot x$ and $D' = -\tan x$, so

$$u'(x) = \frac{-\cot x \ln(\cos x) - \ln(\sin x)\tan x}{[\ln(\cos x)]^2}.$$

The second numerator term is negative because $ND' = \ln(\sin x)\tan x$ is subtracted in $(N'D - ND')/D^2$. Keeping this sign avoids a false cancellation at equal sine and cosine.

Step 3: Evaluate and include the external factor At $x = \pi/4$, $\tan x = \cot x = 1$ and

$$\ln(\sin x) = \ln(\cos x) = -\tfrac{1}{2}\ln 2.$$

Thus the numerator is $\ln 2$, the denominator is $(\ln 2)^2/4$, and

$$u'(\pi/4) = \frac{4}{\ln 2}, \qquad (\ln 2)u'(\pi/4) = 4.$$

The derivative is positive, excluding (A) and (C). No factor $\sqrt{2}$ survives because the logarithmic derivatives involve $\tan x$ and $\cot x$, both equal to 1 here, excluding (B).

Step 4: Conclusion The requested value is $\boxed{4}$. Hence, the correct option is (D).

Quick Tip to Solve: When a logarithm has a variable base, change to natural logarithms before differentiating; both the numerator and denominator contribute.

8.

$$\int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx \textbf{ is equal to}$$

[4 marks]

(A) $10(\pi + 4)$

(B) $10(\pi + 2)$

(C) $20(\pi - 2)$

(D) $20(\pi + 2)$

Correct Answer: (D) $20(\pi + 2)$

Solution:

Concept:

- Expand the square using $\sin^2 x + \cos^2 x = 1$. The absolute value affects only the cross term, giving $(|\sin x| + |\cos x|)^2 = 1 + |\sin 2x|$.
- The function $|\sin 2x|$ has period $\pi/2$. Its integral over one period can be multiplied by the number of complete periods.

Step 1: Separate the constant and periodic contributions The square of each absolute value equals

the square of the corresponding trigonometric function. Therefore,

$$I = \int_0^{20\pi} (1 + 2|\sin x \cos x|)\, dx = 20\pi + \int_0^{20\pi} |\sin 2x|\, dx.$$

The second contribution is positive, so an expression smaller than 20π, such as option (C), cannot be correct.

Step 2: Integrate over one positive arch On $[0, \pi/2]$, $\sin 2x \geq 0$, and hence

$$\int_0^{\pi/2} |\sin 2x|\, dx = \left[-\frac{\cos 2x}{2}\right]_0^{\pi/2} = 1.$$

There are $20\pi/(\pi/2) = 40$ such periods in the given interval. Thus the absolute-sine term contributes 40, not zero; integrating $\sin 2x$ without its absolute value would incorrectly cancel alternate arches.

Step 3: Combine and compare the choices Adding the two contributions gives

$$I = 20\pi + 40 = 20(\pi + 2).$$

Options (A) and (B) have the wrong coefficient of the constant contribution π; option (C) also gives a negative rather than positive correction to 20π. Only option (D) has both contributions correct.

Step 4: Conclusion The integral equals $\boxed{20(\pi + 2)}$. Hence, the correct option is (D).

Quick Tip to Solve: For an absolute trigonometric integral, count the positive arches after expanding the expression; do not cancel positive and negative sine lobes.

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9. Let the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$, $x \in (-1, 1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is equal to [4 marks]

(A) $\frac{\pi}{3} - \frac{1}{4}$

(B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$

(D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

Correct Answer: (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, an integrating factor is $e^{\int P(x)\,dx}$. On the given interval, $1 - x^2 > 0$, so its positive square root is well defined.
- An odd integrand contributes zero over symmetric limits. Use this only after solving the differential equation and applying the initial condition.

Step 1: Apply the integrating factor Here $P(x) = x/(x^2-1)$, and

$$\int P(x)\,dx = \frac{1}{2}\log(1-x^2), \qquad \text{IF} = \sqrt{1-x^2}.$$

Multiplying the entire differential equation by this factor gives

$$\frac{d}{dx}\left(y\sqrt{1-x^2}\right) = x^4 + 2x.$$

The sign is consistent because the derivative of the square root is $-x/\sqrt{1-x^2}$.

Step 2: Use the point through which the curve passes Integrating and substituting $y(0) = 0$ gives

$$y\sqrt{1-x^2} = \frac{x^5}{5} + x^2 + C, \qquad C = 0.$$

Thus $f(x) = (x^5/5 + x^2)/\sqrt{1-x^2}$. The first term is odd, whereas the second is even. Writing $a = \sqrt{3}/2$, the requested integral reduces to $2\int_0^a x^2/\sqrt{1-x^2}\,dx$.

Step 3: Evaluate the remaining even integral Set $x = \sin\theta$, with $0 \le \theta \le \pi/3$. Then

$$2\int_0^a \frac{x^2}{\sqrt{1-x^2}}\,dx = 2\int_0^{\pi/3} \sin^2\theta\,d\theta = \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/3} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}.$$

The angle contribution rules out (C) and (D); the sine endpoint is $\sqrt{3}/2$, not $1/2$, ruling out (A).

Step 4: Conclusion The required value is $\boxed{\pi/3 - \sqrt{3}/4}$. Hence, the correct option is (B).

Quick Tip to Solve: After applying the initial condition, separate the odd and even parts of the solution before integrating over symmetric limits.

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10. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a + b - c)$ is _____. [4 marks]

(A) 12

(B) 13

(C) 14

(D) 16

Correct Answer: (A) 12

Solution:

Concept:

- The centre of a circle is the midpoint of any diameter. For $x^2 + y^2 + 2ax + 2by + c = 0$, its centre is $(-a, -b)$ and its squared radius is $a^2 + b^2 - c$.
- For roots u, v, the identity $(u - v)^2 = (u + v)^2 - 4uv$ finds their squared separation directly from

the coefficients, without solving the quadratic.

Step 1: Find the centre from the sums of roots Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$. The two quadratic equations give

$$x_1 + x_2 = 4, \quad x_1x_2 = -6, \qquad y_1 + y_2 = -2, \quad y_1y_2 = -7.$$

Their midpoint is therefore $(2, -1)$. Comparing this with $(-a, -b)$ gives $a = -2$ and $b = 1$. The pairing of the two ordinate roots with the abscissa roots does not change this midpoint.

Step 2: Calculate the squared diameter and radius The squared coordinate differences are

$$(x_1 - x_2)^2 = 4^2 - 4(-6) = 40, \qquad (y_1 - y_2)^2 = (-2)^2 - 4(-7) = 32.$$

Hence $PQ^2 = 40 + 32 = 72$, so the radius satisfies $r^2 = PQ^2/4 = 18$. The division by four is essential: the length PQ is the diameter, and squaring its half gives one quarter of its squared length.

Step 3: Determine the constant coefficient Using $r^2 = a^2 + b^2 - c$,

$$18 = 4 + 1 - c \quad \Longrightarrow \quad c = -13.$$

The required expression is

$$a + b - c = -2 + 1 - (-13) = 12.$$

As a check, the circle is $(x - 2)^2 + (y + 1)^2 = 18$. The other proposed values $13, 14, 16$ do not follow from these coefficients and fail the computed centre-radius relation.

Step 4: Conclusion The value of $a + b - c$ is $\boxed{12}$. Hence, the correct option is (A).

Quick Tip to Solve: Use root sums for the centre and discriminant-like squared differences for the diameter; explicit radical coordinates are unnecessary.

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11. If the line $x - 1 = 0$ is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point : [4 marks]

(A) $(-2\sqrt{5}, 6)$

(B) $(-\sqrt{5}, 3)$

(C) $(\sqrt{5}, -2)$

(D) $(2\sqrt{5}, 3\sqrt{6})$

Correct Answer: (C) $(\sqrt{5}, -2)$

Solution:

Concept:

- For a horizontal hyperbola $x^2/a^2 - y^2/b^2 = 1$, the eccentricity satisfies $e^2 = 1 + b^2/a^2$, and its directrices are $x = \pm a/e$.
- Once the parameter is fixed by the directrix, a point lies on the hyperbola precisely when its coordinates satisfy the original equation.

Step 1: Write the hyperbola in standard form Since the stated directrix is vertical, the transverse axis

is horizontal. Thus $k > 0$ and

$$\frac{x^2}{6/k} - \frac{y^2}{6} = 1, \qquad a^2 = \frac{6}{k}, \quad b^2 = 6.$$

Consequently $e^2 = 1 + k$. The line $x - 1 = 0$ is $x = 1$, so the positive directrix has $a/e = 1$, or $a^2 = e^2$.

Step 2: Solve for the admissible parameter Substituting the standard-form values gives

$$\frac{6}{k} = 1 + k \quad \Longrightarrow \quad k^2 + k - 6 = 0 \quad \Longrightarrow \quad (k-2)(k+3) = 0.$$

The root $k = -3$ would make the original equation $-3x^2 - y^2 = 6$, which has no real points and is not the specified hyperbola. Hence $k = 2$, and the equation is $2x^2 - y^2 = 6$.

Step 3: Test each listed point Substituting the squared coordinates into $2x^2 - y^2$ yields

$$\text{(A)}: \ 2(20) - 36 = 4, \quad \text{(B)}: \ 2(5) - 9 = 1,$$
$$\text{(C)}: \ 2(5) - 4 = 6, \quad \text{(D)}: \ 2(20) - 54 = -14.$$

Only the point in (C) gives the required right-hand side. The negative ordinate in that option is allowed because the equation contains y^2.

Step 4: Conclusion The hyperbola passes through $\boxed{(\sqrt{5}, -2)}$. Hence, the correct option is (C).

Quick Tip to Solve: Determine the hyperbola parameter from the directrix first, then test squared coordinates; signs of individual coordinates do not affect membership here.

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12. If $0 < x < \frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha} = \frac{\cos^{-1}x}{\beta}$, then the value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is : [4 marks]

(A) $4\sqrt{(1-x^2)}\,(1-2x^2)$
(B) $4x\sqrt{(1-x^2)}\,(1-2x^2)$
(C) $2x\sqrt{(1-x^2)}\,(1-4x^2)$
(D) $4\sqrt{(1-x^2)}\,(1-4x^2)$

Correct Answer: (B) $4x\sqrt{(1-x^2)}\,(1-2x^2)$

Solution:

Concept:

- For $-1 \le x \le 1$, the principal inverse functions satisfy $\sin^{-1}x + \cos^{-1}x = \pi/2$. The given positive range allows us to write $x = \sin\theta$ with $0 < \theta < \pi/4$.
- The identities $\sin 4\theta = 2\sin 2\theta\cos 2\theta$, $\sin 2\theta = 2\sin\theta\cos\theta$, and $\cos 2\theta = 1 - 2\sin^2\theta$ express the answer in terms of x.

Step 1: Convert the proportionality into an angle Put $\theta = \sin^{-1}x$, so that $\cos^{-1}x = \pi/2 - \theta$. The given equality implies

$$\beta\theta = \alpha(\pi/2 - \theta), \qquad (\alpha+\beta)\theta = \frac{\pi\alpha}{2}.$$

It follows that $\alpha/(\alpha+\beta) = 2\theta/\pi$. Thus the angle inside the requested sine is 4θ, rather than 2θ.

Step 2: Expand the sine of four times the angle Because $0 < \theta < \pi/4$, we have $\cos\theta = \sqrt{1-x^2} > 0$. Therefore,

$$\begin{aligned}\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right) &= \sin 4\theta \\ &= 2(2\sin\theta\cos\theta)(1-2\sin^2\theta) \\ &= 4x\sqrt{1-x^2}(1-2x^2).\end{aligned}$$

All factors are positive on the stated open interval, consistent with $0 < 4\theta < \pi$.

Step 3: Distinguish the options Option (B) has precisely this expression. Options (A) and (D) omit the essential factor x, so they do not even tend to zero when x tends to zero from above. Option (C) uses $1-4x^2$, which becomes negative for $x > 1/2$, although the required sine stays positive throughout the allowed interval. These checks independently eliminate the other three choices.

Step 4: Conclusion The value is $\boxed{4x\sqrt{1-x^2}(1-2x^2)}$. Hence, the correct option is (B).

Quick Tip to Solve: When inverse sine and inverse cosine occur in a ratio, use their sum of $\pi/2$ before applying a multiple-angle identity.

13.

The integral $\displaystyle\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ **is equal to**

[4 marks]

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

Correct Answer: (A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

Solution:

Concept:

- The shifted-sine product identity $\sin u\sin v = [\cos(u-v)-\cos(u+v)]/2$ can factor a denominator involving $\sin 2x$.
- The standard primitive is $\int \csc t\,dt = \log|\tan(t/2)| + C$, on each interval where the integrand is defined. Absolute values make the real logarithmic expression valid for either sign of the tangent.

Step 1: Factor the denominator using shifted sines Multiplying numerator and denominator by $\sqrt{3}$

writes the integrand as

$$\frac{(\sqrt{3}-1)(\cos x-\sin x)}{\sqrt{3}+2\sin 2x}.$$

Set $u = x + \pi/6$, $v = x + \pi/3$. The product-to-sum identity gives

$$4\sin u \sin v = \sqrt{3} + 2\sin 2x.$$

Also, expanding the two sines yields $\sin v - \sin u = (\sqrt{3}-1)(\cos x - \sin x)/2$.

Step 2: Split into standard cosecant integrals Substitute both identities into the integrand:

$$\frac{(\sqrt{3}-1)(\cos x-\sin x)}{\sqrt{3}+2\sin 2x} = \frac{1}{2}\left(\csc(x+\pi/6) - \csc(x+\pi/3)\right).$$

The integral is consequently

$$\frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right| - \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| + C.$$

Combining the logarithms gives exactly the ratio printed in (A).

Step 3: Check the sign and coefficient Differentiating $\log|\tan((x+c)/2)|$ gives $\csc(x+c)$, recovering the two terms just obtained. Option (C) reverses the ratio and doubles the coefficient, so its derivative is -2 times the required integrand. Option (B) uses the wrong second shift, and option (D) uses negative shifts. For example, at $x = 0$, (A) has derivative $1 - 1/\sqrt{3}$, whereas (B), (C), and (D) give respectively 0, $-2(1-1/\sqrt{3})$, and $-(1-1/\sqrt{3})$.

Step 4: Conclusion The primitive is $\boxed{\frac{1}{2}\log\left|\frac{\tan(x/2+\pi/12)}{\tan(x/2+\pi/6)}\right| + C}$. Hence, the correct option is (A).

Quick Tip to Solve: For a denominator containing $\sin 2x$, try a product of two shifted sines; their reciprocal difference often integrates directly to logarithms.

14. The area bounded by the curves $y = |x^2 - 1|$ and $y = 1$ is [4 marks]

(A) $\frac{2}{3}(\sqrt{2}+1)$
(B) $\frac{4}{3}(\sqrt{2}-1)$
(C) $2(\sqrt{2}-1)$
(D) $\frac{8}{3}(\sqrt{2}-1)$

Correct Answer: (D) $\frac{8}{3}(\sqrt{2}-1)$

Solution:

Concept:

- The geometric area between two curves is the integral of the upper ordinate minus the lower ordinate. An absolute-value curve must be split where its inner expression changes sign.
- Both $|x^2 - 1|$ and the horizontal line $y = 1$ are even in x. The total bounded area is twice the area

on the nonnegative half-axis, including both bounded lobes.

Step 1: Locate the intersections and split the curve The intersection equation is

$$|x^2 - 1| = 1 \quad \Longrightarrow \quad x^2 - 1 = \pm 1.$$

Thus the curves meet at $x = -\sqrt{2}, 0, \sqrt{2}$. On $[0, 1]$, the absolute-value curve is $y = 1 - x^2$; on $[1, \sqrt{2}]$, it is $y = x^2 - 1$. The horizontal line $y = 1$ is the upper boundary over both intervals. The meeting at zero separates the two symmetric bounded lobes.

Step 2: Set up and evaluate the two area contributions Using symmetry and taking the ordinate differences gives

$$A = 2\left[\int_0^1 x^2\,dx + \int_1^{\sqrt{2}} (2 - x^2)\,dx\right].$$

The first integral equals $1/3$. For the second,

$$\left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}} = \frac{4\sqrt{2}}{3} - \frac{5}{3}.$$

Each area contribution is positive; replacing the absolute-value curve by the same parabola on both intervals would give an incorrect area.

Step 3: Combine the bounded regions It follows that

$$A = 2\left(\frac{1}{3} + \frac{4\sqrt{2}}{3} - \frac{5}{3}\right) = \frac{8}{3}(\sqrt{2} - 1).$$

This is the combined area of the left and right bounded regions. Option (B) is only half the total and misses the symmetry factor. Options (A) and (C) have neither the correctly evaluated integral nor the correct coefficient, so only (D) agrees with the piecewise calculation.

Step 4: Conclusion The bounded area is $\boxed{\frac{8}{3}(\sqrt{2} - 1)}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: For $|x^2 - 1|$, split at $x = 1$ on the positive side, integrate upper minus lower, then double for the symmetric negative side.

15. $\sum_{\substack{i,j=0 \\ i \neq j}}^{n} {}^{n}C_i \, {}^{n}C_j$ **is equal to** [4 marks]

(A) $2^{2n} - {}^{2n}C_n$

(B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^{2n} - \frac{1}{2}{}^{2n}C_n$

(D) $2^{2n-1} + {}^{2n-1}C_n$

Correct Answer: (A) $2^{2n} - {}^{2n}C_n$

Solution:

Concept:

- For independent indices i, j, the sum of products separates: $\sum_i \sum_j a_i b_j = (\sum_i a_i)(\sum_j b_j)$. Here $\sum_{i=0}^{n} \binom{n}{i} = 2^n$.
- The restriction $i \neq j$ removes the diagonal terms $\binom{n}{i}^2$. Vandermonde's identity gives $\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$.

Step 1: Sum over all ordered pairs First include the pairs with equal indices as well:

$$T = \sum_{i=0}^{n} \sum_{j=0}^{n} \binom{n}{i} \binom{n}{j}$$
$$= \left(\sum_{i=0}^{n} \binom{n}{i} \right) \left(\sum_{j=0}^{n} \binom{n}{j} \right) = 2^n 2^n = 2^{2n}.$$

The indices are ordered: a pair (i, j) and its reversed pair (j, i) both occur when $i \neq j$. There is no condition $i < j$, so we must not divide this sum by two.

Step 2: Identify the terms on the diagonal The unwanted terms have $i = j$, hence their sum is

$$D = \sum_{i=0}^{n} \binom{n}{i}^2 = \sum_{i=0}^{n} \binom{n}{i} \binom{n}{n-i}.$$

To verify its value, compare the coefficient of x^n in

$$(1+x)^n (1+x)^n = (1+x)^{2n}.$$

The product on the left yields the displayed convolution, while the right yields $\binom{2n}{n}$. Therefore $D = \binom{2n}{n}$.

Step 3: Remove the diagonal and check the options Subtract each diagonal term once from the unrestricted sum:

$$\sum_{\substack{0 \le i,j \le n \\ i \neq j}} \binom{n}{i} \binom{n}{j} = 2^{2n} - \binom{2n}{n}.$$

Option (B) equals half this result, since $\binom{2n-1}{n-1} = \frac{1}{2}\binom{2n}{n}$; it would apply to unordered pairs. Option (C) removes only half the diagonal, and (D) adds a positive central term rather than removing the

diagonal contribution. Neither matches the specified restriction.

Step 4: Conclusion The ordered off-diagonal sum is $\boxed{2^{2n} - \binom{2n}{n}}$. Hence, the correct option is (A).

The supplied key expression is inconsistent with the printed restriction: it subtracts only half the diagonal sum, whereas all equal-index terms must be removed.

Quick Tip to Solve: For a double sum with $i \neq j$, calculate the unrestricted product of sums and subtract the diagonal. Divide by two only if the condition is $i < j$.

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16. The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point $(1, 3)$ is : [4 marks]

(A) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$

(B) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

(C) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$

(D) $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

Correct Answer: (B) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

Solution:

Concept:

- For the ellipse $x^2/a^2 + y^2/b^2 = 1$, a line $y = mx + c$ is tangent precisely when $c^2 = a^2m^2 + b^2$. Here $a^2 = 5/2$ and $b^2 = 5/3$.
- If the two tangent slopes are m_1, m_2, their acute angle satisfies $\tan\theta = |(m_1 - m_2)/(1 + m_1m_2)|$. The absolute value selects the positive tangent of the acute angle.

Step 1: Find the equation satisfied by the slopes Every nonvertical line through $(1, 3)$ has equation $y = m(x - 1) + 3 = mx + (3 - m)$. Applying the tangent condition gives

$$(3 - m)^2 = \frac{5}{2}m^2 + \frac{5}{3}.$$

Multiplication by 6 and simplification yield

$$54 - 36m + 6m^2 = 15m^2 + 10, \qquad 9m^2 + 36m - 44 = 0.$$

The vertical line through the point is $x = 1$; it is not a vertical tangent, since such tangents occur at $x = \pm\sqrt{5/2}$. Thus both required tangents are represented by the slope equation.

Step 2: Calculate the angle using the two roots Vieta's relations give

$$m_1 + m_2 = -4, \qquad m_1m_2 = -\frac{44}{9}.$$

Hence

$$|m_1 - m_2| = \sqrt{(m_1 + m_2)^2 - 4m_1m_2} = \frac{8\sqrt{5}}{3}.$$

It follows that

$$\tan\theta = \frac{8\sqrt{5}/3}{|1 - 44/9|} = \frac{24\sqrt{5}}{35} = \frac{24}{7\sqrt{5}}.$$

This is positive, so its principal inverse tangent lies between zero and $\pi/2$, as required.

Step 3: Check the printed alternatives The slope difference and the denominator are both fixed by the quadratic. Replacing the numerator 24 by 16 or 32, as in (A) or (C), fails these exact root relations. The expression in (D) is not the tangent of the angle between these two lines.

Step 4: Conclusion The acute angle is $\boxed{\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the sum and product of tangent slopes to find their angle; solving separately for each slope is unnecessary.

17. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is [4 marks]

(A) $y = 4(x - 2)$

(B) $y = 4(x - 1)$

(C) $y = 4(x + 1)$

(D) $y = 4(x + 2)$

Correct Answer: (B) $y = 4(x - 1)$

Solution:

Concept:

- A line is tangent to a parabola when the quadratic formed by substitution has a repeated root. Its discriminant must therefore be zero.
- Write a candidate common tangent as $y = mx + c$. The same slope and intercept must meet the tangency conditions for both parabolas; being tangent to just one is insufficient.

Step 1: Apply tangency to the first parabola Intersecting the line with $y = x^2$ gives

$$x^2 - mx - c = 0.$$

A repeated intersection requires

$$m^2 + 4c = 0, \qquad c = -\frac{m^2}{4}.$$

Every printed candidate has slope 4. In particular, a tangent of that slope to the first parabola must have intercept -4, already distinguishing it from the other printed intercepts.

Step 2: Impose tangency to the second parabola The second intersection equation is

$$mx + c = -(x-2)^2, \qquad x^2 + (m-4)x + (c+4) = 0.$$

Its zero-discriminant condition is

$$(m-4)^2 - 4(c+4) = 0, \qquad c = \frac{m^2}{4} - 2m.$$

Equating this with the first intercept gives

$$-\frac{m^2}{4} = \frac{m^2}{4} - 2m, \qquad m(m-4) = 0.$$

Thus one common tangent is $y = 0$; the other has $m = 4$ and $c = -4$. The question asks for a common tangent among the listed choices, so the second is the relevant one.

Step 3: Verify the listed tangent directly For $y = 4x - 4$, the first intersection is $(x-2)^2 = 0$, and the second is $x^2 = 0$. These repeated roots give contact points $(2, 4)$ and $(0, -4)$, respectively. Choices (A), (C), and (D) have intercepts $-8, 4, 8$, none equal to the required -4 for slope 4, so they cannot be common tangents.

Step 4: Conclusion The listed common tangent is $\boxed{y = 4(x-1)}$. Hence, the correct option is (B).

Quick Tip to Solve: For a proposed tangent, substitute into each parabola and check that both resulting quadratics are perfect squares.

