JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 27 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 July 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by

$a\,R_1\,b \Leftrightarrow ab \geq 0$ **and** $a\,R_2\,b \Leftrightarrow a \geq b$

Then, [4 marks]

(A) R_1 is an equivalence relation but not R_2

(B) R_2 is an equivalence relation but not R_1

(C) both R_1 and R_2 are equivalence relations

(D) neither R_1 nor R_2 is an equivalence relation

Correct Answer: (D) neither R_1 nor R_2 is an equivalence relation

Solution:

Concept:

- An equivalence relation on a set must be reflexive, symmetric and transitive. Failure of even one of these properties is enough to reject equivalence.
- Reflexivity requires every element to be related to itself. Symmetry reverses each related pair, while transitivity joins two related pairs through their common middle element.

Step 1: Test the product relation For every real number a, $a^2 \geq 0$, so R_1 is reflexive. Since $ab = ba$, the condition $ab \geq 0$ is unchanged on interchanging the two entries; hence it is symmetric. However, zero is related to both positive and negative numbers. Take the ordered triple $(1, 0, -1)$. Then $1 \cdot 0 = 0 \geq 0$ and $0 \cdot (-1) = 0 \geq 0$, but $1 \cdot (-1) = -1 < 0$. Thus $1R_10$ and $0R_1(-1)$ do not imply $1R_1(-1)$. Therefore R_1 fails transitivity and is not an equivalence relation.

Step 2: Test the order relation and compare the options The relation R_2 is reflexive because $a \geq a$, and transitive because $a \geq b$ together with $b \geq c$ gives $a \geq c$. It is not symmetric: $1 \geq 0$, whereas $0 \not\geq 1$. Hence R_2 is also not an equivalence relation. Option (A) incorrectly treats R_1 as transitive; option (B) incorrectly treats R_2 as symmetric; option (C) makes both errors. Only option (D) survives all the required tests.

Step 3: Conclusion Neither R_1 nor R_2 is an equivalence relation. Hence, the correct option is (D).

Quick Tip to Solve: For a product condition allowing zero, test transitivity with zero as the middle entry and opposite signs at the ends.

2. Let $f, g : \mathbb{N} - \{1\} \to \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a, and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is [4 marks]

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Correct Answer: (D) neither one-one nor onto

Solution:

Concept:

- If $a = p_1^{e_1} \cdots p_k^{e_k}$ is its prime factorisation, the definition gives $f(a) = \max(e_1, \ldots, e_k)$. Thus $f(a) \geq 1$ for every integer $a \geq 2$.
- A function is one-one if equal outputs force equal inputs. It is onto $\mathbb{N}$ only if every natural number occurs as an output; one missing output disproves onto behaviour.

Step 1: Write the sum and test its range Put $h = f + g$. The plus sign means pointwise addition, not composition, so

$$h(a) = f(a) + g(a) = f(a) + a + 1.$$

The domain is $\mathbb{N} \setminus \{1\}$, so $a \geq 2$. Since every such integer has a prime divisor, its largest prime exponent is at least one. Consequently

$$h(a) \geq 1 + 2 + 1 = 4.$$

In particular, the natural numbers 1, 2 and 3 are absent from the range. Already the missing value 1 proves that h is not onto the stated codomain; we need not determine its entire range.

Step 2: Find two distinct inputs with the same output The exponent term is not monotone, so the increasing summand $a + 1$ does not by itself prove injectivity. Choose $8 = 2^3$ and $10 = 2 \cdot 5$. Their maximum prime exponents are $f(8) = 3$ and $f(10) = 1$. Therefore

$$h(8) = 3 + 9 = 12, \qquad h(10) = 1 + 11 = 12.$$

The inputs 8 and 10 are different, but their outputs coincide. This disproves one-one behaviour. Option (A) fails this collision test, while options (B) and (C) fail the range test; option (C) also fails injectivity. Both properties are therefore absent.

Step 3: Conclusion The function $f + g$ is neither one-one nor onto. Hence, the correct option is (D).

Quick Tip to Solve: To test injectivity of an arithmetic function plus its input, compare a prime power with a nearby square-free number.

3. Let the minimum value v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2, z \in \mathbb{C}$ is attained at $z = z_0$. Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$ is equal to : [4 marks]

(A) 1000

(B) 1024

(C) 1105

(D) 1196

Correct Answer: (A) 1000

Solution:

Concept:

- Write $z = x + iy$, where $x, y \in \mathbb{R}$. Then $|z-u|^2$ is the squared distance between the corresponding complex-plane points.
- Completing squares identifies the minimum of a positive quadratic. For $w = u + iv$, its squared modulus is $|w|^2 = u^2 + v^2$; conjugation changes the sign of the imaginary part.

Step 1: Complete squares to locate the minimum The three squared distances give

$$\begin{aligned} v &= x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-6)^2 \\ &= 3x^2 - 6x + 3y^2 - 12y + 45 \\ &= 3(x-1)^2 + 3(y-2)^2 + 30. \end{aligned}$$

The two squares are nonnegative and vanish simultaneously only at $x = 1, y = 2$. Hence $z_0 = 1 + 2i$ and $v_0 = 30$. In particular, one must minimise the entire sum, rather than separately minimise its three terms at different points.

Step 2: Evaluate the requested complex expression First calculate the power and conjugate carefully:

$$z_0^2 = (1+2i)^2 = -3 + 4i, \qquad \bar{z}_0 = 1 - 2i.$$

Thus

$$\bar{z}_0^3 = (-3-4i)(1-2i) = -11 + 2i.$$

Substitution into the expression inside the modulus gives

$$2z_0^2 - \bar{z}_0^3 + 3 = (-6+8i) - (-11+2i) + 3 = 8 + 6i.$$

Therefore

$$|2z_0^2 - \bar{z}_0^3 + 3|^2 + v_0^2 = 8^2 + 6^2 + 30^2 = 100 + 900 = 1000.$$

The other proposed values 1024, 1105 and 1196 disagree with this exact sum; no rounding or modulus sign ambiguity is involved.

Step 3: Conclusion The required value is $\boxed{1000}$. Hence, the correct option is (A).

Quick Tip to Solve: For a sum of squared distances, complete both coordinate squares first; keep the conjugate sign separate when taking powers.

4. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to [4 marks]

(A) -10

(B) -6

(C) 6

(D) 10

Correct Answer: (D) 10

Solution:

Concept:

- For a two-by-two matrix, the Cayley–Hamilton identity is $A^2 - (\operatorname{tr} A)A + (\det A)I = 0$. It reduces a quadratic polynomial in A to a linear combination of A and I.
- An equality of matrices means equality of corresponding entries. If A is not a scalar multiple of I, the matrices A and I are linearly independent.

Step 1: Reduce the square of the matrix The trace and determinant of the given matrix are

$$\operatorname{tr} A = 1 - 5 = -4, \qquad \det A = (1)(-5) - (2)(-2) = -1.$$

Consequently its characteristic equation gives

$$A^2 + 4A - I = 0, \qquad A^2 = I - 4A.$$

This can also be checked directly:

$$A^2 = \begin{pmatrix} -3 & -8 \\ 8 & 21 \end{pmatrix} = I - 4A.$$

The negative determinant is essential; replacing it by its magnitude would reverse the identity and produce incorrect coefficients.

Step 2: Compare coefficients and verify the result Using the reduced square in the stated equation,

$$\alpha A^2 + \beta A = \alpha I + (\beta - 4\alpha)A = 2I.$$

The off-diagonal entry of A is 2, whereas that of I is zero. Hence the coefficient of A must vanish: $\beta - 4\alpha = 0$. The diagonal terms then give $\alpha = 2$, so $\beta = 8$. Substituting back checks the equation exactly:

$$2A^2 + 8A = 2(I - 4A) + 8A = 2I.$$

Therefore the sum is 10. The proposed sums -10, -6 and 6 cannot satisfy these unique coefficient equations; they arise from incorrect signs or from comparing the wrong matrix entries.

Step 3: Conclusion We obtain $\alpha = 2$, $\beta = 8$, and $\boxed{\alpha + \beta = 10}$. Hence, the correct option is (D).

Quick Tip to Solve: For a quadratic matrix equation, reduce the square with trace and determinant, then use an off-diagonal entry to isolate the coefficient of the matrix.

5. The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is [4 marks]

(A) 0

(B) 1

(C) 2

(D) 6

Correct Answer: (A) 0

Solution:

Concept:

- Congruent integers have congruent positive integer powers. Reducing the bases modulo the divisor before exponentiation avoids handling very large integers.
- Power cycles reduce exponents: $2^3 = 8 \equiv 1 \pmod 7$, while a power of -1 depends only on whether the exponent is even or odd.

Step 1: Reduce both bases modulo seven We have

$$2021 = 7 \cdot 288 + 5, \qquad 2022 = 7 \cdot 288 + 6.$$

Therefore $2021 \equiv -2 \pmod 7$ and $2022 \equiv -1 \pmod 7$. The expression has two different exponents, so retain each exponent with its own base:

$$2021^{2022} + 2022^{2021} \equiv (-2)^{2022} + (-1)^{2021} \pmod 7.$$

The first exponent is even and the second is odd. Thus the first term becomes 2^{2022}, while the second term contributes -1, not $+1$.

Step 2: Use the short power cycle Since $2022 = 3 \cdot 674$, the first term simplifies to

$$2^{2022} = (2^3)^{674} \equiv 1^{674} = 1 \pmod 7.$$

It follows that

$$2021^{2022} + 2022^{2021} \equiv 1 - 1 = 0 \pmod 7.$$

The nonnegative remainder on division by 7 must lie in $\{0, 1, 2, 3, 4, 5, 6\}$; the congruence therefore gives remainder zero exactly. Option (B), remainder 1, would omit the second contribution. Option (C), remainder 2, would incorrectly replace the odd power of -1 by $+1$. Option (D), remainder 6, does not include the first contribution correctly. Only option (A) matches both terms.

Step 3: Conclusion The integer is divisible by 7, and its remainder is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: Replace residues 5 and 6 modulo 7 by -2 and -1; then check exponent parity and the three-term cycle of powers of 2.

6. Suppose $a_1, a_2, \ldots, a_n$, .. be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is $5:17$ and , $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to [4 marks]

(A) 290

(B) 380

(C) 460

(D) 510

Correct Answer: (B) 380

Solution:

Concept:

- For an arithmetic progression with first term a and common difference d, $a_n = a + (n-1)d$ and $S_n = \frac{n}{2}[2a + (n-1)d]$.
- The terms are natural numbers, so the first term is a positive integer. Once a sum ratio relates d to a, the interval for a later term can fix that integer uniquely.

Step 1: Use the ratio to relate the progression parameters The two sums are

$$S_5 = 5(a + 2d), \qquad S_9 = 9(a + 4d).$$

The stated ratio therefore implies

$$\frac{5(a + 2d)}{9(a + 4d)} = \frac{5}{17}.$$

Cross-multiplying and cancelling the common factor 5 gives

$$17(a + 2d) = 9(a + 4d).$$

Expanding both sides, $17a + 34d = 9a + 36d$, and hence $8a = 2d$, so $d = 4a$. The factors 5 and 9 from the sum formula cannot simply be discarded before setting up the ratio; they determine this relation.

Step 2: Apply the term interval and calculate the required sum The fifteenth term becomes

$$a_{15} = a + 14d = a + 56a = 57a.$$

Consequently

$$110 < 57a < 120 \quad \Longrightarrow \quad \frac{110}{57} < a < \frac{120}{57}.$$

The only natural number in this interval is $a = 2$, which gives $d = 8$ and checks $a_{15} = 114$. Thus

$$S_{10} = \frac{10}{2}[2(2) + 9(8)] = 5(4 + 72) = 380.$$

As a further check, $S_5 = 90$ and $S_9 = 306$, whose ratio is $5:17$. Values 290, 460 and 510 do not match the uniquely determined progression, so options (A), (C) and (D) are excluded.

Step 3: Conclusion The sum of the first ten terms is $\boxed{380}$. Hence, the correct option is (B).

Quick Tip to Solve: Eliminate one progression parameter with the sum ratio first, then use the natural-number condition in the strict term interval.

7. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as $f(x) = a\sin\left(\frac{\pi[x]}{2}\right) + [2 - x], a \in \mathbb{R}$ where $[t]$ is the greatest integer less than or equal to t. If $\lim_{x\to -1} f(x)$ exists, then the value of $\int_0^4 f(x)dx$ is equal to [4 marks]

(A) -1

(B) -2

(C) 1

(D) 2

Correct Answer: (B) -2

Solution:

Concept:

- The greatest-integer function is constant between consecutive integers. One-sided limits must use the floor values on the appropriate open intervals, including for negative inputs.
- A two-sided limit exists only when its two one-sided limits agree. A definite integral of a piecewise constant function is the sum of rectangle areas; isolated endpoint values do not affect it.

Step 1: Determine the parameter from the limit For x just below -1, we have $[x] = -2$ and $[2 - x] = 3$. Hence

$$\lim_{x\to -1^-} f(x) = a\sin(-\pi) + 3 = 3.$$

For x just above -1, $[x] = -1$ and $[2 - x] = 2$, giving

$$\lim_{x\to -1^+} f(x) = a\sin(-\pi/2) + 2 = 2 - a.$$

Equating the limits yields $3 = 2 - a$, so $a = -1$. Notice that the value at $x = -1$ itself is unnecessary: existence of the limit compares nearby values on either side.

Step 2: Integrate on the four unit intervals With $a = -1$, the sine contribution for $[x] = 0, 1, 2, 3$ is respectively $0, -1, 0, 1$. On the corresponding intervals $(0, 1), (1, 2), (2, 3), (3, 4)$, the term $[2 - x]$ is respectively $1, 0, -1, -2$. Therefore the values of f on those open intervals are $1, -1, -1, -1$. Each interval has length one, so

$$\int_0^4 f(x)\,dx = 1 - 1 - 1 - 1 = -2.$$

Changing the function at the finitely many integer endpoints does not alter this integral. The other candidates -1, 1 and 2 do not equal this signed area. In particular, using ordinary truncation instead of the floor for negative numbers would give incorrect interval values.

Step 3: Conclusion The parameter is $a = -1$, and the required integral equals $\boxed{-2}$. Hence, the correct option is (B).

Quick Tip to Solve: For floor-function integrals, tabulate values on open unit intervals; determine any parameter with separate left and right floor values first.

8. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to : [4 marks]

(A) 2

(B) $\frac{39}{5}$

(C) 9

(D) $\frac{46}{5}$

Correct Answer: (D) $\frac{46}{5}$

Solution:

Concept:

- The cross product is found by expanding the determinant of the components. Equality of vectors means equality of all three corresponding components.
- The scalar projection of a vector $\vec{u}$ on a nonzero vector $\vec{v}$ is $\vec{u} \cdot \vec{v}/|\vec{v}|$. Its denominator is a length, not a squared length.

Step 1: Find the unknown components Expanding the given cross product gives

$$\vec{a} \times \vec{b} = (4 + 5\beta)\hat{i} + (3\beta - 4\alpha)\hat{j} + (-5\alpha - 3)\hat{k}.$$

Compare the first component with -1: $4 + 5\beta = -1$, so $\beta = -1$. The second then gives $-3 - 4\alpha = 9$, hence $\alpha = -3$. The third component is $-5(-3) - 3 = 12$, which confirms the result. Therefore $\vec{a} = (-3, 1, -1)$.

Step 2: Project in the specified direction Form the two vectors in the order stated in the question:

$$\vec{u} = \vec{b} - 2\vec{a} = (9, -7, 6), \qquad \vec{v} = \vec{b} + \vec{a} = (0, -4, 3).$$

Their dot product is $9(0) + (-7)(-4) + 6(3) = 46$, and $|\vec{v}| = \sqrt{0 + 16 + 9} = 5$. Thus

$$\text{proj}_{\vec{v}}\,\vec{u} = \frac{46}{5}.$$

The positive sign shows the component is along $\vec{v}$. The other values $2, 39/5, 9$ do not equal this dot-product-to-length ratio; using $|\vec{v}|^2$ would instead calculate the coefficient of a vector projection.

Step 3: Conclusion The required scalar projection is $\boxed{46/5}$. Hence, the correct option is (D).

Quick Tip to Solve: Write the vector being projected first in the dot product, then divide by the length of the vector onto which it is projected.

9. The area of the smaller region enclosed by the curves $y^2 = 8x + 4$ and $x^2 + y^2 + 4\sqrt{3}x - 4 = 0$ is equal to [4 marks]

(A) $\frac{1}{3}(2 - 12\sqrt{3} + 8\pi)$

(B) $\frac{1}{3}(2 - 12\sqrt{3} + 6\pi)$

(C) $\frac{1}{3}(4 - 12\sqrt{3} + 8\pi)$

(D) $\frac{1}{3}(4 - 12\sqrt{3} + 6\pi)$

Correct Answer: (C) $\frac{1}{3}(4 - 12\sqrt{3} + 8\pi)$

Solution:

Concept:

- Use horizontal strips when a parabola is given as $y^2 = 8x + 4$. The strip width is the right-hand boundary minus the left-hand boundary.
- Completing squares gives a circle of centre $(-2\sqrt{3}, 0)$ and radius 4. The common region is symmetric about the x-axis.

Step 1: Find the intersections and boundaries Substitute $y^2 = 8x + 4$ in the circle equation:

$$x^2 + (8 + 4\sqrt{3})x = 0.$$

The roots are 0 and $-8 - 4\sqrt{3}$. The latter would make $y^2 = 8x + 4 < 0$, so it gives no real intersection. Thus the intersections are $(0, 2)$ and $(0, -2)$. Completing the square gives

$$(x + 2\sqrt{3})^2 + y^2 = 16.$$

For $-2 \le y \le 2$, the smaller enclosed region has left boundary $x = (y^2 - 4)/8$ and right boundary $x = -2\sqrt{3} + \sqrt{16 - y^2}$. At $y = 0$, these are $-1/2$ and $4 - 2\sqrt{3}$, confirming their order.

Step 2: Integrate the strip width The required area is

$$A = \int_{-2}^{2} \left(-2\sqrt{3} + \sqrt{16 - y^2} - \frac{y^2 - 4}{8} \right) dy.$$

Use $\int \sqrt{16 - y^2}\, dy = \frac{y}{2}\sqrt{16 - y^2} + 8\sin^{-1}(y/4)$. Between the limits its value is $4\sqrt{3} + 8\pi/3$. The constant term contributes $-8\sqrt{3}$, while the parabola term contributes $4/3$. Consequently,

$$A = -8\sqrt{3} + 4\sqrt{3} + \frac{8\pi}{3} + \frac{4}{3} = \frac{4 - 12\sqrt{3} + 8\pi}{3}.$$

Only (C) has both the correct constant term 4 and the circular contribution 8π; (A) loses $2/3$, and (B), (D) have the wrong circular term.

Step 3: Conclusion The smaller enclosed area is $\boxed{(4 - 12\sqrt{3} + 8\pi)/3}$ square units. Hence, the correct option is (C).

Quick Tip to Solve: After eliminating a squared coordinate, reject algebraic intersection roots that make the other coordinate squared negative.

10. Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solutions of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is [4 marks]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

Concept:

- A first-order linear equation $y' - y = x$ has integrating factor e^{-x}. Multiplying by it turns the left side into a product derivative.
- Two graphs intersect only where their values at the same real x agree. The exponential e^x is strictly positive for every finite real x.

Step 1: Find the family of solutions Rewrite the equation as $y' - y = x$. Multiplication by e^{-x} gives

$$\frac{d}{dx}(ye^{-x}) = xe^{-x}.$$

Integration by parts yields $\int xe^{-x}\,dx = -(x+1)e^{-x} + C$. Therefore

$$ye^{-x} = -(x+1)e^{-x} + C, \qquad y = Ce^x - x - 1.$$

This formula is defined on the whole real line. Differentiation checks it directly: $y' = Ce^x - 1 = x + y$, so no roots or domain restrictions have been introduced by the integration.

Step 2: Apply the initial values and compare For $y_1(0) = 0$, we get $C_1 - 1 = 0$, so $C_1 = 1$. For $y_2(0) = 1$, we get $C_2 - 1 = 1$, so $C_2 = 2$. Thus

$$y_1(x) = e^x - x - 1, \qquad y_2(x) = 2e^x - x - 1.$$

Subtract at a common abscissa:

$$y_2(x) - y_1(x) = e^x > 0 \qquad (x \in \mathbb{R}).$$

The second graph lies strictly above the first everywhere, hence equality never occurs. Although the difference tends to zero as $x \to -\infty$, infinity is not a point on either graph. Options (B), (C) and (D) would require at least one finite zero of e^x, which does not exist.

Step 3: Conclusion The number of intersection points is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: For two solutions of the same linear equation, subtract their equations: here their difference satisfies $z' = z$, so a nonzero difference cannot vanish.

11. Let $I = \int_{\pi/4}^{\pi/3} \left(\frac{8\sin x - \sin 2x}{x}\right) dx$. Then [4 marks]

(A) $\frac{\pi}{2} < I < \frac{3\pi}{4}$

(B) $\frac{\pi}{5} < I < \frac{5\pi}{12}$

(C) $\frac{5\pi}{12} < I < \frac{\sqrt{2}}{3}\pi$

(D) $\frac{3\pi}{4} < I < \pi$

Correct Answer: None of the four printed intervals contains the integral.

Solution:

Concept:

- For a continuous decreasing function f on $[a, b]$, its integral lies strictly between $(b-a)f(b)$ and $(b-a)f(a)$, unless it is constant.
- The identity $\sin x/x = \int_0^1 \cos(tx)\, dt$ allows the sign of the derivative of the integrand to be checked without estimating the integral numerically.

Step 1: Establish monotonicity of the integrand Let $f(x) = (8\sin x - \sin 2x)/x$. The identity above gives

$$f(x) = \int_0^1 \{8\cos(tx) - 2\cos(2tx)\}\, dt.$$

Differentiate this finite integral of a smooth function:

$$f\prime(x) = 8\int_0^1 t\sin(tx)\{\cos(tx) - 1\}\, dt < 0.$$

Indeed, $0 < tx \le \pi/3$ for $0 < t \le 1$, so $\sin(tx) > 0$ and $\cos(tx) - 1 < 0$. Hence f decreases throughout the integration interval.

Step 2: Bound the integral and test every interval The interval length is $\pi/12$. Its endpoint values are

$$f(\pi/3) = \frac{21\sqrt{3}}{2\pi}, \qquad f(\pi/4) = \frac{16\sqrt{2} - 4}{\pi}.$$

Therefore the exact bounds are

$$\frac{7\sqrt{3}}{8} < I < \frac{4\sqrt{2} - 1}{3}.$$

Numerically, these bounds are approximately $1.5155 < I < 1.5523$. But the upper end of (C) is $\sqrt{2}\pi/3 \approx 1.4810$, and the lower end of (A) is $\pi/2 \approx 1.5708$. Thus neither (A) nor (C) contains I. Option (B) ends at $5\pi/12 \approx 1.3090$, while (D) starts at $3\pi/4 \approx 2.3562$; these are excluded as well. These disjoint bounds establish the inconsistency without needing an elementary antiderivative.

Step 3: Conclusion For the printed integrand and limits, $\boxed{7\sqrt{3}/8 < I < (4\sqrt{2} - 1)/3}$, so none of the four printed options is correct. The listed key selects (C), but its upper bound is smaller than the proved lower bound for I; therefore that selection cannot hold for this question.

Quick Tip to Solve: Test an integral inequality by bounding its integrand first. Exact endpoint bounds can rule out every option even when the integral has no simple elementary form.

12. Let S be the sample space of all five digit numbers. It p is the probability that a randomly selected number from S, is a multiple of 7 but not divisible by 5 , then $9p$ is equal to : [4 marks]

(A) 1.0146

(B) 1.2085

(C) 1.0285

(D) 1.1521

Correct Answer: (C) 1.0285

Solution:

Concept:

- All five-digit integers from 10000 through 99999 are equally likely, so the sample space has 90000 outcomes.
- The count of multiples of d in $[L, U]$ is $\lfloor U/d \rfloor - \lfloor (L-1)/d \rfloor$. A number divisible by both 7 and 5 is divisible by 35.

Step 1: Count five-digit multiples of seven The first five-digit multiple of 7 is $7 \times 1429 = 10003$, while the last is $7 \times 14285 = 99995$. Including both endpoints, the count is

$$N_7 = 14285 - 1429 + 1 = 12857.$$

Equivalently, this is $\lfloor 99999/7 \rfloor - \lfloor 9999/7 \rfloor = 14285 - 1428$. The endpoint correction matters because the sample space starts at 10000, not at 1; counting all multiples below 99999 would include many shorter integers.

Step 2: Exclude multiples of five and form the probability Among multiples of 7, precisely the multiples of $\text{lcm}(7, 5) = 35$ must be removed. Their number is

$$N_{35} = \left\lfloor \frac{99999}{35} \right\rfloor - \left\lfloor \frac{9999}{35} \right\rfloor = 2857 - 285 = 2572.$$

Hence there are $12857 - 2572 = 10285$ favourable outcomes. Dividing by all 90000 five-digit numbers and multiplying by 9 gives

$$p = \frac{10285}{90000}, \qquad 9p = \frac{10285}{10000} = 1.0285.$$

No independence approximation is needed: the finite range is counted exactly. The values in (A), (B) and (D) differ from this exact count, so they cannot represent the requested probability multiple.

