JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 27 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 27 July 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. The domain of the function $f(x) = \sin^{-1}\left[2x^2 - 3\right] + \log_2\left(\log_{\frac{1}{2}}\left(x^2 - 5x + 5\right)\right)$, where [t] is the greatest integer function, is : [4 marks]

(A) $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2}\right)$

(B) $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$

(C) $\left(1, \frac{5-\sqrt{5}}{2}\right)$

(D) $\left[1, \frac{5+\sqrt{5}}{2}\right)$

Correct Answer: (C) $\left(1, \frac{5-\sqrt{5}}{2}\right)$

Solution:

Concept:

- The square brackets denote the greatest integer function: $[u] = k$ means $k \le u < k+1$. The real inverse sine requires its argument to lie in $[-1, 1]$.
- A real logarithm requires a strictly positive argument. Since the inner logarithm has base $1/2 < 1$, its value is positive precisely when its argument lies strictly between zero and one.

Step 1: Apply the inverse sine restriction The integer $[2x^2 - 3]$ may be only $-1, 0, 1$. Combining the three adjacent intervals gives

$$-1 \le 2x^2 - 3 < 2 \quad \Longleftrightarrow \quad 1 \le x^2 < \frac{5}{2}.$$

Thus x belongs to $(-\sqrt{5/2}, -1] \cup [1, \sqrt{5/2})$. The upper bound is strict because the greatest integer would become 2 there.

Step 2: Apply both logarithmic restrictions Writing $Q = x^2 - 5x + 5$, the outer logarithm requires $\log_{1/2} Q > 0$; together with the inner logarithm this means $0 < Q < 1$. Now

$$Q > 0 \iff x < \frac{5-\sqrt{5}}{2} \text{ or } x > \frac{5+\sqrt{5}}{2},$$

$$Q < 1 \iff (x-1)(x-4) < 0 \iff 1 < x < 4.$$

The logarithmic domain is consequently $(1, (5-\sqrt{5})/2) \cup ((5+\sqrt{5})/2, 4)$.

Step 3: Intersect the restrictions The negative inverse sine interval contributes no point because the logarithmic domain is positive. The second logarithmic interval starts above $\sqrt{5/2}$, so it also contributes nothing. The surviving interval is $(1, (5-\sqrt{5})/2)$. Option A wrongly includes negative values; B includes points with a nonpositive inner logarithm argument; D includes 1 and points beyond the inverse sine restriction.

Step 4: Conclusion The domain is $\left(1, \frac{5-\sqrt{5}}{2}\right)$. Hence, the correct option is (C).

Quick Tip to Solve: For nested logarithms with an inner base below one, first replace positivity of the inner logarithm by the double inequality $0 < Q < 1$.

2. Let S be the set of all $(\alpha, \beta), \pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta, (\alpha, \beta) \in S$. Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}}\right)$ is equal to : [4 marks]

(A) 3

(B) 3 i

(C) 1

(D) 2 − i

Correct Answer: (C) 1

Solution:

Concept:

- Multiply a complex quotient by the conjugate of its denominator to separate its real and imaginary parts. A purely imaginary number has zero real part, whereas a purely real number has zero imaginary part.
- The angular restrictions $\pi < \alpha, \beta < 2\pi$ determine the signs and all permitted angles; each ordered pair must be included exactly once in the requested sum.

Step 1: Determine the permitted angles Put $s = \sin\alpha$, $c = \cos\beta$. Rationalising gives

$$\frac{1-is}{1+2is} = \frac{1-2s^2-3is}{1+4s^2}, \qquad \frac{1+ic}{1-2ic} = \frac{1-2c^2+3ic}{1+4c^2}.$$

Their denominators are positive. Therefore $1-2s^2 = 0$ and $3c = 0$. Since sine is negative on the given interval, $s = -1/\sqrt{2}$, giving $\alpha = 5\pi/4, 7\pi/4$. Also $\beta = 3\pi/2$.

Step 2: Calculate the two complex values For both pairs, $\cos 2\beta = \cos 3\pi = -1$. Also $\sin(5\pi/2) = 1$ and $\sin(7\pi/2) = -1$. Hence the two values of Z are $1-i$ and $-1-i$, each of squared modulus 2. Neither value is zero, so the reciprocal in the sum exists.

Step 3: Evaluate and add the summands Using $1/\overline{Z} = Z/|Z|^2$ and $1/i = -i$,

$$iZ + \frac{1}{i\overline{Z}} = iZ - \frac{iZ}{2} = \frac{iZ}{2}.$$

The two summands are $(1+i)/2$ and $(1-i)/2$, whose sum is 1. Their imaginary parts cancel, ruling out B and D. Their real parts add to 1 rather than 3, ruling out A. Both allowed pairs have been counted; choosing only one angle would leave an erroneous imaginary part.

Step 4: Conclusion The required sum is 1. Hence, the correct option is (C).

Quick Tip to Solve: After finding the possible values of Z, check their common modulus before evaluating reciprocals; $1/\overline{Z} = Z/|Z|^2$ often collapses the sum.

3. If α, β are the roots of the equation

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right) x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0,$$

then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is : [4 marks]

(A) $3x^2 - 20x - 12 = 0$

(B) $3x^2 - 10x - 4 = 0$

(C) $3x^2 - 10x + 2 = 0$

(D) $3x^2 - 20x + 16 = 0$

Correct Answer: (B) $3x^2 - 10x - 4 = 0$

Solution:

Concept:

- For a quadratic $x^2 - Sx + P = 0$, the sum and product of its roots are S and P. A new quadratic is obtained from the sum and product of the transformed roots.
- Compare exponential expressions by taking logarithms. The identities $\log_5 3 = 1/\log_3 5$ and $a^b = e^{b \ln a}$ show when the apparently different powers are equal.

Step 1: Simplify the coefficients Let $t = \log_3 5 > 0$; then $\ln 5 = t \ln 3$. The logarithms of $3^{\sqrt{t}}$ and $5^{\sqrt{1/t}}$ are both $\sqrt{t} \ln 3$. Thus these two powers cancel. Similarly,

$$\ln(3^{t^{1/3}}) = t^{1/3} \ln 3, \qquad \ln(5^{(1/t)^{2/3}}) = t^{1/3} \ln 3,$$

so the powers in the constant coefficient also cancel. The original equation is therefore $x^2 - 5x - 3 = 0$, giving $\alpha + \beta = 5$, $\alpha\beta = -3$.

Step 2: Find the sum of the new roots The nonzero product shows that both reciprocals exist. If $u = \alpha + 1/\beta$ and $v = \beta + 1/\alpha$, then

$$u + v = (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta} = 5 - \frac{5}{3} = \frac{10}{3}.$$

This step uses the reciprocal sum, not the reciprocal of the root sum.

Step 3: Find the product and form the equation Expanding carefully,

$$uv = \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = -3 + 2 - \frac{1}{3} = -\frac{4}{3}.$$

Hence the monic equation is $x^2 - (10/3)x - 4/3 = 0$. Multiplying through by 3 gives $3x^2 - 10x - 4 = 0$. A and D have the wrong root sum $20/3$, while C has the wrong positive product $2/3$.

Step 4: Conclusion The equation is $3x^2 - 10x - 4 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: When two powers with interchanged bases look unrelated, express their logarithms using one variable $t = \log_a b$ before using root formulas.

4. If for $\mathrm{p} \neq \mathrm{q} \neq 0$, the function $f(x) = \frac{\sqrt[7]{\mathrm{p}(729+x)}-3}{\sqrt[3]{729+\mathrm{q}x}-9}$ is continuous at $x = 0$, then : [4 marks]

(A) $7pq\, f(0) - 1 = 0$

(B) $63q\, f(0) - \mathrm{p}^2 = 0$

(C) $21q\, f(0) - \mathrm{p}^2 = 0$

(D) $7pq\, f(0) - 9 = 0$

Correct Answer: (B) $63q\, f(0) - \mathrm{p}^2 = 0$

Solution:

Concept:

- A finite continuous extension at a point where the denominator vanishes requires the numerator to vanish there as well. This first determines the parameter before the limit is evaluated.
- For nonzero differentiable numerator and denominator slopes, the ratio near a common zero tends to the ratio of their derivatives. The power rule applies to the real seventh and cube roots near their positive arguments.

Step 1: Use the zero denominator to determine p At $x = 0$, the denominator is $\sqrt[3]{729} - 9 = 0$. Since $q \neq 0$, it has a simple zero. A finite limit requires

$$\sqrt[7]{729p} - 3 = 0 \quad \Longrightarrow \quad 729p = 3^7 = 2187 \quad \Longrightarrow \quad p = 3.$$

Without this condition the numerator tends to a nonzero constant while the denominator tends to zero. Notice that $p \neq q$ now excludes $q = 3$, in addition to $q = 0$.

Step 2: Evaluate the removable limit Let $N(x) = [p(729 + x)]^{1/7} - 3$, $D(x) = (729 + qx)^{1/3} - 9$. At the common zero,

$$N'(0) = \frac{p}{7(729p)^{6/7}} = \frac{p}{7 \cdot 3^6}, \qquad D'(0) = \frac{q}{3 \cdot 729^{2/3}} = \frac{q}{243}.$$

Thus continuity gives

$$f(0) = \frac{N'(0)}{D'(0)} = \frac{p}{21q} = \frac{1}{7q}.$$

The derivative of the denominator is nonzero because $q \neq 0$.

Step 3: Check the proposed relations With $p = 3$, $63qf(0) = 63/7 = 9 = p^2$, proving B. By contrast $7pqf(0) = p = 3$, so neither A nor D vanishes. Also $21qf(0) = 3 \neq 9 = p^2$, eliminating C. These substitutions check every option without treating p as an arbitrary parameter after the continuity condition has fixed it.

Step 4: Conclusion The required relation is $63qf(0) - p^2 = 0$. Hence, the correct option is (B).

Quick Tip to Solve: In a parameterised zero-denominator limit, impose the numerator zero first; otherwise applying a derivative ratio can produce a relation for an inadmissible parameter.

5. Let $f(x) = 2 + |x| - |x-1| + |x+1|, x \in \mathbf{R}$.

Consider

$$(\text{S1}) : f'\left(-\frac{3}{2}\right) + f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 2$$

$$(\text{S2}) : \int_{-2}^{2} f(x)\mathrm{d}x = 12$$

Then, [4 marks]

(A) both (S1) and (S2) are correct

(B) both (S1) and (S2) are wrong

(C) only (S1) is correct

(D) only (S2) is correct

Correct Answer: (D) only (S2) is correct

Solution:

Concept:

- The sign of $x - a$ determines the derivative of $|x - a|$ away from its corner: it is -1 to the left and 1 to the right. Split the line at all absolute-value zeros.
- On a symmetric interval the integrals of $|x - 1|$ and $|x + 1|$ are equal by the substitution $x \mapsto -x$. They therefore cancel when they appear with opposite signs.

Step 1: Evaluate the four derivatives The breakpoints are $-1, 0, 1$. Away from them,

$$f'(x) = \operatorname{sgn}(x) - \operatorname{sgn}(x-1) + \operatorname{sgn}(x+1).$$

On the four successive intervals $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$, the derivatives are respectively $-1, 1, 3, 1$. Each test point lies strictly inside its interval, so no derivative at a corner is needed. Hence

$$f'(-3/2) + f'(-1/2) + f'(1/2) + f'(3/2) = -1 + 1 + 3 + 1 = 4.$$

Statement S1, which claims 2, is false.

Step 2: Evaluate the definite integral By linearity,

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{2} 2\,dx + \int_{-2}^{2} |x|\,dx - \int_{-2}^{2} |x-1|\,dx + \int_{-2}^{2} |x+1|\,dx.$$

The last two integrals cancel by symmetry. The remaining terms are

$$2(2 - (-2)) + 2\int_{0}^{2} x\,dx = 8 + 4 = 12.$$

Thus S2 is true. Equivalently, both shifted absolute-value integrals individually equal 5, confirming their cancellation.

Step 3: Compare the truth values The derivative test gives false for S1 and the integral gives true for S2. Option A incorrectly accepts S1, B incorrectly rejects S2, and C reverses both truth values. Only the statement pair in D matches the independent calculations. One must not infer the derivative sum from the integral: the two claims concern different properties of the same function.

Step 4: Conclusion Only (S2) is correct. Hence, the correct option is (D).

Quick Tip to Solve: For a sum of shifted absolute values, tabulate derivative signs between the corners; for symmetric integrals, pair the shifts $+a$ and $-a$.

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6. Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5 . Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is $10ar$, n^{th} term is a_n and the common difference is $10ar^2$, is equal to : [4 marks]

(A) $21\,a_{11}$

(B) $22\,a_{11}$

(C) $15\,a_{16}$

(D) $14\,a_{16}$

Correct Answer: (A) $21\,a_{11}$

Solution:

Concept:

- For an AP with first term A and difference d, $a_n = A + (n-1)d$ and $S_n = n[2A + (n-1)d]/2$. An odd number of AP terms has average equal to its middle term.
- An infinite real GP converges for $|r| < 1$, with sum $a/(1-r)$; its first five terms sum to $a(1-r^5)/(1-r)$. These data may be used to check that the other proposed expressions are unequal.

Step 1: Express the sum using the middle term Here $A = 10ar$ and $d = 10ar^2$. Since 21 is odd, its middle index is 11. Direct use of the AP formulas gives

$$S_{21} = \frac{21}{2}(2A + 20d) = 21(A + 10d) = 21a_{11}.$$

The same result follows by pairing the first and last terms: each of ten pairs sums to $2a_{11}$, and the unpaired middle term is a_{11}. No approximation to the GP parameters is required.

Step 2: Use the GP data to check nondegeneracy The given sums imply

$$\frac{a}{1-r} = 5, \qquad 5(1-r^5) = \frac{98}{25},$$

so $r^5 = 27/125$ and $0 < r < 1$. Therefore $a = 5(1-r) > 0$, and both A and d are positive. In particular $a_{11} > 0$, so $22a_{11} \neq 21a_{11}$, excluding B.

Step 3: Rule out the remaining expressions Because $a_{16} = A + 15d$, equality $21a_{11} = 15a_{16}$ would require $6A = 15d$, or $d/A = r = 2/5$. But $(2/5)^5 \neq 27/125$. Equality $21a_{11} = 14a_{16}$ would require $7A = 0$, impossible as $A > 0$. Thus C and D are also excluded. The GP information is consistent with the AP identity and ensures that none of the other expressions coincides accidentally with the sum.

Step 4: Conclusion The sum is $21a_{11}$. Hence, the correct option is (A).

Quick Tip to Solve: For an odd number $2m+1$ of AP terms, use $S_{2m+1} = (2m+1)a_{m+1}$ before trying to calculate the first term or difference.

7. The area of the region enclosed by $y \le 4x^2, x^2 \le 9y$ and $y \le 4$, is equal to : [4 marks]

(A) $\frac{40}{3}$

(B) $\frac{56}{3}$

(C) $\frac{112}{3}$

(D) $\frac{80}{3}$

Correct Answer: (D) $\frac{80}{3}$

Solution:

Concept:

- For a region bounded by inequalities, a horizontal strip is useful when the curves can be written as left and right limits for x in terms of y. Its area is the integral of the total allowed horizontal width.
- The inequalities involve x^2, so the region is symmetric about the y-axis. Both left and right components must be included; subtracting their inner boundaries prevents counting the forbidden central region.

Step 1: Determine the horizontal intervals The condition $x^2 \le 9y$ forces $y \ge 0$ and $|x| \le 3\sqrt{y}$. The condition $y \le 4x^2$ gives $|x| \ge \sqrt{y}/2$. With the cap $y \le 4$, the y-range is $0 \le y \le 4$. At each positive height the region consists of the two intervals

$$-3\sqrt{y} \le x \le -\frac{\sqrt{y}}{2}, \qquad \frac{\sqrt{y}}{2} \le x \le 3\sqrt{y}.$$

They meet only at the origin when $y = 0$.

Step 2: Integrate the combined width The right component has width $3\sqrt{y} - \sqrt{y}/2 = (5/2)\sqrt{y}$. By symmetry the left component has the same width. Thus the total width is $5\sqrt{y}$, and

$$\mathcal{A} = \int_0^4 5\sqrt{y}\,dy = 5\left[\frac{2}{3}y^{3/2}\right]_0^4 = \frac{10}{3}\cdot 8 = \frac{80}{3}.$$

The square roots are real throughout the chosen integration range, and the outer bound always exceeds the inner bound.

Step 3: Check the alternatives and geometry Each individual component has area $40/3$; option A counts only one side. Option B and C do not equal the integral of the allowed width. As a check, the area inside the outer parabola up to height 4 is $\int_0^4 6\sqrt{y}\,dy = 32$; the forbidden central strip has area $\int_0^4 \sqrt{y}\,dy = 16/3$. Their difference is $32 - 16/3 = 80/3$, agreeing with the two-component calculation.

Step 4: Conclusion The enclosed area is $80/3$ square units. Hence, the correct option is (D).

Quick Tip to Solve: When both bounds contain x^2, solve for $|x|$ at fixed y and double one side only after subtracting the inner boundary.

8. $\int_0^2 \left(\left|2x^2-3x\right|+\left[x-\frac{1}{2}\right]\right)\mathrm{d}x$, **where [t] is the greatest integer function, is equal to :** [4 marks]

(A) $\frac{7}{6}$

(B) $\frac{19}{12}$

(C) $\frac{31}{12}$

(D) $\frac{3}{2}$

Correct Answer: (B) $\frac{19}{12}$

Solution:

Concept:

- An absolute value is integrated by splitting at the zeros of its argument. Here $2x^2-3x=x(2x-3)$ changes sign at $x=3/2$ inside the interval.
- The greatest integer function is constant between consecutive integer crossings. Values at isolated jump points do not change a definite integral.

Step 1: Split the absolute-value integral For $0<x<3/2$, the quadratic is negative, whereas for $3/2<x<2$ it is positive. Therefore the absolute value contributes

$$\begin{aligned} I_1 &= \int_0^{3/2}(3x-2x^2)\,dx+\int_{3/2}^{2}(2x^2-3x)\,dx \\ &= \left[\frac{3x^2}{2}-\frac{2x^3}{3}\right]_0^{3/2}+\left[\frac{2x^3}{3}-\frac{3x^2}{2}\right]_{3/2}^{2} \\ &= \frac{9}{8}+\frac{11}{24}=\frac{19}{12}. \end{aligned}$$

The negative portion must be reversed before integrating; integrating the signed quadratic directly would cancel area incorrectly.

Step 2: Integrate the floor function and combine On the three intervals $[0,1/2)$, $[1/2,3/2)$, and $[3/2,2]$, the floor values of $x-1/2$ are respectively $-1,0,1$. Thus

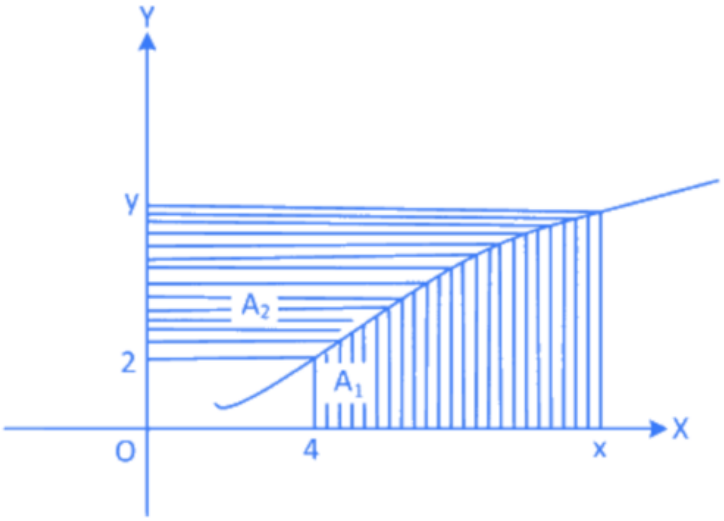
$$I_2=(-1)\left(\frac{1}{2}\right)+0(1)+1\left(\frac{1}{2}\right)=0.$$

The requested integral is consequently $I_1+I_2=19/12$. In particular, the floor contribution is not the integral of $x-1/2$. Options (A), (C) and (D) differ from the exact sum, so none can be selected.

Step 3: Conclusion The integral equals $\boxed{19/12}$. Hence, the correct option is (B).

Quick Tip to Solve: Split at both kinds of breakpoints: zeros for the absolute value and integer crossings for the floor. Here the two nonzero floor rectangles cancel.

9. Consider a curve $y = y(x)$ in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does NOT pass through the point. [4 marks]



(A) (6, 21)

(B) (8, 9)

(C) (10, −4)

(D) (12, −15)

Correct Answer: (C) (10, −4)

Solution:

Concept:

- From the figure, $A_1 = \int_4^x y(t)\,dt$, and $A_2 = \int_2^y x(u)\,du$. These complementary areas fill the rectangle of area xy, apart from the initial rectangle of area $4 \cdot 2 = 8$.
- The derivative gives the tangent slope; a normal has its negative reciprocal slope. Perpendicular nonvertical lines have slope product -1.

Step 1: Derive the curve from the area condition Since $A_1 + A_2 = xy - 8$ and $A_1 = 2A_2$, we have $3A_1 = 2xy - 16$. Differentiate with respect to x:

$$3y = 2y + 2x\frac{dy}{dx}, \qquad \frac{dy}{dx} = \frac{y}{2x}.$$

Separating variables gives $\log y = \frac{1}{2}\log x + \log C$, so $y = C\sqrt{x}$ in the first quadrant. The marked point $(4, 2)$ gives $C = 1$. Hence the relevant curve is $y = \sqrt{x}$, with derivative $1/(2\sqrt{x})$. The initial point is essential: the area relation alone leaves an arbitrary scale factor.

Step 2: Find the normal and test the points The line $2x - 12y = 15$ has slope $1/6$. Its perpendicular normal therefore has slope -6, so the corresponding tangent slope is $1/6$. Thus

$$\frac{1}{2\sqrt{x}} = \frac{1}{6} \quad \Longrightarrow \quad (x, y) = (9, 3).$$

The normal equation is

$$y - 3 = -6(x - 9), \qquad 6x + y = 57.$$

For (A), (B), and (D), the values of $6x + y$ are respectively $36 + 21 = 57$, $48 + 9 = 57$, and $72 - 15 = 57$. For (C), the value is $60 - 4 = 56$, so that point alone is absent from the normal.

Step 3: Conclusion The normal does not pass through $\boxed{(10, -4)}$. Hence, the correct option is (C).

Quick Tip to Solve: For an area-ratio curve, first combine the complementary areas into a rectangle, then differentiate; this avoids evaluating either area explicitly.

10. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0, x + \mathrm{p}y = 39$ and $x - y = 3$ respectively and $\mathrm{P}(2, 3)$ is its circumcentre. Then which of the following is NOT true? [4 marks]

(A) $(\mathrm{AC})^2 = 9\mathrm{p}$

(B) $(\mathrm{AC})^2 + \mathrm{p}^2 = 136$

(C) $32 < \text{area}\,(\Delta\mathrm{ABC}) < 36$

(D) $34 < \text{area}\,(\triangle\mathrm{ABC}) < 38$

Correct Answer: (D) $34 < \text{area}\,(\triangle\mathrm{ABC}) < 38$

Solution:

Concept:

- The circumcentre is equidistant from all three vertices. Once one vertex is found, its distance from the given centre determines the circumcircle.
- The area of a triangle with adjacent vectors (u_1, u_2), (v_1, v_2) is $\frac{1}{2}|u_1v_2 - u_2v_1|$. Each option must be checked because the question asks for the false statement.

Step 1: Find the vertices and parameter The intersection of $2x + y = 0$ and $x - y = 3$ is $A = (1, -2)$. Hence the squared circumradius is

$$PA^2 = (1 - 2)^2 + (-2 - 3)^2 = 26.$$

The circumcircle is $(x - 2)^2 + (y - 3)^2 = 26$, or $x^2 + y^2 - 4x - 6y - 13 = 0$. Substituting $y = -2x$ gives $5x^2 + 8x - 13 = 0$, whose roots are $1, -13/5$. Thus $B = (-13/5, 26/5)$. Substituting $y = x - 3$ gives $2x^2 - 16x + 14 = 0$, whose roots are $1, 7$, so $C = (7, 4)$. Since C lies on $x + py = 39$, $7 + 4p = 39$, giving $p = 8$. These vertices also put B on that side.

Step 2: Check all four statements We obtain

$$AC^2 = (7 - 1)^2 + (4 + 2)^2 = 72 = 9p, \qquad AC^2 + p^2 = 72 + 64 = 136.$$

Thus (A) and (B) are true. Using $B - A = (-18/5, 36/5)$ and $C - A = (6, 6)$,

$$[ABC] = \frac{1}{2}\left|\frac{-18}{5} \cdot 6 - \frac{36}{5} \cdot 6\right| = \frac{162}{5} = 32.4.$$

This lies strictly between 32 and 36, so (C) is true. It does not lie between 34 and 38, making (D) the

sole false statement.

Step 3: Conclusion The false statement is $34 < \text{area}(\triangle ABC) < 38$. Hence, the correct option is (D).

Quick Tip to Solve: Use the known circumcentre to construct the circle first; its intersections with the two known sides determine the vertices before solving for the unknown side parameter.

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11. If the length of the perpendicular drawn from the point $P(a, 4, 2)$, a > 0 on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $\mathrm{a} + \sum_{i=1}^{3} \alpha_i$ is equal to :

[4 marks]

(A) 7

(B) 8

(C) 12

(D) 14

Correct Answer: (B) 8

Solution:

Concept:

- For a line through R with direction vector $\mathbf{d}$, the perpendicular foot is $H = R + t\mathbf{d}$, where $t = ((P - R) \cdot \mathbf{d})/|\mathbf{d}|^2$.
- The squared perpendicular distance is $|P - R|^2 - ((P - R) \cdot \mathbf{d})^2/|\mathbf{d}|^2$. Reflection in the line puts the foot at the midpoint, so $Q = 2H - P$.

Step 1: Use the perpendicular distance to find a Choose $R = (-1, 3, 1)$ and $\mathbf{d} = (2, 3, -1)$. Then $P - R = (a + 1, 1, 1)$, $|\mathbf{d}|^2 = 14$, and

$$(P - R) \cdot \mathbf{d} = 2(a + 1) + 3 - 1 = 2a + 4.$$

Squaring the given distance $2\sqrt{6}$ gives 24. Therefore

$$(a + 1)^2 + 2 - \frac{(2a + 4)^2}{14} = 24.$$

Multiplication by 14 and simplification yield $5a^2 + 6a - 155 = 0$, or $(a - 5)(5a + 31) = 0$. The roots are 5 and $-31/5$; the given restriction $a > 0$ leaves $a = 5$. Keeping that restriction prevents using a spurious geometric configuration.

