JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 28 Shift 1 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 28 July 2022, Shift 1. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1. Let the solution curve of the differential equation $x\,\mathrm{d}y = \left(\sqrt{x^2+y^2}+y\right)\mathrm{d}x, x>0$, intersect the line $x=1$ at $y=0$ and the line $x=2$ at $y=\alpha$. Then the value of α is : [4 marks]

(A) $\frac{1}{2}$
(B) $\frac{3}{2}$
(C) $-\frac{3}{2}$
(D) $\frac{5}{2}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Concept:

- A first-order equation is homogeneous when its derivative depends only on the ratio y/x. Set $v=y/x$, so that $y=xv$ and $dy/dx = v + x\,dv/dx$.
- Because $x>0$, the radical satisfies $\sqrt{x^2+y^2} = x\sqrt{1+v^2}$. The integral $\int dv/\sqrt{1+v^2} = \log(v+\sqrt{1+v^2})$ gives a real expression for every real v.

Step 1: Reduce the equation using the ratio Divide the given equation by $x\,dx$ and substitute $y=xv$. The terms containing v cancel, leaving a separable equation:

$$v + x\frac{dv}{dx} = \sqrt{1+v^2}+v, \qquad \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}.$$

The cancellation is the reason to use the homogeneous substitution rather than try to separate the original variables directly.

Step 2: Integrate and determine the constant Integrating gives $\log(v+\sqrt{1+v^2}) = \log x + C$. At $x=1, y=0$, we have $v=0$, so $C=0$. Exponentiating therefore yields

$$v+\sqrt{1+v^2} = x.$$

Its positive radical requires $\sqrt{1+v^2} = x-v$; squaring and simplifying gives $1 = x^2 - 2xv$.

Step 3: Evaluate the required ordinate Thus $v = (x^2-1)/(2x)$, and multiplying by x gives

$$y = \frac{x^2-1}{2}.$$

This curve meets the initial point, and its derivative $y\prime = x$ satisfies the original equation for positive x. At $x=2$, its ordinate is $3/2$, excluding $1/2, -3/2, 5/2$.

Step 4: Conclusion The intersection with $x=2$ has ordinate $\alpha = 3/2$. Hence, the correct option is (B).

Quick Tip to Solve: For an equation containing $\sqrt{x^2+y^2}$ and $x>0$, substitute $y=xv$ before integrating; retain the sign when extracting x.

2. Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$ is : [4 marks]

(A) $\left(-\infty, \frac{1}{4}\right]$

(B) $\left[-\frac{1}{4}, \infty\right)$

(C) $(-1/3, \infty)$

(D) $\left(-\infty, \frac{1}{3}\right]$

Correct Answer: (B) $\left[-\frac{1}{4}, \infty\right)$

Solution:

Concept:

- The real principal inverse cosine $\cos^{-1} u$ is defined precisely when $-1 \le u \le 1$. Both inequalities must hold simultaneously.
- The denominator $x^2 + 3$ is strictly positive for every real x, so multiplying an inequality by this denominator never reverses its direction.

Step 1: Impose the upper bound on the argument Write the argument as $u = (x^2 - 4x + 2)/(x^2 + 3)$. The condition $u \le 1$ becomes

$$x^2 - 4x + 2 \le x^2 + 3 \quad \Longleftrightarrow \quad -4x \le 1 \quad \Longleftrightarrow \quad x \ge -\frac{1}{4}.$$

Equality is permitted: the argument equals 1 there, and the principal inverse cosine is 0.

Step 2: Impose the lower bound independently The condition $u \ge -1$ becomes

$$x^2 - 4x + 2 \ge -x^2 - 3 \quad \Longleftrightarrow \quad 2x^2 - 4x + 5 \ge 0.$$

Complete the square:

$$2x^2 - 4x + 5 = 2(x - 1)^2 + 3 > 0.$$

Consequently this lower bound excludes no real number, and there are no additional holes caused by the denominator.

Step 3: Intersect the restrictions and check the options The intersection is therefore the closed half-line $[-1/4, \infty)$. Values smaller than $-1/4$ make the argument greater than 1, so an interval extending indefinitely to the left cannot be the domain. Option (C) incorrectly admits some numbers between $-1/3$ and $-1/4$, for which the inverse cosine is not real. Options (A) and (D) also omit valid large positive inputs.

Step 4: Conclusion The domain is $\boxed{[-1/4, \infty)}$. Hence, the correct option is (B).

Quick Tip to Solve: For the domain of $\cos^{-1}(P/Q)$, check the sign of Q first, then impose both bounds rather than only $P \le Q$.

3. Let the vectors $\vec{a} = (1+t)\hat{i} + (1-t)\hat{j} + \hat{k}, \vec{b} = (1-t)\hat{i} + (1+t)\hat{j} + 2\hat{k}$ and $\vec{c} = t\hat{i} - t\hat{j} + \hat{k}, t \in \mathbf{R}$ be such that for $\alpha, \beta, \gamma \in \mathbf{R}, \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \overrightarrow{0} \Rightarrow \alpha = \beta = \gamma = 0$. Then, the set of all values of t is : [4 marks]

(A) a non-empty finite set

(B) equal to $\mathbf{N}$

(C) equal to $\mathbf{R} - \{0\}$

(D) equal to $\mathbf{R}$

Correct Answer: (C) equal to $\mathbf{R} - \{0\}$

Solution:

Concept:

- Three vectors in three-dimensional space are linearly independent if and only if the determinant of their coordinate matrix is nonzero. This is equivalent to their homogeneous linear combination having only the trivial coefficients.
- Elementary determinant row operations can reveal the dependence on the parameter without expanding a long cubic polynomial. A zero determinant signals a nontrivial linear relation.

Step 1: Translate the implication into a determinant test The implication in the question is exactly the definition of linear independence. Form the determinant with the three given vectors as rows:

$$D = \begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix}.$$

We require $D \neq 0$. The parameter is real, so the answer must include every real value meeting this algebraic condition, not just natural-number values.

Step 2: Simplify the determinant without losing parameter values Apply $R_1 \leftarrow R_1 - R_3$, then $R_2 \leftarrow R_2 + R_3$. These operations preserve the determinant and give rows $(1, 1, 0)$, $(1, 1, 3)$, $(t, -t, 1)$. Next subtract the first row from the second:

$$D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 3 \\ t & -t & 1 \end{vmatrix} = -3 \begin{vmatrix} 1 & 1 \\ t & -t \end{vmatrix} = 6t.$$

No division by t was used, so the zero case has not been discarded prematurely.

Step 3: Identify and check the excluded value It follows that $D \neq 0$ exactly when $t \neq 0$. At $t = 0$, the first two coordinates of every vector have equal components and, explicitly, $\vec{b} - \vec{a} - \vec{c} = \vec{0}$, proving dependence. At every nonzero real value the determinant test proves independence. Thus the set is infinite, includes negative and nonintegral numbers, and excludes zero, ruling out (A), (B), and (D).

Step 4: Conclusion The set of allowed parameters is $\mathbb{R} \setminus \{0\}$. Hence, the correct option is (C).

Quick Tip to Solve: When a question states that a vector combination vanishes only trivially, compute a determinant and require it to be nonzero.

4. Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ is equal to : [4 marks]

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Correct Answer: (A) 0

Solution:

Concept:

- For $x \in [-1, 1]$, the principal values satisfy $\cos^{-1} x = \pi/2 - \sin^{-1} x$. The additional term $\cos^{-1}(2x)$ restricts the domain to $[-1/2, 1/2]$.
- The triple-angle identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ reduces this equation to a polynomial. Candidates obtained by taking cosine must be checked in the original principal-value equation.

Step 1: Introduce the principal angle and preserve its range Let $\theta = \sin^{-1} x$. Since $|x| \leq 1/2$, we have $-\pi/6 \leq \theta \leq \pi/6$. The equation becomes

$$\frac{\pi}{2} - 3\theta = \cos^{-1}(2x).$$

Its left-hand side lies in $[0, \pi]$, the exact range of the principal inverse cosine, so taking cosine respects the relevant range.

Step 2: Find all algebraic candidates Taking cosine yields $\sin 3\theta = 2x$. With $\sin\theta = x$, this is

$$3x - 4x^3 = 2x \quad \Longleftrightarrow \quad x(1 - 4x^2) = 0.$$

Hence the only possible solutions are $-1/2, 0, 1/2$. Each lies within the domain; there are no candidates outside it to retain.

Step 3: Verify each principal value and add the roots At $x = -1/2$, the original left side is $2\pi/3 + \pi/3 = \pi$, equal to $\cos^{-1}(-1)$. At $x = 0$, both sides are $\pi/2$. At $x = 1/2$, the left side is $\pi/3 - \pi/3 = 0 = \cos^{-1}(1)$. All three candidates are therefore genuine solutions, and their sum is $-1/2 + 0 + 1/2 = 0$. The other options are not this sum.

Step 4: Conclusion The sum of all solutions is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: After converting an inverse-trigonometric equation to a polynomial, verify every root using the principal-value ranges before adding them.

5. Let a vector $\vec{a}$ has magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in \mathbf{R} \times \mathbf{R} - \{(0, 0)\}$, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\vec{a} - 18x\vec{b})$. Then the value of $|\vec{a} \times \vec{b}|$ is equal to : [4 marks]

(A) $9\sqrt{3}$

(B) $27\sqrt{3}$

(C) 9

(D) 81

Correct Answer: (B) $27\sqrt{3}$

Solution:

Concept:

- Two vectors are perpendicular exactly when their dot product is zero. A condition holding for every nonzero pair (x, y) can be tested at convenient choices of these two scalars.
- The cross-product magnitude is $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\phi$, where ϕ is their angle. Orthogonal vectors have $\sin\phi = 1$.

Step 1: Extract the mutual perpendicularity Choose $x = 1, y = 0$, a pair allowed by the question. The two vectors become $\vec{a}$ and $-18\vec{b}$. Their dot product is $-18\vec{a} \cdot \vec{b} = 0$, so $\vec{a} \cdot \vec{b} = 0$. Thus $\vec{a}$ and $\vec{b}$ are perpendicular, rather than merely the two combinations being perpendicular for some special choice.

Step 2: Determine the magnitude of the second vector For arbitrary x, y, expansion of the required zero dot product gives

$$6xy|\vec{a}|^2 + (6y^2 - 18x^2)\vec{a} \cdot \vec{b} - 18xy|\vec{b}|^2 = 0.$$

The middle term vanishes by the first step. Taking $x = y = 1$ then gives $6 \cdot 81 - 18|\vec{b}|^2 = 0$, so $|\vec{b}|^2 = 27$ and $|\vec{b}| = 3\sqrt{3}$. These conditions also make the expanded expression vanish for every allowed pair, checking consistency.

Step 3: Evaluate the area represented by the cross product Since the mutual angle is a right angle,

$$|\vec{a} \times \vec{b}| = 9(3\sqrt{3}) = 27\sqrt{3}.$$

Option (A) misses a factor of 3 in the second magnitude; (C) gives only the first magnitude; (D) incorrectly treats the two magnitudes as equal. None equals the computed product.

Step 4: Conclusion The required magnitude is $\boxed{27\sqrt{3}}$. Hence, the correct option is (B).

Quick Tip to Solve: When perpendicularity holds for every (x, y), first choose $(1, 0)$ to find the dot product, then $(1, 1)$ to find a magnitude.

6. For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices $A(\sin t, -\cos t), B(\cos t, \sin t)$ and $C(a, b)$ such that its orthocentre lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $\left(a^2 - b^2\right)$ is equal to : [4 marks]

(A) $\frac{8}{3}$

(B) 8

(C) $\frac{77}{9}$

(D) $\frac{80}{9}$

Correct Answer: (B) 8

Solution:

Concept:

- In an equilateral triangle, the orthocentre and centroid coincide. If the vertices have coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, their common point is $((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$.
- A circle written as $(X - h)^2 + (Y - k)^2 = r^2$ has centre (h, k). The identity $(\sin t + \cos t)^2 + (\sin t - \cos t)^2 = 2$ eliminates the parameter here.

Step 1: Write the orthocentre coordinates Let the orthocentre be $H(X, Y)$. Using the equality of orthocentre and centroid in the stated equilateral triangle gives

$$X = \frac{\sin t + \cos t + a}{3}, \qquad Y = \frac{\sin t - \cos t + b}{3}.$$

Thus $3X - a = \sin t + \cos t$ and $3Y - b = \sin t - \cos t$. The signs in the second coordinate follow from the negative cosine in the vertex A.

Step 2: Eliminate the trigonometric parameter Square the two expressions and add. The mixed terms cancel, while $\sin^2 t + \cos^2 t = 1$, giving

$$(3X - a)^2 + (3Y - b)^2 = 2.$$

Divide throughout by 9 to identify the circle:

$$\left(X - \frac{a}{3}\right)^2 + \left(Y - \frac{b}{3}\right)^2 = \frac{2}{9}.$$

Hence the centre obtained from the orthocentre coordinates is $(a/3, b/3)$.

Step 3: Compare centres and evaluate the requested expression The stated centre is $(1, 1/3)$, so coordinate comparison gives $a/3 = 1$ and $b/3 = 1/3$, or $a = 3, b = 1$. Therefore

$$a^2 - b^2 = 3^2 - 1^2 = 8.$$

It is the vertex coordinates, rather than the centre coordinates themselves, that must be squared. Squaring the given centre directly would produce the wrong scale. Only option (B) equals 8; the other displayed values do not satisfy this comparison.

Step 4: Conclusion The required value is $\boxed{a^2 - b^2 = 8}$. Hence, the correct option is (B).

Quick Tip to Solve: For an equilateral triangle, replace the orthocentre by the centroid; then eliminate sine and cosine by squaring and adding.

7. For $\alpha \in \mathbf{N}$, consider a relation R on $\mathbf{N}$ given by $\mathrm{R} = \{(x, y) : 3x + \alpha y$ is a multiple of 7$\}$. The relation R is an equivalence relation if and only if : [4 marks]

(A) $\alpha = 14$

(B) α is a multiple of 4

(C) 4 is the remainder when α is divided by 10

(D) 4 is the remainder when α is divided by 7

Correct Answer: (D) 4 is the remainder when α is divided by 7

Solution:

Concept:

- An equivalence relation is reflexive, symmetric and transitive. Reflexivity alone supplies a necessary arithmetic condition, which must then be shown sufficient for the remaining properties.
- The statement that an integer is a multiple of 7 can be expressed as congruence to zero modulo 7. Since 3 and 7 are coprime, $3u \equiv 0 \pmod 7$ is equivalent to $u \equiv 0 \pmod 7$.

Step 1: Use reflexivity to obtain a necessary condition If R is an equivalence relation on $\mathbb{N}$, every (x, x) must belong to it. In particular $(1, 1) \in R$, giving

$$3 + \alpha \equiv 0 \pmod 7, \qquad \alpha \equiv 4 \pmod 7.$$

This condition also ensures that $(3 + \alpha)x$ is divisible by 7 for every natural x, as reflexivity requires.

Step 2: Prove that this condition is sufficient When $\alpha \equiv 4 \equiv -3 \pmod 7$, the defining expression becomes

$$3x + \alpha y \equiv 3(x - y) \pmod 7.$$

Since 3 has a multiplicative inverse modulo 7, xRy holds exactly when $x \equiv y \pmod 7$. This is reflexive because $x - x = 0$, symmetric because changing the sign preserves divisibility, and transitive because the sum of two multiples of 7 is again a multiple of 7.

Step 3: Distinguish the option statements Therefore the necessary and sufficient condition is remainder 4 upon division by 7. The value $\alpha = 14$ fails even the test at $(1, 1)$. Being a multiple of 4 does not suffice, since $\alpha = 8$ fails that test. Remainder 4 modulo 10 is also insufficient, since $\alpha = 14$ again supplies a counterexample. Option (D) states exactly the condition already proved in both directions.

Step 4: Conclusion The relation is an equivalence relation if and only if $\alpha \equiv 4 \pmod 7$. Hence, the correct option is (D).

Quick Tip to Solve: For relations defined by divisibility of $px + qy$, test $(1, 1)$ first, then rewrite the relation as equality of residue classes.

8. Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is : [4 marks]

(A) $\frac{2}{3}$
(B) $\frac{11}{16}$
(C) $\frac{23}{32}$
(D) $\frac{13}{16}$

Correct Answer: (A) $\frac{2}{3}$

Solution:

Concept:

- When selection is restricted to qualified candidates, the sample space contains only those candidates. The required probability is $P(F \mid Q) = N(F \cap Q)/N(Q)$.
- Counts in a ratio $2 : 1$ form three equal shares: two female shares and one male share. Percentages among all applicants must not be substituted for percentages among qualifiers.

Step 1: Represent the qualified groups Let the total number of candidates appearing be N. The number qualifying is $0.60N$. Write the number of qualified males as k; the given condition then makes the number of qualified females $2k$. Since every qualified candidate belongs to one of these two groups,

$$k + 2k = 0.60N, \qquad k = 0.20N.$$

Consequently the qualified female count is $0.40N$. The available female and male applicant totals are $0.60N$ and $0.40N$, so both derived counts are feasible.

Step 2: Condition on qualification and compare the choices The candidate is chosen from the qualified population, whose size is $0.60N$, rather than from all applicants. Thus

$$P(F \mid Q) = \frac{0.40N}{0.60N} = \frac{2}{3}.$$

Equivalently, among each group of three qualified candidates, two are female. Options (B), (C) and (D) do not equal this ratio and therefore contradict the stated two-to-one split; the original 60 percent female share is not the answer after conditioning.

Step 3: Conclusion The required probability is $\boxed{2/3}$. Hence, the correct option is (A).

Quick Tip to Solve: For a probability within the qualified group, use its internal female-to-male ratio directly: $2 : 1$ gives $2/(2 + 1)$.

9. If $y = y(x), x \in (0, \pi/2)$ be the solution curve of the differential equation $(\sin^2 2x)\frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)\, y = 2e^{-4x}(2\sin 2x + \cos 2x)$, with $y(\pi/4) = e^{-\pi}$, then $y(\pi/6)$ is equal to : [4 marks]

(A) $\frac{2}{\sqrt{3}}e^{-2\pi/3}$

(B) $\frac{2}{\sqrt{3}}e^{2\pi/3}$

(C) $\frac{1}{\sqrt{3}}e^{-2\pi/3}$

(D) $\frac{1}{\sqrt{3}}e^{2\pi/3}$

Correct Answer: (A) $\frac{2}{\sqrt{3}}e^{-2\pi/3}$

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ is solved using an integrating factor $I(x) = e^{\int P(x)\,dx}$, so that $(Iy)' = IQ$.
- Here $0 < x < \pi/2$ implies $\sin 2x > 0$; division by $\sin^2 2x$ is valid and $\int 4\cot 2x\, dx = 2\log(\sin 2x)$.

Step 1: Put the equation in linear form Use $\sin 4x = 2\sin 2x \cos 2x$ and divide the equation by $\sin^2 2x$:

$$y' + (8 + 4\cot 2x)y = \frac{2e^{-4x}(2\sin 2x + \cos 2x)}{\sin^2 2x}.$$

The integrating factor is

$$I(x) = e^{8x + 2\log(\sin 2x)} = e^{8x}\sin^2 2x.$$

Its logarithmic derivative is exactly the coefficient of y, which checks the factor.

Step 2: Integrate and impose the initial value Multiplication by the factor gives

$$\frac{d}{dx}\left(ye^{8x}\sin^2 2x\right) = 2e^{4x}(2\sin 2x + \cos 2x).$$

The right side is the derivative of $e^{4x}\sin 2x$. Therefore

$$ye^{8x}\sin^2 2x = e^{4x}\sin 2x + C.$$

At $x = \pi/4$, both nonconstant sides equal e^{π}, because $y = e^{-\pi}$ and $\sin(\pi/2) = 1$. Hence $C = 0$, giving $y = e^{-4x}/\sin 2x$.

Step 3: Evaluate the requested ordinate At $x = \pi/6$, we have $\sin 2x = \sin(\pi/3) = \sqrt{3}/2$, and hence

$$y(\pi/6) = \frac{e^{-2\pi/3}}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}e^{-2\pi/3}.$$

The positive exponent in (B) and (D) conflicts with the factor e^{-4x}; option (C) omits the factor 2 from the sine denominator.

Step 4: Conclusion $\boxed{y(\pi/6) = \frac{2}{\sqrt{3}}e^{-2\pi/3}}$. Hence, the correct option is (A).

Quick Tip to Solve: After finding the integrating factor, check whether the transformed right side is the derivative of a product of an exponential and a sine.

10. Let C be the centre of the circle $x^2 + y^2 - x + 2y = \frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R. Then the area of the triangle PQR (in unit ^{2}) is : [4 marks]

(A) 2

(B) $2\sqrt{2}$

(C) $8\sin\left(\frac{\pi}{8}\right)$

(D) $8\cos\left(\frac{\pi}{8}\right)$

Correct Answer: (B) $2\sqrt{2}$

Solution:

Concept:

- Completing squares gives a circle's centre and radius. A chord passing through the centre is a diameter and therefore has length $2r$.
- The area of a triangle is $\frac{1}{2} \times$ base $\times$ perpendicular height. If a radius makes angle θ with the base line, its perpendicular projection has length $r\sin\theta$.

Step 1: Find the radius and identify the base Complete the squares without changing the right-hand side incorrectly:

$$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = \frac{11}{4} + \frac{1}{4} + 1 = 4.$$

Thus $C = (1/2, -1)$ and $r = 2$. Because the line meeting the circle at Q and R passes through C, the segment QR is a diameter. Its length is therefore $QR = 4$, independently of the position of P on the circle.

Step 2: Find the altitude and calculate the area Let H be the foot of the perpendicular from P to the line QR. The radius $CP = 2$ makes an angle $\pi/4$ with that line. Its component perpendicular to the line is

$$PH = CP\sin(\pi/4) = 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}.$$

This height is measured to the base line; it is not the full radius. Hence

$$[PQR] = \frac{1}{2} \cdot QR \cdot PH = \frac{1}{2} \cdot 4 \cdot \sqrt{2} = 2\sqrt{2}.$$

Option (A) uses too small an altitude. The quantities in (C) and (D) use half the specified angle and do not equal this base-times-height calculation.

Step 3: Conclusion The area is $\boxed{2\sqrt{2}\ \text{unit}^2}$. Hence, the correct option is (B).

Quick Tip to Solve: When a triangle has a diameter as its base, its area is $r^2\sin\theta$, where θ is the angle between that diameter and the radius to its third vertex.

11. The remainder when $7^{2022} + 3^{2022}$ is divided by 5 is : [4 marks]

(A) 0

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Concept:

- Two integers congruent modulo 5 have powers congruent modulo 5. The remainder must be one of $0, 1, 2, 3, 4$.
- For a number not divisible by 5, fourth powers are congruent to 1 modulo 5; the exponent may therefore be reduced modulo 4. This can also be checked by directly listing the cycle of powers.

Step 1: Reduce the bases and exponent The two bases satisfy $7 \equiv 2 \pmod 5$ and $3 \equiv -2 \pmod 5$. Since 2022 is even, their powers have equal residues. Also

$$2022 = 4 \cdot 505 + 2.$$

The residue cycle of powers of 2 is $2, 4, 3, 1$, repeating after four terms, because $2^4 = 16 \equiv 1 \pmod 5$. It follows that both terms have the same residue as 2^2, namely 4. This checks both the parity and the position within the cycle.

Step 2: Add the residues and choose the allowed remainder Writing the reduction explicitly,

$$7^{2022} \equiv (2^4)^{505} 2^2 \equiv 4 \pmod 5,$$

$$3^{2022} \equiv (-2)^{2022} = 2^{2022} \equiv 4 \pmod 5.$$

Their sum is congruent to $4 + 4 = 8 \equiv 3 \pmod 5$. The value 8 is a congruent integer but is not the remainder, since division by 5 requires a value smaller than 5. Options (A), (B) and (D) correspond to residues 0, 2 and 4 and are excluded by the final addition.

Step 3: Conclusion The remainder is $\boxed{3}$. Hence, the correct option is (C).

Quick Tip to Solve: For powers modulo 5, first reduce the exponent modulo 4, then add the individual residues and reduce the sum once more.

12. Let $S_1 = \{z_1 \in \mathbf{C} : |z_1 - 3| = \frac{1}{2}\}$ and $S_2 = \{z_2 \in \mathbf{C} : |z_2 - |z_2 + 1|| = |z_2 + |z_2 - 1||\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $|z_2 - z_1|$ is : [4 marks]

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

Correct Answer: (C) $\frac{3}{2}$

Solution:

Concept:

- For $z = x + iy$, the moduli $|z - 1|$ and $|z + 1|$ are distances from the points 1 and -1 on the real axis.
- The locus $|z_1 - 3| = 1/2$ is a circle with centre 3 and radius $1/2$. The minimum distance from an exterior point to a circle is its distance from the centre minus the radius.

Step 1: Find every component of the second locus Write $z_2 = x + iy$, $a = |z_2 + 1|$ and $b = |z_2 - 1|$. The given equation is $|z_2 - a| = |z_2 + b|$. Squaring the nonnegative sides gives

$$(x - a)^2 + y^2 = (x + b)^2 + y^2,$$

$$a^2 - b^2 = 2x(a + b).$$

But $a^2 - b^2 = (x + 1)^2 - (x - 1)^2 = 4x$. Thus $2x(2 - a - b) = 0$. Either $x = 0$, or $a + b = 2$. The latter is equality in the triangle inequality between foci -1 and 1, and holds exactly on the real segment $[-1, 1]$. Hence S_2 is the imaginary axis together with that segment.

Step 2: Compare distances to the circle The centre 3 is at distance 3 from the imaginary axis. The circle therefore has minimum distance $3 - 1/2 = 5/2$ from that axis. Its centre is at distance 2 from the segment $[-1, 1]$, attained at its right endpoint 1, so the distance from the circle to this segment is

$$2 - \frac{1}{2} = \frac{3}{2}.$$

This is attained by $z_2 = 1$ and $z_1 = 5/2$, so it is an actual minimum. The sets do not meet, excluding (A); (B) is below their separation; (D) is the distance to the axis alone and misses the additional segment.

Step 3: Conclusion The least value is $\boxed{3/2}$. Hence, the correct option is (C).

Quick Tip to Solve: When a complex-locus calculation factors, retain both factors: here discarding the real segment would incorrectly give $5/2$.