Step 3: Conclusion The required value is $\boxed{9p = 1.0285}$. Hence, the correct option is (C).

Quick Tip to Solve: For divisible by one integer but not another, subtract the count of multiples of their least common multiple before dividing by the sample-space size.

13. Let $A(1,1), B(-4,3), C(-2,-5)$ be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the areas of triangles APB and ABC, respectively. If $\Delta_1 : \Delta_2 = 4 : 7$, then the area enclosed by the lines AP, AC and the x-axis is : [4 marks]

(A) $\frac{1}{4}$
(B) $\frac{3}{4}$
(C) $\frac{1}{2}$
(D) 1

Correct Answer: (C) $\frac{1}{2}$

Solution:

Concept:

- Triangles with a common altitude have areas proportional to their bases. Here $[APB]/[ABC] = BP/BC$, since both bases lie on BC.
- The area of a triangle with a base on the x-axis is half its base length times the absolute ordinate of the opposite vertex.

Step 1: Locate the point on the side From the common altitude from A to line BC,

$$\frac{BP}{BC} = \frac{4}{7}, \qquad BP : PC = 4 : 3.$$

The section formula therefore gives

$$P = \frac{3B + 4C}{7} = \left(\frac{-12-8}{7}, \frac{9-20}{7}\right) = \left(-\frac{20}{7}, -\frac{11}{7}\right).$$

The weights are reversed relative to the adjacent segment names: the coordinate of C has weight 4 because P is four sevenths of the way from B to C. This verifies that P lies inside the side, as required.

Step 2: Find the intercepts and enclosed area The slope of AP is $(-11/7 - 1)/(-20/7 - 1) = 2/3$. Thus

$$AP : \; y - 1 = \frac{2}{3}(x-1), \qquad y = \frac{2x+1}{3}.$$

It meets the x-axis at $(-1/2, 0)$. The slope of AC is $(-5-1)/(-2-1) = 2$, giving $AC : y = 2x - 1$, whose intercept is $(1/2, 0)$. Together with $A(1,1)$, these intercepts form the triangle enclosed by the three lines. Its base length is 1 and its perpendicular height is 1, so

$$\text{Area} = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}.$$

Options (A), (B) and (D) fail this base-height calculation; the original triangle's area is not the requested enclosed area.

Step 3: Conclusion The area enclosed by AP, AC and the x-axis is $\boxed{1/2}$ square unit. Hence, the correct option is (C).

Quick Tip to Solve: Convert an area ratio into a side ratio using the shared altitude, then find the intercepts of the requested lines rather than the original triangle area.

14. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0, g, c \in \mathbb{R}$ passes through the point $(6, 1)$ and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on x-axis is : [4 marks]

(A) $\sqrt{11}$

(B) 4

(C) 3

(D) $2\sqrt{23}$

Correct Answer: (D) $2\sqrt{23}$

Solution:

Concept:

- In $x^2 + y^2 - 2gx + 6y - 19c = 0$, the centre is $(g, -3)$. Its coordinates must satisfy the given line equation.
- The intercept on the x-axis is the distance between the two real roots obtained by setting $y = 0$, not the radius or half of that distance.

Step 1: Use the point and centre conditions Substitute $(6, 1)$ in the circle equation:

$$36 + 1 - 12g + 6 - 19c = 0, \qquad 12g + 19c = 43.$$

The centre $(g, -3)$ lies on $x - 2cy = 8$, so

$$g + 6c = 8, \qquad g = 8 - 6c.$$

Putting this in the first condition gives $12(8 - 6c) + 19c = 43$. Therefore $96 - 53c = 43$, yielding $c = 1$ and $g = 2$. Both conditions are satisfied: the centre is $(2, -3)$, and $2 - 2(1)(-3) = 8$.

Step 2: Calculate the full chord on the axis The circle becomes

$$x^2 + y^2 - 4x + 6y - 19 = 0, \quad \text{or} \quad (x - 2)^2 + (y + 3)^2 = 32.$$

Its radius squared is 32, while the distance of its centre from the x-axis is 3. Hence the half-chord length is $\sqrt{32 - 9} = \sqrt{23}$. Directly setting $y = 0$ confirms the two intersections:

$$x^2 - 4x - 19 = 0, \qquad x = 2 \pm \sqrt{23}.$$

The distance between them is $(2 + \sqrt{23}) - (2 - \sqrt{23}) = 2\sqrt{23}$. Thus neither $\sqrt{11}$, 4 nor 3 is the full intercept. In particular, 3 is merely the centre's distance from the axis and does not measure the chord.

Step 3: Conclusion The length of the intercept on the x-axis is $\boxed{2\sqrt{23}}$ units. Hence, the correct option is (D).

Quick Tip to Solve: When a circle cuts an axis, set the other coordinate to zero and subtract the two roots; this automatically gives the full intercept.

15. Let a function $f : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} \int_0^x (5 - |t-3|)dt, & x > 4 \\ x^2 + bx & , x \leq 4 \end{cases}$$

where $b \in \mathbb{R}$. If f is continuous at $x = 4$, then which of the following statements is NOT true? [4 marks]

(A) f is not differentiable at $x = 4$

(B) $f'(3) + f'(5) = \frac{35}{4}$

(C) f is increasing in $\left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$

(D) f has a local minima at $x = \frac{1}{8}$

Correct Answer: (C) f is increasing in $\left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$

Solution:

Concept:

- Continuity at the joining point requires the value of the polynomial branch to equal the limiting integral. The absolute value changes form at $t = 3$.
- The fundamental theorem of calculus gives the derivative of an integral with a variable upper limit. A positive derivative indicates increase; a negative derivative indicates decrease.

Step 1: Use continuity to determine the parameter For $0 \leq t \leq 3$, the integrand is $2 + t$; for $3 \leq t \leq 4$, it is $8 - t$. Therefore

$$\begin{aligned} \int_0^4 (5 - |t-3|)\, dt &= \int_0^3 (2+t)\, dt + \int_3^4 (8-t)\, dt \\ &= \frac{21}{2} + \frac{9}{2} = 15. \end{aligned}$$

Continuity gives $16 + 4b = 15$, so $b = -1/4$.

Step 2: Check differentiability and the derivative sum Away from the joining point,

$$f'(x) = \begin{cases} 2x - \frac{1}{4}, & x < 4, \\ 8 - x, & x > 4. \end{cases}$$

The left derivative at 4 is $31/4$, whereas the right derivative is 4, so option (A) is true. Also,

$$f'(3) + f'(5) = \frac{23}{4} + 3 = \frac{35}{4},$$

which proves option (B).

Step 3: Test the claimed intervals of increase For $x < 1/8$, the polynomial derivative is negative; it becomes positive for $1/8 < x < 4$. On the integral branch, $8 - x$ is positive until $x = 8$, then negative. Continuity joins the two increasing pieces across 4. Thus the function decreases on both intervals listed in (C), and increases on $(1/8, 8)$. The sign change from negative to positive at $1/8$ proves the local minimum asserted in (D).

Step 4: Conclusion The statement that is NOT true is the claimed increase on $(-\infty, 1/8) \cup (8, \infty)$.

Hence, the correct option is (C).

Quick Tip to Solve: When the question asks for the false statement, mark the derivative signs on every branch before selecting an interval.

16. Let $P(a, b)$ be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b. Then the value of $A + B$ is equal to : [4 marks]

(A) 0

(B) 25

(C) 40

(D) 65

Correct Answer: (D) 65

Solution:

Concept:

- For $y^2 = 4px$, a parametric point is $(pt^2, 2pt)$, and its tangent is $ty = x + pt^2$. Here the focal parameter is $p = 2$.
- The centre of $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(-g, -f)$. Products of the roots can be obtained from the constant and leading coefficients of a quadratic.

Step 1: Find the circle centre and impose the tangent condition Completing squares in the circle equation gives

$$(x - 5)^2 + (y - 7)^2 = 9,$$

so the centre is $(5, 7)$. Write the contact point on $y^2 = 8x$ as $P = (2t^2, 4t)$. The tangent at this point is

$$ty = x + 2t^2.$$

For this tangent to pass through the centre, substitute $x = 5, y = 7$:

$$7t = 5 + 2t^2 \quad \Longrightarrow \quad 2t^2 - 7t + 5 = 0.$$

Step 2: Obtain both contact points Factor the parameter equation:

$$(2t - 5)(t - 1) = 0.$$

Thus $t_1 = 1$, $t_2 = 5/2$. Both are real and yield valid points of tangency. Their coordinates are

$$P_1 = (2, 4), \qquad P_2 = \left(\frac{25}{2}, 10\right).$$

Indeed, each satisfies $b^2 = 8a$, and the corresponding tangent passes through $(5, 7)$.

Step 3: Compute the two products separately The problem defines A as the product of the first

coordinates and B as the product of the second coordinates, not the product of sums. Hence

$$A = 2 \cdot \frac{25}{2} = 25, \qquad B = 4 \cdot 10 = 40,$$

and

$$A + B = 25 + 40 = 65.$$

The choices 25 and 40 are only the separate products, while 0 is neither the required sum nor a consequence of either contact point.

Step 4: Conclusion The required value is $A + B = 65$. Hence, the correct option is (D).

Quick Tip to Solve: Use the tangent parameter: once the two parameter roots are known, their product gives both coordinate products directly.

17. Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$, then $|\vec{b} \times 2\hat{j}|$ is equal to : [4 marks]

(A) 4

(B) 5

(C) $\sqrt{21}$

(D) $\sqrt{17}$

Correct Answer: (B) 5

Solution:

Concept:

- If $\vec{c} = (c_x, c_y, c_z)$, then $\vec{c} \times \hat{i} = (0, c_z, -c_y)$, so its scalar product with $\hat{k}$ is $-c_y$.
- The magnitude of a vector is the square root of the sum of squares of its Cartesian components. A cross product with $\hat{j}$ removes the component parallel to $\hat{j}$.

Step 1: Extract the component fixed by the scalar condition Using $\vec{a} = (2, -1, 5)$ and $\vec{b} = (\alpha, \beta, 2)$, expand

$$\vec{a} \times \vec{b} = (-2 - 5\beta)\hat{i} + (5\alpha - 4)\hat{j} + (2\beta + \alpha)\hat{k}.$$

Crossing this vector with $\hat{i}$ and then taking its $\hat{k}$ component reverses the sign of the $\hat{j}$ component. Consequently the given equation becomes

$$4 - 5\alpha = \frac{23}{2}.$$

Solving gives $-5\alpha = 15/2$, and hence $\alpha = -3/2$. Notice that the condition does not determine β, so the required magnitude must be independent of it.

Step 2: Evaluate the required cross product and its magnitude Since $2\hat{j} = (0, 2, 0)$,

$$\vec{b} \times 2\hat{j} = -4\hat{i} + 2\alpha\hat{k} = -4\hat{i} - 3\hat{k}.$$

Therefore

$$|\vec{b} \times 2\hat{j}| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5.$$

As a check, the $\beta\hat{j}$ term contributes zero because $\hat{j} \times \hat{j} = 0$. The other proposed magnitudes have squares 16, 21 and 17, which disagree with the component sum 25. In particular, using only the magnitude of the $\hat{i}$ component would incorrectly give 4; both perpendicular components must be retained.

Step 3: Conclusion The cross-product magnitude equals 5. Hence, the correct option is (B).

Quick Tip to Solve: In a nested cross product followed by a dot product, identify the single required component before expanding everything.

18. Let a vertical tower AB of height $2h$ stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of elevation α. If $d = \sqrt{7}h$, then $\tan\alpha$ is equal to [4 marks]

(A) $\sqrt{5} - 2$

(B) $\sqrt{3} - 1$

(C) $\sqrt{7} - 2$

(D) $\sqrt{7} - \sqrt{3}$

Correct Answer: (C) $\sqrt{7} - 2$

Solution:

Concept:

- For an angle of elevation θ, $\tan\theta$ is the vertical height divided by the horizontal distance from the tower base.
- The double-angle identity is $\tan 2\alpha = 2t/(1 - t^2)$, where $t = \tan\alpha$. Since 2α is an acute elevation angle, $0 < \alpha < \pi/4$ and $0 < t < 1$.

Step 1: Translate the two observations into distance equations Let $AP = x$. At the original point the visible height is h and the elevation angle is 2α. Thus

$$\tan 2\alpha = \frac{h}{x}, \qquad x = \frac{h}{\tan 2\alpha}.$$

Moving in the direction of $\overrightarrow{AP}$ means moving away from the tower, so the new horizontal distance is $x + d$. At that position the height is the full tower height $2h$, and therefore

$$\tan\alpha = \frac{2h}{x + d}.$$

Using $x - d$ would describe motion towards the tower and would not match the given direction.

Step 2: Eliminate the horizontal distance Put $t = \tan\alpha$. Then

$$x = \frac{h(1-t^2)}{2t}, \qquad x + d = \frac{2h}{t}.$$

Subtract these equations and use $d = \sqrt{7}h$:

$$\frac{d}{h} = \frac{2}{t} - \frac{1-t^2}{2t} = \frac{3+t^2}{2t} = \sqrt{7}.$$

Thus

$$t^2 - 2\sqrt{7}t + 3 = 0,$$

whose roots are $t = \sqrt{7} \pm 2$.

Step 3: Apply the geometric restriction The larger root $\sqrt{7}+2$ exceeds 1 and is excluded by $0 < t < 1$. The smaller root $\sqrt{7} - 2$ is positive and less than 1, so it produces an acute 2α and a positive original distance. This is the only admissible root. The other listed expressions do not solve the derived quadratic; the height $2h$ at the second observation must not be replaced by h.

Step 4: Conclusion We obtain $\tan\alpha = \sqrt{7} - 2$. Hence, the correct option is (C).

Quick Tip to Solve: Fix the direction of motion first, then use the acute-angle bound to reject the extraneous quadratic root.

19. $(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ **is equivalent to** $(\sim p)$ **when** r **is** [4 marks]

(A) p

(B) $\sim p$

(C) q

(D) $\sim q$

Correct Answer: (C) q

Solution:

Concept:

- A biconditional $u \Leftrightarrow v$ is true when its two inputs have the same truth value and false when their truth values differ.
- Two propositions are equivalent only if their truth values agree for every assignment of the independent variables. We can split the check into p false and p true.

Step 1: Handle the case where p is false If p is false, both $p \wedge r$ and $p \wedge (\sim q)$ are false, regardless of r and q. Their biconditional is therefore true. This agrees with $\sim p$, which is also true in this case. Hence this half of the truth table imposes no further restriction on r. The nontrivial requirement occurs when p is true.

Step 2: Handle the case where p is true When p is true, the two conjunctions reduce to r and $\sim q$. To agree with $\sim p$, their biconditional must now be false for both possible values of q. Thus r must have the opposite truth value to $\sim q$, which is exactly $r = q$. If q is true, this makes the pair true and false;

if q is false, it makes the pair false and true. The biconditional is false in both cases, as required.

Step 3: Check the remaining options For $r = p$, take p true and q false: both conjunctions become true, so the biconditional is true although $\sim p$ is false. For $r = \sim p$, take p and q both true: both conjunctions are false, giving the same contradiction. For $r = \sim q$, the conjunctions are identical and the biconditional is always true, which fails whenever p is true. Only option (C) meets every assignment.

Step 4: Conclusion The required proposition is $r = q$. Hence, the correct option is (C).

Quick Tip to Solve: Set the common conjunction factor p to false and true separately; this reduces a three-variable equivalence to a simple two-case check.

20. If the plane P passes through the intersection of two mutually perpendicular planes $2x+ky-5z = 1$ and $3kx - ky + z = 5, k < 3$ and intercepts a unit length on positive x-axis, then the intercept made by the plane P on the y-axis is : [4 marks]

(A) $\frac{1}{11}$

(B) $\frac{5}{11}$

(C) 6

(D) 7

Correct Answer: (D) 7

Solution:

Concept:

- Two planes are perpendicular when their normal vectors have zero dot product. The coefficients of x, y, z are the components of a plane normal.
- A plane through the common line of $F_1 = 0$ and $F_2 = 0$ can be written as $F_1 + \lambda F_2 = 0$, apart from the second plane itself, which can be checked separately.

Step 1: Use perpendicularity to determine k The normal vectors of the given planes are $(2, k, -5)$ and $(3k, -k, 1)$. Their dot product vanishes:

$$6k - k^2 - 5 = 0 \quad \Longrightarrow \quad (k-1)(k-5) = 0.$$

The restriction $k < 3$ retains $k = 1$ and rejects $k = 5$. Thus the two planes become

$$2x + y - 5z - 1 = 0, \qquad 3x - y + z - 5 = 0.$$

Step 2: Impose the unit intercept on the family A unit intercept on the positive x-axis means the desired plane contains $(1, 0, 0)$. Write its equation as

$$(2x + y - 5z - 1) + \lambda(3x - y + z - 5) = 0.$$

At $(1, 0, 0)$, this gives $1 - 2\lambda = 0$, so $\lambda = 1/2$. The second plane alone does not contain $(1, 0, 0)$, so no plane omitted from this parameterization can satisfy the intercept condition. Multiplying by 2 and

collecting terms gives

$$7x + y - 9z - 7 = 0.$$

Step 3: Read the intercept on the y-axis At an intersection with the y-axis, set $x = z = 0$. The equation becomes $y = 7$. It also confirms the first condition: setting $y = z = 0$ gives $x = 1$. Therefore the required intercept is positive and equals 7. The alternatives $1/11$, $5/11$, and 6 do not satisfy the derived plane equation at $(0, y, 0)$; none can be its y-intercept.

Step 4: Conclusion The intercept made on the y-axis is 7. Hence, the correct option is (D).

Quick Tip to Solve: Convert an intercept into a coordinate point before substituting it into the family of planes; this avoids ratio errors.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. For $k \in \mathbb{R}$, let the solutions of the equation $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right) = k, 0 < |x| < \frac{1}{\sqrt{2}}$ be α and β, where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to _____. [4 marks]

Solution:

Concept:

- On the principal branch, $\cos(\sin^{-1}x) = \sqrt{1-x^2}$, and the square root is nonnegative. Also, $\cot(\tan^{-1}u) = 1/u$ for positive u.
- For the monic quadratic $X^2 - bX - 5 = 0$, the sum of its roots is b, and their product is -5. These relations avoid solving that quadratic explicitly.

Step 1: Simplify the nested expression with its domain The given interval ensures $0 < x^2 < 1/2$. Therefore $\sqrt{1-x^2} > 0$, and the argument of the outer inverse sine is

$$x\cot\left(\tan^{-1}\sqrt{1-x^2}\right) = \frac{x}{\sqrt{1-x^2}}.$$

Its absolute value is less than 1, so the principal-value identities apply. The equation reduces to

$$\sqrt{1 - \frac{x^2}{1-x^2}} = \sqrt{\frac{1-2x^2}{1-x^2}} = k.$$

In particular $k > 0$. The expression depends only on x^2, so its two nonzero solutions satisfy $\beta = -\alpha$.

Step 2: Use the product of the quadratic roots Put $s = \alpha^2 = \beta^2$. Then the two specified quadratic roots are

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{2}{s}, \qquad \frac{\alpha}{\beta} = -1.$$

Their product is -5, hence

$$-\frac{2}{s} = -5 \quad \Longrightarrow \quad s = \frac{2}{5}.$$

This lies strictly between 0 and $1/2$, as the original domain requires. The roots are consequently 5 and -1, and their sum gives $b = 4$.

Step 3: Determine the requested ratio Substituting $s = 2/5$ into the simplified trigonometric equation yields

$$k^2 = \frac{1-2s}{1-s} = \frac{1-4/5}{1-2/5} = \frac{1}{3}.$$

The positive value $k = 1/\sqrt{3}$ is consistent with the principal cosine. Finally,

$$\frac{b}{k^2} = \frac{4}{1/3} = 12.$$

All steps use values within the inverse-function domains, and the quadratic with $b = 4$ indeed factors as $(X-5)(X+1)$.

Step 4: Conclusion The required dimensionless numerical value is $\boxed{12}$.

Quick Tip to Solve: Exploit even dependence on x to obtain opposite roots; their ratio is then -1, and Vieta immediately fixes their squared magnitude.

22. The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is _____.

[4 marks]

Solution:

Concept:

- For a population of N observations, $\mu = \sum x_i/N$ and $\sigma^2 = \sum x_i^2/N - \mu^2$. The standard deviation is the nonnegative square root of the variance.
- Correcting one observation changes both the sum and the sum of squares. Subtract the incorrectly entered value before adding the actual observation; do not correct the variance directly.

Step 1: Recover the incorrectly calculated totals The student used ten observations and obtained mean 15, so the incorrect sum is $10(15) = 150$. The reported variance gives the incorrect second moment:

$$15 = \frac{\sum x_i^2}{10} - 15^2.$$

Thus the incorrect sum of squares is

$$\sum x_i^2 = 10(15 + 225) = 2400.$$

These two totals refer to the list containing 25, rather than 15.

Step 2: Replace the mistaken observation After replacing 25 by 15, the correct totals become

$$\sum x_i = 150 - 25 + 15 = 140, \qquad \sum x_i^2 = 2400 - 25^2 + 15^2 = 2000.$$

The sample still contains ten observations; only one recorded value has changed. Therefore the

corrected mean is $140/10 = 14$, while its second raw moment is $2000/10 = 200$.

Step 3: Calculate the corrected standard deviation Using the corrected mean in the population-variance formula,

$$\sigma^2 = 200 - 14^2 = 200 - 196 = 4.$$

Taking the nonnegative square root gives

$$\sigma = \sqrt{4} = 2.$$

In particular, the reduction in the sum of squares alone is not the variance correction, because the mean changes at the same time.

Step 4: Conclusion The correct standard deviation is $\boxed{2}$, in the same units as the observations.

Quick Tip to Solve: When correcting a data entry, update $\sum x$ and $\sum x^2$ separately, then recompute the mean and variance.

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23. An ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H : \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H, respectively. Let the product of the eccentricities of E and H be $\frac{1}{2}$. If l is the length of the latus rectum of the ellipse E, then the value of $113l$ is equal to _____. [4 marks]

Solution:

Concept:

- For a hyperbola $y^2/B^2 - x^2/A^2 = 1$, the transverse axis is vertical, the vertices are $(0, \pm B)$, and its eccentricity is $\sqrt{1 + A^2/B^2}$.
- For an ellipse with vertical semimajor axis b and horizontal semiminor axis a, $e_E^2 = 1 - a^2/b^2$ and the full latus rectum has length $l = 2a^2/b$.

Step 1: Identify the semimajor axis of the ellipse Rewrite the hyperbola as

$$\frac{y^2}{64} - \frac{x^2}{49} = 1.$$

Its transverse axis is the y-axis and its vertices are $(0, \pm 8)$. Since the ellipse has this same major-axis direction and passes through these vertices, its vertical semimajor axis is $b = 8$, so $b^2 = 64$. The horizontal denominator a^2 is the smaller one.

Step 2: Use the product of eccentricities The hyperbola has eccentricity

$$e_H = \sqrt{1 + \frac{49}{64}} = \frac{\sqrt{113}}{8}.$$

Since $e_E e_H = 1/2$,

$$e_E = \frac{4}{\sqrt{113}}, \qquad e_E^2 = \frac{16}{113}.$$

Consequently

$$1-\frac{a^2}{64}=\frac{16}{113}, \qquad a^2=64\frac{97}{113}=\frac{6208}{113}.$$

This is less than 64, consistently with the specified major-axis direction.

Step 3: Evaluate the requested multiple The full latus rectum uses the square of the semiminor axis divided by the semimajor axis:

$$l=\frac{2a^2}{b}=\frac{2}{8}\frac{6208}{113}=\frac{1552}{113}.$$

Therefore

$$113l=1552.$$

Interchanging the two axes would use the wrong eccentricity and the wrong latus-rectum formula.

Step 4: Conclusion The required numerical value is $\boxed{113l=1552}$.

Quick Tip to Solve: Write the hyperbola with its positive squared term first; this immediately identifies its transverse axis and hence the ellipse's major axis.

24. Let $y=y(x)$ be the solution curve of the differential equation $\sin\left(2x^2\right)\log_e\left(\tan x^2\right)dy+\left(4xy-4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)\right)dx=0$, $0<x<\sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}},1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to _____. [4 marks]

Solution:

Concept:

- With $t=x^2$ and $L(t)=\log(\tan t)$, one has $L\sin 2t\,dy/dt+2y=2(\sin t-\cos t)$. On a regular interval, L is an integrating factor since $L\prime(t)=2/\sin 2t$.
- A linear-equation solution is valid on a connected interval where its coefficients are defined. Here L vanishes at $t=\pi/4$, between the initial point and the requested point, so the usual formal answer needs a domain qualification.