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18. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ is : [4 marks]

(A) $\frac{3\pi}{4}$

(B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{5}$

(D) $\frac{5\pi}{6}$

Correct Answer: (A) $\frac{3\pi}{4}$

Solution:

Concept:

- A plane spanned by two nonparallel vectors has a normal equal to their cross product. A direction along the intersection of two planes is perpendicular to both normals.
- The angle between nonzero vectors is determined by $\cos\theta = (\vec{a} \cdot \vec{b})/(|\vec{a}||\vec{b}|)$. A line supplies two opposite directions; the problem explicitly selects the obtuse angle.

Step 1: Determine the intersection direction The first plane is spanned by $\hat{i}$ and $\hat{i} + \hat{j}$, so it is the

xy-plane, with normal $\hat{k}$. For the second plane, take

$$\vec{u} = (1, -1, 0), \qquad \vec{v} = (1, 0, 1).$$

Their cross product is $\vec{n}_2 = (-1, -1, 1)$. A vector in the first plane has zero third component. To lie in the second as well, it must obey $-x - y + z = 0$; consequently, with $z = 0$, it has $y = -x$. A nonzero intersection direction is therefore $\vec{d} = (1, -1, 0)$.

Step 2: Compute the two possible oriented angles The given vector is $\vec{b} = (1, -2, 2)$. Its norm and its scalar product with the chosen direction are

$$|\vec{b}| = \sqrt{1 + 4 + 4} = 3, \quad |\vec{d}| = \sqrt{2}, \quad \vec{d} \cdot \vec{b} = 1 + 2 = 3.$$

Thus the angle for $\vec{d}$ has cosine $1/\sqrt{2}$, so it is $\pi/4$. Reversing the parallel direction to $-\vec{d}$ changes only the sign of the cosine, giving the supplementary angle $3\pi/4$.

Step 3: Select the obtuse alternative The required angle has negative cosine and must equal $\pi - \pi/4$. Its cosine is $-1/\sqrt{2}$, whereas the cosines of $2\pi/3$, $4\pi/5$, and $5\pi/6$ differ from this value. This excludes (B), (C), and (D) without any ambiguity about which direction on the intersection line was chosen.

Step 4: Conclusion The required obtuse angle is $\boxed{3\pi/4}$. Hence, the correct option is (A).

Quick Tip to Solve: After finding an intersection direction, use the word acute or obtuse to choose between the vector angle and its supplement.

19. Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is [4 marks]

(A) $(\sim p) \wedge q$

(B) $p \wedge (\sim q)$

(C) $(\sim p) \vee (\sim q)$

(D) $(\sim p) \wedge (\sim q)$

Correct Answer: (D) $(\sim p) \wedge (\sim q)$

Solution:

Concept:

- The implication $q \Rightarrow p$ is equivalent to $(\sim q) \vee p$. It is false only when q is true and p is false.
- A biconditional $p \Leftrightarrow r$ is false exactly when p and r have different truth values. Its negation is therefore true exactly in those differing cases.

Step 1: Analyse the case where p is true If p is true, the implication $q \Rightarrow p$ is true for either truth value of q, since its consequent is true. Thus both sides of the biconditional are true, and the biconditional is true. Its negation is false whenever p is true. This already rules out an answer that can be true with p true, such as $p \wedge (\sim q)$ in option (B).

Step 2: Analyse the case where p is false If p is false, the implication becomes

$$(q \Rightarrow p) \equiv \sim q.$$

The biconditional now compares false with $\sim q$. It is false precisely when $\sim q$ is true, that is, when q is false. Therefore its negation is true exactly when both p and q are false. This gives $(\sim p) \wedge (\sim q)$.

Step 3: Confirm the full truth pattern and reject distractors For the ordered input pairs $(p, q) = (T, T), (T, F), (F, T), (F, F)$, the original biconditional has truth values T, T, T, F. Its negation has values F, F, F, T. Option (A) instead selects (F, T), and option (C) is true in several cases, including (T, F). Only option (D) reproduces the single true case (F, F). Equivalently, the original expression simplifies to $p \vee q$, whose negation follows directly by De Morgan's law.

Step 4: Conclusion The negation is $\boxed{(\sim p) \wedge (\sim q)}$. Hence, the correct option is (D).

Quick Tip to Solve: For a nested implication inside a biconditional, split into p true and p false before applying the negation.

20. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then, $54\,P(X \leq 2)$ is equal to : [4 marks]

(A) $\frac{73}{27}$
(B) $\frac{146}{27}$
(C) $\frac{146}{81}$
(D) $\frac{126}{81}$

Correct Answer: (B) $\frac{146}{27}$

Solution:

Concept:

- For $X \sim \text{Bin}(n, p)$, the mean is np and the variance is $np(1-p)$, with n the number of trials and p the success probability.
- The probability mass function is $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$. The event $X \leq 2$ includes the three disjoint cases $k = 0, 1, 2$.

Step 1: Recover the binomial parameters The given mean and variance imply

$$np = 4, \qquad np(1-p) = \frac{4}{3}.$$

Dividing the variance equation by the mean equation gives

$$1 - p = \frac{4/3}{4} = \frac{1}{3}.$$

Therefore $p = 2/3$, and the mean equation gives $n = 4/(2/3) = 6$. These parameters are admissible: the success probability lies between zero and one and the trial count is a positive integer.

Step 2: Sum the required three probabilities Using the common denominator $3^6 = 729$, evaluate

$$\begin{aligned} P(X \le 2) &= \sum_{k=0}^{2} \binom{6}{k} \left(\frac{2}{3}\right)^k \left(\frac{1}{3}\right)^{6-k} \\ &= \frac{1 + 6 \cdot 2 + 15 \cdot 4}{729} \\ &= \frac{73}{729}. \end{aligned}$$

The factors 6 and 15 count the ways to place one and two successes. They cannot be dropped: otherwise one would compute probabilities of particular trial sequences rather than the complete events.

Step 3: Apply the multiplier and compare the answers The question asks for 54 times this cumulative probability:

$$54P(X \le 2) = \frac{54 \cdot 73}{729} = \frac{2 \cdot 73}{27} = \frac{146}{27}.$$

Option (A) omits the factor 2 after simplification. Options (C) and (D) do not result from the exact cumulative sum and multiplier. As a check, the probability itself is about 0.1001, so multiplying by 54 gives about 5.41, consistent with $146/27$.

Step 4: Conclusion $\boxed{54P(X \le 2) = 146/27}$. Hence, the correct option is (B).

Quick Tip to Solve: Divide binomial variance by mean to obtain the failure probability immediately, then use one common denominator for the cumulative sum.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is _____. [4 marks]

Solution:

Concept:

- A set with r elements has 2^r subsets: each element is either included or excluded independently. The empty subset is counted in this total.
- It is easier to count the complement of a nonempty-intersection condition. Subsets $C \subseteq A$ satisfying $C \cap B = \varnothing$ can use only elements of $A \setminus B$.

Step 1: Identify the relevant elements of B The intersection and difference are

$$A \cap B = \{3, 6, 7\}, \qquad A \setminus B = \{1, 2, 4, 5\}.$$

Although B has four elements, its element 9 is not in A. Since every eligible C is a subset of A, it can never contain 9. Thus the required nonempty intersection means that at least one of 3, 6, and 7 must be chosen.

Step 2: Subtract the subsets with empty intersection There are $2^7 = 128$ subsets of A altogether. Those containing none of 3, 6, and 7 are exactly the subsets of $\{1, 2, 4, 5\}$, numbering $2^4 = 16$. The required number is therefore

$$128 - 16 = 112.$$

The empty subset belongs among the 16 excluded subsets, because its intersection with every set is empty. This handles the empty case without a separate adjustment.

Step 3: Check by direct selection As an independent count, choose a nonempty subset of $\{3, 6, 7\}$ in $2^3 - 1 = 7$ ways. Then choose any subset of the remaining four elements in $2^4 = 16$ ways. These choices are independent and uniquely determine C, giving

$$(2^3 - 1)2^4 = 7 \cdot 16 = 112.$$

Both methods count subsets themselves, not individual elements of A or B, and agree. The requested entry is an integer count and carries no physical unit.

Step 4: Conclusion The set of eligible subsets has $\boxed{112}$ elements, so the required numerical entry is 112.

Quick Tip to Solve: For subsets intersecting B, first replace B by its intersection with A, then subtract all subsets of the remaining elements.

22. Numbers are to be formed between 1000 and 3000 , which are divisible by 4 , using the digits $1, 2, 3, 4, 5$ and 6 without repetition of digits. Then the total number of such numbers is _____.

[4 marks]

Solution:

Concept:

- A four-digit number lies between 1000 and 3000 only if its thousands digit is 1 or 2. These two choices must be counted separately because the digits cannot repeat.
- Divisibility by 4 depends only on the last two digits. After fixing a valid thousands digit and an ordered two-digit ending, the hundreds digit can be any unused digit.

Step 1: List the admissible endings Using distinct digits from $\{1, 2, 3, 4, 5, 6\}$, the two-digit multiples of 4 are

$$12,\ 16,\ 24,\ 32,\ 36,\ 52,\ 56,\ 64.$$

These are all the possible endings: a multiple of 4 must be even, and checking the possible tens digits for units digits 2, 4 and 6 gives precisely this list. Endings such as 44 are excluded because repetition is forbidden.

Step 2: Count numbers beginning with 1 If the thousands digit is 1, the endings 12 and 16 are unavailable. The six remaining endings are

$$24,\ 32,\ 36,\ 52,\ 56,\ 64.$$

For each ending, three distinct digits have already been used, leaving three of the six digits for the hundreds place. Thus this case contributes

$$6 \times 3 = 18.$$

Step 3: Count numbers beginning with 2 If the thousands digit is 2, exclude every ending containing 2. The four available endings are

$$16,\ 36,\ 56,\ 64.$$

Again, three digits remain for the hundreds place, so this case contributes

$$4 \times 3 = 12.$$

The two cases cannot overlap because their thousands digits differ.

Step 4: Conclusion The total number of admissible numbers is

$$18 + 12 = \boxed{30}.$$

This is a count of numbers, so it has no physical unit.

Quick Tip to Solve: First fix the last two digits for divisibility by 4, then remove endings containing the chosen thousands digit before filling the hundreds place.

23. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to _____. [4 marks]

Solution:

Concept:

- For N observations, the population variance is $s^2 = \frac{\sum x_i^2}{N} - \bar{x}^2$. Hence the mean and standard deviation determine both the sum and the sum of squares.
- Omitting observations changes the number of observations as well as both sums. The mean must therefore be recalculated before finding the new variance.

Step 1: Recover the original sums The original number of observations is 40 and their mean is 30. Consequently,

$$\sum x_i = 40 \times 30 = 1200.$$

Since their standard deviation is 5, their variance is 25. Rearranging the variance formula gives

$$\sum x_i^2 = 40(5^2 + 30^2) = 40(25 + 900) = 37000.$$

The given standard deviation describes the original recorded data, including the two incorrect entries.

Step 2: Remove the two incorrect entries The question asks for omission, rather than replacement, so

the remaining sample contains $40 - 2 = 38$ observations. Its sum and sum of squares are

$$S = 1200 - 12 - 10 = 1178,$$

$$Q = 37000 - 12^2 - 10^2 = 36756.$$

The new mean is therefore

$$\bar{x}_{\text{new}} = \frac{1178}{38} = 31.$$

Removing two observations below the old mean appropriately increases the mean.

Step 3: Compute the requested variance multiple Using the new count and mean,

$$\sigma^2 = \frac{36756}{38} - 31^2.$$

It is simplest to multiply by 38 immediately, since that is exactly what is requested:

$$\begin{aligned} 38\sigma^2 &= 36756 - 38 \times 961 \\ &= 36756 - 36518 = 238. \end{aligned}$$

The result is positive, as required for a nonzero variance.

Step 4: Conclusion Therefore, $\boxed{38\sigma^2 = 238}$. No physical units were assigned to the observations.

Quick Tip to Solve: When observations are deleted, update the count, sum and sum of squares; never subtract their values directly from the standard deviation.

---

24. Suppose $y = y(x)$ be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim\limits_{x\to\infty} y(x)$ is finite. If a and b are respectively the x - and y-intercepts of the tangent to the curve at $x = 0$, then the value of $a - 4b$ is equal to _____. [4 marks]

Solution:

Concept:

- A first-order linear equation $y\prime - y = f(x)$ has integrating factor e^{-x}. Multiplying by it converts the left side into $\frac{d}{dx}(ye^{-x})$.
- The condition that $y(x)$ has a finite limit as $x \to \infty$ removes the growing complementary term. The tangent then follows from its point and slope at $x = 0$.

Step 1: Solve the differential equation Multiplying the equation by e^{-x},

$$\frac{d}{dx}(ye^{-x}) = 2e^{-x} - e^{-2x}.$$

Integrating both terms gives

$$ye^{-x} = -2e^{-x} + \frac{1}{2}e^{-2x} + C.$$

Thus the general solution is

$$y = -2 + \frac{1}{2}e^{-x} + Ce^{x}.$$

The integration constant must still be determined from the stated limiting condition, rather than from an assumed initial value.

Step 2: Apply the finite-limit condition As $x \to \infty$, the term e^{-x} tends to zero, whereas e^{x} grows without bound. A finite limit is therefore possible only when $C = 0$. Hence

$$y(x) = -2 + \frac{1}{2}e^{-x}.$$

At the point of tangency,

$$y(0) = -\frac{3}{2}, \qquad y\prime(0) = -\frac{1}{2}.$$

Substitution in the original equation also checks the slope: $y\prime(0) = y(0) + 2 - 1 = -\frac{1}{2}$.

Step 3: Find the two tangent intercepts The tangent through $(0, -3/2)$ with slope $-1/2$ is

$$y + \frac{3}{2} = -\frac{1}{2}x.$$

Its y-intercept is $b = -3/2$. Setting $y = 0$ gives $x = -3$, so its x-intercept is $a = -3$. These are signed coordinate intercepts, not positive lengths.

Step 4: Conclusion The required value is

$$a - 4b = -3 - 4\left(-\frac{3}{2}\right) = \boxed{3}.$$

Quick Tip to Solve: Use the limit condition to eliminate the growing exponential before differentiating; retain the signs of both axis intercepts.

25. Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is _____. [4 marks]

Solution:

Concept:

- For an arithmetic progression with first term u, common difference d, and n terms, the last term is $u + (n-1)d$.
- The number of terms and common difference are positive integers here. Thus the fixed difference of the end terms must factor as $(n-1)d$.

Step 1: Relate the number of terms to the common difference The first term is 100 and the last term is 199, so

$$199 = 100 + (n-1)d, \qquad (n-1)d = 99.$$

Since the last term exceeds the first and $n \geq 3$, the common difference is positive. Every permitted progression is therefore determined uniquely by a positive divisor d of 99.

Step 2: Impose the bounds on the number of terms The requirement $3 \leq n \leq 33$ is equivalent to

$$2 \leq n - 1 \leq 32.$$

Since $n - 1 = 99/d$, this gives

$$\frac{99}{32} \leq d \leq \frac{99}{2}.$$

We now test only the positive integral divisors of 99, rather than every integer in this interval. Factoring $99 = 3^2 \times 11$, these divisors are

$$1,\ 3,\ 9,\ 11,\ 33,\ 99.$$

Step 3: Check the eligible progressions The divisors in the required interval are $d = 9, 11, 33$. Their corresponding term counts are

d	9	11	33
$n = 1 + 99/d$	12	10	4

All three counts lie between 3 and 33 inclusive. The rejected divisors 1, 3 and 99 give respectively 100, 34 and 2 terms, so none satisfies the stated bounds.

Step 4: Conclusion The sum of the common differences is

$$9 + 11 + 33 = \boxed{53}.$$

The three eligible differences each specify one distinct progression.

Quick Tip to Solve: Turn fixed first and last terms into the factorisation $(n-1)d = 99$; then enforce the term-count limits on its divisor pairs.

---

26. The number of matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \{-1, 0, 1, 2, 3, \ldots\ldots, 10\}$, such that $A = A^{-1}$, is _____. [4 marks]

Solution:

Concept:

- The equality $A = A^{-1}$ is equivalent to $A^2 = I$. Conversely, every matrix satisfying $A^2 = I$ is invertible with inverse A, so no additional determinant check is needed.
- For the allowed set $S = \{-1, 0, 1, \ldots, 10\}$, there are 12 entries, including 11 nonzero entries. Matrix positions are ordered, so (b, c) and (c, b) may represent different matrices.

Step 1: Write the scalar conditions Multiplying the matrix by itself and equating entries to the identity gives

$$a^2 + bc = 1, \quad d^2 + bc = 1, \quad b(a + d) = 0, \quad c(a + d) = 0.$$

We separate the diagonal case $b = c = 0$ from the case in which at least one off-diagonal entry is

nonzero. This partition prevents double counting.

Step 2: Count the diagonal matrices When $b = c = 0$, the first two conditions become $a^2 = d^2 = 1$. Both 1 and -1 belong to S, and the two diagonal entries can be chosen independently. Hence this case contributes

$$2 \times 2 = 4$$

matrices.

Step 3: Count the matrices with a nonzero off-diagonal entry If at least one of b, c is nonzero, the last two scalar conditions force $d = -a$. Both a and $-a$ must lie in S, so $a \in \{-1, 0, 1\}$. For $a = 1$ or $a = -1$, we require $bc = 0$, but not both entries can be zero in this case. There are 11 choices with $b = 0, c \neq 0$, and 11 with $c = 0, b \neq 0$. The two choices of a thus give

$$2(11 + 11) = 44.$$

For $a = 0$, we need $bc = 1$. The only allowed ordered integer pairs are $(1, 1)$ and $(-1, -1)$, contributing 2 more matrices.

Step 4: Conclusion The total number is

$$4 + 44 + 2 = \boxed{50}.$$

Every matrix counted satisfies all four entries of $A^2 = I$.

Quick Tip to Solve: Split into $b = c = 0$ and its complement before using $a + d = 0$; otherwise the diagonal matrices can be counted twice.

---

27. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$ from the point $(2, 1, 4)$ is $\sqrt{3}$, is _____.

[4 marks]

Solution:

Concept:

- The plane containing two intersecting nonparallel lines passes through their common point and has normal vector equal to the cross product of their direction vectors.
- For a plane with normal $\mathbf{n}$ through P, the perpendicular distance of Q is $\frac{|\mathbf{n} \cdot (Q - P)|}{\|\mathbf{n}\|}$. A nonzero normal is essential.

Step 1: Find a normal vector and plane equation The common point is $P = (1, 1, 0)$. The line directions are

$$\mathbf{u} = (1, a, -1), \qquad \mathbf{v} = (-1, 1, -a).$$

Their cross product is

$$\mathbf{u} \times \mathbf{v} = (1 - a^2, a + 1, a + 1).$$

For $a \neq -1$, divide by $a+1$ to take the normal as $(1-a, 1, 1)$. The required plane is therefore

$$(1-a)(x-1) + (y-1) + z = 0.$$

Step 2: Apply the prescribed distance For $Q = (2, 1, 4)$, the displacement from P is $(1, 0, 4)$. Therefore,

$$\frac{|(1-a)+4|}{\sqrt{(1-a)^2+1+1}} = \sqrt{3}.$$

Both sides are nonnegative and the denominator is strictly positive. Squaring is thus valid and gives

$$(5-a)^2 = 3\big((1-a)^2+2\big).$$

Expanding and simplifying,

$$a^2 + 2a - 8 = 0, \qquad (a-2)(a+4) = 0.$$

Step 3: Check the roots and select the largest The roots are $a = 2$ and $a = -4$, both different from -1, so both give nonparallel lines and a unique plane. At $a = 2$, the distance is $3/\sqrt{3} = \sqrt{3}$; at $a = -4$, it is $9/\sqrt{27} = \sqrt{3}$. The exceptional value $a = -1$ makes the directions opposite and the two lines coincident, so it does not define a unique containing plane; in any event it is smaller than 2.

Step 4: Conclusion The largest admissible value is $\boxed{a = 2}$. The parameter is dimensionless.

Quick Tip to Solve: Factor the cross product before applying the distance formula, but identify any parameter value at which that factor vanishes.

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28. If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then $m+n$ is equal to _____. [4 marks]

Solution:

Concept:

- The quartic denominator factors as $k^4 + k^2 + 1 = (k^2 - k + 1)(k^2 + k + 1)$. Its two factors are shifted versions of the same quadratic.
- The identity $\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1} = \frac{2k}{k^4+k^2+1}$ converts the sum into a telescoping difference.

Step 1: Factor and decompose the summand Using a difference of squares,

$$\begin{aligned} k^4 + k^2 + 1 &= (k^2+1)^2 - k^2 \\ &= (k^2 - k + 1)(k^2 + k + 1). \end{aligned}$$

The difference between the two quadratic factors is $2k$. Consequently,

$$\frac{k}{k^4+k^2+1} = \frac{1}{2}\left(\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right).$$

The factor $1/2$ is necessary because the numerator is k, rather than $2k$.

Step 2: Cancel successive terms Notice that

$$k^2 + k + 1 = (k+1)^2 - (k+1) + 1.$$

Thus the negative fraction at index k cancels the positive fraction at index $k+1$. Writing the first few and last terms makes the endpoints clear:

$$S = \frac{1}{2}\left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{91} - \frac{1}{111}\right)\right].$$

All internal fractions cancel; the final denominator is $10^2 + 10 + 1 = 111$.

Step 3: Reduce the remaining fraction Therefore,

$$S = \frac{1}{2}\left(1 - \frac{1}{111}\right) = \frac{55}{111}.$$

The numerator factors as $55 = 5 \times 11$, while $111 = 3 \times 37$. They have no common prime factor, so the fraction is already in lowest terms. The stipulated coprime integers are therefore $m = 55$ and $n = 111$.

Step 4: Conclusion The required sum is

$$m + n = 55 + 111 = \boxed{166}.$$

Quick Tip to Solve: Whenever adjacent quadratic factors differ by $2k$, try a difference of reciprocals; then check the final uncancelled denominator carefully.

29. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _____. [4 marks]

Solution:

Concept:

- The solutions of a system must satisfy both equations simultaneously. The required set is the intersection of their solution sets, not their union.
- Use $\cos 2\theta = 1 - 2\sin^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$ to express both equations in the same variable $s = \sin\theta$.

Step 1: Reduce the first equation Substitute the double-angle identity into the first equation:

$$2s^2 - (1 - 2s^2) = 0 \quad \Rightarrow \quad 4s^2 = 1.$$

Thus $s = 1/2$ or $s = -1/2$. These are only candidates: each still has to satisfy the second equation. Solving the two equations separately and adding all their angles would count values that are not

solutions of the system.

Step 2: Impose the second equation For the second equation, replace $\cos^2\theta$ by $1-s^2$:

$$2(1-s^2)+3s=0 \quad \Rightarrow \quad 2s^2-3s-2=0 \quad \Rightarrow \quad (2s+1)(s-2)=0.$$

The root $s=2$ is impossible for a real angle. The remaining root $s=-1/2$ also satisfies the first equation. In contrast, $s=1/2$ makes the second equation nonzero, so it must be excluded.

Step 3: List and add the common angles In the closed interval $[0,2\pi]$, sine equals $-1/2$ in the third and fourth quadrants, with reference angle $\pi/6$. Therefore the complete common solution set is

$$\theta=\frac{7\pi}{6}, \quad \frac{11\pi}{6}.$$

Neither endpoint satisfies the equations, since its sine is zero. The sum is

$$\frac{7\pi}{6}+\frac{11\pi}{6}=\frac{18\pi}{6}=3\pi.$$

Step 4: Conclusion The sum is $k\pi=3\pi$; hence $\boxed{k=3}$, a dimensionless number.

Quick Tip to Solve: For simultaneous trigonometric equations, first find the common sine values; only then list the angles in the specified interval.