13. If the minimum value of $f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5}, x > 0$, is 14 , then the value of α is equal to : [4 marks]

(A) 32

(B) 64

(C) 128

(D) 256

Correct Answer: (C) 128

Solution:

Concept:

- An interior minimum of a differentiable function satisfies $f\prime(x) = 0$. A positive second derivative confirms a local minimum.
- The stated finite positive minimum forces $\alpha > 0$: for $\alpha < 0$ the function tends to $-\infty$ at zero, while for $\alpha = 0$ its infimum is zero.

Step 1: Express the parameter through the minimizing point Let the minimum occur at $x = t > 0$. Differentiating gives

$$f\prime(x) = 5x - 5\alpha x^{-6}.$$

Thus $f\prime(t) = 0$ implies $t = \alpha t^{-6}$, or

$$\alpha = t^7.$$

For $\alpha > 0$, this critical point is unique. Moreover,

$$f\prime\prime(x) = 5 + 30\alpha x^{-7} > 0 \quad (x > 0),$$

so the function is strictly convex throughout its domain. Together with its divergence to positive infinity at both ends, this confirms that the critical point gives the global minimum.

Step 2: Use the given minimum value Substitute $\alpha = t^7$ into the value at the minimizing point:

$$f(t) = \frac{5}{2}t^2 + \frac{t^7}{t^5} = \frac{7}{2}t^2.$$

The minimum equals 14, hence

$$\frac{7}{2}t^2 = 14, \qquad t^2 = 4.$$

Because $t > 0$, we obtain $t = 2$, and then $\alpha = 2^7 = 128$. As a check, at $x = 2$, the two terms are 10 and 4, whose sum is 14. Each other proposed parameter has a different positive seventh root and hence a different minimum $(7/2)\alpha^{2/7}$.

Step 3: Conclusion $\boxed{\alpha = 128}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the derivative equation to replace the parameter term in the function before inserting the stated minimum; this avoids solving for fractional powers too early.

14. Let α, β and γ be three positive real numbers. Let $f(x) = \alpha x^5 + \beta x^3 + \gamma x, x \in \mathbf{R}$ and $g : \mathbf{R} \to \mathbf{R}$ be such that $g(f(x)) = x$ for all $x \in \mathbf{R}$. If $\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \ldots, \mathrm{a}_\mathrm{n}$ be in arithmetic progression with mean zero, then the value of $f\left(g\left(\frac{1}{\mathrm{n}}\sum_{i=1}^{\mathrm{n}} f\left(\mathrm{a}_i\right)\right)\right)$ is equal to : [4 marks]

(A) 0

(B) 3

(C) 9

(D) 27

Correct Answer: (A) 0

Solution:

Concept:

- The polynomial $f(x) = \alpha x^5 + \beta x^3 + \gamma x$ is odd, so $f(-x) = -f(x)$ and $f(0) = 0$.
- In an arithmetic progression, terms equidistant from the ends have sum twice the mean. When the mean is zero, they occur in opposite pairs.
- The relation $g(f(x)) = x$, applied at $x = 0$, gives $g(0) = 0$. No explicit inverse formula is needed.

Step 1: Pair the terms of the progression Let the common difference be d. Since the mean is $(a_1 + a_n)/2 = 0$, we have $a_1 + a_n = 0$. For every index i, the progression property gives

$$a_i + a_{n+1-i} = a_1 + a_n = 0.$$

Thus $a_{n+1-i} = -a_i$. Oddness of the polynomial now yields

$$f(a_i) + f(a_{n+1-i}) = f(a_i) + f(-a_i) = 0.$$

If n is odd, the central term is zero and also contributes $f(0) = 0$. Therefore the whole sum, and hence its average, is zero.

Step 2: Evaluate the composition at zero The inside of g in the requested expression is

$$\frac{1}{n}\sum_{i=1}^{n} f(a_i) = 0.$$

The given composition relation at $x = 0$ reads $g(f(0)) = 0$, so $g(0) = 0$. Therefore

$$f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right) = f(g(0)) = f(0) = 0.$$

The positive coefficients are consistent with invertibility, since $f\prime(x) = 5\alpha x^4 + 3\beta x^2 + \gamma > 0$, but the value at zero already suffices. Options (B), (C) and (D) are nonzero and violate the exact cancellation.

Step 3: Conclusion The value of the expression is $\boxed{0}$. Hence, the correct option is (A).

Quick Tip to Solve: For an odd function evaluated on a zero-mean arithmetic progression, pair terms from opposite ends before attempting any expansion or inverse calculation.

15. The minimum value of the twice differentiable function $f(x) = \int_0^x \mathrm{e}^{x-\mathrm{t}} f'(\mathrm{t})\mathrm{dt} - \left(x^2 - x + 1\right)\mathrm{e}^x$, $x \in \mathbf{R}$, **is :** [4 marks]

(A) $-\frac{2}{\sqrt{\mathrm{e}}}$

(B) $-2\sqrt{\mathrm{e}}$

(C) $-\sqrt{\mathrm{e}}$

(D) $\frac{2}{\sqrt{\mathrm{e}}}$

Correct Answer: (A) $-\frac{2}{\sqrt{\mathrm{e}}}$

Solution:

Concept:

- Treat the integral as a function of its upper limit and use the fundamental theorem of calculus together with the product rule.
- For a differentiable function on the real line, a derivative changing from negative to positive establishes a minimum. The factor e^x is positive for every real x.

Step 1: Separate the exponential factor Write $I(x) = e^x \int_0^x e^{-t} f\prime(t)\,dt$, so the given equation is $f(x) = I(x) - (x^2 - x + 1)e^x$. Differentiation of the product defining I gives

$$I\prime(x) = I(x) + f\prime(x).$$

This identity remains valid for negative upper limits as well, since the fundamental theorem uses oriented integrals.

Step 2: Differentiate the given identity Differentiate both sides, keeping the derivative of the polynomial as well as that of the exponential:

$$f\prime = I + f\prime - \left[(2x - 1) + (x^2 - x + 1)\right]e^x.$$

The two occurrences of $f\prime$ cancel. Hence

$$I(x) = (x^2 + x)e^x, \qquad f(x) = (2x - 1)e^x.$$

At $x = 0$, this gives $f(0) = -1$, consistent with the original equation whose integral vanishes there.

Step 3: Locate and classify the stationary point Now

$$f\prime(x) = (2x + 1)e^x.$$

Thus the derivative is negative for $x < -1/2$, zero at $x = -1/2$, and positive for $x > -1/2$. This is the global minimum, since the function decreases throughout the first interval and increases throughout the second. Its value is

$$f(-1/2) = (-2)e^{-1/2} = -\frac{2}{\sqrt{e}}.$$

The alternative negative values in (B) and (C) are not attained at the stationary point; (D) even has the wrong sign.

Step 4: Conclusion The minimum value is $-2/\sqrt{e}$, attained at $x = -1/2$. Hence, the correct option is (A).

Quick Tip to Solve: When the kernel is e^{x-t}, factor out e^x; differentiating the integral then reproduces the integral itself plus the endpoint term.

16. Let the operations $*, \odot \in \{\wedge, \vee\}$. If $(\mathrm{p} * \mathrm{q}) \odot (\mathrm{p} \odot \sim \mathrm{q})$ is a tautology, then the ordered pair $(*, \odot)$ is : [4 marks]

(A) $(\vee, \wedge)$

(B) $(\vee, \vee)$

(C) $(\wedge, \wedge)$

(D) $(\wedge, \vee)$

Correct Answer: (B) $(\vee, \vee)$

Solution:

Concept:

- A tautology must be true for every assignment of truth values to p and q. A single false assignment eliminates a proposed pair of connectives.
- Use associativity of disjunction and the complement law $q \vee \sim q = \mathrm{T}$. Conjunction with a statement and its negation is always false.

Step 1: Test the all-disjunction choice For $(*, \odot) = (\vee, \vee)$, the expression becomes

$$(p \vee q) \vee (p \vee \sim q) = p \vee p \vee (q \vee \sim q).$$

The last parenthesis is true whether q is true or false. A disjunction containing a true statement is true, independently of p. Consequently this choice produces a tautology, rather than merely a proposition that is sometimes true.

Step 2: Rule out the conjunction choices For option (A), substitute $* = \vee$ and $\odot = \wedge$. The expression is

$$(p \vee q) \wedge (p \wedge \sim q) = p \wedge \sim q.$$

It is false whenever p is false, so it cannot be a tautology. For option (C), both operations are conjunctions:

$$(p \wedge q) \wedge (p \wedge \sim q) = p \wedge q \wedge \sim q.$$

This is always false, because it requires q and its negation simultaneously.

Step 3: Check the remaining mixed pair For option (D), the expression becomes

$$(p \wedge q) \vee (p \vee \sim q) = p \vee \sim q.$$

The absorption law removes $p \wedge q$, since its truth already implies p. Set $p = \mathrm{F}$ and $q = \mathrm{T}$; then both terms in the final disjunction are false. Thus (D) also fails. Only the all-disjunction choice remains true for all four truth assignments.

Step 4: Conclusion The required ordered pair is $(\vee, \vee)$. Hence, the correct option is (B).

Quick Tip to Solve: For a tautology MCQ, simplify each connective choice; use one false truth assignment to reject any expression that is not identically true.

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17. If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point $R(0, 1)$, then the orthocentre of the triangle PQR is : [4 marks]

(A) (0, 1)

(B) (2, -1)

(C) (6, 3)

(D) (2, 1)

Correct Answer: (B) (2, -1)

Solution:

Concept:

- On $y^2 = 2x - 3$, a point with ordinate t has coordinates $((t^2 + 3)/2, t)$. The tangent follows from implicit differentiation.
- The orthocentre is the intersection of the altitudes. In a right triangle it is the vertex containing the right angle, because the two legs themselves are altitudes.

Step 1: Determine the tangency ordinates Implicit differentiation gives $2y\, dy/dx = 2$, so the tangent slope at ordinate $t \neq 0$ is $1/t$. Its equation is

$$t(y - t) = x - \frac{t^2 + 3}{2}, \quad \text{or} \quad ty = x + \frac{t^2 - 3}{2}.$$

For this tangent to pass through $R(0, 1)$, we require

$$t = \frac{t^2 - 3}{2}, \qquad t^2 - 2t - 3 = 0.$$

Factoring gives $(t - 3)(t + 1) = 0$, hence the two ordinates are 3 and -1. Neither is zero, so the slope form used here is valid.

Step 2: Find the points and the right angle The corresponding points of contact are

$$P = (6, 3), \qquad Q = (2, -1).$$

Indeed, each satisfies $y^2 = 2x - 3$. The slopes of the chord and one tangent side are

$$m_{PQ} = \frac{3 - (-1)}{6 - 2} = 1, \qquad m_{QR} = \frac{-1 - 1}{2 - 0} = -1.$$

Their product is -1, so $PQ \perp QR$. The right angle of the triangle is therefore at Q.

Step 3: Identify the orthocentre The altitude from P to QR is the line PQ, while the altitude from R to PQ is the line RQ. These intersect at $Q = (2, -1)$. Option (A) names the external tangent

intersection, not the right-angle vertex; (C) names the other point of contact. Option (D) has the incorrect ordinate and does not equal the intersection of the altitudes.

Step 4: Conclusion The orthocentre is $(2, -1)$. Hence, the correct option is (B).

Quick Tip to Solve: After locating the two contact points, test perpendicular side slopes before calculating all three altitudes; a right triangle gives its orthocentre immediately.

18. Let the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ **and the matrix** $B_0 = A^{49} + 2A^{98}$**. If** $B_n = Adj\,(B_{n-1})$ **for all** $n \geq 1$**, then** $\det(B_4)$ **is equal to :** [4 marks]

(A) 3^{28}

(B) 3^{30}

(C) 3^{32}

(D) 3^{36}

Correct Answer: (C) 3^{32}

Solution:

Concept:

- The cyclic permutation matrix satisfies $A^3 = I$. Reduce all exponents modulo three before performing any matrix calculation.
- For a 3×3 matrix M, $\det(\text{adj}\, M) = (\det M)^2$. Thus each adjugate operation squares the preceding determinant.

Step 1: Reduce the powers in the initial matrix Applying A three times returns each coordinate to its initial position, so $A^3 = I$. Since $49 = 3 \cdot 16 + 1$ and $98 = 3 \cdot 32 + 2$,

$$B_0 = A + 2A^2 = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}.$$

Keeping the positions of the two nonzero entries in each row avoids an unnecessary expansion of large powers.

Step 2: Evaluate the initial determinant Expand along the first row:

$$\begin{aligned} \det B_0 &= -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} + 2\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} \\ &= -(-1) + 2(4) = 9 = 3^2. \end{aligned}$$

Both contributing cyclic products are positive. This direct computation also checks that the initial exponent of three is two, not one.

Step 3: Iterate the determinant identity four times Write $d_n = \det B_n$. Taking determinants in

$B_n = \text{adj}(B_{n-1})$ gives $d_n = d_{n-1}^2$, because these are matrices of order three. Consequently

$$d_1 = 3^4, \quad d_2 = 3^8, \quad d_3 = 3^{16}, \quad d_4 = 3^{32}.$$

There are four squarings after B_0, so equivalently $d_4 = (3^2)^{2^4}$. The exponent is multiplied by sixteen; it is not increased by a fixed number at each stage. This rules out the exponents 28, 30 and 36 in the other options.

Step 4: Conclusion $\det(B_4) = 3^{32}$. Hence, the correct option is (C).

Quick Tip to Solve: For repeated adjugates, track only the determinant: an order-r adjugate raises it to the power $r-1$ at each iteration.

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19. The foot of the perpendicular from a point on the circle $x^2+y^2=1, z=0$ to the plane $2x+3y+z=6$ lies on which one of the following curves? [4 marks]

(A) $(6x+5y-12)^2+4(3x+7y-8)^2=1, z=6-2x-3y$

(B) $(5x+6y-12)^2+4(3x+5y-9)^2=1, z=6-2x-3y$

(C) $(6x+5y-14)^2+9(3x+5y-7)^2=1, z=6-2x-3y$

(D) $(5x+6y-14)^2+9(3x+7y-8)^2=1, z=6-2x-3y$

Correct Answer: (B) $(5x+6y-12)^2+4(3x+5y-9)^2=1, z=6-2x-3y$

Solution:

Concept:

- The vector perpendicular to the plane $2x+3y+z=6$ is its normal $(2,3,1)$. The original point and its perpendicular foot differ by a multiple of this vector.
- Obtain the locus by expressing the original circle coordinates in terms of the foot coordinates and substituting into the circle equation. The foot also lies on the target plane.

Step 1: Relate a circle point to its foot Let the original point be $P=(u,v,0)$, where $u^2+v^2=1$, and write its foot as $H=(x,y,z)$. Since PH is perpendicular to the plane,

$$(x,y,z) = (u,v,0) + \lambda(2,3,1).$$

The third coordinate gives $\lambda = z$. Thus

$$u = x-2z, \qquad v = y-3z.$$

These relations use the same scalar parameter in every coordinate; treating the shifts independently would not describe perpendicular projection.

Step 2: Eliminate the height using the plane The foot is on the plane, so $z=6-2x-3y$. Substitute

into the preceding inverse relations:

$$\begin{aligned} u &= x - 2(6 - 2x - 3y) = 5x + 6y - 12, \\ v &= y - 3(6 - 2x - 3y) = 6x + 10y - 18 \\ &= 2(3x + 5y - 9). \end{aligned}$$

Both the constant terms and the factor two in the second expression are essential.

Step 3: Substitute into the original circle equation Now $u^2 + v^2 = 1$ becomes

$$(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1,$$

together with $z = 6 - 2x - 3y$. Conversely, any point satisfying these two equations reconstructs a point $(u, v, 0)$ on the unit circle, with displacement $z(2, 3, 1)$. Thus the equations describe exactly the projection locus. The other alternatives have different linear forms or coefficients and do not reproduce the inverse coordinate relations; only (B) has both correct squares.

Step 4: Conclusion The locus is $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1$, with $z = 6 - 2x - 3y$. Hence, the correct option is (B).

Quick Tip to Solve: For projecting a planar curve, recover the original coordinates from the foot along the plane normal, then substitute directly into the original curve equation.

20. Consider the sequence $a_1, a_2, a_3, \ldots$ such that $a_1 = 1, a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for $n = 1, 2, 3, \ldots$. If $\left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right) \cdot \left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right) \cdot \left(\frac{a_3 + \frac{1}{a_4}}{a_5}\right) \ldots \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha}\left({}^{61}C_{31}\right)$, then α is equal to : [4 marks]

(A) -30

(B) -31

(C) -60

(D) -61

Correct Answer: (C) -60

Solution:

Concept:

- When a recurrence contains the reciprocal of a neighbouring term, multiplying by that neighbour can produce a simple recurrence for consecutive products.
- Products of consecutive odd and even integers can be converted to factorials, making a binomial coefficient visible. All terms here are positive by induction from the initial values.

Step 1: Find the consecutive products Multiply the given recurrence by a_{n+1}:

$$a_{n+1}a_{n+2} = a_n a_{n+1} + 2.$$

Define $b_n = a_n a_{n+1}$. Then $b_{n+1} = b_n + 2$, and $b_1 = a_1 a_2 = 2$. Hence $b_n = 2n$ for every positive integer n. Positivity ensures that all divisions in the requested product are valid.

Step 2: Simplify each factor Multiplying its numerator and denominator by a_{n+1} gives

$$\frac{a_n + 1/a_{n+1}}{a_{n+2}} = \frac{a_n a_{n+1} + 1}{a_{n+1} a_{n+2}} = \frac{2n+1}{2n+2}.$$

The requested expression therefore contains precisely the thirty factors with $n = 1, \ldots, 30$:

$$P = \frac{3 \cdot 5 \cdots 61}{4 \cdot 6 \cdots 62}.$$

The final even integer is 62, so the denominator involves $31!$, not $30!$.

Step 3: Convert the odd and even products to factorials The numerator is $61!/(2 \cdot 4 \cdots 60) = 61!/(2^{30}30!)$. The denominator is $(2 \cdot 4 \cdots 62)/2 = 2^{30}31!$. Thus

$$P = \frac{61!}{2^{60}30!31!} = 2^{-60}\binom{61}{31}.$$

Comparison with $2^{\alpha}\binom{61}{31}$ gives $\alpha = -60$. Each of the two separated products contributes a factor 2^{30}, explaining why -30 and -31 are incorrect; excluding the initial even factor two also rules out -61.

Step 4: Conclusion The exponent is $\alpha = -60$. Hence, the correct option is (C).

Quick Tip to Solve: Introduce $b_n = a_n a_{n+1}$ before attempting to solve for individual terms; then count the odd and even factors carefully when extracting powers of two.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from $\{A, B, C, D, E\}$ or a number from $\{1, 2, 3, 4, 5\}$ with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from $\{1, 2, 3, 4, 5\}$ is $\alpha \times 5^6$, then α is equal to _____. [4 marks]

Solution:

Concept:

- For a password of length n, ten symbols are available independently at every position, giving 10^n strings when repetition is allowed.
- The condition at least one numeral is counted by subtracting strings consisting entirely of the five letters. Different allowed lengths form disjoint sets.

Step 1: Count valid passwords at a fixed length The alphabet has five letters and five numerals, so there are ten choices at each position. For a specified length n, the total is 10^n. The invalid strings contain no numeral at all, and therefore have only five choices at each position, giving 5^n. The number meeting the requirement is

$$N_n = 10^n - 5^n.$$

This subtraction includes passwords with two or more numerals exactly once; selecting one position

for a numeral instead would overcount those strings.

Step 2: Add the three permitted lengths The allowed lengths are 6, 7 and 8. A string of one length cannot also belong to either of the other length classes, so add their counts:

$$N = (10^6 - 5^6) + (10^7 - 5^7) + (10^8 - 5^8).$$

Factor 5^6 from the entire expression, remembering that the last two terms contain additional factors of five:

$$\frac{N}{5^6} = (2^6 - 1) + 5(2^7 - 1) + 25(2^8 - 1).$$

The choices are ordered characters, so no division by a factorial is appropriate.

Step 3: Evaluate the coefficient Calculate the three contributions separately:

$$\begin{aligned}\alpha &= (64 - 1) + 5(128 - 1) + 25(256 - 1)\\ &= 63 + 635 + 6375\\ &= 7073.\end{aligned}$$

The result is a dimensionless integer, since it is the coefficient in a count of passwords. The complement method automatically allows every possible distribution of letters and numerals while enforcing the required presence of at least one numeral.

Step 4: Conclusion The required value is $\boxed{\alpha = 7073}$.

Quick Tip to Solve: For an at-least-one requirement, subtract the zero-occurrence class separately for each permitted length, then add the disjoint length classes.

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22. Let $\mathrm{P}(-2, -1, 1)$ and $\mathrm{Q}\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ be the vertices of the rhombus PRQS. If the direction ratios of the diagonal RS are $\alpha, -1, \beta$, where both α and β are integers of minimum absolute values, then $\alpha^2 + \beta^2$ is equal to _____. [4 marks]

Solution:

Concept:

- The diagonals of a rhombus bisect each other at right angles. Since the vertices are in the order PRQS, the diagonals are PQ and RS.
- Direction ratios of perpendicular lines have zero scalar product. Integer direction ratios can be found by solving the resulting linear Diophantine equation.

Step 1: Find the direction of PQ Subtract the coordinates of P from the corresponding coordinates of Q:

$$\overrightarrow{PQ} = \left(\frac{90}{17}, \frac{60}{17}, \frac{94}{17}\right).$$

Thus we may use direction ratios $(45, 30, 47)$ for PQ; multiplying all three ratios by the same nonzero number leaves the direction unchanged.

Step 2: Impose perpendicularity For RS the given ratios are $(\alpha, -1, \beta)$. Their scalar product with

those of PQ must vanish:

$$45\alpha - 30 + 47\beta = 0, \qquad 45\alpha + 47\beta = 30.$$

Reducing modulo 47 gives $-2\alpha \equiv 30 \pmod{47}$, so $\alpha = -15 + 47k$ for an integer k. Substitution then gives $\beta = 15 - 45k$.

Step 3: Choose the smallest absolute integer ratios The solution k=0 gives $\alpha = -15$ and $\beta = 15$. If $k \geq 1$, then $\alpha \geq 32$ and $\beta \leq -30$; if $k \leq -1$, then $\alpha \leq -62$ and $\beta \geq 60$. Thus neither absolute value can be reduced by another integer solution. Finally,

$$\alpha^2 + \beta^2 = (-15)^2 + 15^2 = 225 + 225 = 450.$$

Step 4: Conclusion The required dimensionless integer is $\boxed{450}$.

Quick Tip to Solve: Use the diagonals PQ and RS, rather than adjacent sides, and retain the fixed middle ratio -1 before minimizing the integer solution.

23. Let $f : [0, 1] \to \mathbf{R}$ be a twice differentiable function in $(0, 1)$ such that $f(0) = 3$ and $f(1) = 5$. If the line $y = 2x + 3$ intersects the graph of f at only two distinct points in $(0, 1)$, then the least number of points $x \in (0, 1)$, at which $f''(x) = 0$, is _____. [4 marks]

Solution:

Concept:

- Rolle's theorem requires continuity on each closed interval and differentiability on its interior. Twice differentiability only on $(0, 1)$ does not imply continuity at 0 or 1.
- To test a claimed least number, construct a function satisfying every printed condition and count its second-derivative zeros. The number of such points cannot be negative.

Step 1: Separate the printed hypotheses from the usual intended hypothesis Set $g(x) = f(x) - (2x+3)$. The printed endpoint values give $g(0) = g(1) = 0$, and there are exactly two further zeros in $(0, 1)$. If g were continuous on $[0, 1]$, three applications of Rolle's theorem between consecutive zeros would give three zeros of g', and two more applications would give at least two zeros of $g'' = f''$. The continuous example $f(x) = 2x + 3 + x(x - 1/3)(x - 2/3)(x - 1)$ attains that lower bound: its second derivative is a quadratic, so it has at most two zeros, and Rolle's theorem supplies at least two. However, continuity at the endpoints is not among the printed hypotheses.

Step 2: Construct a counterexample under the literal conditions Define

$$f(x) = \begin{cases} 3, & x = 0, \\ 2x + 3 + (x - \frac{1}{3})(x - \frac{2}{3}), & 0 < x < 1, \\ 5, & x = 1. \end{cases}$$

This function is twice differentiable at every point of $(0, 1)$. In that open interval its intersections with the given line are precisely $x = 1/3$ and $x = 2/3$, since the product vanishes only at those two points.

It also has exactly the stated endpoint values.

Step 3: Count the second-derivative zeros and explain the key For every interior x the quadratic term has second derivative 2, and the straight-line term has second derivative zero. Therefore $f''(x) = 2$ throughout $(0, 1)$, so there are no second-derivative zeros. This attains the absolute lower bound of zero. The answer 2 follows from the customary additional endpoint-continuity assumption, but Rolle's theorem cannot be applied across these endpoint discontinuities.

Step 4: Conclusion The literal printed hypotheses give $\boxed{0}$ points. The supplied key gives 2, which requires the additional assumption that f is continuous on $[0, 1]$; the explicit counterexample above proves why the literal answer differs.

Quick Tip to Solve: Before applying Rolle's theorem to intervals ending at 0 or 1, check continuity at those endpoints; differentiability on an open interval is insufficient.

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24. If $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}\,dx = \alpha\sqrt{2} + \beta\sqrt{3}$**, where** α, β **are integers, then** $\alpha + \beta$ **is equal to _____.**

[4 marks]

Solution:

Concept:

- For the positive quantity $1 + x^2$, write $\sqrt{(1+x^2)^3} = (1+x^2)^{3/2}$. The substitution $t = \sqrt{1+x^2}$ simplifies both nested radicals.
- A definite integral must have its limits transformed at every substitution. The factors $x\,dx = t\,dt$ and $x^2 = t^2 - 1$ account for the entire numerator.