Step 1: Obtain the integrating-factor identity Use $dt=2x\,dx$ and $\sqrt{2}\sin(t-\pi/4)=\sin t-\cos t$. Dividing the transformed equation by $\sin 2t$ yields

$$\frac{d}{dt}(yL)=\sec t-\csc t.$$

Hence, on each regular interval,

$$y(t)L(t)=F(t)+C, \quad F(t)=\log(\sec t+\tan t)-\log\tan(t/2).$$

All logarithm arguments are positive for $0<t<\pi/2$. This identity can also be differentiated directly to recover the given equation.

Step 2: Calculate the conventional endpoint value Let $t_1=\pi/6$ and $t_2=\pi/3$. Direct substitution gives

$$F(t_1)=F(t_2)=\log(3+2\sqrt{3}),$$

while

$$L(t_1) = -\tfrac{1}{2}\log 3, \qquad L(t_2) = \tfrac{1}{2}\log 3.$$

Formally using the same integration constant and $y(t_1) = 1$ therefore gives

$$y(t_2)L(t_2) = L(t_1), \qquad y(t_2) = -1.$$

Its absolute value is 1, which is the numerical answer intended by the question and its key.

Step 3: Check the literal solution-curve requirement The initial condition fixes $C = -\log(6 + 3\sqrt{3})$. At $t_0 = \pi/4$, the numerator of $y = (F + C)/L$ approaches

$$\log \frac{3 + 2\sqrt{2}}{6 + 3\sqrt{3}} \neq 0,$$

whereas $L \to 0$. Thus the initial branch diverges and cannot extend continuously across t_0. In the undivided equation a finite solution at t_0 would require $y(t_0) = 0$, which this branch cannot satisfy. The formal endpoint calculation consequently does not define a single classical solution curve on the entire stated interval.

Step 4: Conclusion The intended numerical response is $\boxed{1}$. Strictly, the stated initial-value problem has no classical solution extending to the requested point, because its initial branch is singular at $x = \sqrt{\pi/4}$.

Quick Tip to Solve: Before transferring an integration constant between two points, check for zeros of the integrating factor or singular coefficients between them.

25. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _____. [4 marks]

Solution:

Concept:

- The absolute-value restriction $|f(n)| \leq 800$ means $-800 \leq f(n) \leq 800$. Complete the square or use the minimum to check the lower bound before solving the upper bound.
- For a consecutive integer interval, sum a polynomial term by term using $\sum_{k=1}^{m} k = m(m+1)/2$ and $\sum_{k=1}^{m} k^2 = m(m+1)(2m+1)/6$.

Step 1: Find every permitted integer The quadratic can be written

$$f(x) = 2\left(x - \frac{1}{4}\right)^2 - \frac{9}{8}.$$

Hence $f(n) \geq -9/8 > -800$, and the lower inequality imposes no restriction. The upper inequality is

$$2n^2 - n - 801 \leq 0.$$

Checking the adjacent integers gives

$$f(-20) = 819,\ f(-19) = 740,\ f(20) = 779,\ f(21) = 860.$$

Since an upward-opening quadratic is at most 800 between its two boundary roots, the permitted integers are exactly $-19, -18, \ldots, 20$. There are 40 of them.

Step 2: Compute the first and second power sums Pair $-19, \ldots, -1$ with $1, \ldots, 19$; their first powers cancel, leaving

$$\sum_{n=-19}^{20} n = 20.$$

Their squares add, and the extra endpoint contributes 20^2:

$$\sum_{n=-19}^{20} n^2 = 2\frac{19 \cdot 20 \cdot 39}{6} + 400 = 4940 + 400 = 5340.$$

The zero contributes to the count but not to either power sum.

Step 3: Sum the quadratic values Use the same interval in all three terms:

$$\begin{aligned}\sum_{n \in S} f(n) &= 2\sum_{n=-19}^{20} n^2 - \sum_{n=-19}^{20} n - \sum_{n=-19}^{20} 1 \\ &= 2(5340) - 20 - 40 \\ &= 10620.\end{aligned}$$

The asymmetry of the interval matters: omitting the extra endpoint 20 would give a different sum.

Step 4: Conclusion The required sum is $\boxed{10620}$.

Quick Tip to Solve: For a quadratic inequality on integers, verify both included endpoints and the immediately excluded neighbours before applying sum formulae.

26. Let S be the set containing all 3×3 matrices with entries from $\{-1, 0, 1\}$. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^{\mathrm{T}}A$ is 6 is _____. [4 marks]

Solution:

Concept:

- The jth diagonal entry of $A^{\mathrm{T}}A$ is the sum of the squares of the entries in column j of A. Thus $\mathrm{tr}(A^{\mathrm{T}}A) = \sum_{i,j} a_{ij}^2$.
- An entry from $\{-1, 0, 1\}$ contributes 1 to this sum precisely when it is nonzero. Count the positions of the nonzero entries first and then count their independent signs.

Step 1: Expand the sum of diagonal elements Let $A = (a_{ij})$, where $1 \le i, j \le 3$. Matrix multiplication

gives

$$(A^{\mathrm{T}}A)_{jj} = \sum_{i=1}^{3} a_{ij}a_{ij} = \sum_{i=1}^{3} a_{ij}^2.$$

Adding the three diagonal entries therefore counts all nine squared entries once. No cross-product terms remain in this trace, and symmetry of A is not required.

Step 2: Translate the condition into nonzero positions Each allowed entry has square 0 or 1: zero has square zero, whereas both signs of 1 have square one. Therefore

$$\mathrm{tr}(A^{\mathrm{T}}A) = 6$$

is equivalent to saying that exactly six of the nine positions of A are nonzero. Select those six positions in

$$\binom{9}{6} = \binom{9}{3} = 84$$

ways. The remaining three positions are necessarily zero.

Step 3: Assign signs independently For each chosen set of positions, every nonzero entry can independently be either -1 or 1, giving $2^6 = 64$ sign patterns. Different positions or sign patterns produce different matrices. By the multiplication principle,

$$N = \binom{9}{6} 2^6 = 84 \cdot 64 = 5376.$$

We are counting arbitrary matrices, so there is no symmetry factor or division by permutations of rows or columns.

Step 4: Conclusion The total number of matrices is $\boxed{5376}$.

Quick Tip to Solve: Recognise $\mathrm{tr}(A^{\mathrm{T}}A)$ as the sum of all squared entries; with entries 0 and ± 1, it simply counts nonzero positions.

---

27. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4 , and l is the length of its major axis, then $\lambda + l$ is equal to _____. [4 marks]

Solution:

Concept:

- Complete squares to put a translated ellipse into standard form. Translation changes the centre but does not change semiaxis lengths or the length of a latus rectum.
- If $a \geq b > 0$ are the semimajor and semiminor axes, the full latus rectum has length $2b^2/a$, whereas the major axis has full length $2a$.

Step 1: Complete the squares and identify the axes Group the terms in x and y:

$$x^2 + 2x + 4(y^2 + 2y) = \lambda.$$

Completing both squares gives

$$(x+1)^2 + 4(y+1)^2 = \lambda + 5.$$

Write $K = \lambda + 5$. A nondegenerate ellipse requires $K > 0$, and its standard equation is

$$\frac{(x+1)^2}{K} + \frac{(y+1)^2}{K/4} = 1.$$

Therefore $a = \sqrt{K}$ and $b^2 = K/4$, with the major axis horizontal.

Step 2: Use the specified latus rectum The latus rectum is the full chord through a focus perpendicular to the major axis. Its length is

$$\frac{2b^2}{a} = \frac{2(K/4)}{\sqrt{K}} = \frac{\sqrt{K}}{2}.$$

Equating this to the given length 4 yields

$$\frac{\sqrt{K}}{2} = 4, \qquad \sqrt{K} = 8, \qquad K = 64.$$

Thus $\lambda = K - 5 = 59$. The positive square root is required since a semiaxis is a length.

Step 3: Calculate the full major-axis length The semimajor axis is $a = 8$, so the specified l, the full major-axis length, is

$$l = 2a = 16.$$

Consequently

$$\lambda + l = 59 + 16 = 75.$$

Notice that the requested major axis is twice the semimajor axis; substituting 8 for l would incorrectly omit this factor of two.

Step 4: Conclusion The requested value is $\boxed{\lambda + l = 75}$.

Quick Tip to Solve: After completing squares, distinguish the semimajor axis a, full major axis $2a$, and latus rectum $2b^2/a$.

28. Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\mathrm{Re}(z) + \mathrm{Im}(z))$ is equal to _____. [4 marks]

Solution:

Concept:

- Write $z = x + iy$, where x, y are real, and $\bar{z} = x - iy$. Two complex numbers are equal precisely when their real and imaginary parts are respectively equal.
- The set contains distinct complex roots, each counted once. Solving the real and imaginary equations by cases ensures that the real root zero is not lost by division.

Step 1: Separate the real and imaginary equations Substitute $z = x + iy$ into the equation:

$$(x + iy)^2 + x - iy = 0.$$

Since $(x+iy)^2 = x^2 - y^2 + 2ixy$, equality to zero gives

$$x^2 - y^2 + x = 0, \qquad y(2x-1) = 0.$$

The second equation provides two exhaustive cases: $y = 0$ or $x = 1/2$. We do not divide by y, since it may be zero.

Step 2: List the roots from both cases If $y = 0$, the real equation becomes $x^2 + x = x(x+1) = 0$, so $z = 0$ or $z = -1$. If $x = 1/2$, the real equation gives

$$\frac{1}{4} - y^2 + \frac{1}{2} = 0, \qquad y^2 = \frac{3}{4}.$$

Thus the other two roots are

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i, \qquad z = \frac{1}{2} - \frac{\sqrt{3}}{2}i.$$

These are distinct and exhaust both cases.

Step 3: Add the requested components For the four roots, the values of $\operatorname{Re} z + \operatorname{Im} z$ are

$$0, \quad -1, \quad \frac{1}{2} + \frac{\sqrt{3}}{2}, \quad \frac{1}{2} - \frac{\sqrt{3}}{2}.$$

Their sum is

$$0 - 1 + \frac{1}{2} + \frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0.$$

The conjugate pair cancels the imaginary contributions, while its combined real part cancels the real root -1.

Step 4: Conclusion The required sum is $\boxed{0}$.

Quick Tip to Solve: For an equation mixing z and $\bar{z}$, real and imaginary parts often factor into simple cases; keep the zero-root case before dividing.

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29. Let the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and $4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3, a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha + \beta + \gamma$ equals _____. [4 marks]

Solution:

Concept:

- A plane through the intersection of two planes belongs to their linear family. A line lies in a plane when its base point satisfies the equation and its direction is perpendicular to the normal.
- Write a symmetric line parametrically before substituting it into a plane. The resulting parameter gives all three coordinates of the intersection.

Step 1: Find the parameter that makes the two lines coplanar Put $A = 4ax - y + 5z - 7a$ and $B = 2x - 5y - z - 3$. The second line is $A = B = 0$, so its containing planes have equation $\lambda A + \mu B = 0$. The first line has base point $(4, -1, 0)$ and direction $(1, -2, 1)$. Substitution of its point

and direction gives

$$\lambda(9a+1)+10\mu=0, \qquad \lambda(4a+7)+11\mu=0.$$

For a nonzero pair (λ,μ), the determinant must vanish:

$$11(9a+1)-10(4a+7)=59(a-1)=0.$$

Thus $a=1$. Neither $\lambda=0$ nor $\mu=0$ alone can satisfy both equations, so this condition captures the required plane.

Step 2: Intersect the given line with the common plane For $a=1$, the equations above give $\mu=-\lambda$. Hence the plane is $A-B=0$, or

$$x+2y+3z-2=0.$$

The line meeting this plane is $(x,y,z)=(3+7t,2-t,3-4t)$. On substitution,

$$3+7t+2(2-t)+3(3-4t)-2=14-7t=0,$$

so $t=2$ and $P=(17,0,-5)$. As a check, this point satisfies $17+0-15-2=0$.

Step 3: Conclusion Therefore $\alpha+\beta+\gamma=17+0-5=\boxed{12}$. This is a dimensionless coordinate sum.

Quick Tip to Solve: For coplanarity of a line and an intersection line, use the family of planes through the latter and impose both a point and a direction condition.

30. Let M and N be the number of points on the curve $y^5-9xy+2x=0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of $M+N$ equals _____.

[4 marks]

Solution:

Concept:

- For a smooth implicit curve $F(x,y)=0$, a horizontal tangent requires $F_x=0$, $F_y\neq 0$; a vertical tangent requires $F_y=0$, $F_x\neq 0$.
- Candidate points must satisfy the original curve as well as the tangent condition. A vanishing numerator or denominator alone is insufficient.

Step 1: Exclude horizontal tangents Implicit differentiation gives

$$5y^4\frac{dy}{dx}-9y-9x\frac{dy}{dx}+2=0, \qquad \frac{dy}{dx}=\frac{9y-2}{5y^4-9x}.$$

A horizontal tangent would require $y=2/9$. But the original equation is $y^5+x(2-9y)=0$. At $y=2/9$ it reduces to $(2/9)^5=0$, which is impossible for any finite x. Hence no point of the curve has a horizontal tangent, so $M=0$.

Step 2: Count the vertical tangents Set the denominator to zero: $x=5y^4/9$. Substitute into the

original curve:

$$y^5 - 5y^5 + \frac{10y^4}{9} = 0 \quad \Longrightarrow \quad y^4\left(\frac{10}{9} - 4y\right) = 0.$$

The distinct real possibilities are $y = 0$ and $y = 5/18$, each giving one x. The numerator $9y - 2$ is respectively -2 and $1/2$, so it is nonzero at both points. Both are smooth points with a vertical tangent; multiplicity four at $y = 0$ does not create four different points. There is no singular point omitted: simultaneous vanishing of numerator and denominator would require the already excluded value $y = 2/9$. Thus $N = 2$.

Step 3: Conclusion Therefore $M + N = 0 + 2 = \boxed{2}$. The requested number counts distinct tangent points.

Quick Tip to Solve: For implicit curves, test every derivative candidate in the original equation and count distinct points rather than algebraic multiplicities.

Physics – Section A: Multiple Choice Questions (80 marks)

31. A torque meter is calibrated to reference standards of mass, length and time each with 5% accuracy. After calibration, the measured torque with this torque meter will have net accuracy of : [4 marks]

(A) 15%

(B) 25%

(C) 75%

(D) 5%

Correct Answer: (B) 25%

Solution:

Concept:

- Torque is force times perpendicular distance, so its dimensional formula is $[\tau] = ML^2T^{-2}$.
- For a product of powers, the maximum fractional uncertainty is the sum of the absolute powers times the fractional uncertainties of the independent reference quantities.

Step 1: Express torque using the reference standards A force has dimensions MLT^{-2}, since it is mass times acceleration. Multiplication by the moment arm introduces one more length factor. Consequently, a measured torque is proportional to

$$\tau \propto m\ell^2t^{-2}.$$

The two factors of length arise from acceleration and the moment arm; the two inverse factors of time arise from acceleration. The negative time exponent affects how torque varies but not the positive contribution of time uncertainty to the maximum error.

Step 2: Add the weighted percentage uncertainties The standard percentage-error rule gives

$$\frac{\Delta\tau}{\tau} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta\ell}{\ell} \times 100 + 2\frac{\Delta t}{t} \times 100.$$

Each reference standard has 5% uncertainty. Thus the net uncertainty is

$$5\% + 2(5\%) + 2(5\%) = 25\%.$$

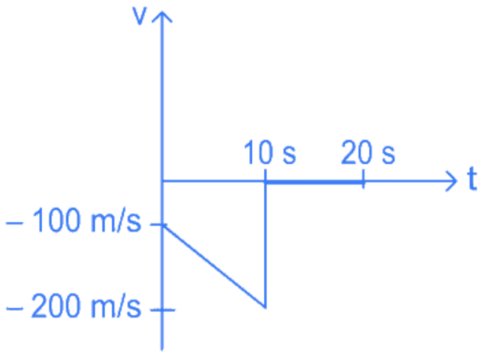
We add magnitudes because the question asks for the limiting accuracy, not a statistical root-mean-square uncertainty. Option (A), 15%, neglects the powers of length and time; option (D), 5%, incorrectly treats torque as a single basic standard. Option (C), 75%, does not follow the dimensional powers.

Step 3: Conclusion The calibrated torque has net accuracy $\boxed{25\%}$ in the percentage-error convention used here. Hence, the correct option is (B).

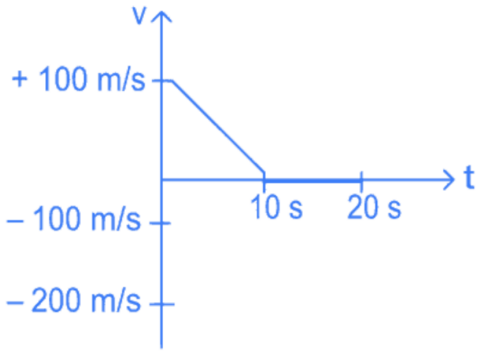
Quick Tip to Solve: Write the dimensional formula first; add percentage errors with the absolute values of the dimensional exponents as weights.

32. A bullet is shot vertically downwards with an initial velocity of $100\,\mathrm{m/s}$ from a certain height. Within $10\,\mathrm{s}$, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time curve for total time $t = 20\,\mathrm{s}$ will be: (Take $g = 10\,\mathrm{m/s^2}$.) [4 marks]

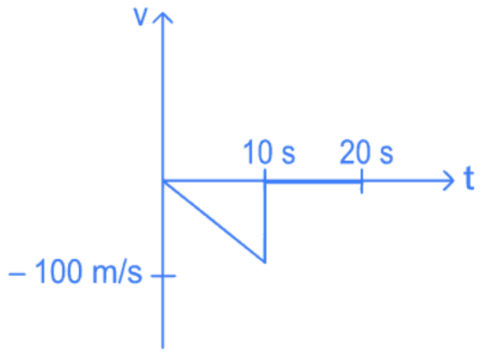
(A)



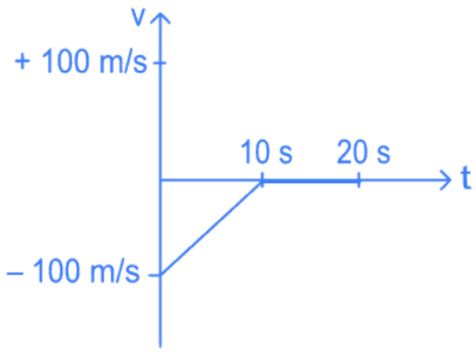
(B)



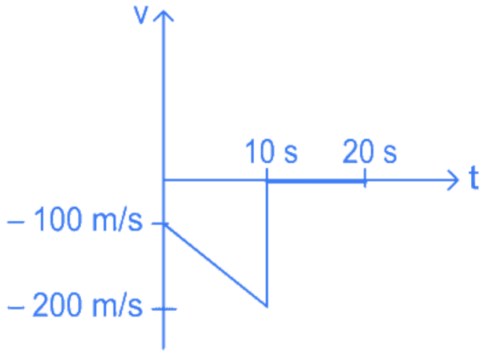
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- With upward chosen positive, a downward launch has negative initial velocity and gravitational acceleration $-g$. Constant acceleration makes the velocity-time graph a straight line.
- An instantaneous perfectly inelastic impact changes the velocity abruptly to zero. After the bullet remains at rest, its graph lies on the time axis.

Step 1: Calculate the velocity during the fall Choose upward as positive, as in the negative-velocity options. The initial velocity and acceleration are

$$u = -100\,\mathrm{m\,s^{-1}}, \qquad a = -10\,\mathrm{m\,s^{-2}}.$$

Before impact the kinematic relation gives

$$v(t) = u + at = -100 - 10t \quad (0 \le t < 10\,\mathrm{s}).$$

The velocity starts at $-100\,\mathrm{m\,s^{-1}}$, and just before the collision at $t = 10\,\mathrm{s}$ it is $-200\,\mathrm{m\,s^{-1}}$. The negative slope expresses increasing downward speed, not slowing down.

Step 2: Include the collision and the remaining interval At $t = 10\,\mathrm{s}$, the perfectly inelastic collision makes the bullet come to rest instantaneously, so the graph jumps vertically from -200 to 0. The bullet then stays on the ground and $v = 0$ for $10 < t \le 20\,\mathrm{s}$. The contact force balances weight during this rest interval; free-fall acceleration no longer applies.

Option (A) displays exactly these three features: the correct initial value, a straight fall to -200, and an abrupt return to zero followed by rest. Option (B) starts positive and slows to zero without impact.

Option (C) incorrectly starts at zero. Option (D) makes the initially downward velocity approach zero during free fall, giving the wrong acceleration direction.

Step 3: Conclusion The required curve is the straight segment from $(0, -100)$ to $(10, -200)$, followed by a jump to zero and rest until $20\,\text{s}$. Hence, the correct option is (A).

Quick Tip to Solve: For a fall followed by impact, draw the free-fall line only up to contact; show the instantaneous velocity jump separately from the later rest interval.

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33. Sand is being dropped from a stationary dropper at a rate of $0.5\,\text{kgs}^{-1}$ on a conveyor belt moving with a velocity of $5\ \text{ms}^{-1}$. The power needed to keep the belt moving with the same velocity will be : [4 marks]

(A) 1.25 W

(B) 2.5 W

(C) 6.25 W

(D) 12.5 W

Correct Answer: (D) 12.5 W

Solution:

Concept:

- An incoming mass stream with no horizontal velocity must acquire the belt velocity. The force on the stream equals the rate of change of its horizontal momentum.
- The motor power at fixed belt speed is $P = Fv$, where F compensates the backward reaction exerted by the incoming material.

Step 1: Find the momentum supplied per second Let $\dot{m} = 0.5\,\text{kg}\,\text{s}^{-1}$ and belt speed $v = 5\,\text{m}\,\text{s}^{-1}$. In a short time Δt, the deposited mass is $\Delta m = \dot{m}\Delta t$. The stationary dropper supplies no horizontal momentum to the sand. On joining the belt this mass acquires horizontal momentum $\Delta m\, v$. Thus

$$F = \frac{\Delta m\, v}{\Delta t} = \dot{m}v = 0.5 \times 5 = 2.5\,\text{N}.$$

The belt exerts this forward force on the sand, and the sand exerts an equal backward force on the belt.

Step 2: Calculate the motor power The motor must supply a forward force of $2.5\,\text{N}$ to keep the belt speed unchanged. Its required power is

$$P = Fv = \dot{m}v^2 = 0.5 \times 5^2 = 12.5\,\text{W}.$$

The sand gains kinetic energy at only $\frac{1}{2}\dot{m}v^2 = 6.25\,\text{W}$; an equal rate is dissipated while the newly arriving sand slips and comes to the belt speed. Therefore option (C) counts only the kinetic-energy increase and omits the dissipative part. Option (B) has the numerical value of the force, not the power, and option (A) also fails $P = Fv$. The dropper's vertical motion does not supply the needed horizontal momentum.

Step 3: Conclusion The required power is $\boxed{12.5\,\mathrm{W}}$. Hence, the correct option is (D).

Quick Tip to Solve: For material landing on a moving belt, use momentum flux to find force and then multiply by belt speed; the kinetic-energy rate alone misses slipping losses.

34. A bag is gently dropped on a conveyor belt moving at a speed of $2\ \mathrm{m/s}$. The coefficient of friction between the conveyor belt and bag is 0.4. Initially, the bag slips on the belt before it stops due to friction. The distance travelled by the bag on the belt during slipping motion, is : [Take $\mathrm{g} = 10\ \mathrm{m/s^{-2}}$] [4 marks]

(A) 2 m

(B) 0.5 m

(C) 3.2 m

(D) 0.8 ms

Correct Answer: (B) 0.5 m

Solution:

Concept:

- While the bag slips, kinetic friction accelerates it in the direction of belt motion with magnitude $a = \mu g$.
- Slipping ends when the bag and belt have equal velocities. The bag stops moving relative to the belt, while it continues moving relative to the ground.

Step 1: Determine the acceleration and slipping time The bag is gently dropped, so its initial horizontal ground velocity is zero. Its weight is balanced vertically by the normal force, $N = mg$. During slipping the horizontal force is $f = \mu N$, giving

$$a = \frac{f}{m} = \mu g = 0.4 \times 10 = 4\,\mathrm{m\,s^{-2}}.$$

Friction accelerates the bag rather than stopping its ground motion. The slipping terminates when its speed reaches the belt speed $2\,\mathrm{m\,s^{-1}}$. Therefore

$$t = \frac{v - u}{a} = \frac{2 - 0}{4} = 0.5\,\mathrm{s}.$$

Step 2: Calculate the distance covered while slipping Using the constant-acceleration equation in the ground frame,

$$s = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(4)(0.5)^2 = 0.5\,\mathrm{m}.$$

Equivalently, $v^2 = u^2 + 2as$ gives $s = 4/8 = 0.5\,\mathrm{m}$. The belt travels $2 \times 0.5 = 1\,\mathrm{m}$ during this interval, so the distance of relative slipping is also $1 - 0.5 = 0.5\,\mathrm{m}$ in this particular case. Thus either natural reading of the distance during slipping gives the same numerical choice.