30. The plane passing through the line $L: lx-y+3(1-l)z=1, x+2y-z=2$ and perpendicular to the plane $3x+2y+z=6$ is $3x-8y+7z=4$. If θ is the acute angle between the line L and the y-axis, then $415\cos^2\theta$ is equal to _____. [4 marks]

Solution:

Concept:

- A plane through the intersection of $P_1=0$ and $P_2=0$ belongs to their linear family $aP_1+bP_2=0$. Its normal is the same linear combination of the two normals.
- A direction vector of their intersection line is the cross product of their normals. If $\mathbf{d}=(d_x,d_y,d_z)$, its angle with the y-axis obeys $\cos^2\theta=d_y^2/|\mathbf{d}|^2$.

Step 1: Determine the parameter of the line Write the defining planes as

$$P_1=lx-y+3(1-l)z-1=0, \qquad P_2=x+2y-z-2=0.$$

Because the stated plane contains their intersection, match $aP_1+bP_2=3x-8y+7z-4$. Comparing the constant and y coefficients gives

$$a+2b=4, \qquad -a+2b=-8.$$

Hence $a=6$, $b=-1$. Comparing the x coefficient gives $6l-1=3$, so $l=2/3$. The z coefficient is then $6\cdot 3(1-2/3)+1=7$, confirming the plane completely.

Step 2: Find the direction ratios Multiply the first plane normal by three to avoid fractions. The two normals may be chosen as

$$\mathbf{n}_1 = (2, -3, 3), \qquad \mathbf{n}_2 = (1, 2, -1).$$

Their cross product is a direction vector of L:

$$\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 = (-3, 5, 7).$$

It is perpendicular to both normals, as the checks $-6 - 15 + 21 = 0$ and $-3 + 10 - 7 = 0$ show. Also, the given plane normal $(3, -8, 7)$ is perpendicular to $(3, 2, 1)$, since $9 - 16 + 7 = 0$, agreeing with the perpendicular-plane condition.

Step 3: Calculate the requested quantity The y-axis has unit direction vector $(0, 1, 0)$. Therefore

$$\cos^2\theta = \frac{5^2}{(-3)^2 + 5^2 + 7^2} = \frac{25}{83}.$$

Using $415 = 5 \times 83$, obtain

$$415\cos^2\theta = 415\frac{25}{83} = 125.$$

Squaring removes the irrelevant choice of orientation of the line, while the problem specifies the acute angle.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{125}$.

Quick Tip to Solve: For the angle of an intersection line with a coordinate axis, cross the plane normals and divide the relevant direction ratio squared by the sum of all three squares.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Two projectiles are thrown with same initial velocity making an angle of $45°$ and $30°$ with the horizontal respectively. The ratio of their respective ranges will be : [4 marks]

(A) $1 : \sqrt{2}$

(B) $\sqrt{2} : 1$

(C) $2 : \sqrt{3}$

(D) $\sqrt{3} : 2$

Correct Answer: (C) $2 : \sqrt{3}$

Solution:

Concept:

- For projection and landing at the same level, horizontal range is $R = u^2 \sin 2\alpha / g$, where u is initial speed, α is the launch angle and g is gravitational acceleration.
- At fixed speed, the common factor u^2/g cancels from a ratio. The angular dependence involves twice the launch angle, not the angle alone.

Step 1: Express the two ranges Resolve the initial velocity into horizontal and vertical components.

The flight time is $T = 2u\sin\alpha/g$, so multiplying by $u\cos\alpha$ gives

$$R = (u\cos\alpha)\frac{2u\sin\alpha}{g} = \frac{u^2\sin 2\alpha}{g}.$$

Both projectiles have the same initial speed and experience the same gravitational acceleration. Thus only the sine factor differs; the different flight times alone do not determine the range ratio.

Step 2: Substitute the two launch angles For the projectile launched at 45°,

$$R_{45} = \frac{u^2\sin 90^\circ}{g} = \frac{u^2}{g}.$$

For the one launched at 30°,

$$R_{30} = \frac{u^2\sin 60^\circ}{g} = \frac{\sqrt{3}\,u^2}{2g}.$$

Consequently,

$$R_{45} : R_{30} = 1 : \frac{\sqrt{3}}{2} = 2 : \sqrt{3}.$$

Step 3: Check the order and reject the distractors The 45° trajectory has the maximum range at this speed, so the first entry must represent the larger range. This rules out (A) and (D), whose first-to-second ratios are less than one. Option (B) has the right order but the wrong value: $\sqrt{2}$ does not follow from $\sin 60^\circ = \sqrt{3}/2$. Only (C) reproduces both the magnitude and the stated order of the ratio.

Step 4: Conclusion The respective ranges have ratio $\boxed{2 : \sqrt{3}}$. Hence, the correct option is (C).

Quick Tip to Solve: With equal launch speeds, take the ratio of $\sin 2\alpha$; remember that 45° produces the greatest range.

32. In a Vernier Calipers, 10 divisions of Vernier scale is equal to the 9 divisions of main scale. When both jaws of Vernier calipers touch each other, the zero of the Vernier scale is shifted to the left of zero of the main scale and 4^{th} Vernier scale division exactly coincides with the main scale reading. One main scale division is equal to $1\ mm$. While measuring diameter of a spherical body, the body is held between two jaws. It is now observed that zero of the Vernier scale lies between 30 and 31 divisions of main scale reading and 6^{th} Vernier scale division exactly coincides with the main scale reading. The diameter of the spherical body will be : [4 marks]

(A) 3.02 cm

(B) 3.06 cm

(C) 3.10 cm

(D) 3.20 cm

Correct Answer: The stated readings give $3.12\,\text{cm}$, which is absent from the options.

Solution:

Concept:

- The least count of a direct vernier is one main-scale division minus one vernier-scale division: $\text{LC} = 1\text{MSD} - 1\text{VSD}$.
- A vernier zero to the left of the main zero gives a negative zero error. True length equals observed length minus the signed zero error.

Step 1: Calculate the least count Here $10\text{VSD} = 9\text{MSD}$, and $1\text{MSD} = 1\,\text{mm}$. Therefore

$$1\text{VSD} = 0.9\,\text{mm}, \qquad \text{LC} = 1.0 - 0.9 = 0.1\,\text{mm}.$$

This is the increment to multiply by the coinciding vernier division, both for the measuring reading and for the zero-error calculation.

Step 2: Find the signed zero error With the jaws closed, the zero is left of the main zero and the fourth of ten vernier divisions coincides. A negative zero error uses the remaining divisions to complete the ten-division cycle:

$$e_0 = -(10 - 4)\text{LC} = -6(0.1) = -0.6\,\text{mm}.$$

Equivalently, the fourth vernier line is $3.6\,\text{mm}$ from its zero; coincidence with the main-scale line at $3\,\text{mm}$ places that zero at $-0.6\,\text{mm}$. Thus the correction is positive $0.6\,\text{mm}$, not negative.

Step 3: Correct the measured diameter The main reading immediately before the vernier zero is $30\,\text{mm}$, and the sixth division coincides. The observed length is

$$D_{\text{obs}} = 30 + 6(0.1) = 30.6\,\text{mm}.$$

Subtract the signed error:

$$D = 30.6 - (-0.6) = 31.2\,\text{mm} = 3.12\,\text{cm}.$$

None of the listed options equals this diameter. In particular, (B) is the uncorrected reading; subtracting a positive error here would have the wrong sign.

Step 4: Conclusion The stated data give $\boxed{3.12\,\text{cm}}$. No listed option is correct; the printed option (C), $3.10\,\text{cm}$, differs from the calculated diameter.

Quick Tip to Solve: For negative zero error, use minus the number of remaining vernier divisions times least count, then subtract that signed error from the observed length.

33. A ball of mass $0.15\ \text{kg}$ hits the wall with its initial speed of $12\ \text{ms}^{-1}$ and bounces back without changing its initial speed. If the force applied by the wall on the ball during the contact is $100\ \text{N}$, calculate the time duration of the contact of ball with the wall. [4 marks]

(A) 0.018 s

(B) 0.036 s

(C) 0.009 s

(D) 0.072 s

Correct Answer: (B) 0.036 s

Solution:

Concept:

- Impulse equals change in momentum: $\mathbf{F}_{\text{av}}\Delta t = \mathbf{p}_f - \mathbf{p}_i$. Momentum is a vector, so a reversal changes its sign even when speed is unchanged.
- For a direct rebound at the same speed u, the magnitude of momentum change is $2mu$, with m the mass. The contact time is that impulse divided by the average force magnitude.

Step 1: Choose a direction and write momenta Take the direction towards the wall as positive. Before impact, the ball has velocity $+12\,\text{m s}^{-1}$, while after impact it has velocity $-12\,\text{m s}^{-1}$. Thus

$$p_i = 0.15(12) = 1.8\,\text{kg m s}^{-1}, \qquad p_f = 0.15(-12) = -1.8\,\text{kg m s}^{-1}.$$

Although the kinetic energy is unchanged in this rebound, momentum is not unchanged because its direction has reversed.

Step 2: Compute the impulse and duration The change in momentum is

$$\Delta p = -1.8 - 1.8 = -3.6\,\text{kg m s}^{-1}.$$

The wall force acts away from the wall, in the negative direction under our convention. Using its stated magnitude,

$$100\Delta t = |\Delta p| = 3.6\,\text{N s}, \qquad \Delta t = \frac{3.6}{100} = 0.036\,\text{s}.$$

The time is positive; the negative momentum change records direction and is not a negative contact time.

Step 3: Compare the alternatives Option (A), $0.018\,\text{s}$, uses mu/F and counts only the stopping impulse, omitting the impulse required to send the ball back. Option (C) halves that already insufficient value. Option (D) doubles the full reversal impulse and is too large. The calculated impulse of $3.6\,\text{N s}$ divided by $100\,\text{N}$ agrees exactly with (B).

Step 4: Conclusion The contact lasts $\boxed{0.036\,\text{s}}$. Hence, the correct option is (B).

Quick Tip to Solve: When a ball rebounds with unchanged speed, use the velocity change $2u$, rather than u, in the impulse equation.

34. A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momentum will be : [4 marks]

(A) 1 : 1

(B) 2 : 1

(C) 1 : 4

(D) 4 : 1

Correct Answer: (B) 2 : 1

Solution:

Concept:

- Nonrelativistic kinetic energy is $K = mv^2/2 = p^2/(2m)$, where momentum magnitude is $p = mv$.
- At fixed kinetic energy, momentum scales as $\sqrt{m}$. Equal energies therefore do not imply equal speeds or equal momenta when the masses differ.

Step 1: Relate momentum to kinetic energy Eliminate the speed between $p = mv$ and $K = mv^2/2$:

$$K = \frac{p^2}{2m}, \qquad p = \sqrt{2mK}.$$

The positive root gives the momentum magnitude. Since the question provides masses and equal kinetic energies but no directions, it asks for the ratio of these magnitudes. No numerical value of the common energy is needed.

Step 2: Use the given masses in the ratio Let the common kinetic energy be K, and let p_8 and p_2 denote the momenta of the $8\,\text{kg}$ and $2\,\text{kg}$ bodies respectively. Then

$$\frac{p_8}{p_2} = \frac{\sqrt{2(8)K}}{\sqrt{2(2)K}} = \sqrt{\frac{8}{2}} = 2.$$

The momentum ratio, in the order asked, is therefore $2 : 1$. As a separate check, equal energies give $8v_8^2 = 2v_2^2$, so $v_2 = 2v_8$. Hence $8v_8/(2v_2) = 2$, the same result.

Step 3: Reject the other ratios Option (A) would require the same momentum for unequal masses, which would produce unequal kinetic energies through $p^2/(2m)$. Option (D) uses the mass ratio $8 : 2 = 4 : 1$, appropriate to equal speeds rather than equal energies. Option (C) reverses that mass ratio and also fails the square-root relation. Only (B) accounts for the lower speed of the heavier body while retaining its larger momentum.

Step 4: Conclusion The respective momentum magnitudes have ratio $\boxed{2 : 1}$. Hence, the correct option is (B).

Quick Tip to Solve: At equal kinetic energy use $p \propto \sqrt{m}$; at equal speed use $p \propto m$. Identify the fixed quantity before forming a ratio.

35. Two uniformly charged spherical conductors A and B of radii $5\ \text{mm}$ and $10\ \text{mm}$ are separated by a distance of $2\ \text{cm}$. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitudes of the electric fields at the surface of the sphere A and B will be : [4 marks]

(A) 1 : 2

(B) 2 : 1

(C) 1 : 1

(D) 1 : 4

Correct Answer: (B) 2 : 1

Solution:

Concept:

- Connecting two conducting spheres by a wire makes their potentials equal at electrostatic equilibrium; charge flows until the potential difference along the conductor vanishes.
- For the spherical-conductor model, $V = kQ/R$ and the surface field magnitude is $E = k|Q|/R^2 = |V|/R$, where R is the sphere radius and $k = 1/(4\pi\varepsilon_0)$.

Step 1: Apply the equilibrium condition The two radii are $R_A = 5\,\text{mm}$ and $R_B = 10\,\text{mm}$. At equilibrium the conducting wire enforces

$$V_A = V_B.$$

Using the spherical-conductor relation,

$$\frac{kQ_A}{R_A} = \frac{kQ_B}{R_B} \quad \Rightarrow \quad \frac{Q_A}{Q_B} = \frac{R_A}{R_B} = \frac{1}{2}.$$

The smaller sphere carries less charge, but its charge is concentrated over a still smaller surface area. Equal potential must not be confused with equal electric field.

Step 2: Calculate the ratio of surface fields Use $E = k|Q|/R^2$, or directly $E = |V|/R$ at the common potential:

$$\frac{E_A}{E_B} = \frac{|Q_A|}{|Q_B|}\frac{R_B^2}{R_A^2} = \frac{1}{2}\left(\frac{10}{5}\right)^2 = 2.$$

Thus the smaller sphere has twice the surface field of the larger one. In this standard spherical-conductor treatment, the stated separation is not substituted for either radius; it does not set the ratio obtained from their common potential.

Step 3: Check the offered ratios Option (A) takes the radius ratio directly and reverses the field trend. Option (C) mistakes equal potential for equal field. Option (D) is an inverse-square radius ratio with no corresponding charge redistribution, and also has the wrong order. The equilibrium charge ratio is $1 : 2$, but the field ratio is $2 : 1$ because the radius enters squared in the field formula.

Step 4: Conclusion The surface electric-field magnitudes satisfy $\boxed{E_A : E_B = 2 : 1}$. Hence, the correct option is (B).

Quick Tip to Solve: For connected spherical conductors at common potential, compare surface fields through $E = V/R$: the smaller radius has the greater field.

36. The oscillating magnetic field in a plane electromagnetic wave is given by $B_y = 5 \times 10^{-6} \sin 1000\pi \left(5x - 4 \times 10^8 t\right) T$. The amplitude of electric field will be : [4 marks]

(A) $15 \times 10^2\ \mathrm{Vm}^{-1}$

(B) $5 \times 10^{-6}\ \mathrm{Vm}^{-1}$

(C) $16 \times 10^{12}\ \mathrm{Vm}^{-1}$

(D) $4 \times 10^2\ \mathrm{Vm}^{-1}$

Correct Answer: (D) $4 \times 10^2\ \mathrm{Vm}^{-1}$

Solution:

Concept:

- A plane wave with phase $kx - \omega t$ travels with phase speed $v = \omega/k$. The coefficients of position and time must be read from the entire phase.
- For a plane electromagnetic wave, the field amplitudes satisfy $E_0 = vB_0$, where E_0 is in $\mathrm{V\,m^{-1}}$ and B_0 is in tesla. Use the speed specified by the wave rather than automatically substituting the vacuum value.

Step 1: Identify the amplitude and phase coefficients The coefficient outside the sine gives the magnetic amplitude, independently of the phase. Expanding the argument only for identifying its coefficients gives

$$B_0 = 5 \times 10^{-6}\,\mathrm{T}, \qquad k = 5000\pi\,\mathrm{m^{-1}}, \qquad \omega = 4 \times 10^{11}\pi\,\mathrm{s^{-1}}.$$

The common factor 1000π multiplies both the spatial and temporal terms; it therefore cancels when their ratio is taken.

Step 2: Calculate the speed and electric amplitude The wave travels along the positive x-direction because its phase has the form $kx - \omega t$. Its speed and electric amplitude are

$$v = \frac{4 \times 10^{11}\pi}{5000\pi} = 8 \times 10^7\,\mathrm{m\,s^{-1}},$$

$$E_0 = vB_0 = (8 \times 10^7)(5 \times 10^{-6}) = 4 \times 10^2\,\mathrm{V\,m^{-1}}.$$

Option (A) results from inserting the vacuum light speed although the phase specifies a different speed. Option (B) merely repeats the magnetic amplitude in electric-field units, while option (C) does not follow from the required amplitude ratio.

Step 3: Conclusion The electric-field amplitude is $\boxed{4 \times 10^2\,\mathrm{V\,m^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Read the wave speed from the ratio of the time and position coefficients before using $E_0 = vB_0$; a common multiplier in the phase cancels.

37. Light travels in two media M_1 and M_2 with speeds 1.5×10^8 ms^{-1} and 2.0×10^8 ms^{-1} respectively. The critical angle between them is : [4 marks]

(A) $\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$

(B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{4}\right)$

(D) $\sin^{-1}\left(\frac{2}{3}\right)$

Correct Answer: (A) $\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$

Solution:

Concept:

- The refractive index is $n = c/v$; a medium in which light travels more slowly is optically denser.
- For incidence from the denser medium into the rarer medium, the critical angle C obeys $n_1 \sin C = n_2$, since the refracted ray makes an angle of 90° with the normal.

Step 1: Identify the direction for total internal reflection The speed in M_1 is $v_1 = 1.5 \times 10^8\,\mathrm{m\,s^{-1}}$, smaller than $v_2 = 2.0 \times 10^8\,\mathrm{m\,s^{-1}}$ in M_2. Thus M_1 has the larger refractive index and the critical angle is defined for incidence from M_1 to M_2. Using Snell's law at grazing refraction,

$$\sin C = \frac{n_2}{n_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2} = \frac{3}{4}.$$

The absolute value of the vacuum light speed is unnecessary because it cancels.

Step 2: Express the angle in the form of an option The critical angle is acute, so the positive square root gives

$$\cos C = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}, \qquad \tan C = \frac{\sin C}{\cos C} = \frac{3}{\sqrt{7}}.$$

Therefore $C = \tan^{-1}(3/\sqrt{7})$, approximately 48.6°. Option (B) has the wrong tangent ratio. Option (C) uses the required sine value as a cosine value and instead gives the complementary angle. Option (D) uses an incorrect speed ratio, $2/3$, rather than $3/4$.

Step 3: Conclusion The critical angle is $\boxed{\tan^{-1}(3/\sqrt{7})}$. Hence, the correct option is (A).

Quick Tip to Solve: First calculate $\sin C = v_{\text{denser}}/v_{\text{rarer}}$. If the options use another inverse function, form a right triangle to convert the ratio.

38. A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be :

(Take radius of earth $= 6400$ km **and** $g = 10\ ms^{-2}$ **)** [4 marks]

(A) 800 km

(B) 1600 km

(C) 2133 km

(D) 4800 km

Correct Answer: (A) 800 km

Solution:

Concept:

- At a distance r from the centre of Earth, gravitational potential energy is $U = -GMm/r$, with zero potential energy at infinity.
- The surface escape speed satisfies $v_e^2 = 2GM/R = 2gR$. Mechanical energy is conserved during the upward motion when air resistance is neglected.

Step 1: Write the initial and final mechanical energies Let R be Earth's radius, m the mass of the body and h its maximum height above the surface. The initial speed is $u = v_e/3$, giving

$$\frac{1}{2}mu^2 = \frac{1}{2}m\frac{v_e^2}{9} = \frac{GMm}{9R}.$$

At the highest point, the speed becomes zero. The radius there is $R + h$, not h. Conservation of energy therefore gives

$$\frac{GMm}{9R} - \frac{GMm}{R} = -\frac{GMm}{R+h}.$$

A constant-g approximation would neglect the appreciable change in gravity over the ascent.

Step 2: Solve for the height and compare the alternatives Cancelling GMm and combining the fractions gives

$$-\frac{8}{9R} = -\frac{1}{R+h}, \qquad R + h = \frac{9}{8}R.$$

Subtracting the surface radius,

$$h = \frac{R}{8} = \frac{6400}{8}\,\text{km} = 800\,\text{km}.$$

The supplied value of g cancels through the escape-speed relation, so no separate numerical value of GM is required. The values 1600, 2133, and $4800\,\text{km}$ do not satisfy $R/(R+h) = 8/9$, and hence fail the energy equation for the specified initial speed.

Step 3: Conclusion The maximum height attained above Earth's surface is $\boxed{800\,\text{km}}$. Hence, the correct option is (A).

Quick Tip to Solve: For launch speed $u = fv_e$, conservation of energy gives $h = Rf^2/(1-f^2)$. Use gravitational potential energy when the height is a significant fraction of Earth's radius.

39. A nucleus of mass M at rest splits into two parts having masses $\frac{M'}{3}$ and $\frac{2M'}{3}(M' < M)$. The ratio of de Broglie wavelength of two parts will be : [4 marks]

(A) 1 : 2

(B) 2 : 1

(C) 1 : 1

(D) 2 : 3

Correct Answer: (C) 1 : 1

Solution:

Concept:

- In the absence of an external impulse, the total momentum of a system is conserved. A nucleus initially at rest has zero total momentum.
- The de Broglie wavelength is $\lambda = h/p$, where p is the magnitude of momentum. The wavelength depends on momentum, not on mass alone.

Step 1: Apply momentum conservation to the two fragments Denote the momenta of the fragments by $\mathbf{p}_1$ and $\mathbf{p}_2$. Since the parent nucleus is at rest,

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{0} \quad \Longrightarrow \quad \mathbf{p}_2 = -\mathbf{p}_1.$$

The momenta are opposite in direction but equal in magnitude, so $p_1 = p_2 = p$. The condition $M' < M$ indicates that some rest energy is released as kinetic energy, but it does not change the zero initial momentum. There is no need to calculate this released energy to obtain the wavelength ratio.

Step 2: Use the equal momentum magnitudes For the two fragments,

$$\lambda_1 = \frac{h}{p_1}, \qquad \lambda_2 = \frac{h}{p_2}, \qquad \frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = 1.$$

In the nonrelativistic description, their masses are $m_1 = M'/3$ and $m_2 = 2M'/3$. Hence their speed magnitudes obey $v_1/v_2 = m_2/m_1 = 2$. Their speeds are unequal precisely so that their momentum magnitudes are equal. Options (A) and (B) incorrectly convert the mass or speed ratio directly into the wavelength ratio; option (D) likewise contradicts equal momenta. The directions of motion do not introduce a negative sign into a ratio of wavelengths.

Step 3: Conclusion The de Broglie wavelengths have ratio $\boxed{\lambda_1 : \lambda_2 = 1 : 1}$. Hence, the correct option is (C).

Quick Tip to Solve: In a two-body breakup from rest, equal and opposite momenta imply equal de Broglie wavelengths, even when the fragment masses and speeds are unequal.

40. An ice cube of dimensions $60\ \text{cm} \times 50\ \text{cm} \times 20\ \text{cm}$ is placed in an insulation box of wall thickness $1\ \text{cm}$. The box keeping the ice cube at 0°C of temperature is brought to a room of temperature 40°C. The rate of melting of ice is approximately :

(Latent heat of fusion of ice is $3.4 \times 10^5\ \text{J kg}^{-1}$ and thermal conducting of insulation wall is $0.05\,\text{Wm}^{-1}{}^\circ\text{C}^{-1}$) [4 marks]

(A) $61 \times 10^{-3}\ \text{kg s}^{-1}$

(B) $61 \times 10^{-5}\ \text{kg s}^{-1}$

(C) $208\ \text{kg s}^{-1}$

(D) $30 \times 10^{-5}\ \text{kg s}^{-1}$

Correct Answer: (B) $61 \times 10^{-5}\ \text{kg s}^{-1}$

Solution:

Concept:

- The conduction rate through a thin wall is $\dot{Q} = kA\Delta T/d$, where k is thermal conductivity, A is the conducting area and d is wall thickness.
- While ice melts at 0°C, heat changes phase rather than temperature. Thus $\dot{Q} = L_f\dot{m}$, where L_f is latent heat of fusion and $\dot{m}$ is the melting rate.