Step 1: Remove the inner radical Let $t = \sqrt{1+x^2}$. The original limits x=0 and $x = \sqrt{3}$ become t=1 and t=2. Since t is positive,

$$\sqrt{1 + x^2 + \sqrt{(1+x^2)^3}} = \sqrt{t^2 + t^3} = t\sqrt{1+t}.$$

Also $x^3 dx = (t^2 - 1)t\,dt$. Thus, after cancellation of t,

$$I = 15\int_1^2 \frac{t^2-1}{\sqrt{1+t}}\,dt = 15\int_1^2 (t-1)\sqrt{1+t}\,dt.$$

Step 2: Integrate a polynomial in a square root Put u=1+t, so the limits are now 2 and 3, and t-1=u-2. Consequently,

$$I = 15\int_2^3 (u^{3/2} - 2u^{1/2})\,du$$
$$= \left[6u^{5/2} - 20u^{3/2}\right]_2^3.$$

Differentiating the bracket gives $15u^{3/2} - 30u^{1/2}$, which checks both integration coefficients.

Step 3: Evaluate the endpoints and identify the integers At u=3 the bracket is $(54 - 60)\sqrt{3} = -6\sqrt{3}$.

At u=2 it is $(24-40)\sqrt{2} = -16\sqrt{2}$. Subtracting lower from upper gives

$$I = 16\sqrt{2} - 6\sqrt{3}.$$

Therefore $\alpha = 16$, $\beta = -6$, and their sum is $16 - 6 = 10$. The positive value of this expression also agrees with the positive integrand on the integration interval.

Step 4: Conclusion The required dimensionless integer is $\boxed{\alpha + \beta = 10}$.

Quick Tip to Solve: For a nested radical involving $(1+x^2)^{3/2}$, first set $t = \sqrt{1+x^2}$; factor before making the second substitution.

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25. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ **and** $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}, \alpha, \beta \in \mathbf{R}$. **Let** α_1 **be the value of** α **which satisfies** $(\mathrm{A}+\mathrm{B})^2 = \mathrm{A}^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ **and** α_2 **be the value of** α **which satisfies** $(\mathrm{A}+\mathrm{B})^2 = \mathrm{B}^2$**. Then** $|\alpha_1 - \alpha_2|$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Matrix equality means equality of corresponding entries. Matrix products must be computed in their stated order; the scalar identity $(A+B)^2 = A^2 + 2AB + B^2$ is not generally valid for matrices.
- Here the upper-right entry of A+B is zero, making its square particularly simple. The two conditions define separate values of alpha and may be solved separately.

Step 1: Compute the three matrix squares Direct multiplication gives

$$(A+B)^2 = \begin{bmatrix} (1+\beta)^2 & 0 \\ 3(1+\beta+\alpha) & \alpha^2 \end{bmatrix},$$

$$A^2 = \begin{bmatrix} -1 & -1-\alpha \\ 2+2\alpha & \alpha^2 - 2 \end{bmatrix},$$

$$B^2 = \begin{bmatrix} \beta^2+1 & \beta \\ \beta & 1 \end{bmatrix}.$$

For example, the lower-left entry of $(A+B)^2$ is $3(1+\beta)+3\alpha$, because $A+B = \begin{bmatrix} 1+\beta & 0 \\ 3 & \alpha \end{bmatrix}$.

Step 2: Solve the first condition Adding the matrix of 2s to A^2 gives upper-right entry $1-\alpha$. Equality with the zero upper-right entry on the left forces $\alpha_1 = 1$. The lower-left equality then becomes $3(2+\beta) = 6$, so $\beta = 0$. The top-left entries are both 1 and the bottom-right entries are both 1, confirming that every entry satisfies the condition.

Step 3: Solve the second condition and subtract The upper-right equality in $(A+B)^2 = B^2$ gives $\beta = 0$. The lower-left equality now gives $3(1+\alpha) = 0$, hence $\alpha_2 = -1$. With these values the diagonal

entries of both squares are 1, so this is a valid solution of the entire matrix equation. Therefore

$$|\alpha_1 - \alpha_2| = |1 - (-1)| = 2.$$

Step 4: Conclusion The required dimensionless value is $\boxed{2}$.

Quick Tip to Solve: Compare the zero upper-right entry first. It identifies alpha or beta immediately and avoids solving unnecessary quadratic equations.

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26. For $\mathrm{p}, \mathrm{q} \in \mathbf{R}$, consider the real valued function $f(x) = (x - \mathrm{p})^2 - \mathrm{q}, x \in \mathbf{R}$ and $\mathrm{q} > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(\mathrm{a}_i)| = 500$ for all $i = 1, 2, 3, 4$, then the absolute difference between the roots of $f(x) = 0$ is _____. [4 marks]

Solution:

Concept:

- Four equally spaced numbers with mean p can be written as $p - 3h, p - h, p + h, p + 3h$, where $h > 0$. Their common difference is 2h.
- For $f(x) = (x - p)^2 - q$, points symmetric about p have the same function value. The roots are $p \pm \sqrt{q}$, so their absolute difference is $2\sqrt{q}$.

Step 1: Use symmetry to reduce the four conditions Write the four progression terms in the symmetric form stated above. Substituting their distances from p into the quadratic gives

$$f(a_1) = f(a_4) = 9h^2 - q, \qquad f(a_2) = f(a_3) = h^2 - q.$$

The printed absolute-value condition says both of these distinct real numbers belong to the set $\{-500, 500\}$. They are distinct because their difference is $8h^2 > 0$.

Step 2: Determine h and q from the signs The larger value must therefore be 500 and the smaller must be -500:

$$9h^2 - q = 500, \qquad h^2 - q = -500.$$

Subtracting the second equation from the first yields $8h^2 = 1000$, hence $h^2 = 125$. The second equation gives

$$q = h^2 + 500 = 625.$$

This is positive as required. Choosing both function values to have the same sign would force h=0, contrary to the positive common difference, so no other sign assignment is possible.

Step 3: Find and check the distance between the roots The equation $(x - p)^2 = 625$ has roots $p - 25$ and $p + 25$. Their absolute difference is

$$|(p + 25) - (p - 25)| = 50.$$

As a consistency check, the outer progression terms give $9(125) - 625 = 500$, while the inner terms give $125 - 625 = -500$. All four required absolute values are therefore exactly 500; the unknown

centre p cancels from the requested distance.

Step 4: Conclusion The absolute difference between the roots is the dimensionless value $\boxed{50}$.

Quick Tip to Solve: Centre an even-length progression at its mean using offsets $-3h, -h, h, 3h$; the quadratic then gives only two distinct values to compare.

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27. Let $x_1, x_2, x_3, \ldots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\bar{x}$ is the mean of new data, then the greatest integer less than or equal to $\bar{x}$ is _____. [4 marks]

Solution:

Concept:

- The original terms are $x_i = 3/2^{i-1}$. The transformed observations are $y_i = (x_i - i)^2 = x_i^2 - 2ix_i + i^2$, and their mean is the sum of the 20 observations divided by 20.
- Use a finite geometric sum for $\sum x_i^2$, a differentiated geometric sum for $\sum ix_i$, and $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$.

Step 1: Compute the three component sums The square sum and integer-square sum are

$$\sum_{i=1}^{20} x_i^2 = 9\frac{1 - 4^{-20}}{1 - 1/4} = 12(1 - 4^{-20}), \qquad \sum_{i=1}^{20} i^2 = 2870.$$

Differentiating the finite geometric series, or multiplying and subtracting its weighted version, gives

$$\sum_{i=1}^{20} ir^{i-1} = \frac{1 - 21r^{20} + 20r^{21}}{(1-r)^2}.$$

At r=1/2, multiplying by 3 therefore gives $\sum ix_i = 12(1 - 11/2^{20})$.

Step 2: Assemble the transformed mean Expand every squared difference before summing. This keeps the cross-term coefficient -2 visible:

$$\begin{aligned} 20\bar{x} &= 12(1 - 4^{-20}) - 24(1 - 11/2^{20}) + 2870 \\ &= 2858 + \frac{264}{2^{20}} - \frac{12}{2^{40}}. \end{aligned}$$

The main term alone gives $2858/20 = 142.9$; the remaining correction is positive and extremely small.

Step 3: Bound the correction before taking the floor Let $\varepsilon = 264/2^{20} - 12/2^{40}$. Both powers are positive, and $0 < \varepsilon < 264/2^{20} < 1$. Thus

$$142.9 < \bar{x} < 142.95 < 143.$$

In particular the mean is above 142 and strictly below 143, so its greatest integer is 142. This bound makes the floor exact without prematurely rounding a sum of small geometric terms.

Step 4: Conclusion The greatest integer less than or equal to the new mean is $\boxed{142}$.

Quick Tip to Solve: Expand the transformed square and sum its three terms separately; bound the small tail correction before applying the greatest-integer operation.

28. $\lim\limits_{x\to 0}\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)^{\frac{100}{x}}$ **is equal to _____.** [4 marks]

Solution:

Concept:

- Write $F(t) = t^3 + 2t^2 + 3\sin t$. Then the base is $F(x + 2\cos x)/F(x + 2)$, and both arguments approach 2.
- Since $F(2) = 16 + 3\sin 2 > 0$, the base is positive near zero. Its real logarithm is therefore defined, and the limit can be obtained by multiplying the logarithmic change by $100/x$.

Step 1: Measure the difference of the two arguments Put $u = x + 2\cos x$ and $v = x + 2$. Their difference is

$$u - v = 2(\cos x - 1) = -x^2 + O(x^4).$$

In particular this perturbation is of second order, whereas the exponent grows only like $1/x$. Also $u, v \to 2$, so F remains positive throughout a sufficiently small neighbourhood containing both arguments.

Step 2: Apply the mean value theorem to the logarithm Define $H(t) = \log F(t)$ near 2. Its derivative

$$H'(t) = \frac{3t^2 + 4t + 3\cos t}{t^3 + 2t^2 + 3\sin t}$$

is continuous and bounded there because its denominator is nonzero. The mean value theorem yields, for some ξ_x between u and v,

$$\log F(u) - \log F(v) = H'(\xi_x)\, 2(\cos x - 1).$$

This controls the ratio without expanding either cubic or the composite sine separately.

Step 3: Evaluate the logarithmic limit and exponentiate The logarithm of the whole expression is

$$\frac{200}{x} H'(\xi_x)(\cos x - 1).$$

The bounded factor $H'(\xi_x)$ tends to $H'(2)$, while $(\cos x - 1)/x \to 0$ from both sides. Thus the logarithm tends to zero. Continuity of the exponential function now gives the original limit as $\exp(0) = 1$. Merely noting that the base tends to 1 would not suffice, because its exponent is unbounded.

Step 4: Conclusion The required dimensionless limit is $\boxed{1}$.

Quick Tip to Solve: Compare the two inner arguments first: an $O(x^2)$ change in a smooth nonzero function is too small to survive an exponent of order $1/x$.

29. The sum of all real values of x for which $\frac{3x^2-9x+17}{x^2+3x+10} = \frac{5x^2-7x+19}{3x^2+5x+12}$ is equal to _____. [4 marks]

Solution:

Concept:

- A rational equation may be cross-multiplied only after checking that its denominators are nonzero. A quadratic with positive leading coefficient and negative discriminant is positive for every real argument.
- After factorisation, retain only real roots. For a quadratic $ax^2 + bx + c = 0$, the sum of its two roots is $-b/a$, provided both roots are real.

Step 1: Check the domain The first denominator is $x^2 + 3x + 10$. Its discriminant is $9 - 40 = -31 < 0$. The second denominator, $3x^2 + 5x + 12$, has discriminant $25 - 144 = -119 < 0$. Both have positive leading coefficients, so neither denominator vanishes for any real x. Cross-multiplication therefore introduces no excluded real value.

Step 2: Cross-multiply and factor Bring the two products to one side and expand carefully:

$$\begin{aligned}(3x^2 - 9x + 17)(3x^2 + 5x + 12) - (5x^2 - 7x + 19)(x^2 + 3x + 10) &= 0,\\ 4x^4 - 20x^3 - 6x^2 - 10x + 14 &= 0,\\ 2(x^2 + x + 1)(2x^2 - 12x + 7) &= 0.\end{aligned}$$

The factorisation can be checked directly by multiplying the two quadratic factors.

Step 3: Select and sum the real roots The factor $x^2 + x + 1$ has discriminant $1 - 4 = -3$, so contributes no real root. For the other factor, the discriminant is $144 - 56 = 88 > 0$, so both its roots are real. They are $3 \pm \sqrt{22}/2$, and their sum is $12/2 = 6$. Summing all four quartic roots would incorrectly include the nonreal pair.

Step 4: Conclusion The sum of all the admissible real values of x is $\boxed{6}$. This is a dimensionless number.

Quick Tip to Solve: Check denominator discriminants first; then use the sum of roots only on the factor whose roots are real.

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30. For the hyperbola $\mathrm{H} : x^2 - y^2 = 1$ and the ellipse $\mathrm{E} : \frac{x^2}{\mathrm{a}^2} + \frac{y^2}{\mathrm{b}^2} = 1$, $\mathrm{a} > \mathrm{b} > 0$, let the
(1) eccentricity of E be reciprocal of the eccentricity of H, and
(2) the line $y = \sqrt{\frac{5}{2}}x + \mathrm{K}$ be a common tangent of E and H.
Then $4\left(\mathrm{a}^2 + \mathrm{b}^2\right)$ is equal to _____. [4 marks]

Solution:

Concept:

- For $x^2/A^2 - y^2/B^2 = 1$, eccentricity is $\sqrt{1 + B^2/A^2}$. For an ellipse with $a > b$, $e^2 = 1 - b^2/a^2$.
- A line $y = mx + k$ is tangent to this ellipse when $k^2 = a^2m^2 + b^2$. For $x^2 - y^2 = 1$, the corresponding condition is $k^2 = m^2 - 1$. These follow from zero discriminant after substitution.

Step 1: Use the eccentricity condition The given hyperbola has both squared semi-axes equal to one, so $e_H = \sqrt{2}$. Hence $e_E = 1/\sqrt{2}$, and

$$1 - \frac{b^2}{a^2} = \frac{1}{2} \quad \Longrightarrow \quad b^2 = \frac{a^2}{2}.$$

This is consistent with the specified inequality $a > b > 0$.

Step 2: Impose tangency to the hyperbola Here $m = \sqrt{5/2}$. Substituting $y = mx + K$ into $x^2 - y^2 = 1$ gives $(1-m^2)x^2 - 2mKx - (K^2+1) = 0$. Its discriminant is $4(K^2+1-m^2)$. A tangent has a repeated intersection, so

$$K^2 = m^2 - 1 = \frac{5}{2} - 1 = \frac{3}{2}.$$

Both signs of K give the same axis lengths.

Step 3: Impose tangency to the ellipse For the ellipse the tangency condition now becomes

$$\frac{3}{2} = \frac{5}{2}a^2 + b^2 = \frac{5}{2}a^2 + \frac{1}{2}a^2 = 3a^2.$$

Therefore $a^2 = 1/2$ and $b^2 = 1/4$. These positive values satisfy the eccentricity and ordering conditions. Thus $a^2 + b^2 = 3/4$, and multiplication by the required factor four gives the result.

Step 4: Conclusion

$$4(a^2 + b^2) = 4\left(\frac{1}{2} + \frac{1}{4}\right) = \boxed{3}.$$

The requested value is dimensionless.

Quick Tip to Solve: For a common slope, equate the squared intercepts from the two tangent conditions; eccentricity supplies the axis ratio.

Physics – Section A: Multiple Choice Questions (80 marks)

31. The dimensions of $\left(\frac{B^2}{\mu_0}\right)$ will be :
(if μ_0 : permeability of free space and B : magnetic field) [4 marks]

(A) $\left[\mathrm{M\,L^2\,T^{-2}}\right]$
(B) $\left[\mathrm{M\,L\,T^{-2}}\right]$
(C) $\left[\mathrm{M\,L^{-1}\,T^{-2}}\right]$
(D) $\left[\mathrm{M\,L^2\,T^{-2}\,A^{-1}}\right]$

Correct Answer: (C) $\left[\mathrm{M\,L^{-1}\,T^{-2}}\right]$

Solution:

Concept:

- Dimensions are obtained by expressing a quantity in base mechanical dimensions and current. The magnetic force on a perpendicular moving charge is $F = qvB$.
- The force per unit length between parallel currents is $F/\ell = \mu_0 I_1 I_2/(2\pi r)$. Numerical constants

such as 2π have no dimensions.

Step 1: Find the magnetic-field dimensions Using $[F] = \mathrm{MLT}^{-2}$, $[q] = \mathrm{AT}$ and $[v] = \mathrm{LT}^{-1}$,

$$[B] = \frac{[F]}{[q][v]} = \frac{\mathrm{MLT}^{-2}}{(\mathrm{AT})(\mathrm{LT}^{-1})} = \mathrm{MT}^{-2}\mathrm{A}^{-1}.$$

The length cancels. This avoids confusing magnetic field with magnetic flux, which has an additional area factor.

Step 2: Find permeability and divide The parallel-current force relation gives

$$[\mu_0] = \frac{[F]}{[I]^2} = \mathrm{MLT}^{-2}\mathrm{A}^{-2}.$$

Here the distance r cancels the length in F/ℓ. Consequently,

$$\left[\frac{B^2}{\mu_0}\right] = \frac{\mathrm{M}^2\mathrm{T}^{-4}\mathrm{A}^{-2}}{\mathrm{MLT}^{-2}\mathrm{A}^{-2}} = \mathrm{ML}^{-1}\mathrm{T}^{-2}.$$

Current cancels completely and the length exponent becomes negative.

Step 3: Check the physical meaning and options The magnetic energy per unit volume is

$$u = \frac{B^2}{2\mu_0}.$$

The factor two cannot change dimensions. Energy divided by volume has dimensions $\mathrm{ML}^2\mathrm{T}^{-2}/\mathrm{L}^3 = \mathrm{ML}^{-1}\mathrm{T}^{-2}$, confirming the algebra. Option (A) is energy itself, option (B) is force, and option (D) retains an unwanted inverse-current dimension. None describes an energy density.

Step 4: Conclusion The required dimensions are $\boxed{[\mathrm{ML}^{-1}\mathrm{T}^{-2}]}$. Hence, the correct option is (C).

Quick Tip to Solve: Recognise $B^2/(2\mu_0)$ as magnetic energy density: divide the dimensions of energy by volume.

32. A NCC parade is going at a uniform speed of $9\ \mathrm{km/h}$ under a mango tree on which a monkey is sitting at a height of $19.6\ \mathrm{m}$. At any particular instant, the monkey drops a mango. A cadet will receive the mango whose distance from the tree at time of drop is: (Given $g = 9.8\ \mathrm{m/s^2}$) [4 marks]

(A) 5 m

(B) 10 m

(C) 19.8 m

(D) 24.5 m

Correct Answer: (A) 5 m

Solution:

Concept:

- A dropped object starts with zero vertical velocity. For a vertical drop through height h under constant gravity, $h = \frac{1}{2}gt^2$.
- The cadet moves horizontally at uniform speed, so the distance covered during the fall is $d = vt$. Convert the speed into metres per second before combining it with SI height and gravity.

Step 1: Calculate the fall time The mango is dropped rather than projected, so its initial vertical velocity is zero. Its fall is governed by gravity and not by the parade speed. With $h = 19.6\,\mathrm{m}$ and $g = 9.8\,\mathrm{m\,s^{-2}}$,

$$19.6 = \frac{1}{2}(9.8)t^2, \qquad t^2 = 4, \qquad t = 2\,\mathrm{s}.$$

We choose the positive root because the catch occurs after the release. The mango falls vertically below its release point.

Step 2: Find the cadet's starting distance Convert the stated speed:

$$v = 9 \times \frac{1000}{3600} = 2.5\,\mathrm{m\,s^{-1}}.$$

To arrive under the tree at the same instant as the mango, the cadet must cover the original horizontal separation in two seconds. Thus

$$d = vt = (2.5)(2) = 5\,\mathrm{m}.$$

The relevant cadet is approaching the tree; a cadet already moving away cannot intercept a mango falling at the tree.

Step 3: Reject the other distances At the given speed, a separation of $10\,\mathrm{m}$ requires four seconds, $19.8\,\mathrm{m}$ requires 7.92 seconds, and $24.5\,\mathrm{m}$ requires 9.8 seconds. Each exceeds the actual two-second fall time. The vertical height is not the distance the cadet runs, and using 9 directly as an SI speed would mix incompatible units.

Step 4: Conclusion The cadet must be $\boxed{5\,\mathrm{m}}$ from the tree when the mango is dropped. Hence, the correct option is (A).

Quick Tip to Solve: Compute the fall time from vertical motion first, then multiply it by the cadet's speed in SI units.

33. In two different experiments, an object of mass $5\ \text{kg}$ moving with a speed of $25\ \text{ms}^{-1}$ hits two different walls and comes to rest within (i) 3 second, (ii) 5 seconds, respectively. Choose the correct option out of the following : [4 marks]

(A) Impulse and average force acting on the object will be same for both the cases.

(B) Impulse will be same for both the cases but the average force will be different.

(C) Average force will be same for both the cases but the impulse will be different.

(D) Average force and impulse will be different for both the cases.

Correct Answer: (B) Impulse will be same for both the cases but the average force will be different.

Solution:

Concept:

- Impulse equals the change in momentum: $J = \Delta p = m(v_f - v_i)$. It depends on the initial and final states, not directly on stopping time.
- The average force is impulse divided by the collision duration: $F_{\text{av}} = \Delta p/\Delta t$. The sign indicates the direction relative to the initial motion.

Step 1: Find the common momentum change Choose the initial direction of motion as positive. In each experiment the object has mass $m = 5\,\text{kg}$, initial speed $v_i = 25\,\text{m}\,\text{s}^{-1}$, and final velocity zero. Therefore

$$J = 5(0 - 25) = -125\,\text{kg}\,\text{m}\,\text{s}^{-1} = -125\,\text{N}\,\text{s}.$$

Both walls produce exactly the same impulse on the object because both bring the same initial momentum to zero. The negative sign signifies an impulse opposite to the original motion.

Step 2: Compare the two average forces For the first collision the stopping time is $3\,\text{s}$, while for the second it is $5\,\text{s}$. Hence

$$F_1 = \frac{-125}{3}\,\text{N} \approx -41.67\,\text{N}, \qquad F_2 = \frac{-125}{5}\,\text{N} = -25\,\text{N}.$$

The shorter collision requires a larger average force magnitude. The instantaneous force may vary during either collision; only its average is determined by the data.

Step 3: Compare with the statements Option (A) wrongly makes the average forces equal. Options (C) and (D) wrongly make the impulses different, despite identical initial and final momenta. Option (B) correctly separates the fixed momentum change from the different rates at which that change occurs. Their average-force magnitude ratio is $5/3$, providing another direct check.

Step 4: Conclusion The impulse is the same in both cases, with magnitude $125\,\text{N}\,\text{s}$, but the average forces differ. Hence, the correct option is (B).

Quick Tip to Solve: Compare momentum endpoints to decide impulse; compare stopping durations only when deciding average force.

34. A balloon has mass of $10\ \mathrm{g}$ in air. The air escapes from the balloon at a uniform rate with velocity $4.5\ \mathrm{cm/s}$. If the balloon shrinks in $5\ \mathrm{s}$ completely. Then, the average force acting on that balloon will be (in dyne). [4 marks]

(A) 3

(B) 9

(C) 12

(D) 18

Correct Answer: (B) 9

Solution:

Concept:

- The thrust produced by a uniform escaping mass stream is $F = v_{\mathrm{rel}}\,|dm/dt|$, the rate of momentum carried away by the air.
- In the centimetre-gram-second system, $1\,\mathrm{dyne} = 1\,\mathrm{g\,cm\,s^{-2}}$. Keeping grams and centimetres throughout gives the force directly in dyne.

Step 1: Calculate the mass flow rate The stated $10\,\mathrm{g}$ of air is expelled uniformly during complete deflation in $5\,\mathrm{s}$. The magnitude of the mass flow rate is consequently

$$\left|\frac{dm}{dt}\right| = \frac{10}{5} = 2\,\mathrm{g\,s^{-1}}.$$

The mass remaining inside decreases, but a positive outflow rate is convenient for calculating the magnitude of thrust. The reaction force acts opposite to the escaping air.

Step 2: Calculate the momentum flux The air escapes with speed $v_{\mathrm{rel}} = 4.5\,\mathrm{cm\,s^{-1}}$. The force due to this outflow is

$$F = \left(2\,\mathrm{g\,s^{-1}}\right)\left(4.5\,\mathrm{cm\,s^{-1}}\right) = 9\,\mathrm{g\,cm\,s^{-2}} = 9\,\mathrm{dyne}.$$

Since the flow rate and escape speed are uniform, this momentum-flux force is also its time average. Equivalently, the outgoing momentum magnitude is $10 \times 4.5 = 45\,\mathrm{g\,cm\,s^{-1}}$; dividing by five seconds gives the same result.

Step 3: Check the units and alternatives In SI units the calculation is $(0.010/5)(0.045) = 9 \times 10^{-5}\,\mathrm{N}$. Since one newton is 10^5 dyne, this again gives nine dyne. Thus options (A), (C) and (D), giving three, twelve and eighteen dyne respectively, do not match the required momentum transfer per second. The requested force here is the reaction thrust associated with the escaping air.

Step 4: Conclusion The average thrust on the balloon has magnitude $\boxed{9\,\mathrm{dyne}}$. Hence, the correct option is (B).

Quick Tip to Solve: For a force requested in dyne, retain grams and centimetres and multiply escape speed by mass lost per second.

35. If the radius of earth shrinks by 2% while its mass remains same. The acceleration due to gravity on the earth's surface will approximately : [4 marks]

(A) decrease by 2%

(B) decrease by 4%

(C) increase by 2%

(D) increase by 4%

Correct Answer: (D) increase by 4%

Solution:

Concept:

- The gravitational acceleration at the surface of a spherical Earth is $g = GM/R^2$, where M is its mass and R its radius.
- At constant mass, the first-order fractional change obeys $\Delta g/g \simeq -2\Delta R/R$. A negative radius change therefore produces a positive change in surface gravity.

Step 1: Express the shrunken radius Let the original radius be R and the original surface acceleration be g. A decrease of two per cent means the new radius is

$$R\prime = R(1 - 0.02) = 0.98R.$$

The mass is explicitly unchanged, so GM in the numerator stays constant. The surface moves closer to the centre and the inverse-square law makes gravity stronger, immediately excluding both options that predict a decrease.