Option (A) incorrectly equates the speed's numerical value with a distance. Option (C) does not satisfy the kinematic relation. Option (D) has the printed unit ms, rather than a unit of distance, and does not match the computed result.

Step 3: Conclusion The distance travelled during slipping is $\boxed{0.5\,\text{m}}$. Hence, the correct option is (B).

Quick Tip to Solve: First find the time needed to reach belt speed; the phrase "stops slipping" means zero relative velocity, not zero ground velocity.

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35. Two cylindrical vessels of equal cross-sectional area $16\ \text{cm}^2$ contain water upto heights $100\ \text{cm}$ and $150\ \text{cm}$ respectively. The vessels are interconnected so that the water levels in them become equal. The work done by the force of gravity during the process, is [Take, density of water $= 10^3\ \text{kg/m}^3$ and $g = 10\ \text{ms}^{-2}$] : [4 marks]

(A) 0.25 J

(B) 1 J

(C) 8 J

(D) 12 J

Correct Answer: (B) 1 J

Solution:

Concept:

- A uniform cylindrical liquid column of area A, density ρ and height h has mass ρAh and centre of mass at $h/2$, so $U = \rho Agh^2/2$.
- Gravity does positive work equal to the decrease in gravitational potential energy. Equal cross-sectional areas make the final common height the arithmetic mean of the initial heights.

Step 1: Find the equilibrium level and convert units Take the common vessel bottoms as the zero of gravitational potential. Conservation of water volume gives

$$A(1.00) + A(1.50) = 2Ah_f, \qquad h_f = 1.25\,\text{m}.$$

The area conversion is essential:

$$A = 16\,\text{cm}^2 = 16 \times 10^{-4}\,\text{m}^2 = 0.0016\,\text{m}^2.$$

The initial and final energies must include both vessels. It is incorrect to treat all the water as concentrated at its upper surface; each uniform column's centre of mass is halfway up.

Step 2: Subtract the final energy from the initial energy For the two initial columns and the two final columns,

$$U_i = \frac{\rho Ag}{2}(h_1^2 + h_2^2), \qquad U_f = \frac{\rho Ag}{2}(2h_f^2).$$

Hence gravity does work

$$\begin{aligned} W_g &= U_i - U_f \\ &= \frac{1000(0.0016)(10)}{2}[1^2 + 1.5^2 - 2(1.25)^2] \\ &= 8(3.25 - 3.125) = 1\,\text{J}. \end{aligned}$$

The positive sign is consistent with water moving from the higher to the lower vessel. Option (C), $8\,\mathrm{J}$, is merely the prefactor before the height-squared difference; options (A) and (D) do not equal the loss of potential energy. No extra external work is needed in calculating the work of gravity.

Step 3: Conclusion The work done by gravity is $\boxed{1\,\mathrm{J}}$. Hence, the correct option is (B).

Quick Tip to Solve: For equal-area vessels use $W_g = \rho A g(h_2 - h_1)^2/4$, obtained by conserving volume and comparing the two column energies.

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36. Two satellites A and B, having masses in the ratio $4:3$, are revolving in circular orbits of radii $3\mathrm{r}$ and $4\mathrm{r}$ respectively around the earth. The ratio of total mechanical energy of A to B is : [4 marks]

(A) 9 : 16

(B) 16 : 9

(C) 1 : 1

(D) 4 : 3

Correct Answer: (B) 16 : 9

Solution:

Concept:

- For a satellite of mass m in a circular orbit of radius R about Earth, the gravitational potential energy is $U = -GMm/R$, taking zero potential energy at infinity.
- The circular-orbit condition gives $K = GMm/(2R)$. Thus the total mechanical energy is $E = K + U = -GMm/(2R)$, where M is the mass of Earth.

Step 1: Express each orbital energy Write the masses as $m_A = 4m_0$ and $m_B = 3m_0$, where m_0 is a common mass factor. The orbital radii are measured from the centre of Earth: $R_A = 3r$ and $R_B = 4r$. The energy of either satellite is negative because its circular orbit is gravitationally bound. Substitution gives

$$E_A = -\frac{GM(4m_0)}{2(3r)} = -\frac{2GMm_0}{3r}, \qquad E_B = -\frac{GM(3m_0)}{2(4r)} = -\frac{3GMm_0}{8r}.$$

Step 2: Take the ratio and compare the options The common factors, including the two negative signs, cancel when the ratio is formed:

$$\frac{E_A}{E_B} = \frac{m_A}{m_B}\frac{R_B}{R_A} = \frac{4}{3}\frac{4}{3} = \frac{16}{9}.$$

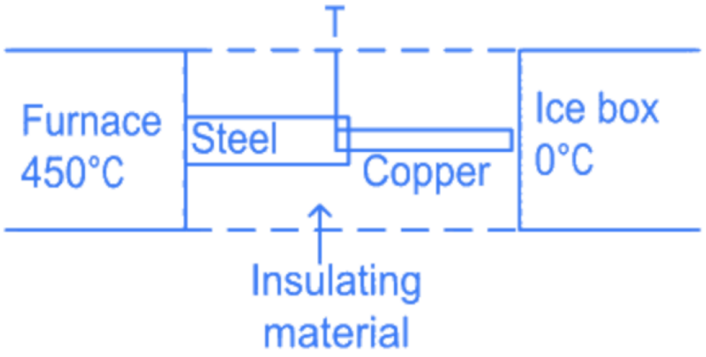
The ratio $9:16$ reverses the required order. The value $4:3$ accounts only for the masses and ignores the different radii; $1:1$ incorrectly treats total energy as independent of satellite mass. Although the ratio is positive, neither individual total energy is positive.

Step 3: Conclusion The ratio of total mechanical energies is $E_A : E_B = 16 : 9$. Hence, the correct option is (B).

Quick Tip to Solve: For two circular satellites about the same planet, use $E_A/E_B = (m_A/m_B)(R_B/R_A)$; total energy scales as mass divided by orbital radius.

---

37. If K_1 and K_2 are the thermal conductivities, L_1 and L_2 are the lengths and A_1 and A_2 are the cross sectional areas of steel and copper rods respectively such that $\frac{K_2}{K_1} = 9, \frac{A_1}{A_2} = 2, \frac{L_1}{L_2} = 2$. Then, for the arrangement as shown in the figure, the value of temperature T of the steel - copper junction in the steady state will be: [4 marks]



(A) $18^\circ C$

(B) $14^\circ C$

(C) $45^\circ C$

(D) $150^\circ C$

Correct Answer: (C) $45^\circ C$

Solution:

Concept:

- In steady conduction, Fourier's law gives the heat current $\dot{Q} = KA\Delta T/L$. The thermal conductance is $G = KA/L$.
- For insulated rods connected in series, no heat accumulates at the junction or escapes through the sides. Therefore the heat current through the steel equals that through the copper.

Step 1: Compare the thermal conductances The figure puts the steel rod between the furnace at $450^\circ C$ and the junction at T, and the copper rod between the junction and the ice box at $0^\circ C$. With subscript 1 for steel and 2 for copper,

$$\frac{G_2}{G_1} = \frac{K_2}{K_1}\frac{A_2}{A_1}\frac{L_1}{L_2} = 9 \times \frac{1}{2} \times 2 = 9.$$

Thus the copper conducts nine times as much heat for the same temperature difference. Its temperature drop must therefore be one-ninth of the steel drop in the steady state.

Step 2: Equate the heat currents and find the junction temperature Conservation of energy at the junction gives

$$G_1(450 - T) = G_2(T - 0) = 9G_1T.$$

After cancelling G_1, solve $450 - T = 9T$, so $10T = 450$ and $T = 45^\circ C$. The drops are $405^\circ C$ across steel and $45^\circ C$ across copper, whose ratio is indeed $9 : 1$. Substituting 18, 14, or $150^\circ C$ would not satisfy the same-current equation, so the other three options fail.

Step 3: Conclusion The steel–copper junction is at $45^\circ C$. Hence, the correct option is (C).

Quick Tip to Solve: In rods connected in series, temperature drops are proportional to thermal resistances $L/(KA)$, not to thermal conductivities alone.

38. Read the following statements :

A. When small temperature difference between a liquid and its surrounding is doubled, the rate of loss of heat of the liquid becomes twice.

B. Two bodies P and Q having equal surface areas are maintained at temperature $10^{\circ}\mathrm{C}$ and $20^{\circ}\mathrm{C}$. The thermal radiation emitted in a given time by P and Q are in the ratio $1 : 1.15$.

C. A Carnot Engine working between 100 K and 400 K has an efficiency of 75%.

D. When small temperature difference between a liquid and its surrounding is quadrupled, the rate of loss of heat of the liquid becomes twice.

Choose the correct answer from the options given below : [4 marks]

(A) A, B, C only

(B) A, B only

(C) A, C only

(D) B, C, D only

Correct Answer: (A) A, B, C only

Solution:

Concept:

- Newton's law of cooling gives a heat-loss rate proportional to the small excess temperature ΔT above the surroundings, with other conditions unchanged.
- Stefan–Boltzmann radiation is $P = e\sigma AT^4$, using absolute temperature. For equal emitting areas and the same emissivity, emitted energies in the same time have ratio $T_1^4 : T_2^4$.
- The efficiency of a reversible Carnot engine is $\eta = 1 - T_c/T_h$, with both reservoir temperatures in kelvin.

Step 1: Test the two cooling statements For a small temperature difference, write the heat-loss rate as $H = C\Delta T$, where C is constant for the given surface and surroundings. Doubling ΔT doubles H, so A is true. Quadrupling ΔT makes the heat-loss rate four times its original value, rather than twice. Thus D is false. This linear relation applies to small temperature differences, as explicitly specified in the question.

Step 2: Test the radiation statement Convert the Celsius temperatures to kelvin before taking the fourth power: $T_P = 283\,\mathrm{K}$ and $T_Q = 293\,\mathrm{K}$. Under the common-emissivity comparison used here,

$$E_P : E_Q = 283^4 : 293^4 = 1 : \left(\frac{293}{283}\right)^4 \simeq 1 : 1.149.$$

This is $1 : 1.15$ to the stated precision, so B is true. A ratio based directly on 10 and 20 degrees Celsius would be physically incorrect.

Step 3: Test the Carnot statement and select the combination The cold reservoir is at $100\,\mathrm{K}$, while the hot reservoir is at $400\,\mathrm{K}$. Therefore

$$\eta = 1 - \frac{100}{400} = \frac{3}{4} = 75\%.$$

Statement C is true. Consequently the true statements are A, B, and C. Option B omits the true C,

option C omits the true B, and option D includes the false D while omitting the true A.

Step 4: Conclusion Statements A, B and C are correct, while D is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Use kelvin in both T^4 radiation ratios and Carnot efficiency; for small-temperature cooling, scale the rate directly with the excess temperature.

39. Same gas is filled in two vessels of the same volume at the same temperature. If the ratio of the number of molecules is $1:4$, then

A. The r.m.s. velocity of gas molecules in two vessels will be the same.

B. The ratio of pressure in these vessels will be $1:4$.

C. The ratio of pressure will be $1:1$.

D. The r.m.s. velocity of gas molecules in two vessels will be in the ratio of $1:4$.

Choose the correct answer from the options given below : [4 marks]

(A) A and C only

(B) B and D only

(C) A and B only

(D) C and D only

Correct Answer: (C) A and B only

Solution:

Concept:

- For molecules of mass m, the root-mean-square speed is $v_{\text{rms}} = \sqrt{3k_BT/m}$. It depends on temperature and molecular mass, not on the total number of molecules.
- The ideal-gas equation in molecular form is $PV = Nk_BT$, where N is the molecule count, V is volume and k_B is Boltzmann's constant.

Step 1: Compare the root-mean-square speeds The two vessels contain the same gas, so each molecule has the same mass m in both vessels. The temperature T is also identical. Substituting into the speed formula gives

$$\frac{v_{\text{rms},1}}{v_{\text{rms},2}} = \sqrt{\frac{T_1}{T_2}} = 1.$$

Adding more molecules at fixed temperature does not change the mean translational kinetic energy per molecule, which remains $3k_BT/2$. Thus statement A is true and statement D, which assigns the count ratio to the speeds, is false.

Step 2: Compare pressures and select the true pair At equal volume and temperature, the molecular ideal-gas equation makes pressure proportional to molecule count:

$$\frac{P_1}{P_2} = \frac{N_1k_BT/V}{N_2k_BT/V} = \frac{N_1}{N_2} = \frac{1}{4}.$$

Therefore statement B is true and statement C is false. The vessel with four times the molecules has

four times as many molecular impacts per unit area per unit time, while the typical molecular speed remains the same. Option A contains false C, option B contains false D, and option D contains both false statements. Only the pair A and B survives.

Step 3: Conclusion The r.m.s. speeds are equal and the pressures have ratio $1:4$; statements A and B are correct. Hence, the correct option is (C).

Quick Tip to Solve: Separate per-molecule quantities from bulk quantities: at fixed temperature the molecular speed stays fixed, while pressure at fixed volume scales with molecule count.

40. Two identical positive charges Q each are fixed at a distance of '2a' apart from each other. Another point charge q_0 with mass 'm' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be : [4 marks]

(A) $\sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

(B) $\sqrt{\frac{q_0 Q}{4\pi^3\varepsilon_0 ma^3}}$

(C) $\sqrt{\frac{2\pi^2\varepsilon_0 ma^3}{q_0 Q}}$

(D) $\sqrt{\frac{8\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

Correct Answer: (A) $\sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

Solution:

Concept:

- Coulomb's force between point charges is kQq_0/r^2, with $k = 1/(4\pi\varepsilon_0)$. Forces from the two fixed charges cancel at their midpoint.
- Simple harmonic motion requires a restoring force $F = -m\omega^2 x$ for a small displacement x. The period is $T = 2\pi/\omega$.

Step 1: Find the force for a small displacement Place the fixed charges at coordinates $-a$ and $+a$. Displace q_0 a small distance x to the right. For positive q_0, the nearer right charge pushes it more strongly to the left, providing the stated restoring motion. The signed net force is

$$F = kQq_0\left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2}\right].$$

Using $(1 \pm x/a)^{-2} \simeq 1 \mp 2x/a$, valid when $|x| \ll a$, gives

$$F \simeq \frac{kQq_0}{a^2}\left(-\frac{4x}{a}\right) = -\frac{4kQq_0}{a^3}x.$$

Step 2: Identify the angular frequency and period Compare this restoring force with $-m\omega^2 x$:

$$\omega^2 = \frac{4kQq_0}{ma^3} = \frac{Qq_0}{\pi\varepsilon_0 ma^3}.$$

It follows that

$$T = 2\pi\sqrt{\frac{\pi\varepsilon_0 ma^3}{Qq_0}} = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{Qq_0}}.$$

The restoring-motion premise requires $Qq_0 > 0$; a negative test charge would be unstable along this line. Option B is the reciprocal of the period expression and has inverse-time dimensions. Options C and D have incorrect numerical factors from the small-displacement force.

Step 3: Conclusion The time period is $T = \sqrt{4\pi^3\varepsilon_0 ma^3/(q_0 Q)}$. Hence, the correct option is (A).

Quick Tip to Solve: Expand the net force to first order in displacement before using the SHM formula; each of the two charge forces contributes to the restoring coefficient.

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41. Two sources of equal emfs are connected in series. This combination is connected to an external resistance R. The internal resistances of the two sources are r_1 and r_2 $(r_1 > r_2)$. If the potential difference across the source of internal resistance r_1 is zero, then the value of R will be : [4 marks]

(A) $r_1 - r_2$

(B) $\frac{r_1 r_2}{r_1 + r_2}$

(C) $\frac{r_1 + r_2}{2}$

(D) $r_2 - r_1$

Correct Answer: (A) $r_1 - r_2$

Solution:

Concept:

- For a cell of emf $\mathcal{E}$ supplying current I, its terminal potential difference is $V = \mathcal{E} - Ir$, where r is its internal resistance.
- Two equal emfs connected in series aiding give total emf $2\mathcal{E}$, with the internal resistances added. The circuit current is $I = 2\mathcal{E}/(R + r_1 + r_2)$.

Step 1: Use the zero terminal-voltage condition The same current flows through both sources and the external resistor because the components form a series circuit. For the source with the larger internal resistance, the condition given is

$$V_1 = \mathcal{E} - Ir_1 = 0.$$

Thus $I = \mathcal{E}/r_1$: its entire emf is balanced by its internal voltage drop. Zero terminal voltage does not mean that either the emf or current is zero. The second source continues to supply a positive terminal voltage because $r_2 < r_1$.

Step 2: Combine this current with the complete loop equation Apply the loop equation with both emfs in series:

$$I(R + r_1 + r_2) = 2\mathcal{E}.$$

Substitute $I = \mathcal{E}/r_1$, cancel the nonzero emf, and rearrange:

$$R + r_1 + r_2 = 2r_1, \qquad R = r_1 - r_2.$$

This is positive, as required by $r_1 > r_2$. As a check, the other source has terminal voltage $\mathcal{E}(1 - r_2/r_1)$, exactly equal to IR. Option D is negative; option B is a parallel-resistance expression unrelated to this series condition; option C does not in general give zero terminal voltage for the first source.

Step 3: Conclusion The required external resistance is $R = r_1 - r_2$. Hence, the correct option is (A).

Quick Tip to Solve: Write a terminal-voltage equation for the specified cell first; zero terminal voltage fixes the common series current as its emf divided by its internal resistance.

---

42. A direct current of 4 A and an alternating current of peak value 4 A flow through resistance of $3\,\Omega$ and $2\,\Omega$ respectively. The ratio of heat produced in the two resistances in same interval of time will be : [4 marks]

(A) 3 : 2

(B) 3 : 1

(C) 3 : 4

(D) 4 : 3

Correct Answer: (B) 3 : 1

Solution:

Concept:

- Joule heat in a resistance carrying a constant current for time t is $H = I^2Rt$. For an alternating current, use the mean-square current over complete cycles.
- For a sinusoidal alternating current of peak value I_0, $I_{\text{rms}} = I_0/\sqrt{2}$, so its average heating power is $I_0^2R/2$.

Step 1: Calculate heating by the direct current The first resistor has resistance $R_1 = 3\,\Omega$ and carries a constant current $I_{\text{dc}} = 4\,\text{A}$. Its power is constant:

$$P_1 = I_{\text{dc}}^2 R_1 = 4^2 \times 3 = 48\,\text{W}.$$

During the stated time interval t, the heat released is therefore $H_1 = 48t$ joules when t is expressed in seconds. This current value is already the heating-equivalent current; it needs no peak-to-rms conversion.

Step 2: Calculate the alternating-current heating and take the ratio The second current has peak value $4\,\text{A}$, so its rms value is $4/\sqrt{2}\,\text{A}$. For the $2\,\Omega$ resistor,

$$P_2 = I_{\text{rms}}^2 R_2 = \left(\frac{4}{\sqrt{2}}\right)^2 \times 2 = 16\,\text{W}.$$

Thus $H_2 = 16t$ joules for the same interval, and

$$H_1 : H_2 = 48t : 16t = 3 : 1.$$

Option A, $3 : 2$, incorrectly uses the peak AC current as if it were the rms current. Options C and D

also fail the mean-square-current calculation. The comparison uses the ordinary AC heating average over complete cycles.

Step 3: Conclusion The ratio of heat produced in the DC resistor to that in the AC resistor is $3 : 1$. Hence, the correct option is (B).

Quick Tip to Solve: For heating comparisons, replace a sinusoidal peak current by $I_0/\sqrt{2}$ before squaring; equal DC and peak AC currents do not produce equal mean-square currents.

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43. A beam of light travelling along X-axis is described by the electric field $E_y = 900 \sin \omega(\mathrm{t} - x/c)$. The ratio of electric force to magnetic force on a charge q moving along Y-axis with a speed of 3×10^7 ms^{-1} will be :

(Given speed of light $= 3 \times 10^8$ ms^{-1}) [4 marks]

(A) 1 : 1

(B) 1 : 10

(C) 10 : 1

(D) 1 : 2

Correct Answer: (C) 10 : 1

Solution:

Concept:

- In a plane electromagnetic wave, the electric and magnetic fields are mutually perpendicular and satisfy $E = cB$, where c is the speed of light.
- The electric force magnitude is $F_E = |q|E$; the magnetic force magnitude is $F_B = |q|vB \sin\theta$, where θ is the angle between velocity and magnetic field.

Step 1: Identify the field and velocity directions The wave travels along the positive X-axis and its electric field is along the Y-axis. Therefore its magnetic field is along the Z-axis, with the sign varying with the wave phase. The charge moves along Y, so its velocity is perpendicular to the magnetic field. Thus $\sin\theta = 1$. The charge also experiences an electric force along the electric-field direction or its opposite, depending on its sign; this sign does not affect the ratio of magnitudes.

Step 2: Cancel the common charge and wave amplitude At the same position and instant, the two field magnitudes obey $B = E/c$. Consequently, away from the instants at which both fields vanish,

$$\frac{F_E}{F_B} = \frac{|q|E}{|q|vB} = \frac{E}{v(E/c)} = \frac{c}{v}.$$

The amplitude 900 and the sine factor cancel; it is unnecessary to know the frequency or the charge value. Substituting the given speeds gives

$$\frac{F_E}{F_B} = \frac{3 \times 10^8}{3 \times 10^7} = 10.$$

A ratio of unity would require $v = c$. The ratios $1 : 10$ and $1 : 2$ would incorrectly make the magnetic

force larger despite $v < c$.

Step 3: Conclusion The electric-force to magnetic-force ratio is $10 : 1$. Hence, the correct option is (C).

Quick Tip to Solve: For a charge moving perpendicular to the magnetic field of a plane light wave, use $F_E/F_B = c/v$; the wave amplitude cancels.

44. A microscope was initially placed in air (refractive index 1). It is then immersed in oil (refractive index 2). For a light whose wavelength in air is λ, calculate the change of microscope's resolving power due to oil and choose the correct option. [4 marks]

(A) Resolving power will be $\frac{1}{4}$ in the oil than it was in the air.

(B) Resolving power will be twice in the oil than it was in the air.

(C) Resolving power will be four times in the oil than it was in the air.

(D) Resolving power will be $\frac{1}{2}$ in the oil than it was in the air.

Correct Answer: (B) Resolving power will be twice in the oil than it was in the air.

Solution:

Concept:

- A microscope resolves two close points when their diffraction images can be distinguished. Its limiting separation is proportional to $\lambda/(\mu \sin\alpha)$, where λ is the wavelength in air, μ is the immersion-medium refractive index, and α is the objective half-angle.
- Resolving power is the reciprocal of limiting separation. At fixed objective aperture and air wavelength, it is proportional to the numerical aperture $\mu \sin\alpha$.

Step 1: Write the ratio for the two media Using the usual microscope expression,

$$d_{\min} = \frac{1.22\lambda}{2\mu \sin\alpha}, \qquad R = \frac{1}{d_{\min}} = \frac{2\mu \sin\alpha}{1.22\lambda}.$$

The same microscope and illumination are used in both cases. Hence λ, α, and the numerical diffraction factor cancel in the ratio. We must compare resolving power, not the minimum separation itself.

Step 2: Substitute the refractive indices and compare the options With $\mu_{\text{air}} = 1$ and $\mu_{\text{oil}} = 2$,

$$\frac{R_{\text{oil}}}{R_{\text{air}}} = \frac{\mu_{\text{oil}}}{\mu_{\text{air}}} = \frac{2}{1} = 2.$$

Equivalently, the wavelength inside the oil is $\lambda/2$, so the diffraction-limited separation is halved and its reciprocal is doubled. These are two descriptions of the same refractive-index effect; multiplying them together would count the effect twice. Thus the factor four in (C) is incorrect. Options (A) and (D) predict a reduction in resolving power, whereas immersion increases it. Only (B) has the correct direction and factor.

Step 3: Conclusion The microscope has twice its original resolving power in oil. Hence, the correct

option is (B).

Quick Tip to Solve: Keep the wavelength convention consistent: use air wavelength with numerical aperture, or medium wavelength, and count the refractive-index factor only once.

---

45. An electron (mass m) with an initial velocity $\vec{v} = v_0\hat{i}\,(v_0 > 0)$ is moving in an electric field $\vec{E} = -E_0\hat{i}\,(E_0 > 0)$ where E_0 is constant. If at t = 0 de Broglie wavelength is $\lambda_0 = \frac{h}{mv_0}$, then its de Broglie wavelength after time t is given by [4 marks]

(A) λ_0

(B) $\lambda_0\left(1 + \frac{eE_0t}{mv_0}\right)$

(C) $\lambda_0 t$

(D) $\frac{\lambda_0}{\left(1+\frac{eE_0t}{mv_0}\right)}$

Correct Answer: (D) $\frac{\lambda_0}{\left(1+\frac{eE_0t}{mv_0}\right)}$

Solution:

Concept:

- The force on a charge in a uniform electric field is $\vec{F} = q\vec{E}$. For an electron, $q = -e$, where $e > 0$ is the elementary charge.
- The de Broglie relation is $\lambda = h/p$. With the nonrelativistic momentum convention specified by $\lambda_0 = h/(mv_0)$, this becomes $\lambda = h/(mv)$.