Step 2: Find the image and the requested sum With $a = 5$, the projection parameter is $t = (2a + 4)/14 = 1$. Thus

$$H = (-1, 3, 1) + (2, 3, -1) = (1, 6, 0).$$

The reflected point is

$$Q = 2H - P = (2, 12, 0) - (5, 4, 2) = (-3, 8, -2).$$

Indeed, H is the midpoint of P, Q, and $(P-H)\cdot\mathbf{d} = (4,-2,2)\cdot(2,3,-1) = 0$, verifying the reflection. Consequently $a+\alpha_1+\alpha_2+\alpha_3 = 5-3+8-2 = 8$. The alternative numbers 7, 12 and 14 do not equal this verified sum.

Step 3: Conclusion The required value is $\boxed{8}$. Hence, the correct option is (B).

Quick Tip to Solve: After finding the perpendicular foot H, reflect by Q=2H-P; do not mistake H itself for the image point.

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12. A six faced die is biased such that

$3 \times \mathrm{P}(\textbf{a prime number}) = 6 \times \mathrm{P}(\textbf{a composite number}) = 2 \times \mathrm{P}(1)$.

Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is : [4 marks]

(A) $\frac{3}{11}$
(B) $\frac{5}{11}$
(C) $\frac{7}{11}$
(D) $\frac{8}{11}$

Correct Answer: The named-event probabilities leave P(4) unspecified; intended equal-category per-face probabilities instead give 8/11 (D).

Solution:

Concept:

- Probabilities of disjoint exhaustive events sum to one. The prime outcomes are $\{2,3,5\}$, the composite outcomes are $\{4,6\}$, and the remaining outcome is 1.
- For two independent throws, the mean count of perfect-square outcomes is $2p_s$, where $p_s = P(1)+P(4)$. A biased die need not assign equal probabilities to different faces.

Step 1: Apply the printed relation to its stated events Let the common value in the printed equation be k. Read literally, it states

$$P(\{2,3,5\}) = \frac{k}{3}, \quad P(\{4,6\}) = \frac{k}{6}, \quad P(1) = \frac{k}{2}.$$

These three events exhaust the six faces, so $k/3 + k/6 + k/2 = 1$, giving $k = 1$. Thus $P(1) = 1/2$ and $P(4)+P(6) = 1/6$. Nothing in this event-probability equation specifies how the composite-event probability is shared between 4 and 6. Consequently, if $u = P(4)$, the requested mean is $E(X) = 2(1/2+u) = 1+2u$, with $0 \le u \le 1/6$. Every listed option is below 1 and therefore fails this literal reading.

Step 2: Identify the intended per-face interpretation The familiar intended answer follows only under a different convention: let each prime face have probability p, each composite face probability q, and face 1 probability r, with $3p = 6q = 2r = c$. Then

$$3p+2q+r = c+\frac{c}{3}+\frac{c}{2} = \frac{11c}{6} = 1.$$

This gives $q = 1/11$, $r = 3/11$, so the perfect-square probability is $r + q = 4/11$, and the mean is $8/11$, option (D). However, this convention treats the displayed probabilities as per-face values rather than probabilities of the named events. The printed event wording does not establish it; a unique mathematical answer requires that additional clarification.

Step 3: Conclusion As literally worded, the mean is $1 + 2P(4) \in [1, 4/3]$, so no listed option is valid and the question is ambiguous. The intended per-face convention gives $\boxed{8/11}$, option (D), but it is an additional interpretation.

Quick Tip to Solve: For a biased die, distinguish the probability of an entire category from the probability of each face in that category before normalizing.

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13. Let $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$.

If $\mathrm{A}^2 + \gamma\mathrm{A} + 18\mathrm{I} = \mathrm{O}$**, then** $\det(\mathrm{A})$ **is equal to _____.** [4 marks]

(A) -18

(B) 18

(C) -50

(D) 50

Correct Answer: (B) 18

Solution:

Concept:

- Every 2×2 matrix satisfies $A^2 - (\operatorname{tr} A)A + (\det A)I = O$, its characteristic equation.
- Here A cannot be a scalar matrix because its top-right entry is -2. Thus A and I are linearly independent as matrices, permitting comparison of the two quadratic identities.

Step 1: Compare the given and characteristic equations Write $t = \operatorname{tr} A = 4 + \beta$ and $d = \det A = 4\beta + 2\alpha$. The Cayley-Hamilton identity becomes

$$A^2 - tA + dI = O.$$

The supplied relation is $A^2 + \gamma A + 18I = O$. Subtracting the characteristic equation from it eliminates A^2 and gives

$$(\gamma + t)A + (18 - d)I = O.$$

This step avoids trying to determine α, β, and γ separately; only the constant coefficient is required.

Step 2: Justify coefficient comparison and obtain the determinant The $(1, 2)$ entry of the last matrix equation is $-2(\gamma + t) = 0$, because the identity matrix has a zero off-diagonal entry. Therefore $\gamma + t = 0$. Substitution then gives $(18 - d)I = O$, which forces $d = 18$. This explicit use of the nonzero off-diagonal entry is necessary: for an arbitrary scalar matrix, a quadratic relation need not be its characteristic polynomial.

For a direct consistency check, the $(1, 1)$ entry of the original relation is $16 - 2\alpha + 4\gamma + 18 = 0$. Using

$\gamma = -(4+\beta)$ reduces it to $2\alpha + 4\beta = 18$, exactly the determinant formula. Thus -18, -50, and 50 are excluded by the matrix equation, rather than merely by a sign convention.

Step 3: Conclusion $\boxed{\det A = 18}$. Hence, the correct option is (B).

Quick Tip to Solve: When comparing a quadratic matrix identity with Cayley-Hamilton, first check that A is not scalar; a nonzero off-diagonal entry makes the comparison valid.

14. A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line $y = 2x$ gives a chord OA of circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then QA : AP is equal to : [4 marks]

(A) 1 : 4

(B) 1 : 5

(C) 2 : 5

(D) 1 : 3

Correct Answer: (A) 1 : 4

Solution:

Concept:

- The first circle has centre $(2, 0)$ and radius 2, so its equation is $x^2 + y^2 = 4x$. The second circle has its centre at the midpoint of its diameter OA.
- The tangent at the end of a diameter is perpendicular to that diameter. A point dividing a straight segment can be located using either coordinate when the segment is not parallel to that coordinate axis.

Step 1: Locate A and determine its tangent Intersect $y = 2x$ with the first circle:

$$x^2 + 4x^2 = 4x, \qquad x(5x - 4) = 0.$$

The root zero gives O, while the other intersection is $A = (4/5, 8/5)$. Since OA is a diameter of C_2, its radius at A has the same slope as OA, namely 2. The tangent slope is therefore $-1/2$. Its equation through A is

$$y - \frac{8}{5} = -\frac{1}{2}\left(x - \frac{4}{5}\right), \qquad x + 2y = 4.$$

This uses the tangent to the second circle, not to the first circle; those generally have different centres and tangent directions at the same point.

Step 2: Find the intercepts and divide the segment Putting $y = 0$ gives $P = (4, 0)$, while putting $x = 0$ gives $Q = (0, 2)$. The point A lies between these intercepts. Along QP, its horizontal displacement from Q is $4/5$, compared with the total horizontal displacement 4. Hence $QA/QP = 1/5$ and $AP/QP = 4/5$, giving $QA : AP = 1 : 4$.

One can also check the distances separately:

$$QA = \sqrt{(4/5)^2 + (-2/5)^2} = \frac{2\sqrt{5}}{5},$$

$$AP = \sqrt{(16/5)^2 + (-8/5)^2} = \frac{8\sqrt{5}}{5}.$$

Their ratio is again $1 : 4$. The ratios $1 : 5$, $2 : 5$, and $1 : 3$ therefore fail the intercept geometry.

Step 3: Conclusion $\boxed{QA : AP = 1 : 4}$. Hence, the correct option is (A).

Quick Tip to Solve: At the endpoint of a diameter, use the diameter direction to get the tangent slope immediately; then compare coordinate displacements along the intercept segment.

15. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then $|a|$ is equal to : [4 marks]

(A) $2\sqrt{2}$

(B) $2\sqrt{3}$

(C) $4\sqrt{2}$

(D) 4

Correct Answer: (C) $4\sqrt{2}$

Solution:

Concept:

- The axis of a parabola is perpendicular to the tangent at its vertex. Hence the perpendicular from the focus to that tangent meets it at the vertex.
- If the distance between focus and vertex is $p > 0$, the length of the latus rectum is $4p$. The point-to-line distance formula gives $d = |Ax_0 + By_0 + C|/\sqrt{A^2 + B^2}$.

Step 1: Identify the focal distance The given latus rectum has length 16, so the focal distance is

$$4p = 16 \quad \Rightarrow \quad p = 4.$$

This is the distance from the focus to the vertex, rather than the distance between the focus and the directrix. The latter would be twice as large. Since the vertex tangent is perpendicular to the axis, its distance from the focus is precisely this value.

Step 2: Apply the perpendicular-distance formula Write the tangent as $x + y - a = 0$ and substitute the focus (a, a). We obtain

$$p = \frac{|a + a - a|}{\sqrt{1^2 + 1^2}} = \frac{|a|}{\sqrt{2}}.$$

Therefore

$$\frac{|a|}{\sqrt{2}} = 4 \quad \Rightarrow \quad |a| = 4\sqrt{2}.$$

Both signs of a give the same focal distance, as required by the absolute value in the question.

Step 3: Check the alternatives For $|a| = 2\sqrt{2}$, the latus rectum would be 8, and for $|a| = 2\sqrt{3}$ it would be $4\sqrt{6}$, neither of which is 16. Taking $|a| = 4$ would give $8\sqrt{2}$. Only $4\sqrt{2}$ gives the specified length, so the other three options are excluded.

Step 4: Conclusion $|a| = 4\sqrt{2}$. Hence, the correct option is (C).

Quick Tip to Solve: When the vertex tangent and focus are given, find their perpendicular distance first; multiplying it by four gives the latus rectum.

16. If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0$, a > 0 makes an angle 30° with the plane $y - z + 2 = 0$, then the direction cosines of the line are : [4 marks]

(A) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

(B) $\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, 0$

(C) $\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0$

(D) $\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0$

Correct Answer: (B) $\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, 0$

Solution:

Concept:

- A direction vector of a line contained in two planes is perpendicular to both plane normals. Subtracting the two plane equations also shows which coordinate is constant.
- For a unit direction vector (l, m, n) and plane normal $\mathbf{N}$, the acute line-plane angle satisfies $\sin\alpha = |\mathbf{N} \cdot (l, m, n)|/|\mathbf{N}|$.

Step 1: Find the form of the direction vector Subtract the equations of the two planes to obtain $cz = -3$. Since an intersection line exists, $c \neq 0$, and z is constant on it. Thus its third direction cosine is zero. A direction vector is $(b, -a, 0)$, since it is perpendicular to $(a, b, 0)$ and (a, b, c). Write its unit direction as $(l, m, 0)$, where

$$l^2 + m^2 = 1.$$

Step 2: Use the angle with the given plane The plane $y - z + 2 = 0$ has normal $(0, 1, -1)$, of length $\sqrt{2}$. Therefore

$$\sin 30^\circ = \frac{|m|}{\sqrt{2}} \quad \Rightarrow \quad |m| = \frac{1}{\sqrt{2}}.$$

Normalisation then gives

$$l^2 = 1 - \frac{1}{2} = \frac{1}{2}.$$

Choose the orientation of the line so that $l = 1/\sqrt{2}$. Its other cosine can have either sign because the angle specifies only its absolute value.

Step 3: Compare the complete family of possibilities The resulting family is $(1/\sqrt{2}, \pm 1/\sqrt{2}, 0)$. Reversing all signs describes the same unoriented line. Option (A) lists only one of the two possible relative signs and therefore does not express the full result. Options (C) and (D) give $|m| = 2/\sqrt{5}$ and $\sqrt{3}/2$, respectively, and neither yields the required sine of the line-plane angle.

Step 4: Conclusion The direction cosines may be written as $\left(\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, 0\right)$, with simultaneous reversal allowed. Hence, the correct option is (B).

Quick Tip to Solve: For an angle with a plane use sine with its normal, and retain sign freedom when the angle determines only an absolute value.

17. Let X have a binomial distribution B(n, p) such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X > n-3) = \frac{k}{2^n}$, then k is equal to : [4 marks]

(A) 528

(B) 529

(C) 629

(D) 630

Correct Answer: (B) 529

Solution:

Concept:

- For $X \sim B(n,p)$, the mean is $\mu = np$ and variance is $v = np(1-p)$. Thus $0 \le v \le \mu$.
- When $p = 1/2$, the probability of $X = r$ is $\binom{n}{r}/2^n$. A strict integer inequality must be converted to its exact permitted values before summing.

Step 1: Recover the mean and variance The two numbers have sum 24 and product 128, so both are roots of

$$t^2 - 24t + 128 = 0 = (t-16)(t-8).$$

Since variance cannot exceed the binomial mean, their assignments are $\mu = 16$ and $v = 8$. Interchanging these would give $1 - p = 2$, which is impossible for a probability.

Step 2: Determine the distribution parameters Dividing variance by mean gives

$$1 - p = \frac{v}{\mu} = \frac{8}{16} = \frac{1}{2}.$$

Hence $p = 1/2$, and $np = 16$ gives $n = 32$. These parameters reproduce both the stated sum and product, so they satisfy the original conditions rather than just the quadratic relation.

Step 3: Evaluate the strict upper tail The event is $X > 29$, so its only values are 30, 31 and 32. Therefore

$$P(X > n-3) = \frac{\binom{32}{30} + \binom{32}{31} + \binom{32}{32}}{2^{32}}.$$

By binomial symmetry the numerator is

$$\binom{32}{2} + 32 + 1 = 496 + 32 + 1 = 529.$$

Thus 528 omits the final endpoint term, while 629 and 630 do not equal the required three-term sum. In particular, including $X = 29$ would violate the strict inequality.

Step 4: Conclusion $k = 529$. Hence, the correct option is (B).

Quick Tip to Solve: For a binomial upper tail near n, use symmetry of binomial coefficients and list the integer endpoints before calculating.

18. The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is $45°$. Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is $60°$. If $\angle BAQ = 30°, AB = d$ and the area of the trapezium PQRB is α, then the ordered pair (d, α) is : [4 marks]

(A) $(10(\sqrt{3}-1), 25)$
(B) $\left(10(\sqrt{3}-1), \frac{25}{2}\right)$
(C) $(10(\sqrt{3}+1), 25)$
(D) $\left(10(\sqrt{3}+1), \frac{25}{2}\right)$

Correct Answer: (A) $(10(\sqrt{3}-1), 25)$

Solution:

Concept:

- In a right triangle, the tangent of an elevation angle equals vertical rise divided by horizontal separation. The sine and cosine of $30°$ resolve the segment AB.
- The parallel sides of trapezium PQRB are the vertical segments PQ and RB; their perpendicular separation is RQ. Its area is half the sum of these sides times RQ.

Step 1: Express all lengths in terms of AB Put $AB = d$. The first elevation gives $AQ = 10/\tan 45° = 10$. Since $\angle BAQ = 30°$, the right triangle ARB gives

$$RB = \frac{d}{2}, \qquad AR = \frac{\sqrt{3}d}{2}.$$

As R lies on AQ,

$$RQ = 10 - \frac{\sqrt{3}d}{2}.$$

The vertical rise from B to P is $10 - d/2$; it is not the full tower height.

Step 2: Apply the elevation from B Using the angle of $60°$ yields

$$\frac{10 - d/2}{10 - \sqrt{3}d/2} = \sqrt{3}.$$

Thus

$$10 - \frac{d}{2} = 10\sqrt{3} - \frac{3d}{2} \quad \Rightarrow \quad d = 10(\sqrt{3}-1).$$

Consequently $RB = 5(\sqrt{3}-1)$ and $RQ = 5(\sqrt{3}-1)$, both positive, consistent with R lying between A and Q.

Step 3: Calculate the area and compare the options The area is

$$\alpha = \frac{1}{2}(PQ + RB)RQ = \frac{1}{2}\{10 + 5(\sqrt{3}-1)\}\,5(\sqrt{3}-1).$$

Simplifying by the difference of squares,

$$\alpha = \frac{25}{2}(\sqrt{3}+1)(\sqrt{3}-1) = 25.$$

The option with the same d but area $25/2$ halves the area twice. The values $10(\sqrt{3}+1)$ would put AR beyond Q and fail the geometric condition.

Step 4: Conclusion $(d, \alpha) = \big(10(\sqrt{3}-1), 25\big)$. Hence, the correct option is (A).

Quick Tip to Solve: Use the height difference from the elevated observer B; the trapezium has vertical parallel sides, not AB and PQ.

---

19. Let $S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right) \sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$**. Then** [4 marks]

(A) $S = \left\{\frac{\pi}{12}\right\}$

(B) $S = \left\{\frac{2\pi}{3}\right\}$

(C) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$

(D) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

Correct Answer: No option is valid on the original domain; the two formal roots are excluded because a secant is undefined.

Solution:

Concept:

- The tangent difference identity is $\tan v - \tan u = \sin(v-u)/(\cos u \cos v)$. It turns products of consecutive secants into a telescoping difference.
- All original secants must be defined. Also $\tan(\theta + 3\pi/2) = -\cot\theta$, and $\tan\theta + \cot\theta = 2/\sin 2\theta$.

Step 1: Telescope the nine terms For $u = \theta + (m-1)\pi/6$ and $v = u + \pi/6$, the sine of the difference is $1/2$. Hence each product of secants is $2(\tan v - \tan u)$. On summing, every interior tangent cancels and the given sum becomes

$$2\{\tan(\theta + 9\pi/6) - \tan\theta\} = -2(\cot\theta + \tan\theta).$$

This cancellation is valid only at angles where the original terms exist.

Step 2: Solve the resulting sine equation The equation therefore reduces to

$$-\frac{4}{\sin 2\theta} = -\frac{8}{\sqrt{3}} \quad \Rightarrow \quad \sin 2\theta = \frac{\sqrt{3}}{2}.$$

Since $0 < 2\theta < \pi$, the two solutions are

$$2\theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}, \qquad \theta = \frac{\pi}{6} \text{ or } \frac{\pi}{3}.$$

Step 3: Check the original domain and the options At $\theta = \pi/6$, one argument in the original secants is $\theta + 2\pi/6 = \pi/2$. At $\theta = \pi/3$, another is $\theta + \pi/6 = \pi/2$. Thus both algebraic candidates make a secant undefined, so neither belongs to the literal set S. Consequently S is empty and its sum is zero. None of the printed options states this. If the telescoped expression is extended through the

removable singularities, its two formal roots sum to $\pi/2$, which is the value represented by option (C), but that extension is not the original domain.

Step 4: Conclusion The literal set is $S = \varnothing$, because both formal roots make an original secant undefined. Thus none of the options is strictly valid; the formal telescoping answer is (C), with sum $\pi/2$, only after extending the expression through its excluded points.

Quick Tip to Solve: After telescoping a trigonometric sum, substitute candidates into the original denominators; cancellation cannot restore excluded angles.

20. If the truth value of the statement $(P \wedge (\sim R)) \rightarrow ((\sim R) \wedge Q)$ is F, then the truth value of which of the following is F ? [4 marks]

(A) $\mathrm{P} \vee \mathrm{Q} \rightarrow \sim \mathrm{R}$

(B) $\mathrm{R} \vee \mathrm{Q} \rightarrow \sim \mathrm{P}$

(C) $\sim (\mathrm{P} \vee \mathrm{Q}) \rightarrow \sim \mathrm{R}$

(D) $\sim (\mathrm{R} \vee \mathrm{Q}) \rightarrow \sim \mathrm{P}$

Correct Answer: (D) $\sim (\mathrm{R} \vee \mathrm{Q}) \rightarrow \sim \mathrm{P}$

Solution:

Concept:

- An implication $U \rightarrow V$ is false precisely when U is true and V is false; in every other case it is true.
- A conjunction is true only when both components are true, whereas a disjunction is false only when both components are false.

Step 1: Extract the forced truth values The given implication is false, so its antecedent $P \wedge \sim R$ is true and its consequent $(\sim R) \wedge Q$ is false. The true antecedent forces

$$P = \mathrm{T}, \qquad R = \mathrm{F}.$$

Hence $\sim R = \mathrm{T}$. The consequent then reduces to $\mathrm{T} \wedge Q$, which can be false only if

$$Q = \mathrm{F}.$$

Thus the original statement fixes all three variables, with no remaining cases to consider.

Step 2: Evaluate the first three alternatives For (A), $P \vee Q = \mathrm{T}$ and $\sim R = \mathrm{T}$, so its implication is $\mathrm{T} \rightarrow \mathrm{T}$, which is true. For (B), $R \vee Q = \mathrm{F}$ and $\sim P = \mathrm{F}$, giving $\mathrm{F} \rightarrow \mathrm{F}$, also true. For (C), $\sim (P \vee Q) = \mathrm{F}$ and $\sim R = \mathrm{T}$, giving $\mathrm{F} \rightarrow \mathrm{T}$, again true. A false antecedent never makes an implication false.

Step 3: Test the remaining alternative For (D), both R and Q are false, so $R \vee Q = \mathrm{F}$ and $\sim (R \vee Q) = \mathrm{T}$. But P is true, so $\sim P = \mathrm{F}$. Therefore this statement is

$$\mathrm{T} \rightarrow \mathrm{F} = \mathrm{F}.$$

It is the unique false alternative under the truth values required by the question.

Step 4: Conclusion $\sim (R \vee Q) \rightarrow \sim P$ is false. Hence, the correct option is (D).

Quick Tip to Solve: Begin with the unique false case of implication, T implies F, and infer the variables before checking each option.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. The number of functions f, from the set $\mathrm{A} = \{x \in \mathbf{N} : x^2 - 10x + 9 \leq 0\}$ to the set $\mathrm{B} = \{\mathrm{n}^2 : \mathrm{n} \in \mathbf{N}\}$ such that $f(x) \leq (x-3)^2 + 1$, for every $x \in \mathrm{A}$, is _____. [4 marks]

Solution:

Concept:

- The domain is obtained by solving the quadratic inequality over positive integers. Each input may independently be assigned any codomain value satisfying its own bound.
- The codomain consists of positive perfect squares. If the bound is M, the number of allowable values is $\lfloor\sqrt{M}\rfloor$; multiply these counts over the domain.

Step 1: Determine the domain exactly Factor the quadratic as

$$x^2 - 10x + 9 = (x-1)(x-9).$$

The inequality holds for $1 \leq x \leq 9$, so

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

The endpoints are included because the inequality is non-strict. The natural numbers here are positive, and the codomain contains $1, 4, 9, 16, 25, 36, \ldots$, rather than all positive integers.

Step 2: Count the permissible squares for each input For successive inputs from 1 to 9, the upper bounds $(x-3)^2 + 1$ are

$$5,\ 2,\ 1,\ 2,\ 5,\ 10,\ 17,\ 26,\ 37.$$

Accordingly the numbers of positive squares at or below these bounds are

$$2,\ 1,\ 1,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6.$$

For example, the bound 17 permits exactly $1, 4, 9, 16$, whereas the bound 37 also permits 25 and 36. At x=3 only the value 1 is possible.

Step 3: Apply the multiplication principle There is no requirement for f to be injective, surjective, increasing or otherwise linked across different inputs. The choices can therefore be made independently, including reuse of the same square at several inputs. The total is

$$2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 1440.$$

The infinite size of the codomain does not make the count infinite, because each input has a finite

upper bound.

Step 4: Conclusion The number of functions satisfying all the restrictions is $\boxed{1440}$. This is a dimensionless count.

Quick Tip to Solve: For functions constrained separately at each input, count the allowed output values input by input and multiply; distinct outputs are not required.

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22. Let for the 9^{th} term in the binomial expansion of $(3+6x)^{\text{n}}$, in the increasing powers of $6x$, to be the greatest for $x=\frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $\text{k}+\text{n}_0$ is equal to : [4 marks]

Solution:

Concept:

- For positive binomial terms, a greatest term is at least as large as each adjacent term. Use ratios to avoid evaluating large binomial coefficients.
- The coefficient of x^r in $(3+6x)^n$ is $\binom{n}{r}3^{n-r}6^r$; the term itself also includes x^r.

Step 1: Compare the ninth term with its neighbours At $x=3/2$, the expansion becomes that of $(3+9)^n$, with $T_{r+1}=\binom{n}{r}3^{n-r}9^r$. Consecutive terms have ratio $T_{r+2}/T_{r+1}=3(n-r)/(r+1)$, which decreases as r increases. Thus the two neighbouring comparisons suffice to identify a greatest term throughout this positive sequence. For the ninth term we require

$$\frac{T_9}{T_8}=\frac{3(n-7)}{8}\geq 1, \qquad \frac{T_{10}}{T_9}=\frac{3(n-8)}{9}\leq 1.$$

These give $n\geq 29/3$ and $n\leq 11$. Hence the least integer is $n_0=10$. At this value the first ratio is $9/8>1$ and the second is $2/3<1$, confirming the ninth term is strictly greatest.

Step 2: Compute the coefficient ratio The requested coefficients belong to the polynomial in x, so retain the powers of 6 and 3 rather than substituting the numerical value of x again. With $n=10$,

$$k=\frac{\binom{10}{6}3^4 6^6}{\binom{10}{3}3^7 6^3}=\frac{210}{120}\left(\frac{6}{3}\right)^3=14.$$

Therefore $k+n_0=14+10=24$. The distinction between the greatest numerical term and the coefficient ratio is essential: the value $x=3/2$ determines n_0, while the coefficient ratio contains no power of x.

Step 3: Conclusion The least exponent is $n_0=10$, and $k=14$. Thus the required dimensionless numerical value is $\boxed{24}$.

Quick Tip to Solve: Use adjacent-term inequalities to find the least integer exponent, then compute coefficients before substituting any value of x.

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23. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is _____. [4 marks]

Solution:

Concept:

- For semi-vertical angle θ, similarity gives $r/h = \tan\theta = 3/4$ at every water level.
- The water volume is $V = \pi r^2 h/3$, while its wet curved surface area is $S = \pi r\ell$, where $\ell = \sqrt{h^2 + r^2}$.

Step 1: Express volume and wet area using depth Let h be the water depth in metres. The vertex is downwards, so the water occupies a smaller cone similar to the tank. The wetted portion is its lateral surface, with no contribution from the free horizontal water surface. From the given angle,

$$r = \frac{3}{4}h, \qquad \ell = \sqrt{h^2 + \frac{9}{16}h^2} = \frac{5}{4}h.$$

Consequently,

$$V = \frac{1}{3}\pi\left(\frac{3}{4}h\right)^2 h = \frac{3\pi}{16}h^3, \qquad S = \pi\frac{3}{4}h\frac{5}{4}h = \frac{15\pi}{16}h^2.$$

Both radius and slant height vary as water enters; holding either constant would give an incorrect rate.

Step 2: Differentiate with respect to time Take time t in hours. Applying the chain rule to both expressions gives

$$\frac{dV}{dt} = \frac{9\pi}{16}h^2\frac{dh}{dt}, \qquad \frac{dS}{dt} = \frac{15\pi}{8}h\frac{dh}{dt}.$$

Since $dV/dt = 6$, at $h = 4$ we obtain

$$6 = 9\pi\frac{dh}{dt}, \qquad \frac{dh}{dt} = \frac{2}{3\pi}\ \mathrm{m\,h^{-1}}.$$

Substituting this value into the area rate,

$$\frac{dS}{dt} = \frac{15\pi}{8}(4)\frac{2}{3\pi} = 5\ \mathrm{m^2\,h^{-1}}.$$

The positive sign agrees with the increasing water level and newly wetted surface. The result describes the curved wall area only, as required.