Step 1: Calculate the area and temperature difference Convert the three dimensions to metres: $a = 0.60$, $b = 0.50$, and $c = 0.20$. Heat enters through all six faces of the closed insulating box. Using the ice dimensions to approximate the thin-wall area,

$$A = 2(ab + bc + ca) = 2(0.30 + 0.10 + 0.12) = 1.04\,\text{m}^2.$$

The wall thickness is $d = 0.01\,\text{m}$, and the temperature difference is $40 - 0 = 40^\circ\text{C}$, numerically the same as $40\,\text{K}$.

Step 2: Convert the heat flow into a mass flow The heat supplied per second by conduction is

$$\dot{Q} = \frac{(0.05)(1.04)(40)}{0.01} = 208\,\text{J s}^{-1}.$$

All this heat melts ice, so

$$\dot{m} = \frac{208}{3.4 \times 10^5} = 6.12 \times 10^{-4}\,\text{kg s}^{-1} \simeq 61 \times 10^{-5}\,\text{kg s}^{-1}.$$

Option (C) mistakes the heat-flow number for a mass-flow rate, omitting division by latent heat. Option (A) is about a hundred times too large, while option (D) is approximately half the result and does not include the full conducting area correctly. The word approximately is consistent with neglecting the small difference between inner and outer wall areas.

Step 3: Conclusion The rate of melting is approximately $\boxed{61 \times 10^{-5}\,\text{kg s}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the total area of all six faces, convert the wall thickness to metres, then divide the conducted power by latent heat to obtain kilograms per second.

41. A gas has n degrees of freedom. The ratio of specific heat of gas at constant volume to the specific heat of gas at constant pressure will be : [4 marks]

(A) $\frac{n}{n+2}$
(B) $\frac{n+2}{n}$
(C) $\frac{n}{2n+2}$
(D) $\frac{n}{n-2}$

Correct Answer: (A) $\frac{n}{n+2}$

Solution:

Concept:

- By equipartition of energy, an ideal gas with n active quadratic degrees of freedom has molar internal energy $U_m = (n/2)RT$. Each degree contributes $RT/2$ per mole.
- For an ideal gas, Mayer's relation is $C_P - C_V = R$, where C_P and C_V are molar heat capacities at constant pressure and volume. Ratios are unchanged if specific heat per unit mass is used.

Step 1: Determine the heat capacity at constant volume At constant volume the gas does no expansion work, and the heat supplied raises its internal energy. Differentiating molar internal energy with respect to temperature for the given number of active degrees of freedom gives

$$C_V = \left(\frac{dU_m}{dT}\right)_V = \frac{n}{2}R.$$

Here n is the number of degrees of freedom, not the number of moles. The same argument per unit mass replaces R with the specific gas constant and leaves the desired ratio unchanged.

Step 2: Add the expansion contribution and take the requested ratio At constant pressure, additional heat is required for expansion against the surroundings. Applying Mayer's relation,

$$C_P = C_V + R = \frac{n+2}{2}R.$$

The question asks for the constant-volume value divided by the constant-pressure value:

$$\frac{C_V}{C_P} = \frac{(n/2)R}{[(n+2)/2]R} = \frac{n}{n+2}.$$

This is below one, as required because $C_P > C_V$. Option (B) is the reciprocal, conventionally called γ. Option (C) has an extra factor of n in the denominator term, and option (D) would make the constant-pressure heat capacity smaller rather than larger.

Step 3: Conclusion The required ratio is $\boxed{C_V/C_P = n/(n+2)}$. Hence, the correct option is (A).

Quick Tip to Solve: Check which heat capacity is in the numerator: $\gamma = C_P/C_V$, but this question asks for $1/\gamma$, which must be less than one.

42. A transverse wave is represented by $y = 2\sin(\omega t - kx)\,\text{cm}$. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be : [4 marks]

(A) 4π

(B) 2π

(C) π

(D) 2

Correct Answer: (A) 4π

Solution:

Concept:

- For a travelling sinusoidal wave $y = A\sin(\omega t - kx)$, the phase speed is $v = \omega/k$, with wave number $k = 2\pi/\lambda$.
- A medium particle oscillates at fixed position. Its transverse velocity is $u_y = \partial y/\partial t = A\omega\cos(\omega t - kx)$, so the maximum particle speed is $u_{\max} = A\omega$.

Step 1: Distinguish the two velocities The displacement amplitude read from the equation is $A = 2\,\text{cm}$. The wave velocity describes propagation of a fixed phase along the medium, whereas particle velocity describes transverse motion of an individual element. Thus their magnitudes are

$$v = \frac{\omega}{k}, \qquad u_{\max} = 2\omega\,\text{cm}\,\text{s}^{-1}.$$

When using centimetres for displacement and wavelength, k must be measured in cm^{-1}; alternatively all lengths can be converted to metres first. The maximum occurs where the magnitude of the cosine factor is one.

Step 2: Impose equality and solve for wavelength For a nonzero oscillation frequency, the common factor ω cancels from the stated equality:

$$\frac{\omega}{k} = A\omega \quad \Longrightarrow \quad \frac{1}{k} = A = 2\,\text{cm}.$$

Since $1/k = \lambda/(2\pi)$,

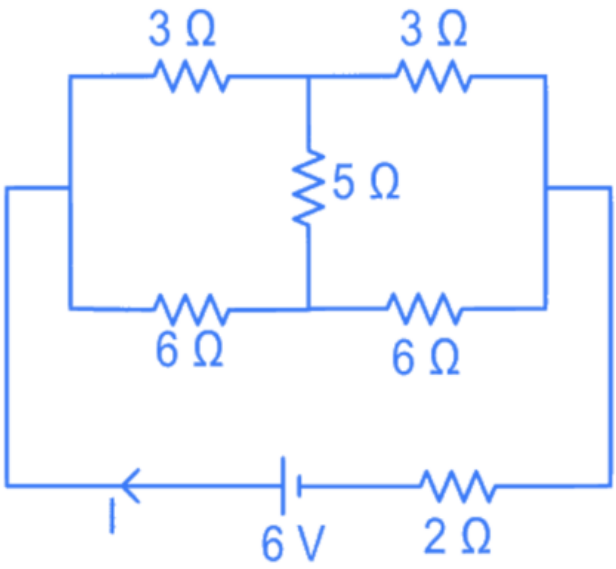
$$\lambda = 2\pi A = 2\pi(2\,\text{cm}) = 4\pi\,\text{cm}.$$

Option (D) confuses $1/k$ with the wavelength and misses 2π. Options (B) and (C) are smaller by factors of two and four respectively and would not make the wave speed equal to the maximum particle speed. The cancellation also explains why no numerical angular frequency is needed.

Step 3: Conclusion The required wavelength is $\boxed{4\pi\,\text{cm}}$. Hence, the correct option is (A).

Quick Tip to Solve: Compare $v = \omega/k$ with $u_{\max} = A\omega$. Equality means $kA = 1$, or $\lambda = 2\pi A$, with consistent length units.

43. A battery of $6\ \mathrm{V}$ is connected to the circuit as shown below. The current I drawn from the battery is : [4 marks]



(A) 1A

(B) 2A

(C) $\frac{6}{11}$ A

(D) $\frac{4}{3}$ A

Correct Answer: (A) 1A

Solution:

Concept:

- A Wheatstone bridge is balanced when the resistance ratios of its two arms are equal. The bridge resistor then joins points at the same potential and carries no current.
- Series resistances add. Two parallel resistances obey $R_{\mathrm{p}} = R_1 R_2/(R_1 + R_2)$, and the battery current is $I = V/R_{\mathrm{total}}$.

Step 1: Check the bridge balance The upper path has two $3\,\Omega$ resistors, while the lower path has two $6\,\Omega$ resistors. Their division ratios are equal:

$$\frac{3}{3} = \frac{6}{6} = 1.$$

Thus the two midpoints are at the same potential. The central $5\,\Omega$ resistor has zero potential difference across it and carries zero current. It can be omitted when calculating the equivalent resistance, without shorting either arm.

Step 2: Reduce the two conducting arms The upper pair is in series and gives $3 + 3 = 6\,\Omega$. The lower pair is also in series and gives $6 + 6 = 12\,\Omega$. These complete paths connect the same two outer nodes, so they are parallel:

$$R_{\mathrm{bridge}} = \frac{6 \times 12}{6 + 12} = 4\,\Omega.$$

The $5\,\Omega$ resistor must not be added to either conducting branch.

Step 3: Include the battery-side resistor The $2\,\Omega$ resistor drawn next to the battery is in series with the entire bridge. Therefore

$$R_{\text{total}} = 4 + 2 = 6\,\Omega, \qquad I = \frac{6}{6} = 1\,\text{A}.$$

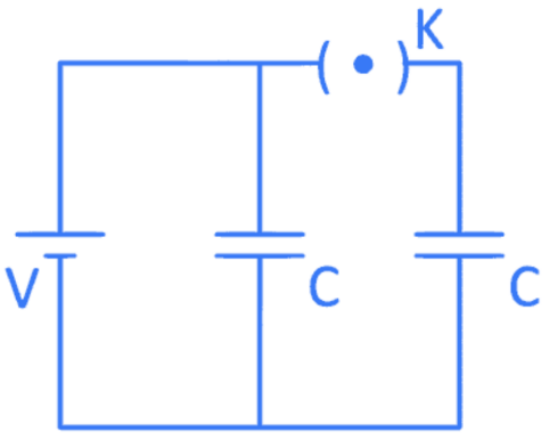
As a check, the bridge voltage is $4\,\text{V}$, giving upper and lower branch currents $2/3\,\text{A}$ and $1/3\,\text{A}$, whose sum is $1\,\text{A}$. The other offered currents do not satisfy this total resistance.

Step 4: Conclusion The current drawn from the battery is $\boxed{1\,\text{A}}$. Hence, the correct option is (A).

Quick Tip to Solve: Check the arm ratios before combining resistors: a balanced bridge removes the central resistor from the current calculation, but a resistor beside the battery still remains in series.

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44. A source of potential difference V is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is E_1. Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now E_2. The ratio E_1/E_2 will be : [4 marks]



(A) $\frac{1}{10}$
(B) $\frac{2}{5}$
(C) $\frac{5}{13}$
(D) $\frac{5}{26}$

Correct Answer: (C) $\frac{5}{13}$

Solution:

Concept:

- For a capacitor, $U = \frac{1}{2}CV^2 = Q^2/(2C)$. A dielectric fully filling the gap multiplies its capacitance by its dielectric constant.
- A capacitor connected across an ideal voltage source retains its voltage. An isolated plate retains its free charge; opening the key does not disconnect the left capacitor from the source.

Step 1: Find the initial energy Let each capacitor initially have capacitance C. With key K closed, both are connected in parallel across the potential difference V. Each has charge CV and energy $\frac{1}{2}CV^2$. The initial total is therefore

$$E_1 = 2\left(\frac{1}{2}CV^2\right) = CV^2.$$

The equality of the capacitors does not imply that their later boundary conditions will remain identical.

Step 2: Treat the capacitor still connected to the source After the key opens, the left capacitor remains directly across the source. Inserting dielectric of constant 5 changes its capacitance to $5C$, while its voltage stays V. Charge can flow to it from the source. Hence

$$U_{\mathrm{L}} = \frac{1}{2}(5C)V^2 = \frac{5}{2}CV^2.$$

Its energy increases, so conserving the combined initial electrical energy would be incorrect.

Step 3: Treat the disconnected capacitor and form the ratio The upper plate of the right capacitor is isolated by the open key, so its charge remains $Q = CV$. Its new capacitance is also $5C$, and its voltage becomes $V/5$. Thus

$$U_{\mathrm{R}} = \frac{(CV)^2}{2(5C)} = \frac{1}{10}CV^2.$$

Adding the two contributions,

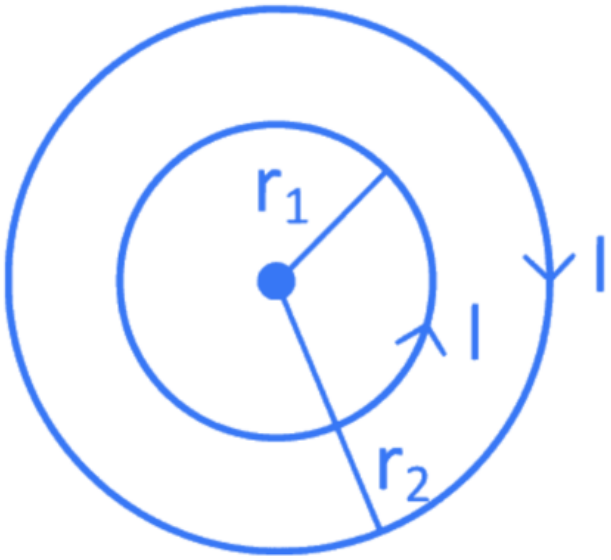
$$E_2 = \left(\frac{5}{2} + \frac{1}{10}\right)CV^2 = \frac{13}{5}CV^2, \qquad \frac{E_1}{E_2} = \frac{5}{13}.$$

The other ratios incorrectly treat both capacitors as charge-fixed or voltage-fixed, or lose a factor of two in the energy sum.

Step 4: Conclusion The required ratio is $\boxed{E_1/E_2 = 5/13}$. Hence, the correct option is (C).

Quick Tip to Solve: After opening a switch, trace each capacitor back to the battery separately. Use the voltage-fixed energy formula for the connected capacitor and the charge-fixed formula for the isolated one.

45. Two concentric circular loops of radii $r_1 = 30$ cm and $r_2 = 50$ cm are placed in X − Y plane as shown in the figure. A current $I = 7$ A is flowing through them in the direction as shown in figure. The net magnetic moment of this system of two circular loops is approximately : [4 marks]



(A) $\frac{7}{2}\hat{k}\,\text{Am}^2$

(B) $-\frac{7}{2}\hat{k}\,\text{Am}^2$

(C) $7\,\hat{k}\,\text{Am}^2$

(D) $-7\,\hat{k}\,\text{Am}^2$

Correct Answer: (B) $-\frac{7}{2}\hat{k}\,\text{Am}^2$

Solution:

Concept:

- The magnetic dipole moment of one circular current loop is $\vec{\mu} = I\pi r^2\hat{n}$, where $\hat{n}$ is determined by the right-hand rule.
- Magnetic moments add as vectors. For coplanar loops carrying opposite senses of current, their moments oppose; the resultant magnitude is the difference of their moments.

Step 1: Determine the moment directions View the loops from the positive z-axis, perpendicular to the X-Y plane. The outer loop carries clockwise current, as its arrow on the right points downward. Its magnetic moment is along $-\hat{k}$. The inner loop carries anticlockwise current, with its right-hand arrow directed upward, so its moment is along $+\hat{k}$. These directions follow by curling the right-hand fingers along the current and reading the thumb direction.

Step 2: Convert the radii and add the vector moments The radii must be expressed in metres:

$$r_1 = 0.30\,\text{m}, \qquad r_2 = 0.50\,\text{m}.$$

With equal current $I = 7\,\text{A}$, the moments are

$$\vec{\mu}_1 = 7\pi(0.30)^2\hat{k}, \qquad \vec{\mu}_2 = -7\pi(0.50)^2\hat{k}.$$

Therefore

$$\vec{\mu}_{\text{net}} = 7\pi\big(0.09 - 0.25\big)\hat{k} = -1.12\pi\hat{k}\,\text{A}\,\text{m}^2.$$

Subtract areas, rather than radii, because magnetic moment is proportional to loop area.

Step 3: Evaluate and compare the options Using $\pi \simeq 22/7$,

$$\vec{\mu}_{\text{net}} \simeq -3.52\hat{k}\,\text{A}\,\text{m}^2 \simeq -\frac{7}{2}\hat{k}\,\text{A}\,\text{m}^2.$$

The negative sign is necessary because the larger outer loop dominates. Option (A) has the correct approximate magnitude but reverses this direction. Options (C) and (D) have magnitude $7\,\text{A}\,\text{m}^2$, which does not equal the difference of the two loop moments.

Step 4: Conclusion The net magnetic moment is approximately $\boxed{-\frac{7}{2}\hat{k}\,\text{A}\,\text{m}^2}$. Hence, the correct option is (B).

Quick Tip to Solve: Assign a signed normal to each loop before adding its moment. With opposite currents and the same current magnitude, the larger loop fixes the direction and the difference of areas fixes the magnitude.

46. A velocity selector consists of electric field $\vec{E} = E\,\hat{k}$ and magnetic field $\vec{B} = B\,\hat{j}$ with $B = 12\,mT$. The value of E required for an electron of energy $728\,eV$ moving along the positive x-axis to pass undeflected is :

(Given, mass of electron $= 9.1 \times 10^{-31}$ kg) [4 marks]

(A) $192\,\text{kVm}^{-1}$

(B) $192\,\text{mVm}^{-1}$

(C) $9600\,\text{kVm}^{-1}$

(D) $16\,\text{kVm}^{-1}$

Correct Answer: (A) $192\,\text{kVm}^{-1}$

Solution:

Concept:

- A particle passes a velocity selector undeflected when its Lorentz force vanishes: $q(\vec{E} + \vec{v} \times \vec{B}) = 0$. For perpendicular fields, the required magnitudes satisfy $E = vB$.
- For a nonrelativistic electron, kinetic energy is $K = \frac{1}{2}m_e v^2$. Convert electronvolts to joules using $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.

Step 1: Convert the kinetic energy The electron energy is $728\,\text{eV}$, not $728\,\text{J}$. Therefore

$$K = 728(1.6 \times 10^{-19}) = 1.1648 \times 10^{-16}\,\text{J}.$$

This energy is very small compared with the electron rest energy, so the usual nonrelativistic kinetic-energy expression is appropriate here.

Step 2: Calculate the electron speed With the given mass $m_e = 9.1 \times 10^{-31}\,\text{kg}$,

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.1648 \times 10^{-16})}{9.1 \times 10^{-31}}} = \sqrt{2.56 \times 10^{14}} = 1.6 \times 10^{7}\,\text{m s}^{-1}.$$

The speed is a scalar magnitude; the electron moves in the specified positive x-direction.

Step 3: Balance the field-force magnitudes Convert the magnetic field to tesla: $B = 12 \times 10^{-3}\,\text{T}$. The required electric-field magnitude is

$$|E| = vB = (1.6 \times 10^{7})(12 \times 10^{-3}) = 1.92 \times 10^{5}\,\text{V m}^{-1} = 192\,\text{kV m}^{-1}.$$

The field unit is kilovolt per metre, ruling out the millivolt option. The values 9600 and $16\,\text{kV m}^{-1}$ do not equal the product of the given magnetic field and calculated speed.

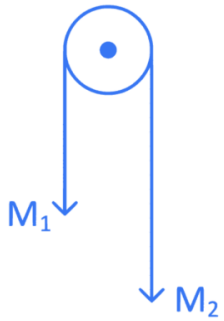
Step 4: Check the direction required for cancellation Since $\hat{i} \times \hat{j} = \hat{k}$, $\vec{v} \times \vec{B}$ points along positive z. The cancelling electric field must therefore point along $-\hat{k}$. If $\vec{E} = E\hat{k}$ uses a signed coefficient, that coefficient is negative; the positive numerical options give the required field magnitude. The negative electron charge reverses both force directions and does not change the cancellation condition.

Step 5: Conclusion The required electric-field magnitude is $\boxed{192\,\text{kV m}^{-1}}$, directed along $-\hat{k}$ for cancellation. Hence, the correct option is (A).

Quick Tip to Solve: Use energy in joules to find the speed, then multiply by the field in tesla. Check the electric-field direction with $\vec{E} = -\vec{v} \times \vec{B}$; the sign of the particle charge cancels.

---

47. Two masses M_1 and M_2 are tied together at the two ends of a light inextensible string that passes over a frictionless pulley. When the mass M_2 is twice that of M_1, the acceleration of the system is a_1. When the mass M_2 is thrice that of M_1, the acceleration of the system is a_2. The ratio $\frac{a_1}{a_2}$ will be : [4 marks]



(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) $\frac{1}{2}$

Correct Answer: (B) $\frac{2}{3}$

Solution:

Concept:

- For two masses joined by a light inextensible string over an ideal pulley, the string tension is the same on both sides and the acceleration magnitudes are equal.
- Adding Newton's second-law equations eliminates the internal tension. The driving force is the weight difference, while both masses contribute to the inertia.

Step 1: Derive the common acceleration Choose downward motion of the heavier mass M_2 and upward motion of M_1 as positive. Let the common tension be T. Newton's law gives

$$M_2 g - T = M_2 a, \qquad T - M_1 g = M_1 a.$$

Add the equations to eliminate T:

$$(M_2 - M_1)g = (M_1 + M_2)a.$$

Hence

$$a = \frac{M_2 - M_1}{M_1 + M_2} g.$$

The denominator is the sum because both bodies accelerate, although their directions are opposite.

Step 2: Evaluate the first mass ratio When $M_2 = 2M_1$, write $M_1 = m$. Substitution gives

$$a_1 = \frac{2m - m}{2m + m} g = \frac{g}{3}.$$

A heavier mass twice the lighter one does not make the acceleration equal to $g/2$: the net driving weight is mg, while the total moving mass is $3m$.

Step 3: Evaluate the second ratio and compare When $M_2 = 3M_1$, the same formula gives

$$a_2 = \frac{3m - m}{3m + m} g = \frac{g}{2}.$$

Therefore

$$\frac{a_1}{a_2} = \frac{g/3}{g/2} = \frac{2}{3}.$$

Both the mass scale and g cancel. Since the greater imbalance produces the larger acceleration, the ratio must be below one; this immediately excludes $3/2$. The values $1/3$ and $1/2$ are individual accelerations as fractions of g, rather than the ratio requested.

Step 4: Conclusion The required acceleration ratio is $\boxed{a_1/a_2 = 2/3}$. Hence, the correct option is (B).

Quick Tip to Solve: For an ideal two-mass pulley, use weight difference divided by total mass. Find each acceleration as a fraction of g first, then divide those fractions in the order asked.

48. The area of cross section of the rope used to lift a load by a crane is 2.5×10^{-4} m^2. The maximum lifting capacity of the crane is 10 metric tons. To increase the lifting capacity of the crane to 25 metric tons, the required area of cross section of the rope should be :

(take $g = 10\, ms^{-2}$) [4 marks]

(A) 6.25×10^{-4} m^2

(B) 10×10^{-4} m^2

(C) 1×10^{-4} m^2

(D) 1.67×10^{-4} m^2

Correct Answer: (A) 6.25×10^{-4} m^2

Solution:

Concept:

- Tensile stress is force per cross-sectional area: $\sigma = F/A$. A rope of the same material has the same limiting permissible tensile stress.
- To lift a load of mass M under the stated capacity condition, the force is its weight Mg. Thus Mg/A remains constant when capacity is scaled.

Step 1: Find the permissible stress from the original rope The initial maximum load is 10 metric tons, or $10^4\,\mathrm{kg}$. With $g = 10\,\mathrm{m\,s^{-2}}$, its weight is

$$F_1 = (10^4)(10) = 10^5\,\mathrm{N}.$$

The original area is $A_1 = 2.5 \times 10^{-4}\,\mathrm{m^2}$, so the limiting stress is

$$\sigma = \frac{10^5}{2.5 \times 10^{-4}} = 4.0 \times 10^8\,\mathrm{Pa}.$$

This is the stress that the replacement rope must not exceed.

Step 2: Determine the force for the new capacity The desired capacity is 25 metric tons, or $2.5 \times 10^4\,\mathrm{kg}$. Its weight is

$$F_2 = (2.5 \times 10^4)(10) = 2.5 \times 10^5\,\mathrm{N}.$$

Keeping the limiting stress unchanged requires

$$A_2 = \frac{F_2}{\sigma} = \frac{2.5 \times 10^5}{4.0 \times 10^8} = 6.25 \times 10^{-4}\,\mathrm{m^2}.$$

The result is an area, so its unit must be square metre.