Step 1: Determine the direction of acceleration The electric field points along negative X, but the electron has negative charge. Therefore its force is along positive X:

$$\vec{F} = (-e)(-E_0\hat{i}) = eE_0\hat{i}, \qquad \vec{a} = \frac{eE_0}{m}\hat{i}.$$

This acceleration is in the same direction as the initial velocity $v_0\hat{i}$. The electron speeds up; it does not slow down or reverse direction for positive time.

Step 2: Calculate the new momentum and wavelength Because the field is constant, its impulse adds linearly to the initial momentum:

$$p(t) = mv_0 + eE_0t.$$

Using this in the de Broglie relation and factoring out mv_0,

$$\lambda(t) = \frac{h}{mv_0 + eE_0t} = \frac{h/(mv_0)}{1 + eE_0t/(mv_0)} = \frac{\lambda_0}{1 + eE_0t/(mv_0)}.$$

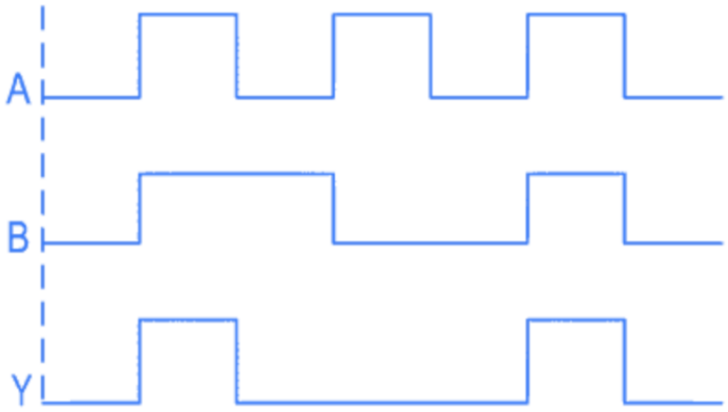
The dimensionless denominator is greater than one for $t > 0$, so the wavelength decreases. Option (A) misses the increase in momentum. Option (B) predicts the opposite trend. Option (C) also has the wrong dimensions, since multiplying a wavelength by time cannot give a wavelength; it fails the initial-value check as well. The result in (D) recovers λ_0 at $t = 0$.

Step 3: Conclusion The wavelength is $\lambda(t) = \dfrac{\lambda_0}{1 + eE_0t/(mv_0)}$. Hence, the correct option is (D).

Quick Tip to Solve: Apply the negative electron charge before deciding whether the field accelerates it. Then use $p = p_0 + Ft$ directly in $\lambda = h/p$.

46. A logic gate circuit has two inputs A and B and output Y. The voltage waveforms of A, B and Y are shown below.

The logic gate circuit is : [4 marks]



(A) AND gate

(B) OR gate

(C) NOR gate

(D) NAND gate

Correct Answer: (A) AND gate

Solution:

Concept:

- A digital voltage waveform is read as logic 1 at its high level and logic 0 at its low level. The inputs must be compared at the same time, away from switching edges.
- An AND gate gives $Y = A \cdot B$: its output is high only when both inputs are high. OR, NOR and NAND have different truth tables.

Step 1: Read the simultaneous input-output states In the initial interval, both inputs are low and the output is low, giving $(A, B, Y) = (0, 0, 0)$. In the first interval where both input pulses are high, the output is high, giving $(1, 1, 1)$. Next, A falls while B remains high; Y becomes low, giving $(0, 1, 0)$. Later A is high while B is low, yet Y remains low, giving $(1, 0, 0)$. The later overlapping pulses reproduce the high output only for the shared high interval.

Step 2: Match the observed states to the gate rules The four observed cases are

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

This is precisely the AND truth table. An OR gate would give a high output during either of the mixed-input intervals $(0, 1)$ and $(1, 0)$, contrary to the graph. A NOR gate would give high output when both inputs are initially low, whereas the graph shows low. A NAND gate would give low output when both inputs are high, which reverses the observed overlapping-pulse output. Thus all three competing gates are excluded by explicit waveform intervals; the conclusion does not depend on how the switching instants themselves are drawn.

Step 3: Conclusion The output is high exactly when both inputs are high, so $Y = A \cdot B$. The circuit is an AND gate. Hence, the correct option is (A).

Quick Tip to Solve: Compare constant-level intervals, not vertical switching lines. A high output only during the overlap of two high input pulses identifies AND.

47. Two bar magnets oscillate in a horizontal plane in earth's magnetic field with time periods of 3 s and 4 s respectively. If their moments of inertia are in the ratio of $3 : 2$, then the ratio of their magnetic moments will be: [4 marks]

(A) 2 : 1

(B) 8 : 3

(C) 1 : 3

(D) 27 : 16

Correct Answer: (B) 8 : 3

Solution:

Concept:

- A magnetic dipole of moment M in a uniform horizontal field B_H experiences restoring torque $\tau = -MB_H \sin\theta$. For small angular oscillations, $\sin\theta \simeq \theta$.
- The small-oscillation period is $T = 2\pi\sqrt{I/(MB_H)}$, where I is the moment of inertia about the suspension axis. Both magnets experience the same horizontal component of the earth's field.

Step 1: Express magnetic moment in terms of period For small displacements, the rotational equation is $I\ddot{\theta} = -MB_H\theta$, so the angular frequency obeys $\omega^2 = MB_H/I$. Rearranging the corresponding period relation gives

$$T^2 = \frac{4\pi^2 I}{MB_H}, \qquad M = \frac{4\pi^2 I}{B_H T^2}.$$

Thus a magnetic moment is proportional to I/T^2, not to IT^2. A shorter period at a given inertia corresponds to a stronger restoring torque and a larger magnetic moment.

Step 2: Take the ratio and substitute the data Let magnet 1 have period $T_1 = 3\,\text{s}$ and magnet 2 have period $T_2 = 4\,\text{s}$. The common field and $4\pi^2$ cancel:

$$\frac{M_1}{M_2} = \frac{I_1}{I_2}\frac{T_2^2}{T_1^2} = \frac{3}{2}\frac{4^2}{3^2} = \frac{48}{18} = \frac{8}{3}.$$

Both inertia and squared-period factors are dimensionless in this ratio. Option (D), $27 : 16$, does not equal the required ratio; reversing the squared-period ratio would instead produce $27 : 32$. Options (A) and (C) also fail the required I/T^2 dependence. The first magnet has both the larger inertia and the shorter period, so its moment must exceed that of the second; the calculated factor $8/3$ satisfies this physical check.

Step 3: Conclusion The ratio of the magnetic moments is $M_1 : M_2 = 8 : 3$. Hence, the correct option is (B).

Quick Tip to Solve: For two magnets in the same field, use $M_1/M_2 = (I_1/I_2)(T_2/T_1)^2$. Reverse the period ratio before squaring it.

48. A magnet hung at 45° with magnetic meridian makes an angle of 60° with the horizontal. The actual value of the angle of dip is - [4 marks]

(A) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$

(B) $\tan^{-1}(\sqrt{6})$

(C) $\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$

(D) $\tan^{-1}\left(\sqrt{\frac{1}{2}}\right)$

Correct Answer: (A) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$

Solution:

Concept:

- The true dip δ is measured in the magnetic meridian, where $\tan\delta = B_V/B_H$. Here B_V and B_H are the vertical and horizontal components of the earth's magnetic field.
- In a vertical plane making azimuth angle θ with the magnetic meridian, the horizontal field component in that plane is $B_H\cos\theta$. The vertical component remains B_V.

Step 1: Relate the observed dip to the true dip The magnet is constrained to a plane at $\theta = 45^\circ$ to the magnetic meridian. Its observed inclination $\delta_a = 60^\circ$ is therefore an apparent dip. In that plane,

$$\tan\delta_a = \frac{B_V}{B_H\cos\theta} = \frac{\tan\delta}{\cos\theta}.$$

Rearranging gives $\tan\delta = \tan\delta_a\cos\theta$. Multiplication by $\cos\theta$, rather than division, is essential because we are recovering the full horizontal component from its smaller projection.

Step 2: Insert the two angles and test the alternatives Using the exact trigonometric values,

$$\tan\delta = \tan 60^\circ \cos 45^\circ = \sqrt{3}\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{3}{2}}.$$

Thus $\delta = \tan^{-1}\sqrt{3/2}$. The true dip must be smaller than the apparent 60°, since the latter uses a reduced horizontal component. Option (B), with $\sqrt{6}$, results from dividing by $\cos 45^\circ$ and predicts an inclination larger than the observed one. Options (C) and (D) give different component ratios and do not satisfy $\tan\delta_a = \tan\delta/\cos 45^\circ$. Direct substitution of option (A) restores $\tan\delta_a = \sqrt{3}$, as required.

Step 3: Conclusion The actual angle of dip is $\delta = \tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: To recover true dip from an apparent dip, multiply the apparent tangent by the cosine of the plane's angle to the magnetic meridian.

49. What is the half-life period of a radioactive material if its activity drops to $1/16^{\text{th}}$ of its initial value in 30 years? [4 marks]

(A) 9.5 years

(B) 8.5 years

(C) 7.5 years

(D) 10.5 years

Correct Answer: (C) 7.5 years

Solution:

Concept:

- The activity of a radioactive sample is proportional to its number of undecayed nuclei. It follows $A(t) = A_0 e^{-kt}$, where k is the decay constant.
- After each half-life $T_{1/2}$, the activity is halved. Therefore $A(t)/A_0 = (1/2)^{t/T_{1/2}}$; repeated half-lives multiply the remaining fractions.

Step 1: Count the number of half-lives in the observed reduction The remaining activity is $A/A_0 = 1/16$, not the fraction that has decayed. Write this remaining fraction as a power of one half:

$$\frac{1}{16} = \frac{1}{2^4} = \left(\frac{1}{2}\right)^4.$$

The sequence is A_0, $A_0/2$, $A_0/4$, $A_0/8$, and $A_0/16$. Hence the elapsed interval contains four half-lives. It would be incorrect to divide the time by sixteen, because decay is exponential rather than a fixed linear loss per year.

Step 2: Divide the elapsed time by the number of half-lives The four half-lives together take 30 years, so

$$4T_{1/2} = 30\,\text{years}, \qquad T_{1/2} = \frac{30}{4}\,\text{years} = 7.5\,\text{years}.$$

As an independent exponential check, $e^{-30k} = 1/16$ implies $30k = \ln 16 = 4 \ln 2$. Since $T_{1/2} = \ln 2/k$, this again gives $30/4$ years. Among the listed values, only 7.5 years makes the given interval exactly four half-lives. Values 9.5, 8.5 and 10.5 years would all give fewer than four half-lives in 30 years, leaving a fraction greater than $1/16$; they cannot satisfy the stated activity reduction.

Step 3: Conclusion The half-life of the material is $7.5\,\text{years}$. Hence, the correct option is (C).

Quick Tip to Solve: When the remaining activity is $1/2^n$ of its initial value, exactly n half-lives have elapsed; divide the total time by n.

50. At a particular station, the TV transmission tower has a height of 100 m. To triple its coverage range, height of the tower should be increased to [4 marks]

(A) 200 m

(B) 300 m

(C) 600 m

(D) 900 m

Correct Answer: (D) 900 m

Solution:

Concept:

- Television waves propagate approximately along straight lines. The curvature of Earth limits direct reception, so the coverage range is set by the distance to the horizon.
- For a tower of height h, with $h \ll R_E$, the horizon distance is $d \simeq \sqrt{2R_E h}$, where R_E is the radius of Earth. Thus range is proportional to the square root of tower height.

Step 1: Relate the range to tower height Take the initial height as $h_1 = 100\,\text{m}$, and call the required final height h_2. Using the same Earth radius for both towers eliminates the constant from the calculation. The question asks for tripling the linear coverage range, so the required distance is $d_2 = 3d_1$. Hence

$$\frac{d_2}{d_1} = \sqrt{\frac{h_2}{h_1}} = 3.$$

Step 2: Square the ratio and compare the options Squaring both sides gives the height ratio, rather than multiplying the original height by three:

$$\frac{h_2}{100} = 9, \qquad h_2 = 900\,\text{m}.$$

The alternatives 200, 300 and $600\,\text{m}$ would increase the range by factors $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{6}$, respectively. None equals three. A height of $900\,\text{m}$ gives precisely $\sqrt{9} = 3$ times the original range. The wording asks for the height increased to, which is the final height; the added height alone would be $800\,\text{m}$.

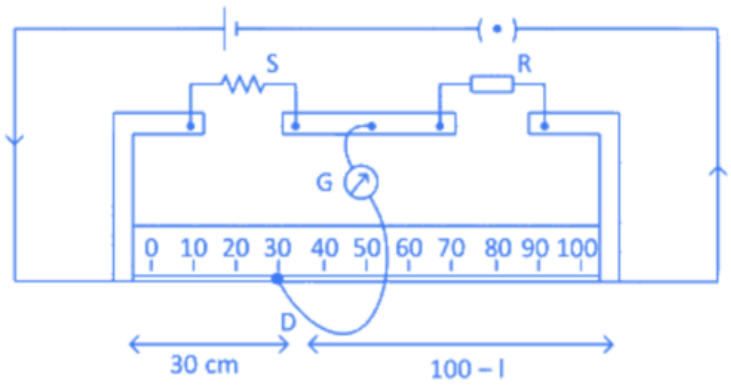
Step 3: Conclusion The tower height should be increased to $\boxed{900\,\text{m}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a specified change in linear horizon range, square the range factor to obtain the height factor. Distinguish the final height from the extra height added.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In a meter bridge experiment, for measuring unknown resistance 'S', the null point is obtained at a distance 30 cm from the left side as shown at point D. If R is 5.6 kΩ, then the value of unknown resistance 'S' will be _____ Ω. [4 marks]



Solution:

Concept:

- A metre bridge is a Wheatstone bridge using a uniform wire. At balance the galvanometer carries no current, and the ratio of gap resistances equals the ratio of the corresponding wire resistances.
- For a uniform bridge wire, resistance is proportional to length. The figure places S in the left gap and R in the right gap, so $S/R = AD/DB$.

Step 1: Identify the correct lengths and gap positions The null point D lies $30\,\text{cm}$ from the left end of a $100\,\text{cm}$ wire. Therefore

$$AD = 30\,\text{cm}, \qquad DB = 100 - 30 = 70\,\text{cm}.$$

The unknown resistance S is above the left segment, while the known resistance R is above the right segment. This correspondence is important: reversing the two lengths would give the reciprocal ratio and an incorrect answer.

Step 2: Apply the balance condition At the null point the two relevant junctions have equal potential, so the Wheatstone balance relation becomes

$$\frac{S}{R} = \frac{30}{70} = \frac{3}{7}.$$

Convert the known resistance to the unit requested in the answer:

$$R = 5.6\,\text{k}\Omega = 5600\,\Omega.$$

Consequently,

$$S = 5600 \times \frac{3}{7} = 800 \times 3 = 2400\,\Omega.$$

As a check, the shorter left wire segment has the smaller resistance; correspondingly $S < R$, as required by the circuit at balance. No current through the galvanometer is needed to find this ratio.

Step 3: Conclusion The unknown resistance is $\boxed{S = 2400\,\Omega}$, so the numerical entry is 2400.

Quick Tip to Solve: Always match each gap resistance with the wire segment on the same side of the balance point before substituting the lengths.

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52. The one division of main scale of Vernier callipers reads 1 mm and 10 divisions of Vernier scale is equal to the 9 divisions on main scale. When the two jaws of the instrument touch each other, the zero of the Vernier lies to the right of zero of the main scale and its fourth division coincides with a main scale division. When a spherical bob is tightly placed between the two jaws, the zero of the Vernier scale lies in between 4.1 cm and 4.2 cm and 6^{th} Vernier division coincides scale division. The diameter of the bob will be _____ $\times$ 10^{-2} cm. [4 marks]

Solution:

Concept:

- The least count of direct vernier calipers is the difference between one main-scale division and one vernier-scale division: $LC = 1\,\text{MSD} - 1\,\text{VSD}$.
- The observed reading is the main-scale reading just before the vernier zero plus the coinciding vernier division times the least count. True reading equals observed reading minus the zero error.

Step 1: Calculate the least count and zero error Ten vernier divisions equal nine main-scale divisions. Since one main-scale division is $1\,\text{mm}$,

$$1\,\text{VSD} = \frac{9}{10}\,\text{mm}, \qquad LC = 1 - 0.9 = 0.1\,\text{mm} = 0.01\,\text{cm}.$$

With the jaws closed, the vernier zero lies to the right of the main-scale zero. Thus the instrument gives a positive reading even for zero actual separation. The fourth vernier division coincides, making the positive zero error

$$e = 4LC = 4(0.01) = 0.04\,\text{cm}.$$

Step 2: Correct the observed diameter With the bob between the jaws, the vernier zero lies between 4.1 and $4.2\,\text{cm}$, so the main-scale contribution is the lower value $4.1\,\text{cm}$. The sixth vernier division contributes $6LC$. Hence

$$d_{\text{obs}} = 4.1 + 6(0.01) = 4.16\,\text{cm}.$$

A positive zero error is subtracted, because the instrument overreads:

$$d = d_{\text{obs}} - e = 4.16 - 0.04 = 4.12\,\text{cm}.$$

Finally write this in the requested power-of-ten form, keeping the physical diameter unchanged:

$$4.12\,\text{cm} = 412 \times 10^{-2}\,\text{cm}.$$

Step 3: Conclusion The diameter is $\boxed{412 \times 10^{-2}\,\text{cm}}$; the required numerical entry is 412.

Quick Tip to Solve: A vernier zero to the right gives a positive zero error: subtract its division count from the measurement division count when both use the same least count.

53. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the two beams are $\pi/2$ and $\pi/3$ at points A and B respectively. The difference between the resultant intensities at the two points is xI. The value of x will be _____.

[4 marks]

Solution:

Concept:

- For two coherent beams with intensities I_1 and I_2, the resultant intensity at phase difference ϕ is $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$.
- Intensity is proportional to the square of wave amplitude. Thus beams of intensities I and $4I$ have amplitudes in the ratio $1:2$, and their interference term has coefficient $4I$.

Step 1: Form the expression for resultant intensity Substitute $I_1 = I$ and $I_2 = 4I$ into the intensity relation. The square root in the cross term must be retained:

$$I_R = I + 4I + 2\sqrt{I(4I)}\cos\phi = 5I + 4I\cos\phi.$$

The first term $5I$ is the sum of the individual intensities and does not depend on position. The phase-dependent cross term accounts for the difference between points A and B. Adding the beam intensities alone would miss this difference.

Step 2: Evaluate both points and take their difference At A, the phase difference is $\pi/2$, whose cosine is zero. Therefore

$$I_A = 5I + 4I\cos\frac{\pi}{2} = 5I.$$

At B, the phase difference is $\pi/3$, whose cosine is $1/2$. Hence

$$I_B = 5I + 4I\cos\frac{\pi}{3} = 5I + 2I = 7I.$$

The positive difference between the two intensities is

$$\Delta I = I_B - I_A = 7I - 5I = 2I.$$

The result also lies within the allowed intensity range: the minimum is $(2\sqrt{I} - \sqrt{I})^2 = I$, and the maximum is $(2\sqrt{I} + \sqrt{I})^2 = 9I$.

Step 3: Conclusion Since the difference is $xI = 2I$, the required dimensionless value is $\boxed{x = 2}$.

Quick Tip to Solve: When comparing two interference points, cancel the common sum of beam intensities first; only the change in the cosine interference term remains.

54. To light, a $50\ \text{W}, 100\ \text{V}$ lamp is connected, in series with a capacitor of capacitance $\frac{50}{\pi\sqrt{x}}\mu F$, with $200\ \text{V}, 50\ \text{Hz}$ AC source. The value of x will be _____. [4 marks]

Solution:

Concept:

- Treat the lamp as a resistor at its rated operating point. Its rated current is $I = P/V_R$, and its resistance is $R = V_R^2/P$, using rms quantities.
- In a series resistor-capacitor circuit, the resistor voltage and capacitor voltage differ in phase by $90°$. Therefore $V_s^2 = V_R^2 + V_C^2$, with $V_C = IX_C$ and $X_C = 1/(2\pi fC)$.

Step 1: Find the current and capacitor voltage The lamp must operate at its rating, so its voltage is $V_R = 100\,\text{V}$ and its power is $50\,\text{W}$. The common series current is therefore

$$I = \frac{50}{100} = 0.5\,\text{A}.$$

The lamp resistance at this operating point is $100^2/50 = 200\,\Omega$. The rms source voltage is $200\,\text{V}$. Since the capacitor voltage is in quadrature with the lamp voltage,

$$V_C = \sqrt{V_s^2 - V_R^2} = \sqrt{200^2 - 100^2} = 100\sqrt{3}\,\text{V}.$$

Subtracting the two voltages arithmetically would incorrectly ignore their phase difference.

Step 2: Obtain the capacitance and compare forms The required capacitive reactance follows from the common current:

$$X_C = \frac{V_C}{I} = \frac{100\sqrt{3}}{0.5} = 200\sqrt{3}\,\Omega.$$

At $f = 50\,\text{Hz}$,

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(200\sqrt{3})}\,\text{F}.$$

Convert farads to microfarads:

$$C = \frac{10^6}{20000\pi\sqrt{3}}\,\mu\text{F} = \frac{50}{\pi\sqrt{3}}\,\mu\text{F}.$$

Comparing this with the supplied expression $50/(\pi\sqrt{x})\,\mu\text{F}$ gives $\sqrt{x} = \sqrt{3}$.

Step 3: Conclusion The required capacitance corresponds to $\boxed{x = 3}$, a dimensionless numerical entry.

Quick Tip to Solve: For a series capacitor used to run a lamp from a higher AC voltage, subtract squared rms voltages to obtain the capacitor voltage, then use the rated lamp current.

55. A $1\ \text{m}$ long copper wire carries a current of $1\ \text{A}$. If the cross section of the wire is $2.0\ \text{mm}^2$ and the resistivity of copper is $1.7 \times 10^{-8}\ \Omega\text{m}$, the force experienced by moving electron in the wire is _____ $\times 10^{-23}\ \text{N}$.

(charge on electorn $= 1.6 \times 10^{-19}\ \text{C}$) [4 marks]

Solution:

Concept:

- The microscopic form of Ohm's law is $E = \rho J$, where $J = I/A$ is current density and ρ is the material resistivity.
- The magnitude of electric force on a charge is $F = |q|E$. An electron has negative charge, so its electric force is opposite to the electric-field direction; the requested blank asks for the magnitude.

Step 1: Calculate the electric field in the wire Convert the area before forming the current density. Since $1\,\text{mm}^2 = 10^{-6}\,\text{m}^2$,

$$A = 2.0 \times 10^{-6}\,\text{m}^2, \qquad J = \frac{1}{2.0 \times 10^{-6}} = 5.0 \times 10^{5}\,\text{A m}^{-2}.$$

The field that sustains this current is

$$E = \rho J = (1.7 \times 10^{-8})(5.0 \times 10^{5}) = 8.5 \times 10^{-3}\,\text{V m}^{-1}.$$

The one-metre length does not appear in this local relation. Equivalently, using $R = \rho L/A$ and $E = IR/L$ cancels the length and gives the same result.

Step 2: Find the force and express the requested coefficient Use the magnitude $e = 1.6 \times 10^{-19}\,\text{C}$:

$$F = eE = (1.6 \times 10^{-19})(8.5 \times 10^{-3}) = 13.6 \times 10^{-22}\,\text{N}.$$

Thus

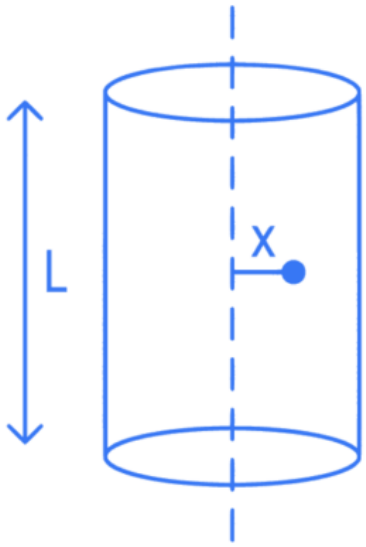
$$F = 1.36 \times 10^{-21}\,\text{N} = 136 \times 10^{-23}\,\text{N}.$$

The electric force acts opposite to the conventional current and the field, in the direction of electron drift. The electrons also undergo collisions; a steady drift velocity does not imply that the electric force is zero. The force calculated here is the applied electric force, not a claim of unbounded electron acceleration through the conductor.

Step 3: Conclusion The force magnitude is $\boxed{136 \times 10^{-23}\,\text{N}}$, so the required numerical entry is 136.

Quick Tip to Solve: Combine the two local relations directly as $F = e\rho I/A$; this avoids unnecessary resistance and potential-difference calculations.

