JEE Main 2022 Paper 1 (B.E./B.Tech) Jul 28 Shift 2 Question Paper with Solutions PDF

Question Paper with Solutions

Conducted by the National Testing Agency (NTA)

Time Allowed: 3 Hours **Maximum Marks: 300**

General Instructions

(i) This paper is Paper 1 (B.E./B.Tech.) of JEE (Main) 2022, Session 2 (July), conducted by the National Testing Agency (NTA) in Computer Based Test (CBT) mode on 28 July 2022, Shift 2. The total duration is 3 hours.

(ii) The paper has 90 questions in three subjects, Mathematics, Physics and Chemistry, in that order, with 30 questions each. Maximum marks: 300.

(iii) Each subject has two sections. Section A has 20 multiple choice questions (MCQs) with four options, of which only one is correct. Section B has 10 numerical value questions; attempt any 5 of them. The answer to each numerical value question is a number.

(iv) Marking scheme (multiple choice questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(v) Marking scheme (numerical value questions): $+4$ for a correct answer, -1 for an incorrect answer and 0 for an unanswered question.

(vi) Calculators, log tables, slide rules and electronic gadgets are not allowed.

Mathematics – Section A: Multiple Choice Questions (80 marks)

1.

Let $S = \left\{x \in [-6,3] - \{-2,2\} : \dfrac{|x+3|-1}{|x|-2} \geq 0\right\}$ **and**

$$T = \left\{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\right\}.$$

Then the number of elements in $\mathrm{S} \cap \mathrm{T}$ **is :** [4 marks]

(A) 7

(B) 5

(C) 4

(D) 3

Correct Answer: (D) 3

Solution:

Concept:

- For a rational expression to be nonnegative, its numerator and denominator must have the same sign, or its numerator must vanish while the denominator remains nonzero.
- To solve an inequality containing $|x|$ and x^2, put $u = |x| \geq 0$, so that $x^2 = u^2$. The intersection contains only integers that satisfy both original conditions.

Step 1: Find the sign intervals for S The numerator vanishes at $x = -4, -2$, since $|x+3| = 1$. It is negative on $(-4,-2)$ and positive outside $[-4,-2]$. The denominator vanishes at $x = -2, 2$; it is positive for $|x| > 2$ and negative for $|x| < 2$. A sign check, followed by the restriction $x \in [-6,3]$, therefore gives

$$S = [-6,-4] \cup (2,3].$$

The zero at -4 is included, whereas -2 and 2 are excluded because the quotient is undefined there.

Step 2: List the integer members of T Putting $u = |x|$, we require

$$u^2 - 7u + 9 \leq 0 \quad \Longrightarrow \quad \frac{7-\sqrt{13}}{2} \leq u \leq \frac{7+\sqrt{13}}{2}.$$

The bounds are approximately 1.697 and 5.303. Since u is an integer, $u = 2,3,4,5$. Thus

$$T = \{-5,-4,-3,-2,2,3,4,5\}.$$

Step 3: Take the intersection and count Only $-5, -4$ from the negative members lie in $[-6,-4]$, and only 3 from the positive members lies in $(2,3]$. Consequently

$$S \cap T = \{-5,-4,3\}.$$

Counting all members of either set alone would include values outside the other set. The proposed counts $7, 5, 4$ do not match this explicit intersection.

Step 4: Conclusion The number of elements of $S \cap T$ is $\boxed{3}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the finite integer set first, then test its members against the sign intervals; always exclude zeros of the denominator.

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2. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1$, $\frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are : [4 marks]

(A) non-real complex numbers

(B) real and both negative

(C) real and both positive

(D) real and exactly one of them is positive

Correct Answer: (B) real and both negative

Solution:

Concept:

- For a monic quadratic $x^2 + px + q = 0$, the sum and product of its roots are $-p$ and q. Symmetric expressions can be obtained without finding the roots separately.
- Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$, and divide by $(\alpha\beta)^2$ to obtain the sum of reciprocal squares.

Step 1: Obtain the original symmetric quantities From the first equation,

$$\alpha + \beta = \sqrt{2}, \qquad \alpha\beta = \sqrt{6}.$$

In particular, neither root is zero. Therefore

$$\alpha^2 + \beta^2 = 2 - 2\sqrt{6}, \qquad \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{2 - 2\sqrt{6}}{6} = \frac{1 - \sqrt{6}}{3}.$$

Step 2: Find the sum of the new coefficients Let the second equation have roots $r = 1 + \alpha^{-2}$ and $s = 1 + \beta^{-2}$. Its coefficient a is minus the root sum, while b is the root product:

$$a = -\left(2 + \frac{1 - \sqrt{6}}{3}\right) = -\frac{7}{3} + \frac{\sqrt{6}}{3},$$

$$b = 1 + \frac{1 - \sqrt{6}}{3} + \frac{1}{6} = \frac{3}{2} - \frac{\sqrt{6}}{3}.$$

The irrational terms cancel, giving $a + b = -5/6$. The extra $1/6$ in the product is $(\alpha\beta)^{-2}$; omitting it would change the final classification.

Step 3: Factor the required quadratic Substitute the coefficient sum into the last equation:

$$x^2 + \frac{17}{6}x + \frac{7}{6} = 0 \quad \Longleftrightarrow \quad (2x + 1)(3x + 7) = 0.$$

Its two roots are $-1/2$ and $-7/3$. Both are real and negative. Their explicit values exclude the non-real

possibility, both-positive possibility, and opposite-sign possibility in the other options.

Step 4: Conclusion The roots are $\boxed{-\frac{1}{2}, -\frac{7}{3}}$, so they are real and both negative. Hence, the correct option is (B).

Quick Tip to Solve: For transformed roots, compute their sum and product first; here the radicals cancel in $a + b$, so solving for α and β is unnecessary.

3. Let A and B be any two 3×3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true? [4 marks]

(A) $\mathrm{A}^4 - \mathrm{B}^4$ is a smmetric matrix

(B) $\mathrm{AB} - \mathrm{BA}$ is a symmetric matrix

(C) $\mathrm{B}^5 - \mathrm{A}^5$ is a skew-symmetric matrix

(D) $\mathrm{AB} + \mathrm{BA}$ is a skew-symmetric matrix

Correct Answer: (C) $\mathrm{B}^5 - \mathrm{A}^5$ is a skew-symmetric matrix

Solution:

Concept:

- A symmetric matrix satisfies $A^{\mathsf{T}} = A$; a skew-symmetric matrix satisfies $B^{\mathsf{T}} = -B$. The transpose reverses products: $(AB)^{\mathsf{T}} = B^{\mathsf{T}} A^{\mathsf{T}}$.
- Every power of a symmetric matrix is symmetric. An even power of a skew-symmetric matrix is symmetric, while an odd power is skew-symmetric.

Step 1: Check the difference of even powers Using the transpose definitions repeatedly,

$$(A^4 - B^4)^{\mathsf{T}} = A^4 - (-B)^4 = A^4 - B^4.$$

Thus option (A) is true. It uses even powers, so the negative sign from the transpose of B disappears. No assumption that A and B commute is needed.

Step 2: Check the two mixed products Transpose each term in its reversed order:

$$(AB - BA)^{\mathsf{T}} = -BA + AB = AB - BA,$$

$$(AB + BA)^{\mathsf{T}} = -BA - AB = -(AB + BA).$$

Therefore the difference is symmetric and the sum is skew-symmetric. Options (B) and (D) are true for every pair of the specified types. Treating the transpose as if it preserved the product order would give the wrong conclusion.

Step 3: Disprove the remaining universal assertion For $C = B^5 - A^5$, the transpose is

$$C^{\mathsf{T}} = -B^5 - A^5, \qquad -C = -B^5 + A^5.$$

These are equal only if $A^5 = 0$, which is not given. For a concrete allowed pair, take $A = I_3$ and $B = 0$. Then $B^5 - A^5 = -I_3$, whose transpose equals itself and does not equal its negative. A real

skew-symmetric matrix must have zero diagonal, which $-I_3$ also violates. Thus (C) cannot hold for arbitrary A, B.

Step 4: Conclusion The assertion that $B^5 - A^5$ is skew-symmetric is NOT true in general. Hence, the correct option is (C).

Quick Tip to Solve: For a matrix-type MCQ, transpose the complete expression and reverse every product; one counterexample disproves a claim asserted for all matrices.

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4. Let $f(x) = ax^2+bx+c$ be such that $f(1) = 3, f(-2) = \lambda$ and $f(3) = 4$. If $f(0)+f(1)+f(-2)+f(3) = 14$, then λ is equal to : [4 marks]

(A) -4

(B) $\frac{13}{2}$

(C) $\frac{23}{2}$

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- The values of a quadratic $f(x) = ax^2 + bx + c$ give linear equations in its coefficients. In particular, $f(0) = c$.
- Use the stated sum of function values as an additional linear relation. The parameter λ is the actual value $f(-2)$, not a coefficient of the polynomial.

Step 1: Translate the two fixed values The conditions at 1 and 3 give

$$a + b + c = 3, \qquad 9a + 3b + c = 4.$$

Subtract the first equation from the second:

$$8a + 2b = 1 \quad \Longrightarrow \quad b = \frac{1}{2} - 4a.$$

Substitute this back into $a + b + c = 3$ to get

$$c = \frac{5}{2} + 3a.$$

This expresses both remaining coefficients in terms of a, without yet using the unknown value at -2.

Step 2: Use the sum condition to determine the coefficients Because $f(1) = 3$, $f(3) = 4$, and $f(-2) = \lambda$, the sum condition is

$$c + 3 + \lambda + 4 = 14 \quad \Longrightarrow \quad c + \lambda = 7.$$

Direct evaluation at -2 gives

$$\lambda = 4a - 2b + c = 15a + \frac{3}{2}.$$

Consequently,

$$\left(\frac{5}{2} + 3a\right) + \left(15a + \frac{3}{2}\right) = 7, \qquad 18a = 3.$$

Hence $a = 1/6$, $b = -1/6$, and $c = 3$. The negative sign in $-2b$ is essential when evaluating the polynomial at a negative argument.

Step 3: Evaluate and verify the parameter With these coefficients,

$$\lambda = f(-2) = \frac{4}{6} + \frac{2}{6} + 3 = 4.$$

The resulting polynomial $f(x) = x^2/6 - x/6 + 3$ indeed has $f(1) = 3$, $f(3) = 4$, and $f(0) = 3$. Its four specified values sum to $3 + 3 + 4 + 4 = 14$. The other proposed values cannot satisfy these simultaneous coefficient relations.

Step 4: Conclusion The required parameter is $\boxed{\lambda = 4}$. Hence, the correct option is (D).

Quick Tip to Solve: Replace $f(0)$ by the constant coefficient immediately; then subtract the equations at two known arguments to eliminate that constant.

5. The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$f(x) = \lim\limits_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$ **is continuous for all x in :** [4 marks]

(A) $R - \{-1\}$

(B) $\mathbb{R} - \{-1, 1\}$

(C) $R - \{1\}$

(D) $R - \{0\}$

Correct Answer: (B) $\mathbb{R} - \{-1, 1\}$

Solution:

Concept:

- For $|x| < 1$, $x^{2n} \to 0$; for $|x| > 1$, divide the numerator and denominator by x^{2n}. The boundary points $x = \pm 1$ must be substituted separately.
- A piecewise-defined limit function is continuous inside each smooth branch. At a boundary point, continuity requires both one-sided limits to equal its actual value.

Step 1: Find the formula away from the boundaries Write the denominator as $1 + (x-1)x^{2n}$. For $|x| < 1$, the powers tend to zero, so

$$f(x) = \cos(2\pi x).$$

For $|x| > 1$, division by x^{2n} gives

$$f(x) = \lim_{n\to\infty} \frac{x^{-2n}\cos(2\pi x) - \sin(x-1)}{x^{-2n} + x - 1} = -\frac{\sin(x-1)}{x-1}.$$

Each expression is continuous on its corresponding open intervals; the denominator in the second branch is nonzero there.

Step 2: Check the point $x = 1$ At $x = 1$, every member of the defining sequence has numerator 1 and denominator 1, hence $f(1) = 1$. However,

$$\lim_{x\to 1^-} f(x) = 1, \qquad \lim_{x\to 1^+} f(x) = -\lim_{t\to 0^+} \frac{\sin t}{t} = -1.$$

The one-sided limits disagree, so the function is discontinuous at 1.

Step 3: Check the point $x = -1$ and select the set At $x = -1$, the numerator is $1 + \sin 2$, while the denominator is -1, so $f(-1) = -1 - \sin 2$. The adjacent branch limits are

$$\lim_{x\to -1^-} f(x) = -\frac{\sin 2}{2}, \qquad \lim_{x\to -1^+} f(x) = 1.$$

They disagree as well. Thus both boundary points must be removed. There is no discontinuity at 0, which lies inside the cosine branch. Options excluding only one boundary, or excluding 0, therefore fail.

Step 4: Conclusion The function is continuous precisely on $\boxed{\mathbb{R} \setminus \{-1, 1\}}$. Hence, the correct option is (B).

Quick Tip to Solve: In limits involving x^{2n}, split at $|x| = 1$, then substitute the two boundary points before testing one-sided continuity.

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6. The function $f(x) = xe^{x(1-x)}, x \in \mathbb{R}$, is : [4 marks]

(A) increasing in $\left(-\frac{1}{2}, 1\right)$
(B) decreasing in $\left(\frac{1}{2}, 2\right)$
(C) increasing in $\left(-1, -\frac{1}{2}\right)$
(D) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Correct Answer: (A) increasing in $\left(-\frac{1}{2}, 1\right)$

Solution:

Concept:

- The sign of $f'(x)$ determines monotonicity: a positive derivative gives increasing behaviour and a negative derivative gives decreasing behaviour on an interval.
- For $f(x) = xe^{g(x)}$, use both the product rule and chain rule. The exponential factor is always positive, so only the remaining polynomial affects the derivative sign.

Step 1: Differentiate and factor Here $g(x) = x(1 - x) = x - x^2$, so $g'(x) = 1 - 2x$. Differentiating gives

$$f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1 - 2x) = e^{x(1-x)}(1 + x - 2x^2).$$

Factor the quadratic:

$$f'(x) = e^{x(1-x)}(1 - x)(2x + 1).$$

The stationary points are $-1/2$ and 1, because the exponential cannot vanish. The chain-rule term is necessary; differentiating only the factor x would miss both changes of sign.

Step 2: Construct the sign intervals For $x < -1/2$, the factors $1 - x$ and $2x + 1$ have opposite signs, so $f'(x) < 0$. For $-1/2 < x < 1$, both factors are positive, giving $f'(x) > 0$. For $x > 1$, only $1 - x$ is negative, again giving $f'(x) < 0$. Thus the function decreases, then increases, then decreases across these three intervals.

Step 3: Compare each proposed interval Option (A) is exactly the interval $(-1/2, 1)$ on which the derivative is positive. Option (B) is not a decreasing interval: on $(1/2, 1)$ the derivative is positive, and only after 1 is it negative. Option (C) lies to the left of $-1/2$, where the function decreases. Option (D) lies inside the positive-derivative region and is increasing rather than decreasing.

Step 4: Conclusion The function is increasing on $\boxed{(-\frac{1}{2}, 1)}$. Hence, the correct option is (A).

Quick Tip to Solve: Factor the derivative after removing its positive exponential factor; test every proposed interval against the roots of the remaining polynomial.

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7. The sum of the absolute maximum and absolute minimum values of the function $f(x) = \tan^{-1}(\sin x - \cos x)$ in the interval $[0, \pi]$ is : [4 marks]

(A) 0

(B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$

(D) $\frac{-\pi}{12}$

Correct Answer: (C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$

Solution:

Concept:

- The principal function $\tan^{-1} t$ is strictly increasing for all real t. Therefore its absolute extrema occur where its argument has its absolute extrema.
- On a closed interval, check stationary points and endpoints. The identity $\sin x - \cos x = \sqrt{2}\sin(x - \pi/4)$ also helps determine the argument range.

Step 1: Find the extrema of the argument Let $h(x) = \sin x - \cos x$. Then

$$h'(x) = \cos x + \sin x.$$

In $[0, \pi]$, this derivative vanishes only at $x = 3\pi/4$. Evaluate the two endpoints and this stationary point:

$$h(0) = -1, \qquad h(\pi) = 1, \qquad h(3\pi/4) = \sqrt{2}.$$

Hence the absolute minimum of the argument is -1, and its absolute maximum is $\sqrt{2}$. Although the amplitude of the sine expression is $\sqrt{2}$, the value $-\sqrt{2}$ is not reached on the given interval.

Step 2: Transfer the extrema through the increasing inverse tangent Since $\tan^{-1}$ preserves order,

$$f_{\min} = \tan^{-1}(-1) = -\frac{\pi}{4}, \qquad f_{\max} = \tan^{-1}\sqrt{2}.$$

Their sum is therefore $\tan^{-1}\sqrt{2} - \pi/4$. Here "absolute minimum" means the least function value on the entire interval, not the absolute value of that minimum.

Step 3: Express the result in the listed form Put $\theta = \tan^{-1}\sqrt{2}$. It lies in $(0, \pi/2)$, so a right triangle with opposite side $\sqrt{2}$, adjacent side 1, and hypotenuse $\sqrt{3}$ gives

$$\cos\theta = \frac{1}{\sqrt{3}}, \qquad \theta = \cos^{-1}\frac{1}{\sqrt{3}}.$$

Thus option (C) is the required sum. It is positive because $\sqrt{2} > 1$, ruling out (A), (B), and (D), whose values are zero or negative.

Step 4: Conclusion The sum is $\boxed{\cos^{-1}(1/\sqrt{3}) - \pi/4}$. Hence, the correct option is (C).

Quick Tip to Solve: Optimize the argument of a strictly increasing inverse function first, and include both endpoints before converting the final inverse-trigonometric form.

8. Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}, t \in \left(0, \frac{\pi}{2}\right)$.

Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to : [4 marks]

(A) $\frac{-2\sqrt{2}}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $\frac{-2}{3}$

Correct Answer: (D) $\frac{-2}{3}$

Solution:

Concept:

- For a parametric curve, the slope is $m = dy/dx = y_t/x_t$, at a point where $x_t \neq 0$. The second derivative is obtained by differentiating this slope with respect to the parameter and dividing by x_t again.
- Write the common radial factor as $r(t) = 2\sqrt{2}\sqrt{\sin 2t}$. Then $x = r\cos t$, $y = r\sin t$, and the product rule lets us evaluate the derivatives at the given parameter without eliminating it.

Step 1: Evaluate the radial derivatives Logarithmic differentiation gives $r_t/r = \cot 2t$, since the interval makes $\sin 2t$ positive. Differentiating this identity gives $r_{tt}/r = \cot^2 2t - 2\csc^2 2t$. Therefore, at $t = \pi/4$,

$$r = 2\sqrt{2}, \qquad r_t = 0, \qquad r_{tt} = -4\sqrt{2}.$$

The vanishing first radial derivative does not make the Cartesian derivatives vanish, because the sine and cosine factors also vary.

Step 2: Calculate the Cartesian slope and curvature derivative The first derivatives are $x_t = r_t \cos t - r \sin t$, $y_t = r_t \sin t + r \cos t$. Their values are $x_t = -2$, $y_t = 2$, so $m = -1$. Differentiating once more,

$$x_{tt} = r_{tt} \cos t - 2r_t \sin t - r \cos t = -6,$$

$$y_{tt} = r_{tt} \sin t + 2r_t \cos t - r \sin t = -6.$$

Consequently,

$$\frac{d^2y}{dx^2} = \frac{y_{tt}x_t - y_t x_{tt}}{x_t^3} = \frac{12 + 12}{-8} = -3.$$

Step 3: Form the required signed ratio The numerator is $1 + m^2 = 2$, and the denominator is -3. Hence the ratio is $-2/3$. A positive answer loses the sign of the second derivative; the extra factor $\sqrt{2}$ in option (A) does not survive the Cartesian derivative evaluations.

Step 4: Conclusion The required value is $\boxed{-2/3}$. Hence, the correct option is (D).

Quick Tip to Solve: For parametric second derivatives, use $(y_{tt}x_t - y_t x_{tt})/x_t^3$; retaining the cube prevents a sign error.

9. Let $I_n(x) = \int_0^x \frac{1}{(t^2+5)^n} dt, n = 1, 2, 3, \ldots$ **. Then :** [4 marks]

(A) $50I_6 - 9I_5 = xI_5'$

(B) $50I_6 - 11I_5 = xI_5'$

(C) $50I_6 - 9I_5 = I_5'$

(D) $50I_6 - 11I_5 = I_5'$

Correct Answer: (A) $50I_6 - 9I_5 = xI_5'$

Solution:

Concept:

- The fundamental theorem of calculus gives $I_n'(x) = (x^2+5)^{-n}$, because the integrand is continuous for every real upper limit.
- A relation between neighbouring powers can be found by differentiating $x/(x^2+5)^5$. The product and chain rules produce exactly the two denominator powers occurring in I_5 and I_6.

Step 1: Differentiate a suitable elementary expression Set $F(x) = x(x^2+5)^{-5}$. Its derivative is

$$F'(x) = (x^2+5)^{-5} - 10x^2(x^2+5)^{-6}.$$

Use $x^2 = (x^2+5) - 5$ to express both terms as neighbouring powers:

$$F'(x) = -9(x^2+5)^{-5} + 50(x^2+5)^{-6}.$$

The coefficient is -9, rather than -11, because the first product-rule term contributes a positive unit.

Step 2: Integrate from the common lower limit Integrate the derivative identity from 0 to x. All integrals exist since the denominator never vanishes. At the lower endpoint, $F(0) = 0$, so there is no integration constant left:

$$\frac{x}{(x^2+5)^5} = 50I_6(x) - 9I_5(x).$$

By the fundamental theorem, the expression on the left is $xI_5'(x)$, proving the identity for every real x.

Step 3: Distinguish the four identities Options (B) and (D) have the incorrect coefficient of I_5. Options (C) and (D) omit the factor x on the right. The latter omission can also be rejected at $x = 0$: both integrals then vanish, whereas $I_5'(0) = 5^{-5} \neq 0$. Thus only the first identity has both the correct coefficients and the correct endpoint behaviour.

Step 4: Conclusion $\boxed{50I_6 - 9I_5 = xI_5'}$. Hence, the correct option is (A).

Quick Tip to Solve: To connect consecutive powers of a quadratic denominator, differentiate $x/(x^2+c)^n$, then integrate from the shared lower limit.

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10. The area enclosed by the curves $y = \log_e\left(x + \mathrm{e}^2\right), x = \log_e\left(\frac{2}{y}\right)$ and $x = \log_{\mathrm{e}} 2$, above the line $y = 1$ is: [4 marks]

(A) $2 + \mathrm{e} - \log_{\mathrm{e}} 2$

(B) $1 + e - \log_e 2$

(C) $e - \log_e 2$

(D) $1 + \log_e 2$

Correct Answer: The literal curves enclose an area of approximately 0.417834, matching none of the listed options.

Solution:

Concept:

- For vertical strips, the area between an upper and a lower graph is the integral of their difference. The integration limits must be determined by the listed bounding curves.
- Here the second curve is $y = 2e^{-x}$. The first, $y = \ln(x + e^2)$, is strictly increasing, while the second is strictly decreasing. Their single intersection is $(0, 2)$.

Step 1: Identify the literal bounded region The curves meet at $x = 0$, since $\ln(e^2) = 2$. Strict monotonicity proves uniqueness. The third boundary is $x = \ln 2 > 0$; over $0 \le x \le \ln 2$, the increasing logarithmic curve is above the decreasing exponential. Moreover, $2e^{-x} \ge 1$, so the entire enclosed region already lies above $y = 1$. The horizontal line therefore does not remove any part of this region.

Step 2: Evaluate its area exactly The literal requested area is

$$A = \int_0^{\ln 2} \left[\ln(x + e^2) - 2e^{-x}\right] dx.$$

Use $\int \ln(x+e^2)dx = (x+e^2)\ln(x+e^2) - (x+e^2)$ and $\int -2e^{-x}dx = 2e^{-x}$. Endpoint substitution gives

$$\begin{aligned} A &= (e^2+\ln 2)\ln(e^2+\ln 2) \\ &\quad - 2e^2 - \ln 2 - 1 \\ &\approx 0.417834. \end{aligned}$$

The positive value agrees with the order of the boundary graphs.

Step 3: Compare with the supplied choices The four listed expressions are approximately 4.0251, 3.0251, 2.0251, and 1.6931, respectively. None equals the literal area. Although the final answer key selects (B), that value results from integrating horizontal widths only from $y=1$ to $y=2$, which uses $y=1$ as a different boundary and does not describe the region enclosed by all three listed curves. The literal equation must be retained rather than modified to force the keyed option.

Step 4: Conclusion For the curves as written, the area is $\boxed{(e^2+\ln 2)\ln(e^2+\ln 2) - 2e^2 - \ln 2 - 1}$, approximately 0.417834 square units. None of the options is correct for the literal question; the final key lists (B).

Quick Tip to Solve: Locate all intersections and check whether the stated horizontal cutoff actually intersects the bounded region before choosing integration limits.

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11. Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \frac{1}{x^2-1}y = \left(\frac{x-1}{x+1}\right)^{1/2}$, $x > 1$ passing through the point $\left(2, \sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}\,y(8)$ is equal to : [4 marks]

(A) $11 + 6\log_e 3$

(B) 19

(C) $12 - 2\log_e 3$

(D) $19 - 6\log_e 3$

Correct Answer: (D) $19 - 6\log_e 3$

Solution:

Concept:

- For $y' + P(x)y = Q(x)$, an integrating factor is $\mu(x) = e^{\int P(x)dx}$. Multiplication by it turns the left side into the derivative of μy.
- The condition $x > 1$ makes both $x-1$ and $x+1$ positive. Therefore the square-root integrating factor and the logarithms below are real and require no sign changes.

Step 1: Find and apply the integrating factor Partial fractions give

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\frac{x-1}{x+1}.$$

Thus $\mu = \sqrt{(x-1)/(x+1)}$. This happens also to equal the right-hand side of the differential equation,

so after multiplication,

$$\frac{d}{dx}(\mu y) = \mu^2 = \frac{x-1}{x+1} = 1 - \frac{2}{x+1}.$$

Integrating gives $\mu y = x - 2\ln(x+1) + C$.

Step 2: Use the prescribed point At $x = 2$, the integrating factor is $1/\sqrt{3}$, and the given value of y is also $1/\sqrt{3}$. Their product is $1/3$, not $1/\sqrt{3}$. Hence

$$\frac{1}{3} = 2 - 2\ln 3 + C, \qquad C = -\frac{5}{3} + 2\ln 3.$$

The initial condition fixes the one integration constant uniquely on $x > 1$.

Step 3: Evaluate at eight and simplify At $x = 8$, $\mu = \sqrt{7/9} = \sqrt{7}/3$. Therefore

$$\frac{\sqrt{7}}{3}y(8) = 8 - 2\ln 9 - \frac{5}{3} + 2\ln 3 = \frac{19}{3} - 2\ln 3.$$

Multiplication by three gives the requested quantity $\sqrt{7}\,y(8) = 19 - 6\ln 3$. The logarithmic term cannot be discarded as in (B), and its sign is fixed by integrating $-2/(x+1)$. The other expressions fail the integrating-factor evaluation with the same initial constant.

Step 4: Conclusion $\boxed{\sqrt{7}\,y(8) = 19 - 6\ln 3}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the product μy intact until the last step; at the initial point both factors here equal $1/\sqrt{3}$.