Step 3: Conclusion The wet curved surface area increases at $\boxed{5\ \mathrm{m^2\,h^{-1}}}$, so the numerical entry is $\boxed{5}$.

Quick Tip to Solve: Write both volume and curved area in terms of the same depth; their derivative ratio eliminates dh/dt immediately.

24. For the curve $C : \left(x^2+y^2-3\right)+\left(x^2-y^2-1\right)^5=0$, the value of $3y'-y^3y''$, at the point (α,α), $\alpha>0$, on C, is equal to _____. [4 marks]

Solution:

Concept:

- For an implicitly defined curve, differentiate every occurrence of y using $y=y(x)$, so $d(y^2)/dx = 2yy'$.
- When differentiating a fifth power twice, both the derivative of its inner factor and the derivative of y' must be retained.

Step 1: Locate the point and find the first derivative Put $x=y=\alpha$ into the curve equation. Then $x^2-y^2-1=-1$, so

$$2\alpha^2-3+(-1)^5=0, \qquad \alpha=\sqrt{2}$$

using $\alpha>0$. Define $u=x^2-y^2-1$ and $p=y'$. Differentiating once gives

$$2x+2yp+5u^4(2x-2yp)=0.$$

At the required point $u=-1$, hence $12x-8yp=0$. Since $x=y=\sqrt{2}\neq 0$, it follows that $p=3/2$. The partial derivative with respect to y is $2y-10yu^4=-8y\neq 0$, so a differentiable local branch $y(x)$ is valid here.

Step 2: Differentiate again and evaluate the expression Write $q=y''$. The second derivative equation, before evaluating at the point, is

$$2+2p^2+2yq+20u^3(2x-2yp)^2+5u^4(2-2p^2-2yq)=0.$$

At $u=-1$ this reduces to

$$12-8p^2-8yq-80(x-yp)^2=0.$$

Now $p=3/2$, $x=y=\sqrt{2}$, and $(x-yp)^2=(-\sqrt{2}/2)^2=1/2$. Therefore

$$12-18-8yq-40=0, \qquad yq=-\frac{23}{4}.$$

Use $y^2=2$ to evaluate the requested combination without introducing unnecessary radicals:

$$3p-y^3q=\frac{9}{2}-y^2(yq)=\frac{9}{2}-2\left(-\frac{23}{4}\right)=16.$$

In particular, replacing u by -1 before the second differentiation would lose the nonzero derivative of u.

Step 3: Conclusion At $(\sqrt{2},\sqrt{2})$, the required dimensionless value is $\boxed{3y'-y^3y''=16}$.

Quick Tip to Solve: Differentiate the full implicit equation twice before substituting the point; retain the inner-derivative square term.

25. **Let $f(x) = \min\{[x-1],[x-2],\ldots,[x-10]\}$ where [t] denotes the greatest integer $\leq$ t. Then $\int_0^{10} f(x)\mathrm{d}x + \int_0^{10}(f(x))^2\,\mathrm{d}x + \int_0^{10}|f(x)|\mathrm{d}x$ is equal to _____.** [4 marks]

Solution:

Concept:

- The floor function is nondecreasing. Thus the smallest of $[x-1],\ldots,[x-10]$ is always $[x-10]$.
- A change at finitely many interval endpoints does not change a definite integral; integrate a step function by summing its constant value times each interval length.

Step 1: Find the values of the minimum function For every real x, $x-10 \leq x-j$ for $1 \leq j \leq 10$. Taking floors preserves this inequality, so

$$f(x) = [x-10].$$

On each unit interval $[j, j+1)$, where $j = 0, 1, \ldots, 9$, we have $[x] = j$, and the integer translation identity yields

$$f(x) = [x] - 10 = j - 10.$$

Hence the ten values across the integration range are $-10, -9, \ldots, -1$, each persisting for an interval of length one. Although $f(10) = 0$, that single endpoint has zero contribution to every integral. In particular $f(x) \leq 0$ throughout the full integration range.

Step 2: Combine and integrate the three terms Since $f \leq 0$, its absolute value is $|f| = -f$. By linearity, the sum of the first and third integrals cancels exactly. The requested quantity therefore equals

$$\int_0^{10} \left(f(x) + f(x)^2 + |f(x)|\right) dx = \int_0^{10} f(x)^2\,dx.$$

Using the unit-interval values gives

$$\int_0^{10} f(x)^2\,dx = \sum_{j=0}^{9}(j-10)^2 = \sum_{m=1}^{10} m^2.$$

Now apply the finite-square sum formula:

$$\sum_{m=1}^{10} m^2 = \frac{10(10+1)(2\cdot 10+1)}{6} = \frac{10\cdot 11\cdot 21}{6} = 385.$$

As a separate sign check, $\int_0^{10} f = -55$, while $\int_0^{10} |f| = 55$, confirming the cancellation rather than addition of these contributions.

Step 3: Conclusion The sum of the three integrals is the dimensionless value $\boxed{385}$.

Quick Tip to Solve: First determine the sign of a floor-function minimum: here $f + |f| = 0$, leaving only a sum of squares.

26. Let f be a differentiable function satisfying $f(x) = \frac{2}{\sqrt{3}} \int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right) d\lambda, x > 0$ and $f(1) = \sqrt{3}$. If $y = f(x)$ passes through the point $(\alpha, 6)$, then α is equal to _____. [4 marks]

Solution:

Concept:

- A variable substitution converts the fixed upper integration limit into a variable limit, allowing the fundamental theorem of calculus to be applied.
- The resulting separable equation $2xf'(x) = f(x)$ has solutions proportional to $\sqrt{x}$ for $x > 0$.

Step 1: Transform the integral equation Put $t = \lambda\sqrt{x/3}$ for a fixed $x > 0$. Then $d\lambda = \sqrt{3/x}\, dt$, and the limits $\lambda = 0, \sqrt{3}$ become $t = 0, \sqrt{x}$. The given equation becomes

$$f(x) = \frac{2}{\sqrt{x}} \int_0^{\sqrt{x}} f(t^2)\, dt.$$

Multiplying by $\sqrt{x}$ makes the differentiation particularly direct:

$$\sqrt{x}\, f(x) = 2 \int_0^{\sqrt{x}} f(t^2)\, dt.$$

Differentiate with respect to x. The upper endpoint contributes its derivative $1/(2\sqrt{x})$, so

$$\frac{f(x)}{2\sqrt{x}} + \sqrt{x}\, f'(x) = \frac{f(x)}{\sqrt{x}}.$$

Therefore $2xf'(x) = f(x)$. The factor of two and the square-root endpoint derivative are both needed.

Step 2: Determine the function and the required abscissa The equation can be written as

$$\frac{d}{dx}\left(\frac{f(x)}{\sqrt{x}}\right) = 0.$$

Thus $f(x) = C\sqrt{x}$ throughout $x > 0$. The condition $f(1) = \sqrt{3}$ yields $C = \sqrt{3}$, so $f(x) = \sqrt{3x}$. This candidate also satisfies the original integral: its integrand is $f(\lambda^2 x/3) = \lambda\sqrt{x}$ because $\lambda \geq 0$, and

$$\frac{2}{\sqrt{3}} \int_0^{\sqrt{3}} \lambda\sqrt{x}\, d\lambda = \sqrt{3x}.$$

Finally, the point $(\alpha, 6)$ lies on the graph if

$$\sqrt{3\alpha} = 6, \qquad 3\alpha = 36, \qquad \alpha = 12.$$

The answer lies in the required positive domain, and no negative root arises from the positive square root.

Step 3: Conclusion The function is $f(x) = \sqrt{3x}$, and the required dimensionless abscissa is $\boxed{\alpha = 12}$.

Quick Tip to Solve: Rescale the integration variable so that its square is the argument of f; the moving upper endpoint then produces a simple differential equation.

27. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-coplanar vectors such that $\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}$, $\alpha > 0$. If $|\vec{a}| + \left|\vec{b}\right| + |\vec{c}| = \frac{1}{36}$, then α is equal to _____. [4 marks]

Solution:

Concept:

- A cross product is perpendicular to both factors. Since the three nonzero vectors satisfy the stated cross-product relations, they are pairwise perpendicular.
- For perpendicular vectors of lengths A, B, C, cross-product magnitudes give $AB = 4C$, $BC = 9A$, and $CA = \alpha B$.

Step 1: Deduce the fixed length forced by the first two relations Let $A = |\vec{a}|$, $B = |\vec{b}|$, and $C = |\vec{c}|$. Non-coplanarity guarantees that all three lengths are positive. The equality $\vec{a} \times \vec{b} = 4\vec{c}$ shows $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$. Similarly $\vec{b} \times \vec{c} = 9\vec{a}$ shows $\vec{a} \perp \vec{b}$, so all pairwise angles are right angles. Taking magnitudes therefore gives

$$AB = 4C, \qquad BC = 9A.$$

Multiplying these equations and cancelling the positive factor AC yields $B^2 = 36$, hence $B = 6$. In particular, the three positive lengths must satisfy $A + B + C > 6$, independently of the third relation or the value of α.

Step 2: Check the printed sum condition The question as printed requires

$$A + B + C = \frac{1}{36}.$$

This is impossible, because $1/36 < 6 = B$. The contradiction already follows from the first two vector relations, so no choice of a positive α can repair it. To check the key value directly, the remaining magnitude equations would give $A = 2\sqrt{\alpha}$ and $C = 3\sqrt{\alpha}$, so

$$A + B + C = 5\sqrt{\alpha} + 6 > 6 \qquad (\alpha > 0).$$

At the listed key value $\alpha = 36$, the sum would be $12 + 6 + 18 = 36$, rather than $1/36$. This comparison identifies the exact mismatch but does not authorize replacing the stated condition. The literal question must retain its own sum and has no admissible solution.

Step 3: Conclusion For the printed condition $|\vec{a}| + |\vec{b}| + |\vec{c}| = 1/36$, no positive value of α exists. The listed answer-key value 36 does not satisfy the printed condition, because it gives the length sum 36.

Quick Tip to Solve: Before solving for a parameter in cross-product equations, check whether any vector length is fixed; that can immediately expose an inconsistent scalar condition.

28. Consider a matrix $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$, where α, β, γ are three distinct natural numbers. If $\frac{\det(\text{adj}(\text{adj}(\text{adj}(\text{adj}\, A))))}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}} = 2^{32} \times 3^{16}$, then the number of such 3 - tuples (α, β, γ) is _____. [4 marks]

Solution:

Concept:

- For a 3×3 matrix M, $\det(\text{adj}\, M) = (\det M)^2$; four adjugates therefore raise the original determinant to the sixteenth power.
- The Vandermonde determinant factors into pairwise differences. The positive sum $s = \alpha + \beta + \gamma$ is the remaining factor here.

Step 1: Reduce the determinant identity to a sum Put $s = \alpha + \beta + \gamma$. Adding the first row to the third row changes the third row into (s, s, s) without changing the determinant. Hence

$$\det A = s \det \begin{pmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{pmatrix} = s(\beta-\alpha)(\gamma-\alpha)(\gamma-\beta).$$

The last equality follows by a cyclic row permutation of the standard Vandermonde matrix, which has positive sign. The product of differences can equivalently be written $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$. Since the entries are distinct, the denominator in the question is nonzero. Four applications of the adjugate determinant identity give

$$\det(\text{adj}^4\, A) = (\det A)^{16}.$$

After cancellation of the difference factors, the displayed condition becomes

$$s^{16} = 2^{32}3^{16} = 12^{16}.$$

Natural numbers are positive here, so $s = 12$.

Step 2: Count ordered positive triples with distinct entries The number of positive integer solutions of $\alpha + \beta + \gamma = 12$, without distinctness imposed, is $\binom{11}{2} = 55$. For one specified equality, say $\alpha = \beta$, the equation becomes $2\alpha + \gamma = 12$. It has five positive solutions, with $\alpha = 1, 2, 3, 4, 5$. There are three possible equal pairs. Their pairwise and triple intersections each consist of the single triple $(4, 4, 4)$. Inclusion-exclusion therefore gives $3 \cdot 5 - 3 + 1 = 13$ triples with at least two equal entries. The required distinct ordered triples number

$$55 - 13 = 42.$$

Order matters because the question asks for 3-tuples; each set of three distinct entries contributes six permutations.

Step 3: Conclusion There are $\boxed{42}$ ordered triples of distinct natural numbers satisfying the determinant condition.

Quick Tip to Solve: Each adjugate doubles the determinant exponent for a matrix of order three; cancel first, then count distinct positive compositions of the resulting sum.

29. $\frac{2^3-1^3}{1\times7}+\frac{4^3-3^3+2^3-1^3}{2\times11}+\frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times15}+\cdots+\frac{30^3-29^3+28^3-27^3+\ldots+2^3-1^3}{15\times63}$ **is equal to _____.**

[4 marks]

Solution:

Concept:

- Group each consecutive even cube with the odd cube immediately preceding it. The numerator of the nth fraction is a finite sum over these pairs.
- The identities $\sum_{k=1}^{n} k = n(n+1)/2$ and $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$ reduce this sum to a polynomial.

Step 1: Find the general numerator Write $S_n = \sum_{k=1}^{n}[(2k)^3-(2k-1)^3]$. Expand the odd cube carefully, retaining its negative sign outside the bracket. Each pair becomes $12k^2 - 6k + 1$. Consequently,

$$S_n = 12\frac{n(n+1)(2n+1)}{6} - 6\frac{n(n+1)}{2} + n.$$

Factoring out n gives

$$S_n = n[2(n+1)(2n+1) - 3(n+1) + 1] = n^2(4n+3).$$

For example, the first numerator is 7, agreeing with this expression at n=1.

Step 2: Simplify and sum the fractions The second denominator factor follows the progression 7, 11, 15, and is $4n+3$; hence the complete denominator is $n(4n+3)$. All its factors are nonzero for the positive integers used here. The nth summand is therefore

$$\frac{S_n}{n(4n+3)} = n.$$

The last numerator ends at 30^3, so it corresponds to $2n = 30$, or n=15. Its denominator is indeed $15(4\cdot15+3) = 15\cdot63$, confirming the endpoint. Thus the requested sum is

$$1+2+\cdots+15 = \frac{15\cdot16}{2} = 120.$$

Step 3: Conclusion The value of the given series is $\boxed{120}$, a dimensionless number.

Quick Tip to Solve: For alternating consecutive cubes, pair the even and odd terms before summing; the resulting quadratic often cancels the denominator.

30. A common tangent T to the curves $C_1 : \frac{x^2}{4}+\frac{y^2}{9} = 1$ and $C_2 : \frac{x^2}{42}-\frac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1 + x_2|$ is equal to _____.

[4 marks]

Solution:

Concept:

- A nonvertical line $y = mx + c$ is tangent to $x^2/a^2 + y^2/b^2 = 1$ when $c^2 = a^2m^2 + b^2$.

- For $x^2/a^2 - y^2/b^2 = 1$, the corresponding condition is $c^2 = a^2m^2 - b^2$. The tangency point follows by comparing with the tangent equation at a point.

Step 1: Determine the admissible line The two tangency conditions give

$$c^2 = 4m^2 + 9 = 42m^2 - 143.$$

Hence $38m^2 = 152$, so $m = \pm 2$, and $c = \pm 5$. Vertical tangents to the ellipse are $x = \pm 2$, whereas those to the hyperbola are $x = \pm\sqrt{42}$, so no common vertical tangent has been missed. A negative-slope line enters the fourth quadrant for sufficiently large positive x. For slope +2, the intercept -5 also gives negative y at small positive x. Only $T : y = 2x + 5$ avoids that quadrant.

Step 2: Calculate both points of contact The ellipse tangent at (x_1, y_1) is $xx_1/4 + yy_1/9 = 1$. Rewrite T as $-2x/5 + y/5 = 1$, then compare coefficients:

$$x_1 = -\frac{8}{5}, \qquad y_1 = \frac{9}{5}.$$

For the hyperbola the tangent is $xx_2/42 - yy_2/143 = 1$, giving

$$x_2 = -\frac{84}{5}, \qquad y_2 = -\frac{143}{5}.$$

Both points satisfy $y = 2x + 5$; the negative x-coordinate of the second point is compatible with avoiding the fourth quadrant. Finally,

$$|2x_1 + x_2| = \left|-\frac{16}{5} - \frac{84}{5}\right| = 20.$$

Step 3: Conclusion The requested dimensionless value is $\boxed{20}$.

Quick Tip to Solve: First equate the squared intercept conditions, then use the quadrant condition to select the signs of both slope and intercept.

Physics – Section A: Multiple Choice Questions (80 marks)

31. An expression of energy density is given by $u = \frac{\alpha}{\beta}\sin\left(\frac{\alpha x}{kt}\right)$, where α, β are constants, x is displacement, k is Boltzmann constant and t is the temperature. The dimensions of β will be :

[4 marks]

(A) $\left[\mathrm{ML}^2\ \mathrm{T}^{-2}\theta^{-1}\right]$
(B) $\left[\mathrm{M}^0\ \mathrm{L}^2\ \mathrm{T}^{-2}\right]$
(C) $\left[\mathrm{M}^0\ \mathrm{L}^0\ \mathrm{T}^0\right]$
(D) $\left[\mathrm{M}^0\ \mathrm{L}^2\ \mathrm{T}^0\right]$

Correct Answer: (D) $\left[\mathrm{M}^0\ \mathrm{L}^2\ \mathrm{T}^0\right]$

Solution:

Concept:

- The argument of a trigonometric function must be dimensionless. This condition determines the dimensions of the constant inside the sine.
- Energy density is energy per unit volume: $[u] = \mathrm{ML}^{-1}\mathrm{T}^{-2}$. Boltzmann constant has dimensions energy per temperature.

Step 1: Use the sine argument The symbol t here denotes temperature, not time; use Θ for its dimension to keep that distinction explicit. Boltzmann constant has

$$[k] = \mathrm{ML}^2\mathrm{T}^{-2}\Theta^{-1}.$$

Thus $[kt] = \mathrm{ML}^2\mathrm{T}^{-2}$, which is an energy. Since $\alpha x/(kt)$ is dimensionless and x is displacement,

$$[\alpha][x] = [kt], \qquad [\alpha] = \mathrm{MLT}^{-2}.$$

Therefore alpha has the dimensions of force, despite the fact that the expression describes an energy density.

Step 2: Use the amplitude and compare options The sine factor is dimensionless, so the prefactor α/β must have precisely the same dimensions as u. Divide the dimensions of alpha by those of energy density:

$$[\beta] = \frac{[\alpha]}{[u]} = \frac{\mathrm{MLT}^{-2}}{\mathrm{ML}^{-1}\mathrm{T}^{-2}} = \mathrm{M}^0\mathrm{L}^2\mathrm{T}^0.$$

Mass and time cancel, and there is no remaining temperature factor. Option A instead has the dimensions of Boltzmann constant and retains mass and temperature. Option B has an unwanted inverse-square time factor. Option C is dimensionless and omits the required area factor. Only option D has the correct length-squared dependence.

Step 3: Conclusion The dimensions are $\boxed{[\beta] = [\mathrm{M}^0\mathrm{L}^2\mathrm{T}^0]}$. Hence, the correct option is (D).

Quick Tip to Solve: For a sine expression, first make its argument dimensionless, then match the prefactor to the physical quantity on the left.

32. A body of mass $10\ \mathrm{kg}$ is projected at an angle of 45° with the horizontal. The trajectory of the body is observed to pass through a point $(20, 10)$. If T is the time of flight, then its momentum vector, at time $\mathrm{t} = \frac{\mathrm{T}}{\sqrt{2}}$, is _____.

[Take $\mathrm{g} = 10\ \mathrm{m/s^2}$] [4 marks]

(A) $100\hat{i} + (100\sqrt{2} - 200)\hat{j}$

(B) $100\sqrt{2}\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(C) $100\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(D) $100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$

Correct Answer: (D) $100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$

Solution:

Concept:

- For projectile motion without air resistance, horizontal velocity is constant and vertical velocity is $v_y = u_y - gt$. Momentum is mass times instantaneous velocity.
- At a launch angle of 45°, $u_x = u_y$. The trajectory equation determines their common value from a known point.

Step 1: Find the initial components Let $u_x = u_y = v$. Eliminating time from $x = vt$ and $y = vt - gt^2/2$ gives

$$y = x - \frac{gx^2}{2v^2}.$$

Substitute the point (20,10) and $g = 10\,\mathrm{m\,s^{-2}}$:

$$10 = 20 - \frac{10(20)^2}{2v^2}, \qquad v^2 = 200.$$

Therefore $u_x = u_y = 10\sqrt{2}\,\mathrm{m\,s^{-1}}$. This result fixes both components, rather than just the magnitude of the launch velocity.

Step 2: Evaluate the momentum at the specified time The complete flight time to the launch level is

$$T = \frac{2u_y}{g} = 2\sqrt{2}\,\mathrm{s}.$$

Hence the specified instant is $t = T/\sqrt{2} = 2\,\mathrm{s}$. The instantaneous velocity components become

$$v_x = 10\sqrt{2}, \qquad v_y = 10\sqrt{2} - 20 \quad (\mathrm{m\,s^{-1}}).$$

Multiplying each component by the 10 kg mass yields

$$\vec{p} = 100\sqrt{2}\,\hat{i} + (100\sqrt{2} - 200)\,\hat{j} \quad \mathrm{kg\,m\,s^{-1}}.$$

The vertical component is negative because this time is after the apex at $\sqrt{2}$ seconds. Options A and C give the wrong horizontal component; B has the correct horizontal component but the wrong vertical one.

Step 3: Conclusion $\boxed{\vec{p} = [100\sqrt{2}\,\hat{i} + (100\sqrt{2} - 200)\,\hat{j}]\,\mathrm{kg\,m\,s^{-1}}}$. Hence, the correct option is (D).

Quick Tip to Solve: Use the known trajectory point to find the horizontal launch component, then calculate the requested time before forming the momentum vector.

33. A block of mass M slides down on a rough inclined plane with constant velocity. The angle made by the incline plane with horizontal is θ. The magnitude of the contact force will be : [4 marks]

(A) Mg

(B) $\mathrm{Mg}\cos\theta$

(C) $\sqrt{\mathrm{Mg}\sin\theta + \mathrm{Mg}\cos\theta}$

(D) $\mathrm{Mg}\sin\theta\sqrt{1+\mu}$

Correct Answer: (A) Mg

Solution:

Concept:

- Constant velocity means zero acceleration, so Newton's second law requires the vector sum of all forces to vanish.
- The contact force from a rough surface includes both the normal reaction and friction. These two perpendicular components must be combined vectorially.

Step 1: Resolve the forces on the block Take the direction down the incline as positive along the plane. Gravity has component $Mg\sin\theta$ down the plane and $Mg\cos\theta$ into it. Since the block slides with constant velocity, there is no acceleration along either axis. Therefore the upward friction magnitude and normal reaction are

$$f = Mg\sin\theta, \qquad N = Mg\cos\theta.$$

Friction is not zero simply because velocity is constant: it is precisely what balances the downhill gravitational component. The coefficient, if required, would satisfy $\mu = f/N = \tan\theta$.

Step 2: Combine the contact components The normal reaction acts perpendicular to the surface whereas friction acts along it. Their resultant is the complete contact force $\vec{R}$, with magnitude

$$R = \sqrt{N^2 + f^2} = Mg\sqrt{\cos^2\theta + \sin^2\theta} = Mg.$$

Equivalently, the only noncontact force is weight; the total contact force must be equal and opposite to that weight for equilibrium. Option B gives only the normal component and leaves out friction. Option C adds unsquared force components under a square root, giving incorrect dimensions. Option D does not reproduce the vector sum of the normal and friction forces or the equilibrium requirement.

Step 3: Conclusion The magnitude of the resultant contact force is $\boxed{Mg}$, measured in newtons. Hence, the correct option is (A).

Quick Tip to Solve: On a rough surface, contact force means the resultant of normal reaction and friction; for constant velocity with gravity as the only other force, it equals the weight.

34. A block 'A' takes 2 s to slide down a frictionless incline of 30° and length 'l', kept inside a lift going up with uniform velocity 'v'. If the incline is changed to 45°, the time taken by the block, to slide down the incline, will be approximately : [4 marks]

(A) 2.66 s

(B) 0.83 s

(C) 1.68 s

(D) 0.70 s

Correct Answer: (C) 1.68 s

Solution:

Concept:

- A lift moving with uniform velocity is an inertial frame. There is no additional inertial force, so the effective gravitational acceleration remains g.
- For a frictionless incline at angle ϕ, acceleration along the plane is $g\sin\phi$. From rest over fixed length l, $l = \frac{1}{2}g\sin\phi\, t^2$.

Step 1: Compare the two accelerations The upward velocity v of the lift does not imply an upward acceleration. Consequently its value does not enter the equation of relative motion along the incline. At $30°$, the acceleration is $a_1 = g\sin 30° = g/2$, and the given sliding time is $t_1 = 2\,\text{s}$. At $45°$, the acceleration becomes $a_2 = g\sin 45° = g/\sqrt{2}$. This second acceleration is larger, so the new time over the same incline length must be smaller than two seconds.

Step 2: Use the fixed length to find the time The length and the initial condition of release from rest are the same in both cases. Equating the two distance expressions,

$$\tfrac{1}{2}a_1 t_1^2 = \tfrac{1}{2}a_2 t_2^2.$$

Thus

$$t_2 = t_1\sqrt{\frac{a_1}{a_2}} = 2\sqrt{\frac{1/2}{1/\sqrt{2}}} = 2\,2^{-1/4}\,\text{s}.$$

Since $2^{1/4} \approx 1.1892$, the time is $t_2 \approx 1.682\,\text{s}$. Option A incorrectly makes the time longer despite the larger acceleration. Options B and D give reductions far greater than the square-root acceleration ratio allows; they do not satisfy the same-length equation.

Step 3: Conclusion The new sliding time is approximately $\boxed{1.68\,\text{s}}$. Hence, the correct option is (C).

Quick Tip to Solve: For the same frictionless incline length, time varies as $1/\sqrt{\sin\phi}$; a lift velocity matters only if it changes with time.

35. The velocity of the bullet becomes one third after it penetrates 4 cm in a wooden block. Assuming that bullet is facing a constant resistance during its motion in the block. The bullet stops completely after travelling at (4 + x) cm inside the block. The value of x is : [4 marks]

(A) 2.0

(B) 1.0

(C) 0.5

(D) 1.5

Correct Answer: (C) 0.5

Solution:

Concept:

- A constant resisting force on a bullet of fixed mass gives constant retardation. Its squared speed then varies linearly with penetration distance.
- The kinematic equation $v^2 = u^2 + 2as$, or equivalently the work-energy theorem, relates the lost kinetic energy to the distance travelled in the wood.