Step 2: Compute the fractional change Using the new radius in the inverse-square relation,

$$\frac{g\prime}{g} = \left(\frac{R}{R\prime}\right)^2 = \frac{1}{(0.98)^2} = \frac{1}{0.9604} \approx 1.04123.$$

Thus the exact increase for this radius change is about 4.123%. To the first order appropriate for the word approximately,

$$\frac{\Delta g}{g} \simeq -2(-0.02) = 0.04,$$

which means an increase of four per cent.

Step 3: Select the approximation The sign follows from the smaller distance to Earth's centre, and the factor two follows from the power of the radius. Option (C), an increase of two per cent, would ignore this inverse-square exponent. Options (A) and (B) have the wrong sign. Option (D) agrees with the linear approximation and is the nearest offered value to the exact percentage. No change in mass or in the gravitational constant is involved.

Step 4: Conclusion The acceleration due to gravity approximately increases by $\boxed{4\%}$. Hence, the correct option is (D).

Quick Tip to Solve: For fixed mass, reverse the sign of the radius percentage change and double it to estimate the change in surface gravity.

36. The force required to stretch a wire of cross-section $1\ \text{cm}^2$ to double its length will be : (Given Yong's modulus of the wire $= 2 \times 10^{11}\ \text{N/m}^2$) [4 marks]

(A) 1×10^7 N

(B) 1.5×10^7 N

(C) 2×10^7 N

(D) 2.5×10^7 N

Correct Answer: (C) 2×10^7 N

Solution:

Concept:

- Young's modulus relates longitudinal stress to longitudinal strain: $Y = (F/A)/(\Delta L/L)$, where A is the stated cross-sectional area and L the original length.
- An increase to double the original length means $L_f = 2L$, so the extension is $\Delta L = L$. Use the linear elastic relation requested by the problem.

Step 1: Convert the area and identify the strain The given area must be converted to square metres before using the modulus in SI units. Since $1\,\text{cm} = 10^{-2}\,\text{m}$, the area conversion is squared:

$$A = 1\,\text{cm}^2 = 10^{-4}\,\text{m}^2.$$

The strain is extension divided by original length, not final length divided by original length:

$$\frac{\Delta L}{L} = \frac{2L - L}{L} = 1.$$

Step 2: Calculate and check the stretching force Rearrange the modulus equation to isolate the applied force:

$$F = YA\frac{\Delta L}{L} = (2 \times 10^{11})(10^{-4})(1) = 2 \times 10^7\,\text{N}.$$

The dimensions are $(\text{N}\,\text{m}^{-2})(\text{m}^2) = \text{N}$. Option (A) corresponds to half this strain, while (B) and (D) give strains 0.75 and 1.25 with the stated area and modulus. None gives the required unit strain. Doubling the length is a very large strain for a real wire, but the question explicitly supplies a constant Young modulus and calls for this ideal elastic calculation.

Step 3: Conclusion The force in the stated elastic model is 2×10^7 N. Hence, the correct option is (C).

Quick Tip to Solve: When a length doubles, its extension equals its original length. Convert cm^2 to m^2 with a factor of 10^{-4}, not 10^{-2}.

37. Given below are two statements :

Statement I : The average momentum of a molecule in a sample of an ideal gas depends on temperature.

Statement II : The rms speed of oxygen molecules in a gas is v. If the temperature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become $2v$.

In the light of the above statements, choose the correct answer from the options given below :

[4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Momentum is a vector. In an ideal gas at equilibrium with no bulk flow, molecular motions are isotropic, so $\langle \mathbf{p} \rangle = \mathbf{0}$ at every temperature.
- The root-mean-square speed is $v_{\text{rms}} = \sqrt{3k_BT/m}$, where m is the mass of one moving particle. It depends on both temperature and particle mass.

Step 1: Evaluate Statement I using vector averaging For every direction of molecular motion there is an equally likely opposite direction. Thus each average velocity component vanishes:

$$\langle p_x \rangle = \langle p_y \rangle = \langle p_z \rangle = 0.$$

Consequently the average momentum vector remains zero when temperature changes. Temperature does change the mean kinetic energy and the typical momentum magnitude, but neither is the average vector momentum stated here. Statement I is therefore false.

Step 2: Evaluate Statement II and select the combination Let the original oxygen molecule have mass m and the original temperature be T. On dissociation each oxygen atom has mass $m/2$, while the new temperature is $2T$. Therefore

$$\frac{v_{\text{rms,new}}}{v_{\text{rms,old}}} = \sqrt{\frac{(2T)/(m/2)}{T/m}} = \sqrt{4} = 2.$$

The new rms speed is $2v$, so Statement II is true. Options (A) and (C) incorrectly make Statement I true, and option (B) incorrectly makes Statement II false. Only (D) has the required false-true combination.

Step 3: Conclusion Statement I is false and Statement II is true: the average momentum is zero, while the rms speed becomes $2v$. Hence, the correct option is (D).

Quick Tip to Solve: Keep vector average momentum separate from rms momentum. For rms-speed ratios, include the change in particle mass as well as the change in temperature.

38. In the wave equation

$$y = 0.5 \sin \frac{2\pi}{\lambda}(400\mathrm{t} - x)\,\mathrm{m}$$

the velocity of the wave will be: [4 marks]

(A) 200 m/s

(B) $200\sqrt{2}$ m/s

(C) 400 m/s

(D) $400\sqrt{2}$ m/s

Correct Answer: (C) 400 m/s

Solution:

Concept:

- A progressive sinusoidal wave can be written as $y = A\sin(\omega t - kx)$. Its phase speed is $u = \omega/k$, while A is the displacement amplitude.
- A point of fixed phase obeys $\omega t - kx =$ constant. Differentiating this condition gives the direction and speed of propagation.

Step 1: Identify the time and position coefficients Expand the phase of the supplied wave without changing its argument:

$$y = 0.5 \sin\left(\frac{800\pi}{\lambda}t - \frac{2\pi}{\lambda}x\right)\,\mathrm{m}.$$

Comparison with $A\sin(\omega t - kx)$ gives

$$A = 0.5\,\mathrm{m}, \qquad k = \frac{2\pi}{\lambda}, \qquad \omega = \frac{800\pi}{\lambda}.$$

The same factor $2\pi/\lambda$ multiplies both terms in the original phase.

Step 2: Determine the propagation speed and reject amplitude factors The ratio of the angular frequency to the wave number is

$$u = \frac{\omega}{k} = \frac{800\pi/\lambda}{2\pi/\lambda} = 400\,\mathrm{m\,s^{-1}}.$$

Equivalently, holding the original phase constant gives $400t - x =$ constant, hence $dx/dt = +400\,\mathrm{m\,s^{-1}}$. Thus the wave travels along positive x. The amplitude 0.5 does not multiply the phase speed, so option (A), obtained by multiplying 400 by 0.5, is incorrect. There is no rms or diagonal-component operation here, so the factors $\sqrt{2}$ in (B) and (D) have no role.

Step 3: Conclusion The velocity of propagation is $+400\,\mathrm{m\,s^{-1}}$ along the positive x-direction. Hence, the correct option is (C).

Quick Tip to Solve: For $A\sin[k(ut - x)]$, read the phase speed directly as u; the displacement amplitude A affects particle motion, not wave speed.

39. Two capacitors, each having capacitance $40\,\mu\mathrm{F}$ are connected in series. The space between one of the capacitors is filled with dielectric material of dielectric constant K such that the equivalence capacitance of the system became $24\,\mu\mathrm{F}$. The value of K will be : [4 marks]

(A) 1.5

(B) 2.5

(C) 1.2

(D) 3

Correct Answer: (A) 1.5

Solution:

Concept:

- A dielectric completely filling a capacitor multiplies its capacitance by its dielectric constant: $C \to KC$. The other capacitor remains unchanged.
- For two capacitors in series, the charge is equal on both and $1/C_{\text{eq}} = 1/C_1 + 1/C_2$. Capacitances may all be expressed in the same unit during this calculation.

Step 1: Write the equivalent capacitance after inserting the dielectric Initially each capacitor has capacitance $C = 40\,\mu\mathrm{F}$. The capacitor filled with the dielectric becomes $40K\,\mu\mathrm{F}$, whereas the other stays at $40\,\mu\mathrm{F}$. Thus

$$C_{\text{eq}} = \frac{(40)(40K)}{40 + 40K}\,\mu\mathrm{F} = \frac{40K}{1+K}\,\mu\mathrm{F}.$$

Adding the capacitances directly would apply to a parallel connection, not the series arrangement given.

Step 2: Solve for the dielectric constant and verify it Use the final equivalent capacitance:

$$24 = \frac{40K}{1+K}, \qquad 24 + 24K = 40K.$$

$$K = \frac{24}{16} = 1.5.$$

For this value the modified capacitor is $60\,\mu\mathrm{F}$, and the series combination is $40 \times 60/(40+60) = 24\,\mu\mathrm{F}$, as required. For the other offered values, $K = 2.5, 1.2, 3$, the equivalent capacitances are respectively approximately $28.57, 21.82, 30\,\mu\mathrm{F}$, so options (B), (C) and (D) do not satisfy the given condition. The answer also lies between the initial equivalent value 20 and the limiting value 40.

Step 3: Conclusion The dielectric constant is $K = 1.5$, a dimensionless quantity. Hence, the correct option is (A).

Quick Tip to Solve: For two initially equal series capacitors with a dielectric in only one, use $C_{\text{eq}} = CK/(1+K)$, not $KC/2$.

40. A wire of resistance R_1 is drawn out so that its length is increased by twice of its original length. The ratio of new resistance to original resistance is : [4 marks]

(A) 9 : 1

(B) 1 : 9

(C) 4 : 1

(D) 3 : 1

Correct Answer: (A) 9 : 1

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L length and A cross-sectional area.
- Drawing out a wire preserves its material volume: $AL = A'L'$. At unchanged temperature its resistivity is treated as constant, giving $R \propto L^2$.

Step 1: Interpret the increase and conserve volume The length is increased *by* twice its original length. Therefore the added length is $2L$ and the final length is

$$L' = L + 2L = 3L.$$

Conservation of the wire's volume gives

$$A'L' = AL \quad \Rightarrow \quad A' = \frac{AL}{3L} = \frac{A}{3}.$$

The area decreases as the wire becomes longer; keeping it unchanged would violate conservation of material.

Step 2: Form the ratio of the two resistances Using the same resistivity in the original and drawn-out wire,

$$\frac{R'}{R_1} = \frac{\rho L'/A'}{\rho L/A} = \frac{L'}{L}\frac{A}{A'} = 3 \times 3 = 9.$$

The wire is longer and thinner, and each change increases its resistance. Thus $R' : R_1 = 9 : 1$. Option (B) reverses the required ratio. Option (C) would apply if the final length were $2L$, rather than increasing by $2L$. Option (D) accounts for the longer length but misses the reduced area. Neither the original numerical resistance nor the original dimensions need to be known, since they cancel in the ratio.

Step 3: Conclusion The ratio of new resistance to original resistance is $9 : 1$. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish an increase *by* $2L$ from a final length $2L$. For a volume-preserving stretch, square the final-to-original length ratio.

41. The current sensitivity of a galvanometer can be increased by :

(A) decreasing the number of turns

(B) increasing the magnetic field

(C) decreasing the area of the coil

(D) decreasing the torsional constant of the spring

Choose the most appropriate answer from the options given below : [4 marks]

(A) (B) and (C) only

(B) (C) and (D) only

(C) (A) and (C) only

(D) (B) and (D) only

Correct Answer: (D) (B) and (D) only

Solution:

Concept:

- In a radial magnetic field, a moving coil of N turns and area A carries current I and experiences magnetic torque $NIAB$.
- The restoring torque of the suspension is $\kappa\theta$, with torsional constant κ and deflection θ. Current sensitivity means deflection per unit current, θ/I.

Step 1: Derive the sensitivity relation At the equilibrium deflection, magnetic torque balances the restoring torque:

$$NIAB = \kappa\theta.$$

Dividing by $I\kappa$ yields

$$S_I = \frac{\theta}{I} = \frac{NAB}{\kappa}.$$

This formula shows directly which changes produce a larger deflection for the same current. It is current sensitivity rather than voltage sensitivity; the resistance of the coil is not needed for this comparison.

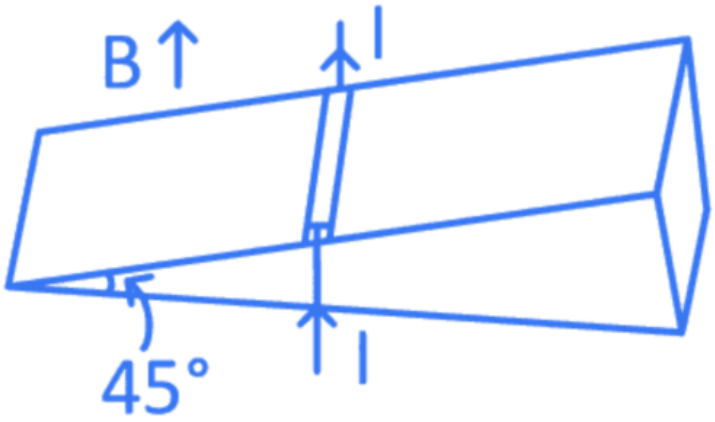
Step 2: Test all four proposed changes and the printed combinations Decreasing the number of turns (A) reduces N and hence reduces sensitivity. Increasing the magnetic field (B) increases B, increasing sensitivity. Decreasing the coil area (C) reduces A, reducing sensitivity. Decreasing the torsional constant (D) reduces the denominator, increasing sensitivity because the spring offers less restoring torque for a given angle. Therefore only (B) and (D) are suitable. The printed combinations (B,C), (C,D) and (A,C) each include at least one sensitivity-reducing change. The combination (B,D) includes exactly the two sensitivity-increasing changes, and it is option (D).

Step 3: Conclusion Increasing the magnetic field and decreasing the torsional constant increase current sensitivity. Hence, the correct option is (D).

Quick Tip to Solve: Use $S_I = NAB/\kappa$: increasing a numerator factor raises sensitivity, whereas increasing the spring's torsional constant lowers it.

42. As shown in the figure, a metallic rod of linear density $0.45\ \text{kg m}^{-1}$ is lying horizontally on a smooth inclined plane which makes an angle of 45° with the horizontal. The minimum current flowing in the rod required to keep it stationary, when $0.15\ \text{T}$ magnetic field is acting on it in the vertical upward direction, will be :

{Use $g = 10\ \text{m/s}^2$} [4 marks]



(A) 30 A

(B) 15 A

(C) 10 A

(D) 3 A

Correct Answer: (A) 30 A

Solution:

Concept:

- A straight current-carrying rod of length ℓ, perpendicular to a uniform field B, experiences force $F_B = IB\ell$. Its direction follows the cross product of current direction and magnetic field.
- For a smooth plane inclined by α, weight has downslope component $mg\sin\alpha$. The plane's normal reaction has no component along the incline.

Step 1: Resolve forces along the smooth incline The rod lies horizontally across the incline while the magnetic field is vertically upward, so the rod is perpendicular to the field. The magnetic force is horizontal. Choose the current direction so this force has an upslope component. If the plane angle is α, the horizontal magnetic force projects onto the plane as $IB\ell\cos\alpha$. With linear mass density μ, the rod's mass is $m = \mu\ell$; its downslope weight is $\mu\ell g\sin\alpha$. There is no friction to supply a missing tangential force.

Step 2: Balance tangential forces and calculate the threshold current Equilibrium along the plane requires

$$IB\ell\cos\alpha = \mu\ell g\sin\alpha.$$

Cancel the length and solve:

$$I = \frac{\mu g}{B}\tan\alpha = \frac{0.45\times 10}{0.15}\tan 45^\circ = 30\ \text{A}.$$

The normal reaction adjusts to balance the perpendicular components; it does not change this tangential condition. Currents below 30 A leave a net downslope force, whereas the opposite current direction would reinforce the sliding tendency. Options (B), (C) and (D), namely 15 A, 10 A and 3 A, provide only one-half, one-third and one-tenth of the required upslope magnetic component, respectively. They therefore cannot keep the rod stationary on the smooth plane.

Step 3: Conclusion The minimum current, with the direction chosen to oppose sliding, is $30\,\text{A}$. Hence, the correct option is (A).

Quick Tip to Solve: When the field is vertical, the magnetic force on a horizontal rod is horizontal. Project it onto the incline before equating it to the downslope weight.

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43. The equation of current in a purely inductive circuit is $5\sin(49\,\pi t - 30^\circ)$. If the inductance is $30\,\text{mH}$ then the equation for the voltage across the inductor, will be :

{ **Let** $\pi = \frac{22}{7}$ } [4 marks]

(A) $1.47\sin(49\pi t - 30^\circ)$

(B) $1.47\sin(49\pi t + 60^\circ)$

(C) $23.1\sin(49\pi t - 30^\circ)$

(D) $23.1\sin(49\pi t + 60^\circ)$

Correct Answer: (D) $23.1\sin(49\pi t + 60^\circ)$

Solution:

Concept:

- For an ideal inductor, the applied potential difference obeys $v = L\,di/dt$. Differentiating a sinusoidal current advances its phase by 90°.
- The voltage amplitude is $V_0 = \omega L I_0$, where ω is angular frequency, L is inductance and I_0 is peak current. Inductance must be expressed in henry.

Step 1: Read the current and convert the inductance The given current is $i = 5\sin(49\pi t - 30^\circ)$. Therefore $I_0 = 5\,\text{A}$, $\omega = 49\pi\,\text{rad}\,\text{s}^{-1}$, and $L = 30\,\text{mH} = 0.030\,\text{H}$. The coefficient of time, rather than the initial phase, sets the frequency.

Step 2: Differentiate and calculate the peak voltage Using the chain rule, the derivative of the sine contains its angular frequency. Thus

$$v = 0.030(5)(49\pi)\cos(49\pi t - 30^\circ).$$

With the specified $\pi = 22/7$, the amplitude is

$$V_0 = 0.030 \times 5 \times 49 \times \frac{22}{7} = 23.1\,\text{V}.$$

The quantity ωL alone is the inductive reactance, so it must still be multiplied by the current amplitude.

Step 3: Express the voltage in the required sine form Use $\cos\theta = \sin(\theta + 90^\circ)$. The voltage phase is consequently $-30^\circ + 90^\circ = 60^\circ$. Options A and B have the wrong amplitude; option C keeps the

current phase instead of advancing it. Only D combines the required amplitude and phase.

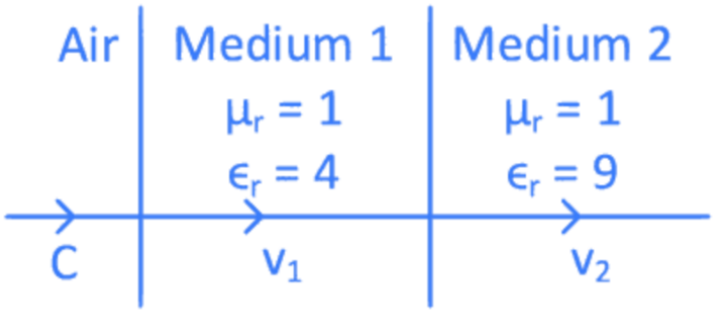
Step 4: Conclusion The voltage is $v = 23.1\sin(49\pi t + 60^\circ)$ V. Hence, the correct option is (D).

Quick Tip to Solve: For a pure inductor, calculate $\omega L I_0$ and add 90° to the current phase; these two checks select the voltage independently.

---

44. As shown in the figure, after passing through the medium 1 . The speed of light v_2 in medium 2 will be :

(**Given** $c = 3 \times 10^8$ ms^{-1}) [4 marks]



(A) 1.0×10^8 ms^{-1}

(B) 0.5×10^8 ms^{-1}

(C) 1.5×10^8 ms^{-1}

(D) 3.0×10^8 ms^{-1}

Correct Answer: (A) 1.0×10^8 ms^{-1}

Solution:

Concept:

- The speed of an electromagnetic wave in a medium is $v = 1/\sqrt{\mu\varepsilon}$. In terms of relative permeability and permittivity this becomes $v = c/\sqrt{\mu_r\varepsilon_r}$.
- The propagation speed is a property of the medium currently occupied. Passing through a previous medium changes the wavelength there, but does not multiply the reductions in speed of successive media.

Step 1: Read the electromagnetic properties of medium 2 The figure gives $\mu_r = 1$ and $\varepsilon_r = 9$ for medium 2. For medium 1 the corresponding quantities are 1 and 4. These are relative material constants, so the refractive indices are dimensionless. The required speed is in medium 2, and the vacuum speed supplied is $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$.

Step 2: Calculate its refractive index and wave speed The refractive index of medium 2 is

$$n_2 = \sqrt{\mu_{r2}\varepsilon_{r2}} = \sqrt{1 \times 9} = 3.$$

Therefore

$$v_2 = \frac{c}{n_2} = \frac{3 \times 10^8}{3} = 1.0 \times 10^8\,\mathrm{m\,s^{-1}}.$$

The frequency stays fixed at the boundaries; the wavelength adapts so that $v = f\lambda$ holds in each medium.

Step 3: Check the alternatives against the medium labels Medium 1 has $n_1 = 2$, giving $v_1 = 1.5 \times 10^8\,\mathrm{m\,s^{-1}}$; that is option C and is not the requested medium. Option B incorrectly applies both refractive indices as successive independent reductions from c. Option D is the vacuum speed and ignores the non-unit permittivity of medium 2.

Step 4: Conclusion The speed in medium 2 is $v_2 = 1.0 \times 10^8\,\mathrm{m\,s^{-1}}$. Hence, the correct option is (A).

Quick Tip to Solve: Use $n = \sqrt{\mu_r \varepsilon_r}$ for the destination medium. Do not divide by every refractive index along the ray path.

---

45. In normal adjustment, for a refracting telescope, the distance between objective and eye piece is 30 cm. The focal length of the objective, when the angular magnification of the telescope is 2 , will be : [4 marks]

(A) 20 cm

(B) 30 cm

(C) 10 cm

(D) 15 cm

Correct Answer: (A) 20 cm

Solution:

Concept:

- In normal adjustment, the final image of an astronomical telescope is at infinity. The real image formed by the objective lies at the first focal plane of the eyepiece.
- The lens separation is $L = f_o + f_e$, and the magnitude of angular magnification is $|M| = f_o/f_e$, with f_o and f_e the positive focal lengths of the objective and eyepiece.

Step 1: Write the normal-adjustment relations The separation is $30\,\mathrm{cm}$, so $f_o + f_e = 30\,\mathrm{cm}$. The given angular magnification is its magnitude, 2. The image inversion contributes a negative sign to signed magnification but does not make either focal length negative for the two converging lenses. Hence

$$\frac{f_o}{f_e} = 2, \qquad f_o = 2f_e.$$

Step 2: Solve the sum and ratio together Substituting the ratio into the separation relation gives

$$2f_e + f_e = 30\,\mathrm{cm}, \qquad f_e = 10\,\mathrm{cm}.$$

The objective focal length is then

$$f_o = 2(10) = 20\,\mathrm{cm}.$$

As a check, these lengths add to $30\,\mathrm{cm}$ and have the required ratio $20/10 = 2$.

Step 3: Identify why the remaining focal lengths fail Option C, $10\,\mathrm{cm}$, is the eyepiece focal length rather than the objective focal length. Option B uses the full separation as the objective focal length, leaving no positive focal length for the eyepiece. Option D would give two equal $15\,\mathrm{cm}$ focal lengths and therefore unit magnification. Thus each of those alternatives violates one of the two independent conditions.

Step 4: Conclusion The objective has focal length $f_o = 20\,\mathrm{cm}$. Hence, the correct option is (A).

Quick Tip to Solve: For normal adjustment, combine the focal-length sum with their ratio: $f_o = |M|L/(|M|+1)$. Check that the longer focal length belongs to the objective.

---

46. The equation $\lambda = \frac{1.227}{x}\ \mathrm{nm}$ can be used to find the de-Brogli wavelength of an electron. In this equation x stands for :

Where

m = **mass of electron**

P = **momentum of electron**

K = **Kinetic energy of electron**

V = **Accelerating potential in volts for electron** [4 marks]

(A) $\sqrt{\mathrm{mK}}$

(B) $\sqrt{\mathrm{P}}$

(C) $\sqrt{\mathrm{K}}$

(D) $\sqrt{\mathrm{V}}$

Correct Answer: (D) $\sqrt{\mathrm{V}}$

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$. A nonrelativistic electron accelerated from rest through a potential difference V gains kinetic energy $K = eV$.
- Using $K = p^2/(2m)$ gives $p = \sqrt{2meV}$, so $\lambda = h/\sqrt{2meV}$. The numerical coefficient depends on the units in which voltage and wavelength are expressed.

Step 1: Express momentum through the accelerating potential The electric field does work eV on the electron. Setting this equal to its nonrelativistic kinetic energy gives

$$\frac{p^2}{2m} = eV, \qquad p = \sqrt{2meV}.$$

Here e is the positive magnitude of electronic charge; the electron charge sign does not make the gained kinetic energy negative.

Step 2: Evaluate the coefficient in nanometres Substituting $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$, $m = 9.11 \times 10^{-31}\,\mathrm{kg}$

and $e = 1.602 \times 10^{-19}$ C, one obtains

$$\lambda = \frac{1.227 \times 10^{-9}}{\sqrt{V}} \text{ m} = \frac{1.227}{\sqrt{V}} \text{ nm},$$

where V is the numerical accelerating potential measured in volts. Comparing this expression with the given equation identifies the denominator.

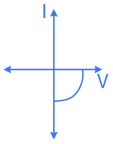
Step 3: Distinguish the other physical quantities Option A contains $\sqrt{mK}$; the corresponding exact expression would have coefficient $h/\sqrt{2}$, with consistent SI units, rather than the quoted voltage-based coefficient. Option B predicts inverse square-root momentum dependence, whereas de Broglie wavelength is inversely proportional to momentum itself. Option C misses the mass and unit conversion factors needed when kinetic energy is used as the independent quantity.

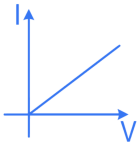
Step 4: Conclusion In the quoted numerical formula, $x = \sqrt{V}$, with V in volts. Hence, the correct option is (D).

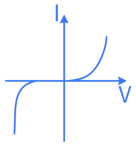
Quick Tip to Solve: The constants 1.227 in nanometres and 12.27 in angstroms both pair with $\sqrt{V}$ for a nonrelativistic electron accelerated from rest.