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12. Let the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ pass through the point $(2\sqrt{2}, -2\sqrt{2})$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola? [4 marks]

(A) $(2\sqrt{3}, 3\sqrt{2})$
(B) $(3\sqrt{3}, -6\sqrt{2})$
(C) $(\sqrt{3}, -\sqrt{6})$
(D) $(3\sqrt{6}, 6\sqrt{2})$

Correct Answer: (B) $(3\sqrt{3}, -6\sqrt{2})$

Solution:

Concept:

- For the hyperbola $x^2/a^2 - y^2/b^2 = 1$, its focal distance is $c = ae = \sqrt{a^2 + b^2}$, and the length of its latus rectum is $2b^2/a$.
- Under the intended axial convention, the directrix through $(-c, 0)$ is $x = -c$. A parabola with this directrix and focus $(c, 0)$ has vertex at the origin, equation $y^2 = 4cx$, and latus-rectum length $4c$.

Step 1: Use the latus-rectum condition The stated common focus is the positive focus $(c, 0)$ of the

hyperbola, while the directrix passes through its negative focus $(-c, 0)$. Using the standard axial directrix, the parabola has parameter c. The given length relation becomes

$$4c = e\frac{2b^2}{a}.$$

Since $c = ae$ and $a, e > 0$, cancellation gives $4a = 2b^2/a$, or $b^2 = 2a^2$. This relation determines the shape before the point on the hyperbola fixes the scale.

Step 2: Find the focal distance and parabola Substitute $(2\sqrt{2}, -2\sqrt{2})$ into the hyperbola:

$$\frac{8}{a^2} - \frac{8}{b^2} = 1.$$

Using $b^2 = 2a^2$, this reduces to $4/a^2 = 1$, so $a^2 = 4$, $b^2 = 8$, and $c = \sqrt{12} = 2\sqrt{3}$. Consequently the parabola is

$$y^2 = 8\sqrt{3}\,x.$$

Step 3: Test each proposed point For (B), $y^2 = (-6\sqrt{2})^2 = 72$, and $8\sqrt{3}\,x = 8\sqrt{3}(3\sqrt{3}) = 72$, so the point lies on the curve. For (A), the two sides are 18 and 48; for (C), they are 6 and 24; for (D), they are 72 and $72\sqrt{2}$. Each of these unequal pairs rejects that alternative. The sign of y is immaterial because the equation contains y^2.

Step 4: State the geometric qualification The wording does not explicitly require the directrix to be perpendicular to the hyperbola axis. If an oblique directrix through $(-c, 0)$ has unit normal $(\cos\theta, \sin\theta)$, the focus-to-directrix distance is $2c|\cos\theta|$, so its parabola has latus-rectum length $4c|\cos\theta|$. The supplied relation then gives $b^2 = 2a^2|\cos\theta|$, allowing different orientations and hyperbolas. Thus (B) is the keyed answer under the usual axial interpretation; the literal condition about passing through a focus alone does not specify a unique parabola.

Step 5: Conclusion Under the intended axial-directrix convention, the required point is $\boxed{(3\sqrt{3}, -6\sqrt{2})}$. Hence, the correct option is (B) with that convention. Without a restriction on directrix orientation, the literal wording is underdetermined.

Quick Tip to Solve: First cancel the eccentricity in the latus-rectum relation; this immediately yields $b^2 = 2a^2$, reducing the remaining algebra.

13. Let S be the set of all $a \in R$ for which the angle between the vectors $\vec{u} = a\,(\log_e b)\,\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\,\hat{i} + 2\hat{j} + 2a\,(\log_e b)\,\hat{k}$, $(b > 1)$ is acute. Then S is equal to : [4 marks]

(A) $\left(-\infty, -\frac{4}{3}\right)$

(B) Φ

(C) $\left(-\frac{4}{3}, 0\right)$

(D) $\left(\frac{12}{7}, \infty\right)$

Correct Answer: (B) Φ

Solution:

Concept:

- For two nonzero real vectors, their angle is acute exactly when their scalar product is positive, since the product of their magnitudes is positive.
- Put $L = \ln b$. As the allowed parameter b ranges over $b > 1$, L ranges over all positive real numbers, including arbitrarily small positive values. The keyed interpretation requires one fixed value of a to satisfy the condition for every allowed b. For a single fixed b, the answer instead depends on that parameter.

Step 1: Compute the scalar product The vectors are $(aL, -6, 3)$ and $(L, 2, 2aL)$. Both are nonzero because their second components are nonzero. Thus their scalar product is

$$\vec{u} \cdot \vec{v} = aL^2 - 12 + 6aL = a(L^2 + 6L) - 12.$$

The required inequality is therefore $a(L^2+6L) > 12$. In particular, no nonpositive a can work, because $L^2 + 6L > 0$.

Step 2: Check positive values of the proposed parameter If $a > 0$ is fixed, the inequality for an individual b is $a > 12/(L^2 + 6L)$. However, b is allowed to approach 1 from above, so L can approach zero from above. For every finite positive a, choose a sufficiently small $L > 0$ that $a(L^2 + 6L) < 12$. Then the scalar product is negative and the angle is obtuse. Equivalently, its limiting value as $b \to 1^+$ is -12. Hence no fixed real a ensures an acute angle throughout the stated parameter range.

Step 3: Compare the proposed sets Options (A) and (C) contain negative values, already ruled out by the sign of the scalar product. Option (D) corresponds to the special choice $L = 1$, or $b = e$, for which the threshold is $12/7$; that special value cannot replace the full range $b > 1$. The set for the allowed unrestricted parameter range is therefore empty.

Step 4: Conclusion Under the universal interpretation, no real a satisfies the condition for all allowed $b > 1$, so $\boxed{S = \varnothing}$. Hence, the correct option is (B) under this interpretation. For a single fixed $b > 1$, the set would instead be $\left(12/[(\ln b)^2 + 6\ln b], \infty\right)$.

Quick Tip to Solve: When a positive parameter has no lower bound away from zero, test its approach to zero before accepting a fixed threshold such as $12/7$.

14. Let A and B be two events such that $P(B \mid A) = \frac{2}{5}, P(A \mid B) = \frac{1}{7}$ and $P(A \cap B) = \frac{1}{9}$. Consider
(S1) $P(A' \cup B) = \frac{5}{6}$,
(S2) $P(A' \cap B') = \frac{1}{18}$
Then : [4 marks]

(A) Both (S1) and (S2) are true
(B) Both (S1) and (S2) are false
(C) Only (S1) is true
(D) Only (S2) is true

Correct Answer: (A) Both (S1) and (S2) are true

Solution:

Concept:

- Conditional probability gives $P(B \mid A) = P(A \cap B)/P(A)$ and $P(A \mid B) = P(A \cap B)/P(B)$. These identify the two marginal probabilities from their common intersection.
- Use complements to evaluate the stated events: $(A' \cup B)' = A \cap B'$, while $A' \cap B' = (A \cup B)'$. Inclusion-exclusion gives the probability of the union.

Step 1: Recover the marginal probabilities From the first conditional probability,

$$P(A) = \frac{P(A \cap B)}{P(B \mid A)} = \frac{1/9}{2/5} = \frac{5}{18}.$$

From the second,

$$P(B) = \frac{P(A \cap B)}{P(A \mid B)} = \frac{1/9}{1/7} = \frac{7}{9} = \frac{14}{18}.$$

The intersection is $2/18$, which is smaller than both marginal probabilities as required.

Step 2: Evaluate statement S1 by a complement The part of A outside B has probability

$$P(A \cap B') = P(A) - P(A \cap B) = \frac{5}{18} - \frac{2}{18} = \frac{1}{6}.$$

Its complement is $A' \cup B$, so

$$P(A' \cup B) = 1 - \frac{1}{6} = \frac{5}{6}.$$

Thus (S1) is true; subtracting $P(B)$ alone would refer to a different complement.

Step 3: Evaluate statement S2 by inclusion-exclusion The probability of at least one event is

$$P(A \cup B) = \frac{5}{18} + \frac{14}{18} - \frac{2}{18} = \frac{17}{18}.$$

De Morgan's law now gives

$$P(A' \cap B') = 1 - \frac{17}{18} = \frac{1}{18}.$$

Thus (S2) is also true. The four disjoint regions have probabilities $3/18$, $12/18$, $2/18$, and $1/18$, summing to one, which checks the arithmetic. Every alternative claiming one or both statements false is therefore excluded.

Step 4: Conclusion Both (S1) and (S2) are true. Hence, the correct option is (A).

Quick Tip to Solve: Obtain both marginals from the conditional probabilities first, then use a complement for $A' \cup B$ to avoid double counting.

15. The differential equation of the family of circles passing through the points $(0, 2)$ and $(0, -2)$ is : [4 marks]

(A) $2xy\frac{dy}{dx} + \left(x^2 - y^2 + 4\right) = 0$
(B) $2xy\frac{dy}{dx} + \left(x^2 + y^2 - 4\right) = 0$
(C) $2xy\frac{dy}{dx} + \left(y^2 - x^2 + 4\right) = 0$
(D) $2xy\frac{dy}{dx} - \left(x^2 - y^2 + 4\right) = 0$

Correct Answer: (A) $2xy\frac{dy}{dx} + \left(x^2 - y^2 + 4\right) = 0$

Solution:

Concept:

- A circle has general equation $x^2 + y^2 + Dx + Ey + F = 0$. The two specified points impose two conditions, leaving one arbitrary constant.
- Differentiate the one-parameter family once and eliminate its constant to obtain a first-order differential equation. The chain rule gives $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$.

Step 1: Determine the family of circles Substitute the two given points separately into the general equation. At $(0, 2)$, we get $4 + 2E + F = 0$; at $(0, -2)$, we get $4 - 2E + F = 0$. Subtracting gives $4E = 0$, so $E = 0$ and then $F = -4$. Thus every circle in the family is represented by

$$x^2 + y^2 + Dx - 4 = 0.$$

The coefficient D remains arbitrary, as expected because the centre can move along the perpendicular bisector of the fixed chord.

Step 2: Differentiate and eliminate the constant Differentiating with respect to x while treating D as constant gives

$$2x + 2y\frac{dy}{dx} + D = 0.$$

Multiply this equation by x. From the original family, $Dx = 4 - x^2 - y^2$. Substitution therefore gives

$$2x^2 + 2xy\frac{dy}{dx} + 4 - x^2 - y^2 = 0.$$

Combining the squared terms leaves

$$2xy\frac{dy}{dx} + x^2 - y^2 + 4 = 0.$$

This elimination avoids division by x, so it introduces no unnecessary denominator.

Step 3: Compare the signs in the options The required coefficient pattern outside the derivative is

$+x^2 - y^2 + 4$. Option (A) has exactly these signs. Options (B) and (C) change the relative signs of the squared terms, while option (D) negates the entire bracket without negating the derivative term; none is equivalent to the equation derived above.

Step 4: Conclusion The differential equation is $\boxed{2xy\frac{dy}{dx} + x^2 - y^2 + 4 = 0}$. Hence, the correct option is (A).

Quick Tip to Solve: For circles through two points on the y-axis symmetric about the origin, first set the linear y-term to zero, then eliminate the remaining x-coefficient by differentiation.

16. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin $O(0, 0)$. Then the area of the triangle OAB is : [4 marks]

(A) $\frac{3\sqrt{3}}{2}$

(B) $\frac{3\sqrt{3}}{4}$

(C) $\frac{3}{2\sqrt{3}}$

(D) $\frac{3}{4\sqrt{3}}$

Correct Answer: (B) $\frac{3\sqrt{3}}{4}$

Solution:

Concept:

- The radius to a point of tangency is perpendicular to its tangent. Consequently, the point of contact lies both on the circle and on the polar of the external point.
- The area of a triangle is half its base times its perpendicular height. The two contact points here are symmetric about the x-axis.

Step 1: Locate the centre and the contact chord Complete the square in the circle equation:

$$(x - 2)^2 + y^2 = 1.$$

Thus the centre is $C = (2, 0)$ and the radius is 1. Let a point of tangency be $T = (a, b)$. Since the tangent passes through $O = (0, 0)$, the vectors $CT = (a - 2, b)$ and $OT = (a, b)$ are perpendicular. Their dot product gives

$$a(a - 2) + b^2 = 0.$$

Meanwhile, the circle equation gives $a^2 + b^2 - 4a + 3 = 0$. Subtracting the perpendicularity equation from this one yields $-2a + 3 = 0$, hence $a = 3/2$.

Step 2: Find the two points and the triangle dimensions Substituting $x = 3/2$ into $(x - 2)^2 + y^2 = 1$ gives

$$\frac{1}{4} + y^2 = 1, \qquad y = \pm\frac{\sqrt{3}}{2}.$$

Hence the points of tangency are

$$A = \left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \quad B = \left(\frac{3}{2}, -\frac{\sqrt{3}}{2}\right).$$

The segment AB is vertical, has length $\sqrt{3}$, and lies on $x = 3/2$. Its perpendicular distance from the origin is $3/2$. These are the base and height of the required triangle.

Step 3: Calculate the area and compare Using the contact chord as the base,

$$[OAB] = \frac{1}{2} \times \sqrt{3} \times \frac{3}{2} = \frac{3\sqrt{3}}{4}.$$

Option (A) is twice this value, while options (C) and (D) are respectively $\sqrt{3}/2$ and $\sqrt{3}/4$, so they do not match the base-height calculation.

Step 4: Conclusion The area of $\triangle OAB$ is $\boxed{3\sqrt{3}/4}$ square units. Hence, the correct option is (B).

Quick Tip to Solve: For two tangents from a given point, find the contact chord first; its length and its distance from the external point give the triangle area directly.

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17. Let the lines
$\frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$ **and**
$\frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$ **be coplanar**
and P **be the plane containing these two lines.**
Then which of the following points does NOT lie on P? [4 marks]

(A) $(0, -2, -2)$

(B) $(-5, 0, -1)$

(C) $(3, -1, 0)$

(D) $(0, 4, 5)$

Correct Answer: (D) $(0, 4, 5)$

Solution:

Concept:

- Two nonparallel lines with direction vectors $\mathbf{u}, \mathbf{v}$ are coplanar when the displacement between points on them has zero scalar triple product with $\mathbf{u}, \mathbf{v}$.
- The cross product $\mathbf{u} \times \mathbf{v}$ is normal to their plane. A test point belongs to the plane precisely when it satisfies its Cartesian equation.

Step 1: Apply the coplanarity condition Take $A = (1, 2, 3)$ on the first line and $B = (-26, -18, -28)$ on the second. The direction vectors are $\mathbf{u} = (\lambda, 1, 2)$ and $\mathbf{v} = (-2, 3, \lambda)$. Then

$$\mathbf{u} \times \mathbf{v} = (\lambda - 6, -\lambda^2 - 4, 3\lambda + 2).$$

Since $B - A = (-27, -20, -31)$, coplanarity gives

$$\begin{aligned} 0 &= -27(\lambda - 6) + 20(\lambda^2 + 4) - 31(3\lambda + 2) \\ &= 20\lambda^2 - 120\lambda + 180 \\ &= 20(\lambda - 3)^2. \end{aligned}$$

Therefore $\lambda = 3$, with no second possible parameter value.

Step 2: Write the plane equation For $\lambda = 3$, the cross product is $(-3, -13, 11)$, which is nonzero. The unique plane through the first line and parallel to both directions therefore has equation

$$-3(x - 1) - 13(y - 2) + 11(z - 3) = 0,$$

or, equivalently,

$$3x + 13y - 11z + 4 = 0.$$

The coplanarity condition ensures that the second line also lies in this plane, rather than merely being parallel to it.

Step 3: Test all four points Evaluate $L(x, y, z) = 3x + 13y - 11z + 4$. At the first three listed points,

$$\begin{aligned} L(0, -2, -2) &= -26 + 22 + 4 = 0, \\ L(-5, 0, -1) &= -15 + 11 + 4 = 0, \\ L(3, -1, 0) &= 9 - 13 + 4 = 0. \end{aligned}$$

At the final point, $L(0, 4, 5) = 52 - 55 + 4 = 1 \neq 0$. Thus the first three points lie on the plane, while the fourth fails its equation.

Step 4: Conclusion The point $\boxed{(0, 4, 5)}$ does not lie on P. Hence, the correct option is (D).

Quick Tip to Solve: After finding the plane, substitute every option into one linear expression; keep its constant term when testing whether the result is zero.

18. A plane P is parallel to two lines whose direction ratios are $-2, 1, -3$ and $-1, 2, -2$ and it contains the point $(2, 2, -2)$. Let P intersect the co-ordinate axes at the points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin, and $\mathrm{p} = \alpha + \beta + \gamma$, then the ordered pair (V, p) is equal to : [4 marks]

(A) $(48, -13)$

(B) $(24, -13)$

(C) $(48, 11)$

(D) $(24, -5)$

Correct Answer: (B) $(24, -13)$

Solution:

Concept:

- A plane parallel to two nonparallel directions has a normal vector equal to their cross product. Through a known point, its equation is $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$.
- For axis intercepts α, β, γ, the tetrahedron formed with the origin has volume $V = |\alpha\beta\gamma|/6$. The intercepts themselves retain their signs.

Step 1: Find a normal and the plane equation The two directions are $\mathbf{u} = (-2, 1, -3)$ and $\mathbf{v} = (-1, 2, -2)$. Their cross product is

$$\mathbf{u} \times \mathbf{v} = (4, -1, -3).$$

It is perpendicular to both: $-8 - 1 + 9 = 0$ and $-4 - 2 + 6 = 0$. Since P passes through $(2, 2, -2)$,

$$4(x - 2) - (y - 2) - 3(z + 2) = 0.$$

Simplifying gives

$$4x - y - 3z = 12.$$

This equation incorporates both parallel directions and the specified point, so no additional plane parameter remains.

Step 2: Determine the signed intercepts To obtain the x-intercept set $y = z = 0$, giving $4x = 12$ and $\alpha = 3$. On the y-axis set $x = z = 0$, giving $-y = 12$ and $\beta = -12$. On the z-axis set $x = y = 0$, giving $-3z = 12$ and $\gamma = -4$. Thus the intercept form can also be written

$$\frac{x}{3} + \frac{y}{-12} + \frac{z}{-4} = 1.$$

Negative intercepts locate the vertices on the negative coordinate rays; they must not be replaced by positive numbers in the requested sum.

Step 3: Calculate the volume and intercept sum The three axis edges at the origin are mutually perpendicular. Therefore

$$V = \frac{|3(-12)(-4)|}{6} = \frac{144}{6} = 24, \qquad p = 3 - 12 - 4 = -13.$$

Options (A) and (C) incorrectly give volume 48. Option (D) has the correct volume but the wrong signed intercept sum. Only option (B) has both quantities correct.

Step 4: Conclusion The ordered pair is $\boxed{(V, p) = (24, -13)}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep signs when adding axis intercepts, but take the absolute value of their product when calculating a geometric volume.

19. A horizontal park is in the shape of a triangle OAB **with** AB $= 16$**. A vertical lamp post** OP **is erected at the point** O **such that** $\angle$PAO $= \angle$PBO $= 15^\circ$ **and** $\angle$PCO $= 45^\circ$**, where** C **is the midpoint of** AB**. Then** $(\text{OP})^2$ **is equal to :** [4 marks]

(A) $\frac{32}{\sqrt{3}}(\sqrt{3}-1)$

(B) $\frac{32}{\sqrt{3}}(2-\sqrt{3})$

(C) $\frac{16}{\sqrt{3}}(\sqrt{3}-1)$

(D) $\frac{16}{\sqrt{3}}(2-\sqrt{3})$

Correct Answer: (B) $\frac{32}{\sqrt{3}}(2-\sqrt{3})$

Solution:

Concept:

- Because the post is vertical and the park is horizontal, each triangle formed by the post and a point of the park is right-angled at O. Thus $\tan\theta = OP/\text{horizontal distance}$.
- Equal sides in a triangle make the median to their common base perpendicular to that base. Also, $\cot 15^\circ = 2+\sqrt{3}$.

Step 1: Express the ground distances using the height Put $OP = h > 0$. From $\angle PAO = \angle PBO = 15^\circ$, the right triangles AOP and BOP give

$$OA = OB = h\cot 15^\circ = h(2+\sqrt{3}).$$

From $\angle PCO = 45^\circ$, the right triangle COP gives

$$OC = \frac{h}{\tan 45^\circ} = h.$$

In particular, OAB is isosceles. Since C is the midpoint of $AB = 16$, we have $AC = 8$, and the median OC is perpendicular to AB.

Step 2: Apply Pythagoras in the ground triangle The triangle OCA is right-angled at C, so

$$OA^2 = OC^2 + AC^2.$$

Substitute the expressions already found:

$$h^2(2+\sqrt{3})^2 = h^2 + 64.$$

Expanding the square gives

$$(7+4\sqrt{3})h^2 = h^2 + 64,$$

hence

$$(6+4\sqrt{3})h^2 = 64.$$

This relation uses the horizontal median, not the slant distance PC; confusing those two lengths would change the answer.

Step 3: Simplify and match the squared height It follows that

$$h^2 = \frac{64}{6+4\sqrt{3}} = \frac{32}{3+2\sqrt{3}} = \frac{32}{\sqrt{3}}(2-\sqrt{3}).$$

For the last equality, factor the denominator as $\sqrt{3}(\sqrt{3}+2)$ and use $1/(2+\sqrt{3}) = 2-\sqrt{3}$. Option (B) agrees. Option (D) is half the required value, and options (A) and (C) use $\sqrt{3}-1$ instead of the necessary rationalising factor.

Step 4: Conclusion The required squared height is $\boxed{(OP)^2 = \frac{32}{\sqrt{3}}(2-\sqrt{3})}$. Hence, the correct option is (B).

Quick Tip to Solve: Equal angles of elevation from A and B imply equal ground distances from O; this makes the midpoint C useful through a right-angled ground triangle.

20. Let

p **: Ramesh listens to music.**

q **: Ramesh is out of his village.**

r **: It is Sunday.**

s **: It is Saturday.**

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as [4 marks]

(A) $((\sim q) \wedge (r \vee s)) \Rightarrow p$

(B) $(\mathrm{q} \wedge (\mathrm{r} \vee \mathrm{s})) \Rightarrow \mathrm{p}$

(C) $p \Rightarrow (q \wedge (r \vee s))$

(D) $\mathrm{p} \Rightarrow ((\sim \mathrm{q}) \wedge (\mathrm{r} \vee \mathrm{s}))$

Correct Answer: (D) $\mathrm{p} \Rightarrow ((\sim \mathrm{q}) \wedge (\mathrm{r} \vee \mathrm{s}))$

Solution:

Concept:

- The statement "p only if t" means $p \Rightarrow t$: the condition t is necessary for p, not a guarantee that p occurs.
- Negation reverses a statement, conjunction $\wedge$ means "and", and disjunction $\vee$ means "or". The grouping of these connectives determines the condition precisely.

Step 1: Translate each part of the required condition The given proposition q says that Ramesh is out of his village. Therefore "he is in his village" is represented by $\sim q$. The phrase "it is Sunday or Saturday" is represented by $r \vee s$. Both the location condition and this day condition are required, so

their combined statement is

$$t = (\sim q) \wedge (r \vee s).$$

The parentheses ensure that the choice between Sunday and Saturday is made within the day condition, while being in the village is required in either case.

Step 2: Use the direction of only if The full sentence says that listening to music can occur only when the combined condition holds. Hence, whenever p is true, t must be true:

$$p \Rightarrow \big((\sim q) \wedge (r \vee s)\big).$$

It does not assert that every Sunday or Saturday spent in the village necessarily includes listening to music. Thus the reverse implication would make a stronger and different claim than the one printed.

Step 3: Reject the other translations Option (A) reverses the implication: it makes the combined condition sufficient for music. Option (B) also reverses the implication and uses q, incorrectly placing Ramesh outside the village. Option (C) has the correct implication direction but again uses q instead of $\sim q$. Option (D) alone has the correct direction, location negation, and conjunction with the day disjunction.

Step 4: Conclusion The symbolic statement is $\boxed{p \Rightarrow ((\sim q) \wedge (r \vee s))}$. Hence, the correct option is (D).

Quick Tip to Solve: In "p only if t", place p before the implication arrow. Then translate the necessary condition t separately before combining the clauses.

Mathematics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

21. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}} + \beta x\right)^4, (1 - 3\beta x)^2$ and $\left(1 - \frac{\beta}{2}x\right)^6, \beta > 0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P. , then $50 - \frac{2d}{\beta^2}$ is equal to _____. [4 marks]

Solution:

Concept:

- For an even power n, the unique middle term of $(a + bx)^n$ is the $(n/2 + 1)$-th term, with coefficient $\binom{n}{n/2} a^{n/2} b^{n/2}$.
- Three numbers A, B, C are in an arithmetic progression precisely when $2B = A + C$. The common difference in their stated order is $d = B - A$.

Step 1: Extract the three middle coefficients In the fourth-power expansion the middle term is the coefficient of x^2; in the square it is the coefficient of x; in the sixth power it is the coefficient of x^3.

Therefore the three coefficients, in their given order, are

$$A = \binom{4}{2}\left(\frac{1}{\sqrt{6}}\right)^2 \beta^2 = \beta^2,$$
$$B = \binom{2}{1}(-3\beta) = -6\beta,$$
$$C = \binom{6}{3}\left(-\frac{\beta}{2}\right)^3 = -\frac{5}{2}\beta^3.$$

The negative signs in the second and third coefficients come from odd powers of their negative x-coefficients.

Step 2: Impose the arithmetic-progression condition Using $2B = A + C$,

$$-12\beta = \beta^2 - \frac{5}{2}\beta^3.$$

Since $\beta > 0$, division by β is allowed. Rearranging and clearing the denominator gives

$$5\beta^2 - 2\beta - 24 = 0, \qquad (5\beta - 12)(\beta + 2) = 0.$$

Thus the roots are $12/5$ and -2. The positivity condition excludes the negative root, leaving $\beta = 12/5$.

Step 3: Find the requested expression The common difference is $d = B - A = -6\beta - \beta^2$. Dividing this relation by β^2 avoids expanding large fractions:

$$\frac{d}{\beta^2} = -\frac{6}{\beta} - 1 = -\frac{6}{12/5} - 1 = -\frac{7}{2}.$$

Consequently,

$$50 - \frac{2d}{\beta^2} = 50 - 2\left(-\frac{7}{2}\right) = 57.$$

As a check, $A = 144/25$, $B = -360/25$, and $C = -864/25$, whose consecutive differences both equal $-504/25$.

Step 4: Conclusion The required numerical value is $\boxed{57}$.

Quick Tip to Solve: For middle coefficients, keep the signs from the binomial factors. After finding beta, calculate d divided by beta squared directly to simplify the final substitution.

22. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168 , then $\mathrm{b} + 3\ \mathrm{g}$ is equal to _____. [4 marks]

Solution:

Concept:

- The boys and girls are selected independently, so the multiplication principle gives $\binom{b}{3}\binom{g}{2} = 168$. Here $b \geq 3$ and $g \geq 2$ are integers.

- The binomial coefficients $\binom{b}{3}$ increase with b. Each admissible coefficient must divide 168, and the remaining factor must be a triangular number $\binom{g}{2}$.

Step 1: Bound the number of boys Since at least one pair of girls can be selected, $\binom{b}{3} \leq 168$. Now $\binom{11}{3} = 165$, whereas $\binom{12}{3} = 220 > 168$, so only $3 \leq b \leq 11$ need consideration. This finite bound prevents overlooking larger classes.

Step 2: List the possible divisors For $b = 3, 4, 5, 6, 7, 8, 9, 10, 11$, the values of $\binom{b}{3}$ are respectively $1, 4, 10, 20, 35, 56, 84, 120, 165$. Of these, only $1, 4, 56, 84$ divide 168. Thus the possible pairs $(b, \binom{g}{2})$ are $(3, 168), (4, 42), (8, 3), (9, 2)$.

Step 3: Require an integral number of girls For $\binom{g}{2} = T$, the equation $g(g-1) = 2T$ requires $1 + 8T$ to be an odd perfect square. The four values of this discriminant are $1345, 337, 25, 17$. Only 25 is a square: $36^2 < 1345 < 37^2$, $18^2 < 337 < 19^2$, and $4^2 < 17 < 5^2$. Hence $b = 8$ and $g = (1+5)/2 = 3$. Check: $\binom{8}{3}\binom{3}{2} = 56 \cdot 3 = 168$.