56. A long cylindrical volume contains a uniformly distributed charge of density $\rho\,\mathrm{Cm}^{-3}$. The electric field inside the cylindrical volume at a distance $x = \frac{2\varepsilon_0}{\rho}\mathrm{m}$ from its axis is _____ Vm^{-1}. [4 marks]



Solution:

Concept:

- For a long uniformly charged cylindrical volume, cylindrical symmetry makes the internal electric field radial and dependent only on the distance r from the axis.
- Gauss's law states $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enclosed}}/\varepsilon_0$. For an internal coaxial Gaussian cylinder, enclosed charge is $\rho\pi r^2 L$, not the charge of the entire physical cylinder.

Step 1: Choose a cylindrical Gaussian surface Take a coaxial Gaussian cylinder of radius r at the observation point and arbitrary length L. The field has constant magnitude on its curved surface and points normal to that surface. On the two flat ends the field is parallel to each end face, so their electric flux is zero. The total flux is consequently

$$\Phi = E(2\pi r L).$$

Because the observation point is inside the charged volume, the enclosed charge occupies just the Gaussian volume:

$$Q_{\text{enclosed}} = \rho\pi r^2 L.$$

The assumption of a long cylinder lets us neglect the field distortion associated with the physical ends.

Step 2: Apply Gauss law and substitute the stated distance Equating the flux to enclosed charge divided by permittivity gives

$$E(2\pi r L) = \frac{\rho\pi r^2 L}{\varepsilon_0}.$$

Cancel $\pi r L$ to obtain the internal field:

$$E = \frac{\rho r}{2\varepsilon_0}.$$

Unlike the external field of a fixed line charge, this internal field grows linearly with distance because

the enclosed charge grows as r^2. At the specified distance $r = x = 2\varepsilon_0/\rho$,

$$E = \frac{\rho}{2\varepsilon_0}\left(\frac{2\varepsilon_0}{\rho}\right) = 1\,\mathrm{V\,m^{-1}}.$$

Both the charge density and the arbitrary Gaussian length cancel, as they must for the stated substitution.

Step 3: Conclusion The electric field at the given distance has magnitude $\boxed{1\,\mathrm{V\,m^{-1}}}$, giving the numerical entry 1.

Quick Tip to Solve: Inside a uniformly charged cylinder, first use the enclosed radius to derive $E = \rho r/(2\varepsilon_0)$; do not apply the external line-charge expression with a fixed total charge.

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57. A mass 0.9 kg, attached to a horizontal spring, executes SHM with an amplitude A_1. When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move together with amplitude A_2. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha-1}$, then the value of α will be _____.

[4 marks]

Solution:

Concept:

- At the mean position the spring displacement is zero and the speed is maximum: $v = \omega A$, where $\omega = \sqrt{k/m}$.
- During the brief loading, horizontal momentum is conserved. The added mass has no initial horizontal velocity, and the spring force is zero at that instant. Mechanical energy is not conserved during this inelastic joining.

Step 1: Find the speed immediately after loading Let the original mass be $m = 0.900\,\mathrm{kg}$, and the combined mass be $M = 0.900 + 0.124 = 1.024\,\mathrm{kg}$. Just before loading, $v_1 = A_1\sqrt{k/m}$. Momentum conservation gives

$$mv_1 = Mv_2, \qquad v_2 = \frac{m}{M}v_1.$$

The added mass slows the block. Keeping the speed unchanged would increase horizontal momentum and kinetic energy despite the added mass bringing neither. The joining is inelastic: its kinetic energy falls to m/M times the original value, while horizontal momentum is conserved.

Step 2: Relate the two amplitudes The equilibrium position is unchanged for a horizontal spring. Thus the joined masses are also at their mean position, with angular frequency $\omega_2 = \sqrt{k/M}$ and maximum speed $v_2 = \omega_2 A_2$. Therefore

$$\frac{A_2}{A_1} = \frac{m}{M}\sqrt{\frac{M}{m}} = \sqrt{\frac{m}{M}}.$$

Hence

$$\frac{A_1}{A_2} = \sqrt{\frac{1.024}{0.900}} = \sqrt{\frac{1024}{900}} = \frac{16}{15}.$$

Step 3: Determine the requested parameter Compare the given form with this ratio:

$$\frac{\alpha}{\alpha-1}=\frac{16}{15} \quad \Rightarrow \quad 15\alpha = 16\alpha - 16.$$

It follows that $\alpha = 16$. This parameter is dimensionless, as is the amplitude ratio.

Step 4: Conclusion The requested value is $\boxed{\alpha = 16}$.

Quick Tip to Solve: For sudden loading at the mean position, conserve momentum first; then use the new maximum speed and new angular frequency to find the amplitude.

---

58. A square aluminum (shear modulus is $25 \times 10^9\,\mathrm{Nm}^{-2}$) slab of side $60\ \mathrm{cm}$ and thickness $15\ \mathrm{cm}$ is subjected to a shearing force (on its narrow face) of $18.0 \times 10^4\ \mathrm{N}$. The lower edge is riveted to the floor. The displacement of the upper edge is _____ μm. [4 marks]

Solution:

Concept:

- Shear modulus is the ratio of tangential stress to shear strain: $G = (F/A)/(\Delta x/h)$, where A is the loaded face area and h the separation of the fixed and displaced faces.
- For a small sideways displacement, $\Delta x = Fh/(AG)$. The thickness enters the area of the narrow face; it is not the vertical height of this slab.

Step 1: Identify the geometry and convert to SI units The square slab has side $L = 60\,\mathrm{cm} = 0.60\,\mathrm{m}$ and thickness $b = 15\,\mathrm{cm} = 0.15\,\mathrm{m}$. Its lower edge is fixed and its upper edge is displaced, so the separation is $h = L = 0.60\,\mathrm{m}$. The tangential force acts on the narrow face, whose area is

$$A = Lb = (0.60)(0.15) = 0.090\,\mathrm{m}^2.$$

Using the broad square area would describe a different loading geometry.

Step 2: Calculate the shear strain The force is $F = 18.0 \times 10^4\,\mathrm{N}$, and $G = 25 \times 10^9\,\mathrm{N\,m^{-2}}$. The shear stress is

$$\frac{F}{A} = \frac{18.0 \times 10^4}{0.090} = 2.0 \times 10^6\,\mathrm{N\,m^{-2}}.$$

Consequently the dimensionless strain is

$$\frac{\Delta x}{h} = \frac{2.0 \times 10^6}{25 \times 10^9} = 8.0 \times 10^{-5}.$$

Step 3: Find the upper-edge displacement Multiply the strain by the height:

$$\Delta x = (8.0 \times 10^{-5})(0.60) = 4.8 \times 10^{-5}\,\mathrm{m}.$$

Since $1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$, this becomes

$$\Delta x = 48\,\mu\mathrm{m}.$$

The very small strain is consistent with the small-angle shear formula used above.

Step 4: Conclusion The upper edge moves by $\boxed{48\,\mu\text{m}}$; the numerical entry is $\boxed{48}$.

Quick Tip to Solve: In a sheared slab, label the loaded area and the distance between its fixed and displaced edges before substituting into the modulus formula.

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59. A pulley of radius $1.5\ \text{m}$ is rotated about its axis by a force $F = \left(12\text{t} - 3\text{t}^2\right) N$ applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is $4.5\ \text{kg m}^2$, the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is _____. [4 marks]

Solution:

Concept:

- A tangential force produces torque $\tau = rF$. For constant moment of inertia, angular acceleration is $\dot{\omega} = \tau/I$.
- A reversal occurs when angular velocity passes through zero, rather than when torque first changes sign. Angular displacement follows by integrating angular velocity, and one turn is 2π radians.

Step 1: Find the angular acceleration and velocity Take the initial direction of the applied torque as positive and the pulley initially at rest. With $r = 1.5\,\text{m}$ and $I = 4.5\,\text{kg m}^2$,

$$\dot{\omega} = \frac{1.5}{4.5}(12t - 3t^2) = 4t - t^2.$$

Integrating from zero and using $\omega(0) = 0$ gives

$$\omega(t) = \int_0^t (4u - u^2)\,du = 2t^2 - \frac{t^3}{3}.$$

The force changes sign at $t = 4\,\text{s}$, but the pulley still has positive angular velocity then.

Step 2: Locate the first reversal Factor the angular velocity:

$$\omega = t^2\left(2 - \frac{t}{3}\right).$$

Apart from its initial zero, the next zero occurs at $t = 6\,\text{s}$. The angular velocity is positive for $0 < t < 6$ and negative just after $6\,\text{s}$, so this is the reversal time. Until then the total angle travelled equals the positive angular displacement.

Step 3: Integrate the angle and convert to turns The displacement up to reversal is

$$\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right) dt = \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6.$$

Thus $\theta = 144 - 108 = 36\,\text{rad}$, and the number of rotations is

$$N = \frac{36}{2\pi} = \frac{18}{\pi}.$$

Comparison with K/π gives $K = 18$.

Step 4: Conclusion The pulley makes $18/\pi$ rotations before reversing, so $\boxed{K = 18}$.

Quick Tip to Solve: For time-dependent torque, integrate once to locate the zero of angular velocity; the zero of torque identifies an extremum of speed instead.

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60. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown at an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is _____. [4 marks]

Solution:

Concept:

- For a projectile returning to its launch level, the time of flight is $T = 2u_y/g$, where u_y is its initial vertical velocity.
- The maximum rise above the launch level is $H = u_y^2/(2g)$. In uniform gravity without air resistance, both flight time and maximum height are independent of mass and horizontal velocity.

Step 1: Compare the vertical launch components Let the speed of the first ball be u, and the speed of the second ball be v. The first is projected straight upward, so its vertical component is u. Because the second projection angle is measured from the vertical, its vertical component is $v\cos\theta$, not $v\sin\theta$. Their times in air are therefore

$$T_1 = \frac{2u}{g}, \qquad T_2 = \frac{2v\cos\theta}{g}.$$

Equal flight times imply $u = v\cos\theta$. The second ball may have a different total speed, but it must have the same vertical component.

Step 2: Compare the maximum heights At the highest point the vertical speed vanishes. Applying $v_y^2 = u_y^2 - 2gH$ to each projectile gives

$$H_1 = \frac{u^2}{2g}, \qquad H_2 = \frac{v^2\cos^2\theta}{2g}.$$

Using $u = v\cos\theta$, we obtain $H_1 = H_2$. The horizontal motion of the second ball contributes to its range but cannot change its vertical rise.

Step 3: Extract the numerical value The stated ratio is $H_1/H_2 = 1/x$. Since the two heights are equal,

$$\frac{1}{x} = 1 \quad \Rightarrow \quad x = 1.$$

The different masses m and $2m$ do not enter this result because their gravitational accelerations are the same.

Step 4: Conclusion The ratio of the heights is $1 : 1$, and the requested dimensionless value is $\boxed{x = 1}$.

Quick Tip to Solve: Equal flight times to the same level mean equal initial vertical components; use this fact before considering launch angle or mass.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. $250\ \mathrm{g}$ **solution of** D**-glucose in water contains** 10.8% **of carbon by weight. The molality of the solution is nearest to**

(Given: Atomic Weights are, $\mathrm{H}, 1\ \mathrm{u}; \mathrm{C}, 12\ \mathrm{u}; \mathrm{O}, 16\ \mathrm{u}$**)** [4 marks]

(A) 1.03

(B) 2.06

(C) 3.09

(D) 5.40

Correct Answer: (B) 2.06

Solution:

Concept:

- Molality is moles of solute per kilogram of solvent: $b = n_{\text{solute}}/m_{\text{solvent}}$. The solvent mass excludes the solute.
- D-glucose is $\mathrm{C_6H_{12}O_6}$, with molar mass $180\,\mathrm{g\,mol^{-1}}$. Its carbon mass fraction is $72/180 = 0.40$, using the supplied atomic weights.

Step 1: Find the glucose mass from carbon The solution contains 10.8% carbon by mass, so its carbon mass is

$$m_C = \frac{10.8}{100}(250) = 27.0\,\mathrm{g}.$$

Water contains no carbon; all of this carbon belongs to glucose. Each $180\,\mathrm{g}$ of glucose contains $6 \times 12 = 72\,\mathrm{g}$ carbon. Thus

$$m_{\text{glucose}} = 27.0\frac{180}{72} = 67.5\,\mathrm{g}.$$

The carbon percentage must first be converted to glucose mass; it is not the percentage of glucose itself.

Step 2: Determine moles and solvent mass The number of moles of glucose is

$$n = \frac{67.5}{180} = 0.375\,\mathrm{mol}.$$

Subtract its mass from the total solution mass to obtain the water mass:

$$m_{\text{water}} = 250 - 67.5 = 182.5\,\mathrm{g} = 0.1825\,\mathrm{kg}.$$

The denominator for molality is this water mass, rather than $0.250\,\mathrm{kg}$ of solution.

Step 3: Calculate and compare with all options Therefore

$$b = \frac{0.375}{0.1825} = 2.05479\ldots\ \mathrm{mol\,kg^{-1}}.$$

This is nearest to 2.06. Option (A), 1.03, is approximately half the computed value; (C), 3.09, is too large; and (D), 5.40, also fails the molality calculation. Only (B) agrees with both the carbon balance and the solvent mass.

Step 4: Conclusion The molality is approximately $\boxed{2.06\,\text{mol kg}^{-1}}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert the elemental mass into the full solute mass first, then subtract that solute from the solution mass before calculating molality.

62. Given below are two statements.

Statement I: O_2, Cu^{2+}, and Fe^{3+} are weakly attracted by magnetic field and are magnetized in the same direction as magnetic field.

Statement II: $NaCl$ and H_2O are weakly magnetized in opposite direction to magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Correct Answer: (A) Both Statement I and Statement II are correct.

Solution:

Concept:

- Paramagnetic species possess unpaired electrons and have a weak induced magnetization parallel to the applied field, so they are attracted into it.
- Diamagnetic species have all electrons paired. Their induced magnetization opposes the applied field, and they are weakly repelled; this is distinct from the strong alignment associated with ferromagnetism.

Step 1: Test Statement I using electron configurations In molecular oxygen, O_2, the two highest occupied antibonding π^* orbitals each contain one unpaired electron. Oxygen is therefore paramagnetic. For the ions, removal of the outer electrons gives

$$Cu^{2+} : [Ar]\,3d^9, \qquad Fe^{3+} : [Ar]\,3d^5.$$

The copper ion has one unpaired electron, and the free ferric ion has five. Both ions are paramagnetic as well. Consequently all three listed species are weakly attracted and magnetize along the field, exactly as Statement I says.

Step 2: Test Statement II using electron pairing In $NaCl$, the constituent ions Na^+ and Cl^- have closed-shell configurations, $[Ne]$ and $[Ar]$, respectively. Their electrons are paired. A water molecule, H_2O, is also a closed-shell molecule with paired electrons. These substances are diamagnetic, so their weak induced magnetization is opposite to the external field. Statement II is therefore also correct.

Step 3: Select the combination of truth values The independent checks establish the combination (I true, II true). Option (B) incorrectly rejects both statements. Option (C) incorrectly calls the diamagnetic description false, and option (D) incorrectly rejects the paramagnetic examples. Only

option (A) retains both verified statements.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: Check electron pairing first: unpaired electrons indicate paramagnetism, while closed shells generally give weak magnetization opposite to the field.

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63. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Energy of 2s orbital of hydrogen atom is greater than that of 2s orbital of lithium.

Reason R: Energies of the orbitals in the same subshell decrease with increase in the atomic number.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Correct Answer: (A) Both A and R are true and R is the correct explanation of A.

Solution:

Concept:

- Bound orbital energies are negative relative to a free electron. A more strongly bound electron has a lower, more negative energy.
- For corresponding orbitals, an increase in effective nuclear attraction stabilizes the orbital. Shielding reduces the nuclear attraction but does not cancel the increase in attraction relevant to the comparison of hydrogen and lithium $2s$ orbitals.

Step 1: Evaluate the assertion about the two 2s orbitals For the hydrogen atom, the orbital energy depends on the principal quantum number:

$$E_n = -\frac{13.6}{n^2}\,\text{eV}, \qquad E_{2s}(\text{H}) = -3.4\,\text{eV}.$$

The lithium $2s$ electron experiences the attraction of a nucleus with three protons, partly screened by the two $1s$ electrons. Its $2s$ orbital is more strongly bound than the hydrogen $2s$ orbital. Its energy is therefore more negative. Thus the hydrogen $2s$ energy is greater, and Assertion A is true. Greater energy here does not mean a larger magnitude of negative binding energy.

Step 2: Evaluate the reason and its connection For the same type of subshell, increasing atomic number increases nuclear attraction; the accompanying shielding is accounted for through effective nuclear charge. The corresponding orbital becomes more stable and its energy decreases. This is the stated orbital-energy trend, so Reason R is true in the comparison being made. Applying it to the $2s$ orbital as the atomic number increases from hydrogen to lithium directly gives the inequality asserted in A.

Step 3: Reject the alternative classifications The reason describes the physical cause of the stabilization of lithium $2s$, so it explains the assertion rather than merely accompanying it. Option (B) incorrectly denies this explanatory connection. Option (C) calls the valid energy trend false, and option (D) reverses the verified assertion. Hence the correct combination is that both statements are true and R explains A.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: Compare negative energies with their signs: stronger nuclear binding lowers an orbital energy, making it more negative rather than greater.

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64. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Activated charcoal adsorbs SO_2 more efficiently than CH_4.

Reason R : Gases with lower critical temperatures are readily adsorbed by activated charcoal.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (C) A is correct but R is not correct.

Solution:

Concept:

- Physical adsorption on activated charcoal is favoured by strong intermolecular attractions between the gas and the surface.
- The critical temperature measures the ease of liquefaction: a gas with a higher critical temperature is generally more readily adsorbed at a given temperature.

Step 1: Evaluate the assertion SO_2 is more easily liquefied than CH_4, reflecting its stronger intermolecular attractions and higher critical temperature. It therefore experiences stronger attractive interactions with the large surface of activated charcoal. Consequently, charcoal adsorbs SO_2 more efficiently than CH_4, and Assertion A is correct.

Step 2: Evaluate the reason Reason R reverses the critical-temperature trend. Gases with higher, rather than lower, critical temperatures are readily adsorbed. A low critical temperature indicates a gas that is difficult to liquefy, so it is not the explanation of greater adsorption. Thus R is incorrect. Both alternatives saying that A and R are correct fail, while the alternative denying A also fails.

Step 3: Conclusion Assertion A is correct but Reason R is not correct. Hence, the correct option is (C).

Quick Tip to Solve: For physisorption, connect stronger adsorption with easier liquefaction and a higher critical temperature.

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65. Boiling point of a 2% aqueous solution of a non-volatile solute A is equal to the boiling point of 8% aqueous solution of a non-volatile solute B. The relation between molecular weights of A and B is [4 marks]

(A) $M_A = 4M_B$

(B) $M_B = 4M_A$

(C) $M_A = 8M_B$

(D) $M_B = 8M_A$

Correct Answer: (B) $M_B = 4M_A$

Solution:

Concept:

- For a dilute solution of a non-volatile, non-electrolyte solute, elevation of boiling point is $\Delta T_b = K_b m$, where m is the molality.
- Both solutions have water as solvent; equal boiling points therefore correspond to equal boiling-point elevations and equal molalities under the dilute-solution convention.

Step 1: Compare solute amounts Use the usual dilute-solution interpretation of the stated percentages: compare 2 mass units of A with 8 mass units of B for the same reference amount of aqueous solvent. Their respective mole amounts are $2/M_A$ and $8/M_B$. The fourfold greater mass concentration of B must be compensated by a fourfold greater molar mass if the number of solute particles is to remain equal.

Step 2: Equate the molalities The common solvent amount and the same value of K_b cancel. Thus

$$\frac{2}{M_A} = \frac{8}{M_B} \quad \Longrightarrow \quad 2M_B = 8M_A \quad \Longrightarrow \quad M_B = 4M_A.$$

The relation $M_A = 4M_B$ reverses the required ratio, and either factor of eight is inconsistent with the concentration ratio $8/2 = 4$. The result uses the dilute-solution convention implicit in the options; the question does not specify a strict mass-percent definition.

Step 3: Conclusion The required relation is $M_B = 4M_A$. Hence, the correct option is (B).

Quick Tip to Solve: For equal colligative effects in the same dilute solvent, equate concentration divided by molar mass.

66. The incorrect statement is [4 marks]

(A) The first ionization enthalpy of K is less than that of Na and Li.

(B) Xe does not have the lowest first ionization enthalpy in its group.

(C) The first ionization enthalpy of element with atomic number 37 is lower than that of the element with atomic number 38.

(D) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

Correct Answer: (D) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

Solution:

Concept:

- First ionization enthalpy is the energy needed to remove the most loosely bound electron from an isolated gaseous atom.
- Down a group, increasing size and shielding generally lower ionization enthalpy. Across a period, increasing effective nuclear charge generally raises it, with electronic-configuration exceptions.

Step 1: Check the first three statements For alkali metals, the first ionization enthalpy decreases from Li to Na to K, so statement A is correct. Xenon is not the lowest member of its group: the heavier noble gas radon has a lower first ionization enthalpy, making statement B correct. Atomic numbers 37 and 38 are rubidium and strontium. Removing the sole $5s$ electron from Rb is easier than removing a $5s$ electron from Sr, so statement C is correct.

Step 2: Compare gallium and zinc Zinc, atomic number 30, has configuration $[\mathrm{Ar}]3d^{10}4s^2$, whereas gallium has $[\mathrm{Ar}]3d^{10}4s^24p^1$. Gallium loses the higher-energy $4p$ electron more easily. Its first ionization enthalpy is therefore lower than that of zinc, not higher as statement D claims. The stable filled $4s$ subshell in zinc reinforces this comparison.

Step 3: Conclusion The incorrect statement is that the first ionization enthalpy of Ga is higher than that of Zn. Hence, the correct option is (D).

Quick Tip to Solve: At the start of a p block, compare the new p electron with the filled s subshell of the preceding element.

67. Given below are two statements.

Statement I : Iron (III) catalyst, acidified $K_2Cr_2O_7$ and neutral $KMnO_4$ have the ability to oxidise I^- to I_2 independently.

Statement II : Manganate ion is paramagnetic in nature and involves $p\pi - p\pi$ bonding.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (B) Both Statement I and Statement II are false.

Solution:

Concept:

- An oxidizing agent accepts electrons. The iodine oxidation product depends on the reagent and the reaction medium, so oxidation of iodide need not stop at iodine.
- In $MnO_4{}^{2-}$, manganese is in oxidation state $+6$ and has a $3d^1$ configuration, which gives an unpaired electron.

Step 1: Test the product specified in Statement I Iron(III) can oxidize iodide to iodine: $2\,Fe^{3+} + 2\,I^- \longrightarrow 2\,Fe^{2+} + I_2$, and acidified dichromate also oxidizes iodide to iodine. However, neutral permanganate oxidizes iodide to iodate rather than giving iodine as the specified final product:

$$2\,MnO_4{}^- + I^- + H_2O \longrightarrow 2\,MnO_2 + IO_3{}^- + 2\,OH^-.$$

All atoms and charges balance: the net charge is -3 on each side. Therefore the claim that all three independently give I_2 is false.

Step 2: Separate magnetism from the bonding claim For manganate, $x + 4(-2) = -2$, giving $x = +6$. The resulting d^1 ion is paramagnetic, so the first clause of Statement II is true. However, in the conventional description of manganese oxo bonding used here, oxygen p orbitals overlap with manganese d orbitals to give $p\pi - d\pi$ bonding. Calling it $p\pi - p\pi$ bonding is incorrect. Because the two clauses are joined by "and", Statement II is false. Thus both-true and the two mixed truth-value alternatives are excluded.

Step 3: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: Check the final oxidation product in the stated medium; neutral permanganate takes iodide to iodate.

68. The total number of $\mathrm{Mn}=\mathrm{O}$ bonds in $\mathrm{Mn_2O_7}$ is _____. [4 marks]

(A) 4

(B) 5

(C) 6

(D) 3

Correct Answer: (C) 6

Solution:

Concept:

- $\mathrm{Mn_2O_7}$ is a covalent molecular oxide consisting of two manganese-centred tetrahedral units sharing one oxygen atom.
- The shared oxygen is a bridge with two single $\mathrm{Mn-O}$ bonds; the other oxygen atoms are terminal oxo groups.

Step 1: Identify terminal and bridging oxygen atoms Write the connectivity in condensed structural form as $\mathrm{O_3Mn-O-MnO_3}$. Of the seven oxygen atoms, one joins the two manganese centres. The six remaining oxygen atoms are terminal, three attached to each manganese. The bridging oxygen has two single bonds and must not be counted as a terminal double-bonded oxygen.

Step 2: Count the double bonds Each terminal oxo group contributes one $\mathrm{Mn}=\mathrm{O}$ bond. Therefore,

$$N(\mathrm{Mn}=\mathrm{O})=3+3=6.$$

This is also consistent with oxidation state $+7$ for each manganese: the conventional structural bond-order total at a centre is three double bonds plus one single bridge bond. Counting only one manganese gives 3, while 4 or 5 misses terminal oxo groups; none of those alternatives represents the complete molecule.