47. Identify the solar cell characteristics from the following options : [4 marks]

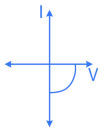
(A) 

(B) 

(C) 

(D) 

Correct Answer: (B)



Solution:

Concept:

- A solar cell is an illuminated p-n junction that converts radiant energy into electrical energy. Its useful operating region supplies power to an external load.
- With the usual diode sign convention, illumination shifts the diode current downward: $I = I_0[\exp(eV/kT) - 1] - I_{\text{ph}}$. Here I_{ph} is the positive photocurrent magnitude.

Step 1: Locate the short-circuit and open-circuit intercepts At zero terminal voltage the load is a short circuit. The diode contribution vanishes in the ideal model and $I = -I_{\text{ph}}$, so the current intercept lies below the origin. At open circuit the net current is zero; a positive terminal voltage V_{oc} balances the photocurrent. These intercepts place the useful characteristic between the negative current axis and the positive voltage axis.

Step 2: Identify the power-generating quadrant For voltages between zero and V_{oc}, the current in the diode sign convention remains negative. Thus $VI < 0$: the device delivers electrical power rather than absorbing it. The curve rises towards the positive voltage-axis intercept because the forward diode current increases with voltage. This is the fourth-quadrant curve shown in option B.

Step 3: Compare all four plotted curves Option A is an unshifted forward diode characteristic in the first quadrant, with no negative short-circuit current. Option C is a straight line through the origin, characteristic of an ohmic resistor. Option D is a dark diode-type curve with forward conduction and reverse breakdown, rather than an illuminated junction delivering power. Only B shows a nonzero negative current at zero voltage together with a positive open-circuit voltage.

Step 4: Conclusion The solar cell characteristic is the fourth-quadrant current-voltage curve. Hence, the correct option is (B).

Quick Tip to Solve: Find the two intercepts first: solar-cell short-circuit current is below the origin and open-circuit voltage is positive under the diode sign convention.

48. A Carnot engine has efficiency of 50%. If the temperature of sink is reduced by $40^{\circ}\mathrm{C}$, its efficiency increases by 30%. The temperature of the source will be: [4 marks]

(A) 166.7 K

(B) 255.1 K

(C) 266.7 K

(D) 367.7 K

Correct Answer: (C) 266.7 K

Solution:

Concept:

- The efficiency of a Carnot engine is $\eta = 1 - T_c/T_h$, where source and sink temperatures T_h and T_c must be absolute temperatures in kelvin.
- With the source temperature fixed, lowering the sink by ΔT increases the efficiency by $\Delta\eta = \Delta T/T_h$. A temperature difference of $40^{\circ}\mathrm{C}$ equals $40\,\mathrm{K}$.

Step 1: Apply the stated percentage increase The initial efficiency is $\eta_1 = 0.50$. An increase by 30% means an increase equal to 30% of this initial value, so

$$\eta_2 = 0.50(1 + 0.30) = 0.65, \qquad \eta_2 - \eta_1 = 0.15.$$

This differs from an increase of thirty percentage points. The relative-percentage reading is also the one consistent with the offered source temperatures.

Step 2: Subtract the two Carnot relations Initially $\eta_1 = 1 - T_c/T_h$; after cooling the sink,

$$\eta_2 = 1 - \frac{T_c - 40}{T_h}.$$

Subtracting eliminates the unknown initial sink temperature:

$$0.65 - 0.50 = \frac{40}{T_h}, \qquad T_h = \frac{40}{0.15} = 266.7\,\text{K}.$$

There is no Celsius-to-kelvin offset to add to a temperature difference.

Step 3: Verify the temperatures and reject the alternatives The original efficiency gives $T_c = 0.50T_h \approx 133.3\,\text{K}$. The cooled sink is approximately $93.3\,\text{K}$, for which $1 - 93.3/266.7 \approx 0.65$, as required. The other offered source temperatures, 166.7, 255.1 and $367.7\,\text{K}$, give efficiency increases $40/T_h$ different from 0.15, so none meets both conditions.

Step 4: Conclusion The source temperature is approximately $266.7\,\text{K}$. Hence, the correct option is (C).

Quick Tip to Solve: For a fixed Carnot source, subtract efficiencies to get $T_h = \Delta T_c/\Delta\eta$. Convert a relative percentage increase to a change in efficiency first.

49. The half life period of a radioactive substance is 60 days. The time taken for $\frac{7}{8}$th of its original mass to disintegrate will be : [4 marks]

(A) 120 days

(B) 130 days

(C) 180 days

(D) 20 days

Correct Answer: (C) 180 days

Solution:

Concept:

- Radioactive decay reduces the number of undecayed nuclei according to $N = N_0(1/2)^{t/T_{1/2}}$. The mass of the remaining original radioactive substance follows the same fraction.
- The fraction disintegrated and the fraction remaining add to one. Half-life counts the time for the remaining undecayed sample to halve, not the time for a fixed amount of the original sample to disappear.

Step 1: Convert the disintegrated fraction into the remaining fraction The requested time is when $7/8$ of the original mass has disintegrated. The fraction of the original radioactive substance still undecayed is therefore

$$\frac{N}{N_0} = 1 - \frac{7}{8} = \frac{1}{8}.$$

The material produced by decay is not included as surviving original radioactive nuclei. This distinction is essential before applying the exponential law.

Step 2: Count the required half-lives Each half-life is 60 days. After one, two and three half-lives, the

undecayed fractions are respectively $1/2$, $1/4$ and $1/8$. Equivalently,

$$\left(\frac{1}{2}\right)^{t/60} = \frac{1}{8} = \left(\frac{1}{2}\right)^{3},$$

so equality of the exponents gives

$$t = 3 \times 60 = 180\,\text{days}.$$

The decay rate falls as the number of surviving nuclei falls, so a linear fraction-of-time calculation would be invalid.

Step 3: Test the other proposed times At 120 days only two half-lives have elapsed: $1/4$ remains and $3/4$ has disintegrated, so A is too small. At 130 days the remaining fraction is $2^{-130/60}$, still larger than $1/8$, so B is too small. At 20 days less than one half-life has passed, so D cannot account for loss of most of the original sample.

Step 4: Conclusion The required time is $180\,\text{days}$, corresponding to three half-lives. Hence, the correct option is (C).

Quick Tip to Solve: Always subtract the decayed fraction from one. If the remainder is $1/2^n$, the elapsed time is exactly n half-lives.

50. In the case of amplitude modulation to avoid distortion the modulation index (μ) should be :

[4 marks]

(A) $\mu \leq 1$

(B) $\mu \geq 1$

(C) $\mu = 2$

(D) $\mu = 0$

Correct Answer: (A) $\mu \leq 1$

Solution:

Concept:

- The modulation index is the ratio of message amplitude to carrier amplitude: $\mu = A_m/A_c$. Its magnitude is non-negative.
- For sinusoidal amplitude modulation the envelope factor is $1 + \mu\cos\omega_m t$. An envelope detector reproduces the message without envelope reversal only when this factor never becomes negative.

Step 1: Apply the envelope condition The modulated signal can be written as

$$s(t) = A_c(1 + \mu\cos\omega_m t)\cos\omega_c t.$$

Since the cosine of the message phase ranges from -1 to $+1$, the smallest envelope factor is $1 - \mu$. Requiring this minimum to be non-negative gives

$$1 - \mu \geq 0 \quad \Longrightarrow \quad \mu \leq 1.$$

At the limiting value $\mu = 1$, the envelope just touches zero; for a value greater than one it crosses zero and envelope detection gives distortion.

Step 2: Compare the four alternatives Option (A) states the required upper bound. Option (B) permits all values above one, so it includes overmodulation and cannot be the condition. Option (C), $\mu = 2$, is specifically an overmodulated case. Option (D), $\mu = 0$, gives an unmodulated carrier with no message information; although it lies in the allowed interval, it is not the general condition for undistorted amplitude modulation. For useful modulation the range is $0 < \mu \leq 1$.

Step 3: Conclusion The modulation index must satisfy $\boxed{\mu \leq 1}$, with $\mu \geq 0$ by definition. Hence, the correct option is (A).

Quick Tip to Solve: Check the minimum envelope factor $1 - \mu$. A negative minimum signals overmodulation, so the upper limit is one.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. If the projection of $2\hat{i} + 4\hat{j} - 2\hat{k}$ on $\hat{i} + 2\hat{j} + \alpha\hat{k}$ is zero. Then, the value of α will be _____.

[4 marks]

Solution:

Concept:

- The scalar projection of a vector $\mathbf{a}$ on a nonzero vector $\mathbf{b}$ is $\mathbf{a} \cdot \mathbf{b}/|\mathbf{b}|$. A zero projection means the vectors are perpendicular.
- In Cartesian components the dot product is the sum of the products of corresponding components. The unit vectors satisfy $\hat{i} \cdot \hat{j} = 0$ and $\hat{i} \cdot \hat{i} = 1$.

Step 1: Translate zero projection into an equation Put $\mathbf{a} = 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\mathbf{b} = \hat{i} + 2\hat{j} + \alpha\hat{k}$. The second vector is never the zero vector because its first component is one. Thus its magnitude $\sqrt{5 + \alpha^2}$ is positive and the zero-projection condition is

$$\frac{\mathbf{a} \cdot \mathbf{b}}{\sqrt{5 + \alpha^2}} = 0 \quad \Longrightarrow \quad \mathbf{a} \cdot \mathbf{b} = 0.$$

This avoids unnecessarily evaluating the magnitude.

Step 2: Evaluate the components and solve Multiply corresponding Cartesian components, keeping the negative sign of the third component:

$$2(1) + 4(2) + (-2)\alpha = 0.$$

Consequently,

$$10 - 2\alpha = 0, \qquad \alpha = 5.$$

As a check, substituting this value makes the dot product $2 + 8 - 10 = 0$, while $|\mathbf{b}| = \sqrt{30} \neq 0$. Hence the original projection is indeed zero, rather than an undefined expression. The parameter is a

dimensionless coefficient of a unit vector, so no physical unit is attached to its value.

Step 3: Conclusion The required dimensionless value is $\boxed{\alpha = 5}$.

Quick Tip to Solve: For zero projection on a nonzero vector, equate the dot product directly to zero; no square root or trigonometry is needed.

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52. In a Young's double slit experiment, a laser light of 560 nm produces an interference pattern with consecutive bright fringes' separation of 7.2 mm. Now another light is used to produce an interference pattern with consecutive bright fringes' separation of 8.1 mm. The wavelength of second light is _____ nm. [4 marks]

Solution:

Concept:

- The separation of consecutive bright fringes is the fringe width $\beta = \lambda D/d$, where D is the slit-to-screen distance and d the slit separation.
- If the same apparatus is used with a different light source, D and d remain unchanged. Therefore fringe width is directly proportional to wavelength.

Step 1: Eliminate the fixed apparatus dimensions For the original light, $\beta_1 = 7.2\,\text{mm}$ and $\lambda_1 = 560\,\text{nm}$. For the second light, $\beta_2 = 8.1\,\text{mm}$. Write the two interference relations and divide:

$$\frac{\beta_2}{\beta_1} = \frac{\lambda_2 D/d}{\lambda_1 D/d} = \frac{\lambda_2}{\lambda_1}.$$

Both fringe widths are measured in millimetres, so their ratio is dimensionless; conversion to metres is unnecessary when taking this ratio.

Step 2: Calculate the new wavelength and check its trend Rearrange the proportionality and substitute the values:

$$\lambda_2 = \lambda_1 \frac{\beta_2}{\beta_1} = 560 \frac{8.1}{7.2}\,\text{nm}.$$

Since $8.1/7.2 = 81/72 = 9/8$,

$$\lambda_2 = 560 \left(\frac{9}{8}\right)\,\text{nm} = 630\,\text{nm}.$$

The fringe width has increased by a factor of 1.125, and the wavelength increases by the same factor, as required. This is the separation of adjacent bright fringes, so there is no additional factor of two from bright-to-dark spacing.

Step 3: Conclusion The second light has wavelength $\boxed{630\,\text{nm}}$. The numerical entry in nm is $\boxed{630}$.

Quick Tip to Solve: For unchanged slit geometry, use $\lambda_2/\lambda_1 = \beta_2/\beta_1$; keep both fringe widths in the same unit.

---

53. The frequencies at which the current amplitude in an LCR series circuit becomes $\frac{1}{\sqrt{2}}$ times its maximum value, are $212\,\mathrm{rad\,s^{-1}}$ and $232\,\mathrm{rad\,s^{-1}}$. The value of resistance in the circuit is $R = 5\,\Omega$. The self inductance in the circuit is _____ mH. [4 marks]

Solution:

Concept:

- The current amplitude is $I = V_0/\sqrt{R^2 + (\omega L - 1/(\omega C))^2}$. At resonance the reactance vanishes and $I_{\max} = V_0/R$.
- The half-power frequencies have current amplitude $I_{\max}/\sqrt{2}$. The exact angular-frequency bandwidth of a series LCR circuit is $\omega_2 - \omega_1 = R/L$.

Step 1: Identify the two half-power conditions At the specified current amplitude, the impedance magnitude must be $\sqrt{2}R$, so

$$\left|\omega L - \frac{1}{\omega C}\right| = R.$$

The lower frequency has negative net reactance and the upper has positive net reactance. Thus

$$L\omega_1^2 + R\omega_1 - \frac{1}{C} = 0, \qquad L\omega_2^2 - R\omega_2 - \frac{1}{C} = 0.$$

Subtracting gives

$$(\omega_1 + \omega_2)\{L(\omega_2 - \omega_1) - R\} = 0.$$

Both frequencies are positive, so $L(\omega_2 - \omega_1) = R$.

Step 2: Substitute the angular-frequency bandwidth The values are already angular frequencies because their unit is $\mathrm{rad\,s^{-1}}$. Therefore

$$\Delta\omega = 232 - 212 = 20\,\mathrm{rad\,s^{-1}}, \qquad L = \frac{5}{20}\,\mathrm{H} = 0.25\,\mathrm{H}.$$

Converting henries to millihenries,

$$L = 0.25 \times 1000\,\mathrm{mH} = 250\,\mathrm{mH}.$$

No factor of 2π is used: it would be needed only if the given frequencies were ordinary frequencies in hertz.

Step 3: Conclusion The self-inductance is $\boxed{250\,\mathrm{mH}}$, so the numerical entry is $\boxed{250}$.

Quick Tip to Solve: Use $\Delta\omega = R/L$ for rad/s, but $\Delta f = R/(2\pi L)$ for Hz. Read the frequency unit before substituting.

54. Two electric dipoles of dipole moments $1.2 \times 10^{-30}\,\text{Cm}$ and $2.4 \times 10^{-30}\,\text{Cm}$ are placed in two different uniform electric fields of strengths $5 \times 10^4\,\text{NC}^{-1}$ and $15 \times 10^4\,\text{NC}^{-1}$ respectively. The ratio of maximum torque experienced by the electric dipoles will be $\frac{1}{x}$. The value of x is _____. [4 marks]

Solution:

Concept:

- An electric dipole of moment $\mathbf{p}$ in a uniform electric field $\mathbf{E}$ experiences torque $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$. Its magnitude is $pE\sin\theta$.
- The maximum torque occurs at $\theta = 90^\circ$, so $\tau_{\max} = pE$. Both dipoles must be compared using their own field strengths.

Step 1: Calculate the maximum torque for each dipole For the first dipole, substitute $p_1 = 1.2 \times 10^{-30}\,\text{C m}$ and $E_1 = 5 \times 10^4\,\text{N C}^{-1}$:

$$\tau_{1,\max} = p_1 E_1 = 6.0 \times 10^{-26}\,\text{N m}.$$

For the second, $p_2 = 2.4 \times 10^{-30}\,\text{C m}$ and $E_2 = 15 \times 10^4\,\text{N C}^{-1}$, giving

$$\tau_{2,\max} = p_2 E_2 = 36 \times 10^{-26}\,\text{N m}.$$

The coulomb units cancel, leaving the correct torque unit.

Step 2: Form the ratio in the printed order The requested ratio is the first maximum torque divided by the second:

$$\frac{\tau_{1,\max}}{\tau_{2,\max}} = \frac{6.0 \times 10^{-26}}{36 \times 10^{-26}} = \frac{1}{6}.$$

Comparing this with the given form $1/x$ gives $x = 6$. The second dipole has twice the dipole moment and three times the field strength, so its maximum torque is six times the first. This independently checks both the ratio and its ordering; reversing the order would give six rather than $1/6$.

Step 3: Conclusion The torque ratio is $1 : 6$, and the required dimensionless value is $\boxed{x = 6}$.

Quick Tip to Solve: For maximum dipole torque compare the products pE. The second moment doubles and its field triples, giving a sixfold torque.

55. The diameter of an air bubble which was initially $2\ \text{mm}$, rises steadily through a solution of density $1750\ \text{kg m}^{-3}$ at the rate of $0.35\,\text{cms}^{-1}$. The coefficient of viscosity of the solution is _____ poise (in nearest integer). (the density of air is negligible). [4 marks]

Solution:

Concept:

- At steady terminal speed the upward buoyant force balances viscous drag and weight. Neglecting air density makes the bubble weight negligible compared with buoyancy.
- Using Stokes drag $F_d = 6\pi\eta r v$ and buoyancy $F_b = \frac{4}{3}\pi r^3 \rho g$ gives $\eta = 2r^2\rho g/(9v)$, where r is the bubble radius.

Step 1: Convert the radius and terminal speed to SI units The stated diameter is $2\,\text{mm}$, so the radius to use in the drag relation is

$$r = 1\,\text{mm} = 10^{-3}\,\text{m}.$$

The steady rise speed is

$$v = 0.35\,\text{cm}\,\text{s}^{-1} = 3.5 \times 10^{-3}\,\text{m}\,\text{s}^{-1}.$$

The solution density is $\rho = 1750\,\text{kg}\,\text{m}^{-3}$. At terminal speed the acceleration is zero; buoyancy is balanced by the downward viscous force, rather than equated to a nonzero resultant force.

Step 2: Evaluate the viscosity and convert to poise With $g = 9.8\,\text{m}\,\text{s}^{-2}$,

$$\eta = \frac{2(10^{-3})^2(1750)(9.8)}{9(3.5 \times 10^{-3})}\,\text{Pa}\,\text{s} = 1.0889\,\text{Pa}\,\text{s}.$$

Since $1\,\text{Pa}\,\text{s} = 10\,\text{poise}$,

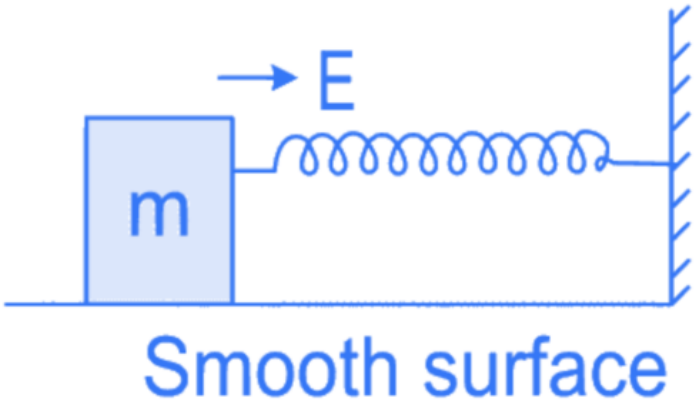
$$\eta = 10.889\,\text{poise} \approx 11\,\text{poise}.$$

Even using the common approximation $g = 10\,\text{m}\,\text{s}^{-2}$ gives $11.11\,\text{poise}$, which rounds to the same integer. The factor of ten in the unit conversion must be applied after the SI calculation.

Step 3: Conclusion The coefficient of viscosity, to the nearest integer, is $\boxed{11\,\text{poise}}$. The requested numerical entry is $\boxed{11}$.

Quick Tip to Solve: Use the radius, half the given diameter, in Stokes law; then multiply Pa s by ten to convert the result into poise.

56. A block of mass 'm' (as shown in figure) moving with kinetic energy E compresses a spring through a distance $25\ \text{cm}$ when, its speed is halved. The value of spring constant of used spring will be $\text{nE}\ \text{Nm}^{-1}$ for $\text{n} =$ _____. [4 marks]



Solution:

Concept:

- On the smooth surface there is no frictional loss. The decrease in the kinetic energy of the block equals the increase in elastic potential energy $\frac{1}{2}kx^2$ of the spring.

- Kinetic energy is proportional to the square of speed: if the speed is halved, the kinetic energy becomes one quarter of its initial value, not one half.

Step 1: Find how much kinetic energy goes into the spring Let the initial speed be v. The given initial kinetic energy is

$$E = \frac{1}{2}mv^2.$$

At the instant when the spring compression is $25\,\text{cm}$, the block speed is $v/2$, so its remaining kinetic energy is

$$K_f = \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{E}{4}.$$

The energy transferred to the initially uncompressed spring is therefore

$$U_s = E - \frac{E}{4} = \frac{3E}{4}.$$

The block is still moving at this instant; equating all of E to spring energy would incorrectly treat this compression as the maximum compression.

Step 2: Use the measured compression to obtain the spring constant Convert the compression to metres: $x = 25\,\text{cm} = 0.25\,\text{m}$. Equate elastic energy to the calculated loss of kinetic energy:

$$\frac{1}{2}k(0.25)^2 = \frac{3E}{4}.$$

Since $(0.25)^2 = 1/16$, this gives

$$\frac{k}{32} = \frac{3E}{4}, \qquad k = 24E.$$

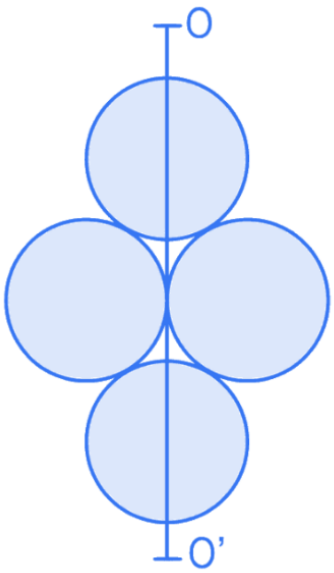
When the numerical value of E is expressed in joules, $k = 24E\,\text{N m}^{-1}$. Equivalently $k/E = 24\,\text{m}^{-2}$, which confirms the required physical dimensions.

Step 3: Conclusion Comparing $k = 24E\,\text{N m}^{-1}$ with the specified form gives the required coefficient $\boxed{n = 24}$.

Quick Tip to Solve: Halving speed leaves one quarter of the kinetic energy; only the lost three quarters is stored in the spring at the stated compression.

57. Four identical discs each of mass 'M' and diameter 'a' are arranged in a small plane as shown in figure. If the moment of inertia of the system about OO' is $\frac{x}{4}Ma^2$. Then, the value of x will be _____.

[4 marks]



Solution:

Concept:

- For a uniform thin disc of mass M and radius r, the moment of inertia about any diameter in its plane is $I_d = Mr^2/4$. The symmetry of the disc and the perpendicular-axis theorem give this result.
- For an axis parallel to that diameter at perpendicular distance d, the parallel-axis theorem gives $I = I_d + Md^2$. Moments of inertia about the same axis add for distinct discs.

Step 1: Read the geometry of the four discs The diameter of each disc is a, so its radius is $r = a/2$. The vertical axis OO' passes through the centres of the upper and lower discs. The centres of the left and right discs lie at perpendicular distance $a/2$ from this axis. Displacements along the axis do not contribute to the parallel-axis term.

Step 2: Evaluate the contributions of the two pairs For the upper and lower discs the axis is a diameter, giving a combined contribution

$$I_{\text{vertical}} = 2\left(\frac{M(a/2)^2}{4}\right) = \frac{Ma^2}{8}.$$

For each side disc the corresponding central diameter is parallel to the given axis. Therefore

$$I_{\text{side}} = \frac{Ma^2}{16} + M\left(\frac{a}{2}\right)^2 = \frac{5Ma^2}{16}.$$

Both side discs together contribute $5Ma^2/8$.

Step 3: Add and compare the specified coefficient The total moment of inertia is

$$I = \frac{Ma^2}{8} + \frac{5Ma^2}{8} = \frac{3Ma^2}{4}.$$

Comparing with $I = (x/4)Ma^2$ gives $x/4 = 3/4$. The total has the required mass-times-length-squared dimensions.

Step 4: Conclusion The system has $I_{OO'} = 3Ma^2/4$, so the required dimensionless value is $\boxed{x = 3}$.

Quick Tip to Solve: For an axis in the plane of a disc, start with its diameter moment $Mr^2/4$, and use only the perpendicular centre-to-axis distance in Md^2.

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58. A freshly prepared radioactive source of half life 2 hours 30 minutes emits radiation which is 64 times the permissible safe level. The minimum time, after which it would be possible to work safely with source, will be _____ hours. [4 marks]

Solution:

Concept:

- The activity of a radioactive source is proportional to the number of undecayed nuclei: $A = \lambda N$. For a single radionuclide, the decay constant λ stays fixed.
- After time t, the activity is $A(t) = A_0(1/2)^{t/T_{1/2}}$, where $T_{1/2}$ is the half-life. The radiation level decreases by the same factor as the activity.

Step 1: Express the half-life in hours The given half-life is 2 hours 30 minutes. Since 30 minutes equals half an hour,

$$T_{1/2} = 2 + \frac{30}{60} = 2.5 \text{ h}.$$

Let the permissible safe radiation level be A_s. Initially the source has radiation level $A_0 = 64A_s$. The source becomes safe when its level first reaches A_s; any later time also satisfies the safety condition.

Step 2: Find the required number of halvings At the threshold, the decay law requires

$$A_s = 64A_s \left(\frac{1}{2}\right)^{t/(2.5\,\text{h})}.$$

Cancelling the nonzero safe level gives

$$2^{t/(2.5\,\text{h})} = 64 = 2^6.$$

Thus exactly six half-lives are needed. The successive ratios to the safe level are $64, 32, 16, 8, 4, 2, 1$, confirming that five half-lives would still leave the source above the permitted level.