Step 4: Conclusion Therefore the required integer is

$$b + 3g = 8 + 3(3) = \boxed{17}.$$

Quick Tip to Solve: Factor the total number of selections first, then test which remaining factors are triangular numbers; this makes the integer search short and exhaustive.

23. Let $z = a + ib, b \neq 0$ be complex numbers satisfying $z^2 = \bar{z} \cdot 2^{1-z}$. Then the least value of $n \in N$, such that $z^n = (z+1)^n$, is equal to _____. [4 marks]

Solution:

Concept:

- For a positive real base, $2^{1-z} = \exp((1-z)\ln 2)$, so its modulus is 2^{1-a} when $z = a + ib$. The exponential with a real positive base is single-valued under this definition.
- If two nonzero complex numbers have equal positive integral powers, their moduli are equal. This necessary condition must be compatible with the equation defining z before an argument or root-of-unity calculation can determine n.

Step 1: Derive the necessary real part Assume that a positive integer n satisfies $z^n = (z+1)^n$. Taking moduli and the positive n-th root gives $|z| = |z+1|$. Squaring yields $a^2 + b^2 = (a+1)^2 + b^2$, hence $a = -1/2$. Since $b \neq 0$, neither z nor its conjugate is zero.

Step 2: Use the modulus of the defining equation Write $\rho = |z| > 0$. From $z^2 = \bar{z}\, 2^{1-z}$, taking moduli gives $\rho^2 = \rho\, 2^{1-a}$. Thus $\rho = 2^{3/2} = 2\sqrt{2}$. It follows that

$$b^2 = \rho^2 - a^2 = 8 - \frac{1}{4} = \frac{31}{4}.$$

Therefore the only candidates compatible with equality of powers are $z = -1/2 \pm i\sqrt{31}/2$.

Step 3: Show that neither candidate satisfies the equation Divide the defining equation by $\bar{z}$. Because

$z/\bar{z} = z^2/|z|^2$, its left side becomes $z^3/|z|^2$. At either candidate,

$$\mathrm{Re}\left(\frac{z^3}{|z|^2}\right) = \frac{a^3 - 3ab^2}{8} = \frac{23}{16} > 0.$$

The right side has real part $2\sqrt{2}\cos(b\ln 2)$. But $|b|\ln 2 = \sqrt{31}\ln 2/2$ lies strictly between $\pi/2$ and π: the elementary bounds $5.5 < \sqrt{31} < 6$ and $0.69 < \ln 2 < 0.70$ place it between 1.8975 and 2.1. Its cosine is negative, contradicting the positive real part on the left. The contradiction holds for either sign of b.

Step 4: Conclusion For the displayed equation, no such positive integer n exists. The supplied answer key lists 6, but that value is incompatible with the stated complex exponential equation, as the necessary-modulus and real-part checks prove.

Quick Tip to Solve: Before equating complex arguments, compare moduli and substitute the resulting candidates into the original equation; a root-of-unity shortcut is invalid if those candidates fail.

---

24. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\sigma^2$ is equal to _____.

[4 marks]

Solution:

Concept:

- Drawing three balls without replacement gives a hypergeometric random variable. For a population of N containing K white balls and a sample of size n, $\mathrm{Var}(X) = np(1-p)(N-n)/(N-1)$, where $p = K/N$.
- The finite-population factor $(N-n)/(N-1)$ is essential because the successive draws are dependent. A direct probability distribution provides an independent check.

Step 1: Identify the parameters There are $N = 10$ balls, $K = 4$ white balls and $n = 3$ draws. Thus $p = 4/10 = 2/5$ and $1 - p = 3/5$. The mean is $E[X] = np = 6/5$. No replacement is stated, so the binomial variance alone would be inappropriate.

Step 2: Compute the variance Using the without-replacement correction gives

$$\sigma^2 = 3\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)\left(\frac{7}{9}\right) = \frac{14}{25}.$$

Thus $100\sigma^2 = 56$. This is exact, so rounding does not change the integer.

Step 3: Verify from the full distribution The denominator is $\binom{10}{3} = 120$. For $X = 0, 1, 2, 3$, the numbers of selections are $\binom{4}{0}\binom{6}{3} = 20$, $\binom{4}{1}\binom{6}{2} = 60$, $\binom{4}{2}\binom{6}{1} = 36$, and $\binom{4}{3}\binom{6}{0} = 4$. They sum to 120. Consequently

$$E[X^2] = \frac{60 + 4(36) + 9(4)}{120} = 2, \qquad E[X]^2 = \frac{36}{25}.$$

Their difference is again $14/25$.

Step 4: Conclusion The printed data give $\boxed{100\sigma^2 = 56}$. The supplied answer key lists 57, but both the exact hypergeometric formula and the full distribution give 56; hence 56 is the value for the question as stated.

Quick Tip to Solve: For draws without replacement, multiply the binomial variance by the finite-population correction; verify a disputed numerical key using the full distribution.

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25. The value of the integral $\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}dx$ is equal to _____. [4 marks]

Solution:

Concept:

- A finite cosine sum removes the apparent quotient singularity: $\sin(6x)/\sin x = 2(\cos x + \cos 3x + \cos 5x)$.
- Integrate each cosine term using $\int \cos(kx)\,dx = \sin(kx)/k$. At the upper endpoint the three sine values alternate in sign.

Step 1: Derive a useful trigonometric identity Use $2\sin x\cos(kx) = \sin((k+1)x) - \sin((k-1)x)$. Adding this identity for $k = 1, 3, 5$ makes all intermediate sine terms cancel:

$$2\sin x(\cos x + \cos 3x + \cos 5x) = \sin 6x.$$

Division is valid for $0 < x \le \pi/2$. At zero the quotient has the finite limit 6, so this endpoint causes no improper divergence.

Step 2: Integrate the finite sum Writing the original integral as I, the identity gives

$$I = 120\int_0^{\pi/2}(\cos x + \cos 3x + \cos 5x)\,dx.$$

Therefore

$$I = 120\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5}\right]_0^{\pi/2}.$$

The lower endpoint contributes zero in all three terms.

Step 3: Evaluate the endpoint signs carefully At $x = \pi/2$, the sine values are $1, -1, 1$, respectively. In particular, $\sin(3\pi/2) = -1$, so the middle contribution is negative. Thus

$$I = 120\left(1 - \frac{1}{3} + \frac{1}{5}\right) = 120\left(\frac{15-5+3}{15}\right) = 120\left(\frac{13}{15}\right) = 104.$$

The prefactor 60 in the question has already combined with the factor 2 in the identity.

Step 4: Conclusion The value of the given definite integral is $\boxed{104}$.

Quick Tip to Solve: For an even multiple in $\sin(2mx)/\sin x$, telescope into odd cosines; integrate the finite sum and track the alternating endpoint signs.

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26. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point $R(\sqrt{2}, 2\sqrt{2}-2)$. If S is the focus of the ellipse on its negative major axis, then SP^2+SQ^2 is equal to _____. [4 marks]

Solution:

Concept:

- The ellipse has vertical major semiaxis $a=2$, horizontal semiaxis $b=\sqrt{2}$, and focal distance $c=\sqrt{a^2-b^2}=\sqrt{2}$. Its negative-major-axis focus is $S=(0,-\sqrt{2})$.
- If tangents from $R=(x_1,y_1)$ meet an ellipse, their contact points lie on the chord of contact $xx_1/2+yy_1/4=1$. Intersecting this line with the ellipse gives both contacts.

Step 1: Write the chord of contact Here $R=(\sqrt{2}, 2\sqrt{2}-2)$, so the contact points satisfy

$$\frac{x}{\sqrt{2}}+\frac{(\sqrt{2}-1)y}{2}=1.$$

Introduce $X=x/\sqrt{2}$, $Y=y/2$, and $k=\sqrt{2}-1$. The ellipse becomes $X^2+Y^2=1$, and the contact chord becomes $X+kY=1$.

Step 2: Find the two points of tangency Substitute $X=1-kY$ into the unit-circle equation:

$$(1-kY)^2+Y^2=1, \qquad Y((1+k^2)Y-2k)=0.$$

One solution is $(X,Y)=(1,0)$, giving $P=(\sqrt{2},0)$. The other has $Y=2k/(1+k^2)=1/\sqrt{2}$ and $X=1-k/\sqrt{2}=1/\sqrt{2}$. Hence $Q=(1,\sqrt{2})$. Both points satisfy the original ellipse and contact chord; swapping their names does not affect the required sum.

Step 3: Calculate squared distances to the lower focus With $S=(0,-\sqrt{2})$, the Cartesian distance formula gives

$$SP^2=(\sqrt{2})^2+(0+\sqrt{2})^2=4,$$

and

$$SQ^2=(1-0)^2+(\sqrt{2}+\sqrt{2})^2=1+8=9.$$

The sign of the focus matters: the question specifies the negative major axis, so use $-\sqrt{2}$ as its ordinate.

Step 4: Conclusion Therefore $\boxed{SP^2+SQ^2=4+9=13}$.

Quick Tip to Solve: Scale an ellipse to a unit circle before intersecting its chord of contact; the algebra becomes a simple quadratic with one readily visible root.

27. If $1+(2+{}^{49}C_1+{}^{49}C_2+\ \ldots\ +\ {}^{49}C_{49})({}^{50}C_2+{}^{50}C_4+\ \ldots\ +\ {}^{50}C_{50})$ is equal to $2^{\mathrm{n}}\cdot \mathrm{m}$, where m is odd, then $\mathrm{n}+\mathrm{m}$ is equal to _____. [4 marks]

Solution:

Concept:

- The binomial theorem gives $\sum_{k=0}^{n}\binom{n}{k}=2^n$. The sum of the even-indexed coefficients is 2^{n-1}, obtained by adding the expansions of $(1+1)^n$ and $(1-1)^n$.
- In a representation $2^n m$ with m odd, n is the full power of 2 dividing the integer. Carefully include or exclude the constant coefficient before multiplying.

Step 1: Evaluate the first factor The first parenthesis starts with 2, followed by every coefficient from $\binom{49}{1}$ through $\binom{49}{49}$. Since the omitted coefficient is $\binom{49}{0}=1$,

$$A=2+\left(2^{49}-1\right)=2^{49}+1.$$

The initial 2 is therefore not simply the constant term of the usual binomial sum.

Step 2: Evaluate the second factor Adding the two binomial expansions yields

$$2\left(\binom{50}{0}+\binom{50}{2}+\cdots+\binom{50}{50}\right)=2^{50}.$$

Therefore the sum including $\binom{50}{0}$ equals 2^{49}. The displayed second parenthesis starts at index 2, so

$$B=\binom{50}{2}+\binom{50}{4}+\cdots+\binom{50}{50}=2^{49}-1.$$

Step 3: Use the difference of squares Keep the separate 1 outside the product. The complete expression is

$$1+AB=1+(2^{49}+1)(2^{49}-1)=1+2^{98}-1=2^{98}.$$

Thus its odd part is exactly 1. Taking a smaller exponent would leave an even factor, and taking a larger exponent would not leave an integer odd factor, so the required representation has $n=98$, $m=1$.

Step 4: Conclusion Hence $\boxed{n+m=98+1=99}$.

Quick Tip to Solve: Before multiplying binomial sums, check whether the constant coefficient is included; conjugate factors 2^k+1 and 2^k-1 often collapse immediately.

28. Two tangent lines l_1 and l_2 are drawn from the point $(2, 0)$ to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x - 5)^2 + y^2 = r$, then 17r is equal to _____. [4 marks]

Solution:

Concept:

- A nonvertical line through $(2, 0)$ can be written $y = m(x - 2)$. It is tangent to a parabola when substitution gives a quadratic with zero discriminant.
- A line is tangent to a circle when its perpendicular distance from the centre equals the radius. In $(x - 5)^2 + y^2 = r$, the symbol r denotes the squared radius, not the radius itself.

Step 1: Find the slopes of the parabola tangents Substitute $y = m(x - 2)$ into $2y^2 = -x$. Rearranging gives

$$2m^2x^2 + (1 - 8m^2)x + 8m^2 = 0.$$

Tangency means a repeated root, so

$$(1 - 8m^2)^2 - 4(2m^2)(8m^2) = 0.$$

The quartic terms cancel, leaving $1 - 16m^2 = 0$, hence $m = \pm 1/4$. The vertical line through the given point is $x = 2$, which has no real intersection with this left-opening parabola, so no tangent has been omitted.

Step 2: Write the two line equations The positive-slope tangent is $y = (x - 2)/4$, or $x - 4y - 2 = 0$. The negative-slope tangent is $y = -(x - 2)/4$, or $x + 4y - 2 = 0$. These lines are reflections in the x-axis, consistent with the symmetry of the parabola and the position of the exterior point.

Step 3: Impose circle tangency The centre of the circle is $C = (5, 0)$. Its perpendicular distance to either line is

$$d = \frac{|5 \mp 4(0) - 2|}{\sqrt{1^2 + 4^2}} = \frac{3}{\sqrt{17}}.$$

Since both distances agree, a single circle centred at C is tangent to both lines. Its radius squared is $d^2 = 9/17$, which is precisely the parameter r in the displayed circle equation.

Step 4: Conclusion Thus $r = 9/17$, and the required value is $\boxed{17r = 9}$.

Quick Tip to Solve: After finding tangent slopes with a discriminant, use the centre-to-line distance. Read the circle equation carefully to distinguish radius from radius squared.

29. $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \ldots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then m . n is equal to _____.

[4 marks]

Solution:

Concept:

- The finite geometric-series formula is $a + ar + \cdots + ar^{N-1} = a(r^N - 1)/(r - 1)$, for $r \neq 1$. Here the series from the second term onward has a common ratio.
- Every positive integer can be expressed uniquely as $2^n m$, where m is odd; remove all powers of

two before reading off m.

Step 1: Separate the exceptional first term The first numerator is 6, whereas the remaining numerators double: $10, 20, 40, \ldots, 10240$. From the second term to the last, the denominator exponent decreases from 11 to 1. There are therefore 11 terms in that geometric progression. Its first term is $10/3^{11}$, and each next term is obtained by multiplying by $2 \times 3 = 6$. The initial term $6/3^{12}$ is kept separate; including it in the same progression would use the wrong ratio.

Step 2: Sum the progression and cancel the residual term Writing the given sum as S, we obtain

$$\begin{aligned} S &= \frac{6}{3^{12}} + \frac{10}{3^{11}} \frac{6^{11} - 1}{6 - 1} \\ &= \frac{2}{3^{11}} + \frac{2(6^{11} - 1)}{3^{11}} \\ &= \frac{2 \cdot 6^{11}}{3^{11}} = 2 \cdot 2^{11} = 2^{12}. \end{aligned}$$

The cancellation is exact; no numerical rounding of the small fractions is needed.

Step 3: Identify the odd factor Comparing $S = 2^{12}$ with $2^n m$, the odd factor is $m = 1$. Any smaller exponent of 2 would leave an even factor and violate the condition that m is odd. Thus $n = 12$.

Step 4: Conclusion The required product is $mn = 1 \times 12 = \boxed{12}$.

Quick Tip to Solve: When one initial term breaks a geometric pattern, sum the regular tail separately; the exceptional term often cancels the tail formula's residual constant.

30. Let $S = \left[-\pi, \frac{\pi}{2}\right) - \left\{-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right\}$. Then the number of elements in the set $A = \{\theta \in S : \tan\theta(1 + \sqrt{5}\tan(2\theta)) = \sqrt{5} - \tan(2\theta)\}$ is _____. [4 marks]

Solution:

Concept:

- The addition formula gives $\tan(\theta + 2\theta) = (\tan\theta + \tan 2\theta)/(1 - \tan\theta\tan 2\theta)$, when the denominator is nonzero.
- For a tangent equation $\tan u = c$, all solutions are $u = \tan^{-1} c + k\pi$, with integer k. Count only solutions inside the specified interval and then check exclusions.

Step 1: Recognise the tangent addition formula Rearrange the given equation without dividing by either tangent:

$$\tan\theta + \tan 2\theta = \sqrt{5}(1 - \tan\theta\tan 2\theta).$$

Both tangents exist on the supplied set S. Also, the factor on the right cannot vanish for a solution. If it vanished, put $u = \tan\theta$, $v = \tan 2\theta$. We would have $uv = 1$ and $u + v = 0$, giving $-u^2 = 1$, impossible for real u. Division is consequently valid at every solution, and the equation becomes $\tan 3\theta = \sqrt{5}$.

Step 2: Count the solutions in the interval Let $\alpha = \tan^{-1}\sqrt{5}$, so $0 < \alpha < \pi/2$. Then

$$\theta = \frac{\alpha + k\pi}{3}, \qquad k \in \mathbb{Z}.$$

The interval restriction yields

$$-3\pi \le \alpha + k\pi < \frac{3\pi}{2}, \qquad -3 - \frac{\alpha}{\pi} \le k < \frac{3}{2} - \frac{\alpha}{\pi}.$$

Since $0 < \alpha/\pi < 1/2$, the integers are exactly $k = -3, -2, -1, 0, 1$.

Step 3: Check the removed angles The removed angles are $-\pi/2, -\pi/4, -3\pi/4, \pi/4$. At these angles, $\tan 3\theta$ is either undefined or equals 1 or -1, never $\sqrt{5}$. Thus none of the five counted angles is removed from S, and every counted solution satisfies the original addition-form equation.

Step 4: Conclusion The set A has $\boxed{5}$ elements.

Quick Tip to Solve: Before replacing a rearranged equation by a tangent addition formula, check that its denominator cannot vanish; then count integer parameters using the original half-open interval.

Physics – Section A: Multiple Choice Questions (80 marks)

31. Consider the efficiency of carnot's engine is given by $\eta = \frac{\alpha\beta}{\sin\theta}\log_e\frac{\beta x}{kT}$, where α and β are constants. If T is temperature, k is Boltzmann constant, θ is angular displacement and x has the dimensions of length. Then, choose the incorrect option : [4 marks]

(A) Dimensions of β is same as that of force.

(B) Dimensions of $\alpha^{-1}x$ is same as that of energy.

(C) Dimensions of $\eta^{-1}\sin\theta$ is same as that of $\alpha\beta$.

(D) Dimensions of α is same as that of β.

Correct Answer: (D) Dimensions of α is same as that of β.

Solution:

Concept:

- The argument of a logarithm is dimensionless. Since Boltzmann's constant times temperature is an energy, $[kT] = ML^2\mathsf{T}^{-2}$.
- Efficiency, angle and its sine are dimensionless. Products and inverse powers of dimensional constants must therefore be checked independently of their numerical values.

Step 1: Determine the dimension of beta The logarithmic argument is $\beta x/(kT)$. With $[x] = L$, dimensional consistency demands

$$\frac{[\beta]L}{ML^2\mathsf{T}^{-2}} = 1 \quad \Rightarrow \quad [\beta] = ML\mathsf{T}^{-2}.$$

This is precisely the dimension of force. Hence statement (A) is correct. The temperature symbol T in the question should not be confused with the time dimension denoted here by T.

Step 2: Determine the dimension of alpha The logarithm and $\sin\theta$ are dimensionless, and so is η. Therefore the prefactor requires $[\alpha\beta] = 1$, giving

$$[\alpha] = M^{-1}L^{-1}\mathsf{T}^2, \qquad [\alpha^{-1}x] = (ML\mathsf{T}^{-2})L = ML^2\mathsf{T}^{-2}.$$

Thus statement (B) correctly identifies an energy. Alpha is an inverse force, not a force.

Step 3: Test the remaining statements The combination $\eta^{-1}\sin\theta$ is dimensionless, as is $\alpha\beta$; statement (C) is consequently correct. Statement (D) asserts that alpha and beta have identical dimensions, but their dimensions are reciprocal force and force, respectively. These differ in every mass, length and time exponent. The question asks for the incorrect statement, so select this mismatch rather than one of the three valid dimensional relations.

Step 4: Conclusion The dimensions of α and β are different. Hence, the correct option is (D).

Quick Tip to Solve: In a dimensional formula involving a logarithm, first make its argument dimensionless, then use the dimensions of the entire prefactor.

32. At time $t = 0$ a particle starts travelling from a height $7\hat{z}$ cm in a plane keeping z coordinate constant. At any instant of time it's position along the $\hat{x}$ and $\hat{y}$ directions are defined as $3\mathrm{t}$ and $5\mathrm{t}^3$ respectively. At t = 1s acceleration of the particle will be [4 marks]

(A) $-30\hat{y}$

(B) $30\hat{y}$

(C) $3\hat{x} + 15\hat{y}$

(D) $3\hat{x} + 15\hat{y} + 7\hat{z}$

Correct Answer: (B) $30\hat{y}$

Solution:

Concept:

- For motion in fixed Cartesian directions, velocity is $\vec{v} = d\vec{r}/dt$ and acceleration is $\vec{a} = d^2\vec{r}/dt^2$. Differentiate each coordinate separately.
- A constant coordinate contributes neither velocity nor acceleration. A coordinate linear in time gives a constant velocity but zero acceleration in that direction.

Step 1: Write the position and differentiate once The position in the coordinate units used in the question is

$$\vec{r}(t) = 3t\,\hat{x} + 5t^3\,\hat{y} + 7\,\hat{z}.$$

The height remains fixed throughout the motion. The unit vectors are fixed in Cartesian space, so only the scalar coefficients need differentiation. Therefore

$$\vec{v}(t) = 3\,\hat{x} + 15t^2\,\hat{y}.$$

There is no $\hat{z}$ term in velocity because the derivative of the constant height is zero.

Step 2: Differentiate again and substitute the time The x component of velocity is constant, while its

y component changes with time. A second derivative gives

$$\vec{a}(t) = 0\,\hat{x} + 30t\,\hat{y} + 0\,\hat{z}.$$

At $t = 1\,\text{s}$, this becomes $\vec{a} = 30\,\hat{y}$, directed along positive y. Using centimetres for the given coordinates, its unit is $\text{cm}\,\text{s}^{-2}$. The motion occurs in a plane, but that does not force its acceleration to have nonzero components in every coordinate direction.

Step 3: Reject the other vectors Option (A) reverses the sign of the second derivative and is incorrect. Option (C) reproduces the velocity components at $t = 1$, not the acceleration; it retains a spurious x component and uses 15 instead of 30 for y. Option (D) additionally retains the constant height as though it were a derivative. Only option (B) contains the second derivative of the position vector.

Step 4: Conclusion At $t = 1\,\text{s}$, $\vec{a} = 30\hat{y}\,\text{cm}\,\text{s}^{-2}$ in the stated coordinate units. Hence, the correct option is (B).

Quick Tip to Solve: Label each line as position, velocity or acceleration: the powers of time decrease at each differentiation, and constant coordinates disappear immediately.

33. A pressure-pump has a horizontal tube of cross sectional area $10\ \text{cm}^2$ for the outflow of water at a speed of $20\ \text{m/s}$. The force exerted on the vertical wall just in front of the tube which stops water horizontally flowing out of the tube, is :

[given: density of water $= 1000\ \text{kg/m}^3$] [4 marks]

(A) 300 N

(B) 500 N

(C) 250 N

(D) 400 N

Correct Answer: (D) 400 N

Solution:

Concept:

- The mass flow rate of a steady jet is $\dot{m} = \rho A v$, where ρ is density, A the cross-sectional area and v the incident speed.
- The force equals the rate of change of momentum. When the horizontal jet is stopped in its original direction, the horizontal momentum change per unit mass has magnitude v.

Step 1: Convert the area and find mass flow rate The flow area must be in square metres to match the density and speed:

$$A = 10\,\text{cm}^2 = 10 \times 10^{-4}\,\text{m}^2 = 10^{-3}\,\text{m}^2.$$

In a time interval Δt, a water column of length $v\Delta t$ strikes the wall, so its mass is $\rho A v \Delta t$. Thus

$$\dot{m} = 1000 \times 10^{-3} \times 20 = 20\,\text{kg}\,\text{s}^{-1}.$$

This is a continuous flow, so the mass reaching the wall each second is the relevant mass; the total

amount of water in the pump is unnecessary.

Step 2: Calculate the horizontal momentum transfer The wall brings the incident horizontal velocity from $20\,\mathrm{m\,s^{-1}}$ to zero. The magnitude of the wall's force on the water is

$$F = \dot{m}(v - 0) = 20 \times 20 = 400\,\mathrm{N}.$$

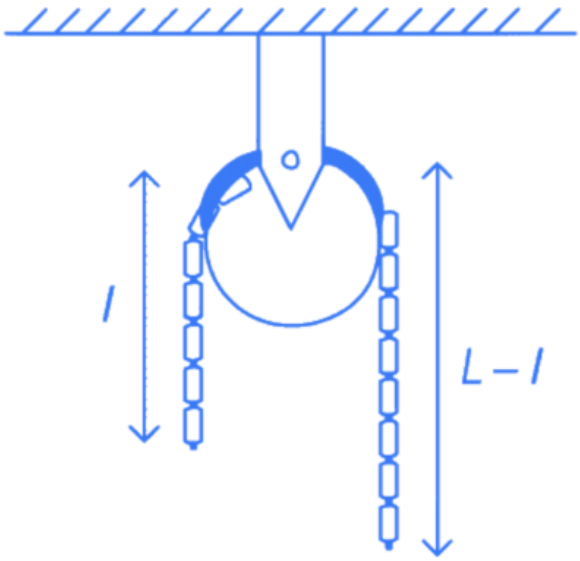
By Newton's third law, the force exerted by the water on the wall has the same magnitude and points in the incident jet direction. Equivalently, combine the two steps as $F = \rho A v^2$. There is no factor of two because the stream is stopped horizontally, not reflected straight back with equal speed.

Step 3: Compare with the options The momentum-flux result is exactly $400\,\mathrm{N}$. Values 300, 500 and $250\,\mathrm{N}$ do not equal $\rho A v^2$ for the supplied area, density and speed, so (A), (B) and (C) are excluded. Density times area times speed squared has units $\mathrm{kg\,m\,s^{-2}}$, confirming that it represents a force.

Step 4: Conclusion The water exerts a force of $\boxed{400\,\mathrm{N}}$ on the wall. Hence, the correct option is (D).

Quick Tip to Solve: For a jet striking a wall, multiply mass arriving per second by the change in velocity; double the speed change only for a jet reflected straight backwards.

34. A uniform metal chain of mass m and length 'L' passes over a massless and frictionless pulley. It is released from rest with a part of its length 'l' is hanging on one side and rest of its length 'L − l' is hanging on the other side of the pully. At a certain point of time, when $l = \frac{L}{x}$, the acceleration of the chain is $\frac{g}{2}$. The value of x is _____. [4 marks]



(A) 6

(B) 2

(C) 1.5

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- A uniform chain has linear mass density $\lambda = m/L$; hanging lengths l and $L - l$ therefore have masses λl and $\lambda(L - l)$.
- For an ideal massless, frictionless pulley, the difference of the hanging weights drives the chain, while the entire mass m is accelerated.

Step 1: Choose the direction from the figure The supplied figure labels l on the shorter hanging side and $L - l$ on the longer side. The longer side descends and pulls the shorter side upward. Both sides have the same speed and acceleration magnitude because the chain is inextensible. The driving force in that direction is

$$F = \lambda(L - l)g - \lambda l g = \lambda g(L - 2l).$$

The uniform-density condition is essential: the ratio of the two hanging masses is the ratio of their lengths.