Step 3: Check the scaling against the choices Equivalently,

$$\frac{A_2}{A_1} = \frac{M_2}{M_1} = \frac{25}{10} = 2.5.$$

The area must increase by a factor of 2.5, exactly as the supported load increases. Option (B) increases the area fourfold and is not the required value for the stated capacity. Options (C) and (D) reduce the area, raising the stress for an even larger load; they cannot provide the proposed increase in safe lifting capacity.

Step 4: Conclusion The required rope area is $\boxed{6.25 \times 10^{-4}\,\mathrm{m}^2}$. Hence, the correct option is (A).

Quick Tip to Solve: For the same rope material and limiting stress, capacity is proportional to cross-sectional area. Scale the area by the load ratio; do not scale the radius by that ratio.

49. The maximum and minimum voltage of an amplitude modulated signal are $60\ \mathrm{V}$ and $20\ \mathrm{V}$ respectively. The percentage modulation index will be : [4 marks]

(A) 0.5%

(B) 50%

(C) 2%

(D) 30%

Correct Answer: (B) 50%

Solution:

Concept:

- For a single-tone amplitude-modulated wave, the envelope extremes are $V_{\max} = V_c(1+m)$ and $V_{\min} = V_c(1-m)$, where V_c is carrier amplitude and m is the modulation index.
- The modulation index is dimensionless. Percentage modulation is $100m$, so a fractional index must be multiplied by 100 before attaching a percent sign.

Step 1: Separate carrier and modulating amplitudes Add the envelope-maximum and envelope-minimum equations. The modulation terms cancel, giving

$$2V_c = V_{\max} + V_{\min} = 60 + 20 = 80\,\mathrm{V}.$$

Hence the carrier amplitude is $V_c = 40\,\mathrm{V}$. Subtracting the equations gives the modulating-signal amplitude:

$$2V_m = V_{\max} - V_{\min} = 60 - 20 = 40\,\mathrm{V}, \qquad V_m = 20\,\mathrm{V}.$$

These amplitudes reconstruct the given extremes, since $40 + 20 = 60$ and $40 - 20 = 20$.

Step 2: Calculate the fractional modulation index The ratio of modulating amplitude to carrier amplitude is

$$m = \frac{V_m}{V_c} = \frac{20}{40} = 0.5.$$

Directly in terms of the observed envelope extremes, the same calculation is

$$m = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} = \frac{60 - 20}{60 + 20} = \frac{40}{80} = 0.5.$$

Using only the difference or only the maximum would omit the carrier-amplitude normalization.

Step 3: Express the result as a percentage The question asks for a percentage rather than the dimensionless fraction. Therefore

$$\text{percentage modulation} = m \times 100\% = 0.5 \times 100\% = 50\%.$$

The value $m = 0.5 < 1$ is consistent with an envelope whose minimum is positive and which is not overmodulated. Option (A) confuses the fraction 0.5 with 0.5%. Options (C) and (D) would correspond to much smaller modulation ratios and do not reproduce the given envelope extremes.

Step 4: Conclusion The percentage modulation index is $\boxed{50\%}$. Hence, the correct option is (B).

Quick Tip to Solve: Use difference over sum of the envelope extremes, then multiply by 100 for percentage modulation. Keep the fractional index 0.5 distinct from the percentage 50%.

50. Mass numbers of two nuclei are in the ratio of $4 : 3$. Their nuclear densities will be in the ratio of [4 marks]

(A) 4 : 3
(B) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$
(C) 1 : 1
(D) $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

Correct Answer: (C) 1 : 1

Solution:

Concept:

- The mass number A counts the nucleons. A nucleus has approximate mass $M = Am_N$, where m_N is the mean nucleon mass.
- The nuclear-radius law is $R = R_0 A^{1/3}$, with the same empirical constant R_0 for the nuclei being compared. Density is mass divided by volume.

Step 1: Write the nuclear volume Approximating a nucleus by a sphere, its volume is $V = 4\pi R^3/3$. Substituting the radius law gives

$$V = \frac{4\pi}{3} R_0^3 A.$$

Thus volume is directly proportional to mass number, rather than to its cube root. For the two nuclei in the question, the volume ratio is therefore $V_1 : V_2 = 4 : 3$.

Step 2: Cancel the mass-number dependence The corresponding masses are also in the ratio $4 : 3$. Hence

$$\rho = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3}, \qquad \frac{\rho_1}{\rho_2} = \frac{4/3}{4/3} = 1.$$

The factor A cancels, so a larger nucleus contains more nucleons but also occupies proportionately more volume. Nuclear density is consequently approximately independent of the mass number.

Step 3: Compare the offered ratios Option (A) uses the mass ratio alone and neglects the accompanying increase in volume. Options (B) and (D) are cube-root ratios associated with radii or their reciprocals, not mass per unit volume. Only the equal-density ratio uses both the mass law and the volume law consistently.

Step 4: Conclusion The nuclear densities are in the ratio $\boxed{1 : 1}$. Hence, the correct option is (C).

Quick Tip to Solve: Cube the radius relation before comparing densities: nuclear radius varies as $A^{1/3}$, but nuclear volume and mass both vary as A.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. If $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})\,\mathrm{m}$ and $\vec{B} = (\hat{i} + 2\hat{j} + 2\hat{k})\,\mathrm{m}$. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be _____ m. [4 marks]

Solution:

Concept:

- The component of $\vec{A}$ along $\vec{B}$ is the scalar projection $\vec{A} \cdot \hat{B}$, where $\hat{B} = \vec{B}/|\vec{B}|$ is a unit vector.
- The requested magnitude is $|\vec{A} \cdot \vec{B}|/|\vec{B}|$. In Cartesian coordinates, a dot product is the sum of products of corresponding components.

Step 1: Evaluate the dot product Use the components $(2, 3, -1)$ and $(1, 2, 2)$, retaining the negative sign of the last component of $\vec{A}$:

$$\vec{A} \cdot \vec{B} = 2(1) + 3(2) + (-1)(2) = 2 + 6 - 2 = 6\,\mathrm{m}^2.$$

The dot product is positive, so the component points along the chosen direction of $\vec{B}$. The dimensions are length squared at this stage because both supplied vectors have length units.

Step 2: Normalise the direction vector The magnitude of the direction-defining vector is

$$|\vec{B}| = \sqrt{1^2 + 2^2 + 2^2} = 3\,\mathrm{m}.$$

Consequently its unit vector is $\hat{B} = (\hat{i} + 2\hat{j} + 2\hat{k})/3$. Dividing by this magnitude is essential: a dot product with $\vec{B}$ itself is not yet a scalar component, because $\vec{B}$ is not a unit vector.

Step 3: Calculate and check the projection The magnitude of the required component is therefore

$$A_{\parallel B} = \frac{|6\,\mathrm{m}^2|}{3\,\mathrm{m}} = 2\,\mathrm{m}.$$

As a consistency check, $|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}\,\mathrm{m}$. The calculated component is smaller than the vector magnitude, as every projection must be. Its unit is metres after the normalisation.

Step 4: Conclusion The magnitude of the component of $\vec{A}$ along $\vec{B}$ is $\boxed{2\,\mathrm{m}}$; the numerical entry is 2.

Quick Tip to Solve: To find the component of A along B, divide A dot B by the magnitude of B, the vector defining the direction; divide by the magnitude of A only for the reverse projection.

52. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be _____ m.

Given, the length of the rod is $10\sqrt{3}$ m. [4 marks]

Solution:

Concept:

- The radius of gyration k is defined by $I = Mk^2$, where I is the moment of inertia about the specified axis and M is the total mass.
- For a uniform thin rod of length L, the moment of inertia about a transverse axis through its centre is $I = ML^2/12$. The thin-rod model applies when only its length is specified.

Step 1: Identify the central transverse axis The stated axis passes through the centre and is perpendicular to the length, so distances along the rod range symmetrically from $-L/2$ to $L/2$. With uniform linear mass density M/L, the moment of inertia follows from

$$I = \frac{M}{L}\int_{-L/2}^{L/2} s^2\,ds = \frac{M}{L}\left[\frac{s^3}{3}\right]_{-L/2}^{L/2} = \frac{ML^2}{12}.$$

This is the centre-axis result; using $ML^2/3$ would instead describe a transverse axis through an end.

Step 2: Convert moment of inertia to radius of gyration Equate the defining form Mk^2 to the central-axis expression:

$$Mk^2 = \frac{ML^2}{12}, \qquad k^2 = \frac{L^2}{12}, \qquad k = \frac{L}{\sqrt{12}}.$$

The mass cancels because the geometric distribution, rather than the total amount of mass, fixes the radius of gyration of this uniform rod. Take the positive root since k is a distance.

Step 3: Insert the given rod length Using $L = 10\sqrt{3}\,\text{m}$ and $\sqrt{12} = 2\sqrt{3}$,

$$k = \frac{10\sqrt{3}}{2\sqrt{3}}\,\text{m} = 5\,\text{m}.$$

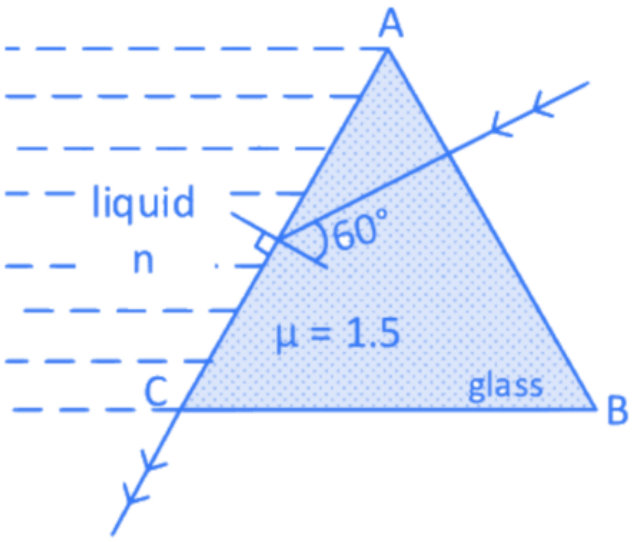
The result is less than the half-length $5\sqrt{3}\,\text{m}$, consistent with k being the root-mean-square distance of the mass elements from the central axis.

Step 4: Conclusion The radius of gyration is $\boxed{5\,\text{m}}$, so the numerical entry is 5.

Quick Tip to Solve: First identify whether the transverse axis passes through the centre or an end: the corresponding thin-rod factors in I are 1/12 and 1/3.

53. In the given figure, the face AC of the equilateral prism is immersed in a liquid of refractive index 'n'. For incident angle 60° at the side AC, the refractive light beam just grazes along face AC. The refractive index of the liquid $n = \frac{\sqrt{x}}{4}$. The value of x is _____.

(Given refractive index of glass $= 1.5$) [4 marks]



Solution:

Concept:

- Snell's law at the glass-liquid boundary is $n_g \sin i = n \sin r$, where angles are measured from the normal and $n_g = 1.5$.
- A refracted ray that grazes the boundary has $r = 90^\circ$. The corresponding internal incidence angle is the critical angle for the glass-liquid pair.

Step 1: Read the angle at face AC The figure places the 60° angle between the ray inside the glass and the normal to face AC. The ray subsequently travels along that face in the liquid. Thus the angle to use at the glass-liquid boundary is $i = 60^\circ$, not the complementary angle to the surface. Since the ray grazes AC, its angle from the normal in the liquid is $r = 90^\circ$. No additional refraction calculation at the other face is needed to determine the liquid index.

Step 2: Apply the grazing-emergence condition Substitute these angles into Snell's law, taking the incident medium to be glass and the refracted medium to be liquid:

$$1.5 \sin 60^\circ = n \sin 90^\circ.$$

Since $\sin 60^\circ = \sqrt{3}/2$ and $\sin 90^\circ = 1$,

$$n = \frac{3}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}.$$

This value is less than the glass index, as required for a critical angle and grazing emergence from the denser medium.

Step 3: Match the prescribed form The question writes the liquid index as $n = \sqrt{x}/4$. Comparing this

with the calculated expression gives

$$\sqrt{x} = 3\sqrt{3}, \qquad x = (3\sqrt{3})^2 = 9 \times 3 = 27.$$

Equivalently, $n^2 = x/16 = 27/16$. Refractive index and the parameter x are both dimensionless, so no length or angular unit is attached to the numerical entry.

Step 4: Conclusion The liquid has refractive index $n = 3\sqrt{3}/4 = \sqrt{27}/4$, and hence $\boxed{x = 27}$.

Quick Tip to Solve: For grazing emergence, put the refracted angle equal to 90 degrees in Snell's law; use the incidence angle measured from the normal in the denser medium.

54. Two lighter nuclei combine to form a comparatively heavier nucleus by the relation given below :

$$^{2}_{1}X + {}^{2}_{1}X = {}^{4}_{2}Y$$

The binding energies per nucleon for $^{2}_{1}X$ and $^{4}_{2}Y$ are $1.1\,\text{MeV}$ and $7.6\,\text{MeV}$ respectively. The energy released in this process is _____ MeV. [4 marks]

Solution:

Concept:

- Total binding energy equals the binding energy per nucleon multiplied by the mass number. The per-nucleon values cannot be subtracted directly to obtain a reaction energy.
- When nucleon numbers are conserved, energy released equals the increase in total binding energy: $Q = B_{\text{products}} - B_{\text{reactants}}$. Greater binding corresponds to lower rest energy.

Step 1: Count the nucleons on each side The reaction combines two nuclei, each with mass number 2, into one nucleus of mass number 4. Both sides therefore contain four nucleons in total, and the charge number is also conserved. The initial binding-energy value $1.1\,\text{MeV}$ applies to each nucleon of each of the two initial nuclei; the final value $7.6\,\text{MeV}$ applies to each of the four nucleons in the product.

Step 2: Calculate both total binding energies For one initial nucleus,

$$B_X = 2(1.1) = 2.2\,\text{MeV}.$$

There are two such nuclei, so

$$B_{\text{initial}} = 2B_X = 4.4\,\text{MeV}.$$

The final nucleus has total binding energy

$$B_{\text{final}} = 4(7.6) = 30.4\,\text{MeV}.$$

Multiplying by the mass numbers avoids confusing a per-nucleon energy with the energy of a whole nucleus.

Step 3: Use the increase in binding energy The product nucleons are more tightly bound, so their

combined rest energy is lower. The difference is released as reaction energy:

$$Q = B_{\text{final}} - B_{\text{initial}} = 30.4 - 4.4 = 26.0\,\text{MeV}.$$

The positive sign denotes release rather than absorption. As a direct check, all four nucleons gain $7.6 - 1.1 = 6.5\,\text{MeV}$ of binding energy each; multiplying by four gives the same value.

Step 4: Conclusion The energy released in the fusion process is $\boxed{26\,\text{MeV}}$; the numerical entry is 26.

Quick Tip to Solve: Convert every binding energy per nucleon to a total first; alternatively, when the same number of nucleons is present on both sides, multiply the per-nucleon increase by that number.

---

55. A uniform heavy rod of mass $20\ \text{kg}$, cross sectional area $0.4\ \text{m}^2$ and length $20\ \text{m}$ is hanging from a fixed support. Neglecting the lateral contraction, the elongation in the rod due to its own weight is $x \times 10^{-9}\ \text{m}$. The value of x is _____.

(Given, young modulus Y = $2 \times 10^{11}\ \text{Nm}^{-2}$ and g = $10\ \text{ms}^{-2}$) [4 marks]

Solution:

Concept:

- Young's modulus obeys $Y = (F/A)/(d\ell/\ell)$; for a small element of length ds, its extension is $d\ell = T(s)ds/(AY)$.
- In a uniform rod hanging vertically, tension increases from zero at the free lower end to Mg at the support. It is not equal to the full weight at every cross-section.

Step 1: Express the tension at a general section Let s be distance upwards from the free lower end, and let L denote the full rod length. The section at s supports only the rod material below it. Since mass per unit length is M/L,

$$T(s) = \frac{M}{L}sg, \qquad 0 \le s \le L.$$

This correctly gives zero tension at the bottom and the full weight at the top. The cross-sectional area is constant because lateral contraction is neglected.

Step 2: Integrate the elastic extension For each small element use its local tension, then add the extensions along the rod:

$$\Delta L = \int_0^L \frac{T(s)}{AY}\,ds = \frac{Mg}{LAY}\int_0^L s\,ds = \frac{MgL}{2AY}.$$

The factor $1/2$ reflects the linearly varying load: the mean tension is half the weight. Using $MgL/(AY)$ would describe a rod pulled by an external end load equal to its weight, not stretching solely under its own weight.

Step 3: Substitute the supplied values With $M = 20\,\text{kg}$, $g = 10\,\text{m s}^{-2}$, $L = 20\,\text{m}$, $A = 0.4\,\text{m}^2$, and $Y = 2 \times 10^{11}\,\text{N m}^{-2}$,

$$\Delta L = \frac{20 \times 10 \times 20}{2(0.4)(2 \times 10^{11})}\,\text{m} = 2.5 \times 10^{-8}\,\text{m} = 25 \times 10^{-9}\,\text{m}.$$

Comparing coefficients in the requested form determines the dimensionless value of x.

Step 4: Conclusion The elongation is $25 \times 10^{-9}\,\text{m}$, so $\boxed{x = 25}$.

Quick Tip to Solve: For a uniform rod stretched only by its own weight, use half its weight as the average tension; this supplies the factor $1/2$ in the extension.

---

56. Three point charges of magnitude $5\mu\text{C}, 0.16\mu\text{C}$ and $0.3\mu\text{C}$ are located at the vertices A, B, C of a right angled triangle whose sides are $AB = 3\text{ cm}, BC = 3\sqrt{2}\text{ cm}$ and $CA = 3\text{ cm}$ and point A is the right angle corner. Charge at point A experiences _____ N of electrostatic force due to the other two charges. [4 marks]

Solution:

Concept:

- Coulomb's law gives the magnitude of the force between two point charges: $F = k|q_1 q_2|/r^2$, with $k = 9 \times 10^9\,\text{N m}^2\,\text{C}^{-2}$.
- The net force is the vector sum of the individual forces. Here AB and AC meet at a right angle, so their force magnitudes combine by Pythagoras.

Step 1: Assign the charges and convert to SI units The charges at A, B, C, respectively, are $q_A = 5 \times 10^{-6}\,\text{C}$, $q_B = 0.16 \times 10^{-6}\,\text{C}$, and $q_C = 0.3 \times 10^{-6}\,\text{C}$. For the force on A, both relevant separations are $AB = AC = 3 \times 10^{-2}\,\text{m}$. The hypotenuse $BC = 3\sqrt{2}\,\text{cm}$ is not a separation involving charge A, so it does not enter either Coulomb-force denominator.

Step 2: Calculate the force from each other charge For the interaction with the charge at B,

$$F_{AB} = \frac{9 \times 10^9 (5 \times 10^{-6})(0.16 \times 10^{-6})}{(3 \times 10^{-2})^2} = 8\,\text{N}.$$

Similarly, the charge at C exerts

$$F_{AC} = \frac{9 \times 10^9 (5 \times 10^{-6})(0.3 \times 10^{-6})}{(3 \times 10^{-2})^2} = 15\,\text{N}.$$

Each force lies on the line joining the corresponding pair of charges.

Step 3: Add the perpendicular forces Since A is the right-angle vertex, the two force lines are perpendicular. Hence the resultant magnitude is

$$F_A = \sqrt{F_{AB}^2 + F_{AC}^2} = \sqrt{8^2 + 15^2} = \sqrt{289} = 17\,\text{N}.$$

Adding the magnitudes directly would give $23\,\text{N}$, which is appropriate only for collinear forces in the same direction. Here the right-angle geometry instead produces the familiar $8, 15, 17$ combination.

Step 4: Conclusion The magnitude of the electrostatic force experienced by the charge at A is $\boxed{17\,\text{N}}$; the numerical entry is 17.

Quick Tip to Solve: Use the two distances from the charge whose force is required. If their lines are perpendicular, calculate the individual forces and combine their squares.

---

57. In a coil of resistance $8\,\Omega$, the magnetic flux due to an external magnetic field varies with time as $\phi = \frac{2}{3}\left(9 - t^2\right)$. The value of total heat produced in the coil, till the flux becomes zero, will be _____ J. [4 marks]

Solution:

Concept:

- Faraday's law gives the induced emf as $\mathcal{E} = -d\phi/dt$; the sign specifies the direction required by Lenz's law.
- For a coil of resistance R, the instantaneous heating power is $P = I^2R = \mathcal{E}^2/R$. Total heat is the time integral of this power.

Step 1: Find the time interval The prescribed flux is $\phi = (2/3)(9 - t^2)$. We start at $t = 0$, when the flux is $6\,\mathrm{Wb}$, and stop at its first zero for nonnegative time:

$$\frac{2}{3}(9 - t_f^2) = 0 \quad \Rightarrow \quad t_f = 3\,\mathrm{s}.$$

The negative root is outside the elapsed-time interval, so it is not another integration limit.

Step 2: Obtain the induced current and power Differentiate the entire flux expression with respect to time. The emf and current grow linearly with time:

$$\mathcal{E} = -\frac{d}{dt}\left[\frac{2}{3}(9 - t^2)\right] = \frac{4t}{3}, \qquad I = \frac{\mathcal{E}}{8} = \frac{t}{6}.$$

Thus $P = I^2R = (t^2/36)8 = 2t^2/9$. The instantaneous power is nonnegative, independent of the current's chosen sign convention. A constant-current heating formula cannot be used with just the final current because this current changes throughout the interval.

Step 3: Integrate the heat produced Add the heat contributions $dQ = P\,dt$ over the whole interval:

$$Q = \int_0^3 \frac{2t^2}{9}\,dt = \frac{2}{9}\left[\frac{t^3}{3}\right]_0^3 = \frac{2}{27}(27) = 2\,\mathrm{J}.$$

The result uses the changing emf, rather than treating the total flux change as the emf itself.

Step 4: Conclusion The total heat produced before the flux first becomes zero is $\boxed{2\,\mathrm{J}}$. The required numerical entry is 2.

Quick Tip to Solve: For time-dependent flux, differentiate first, square the emf and then integrate $\mathcal{E}^2/R$; integrating the emf alone gives charge information, not heat.

58. As per given figures, two springs of spring constants k and $2k$ are connected to mass m. If the period of oscillation in figure (a) is 3s, then the period of oscillation in figure (b) will be $\sqrt{x}$ s. The value of x is _____. [4 marks]

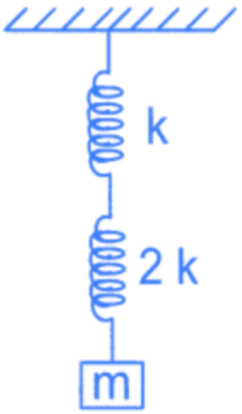


figure (a)

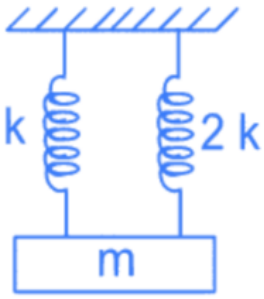


figure (b)

Solution:

Concept:

- The period of a mass on a spring is $T = 2\pi\sqrt{m/k_{\text{eff}}}$, where k_{eff} is the restoring-force constant about equilibrium.
- Springs in series carry the same force and their extensions add. Springs in parallel have the same extension and their restoring forces add.

Step 1: Reduce the series arrangement in figure (a) The two springs are attached one after the other. Under a force F, their extensions are F/k and $F/(2k)$. Therefore the total extension is

$$y = \frac{F}{k} + \frac{F}{2k} = \frac{3F}{2k}, \qquad k_a = \frac{F}{y} = \frac{2k}{3}.$$

This effective constant is less than either individual spring constant, as expected for a series combination.

Step 2: Reduce the parallel arrangement in figure (b) Both upper ends are fixed, and both lower ends move with the same mass. For a displacement y from equilibrium, the restoring forces add:

$$F_{\text{restoring}} = -(k + 2k)y = -3ky, \qquad k_b = 3k.$$

Gravity determines the static extension but does not change these small-oscillation spring constants. The same mass m is present in both arrangements.