Step 1: Determine the retardation from the first segment Let the entry speed be u and let the positive magnitude of the retardation be b. It is convenient to use centimetres consistently for the two distances, because only a ratio is needed. After travelling 4 cm the speed is u/3, so

$$\frac{u^2}{9} = u^2 - 2b(4\,\text{cm}).$$

Therefore

$$2b(4\,\text{cm}) = \frac{8u^2}{9}.$$

The bullet has lost eight ninths of its original kinetic energy in this first segment; only one ninth remains. The time elapsed is not needed.

Step 2: Find the remaining stopping distance Let the remaining distance be x cm. During this part the speed decreases from u/3 to zero under the same retardation, so

$$0 = \frac{u^2}{9} - 2b(x\,\text{cm}).$$

Dividing this equation by the energy-loss equation for the first segment removes both b and u:

$$\frac{x}{4} = \frac{u^2/9}{8u^2/9} = \frac{1}{8}.$$

Hence $x = 0.5$. The total penetration distance is $4 + x = 4.5\,\text{cm}$, but the question asks for the additional distance x. Options A, B and D would require more work than the remaining kinetic energy permits at this fixed resistance; each exceeds the computed 0.5 cm.

Step 3: Conclusion The additional stopping distance is $\boxed{0.5\,\text{cm}}$, so the requested value of x is $\boxed{0.5}$. Hence, the correct option is (C).

Quick Tip to Solve: With constant resistance, distance is proportional to kinetic energy lost; reducing speed to one third leaves only one ninth of the initial stopping distance.

36. A body of mass m is projected with velocity $\lambda\, v_{\mathrm{e}}$ in vertically upward direction from the surface of the earth into space. It is given that v_{e} is escape velocity and $\lambda < 1$. If air resistance is considered to be negligible, then the maximum height from the centre of earth, to which the body can go, will be : (R : radius of earth) [4 marks]

(A) $\frac{\mathrm{R}}{1+\lambda^2}$

(B) $\frac{R}{1-\lambda^2}$

(C) $\frac{R}{1-\lambda}$

(D) $\frac{\lambda^2 \mathrm{R}}{1-\lambda^2}$

Correct Answer: (B) $\frac{R}{1-\lambda^2}$

Solution:

Concept:

- With negligible air resistance, the sum of kinetic energy and gravitational potential energy is conserved. At distance r from Earth's centre the potential energy is $-GMm/r$, with zero potential at infinity.
- The surface escape speed obeys $v_e^2 = 2GM/R$. At the highest point of a radial trajectory, the speed is zero; use the varying gravitational potential rather than constant surface gravity.

Step 1: Write the initial and final energies Let $r_{\max}$ denote the greatest distance from the centre, so the height above the surface is $r_{\max} - R$. The launch point is at radius R and the launch speed is λv_e. Energy conservation gives

$$\frac{1}{2}m\lambda^2 v_e^2 - \frac{GMm}{R} = -\frac{GMm}{r_{\max}}.$$

The final kinetic energy vanishes, but the final potential energy remains negative because the body does not escape.

Step 2: Substitute the escape-speed relation Use $v_e^2 = 2GM/R$ and cancel the common nonzero factor GMm:

$$\frac{\lambda^2 - 1}{R} = -\frac{1}{r_{\max}}, \qquad r_{\max} = \frac{R}{1-\lambda^2}.$$

This is positive and larger than R for $0 < \lambda < 1$. Option (A) would incorrectly put the turning point inside Earth. Option (C) incorrectly makes energy proportional to speed rather than speed squared. Option (D) is $r_{\max} - R$, the height above the surface, whereas the question asks for distance from the centre.

Step 3: Conclusion The maximum distance from Earth's centre is $\boxed{R/(1-\lambda^2)}$. Hence, the correct option is (B).

Quick Tip to Solve: First decide whether the requested height is measured from the surface or the centre; subtract R only for surface altitude.

37. A steel wire of length $3.2\ \text{m}$ $\left(\text{Y}_\text{s} = 2.0 \times 10^{11}\ \text{Nm}^{-2}\right)$
and a copper wire of length $4.4\ \text{m}$ $\left(\text{Y}_\text{c} = 1.1 \times 10^{11}\ \text{Nm}^{-2}\right)$,
both of radius $1.4\ \text{mm}$ **are connected end to end. When stretched by a load, the net elongation is found to be** $1.4\ \text{mm}$**. The load applied, in Newton, will be:** (**Given** $\pi = \frac{22}{7}$**)** [4 marks]

(A) 360

(B) 180

(C) 1080

(D) 154

Correct Answer: (D) 154

Solution:

Concept:

- For a wire obeying Hooke's law, its extension is $\Delta L = FL/(AY)$, where F is the tensile load, A the cross-sectional area and Y Young's modulus.
- Wires joined end to end carry the same tensile force in static equilibrium. Their extensions add, so the total extension is $(F/A)(L_s/Y_s + L_c/Y_c)$.

Step 1: Calculate the area and combined compliance The common radius is $1.4 \times 10^{-3}\ \text{m}$, giving

$$A = \frac{22}{7}(1.4 \times 10^{-3})^2 = 6.16 \times 10^{-6}\ \text{m}^2.$$

For the two wires,

$$\frac{L_s}{Y_s} + \frac{L_c}{Y_c} = \frac{3.2}{2.0 \times 10^{11}} + \frac{4.4}{1.1 \times 10^{11}} = 5.6 \times 10^{-11}\ \text{m}^3/\text{N}.$$

The copper contributes more extension per unit force because of its greater length and smaller modulus.

Step 2: Use the total extension to obtain the load The given extension is $1.4 \times 10^{-3}\ \text{m}$. Therefore

$$F = \frac{A\Delta L}{L_s/Y_s + L_c/Y_c} = \frac{(6.16 \times 10^{-6})(1.4 \times 10^{-3})}{5.6 \times 10^{-11}} = 154\,\text{N}.$$

As a check, this force produces steel and copper extensions of $0.4\,\text{mm}$ and $1.0\,\text{mm}$, respectively, whose sum is the required $1.4\,\text{mm}$. The other proposed loads, 360 N, 180 N and 1080 N, give different extensions because the extension is directly proportional to load.

Step 3: Conclusion The common tensile load applied to the joined wires is $\boxed{154\,\text{N}}$. Hence, the correct option is (D).

Quick Tip to Solve: For wires in series, add $L/(AY)$, not the Young moduli; the force is common and the extensions add.

38. Which statements are correct about degrees of freedom ?

(A) A molecule with n degrees of freedom has n^2 different ways of storing energy.

(B) Each degree of freedom is associated with $\frac{1}{2}$ RT average energy per mole.

(C) A monatomic gas molecule has 1 rotational degree of freedom where as diatomic molecule has 2 rotational degrees of freedom.

(D) CH_4 has a total of 6 degrees of freedom.

Choose the correct answer from the options given below : [4 marks]

(A) (B) and (C) only

(B) (B) and (D) only

(C) (A) and (B) only

(D) (C) and (D) only

Correct Answer: (B) (B) and (D) only

Solution:

Concept:

- A degree of freedom is an independent coordinate or quadratic energy contribution available to a molecule. The equipartition theorem assigns average energy $k_BT/2$ per molecule to each active quadratic degree of freedom.
- For a mole this contribution is $RT/2$. In the usual rigid-molecule model, a nonlinear polyatomic molecule has three translational and three rotational degrees of freedom; vibrational modes are neglected when not excited.

Step 1: Test the first two statements Statement (A) is false: n degrees of freedom mean n independent modes, not n^2 ways of storing energy. If all these modes contribute quadratically, their total average energy is $nk_BT/2$ per molecule. Statement (B) is true because multiplying the single-molecule contribution by Avogadro's number gives

$$N_A\frac{1}{2}k_BT = \frac{1}{2}RT.$$

This is an energy per mole for each active degree of freedom, not the total for an arbitrary molecule.

Step 2: Test rotations and identify the combination Statement (C) is false. A monatomic gas is treated as a point particle and has no active rotational degree of freedom in this model; a rigid diatomic molecule has two rotational degrees. Statement (D) is true in the rigid-molecule approximation: methane is a nonlinear molecule, so its three translational and three rotational degrees give

$$f = 3 + 3 = 6.$$

Thus only (B) and (D) are correct. The combinations (B),(C), (A),(B), and (C),(D) each include at least one false statement, so they are excluded. Vibrational contributions at sufficiently high temperature are outside the stated model used in this question.

Step 3: Conclusion Only the equipartition and methane statements are correct. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the statement letters from the answer-option letters; first mark every statement true or false, then select its combination.

39. A charge of $4\,\mu C$ is to be divided into two. The distance between the two divided charges is constant. The magnitude of the divided charges so that the force between them is maximum, will be : [4 marks]

(A) $1\,\mu C$ and $3\,\mu C$

(B) $2\,\mu C$ and $2\,\mu C$

(C) 0 and $4\,\mu\,C$

(D) $1.5\,\mu C$ and $2.5\,\mu C$

Correct Answer: (B) $2\,\mu C$ and $2\,\mu C$

Solution:

Concept:

- Coulomb's law gives $F = kq_1q_2/r^2$ for two positive point charges a fixed distance r apart. Since k/r^2 is fixed, maximising the force is equivalent to maximising the charge product.
- For a fixed sum Q, the product of two nonnegative parts is greatest when they are equal. This follows either by completing a square or by differentiating $q(Q-q)$.

Step 1: Express the force in terms of one part Let one charge be q and the other $Q-q$, where $Q = 4\,\mu C$. For an actual division of this positive charge, $0 \le q \le Q$. Then

$$F = \frac{k}{r^2}q(Q-q) = \frac{k}{r^2}\left[\frac{Q^2}{4} - \left(q - \frac{Q}{2}\right)^2\right].$$

The squared term can never be negative, so the bracket has its greatest possible value when the squared term is zero.

Step 2: Evaluate the optimal charges and compare the options The maximum therefore occurs at

$$q = \frac{Q}{2} = 2\,\mu C, \qquad Q - q = 2\,\mu C.$$

Their product is $4\,(\mu C)^2$. For the alternatives, the charge products in the same units are $1 \times 3 = 3$, $0 \times 4 = 0$, and $1.5 \times 2.5 = 3.75$, respectively. Each is below 4, and hence each gives a smaller force at the same separation. The equal split is an interior maximum, not a zero-force endpoint; checking the products also directly confirms that the result has the required physical meaning.

Step 3: Conclusion The two charges must be $\boxed{2\,\mu C \text{ and } 2\,\mu C}$. Hence, the correct option is (B).

Quick Tip to Solve: With separation fixed and charge sum fixed, maximise the product; completing the square immediately gives an equal split.

40. (A) The drift velocity of electrons decreases with the increase in the temperature of conductor.
(B) The drift velocity is inversely proportional to the area of cross-section of given conductor.
(C) The drift velocity does not depend on the applied potential difference to the conductor.
(D) The drift velocity of electron is inversely proportional to the length of the conductor.
(E) The drift velocity increases with the increase in the temperature of conductor.
Choose the correct answer from the options given below : [4 marks]

(A) (A) and (B) only
(B) (A) and (D) only
(C) (B) and (E) only
(D) (B) and (C) only

Correct Answer: (B) (A) and (D) only

Solution:

Concept:

- The magnitude of electron drift velocity in a metallic conductor is $v_d = eE\tau/m_e = eV\tau/(m_e\ell)$, where τ is relaxation time, V the applied voltage and ℓ the length.
- Current is $I = neAv_d$. Its rearrangement $v_d = I/(neA)$ shows an inverse area dependence only when current is held fixed; the electrical conditions must be specified when comparing conductors.

Step 1: Check temperature, voltage and length dependence Under a fixed applied potential difference, heating a metal increases lattice scattering and decreases the relaxation time. Thus v_d decreases and statement (A) is true, while (E) is false. Also

$$v_d = \frac{e\tau}{m_e}\frac{V}{\ell}$$

shows directly that changing V changes the drift velocity: statement (C) is false. With material, temperature and voltage fixed, the same equation gives $v_d \propto 1/\ell$, so statement (D) is true.

Step 2: Assess the area statement and select the answer At fixed voltage and length, the electric field V/ℓ does not depend on cross-sectional area. Increasing the area increases the current in the same proportion, leaving I/A, and hence drift velocity, unchanged. Therefore statement (B), presented as an unconditional dependence on area, is false in this voltage-driven comparison. Its familiar inverse-area relation requires a separate fixed-current condition. Only (A) and (D) form the correct combination. Option (A) includes the incorrect area statement; option (C) includes the incorrect rising-temperature statement; option (D) includes the incorrect voltage-independence statement.

Step 3: Conclusion The temperature-decrease and inverse-length statements are correct under the usual fixed-voltage comparison. Hence, the correct option is (B).

Quick Tip to Solve: Before using $v_d = I/(neA)$, ask what is fixed; at fixed voltage use $v_d = eV\tau/(m_e\ell)$.

41. A cyclotron is used to accelerate protons. If the operating magnetic field is $1.0\ \mathrm{T}$ and the radius of the cyclotron 'dees' is $60\ \mathrm{cm}$, the kinetic energy of the accelerated protons in MeV will be :

[use $m_p = 1.6 \times 10^{-27}\ \mathrm{kg}, e = 1.6 \times 10^{-19}\ \mathrm{C}$] [4 marks]

(A) 12

(B) 18

(C) 16

(D) 32

Correct Answer: (B) 18

Solution:

Concept:

- At the outermost orbit of a nonrelativistic cyclotron, magnetic force supplies the centripetal force: $qvB = mv^2/r$, so the momentum is $p = qBr$. The magnetic field bends the path while the gap electric field supplies energy.
- The final kinetic energy is $K = p^2/(2m) = q^2B^2r^2/(2m)$. Convert joules to electronvolts with $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$.

Step 1: Calculate the momentum at the edge of the dees Use the dee radius as the orbit radius, $r = 60\,\mathrm{cm} = 0.60\,\mathrm{m}$, and take the proton charge as e. Then

$$p = eBr = (1.6 \times 10^{-19})(1.0)(0.60) = 9.6 \times 10^{-20}\,\mathrm{kg\,m\,s^{-1}}.$$

The radius is not the diameter, and its conversion to metres must be made before squaring. The supplied mass $1.6 \times 10^{-27}\,\mathrm{kg}$ is used as the problem requests.

Step 2: Find the kinetic energy and convert to MeV From the momentum,

$$K = \frac{(9.6 \times 10^{-20})^2}{2(1.6 \times 10^{-27})} = 2.88 \times 10^{-12}\,\mathrm{J}.$$

Since $1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}$,

$$K = \frac{2.88 \times 10^{-12}}{1.6 \times 10^{-13}} = 18\,\mathrm{MeV}.$$

The energy corresponding to this specified radius and field is unique; 12, 16 and 32 MeV do not satisfy the orbit-energy equation with the supplied mass. No factor of two for a full circular path is required, because radius determines the instantaneous momentum directly.

Step 3: Conclusion The accelerated protons have kinetic energy $\boxed{18\,\mathrm{MeV}}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $p = qBr$ at the outer radius and then $K = p^2/(2m)$; one MeV is 1.6×10^{-13} joule.

42. A series LCR circuit has $\mathrm{L} = 0.01\,\mathrm{H}, \mathrm{R} = 10\,\Omega$ and $\mathrm{C} = 1\mu\mathrm{F}$ and it is connected to ac voltage of amplitude $(\mathrm{V_m})\,50$ V. At frequency 60% lower than resonant frequency, the amplitude of current will be approximately : [4 marks]

(A) 466 mA

(B) 312 mA

(C) 238 mA

(D) 196 mA

Correct Answer: (C) 238 mA

Solution:

Concept:

- For a series LCR circuit the impedance magnitude is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$, and the current amplitude is $I_m = V_m/Z$. Voltage and current must both be amplitudes here.
- Resonance occurs at $\omega_0 = 1/\sqrt{LC}$. A frequency 60 percent lower than resonance is $0.40\omega_0$, rather than $0.60\omega_0$; the same ratio applies to ordinary and angular frequencies.

Step 1: Calculate the operating angular frequency and reactances Convert the capacitance to $C = 1.0 \times 10^{-6}\,\mathrm{F}$. The resonant angular frequency is

$$\omega_0 = \frac{1}{\sqrt{(0.01)(10^{-6})}} = 10^4\,\mathrm{rad\,s^{-1}}.$$

Therefore the operating value is $\omega = 4000\,\mathrm{rad\,s^{-1}}$, giving

$$X_L = \omega L = 40\,\Omega, \qquad X_C = \frac{1}{\omega C} = 250\,\Omega.$$

Below resonance the circuit is capacitive, so $X_L - X_C = -210\,\Omega$.

Step 2: Find impedance and current amplitude The impedance includes the resistance and the net reactance in quadrature:

$$Z = \sqrt{10^2 + (-210)^2} = \sqrt{44200}\,\Omega \simeq 210.24\,\Omega.$$

Thus

$$I_m = \frac{50}{210.24}\,\mathrm{A} \simeq 0.2378\,\mathrm{A} = 238\,\mathrm{mA}.$$

Since R is small compared with the net reactance, the check $50/210$ A gives nearly the same value. The other amplitudes 466, 312 and 196 mA imply different impedances and do not match this frequency. Using rms conversion on only the voltage would incorrectly mix amplitudes with rms quantities.

Step 3: Conclusion The current amplitude is approximately $\boxed{238\,\mathrm{mA}}$. Hence, the correct option is (C).

Quick Tip to Solve: Translate percent lower into a frequency multiplier first, then subtract the reactances before adding resistance in quadrature.

43. Identify the correct statements from the following descriptions of various properties of electromagnetic waves.

(A) In a plane electromagnetic wave electric field and magnetic field must be perpendicular to each other and direction of propagation of wave should be along electric field or magnetic field.

(B) The energy in electromagnetic wave is divided equally between electric and magnetic fields.

(C) Both electric field and magnetic field are parallel to each other and perpendicular to the direction of propagation of wave.

(D) The electric field, magnetic field and direction of propagation of wave must be perpendicular to each other.

(E) The ratio of amplitude of magnetic field to the amplitude of electric field is equal to speed of light.

Choose the most appropriate answer from the options given below : [4 marks]

(A) (D) only

(B) (B) and (D) only

(C) (B), (C) and (E) only

(D) (A), (B) and (E) only

Correct Answer: (B) (B) and (D) only

Solution:

Concept:

- A plane electromagnetic wave is transverse: $\mathbf{E}$, $\mathbf{B}$, and its propagation vector are mutually perpendicular. The energy flow is along $\mathbf{E} \times \mathbf{B}$.
- In vacuum the amplitudes satisfy $E_0 = cB_0$. The instantaneous energy densities are $u_E = \varepsilon_0 E^2/2$ and $u_B = B^2/(2\mu_0)$, with $c^2 = 1/(\mu_0\varepsilon_0)$.

Step 1: Check the directions Statement (A) starts correctly by making the two fields perpendicular, but incorrectly puts propagation along one of them. Propagation is perpendicular to both, so (A) is false. Statement (C) incorrectly makes the electric and magnetic fields parallel and is also false. Statement (D) gives the mutually perpendicular directions correctly and is true.

Step 2: Compare the field energies Substitute $E = cB$ into the electric energy density:

$$u_E = \frac{\varepsilon_0 c^2 B^2}{2} = \frac{B^2}{2\mu_0} = u_B.$$

Thus the wave energy is shared equally between the electric and magnetic fields; statement (B) is true. This equality holds instantaneously for a travelling plane wave, and hence also for the time averages.

Step 3: Check the amplitude ratio and options The amplitude relation gives

$$\frac{B_0}{E_0} = \frac{1}{c}, \qquad \frac{E_0}{B_0} = c.$$

Statement (E) reverses the ratio and is false. The first option omits the true statement (B); the third includes false statements (C) and (E); the fourth includes false statements (A) and (E). Only the second option contains exactly the two true statements.

Step 4: Conclusion Only the equal-energy statement and the mutually perpendicular direction

statement are correct. Hence, the correct option is (B).

Quick Tip to Solve: Remember the ordered triad: electric field, magnetic field, and propagation are mutually perpendicular, while the larger amplitude ratio is $E_0/B_0 = c$.

44. Two coherent sources of light interfere. The intensity ratio of two sources is $1 : 4$. For this interference pattern if the value of $\frac{I_{max}+I_{min}}{I_{max}-I_{min}}$ is equal to $\frac{2\alpha+1}{\beta+3}$, then $\frac{\alpha}{\beta}$ will be : [4 marks]

(A) 1.5

(B) 2

(C) 0.5

(D) 1

Correct Answer: The intended reduced-fraction answer is 2, but the unrestricted equation also permits other listed ratios.

Solution:

Concept:

- For coherent waves with intensities I_1 and I_2, the extreme intensities are $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$.
- Intensity is proportional to amplitude squared, so the intensity ratio $1 : 4$ corresponds to the amplitude ratio $1 : 2$. Equality of fractions alone does not uniquely identify their numerators and denominators.

Step 1: Find the two extreme intensities Write $I_1 = I_0$ and $I_2 = 4I_0$, where $I_0 > 0$. The two amplitudes are proportional to $\sqrt{I_0}$ and $2\sqrt{I_0}$. Therefore

$$I_{max} = 9I_0, \qquad I_{min} = I_0.$$

The nonzero minimum is expected because the two sources have unequal amplitudes; destructive interference cannot cancel them completely.

Step 2: Evaluate the measured ratio The required ratio is

$$\frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{9I_0 + I_0}{9I_0 - I_0} = \frac{10}{8} = \frac{5}{4}.$$

Using the conventional identification with this reduced fraction gives $2\alpha + 1 = 5$ and $\beta + 3 = 4$, hence $\alpha = 2$, $\beta = 1$, and $\alpha/\beta = 2$, the listed option (B).

Step 3: State the limitation in the printed wording Strictly, the question does not require $(2\alpha + 1)/(\beta + 3)$ to be in lowest terms or otherwise restrict α, β. Cross-multiplication gives only

$$8\alpha - 5\beta = 11.$$

For example $(\alpha, \beta) = (2, 1)$ gives ratio 2, whereas $(11, 11)$ gives ratio 1; both satisfy the printed equality. Thus option (D) is also compatible with the unrestricted equation. The optical result is unique, but the requested parameter ratio is not.

Step 4: Conclusion The intensity ratio expression equals $5/4$. The conventional reduced-fraction interpretation gives $\alpha/\beta = 2$, option (B), but the literal printed question lacks a condition ensuring a unique answer.

Quick Tip to Solve: First take square roots of source intensities. Match numerator and denominator separately only if a reduced-fraction convention or another normalization is specified.

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45. With reference to the observations in photo-electric effect, identify the correct statements from below :

(A) The square of maximum velocity of photoelectrons varies linearly with frequency of incident light.

(B) The value of saturation current increases on moving the source of light away from the metal surface.

(C) The maximum kinetic energy of photo-electrons decreases on decreasing the power of LED (light emitting diode) source of light.

(D) The immediate emission of photo-electrons out of metal surface can not be explained by particle nature of light/electromagnetic waves.

(E) Existence of threshold wavelength can not be explained by wave nature of light/ electromagnetic waves.

Choose the correct answer from the options given below : [4 marks]

(A) (A) and (B) only

(B) (A) and (E) only

(C) (C) and (E) only

(D) (D) and (E) only

Correct Answer: (B) (A) and (E) only

Solution:

Concept:

- Einstein's equation is $K_{\max} = h\nu - \phi = \frac{1}{2}m_e v_{\max}^2$, where ϕ is the work function. It separates the effect of photon frequency from the effect of photon flux.
- Above threshold, saturation current measures the number of emitted electrons per second and increases with incident intensity. The threshold condition is $h\nu_0 = \phi$, or $\lambda_0 = hc/\phi$.

Step 1: Test the velocity and current statements From Einstein's equation,

$$v_{\max}^2 = \frac{2h}{m_e}\nu - \frac{2\phi}{m_e}.$$

This is a straight-line dependence on frequency, so statement (A) is true. Moving a source farther away reduces the light intensity received by the metal, and hence reduces the photoelectron emission rate and saturation current. Statement (B), which claims an increase, is false.

Step 2: Separate photon energy from photon number For light of unchanged frequency, decreasing LED power decreases the number of incident photons per second; it does not reduce the energy

$h\nu$ of each photon. Therefore the saturation current decreases, but the maximum kinetic energy remains $h\nu - \phi$. Statement (C) is false. The one-photon transfer of energy also explains essentially instantaneous emission without waiting for gradual energy accumulation, so statement (D) is false.

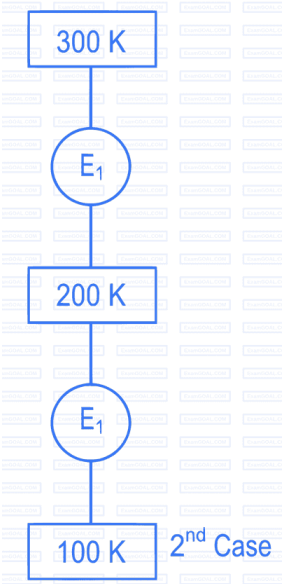
Step 3: Check the threshold observation A photoelectron can escape only if $h\nu \geq \phi$, equivalently $\lambda \leq \lambda_0$. Classical wave theory has no photon energy cutoff and cannot explain a sharp threshold wavelength independent of intensity; statement (E) is true. Thus the first option includes false statement (B), while the third and fourth include false statements (C) and (D), respectively.

Step 4: Conclusion The velocity-squared dependence and the failure of classical waves to explain threshold wavelength are the two correct statements. Hence, the correct option is (B).

Quick Tip to Solve: Use frequency to decide maximum photoelectron energy and use intensity to decide saturation current; never interchange these two roles.

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46. In 1^{st} case, Carnot engine operates between temperatures 300 K and 100 K. In 2^{nd} case, as shown in the figure, a combination of two engines is used. The efficiency of this combination (in 2^{nd} case) will be : [4 marks]



(A) same as the 1^{st} case.

(B) always greater than the 1^{st} case.

(C) always less than the 1^{st} case.

(D) may increase or decrease with respect to the 1^{st} case.

Correct Answer: (A) same as the 1^{st} case.

Solution:

Concept:

- A reversible Carnot engine operating between absolute temperatures T_H and T_C has efficiency $\eta = 1 - T_C/T_H$. It rejects the fraction T_C/T_H of its absorbed heat.

- For two engines in series, the first engine's rejected heat is the second engine's input. The combination's efficiency is total work divided by the heat drawn from the hottest reservoir, not the sum of the separate efficiencies.

Step 1: Calculate the single-engine efficiency In the first case the reservoirs are at $300\,\mathrm{K}$ and $100\,\mathrm{K}$. Its efficiency is

$$\eta = 1 - \frac{100}{300} = \frac{2}{3}.$$

Take the heat drawn from the hot reservoir to be Q; the single engine delivers work $2Q/3$ and rejects $Q/3$. The heat input is the appropriate common basis for comparing the two cases.