Step 2: Apply the equation of motion to the whole chain The total accelerated mass is $\lambda L = m$. Summing the equations for the two sides eliminates the common tension and gives

$$\lambda g(L - 2l) = \lambda L a, \qquad a = g\left(1 - \frac{2l}{L}\right).$$

Use the specified instant, for which $a = g/2$ and $l = L/x$:

$$\frac{1}{2} = 1 - \frac{2}{x} \quad \Rightarrow \quad \frac{2}{x} = \frac{1}{2} \quad \Rightarrow \quad x = 4.$$

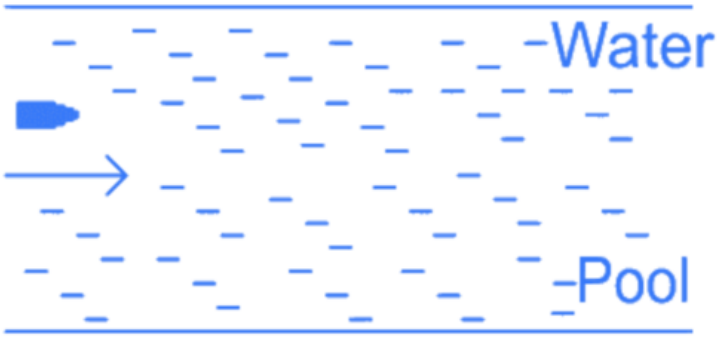
The chain's initial release from rest is irrelevant to this instantaneous force calculation; there is no need to find its speed.

Step 3: Check the resulting lengths and alternatives For $x = 4$, the two hanging lengths are $L/4$ and $3L/4$. Their masses differ by $m/2$, so the weight difference is $mg/2$, which accelerates total mass m at exactly $g/2$. Option (A), $x = 6$, would give $2g/3$; option (B), $x = 2$, gives equal lengths and zero acceleration. Option (C), $x = 1.5$, also contradicts the shorter-side label and gives an acceleration magnitude $g/3$, not $g/2$.

Step 4: Conclusion The required value is $\boxed{x = 4}$. Hence, the correct option is (D).

Quick Tip to Solve: For an ideal uniform hanging chain, divide the difference of the hanging lengths by their sum to obtain a/g; identify the shorter side from the figure.

35. A bullet of mass $200\ \mathrm{g}$ having initial kinetic energy $90\ \mathrm{J}$ is shot inside a long swimming pool as shown in the figure. If it's kinetic energy reduces to $40\ \mathrm{J}$ within $1\ \mathrm{s}$, the minimum length of the pool, the bullet has to travel so that it completely comes to rest is [4 marks]



(A) 45 m

(B) 90 m

(C) 125 m

(D) 25 m

Correct Answer: (A) 45 m

Solution:

Concept:

- Translational kinetic energy is $K = \frac{1}{2}mv^2$; use it to convert each given energy into a speed before applying the kinematic equations.
- In the uniform-retardation model for the bullet in the water, $v = u + at$ and $v^2 = u^2 + 2as$. The required pool length is the distance from entry until speed reaches zero.

Step 1: Determine the two speeds The mass is $m = 200\,\mathrm{g} = 0.200\,\mathrm{kg}$. The entry kinetic energy gives

$$u = \sqrt{\frac{2(90)}{0.200}} = \sqrt{900} = 30\,\mathrm{m\,s^{-1}}.$$

After one second, the kinetic energy is $40\,\mathrm{J}$, so

$$v_1 = \sqrt{\frac{2(40)}{0.200}} = \sqrt{400} = 20\,\mathrm{m\,s^{-1}}.$$

Energy is proportional to speed squared, so the ratio of the two speeds is not the ratio $40/90$.

Step 2: Find the retardation Using the speeds separated by $1\,\mathrm{s}$, the uniform acceleration is

$$a = \frac{v_1 - u}{1} = 20 - 30 = -10\,\mathrm{m\,s^{-2}}.$$

The negative sign shows that the water resists the motion. As a check, the distance during the first second is $(30 + 20)/2 = 25\,\mathrm{m}$, and the resisting force magnitude is $m|a| = 2\,\mathrm{N}$. Its work over $25\,\mathrm{m}$ is $50\,\mathrm{J}$, exactly the observed energy loss.

Step 3: Calculate the full stopping distance At the stopping point the final speed is zero. Using the

entry speed, not merely the speed after one second, gives

$$0 = 30^2 + 2(-10)s \quad \Rightarrow \quad s = \frac{900}{20} = 45\,\text{m}.$$

The remaining distance after the first second is $20^2/(2\times10) = 20\,\text{m}$, so the total is again $25+20 = 45\,\text{m}$. Option (D) counts only the first-second distance. Options (B) and (C) do not satisfy the stopping equation with the calculated retardation.

Step 4: Conclusion The minimum required pool length in this uniform-retardation model is $\boxed{45\,\text{m}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert energies into speeds using square roots, then calculate the stopping distance from the initial speed; the first timed interval is only part of the total travel.

36. Assume there are two identical simple pendulum clocks. Clock - 1 is placed on the earth and Clock - 2 is placed on a space station located at a height h above the earth surface. Clock - 1 and Clock - 2 operate at time periods 4 s and 6 s respectively. Then the value of h is -
(consider radius of earth $R_E = 6400$ km and g on earth $10\ \text{m/s}^2$) [4 marks]

(A) 1200 km

(B) 1600 km

(C) 3200 km

(D) 4800 km

Correct Answer: (C) 3200 km

Solution:

Concept:

- For identical simple pendulums, $T = 2\pi\sqrt{L/g}$, where L is the common length and g is the local gravitational acceleration.
- At height h above a spherical Earth, $g_h = g_0[R_E/(R_E+h)]^2$. Hence the period increases in direct proportion to distance from the Earth's centre.

Step 1: Compare the two periods The clocks have the same pendulum length, so their period ratio removes the common factors 2π and $\sqrt{L}$. The gravitational acceleration at the elevated clock must be smaller because its period is longer.

$$\frac{T_h}{T_0} = \sqrt{\frac{g_0}{g_h}} = \frac{R_E + h}{R_E}.$$

The ratio is $6/4 = 3/2$, not $(6/4)^2$; the square root in the pendulum formula cancels the inverse square dependence of gravity.

Step 2: Calculate the altitude Substitute the measured periods and the given Earth radius into the

exact height relation, without using a small-height approximation:

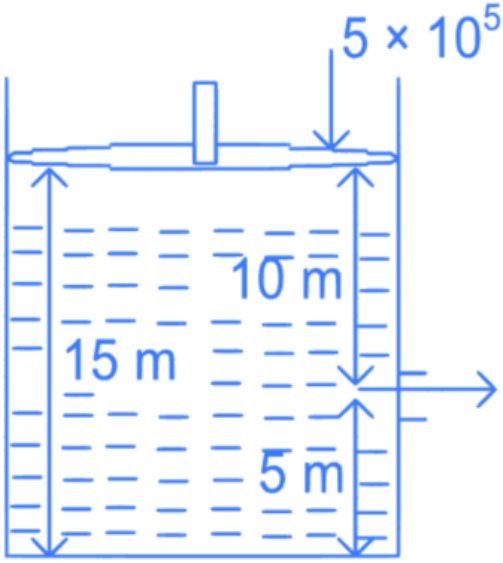
$$\frac{6}{4} = 1 + \frac{h}{6400}, \qquad h = 6400\left(\frac{3}{2} - 1\right) = 3200\,\text{km}.$$

As a check, $g_h = 10(2/3)^2 = 40/9\,\text{m s}^{-2}$, giving $T_h/T_0 = 3/2$. The other proposed heights produce period ratios 1.1875, 1.25, and 1.75, respectively, so they cannot match the observed ratio.

Step 3: Conclusion The required height is $\boxed{h = 3200\,\text{km}}$. Hence, the correct option is (C).

Quick Tip to Solve: For identical pendulums at different heights, use $T_h/T_0 = (R_E + h)/R_E$ directly; keep the height and radius in the same units.

37. Consider a cylindrical tank of radius 1 m is filled with water. The top surface of water is at 15 m from the bottom of the cylinder. There is a hole on the wall of cylinder at a height of 5 m from the bottom. A force of 5×10^5 N is applied an the top surface of water using a piston. The speed of ifflux from the hole will be : (given atmospheric pressure $\text{P}_\text{A} = 1.01 \times 10^5$ Pa, density of water $\rho_\text{W} = 1000$ kg/m^3 and gravitational acceleration $\text{g} = 10$ m/s^2) [4 marks]



(A) 11.6 m/s

(B) 10.8 m/s

(C) 17.8 m/s

(D) 14.4 m/s

Correct Answer: (C) 17.8 m/s

Solution:

Concept:

- Bernoulli's equation relates pressure energy, kinetic energy and gravitational potential energy per unit volume along the water flow.

- The pressure on the water under the piston is $P_1 = F/A$, with piston area $A = \pi r^2$. At the outlet the water pressure is atmospheric, $P_2 = P_A$.

Step 1: Identify pressure and height differences The tank radius is $1\,\text{m}$, so the piston area is $A = \pi\,\text{m}^2$. The given piston force establishes

$$P_1 = \frac{5 \times 10^5}{\pi} \simeq 1.592 \times 10^5\,\text{Pa}.$$

The hole is $15 - 5 = 10\,\text{m}$ below the top water surface. Its height above the bottom is not the head to use. The jet opens into air, so subtract $P_A = 1.01 \times 10^5\,\text{Pa}$ from the piston pressure.

Step 2: Apply Bernoulli and evaluate the speed The upper surface moves negligibly compared with the narrow outlet jet. Thus Bernoulli between the top surface and the hole gives

$$\frac{1}{2}\rho v^2 = P_1 - P_A + \rho g(10).$$

Using the stated density and gravity,

$$v^2 = \frac{2}{1000}\left[\frac{5 \times 10^5}{\pi} - 1.01 \times 10^5 + 1000(10)(10)\right] \simeq 316.3.$$

Therefore $v \simeq 17.8\,\text{m s}^{-1}$. The pressure head is positive as well as the gravitational head. The other speeds do not satisfy this energy balance; using only the height head would also omit the piston's pressure contribution.

Step 3: Conclusion The efflux speed is $\boxed{17.8\,\text{m s}^{-1}}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the vertical distance from the water surface to the hole, and compare the piston pressure F/A with the atmospheric pressure at the jet.

38. A vessel contains $14\,\text{g}$ of nitrogen gas at a temperature of 27°C. The amount of heat to be transferred to the gas to double the r.m.s speed of its molecules will be :

Take $\text{R} = 8.32\,\text{J mol}^{-1}\,\text{k}^{-1}$**.** [4 marks]

(A) 2229 J

(B) 5616 J

(C) 9360 J

(D) 13,104 J

Correct Answer: (C) 9360 J

Solution:

Concept:

- The r.m.s. speed of a gas is $v_{\text{rms}} = \sqrt{3RT/M}$; for a fixed gas its square is proportional to absolute temperature.
- In a rigid vessel, no expansion work is done, so the supplied heat equals the rise in internal energy:

$Q = nC_V\Delta T$. For diatomic nitrogen use $C_V = 5R/2$.

Step 1: Determine the temperature rise The initial temperature must first be converted to kelvin:

$$T_1 = 27 + 273 = 300\,\mathrm{K}.$$

Doubling the molecular r.m.s. speed requires a fourfold temperature, because temperature depends on the square of the speed:

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^2 = 4, \qquad T_2 = 1200\,\mathrm{K}.$$

The relevant increase is therefore $\Delta T = 1200 - 300 = 900\,\mathrm{K}$, rather than $300\,\mathrm{K}$ or $1200\,\mathrm{K}$.

Step 2: Find the amount of gas and its heat gain For nitrogen molecules the molar mass is $28\,\mathrm{g\,mol^{-1}}$, so

$$n = \frac{14}{28} = \frac{1}{2}\,\mathrm{mol}.$$

With the diatomic ideal-gas heat capacity and the given gas constant,

$$Q = \frac{1}{2} \times \frac{5}{2} \times 8.32 \times 900 = 9360\,\mathrm{J}.$$

This is the increase in translational plus rotational internal energy. The value $5616\,\mathrm{J}$ would follow from the monatomic heat capacity $3R/2$, while $13{,}104\,\mathrm{J}$ uses $C_P = 7R/2$ and incorrectly includes constant-pressure expansion work. The remaining smaller value also fails the required fourfold-temperature condition.

Step 3: Conclusion The heat to be supplied is $\boxed{9360\,\mathrm{J}}$. Hence, the correct option is (C).

Quick Tip to Solve: First square the speed ratio to obtain the temperature ratio, then use the temperature difference in $nC_V\Delta T$ for a rigid vessel.

39. A slab of dielectric constant K has the same cross-sectional area as the plates of a parallel plate capacitor and thickness $\frac{3}{4}$ d, where d is the separation of the plates. The capacitance of the capacitor when the slab is inserted between the plates will be :

(Given C_0 = capacitance of capacitor with air as medium between plates.) [4 marks]

(A) $\frac{4KC_0}{3+K}$

(B) $\frac{3KC_0}{3+K}$

(C) $\frac{3+K}{4KC_0}$

(D) $\frac{K}{4+K}$

Correct Answer: (A) $\frac{4KC_0}{3+K}$

Solution:

Concept:

- A dielectric layer and an air layer stacked along the field act as capacitors in series, since the same electric displacement passes through both layers.

- For plate area A, the empty capacitance is $C_0 = \varepsilon_0 A/d$. A dielectric layer of thickness t contributes an effective air thickness t/K.

Step 1: Identify the two series layers The slab covers the complete plate area but only three quarters of the separation. Thus its thickness and the total air thickness are

$$t_d = \frac{3d}{4}, \qquad t_a = d - t_d = \frac{d}{4}.$$

There is no side-by-side division of area, so the parallel-capacitor formula is inappropriate. The two layer capacitances are

$$C_a = \frac{\varepsilon_0 A}{d/4} = 4C_0, \qquad C_d = \frac{K\varepsilon_0 A}{3d/4} = \frac{4K}{3}C_0.$$

Step 2: Combine the layers and check limiting cases Adding their inverse capacitances gives

$$\frac{1}{C} = \frac{1}{4C_0} + \frac{3}{4KC_0} = \frac{K+3}{4KC_0}.$$

Therefore

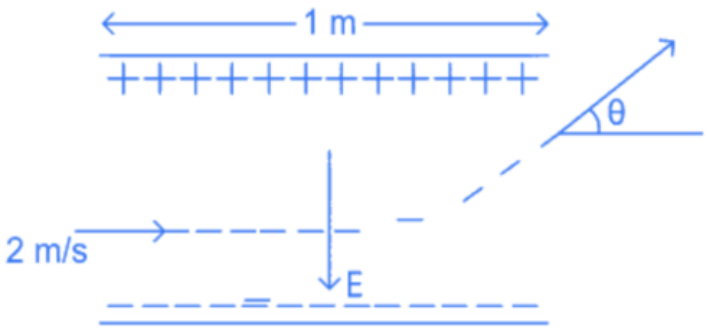
$$C = \frac{4KC_0}{K+3}.$$

For $K = 1$, the slab behaves like air and this reduces to C_0, as required. For very large K, the air gap limits the capacitance to $4C_0$. Option (B) already fails the $K = 1$ check; option (C) has inverse-capacitance units, and option (D) contains no capacitance scale at all. The result is independent of where the slab lies within the gap because the combined air thickness is unchanged.

Step 3: Conclusion The capacitance is $\boxed{C = 4KC_0/(3+K)}$. Hence, the correct option is (A).

Quick Tip to Solve: For layers along the field, add effective thicknesses: $d_{\text{eff}} = d_{\text{air}} + d_{\text{dielectric}}/K$, then use $C = \varepsilon_0 A/d_{\text{eff}}$.

40. A uniform electric field $\mathrm{E} = (8\ \mathrm{m/e})\ \mathrm{V/m}$ is created between two parallel plates of length $1\ \mathrm{m}$ as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of $2\ \mathrm{m/s}$. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be _____. [4 marks]



(A) $\tan^{-1}(4)$

(B) $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{3}\right)$

(D) $\tan^{-1}(3)$

Correct Answer: (B) $\tan^{-1}(2)$

Solution:

Concept:

- The electric force magnitude on an electron is eE, so its transverse acceleration is $a_y = eE/m$; its direction is opposite to the electric field.
- The field has no component along the initial motion, so the longitudinal velocity remains constant. The exit angle obeys $\tan\theta = |v_y|/v_x$.

Step 1: Find the acceleration and transit time Take the initial velocity as horizontal. The field points downward in the figure, so the negatively charged electron accelerates upward. Using e for the magnitude of its charge,

$$a_y = \frac{e}{m}\left(\frac{8m}{e}\right) = 8\,\mathrm{m\,s^{-2}}.$$

There is no horizontal force. Hence the electron crosses the plate length $L = 1\,\mathrm{m}$ at its unchanged horizontal speed $v_x = 2\,\mathrm{m\,s^{-1}}$, taking

$$t = \frac{L}{v_x} = \frac{1}{2}\,\mathrm{s}.$$

Step 2: Compute the final velocity direction The electron enters midway between the plates with zero vertical velocity. Its upward exit component is therefore

$$v_y = a_y t = 8 \times \frac{1}{2} = 4\,\mathrm{m\,s^{-1}}.$$

The angle is determined by the direction of the velocity at the exit, rather than the straight line joining the entry and exit points:

$$\tan\theta = \frac{v_y}{v_x} = \frac{4}{2} = 2, \qquad \theta = \tan^{-1}(2).$$

Option (A) mistakes the numerical vertical speed for the component ratio. Options (C) and (D) likewise do not give v_y/v_x. The electron bends upward as the shown path indicates; this reverses the force relative to the downward field but does not make the angle magnitude negative.

Step 3: Conclusion The deviation is $\boxed{\theta = \tan^{-1}(2)}$, upward from the initial direction. Hence, the correct option is (B).

Quick Tip to Solve: For transverse deflection across plates, use $\tan\theta = eEL/(mv_x^2)$; one factor of v_x enters through the transit time.

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41. Given below are two statements :

Statement I : A uniform wire of resistance $80\,\Omega$ is cut into four equal parts. These parts are now connected in parallel. The equivalent resistance of the combination will be $5\,\Omega$.

Statement II: Two resistances 2R and 3R are connected in parallel in a electric circuit. The value of thermal energy developed in 3R and 2R will be in the ratio $3:2$.

In the light of the above statements, choose the most appropriate answer from the option given below [4 marks]

(A) Both statement I and statement II are correct

(B) Both statement I and statement II are incorrect

(C) Statement I is correct but statement II is incorrect

(D) Statement I is incorrect but statement II is correct

Correct Answer: (C) Statement I is correct but statement II is incorrect

Solution:

Concept:

- For a uniform wire, resistance is proportional to length when material and cross-sectional area are unchanged: $R = \rho L/A$.
- Parallel branches have equal potential difference. For equal operating times their heat production follows $H = V^2t/R$, so it is inversely proportional to resistance.

Step 1: Evaluate Statement I Cutting the wire into four equal lengths does not alter its material or cross-sectional area. Each piece therefore has resistance

$$r = \frac{80}{4} = 20\,\Omega.$$

The four identical pieces are then connected in parallel. Their conductances add:

$$\frac{1}{R_{\text{eq}}} = 4\left(\frac{1}{20}\right) = \frac{1}{5}, \qquad R_{\text{eq}} = 5\,\Omega.$$

Thus Statement I is correct. Equivalently, making n equal pieces and connecting them in parallel reduces the original resistance by n^2.

Step 2: Evaluate Statement II and select the combination The $2R$ and $3R$ branches share the same

voltage. Over the same interval, the ratio in the stated order, heat in $3R$ to heat in $2R$, is

$$\frac{H_{3R}}{H_{2R}} = \frac{V^2 t/(3R)}{V^2 t/(2R)} = \frac{2}{3}.$$

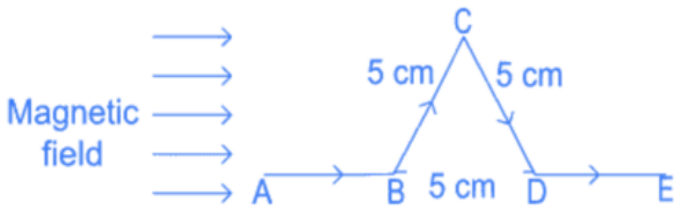
Therefore the ratio is $2:3$, not $3:2$, and Statement II is incorrect. The larger resistance dissipates less energy here because its current is smaller. Using I^2Rt with the same current in both branches would be invalid for this parallel connection. Options (A), (B) and (D) each assign the wrong truth value to at least one statement.

Step 3: Conclusion Statement I is correct but Statement II is incorrect. Hence, the correct option is (C).

Quick Tip to Solve: Before using a heating ratio, identify what is common: voltage in parallel gives $H \propto 1/R$, while current in series gives $H \propto R$.

42. A triangular shaped wire carrying 10 A current is placed in a uniform magnetic field of 0.5 T, as shown in figure. The magnetic force on segment CD is

(Given $\mathrm{BC} = \mathrm{CD} = \mathrm{BD} = 5$ cm.) [4 marks]



(A) 0.126 N

(B) 0.312 N

(C) 0.216 N

(D) 0.245 N

Correct Answer: (C) 0.216 N

Solution:

Concept:

- The magnetic force magnitude on a straight wire segment is $F = ILB\sin\alpha$, where α is the angle between the current direction and the magnetic field.
- The three equal sides $BC = CD = BD$ form an equilateral triangle, so each internal angle is $60°$. Only the requested segment CD is included in its force.

Step 1: Determine the angle and length for CD The field arrows point horizontally to the right, parallel to the base BD. The current on CD flows from C down toward D. Since the triangle is equilateral, this current direction makes $60°$ below the rightward field. Consequently $\sin\alpha = \sin 60° = \sqrt{3}/2$. The segment length in SI units is

$$L_{CD} = 5\,\mathrm{cm} = 0.05\,\mathrm{m}.$$

The angle at the upper vertex is not an angle to the field that needs any further division; the equal-side geometry fixes the required sine directly.

Step 2: Calculate and check the force Substitute the segment current, length and field strength:

$$F_{CD} = 10(0.05)(0.5)\frac{\sqrt{3}}{2} = 0.125\sqrt{3}\,\text{N} \simeq 0.2165\,\text{N}.$$

This matches $0.216\,\text{N}$ to the precision of the options. The maximum possible force for this length is $ILB = 0.25\,\text{N}$, excluding $0.312\,\text{N}$ immediately. The other listed values fail the $\sin 60^\circ$ factor. By the right-hand rule for $I\boldsymbol{L} \times \boldsymbol{B}$, the force on CD points out of the page. Forces on the other parts of the wire are irrelevant to the requested segment magnitude.

Step 3: Conclusion The magnetic force on CD has magnitude $\boxed{0.216\,\text{N}}$, directed out of the page. Hence, the correct option is (C).

Quick Tip to Solve: For a straight side in a magnetic field, first measure the angle to the field direction; the triangle's side length alone is insufficient without the sine factor.

---

43. The magnetic field at the center of current carrying circular loop is B_1. The magnetic field at a distance of $\sqrt{3}$ times radius of the given circular loop from the center on its axis is B_2. The value of B_1/B_2 will be [4 marks]

(A) 9 : 4

(B) 12 : $\sqrt{5}$

(C) 8 : 1

(D) 5 : $\sqrt{3}$

Correct Answer: (C) 8 : 1

Solution:

Concept:

- For a circular loop of radius R carrying current I, the axial magnetic field at distance x from its centre is $B(x) = \mu_0 I R^2/[2(R^2 + x^2)^{3/2}]$. The same current and radius occur in both fields.
- The central field is obtained by setting $x = 0$. Dividing the two fields cancels the current, permeability and radius scale; the ratio depends only on x/R.

Step 1: Evaluate the field at the centre At the centre, the denominator becomes $2R^3$, so the magnetic field is

$$B_1 = \frac{\mu_0 I R^2}{2R^3} = \frac{\mu_0 I}{2R}.$$

This is the maximum axial field of the loop. The field farther along its axis must therefore be smaller, which provides a useful check on the final ratio.

Step 2: Insert the specified axial distance The second point has $x = \sqrt{3}R$, not a distance measured from the wire. Thus

$$R^2 + x^2 = R^2 + 3R^2 = 4R^2, \qquad (4R^2)^{3/2} = 8R^3.$$

Consequently,

$$B_2 = \frac{\mu_0 I R^2}{16R^3} = \frac{\mu_0 I}{16R}.$$

The exponent $3/2$ must be applied to the whole quantity; replacing it by a square root alone gives a wrong factor.

Step 3: Take the ratio and check the choices Dividing the central field by the field at the second point gives

$$\frac{B_1}{B_2} = \frac{\mu_0 I/(2R)}{\mu_0 I/(16R)} = 8.$$

Equivalently, the normalized axial field is $[1 + (x/R)^2]^{-3/2}$; here it equals $1/8$. Options (A), (B) and (D) do not equal this factor and fail the same substitution.

Step 4: Conclusion The required ratio is $\boxed{B_1 : B_2 = 8 : 1}$. Hence, the correct option is (C).

Quick Tip to Solve: For an axial point at $x = kR$, use $B(0)/B(x) = (1 + k^2)^{3/2}$; the loop current cancels immediately.

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44. A transformer operating at primary voltage $8\,\text{kV}$ and secondary voltage $160\ \text{V}$ serves a load of $80\ \text{kW}$. Assuming the transformer to be ideal with purely resistive load and working on unity power factor, the loads in the primary and secondary circuit would be [4 marks]

(A) $800\,\Omega$ and $1.06\,\Omega$

(B) $10\,\Omega$ and $500\,\Omega$

(C) $800\,\Omega$ and $0.32\,\Omega$

(D) $1.06\,\Omega$ and $500\,\Omega$

Correct Answer: (C) $800\,\Omega$ and $0.32\,\Omega$

Solution:

Concept:

- An ideal transformer conserves average power: $P_p = P_s = P$. At unity power factor, $P = VI$, and for a purely resistive equivalent load $P = V^2/R$.
- The resistance seen at the primary is the secondary load reflected through the transformer. It is not the winding resistance; its ratio to the secondary resistance is $(V_p/V_s)^2$.

Step 1: Express the operating values in SI units The rms voltages and real output power are

$$V_p = 8\,\text{kV} = 8000\,\text{V}, \quad V_s = 160\,\text{V}, \quad P = 80\,\text{kW} = 80000\,\text{W}.$$

Because there are no transformer losses, this same power must be supplied at the primary. Unity power factor means that no extra factor of $\cos\phi$ reduces VI.

Step 2: Find the equivalent load on each side Apply $R = V^2/P$ separately at the two terminals:

$$R_p = \frac{8000^2}{80000} = 800\,\Omega, \qquad R_s = \frac{160^2}{80000} = 0.32\,\Omega.$$

The much larger resistance on the primary side is consistent with its higher voltage and lower current for the same transferred power.

Step 3: Verify through currents and the turns ratio The primary and secondary currents are

$$I_p = \frac{80000}{8000} = 10\,\text{A}, \qquad I_s = \frac{80000}{160} = 500\,\text{A}.$$

These are currents, so the numbers in option (B) cannot be taken as resistances. Also, $V_p/V_s = 50$, and

$$\frac{R_p}{R_s} = \frac{800}{0.32} = 2500 = 50^2.$$

Options (A) and (D) use $1.06\,\Omega$, which does not dissipate the stated power at $160\,\text{V}$.

Step 4: Conclusion The primary and secondary equivalent loads are $\boxed{800\,\Omega \text{ and } 0.32\,\Omega}$, respectively. Hence, the correct option is (C).

Quick Tip to Solve: Find transformer loads directly from $R = V^2/P$. Do not confuse the associated values of P/V with resistances.

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45. Sun light falls normally on a surface of area $36\ \text{cm}^2$ and exerts an average force of 7.2×10^{-9} N within a time period of 20 minutes. Considering a case of complete absorption, the energy flux of incident light is [4 marks]

(A) $25.92 \times 10^2\ \text{W/cm}^2$

(B) $8.64 \times 10^{-6}\ \text{W/cm}^2$

(C) $6.0\ \text{W/cm}^2$

(D) $0.06\ \text{W/cm}^2$

Correct Answer: (D) $0.06\ \text{W/cm}^2$

Solution:

Concept:

- Light carries momentum. For normal incidence and complete absorption, a light pulse of energy E transfers momentum E/c to the surface, where $c = 3.0 \times 10^8\,\text{m s}^{-1}$.
- The average force is $F = E/(ct)$. Irradiance or energy flux is the incident energy per unit area per unit time, $\mathcal{I} = E/(At)$, giving $F = \mathcal{I}A/c$. Reflection would introduce a different momentum-transfer factor.