Step 3: Conclusion The molecule contains 6 $\mathrm{Mn}=\mathrm{O}$ bonds. Hence, the correct option is (C).

Quick Tip to Solve: In a bridged oxide, first separate shared oxygen atoms from terminal oxo groups before counting double bonds.

69. Given below are two statements: one is labelled as Assertion A and, the other is labelled as Reason R.

Assertion A: [6] Annulene, [8] Annulene and cis-[10] Annulene, are respectively aromatic, not-aromatic and aromatic.

Reason R: Planarity is one of the requirements of aromatic systems.

In the light of the above statements, choose the most appropriate answer from the options given below. [4 marks]



(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Correct Answer: (D) A is not correct but R is correct.

Solution:

Concept:

- An aromatic molecule must be cyclic, planar and continuously conjugated, with $(4n+2)$ delocalized π electrons.
- The electron-count condition alone is insufficient: loss of planarity interrupts the effective cyclic overlap of adjacent p orbitals.

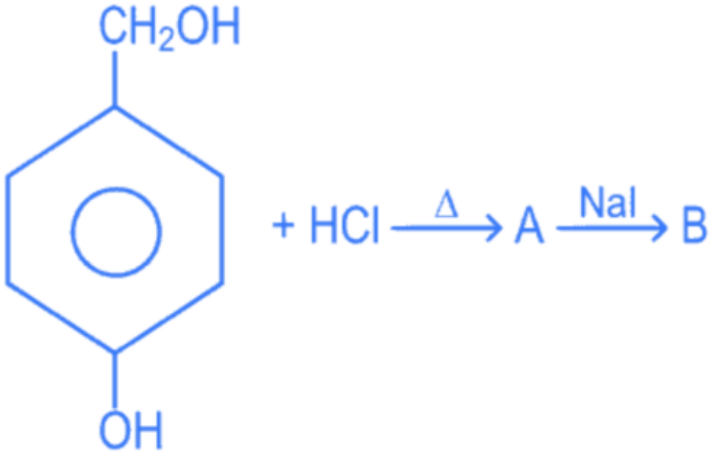
Step 1: Check the three annulenes in the assertion [6]Annulene is benzene: it is planar and has six π electrons, so it is aromatic. [8]Annulene has eight π electrons and adopts a nonplanar tub conformation, avoiding the antiaromatic planar arrangement; it is non-aromatic. In the illustrated cis-[10]annulene, inward-facing hydrogen atoms cause steric crowding. The ring cannot maintain the planar geometry required for effective cyclic conjugation, despite having ten π electrons.

Step 2: Evaluate the assertion and reason separately The first two classifications in Assertion A are correct, but the claim that the illustrated cis-[10]annulene is aromatic is false. Thus the assertion as a whole is false. Reason R is true because planarity is a necessary condition for aromaticity. The options saying that both statements are correct, or that A is correct and R is incorrect, therefore fail. The inward-facing hydrogens in the given structure are the decisive reason not to apply the ten-electron count in isolation.

Step 3: Conclusion Assertion A is not correct but Reason R is correct. Hence, the correct option is (D).

Quick Tip to Solve: Check planarity before applying the $(4n+2)$ rule; ten π electrons do not by themselves guarantee aromaticity.

70. In the above reaction product B is : [4 marks]



(A) $\text{p-HO}-C_6H_4-CH_2I$

(B) $\text{p-I}-C_6H_4-CH_2I$

(C) $\text{p-HO}-C_6H_4-CH_2Cl$

(D) $\text{p-I}-C_6H_4-CH_2Cl$

Correct Answer: (A) $\text{p-HO}-C_6H_4-CH_2I$

Solution:

Concept:

- A benzylic alcohol reacts with hydrogen chloride on heating to replace its alcoholic $-OH$ group with $-Cl$.
- Sodium iodide exchanges a benzylic chloride for iodide, whereas a phenolic $C-OH$ bond is not replaced by these reagents under the stated conditions.

Step 1: Identify intermediate A The starting compound has a hydroxymethyl group and a phenolic hydroxyl group in para positions. The side-chain $-CH_2OH$ is a benzyl alcohol group and reacts with HCl, while the hydroxyl group attached directly to the benzene ring remains intact. Thus

$$\text{p-HO}-C_6H_4-CH_2OH + HCl \xrightarrow{\Delta} \text{p-HO}-C_6H_4-CH_2Cl + H_2O.$$

Intermediate A is 4-(chloromethyl)phenol.

Step 2: Apply iodide substitution and compare options Iodide is a good nucleophile and replaces chloride at the primary benzylic carbon:

$$\text{p-HO}-C_6H_4-CH_2Cl + NaI \longrightarrow \text{p-HO}-C_6H_4-CH_2I + NaCl.$$

Consequently B retains phenolic $-OH$ and contains $-CH_2I$. The structure with iodine directly attached to the aromatic ring wrongly replaces the phenolic group. A chloromethyl structure represents intermediate A rather than the product after the second reagent. The option retaining chloride and replacing the phenolic group makes both errors.

Step 3: Conclusion Product B is 4-(iodomethyl)phenol, $\text{p-HO}-C_6H_4-CH_2I$. Hence, the correct

option is (A).

Quick Tip to Solve: Distinguish benzylic C−OH from phenolic C−OH: these reagents substitute the side chain, not the aryl hydroxyl group.

71. A sugar 'X' dehydrates very slowly under acidic condition to give furfural which on further reaction with resorcinol gives the coloured product after sometime. Sugar 'X' is [4 marks]

(A) Aldopentose

(B) Aldotetrose

(C) Oxalic acid

(D) Ketotetrose

Correct Answer: (A) Aldopentose

Solution:

Concept:

- Pentoses have five carbon atoms; acidic dehydration converts a pentose into the five-carbon heterocycle furfural. The carbon count of the product helps identify the carbohydrate.
- In the resorcinol test, ketoses undergo acid-catalysed dehydration more readily than aldoses. A delayed coloured condensation product is consistent with an aldose rather than a rapidly responding ketose.

Step 1: Identify the carbon skeleton from the product Furfural contains a four-carbon furan ring and a one-carbon aldehyde substituent, so it contains five carbon atoms in total. Formation of this product by loss of water points to a pentose. A representative overall dehydration is

$$C_5H_{10}O_5 \xrightarrow{H^+} C_5H_4O_2 + 3\,H_2O.$$

The oxygen-containing ring is formed during dehydration; the number of carbon atoms is retained. The product therefore excludes both of the four-carbon sugars listed.

Step 2: Use the slow reaction to distinguish the sugar type The question specifically says that dehydration is very slow and that colour appears only after some time on reaction with resorcinol. This delayed response supports an aldose. Combining this behaviour with the five-carbon skeleton gives an aldopentose. Aldotetrose and ketotetrose have only four carbon atoms, while oxalic acid is a two-carbon dicarboxylic acid and is not a sugar. None can account for the stated pentose-to-furfural transformation.

Step 3: Conclusion Sugar X is an aldopentose. Hence, the correct option is (A).

Quick Tip to Solve: First count the carbon atoms in the dehydration product: furfural points to a pentose; then use slow versus rapid colour formation to distinguish aldose behaviour.

72. In Carius method of estimation of halogen, 0.45 g of an organic compound gave 0.36 g of AgBr. Find out the percentage of bromine in the compound.

(Molar masses : $AgBr = 188\ g\ mol^{-1}; Br = 80\ g\ mol^{-1}$**)** [4 marks]

(A) 34.04%

(B) 40.04%

(C) 36.03%

(D) 38.04%

Correct Answer: (A) 34.04%

Solution:

Concept:

- In the Carius method, halogen in the organic compound is converted into a silver halide precipitate. Each mole of AgBr contains one mole of bromine atoms.
- The bromine mass fraction in pure AgBr is $M(\mathrm{Br})/M(\mathrm{AgBr}) = 80/188$. Percentage bromine is the recovered bromine mass divided by the original sample mass, multiplied by 100.

Step 1: Calculate the mass of bromine in the precipitate The precipitate mass includes both silver and bromine, so it cannot be used directly as the bromine mass. Using the supplied molar masses,

$$n(\mathrm{AgBr}) = \frac{0.36}{188}\ \mathrm{mol}.$$

The one-to-one relation between AgBr and bromine atoms gives

$$m(\mathrm{Br}) = \frac{0.36}{188} \times 80 = 0.15319\ \mathrm{g}.$$

This is the mass of bromine originally present in the 0.45 g sample, assuming complete conversion and collection as in the Carius estimation.

Step 2: Convert this mass to a percentage of the sample The denominator must be the mass of the original organic compound, not that of the silver bromide precipitate. Thus

$$\%\mathrm{Br} = \frac{0.36 \times 80}{188 \times 0.45} \times 100 \approx 34.04255\%.$$

Rounding to two decimal places gives 34.04%. The alternatives 40.04%, 36.03% and 38.04% do not follow from the given masses and the required 80/188 conversion factor; selecting any of them would require a different bromine mass.

Step 3: Conclusion The compound contains 34.04% bromine by mass. Hence, the correct option is (A).

Quick Tip to Solve: For a silver halide estimate, multiply precipitate mass by halogen atomic mass divided by silver-halide molar mass before dividing by sample mass.

73. Match List I with List II.

	List I		List II
(A)	Benzenesulphonyl chloride	(I)	Test for primary amines
(B)	Hoffmann bromamide reaction	(II)	Anti Saytzeff
(C)	Carbylamine reaction	(III)	Hinsberg reagent
(D)	Hoffmann orientation	(IV)	Known reaction of Isocyanates.

Choose the correct answer from the options given below: [4 marks]

(A) A-IV, B-III, C-II, D-I

(B) A-IV, B-II, C-I, D-III

(C) A-III, B-IV, C-I, D-II

(D) A-IV, B-III, C-I, D-II

Correct Answer: (C) A-III, B-IV, C-I, D-II

Solution:

Concept:

- Benzenesulphonyl chloride, $C_6H_5SO_2Cl$, is Hinsberg reagent. Its reactions with amines distinguish primary, secondary and tertiary amines through their different sulphonamide behaviour.
- Hofmann bromamide degradation proceeds through an isocyanate intermediate, the carbylamine reaction detects primary amines, and Hofmann elimination favours the less substituted alkene.

Step 1: Match the reagent and the rearrangement Entry A, benzenesulphonyl chloride, is the reagent used in the Hinsberg test. Therefore A matches III. In the Hoffmann bromamide reaction a primary amide reacts with bromine and alkali, loses its carbonyl carbon and gives a primary amine with one carbon fewer. Rearrangement produces an isocyanate intermediate, $R-N=C=O$, which is hydrolysed to the amine. Thus B matches IV. The printed phrase about isocyanates refers to this mechanistic association, rather than to the identity of Hinsberg reagent.

Step 2: Match the diagnostic test and elimination orientation In the carbylamine reaction, a primary amine reacts with chloroform and alcoholic alkali to give an offensive-smelling isocyanide. Secondary and tertiary amines do not give this test, so C matches I. Hoffmann orientation favours the less substituted alkene and is consequently described as anti-Saytzeff; hence D matches II. The full pattern is A-III, B-IV, C-I, D-II. Every other offered pattern assigns A to IV instead of III, so each is already excluded by the unambiguous identity of Hinsberg reagent.

Step 3: Conclusion The matching is A-III, B-IV, C-I, D-II. Hence, the correct option is (C).

Quick Tip to Solve: Anchor this matching with Hinsberg reagent = benzenesulphonyl chloride; then connect bromamide rearrangement with isocyanate and carbylamine with primary amines.

74. **Which of the following methods are not used to refine any metal?**

A. Liquation

B. Calcination

C. Electrolysis

D. Leaching

E. Distillation

Choose the correct answer from the options given below: [4 marks]

(A) B and D only

(B) A, B, D and E only

(C) B, D and E only

(D) A, C and E only

Correct Answer: (A) B and D only

Solution:

Concept:

- Refining means purifying an already obtained crude metal. Liquation, distillation and electrolytic refining separate the metal from impurities using physical properties or selective electrode processes.
- Calcination converts suitable ores into oxides by heating in limited or absent air; leaching selectively dissolves an ore constituent. These belong to ore processing rather than refining the crude metal.

Step 1: Classify each listed process by its purpose A, liquation, refines low-melting metals when the metal melts and flows away from higher-melting impurities. C, electrolysis, is used in electrolytic refining: impure metal forms the anode and pure metal deposits at the cathode under suitable conditions. E, distillation, refines volatile metals such as zinc by vaporising and condensing the metal while nonvolatile impurities remain behind. All three can therefore be used as metal-refining methods.

Step 2: Identify the processes before metal extraction B, calcination, removes volatile components and converts ores such as carbonates or hydrated oxides into oxides. For example, $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$; this step prepares an oxide for subsequent reduction rather than purifying zinc metal. D, leaching, relies on selective dissolution of an ore in a suitable reagent and separates it from gangue. Although electrolysis can also be used in extraction, it is not excluded here because it is certainly used in refining. Therefore the requested pair is B and D only. Options including A, C or E wrongly reject genuine refining methods.

Step 3: Conclusion Calcination and leaching, B and D only, are not metal-refining methods in this classification. Hence, the correct option is (A).

Quick Tip to Solve: Ask what enters the process: refining starts with crude metal; concentration and oxide preparation start with an ore or an ore-derived compound.

75. Given below are two statements.

Statement I : Hydrogen peroxide can act as an oxidizing agent in both acidic and basic conditions.

Statement II : Density of hydrogen peroxide at $298\ \mathrm{K}$ is lower than that of D_2O.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (C) Statement I is true but Statement II is false.

Solution:

Concept:

- Oxygen in H_2O_2 has oxidation number -1. Reduction to oxygen at -2 allows hydrogen peroxide to oxidise another substance in either acidic or alkaline medium.
- At approximately 298 K, pure hydrogen peroxide has density about $1.44\ \mathrm{g\,cm^{-3}}$, whereas heavy water has density about $1.10\ \mathrm{g\,cm^{-3}}$. Density comparisons must use the specified substances at the same temperature.

Step 1: Test the redox statement in both media An oxidising agent is reduced while oxidising the other reactant. In acid the reduction half-reaction is

$$\mathrm{H_2O_2 + 2\,H^+ + 2\,e^- \longrightarrow 2\,H_2O.}$$

In alkali it can be written as

$$\mathrm{H_2O_2 + 2\,e^- \longrightarrow 2\,OH^-.}$$

In each equation oxygen changes from -1 in peroxide to -2 in the product. Thus the ability to act as an oxidising agent is not restricted to acidic conditions, and Statement I is true. Its additional ability to act as a reducing agent in other reactions does not invalidate this statement.

Step 2: Compare the densities and eliminate the alternatives The density of pure hydrogen peroxide at room temperature exceeds that of heavy water: approximately 1.44 is greater than 1.10 in the same density units. Statement II says the opposite, so it is false. The greater molecular mass of heavy water than ordinary water does not make heavy water denser than every other oxygen-containing liquid. Option A requires both statements to be true, option B wrongly rejects Statement I, and option D reverses the established truth values; all three are excluded.

Step 3: Conclusion Statement I is true but Statement II is false. Hence, the correct option is (C).

Quick Tip to Solve: For peroxide redox questions, track oxygen from -1; for the density statement, compare pure hydrogen peroxide with heavy water rather than with ordinary water.

76. Given below are two statements.

Statement I: The chlorides of Be and Al have Cl-bridged structure. Both are soluble in organic solvents and act as Lewis bases.

Statement II: Hydroxides of Be and Al dissolve in excess alkali to give beryllate and aluminate ions.

In the light of the above statements, choose the correct answer from the options given below.

[4 marks]

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Correct Answer: (D) Statement I is false but Statement II is true.

Solution:

Concept:

- Electron-deficient chlorides of beryllium and aluminium form chlorine-bridged structures. Lewis acids accept an electron pair, whereas Lewis bases donate an electron pair.
- Beryllium hydroxide and aluminium hydroxide are amphoteric: they react with acids and also dissolve in excess alkali by forming hydroxo complex anions.

Step 1: Check every clause of Statement I The structural and solubility descriptions reflect the covalent character and diagonal relationship of beryllium and aluminium. Bridging chlorine atoms help complete the coordination of the electron-deficient centres, and their chlorides are soluble in suitable organic solvents. However, the assertion that these chlorides act as Lewis bases is incorrect. The metal centres can accept an electron pair from a donor, so $BeCl_2$ and $AlCl_3$ act as Lewis acids. One false clause makes the combined Statement I false even though its preceding structural observations are correct.

Step 2: Check dissolution of the hydroxides in excess alkali Both hydroxides dissolve in excess hydroxide through complex formation. Representative aqueous equations are

$$Be(OH)_2 + 2\,OH^- \longrightarrow [Be(OH)_4]^{2-},$$

$$Al(OH)_3 + OH^- \longrightarrow [Al(OH)_4]^-.$$

These are beryllate and aluminate hydroxo species, respectively, and establish the truth of Statement II. Options A and C incorrectly regard Statement I as true; option B incorrectly regards Statement II as false. This leaves the combination false, true, without needing to deny the genuine chlorine bridging in the first statement.

Step 3: Conclusion Statement I is false because the chlorides are Lewis acids; Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: In a compound statement, check the acid/base word separately: electron-deficient metal chlorides accept electron pairs, while amphoteric hydroxides dissolve in excess alkali.

77. Which oxoacid of phosphorous has the highest number of oxygen atoms present in its chemical formula? [4 marks]

(A) Pyrophosphorus acid

(B) Hypophosphoric acid

(C) Phosphoric acid

(D) Pyrophosphoric acid

Correct Answer: (D) Pyrophosphoric acid

Solution:

Concept:

- The number of oxygen atoms must be read from the complete molecular formula of each named acid. The oxidation state or the number of ionisable hydrogens is not itself the requested quantity.
- Pyro acids can be formed by condensation of two appropriate acid molecules with loss of one water molecule. This preserves both phosphorus atoms but reduces the combined oxygen count by one.

Step 1: Write the formula of each listed acid The four choices correspond to the following molecular formulae:

$$\begin{aligned} \text{Pyrophosphorus (pyrophosphorous) acid: } & \mathrm{H_4P_2O_5}, \\ \text{Hypophosphoric acid: } & \mathrm{H_4P_2O_6}, \\ \text{Phosphoric acid: } & \mathrm{H_3PO_4}, \\ \text{Pyrophosphoric acid: } & \mathrm{H_4P_2O_7}. \end{aligned}$$

The printed first name refers to the pyro derivative of phosphorous acid. It must not be confused with pyrophosphoric acid, which is a different option and contains two additional oxygen atoms.

Step 2: Compare the oxygen subscripts directly The oxygen counts in options A through D are 5, 6, 4 and 7, respectively. The largest is 7, belonging to pyrophosphoric acid. Its formula also follows from condensation of phosphoric acid:

$$2\,\mathrm{H_3PO_4} \longrightarrow \mathrm{H_4P_2O_7} + \mathrm{H_2O}.$$

Two molecules initially provide eight oxygen atoms, and removal of one water leaves seven in the acid. Hypophosphoric acid has six, so it is below the required maximum; the other two have five and four. Compare oxygen atoms per formula molecule, not per phosphorus atom, because the question asks about the chemical formula.

Step 3: Conclusion Pyrophosphoric acid, $\mathrm{H_4P_2O_7}$, contains seven oxygen atoms, the highest among these choices. Hence, the correct option is (D).

Quick Tip to Solve: Write all four molecular formulae before comparing; the similar names pyrophosphorous and pyrophosphoric correspond to five and seven oxygen atoms in their pyro acids.

78. Match List I with List II.

	List I Pollutant		List II Disease/sickness
(A)	Sulphate (>500 ppm)	(I)	Methemoglobinemia
(B)	Nitrate (>50 ppm)	(II)	Brown mottling of teeth
(C)	Lead (>50 ppb)	(III)	Laxative effect
(D)	Fluoride (>2 ppm)	(IV)	Kidney damage

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-I, C-II, D-III

(B) A-III, B-I, C-IV, D-II

(C) A-II, B-IV, C-I, D-III

(D) A-II, B-IV, C-III, D-I

Correct Answer: (B) A-III, B-I, C-IV, D-II

Solution:

Concept:

- Water pollutants produce characteristic physiological effects; identify each pollutant rather than comparing the numerical concentration alone.
- Sulphate causes a laxative effect; excess nitrate causes methemoglobinemia, while lead damages kidneys and excess fluoride mottles teeth.

Step 1: Match sulphate and nitrate Sulphate concentrations above $500\,\text{ppm}$ give a laxative effect, so A matches III. Nitrate above $50\,\text{ppm}$ is associated with methemoglobinemia (blue-baby syndrome), so B matches I. Nitrate can be reduced to nitrite, which oxidises haemoglobin iron and reduces its oxygen-carrying ability.

Step 2: Match lead and fluoride Lead at the stated concentration is associated with kidney damage, giving C–IV. Fluoride at low controlled concentrations helps protect teeth, but excess fluoride above $2\,\text{ppm}$ causes brown mottling of teeth, giving D–II. The complete association is therefore A–III, B–I, C–IV, D–II.

Step 3: Eliminate the remaining codes Option (A) assigns kidney damage to sulphate and mottling to lead, contradicting the characteristic effects. Options (C) and (D) both assign brown mottling to sulphate and kidney damage to nitrate; those assignments already rule them out. Only option (B) preserves all four correct pollutant–effect pairs.

Step 4: Conclusion The correct matching is A–III, B–I, C–IV, D–II. Hence, the correct option is (B).

Quick Tip to Solve: Anchor nitrate to blue-baby syndrome and fluoride to mottled teeth; then check sulphate and lead before selecting the matching code.

79. Match List I with List II.

	List I Polymers		List II Commercial names
(A)	Phenol-formaldehyde resin	(I)	Glyptal
(B)	Copolymer of 1,3-butadiene and sterene	(II)	Novolac
(C)	Polyester of glycol and phthalic acid	(III)	Buna-S
(D)	Polyester of glycol and terephthalic acid	(IV)	Dacron

Choose the correct answer from the options given below: [4 marks]

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-III, C-I, D-IV

(C) A-II, B-I, C-III, D-IV

(D) A-III, B-II, C-IV, D-I

Correct Answer: (B) A-II, B-III, C-I, D-IV

Solution:

Concept:

- Commercial polymer names are identified from the monomer combination and the type of polymerisation.
- Novolac is a phenol–formaldehyde resin; Buna-S is a butadiene–styrene copolymer; Glyptal and Dacron are polyesters with different aromatic diacids.

Step 1: Identify the resin and the rubber Phenol and formaldehyde first form a linear condensation resin known as Novolac, giving A–II. Further cross-linking can produce Bakelite, but the listed name for the stated resin is Novolac. Copolymerising 1,3-butadiene with styrene gives Buna-S (styrene-butadiene rubber), so B–III. The S refers to styrene and distinguishes it from Buna-N.

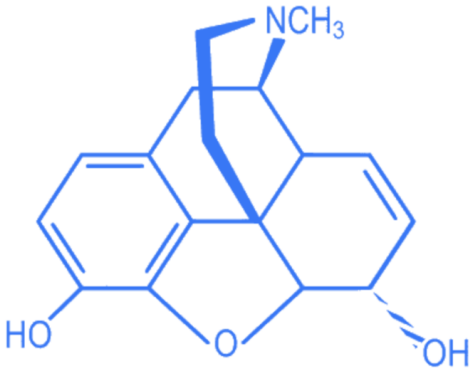
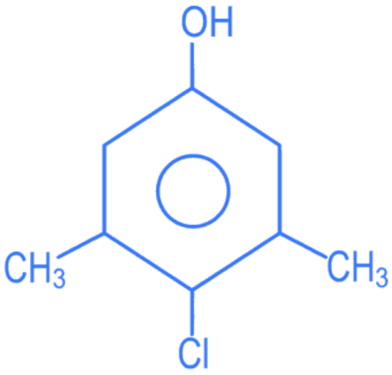
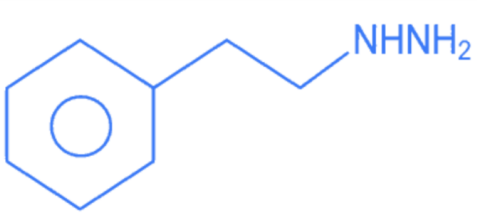
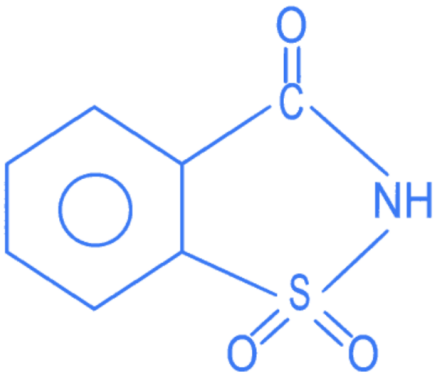
Step 2: Distinguish the two polyesters Glycol and phthalic acid form the polyester Glyptal, giving C–I. Glycol and terephthalic acid form polyethylene terephthalate, commercially called Dacron, giving D–IV. Although both products contain ester links, the position of the carboxyl groups in the aromatic diacid distinguishes their monomer identities.