Step 2: Follow the heat through the intermediate reservoir The figure shows the first engine between $300\,\mathrm{K}$ and $200\,\mathrm{K}$, and the second between $200\,\mathrm{K}$ and $100\,\mathrm{K}$. For the first engine, $\eta_1 = 1 - 200/300 = 1/3$, so $W_1 = Q/3$ and the rejected heat is $Q_2 = 2Q/3$. The second engine has $\eta_2 = 1 - 100/200 = 1/2$, so

$$W_2 = \eta_2 Q_2 = \frac{1}{2}\frac{2Q}{3} = \frac{Q}{3}.$$

Step 3: Compare the total efficiencies Total work is $W_1 + W_2 = 2Q/3$, and therefore

$$\eta_{\text{combination}} = \frac{W_1 + W_2}{Q} = \frac{2}{3}.$$

The intermediate temperature cancels because the heat-rejection ratios multiply: $(200/300)(100/200) = 100/300$. Hence none of the claims of a necessarily larger, necessarily smaller, or variable efficiency applies to this reversible series combination.

Step 4: Conclusion The combination has efficiency $2/3$, the same as the first case. Hence, the correct option is (A).

Quick Tip to Solve: For engines in series, multiply their rejected-heat fractions, or use $\eta_{\text{net}} = 1-(1-\eta_1)(1-\eta_2)$; do not add efficiencies.

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47. A compass needle of oscillation magnetometer oscillates 20 times per minute at a place P of dip $30°$. The number of oscillations per minute become 10 at another place Q of $60°$ dip. The ratio of the total magnetic field at the two places $(B_Q : B_P)$ is : [4 marks]

(A) $\sqrt{3} : 4$

(B) $4 : \sqrt{3}$

(C) $\sqrt{3} : 2$

(D) $2 : \sqrt{3}$

Correct Answer: (A) $\sqrt{3} : 4$

Solution:

Concept:

- For small horizontal oscillations of a compass magnet, $f = (2\pi)^{-1}\sqrt{mB_H/I}$, where m is magnetic moment, I is moment of inertia, and B_H is the horizontal magnetic field.

- At a place with magnetic dip δ, the horizontal component is $B_H = B\cos\delta$, where B is the total magnetic field. The same compass has unchanged m and I.

Step 1: Compare the horizontal fields using the counts The oscillation counts are taken over the same one-minute interval, so their ratio equals the frequency ratio. Consequently

$$\frac{f_Q}{f_P} = \frac{10}{20} = \frac{1}{2}, \qquad \frac{B_{H,Q}}{B_{H,P}} = \left(\frac{f_Q}{f_P}\right)^2 = \frac{1}{4}.$$

The field ratio follows from the square of frequency, not from frequency itself. The vertical field does not provide the restoring torque for horizontal compass motion.

Step 2: Recover the total fields using the dip angles Insert $B_{H,Q} = B_Q\cos 60^\circ$ and $B_{H,P} = B_P\cos 30^\circ$:

$$\frac{B_Q\cos 60^\circ}{B_P\cos 30^\circ} = \frac{1}{4}.$$

Thus

$$\frac{B_Q}{B_P} = \frac{1}{4}\frac{\cos 30^\circ}{\cos 60^\circ} = \frac{1}{4}\frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{4}.$$

The greater dip at Q makes its horizontal component a smaller fraction of its total field.

Step 3: Check the listed ratios Only $\sqrt{3} : 4$ gives the computed horizontal-field ratio $1 : 4$ after the respective cosine factors are included. Option (B) reverses the requested order of total fields. Option (C) is too large by a factor of two, while option (D) gives the inverse of option (C). These alternatives do not reproduce both the oscillation counts and the dip angles.

Step 4: Conclusion The total-field ratio is $B_Q : B_P = \sqrt{3} : 4$. Hence, the correct option is (A).

Quick Tip to Solve: Square the oscillation-frequency ratio to obtain horizontal fields, then divide each horizontal field by the cosine of its local dip.

48. The activity of a radioactive material is 6.4×10^{-4} curie. Its half life is 5 days. The activity will become 5×10^{-6} curie after : [4 marks]

(A) 7 days

(B) 15 days

(C) 25 days

(D) 35 days

Correct Answer: (D) 35 days

Solution:

Concept:

- The activity of a radioactive sample follows the same exponential decay as its number of undecayed nuclei: $A = A_0e^{-\lambda t}$, where λ is the decay constant.
- With half-life $T_{1/2}$, the convenient form is $A/A_0 = 2^{-t/T_{1/2}}$. The time is the number of required halvings multiplied by the half-life.

Step 1: Find the activity reduction factor Both activities are expressed in curie, so their units cancel in the ratio and no conversion to becquerel is needed. The reduction factor is

$$\frac{A_0}{A} = \frac{6.4 \times 10^{-4}}{5 \times 10^{-6}} = \frac{6.4}{5} \times 10^2 = 128.$$

Keeping the powers of ten in this calculation is essential: the final activity is one hundred and twenty-eight times smaller, rather than merely a little smaller.

Step 2: Count the required half-lives Express the factor as a power of two:

$$128 = 2^7, \qquad \frac{A}{A_0} = \left(\frac{1}{2}\right)^7.$$

Thus seven half-lives must elapse. Each half-life lasts five days, so

$$t = 7T_{1/2} = 7 \times 5\,\text{days} = 35\,\text{days}.$$

As a direct check, $6.4 \times 10^{-4}/128 = 5 \times 10^{-6}$ curie, exactly the required final activity.

Step 3: Distinguish the remaining options The value seven counts half-lives and is not itself a number of days. Fifteen days corresponds to three half-lives and an activity 8.0×10^{-5} curie; twenty-five days corresponds to five half-lives and an activity 2.0×10^{-5} curie. Neither is the requested 5.0×10^{-6} curie. Only the thirty-five-day option produces the required reduction.

Step 4: Conclusion The required elapsed time is $35\,\text{days}$. Hence, the correct option is (D).

Quick Tip to Solve: Before using logarithms, simplify the activity ratio and look for a power of two; each factor of two corresponds to one half-life.

49. For a constant collector-emitter voltage of 8 V, the collector current of a transistor reached to the value of 6 mA from 4 mA, whereas base current changed from $20\,\mu$A to $25\,\mu$A value. If transistor is in active state, small signal current gain (current amplification factor) will be : [4 marks]

(A) 240

(B) 400

(C) 0.0025

(D) 200

Correct Answer: (B) 400

Solution:

Concept:

- The small signal common-emitter current gain is $\beta_{\text{ac}} = (\Delta I_C/\Delta I_B)_{V_{CE}}$. It is a ratio of changes measured at a fixed collector-emitter voltage.
- Convert the two current changes to the same unit before dividing. The gain is dimensionless and differs in general from the direct-current ratio I_C/I_B at a single operating point.

Step 1: Calculate the changes in current The collector current rises from $4\,\text{mA}$ to $6\,\text{mA}$, giving

$$\Delta I_C = 6 - 4 = 2\,\text{mA} = 2000\,\mu\text{A}.$$

The base current rises from $20\,\mu\text{A}$ to $25\,\mu\text{A}$, giving

$$\Delta I_B = 25 - 20 = 5\,\mu\text{A}.$$

Both increments are positive, so the small signal current gain will also be positive.

Step 2: Form the fixed-voltage current gain The collector-emitter voltage is held at the specified $8\,\text{V}$; this is precisely the condition in the definition of the gain. Hence

$$\beta_{\text{ac}} = \frac{2000\,\mu\text{A}}{5\,\mu\text{A}} = 400.$$

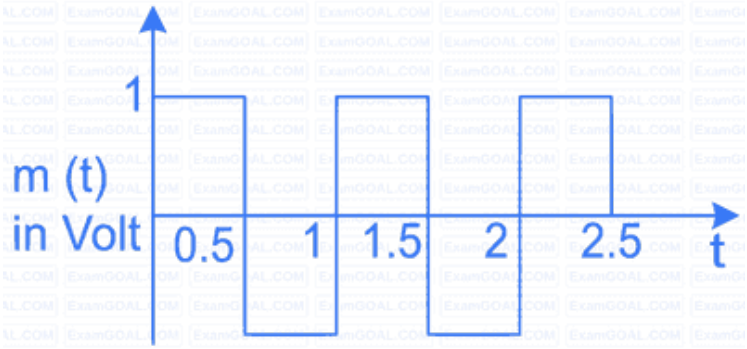
The microampere units cancel, leaving a pure number. The voltage establishes the measurement condition and is not another factor to multiply into the current ratio.

Step 3: Check the other numerical choices The value 240 is the final direct-current ratio $6\,\text{mA}/25\,\mu\text{A}$, not the slope between the two operating points. The value 200 is the initial direct-current ratio $4\,\text{mA}/20\,\mu\text{A}$, again not the requested small signal gain. The value 0.0025 is $\Delta I_B/\Delta I_C = 1/400$, the reciprocal of the required gain. Only 400 uses the correct increments in the required order.

Step 4: Conclusion The small signal current amplification factor is 400, dimensionless. Hence, the correct option is (B).

Quick Tip to Solve: For small signal gain use differences of the currents, and convert milliampere to microampere before dividing; endpoint current ratios give DC gains.

50. A square wave of the modulating signal is shown in the figure. The carrier wave is given by $C(t) = 5\sin(8\pi t)$ Volt. The modulation index is : [4 marks]



(A) 0.2

(B) 0.1

(C) 0.3

(D) 0.4

Correct Answer: (A) 0.2

Solution:

Concept:

- The modulation index is the ratio of the peak modulating voltage to the unmodulated carrier amplitude: $\mu = A_m/A_c$. Both amplitudes must be measured from their respective mean levels.
- The coefficient multiplying the sine in the carrier equation is its peak amplitude. The angular frequency determines how rapidly it oscillates and does not enter the amplitude ratio.

Step 1: Read the two amplitudes The square-wave graph reaches a positive peak of $1\,\mathrm{V}$, so its modulating amplitude is $A_m = 1\,\mathrm{V}$. The negative level is the other extreme of the same waveform, not an additional amplitude to add. From $C(t) = 5\sin(8\pi t)$, the carrier peak amplitude is $A_c = 5\,\mathrm{V}$. The factor 8π is an angular frequency, not a voltage.

Step 2: Form the amplitude ratio and check the options Substitution gives

$$\mu = \frac{A_m}{A_c} = \frac{1}{5} = 0.2.$$

The units cancel, so the index is dimensionless. Option (B), 0.1, would result from incorrectly comparing the modulating peak with the carrier peak-to-peak voltage. Option (D), 0.4, incorrectly uses the square wave peak-to-peak voltage with the carrier peak. Option (C), 0.3, is neither the stated amplitude ratio nor a quantity obtained from the given peaks. Also, $\mu < 1$, consistent with modulation without overmodulation.

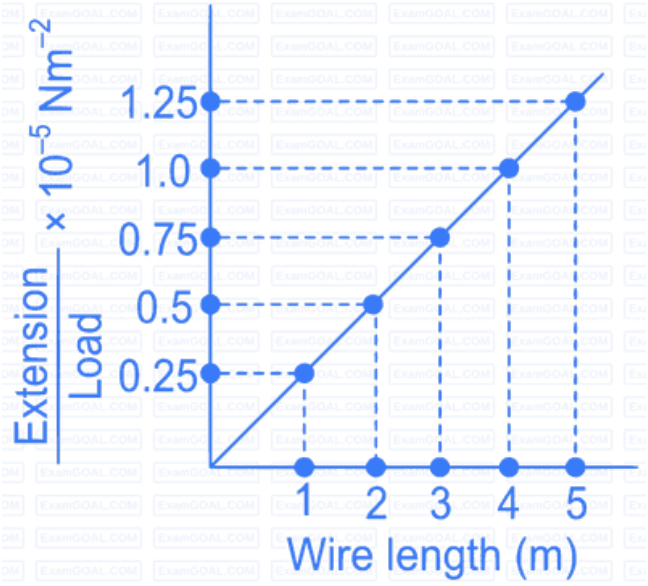
Step 3: Conclusion The modulation index is $\boxed{0.2}$. Hence, the correct option is (A).

Quick Tip to Solve: Use peak with peak, or peak-to-peak with peak-to-peak; mixing the two conventions creates a factor-of-two error.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. In an experiment to determine the Young's modulus, steel wires of five different lengths $(1, 2, 3, 4,$ and $5\ \text{m})$ but of same cross section $(2\ \text{mm}^2)$ were taken and curves between extension and load were obtained. The slope (extension/load) of the curves were plotted with the wire length and the following graph is obtained. If the Young's modulus of given steel wires is $x \times 10^{11}\ \text{Nm}^{-2}$, then the value of x is _____. [4 marks]



Solution:

Concept:

- For an elastic wire, Young's modulus is $Y = (F/A)/(\Delta L/L) = FL/(A\Delta L)$, where F is the tensile load, A the cross-sectional area and L the original length.
- Consequently the slope of extension against load is $s = \Delta L/F = L/(AY)$. A graph of this slope against wire length has gradient $g = 1/(AY)$.

Step 1: Read the gradient from the plotted points The line passes through the origin. For a wire of length $1\,\text{m}$, the extension/load value is $0.25 \times 10^{-5}\,\text{m N}^{-1}$. Equivalently, the point at $5\,\text{m}$ gives $1.25 \times 10^{-5}\,\text{m N}^{-1}$. Thus

$$g = \frac{1.25 \times 10^{-5}}{5} = 2.5 \times 10^{-6}\,\text{N}^{-1}.$$

The power-of-ten scale on the vertical axis must be included when calculating this gradient.

Step 2: Calculate the modulus in SI units Convert the common cross-sectional area before substitution:

$$A = 2\,\text{mm}^2 = 2 \times 10^{-6}\,\text{m}^2.$$

Since $g = 1/(AY)$,

$$Y = \frac{1}{Ag} = \frac{1}{(2 \times 10^{-6})(2.5 \times 10^{-6})} = 2 \times 10^{11}\,\text{N m}^{-2}.$$

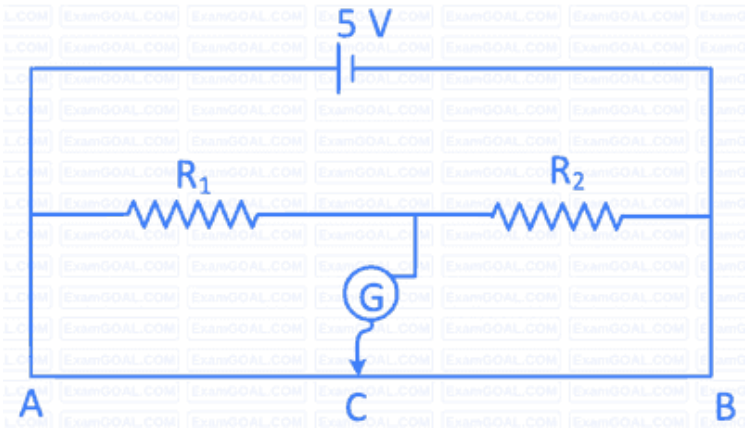
This is a material property, so every point on the straight line yields the same value. Comparing this result with the specified form $x \times 10^{11}\,\mathrm{N\,m^{-2}}$ identifies the requested coefficient.

Step 3: Conclusion Young's modulus is $2 \times 10^{11}\,\mathrm{N\,m^{-2}}$, so $\boxed{x = 2}$. The coefficient x is dimensionless.

Quick Tip to Solve: When the graph plots a slope against length, first express that slope algebraically; its gradient is not Young's modulus itself.

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52. In the given figure of meter bridge experiment, the balancing length AC corresponding to null deflection of the galvanometer is 40 cm. The balancing length, if the radius of the wire AB is doubled, will be _____ cm. [4 marks]



Solution:

Concept:

- At the null point of a metre bridge, the galvanometer carries no current and the bridge balance condition is $R_1/R_2 = R_{AC}/R_{CB}$.
- For a uniform bridge wire, $R = \rho l/A$. Since both wire segments have the same resistivity and area, their resistance ratio equals their length ratio, irrespective of the common wire radius.

Step 1: Write the original balance condition The total length of the bridge wire AB is $100\,\mathrm{cm}$. The given balance point is $AC = 40\,\mathrm{cm}$, hence $CB = 60\,\mathrm{cm}$. Therefore

$$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}.$$

This ratio is a property of the two resistances in the gaps. Changing the bridge wire does not change those two resistances.

Step 2: Apply the change to both wire segments Doubling the radius of the entire uniform wire AB makes its area four times larger. Consequently, at any fixed point C, both R_{AC} and R_{CB} become one-fourth of their old values. Their ratio is unchanged:

$$\frac{R_{AC}/4}{R_{CB}/4} = \frac{R_{AC}}{R_{CB}}.$$

If the new balancing length is l in centimetres, it must satisfy

$$\frac{l}{100-l}=\frac{2}{3}, \qquad 3l=200-2l, \qquad l=40.$$

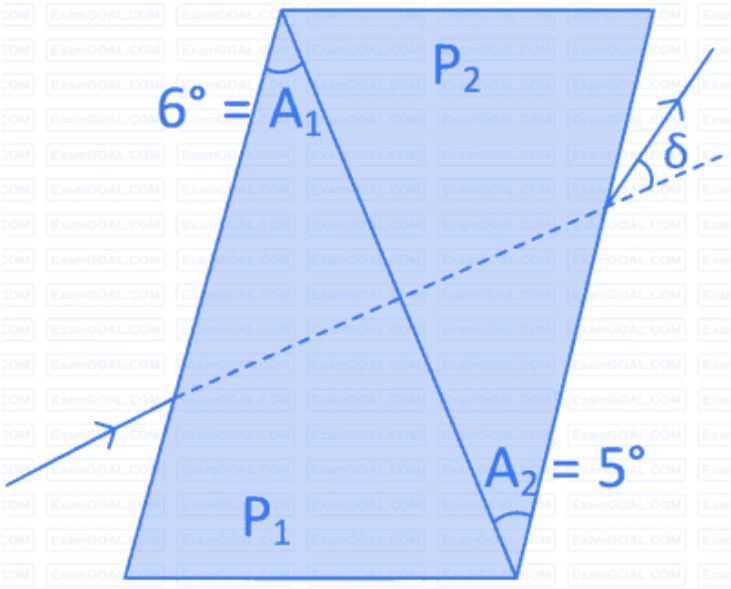
The bridge current can change because the total wire resistance changes, but the point of equal potentials, and hence the null position, remains the same.

Step 3: Conclusion The new balancing length is $\boxed{40\,\text{cm}}$.

Quick Tip to Solve: A uniform change of bridge-wire thickness cancels from the resistance ratio; it changes sensitivity or current, not the ideal balance length.

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53. A thin prism of angle 6° and refractive index for yellow light $(\mathrm{n_Y})\,1.5$ is combined with another prism of angle 5° and $\mathrm{n_Y}=1.55$. The combination produces no dispersion. The net average deviation (δ) produced by the combination is $\left(\frac{1}{x}\right)^\circ$. The value of x is _____. [4 marks]



Solution:

Concept:

- For a thin prism in air, the small-angle deviation for yellow light is $\delta_Y=(n_Y-1)A$. The prism angle and deviation may both be expressed in degrees in this linear relation.
- In an achromatic pair the prisms are oppositely oriented, so their dispersions cancel while their mean deviations subtract. Zero dispersion does not require zero mean deviation.

Step 1: Calculate the individual yellow-light deviations For the first prism, use $A_1=6^\circ$ and $n_{Y1}=1.5$:

$$\delta_1=(1.5-1)6^\circ=3^\circ.$$

For the second prism, use $A_2 = 5^\circ$ and $n_{Y2} = 1.55$:

$$\delta_2 = (1.55 - 1)5^\circ = 2.75^\circ.$$

These are the magnitudes that each prism would produce alone. The given no-dispersion condition identifies an achromatic arrangement, rather than two prisms deviating light in the same direction.

Step 2: Subtract the opposed deviations and identify the coefficient The bases of the two prisms point oppositely, as shown in the combination. Therefore the net average deviation is the magnitude of the difference:

$$\delta = |\delta_1 - \delta_2| = |3 - 2.75|^\circ = 0.25^\circ = \left(\frac{1}{4}\right)^\circ.$$

Adding the magnitudes would instead give 5.75°, corresponding to a reinforcing arrangement and inconsistent with the achromatic pair specified here. Compare the calculated result with $\delta = (1/x)^\circ$; hence $1/x = 1/4$. No conversion to radians is necessary because both prism angles were consistently used in degrees.

Step 3: Conclusion The net average deviation is 0.25°, giving $\boxed{x = 4}$, a dimensionless coefficient.

Quick Tip to Solve: Achromatic means the colours recombine; calculate the remaining yellow-light deviation by subtracting the two thin-prism deviations.

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54. A conducting circular loop is placed in $X - Y$ plane in presence of magnetic field $\vec{\mathrm{B}} = \left(3\mathrm{t}^3\,\hat{j} + 3\mathrm{t}^2\,\hat{k}\right)$ in SI unit. If the radius of the loop is $1\ \mathrm{m}$, the induced emf in the loop, at time, $\mathrm{t} = 2\ \mathrm{s}$ is $\mathrm{n}\pi\ \mathrm{V}$. The value of n is _____. [4 marks]

Solution:

Concept:

- Magnetic flux through a flat loop is $\Phi_B = \vec{B} \cdot \vec{A}$. The area vector is perpendicular to the plane of the loop, so only the normal component of magnetic field contributes.
- Faraday's law gives the induced emf $\mathcal{E} = -d\Phi_B/dt$. Its magnitude is the absolute rate of change of flux; the sign expresses Lenz's law for a chosen loop orientation.

Step 1: Project the field onto the loop normal The loop lies in the XY plane, so choose its positive area vector along $\hat{k}$. Its radius is $1\,\mathrm{m}$, giving

$$A = \pi r^2 = \pi\,\mathrm{m}^2.$$

In the given field $\vec{B} = 3t^3\hat{j} + 3t^2\hat{k}$, the $\hat{j}$ component is parallel to the loop plane. It therefore has zero scalar product with the area vector and contributes no flux, despite varying with time. Only $B_z = 3t^2$ is required.

Step 2: Differentiate the flux and evaluate the magnitude The loop area is fixed, so the time dependence comes entirely from the normal magnetic field:

$$\Phi_B = (3t^2)\pi.$$

Differentiate before inserting the time:

$$\mathcal{E} = -\frac{d}{dt}(3\pi t^2) = -6\pi t.$$

At $t = 2\,\text{s}$, this gives $\mathcal{E} = -12\pi\,\text{V}$ for the chosen positive area orientation. The question asks for the emf magnitude as $n\pi\,\text{V}$, so

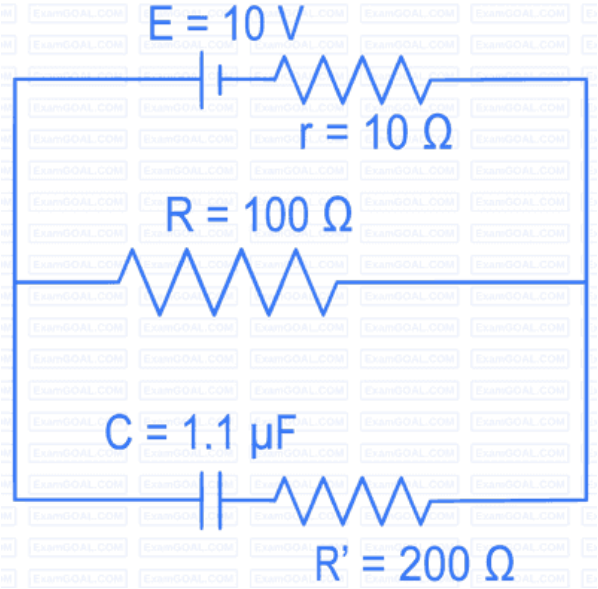
$$|\mathcal{E}| = 12\pi\,\text{V}.$$

Reversing the area-vector convention would reverse the sign but would not affect this magnitude.

Step 3: Conclusion The induced emf has magnitude $12\pi\,\text{V}$, hence $\boxed{n = 12}$.

Quick Tip to Solve: For a planar loop, discard magnetic-field components parallel to its plane before differentiating the flux.

55. As show in the figure, in steady state, the charge stored in the capacitor is _____ $\times 10^{-6}$ C. [4 marks]



Solution:

Concept:

- In the steady state of a DC circuit, an ideal capacitor carries no current. It acts as an open circuit, although a nonzero charge and potential difference remain on its plates.
- The capacitor charge is $Q = CV_C$. With no current through a resistor in series with the capacitor, that resistor has zero voltage drop in the steady state.

Step 1: Reduce the conducting part of the circuit The lower branch contains the $1.1\,\mu\text{F}$ capacitor in series with $R\prime = 200\,\Omega$. Its steady current is zero, so the voltage across $R\prime$ is zero. Thus the capacitor voltage equals the potential difference between the left and right rails. The only closed conducting loop contains the $10\,\text{V}$ source, its $10\,\Omega$ resistance, and the middle $100\,\Omega$ resistor. These two resistances

are in series in that loop.

Step 2: Find the rail voltage and the stored charge The steady loop current and rail voltage are

$$I = \frac{E}{r+R} = \frac{10}{10+100} = \frac{1}{11}\,\text{A},$$

$$V_C = IR = \frac{100}{11}\,\text{V}.$$

The capacitor is therefore not charged to the full emf, because current through the $100\,\Omega$ branch causes a voltage drop across the source resistance. Substitution gives

$$Q = CV_C = (1.1\times10^{-6})\frac{100}{11} = 10\times10^{-6}\,\text{C}.$$

The $200\,\Omega$ resistor can affect the charging transient, but it does not change this final charge since its final current is zero.

Step 3: Conclusion The stored charge is $\boxed{10\times10^{-6}\,\text{C}}$. The numerical entry is $\boxed{10}$.

Quick Tip to Solve: Open the capacitor branch for steady-current analysis, then use the rail voltage to calculate its charge; a series resistor in that open branch drops no voltage.

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56. A parallel plate capacitor with width 4 cm, length 8 cm and separation between the plates of 4 mm is connected to a battery of 20 V. A dielectric slab of dielectric constant 5 having length 1 cm, width 4 cm and thickness 4 mm is inserted between the plates of parallel plate capacitor. The electrostatic energy of this system will be _____ ϵ_0 J. (Where ϵ_0 is the permittivity of free space) [4 marks]

Solution:

Concept:

- The slab fills the whole plate separation over only part of the plate area. The dielectric-filled region and the remaining air region act as capacitors in parallel because both span the same two plates.
- For parallel regions, $C = \epsilon_0(A_a + KA_d)/d$, where A_a and A_d are the air and dielectric areas. Since the battery stays connected, the voltage is fixed and stored energy is $U = \frac{1}{2}CV^2$.

Step 1: Calculate the two parallel areas The total plate area is

$$(0.04)(0.08) = 0.0032\,\text{m}^2.$$

The dielectric slab covers width $4\,\text{cm}$ and length $1\,\text{cm}$, so

$$A_d = (0.04)(0.01) = 0.0004\,\text{m}^2.$$

The uncovered area is

$$A_a = 0.0032 - 0.0004 = 0.0028\,\text{m}^2.$$

Its thickness equals the $4\,\text{mm}$ plate separation, so there is no air layer in series with the slab in the covered region.