Step 1: Relate the measured force to incident power For complete absorption, the incident power is

$$P_{\text{inc}} = Fc = (7.2 \times 10^{-9})(3.0 \times 10^8) = 2.16\,\text{W}.$$

The force is already an average force, so it determines average power directly. A factor of two appropriate to perfect reflection must not be used for this absorbing surface.

Step 2: Divide by the illuminated area The required options express flux per square centimetre.

Keeping the area as $36\,\mathrm{cm}^2$,

$$\mathcal{I} = \frac{P_{\text{inc}}}{A} = \frac{2.16}{36} = 0.06\,\mathrm{W\,cm^{-2}}.$$

As an SI check, $36\,\mathrm{cm}^2 = 3.6 \times 10^{-3}\,\mathrm{m}^2$, so $\mathcal{I} = 600\,\mathrm{W\,m^{-2}}$. Dividing this by 10^4 returns the same value per square centimetre.

Step 3: Check why the stated duration cancels The interval is $20\,\mathrm{min} = 1200\,\mathrm{s}$. The incident energy in that interval is

$$E = Fct = 2.16 \times 1200 = 2592\,\mathrm{J}.$$

Dividing by 36×1200 again gives $0.06\,\mathrm{W\,cm^{-2}}$. Option (A) mistakes an energy-sized number for flux; options (B) and (C) also fail $F = \mathcal{I}A/c$. The duration cannot remain in an average-flux result once force is given.

Step 4: Conclusion The incident energy flux is $\boxed{0.06\,\mathrm{W\,cm^{-2}}}$. Hence, the correct option is (D).

Quick Tip to Solve: For a perfectly absorbing surface at normal incidence use $\mathcal{I} = Fc/A$. The exposure time cancels between energy and energy per unit time.

---

46. The power of a lens (biconvex) is $1.25\ \mathrm{m^{-1}}$ in particular medium. Refractive index of the lens is 1.5 and radii of curvature are $20\ \mathrm{cm}$ and $40\ \mathrm{cm}$ respectively. The refractive index of surrounding medium: [4 marks]

(A) 1.0

(B) $\frac{9}{7}$

(C) $\frac{3}{2}$

(D) $\frac{4}{3}$

Correct Answer: (B) $\frac{9}{7}$

Solution:

Concept:

- For a thin lens immersed in a surrounding medium, its reciprocal focal length is $P = (n_l/n_m - 1)(1/R_1 - 1/R_2)$, where n_l and n_m are the absolute refractive indices of lens and medium.
- Use signed radii in the Cartesian convention. For light travelling left to right through a biconvex lens, $R_1 > 0$ and $R_2 < 0$. Radii must be in metres when power is in $\mathrm{m^{-1}}$.

Step 1: Set the curvature signs and convert the radii The first convex surface has its centre on the right, while the second has its centre on the left. Hence

$$R_1 = +0.20\,\mathrm{m}, \qquad R_2 = -0.40\,\mathrm{m}.$$

The curvature combination is

$$\frac{1}{R_1} - \frac{1}{R_2} = 5 - (-2.5) = 7.5\,\mathrm{m^{-1}}.$$

Using both radii as positive would incorrectly subtract their magnitudes.

Step 2: Solve for the relative refractive index Insert the measured power and $n_l = 1.5$:

$$1.25 = \left(\frac{1.5}{n_m} - 1\right) 7.5.$$

Therefore,

$$\frac{1.5}{n_m} - 1 = \frac{1.25}{7.5} = \frac{1}{6}, \qquad \frac{1.5}{n_m} = \frac{7}{6}.$$

Solving this relation gives

$$n_m = 1.5 \times \frac{6}{7} = \frac{9}{7}.$$

This value is lower than the lens index, as expected for a converging biconvex lens of positive power.

Step 3: Check the other proposed media In air, $n_m = 1$ would give power $3.75\,\mathrm{m}^{-1}$, so option (A) is too large in power. If $n_m = 3/2$, lens and medium have equal index and the power is zero. For $n_m = 4/3$, the relative index is $9/8$, giving power $0.9375\,\mathrm{m}^{-1}$. Only $9/7$ reproduces the stated value.

Step 4: Conclusion The refractive index of the surrounding medium is $\boxed{n_m = 9/7}$. Hence, the correct option is (B).

Quick Tip to Solve: For an immersed lens replace the air factor $n_l - 1$ by $n_l/n_m - 1$, and assign opposite signs to the radii of a biconvex lens.

---

47. Two streams of photons, possessing energies equal to five and ten times the work function of metal are incident on the metal surface successively. The ratio of maximum velocities of the photoelectron emitted, in the two cases respectively, will be [4 marks]

(A) 1 : 2

(B) 1 : 3

(C) 2 : 3

(D) 3 : 2

Correct Answer: (C) 2 : 3

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_\gamma - \phi$, where ϕ is the work function, the minimum energy required to release an electron from the metal.
- For the nonrelativistic photoelectrons in the photoelectric model, $K_{\max} = m_e v_{\max}^2/2$. Thus the maximum speed is proportional to the square root of the photon energy remaining after the work function is subtracted.

Step 1: Determine the kinetic energies after escape Both streams strike the same metal, so the work function is the same in the two experiments. Their photon energies are $E_1 = 5\phi$ and $E_2 = 10\phi$. After overcoming the surface barrier,

$$K_1 = 5\phi - \phi = 4\phi, \qquad K_2 = 10\phi - \phi = 9\phi.$$

The incident photon-energy ratio $1:2$ is not the kinetic-energy ratio because the same nonzero energy is spent in escape in each case.

Step 2: Convert the kinetic-energy ratio to a speed ratio The electron mass is identical in both cases. Dividing the two kinetic-energy equations yields

$$\frac{v_1^2}{v_2^2} = \frac{K_1}{K_2} = \frac{4}{9}.$$

Taking the positive square root, since the question asks for maximum speed magnitudes, gives

$$\frac{v_1}{v_2} = \sqrt{\frac{4}{9}} = \frac{2}{3}.$$

The higher-energy photon therefore produces the larger maximum electron speed, consistent with this ratio being less than unity.

Step 3: Compare with all options Option (A) copies the photon-energy ratio rather than subtracting the work function and taking a square root. Option (B) gives a speed ratio whose square is $1/9$, inconsistent with $K_1/K_2 = 4/9$. Option (D) reverses the requested order: the question asks for the five-work-function stream first and the ten-work-function stream second. This leaves only $2:3$.

Step 4: Conclusion The ratio of maximum photoelectron speeds is $\boxed{v_1 : v_2 = 2:3}$. Hence, the correct option is (C).

Quick Tip to Solve: First subtract the work function from each photon energy; only then take the square root of the kinetic-energy ratio to obtain the speed ratio.

---

48. A radioactive sample decays $\frac{7}{8}$ times its original quantity in 15 minutes. The half-life of the sample is [4 marks]

(A) 5 min

(B) 7.5 min

(C) 15 min

(D) 30 min

Correct Answer: (A) 5 min

Solution:

Concept:

- Radioactive decay leaves $N = N_0(1/2)^{t/T_{1/2}}$ undecayed nuclei after time t, where $T_{1/2}$ is the half-life and N_0 the original number.
- The amount that has decayed is $N_0 - N$. A statement that $7/8$ of the original sample decays means the remaining fraction is $1/8$, not $7/8$. The mass or number of atoms may be used because both have the same fractional decay.

Step 1: Find the fraction that remains Let the initial quantity be N_0. In fifteen minutes the decayed

quantity is $7N_0/8$, leaving

$$N = N_0 - \frac{7}{8}N_0 = \frac{1}{8}N_0.$$

Thus the remaining sample is one-eighth of its original size. Distinguishing the decayed fraction from the remaining fraction is essential before applying the half-life law.

Step 2: Count the elapsed half-lives Each half-life halves the quantity still present. The successive remaining fractions are

$$1 \longrightarrow \frac{1}{2} \longrightarrow \frac{1}{4} \longrightarrow \frac{1}{8}.$$

Exactly three half-lives have therefore elapsed. Algebraically,

$$\left(\frac{1}{2}\right)^{15/T_{1/2}} = \frac{1}{8} = \left(\frac{1}{2}\right)^{3}, \qquad \frac{15}{T_{1/2}} = 3.$$

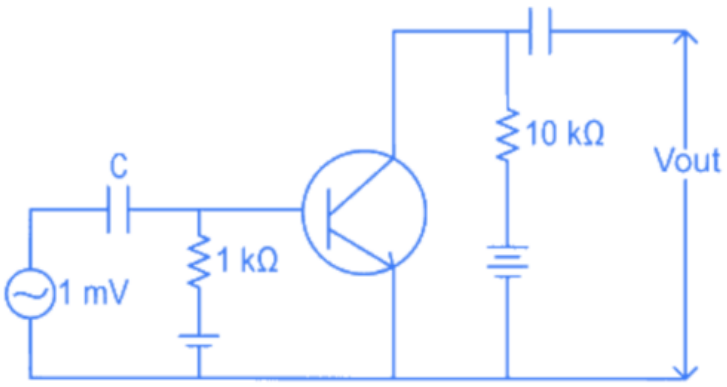
Consequently, $T_{1/2} = 15/3 = 5\,\text{min}$.

Step 3: Check the proposed times With a half-life of five minutes, fifteen minutes leaves $1/8$ and hence removes $7/8$, exactly as required. A half-life of $7.5\,\text{min}$ would leave $1/4$, while $15\,\text{min}$ would leave $1/2$. A half-life of thirty minutes would leave $2^{-1/2}$, still more than half. Thus none of the other choices produces the specified decay fraction.

Step 4: Conclusion The half-life of the sample is $\boxed{5\,\text{min}}$. Hence, the correct option is (A).

Quick Tip to Solve: Convert the decayed fraction into the remaining fraction first. Recognizing $1/8 = 2^{-3}$ immediately identifies three elapsed half-lives.

49. An n.p.n transistor with current gain $\beta = 100$ in common emitter configuration is shown in figure. The output voltage of the amplifier will be [4 marks]



(A) 0.1 V

(B) 1.0 V

(C) 10 V

(D) 100 V

Correct Answer: (B) 1.0 V

Solution:

Concept:

- In the elementary common-emitter amplifier model, a small input-current change produces a collector-current change $\Delta I_C = \beta \Delta I_B$. Here β is the current gain.
- The corresponding voltage-gain magnitude is $|A_v| = \beta R_C / R_{\text{in}}$. The input and collector load resistances indicated for this calculation are $1\,\text{k}\Omega$ and $10\,\text{k}\Omega$, respectively. Coupling capacitors pass the signal while blocking the DC bias.

Step 1: Read the signal and resistance values The diagram gives an input-signal amplitude $V_{\text{in}} = 1\,\text{mV} = 10^{-3}\,\text{V}$, an input resistance $R_{\text{in}} = 1\,\text{k}\Omega = 10^3\,\Omega$, and a collector resistance $R_C = 10\,\text{k}\Omega = 10^4\,\Omega$. The current gain is $\beta = 100$. Use the resistance ratio as well as the current gain: current gain alone is not voltage gain.

Step 2: Calculate the signal-current amplification The input signal-current amplitude in the elementary amplifier calculation is

$$\Delta I_B = \frac{V_{\text{in}}}{R_{\text{in}}} = \frac{10^{-3}}{10^3} = 10^{-6}\,\text{A}.$$

The collector-current amplitude is then

$$\Delta I_C = 100 \times 10^{-6} = 10^{-4}\,\text{A}.$$

This is the change in current caused by the AC input; it is not the steady bias current set by the DC supply.

Step 3: Find the output voltage and its phase The signal across the collector load has amplitude

$$V_{\text{out}} = \Delta I_C R_C = 10^{-4} \times 10^4 = 1\,\text{V}.$$

Equivalently, $|A_v| = 100(10/1) = 1000$, so $V_{\text{out}} = 1000 \times 1\,\text{mV}$. The common-emitter output is inverted relative to the input, but the listed options ask for its positive amplitude. Option (A) omits the resistance-ratio factor; options (C) and (D) exceed the calculated gain by factors of ten and one hundred.

Step 4: Conclusion The output-signal voltage amplitude is $\boxed{1.0\,\text{V}}$, with phase inversion relative to the input. Hence, the correct option is (B).

Quick Tip to Solve: For this amplifier model, multiply current gain by R_C/R_{in} to obtain voltage-gain magnitude, then convert the millivolt input to volts.

50. A FM Broadcast transmitter, using modulating signal of frequency 20 kHz has a deviation ratio of 10. The Bandwidth required for transmission is : [4 marks]

(A) 220 kHz

(B) 180 kHz

(C) 360 kHz

(D) 440 khz

Correct Answer: (D) 440 khz

Solution:

Concept:

- For frequency modulation, the deviation ratio is $\beta = \Delta f / f_m$, where Δf is the maximum frequency deviation and f_m is the highest modulating frequency.
- Carson's rule estimates the required transmission bandwidth as $B = 2(\Delta f + f_m) = 2(\beta + 1)f_m$. It includes significant sidebands on both sides of the carrier.

Step 1: Find the peak frequency deviation The given modulating frequency is $f_m = 20\,\text{kHz}$, and the deviation ratio is $\beta = 10$. Hence the carrier frequency varies by up to

$$\Delta f = \beta f_m = 10(20) = 200\,\text{kHz}.$$

This deviation is measured from the carrier frequency to one extreme; it is not itself the total occupied bandwidth.

Step 2: Apply the FM bandwidth rule Substitute the deviation and highest modulating frequency into Carson's rule:

$$B = 2(200 + 20) = 440\,\text{kHz}.$$

The factor of two accounts for frequencies on both sides of the carrier. The value $220\,\text{kHz}$ in option (A) is only the one-sided extent $\Delta f + f_m$. Option (B), $180\,\text{kHz}$, subtracts the modulating frequency from the deviation, which is not the bandwidth rule. Option (C), $360\,\text{kHz}$, doubles that incorrect difference. Thus only the sum followed by doubling gives the required FM transmission band.

Step 3: Conclusion The required bandwidth is $\boxed{440\,\text{kHz}}$. Hence, the correct option is (D).

Quick Tip to Solve: Compute the one-sided extent $(\beta + 1)f_m$ first, then double it; a deviation ratio does not directly equal bandwidth divided by modulating frequency.

Physics – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

51. A ball is thrown vertically upwards with a velocity of $19.6\ \mathrm{ms}^{-1}$ from the top of a tower. The ball strikes the ground after $6\ \mathrm{s}$. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)\ \mathrm{m}$. The value of k is _____. (use $g = 9.8\ \mathrm{m/s^2}$) [4 marks]

Solution:

Concept:

- With upward displacement positive, vertical motion satisfies $s = ut - \frac{1}{2}gt^2$. The launch point and the ground are different reference levels in this question.
- At the highest point the vertical velocity is zero, so the rise above the launch point is $h = u^2/(2g)$. The maximum height above ground is the tower height plus this rise.

Step 1: Determine the height of the tower Let the tower height be H. The ground is at displacement $-H$ relative to the top of the tower. Use $u = 19.6\,\mathrm{m\,s^{-1}}$, $t = 6\,\mathrm{s}$, and $g = 9.8\,\mathrm{m\,s^{-2}}$:

$$-H = 19.6(6) - \frac{1}{2}(9.8)(6^2) = 117.6 - 176.4 = -58.8\,\mathrm{m}.$$

Therefore the tower is $58.8\,\mathrm{m}$ tall. A negative displacement is expected because the ball eventually lands below its launch point.

Step 2: Add the rise above the tower The time to the highest point is $u/g = 2\,\mathrm{s}$, which is less than the total flight time. Its additional rise is

$$h = \frac{(19.6)^2}{2(9.8)} = 19.6\,\mathrm{m}.$$

Thus the highest position above ground is

$$H_{\max} = H + h = 58.8 + 19.6 = 78.4\,\mathrm{m}.$$

The question expresses this height as $k/5$ metres, so

$$\frac{k}{5} = 78.4 \quad \Rightarrow \quad k = 392.$$

The tower height alone would omit the initial upward part of the motion.

Step 3: Conclusion The ball reaches $78.4\,\mathrm{m}$ above ground; therefore $\boxed{k = 392}$.

Quick Tip to Solve: Keep the launch level separate from ground: first find the tower height from the full flight, then add the rise before the velocity becomes zero.

52. The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\rho = \rho_0\left(1 - \frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}$m. The value of α is _____. (where x is the distance from end A) [4 marks]

Solution:

Concept:

- For a rod with linear mass density $\rho(x)$, a small element has mass $dm = \rho(x)\,dx$. Its contribution to the first moment about end A is $x\,dm$.
- The centre-of-mass coordinate measured from A is $x_{\text{cm}} = \int_0^L x\rho(x)\,dx / \int_0^L \rho(x)\,dx$. A nonuniform rod need not balance at its geometrical midpoint.

Step 1: Integrate the total mass Here $\rho(x) = \rho_0(1 - x^2/L^2)$, with $x = 0$ at A and $x = L$ at B. The density decreases toward B, so the centre of mass must lie closer to A than $L/2$. The total mass is

$$M = \rho_0 \int_0^L \left(1 - \frac{x^2}{L^2}\right) dx = \rho_0 \left[x - \frac{x^3}{3L^2}\right]_0^L = \frac{2\rho_0 L}{3}.$$

Step 2: Evaluate the first moment and take the ratio The moment of the mass distribution about A is

$$\int_0^L x\,dm = \rho_0 \int_0^L \left(x - \frac{x^3}{L^2}\right) dx = \rho_0 \left[\frac{x^2}{2} - \frac{x^4}{4L^2}\right]_0^L = \frac{\rho_0 L^2}{4}.$$

Divide by the total mass:

$$x_{\text{cm}} = \frac{\rho_0 L^2/4}{2\rho_0 L/3} = \frac{3L}{8}.$$

The density scale ρ_0 cancels, as changing every mass by the same factor cannot shift the balancing point. Comparing with $3L/\alpha$ gives $\alpha = 8$, and $3L/8 < L/2$ confirms the expected direction.

Step 3: Conclusion The centre of mass is at $3L/8$ from A, so $\boxed{\alpha = 8}$.

Quick Tip to Solve: Use the density in both integrals; dividing the first moment by the rod length instead of its total mass gives the wrong centre of mass.

53. A string of area of cross-section $4\ \text{mm}^2$ and length 0.5 m is connected with a rigid body of mass 2 kg. The body is rotated in a vertical circular path of radius 0.5 m. The body acquires a speed of 5 m/s at the bottom of the circular path. Strain produced in the string when the body is at the bottom of the circle is _____ $\times 10^{-5}$.
(use Young's modulus 10^{11} N/m^2 and $g = 10$ m/s^2) [4 marks]

Solution:

Concept:

- At the bottom of a vertical circle, the centre is upward. The net upward radial force is $T - mg = mv^2/r$, where T is the string tension.
- Young's modulus is $Y =$ stress/strain. For string cross-sectional area A, the tensile stress is T/A,

and hence strain is $\epsilon = T/(AY)$.

Step 1: Calculate the bottom-point tension The tension points upward along the string while weight points downward. Their difference supplies the centripetal force, so they must not be added on the net-force side:

$$T = \frac{mv^2}{r} + mg = \frac{2(5)^2}{0.5} + 2(10) = 100 + 20 = 120\,\text{N}.$$

This exceeds the weight because the mass also needs upward centripetal acceleration. The given bottom-point speed is already sufficient to determine the instantaneous tension; no energy calculation around the rest of the circle is necessary.

Step 2: Convert the area and find strain The area must be converted from square millimetres to square metres:

$$A = 4\,\text{mm}^2 = 4 \times 10^{-6}\,\text{m}^2.$$

Thus

$$\text{stress} = \frac{120}{4 \times 10^{-6}} = 3.0 \times 10^7\,\text{N m}^{-2},$$

and

$$\epsilon = \frac{3.0 \times 10^7}{10^{11}} = 3.0 \times 10^{-4} = 30 \times 10^{-5}.$$

Strain is dimensionless. The length $0.5\,\text{m}$ would be needed to calculate extension, but cancels from stress divided by modulus when strain itself is asked.

Step 3: Conclusion The strain is 30×10^{-5}; the required numerical entry is $\boxed{30}$.

Quick Tip to Solve: At the bottom use $T = mv^2/r + mg$, then divide tension by AY; remember that $1\,\text{mm}^2 = 10^{-6}\,\text{m}^2$.

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54. At a certain temperature, the degrees of freedom per molecule for gas is 8. The gas performs 150 J of work when it expands under constant pressure. The amount of heat absorbed by the gas will be _____ J. [4 marks]

Solution:

Concept:

- For an ideal gas with f active quadratic degrees of freedom per molecule, the internal-energy change is $\Delta U = (f/2)nR\Delta T$.
- For a constant-pressure ideal-gas process, work done by the gas is $W = P\Delta V = nR\Delta T$. With this sign convention the first law gives $Q = \Delta U + W$.

Step 1: Relate the temperature change to the work The gas expands and performs $150\,\text{J}$ of work on its surroundings, so $W = +150\,\text{J}$. At fixed pressure the ideal-gas equation gives

$$P(V_2 - V_1) = nR(T_2 - T_1),$$

and therefore

$$nR\Delta T = 150\,\text{J}.$$

This relation avoids needing either the amount of gas or its absolute temperature. Since the volume increases at constant pressure, its temperature also increases, so its internal energy rises.

Step 2: Add the internal-energy increase and expansion work There are $f = 8$ degrees of freedom per molecule. Treating these as active for the stated process, the internal-energy increase is

$$\Delta U = \frac{8}{2} nR\Delta T = 4(150) = 600\,\text{J}.$$

The supplied heat must provide both this energy increase and the energy transferred out as work:

$$Q = \Delta U + W = 600 + 150 = 750\,\text{J}.$$

Equivalently, $C_P = (f/2+1)R = 5R$, so $Q/W = C_P/R = 5$. Using only $600\,\text{J}$ would account for the temperature rise but miss the expansion work; using $150\,\text{J}$ would miss the energy stored in molecular motion.

Step 3: Conclusion The heat absorbed by the gas is $\boxed{750\,\text{J}}$. The numerical entry is 750.

Quick Tip to Solve: At constant pressure, combine equipartition with $W = nR\Delta T$ to obtain $Q = (f/2+1)W$ directly.

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55. The potential energy of a particle of mass $4\,\text{kg}$ in motion along the x-axis is given by $\text{U} = 4(1-\cos 4x)$ J. The time period of the particle for small oscillation $(\sin\theta \simeq \theta)$ is $\left(\frac{\pi}{K}\right)$ s. The value of K is _____. [4 marks]

Solution:

Concept:

- In one-dimensional conservative motion, the restoring force is $F = -dU/dx$. Stable equilibrium occurs at a minimum of the potential.
- Near a stable equilibrium, $F \simeq -k_{\text{eff}}\xi$ gives simple harmonic motion with $\omega = \sqrt{k_{\text{eff}}/m}$ and period $T = 2\pi/\omega$, where ξ is the small displacement from equilibrium.

Step 1: Find and linearise the restoring force The potential is $U = 4(1-\cos 4x)\,\text{J}$. Its minima satisfy $4x = 2\pi n$. Choosing the minimum at $x = 0$, the force is

$$F = -\frac{dU}{dx} = -16\sin 4x.$$

For a small displacement, $|4x| \ll 1$, use the stated approximation $\sin 4x \simeq 4x$:

$$F \simeq -64x.$$

The negative sign shows that the force points back toward the minimum. Around any other minimum the same result holds in terms of displacement from that minimum, because the potential repeats periodically.

Step 2: Calculate the frequency and compare the period The effective force constant is consequently

$k_{\text{eff}} = 64\,\text{N m}^{-1}$. With mass $m = 4\,\text{kg}$,

$$\omega^2 = \frac{k_{\text{eff}}}{m} = \frac{64}{4} = 16\,\text{s}^{-2}, \qquad \omega = 4\,\text{rad s}^{-1}.$$

Hence

$$T = \frac{2\pi}{4} = \frac{\pi}{2}\,\text{s}.$$

The question writes the period as $(\pi/K)\,\text{s}$, so $K = 2$. The factor four inside the trigonometric argument must appear in differentiation and again in the small-angle approximation.

Step 3: Conclusion The small-oscillation period is $\pi/2\,\text{s}$, giving $\boxed{K = 2}$.

Quick Tip to Solve: For small oscillations, read the effective stiffness from $U \simeq \frac{1}{2}k_{\text{eff}}x^2$, or evaluate the potential curvature at its minimum.

56. An electrical bulb rated 220 V, 100 W, is connected in series with another bulb rated 220 V, 60 W. If the voltage across combination is 220 V, the power consumed by the 100 W bulb will be about _____ W. [4 marks]

Solution:

Concept:

- A bulb rated at voltage V_r and power P_r has operating resistance $R = V_r^2/P_r$. Use the standard constant-resistance model for the two rated bulbs.
- Series-connected bulbs carry the same current. For supply voltage V, the current is $I = V/(R_1+R_2)$, and the actual power dissipated in either bulb is I^2R, not necessarily its rated power.

Step 1: Convert the ratings to resistances Both bulbs are rated at $220\,\text{V}$. Label the $100\,\text{W}$ bulb as bulb 1 and the $60\,\text{W}$ bulb as bulb 2:

$$R_1 = \frac{220^2}{100} = 484\,\Omega, \qquad R_2 = \frac{220^2}{60} = \frac{2420}{3}\,\Omega.$$

Thus the lower-rated-power bulb has greater resistance at the same rated voltage. In a series combination the full supply is shared between the two resistances; neither bulb individually receives $220\,\text{V}$.

Step 2: Find the common current and the requested power The series resistance and common current are

$$R_1 + R_2 = \frac{3872}{3}\,\Omega, \qquad I = \frac{220}{3872/3} = \frac{15}{88}\,\text{A}.$$

Therefore the actual power in the bulb rated $100\,\text{W}$ is

$$P_1 = I^2R_1 = \left(\frac{15}{88}\right)^2(484) = \frac{225}{16} = 14.0625\,\text{W}.$$

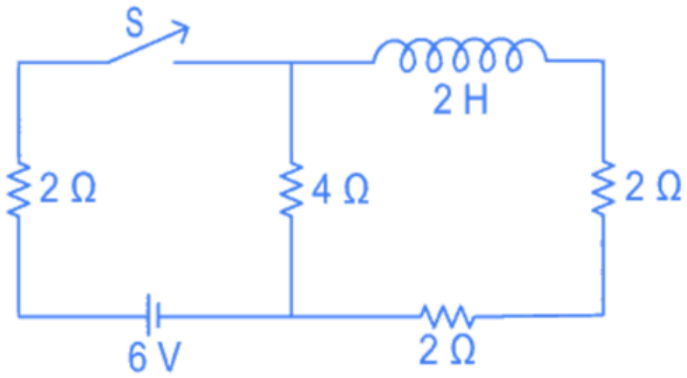
As a voltage check, this bulb receives $IR_1 = 82.5\,\text{V}$, and $82.5^2/484$ gives the same power. Its actual power is much less than its rating because its applied voltage is substantially reduced. Rounding the

requested approximate value gives 14.

Step 3: Conclusion The 100 W bulb consumes about $\boxed{14\,\text{W}}$.

Quick Tip to Solve: Use the ratings only to find resistances, then calculate the shared current; in series the higher rated-power bulb has the smaller resistance.

57. For the given circuit the current through battery of 6 V just after closing the switch 'S' will be _____ A. [4 marks]



Solution:

Concept:

- The current through an ideal inductor cannot change instantaneously. Its voltage obeys $V_L = L\,di_L/dt$, so a finite applied voltage produces a finite rate of change rather than an instantaneous current jump.
- Immediately after a switch is closed, an initially unenergised inductor has $i_L(0^+) = i_L(0^-) = 0$. Its branch is therefore treated as open only for evaluating the initial current.
- Ohm's law gives the battery current as $I = V/R_{\text{eq}}$ once the conducting paths at the specified instant have been identified.