Step 3: Convert the number of half-lives to time The required minimum elapsed time is

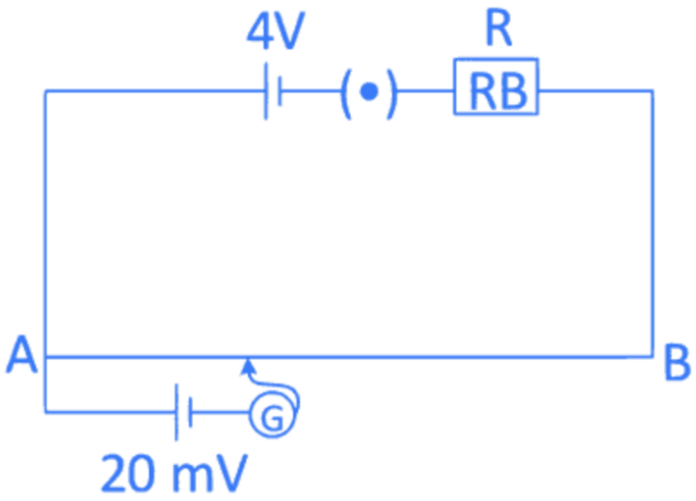
$$t = 6T_{1/2} = 6 \times 2.5 = 15 \text{ h}.$$

Because exponential radioactive decay is monotonic, the threshold cannot be reached earlier. The calculation concerns the emitted radiation level itself; no additional inverse-square factor is involved because no change in working distance is specified.

Step 4: Conclusion The minimum waiting time is $\boxed{15 \text{ hours}}$.

Quick Tip to Solve: Write the initial-to-safe ratio as a power of two; its exponent gives the number of half-lives before multiplying by the half-life in hours.

59. As shown in the figure, a potentiometer wire of resistance $20\,\Omega$ and length $300\ \text{cm}$ is connected with resistance box (R.B.) and a standard cell of emf $4\ \text{V}$. For a resistance 'R' of resistance box introduced into the circuit, the null point for a cell of $20\,\text{mV}$ is found to be $60\ \text{cm}$. The value of 'R' is _____ Ω. [4 marks]



Solution:

Concept:

- For a uniform potentiometer wire, the potential gradient is constant: $k = V_{AB}/L$. At a null point, the test-cell emf equals the potential drop along its balancing length: $E = k\ell$.
- At balance, no current is drawn from the test cell. The driver cell sends current through the resistance box and potentiometer wire in series, so $I = E_d/(R + R_w)$.

Step 1: Determine the voltage across the potentiometer wire The test-cell emf is $20\ \text{mV} = 0.020\ \text{V}$, balanced by $60\ \text{cm}$ of wire. Hence

$$k = \frac{0.020}{60}\ \text{V cm}^{-1} = \frac{1}{3000}\ \text{V cm}^{-1}.$$

The full length is $300\ \text{cm}$, so its end-to-end voltage is

$$V_{AB} = kL = \frac{300}{3000} = 0.100\ \text{V}.$$

The length ratio is dimensionless because both lengths are expressed in centimetres.

Step 2: Use the wire resistance to find the circuit current The resistance of the complete potentiometer wire is $R_w = 20\ \Omega$. By Ohm's law, the current through it is

$$I = \frac{V_{AB}}{R_w} = \frac{0.100}{20} = 0.005\ \text{A}.$$

At null the galvanometer branch carries zero current, so this is also the current through the resistance box. The test cell does not add its emf to the primary circuit.

Step 3: Subtract the wire resistance from the total resistance The driver emf is $4\ \text{V}$. With no internal

resistance specified, the total series resistance is

$$R_{\text{total}} = \frac{4}{0.005} = 800\ \Omega.$$

This includes the wire, so

$$R = 800 - 20 = 780\ \Omega.$$

As a check, $4/(780 + 20) = 0.005$ A, which produces the required 0.020 V drop along one fifth of the wire.

Step 4: Conclusion The resistance introduced from the box is $\boxed{780\ \Omega}$.

Quick Tip to Solve: First scale the balanced emf by full length divided by balancing length; then find the wire current and subtract the wire resistance from the total.

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60. The frequency of echo will be _____ Hz if the train blowing a whistle of frequency 320 Hz is moving with a velocity of 36 km/h towards a hill from which an echo is heard by the train driver. Velocity of sound in air is 330 m/s. [4 marks]

Solution:

Concept:

- For sound in stationary air, a source approaching a stationary receiver at speed u produces frequency $f_1 = fv/(v - u)$, where v is the speed of sound.
- Reflection at a stationary hill preserves the frequency measured at the hill. A receiver approaching that reflected wave at speed u hears $f_{\text{echo}} = f_1(v + u)/v$. The outward and returning Doppler shifts both matter.

Step 1: Convert the train speed and find the frequency at the hill The train speed is

$$u = 36 \times \frac{1000}{3600} = 10\ \text{m s}^{-1}.$$

The whistle emits $f = 320$ Hz, while sound travels at $v = 330\ \text{m s}^{-1}$. Since the whistle approaches the stationary hill, its wavefront spacing ahead is reduced. The hill receives

$$f_1 = 320\frac{330}{330 - 10} = 330\ \text{Hz}.$$

This is the frequency that is reflected in the rest frame of the hill.

Step 2: Apply the shift for the returning wave The hill is stationary, so it reflects a 330 Hz wave rather than introducing a further source-motion shift. The train driver now moves toward that reflected wave at $10\ \text{m s}^{-1}$. Therefore

$$f_{\text{echo}} = 330\frac{330 + 10}{330} = 340\ \text{Hz}.$$

Both shifts increase the frequency: the source approaches the hill on emission, and the driver approaches the wave on reception.

Step 3: Check the combined expression Equivalently the two factors combine as

$$f_{\text{echo}} = f\frac{v+u}{v-u} = 320\frac{340}{320} = 340 \text{ Hz}.$$

At zero train speed this expression reduces to f, as it should. Applying only a single Doppler shift would omit either the moving whistle or the moving listener and would give the wrong echo frequency.

Step 4: Conclusion The driver hears the echo at $\boxed{340 \text{ Hz}}$.

Quick Tip to Solve: For an echo from a fixed wall while the same train emits and receives, multiply the two Doppler factors to get $f(v+u)/(v-u)$.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Identify the incorrect statement from the following. [4 marks]

(A) A circular path around the nucleus in which an electron moves is proposed as Bohr's orbit.

(B) An orbital is the one electron wave function (ψ) in an atom.

(C) The existence of Bohr's orbits is supported by hydrogen spectrum.

(D) Atomic orbital is characterised by the quantum numbers n and l only.

Correct Answer: (D) Atomic orbital is characterised by the quantum numbers n and l only.

Solution:

Concept:

- A Bohr orbit is a definite circular electron path in the older atomic model. A quantum-mechanical orbital is a spatial one-electron wave function ψ, whose squared magnitude gives a probability density.
- A particular spatial atomic orbital is specified by three quantum numbers: principal n, azimuthal l, and magnetic m_l. The pair (n, l) identifies a subshell rather than generally identifying one orbital.

Step 1: Check the statements about orbits and wave functions Statement (A) correctly describes what Bohr proposed: an electron moves in a circular orbit around the nucleus. This is a statement about the Bohr model, so the modern absence of a definite trajectory does not make the historical description incorrect. Statement (B) is also the standard definition of an orbital as a one-electron wave function. Thus neither (A) nor (B) answers the request for an incorrect statement.

Step 2: Check the hydrogen-spectrum statement Bohr's quantised energy levels account for the main line spectrum of hydrogen through $h\nu = E_i - E_f$. The hydrogen spectrum therefore supports the quantisation underlying Bohr's orbits in the sense used in the question. Statement (C) is acceptable in this model context; it does not claim that the model explains every fine detail of atomic spectra or provides the modern spatial probability description.

Step 3: Identify the missing quantum number Statement (D) says that n and l alone characterise an atomic orbital. This omits m_l, which takes values $-l, -l+1, \ldots, +l$ and distinguishes the $2l+1$ orbitals within a subshell. For example, $n = 2, l = 1$ describes the 2p subshell, containing three spatial orbitals. Electron spin m_s labels an electron spin state and is not needed to specify the spatial orbital

itself.

Step 4: Conclusion The incorrect statement is that an orbital is characterised by n and l only; m_l is also needed. Hence, the correct option is (D).

Quick Tip to Solve: Distinguish shell n, subshell (n, l), and individual spatial orbital (n, l, m_l); the word "only" makes the last statement false.

62. Which of the following relation is not correct? [4 marks]

(A) $\Delta \mathrm{H} = \Delta \mathrm{U} - \mathrm{P}\Delta \mathrm{V}$

(B) $\Delta \mathrm{U} = \mathrm{q} + \mathrm{W}$

(C) $\Delta \mathrm{S}_{\text{sys}} + \Delta \mathrm{S}_{\text{surr}} \geqslant 0$

(D) $\Delta \mathrm{G} = \Delta \mathrm{H} - \mathrm{T}\Delta \mathrm{S}$

Correct Answer: (A) $\Delta \mathrm{H} = \Delta \mathrm{U} - \mathrm{P}\Delta \mathrm{V}$

Solution:

Concept:

- Enthalpy is defined by $H = U + PV$. Consequently $\Delta H = \Delta U + \Delta(PV)$, which becomes $\Delta H = \Delta U + P\Delta V$ for constant pressure.
- With the chemistry convention, heat supplied to a system and work done on it are positive, giving $\Delta U = q + W$. Gibbs energy is $G = H - TS$, and at constant temperature $\Delta G = \Delta H - T\Delta S$.

Step 1: Test the sign of the enthalpy relation At constant pressure, taking a difference of the defining equation gives

$$H_2 - H_1 = (U_2 - U_1) + P(V_2 - V_1).$$

Thus the pressure-volume term enters with a plus sign. Option (A) instead has $\Delta H = \Delta U - P\Delta V$, so it has the wrong sign. The first-law work expression $W = -P_{\text{ext}}\Delta V$ for expansion must not be confused with the $+PV$ term defining enthalpy.

Step 2: Check the first-law and entropy statements Option (B), $\Delta U = q + W$, is the first law using work done on the system as positive. It is therefore correct under the standard chemistry sign convention. In option (C), the system and surroundings together form the thermodynamic universe; the second law requires

$$\Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0.$$

Equality holds for a reversible process, while a spontaneous irreversible process gives a positive sum. This is the intended physically allowed-process condition.

Step 3: Check the Gibbs-energy relation From $G = H - TS$, at the same constant temperature in the initial and final states,

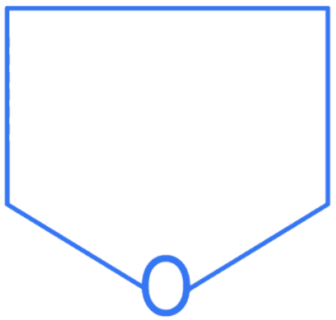
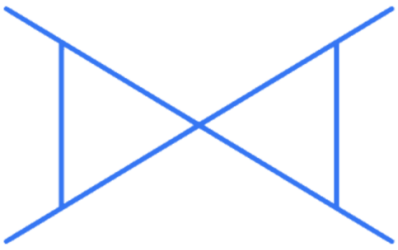
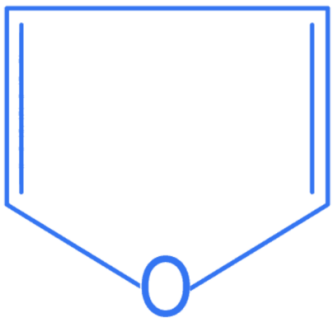
$$\Delta G = \Delta H - T\Delta S.$$

Hence option (D) is the familiar constant-temperature identity. These usual thermodynamic conditions are implicit in the listed relations. Among the four choices, (A) is the one that contradicts its defining thermodynamic identity even under its standard constant-pressure interpretation.

Step 4: Conclusion The enthalpy relation needs $+P\Delta V$, not $-P\Delta V$, at constant pressure. Hence, the correct option is (A).

Quick Tip to Solve: Rederive the sign directly from $H = U + PV$; the negative sign in expansion work belongs to the first law and does not change the definition of enthalpy.

63. Match List - I with List - II. [4 marks]

	List - I		List - II
(A)		(I)	Spiro compound
(B)		(II)	Aromatic compound
(C)		(III)	Non-planar Heterocyclic compound
(D)		(IV)	Bicyclo compound

(A) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(B) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(C) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

(D) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

Correct Answer: (C) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Solution:

Concept:

- A heterocycle contains a ring atom other than carbon. A saturated five-membered oxygen heterocycle is non-aromatic and adopts a non-planar conformation to reduce eclipsing strain.
- Two rings sharing exactly one atom form a spiro system. A bicyclic framework has two bridgehead atoms connected by three distinct paths.
- Aromaticity requires a planar, cyclic, continuously conjugated system with $4n + 2$ delocalised electrons. In furan, two double bonds and one oxygen lone pair provide six such electrons.

Step 1: Identify the two oxygen heterocycles Structure (A) is the saturated five-membered ring containing oxygen. Its ring carbons are tetrahedral and the ring puckers; it is therefore the non-planar heterocyclic compound, category (III). Structure (D) has two double bonds in the oxygen-containing five-membered ring. Four electrons from those double bonds and two from one oxygen lone pair form a conjugated six-electron aromatic circuit, so (D) belongs to category (II). The second oxygen lone pair does not add to this aromatic circuit.

Step 2: Distinguish the bicyclic and spiro skeletons In (B), the central bond joins two bridgehead carbons. The two outer routes and that central bond give three paths between the bridgeheads, making the framework bicyclic, category (IV). In (C), the two triangular rings meet at only the central carbon. Sharing one atom rather than a bond identifies a spiro compound, category (I). The terminal lines attached to the rings do not alter these ring-sharing relationships.

Step 3: Compare the complete mapping with the options The mapping is (A)–(III), (B)–(IV), (C)–(I), $(D)-(II)$. Only the third option contains all four assignments. The first incorrectly calls the saturated oxygen ring aromatic; the second calls that ring bicyclic; the fourth incorrectly assigns the spiro framework to the aromatic category. These mismatches rule out the other three choices.

Step 4: Conclusion In the order of the four structures, the categories are III, IV, I and II. Hence, the correct option is (C).

Quick Tip to Solve: Count shared ring atoms before inspecting substituents: one shared atom signals spiro; two bridgeheads with three connecting paths signal a bicyclic framework.

64. In which of the following pairs, electron gain enthalpies of constituent elements are nearly the same or identical?

(A) Rb and Cs

(B) Na and K

(C) Ar and Kr

(D) I and At

Choose the correct answer from the options given below : [4 marks]

(A) (A) and (B) only

(B) (B) and (C) only

(C) (A) and (C) only

(D) (C) and (D) only

Correct Answer: (C) (A) and (C) only

Solution:

Concept:

- Electron gain enthalpy is the enthalpy change for adding an electron to an isolated gaseous atom: $\mathrm{X(g) + e^- \longrightarrow X^-(g)}$. A negative value means energy is released.
- Compare tabulated values rather than ionisation energies. Down a group, size, shielding and electronic configuration jointly affect electron addition; similar values need not imply identical atomic sizes.

Step 1: Compare the alkali-metal pairs For rubidium and caesium the electron gain enthalpies are approximately -47 and $-46\,\mathrm{kJ\,mol^{-1}}$, respectively. Their separation is only about $1\,\mathrm{kJ\,mol^{-1}}$, so pair (A) qualifies as nearly the same. Sodium and potassium instead have values near -53 and $-48\,\mathrm{kJ\,mol^{-1}}$; they are not the nearly identical pair selected in this comparison. Thus pair (B) is not selected.

Step 2: Check the noble gases and halogens Argon and krypton both have an electron gain enthalpy close to $+96\,\mathrm{kJ\,mol^{-1}}$. The positive sign reflects the unfavourable addition of an electron beyond a filled valence shell, but equality of the values makes pair (C) correct. Iodine and astatine have distinctly different negative electron gain enthalpies, approximately -295 and $-270\,\mathrm{kJ\,mol^{-1}}$, so pair (D) does not qualify. The options containing (B) or (D) therefore fail; only the combination of (A) and (C) satisfies both comparisons.

Step 3: Conclusion The matching element pairs are Rb and Cs, and Ar and Kr only. Hence, the correct option is (C).

Quick Tip to Solve: For this comparison, remember the close Rb/Cs values and the equal positive Ar/Kr values; do not substitute electron affinity trends for the actual pairwise comparison.

65. Match List - I with List - II, match the gas evolved during each reaction.

	List - I		List - II
(A)	$(NH_4)_2\,Cr_2O_7 \xrightarrow{\Delta}$	**(I)**	H_2
(B)	$KMnO_4 + HCl \rightarrow$	**(II)**	N_2
(C)	$Al + NaOH + H_2O \rightarrow$	**(III)**	O_2
(D)	$NaNO_3 \xrightarrow{\Delta}$	**(IV)**	Cl_2

Choose the correct answer from the options given below : [4 marks]

(A) (A) − (II), (B) − (III), (C) − (I), (D) − (IV)

(B) (A) − (III), (B) − (I), (C) − (IV), (D) − (II)

(C) (A) − (II), (B) − (IV), (C) − (I), (D) − (III)

(D) (A) − (III), (B) − (IV), (C) − (I), (D) − (II)

Correct Answer: (C) (A) − (II), (B) − (IV), (C) − (I), (D) − (III)

Solution:

Concept:

- Identify the gas by balancing the reaction, rather than associating every oxygen-containing salt with oxygen evolution. Internal redox changes determine the products of ammonium dichromate decomposition.
- Acidified permanganate oxidises chloride to chlorine; aluminium in aqueous alkali releases hydrogen; sodium nitrate decomposes to nitrite and oxygen.

Step 1: Identify the products in the first two reactions On heating ammonium dichromate, ammonium nitrogen is oxidised to nitrogen while chromium(VI) is reduced to chromium(III):

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4\,H_2O.$$

The gas to match is N_2, giving (A)–(II). Permanganate and hydrochloric acid give

$$2\,KMnO_4 + 16\,HCl \longrightarrow 2\,KCl + 2\,MnCl_2 + 5\,Cl_2 + 8\,H_2O.$$

Chloride supplies electrons to manganese(VII), so the evolved gas is chlorine, giving (B)–(IV), rather than oxygen.

Step 2: Complete the aluminium and nitrate matches Aluminium dissolves in aqueous sodium hydroxide with evolution of hydrogen:

$$2\,Al + 2\,NaOH + 6\,H_2O \longrightarrow 2\,Na[Al(OH)_4] + 3\,H_2.$$

Hence (C)–(I). Heating sodium nitrate leaves sodium nitrite and releases oxygen:

$$2\,NaNO_3 \xrightarrow{\Delta} 2\,NaNO_2 + O_2.$$

Hence (D)–(III). The first option incorrectly assigns oxygen to permanganate/HCl and chlorine to nitrate. The second and fourth options incorrectly assign oxygen to ammonium dichromate; nitrogen

from that salt fixes the first match immediately.

Step 3: Conclusion The evolved gases, in order, are nitrogen, chlorine, hydrogen and oxygen. Hence, the correct option is (C).

Quick Tip to Solve: For a gas-matching problem, balance one distinctive reaction first: ammonium dichromate gives nitrogen, while permanganate with HCl gives chlorine.

66. Which of the following has least tendency to liberate H_2 from mineral acids? [4 marks]

(A) Cu

(B) Mn

(C) Ni

(D) Zn

Correct Answer: (A) Cu

Solution:

Concept:

- A metal liberates hydrogen from a non-oxidising mineral acid when its oxidation can drive the reduction $2\,H^+ + 2\,e^- \longrightarrow H_2$. This is a redox displacement of hydrogen, not simply a test of whether the metal reacts with any acid.
- The standard hydrogen electrode has $E^\circ = 0\,V$. For $M + 2\,H^+ \longrightarrow M^{2+} + H_2$, the standard cell potential is $E^\circ_{cell} = -E^\circ(M^{2+}/M)$.

Step 1: Compare the reduction potentials Copper has $E^\circ(Cu^{2+}/Cu) \approx +0.34\,V$. Consequently, its oxidation coupled to proton reduction would give $E^\circ_{cell} \approx -0.34\,V$, which is not favourable under standard conditions. Copper lies below hydrogen in the metal activity series and does not normally release hydrogen from dilute hydrochloric or dilute sulphuric acid.

Step 2: Eliminate the other metals and distinguish oxidising acids Manganese, nickel and zinc have negative standard reduction potentials, approximately -1.18, -0.25 and $-0.76\,V$, respectively. Their oxidation coupled to proton reduction therefore has a positive standard cell potential. They have a greater tendency than copper to liberate hydrogen, although rates can depend on surface conditions. Copper can dissolve in oxidising mineral acids such as nitric acid or hot concentrated sulphuric acid, but the acid is then reduced to nitrogen oxides or sulphur dioxide; that behaviour does not establish hydrogen evolution. Thus none of Mn, Ni or Zn is the least-tendency choice.

Step 3: Conclusion Copper has the least tendency to liberate H_2 from mineral acids among the listed metals. Hence, the correct option is (A).

Quick Tip to Solve: Compare each metal with hydrogen in the activity series; a metal dissolving in an oxidising acid need not produce hydrogen gas.

67. Match List - I with List - II.

	List - I		List - II
(A)	$Cd(s) + 2Ni(OH)_3(s) \rightarrow CdO(s) + 2Ni(OH)_2(s) + H_2O(l)$	**(I)**	**Primary battery**
(B)	$Zn(Hg) + HgO(s) \rightarrow ZnO(s) + Hg(l)$	**(II)**	**Discharging of secondary battery**
(C)	$2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4(aq)$	**(III)**	**Fuel cell**
(D)	$2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$	**(IV)**	**Charging of secondary battery**

Choose the correct answer from the options given below: [4 marks]

(A) $(A) - (I), (B) - (II), (C) - (III), (D) - (IV)$

(B) $(A) - (IV), (B) - (I), (C) - (II), (D) - (III)$

(C) $(A) - (II), (B) - (I), (C) - (IV), (D) - (III)$

(D) $(A) - (II), (B) - (I), (C) - (III), (D) - (IV)$

Correct Answer: (C) $(A) - (II), (B) - (I), (C) - (IV), (D) - (III)$

Solution:

Concept:

- A primary cell is used without ordinary recharging. A secondary cell operates reversibly: discharge converts stored chemical energy to electrical energy, while charging reverses that reaction using external electrical energy.
- A fuel cell receives fuel and oxidant continuously from outside. Identify charging and discharging from the direction of the printed reaction, not merely from the chemical names.

Step 1: Classify the cadmium and mercury reactions In (A), cadmium metal is oxidised and nickel(III) hydroxide is reduced to nickel(II) hydroxide. These changes are the energy-delivering direction of a nickel–cadmium secondary battery, so (A) matches discharging of a secondary battery, (II). In (B), the zinc amalgam/mercuric oxide reaction represents the mercury cell:

$$\mathrm{Zn(Hg) + HgO \longrightarrow ZnO + Hg.}$$

It is a primary battery, giving (B)–(I). The participation of an amalgam does not make it a rechargeable or fuel cell.

Step 2: Read the direction of lead storage and fuel-cell reactions During discharge of a lead storage battery, lead and lead dioxide become lead sulphate and sulphuric acid is consumed. Reaction (C) runs in the opposite direction:

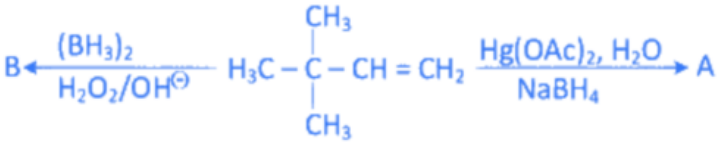
$$\mathrm{2\,PbSO_4 + 2\,H_2O \longrightarrow Pb + PbO_2 + 2\,H_2SO_4.}$$

It regenerates the electrodes and acid, identifying charging of the secondary battery, (IV). Reaction (D), $\mathrm{2\,H_2 + O_2 \longrightarrow 2\,H_2O}$, is the hydrogen–oxygen fuel-cell reaction, giving (III). Option (A) misclassifies the cadmium cell as primary; option (B) reverses charging and discharging; option (D) interchanges the last two categories.

Step 3: Conclusion The four reactions represent discharging of a secondary battery, a primary battery, charging of a secondary battery, and a fuel cell, respectively. Hence, the correct option is (C).

Quick Tip to Solve: In a lead storage battery, regeneration of Pb, PbO2 and sulphuric acid signals charging; formation of lead sulphate signals discharge.

68. Choose the correct option for the following reactions. [4 marks]



(A) 'A' and 'B' are both Markovnikov addition products.

(B) 'A' is Markovnikov product and 'B' is anti-Markovnikov product.

(C) 'A' and 'B' are both anti-Markovnikov products.

(D) 'B' is Markovnikov and 'A' is anti-Markovnikov product.

Correct Answer: (B) 'A' is Markovnikov product and 'B' is anti-Markovnikov product.

Solution:

Concept:

- Markovnikov hydration places the hydroxyl group on the more substituted carbon of the original carbon–carbon double bond. Anti-Markovnikov hydration places it on the less substituted carbon.
- Oxymercuration–demercuration gives Markovnikov hydration without the free-carbocation rearrangements typical of acid hydration. Hydroboration–oxidation gives anti-Markovnikov hydration.

Step 1: Identify product A from the mercury reagents The alkene is $(CH_3)_3C{-}CH{=}CH_2$. In the route labelled A, $Hg(OAc)_2, H_2O$, followed by $NaBH_4$, adds water across the double bond. Water attacks the more substituted alkene carbon in the bridged mercurinium-ion process. Demercuration replaces the carbon–mercury bond by hydrogen, giving

$$A = (CH_3)_3C{-}CH(OH){-}CH_3.$$

Thus A is the Markovnikov product; the adjacent highly substituted carbon does not require a skeletal rearrangement in this route.

Step 2: Identify product B and compare every option The route labelled B uses borane followed by alkaline hydrogen peroxide. Boron bonds preferentially to the terminal, less hindered carbon while hydrogen is delivered to the internal carbon. Oxidation replaces the C–B bond by C–OH without moving the carbon skeleton:

$$B = (CH_3)_3C{-}CH_2{-}CH_2OH.$$

This is anti-Markovnikov hydration. Option (A) incorrectly calls B Markovnikov; option (C) incorrectly calls A anti-Markovnikov; option (D) reverses the two labels. Only option (B) assigns both reaction outcomes correctly.

Step 3: Conclusion A is the Markovnikov product and B is the anti-Markovnikov product. Hence, the correct option is (B).