Step 3: Check all option codes The complete code is A–II, B–III, C–I, D–IV. Option (A) interchanges the two polyester names. Option (C) assigns Glyptal to the butadiene–styrene rubber and Buna-S to a polyester. Option (D) misidentifies the phenolic resin as Buna-S. These errors leave only option (B).

Step 4: Conclusion The matching is A–II, B–III, C–I, D–IV. Hence, the correct option is (B).

Quick Tip to Solve: For the polyester pair, remember phthalic acid gives Glyptal whereas terephthalic acid gives Dacron; then use S in Buna-S for styrene.

80. Match List I with List II. Choose the correct answer from the options given below: [4 marks]

	List I		List II
(A)		(I)	Anti-depressant
(B)		(II)	550 times sweeter than cane sugar.
(C)		(III)	Narcotic analgesic
(D)		(IV)	Antiseptic

(A) A-IV, B-III, C-II, D-I

(B) A-III, B-I, C-II, D-IV

(C) A-III, B-IV, C-I, D-II

(D) A-III, B-I, C-IV, D-II

Correct Answer: (C) A-III, B-IV, C-I, D-II

Solution:

Concept:

- A chemical structure can be matched with a therapeutic or other use by recognising the characteristic molecular skeleton and functional groups.
- The structures shown are morphine (A), chloroxylenol (B), phenelzine (C) and saccharin (D); their uses are distinct.

Step 1: Identify morphine and chloroxylenol Structure A has the bridged morphine skeleton with a tertiary N-methyl amine, an ether bridge and hydroxyl groups. Morphine is a narcotic analgesic, so A–III. Structure B is a substituted phenol with two methyl groups and chlorine, identifying chloroxylenol. It is used as an antiseptic, so B–IV. The presence of a phenolic group alone does not make these two molecules interchangeable; their complete structures determine their functions.

Step 2: Identify phenelzine and saccharin Structure C contains the phenylalkyl hydrazine group, ending in $-NHNH_2$, and is phenelzine, an antidepressant. Thus C–I. Structure D is the fused cyclic sulfonimide saccharin, recognisable from the carbonyl and sulfonyl groups joined through nitrogen. Saccharin is an artificial sweetener about 550 times sweeter than cane sugar, giving D–II. Sweetness is the use being tested here rather than a therapeutic classification.

Step 3: Compare the complete matching with the options The complete set is A–III, B–IV, C–I, D–II. Option (A) incorrectly calls morphine an antiseptic; option (B) correctly starts with morphine but assigns chloroxylenol as an antidepressant and saccharin as an antiseptic. Option (D) interchanges the uses of chloroxylenol and phenelzine. Option (C) alone preserves all four correct assignments.

Step 4: Conclusion The correct matching is A–III, B–IV, C–I, D–II. Hence, the correct option is (C).

Quick Tip to Solve: Recognise saccharin by its carbonyl–sulfonimide ring and chloroxylenol by its chlorinated dimethylphenol ring; these two matches quickly isolate the code.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. $20\ \text{mL}$ **of** $0.02\ \text{M}\ K_2Cr_2O_7$ **solution is used for the titration of** $10\ \text{mL}$ **of** Fe^{2+} **solution in the acidic medium.**

The molarity of Fe^{2+} **solution is** _____ $\times 10^{-2}\ \text{M}$**. (Nearest Integer)** [4 marks]

Solution:

Concept:

- In acidic medium, one dichromate ion gains six electrons as its two chromium atoms change from oxidation state $+6$ to $+3$.
- Each iron(II) ion loses one electron to become iron(III); therefore one mole of dichromate oxidises six moles of iron(II).

Step 1: Balance the electron transfer The reduction and oxidation half-reactions are

$$\mathrm{Cr_2O_7^{2-} + 14\,H^+ + 6\,e^- \longrightarrow 2\,Cr^{3+} + 7\,H_2O}, \qquad \mathrm{Fe^{2+} \longrightarrow Fe^{3+} + e^-}.$$

Multiplying the iron half-reaction by six gives the mole ratio

$$n(\mathrm{Fe^{2+}}) = 6n(\mathrm{Cr_2O_7^{2-}}).$$

The acidic medium is essential for using this dichromate reduction stoichiometry.

Step 2: Calculate the dichromate amount Convert the titrant volume to litres before multiplying by molarity:

$$n(\mathrm{K_2Cr_2O_7}) = 0.02\,\mathrm{mol\,L^{-1}} \times 20 \times 10^{-3}\,\mathrm{L} = 4.0 \times 10^{-4}\,\mathrm{mol}.$$

Each formula unit supplies one dichromate ion, so the amount of iron(II) in the aliquot is

$$n(\mathrm{Fe^{2+}}) = 6(4.0 \times 10^{-4}) = 2.4 \times 10^{-3}\,\mathrm{mol}.$$

Step 3: Find the molarity and requested coefficient This iron amount belongs to the $10\,\mathrm{mL}$ aliquot, not the titrant volume. Hence

$$M_{\mathrm{Fe^{2+}}} = \frac{2.4 \times 10^{-3}}{10 \times 10^{-3}} = 0.24\,\mathrm{mol\,L^{-1}} = 24 \times 10^{-2}\,\mathrm{M}.$$

The blank asks for the coefficient multiplying $10^{-2}\,\mathrm{M}$, so no additional rounding changes it.

Step 4: Conclusion The iron(II) molarity is $0.24\,\mathrm{M} = 24 \times 10^{-2}\,\mathrm{M}$. The required nearest-integer entry is $\boxed{24}$.

Quick Tip to Solve: Balance electrons first: dichromate has a factor of six, and the final iron concentration uses the aliquot volume of 10 mL.

82.

$$2NO + 2H_2 \rightarrow N_2 + 2H_2O$$

The above reaction has been studied at $800^\circ C$. The related data are given in the table below

Reaction serial number	Initial Pressure of H_2/kPa	Initial Pressure of NO/kPa	Initial rate $(-dp/dt)/(kPa/s)$
1	65.6	40.0	0.135
2	65.6	20.1	0.033
3	38.6	65.6	0.214
4	19.2	65.6	0.106

The order of the reaction with respect to NO is _____. [4 marks]

Solution:

Concept:

- An experimental rate law has the form $r = kp_{NO}^m p_{H_2}^n$ at a fixed temperature; partial pressure is proportional to concentration.
- To isolate the order in one reactant, compare experiments in which the other reactant has the same initial pressure. Reaction coefficients alone do not establish an experimental order.

Step 1: Choose experiments with fixed hydrogen pressure Experiments 1 and 2 both have hydrogen pressure $65.6\,\text{kPa}$. Thus the hydrogen term and the constant k cancel when their rates are divided. The NO pressures are 40.0 and $20.1\,\text{kPa}$, with initial rates 0.135 and $0.033\,\text{kPa}\,\text{s}^{-1}$, respectively.

Step 2: Compare the pressure ratio with the rate ratio The rate-law ratio gives

$$\frac{0.135}{0.033} = \left(\frac{40.0}{20.1}\right)^m.$$

Numerically,

$$\frac{0.135}{0.033} \simeq 4.09, \qquad \frac{40.0}{20.1} \simeq 1.99.$$

Doubling the NO pressure therefore approximately quadruples the rate. Since $2^2 = 4$, the order in NO is two. Equivalently, logarithms give

$$m = \frac{\ln(0.135/0.033)}{\ln(40.0/20.1)} \simeq 2.05,$$

which identifies the integer order two within the precision of the experimental data.

Step 3: Separate this result from the hydrogen order Experiments 3 and 4 keep NO pressure fixed and roughly double both hydrogen pressure and rate; they would identify first order in hydrogen. That does not change the requested order with respect to NO. The answer is the exponent m, a dimensionless number, rather than the overall order or a stoichiometric coefficient inferred without evidence.

Step 4: Conclusion The reaction is second order with respect to NO. The required dimensionless entry is $\boxed{2}$.

Quick Tip to Solve: Use rows 1 and 2 for the NO order because hydrogen pressure is unchanged; a twofold pressure change causing a fourfold rate change means order two.

83. Amongst the following, the number of oxide(s) which are paramagnetic in nature is

$$Na_2O, KO_2, NO_2,\ N_2O, ClO_2, NO, SO_2, Cl_2O$$

[4 marks]

Solution:

Concept:

- An oxide is paramagnetic when it contains an unpaired electron; a closed-shell species with all electrons paired is diamagnetic.
- Odd-electron molecules such as NO, NO_2 and ClO_2 are paramagnetic. The superoxide ion O_2^- also has one unpaired electron.

Step 1: Identify the superoxide In KO_2, potassium is present as K^+, so the oxygen species is superoxide, O_2^-. Its molecular-orbital occupancy leaves one electron unpaired in an antibonding π^* orbital. Thus KO_2 is paramagnetic. By contrast, Na_2O contains closed-shell oxide ions O^{2-}, and it is diamagnetic.

Step 2: Count the odd-electron molecular oxides Count valence electrons for the relevant neutral molecules. For NO, the total is $5 + 6 = 11$; for NO_2, it is $5 + 2(6) = 17$; for ClO_2, it is $7 + 2(6) = 19$. Each has an odd electron count and an unpaired electron, so all three are paramagnetic. These add three to the superoxide already identified.

Step 3: Check the remaining species N_2O has $2(5) + 6 = 16$ valence electrons and a closed-shell ground state. SO_2 has $6 + 2(6) = 18$ and is diamagnetic. Cl_2O has $2(7) + 6 = 20$ and also has all electrons paired in its ground state. Therefore none of these three increases the count. An even electron count alone is not a universal proof of diamagnetism, but these particular ground states are closed shell.

Step 4: Conclusion The paramagnetic oxides are KO_2, NO_2, ClO_2 and NO. Their number is $\boxed{4}$, a dimensionless count.

Quick Tip to Solve: Check ionic superoxides separately from molecular oxides: KO_2 contains the superoxide ion with one unpaired electron.

84. The molar heat capacity for an ideal gas at constant pressure is $20.785\ J\ K^{-1}\ mol^{-1}$. The change in internal energy is $5000\ J$ upon heating it from $300\ K$ to $500\ K$. The number of moles of the gas at constant volume is _____. [Nearest integer] (Given: $R = 8.314\ J\ K^{-1}\ mol^{-1}$) [4 marks]

Solution:

Concept:

- For an ideal gas, molar heat capacities satisfy Mayer relation $C_{P,m} - C_{V,m} = R$.
- The internal-energy change of an ideal gas is $\Delta U = nC_{V,m}\Delta T$ when the molar heat capacity is constant over the temperature interval.

Step 1: Calculate the molar constant-volume heat capacity The given value is $C_{P,m} =$

$20.785\,\mathrm{J\,K^{-1}\,mol^{-1}}$, while $R = 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$. Therefore

$$C_{V,m} = C_{P,m} - R = 20.785 - 8.314 = 12.471\,\mathrm{J\,K^{-1}\,mol^{-1}}.$$

Using $C_{P,m}$ directly for internal energy would overestimate the energy per mole: at constant pressure part of the heat supplies expansion work.

Step 2: Use the temperature rise in the energy equation The temperature increases from $300\,\mathrm{K}$ to $500\,\mathrm{K}$, so

$$\Delta T = 500 - 300 = 200\,\mathrm{K}.$$

Substitute this difference, rather than either absolute temperature, into

$$5000 = n(12.471)(200).$$

The product $C_{V,m}\Delta T$ is

$$12.471 \times 200 = 2494.2\,\mathrm{J\,mol^{-1}}.$$

The sign is positive because heating raises the internal energy.

Step 3: Determine and round the amount of gas Solve for the number of moles:

$$n = \frac{5000\,\mathrm{J}}{2494.2\,\mathrm{J\,mol^{-1}}} \simeq 2.00465\,\mathrm{mol}.$$

Rounding to the nearest integer gives two moles. The supplied decimal constants make the unrounded result slightly above two; the requested integer convention resolves this without changing the governing equation.

Step 4: Conclusion The gas amount is approximately $2.00465\,\mathrm{mol}$, so the nearest-integer entry is $\boxed{2}$ (moles).

Quick Tip to Solve: Convert Cp to Cv by subtracting R before using an internal-energy change, and use the temperature difference of 200 K.

85. According to MO theory, number of species/ions from the following having identical bond order is _____.

$$\mathrm{CN^-, NO^+, O_2, O_2^+, O_2^{2+}}$$

[4 marks]

Solution:

Concept:

- Bond order is $\mathrm{BO} = (N_b - N_a)/2$, where N_b and N_a are the numbers of electrons occupying bonding and antibonding molecular orbitals.
- Isoelectronic diatomic species have the same valence-electron count and, for the species here, the same bond order. Removing an electron from an antibonding orbital increases bond order by

one-half.

Step 1: Count the valence electrons of the isoelectronic species For CN^-, carbon supplies four valence electrons, nitrogen supplies five, and the negative charge adds one. Thus its valence-electron count is $4 + 5 + 1 = 10$. For NO^+, the corresponding count is $5 + 6 - 1 = 10$. In the ten-valence-electron configuration the two $2s$ orbitals contain two bonding and two antibonding electrons; the six $2p$ electrons occupy bonding orbitals. Hence $N_b = 8$, $N_a = 2$, and both species have bond order $(8 - 2)/2 = 3$.

Step 2: Compare the oxygen molecule and its positive ions Neutral O_2 has twelve valence electrons, including two electrons in antibonding π^* orbitals. Its bond order is $(8 - 4)/2 = 2$. The first ionisation removes one of these antibonding electrons, whereas the second removes the other. Therefore

$$\mathrm{BO}(O_2^+) = \frac{8-3}{2} = 2.5,$$
$$\mathrm{BO}(O_2^{2+}) = \frac{8-2}{2} = 3.$$

The five bond orders, in the given order, are therefore $3, 3, 2, 2.5, 3$. Only the group with bond order three contains mutually identical bond orders; the other two values occur once each.

Step 3: Conclusion The species CN^-, NO^+ and O_2^{2+} all have bond order 3. Thus the required number of species is $\boxed{3}$.

Quick Tip to Solve: Count valence electrons first: the three species with ten valence electrons form the bond-order-three group; ionising O_2 removes antibonding electrons.

86. At 310 K, the solubility of CaF_2 in water is 2.34×10^{-3} g/100 mL. The solubility product of CaF_2 is _____ $\times 10^{-8}$(mol/L)^3. (Give molar mass : $CaF_2 = 78$ g mol^{-1}) [4 marks]

Solution:

Concept:

- For a sparingly soluble salt, the solubility product is the product of equilibrium ionic concentrations raised to their stoichiometric powers.
- For $CaF_2(s) \rightleftharpoons Ca^{2+}(aq) + 2\,F^-(aq)$, molar solubility s gives $[Ca^{2+}] = s$ and $[F^-] = 2s$, so $K_{sp} = 4s^3$.

Step 1: Convert the given mass solubility to molar solubility The stated solubility is per $100\,\mathrm{mL}$, so multiply by ten to express the mass dissolved per litre. Divide this mass concentration by the molar mass, keeping grams and litres consistent:

$$c_m = 10(2.34 \times 10^{-3}) = 2.34 \times 10^{-2}\,\mathrm{g\,L^{-1}},$$
$$s = \frac{2.34 \times 10^{-2}\,\mathrm{g\,L^{-1}}}{78\,\mathrm{g\,mol^{-1}}} = 3.00 \times 10^{-4}\,\mathrm{mol\,L^{-1}}.$$

This is the concentration of dissolved formula units, not the fluoride concentration.

Step 2: Use the dissociation stoichiometry and express the requested coefficient Every mole of

dissolved salt supplies one mole of calcium ions and two moles of fluoride ions. In water with no added common ion, use the concentrations arising from this dissociation:

$$\begin{aligned} K_{sp} &= (3.00 \times 10^{-4})(6.00 \times 10^{-4})^2 \\ &= 4(3.00 \times 10^{-4})^3 \\ &= 1.08 \times 10^{-10}\,(\text{mol L}^{-1})^3 \\ &= 0.0108 \times 10^{-8}\,(\text{mol L}^{-1})^3. \end{aligned}$$

Thus the coefficient in the printed blank is 0.0108. For the integer-valued numerical response, this coefficient rounds to zero. Rounding applies to the requested coefficient only; the physical solubility product remains positive.

Step 3: Conclusion $K_{sp} = 1.08 \times 10^{-10}\,(\text{mol L}^{-1})^3 = 0.0108 \times 10^{-8}\,(\text{mol L}^{-1})^3$. The nearest-integer entry for the blank is $\boxed{0}$, agreeing with the numerical answer key.

Quick Tip to Solve: Convert $\text{g}/100\,\text{mL}$ to mol/L before cubing; then express the answer using the power of ten already printed beside the blank.

87. The conductivity of a solution of complex with formula $CoCl_3\,(NH_3)_4$ corresponds to 1 : 1 electrolyte, then the primary valency of central metal ion is _____. [4 marks]

Solution:

Concept:

- In Werner theory, primary valency is the oxidation state of the central metal; secondary valency is its coordination number.
- A $1:1$ electrolyte dissociates into one complex cation and one counter-anion per formula unit. Coordinated ligands remain inside the coordination sphere during this dissociation.

Step 1: Identify the coordination sphere using conductivity The formula contains one cobalt atom, three chloride ions and four neutral ammonia molecules. Conductivity corresponding to a $1:1$ electrolyte requires two ions in solution, one cation and one anion. Therefore one chloride is outside the coordination sphere, while the other two chlorides are coordinated to cobalt together with the four ammonia ligands. The formulation is

$$[Co(NH_3)_4Cl_2]Cl \longrightarrow [Co(NH_3)_4Cl_2]^+ + Cl^-.$$

The complex ion has charge $+1$, balancing the charge -1 of the single outer-sphere chloride. Writing all three chlorides outside would instead produce a different electrolyte type.

Step 2: Calculate the metal oxidation state Let the oxidation state of cobalt in the complex cation be x. Each ammonia ligand is neutral and each coordinated chloride contributes -1. The charge balance inside the brackets gives

$$x + 4(0) + 2(-1) = +1, \qquad x = +3.$$

The same result follows from the overall neutral formula: the three chloride ions together contribute

-3, so cobalt must contribute $+3$. The metal has six donor atoms around it, four from ammonia and two from chloride, so its secondary valency is six. This coordination number must not be confused with the primary valency asked for.

Step 3: Conclusion The cobalt ion is in oxidation state $+3$. Its primary valency is therefore $\boxed{3}$.

Quick Tip to Solve: Use conductivity to count ions outside the brackets, then use charge balance for primary valency; count coordinated donor atoms only for secondary valency.

88. In the titration of $KMnO_4$ and oxalic acid in acidic medium, the change in oxidation number of carbon at the end point is _____. [4 marks]

Solution:

Concept:

- Oxidation number measures the formal electron allocation to an atom. The algebraic sum of oxidation numbers equals the charge of the molecule or ion.
- In acidic permanganate titration, oxalic acid is oxidised to carbon dioxide while permanganate is reduced to Mn^{2+}. The requested change concerns carbon, not manganese.

Step 1: Find the initial oxidation number of carbon in oxalic acid Oxalic acid is $H_2C_2O_4$. Its two carbon atoms are equivalent. Assign the usual oxidation numbers $+1$ to hydrogen and -2 to oxygen. If the oxidation number of each carbon is x, neutrality requires

$$2(+1) + 2x + 4(-2) = 0.$$

Hence $2x = 6$, giving $x = +3$. The total oxidation-number contribution of both carbon atoms is $+6$, but the oxidation number of one carbon atom is $+3$.

Step 2: Compare with carbon in the oxidation product The carbon-containing product is CO_2. Writing the oxidation number of its carbon as y,

$$y + 2(-2) = 0, \qquad y = +4.$$

Thus each carbon increases its oxidation number by one unit. The half-reaction confirms the electron count:

$$H_2C_2O_4 \longrightarrow 2\,CO_2 + 2\,H^+ + 2\,e^-.$$

Two carbon atoms lose two electrons in total, so each carbon loses one electron. At the end point, the oxidation of the oxalic acid has been completed; this does not make carbon undergo any further change. The manganese change from $+7$ to $+2$ belongs to the oxidising agent and is irrelevant to the requested carbon change.

Step 3: Conclusion Carbon changes from $+3$ in $H_2C_2O_4$ to $+4$ in CO_2. The increase in oxidation number per carbon atom is $\boxed{1}$.

Quick Tip to Solve: For a redox question, identify the named atom first. Oxalic acid loses two electrons per molecule because it contains two carbons, each changing by only one unit.

89. Optical activity of an enantiomeric mixture is $+12.6^\circ$ and the specific rotation of $(+)$ isomer is $+30^\circ$. The optical purity is _____%. [4 marks]

Solution:

Concept:

- Optical purity is the observed rotation of an enantiomeric mixture divided by the rotation of the pure enantiomer under equivalent measurement conditions, multiplied by 100.
- Enantiomers rotate plane-polarised light equally in opposite directions. Their contributions cancel in equal amounts, so the net rotation measures the excess of one enantiomer over the other.

Step 1: Set up the rotation ratio The mixture has positive rotation $+12.6^\circ$, and the pure positive enantiomer has rotation $+30^\circ$. The positive sign shows that the positive enantiomer is present in excess. Using the rotations on the common basis intended in the question, the optical purity is

$$\text{Optical purity} = \frac{\text{rotation of mixture}}{\text{rotation of pure positive enantiomer}} \times 100.$$

Both rotations have the same angular unit, so the units cancel. The result is a percentage rather than an angle.

Step 2: Calculate the purity and distinguish it from composition Substitution gives

$$\text{Optical purity} = \frac{12.6}{30} \times 100 = 0.42 \times 100 = 42\%.$$

This is also the enantiomeric excess of the positive form. To check the meaning, let its mole fraction be f, with the negative form occupying fraction $1 - f$. Add their opposite rotations:

$$12.6 = 30f - 30(1 - f) = 30(2f - 1).$$

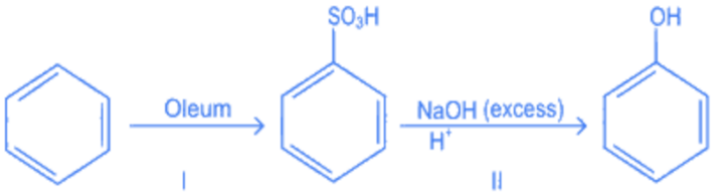
Thus $2f - 1 = 0.42$, giving $f = 0.71$. A mixture with 71% positive form and 29% negative form has excess $71 - 29 = 42\%$, confirming the rotation ratio. The fraction of the major enantiomer is therefore not itself the optical purity; the equal portions cancel.

Step 3: Conclusion The optical purity of the mixture is $\boxed{42\%}$, so the numerical entry is $\boxed{42}$. The positive enantiomer is in excess.

Quick Tip to Solve: Optical purity equals enantiomeric excess, not the percentage of the major enantiomer. Divide observed rotation by pure-enantiomer rotation before multiplying by 100.

90. In the following reaction

The % yield for reaction I is 60% and that of reaction II is 50%. The overall yield of the complete reaction is _____ %. [nearest integer] [4 marks]



Solution:

Concept:

- For consecutive reactions, each step acts on the material actually obtained from the previous step. The overall fractional yield is the product of the individual fractional yields.
- The displayed sequence converts benzene into benzenesulfonic acid and then phenol. Each stage preserves one aromatic ring per molecule, so the theoretical molar conversion for the full sequence is one-to-one.

Step 1: Apply the first percentage yield to a convenient starting amount Choose $100\,\text{mol}$ of benzene as the basis. Reaction I is sulfonation by oleum to give benzenesulfonic acid. The theoretical amount of this intermediate is $100\,\text{mol}$, since one benzene molecule gives one intermediate molecule. At a 60% yield, the actual amount available for the next reaction is

$$n_{\text{intermediate}} = 100 \times \frac{60}{100} = 60\,\text{mol}.$$

The unconverted or lost portion from the first stage cannot contribute to phenol in the second stage.

Step 2: Apply the second yield to the intermediate actually formed Reaction II uses excess sodium hydroxide followed by acidification to convert the intermediate into phenol. Its 50% yield applies to the $60\,\text{mol}$ of intermediate obtained, rather than to the original $100\,\text{mol}$ of benzene. Therefore

$$n_{\text{phenol}} = 60 \times \frac{50}{100} = 30\,\text{mol}.$$

The theoretical yield of phenol from the original basis is $100\,\text{mol}$. Hence

$$\text{Overall yield} = \frac{30}{100} \times 100 = 30\%.$$

Equivalently, multiply the fractional yields directly: $0.60 \times 0.50 = 0.30$. Adding or averaging the two percentages would not account for material lost before the second reaction begins.

Step 3: Conclusion The overall percentage yield is $60 \times 50/100 = \boxed{30\%}$. The nearest-integer numerical entry is $\boxed{30}$.

Quick Tip to Solve: For successive steps, multiply fractional yields. A 50% second-step yield means half of the first-step product, not half of the original starting material.