Step 1: Identify the initial inductor current Before closing S, the battery is disconnected from the upper part of the network. There is no source driving the right-hand loop, so the 2 H inductor carries zero current. Just after closing the switch, continuity of inductor current requires

$$i_L(0^+) = 0.$$

Consequently, no current initially passes through the inductor or either of the two right-hand 2 ohm resistors in its branch.

Step 2: Find the conducting path and its resistance The initial battery-current path runs through the left 2 ohm resistor, the closed switch and the central 4 ohm resistor. These resistors are in series for this initial circuit, giving

$$R_{\text{eq}}(0^+) = 2 + 4 = 6\,\Omega.$$

The 4 ohm resistor is not initially in parallel with a conducting 4 ohm branch: that branch is

interrupted by the zero-current inductor.

Step 3: Apply the battery voltage With the 6 V battery across this 6 ohm path,

$$I_{\text{battery}}(0^+) = \frac{6}{6} = 1\,\text{A}.$$

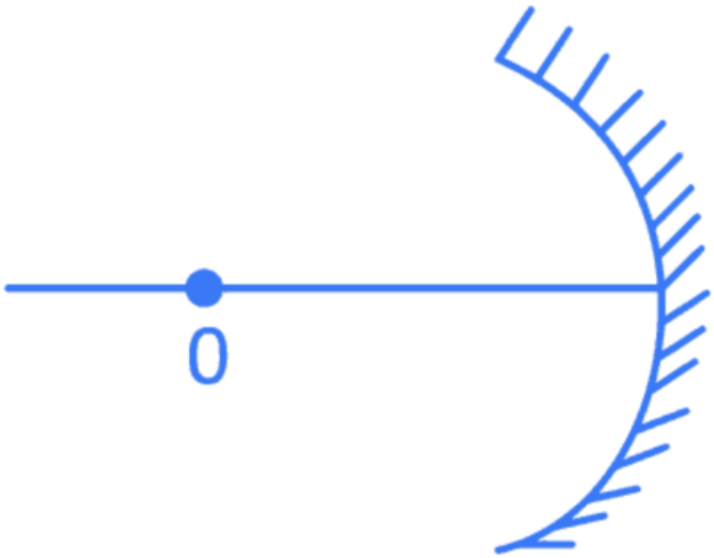
This initial value differs from the eventual steady-state current, when the ideal inductor acts as a connecting wire.

Step 4: Conclusion The current through the battery immediately after closing S is $\boxed{1\,\text{A}}$. The numerical entry is $\boxed{1}$.

Quick Tip to Solve: For the instant after switching, preserve the inductor's pre-switch current; only at long times in a DC circuit may it be replaced by a wire.

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58. An object 'O' is placed at a distance of $100\ \text{cm}$ in front of a concave mirror of radius of curvature $200\ \text{cm}$ as shown in the figure. The object starts moving towards the mirror at a speed $2\ \text{cm/s}$. The position of the image from the mirror after $10\ \text{s}$ will be at _____ cm. [4 marks]



Solution:

Concept:

- For a spherical mirror, the focal length has magnitude $R/2$. With the Cartesian sign convention a concave mirror has $f = -R/2$.
- The mirror equation is $1/f = 1/v + 1/u$, where u and v are measured from the pole and carry their Cartesian signs.
- The object displacement during uniform motion is speed multiplied by elapsed time. First update the object position, then apply the mirror equation to that new position.

Step 1: Locate the object after ten seconds The object initially lies 100 cm in front of the mirror and

moves towards it at 2 cm per second. In 10 s its displacement towards the mirror is

$$s = 2 \times 10 = 20\,\text{cm}.$$

Hence its final distance in front of the pole is

$$d = 100 - 20 = 80\,\text{cm}, \qquad u = -80\,\text{cm}.$$

The negative sign denotes the incident-light side of the mirror.

Step 2: Use the signed focal length The radius of curvature is 200 cm, so the concave mirror has

$$f = -\frac{200}{2} = -100\,\text{cm}.$$

Initially the object was at the focus and its image was at infinity. After moving towards the mirror, it lies between the focus and the pole, so a virtual image behind the mirror is expected.

Step 3: Calculate the image position Substitute the updated object distance into the mirror formula:

$$\begin{aligned}\frac{1}{v} &= \frac{1}{f} - \frac{1}{u} \\ &= -\frac{1}{100} + \frac{1}{80} = \frac{1}{400}.\end{aligned}$$

Thus $v = +400\,\text{cm}$. The positive result agrees with a virtual image behind the reflecting surface; it is not a real image in front.

Step 4: Conclusion After 10 s the image is $\boxed{400\,\text{cm}}$ behind the mirror. The required numerical value is $\boxed{400}$.

Quick Tip to Solve: When an object moves across a mirror's focus, recompute the image with signed distances; the image does not move by the same displacement as the object.

59. In an experiment with a convex lens, The plot of the image distance (v') against the object distance (μ') measured from the focus gives a curve $v'\mu' = 225$. If all the distances are measured in cm. The magnitude of the focal length of the lens is _____ cm. [4 marks]

Solution:

Concept:

- For a thin convex lens, let U and V be positive object and real-image distances from the optical centre. The lens equation is $1/f = 1/U + 1/V$.
- The corresponding distances measured from the object-side and image-side focal points are $x = U - f$ and $y = V - f$. Newton's lens formula is $xy = f^2$.
- Here the printed symbols μ' and v' denote the focus-referenced distances. Their product has dimensions of length squared.

Step 1: Relate the focus-based distances to the lens equation The stated distances are measured from

the foci, rather than from the optical centre. Put $U = f + x$ and $V = f + y$. On multiplying the lens equation by fUV, we obtain

$$UV = f(U + V).$$

This equation uses the central distances; replacing x directly by an optical-centre object distance would therefore be incorrect.

Step 2: Derive the product relation Insert the shifted distances and expand both sides:

$$(f + x)(f + y) = f[(f + x) + (f + y)],$$
$$f^2 + f(x + y) + xy = 2f^2 + f(x + y).$$

Cancel the common terms to obtain

$$xy = f^2.$$

Thus the rectangular-hyperbola constant in a plot of the two focus-based distances is the square of the focal length.

Step 3: Read the constant and take its positive root The given curve is $v'\mu' = 225$, with both lengths expressed in centimetres. Therefore

$$f^2 = 225\,\text{cm}^2, \qquad |f| = \sqrt{225}\,\text{cm} = 15\,\text{cm}.$$

The question asks for the magnitude, so the positive square root is the required entry.

Step 4: Conclusion The magnitude of the focal length is $\boxed{15\,\text{cm}}$, so the numerical answer is $\boxed{15}$.

Quick Tip to Solve: When both lens distances are measured from the foci, use their product f^2; the ordinary lens formula instead uses distances from the optical centre.

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60. In an experiment to find acceleration due to gravity (g) using simple pendulum, time period of 0.5 s is measured from time of 100 oscillation with a watch of 1 s resolution. If measured value of length is 10 cm known to 1 mm accuracy, The accuracy in the determination of g is found to be $x\%$. The value of x is _____. [4 marks]

Solution:

Concept:

- For small oscillations, the period of a simple pendulum is $T = 2\pi\sqrt{l/g}$. Hence $g = 4\pi^2 l/T^2$.
- For the maximum fractional uncertainty in a product or quotient, add the absolute fractional errors weighted by their powers: $\Delta g/g = \Delta l/l + 2\Delta T/T$.
- Timing many oscillations reduces the uncertainty in a single period. If $t = NT$ and the oscillation count N is exact, then $\Delta T/T = \Delta t/t$.

Step 1: Convert the length uncertainty consistently The measured length is 10 cm, or 100 mm, and

its stated uncertainty is 1 mm. Its fractional and percentage uncertainties are therefore

$$\frac{\Delta l}{l} = \frac{1}{100} = 0.01, \qquad \frac{\Delta l}{l} \times 100 = 1\%.$$

The millimetre uncertainty must not be divided directly by a length written in centimetres.

Step 2: Find the fractional timing uncertainty The measured period is 0.5 s and the watch times 100 oscillations. Thus the full timing interval is

$$t = 100 \times 0.5 = 50\,\text{s}.$$

Using the stated 1 s watch resolution as the time uncertainty gives

$$\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{50} = 0.02.$$

Equivalently the uncertainty per period is 0.01 s, rather than 1 s.

Step 3: Propagate both contributions to gravity Since g is proportional to lT^{-2}, the period contribution carries a factor of two. The maximum percentage uncertainty is

$$\frac{\Delta g}{g} \times 100 = 1 + 2(2) = 5\%.$$

The two contributions add in this standard maximum-error estimate; their signs are not subtracted even though time occurs in the denominator.

Step 4: Conclusion The accuracy expressed as the percentage uncertainty is 5%. Hence $\boxed{x = 5}$.

Quick Tip to Solve: Apply the stopwatch resolution to the total time of 100 oscillations, then use the exponent two on the period when propagating the error in gravity.

Chemistry – Section A: Multiple Choice Questions (80 marks)

61. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Zero orbital overlap is an out of phase overlap.

Reason R : It results due to different orientation / direction of approach of orbitals.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Correct Answer: (D) A is false but R is true

Solution:

Concept:

- The overlap integral $S = \int \psi_A \psi_B \, d\tau$ describes the net overlap of two atomic-orbital wavefunctions. Its sign depends on their relative phases and its magnitude on their spatial arrangement.
- In-phase overlap gives a positive overlap integral and favours bonding. Out-of-phase overlap gives negative overlap and antibonding interaction; zero overlap is a distinct case.
- Zero overlap can result from symmetry or a perpendicular orientation: positive and negative contributions to the overlap integral cancel exactly.

Step 1: Test the assertion about zero overlap Out-of-phase lobes have wavefunctions of opposite sign in the overlap region. Their product is negative and the associated interaction is antibonding. This is negative overlap, not zero overlap. Zero overlap means the net integral vanishes. Thus the assertion equating zero overlap with out-of-phase overlap is false.

Step 2: Test the stated reason The relative orientation and approach direction of orbitals determine whether their lobes overlap with compatible symmetry. For example, an s orbital approaching a p orbital along a direction perpendicular to the p axis overlaps equally with its positive and negative lobes. The two contributions cancel and the total overlap is zero. Therefore the reason correctly identifies orientation or direction of approach as a cause of zero overlap.

Step 3: Compare the truth values with the choices The correct pair of truth values is: assertion false, reason true. Options (A) and (B) both demand that the assertion be true, and option (C) also makes the assertion true while rejecting the valid reason. None describes zero overlap correctly. Option (D) alone combines the false assertion with the true reason.

Step 4: Conclusion Assertion A is false, whereas Reason R is true. Hence, the correct option is (D). The supplied answer key lists A, but the printed assertion confuses negative overlap with zero overlap; the explicit truth-value analysis supports D.

Quick Tip to Solve: Distinguish the sign of overlap from its absence: opposite phases give negative overlap, while symmetry cancellation gives zero overlap.

62. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R

Assertion A : The reduction of a metal oxide is easier if the metal formed is in liquid state than solid state.

Reason R : The value of ΔG^{Θ} becomes more on negative side as entropy is higher in liquid state than solid state.

In the light of the above statements, choose the most appropriate answer from the options given below [4 marks]

(A) Both A and R are correct and R is the correct explanation of A

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Correct Answer: (A) Both A and R are correct and R is the correct explanation of A

Solution:

Concept:

- The thermodynamic criterion for a favourable reaction at constant temperature and pressure is negative Gibbs energy change: $\Delta G = \Delta H - T\Delta S$.
- A liquid has higher molar entropy than the corresponding solid. Forming a liquid metal therefore gives a larger product-entropy contribution than forming its solid form.
- At the melting temperature the two phases have equal molar Gibbs energies. Above it the liquid is thermodynamically stable and its Gibbs energy is lower than that of the extrapolated solid.

Step 1: Examine the entropy change in reduction Consider reduction in which a metal oxide yields the metal and the other reactants and products are unchanged. Replacing solid metal among the products by liquid metal increases the entropy of the products, because a liquid permits greater configurational freedom. The reaction entropy consequently shifts in the positive direction. This is the phase contribution relevant to the assertion.

Step 2: Connect the phase change to Gibbs energy The larger entropy makes the term $-T\Delta S$ more negative. In the temperature range where the metal is molten, this lowers the Gibbs energy required for its formation and favours the reduction. More precisely, the liquid and solid have equal Gibbs energy at melting, while above melting the entropy advantage makes the liquid the stable phase. This is the thermodynamic basis of the easier reduction when molten metal is formed.

Step 3: Judge the assertion and reason together The assertion expresses this favourable phase effect on reduction and is correct in the intended thermodynamic comparison. The reason also correctly connects higher liquid-state entropy with a shift of reaction Gibbs energy towards more negative values, and directly explains the assertion. Option (B) incorrectly denies that explanatory connection; (C) and (D) each reject a correct statement.

Step 4: Conclusion Both A and R are correct, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: In a phase-dependent reduction question, inspect the product entropy: forming molten metal adds an entropy advantage that favours reduction above its melting point.

63. Which of the following pair is not isoelectronic species?

(At. no. Sm, 62; Er, 68; Yb, 70; Lu, 71; Eu, 63; Tb, 65; Tm, 69) [4 marks]

(A) Sm^{2+} and Er^{3+}

(B) Yb^{2+} and Lu^{3+}

(C) Eu^{2+} and Tb^{4+}

(D) Tb^{2+} and Tm^{4+}

Correct Answer: (A) Sm^{2+} and Er^{3+}

Solution:

Concept:

- Isoelectronic species have the same total number of electrons. For a monatomic cation M^{n+} with atomic number Z, the electron count is $Z - n$.
- The comparison here is between each pair of ions, so subtract the given positive charge separately from the atomic number of each element.
- Equal ionic charges are not required for isoelectronic species. What must agree is the resulting electron count, not the identity of the element.

Step 1: Count the electrons in the first two pairs For option (A), the counts are

$$Sm^{2+} : 62 - 2 = 60, \qquad Er^{3+} : 68 - 3 = 65.$$

These numbers are unequal, so (A) is a pair of non-isoelectronic species. For option (B),

$$Yb^{2+} : 70 - 2 = 68, \qquad Lu^{3+} : 71 - 3 = 68.$$

Both ions have 68 electrons, so (B) does not satisfy the question.

Step 2: Check the remaining two pairs For option (C),

$$Eu^{2+} : 63 - 2 = 61, \qquad Tb^{4+} : 65 - 4 = 61.$$

These are isoelectronic. For option (D),

$$Tb^{2+} : 65 - 2 = 63, \qquad Tm^{4+} : 69 - 4 = 65.$$

Their counts differ, so (D) is also a non-isoelectronic pair. No assumption about a common ionic charge or neighbouring elements can replace these explicit counts.

Step 3: Account for the two valid choices Options (A) and (D) each have unequal electron counts within the pair, whereas (B) and (C) each have equal counts. The question as printed therefore has two acceptable choices. Selecting (A) provides a correct pair, and the official final answer key also accepts (D). This multiple acceptance is consistent with direct electron counting rather than a disagreement with the key.

Step 4: Conclusion Sm^{2+} and Er^{3+} are not isoelectronic. Hence, the correct option is (A). Option (D) is also correct; the official final key accepts both (A) and (D).

Quick Tip to Solve: For every cation write $Z -$ charge beside it before comparing the pair; this immediately exposes both accepted choices here.

64. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Permanganate titrations are not performed in presence of hydrochloric acid.

Reason R : Chlorine is formed as a consequence of oxidation of hydrochloric acid.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Correct Answer: (A) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Acidified permanganate is a strong oxidising agent. In acidic medium, MnO_4^- accepts five electrons to form Mn^{2+}.
- The acid used to supply H^+ must not itself react with the oxidising titrant; otherwise the measured titrant volume is not due solely to the analyte.

Step 1: Evaluate the assertion In a permanganate titration the volume of $KMnO_4$ should measure the amount of the reducing analyte, such as oxalate or Fe^{2+}. Hydrochloric acid supplies chloride ions as well as hydrogen ions. Chloride can be oxidised by acidified permanganate, so hydrochloric acid introduces a competing reducing species into the flask. Consequently it is not a suitable acid for these titrations. Assertion A is true.

Step 2: Evaluate the reason The chloride ions supplied by hydrochloric acid lose electrons and form chlorine. The balanced competing reaction is

$$2\,MnO_4^- + 16\,H^+ + 10\,Cl^- \longrightarrow 2\,Mn^{2+} + 8\,H_2O + 5\,Cl_2.$$

Chloride is oxidised from oxidation state -1 to 0, while manganese is reduced from $+7$ to $+2$. Thus chlorine formation is an oxidation of hydrochloric acid, and Reason R is true.

Step 3: Connect the competing reaction to the endpoint The chloride reaction consumes additional permanganate. Its consumption is unrelated to the amount of analyte, so it can increase the titre and distort the endpoint and the calculated concentration. This is precisely why hydrochloric acid is avoided; dilute sulphuric acid is normally used instead. Option B wrongly denies this causal link, and options C and D incorrectly declare one of the true statements false.

Step 4: Conclusion Both A and R are true, and R correctly explains why hydrochloric acid is avoided in permanganate titrations. Hence, the correct option is (A).

Quick Tip to Solve: For a redox titration, check that the acid supplies H^+ without supplying another species that the titrant can oxidise.

65. Match List I with List II

	List - I (Complex)		List - II (Hybridization)
(A)	$Ni(CO)_4$	**(I)**	sp^3
(B)	$[Ni(CN)_4]^{2-}$	**(II)**	sp^3d^2
(C)	$[Co(CN)_6]^{3-}$	**(III)**	d^2sp^3
(D)	$[CoF_6]^{3-}$	**(IV)**	dsp^2

Choose the correct answer from the options given below : [4 marks]

(A) A-IV, B-I, C-III, D-II

(B) A-I, B-IV, C-III, D-II

(C) A-I, B-IV, C-II, D-III

(D) A-IV, B-I, C-II, D-III

Correct Answer: (B) A-I, B-IV, C-III, D-II

Solution:

Concept:

- In valence bond theory, tetrahedral, square-planar, inner-orbital octahedral and outer-orbital octahedral complexes correspond to sp^3, dsp^2, d^2sp^3 and sp^3d^2, respectively.
- Find the metal oxidation state and its d-electron count first. Strong-field ligands promote pairing when necessary, while weak-field fluoride gives the high-spin cobalt(III) complex.

Step 1: Match the nickel complexes In $\mathrm{Ni(CO)_4}$, nickel is in oxidation state zero. Carbon monoxide produces the paired $3d^{10}$ arrangement, leaving the $4s$ and three $4p$ orbitals for tetrahedral sp^3 hybridisation. Therefore A matches I. In $[\mathrm{Ni(CN)_4}]^{2-}$, nickel is $+2$, hence $3d^8$. The strong-field cyanide ligands pair the electrons so that one $3d$ orbital is vacant. Its combination with $4s$ and two $4p$ orbitals gives square-planar dsp^2 hybridisation: B matches IV.

Step 2: Match the cobalt complexes Both $[\mathrm{Co(CN)_6}]^{3-}$ and $[\mathrm{CoF_6}]^{3-}$ contain cobalt(III), which is $3d^6$. Cyanide is a strong-field ligand, giving a low-spin arrangement with two vacant inner $3d$ orbitals. The octahedral hybridisation is d^2sp^3, so C matches III. Fluoride is weak-field and does not produce this pairing. The high-spin complex uses two outer $4d$ orbitals together with $4s$ and $4p$, giving sp^3d^2, so D matches II.

Step 3: Compare all four matches The complete sequence is A-I, B-IV, C-III, D-II. Option A reverses the tetrahedral and square-planar nickel assignments. Option C interchanges the inner-orbital cyanide and outer-orbital fluoride cobalt assignments. Option D makes both of those errors. Only option B preserves the geometry and orbital choice for all four complexes.

Step 4: Conclusion The matching is A-I, B-IV, C-III, D-II. Hence, the correct option is (B).

Quick Tip to Solve: Use dsp^2 for square-planar nickel cyanide, then distinguish cobalt cyanide (inner orbital) from cobalt fluoride (outer orbital).

66. Dinitrogen and dioxygen, the main constituents of air do not react with each other in atmosphere to form oxides of nitrogen because : [4 marks]

(A) N_2 is unreactive in the condition of atmosphere.

(B) Oxides of nitrogen are unstable.

(C) Reaction between them can occur in the presence of a catalyst.

(D) The reaction is endothermic and require very high temperature.

Correct Answer: (D) The reaction is endothermic and require very high temperature.

Solution:

Concept:

- The initial formation of nitric oxide from the major atmospheric gases is $N_2(g)+O_2(g) \longrightarrow 2\,NO\,(g)$. This reaction is endothermic.
- The very strong $N{\equiv}N$ bond makes the process kinetically difficult at ordinary atmospheric temperatures; sufficient energy is needed to initiate reaction.

Step 1: Identify the energetic barrier Dinitrogen has a strong triple bond with a large bond-dissociation enthalpy. Reacting it directly with dioxygen therefore requires a substantial energy input. At ordinary atmospheric temperature most collisions cannot overcome the activation barrier, and there is no appreciable direct formation of nitric oxide. The statement concerns ordinary atmospheric conditions, rather than the high-energy conditions in an electric discharge.

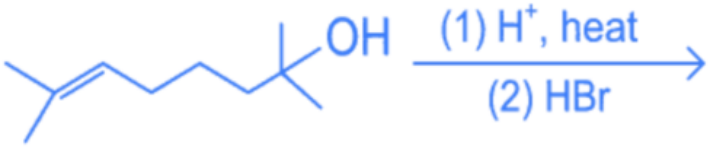
Step 2: Relate temperature to nitric oxide formation For the direct reaction, ΔH is positive, approximately $+180\,kJ\,mol^{-1}$ for the equation as written. High temperature supplies energy for bond rearrangement and favours the endothermic formation of nitric oxide. This is why atmospheric nitrogen oxides can form during lightning, and why very hot combustion gases can contain nitric oxide. At ordinary temperature the gases do not appreciably react with one another.

Step 3: Select the most complete explanation Option D explicitly states the endothermic nature of the reaction and its requirement for very high temperature. Option A describes nitrogen as unreactive but does not give the energetic explanation asked for. Option B is not a general reason: nitrogen oxides do exist and can persist under suitable conditions. Option C merely proposes a catalyst and does not explain the lack of reaction in the atmosphere; a catalyst also does not reverse the reaction enthalpy.

Step 4: Conclusion The reaction is endothermic and requires very high temperature. Hence, the correct option is (D).

Quick Tip to Solve: For direct N_2-O_2 reaction, connect the strong nitrogen triple bond with the high-energy conditions of lightning.

67. The major product in the given reaction is [4 marks]



(A) $(CH_3)_2C{=}CH(CH_2)_3C(CH_3)_2B_2$

(B) 1-(B_2)-1,2,2-trimethylcyclopentane

(C) 1-(2-bromopropan-2-yl)-2,2-dimethylcyclopentane

(D) 3-bromo-2,7-dimethyloctane

Correct Answer: (C) 1-(2-bromopropan-2-yl)-2,2-dimethylcyclopentane

Solution:

Concept:

- On heating with acid, a tertiary alcohol loses water after protonation to generate a tertiary carbocation. A nearby carbon-carbon double bond can act as an intramolecular nucleophile.
- Formation of a five-membered carbon ring is favourable. Track which alkene carbon forms the new bond and which alkene carbon retains the positive charge before bromide capture.

Step 1: Generate the tertiary carbocation The starting alcohol has the skeleton

$$(CH_3)_2C{=}CH(CH_2)_3C(OH)(CH_3)_2.$$

Protonation of the hydroxyl group makes water the leaving group. Its departure gives a tertiary carbocation at the carbon that originally bore the hydroxyl group. This reactive centre lies within the same molecule as the alkene, allowing intramolecular attack rather than requiring a collision with a second alkene molecule.

Step 2: Close the five-membered ring The alkene carbon bearing one hydrogen forms a new bond to the alcohol-derived carbocation. Count the resulting ring: that alkene carbon, the three intervening methylene carbons, and the former alcohol carbon make five ring atoms. The other alkene carbon now carries the positive charge. Since it bears two methyl groups and is attached to the ring, it is a tertiary carbocation. The ring carbon originating from the alcohol also retains its two methyl substituents.

Step 3: Add bromide and identify the structure Bromide from the subsequent HBr treatment captures the external tertiary carbocation. The product is a cyclopentane bearing adjacent gem-dimethyl and $C(Br)(CH_3)_2$ substituents, namely 1-(2-bromopropan-2-yl)-2,2-dimethylcyclopentane. This is the structure in option C. Option A is acyclic, option B places the halogen-labelled group on a ring carbon, and option D is also acyclic; none matches the five-membered cyclisation followed by capture of the external cation.

Step 4: Conclusion The major product is 1-(2-bromopropan-2-yl)-2,2-dimethylcyclopentane, the cyclic structure with bromine on the external tertiary carbon. Hence, the correct option is (C).

Quick Tip to Solve: Count the ring atoms before placing bromine: cyclisation uses the less-substituted alkene carbon, leaving the more-substituted external carbon to capture bromide.

68. Arrange the following in increasing order of reactivity towards nitration

A. p-xylene

B. bromobenzene

C. mesitylene

D. nitrobenzene

E. benzene

Choose the correct answer from the options given below [4 marks]

(A) C < D < E < A < B

(B) D < B < E < A < C

(C) D < C < E < A < B

(D) C < D < E < B < A

Correct Answer: (B) D < B < E < A < C

Solution:

Concept:

- Nitration is an electrophilic aromatic substitution. Electron donation to the aromatic ring generally increases its reactivity, while electron withdrawal decreases it.
- Methyl groups activate by hyperconjugation and the positive inductive effect. Nitro strongly deactivates; bromine is a net deactivator despite its ortho/para directing resonance effect.

Step 1: Place the deactivated aromatic rings The nitro group withdraws electron density through both its negative inductive and negative resonance effects. Nitrobenzene is therefore much less reactive than benzene towards the electrophilic nitrating species $NO_2{}^+$. Bromobenzene is also less reactive than benzene: bromine donates by resonance, but its electron-withdrawing inductive effect dominates the overall rate. Nitrobenzene is more strongly deactivated, giving

$$\text{nitrobenzene} < \text{bromobenzene} < \text{benzene}.$$

Step 2: Place the methyl-substituted rings Each methyl group donates electron density and stabilises the positively charged intermediate formed during electrophilic substitution. The two methyl groups in p-xylene make its ring more reactive than benzene. Mesitylene is 1,3,5-trimethylbenzene, and its three methyl groups give still stronger activation. Thus the remaining order is benzene, p-xylene, then mesitylene. This compares overall reactivity, not simply the position at which substitution is directed.

Step 3: Translate the names into list letters The complete increasing order is nitrobenzene, bromobenzene, benzene, p-xylene, mesitylene. With the given labels, this becomes $D < B < E < A < C$, exactly option B. Options A and D incorrectly put strongly activated mesitylene at the least-reactive end. Option C also places mesitylene below benzene and puts bromobenzene above the methyl-activated rings, contrary to the substituent effects.

Step 4: Conclusion The increasing reactivity order is $D < B < E < A < C$. Hence, the correct option is (B).

Quick Tip to Solve: For nitration rates, separate directing effect from activation: a halogen directs ortho/para but still deactivates the ring overall.

69. Compound I is heated with Conc. HI to give a hydroxy compound A which is further heated with Zn dust to give compound B. Identify A and B. [4 marks]



(A) A = propan-2-ol; B = propane

(B) A = phenol; B = 1,4-benzoquinone

(C) A = 2-isopropylphenol; B = isopropylbenzene

(D) A = phenol; B = benzene

Correct Answer: (D) A = phenol; B = benzene

Solution:

Concept:

- Concentrated HI cleaves an aryl-alkyl ether at the alkyl-oxygen bond after oxygen protonation. The aromatic sp^2 carbon does not undergo the ordinary backside substitution of an alkyl carbon.
- Heating phenol with zinc dust replaces its hydroxyl group by hydrogen: $C_6H_5OH + Zn \longrightarrow C_6H_6 + ZnO$.