Step 3: Compare the periods without finding mass Using the ratio of the two period formulae cancels both m and 2π:

$$\frac{T_b}{T_a} = \sqrt{\frac{k_a}{k_b}} = \sqrt{\frac{2k/3}{3k}} = \frac{\sqrt{2}}{3}.$$

Since $T_a = 3\,\text{s}$,

$$T_b = 3\left(\frac{\sqrt{2}}{3}\right) = \sqrt{2}\,\text{s}.$$

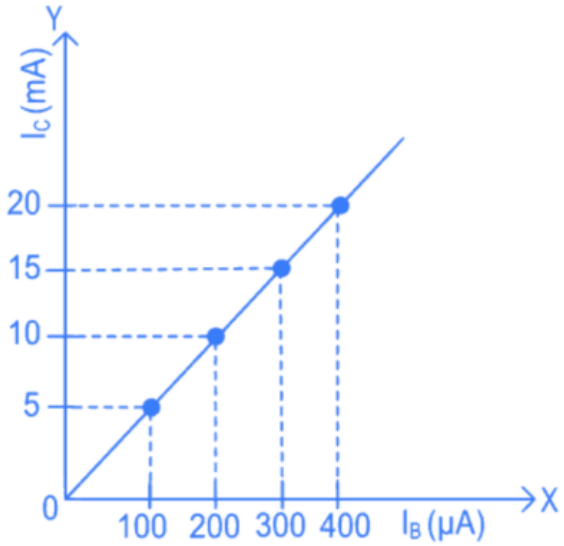
The parallel arrangement is stiffer, so its smaller period is physically consistent. Comparing this with the specified $\sqrt{x}\,\text{s}$ identifies the requested number.

Step 4: Conclusion The period in figure (b) is $\sqrt{2}\,\text{s}$, so $\boxed{x = 2}$ (dimensionless).

Quick Tip to Solve: For the same mass, use $T_b/T_a = \sqrt{k_a/k_b}$. First decide whether the springs share force (series) or displacement (parallel).

---

59. The typical transfer characteristics of a transistor in CE configuration is shown in figure. A load resistor of $2\,k\Omega$ is connected in the collector branch of the circuit used. The input resistance of the transistor is $0.50\,\text{k}\Omega$. The voltage gain of the transistor is _____. [4 marks]



Solution:

Concept:

- The small-signal current gain in a common-emitter configuration is the slope $\beta = \Delta I_C/\Delta I_B$ of its transfer characteristic.
- The voltage-gain magnitude is $|A_v| = \beta R_L/r_{\text{in}}$. The signed common-emitter gain is negative because the output voltage is inverted.

Step 1: Read the current increments from the graph Use two marked points on the straight-line part of the characteristic, for example $(I_B, I_C) = (100\,\mu\text{A}, 5\,\text{mA})$ and $(400\,\mu\text{A}, 20\,\text{mA})$. The increments are

$$\Delta I_C = (20-5)\,\text{mA} = 15\times10^{-3}\,\text{A}, \qquad \Delta I_B = (400-100)\,\mu\text{A} = 300\times10^{-6}\,\text{A}.$$

Taking differences makes clear that we require the transfer slope, not an axis value on its own.

Step 2: Calculate the current gain Convert both current units to amperes before dividing:

$$\beta = \frac{15 \times 10^{-3}}{300 \times 10^{-6}} = 50.$$

The other labelled points give the same slope: a $100\,\mu\text{A}$ increase in base current produces a $5\,\text{mA}$ increase in collector current. Failing to account for the milliampere and microampere scales would give an incorrect gain.

Step 3: Convert current gain to voltage gain An input increment obeys $\Delta V_{\text{in}} = r_{\text{in}}\Delta I_B$, while the magnitude of the load-voltage increment is $|\Delta V_{\text{out}}| = R_L\Delta I_C$. Hence

$$|A_v| = \frac{|\Delta V_{\text{out}}|}{\Delta V_{\text{in}}} = \frac{\beta R_L}{r_{\text{in}}} = 50\frac{2\,\text{k}\Omega}{0.50\,\text{k}\Omega} = 200.$$

The resistance units cancel. The usual numerical voltage-gain question requests this positive magnitude; the minus sign describes the phase inversion.

Step 4: Conclusion The voltage-gain magnitude is $\boxed{200}$, a dimensionless ratio. The signed common-emitter gain would be -200; the required numerical entry is 200.

Quick Tip to Solve: In a transistor transfer graph, convert the current-axis units before finding β, then multiply by the load-to-input resistance ratio.

---

60. A potentiometer wire of length $300\ \text{cm}$ is connected in series with a resistance $780\ \Omega$ and a standard cell of emf 4V. A constant current flows through potentiometer wire. The length of the null point for cell of emf $20\,\text{mV}$ is found to be $60\ \text{cm}$. The resistance of the potentiometer wire is _____ Ω. [4 marks]

Solution:

Concept:

- For a uniform potentiometer wire carrying constant current, the potential drop is proportional to length: $V = k_\ell l$. At balance this drop equals the test-cell emf.
- The wire and the $780\,\Omega$ resistor form a series voltage divider across the $4\,\text{V}$ driving cell. The null-point branch draws no current at balance.

Step 1: Determine the potential gradient The test-cell emf is $20\,\text{mV} = 0.020\,\text{V}$, and its balance length is $60\,\text{cm}$. Thus

$$k_\ell = \frac{0.020}{60}\,\text{V cm}^{-1} = \frac{1}{3000}\,\text{V cm}^{-1}.$$

This is the voltage drop per unit length along the potentiometer wire, not along the entire series circuit. At null, the test cell does not alter the current in that wire.

Step 2: Find the voltage across the entire wire The whole wire is $300\,\text{cm}$ long, five times the balance length, so

$$V_w = k_\ell(300) = \frac{0.020 \times 300}{60} = 0.100\,\text{V}.$$

Only this portion of the driving cell's $4\,\text{V}$ falls across the wire. The remaining drop across the series

resistance is

$$V_s = 4.000 - 0.100 = 3.900\,\text{V}.$$

Both drops correspond to the same steady current because the wire and resistor are in series.

Step 3: Use the series current to determine resistance The known resistor fixes the current:

$$I = \frac{V_s}{780\,\Omega} = \frac{3.9}{780} = 0.005\,\text{A}.$$

Applying Ohm's law to the wire then gives

$$R_w = \frac{V_w}{I} = \frac{0.100}{0.005} = 20\,\Omega.$$

As a check, the total circuit resistance is $780 + 20 = 800\,\Omega$, and $4/800 = 0.005\,\text{A}$, consistent with the current used.

Step 4: Conclusion The resistance of the potentiometer wire is $\boxed{20\,\Omega}$. The numerical entry is 20.

Quick Tip to Solve: First scale the balance emf by total length divided by balance length; subtract that wire drop from the driving emf before calculating the series current.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Hemoglobin contains 0.34% of iron by mass. The number of Fe atoms in 3.3 g of hemoglobin is (Given: Atomic mass of Fe is 56 u, $\text{N}_\text{A} = 6.022 \times 10^{23}$ mol^{-1}.) [4 marks]

(A) 1.21×10^5

(B) 12.0×10^{16}

(C) 1.21×10^{20}

(D) 3.4×10^{22}

Correct Answer: (C) 1.21×10^{20}

Solution:

Concept:

- A mass percentage gives the mass of a component per 100 units of sample mass. Thus the iron mass fraction here is $0.34/100 = 0.0034$.
- The number of atoms is $N = (m/M)N_A$, where m is the mass of that element, M its molar mass, and N_A Avogadro's constant.

Step 1: Extract the mass of iron from the sample The given 3.3 g is the mass of hemoglobin, not the mass of iron alone. Use the stated percentage to isolate the iron contribution:

$$m_{\text{Fe}} = \frac{0.34}{100} \times 3.3 = 0.01122\,\text{g} = 1.122 \times 10^{-2}\,\text{g}.$$

The iron content is much less than a gram, which provides an immediate check on the decimal placement. Treating 0.34% as 0.34 would overestimate the iron mass by a factor of 100.

Step 2: Convert the iron mass to moles and atoms The stated atomic mass corresponds to the molar mass $M_{\mathrm{Fe}} = 56\,\mathrm{g\,mol^{-1}}$. Therefore

$$n_{\mathrm{Fe}} = \frac{0.01122}{56} = 2.0036 \times 10^{-4}\,\mathrm{mol}.$$

Multiplying by the number of atoms in one mole gives

$$N_{\mathrm{Fe}} = (2.0036 \times 10^{-4})(6.022 \times 10^{23}) = 1.2065 \times 10^{20} \simeq 1.21 \times 10^{20}.$$

No molecular mass of hemoglobin is required because its iron mass fraction directly supplies the mass of iron.

Step 3: Compare the printed options The calculated count has order 10^{20}. Option (C) agrees with both the coefficient and exponent. Option (A), 1.21×10^{5}, is far too small for even this small macroscopic iron sample; option (B), 12.0×10^{16}, is also too small. Option (D), 3.4×10^{22}, is too large and does not follow from the given iron mass.

Step 4: Conclusion There are approximately $\boxed{1.21 \times 10^{20}}$ iron atoms in the sample. Hence, the correct option is (C).

Quick Tip to Solve: For an element present by mass percentage, use $N = (\text{sample mass})(\%/100)N_A/M$; the molar mass of the entire compound is unnecessary.

62. Arrange the following in increasing order of their covalent character.

A. CaF_2

B. $CaCl_2$

C. $CaBr_2$

D. CaI_2

Choose the correct answer from the options given below. [4 marks]

(A) B < A < C < D

(B) A < B < C < D

(C) A < B < D < C

(D) A < C < B < D

Correct Answer: (B) A < B < C < D

Solution:

Concept:

- Fajans' rule relates covalent character to distortion of an anion's electron cloud by a cation. A more easily polarised anion produces more covalent character.
- For salts with the same cation and equal anion charge, larger anions are more polarisable. The halide-ion size increases from fluoride through chloride and bromide to iodide.

Step 1: Identify the factor held constant All four salts contain the same cation, Ca^{2+}, and all four halide anions carry a single negative charge. Consequently the cation's charge and size, and hence

its polarising power, are unchanged across this comparison. The factor to compare is the ability of each halide electron cloud to be distorted. Comparing only formula-unit masses would not explain the bonding trend.

Step 2: Order the anion polarisabilities Going down the halogen group adds electron shells. The outer electrons are farther from the nucleus and more shielded, so the electron cloud becomes larger and easier to distort:

$$F^- < Cl^- < Br^- < I^- \quad \text{in size and polarisability.}$$

Fluoride is the smallest and least polarisable member, while iodide is the largest and most polarisable. Distortion toward the calcium ion increases electron sharing, so covalent character increases in the same order.

Step 3: Translate the trend into the question labels The resulting compound order is

$$CaF_2 < CaCl_2 < CaBr_2 < CaI_2.$$

Using the assigned labels, this is $A < B < C < D$. Option (A) incorrectly places chloride below fluoride. Option (C) incorrectly puts iodide before bromide, and option (D) incorrectly puts bromide before chloride. These reversed adjacent comparisons contradict the anion-polarisability trend.

Step 4: Conclusion The increasing covalent-character order is $\boxed{A < B < C < D}$. Hence, the correct option is (B).

Quick Tip to Solve: When the cation is identical, compare anion size: the larger halide is more polarisable and its salt has greater covalent character.

---

63. Class XII students were asked to prepare one litre of buffer solution of $\mathrm{pH}\ 8.26$ by their Chemistry teacher: The amount of ammonium chloride to be dissolved by the student in $0.2\ \mathrm{M}$ ammonia solution to make one litre of the buffer is :

(Given: $\mathrm{pK_b}\,(NH_3) = 4.74$

Molar mass of $NH_3 = 17\ \mathrm{g\ mol^{-1}}$

Molar mass of $NH_4Cl = 53.5\ \mathrm{g\ mol^{-1}}$ **)** [4 marks]

(A) 53.5 g

(B) 72.3 g

(C) 107.0 g

(D) 126.0 g

Correct Answer: (C) 107.0 g

Solution:

Concept:

- A weak base and a salt of its conjugate acid form a basic buffer. Here the base is NH_3 and the conjugate-acid ion is NH_4^+ from NH_4Cl.

- For a basic buffer, $\mathrm{pOH} = \mathrm{p}K_b + \log([\text{salt}]/[\text{base}])$. At the usual aqueous temperature, $\mathrm{pH} + \mathrm{pOH} = 14$.

Step 1: Convert the desired pH into pOH The target pH is 8.26, so the required hydroxide-ion logarithmic concentration is

$$\mathrm{pOH} = 14.00 - 8.26 = 5.74.$$

We use the basic-buffer equation because the supplied dissociation constant is K_b of ammonia. Using the pH directly with $\mathrm{p}K_b$ would mix the acid and base forms of the buffer equation.

Step 2: Determine the salt-to-base ratio The equilibrium $\mathrm{NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-}$ gives the buffer relation

$$5.74 = 4.74 + \log \frac{[\mathrm{NH_4^+}]}{[\mathrm{NH_3}]}.$$

Therefore

$$\log \frac{[\mathrm{NH_4^+}]}{[\mathrm{NH_3}]} = 1, \qquad \frac{[\mathrm{NH_4^+}]}{[\mathrm{NH_3}]} = 10.$$

With the stipulated ammonia concentration $0.2\,\mathrm{mol\,L^{-1}}$, the required ammonium concentration is $2.0\,\mathrm{mol\,L^{-1}}$. The salt supplies one ammonium ion per formula unit.

Step 3: Convert the required amount of salt to mass For one litre of final buffer, the required salt amount is

$$n_{\mathrm{NH_4Cl}} = 2.0 \times 1.0 = 2.0\,\mathrm{mol}, \qquad m = 2.0 \times 53.5 = 107.0\,\mathrm{g}.$$

Option (C) has this mass. Option (A) supplies only one mole and gives a salt/base ratio of 5, not 10. Options (B) and (D) give ratios $72.3/10.7$ and $126.0/10.7$, respectively, also different from the required ratio. The molar mass of ammonia is not needed because its concentration is already given in moles per litre.

Step 4: Conclusion The required mass of $\mathrm{NH_4Cl}$ is $\boxed{107.0\,\mathrm{g}}$. Hence, the correct option is (C).

Quick Tip to Solve: For an ammonia buffer, find pOH first; a one-unit excess of pOH over $\mathrm{p}K_b$ requires ten times as much conjugate-acid salt as base.

64. At $30^\circ C$, the half life for the decomposition of AB_2 is 200 s and is independent of the initial concentration of AB_2. The time required for 80% of the AB_2 to decompose is

Given: $\log 2 = 0.30 \quad \log 3 = 0.48$ [4 marks]

(A) 200 s

(B) 323 s

(C) 467 s

(D) 532 s

Correct Answer: (C) 467 s

Solution:

Concept:

- A concentration-independent half-life identifies a first-order reaction: $t_{1/2} = \ln 2/k$, where k is its rate constant.
- The integrated first-order rate equation is $kt = 2.303 \log([A]_0/[A]_t)$. Use the remaining concentration, rather than the decomposed fraction, in its denominator.

Step 1: Identify the order and remaining fraction The half-life is independent of the initial concentration of AB_2. This is the characteristic first-order dependence, not zero-order or second-order kinetics. If the initial concentration is a, decomposition of 80% leaves $0.20a = a/5$. Thus the required concentration ratio is $a/(0.20a) = 5$. The temperature fixes the rate constant during this calculation; it is not necessary to use an Arrhenius equation.

Step 2: Eliminate the rate constant For the half-life and the requested time, respectively,

$$k(200) = 2.303 \log 2, \qquad kt = 2.303 \log 5.$$

Dividing cancels the rate constant and the logarithm-conversion factor:

$$t = 200 \frac{\log 5}{\log 2}.$$

The supplied logarithms give $\log 5 = \log(10/2) = 1 - 0.30 = 0.70$. Therefore,

$$t = 200 \frac{0.70}{0.30} = 466.67\,\text{s} \simeq 467\,\text{s}.$$

Two half-lives would decompose only 75%, so a time somewhat greater than $400\,\text{s}$ is also a useful check. Option A is only one half-life, option B is below two half-lives, and option D is not the logarithmic time for a remaining fraction of one fifth.

Step 3: Conclusion The time needed for 80% decomposition is approximately $467\,\text{s}$. Hence, the correct option is (C).

Quick Tip to Solve: For a first-order reaction, divide the integrated equation by the half-life equation: $t/t_{1/2} = \log(1/\text{fraction left})/\log 2$.

65. The metal complex that is diamagnetic is (Atomic number: $\mathrm{Fe}, 26; \mathrm{Cu}, 29$) [4 marks]

(A) $\mathrm{K_3\,[Cu(CN)_4]}$

(B) $\mathrm{K_2\,[Cu(CN)_4]}$

(C) $\mathrm{K_3\,[Fe(CN)_4]}$

(D) $\mathrm{K_4\,[FeCl_6]}$

Correct Answer: (A) $\mathrm{K_3\,[Cu(CN)_4]}$

Solution:

Concept:

- A coordination compound is diamagnetic when all its electrons are paired. An unpaired electron produces paramagnetism.
- Calculate the oxidation state by adding the ligand charges. Each $\mathrm{CN^-}$ or $\mathrm{Cl^-}$ ligand contributes charge -1; each potassium ion contributes $+1$.

Step 1: Examine the copper complexes For $\mathrm{K_3[Cu(CN)_4]}$, the complex ion has charge -3, hence $x - 4 = -3$ and copper is in oxidation state $+1$. Neutral copper has configuration $[\mathrm{Ar}]3d^{10}4s^1$, so $\mathrm{Cu^+}$ has configuration $3d^{10}$. All five d orbitals are fully occupied and all electrons are paired. This complex is therefore diamagnetic. By contrast, in $\mathrm{K_2[Cu(CN)_4]}$, $x - 4 = -2$, so the metal is $\mathrm{Cu^{2+}}$, with configuration $3d^9$. Nine d electrons cannot all be paired, so option B is paramagnetic.

Step 2: Reject the iron alternatives In the formula $\mathrm{K_3[Fe(CN)_4]}$ as printed, $x - 4 = -3$ gives iron in oxidation state $+1$. Its d-electron count is $3d^7$. An odd number of d electrons necessarily leaves an unpaired electron, irrespective of how strongly the ligand field rearranges their distribution. Option C therefore cannot be diamagnetic. For $\mathrm{K_4[FeCl_6]}$, $x - 6 = -4$ gives $\mathrm{Fe^{2+}}$, or $3d^6$. Chloride is a weak-field ligand, producing a high-spin octahedral configuration $t_{2g}^4 e_g^2$ with four unpaired electrons. Option D is paramagnetic as well. Only the filled d shell of the copper(I) compound satisfies the requirement.

Step 3: Conclusion $\mathrm{K_3[Cu(CN)_4]}$ contains $\mathrm{Cu^+}$ with a fully paired $3d^{10}$ configuration. Hence, the correct option is (A).

Quick Tip to Solve: First obtain the metal oxidation state from the entire complex charge; a strong-field ligand does not automatically make every d-electron count diamagnetic.

66. The correct decreasing order of priority of functional groups in naming an organic compound as per IUPAC system of nomenclature is [4 marks]

(A) $-COOH > -CONH_2 > -COCl > -CHO$

(B) $-SO_3H > -COCl > -CONH_2 > -CN$

(C) $-COOR > -COCl > -NH_2 > > C{=}O$

(D) $-COOH > -COOR > -CONH_2 > -COCl$

Correct Answer: (B) $-SO_3H > -COCl > -CONH_2 > -CN$

Solution:

Concept:

- When several functional groups occur in one compound, the senior group supplies the principal suffix and lower-priority groups are expressed by prefixes.
- The relevant decreasing seniority is carboxylic acid, sulfonic acid, ester, acid halide, amide, nitrile, aldehyde, ketone and amine. Only the groups actually compared in an option need be retained.

Step 1: Select the correctly ordered subsequence Among the four groups in option B, sulfonic acid $-SO_3H$ has higher naming priority than an acid chloride $-COCl$. An acid chloride has higher priority than an amide $-CONH_2$, and the amide has higher priority than a nitrile $-CN$. Therefore the complete sequence in that option is consistent:

$$-SO_3H > -COCl > -CONH_2 > -CN.$$

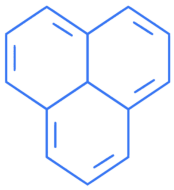
This is a nomenclature seniority rule. It should not be inferred from a comparison of acid strengths, chemical reactivities or oxidation states alone.

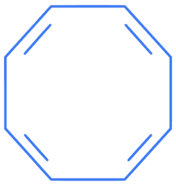
Step 2: Check the errors in the other sequences Option A places the amide group $-CONH_2$ above the acid chloride group $-COCl$. Their required order is the reverse, even though the carboxylic acid and aldehyde positions in that sequence are consistent. Option C puts the amino group $-NH_2$ before a ketone carbonyl $> C{=}O$, whereas a ketone is senior to an amine. Option D again puts an amide before an acid chloride; acid chloride must precede amide. Thus every rejected option contains a specific reversed pair. The correct sequence does not require all possible functional groups to appear, only that every displayed comparison respect the seniority order.

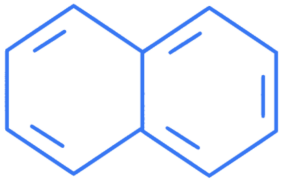
Step 3: Conclusion The correct decreasing priority is $-SO_3H > -COCl > -CONH_2 > -CN$. Hence, the correct option is (B).

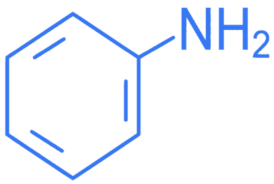
Quick Tip to Solve: Test adjacent pairs in each proposed priority sequence. The acid halide-before-amide and ketone-before-amine rules immediately eliminate the three incorrect sequences.

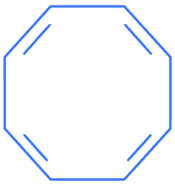
67. Which of the following is not an example of benzenoid compound? [4 marks]

(A) 

(B) 

(C) 

(D) 

Correct Answer: (B) 

Solution:

Concept:

- A benzenoid compound contains at least one benzene-type six-membered aromatic ring. The mere presence of a drawn six-membered carbon ring does not establish benzenoid character.
- Aromatic conjugation requires an uninterrupted cyclic set of p orbitals. A saturated ring-junction carbon interrupts that conjugation; an eight-membered ring is not a benzene ring.

Step 1: Identify the unequivocal nonbenzenoid option Option B is cycloocta-1,3,5,7-tetraene: one eight-membered ring bearing four alternating double bonds. It contains no benzene ring, so it is not benzenoid. Its eight pi electrons also do not give a planar aromatic $4n + 2$ system. The actual molecule adopts a nonplanar tub conformation and is nonaromatic. These observations distinguish the meaning of nonbenzenoid from the broader question of whether any molecule is aromatic; lack of a benzene ring alone already settles option B for the printed demand.

Step 2: Examine the remaining drawings and the accepted alternatives Option C is naphthalene, with two fused benzene-type rings and an extended conjugated pi system; it is benzenoid. Option D is aniline, where an amino group is attached to an intact benzene ring, so it is also benzenoid. In option A the three drawn six-membered rings meet at a central carbon joined to its three neighbours by single bonds. That central saturated junction interrupts conjugation through each of those six-membered rings. Thus the drawing does not supply an intact benzene ring and A is also accepted as nonbenzenoid. This explains why a single-answer reading cannot reject A simply because its outline resembles fused aromatic rings. The final key accepts both A and B; C and D remain benzenoid.

Step 3: Conclusion Cyclooctatetraene is not benzenoid. The official answer key accepts both A and B,

consistent with the structures shown. Hence, the correct option is (B), with A also accepted.

Quick Tip to Solve: Count the atoms and inspect the bonding of each ring. Do not identify every six-membered polygon as benzene: a saturated junction can break the required conjugation.

68. Hydrolysis of which compound will give carbolic acid? [4 marks]

(A) Cumene

(B) Benzenediazonium chloride

(C) Benzal chloride

(D) Ethylene glycol ketal

Correct Answer: (B) Benzenediazonium chloride

Solution:

Concept:

- Carbolic acid is the common name of phenol, C_6H_5OH, in which the hydroxyl group is attached directly to a benzene-ring carbon.
- An arenediazonium salt undergoes hydrolysis on warming with water, replacing its diazonium group by hydroxyl and liberating nitrogen gas.