Step 2: Combine the capacitances and calculate the energy With $K = 5$ and $d = 0.004\,\text{m}$,

$$C = \frac{\epsilon_0}{0.004}\big(0.0028 + 5 \times 0.0004\big) = 1.2\epsilon_0.$$

Here the numerical coefficient carries the geometrical length factor needed to turn permittivity into capacitance. The connected battery maintains $V = 20\,\text{V}$, so

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(1.2\epsilon_0)(20)^2 = 240\epsilon_0\,\text{J}.$$

The constant-charge energy formula must not be applied with the original charge, because charge flows from the battery as the slab enters and the capacitance increases.

Step 3: Conclusion The stored electrostatic energy is $\boxed{240\epsilon_0\,\text{J}}$, so the requested numerical entry is $\boxed{240}$.

Quick Tip to Solve: A slab covering part of the area gives parallel capacitors; a slab filling only part of the gap gives series layers. Check whether voltage or charge remains fixed.

57. A wire of length 30 cm, stretched between rigid supports, has it's nth and (n + 1)th harmonics at 400 Hz and 450 Hz, respectively. If tension in the string is 2700 N, it's linear mass density is _____ kg/m. [4 marks]

Solution:

Concept:

- A string fixed at both ends supports standing waves with frequencies $f_n = nv/(2L)$, where v is wave speed and L is vibrating length.
- The transverse wave speed obeys $v = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length. Adjacent harmonic frequencies differ by the fundamental frequency.

Step 1: Find the fundamental frequency The two given frequencies correspond to consecutive harmonics, not two arbitrary modes. Subtracting eliminates the unknown integer harmonic number:

$$f_1 = f_{n+1} - f_n = 450 - 400 = 50\,\text{Hz}.$$

Thus $n = 400/50 = 8$, so the stated pair is the eighth and ninth harmonics. This also checks that the frequency assignment is consistent.

Step 2: Calculate the wave speed Convert the length before using the standing-wave formula: $L = 30\,\text{cm} = 0.30\,\text{m}$. The fundamental wavelength is twice this length because there are nodes at both fixed ends. Hence

$$v = 2Lf_1 = 2(0.30)(50) = 30\,\text{m}\,\text{s}^{-1}.$$

Using 400 Hz as the fundamental would give an incorrect speed because that frequency is the eighth harmonic.

Step 3: Determine the linear mass density Rearrange the speed-tension relation rather than multiply-

ing tension by speed:

$$\mu = \frac{T}{v^2} = \frac{2700}{30^2} = \frac{2700}{900} = 3\,\mathrm{kg\,m^{-1}}.$$

The units are consistent: a newton divided by a squared speed is kilograms per metre. The value is positive, as required for a mass density.

Step 4: Conclusion The linear mass density is $\boxed{3\,\mathrm{kg\,m^{-1}}}$. The numerical entry is $\boxed{3}$.

Quick Tip to Solve: For adjacent harmonics, subtract the frequencies first; this avoids solving separately for the harmonic number.

58. A spherical soap bubble of radius 3 cm is formed inside another spherical soap bubble of radius 6 cm. If the internal pressure of the smaller bubble of radius 3 cm in the above system is equal to the internal pressure of the another single soap bubble of radius r cm. The value of r is _____.

[4 marks]

Solution:

Concept:

- A soap bubble has two liquid-air surfaces, giving excess pressure $\Delta P = 4S/R$, where S is surface tension and R is radius.
- For nested bubbles, pressure increases at each interface when moving inward. The pressure outside the smaller bubble is the pressure inside the larger bubble, not atmospheric pressure.

Step 1: Write the pressure inside the larger bubble Let the pressure outside the complete system be P_0, and let the common surface tension of the soap solution be S. For the outer bubble of radius $R_2 = 6\,\mathrm{cm}$, the pressure in the space between bubbles is

$$P_2 = P_0 + \frac{4S}{R_2}.$$

It is this raised pressure that acts on the outside of the inner bubble.

Step 2: Add the inner bubble pressure rise The smaller bubble has radius $R_1 = 3\,\mathrm{cm}$. Crossing its two surfaces produces another excess pressure. Therefore

$$P_1 = P_2 + \frac{4S}{R_1} = P_0 + 4S\left(\frac{1}{R_2} + \frac{1}{R_1}\right).$$

The two rises add; subtracting them would describe a different pressure difference and would miss the outer bubble contribution.

Step 3: Compare with the single bubble A single bubble of radius r, surrounded by the same external pressure, has internal pressure $P_0 + 4S/r$. Equating this with P_1, canceling P_0 and $4S$, and using centimetres consistently gives

$$\frac{1}{r} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}\,\mathrm{cm^{-1}}, \qquad r = 2\,\mathrm{cm}.$$

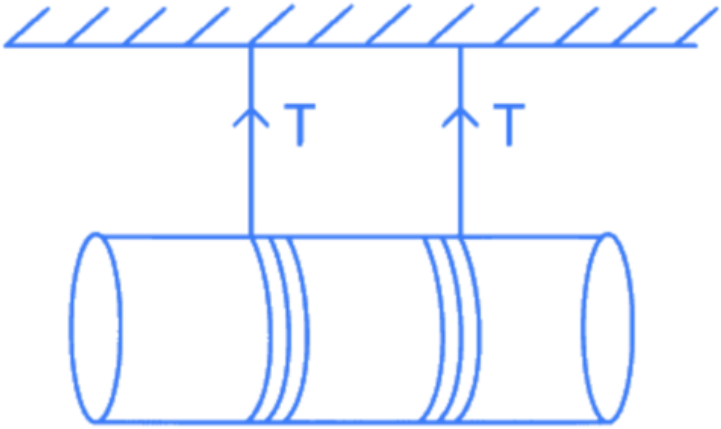
This radius is smaller than either original radius, as expected because its excess pressure equals two positive contributions.

Step 4: Conclusion The radius of the equivalent single soap bubble is $\boxed{2\,\text{cm}}$; the numerical entry is $\boxed{2}$.

Quick Tip to Solve: For nested soap bubbles, add reciprocal radii when comparing the innermost pressure with atmospheric pressure.

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59. A solid cylinder length is suspended symmetrically through two massless strings, as shown in the figure. The distance from the initial rest position, the cylinder should be unbinding the strings to achieve a speed of $4\ \text{ms}^{-1}$, is _____ cm. (take g = $10\ \text{ms}^{-2}$) [4 marks]



Solution:

Concept:

- A falling cylinder gains both translational kinetic energy $Mv^2/2$ and rotational kinetic energy $I\omega^2/2$. A solid cylinder has $I = MR^2/2$ about its symmetry axis.
- For strings unwinding without slipping, $v = R\omega$. With massless strings and fixed supports, the loss of gravitational potential energy becomes the two forms of kinetic energy.

Step 1: Relate translation and rotation Let the cylinder mass be M, its radius R, and its downward displacement h. Both strings unwind symmetrically, so the cylinder remains horizontal. The speed of the string contact point is zero at the fixed string, giving

$$\omega = \frac{v}{R}.$$

There are two tensions, but only one cylinder and one rotational kinetic-energy term; the energy must not be doubled merely because two strings support it.

Step 2: Apply conservation of mechanical energy The cylinder starts from rest. Its gravitational potential-energy loss is Mgh. The strings do not supply energy, so

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2.$$

Substitute $I = MR^2/2$ and $\omega = v/R$:

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2.$$

The radius cancels, explaining why its numerical value is unnecessary.

Step 3: Evaluate the required drop Cancel the mass and insert the specified speed and acceleration due to gravity:

$$h = \frac{3v^2}{4g} = \frac{3(4)^2}{4(10)} = \frac{48}{40} = 1.2\,\text{m}.$$

Since the requested distance is in centimetres,

$$h = 1.2 \times 100 = 120\,\text{cm}.$$

The drop exceeds the free-fall value for the same speed because some available energy is stored in rotation.

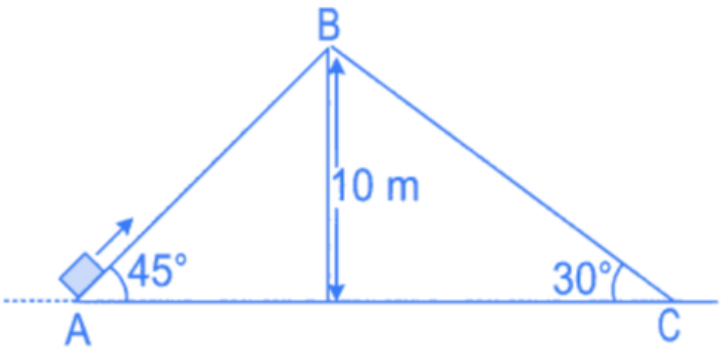
Step 4: Conclusion The cylinder must descend $\boxed{120\,\text{cm}}$, so the numerical entry is $\boxed{120}$.

Quick Tip to Solve: Include both kinetic energies for an unwinding cylinder; use the no-slip relation to eliminate its angular speed and radius.

---

60. Two inclined planes are placed as shown in figure. A block is projected from the Point A of inclined plane AB along its surface with a velocity just sufficient to carry it to the top Point B at a height 10 m. After reaching the Point B the block slides down on inclined plane BC. Time it takes to reach to the point C from point A is $t(\sqrt{2}+1)$ s. The value of t is _____.

(use $g = 10\ \text{m/s}^2$ **)** [4 marks]



Solution:

Concept:

- On a smooth plane inclined at angle θ, the component of gravitational acceleration along it is $g\sin\theta$. The path length corresponding to height h is $h/\sin\theta$.
- When the block just reaches the summit, its speed there is zero. Energy conservation gives its launch speed $u = \sqrt{2gh}$; ascent and descent times must be calculated with their respective inclinations.

Step 1: Calculate the ascent time on AB The figure gives $\angle A = 45^\circ$, $h = 10\,\text{m}$, and $g = 10\,\text{m s}^{-2}$. The block just stops at B, so

$$u = \sqrt{2gh} = \sqrt{200} = 10\sqrt{2}\,\text{m s}^{-1}.$$

Its retardation on AB is $g\sin 45^\circ = 5\sqrt{2}\,\text{m s}^{-2}$. Using final speed zero,

$$t_{AB} = \frac{u}{g\sin 45^\circ} = \frac{10\sqrt{2}}{5\sqrt{2}} = 2\,\text{s}.$$

Step 2: Calculate the descent time on BC The other slope makes 30° with the horizontal. Its length is

$$BC = \frac{10}{\sin 30^\circ} = 20\,\text{m}.$$

Starting from rest at B, the block accelerates down this slope with $a = g\sin 30^\circ = 5\,\text{m s}^{-2}$. Hence

$$20 = \frac{1}{2}(5)t_{BC}^2, \qquad t_{BC} = \sqrt{8} = 2\sqrt{2}\,\text{s}.$$

The positive root is the physically meaningful time.

Step 3: Compare the total time with the stated form Add the times for the two separate parts of the path:

$$T = t_{AB} + t_{BC} = 2 + 2\sqrt{2} = 2(\sqrt{2}+1)\,\text{s}.$$

The question writes this as $t(\sqrt{2}+1)\,\text{s}$. Comparing coefficients gives $t = 2$. The steeper ascent has the shorter time, consistent with its larger acceleration magnitude.

Step 4: Conclusion The required coefficient is $\boxed{t = 2}$; the total travel time is $2(\sqrt{2}+1)\,\text{s}$.

Quick Tip to Solve: Find a separate time on each slope; the common height does not imply equal times when the inclinations differ.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. The correct decreasing order of energy for the orbitals having, following set of quantum numbers :

(A) n = 3, l = 0, m = 0

(B) n = 4, l = 0, m = 0

(C) n = 3, l = 1, m = 0

(D) n = 3, l = 2, m = 1

is : [4 marks]

(A) $(D) > (B) > (C) > (A)$

(B) $(B) > (D) > (C) > (A)$

(C) $(C) > (B) > (D) > (A)$

(D) $(B) > (C) > (D) > (A)$

Correct Answer: (A) $(D) > (B) > (C) > (A)$

Solution:

Concept:

- For ground-state filling in a many-electron atom, the Aufbau ordering follows the $(n + l)$ rule: the orbital with smaller $n + l$ has lower energy.
- If two orbitals have equal $n + l$, the one with lower principal quantum number n lies lower. The magnetic quantum number m does not alter this ordering in the absence of an external field.

Step 1: Identify the four orbitals The angular momentum quantum number specifies the subshell: $l = 0, 1, 2$ mean s, p and d, respectively. Thus the listed sets correspond to

$$A : 3s, \qquad B : 4s, \qquad C : 3p, \qquad D : 3d.$$

The given magnetic quantum numbers are allowed values for these subshells. They indicate orientation, rather than changing the relative energy used in the filling rule.

Step 2: Apply the energy rule Compute the sums for each subshell:

$$(n + l)_A = 3, \quad (n + l)_B = 4, \quad (n + l)_C = 4, \quad (n + l)_D = 5.$$

Therefore A is lowest and D is highest. To resolve the tie between B and C, compare n: C has $n = 3$, whereas B has $n = 4$. Hence $3p$ lies below $4s$, and the increasing order is

$$3s < 3p < 4s < 3d.$$

Step 3: Select the decreasing order Reversing the increasing sequence gives

$$D > B > C > A.$$

This is the first printed option. The second option incorrectly places $4s$ above $3d$. The third puts $3p$ highest, despite its smaller $n + l$. The fourth puts $3d$ below $3p$, again violating the rule. Thus only one

printed sequence agrees with both the sum rule and its tie-breaker.

Step 4: Conclusion The decreasing energy order is $\boxed{3d > 4s > 3p > 3s}$. Hence, the correct option is (A).

Quick Tip to Solve: Write the subshell names and their n + l values before reversing the sequence for a decreasing-order question.

62. Match List - I with List - II.

	List - I		List - II
(A)	$\psi_{MO} = \psi_A - \psi_B$	**(I)**	**Dipole moment**
(B)	$\mu = Q \times r$	**(II)**	**Bonding molecular orbital**
(C)	$\frac{N_b - N_a}{2}$	**(III)**	**Anti-bonding molecular orbital**
(D)	$\psi_{MO} = \psi_A + \psi_B$	**(IV)**	**Bond order**

Choose the correct answer from the options given below : [4 marks]

(A) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

(B) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

(C) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

(D) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

Correct Answer: (C) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

Solution:

Concept:

- Constructive combination of atomic wavefunctions gives a bonding molecular orbital; destructive combination gives an antibonding molecular orbital.
- The electric dipole moment is $\mu = Qr$, and bond order is $(N_b - N_a)/2$, where N_b and N_a are the numbers of electrons in bonding and antibonding molecular orbitals.

Step 1: Match the orbital combinations For the combinations written in the list, the difference $\psi_A - \psi_B$ represents destructive interference. It creates a node between nuclei and corresponds to the antibonding molecular orbital, entry III. The sum $\psi_A + \psi_B$ represents constructive interference and enhanced electron density between nuclei, so it matches the bonding molecular orbital, entry II. Thus

$$A \to \text{III}, \qquad D \to \text{II}.$$

Step 2: Match the two physical quantities The product of separated charge magnitude Q and separation r is the magnitude of the dipole moment. Therefore entry B matches I, not bond order. In contrast, the difference between bonding and antibonding electron populations, divided by two, counts net bonds. Therefore

$$B \to \text{I}, \qquad C \to \text{IV}.$$

These formulas describe different quantities and cannot be interchanged.

Step 3: Check the complete option Combining all four matches gives

$$A - \text{III}, \quad B - \text{I}, \quad C - \text{IV}, \quad D - \text{II}.$$

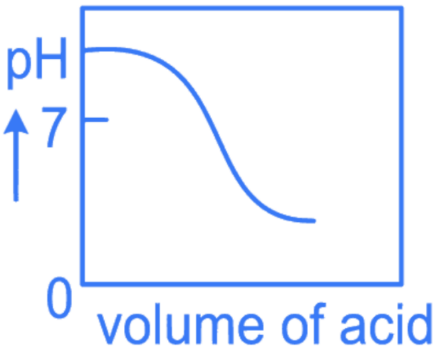
The first option interchanges bonding and antibonding assignments. The second interchanges dipole moment and bond order. The fourth assigns the wavefunction sum to dipole moment and also mismatches the formulas. Only the third printed option preserves every definition simultaneously.

Step 4: Conclusion The correct matching is $A - \text{III}, B - \text{I}, C - \text{IV}, D - \text{II}$. Hence, the correct option is (C).

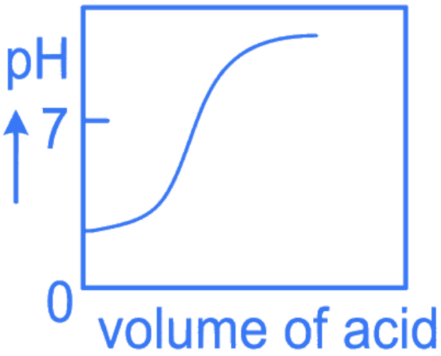
Quick Tip to Solve: Match the unmistakable formulas for dipole moment and bond order first, then distinguish the wavefunction sum and difference.

63. The plot of pH-metric titration of weak base NH_4OH vs strong acid HCl looks like : [4 marks]

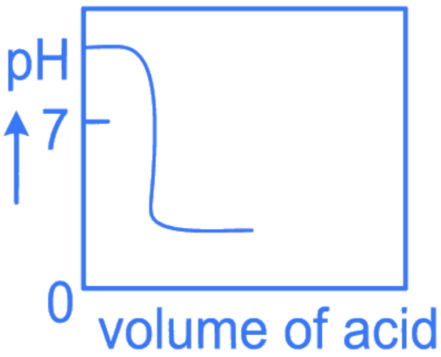
(A)



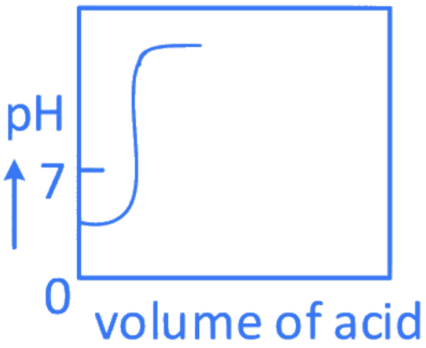
(B)



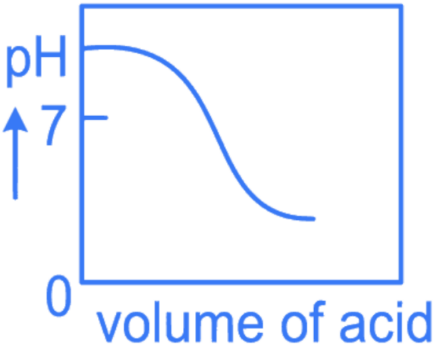
(C)



(D)



Correct Answer: (A)



Solution:

Concept:

- A weak base initially gives a basic solution. Adding a strong acid consumes the base and lowers pH, so a plot against volume of added acid must descend.
- At equivalence, a weak base and strong acid form a salt whose cation hydrolyses to make the solution acidic. The equivalence-point pH is therefore below 7, unlike a strong-base strong-acid titration.

Step 1: Identify the direction of change The flask initially contains the weak base NH_4OH, so its pH is above 7. On adding HCl, neutralisation converts it into ammonium chloride:

$$NH_4OH + HCl \longrightarrow NH_4Cl + H_2O.$$

The horizontal axis is volume of acid added. Increasing this volume cannot produce a rising basic curve. Consequently the increasing curves in options B and D are ruled out immediately.

Step 2: Examine the equivalence region Before equivalence, remaining weak base and its ammonium salt form a buffer mixture, so pH decreases gradually. At equivalence, the base has been consumed and the solution contains $NH_4{}^+$. Its hydrolysis produces acidity:

$$NH_4{}^+ + H_2O \rightleftharpoons NH_3 + H_3O^+.$$

Thus the equivalence region lies below neutral pH and the sharp change is less pronounced than for a strong base titrated with a strong acid.

Step 3: Compare the two decreasing curves Option A shows the appropriate gradual descent from a

basic initial pH into the acidic region, with a relatively subdued titration transition. Option C shows a much sharper, nearly vertical transition characteristic of strong-base strong-acid neutralisation. After equivalence, excess strong acid controls the pH, so the curve remains on the acidic side. All these features agree with the first plotted curve.

Step 4: Conclusion The appropriate titration curve is the first decreasing curve. Hence, the correct option is (A).

Quick Tip to Solve: Check which reagent is being added, then use whether the salt at equivalence is acidic, basic or neutral to select the curve.

64. Given below are two statements :

Statement I : For KI, molar conductivity increases steeply with dilution

Statement II : For carbonic acid, molar conductivity increases slowly with dilution

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- Molar conductivity is the conductance associated with one mole of dissolved electrolyte. It is $\Lambda_m = \kappa/c$ when consistent SI units are used; κ is conductivity and c is concentration.
- A strong electrolyte is almost completely ionised already, whereas a weak electrolyte undergoes substantially greater ionisation as its solution is diluted. This difference determines how rapidly molar conductivity rises.

Step 1: Evaluate Statement I KI is a strong electrolyte and dissociates essentially completely into K^+ and I^-. Dilution reduces interionic attractions and improves ionic mobility, so its molar conductivity rises gradually, rather than steeply. Near infinite dilution its behaviour is represented by

$$\Lambda_m = \Lambda_m^\circ - A\sqrt{c}.$$

The finite limiting value is approached smoothly. Hence the claim of a steep rise for KI is false.

Step 2: Evaluate Statement II Carbonic acid, H_2CO_3, is a weak electrolyte. Dilution shifts its ionisation equilibria towards ions; the fraction ionised increases considerably. Thus both increased ionisation and improved mobility contribute to its molar conductivity. The increase is pronounced, not slow. Statement II is therefore also false. A weak electrolyte must not be confused with a weak conductor whose molar conductivity remains nearly unchanged.

Step 3: Eliminate the other combinations Option (A) would require both descriptions to be correct;

each reverses the usual strong/weak electrolyte trend. Option (C) incorrectly calls the KI statement true, and option (D) incorrectly calls the carbonic-acid statement true. Only the combination in which both statements are false survives.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: First classify the electrolyte: dilution mainly improves mobility for a strong electrolyte but also greatly increases ionisation for a weak one.

---

65. Outermost electronic configurations of four elements A, B, C, D are given below :

(A) $3s^2$

(B) $3s^2 3p^1$

(C) $3s^2 3p^3$

(D) $3s^2 3p^4$

The correct order of first ionization enthalpy for them is : [4 marks]

(A) $(A) < (B) < (C) < (D)$

(B) $(B) < (A) < (D) < (C)$

(C) $(B) < (D) < (A) < (C)$

(D) $(B) < (A) < (C) < (D)$

Correct Answer: (B) $(B) < (A) < (D) < (C)$

Solution:

Concept:

- First ionization enthalpy is the energy required to remove one electron from an isolated gaseous atom. Across a period it generally rises with effective nuclear charge, but subshell energies and electron pairing cause predictable exceptions.
- A filled $3s^2$ subshell and a half-filled $3p^3$ subshell are relatively stable. A $3p$ electron is easier to remove than a $3s$ electron of the preceding element, and repulsion in a paired p orbital favours removal.

Step 1: Identify the four configurations The period-three configurations identify A as magnesium, B as aluminium, C as phosphorus and D as sulfur. The electron removed from A is a $3s$ electron, whereas that removed from B, C and D is in a $3p$ orbital. We must therefore apply the general period trend together with the two known subshell exceptions, rather than rank the configurations simply by the number of electrons.

Step 2: Compare the two exceptional pairs For aluminium, the single higher-energy $3p$ electron is less penetrating and easier to remove than an electron from the filled $3s^2$ subshell of magnesium. Thus $B < A$. Phosphorus has three singly occupied $3p$ orbitals; sulfur has one paired orbital. Electron-pair repulsion makes sulfur lose an electron more readily, giving $D < C$. The rise in effective nuclear charge from Mg to S still makes sulfur harder to ionise than magnesium.

Step 3: Assemble and check the order The complete increasing order is

$$B < A < D < C.$$

Option (A) misses both exceptions. Option (C) places sulfur below magnesium, contradicting the overall rise across period three. Option (D) places phosphorus below sulfur and misses the half-filled-subshell stability. The sequence in option (B) includes both exceptions correctly.

Step 4: Conclusion The increasing first ionization enthalpy is $B < A < D < C$. Hence, the correct option is (B).

Quick Tip to Solve: For period-three ionisation energies remember the two downward exceptions: Mg to Al and P to S; then insert them into the overall increasing trend.

66. In neutral or alkaline solution, MnO_4^- oxidises thiosulphate to : [4 marks]

(A) $S_2O_7^{2-}$
(B) $S_2O_8^{2-}$
(C) SO_3^{2-}
(D) SO_4^{2-}

Correct Answer: (D) SO_4^{2-}

Solution:

Concept:

- Permanganate is an oxidising agent. In neutral or alkaline medium its manganese is commonly reduced from oxidation state $+7$ to $+4$, producing MnO_2.
- Thiosulfate, ${S_2O_3}^{2-}$, contains two inequivalent sulfur atoms. Complete oxidation of these sulfur atoms gives sulfate, ${SO_4}^{2-}$, in which sulfur is in oxidation state $+6$.

Step 1: Write the sulfur oxidation half-reaction The sulfur atoms must be conserved: one thiosulfate ion produces two sulfate ions. In alkaline medium balance oxygen and hydrogen using hydroxide and water, then balance charge with electrons:

$$ {S_2O_3}^{2-} + 10\,OH^- \longrightarrow 2\,{SO_4}^{2-} + 5\,H_2O + 8\,e^-. $$

The total sulfur oxidation number changes from $+4$ in thiosulfate to $+12$ in two sulfate ions, corresponding to loss of eight electrons.

Step 2: Combine with permanganate reduction The reduction half-reaction is

$$ {MnO_4}^- + 2\,H_2O + 3\,e^- \longrightarrow MnO_2 + 4\,OH^-. $$

Multiplying the oxidation half by three and the reduction half by eight gives the overall balanced equation

$$ 8\,{MnO_4}^- + 3\,{S_2O_3}^{2-} + H_2O \longrightarrow 8\,MnO_2 + 6\,{SO_4}^{2-} + 2\,OH^-. $$

This explicitly identifies sulfate as the sulfur-containing oxidation product and conserves atoms and charge.

Step 3: Distinguish the listed ions Pyrosulfate and peroxodisulfate, the ions in (A) and (B), are not the products of this standard permanganate oxidation. Sulfite in (C) has sulfur at $+4$ and does not represent the complete oxidation to the stable $+6$ product. The sulfate ion in (D) agrees with the balanced redox equation.

Step 4: Conclusion In neutral or alkaline solution permanganate oxidises thiosulfate to $SO_4{}^{2-}$. Hence, the correct option is (D).

Quick Tip to Solve: For thiosulfate oxidation to sulfate, conserve both sulfur atoms: one $S_2O_3{}^{2-}$ forms two $SO_4{}^{2-}$ and releases eight electrons.