Step 1: Identify the two sides of the ether Compound I is phenyl isopropyl ether, $C_6H_5{-}O{-}CH(CH_3)_2$. One side of oxygen is bonded directly to an aromatic ring, while the other is bonded to the secondary isopropyl carbon. The position of oxygen matters: there is no carbon-carbon bond between the isopropyl group and the benzene ring in the reactant.

Step 2: Cleavage gives the hydroxy compound Protonation of oxygen by the strong acid activates the ether for cleavage. The alkyl-oxygen bond breaks, giving phenol as the hydroxy compound A and an isopropyl iodide coproduct. The overall change can be written

$$C_6H_5{-}O{-}CH(CH_3)_2 + HI \longrightarrow C_6H_5OH + (CH_3)_2CHI.$$

Cleavage at the aryl-oxygen bond is disfavoured because the aryl carbon is sp^2 and that bond has partial double-bond character. Therefore A is phenol, not propan-2-ol or an isopropyl-substituted phenol.

Step 3: Apply the zinc-dust reaction When A is heated with zinc dust, the oxygen of the phenolic hydroxyl group is removed as zinc oxide and the ring gains hydrogen. Hence B is benzene. Option A uses the wrong hydroxy fragment, option B incorrectly gives an oxidation product (benzoquinone) for a reducing zinc-dust treatment, and option C wrongly retains the isopropyl group as a ring substituent. Only option D gives phenol followed by benzene.

Step 4: Conclusion A is phenol (C_6H_5OH) and B is benzene (C_6H_6). Hence, the correct option is (D).

Quick Tip to Solve: In an aryl-alkyl ether, leave oxygen on the aryl side during HI cleavage; zinc dust then converts the resulting phenol to benzene.

70. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Aniline on nitration yields ortho, meta & para nitro derivatives of aniline.

Reason R : Nitrating mixture is a strong acidic mixture.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Correct Answer: (A) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- An unprotonated amino group donates its lone pair into the benzene ring and directs electrophilic substitution mainly to the ortho and para positions.
- The strongly acidic nitrating mixture protonates aniline to the anilinium ion, $C_6H_5NH_3^+$, whose positively charged group is deactivating and meta directing.

Step 1: Evaluate the assertion Direct nitration of aniline gives a mixture containing ortho-, meta- and para-nitroaniline rather than exclusively the ortho and para products expected from an unprotonated amino group. Both the free amine and the protonated amine need to be considered under the reaction conditions. Thus Assertion A, which states that all three positional derivatives are obtained, is true.

Step 2: Evaluate the reason and its chemical effect The nitrating mixture contains concentrated nitric acid and sulphuric acid and is strongly acidic, so Reason R is true. In this medium aniline is protonated according to

$$C_6H_5NH_2 + H^+ \rightleftharpoons C_6H_5NH_3^+.$$

Protonation removes the nitrogen lone pair from conjugation with the ring. The $-NH_3^+$ group withdraws electron density and directs nitration to the meta position. Consequently a substantial meta product appears, even though free aniline itself is ortho/para directing.

Step 3: Explain why the reason accounts for the product mixture The free-aniline pathway supplies ortho and para substitution, while the anilinium-ion pathway accounts for meta substitution. It is the strong acidity specified in R that creates the protonated form responsible for the otherwise unexpected meta product. R therefore explains the assertion rather than merely accompanying it. Option B incorrectly denies the causal connection; options C and D incorrectly reject a true statement. Protecting the amino group before nitration is the usual way to avoid the complications of direct nitration.

Step 4: Conclusion Both A and R are true, and the strong acidity in R correctly explains the mixture described in A. Hence, the correct option is (A).

Quick Tip to Solve: Before predicting an amino group's directing effect, check the medium: in strong acid, use $-NH_3^+$ as well as $-NH_2$.

71. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R

Assertion A : Thin layer chromatography is an adsorption chromatography.

Reason R : A thin layer of silica gel is spread over a glass plate of suitable size in thin layer chromatography which acts as an adsorbent.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Correct Answer: (A) Both A and R are true and R is the correct explanation of A

Solution:

Concept:

- Thin-layer chromatography (TLC) separates components by their different adsorption strengths on a solid stationary phase.
- Silica gel is a porous solid adsorbent. A suitable solvent moves over its surface as the mobile phase.

Step 1: Evaluate Assertion A In ordinary TLC, the stationary phase is a thin coating of an active solid such as silica gel or alumina. The components of a mixture are reversibly adsorbed on that solid. As the solvent rises by capillary action, each component repeatedly adsorbs and desorbs. Stronger adsorption delays a component more, whereas weaker adsorption lets it travel farther. Thus the separation is based on adsorption, so Assertion A is true.

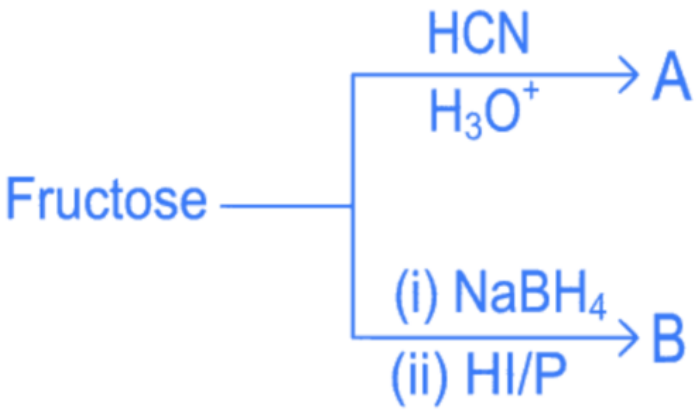
Step 2: Evaluate Reason R Silica gel can be spread as a thin, uniform layer on a glass plate, which provides mechanical support. The silica gel, rather than the glass itself, offers the active adsorbing surface. The developing solvent carries the dissolved components along this coating. Therefore the stated role of the silica layer as an adsorbent is correct, and Reason R is true.

Step 3: Connect the reason with the assertion Reason R identifies the solid adsorbent that causes the differential retention described in Assertion A. It therefore gives the physical basis for calling this technique adsorption chromatography. This is a direct explanation, rather than two unrelated correct facts. Option B wrongly denies this connection, while C and D each incorrectly reject one of the true statements.

Step 4: Conclusion Both A and R are true, and R is the correct explanation of A. Hence, the correct option is (A).

Quick Tip to Solve: For a chromatography assertion, identify the stationary phase first: a solid silica or alumina surface signals adsorption.

72. The formulas of A and B for the following reaction sequence are [4 marks]



(A) $A = C_7H_{14}O_8, \quad B = C_6H_{14}$
(B) $A = C_7H_{13}O_7, \quad B = C_7H_{14}O$
(C) $A = C_7H_{12}O_8, \quad B = C_6H_{14}$
(D) $A = C_7H_{14}O_8, \quad B = C_6H_{14}O_6$

Correct Answer: (A) $A = C_7H_{14}O_8, \quad B = C_6H_{14}$

Solution:

Concept:

- Open-chain fructose has formula $C_6H_{12}O_6$ and contains a ketone group. Addition of HCN makes a cyanohydrin and adds one carbon atom.
- Acidic hydrolysis changes $-CN$ to $-COOH$. Reduction with $NaBH_4$, followed by hot HI/phosphorus, removes oxygen without changing the six-carbon skeleton.

Step 1: Obtain the formula of A The ketone carbon of fructose undergoes nucleophilic addition with cyanide, followed by protonation. The intermediate therefore contains the atoms of fructose plus those of HCN: $C_7H_{13}NO_6$. Hydrolysis of its nitrile consumes two water molecules and releases ammonia. The overall formula balance is

$$C_6H_{12}O_6 + HCN + 2\,H_2O \longrightarrow C_7H_{14}O_8 + NH_3.$$

Thus A has seven carbons and eight oxygens, with molecular formula $C_7H_{14}O_8$.

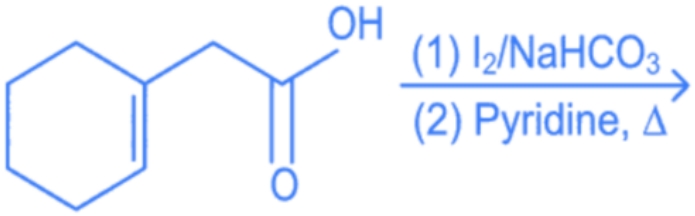
Step 2: Obtain the formula of B In the second route, $NaBH_4$ reduces the ketone group to a secondary alcohol, giving the hexitol formula $C_6H_{14}O_6$. The subsequent HI/phosphorus treatment replaces the oxygen-containing groups by hydrogen. The carbon chain is neither lengthened nor shortened, so the final hydrocarbon is normal hexane, C_6H_{14}. Stopping at the hexitol would miss the second reagent.

Step 3: Compare the proposed pairs Only option A combines the correct seven-carbon hydroxy acid with the six-carbon hydrocarbon. Option B gives the wrong formulas for both routes; option C has two fewer hydrogens in A than the balanced hydrolysis gives. Option D has the right A but gives the intermediate hexitol rather than the final oxygen-free product B.

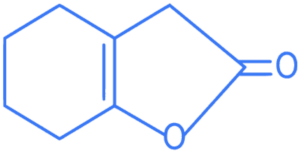
Step 4: Conclusion $A = C_7H_{14}O_8$ and $B = C_6H_{14}$. Hence, the correct option is (A).

Quick Tip to Solve: Count carbons before tracing every group: cyanide adds one carbon, whereas reduction by $NaBH_4$ followed by HI/P preserves the original six.

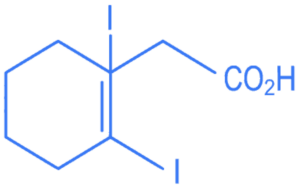
73. Find out the major product for the above reaction. [4 marks]



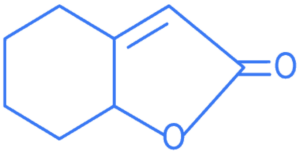
(A) Fused lactone with the double bond on the shared ring edge.



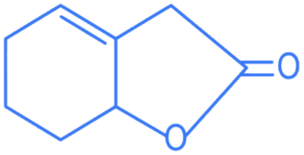
(B) Diiodinated carboxylic-acid structure shown below.



(C) Fused lactone with the double bond conjugated to its carbonyl group.

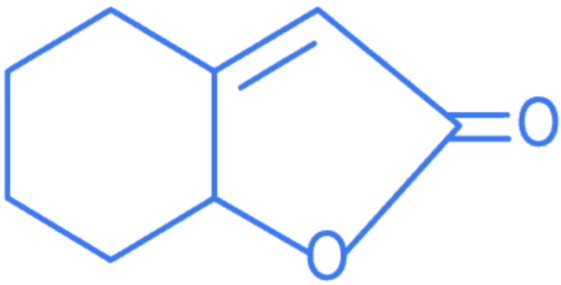


(D) Fused lactone with the double bond in the six-membered ring.



Correct Answer: (C)

Fused lactone with the double bond conjugated to its carbonyl group.



Solution:

Concept:

- An alkenoic acid can undergo intramolecular iodolactonisation: the carboxylate oxygen opens an iodonium ion to make a cyclic ester.

- Heated pyridine promotes elimination of HI. A double bond conjugated with a lactone carbonyl is stabilised relative to an isolated alkene.

Step 1: Form the carboxylate and activate the alkene $NaHCO_3$ converts the carboxylic acid into its carboxylate, which is the intramolecular nucleophile. Iodine adds electrophilically to the cyclohexene double bond and creates an iodonium intermediate. The carboxylate oxygen can attack the neighbouring alkene carbon to close a five-membered ring. This favourable closure gives a fused gamma-lactone, with iodine on the ring-junction carbon bearing the side chain.

Step 2: Eliminate hydrogen iodide Treatment with pyridine and heat removes a beta hydrogen and iodide from the iodolactone. Removal of a hydrogen from the lactone methylene creates a double bond between that carbon and the adjacent ring-junction carbon. This is the alpha,beta double bond of an unsaturated lactone: it lies directly next to the carbonyl and is conjugated with it. Formation of this stabilised unsaturated cyclic ester gives the major product.

Step 3: Locate the correct drawn structure Option C places the double bond in the five-membered lactone ring adjacent to its carbonyl, exactly as required. Option A places it on the shared ring edge, and D leaves an isolated alkene in the six-membered ring; neither gives the same carbonyl conjugation. Option B retains the acid and iodine substituents rather than undergoing the intramolecular cyclisation and subsequent elimination.

Step 4: Conclusion The major product is the fused alpha,beta-unsaturated gamma-lactone shown in option C. Hence, the correct option is (C).

Quick Tip to Solve: With iodine/bicarbonate on an alkenoic acid, test five-membered lactone closure first; after pyridine/heat, look for the alkene conjugated to the carbonyl.

74. The correct decreasing order for metallic character is : [4 marks]

(A) Na $>$ Mg $>$ Be $>$ Si $>$ P

(B) P $>$ Si $>$ Be $>$ Mg $>$ Na

(C) Si $>$ P $>$ Be $>$ Na $>$ Mg

(D) Be $>$ Na $>$ Mg $>$ Si $>$ P

Correct Answer: (A) Na $>$ Mg $>$ Be $>$ Si $>$ P

Solution:

Concept:

- Metallic character measures the tendency of an atom to lose electrons and form a cation; loosely held valence electrons favour metallic behaviour.
- Across a period, increasing effective nuclear charge generally decreases metallic character. Down a group, greater size and shielding generally increase it.

Step 1: Compare the elements in period 3 Sodium, magnesium, silicon and phosphorus all belong to period 3. Their progression from left to right is Na, Mg, Si, P. The valence electrons become more strongly attracted as effective nuclear charge rises. Hence the metallic character decreases in the order $Na > Mg > Si > P$. Sodium is an alkali metal, magnesium an alkaline-earth metal, silicon a

metalloid and phosphorus a nonmetal.

Step 2: Insert beryllium into the sequence Beryllium lies above magnesium in group 2. Its smaller atomic radius and less shielded outer electrons make electron loss more difficult than in magnesium, so $\mathrm{Mg} > \mathrm{Be}$ in metallic character. Beryllium is nevertheless a metal, and is more metallic than the metalloid silicon and the nonmetal phosphorus in the present comparison. Combining these comparisons gives $\mathrm{Na} > \mathrm{Mg} > \mathrm{Be} > \mathrm{Si} > \mathrm{P}$.

Step 3: Reject the reversed trends Option B reverses the essential metal-to-nonmetal trend. Option C places silicon and phosphorus above the strongly electropositive metals, while option D incorrectly places beryllium above sodium and magnesium. These contradict the known positions and electron-loss tendencies of the elements. Only option A obeys both the across-period and down-group comparisons.

Step 4: Conclusion The correct decreasing order is $\mathrm{Na} > \mathrm{Mg} > \mathrm{Be} > \mathrm{Si} > \mathrm{P}$. Hence, the correct option is (A).

Quick Tip to Solve: Order the same-period elements from left to right first; then use the down-group increase to place Be below Mg in metallic character.

75. The products obtained during treatment of hard water using Clark's method are : [4 marks]

(A) $\mathrm{CaCO_3}$ and $\mathrm{MgCO_3}$

(B) $\mathrm{Ca(OH)_2}$ and $\mathrm{Mg(OH)_2}$

(C) $\mathrm{CaCO_3}$ and $\mathrm{Mg(OH)_2}$

(D) $\mathrm{Ca(OH)_2}$ and $\mathrm{MgCO_3}$

Correct Answer: (C) $\mathrm{CaCO_3}$ and $\mathrm{Mg(OH)_2}$

Solution:

Concept:

- Clark's method removes temporary hardness by adding a calculated amount of lime, $\mathrm{Ca(OH)_2}$, to water containing calcium or magnesium bicarbonate.
- The dissolved bicarbonates are converted into sparingly soluble solids. Calcium is removed as $\mathrm{CaCO_3}$, while magnesium is removed as $\mathrm{Mg(OH)_2}$.

Step 1: Treat calcium bicarbonate Temporary calcium hardness is caused by dissolved $\mathrm{Ca(HCO_3)_2}$. Lime neutralises its bicarbonate ions and gives insoluble calcium carbonate:

$$\mathrm{Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2\,CaCO_3\downarrow + 2\,H_2O.}$$

Both the calcium originally dissolved in the water and the calcium supplied by lime appear in the precipitate. Calcium hydroxide is therefore the treating reagent, rather than the final calcium precipitate.

Step 2: Treat magnesium bicarbonate Magnesium bicarbonate needs more lime because magnesium

is finally precipitated as hydroxide. The net balanced change is

$$\mathrm{Mg(HCO_3)_2 + 2\,Ca(OH)_2 \longrightarrow Mg(OH)_2\downarrow + 2\,CaCO_3\downarrow + 2\,H_2O.}$$

The excess hydroxide converts the magnesium species to the poorly soluble hydroxide, while carbonate is captured by calcium. The solid products can then be allowed to settle and removed from the softened water.

Step 3: Select the pair of precipitates The characteristic products are thus calcium carbonate and magnesium hydroxide. Option A incorrectly retains magnesium carbonate as the final magnesium product. Options B and D incorrectly list calcium hydroxide as a product, although it is added to accomplish the treatment. The balanced equations select the pair in option C unambiguously.

Step 4: Conclusion The products are $\mathrm{CaCO_3}$ and $\mathrm{Mg(OH)_2}$. Hence, the correct option is (C).

Quick Tip to Solve: Remember lime as the reagent: in the final precipitates, calcium takes carbonate and magnesium takes hydroxide.

76. Statement I : An alloy of lithium and magnesium is used to make aircraft plates.
Statement II : The magnesium ions are important for cell-membrane integrity.
In the light the above statements, choose the correct answer from the options given below : [4 marks]

(A) Both Statement I and Statement II are true
(B) Both Statement I and Statement II are false
(C) Statement I is true but Statement II is false
(D) Statement I is false but Statement II is true

Correct Answer: (B) Both Statement I and Statement II are false

Solution:

Concept:

- The uses of light-metal alloys depend on the specific pair of metals; a lithium alloy must not be identified merely from the low density of lithium.
- $\mathrm{Ca^{2+}}$ is associated with maintaining cell-membrane integrity. $\mathrm{Mg^{2+}}$ has major roles in enzyme activation and is the central metal ion in chlorophyll.

Step 1: Check Statement I The standard use of lithium relevant here is that its alloys with lead make white-metal bearings, with aluminium make aircraft parts, and with magnesium make armour plates. The statement interchanges the magnesium-alloy use with the aluminium-alloy use. Aircraft applications cannot be transferred from one listed alloy to another simply because both constituent metals are light. Therefore Statement I, as stated for the lithium-magnesium alloy and aircraft plates, is false.

Step 2: Check Statement II The biological association in the statement is also interchanged. Calcium ions are important for the integrity of cell membranes and for blood clotting and neuromuscular functions. Magnesium ions are important in numerous enzyme-catalysed biochemical reactions; magnesium also occurs in chlorophyll. Thus the textbook statement about cell-membrane integrity

applies to Ca^{2+}, rather than the magnesium ions named here. Statement II is therefore false.

Step 3: Combine the truth values Since Statement I is false and Statement II is false, the correct combination is the one stating that both are false. Option A accepts both interchanged associations. Option C wrongly accepts the alloy application, whereas option D wrongly accepts the biological assignment. Neither error is corrected by the truth of other uses or functions of those metals.

Step 4: Conclusion Both Statement I and Statement II are false. Hence, the correct option is (B).

Quick Tip to Solve: Keep the two pairs distinct: Li-Al for aircraft parts and Li-Mg for armour plates; calcium for membrane integrity and magnesium for chlorophyll.

77. White phosphorus reacts with thionyl chloride to give : [4 marks]

(A) PCl_5, SO_2 and S_2Cl_2

(B) PCl_3, SO_2 and S_2Cl_2

(C) PCl_3, SO_2 and Cl_2

(D) PCl_5, SO_2 and Cl_2

Correct Answer: (B) PCl_3, SO_2 and S_2Cl_2

Solution:

Concept:

- White phosphorus is molecular P_4. With thionyl chloride, it forms phosphorus trichloride, sulfur dioxide and disulfur dichloride.
- A balanced reaction must conserve phosphorus, sulfur, oxygen and chlorine. The formula $SOCl_2$ contains one sulfur, one oxygen and two chlorine atoms.

Step 1: Identify the phosphorus chloride The characteristic phosphorus product of the reaction of white phosphorus with thionyl chloride is PCl_3, not PCl_5. The sulfur-containing coproducts are SO_2 and S_2Cl_2. The complete balanced equation is

$$P_4 + 8\,SOCl_2 \longrightarrow 4\,PCl_3 + 4\,SO_2 + 2\,S_2Cl_2.$$

This identifies the actual set of products; merely knowing that a reagent contains chlorine does not mean it releases free chlorine as a final product.

Step 2: Check the atom balance The left side has four phosphorus atoms, eight sulfur atoms, eight oxygen atoms and sixteen chlorine atoms. On the right, $4PCl_3$ supplies four phosphorus and twelve chlorine atoms; $4SO_2$ supplies four sulfur and eight oxygen atoms; $2S_2Cl_2$ supplies four further sulfur and four further chlorine atoms. Thus every element balances exactly, including sulfur in both its products.

Step 3: Compare all listed sets Option B lists precisely these three products. Options A and D incorrectly replace phosphorus trichloride by phosphorus pentachloride. Option C has the correct phosphorus chloride and sulfur dioxide, but substitutes free Cl_2 for disulfur dichloride, thereby omitting the sulfur chloride actually formed. Atom accounting supports the balanced known reaction, rather than a guess based solely on a possible phosphorus oxidation state.

Step 4: Conclusion The products are PCl_3, SO_2 and S_2Cl_2. Hence, the correct option is (B).

Quick Tip to Solve: For P_4 with $SOCl_2$, recall the trio trichloride, sulfur dioxide and disulfur dichloride; verify it with the 1:8:4:4:2 equation.

78. Concentrated HNO_3 reacts with Iodine to give : [4 marks]

(A) HI, NO_2 and H_2O

(B) HIO_2, N_2O and H_2O

(C) HIO_3, NO_2 and H_2O

(D) HIO_4, NO_2 and H_2O

Correct Answer: (C) HIO_3, NO_2 and H_2O

Solution:

Concept:

- Concentrated HNO_3 is a strong oxidising agent. In its reaction with iodine, iodine is oxidised from oxidation state zero to the positive oxidation state of iodic acid.
- The nitrogen atom in nitrate has oxidation state $+5$. Formation of NO_2 lowers it to $+4$, so each nitrogen atom accepts one electron.

Step 1: Identify the iodine product Iodine is not converted into the reducing acid HI. Concentrated nitric acid oxidises I_2 to iodic acid, HIO_3. In this acid, hydrogen contributes $+1$ and three oxygens contribute -6, so iodine must be $+5$. Each iodine atom therefore loses five electrons, and one molecule of iodine loses ten electrons in all.

Step 2: Balance the oxidation and reduction Ten nitrate nitrogen atoms are required to accept these ten electrons. Balancing hydrogen and oxygen then gives the molecular equation

$$I_2 + 10\,HNO_3 \longrightarrow 2\,HIO_3 + 10\,NO_2 + 4\,H_2O.$$

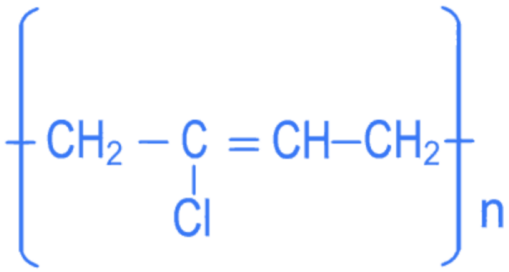
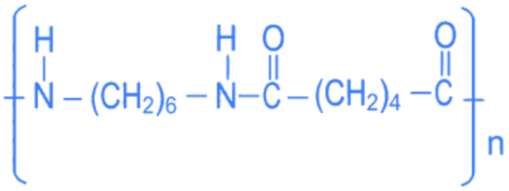
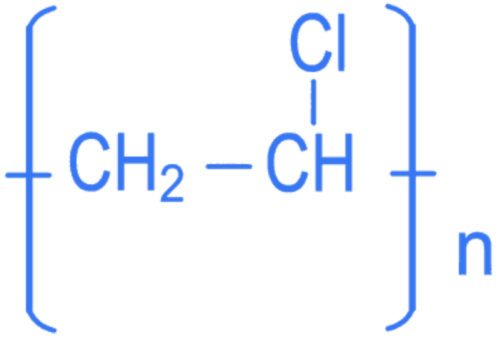
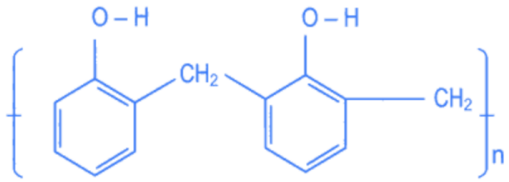
There are two iodine atoms, ten nitrogen atoms, ten hydrogen atoms and thirty oxygen atoms on each side. The brown nitrogen dioxide is the reduction product of concentrated nitric acid.

Step 3: Check the alternatives Option (A) would reduce iodine to iodide, opposite to the oxidising action of concentrated nitric acid. Option (B) gives iodous acid and nitrous oxide, which are not the products of this standard reaction. Option (D) gives periodic acid rather than the iodic acid formed here. Only option (C) contains all three products in the balanced reaction.

Step 4: Conclusion The products are HIO_3, NO_2 and H_2O. Hence, the correct option is (C).

Quick Tip to Solve: Use iodine changing from zero to $+5$ and nitrogen changing from $+5$ to $+4$ to check the ten-to-one nitric-acid/iodine coefficient.

79. Match List I with List II. Choose the correct answer from the options given below: [4 marks]

	List - I (Polymer)		List - II (Nature)
(A)		(I)	Thermosetting polymer
(B)		(II)	Fibers
(C)		(III)	Elastomer
(D)		(IV)	Thermoplastic polymer

(A) A-II, B-III, C-IV, D-I

(B) A-III, B-II, C-IV, D-I

(C) A-III, B-I, C-IV, D-II

(D) A-I, B-III, C-IV, D-II

Correct Answer: (B) A-III, B-II, C-IV, D-I

Solution:

Concept:

- Classify a polymer by its characteristic mechanical behaviour and the intermolecular forces between its chains. Elastomers have flexible chains, fibres have strong attractions and ordered chains, and thermoplastics soften on heating.

- Thermosetting materials develop a rigid cross-linked network on curing and cannot be repeatedly melted and remoulded. Recognising the repeat unit identifies the standard polymer and hence its usual classification.

Step 1: Identify the rubber and the polyamide Polymer A has the chloroprene repeat unit, $[-CH_2-C(Cl)=CH-CH_2-]_n$, so it is neoprene, a synthetic rubber. Its flexible chains allow elastic deformation and recovery; match A with III, elastomer. Polymer B contains the repeating amide link $-CO-NH-$ and the six-carbon diamine/four-methylene diacid residues of nylon-6,6. Hydrogen bonds between its amide groups strengthen and align its chains, giving a high-tensile-strength fibre. Thus B matches II.

Step 2: Identify the vinyl polymer and the phenolic resin Polymer C has the repeat unit $[-CH_2-CHCl-]_n$, identifying polyvinyl chloride, PVC. It is a thermoplastic: heating softens the material so it can be shaped again. Hence C matches IV. Polymer D shows phenolic rings joined through methylene bridges, identifying the phenol-formaldehyde resin associated with Bakelite. On curing, such resin forms a rigid network that does not soften for remoulding; its listed nature is thermosetting, I. Hence D matches I.

Step 3: Compare the complete matching with the options The required sequence is A-III, B-II, C-IV, D-I. Option A incorrectly exchanges the elastomer and fibre classifications of A and B. Option C incorrectly labels nylon-6,6 as thermosetting and the phenolic resin as a fibre. Option D also assigns those polymers incorrect categories. Only option B retains all four correct matches.