Step 1: Apply diazonium-salt hydrolysis Benzenediazonium chloride contains the $C_6H_5{-}N_2^+$ ion. Warming its aqueous solution replaces the diazonium group at the same aromatic carbon by $-OH$. The overall transformation is

$$C_6H_5N_2Cl + H_2O \xrightarrow{\text{warm}} C_6H_5OH + N_2\uparrow + HCl.$$

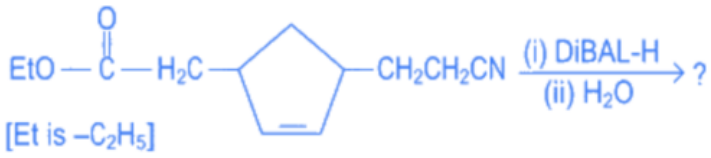
The product C_6H_5OH is phenol, which is precisely carbolic acid. Nitrogen escapes as a stable gas, making diazonium chemistry a useful route for replacing an amino-derived substituent by another group on an aromatic ring.

Step 2: Distinguish hydrolysis from other routes Cumene can furnish phenol industrially, but it must first undergo air oxidation to cumene hydroperoxide, followed by acid-catalysed cleavage. Simple hydrolysis of cumene does not make phenol, so option A does not answer the specific process requested. Hydrolysis of benzal chloride, $C_6H_5CHCl_2$, gives benzaldehyde, C_6H_5CHO; it does not replace an aryl substituent by hydroxyl. An ethylene glycol ketal is a carbonyl-protecting derivative, and its acidic hydrolysis regenerates the corresponding aldehyde or ketone together with ethylene glycol. Neither option C nor option D directly produces carbolic acid by the hydrolysis described. The distinction is between an actual hydrolysis substrate and a compound used somewhere in a different multistep phenol synthesis.

Step 3: Conclusion Hydrolysis of benzenediazonium chloride on warming gives phenol (carbolic acid). Hence, the correct option is (B).

Quick Tip to Solve: When carbolic acid is named, translate it to phenol first; then look for a reaction that installs hydroxyl directly on the benzene ring.

69. Consider the above reaction and predict the major product. [4 marks]



(A) 

(B) 

(C) 

(D) 

Correct Answer: (A)



Solution:

Concept:

- Diisobutylaluminium hydride (DIBAL-H), followed by aqueous work-up under controlled reduction conditions, converts an ester to an aldehyde and a nitrile to an aldehyde.
- In this transformation the isolated carbon-carbon double bond of the cyclopentene ring remains unchanged. Track both side chains independently rather than reducing only the nitrile.

Step 1: Reduce the ester side chain The left side chain is an ethyl ester with the arrangement

$$\text{ring}-\text{CH}_2-\text{C}(=\text{O})-\text{OEt}.$$

Controlled reduction with DIBAL-H gives an aluminium-bound intermediate that releases the aldehyde on aqueous work-up. Therefore this side chain becomes $\text{ring}-\text{CH}_2-\text{CHO}$. The carbonyl carbon remains part of the carbon skeleton, while the ethoxy group is no longer attached to it. This is the left-hand $\text{OHC}-\text{CH}_2^-$ group drawn in option A, not the unchanged ethyl ester drawn in options B

and C.

Step 2: Reduce the nitrile The right side chain contains the nitrile group:

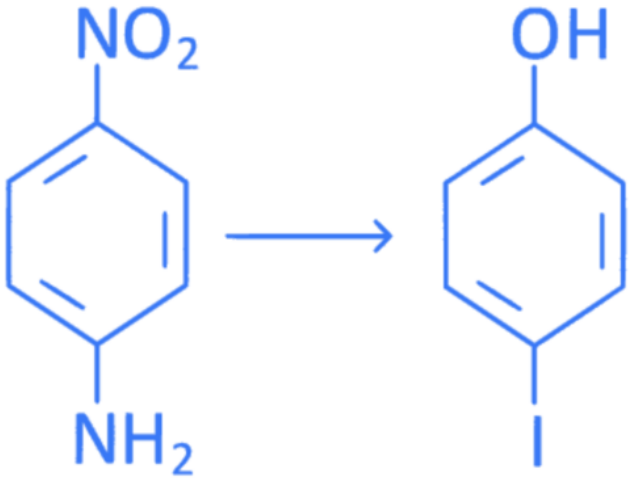
$$\text{ring}-CH_2-CH_2-CN.$$

DIBAL-H partially reduces the nitrile to an imine-type aluminium intermediate; hydrolysis then gives $\text{ring}-CH_2-CH_2-CHO$. The nitrile carbon is retained as the new aldehyde carbon. The ring double bond is not hydrogenated by these conditions. Combining the two transformations gives the dialdehyde with one $-CH_2CHO$ side chain and one $-CH_2CH_2CHO$ side chain on the same unchanged cyclopentene skeleton. Option A displays both changes. Option B reduces only the nitrile and incorrectly leaves the ester intact. Option C leaves the ester intact and gives a carboxylic acid at the nitrile position, while option D also shows that acid. Formation of that acid is not the outcome of the stated partial reduction followed by work-up.

Step 3: Conclusion Both the ester and nitrile groups become aldehydes while the ring double bond is retained, giving the dialdehyde shown in option (A). Hence, the correct option is (A).

Quick Tip to Solve: For DIBAL-H followed by water, mark an ester and a nitrile separately: each can stop at the aldehyde stage, whereas an isolated alkene remains intact.

70. The correct sequential order of the reagents for the given reaction is [4 marks]



(A) $HNO_2, Fe/H^+, HNO_2, KI, H_2O/H^+$

(B) $HNO_2, KI, Fe/H^+, HNO_2, H_2O/$ warm

(C) $HNO_2, KI, HNO_2, Fe/H^+, H_2O/H^+$

(D) $HNO_2, Fe/H^+, KI, HNO_2, H_2O/$ warm

Correct Answer: (B) $HNO_2, KI, Fe/H^+, HNO_2, H_2O/$ warm

Solution:

Concept:

- An aromatic primary amino group is diazotised by nitrous acid in the cold. The diazonium group can then be replaced by iodide using KI or by hydroxyl using warm water.
- Iron in acid reduces an aromatic nitro group to an amino group. A nitro group cannot be directly diazotised, so reduction must precede its conversion to a second diazonium group.

Step 1: Replace the original amino group by iodine The starting compound has an amino group and a nitro group para to one another. The product has iodine at the original amino position and hydroxyl at the original nitro position. First use HNO_2 under diazotisation conditions to convert the existing $-NH_2$ into $-N_2{}^+$. Treat that diazonium salt with KI, replacing the diazonium group by iodine with evolution of nitrogen. The nitro group is retained through these two operations. This sequence fixes the iodine substituent before the nitro group is reduced, thereby avoiding diazotisation of both amino positions after a premature reduction.

Step 2: Convert the nitro position to hydroxyl Next, Fe/H^+ reduces the remaining $-NO_2$ group to $-NH_2$. A second treatment with HNO_2 diazotises this newly formed aromatic amine. Finally, warming with water hydrolyses that diazonium group to $-OH$, yielding the required para-iodophenol. The sequence is therefore

$$HNO_2, \quad KI, \quad Fe/H^+, \quad HNO_2, \quad \text{warm water.}$$

Option A reduces the nitro group before replacing the first diazonium group. Option C attempts the second diazotisation before generating the needed second amino group. Option D also puts the reduction before the iodide substitution and does not give the required controlled sequence. Only option B follows the substrate changes in their proper order.

Step 3: Conclusion The required sequence is HNO_2, KI, Fe/H^+, HNO_2, followed by warm water. Hence, the correct option is (B).

Quick Tip to Solve: Assign each product substituent to its starting position: amino to iodine first, then nitro to amino to diazonium to hydroxyl.

71. Animal starch is the other name of [4 marks]

(A) amylose.

(B) maltose.

(C) glycogen.

(D) amylopectin.

Correct Answer: (C) glycogen.

Solution:

Concept:

- Polysaccharides contain many monosaccharide units joined through glycosidic linkages. The main glucose-storage polysaccharide in animals is glycogen.

- Plant starch contains amylose and amylopectin. Glycogen resembles amylopectin in having branched chains, but its branching is more extensive.

Step 1: Identify what the name describes The expression animal starch refers to the reserve carbohydrate of animals, rather than to a component isolated from plant starch. Animals store excess glucose as glycogen, particularly in the liver and skeletal muscles. When glucose is required, this polymer can be broken down to supply glucose units. Its storage function is therefore analogous to that of starch in plants, which explains the common name.

Step 2: Compare the four carbohydrates Amylose is the essentially unbranched component of plant starch, with glucose units joined mainly by $\alpha(1 \rightarrow 4)$ linkages. Amylopectin is the branched component of plant starch and contains additional $\alpha(1 \rightarrow 6)$ branch points. Neither is the name of the characteristic animal reserve. Maltose is a disaccharide containing only two glucose units, so it cannot be identified as the animal storage polysaccharide. Glycogen is the highly branched glucose polymer that satisfies both the biological source and the storage role in the question.

Step 3: Conclusion Animal starch is **glycogen**. Hence, the correct option is (C).

Quick Tip to Solve: Connect the common name with the organism: starch is the plant glucose reserve, while glycogen is the animal glucose reserve.

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72. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Phenolphthalein is a pH dependent indicator, remains colourless in acidic solution and gives pink colour in basic medium.

Reason R: Phenolphthalein is a weak acid. It doesn't dissociate in basic medium.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (C) A is true but R is false.

Solution:

Concept:

- An acid-base indicator has differently coloured forms in acid-base equilibrium. A change in pH changes their relative concentrations.
- Phenolphthalein is colourless in an acidic solution and pink in the ordinary basic indicator range. Removing protons in basic medium favours its coloured ionised form.

Step 1: Evaluate the assertion The assertion describes the familiar behaviour of phenolphthalein correctly. In an acidic solution its colourless form predominates; as the solution becomes sufficiently basic, the pink form becomes appreciable. Its colour therefore depends on the acid-base conditions,

which are expressed by pH. Assertion A is true. This is the usual indicator behaviour relevant to an acid-base titration.

Step 2: Evaluate the reason Phenolphthalein is a weak acid, but the reason adds that it does not dissociate in basic medium. That second claim is false. In the simplified weak-acid indicator description, write

$$HIn \rightleftharpoons H^+ + In^-.$$

A base removes H^+, so the equilibrium moves towards the ionised indicator. Basic medium thus favours ionisation instead of preventing it. The colour change is associated with this change of predominant form.

Step 3: Choose the logical combination A reason joined from two claims is not true merely because its first claim is correct. Since its statement about dissociation is incorrect, R as a whole is false. Options (A) and (B) wrongly treat R as true, while option (D) incorrectly rejects the observed behaviour stated in A.

Step 4: Conclusion Assertion A is true, but Reason R is false. Hence, the correct option is (C).

Quick Tip to Solve: For a weak-acid indicator, adding base removes protons and favours the ionised form; do not confuse weak ionisation with absence of ionisation.

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73. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Finest gold is red in colour, as the size of the particles increases, it appears purple then blue and finally gold.

Reason R : The colour of the colloidal solution depends on the wavelength of light scattered by the dispersed particles.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (A) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- The observed colour of a colloidal sol depends on how its dispersed particles interact with light, including the wavelengths scattered.
- Particle size influences this optical response. Colloidal gold can consequently show different colours even though the dispersed substance remains gold.

Step 1: Evaluate the assertion The assertion gives the standard size-dependent colour sequence for gold particles: the finest gold sol is red, and increasing particle size can give purple and then blue

colours, ultimately approaching the appearance of bulk gold. This change is an optical consequence of the dispersed particles, rather than evidence that gold has changed into another chemical substance. Assertion A is therefore true in the stated colloidal context.

Step 2: Evaluate the reason The colour seen in a colloidal system depends on the wavelengths of light scattered by its dispersed particles. Different wavelengths do not contribute equally to the observed light. Since the scattering response depends on particle size, altering the size distribution alters the colour perceived by the observer. Reason R correctly identifies the optical property relevant to colloidal colour and is true.

Step 3: Test the explanatory connection The assertion specifically changes the size of the gold particles, while keeping their chemical identity fixed. That size change changes their interaction with the incident light and hence the wavelengths contributing to the observed colour. Thus R supplies the physical basis of the sequence described in A. Option (B) unnecessarily denies this connection, and options (C) and (D) incorrectly declare one of the two statements false.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: When a colloidal substance changes colour without a change of composition, check particle size and its effect on light scattering first.

74. The metal that has very low melting point and its periodic position is closer to a metalloid is : [4 marks]

(A) Al

(B) Ga

(C) Se

(D) In

Correct Answer: (B) Ga

Solution:

Concept:

- The required choice must satisfy both conditions: it must be a metal with an unusually low melting point and lie close to a metalloid in the periodic table.
- Gallium is a group 13 metal in period 4. Germanium, immediately to its right in group 14, is a metalloid.

Step 1: Identify the low-melting metal Gallium has an exceptionally low melting point of approximately $29.8\,^{\circ}\mathrm{C}$. Thus a sample can melt near room temperature, although it is still classified as a metal. This unusually low melting point is a characteristic property used to distinguish gallium from the other group 13 metals listed. The question concerns melting, so a high boiling point would not disqualify it.

Step 2: Check the periodic position Gallium lies in period 4, group 13. Its neighbour to the right is germanium, a familiar metalloid of group 14. The second clue therefore also agrees directly with

gallium. Both clues are needed; proximity to the metalloid region alone does not establish that an element has a very low melting point.

Step 3: Eliminate the remaining options Aluminium is a metal, but its melting point is about $660\,^{\circ}\mathrm{C}$, much higher than that of gallium. Indium is also a metal of group 13, but melts near $157\,^{\circ}\mathrm{C}$, rather than near room temperature. Selenium is a nonmetal, so it fails the explicit requirement that the answer be a metal. These comparisons leave gallium as the choice satisfying the combined description.

Step 4: Conclusion The metal is gallium, Ga. Hence, the correct option is (B).

Quick Tip to Solve: Remember the near-room-temperature melting point of gallium, then verify its position immediately beside the metalloid germanium.

75. The metal that is not extracted from its sulfide ore is : [4 marks]

(A) Aluminium

(B) Iron

(C) Lead

(D) Zinc

Correct Answer: (A) Aluminium

Solution:

Concept:

- Ore occurrence and commercial extraction are different facts. A mineral can contain a metal without being the usual practical source used to produce that metal.
- Aluminium is obtained from bauxite, a hydrated oxide ore, by purification to alumina followed by electrolysis. Lead and zinc have important sulfide-ore extraction routes.

Step 1: Identify the intended aluminium choice Aluminium is conventionally associated with bauxite, a hydrated aluminium oxide ore. Bauxite is purified to alumina, and the alumina is reduced electrolytically in a molten electrolyte. This standard extraction route does not start from an aluminium sulfide ore. The intended textbook comparison therefore selects aluminium, option (A), rather than the sulfide ores galena and zinc blende.

Step 2: Check the lead and zinc alternatives Lead is obtained from galena, PbS, and zinc from zinc blende, ZnS. Their sulfide ores are converted through suitable roasting and reduction processes. These two alternatives are consequently excluded on the stated criterion. Their extraction provides the unambiguous sulfide examples in the options.

Step 3: Address the limitation in the iron alternative Iron is normally extracted from oxide ores such as haematite, $\mathrm{Fe_2O_3}$, and magnetite, $\mathrm{Fe_3O_4}$. Although iron pyrites, $\mathrm{FeS_2}$, exists, its occurrence does not prove that it is a normal commercial ore for iron extraction. Thus a strictly literal reading of not extracted from its sulfide ore also raises iron as a competing choice. The broad wording does not uniquely distinguish aluminium from iron on normal industrial extraction alone; the intended answer must be stated with that qualification.

Step 4: Conclusion The intended answer is aluminium, option (A), matching the supplied answer

key. However, iron is also normally extracted from oxide ores rather than pyrites, so the literal industrial-extraction wording is not uniquely discriminating.

Quick Tip to Solve: Distinguish an element occurring in a sulfide mineral from that mineral actually being used for commercial metal extraction.

76. The products obtained from a reaction of hydrogen peroxide and acidified potassium permanganate are :

[4 marks]

(A) Mn^{4+}, H_2O only

(B) Mn^{2+}, H_2O only

(C) Mn^{4+}, H_2O, O_2 only

(D) Mn^{2+}, H_2O, O_2 only

Correct Answer: (D) Mn^{2+}, H_2O, O_2 only

Solution:

Concept:

- Acidified permanganate is a strong oxidising agent. In acidic medium manganese is reduced from oxidation state $+7$ in MnO_4^- to $+2$.
- Here hydrogen peroxide acts as a reducing agent: its oxygen is oxidised from -1 in H_2O_2 to 0 in O_2.

Step 1: Write the two half-reactions The reduction half-reaction of permanganate in acid is

$$MnO_4^- + 8\,H^+ + 5\,e^- \longrightarrow Mn^{2+} + 4\,H_2O.$$

Hydrogen peroxide supplies electrons through

$$H_2O_2 \longrightarrow O_2 + 2\,H^+ + 2\,e^-.$$

These equations already establish that oxygen gas must be included among the products, together with manganese(II) ions.

Step 2: Balance the overall reaction Multiply the permanganate half-reaction by two and the peroxide half-reaction by five. Add them and cancel the ten electrons and the common protons. The resulting net ionic equation is

$$2\,MnO_4^- + 5\,H_2O_2 + 6\,H^+ \longrightarrow 2\,Mn^{2+} + 5\,O_2 + 8\,H_2O.$$

The charges are $+4$ on each side, and manganese, hydrogen and oxygen atoms are conserved. Potassium and the acid anion are spectators in this net redox description.

Step 3: Check the listed products The reacting medium is acidic, so the manganese product is Mn^{2+}, not a manganese(IV) product. This excludes (A) and (C). Option (B) has the correct manganese ion but omits the oxygen gas formed by oxidation of peroxide. Only option (D) includes all three products

of the net ionic reaction.

Step 4: Conclusion The products are Mn^{2+}, H_2O and O_2. Hence, the correct option is (D).

Quick Tip to Solve: First use the medium to select the manganese product, then track peroxide oxygen from oxidation state minus one to oxygen gas.

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77. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: LiF is sparingly soluble in water.

Reason R: The ionic radius of Li^+ ion is smallest among its group members, hence has least hydration enthalpy.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (C) A is true but R is false.

Solution:

Concept:

- Dissolving an ionic solid involves opposing energy changes: separating its lattice requires energy, while hydration of the separated ions releases energy.
- A smaller ion has greater charge density and is hydrated more strongly. Among alkali-metal cations, Li^+ has the greatest magnitude of hydration enthalpy.

Step 1: Evaluate the assertion Lithium fluoride is sparingly soluble in water. Its small Li^+ and F^- ions form a strongly bound lattice. The large energy needed to separate these ions is not sufficiently offset by hydration to make the salt highly soluble. Therefore Assertion A is true. Solubility must be considered from the balance of lattice and hydration effects, rather than by looking at the cation alone.

Step 2: Evaluate the reason The first part of R correctly says that the lithium ion is the smallest among its group members. The claimed consequence is reversed, however. A small cation attracts polar water molecules strongly, so Li^+ has the most strongly exothermic hydration enthalpy among the alkali-metal ions, or the greatest hydration-enthalpy magnitude. It does not have the least hydration enthalpy in the intended sense of weakest hydration. R as a whole is false.

Step 3: Select the assertion-reason outcome Strong hydration of lithium does not contradict the low solubility of lithium fluoride, since the lattice is also particularly strong. Thus the correct explanation of low solubility involves their competition. Options (A) and (B) treat the false reason as true. Option (D) rejects the true assertion. The only listed combination retaining the correct truth values is (C).

Step 4: Conclusion Assertion A is true, but Reason R is false. Hence, the correct option is (C).

Quick Tip to Solve: For hydration strength use charge density: the smaller alkali-metal ion is hydrated more strongly. For solubility, compare that gain with lattice separation.

78. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion : Boric acid is a weak acid.

Reason R : Boric acid is not able to release H^+ ion on its own. It receives OH^- ion from water and releases H^+ ion.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (A) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- Boric acid, $B(OH)_3$, is a weak monobasic Lewis acid in water. It accepts a hydroxide ion at its electron-deficient boron atom rather than directly donating one of its own hydrogen atoms as a proton.
- Acid strength refers to the extent of formation of hydronium ions in water. A weak acid establishes an equilibrium with relatively little ionisation; the presence of three hydroxyl groups does not make boric acid tribasic.

Step 1: Evaluate the assertion Boric acid is only weakly acidic in aqueous solution. The equilibrium producing hydronium ions lies predominantly toward undissociated boric acid. Thus Assertion A is true; the word weak describes the small extent of the acid reaction in water, rather than the number of hydrogen atoms written in the formula.

Step 2: Evaluate the reason The vacant orbital on boron can accept an electron pair from hydroxide. Water supplies the hydroxide ion and the corresponding proton appears as hydronium in solution. The net reaction is

$$B(OH)_3 + 2\,H_2O \rightleftharpoons B(OH)_4{}^- + H_3O^+.$$

Therefore R correctly describes the origin of the acidity. The proton is obtained from water in this process; boric acid is not acting as a direct proton donor.

Step 3: Connect the two statements The stated hydroxide-accepting mechanism is the standard explanation of the weak acidic behaviour of boric acid. Both statements are true and their explanatory connection holds. Option (B) denies this connection, while (C) and (D) incorrectly reject one of the true statements.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For boric acid, write the hydroxide-acceptance equilibrium first; count the hydronium formed, not the three hydroxyl groups in its formula.

79. Match List I with List II.

	List I Pollutant		List II Source
(A)	Microorganisms	(I)	Strip mining
(B)	Plant nutrients	(II)	Domestic sewage
(C)	Toxic heavy metals	(III)	Chemical fertilizer
(D)	Sediment	(IV)	Chemical factory

Choose the correct answer from the options given below : [4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-I, C-IV D-III

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-IV, C-III, D-II

Correct Answer: (A) A-II, B-III, C-IV, D-I

Solution:

Concept:

- Water pollutants can be identified from the process that introduces them: biological waste brings microorganisms, fertilizers bring plant nutrients, industrial effluent can bring heavy metals, and soil disturbance adds sediment.
- In a matching question, form each pollutant–source pair independently before choosing a complete option. A single correct pair is insufficient if another pair in that option is incorrect.

Step 1: Match the biological and nutrient pollutants Domestic sewage contains human waste and organic matter and can introduce disease-causing microorganisms into water. Thus microorganisms correspond to domestic sewage: A–II. Chemical fertilizers supply plant nutrients such as nitrogen and phosphorus compounds; their runoff can promote excessive algal growth. Hence plant nutrients correspond to chemical fertilizer: B–III.

Step 2: Match the industrial and physical pollutants Chemical-factory discharges can contain toxic heavy metals, which persist in water and can accumulate through food chains. Therefore toxic heavy metals correspond to chemical factory: C–IV. Strip mining removes vegetation and overburden and exposes loose earth to erosion. Runoff then transports suspended soil and rock particles into water bodies, so sediment corresponds to strip mining: D–I.

Step 3: Check the complete option The complete matching is A–II, B–III, C–IV, D–I. Option (B) incorrectly links plant nutrients with strip mining and sediment with fertilizer. Options (C) and (D) incorrectly associate microorganisms with strip mining and plant nutrients with a chemical factory. Only option (A) retains all four direct source relationships.

Step 4: Conclusion The matching is A–II, B–III, C–IV, D–I. Hence, the correct option is (A).

Quick Tip to Solve: Use the distinctive pair sediment–strip mining first, then confirm microorganisms–sewage and nutrients–fertilizer before selecting the full option.