67. Low oxidation state of metals in their complexes are common when ligands : [4 marks]

(A) have good π-accepting character

(B) have good σ-donor character

(C) are having good π-donating ability

(D) are having poor σ-donating ability

Correct Answer: (A) have good π-accepting character

Solution:

Concept:

- A low-oxidation-state metal has relatively high electron density in its occupied orbitals. Ligands with vacant orbitals of suitable symmetry can accept part of this density through metal-to-ligand π back-donation.
- In a metal carbonyl, ligand-to-metal σ donation and metal-to-ligand π back-bonding reinforce each other. The accepting orbitals of CO are its vacant antibonding π^* orbitals.

Step 1: Identify what stabilises an electron-rich metal A metal in a low oxidation state is not strongly electron deficient. A ligand that only adds electron density cannot explain the special stabilisation of such a centre. A good π acceptor can remove some metal electron density into a ligand orbital. This lowers the electron concentration on the metal and creates an additional bonding interaction between metal and ligand, favouring the low oxidation state.

Step 2: Relate the argument to metal carbonyls Carbon monoxide first donates a lone pair into a suitable metal orbital to form a σ bond. Occupied metal d orbitals can simultaneously overlap with empty CO π^* orbitals and transfer electron density back to the ligand. This synergic interaction explains why neutral metal carbonyls, with metal oxidation state zero, can be stable. The example directly illustrates the property named in option (A).

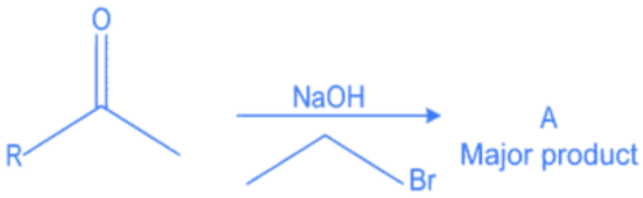
Step 3: Reject properties that do not explain the effect Good σ donation alone, option (B), supplies more density to an already electron-rich centre and is not the distinguishing reason. A π-donor ligand in (C) also donates density instead of accepting it. Poor σ donation in (D) does not itself provide the stabilising back-bonding interaction. The required characteristic is specifically strong π-acceptor

ability.

Step 4: Conclusion Low oxidation states are favoured by ligands having good π-accepting character. Hence, the correct option is (A).

Quick Tip to Solve: Associate low metal oxidation state with electron-rich metal orbitals; then look for a ligand able to accept that density by back-bonding.

68. The structure of A in the given reaction is : [4 marks]



(A) $R-C(OH)(CH_3)-CH(Br)-CH_3$

(B) $R-C(OH)(CH_3)-CH_2CH_3$

(C) $R-CO-CH_2CH_2CH_3$

(D) $R-CO-CH_2CH_2CH_2OH$

Correct Answer: (C) $R-CO-CH_2CH_2CH_3$

Solution:

Concept:

- A ketone with an alpha hydrogen can form an enolate in base. The alpha carbon of the enolate acts as a nucleophile and forms a carbon-carbon bond with a suitable primary alkyl halide.
- The reactant shown is $RCOCH_3$, and the alkylating agent is ethyl bromide, CH_3CH_2Br. Replacing one alpha hydrogen by an ethyl group preserves the carbonyl group.

Step 1: Generate the carbon nucleophile Hydroxide removes an alpha hydrogen from the methyl group adjacent to the carbonyl. The resulting enolate is resonance-stabilised, with negative charge distributed between oxygen and the alpha carbon. For carbon alkylation its useful representation is

$$RCOCH_3 + OH^- \rightleftharpoons RCOCH_2{}^- + H_2O.$$

This step makes the alpha carbon nucleophilic; it does not convert the carbonyl carbon into an alcohol.

Step 2: Form the new carbon-carbon bond The nucleophilic alpha carbon attacks the primary carbon bearing bromine in ethyl bromide, displacing bromide by substitution:

$$RCOCH_2{}^- + CH_3CH_2Br \longrightarrow RCOCH_2CH_2CH_3 + Br^-.$$

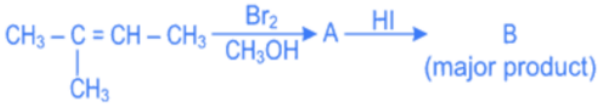
Count the new chain carefully: the original alpha carbon remains, and the two carbons of the ethyl group are attached to it. Thus the carbonyl is followed by $CH_2CH_2CH_3$, the structure shown in option (C).

Step 3: Check each alternative against the reagents Options (A) and (B) are alcohols obtained by adding a carbon nucleophile to the carbonyl carbon; the stated reagents do not supply an ethyl organometallic reagent for that process. Option (A) also retains bromine on the added group. Option (D) contains a terminal hydroxyl group, but substitution of the alpha enolate with ethyl bromide gives no such hydroxyl group. Only (C) preserves the ketone and gives the expected carbon alkylation product.

Step 4: Conclusion The product is $\mathrm{RCOCH_2CH_2CH_3}$, the alpha-ethylated ketone. Hence, the correct option is (C).

Quick Tip to Solve: For enolate alkylation, retain the original alpha carbon and attach the entire alkyl group from the halide; do not count the ethyl group as replacing the alpha carbon.

69. Major product 'B' of the following reaction sequence is : [4 marks]



(A) $(CH_3)_2C(OH)-CH(Br)-CH_3$

(B) $(CH_3)_2C(I)-CH(Br)-CH_3$

(C) $(CH_3)_2C(Br)-CH(OH)-CH_3$

(D) $(CH_3)_2C(Br)-CH(I)-CH_3$

Correct Answer: (B) $(CH_3)_2C(I)-CH(Br)-CH_3$

Solution:

Concept:

- Addition of bromine to an alkene in methanol proceeds through a bromonium intermediate. Methanol attacks the more substituted carbon, giving an alkoxy group there and bromine at the other carbon.
- Hydrogen iodide protonates an ether oxygen and cleaves the carbon-oxygen bond. When one ether carbon is tertiary, cleavage can occur through the stable tertiary carbocation, giving a tertiary iodide.

Step 1: Determine the intermediate A The alkene is $(CH_3)_2C{=}CHCH_3$. Bromine first forms a bridged bromonium ion. Methanol, present as solvent and nucleophile, attacks the more substituted alkene carbon. Deprotonation gives the bromoether

$$\mathrm{A} = (CH_3)_2C(OCH_3)-CH(Br)-CH_3.$$

Thus the methoxy group is attached to the carbon already bearing two methyl groups, while bromine is attached to the adjacent secondary carbon.

Step 2: Apply cleavage by HI The ether oxygen is protonated by HI, making its carbon-oxygen bond easier to break. Cleavage at the tertiary carbon produces a stable tertiary carbocation, which is captured by iodide. The original carbon-bromine bond is retained. The organic product therefore has

the structure

$$B = (CH_3)_2C(I)-CH(Br)-CH_3.$$

This places iodine at the more substituted carbon and bromine at the neighbouring carbon, precisely as in option (B).

Step 3: Eliminate the other structures Options (A) and (C) contain an alcohol group and fail to show the iodide produced by the HI cleavage under the stated sequence. Option (C) also reverses the regiochemistry of the initial bromine addition in methanol. Option (D) puts bromine at the tertiary carbon and iodine at the secondary carbon, which is the reverse of the product obtained by forming and cleaving the actual bromoether.

Step 4: Conclusion The major product is $(CH_3)_2C(I)-CH(Br)-CH_3$. Hence, the correct option is (B).

Quick Tip to Solve: In bromine/methanol addition put methoxy on the more substituted alkene carbon; when that ether is cleaved by HI, replace its tertiary methoxy group by iodine.

70. Match List I with List II.

	List I		List II
(A)	$C_6H_5COCl \xrightarrow{H_2/Pd-BaSO_4} C_6H_5CHO$	**(I)**	**Gatterman Koch reaction**
(B)	$CH_3CN \xrightarrow[H_3O^+]{SnCl_2/HCl} CH_3CHO$	**(II)**	**Etard reaction**
(C)	$C_6H_5CH_3 \xrightarrow[H_3O^+]{CrO_2Cl_2} C_6H_5CHO$	**(III)**	**Stephen reaction**
(D)	$C_6H_6 \xrightarrow[AlCl_3\ (anhyd)]{CO,\ HCl} C_6H_5CHO$	**(IV)**	**Rosenmund reaction**

Choose the correct answer from the options given below: [4 marks]

(A) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

(B) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(C) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(D) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

Correct Answer: (A) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

Solution:

Concept:

- The reagents identify the named reaction more reliably than the aldehyde product alone. Several different transformations in the list give benzaldehyde, but each uses a different starting functional group.
- Rosenmund reduction converts an acyl chloride into an aldehyde; Stephen reduction converts a nitrile into an aldehyde after hydrolysis. Etard oxidation changes a benzylic methyl group into an aldehyde, and Gattermann-Koch reaction introduces a formyl group into an aromatic ring.

Step 1: Match the two reductions In reaction (A), benzoyl chloride is hydrogenated over poisoned

palladium on barium sulfate:

$$C_6H_5COCl \xrightarrow{H_2/Pd-BaSO_4} C_6H_5CHO.$$

This is Rosenmund reduction, so A matches IV. In reaction (B), $SnCl_2/HCl$ reduces the nitrile to an iminium salt, whose hydrolysis gives the aldehyde. This is Stephen reaction, so B matches III. The hydrolysis step is essential to the conversion of the nitrogen-containing intermediate into acetaldehyde.

Step 2: Match oxidation and aromatic formylation Reaction (C) uses chromyl chloride followed by hydrolysis to oxidise the benzylic methyl group of toluene selectively to an aldehyde. This is Etard reaction, hence C matches II. Reaction (D) uses carbon monoxide and hydrogen chloride with anhydrous aluminium chloride to formylate benzene. This is Gattermann-Koch reaction, hence D matches I.

Step 3: Compare the complete matching with the options Together the four matches are

$$A:IV, \qquad B:III, \qquad C:II, \qquad D:I.$$

Option (A) has all four. Option (B) mistakes the acid-chloride hydrogenation for aromatic formylation; option (C) confuses Rosenmund and Etard transformations; option (D) assigns the nitrile and aromatic formylation reactions incorrectly. Each reaction must be matched by both starting group and reagent.

Step 4: Conclusion The matching is A-IV, B-III, C-II, D-I. Hence, the correct option is (A).

Quick Tip to Solve: Identify the starting group first: acyl chloride gives Rosenmund, nitrile gives Stephen, benzylic methyl gives Etard, and benzene formylation gives Gattermann-Koch.

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71. An organic compound $'A'$ contains nitrogen and chlorine. It dissolves readily in water to give a solution that turns litmus red. Titration of compound $'A'$ with standard base indicates that the molecular weight of $'A'$ is 131 ± 2. When a sample of $'A'$ is treated with aq. $NaOH$, a liquid separates which contains N but not Cl. Treatment of the obtained liquid with nitrous acid followed by phenol gives orange precipitate. The compound $'A'$ is : [4 marks]

(A) Cyclohexa-2,5-dien-1-aminium chloride

(B) 2-Chloroaniline

(C) (Cyclopenta-2,4-dien-1-yl)methanaminium chloride

(D) Anilinium chloride, $C_6H_5NH_3^+ Cl^-$

Correct Answer: (D) Anilinium chloride, $C_6H_5NH_3^+ Cl^-$

Solution:

Concept:

- An amine hydrochloride is an ionic salt containing a protonated amine and chloride; aqueous alkali liberates the corresponding neutral amine.
- A primary aromatic amine undergoes diazotisation with nitrous acid at low temperature. Its diazonium ion couples with phenol to give a coloured azo compound.

Step 1: Identify the nature of the chlorine The compound dissolves readily in water, gives an acidic solution and reacts with base to release a chlorine-free nitrogen compound. These observations indicate chloride as a counter-ion rather than a chlorine atom bonded to the aromatic ring. The protonated amine is deprotonated by hydroxide. Thus 2-chloroaniline, option (B), is unsuitable: its ring-bound chlorine would remain in the organic product.

Step 2: Use the colour test to identify the free amine The liberated liquid must be aniline, $C_6H_5NH_2$. Nitrous acid converts it into benzenediazonium salt, which couples with phenol to give an orange azo precipitate. The amino groups in options (A) and (C) are attached to saturated carbon, rather than directly to an aromatic benzene ring. They therefore do not supply the stable aromatic diazonium species required for this coupling test.

Step 3: Check the molecular mass and salt formula Anilinium chloride has formula $C_6H_5NH_3Cl$, or C_6H_8NCl. Its molar mass is

$$6(12) + 8(1) + 14 + 35.5 = 129.5\ \text{g mol}^{-1},$$

within the stated interval 129–133. Alkali liberates aniline according to

$$C_6H_5NH_3Cl + NaOH \longrightarrow C_6H_5NH_2 + NaCl + H_2O.$$

All three observations are therefore consistent with the same salt.

Step 4: Conclusion The compound is anilinium chloride, $C_6H_5NH_3^+\,Cl^-$. Hence, the correct option is (D).

Quick Tip to Solve: First distinguish ionic chloride from ring-bound chlorine using the alkali test; then use azo coupling to identify a primary aromatic amine.

72. Match List - I with Match List - II.

	List - I		List - II
(A)	Glucose + HI	(I)	Gluconic acid
(B)	Glucose + Br_2 water	(II)	Glucose pentacetate
(C)	Glucose + acetic anhydride	(III)	Saccharic acid
(D)	Glucose + HNO_3	(IV)	Hexane

Choose the correct answer from the options given below: [4 marks]

(A) (A) - (IV), (B) - (I), (C) -(II), (D) - (III)

(B) (A) - (IV), (B) - (III), (C) -(II), (D) - (I)

(C) (A) - (III), (B) - (I), (C) -(IV), (D) - (II)

(D) (A) - (I), (B) - (III), (C) -(IV), (D) - (II)

Correct Answer: (A) (A) - (IV), (B) - (I), (C) -(II), (D) - (III)

Solution:

Concept:

- Glucose has a straight six-carbon skeleton, an aldehyde group in its open-chain form and five hydroxyl groups.
- Reducing, mildly oxidising, acetylating and strongly oxidising reagents probe different parts of this structure; match the reagent before comparing option patterns.

Step 1: Match reduction and mild oxidation On prolonged heating with HI, glucose is completely reduced to hexane. The six carbon atoms remain in an unbranched chain, so (A) matches (IV). Bromine water is a mild oxidising agent: it converts the aldehyde end into a carboxylic acid without oxidising the terminal primary alcohol. The product is gluconic acid, so (B) matches (I). It is not saccharic acid because the second terminal group has not been oxidised.

Step 2: Match acetylation and strong oxidation Acetic anhydride acetylates all five hydroxyl groups of glucose, giving glucose pentaacetate. Thus (C) matches (II); the reaction does not change the six-carbon skeleton into hexane. Nitric acid oxidises both the aldehyde group and the terminal primary alcohol into carboxyl groups. The resulting dicarboxylic acid is saccharic acid, so (D) matches (III).

Step 3: Compare all four options The complete sequence is

$$(A)\rightarrow(IV),\quad (B)\rightarrow(I),\quad (C)\rightarrow(II),\quad (D)\rightarrow(III).$$

Option (B) exchanges the two oxidation products, assigning saccharic acid to bromine water. Options (C) and (D) incorrectly assign hexane to acetylation and pentaacetate to nitric-acid oxidation. Only option (A) preserves every reagent-product relationship.

Step 4: Conclusion The four reagent-product assignments give the matching in the first option. Hence, the correct option is (A).

Quick Tip to Solve: Remember: bromine water oxidises one end of glucose; nitric acid oxidises both ends, while acetic anhydride counts its five hydroxyl groups.

73. Match List - I with List - II.

	List - I (Mixture)		List - II (Purification Process)
(A)	Chloroform & Aniline	(I)	Steam distillation
(B)	Benzoic acid & Napthalene	(II)	Sublimation
(C)	Water & Aniline	(III)	Distillation
(D)	Napthalene & Sodium chloride	(IV)	Crystallisation

Choose the correct answer from the options given below : [4 marks]

(A) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(B) (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

(C) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

(D) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (D) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Solution:

Concept:

- Choose a separation method from the distinguishing physical property: boiling point, solubility, volatility with steam or sublimability.
- Ordinary distillation separates liquids with sufficiently different boiling points. Steam distillation is useful for a water-immiscible, steam-volatile organic substance.

Step 1: Assign the liquid mixtures Chloroform is much more volatile than aniline: their boiling points are approximately 61 and 184 degrees Celsius. Distillation therefore separates them, so (A) matches (III). Aniline is only sparingly soluble in water and is steam volatile. Steam distillation separates it from the aqueous mixture at a temperature below its normal boiling point; hence (C) matches (I). This assignment distinguishes steam distillation from ordinary distillation in the available list.

Step 2: Assign the solid mixtures Benzoic acid and naphthalene can be separated by crystallisation using a solvent in which their solubilities and their temperature dependence differ suitably. Benzoic acid is much more soluble in hot water than cold water, whereas naphthalene is practically insoluble in water. Thus (B) matches (IV). Naphthalene readily sublimes, while sodium chloride does not; sublimation separates the volatile solid from the nonvolatile salt. Hence (D) matches (II).

Step 3: Eliminate the alternative matchings Together the four assignments give

$$(A)\to(III),\quad (B)\to(IV),\quad (C)\to(I),\quad (D)\to(II).$$

Option (A) incorrectly uses crystallisation for the two-liquid mixture. Option (B) interchanges the appropriate processes for the benzoic-acid mixture and aqueous aniline. Option (C) incorrectly assigns sublimation to water and aniline and steam distillation to the sodium-chloride mixture.

Step 4: Conclusion The four mixture-process assignments give the matching in the fourth option. Hence, the correct option is (D).

Quick Tip to Solve: Start with the unmistakable pair naphthalene and sodium chloride: only naphthalene sublimes. Then distinguish steam distillation from ordinary distillation.

74. Fe^{3+} **cation gives a prussian blue precipitate on addition of potassium ferrocyanide solution due to the formation of :** [4 marks]

(A) $[Fe\,(H_2O)_6]_2\,[Fe(CN)_6]$

(B) $Fe_2\,[Fe(CN)_6]_2$

(C) $Fe_3\,[Fe(OH)_2(CN)_4]_2$

(D) $Fe_4\,[Fe(CN)_6]_3$

Correct Answer: (D) $Fe_4\,[Fe(CN)_6]_3$

Solution:

Concept:

- Potassium ferrocyanide supplies the complex anion $[Fe(CN)_6]^{4-}$, called hexacyanidoferrate(II).
- Ferric ions are Fe^{3+}. A neutral precipitate requires total positive charge to equal total negative charge, while the ferrocyanide complex remains intact.

Step 1: Identify the charges of the reacting species In ferrocyanide, six cyanide ligands contribute a total charge of -6, and the central iron is in oxidation state $+2$. The complex ion therefore carries charge -4:

$$+2 + 6(-1) = -4.$$

The external iron ions supplied in the question have charge $+3$. Do not confuse these ferric ions with the iron already coordinated inside the ferrocyanide ion.

Step 2: Balance the ions to form the precipitate The least common multiple of 3 and 4 is 12. Four ferric ions contribute $+12$, while three ferrocyanide ions contribute -12. Their neutral combination is therefore $Fe_4[Fe(CN)_6]_3$. The precipitation equation is

$$4\,Fe^{3+} + 3\,[Fe(CN)_6]^{4-} \longrightarrow Fe_4[Fe(CN)_6]_3\downarrow.$$

This is ferric ferrocyanide, the characteristic Prussian blue precipitate.

Step 3: Check the competing formulae Option (A) substitutes an aquated-iron formulation with a different ratio; it is not the ferric ferrocyanide formula required here. Option (B), with two ferric ions and two ferrocyanide ions, would give net charge $+6 - 8 = -2$, rather than the neutral precipitate. Option (C) introduces hydroxide ligands and changes the complex anion even though the reagent supplies ferrocyanide. Option (D) alone retains the correct complex and balances the charges.

Step 4: Conclusion The Prussian blue precipitate is $Fe_4[Fe(CN)_6]_3$. Hence, the correct option is (D).

Quick Tip to Solve: For ferric ferrocyanide, balance external $+3$ iron against the -4 complex ion: the subscripts are 4 and 3.

75. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Dissolved substances can be removed from a colloidal solution by diffusion through a parchment paper.

Reason (R) : Particles in a true solution cannot pass through parchment paper but the colloidal particles can pass through the parchment paper.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(C) (A) is correct but (R) is not correct

(D) (A) is not correct but (R) is correct

Correct Answer: (C) (A) is correct but (R) is not correct

Solution:

Concept:

- Dialysis removes dissolved electrolytes or other small solute particles from a colloidal dispersion through a semipermeable membrane.
- The small ions and molecules of a true solution can diffuse through parchment, whereas the much larger colloidal particles are retained.

Step 1: Evaluate the assertion A colloidal solution can contain dissolved salts and other small solute particles as impurities. When the dispersion is enclosed in a parchment membrane and placed in water, the dissolved species diffuse into the surrounding water down their concentration gradient. Replacing the outer water sustains this gradient and removes more impurity. This purification process is dialysis, so the assertion is correct.

Step 2: Evaluate the reason The reason states that true-solution particles cannot pass through parchment while colloidal particles can. This reverses the actual size-selective behaviour of the membrane. Small dissolved ions and molecules pass through its pores; colloidal particles are too large to do so. Thus the reason is incorrect. If the reason were true, dialysis would remove the dispersed colloidal material and retain the dissolved impurities, the opposite of its purpose.

Step 3: Select the corresponding statement pair Because the assertion is true and the reason is false, neither of the first two options can be selected: both require the reason to be correct. The last option is also excluded because it calls the assertion incorrect and the reason correct. The third option states precisely the observed truth values. The correct explanation of the assertion is diffusion of small dissolved species through parchment, combined with retention of the larger dispersed particles.

Step 4: Conclusion The assertion is correct, but the reason is not correct. Hence, the correct option is (C).

Quick Tip to Solve: For dialysis, remember the membrane retains the colloid and releases the small dissolved impurities; reverse this statement and the reason becomes false.

76. An element A of group 1 shows similarity to an element B belonging to group 2. If A has maximum hydration enthalpy in group 1 then B is : [4 marks]

(A) Mg

(B) Be

(C) Ca

(D) Sr

Correct Answer: (A) Mg

Solution:

Concept:

- Hydration enthalpy measures the enthalpy change when gaseous ions are hydrated. For ions of the same charge, a smaller ionic radius gives stronger ion-water attraction and a greater hydration-enthalpy magnitude.
- Lithium and magnesium exhibit a diagonal relationship because their ionic sizes and polarising powers are relatively similar.

Step 1: Identify element A from hydration Among group 1 ions, Li^+ is the smallest. Its positive charge is concentrated into the smallest volume, producing the strongest attraction for water molecules and the largest magnitude of hydration enthalpy. The phrase maximum hydration enthalpy is used here in this usual magnitude sense, because hydration itself is exothermic. Hence the group 1 element A is lithium, rather than sodium, potassium or a heavier alkali metal.

Step 2: Apply the diagonal relationship Lithium lies diagonally above magnesium in the periodic table. Their similar charge densities and polarising powers lead to chemical similarities. For example, both form nitrides directly with nitrogen, and their carbonates decompose on heating to the corresponding oxides and carbon dioxide. These related behaviours identify the group 2 partner B as magnesium. The clue concerns a diagonal relationship rather than simply selecting the smallest group 2 atom.

Step 3: Rule out the remaining group 2 elements Beryllium is diagonally related to aluminium in group 13, not to the group 1 element identified here. Calcium and strontium are larger group 2 elements and are not the characteristic diagonal partners of lithium. Thus merely belonging to group 2 is insufficient: the partner must reproduce the anomalous lithium chemistry. Magnesium is the only listed element that meets this requirement.

Step 4: Conclusion Element A is lithium and its group 2 diagonal partner B is magnesium. Hence, the correct option is (A).

Quick Tip to Solve: Use the two common diagonal pairs: lithium–magnesium and beryllium–aluminium. The hydration clue selects lithium first.

77. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Boron is unable to form BF_6^{3-}.

Reason (R) : Size of B is very small.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Correct Answer: (B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Solution:

Concept:

- Boron is a second-period atom with only 2s and 2p orbitals in its valence shell; it cannot expand its octet to form six ordinary covalent bonds.
- Boron has a very small atomic size. A true physical observation is not necessarily the electronic explanation of a proposed bonding limitation.

Step 1: Evaluate the assertion The proposed ion has six fluoride ligands surrounding one boron atom. In the elementary valence-shell description, this would require six electron-pair bonds and an expanded valence shell. Boron forms three bonds in BF_3, and can accept a fluoride ion to form tetrahedral $BF_4{}^-$, attaining an octet. It cannot expand that octet to form $BF_6{}^{3-}$. Thus the assertion is true.

Step 2: Evaluate the reason independently Boron is indeed a very small atom: it lies in the second period and has a compact valence shell. Therefore the reason is also true as a statement about atomic size. However, truth of the size statement alone does not establish that it explains the absence of the six-coordinate fluoride ion. In this question the decisive distinction is electronic: boron has no d orbitals in its valence shell, so the usual expanded-octet bonding scheme is unavailable.

Step 3: Identify the required explanatory relationship The standard valence-shell explanation is that the second shell contains only one s and three p orbitals, supporting an octet rather than the twelve valence electrons of six ordinary electron-pair bonds. Larger group 13 atoms are described at this level as able to attain higher covalency; the absence of suitable valence d orbitals is the conventional explanation tested for boron. Thus both statements are true, but the size statement is not the correct explanation in this assertion-reason pairing. The first option incorrectly treats it as the explanation. The third and fourth options each wrongly declare one of the statements false.

Step 4: Conclusion Both the assertion and the reason are true, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B).

Quick Tip to Solve: In this assertion-reason pair, distinguish a true size statement from the standard electronic explanation: second-period boron cannot expand its octet.

78. Given below are two statements:

Statement I : The non bio-degradable fly ash and slag from steel industry can be used by cement industry.

Statement II : The fuel obtained from plastic waste is lead free.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Correct Answer: (A) Both Statement I and Statement II are correct

Solution:

Concept:

- Industrial waste management uses the waste of one process as a useful input to another. Non-biodegradable does not mean chemically useless: inorganic solids can still provide suitable components for construction materials.
- Plastic-waste conversion into useful fuel is a recycling route. The fuel obtained in this treatment is described as lead free; lead-containing antiknock additives are not required to create the fuel.

Step 1: Assess Statement I Fly ash and steel-industry slag contain inorganic constituents that can be used in cement manufacture. Their utilization reduces the amount of solid waste requiring disposal and also replaces part of the raw material used by the cement industry. Therefore the stated possibility of using these wastes is correct. The fact that they do not biodegrade is precisely why useful reuse is preferable to simply accumulating them.

Step 2: Assess Statement II Plastic waste can be converted to liquid fuel by suitable thermal processing. In the environmental-chemistry treatment of this recycling process, the resulting fuel is lead free. Thus Statement II is also correct. The statement does not say that the fuel is free from every possible pollutant; absence of lead is the particular property being tested.