Quick Tip to Solve: Tag the reagents before inspecting the crowded skeleton: mercury/water then NaBH4 means Markovnikov; borane then peroxide/alkali means anti-Markovnikov.

69. Among the following marked proton of which compound shows lowest $\mathrm{pK_a}$ value? [4 marks]

(A) CH_3COOH, with an alpha C–H proton marked.

(B) CH_3COCH_3, with an alpha C–H proton marked.

(C) $C_6H_5CH_2COCH_3$, with a benzylic alpha C–H proton marked.

(D) $C_6H_5CH_2COOH$, with a benzylic alpha C–H proton marked.

Correct Answer: (C) $C_6H_5CH_2COCH_3$, with a benzylic alpha C–H proton marked.

Solution:

Concept:

- For the specified proton, lower pK_a means greater acidity. Compare the conjugate bases obtained by removing that exact marked proton; the acidic O–H proton in a carboxyl group is not the marked carbon-bound proton here.
- An alpha-carbonyl carbanion is stabilised as an enolate. A benzylic alpha-carbonyl anion can additionally delocalise charge into the aromatic ring, increasing its stabilisation.

Step 1: Form the anion from each marked C–H bond Option (A) marks an alpha C–H bond of acetic acid, not its O–H bond. Option (B) marks an alpha C–H bond of acetone, so its conjugate base is an enolate with resonance between carbon and oxygen. Option (C) marks the methylene group between a phenyl ring and a ketone carbonyl:

$$C_6H_5CH_2COCH_3 \xrightarrow{-H^+} C_6H_5CH^-COCH_3.$$

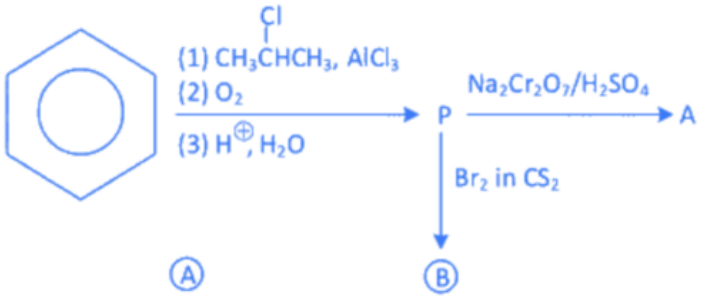
The resulting centre is both benzylic and alpha to a ketone.

Step 2: Compare resonance and reject the tempting acid choices In (C), the negative charge can be delocalised into the ketone oxygen through enolate resonance and into the phenyl ring through benzylic resonance. This dual stabilisation favours removal of the marked proton over the acetone proton in (B), whose anion lacks the phenyl contribution. Options (A) and (D) contain carboxylic acid groups, but their marked protons are on carbon. Resonance donation from the hydroxyl oxygen makes the carboxyl carbonyl less effective than a ketone at stabilising an alpha carbanion. Thus a carboxyl O–H acidity comparison would answer a different question. The ketone with the additional benzylic stabilisation has the lowest marked-proton pKa.

Step 3: Conclusion The marked benzylic alpha proton of $C_6H_5CH_2COCH_3$ in option (C) has the lowest pK_a. Hence, the correct option is (C).

Quick Tip to Solve: Circle the marked hydrogen first. For carbon acids, compare resonance of the carbanions rather than selecting a molecule merely because it contains COOH.

70. Identify the major products A and B for the below given reaction sequence. [4 marks]



(A) A: phenol; B: 4-bromophenol.

(B) A: 1,4-benzoquinone; B: 4-bromophenol.

(C) A: 1,4-benzoquinone; B: 2,4,6-tribromophenol.

(D) A: phenol; B: 2,4,6-tribromophenol.

Correct Answer: (B) A: 1,4-benzoquinone; B: 4-bromophenol.

Solution:

Concept:

- The cumene process converts benzene to isopropylbenzene, then to cumene hydroperoxide, whose acid cleavage gives phenol and acetone. The product P entering the two branches is therefore phenol.
- Phenol is oxidised by acidified dichromate to a quinone. Its activated aromatic ring undergoes electrophilic bromination; the solvent controls whether monobromination or extensive bromination predominates.

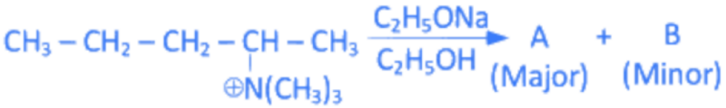
Step 1: Identify P and its oxidation product A Friedel–Crafts alkylation with isopropyl chloride and $AlCl_3$ gives cumene. Oxygen forms the hydroperoxide, and acid/water cleavage gives C_6H_5OH together with acetone. Thus P is phenol. On treatment with $Na_2Cr_2O_{7/}H_2SO_4$, phenol is oxidised to 1,4-benzoquinone, with carbonyl groups at opposite ring positions. Product A must therefore be the quinone, eliminating the choices in which A remains phenol.

Step 2: Apply the bromination conditions to product B The second branch specifies Br_2 in CS_2, a non-polar solvent. Under these conditions phenol undergoes controlled monobromination, with substitution at the ortho and para positions favoured by the electron-donating hydroxyl group. The less hindered para product is major, so B is 4-bromophenol. The 2,4,6-tribromophenol product is characteristic of bromine water and does not match the stated non-polar medium. Option (A) has the right B but wrong A; option (C) has the right A but wrong B; option (D) has both assignments wrong. The quinone/para-bromophenol pair is uniquely correct.

Step 3: Conclusion A is 1,4-benzoquinone and B is 4-bromophenol. Hence, the correct option is (B).

Quick Tip to Solve: For phenol bromination, read the solvent: bromine water gives tribromination, whereas bromine in CS2 favours monobromination, chiefly para.

71. Identify the correct statement for the below given transformation. [4 marks]



(A) A: $CH_3CH_2CH{=}CHCH_3$; B: $CH_3CH_2CH_2CH{=}CH_2$, Saytzeff products

(B) A: $CH_3CH_2CH{=}CHCH_3$; B: $CH_3CH_2CH_2CH{=}CH_2$, Hofmann products

(C) A: $CH_3CH_2CH_2CH{=}CH_2$; B: $CH_3CH_2CH{=}CHCH_3$, Hofmann products

(D) A: $CH_3CH_2CH_2CH{=}CH_2$; B: $CH_3CH_2CH{=}CHCH_3$, Saytzeff products

Correct Answer: (C) A: $CH_3CH_2CH_2CH{=}CH_2$; B: $CH_3CH_2CH{=}CHCH_3$, Hofmann products

Solution:

Concept:

- A quaternary ammonium group, $-N(CH_3)_3{}^+$, leaves as neutral trimethylamine during base-induced elimination of a neighbouring hydrogen.
- Hofmann elimination preferentially forms the less substituted alkene. The two possible beta positions must be compared rather than applying the usual Saytzeff rule for an alkyl halide.

Step 1: Identify the alpha and beta carbons The carbon carrying the trimethylammonium group is the alpha carbon. It is bonded to a methyl group on one side and to a propyl group on the other. A hydrogen can be removed from either adjacent beta carbon. Removal from the terminal methyl group puts the double bond at the end of the five-carbon chain; removal from the other beta carbon puts it inside the chain.

Step 2: Form the two possible alkenes Elimination towards the terminal carbon gives pent-1-ene:

$$CH_3CH_2CH_2CH{=}CH_2.$$

Elimination towards the internal carbon gives pent-2-ene:

$$CH_3CH_2CH{=}CHCH_3.$$

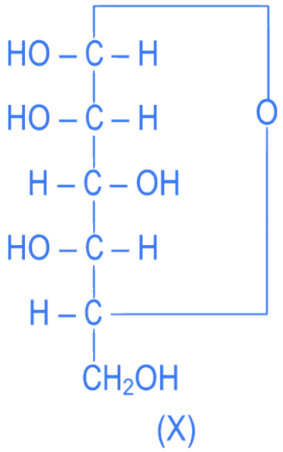
No carbon skeleton rearrangement is required: both products retain the same five-carbon chain, and trimethylamine is expelled.

Step 3: Apply the product preference and compare the options The bulky ammonium leaving group favours removal of a hydrogen at the less hindered beta site, giving the Hofmann orientation. Thus the less substituted terminal alkene is A, the major product; the internal alkene is B, the minor product. Options (A) and (B) reverse the major and minor structures. Option (D) gives their structures in the correct order but incorrectly calls the orientation Saytzeff.

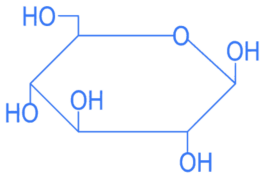
Step 4: Conclusion $A = CH_3CH_2CH_2CH{=}CH_2$ and $B = CH_3CH_2CH{=}CHCH_3$; the preference is Hofmann orientation. Hence, the correct option is (C).

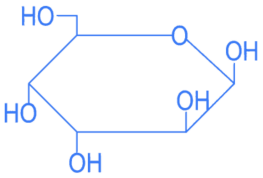
Quick Tip to Solve: For a quaternary ammonium elimination, first mark both beta positions and select the less substituted alkene as the major Hofmann product.

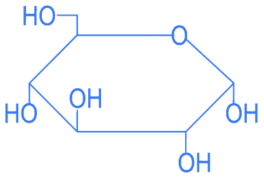
72. For the below given cyclic hemiacetal (X), the correct pyranose structure is : [4 marks]

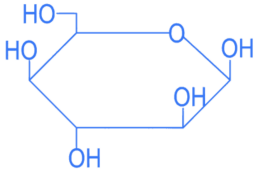


(X)

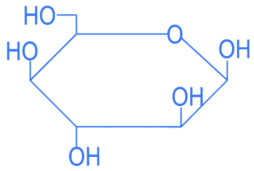
(A) 

(B) 

(C) 

(D) 

Correct Answer: (D)



Solution:

Concept:

- A pyranose is a six-membered cyclic hemiacetal ring containing five carbon atoms and one oxygen atom. Its stereochemistry must be preserved when changing the drawing convention.
- In the conventional Haworth orientation with ring oxygen at the upper right, substituents on the right of the vertical Fischer chain point down, and those on the left point up.

Step 1: Number the cyclic hemiacetal Begin at the top hemiacetal carbon, C1, which is bonded to both the ring oxygen and a hydroxyl group. Continue down the vertical chain through C2, C3 and C4 to C5. The lower exocyclic group is CH_2OH, C6. The oxygen bridge joining C1 and C5 is already present, so this conversion changes the representation rather than opening or altering the sugar.

Step 2: Transfer the hydroxyl orientations The hydroxyl groups at C1, C2 and C4 lie on the left of the vertical chain; their Haworth positions are therefore above the ring. The hydroxyl group at C3 lies on the right and must be below the ring. The terminal CH_2OH substituent at C5 points upward in the corresponding conventional pyranose drawing. Therefore the required sequence at C1 through C4 is

up, up, down, up.

Step 3: Inspect each candidate Option (D) has the required upward anomeric hydroxyl, upward C2 hydroxyl, downward C3 hydroxyl and upward C4 hydroxyl. Option (B) agrees at C1, C2 and C3 but puts the C4 hydroxyl down. Option (A) reverses the C2 and C3 hydroxyl orientations and also has C4 down. Option (C) additionally places the anomeric hydroxyl down. These differences change stereochemistry, so the ring shape alone is insufficient.

Step 4: Conclusion The pyranose retaining every stereocentre of X is the structure in (D). Hence, the correct option is (D).

Quick Tip to Solve: Number the anomeric carbon first, then check every hydroxyl independently: left in the vertical chain becomes up in the standard Haworth drawing.

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73. For kinetic study of the reaction of iodide ion with H_2O_2 at room temperature :

(A) Always use freshly prepared starch solution.

(B) Always keep the concentration of sodium thiosulphate solution less than that of KI solution.

(C) Record the time immediately after the appearance of blue colour.

(D) Record the time immediately before the appearance of blue colour.

(E) Always keep the concentration of sodium thiosulphate solution more than that of KI solution.

Choose the correct answer from the options given below : [4 marks]

(A) (A), (B), (C) only

(B) (A), (D), (E) only

(C) (D), (E) only

(D) (A), (B), (E) only

Correct Answer: (A) (A), (B), (C) only

Solution:

Concept:

- In acidic solution, iodide is oxidised by hydrogen peroxide: $H_2O_2 + 2\,I^- + 2\,H^+ \longrightarrow I_2 + 2\,H_2O$. The iodine production rate is measured indirectly with a clock endpoint.
- Thiosulphate removes iodine until it is exhausted: $I_2 + 2\,{S_2O_3}^{2-} \longrightarrow 2\,I^- + {S_4O_6}^{2-}$. Free iodine subsequently gives the blue starch complex.

Step 1: Check the starch and thiosulphate conditions Freshly prepared starch gives a reliable, sharp colour response; stored starch solution can deteriorate and obscure the endpoint. Statement (A) is therefore correct. Sodium thiosulphate must be present in a small, controlled amount compared with the iodide reagent, so only a small extent of the main reaction is needed to exhaust the iodine scavenger. This makes statement (B) correct and statement (E), which reverses the required concentration relation, incorrect.

Step 2: Identify the observable clock endpoint As long as thiosulphate remains, the iodine formed is immediately reduced back to iodide and cannot accumulate to produce the blue starch complex. When thiosulphate is consumed, the first persistent blue colour appears. The elapsed time is recorded

immediately on observing this appearance, corresponding to statement (C). Recording the time before its appearance, statement (D), would not identify an observable endpoint and would not give the prescribed clock time.

Step 3: Select the complete statement set The correct statements are (A), (B) and (C). Option (A) contains exactly this set. Options (B) and (C) include both the incorrect timing statement (D) and the incorrect concentration statement (E). Option (D) contains (B) and (E), which are contradictory concentration requirements, and omits the correct timing condition.

Step 4: Conclusion Fresh starch, lower thiosulphate concentration and recording immediately upon blue colour appearance are the correct conditions. Hence, the correct option is (A).

Quick Tip to Solve: In an iodine clock experiment, blue colour marks exhaustion of thiosulphate; stop timing at the first persistent blue appearance.

74. Match List - I with List - II.

	List I: Reaction		List II: Catalyst
(A)	$4\,NH_3(g) + 5\,O_2(g) \longrightarrow 4\,NO\,(g) + 6\,H_2O\,(g)$	**(I)**	$NO\,(g)$
(B)	$N_2(g) + 3\,H_2(g) \longrightarrow 2\,NH_3(g)$	**(II)**	$H_2SO_4(l)$
(C)	$C_{12}H_{22}O_{11}(aq) + H_2O\,(l) \longrightarrow \underbrace{C_6H_{12}O_6}_{\text{Glucose}} + \underbrace{C_6H_{12}O_6}_{\text{Fructose}}$	**(III)**	$Pt\,(s)$
(D)	$2\,SO_2(g) + O_2(g) \longrightarrow 2\,SO_3(g)$	**(IV)**	$Fe\,(s)$

Choose the correct answer from the options given below: [4 marks]

(A) (A) − (II), (B) − (III), (C) − (I), (D) − (IV)

(B) (A) − (III), (B) − (II), (C) − (I), (D) − (IV)

(C) (A) − (III), (B) − (IV), (C) − (II), (D) − (I)

(D) (A) − (III), (B) − (II), (C) − (IV), (D) − (I)

Correct Answer: (C) (A) − (III), (B) − (IV), (C) − (II), (D) − (I)

Solution:

Concept:

- A catalyst provides an alternative reaction pathway and is regenerated overall. Match each reaction to its characteristic catalyst rather than treating all oxidation catalysts as interchangeable.
- The listed examples include heterogeneous catalysis by solid platinum or iron and homogeneous catalysis by acid or a gaseous nitrogen oxide.

Step 1: Match ammonia oxidation and synthesis Reaction (A), oxidation of ammonia to nitric oxide, is the catalytic oxidation step of the Ostwald process. Platinum is the appropriate listed catalyst, so (A) matches (III). Reaction (B), synthesis of ammonia from nitrogen and hydrogen, is the Haber process. Finely divided iron is used as the catalyst; hence (B) matches (IV). Iron here catalyses ammonia formation, whereas platinum catalyses ammonia oxidation.

Step 2: Match sucrose hydrolysis Reaction (C) converts sucrose and water into glucose and fructose. This hydrolysis is acid catalysed. The listed sulphuric acid provides hydrogen ions and therefore

corresponds to (II). The acid accelerates cleavage of the glycosidic linkage and is regenerated; it is not one of the two monosaccharide products. Thus (C) matches (II).

Step 3: Match sulphur dioxide oxidation and eliminate alternatives For reaction (D), the listed gaseous catalyst is nitric oxide, (I), as in the nitrogen-oxide catalytic cycle of the lead chamber process. Nitric oxide is oxidised by oxygen to nitrogen dioxide, which transfers oxygen to sulphur dioxide and regenerates nitric oxide. The usual contact-process catalyst is not among these choices. The resulting mapping is (A)-(III), (B)-(IV), (C)-(II), (D)-(I). Only option (C) contains all four matches. Options (A), (B) and (D) assign an incorrect catalyst to at least the Haber reaction or sucrose hydrolysis.

Step 4: Conclusion Platinum, iron, sulphuric acid and nitric oxide catalyse the four listed reactions respectively. Hence, the correct option is (C).

Quick Tip to Solve: Fix the two distinctive pairs first: Haber synthesis uses iron, while sucrose hydrolysis uses acid. Then match the oxidation reactions.

75. Which of the reaction is suitable for concentrating ore by leaching process ? [4 marks]

(A) $2Cu_2\,S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(B) $Fe_3O_4 + CO \rightarrow 3FeO + CO_2$

(C) $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na\,[Al(OH)_4]$

(D) $Al_2O_3 + 6Mg \rightarrow 6MgO + 4Al$

Correct Answer: (C) $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na\,[Al(OH)_4]$

Solution:

Concept:

- Leaching selectively dissolves the desired ore constituent in a suitable reagent while much of the gangue remains insoluble and can be separated by filtration.
- Amphoteric alumina dissolves in aqueous sodium hydroxide to form soluble sodium tetrahydroxoaluminate. This is the chemical basis of concentrating bauxite by alkaline leaching.

Step 1: Identify the soluble complex-forming reaction In option (C), alumina reacts with aqueous sodium hydroxide in the presence of water:

$$Al_2O_3 + 2\,NaOH + 3\,H_2O \longrightarrow 2\,Na[Al(OH)_4].$$

The aluminium-containing component passes into solution as sodium aluminate. Insoluble impurities can then be removed. This is selective dissolution, which is precisely what distinguishes leaching from reduction or roasting.

Step 2: Check the roles of the other reactions Option (A) oxidises copper sulphide using oxygen and releases sulphur dioxide. It represents roasting or oxidative treatment, not dissolution in a leaching solution. Option (B) uses carbon monoxide to reduce iron oxide to a lower oxide; the iron oxidation state changes and carbon monoxide is oxidised to carbon dioxide. Neither reaction concentrates an ore by forming a selectively soluble species.

Step 3: Distinguish extraction from concentration Option (D) describes a metallothermic reduction

in which magnesium removes oxygen from alumina to produce aluminium. Such a reaction concerns extraction of the metal, not concentration by selective dissolution. Its printed coefficients also do not balance the oxygen atoms, but the process classification is enough to exclude it. Only option (C) shows the ore component entering an aqueous soluble complex without direct metal formation.

Step 4: Conclusion The reaction of alumina with aqueous sodium hydroxide to form $\mathrm{Na[Al(OH)_4]}$ is suitable for leaching. Hence, the correct option is (C).

Quick Tip to Solve: Look for selective dissolution into a soluble complex when identifying leaching; roasting and metal-oxide reduction belong to different stages of metallurgy.

76. The metal salts formed during softening of hardwater using Clark's method are : [4 marks]

(A) $\mathrm{Ca(OH)_2}$ and $\mathrm{Mg(OH)_2}$

(B) $\mathrm{CaCO_3}$ and $\mathrm{Mg(OH)_2}$

(C) $\mathrm{Ca(OH)_2}$ and $\mathrm{MgCO_3}$

(D) $\mathrm{CaCO_3}$ and $\mathrm{MgCO_3}$

Correct Answer: (B) $\mathrm{CaCO_3}$ and $\mathrm{Mg(OH)_2}$

Solution:

Concept:

- Clark's method removes temporary hardness by adding a calculated amount of slaked lime, $\mathrm{Ca(OH)_2}$, to water containing calcium or magnesium bicarbonate.
- The dissolved hardness-producing ions are removed as sparingly soluble precipitates. Calcium precipitates as carbonate, whereas magnesium is finally removed as hydroxide.

Step 1: Remove calcium bicarbonate hardness Calcium bicarbonate reacts with one equivalent of slaked lime:

$$\mathrm{Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2\,CaCO_3\downarrow + 2\,H_2O.}$$

Both the calcium already present in the water and the calcium added with lime are precipitated as calcium carbonate. Thus calcium hydroxide is the reagent and calcium carbonate is the calcium-containing product; confusing these roles would lead to an incorrect option.

Step 2: Remove magnesium bicarbonate hardness Magnesium bicarbonate requires two equivalents of lime for complete removal:

$$\mathrm{Mg(HCO_3)_2 + 2\,Ca(OH)_2 \longrightarrow Mg(OH)_2\downarrow + 2\,CaCO_3\downarrow + 2\,H_2O.}$$

Although magnesium carbonate may be considered in an intermediate stage, treatment with lime ultimately precipitates magnesium hydroxide. The final solids are therefore calcium carbonate and magnesium hydroxide, which can be separated from the softened water.

Step 3: Compare the proposed product pairs Option (B) gives exactly these two final precipitates. Option (A) includes calcium hydroxide, which is added as the softening reagent rather than identified as the removed calcium product. Option (C) gives both the wrong calcium species and magnesium

carbonate. Option (D) gives calcium carbonate correctly but incorrectly retains magnesium carbonate instead of the hydroxide formed on complete lime treatment.

Step 4: Conclusion The precipitates obtained in Clark's method are $CaCO_3$ and $Mg(OH)_2$. Hence, the correct option is (B).

Quick Tip to Solve: For lime softening, remember the final solids as calcium carbonate and magnesium hydroxide; magnesium bicarbonate consumes twice as much lime.

77. Which of the following statement is incorrect? [4 marks]

(A) Low solubility of LiF in water is due to its small hydration enthalpy.

(B) KO_2 is paramagnetic.

(C) Solution of sodium in liquid ammonia is conducting in nature.

(D) Sodium metal has higher density than potassium metal.

Correct Answer: (A) Low solubility of LiF in water is due to its small hydration enthalpy.

Solution:

Concept:

- The solubility of an ionic solid reflects the competition between the energy needed to separate its lattice and the energy released when its ions are hydrated.
- Lithium fluoride contains small ions and has a strong ionic lattice. The lithium ion also has a large magnitude of hydration enthalpy, so its low solubility cannot be attributed to weak hydration.

Step 1: Test the explanation for lithium fluoride Both Li^+ and F^- are small and strongly attract each other in the crystal. The resulting high lattice enthalpy strongly opposes dissolution. Their small size also gives strong interactions with water; in particular, lithium has the greatest magnitude of hydration enthalpy among the alkali-metal cations. Therefore option (A), which explains the low solubility using a small hydration enthalpy, is incorrect.

Step 2: Check superoxide and the ammonia solution In KO_2, potassium is present as K^+ and the anion is superoxide, $O_2{}^-$. The superoxide ion contains one unpaired electron and is therefore paramagnetic, so statement (B) is correct. Sodium dissolves in liquid ammonia to give solvated sodium cations and solvated electrons. These mobile charged species make the solution electrically conducting, confirming statement (C).

Step 3: Check the density comparison and select the exception Potassium has a lower density than sodium despite having the larger atomic mass, because its atomic volume increases sufficiently down the group. The approximate densities are $0.86\,g\,cm^{-3}$ for potassium and $0.97\,g\,cm^{-3}$ for sodium. Thus statement (D) is correct. Since the question asks for the incorrect statement, the three verified statements are excluded and (A) is selected.

Step 4: Conclusion Low solubility of LiF is associated with its high lattice enthalpy, not a small hydration enthalpy. Hence, the correct option is (A).

Quick Tip to Solve: For lithium salts, distinguish large lattice enthalpy from large hydration enthalpy; low solubility does not imply weak ion-water attraction.

78. Given below are two statements:

Statement I : In polluted water values of both dissolved oxygen and BOD are very low.

Statement II : Eutrophication results in decrease in the amount of dissolved oxygen.

In the light of the above statements, choose the most appropriate answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- Biochemical oxygen demand (BOD) is the oxygen required by microorganisms to oxidise biodegradable organic matter in a water sample. A larger biodegradable organic load normally produces a larger BOD.
- Dissolved oxygen (DO) is oxygen already present in water. Microbial decomposition consumes this oxygen, so strong organic pollution lowers DO while increasing BOD.
- Eutrophication is nutrient enrichment that promotes excessive algal growth; decomposition of the resulting biomass consumes dissolved oxygen.

Step 1: Evaluate Statement I Polluted water containing a large amount of biodegradable waste supports intense microbial activity. The bacteria use dissolved oxygen while breaking down the organic material. Consequently the dissolved-oxygen concentration becomes low, but the oxygen demand of the sample becomes high. Statement I incorrectly says that both DO and BOD are very low. Since its claim about BOD is opposite to the expected trend, Statement I is false.

Step 2: Evaluate Statement II Excess nitrates and phosphates stimulate an algal bloom. When algae and other aquatic organisms die, aerobic decomposers consume oxygen in decomposing their remains. This can severely deplete the oxygen available to fish and other aquatic animals. Thus eutrophication results in a decrease in dissolved oxygen, making Statement II true.

Step 3: Match the truth values to the options The required pair is false for Statement I and true for Statement II. Option (A) incorrectly accepts Statement I; option (B) incorrectly rejects Statement II; and option (C) reverses both truth values. Only option (D) matches the two independent evaluations.

Step 4: Conclusion Statement I is false because polluted water generally has low dissolved oxygen but high BOD; Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: Remember the opposite trends: more biodegradable waste means greater BOD and less dissolved oxygen.