Step 4: Conclusion $\boxed{\text{A-III, B-II, C-IV, D-I}}$. Hence, the correct option is (B).

Quick Tip to Solve: Recognise chloroprene as neoprene, amide links as nylon, and the vinyl chloride unit as PVC; these three matches usually identify the correct sequence before checking the phenolic resin.

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80. Two statements in respect of drug-enzyme interaction are given below

Statement I : Action of an enzyme can be blocked only when an inhibitor blocks the active site of the enzyme.

Statement II : An inhibitor can form a strong covalent bond with the enzyme.

In the light of the above statements, choose the correct answer from the options given below [4 marks]

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Correct Answer: (D) Statement I is false but Statement II is true

Solution:

Concept:

- An enzyme catalyses a reaction by binding substrates at an active site. Inhibitors can suppress this activity by more than one mechanism.
- Competitive inhibition involves the active site; an inhibitor binding at a separate allosteric site can

instead alter the active-site shape. Irreversible inhibitors can form covalent bonds with the enzyme.

Step 1: Test the restriction in Statement I Statement I uses the word only. A competitive inhibitor occupies the active site and prevents the substrate from binding there. However, an allosteric inhibitor binds at a site different from the active site. The resulting change in enzyme conformation can make the active site unsuitable for its substrate or impair its catalytic activity. Thus direct occupation or blocking of the active site is not the only way to inhibit enzyme action. Statement I is false.

Step 2: Evaluate Statement II An inhibitor can react with a functional group of the enzyme and form a strong covalent bond. Such binding is typically irreversible on the timescale of enzyme action: the inhibitor cannot simply dissociate like a reversibly bound competitive inhibitor. This persistent chemical modification can destroy catalytic activity. The word can expresses a possible mechanism rather than claiming that every inhibitor binds covalently. Statement II is therefore true.

Step 3: Select the matching combination The required truth values are false for Statement I and true for Statement II. Option (A) incorrectly accepts the active-site-only restriction. Option (B) incorrectly rejects covalent inhibition. Option (C) reverses both truth values. Option (D) alone gives the correct combination.

Step 4: Conclusion Statement I is false but Statement II is true. Hence, the correct option is (D).

Quick Tip to Solve: In statements containing only, look for another established mechanism: allosteric inhibition disproves the restriction to direct active-site blocking.

Chemistry – Section B: Numerical Value Questions (20 marks)

Attempt any 5 of the 10 questions in this section.

81. 2L of 0.2M H_2SO_4 is reacted with 2L of 0.1M NaOH solution, the molarity of the resulting product Na_2SO_4 in the solution is _____ millimolar. (Nearest integer) [4 marks]

Solution:

Concept:

- The stoichiometric equation for formation of sodium sulphate is $\mathrm{H_2SO_4} + 2\,\mathrm{NaOH} \longrightarrow \mathrm{Na_2SO_4} + 2\,\mathrm{H_2O}$. Two moles of sodium hydroxide correspond to one mole of sodium sulphate.
- The amount of a solute is $n = MV$, with volume in litres. Product molarity is the amount of product divided by the total mixed solution volume.

Step 1: Calculate the reactant amounts The given acid and base amounts are

$$n(\mathrm{H_2SO_4}) = 0.2 \times 2 = 0.40 \text{ mol},$$

$$n(\mathrm{NaOH}) = 0.1 \times 2 = 0.20 \text{ mol}.$$

The acid would require 0.80 mol of base for complete neutralisation, but only 0.20 mol is present. Sodium hydroxide is therefore the limiting reactant for the stated product calculation; the initial acid amount cannot be used as the product amount.

Step 2: Find the amount of the specified product Use the ratio of coefficients in the balanced equation:

$$n(\mathrm{Na_2SO_4}) = \frac{n(\mathrm{NaOH})}{2} = \frac{0.20}{2} = 0.10 \text{ mol}.$$

The stoichiometric acid consumption corresponding to this quantity is 0.10 mol, less than the available 0.40 mol. This independently confirms that the base controls the calculated yield.

Step 3: Divide by the combined volume Taking the mixed volumes as additive, the final solution volume is

$$V = 2 + 2 = 4 \text{ L}.$$

Thus

$$M(\mathrm{Na_2SO_4}) = \frac{0.10}{4} = 0.025 \text{ mol L}^{-1}.$$

Since one molar equals one thousand millimolar, this is 25 mmol L^{-1}. Use the total four-litre volume, not the original two-litre base volume.

Step 4: Conclusion The molarity of the specified product is 25 millimolar. The nearest-integer answer is $\boxed{25}$.

Quick Tip to Solve: First use the limiting base and the coefficient ratio $2:1$; then divide the product moles by the sum of both solution volumes.

---

82. If the wavelength for an electron emitted from H-atom is 3.3×10^{-10} m, then energy absorbed by the electron in its ground state compared to minimum energy required for its escape from the atom, is _____ times. (Nearest integer)

[**Given** : $\mathrm{h} = 6.626 \times 10^{-34}$ J s **]**

Mass of electron $= 9.1 \times 10^{-31}$ kg [4 marks]

Solution:

Concept:

- The de Broglie wavelength of an emitted electron obeys $\lambda = h/p$. Its nonrelativistic kinetic energy is $K = p^2/(2m_e) = h^2/(2m_e\lambda^2)$.
- To eject the ground-state electron of hydrogen requires the ionisation energy $I = 13.6$ eV $= 2.179 \times 10^{-18}$ J. The absorbed energy supplies both the binding energy and the outgoing kinetic energy: $E = I + K$.

Step 1: Calculate the outgoing kinetic energy The given wavelength is that of the electron, so use the matter-wave expression rather than the photon formula hc/λ. Substitution gives

$$K = \frac{(6.626 \times 10^{-34})^2}{2(9.1 \times 10^{-31})(3.3 \times 10^{-10})^2}.$$

The numerator is 4.3904×10^{-67} and the denominator is 1.9820×10^{-49} in consistent SI units. Therefore

$$K \simeq 2.215 \times 10^{-18} \text{ J}.$$

This is approximately 13.83 eV, comparable to the ionisation energy.

Step 2: Include the energy needed to escape The electron initially has energy -13.6 eV, while the electron at infinity has positive kinetic energy K. The required change in energy is therefore $K-(-I)=K+I$, not merely K. Consequently,

$$\frac{E}{I}=1+\frac{K}{I}=1+\frac{2.215\times10^{-18}}{2.179\times10^{-18}}\simeq2.017.$$

The small departure from two reflects the rounded wavelength and physical constants.

Step 3: Apply the requested rounding The ratio is dimensionless because both energies are measured in joules. Its nearest integer is two. Rounding the kinetic-energy ratio alone would produce one and would omit the energy consumed in overcoming the initial hydrogen binding.

Step 4: Conclusion The absorbed energy is approximately 2.017 times the minimum escape energy. The nearest-integer answer is $\boxed{2}$.

Quick Tip to Solve: For an emitted electron, compute $h^2/(2m\lambda^2)$, then add the ionisation energy before taking the requested ratio.

---

83. A gaseous mixture of two substances A and B, under a total pressure of 0.8 atm is in equilibrium with an ideal liquid solution. The mole fraction of substance A is 0.5 in the vapour phase and 0.2 in the liquid phase. The vapour pressure of pure liquid A is _____ atm. (Nearest integer) [4 marks]

Solution:

Concept:

- For an ideal liquid mixture, Raoult's law gives $p_A=x_AP_A^\circ$, where x_A is the liquid-phase mole fraction and P_A° is the pure-liquid vapour pressure.
- In the equilibrium vapour, Dalton's law gives $p_A=y_AP$, where y_A is the vapour-phase mole fraction and P is the total vapour pressure. The same partial pressure appears in both laws.

Step 1: Find the partial pressure from the vapour composition The total pressure is 0.8 atm, and the vapour contains mole fraction $y_A=0.5$ of A. Hence

$$p_A=y_AP=0.5\times0.8=0.4\text{ atm}.$$

This is the pressure contributed by A in the mixture, rather than the vapour pressure of pure A. The other component supplies the remaining 0.4 atm, consistent with the stated total pressure.

Step 2: Use the liquid composition The liquid-phase mole fraction is $x_A=0.2$. Therefore

$$P_A^\circ=\frac{p_A}{x_A}=\frac{0.4}{0.2}=2\text{ atm}.$$

Combining the laws directly gives the same result:

$$P_A^\circ=\frac{y_AP}{x_A}=\frac{(0.5)(0.8)}{0.2}.$$

The liquid and vapour mole fractions differ because A is relatively more volatile and becomes enriched in the vapour.

Step 3: Check the interpretation and units Substitution back into Raoult's law gives $(0.2)(2) = 0.4\ \text{atm}$, exactly the partial pressure obtained from Dalton's law. A pure-liquid vapour pressure exceeding the mixture total pressure is entirely possible: mixing reduces the partial pressure by the liquid mole fraction. No temperature or amount calculation is needed because both phases are already at equilibrium.

Step 4: Conclusion The vapour pressure of pure liquid A is $2\ \text{atm}$. The nearest-integer answer is $\boxed{2}$.

Quick Tip to Solve: Keep x_A for the liquid and y_A for the vapour; equating $x_A P_A^\circ = y_A P$ avoids interchanging the two compositions.

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84. At $600\ \text{K}, 2\ \text{mol}$ of NO are mixed with $1\ \text{mol}$ of O_2.

$$2\text{NO}_{(g)} + \text{O}_2(g) \rightleftarrows 2\text{NO}_2(g)$$

The reaction occurring as above comes to equilibrium under a total pressure of 1 atm. Analysis of the system shows that $0.6\ \text{mol}$ of oxygen are present at equilibrium. The equilibrium constant for the reaction is _____. (Nearest integer) [4 marks]

Solution:

Concept:

- For $2\,\text{NO}\,(g) + \text{O}_2(g) \rightleftharpoons 2\,\text{NO}_2(g)$, the pressure equilibrium constant is $K_p = p_{\text{NO}_2}^2/(p_{\text{NO}}^2 p_{\text{O}_2})$. Each partial pressure equals its equilibrium mole fraction times the total pressure.
- The reaction consumes two moles of NO per mole of O_2 and forms two moles of NO_2. Equilibrium amounts must be calculated before converting them to partial pressures.

Step 1: Set up the equilibrium amounts Oxygen decreases from 1.0 to $0.6\ \text{mol}$, so $0.4\ \text{mol}$ has reacted. Accordingly,

$$n_{\text{NO}} = 2 - 2(0.4) = 1.2\ \text{mol},$$

$$n_{\text{NO}_2} = 2(0.4) = 0.8\ \text{mol}.$$

The equilibrium total is $1.2 + 0.6 + 0.8 = 2.6\ \text{mol}$. The initial total of three moles must not be used because the reaction decreases the total gas amount.

Step 2: Calculate the partial pressures At total pressure $1\ \text{atm}$, the equilibrium partial pressures are

$$p_{\text{NO}} = \frac{1.2}{2.6} = \frac{6}{13}\ \text{atm},$$

$$p_{\text{O}_2} = \frac{0.6}{2.6} = \frac{3}{13}\ \text{atm}, \qquad p_{\text{NO}_2} = \frac{0.8}{2.6} = \frac{4}{13}\ \text{atm}.$$

Their sum is one atmosphere, which checks the mole-fraction conversion.

Step 3: Evaluate the pressure equilibrium constant Using these equilibrium partial pressures,

$$K_p = \frac{(4/13)^2}{(6/13)^2(3/13)} = \frac{16 \times 13}{36 \times 3} = \frac{52}{27} \simeq 1.926.$$

The supplied pressure selects the pressure-based expression; no volume is needed. The temperature specifies the condition at which this constant applies. Rounding the final value, rather than intermediate mole fractions, gives the requested integer.

Step 4: Conclusion The equilibrium constant K_p is approximately 1.926 when pressures are expressed in atmospheres. The nearest-integer answer is $\boxed{2}$.

Quick Tip to Solve: Use the loss of oxygen as the reaction extent, update the total moles, and only then form the partial-pressure expression with the stoichiometric powers.

---

85. A sample of 0.125 g of an organic compound when analyzed by Duma's method yields 22.78 mL of nitrogen gas collected over KOH solution at 280 K and 759 mm Hg. The percentage of nitrogen in the given organic compound is _____. (Nearest integer)

Given :

(a) The vapour pressure of water of 280 K is 14.2 mm Hg.

(b) $R = 0.082$ L atm K^{-1} mol^{-1} [4 marks]

Solution:

Concept:

- In Dumas' method, the nitrogen in an organic compound is converted into N_2. Its measured amount therefore gives the mass of nitrogen originally present in the sample.
- For a gas collected over an aqueous liquid, subtract the stated water vapour pressure from the total pressure. Then use $PV = nRT$, with pressure in atmospheres and volume in litres for the supplied value of R.

Step 1: Find the dry nitrogen pressure The measured pressure includes both nitrogen and water vapour. Dalton's law gives the partial pressure of dry nitrogen as

$$P_{N_2} = 759 - 14.2 = 744.8 \text{ mm Hg}.$$

Convert this to atmospheres before substitution:

$$P_{N_2} = \frac{744.8}{760} \text{ atm}.$$

The collected nitrogen volume is 22.78 mL $= 0.02278$ L, and its temperature is already in kelvin.

Step 2: Calculate the amount and mass of nitrogen Using the ideal-gas equation at the collection conditions,

$$n(N_2) = \frac{PV}{RT} = \frac{(744.8/760)(0.02278)}{0.082(280)} = 9.7232 \times 10^{-4} \text{ mol}.$$

One mole of nitrogen gas contains two nitrogen atoms and has mass 28 g. Thus the mass of nitrogen

in the sample is

$$m_N = (9.7232 \times 10^{-4})(28) = 0.0272249 \text{ g}.$$

There is no additional factor of two: it is already included in the molar mass of N_2.

Step 3: Express the result as a mass percentage Divide the nitrogen mass by the complete sample mass, rather than by the gas volume:

$$\%N = \frac{0.0272249}{0.125} \times 100 = 21.7799\%.$$

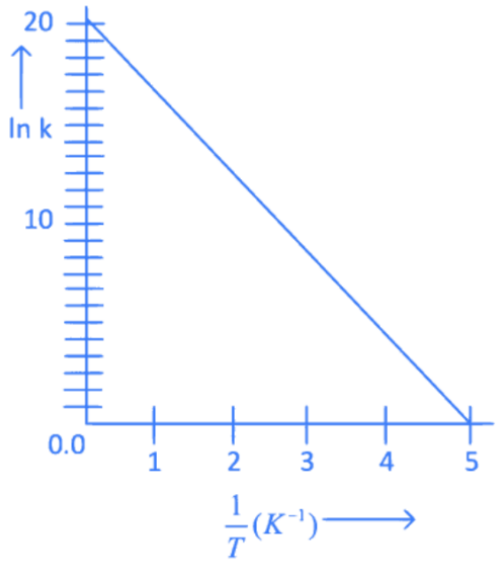
Rounding is performed only after this final calculation, because the question asks for the nearest integer.

Step 4: Conclusion The percentage of nitrogen is approximately 21.78%, so the required nearest-integer answer is $\boxed{22}$.

Quick Tip to Solve: Subtract aqueous vapour pressure first; use 28 g per mole of nitrogen gas and postpone rounding until the mass percentage is obtained.

86. For a reaction, given below is the graph of $\ln k$ vs $\frac{1}{T}$. The activation energy for the reaction is equal to _____ cal mol^{-1}. (nearest integer)

(Given : $R = 2 \text{ cal K}^{-1} \text{ mol}^{-1}$) [4 marks]



Solution:

Concept:

- The Arrhenius equation is $k = A\exp[-E_a/(RT)]$, where E_a is the activation energy, A is the pre-exponential factor and T is absolute temperature.
- Taking natural logarithms gives $\ln k = \ln A - (E_a/R)(1/T)$. Therefore the slope of a plot of $\ln k$ against $1/T$ is $-E_a/R$, not $-E_a/(2.303R)$.

Step 1: Read the two intercepts correctly The vertical axis is $\ln k$ and the horizontal axis is $1/T$, in K^{-1}. The straight line passes through the intercepts $(0, 20)$ and $(5, 0)$. The numerical horizontal markings have no additional power-of-ten multiplier. To calculate the slope we must divide the change in the vertical coordinate by the change in the horizontal coordinate, keeping the same order for both differences.

Step 2: Determine the signed slope Using the intercepts shown,

$$m = \frac{\Delta(\ln k)}{\Delta(1/T)} = \frac{0 - 20}{5 - 0} = -4 \text{ K}.$$

The negative sign is expected: as reciprocal temperature increases, temperature decreases and the rate constant becomes smaller. The vertical intercept gives $\ln A$, whereas activation energy comes from the slope; using the intercept itself as the energy would therefore be incorrect.

Step 3: Obtain the activation energy Compare the straight-line equation with the logarithmic Arrhenius equation:

$$m = -\frac{E_a}{R} \quad \Longrightarrow \quad E_a = -mR.$$

With the supplied gas constant,

$$E_a = -(-4 \text{ K})(2 \text{ cal K}^{-1} \text{ mol}^{-1}) = 8 \text{ cal mol}^{-1}.$$

The kelvin units cancel, leaving energy per mole. No factor of 2.303 is needed because the question explicitly uses $\ln$, rather than the common logarithm.

Step 4: Conclusion The activation energy is $\boxed{8 \text{ cal mol}^{-1}}$; the nearest-integer entry is $\boxed{8}$.

Quick Tip to Solve: For a natural-log Arrhenius plot, multiply the magnitude of the negative slope by R; add 2.303 only for a base-10 logarithm plot.

87. Among the following the number of state variables is _____.

Internal energy (U)

Volume (V)

Heat (q)

Enthalpy (H) [4 marks]

Solution:

Concept:

- A state variable has a value determined by the thermodynamic state of a system. Its change between fixed initial and final states is independent of the process used to connect them.
- Heat is energy transferred during a process because of a temperature difference. Unlike a state function, the heat transferred depends on the path and is not a property stored in the system.

Step 1: Classify internal energy and volume Internal energy U is a state function: once the equilibrium state and the amount of material are fixed, the system has a definite internal energy. Different processes

connecting the same end states give the same ΔU. Volume V is also a state variable, because it specifies the space occupied in a given state; it is not defined by how the system arrived at that state. Thus both U and V must be included in the count.

Step 2: Distinguish heat from a state property Heat q describes energy crossing the system boundary during a particular process. For example, the first law in the chemistry sign convention is $\Delta U = q + w$, where w is work done on the system. Two paths can have different values of q and w while giving the same ΔU. Consequently, heat cannot be assigned as a unique value to an equilibrium state and must be excluded. Although heat can equal ΔH under suitable constant-pressure conditions with only pressure-volume work, that special relation does not make heat a state function.

Step 3: Classify enthalpy and count the entries Enthalpy is defined by

$$H = U + PV.$$

Internal energy, pressure and volume are state properties, so the combination $U + PV$ is also fixed by the state. Hence H is included. The accepted entries are therefore U, V and H, while the sole excluded entry is q. Counting the three accepted entries gives the requested numerical result.

Step 4: Conclusion There are $\boxed{3}$ state variables in the list: internal energy, volume and enthalpy.

Quick Tip to Solve: Ask whether a quantity belongs to the state or describes a transfer during a process: U, V and H belong to the state, while heat and work describe transfers.

---

88. Metal M crystallizes into a fcc lattice with the edge length of 4.0×10^{-8} cm. The atomic mass of the metal is _____ g/mol. (Nearest integer)

(**Use :** $N_A = 6.02 \times 10^{23}$ mol^{-1}**, density of metal,** $M = 9.03$ g cm^{-3} **)** [4 marks]

Solution:

Concept:

- The density of a crystalline metal is the mass of one unit cell divided by its volume. For a cubic cell of edge a, this gives $\rho = ZM/(N_A a^3)$, where M is molar mass and Z is the number of atoms per cell.
- A face-centred cubic cell has eight corner atoms contributing one-eighth each and six face-centred atoms contributing one-half each. Therefore its effective number of atoms is $Z = 4$.

Step 1: Count the atoms in the fcc cell An atom at a corner is shared among eight adjoining cells, while an atom at a face centre is shared between two. The effective number belonging to one fcc cell is

$$Z = 8\left(\frac{1}{8}\right) + 6\left(\frac{1}{2}\right) = 1 + 3 = 4.$$

There is no atom at the body centre in an fcc structure. Using the number of visible atom positions without their sharing fractions would overestimate the cell mass.

Step 2: Calculate the cell volume and mass The edge length is already in centimetres, consistent with

the density in $\mathrm{g\,cm^{-3}}$. Hence

$$a^3 = (4.0 \times 10^{-8})^3 = 6.4 \times 10^{-23}\ \mathrm{cm^3}.$$

Multiplying by the given density gives the mass of a single unit cell:

$$m_{\text{cell}} = \rho a^3 = (9.03)(6.4 \times 10^{-23}) = 5.7792 \times 10^{-22}\ \mathrm{g}.$$

The exponent must be cubed along with the numerical edge length.

Step 3: Convert the cell mass to molar mass Each cell contains four atoms, so its mass is $4M/N_A$. Rearranging this relation gives

$$M = \frac{\rho a^3 N_A}{4}.$$

Substitute the calculated cell mass and the supplied Avogadro constant:

$$M = \frac{(5.7792 \times 10^{-22})(6.02 \times 10^{23})}{4} = 86.97696\ \mathrm{g\,mol^{-1}}.$$

This is the molar atomic mass of the metal. Keep the intermediate digits and round only the final value as requested.

Step 4: Conclusion The atomic molar mass is approximately $86.98\ \mathrm{g\,mol^{-1}}$, giving the nearest-integer answer $\boxed{87}$.

Quick Tip to Solve: For fcc density problems use Z = 4, cube the edge length including its power of ten, and keep the edge units consistent with the density units.

89. On reaction with stronger oxidizing agent like KIO_4, hydrogen peroxide oxidizes with the evolution of O_2. The oxidation number of I in KIO_4 changes to _____. [4 marks]

Solution:

Concept:

- Hydrogen peroxide can be either an oxidizing agent or a reducing agent. Against a stronger oxidizing agent such as periodate, it acts as the reducing agent and is itself oxidized to oxygen.
- The oxygen oxidation number in H_2O_2 is -1, while it is zero in O_2. This oxidation releases electrons, which are accepted by the iodine-containing oxidizing agent.

Step 1: Find iodine in the initial periodate Potassium periodate is KIO_4. With potassium assigned $+1$ and each oxide oxygen assigned -2, electrical neutrality requires

$$1 + x + 4(-2) = 0 \quad \Longrightarrow \quad x = +7.$$

Thus iodine initially occupies the $+7$ oxidation state. The question asks for its new oxidation number after reaction, rather than this initial value or the size of the decrease.

Step 2: Identify the reduction product In this reaction, peroxide is oxidized to molecular oxygen and

periodate is reduced to iodate. The balanced molecular equation is

$$KIO_4 + H_2O_2 \longrightarrow KIO_3 + H_2O + O_2.$$

It conserves one potassium atom, one iodine atom, two hydrogen atoms and six oxygen atoms on each side. Equivalently, the iodine-containing ion changes from IO_4^- to IO_3^-. Formation of oxygen therefore accompanies reduction of iodine, not an increase beyond its original high oxidation state.

Step 3: Determine the final oxidation number For potassium iodate, let the final iodine oxidation number be y:

$$1 + y + 3(-2) = 0 \quad \Longrightarrow \quad y = +5.$$

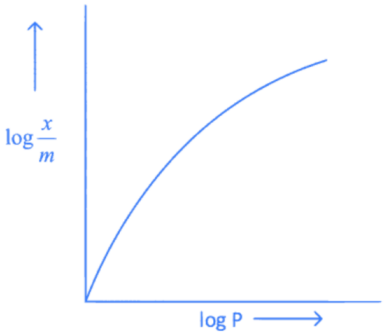
The change $+7$ to $+5$ corresponds to a gain of two electrons by iodine. This matches the loss of two electrons when the two peroxide oxygen atoms each change from -1 to zero. The electron balance confirms the reaction and distinguishes the final value $+5$ from the two-unit change in oxidation number.

Step 4: Conclusion The oxidation number of iodine changes from $+7$ to $\boxed{+5}$. The required numerical entry is $\boxed{5}$.

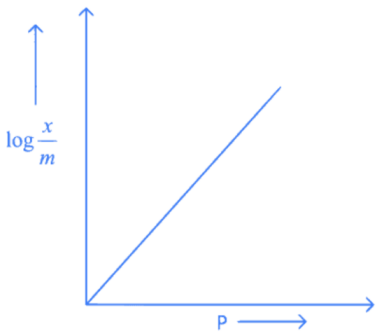
Quick Tip to Solve: Write periodate turning into iodate before assigning oxidation numbers; distinguish the final iodine value of $+5$ from the decrease of two units.

90. Among the following the number of curves not in accordance with Freundlich adsorption isotherm is _____. [4 marks]

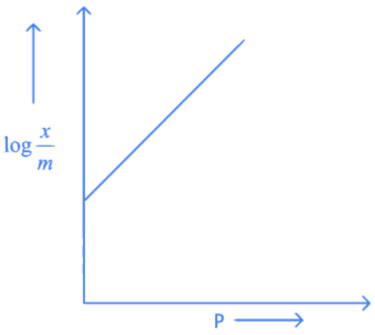
(a)



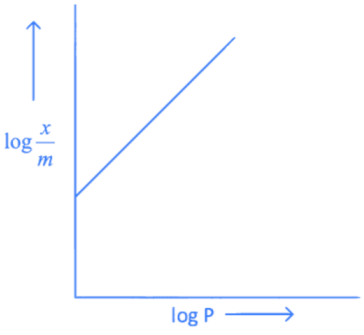
(b)



(c)



(d)



Solution:

Concept:

- At a fixed temperature, the Freundlich adsorption relation is $x/m = kP^{1/n}$, where x/m is adsorbed mass per unit mass of adsorbent, P is gas pressure and k, n are empirical constants.
- Taking logarithms gives $\log(x/m) = \log k + (1/n)\log P$. Thus a graph of $\log(x/m)$ against $\log P$ must be straight, with slope $1/n$ and intercept $\log k$.

Step 1: Check curve (a) using both axis labels Curve (a) places $\log(x/m)$ on the vertical axis and $\log P$ on the horizontal axis. In these coordinates the Freundlich relation is linear. The given graph is curved, with its slope decreasing as the horizontal coordinate increases, so it cannot represent the required straight-line logarithmic form. It would be inappropriate to accept it merely because an ordinary adsorption curve often rises with decreasing slope: the logarithmic axis labels matter.

Step 2: Check curves (b) and (c) Both (b) and (c) use $\log(x/m)$ vertically but ordinary pressure P horizontally. The relation for these coordinates is

$$\log(x/m) = \log k + \frac{1}{n}\log P.$$

It is logarithmic in P, not a nonhorizontal straight line in P. Graph (b), a straight line through the origin, therefore fails. Graph (c), a straight line with a nonzero vertical intercept, also fails: changing the intercept cannot correct the horizontal variable. In fact the slope against P varies as $1/P$ for positive pressure, whereas both drawn slopes are constant.

Step 3: Check curve (d) and count the failures Curve (d) has the correct pair of logarithmic axes and shows a straight line with a vertical intercept. Its slope represents $1/n$ and its intercept represents $\log k$, exactly as required by the logarithmic equation. It is therefore consistent. The inconsistent curves are (a), (b) and (c); there are three of them. We count the curves that are not in accordance with the isotherm, rather than the one acceptable curve.

Step 4: Conclusion The number of curves not in accordance with the Freundlich adsorption isotherm is $\boxed{3}$, namely (a), (b) and (c).

Quick Tip to Solve: Read both axes before judging an adsorption plot: the Freundlich straight line is log(x/m) against log P, not log(x/m) against ordinary P.