Step 3: Choose the combination The truth values are I: correct and II: correct. Option (B) wrongly rejects both statements. Option (C) rejects the valid statement about lead-free fuel, while option (D) rejects the valid reuse of fly ash and slag. Only option (A) retains both correct statements.

Step 4: Conclusion Both Statement I and Statement II are correct. Hence, the correct option is (A).

Quick Tip to Solve: For waste-reuse statements, distinguish biodegradability from usability: an inorganic waste can remain useful as a cement raw material.

79. Match List - I with List - II.

	List - I(Polymer)		List - II(Monomer)
(A)	Neoprene	(I)	Acrylonitrile
(B)	Teflon	(II)	Chloroprene
(C)	Acrilan	(III)	Tetrafluoroethene
(D)	Natural rubber	(IV)	Isoprene

Choose the correct answer from the options given below : [4 marks]

(A) $(A)-(II),(B)-(III),(C)-(I),(D)-(IV)$

(B) $(A)-(II),(B)-(I),(C)-(III),(D)-(IV)$

(C) $(A)-(II),(B)-(I),(C)-(IV),(D)-(III)$

(D) $(A)-(I),(B)-(II),(C)-(III),(D)-(IV)$

Correct Answer: (A) $(A)-(II),(B)-(III),(C)-(I),(D)-(IV)$

Solution:

Concept:

- An addition polymer is identified by the monomer whose carbon-carbon double bond opens to form the repeating chain. Substituents on that monomer are retained on the polymer backbone.
- Neoprene, Teflon, Acrilan and natural rubber have characteristic monomers: chloroprene, tetrafluoroethene, acrylonitrile and isoprene respectively.

Step 1: Match the synthetic elastomer and fluoropolymer Neoprene is formed from chloroprene, or 2-chlorobuta-1,3-diene, $CH_2{=}C(Cl){-}CH{=}CH_2$. Thus (A) matches (II). Teflon is polytetrafluoroethene; its monomer is $CF_2{=}CF_2$, tetrafluoroethene. Thus (B) matches (III). The chlorine substituent identifies neoprene, whereas complete fluorination identifies Teflon.

Step 2: Match the fibre and natural elastomer Acrilan is the acrylic polymer associated with acrylonitrile, $CH_2{=}CH{-}CN$, so (C) matches (I). Natural rubber is cis-1,4-polyisoprene, formed from isoprene, or 2-methylbuta-1,3-diene, $CH_2{=}C(CH_3){-}CH{=}CH_2$. Hence (D) matches (IV). The nitrile group belongs to the acrylic fibre monomer, not to Teflon.

Step 3: Compare all proposed matchings The complete sequence is (A)-(II), (B)-(III), (C)-(I), (D)-(IV). Option (B) incorrectly assigns acrylonitrile to Teflon and tetrafluoroethene to Acrilan. Option (C) also confuses natural rubber with the fluorinated monomer. Option (D) assigns acrylonitrile to neoprene and chloroprene to Teflon, so it fails the first two direct identifications.

Step 4: Conclusion Neoprene matches chloroprene, Teflon matches tetrafluoroethene, Acrilan matches acrylonitrile, and natural rubber matches isoprene. Hence, the correct option is (A).

Quick Tip to Solve: Start with Teflon: its fluorine atoms immediately identify tetrafluoroethene and eliminate every option except the correct sequence.

80. Which of the following enhances the lathering property of soap? [4 marks]

(A) Sodium stearate

(B) Sodium carbonate

(C) Sodium rosinate

(D) Trisodium phosphate

Correct Answer: (C) Sodium rosinate

Solution:

Concept:

- Soaps are sodium or potassium salts of long-chain fatty acids. Their molecules have a water-attracting ionic head and a water-repelling hydrocarbon portion, enabling cleansing through micelle formation.
- Commercial soap formulations contain additives for specific purposes. Sodium rosinate, obtained by adding rosin, enhances lather formation; this function differs from the builder or water-softening role of inorganic salts.

Step 1: Identify the lather-enhancing additive Rosin is added during soap manufacture to form sodium rosinate. This constituent increases the lathering property of soap. The question asks for the additive associated specifically with enhanced lather, rather than for a salt that is itself an ordinary fatty-acid soap or a general cleaning aid.

Step 2: Separate the roles of the other salts Sodium stearate is a typical soap itself: its long-chain carboxylate ion provides the basic cleansing action. Sodium carbonate is commonly used as a washing aid and helps deal with hardness or alkalinity. Trisodium phosphate is a builder and cleaning aid. These different functions should not be confused with the characteristic lather-enhancing use of sodium rosinate.

Step 3: Select the unique named function Among the listed compounds, sodium rosinate is the standard answer linked to rosin and increased lather. Selecting sodium stearate merely because it is a soap overlooks the word enhances. Similarly, selecting carbonate or phosphate confuses improved washing conditions with the particular foam-promoting additive asked for here.

Step 4: Conclusion Sodium rosinate enhances the lathering property of soap. Hence, the correct option is (C).

Quick Tip to Solve: Associate rosin with sodium rosinate and increased lather; carbonate and phosphate are usually identified as washing aids or builders.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. The normality of H_2SO_4 in the solution obtained on mixing 100 mL of 0.1 M H_2SO_4 with 50 mL of 0.1 M NaOH is _____ $\times 10^{-1}$ N. (Nearest Integer) [4 marks]

Solution:

Concept:

- Normality measures reactive equivalents per litre. For complete neutralisation, one mole of sulphuric acid supplies two acid equivalents, while one mole of sodium hydroxide supplies one base equivalent.
- Neutralisation subtracts base equivalents from the initial acid equivalents. The remaining acid equivalents are divided by the final mixture volume, using the sum of the stated volumes.

Step 1: Calculate initial acid and base equivalents The acid amount is $0.100 \times 0.1 = 0.010\,\text{mol}$. Since H_2SO_4 is dibasic, this corresponds to $2 \times 0.010 = 0.020$ acid equivalents. The base amount is $0.050 \times 0.1 = 0.005\,\text{mol}$, or 0.005 base equivalents. Working in equivalents accounts directly for the two acidic protons without treating acid and base molarities as interchangeable.

Step 2: Find the remaining acidity The hydroxide consumes an equal number of acid equivalents. Therefore the excess acidity is

$$E_{\text{acid}} = 0.020 - 0.005 = 0.015 \text{ equiv.}$$

The solution remains acidic because the initial acid equivalents exceed the base equivalents. It would be incorrect to subtract the number of acid molecules and base molecules and then forget the acid valence factor.

Step 3: Divide by total volume and express the entry The final volume is $100 + 50 = 150\,\text{mL} = 0.150\,\text{L}$. Thus

$$N = \frac{0.015}{0.150} = 0.100\,\text{N} = 1 \times 10^{-1}\,\text{N}.$$

The requested entry is the coefficient multiplying 10^{-1}, not the decimal normality itself. It is already an integer, so rounding leaves it unchanged.

Step 4: Conclusion The residual acidity is $0.10\,\text{N}$; hence the required nearest-integer coefficient is $\boxed{1}$.

Quick Tip to Solve: Use acid and base equivalents first, then divide their difference by the total volume in litres; sulphuric acid contributes two equivalents per mole.

82. A gas (Molar mass = 280 $\mathrm{g\ mol^{-1}}$) was burnt in excess O_2 in a constant volume calorimeter and during combustion the temperature of calorimeter increased from $298.0\ \mathrm{K}$ to $298.45\ \mathrm{K}$. If the heat capacity of calorimeter is $2.5\ \mathrm{kJ\ K^{-1}}$ and enthalpy of combustion of gas is $9\ \mathrm{kJ\ mol^{-1}}$ then amount of gas burnt is _____ g. (Nearest Integer) [4 marks]

Solution:

Concept:

- The heat absorbed by a calorimeter is its heat capacity multiplied by its temperature rise: $q_{\text{cal}} = C_{\text{cal}}\Delta T$. A positive calorimeter temperature change indicates that the burning sample released heat.
- The supplied molar combustion energy gives the heat released per mole in the intended calorimetric calculation. Divide the measured heat magnitude by that value, then multiply the number of moles by molar mass.

Step 1: Find the temperature rise and heat absorbed The temperature rise is

$$\Delta T = 298.45 - 298.00 = 0.45\,\mathrm{K}.$$

Consequently,

$$q_{\text{cal}} = (2.5\,\mathrm{kJ\,K^{-1}})(0.45\,\mathrm{K}) = 1.125\,\mathrm{kJ}.$$

The gas has released this magnitude of heat; the opposite signs of sample heat and calorimeter heat follow from energy conservation.

Step 2: Calculate the amount of gas Using the supplied combustion-energy magnitude of $9\,\mathrm{kJ\,mol^{-1}}$, the calorimetric amount is

$$n = \frac{1.125\,\mathrm{kJ}}{9\,\mathrm{kJ\,mol^{-1}}} = 0.125\,\mathrm{mol}.$$

The units cancel to moles, so this quantity is not yet the requested mass. For the numerical calculation the given molar heat is taken as the heat released per mole. Strictly, a constant-volume measurement gives the change in internal energy, whereas the question supplies enthalpy. The intended calculation uses the approximation $\Delta U \approx \Delta H$; no combustion equation or gas-mole change is supplied for a correction.

Step 3: Convert moles to grams The molar mass is $280\,\mathrm{g\,mol^{-1}}$. Hence

$$m = nM = (0.125)(280) = 35\,\mathrm{g}.$$

The temperature rise must be used rather than either absolute temperature. Multiplying heat by molar mass before dividing by molar combustion heat gives the same dimensional result. The computed mass is integral, so the nearest-integer instruction does not alter it.

Step 4: Conclusion The calorimetric mass of gas burnt is $\boxed{35\,\mathrm{g}}$; the required integer is 35.

Quick Tip to Solve: Calculate the small temperature rise first; heat capacity times that rise gives the heat in kJ, which must be divided by the molar heat before converting to grams.

83. When a certain amount of solid A is dissolved in $100\ \mathrm{g}$ of water at $25^\circ\mathrm{C}$ to make a dilute solution, the vapour pressure of the solution is reduced to one-half of that of pure water. The vapour pressure of pure water is $23.76\,\mathrm{mmHg}$. The number of moles of solute A added is _____. (Nearest Integer) [4 marks]

Solution:

Concept:

- For an ideal solution containing a nonvolatile solute, Raoult law gives $p = x_{\text{water}}p^\circ$. The ratio of solution vapour pressure to pure-solvent vapour pressure therefore equals the solvent mole fraction.
- The solvent mole fraction is $n_w/(n_w + n_A)$. Use this exact expression when the pressure change is large; replacing the total number of moles by solvent moles would be an invalid dilute-limit approximation here.

Step 1: Determine the water mole fraction The vapour pressure falls to one-half, so

$$p = \frac{23.76}{2} = 11.88\,\mathrm{mmHg}, \qquad x_w = \frac{11.88}{23.76} = \frac{1}{2}.$$

The absolute pressure value cancels in the ratio. The solute contributes no vapour in the usual Raoult-law treatment of a dissolved solid, so the observed vapour pressure is attributed to water.

Step 2: Relate solute moles to solvent moles Using $x_w = n_w/(n_w + n_A)$,

$$\frac{n_w}{n_w + n_A} = \frac{1}{2} \quad \Longrightarrow \quad 2n_w = n_w + n_A \quad \Longrightarrow \quad n_A = n_w.$$

A half-pressure solution therefore contains equal numbers of solvent and solute particles in this model. Although the wording calls the solution dilute, a one-half lowering is not small, so the exact mole-fraction form is essential.

Step 3: Calculate the amount and round The molar mass of water is $18\,\mathrm{g\,mol^{-1}}$, giving

$$n_w = \frac{100}{18} = 5.555\ldots\ \mathrm{mol}.$$

Thus $n_A = 5.555\ldots\ \mathrm{mol}$, which rounds to 6 as the nearest integer. The unrounded amount is retained until the final step; rounding the solvent mass or using a half-solvent-moles shortcut would change the result.

Step 4: Conclusion The solute amount is approximately $5.56\,\mathrm{mol}$; the required nearest integer is $\boxed{6}$.

Quick Tip to Solve: For a large vapour-pressure lowering, use the exact solvent mole fraction; half the vapour pressure means equal solute and solvent moles.

84.

$$\underset{\text{Reactant}}{[A]} \quad \rightarrow \quad \underset{\text{Product}}{[B]}$$

If formation of compound [B] follows the first order of kinetics and after 70 minutes the concentration of [A] was found to be half of its initial concentration. Then the rate constant of the reaction is $x \times 10^{-6}\ \text{s}^{-1}$. The value of x is _____. (Nearest Integer) [4 marks]

Solution:

Concept:

- For first-order disappearance of a reactant, $[A]_t = [A]_0 e^{-kt}$, where k is the rate constant and t is elapsed time. The half-life is the time at which the reactant concentration becomes half its initial value.
- The first-order half-life relation is $k = \ln 2/t_{1/2}$. The time unit determines the rate-constant unit, so a result requested in inverse seconds requires converting minutes to seconds before substitution.

Step 1: Recognise and convert the half-life After 70 minutes the reactant concentration is $[A]_0/2$; therefore $t_{1/2} = 70\,\text{min}$. Converting to seconds,

$$t_{1/2} = 70 \times 60 = 4200\,\text{s}.$$

The concentration of product is not needed because the first-order integrated equation can be applied directly to the remaining reactant.

Step 2: Substitute in the integrated rate law At half-life,

$$\ln \frac{[A]_0}{[A]_t} = kt_{1/2} \quad \Longrightarrow \quad \ln 2 = k(4200).$$

Hence

$$k = \frac{0.693147}{4200} = 1.65035 \times 10^{-4}\,\text{s}^{-1}.$$

This positive value is consistent with a reactant concentration that decreases exponentially. No initial numerical concentration is required because the ratio is specified.

Step 3: Express the coefficient in the requested scale The question writes the constant as $x \times 10^{-6}\,\text{s}^{-1}$. Therefore

$$1.65035 \times 10^{-4} = 165.035 \times 10^{-6}, \qquad x = 165.035.$$

Rounding this coefficient to the nearest integer gives 165. Using 70 without conversion would give a constant in inverse minutes and would be incompatible with the stated inverse-second scale.

Step 4: Conclusion $k \approx 165.035 \times 10^{-6}\,\text{s}^{-1}$, so the required nearest-integer value is $\boxed{165}$.

Quick Tip to Solve: Convert the half-life to seconds before calculating the rate constant, and only then rewrite the power of ten requested for the numerical entry.

85. The number of molecule(s) or ion(s) from the following having non-planar structure is _____.

$$NO_3^-, H_2O_2, BF_3, PCl_3, XeF_4, SF_4, XeO_3, PH_4^+, SO_3, [Al(OH)_4]^-$$

[4 marks]

Solution:

Concept:

- VSEPR theory predicts shape from the number of bonding regions and lone pairs around the central atom. Multiple bonds count as one electron-density region.
- A molecule is planar only when all its atoms lie in one plane. Trigonal planar and square planar structures qualify; tetrahedral, pyramidal and seesaw structures do not.

Step 1: Classify the planar species NO_3^-, BF_3 and SO_3 have three bonding regions and no central lone pair, giving trigonal planar structures. Resonance in nitrate and sulfur trioxide does not change their planar geometry. In XeF_4, six electron pairs occupy an octahedral arrangement, but the two lone pairs occupy opposite positions; the four fluorine atoms and xenon therefore lie in a square plane. Thus these four species are excluded from the required count.

Step 2: Classify and count the non-planar species H_2O_2 has a non-planar, skew arrangement about the O–O bond. PCl_3 has three bonds and one lone pair, so it is trigonal pyramidal. SF_4 has four bonds and one lone pair in a trigonal-bipyramidal electron arrangement and adopts a seesaw shape. XeO_3 is trigonal pyramidal, whereas PH_4^+ and $[Al(OH)_4]^-$ have tetrahedral arrangements. Each of these six species has atoms extending outside a single plane; a lone pair is not counted as an atom when checking planarity.

Step 3: Conclusion The non-planar species are H_2O_2, PCl_3, SF_4, XeO_3, PH_4^+ and $[Al(OH)_4]^-$. The required number is $\boxed{6}$ (dimensionless).

Quick Tip to Solve: First remove the familiar trigonal planar and square planar species, then count the skew, pyramidal, seesaw and tetrahedral structures.

86. The spin only magnetic moment of the complex present in Fehling's reagent is _____ B.M. (Nearest integer).

[4 marks]

Solution:

Concept:

- Fehling's reagent contains an alkaline copper(II)–tartrate complex. Tartrate stabilises dissolved copper(II), so the metal oxidation state before the reagent reacts is $+2$.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ B.M., where n is the number of unpaired electrons and B.M. denotes Bohr magnetons.

Step 1: Determine the metal electron configuration Neutral copper has atomic number 29 and configuration $[Ar]3d^{10}4s^1$. On forming Cu^{2+}, the outer $4s$ electron and one $3d$ electron are removed, leaving $[Ar]3d^9$. This is the copper oxidation state in the blue complex of Fehling's reagent. It must

not be confused with the copper(I) oxide precipitate formed after the reagent oxidises an aldehyde.

Step 2: Count unpaired electrons and calculate the moment Nine electrons in five d orbitals give four paired orbitals and one singly occupied orbital. Therefore a mononuclear d^9 copper(II) centre has one unpaired electron; changing the splitting pattern cannot pair all nine electrons. Substituting $n = 1$ gives

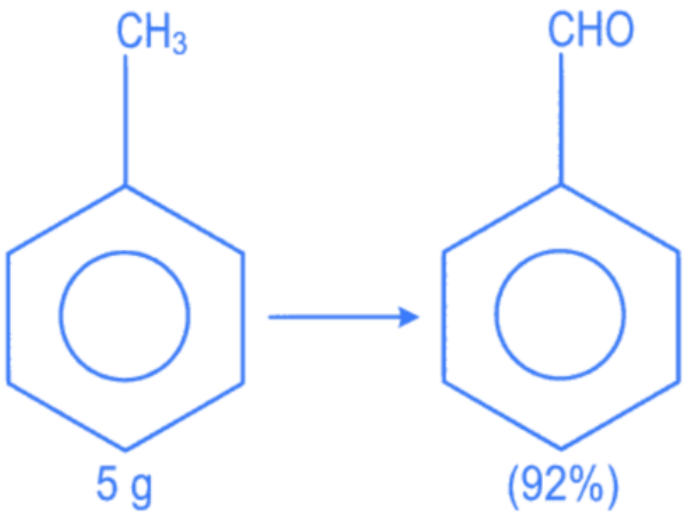
$$\mu = \sqrt{1(1+2)} = \sqrt{3} = 1.732 \text{ B.M.}$$

The question explicitly asks for the spin-only value, so orbital contributions and any measured deviations are not included. Rounding is performed on the final moment, rather than on any intermediate quantity.

Step 3: Conclusion The spin-only moment is 1.732 B.M., whose nearest integer is $\boxed{2}$ B.M..

Quick Tip to Solve: Identify copper(II) in the reagent before considering its reaction product: a d-nine ion always leaves one unpaired electron.

87. In the above reaction, 5 g of toluene is converted into benzaldehyde with 92% yield. The amount of benzaldehyde produced is _____ $\times 10^{-2}$ g. (Nearest integer) [4 marks]



Solution:

Concept:

- Conversion of a toluene molecule into a benzaldehyde molecule preserves its seven-carbon skeleton. The mole ratio of toluene consumed to benzaldehyde formed is $1 : 1$.
- Percentage yield equals actual product mass divided by theoretical product mass, multiplied by 100. Use molar masses to convert the given reactant mass into the theoretical product mass.

Step 1: Calculate the theoretical benzaldehyde mass Toluene is C_7H_8, with molar mass $7(12) + 8(1) =$

$92\ \mathrm{g\,mol^{-1}}$. Benzaldehyde is C_7H_6O, with molar mass $7(12)+6(1)+16=106\ \mathrm{g\,mol^{-1}}$. Thus

$$n(\text{toluene})=\frac{5}{92}\ \text{mol}, \qquad m_{\text{theoretical}}=\frac{5}{92}(106)\ \text{g}.$$

The product is heavier per mole because oxidation adds oxygen while removing hydrogen; equal mole amounts do not mean equal masses.

Step 2: Apply the stated yield and extract the coefficient Only 92 percent of the theoretical benzaldehyde is obtained. Therefore

$$m_{\text{actual}}=\frac{92}{100}\frac{5}{92}(106)=5.30\ \text{g}.$$

The printed blank multiplies 10^{-2} g, so the number to enter is not 5.30. Writing the result in that requested form gives

$$5.30\ \text{g}=530\times10^{-2}\ \text{g}.$$

This value is already an integer coefficient. The yield factor is used once, after the stoichiometric mass conversion; applying it to the product mass again would incorrectly reduce the yield twice.

Step 3: Conclusion The benzaldehyde mass produced is $5.30\ \text{g}=\boxed{530}\times10^{-2}\ \text{g}$. The required nearest-integer entry is 530.

Quick Tip to Solve: For a one-to-one conversion, multiply reactant mass by the product/reactant molar-mass ratio and then by the fractional yield.

88. For a real gas at 25°C temperature and high pressure (99 bar) the value of compressibility factor is 2, so the value of Vander Waal's constant 'b' should be _____ $\times10^{-2}$ L mol^{-1} (Nearest integer) (Given R $=0.083$ L bar K^{-1} mol^{-1}) [4 marks]

Solution:

Concept:

- The compressibility factor is $Z=PV_m/(RT)$, where V_m is molar volume. Thus Z compares the measured molar volume with the ideal-gas value at the same temperature and pressure.
- In the high-pressure approximation for a van der Waals gas, the excluded-volume term dominates the deviation and $P(V_m-b)\simeq RT$. Hence $Z\simeq1+Pb/(RT)$.

Step 1: Express the excluded-volume constant in terms of Z For one mole, the full van der Waals equation is $(P+a/V_m^2)(V_m-b)=RT$. In the stated high-pressure treatment the attraction correction is neglected compared with the pressure, giving

$$V_m\simeq b+\frac{RT}{P}.$$

Multiplying by $P/(RT)$ gives $Z\simeq1+Pb/(RT)$, so

$$b\simeq\frac{(Z-1)RT}{P}.$$

The subtraction of 1 is essential: the ideal-gas contribution already accounts for one unit of Z.

Step 2: Substitute consistent units and round The absolute temperature is $T = 25 + 273 = 298$ K. Using the supplied litre-bar gas constant ensures that b is obtained in litres per mole:

$$b = \frac{(2-1)(0.083)(298)}{99} = \frac{24.734}{99} = 0.24984\ \mathrm{L\,mol^{-1}}.$$

In the requested scale this is $24.984 \times 10^{-2}\ \mathrm{L\,mol^{-1}}$, which rounds to a coefficient of 25. This numerical determination uses the high-pressure approximation intended by the problem; without neglecting attraction, Z alone would not determine b unless a were also specified.

Step 3: Conclusion Using the high-pressure van der Waals approximation, $b \simeq \boxed{25} \times 10^{-2}\ \mathrm{L\,mol^{-1}}$. The nearest-integer entry is 25.

Quick Tip to Solve: At high pressure use Z minus one for the excluded-volume contribution, and convert Celsius to kelvin before substituting.

89. Among the following ores Bauxite, Siderite, Cuprite, Calamine, Haematite, Kaolinite, Malachite, Magnetite, Sphalerite, Limonite, Cryolite, the number of principal ores if iron is _____.

[4 marks]

Solution:

Concept:

- An ore is a naturally occurring mineral from which a metal can be extracted economically. For this question, identify the principal iron minerals in the supplied list rather than counting every familiar ore.
- The common principal iron ores are haematite, magnetite, limonite and siderite. Their formulas contain iron in oxide, hydrated oxide or carbonate form.

Step 1: Identify the iron-bearing principal ores Siderite is $\mathrm{FeCO_3}$, an iron carbonate. Haematite is $\mathrm{Fe_2O_3}$, the principal ferric oxide ore. Magnetite is $\mathrm{Fe_3O_4}$, a mixed iron oxide. Limonite is hydrated ferric oxide, commonly represented as $\mathrm{Fe_2O_3 \cdot \mathit{x}H_2O}$. These four names in the supplied list are therefore included. The different oxidation states or presence of water do not change the identity of the metal obtained from these iron ores.

Step 2: Exclude the ores of other metals Bauxite is a hydrated aluminium oxide ore; kaolinite is an aluminium silicate, and cryolite is $\mathrm{Na_3AlF_6}$, associated with aluminium metallurgy. Cuprite, $\mathrm{Cu_2O}$, and malachite, $\mathrm{CuCO_3 \cdot Cu(OH)_2}$, are copper ores. Calamine, $\mathrm{ZnCO_3}$, and sphalerite, ZnS, are zinc ores. Thus none of these seven entries adds another principal iron ore. Classifying by the metal in the mineral prevents confusing similarly familiar carbonate names, particularly siderite and calamine, which belong to different metals.

Step 3: Conclusion The principal iron ores listed are siderite, haematite, magnetite and limonite. Their number is $\boxed{4}$ (dimensionless).

Quick Tip to Solve: Remember iron ores as two oxides, one hydrated oxide and one carbonate: haematite, magnetite, limonite and siderite.

90. The oxidation state of manganese in the product obtained in a reaction of potassium permanganate and hydrogen peroxide in basic medium is _____. [4 marks]

Solution:

Concept:

- Potassium permanganate contains MnO_4^-, with manganese in oxidation state $+7$. Hydrogen peroxide acts as the reducing agent toward permanganate and is oxidised to oxygen.
- In the usual neutral or basic permanganate–peroxide reaction, the manganese product is MnO_2. Oxidation numbers are determined by making their sum equal to the charge of the species.

Step 1: Identify and balance the manganese reduction The manganese reduction half-reaction in basic medium is

$$MnO_4^- + 2\,H_2O + 3\,e^- \longrightarrow MnO_2 + 4\,OH^-.$$

It balances one manganese and four oxygen atoms on each side; hydrogen and net charge also balance. Peroxide is oxidised according to

$$H_2O_2 + 2\,OH^- \longrightarrow O_2 + 2\,H_2O + 2\,e^-.$$

The product here is manganese dioxide, rather than the Mn^{2+} formed by permanganate reduction in strongly acidic conditions.

Step 2: Combine the reactions and obtain the oxidation number Multiplying the reduction by two and the peroxide oxidation by three gives the net equation

$$2\,MnO_4^- + 3\,H_2O_2 \longrightarrow 2\,MnO_2 + 3\,O_2 + 2\,OH^- + 2\,H_2O.$$

Potassium ions are spectators and do not affect the oxidation number. In neutral MnO_2, each oxygen has oxidation number -2. If manganese has oxidation number x,

$$x + 2(-2) = 0 \quad \Rightarrow \quad x = +4.$$

Thus manganese gains three electrons per atom on going from permanganate to manganese dioxide, consistent with the reduction half-reaction and confirming the assigned product oxidation state.

Step 3: Conclusion The product is MnO_2, so the oxidation state of manganese is $\boxed{+4}$. The numerical entry is 4.

Quick Tip to Solve: Identify the manganese product for the medium first; only then use oxygen as minus two to calculate the metal oxidation state.