80. Vulcanization of rubber is carried out by heating a mixture of [4 marks]

(A) isoprene and styrene

(B) neoprene and sulphur

(C) isoprene and sulphur

(D) neoprene and styrene

Correct Answer: (C) isoprene and sulphur

Solution:

Concept:

- Natural rubber is cis-1,4-polyisoprene, made from the monomer isoprene (2-methylbuta-1,3-diene). Vulcanization is a treatment of this rubber with sulphur, rather than a fresh copolymerisation with styrene.
- Sulphur forms cross-links between neighbouring polyisoprene chains at suitable sites associated with their unsaturation. The resulting network reduces unrestricted slipping of the chains and improves useful mechanical properties.

Step 1: Identify the rubber referred to by the options The isoprene-based material in the options refers to natural rubber, whose repeating chains are polyisoprene. In the actual industrial process the material heated is raw natural rubber, not a bottle of unpolymerised isoprene. The question uses isoprene as the identifying constituent of that rubber, so we must select the isoprene-based rubber together with the vulcanizing agent.

Step 2: Identify the vulcanizing agent Raw natural rubber is heated with sulphur. Sulphur bridges connect adjacent polymer chains, producing a three-dimensional network. This raises tensile strength and elasticity and reduces the tendency of the raw polymer to become sticky at high temperature or brittle at low temperature. It is the formation of interchain links, rather than simply mixing two monomers, that characterises vulcanization.

Step 3: Eliminate the other combinations Option (A) and option (D) contain styrene but omit sulphur, so they do not describe sulphur vulcanization. Neoprene in option (B) is polychloroprene, a different synthetic rubber, rather than the isoprene-based natural rubber intended here. Option (C) is therefore the intended natural-rubber–sulphur combination.

Step 4: Conclusion Vulcanization heats isoprene-based natural rubber (cis-polyisoprene) with sulphur. Hence, the correct option is (C).

Quick Tip to Solve: Associate vulcanization with sulphur cross-links in already formed polyisoprene; do not confuse it with polymerisation of the isoprene monomer.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. Consider an imaginary ion $^{48}_{22}X^{3-}$. The nucleus contains 'a'% more neutrons than the number of electrons in the ion. The value of 'a' is _____. [nearest integer] [4 marks]

Solution:

Concept:

- The atomic number Z gives the number of protons, while the mass number A gives the total number of protons and neutrons. Therefore the neutron count is $N = A - Z$.
- A negative ionic charge means that electrons have been gained. For a $3-$ ion the electron count is $Z + 3$, while the nucleus and its neutron count remain unchanged.
- The percentage by which one quantity exceeds another is the difference divided by the reference quantity, multiplied by 100. Here the reference is explicitly the number of electrons in the ion.

Step 1: Count the neutrons For $^{48}_{22}X^{3-}$, the mass number is 48 and the atomic number is 22. Subtract the proton count from the total nucleon count:

$$N = 48 - 22 = 26.$$

The isotope therefore contains 26 neutrons. The ionic charge does not enter this subtraction because electron gain does not change the nucleus.

Step 2: Count the electrons A neutral atom of atomic number 22 would contain 22 electrons. The charge $3-$ represents three additional electrons, so the ion contains

$$E = 22 + 3 = 25.$$

Using 19 would describe a $3+$ cation and would reverse the meaning of the printed charge.

Step 3: Calculate the stated percentage The neutron count exceeds the electron count by $26 - 25 = 1$. Since the comparison is with the electrons, divide by 25:

$$a = \frac{N - E}{E} \times 100 = \frac{26 - 25}{25} \times 100 = 4.$$

The result is already an integer, so rounding does not change it. The percentage itself is dimensionless.

Step 4: Conclusion There are 26 neutrons and 25 electrons. The excess is 4%, so the requested nearest-integer value is $\boxed{a = 4}$.

Quick Tip to Solve: Apply the charge only to the electron count, then use the electron count as the denominator because the wording says more than the electrons.

82. For the reaction

$$H_2F_2(\ g) \rightarrow H_2(\ g) + F_2(\ g)$$

$\Delta U = -59.6\ \text{kJ mol}^{-1}$ **at** 27°C**.**

The enthalpy change for the above reaction is (−) _____ kJ mol^{-1} **[nearest integer]**

Given: $\text{R} = 8.314\ \text{J K}^{-1}\ \text{mol}^{-1}$**.** [4 marks]

Solution:

Concept:

- For a reaction involving ideal gases at temperature T, the molar reaction enthalpy and internal-energy change are related by $\Delta H = \Delta U + \Delta n_g RT$. Here Δn_g counts gaseous product moles minus gaseous reactant moles.
- Temperature must be expressed in kelvin, and RT must use the same energy unit as ΔU. The given R is in joules, whereas the supplied internal-energy change is in kilojoules per mole.

Step 1: Count the gaseous moles The written reaction is $H_2F_2(g) \longrightarrow H_2(g) + F_2(g)$. There is one mole of gaseous reactant and two moles of gaseous products for the reaction as written. Therefore

$$\Delta n_g = (1 + 1) - 1 = 1.$$

The correction to internal energy is positive because the gas mole count increases.

Step 2: Convert the temperature and evaluate the correction The temperature is $27^\circ\text{C} = 300\ \text{K}$. With the specified gas constant,

$$\Delta n_g RT = 1 \times 8.314 \times 300 = 2494.2\ \text{J mol}^{-1}.$$

Convert joules to kilojoules by dividing by 1000:

$$\Delta n_g RT = 2.4942\ \text{kJ mol}^{-1}.$$

Combining 2494.2 directly with a kilojoule value would introduce a factor-of-1000 error.

Step 3: Find the enthalpy and interpret the blank Substitute the negative internal-energy change without losing its sign:

$$\Delta H = -59.6 + 2.4942 = -57.1058\ \text{kJ mol}^{-1}.$$

The enthalpy is less negative than the internal-energy change. The question places a minus sign before the blank, so the blank asks for the positive magnitude rounded to the nearest integer, namely 57.

Step 4: Conclusion $\Delta H \approx -57\ \text{kJ mol}^{-1}$. With the printed minus sign already supplied, the requested entry is $\boxed{57}$.

Quick Tip to Solve: Count gaseous moles before using the formula, and convert the RT correction to kilojoules before adding it to the given ΔU.

83. The elevation in boiling point for 1 molal solution of non-volatile solute A is $3\ \mathrm{K}$. The depression in freezing point for 2 molal solution of A in the same solvent is 6 K. The ratio of K_b and K_f i.e., K_b/K_f is $1 : X$. The value of X is [nearest integer] [4 marks]

Solution:

Concept:

- For a dilute solution of a non-volatile solute, boiling-point elevation is $\Delta T_b = K_b m$, where K_b is the molal elevation constant and m is molality.
- Freezing-point depression is $\Delta T_f = K_f m$, where K_f is the molal depression constant. Each temperature change must be divided by its own molality before the two constants are compared.
- If a common van't Hoff factor i is retained for the same solute, it multiplies both expressions and cancels in the ratio of the constants under the stated dilute-solution treatment.

Step 1: Find the boiling-point constant The boiling-point elevation is $3\,\mathrm{K}$ for a molality of $1\,\mathrm{mol\,kg^{-1}}$. Hence

$$K_b = \frac{\Delta T_b}{m_b} = \frac{3}{1} = 3\,\mathrm{K\,kg\,mol^{-1}}.$$

The temperature rise is already a difference in kelvin, so no conversion involving 273 is needed.

Step 2: Find the freezing-point constant The second solution has twice the molality: $m_f = 2\,\mathrm{mol\,kg^{-1}}$. Its depression is $6\,\mathrm{K}$, giving

$$K_f = \frac{\Delta T_f}{m_f} = \frac{6}{2} = 3\,\mathrm{K\,kg\,mol^{-1}}.$$

Although the observed depression is twice the elevation, the solution used for that measurement is also twice as concentrated.

Step 3: Compare the constants The constants therefore have the ratio

$$\frac{K_b}{K_f} = \frac{3}{3} = 1 = \frac{1}{X}.$$

Thus $X = 1$. Comparing $3 : 6$ directly would give the ratio of the observed changes rather than the requested constants and would miss the difference in molality.

Step 4: Conclusion $K_b : K_f = 1 : 1$, so the nearest-integer entry is $\boxed{X = 1}$. The ratio and X are dimensionless.

Quick Tip to Solve: Calculate each constant as temperature change divided by molality; only then reduce the ratio to the form $1 : X$.

84. $20\ \text{mL}$ **of** $0.02\ \text{M}$ **hypo solution is used for the titration of** $10\ \text{mL}$ **of copper sulphate solution, in the presence of excess of KI using starch as an indicator. The molarity of** Cu^{2+} **is found to be _____** $\times 10^{-2}\ \text{M}$**. [nearest integer]**

Given : $2\,Cu^{2+} + 4\,I^- \rightarrow Cu_2I_2 + I_2$

$$I_2 + 2\ S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$$

[4 marks]

Solution:

Concept:

- In iodometry, copper(II) ions oxidise iodide to iodine, and the liberated iodine is titrated with sodium thiosulphate (hypo). The reaction coefficients must be combined to relate copper directly to thiosulphate.
- For a solution of molarity M and volume V in litres, the number of moles is $n = MV$. Starch indicates the iodine endpoint; it does not enter the mole ratio.

Step 1: Combine the two reaction ratios The supplied reactions are

$$2\,Cu^{2+} + 4\,I^- \longrightarrow Cu_2I_2 + I_2$$

and

$$I_2 + 2\,S_2O_3{}^{2-} \longrightarrow 2\,I^- + S_4O_6{}^{2-}.$$

Two moles of copper(II) produce one mole of iodine, which consumes two moles of thiosulphate. Therefore the moles of copper(II) in the aliquot equal the moles of thiosulphate delivered. Neither of the individual factors of two survives in the final copper–thiosulphate relation.

Step 2: Calculate the copper amount The thiosulphate volume is $20\,\text{mL} = 0.020\,\text{L}$, and its molarity is $0.020\,\text{mol}\,\text{L}^{-1}$. Thus

$$n(S_2O_3{}^{2-}) = 0.020 \times 0.020 = 4.0 \times 10^{-4}\,\text{mol}.$$

By the combined stoichiometry, the $10\,\text{mL}$ copper solution contains $4.0 \times 10^{-4}\,\text{mol}$ of Cu^{2+}. Excess iodide ensures that copper can liberate the corresponding iodine quantitatively.

Step 3: Find the molarity and requested coefficient The copper solution volume is $10\,\text{mL} = 0.010\,\text{L}$. Consequently

$$M(Cu^{2+}) = \frac{4.0 \times 10^{-4}}{0.010} = 4.0 \times 10^{-2}\,\text{mol}\,\text{L}^{-1}.$$

The blank is the coefficient of $10^{-2}\,\text{M}$, so its nearest-integer value is 4. Using the titrant volume in this final denominator would calculate the concentration of the wrong solution.

Step 4: Conclusion The copper(II) molarity is $4 \times 10^{-2}\,\text{M}$. Therefore the requested coefficient is $\boxed{4}$.

Quick Tip to Solve: Cancel iodine between the two reactions first: copper(II) and thiosulphate are equimolar here, even though both separate equations contain a factor of two.

85. The spin-only magnetic moment value of the compound with strongest oxidizing ability among MnF_4, MnF_3 and MnF_2 is _____ B.M. [nearest integer] [4 marks]

Solution:

Concept:

- An oxidising species accepts electrons and is reduced. For the manganese fluorides, compare the stability of the manganese oxidation state before and after reduction, rather than ranking only by the number of fluorine atoms.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{B.M.}$, where n is the number of unpaired electrons. The $4s$ electrons are removed before the $3d$ electrons when manganese forms cations.

Step 1: Identify the strongest oxidising member Manganese has ground-state configuration $[Ar]3d^5 4s^2$. Thus MnF_2, MnF_3 and MnF_4 contain Mn^{2+}, Mn^{3+} and Mn^{4+}, with configurations $3d^5$, $3d^4$ and $3d^3$, respectively. The divalent ion has a particularly stable half-filled d shell. The trivalent ion readily gains one electron to reach this stable configuration; consequently MnF_3 is the strongest oxidising member of this set. The tetravalent d^3 configuration has the stabilised t_{2g}^3 arrangement, so the highest oxidation number alone is not the criterion.

Step 2: Count the unpaired electrons and calculate For Mn^{3+}, the four $3d$ electrons remain unpaired with fluoride, a weak-field ligand. Therefore $n = 4$; do not use the five unpaired electrons of the reduced product Mn^{2+}. Substitution gives

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.899\,\text{B.M.}$$

The requested value is rounded only after computing the square root.

Step 3: Conclusion The compound is MnF_3, whose spin-only magnetic moment is approximately $4.90\,\text{B.M.}$. The nearest-integer answer is $\boxed{5}$.

Quick Tip to Solve: First choose the oxidant from oxidation-state stability; then calculate the magnetic moment of that compound, not of its reduction product.

86. Total number of isomers (including stereoisomers) obtained on monochlorination of methylcyclohexane is _____.

[4 marks]

Solution:

Concept:

- Monochlorination replaces one hydrogen atom by chlorine without changing the carbon skeleton. Equivalent carbon positions give the same constitutional product.
- Count configurational stereoisomers separately, including both enantiomers of a chiral cis or trans form. Chair interconversion produces conformers, not additional isomers.

Step 1: List the inequivalent substitution sites Number the ring carbon bearing the methyl group as C1. The methyl carbon is one site; the hydrogen on C1 is another. Ring symmetry makes C2 equivalent to C6 and C3 equivalent to C5, while C4 is distinct. Thus there are five constitutional

products: chloromethylcyclohexane, 1-chloro-1-methylcyclohexane, 1-chloro-2-methylcyclohexane, 1-chloro-3-methylcyclohexane and 1-chloro-4-methylcyclohexane. Substitution at another hydrogen on an already listed carbon must not be counted as a new constitution.

Step 2: Count the stereoisomers for each constitution Chloromethylcyclohexane is achiral and contributes one. The geminal 1-chloro-1-methyl compound is also achiral because the two ring paths from C1 are equivalent, so it contributes one. The 1,2-disubstituted compound has cis and trans forms; since chlorine and methyl are different substituents, each form has an enantiomer, giving four. The 1,3-disubstituted compound similarly gives two enantiomers for each of its cis and trans forms, again four. The 1,4-disubstituted compound gives cis and trans forms, each achiral, contributing two. Hence

$$N = 1 + 1 + 4 + 4 + 2 = 12.$$

Axial and equatorial chair forms of any one configuration interconvert and are not counted twice.

Step 3: Conclusion The total number of distinct monochlorination products, including configurational stereoisomers, is $\boxed{12}$. This is a dimensionless count.

Quick Tip to Solve: Group the five substitution sites first, then attach the stereoisomer counts $1, 1, 4, 4, 2$; never count a chair flip as a new product.

87. A $100\ \mathrm{mL}$ solution of CH_3CH_2MgBr on treatment with methanol produces $2.24\ \mathrm{mL}$ of a gas at STP. The weight of gas produced is _____ mg. [nearest integer] [4 marks]

Solution:

Concept:

- A Grignard reagent RMgX is strongly basic. A protic alcohol transfers its hydroxyl proton to the carbon group, producing the hydrocarbon RH.
- At STP one mole of an ideal gas occupies $22.4\,\mathrm{L}$, or $22400\,\mathrm{mL}$. Gas mass follows from $m = nM$, where M is its molar mass.

Step 1: Identify the gas formed The carbon group in CH_3CH_2MgBr is ethyl, C_2H_5. Methanol supplies a proton from its $\mathrm{O{-}H}$ bond, so the gas is ethane, not methane:

$$C_2H_5MgBr + CH_3OH \longrightarrow C_2H_6 + CH_3OMgBr.$$

The ethyl group is retained intact and receives one hydrogen. Methanol supplies the proton rather than determining the carbon skeleton of the gas. One mole of the reagent consequently gives one mole of ethane. The stated solution volume is not needed to determine the mass of a gas whose own volume is already measured.

Step 2: Convert the gas volume to mass The measured volume is $2.24\,\mathrm{mL}$, not $2.24\,\mathrm{L}$. Thus

$$n(C_2H_6) = \frac{2.24}{22400} = 1.00 \times 10^{-4}\,\mathrm{mol}.$$

Using carbon and hydrogen molar masses of 12 and $1\,\mathrm{g\,mol^{-1}}$, the molar mass of ethane is

$$M(\mathrm{C_2H_6}) = 2(12) + 6(1) = 30\,\mathrm{g\,mol^{-1}}.$$

Therefore

$$m = (1.00 \times 10^{-4})(30) = 3.00 \times 10^{-3}\,\mathrm{g} = 3.00\,\mathrm{mg}.$$

The conversion from grams to milligrams introduces a factor of 1000.

Step 3: Conclusion The gas is ethane, and its mass is $\boxed{3\,\mathrm{mg}}$. The required nearest-integer numerical entry is $\boxed{3}$.

Quick Tip to Solve: In protonolysis of RMgX, write RH first. Use $22400\,\mathrm{mL\,mol^{-1}}$ when the measured gas volume is in millilitres.

88. A 10 g mixture of hydrogen and helium is contained in a vessel of capacity $0.0125\ \mathrm{m^3}$ at 6 bar and $27^\circ\mathrm{C}$. The mass of helium in the mixture is _____ g. (nearest integer)

Given: $\mathrm{R} = 8.3\ \mathrm{J\ K^{-1}\ mol^{-1}}$

(Atomic masses of H and He are 1u and 4u, respectively) [4 marks]

Solution:

Concept:

- For an ideal gas mixture, $PV = n_{\text{total}}RT$. All constituents share the stated pressure, volume and temperature through the total number of moles.
- The mass balance and mole balance are different: molecular hydrogen has molar mass $2\,\mathrm{g\,mol^{-1}}$, while helium has molar mass $4\,\mathrm{g\,mol^{-1}}$. Use kelvin and SI pressure with the given value of R.

Step 1: Calculate the total amount of gas Convert the given conditions to SI units:

$$P = 6 \times 10^5\,\mathrm{Pa}, \qquad V = 0.0125\,\mathrm{m^3}, \qquad T = 27 + 273 = 300\,\mathrm{K}.$$

The gas law then gives

$$n_{\text{total}} = \frac{(6 \times 10^5)(0.0125)}{8.3(300)} = \frac{7500}{2490} = \frac{250}{83}\,\mathrm{mol}.$$

The total is approximately $3.012\,\mathrm{mol}$. Do not substitute 27 for the absolute temperature or use pressure in bar with R expressed in joules.

Step 2: Apply the mass balance to find helium Let $x\,\mathrm{g}$ be the mass of helium. Then hydrogen has mass $(10 - x)\,\mathrm{g}$, and the total moles satisfy

$$\frac{x}{4} + \frac{10 - x}{2} = \frac{250}{83}.$$

Simplifying,

$$5 - \frac{x}{4} = \frac{250}{83}, \qquad x = 4\left(5 - \frac{250}{83}\right) = \frac{660}{83} = 7.9518\,\mathrm{g}.$$

This lies between zero and the total mass of $10\,\mathrm{g}$, as required. The remaining approximately $2.048\,\mathrm{g}$ of hydrogen contributes more moles per gram than helium; together the amounts reproduce the gas-law total.

Step 3: Conclusion The mass of helium is approximately $7.95\,\mathrm{g}$, giving the nearest-integer answer $\boxed{8\,\mathrm{g}}$. The numerical entry is $\boxed{8}$.

Quick Tip to Solve: For a two-gas mixture, calculate total moles first, then use $m_1/M_1 + m_2/M_2 = n$ together with the given total mass.

89. The number of non-ionisable protons present in the product B obtained from the following reactions is _____.

$$\mathrm{C_2H_5OH + PCl_3 \rightarrow C_2H_5Cl + A}$$

$$\mathrm{A + PCl_3 \rightarrow B}$$

[4 marks]

Solution:

Concept:

- Phosphorus trichloride converts an alcohol to an alkyl chloride and produces phosphorous acid. Further treatment of phosphorous acid with phosphorus trichloride produces pyrophosphorous acid by condensation.
- In phosphorus oxyacids, hydrogen attached to oxygen is ionisable in ordinary acid dissociation. Hydrogen bonded directly to phosphorus is non-ionisable; count the $\mathrm{P-H}$ bonds rather than every hydrogen in the molecular formula.

Step 1: Identify products A and B The balanced first reaction is

$$\mathrm{3\,C_2H_5OH + PCl_3 \longrightarrow 3\,C_2H_5Cl + H_3PO_3.}$$

Thus A is phosphorous acid, $\mathrm{H_3PO_3}$. Further treatment with $\mathrm{PCl_3}$ forms pyrophosphorous acid B:

$$\mathrm{5\,H_3PO_3 + PCl_3 \longrightarrow 3\,H_4P_2O_5 + 3\,HCl.}$$

This equation conserves six phosphorus atoms, fifteen hydrogen atoms, fifteen oxygen atoms and three chlorine atoms. The condensation relation $\mathrm{2\,H_3PO_3 \longrightarrow H_4P_2O_5 + H_2O}$, combined with consumption of water by $\mathrm{PCl_3}$, accounts for formation of the condensed acid. B is therefore $\mathrm{H_4P_2O_5}$, not the original acid A.

Step 2: Count hydrogens attached directly to phosphorus Pyrophosphorous acid has the connectivity

$$\mathrm{HO-P(H)(=O)-O-P(H)(=O)-OH.}$$

Each of its two phosphorus atoms carries one directly bonded hydrogen, giving two $P-H$ bonds in total. Its other two hydrogens belong to terminal hydroxyl groups. Those hydroxyl hydrogens are ionisable, whereas the two hydrogens bonded directly to phosphorus are non-ionisable in ordinary aqueous acid dissociation. Thus the four hydrogens in the formula split into two ionisable and two non-ionisable hydrogens. Counting all four would ignore their bonding environments; counting only one would mistakenly use the single $P-H$ bond of phosphorous acid A instead of product B.

Step 3: Conclusion Product B is pyrophosphorous acid, $H_4P_2O_5$, with one non-ionisable hydrogen on each phosphorus atom. The required dimensionless number is $\boxed{2}$.

Quick Tip to Solve: For a phosphorus oxyacid, rewrite its structure as $P(=O)$ with explicit OH and $P-H$ groups before counting acidic and non-acidic hydrogens.

90. How many of the following drugs is/are example(s) of broad spectrum antibiotics?
Ofloxacin, Penicillin G, Terpineol, Salvarsan. [4 marks]

Solution:

Concept:

- Broad-spectrum antibiotics act against a wide range of bacteria, including both Gram-positive and Gram-negative organisms. A compound with antimicrobial activity is not automatically a broad-spectrum antibiotic.
- Distinguish antibiotics from antiseptics and from older synthetic agents used for a particular infection. Count only members satisfying the category named in the question.

Step 1: Classify the antibiotic entries Ofloxacin is a broad-spectrum antibacterial antibiotic of the fluoroquinolone class. It is active against a wide range of Gram-positive and Gram-negative bacteria, so it is included. Penicillin G is an antibiotic, but it has a relatively narrow antibacterial spectrum and is particularly associated with susceptible Gram-positive bacteria. It is therefore not counted as a broad-spectrum antibiotic. The word antibiotic alone is insufficient: the qualifier broad-spectrum determines which entries belong in this count.

Step 2: Exclude the other agents and total the count Terpineol is used as an antiseptic constituent in formulations such as Dettol; it is not classified as a broad-spectrum antibiotic. Salvarsan, an organoarsenic drug historically used to treat syphilis, is a synthetic chemotherapeutic agent and is not an example of the broad-spectrum antibiotics requested here. Its antimicrobial use does not turn it into a broad-spectrum antibiotic. Hence the four entries contribute, in their printed order,

$$1 + 0 + 0 + 0 = 1.$$

There is no need to count the number of different bacteria each drug can affect: the question asks for the number of listed drugs belonging to the specified class.

Step 3: Conclusion Only ofloxacin qualifies among the four listed drugs. The number of broad-spectrum antibiotics is $\boxed{1}$, a dimensionless count.

Quick Tip to Solve: Check both parts of the classification: Penicillin G is an antibiotic but narrow-spectrum, while terpineol is an antiseptic. Only ofloxacin meets the full description.