79. Terylene polymer is obtained by condensation of : [4 marks]

(A) Ethane-1,2-diol and Benzene-1,3 dicarboxylic acid

(B) Propane-1, 2-diol and Benzene-1, 4 dicarboxylic acid

(C) Ethane-1,2-diol and Benzene-1, 4 dicarboxylic acid

(D) Ethane-1,2-diol and Benzene-1, 2 dicarboxylic acid

Correct Answer: (C) Ethane-1,2-diol and Benzene-1, 4 dicarboxylic acid

Solution:

Concept:

- Terylene is polyethylene terephthalate (PET), a polyester formed by condensation of a diol and a dicarboxylic acid.
- The diol is ethylene glycol, or ethane-1,2-diol, with formula $HOCH_2CH_2OH$. The acid is terephthalic acid, or benzene-1,4-dicarboxylic acid, with formula $HOOC-C_6H_4-COOH$.
- An alcohol group reacts with a carboxylic acid group to form an ester linkage $-COO-$, eliminating water during chain formation.

Step 1: Identify the glycol component Ethane-1,2-diol has two hydroxyl groups, one on each carbon of an ethylene group. Because it is bifunctional, each molecule can connect to acid-derived units at both ends, allowing a long chain to form. Propane-1,2-diol contains an additional methyl substituent and is not the glycol component of terylene.

Step 2: Identify the acid component and repeat unit Terephthalic acid carries its two carboxyl groups at the para positions, that is, positions 1 and 4 of the benzene ring. Reaction with ethylene glycol gives the polyester repeat unit

$$[-OCH_2CH_2OCO-C_6H_4-CO-]_n .$$

The linking carbonyl and oxygen groups come from condensation of the acid and alcohol functionalities; the benzene ring remains part of the polymer backbone.

Step 3: Eliminate the alternative pairs Option (A) uses benzene-1,3-dicarboxylic acid, which is isophthalic acid rather than terephthalic acid. Option (D) uses benzene-1,2-dicarboxylic acid, the ortho isomer. Option (B) has the correct para acid but the wrong diol. Option (C) contains both required monomers in their correct identities.

Step 4: Conclusion Terylene is obtained by condensation of ethane-1,2-diol and benzene-1,4-dicarboxylic acid. Hence, the correct option is (C).

Quick Tip to Solve: Link the name terylene to terephthalic acid: its carboxyl groups are para, at positions 1 and 4.

80. Statements about Enzyme Inhibitor Drugs are given below :

(A) There are Competitive and Non-competitive inhibitor drugs.

(B) These can bind at the active sites and allosteric sites.

(C) Competitive Drugs are allosteric site blocking drugs.

(D) Non-competitive Drugs are active site blocking drugs.

Choose the correct answer from the options given below: [4 marks]

(A) (A), (D) only

(B) (A), (C) only

(C) (A), (B) only

(D) (A), (B), (C) only

Correct Answer: (C) (A), (B) only

Solution:

Concept:

- Enzymes possess active sites where substrates bind and undergo reaction. A competitive inhibitor competes with the substrate for occupation of this active site.
- An allosteric site is a binding site distinct from the active site. In the standard non-competitive inhibition model, inhibitor binding there changes the enzyme and suppresses catalytic activity.
- The distinction concerns where inhibition occurs and whether substrate binding is directly competed for; both mechanisms are used in drug action.

Step 1: Check the existence and binding-site statements Statement (A) is correct: competitive and non-competitive inhibitors are recognised classes of enzyme-inhibitor drugs. Statement (B) is also correct when it refers collectively to these drugs: some bind at active sites, while others bind at allosteric sites. It does not require every inhibitor to bind at both sites simultaneously.

Step 2: Test the two proposed assignments Statement (C) assigns competitive drugs to the allosteric site. This is false: competitive inhibition involves the active site, because inhibitor and substrate compete for the same binding location. Statement (D) assigns non-competitive drugs to active-site blocking. This is also false in the classification tested here: non-competitive drugs bind at a different, allosteric site and alter enzyme activity.

Step 3: Select the correct combination Only statements (A) and (B) survive the independent checks. Option (A) includes the false statement (D), and option (B) includes the false statement (C). Option (D) likewise includes (C). The third option contains exactly (A) and (B), without either incorrect reversal of the binding-site assignments.

Step 4: Conclusion Only the first two statements are correct. Competitive inhibitors act at the active site, while non-competitive inhibitors act at an allosteric site. Hence, the correct option is (C).

Quick Tip to Solve: For competitive inhibition, ask what competes: inhibitor and substrate compete for the same active site.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. In the given reaction,

$$X + Y + 3Z \leftrightarrows XYZ_3$$

if one mole of each of X and Y with 0.05 mol of Z gives compound XYZ_3. (Given : Atomic masses of X, Y and Z are 10, 20 and 30 amu, respectively.) The yield of XYZ_3 is _____ g. (Nearest integer) [4 marks]

Solution:

Concept:

- For $\mathrm{X + Y + 3\,Z \longrightarrow XYZ_3}$, one mole of product requires one mole each of X and Y and three moles of Z. The stoichiometric coefficients determine the mole ratios, not the atomic masses.
- The limiting reagent is found by comparing available moles divided by the corresponding stoichiometric coefficient.
- The mass yield is $m = nM$, where n is the amount of product and M is its molar mass; round only the final requested value.

Step 1: Identify the limiting reagent The available amounts are $n_X = 1\,\mathrm{mol}$, $n_Y = 1\,\mathrm{mol}$ and $n_Z = 0.05\,\mathrm{mol}$. Divide by the respective coefficients:

$$\frac{n_X}{1} = 1, \qquad \frac{n_Y}{1} = 1, \qquad \frac{n_Z}{3} = \frac{0.05}{3}.$$

The smallest value belongs to Z, so Z limits the amount of compound that can form. X and Y are present in large excess relative to the amount required by Z.

Step 2: Calculate the moles and molar mass of product Three moles of Z make one mole of $\mathrm{XYZ_3}$. Therefore

$$n_{\mathrm{XYZ_3}} = \frac{0.05}{3} = \frac{1}{60}\,\mathrm{mol}.$$

Using the given atomic masses, the product molar mass is

$$M_{\mathrm{XYZ_3}} = 10 + 20 + 3(30) = 120\,\mathrm{g\,mol^{-1}}.$$

Only 1/60 mole each of X and Y is required, which confirms that their initial amounts are sufficient.

Step 3: Convert the amount to the requested mass Multiply the product amount by its molar mass:

$$m = \frac{1}{60} \times 120 = 2\,\mathrm{g}.$$

This is already an integer, so rounding does not alter the answer. The calculation gives the stoichiometric yield; no equilibrium constant or percentage-conversion factor is supplied for a separate equilibrium-yield calculation.

Step 4: Conclusion The yield of $\mathrm{XYZ_3}$ is $\boxed{2\,\mathrm{g}}$. The requested nearest-integer entry is $\boxed{2}$.

Quick Tip to Solve: Compare moles divided by coefficients before using molar masses; the smallest quotient fixes the product amount.

82. The number of paramagnetic species among the following is _____.

$B_2, Li_2, C_2, C_2^-, O_2^{2-}, O_2^+$ **and** He_2^+ [4 marks]

Solution:

Concept:

- A species is paramagnetic if its ground-state molecular-orbital configuration contains at least one unpaired electron. Completely paired configurations are diamagnetic.
- For the lighter second-period homonuclear molecules up to N_2, the π_{2p} bonding orbitals lie below σ_{2p}. Hund's rule puts electrons singly into degenerate orbitals before pairing.
- For oxygen species the π^*_{2p} antibonding orbitals control the magnetic behaviour. Core orbitals are filled and paired and do not affect the count.

Step 1: Check the lithium, boron and carbon species Li_2 has two valence electrons in σ^2_{2s}; both are paired, so it is diamagnetic. B_2 has six valence electrons: $\sigma^2_{2s}\sigma^{*2}_{2s}\pi^1_{2p_x}\pi^1_{2p_y}$. Its two degenerate pi orbitals each contain one electron, so it is paramagnetic. C_2 has eight valence electrons and both bonding pi orbitals are doubly occupied, making it diamagnetic. Adding one electron to give $C_2{}^-$ places it in the next orbital, σ_{2p}, leaving one unpaired electron.

Step 2: Check the oxygen and helium ions $O_2{}^{2-}$ has fourteen valence electrons. Its two antibonding pi orbitals are fully occupied, with all electrons paired, so it is diamagnetic. $O_2{}^+$ has eleven valence electrons and only one electron in the pair of antibonding pi orbitals, so it is paramagnetic. $He_2{}^+$ has three total electrons, with configuration $\sigma^2_{1s}\sigma^{*1}_{1s}$; its antibonding electron is unpaired, so it too is paramagnetic.

Step 3: Count the paramagnetic members The paramagnetic species are B_2, $C_2{}^-$, $O_2{}^+$ and $He_2{}^+$. The remaining three species have completely paired configurations. Count species rather than unpaired electrons: a molecule with two unpaired electrons still contributes only one species.

Step 4: Conclusion There are $\boxed{4}$ paramagnetic species: B_2, $C_2{}^-$, $O_2{}^+$ and $He_2{}^+$. This is a dimensionless count.

Quick Tip to Solve: Do not classify by total electron parity alone: the even-electron molecule B2 is paramagnetic because its pi orbitals are degenerate.

83. $150\ g$ **of acetic acid was contaminated with** $10.2\ g$ **ascorbic acid** $(C_6H_8O_6)$ **to lower down its freezing point by** $(x \times 10^{-1})^\circ C$**. The value of** x **is _____. (Nearest integer)**

[Given $K_f = 3.9\ K\ kg\ mol^{-1}$**; molar mass of ascorbic acid** $= 176\ g\ mol^{-1}$**]** [4 marks]

Solution:

Concept:

- For a dilute solution of a non-electrolyte, the freezing-point depression is $\Delta T_f = K_f m$, where m is the molality in moles of solute per kilogram of solvent.
- Ascorbic acid is the solute and the 150 g of acetic acid is the solvent. Use the solvent mass, rather than the combined solution mass, in calculating molality.

- A temperature difference has the same numerical value in kelvin and degrees Celsius. After finding the depression, compare it with the stated form $x \times 10^{-1}\,^{\circ}\mathrm{C}$.

Step 1: Convert the solute amount and solvent mass The molar mass of ascorbic acid is given as $176\,\mathrm{g\,mol^{-1}}$, so its amount is

$$n = \frac{10.2}{176}\,\mathrm{mol}.$$

The acetic-acid solvent mass is

$$150\,\mathrm{g} = 0.150\,\mathrm{kg}.$$

The stated freezing-point constant already corresponds to the solvent, so no molar-mass conversion of acetic acid is needed.

Step 2: Evaluate the molality and freezing-point depression The molality is

$$m = \frac{10.2/176}{0.150} = 0.38636\,\mathrm{mol\,kg^{-1}}.$$

Ascorbic acid is treated as the molecular solute in this colligative-property calculation. Substitution gives

$$\Delta T_f = 3.9 \times 0.38636 = 1.5068\,\mathrm{K} = 1.5068\,^{\circ}\mathrm{C}.$$

The units cancel as $\mathrm{K\,kg\,mol^{-1}} \times \mathrm{mol\,kg^{-1}} = \mathrm{K}$.

Step 3: Extract and round the coefficient x The question expresses the depression as $x \times 10^{-1}\,^{\circ}\mathrm{C}$. Hence

$$x = \frac{1.5068}{10^{-1}} = 15.068.$$

Rounding this coefficient to the nearest integer gives 15. Rounding the temperature itself first would lose information, so the rounding is performed only after extracting x.

Step 4: Conclusion The depression is approximately $1.5068\,^{\circ}\mathrm{C}$, so the required nearest-integer coefficient is $\boxed{x = 15}$ (dimensionless).

Quick Tip to Solve: Use kilograms of solvent in molality, then divide the final depression by the printed power of ten before rounding.

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84. $\mathrm{K_a}$ **for butyric acid** $(\mathrm{C_3H_7COOH})$ **is** 2×10^{-5}**. The** pH **of** $0.2\,\mathrm{M}$ **solution of butyric acid is** _____ $\times 10^{-1}$**. (Nearest integer)**

[Given $\log 2 = 0.30$**]** [4 marks]

Solution:

Concept:

- Butyric acid is a monoprotic weak acid. For $\mathrm{HA} \rightleftharpoons \mathrm{H^+} + \mathrm{A^-}$, the dissociation constant is $K_a = [\mathrm{H^+}][\mathrm{A^-}]/[\mathrm{HA}]$.
- If dissociation is small at initial concentration C, then $[\mathrm{H^+}] \simeq \sqrt{K_a C}$. Check the small-dissociation approximation after substitution.
- The pH is $-\log_{10}[\mathrm{H^+}]$, with concentration expressed numerically in molarity. The requested blank

is ten times the pH because the printed expression multiplies it by 10^{-1}.

Step 1: Apply the weak-acid equilibrium relation Let the equilibrium hydrogen-ion concentration be h. Neglecting water ionisation, $[A^-] = h$ and $[HA] = 0.2 - h$. Thus

$$2 \times 10^{-5} = \frac{h^2}{0.2 - h}.$$

For small h compared with 0.2,

$$h \simeq \sqrt{(2 \times 10^{-5})(0.2)} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3}\,\text{mol}\,\text{L}^{-1}.$$

Step 2: Check the approximation and calculate pH The fractional dissociation is $h/C = 0.002/0.2 = 0.01$, or only one percent, so replacing $0.2 - h$ with 0.2 is appropriate. Using the supplied logarithm,

$$\text{pH} = -\log(2 \times 10^{-3}) = 3 - \log 2 = 3 - 0.30 = 2.70.$$

The hydrogen-ion concentration is much larger than that from water, so neglecting water autoionisation is also consistent.

Step 3: Convert pH to the requested coefficient If the blank is y, the printed relation is $\text{pH} = y \times 10^{-1}$. Therefore

$$y = \frac{2.70}{0.1} = 27.$$

The exact quadratic gives a pH very close to 2.70 and leaves the rounded coefficient unchanged. The pH and this coefficient are dimensionless quantities, rather than concentrations.

Step 4: Conclusion The pH is approximately 2.70, so the required nearest-integer entry in $\text{pH} = y \times 10^{-1}$ is $\boxed{27}$ (dimensionless).

Quick Tip to Solve: First compute the pH using sqrt(Ka C), then undo the printed factor of 10 to the power minus one to fill the blank.

85. For the given first order reaction

$$A \rightarrow B$$

the half life of the reaction is 0.3010 min. The ratio of the initial concentration of reactant to the concentration of reactant at time 2.0 min will be equal to _____. (Nearest integer) [4 marks]

Solution:

Concept:

- A first-order reaction has rate proportional to the instantaneous reactant concentration: $-d[A]/dt = k[A]$, where k is the rate constant.
- The integrated law is $\ln([A]_0/[A]_t) = kt$, and the half-life is $t_{1/2} = \ln 2/k$. The concentration ratio is dimensionless.

Step 1: Find the first-order rate constant The supplied half-life is $0.3010\,\text{min}$. Use minutes throughout, since the elapsed time is also in minutes. Thus

$$k = \frac{0.693147}{0.3010} = 2.30281\,\text{min}^{-1}.$$

The rate constant is positive, as required for disappearance of the reactant. First-order half-life is independent of the initial concentration, so no numerical starting concentration is needed.

Step 2: Calculate the concentration ratio after two minutes Put $t = 2.0\,\text{min}$ into the integrated law:

$$\frac{[\text{A}]_0}{[\text{A}]_t} = e^{kt} = e^{(2.30281)(2.0)} \simeq 100.05.$$

Equivalently, $\log_{10}([\text{A}]_0/[\text{A}]_t) = (t/t_{1/2})\log_{10} 2$, which is approximately 2. About 6.645 half-lives have elapsed, so the remaining concentration is about one hundredth of its initial value. Round only the final ratio to the nearest integer, rather than rounding the number of half-lives first.

Step 3: Conclusion The requested initial-to-final concentration ratio, rounded to the nearest integer, is $\boxed{100}$ (dimensionless).

Quick Tip to Solve: Combine the two first-order formulae directly: $[\text{A}]_0/[\text{A}]_t = 2^{t/t_{1/2}}$. Check that an initial-to-final ratio exceeds one.

86. The number of interhalogens from the following having square pyramidal structure is :

$$\text{ClF}_3, \text{IF}_7, \text{BrF}_5, \text{BrF}_3, \text{I}_2\text{Cl}_6, \text{IF}_5, \text{ClF}, \text{ClF}_5$$

[4 marks]

Solution:

Concept:

- VSEPR theory assigns molecular shape from the numbers of bonding pairs and lone pairs around the central atom. A terminal halogen makes one single bond in these interhalogens.
- An AX_5E arrangement has six electron domains: an octahedral electron-pair arrangement and a square-pyramidal molecular shape. Only atom positions determine the named molecular shape.

Step 1: Identify the five-coordinate interhalogens The central halogen has seven valence electrons. For BrF_5, IF_5 and ClF_5, five electrons form five single bonds to fluorine and the remaining two electrons form one lone pair. Each therefore has five bond pairs plus one lone pair. Four fluorine atoms form a square base, the fifth occupies an apex, and the lone pair occupies the opposite octahedral position. All three are AX_5E, hence square pyramidal.

Step 2: Exclude the remaining species and count In ClF_3 and BrF_3, there are three bond pairs and two lone pairs: AX_3E_2, giving a T shape. IF_7 has seven bond pairs and no central lone pair, giving a pentagonal-bipyramidal shape. ClF is diatomic and therefore linear. I_2Cl_6, the dimeric form of iodine trichloride, has bridging chlorines and a planar dimer structure; it is not a five-terminal-ligand square pyramid. Thus none of these five additional entries belongs in the square-pyramidal count. Count

compounds, rather than lone pairs or the two iodine centres in the dimer.

Step 3: Conclusion The square-pyramidal interhalogens are BrF_5, IF_5 and ClF_5. The required number is $\boxed{3}$.

Quick Tip to Solve: For halogen pentafluorides, five bonds plus one lone pair immediately give AX_5E: distinguish square-pyramidal molecular shape from octahedral electron-domain geometry.

87. The disproportionation of MnO_4^{2-} in acidic medium resulted in the formation of two manganese compounds A and B. If the oxidation state of Mn in B is smaller than that of A, then the spin-only magnetic moment (μ) value of B in BM is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- Disproportionation converts the same oxidation state into a higher and a lower oxidation state. In $MnO_4{}^{2-}$, manganese is in the $+6$ state.
- The spin-only magnetic moment is $\mu = \sqrt{n(n+2)}\,\text{BM}$, where n is the number of unpaired electrons. Electron count must correspond to the product, not the starting manganate ion.

Step 1: Identify the lower-oxidation-state product In acidic solution, manganate disproportionates into permanganate and manganese dioxide:

$$3\,MnO_4{}^{2-} + 4\,H^+ \longrightarrow 2\,MnO_4{}^- + MnO_2 + 2\,H_2O.$$

The equation balances three manganese atoms, twelve oxygen atoms, four hydrogen atoms and charge -2 on each side. Manganese is $+7$ in $MnO_4{}^-$ and $+4$ in MnO_2. Since B has the smaller manganese oxidation state, B$=MnO_2$, while A contains permanganate. Do not identify B as a $+2$ product: that belongs to a different reduction reaction.

Step 2: Count unpaired electrons and calculate the moment Neutral manganese has configuration $[Ar]\,3d^5 4s^2$. Forming Mn^{4+} removes the two $4s$ electrons followed by two $3d$ electrons, leaving $3d^3$. These three electrons remain unpaired; in an octahedral description they occupy the three separate t_{2g} orbitals. Therefore

$$n = 3, \qquad \mu = \sqrt{3(3+2)} = \sqrt{15} = 3.873\ldots\ \text{BM}.$$

Round this moment to the nearest integer as requested, rather than using the oxidation number itself as the magnetic moment.

Step 3: Conclusion For B, MnO_2, the spin-only moment is $\sqrt{15}\,\text{BM}$, so the required nearest-integer value is $\boxed{4}\,\text{BM}$.

Quick Tip to Solve: First write the disproportionation products and choose the lower oxidation state. Only then find the d-electron count and use $\sqrt{n(n+2)}$.

88. Total number of relatively more stable isomer(s) possible for octahedral complex $[Cu(en)_2(SCN)_2]$ will be _____. [4 marks]

Solution:

Concept:

- Ethylenediamine, en, is a neutral bidentate ligand. Two en ligands supply four donor atoms; two monodentate thiocyanate ligands complete the octahedral coordination sphere.
- Cu(II) is a d^9 ion and undergoes Jahn–Teller distortion. Relatively stable arrangements place the four strongly bound en donor atoms in the equatorial plane and the two monodentate ligands at opposite axial positions.
- Thiocyanate is ambidentate: NCS^- denotes N coordination and SCN^- denotes S coordination. Different choices of donor atom give linkage isomers.

Step 1: Select the relatively stable geometric arrangement Both en ligands are neutral and the two thiocyanate ligands each have charge -1, so the neutral complex contains Cu^{2+}, a $3d^9$ ion. Its Jahn–Teller elongation is accommodated most readily when the four en nitrogen donors occupy the equatorial positions while the two monodentate thiocyanate donors occupy the elongated axial positions. The two thiocyanate ligands are therefore trans in this relatively stable family. Including the cis optical pair would count a different geometric family and would miss the role of the stability qualifier.

Step 2: Enumerate the trans linkage possibilities Each axial thiocyanate can attach through nitrogen or sulfur. The distinct pairs of donor atoms are N,N; S,S; and N,S. They correspond to

$$\text{trans-}[Cu(en)_2(NCS)_2],$$
$$\text{trans-}[Cu(en)_2(SCN)_2],$$
$$\text{trans-}[Cu(en)_2(NCS)(SCN)].$$

Choosing S,N instead of N,S does not create a fourth isomer: rotation interchanges the two opposite axial positions. These trans arrangements do not contribute separate optical pairs. Thus the linkage choices give exactly three distinct relatively stable isomers.

Step 3: Conclusion The relatively stable trans family contains three linkage arrangements, N,N; S,S; and N,S. The requested total is $\boxed{3}$.

Quick Tip to Solve: For this Cu(II) complex, apply the Jahn–Teller preference first, then count the unordered axial donor pairs from the two possible thiocyanate donor atoms.

89. On complete combustion of 0.492 g of an organic compound containing C, H and O, 0.7938 g of CO_2 and 0.4428 g of H_2O was produced. The % composition of oxygen in the compound is _____.

[4 marks]

Solution:

Concept:

- In complete combustion, all carbon in a C/H/O compound appears in CO_2 and all hydrogen appears in H_2O. Their masses follow the respective elemental mass fractions.
- The oxygen originally in the compound is found by subtracting the carbon and hydrogen masses from the sample mass. Oxygen in the combustion products also comes from the external oxidant.

Step 1: Calculate the carbon and hydrogen masses Using molar masses $M(CO_2) = 44\,\text{g mol}^{-1}$ and $M(H_2O) = 18\,\text{g mol}^{-1}$,

$$m_C = 0.7938 \times \frac{12}{44} = 0.2164909\ldots\ \text{g},$$

$$m_H = 0.4428 \times \frac{2}{18} = 0.0492\,\text{g}.$$

The numerator 2 in the water calculation accounts for two hydrogen atoms per water molecule. Carbon contributes $12/44$ of the carbon dioxide mass, not its entire mass. These two values refer to the elements in the original $0.492\,\text{g}$ sample.

Step 2: Obtain oxygen by difference and convert to percentage The sample contains only carbon, hydrogen and oxygen, so their masses sum to the sample mass. Consequently,

$$m_O = 0.492 - 0.2164909\ldots - 0.0492 = 0.2263091\ldots\ \text{g}.$$

Divide this mass by the original sample mass:

$$\%O = \frac{0.2263091\ldots}{0.492} \times 100 = 45.9978\ldots\%.$$

The complementary carbon and hydrogen percentages sum to about 54.0022%, which checks that all three percentages total 100%. Using oxygen directly from the product masses would overcount oxygen supplied during combustion.

Step 3: Conclusion The percentage composition of oxygen is approximately $\boxed{46\%}$.

Quick Tip to Solve: Find C from $12m(CO_2)/44$ and H from $2m(H_2O)/18$; obtain sample oxygen by mass difference, never from total product oxygen.

90. An element M crystallises in a body centred cubic unit cell with a cell edge of $300\,\text{pm}$. The density of the element is $6.0\ \text{g cm}^{-3}$. The number of atoms present in $180\ \text{g}$ of the element is _____ $\times 10^{23}$. (Nearest integer) [4 marks]

Solution:

Concept:

- A body-centred cubic unit cell contains $8 \times \frac{1}{8} + 1 = 2$ atoms: one atom effectively from its corners and one whole atom at the body centre.
- For a cubic cell, $V_{\text{cell}} = a^3$. The sample volume is m/ρ, so the number of atoms is $N = Zm/(\rho a^3)$, with all lengths and density units made consistent.

Step 1: Convert the cell edge and obtain the cell volume The density is expressed in g cm^{-3}, so convert picometres to centimetres. Since $1\,\text{pm} = 10^{-10}\,\text{cm}$,

$$a = 300 \times 10^{-10} = 3.00 \times 10^{-8}\,\text{cm},$$

$$a^3 = (3.00 \times 10^{-8})^3 = 2.70 \times 10^{-23}\,\text{cm}^3.$$

The mass of one unit cell is therefore

$$m_{\text{cell}} = \rho a^3 = 6.0(2.70 \times 10^{-23}) = 1.62 \times 10^{-22}\,\text{g}.$$

This cell mass corresponds to two atoms, not to a single atom.

Step 2: Count the cells and then the atoms in the sample For the $180\,\text{g}$ sample,

$$N_{\text{cells}} = \frac{180}{1.62 \times 10^{-22}} = 1.1111\ldots \times 10^{24}.$$

Multiplying by the bcc occupancy gives

$$N = 2N_{\text{cells}} = 2.2222\ldots \times 10^{24} = 22.2222\ldots \times 10^{23}\ \text{atoms}.$$

The blank asks for the coefficient multiplying 10^{23}, so round $22.2222\ldots$, rather than rounding the atom count or reporting 2. Avogadro's constant is unnecessary because the sample mass and the unit-cell mass already give a direct cell count.

Step 3: Conclusion The sample contains approximately 22.2222×10^{23} atoms. The nearest-integer coefficient requested in the blank is $\boxed{22}$.

Quick Tip to Solve: Use $N = 2m/(\rho a^3)$ for bcc, convert a to cm before cubing, and express the final count in the exact power of ten printed after the blank.